

Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.2-Cosine/89-4.2.2.1-a+b-cos^m-c+d-cosⁿ

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Contents

1	Introduction	3
2	detailed summary tables of results	21
3	Listing of integrals	247
4	Appendix	7695

CHAPTER 1

INTRODUCTION

1.1	Listing of CAS systems tested	4
1.2	Results	5
1.3	Time and leaf size Performance	8
1.4	Performance based on number of rules Rubi used	10
1.5	Performance based on number of steps Rubi used	11
1.6	Solved integrals histogram based on leaf size of result	12
1.7	Solved integrals histogram based on CPU time used	13
1.8	Leaf size vs. CPU time used	14
1.9	list of integrals with no known antiderivative	15
1.10	List of integrals solved by CAS but has no known antiderivative	15
1.11	list of integrals solved by CAS but failed verification	15
1.12	Timing	16
1.13	Verification	16
1.14	Important notes about some of the results	16
1.15	Design of the test system	19

This report gives the result of running the computer algebra independent integration test. The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [932]. This is test number [89].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (932)	0.00 (0)
Mathematica	99.68 (929)	0.32 (3)
Maple	91.63 (854)	8.37 (78)
Fricas	72.42 (675)	27.58 (257)
Maxima	34.87 (325)	65.13 (607)
Mupad	33.26 (310)	66.74 (622)
Giac	30.15 (281)	69.85 (651)
Sympy	10.84 (101)	89.16 (831)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

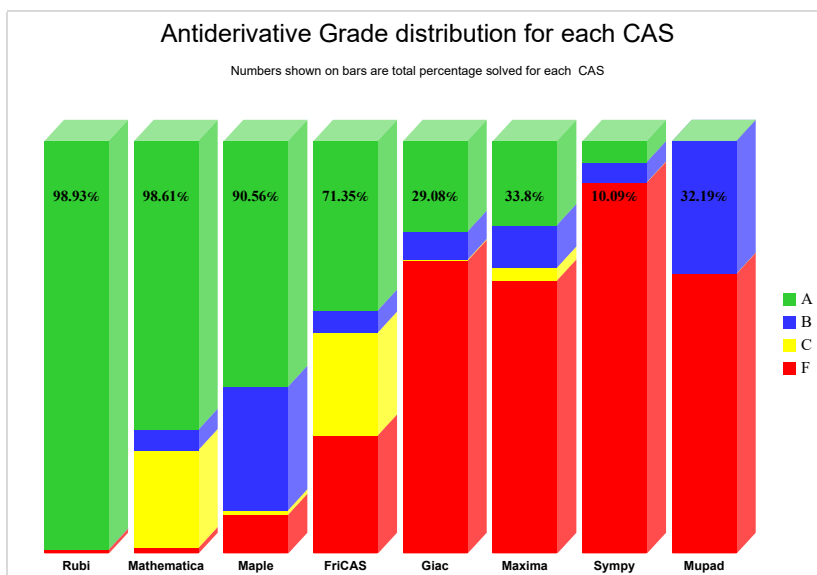
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

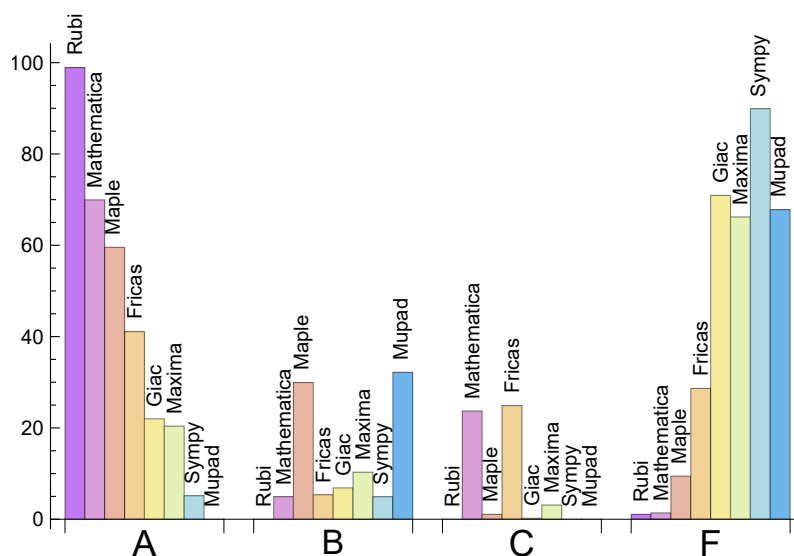
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.927	0.000	0.000	1.073
Mathematica	69.957	4.936	23.712	1.395
Maple	59.549	29.936	1.073	9.442
Fricas	41.094	5.365	24.893	28.648
Giac	21.996	6.867	0.215	70.923
Maxima	20.386	10.300	3.112	66.202
Sympy	5.150	4.936	0.000	89.914
Mupad	0.000	32.189	0.000	67.811

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Maple	78	100.00	0.00	0.00
Fricas	257	76.26	23.74	0.00
Maxima	607	89.95	2.80	7.25
Mupad	622	0.00	100.00	0.00
Giac	651	89.09	9.52	1.38
Sympy	831	48.38	51.62	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.24
Rubi	0.32
Maxima	1.35
Mathematica	2.87
Giac	2.90
Sympy	6.37
Mupad	14.55
Maple	15.48

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	154.08	1.00	131.00	1.00
Fricas	205.82	1.63	161.00	1.33
Sympy	223.22	2.51	129.00	1.75
Mathematica	234.01	1.44	133.00	0.96
Mupad	402.21	2.47	102.00	1.07
Maple	430.47	2.17	194.00	1.63
Giac	820.91	5.71	116.00	1.23
Maxima	6483.49	48.36	127.00	1.47

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

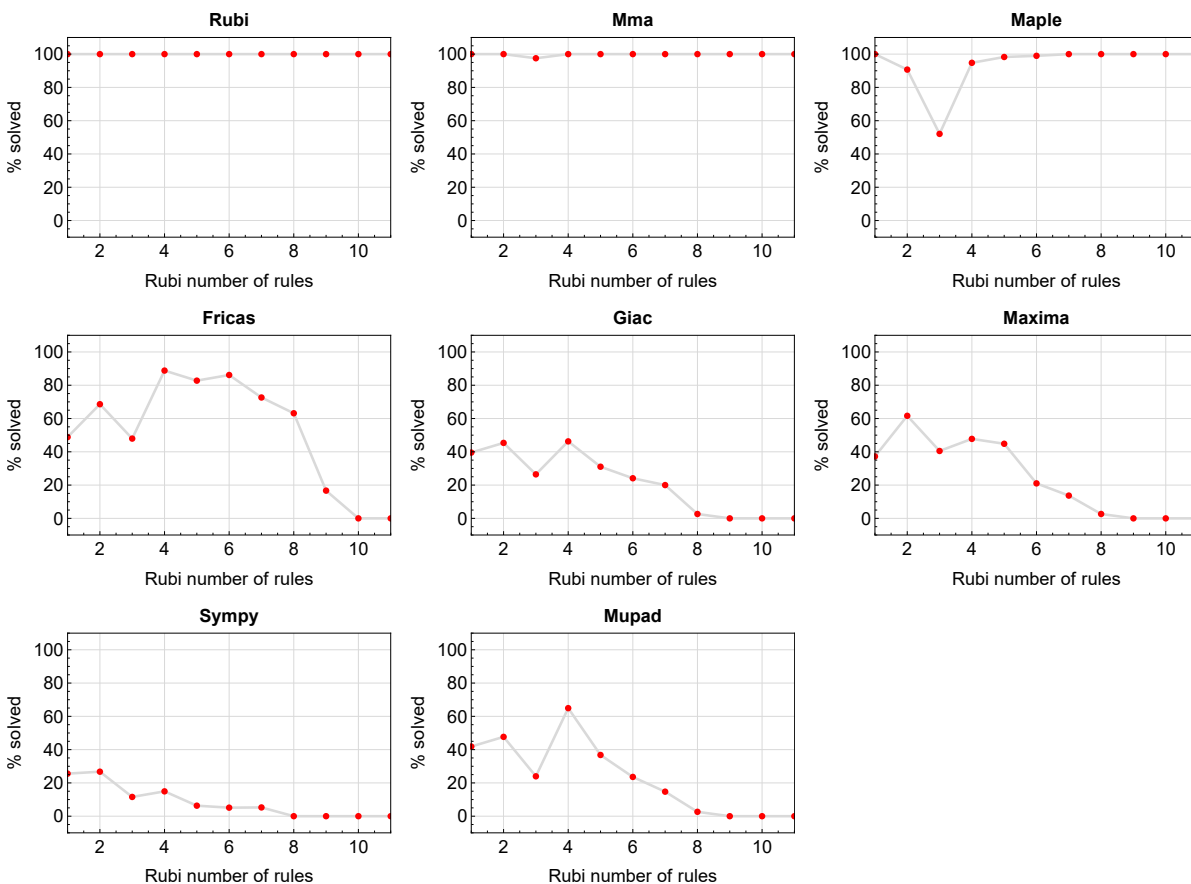


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

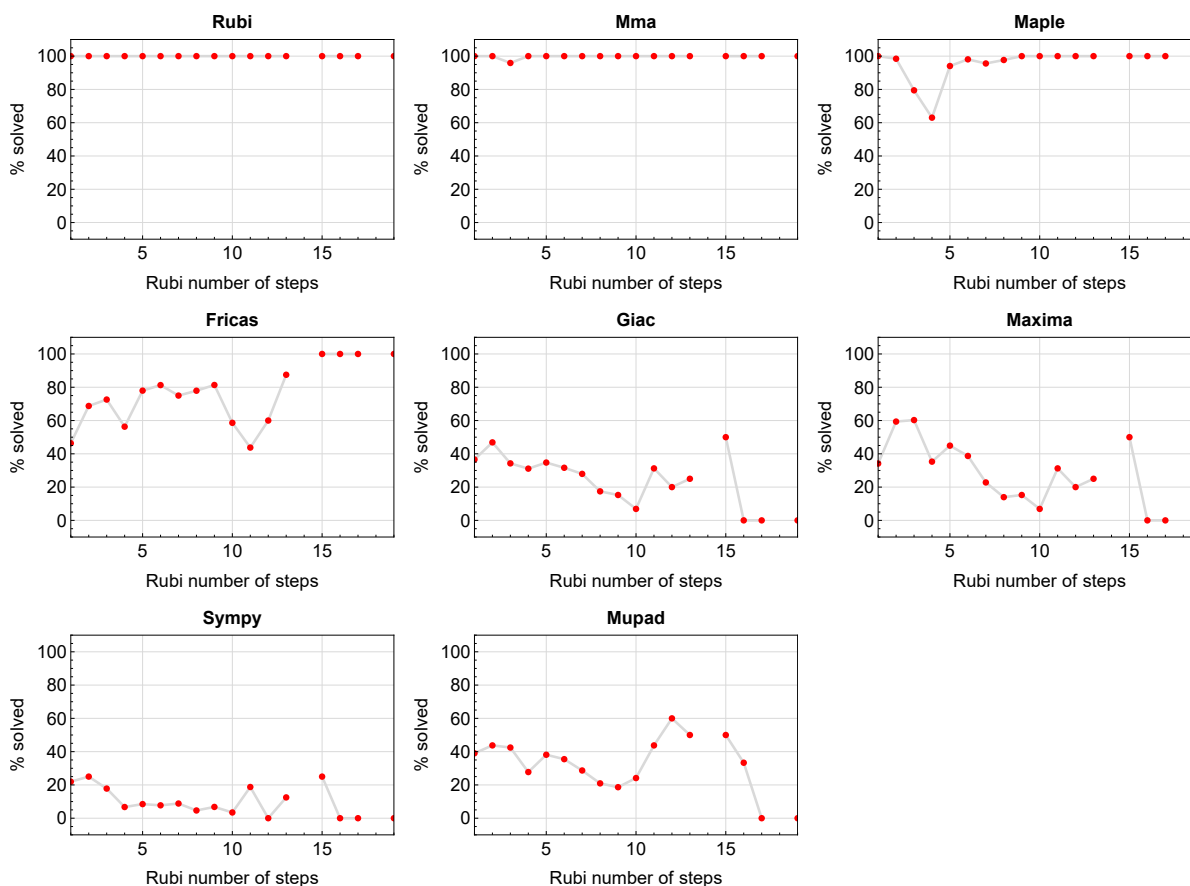


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the percentage of solved integrals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

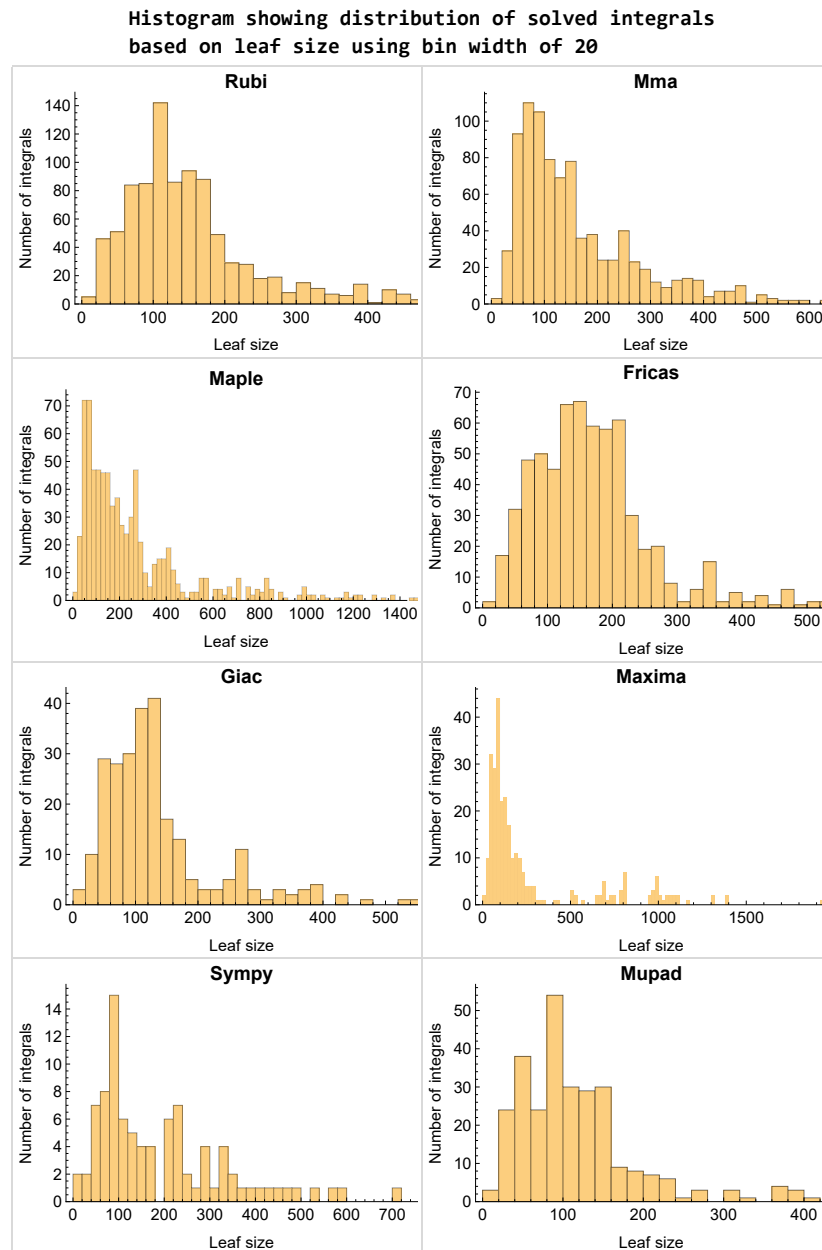


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

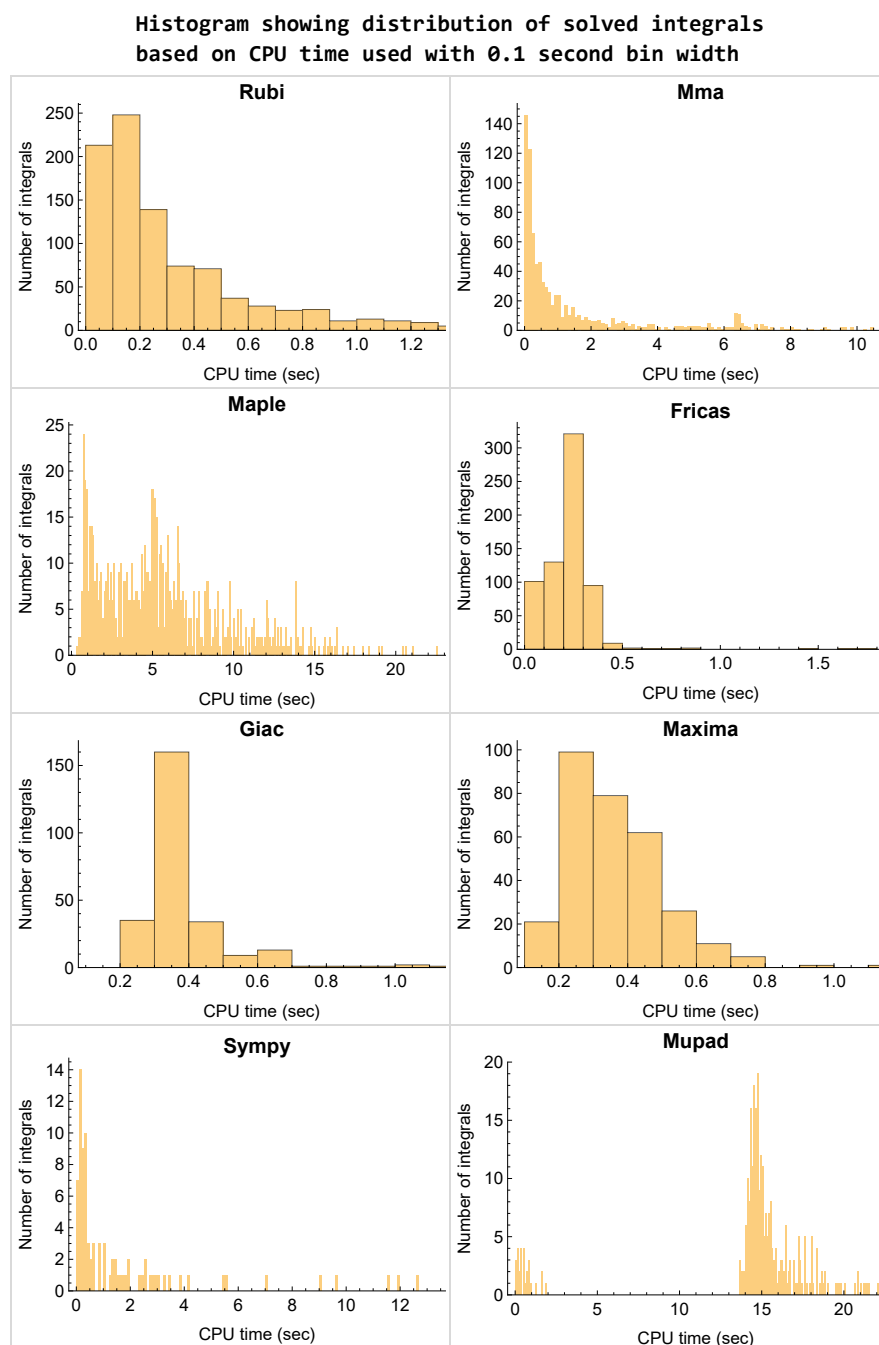


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fracas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

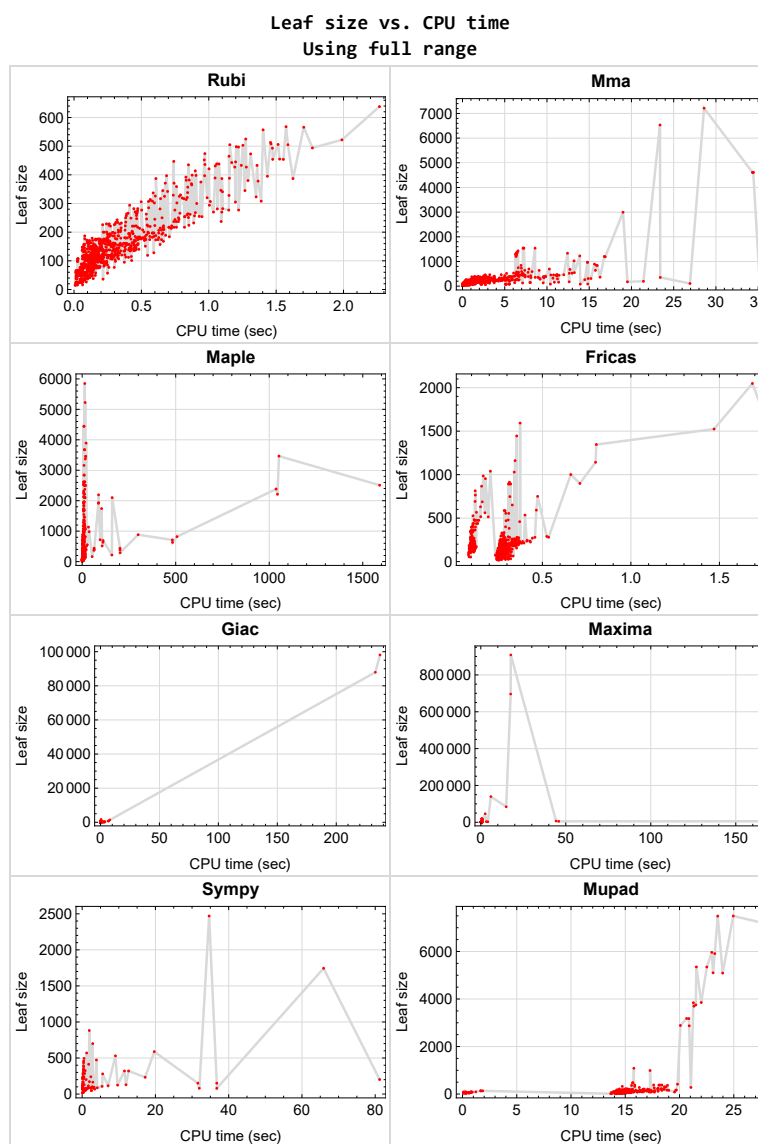


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{680, 681, 682, 683, 684, 685, 686, 687, 688, 689}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {117, 146, 147, 148, 150, 151, 152, 155, 156, 161, 162, 167, 169, 195, 213, 214, 215, 216, 217, 228, 229, 230, 232, 235, 236, 237, 242, 243, 249, 250, 253, 257, 258, 356, 357, 358, 359, 360, 361, 362, 363, 367, 368, 369, 373, 374, 379, 380, 386, 387, 610, 618, 638, 676, 677, 678, 679, 721, 724, 734, 735, 751, 755, 757, 763, 764, 765, 766, 773, 774}

Maple {126, 392, 631, 632, 638, 639, 732, 739, 746, 747, 752, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
```

```

Return the tree size of this expression.
"""
if expr not in SR:
    # deal with lists, tuples, vectors
    return 1 + sum(tree_size(a) for a in expr)
expr = SR(expr)
x, aa = expr.operator(), expr.operands()
if x is None:
    return 1
else:
    return 1 + sum(tree_size(a) for a in aa)

```

For SymPy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```

try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1

```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```

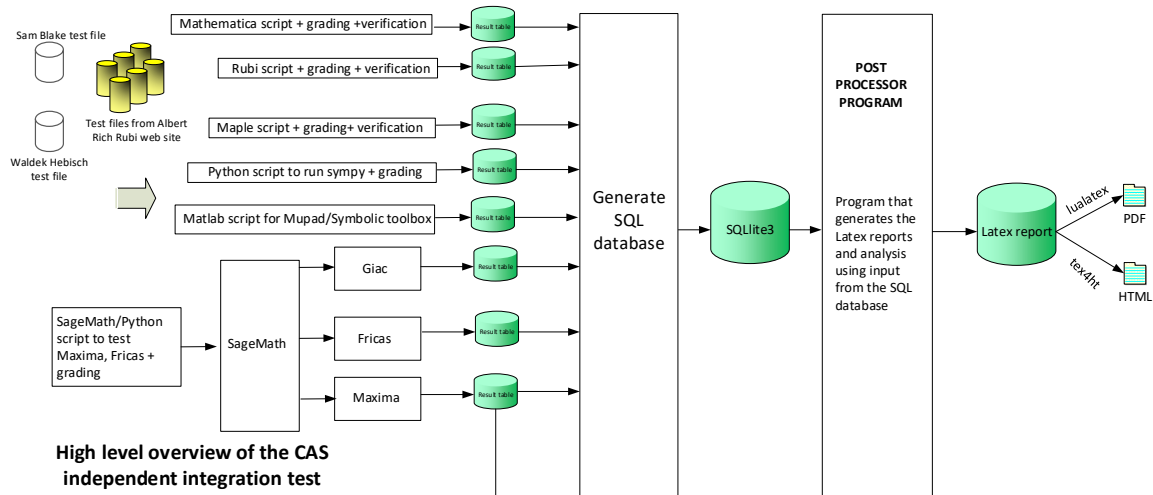
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand,the_variable)

```

Which gives $\sin(x)^2/2$

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in *Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

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June 27, 2023
Design v1.0a

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1	List of integrals sorted by grade for each CAS	22
2.2	Detailed conclusion table per each integral for all CAS systems	33
2.3	Detailed conclusion table specific for Rubi results	220

2.1 List of integrals sorted by grade for each CAS

Rubi	22
Mma	23
Maple	24
Fricas	26
Maxima	27
Giac	28
Mupad	29
Sympy	31

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624,

625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 48, 53, 54, 55, 56, 57, 58, 63, 64, 65, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 233, 234, 238, 239, 240, 241, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 259, 260, 261, 262, 266, 267, 268, 272, 273, 274, 300, 313, 327, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 364, 370, 375, 376, 381, 382, 383, 388, 389, 390, 391, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 493, 494, 495, 496, 497, 500, 501, 502, 503, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 538, 539, 540, 541, 542, 543, 546, 547, 548, 549, 550, 551, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 611, 612, 613, 614, 615, 619, 620, 621, 622, 626, 627, 628, 629, 630, 632, 633, 634, 639, 647, 650, 651, 656, 657, 661, 663, 668, 669, 671, 690, 691,

692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 769, 770, 771, 772, 775, 776, 777, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade { 28, 37, 38, 46, 47, 49, 50, 51, 52, 59, 60, 61, 62, 69, 70, 71, 80, 81, 91, 92, 586, 644, 645, 646, 648, 649, 652, 653, 654, 655, 658, 659, 660, 662, 670, 676, 677, 678, 679, 721, 734, 768, 773, 774, 788, 790 }

C grade { 117, 118, 119, 120, 128, 129, 130, 136, 138, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 213, 214, 215, 216, 217, 228, 229, 230, 235, 236, 237, 242, 243, 249, 250, 257, 258, 263, 264, 265, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 297, 298, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 371, 372, 373, 374, 377, 378, 379, 380, 384, 385, 386, 387, 392, 393, 459, 491, 492, 498, 499, 504, 505, 506, 507, 514, 515, 521, 522, 528, 529, 535, 536, 537, 544, 545, 552, 553, 559, 560, 609, 610, 616, 617, 618, 623, 624, 625, 631, 635, 636, 637, 638, 640, 641, 642, 643, 664, 665, 666, 667, 672, 673, 674, 675, 778 }

F normal fail { 288, 289, 290 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 112, 113, 114, 115, 121, 122, 123, 124, 125, 131, 132, 133, 139, 140, 141, 142, 143, 146, 147, 149, 150, 153, 154, 155, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 186, 189, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 236, 237, 238, 239, 240, 242, 243, 244, 246, 247, 249, 250, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 286, 287,

293, 294, 295, 296, 297, 301, 302, 303, 307, 308, 309, 310, 314, 315, 316, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 490, 497, 504, 509, 510, 511, 513, 516, 517, 518, 520, 525, 527, 528, 533, 534, 535, 543, 546, 547, 548, 549, 554, 555, 556, 561, 564, 565, 572, 578, 583, 584, 605, 627, 628, 648, 650, 651, 656, 657, 658, 660, 661, 662, 663, 664, 666, 667, 668, 669, 672, 673, 674, 692, 693, 694, 695, 696, 700, 701, 703, 704, 708, 710, 711, 712, 715, 716, 733, 753, 754, 755, 779, 780, 781, 782, 783, 784, 785, 786, 791, 792, 793, 799, 800, 801, 802, 803, 806, 807, 808, 809, 810, 813, 814, 815, 816, 817, 820, 821, 822, 823, 824, 827, 828, 829, 830, 831, 834, 835, 836, 837, 838, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886 }

B grade { 100, 101, 102, 103, 108, 109, 110, 111, 116, 117, 118, 119, 120, 127, 128, 129, 130, 134, 135, 136, 137, 138, 144, 145, 148, 151, 152, 156, 157, 158, 159, 164, 165, 166, 171, 172, 173, 180, 185, 187, 188, 196, 201, 208, 209, 222, 223, 233, 234, 235, 241, 245, 248, 251, 252, 261, 271, 285, 291, 292, 298, 299, 300, 304, 305, 306, 311, 312, 313, 317, 327, 332, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 512, 514, 515, 519, 521, 522, 523, 524, 529, 530, 531, 532, 536, 537, 538, 539, 540, 541, 542, 544, 545, 551, 552, 553, 558, 559, 560, 562, 563, 566, 567, 568, 569, 570, 571, 574, 575, 576, 577, 580, 581, 582, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 652, 653, 654, 655, 659, 665, 670, 671, 675, 690, 691, 697, 698, 699, 702, 705, 706, 707, 709, 713, 714, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 778, 794, 804, 805, 811, 812, 818, 819, 825, 826, 832, 833, 839, 840, 841 }

C grade { 32, 41, 42, 89, 126, 526, 550, 557, 573, 579 }

F normal fail { 221, 288, 289, 290, 396, 397, 398, 399, 400, 401, 402, 676, 677, 678, 679, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 795, 796, 797, 798, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

F(-1) timeout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 138, 139, 140, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 403, 404, 405, 406, 407, 408, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 463, 464, 468, 469, 472, 480, 778, 779, 780, 782, 784, 785, 791, 792, 793, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886 }

B grade { 7, 8, 19, 101, 108, 109, 127, 128, 129, 134, 135, 136, 137, 141, 142, 143, 144, 145, 234, 235, 261, 283, 284, 285, 411, 412, 423, 456, 462, 465, 466, 467, 470, 471, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 781, 783, 786, 869 }

C grade { 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 486, 487, 488, 489, 493, 494, 495, 496, 500, 501, 502, 503, 508, 509, 510, 511, 512, 516, 517, 518, 519, 523, 524, 525, 526, 530, 531, 532, 533, 534, 538, 539, 540, 541, 542, 543, 546, 547, 548, 549, 550, 554, 555, 556, 557, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 794, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841 }

F normal fail { 288, 290, 397, 398, 399, 400, 401, 402, 497, 513, 514, 515, 520, 521, 522, 551, 552, 553, 558, 559, 560, 595, 596, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 730, 731, 732, 733, 734, 736, 737, 738, 739, 740, 741, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 795, 796, 797, 798, 887, 888, 889, 890, 891, 892, 893, }

894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

F(-1) timeout fail { 289, 490, 491, 492, 498, 499, 504, 505, 506, 507, 527, 528, 529, 535, 536, 537, 544, 545, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 597, 598, 599, 600, 601, 602, 676, 677, 678, 679, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 735, 742 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 43, 44, 45, 47, 48, 49, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 112, 113, 114, 115, 121, 210, 212, 218, 219, 220, 346, 348, 353, 354, 355, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 780, 793, 842, 843, 844, 845, 846, 847, 850, 851, 852, 853, 854, 855, 858, 859, 860, 861, 862, 863, 866, 867, 868, 869, 873, 874, 875, 876, 880, 881, 882, 883 }

B grade { 40, 42, 46, 50, 51, 52, 101, 102, 103, 109, 110, 111, 117, 118, 119, 122, 123, 124, 125, 126, 128, 134, 135, 137, 143, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 221, 222, 223, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 339, 340, 341, 342, 343, 344, 345, 347, 349, 350, 351, 352, 356, 357, 358, 359, 360, 396, 781, 782, 783, 784, 785, 848, 849, 856, 857, 864, 865, 870, 871, 872, 877, 878, 879, 884, 885, 886 }

C grade { 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 261, 262, 278, 279, 280, 281, 285, 286, 287, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371 }

F normal fail { 100, 108, 116, 127, 136, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 224, 225, 231, 232, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 259, 260, 275, 276, 277, 282, 283, 284, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 366, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654,

655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

F(-1) timedout fail { 120, 129, 130, 131, 132, 133, 139, 140, 141, 142, 145, 258, 386, 602, 724, 725, 786 }

F(-2) exception fail { 138, 196, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 601, 778, 779, 791, 792 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 131, 132, 133, 134, 135, 136, 139, 140, 141, 142, 143, 202, 203, 204, 205, 266, 267, 268, 272, 273, 274, 281, 339, 340, 341, 342, 403, 404, 405, 406, 407, 408, 409, 410, 416, 417, 418, 419, 420, 421, 428, 429, 430, 431, 433, 434, 438, 439, 440, 441, 443, 444, 450, 451, 452, 454, 457, 459, 460, 462, 463, 464, 465, 467, 468, 470, 471, 472, 476, 478, 480, 779, 780, 783, 785, 786, 793, 842, 843, 844, 850, 851, 858 }

B grade { 7, 8, 18, 19, 126, 223, 265, 271, 275, 276, 277, 278, 279, 280, 282, 283, 284, 285, 286, 287, 411, 412, 413, 414, 415, 422, 423, 424, 425, 426, 427, 432, 435, 436, 437, 442, 445, 446, 447, 448, 449, 453, 455, 456, 458, 461, 466, 469, 473, 474, 475, 477, 479, 481, 482, 483, 484, 485, 778, 781, 782, 784, 791, 792 }

C grade { 212, 219 }

F normal fail { 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 206, 207, 209, 213, 214, 222, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 242, 243, 249, 250, 257, 258, 261, 262, 263, 264, 269, 270, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 343, 344, 345, 349, 351, 352, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 373, 374, 379, 380, 386, 387, 397, 398, 399, 400, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, }

501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 845, 846, 847, 848, 849, 852, 853, 854, 855, 856, 857, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

F(-1) timeout fail { 208, 210, 211, 215, 216, 217, 218, 220, 221, 224, 225, 231, 232, 238, 239, 240, 241, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 259, 260, 289, 346, 347, 348, 350, 353, 354, 355, 356, 357, 358, 366, 372, 375, 376, 377, 378, 381, 382, 383, 384, 385, 388, 389, 390, 391, 392, 393, 394, 395, 396, 603 }

F(-2) exception fail { 128, 129, 130, 137, 138, 144, 145, 401, 402 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 124, 125, 126, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 202, 203, 204, 205, 210, 211, 212, 218, 219, 220, 221, 266, 267, 268, 272, 273, 274, 339, 340, 341, 342, 346, 347, 348, 353, 354, 355, 396, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 524, 525, 526, 548, 549, 550, 555, 556, 557, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 780, 781, 782, 783, 785, 786, 791, 792, 793, 794, 822, 823, 842, 843, 844, 845, 850, 851, 852, 853, 858, 859, 860, 861, 866, 867, 868, 873, 874, 875, 880, 881, 882 }

C grade { }

F normal fail { }

F(-1) timedout fail { 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 206, 207, 208, 209, 213, 214, 215, 216, 217, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 343, 344, 345, 349, 350, 351, 352, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 551, 552, 553, 554, 558, 559, 560, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 784, 787, 788, 789, 790, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 846, 847, 848, 849, 854, 855, 856, 857, 862, 863, 864, 865, 869, 870, 871, 872, 876, 877, 878, 879, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

F(-2) exception fail { }

Sympy

- A grade** { 2, 4, 6, 7, 17, 47, 48, 56, 57, 58, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 93, 94, 404, 406, 408, 410, 420, 421, 429, 431, 440, 441, 793, 843, 844, 845, 852, 853, 867, 868, 875 }
- B grade** { 1, 3, 5, 13, 14, 15, 16, 23, 24, 25, 26, 33, 34, 35, 43, 44, 45, 46, 53, 54, 55, 63, 64, 72, 82, 403, 405, 407, 409, 411, 417, 418, 419, 428, 430, 438, 439, 452, 453, 454, 464, 780, 781, 782, 783, 791 }
- C grade** { }
- F normal fail** { 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 27, 28, 29, 30, 36, 37, 38, 39, 49, 50, 51, 52, 59, 60, 61, 62, 69, 70, 71, 79, 80, 81, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 106, 107, 108, 115, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 138, 142, 143, 144, 145, 148, 149, 150, 156, 176, 177, 178, 179, 180, 185, 186, 187, 188, 194, 195, 196, 197, 199, 200, 201, 202, 203, 207, 208, 209, 210, 221, 222, 223, 225, 226, 227, 228, 229, 232, 233, 234, 235, 236, 239, 240, 241, 242, 246, 247, 248, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 273, 276, 277, 278, 279, 280, 283, 284, 285, 286, 287, 293, 294, 295, 296, 297, 301, 302, 303, 308, 309, 310, 315, 316, 318, 319, 320, 321, 325, 326, 327, 328, 332, 333, 334, 343, 344, 345, 351, 352, 363, 364, 365, 366, 369, 370, 371, 372, 374, 375, 376, 399, 400, 401, 402, 412, 413, 414, 415, 416, 422, 423, 424, 425, 426, 432, 433, 434, 435, 442, 443, 444, 445, 455, 456, 457, 458, 465, 466, 467, 468, 475, 476, 477, 484, 485, 487, 488, 489, 490, 491, 492, 495, 496, 497, 503, 510, 511, 512, 513, 514, 515, 517, 518, 519, 520, 521, 522, 524, 525, 526, 527, 528, 529, 532, 533, 534, 535, 536, 537, 542, 543, 544, 545, 548, 549, 550, 551, 552, 553, 555, 556, 557, 558, 559, 560, 563, 564, 565, 571, 603, 604, 605, 606, 607, 611, 612, 613, 614, 626, 627, 628, 629, 630, 632, 633, 634, 635, 639, 640, 641, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 692, 693, 694, 695, 696, 701, 702, 703, 709, 710, 711, 713, 714, 715, 716, 719, 720, 721, 722, 725, 726, 727, 733, 734, 735, 741, 742, 752, 753, 754, 755, 758, 759, 760, 771, 772, 775, 776, 777, 778, 779, 784, 785, 786, 787, 788, 789, 790, 794, 795, 796, 797, 798, 801, 802, 803, 823, 824, 825, 826, 832, 833, 846, 869, 870, 876, 877, 889, 890, 901, 902, 903, 904, 908, 909, 910, 911, 913, 914, 915, 916, 921, 922, 923, 928, 929, 930, 931, 932 }
- F(-1) timedout fail** { 31, 32, 40, 41, 42, 95, 96, 104, 105, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 139, 140, 141, 146, 147, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 189, 190, 191, 192, 193, 198, 204, 205, 206, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 224, 230, 231, 237, 238, 243, 244, 245, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 268, 274, 275, 281, 282, 288, 289, 290, 291, 292, 298, 299, 300, 304, 305, 306, 307, 311, 312, 313, 314, 317, 322, 323, 324, 329, 330, 331, 335, 336, 337, 338, 339, 340, 341, 342, 346, 347, 348, 349, 350, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 367, 368, 373, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 427, 436, 437, 446, 447, 448, 449, 450, 451, 459, 460, 461, 462, 463, 469, 470, 471, 472, 473, 474, 478, 479, 480, 481, 482, 483, 486, 493, 494, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 509, 516, 523, 530, 531, 538, 539, 540, 541, 546, 547, 554, 561, 562, 566, 567, 568, 569, 570, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 631,

636, 637, 638, 642, 643, 676, 677, 678, 679, 680, 681, 689, 690, 691, 697, 698, 699, 700, 704, 705, 706, 707, 708, 712, 717, 718, 723, 724, 728, 729, 730, 731, 732, 736, 737, 738, 739, 740, 743, 744, 745, 746, 747, 748, 749, 750, 751, 756, 757, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 773, 774, 792, 799, 800, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 827, 828, 829, 830, 831, 834, 835, 836, 837, 838, 839, 840, 841, 842, 847, 848, 849, 850, 851, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 871, 872, 873, 874, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 905, 906, 907, 912, 917, 918, 919, 920, 924, 925, 926, 927 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column **N.S.** means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	75	73	84	75	216	92	107
N.S.	1	1.00	0.66	0.64	0.74	0.66	1.89	0.81	0.94
time (sec)	N/A	0.090	0.138	2.449	0.221	0.271	0.343	0.309	16.482

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	65	64	69	64	168	77	93
N.S.	1	1.00	0.71	0.70	0.75	0.70	1.83	0.84	1.01
time (sec)	N/A	0.073	0.101	2.254	0.242	0.335	0.228	0.331	16.631

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	53	57	53	144	62	79
N.S.	1	1.00	0.96	0.70	0.75	0.70	1.89	0.82	1.04
time (sec)	N/A	0.064	0.065	2.191	0.220	0.277	0.167	0.320	17.686

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	57	40	46	42	92	47	55
N.S.	1	1.00	1.06	0.74	0.85	0.78	1.70	0.87	1.02
time (sec)	N/A	0.056	0.071	1.662	0.225	0.261	0.120	0.308	14.560

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	29	34	29	66	31	50
N.S.	1	1.00	0.84	0.76	0.89	0.76	1.74	0.82	1.32
time (sec)	N/A	0.017	0.048	0.714	0.217	0.263	0.100	0.301	15.134

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	15	15
N.S.	1	1.00	1.73	1.07	1.00	1.13	1.13	1.00	1.00
time (sec)	N/A	0.010	0.003	0.333	0.243	0.256	0.059	0.304	13.686

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	28	36	49	43	20
N.S.	1	1.00	1.00	1.81	1.75	2.25	3.06	2.69	1.25
time (sec)	N/A	0.024	0.007	1.144	0.233	0.268	2.511	0.337	13.698

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	38	60	0	63	47
N.S.	1	1.00	1.00	1.25	1.58	2.50	0.00	2.62	1.96
time (sec)	N/A	0.042	0.008	1.781	0.220	0.277	0.000	0.324	13.797

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	47	58	74	0	80	75
N.S.	1	1.00	1.00	1.00	1.23	1.57	0.00	1.70	1.60
time (sec)	N/A	0.052	0.011	2.257	0.242	0.278	0.000	0.337	14.610

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	60	70	88	0	96	102
N.S.	1	1.00	0.95	0.95	1.11	1.40	0.00	1.52	1.62
time (sec)	N/A	0.058	0.103	2.284	0.212	0.271	0.000	0.341	16.521

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	56	73	95	99	0	110	130
N.S.	1	1.00	0.66	0.86	1.12	1.16	0.00	1.29	1.53
time (sec)	N/A	0.080	0.129	2.684	0.231	0.257	0.000	0.321	17.611

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	65	83	107	110	0	124	158
N.S.	1	1.00	0.64	0.82	1.06	1.09	0.00	1.23	1.56
time (sec)	N/A	0.086	0.199	2.496	0.232	0.261	0.000	0.307	19.005

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	73	75	121	89	343	106	121
N.S.	1	1.00	0.57	0.58	0.94	0.69	2.66	0.82	0.94
time (sec)	N/A	0.151	0.155	2.974	0.233	0.268	0.370	0.307	16.985

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	61	64	95	76	221	89	105
N.S.	1	1.00	0.59	0.62	0.92	0.74	2.15	0.86	1.02
time (sec)	N/A	0.137	0.075	2.761	0.217	0.264	0.254	0.342	17.331

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	53	55	83	63	211	72	89
N.S.	1	1.00	0.61	0.63	0.95	0.72	2.43	0.83	1.02
time (sec)	N/A	0.111	0.084	2.258	0.221	0.259	0.183	0.309	17.201

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	69	41	42	61	49	107	54	61
N.S.	1	1.21	0.72	0.74	1.07	0.86	1.88	0.95	1.07
time (sec)	N/A	0.043	0.055	1.422	0.225	0.254	0.126	0.306	13.706

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	31	45	36	78	38	57
N.S.	1	1.00	0.76	0.69	1.00	0.80	1.73	0.84	1.27
time (sec)	N/A	0.015	0.083	0.701	0.227	0.252	0.092	0.301	14.395

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	47	44	43	53	0	79	33
N.S.	1	1.00	1.38	1.29	1.26	1.56	0.00	2.32	0.97
time (sec)	N/A	0.068	0.026	1.226	0.213	0.254	0.000	0.330	14.234

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	44	49	76	0	79	56
N.S.	1	1.00	0.82	1.29	1.44	2.24	0.00	2.32	1.65
time (sec)	N/A	0.068	0.017	1.887	0.219	0.257	0.000	0.329	14.195

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	70	88	83	0	90	83
N.S.	1	1.00	1.00	1.30	1.63	1.54	0.00	1.67	1.54
time (sec)	N/A	0.086	0.011	2.236	0.210	0.256	0.000	0.340	14.145

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	75	85	96	0	106	112
N.S.	1	1.00	1.00	1.14	1.29	1.45	0.00	1.61	1.70
time (sec)	N/A	0.107	0.219	3.313	0.245	0.262	0.000	0.352	15.599

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	58	112	145	111	0	122	141
N.S.	1	1.00	0.60	1.17	1.51	1.16	0.00	1.27	1.47
time (sec)	N/A	0.126	0.297	2.931	0.217	0.263	0.000	0.354	16.990

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	73	75	143	89	379	106	121
N.S.	1	1.00	0.57	0.58	1.11	0.69	2.94	0.82	0.94
time (sec)	N/A	0.167	0.112	3.378	0.230	0.260	0.385	0.332	16.744

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	63	66	117	76	272	88	105
N.S.	1	1.00	0.60	0.63	1.11	0.72	2.59	0.84	1.00
time (sec)	N/A	0.130	0.086	2.467	0.204	0.254	0.271	0.319	17.432

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	88	51	53	94	63	224	71	89
N.S.	1	1.04	0.60	0.62	1.11	0.74	2.64	0.84	1.05
time (sec)	N/A	0.085	0.080	2.311	0.240	0.255	0.195	0.330	17.364

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	42	70	50	121	55	63
N.S.	1	1.00	0.70	0.67	1.11	0.79	1.92	0.87	1.00
time (sec)	N/A	0.062	0.102	1.619	0.218	0.280	0.126	0.328	14.110

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	59	67	65	0	100	88
N.S.	1	1.00	1.37	1.00	1.14	1.10	0.00	1.69	1.49
time (sec)	N/A	0.070	0.408	1.308	0.214	0.269	0.000	0.371	14.531

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	211	55	64	91	0	80	57
N.S.	1	1.00	4.40	1.15	1.33	1.90	0.00	1.67	1.19
time (sec)	N/A	0.080	1.078	2.257	0.224	0.301	0.000	0.447	14.895

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	50	80	99	98	0	100	88
N.S.	1	1.00	0.85	1.36	1.68	1.66	0.00	1.69	1.49
time (sec)	N/A	0.090	0.025	2.544	0.341	0.263	0.000	0.430	14.626

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	94	111	98	0	106	112
N.S.	1	1.00	1.00	1.31	1.54	1.36	0.00	1.47	1.56
time (sec)	N/A	0.109	0.228	3.446	0.238	0.254	0.000	0.367	16.088

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	123	156	111	0	122	141
N.S.	1	1.00	1.00	1.32	1.68	1.19	0.00	1.31	1.52
time (sec)	N/A	0.142	0.314	3.794	0.240	0.267	0.000	0.409	16.968

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	114	145	179	124	0	138	170
N.S.	1	1.00	1.00	1.27	1.57	1.09	0.00	1.21	1.49
time (sec)	N/A	0.156	0.433	4.074	0.242	0.252	0.000	0.355	18.351

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	73	75	165	89	434	106	121
N.S.	1	1.00	0.57	0.59	1.30	0.70	3.42	0.83	0.95
time (sec)	N/A	0.183	0.128	3.733	0.220	0.252	0.391	0.359	16.436

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	114	63	66	128	76	280	89	105
N.S.	1	1.12	0.62	0.65	1.25	0.75	2.75	0.87	1.03
time (sec)	N/A	0.139	0.091	2.794	0.316	0.243	0.277	0.349	17.628

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	56	55	106	63	224	72	89
N.S.	1	1.00	0.64	0.63	1.22	0.72	2.57	0.83	1.02
time (sec)	N/A	0.102	0.139	2.109	0.223	0.254	0.196	0.306	17.281

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	91	70	89	80	0	116	93
N.S.	1	1.00	1.25	0.96	1.22	1.10	0.00	1.59	1.27
time (sec)	N/A	0.096	0.750	2.447	0.270	0.255	0.000	0.370	13.996

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	241	82	85	105	0	129	117
N.S.	1	1.00	3.30	1.12	1.16	1.44	0.00	1.77	1.60
time (sec)	N/A	0.105	2.997	2.186	0.233	0.321	0.000	0.359	14.568

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	272	91	110	111	0	129	115
N.S.	1	1.00	3.73	1.25	1.51	1.52	0.00	1.77	1.58
time (sec)	N/A	0.105	3.459	2.970	0.251	0.264	0.000	0.379	14.792

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	96	120	110	0	116	117
N.S.	1	1.00	0.84	1.32	1.64	1.51	0.00	1.59	1.60
time (sec)	N/A	0.123	0.029	3.296	0.298	0.275	0.000	0.349	14.642

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	135	182	111	0	122	141
N.S.	1	1.00	1.00	1.41	1.90	1.16	0.00	1.27	1.47
time (sec)	N/A	0.163	1.276	3.954	0.273	0.298	0.000	0.377	17.745

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	111	156	190	124	0	138	170
N.S.	1	1.00	1.00	1.41	1.71	1.12	0.00	1.24	1.53
time (sec)	N/A	0.162	1.574	4.382	0.239	0.272	0.000	0.380	19.710

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	136	178	270	137	0	154	199
N.S.	1	1.00	1.00	1.31	1.99	1.01	0.00	1.13	1.46
time (sec)	N/A	0.239	3.760	4.540	0.264	0.278	0.000	0.381	18.603

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	173	67	217	79	882	101	98
N.S.	1	1.00	1.47	0.57	1.84	0.67	7.47	0.86	0.83
time (sec)	N/A	0.124	0.842	0.805	0.315	0.260	1.934	0.341	16.481

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	143	54	176	70	570	88	70
N.S.	1	1.00	1.52	0.57	1.87	0.74	6.06	0.94	0.74
time (sec)	N/A	0.127	0.743	0.780	0.490	0.321	1.225	0.329	14.751

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	117	43	133	57	325	73	89
N.S.	1	1.00	1.54	0.57	1.75	0.75	4.28	0.96	1.17
time (sec)	N/A	0.069	0.722	0.761	0.316	0.256	0.817	0.328	14.376

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	89	31	92	46	129	58	66
N.S.	1	1.00	2.07	0.72	2.14	1.07	3.00	1.35	1.53
time (sec)	N/A	0.093	0.474	0.735	0.309	0.259	0.560	0.381	14.916

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	69	23	49	37	27	28	23
N.S.	1	1.00	2.38	0.79	1.69	1.28	0.93	0.97	0.79
time (sec)	N/A	0.046	0.141	0.717	0.315	0.248	0.359	0.360	14.640

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	17	17	23	22	20	16	16
N.S.	1	1.00	0.77	0.77	1.05	1.00	0.91	0.73	0.73
time (sec)	N/A	0.015	0.012	0.699	0.214	0.243	0.297	0.356	14.196

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	103	46	75	65	0	54	31
N.S.	1	1.00	2.71	1.21	1.97	1.71	0.00	1.42	0.82
time (sec)	N/A	0.053	0.282	0.868	0.220	0.322	0.000	0.438	14.482

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	188	74	119	97	0	84	67
N.S.	1	1.00	3.55	1.40	2.25	1.83	0.00	1.58	1.26
time (sec)	N/A	0.092	0.783	0.933	0.231	0.254	0.000	0.319	14.798

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	244	105	162	112	0	101	95
N.S.	1	1.00	2.94	1.27	1.95	1.35	0.00	1.22	1.14
time (sec)	N/A	0.105	1.167	0.992	0.243	0.260	0.000	0.320	14.756

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	368	132	205	124	0	114	96
N.S.	1	1.00	3.57	1.28	1.99	1.20	0.00	1.11	0.93
time (sec)	N/A	0.119	3.134	1.044	0.208	0.270	0.000	0.314	14.585

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	115	77	207	108	700	108	135
N.S.	1	1.00	0.93	0.62	1.67	0.87	5.65	0.87	1.09
time (sec)	N/A	0.213	0.528	0.872	0.309	0.257	2.879	0.332	14.881

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	107	66	164	99	413	95	113
N.S.	1	1.00	0.94	0.58	1.44	0.87	3.62	0.83	0.99
time (sec)	N/A	0.176	0.423	0.789	0.353	0.306	1.774	0.454	14.579

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	97	55	118	90	201	79	91
N.S.	1	1.00	1.21	0.69	1.48	1.12	2.51	0.99	1.14
time (sec)	N/A	0.193	0.450	0.791	0.304	0.255	1.080	0.319	14.330

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	87	36	72	80	56	50	35
N.S.	1	1.00	1.53	0.63	1.26	1.40	0.98	0.88	0.61
time (sec)	N/A	0.096	0.256	0.703	0.313	0.256	0.650	0.303	14.215

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	31	47	51	48	31	30
N.S.	1	1.00	0.65	0.56	0.85	0.93	0.87	0.56	0.55
time (sec)	N/A	0.044	0.079	0.677	0.232	0.242	0.491	0.298	14.362

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	31	46	49	44	31	30
N.S.	1	1.00	0.96	0.56	0.84	0.89	0.80	0.56	0.55
time (sec)	N/A	0.032	0.036	0.629	0.238	0.245	0.427	0.304	14.060

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	152	62	98	114	0	77	43
N.S.	1	1.00	2.30	0.94	1.48	1.73	0.00	1.17	0.65
time (sec)	N/A	0.122	0.437	0.900	0.227	0.325	0.000	0.332	14.080

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	239	92	145	146	0	106	92
N.S.	1	1.00	2.95	1.14	1.79	1.80	0.00	1.31	1.14
time (sec)	N/A	0.187	1.073	0.839	0.250	0.297	0.000	0.312	14.369

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	292	120	190	162	0	122	122
N.S.	1	1.00	2.45	1.01	1.60	1.36	0.00	1.03	1.03
time (sec)	N/A	0.208	1.590	1.052	0.279	0.273	0.000	0.322	14.364

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	343	152	234	172	0	135	153
N.S.	1	1.00	2.58	1.14	1.76	1.29	0.00	1.02	1.15
time (sec)	N/A	0.216	2.911	1.261	0.240	0.262	0.000	0.329	14.297

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	127	77	184	135	473	113	137
N.S.	1	1.00	0.83	0.50	1.20	0.88	3.09	0.74	0.90
time (sec)	N/A	0.298	2.026	0.838	0.332	0.267	3.889	0.313	14.301

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	107	66	137	126	240	96	113
N.S.	1	1.00	0.90	0.55	1.15	1.06	2.02	0.81	0.95
time (sec)	N/A	0.329	0.488	0.785	0.308	0.252	2.377	0.315	14.279

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	108	51	92	116	75	68	81
N.S.	1	1.00	1.12	0.53	0.96	1.21	0.78	0.71	0.84
time (sec)	N/A	0.223	0.816	0.808	0.318	0.269	1.362	0.308	14.227

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	46	45	67	75	68	46	45
N.S.	1	1.00	0.55	0.54	0.81	0.90	0.82	0.55	0.54
time (sec)	N/A	0.105	0.130	0.707	0.232	0.245	1.035	0.307	14.536

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	44	31	47	73	48	31	30
N.S.	1	1.00	0.53	0.37	0.57	0.88	0.58	0.37	0.36
time (sec)	N/A	0.066	0.074	0.837	0.575	0.295	0.808	0.305	14.316

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	45	67	75	63	46	45
N.S.	1	1.00	0.78	0.54	0.81	0.90	0.76	0.55	0.54
time (sec)	N/A	0.057	0.056	0.659	0.229	0.234	0.687	0.293	14.604

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	201	75	119	158	0	94	58
N.S.	1	1.00	2.07	0.77	1.23	1.63	0.00	0.97	0.60
time (sec)	N/A	0.224	0.550	0.894	0.228	0.254	0.000	0.331	14.175

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	286	105	165	190	0	122	111
N.S.	1	1.00	2.55	0.94	1.47	1.70	0.00	1.09	0.99
time (sec)	N/A	0.323	1.129	1.025	0.344	0.259	0.000	0.322	14.306

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	343	135	211	206	0	139	141
N.S.	1	1.00	2.20	0.87	1.35	1.32	0.00	0.89	0.90
time (sec)	N/A	0.338	2.914	1.102	0.289	0.259	0.000	0.322	14.377

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	137	88	204	171	530	128	159
N.S.	1	1.00	0.74	0.48	1.11	0.93	2.88	0.70	0.86
time (sec)	N/A	0.412	6.175	0.934	0.356	0.266	9.079	0.321	14.851

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	135	77	158	162	280	112	137
N.S.	1	1.00	0.90	0.51	1.05	1.08	1.87	0.75	0.91
time (sec)	N/A	0.421	1.664	0.830	0.327	0.267	5.596	0.337	14.974

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	117	64	112	152	95	83	102
N.S.	1	1.00	0.92	0.50	0.88	1.20	0.75	0.65	0.80
time (sec)	N/A	0.308	3.187	0.650	0.330	0.269	3.261	0.343	14.835

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	56	57	87	99	88	59	58
N.S.	1	1.00	0.49	0.50	0.76	0.87	0.77	0.52	0.51
time (sec)	N/A	0.214	1.227	0.692	0.248	0.258	2.401	0.320	15.080

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	56	58	87	99	87	59	58
N.S.	1	1.00	0.50	0.52	0.78	0.88	0.78	0.53	0.52
time (sec)	N/A	0.126	0.877	0.777	0.258	0.252	1.890	0.317	14.941

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	56	57	87	99	85	59	58
N.S.	1	1.00	0.50	0.51	0.78	0.88	0.76	0.53	0.52
time (sec)	N/A	0.090	0.163	0.780	0.230	0.243	1.585	0.307	14.939

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	77	56	87	99	83	59	58
N.S.	1	1.00	0.69	0.50	0.78	0.88	0.74	0.53	0.52
time (sec)	N/A	0.071	0.120	0.630	0.228	0.283	1.431	0.297	15.283

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	185	88	139	202	0	110	83
N.S.	1	1.00	1.54	0.73	1.16	1.68	0.00	0.92	0.69
time (sec)	N/A	0.320	0.901	1.026	0.229	0.265	0.000	0.333	14.901

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	341	118	186	234	0	139	130
N.S.	1	1.00	2.53	0.87	1.38	1.73	0.00	1.03	0.96
time (sec)	N/A	0.417	3.376	1.065	0.249	0.260	0.000	0.326	14.974

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	455	148	231	250	0	155	160
N.S.	1	1.00	2.46	0.80	1.25	1.35	0.00	0.84	0.86
time (sec)	N/A	0.453	6.781	1.271	0.329	0.261	0.000	0.337	15.503

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	143	98	224	207	588	145	181
N.S.	1	1.00	0.64	0.44	1.00	0.92	2.61	0.64	0.80
time (sec)	N/A	0.543	8.033	0.997	0.339	0.257	19.695	0.346	15.425

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	149	88	178	198	320	129	159
N.S.	1	1.00	0.78	0.46	0.93	1.04	1.68	0.68	0.83
time (sec)	N/A	0.539	7.982	0.743	0.324	0.268	11.539	0.432	15.056

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	127	77	132	188	116	100	125
N.S.	1	1.00	0.76	0.46	0.79	1.12	0.69	0.60	0.74
time (sec)	N/A	0.449	7.118	0.713	0.309	0.258	7.073	0.420	14.852

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	66	71	107	123	107	72	127
N.S.	1	1.00	0.43	0.46	0.69	0.79	0.69	0.46	0.82
time (sec)	N/A	0.339	2.737	0.738	0.222	0.247	5.436	0.425	14.121

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	66	57	87	123	87	59	58
N.S.	1	1.00	0.45	0.39	0.59	0.84	0.59	0.40	0.39
time (sec)	N/A	0.239	2.694	0.843	0.221	0.300	4.125	0.434	14.414

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	66	45	67	123	68	46	45
N.S.	1	1.00	0.47	0.32	0.48	0.88	0.49	0.33	0.32
time (sec)	N/A	0.165	2.261	0.818	0.261	0.250	3.418	0.358	14.012

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	66	57	87	123	85	59	58
N.S.	1	1.00	0.46	0.40	0.61	0.86	0.59	0.41	0.41
time (sec)	N/A	0.126	0.224	0.767	0.263	0.253	3.037	0.438	14.056

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	89	69	107	123	102	72	127
N.S.	1	1.00	0.62	0.48	0.75	0.86	0.71	0.50	0.89
time (sec)	N/A	0.113	0.117	0.780	0.213	0.247	2.758	0.388	14.571

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	211	101	159	246	0	126	99
N.S.	1	1.00	1.38	0.66	1.04	1.61	0.00	0.82	0.65
time (sec)	N/A	0.451	1.622	1.112	0.246	0.269	0.000	0.437	14.541

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	393	131	206	278	0	155	149
N.S.	1	1.00	2.34	0.78	1.23	1.65	0.00	0.92	0.89
time (sec)	N/A	0.663	5.518	1.174	0.302	0.289	0.000	0.443	14.776

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	507	160	251	294	0	171	179
N.S.	1	1.00	2.26	0.71	1.12	1.31	0.00	0.76	0.80
time (sec)	N/A	0.852	7.091	1.223	0.257	0.266	0.000	0.415	14.231

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	76	83	127	147	129	85	75
N.S.	1	1.00	0.41	0.45	0.69	0.80	0.70	0.46	0.41
time (sec)	N/A	0.707	5.085	0.742	0.233	0.253	11.966	0.422	15.305

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	76	84	127	147	124	85	151
N.S.	1	1.00	0.43	0.48	0.72	0.84	0.70	0.48	0.86
time (sec)	N/A	0.422	5.085	0.869	0.227	0.250	9.672	0.414	14.791

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	92	97	79	72	0	117	0
N.S.	1	1.00	0.58	0.61	0.50	0.46	0.00	0.74	0.00
time (sec)	N/A	0.306	0.202	0.971	0.391	0.243	0.000	0.659	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	84	65	62	0	96	0
N.S.	1	1.00	0.66	0.69	0.53	0.51	0.00	0.79	0.00
time (sec)	N/A	0.212	0.096	0.797	0.365	0.243	0.000	0.619	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	68	71	51	52	0	75	0
N.S.	1	1.00	0.79	0.83	0.59	0.60	0.00	0.87	0.00
time (sec)	N/A	0.138	0.064	0.917	0.439	0.303	0.000	0.484	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	58	36	40	0	53	0
N.S.	1	1.00	0.96	1.04	0.64	0.71	0.00	0.95	0.00
time (sec)	N/A	0.057	0.046	0.776	0.388	0.261	0.000	0.547	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	20	32	0	30	33
N.S.	1	1.00	1.12	1.65	0.77	1.23	0.00	1.15	1.27
time (sec)	N/A	0.017	0.020	0.942	0.373	0.242	0.000	0.372	14.123

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	50	182	0	146	0	58	0
N.S.	1	1.00	1.35	4.92	0.00	3.95	0.00	1.57	0.00
time (sec)	N/A	0.060	0.035	1.415	0.000	0.270	0.000	0.381	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	79	383	1170	140	0	104	0
N.S.	1	1.00	1.27	6.18	18.87	2.26	0.00	1.68	0.00
time (sec)	N/A	0.140	0.071	1.221	0.402	0.263	0.000	0.422	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	94	551	2642	155	0	131	0
N.S.	1	1.00	0.92	5.40	25.90	1.52	0.00	1.28	0.00
time (sec)	N/A	0.194	0.122	1.560	4.163	0.265	0.000	0.438	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	109	717	5115	165	0	154	0
N.S.	1	1.00	0.79	5.20	37.07	1.20	0.00	1.12	0.00
time (sec)	N/A	0.265	0.224	1.779	45.833	0.264	0.000	0.425	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	93	99	84	78	0	122	0
N.S.	1	1.00	0.57	0.61	0.52	0.48	0.00	0.75	0.00
time (sec)	N/A	0.291	0.165	0.714	0.388	0.263	0.000	0.681	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	81	86	69	67	0	100	0
N.S.	1	1.00	0.70	0.74	0.59	0.58	0.00	0.86	0.00
time (sec)	N/A	0.162	0.106	0.867	0.385	0.254	0.000	0.428	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	67	71	53	55	0	77	0
N.S.	1	1.00	0.78	0.83	0.62	0.64	0.00	0.90	0.00
time (sec)	N/A	0.078	0.063	0.970	0.393	0.246	0.000	0.333	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	38	44	0	55	0
N.S.	1	1.00	0.93	0.98	0.64	0.75	0.00	0.93	0.00
time (sec)	N/A	0.032	0.041	0.772	0.408	0.309	0.000	0.308	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	209	0	127	0	89	0
N.S.	1	1.00	0.98	3.17	0.00	1.92	0.00	1.35	0.00
time (sec)	N/A	0.131	0.058	1.335	0.000	0.255	0.000	0.312	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	81	385	1314	146	0	107	0
N.S.	1	1.00	1.25	5.92	20.22	2.25	0.00	1.65	0.00
time (sec)	N/A	0.141	0.079	1.370	0.413	0.251	0.000	0.329	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	97	551	3216	162	0	134	0
N.S.	1	1.00	0.92	5.20	30.34	1.53	0.00	1.26	0.00
time (sec)	N/A	0.199	0.159	1.557	0.931	0.317	0.000	0.328	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	110	718	5542	173	0	158	0
N.S.	1	1.00	0.76	4.99	38.49	1.20	0.00	1.10	0.00
time (sec)	N/A	0.294	0.236	1.901	164.443	0.267	0.000	0.333	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	107	112	111	101	0	156	0
N.S.	1	1.00	0.53	0.55	0.55	0.50	0.00	0.77	0.00
time (sec)	N/A	0.432	0.314	5.543	0.356	0.264	0.000	1.757	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	95	99	94	88	0	132	0
N.S.	1	1.00	0.65	0.68	0.64	0.60	0.00	0.90	0.00
time (sec)	N/A	0.199	0.192	2.147	0.470	0.308	0.000	0.809	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	84	86	77	75	0	108	0
N.S.	1	1.00	0.72	0.74	0.66	0.65	0.00	0.93	0.00
time (sec)	N/A	0.100	0.146	1.163	0.372	0.265	0.000	0.442	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	71	73	60	62	0	84	0
N.S.	1	1.00	0.80	0.82	0.67	0.70	0.00	0.94	0.00
time (sec)	N/A	0.054	0.068	0.885	0.342	0.257	0.000	0.356	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	89	246	0	147	0	120	0
N.S.	1	1.00	0.91	2.51	0.00	1.50	0.00	1.22	0.00
time (sec)	N/A	0.242	0.279	1.973	0.000	0.276	0.000	0.386	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	217	432	10847	164	0	135	0
N.S.	1	1.00	2.36	4.70	117.90	1.78	0.00	1.47	0.00
time (sec)	N/A	0.243	5.580	6.561	0.673	0.270	0.000	0.375	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	267	551	3667	170	0	140	0
N.S.	1	1.00	2.52	5.20	34.59	1.60	0.00	1.32	0.00
time (sec)	N/A	0.283	6.367	27.530	3.255	0.280	0.000	0.526	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	284	717	6703	183	0	166	0
N.S.	1	1.00	1.97	4.98	46.55	1.27	0.00	1.15	0.00
time (sec)	N/A	0.419	6.465	96.056	44.297	0.270	0.000	0.416	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	388	882	0	196	0	192	0
N.S.	1	1.00	2.13	4.85	0.00	1.08	0.00	1.05	0.00
time (sec)	N/A	0.581	6.688	299.260	0.000	0.266	0.000	0.422	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	83	86	77	75	0	108	0
N.S.	1	1.00	0.70	0.72	0.65	0.63	0.00	0.91	0.00
time (sec)	N/A	0.117	0.161	0.983	0.371	0.250	0.000	0.443	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	112	194	696204	153	0	141	0
N.S.	1	1.00	0.64	1.11	4001.17	0.88	0.00	0.81	0.00
time (sec)	N/A	0.649	0.228	1.163	17.570	0.293	0.000	0.577	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	101	183	908518	143	0	139	0
N.S.	1	1.00	0.72	1.31	6489.41	1.02	0.00	0.99	0.00
time (sec)	N/A	0.461	0.170	1.293	17.657	0.257	0.000	0.415	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	84	132	19437	131	0	103	97
N.S.	1	1.00	0.81	1.27	186.89	1.26	0.00	0.99	0.93
time (sec)	N/A	0.158	0.095	1.414	0.662	0.265	0.000	0.360	14.229

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	53	120	18948	122	0	100	60
N.S.	1	1.00	0.73	1.64	259.56	1.67	0.00	1.37	0.82
time (sec)	N/A	0.069	0.032	1.223	0.575	0.312	0.000	0.341	0.128

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	40	56	90	126	0	93	45
N.S.	1	1.00	0.87	1.22	1.96	2.74	0.00	2.02	0.98
time (sec)	N/A	0.028	0.008	0.441	0.352	0.262	0.000	0.298	14.131

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	226	0	164	0	121	0
N.S.	1	1.00	0.76	2.66	0.00	1.93	0.00	1.42	0.00
time (sec)	N/A	0.158	0.040	1.639	0.000	0.271	0.000	0.343	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	192	466	18435	236	0	0	0
N.S.	1	1.00	1.78	4.31	170.69	2.19	0.00	0.00	0.00
time (sec)	N/A	0.277	0.855	1.746	0.548	0.267	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	360	677	0	251	0	0	0
N.S.	1	1.00	2.45	4.61	0.00	1.71	0.00	0.00	0.00
time (sec)	N/A	0.460	1.261	1.802	0.000	0.278	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	385	883	0	263	0	0	0
N.S.	1	1.00	2.13	4.88	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.629	1.416	2.039	0.000	0.281	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	118	265	0	184	0	167	0
N.S.	1	1.00	0.64	1.45	0.00	1.01	0.00	0.91	0.00
time (sec)	N/A	0.489	0.264	1.159	0.000	0.268	0.000	1.139	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	110	234	0	174	0	101	0
N.S.	1	1.00	0.76	1.61	0.00	1.20	0.00	0.70	0.00
time (sec)	N/A	0.334	0.193	1.264	0.000	0.261	0.000	0.689	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	100	173	0	164	0	145	0
N.S.	1	1.00	0.95	1.65	0.00	1.56	0.00	1.38	0.00
time (sec)	N/A	0.176	0.133	1.561	0.000	0.255	0.000	0.464	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	54	140	46532	154	0	116	0
N.S.	1	1.00	0.70	1.82	604.31	2.00	0.00	1.51	0.00
time (sec)	N/A	0.067	0.077	1.283	2.590	0.250	0.000	0.375	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	138	15721	153	0	112	0
N.S.	1	1.00	0.82	1.79	204.17	1.99	0.00	1.45	0.00
time (sec)	N/A	0.045	0.048	1.105	1.142	0.254	0.000	0.307	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	227	290	0	254	0	137	0
N.S.	1	1.00	1.99	2.54	0.00	2.23	0.00	1.20	0.00
time (sec)	N/A	0.274	1.266	1.948	0.000	0.265	0.000	0.359	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	103	567	139280	286	0	0	0
N.S.	1	1.00	0.72	3.94	967.22	1.99	0.00	0.00	0.00
time (sec)	N/A	0.445	0.338	1.684	5.926	0.308	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	841	807	0	302	0	0	0
N.S.	1	1.00	4.55	4.36	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.594	6.928	1.809	0.000	0.281	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	123	242	0	208	0	122	0
N.S.	1	1.00	0.67	1.32	0.00	1.14	0.00	0.67	0.00
time (sec)	N/A	0.477	0.422	1.425	0.000	0.268	0.000	3.304	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	113	208	0	198	0	161	0
N.S.	1	1.00	0.78	1.43	0.00	1.37	0.00	1.11	0.00
time (sec)	N/A	0.319	0.306	1.398	0.000	0.256	0.000	2.120	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	104	174	0	188	0	138	0
N.S.	1	1.00	0.97	1.63	0.00	1.76	0.00	1.29	0.00
time (sec)	N/A	0.167	0.269	1.276	0.000	0.263	0.000	1.090	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	174	0	188	0	69	0
N.S.	1	1.00	0.61	1.63	0.00	1.76	0.00	0.64	0.00
time (sec)	N/A	0.097	0.170	1.289	0.000	0.262	0.000	0.643	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	174	84332	188	0	129	0
N.S.	1	1.00	0.61	1.63	788.15	1.76	0.00	1.21	0.00
time (sec)	N/A	0.074	0.106	1.308	14.940	0.259	0.000	0.340	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	279	325	0	298	0	0	0
N.S.	1	1.00	1.94	2.26	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.409	4.764	1.749	0.000	0.276	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	731	601	0	330	0	0	0
N.S.	1	1.00	4.20	3.45	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.624	6.972	1.693	0.000	0.271	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	241	270	0	148	0	0	87
N.S.	1	1.00	2.17	2.43	0.00	1.33	0.00	0.00	0.78
time (sec)	N/A	0.087	4.640	8.852	0.000	0.093	0.000	0.000	15.493

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	232	219	0	137	0	0	80
N.S.	1	1.00	2.67	2.52	0.00	1.57	0.00	0.00	0.92
time (sec)	N/A	0.068	3.560	6.664	0.000	0.091	0.000	0.000	0.167

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	222	225	0	125	0	0	53
N.S.	1	1.00	3.64	3.69	0.00	2.05	0.00	0.00	0.87
time (sec)	N/A	0.062	3.194	5.078	0.000	0.089	0.000	0.000	0.134

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	155	150	0	107	0	0	27
N.S.	1	1.00	4.43	4.29	0.00	3.06	0.00	0.00	0.77
time (sec)	N/A	0.048	1.333	2.605	0.000	0.084	0.000	0.000	14.942

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	209	148	0	156	0	0	60
N.S.	1	1.00	3.67	2.60	0.00	2.74	0.00	0.00	1.05
time (sec)	N/A	0.059	3.235	3.096	0.000	0.087	0.000	0.000	15.552

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	247	368	0	175	0	0	87
N.S.	1	1.00	2.98	4.43	0.00	2.11	0.00	0.00	1.05
time (sec)	N/A	0.070	4.506	4.266	0.000	0.093	0.000	0.000	15.316

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	477	384	0	188	0	0	87
N.S.	1	1.00	4.30	3.46	0.00	1.69	0.00	0.00	0.78
time (sec)	N/A	0.088	6.318	6.212	0.000	0.094	0.000	0.000	15.449

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	255	260	0	175	0	0	136
N.S.	1	1.00	1.73	1.77	0.00	1.19	0.00	0.00	0.93
time (sec)	N/A	0.170	5.277	12.147	0.000	0.100	0.000	0.000	15.070

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	245	272	0	162	0	0	129
N.S.	1	1.00	2.02	2.25	0.00	1.34	0.00	0.00	1.07
time (sec)	N/A	0.149	4.736	9.442	0.000	0.094	0.000	0.000	14.544

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	235	250	0	149	0	0	104
N.S.	1	1.00	2.47	2.63	0.00	1.57	0.00	0.00	1.09
time (sec)	N/A	0.128	3.865	7.076	0.000	0.092	0.000	0.000	14.623

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	224	228	0	134	0	0	59
N.S.	1	1.00	3.34	3.40	0.00	2.00	0.00	0.00	0.88
time (sec)	N/A	0.100	3.856	3.940	0.000	0.089	0.000	0.000	14.886

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	112	185	0	97	0	0	82
N.S.	1	1.00	2.55	4.20	0.00	2.20	0.00	0.00	1.86
time (sec)	N/A	0.106	0.264	4.041	0.000	0.084	0.000	0.000	14.903

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	111	371	0	187	0	0	109
N.S.	1	1.00	1.22	4.08	0.00	2.05	0.00	0.00	1.20
time (sec)	N/A	0.118	0.218	4.827	0.000	0.092	0.000	0.000	14.961

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	114	386	0	202	0	0	114
N.S.	1	1.00	0.94	3.19	0.00	1.67	0.00	0.00	0.94
time (sec)	N/A	0.150	0.282	7.274	0.000	0.094	0.000	0.000	15.266

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	255	260	0	175	0	0	206
N.S.	1	1.00	1.73	1.77	0.00	1.19	0.00	0.00	1.40
time (sec)	N/A	0.190	6.712	12.954	0.000	0.100	0.000	0.000	15.005

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	245	272	0	162	0	0	143
N.S.	1	1.00	2.02	2.25	0.00	1.34	0.00	0.00	1.18
time (sec)	N/A	0.186	5.460	10.527	0.000	0.099	0.000	0.000	14.746

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	233	250	0	148	0	0	104
N.S.	1	1.00	2.56	2.75	0.00	1.63	0.00	0.00	1.14
time (sec)	N/A	0.136	5.014	5.943	0.000	0.096	0.000	0.000	13.916

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	146	172	0	180	0	0	104
N.S.	1	1.00	1.60	1.89	0.00	1.98	0.00	0.00	1.14
time (sec)	N/A	0.134	0.401	5.967	0.000	0.092	0.000	0.000	14.643

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	139	371	0	187	0	0	126
N.S.	1	1.00	1.53	4.08	0.00	2.05	0.00	0.00	1.38
time (sec)	N/A	0.128	0.323	6.454	0.000	0.097	0.000	0.000	14.487

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	138	386	0	200	0	0	154
N.S.	1	1.00	1.18	3.30	0.00	1.71	0.00	0.00	1.32
time (sec)	N/A	0.153	0.437	7.484	0.000	0.093	0.000	0.000	15.240

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	140	439	0	215	0	0	145
N.S.	1	1.00	0.95	2.99	0.00	1.46	0.00	0.00	0.99
time (sec)	N/A	0.198	0.644	10.407	0.000	0.117	0.000	0.000	15.566

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	271	273	0	188	0	0	221
N.S.	1	1.00	1.57	1.58	0.00	1.09	0.00	0.00	1.28
time (sec)	N/A	0.251	3.087	15.950	0.000	0.102	0.000	0.000	15.027

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	255	260	0	175	0	0	223
N.S.	1	1.00	1.73	1.77	0.00	1.19	0.00	0.00	1.52
time (sec)	N/A	0.204	5.935	12.700	0.000	0.113	0.000	0.000	15.160

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	245	272	0	162	0	0	146
N.S.	1	1.00	2.02	2.25	0.00	1.34	0.00	0.00	1.21
time (sec)	N/A	0.180	5.784	9.147	0.000	0.094	0.000	0.000	15.011

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	157	194	0	194	0	0	149
N.S.	1	1.00	1.32	1.63	0.00	1.63	0.00	0.00	1.25
time (sec)	N/A	0.156	0.936	8.785	0.000	0.106	0.000	0.000	15.017

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	186	292	0	125	0	0	145
N.S.	1	1.00	1.90	2.98	0.00	1.28	0.00	0.00	1.48
time (sec)	N/A	0.159	0.688	8.245	0.000	0.092	0.000	0.000	15.024

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	169	386	0	202	0	0	202
N.S.	1	1.00	1.40	3.19	0.00	1.67	0.00	0.00	1.67
time (sec)	N/A	0.173	0.663	8.786	0.000	0.101	0.000	0.000	15.642

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	171	439	0	215	0	0	199
N.S.	1	1.00	1.16	2.99	0.00	1.46	0.00	0.00	1.35
time (sec)	N/A	0.206	1.042	10.460	0.000	0.093	0.000	0.000	15.673

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	315	229	0	208	0	0	0
N.S.	1	1.00	2.46	1.79	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.130	1.859	5.279	0.000	0.100	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	289	215	0	198	0	0	0
N.S.	1	1.00	2.89	2.15	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.121	1.346	4.191	0.000	0.099	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	264	199	0	186	0	0	0
N.S.	1	1.00	3.67	2.76	0.00	2.58	0.00	0.00	0.00
time (sec)	N/A	0.101	2.196	3.261	0.000	0.104	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	256	198	0	184	0	0	0
N.S.	1	1.00	3.66	2.83	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.098	1.000	2.405	0.000	0.093	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	257	200	0	184	0	0	0
N.S.	1	1.00	3.67	2.86	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.103	1.070	1.704	0.000	0.086	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	297	253	0	236	0	0	0
N.S.	1	1.00	3.09	2.64	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.126	1.791	2.536	0.000	0.099	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	332	413	0	258	0	0	0
N.S.	1	1.00	2.68	3.33	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.130	2.910	3.731	0.000	0.095	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	148	283	0	288	0	0	0
N.S.	1	1.00	0.92	1.77	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	0.263	0.944	6.383	0.000	0.101	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	136	270	0	278	0	0	0
N.S.	1	1.00	0.99	1.96	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.251	0.614	6.199	0.000	0.101	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	128	257	0	268	0	0	0
N.S.	1	1.00	1.14	2.29	0.00	2.39	0.00	0.00	0.00
time (sec)	N/A	0.221	0.496	5.039	0.000	0.096	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	114	257	0	268	0	0	0
N.S.	1	1.00	1.05	2.36	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.209	0.384	3.260	0.000	0.101	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	71	188	0	150	0	0	0
N.S.	1	1.00	1.25	3.30	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	0.062	0.278	2.901	0.000	0.102	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	304	257	0	268	0	0	0
N.S.	1	1.00	2.79	2.36	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.206	2.415	1.870	0.000	0.113	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	334	405	0	318	0	0	0
N.S.	1	1.00	2.46	2.98	0.00	2.34	0.00	0.00	0.00
time (sec)	N/A	0.231	2.370	3.289	0.000	0.098	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	364	413	0	338	0	0	0
N.S.	1	1.00	2.25	2.55	0.00	2.09	0.00	0.00	0.00
time (sec)	N/A	0.255	4.964	4.549	0.000	0.105	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	175	296	0	364	0	0	0
N.S.	1	1.00	0.85	1.43	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.378	2.101	11.214	0.000	0.114	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	166	283	0	354	0	0	0
N.S.	1	1.00	0.92	1.56	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.376	1.338	10.449	0.000	0.108	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	157	270	0	344	0	0	0
N.S.	1	1.00	1.01	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.336	1.290	10.244	0.000	0.107	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	146	270	0	344	0	0	0
N.S.	1	1.00	0.94	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.346	1.021	9.678	0.000	0.097	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	146	270	0	344	0	0	0
N.S.	1	1.00	0.94	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.328	1.081	4.295	0.000	0.106	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	139	270	0	344	0	0	0
N.S.	1	1.00	0.90	1.74	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.356	2.240	4.188	0.000	0.093	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	304	268	0	344	0	0	0
N.S.	1	1.00	1.96	1.73	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.465	4.941	2.309	0.000	0.095	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	364	555	0	394	0	0	0
N.S.	1	1.00	2.01	3.07	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.511	2.442	4.149	0.000	0.100	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	394	453	0	414	0	0	0
N.S.	1	1.00	1.90	2.19	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.649	2.834	5.208	0.000	0.107	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	105	180	1921	108	0	0	0
N.S.	1	1.00	0.68	1.17	12.47	0.70	0.00	0.00	0.00
time (sec)	N/A	0.287	0.230	11.510	0.581	0.265	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	91	145	1059	98	0	0	0
N.S.	1	1.00	0.78	1.25	9.13	0.84	0.00	0.00	0.00
time (sec)	N/A	0.216	0.137	11.604	0.465	0.263	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	77	108	791	88	0	0	0
N.S.	1	1.00	1.07	1.50	10.99	1.22	0.00	0.00	0.00
time (sec)	N/A	0.145	0.063	11.620	0.436	0.264	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	50	72	146	119	0	0	0
N.S.	1	1.00	1.35	1.95	3.95	3.22	0.00	0.00	0.00
time (sec)	N/A	0.076	0.034	4.028	0.409	0.287	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	42	98	49	0	58	41
N.S.	1	1.00	1.08	1.17	2.72	1.36	0.00	1.61	1.14
time (sec)	N/A	0.069	0.034	5.368	0.359	0.255	0.000	0.352	14.471

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	51	52	190	61	0	87	82
N.S.	1	1.00	0.66	0.68	2.47	0.79	0.00	1.13	1.06
time (sec)	N/A	0.137	0.060	4.066	0.369	0.259	0.000	0.475	14.767

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	61	62	237	71	0	116	132
N.S.	1	1.00	0.53	0.54	2.06	0.62	0.00	1.01	1.15
time (sec)	N/A	0.211	0.080	5.248	0.336	0.272	0.000	0.449	16.181

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	71	72	283	81	0	143	415
N.S.	1	1.00	0.46	0.47	1.85	0.53	0.00	0.93	2.71
time (sec)	N/A	0.274	0.118	4.549	0.366	0.246	0.000	0.548	19.816

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	106	181	1942	114	0	0	0
N.S.	1	1.00	0.66	1.13	12.14	0.71	0.00	0.00	0.00
time (sec)	N/A	0.306	0.245	11.908	0.594	0.267	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	92	146	1080	103	0	0	0
N.S.	1	1.00	0.77	1.22	9.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.245	0.147	11.744	0.470	0.265	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	152	803	90	0	0	0
N.S.	1	1.00	1.05	2.03	10.71	1.20	0.00	0.00	0.00
time (sec)	N/A	0.162	0.069	13.854	0.450	0.270	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	85	144	997	109	0	0	0
N.S.	1	1.00	1.12	1.89	13.12	1.43	0.00	0.00	0.00
time (sec)	N/A	0.163	0.102	5.444	0.468	0.291	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	52	53	125	62	0	0	89
N.S.	1	1.00	0.64	0.65	1.54	0.77	0.00	0.00	1.10
time (sec)	N/A	0.153	0.072	5.233	0.354	0.256	0.000	0.000	15.377

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	62	63	217	73	0	0	133
N.S.	1	1.00	0.51	0.52	1.79	0.60	0.00	0.00	1.10
time (sec)	N/A	0.238	0.097	4.949	0.354	0.265	0.000	0.000	16.402

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	72	73	263	86	0	87931	157
N.S.	1	1.00	0.45	0.45	1.63	0.53	0.00	546.16	0.98
time (sec)	N/A	0.344	0.146	5.291	0.331	0.262	0.000	233.170	19.641

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	182	218	7450	137	0	0	0
N.S.	1	1.00	0.91	1.09	37.25	0.68	0.00	0.00	0.00
time (sec)	N/A	0.548	2.992	12.294	0.758	0.271	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	182	183	1964	124	0	0	0
N.S.	1	1.00	1.14	1.14	12.28	0.78	0.00	0.00	0.00
time (sec)	N/A	0.475	2.869	12.504	0.561	0.302	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	182	172	1106	111	0	0	0
N.S.	1	1.00	1.52	1.43	9.22	0.92	0.00	0.00	0.00
time (sec)	N/A	0.354	2.802	13.842	0.471	0.277	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	182	162	973	127	0	0	0
N.S.	1	1.00	1.60	1.42	8.54	1.11	0.00	0.00	0.00
time (sec)	N/A	0.292	2.779	13.943	0.500	0.282	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	356	170	1395	131	0	0	0
N.S.	1	1.00	3.02	1.44	11.82	1.11	0.00	0.00	0.00
time (sec)	N/A	0.276	8.943	5.563	0.480	0.280	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	64	65	151	81	0	0	135
N.S.	1	1.00	0.53	0.54	1.25	0.67	0.00	0.00	1.12
time (sec)	N/A	0.306	0.140	5.093	0.335	0.272	0.000	0.000	16.351

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	74	75	243	94	0	98101	163
N.S.	1	1.00	0.46	0.47	1.51	0.58	0.00	609.32	1.01
time (sec)	N/A	0.367	5.183	5.492	0.336	0.291	0.000	237.097	18.395

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	84	85	289	107	0	0	279
N.S.	1	1.00	0.42	0.42	1.44	0.53	0.00	0.00	1.39
time (sec)	N/A	0.430	5.206	5.270	0.357	0.267	0.000	0.000	21.049

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	51	0	121	50	0	0	42
N.S.	1	1.00	1.34	0.00	3.18	1.32	0.00	0.00	1.11
time (sec)	N/A	0.077	0.067	0.000	0.329	0.271	0.000	0.000	0.627

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	50	72	146	119	0	0	0
N.S.	1	1.00	1.35	1.95	3.95	3.22	0.00	0.00	0.00
time (sec)	N/A	0.074	0.050	4.896	0.388	0.275	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	55	83	420	164	0	83	0
N.S.	1	1.00	1.45	2.18	11.05	4.32	0.00	2.18	0.00
time (sec)	N/A	0.088	0.124	3.370	0.410	0.291	0.000	0.501	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	151	178	0	155	0	0	0
N.S.	1	1.00	0.88	1.04	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.522	0.201	12.770	0.000	0.354	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	122	141	0	143	0	0	0
N.S.	1	1.00	0.95	1.10	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.339	0.119	12.043	0.000	0.330	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	89	108	698	89	0	0	0
N.S.	1	1.00	0.94	1.14	7.35	0.94	0.00	0.00	0.00
time (sec)	N/A	0.204	0.071	3.732	0.700	0.287	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	51	67	522	159	0	0	0
N.S.	1	1.00	0.91	1.20	9.32	2.84	0.00	0.00	0.00
time (sec)	N/A	0.077	0.036	4.497	0.706	0.287	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	180	130	665	132	0	0	0
N.S.	1	1.00	1.94	1.40	7.15	1.42	0.00	0.00	0.00
time (sec)	N/A	0.153	1.552	5.281	0.652	0.274	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	473	158	818	145	0	0	0
N.S.	1	1.00	3.61	1.21	6.24	1.11	0.00	0.00	0.00
time (sec)	N/A	0.303	7.163	5.850	0.650	0.274	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	1540	180	1006	157	0	0	0
N.S.	1	1.00	9.11	1.07	5.95	0.93	0.00	0.00	0.00
time (sec)	N/A	0.456	8.592	5.849	0.654	0.287	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	135	169	0	135	0	0	0
N.S.	1	1.00	1.07	1.34	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.327	0.169	12.564	0.000	0.276	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	106	133	0	125	0	0	0
N.S.	1	1.00	1.25	1.56	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.225	0.120	12.858	0.000	0.286	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	101	108	689	70	0	0	0
N.S.	1	1.00	1.87	2.00	12.76	1.30	0.00	0.00	0.00
time (sec)	N/A	0.134	0.101	2.635	0.674	0.280	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	49	61	505	54	0	0	0
N.S.	1	1.00	1.81	2.26	18.70	2.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.035	4.221	0.663	0.286	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	C	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	178	132	648	121	0	0	0
N.S.	1	1.00	2.87	2.13	10.45	1.95	0.00	0.00	0.00
time (sec)	N/A	0.104	1.271	4.940	0.584	0.276	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	471	156	801	134	0	0	0
N.S.	1	1.00	4.81	1.59	8.17	1.37	0.00	0.00	0.00
time (sec)	N/A	0.203	6.446	4.826	0.574	0.313	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	1538	174	989	146	0	0	0
N.S.	1	1.00	11.48	1.30	7.38	1.09	0.00	0.00	0.00
time (sec)	N/A	0.286	7.169	5.379	0.586	0.288	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	212	240	0	192	0	0	0
N.S.	1	1.00	1.22	1.38	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.561	0.305	11.473	0.000	0.365	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	158	204	0	182	0	0	0
N.S.	1	1.00	1.18	1.52	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.369	0.332	3.315	0.000	0.353	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	118	131	0	145	0	0	0
N.S.	1	1.00	1.22	1.35	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.166	0.213	3.483	0.000	0.324	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	106	181	0	146	0	0	0
N.S.	1	1.00	1.09	1.87	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.166	0.386	4.203	0.000	0.292	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	247	196	0	171	0	0	0
N.S.	1	1.00	1.80	1.43	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.326	6.605	5.566	0.000	0.286	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	589	223	0	185	0	0	0
N.S.	1	1.00	3.33	1.26	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.490	8.234	4.680	0.000	0.296	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	217	344	0	236	0	0	0
N.S.	1	1.00	1.01	1.61	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.721	0.704	12.741	0.000	0.429	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	191	330	0	226	0	0	0
N.S.	1	1.00	1.10	1.90	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.533	0.535	3.850	0.000	0.433	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	149	196	0	180	0	0	0
N.S.	1	1.00	1.09	1.43	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.319	0.541	3.629	0.000	0.281	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	122	195	0	178	0	0	0
N.S.	1	1.00	0.89	1.42	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.310	0.741	3.357	0.000	0.295	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	134	229	0	180	0	0	0
N.S.	1	1.00	0.98	1.67	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.322	0.849	5.670	0.000	0.306	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	506	260	0	205	0	0	0
N.S.	1	1.00	2.86	1.47	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.480	7.206	5.649	0.000	0.279	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	639	287	0	219	0	0	0
N.S.	1	1.00	2.94	1.32	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.703	9.073	5.452	0.000	0.281	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	256	448	0	280	0	0	0
N.S.	1	1.00	1.01	1.76	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.949	1.104	12.472	0.000	0.535	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	230	378	0	270	0	0	0
N.S.	1	1.00	1.07	1.77	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.722	1.009	3.923	0.000	0.444	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	176	260	0	214	0	0	0
N.S.	1	1.00	0.99	1.47	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.482	1.791	3.605	0.000	0.293	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	148	260	0	214	0	0	0
N.S.	1	1.00	0.84	1.47	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.497	1.273	3.737	0.000	0.293	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	149	260	0	214	0	0	0
N.S.	1	1.00	0.84	1.47	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.500	1.819	3.240	0.000	0.283	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	148	277	0	214	0	0	0
N.S.	1	1.00	0.84	1.56	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.520	1.451	5.661	0.000	0.285	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	559	324	0	239	0	0	0
N.S.	1	1.00	2.58	1.49	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.707	7.760	5.442	0.000	0.294	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	694	351	0	253	0	0	0
N.S.	1	1.00	2.70	1.37	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.894	10.280	5.859	0.000	0.298	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	190	350	0	248	0	0	0
N.S.	1	1.00	0.88	1.61	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.660	5.541	3.894	0.000	0.289	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	158	350	0	248	0	0	0
N.S.	1	1.00	0.73	1.61	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.820	1.438	3.502	0.000	0.308	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	30	34	306	31	0	0	0
N.S.	1	1.00	1.88	2.12	19.12	1.94	0.00	0.00	0.00
time (sec)	N/A	0.063	0.020	1.480	0.505	0.259	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	40	323	105	0	0	0
N.S.	1	1.00	0.78	0.98	7.88	2.56	0.00	0.00	0.00
time (sec)	N/A	0.117	0.014	1.464	0.566	0.273	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	186	152	1063	155	0	0	0
N.S.	1	1.00	1.44	1.18	8.24	1.20	0.00	0.00	0.00
time (sec)	N/A	0.327	0.762	12.560	0.426	0.306	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	152	120	795	142	0	0	0
N.S.	1	1.00	1.79	1.41	9.35	1.67	0.00	0.00	0.00
time (sec)	N/A	0.248	0.530	12.954	0.410	0.283	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	125	79	148	155	0	122	0
N.S.	1	1.00	2.60	1.65	3.08	3.23	0.00	2.54	0.00
time (sec)	N/A	0.130	0.418	4.548	0.363	0.303	0.000	0.454	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	40	40	82	42	0	62	42
N.S.	1	1.00	1.08	1.08	2.22	1.14	0.00	1.68	1.14
time (sec)	N/A	0.102	0.033	5.457	0.300	0.247	0.000	0.400	14.265

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	52	51	174	52	0	90	85
N.S.	1	1.00	0.66	0.65	2.20	0.66	0.00	1.14	1.08
time (sec)	N/A	0.175	0.330	5.577	0.307	0.252	0.000	0.442	14.732

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	62	63	221	64	0	120	158
N.S.	1	1.00	0.53	0.53	1.87	0.54	0.00	1.02	1.34
time (sec)	N/A	0.267	0.402	5.448	0.306	0.268	0.000	0.487	15.966

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	181	154	1305	124	0	0	0
N.S.	1	1.00	1.59	1.35	11.45	1.09	0.00	0.00	0.00
time (sec)	N/A	0.188	0.286	13.344	0.399	0.281	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	155	122	966	111	0	0	0
N.S.	1	1.00	2.15	1.69	13.42	1.54	0.00	0.00	0.00
time (sec)	N/A	0.107	0.334	13.254	0.400	0.299	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	124	81	221	64	0	119	0
N.S.	1	1.00	3.35	2.19	5.97	1.73	0.00	3.22	0.00
time (sec)	N/A	0.058	0.174	4.746	0.355	0.261	0.000	0.605	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	39	41	75	41	0	59	31
N.S.	1	1.00	1.11	1.17	2.14	1.17	0.00	1.69	0.89
time (sec)	N/A	0.056	0.026	5.497	0.301	0.255	0.000	0.608	14.709

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	51	53	164	51	0	87	84
N.S.	1	1.00	0.68	0.71	2.19	0.68	0.00	1.16	1.12
time (sec)	N/A	0.111	0.161	4.403	0.303	0.255	0.000	0.631	14.631

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	61	65	209	63	0	117	156
N.S.	1	1.00	0.54	0.58	1.87	0.56	0.00	1.04	1.39
time (sec)	N/A	0.174	0.188	5.573	0.307	0.260	0.000	0.663	15.422

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	256	203	0	226	0	779	0
N.S.	1	1.00	1.38	1.10	0.00	1.22	0.00	4.21	0.00
time (sec)	N/A	0.563	1.112	12.350	0.000	0.272	0.000	6.911	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	228	169	0	212	0	581	0
N.S.	1	1.00	1.62	1.20	0.00	1.50	0.00	4.12	0.00
time (sec)	N/A	0.426	0.917	13.858	0.000	0.277	0.000	1.009	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	161	116	0	162	0	269	0
N.S.	1	1.00	1.50	1.08	0.00	1.51	0.00	2.51	0.00
time (sec)	N/A	0.331	0.518	2.077	0.000	0.261	0.000	0.670	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	118	80	209	144	0	162	0
N.S.	1	1.00	2.03	1.38	3.60	2.48	0.00	2.79	0.00
time (sec)	N/A	0.135	0.472	5.992	0.428	0.291	0.000	0.567	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	157	85	351	152	0	279	0
N.S.	1	1.00	1.65	0.89	3.69	1.60	0.00	2.94	0.00
time (sec)	N/A	0.240	0.531	5.449	0.405	0.291	0.000	0.696	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	171	102	504	165	0	235	0
N.S.	1	1.00	1.27	0.76	3.73	1.22	0.00	1.74	0.00
time (sec)	N/A	0.444	0.471	4.721	0.413	0.286	0.000	0.689	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	218	120	692	177	0	265	0
N.S.	1	1.00	1.26	0.69	4.00	1.02	0.00	1.53	0.00
time (sec)	N/A	0.606	0.745	6.346	0.437	0.285	0.000	0.787	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	255	194	0	239	0	747	0
N.S.	1	1.00	1.58	1.20	0.00	1.48	0.00	4.64	0.00
time (sec)	N/A	0.379	0.441	13.829	0.000	0.261	0.000	6.634	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	227	159	0	225	0	560	0
N.S.	1	1.00	1.92	1.35	0.00	1.91	0.00	4.75	0.00
time (sec)	N/A	0.263	0.368	13.240	0.000	0.264	0.000	0.937	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.153	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	268	384	0	188	0	0	0
N.S.	1	1.00	1.77	2.54	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.139	1.257	9.734	0.000	0.106	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	255	368	0	167	0	0	0
N.S.	1	1.00	2.07	2.99	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.130	0.864	8.435	0.000	0.092	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	124	148	0	124	0	0	0
N.S.	1	1.00	1.28	1.53	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.106	0.551	3.839	0.000	0.090	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	141	150	0	107	0	0	0
N.S.	1	1.00	1.88	2.00	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.098	0.586	3.627	0.000	0.092	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	140	225	0	125	0	0	0
N.S.	1	1.00	1.39	2.23	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.120	0.926	5.655	0.000	0.098	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	224	219	0	145	0	0	0
N.S.	1	1.00	1.76	1.72	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.143	0.739	6.652	0.000	0.094	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	198	270	0	156	0	0	0
N.S.	1	1.00	1.31	1.79	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.163	1.603	9.768	0.000	0.103	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	261	386	0	202	0	0	0
N.S.	1	1.00	1.62	2.40	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.196	1.472	20.620	0.000	0.098	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	250	371	0	179	0	0	0
N.S.	1	1.00	1.91	2.83	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.177	1.064	19.145	0.000	0.100	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	185	0	77	0	0	0
N.S.	1	1.00	0.75	2.89	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.135	0.708	5.010	0.000	0.083	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	127	228	0	134	0	0	0
N.S.	1	1.00	1.19	2.13	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.163	1.190	5.629	0.000	0.088	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	136	250	0	157	0	0	0
N.S.	1	1.00	1.01	1.85	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.172	1.465	8.363	0.000	0.093	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	149	272	0	170	0	0	0
N.S.	1	1.00	0.93	1.69	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.188	1.692	10.485	0.000	0.098	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	279	439	0	215	0	0	0
N.S.	1	1.00	1.49	2.35	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.289	2.126	64.021	0.000	0.094	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	259	386	0	200	0	0	0
N.S.	1	1.00	1.65	2.46	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.248	1.494	64.270	0.000	0.092	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	157	371	0	179	0	0	0
N.S.	1	1.00	1.20	2.83	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.222	1.387	62.794	0.000	0.094	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	135	172	0	148	0	0	0
N.S.	1	1.00	1.03	1.31	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.222	1.390	7.908	0.000	0.091	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	137	250	0	156	0	0	0
N.S.	1	1.00	1.05	1.91	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.224	1.306	6.865	0.000	0.092	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	170	0	0	0
N.S.	1	1.00	0.91	1.69	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.257	2.209	11.248	0.000	0.094	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	183	0	0	0
N.S.	1	1.00	0.83	1.39	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.299	2.622	12.571	0.000	0.100	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	271	439	0	215	0	0	0
N.S.	1	1.00	1.45	2.35	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.331	2.668	202.273	0.000	0.100	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	278	386	0	202	0	0	0
N.S.	1	1.00	1.73	2.40	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.291	3.663	202.294	0.000	0.101	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	70	292	0	121	0	0	0
N.S.	1	1.00	0.59	2.47	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.268	2.179	202.988	0.000	0.093	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	150	194	0	162	0	0	0
N.S.	1	1.00	0.94	1.22	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.273	3.027	10.709	0.000	0.099	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	170	0	0	0
N.S.	1	1.00	0.91	1.69	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.302	3.028	11.147	0.000	0.104	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	183	0	0	0
N.S.	1	1.00	0.83	1.39	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.326	3.238	16.315	0.000	0.102	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	285	413	0	248	0	0	0
N.S.	1	1.00	1.74	2.52	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.216	2.515	5.166	0.000	0.095	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	256	253	0	196	0	0	0
N.S.	1	1.00	1.88	1.86	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.203	1.597	3.144	0.000	0.093	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	180	200	0	184	0	0	0
N.S.	1	1.00	1.64	1.82	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.186	0.921	2.104	0.000	0.094	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	181	198	0	184	0	0	0
N.S.	1	1.00	1.65	1.80	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.187	0.912	2.932	0.000	0.094	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	311	199	0	186	0	0	0
N.S.	1	1.00	2.78	1.78	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.185	1.472	3.441	0.000	0.092	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	312	215	0	207	0	0	0
N.S.	1	1.00	2.23	1.54	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.203	3.073	4.520	0.000	0.097	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	341	229	0	217	0	0	0
N.S.	1	1.00	2.03	1.36	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.238	2.221	5.244	0.000	0.108	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	287	413	0	328	0	0	0
N.S.	1	1.00	1.42	2.04	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.345	2.010	9.834	0.000	0.100	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	252	405	0	278	0	0	0
N.S.	1	1.00	1.43	2.30	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.318	1.181	3.487	0.000	0.094	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	242	257	0	277	0	0	0
N.S.	1	1.00	1.62	1.72	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.276	1.084	2.584	0.000	0.098	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	98	188	0	150	0	0	0
N.S.	1	1.00	1.27	2.44	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.120	0.538	3.280	0.000	0.087	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	239	257	0	277	0	0	0
N.S.	1	1.00	1.60	1.72	0.00	1.86	0.00	0.00	0.00
time (sec)	N/A	0.285	1.262	3.717	0.000	0.094	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	259	257	0	277	0	0	0
N.S.	1	1.00	1.70	1.69	0.00	1.82	0.00	0.00	0.00
time (sec)	N/A	0.289	1.661	4.526	0.000	0.100	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	257	270	0	287	0	0	0
N.S.	1	1.00	1.44	1.52	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.306	1.634	5.271	0.000	0.103	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	271	283	0	297	0	0	0
N.S.	1	1.00	1.36	1.42	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.329	1.883	5.860	0.000	0.103	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	363	555	0	354	0	0	0
N.S.	1	1.00	1.64	2.51	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.433	1.853	4.375	0.000	0.105	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	274	268	0	353	0	0	0
N.S.	1	1.00	1.41	1.37	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.409	1.782	2.813	0.000	0.099	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	363	270	0	353	0	0	0
N.S.	1	1.00	1.86	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.405	1.619	4.513	0.000	0.096	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	363	270	0	353	0	0	0
N.S.	1	1.00	1.86	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.413	1.737	4.796	0.000	0.104	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	272	270	0	353	0	0	0
N.S.	1	1.00	1.39	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.437	2.304	5.168	0.000	0.100	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	378	270	0	353	0	0	0
N.S.	1	1.00	1.94	1.38	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.437	2.059	5.615	0.000	0.108	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	285	283	0	363	0	0	0
N.S.	1	1.00	1.29	1.28	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.446	2.353	5.571	0.000	0.120	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	71	72	283	81	0	143	163
N.S.	1	1.00	0.46	0.47	1.85	0.53	0.00	0.93	1.07
time (sec)	N/A	0.425	0.139	6.550	0.327	0.266	0.000	0.401	18.852

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	61	62	237	71	0	116	134
N.S.	1	1.00	0.53	0.54	2.06	0.62	0.00	1.01	1.17
time (sec)	N/A	0.425	0.087	6.084	0.314	0.269	0.000	0.397	1.859

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	51	52	190	59	0	87	84
N.S.	1	1.00	0.66	0.68	2.47	0.77	0.00	1.13	1.09
time (sec)	N/A	0.299	0.068	6.507	0.346	0.269	0.000	0.370	0.825

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	40	98	40	0	58	43
N.S.	1	1.00	1.08	1.11	2.72	1.11	0.00	1.61	1.19
time (sec)	N/A	0.215	0.045	5.780	0.316	0.258	0.000	0.388	0.301

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	70	88	146	119	0	0	0
N.S.	1	1.00	1.23	1.54	2.56	2.09	0.00	0.00	0.00
time (sec)	N/A	0.249	0.065	6.852	0.365	0.284	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	108	791	88	0	0	0
N.S.	1	1.00	1.05	1.17	8.60	0.96	0.00	0.00	0.00
time (sec)	N/A	0.236	0.093	14.159	0.407	0.260	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	111	145	1059	108	0	0	0
N.S.	1	1.00	0.82	1.07	7.79	0.79	0.00	0.00	0.00
time (sec)	N/A	0.302	0.178	14.202	0.433	0.278	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	72	73	263	86	0	0	221
N.S.	1	1.00	0.45	0.45	1.63	0.53	0.00	0.00	1.37
time (sec)	N/A	0.391	0.177	5.970	0.319	0.270	0.000	0.000	18.006

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	62	63	217	73	0	0	135
N.S.	1	1.00	0.51	0.52	1.79	0.60	0.00	0.00	1.12
time (sec)	N/A	0.298	0.117	6.283	0.316	0.285	0.000	0.000	1.695

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	52	53	125	60	0	0	91
N.S.	1	1.00	0.64	0.65	1.54	0.74	0.00	0.00	1.12
time (sec)	N/A	0.222	0.090	4.945	0.319	0.268	0.000	0.000	0.813

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	85	150	997	91	0	0	0
N.S.	1	1.00	0.89	1.56	10.39	0.95	0.00	0.00	0.00
time (sec)	N/A	0.231	0.130	6.610	0.434	0.286	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	99	117	803	90	0	0	0
N.S.	1	1.00	1.04	1.23	8.45	0.95	0.00	0.00	0.00
time (sec)	N/A	0.219	0.107	15.563	0.425	0.268	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	112	146	1080	112	0	0	0
N.S.	1	1.00	0.80	1.04	7.71	0.80	0.00	0.00	0.00
time (sec)	N/A	0.305	0.205	14.245	0.429	0.267	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	126	179	1942	123	0	0	0
N.S.	1	1.00	0.70	0.99	10.79	0.68	0.00	0.00	0.00
time (sec)	N/A	0.396	0.353	14.720	0.533	0.272	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	84	85	289	107	0	0	306
N.S.	1	1.00	0.42	0.42	1.44	0.53	0.00	0.00	1.52
time (sec)	N/A	0.519	5.580	1.563	0.322	0.273	0.000	0.000	18.609

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	74	75	243	94	0	0	227
N.S.	1	1.00	0.46	0.47	1.51	0.58	0.00	0.00	1.41
time (sec)	N/A	0.442	5.519	1.375	0.320	0.262	0.000	0.000	18.307

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	64	65	151	81	0	0	137
N.S.	1	1.00	0.53	0.54	1.25	0.67	0.00	0.00	1.13
time (sec)	N/A	0.377	0.199	1.156	0.312	0.264	0.000	0.000	1.667

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	404	181	1395	128	0	0	0
N.S.	1	1.00	2.93	1.31	10.11	0.93	0.00	0.00	0.00
time (sec)	N/A	0.405	6.194	1.310	0.445	0.271	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	202	168	973	111	0	0	0
N.S.	1	1.00	1.51	1.25	7.26	0.83	0.00	0.00	0.00
time (sec)	N/A	0.376	1.967	16.283	0.426	0.267	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	202	154	1106	120	0	0	0
N.S.	1	1.00	1.44	1.10	7.90	0.86	0.00	0.00	0.00
time (sec)	N/A	0.381	1.953	16.027	0.432	0.265	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	202	183	1964	133	0	0	0
N.S.	1	1.00	1.12	1.02	10.91	0.74	0.00	0.00	0.00
time (sec)	N/A	0.463	2.000	14.777	0.538	0.279	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	202	216	7450	146	0	0	0
N.S.	1	1.00	0.92	0.98	33.86	0.66	0.00	0.00	0.00
time (sec)	N/A	0.565	2.035	13.831	0.703	0.274	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	1540	182	989	130	0	0	0
N.S.	1	1.00	10.00	1.18	6.42	0.84	0.00	0.00	0.00
time (sec)	N/A	0.358	7.271	5.937	0.561	0.280	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	473	166	801	114	0	0	0
N.S.	1	1.00	4.01	1.41	6.79	0.97	0.00	0.00	0.00
time (sec)	N/A	0.252	6.451	6.408	0.559	0.289	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	178	138	648	86	0	0	0
N.S.	1	1.00	2.17	1.68	7.90	1.05	0.00	0.00	0.00
time (sec)	N/A	0.159	1.317	5.883	0.545	0.297	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	68	77	505	39	0	0	0
N.S.	1	1.00	1.45	1.64	10.74	0.83	0.00	0.00	0.00
time (sec)	N/A	0.095	0.082	5.918	0.580	0.271	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	171	108	689	70	0	0	0
N.S.	1	1.00	1.82	1.15	7.33	0.74	0.00	0.00	0.00
time (sec)	N/A	0.203	0.553	4.446	0.597	0.278	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	257	133	0	125	0	0	0
N.S.	1	1.00	2.06	1.06	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.656	13.543	0.000	0.278	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	1542	188	1006	141	0	0	0
N.S.	1	1.00	8.16	0.99	5.32	0.75	0.00	0.00	0.00
time (sec)	N/A	0.680	7.214	6.083	0.604	0.296	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	475	168	818	125	0	0	0
N.S.	1	1.00	3.15	1.11	5.42	0.83	0.00	0.00	0.00
time (sec)	N/A	0.529	6.456	6.797	0.595	0.285	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	180	136	665	98	0	0	0
N.S.	1	1.00	1.59	1.20	5.88	0.87	0.00	0.00	0.00
time (sec)	N/A	0.385	1.543	6.069	0.583	0.283	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	76	71	83	522	144	0	0	0
N.S.	1	1.36	1.27	1.48	9.32	2.57	0.00	0.00	0.00
time (sec)	N/A	0.190	0.070	6.867	0.630	0.286	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	135	173	108	698	89	0	0	0
N.S.	1	1.29	1.65	1.03	6.65	0.85	0.00	0.00	0.00
time (sec)	N/A	0.346	0.343	3.976	0.596	0.290	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	259	141	0	143	0	0	0
N.S.	1	1.00	1.54	0.84	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.444	0.425	13.815	0.000	0.293	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	591	233	0	161	0	0	0
N.S.	1	1.00	3.00	1.18	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.581	6.998	6.635	0.000	0.282	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	249	202	0	136	0	0	0
N.S.	1	1.00	1.59	1.29	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.400	6.310	6.533	0.000	0.280	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	99	139	0	126	0	0	0
N.S.	1	1.00	0.85	1.19	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.253	0.359	6.103	0.000	0.283	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	140	131	0	125	0	0	0
N.S.	1	1.00	1.20	1.12	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.252	0.294	4.331	0.000	0.291	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	248	203	0	182	0	0	0
N.S.	1	1.00	1.43	1.17	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.472	4.669	4.755	0.000	0.333	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	243	240	0	201	0	0	0
N.S.	1	1.00	1.14	1.12	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.663	4.705	15.046	0.000	0.349	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	641	297	0	195	0	0	0
N.S.	1	1.00	2.70	1.25	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.741	7.374	6.395	0.000	0.301	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	508	266	0	170	0	0	0
N.S.	1	1.00	2.58	1.35	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.553	6.535	6.128	0.000	0.294	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	131	202	0	169	0	0	0
N.S.	1	1.00	0.83	1.29	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.409	0.681	6.546	0.000	0.285	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	122	195	0	167	0	0	0
N.S.	1	1.00	0.78	1.24	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.390	0.477	4.358	0.000	0.282	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	164	188	0	169	0	0	0
N.S.	1	1.00	1.04	1.20	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.408	0.500	4.306	0.000	0.277	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	373	306	0	235	0	0	0
N.S.	1	1.00	1.74	1.43	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.639	1.867	4.557	0.000	0.412	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	412	344	0	245	0	0	0
N.S.	1	1.00	1.62	1.35	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.823	2.497	14.710	0.000	0.414	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	696	361	0	229	0	0	0
N.S.	1	1.00	2.51	1.30	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	1.148	7.864	6.681	0.000	0.292	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	561	330	0	204	0	0	0
N.S.	1	1.00	2.37	1.39	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	1.092	6.639	6.649	0.000	0.280	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	153	266	0	203	0	0	0
N.S.	1	1.00	0.78	1.35	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.660	2.481	6.224	0.000	0.302	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	125	260	0	203	0	0	0
N.S.	1	1.00	0.63	1.32	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.628	0.796	4.577	0.000	0.300	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	153	250	0	203	0	0	0
N.S.	1	1.00	0.78	1.27	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.593	2.506	4.630	0.000	0.292	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	196	250	0	203	0	0	0
N.S.	1	1.00	0.99	1.27	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.582	2.854	3.997	0.000	0.290	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	254	254	454	401	0	279	0	0	0
N.S.	1	1.00	1.79	1.58	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.820	6.944	4.382	0.000	0.458	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	460	448	0	289	0	0	0
N.S.	1	1.00	1.56	1.52	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	1.004	2.683	15.022	0.000	0.525	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	138	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.352	0.000	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	111	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	141	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	0.409	0.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	188	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	1.942	0.000	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	135	100	105	97	286	122	175
N.S.	1	1.00	0.90	0.67	0.70	0.65	1.91	0.81	1.17
time (sec)	N/A	0.106	0.196	3.052	0.194	0.274	0.671	0.313	17.245

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	89	90	94	86	238	107	154
N.S.	1	1.00	0.70	0.70	0.73	0.67	1.86	0.84	1.20
time (sec)	N/A	0.091	0.119	2.934	0.200	0.265	0.483	0.305	17.379

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	78	80	84	75	216	92	115
N.S.	1	1.00	0.68	0.70	0.74	0.66	1.89	0.81	1.01
time (sec)	N/A	0.085	0.087	2.571	0.206	0.261	0.344	0.297	14.309

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	69	69	64	168	77	115
N.S.	1	1.00	0.97	0.75	0.75	0.70	1.83	0.84	1.25
time (sec)	N/A	0.073	0.089	2.679	0.203	0.256	0.241	0.286	18.031

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	57	57	53	144	62	75
N.S.	1	1.00	0.96	0.75	0.75	0.70	1.89	0.82	0.99
time (sec)	N/A	0.062	0.084	2.678	0.208	0.254	0.182	0.278	14.194

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	57	44	46	42	92	47	55
N.S.	1	1.00	1.06	0.81	0.85	0.78	1.70	0.87	1.02
time (sec)	N/A	0.047	0.043	1.812	0.203	0.252	0.116	0.292	14.051

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	32	34	29	66	31	31
N.S.	1	1.00	0.92	0.84	0.89	0.76	1.74	0.82	0.82
time (sec)	N/A	0.016	0.066	0.802	0.197	0.249	0.087	0.288	14.373

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	15	17
N.S.	1	1.00	1.73	1.07	1.00	1.13	1.13	1.00	1.13
time (sec)	N/A	0.009	0.018	0.583	0.183	0.252	0.054	0.268	13.692

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	28	36	49	43	57
N.S.	1	1.00	1.00	1.81	1.75	2.25	3.06	2.69	3.56
time (sec)	N/A	0.024	0.004	1.315	0.189	0.274	2.525	0.287	13.888

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	30	38	60	0	63	47
N.S.	1	1.00	1.00	1.25	1.58	2.50	0.00	2.62	1.96
time (sec)	N/A	0.042	0.006	2.211	0.213	0.284	0.000	0.296	13.814

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	47	58	74	0	105	81
N.S.	1	1.00	1.00	1.00	1.23	1.57	0.00	2.23	1.72
time (sec)	N/A	0.052	0.008	2.986	0.199	0.271	0.000	0.295	14.658

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	60	70	88	0	122	111
N.S.	1	1.00	0.95	0.95	1.11	1.40	0.00	1.94	1.76
time (sec)	N/A	0.065	0.117	3.061	0.199	0.289	0.000	0.296	16.442

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	59	73	95	99	0	164	150
N.S.	1	1.00	0.69	0.86	1.12	1.16	0.00	1.93	1.76
time (sec)	N/A	0.077	0.151	3.071	0.201	0.286	0.000	0.291	16.755

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	94	83	107	110	0	178	180
N.S.	1	1.00	0.93	0.82	1.06	1.09	0.00	1.76	1.78
time (sec)	N/A	0.080	0.225	3.284	0.205	0.298	0.000	0.302	17.205

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	123	112	120	110	343	127	143
N.S.	1	1.00	0.82	0.75	0.80	0.73	2.29	0.85	0.95
time (sec)	N/A	0.121	0.275	3.465	0.289	0.268	0.370	0.287	14.390

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	85	95	94	86	221	102	117
N.S.	1	1.00	0.77	0.86	0.85	0.77	1.99	0.92	1.05
time (sec)	N/A	0.116	0.152	3.874	0.203	0.269	0.250	0.292	14.089

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	86	75	82	77	211	82	93
N.S.	1	1.00	0.85	0.74	0.81	0.76	2.09	0.81	0.92
time (sec)	N/A	0.103	0.170	2.718	0.207	0.259	0.182	0.293	14.965

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	59	61	60	52	107	60	72
N.S.	1	1.00	0.83	0.86	0.85	0.73	1.51	0.85	1.01
time (sec)	N/A	0.058	0.275	1.945	0.197	0.257	0.116	0.275	14.685

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	43	44	40	78	43	42
N.S.	1	1.00	0.92	0.86	0.88	0.80	1.56	0.86	0.84
time (sec)	N/A	0.017	0.114	0.954	0.197	0.249	0.090	0.273	14.409

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	43	42	52	0	78	73
N.S.	1	1.00	1.39	1.30	1.27	1.58	0.00	2.36	2.21
time (sec)	N/A	0.065	0.023	1.521	0.203	0.280	0.000	0.307	14.450

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	43	48	74	0	77	181
N.S.	1	1.00	0.97	1.30	1.45	2.24	0.00	2.33	5.48
time (sec)	N/A	0.081	0.063	2.367	0.195	0.260	0.000	0.295	14.686

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	67	69	87	93	0	127	99
N.S.	1	1.00	1.14	1.17	1.47	1.58	0.00	2.15	1.68
time (sec)	N/A	0.092	0.008	3.077	0.191	0.263	0.000	0.302	15.289

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	74	84	100	0	178	141
N.S.	1	1.00	0.89	0.92	1.05	1.25	0.00	2.22	1.76
time (sec)	N/A	0.110	0.156	4.016	0.190	0.274	0.000	0.305	17.132

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	82	111	144	133	0	258	184
N.S.	1	1.00	0.75	1.01	1.31	1.21	0.00	2.35	1.67
time (sec)	N/A	0.116	0.198	3.764	0.211	0.259	0.000	0.321	18.044

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	90	110	132	136	0	272	221
N.S.	1	1.00	0.67	0.81	0.98	1.01	0.00	2.01	1.64
time (sec)	N/A	0.135	0.394	4.334	0.201	0.281	0.000	0.330	18.544

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	193	159	135	145	132	393	150	380
N.S.	1	1.14	0.94	0.79	0.85	0.78	2.31	0.88	2.24
time (sec)	N/A	0.248	0.518	4.427	0.220	0.267	0.367	0.299	15.875

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	130	113	119	110	284	124	319
N.S.	1	1.00	0.72	0.63	0.66	0.61	1.58	0.69	1.77
time (sec)	N/A	0.243	0.426	3.545	0.208	0.264	0.260	0.312	15.867

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	100	102	95	84	233	96	279
N.S.	1	1.00	0.83	0.84	0.79	0.69	1.93	0.79	2.31
time (sec)	N/A	0.127	0.345	2.627	0.290	0.271	0.179	0.292	16.293

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	90	80	67	72	66	128	72	77
N.S.	1	1.18	1.05	0.88	0.95	0.87	1.68	0.95	1.01
time (sec)	N/A	0.078	0.185	2.005	0.199	0.273	0.126	0.291	14.439

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	105	73	69	72	0	137	123
N.S.	1	1.00	1.44	1.00	0.95	0.99	0.00	1.88	1.68
time (sec)	N/A	0.125	0.389	1.570	0.223	0.272	0.000	0.310	14.793

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	88	57	66	94	0	129	97
N.S.	1	1.00	1.29	0.84	0.97	1.38	0.00	1.90	1.43
time (sec)	N/A	0.144	0.533	2.686	0.203	0.272	0.000	0.305	14.828

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	55	82	101	112	0	143	136
N.S.	1	1.00	0.70	1.04	1.28	1.42	0.00	1.81	1.72
time (sec)	N/A	0.155	0.125	3.592	0.191	0.271	0.000	0.314	14.491

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	70	96	113	126	0	205	157
N.S.	1	1.00	0.64	0.88	1.04	1.16	0.00	1.88	1.44
time (sec)	N/A	0.210	0.182	4.371	0.227	0.266	0.000	0.310	16.338

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	90	125	158	140	0	330	224
N.S.	1	1.00	0.68	0.94	1.19	1.05	0.00	2.48	1.68
time (sec)	N/A	0.241	0.319	4.997	0.199	0.298	0.000	0.315	17.994

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	120	148	181	170	0	367	260
N.S.	1	1.00	0.71	0.88	1.07	1.01	0.00	2.17	1.54
time (sec)	N/A	0.264	0.570	4.909	0.249	0.278	0.000	0.355	17.674

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	181	176	192	171	495	197	476
N.S.	1	1.00	0.73	0.71	0.78	0.69	2.00	0.80	1.93
time (sec)	N/A	0.465	0.709	5.947	0.201	0.275	0.519	0.313	15.709

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	156	153	170	150	459	168	214
N.S.	1	1.00	0.66	0.65	0.72	0.64	1.95	0.71	0.91
time (sec)	N/A	0.384	0.623	4.313	0.186	0.264	0.391	0.318	14.464

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	133	125	133	121	301	134	363
N.S.	1	1.00	0.78	0.74	0.78	0.71	1.77	0.79	2.14
time (sec)	N/A	0.249	0.607	3.735	0.201	0.280	0.274	0.301	15.577

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	104	101	111	96	240	107	123
N.S.	1	1.00	0.76	0.74	0.81	0.70	1.75	0.78	0.90
time (sec)	N/A	0.174	0.353	2.433	0.195	0.266	0.195	0.285	14.748

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	128	98	95	98	0	212	158
N.S.	1	1.00	1.20	0.92	0.89	0.92	0.00	1.98	1.48
time (sec)	N/A	0.272	0.553	2.617	0.200	0.276	0.000	0.316	14.752

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	119	87	90	116	0	170	150
N.S.	1	1.00	1.04	0.76	0.79	1.02	0.00	1.49	1.32
time (sec)	N/A	0.278	0.889	2.476	0.192	0.263	0.000	0.308	14.551

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	174	96	115	130	0	177	152
N.S.	1	1.00	1.61	0.89	1.06	1.20	0.00	1.64	1.41
time (sec)	N/A	0.283	2.001	3.341	0.182	0.276	0.000	0.337	14.529

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	77	109	125	138	0	221	185
N.S.	1	1.00	0.67	0.95	1.09	1.20	0.00	1.92	1.61
time (sec)	N/A	0.298	0.231	4.248	0.191	0.269	0.000	0.337	14.883

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	101	147	187	163	0	360	245
N.S.	1	1.00	0.66	0.95	1.21	1.06	0.00	2.34	1.59
time (sec)	N/A	0.386	0.344	4.676	0.214	0.272	0.000	0.337	18.016

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	125	162	195	182	0	461	304
N.S.	1	1.00	0.66	0.86	1.04	0.97	0.00	2.45	1.62
time (sec)	N/A	0.414	0.503	6.155	0.209	0.261	0.000	0.327	18.752

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	154	209	275	217	0	592	370
N.S.	1	1.00	0.69	0.94	1.24	0.98	0.00	2.67	1.67
time (sec)	N/A	0.461	0.654	6.021	0.195	0.279	0.000	0.363	18.890

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	153	256	0	479	0	393	474
N.S.	1	1.00	0.79	1.33	0.00	2.48	0.00	2.04	2.46
time (sec)	N/A	0.652	0.633	1.319	0.000	0.307	0.000	0.292	15.665

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	122	179	0	400	0	249	203
N.S.	1	1.00	0.82	1.21	0.00	2.70	0.00	1.68	1.37
time (sec)	N/A	0.376	0.398	1.176	0.000	0.292	0.000	0.299	15.341

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	138	0	334	0	177	168
N.S.	1	1.00	0.88	1.25	0.00	3.04	0.00	1.61	1.53
time (sec)	N/A	0.281	0.302	1.179	0.000	0.284	0.000	0.297	15.565

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	72	97	0	269	1744	126	190
N.S.	1	1.00	0.95	1.28	0.00	3.54	22.95	1.66	2.50
time (sec)	N/A	0.160	0.204	0.980	0.000	0.281	65.916	0.283	14.407

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	58	65	0	223	320	240	99
N.S.	1	1.00	0.98	1.10	0.00	3.78	5.42	4.07	1.68
time (sec)	N/A	0.087	0.090	0.835	0.000	0.286	12.669	0.301	14.596

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	175	172	78	43
N.S.	1	1.00	0.98	0.90	0.00	3.57	3.51	1.59	0.88
time (sec)	N/A	0.040	0.024	0.891	0.000	0.282	1.983	0.309	14.513

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	102	83	0	278	0	119	99
N.S.	1	1.00	1.50	1.22	0.00	4.09	0.00	1.75	1.46
time (sec)	N/A	0.127	0.199	0.941	0.000	0.316	0.000	0.291	15.068

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	115	123	0	382	0	153	324
N.S.	1	1.00	1.35	1.45	0.00	4.49	0.00	1.80	3.81
time (sec)	N/A	0.225	0.455	1.231	0.000	0.312	0.000	0.317	14.723

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	238	192	0	459	0	211	1087
N.S.	1	1.00	2.00	1.61	0.00	3.86	0.00	1.77	9.13
time (sec)	N/A	0.546	0.942	1.579	0.000	0.372	0.000	0.315	15.792

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	258	252	0	535	0	286	991
N.S.	1	1.00	1.64	1.61	0.00	3.41	0.00	1.82	6.31
time (sec)	N/A	0.639	1.826	1.529	0.000	0.402	0.000	0.337	17.283

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	176	257	0	747	0	333	3852
N.S.	1	1.00	0.66	0.97	0.00	2.81	0.00	1.25	14.48
time (sec)	N/A	0.786	0.907	1.523	0.000	0.316	0.000	0.311	22.001

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	213	144	218	0	651	0	262	3751
N.S.	1	1.28	0.87	1.31	0.00	3.92	0.00	1.58	22.60
time (sec)	N/A	0.487	0.721	1.565	0.000	0.330	0.000	0.310	21.497

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	113	178	0	554	0	847	3180
N.S.	1	1.00	0.73	1.15	0.00	3.57	0.00	5.46	20.52
time (sec)	N/A	0.306	0.637	1.199	0.000	0.310	0.000	0.372	20.653

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	103	143	0	470	0	175	2872
N.S.	1	1.00	0.95	1.32	0.00	4.35	0.00	1.62	26.59
time (sec)	N/A	0.181	0.403	0.999	0.000	0.295	0.000	0.315	20.892

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	114	0	321	0	135	99
N.S.	1	1.00	0.98	1.34	0.00	3.78	0.00	1.59	1.16
time (sec)	N/A	0.084	0.265	0.959	0.000	0.273	0.000	0.311	15.171

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	84	114	0	320	2470	135	99
N.S.	1	1.00	0.98	1.33	0.00	3.72	28.72	1.57	1.15
time (sec)	N/A	0.062	0.171	0.829	0.000	0.280	34.660	0.293	14.363

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	146	164	0	592	0	198	2886
N.S.	1	1.00	1.24	1.39	0.00	5.02	0.00	1.68	24.46
time (sec)	N/A	0.263	0.400	1.490	0.000	0.466	0.000	0.311	20.080

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	163	205	0	750	0	332	3176
N.S.	1	1.00	1.05	1.32	0.00	4.84	0.00	2.14	20.49
time (sec)	N/A	0.443	0.857	1.734	0.000	0.473	0.000	0.327	20.860

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	285	273	0	899	0	293	3699
N.S.	1	1.00	1.31	1.26	0.00	4.14	0.00	1.35	17.05
time (sec)	N/A	0.742	3.814	1.826	0.000	0.712	0.000	0.316	21.327

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	499	330	0	1001	0	368	3843
N.S.	1	1.00	1.85	1.22	0.00	3.71	0.00	1.36	14.23
time (sec)	N/A	1.097	6.481	2.063	0.000	0.660	0.000	0.341	21.276

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	199	306	0	1161	0	1735	5962
N.S.	1	1.00	0.66	1.02	0.00	3.87	0.00	5.78	19.87
time (sec)	N/A	0.911	1.613	1.881	0.000	0.346	0.000	0.500	22.974

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	177	266	0	1029	0	354	5350
N.S.	1	1.00	0.80	1.20	0.00	4.66	0.00	1.60	24.21
time (sec)	N/A	0.549	1.223	1.385	0.000	0.341	0.000	0.321	21.552

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	149	233	0	913	0	319	5102
N.S.	1	1.00	0.83	1.30	0.00	5.10	0.00	1.78	28.50
time (sec)	N/A	0.347	0.945	1.443	0.000	0.314	0.000	0.322	23.091

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	115	180	0	587	0	250	203
N.S.	1	1.00	0.77	1.21	0.00	3.94	0.00	1.68	1.36
time (sec)	N/A	0.203	0.546	1.157	0.000	0.285	0.000	0.317	17.064

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	115	191	0	555	0	271	207
N.S.	1	1.00	0.86	1.43	0.00	4.14	0.00	2.02	1.54
time (sec)	N/A	0.148	0.347	1.081	0.000	0.293	0.000	0.317	16.757

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	113	182	0	585	0	251	203
N.S.	1	1.00	0.85	1.37	0.00	4.40	0.00	1.89	1.53
time (sec)	N/A	0.141	0.305	1.168	0.000	0.289	0.000	0.296	16.627

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	192	251	0	1142	0	344	5090
N.S.	1	1.00	1.05	1.38	0.00	6.27	0.00	1.89	27.97
time (sec)	N/A	0.488	0.948	1.669	0.000	0.800	0.000	0.316	23.975

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	205	292	0	1346	0	380	5347
N.S.	1	1.00	0.88	1.26	0.00	5.80	0.00	1.64	23.05
time (sec)	N/A	0.888	3.014	2.045	0.000	0.805	0.000	0.335	22.513

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	427	360	0	1524	0	801	5910
N.S.	1	1.00	1.40	1.18	0.00	5.00	0.00	2.63	19.38
time (sec)	N/A	1.198	6.522	2.439	0.000	1.470	0.000	0.348	23.246

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	240	395	0	1593	0	563	7494
N.S.	1	1.00	0.78	1.29	0.00	5.19	0.00	1.83	24.41
time (sec)	N/A	1.012	4.081	2.253	0.000	0.374	0.000	0.377	24.952

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	227	361	0	1445	0	531	7247
N.S.	1	1.00	0.91	1.44	0.00	5.78	0.00	2.12	28.99
time (sec)	N/A	0.653	2.034	1.749	0.000	0.355	0.000	0.354	27.411

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	158	281	0	893	0	399	378
N.S.	1	1.00	0.71	1.27	0.00	4.02	0.00	1.80	1.70
time (sec)	N/A	0.380	1.105	1.372	0.000	0.321	0.000	0.351	18.097

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	162	290	0	893	0	427	381
N.S.	1	1.00	0.79	1.41	0.00	4.33	0.00	2.07	1.85
time (sec)	N/A	0.346	1.024	1.385	0.000	0.309	0.000	0.328	18.372

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	164	293	0	891	0	427	382
N.S.	1	1.00	0.85	1.53	0.00	4.64	0.00	2.22	1.99
time (sec)	N/A	0.263	0.833	1.331	0.000	0.315	0.000	0.344	17.617

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	159	280	0	895	0	399	378
N.S.	1	1.00	0.86	1.52	0.00	4.86	0.00	2.17	2.05
time (sec)	N/A	0.261	0.673	1.230	0.000	0.310	0.000	0.312	18.194

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	274	379	0	1815	0	554	7235
N.S.	1	1.00	1.09	1.51	0.00	7.23	0.00	2.21	28.82
time (sec)	N/A	0.849	2.106	2.319	0.000	1.729	0.000	0.348	27.374

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	416	421	0	2048	0	587	7490
N.S.	1	1.00	1.35	1.37	0.00	6.65	0.00	1.91	24.32
time (sec)	N/A	1.389	6.575	2.624	0.000	1.686	0.000	0.378	23.520

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	214	827	0	474	0	0	0
N.S.	1	1.00	0.81	3.13	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	0.465	0.904	7.526	0.000	0.121	0.000	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	180	665	0	436	0	0	0
N.S.	1	1.00	0.87	3.21	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	0.329	0.666	6.686	0.000	0.108	0.000	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	137	452	0	398	0	0	0
N.S.	1	1.00	0.85	2.79	0.00	2.46	0.00	0.00	0.00
time (sec)	N/A	0.215	0.453	5.452	0.000	0.106	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	170	0	355	0	0	0
N.S.	1	1.00	1.00	2.98	0.00	6.23	0.00	0.00	0.00
time (sec)	N/A	0.048	0.035	3.005	0.000	0.098	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	81	194	0	0	0	0	0
N.S.	1	1.00	0.69	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.277	13.944	3.468	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	307	622	0	0	0	0	0
N.S.	1	1.00	1.56	3.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.591	14.873	4.964	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	393	977	0	0	0	0	0
N.S.	1	1.00	1.50	3.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.862	5.681	5.456	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	262	995	0	516	0	0	0
N.S.	1	1.00	0.83	3.17	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.832	1.057	8.671	0.000	0.152	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	214	827	0	474	0	0	0
N.S.	1	1.00	0.83	3.21	0.00	1.84	0.00	0.00	0.00
time (sec)	N/A	0.463	0.883	7.814	0.000	0.141	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	174	663	0	438	0	0	0
N.S.	1	1.00	0.87	3.33	0.00	2.20	0.00	0.00	0.00
time (sec)	N/A	0.289	0.614	6.557	0.000	0.135	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	134	450	0	398	0	0	0
N.S.	1	1.00	0.85	2.87	0.00	2.54	0.00	0.00	0.00
time (sec)	N/A	0.192	0.359	4.679	0.000	0.100	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	107	249	0	0	0	0	0
N.S.	1	1.00	0.60	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	26.932	4.426	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	363	740	0	0	0	0	0
N.S.	1	1.00	1.74	3.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.666	16.283	5.118	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	386	980	0	0	0	0	0
N.S.	1	1.00	1.51	3.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.929	5.854	6.467	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	268	1140	0	561	0	0	0
N.S.	1	1.00	0.72	3.07	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.742	0.987	14.836	0.000	0.176	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	263	995	0	514	0	0	0
N.S.	1	1.00	0.85	3.23	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.582	1.027	10.019	0.000	0.194	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	214	827	0	474	0	0	0
N.S.	1	1.00	0.86	3.32	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.422	0.671	7.885	0.000	0.116	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	177	662	0	437	0	0	0
N.S.	1	1.00	0.90	3.36	0.00	2.22	0.00	0.00	0.00
time (sec)	N/A	0.305	0.498	6.553	0.000	0.127	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	379	528	0	0	0	0	0
N.S.	1	1.00	1.71	2.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.664	1.095	6.062	0.000	0.000	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	390	960	0	0	0	0	0
N.S.	1	1.00	1.76	4.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.725	1.465	10.753	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	395	1134	0	0	0	0	0
N.S.	1	1.00	1.46	4.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.035	1.843	33.461	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	434	1742	0	0	0	0	0
N.S.	1	1.00	1.34	5.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.351	2.666	103.948	0.000	0.000	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	211	824	0	474	0	0	0
N.S.	1	1.00	0.86	3.35	0.00	1.93	0.00	0.00	0.00
time (sec)	N/A	0.539	0.787	7.396	0.000	0.131	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	92	275	0	148	0	0	0
N.S.	1	1.00	0.67	1.99	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.294	0.282	5.999	0.000	0.101	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	81	253	0	138	0	0	0
N.S.	1	1.00	0.77	2.41	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.214	0.182	3.357	0.000	0.111	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	231	0	128	0	0	0
N.S.	1	1.00	0.88	2.96	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.128	0.096	2.727	0.000	0.101	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	137	0	108	0	0	0
N.S.	1	1.00	1.00	5.96	0.00	4.70	0.00	0.00	0.00
time (sec)	N/A	0.014	0.042	3.663	0.000	0.112	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	158	0	0	0	0	0
N.S.	1	1.00	0.85	3.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.137	0.101	2.309	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	157	350	0	0	0	0	0
N.S.	1	1.00	1.65	3.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.373	10.874	3.777	0.000	0.000	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	194	408	0	0	0	0	0
N.S.	1	1.00	1.44	3.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.418	0.901	4.085	0.000	0.000	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	114	276	0	148	0	0	0
N.S.	1	1.00	0.81	1.97	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.215	0.266	7.311	0.000	0.121	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	104	253	0	138	0	0	0
N.S.	1	1.00	0.97	2.36	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.159	0.231	5.154	0.000	0.100	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	94	231	0	128	0	0	0
N.S.	1	1.00	1.18	2.89	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.100	0.115	3.546	0.000	0.094	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	44	138	0	106	0	0	0
N.S.	1	1.00	1.83	5.75	0.00	4.42	0.00	0.00	0.00
time (sec)	N/A	0.015	0.050	3.814	0.000	0.088	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	61	159	0	0	0	0	0
N.S.	1	1.00	1.22	3.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	0.111	3.009	0.000	0.000	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	178	351	0	0	0	0	0
N.S.	1	1.00	1.82	3.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	0.970	4.613	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	237	408	0	0	0	0	0
N.S.	1	1.00	1.72	2.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.442	1.416	5.865	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	182	665	0	439	0	0	0
N.S.	1	1.00	0.85	3.09	0.00	2.04	0.00	0.00	0.00
time (sec)	N/A	0.349	0.753	5.593	0.000	0.111	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	137	453	0	398	0	0	116
N.S.	1	1.00	0.83	2.75	0.00	2.41	0.00	0.00	0.70
time (sec)	N/A	0.248	0.538	4.109	0.000	0.122	0.000	0.000	14.342

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	86	220	0	355	0	0	80
N.S.	1	1.00	0.70	1.80	0.00	2.91	0.00	0.00	0.66
time (sec)	N/A	0.142	2.301	3.405	0.000	0.102	0.000	0.000	14.600

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	78	0	146	0	0	52
N.S.	1	1.00	1.00	1.37	0.00	2.56	0.00	0.00	0.91
time (sec)	N/A	0.049	0.021	0.922	0.000	0.087	0.000	0.000	14.557

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	166	0	0	0	0	0
N.S.	1	1.00	1.00	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.160	0.169	2.073	0.000	0.000	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	310	532	0	0	0	0	0
N.S.	1	1.00	1.50	2.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.629	15.204	3.057	0.000	0.000	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	518	710	0	0	0	0	0
N.S.	1	1.00	1.93	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.901	6.374	3.667	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	242	1285	0	687	0	0	0
N.S.	1	1.00	0.74	3.94	0.00	2.11	0.00	0.00	0.00
time (sec)	N/A	0.603	1.028	6.158	0.000	0.161	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	197	743	0	630	0	0	0
N.S.	1	1.00	0.77	2.89	0.00	2.45	0.00	0.00	0.00
time (sec)	N/A	0.388	0.740	5.738	0.000	0.151	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	159	530	0	567	0	0	0
N.S.	1	1.00	0.85	2.85	0.00	3.05	0.00	0.00	0.00
time (sec)	N/A	0.252	0.594	5.251	0.000	0.128	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	137	373	0	527	0	0	0
N.S.	1	1.00	0.81	2.19	0.00	3.10	0.00	0.00	0.00
time (sec)	N/A	0.217	0.417	3.541	0.000	0.108	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	83	217	0	482	0	0	0
N.S.	1	1.00	0.78	2.05	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.088	0.131	3.013	0.000	0.115	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	402	376	0	0	0	0	0
N.S.	1	1.00	2.28	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.531	3.773	4.447	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	441	898	0	0	0	0	0
N.S.	1	1.00	1.59	3.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.221	2.700	6.441	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	466	1546	0	0	0	0	0
N.S.	1	1.00	1.35	4.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.269	4.585	8.819	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	272	1688	0	1040	0	0	0
N.S.	1	1.00	0.62	3.87	0.00	2.39	0.00	0.00	0.00
time (sec)	N/A	0.972	1.525	12.064	0.000	0.206	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	237	1295	0	954	0	0	0
N.S.	1	1.00	0.69	3.75	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	0.659	1.254	10.288	0.000	0.178	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	188	911	0	867	0	0	0
N.S.	1	1.00	0.67	3.24	0.00	3.09	0.00	0.00	0.00
time (sec)	N/A	0.638	1.003	9.852	0.000	0.157	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	175	850	0	814	0	0	0
N.S.	1	1.00	0.67	3.23	0.00	3.10	0.00	0.00	0.00
time (sec)	N/A	0.536	0.919	8.480	0.000	0.120	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	154	746	0	765	0	0	0
N.S.	1	1.00	0.63	3.07	0.00	3.15	0.00	0.00	0.00
time (sec)	N/A	0.320	0.770	7.803	0.000	0.122	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	158	489	0	692	0	0	0
N.S.	1	1.00	0.71	2.21	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	0.270	0.609	5.956	0.000	0.120	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	464	849	0	0	0	0	0
N.S.	1	1.00	1.45	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.988	3.231	9.017	0.000	0.000	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	517	1324	0	0	0	0	0
N.S.	1	1.00	1.36	3.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.291	5.220	12.070	0.000	0.000	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	189	616	0	985	0	0	0
N.S.	1	1.00	0.67	2.18	0.00	3.49	0.00	0.00	0.00
time (sec)	N/A	0.444	0.923	7.513	0.000	0.166	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	81	231	0	136	0	0	0
N.S.	1	1.00	0.73	2.08	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.247	0.254	3.845	0.000	0.099	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	231	0	128	0	0	78
N.S.	1	1.00	0.90	2.96	0.00	1.64	0.00	0.00	1.00
time (sec)	N/A	0.150	0.154	2.600	0.000	0.098	0.000	0.000	0.092

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	155	0	108	0	0	54
N.S.	1	1.00	0.84	3.04	0.00	2.12	0.00	0.00	1.06
time (sec)	N/A	0.075	0.101	2.286	0.000	0.097	0.000	0.000	14.495

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	0	54	0	0	39
N.S.	1	1.00	1.00	1.00	0.00	2.35	0.00	0.00	1.70
time (sec)	N/A	0.016	0.048	0.440	0.000	0.086	0.000	0.000	14.816

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	138	0	0	0	0	0
N.S.	1	1.00	1.00	5.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.107	1.839	0.000	0.000	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	158	350	0	0	0	0	0
N.S.	1	1.00	1.56	3.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.428	0.786	2.526	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	195	408	0	0	0	0	0
N.S.	1	1.00	1.42	2.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	0.890	3.092	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	102	254	0	136	0	0	0
N.S.	1	1.00	0.90	2.25	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.168	0.254	4.359	0.000	0.097	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	94	232	0	128	0	0	78
N.S.	1	1.00	1.18	2.90	0.00	1.60	0.00	0.00	0.98
time (sec)	N/A	0.114	0.191	2.926	0.000	0.096	0.000	0.000	0.086

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	60	158	0	106	0	0	52
N.S.	1	1.00	1.13	2.98	0.00	2.00	0.00	0.00	0.98
time (sec)	N/A	0.059	0.109	2.282	0.000	0.091	0.000	0.000	15.075

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	44	54	0	52	0	0	39
N.S.	1	1.00	1.83	2.25	0.00	2.17	0.00	0.00	1.62
time (sec)	N/A	0.015	0.051	0.524	0.000	0.095	0.000	0.000	0.097

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	45	139	0	0	0	0	0
N.S.	1	1.00	1.80	5.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.126	1.745	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	179	351	0	0	0	0	0
N.S.	1	1.00	1.72	3.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	1.030	2.894	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	236	408	0	0	0	0	0
N.S.	1	1.00	1.69	2.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.481	1.302	3.447	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	77	290	0	148	0	0	87
N.S.	1	1.00	0.69	2.61	0.00	1.33	0.00	0.00	0.78
time (sec)	N/A	0.095	0.400	9.743	0.000	0.102	0.000	0.000	15.231

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	66	262	0	137	0	0	80
N.S.	1	1.00	0.76	3.01	0.00	1.57	0.00	0.00	0.92
time (sec)	N/A	0.081	0.192	7.898	0.000	0.098	0.000	0.000	15.098

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	229	0	125	0	0	53
N.S.	1	1.00	0.87	3.75	0.00	2.05	0.00	0.00	0.87
time (sec)	N/A	0.064	0.104	6.597	0.000	0.094	0.000	0.000	0.178

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	152	0	107	0	0	33
N.S.	1	1.00	1.00	4.34	0.00	3.06	0.00	0.00	0.94
time (sec)	N/A	0.046	0.078	3.369	0.000	0.093	0.000	0.000	0.232

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	150	0	156	0	0	60
N.S.	1	1.00	0.89	2.63	0.00	2.74	0.00	0.00	1.05
time (sec)	N/A	0.060	0.136	4.349	0.000	0.092	0.000	0.000	15.478

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	397	0	175	0	0	87
N.S.	1	1.00	0.78	4.78	0.00	2.11	0.00	0.00	1.05
time (sec)	N/A	0.069	0.306	6.824	0.000	0.101	0.000	0.000	15.617

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	502	0	188	0	0	87
N.S.	1	1.00	0.86	4.52	0.00	1.69	0.00	0.00	0.78
time (sec)	N/A	0.089	0.248	9.332	0.000	0.106	0.000	0.000	15.756

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	113	398	0	195	0	0	135
N.S.	1	1.00	0.71	2.49	0.00	1.22	0.00	0.00	0.84
time (sec)	N/A	0.143	0.797	17.908	0.000	0.109	0.000	0.000	15.230

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	98	362	0	180	0	0	128
N.S.	1	1.00	0.73	2.68	0.00	1.33	0.00	0.00	0.95
time (sec)	N/A	0.137	0.639	10.986	0.000	0.103	0.000	0.000	15.164

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	79	357	0	162	0	0	102
N.S.	1	1.00	0.78	3.53	0.00	1.60	0.00	0.00	1.01
time (sec)	N/A	0.115	0.402	9.503	0.000	0.099	0.000	0.000	15.364

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	283	0	147	0	0	76
N.S.	1	1.00	0.89	3.93	0.00	2.04	0.00	0.00	1.06
time (sec)	N/A	0.099	0.577	7.045	0.000	0.098	0.000	0.000	14.717

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	202	0	178	0	0	81
N.S.	1	1.00	0.91	2.97	0.00	2.62	0.00	0.00	1.19
time (sec)	N/A	0.109	0.438	6.907	0.000	0.100	0.000	0.000	15.294

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	73	420	0	198	0	0	108
N.S.	1	1.00	0.77	4.42	0.00	2.08	0.00	0.00	1.14
time (sec)	N/A	0.126	0.664	8.408	0.000	0.112	0.000	0.000	15.576

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	124	633	0	223	0	0	113
N.S.	1	1.00	0.92	4.69	0.00	1.65	0.00	0.00	0.84
time (sec)	N/A	0.134	0.503	11.749	0.000	0.105	0.000	0.000	15.457

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	137	470	0	227	0	0	178
N.S.	1	1.00	0.71	2.42	0.00	1.17	0.00	0.00	0.92
time (sec)	N/A	0.262	1.002	13.529	0.000	0.115	0.000	0.000	14.771

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	110	421	0	205	0	0	146
N.S.	1	1.00	0.69	2.65	0.00	1.29	0.00	0.00	0.92
time (sec)	N/A	0.232	0.777	12.034	0.000	0.113	0.000	0.000	14.575

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	84	412	0	185	0	0	125
N.S.	1	1.00	0.72	3.55	0.00	1.59	0.00	0.00	1.08
time (sec)	N/A	0.199	0.571	9.484	0.000	0.115	0.000	0.000	14.750

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	86	303	0	214	0	0	124
N.S.	1	1.00	0.69	2.44	0.00	1.73	0.00	0.00	1.00
time (sec)	N/A	0.217	0.601	8.831	0.000	0.103	0.000	0.000	15.140

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	85	558	0	222	0	0	128
N.S.	1	1.00	0.71	4.65	0.00	1.85	0.00	0.00	1.07
time (sec)	N/A	0.220	1.093	9.727	0.000	0.111	0.000	0.000	15.466

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	125	711	0	244	0	0	156
N.S.	1	1.00	0.84	4.77	0.00	1.64	0.00	0.00	1.05
time (sec)	N/A	0.236	0.936	12.050	0.000	0.107	0.000	0.000	15.553

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	177	820	0	270	0	0	147
N.S.	1	1.00	0.91	4.23	0.00	1.39	0.00	0.00	0.76
time (sec)	N/A	0.270	0.905	14.691	0.000	0.114	0.000	0.000	16.465

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	158	552	0	0	0	0	0
N.S.	1	1.00	1.41	4.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.404	11.380	5.135	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	81	227	0	0	0	0	0
N.S.	1	1.00	1.08	3.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	10.404	4.180	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	188	0	0	0	0	0
N.S.	1	1.00	0.91	3.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	0.158	2.918	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	150	0	0	0	0	0
N.S.	1	1.00	1.00	5.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.150	2.174	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	195	354	0	0	0	0	0
N.S.	1	1.00	2.53	4.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	1.959	3.900	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	210	425	0	0	0	0	0
N.S.	1	1.00	1.64	3.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.592	2.818	5.968	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	266	1070	0	0	0	0	0
N.S.	1	1.00	1.09	4.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.779	1.251	17.418	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	251	815	0	0	0	0	0
N.S.	1	1.00	1.36	4.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.528	1.238	17.084	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	194	794	0	0	0	0	0
N.S.	1	1.00	1.19	4.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.431	2.208	5.815	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	229	713	0	0	0	0	0
N.S.	1	1.00	1.55	4.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	2.605	5.603	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	238	612	0	0	0	0	0
N.S.	1	1.00	1.52	3.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	2.334	4.747	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	278	847	0	0	0	0	0
N.S.	1	1.00	1.28	3.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.760	1.784	6.607	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	294	981	0	0	0	0	0
N.S.	1	1.00	1.05	3.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.063	2.273	9.053	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	354	2194	0	0	0	0	0
N.S.	1	1.00	1.02	6.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.090	2.160	87.674	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	309	1935	0	0	0	0	0
N.S.	1	1.00	1.10	6.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.103	1.922	88.056	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	284	1914	0	0	0	0	0
N.S.	1	1.00	1.08	7.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.846	1.435	86.514	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	272	1836	0	0	0	0	0
N.S.	1	1.00	1.11	7.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.749	1.352	9.605	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	291	1736	0	0	0	0	0
N.S.	1	1.00	1.16	6.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.775	1.880	8.228	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	301	1176	0	0	0	0	0
N.S.	1	1.00	1.15	4.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.839	1.812	6.320	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	334	1965	0	0	0	0	0
N.S.	1	1.00	1.02	5.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.150	2.658	11.478	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	349	2101	0	0	0	0	0
N.S.	1	1.00	0.88	5.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.435	3.732	15.663	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	429	1688	0	0	0	0	0
N.S.	1	1.00	0.98	3.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.076	7.175	7.062	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	314	1085	0	0	0	0	0
N.S.	1	1.00	0.85	2.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.677	5.867	6.418	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	137	181	0	0	0	0	0
N.S.	1	1.00	1.01	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	1.233	8.260	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	203	770	0	0	0	0	0
N.S.	1	1.00	0.89	3.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	2.221	10.104	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	247	1179	0	0	0	0	0
N.S.	1	1.00	0.91	4.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.454	5.472	12.654	0.000	0.000	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	453	2105	0	0	0	0	0
N.S.	1	1.00	1.38	6.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.737	10.464	12.110	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	1304	2499	0	0	0	0	0
N.S.	1	1.00	3.35	6.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.046	6.333	16.384	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	508	508	1189	2311	0	0	0	0	0
N.S.	1	1.00	2.34	4.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.462	16.899	8.065	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	437	1946	0	0	0	0	0
N.S.	1	1.00	1.01	4.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.358	7.427	7.200	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	339	1363	0	0	0	0	0
N.S.	1	1.00	0.90	3.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.941	5.196	9.362	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	357	1016	0	0	0	0	0
N.S.	1	1.00	1.06	3.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.588	8.605	10.740	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	256	1450	0	0	0	0	0
N.S.	1	1.00	0.92	5.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.502	3.255	12.161	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	399	2104	0	0	0	0	0
N.S.	1	1.00	1.23	6.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.782	9.805	13.863	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	1302	2499	0	0	0	0	0
N.S.	1	1.00	3.36	6.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.097	6.331	15.871	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	1368	3454	0	0	0	0	0
N.S.	1	1.00	3.01	7.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.474	6.431	18.996	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	506	506	1203	2568	0	0	0	0	0
N.S.	1	1.00	2.38	5.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.514	16.788	8.334	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	329	2232	0	0	0	0	0
N.S.	1	1.00	0.74	5.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.170	4.361	10.170	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	445	462	2508	0	0	0	0	0
N.S.	1	1.00	1.04	5.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.199	11.992	10.914	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	328	2211	0	0	0	0	0
N.S.	1	1.00	0.84	5.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.831	4.624	12.265	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	386	2373	0	0	0	0	0
N.S.	1	1.00	1.14	7.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.843	7.799	14.093	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	1302	2499	0	0	0	0	0
N.S.	1	1.00	3.36	6.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.625	6.365	16.335	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	1368	3454	0	0	0	0	0
N.S.	1	1.00	3.01	7.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.554	6.479	18.321	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	1431	3891	0	0	0	0	0
N.S.	1	1.00	2.74	7.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.988	6.572	21.011	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	414	224	834	0	0	0	0	0
N.S.	1	1.09	0.59	2.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.900	4.084	8.934	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	130	145	0	0	0	0	0
N.S.	1	1.00	1.12	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.918	7.706	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	170	111	0	0	0	0	0
N.S.	1	1.00	1.56	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	1.236	8.486	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	211	658	0	0	0	0	0
N.S.	1	1.00	0.94	2.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.260	3.953	11.278	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	339	1184	0	0	0	0	0
N.S.	1	1.00	1.24	4.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.471	9.646	13.138	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	465	465	1201	2526	0	0	0	0	0
N.S.	1	1.00	2.58	5.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.150	6.337	9.310	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	387	387	282	1036	0	0	0	0	0
N.S.	1	1.00	0.73	2.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.608	9.194	9.773	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	196	783	0	0	0	0	0
N.S.	1	1.00	0.74	2.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.401	3.647	6.676	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	202	750	0	0	0	0	0
N.S.	1	1.00	0.76	2.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	4.254	10.474	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	1233	1228	0	0	0	0	0
N.S.	1	1.00	4.33	4.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.554	6.355	12.915	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	1269	2618	0	0	0	0	0
N.S.	1	1.00	3.55	7.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.844	6.360	14.293	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	1314	3372	0	0	0	0	0
N.S.	1	1.00	3.03	7.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.241	6.418	16.783	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	497	497	1282	4445	0	0	0	0	0
N.S.	1	1.00	2.58	8.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.221	6.369	9.722	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	342	342	277	2129	0	0	0	0	0
N.S.	1	1.00	0.81	6.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.694	4.267	9.306	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	1273	2614	0	0	0	0	0
N.S.	1	1.00	3.55	7.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.761	6.330	8.029	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	1296	2841	0	0	0	0	0
N.S.	1	1.00	3.40	7.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.833	6.292	11.185	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	1321	3675	0	0	0	0	0
N.S.	1	1.00	3.32	9.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.905	6.404	13.490	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	1351	5224	0	0	0	0	0
N.S.	1	1.00	2.86	11.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.311	6.444	15.914	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	131	103	0	0	0	0	0
N.S.	1	1.00	4.09	3.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	9.556	7.877	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	156	95	0	0	0	0	0
N.S.	1	1.00	6.24	3.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.752	6.780	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	143	107	0	0	0	0	0
N.S.	1	1.00	2.55	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	1.714	7.702	0.000	0.000	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	72	121	0	0	0	0	0
N.S.	1	1.00	1.47	2.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	1.482	7.794	0.000	0.000	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	140	104	0	0	0	0	0
N.S.	1	1.00	2.41	1.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	1.164	7.247	0.000	0.000	0.000	0.000	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	144	113	0	0	0	0	0
N.S.	1	1.00	2.40	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	1.436	7.540	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	144	112	0	0	0	0	0
N.S.	1	1.00	1.71	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	1.039	7.300	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	153	126	0	0	0	0	0
N.S.	1	1.00	1.87	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	1.435	7.385	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	150	111	0	0	0	0	0
N.S.	1	1.00	2.78	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.459	7.248	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	158	97	0	0	0	0	0
N.S.	1	1.00	3.36	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.127	0.359	6.589	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	145	109	0	0	0	0	0
N.S.	1	1.00	4.26	3.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.517	6.040	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	74	117	0	0	0	0	0
N.S.	1	1.00	2.74	4.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.205	6.842	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	154	116	0	0	0	0	0
N.S.	1	1.00	1.92	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	0.537	7.545	0.000	0.000	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	146	106	0	0	0	0	0
N.S.	1	1.00	1.78	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	0.456	6.398	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	160	96	0	0	0	0	0
N.S.	1	1.00	2.58	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.565	4.673	0.000	0.000	0.000	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	155	118	0	0	0	0	0
N.S.	1	1.00	2.58	1.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	0.436	6.838	0.000	0.000	0.000	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	175	128	0	0	0	0	0
N.S.	1	1.00	2.27	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	12.626	6.267	0.000	0.000	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	140	118	0	0	0	0	0
N.S.	1	1.00	1.87	1.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.719	5.674	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	201	130	0	0	0	0	0
N.S.	1	1.00	2.03	1.31	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.135	1.623	6.706	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	155	149	0	0	0	0	0
N.S.	1	1.00	1.53	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	1.498	6.592	0.000	0.000	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	128	130	0	0	0	0	0
N.S.	1	1.00	1.75	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	1.070	6.726	0.000	0.000	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	128	140	0	0	0	0	0
N.S.	1	1.00	1.71	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	0.747	5.622	0.000	0.000	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	135	144	0	0	0	0	0
N.S.	1	1.00	1.36	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.825	7.061	0.000	0.000	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	126	158	0	0	0	0	0
N.S.	1	1.00	1.30	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.659	6.820	0.000	0.000	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	194	139	0	0	0	0	0
N.S.	1	1.00	1.96	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.624	6.914	0.000	0.000	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	142	120	0	0	0	0	0
N.S.	1	1.00	1.46	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.226	6.783	0.000	0.000	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	203	132	0	0	0	0	0
N.S.	1	1.00	2.64	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.103	0.737	5.402	0.000	0.000	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	156	142	0	0	0	0	0
N.S.	1	1.00	1.97	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	0.421	6.503	0.000	0.000	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	130	148	0	0	0	0	0
N.S.	1	1.00	1.37	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	0.386	6.620	0.000	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	130	160	0	0	0	0	0
N.S.	1	1.00	1.34	1.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.237	7.030	0.000	0.000	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	4608	0	0	0	0	0	0
N.S.	1	1.00	26.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	34.392	0.000	0.000	0.000	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	23
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	0.92
time (sec)	N/A	0.064	122.063	0.694	1.341	0.805	0.000	22.414	16.167

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	0	23	23
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.00	0.92	0.92
time (sec)	N/A	0.062	55.730	0.502	0.821	1.055	0.000	23.568	15.362

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	23
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.92
time (sec)	N/A	0.064	33.042	0.579	0.790	0.768	102.892	20.086	15.245

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	23
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.92
time (sec)	N/A	0.064	24.782	0.559	0.788	0.992	2.462	18.687	15.243

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	23	24	23	23
N.S.	1	1.00	1.08	0.84	0.92	0.92	0.96	0.92	0.92
time (sec)	N/A	0.062	15.107	0.620	0.775	0.505	1.003	18.064	15.896

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	44	26	23	23
N.S.	1	1.00	1.08	0.84	0.92	1.76	1.04	0.92	0.92
time (sec)	N/A	0.062	13.959	0.694	0.807	0.372	0.559	19.508	16.235

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	44	26	23	23
N.S.	1	1.00	1.08	0.84	0.92	1.76	1.04	0.92	0.92
time (sec)	N/A	0.066	0.320	0.578	0.804	0.348	1.263	19.047	14.698

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	26	23	23
N.S.	1	1.00	1.08	0.84	0.92	1.84	1.04	0.92	0.92
time (sec)	N/A	0.063	101.449	0.540	0.834	0.344	9.844	20.043	16.782

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	26	23	23
N.S.	1	1.00	1.08	0.84	0.92	1.84	1.04	0.92	0.92
time (sec)	N/A	0.065	83.833	0.573	0.822	0.321	23.652	19.521	14.684

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	21	23	46	0	23	23
N.S.	1	1.00	1.08	0.84	0.92	1.84	0.00	0.92	0.92
time (sec)	N/A	0.065	103.350	0.431	0.827	0.373	0.000	19.321	16.491

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	97	502	0	188	0	0	0
N.S.	1	1.00	0.64	3.32	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.136	0.397	22.510	0.000	0.098	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	85	397	0	167	0	0	0
N.S.	1	1.00	0.69	3.23	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.123	0.276	20.420	0.000	0.094	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	71	150	0	124	0	0	0
N.S.	1	1.00	0.73	1.55	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.111	0.201	5.717	0.000	0.088	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	152	0	107	0	0	0
N.S.	1	1.00	0.69	2.03	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.096	0.231	4.858	0.000	0.094	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	76	229	0	125	0	0	0
N.S.	1	1.00	0.75	2.27	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.108	0.246	7.762	0.000	0.099	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	88	262	0	145	0	0	0
N.S.	1	1.00	0.69	2.06	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.126	0.393	8.597	0.000	0.098	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	99	290	0	156	0	0	0
N.S.	1	1.00	0.66	1.92	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.153	0.561	9.770	0.000	0.105	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	139	689	0	235	0	0	0
N.S.	1	1.00	0.70	3.44	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.224	0.931	112.041	0.000	0.108	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	126	633	0	223	0	0	0
N.S.	1	1.00	0.72	3.62	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.219	1.185	111.078	0.000	0.098	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	93	513	0	190	0	0	0
N.S.	1	1.00	0.69	3.80	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.168	0.490	106.691	0.000	0.101	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	83	202	0	146	0	0	0
N.S.	1	1.00	0.77	1.87	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.161	0.747	8.490	0.000	0.094	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	283	0	147	0	0	0
N.S.	1	1.00	0.78	2.53	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.168	0.618	8.316	0.000	0.094	0.000	0.000	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	100	357	0	170	0	0	0
N.S.	1	1.00	0.71	2.53	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.186	0.852	10.214	0.000	0.096	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	120	362	0	191	0	0	0
N.S.	1	1.00	0.69	2.07	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.196	1.079	12.079	0.000	0.100	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	135	398	0	203	0	0	0
N.S.	1	1.00	0.68	1.99	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.216	1.437	13.890	0.000	0.105	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	191	820	0	270	0	0	0
N.S.	1	1.00	0.82	3.50	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.310	1.248	506.964	0.000	0.109	0.000	0.000	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	134	711	0	244	0	0	0
N.S.	1	1.00	0.71	3.76	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.272	1.377	482.095	0.000	0.115	0.000	0.000	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	106	630	0	214	0	0	0
N.S.	1	1.00	0.66	3.94	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.263	1.012	483.020	0.000	0.103	0.000	0.000	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	108	303	0	182	0	0	0
N.S.	1	1.00	0.65	1.83	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.268	0.906	8.457	0.000	0.104	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	106	412	0	193	0	0	0
N.S.	1	1.00	0.68	2.64	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.260	0.816	9.709	0.000	0.108	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	132	421	0	216	0	0	0
N.S.	1	1.00	0.66	2.12	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.286	1.228	11.241	0.000	0.113	0.000	0.000	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	159	470	0	238	0	0	0
N.S.	1	1.00	0.68	2.01	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.324	1.532	13.290	0.000	0.115	0.000	0.000	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	165	425	0	0	0	0	0
N.S.	1	1.00	0.88	2.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.607	35.183	11.163	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	83	354	0	0	0	0	0
N.S.	1	1.00	0.71	3.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	14.833	3.503	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	63	150	0	0	0	0	0
N.S.	1	1.00	1.29	3.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.277	1.816	0.000	0.000	0.000	0.000	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	47	188	0	0	0	0	0
N.S.	1	1.00	0.51	2.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.240	2.500	0.000	0.000	0.000	0.000	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	176	227	0	0	0	0	0
N.S.	1	1.00	1.30	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	19.526	3.782	0.000	0.000	0.000	0.000	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	196	552	0	0	0	0	0
N.S.	1	1.00	1.14	3.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.449	21.422	3.933	0.000	0.000	0.000	0.000	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	294	981	0	0	0	0	0
N.S.	1	1.00	0.86	2.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.102	4.854	37.465	0.000	0.000	0.000	0.000	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	351	847	0	0	0	0	0
N.S.	1	1.00	1.27	3.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.748	2.777	5.975	0.000	0.000	0.000	0.000	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	297	612	0	0	0	0	0
N.S.	1	1.00	1.37	2.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	5.340	4.028	0.000	0.000	0.000	0.000	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	574	713	0	0	0	0	0
N.S.	1	1.00	2.76	3.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.509	6.408	5.332	0.000	0.000	0.000	0.000	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	250	794	0	0	0	0	0
N.S.	1	1.00	1.12	3.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.599	3.403	5.528	0.000	0.000	0.000	0.000	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	319	815	0	0	0	0	0
N.S.	1	1.00	1.30	3.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.600	3.830	6.506	0.000	0.000	0.000	0.000	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	455	455	747	2101	0	0	0	0	0
N.S.	1	1.00	1.64	4.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.526	6.599	160.230	0.000	0.000	0.000	0.000	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	532	1965	0	0	0	0	0
N.S.	1	1.00	1.37	5.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.153	6.049	10.030	0.000	0.000	0.000	0.000	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	341	1176	0	0	0	0	0
N.S.	1	1.00	1.06	3.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.840	5.355	5.524	0.000	0.000	0.000	0.000	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	395	1736	0	0	0	0	0
N.S.	1	1.00	1.25	5.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.815	3.709	8.232	0.000	0.000	0.000	0.000	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	429	1836	0	0	0	0	0
N.S.	1	1.00	1.42	6.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.763	4.998	8.523	0.000	0.000	0.000	0.000	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	280	1914	0	0	0	0	0
N.S.	1	1.00	0.88	6.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.803	3.554	9.079	0.000	0.000	0.000	0.000	0.000

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	353	2115	0	0	0	0	0
N.S.	1	1.00	0.96	5.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.872	23.410	10.359	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	261	1201	0	0	0	0	0
N.S.	1	1.00	0.84	3.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.565	14.474	11.341	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	269	269	215	770	0	0	0	0	0
N.S.	1	1.00	0.80	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.458	4.531	9.088	0.000	0.000	0.000	0.000	0.000

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	146	181	0	0	0	0	0
N.S.	1	1.00	0.94	1.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	1.510	8.205	0.000	0.000	0.000	0.000	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	2995	1069	0	0	0	0	0
N.S.	1	1.00	6.95	2.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.048	19.003	6.164	0.000	0.000	0.000	0.000	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	498	498	837	1655	0	0	0	0	0
N.S.	1	1.00	1.68	3.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.205	15.953	7.518	0.000	0.000	0.000	0.000	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	441	2509	0	0	0	0	0
N.S.	1	1.00	1.03	5.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.193	11.054	15.277	0.000	0.000	0.000	0.000	0.000

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	345	2114	0	0	0	0	0
N.S.	1	1.00	0.95	5.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.874	8.106	13.080	0.000	0.000	0.000	0.000	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	291	1469	0	0	0	0	0
N.S.	1	1.00	0.92	4.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.642	4.896	11.981	0.000	0.000	0.000	0.000	0.000

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	397	397	639	1016	0	0	0	0	0
N.S.	1	1.00	1.61	2.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.687	15.736	10.266	0.000	0.000	0.000	0.000	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	322	1362	0	0	0	0	0
N.S.	1	1.00	0.74	3.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.849	9.097	8.994	0.000	0.000	0.000	0.000	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	493	493	845	1916	0	0	0	0	0
N.S.	1	1.00	1.71	3.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.472	15.845	7.583	0.000	0.000	0.000	0.000	0.000

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	568	568	961	2265	0	0	0	0	0
N.S.	1	1.00	1.69	3.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.573	14.804	8.304	0.000	0.000	0.000	0.000	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	494	521	3464	0	0	0	0	0
N.S.	1	1.00	1.05	7.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.769	12.638	1052.186	0.000	0.000	0.000	0.000	0.000

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	427	387	2509	0	0	0	0	0
N.S.	1	1.00	0.91	5.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.279	9.851	1590.088	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	376	2383	0	0	0	0	0
N.S.	1	1.00	0.99	6.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.366	9.551	1036.659	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	452	452	377	2211	0	0	0	0	0
N.S.	1	1.00	0.83	4.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.967	9.618	1044.133	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	505	505	421	2508	0	0	0	0	0
N.S.	1	1.00	0.83	4.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.587	11.190	10.072	0.000	0.000	0.000	0.000	0.000

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	503	421	2232	0	0	0	0	0
N.S.	1	1.00	0.84	4.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.253	8.072	9.309	0.000	0.000	0.000	0.000	0.000

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	566	566	970	2526	0	0	0	0	0
N.S.	1	1.00	1.71	4.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.705	14.793	9.098	0.000	0.000	0.000	0.000	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	638	1226	3159	0	0	0	0	0
N.S.	1	1.00	1.92	4.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.267	13.883	10.007	0.000	0.000	0.000	0.000	0.000

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	322	1202	0	0	0	0	0
N.S.	1	1.00	1.03	3.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.566	10.703	12.187	0.000	0.000	0.000	0.000	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	264	264	296	658	0	0	0	0	0
N.S.	1	1.00	1.12	2.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.391	11.238	10.203	0.000	0.000	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	103	111	0	0	0	0	0
N.S.	1	1.00	0.80	0.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.155	0.813	7.814	0.000	0.000	0.000	0.000	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	146	137	0	0	0	0	0
N.S.	1	1.00	1.07	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.170	1.714	7.100	0.000	0.000	0.000	0.000	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	235	820	0	0	0	0	0
N.S.	1	1.00	0.50	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.971	5.159	7.859	0.000	0.000	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	505	505	678	1689	0	0	0	0	0
N.S.	1	1.00	1.34	3.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.156	12.713	8.534	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	397	397	440	2618	0	0	0	0	0
N.S.	1	1.00	1.11	6.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.969	11.415	12.458	0.000	0.000	0.000	0.000	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	325	325	369	1228	0	0	0	0	0
N.S.	1	1.00	1.14	3.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.672	7.424	11.515	0.000	0.000	0.000	0.000	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	307	307	237	750	0	0	0	0	0
N.S.	1	1.00	0.77	2.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.552	6.229	9.047	0.000	0.000	0.000	0.000	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	306	306	235	783	0	0	0	0	0
N.S.	1	1.00	0.77	2.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.498	2.619	6.090	0.000	0.000	0.000	0.000	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	447	447	893	1036	0	0	0	0	0
N.S.	1	1.00	2.00	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.740	15.624	8.918	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	525	525	1025	2526	0	0	0	0	0
N.S.	1	1.00	1.95	4.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.275	13.206	8.872	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	513	513	546	5849	0	0	0	0	0
N.S.	1	1.00	1.06	11.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.458	13.243	13.994	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	438	438	525	3675	0	0	0	0	0
N.S.	1	1.00	1.20	8.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.059	12.965	11.878	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	421	421	471	2841	0	0	0	0	0
N.S.	1	1.00	1.12	6.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.002	11.201	9.684	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	399	399	455	2614	0	0	0	0	0
N.S.	1	1.00	1.14	6.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.877	10.780	7.944	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	382	382	359	2129	0	0	0	0	0
N.S.	1	1.00	0.94	5.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.855	5.653	8.396	0.000	0.000	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	557	557	1335	4445	0	0	0	0	0
N.S.	1	1.00	2.40	7.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.403	12.430	9.520	0.000	0.000	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	294	294	7214	0	0	0	0	0	0
N.S.	1	1.00	24.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	28.598	0.000	0.000	0.000	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	222	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	0.537	0.000	0.000	0.000	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	159	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	0.217	0.000	0.000	0.000	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	107	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.129	0.000	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	47	65	0	30	0	71	0
N.S.	1	1.00	1.81	2.50	0.00	1.15	0.00	2.73	0.00
time (sec)	N/A	0.091	0.101	2.118	0.000	0.307	0.000	0.343	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	64	63	0	32	0	46	0
N.S.	1	1.00	0.98	0.97	0.00	0.49	0.00	0.71	0.00
time (sec)	N/A	0.114	0.098	1.268	0.000	0.321	0.000	0.336	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	26	45	24	87	30	25
N.S.	1	1.00	0.78	0.70	1.22	0.65	2.35	0.81	0.68
time (sec)	N/A	0.022	0.088	0.795	0.214	0.292	0.097	0.297	14.611

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	34	144	70	333	88	29
N.S.	1	1.00	1.04	1.31	5.54	2.69	12.81	3.38	1.12
time (sec)	N/A	0.034	0.496	3.221	0.222	0.312	0.344	0.324	14.500

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	143	27	114	1370	28
N.S.	1	1.00	1.00	1.04	5.11	0.96	4.07	48.93	1.00
time (sec)	N/A	0.050	0.807	5.983	0.449	0.308	1.694	7.946	14.649

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	30	115	56	80	47	33
N.S.	1	1.00	1.04	1.15	4.42	2.15	3.08	1.81	1.27
time (sec)	N/A	0.041	0.503	0.951	0.224	0.275	0.818	0.303	14.141

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	133	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.383	0.000	0.000	0.000	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	254	0	0	0	0	0	0
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	1.988	0.000	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	77	0	194	233	281	93
N.S.	1	1.00	1.02	1.22	0.00	3.08	3.70	4.46	1.48
time (sec)	N/A	0.119	0.148	0.902	0.000	0.301	17.197	0.305	14.595

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	22	0	50	37
N.S.	1	1.00	1.00	1.05	0.00	1.00	0.00	2.27	1.68
time (sec)	N/A	0.032	0.225	0.846	0.000	0.269	0.000	0.295	14.610

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	31	37	52	43	56	72	74
N.S.	1	1.00	0.66	0.79	1.11	0.91	1.19	1.53	1.57
time (sec)	N/A	0.079	0.084	1.019	0.287	0.293	1.041	0.287	14.308

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	100	299	0	162	0	0	0
N.S.	1	1.00	0.60	1.78	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.187	0.696	8.566	0.000	0.110	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	91	271	0	151	0	0	0
N.S.	1	1.00	0.65	1.95	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.148	0.285	6.955	0.000	0.106	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	75	238	0	139	0	0	0
N.S.	1	1.00	0.69	2.20	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.100	0.153	5.516	0.000	0.104	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	55	161	0	119	0	0	0
N.S.	1	1.00	0.69	2.01	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.094	0.136	4.773	0.000	0.110	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	73	215	0	170	0	0	0
N.S.	1	1.00	0.70	2.05	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.145	0.406	5.150	0.000	0.098	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	85	403	0	189	0	0	0
N.S.	1	1.00	0.62	2.96	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.186	0.413	6.514	0.000	0.108	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	107	576	0	202	0	0	0
N.S.	1	1.00	0.63	3.41	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.228	0.615	8.326	0.000	0.110	0.000	0.000	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	103	301	0	165	0	0	0
N.S.	1	1.00	0.61	1.78	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.261	0.457	8.392	0.000	0.111	0.000	0.000	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	88	273	0	153	0	0	0
N.S.	1	1.00	0.63	1.95	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.181	0.101	6.963	0.000	0.116	0.000	0.000	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	76	240	0	140	0	0	0
N.S.	1	1.00	0.68	2.14	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.157	0.054	5.895	0.000	0.112	0.000	0.000	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	57	163	0	119	0	0	0
N.S.	1	1.00	0.69	1.96	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.167	0.045	5.002	0.000	0.111	0.000	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	73	217	0	171	0	0	0
N.S.	1	1.00	0.66	1.97	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.250	0.461	5.147	0.000	0.122	0.000	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	87	404	0	192	0	0	0
N.S.	1	1.00	0.62	2.87	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.314	0.342	6.386	0.000	0.119	0.000	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	107	577	0	205	0	0	0
N.S.	1	1.00	0.61	3.32	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.237	0.550	8.298	0.000	0.108	0.000	0.000	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	100	301	0	171	0	0	0
N.S.	1	1.00	0.58	1.76	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.138	0.107	9.010	0.000	0.127	0.000	0.000	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	89	273	0	157	0	0	0
N.S.	1	1.00	0.61	1.88	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.152	0.146	8.884	0.000	0.117	0.000	0.000	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	78	240	0	142	0	0	0
N.S.	1	1.00	0.67	2.07	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.134	0.056	15.668	0.000	0.117	0.000	0.000	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	54	163	0	119	0	0	0
N.S.	1	1.00	0.64	1.92	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.117	0.391	53.232	0.000	0.111	0.000	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	73	217	0	173	0	0	0
N.S.	1	1.00	0.65	1.94	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.147	0.371	158.889	0.000	0.106	0.000	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	87	406	0	196	0	0	0
N.S.	1	1.00	0.61	2.84	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.165	0.376	2.429	0.000	0.117	0.000	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	102	579	0	211	0	0	0
N.S.	1	1.00	0.58	3.29	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.192	0.665	3.174	0.000	0.115	0.000	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	101	298	0	165	0	0	0
N.S.	1	1.00	0.58	1.72	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.164	0.667	7.524	0.000	0.120	0.000	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	88	270	0	154	0	0	0
N.S.	1	1.00	0.61	1.88	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.136	0.413	6.289	0.000	0.105	0.000	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	78	237	0	142	0	0	94
N.S.	1	1.00	0.69	2.10	0.00	1.26	0.00	0.00	0.83
time (sec)	N/A	0.116	0.067	5.129	0.000	0.110	0.000	0.000	0.290

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	160	0	122	0	0	48
N.S.	1	1.00	0.66	1.95	0.00	1.49	0.00	0.00	0.59
time (sec)	N/A	0.088	0.037	3.629	0.000	0.094	0.000	0.000	0.347

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	73	214	0	173	0	0	0
N.S.	1	1.00	0.69	2.02	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.141	0.286	5.145	0.000	0.115	0.000	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	84	406	0	192	0	0	0
N.S.	1	1.00	0.62	3.01	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.183	0.239	6.363	0.000	0.099	0.000	0.000	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	101	579	0	205	0	0	0
N.S.	1	1.00	0.60	3.45	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.207	0.409	8.488	0.000	0.105	0.000	0.000	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	104	301	0	165	0	0	0
N.S.	1	1.00	0.59	1.71	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.184	0.626	8.567	0.000	0.123	0.000	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	88	273	0	154	0	0	0
N.S.	1	1.00	0.60	1.86	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.154	0.639	6.761	0.000	0.109	0.000	0.000	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	75	240	0	142	0	0	0
N.S.	1	1.00	0.65	2.07	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.132	0.406	5.583	0.000	0.109	0.000	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	57	163	0	122	0	0	0
N.S.	1	1.00	0.67	1.92	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.101	0.040	4.540	0.000	0.102	0.000	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	76	217	0	173	0	0	0
N.S.	1	1.00	0.68	1.94	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.105	0.141	4.831	0.000	0.105	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	87	406	0	192	0	0	0
N.S.	1	1.00	0.62	2.90	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.155	0.117	6.595	0.000	0.101	0.000	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	104	579	0	205	0	0	0
N.S.	1	1.00	0.61	3.39	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.203	0.239	8.145	0.000	0.118	0.000	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	104	301	0	165	0	0	0
N.S.	1	1.00	0.59	1.71	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.175	0.884	8.294	0.000	0.119	0.000	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	91	273	0	154	0	0	0
N.S.	1	1.00	0.62	1.86	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.134	0.633	7.046	0.000	0.122	0.000	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	78	240	0	142	0	0	0
N.S.	1	1.00	0.67	2.07	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.113	0.465	5.699	0.000	0.107	0.000	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	57	163	0	122	0	0	0
N.S.	1	1.00	0.67	1.92	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.089	0.038	4.594	0.000	0.117	0.000	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	76	217	0	173	0	0	0
N.S.	1	1.00	0.68	1.94	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.118	0.059	4.664	0.000	0.103	0.000	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	87	406	0	192	0	0	0
N.S.	1	1.00	0.61	2.84	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.121	0.097	6.412	0.000	0.109	0.000	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	104	579	0	205	0	0	0
N.S.	1	1.00	0.60	3.35	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.193	0.098	8.424	0.000	0.116	0.000	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	104	579	0	205	0	0	0
N.S.	1	1.00	0.59	3.29	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.160	0.036	8.385	0.000	0.118	0.000	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	81	91	93	252	0	278	105
N.S.	1	1.00	0.47	0.53	0.54	1.47	0.00	1.62	0.61
time (sec)	N/A	0.096	0.404	5.223	0.481	0.363	0.000	3.404	16.238

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	69	74	68	230	202	194	92
N.S.	1	1.00	0.51	0.54	0.50	1.69	1.49	1.43	0.68
time (sec)	N/A	0.071	0.281	4.900	0.417	0.326	81.203	2.583	16.086

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	55	40	204	165	142	79
N.S.	1	1.00	0.58	0.56	0.41	2.08	1.68	1.45	0.81
time (sec)	N/A	0.028	0.224	5.025	0.410	0.343	2.901	1.878	14.954

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	42	39	40	181	80	0	35
N.S.	1	1.00	0.71	0.66	0.68	3.07	1.36	0.00	0.59
time (sec)	N/A	0.014	0.036	4.966	0.368	0.341	1.354	0.000	0.319

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	40	52	92	210	0	0	0
N.S.	1	1.00	0.67	0.87	1.53	3.50	0.00	0.00	0.00
time (sec)	N/A	0.027	0.026	4.677	0.375	0.357	0.000	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	57	120	205	0	0	0
N.S.	1	1.00	0.74	0.84	1.76	3.01	0.00	0.00	0.00
time (sec)	N/A	0.051	0.037	5.058	0.421	0.333	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	103	716	225	0	0	0
N.S.	1	1.00	0.61	0.96	6.69	2.10	0.00	0.00	0.00
time (sec)	N/A	0.064	0.084	5.139	0.617	0.339	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	76	122	957	253	0	0	0
N.S.	1	1.00	0.52	0.84	6.60	1.74	0.00	0.00	0.00
time (sec)	N/A	0.074	0.222	4.929	0.719	0.328	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	81	92	100	261	0	279	106
N.S.	1	1.00	0.46	0.52	0.56	1.47	0.00	1.58	0.60
time (sec)	N/A	0.080	0.365	5.076	0.466	0.351	0.000	3.459	16.238

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	69	75	74	237	0	195	93
N.S.	1	1.00	0.49	0.54	0.53	1.69	0.00	1.39	0.66
time (sec)	N/A	0.063	0.231	5.034	0.447	0.351	0.000	2.590	1.226

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	58	56	43	209	151	0	50
N.S.	1	1.00	0.57	0.55	0.43	2.07	1.50	0.00	0.50
time (sec)	N/A	0.025	0.033	4.899	0.443	0.320	31.582	0.000	0.549

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	42	40	40	184	80	0	36
N.S.	1	1.00	0.69	0.66	0.66	3.02	1.31	0.00	0.59
time (sec)	N/A	0.015	0.048	5.104	0.364	0.341	31.981	0.000	14.774

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	40	53	95	212	0	0	0
N.S.	1	1.00	0.65	0.85	1.53	3.42	0.00	0.00	0.00
time (sec)	N/A	0.029	0.031	4.959	0.439	0.383	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	50	58	123	208	0	0	0
N.S.	1	1.00	0.71	0.83	1.76	2.97	0.00	0.00	0.00
time (sec)	N/A	0.051	0.043	4.567	0.547	0.358	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	65	104	747	232	0	0	0
N.S.	1	1.00	0.59	0.95	6.79	2.11	0.00	0.00	0.00
time (sec)	N/A	0.073	0.079	4.993	0.548	0.371	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	77	123	992	260	0	0	0
N.S.	1	1.00	0.52	0.83	6.66	1.74	0.00	0.00	0.00
time (sec)	N/A	0.070	0.013	5.112	0.538	0.346	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	81	94	110	279	0	278	108
N.S.	1	1.00	0.43	0.50	0.59	1.49	0.00	1.49	0.58
time (sec)	N/A	0.079	0.396	5.095	0.510	0.357	0.000	3.702	15.839

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	69	77	82	251	0	0	64
N.S.	1	1.00	0.47	0.52	0.55	1.70	0.00	0.00	0.43
time (sec)	N/A	0.069	0.994	5.217	0.484	0.347	0.000	0.000	0.763

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	57	58	47	219	0	0	52
N.S.	1	1.00	0.53	0.54	0.44	2.05	0.00	0.00	0.49
time (sec)	N/A	0.030	0.268	5.108	0.471	0.343	0.000	0.000	14.910

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	42	40	190	0	0	38
N.S.	1	1.00	0.65	0.65	0.62	2.92	0.00	0.00	0.58
time (sec)	N/A	0.016	0.059	4.914	0.445	0.318	0.000	0.000	0.373

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	40	55	99	216	0	0	0
N.S.	1	1.00	0.61	0.83	1.50	3.27	0.00	0.00	0.00
time (sec)	N/A	0.030	0.049	4.920	0.402	0.360	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	50	60	127	214	0	0	0
N.S.	1	1.00	0.68	0.81	1.72	2.89	0.00	0.00	0.00
time (sec)	N/A	0.051	0.060	4.875	0.488	0.328	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	65	106	803	242	0	0	0
N.S.	1	1.00	0.56	0.91	6.92	2.09	0.00	0.00	0.00
time (sec)	N/A	0.064	0.109	4.719	0.440	0.340	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	76	125	1060	274	0	0	0
N.S.	1	1.00	0.48	0.80	6.75	1.75	0.00	0.00	0.00
time (sec)	N/A	0.073	0.222	5.031	0.443	0.367	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	69	74	68	236	0	0	95
N.S.	1	1.00	0.51	0.54	0.50	1.74	0.00	0.00	0.70
time (sec)	N/A	0.060	0.418	5.041	0.430	0.348	0.000	0.000	16.110

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	55	40	210	151	0	82
N.S.	1	1.00	0.58	0.56	0.41	2.14	1.54	0.00	0.84
time (sec)	N/A	0.025	0.350	4.762	0.415	0.313	36.742	0.000	0.902

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	42	39	40	187	80	0	61
N.S.	1	1.00	0.71	0.66	0.68	3.17	1.36	0.00	1.03
time (sec)	N/A	0.016	0.033	5.020	0.395	0.338	1.431	0.000	0.593

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	40	52	92	215	0	0	0
N.S.	1	1.00	0.67	0.87	1.53	3.58	0.00	0.00	0.00
time (sec)	N/A	0.031	0.027	4.914	0.413	0.368	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	57	125	211	0	0	0
N.S.	1	1.00	0.74	0.84	1.84	3.10	0.00	0.00	0.00
time (sec)	N/A	0.045	0.034	5.027	0.419	0.331	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	103	722	231	0	0	0
N.S.	1	1.00	0.61	0.96	6.75	2.16	0.00	0.00	0.00
time (sec)	N/A	0.063	0.060	5.033	0.421	0.344	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	76	122	957	259	0	0	0
N.S.	1	1.00	0.52	0.84	6.60	1.79	0.00	0.00	0.00
time (sec)	N/A	0.077	0.113	5.042	0.424	0.343	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	69	77	68	236	0	0	95
N.S.	1	1.00	0.47	0.52	0.46	1.59	0.00	0.00	0.64
time (sec)	N/A	0.061	0.505	5.014	0.417	0.368	0.000	0.000	15.809

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	57	58	40	210	0	0	82
N.S.	1	1.00	0.53	0.54	0.37	1.96	0.00	0.00	0.77
time (sec)	N/A	0.027	0.346	4.952	0.415	0.335	0.000	0.000	0.829

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	42	40	187	80	0	61
N.S.	1	1.00	0.65	0.65	0.62	2.88	1.23	0.00	0.94
time (sec)	N/A	0.015	0.043	4.902	0.371	0.346	36.756	0.000	0.510

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	40	55	92	215	0	0	0
N.S.	1	1.00	0.61	0.83	1.39	3.26	0.00	0.00	0.00
time (sec)	N/A	0.030	0.032	5.212	0.384	0.377	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	50	60	133	211	0	0	0
N.S.	1	1.00	0.68	0.81	1.80	2.85	0.00	0.00	0.00
time (sec)	N/A	0.046	0.042	5.048	0.409	0.333	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	65	106	739	231	0	0	0
N.S.	1	1.00	0.56	0.91	6.37	1.99	0.00	0.00	0.00
time (sec)	N/A	0.070	0.060	4.837	0.443	0.349	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	76	125	983	259	0	0	0
N.S.	1	1.00	0.48	0.80	6.26	1.65	0.00	0.00	0.00
time (sec)	N/A	0.076	0.118	5.118	0.439	0.330	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	72	77	68	236	0	0	95
N.S.	1	1.00	0.49	0.52	0.46	1.59	0.00	0.00	0.64
time (sec)	N/A	0.067	0.775	5.200	0.422	0.369	0.000	0.000	16.106

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	60	58	40	210	0	0	82
N.S.	1	1.00	0.56	0.54	0.37	1.96	0.00	0.00	0.77
time (sec)	N/A	0.028	0.463	4.760	0.411	0.370	0.000	0.000	0.742

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	42	40	187	0	0	61
N.S.	1	1.00	0.69	0.65	0.62	2.88	0.00	0.00	0.94
time (sec)	N/A	0.016	0.040	4.957	0.362	0.325	0.000	0.000	0.521

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	43	55	92	215	0	0	0
N.S.	1	1.00	0.65	0.83	1.39	3.26	0.00	0.00	0.00
time (sec)	N/A	0.034	0.034	4.964	0.386	0.396	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. The column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [503] had the largest ratio of [.5000000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	4	1.00	19	0.210
2	A	6	4	1.00	19	0.210
3	A	6	4	1.00	19	0.210
4	A	5	4	1.00	19	0.210
5	A	1	1	1.00	17	0.059
6	A	2	1	1.00	10	0.100
7	A	2	2	1.00	17	0.118
8	A	4	4	1.00	19	0.210
9	A	5	5	1.00	19	0.263
10	A	5	4	1.00	19	0.210
11	A	6	4	1.00	19	0.210
12	A	6	4	1.00	19	0.210
13	A	11	4	1.00	21	0.190
14	A	9	4	1.00	21	0.190
15	A	9	4	1.00	21	0.190
16	A	2	2	1.21	19	0.105
17	A	1	1	1.00	12	0.083
18	A	3	3	1.00	19	0.158
19	A	5	4	1.00	21	0.190
20	A	7	5	1.00	21	0.238
21	A	8	5	1.00	21	0.238
22	A	9	4	1.00	21	0.190
23	A	13	4	1.00	21	0.190
24	A	11	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	8	6	1.04	19	0.316
26	A	7	5	1.00	12	0.417
27	A	6	5	1.00	19	0.263
28	A	6	5	1.00	21	0.238
29	A	7	5	1.00	21	0.238
30	A	9	5	1.00	21	0.238
31	A	11	5	1.00	21	0.238
32	A	11	4	1.00	21	0.190
33	A	15	4	1.00	21	0.190
34	A	11	6	1.12	19	0.316
35	A	10	5	1.00	12	0.417
36	A	8	6	1.00	19	0.316
37	A	8	6	1.00	21	0.286
38	A	8	6	1.00	21	0.286
39	A	9	5	1.00	21	0.238
40	A	12	5	1.00	21	0.238
41	A	13	5	1.00	21	0.238
42	A	15	4	1.00	21	0.190
43	A	7	5	1.00	21	0.238
44	A	6	5	1.00	21	0.238
45	A	2	2	1.00	21	0.095
46	A	4	4	1.00	21	0.190
47	A	2	2	1.00	19	0.105
48	A	1	1	1.00	12	0.083
49	A	3	3	1.00	19	0.158
50	A	5	5	1.00	21	0.238
51	A	6	6	1.00	21	0.286
52	A	6	5	1.00	21	0.238
53	A	7	6	1.00	21	0.286
54	A	3	3	1.00	21	0.143
55	A	6	6	1.00	21	0.286
56	A	3	3	1.00	21	0.143
57	A	2	2	1.00	19	0.105
58	A	2	2	1.00	12	0.167
59	A	4	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	6	6	1.00	21	0.286
61	A	7	7	1.00	21	0.333
62	A	7	6	1.00	21	0.286
63	A	4	3	1.00	21	0.143
64	A	7	7	1.00	21	0.333
65	A	5	5	1.00	21	0.238
66	A	3	3	1.00	21	0.143
67	A	3	3	1.00	19	0.158
68	A	3	2	1.00	12	0.167
69	A	5	4	1.00	19	0.210
70	A	7	6	1.00	21	0.286
71	A	8	7	1.00	21	0.333
72	A	5	3	1.00	21	0.143
73	A	8	7	1.00	21	0.333
74	A	6	6	1.00	21	0.286
75	A	5	5	1.00	21	0.238
76	A	4	4	1.00	21	0.190
77	A	4	3	1.00	19	0.158
78	A	4	2	1.00	12	0.167
79	A	6	4	1.00	19	0.210
80	A	8	6	1.00	21	0.286
81	A	9	7	1.00	21	0.333
82	A	6	3	1.00	21	0.143
83	A	9	7	1.00	21	0.333
84	A	7	6	1.00	21	0.286
85	A	6	6	1.00	21	0.286
86	A	6	6	1.00	21	0.286
87	A	5	4	1.00	21	0.190
88	A	5	3	1.00	19	0.158
89	A	5	2	1.00	12	0.167
90	A	7	4	1.00	19	0.210
91	A	9	6	1.00	21	0.286
92	A	10	7	1.00	21	0.333
93	A	7	6	1.00	21	0.286
94	A	7	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	5	4	1.00	23	0.174
96	A	4	4	1.00	23	0.174
97	A	3	3	1.00	23	0.130
98	A	2	2	1.00	21	0.095
99	A	1	1	1.00	14	0.071
100	A	2	2	1.00	21	0.095
101	A	3	3	1.00	23	0.130
102	A	4	3	1.00	23	0.130
103	A	5	3	1.00	23	0.130
104	A	6	6	1.00	23	0.261
105	A	4	4	1.00	23	0.174
106	A	3	3	1.00	21	0.143
107	A	2	2	1.00	14	0.143
108	A	4	4	1.00	21	0.190
109	A	4	4	1.00	23	0.174
110	A	5	5	1.00	23	0.217
111	A	6	5	1.00	23	0.217
112	A	6	6	1.00	23	0.261
113	A	5	4	1.00	23	0.174
114	A	4	3	1.00	21	0.143
115	A	3	2	1.00	14	0.143
116	A	4	4	1.00	21	0.190
117	A	4	4	1.00	23	0.174
118	A	4	4	1.00	23	0.174
119	A	5	5	1.00	23	0.217
120	A	6	5	1.00	23	0.217
121	A	4	2	1.00	14	0.143
122	A	7	7	1.00	23	0.304
123	A	6	6	1.00	23	0.261
124	A	4	4	1.00	23	0.174
125	A	3	3	1.00	21	0.143
126	A	2	2	1.00	14	0.143
127	A	5	4	1.00	21	0.190
128	A	6	5	1.00	23	0.217
129	A	7	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	8	6	1.00	23	0.261
131	A	7	7	1.00	23	0.304
132	A	6	6	1.00	23	0.261
133	A	4	4	1.00	23	0.174
134	A	3	3	1.00	21	0.143
135	A	3	3	1.00	14	0.214
136	A	6	5	1.00	21	0.238
137	A	7	6	1.00	23	0.261
138	A	8	6	1.00	23	0.261
139	A	7	7	1.00	23	0.304
140	A	6	6	1.00	23	0.261
141	A	4	4	1.00	23	0.174
142	A	4	4	1.00	21	0.190
143	A	4	3	1.00	14	0.214
144	A	7	6	1.00	21	0.286
145	A	8	7	1.00	23	0.304
146	A	6	4	1.00	21	0.190
147	A	5	4	1.00	21	0.190
148	A	4	4	1.00	21	0.190
149	A	3	3	1.00	21	0.143
150	A	4	4	1.00	21	0.190
151	A	5	4	1.00	21	0.190
152	A	6	4	1.00	21	0.190
153	A	10	4	1.00	23	0.174
154	A	9	4	1.00	23	0.174
155	A	7	4	1.00	23	0.174
156	A	6	4	1.00	23	0.174
157	A	6	4	1.00	23	0.174
158	A	7	4	1.00	23	0.174
159	A	9	4	1.00	23	0.174
160	A	12	4	1.00	23	0.174
161	A	10	4	1.00	23	0.174
162	A	8	4	1.00	23	0.174
163	A	8	5	1.00	23	0.217
164	A	8	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	10	4	1.00	23	0.174
166	A	12	4	1.00	23	0.174
167	A	16	4	1.00	23	0.174
168	A	13	4	1.00	23	0.174
169	A	11	4	1.00	23	0.174
170	A	10	5	1.00	23	0.217
171	A	10	5	1.00	23	0.217
172	A	11	4	1.00	23	0.174
173	A	13	4	1.00	23	0.174
174	A	6	5	1.00	23	0.217
175	A	5	5	1.00	23	0.217
176	A	4	4	1.00	23	0.174
177	A	4	4	1.00	23	0.174
178	A	4	4	1.00	23	0.174
179	A	5	5	1.00	23	0.217
180	A	6	5	1.00	23	0.217
181	A	7	6	1.00	23	0.261
182	A	6	6	1.00	23	0.261
183	A	5	5	1.00	23	0.217
184	A	5	5	1.00	23	0.217
185	A	3	3	1.00	23	0.130
186	A	5	5	1.00	23	0.217
187	A	6	6	1.00	23	0.261
188	A	7	6	1.00	23	0.261
189	A	8	6	1.00	23	0.261
190	A	7	6	1.00	23	0.261
191	A	6	5	1.00	23	0.217
192	A	6	6	1.00	23	0.261
193	A	6	5	1.00	23	0.217
194	A	6	5	1.00	23	0.217
195	A	6	5	1.00	23	0.217
196	A	7	6	1.00	23	0.261
197	A	8	6	1.00	23	0.261
198	A	5	3	1.00	25	0.120
199	A	4	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	3	3	1.00	25	0.120
201	A	2	2	1.00	25	0.080
202	A	1	1	1.00	25	0.040
203	A	2	2	1.00	25	0.080
204	A	3	2	1.00	25	0.080
205	A	4	2	1.00	25	0.080
206	A	6	5	1.00	25	0.200
207	A	5	5	1.00	25	0.200
208	A	4	4	1.00	25	0.160
209	A	4	4	1.00	25	0.160
210	A	3	3	1.00	25	0.120
211	A	4	4	1.00	25	0.160
212	A	5	4	1.00	25	0.160
213	A	6	5	1.00	25	0.200
214	A	5	5	1.00	25	0.200
215	A	4	4	1.00	25	0.160
216	A	4	4	1.00	25	0.160
217	A	4	4	1.00	25	0.160
218	A	3	3	1.00	25	0.120
219	A	4	4	1.00	25	0.160
220	A	5	4	1.00	25	0.160
221	A	2	2	1.00	25	0.080
222	A	2	2	1.00	25	0.080
223	A	2	2	1.00	28	0.071
224	A	7	7	1.00	25	0.280
225	A	6	6	1.00	25	0.240
226	A	5	5	1.00	25	0.200
227	A	2	2	1.00	25	0.080
228	A	4	4	1.00	25	0.160
229	A	5	5	1.00	25	0.200
230	A	6	5	1.00	25	0.200
231	A	7	6	1.00	23	0.261
232	A	6	5	1.00	23	0.217
233	A	5	4	1.00	23	0.174
234	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	3	3	1.00	23	0.130
236	A	5	5	1.00	23	0.217
237	A	6	5	1.00	23	0.217
238	A	7	7	1.00	25	0.280
239	A	6	6	1.00	25	0.240
240	A	4	4	1.00	25	0.160
241	A	4	4	1.00	25	0.160
242	A	5	5	1.00	25	0.200
243	A	6	5	1.00	25	0.200
244	A	8	8	1.00	25	0.320
245	A	7	7	1.00	25	0.280
246	A	5	5	1.00	25	0.200
247	A	5	5	1.00	25	0.200
248	A	5	5	1.00	25	0.200
249	A	6	6	1.00	25	0.240
250	A	7	6	1.00	25	0.240
251	A	9	8	1.00	25	0.320
252	A	8	7	1.00	25	0.280
253	A	6	6	1.00	25	0.240
254	A	6	5	1.00	25	0.200
255	A	6	5	1.00	25	0.200
256	A	6	5	1.00	25	0.200
257	A	7	6	1.00	25	0.240
258	A	8	6	1.00	25	0.240
259	A	7	6	1.00	25	0.240
260	A	7	6	1.00	25	0.240
261	A	2	2	1.00	15	0.133
262	A	2	2	1.00	17	0.118
263	A	4	3	1.00	26	0.115
264	A	3	3	1.00	26	0.115
265	A	2	2	1.00	26	0.077
266	A	1	1	1.00	26	0.038
267	A	2	2	1.00	26	0.077
268	A	3	2	1.00	26	0.077
269	A	4	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	3	3	1.00	25	0.120
271	A	2	2	1.00	25	0.080
272	A	1	1	1.00	25	0.040
273	A	2	2	1.00	25	0.080
274	A	3	2	1.00	25	0.080
275	A	7	7	1.00	26	0.269
276	A	6	6	1.00	26	0.231
277	A	5	5	1.00	26	0.192
278	A	2	2	1.00	26	0.077
279	A	4	4	1.00	26	0.154
280	A	5	5	1.00	26	0.192
281	A	6	5	1.00	26	0.192
282	A	7	7	1.00	25	0.280
283	A	6	6	1.00	25	0.240
284	A	5	5	1.00	25	0.200
285	A	2	2	1.00	25	0.080
286	A	3	3	1.00	25	0.120
287	A	5	5	1.00	25	0.200
288	A	3	3	1.00	25	0.120
289	A	3	3	1.00	25	0.120
290	A	3	3	1.00	25	0.120
291	A	9	6	1.00	21	0.286
292	A	8	6	1.00	21	0.286
293	A	7	6	1.00	21	0.286
294	A	6	5	1.00	21	0.238
295	A	7	6	1.00	21	0.286
296	A	8	6	1.00	21	0.286
297	A	9	6	1.00	21	0.286
298	A	9	7	1.00	23	0.304
299	A	8	7	1.00	23	0.304
300	A	5	5	1.00	23	0.217
301	A	7	6	1.00	23	0.261
302	A	8	7	1.00	23	0.304
303	A	9	7	1.00	23	0.304
304	A	17	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	15	6	1.00	23	0.261
306	A	13	6	1.00	23	0.261
307	A	13	7	1.00	23	0.304
308	A	13	6	1.00	23	0.261
309	A	15	6	1.00	23	0.261
310	A	17	6	1.00	23	0.261
311	A	19	6	1.00	23	0.261
312	A	17	6	1.00	23	0.261
313	A	16	7	1.00	23	0.304
314	A	16	7	1.00	23	0.304
315	A	17	6	1.00	23	0.261
316	A	19	6	1.00	23	0.261
317	A	9	7	1.00	23	0.304
318	A	8	7	1.00	23	0.304
319	A	7	6	1.00	23	0.261
320	A	7	6	1.00	23	0.261
321	A	7	6	1.00	23	0.261
322	A	8	7	1.00	23	0.304
323	A	9	7	1.00	23	0.304
324	A	10	8	1.00	23	0.348
325	A	9	8	1.00	23	0.348
326	A	8	7	1.00	23	0.304
327	A	5	5	1.00	23	0.217
328	A	8	7	1.00	23	0.304
329	A	8	7	1.00	23	0.304
330	A	9	8	1.00	23	0.348
331	A	10	8	1.00	23	0.348
332	A	10	8	1.00	23	0.348
333	A	9	7	1.00	23	0.304
334	A	9	8	1.00	23	0.348
335	A	9	8	1.00	23	0.348
336	A	9	8	1.00	23	0.348
337	A	9	7	1.00	23	0.304
338	A	10	8	1.00	23	0.348
339	A	5	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	4	3	1.00	25	0.120
341	A	3	3	1.00	25	0.120
342	A	2	2	1.00	25	0.080
343	A	3	3	1.00	25	0.120
344	A	4	4	1.00	25	0.160
345	A	5	4	1.00	25	0.160
346	A	6	5	1.00	25	0.200
347	A	5	5	1.00	25	0.200
348	A	4	4	1.00	25	0.160
349	A	5	5	1.00	25	0.200
350	A	5	5	1.00	25	0.200
351	A	6	6	1.00	25	0.240
352	A	7	6	1.00	25	0.240
353	A	6	5	1.00	25	0.200
354	A	5	5	1.00	25	0.200
355	A	4	4	1.00	25	0.160
356	A	5	5	1.00	25	0.200
357	A	5	5	1.00	25	0.200
358	A	5	5	1.00	25	0.200
359	A	6	6	1.00	25	0.240
360	A	7	6	1.00	25	0.240
361	A	7	6	1.00	23	0.261
362	A	6	6	1.00	23	0.261
363	A	4	4	1.00	23	0.174
364	A	3	3	1.00	23	0.130
365	A	6	5	1.00	23	0.217
366	A	7	6	1.00	23	0.261
367	A	7	6	1.00	25	0.240
368	A	6	6	1.00	25	0.240
369	A	5	5	1.00	25	0.200
370	A	3	3	1.36	25	0.120
371	A	6	6	1.29	25	0.240
372	A	7	7	1.00	25	0.280
373	A	7	6	1.00	25	0.240
374	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	5	5	1.00	25	0.200
376	A	5	5	1.00	25	0.200
377	A	7	7	1.00	25	0.280
378	A	8	8	1.00	25	0.320
379	A	8	7	1.00	25	0.280
380	A	7	7	1.00	25	0.280
381	A	6	6	1.00	25	0.240
382	A	6	6	1.00	25	0.240
383	A	6	6	1.00	25	0.240
384	A	8	8	1.00	25	0.320
385	A	9	9	1.00	25	0.360
386	A	9	7	1.00	25	0.280
387	A	8	7	1.00	25	0.280
388	A	7	6	1.00	25	0.240
389	A	7	6	1.00	25	0.240
390	A	7	6	1.00	25	0.240
391	A	7	7	1.00	25	0.280
392	A	9	8	1.00	25	0.320
393	A	10	9	1.00	25	0.360
394	A	8	7	1.00	25	0.280
395	A	8	7	1.00	25	0.280
396	A	3	3	1.00	25	0.120
397	A	7	6	1.00	21	0.286
398	A	6	5	1.00	21	0.238
399	A	4	3	1.00	21	0.143
400	A	3	2	1.00	19	0.105
401	A	4	3	1.00	21	0.143
402	A	5	4	1.00	21	0.190
403	A	8	4	1.00	19	0.210
404	A	7	4	1.00	19	0.210
405	A	7	4	1.00	19	0.210
406	A	6	4	1.00	19	0.210
407	A	6	4	1.00	19	0.210
408	A	5	4	1.00	19	0.210
409	A	1	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	2	1	1.00	10	0.100
411	A	2	2	1.00	17	0.118
412	A	4	4	1.00	19	0.210
413	A	5	5	1.00	19	0.263
414	A	5	4	1.00	19	0.210
415	A	6	4	1.00	19	0.210
416	A	6	4	1.00	19	0.210
417	A	7	5	1.00	21	0.238
418	A	7	5	1.00	21	0.238
419	A	6	5	1.00	21	0.238
420	A	2	2	1.00	19	0.105
421	A	1	1	1.00	12	0.083
422	A	3	3	1.00	19	0.158
423	A	4	4	1.00	21	0.190
424	A	5	5	1.00	21	0.238
425	A	6	6	1.00	21	0.286
426	A	6	5	1.00	21	0.238
427	A	7	5	1.00	21	0.238
428	A	8	6	1.14	21	0.286
429	A	4	3	1.00	21	0.143
430	A	3	2	1.00	19	0.105
431	A	2	2	1.18	12	0.167
432	A	4	4	1.00	19	0.210
433	A	4	4	1.00	21	0.190
434	A	4	4	1.00	21	0.190
435	A	6	6	1.00	21	0.286
436	A	7	7	1.00	21	0.333
437	A	7	6	1.00	21	0.286
438	A	9	7	1.00	21	0.333
439	A	5	3	1.00	21	0.143
440	A	4	2	1.00	19	0.105
441	A	3	3	1.00	12	0.250
442	A	5	5	1.00	19	0.263
443	A	5	5	1.00	21	0.238
444	A	5	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	5	5	1.00	21	0.238
446	A	7	7	1.00	21	0.333
447	A	8	8	1.00	21	0.381
448	A	8	7	1.00	21	0.333
449	A	7	6	1.00	21	0.286
450	A	6	6	1.00	21	0.286
451	A	5	5	1.00	21	0.238
452	A	5	5	1.00	21	0.238
453	A	3	3	1.00	19	0.158
454	A	2	2	1.00	12	0.167
455	A	4	4	1.00	19	0.210
456	A	6	6	1.00	21	0.286
457	A	6	6	1.00	21	0.286
458	A	7	6	1.00	21	0.286
459	A	7	6	1.00	21	0.286
460	A	6	6	1.28	21	0.286
461	A	5	5	1.00	21	0.238
462	A	4	4	1.00	21	0.190
463	A	4	4	1.00	19	0.210
464	A	4	4	1.00	12	0.333
465	A	5	5	1.00	19	0.263
466	A	6	6	1.00	21	0.286
467	A	7	6	1.00	21	0.286
468	A	8	6	1.00	21	0.286
469	A	7	7	1.00	21	0.333
470	A	6	6	1.00	21	0.286
471	A	5	5	1.00	21	0.238
472	A	5	5	1.00	21	0.238
473	A	5	4	1.00	19	0.210
474	A	5	5	1.00	12	0.417
475	A	6	6	1.00	19	0.316
476	A	7	6	1.00	21	0.286
477	A	8	6	1.00	21	0.286
478	A	7	7	1.00	21	0.333
479	A	6	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
480	A	6	6	1.00	21	0.286
481	A	6	5	1.00	21	0.238
482	A	6	4	1.00	19	0.210
483	A	6	5	1.00	12	0.417
484	A	7	6	1.00	19	0.316
485	A	8	6	1.00	21	0.286
486	A	8	8	1.00	23	0.348
487	A	7	7	1.00	23	0.304
488	A	6	6	1.00	21	0.286
489	A	2	2	1.00	14	0.143
490	A	5	5	1.00	21	0.238
491	A	9	9	1.00	23	0.391
492	A	10	10	1.00	23	0.435
493	A	9	8	1.00	23	0.348
494	A	8	7	1.00	23	0.304
495	A	7	6	1.00	21	0.286
496	A	6	6	1.00	14	0.429
497	A	8	8	1.00	21	0.381
498	A	9	9	1.00	23	0.391
499	A	10	10	1.00	23	0.435
500	A	10	8	1.00	23	0.348
501	A	9	7	1.00	23	0.304
502	A	8	6	1.00	21	0.286
503	A	7	7	1.00	14	0.500
504	A	9	9	1.00	21	0.429
505	A	9	9	1.00	23	0.391
506	A	10	10	1.00	23	0.435
507	A	11	10	1.00	23	0.435
508	A	8	7	1.00	14	0.500
509	A	6	6	1.00	23	0.261
510	A	5	5	1.00	23	0.217
511	A	4	4	1.00	21	0.190
512	A	1	1	1.00	14	0.071
513	A	3	3	1.00	21	0.143
514	A	6	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
515	A	7	7	1.00	23	0.304
516	A	6	6	1.00	23	0.261
517	A	5	5	1.00	23	0.217
518	A	4	4	1.00	21	0.190
519	A	1	1	1.00	14	0.071
520	A	3	3	1.00	21	0.143
521	A	6	6	1.00	23	0.261
522	A	7	7	1.00	23	0.304
523	A	7	7	1.00	23	0.304
524	A	6	6	1.00	23	0.261
525	A	5	5	1.00	21	0.238
526	A	2	2	1.00	14	0.143
527	A	2	2	1.00	21	0.095
528	A	9	9	1.00	23	0.391
529	A	10	10	1.00	23	0.435
530	A	8	8	1.00	23	0.348
531	A	7	7	1.00	23	0.304
532	A	6	6	1.00	23	0.261
533	A	6	6	1.00	21	0.286
534	A	4	4	1.00	14	0.286
535	A	7	7	1.00	21	0.333
536	A	10	10	1.00	23	0.435
537	A	11	10	1.00	23	0.435
538	A	9	9	1.00	23	0.391
539	A	8	8	1.00	23	0.348
540	A	7	7	1.00	23	0.304
541	A	7	7	1.00	23	0.304
542	A	7	6	1.00	21	0.286
543	A	7	7	1.00	14	0.500
544	A	10	10	1.00	21	0.476
545	A	11	11	1.00	23	0.478
546	A	8	7	1.00	14	0.500
547	A	5	5	1.00	23	0.217
548	A	4	4	1.00	23	0.174
549	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
550	A	1	1	1.00	14	0.071
551	A	1	1	1.00	21	0.048
552	A	6	6	1.00	23	0.261
553	A	7	7	1.00	23	0.304
554	A	5	5	1.00	23	0.217
555	A	4	4	1.00	23	0.174
556	A	3	3	1.00	21	0.143
557	A	1	1	1.00	14	0.071
558	A	1	1	1.00	21	0.048
559	A	6	6	1.00	23	0.261
560	A	7	7	1.00	23	0.304
561	A	6	4	1.00	21	0.190
562	A	5	4	1.00	21	0.190
563	A	4	4	1.00	21	0.190
564	A	3	3	1.00	21	0.143
565	A	4	4	1.00	21	0.190
566	A	5	4	1.00	21	0.190
567	A	6	4	1.00	21	0.190
568	A	7	5	1.00	23	0.217
569	A	6	5	1.00	23	0.217
570	A	5	5	1.00	23	0.217
571	A	4	4	1.00	23	0.174
572	A	4	4	1.00	23	0.174
573	A	5	5	1.00	23	0.217
574	A	6	5	1.00	23	0.217
575	A	7	6	1.00	23	0.261
576	A	6	6	1.00	23	0.261
577	A	5	5	1.00	23	0.217
578	A	5	5	1.00	23	0.217
579	A	5	5	1.00	23	0.217
580	A	6	6	1.00	23	0.261
581	A	7	6	1.00	23	0.261
582	A	6	6	1.00	23	0.261
583	A	5	5	1.00	23	0.217
584	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	1	1	1.00	23	0.043
586	A	5	5	1.00	23	0.217
587	A	7	7	1.00	23	0.304
588	A	7	7	1.00	23	0.304
589	A	6	6	1.00	23	0.261
590	A	6	6	1.00	23	0.261
591	A	6	6	1.00	23	0.261
592	A	6	6	1.00	23	0.261
593	A	7	7	1.00	23	0.304
594	A	8	7	1.00	23	0.304
595	A	8	8	1.00	23	0.348
596	A	7	7	1.00	23	0.304
597	A	7	7	1.00	23	0.304
598	A	7	7	1.00	23	0.304
599	A	7	7	1.00	23	0.304
600	A	7	7	1.00	23	0.304
601	A	8	7	1.00	23	0.304
602	A	9	7	1.00	23	0.304
603	A	7	7	1.00	25	0.280
604	A	7	7	1.00	25	0.280
605	A	1	1	1.00	25	0.040
606	A	3	3	1.00	25	0.120
607	A	4	4	1.00	25	0.160
608	A	5	5	1.00	25	0.200
609	A	6	5	1.00	25	0.200
610	A	8	8	1.00	25	0.320
611	A	8	8	1.00	25	0.320
612	A	6	6	1.00	25	0.240
613	A	5	5	1.00	25	0.200
614	A	4	4	1.00	25	0.160
615	A	5	5	1.00	25	0.200
616	A	6	5	1.00	25	0.200
617	A	7	5	1.00	25	0.200
618	A	8	8	1.00	25	0.320
619	A	7	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
620	A	7	7	1.00	25	0.280
621	A	6	6	1.00	25	0.240
622	A	5	5	1.00	25	0.200
623	A	6	5	1.00	25	0.200
624	A	7	5	1.00	25	0.200
625	A	8	5	1.00	25	0.200
626	A	8	8	1.09	25	0.320
627	A	1	1	1.00	25	0.040
628	A	1	1	1.00	25	0.040
629	A	3	3	1.00	25	0.120
630	A	4	4	1.00	25	0.160
631	A	7	7	1.00	25	0.280
632	A	6	6	1.00	25	0.240
633	A	4	4	1.00	25	0.160
634	A	4	4	1.00	25	0.160
635	A	4	4	1.00	25	0.160
636	A	5	5	1.00	25	0.200
637	A	6	5	1.00	25	0.200
638	A	7	7	1.00	25	0.280
639	A	5	5	1.00	25	0.200
640	A	5	5	1.00	25	0.200
641	A	5	5	1.00	25	0.200
642	A	5	5	1.00	25	0.200
643	A	6	5	1.00	25	0.200
644	A	1	1	1.00	25	0.040
645	A	1	1	1.00	25	0.040
646	A	2	2	1.00	25	0.080
647	A	2	2	1.00	25	0.080
648	A	1	1	1.00	25	0.040
649	A	1	1	1.00	25	0.040
650	A	2	2	1.00	25	0.080
651	A	2	2	1.00	25	0.080
652	A	2	2	1.00	27	0.074
653	A	2	2	1.00	27	0.074
654	A	1	1	1.00	27	0.037

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
655	A	1	1	1.00	27	0.037
656	A	2	2	1.00	27	0.074
657	A	2	2	1.00	27	0.074
658	A	1	1	1.00	27	0.037
659	A	1	1	1.00	27	0.037
660	A	1	1	1.00	25	0.040
661	A	1	1	1.00	25	0.040
662	A	2	2	1.00	25	0.080
663	A	2	2	1.00	25	0.080
664	A	1	1	1.00	25	0.040
665	A	1	1	1.00	25	0.040
666	A	2	2	1.00	25	0.080
667	A	2	2	1.00	25	0.080
668	A	2	2	1.00	27	0.074
669	A	2	2	1.00	27	0.074
670	A	1	1	1.00	27	0.037
671	A	1	1	1.00	27	0.037
672	A	2	2	1.00	27	0.074
673	A	2	2	1.00	27	0.074
674	A	1	1	1.00	27	0.037
675	A	1	1	1.00	27	0.037
676	A	5	3	1.00	23	0.130
677	A	5	3	1.00	23	0.130
678	A	5	3	1.00	23	0.130
679	A	5	3	1.00	23	0.130
680	N/A	0	0	1.00	25	0.000
681	N/A	0	0	1.00	25	0.000
682	N/A	0	0	1.00	25	0.000
683	N/A	0	0	1.00	25	0.000
684	N/A	0	0	1.00	25	0.000
685	N/A	0	0	1.00	25	0.000
686	N/A	0	0	1.00	25	0.000
687	N/A	0	0	1.00	25	0.000
688	N/A	0	0	1.00	25	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
689	N/A	0	0	1.00	25	0.000
690	A	9	6	1.00	21	0.286
691	A	8	6	1.00	21	0.286
692	A	7	6	1.00	21	0.286
693	A	6	5	1.00	21	0.238
694	A	7	6	1.00	21	0.286
695	A	8	6	1.00	21	0.286
696	A	9	6	1.00	21	0.286
697	A	10	7	1.00	23	0.304
698	A	9	7	1.00	23	0.304
699	A	8	7	1.00	23	0.304
700	A	7	6	1.00	23	0.261
701	A	7	6	1.00	23	0.261
702	A	8	7	1.00	23	0.304
703	A	9	7	1.00	23	0.304
704	A	10	7	1.00	23	0.304
705	A	10	8	1.00	23	0.348
706	A	9	8	1.00	23	0.348
707	A	8	7	1.00	23	0.304
708	A	8	7	1.00	23	0.304
709	A	8	7	1.00	23	0.304
710	A	9	8	1.00	23	0.348
711	A	10	8	1.00	23	0.348
712	A	11	10	1.00	23	0.435
713	A	7	7	1.00	23	0.304
714	A	3	3	1.00	23	0.130
715	A	5	5	1.00	23	0.217
716	A	9	8	1.00	23	0.348
717	A	10	9	1.00	23	0.391
718	A	12	10	1.00	23	0.435
719	A	11	10	1.00	23	0.435
720	A	10	9	1.00	23	0.391
721	A	10	9	1.00	23	0.391
722	A	10	9	1.00	23	0.391
723	A	10	9	1.00	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
724	A	13	11	1.00	23	0.478
725	A	12	11	1.00	23	0.478
726	A	11	10	1.00	23	0.435
727	A	11	10	1.00	23	0.435
728	A	11	10	1.00	23	0.435
729	A	11	10	1.00	23	0.435
730	A	6	6	1.00	25	0.240
731	A	5	5	1.00	25	0.200
732	A	4	4	1.00	25	0.160
733	A	2	2	1.00	25	0.080
734	A	8	8	1.00	25	0.320
735	A	8	8	1.00	25	0.320
736	A	7	6	1.00	25	0.240
737	A	6	6	1.00	25	0.240
738	A	5	5	1.00	25	0.200
739	A	6	6	1.00	25	0.240
740	A	7	7	1.00	25	0.280
741	A	9	9	1.00	25	0.360
742	A	9	9	1.00	25	0.360
743	A	8	6	1.00	25	0.240
744	A	7	6	1.00	25	0.240
745	A	6	6	1.00	25	0.240
746	A	7	7	1.00	25	0.280
747	A	8	8	1.00	25	0.320
748	A	8	8	1.00	25	0.320
749	A	9	9	1.00	25	0.360
750	A	10	9	1.00	25	0.360
751	A	5	5	1.00	25	0.200
752	A	4	4	1.00	25	0.160
753	A	2	2	1.00	25	0.080
754	A	2	2	1.00	25	0.080
755	A	9	9	1.00	25	0.360
756	A	8	8	1.00	25	0.320
757	A	6	6	1.00	25	0.240
758	A	5	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
759	A	5	5	1.00	25	0.200
760	A	5	5	1.00	25	0.200
761	A	7	7	1.00	25	0.280
762	A	8	8	1.00	25	0.320
763	A	7	6	1.00	25	0.240
764	A	6	6	1.00	25	0.240
765	A	6	6	1.00	25	0.240
766	A	6	6	1.00	25	0.240
767	A	6	6	1.00	25	0.240
768	A	8	8	1.00	25	0.320
769	A	6	5	1.00	21	0.238
770	A	5	4	1.00	21	0.190
771	A	4	3	1.00	21	0.143
772	A	3	2	1.00	19	0.105
773	A	5	3	1.00	21	0.143
774	A	8	3	1.00	21	0.143
775	A	8	6	1.00	21	0.286
776	A	7	5	1.00	21	0.238
777	A	6	4	1.00	19	0.210
778	A	2	2	1.00	21	0.095
779	A	3	3	1.00	19	0.158
780	A	1	1	1.00	25	0.040
781	A	1	1	1.00	27	0.037
782	A	1	1	1.00	31	0.032
783	A	1	1	1.00	27	0.037
784	A	1	1	1.00	29	0.034
785	A	2	2	1.00	25	0.080
786	A	1	1	1.00	29	0.034
787	A	3	3	1.00	25	0.120
788	A	3	3	1.00	25	0.120
789	A	3	3	1.00	25	0.120
790	A	3	3	1.00	25	0.120
791	A	3	3	1.00	28	0.107
792	A	2	2	1.00	23	0.087
793	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
794	A	3	3	1.00	28	0.107
795	A	7	4	1.00	25	0.160
796	A	7	4	1.00	25	0.160
797	A	7	4	1.00	25	0.160
798	A	7	4	1.00	25	0.160
799	A	9	6	1.00	31	0.194
800	A	8	6	1.00	29	0.207
801	A	6	5	1.00	23	0.217
802	A	6	5	1.00	29	0.172
803	A	7	6	1.00	31	0.194
804	A	8	6	1.00	31	0.194
805	A	9	6	1.00	31	0.194
806	A	9	6	1.00	29	0.207
807	A	7	5	1.00	23	0.217
808	A	7	6	1.00	29	0.207
809	A	6	5	1.00	31	0.161
810	A	7	6	1.00	31	0.194
811	A	8	6	1.00	31	0.194
812	A	9	6	1.00	31	0.194
813	A	8	5	1.00	23	0.217
814	A	8	6	1.00	29	0.207
815	A	7	6	1.00	31	0.194
816	A	6	5	1.00	31	0.161
817	A	7	6	1.00	31	0.194
818	A	8	6	1.00	31	0.194
819	A	9	6	1.00	31	0.194
820	A	9	6	1.00	31	0.194
821	A	8	6	1.00	31	0.194
822	A	7	6	1.00	29	0.207
823	A	5	4	1.00	23	0.174
824	A	7	6	1.00	29	0.207
825	A	8	6	1.00	31	0.194
826	A	9	6	1.00	31	0.194
827	A	9	6	1.00	31	0.194
828	A	8	6	1.00	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
829	A	7	6	1.00	31	0.194
830	A	6	5	1.00	29	0.172
831	A	6	5	1.00	23	0.217
832	A	8	6	1.00	29	0.207
833	A	9	6	1.00	31	0.194
834	A	9	6	1.00	31	0.194
835	A	8	6	1.00	31	0.194
836	A	7	6	1.00	31	0.194
837	A	6	5	1.00	31	0.161
838	A	7	6	1.00	29	0.207
839	A	7	5	1.00	23	0.217
840	A	9	6	1.00	29	0.207
841	A	8	5	1.00	23	0.217
842	A	7	5	1.00	33	0.152
843	A	6	5	1.00	33	0.152
844	A	2	2	1.00	33	0.061
845	A	3	2	1.00	33	0.061
846	A	3	3	1.00	33	0.091
847	A	5	5	1.00	33	0.152
848	A	6	6	1.00	33	0.182
849	A	6	5	1.00	33	0.152
850	A	7	5	1.00	33	0.152
851	A	6	5	1.00	33	0.152
852	A	2	2	1.00	33	0.061
853	A	3	2	1.00	33	0.061
854	A	3	3	1.00	33	0.091
855	A	5	5	1.00	33	0.152
856	A	6	6	1.00	33	0.182
857	A	6	5	1.00	33	0.152
858	A	7	5	1.00	33	0.152
859	A	6	5	1.00	33	0.152
860	A	2	2	1.00	33	0.061
861	A	3	2	1.00	33	0.061
862	A	3	3	1.00	33	0.091
863	A	5	5	1.00	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
864	A	6	6	1.00	33	0.182
865	A	6	5	1.00	33	0.152
866	A	6	5	1.00	33	0.152
867	A	2	2	1.00	33	0.061
868	A	3	2	1.00	33	0.061
869	A	3	3	1.00	33	0.091
870	A	5	5	1.00	33	0.152
871	A	6	6	1.00	33	0.182
872	A	6	5	1.00	33	0.152
873	A	6	5	1.00	33	0.152
874	A	2	2	1.00	33	0.061
875	A	3	2	1.00	33	0.061
876	A	3	3	1.00	33	0.091
877	A	5	5	1.00	33	0.152
878	A	6	6	1.00	33	0.182
879	A	6	5	1.00	33	0.152
880	A	6	5	1.00	33	0.152
881	A	2	2	1.00	33	0.061
882	A	3	2	1.00	33	0.061
883	A	3	3	1.00	33	0.091
884	A	5	5	1.00	33	0.152
885	A	6	6	1.00	33	0.182
886	A	6	5	1.00	33	0.152
887	A	4	3	1.00	31	0.097
888	A	4	3	1.00	29	0.103
889	A	3	2	1.00	23	0.087
890	A	4	3	1.00	29	0.103
891	A	4	3	1.00	31	0.097
892	A	4	3	1.00	31	0.097
893	A	4	3	1.00	31	0.097
894	A	4	3	1.00	29	0.103
895	A	3	2	1.00	23	0.087
896	A	4	3	1.00	29	0.103
897	A	4	3	1.00	31	0.097
898	A	4	3	1.00	31	0.097

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
899	A	4	3	1.00	31	0.097
900	A	4	3	1.00	29	0.103
901	A	3	2	1.00	23	0.087
902	A	4	3	1.00	29	0.103
903	A	4	3	1.00	31	0.097
904	A	4	3	1.00	31	0.097
905	A	4	3	1.00	31	0.097
906	A	4	3	1.00	29	0.103
907	A	3	2	1.00	23	0.087
908	A	4	3	1.00	29	0.103
909	A	4	3	1.00	31	0.097
910	A	4	3	1.00	31	0.097
911	A	4	3	1.00	29	0.103
912	A	4	3	1.00	29	0.103
913	A	4	3	1.00	27	0.111
914	A	3	2	1.00	21	0.095
915	A	4	3	1.00	27	0.111
916	A	4	3	1.00	29	0.103
917	A	4	3	1.00	29	0.103
918	A	4	3	1.00	29	0.103
919	A	4	3	1.00	31	0.097
920	A	4	3	1.00	31	0.097
921	A	4	3	1.00	31	0.097
922	A	4	3	1.00	31	0.097
923	A	4	3	1.00	31	0.097
924	A	4	3	1.00	31	0.097
925	A	4	3	1.00	31	0.097
926	A	4	3	1.00	31	0.097
927	A	4	3	1.00	31	0.097
928	A	4	3	1.00	31	0.097
929	A	4	3	1.00	31	0.097
930	A	4	3	1.00	31	0.097
931	A	4	3	1.00	31	0.097
932	A	4	3	1.00	31	0.097

CHAPTER 3

LISTING OF INTEGRALS

3.1	$\int \cos^5(c + dx)(a + a \cos(c + dx)) dx$	275
3.2	$\int \cos^4(c + dx)(a + a \cos(c + dx)) dx$	281
3.3	$\int \cos^3(c + dx)(a + a \cos(c + dx)) dx$	286
3.4	$\int \cos^2(c + dx)(a + a \cos(c + dx)) dx$	291
3.5	$\int \cos(c + dx)(a + a \cos(c + dx)) dx$	296
3.6	$\int (a + a \cos(c + dx)) dx$	300
3.7	$\int (a + a \cos(c + dx)) \sec(c + dx) dx$	303
3.8	$\int (a + a \cos(c + dx)) \sec^2(c + dx) dx$	307
3.9	$\int (a + a \cos(c + dx)) \sec^3(c + dx) dx$	311
3.10	$\int (a + a \cos(c + dx)) \sec^4(c + dx) dx$	316
3.11	$\int (a + a \cos(c + dx)) \sec^5(c + dx) dx$	321
3.12	$\int (a + a \cos(c + dx)) \sec^6(c + dx) dx$	326
3.13	$\int \cos^4(c + dx)(a + a \cos(c + dx))^2 dx$	331
3.14	$\int \cos^3(c + dx)(a + a \cos(c + dx))^2 dx$	337
3.15	$\int \cos^2(c + dx)(a + a \cos(c + dx))^2 dx$	342
3.16	$\int \cos(c + dx)(a + a \cos(c + dx))^2 dx$	347
3.17	$\int (a + a \cos(c + dx))^2 dx$	352
3.18	$\int (a + a \cos(c + dx))^2 \sec(c + dx) dx$	356
3.19	$\int (a + a \cos(c + dx))^2 \sec^2(c + dx) dx$	360
3.20	$\int (a + a \cos(c + dx))^2 \sec^3(c + dx) dx$	364
3.21	$\int (a + a \cos(c + dx))^2 \sec^4(c + dx) dx$	369
3.22	$\int (a + a \cos(c + dx))^2 \sec^5(c + dx) dx$	374
3.23	$\int \cos^3(c + dx)(a + a \cos(c + dx))^3 dx$	379
3.24	$\int \cos^2(c + dx)(a + a \cos(c + dx))^3 dx$	385
3.25	$\int \cos(c + dx)(a + a \cos(c + dx))^3 dx$	391
3.26	$\int (a + a \cos(c + dx))^3 dx$	397
3.27	$\int (a + a \cos(c + dx))^3 \sec(c + dx) dx$	402
3.28	$\int (a + a \cos(c + dx))^3 \sec^2(c + dx) dx$	407

3.29	$\int (a + a \cos(c + dx))^3 \sec^3(c + dx) dx$	412
3.30	$\int (a + a \cos(c + dx))^3 \sec^4(c + dx) dx$	417
3.31	$\int (a + a \cos(c + dx))^3 \sec^5(c + dx) dx$	422
3.32	$\int (a + a \cos(c + dx))^3 \sec^6(c + dx) dx$	428
3.33	$\int \cos^2(c + dx)(a + a \cos(c + dx))^4 dx$	434
3.34	$\int \cos(c + dx)(a + a \cos(c + dx))^4 dx$	440
3.35	$\int (a + a \cos(c + dx))^4 dx$	446
3.36	$\int (a + a \cos(c + dx))^4 \sec(c + dx) dx$	451
3.37	$\int (a + a \cos(c + dx))^4 \sec^2(c + dx) dx$	456
3.38	$\int (a + a \cos(c + dx))^4 \sec^3(c + dx) dx$	462
3.39	$\int (a + a \cos(c + dx))^4 \sec^4(c + dx) dx$	468
3.40	$\int (a + a \cos(c + dx))^4 \sec^5(c + dx) dx$	474
3.41	$\int (a + a \cos(c + dx))^4 \sec^6(c + dx) dx$	480
3.42	$\int (a + a \cos(c + dx))^4 \sec^7(c + dx) dx$	486
3.43	$\int \frac{\cos^5(c+dx)}{a+a \cos(c+dx)} dx$	492
3.44	$\int \frac{\cos^4(c+dx)}{a+a \cos(c+dx)} dx$	498
3.45	$\int \frac{\cos^3(c+dx)}{a+a \cos(c+dx)} dx$	504
3.46	$\int \frac{\cos^2(c+dx)}{a+a \cos(c+dx)} dx$	509
3.47	$\int \frac{\cos(c+dx)}{a+a \cos(c+dx)} dx$	514
3.48	$\int \frac{1}{a+a \cos(c+dx)} dx$	518
3.49	$\int \frac{\sec(c+dx)}{a+a \cos(c+dx)} dx$	522
3.50	$\int \frac{\sec^2(c+dx)}{a+a \cos(c+dx)} dx$	526
3.51	$\int \frac{\sec^3(c+dx)}{a+a \cos(c+dx)} dx$	531
3.52	$\int \frac{\sec^4(c+dx)}{a+a \cos(c+dx)} dx$	536
3.53	$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^2} dx$	541
3.54	$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^2} dx$	547
3.55	$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^2} dx$	553
3.56	$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^2} dx$	558
3.57	$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^2} dx$	563
3.58	$\int \frac{1}{(a+a \cos(c+dx))^2} dx$	567
3.59	$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^2} dx$	571
3.60	$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^2} dx$	576
3.61	$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^2} dx$	582
3.62	$\int \frac{\sec^4(c+dx)}{(a+a \cos(c+dx))^2} dx$	588
3.63	$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^3} dx$	594
3.64	$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^3} dx$	600
3.65	$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^3} dx$	606

3.66	$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^3} dx$	611
3.67	$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^3} dx$	616
3.68	$\int \frac{1}{(a+a \cos(c+dx))^3} dx$	621
3.69	$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^3} dx$	625
3.70	$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^3} dx$	630
3.71	$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^3} dx$	636
3.72	$\int \frac{\cos^6(c+dx)}{(a+a \cos(c+dx))^4} dx$	642
3.73	$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^4} dx$	648
3.74	$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^4} dx$	655
3.75	$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^4} dx$	661
3.76	$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^4} dx$	666
3.77	$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^4} dx$	671
3.78	$\int \frac{1}{(a+a \cos(c+dx))^4} dx$	676
3.79	$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^4} dx$	681
3.80	$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^4} dx$	687
3.81	$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^4} dx$	693
3.82	$\int \frac{\cos^7(c+dx)}{(a+a \cos(c+dx))^5} dx$	700
3.83	$\int \frac{\cos^6(c+dx)}{(a+a \cos(c+dx))^5} dx$	707
3.84	$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^5} dx$	714
3.85	$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^5} dx$	721
3.86	$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^5} dx$	727
3.87	$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^5} dx$	733
3.88	$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^5} dx$	738
3.89	$\int \frac{1}{(a+a \cos(c+dx))^5} dx$	743
3.90	$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^5} dx$	748
3.91	$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^5} dx$	754
3.92	$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^5} dx$	761
3.93	$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^6} dx$	769
3.94	$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^6} dx$	775
3.95	$\int \cos^4(c+dx) \sqrt{a+a \cos(c+dx)} dx$	781
3.96	$\int \cos^3(c+dx) \sqrt{a+a \cos(c+dx)} dx$	786
3.97	$\int \cos^2(c+dx) \sqrt{a+a \cos(c+dx)} dx$	791
3.98	$\int \cos(c+dx) \sqrt{a+a \cos(c+dx)} dx$	796
3.99	$\int \sqrt{a+a \cos(c+dx)} dx$	800
3.100	$\int \sqrt{a+a \cos(c+dx)} \sec(c+dx) dx$	803

3.101	$\int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx$	807
3.102	$\int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx$	812
3.103	$\int \sqrt{a + a \cos(c + dx)} \sec^4(c + dx) dx$	819
3.104	$\int \cos^3(c + dx)(a + a \cos(c + dx))^{3/2} dx$	828
3.105	$\int \cos^2(c + dx)(a + a \cos(c + dx))^{3/2} dx$	834
3.106	$\int \cos(c + dx)(a + a \cos(c + dx))^{3/2} dx$	839
3.107	$\int (a + a \cos(c + dx))^{3/2} dx$	843
3.108	$\int (a + a \cos(c + dx))^{3/2} \sec(c + dx) dx$	847
3.109	$\int (a + a \cos(c + dx))^{3/2} \sec^2(c + dx) dx$	852
3.110	$\int (a + a \cos(c + dx))^{3/2} \sec^3(c + dx) dx$	858
3.111	$\int (a + a \cos(c + dx))^{3/2} \sec^4(c + dx) dx$	865
3.112	$\int \cos^3(c + dx)(a + a \cos(c + dx))^{5/2} dx$	874
3.113	$\int \cos^2(c + dx)(a + a \cos(c + dx))^{5/2} dx$	880
3.114	$\int \cos(c + dx)(a + a \cos(c + dx))^{5/2} dx$	885
3.115	$\int (a + a \cos(c + dx))^{5/2} dx$	890
3.116	$\int (a + a \cos(c + dx))^{5/2} \sec(c + dx) dx$	894
3.117	$\int (a + a \cos(c + dx))^{5/2} \sec^2(c + dx) dx$	899
3.118	$\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx$	911
3.119	$\int (a + a \cos(c + dx))^{5/2} \sec^4(c + dx) dx$	919
3.120	$\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx$	930
3.121	$\int (a + a \cos(c + dx))^{7/2} dx$	937
3.122	$\int \frac{\cos^4(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	941
3.123	$\int \frac{\cos^3(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	1395
3.124	$\int \frac{\cos^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	1990
3.125	$\int \frac{\cos(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	2007
3.126	$\int \frac{1}{\sqrt{a+a \cos(c+dx)}} dx$	2024
3.127	$\int \frac{\sec(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	2028
3.128	$\int \frac{\sec^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	2033
3.129	$\int \frac{\sec^3(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	2050
3.130	$\int \frac{\sec^4(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	2056
3.131	$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$	2062
3.132	$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$	2068
3.133	$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$	2073
3.134	$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$	2078
3.135	$\int \frac{1}{(a+a \cos(c+dx))^{3/2}} dx$	2112
3.136	$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$	2126
3.137	$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$	2131
3.138	$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$	2228

3.139	$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$	2235
3.140	$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$	2241
3.141	$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$	2247
3.142	$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$	2252
3.143	$\int \frac{1}{(a+a \cos(c+dx))^{5/2}} dx$	2256
3.144	$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$	2311
3.145	$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$	2317
3.146	$\int \cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx)) dx$	2324
3.147	$\int \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx)) dx$	2329
3.148	$\int \sqrt{\cos(c+dx)}(a+a \cos(c+dx)) dx$	2334
3.149	$\int \frac{a+a \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx$	2338
3.150	$\int \frac{a+a \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx$	2342
3.151	$\int \frac{a+a \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} dx$	2346
3.152	$\int \frac{a+a \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)} dx$	2351
3.153	$\int \cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^2 dx$	2357
3.154	$\int \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^2 dx$	2363
3.155	$\int \sqrt{\cos(c+dx)}(a+a \cos(c+dx))^2 dx$	2369
3.156	$\int \frac{(a+a \cos(c+dx))^2}{\sqrt{\cos(c+dx)}} dx$	2374
3.157	$\int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)} dx$	2379
3.158	$\int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)} dx$	2383
3.159	$\int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)} dx$	2388
3.160	$\int \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^3 dx$	2393
3.161	$\int \sqrt{\cos(c+dx)}(a+a \cos(c+dx))^3 dx$	2399
3.162	$\int \frac{(a+a \cos(c+dx))^3}{\sqrt{\cos(c+dx)}} dx$	2405
3.163	$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)} dx$	2410
3.164	$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)} dx$	2415
3.165	$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)} dx$	2420
3.166	$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)} dx$	2425
3.167	$\int \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^4 dx$	2430
3.168	$\int \sqrt{\cos(c+dx)}(a+a \cos(c+dx))^4 dx$	2437
3.169	$\int \frac{(a+a \cos(c+dx))^4}{\sqrt{\cos(c+dx)}} dx$	2443
3.170	$\int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)} dx$	2449
3.171	$\int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)} dx$	2454

3.172	$\int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)} dx$	2459
3.173	$\int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)} dx$	2464
3.174	$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a \cos(c+dx)} dx$	2470
3.175	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a \cos(c+dx)} dx$	2475
3.176	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a \cos(c+dx)} dx$	2480
3.177	$\int \frac{\sqrt{\cos(c+dx)}}{a+a \cos(c+dx)} dx$	2485
3.178	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))} dx$	2489
3.179	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))} dx$	2493
3.180	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))} dx$	2498
3.181	$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$	2503
3.182	$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$	2509
3.183	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$	2514
3.184	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$	2519
3.185	$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^2} dx$	2524
3.186	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^2} dx$	2528
3.187	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^2} dx$	2533
3.188	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^2} dx$	2539
3.189	$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx$	2545
3.190	$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx$	2551
3.191	$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx$	2557
3.192	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx$	2562
3.193	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx$	2568
3.194	$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^3} dx$	2573
3.195	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^3} dx$	2578
3.196	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^3} dx$	2584
3.197	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^3} dx$	2591
3.198	$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} dx$	2598
3.199	$\int \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} dx$	2605
3.200	$\int \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)} dx$	2610
3.201	$\int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx$	2615
3.202	$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx$	2619

3.203	$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx$	2623
3.204	$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx$	2627
3.205	$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)} dx$	2632
3.206	$\int \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{\frac{3}{2}} dx$	2638
3.207	$\int \sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{\frac{3}{2}} dx$	2645
3.208	$\int \frac{(a+a \cos(c+dx))^{\frac{3}{2}}}{\sqrt{\cos(c+dx)}} dx$	2650
3.209	$\int \frac{(a+a \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)} dx$	2655
3.210	$\int \frac{(a+a \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{5}{2}}(c+dx)} dx$	2660
3.211	$\int \frac{(a+a \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{7}{2}}(c+dx)} dx$	2664
3.212	$\int \frac{(a+a \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{9}{2}}(c+dx)} dx$	2669
3.213	$\int \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{\frac{5}{2}} dx$	3116
3.214	$\int \sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{\frac{5}{2}} dx$	3126
3.215	$\int \frac{(a+a \cos(c+dx))^{\frac{5}{2}}}{\sqrt{\cos(c+dx)}} dx$	3133
3.216	$\int \frac{(a+a \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{3}{2}}(c+dx)} dx$	3138
3.217	$\int \frac{(a+a \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{5}{2}}(c+dx)} dx$	3143
3.218	$\int \frac{(a+a \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{7}{2}}(c+dx)} dx$	3149
3.219	$\int \frac{(a+a \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{9}{2}}(c+dx)} dx$	3154
3.220	$\int \frac{(a+a \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{11}{2}}(c+dx)} dx$	3580
3.221	$\int \frac{(a+a \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{5}{4}}(c+dx)} dx$	3586
3.222	$\int \frac{\sqrt{a+a \cos(e+fx)}}{\sqrt{\cos(e+fx)}} dx$	3590
3.223	$\int \frac{\sqrt{a-a \cos(e+fx)}}{\sqrt{-\cos(e+fx)}} dx$	3594
3.224	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	3598
3.225	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	3604
3.226	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a \cos(c+dx)}} dx$	3609
3.227	$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} dx$	3614
3.228	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} dx$	3618
3.229	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} dx$	3623
3.230	$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} dx$	3629
3.231	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$	3636

3.232	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$	3641
3.233	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx$	3646
3.234	$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx$	3651
3.235	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx$	3655
3.236	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx$	3660
3.237	$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx$	3666
3.238	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx$	3673
3.239	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx$	3679
3.240	$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx$	3684
3.241	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx$	3688
3.242	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx$	3692
3.243	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx$	3697
3.244	$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx$	3703
3.245	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx$	3709
3.246	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx$	3715
3.247	$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx$	3720
3.248	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx$	3725
3.249	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} dx$	3730
3.250	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} dx$	3736
3.251	$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx$	3743
3.252	$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx$	3750
3.253	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx$	3756
3.254	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx$	3762
3.255	$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}} dx$	3767
3.256	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} dx$	3772
3.257	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx$	3778
3.258	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx$	3785
3.259	$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}} dx$	3792
3.260	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}} dx$	3798
3.261	$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}} dx$	3804

3.262	$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}} dx$	3808
3.263	$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)} dx$	3812
3.264	$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)} dx$	3817
3.265	$\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx$	3822
3.266	$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx$	3826
3.267	$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx$	3830
3.268	$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx$	3834
3.269	$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx) dx$	3839
3.270	$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)} dx$	3844
3.271	$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx$	3849
3.272	$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx$	3853
3.273	$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx$	3857
3.274	$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx$	3861
3.275	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}} dx$	3866
3.276	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}} dx$	3873
3.277	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}} dx$	3879
3.278	$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx$	3884
3.279	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx$	3888
3.280	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx$	3893
3.281	$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx$	3899
3.282	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx$	3906
3.283	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx$	3912
3.284	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}} dx$	3918
3.285	$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$	3923
3.286	$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)} dx$	3927
3.287	$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)} dx$	3932
3.288	$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)} dx$	3938
3.289	$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3} dx$	3942
3.290	$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3} dx$	3946
3.291	$\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx) dx$	3950
3.292	$\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx) dx$	3956

3.293	$\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$	3962
3.294	$\int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx$	3967
3.295	$\int \frac{a+a \cos(c+dx)}{\sqrt{\sec(c+dx)}} dx$	3972
3.296	$\int \frac{a+a \cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx$	3977
3.297	$\int \frac{a+a \cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx$	3982
3.298	$\int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx$	3987
3.299	$\int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx$	3993
3.300	$\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$	3999
3.301	$\int (a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$	4004
3.302	$\int \frac{(a+a \cos(c+dx))^2}{\sqrt{\sec(c+dx)}} dx$	4009
3.303	$\int \frac{(a+a \cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)} dx$	4015
3.304	$\int (a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx$	4021
3.305	$\int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx$	4028
3.306	$\int (a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx$	4034
3.307	$\int (a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx$	4040
3.308	$\int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx$	4046
3.309	$\int \frac{(a+a \cos(c+dx))^3}{\sqrt{\sec(c+dx)}} dx$	4052
3.310	$\int \frac{(a+a \cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)} dx$	4058
3.311	$\int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx$	4064
3.312	$\int (a + a \cos(c + dx))^4 \sec^{\frac{7}{2}}(c + dx) dx$	4071
3.313	$\int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx$	4078
3.314	$\int (a + a \cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx) dx$	4084
3.315	$\int (a + a \cos(c + dx))^4 \sqrt{\sec(c + dx)} dx$	4090
3.316	$\int \frac{(a+a \cos(c+dx))^4}{\sqrt{\sec(c+dx)}} dx$	4097
3.317	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a \cos(c+dx)} dx$	4104
3.318	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a \cos(c+dx)} dx$	4110
3.319	$\int \frac{\sqrt{\sec(c+dx)}}{a+a \cos(c+dx)} dx$	4116
3.320	$\int \frac{1}{(a+a \cos(c+dx)) \sqrt{\sec(c+dx)}} dx$	4121
3.321	$\int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{3}{2}}(c+dx)} dx$	4126
3.322	$\int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{5}{2}}(c+dx)} dx$	4131
3.323	$\int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{7}{2}}(c+dx)} dx$	4137
3.324	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$	4143
3.325	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$	4150
3.326	$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^2} dx$	4156

3.327	$\int \frac{1}{(a+a \cos(c+dx))^2 \sqrt{\sec(c+dx)}} dx$	4162
3.328	$\int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)} dx$	4167
3.329	$\int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)} dx$	4173
3.330	$\int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{7}{2}}(c+dx)} dx$	4179
3.331	$\int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{9}{2}}(c+dx)} dx$	4185
3.332	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx$	4192
3.333	$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^3} dx$	4199
3.334	$\int \frac{1}{(a+a \cos(c+dx))^3 \sqrt{\sec(c+dx)}} dx$	4205
3.335	$\int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)} dx$	4212
3.336	$\int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)} dx$	4219
3.337	$\int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{7}{2}}(c+dx)} dx$	4225
3.338	$\int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{9}{2}}(c+dx)} dx$	4231
3.339	$\int \sqrt{a+a \cos(c+dx)} \sec^{\frac{9}{2}}(c+dx) dx$	4238
3.340	$\int \sqrt{a+a \cos(c+dx)} \sec^{\frac{7}{2}}(c+dx) dx$	4244
3.341	$\int \sqrt{a+a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx) dx$	4249
3.342	$\int \sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) dx$	4254
3.343	$\int \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)} dx$	4258
3.344	$\int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\sec(c+dx)}} dx$	4262
3.345	$\int \frac{\sqrt{a+a \cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)} dx$	4267
3.346	$\int (a+a \cos(c+dx))^{\frac{3}{2}} \sec^{\frac{9}{2}}(c+dx) dx$	4273
3.347	$\int (a+a \cos(c+dx))^{\frac{3}{2}} \sec^{\frac{7}{2}}(c+dx) dx$	4279
3.348	$\int (a+a \cos(c+dx))^{\frac{3}{2}} \sec^{\frac{5}{2}}(c+dx) dx$	4284
3.349	$\int (a+a \cos(c+dx))^{\frac{3}{2}} \sec^{\frac{3}{2}}(c+dx) dx$	4289
3.350	$\int (a+a \cos(c+dx))^{\frac{3}{2}} \sqrt{\sec(c+dx)} dx$	4295
3.351	$\int \frac{(a+a \cos(c+dx))^{\frac{3}{2}}}{\sqrt{\sec(c+dx)}} dx$	4300
3.352	$\int \frac{(a+a \cos(c+dx))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(c+dx)} dx$	4306
3.353	$\int (a+a \cos(c+dx))^{\frac{5}{2}} \sec^{\frac{11}{2}}(c+dx) dx$	4313
3.354	$\int (a+a \cos(c+dx))^{\frac{5}{2}} \sec^{\frac{9}{2}}(c+dx) dx$	4319
3.355	$\int (a+a \cos(c+dx))^{\frac{5}{2}} \sec^{\frac{7}{2}}(c+dx) dx$	4325
3.356	$\int (a+a \cos(c+dx))^{\frac{5}{2}} \sec^{\frac{5}{2}}(c+dx) dx$	4330
3.357	$\int (a+a \cos(c+dx))^{\frac{5}{2}} \sec^{\frac{3}{2}}(c+dx) dx$	4336
3.358	$\int (a+a \cos(c+dx))^{\frac{5}{2}} \sqrt{\sec(c+dx)} dx$	4342
3.359	$\int \frac{(a+a \cos(c+dx))^{\frac{5}{2}}}{\sqrt{\sec(c+dx)}} dx$	4348
3.360	$\int \frac{(a+a \cos(c+dx))^{\frac{5}{2}}}{\sec^{\frac{3}{2}}(c+dx)} dx$	4355

3.361	$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$	4366
3.362	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$	4373
3.363	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$	4379
3.364	$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx$	4384
3.365	$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} dx$	4388
3.366	$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} dx$	4393
3.367	$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$	4399
3.368	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$	4406
3.369	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$	4412
3.370	$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}} dx$	4418
3.371	$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} dx$	4423
3.372	$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} dx$	4429
3.373	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx$	4435
3.374	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx$	4441
3.375	$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx$	4447
3.376	$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} dx$	4452
3.377	$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} dx$	4457
3.378	$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)} dx$	4463
3.379	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx$	4469
3.380	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx$	4476
3.381	$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx$	4482
3.382	$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} dx$	4488
3.383	$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} dx$	4493
3.384	$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} dx$	4499
3.385	$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)} dx$	4505
3.386	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx$	4512
3.387	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx$	4519
3.388	$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}} dx$	4526
3.389	$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} dx$	4532
3.390	$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} dx$	4538

3.391	$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{5}{2}}(c+dx)} dx$	4544
3.392	$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{7}{2}}(c+dx)} dx$	4550
3.393	$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{9}{2}}(c+dx)} dx$	4557
3.394	$\int \frac{1}{(a+a \cos(c+dx))^{9/2} \sec^{\frac{5}{2}}(c+dx)} dx$	4565
3.395	$\int \frac{1}{(a+a \cos(c+dx))^{9/2} \sec^{\frac{7}{2}}(c+dx)} dx$	4572
3.396	$\int (a+a \cos(c+dx))^{3/2} \sec^{\frac{5}{4}}(c+dx) dx$	4579
3.397	$\int \cos^m(c+dx)(a+a \cos(c+dx))^4 dx$	4583
3.398	$\int \cos^m(c+dx)(a+a \cos(c+dx))^3 dx$	4589
3.399	$\int \cos^m(c+dx)(a+a \cos(c+dx))^2 dx$	4594
3.400	$\int \cos^m(c+dx)(a+a \cos(c+dx)) dx$	4599
3.401	$\int \frac{\cos^m(c+dx)}{a+a \cos(c+dx)} dx$	4603
3.402	$\int \frac{\cos^m(c+dx)}{(a+a \cos(c+dx))^2} dx$	4607
3.403	$\int \cos^7(c+dx)(a+b \cos(c+dx)) dx$	4612
3.404	$\int \cos^6(c+dx)(a+b \cos(c+dx)) dx$	4619
3.405	$\int \cos^5(c+dx)(a+b \cos(c+dx)) dx$	4625
3.406	$\int \cos^4(c+dx)(a+b \cos(c+dx)) dx$	4631
3.407	$\int \cos^3(c+dx)(a+b \cos(c+dx)) dx$	4636
3.408	$\int \cos^2(c+dx)(a+b \cos(c+dx)) dx$	4641
3.409	$\int \cos(c+dx)(a+b \cos(c+dx)) dx$	4646
3.410	$\int (a+b \cos(c+dx)) dx$	4650
3.411	$\int (a+b \cos(c+dx)) \sec(c+dx) dx$	4653
3.412	$\int (a+b \cos(c+dx)) \sec^2(c+dx) dx$	4657
3.413	$\int (a+b \cos(c+dx)) \sec^3(c+dx) dx$	4661
3.414	$\int (a+b \cos(c+dx)) \sec^4(c+dx) dx$	4666
3.415	$\int (a+b \cos(c+dx)) \sec^5(c+dx) dx$	4671
3.416	$\int (a+b \cos(c+dx)) \sec^6(c+dx) dx$	4676
3.417	$\int \cos^4(c+dx)(a+b \cos(c+dx))^2 dx$	4681
3.418	$\int \cos^3(c+dx)(a+b \cos(c+dx))^2 dx$	4687
3.419	$\int \cos^2(c+dx)(a+b \cos(c+dx))^2 dx$	4693
3.420	$\int \cos(c+dx)(a+b \cos(c+dx))^2 dx$	4698
3.421	$\int (a+b \cos(c+dx))^2 dx$	4703
3.422	$\int (a+b \cos(c+dx))^2 \sec(c+dx) dx$	4707
3.423	$\int (a+b \cos(c+dx))^2 \sec^2(c+dx) dx$	4711
3.424	$\int (a+b \cos(c+dx))^2 \sec^3(c+dx) dx$	4716
3.425	$\int (a+b \cos(c+dx))^2 \sec^4(c+dx) dx$	4721
3.426	$\int (a+b \cos(c+dx))^2 \sec^5(c+dx) dx$	4727
3.427	$\int (a+b \cos(c+dx))^2 \sec^6(c+dx) dx$	4733
3.428	$\int \cos^3(c+dx)(a+b \cos(c+dx))^3 dx$	4739
3.429	$\int \cos^2(c+dx)(a+b \cos(c+dx))^3 dx$	4747
3.430	$\int \cos(c+dx)(a+b \cos(c+dx))^3 dx$	4753
3.431	$\int (a+b \cos(c+dx))^3 dx$	4758
3.432	$\int (a+b \cos(c+dx))^3 \sec(c+dx) dx$	4763

3.433	$\int (a + b \cos(c + dx))^3 \sec^2(c + dx) dx$	4768
3.434	$\int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx$	4773
3.435	$\int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx$	4778
3.436	$\int (a + b \cos(c + dx))^3 \sec^5(c + dx) dx$	4784
3.437	$\int (a + b \cos(c + dx))^3 \sec^6(c + dx) dx$	4791
3.438	$\int \cos^3(c + dx)(a + b \cos(c + dx))^4 dx$	4798
3.439	$\int \cos^2(c + dx)(a + b \cos(c + dx))^4 dx$	4807
3.440	$\int \cos(c + dx)(a + b \cos(c + dx))^4 dx$	4814
3.441	$\int (a + b \cos(c + dx))^4 dx$	4821
3.442	$\int (a + b \cos(c + dx))^4 \sec(c + dx) dx$	4827
3.443	$\int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx$	4833
3.444	$\int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx$	4839
3.445	$\int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx$	4845
3.446	$\int (a + b \cos(c + dx))^4 \sec^5(c + dx) dx$	4851
3.447	$\int (a + b \cos(c + dx))^4 \sec^6(c + dx) dx$	4858
3.448	$\int (a + b \cos(c + dx))^4 \sec^7(c + dx) dx$	4865
3.449	$\int \frac{\cos^5(c+dx)}{a+b \cos(c+dx)} dx$	4873
3.450	$\int \frac{\cos^4(c+dx)}{a+b \cos(c+dx)} dx$	4880
3.451	$\int \frac{\cos^3(c+dx)}{a+b \cos(c+dx)} dx$	4886
3.452	$\int \frac{\cos^2(c+dx)}{a+b \cos(c+dx)} dx$	4892
3.453	$\int \frac{\cos(c+dx)}{a+b \cos(c+dx)} dx$	4898
3.454	$\int \frac{1}{a+b \cos(c+dx)} dx$	4903
3.455	$\int \frac{\sec(c+dx)}{a+b \cos(c+dx)} dx$	4907
3.456	$\int \frac{\sec^2(c+dx)}{a+b \cos(c+dx)} dx$	4912
3.457	$\int \frac{\sec^3(c+dx)}{a+b \cos(c+dx)} dx$	4918
3.458	$\int \frac{\sec^4(c+dx)}{a+b \cos(c+dx)} dx$	4924
3.459	$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^2} dx$	4931
3.460	$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^2} dx$	4941
3.461	$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^2} dx$	4949
3.462	$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^2} dx$	4957
3.463	$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^2} dx$	4963
3.464	$\int \frac{1}{(a+b \cos(c+dx))^2} dx$	4968
3.465	$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^2} dx$	4974
3.466	$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^2} dx$	4981
3.467	$\int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^2} dx$	4989
3.468	$\int \frac{\sec^4(c+dx)}{(a+b \cos(c+dx))^2} dx$	4998
3.469	$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^3} dx$	5008
3.470	$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^3} dx$	5020

3.471	$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^3} dx$	5029
3.472	$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^3} dx$	5038
3.473	$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^3} dx$	5044
3.474	$\int \frac{1}{(a+b \cos(c+dx))^3} dx$	5050
3.475	$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^3} dx$	5056
3.476	$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^3} dx$	5065
3.477	$\int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^3} dx$	5075
3.478	$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^4} dx$	5087
3.479	$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^4} dx$	5099
3.480	$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^4} dx$	5110
3.481	$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^4} dx$	5118
3.482	$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^4} dx$	5125
3.483	$\int \frac{1}{(a+b \cos(c+dx))^4} dx$	5132
3.484	$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^4} dx$	5139
3.485	$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^4} dx$	5150
3.486	$\int \cos^3(c+dx) \sqrt{a+b \cos(c+dx)} dx$	5163
3.487	$\int \cos^2(c+dx) \sqrt{a+b \cos(c+dx)} dx$	5170
3.488	$\int \cos(c+dx) \sqrt{a+b \cos(c+dx)} dx$	5177
3.489	$\int \sqrt{a+b \cos(c+dx)} dx$	5183
3.490	$\int \sqrt{a+b \cos(c+dx)} \sec(c+dx) dx$	5187
3.491	$\int \sqrt{a+b \cos(c+dx)} \sec^2(c+dx) dx$	5192
3.492	$\int \sqrt{a+b \cos(c+dx)} \sec^3(c+dx) dx$	5199
3.493	$\int \cos^3(c+dx)(a+b \cos(c+dx))^{3/2} dx$	5207
3.494	$\int \cos^2(c+dx)(a+b \cos(c+dx))^{3/2} dx$	5215
3.495	$\int \cos(c+dx)(a+b \cos(c+dx))^{3/2} dx$	5222
3.496	$\int (a+b \cos(c+dx))^{3/2} dx$	5228
3.497	$\int (a+b \cos(c+dx))^{3/2} \sec(c+dx) dx$	5233
3.498	$\int (a+b \cos(c+dx))^{3/2} \sec^2(c+dx) dx$	5239
3.499	$\int (a+b \cos(c+dx))^{3/2} \sec^3(c+dx) dx$	5246
3.500	$\int \cos^3(c+dx)(a+b \cos(c+dx))^{5/2} dx$	5254
3.501	$\int \cos^2(c+dx)(a+b \cos(c+dx))^{5/2} dx$	5263
3.502	$\int \cos(c+dx)(a+b \cos(c+dx))^{5/2} dx$	5271
3.503	$\int (a+b \cos(c+dx))^{5/2} dx$	5278
3.504	$\int (a+b \cos(c+dx))^{5/2} \sec(c+dx) dx$	5285
3.505	$\int (a+b \cos(c+dx))^{5/2} \sec^2(c+dx) dx$	5292
3.506	$\int (a+b \cos(c+dx))^{5/2} \sec^3(c+dx) dx$	5299
3.507	$\int (a+b \cos(c+dx))^{5/2} \sec^4(c+dx) dx$	5307
3.508	$\int (a+b \cos(c+dx))^{7/2} dx$	5316
3.509	$\int \cos^3(c+dx) \sqrt{3+4 \cos(c+dx)} dx$	5323

3.510	$\int \cos^2(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$	5329
3.511	$\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$	5334
3.512	$\int \sqrt{3 + 4 \cos(c + dx)} dx$	5339
3.513	$\int \sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) dx$	5343
3.514	$\int \sqrt{3 + 4 \cos(c + dx)} \sec^2(c + dx) dx$	5347
3.515	$\int \sqrt{3 + 4 \cos(c + dx)} \sec^3(c + dx) dx$	5352
3.516	$\int \sqrt{3 - 4 \cos(c + dx)} \cos^3(c + dx) dx$	5358
3.517	$\int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx$	5364
3.518	$\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx$	5369
3.519	$\int \sqrt{3 - 4 \cos(c + dx)} dx$	5374
3.520	$\int \sqrt{3 - 4 \cos(c + dx)} \sec(c + dx) dx$	5378
3.521	$\int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx$	5382
3.522	$\int \sqrt{3 - 4 \cos(c + dx)} \sec^3(c + dx) dx$	5388
3.523	$\int \frac{\cos^3(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$	5394
3.524	$\int \frac{\cos^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$	5401
3.525	$\int \frac{\cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$	5407
3.526	$\int \frac{1}{\sqrt{a+b \cos(c+dx)}} dx$	5412
3.527	$\int \frac{\sec(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$	5416
3.528	$\int \frac{\sec^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$	5420
3.529	$\int \frac{\sec^3(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$	5427
3.530	$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$	5435
3.531	$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$	5443
3.532	$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$	5450
3.533	$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$	5456
3.534	$\int \frac{1}{(a+b \cos(c+dx))^{3/2}} dx$	5461
3.535	$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$	5466
3.536	$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$	5472
3.537	$\int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$	5480
3.538	$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	5488
3.539	$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	5497
3.540	$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	5505
3.541	$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	5512
3.542	$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	5519
3.543	$\int \frac{1}{(a+b \cos(c+dx))^{5/2}} dx$	5525
3.544	$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	5531

3.545	$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	5539
3.546	$\int \frac{1}{(a+b \cos(c+dx))^{7/2}} dx$	5548
3.547	$\int \frac{\cos^3(c+dx)}{\sqrt{3+4 \cos(c+dx)}} dx$	5555
3.548	$\int \frac{\cos^2(c+dx)}{\sqrt{3+4 \cos(c+dx)}} dx$	5560
3.549	$\int \frac{\cos(c+dx)}{\sqrt{3+4 \cos(c+dx)}} dx$	5565
3.550	$\int \frac{1}{\sqrt{3+4 \cos(c+dx)}} dx$	5569
3.551	$\int \frac{\sec(c+dx)}{\sqrt{3+4 \cos(c+dx)}} dx$	5572
3.552	$\int \frac{\sec^2(c+dx)}{\sqrt{3+4 \cos(c+dx)}} dx$	5575
3.553	$\int \frac{\sec^3(c+dx)}{\sqrt{3+4 \cos(c+dx)}} dx$	5580
3.554	$\int \frac{\cos^3(c+dx)}{\sqrt{3-4 \cos(c+dx)}} dx$	5586
3.555	$\int \frac{\cos^2(c+dx)}{\sqrt{3-4 \cos(c+dx)}} dx$	5591
3.556	$\int \frac{\cos(c+dx)}{\sqrt{3-4 \cos(c+dx)}} dx$	5596
3.557	$\int \frac{1}{\sqrt{3-4 \cos(c+dx)}} dx$	5600
3.558	$\int \frac{\sec(c+dx)}{\sqrt{3-4 \cos(c+dx)}} dx$	5603
3.559	$\int \frac{\sec^2(c+dx)}{\sqrt{3-4 \cos(c+dx)}} dx$	5606
3.560	$\int \frac{\sec^3(c+dx)}{\sqrt{3-4 \cos(c+dx)}} dx$	5611
3.561	$\int \cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx)) dx$	5617
3.562	$\int \cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx)) dx$	5622
3.563	$\int \sqrt{\cos(c+dx)}(A+B \cos(c+dx)) dx$	5627
3.564	$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx$	5631
3.565	$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx$	5635
3.566	$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} dx$	5639
3.567	$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)} dx$	5643
3.568	$\int \cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^2 dx$	5648
3.569	$\int \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^2 dx$	5654
3.570	$\int \sqrt{\cos(c+dx)}(a+b \cos(c+dx))^2 dx$	5660
3.571	$\int \frac{(a+b \cos(c+dx))^2}{\sqrt{\cos(c+dx)}} dx$	5665
3.572	$\int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)} dx$	5669
3.573	$\int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)} dx$	5673
3.574	$\int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)} dx$	5678
3.575	$\int \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^3 dx$	5683
3.576	$\int \sqrt{\cos(c+dx)}(a+b \cos(c+dx))^3 dx$	5689
3.577	$\int \frac{(a+b \cos(c+dx))^3}{\sqrt{\cos(c+dx)}} dx$	5695

3.578	$\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)} dx$	5701
3.579	$\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)} dx$	5707
3.580	$\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)} dx$	5713
3.581	$\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)} dx$	5719
3.582	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b \cos(c+dx)} dx$	5725
3.583	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b \cos(c+dx)} dx$	5730
3.584	$\int \frac{\sqrt{\cos(c+dx)}}{a+b \cos(c+dx)} dx$	5735
3.585	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx$	5739
3.586	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} dx$	5742
3.587	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))} dx$	5747
3.588	$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx$	5753
3.589	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx$	5760
3.590	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx$	5766
3.591	$\int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^2} dx$	5771
3.592	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^2} dx$	5776
3.593	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^2} dx$	5781
3.594	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^2} dx$	5788
3.595	$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$	5795
3.596	$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$	5804
3.597	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$	5812
3.598	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$	5820
3.599	$\int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^3} dx$	5827
3.600	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^3} dx$	5834
3.601	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^3} dx$	5841
3.602	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^3} dx$	5849
3.603	$\int \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)} dx$	5858
3.604	$\int \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)} dx$	5866
3.605	$\int \frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx$	5873
3.606	$\int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx$	5877
3.607	$\int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx$	5882

3.608	$\int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx$	5888
3.609	$\int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)} dx$	5895
3.610	$\int \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^{\frac{3}{2}} dx$	5903
3.611	$\int \sqrt{\cos(c+dx)}(a+b \cos(c+dx))^{\frac{3}{2}} dx$	5913
3.612	$\int \frac{(a+b \cos(c+dx))^{\frac{3}{2}}}{\sqrt{\cos(c+dx)}} dx$	5921
3.613	$\int \frac{(a+b \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c+dx)} dx$	5928
3.614	$\int \frac{(a+b \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{5}{2}}(c+dx)} dx$	5934
3.615	$\int \frac{(a+b \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{7}{2}}(c+dx)} dx$	5940
3.616	$\int \frac{(a+b \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{9}{2}}(c+dx)} dx$	5947
3.617	$\int \frac{(a+b \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{11}{2}}(c+dx)} dx$	5955
3.618	$\int \sqrt{\cos(c+dx)}(a+b \cos(c+dx))^{\frac{5}{2}} dx$	5964
3.619	$\int \frac{(a+b \cos(c+dx))^{\frac{5}{2}}}{\sqrt{\cos(c+dx)}} dx$	5974
3.620	$\int \frac{(a+b \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{3}{2}}(c+dx)} dx$	5982
3.621	$\int \frac{(a+b \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{5}{2}}(c+dx)} dx$	5990
3.622	$\int \frac{(a+b \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{7}{2}}(c+dx)} dx$	5997
3.623	$\int \frac{(a+b \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{9}{2}}(c+dx)} dx$	6004
3.624	$\int \frac{(a+b \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{11}{2}}(c+dx)} dx$	6012
3.625	$\int \frac{(a+b \cos(c+dx))^{\frac{5}{2}}}{\cos^{\frac{13}{2}}(c+dx)} dx$	6022
3.626	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$	6032
3.627	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx$	6039
3.628	$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b \cos(c+dx)}} dx$	6043
3.629	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b \cos(c+dx)}} dx$	6047
3.630	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b \cos(c+dx)}} dx$	6052
3.631	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^{\frac{3}{2}}} dx$	6058
3.632	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^{\frac{3}{2}}} dx$	6067
3.633	$\int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^{\frac{3}{2}}} dx$	6073
3.634	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^{\frac{3}{2}}} dx$	6078
3.635	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^{\frac{3}{2}}} dx$	6083
3.636	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^{\frac{3}{2}}} dx$	6090
3.637	$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b \cos(c+dx))^{\frac{3}{2}}} dx$	6098

3.638	$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	6107
3.639	$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	6117
3.640	$\int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^{5/2}} dx$	6124
3.641	$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^{5/2}} dx$	6132
3.642	$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^{5/2}} dx$	6140
3.643	$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^{5/2}} dx$	6149
3.644	$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3 \cos(c+dx)}} dx$	6157
3.645	$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3 \cos(c+dx)}} dx$	6160
3.646	$\int \frac{1}{\sqrt{2-3 \cos(c+dx)}\sqrt{\cos(c+dx)}} dx$	6163
3.647	$\int \frac{1}{\sqrt{-2-3 \cos(c+dx)}\sqrt{\cos(c+dx)}} dx$	6167
3.648	$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2 \cos(c+dx)}} dx$	6171
3.649	$\int \frac{1}{\sqrt{3-2 \cos(c+dx)}\sqrt{\cos(c+dx)}} dx$	6175
3.650	$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2 \cos(c+dx)}} dx$	6179
3.651	$\int \frac{1}{\sqrt{-3-2 \cos(c+dx)}\sqrt{\cos(c+dx)}} dx$	6183
3.652	$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3 \cos(c+dx)}} dx$	6187
3.653	$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3 \cos(c+dx)}} dx$	6191
3.654	$\int \frac{1}{\sqrt{2-3 \cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$	6195
3.655	$\int \frac{1}{\sqrt{-2-3 \cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$	6198
3.656	$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2 \cos(c+dx)}} dx$	6201
3.657	$\int \frac{1}{\sqrt{3-2 \cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$	6205
3.658	$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2 \cos(c+dx)}} dx$	6209
3.659	$\int \frac{1}{\sqrt{-3-2 \cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$	6213
3.660	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3 \cos(c+dx)}} dx$	6217
3.661	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3 \cos(c+dx)}} dx$	6221
3.662	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3 \cos(c+dx)}} dx$	6225
3.663	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3 \cos(c+dx)}} dx$	6229
3.664	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2 \cos(c+dx)}} dx$	6233
3.665	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2 \cos(c+dx)}} dx$	6237
3.666	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2 \cos(c+dx)}} dx$	6241
3.667	$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2 \cos(c+dx)}} dx$	6245
3.668	$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3 \cos(c+dx)}} dx$	6249
3.669	$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3 \cos(c+dx)}} dx$	6253

3.670	$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx$	6257
3.671	$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx$	6261
3.672	$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx$	6265
3.673	$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$	6269
3.674	$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx$	6273
3.675	$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx$	6277
3.676	$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)} dx$	6281
3.677	$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)} dx$	6288
3.678	$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))} dx$	6295
3.679	$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))} dx$	6302
3.680	$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx$	6309
3.681	$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx$	6312
3.682	$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx$	6315
3.683	$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx$	6319
3.684	$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx$	6323
3.685	$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx$	6327
3.686	$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$	6331
3.687	$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$	6335
3.688	$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$	6339
3.689	$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$	6343
3.690	$\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx) dx$	6346
3.691	$\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx) dx$	6352
3.692	$\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx) dx$	6357
3.693	$\int (A+B\cos(c+dx))\sqrt{\sec(c+dx)} dx$	6362
3.694	$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}} dx$	6367
3.695	$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx$	6372
3.696	$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx$	6378
3.697	$\int (a+b\cos(c+dx))^2 \sec^{\frac{9}{2}}(c+dx) dx$	6384
3.698	$\int (a+b\cos(c+dx))^2 \sec^{\frac{7}{2}}(c+dx) dx$	6390
3.699	$\int (a+b\cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx) dx$	6396
3.700	$\int (a+b\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx) dx$	6402

3.701	$\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$	6407
3.702	$\int \frac{(a+b \cos(c+dx))^2}{\sqrt{\sec(c+dx)}} dx$	6412
3.703	$\int \frac{(a+b \cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)} dx$	6418
3.704	$\int \frac{(a+b \cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)} dx$	6424
3.705	$\int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx$	6430
3.706	$\int (a + b \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx$	6437
3.707	$\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx$	6444
3.708	$\int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx$	6450
3.709	$\int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx$	6456
3.710	$\int \frac{(a+b \cos(c+dx))^3}{\sqrt{\sec(c+dx)}} dx$	6462
3.711	$\int \frac{(a+b \cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)} dx$	6468
3.712	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b \cos(c+dx)} dx$	6475
3.713	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b \cos(c+dx)} dx$	6482
3.714	$\int \frac{\sqrt{\sec(c+dx)}}{a+b \cos(c+dx)} dx$	6487
3.715	$\int \frac{1}{(a+b \cos(c+dx)) \sqrt{\sec(c+dx)}} dx$	6491
3.716	$\int \frac{1}{(a+b \cos(c+dx)) \sec^{\frac{3}{2}}(c+dx)} dx$	6496
3.717	$\int \frac{1}{(a+b \cos(c+dx)) \sec^{\frac{5}{2}}(c+dx)} dx$	6502
3.718	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx$	6508
3.719	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx$	6516
3.720	$\int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^2} dx$	6523
3.721	$\int \frac{1}{(a+b \cos(c+dx))^2 \sqrt{\sec(c+dx)}} dx$	6530
3.722	$\int \frac{1}{(a+b \cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)} dx$	6537
3.723	$\int \frac{1}{(a+b \cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)} dx$	6544
3.724	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$	6551
3.725	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$	6561
3.726	$\int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^3} dx$	6570
3.727	$\int \frac{1}{(a+b \cos(c+dx))^3 \sqrt{\sec(c+dx)}} dx$	6578
3.728	$\int \frac{1}{(a+b \cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)} dx$	6586
3.729	$\int \frac{1}{(a+b \cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)} dx$	6594
3.730	$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx$	6602
3.731	$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx$	6610
3.732	$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx$	6616
3.733	$\int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx$	6621

3.734	$\int \frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{\sec(c+dx)}} dx$	6625
3.735	$\int \frac{\sqrt{a+b \cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)} dx$	6634
3.736	$\int (a+b \cos(c+dx))^{3/2} \sec^{\frac{9}{2}}(c+dx) dx$	6642
3.737	$\int (a+b \cos(c+dx))^{3/2} \sec^{\frac{7}{2}}(c+dx) dx$	6650
3.738	$\int (a+b \cos(c+dx))^{3/2} \sec^{\frac{5}{2}}(c+dx) dx$	6658
3.739	$\int (a+b \cos(c+dx))^{3/2} \sec^{\frac{3}{2}}(c+dx) dx$	6664
3.740	$\int (a+b \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)} dx$	6671
3.741	$\int \frac{(a+b \cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}} dx$	6678
3.742	$\int \frac{(a+b \cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)} dx$	6687
3.743	$\int (a+b \cos(c+dx))^{5/2} \sec^{\frac{11}{2}}(c+dx) dx$	6696
3.744	$\int (a+b \cos(c+dx))^{5/2} \sec^{\frac{9}{2}}(c+dx) dx$	6705
3.745	$\int (a+b \cos(c+dx))^{5/2} \sec^{\frac{7}{2}}(c+dx) dx$	6713
3.746	$\int (a+b \cos(c+dx))^{5/2} \sec^{\frac{5}{2}}(c+dx) dx$	6721
3.747	$\int (a+b \cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx) dx$	6729
3.748	$\int (a+b \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)} dx$	6738
3.749	$\int \frac{(a+b \cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}} dx$	6747
3.750	$\int \frac{(a+b \cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)} dx$	6756
3.751	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$	6767
3.752	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$	6774
3.753	$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx$	6779
3.754	$\int \frac{1}{\sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)}} dx$	6783
3.755	$\int \frac{1}{\sqrt{a+b \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} dx$	6787
3.756	$\int \frac{1}{\sqrt{a+b \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} dx$	6795
3.757	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$	6803
3.758	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$	6811
3.759	$\int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^{3/2}} dx$	6818
3.760	$\int \frac{1}{(a+b \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} dx$	6824
3.761	$\int \frac{1}{(a+b \cos(c+dx))^{3/2} \sec^{\frac{3}{2}}(c+dx)} dx$	6830
3.762	$\int \frac{1}{(a+b \cos(c+dx))^{3/2} \sec^{\frac{5}{2}}(c+dx)} dx$	6837
3.763	$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	6846
3.764	$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$	6853
3.765	$\int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^{5/2}} dx$	6861

3.766	$\int \frac{1}{(a+b \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} dx$	6869
3.767	$\int \frac{1}{(a+b \cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} dx$	6877
3.768	$\int \frac{1}{(a+b \cos(c+dx))^{5/2} \sec^{\frac{5}{2}}(c+dx)} dx$	6884
3.769	$\int \cos^m(c+dx)(a+b \cos(c+dx))^4 dx$	6894
3.770	$\int \cos^m(c+dx)(a+b \cos(c+dx))^3 dx$	6900
3.771	$\int \cos^m(c+dx)(a+b \cos(c+dx))^2 dx$	6905
3.772	$\int \cos^m(c+dx)(a+b \cos(c+dx)) dx$	6909
3.773	$\int \frac{\cos^m(c+dx)}{a+b \cos(c+dx)} dx$	6913
3.774	$\int \frac{\cos^m(c+dx)}{(a+b \cos(c+dx))^2} dx$	6917
3.775	$\int (a+b \cos(c+dx))^3 \sec^m(c+dx) dx$	6922
3.776	$\int (a+b \cos(c+dx))^2 \sec^m(c+dx) dx$	6928
3.777	$\int (a+b \cos(c+dx)) \sec^m(c+dx) dx$	6933
3.778	$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}} dx$	6937
3.779	$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}} dx$	6941
3.780	$\int (a+a \cos(c+dx)) \left(-\frac{B}{2} + B \cos(c+dx)\right) dx$	6945
3.781	$\int (a+a \cos(c+dx))^4 \left(-\frac{4B}{5} + B \cos(c+dx)\right) dx$	6949
3.782	$\int (a+a \cos(c+dx))^n \left(-\frac{Bn}{1+n} + B \cos(c+dx)\right) dx$	6953
3.783	$\int \frac{-\frac{3B}{2} + B \cos(c+dx)}{(a+a \cos(c+dx))^3} dx$	6958
3.784	$\int (a+a \cos(c+dx))^{3/2} \left(-\frac{3B}{5} + B \cos(c+dx)\right) dx$	6962
3.785	$\int \frac{B+B \cos(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$	6966
3.786	$\int \frac{-\frac{5B}{3} + B \cos(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$	6982
3.787	$\int (a+a \cos(c+dx))^{2/3} (A+B \cos(c+dx)) dx$	6986
3.788	$\int \sqrt[3]{a+a \cos(c+dx)} (A+B \cos(c+dx)) dx$	6990
3.789	$\int \frac{A+B \cos(c+dx)}{\sqrt[3]{a+a \cos(c+dx)}} dx$	6994
3.790	$\int \frac{A+B \cos(c+dx)}{(a+a \cos(c+dx))^{2/3}} dx$	6998
3.791	$\int \frac{\frac{bB}{a} + B \cos(c+dx)}{a+b \cos(c+dx)} dx$	7002
3.792	$\int \frac{a+b \cos(c+dx)}{(b+a \cos(c+dx))^2} dx$	7007
3.793	$\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)} dx$	7011
3.794	$\int \frac{aB+bB \cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$	7015
3.795	$\int (a+b \cos(c+dx))^{2/3} (A+B \cos(c+dx)) dx$	7019
3.796	$\int \sqrt[3]{a+b \cos(c+dx)} (A+B \cos(c+dx)) dx$	7024
3.797	$\int \frac{A+B \cos(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx$	7029
3.798	$\int \frac{A+B \cos(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx$	7034
3.799	$\int \cos^2(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$	7039
3.800	$\int \cos(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$	7045
3.801	$\int \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$	7050

3.802	$\int \sqrt{b \cos(c+dx)}(A+B \cos(c+dx)) \sec(c+dx) dx$	7055
3.803	$\int \sqrt{b \cos(c+dx)}(A+B \cos(c+dx)) \sec^2(c+dx) dx$	7060
3.804	$\int \sqrt{b \cos(c+dx)}(A+B \cos(c+dx)) \sec^3(c+dx) dx$	7065
3.805	$\int \sqrt{b \cos(c+dx)}(A+B \cos(c+dx)) \sec^4(c+dx) dx$	7071
3.806	$\int \cos(c+dx)(b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) dx$	7077
3.807	$\int (b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) dx$	7083
3.808	$\int (b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) \sec(c+dx) dx$	7088
3.809	$\int (b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) \sec^2(c+dx) dx$	7093
3.810	$\int (b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) \sec^3(c+dx) dx$	7098
3.811	$\int (b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) \sec^4(c+dx) dx$	7103
3.812	$\int (b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) \sec^5(c+dx) dx$	7109
3.813	$\int (b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) dx$	7115
3.814	$\int (b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) \sec(c+dx) dx$	7120
3.815	$\int (b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) \sec^2(c+dx) dx$	7126
3.816	$\int (b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) \sec^3(c+dx) dx$	7132
3.817	$\int (b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) \sec^4(c+dx) dx$	7137
3.818	$\int (b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) \sec^5(c+dx) dx$	7142
3.819	$\int (b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) \sec^6(c+dx) dx$	7147
3.820	$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$	7153
3.821	$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$	7159
3.822	$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$	7164
3.823	$\int \frac{A+B \cos(c+dx)}{\sqrt{b \cos(c+dx)}} dx$	7169
3.824	$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{\sqrt{b \cos(c+dx)}} dx$	7173
3.825	$\int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx$	7178
3.826	$\int \frac{(A+B \cos(c+dx)) \sec^3(c+dx)}{\sqrt{b \cos(c+dx)}} dx$	7184
3.827	$\int \frac{\cos^4(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	7190
3.828	$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	7195
3.829	$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	7200
3.830	$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	7205
3.831	$\int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{3/2}} dx$	7210
3.832	$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{3/2}} dx$	7215
3.833	$\int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx$	7220
3.834	$\int \frac{\cos^5(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	7226
3.835	$\int \frac{\cos^4(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	7231
3.836	$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	7236
3.837	$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	7241
3.838	$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	7246

3.839	$\int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{5/2}} dx$	7251
3.840	$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{5/2}} dx$	7256
3.841	$\int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{7/2}} dx$	7262
3.842	$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$	7268
3.843	$\int \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$	7274
3.844	$\int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$	7280
3.845	$\int \frac{\sqrt{b \cos(c+dx)} (A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$	7285
3.846	$\int \frac{\sqrt{b \cos(c+dx)} (A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$	7289
3.847	$\int \frac{\sqrt{b \cos(c+dx)} (A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$	7294
3.848	$\int \frac{\sqrt{b \cos(c+dx)} (A+B \cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$	7299
3.849	$\int \frac{\sqrt{b \cos(c+dx)} (A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$	7305
3.850	$\int \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) dx$	7311
3.851	$\int \sqrt{\cos(c+dx)} (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) dx$	7317
3.852	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$	7322
3.853	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$	7327
3.854	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$	7331
3.855	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$	7336
3.856	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$	7341
3.857	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$	7346
3.858	$\int \sqrt{\cos(c+dx)} (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) dx$	7352
3.859	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$	7358
3.860	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$	7363
3.861	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$	7368
3.862	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$	7372
3.863	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$	7377
3.864	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$	7382
3.865	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)} dx$	7387
3.866	$\int \frac{\cos^{\frac{5}{2}}(c+dx) (A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$	7393
3.867	$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$	7398
3.868	$\int \frac{\sqrt{\cos(c+dx)} (A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$	7403

3.869	$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} dx$	7407
3.870	$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx$	7412
3.871	$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx$	7417
3.872	$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx$	7423
3.873	$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	7429
3.874	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	7434
3.875	$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	7438
3.876	$\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	7442
3.877	$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}} dx$	7447
3.878	$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx$	7452
3.879	$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx$	7457
3.880	$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	7463
3.881	$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	7468
3.882	$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	7472
3.883	$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	7476
3.884	$\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	7481
3.885	$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}} dx$	7486
3.886	$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{5/2}} dx$	7491
3.887	$\int \cos^2(c+dx) \sqrt[3]{b \cos(c+dx)}(A+B \cos(c+dx)) dx$	7497
3.888	$\int \cos(c+dx) \sqrt[3]{b \cos(c+dx)}(A+B \cos(c+dx)) dx$	7501
3.889	$\int \sqrt[3]{b \cos(c+dx)}(A+B \cos(c+dx)) dx$	7505
3.890	$\int \sqrt[3]{b \cos(c+dx)}(A+B \cos(c+dx)) \sec(c+dx) dx$	7509
3.891	$\int \sqrt[3]{b \cos(c+dx)}(A+B \cos(c+dx)) \sec^2(c+dx) dx$	7514
3.892	$\int \sqrt[3]{b \cos(c+dx)}(A+B \cos(c+dx)) \sec^3(c+dx) dx$	7519
3.893	$\int \cos^2(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx$	7524
3.894	$\int \cos(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx$	7528
3.895	$\int (b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx$	7532
3.896	$\int (b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) \sec(c+dx) dx$	7536
3.897	$\int (b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) \sec^2(c+dx) dx$	7540
3.898	$\int (b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) \sec^3(c+dx) dx$	7545
3.899	$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx$	7550
3.900	$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx$	7554
3.901	$\int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{2/3}} dx$	7558

3.902	$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{2/3}} dx$	7562
3.903	$\int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{2/3}} dx$	7566
3.904	$\int \frac{(A+B \cos(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{2/3}} dx$	7570
3.905	$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx$	7574
3.906	$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx$	7578
3.907	$\int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{4/3}} dx$	7582
3.908	$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{4/3}} dx$	7586
3.909	$\int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx$	7590
3.910	$\int \frac{(A+B \cos(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{4/3}} dx$	7594
3.911	$\int \cos^m(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$	7598
3.912	$\int \cos^2(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$	7603
3.913	$\int \cos(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$	7607
3.914	$\int (b \cos(c+dx))^n(A+B \cos(c+dx)) dx$	7611
3.915	$\int (b \cos(c+dx))^n(A+B \cos(c+dx)) \sec(c+dx) dx$	7615
3.916	$\int (b \cos(c+dx))^n(A+B \cos(c+dx)) \sec^2(c+dx) dx$	7619
3.917	$\int (b \cos(c+dx))^n(A+B \cos(c+dx)) \sec^3(c+dx) dx$	7623
3.918	$\int (b \cos(c+dx))^n(A+B \cos(c+dx)) \sec^4(c+dx) dx$	7627
3.919	$\int \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$	7632
3.920	$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$	7637
3.921	$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$	7642
3.922	$\int \frac{(b \cos(c+dx))^n(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$	7647
3.923	$\int \frac{(b \cos(c+dx))^n(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$	7651
3.924	$\int \frac{(b \cos(c+dx))^n(A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$	7655
3.925	$\int \frac{(b \cos(c+dx))^n(A+B \cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$	7659
3.926	$\int \frac{(b \cos(c+dx))^n(A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$	7663
3.927	$\int \cos^m(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx$	7667
3.928	$\int \cos^m(c+dx)(b \cos(c+dx))^{2/3}(A+B \cos(c+dx)) dx$	7672
3.929	$\int \cos^m(c+dx) \sqrt[3]{b \cos(c+dx)}(A+B \cos(c+dx)) dx$	7677
3.930	$\int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx$	7682
3.931	$\int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx$	7686
3.932	$\int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx$	7690

3.1 $\int \cos^5(c + dx)(a + a \cos(c + dx)) dx$

Optimal result	275
Rubi [A] (verified)	275
Mathematica [A] (verified)	277
Maple [A] (verified)	277
Fricas [A] (verification not implemented)	278
Sympy [B] (verification not implemented)	278
Maxima [A] (verification not implemented)	279
Giac [A] (verification not implemented)	279
Mupad [B] (verification not implemented)	279

Optimal result

Integrand size = 19, antiderivative size = 114

$$\int \cos^5(c + dx)(a + a \cos(c + dx)) dx = \frac{5ax}{16} + \frac{a \sin(c + dx)}{d} + \frac{5a \cos(c + dx) \sin(c + dx)}{16d} + \frac{5a \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{a \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin^5(c + dx)}{5d}$$

[Out] 5/16*a*x+a*sin(d*x+c)/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^5/d

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 2713, 2715, 8}

$$\int \cos^5(c + dx)(a + a \cos(c + dx)) dx = \frac{a \sin^5(c + dx)}{5d} - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin(c + dx)}{d} + \frac{a \sin(c + dx) \cos^5(c + dx)}{6d} + \frac{5a \sin(c + dx) \cos^3(c + dx)}{24d} + \frac{5a \sin(c + dx) \cos(c + dx)}{16d} + \frac{5ax}{16}$$

[In] Int[Cos[c + d*x]^5*(a + a*cos[c + d*x]),x]

[Out] (5*a*x)/16 + (a*Sin[c + d*x])/d + (5*a*cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a*cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^5)/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \cos^5(c + dx) dx + a \int \cos^6(c + dx) dx \\
 &= \frac{a \cos^5(c + dx) \sin(c + dx)}{6d} + \frac{1}{6}(5a) \int \cos^4(c + dx) dx \\
 &\quad - \frac{a \text{Subst}(\int (1 - 2x^2 + x^4) dx, x, -\sin(c + dx))}{d} \\
 &= \frac{a \sin(c + dx)}{d} + \frac{5a \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{a \cos^5(c + dx) \sin(c + dx)}{6d} \\
 &\quad - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin^5(c + dx)}{5d} + \frac{1}{8}(5a) \int \cos^2(c + dx) dx \\
 &= \frac{a \sin(c + dx)}{d} + \frac{5a \cos(c + dx) \sin(c + dx)}{16d} + \frac{5a \cos^3(c + dx) \sin(c + dx)}{24d} \\
 &\quad + \frac{a \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin^5(c + dx)}{5d} + \frac{1}{16}(5a) \int 1 dx
 \end{aligned}$$

$$= \frac{5ax}{16} + \frac{a \sin(c+dx)}{d} + \frac{5a \cos(c+dx) \sin(c+dx)}{16d} + \frac{5a \cos^3(c+dx) \sin(c+dx)}{24d}$$

$$+ \frac{a \cos^5(c+dx) \sin(c+dx)}{6d} - \frac{2a \sin^3(c+dx)}{3d} + \frac{a \sin^5(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \cos^5(c+dx)(a+a\cos(c+dx)) dx$$

$$= \frac{a(960 \sin(c+dx) - 640 \sin^3(c+dx) + 192 \sin^5(c+dx) + 5(60c + 60dx + 45 \sin(2(c+dx))) + 9 \sin(4(c+dx)))}{960d}$$

[In] Integrate[Cos[c + d*x]^5*(a + a*Cos[c + d*x]), x]

[Out] (a*(960*Sin[c + d*x] - 640*Sin[c + d*x]^3 + 192*Sin[c + d*x]^5 + 5*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])))/(960*d)

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.64

method	result
parallelerisch	$\frac{(60dx + \sin(6dx+6c) + 120 \sin(dx+c) + 45 \sin(2dx+2c) + 20 \sin(3dx+3c) + 9 \sin(4dx+4c) + \frac{12 \sin(5dx+5c)}{5})a}{192d}$
derivativedivides	$a \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{a \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5}$
default	$a \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{a \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5}$
parts	$a \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c) + \frac{a \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right)}{d}$
risch	$\frac{5ax}{16} + \frac{5a \sin(dx+c)}{8d} + \frac{a \sin(6dx+6c)}{192d} + \frac{a \sin(5dx+5c)}{80d} + \frac{3a \sin(4dx+4c)}{64d} + \frac{5a \sin(3dx+3c)}{48d} + \frac{15a \sin(2dx+2c)}{64d}$
norman	$\frac{5ax}{16} + \frac{27a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d} + \frac{107a \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24d} + \frac{283a \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{20d} + \frac{133a \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{20d} + \frac{39a \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d} + \frac{5a \left(\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d}$

[In] int(cos(d*x+c)^5*(a+cos(d*x+c)*a), x, method=_RETURNVERBOSE)

[Out] $1/192*(60*d*x+\sin(6*d*x+6*c)+120*\sin(d*x+c)+45*\sin(2*d*x+2*c)+20*\sin(3*d*x+3*c)+9*\sin(4*d*x+4*c)+12/5*\sin(5*d*x+5*c))*a/d$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \cos^5(c+dx)(a+a\cos(c+dx)) dx$$

$$= \frac{75 adx + (40 a \cos(dx+c))^5 + 48 a \cos(dx+c)^4 + 50 a \cos(dx+c)^3 + 64 a \cos(dx+c)^2 + 75 a \cos(dx+c)}{240 d}$$

[In] `integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="fricas")`

[Out] $1/240*(75*a*d*x + (40*a*\cos(d*x + c))^5 + 48*a*\cos(d*x + c)^4 + 50*a*\cos(d*x + c)^3 + 64*a*\cos(d*x + c)^2 + 75*a*\cos(d*x + c) + 128*a)*\sin(d*x + c))/d$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(107) = 214.

Time = 0.34 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.89

$$\int \cos^5(c+dx)(a+a\cos(c+dx)) dx$$

$$= \begin{cases} \frac{5ax \sin^6(c+dx)}{16} + \frac{15ax \sin^4(c+dx) \cos^2(c+dx)}{16} + \frac{15ax \sin^2(c+dx) \cos^4(c+dx)}{16} + \frac{5ax \cos^6(c+dx)}{16} + \frac{5a \sin^5(c+dx) \cos(c+dx)}{16d} + \frac{8a \sin^4(c+dx) \cos^2(c+dx)}{16d} \\ x(a \cos(c) + a) \cos^5(c) \end{cases}$$

[In] `integrate(cos(d*x+c)**5*(a+a*cos(d*x+c)),x)`

[Out] `Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8*a*sin(c + d*x)**5/(15*d) + 5*a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) + a*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**5, True))`

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \cos^5(c + dx)(a + a \cos(c + dx)) dx$$

$$= \frac{64 (3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))a - 5 (4 \sin(2dx + 2c)^3 - 60 dx - 60c - 9 \sin(4dx + 4c) - 48 \sin(2dx + 2c))a}{960 d}$$

[In] integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int \cos^5(c + dx)(a + a \cos(c + dx)) dx = \frac{5}{16} ax + \frac{a \sin(6 dx + 6 c)}{192 d} + \frac{a \sin(5 dx + 5 c)}{80 d}$$

$$+ \frac{3 a \sin(4 dx + 4 c)}{64 d} + \frac{5 a \sin(3 dx + 3 c)}{48 d}$$

$$+ \frac{15 a \sin(2 dx + 2 c)}{64 d} + \frac{5 a \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] 5/16*a*x + 1/192*a*sin(6*d*x + 6*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 3/64*a*sin(4*d*x + 4*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 15/64*a*sin(2*d*x + 2*c)/d + 5/8*a*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 16.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \cos^5(c + dx)(a + a \cos(c + dx)) dx = \frac{5 a x}{16}$$

$$+ \frac{5 a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11}}{8} + \frac{39 a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{8} + \frac{133 a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{20} + \frac{283 a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{20} + \frac{107 a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{24} + \frac{27 a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8}$$

$$d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)^6$$

```
[In] int(cos(c + d*x)^5*(a + a*cos(c + d*x)),x)
```

```
[Out] (5*a*x)/16 + ((27*a*tan(c/2 + (d*x)/2))/8 + (107*a*tan(c/2 + (d*x)/2)^3)/24  
+ (283*a*tan(c/2 + (d*x)/2)^5)/20 + (133*a*tan(c/2 + (d*x)/2)^7)/20 + (39*  
a*tan(c/2 + (d*x)/2)^9)/8 + (5*a*tan(c/2 + (d*x)/2)^11)/8)/(d*(tan(c/2 + (d  
*x)/2)^2 + 1)^6)
```

3.2 $\int \cos^4(c + dx)(a + a \cos(c + dx)) dx$

Optimal result	281
Rubi [A] (verified)	281
Mathematica [A] (verified)	283
Maple [A] (verified)	283
Fricas [A] (verification not implemented)	284
Sympy [A] (verification not implemented)	284
Maxima [A] (verification not implemented)	284
Giac [A] (verification not implemented)	285
Mupad [B] (verification not implemented)	285

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \cos^4(c + dx)(a + a \cos(c + dx)) dx = \frac{3ax}{8} + \frac{a \sin(c + dx)}{d} + \frac{3a \cos(c + dx) \sin(c + dx)}{8d} + \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin^5(c + dx)}{5d}$$

[Out] $3/8*a*x+a*\sin(d*x+c)/d+3/8*a*\cos(d*x+c)*\sin(d*x+c)/d+1/4*a*\cos(d*x+c)^3*\sin(d*x+c)/d-2/3*a*\sin(d*x+c)^3/d+1/5*a*\sin(d*x+c)^5/d$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 2715, 8, 2713}

$$\int \cos^4(c + dx)(a + a \cos(c + dx)) dx = \frac{a \sin^5(c + dx)}{5d} - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin(c + dx)}{d} + \frac{a \sin(c + dx) \cos^3(c + dx)}{4d} + \frac{3a \sin(c + dx) \cos(c + dx)}{8d} + \frac{3ax}{8}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^4*(a + a*\text{Cos}[c + d*x]), x]$

[Out] $(3*a*x)/8 + (a*\text{Sin}[c + d*x])/d + (3*a*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (a*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(4*d) - (2*a*\text{Sin}[c + d*x]^3)/(3*d) + (a*\text{Sin}[c + d*x]^5)/(5*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2715

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \cos^4(c + dx) dx + a \int \cos^5(c + dx) dx \\
 &= \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{1}{4}(3a) \int \cos^2(c + dx) dx \\
 &\quad - \frac{a \text{Subst}(\int (1 - 2x^2 + x^4) dx, x, -\sin(c + dx))}{d} \\
 &= \frac{a \sin(c + dx)}{d} + \frac{3a \cos(c + dx) \sin(c + dx)}{8d} + \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} \\
 &\quad - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin^5(c + dx)}{5d} + \frac{1}{8}(3a) \int 1 dx \\
 &= \frac{3ax}{8} + \frac{a \sin(c + dx)}{d} + \frac{3a \cos(c + dx) \sin(c + dx)}{8d} \\
 &\quad + \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin^5(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \cos^4(c + dx)(a + a \cos(c + dx)) dx$$

$$= \frac{a(480 \sin(c + dx) - 320 \sin^3(c + dx) + 96 \sin^5(c + dx) + 15(12(c + dx) + 8 \sin(2(c + dx))) + \sin(4(c + dx)))}{480d}$$

[In] Integrate[Cos[c + d*x]^4*(a + a*Cos[c + d*x]),x]

[Out] (a*(480*Sin[c + d*x] - 320*Sin[c + d*x]^3 + 96*Sin[c + d*x]^5 + 15*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])))/(480*d)

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{a(180dx+300 \sin(dx+c)+6 \sin(5dx+5c)+15 \sin(4dx+4c)+50 \sin(3dx+3c)+120 \sin(2dx+2c))}{480d}$
derivativedivides	$\frac{a \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + a \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right)}{d}$
default	$\frac{a \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + a \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right)}{d}$
parts	$\frac{a \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right)}{d} + \frac{a \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5d}$
risch	$\frac{3ax}{8} + \frac{5a \sin(dx+c)}{8d} + \frac{a \sin(5dx+5c)}{80d} + \frac{a \sin(4dx+4c)}{32d} + \frac{5a \sin(3dx+3c)}{48d} + \frac{a \sin(2dx+2c)}{4d}$
norman	$\frac{3ax}{8} + \frac{13a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} + \frac{19a \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d} + \frac{116a \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15d} + \frac{13a \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d} + \frac{3a \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} + \frac{15ax \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8} \frac{1}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^5}$

[In] int(cos(d*x+c)^4*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] 1/480*a*(180*d*x+300*sin(d*x+c)+6*sin(5*d*x+5*c)+15*sin(4*d*x+4*c)+50*sin(3*d*x+3*c)+120*sin(2*d*x+2*c))/d

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \cos^4(c + dx)(a + a \cos(c + dx)) dx$$

$$= \frac{45 adx + (24 a \cos(dx + c)^4 + 30 a \cos(dx + c)^3 + 32 a \cos(dx + c)^2 + 45 a \cos(dx + c) + 64 a) \sin(dx + c)}{120 d}$$

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(45*a*d*x + (24*a*cos(d*x + c)^4 + 30*a*cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 + 45*a*cos(d*x + c) + 64*a)*sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

$$\int \cos^4(c + dx)(a + a \cos(c + dx)) dx$$

$$= \begin{cases} \frac{3ax \sin^4(c+dx)}{8} + \frac{3ax \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{3ax \cos^4(c+dx)}{8} + \frac{8a \sin^5(c+dx)}{15d} + \frac{4a \sin^3(c+dx) \cos^2(c+dx)}{3d} + \frac{3a \sin^3(c+dx) \cos(c+dx)}{8d} \\ x(a \cos(c) + a) \cos^4(c) \end{cases}$$

```
[In] integrate(cos(d*x+c)**4*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*x*cos(c + d*x)**4/8 + 8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + a*sin(c + d*x)*cos(c + d*x)**4/d + 5*a*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \cos^4(c + dx)(a + a \cos(c + dx)) dx$$

$$= \frac{32 (3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))a + 15 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c))a}{480 d}$$

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```


Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int \cos^4(c + dx)(a + a \cos(c + dx)) dx = \frac{3}{8} ax + \frac{a \sin(5 dx + 5 c)}{80 d} + \frac{a \sin(4 dx + 4 c)}{32 d} + \frac{5 a \sin(3 dx + 3 c)}{48 d} + \frac{a \sin(2 dx + 2 c)}{4 d} + \frac{5 a \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] 3/8*a*x + 1/80*a*sin(5*d*x + 5*c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 5/8*a*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 16.63 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \cos^4(c + dx)(a + a \cos(c + dx)) dx = \frac{3 a x}{8} + \frac{\frac{3 a \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^9}{4} + \frac{13 a \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^7}{6} + \frac{116 a \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^5}{15} + \frac{19 a \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^3}{6} + \frac{13 a \tan\left(\frac{c}{2} + \frac{d x}{2}\right)}{4}}{d \left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right)^2 + 1\right)^5}$$

[In] int(cos(c + d*x)^4*(a + a*cos(c + d*x)),x)

[Out] (3*a*x)/8 + ((13*a*tan(c/2 + (d*x)/2))/4 + (19*a*tan(c/2 + (d*x)/2)^3)/6 + (116*a*tan(c/2 + (d*x)/2)^5)/15 + (13*a*tan(c/2 + (d*x)/2)^7)/6 + (3*a*tan(c/2 + (d*x)/2)^9)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)

3.3 $\int \cos^3(c + dx)(a + a \cos(c + dx)) dx$

Optimal result	286
Rubi [A] (verified)	286
Mathematica [A] (verified)	288
Maple [A] (verified)	288
Fricas [A] (verification not implemented)	289
Sympy [B] (verification not implemented)	289
Maxima [A] (verification not implemented)	289
Giac [A] (verification not implemented)	290
Mupad [B] (verification not implemented)	290

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \cos^3(c + dx)(a + a \cos(c + dx)) dx = \frac{3ax}{8} + \frac{a \sin(c + dx)}{d} + \frac{3a \cos(c + dx) \sin(c + dx)}{8d} + \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{a \sin^3(c + dx)}{3d}$$

[Out] $3/8*a*x+a*\sin(d*x+c)/d+3/8*a*\cos(d*x+c)*\sin(d*x+c)/d+1/4*a*\cos(d*x+c)^3*\sin(d*x+c)/d-1/3*a*\sin(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 2713, 2715, 8}

$$\int \cos^3(c + dx)(a + a \cos(c + dx)) dx = -\frac{a \sin^3(c + dx)}{3d} + \frac{a \sin(c + dx)}{d} + \frac{a \sin(c + dx) \cos^3(c + dx)}{4d} + \frac{3a \sin(c + dx) \cos(c + dx)}{8d} + \frac{3ax}{8}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^3*(a + a*\text{Cos}[c + d*x]), x]$

[Out] $(3*a*x)/8 + (a*\text{Sin}[c + d*x])/d + (3*a*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (a*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(4*d) - (a*\text{Sin}[c + d*x]^3)/(3*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \cos^3(c + dx) dx + a \int \cos^4(c + dx) dx \\
 &= \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{1}{4}(3a) \int \cos^2(c + dx) dx \\
 &\quad - \frac{a \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{d} \\
 &= \frac{a \sin(c + dx)}{d} + \frac{3a \cos(c + dx) \sin(c + dx)}{8d} \\
 &\quad + \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{a \sin^3(c + dx)}{3d} + \frac{1}{8}(3a) \int 1 dx \\
 &= \frac{3ax}{8} + \frac{a \sin(c + dx)}{d} + \frac{3a \cos(c + dx) \sin(c + dx)}{8d} \\
 &\quad + \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{a \sin^3(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \cos^3(c + dx)(a + a \cos(c + dx)) dx = \frac{3a(c + dx)}{8d} + \frac{a \sin(c + dx)}{d} - \frac{a \sin^3(c + dx)}{3d} + \frac{a \sin(2(c + dx))}{4d} + \frac{a \sin(4(c + dx))}{32d}$$

[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x]),x]

[Out] (3*a*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result
parallelrisc	$\frac{a(36dx+3 \sin(4dx+4c)+8 \sin(3dx+3c)+24 \sin(2dx+2c)+72 \sin(dx+c))}{96d}$
derivativedivides	$\frac{a \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{a(2+\cos^2(dx+c)) \sin(dx+c)}{3}}{d}$
default	$\frac{a \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{a(2+\cos^2(dx+c)) \sin(dx+c)}{3}}{d}$
parts	$\frac{a(2+\cos^2(dx+c)) \sin(dx+c)}{3d} + \frac{a \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right)}{d}$
risc	$\frac{3ax}{8} + \frac{3a \sin(dx+c)}{4d} + \frac{a \sin(4dx+4c)}{32d} + \frac{a \sin(3dx+3c)}{12d} + \frac{a \sin(2dx+2c)}{4d}$
norman	$\frac{3ax}{8} + \frac{13a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} + \frac{31a \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} + \frac{49a \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} + \frac{3a \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} + \frac{3ax \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + \frac{9ax \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4}$

[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] 1/96*a*(36*d*x+3*sin(4*d*x+4*c)+8*sin(3*d*x+3*c)+24*sin(2*d*x+2*c)+72*sin(d*x+c))/d

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \cos^3(c+dx)(a+a\cos(c+dx)) dx$$

$$= \frac{9adx + (6a\cos(dx+c))^3 + 8a\cos(dx+c)^2 + 9a\cos(dx+c) + 16a\sin(dx+c)}{24d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(9*a*d*x + (6*a*cos(d*x + c))^3 + 8*a*cos(d*x + c)^2 + 9*a*cos(d*x + c) + 16*a)*sin(d*x + c)/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(70) = 140.

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.89

$$\int \cos^3(c+dx)(a+a\cos(c+dx)) dx$$

$$= \begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2a\sin^3(c+dx)}{3d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{8d} \\ x(a\cos(c) + a)\cos^3(c) \end{cases}$$

[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c)),x)

[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a*sin(c + d*x)**3/(3*d) + 5*a*sin(c + d*x)*cos(c + d*x)**3/(8*d) + a*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \cos^3(c+dx)(a+a\cos(c+dx)) dx =$$

$$\frac{32(\sin(dx+c)^3 - 3\sin(dx+c))a - 3(12dx + 12c + \sin(4dx + 4c) + 8\sin(2dx + 2c))a}{96d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \cos^3(c + dx)(a + a \cos(c + dx)) dx = \frac{3}{8} ax + \frac{a \sin(4 dx + 4 c)}{32 d} + \frac{a \sin(3 dx + 3 c)}{12 d} + \frac{a \sin(2 dx + 2 c)}{4 d} + \frac{3 a \sin(dx + c)}{4 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] 3/8*a*x + 1/32*a*sin(4*d*x + 4*c)/d + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 3/4*a*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \cos^3(c + dx)(a + a \cos(c + dx)) dx = \frac{3 a x}{8} + \frac{\frac{3 a \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^7}{4} + \frac{49 a \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^5}{12} + \frac{31 a \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^3}{12} + \frac{13 a \tan\left(\frac{c}{2} + \frac{d x}{2}\right)}{4}}{d \left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right)^2 + 1\right)^4}$$

[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x)),x)

[Out] (3*a*x)/8 + ((13*a*tan(c/2 + (d*x)/2))/4 + (31*a*tan(c/2 + (d*x)/2)^3)/12 + (49*a*tan(c/2 + (d*x)/2)^5)/12 + (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)

3.4 $\int \cos^2(c + dx)(a + a \cos(c + dx)) dx$

Optimal result	291
Rubi [A] (verified)	291
Mathematica [A] (verified)	292
Maple [A] (verified)	293
Fricas [A] (verification not implemented)	293
Sympy [A] (verification not implemented)	294
Maxima [A] (verification not implemented)	294
Giac [A] (verification not implemented)	294
Mupad [B] (verification not implemented)	295

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cos^2(c + dx)(a + a \cos(c + dx)) dx = \frac{ax}{2} + \frac{a \sin(c + dx)}{d} + \frac{a \cos(c + dx) \sin(c + dx)}{2d} - \frac{a \sin^3(c + dx)}{3d}$$

[Out] $1/2*a*x+a*\sin(d*x+c)/d+1/2*a*\cos(d*x+c)*\sin(d*x+c)/d-1/3*a*\sin(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 2715, 8, 2713}

$$\int \cos^2(c + dx)(a + a \cos(c + dx)) dx = -\frac{a \sin^3(c + dx)}{3d} + \frac{a \sin(c + dx)}{d} + \frac{a \sin(c + dx) \cos(c + dx)}{2d} + \frac{ax}{2}$$

[In] `Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x]),x]`

[Out] $(a*x)/2 + (a*\sin[c + d*x])/d + (a*\cos[c + d*x]*\sin[c + d*x])/(2*d) - (a*\sin[c + d*x]^3)/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \cos^2(c + dx) dx + a \int \cos^3(c + dx) dx \\ &= \frac{a \cos(c + dx) \sin(c + dx)}{2d} + \frac{1}{2}a \int 1 dx - \frac{a \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{d} \\ &= \frac{ax}{2} + \frac{a \sin(c + dx)}{d} + \frac{a \cos(c + dx) \sin(c + dx)}{2d} - \frac{a \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \cos^2(c + dx)(a + a \cos(c + dx)) dx = \frac{a(c + dx)}{2d} + \frac{a \sin(c + dx)}{d} - \frac{a \sin^3(c + dx)}{3d} + \frac{a \sin(2(c + dx))}{4d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[2*(c + d*x)])/(4*d)
```


Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{a(6dx+\sin(3dx+3c))+3\sin(2dx+2c)+9\sin(dx+c)}{12d}$
risch	$\frac{ax}{2} + \frac{3a\sin(dx+c)}{4d} + \frac{a\sin(3dx+3c)}{12d} + \frac{a\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a(2+\cos^2(dx+c))\sin(dx+c)}{3} + a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)$ d
default	$\frac{a(2+\cos^2(dx+c))\sin(dx+c)}{3} + a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)$ d
parts	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{a(2+\cos^2(dx+c))\sin(dx+c)}{3d}$
norman	$\frac{a\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} + \frac{ax}{2} + \frac{3a\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{4a\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3d} + \frac{3ax\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{3ax\left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{ax\left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2}$ $(1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^3$

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*a*(6*d*x+sin(3*d*x+3*c))+3*sin(2*d*x+2*c)+9*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos^2(c+dx)(a+a\cos(c+dx))dx$$

$$= \frac{3adx + (2a\cos(dx+c))^2 + 3a\cos(dx+c) + 4a)\sin(dx+c)}{6d}$$

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*d*x + (2*a*cos(d*x + c))^2 + 3*a*cos(d*x + c) + 4*a)*sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \cos^2(c + dx)(a + a \cos(c + dx)) dx$$

$$= \begin{cases} \frac{ax \sin^2(c+dx)}{2} + \frac{ax \cos^2(c+dx)}{2} + \frac{2a \sin^3(c+dx)}{3d} + \frac{a \sin(c+dx) \cos^2(c+dx)}{d} + \frac{a \sin(c+dx) \cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a \cos(c) + a) \cos^2(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + 2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d + a*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \cos^2(c + dx)(a + a \cos(c + dx)) dx$$

$$= -\frac{4(\sin(dx + c)^3 - 3 \sin(dx + c))a - 3(2dx + 2c + \sin(2dx + 2c))a}{12d}$$

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \cos^2(c + dx)(a + a \cos(c + dx)) dx = \frac{1}{2}ax + \frac{a \sin(3dx + 3c)}{12d}$$

$$+ \frac{a \sin(2dx + 2c)}{4d} + \frac{3a \sin(dx + c)}{4d}$$

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 3/4*a*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \cos^2(c + dx)(a + a \cos(c + dx)) dx = \frac{ax}{2} + \frac{2a \sin(c + dx)}{3d} + \frac{a \cos(c + dx) \sin(c + dx)}{2d} + \frac{a \cos(c + dx)^2 \sin(c + dx)}{3d}$$

[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x)),x)

[Out] (a*x)/2 + (2*a*sin(c + d*x))/(3*d) + (a*cos(c + d*x)*sin(c + d*x))/(2*d) + (a*cos(c + d*x)^2*sin(c + d*x))/(3*d)

3.5 $\int \cos(c + dx)(a + a \cos(c + dx)) dx$

Optimal result	296
Rubi [A] (verified)	296
Mathematica [A] (verified)	297
Maple [A] (verified)	297
Fricas [A] (verification not implemented)	297
Sympy [B] (verification not implemented)	298
Maxima [A] (verification not implemented)	298
Giac [A] (verification not implemented)	298
Mupad [B] (verification not implemented)	299

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \cos(c + dx)(a + a \cos(c + dx)) dx = \frac{ax}{2} + \frac{a \sin(c + dx)}{d} + \frac{a \cos(c + dx) \sin(c + dx)}{2d}$$

[Out] $1/2*a*x+a*\sin(d*x+c)/d+1/2*a*\cos(d*x+c)*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.02 (sec), antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2813}

$$\int \cos(c + dx)(a + a \cos(c + dx)) dx = \frac{a \sin(c + dx)}{d} + \frac{a \sin(c + dx) \cos(c + dx)}{2d} + \frac{ax}{2}$$

[In] `Int[Cos[c + d*x]*(a + a*Cos[c + d*x]),x]`

[Out] `(a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)`

Rule 2813

`Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Cos[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]`

Rubi steps

$$\text{integral} = \frac{ax}{2} + \frac{a \sin(c + dx)}{d} + \frac{a \cos(c + dx) \sin(c + dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \cos(c + dx)(a + a \cos(c + dx)) dx = \frac{a(2(c + dx) + 4 \sin(c + dx) + \sin(2(c + dx)))}{4d}$$

[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x]),x]

[Out] (a*(2*(c + d*x) + 4*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\frac{a(2dx+4 \sin(dx+c)+\sin(2dx+2c))}{4d}$	29
risch	$\frac{ax}{2} + \frac{a \sin(dx+c)}{d} + \frac{a \sin(2dx+2c)}{4d}$	32
derivativedivides	$\frac{a\left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + a \sin(dx+c)}{d}$	38
default	$\frac{a\left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + a \sin(dx+c)}{d}$	38
parts	$\frac{a\left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{a \sin(dx+c)}{d}$	40
norman	$\frac{\frac{a \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} + ax \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{ax}{2} + \frac{3a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{ax \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}$	82

[In] int(cos(d*x+c)*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] 1/4*a*(2*d*x+4*sin(d*x+c)+sin(2*d*x+2*c))/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \cos(c + dx)(a + a \cos(c + dx)) dx = \frac{adx + (a \cos(dx + c) + 2a) \sin(dx + c)}{2d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(a*d*x + (a*cos(d*x + c) + 2*a)*sin(d*x + c))/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \cos(c + dx)(a + a \cos(c + dx)) dx$$

$$= \begin{cases} \frac{ax \sin^2(c+dx)}{2} + \frac{ax \cos^2(c+dx)}{2} + \frac{a \sin(c+dx) \cos(c+dx)}{2d} + \frac{a \sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(a \cos(c) + a) \cos(c) & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x)

[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*cos(c + d*x)/(2*d) + a*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c), True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \cos(c + dx)(a + a \cos(c + dx)) dx = \frac{(2 dx + 2 c + \sin(2 dx + 2 c))a + 4 a \sin(dx + c)}{4 d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*a*sin(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos(c + dx)(a + a \cos(c + dx)) dx = \frac{1}{2} ax + \frac{a \sin(2 dx + 2 c)}{4 d} + \frac{a \sin(dx + c)}{d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] 1/2*a*x + 1/4*a*sin(2*d*x + 2*c)/d + a*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 15.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \cos(c + dx)(a + a \cos(c + dx)) dx = \frac{ax}{2} + \frac{a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 3a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)^2}$$

[In] int(cos(c + d*x)*(a + a*cos(c + d*x)),x)

[Out] (a*x)/2 + (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^2)

3.6 $\int (a + a \cos(c + dx)) dx$

Optimal result	300
Rubi [A] (verified)	300
Mathematica [A] (verified)	301
Maple [A] (verified)	301
Fricas [A] (verification not implemented)	301
Sympy [A] (verification not implemented)	302
Maxima [A] (verification not implemented)	302
Giac [A] (verification not implemented)	302
Mupad [B] (verification not implemented)	302

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int (a + a \cos(c + dx)) dx = ax + \frac{a \sin(c + dx)}{d}$$

[Out] `a*x+a*sin(d*x+c)/d`

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2717}

$$\int (a + a \cos(c + dx)) dx = \frac{a \sin(c + dx)}{d} + ax$$

[In] `Int[a + a*Cos[c + d*x],x]`

[Out] `a*x + (a*Sin[c + d*x])/d`

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= ax + a \int \cos(c + dx) dx \\ &= ax + \frac{a \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int (a + a \cos(c + dx)) dx = ax + \frac{a \cos(dx) \sin(c)}{d} + \frac{a \cos(c) \sin(dx)}{d}$$

[In] Integrate[a + a*Cos[c + d*x],x]

[Out] a*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax + \frac{a \sin(dx+c)}{d}$	16
risch	$ax + \frac{a \sin(dx+c)}{d}$	16
parallelrisch	$ax + \frac{a \sin(dx+c)}{d}$	16
parts	$ax + \frac{a \sin(dx+c)}{d}$	16
derivativedivides	$\frac{a(dx+c)+a \sin(dx+c)}{d}$	21
norman	$\frac{ax+ax \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \frac{2a \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d}}{1+\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right)}$	50

[In] int(a*cos(d*x+c)*a,x,method=_RETURNVERBOSE)

[Out] a*x+a*sin(d*x+c)/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + a \cos(c + dx)) dx = \frac{adx + a \sin(dx + c)}{d}$$

[In] integrate(a+a*cos(d*x+c),x, algorithm="fricas")

[Out] (a*d*x + a*sin(d*x + c))/d

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + a \cos(c + dx)) dx = ax + a \begin{cases} \frac{\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x \cos(c) & \text{otherwise} \end{cases}$$

[In] integrate(a+a*cos(d*x+c),x)

[Out] a*x + a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx)) dx = ax + \frac{a \sin(dx + c)}{d}$$

[In] integrate(a+a*cos(d*x+c),x, algorithm="maxima")

[Out] a*x + a*sin(d*x + c)/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx)) dx = ax + \frac{a \sin(dx + c)}{d}$$

[In] integrate(a+a*cos(d*x+c),x, algorithm="giac")

[Out] a*x + a*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx)) dx = ax + \frac{a \sin(c + dx)}{d}$$

[In] int(a + a*cos(c + d*x),x)

[Out] a*x + (a*sin(c + d*x))/d

3.7 $\int (a + a \cos(c + dx)) \sec(c + dx) dx$

Optimal result	303
Rubi [A] (verified)	303
Mathematica [A] (verified)	304
Maple [A] (verified)	304
Fricas [B] (verification not implemented)	305
Sympy [A] (verification not implemented)	305
Maxima [A] (verification not implemented)	305
Giac [B] (verification not implemented)	306
Mupad [B] (verification not implemented)	306

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int (a + a \cos(c + dx)) \sec(c + dx) dx = ax + \frac{a \operatorname{arctanh}(\sin(c + dx))}{d}$$

[Out] `a*x+a*arctanh(sin(d*x+c))/d`

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2814, 3855}

$$\int (a + a \cos(c + dx)) \sec(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{d} + ax$$

[In] `Int[(a + a*Cos[c + d*x])*Sec[c + d*x],x]`

[Out] `a*x + (a*ArcTanh[Sin[c + d*x]])/d`

Rule 2814

`Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \text{integral} &= ax + a \int \sec(c + dx) dx \\ &= ax + \frac{a \operatorname{arctanh}(\sin(c + dx))}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx)) \sec(c + dx) dx = ax + \frac{a \operatorname{arctanh}(\sin(c + dx))}{d}$$

[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x],x]

[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result	size
derivativdivides	$\frac{a(dx+c)+a \ln(\sec(dx+c)+\tan(dx+c))}{d}$	29
default	$\frac{a(dx+c)+a \ln(\sec(dx+c)+\tan(dx+c))}{d}$	29
parts	$\frac{a \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a(dx+c)}{d}$	31
parallelrisch	$\frac{a \left(dx + \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) - \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) \right)}{d}$	36
risch	$ax + \frac{a \ln(e^{i(dx+c)} + i)}{d} - \frac{a \ln(e^{i(dx+c)} - i)}{d}$	42
norman	$\frac{ax+ax \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{1+\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} + \frac{a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)}{d} - \frac{a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)}{d}$	71

[In] int((a+cos(d*x+c)*a)*sec(d*x+c),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int (a + a \cos(c + dx)) \sec(c + dx) dx$$

$$= \frac{2 a dx + a \log(\sin(dx + c) + 1) - a \log(-\sin(dx + c) + 1)}{2 d}$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")

[Out] 1/2*(2*a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d

Sympy [A] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int (a + a \cos(c + dx)) \sec(c + dx) dx = ax + a \left(\begin{cases} \frac{x \tan(c) \sec(c)}{\tan(c) + \sec(c)} + \frac{x \sec^2(c)}{\tan(c) + \sec(c)} & \text{for } d = 0 \\ \frac{\log(\tan(c + dx) + \sec(c + dx))}{d} & \text{otherwise} \end{cases} \right)$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x)

[Out] a*x + a*Piecewise((x*tan(c)*sec(c)/(tan(c) + sec(c)) + x*sec(c)**2/(tan(c) + sec(c)), Eq(d, 0)), (log(tan(c + d*x) + sec(c + d*x))/d, True))

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int (a + a \cos(c + dx)) \sec(c + dx) dx = \frac{(dx + c)a + a \log(\sec(dx + c) + \tan(dx + c))}{d}$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")

[Out] ((d*x + c)*a + a*log(sec(d*x + c) + tan(d*x + c)))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(16) = 32$.

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.69

$$\int (a + a \cos(c + dx)) \sec(c + dx) dx$$

$$= \frac{(dx + c)a + a \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - a \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right)}{d}$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")

[Out] ((d*x + c)*a + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)))/d

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (a + a \cos(c + dx)) \sec(c + dx) dx = ax + \frac{2a \operatorname{atanh} \left(\tan \left(\frac{c}{2} + \frac{dx}{2} \right) \right)}{d}$$

[In] int((a + a*cos(c + d*x))/cos(c + d*x),x)

[Out] a*x + (2*a*atanh(tan(c/2 + (d*x)/2)))/d

3.8 $\int (a + a \cos(c + dx)) \sec^2(c + dx) dx$

Optimal result	307
Rubi [A] (verified)	307
Mathematica [A] (verified)	308
Maple [A] (verified)	308
Fricas [B] (verification not implemented)	309
Sympy [F]	309
Maxima [A] (verification not implemented)	310
Giac [B] (verification not implemented)	310
Mupad [B] (verification not implemented)	310

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (a + a \cos(c + dx)) \sec^2(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a \tan(c + dx)}{d}$$

[Out] `a*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d`

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 3852, 8, 3855}

$$\int (a + a \cos(c + dx)) \sec^2(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a \tan(c + dx)}{d}$$

[In] `Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^2,x]`

[Out] `(a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d`

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2827

`Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \sec(c + dx) dx + a \int \sec^2(c + dx) dx \\ &= \frac{a \operatorname{arctanh}(\sin(c + dx))}{d} - \frac{a \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\ &= \frac{a \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a \tan(c + dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx)) \sec^2(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a \tan(c + dx)}{d}$$

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{a \ln(\sec(dx+c)+\tan(dx+c))+a \tan(dx+c)}{d}$	30
default	$\frac{a \ln(\sec(dx+c)+\tan(dx+c))+a \tan(dx+c)}{d}$	30
parts	$\frac{a \tan(dx+c)}{d} + \frac{a \ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
risch	$\frac{2ia}{d(e^{2i(dx+c)}+1)} + \frac{a \ln(e^{i(dx+c)}+i)}{d} - \frac{a \ln(e^{i(dx+c)}-i)}{d}$	59
parallelrisc	$\frac{a \left(-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) \cos(dx+c) + \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) \cos(dx+c) + \sin(dx+c) \right)}{d \cos(dx+c)}$	60
norman	$\frac{-\frac{2a \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} - \frac{2a \left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right) \left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{a \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d} - \frac{a \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	101

```
[In] int((a*cos(d*x+c)*a)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*ln(sec(d*x+c)+tan(d*x+c))+a*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int (a + a \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{a \cos(dx + c) \log(\sin(dx + c) + 1) - a \cos(dx + c) \log(-\sin(dx + c) + 1) + 2a \sin(dx + c)}{2d \cos(dx + c)}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a + a \cos(c + dx)) \sec^2(c + dx) dx = a \left(\int \cos(c + dx) \sec^2(c + dx) dx + \int \sec^2(c + dx) dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int (a + a \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{a(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 2a \tan(dx + c)}{2d}$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")

[Out] 1/2*(a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*a*tan(d*x + c))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int (a + a \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{a \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|) - a \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|) - \frac{2a \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1}}{d}$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")

[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d

Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int (a + a \cos(c + dx)) \sec^2(c + dx) dx = \frac{2a \operatorname{atanh}(\tan(\frac{c}{2} + \frac{dx}{2}))}{d} - \frac{2a \tan(\frac{c}{2} + \frac{dx}{2})}{d \left(\tan(\frac{c}{2} + \frac{dx}{2})^2 - 1 \right)}$$

[In] int((a + a*cos(c + d*x))/cos(c + d*x)^2,x)

[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*a*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))

3.9 $\int (a + a \cos(c + dx)) \sec^3(c + dx) dx$

Optimal result	311
Rubi [A] (verified)	311
Mathematica [A] (verified)	312
Maple [A] (verified)	313
Fricas [A] (verification not implemented)	313
Sympy [F]	314
Maxima [A] (verification not implemented)	314
Giac [A] (verification not implemented)	314
Mupad [B] (verification not implemented)	315

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int (a + a \cos(c + dx)) \sec^3(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a \tan(c + dx)}{d} + \frac{a \sec(c + dx) \tan(c + dx)}{2d}$$

[Out] $1/2*a*\operatorname{arctanh}(\sin(d*x+c))/d+a*\tan(d*x+c)/d+1/2*a*\sec(d*x+c)*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.05 (sec), antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2827, 3853, 3855, 3852, 8}

$$\int (a + a \cos(c + dx)) \sec^3(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a \tan(c + dx)}{d} + \frac{a \tan(c + dx) \sec(c + dx)}{2d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])* \operatorname{Sec}[c + d*x]^3, x]$

[Out] $(a*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (a*\operatorname{Tan}[c + d*x])/d + (a*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \sec^2(c + dx) dx + a \int \sec^3(c + dx) dx \\ &= \frac{a \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2}a \int \sec(c + dx) dx - \frac{a \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\ &= \frac{a \arctanh(\sin(c + dx))}{2d} + \frac{a \tan(c + dx)}{d} + \frac{a \sec(c + dx) \tan(c + dx)}{2d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx)) \sec^3(c + dx) dx = \frac{a \arctanh(\sin(c + dx))}{2d} + \frac{a \tan(c + dx)}{d} + \frac{a \sec(c + dx) \tan(c + dx)}{2d}$$

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	s
derivativedivides	$\frac{a \tan(dx+c) + a \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right)}{d}$	4
default	$\frac{a \tan(dx+c) + a \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right)}{d}$	4
parts	$\frac{a \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right)}{d} + \frac{a \tan(dx+c)}{d}$	4
parallelrisch	$\frac{\left((-1 - \cos(2dx+2c)) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + (1 + \cos(2dx+2c)) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + 2 \sin(dx+c) + 2 \sin(2dx+2c) \right) a}{2d(1 + \cos(2dx+2c))}$	9
risch	$-\frac{ia(e^{3i(dx+c)} - 2e^{2i(dx+c)} - e^{i(dx+c)} - 2)}{d(e^{2i(dx+c)} + 1)^2} + \frac{a \ln(e^{i(dx+c)} + i)}{2d} - \frac{a \ln(e^{i(dx+c)} - i)}{2d}$	9
norman	$\frac{3a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{2a \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} - \frac{a \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} - \frac{a \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2d} + \frac{a \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2d}$	1

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*tan(d*x+c)+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int (a + a \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \frac{a \cos(dx + c)^2 \log(\sin(dx + c) + 1) - a \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2(2a \cos(dx + c) + a) \sin(dx + c)}{4d \cos(dx + c)^2}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
+ c) + 1) + 2*(2*a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a + a \cos(c + dx)) \sec^3(c + dx) dx = a \left(\int \cos(c + dx) \sec^3(c + dx) dx + \int \sec^3(c + dx) dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int (a + a \cos(c + dx)) \sec^3(c + dx) dx$$

$$= - \frac{a \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) \right) - 4a \tan(dx+c)}{4d}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int (a + a \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \frac{a \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - a \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 - 3 a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)^2}}{2d}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int (a + a \cos(c + dx)) \sec^3(c + dx) dx = \frac{3 a \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)} + \frac{a \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d}$$

`[In] int((a + a*cos(c + d*x))/cos(c + d*x)^3,x)`

```
[Out] (3*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4
- 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2 + (d*x)/2)))/d
```

3.10 $\int (a + a \cos(c + dx)) \sec^4(c + dx) dx$

Optimal result	316
Rubi [A] (verified)	316
Mathematica [A] (verified)	317
Maple [A] (verified)	318
Fricas [A] (verification not implemented)	318
Sympy [F]	319
Maxima [A] (verification not implemented)	319
Giac [A] (verification not implemented)	319
Mupad [B] (verification not implemented)	320

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (a + a \cos(c + dx)) \sec^4(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a \tan(c + dx)}{d} + \frac{a \sec(c + dx) \tan(c + dx)}{2d} + \frac{a \tan^3(c + dx)}{3d}$$

[Out] $1/2*a*\operatorname{arctanh}(\sin(d*x+c))/d+a*\tan(d*x+c)/d+1/2*a*\sec(d*x+c)*\tan(d*x+c)/d+1/3*a*\tan(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 3852, 3853, 3855}

$$\int (a + a \cos(c + dx)) \sec^4(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a \tan^3(c + dx)}{3d} + \frac{a \tan(c + dx)}{d} + \frac{a \tan(c + dx) \sec(c + dx)}{2d}$$

[In] $\operatorname{Int}[(a + a \operatorname{Cos}[c + d*x])*\operatorname{Sec}[c + d*x]^4, x]$

[Out] $(a*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (a*\operatorname{Tan}[c + d*x])/d + (a*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d) + (a*\operatorname{Tan}[c + d*x]^3)/(3*d)$

Rule 2827

$\operatorname{Int}[(b*\sin[e + f*x] + (f_*)*(x_))^{(m)}*((c_*) + (d_*)*\sin[e + f*x] + (f_*)*(x_))], x_Symbol] \rightarrow \operatorname{Dist}[c, \operatorname{Int}[(b*\sin[e + f*x])^m, x], x] + \operatorname{Dist}[d/b, \operatorname{Int}[($

$b \sin[e + f x]^{m+1}, x, x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 3852

$\text{Int}[\text{csc}[(c_.) + (d_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rule 3853

$\text{Int}[(\text{csc}[(c_.) + (d_.)(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d x]*(b*\text{Csc}[c + d x])^{(n - 1)}/(d*(n - 1)), x] + \text{Dist}[b^2*((n - 2)/(n - 1)), \text{Int}[(b*\text{Csc}[c + d x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \& \& \text{IntegerQ}[2*n]$

Rule 3855

$\text{Int}[\text{csc}[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{ArcTanh}[\text{Cos}[c + d x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \sec^3(c + dx) dx + a \int \sec^4(c + dx) dx \\ &= \frac{a \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2} a \int \sec(c + dx) dx - \frac{a \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d} \\ &= \frac{a \arctanh(\sin(c + dx))}{2d} + \frac{a \tan(c + dx)}{d} + \frac{a \sec(c + dx) \tan(c + dx)}{2d} + \frac{a \tan^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int (a + a \cos(c + dx)) \sec^4(c + dx) dx = \frac{a \arctanh(\sin(c + dx))}{2d} + \frac{a \sec(c + dx) \tan(c + dx)}{2d} + \frac{a(\tan(c + dx) + \frac{1}{3} \tan^3(c + dx))}{d}$$

[In] Integrate[(a + a Cos[c + d x])*Sec[c + d x]^4, x]

[Out] (a*ArcTanh[Sin[c + d x]])/(2*d) + (a*Sec[c + d x]*Tan[c + d x])/(2*d) + (a*(Tan[c + d x] + Tan[c + d x]^3/3))/d

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right) - a\left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3}\right)\tan(dx+c)}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right) - a\left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3}\right)\tan(dx+c)}{d}$
parts	$-\frac{a\left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3}\right)\tan(dx+c)}{d} + \frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$-\frac{ia(3e^{5i(dx+c)} - 12e^{2i(dx+c)} - 3e^{i(dx+c)} - 4)}{3d(e^{2i(dx+c)} + 1)^3} + \frac{a \ln(e^{i(dx+c)} + i)}{2d} - \frac{a \ln(e^{i(dx+c)} - i)}{2d}$
norman	$\frac{-\frac{3a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} - \frac{5a \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3d} + \frac{a \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3d} - \frac{a \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} - \frac{a \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2d} + \frac{a \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2d}$
parallelrisc	$-\frac{a\left(3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \cos(3dx+3c) + 9 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \cos(dx+c) - 3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) \cos(3dx+3c) - 9 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) \cos(dx+c)\right)}{6d(\cos(3dx+3c)+3 \cos(dx+c))}$

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int (a + a \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{3a \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 3a \cos(dx + c)^3 \log(-\sin(dx + c) + 1) + 2(4a \cos(dx + c)^2 + 3a \cos(dx + c) + 2a) \sin(dx + c)}{12d \cos(dx + c)^3}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a + a \cos(c + dx)) \sec^4(c + dx) dx = a \left(\int \cos(c + dx) \sec^4(c + dx) dx + \int \sec^4(c + dx) dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int (a + a \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{4 (\tan(dx + c)^3 + 3 \tan(dx + c))a - 3a \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right)}{12d}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.52

$$\int (a + a \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{3a \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 3a \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(3a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 4a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 9a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)^3}}{6d}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 4*a*tan(1/2*d*x + 1/2*c)^3 + 9*
a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 16.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int (a + a \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{a \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 - \frac{4a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3} + 3a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1 \right)}$$

[In] `int((a + a*cos(c + d*x))/cos(c + d*x)^4,x)`

[Out] `(a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2) - (4*a*tan(c/2 + (d*x)/2)^3)/3 + a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))`

3.11 $\int (a + a \cos(c + dx)) \sec^5(c + dx) dx$

Optimal result	321
Rubi [A] (verified)	321
Mathematica [A] (verified)	323
Maple [A] (verified)	323
Fricas [A] (verification not implemented)	324
Sympy [F]	324
Maxima [A] (verification not implemented)	324
Giac [A] (verification not implemented)	325
Mupad [B] (verification not implemented)	325

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int (a + a \cos(c + dx)) \sec^5(c + dx) dx = \frac{3a \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a \tan(c + dx)}{d} + \frac{3a \sec(c + dx) \tan(c + dx)}{8d} + \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{a \tan^3(c + dx)}{3d}$$

[Out] $3/8*a*\operatorname{arctanh}(\sin(d*x+c))/d+a*\tan(d*x+c)/d+3/8*a*\sec(d*x+c)*\tan(d*x+c)/d+1/4*a*\sec(d*x+c)^3*\tan(d*x+c)/d+1/3*a*\tan(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 3853, 3855, 3852}

$$\int (a + a \cos(c + dx)) \sec^5(c + dx) dx = \frac{3a \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a \tan^3(c + dx)}{3d} + \frac{a \tan(c + dx)}{d} + \frac{a \tan(c + dx) \sec^3(c + dx)}{4d} + \frac{3a \tan(c + dx) \sec(c + dx)}{8d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])* \operatorname{Sec}[c + d*x]^5, x]$

[Out] $(3*a*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(8*d) + (a*\operatorname{Tan}[c + d*x])/d + (3*a*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(8*d) + (a*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(4*d) + (a*\operatorname{Tan}[c + d*x]^3)/(3*d)$

Rule 2827

$\text{Int}[(b \cdot \sin[e] + f \cdot x)^m \cdot (c + d \cdot \sin[e] + f \cdot x)], x_{\text{Symbol}}] \rightarrow \text{Dist}[c, \text{Int}[(b \cdot \sin[e + f \cdot x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b \cdot \sin[e + f \cdot x])^{m+1}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 3852

$\text{Int}[\csc[c + d \cdot x]^n, x_{\text{Symbol}}] \rightarrow \text{Dist}[-d^{-1}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}, x], x], x, \text{Cot}[c + d \cdot x]], x] /; \text{FreeQ}\{c, d\}, x \ \&\& \ \text{IGtQ}[n/2, 0]$

Rule 3853

$\text{Int}[(\csc[c + d \cdot x] \cdot (b \cdot \sin[c + d \cdot x]))^n, x_{\text{Symbol}}] \rightarrow \text{Simp}[(-b) \cdot \cos[c + d \cdot x] \cdot (b \cdot \csc[c + d \cdot x])^{n-1} / (d \cdot (n-1)), x] + \text{Dist}[b^2 \cdot ((n-2)/(n-1)), \text{Int}[(b \cdot \csc[c + d \cdot x])^{n-2}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \& \ \text{IntegerQ}[2 \cdot n]$

Rule 3855

$\text{Int}[\csc[c + d \cdot x], x_{\text{Symbol}}] \rightarrow \text{Simp}[-\text{ArcTanh}[\cos[c + d \cdot x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \sec^4(c + dx) dx + a \int \sec^5(c + dx) dx \\
 &= \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{4}(3a) \int \sec^3(c + dx) dx \\
 &\quad - \frac{a \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d} \\
 &= \frac{a \tan(c + dx)}{d} + \frac{3a \sec(c + dx) \tan(c + dx)}{8d} + \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} \\
 &\quad + \frac{a \tan^3(c + dx)}{3d} + \frac{1}{8}(3a) \int \sec(c + dx) dx \\
 &= \frac{3a \arctanh(\sin(c + dx))}{8d} + \frac{a \tan(c + dx)}{d} + \frac{3a \sec(c + dx) \tan(c + dx)}{8d} \\
 &\quad + \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{a \tan^3(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int (a + a \cos(c + dx)) \sec^5(c + dx) dx$$

$$= \frac{a(9 \operatorname{arctanh}(\sin(c + dx)) + \tan(c + dx) (9 \sec(c + dx) + 6 \sec^3(c + dx) + 8(3 + \tan^2(c + dx))))}{24d}$$

[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^5,x]

[Out] (a*(9*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(9*Sec[c + d*x] + 6*Sec[c + d*x]^3 + 8*(3 + Tan[c + d*x]^2))))/(24*d)

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-a \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + a \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right)}{d}$
default	$\frac{-a \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + a \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right)}{d}$
parts	$\frac{a \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right)}{d} - \frac{a \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d}$
risch	$\frac{ia(9e^{7i(dx+c)} + 33e^{5i(dx+c)} - 48e^{4i(dx+c)} - 33e^{3i(dx+c)} - 64e^{2i(dx+c)} - 9e^{i(dx+c)} - 16)}{12d(e^{2i(dx+c)} + 1)^4} - \frac{3a \ln(e^{i(dx+c)} - i)}{8d} + \frac{3a}{8d}$
parallelrisc	$\frac{a(9(-\cos(4dx+4c) - 4\cos(2dx+2c) - 3) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + 9(\cos(4dx+4c) + 4\cos(2dx+2c) + 3) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right))}{24d(\cos(4dx+4c) + 4\cos(2dx+2c) + 3)}$
norman	$\frac{\frac{13a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} + \frac{2a \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3d} + \frac{3a \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2d} + \frac{10a \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3d} - \frac{3a \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^4} - \frac{3a \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d}$

[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int (a + a \cos(c + dx)) \sec^5(c + dx) dx$$

$$= \frac{9 a \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 9 a \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2(16 a \cos(dx + c)^3 + 9 a \cos(dx + c)^2 + 8 a \cos(dx + c) + 6 a) \sin(dx + c)}{48 d \cos(dx + c)^4}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*a*cos(d*x + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int (a + a \cos(c + dx)) \sec^5(c + dx) dx = a \left(\int \cos(c + dx) \sec^5(c + dx) dx + \int \sec^5(c + dx) dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int (a + a \cos(c + dx)) \sec^5(c + dx) dx$$

$$= \frac{16(\tan(dx + c)^3 + 3 \tan(dx + c))a - 3a \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right)}{48 d}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```


Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int (a + a \cos(c + dx)) \sec^5(c + dx) dx$$

$$= \frac{9a \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 9a \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - \frac{2\left(9a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 49a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 31a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 39a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)^4}}{24d}$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")

[Out] 1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 49*a*tan(1/2*d*x + 1/2*c)^5 + 31*a*tan(1/2*d*x + 1/2*c)^3 - 39*a*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d

Mupad [B] (verification not implemented)

Time = 17.61 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int (a + a \cos(c + dx)) \sec^5(c + dx) dx$$

$$= \frac{-\frac{3a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{4} + \frac{49a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{12} - \frac{31a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{12} + \frac{13a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)} + \frac{3a \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{4d}$$

[In] int((a + a*cos(c + d*x))/cos(c + d*x)^5,x)

[Out] ((13*a*tan(c/2 + (d*x)/2))/4 - (31*a*tan(c/2 + (d*x)/2)^3)/12 + (49*a*tan(c/2 + (d*x)/2)^5)/12 - (3*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)

3.12 $\int (a + a \cos(c + dx)) \sec^6(c + dx) dx$

Optimal result	326
Rubi [A] (verified)	326
Mathematica [A] (verified)	328
Maple [A] (verified)	328
Fricas [A] (verification not implemented)	329
Sympy [F]	329
Maxima [A] (verification not implemented)	329
Giac [A] (verification not implemented)	330
Mupad [B] (verification not implemented)	330

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int (a + a \cos(c + dx)) \sec^6(c + dx) dx = \frac{3a \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a \tan(c + dx)}{d} + \frac{3a \sec(c + dx) \tan(c + dx)}{8d} + \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{2a \tan^3(c + dx)}{3d} + \frac{a \tan^5(c + dx)}{5d}$$

[Out] $3/8*a*\operatorname{arctanh}(\sin(d*x+c))/d+a*\tan(d*x+c)/d+3/8*a*\sec(d*x+c)*\tan(d*x+c)/d+1/4*a*\sec(d*x+c)^3*\tan(d*x+c)/d+2/3*a*\tan(d*x+c)^3/d+1/5*a*\tan(d*x+c)^5/d$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 3852, 3853, 3855}

$$\int (a + a \cos(c + dx)) \sec^6(c + dx) dx = \frac{3a \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a \tan^5(c + dx)}{5d} + \frac{2a \tan^3(c + dx)}{3d} + \frac{a \tan(c + dx)}{d} + \frac{a \tan(c + dx) \sec^3(c + dx)}{4d} + \frac{3a \tan(c + dx) \sec(c + dx)}{8d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])* \text{Sec}[c + d*x]^6, x]$

[Out] $(3a \operatorname{ArcTanh}[\sin[c + dx]])/(8d) + (a \tan[c + dx])/d + (3a \operatorname{Sec}[c + dx] * \tan[c + dx])/(8d) + (a \operatorname{Sec}[c + dx]^3 \tan[c + dx])/(4d) + (2a \tan[c + dx]^3)/(3d) + (a \tan[c + dx]^5)/(5d)$

Rule 2827

$\operatorname{Int}[(b \sin[e + fx] + (f \cdot x))^m ((c + d \sin[e + fx] + (f \cdot x)))], x_Symbol] := \operatorname{Dist}[c, \operatorname{Int}[(b \sin[e + fx])^m, x], x] + \operatorname{Dist}[d/b, \operatorname{Int}[(b \sin[e + fx])^{m+1}, x], x] /;$ $\operatorname{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 3852

$\operatorname{Int}[\operatorname{csc}[c + d \cdot x] (c + d \cdot x)^n], x_Symbol] := \operatorname{Dist}[-d^{-1}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}], x], x], x, \operatorname{Cot}[c + dx]], x] /;$ $\operatorname{FreeQ}\{c, d\}, x] \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3853

$\operatorname{Int}[(\operatorname{csc}[c + d \cdot x] + d \cdot x) (b \cdot x)^n], x_Symbol] := \operatorname{Simp}[(-b) \operatorname{Cos}[c + dx] * ((b \operatorname{Csc}[c + dx])^{n-1} / (d(n-1))), x] + \operatorname{Dist}[b^2 * ((n-2)/(n-1)), \operatorname{Int}[(b \operatorname{Csc}[c + dx])^{n-2}, x], x] /;$ $\operatorname{FreeQ}\{b, c, d\}, x] \&\& \operatorname{GtQ}[n, 1] \& \& \operatorname{IntegerQ}[2 * n]$

Rule 3855

$\operatorname{Int}[\operatorname{csc}[c + d \cdot x] (c + d \cdot x)], x_Symbol] := \operatorname{Simp}[-\operatorname{ArcTanh}[\operatorname{Cos}[c + dx]]/d, x] /;$ $\operatorname{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \sec^5(c + dx) dx + a \int \sec^6(c + dx) dx \\
 &= \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{4}(3a) \int \sec^3(c + dx) dx \\
 &\quad - \frac{a \operatorname{Subst}(\int (1 + 2x^2 + x^4) dx, x, -\tan(c + dx))}{d} \\
 &= \frac{a \tan(c + dx)}{d} + \frac{3a \sec(c + dx) \tan(c + dx)}{8d} + \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} \\
 &\quad + \frac{2a \tan^3(c + dx)}{3d} + \frac{a \tan^5(c + dx)}{5d} + \frac{1}{8}(3a) \int \sec(c + dx) dx \\
 &= \frac{3a \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a \tan(c + dx)}{d} + \frac{3a \sec(c + dx) \tan(c + dx)}{8d} \\
 &\quad + \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{2a \tan^3(c + dx)}{3d} + \frac{a \tan^5(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int (a + a \cos(c + dx)) \sec^6(c + dx) dx$$

$$= \frac{a(45 \operatorname{arctanh}(\sin(c + dx)) + \tan(c + dx) (120 + 45 \sec(c + dx) + 30 \sec^3(c + dx) + 80 \tan^2(c + dx) + 24 \tan^4(c + dx)))}{120d}$$

[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^6,x]

[Out] (a*(45*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(120 + 45*Sec[c + d*x] + 30*Sec[c + d*x]^3 + 80*Tan[c + d*x]^2 + 24*Tan[c + d*x]^4)))/(120*d)

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right) - a \left(- \frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c)}{d}$
default	$\frac{a \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right) - a \left(- \frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c)}{d}$
parts	$\frac{a \left(- \frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c)}{d} + \frac{a \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right)}{d}$
risch	$-\frac{ia(45 e^{9i(dx+c)} + 210 e^{7i(dx+c)} - 640 e^{4i(dx+c)} - 210 e^{3i(dx+c)} - 320 e^{2i(dx+c)} - 45 e^{i(dx+c)} - 64)}{60d(e^{2i(dx+c)} + 1)^5} + \frac{3a \ln(e^{i(dx+c)} + i)}{8d}$
norman	$\frac{-\frac{13a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} - \frac{a \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} - \frac{137a \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{30d} - \frac{167a \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{30d} + \frac{17a \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} - \frac{3a \left(\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^5}$
parallelrisch	$\frac{8 \left(\left(- \frac{45 \cos(dx+c)}{32} - \frac{45 \cos(3dx+3c)}{64} - \frac{9 \cos(5dx+5c)}{64} \right) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + \left(\frac{45 \cos(dx+c)}{32} + \frac{45 \cos(3dx+3c)}{64} + \frac{9 \cos(5dx+5c)}{64} \right) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) \right)}{3d(\cos(5dx+5c) + 5 \cos(3dx+3c) + 10 \cos(dx+c) + 5)}$

[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^6,x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int (a + a \cos(c + dx)) \sec^6(c + dx) dx$$

$$= \frac{45 a \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 45 a \cos(dx + c)^5 \log(-\sin(dx + c) + 1) + 2(64 a \cos(dx + c)^4 + 45 a \cos(dx + c)^3 + 32 a \cos(dx + c)^2 + 30 a \cos(dx + c) + 24 a) \sin(dx + c)}{240 d \cos(dx + c)^5}$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fricas")

```
[Out] 1/240*(45*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(64*a*cos(d*x + c)^4 + 45*a*cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 + 30*a*cos(d*x + c) + 24*a)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int (a + a \cos(c + dx)) \sec^6(c + dx) dx = a \left(\int \cos(c + dx) \sec^6(c + dx) dx + \int \sec^6(c + dx) dx \right)$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**6,x)

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int (a + a \cos(c + dx)) \sec^6(c + dx) dx$$

$$= \frac{16(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c))a - 15a \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log(\sin(dx+c) + 1) + 3 \log(\sin(dx+c) - 1) \right)}{240 d}$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="maxima")

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 15*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.23

$$\int (a + a \cos(c + dx)) \sec^6(c + dx) dx$$

$$= \frac{45 a \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 45 a \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(45 a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^9 - 130 a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^7 + 464 a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 190 a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 195 a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{120 d}$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="giac")

```
[Out] 1/120*(45*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*a*tan(1/2*d*x + 1/2*c)^9 - 130*a*tan(1/2*d*x + 1/2*c)^7 + 464*a*tan(1/2*d*x + 1/2*c)^5 - 190*a*tan(1/2*d*x + 1/2*c)^3 + 195*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5/d
```

Mupad [B] (verification not implemented)

Time = 19.01 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.56

$$\int (a + a \cos(c + dx)) \sec^6(c + dx) dx = \frac{3 a \operatorname{atanh} \left(\tan \left(\frac{c}{2} + \frac{dx}{2} \right) \right)}{4 d}$$

$$- \frac{\frac{3 a \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^9}{4} - \frac{13 a \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^7}{6} + \frac{116 a \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^5}{15} - \frac{19 a \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^3}{6} + \frac{13 a \tan \left(\frac{c}{2} + \frac{dx}{2} \right)}{4}}{d \left(\tan \left(\frac{c}{2} + \frac{dx}{2} \right)^{10} - 5 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^8 + 10 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^6 - 10 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^4 + 5 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^2 - 1 \right)}$$

[In] int((a + a*cos(c + d*x))/cos(c + d*x)^6,x)

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((13*a*tan(c/2 + (d*x)/2))/4 - (19*a*tan(c/2 + (d*x)/2)^3)/6 + (116*a*tan(c/2 + (d*x)/2)^5)/15 - (13*a*tan(c/2 + (d*x)/2)^7)/6 + (3*a*tan(c/2 + (d*x)/2)^9)/4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.13 $\int \cos^4(c + dx)(a + a \cos(c + dx))^2 dx$

Optimal result	331
Rubi [A] (verified)	331
Mathematica [A] (verified)	333
Maple [A] (verified)	334
Fricas [A] (verification not implemented)	334
Sympy [B] (verification not implemented)	335
Maxima [A] (verification not implemented)	335
Giac [A] (verification not implemented)	336
Mupad [B] (verification not implemented)	336

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \cos^4(c + dx)(a + a \cos(c + dx))^2 dx = \frac{11a^2x}{16} + \frac{2a^2 \sin(c + dx)}{d} + \frac{11a^2 \cos(c + dx) \sin(c + dx)}{16d} + \frac{11a^2 \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{a^2 \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{4a^2 \sin^3(c + dx)}{3d} + \frac{2a^2 \sin^5(c + dx)}{5d}$$

[Out] $11/16*a^2*x+2*a^2*\sin(d*x+c)/d+11/16*a^2*\cos(d*x+c)*\sin(d*x+c)/d+11/24*a^2*\cos(d*x+c)^3*\sin(d*x+c)/d+1/6*a^2*\cos(d*x+c)^5*\sin(d*x+c)/d-4/3*a^2*\sin(d*x+c)^3/d+2/5*a^2*\sin(d*x+c)^5/d$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 2715, 8, 2713}

$$\int \cos^4(c + dx)(a + a \cos(c + dx))^2 dx = \frac{2a^2 \sin^5(c + dx)}{5d} - \frac{4a^2 \sin^3(c + dx)}{3d} + \frac{2a^2 \sin(c + dx)}{d} + \frac{a^2 \sin(c + dx) \cos^5(c + dx)}{6d} + \frac{11a^2 \sin(c + dx) \cos^3(c + dx)}{24d} + \frac{11a^2 \sin(c + dx) \cos(c + dx)}{16d} + \frac{11a^2x}{16}$$

[In] Int[Cos[c + d*x]^4*(a + a*Cos[c + d*x])^2,x]

[Out] (11*a^2*x)/16 + (2*a^2*Sin[c + d*x])/d + (11*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (11*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (4*a^2*Sin[c + d*x]^3)/(3*d) + (2*a^2*Sin[c + d*x]^5)/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^2 \cos^4(c + dx) + 2a^2 \cos^5(c + dx) + a^2 \cos^6(c + dx)) dx \\
 &= a^2 \int \cos^4(c + dx) dx + a^2 \int \cos^6(c + dx) dx + (2a^2) \int \cos^5(c + dx) dx \\
 &= \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{a^2 \cos^5(c + dx) \sin(c + dx)}{6d} + \frac{1}{4}(3a^2) \int \cos^2(c + dx) dx \\
 &\quad + \frac{1}{6}(5a^2) \int \cos^4(c + dx) dx - \frac{(2a^2) \text{Subst}(\int (1 - 2x^2 + x^4) dx, x, -\sin(c + dx))}{d} \\
 &= \frac{2a^2 \sin(c + dx)}{d} + \frac{3a^2 \cos(c + dx) \sin(c + dx)}{8d} + \frac{11a^2 \cos^3(c + dx) \sin(c + dx)}{24d} \\
 &\quad + \frac{a^2 \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{4a^2 \sin^3(c + dx)}{3d} \\
 &\quad + \frac{2a^2 \sin^5(c + dx)}{5d} + \frac{1}{8}(3a^2) \int 1 dx + \frac{1}{8}(5a^2) \int \cos^2(c + dx) dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{3a^2x}{8} + \frac{2a^2 \sin(c+dx)}{d} + \frac{11a^2 \cos(c+dx) \sin(c+dx)}{16d} + \frac{11a^2 \cos^3(c+dx) \sin(c+dx)}{24d} \\
&\quad + \frac{a^2 \cos^5(c+dx) \sin(c+dx)}{6d} - \frac{4a^2 \sin^3(c+dx)}{3d} + \frac{2a^2 \sin^5(c+dx)}{5d} + \frac{1}{16}(5a^2) \int 1 dx \\
&= \frac{11a^2x}{16} + \frac{2a^2 \sin(c+dx)}{d} + \frac{11a^2 \cos(c+dx) \sin(c+dx)}{16d} \\
&\quad + \frac{11a^2 \cos^3(c+dx) \sin(c+dx)}{24d} + \frac{a^2 \cos^5(c+dx) \sin(c+dx)}{6d} \\
&\quad - \frac{4a^2 \sin^3(c+dx)}{3d} + \frac{2a^2 \sin^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\begin{aligned}
&\int \cos^4(c+dx)(a+a\cos(c+dx))^2 dx \\
&= \frac{a^2(660dx + 1200 \sin(c+dx) + 465 \sin(2(c+dx)) + 200 \sin(3(c+dx)) + 75 \sin(4(c+dx)) + 24 \sin(5(c+dx)) + 5 \sin(6(c+dx)))}{960d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^4*(a + a*Cos[c + d*x])^2,x]

[Out] (a^2*(660*d*x + 1200*Sin[c + d*x] + 465*Sin[2*(c + d*x)] + 200*Sin[3*(c + d*x)] + 75*Sin[4*(c + d*x)] + 24*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(960*d)

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{(132dx + \sin(6dx+6c) + 240 \sin(dx+c) + 93 \sin(2dx+2c) + 40 \sin(3dx+3c) + 15 \sin(4dx+4c) + \frac{24 \sin(5dx+5c)}{5}) a^2}{192d}$
risch	$\frac{11a^2x}{16} + \frac{5a^2 \sin(dx+c)}{4d} + \frac{a^2 \sin(6dx+6c)}{192d} + \frac{a^2 \sin(5dx+5c)}{40d} + \frac{5a^2 \sin(4dx+4c)}{64d} + \frac{5a^2 \sin(3dx+3c)}{24d} + \frac{31a^2 \sin(2dx+2c)}{64d}$
derivativdivides	$a^2 \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{2a^2 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + a^2 \frac{d}{d}$
default	$a^2 \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{2a^2 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + a^2 \frac{d}{d}$
parts	$a^2 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{a^2 \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right)}{d}$
norman	$\frac{11a^2x}{16} + \frac{53a^2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d} + \frac{87a^2 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d} + \frac{501a^2 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{20d} + \frac{331a^2 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{20d} + \frac{187a^2 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24d} + \frac{11a^2 \sin(dx+c)}{16d}$

[In] int(cos(d*x+c)^4*(a+cos(d*x+c))*a^2,x,method=_RETURNVERBOSE)

[Out] 1/192*(132*d*x+sin(6*d*x+6*c)+240*sin(d*x+c)+93*sin(2*d*x+2*c)+40*sin(3*d*x+3*c)+15*sin(4*d*x+4*c)+24/5*sin(5*d*x+5*c))*a^2/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \cos^4(c+dx)(a+a\cos(c+dx))^2 dx$$

$$= \frac{165a^2dx + (40a^2\cos(dx+c))^5 + 96a^2\cos(dx+c)^4 + 110a^2\cos(dx+c)^3 + 128a^2\cos(dx+c)^2 + 165a^2\cos(dx+c) + 256a^2}{240d}$$

[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/240*(165*a^2*d*x + (40*a^2*cos(d*x + c))^5 + 96*a^2*cos(d*x + c)^4 + 110*a^2*cos(d*x + c)^3 + 128*a^2*cos(d*x + c)^2 + 165*a^2*cos(d*x + c) + 256*a^2)*sin(d*x + c)/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(122) = 244$.

Time = 0.37 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.66

$$\int \cos^4(c + dx)(a + a \cos(c + dx))^2 dx$$

$$= \left\{ \begin{array}{l} \frac{5a^2x \sin^6(c+dx)}{16} + \frac{15a^2x \sin^4(c+dx) \cos^2(c+dx)}{16} + \frac{3a^2x \sin^4(c+dx)}{8} + \frac{15a^2x \sin^2(c+dx) \cos^4(c+dx)}{16} + \frac{3a^2x \sin^2(c+dx) \cos^2(c+dx)}{4} \\ x(a \cos(c) + a)^2 \cos^4(c) \end{array} \right.$$

[In] integrate(cos(d*x+c)**4*(a+a*cos(d*x+c))**2,x)

[Out] Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*a**2*x*sin(c + d*x)**4/8 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*a**2*x*cos(c + d*x)**6/16 + 3*a**2*x*cos(c + d*x)**4/8 + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 16*a**2*sin(c + d*x)**5/(15*d) + 5*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 8*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 2*a**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c)**4, True))

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \cos^4(c + dx)(a + a \cos(c + dx))^2 dx$$

$$= \frac{128 (3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))a^2 - 5 (4 \sin(2dx + 2c)^3 - 60dx - 60c - 9 \sin(4dx + 4c) - 48 \sin(2dx + 2c))a^2 + 30(12dx + 12c + \sin(4dx + 4c) + 8 \sin(2dx + 2c))a^2}{960d}$$

[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] 1/960*(128*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2)/d

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \cos^4(c + dx)(a + a \cos(c + dx))^2 dx = \frac{11}{16} a^2 x + \frac{a^2 \sin(6 dx + 6 c)}{192 d} + \frac{a^2 \sin(5 dx + 5 c)}{40 d} + \frac{5 a^2 \sin(4 dx + 4 c)}{64 d} + \frac{5 a^2 \sin(3 dx + 3 c)}{24 d} + \frac{31 a^2 \sin(2 dx + 2 c)}{64 d} + \frac{5 a^2 \sin(dx + c)}{4 d}$$

[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] 11/16*a^2*x + 1/192*a^2*sin(6*d*x + 6*c)/d + 1/40*a^2*sin(5*d*x + 5*c)/d + 5/64*a^2*sin(4*d*x + 4*c)/d + 5/24*a^2*sin(3*d*x + 3*c)/d + 31/64*a^2*sin(2*d*x + 2*c)/d + 5/4*a^2*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 16.98 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \cos^4(c + dx)(a + a \cos(c + dx))^2 dx = \frac{11 a^2 x}{16} + \frac{\frac{11 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11}}{8} + \frac{187 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{24} + \frac{331 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{20} + \frac{501 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{20} + \frac{87 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{8} + \frac{53 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)^6}$$

[In] int(cos(c + d*x)^4*(a + a*cos(c + d*x))^2,x)

[Out] (11*a^2*x)/16 + ((87*a^2*tan(c/2 + (d*x)/2)^3)/8 + (501*a^2*tan(c/2 + (d*x)/2)^5)/20 + (331*a^2*tan(c/2 + (d*x)/2)^7)/20 + (187*a^2*tan(c/2 + (d*x)/2)^9)/24 + (11*a^2*tan(c/2 + (d*x)/2)^11)/8 + (53*a^2*tan(c/2 + (d*x)/2))/8)/d*(tan(c/2 + (d*x)/2)^2 + 1)^6

3.14 $\int \cos^3(c + dx)(a + a \cos(c + dx))^2 dx$

Optimal result	337
Rubi [A] (verified)	337
Mathematica [A] (verified)	339
Maple [A] (verified)	339
Fricas [A] (verification not implemented)	340
Sympy [B] (verification not implemented)	340
Maxima [A] (verification not implemented)	340
Giac [A] (verification not implemented)	341
Mupad [B] (verification not implemented)	341

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^2 dx = \frac{3a^2x}{4} + \frac{2a^2 \sin(c + dx)}{d} + \frac{3a^2 \cos(c + dx) \sin(c + dx)}{4d} + \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{2d} - \frac{a^2 \sin^3(c + dx)}{d} + \frac{a^2 \sin^5(c + dx)}{5d}$$

[Out] $3/4*a^2*x+2*a^2*\sin(d*x+c)/d+3/4*a^2*\cos(d*x+c)*\sin(d*x+c)/d+1/2*a^2*\cos(d*x+c)^3*\sin(d*x+c)/d-a^2*\sin(d*x+c)^3/d+1/5*a^2*\sin(d*x+c)^5/d$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 2713, 2715, 8}

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^2 dx = \frac{a^2 \sin^5(c + dx)}{5d} - \frac{a^2 \sin^3(c + dx)}{d} + \frac{2a^2 \sin(c + dx)}{d} + \frac{a^2 \sin(c + dx) \cos^3(c + dx)}{2d} + \frac{3a^2 \sin(c + dx) \cos(c + dx)}{4d} + \frac{3a^2x}{4}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^3*(a + a*\text{Cos}[c + d*x])^2,x]$

[Out] $(3a^2x)/4 + (2a^2\sin[c + dx])/d + (3a^2\cos[c + dx]\sin[c + dx])/(4d) + (a^2\cos[c + dx]^3\sin[c + dx])/(2d) - (a^2\sin[c + dx]^3)/d + (a^2\sin[c + dx]^5)/(5d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^2 \cos^3(c + dx) + 2a^2 \cos^4(c + dx) + a^2 \cos^5(c + dx)) dx \\
 &= a^2 \int \cos^3(c + dx) dx + a^2 \int \cos^5(c + dx) dx + (2a^2) \int \cos^4(c + dx) dx \\
 &= \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{2d} + \frac{1}{2}(3a^2) \int \cos^2(c + dx) dx \\
 &\quad - \frac{a^2 \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{d} \\
 &\quad - \frac{a^2 \text{Subst}(\int (1 - 2x^2 + x^4) dx, x, -\sin(c + dx))}{d} \\
 &= \frac{2a^2 \sin(c + dx)}{d} + \frac{3a^2 \cos(c + dx) \sin(c + dx)}{4d} + \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{2d} \\
 &\quad - \frac{a^2 \sin^3(c + dx)}{d} + \frac{a^2 \sin^5(c + dx)}{5d} + \frac{1}{4}(3a^2) \int 1 dx
 \end{aligned}$$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^2 dx$$

$$= \frac{15 a^2 dx + (4 a^2 \cos(dx + c))^4 + 10 a^2 \cos(dx + c)^3 + 12 a^2 \cos(dx + c)^2 + 15 a^2 \cos(dx + c) + 24 a^2 \sin(dx + c)}{20 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/20*(15*a^2*d*x + (4*a^2*cos(d*x + c)^4 + 10*a^2*cos(d*x + c)^3 + 12*a^2*cos(d*x + c)^2 + 15*a^2*cos(d*x + c) + 24*a^2)*sin(d*x + c))/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(94) = 188.

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.15

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^2 dx$$

$$= \begin{cases} \frac{3a^2 x \sin^4(c+dx)}{4} + \frac{3a^2 x \sin^2(c+dx) \cos^2(c+dx)}{2} + \frac{3a^2 x \cos^4(c+dx)}{4} + \frac{8a^2 \sin^5(c+dx)}{15d} + \frac{4a^2 \sin^3(c+dx) \cos^2(c+dx)}{3d} + \frac{3a^2 \sin^3(c+dx)}{4} \\ x(a \cos(c) + a)^2 \cos^3(c) \end{cases}$$

[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**2,x)

[Out] Piecewise(((3*a**2*x*sin(c + d*x)**4/4 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 3*a**2*x*cos(c + d*x)**4/4 + 8*a**2*sin(c + d*x)**5/(15*d) + 4*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a**2*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^2 dx$$

$$= \frac{16 (3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c)) a^2 - 80 (\sin(dx + c)^3 - 3 \sin(dx + c)) a^2 + 15 (12 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c)) a^2}{240 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] $\frac{1}{240}*(16*(3*\sin(d*x + c)^5 - 10*\sin(d*x + c)^3 + 15*\sin(d*x + c))*a^2 - 80*(\sin(d*x + c)^3 - 3*\sin(d*x + c))*a^2 + 15*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*a^2)/d$

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^2 dx = \frac{3}{4} a^2 x + \frac{a^2 \sin(5 dx + 5 c)}{80 d} + \frac{a^2 \sin(4 dx + 4 c)}{16 d} + \frac{3 a^2 \sin(3 dx + 3 c)}{16 d} + \frac{a^2 \sin(2 dx + 2 c)}{2 d} + \frac{11 a^2 \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{3}{4}a^2x + \frac{1}{80}a^2\sin(5d*x + 5c)/d + \frac{1}{16}a^2\sin(4d*x + 4c)/d + \frac{3}{16}a^2\sin(3d*x + 3c)/d + \frac{1}{2}a^2\sin(2d*x + 2c)/d + \frac{11}{8}a^2\sin(dx + c)/d$

Mupad [B] (verification not implemented)

Time = 17.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^2 dx = \frac{3 a^2 x}{4} + \frac{\frac{3 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{2} + 7 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \frac{72 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{5} + 9 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \frac{13 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)^5}$$

[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^2,x)

[Out] $\frac{3*a^2*x}{4} + \frac{9*a^2*\tan(c/2 + (d*x)/2)^3}{5} + \frac{72*a^2*\tan(c/2 + (d*x)/2)^5}{5} + \frac{7*a^2*\tan(c/2 + (d*x)/2)^7}{2} + \frac{3*a^2*\tan(c/2 + (d*x)/2)^9}{2} + \frac{13*a^2*\tan(c/2 + (d*x)/2)}{2}/(d*(\tan(c/2 + (d*x)/2)^2 + 1)^5)$

3.15 $\int \cos^2(c + dx)(a + a \cos(c + dx))^2 dx$

Optimal result	342
Rubi [A] (verified)	342
Mathematica [A] (verified)	344
Maple [A] (verified)	344
Fricas [A] (verification not implemented)	345
Sympy [B] (verification not implemented)	345
Maxima [A] (verification not implemented)	345
Giac [A] (verification not implemented)	346
Mupad [B] (verification not implemented)	346

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^2 dx = \frac{7a^2x}{8} + \frac{2a^2 \sin(c + dx)}{d} + \frac{7a^2 \cos(c + dx) \sin(c + dx)}{8d} + \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{2a^2 \sin^3(c + dx)}{3d}$$

[Out] $7/8*a^2*x+2*a^2*\sin(d*x+c)/d+7/8*a^2*\cos(d*x+c)*\sin(d*x+c)/d+1/4*a^2*\cos(d*x+c)^3*\sin(d*x+c)/d-2/3*a^2*\sin(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 2715, 8, 2713}

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^2 dx = -\frac{2a^2 \sin^3(c + dx)}{3d} + \frac{2a^2 \sin(c + dx)}{d} + \frac{a^2 \sin(c + dx) \cos^3(c + dx)}{4d} + \frac{7a^2 \sin(c + dx) \cos(c + dx)}{8d} + \frac{7a^2x}{8}$$

[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2,x]

[Out] $(7*a^2*x)/8 + (2*a^2*\sin[c + d*x])/d + (7*a^2*\cos[c + d*x]*\sin[c + d*x])/(8*d) + (a^2*\cos[c + d*x]^3*\sin[c + d*x])/(4*d) - (2*a^2*\sin[c + d*x]^3)/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2715

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 2836

`Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^2 \cos^2(c + dx) + 2a^2 \cos^3(c + dx) + a^2 \cos^4(c + dx)) dx \\
 &= a^2 \int \cos^2(c + dx) dx + a^2 \int \cos^4(c + dx) dx + (2a^2) \int \cos^3(c + dx) dx \\
 &= \frac{a^2 \cos(c + dx) \sin(c + dx)}{2d} + \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{1}{2} a^2 \int 1 dx \\
 &\quad + \frac{1}{4} (3a^2) \int \cos^2(c + dx) dx - \frac{(2a^2) \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{d} \\
 &= \frac{a^2 x}{2} + \frac{2a^2 \sin(c + dx)}{d} + \frac{7a^2 \cos(c + dx) \sin(c + dx)}{8d} \\
 &\quad + \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{2a^2 \sin^3(c + dx)}{3d} + \frac{1}{8} (3a^2) \int 1 dx \\
 &= \frac{7a^2 x}{8} + \frac{2a^2 \sin(c + dx)}{d} + \frac{7a^2 \cos(c + dx) \sin(c + dx)}{8d} \\
 &\quad + \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{2a^2 \sin^3(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^2 dx$$

$$= \frac{a^2(84dx + 144 \sin(c + dx) + 48 \sin(2(c + dx)) + 16 \sin(3(c + dx)) + 3 \sin(4(c + dx)))}{96d}$$

[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2,x]

[Out] (a^2*(84*d*x + 144*Sin[c + d*x] + 48*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)] + 3*Sin[4*(c + d*x)])/(96*d)

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^2(84dx+144 \sin(dx+c)+3 \sin(4dx+4c)+16 \sin(3dx+3c)+48 \sin(2dx+2c))}{96d}$
risch	$\frac{7a^2x}{8} + \frac{3a^2 \sin(dx+c)}{2d} + \frac{a^2 \sin(4dx+4c)}{32d} + \frac{a^2 \sin(3dx+3c)}{6d} + \frac{a^2 \sin(2dx+2c)}{2d}$
derivativdivides	$a^2 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{2a^2(2+\cos^2(dx+c)) \sin(dx+c)}{3} + a^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)$
default	$a^2 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{2a^2(2+\cos^2(dx+c)) \sin(dx+c)}{3} + a^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)$
parts	$a^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + \frac{a^2 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right)}{d} + \frac{2a^2(2+\cos^2(dx+c)) \sin(dx+c)}{3d}$
norman	$\frac{7a^2x}{8} + \frac{25a^2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} + \frac{83a^2 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} + \frac{77a^2 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} + \frac{7a^2 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} + \frac{7a^2x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{21a^2x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^4}$

[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] 1/96*a^2*(84*d*x+144*sin(d*x+c)+3*sin(4*d*x+4*c)+16*sin(3*d*x+3*c)+48*sin(2*d*x+2*c))/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^2 dx$$

$$= \frac{21 a^2 dx + (6 a^2 \cos(dx + c))^3 + 16 a^2 \cos(dx + c)^2 + 21 a^2 \cos(dx + c) + 32 a^2 \sin(dx + c)}{24 d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/24*(21*a^2*d*x + (6*a^2*cos(d*x + c)^3 + 16*a^2*cos(d*x + c)^2 + 21*a^2*cos(d*x + c) + 32*a^2)*sin(d*x + c))/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(82) = 164.

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.43

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^2 dx$$

$$= \begin{cases} \frac{3a^2 x \sin^4(c+dx)}{8} + \frac{3a^2 x \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{a^2 x \sin^2(c+dx)}{2} + \frac{3a^2 x \cos^4(c+dx)}{8} + \frac{a^2 x \cos^2(c+dx)}{2} + \frac{3a^2 \sin^3(c+dx) \cos(c+dx)}{8d} \\ x(a \cos(c) + a)^2 \cos^2(c) \end{cases}$$

[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**2,x)

[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**2*x*sin(c + d*x)**2/2 + 3*a**2*x*cos(c + d*x)**4/8 + a**2*x*cos(c + d*x)**2/2 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*a**2*sin(c + d*x)**3/(3*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*a**2*sin(c + d*x)*cos(c + d*x)**2/d + a**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c)**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^2 dx =$$

$$\frac{64 (\sin(dx + c)^3 - 3 \sin(dx + c))a^2 - 3(12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c))a^2 - 24(2 \sin(dx + c) - 1)}{96 d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] $-1/96*(64*(\sin(d*x + c))^3 - 3*\sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*a^2 - 24*(2*d*x + 2*c + \sin(2*d*x + 2*c))*a^2/d$

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^2 dx = \frac{7}{8} a^2 x + \frac{a^2 \sin(4 dx + 4 c)}{32 d} + \frac{a^2 \sin(3 dx + 3 c)}{6 d} + \frac{a^2 \sin(2 dx + 2 c)}{2 d} + \frac{3 a^2 \sin(dx + c)}{2 d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] $7/8*a^2*x + 1/32*a^2*\sin(4*d*x + 4*c)/d + 1/6*a^2*\sin(3*d*x + 3*c)/d + 1/2*a^2*\sin(2*d*x + 2*c)/d + 3/2*a^2*\sin(d*x + c)/d$

Mupad [B] (verification not implemented)

Time = 17.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^2 dx = \frac{7 a^2 x}{8} + \frac{7 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{4} + \frac{77 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{12} + \frac{83 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{12} + \frac{25 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4} \frac{1}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)^4}$$

[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^2,x)

[Out] $(7*a^2*x)/8 + ((83*a^2*\tan(c/2 + (d*x)/2)^3)/12 + (77*a^2*\tan(c/2 + (d*x)/2)^5)/12 + (7*a^2*\tan(c/2 + (d*x)/2)^7)/4 + (25*a^2*\tan(c/2 + (d*x)/2))/4)/(d*(\tan(c/2 + (d*x)/2)^2 + 1)^4)$

3.16 $\int \cos(c + dx)(a + a \cos(c + dx))^2 dx$

Optimal result	347
Rubi [A] (verified)	347
Mathematica [A] (verified)	348
Maple [A] (verified)	349
Fricas [A] (verification not implemented)	349
Sympy [B] (verification not implemented)	350
Maxima [A] (verification not implemented)	350
Giac [A] (verification not implemented)	350
Mupad [B] (verification not implemented)	351

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \cos(c + dx)(a + a \cos(c + dx))^2 dx = a^2x + \frac{2a^2 \sin(c + dx)}{d} + \frac{a^2 \cos(c + dx) \sin(c + dx)}{d} - \frac{a^2 \sin^3(c + dx)}{3d}$$

[Out] $a^2x + 2a^2 \sin(dx+c)/d + a^2 \cos(dx+c) \sin(dx+c)/d - 1/3 a^2 \sin(dx+c)^3/d$

Rubi [A] (verified)

Time = 0.04 (sec), antiderivative size = 69, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2830, 2723}

$$\int \cos(c + dx)(a + a \cos(c + dx))^2 dx = \frac{4a^2 \sin(c + dx)}{3d} + \frac{a^2 \sin(c + dx) \cos(c + dx)}{3d} + a^2x + \frac{\sin(c + dx)(a \cos(c + dx) + a)^2}{3d}$$

[In] `Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2,x]`

[Out] $a^2x + (4a^2 \sin[c + d*x])/(3d) + (a^2 \cos[c + d*x] \sin[c + d*x])/(3d) + ((a + a \cos[c + d*x])^2 \sin[c + d*x])/(3d)$

Rule 2723

`Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^2, x_Symbol] := Simp[(2*a^2 + b^2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(Sin[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]`

Rule 2830

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(a + a \cos(c + dx))^2 \sin(c + dx)}{3d} + \frac{2}{3} \int (a + a \cos(c + dx))^2 dx \\ &= a^2 x + \frac{4a^2 \sin(c + dx)}{3d} + \frac{a^2 \cos(c + dx) \sin(c + dx)}{3d} + \frac{(a + a \cos(c + dx))^2 \sin(c + dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\begin{aligned} &\int \cos(c + dx)(a + a \cos(c + dx))^2 dx \\ &= \frac{a^2(12dx + 21 \sin(c + dx) + 6 \sin(2(c + dx)) + \sin(3(c + dx)))}{12d} \end{aligned}$$

[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2,x]

[Out] (a^2*(12*d*x + 21*Sin[c + d*x] + 6*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(12*d)

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result
parallelrisc	$\frac{a^2(12dx+21\sin(dx+c)+\sin(3dx+3c)+6\sin(2dx+2c))}{12d}$
risc	$a^2x + \frac{7a^2\sin(dx+c)}{4d} + \frac{a^2\sin(3dx+3c)}{12d} + \frac{a^2\sin(2dx+2c)}{2d}$
derivativdivides	$\frac{\frac{a^2(2+\cos^2(dx+c))\sin(dx+c)}{3} + 2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + a^2\sin(dx+c)}{d}$
default	$\frac{\frac{a^2(2+\cos^2(dx+c))\sin(dx+c)}{3} + 2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + a^2\sin(dx+c)}{d}$
parts	$\frac{a^2(2+\cos^2(dx+c))\sin(dx+c)}{3d} + \frac{a^2\sin(dx+c)}{d} + \frac{2a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$
norman	$\frac{a^2x + a^2x\left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{6a^2\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{16a^2\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3d} + \frac{2a^2\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} + 3a^2x\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 3a^2x\left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3}$

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*a^2*(12*d*x+21*sin(d*x+c)+sin(3*d*x+3*c)+6*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \cos(c+dx)(a+a\cos(c+dx))^2 dx$$

$$= \frac{3a^2dx + (a^2\cos(dx+c))^2 + 3a^2\cos(dx+c) + 5a^2\sin(dx+c)}{3d}$$

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*d*x + (a^2*cos(d*x + c))^2 + 3*a^2*cos(d*x + c) + 5*a^2)*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.88

$$\int \cos(c + dx)(a + a \cos(c + dx))^2 dx$$

$$= \begin{cases} a^2 x \sin^2(c + dx) + a^2 x \cos^2(c + dx) + \frac{2a^2 \sin^3(c + dx)}{3d} + \frac{a^2 \sin(c + dx) \cos^2(c + dx)}{d} + \frac{a^2 \sin(c + dx) \cos(c + dx)}{d} + \frac{a^2 \sin(c + dx)}{d} \\ x(a \cos(c) + a)^2 \cos(c) \end{cases}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2,x)

[Out] Piecewise((a**2*x*sin(c + d*x)**2 + a**2*x*cos(c + d*x)**2 + 2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2/d + a**2*sin(c + d*x)*cos(c + d*x)/d + a**2*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c), True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \cos(c + dx)(a + a \cos(c + dx))^2 dx =$$

$$\frac{2(\sin(dx + c)^3 - 3\sin(dx + c))a^2 - 3(2dx + 2c + \sin(2dx + 2c))a^2 - 6a^2 \sin(dx + c)}{6d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 6*a^2*sin(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \cos(c + dx)(a + a \cos(c + dx))^2 dx = a^2 x + \frac{a^2 \sin(3dx + 3c)}{12d}$$

$$+ \frac{a^2 \sin(2dx + 2c)}{2d} + \frac{7a^2 \sin(dx + c)}{4d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] a^2*x + 1/12*a^2*sin(3*d*x + 3*c)/d + 1/2*a^2*sin(2*d*x + 2*c)/d + 7/4*a^2*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 13.71 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \cos(c + dx)(a + a \cos(c + dx))^2 dx = a^2 x + \frac{5 a^2 \sin(c + dx)}{3 d} + \frac{a^2 \cos(c + dx)^2 \sin(c + dx)}{3 d} + \frac{a^2 \cos(c + dx) \sin(c + dx)}{d}$$

`[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^2,x)`

```
[Out] a^2*x + (5*a^2*sin(c + d*x))/(3*d) + (a^2*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (a^2*cos(c + d*x)*sin(c + d*x))/d
```

3.17 $\int (a + a \cos(c + dx))^2 dx$

Optimal result	352
Rubi [A] (verified)	352
Mathematica [A] (verified)	353
Maple [A] (verified)	353
Fricas [A] (verification not implemented)	353
Sympy [A] (verification not implemented)	354
Maxima [A] (verification not implemented)	354
Giac [A] (verification not implemented)	354
Mupad [B] (verification not implemented)	355

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int (a + a \cos(c + dx))^2 dx = \frac{3a^2x}{2} + \frac{2a^2 \sin(c + dx)}{d} + \frac{a^2 \cos(c + dx) \sin(c + dx)}{2d}$$

[Out] $3/2*a^2*x+2*a^2*\sin(d*x+c)/d+1/2*a^2*\cos(d*x+c)*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.02 (sec), antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2723}

$$\int (a + a \cos(c + dx))^2 dx = \frac{2a^2 \sin(c + dx)}{d} + \frac{a^2 \sin(c + dx) \cos(c + dx)}{2d} + \frac{3a^2x}{2}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^2, x]$

[Out] $(3*a^2*x)/2 + (2*a^2*\text{Sin}[c + d*x])/d + (a^2*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rule 2723

$\text{Int}[(a + b*\text{sin}[(c + d)*(x)])^2, x_Symbol] \rightarrow \text{Simp}[(2*a^2 + b^2)*(x/2), x] + (-\text{Simp}[2*a*b*(\text{Cos}[c + d*x]/d), x] - \text{Simp}[b^2*\text{Cos}[c + d*x]*(\text{Sin}[c + d*x]/(2*d)), x]) /; \text{FreeQ}\{a, b, c, d\}, x]$

Rubi steps

$$\text{integral} = \frac{3a^2x}{2} + \frac{2a^2 \sin(c + dx)}{d} + \frac{a^2 \cos(c + dx) \sin(c + dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int (a + a \cos(c + dx))^2 dx = \frac{a^2(6(c + dx) + 8 \sin(c + dx) + \sin(2(c + dx)))}{4d}$$

`[In] Integrate[(a + a*Cos[c + d*x])^2,x]``[Out] (a^2*(6*(c + d*x) + 8*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)`**Maple [A] (verified)**

Time = 0.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$\frac{a^2(6dx+8\sin(dx+c)+\sin(2dx+2c))}{4d}$	31
risch	$\frac{3a^2x}{2} + \frac{2a^2\sin(dx+c)}{d} + \frac{a^2\sin(2dx+2c)}{4d}$	39
parts	$a^2x + \frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{2a^2\sin(dx+c)}{d}$	50
derivativedivides	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + 2a^2\sin(dx+c) + a^2(dx+c)}{d}$	52
default	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + 2a^2\sin(dx+c) + a^2(dx+c)}{d}$	52
norman	$\frac{\frac{3a^2x}{2} + \frac{5a^2\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{3a^2\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} + 3a^2x\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{3a^2x\left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}$	94

`[In] int((a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)``[Out] 1/4*a^2*(6*d*x+8*sin(d*x+c)+sin(2*d*x+2*c))/d`**Fricas [A] (verification not implemented)**

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int (a + a \cos(c + dx))^2 dx = \frac{3a^2dx + (a^2 \cos(dx + c) + 4a^2) \sin(dx + c)}{2d}$$

`[In] integrate((a+a*cos(d*x+c))^2,x, algorithm="fricas")``[Out] 1/2*(3*a^2*d*x + (a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/d`

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int (a + a \cos(c + dx))^2 dx$$

$$= \begin{cases} \frac{a^2 x \sin^2(c+dx)}{2} + \frac{a^2 x \cos^2(c+dx)}{2} + a^2 x + \frac{a^2 \sin(c+dx) \cos(c+dx)}{2d} + \frac{2a^2 \sin(c+dx)}{d} & \text{for } d \neq 0 \\ x(a \cos(c) + a)^2 & \text{otherwise} \end{cases}$$

[In] integrate((a+a*cos(d*x+c))**2,x)

[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*x + a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*a**2*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx))^2 dx = a^2 x + \frac{(2 dx + 2 c + \sin(2 dx + 2 c))a^2}{4 d} + \frac{2 a^2 \sin(dx + c)}{d}$$

[In] integrate((a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] a^2*x + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 2*a^2*sin(d*x + c)/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int (a + a \cos(c + dx))^2 dx = \frac{3}{2} a^2 x + \frac{a^2 \sin(2 dx + 2 c)}{4 d} + \frac{2 a^2 \sin(dx + c)}{d}$$

[In] integrate((a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] 3/2*a^2*x + 1/4*a^2*sin(2*d*x + 2*c)/d + 2*a^2*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 14.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int (a + a \cos(c + dx))^2 dx = \frac{3a^2 x}{2} + \frac{3a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 5a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)^2}$$

[In] `int((a + a*cos(c + d*x))^2,x)`

[Out] `(3*a^2*x)/2 + (3*a^2*tan(c/2 + (d*x)/2)^3 + 5*a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^2)`

3.18 $\int (a + a \cos(c + dx))^2 \sec(c + dx) dx$

Optimal result	356
Rubi [A] (verified)	356
Mathematica [A] (verified)	357
Maple [A] (verified)	357
Fricas [A] (verification not implemented)	358
Sympy [F]	358
Maxima [A] (verification not implemented)	359
Giac [B] (verification not implemented)	359
Mupad [B] (verification not implemented)	359

Optimal result

Integrand size = 19, antiderivative size = 34

$$\int (a + a \cos(c + dx))^2 \sec(c + dx) dx = 2a^2x + \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \sin(c + dx)}{d}$$

[Out] $2*a^2*x + a^2*\operatorname{arctanh}(\sin(d*x+c))/d + a^2*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.07 (sec), antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2825, 2814, 3855}

$$\int (a + a \cos(c + dx))^2 \sec(c + dx) dx = \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \sin(c + dx)}{d} + 2a^2x$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^2*\operatorname{Sec}[c + d*x], x]$

[Out] $2*a^2*x + (a^2*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (a^2*\operatorname{Sin}[c + d*x])/d$

Rule 2814

$\operatorname{Int}[(a + b*\sin[e + f*x])^2/(c + d*\sin[e + f*x]), x_Symbol] \rightarrow \operatorname{Simp}[b*(x/d), x] - \operatorname{Dist}[(b*c - a*d)/d, \operatorname{Int}[1/(c + d*\sin[e + f*x]), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0]$

Rule 2825

$\operatorname{Int}[(a + b*\sin[e + f*x])^2/(c + d*\sin[e + f*x]), x_Symbol] \rightarrow \operatorname{Simp}[(-b^2)*(\operatorname{Cos}[e + f*x]/(d*f)), x] + \operatorname{Dist}[1/d, \operatorname{Int}[\operatorname{Simp}[a^2*d - b*(b*c - 2*a*d)*\sin[e + f*x], x]/(c + d*\sin[e + f*x]), x],$

`x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{a^2 \sin(c + dx)}{d} + \int (a^2 + 2a^2 \cos(c + dx)) \sec(c + dx) dx \\ &= 2a^2 x + \frac{a^2 \sin(c + dx)}{d} + a^2 \int \sec(c + dx) dx \\ &= 2a^2 x + \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\begin{aligned} \int (a + a \cos(c + dx))^2 \sec(c + dx) dx &= 2a^2 x + \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} \\ &\quad + \frac{a^2 \cos(dx) \sin(c)}{d} + \frac{a^2 \cos(c) \sin(dx)}{d} \end{aligned}$$

`[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x], x]`

`[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*Cos[c]*Sin[d*x])/d`

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a^2 \sin(dx+c) + 2a^2(dx+c) + a^2 \ln(\sec(dx+c) + \tan(dx+c))}{d}$
default	$\frac{a^2 \sin(dx+c) + 2a^2(dx+c) + a^2 \ln(\sec(dx+c) + \tan(dx+c))}{d}$
parallelrisc	$\frac{a^2 \left(2dx + \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + \sin(dx+c) \right)}{d}$
parts	$\frac{a^2 \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{a^2 \sin(dx+c)}{d} + \frac{2a^2(dx+c)}{d}$
risc	$2a^2x - \frac{ia^2e^{i(dx+c)}}{2d} + \frac{ia^2e^{-i(dx+c)}}{2d} + \frac{a^2 \ln(e^{i(dx+c)} + i)}{d} - \frac{a^2 \ln(e^{i(dx+c)} - i)}{d}$
norman	$\frac{2a^2x + \frac{2a^2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{2a^2 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} + 4a^2x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2a^2x \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + \frac{a^2 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d}$

```
[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*sin(d*x+c)+2*a^2*(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int (a + a \cos(c + dx))^2 \sec(c + dx) dx$$

$$= \frac{4a^2 dx + a^2 \log(\sin(dx + c) + 1) - a^2 \log(-\sin(dx + c) + 1) + 2a^2 \sin(dx + c)}{2d}$$

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) + 2*a^2*sin(d*x + c))/d
```

Sympy [F]

$$\int (a + a \cos(c + dx))^2 \sec(c + dx) dx = a^2 \left(\int 2 \cos(c + dx) \sec(c + dx) dx \right.$$

$$\left. + \int \cos^2(c + dx) \sec(c + dx) dx \right.$$

$$\left. + \int \sec(c + dx) dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c),x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x), x) + Integral(cos(c + d*x)**2*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int (a + a \cos(c + dx))^2 \sec(c + dx) dx$$

$$= \frac{2(dx + c)a^2 + a^2 \log(\sec(dx + c) + \tan(dx + c)) + a^2 \sin(dx + c)}{d}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="maxima")

[Out] (2*(d*x + c)*a^2 + a^2*log(sec(d*x + c) + tan(d*x + c)) + a^2*sin(d*x + c)) /d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(34) = 68.

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int (a + a \cos(c + dx))^2 \sec(c + dx) dx$$

$$= \frac{2(dx + c)a^2 + a^2 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|) - a^2 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|) + \frac{2a^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1}}{d}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="giac")

[Out] (2*(d*x + c)*a^2 + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d

Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int (a + a \cos(c + dx))^2 \sec(c + dx) dx = 2a^2 x + \frac{a^2 (2 \operatorname{atanh}(\tan(\frac{c}{2} + \frac{dx}{2})) + \sin(c + dx))}{d}$$

[In] int((a + a*cos(c + d*x))^2/cos(c + d*x),x)

[Out] 2*a^2*x + (a^2*(2*atanh(tan(c/2 + (d*x)/2)) + sin(c + d*x)))/d

3.19 $\int (a + a \cos(c + dx))^2 \sec^2(c + dx) dx$

Optimal result	360
Rubi [A] (verified)	360
Mathematica [A] (verified)	361
Maple [A] (verified)	361
Fricas [B] (verification not implemented)	362
Sympy [F]	362
Maxima [A] (verification not implemented)	363
Giac [B] (verification not implemented)	363
Mupad [B] (verification not implemented)	363

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int (a + a \cos(c + dx))^2 \sec^2(c + dx) dx = a^2 x + \frac{2a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \tan(c + dx)}{d}$$

[Out] $a^2 x + 2a^2 \operatorname{arctanh}(\sin(dx+c))/d + a^2 \tan(dx+c)/d$

Rubi [A] (verified)

Time = 0.07 (sec), antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 3855, 3852, 8}

$$\int (a + a \cos(c + dx))^2 \sec^2(c + dx) dx = \frac{2a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \tan(c + dx)}{d} + a^2 x$$

[In] $\text{Int}[(a + a \cos[c + d*x])^2 \sec[c + d*x]^2, x]$

[Out] $a^2 x + (2a^2 \operatorname{ArcTanh}[\sin[c + d*x]])/d + (a^2 \tan[c + d*x])/d$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2836

$\text{Int}[(d \sin[e] + f x)^n ((a + b \sin[e + f x])^m (d \sin[e + f x])^n), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[(a + b \sin[e + f x])^m (d \sin[e + f x])^n, x], x] /; \text{FreeQ}\{a, b, d, e, f, n\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IGT}[m, 0] \ \&\& \ \text{RationalQ}[n]$

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^2 + 2a^2 \sec(c + dx) + a^2 \sec^2(c + dx)) dx \\
 &= a^2 x + a^2 \int \sec^2(c + dx) dx + (2a^2) \int \sec(c + dx) dx \\
 &= a^2 x + \frac{2a^2 \arctanh(\sin(c + dx))}{d} - \frac{a^2 \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\
 &= a^2 x + \frac{2a^2 \arctanh(\sin(c + dx))}{d} + \frac{a^2 \tan(c + dx)}{d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int (a + a \cos(c + dx))^2 \sec^2(c + dx) dx = a^2 \left(x + \frac{2 \arctanh(\sin(c + dx))}{d} + \frac{\tan(c + dx)}{d} \right)$$

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

```
[Out] a^2*(x + (2*ArcTanh[Sin[c + d*x]]))/d + Tan[c + d*x]/d
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

method	result
derivativdivides	$\frac{a^2(dx+c)+2a^2 \ln(\sec(dx+c)+\tan(dx+c))+a^2 \tan(dx+c)}{d}$
default	$\frac{a^2(dx+c)+2a^2 \ln(\sec(dx+c)+\tan(dx+c))+a^2 \tan(dx+c)}{d}$
parts	$\frac{a^2 \tan(dx+c)}{d} + \frac{a^2(dx+c)}{d} + \frac{2a^2 \ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$a^2x + \frac{2ia^2}{d(e^{2i(dx+c)}+1)} + \frac{2a^2 \ln(e^{i(dx+c)}+i)}{d} - \frac{2a^2 \ln(e^{i(dx+c)}-i)}{d}$
parallelrisc	$\frac{a^2(dx \cos(dx+c)-2 \ln(\tan(\frac{dx}{2}+\frac{c}{2}))-1) \cos(dx+c)+2 \ln(\tan(\frac{dx}{2}+\frac{c}{2}))+1) \cos(dx+c)+\sin(dx+c)}{d \cos(dx+c)}$
norman	$\frac{a^2x(\tan^4(\frac{dx}{2}+\frac{c}{2}))+a^2x(\tan^6(\frac{dx}{2}+\frac{c}{2}))-a^2x-\frac{2a^2 \tan(\frac{dx}{2}+\frac{c}{2})}{d}-\frac{4a^2(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{d}-\frac{2a^2(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{d}-a^2x(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{(1+\tan^2(\frac{dx}{2}+\frac{c}{2}))^2(\tan^2(\frac{dx}{2}+\frac{c}{2}))-1)}$

[In] `int((a+cos(d*x+c))*a^2*sec(d*x+c)^2,x,method=_RETURNVERBOSE)`

[Out] `1/d*(a^2*(d*x+c)+2*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*tan(d*x+c))`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int (a + a \cos(c + dx))^2 \sec^2(c + dx) dx$$

$$= \frac{a^2 dx \cos(dx + c) + a^2 \cos(dx + c) \log(\sin(dx + c) + 1) - a^2 \cos(dx + c) \log(-\sin(dx + c) + 1) + a^2 \sin(dx + c)}{d \cos(dx + c)}$$

[In] `integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="fricas")`

[Out] `(a^2*d*x*cos(d*x + c) + a^2*cos(d*x + c)*log(sin(d*x + c) + 1) - a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))`

Sympy [F]

$$\int (a + a \cos(c + dx))^2 \sec^2(c + dx) dx = a^2 \left(\int 2 \cos(c + dx) \sec^2(c + dx) dx \right. \\ \left. + \int \cos^2(c + dx) \sec^2(c + dx) dx \right. \\ \left. + \int \sec^2(c + dx) dx \right)$$

[In] `integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**2,x)`

[Out] `a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))`

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int (a + a \cos(c + dx))^2 \sec^2(c + dx) dx$$

$$= \frac{(dx + c)a^2 + a^2(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + a^2 \tan(dx + c)}{d}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="maxima")

[Out] ((d*x + c)*a^2 + a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + a^2*tan(d*x + c))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(34) = 68.

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.32

$$\int (a + a \cos(c + dx))^2 \sec^2(c + dx) dx$$

$$= \frac{(dx + c)a^2 + 2a^2 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 2a^2 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - \frac{2a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1}}{d}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="giac")

[Out] ((d*x + c)*a^2 + 2*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d

Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int (a + a \cos(c + dx))^2 \sec^2(c + dx) dx = a^2 x + \frac{4a^2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d}$$

$$- \frac{2a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^2,x)

[Out] a^2*x + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))

3.20 $\int (a + a \cos(c + dx))^2 \sec^3(c + dx) dx$

Optimal result	364
Rubi [A] (verified)	364
Mathematica [A] (verified)	366
Maple [A] (verified)	366
Fricas [A] (verification not implemented)	367
Sympy [F]	367
Maxima [A] (verification not implemented)	367
Giac [A] (verification not implemented)	368
Mupad [B] (verification not implemented)	368

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int (a + a \cos(c + dx))^2 \sec^3(c + dx) dx = \frac{3a^2 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d}$$

[Out] $3/2*a^2*\operatorname{arctanh}(\sin(d*x+c))/d+2*a^2*\tan(d*x+c)/d+1/2*a^2*\sec(d*x+c)*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2836, 3855, 3852, 8, 3853}

$$\int (a + a \cos(c + dx))^2 \sec^3(c + dx) dx = \frac{3a^2 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{a^2 \tan(c + dx) \sec(c + dx)}{2d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^2*\text{Sec}[c + d*x]^3,x]$

[Out] $(3*a^2*\text{ArcTanh}[\text{Sin}[c + d*x]])/(2*d) + (2*a^2*\text{Tan}[c + d*x])/d + (a^2*\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/(2*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2836


```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^2 \sec(c + dx) + 2a^2 \sec^2(c + dx) + a^2 \sec^3(c + dx)) dx \\
 &= a^2 \int \sec(c + dx) dx + a^2 \int \sec^3(c + dx) dx + (2a^2) \int \sec^2(c + dx) dx \\
 &= \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d} \\
 &\quad + \frac{1}{2} a^2 \int \sec(c + dx) dx - \frac{(2a^2) \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\
 &= \frac{3a^2 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx))^2 \sec^3(c + dx) dx = \frac{3a^2 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d}$$

[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3,x]

[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

method	result
derivativdivides	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2a^2 \tan(dx+c)+a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
default	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2a^2 \tan(dx+c)+a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
parts	$\frac{a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d} + \frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{2a^2 \tan(dx+c)}{d}$
risch	$-\frac{ia^2(e^{3i(dx+c)}-4e^{2i(dx+c)}-e^{i(dx+c)}-4)}{d(e^{2i(dx+c)}+1)^2} + \frac{3a^2 \ln(e^{i(dx+c)}+i)}{2d} - \frac{3a^2 \ln(e^{i(dx+c)}-i)}{2d}$
parallelrisc	$-\frac{a^2 \left(3 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) \cos(2dx+2c) - 3 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) \cos(2dx+2c) - 2 \sin(dx+c) + 3 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) - 3 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) \right)}{2d(1+\cos(2dx+2c))}$
norman	$\frac{5a^2 \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d} + \frac{7a^2 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{a^2 \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{3a^2 \left(\tan^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{3a^2 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)}{2d} + \frac{3a^2 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)}{2d}$

[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*tan(d*x+c)+a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int (a + a \cos(c + dx))^2 \sec^3(c + dx) dx$$

$$= \frac{3 a^2 \cos(dx + c)^2 \log(\sin(dx + c) + 1) - 3 a^2 \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2(4 a^2 \cos(dx + c) + a^2 \sin(dx + c))}{4 d \cos(dx + c)^2}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="fricas")

```
[Out] 1/4*(3*a^2*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(4*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a + a \cos(c + dx))^2 \sec^3(c + dx) dx = a^2 \left(\int 2 \cos(c + dx) \sec^3(c + dx) dx \right. \\ \left. + \int \cos^2(c + dx) \sec^3(c + dx) dx \right. \\ \left. + \int \sec^3(c + dx) dx \right)$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x)

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(cos(c + d*x)**2*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.63

$$\int (a + a \cos(c + dx))^2 \sec^3(c + dx) dx =$$

$$\frac{a^2 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1) \right) - 2 a^2 (\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1))}{4 d}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="maxima")

```
[Out] -1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 8*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int (a + a \cos(c + dx))^2 \sec^3(c + dx) dx$$

$$= \frac{3a^2 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3a^2 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(3a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 5a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)^2}}{2d}$$

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2/d
```

Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int (a + a \cos(c + dx))^2 \sec^3(c + dx) dx = \frac{3a^2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{3a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 - 5a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)}$$

```
[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (3*a^2*tan(c/2 + (d*x)/2)^3 - 5*a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.21 $\int (a + a \cos(c + dx))^2 \sec^4(c + dx) dx$

Optimal result	369
Rubi [A] (verified)	369
Mathematica [A] (verified)	371
Maple [A] (verified)	371
Fricas [A] (verification not implemented)	372
Sympy [F]	372
Maxima [A] (verification not implemented)	372
Giac [A] (verification not implemented)	373
Mupad [B] (verification not implemented)	373

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (a + a \cos(c + dx))^2 \sec^4(c + dx) dx = \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{d} + \frac{a^2 \tan^3(c + dx)}{3d}$$

[Out] $a^2 \operatorname{arctanh}(\sin(d*x+c))/d + 2*a^2*\tan(d*x+c)/d + a^2*\sec(d*x+c)*\tan(d*x+c)/d + 1/3*a^2*\tan(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.11 (sec), antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2836, 3852, 8, 3853, 3855}

$$\int (a + a \cos(c + dx))^2 \sec^4(c + dx) dx = \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \tan^3(c + dx)}{3d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{a^2 \tan(c + dx) \sec(c + dx)}{d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^2*\text{Sec}[c + d*x]^4, x]$

[Out] $(a^2*\text{ArcTanh}[\text{Sin}[c + d*x]])/d + (2*a^2*\text{Tan}[c + d*x])/d + (a^2*\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/d + (a^2*\text{Tan}[c + d*x]^3)/(3*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> Simp}[a*x, x] \text{ /; FreeQ}[a, x]$

Rule 2836

```
Int[((d_)*sin[(e_) + (f_)*(x_)]^(n_))*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(m_)), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]
```

Rule 3852

```
Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^2 \sec^2(c + dx) + 2a^2 \sec^3(c + dx) + a^2 \sec^4(c + dx)) dx \\
 &= a^2 \int \sec^2(c + dx) dx + a^2 \int \sec^4(c + dx) dx + (2a^2) \int \sec^3(c + dx) dx \\
 &= \frac{a^2 \sec(c + dx) \tan(c + dx)}{d} + a^2 \int \sec(c + dx) dx \\
 &\quad - \frac{a^2 \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} - \frac{a^2 \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d} \\
 &= \frac{a^2 \arctanh(\sin(c + dx))}{d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{d} + \frac{a^2 \tan^3(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx))^2 \sec^4(c + dx) dx = \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{d} + \frac{a^2 \tan^3(c + dx)}{3d}$$

`[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^4,x]``[Out] (a^2*ArcTanh[Sin[c + d*x]])/d + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c + d*x]*Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)`**Maple [A] (verified)**

Time = 3.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

method	result
derivativdivides	$\frac{a^2 \tan(dx+c) + 2a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) - a^2 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d}$
default	$\frac{a^2 \tan(dx+c) + 2a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) - a^2 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d}$
parts	$-\frac{a^2 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d} + \frac{a^2 \tan(dx+c)}{d} + \frac{a^2 \sec(dx+c) \tan(dx+c)}{d} + \frac{a^2 \ln(\sec(dx+c) + \tan(dx+c))}{d}$
risch	$-\frac{2ia^2 (3e^{5i(dx+c)} - 3e^{4i(dx+c)} - 12e^{2i(dx+c)} - 3e^{i(dx+c)} - 5)}{3d(e^{2i(dx+c)} + 1)^3} + \frac{a^2 \ln(e^{i(dx+c)} + i)}{d} - \frac{a^2 \ln(e^{i(dx+c)} - i)}{d}$
parallelrisch	$-\frac{3 \left(\left(\cos(dx+c) + \frac{\cos(3dx+3c)}{3} \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + \left(-\cos(dx+c) - \frac{\cos(3dx+3c)}{3} \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) - \sin(dx+c) \right)}{d(\cos(3dx+3c) + 3\cos(dx+c))}$
norman	$-\frac{6a^2 \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d} - \frac{20a^2 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} + \frac{8a^2 \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} + \frac{4a^2 \left(\tan^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} - \frac{2a^2 \left(\tan^9 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} + \frac{a^2 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d}$

`[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^4,x,method=_RETURNVERBOSE)``[Out] 1/d*(a^2*tan(d*x+c)+2*a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))`

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.45

$$\int (a + a \cos(c + dx))^2 \sec^4(c + dx) dx$$

$$= \frac{3 a^2 \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 3 a^2 \cos(dx + c)^3 \log(-\sin(dx + c) + 1) + 2 (5 a^2 \cos(dx + c)^2 + 3 a^2 \cos(dx + c) + a^2) \sin(dx + c)}{6 d \cos(dx + c)^3}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="fricas")

```
[Out] 1/6*(3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(5*a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a + a \cos(c + dx))^2 \sec^4(c + dx) dx = a^2 \left(\int 2 \cos(c + dx) \sec^4(c + dx) dx \right. \\ \left. + \int \cos^2(c + dx) \sec^4(c + dx) dx \right. \\ \left. + \int \sec^4(c + dx) dx \right)$$

[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**4,x)

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**2*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29

$$\int (a + a \cos(c + dx))^2 \sec^4(c + dx) dx$$

$$= \frac{2 (\tan(dx + c)^3 + 3 \tan(dx + c)) a^2 - 3 a^2 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right)}{6 d}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="maxima")

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*tan(d*x + c))/d
```


Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.61

$$\int (a + a \cos(c + dx))^2 \sec^4(c + dx) dx$$

$$= \frac{3a^2 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3a^2 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(3a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 8a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 9\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)^3}}{3d}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="giac")

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 8*a^2*tan(1/2*d*x + 1/2*c
)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 15.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.70

$$\int (a + a \cos(c + dx))^2 \sec^4(c + dx) dx$$

$$= \frac{2a^2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{2a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 - \frac{16a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3} + 6a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^4,x)

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (2*a^2*tan(c/2 + (d*x)/2)^5 - (16*a^2
*tan(c/2 + (d*x)/2)^3)/3 + 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/
2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.22 $\int (a + a \cos(c + dx))^2 \sec^5(c + dx) dx$

Optimal result	374
Rubi [A] (verified)	374
Mathematica [A] (verified)	376
Maple [A] (verified)	376
Fricas [A] (verification not implemented)	377
Sympy [F]	377
Maxima [A] (verification not implemented)	377
Giac [A] (verification not implemented)	378
Mupad [B] (verification not implemented)	378

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int (a + a \cos(c + dx))^2 \sec^5(c + dx) dx = \frac{7a^2 \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{7a^2 \sec(c + dx) \tan(c + dx)}{8d} + \frac{a^2 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{2a^2 \tan^3(c + dx)}{3d}$$

[Out] $7/8*a^2*\operatorname{arctanh}(\sin(d*x+c))/d+2*a^2*\tan(d*x+c)/d+7/8*a^2*\sec(d*x+c)*\tan(d*x+c)/d+1/4*a^2*\sec(d*x+c)^3*\tan(d*x+c)/d+2/3*a^2*\tan(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 3853, 3855, 3852}

$$\int (a + a \cos(c + dx))^2 \sec^5(c + dx) dx = \frac{7a^2 \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{2a^2 \tan^3(c + dx)}{3d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{a^2 \tan(c + dx) \sec^3(c + dx)}{4d} + \frac{7a^2 \tan(c + dx) \sec(c + dx)}{8d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^2*\text{Sec}[c + d*x]^5,x]$

[Out] $(7*a^2*\text{ArcTanh}[\text{Sin}[c + d*x]])/(8*d) + (2*a^2*\text{Tan}[c + d*x])/d + (7*a^2*\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/(8*d) + (a^2*\text{Sec}[c + d*x]^3*\text{Tan}[c + d*x])/(4*d) + (2*a^2*\text{Tan}[c + d*x]^3)/(3*d)$

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^2 \sec^3(c + dx) + 2a^2 \sec^4(c + dx) + a^2 \sec^5(c + dx)) dx \\
 &= a^2 \int \sec^3(c + dx) dx + a^2 \int \sec^5(c + dx) dx + (2a^2) \int \sec^4(c + dx) dx \\
 &= \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d} + \frac{a^2 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{2} a^2 \int \sec(c + dx) dx \\
 &\quad + \frac{1}{4} (3a^2) \int \sec^3(c + dx) dx - \frac{(2a^2) \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d} \\
 &= \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{7a^2 \sec(c + dx) \tan(c + dx)}{8d} \\
 &\quad + \frac{a^2 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{2a^2 \tan^3(c + dx)}{3d} + \frac{1}{8} (3a^2) \int \sec(c + dx) dx \\
 &= \frac{7a^2 \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{2a^2 \tan(c + dx)}{d} + \frac{7a^2 \sec(c + dx) \tan(c + dx)}{8d} \\
 &\quad + \frac{a^2 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{2a^2 \tan^3(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int (a + a \cos(c + dx))^2 \sec^5(c + dx) dx$$

$$= \frac{a^2(21 \operatorname{arctanh}(\sin(c + dx)) + \tan(c + dx) (21 \sec(c + dx) + 6 \sec^3(c + dx) + 16(3 + \tan^2(c + dx))))}{24d}$$

[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^5,x]

[Out] (a^2*(21*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(21*Sec[c + d*x] + 6*Sec[c + d*x]^3 + 16*(3 + Tan[c + d*x]^2))))/(24*d)

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right) - 2a^2 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + a^2 \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right)}{d}$
default	$\frac{a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right) - 2a^2 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + a^2 \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right)}{d}$
parts	$\frac{a^2 \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right)}{d} + \frac{a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
risch	$-\frac{ia^2(21e^{7i(dx+c)} + 45e^{5i(dx+c)} - 96e^{4i(dx+c)} - 45e^{3i(dx+c)} - 128e^{2i(dx+c)} - 21e^{i(dx+c)} - 32)}{12d(e^{2i(dx+c)} + 1)^4} + \frac{7a^2 \ln(e^{i(dx+c)} + i)}{8d}$
parallelrisc	$\frac{a^2 \left(21(-\cos(4dx+4c) - 4\cos(2dx+2c) - 3) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + 21(\cos(4dx+4c) + 4\cos(2dx+2c) + 3) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) \right)}{24d(\cos(4dx+4c) + 4\cos(2dx+2c) + 3)}$
norman	$\frac{\frac{25a^2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} + \frac{67a^2 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} - \frac{7a^2 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d} + \frac{25a^2 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d} + \frac{35a^2 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} - \frac{7a^2 \left(\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^4}$

[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-2*a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))

[Out] $\frac{1}{48} \cdot (32 \cdot (\tan(dx + c))^3 + 3 \cdot \tan(dx + c)) \cdot a^2 - 3 \cdot a^2 \cdot (2 \cdot (3 \cdot \sin(dx + c))^3 - 5 \cdot \sin(dx + c)) / (\sin(dx + c)^4 - 2 \cdot \sin(dx + c)^2 + 1) - 3 \cdot \log(\sin(dx + c) + 1) + 3 \cdot \log(\sin(dx + c) - 1) - 12 \cdot a^2 \cdot (2 \cdot \sin(dx + c)) / (\sin(dx + c)^2 - 1) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1)) / d$

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int (a + a \cos(c + dx))^2 \sec^5(c + dx) dx$$

$$= \frac{21 a^2 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 21 a^2 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(21 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^7 - 77 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 \right)}{\left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)^2 - 1}}{24 d}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="giac")

[Out] $\frac{1}{24} \cdot (21 \cdot a^2 \cdot \log(\text{abs}(\tan(1/2 \cdot dx + 1/2 \cdot c) + 1)) - 21 \cdot a^2 \cdot \log(\text{abs}(\tan(1/2 \cdot dx + 1/2 \cdot c) - 1)) - 2 \cdot (21 \cdot a^2 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^7 - 77 \cdot a^2 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^5 + 83 \cdot a^2 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)^3 - 75 \cdot a^2 \cdot \tan(1/2 \cdot dx + 1/2 \cdot c)) / (\tan(1/2 \cdot dx + 1/2 \cdot c)^2 - 1)^4) / d$

Mupad [B] (verification not implemented)

Time = 16.99 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

$$\int (a + a \cos(c + dx))^2 \sec^5(c + dx) dx$$

$$= \frac{7 a^2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{4 d} - \frac{\frac{7 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{4} - \frac{77 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{12} + \frac{83 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{12} - \frac{25 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^5,x)

[Out] $\frac{(7 \cdot a^2 \cdot \operatorname{atanh}(\tan(c/2 + (d \cdot x)/2))) / (4 \cdot d) - ((83 \cdot a^2 \cdot \tan(c/2 + (d \cdot x)/2)^3) / 12 - (77 \cdot a^2 \cdot \tan(c/2 + (d \cdot x)/2)^5) / 12 + (7 \cdot a^2 \cdot \tan(c/2 + (d \cdot x)/2)^7) / 4 - (25 \cdot a^2 \cdot \tan(c/2 + (d \cdot x)/2)) / 4) / (d \cdot (6 \cdot \tan(c/2 + (d \cdot x)/2)^4 - 4 \cdot \tan(c/2 + (d \cdot x)/2)^2 - 4 \cdot \tan(c/2 + (d \cdot x)/2)^6 + \tan(c/2 + (d \cdot x)/2)^8 + 1))$

3.23 $\int \cos^3(c + dx)(a + a \cos(c + dx))^3 dx$

Optimal result	379
Rubi [A] (verified)	379
Mathematica [A] (verified)	381
Maple [A] (verified)	381
Fricas [A] (verification not implemented)	382
Sympy [B] (verification not implemented)	383
Maxima [A] (verification not implemented)	383
Giac [A] (verification not implemented)	384
Mupad [B] (verification not implemented)	384

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^3 dx = \frac{23a^3x}{16} + \frac{4a^3 \sin(c + dx)}{d} + \frac{23a^3 \cos(c + dx) \sin(c + dx)}{16d} + \frac{23a^3 \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{a^3 \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{7a^3 \sin^3(c + dx)}{3d} + \frac{3a^3 \sin^5(c + dx)}{5d}$$

[Out] $\frac{23}{16}a^3x + \frac{4a^3 \sin(d*x+c)}{d} + \frac{23}{16}a^3 \cos(d*x+c) \sin(d*x+c) / d + \frac{23}{24}a^3 \cos(d*x+c)^3 \sin(d*x+c) / d + \frac{1}{6}a^3 \cos(d*x+c)^5 \sin(d*x+c) / d - \frac{7}{3}a^3 \sin(d*x+c)^3 / d + \frac{3}{5}a^3 \sin(d*x+c)^5 / d$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 2713, 2715, 8}

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^3 dx = \frac{3a^3 \sin^5(c + dx)}{5d} - \frac{7a^3 \sin^3(c + dx)}{3d} + \frac{4a^3 \sin(c + dx)}{d} + \frac{a^3 \sin(c + dx) \cos^5(c + dx)}{6d} + \frac{23a^3 \sin(c + dx) \cos^3(c + dx)}{24d} + \frac{23a^3 \sin(c + dx) \cos(c + dx)}{16d} + \frac{23a^3x}{16}$$

[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3,x]

[Out] (23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^5)/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^3 \cos^3(c + dx) + 3a^3 \cos^4(c + dx) + 3a^3 \cos^5(c + dx) + a^3 \cos^6(c + dx)) dx \\
 &= a^3 \int \cos^3(c + dx) dx + a^3 \int \cos^6(c + dx) dx \\
 &\quad + (3a^3) \int \cos^4(c + dx) dx + (3a^3) \int \cos^5(c + dx) dx \\
 &= \frac{3a^3 \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{a^3 \cos^5(c + dx) \sin(c + dx)}{6d} \\
 &\quad + \frac{1}{6}(5a^3) \int \cos^4(c + dx) dx + \frac{1}{4}(9a^3) \int \cos^2(c + dx) dx \\
 &\quad - \frac{a^3 \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{d} \\
 &\quad - \frac{(3a^3) \text{Subst}(\int (1 - 2x^2 + x^4) dx, x, -\sin(c + dx))}{d}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{4a^3 \sin(c+dx)}{d} + \frac{9a^3 \cos(c+dx) \sin(c+dx)}{8d} + \frac{23a^3 \cos^3(c+dx) \sin(c+dx)}{24d} \\
&\quad + \frac{a^3 \cos^5(c+dx) \sin(c+dx)}{6d} - \frac{7a^3 \sin^3(c+dx)}{3d} \\
&\quad + \frac{3a^3 \sin^5(c+dx)}{5d} + \frac{1}{8}(5a^3) \int \cos^2(c+dx) dx + \frac{1}{8}(9a^3) \int 1 dx \\
&= \frac{9a^3 x}{8} + \frac{4a^3 \sin(c+dx)}{d} + \frac{23a^3 \cos(c+dx) \sin(c+dx)}{16d} + \frac{23a^3 \cos^3(c+dx) \sin(c+dx)}{24d} \\
&\quad + \frac{a^3 \cos^5(c+dx) \sin(c+dx)}{6d} - \frac{7a^3 \sin^3(c+dx)}{3d} + \frac{3a^3 \sin^5(c+dx)}{5d} + \frac{1}{16}(5a^3) \int 1 dx \\
&= \frac{23a^3 x}{16} + \frac{4a^3 \sin(c+dx)}{d} + \frac{23a^3 \cos(c+dx) \sin(c+dx)}{16d} \\
&\quad + \frac{23a^3 \cos^3(c+dx) \sin(c+dx)}{24d} + \frac{a^3 \cos^5(c+dx) \sin(c+dx)}{6d} \\
&\quad - \frac{7a^3 \sin^3(c+dx)}{3d} + \frac{3a^3 \sin^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\begin{aligned}
&\int \cos^3(c+dx)(a+a\cos(c+dx))^3 dx \\
&= \frac{a^3(1380dx + 2520 \sin(c+dx) + 945 \sin(2(c+dx)) + 380 \sin(3(c+dx)) + 135 \sin(4(c+dx)) + 36 \sin(5(c+dx)))}{960d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3,x]

[Out] (a^3*(1380*d*x + 2520*Sin[c + d*x] + 945*Sin[2*(c + d*x)] + 380*Sin[3*(c + d*x)] + 135*Sin[4*(c + d*x)] + 36*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(960*d)

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

method	result
parallelr risch	$\frac{(276dx + \sin(6dx+6c) + 504 \sin(dx+c) + 189 \sin(2dx+2c) + 76 \sin(3dx+3c) + 27 \sin(4dx+4c) + \frac{36 \sin(5dx+5c)}{5}) a^3}{192d}$
derivativ divides	$\frac{23a^3x}{16} + \frac{21a^3 \sin(dx+c)}{8d} + \frac{a^3 \sin(6dx+6c)}{192d} + \frac{3a^3 \sin(5dx+5c)}{80d} + \frac{9a^3 \sin(4dx+4c)}{64d} + \frac{19a^3 \sin(3dx+3c)}{48d} + \frac{63a^3 \sin(2dx+2c)}{32d} + \frac{27a^3 \sin(dx+c)}{16d} + \frac{3a^3}{16d}$
default	$a^3 \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{3a^3 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + 3a^3$
parts	$\frac{a^3(2 + \cos^2(dx+c)) \sin(dx+c)}{3d} + \frac{a^3 \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right)}{d} + \frac{3a^3 \left(\frac{\cos^3(dx+c)}{3} \right)}{d}$
norman	$\frac{23a^3x}{16} + \frac{105a^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d} + \frac{211a^3 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{8d} + \frac{969a^3 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{20d} + \frac{759a^3 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{20d} + \frac{391a^3 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{24d} + \frac{23a^3}{24d}$

[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] 1/192*(276*d*x+sin(6*d*x+6*c)+504*sin(d*x+c)+189*sin(2*d*x+2*c)+76*sin(3*d*x+3*c)+27*sin(4*d*x+4*c)+36/5*sin(5*d*x+5*c))*a^3/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^3 dx$$

$$= \frac{345 a^3 dx + (40 a^3 \cos(dx+c))^5 + 144 a^3 \cos(dx+c)^4 + 230 a^3 \cos(dx+c)^3 + 272 a^3 \cos(dx+c)^2 + 345 a^3 \cos(dx+c) + 544 a^3}{240 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/240*(345*a^3*d*x + (40*a^3*cos(d*x + c))^5 + 144*a^3*cos(d*x + c)^4 + 230*a^3*cos(d*x + c)^3 + 272*a^3*cos(d*x + c)^2 + 345*a^3*cos(d*x + c) + 544*a^3)*sin(d*x + c)/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(122) = 244$.

Time = 0.39 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.94

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^3 dx$$

$$= \left\{ \begin{array}{l} \frac{5a^3 x \sin^6(c+dx)}{16} + \frac{15a^3 x \sin^4(c+dx) \cos^2(c+dx)}{16} + \frac{9a^3 x \sin^4(c+dx)}{8} + \frac{15a^3 x \sin^2(c+dx) \cos^4(c+dx)}{16} + \frac{9a^3 x \sin^2(c+dx) \cos^2(c+dx)}{4} \\ x(a \cos(c) + a)^3 \cos^3(c) \end{array} \right.$$

[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**3,x)

[Out] Piecewise((5*a**3*x*sin(c + d*x)**6/16 + 15*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a**3*x*sin(c + d*x)**4/8 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*a**3*x*cos(c + d*x)**6/16 + 9*a**3*x*cos(c + d*x)**4/8 + 5*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8*a**3*sin(c + d*x)**5/(5*d) + 5*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 4*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 9*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a**3*sin(c + d*x)**3/(3*d) + 11*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 3*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^3 dx$$

$$= \frac{192 (3 \sin(dx + c))^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c) a^3 - 5 (4 \sin(2dx + 2c))^3 - 60 dx - 60 c - 9 \sin(4dx + 4c) - 48 \sin(2dx + 2c) a^3 - 320 (\sin(dx + c)^3 - 3 \sin(dx + c)) a^3 + 90 (12 dx + 12 c + \sin(4dx + 4c) + 8 \sin(2dx + 2c)) a^3}{d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^3 dx = \frac{23}{16} a^3 x + \frac{a^3 \sin(6 dx + 6 c)}{192 d} + \frac{3 a^3 \sin(5 dx + 5 c)}{80 d} + \frac{9 a^3 \sin(4 dx + 4 c)}{64 d} + \frac{19 a^3 \sin(3 dx + 3 c)}{48 d} + \frac{63 a^3 \sin(2 dx + 2 c)}{64 d} + \frac{21 a^3 \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] 23/16*a^3*x + 1/192*a^3*sin(6*d*x + 6*c)/d + 3/80*a^3*sin(5*d*x + 5*c)/d + 9/64*a^3*sin(4*d*x + 4*c)/d + 19/48*a^3*sin(3*d*x + 3*c)/d + 63/64*a^3*sin(2*d*x + 2*c)/d + 21/8*a^3*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 16.74 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^3 dx = \frac{23 a^3 x}{16} + \frac{\frac{23 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11}}{8} + \frac{391 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{24} + \frac{759 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{20} + \frac{969 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{20} + \frac{211 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{8} + \frac{105 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)^6}$$

[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^3,x)

[Out] (23*a^3*x)/16 + ((211*a^3*tan(c/2 + (d*x)/2)^3)/8 + (969*a^3*tan(c/2 + (d*x)/2)^5)/20 + (759*a^3*tan(c/2 + (d*x)/2)^7)/20 + (391*a^3*tan(c/2 + (d*x)/2)^9)/24 + (23*a^3*tan(c/2 + (d*x)/2)^11)/8 + (105*a^3*tan(c/2 + (d*x)/2))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)

3.24 $\int \cos^2(c + dx)(a + a \cos(c + dx))^3 dx$

Optimal result	385
Rubi [A] (verified)	385
Mathematica [A] (verified)	387
Maple [A] (verified)	387
Fricas [A] (verification not implemented)	388
Sympy [B] (verification not implemented)	388
Maxima [A] (verification not implemented)	389
Giac [A] (verification not implemented)	389
Mupad [B] (verification not implemented)	389

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^3 dx = \frac{13a^3x}{8} + \frac{4a^3 \sin(c + dx)}{d} + \frac{13a^3 \cos(c + dx) \sin(c + dx)}{8d} + \frac{3a^3 \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{5a^3 \sin^3(c + dx)}{3d} + \frac{a^3 \sin^5(c + dx)}{5d}$$

[Out] $13/8*a^3*x+4*a^3*\sin(d*x+c)/d+13/8*a^3*\cos(d*x+c)*\sin(d*x+c)/d+3/4*a^3*\cos(d*x+c)^3*\sin(d*x+c)/d-5/3*a^3*\sin(d*x+c)^3/d+1/5*a^3*\sin(d*x+c)^5/d$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 2715, 8, 2713}

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^3 dx = \frac{a^3 \sin^5(c + dx)}{5d} - \frac{5a^3 \sin^3(c + dx)}{3d} + \frac{4a^3 \sin(c + dx)}{d} + \frac{3a^3 \sin(c + dx) \cos^3(c + dx)}{4d} + \frac{13a^3 \sin(c + dx) \cos(c + dx)}{8d} + \frac{13a^3x}{8}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^2*(a + a*\text{Cos}[c + d*x])^3, x]$

```
[Out] (13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3)/
(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a^3 \cos^2(c + dx) + 3a^3 \cos^3(c + dx) + 3a^3 \cos^4(c + dx) + a^3 \cos^5(c + dx)) dx \\
&= a^3 \int \cos^2(c + dx) dx + a^3 \int \cos^5(c + dx) dx \\
&\quad + (3a^3) \int \cos^3(c + dx) dx + (3a^3) \int \cos^4(c + dx) dx \\
&= \frac{a^3 \cos(c + dx) \sin(c + dx)}{2d} + \frac{3a^3 \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{1}{2} a^3 \int 1 dx \\
&\quad + \frac{1}{4} (9a^3) \int \cos^2(c + dx) dx - \frac{a^3 \text{Subst}(\int (1 - 2x^2 + x^4) dx, x, -\sin(c + dx))}{d} \\
&\quad - \frac{(3a^3) \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{d} \\
&= \frac{a^3 x}{2} + \frac{4a^3 \sin(c + dx)}{d} + \frac{13a^3 \cos(c + dx) \sin(c + dx)}{8d} + \frac{3a^3 \cos^3(c + dx) \sin(c + dx)}{4d} \\
&\quad - \frac{5a^3 \sin^3(c + dx)}{3d} + \frac{a^3 \sin^5(c + dx)}{5d} + \frac{1}{8} (9a^3) \int 1 dx
\end{aligned}$$

$$= \frac{13a^3x}{8} + \frac{4a^3 \sin(c+dx)}{d} + \frac{13a^3 \cos(c+dx) \sin(c+dx)}{8d} + \frac{3a^3 \cos^3(c+dx) \sin(c+dx)}{4d} - \frac{5a^3 \sin^3(c+dx)}{3d} + \frac{a^3 \sin^5(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3 dx = \frac{a^3(780dx + 1380 \sin(c+dx) + 480 \sin(2(c+dx)) + 170 \sin(3(c+dx)) + 45 \sin(4(c+dx)) + 6 \sin(5(c+dx)))}{480d}$$

[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3,x]

[Out] (a^3*(780*d*x + 1380*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 170*Sin[3*(c + d*x)] + 45*Sin[4*(c + d*x)] + 6*Sin[5*(c + d*x)])/(480*d)

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{a^3(780dx+1380 \sin(dx+c)+6 \sin(5dx+5c)+45 \sin(4dx+4c)+170 \sin(3dx+3c)+480 \sin(2dx+2c))}{480d}$
risch	$\frac{13a^3x}{8} + \frac{23a^3 \sin(dx+c)}{8d} + \frac{a^3 \sin(5dx+5c)}{80d} + \frac{3a^3 \sin(4dx+4c)}{32d} + \frac{17a^3 \sin(3dx+3c)}{48d} + \frac{a^3 \sin(2dx+2c)}{d}$
derivativedivides	$\frac{a^3 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + 3a^3 \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + a^3(2 + \cos^2(dx+c))}{d}$
default	$\frac{a^3 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + 3a^3 \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + a^3(2 + \cos^2(dx+c))}{d}$
parts	$\frac{a^3 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d} + \frac{a^3 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5d} + \frac{a^3(2 + \cos^2(dx+c)) \sin(dx+c)}{d}$
norman	$\frac{13a^3x}{8} + \frac{51a^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} + \frac{133a^3 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d} + \frac{416a^3 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15d} + \frac{91a^3 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d} + \frac{13a^3 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} + \frac{65a^3}{4d} \frac{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}$

[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] 1/480*a^3*(780*d*x+1380*sin(d*x+c)+6*sin(5*d*x+5*c)+45*sin(4*d*x+4*c)+170*sin(3*d*x+3*c)+480*sin(2*d*x+2*c))/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^3 dx$$

$$= \frac{195 a^3 dx + (24 a^3 \cos(dx + c))^4 + 90 a^3 \cos(dx + c)^3 + 152 a^3 \cos(dx + c)^2 + 195 a^3 \cos(dx + c) + 304 a^3}{120 d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/120*(195*a^3*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^3*cos(d*x + c)^3 + 152*a^3*cos(d*x + c)^2 + 195*a^3*cos(d*x + c) + 304*a^3)*sin(d*x + c))/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(99) = 198.

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.59

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^3 dx$$

$$= \begin{cases} \frac{9a^3 x \sin^4(c+dx)}{8} + \frac{9a^3 x \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{a^3 x \sin^2(c+dx)}{2} + \frac{9a^3 x \cos^4(c+dx)}{8} + \frac{a^3 x \cos^2(c+dx)}{2} + \frac{8a^3 \sin^5(c+dx)}{15d} + \frac{4a^3 \sin^3(c+dx)}{15d} \\ x(a \cos(c) + a)^3 \cos^2(c) \end{cases}$$

[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**3,x)

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**4/8 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**3*x*sin(c + d*x)**2/2 + 9*a**3*x*cos(c + d*x)**4/8 + a**3*x*cos(c + d*x)**2/2 + 8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a**3*sin(c + d*x)**3/d + a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*a**3*sin(c + d*x)*cos(c + d*x)**2/d + a**3*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c)**2, True))
```


Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.11

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^3 dx$$

$$= \frac{32 (3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))a^3 - 480 (\sin(dx + c)^3 - 3 \sin(dx + c))a^3 + 45 (12dx + 12c + \sin(4dx + 4c) + 8\sin(2dx + 2c))a^3 + 120(2dx + 2c + \sin(2dx + 2c))a^3}{480d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3)/d

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^3 dx = \frac{13}{8} a^3 x + \frac{a^3 \sin(5 dx + 5 c)}{80 d}$$

$$+ \frac{3 a^3 \sin(4 dx + 4 c)}{32 d} + \frac{17 a^3 \sin(3 dx + 3 c)}{48 d}$$

$$+ \frac{a^3 \sin(2 dx + 2 c)}{d} + \frac{23 a^3 \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] 13/8*a^3*x + 1/80*a^3*sin(5*d*x + 5*c)/d + 3/32*a^3*sin(4*d*x + 4*c)/d + 17/48*a^3*sin(3*d*x + 3*c)/d + a^3*sin(2*d*x + 2*c)/d + 23/8*a^3*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 17.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^3 dx$$

$$= \frac{13 a^3 x}{8} + \frac{13 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{4} + \frac{91 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{6} + \frac{416 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{15} + \frac{133 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{6} + \frac{51 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4}$$

$$d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)^5$$

[In] `int(cos(c + d*x)^2*(a + a*cos(c + d*x))^3,x)`

[Out] $(13*a^3*x)/8 + ((133*a^3*\tan(c/2 + (d*x)/2)^3)/6 + (416*a^3*\tan(c/2 + (d*x)/2)^5)/15 + (91*a^3*\tan(c/2 + (d*x)/2)^7)/6 + (13*a^3*\tan(c/2 + (d*x)/2)^9)/4 + (51*a^3*\tan(c/2 + (d*x)/2))/4)/(d*(\tan(c/2 + (d*x)/2)^2 + 1)^5)$

3.25 $\int \cos(c + dx)(a + a \cos(c + dx))^3 dx$

Optimal result	391
Rubi [A] (verified)	391
Mathematica [A] (verified)	393
Maple [A] (verified)	393
Fricas [A] (verification not implemented)	394
Sympy [B] (verification not implemented)	395
Maxima [A] (verification not implemented)	395
Giac [A] (verification not implemented)	396
Mupad [B] (verification not implemented)	396

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \cos(c + dx)(a + a \cos(c + dx))^3 dx = \frac{15a^3x}{8} + \frac{4a^3 \sin(c + dx)}{d} + \frac{15a^3 \cos(c + dx) \sin(c + dx)}{8d} + \frac{a^3 \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{a^3 \sin^3(c + dx)}{d}$$

[Out] 15/8*a^3*x+4*a^3*sin(d*x+c)/d+15/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2830, 2724, 2717, 2715, 8, 2713}

$$\int \cos(c + dx)(a + a \cos(c + dx))^3 dx = -\frac{a^3 \sin^3(c + dx)}{4d} + \frac{3a^3 \sin(c + dx)}{d} + \frac{9a^3 \sin(c + dx) \cos(c + dx)}{8d} + \frac{15a^3x}{8} + \frac{\sin(c + dx)(a \cos(c + dx) + a)^3}{4d}$$

[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3,x]

[Out] (15*a^3*x)/8 + (3*a^3*Sin[c + d*x])/d + (9*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + ((a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) - (a^3*Sin[c + d*x]^3)/(4*d)

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2715

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 2717

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 2724

`Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Int[ExpandTrig[(a + b*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]`

Rule 2830

`Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]`

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(a + a \cos(c + dx))^3 \sin(c + dx)}{4d} + \frac{3}{4} \int (a + a \cos(c + dx))^3 dx \\ &= \frac{(a + a \cos(c + dx))^3 \sin(c + dx)}{4d} \\ &\quad + \frac{3}{4} \int (a^3 + 3a^3 \cos(c + dx) + 3a^3 \cos^2(c + dx) + a^3 \cos^3(c + dx)) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{3a^3x}{4} + \frac{(a + a \cos(c + dx))^3 \sin(c + dx)}{4d} + \frac{1}{4}(3a^3) \int \cos^3(c + dx) dx \\
&\quad + \frac{1}{4}(9a^3) \int \cos(c + dx) dx + \frac{1}{4}(9a^3) \int \cos^2(c + dx) dx \\
&= \frac{3a^3x}{4} + \frac{9a^3 \sin(c + dx)}{4d} + \frac{9a^3 \cos(c + dx) \sin(c + dx)}{8d} \\
&\quad + \frac{(a + a \cos(c + dx))^3 \sin(c + dx)}{4d} + \frac{1}{8}(9a^3) \int 1 dx \\
&\quad - \frac{(3a^3) \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{4d} \\
&= \frac{15a^3x}{8} + \frac{3a^3 \sin(c + dx)}{d} + \frac{9a^3 \cos(c + dx) \sin(c + dx)}{8d} \\
&\quad + \frac{(a + a \cos(c + dx))^3 \sin(c + dx)}{4d} - \frac{a^3 \sin^3(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\begin{aligned}
&\int \cos(c + dx)(a + a \cos(c + dx))^3 dx \\
&= \frac{a^3(60dx + 104 \sin(c + dx) + 32 \sin(2(c + dx)) + 8 \sin(3(c + dx)) + \sin(4(c + dx)))}{32d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3,x]

[Out] (a^3*(60*d*x + 104*Sin[c + d*x] + 32*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)] + Sin[4*(c + d*x)]))/(32*d)

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

method	result
parallelrisc	$\frac{a^3(60dx+\sin(4dx+4c)+8\sin(3dx+3c)+32\sin(2dx+2c)+104\sin(dx+c))}{32d}$
risc	$\frac{15a^3x}{8} + \frac{13a^3\sin(dx+c)}{4d} + \frac{a^3\sin(4dx+4c)}{32d} + \frac{a^3\sin(3dx+3c)}{4d} + \frac{a^3\sin(2dx+2c)}{d}$
derivativdivides	$\frac{a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + a^3(2+\cos^2(dx+c))\sin(dx+c) + 3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$
default	$\frac{a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + a^3(2+\cos^2(dx+c))\sin(dx+c) + 3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$
parts	$\frac{a^3\sin(dx+c)}{d} + \frac{a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right)}{d} + \frac{3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{a^3(2+\cos^2(dx+c))\sin(dx+c)}{d}$
norman	$\frac{\frac{15a^3x}{8} + \frac{49a^3\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} + \frac{73a^3\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} + \frac{55a^3\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} + \frac{15a^3\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} + \frac{15a^3x\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{45a^3x}{2}}{\left(1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^4}$

[In] `int(cos(d*x+c)*(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{32}a^3(60d*x+\sin(4*d*x+4*c)+8*\sin(3*d*x+3*c)+32*\sin(2*d*x+2*c)+104*\sin(d*x+c))/d$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \cos(c+dx)(a+a\cos(c+dx))^3 dx$$

$$= \frac{15a^3dx + (2a^3\cos(dx+c))^3 + 8a^3\cos(dx+c)^2 + 15a^3\cos(dx+c) + 24a^3\sin(dx+c)}{8d}$$

[In] `integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] $\frac{1}{8}*(15*a^3*d*x + (2*a^3*\cos(d*x + c))^3 + 8*a^3*\cos(d*x + c)^2 + 15*a^3*\cos(d*x + c) + 24*a^3)*\sin(d*x + c))/d$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(78) = 156.

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.64

$$\int \cos(c + dx)(a + a \cos(c + dx))^3 dx$$

$$= \begin{cases} \frac{3a^3 x \sin^4(c+dx)}{8} + \frac{3a^3 x \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{3a^3 x \sin^2(c+dx)}{2} + \frac{3a^3 x \cos^4(c+dx)}{8} + \frac{3a^3 x \cos^2(c+dx)}{2} + \frac{3a^3 \sin^3(c+dx) \cos(c+dx)}{8d} \\ x(a \cos(c) + a)^3 \cos(c) \end{cases}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3,x)

[Out] Piecewise(((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*x*cos(c + d*x)**2/2 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a**3*sin(c + d*x)**3/d + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + a**3*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c), True))

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \cos(c + dx)(a + a \cos(c + dx))^3 dx =$$

$$\frac{32 (\sin(dx + c)^3 - 3 \sin(dx + c))a^3 - (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c))a^3 - 24 (2 dx + 2 c + \sin(2 dx + 2 c))a^3 - 32 a^3 \sin(dx + c)}{32 d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] -1/32*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 - 32*a^3*sin(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \cos(c + dx)(a + a \cos(c + dx))^3 dx = \frac{15}{8} a^3 x + \frac{a^3 \sin(4 dx + 4 c)}{32 d} + \frac{a^3 \sin(3 dx + 3 c)}{4 d} + \frac{a^3 \sin(2 dx + 2 c)}{d} + \frac{13 a^3 \sin(dx + c)}{4 d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] 15/8*a^3*x + 1/32*a^3*sin(4*d*x + 4*c)/d + 1/4*a^3*sin(3*d*x + 3*c)/d + a^3*sin(2*d*x + 2*c)/d + 13/4*a^3*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 17.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \cos(c + dx)(a + a \cos(c + dx))^3 dx = \frac{15 a^3 x}{8} + \frac{15 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{4} + \frac{55 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{4} + \frac{73 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{4} + \frac{49 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4} \frac{1}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)^4}$$

[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^3,x)

[Out] (15*a^3*x)/8 + ((73*a^3*tan(c/2 + (d*x)/2)^3)/4 + (55*a^3*tan(c/2 + (d*x)/2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 + (49*a^3*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)

3.26 $\int (a + a \cos(c + dx))^3 dx$

Optimal result	397
Rubi [A] (verified)	397
Mathematica [A] (verified)	399
Maple [A] (verified)	399
Fricas [A] (verification not implemented)	400
Sympy [B] (verification not implemented)	400
Maxima [A] (verification not implemented)	400
Giac [A] (verification not implemented)	401
Mupad [B] (verification not implemented)	401

Optimal result

Integrand size = 12, antiderivative size = 63

$$\int (a + a \cos(c + dx))^3 dx = \frac{5a^3 x}{2} + \frac{4a^3 \sin(c + dx)}{d} + \frac{3a^3 \cos(c + dx) \sin(c + dx)}{2d} - \frac{a^3 \sin^3(c + dx)}{3d}$$

[Out] $5/2*a^3*x+4*a^3*\sin(d*x+c)/d+3/2*a^3*\cos(d*x+c)*\sin(d*x+c)/d-1/3*a^3*\sin(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2724, 2717, 2715, 8, 2713}

$$\int (a + a \cos(c + dx))^3 dx = -\frac{a^3 \sin^3(c + dx)}{3d} + \frac{4a^3 \sin(c + dx)}{d} + \frac{3a^3 \sin(c + dx) \cos(c + dx)}{2d} + \frac{5a^3 x}{2}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^3, x]$

[Out] $(5*a^3*x)/2 + (4*a^3*\text{Sin}[c + d*x])/d + (3*a^3*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d) - (a^3*\text{Sin}[c + d*x]^3)/(3*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] \text{ /; } \text{FreeQ}[a, x]$

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2724

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrig[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^3 + 3a^3 \cos(c + dx) + 3a^3 \cos^2(c + dx) + a^3 \cos^3(c + dx)) dx \\
 &= a^3 x + a^3 \int \cos^3(c + dx) dx + (3a^3) \int \cos(c + dx) dx + (3a^3) \int \cos^2(c + dx) dx \\
 &= a^3 x + \frac{3a^3 \sin(c + dx)}{d} + \frac{3a^3 \cos(c + dx) \sin(c + dx)}{2d} \\
 &\quad + \frac{1}{2}(3a^3) \int 1 dx - \frac{a^3 \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{d} \\
 &= \frac{5a^3 x}{2} + \frac{4a^3 \sin(c + dx)}{d} + \frac{3a^3 \cos(c + dx) \sin(c + dx)}{2d} - \frac{a^3 \sin^3(c + dx)}{3d}
 \end{aligned}$$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int (a + a \cos(c + dx))^3 dx$$

$$= \frac{15 a^3 dx + (2 a^3 \cos(dx + c)^2 + 9 a^3 \cos(dx + c) + 22 a^3) \sin(dx + c)}{6 d}$$

[In] integrate((a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/6*(15*a^3*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 22*a^3)*sin(d*x + c))/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(58) = 116.

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.92

$$\int (a + a \cos(c + dx))^3 dx$$

$$= \begin{cases} \frac{3a^3 x \sin^2(c+dx)}{2} + \frac{3a^3 x \cos^2(c+dx)}{2} + a^3 x + \frac{2a^3 \sin^3(c+dx)}{3d} + \frac{a^3 \sin(c+dx) \cos^2(c+dx)}{d} + \frac{3a^3 \sin(c+dx) \cos(c+dx)}{2d} + \frac{3a^3 \sin(c+dx)}{d} \\ x(a \cos(c) + a)^3 \end{cases}$$

[In] integrate((a+a*cos(d*x+c))**3,x)

[Out] Piecewise(((3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**2/2 + a**3*x + 2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*a**3*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int (a + a \cos(c + dx))^3 dx = a^3 x - \frac{(\sin(dx + c))^3 - 3 \sin(dx + c)}{3 d} a^3$$

$$+ \frac{3(2 dx + 2 c + \sin(2 dx + 2 c)) a^3}{4 d} + \frac{3 a^3 \sin(dx + c)}{d}$$

[In] integrate((a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] a^3*x - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3/d + 3/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3/d + 3*a^3*sin(d*x + c)/d

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int (a + a \cos(c + dx))^3 dx = \frac{5}{2} a^3 x + \frac{a^3 \sin(3 dx + 3 c)}{12 d} + \frac{3 a^3 \sin(2 dx + 2 c)}{4 d} + \frac{15 a^3 \sin(dx + c)}{4 d}$$

[In] integrate((a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] 5/2*a^3*x + 1/12*a^3*sin(3*d*x + 3*c)/d + 3/4*a^3*sin(2*d*x + 2*c)/d + 15/4*a^3*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx))^3 dx = \frac{5 a^3 x}{2} + \frac{11 a^3 \sin(c + dx)}{3 d} + \frac{a^3 \cos(c + dx)^2 \sin(c + dx)}{3 d} + \frac{3 a^3 \cos(c + dx) \sin(c + dx)}{2 d}$$

[In] int((a + a*cos(c + d*x))^3,x)

[Out] (5*a^3*x)/2 + (11*a^3*sin(c + d*x))/(3*d) + (a^3*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (3*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)

3.27 $\int (a + a \cos(c + dx))^3 \sec(c + dx) dx$

Optimal result	402
Rubi [A] (verified)	402
Mathematica [A] (verified)	403
Maple [A] (verified)	404
Fricas [A] (verification not implemented)	404
Sympy [F]	405
Maxima [A] (verification not implemented)	405
Giac [A] (verification not implemented)	405
Mupad [B] (verification not implemented)	406

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int (a + a \cos(c + dx))^3 \sec(c + dx) dx = \frac{7a^3 x}{2} + \frac{a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{3a^3 \sin(c + dx)}{d} + \frac{a^3 \cos(c + dx) \sin(c + dx)}{2d}$$

[Out] $7/2*a^3*x+a^3*\operatorname{arctanh}(\sin(d*x+c))/d+3*a^3*\sin(d*x+c)/d+1/2*a^3*\cos(d*x+c)*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2836, 2717, 2715, 8, 3855}

$$\int (a + a \cos(c + dx))^3 \sec(c + dx) dx = \frac{a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{3a^3 \sin(c + dx)}{d} + \frac{a^3 \sin(c + dx) \cos(c + dx)}{2d} + \frac{7a^3 x}{2}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^3*\operatorname{Sec}[c + d*x], x]$

[Out] $(7*a^3*x)/2 + (a^3*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (3*a^3*\operatorname{Sin}[c + d*x])/d + (a^3*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/(2*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[SIN[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*SIN[e + f*x])^m*(d*SIN[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (3a^3 + 3a^3 \cos(c + dx) + a^3 \cos^2(c + dx) + a^3 \sec(c + dx)) dx \\
 &= 3a^3 x + a^3 \int \cos^2(c + dx) dx + a^3 \int \sec(c + dx) dx + (3a^3) \int \cos(c + dx) dx \\
 &= 3a^3 x + \frac{a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{3a^3 \sin(c + dx)}{d} + \frac{a^3 \cos(c + dx) \sin(c + dx)}{2d} + \frac{1}{2} a^3 \int 1 dx \\
 &= \frac{7a^3 x}{2} + \frac{a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{3a^3 \sin(c + dx)}{d} + \frac{a^3 \cos(c + dx) \sin(c + dx)}{2d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\begin{aligned}
 &\int (a + a \cos(c + dx))^3 \sec(c + dx) dx \\
 &= \frac{a^3 (14dx - 4 \log(\cos(\frac{1}{2}(c + dx)) - \sin(\frac{1}{2}(c + dx))) + 4 \log(\cos(\frac{1}{2}(c + dx)) + \sin(\frac{1}{2}(c + dx))) + 12 \sin(c + dx))}{4d}
 \end{aligned}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x], x]
```

```
[Out] (a^3*(14*d*x - 4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*Sin[c + d*x] + Sin[2*(c + d*x)])/(4*d)
```


Sympy [F]

$$\int (a + a \cos(c + dx))^3 \sec(c + dx) dx = a^3 \left(\int 3 \cos(c + dx) \sec(c + dx) dx \right. \\ \left. + \int 3 \cos^2(c + dx) \sec(c + dx) dx \right. \\ \left. + \int \cos^3(c + dx) \sec(c + dx) dx \right. \\ \left. + \int \sec(c + dx) dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c),x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x), x) + Integral(3*cos(c + d*x)**2
*sec(c + d*x), x) + Integral(cos(c + d*x)**3*sec(c + d*x), x) + Integral(se
c(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int (a + a \cos(c + dx))^3 \sec(c + dx) dx \\ = \frac{(2 dx + 2 c + \sin(2 dx + 2 c))a^3 + 12(dx + c)a^3 + 4a^3 \log(\sec(dx + c) + \tan(dx + c)) + 12a^3 \sin(dx + c)}{4d}$$

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a^3 + 4*a^3*log(se
c(d*x + c) + tan(d*x + c)) + 12*a^3*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int (a + a \cos(c + dx))^3 \sec(c + dx) dx \\ = \frac{7(dx + c)a^3 + 2a^3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 2a^3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) + \frac{2\left(5a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 7a^3\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)^2}}{2d}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="giac")

[Out] $\frac{1}{2}*(7*(d*x + c)*a^3 + 2*a^3*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*\tan(1/2*d*x + 1/2*c)^3 + 7*a^3*\tan(1/2*d*x + 1/2*c)))/(\tan(1/2*d*x + 1/2*c)^2 + 1)^2/d$

Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int (a + a \cos(c + dx))^3 \sec(c + dx) dx = \frac{7a^3 x}{2} + \frac{2a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} + \frac{5a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 7a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x),x)

[Out] $(7*a^3*x)/2 + (2*a^3*\operatorname{atanh}(\tan(c/2 + (d*x)/2)))/d + (5*a^3*\tan(c/2 + (d*x)/2)^3 + 7*a^3*\tan(c/2 + (d*x)/2))/(d*(2*\tan(c/2 + (d*x)/2)^2 + \tan(c/2 + (d*x)/2)^4 + 1))$

3.28 $\int (a + a \cos(c + dx))^3 \sec^2(c + dx) dx$

Optimal result	407
Rubi [A] (verified)	407
Mathematica [B] (verified)	408
Maple [A] (verified)	409
Fricas [A] (verification not implemented)	410
Sympy [F]	410
Maxima [A] (verification not implemented)	410
Giac [A] (verification not implemented)	411
Mupad [B] (verification not implemented)	411

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a + a \cos(c + dx))^3 \sec^2(c + dx) dx = 3a^3x + \frac{3a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^3 \sin(c + dx)}{d} + \frac{a^3 \tan(c + dx)}{d}$$

[Out] $3*a^3*x + 3*a^3*\operatorname{arctanh}(\sin(d*x+c))/d + a^3*\sin(d*x+c)/d + a^3*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.08 (sec), antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2836, 2717, 3855, 3852, 8}

$$\int (a + a \cos(c + dx))^3 \sec^2(c + dx) dx = \frac{3a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^3 \sin(c + dx)}{d} + \frac{a^3 \tan(c + dx)}{d} + 3a^3x$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^3*\operatorname{Sec}[c + d*x]^2, x]$

[Out] $3*a^3*x + (3*a^3*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (a^3*\operatorname{Sin}[c + d*x])/d + (a^3*\operatorname{Tan}[c + d*x])/d$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (3a^3 + a^3 \cos(c + dx) + 3a^3 \sec(c + dx) + a^3 \sec^2(c + dx)) dx \\
&= 3a^3 x + a^3 \int \cos(c + dx) dx + a^3 \int \sec^2(c + dx) dx + (3a^3) \int \sec(c + dx) dx \\
&= 3a^3 x + \frac{3a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^3 \sin(c + dx)}{d} - \frac{a^3 \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\
&= 3a^3 x + \frac{3a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^3 \sin(c + dx)}{d} + \frac{a^3 \tan(c + dx)}{d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 211 vs. 2(48) = 96.

Time = 1.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.40

$$\int (a + a \cos(c + dx))^3 \sec^2(c + dx) dx$$

$$= \frac{1}{8} a^3 (1 + \cos(c + dx))^3 \sec^6\left(\frac{1}{2}(c + dx)\right) \left(3x - \frac{3 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} \right. \\ \left. + \frac{3 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} + \frac{\cos(dx) \sin(c)}{d} + \frac{\cos(c) \sin(dx)}{d} \right. \\ \left. + \frac{\sin\left(\frac{dx}{2}\right)}{d \left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right) \left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)} \right. \\ \left. + \frac{\sin\left(\frac{dx}{2}\right)}{d \left(\cos\left(\frac{c}{2}\right) + \sin\left(\frac{c}{2}\right)\right) \left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)} \right)$$

[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2,x]

[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(3*x - (3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d + Sin[(d*x)/2]/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(d*x)/2]/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{a^3 \sin(dx+c) + 3a^3(dx+c) + 3a^3 \ln(\sec(dx+c) + \tan(dx+c)) + a^3 \tan(dx+c)}{d}$
default	$\frac{a^3 \sin(dx+c) + 3a^3(dx+c) + 3a^3 \ln(\sec(dx+c) + \tan(dx+c)) + a^3 \tan(dx+c)}{d}$
parts	$\frac{a^3 \tan(dx+c)}{d} + \frac{a^3 \sin(dx+c)}{d} + \frac{3a^3 \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{3a^3(dx+c)}{d}$
parallelrisc	$\frac{a^3 \left(6dx \cos(dx+c) - 6 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \cos(dx+c) + 6 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) \cos(dx+c) + 2 \sin(dx+c) + \sin(2dx+2c) \right)}{2d \cos(dx+c)}$
risc	$3a^3 x - \frac{ia^3 e^{i(dx+c)}}{2d} + \frac{ia^3 e^{-i(dx+c)}}{2d} + \frac{2ia^3}{d(e^{2i(dx+c)}+1)} - \frac{3a^3 \ln(e^{i(dx+c)}-i)}{d} + \frac{3a^3 \ln(e^{i(dx+c)}+i)}{d}$
norman	$\frac{-3a^3 x - \frac{4a^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} - \frac{8a^3 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} - \frac{4a^3 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} - 6a^3 x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 6a^3 x \left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 3a^3 x}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}$

[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*sin(d*x+c)+3*a^3*(d*x+c)+3*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*tan(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.90

$$\int (a + a \cos(c + dx))^3 \sec^2(c + dx) dx$$

$$= \frac{6 a^3 dx \cos(dx + c) + 3 a^3 \cos(dx + c) \log(\sin(dx + c) + 1) - 3 a^3 \cos(dx + c) \log(-\sin(dx + c) + 1) + 2 a^3 \sin(dx + c)}{2 d \cos(dx + c)}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="fricas")

```
[Out] 1/2*(6*a^3*d*x*cos(d*x + c) + 3*a^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + a^3)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a + a \cos(c + dx))^3 \sec^2(c + dx) dx = a^3 \left(\int 3 \cos(c + dx) \sec^2(c + dx) dx \right. \\ \left. + \int 3 \cos^2(c + dx) \sec^2(c + dx) dx \right. \\ \left. + \int \cos^3(c + dx) \sec^2(c + dx) dx \right. \\ \left. + \int \sec^2(c + dx) dx \right)$$

[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**2,x)

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(3*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int (a + a \cos(c + dx))^3 \sec^2(c + dx) dx$$

$$= \frac{6(dx + c)a^3 + 3a^3(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 2a^3 \sin(dx + c) + 2a^3 \tan(dx + c)}{2d}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="maxima")

```
[Out] 1/2*(6*(d*x + c)*a^3 + 3*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) + 2*a^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int (a + a \cos(c + dx))^3 \sec^2(c + dx) dx$$

$$= \frac{3(dx + c)a^3 + 3a^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3a^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{4a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 1}}{d}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="giac")

[Out] (3*(d*x + c)*a^3 + 3*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^4 - 1))/d

Mupad [B] (verification not implemented)

Time = 14.89 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (a + a \cos(c + dx))^3 \sec^2(c + dx) dx = 3a^3 x + \frac{6a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{4a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 1\right)}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^2,x)

[Out] 3*a^3*x + (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (4*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 1))

3.29 $\int (a + a \cos(c + dx))^3 \sec^3(c + dx) dx$

Optimal result	412
Rubi [A] (verified)	412
Mathematica [A] (verified)	414
Maple [A] (verified)	414
Fricas [A] (verification not implemented)	415
Sympy [F]	415
Maxima [A] (verification not implemented)	415
Giac [A] (verification not implemented)	416
Mupad [B] (verification not implemented)	416

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int (a + a \cos(c + dx))^3 \sec^3(c + dx) dx = a^3 x + \frac{7a^3 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{3a^3 \tan(c + dx)}{d} + \frac{a^3 \sec(c + dx) \tan(c + dx)}{2d}$$

[Out] $a^3 x + 7/2 a^3 \operatorname{arctanh}(\sin(dx+c))/d + 3 a^3 \tan(dx+c)/d + 1/2 a^3 \sec(dx+c) \tan(dx+c)/d$

Rubi [A] (verified)

Time = 0.09 (sec), antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2836, 3855, 3852, 8, 3853}

$$\int (a + a \cos(c + dx))^3 \sec^3(c + dx) dx = \frac{7a^3 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{3a^3 \tan(c + dx)}{d} + \frac{a^3 \tan(c + dx) \sec(c + dx)}{2d} + a^3 x$$

[In] $\text{Int}[(a + a \cos[c + dx])^3 \sec[c + dx]^3, x]$

[Out] $a^3 x + (7 a^3 \operatorname{ArcTanh}[\sin[c + dx]])/(2d) + (3 a^3 \tan[c + dx])/d + (a^3 \sec[c + dx] \tan[c + dx])/(2d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a x, x] /; \text{FreeQ}[a, x]$

Rule 2836


```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^3 + 3a^3 \sec(c + dx) + 3a^3 \sec^2(c + dx) + a^3 \sec^3(c + dx)) dx \\
 &= a^3 x + a^3 \int \sec^3(c + dx) dx + (3a^3) \int \sec(c + dx) dx + (3a^3) \int \sec^2(c + dx) dx \\
 &= a^3 x + \frac{3a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^3 \sec(c + dx) \tan(c + dx)}{2d} \\
 &\quad + \frac{1}{2} a^3 \int \sec(c + dx) dx - \frac{(3a^3) \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\
 &= a^3 x + \frac{7a^3 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{3a^3 \tan(c + dx)}{d} + \frac{a^3 \sec(c + dx) \tan(c + dx)}{2d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int (a + a \cos(c + dx))^3 \sec^3(c + dx) dx = a^3 \left(x + \frac{7 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{3 \tan(c + dx)}{d} + \frac{\sec(c + dx) \tan(c + dx)}{2d} \right)$$

`[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3,x]``[Out] a^3*(x + (7*ArcTanh[Sin[c + d*x]])/(2*d) + (3*Tan[c + d*x])/d + (Sec[c + d*x]*Tan[c + d*x])/(2*d))`**Maple [A] (verified)**

Time = 2.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{a^3(dx+c)+3a^3 \ln(\sec(dx+c)+\tan(dx+c))+3a^3 \tan(dx+c)+a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
default	$\frac{a^3(dx+c)+3a^3 \ln(\sec(dx+c)+\tan(dx+c))+3a^3 \tan(dx+c)+a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
parts	$\frac{a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d} + \frac{a^3(dx+c)}{d} + \frac{3a^3 \tan(dx+c)}{d} + \frac{3a^3 \ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$a^3 x - \frac{ia^3(e^{3i(dx+c)} - 6e^{2i(dx+c)} - e^{i(dx+c)} - 6)}{d(e^{2i(dx+c)} + 1)^2} - \frac{7a^3 \ln(e^{i(dx+c)} - i)}{2d} + \frac{7a^3 \ln(e^{i(dx+c)} + i)}{2d}$
parallelrisch	$\frac{a^3(2dx \cos(2dx+2c) + 7 \ln(\tan(\frac{dx}{2} + \frac{c}{2}) + 1) \cos(2dx+2c) - 7 \ln(\tan(\frac{dx}{2} + \frac{c}{2}) - 1) \cos(2dx+2c) + 2dx + 2 \sin(dx+c) + 7 \ln(\tan(\frac{dx}{2} + \frac{c}{2})))}{2d(1+\cos(2dx+2c))}$
norman	$\frac{a^3 x + a^3 x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + a^3 x \left(\tan^8\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + a^3 x \left(\tan^{10}\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + \frac{7a^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{16a^3 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{d} + \frac{6a^3 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$

`[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^3,x,method=_RETURNVERBOSE)``[Out] 1/d*(a^3*(d*x+c)+3*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)+a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))`

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int (a + a \cos(c + dx))^3 \sec^3(c + dx) dx$$

$$= \frac{4 a^3 dx \cos(dx + c)^2 + 7 a^3 \cos(dx + c)^2 \log(\sin(dx + c) + 1) - 7 a^3 \cos(dx + c)^2 \log(-\sin(dx + c) + 1)}{4 d \cos(dx + c)^2}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="fricas")

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 + 7*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)
- 7*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^3*cos(d*x + c) + a^3
)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a + a \cos(c + dx))^3 \sec^3(c + dx) dx = a^3 \left(\int 3 \cos(c + dx) \sec^3(c + dx) dx \right.$$

$$+ \int 3 \cos^2(c + dx) \sec^3(c + dx) dx$$

$$+ \int \cos^3(c + dx) \sec^3(c + dx) dx$$

$$\left. + \int \sec^3(c + dx) dx \right)$$

[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**3,x)

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(3*cos(c + d*x)
**2*sec(c + d*x)**3, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int (a + a \cos(c + dx))^3 \sec^3(c + dx) dx$$

$$= \frac{4(dx + c)a^3 - a^3 \left(\frac{2 \sin(dx + c)}{\sin(dx + c)^2 - 1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right) + 6a^3(\log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1))}{4d}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="maxima")

[Out] 1/4*(4*(d*x + c)*a^3 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^3*tan(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int (a + a \cos(c + dx))^3 \sec^3(c + dx) dx$$

$$= \frac{2(dx + c)a^3 + 7a^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 7a^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(5a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 7a^3\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2 - 1}}{2d}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="giac")

[Out] 1/2*(2*(d*x + c)*a^3 + 7*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 7*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2/d

Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int (a + a \cos(c + dx))^3 \sec^3(c + dx) dx = a^3 x + \frac{7a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{5a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 - 7a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^3,x)

[Out] a^3*x + (7*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^3 - 7*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))

3.30 $\int (a + a \cos(c + dx))^3 \sec^4(c + dx) dx$

Optimal result	417
Rubi [A] (verified)	417
Mathematica [A] (verified)	419
Maple [A] (verified)	419
Fricas [A] (verification not implemented)	420
Sympy [F]	420
Maxima [A] (verification not implemented)	420
Giac [A] (verification not implemented)	421
Mupad [B] (verification not implemented)	421

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (a + a \cos(c + dx))^3 \sec^4(c + dx) dx = \frac{5a^3 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{3a^3 \sec(c + dx) \tan(c + dx)}{2d} + \frac{a^3 \tan^3(c + dx)}{3d}$$

[Out] $5/2*a^3*\operatorname{arctanh}(\sin(d*x+c))/d+4*a^3*\tan(d*x+c)/d+3/2*a^3*\sec(d*x+c)*\tan(d*x+c)/d+1/3*a^3*\tan(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.11 (sec), antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2836, 3855, 3852, 8, 3853}

$$\int (a + a \cos(c + dx))^3 \sec^4(c + dx) dx = \frac{5a^3 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a^3 \tan^3(c + dx)}{3d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{3a^3 \tan(c + dx) \sec(c + dx)}{2d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^3*\operatorname{Sec}[c + d*x]^4, x]$

[Out] $(5*a^3*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (4*a^3*\operatorname{Tan}[c + d*x])/d + (3*a^3*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d) + (a^3*\operatorname{Tan}[c + d*x]^3)/(3*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^3 \sec(c + dx) + 3a^3 \sec^2(c + dx) + 3a^3 \sec^3(c + dx) + a^3 \sec^4(c + dx)) dx \\
 &= a^3 \int \sec(c + dx) dx + a^3 \int \sec^4(c + dx) dx + (3a^3) \int \sec^2(c + dx) dx + (3a^3) \int \sec^3(c + dx) dx \\
 &= \frac{a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{3a^3 \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2}(3a^3) \int \sec(c + dx) dx \\
 &\quad - \frac{a^3 \operatorname{Subst}\left(\int (1 + x^2) dx, x, -\tan(c + dx)\right)}{d} - \frac{(3a^3) \operatorname{Subst}\left(\int 1 dx, x, -\tan(c + dx)\right)}{d} \\
 &= \frac{5a^3 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{3a^3 \sec(c + dx) \tan(c + dx)}{2d} + \frac{a^3 \tan^3(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx))^3 \sec^4(c + dx) dx = \frac{5a^3 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{3a^3 \sec(c + dx) \tan(c + dx)}{2d} + \frac{a^3 \tan^3(c + dx)}{3d}$$

`[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4,x]`

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+3a^3 \tan(dx+c)+3a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d} - a^3 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right)$
default	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+3a^3 \tan(dx+c)+3a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d} - a^3 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right)$
parts	$-\frac{a^3 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d} + \frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{3a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
risch	$-\frac{ia^3(9e^{5i(dx+c)}-18e^{4i(dx+c)}-48e^{2i(dx+c)}-9e^{i(dx+c)}-22)}{3d(e^{2i(dx+c)}+1)^3} + \frac{5a^3 \ln(e^{i(dx+c)}+i)}{2d} - \frac{5a^3 \ln(e^{i(dx+c)}-i)}{2d}$
parallelrisch	$-\frac{a^3(15 \ln(\tan(\frac{dx}{2}+\frac{c}{2}))-1) \cos(3dx+3c)-15 \ln(\tan(\frac{dx}{2}+\frac{c}{2})+1) \cos(3dx+3c)+45 \ln(\tan(\frac{dx}{2}+\frac{c}{2}))-1) \cos(dx+c)-45 \ln(\tan(\frac{dx}{2}+\frac{c}{2})+1) \cos(dx+c)}{6d(\cos(3dx+3c)+3 \cos(dx+c))}$
norman	$\frac{-\frac{11a^3 \tan(\frac{dx}{2}+\frac{c}{2})}{d} - \frac{59a^3 (\tan^3(\frac{dx}{2}+\frac{c}{2}))}{3d} + \frac{2a^3 (\tan^5(\frac{dx}{2}+\frac{c}{2}))}{d} + \frac{14a^3 (\tan^7(\frac{dx}{2}+\frac{c}{2}))}{d} - \frac{5a^3 (\tan^9(\frac{dx}{2}+\frac{c}{2}))}{3d} - \frac{5a^3 (\tan^{11}(\frac{dx}{2}+\frac{c}{2}))}{d}}{(1+\tan^2(\frac{dx}{2}+\frac{c}{2}))^3 (\tan^2(\frac{dx}{2}+\frac{c}{2})-1)^3}$

`[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^4,x,method=_RETURNVERBOSE)`

```
[Out] 1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)+3*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

$$\int (a + a \cos(c + dx))^3 \sec^4(c + dx) dx$$

$$= \frac{15 a^3 \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 15 a^3 \cos(dx + c)^3 \log(-\sin(dx + c) + 1) + 2(22 a^3 \cos(dx + c)^2 + 9 a^3 \cos(dx + c) + 2 a^3) \sin(dx + c)}{12 d \cos(dx + c)^3}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="fricas")

```
[Out] 1/12*(15*a^3*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(22*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a + a \cos(c + dx))^3 \sec^4(c + dx) dx = a^3 \left(\int 3 \cos(c + dx) \sec^4(c + dx) dx \right. \\ \left. + \int 3 \cos^2(c + dx) \sec^4(c + dx) dx \right. \\ \left. + \int \cos^3(c + dx) \sec^4(c + dx) dx \right. \\ \left. + \int \sec^4(c + dx) dx \right)$$

[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**4,x)

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(3*cos(c + d*x)**2*sec(c + d*x)**4, x) + Integral(cos(c + d*x)**3*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int (a + a \cos(c + dx))^3 \sec^4(c + dx) dx$$

$$= \frac{4(\tan(dx + c)^3 + 3 \tan(dx + c))a^3 - 9a^3 \left(\frac{2 \sin(dx + c)}{\sin(dx + c)^2 - 1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right)}{12 d}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="maxima")

[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*a^3*tan(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int (a + a \cos(c + dx))^3 \sec^4(c + dx) dx$$

$$= \frac{15 a^3 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 15 a^3 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(15 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 40 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 33 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)} d}{6 d}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="giac")

[Out] 1/6*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3/d

Mupad [B] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

$$\int (a + a \cos(c + dx))^3 \sec^4(c + dx) dx$$

$$= \frac{5 a^3 \operatorname{atanh} \left(\tan \left(\frac{c}{2} + \frac{dx}{2} \right) \right)}{d} - \frac{5 a^3 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^5 - \frac{40 a^3 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^3}{3} + 11 a^3 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)}{d \left(\tan \left(\frac{c}{2} + \frac{dx}{2} \right)^6 - 3 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^4 + 3 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^2 - 1 \right)}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^4,x)

[Out] (5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2)^5 - (40*a^3*tan(c/2 + (d*x)/2)^3)/3 + 11*a^3*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))

3.31 $\int (a + a \cos(c + dx))^3 \sec^5(c + dx) dx$

Optimal result	422
Rubi [A] (verified)	422
Mathematica [A] (verified)	424
Maple [A] (verified)	425
Fricas [A] (verification not implemented)	425
Sympy [F(-1)]	426
Maxima [A] (verification not implemented)	426
Giac [A] (verification not implemented)	426
Mupad [B] (verification not implemented)	427

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int (a + a \cos(c + dx))^3 \sec^5(c + dx) dx = \frac{15a^3 \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{15a^3 \sec(c + dx) \tan(c + dx)}{8d} + \frac{a^3 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{a^3 \tan^3(c + dx)}{d}$$

[Out] $15/8*a^3*\operatorname{arctanh}(\sin(d*x+c))/d+4*a^3*\tan(d*x+c)/d+15/8*a^3*\sec(d*x+c)*\tan(d*x+c)/d+1/4*a^3*\sec(d*x+c)^3*\tan(d*x+c)/d+a^3*\tan(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2836, 3852, 8, 3853, 3855}

$$\int (a + a \cos(c + dx))^3 \sec^5(c + dx) dx = \frac{15a^3 \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a^3 \tan^3(c + dx)}{d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{a^3 \tan(c + dx) \sec^3(c + dx)}{4d} + \frac{15a^3 \tan(c + dx) \sec(c + dx)}{8d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^3*\operatorname{Sec}[c + d*x]^5,x]$

[Out] $(15*a^3*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(8*d) + (4*a^3*\operatorname{Tan}[c + d*x])/d + (15*a^3*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(8*d) + (a^3*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(4*d) + (a^3*\operatorname{Tan}[c + d*x]^3)/d$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2836

`Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]`

Rule 3852

`Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3853

`Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3855

`Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^3 \sec^2(c + dx) + 3a^3 \sec^3(c + dx) + 3a^3 \sec^4(c + dx) + a^3 \sec^5(c + dx)) dx \\
 &= a^3 \int \sec^2(c + dx) dx + a^3 \int \sec^5(c + dx) dx \\
 &\quad + (3a^3) \int \sec^3(c + dx) dx + (3a^3) \int \sec^4(c + dx) dx \\
 &= \frac{3a^3 \sec(c + dx) \tan(c + dx)}{2d} + \frac{a^3 \sec^3(c + dx) \tan(c + dx)}{4d} \\
 &\quad + \frac{1}{4}(3a^3) \int \sec^3(c + dx) dx + \frac{1}{2}(3a^3) \int \sec(c + dx) dx \\
 &\quad - \frac{a^3 \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\
 &\quad - \frac{(3a^3) \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{3a^3 \operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{4a^3 \tan(c+dx)}{d} + \frac{15a^3 \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{a^3 \sec^3(c+dx) \tan(c+dx)}{4d} + \frac{a^3 \tan^3(c+dx)}{d} + \frac{1}{8}(3a^3) \int \sec(c+dx) dx \\
&= \frac{15a^3 \operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{4a^3 \tan(c+dx)}{d} + \frac{15a^3 \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{a^3 \sec^3(c+dx) \tan(c+dx)}{4d} + \frac{a^3 \tan^3(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int (a + a \cos(c+dx))^3 \sec^5(c+dx) dx &= \frac{15a^3 \operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{4a^3 \tan(c+dx)}{d} \\
&\quad + \frac{15a^3 \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{a^3 \sec^3(c+dx) \tan(c+dx)}{4d} + \frac{a^3 \tan^3(c+dx)}{d}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5,x]

[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a^3*Tan[c + d*x]^3)/d

Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{a^3 \tan(dx+c) + 3a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) - 3a^3 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + a^3 \left(-\left(-\frac{\sec^2(dx+c)}{3} \right) \right)}{d}$
default	$\frac{a^3 \tan(dx+c) + 3a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) - 3a^3 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + a^3 \left(-\left(-\frac{\sec^2(dx+c)}{3} \right) \right)}{d}$
parts	$\frac{a^3 \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right)}{d} + \frac{a^3 \tan(dx+c)}{d} - \frac{3a^3 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right)}{d}$
risch	$-\frac{ia^3(15e^{7i(dx+c)} - 8e^{6i(dx+c)} + 23e^{5i(dx+c)} - 72e^{4i(dx+c)} - 23e^{3i(dx+c)} - 88e^{2i(dx+c)} - 15e^{i(dx+c)} - 24)}{4d(e^{2i(dx+c)} + 1)^4} - \frac{15a^3 \ln(e^{i(dx+c)} + 1)}{8d}$
parallelrisch	$\frac{a^3 \left(15(-\cos(4dx+4c) - 4\cos(2dx+2c) - 3) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + 15(\cos(4dx+4c) + 4\cos(2dx+2c) + 3) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) \right)}{8d(\cos(4dx+4c) + 4\cos(2dx+2c) + 3)}$
norman	$\frac{\frac{49a^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} + \frac{37a^3 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2d} - \frac{17a^3 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} - \frac{5a^3 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} + \frac{47a^3 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} + \frac{5a^3 \left(\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^4}$

[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*tan(d*x+c)+3*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-3*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int (a + a \cos(c + dx))^3 \sec^5(c + dx) dx$$

$$= \frac{15 a^3 \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 15 a^3 \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2(24 a^3 \cos(dx + c)^3 + 15 a^3 \cos(dx + c)^2 + 8 a^3 \cos(dx + c) + 2 a^3) \sin(dx + c)}{16 d \cos(dx + c)^4}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="fricas")

[Out] 1/16*(15*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(24*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c)^2 + 8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sec^5(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**5,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int (a + a \cos(c + dx))^3 \sec^5(c + dx) dx$$

$$= \frac{16 (\tan(dx + c)^3 + 3 \tan(dx + c)) a^3 - a^3 \left(\frac{2 (3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right)}{16}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="maxima")

[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int (a + a \cos(c + dx))^3 \sec^5(c + dx) dx$$

$$= \frac{15 a^3 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 15 a^3 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(15 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^7 - 55 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 \right)}{\left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)^4 - 2 \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)^2 + 1}}{8 d}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="giac")

[Out] 1/8*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 - 55*a^3*tan(1/2*d*x + 1/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 - 49*a^3*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4/d

Mupad [B] (verification not implemented)

Time = 16.97 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.52

$$\int (a + a \cos(c + dx))^3 \sec^5(c + dx) dx$$

$$= \frac{15 a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{4 d}$$

$$- \frac{\frac{15 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{4} - \frac{55 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{4} + \frac{73 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{4} - \frac{49 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^5,x)

```
[Out] (15*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((73*a^3*tan(c/2 + (d*x)/2)^3)/4
- (55*a^3*tan(c/2 + (d*x)/2)^5)/4 + (15*a^3*tan(c/2 + (d*x)/2)^7)/4 - (49*
a^3*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.32 $\int (a + a \cos(c + dx))^3 \sec^6(c + dx) dx$

Optimal result	428
Rubi [A] (verified)	428
Mathematica [A] (verified)	430
Maple [C] (verified)	430
Fricas [A] (verification not implemented)	431
Sympy [F(-1)]	432
Maxima [A] (verification not implemented)	432
Giac [A] (verification not implemented)	432
Mupad [B] (verification not implemented)	433

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int (a + a \cos(c + dx))^3 \sec^6(c + dx) dx = \frac{13a^3 \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{13a^3 \sec(c + dx) \tan(c + dx)}{8d} + \frac{3a^3 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{5a^3 \tan^3(c + dx)}{3d} + \frac{a^3 \tan^5(c + dx)}{5d}$$

[Out] 13/8*a^3*arctanh(sin(d*x+c))/d+4*a^3*tan(d*x+c)/d+13/8*a^3*sec(d*x+c)*tan(d*x+c)/d+3/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+5/3*a^3*tan(d*x+c)^3/d+1/5*a^3*tan(d*x+c)^5/d

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 3853, 3855, 3852}

$$\int (a + a \cos(c + dx))^3 \sec^6(c + dx) dx = \frac{13a^3 \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a^3 \tan^5(c + dx)}{5d} + \frac{5a^3 \tan^3(c + dx)}{3d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{3a^3 \tan(c + dx) \sec^3(c + dx)}{4d} + \frac{13a^3 \tan(c + dx) \sec(c + dx)}{8d}$$

[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^6,x]

[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)

Rule 2836

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rule 3852

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^3 \sec^3(c + dx) + 3a^3 \sec^4(c + dx) + 3a^3 \sec^5(c + dx) + a^3 \sec^6(c + dx)) dx \\
 &= a^3 \int \sec^3(c + dx) dx + a^3 \int \sec^6(c + dx) dx \\
 &\quad + (3a^3) \int \sec^4(c + dx) dx + (3a^3) \int \sec^5(c + dx) dx \\
 &= \frac{a^3 \sec(c + dx) \tan(c + dx)}{2d} + \frac{3a^3 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{2} a^3 \int \sec(c + dx) dx \\
 &\quad + \frac{1}{4} (9a^3) \int \sec^3(c + dx) dx - \frac{a^3 \text{Subst}(\int (1 + 2x^2 + x^4) dx, x, -\tan(c + dx))}{d} \\
 &\quad - \frac{(3a^3) \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{a^3 \operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{4a^3 \tan(c+dx)}{d} + \frac{13a^3 \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{3a^3 \sec^3(c+dx) \tan(c+dx)}{4d} + \frac{5a^3 \tan^3(c+dx)}{3d} \\
&\quad + \frac{a^3 \tan^5(c+dx)}{5d} + \frac{1}{8}(9a^3) \int \sec(c+dx) dx \\
&= \frac{13a^3 \operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{4a^3 \tan(c+dx)}{d} + \frac{13a^3 \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{3a^3 \sec^3(c+dx) \tan(c+dx)}{4d} + \frac{5a^3 \tan^3(c+dx)}{3d} + \frac{a^3 \tan^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int (a + a \cos(c+dx))^3 \sec^6(c+dx) dx &= \frac{13a^3 \operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{4a^3 \tan(c+dx)}{d} \\
&\quad + \frac{13a^3 \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{3a^3 \sec^3(c+dx) \tan(c+dx)}{4d} \\
&\quad + \frac{5a^3 \tan^3(c+dx)}{3d} + \frac{a^3 \tan^5(c+dx)}{5d}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^6,x]

[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27

method	result
risch	$\frac{ia^3(195e^{9i(dx+c)} + 750e^{7i(dx+c)} - 720e^{6i(dx+c)} - 2320e^{4i(dx+c)} - 750e^{3i(dx+c)} - 1520e^{2i(dx+c)} - 195e^{i(dx+c)} - 304)}{60d(e^{2i(dx+c)} + 1)^5}$
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right) - 3a^3\left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3}\right)\tan(dx+c) + 3a^3\left(-\left(-\frac{\sec^3(dx+c)}{4} - 3\right)\right)}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right) - 3a^3\left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3}\right)\tan(dx+c) + 3a^3\left(-\left(-\frac{\sec^3(dx+c)}{4} - 3\right)\right)}{d}$
parts	$\frac{a^3\left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15}\right)\tan(dx+c)}{d} + \frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d} + \frac{3a^3\left(\frac{\sec^3(dx+c)}{4} + 3\right)}{d}$
norman	$\frac{-\frac{51a^3\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} - \frac{193a^3\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} + \frac{31a^3\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{60d} - \frac{857a^3\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{60d} - \frac{1127a^3\left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{60d} + \frac{481a^3\left(\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{60d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^5}$
parallelrisch	$\frac{a^3\left(975\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\cos(3dx+3c) - 975\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)\cos(3dx+3c) + 1950\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\cos(dx+c) - 1950\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)\cos(dx+c)\right)}{d}$

[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^6,x,method=_RETURNVERBOSE)

[Out]
$$\frac{-1/60*I*a^3*(195*\exp(9*I*(d*x+c))+750*\exp(7*I*(d*x+c))-720*\exp(6*I*(d*x+c))-2320*\exp(4*I*(d*x+c))-750*\exp(3*I*(d*x+c))-1520*\exp(2*I*(d*x+c))-195*\exp(I*(d*x+c))-304)/d/(\exp(2*I*(d*x+c))+1)^5+13/8*a^3/d*\ln(\exp(I*(d*x+c))+I)-13/8*a^3/d*\ln(\exp(I*(d*x+c))-I)}$$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int (a + a \cos(c + dx))^3 \sec^6(c + dx) dx$$

$$= \frac{195 a^3 \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 195 a^3 \cos(dx + c)^5 \log(-\sin(dx + c) + 1) + 2(304 a^3 \cos(dx + c)^4 + 195 a^3 \cos(dx + c)^3 + 152 a^3 \cos(dx + c)^2 + 90 a^3 \cos(dx + c) + 24 a^3) \sin(dx + c)}{240 d \cos(dx + c)^5}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="fricas")

[Out]
$$\frac{1/240*(195*a^3*\cos(d*x + c)^5*\log(\sin(d*x + c) + 1) - 195*a^3*\cos(d*x + c)^5*\log(-\sin(d*x + c) + 1) + 2*(304*a^3*\cos(d*x + c)^4 + 195*a^3*\cos(d*x + c)^3 + 152*a^3*\cos(d*x + c)^2 + 90*a^3*\cos(d*x + c) + 24*a^3)*\sin(d*x + c)}{d*\cos(d*x + c)^5}$$

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sec^6(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**6,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.57

$$\int (a + a \cos(c + dx))^3 \sec^6(c + dx) dx$$

$$= \frac{16 (3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c)) a^3 + 240 (\tan(dx + c)^3 + 3 \tan(dx + c)) a^3 - 45 \dots}{\dots}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="maxima")

[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 45*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.21

$$\int (a + a \cos(c + dx))^3 \sec^6(c + dx) dx$$

$$= \frac{195 a^3 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 195 a^3 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(195 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^9 - 910 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^7 + 1664 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 1330 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 765 a^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{120 d}$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="giac")

[Out] 1/120*(195*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 195*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d

Mupad [B] (verification not implemented)

Time = 18.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

$$\int (a + a \cos(c + dx))^3 \sec^6(c + dx) dx = \frac{13 a^3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{4 d} - \frac{\frac{13 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{4} - \frac{91 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{6} + \frac{416 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{15} - \frac{133 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{6} + \frac{51 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1 \right)}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^6,x)

[Out] (13*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((416*a^3*tan(c/2 + (d*x)/2)^5)/15 - (133*a^3*tan(c/2 + (d*x)/2)^3)/6 - (91*a^3*tan(c/2 + (d*x)/2)^7)/6 + (13*a^3*tan(c/2 + (d*x)/2)^9)/4 + (51*a^3*tan(c/2 + (d*x)/2))/4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))

3.33 $\int \cos^2(c + dx)(a + a \cos(c + dx))^4 dx$

Optimal result	434
Rubi [A] (verified)	434
Mathematica [A] (verified)	436
Maple [A] (verified)	437
Fricas [A] (verification not implemented)	437
Sympy [B] (verification not implemented)	438
Maxima [A] (verification not implemented)	438
Giac [A] (verification not implemented)	439
Mupad [B] (verification not implemented)	439

Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^4 dx = \frac{49a^4x}{16} + \frac{8a^4 \sin(c + dx)}{d} + \frac{49a^4 \cos(c + dx) \sin(c + dx)}{16d} + \frac{41a^4 \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{a^4 \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{4a^4 \sin^3(c + dx)}{d} + \frac{4a^4 \sin^5(c + dx)}{5d}$$

[Out] 49/16*a^4*x+8*a^4*sin(d*x+c)/d+49/16*a^4*cos(d*x+c)*sin(d*x+c)/d+41/24*a^4*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^4*cos(d*x+c)^5*sin(d*x+c)/d-4*a^4*sin(d*x+c)^3/d+4/5*a^4*sin(d*x+c)^5/d

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2836, 2715, 8, 2713}

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^4 dx = \frac{4a^4 \sin^5(c + dx)}{5d} - \frac{4a^4 \sin^3(c + dx)}{d} + \frac{8a^4 \sin(c + dx)}{d} + \frac{a^4 \sin(c + dx) \cos^5(c + dx)}{6d} + \frac{41a^4 \sin(c + dx) \cos^3(c + dx)}{24d} + \frac{49a^4 \sin(c + dx) \cos(c + dx)}{16d} + \frac{49a^4x}{16}$$

[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4,x]

[Out] (49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned} \text{integral} &= \int (a^4 \cos^2(c + dx) + 4a^4 \cos^3(c + dx) + 6a^4 \cos^4(c + dx) + 4a^4 \cos^5(c + dx) \\ &\quad + a^4 \cos^6(c + dx)) dx \\ &= a^4 \int \cos^2(c + dx) dx + a^4 \int \cos^6(c + dx) dx + (4a^4) \int \cos^3(c + dx) dx \\ &\quad + (4a^4) \int \cos^5(c + dx) dx + (6a^4) \int \cos^4(c + dx) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{a^4 \cos(c+dx) \sin(c+dx)}{2d} + \frac{3a^4 \cos^3(c+dx) \sin(c+dx)}{2d} \\
&\quad + \frac{a^4 \cos^5(c+dx) \sin(c+dx)}{6d} + \frac{1}{2}a^4 \int 1 dx + \frac{1}{6}(5a^4) \int \cos^4(c+dx) dx \\
&\quad + \frac{1}{2}(9a^4) \int \cos^2(c+dx) dx - \frac{(4a^4) \text{Subst}(\int (1-x^2) dx, x, -\sin(c+dx))}{d} \\
&\quad - \frac{(4a^4) \text{Subst}(\int (1-2x^2+x^4) dx, x, -\sin(c+dx))}{d} \\
&= \frac{a^4 x}{2} + \frac{8a^4 \sin(c+dx)}{d} + \frac{11a^4 \cos(c+dx) \sin(c+dx)}{4d} \\
&\quad + \frac{41a^4 \cos^3(c+dx) \sin(c+dx)}{24d} + \frac{a^4 \cos^5(c+dx) \sin(c+dx)}{6d} - \frac{4a^4 \sin^3(c+dx)}{d} \\
&\quad + \frac{4a^4 \sin^5(c+dx)}{5d} + \frac{1}{8}(5a^4) \int \cos^2(c+dx) dx + \frac{1}{4}(9a^4) \int 1 dx \\
&= \frac{11a^4 x}{4} + \frac{8a^4 \sin(c+dx)}{d} + \frac{49a^4 \cos(c+dx) \sin(c+dx)}{16d} + \frac{41a^4 \cos^3(c+dx) \sin(c+dx)}{24d} \\
&\quad + \frac{a^4 \cos^5(c+dx) \sin(c+dx)}{6d} - \frac{4a^4 \sin^3(c+dx)}{d} + \frac{4a^4 \sin^5(c+dx)}{5d} + \frac{1}{16}(5a^4) \int 1 dx \\
&= \frac{49a^4 x}{16} + \frac{8a^4 \sin(c+dx)}{d} + \frac{49a^4 \cos(c+dx) \sin(c+dx)}{16d} \\
&\quad + \frac{41a^4 \cos^3(c+dx) \sin(c+dx)}{24d} + \frac{a^4 \cos^5(c+dx) \sin(c+dx)}{6d} \\
&\quad - \frac{4a^4 \sin^3(c+dx)}{d} + \frac{4a^4 \sin^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\begin{aligned}
&\int \cos^2(c+dx)(a+a\cos(c+dx))^4 dx \\
&= \frac{a^4(2940dx + 5280 \sin(c+dx) + 1905 \sin(2(c+dx)) + 720 \sin(3(c+dx)) + 225 \sin(4(c+dx)) + 48 \sin(5(c+dx)))}{960d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^2*(a + a*cos[c + d*x])^4,x]

[Out] (a^4*(2940*d*x + 5280*Sin[c + d*x] + 1905*Sin[2*(c + d*x)] + 720*Sin[3*(c + d*x)] + 225*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/ (960*d)

Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

method	result
parallelrisc	$\frac{(588dx + \sin(6dx+6c) + 1056 \sin(dx+c) + 381 \sin(2dx+2c) + 144 \sin(3dx+3c) + 45 \sin(4dx+4c) + \frac{48 \sin(5dx+5c)}{5}) a^4}{192d}$
risc	$\frac{49a^4x}{16} + \frac{11a^4 \sin(dx+c)}{2d} + \frac{a^4 \sin(6dx+6c)}{192d} + \frac{a^4 \sin(5dx+5c)}{20d} + \frac{15a^4 \sin(4dx+4c)}{64d} + \frac{3a^4 \sin(3dx+3c)}{4d} + \frac{127a^4 \sin(2dx+2c)}{16d} + \frac{49a^4 \sin(dx+c)}{16d}$
derivativdivides	$a^4 \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{4a^4 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5}$
default	$a^4 \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{4a^4 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5}$
parts	$a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + \frac{a^4 \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right)}{d} + \frac{4a^4(2 + \cos(dx+c)) \sin(dx+c)}{d}$
norman	$\frac{49a^4x}{16} + \frac{207a^4 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d} + \frac{1471a^4 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24d} + \frac{1967a^4 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{20d} + \frac{1617a^4 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{20d} + \frac{833a^4 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24d}$

[In] `int(cos(d*x+c)^2*(a+cos(d*x+c))*a^4,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{192} * (588 * d * x + \sin(6 * d * x + 6 * c) + 1056 * \sin(d * x + c) + 381 * \sin(2 * d * x + 2 * c) + 144 * \sin(3 * d * x + 3 * c) + 45 * \sin(4 * d * x + 4 * c) + 48 / 5 * \sin(5 * d * x + 5 * c)) * a^4 / d$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^4 dx$$

$$= \frac{735 a^4 dx + (40 a^4 \cos(dx + c))^5 + 192 a^4 \cos(dx + c)^4 + 410 a^4 \cos(dx + c)^3 + 576 a^4 \cos(dx + c)^2 + 735 a^4 \cos(dx + c) + 1152 a^4 \sin(dx + c)}{240 d}$$

[In] `integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="fricas")`

[Out] $\frac{1}{240} * (735 * a^4 * d * x + (40 * a^4 * \cos(d * x + c))^5 + 192 * a^4 * \cos(d * x + c)^4 + 410 * a^4 * \cos(d * x + c)^3 + 576 * a^4 * \cos(d * x + c)^2 + 735 * a^4 * \cos(d * x + c) + 1152 * a^4 * \sin(d * x + c)) / d$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(121) = 242.

Time = 0.39 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.42

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^4 dx$$

$$= \begin{cases} \frac{5a^4 x \sin^6(c+dx)}{16} + \frac{15a^4 x \sin^4(c+dx) \cos^2(c+dx)}{16} + \frac{9a^4 x \sin^4(c+dx)}{4} + \frac{15a^4 x \sin^2(c+dx) \cos^4(c+dx)}{16} + \frac{9a^4 x \sin^2(c+dx) \cos^2(c+dx)}{2} \\ x(a \cos(c) + a)^4 \cos^2(c) \end{cases}$$

[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**4,x)

[Out] Piecewise((5*a**4*x*sin(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a**4*x*sin(c + d*x)**4/4 + 15*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 9*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + a**4*x*sin(c + d*x)**2/2 + 5*a**4*x*cos(c + d*x)**6/16 + 9*a**4*x*cos(c + d*x)**4/4 + a**4*x*cos(c + d*x)**2/2 + 5*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 32*a**4*sin(c + d*x)**5/(15*d) + 5*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 16*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*a**4*sin(c + d*x)**3/(3*d) + 11*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 4*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 15*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 4*a**4*sin(c + d*x)*cos(c + d*x)**2/d + a**4*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**4*cos(c)**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.30

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^4 dx$$

$$= \frac{256 (3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))a^4 - 5 (4 \sin(2dx + 2c)^3 - 60dx - 60c - 9 \sin(2dx + 2c))a^4 - 1280 (\sin(dx + c)^3 - 3 \sin(dx + c))a^4 + 180 (12dx + 12c + \sin(4dx + 4c) + 8 \sin(2dx + 2c))a^4 + 240 (2dx + 2c + \sin(2dx + 2c))a^4}{d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4)/d

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^4 dx = \frac{49}{16} a^4 x + \frac{a^4 \sin(6 dx + 6 c)}{192 d} + \frac{a^4 \sin(5 dx + 5 c)}{20 d} + \frac{15 a^4 \sin(4 dx + 4 c)}{64 d} + \frac{3 a^4 \sin(3 dx + 3 c)}{4 d} + \frac{127 a^4 \sin(2 dx + 2 c)}{64 d} + \frac{11 a^4 \sin(dx + c)}{2 d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] 49/16*a^4*x + 1/192*a^4*sin(6*d*x + 6*c)/d + 1/20*a^4*sin(5*d*x + 5*c)/d + 15/64*a^4*sin(4*d*x + 4*c)/d + 3/4*a^4*sin(3*d*x + 3*c)/d + 127/64*a^4*sin(2*d*x + 2*c)/d + 11/2*a^4*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^4 dx = \frac{49 a^4 x}{16} + \frac{49 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11}}{8} + \frac{833 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{24} + \frac{1617 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{20} + \frac{1967 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{20} + \frac{1471 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{24} + \frac{207 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8} + \frac{1}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)^6}$$

[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^4,x)

[Out] (49*a^4*x)/16 + ((1471*a^4*tan(c/2 + (d*x)/2)^3)/24 + (1967*a^4*tan(c/2 + (d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/20 + (833*a^4*tan(c/2 + (d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 + (207*a^4*tan(c/2 + (d*x)/2))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)

3.34 $\int \cos(c + dx)(a + a \cos(c + dx))^4 dx$

Optimal result	440
Rubi [A] (verified)	440
Mathematica [A] (verified)	442
Maple [A] (verified)	443
Fricas [A] (verification not implemented)	443
Sympy [B] (verification not implemented)	444
Maxima [A] (verification not implemented)	444
Giac [A] (verification not implemented)	445
Mupad [B] (verification not implemented)	445

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int \cos(c + dx)(a + a \cos(c + dx))^4 dx = \frac{7a^4x}{2} + \frac{8a^4 \sin(c + dx)}{d} + \frac{7a^4 \cos(c + dx) \sin(c + dx)}{2d} + \frac{a^4 \cos^3(c + dx) \sin(c + dx)}{d} - \frac{8a^4 \sin^3(c + dx)}{3d} + \frac{a^4 \sin^5(c + dx)}{5d}$$

[Out] $7/2*a^4*x+8*a^4*\sin(d*x+c)/d+7/2*a^4*\cos(d*x+c)*\sin(d*x+c)/d+a^4*\cos(d*x+c)^3*\sin(d*x+c)/d-8/3*a^4*\sin(d*x+c)^3/d+1/5*a^4*\sin(d*x+c)^5/d$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2830, 2724, 2717, 2715, 8, 2713}

$$\int \cos(c + dx)(a + a \cos(c + dx))^4 dx = -\frac{16a^4 \sin^3(c + dx)}{15d} + \frac{32a^4 \sin(c + dx)}{5d} + \frac{a^4 \sin(c + dx) \cos^3(c + dx)}{5d} + \frac{27a^4 \sin(c + dx) \cos(c + dx)}{10d} + \frac{7a^4x}{2} + \frac{\sin(c + dx)(a \cos(c + dx) + a)^4}{5d}$$

[In] $\text{Int}[\text{Cos}[c + d*x]*(a + a*\text{Cos}[c + d*x])^4,x]$

[Out] $(7a^4x)/2 + (32a^4\sin[c + dx])/(5d) + (27a^4\cos[c + dx]\sin[c + dx])/ (10d) + (a^4\cos[c + dx]^3\sin[c + dx])/(5d) + ((a + a\cos[c + dx])^4\sin[c + dx])/(5d) - (16a^4\sin[c + dx]^3)/(15d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2713

$\text{Int}[\sin[(c_) + (d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + dx]], x] /; \text{FreeQ}\{c, d, x\} \&\& \text{IGtQ}[(n - 1)/2, 0]$

Rule 2715

$\text{Int}[(b_)*\sin[(c_) + (d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + dx]*((b*\sin[c + dx])^{(n - 1)})/(d*n), x] + \text{Dist}[b^2*((n - 1)/n), \text{Int}[(b*\sin[c + dx])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d, x\} \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2717

$\text{Int}[\sin[\text{Pi}/2 + (c_) + (d_)*(x_)], x_Symbol] \rightarrow \text{Simp}[\sin[c + dx]/d, x] /; \text{FreeQ}\{c, d, x\}$

Rule 2724

$\text{Int}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[(a + b*\sin[c + dx])^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[n, 0]$

Rule 2830

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)]), x_Symbol] \rightarrow \text{Simp}[(-d)*\text{Cos}[e + f*x]*((a + b*\sin[e + f*x])^m/(f*(m + 1))), x] + \text{Dist}[(a*d*m + b*c*(m + 1))/(b*(m + 1)), \text{Int}[(a + b*\sin[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{!LtQ}[m, -2^{(-1)}]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(a + a \cos(c + dx))^4 \sin(c + dx)}{5d} + \frac{4}{5} \int (a + a \cos(c + dx))^4 dx \\ &= \frac{(a + a \cos(c + dx))^4 \sin(c + dx)}{5d} \\ &\quad + \frac{4}{5} \int (a^4 + 4a^4 \cos(c + dx) + 6a^4 \cos^2(c + dx) + 4a^4 \cos^3(c + dx) + a^4 \cos^4(c + dx)) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{4a^4x}{5} + \frac{(a + a \cos(c + dx))^4 \sin(c + dx)}{5d} \\
&\quad + \frac{1}{5}(4a^4) \int \cos^4(c + dx) dx + \frac{1}{5}(16a^4) \int \cos(c + dx) dx \\
&\quad + \frac{1}{5}(16a^4) \int \cos^3(c + dx) dx + \frac{1}{5}(24a^4) \int \cos^2(c + dx) dx \\
&= \frac{4a^4x}{5} + \frac{16a^4 \sin(c + dx)}{5d} + \frac{12a^4 \cos(c + dx) \sin(c + dx)}{5d} \\
&\quad + \frac{a^4 \cos^3(c + dx) \sin(c + dx)}{5d} + \frac{(a + a \cos(c + dx))^4 \sin(c + dx)}{5d} \\
&\quad + \frac{1}{5}(3a^4) \int \cos^2(c + dx) dx + \frac{1}{5}(12a^4) \int 1 dx \\
&\quad - \frac{(16a^4) \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{5d} \\
&= \frac{16a^4x}{5} + \frac{32a^4 \sin(c + dx)}{5d} + \frac{27a^4 \cos(c + dx) \sin(c + dx)}{10d} \\
&\quad + \frac{a^4 \cos^3(c + dx) \sin(c + dx)}{5d} + \frac{(a + a \cos(c + dx))^4 \sin(c + dx)}{5d} \\
&\quad - \frac{16a^4 \sin^3(c + dx)}{15d} + \frac{1}{10}(3a^4) \int 1 dx \\
&= \frac{7a^4x}{2} + \frac{32a^4 \sin(c + dx)}{5d} + \frac{27a^4 \cos(c + dx) \sin(c + dx)}{10d} \\
&\quad + \frac{a^4 \cos^3(c + dx) \sin(c + dx)}{5d} \\
&\quad + \frac{(a + a \cos(c + dx))^4 \sin(c + dx)}{5d} - \frac{16a^4 \sin^3(c + dx)}{15d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\begin{aligned}
&\int \cos(c + dx)(a + a \cos(c + dx))^4 dx \\
&= \frac{a^4(840dx + 1470 \sin(c + dx) + 480 \sin(2(c + dx)) + 145 \sin(3(c + dx)) + 30 \sin(4(c + dx)) + 3 \sin(5(c + dx)))}{240d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^4,x]

[Out] (a^4*(840*d*x + 1470*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 145*Sin[3*(c + d*x)] + 30*Sin[4*(c + d*x)] + 3*Sin[5*(c + d*x)]))/(240*d)

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

method	result
parallelrisc	$\frac{a^4(840dx+3\sin(5dx+5c)+30\sin(4dx+4c)+145\sin(3dx+3c)+480\sin(2dx+2c)+1470\sin(dx+c))}{240d}$
risc	$\frac{7a^4x}{2} + \frac{49a^4\sin(dx+c)}{8d} + \frac{a^4\sin(5dx+5c)}{80d} + \frac{a^4\sin(4dx+4c)}{8d} + \frac{29a^4\sin(3dx+3c)}{48d} + \frac{2a^4\sin(2dx+2c)}{d}$
derivativdivides	$\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5} + 4a^4\left(\frac{(\cos^3(dx+c)+\frac{3\cos(\frac{dx+c}{2}))\sin(dx+c)}{4})}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + 2a^4(2+\cos^2(dx+c))$
default	$\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5} + 4a^4\left(\frac{(\cos^3(dx+c)+\frac{3\cos(\frac{dx+c}{2}))\sin(dx+c)}{4})}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + 2a^4(2+\cos^2(dx+c))$
parts	$\frac{a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5d} + \frac{a^4\sin(dx+c)}{d} + \frac{4a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{2a^4(2+\cos^2(dx+c))}{d}$
norman	$\frac{7a^4x}{2} + \frac{25a^4\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{158a^4\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3d} + \frac{896a^4\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15d} + \frac{98a^4\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3d} + \frac{7a^4\left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} + \frac{35a^4x}{d} + \frac{35a^4c}{d}$

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*a^4*(840*d*x+3*sin(5*d*x+5*c)+30*sin(4*d*x+4*c)+145*sin(3*d*x+3*c)+480*sin(2*d*x+2*c)+1470*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \cos(c+dx)(a+a\cos(c+dx))^4 dx$$

$$= \frac{105a^4dx + (6a^4\cos(dx+c))^4 + 30a^4\cos(dx+c)^3 + 68a^4\cos(dx+c)^2 + 105a^4\cos(dx+c) + 166a^4}{30d}$$

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(105*a^4*d*x + (6*a^4*cos(d*x + c))^4 + 30*a^4*cos(d*x + c)^3 + 68*a^4*cos(d*x + c)^2 + 105*a^4*cos(d*x + c) + 166*a^4)*sin(d*x + c)/d
```


Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \cos(c + dx)(a + a \cos(c + dx))^4 dx = \frac{7}{2} a^4 x + \frac{a^4 \sin(5 dx + 5 c)}{80 d} + \frac{a^4 \sin(4 dx + 4 c)}{8 d} + \frac{29 a^4 \sin(3 dx + 3 c)}{48 d} + \frac{2 a^4 \sin(2 dx + 2 c)}{d} + \frac{49 a^4 \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] 7/2*a^4*x + 1/80*a^4*sin(5*d*x + 5*c)/d + 1/8*a^4*sin(4*d*x + 4*c)/d + 29/48*a^4*sin(3*d*x + 3*c)/d + 2*a^4*sin(2*d*x + 2*c)/d + 49/8*a^4*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 17.63 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \cos(c + dx)(a + a \cos(c + dx))^4 dx = \frac{7 a^4 x}{2} + \frac{7 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \frac{98 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{3} + \frac{896 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{15} + \frac{158 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3} + 25 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)^5}$$

[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^4,x)

[Out] (7*a^4*x)/2 + ((158*a^4*tan(c/2 + (d*x)/2)^3)/3 + (896*a^4*tan(c/2 + (d*x)/2)^5)/15 + (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*tan(c/2 + (d*x)/2)^9 + 25*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)

3.35 $\int (a + a \cos(c + dx))^4 dx$

Optimal result	446
Rubi [A] (verified)	446
Mathematica [A] (verified)	448
Maple [A] (verified)	448
Fricas [A] (verification not implemented)	449
Sympy [B] (verification not implemented)	449
Maxima [A] (verification not implemented)	449
Giac [A] (verification not implemented)	450
Mupad [B] (verification not implemented)	450

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int (a + a \cos(c + dx))^4 dx = \frac{35a^4x}{8} + \frac{8a^4 \sin(c + dx)}{d} + \frac{27a^4 \cos(c + dx) \sin(c + dx)}{8d} + \frac{a^4 \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{4a^4 \sin^3(c + dx)}{3d}$$

[Out] $35/8*a^4*x+8*a^4*\sin(d*x+c)/d+27/8*a^4*\cos(d*x+c)*\sin(d*x+c)/d+1/4*a^4*\cos(d*x+c)^3*\sin(d*x+c)/d-4/3*a^4*\sin(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2724, 2717, 2715, 8, 2713}

$$\int (a + a \cos(c + dx))^4 dx = -\frac{4a^4 \sin^3(c + dx)}{3d} + \frac{8a^4 \sin(c + dx)}{d} + \frac{a^4 \sin(c + dx) \cos^3(c + dx)}{4d} + \frac{27a^4 \sin(c + dx) \cos(c + dx)}{8d} + \frac{35a^4x}{8}$$

[In] Int[(a + a*Cos[c + d*x])^4,x]

[Out] $(35*a^4*x)/8 + (8*a^4*\sin[c + d*x])/d + (27*a^4*\cos[c + d*x]*\sin[c + d*x])/(8*d) + (a^4*\cos[c + d*x]^3*\sin[c + d*x])/(4*d) - (4*a^4*\sin[c + d*x]^3)/(3*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1)/(d*n), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[SIN[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2724

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Int[ExpandTrig[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^4 + 4a^4 \cos(c + dx) + 6a^4 \cos^2(c + dx) + 4a^4 \cos^3(c + dx) + a^4 \cos^4(c + dx)) dx \\
 &= a^4 x + a^4 \int \cos^4(c + dx) dx + (4a^4) \int \cos(c + dx) dx \\
 &\quad + (4a^4) \int \cos^3(c + dx) dx + (6a^4) \int \cos^2(c + dx) dx \\
 &= a^4 x + \frac{4a^4 \sin(c + dx)}{d} + \frac{3a^4 \cos(c + dx) \sin(c + dx)}{d} + \frac{a^4 \cos^3(c + dx) \sin(c + dx)}{4d} \\
 &\quad + \frac{1}{4}(3a^4) \int \cos^2(c + dx) dx + (3a^4) \int 1 dx - \frac{(4a^4) \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{d} \\
 &= 4a^4 x + \frac{8a^4 \sin(c + dx)}{d} + \frac{27a^4 \cos(c + dx) \sin(c + dx)}{8d} \\
 &\quad + \frac{a^4 \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{4a^4 \sin^3(c + dx)}{3d} + \frac{1}{8}(3a^4) \int 1 dx
 \end{aligned}$$

$$= \frac{35a^4x}{8} + \frac{8a^4 \sin(c+dx)}{d} + \frac{27a^4 \cos(c+dx) \sin(c+dx)}{8d} + \frac{a^4 \cos^3(c+dx) \sin(c+dx)}{4d} - \frac{4a^4 \sin^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int (a + a \cos(c+dx))^4 dx = \frac{a^4(420c + 420dx + 672 \sin(c+dx) + 168 \sin(2(c+dx)) + 32 \sin(3(c+dx)) + 3 \sin(4(c+dx)))}{96d}$$

[In] Integrate[(a + a*Cos[c + d*x])^4,x]

[Out] (a^4*(420*c + 420*d*x + 672*Sin[c + d*x] + 168*Sin[2*(c + d*x)] + 32*Sin[3*(c + d*x)] + 3*Sin[4*(c + d*x)]))/(96*d)

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

method	result
parallelrisc	$\frac{a^4(420dx+672 \sin(dx+c)+3 \sin(4dx+4c)+32 \sin(3dx+3c)+168 \sin(2dx+2c))}{96d}$
risc	$\frac{35a^4x}{8} + \frac{7a^4 \sin(dx+c)}{d} + \frac{a^4 \sin(4dx+4c)}{32d} + \frac{a^4 \sin(3dx+3c)}{3d} + \frac{7a^4 \sin(2dx+2c)}{4d}$
derivativdivides	$a^4 \left(\frac{\left(\frac{\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}}{4} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{4a^4(2+\cos^2(dx+c)) \sin(dx+c)}{3} + 6a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 4a^4 \sin^3(dx+c)$
default	$a^4 \left(\frac{\left(\frac{\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}}{4} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{4a^4(2+\cos^2(dx+c)) \sin(dx+c)}{3} + 6a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 4a^4 \sin^3(dx+c)$
parts	$a^4x + \frac{a^4 \left(\frac{\left(\frac{\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}}{4} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right)}{d} + \frac{6a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d} + \frac{4a^4(2+\cos^2(dx+c)) \sin(dx+c)}{3d}$
norman	$\frac{35a^4x}{8} + \frac{93a^4 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} + \frac{511a^4 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} + \frac{385a^4 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} + \frac{35a^4 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} + \frac{35a^4x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{12}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{14}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{16}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{18}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{20}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{22}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{24}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{26}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{28}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{30}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{32}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{34}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{36}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{38}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{40}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{42}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{44}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{46}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{48}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{50}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{52}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{54}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{56}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{58}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{60}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{62}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{64}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{66}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{68}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{70}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{72}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{74}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{76}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{78}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{80}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{82}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{84}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{86}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{88}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{90}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{92}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{94}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{96}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{98}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + \frac{105a^4 \left(\tan^{100}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2}$

[In] int((a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)

[Out] 1/96*a^4*(420*d*x+672*sin(d*x+c)+3*sin(4*d*x+4*c)+32*sin(3*d*x+3*c)+168*sin(2*d*x+2*c))/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int (a + a \cos(c + dx))^4 dx$$

$$= \frac{105 a^4 dx + (6 a^4 \cos(dx + c))^3 + 32 a^4 \cos(dx + c)^2 + 81 a^4 \cos(dx + c) + 160 a^4 \sin(dx + c)}{24 d}$$

[In] integrate((a+a*cos(d*x+c))^4,x, algorithm="fricas")

[Out] 1/24*(105*a^4*d*x + (6*a^4*cos(d*x + c))^3 + 32*a^4*cos(d*x + c)^2 + 81*a^4*cos(d*x + c) + 160*a^4*sin(d*x + c))/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(82) = 164.

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.57

$$\int (a + a \cos(c + dx))^4 dx$$

$$= \begin{cases} \frac{3a^4 x \sin^4(c+dx)}{8} + \frac{3a^4 x \sin^2(c+dx) \cos^2(c+dx)}{4} + 3a^4 x \sin^2(c+dx) + \frac{3a^4 x \cos^4(c+dx)}{8} + 3a^4 x \cos^2(c+dx) + a^4 x + \\ x(a \cos(c) + a)^4 \end{cases}$$

[In] integrate((a+a*cos(d*x+c))**4,x)

[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*x*cos(c + d*x)**2 + a**4*x + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 8*a**4*sin(c + d*x)**3/(3*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 4*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 3*a**4*sin(c + d*x)*cos(c + d*x)/d + 4*a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**4, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.22

$$\int (a + a \cos(c + dx))^4 dx = a^4 x - \frac{4 (\sin(dx + c)^3 - 3 \sin(dx + c)) a^4}{3 d}$$

$$+ \frac{(12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c)) a^4}{32 d}$$

$$+ \frac{3 (2 dx + 2 c + \sin(2 dx + 2 c)) a^4}{2 d} + \frac{4 a^4 \sin(dx + c)}{d}$$

[In] integrate((a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] $a^4*x - 4/3*(\sin(d*x + c))^3 - 3*\sin(d*x + c))*a^4/d + 1/32*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(2*d*x + 2*c))*a^4/d + 3/2*(2*d*x + 2*c + \sin(2*d*x + 2*c))*a^4/d + 4*a^4*\sin(d*x + c)/d$

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int (a + a \cos(c + dx))^4 dx = \frac{35}{8} a^4 x + \frac{a^4 \sin(4 dx + 4 c)}{32 d} + \frac{a^4 \sin(3 dx + 3 c)}{3 d} + \frac{7 a^4 \sin(2 dx + 2 c)}{4 d} + \frac{7 a^4 \sin(dx + c)}{d}$$

[In] integrate((a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] $35/8*a^4*x + 1/32*a^4*\sin(4*d*x + 4*c)/d + 1/3*a^4*\sin(3*d*x + 3*c)/d + 7/4*a^4*\sin(2*d*x + 2*c)/d + 7*a^4*\sin(d*x + c)/d$

Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int (a + a \cos(c + dx))^4 dx = \frac{35 a^4 x}{8} + \frac{35 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{4} + \frac{385 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{12} + \frac{511 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{12} + \frac{93 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4} \frac{1}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)^4}$$

[In] int((a + a*cos(c + d*x))^4,x)

[Out] $(35*a^4*x)/8 + ((511*a^4*\tan(c/2 + (d*x)/2)^3)/12 + (385*a^4*\tan(c/2 + (d*x)/2)^5)/12 + (35*a^4*\tan(c/2 + (d*x)/2)^7)/4 + (93*a^4*\tan(c/2 + (d*x)/2)))/4/(d*(\tan(c/2 + (d*x)/2)^2 + 1)^4)$

3.36 $\int (a + a \cos(c + dx))^4 \sec(c + dx) dx$

Optimal result	451
Rubi [A] (verified)	451
Mathematica [A] (verified)	453
Maple [A] (verified)	453
Fricas [A] (verification not implemented)	454
Sympy [F]	454
Maxima [A] (verification not implemented)	454
Giac [A] (verification not implemented)	455
Mupad [B] (verification not implemented)	455

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (a + a \cos(c + dx))^4 \sec(c + dx) dx = 6a^4x + \frac{a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{7a^4 \sin(c + dx)}{d} + \frac{2a^4 \cos(c + dx) \sin(c + dx)}{d} - \frac{a^4 \sin^3(c + dx)}{3d}$$

[Out] $6*a^4*x + a^4*\operatorname{arctanh}(\sin(d*x+c))/d + 7*a^4*\sin(d*x+c)/d + 2*a^4*\cos(d*x+c)*\sin(d*x+c)/d - 1/3*a^4*\sin(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2836, 2717, 2715, 8, 2713, 3855}

$$\int (a + a \cos(c + dx))^4 \sec(c + dx) dx = \frac{a^4 \operatorname{arctanh}(\sin(c + dx))}{d} - \frac{a^4 \sin^3(c + dx)}{3d} + \frac{7a^4 \sin(c + dx)}{d} + \frac{2a^4 \sin(c + dx) \cos(c + dx)}{d} + 6a^4x$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^4*\operatorname{Sec}[c + d*x], x]$

[Out] $6*a^4*x + (a^4*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (7*a^4*\operatorname{Sin}[c + d*x])/d + (2*a^4*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/d - (a^4*\operatorname{Sin}[c + d*x]^3)/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2715

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 2717

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 2836

`Int[((d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (4a^4 + 6a^4 \cos(c + dx) + 4a^4 \cos^2(c + dx) + a^4 \cos^3(c + dx) + a^4 \sec(c + dx)) dx \\
 &= 4a^4 x + a^4 \int \cos^3(c + dx) dx + a^4 \int \sec(c + dx) dx \\
 &\quad + (4a^4) \int \cos^2(c + dx) dx + (6a^4) \int \cos(c + dx) dx \\
 &= 4a^4 x + \frac{a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{6a^4 \sin(c + dx)}{d} + \frac{2a^4 \cos(c + dx) \sin(c + dx)}{d} \\
 &\quad + (2a^4) \int 1 dx - \frac{a^4 \operatorname{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{d}
 \end{aligned}$$

$$= 6a^4x + \frac{a^4 \operatorname{arctanh}(\sin(c+dx))}{d} + \frac{7a^4 \sin(c+dx)}{d} + \frac{2a^4 \cos(c+dx) \sin(c+dx)}{d} - \frac{a^4 \sin^3(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int (a + a \cos(c+dx))^4 \sec(c+dx) dx = \frac{a^4(72dx - 12 \log(\cos(\frac{1}{2}(c+dx)) - \sin(\frac{1}{2}(c+dx))) + 12 \log(\cos(\frac{1}{2}(c+dx)) + \sin(\frac{1}{2}(c+dx))) + 81 \sin^3(c+dx) + 12 \sin^2(c+dx) + \sin(c+dx))}{12d}$$

[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x],x]

[Out] (a^4*(72*d*x - 12*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 81*Sin[c + d*x] + 12*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(12*d)

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{a^4(72dx + \sin(3dx+3c) + 12 \sin(2dx+2c) + 81 \sin(dx+c) + 12 \ln(\tan(\frac{dx}{2} + \frac{c}{2}) + 1) - 12 \ln(\tan(\frac{dx}{2} + \frac{c}{2}) - 1))}{12d}$
derivativedivides	$\frac{a^4(2 + \cos^2(dx+c)) \sin(dx+c)}{3} + 4a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 6a^4 \sin(dx+c) + 4a^4(dx+c) + a^4 \ln(\sec(dx+c) + \tan(dx+c))$
default	$\frac{a^4(2 + \cos^2(dx+c)) \sin(dx+c)}{3} + 4a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 6a^4 \sin(dx+c) + 4a^4(dx+c) + a^4 \ln(\sec(dx+c) + \tan(dx+c))$
parts	$\frac{a^4 \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{a^4(2 + \cos^2(dx+c)) \sin(dx+c)}{3d} + \frac{4a^4(dx+c)}{d} + \frac{6a^4 \sin(dx+c)}{d} + \frac{4a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d}$
risch	$6a^4x - \frac{27ia^4 e^{i(dx+c)}}{8d} + \frac{27ia^4 e^{-i(dx+c)}}{8d} + \frac{a^4 \ln(e^{i(dx+c)} + i)}{d} - \frac{a^4 \ln(e^{i(dx+c)} - i)}{d} + \frac{a^4 \sin(3dx+3c)}{12d} + \frac{a^4 \sin(dx+c)}{d}$
norman	$\frac{6a^4x + \frac{18a^4 \tan(\frac{dx}{2} + \frac{c}{2})}{d} + \frac{130a^4 \left(\tan^3(\frac{dx}{2} + \frac{c}{2}) \right)}{3d} + \frac{106a^4 \left(\tan^5(\frac{dx}{2} + \frac{c}{2}) \right)}{3d} + \frac{10a^4 \left(\tan^7(\frac{dx}{2} + \frac{c}{2}) \right)}{d} + 24a^4x \left(\tan^2(\frac{dx}{2} + \frac{c}{2}) \right) + 36a^4 \left(\tan^4(\frac{dx}{2} + \frac{c}{2}) \right)}{\left(1 + \tan^2(\frac{dx}{2} + \frac{c}{2}) \right)^4}$

[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c),x,method=_RETURNVERBOSE)

[Out] 1/12*a^4*(72*d*x+sin(3*d*x+3*c))+12*sin(2*d*x+2*c)+81*sin(d*x+c)+12*ln(tan(1/2*d*x+1/2*c)+1)-12*ln(tan(1/2*d*x+1/2*c)-1))/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int (a + a \cos(c + dx))^4 \sec(c + dx) dx$$

$$= \frac{36 a^4 dx + 3 a^4 \log(\sin(dx + c) + 1) - 3 a^4 \log(-\sin(dx + c) + 1) + 2 (a^4 \cos(dx + c)^2 + 6 a^4 \cos(dx + c))}{6 d}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="fricas")

[Out] 1/6*(36*a^4*d*x + 3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) + 1) + 2*(a^4*cos(d*x + c)^2 + 6*a^4*cos(d*x + c) + 20*a^4)*sin(d*x + c))/d

Sympy [F]

$$\int (a + a \cos(c + dx))^4 \sec(c + dx) dx = a^4 \left(\int 4 \cos(c + dx) \sec(c + dx) dx \right.$$

$$+ \int 6 \cos^2(c + dx) \sec(c + dx) dx$$

$$+ \int 4 \cos^3(c + dx) \sec(c + dx) dx$$

$$+ \int \cos^4(c + dx) \sec(c + dx) dx$$

$$\left. + \int \sec(c + dx) dx \right)$$

[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c),x)

[Out] a**4*(Integral(4*cos(c + d*x)*sec(c + d*x), x) + Integral(6*cos(c + d*x)**2*sec(c + d*x), x) + Integral(4*cos(c + d*x)**3*sec(c + d*x), x) + Integral(cos(c + d*x)**4*sec(c + d*x), x) + Integral(sec(c + d*x), x))

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int (a + a \cos(c + dx))^4 \sec(c + dx) dx =$$

$$\frac{(\sin(dx + c))^3 - 3 \sin(dx + c) a^4 - 3(2 dx + 2 c + \sin(2 dx + 2 c)) a^4 - 12(dx + c) a^4 - 3 a^4 \log(\sec(dx + c))}{3 d}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="maxima")

[Out] $-1/3*((\sin(d*x + c))^3 - 3*\sin(d*x + c))*a^4 - 3*(2*d*x + 2*c + \sin(2*d*x + 2*c))*a^4 - 12*(d*x + c)*a^4 - 3*a^4*\log(\sec(d*x + c) + \tan(d*x + c)) - 18*a^4*\sin(d*x + c))/d$

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int (a + a \cos(c + dx))^4 \sec(c + dx) dx = \frac{18(dx + c)a^4 + 3a^4 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3a^4 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2\left(15a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + \dots\right)}{3d}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="giac")

[Out] $1/3*(18*(d*x + c)*a^4 + 3*a^4*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^4*\tan(1/2*d*x + 1/2*c)^5 + 38*a^4*\tan(1/2*d*x + 1/2*c)^3 + 27*a^4*\tan(1/2*d*x + 1/2*c)))/(\tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d$

Mupad [B] (verification not implemented)

Time = 14.00 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int (a + a \cos(c + dx))^4 \sec(c + dx) dx = 6a^4 x + \frac{20a^4 \sin(c + dx)}{3d} + \frac{2a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{a^4 \cos(c + dx)^2 \sin(c + dx)}{3d} + \frac{2a^4 \cos(c + dx) \sin(c + dx)}{d}$$

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x),x)

[Out] $6*a^4*x + (20*a^4*\sin(c + d*x))/(3*d) + (2*a^4*\operatorname{atanh}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (a^4*\cos(c + d*x)^2*\sin(c + d*x))/(3*d) + (2*a^4*\cos(c + d*x)*\sin(c + d*x))/d$

3.37 $\int (a + a \cos(c + dx))^4 \sec^2(c + dx) dx$

Optimal result	456
Rubi [A] (verified)	456
Mathematica [B] (verified)	458
Maple [A] (verified)	459
Fricas [A] (verification not implemented)	459
Sympy [F]	460
Maxima [A] (verification not implemented)	460
Giac [A] (verification not implemented)	461
Mupad [B] (verification not implemented)	461

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (a + a \cos(c + dx))^4 \sec^2(c + dx) dx = \frac{13a^4x}{2} + \frac{4a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{4a^4 \sin(c + dx)}{d} + \frac{a^4 \cos(c + dx) \sin(c + dx)}{2d} + \frac{a^4 \tan(c + dx)}{d}$$

[Out] $13/2*a^4*x+4*a^4*\operatorname{arctanh}(\sin(d*x+c))/d+4*a^4*\sin(d*x+c)/d+1/2*a^4*\cos(d*x+c)*\sin(d*x+c)/d+a^4*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2836, 2717, 2715, 8, 3855, 3852}

$$\int (a + a \cos(c + dx))^4 \sec^2(c + dx) dx = \frac{4a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{4a^4 \sin(c + dx)}{d} + \frac{a^4 \tan(c + dx)}{d} + \frac{a^4 \sin(c + dx) \cos(c + dx)}{2d} + \frac{13a^4x}{2}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^4*\operatorname{Sec}[c + d*x]^2,x]$

[Out] $(13*a^4*x)/2 + (4*a^4*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (4*a^4*\operatorname{Sin}[c + d*x])/d + (a^4*\operatorname{Cos}[c + d*x]*\operatorname{Sin}[c + d*x])/(2*d) + (a^4*\operatorname{Tan}[c + d*x])/d$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2717

Int[sin[Pi/2 + (c_) + (d_)*(x_)], x_Symbol] := Simp[SIN[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rule 3852

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3855

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (6a^4 + 4a^4 \cos(c + dx) + a^4 \cos^2(c + dx) + 4a^4 \sec(c + dx) + a^4 \sec^2(c + dx)) dx \\
 &= 6a^4 x + a^4 \int \cos^2(c + dx) dx + a^4 \int \sec^2(c + dx) dx \\
 &\quad + (4a^4) \int \cos(c + dx) dx + (4a^4) \int \sec(c + dx) dx \\
 &= 6a^4 x + \frac{4a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{4a^4 \sin(c + dx)}{d} + \frac{a^4 \cos(c + dx) \sin(c + dx)}{2d} \\
 &\quad + \frac{1}{2} a^4 \int 1 dx - \frac{a^4 \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{d}
 \end{aligned}$$

$$= \frac{13a^4x}{2} + \frac{4a^4 \operatorname{arctanh}(\sin(c+dx))}{d} + \frac{4a^4 \sin(c+dx)}{d} + \frac{a^4 \cos(c+dx) \sin(c+dx)}{2d} + \frac{a^4 \tan(c+dx)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 241 vs. $2(73) = 146$.

Time = 3.00 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.30

$$\int (a + a \cos(c + dx))^4 \sec^2(c + dx) dx$$

$$= \frac{1}{64} a^4 (1 + \cos(c + dx))^4 \sec^8\left(\frac{1}{2}(c + dx)\right) \left(26x - \frac{16 \log(\cos(\frac{1}{2}(c + dx)) - \sin(\frac{1}{2}(c + dx)))}{d} + \frac{16 \log(\cos(\frac{1}{2}(c + dx)) + \sin(\frac{1}{2}(c + dx)))}{d} + \frac{16 \cos(dx) \sin(c)}{d} + \frac{\cos(2dx) \sin(2c)}{d} + \frac{16 \cos(c) \sin(dx)}{d} + \frac{\cos(2c) \sin(2dx)}{d} + \frac{4 \sin(\frac{dx}{2})}{d(\cos(\frac{c}{2}) - \sin(\frac{c}{2}))(\cos(\frac{1}{2}(c + dx)) - \sin(\frac{1}{2}(c + dx)))} + \frac{4 \sin(\frac{dx}{2})}{d(\cos(\frac{c}{2}) + \sin(\frac{c}{2}))(\cos(\frac{1}{2}(c + dx)) + \sin(\frac{1}{2}(c + dx)))} \right)$$

[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^2,x]

[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(26*x - (16*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (16*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (16*Cos[d*x]*Sin[c])/d + (Cos[2*d*x]*Sin[2*c])/d + (16*Cos[c]*Sin[d*x])/d + (Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/64

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^4 \left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 4a^4 \sin(dx+c) + 6a^4(dx+c) + 4a^4 \ln(\sec(dx+c) + \tan(dx+c)) + a^4 \tan(dx+c)}{d}$
default	$\frac{a^4 \left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 4a^4 \sin(dx+c) + 6a^4(dx+c) + 4a^4 \ln(\sec(dx+c) + \tan(dx+c)) + a^4 \tan(dx+c)}{d}$
parts	$\frac{a^4 \tan(dx+c)}{d} + \frac{a^4 \left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d} + \frac{4a^4 \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{6a^4(dx+c)}{d} + \frac{4a^4 \sin(dx+c)}{d}$
parallelrisch	$\frac{a^4 \left(52dx \cos(dx+c) - 32 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) \cos(dx+c) + 32 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) \cos(dx+c) + 9 \sin(dx+c) + \sin(3dx+3c) \right)}{8d \cos(dx+c)}$
risch	$\frac{13a^4 x}{2} - \frac{ia^4 e^{2i(dx+c)}}{8d} - \frac{2ia^4 e^{i(dx+c)}}{d} + \frac{2ia^4 e^{-i(dx+c)}}{d} + \frac{ia^4 e^{-2i(dx+c)}}{8d} + \frac{2ia^4}{d(e^{2i(dx+c)}+1)} + \frac{4a^4 \ln(e^{i(dx+c)})}{d}$
norman	$\frac{-\frac{13a^4 x}{2} - \frac{11a^4 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 24a^4 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 10a^4 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 8a^4 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 5a^4 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 39a^4 x}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^4 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a^4*sin(d*x+c)+6*a^4*(d*x+c)+4*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int (a + a \cos(c + dx))^4 \sec^2(c + dx) dx$$

$$= \frac{13 a^4 dx \cos(dx + c) + 4 a^4 \cos(dx + c) \log(\sin(dx + c) + 1) - 4 a^4 \cos(dx + c) \log(-\sin(dx + c) + 1) + \dots}{2 d \cos(dx + c)}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(13*a^4*d*x*cos(d*x + c) + 4*a^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 4*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^4*cos(d*x + c)^2 + 8*a^4*cos(d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

SymPy [F]

$$\int (a + a \cos(c + dx))^4 \sec^2(c + dx) dx = a^4 \left(\int 4 \cos(c + dx) \sec^2(c + dx) dx \right. \\ \left. + \int 6 \cos^2(c + dx) \sec^2(c + dx) dx \right. \\ \left. + \int 4 \cos^3(c + dx) \sec^2(c + dx) dx \right. \\ \left. + \int \cos^4(c + dx) \sec^2(c + dx) dx \right. \\ \left. + \int \sec^2(c + dx) dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**2,x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(6*cos(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(4*cos(c + d*x)**3*sec(c + d*x)**2, x) +
Integral(cos(c + d*x)**4*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16

$$\int (a + a \cos(c + dx))^4 \sec^2(c + dx) dx \\ = \frac{(2 dx + 2 c + \sin(2 dx + 2 c))a^4 + 24(dx + c)a^4 + 8a^4(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 16}{4d}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^4 + 8*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^4*sin(d*x + c) + 4*a^4*tan
(d*x + c))/d
```


Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int (a + a \cos(c + dx))^4 \sec^2(c + dx) dx$$

$$= \frac{13(dx + c)a^4 + 8a^4 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 8a^4 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{4a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1} + \frac{2}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1}}{2d}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="giac")

```
[Out] 1/2*(13*(d*x + c)*a^4 + 8*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^4*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int (a + a \cos(c + dx))^4 \sec^2(c + dx) dx$$

$$= \frac{13a^4x}{2} + \frac{8a^4 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d}$$

$$+ \frac{-5a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + 2a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 11a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(-\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)}$$

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^2,x)

```
[Out] (13*a^4*x)/2 + (8*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)/2)^3 - 5*a^4*tan(c/2 + (d*x)/2)^5 + 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

3.38 $\int (a + a \cos(c + dx))^4 \sec^3(c + dx) dx$

Optimal result	462
Rubi [A] (verified)	462
Mathematica [B] (verified)	464
Maple [A] (verified)	465
Fricas [A] (verification not implemented)	465
Sympy [F]	466
Maxima [A] (verification not implemented)	466
Giac [A] (verification not implemented)	467
Mupad [B] (verification not implemented)	467

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (a + a \cos(c + dx))^4 \sec^3(c + dx) dx = 4a^4x + \frac{13a^4 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a^4 \sin(c + dx)}{d} + \frac{4a^4 \tan(c + dx)}{d} + \frac{a^4 \sec(c + dx) \tan(c + dx)}{2d}$$

[Out] $4*a^4*x + 13/2*a^4*\operatorname{arctanh}(\sin(d*x+c))/d + a^4*\sin(d*x+c)/d + 4*a^4*\tan(d*x+c)/d + 1/2*a^4*\sec(d*x+c)*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2836, 2717, 3855, 3852, 8, 3853}

$$\int (a + a \cos(c + dx))^4 \sec^3(c + dx) dx = \frac{13a^4 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a^4 \sin(c + dx)}{d} + \frac{4a^4 \tan(c + dx)}{d} + \frac{a^4 \tan(c + dx) \sec(c + dx)}{2d} + 4a^4x$$

[In] $\operatorname{Int}[(a + a \operatorname{Cos}[c + d*x])^4 \operatorname{Sec}[c + d*x]^3, x]$

[Out] $4*a^4*x + (13*a^4*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (a^4*\operatorname{Sin}[c + d*x])/d + (4*a^4*\operatorname{Tan}[c + d*x])/d + (a^4*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (4a^4 + a^4 \cos(c + dx) + 6a^4 \sec(c + dx) + 4a^4 \sec^2(c + dx) + a^4 \sec^3(c + dx)) dx \\
 &= 4a^4 x + a^4 \int \cos(c + dx) dx + a^4 \int \sec^3(c + dx) dx \\
 &\quad + (4a^4) \int \sec^2(c + dx) dx + (6a^4) \int \sec(c + dx) dx \\
 &= 4a^4 x + \frac{6a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^4 \sin(c + dx)}{d} + \frac{a^4 \sec(c + dx) \tan(c + dx)}{2d} \\
 &\quad + \frac{1}{2} a^4 \int \sec(c + dx) dx - \frac{(4a^4) \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{d}
 \end{aligned}$$

$$= 4a^4x + \frac{13a^4 \operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{a^4 \sin(c+dx)}{d} \\ + \frac{4a^4 \tan(c+dx)}{d} + \frac{a^4 \sec(c+dx) \tan(c+dx)}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 272 vs. $2(73) = 146$.

Time = 3.46 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.73

$$\int (a + a \cos(c+dx))^4 \sec^3(c+dx) dx \\ = \frac{1}{64} a^4 (1 + \cos(c+dx))^4 \sec^8\left(\frac{1}{2}(c+dx)\right) \left(16x \right. \\ \left. - \frac{26 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} \right. \\ \left. + \frac{26 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{4 \cos(dx) \sin(c)}{d} + \frac{4 \cos(c) \sin(dx)}{d} \right. \\ \left. + \frac{1}{d \left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \right. \\ \left. + \frac{16 \sin\left(\frac{dx}{2}\right)}{d \left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right) \left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)} \right. \\ \left. - \frac{1}{d \left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \right. \\ \left. + \frac{16 \sin\left(\frac{dx}{2}\right)}{d \left(\cos\left(\frac{c}{2}\right) + \sin\left(\frac{c}{2}\right)\right) \left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)} \right)$$

[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^3,x]

[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(16*x - (26*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d + (26*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*Cos[d*x]*Sin[c])/d + (4*Cos[c]*Sin[d*x])/d + 1/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/64

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{a^4 \sin(dx+c)+4a^4(dx+c)+6a^4 \ln(\sec(dx+c)+\tan(dx+c))+4a^4 \tan(dx+c)+a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
default	$\frac{a^4 \sin(dx+c)+4a^4(dx+c)+6a^4 \ln(\sec(dx+c)+\tan(dx+c))+4a^4 \tan(dx+c)+a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
parts	$\frac{a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d} + \frac{a^4 \sin(dx+c)}{d} + \frac{4a^4 \tan(dx+c)}{d} + \frac{6a^4 \ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$\frac{a^4 \left(-13(1+\cos(2dx+2c)) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 8dx \cos(2dx+2c) + 13 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) \cos(2dx+2c) + 8dx + 13 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \right)}{2d(1+\cos(2dx+2c))}$
risch	$4a^4 x - \frac{ia^4 e^{i(dx+c)}}{2d} + \frac{ia^4 e^{-i(dx+c)}}{2d} - \frac{ia^4 (e^{3i(dx+c)} - 8e^{2i(dx+c)} - e^{i(dx+c)} - 8)}{d(e^{2i(dx+c)} + 1)^2} - \frac{13a^4 \ln(e^{i(dx+c)} - i)}{2d} + \frac{13a^4 \ln(e^{-i(dx+c)} + i)}{2d}$
norman	$\frac{4a^4 x + \frac{11a^4 \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d} + \frac{31a^4 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} + \frac{22a^4 \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{10a^4 \left(\tan^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{17a^4 \left(\tan^9 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{5a^4 \left(\tan^{11} \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d}}{d}$

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*sin(d*x+c)+4*a^4*(d*x+c)+6*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^4*tan(d*x+c)+a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.52

$$\int (a + a \cos(c + dx))^4 \sec^3(c + dx) dx$$

$$= \frac{16 a^4 dx \cos(dx + c)^2 + 13 a^4 \cos(dx + c)^2 \log(\sin(dx + c) + 1) - 13 a^4 \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 8 a^4 \cos(dx + c) + a^4 \sin(dx + c)}{4 d \cos(dx + c)^2}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a^4*d*x*cos(d*x + c)^2 + 13*a^4*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 13*a^4*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^4*cos(d*x + c)^2 + 8*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

SymPy [F]

$$\int (a + a \cos(c + dx))^4 \sec^3(c + dx) dx = a^4 \left(\int 4 \cos(c + dx) \sec^3(c + dx) dx \right. \\ \left. + \int 6 \cos^2(c + dx) \sec^3(c + dx) dx \right. \\ \left. + \int 4 \cos^3(c + dx) \sec^3(c + dx) dx \right. \\ \left. + \int \cos^4(c + dx) \sec^3(c + dx) dx \right. \\ \left. + \int \sec^3(c + dx) dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**3,x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(6*cos(c + d*x)
**2*sec(c + d*x)**3, x) + Integral(4*cos(c + d*x)**3*sec(c + d*x)**3, x) +
Integral(cos(c + d*x)**4*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.51

$$\int (a + a \cos(c + dx))^4 \sec^3(c + dx) dx \\ = \frac{16(dx + c)a^4 - a^4 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right) + 12a^4(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 4a^4 \sin(dx + c) + 16a^4 \tan(dx + c)}{4d}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a^4 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^4*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

$$\int (a + a \cos(c + dx))^4 \sec^3(c + dx) dx$$

$$= \frac{8(dx + c)a^4 + 13a^4 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 13a^4 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{4a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1} - \frac{2a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1}}{2d}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="giac")

[Out] 1/2*(8*(d*x + c)*a^4 + 13*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^4*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2/d

Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.58

$$\int (a + a \cos(c + dx))^4 \sec^3(c + dx) dx$$

$$= 4a^4 x + \frac{13a^4 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} + \frac{5a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + 2a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 - 11a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(-\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^3,x)

[Out] 4*a^4*x + (13*a^4*atanh(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)/2)^3 + 5*a^4*tan(c/2 + (d*x)/2)^5 - 11*a^4*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))

3.39 $\int (a + a \cos(c + dx))^4 \sec^4(c + dx) dx$

Optimal result	468
Rubi [A] (verified)	468
Mathematica [A] (verified)	470
Maple [A] (verified)	470
Fricas [A] (verification not implemented)	471
Sympy [F]	471
Maxima [A] (verification not implemented)	472
Giac [A] (verification not implemented)	472
Mupad [B] (verification not implemented)	472

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (a + a \cos(c + dx))^4 \sec^4(c + dx) dx = a^4 x + \frac{6a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{7a^4 \tan(c + dx)}{d} + \frac{2a^4 \sec(c + dx) \tan(c + dx)}{d} + \frac{a^4 \tan^3(c + dx)}{3d}$$

[Out] $a^4 x + 6a^4 \operatorname{arctanh}(\sin(dx+c))/d + 7a^4 \tan(dx+c)/d + 2a^4 \sec(dx+c) \tan(dx+c)/d + 1/3 a^4 \tan(dx+c)^3/d$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2836, 3855, 3852, 8, 3853}

$$\int (a + a \cos(c + dx))^4 \sec^4(c + dx) dx = \frac{6a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^4 \tan^3(c + dx)}{3d} + \frac{7a^4 \tan(c + dx)}{d} + \frac{2a^4 \tan(c + dx) \sec(c + dx)}{d} + a^4 x$$

[In] $\text{Int}[(a + a \cos[c + d*x])^4 \sec[c + d*x]^4, x]$

[Out] $a^4 x + (6a^4 \operatorname{ArcTanh}[\sin[c + d*x]])/d + (7a^4 \tan[c + d*x])/d + (2a^4 \sec[c + d*x] \tan[c + d*x])/d + (a^4 \tan[c + d*x]^3)/(3d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2836

Int[((d_)*sin[(e_) + (f_)*(x_)]^(n_))*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(m_)), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rule 3852

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^4 + 4a^4 \sec(c + dx) + 6a^4 \sec^2(c + dx) + 4a^4 \sec^3(c + dx) + a^4 \sec^4(c + dx)) dx \\
 &= a^4 x + a^4 \int \sec^4(c + dx) dx + (4a^4) \int \sec(c + dx) dx \\
 &\quad + (4a^4) \int \sec^3(c + dx) dx + (6a^4) \int \sec^2(c + dx) dx \\
 &= a^4 x + \frac{4a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{2a^4 \sec(c + dx) \tan(c + dx)}{d} + (2a^4) \int \sec(c + dx) dx \\
 &\quad - \frac{a^4 \operatorname{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d} - \frac{(6a^4) \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\
 &= a^4 x + \frac{6a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{7a^4 \tan(c + dx)}{d} \\
 &\quad + \frac{2a^4 \sec(c + dx) \tan(c + dx)}{d} + \frac{a^4 \tan^3(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (a + a \cos(c + dx))^4 \sec^4(c + dx) dx = a^4 \left(x + \frac{6 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{7 \tan(c + dx)}{d} + \frac{2 \sec(c + dx) \tan(c + dx)}{d} + \frac{\tan^3(c + dx)}{3d} \right)$$

[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^4,x]

[Out] a^4*(x + (6*ArcTanh[Sin[c + d*x]])/d + (7*Tan[c + d*x])/d + (2*Sec[c + d*x]*Tan[c + d*x])/d + Tan[c + d*x]^3/(3*d))

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

method	result
parts	$-\frac{a^4 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d} + \frac{a^4(dx+c)}{d} + \frac{2a^4 \sec(dx+c) \tan(dx+c)}{d} + \frac{6a^4 \ln(\sec(dx+c)+\tan(dx+c))}{d} +$
derivativdivides	$\frac{a^4(dx+c)+4a^4 \ln(\sec(dx+c)+\tan(dx+c))+6a^4 \tan(dx+c)+4a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right) - a^4 \left(-\frac{2}{3} \right)}{d}$
default	$\frac{a^4(dx+c)+4a^4 \ln(\sec(dx+c)+\tan(dx+c))+6a^4 \tan(dx+c)+4a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right) - a^4 \left(-\frac{2}{3} \right)}{d}$
risch	$a^4 x - \frac{4ia^4(3e^{5i(dx+c)} - 9e^{4i(dx+c)} - 21e^{2i(dx+c)} - 3e^{i(dx+c)} - 10)}{3d(e^{2i(dx+c)} + 1)^3} + \frac{6a^4 \ln(e^{i(dx+c)} + i)}{d} - \frac{6a^4 \ln(e^{i(dx+c)} - i)}{d}$
parallelrisc	$-\frac{18 \left(\left(\cos(dx+c) + \frac{\cos(3dx+3c)}{3} \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + \left(-\cos(dx+c) - \frac{\cos(3dx+3c)}{3} \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) - \frac{dx \cos(dx+c)}{6} \right)}{d(\cos(3dx+3c) + 3 \cos(dx+c))}$
norman	$\frac{a^4 x \left(\tan^{12} \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + a^4 x \left(\tan^{14} \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - a^4 x - \frac{18a^4 \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d} - \frac{140a^4 \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} - \frac{50a^4 \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} + \frac{40a^4 \left(\tan^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d}}{d}$

[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] -a^4/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^4/d*(d*x+c)+2*a^4*sec(d*x+c)*tan(d*x+c)/d+6*a^4/d*ln(sec(d*x+c)+tan(d*x+c))+6*a^4*tan(d*x+c)/d

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.51

$$\int (a + a \cos(c + dx))^4 \sec^4(c + dx) dx$$

$$= \frac{3 a^4 dx \cos(dx + c)^3 + 9 a^4 \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 9 a^4 \cos(dx + c)^3 \log(-\sin(dx + c) + 1)}{3 d \cos(dx + c)^3}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cos(d*x + c)^3 + 9*a^4*cos(d*x + c)^3*log(sin(d*x + c) + 1)
- 9*a^4*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (20*a^4*cos(d*x + c)^2 + 6*
a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a + a \cos(c + dx))^4 \sec^4(c + dx) dx = a^4 \left(\int 4 \cos(c + dx) \sec^4(c + dx) dx \right.$$

$$+ \int 6 \cos^2(c + dx) \sec^4(c + dx) dx$$

$$+ \int 4 \cos^3(c + dx) \sec^4(c + dx) dx$$

$$+ \int \cos^4(c + dx) \sec^4(c + dx) dx$$

$$\left. + \int \sec^4(c + dx) dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**4,x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)*sec(c + d*x)**4, x) + Integral(6*cos(c + d*x)
**2*sec(c + d*x)**4, x) + Integral(4*cos(c + d*x)**3*sec(c + d*x)**4, x) +
Integral(cos(c + d*x)**4*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int (a + a \cos(c + dx))^4 \sec^4(c + dx) dx$$

$$= \frac{(\tan(dx + c))^3 + 3 \tan(dx + c)a^4 + 3(dx + c)a^4 - 3a^4 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right) + 6a^4(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 18a^4 \tan(dx + c)}{3d}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 3*(d*x + c)*a^4 - 3*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*a^4*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int (a + a \cos(c + dx))^4 \sec^4(c + dx) dx$$

$$= \frac{3(dx + c)a^4 + 18a^4 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|) - 18a^4 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|) - \frac{2(15a^4 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 3a^4)}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^3}}{3d}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 - 3*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 14.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int (a + a \cos(c + dx))^4 \sec^4(c + dx) dx$$

$$= a^4 x + \frac{12 a^4 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{10 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 - \frac{76 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3} + 18 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1 \right)}$$

[In] $\text{int}((a + a\cos(c + d*x))^4/\cos(c + d*x)^4,x)$

[Out] $a^4*x + (12*a^4*\text{atanh}(\tan(c/2 + (d*x)/2)))/d - (10*a^4*\tan(c/2 + (d*x)/2)^5 - (76*a^4*\tan(c/2 + (d*x)/2)^3)/3 + 18*a^4*\tan(c/2 + (d*x)/2))/(d*(3*\tan(c/2 + (d*x)/2)^2 - 3*\tan(c/2 + (d*x)/2)^4 + \tan(c/2 + (d*x)/2)^6 - 1))$

3.40 $\int (a + a \cos(c + dx))^4 \sec^5(c + dx) dx$

Optimal result	474
Rubi [A] (verified)	474
Mathematica [A] (verified)	476
Maple [A] (verified)	477
Fricas [A] (verification not implemented)	477
Sympy [F(-1)]	478
Maxima [B] (verification not implemented)	478
Giac [A] (verification not implemented)	478
Mupad [B] (verification not implemented)	479

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int (a + a \cos(c + dx))^4 \sec^5(c + dx) dx = \frac{35a^4 \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{8a^4 \tan(c + dx)}{d} + \frac{27a^4 \sec(c + dx) \tan(c + dx)}{8d} + \frac{a^4 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{4a^4 \tan^3(c + dx)}{3d}$$

[Out] $35/8*a^4*\operatorname{arctanh}(\sin(d*x+c))/d+8*a^4*\tan(d*x+c)/d+27/8*a^4*\sec(d*x+c)*\tan(d*x+c)/d+1/4*a^4*\sec(d*x+c)^3*\tan(d*x+c)/d+4/3*a^4*\tan(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2836, 3855, 3852, 8, 3853}

$$\int (a + a \cos(c + dx))^4 \sec^5(c + dx) dx = \frac{35a^4 \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{4a^4 \tan^3(c + dx)}{3d} + \frac{8a^4 \tan(c + dx)}{d} + \frac{a^4 \tan(c + dx) \sec^3(c + dx)}{4d} + \frac{27a^4 \tan(c + dx) \sec(c + dx)}{8d}$$

[In] $\operatorname{Int}[(a + a \operatorname{Cos}[c + d*x])^4 \operatorname{Sec}[c + d*x]^5, x]$

[Out] $(35*a^4*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(8*d) + (8*a^4*\operatorname{Tan}[c + d*x])/d + (27*a^4*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(8*d) + (a^4*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(4*d) + (4*a^4*\operatorname{Tan}[c + d*x]^3)/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2836

`Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]`

Rule 3852

`Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3853

`Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3855

`Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^4 \sec(c + dx) + 4a^4 \sec^2(c + dx) + 6a^4 \sec^3(c + dx) + 4a^4 \sec^4(c + dx) \\
 &\quad + a^4 \sec^5(c + dx)) dx \\
 &= a^4 \int \sec(c + dx) dx + a^4 \int \sec^5(c + dx) dx + (4a^4) \int \sec^2(c + dx) dx \\
 &\quad + (4a^4) \int \sec^4(c + dx) dx + (6a^4) \int \sec^3(c + dx) dx \\
 &= \frac{a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{3a^4 \sec(c + dx) \tan(c + dx)}{d} \\
 &\quad + \frac{a^4 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{4} (3a^4) \int \sec^3(c + dx) dx \\
 &\quad + (3a^4) \int \sec(c + dx) dx - \frac{(4a^4) \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\
 &\quad - \frac{(4a^4) \operatorname{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{4a^4 \operatorname{arctanh}(\sin(c+dx))}{d} + \frac{8a^4 \tan(c+dx)}{d} + \frac{27a^4 \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{a^4 \sec^3(c+dx) \tan(c+dx)}{4d} + \frac{4a^4 \tan^3(c+dx)}{3d} + \frac{1}{8}(3a^4) \int \sec(c+dx) dx \\
&= \frac{35a^4 \operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{8a^4 \tan(c+dx)}{d} + \frac{27a^4 \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{a^4 \sec^3(c+dx) \tan(c+dx)}{4d} + \frac{4a^4 \tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int (a + a \cos(c+dx))^4 \sec^5(c+dx) dx &= \frac{35a^4 \operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{8a^4 \tan(c+dx)}{d} \\
&\quad + \frac{27a^4 \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{a^4 \sec^3(c+dx) \tan(c+dx)}{4d} + \frac{4a^4 \tan^3(c+dx)}{3d}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^5,x]

[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (4*a^4*Tan[c + d*x]^3)/(3*d)

Maple [A] (verified)

Time = 3.95 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.41

method	result
parts	$a^4 \left(- \left(- \frac{\left(\sec^3(dx+c) \right)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right) \frac{1}{d} + \frac{4a^4 \ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{4a^4 \ln(\sec(dx+c)+\tan(dx+c))}{d}$
derivativedivides	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c)) + 4a^4 \tan(dx+c) + 6a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right) - 4a^4 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right)}{d}$
default	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c)) + 4a^4 \tan(dx+c) + 6a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right) - 4a^4 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right)}{d}$
risch	$- \frac{ia^4 (81 e^{7i(dx+c)} - 96 e^{6i(dx+c)} + 105 e^{5i(dx+c)} - 480 e^{4i(dx+c)} - 105 e^{3i(dx+c)} - 544 e^{2i(dx+c)} - 81 e^{i(dx+c)} - 160)}{12d(e^{2i(dx+c)}+1)^4} + 35 \frac{a^4}{d}$
parallelrisch	$\frac{a^4 \left(105(-\cos(4dx+4c) - 4\cos(2dx+2c) - 3) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + 105(\cos(4dx+4c) + 4\cos(2dx+2c) + 3) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) \right)}{24d(\cos(4dx+4c) + 4\cos(2dx+2c) + 3)}$
norman	$\frac{\frac{93a^4 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4d} + \frac{605a^4 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} + \frac{5a^4 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d} - \frac{515a^4 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} + \frac{125a^4 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12d} + \frac{133a^4 \left(\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^4 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^4}$

[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] a^4/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+4*a^4/d*ln(sec(d*x+c)+tan(d*x+c))-4*a^4/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^4*sec(d*x+c)*tan(d*x+c)/d+4*a^4*tan(d*x+c)/d

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16

$$\int (a + a \cos(c + dx))^4 \sec^5(c + dx) dx$$

$$= \frac{105 a^4 \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 105 a^4 \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2(160 a^4 \cos(dx + c)^3 + 81 a^4 \cos(dx + c)^2 + 32 a^4 \cos(dx + c) + 6 a^4) \sin(dx + c)}{48 d \cos(dx + c)^4}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="fricas")

[Out] 1/48*(105*a^4*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(160*a^4*cos(d*x + c)^3 + 81*a^4*cos(d*x + c)^2 + 32*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^5(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**5,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(88) = 176.

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.90

$$\int (a + a \cos(c + dx))^4 \sec^5(c + dx) dx$$

$$= \frac{64 (\tan(dx + c)^3 + 3 \tan(dx + c)) a^4 - 3 a^4 \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right)}{24 d}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="maxima")

[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 3*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*a^4*tan(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int (a + a \cos(c + dx))^4 \sec^5(c + dx) dx$$

$$= \frac{105 a^4 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 105 a^4 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(105 a^4 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^7 - 385 a^4 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 + 511 a^4 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 - 279 a^4 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{24 d}}{24 d}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="giac")

[Out] 1/24*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*a^4*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4/d

Mupad [B] (verification not implemented)

Time = 17.75 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.47

$$\int (a + a \cos(c + dx))^4 \sec^5(c + dx) dx$$

$$= \frac{35 a^4 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{4 d}$$

$$- \frac{\frac{35 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{4} - \frac{385 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{12} + \frac{511 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{12} - \frac{93 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^5,x)

```
[Out] (35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((511*a^4*tan(c/2 + (d*x)/2)^3)/
12 - (385*a^4*tan(c/2 + (d*x)/2)^5)/12 + (35*a^4*tan(c/2 + (d*x)/2)^7)/4 -
(93*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*
x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.41 $\int (a + a \cos(c + dx))^4 \sec^6(c + dx) dx$

Optimal result	480
Rubi [A] (verified)	480
Mathematica [A] (verified)	482
Maple [C] (verified)	483
Fricas [A] (verification not implemented)	483
Sympy [F(-1)]	484
Maxima [A] (verification not implemented)	484
Giac [A] (verification not implemented)	484
Mupad [B] (verification not implemented)	485

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int (a + a \cos(c + dx))^4 \sec^6(c + dx) dx = \frac{7a^4 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{8a^4 \tan(c + dx)}{d} + \frac{7a^4 \sec(c + dx) \tan(c + dx)}{2d} + \frac{a^4 \sec^3(c + dx) \tan(c + dx)}{d} + \frac{8a^4 \tan^3(c + dx)}{3d} + \frac{a^4 \tan^5(c + dx)}{5d}$$

[Out] $\frac{7}{2}a^4 \operatorname{arctanh}(\sin(dx+c))/d + 8a^4 \tan(dx+c)/d + \frac{7}{2}a^4 \sec(dx+c) \tan(dx+c)/d + a^4 \sec^3(dx+c) \tan(dx+c)/d + \frac{8}{3}a^4 \tan^3(dx+c)/d + \frac{1}{5}a^4 \tan^5(dx+c)/d$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2836, 3852, 8, 3853, 3855}

$$\int (a + a \cos(c + dx))^4 \sec^6(c + dx) dx = \frac{7a^4 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a^4 \tan^5(c + dx)}{5d} + \frac{8a^4 \tan^3(c + dx)}{3d} + \frac{8a^4 \tan(c + dx)}{d} + \frac{a^4 \tan(c + dx) \sec^3(c + dx)}{d} + \frac{7a^4 \tan(c + dx) \sec(c + dx)}{2d}$$

[In] Int[(a + a*cos[c + d*x])^4*Sec[c + d*x]^6,x]

[Out] (7*a^4*ArcTanh[Sin[c + d*x]]/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^4*Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2836

Int[((d_)*sin[(e_) + (f_)*(x_)]^(n_))*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(m_)), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rule 3852

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int (a^4 \sec^2(c + dx) + 4a^4 \sec^3(c + dx) + 6a^4 \sec^4(c + dx) + 4a^4 \sec^5(c + dx) \\
 &\quad + a^4 \sec^6(c + dx)) dx \\
 &= a^4 \int \sec^2(c + dx) dx + a^4 \int \sec^6(c + dx) dx + (4a^4) \int \sec^3(c + dx) dx \\
 &\quad + (4a^4) \int \sec^5(c + dx) dx + (6a^4) \int \sec^4(c + dx) dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2a^4 \sec(c+dx) \tan(c+dx)}{d} + \frac{a^4 \sec^3(c+dx) \tan(c+dx)}{d} + (2a^4) \int \sec(c+dx) dx \\
&\quad + (3a^4) \int \sec^3(c+dx) dx - \frac{a^4 \text{Subst}(\int 1 dx, x, -\tan(c+dx))}{d} \\
&\quad - \frac{a^4 \text{Subst}(\int (1+2x^2+x^4) dx, x, -\tan(c+dx))}{d} \\
&\quad - \frac{(6a^4) \text{Subst}(\int (1+x^2) dx, x, -\tan(c+dx))}{d} \\
&= \frac{2a^4 \operatorname{arctanh}(\sin(c+dx))}{d} + \frac{8a^4 \tan(c+dx)}{d} + \frac{7a^4 \sec(c+dx) \tan(c+dx)}{2d} \\
&\quad + \frac{a^4 \sec^3(c+dx) \tan(c+dx)}{d} + \frac{8a^4 \tan^3(c+dx)}{3d} \\
&\quad + \frac{a^4 \tan^5(c+dx)}{5d} + \frac{1}{2} (3a^4) \int \sec(c+dx) dx \\
&= \frac{7a^4 \operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{8a^4 \tan(c+dx)}{d} + \frac{7a^4 \sec(c+dx) \tan(c+dx)}{2d} \\
&\quad + \frac{a^4 \sec^3(c+dx) \tan(c+dx)}{d} + \frac{8a^4 \tan^3(c+dx)}{3d} + \frac{a^4 \tan^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int (a + a \cos(c+dx))^4 \sec^6(c+dx) dx &= \frac{7a^4 \operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{8a^4 \tan(c+dx)}{d} \\
&\quad + \frac{7a^4 \sec(c+dx) \tan(c+dx)}{2d} \\
&\quad + \frac{a^4 \sec^3(c+dx) \tan(c+dx)}{d} \\
&\quad + \frac{8a^4 \tan^3(c+dx)}{3d} + \frac{a^4 \tan^5(c+dx)}{5d}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^6,x]

[Out] (7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^4*Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.41

method	result
risch	$\frac{ia^4(105e^{9i(dx+c)} - 30e^{8i(dx+c)} + 330e^{7i(dx+c)} - 480e^{6i(dx+c)} - 1180e^{4i(dx+c)} - 330e^{3i(dx+c)} - 800e^{2i(dx+c)} - 105e^{i(dx+c)})}{15d(e^{2i(dx+c)} + 1)^5}$
derivativedivides	$a^4 \tan(dx+c) + 4a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) - 6a^4 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + 4a^4 \left(-\left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \right)$
default	$a^4 \tan(dx+c) + 4a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) - 6a^4 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + 4a^4 \left(-\left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \right)$
parts	$-\frac{a^4 \left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c)}{d} + \frac{a^4 \tan(dx+c)}{d} + \frac{4a^4 \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3\sec(dx+c)}{8} \right) \right) \tan(dx+c)}{d}$
parallelrisch	$\frac{77 \left(\left(-\frac{15 \cos(dx+c)}{11} - \frac{15 \cos(3dx+3c)}{22} - \frac{3 \cos(5dx+5c)}{22} \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + \left(\frac{15 \cos(dx+c)}{11} + \frac{15 \cos(3dx+3c)}{22} + \frac{3 \cos(5dx+5c)}{22} \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) \right)}{3d(\cos(5dx+5c) + 5 \cos(3dx+3c) + 5)}$

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*I*a^4*(105*exp(9*I*(d*x+c))-30*exp(8*I*(d*x+c))+330*exp(7*I*(d*x+c))-
480*exp(6*I*(d*x+c))-1180*exp(4*I*(d*x+c))-330*exp(3*I*(d*x+c))-800*exp(2*I
*(d*x+c))-105*exp(I*(d*x+c))-166)/d/(exp(2*I*(d*x+c))+1)^5+7/2*a^4/d*ln(exp
(I*(d*x+c))+I)-7/2*a^4/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12

$$\int (a + a \cos(c + dx))^4 \sec^6(c + dx) dx$$

$$= \frac{105 a^4 \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 105 a^4 \cos(dx + c)^5 \log(-\sin(dx + c) + 1) + 2(166 a^4 \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 166 a^4 \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 68 a^4 \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 68 a^4 \cos(dx + c)^3 \log(-\sin(dx + c) + 1) + 30 a^4 \cos(dx + c)^2 \log(\sin(dx + c) + 1) - 30 a^4 \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 6 a^4 \cos(dx + c) \log(\sin(dx + c) + 1) - 6 a^4 \cos(dx + c) \log(-\sin(dx + c) + 1) + a^4 \log(\sin(dx + c) + 1) - a^4 \log(-\sin(dx + c) + 1))}{60 d \cos(dx + c)}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/60*(105*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^5
*log(-sin(d*x + c) + 1) + 2*(166*a^4*cos(d*x + c)^4 + 105*a^4*cos(d*x + c)^
3 + 68*a^4*cos(d*x + c)^2 + 30*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*c
os(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^6(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**6,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.71

$$\int (a + a \cos(c + dx))^4 \sec^6(c + dx) dx$$

$$= \frac{4(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c))a^4 + 120(\tan(dx + c)^3 + 3 \tan(dx + c))a^4 - 15a^4}{\dots}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="maxima")

[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 15*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.24

$$\int (a + a \cos(c + dx))^4 \sec^6(c + dx) dx$$

$$= \frac{105 a^4 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 105 a^4 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - \frac{2\left(105 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 - 490 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 896 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 790 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 375 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)^5}{30 d}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="giac")

[Out] 1/30*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 - 490*a^4*tan(1/2*d*x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 - 790*a^4*tan(1/2*d*x + 1/2*c)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d

Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.53

$$\int (a + a \cos(c + dx))^4 \sec^6(c + dx) dx = \frac{7 a^4 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{7 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 - \frac{98 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{3} + \frac{896 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{15} - \frac{158 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3} + 25 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1 \right)}$$

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^6,x)

[Out] (7*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((896*a^4*tan(c/2 + (d*x)/2)^5)/15 - (158*a^4*tan(c/2 + (d*x)/2)^3)/3 - (98*a^4*tan(c/2 + (d*x)/2)^7)/3 + 7*a^4*tan(c/2 + (d*x)/2)^9 + 25*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))

3.42 $\int (a + a \cos(c + dx))^4 \sec^7(c + dx) dx$

Optimal result	486
Rubi [A] (verified)	486
Mathematica [A] (verified)	489
Maple [C] (verified)	489
Fricas [A] (verification not implemented)	490
Sympy [F(-1)]	490
Maxima [B] (verification not implemented)	490
Giac [A] (verification not implemented)	491
Mupad [B] (verification not implemented)	491

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int (a + a \cos(c + dx))^4 \sec^7(c + dx) dx = \frac{49a^4 \operatorname{arctanh}(\sin(c + dx))}{16d} + \frac{8a^4 \tan(c + dx)}{d} + \frac{49a^4 \sec(c + dx) \tan(c + dx)}{16d} + \frac{41a^4 \sec^3(c + dx) \tan(c + dx)}{24d} + \frac{a^4 \sec^5(c + dx) \tan(c + dx)}{6d} + \frac{4a^4 \tan^3(c + dx)}{d} + \frac{4a^4 \tan^5(c + dx)}{5d}$$

[Out] $49/16*a^4*\operatorname{arctanh}(\sin(d*x+c))/d+8*a^4*\tan(d*x+c)/d+49/16*a^4*\sec(d*x+c)*\tan(d*x+c)/d+41/24*a^4*\sec(d*x+c)^3*\tan(d*x+c)/d+1/6*a^4*\sec(d*x+c)^5*\tan(d*x+c)/d+4*a^4*\tan(d*x+c)^3/d+4/5*a^4*\tan(d*x+c)^5/d$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

= {2836, 3853, 3855, 3852}

$$\int (a + a \cos(c + dx))^4 \sec^7(c + dx) dx = \frac{49a^4 \operatorname{arctanh}(\sin(c + dx))}{16d} + \frac{4a^4 \tan^5(c + dx)}{5d} + \frac{4a^4 \tan^3(c + dx)}{d} + \frac{8a^4 \tan(c + dx)}{d} + \frac{a^4 \tan(c + dx) \sec^5(c + dx)}{6d} + \frac{41a^4 \tan(c + dx) \sec^3(c + dx)}{24d} + \frac{49a^4 \tan(c + dx) \sec(c + dx)}{16d}$$

[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^7,x]

[Out] (49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d + (4*a^4*Tan[c + d*x]^5)/(5*d)

Rule 2836

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rule 3852

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (a^4 \sec^3(c + dx) + 4a^4 \sec^4(c + dx) + 6a^4 \sec^5(c + dx) + 4a^4 \sec^6(c + dx) \\
&\quad + a^4 \sec^7(c + dx)) dx \\
&= a^4 \int \sec^3(c + dx) dx + a^4 \int \sec^7(c + dx) dx + (4a^4) \int \sec^4(c + dx) dx \\
&\quad + (4a^4) \int \sec^6(c + dx) dx + (6a^4) \int \sec^5(c + dx) dx \\
&= \frac{a^4 \sec(c + dx) \tan(c + dx)}{2d} + \frac{3a^4 \sec^3(c + dx) \tan(c + dx)}{2d} \\
&\quad + \frac{a^4 \sec^5(c + dx) \tan(c + dx)}{6d} + \frac{1}{2} a^4 \int \sec(c + dx) dx + \frac{1}{6} (5a^4) \int \sec^5(c + dx) dx \\
&\quad + \frac{1}{2} (9a^4) \int \sec^3(c + dx) dx - \frac{(4a^4) \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d} \\
&\quad - \frac{(4a^4) \text{Subst}(\int (1 + 2x^2 + x^4) dx, x, -\tan(c + dx))}{d} \\
&= \frac{a^4 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{8a^4 \tan(c + dx)}{d} + \frac{11a^4 \sec(c + dx) \tan(c + dx)}{4d} \\
&\quad + \frac{41a^4 \sec^3(c + dx) \tan(c + dx)}{24d} + \frac{a^4 \sec^5(c + dx) \tan(c + dx)}{6d} + \frac{4a^4 \tan^3(c + dx)}{d} \\
&\quad + \frac{4a^4 \tan^5(c + dx)}{5d} + \frac{1}{8} (5a^4) \int \sec^3(c + dx) dx + \frac{1}{4} (9a^4) \int \sec(c + dx) dx \\
&= \frac{11a^4 \operatorname{arctanh}(\sin(c + dx))}{4d} + \frac{8a^4 \tan(c + dx)}{d} + \frac{49a^4 \sec(c + dx) \tan(c + dx)}{16d} \\
&\quad + \frac{41a^4 \sec^3(c + dx) \tan(c + dx)}{24d} + \frac{a^4 \sec^5(c + dx) \tan(c + dx)}{6d} \\
&\quad + \frac{4a^4 \tan^3(c + dx)}{d} + \frac{4a^4 \tan^5(c + dx)}{5d} + \frac{1}{16} (5a^4) \int \sec(c + dx) dx \\
&= \frac{49a^4 \operatorname{arctanh}(\sin(c + dx))}{16d} + \frac{8a^4 \tan(c + dx)}{d} \\
&\quad + \frac{49a^4 \sec(c + dx) \tan(c + dx)}{16d} + \frac{41a^4 \sec^3(c + dx) \tan(c + dx)}{24d} \\
&\quad + \frac{a^4 \sec^5(c + dx) \tan(c + dx)}{6d} + \frac{4a^4 \tan^3(c + dx)}{d} + \frac{4a^4 \tan^5(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.76 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx))^4 \sec^7(c + dx) dx = \frac{49a^4 \operatorname{arctanh}(\sin(c + dx))}{16d} + \frac{8a^4 \tan(c + dx)}{d} + \frac{49a^4 \sec(c + dx) \tan(c + dx)}{16d} + \frac{41a^4 \sec^3(c + dx) \tan(c + dx)}{24d} + \frac{a^4 \sec^5(c + dx) \tan(c + dx)}{6d} + \frac{4a^4 \tan^3(c + dx)}{d} + \frac{4a^4 \tan^5(c + dx)}{5d}$$

`[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^7, x]`

```
[Out] (49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d + (4*a^4*Tan[c + d*x]^5)/(5*d)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.54 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.31

method	result
risch	$\frac{ia^4(735e^{11i(dx+c)} + 3845e^{9i(dx+c)} - 1920e^{8i(dx+c)} + 3750e^{7i(dx+c)} - 11520e^{6i(dx+c)} - 3750e^{5i(dx+c)} - 15360e^{4i(dx+c)} - 3750e^{3i(dx+c)} - 1920e^{2i(dx+c)} - 375e^{i(dx+c)} - 375)}{120d(e^{2i(dx+c)} + 1)^6}$
derivativedivides	$a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) - 4a^4 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + 6a^4 \left(-\left(-\frac{\sec^3(dx+c)}{4} - 3 \right) \right)$
default	$a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) - 4a^4 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + 6a^4 \left(-\left(-\frac{\sec^3(dx+c)}{4} - 3 \right) \right)$
parallelrisch	$\frac{a^4 \left(735(-\cos(6dx+6c)) - 6\cos(4dx+4c) - 15\cos(2dx+2c) - 10 \right) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + 735(\cos(6dx+6c) + 6\cos(4dx+4c) + 15\cos(2dx+2c) + 10)}{240d(\cos(dx+c) - 1)}$
parts	$\frac{a^4 \left(-\left(-\frac{\sec^5(dx+c)}{6} - \frac{5(\sec^3(dx+c))}{24} - \frac{5\sec(dx+c)}{16} \right) \tan(dx+c) + \frac{5\ln(\sec(dx+c) + \tan(dx+c))}{16} \right)}{d} + \frac{a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} \right)}{d}$

`[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^7, x, method=_RETURNVERBOSE)`

```
[Out] -1/120*I*a^4*(735*exp(11*I*(d*x+c))+3845*exp(9*I*(d*x+c))-1920*exp(8*I*(d*x+c))+3750*exp(7*I*(d*x+c))-11520*exp(6*I*(d*x+c))-3750*exp(5*I*(d*x+c))-15360*exp(4*I*(d*x+c))-3750*exp(3*I*(d*x+c))-1920*exp(2*I*(d*x+c))-375*exp(I*(d*x+c))-375)
```

60*exp(4*I*(d*x+c))-3845*exp(3*I*(d*x+c))-6912*exp(2*I*(d*x+c))-735*exp(I*(d*x+c))-1152)/d/(exp(2*I*(d*x+c))+1)^6+49/16*a^4/d*ln(exp(I*(d*x+c))+I)-49/16*a^4/d*ln(exp(I*(d*x+c))-I)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int (a + a \cos(c + dx))^4 \sec^7(c + dx) dx$$

$$= \frac{735 a^4 \cos(dx + c)^6 \log(\sin(dx + c) + 1) - 735 a^4 \cos(dx + c)^6 \log(-\sin(dx + c) + 1) + 2(1152 a^4 \cos(dx + c)^5 + 576 a^4 \cos(dx + c)^4 + 410 a^4 \cos(dx + c)^3 + 192 a^4 \cos(dx + c)^2 + 40 a^4 \sin(dx + c))}{480 a^4 \cos(dx + c)^6}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="fricas")

[Out] 1/480*(735*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 735*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(1152*a^4*cos(d*x + c)^5 + 735*a^4*cos(d*x + c)^4 + 576*a^4*cos(d*x + c)^3 + 410*a^4*cos(d*x + c)^2 + 192*a^4*cos(d*x + c) + 40*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^7(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**7,x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(126) = 252.

Time = 0.26 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.99

$$\int (a + a \cos(c + dx))^4 \sec^7(c + dx) dx$$

$$= \frac{128(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c))a^4 + 640(\tan(dx + c)^3 + 3 \tan(dx + c))a^4 - 5}{480 a^4 \cos(dx + c)^6}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="maxima")

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 6
40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 5*a^4*(2*(15*sin(d*x + c)^5 - 40
*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*s
in(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) -
180*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*a^4
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
+ c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.13

$$\int (a + a \cos(c + dx))^4 \sec^7(c + dx) dx$$

$$= \frac{735 a^4 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 735 a^4 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - \frac{2\left(735 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{11} - 4165 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 + 9702 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 11802 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 7355 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 3105 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)}{240 d}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/240*(735*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 735*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*a^4*tan(1/2
*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*a^4*tan(1/2*d*x +
1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*a^4*tan(1/2*d*x + 1/2*c)
^2 - 1)/d
```

Mupad [B] (verification not implemented)

Time = 18.60 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

$$\int (a + a \cos(c + dx))^4 \sec^7(c + dx) dx = \frac{49 a^4 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{8 d}$$

$$- \frac{\frac{49 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11}}{8} - \frac{833 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{24} + \frac{1617 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{20} - \frac{1967 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{20} + \frac{1471 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{24} - \frac{207 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} - 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 20 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

```
[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^7,x)
```

```
[Out] (49*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((1471*a^4*tan(c/2 + (d*x)/2)^3)
/24 - (1967*a^4*tan(c/2 + (d*x)/2)^5)/20 + (1617*a^4*tan(c/2 + (d*x)/2)^7)/
20 - (833*a^4*tan(c/2 + (d*x)/2)^9)/24 + (49*a^4*tan(c/2 + (d*x)/2)^11)/8 -
(207*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 +
(d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2
+ (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.43 $\int \frac{\cos^5(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	492
Rubi [A] (verified)	492
Mathematica [A] (verified)	494
Maple [A] (verified)	494
Fricas [A] (verification not implemented)	495
Sympy [B] (verification not implemented)	495
Maxima [A] (verification not implemented)	496
Giac [A] (verification not implemented)	496
Mupad [B] (verification not implemented)	497

Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{\cos^5(c+dx)}{a+a \cos(c+dx)} dx = \frac{15x}{8a} - \frac{4 \sin(c+dx)}{ad} + \frac{15 \cos(c+dx) \sin(c+dx)}{8ad} + \frac{5 \cos^3(c+dx) \sin(c+dx)}{4ad} - \frac{\cos^4(c+dx) \sin(c+dx)}{d(a+a \cos(c+dx))} + \frac{4 \sin^3(c+dx)}{3ad}$$

[Out] 15/8*x/a-4*sin(d*x+c)/a/d+15/8*cos(d*x+c)*sin(d*x+c)/a/d+5/4*cos(d*x+c)^3*sin(d*x+c)/a/d-cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))+4/3*sin(d*x+c)^3/a/d

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2846, 2827, 2713, 2715, 8}

$$\int \frac{\cos^5(c+dx)}{a+a \cos(c+dx)} dx = \frac{4 \sin^3(c+dx)}{3ad} - \frac{4 \sin(c+dx)}{ad} - \frac{\sin(c+dx) \cos^4(c+dx)}{d(a \cos(c+dx) + a)} + \frac{5 \sin(c+dx) \cos^3(c+dx)}{4ad} + \frac{15 \sin(c+dx) \cos(c+dx)}{8ad} + \frac{15x}{8a}$$

[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x]),x]

[Out] (15*x)/(8*a) - (4*Sin[c + d*x])/(a*d) + (15*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - (Cos[c + d*x]^4*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) + (4*Sin[c + d*x]^3)/(3*a*d)

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^4(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{\int \cos^3(c + dx)(4a - 5a \cos(c + dx)) dx}{a^2} \\
 &= -\frac{\cos^4(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{4 \int \cos^3(c + dx) dx}{a} + \frac{5 \int \cos^4(c + dx) dx}{a} \\
 &= \frac{5 \cos^3(c + dx) \sin(c + dx)}{4ad} - \frac{\cos^4(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} \\
 &\quad + \frac{15 \int \cos^2(c + dx) dx}{4a} + \frac{4 \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{ad}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{4 \sin(c+dx)}{ad} + \frac{15 \cos(c+dx) \sin(c+dx)}{8ad} + \frac{5 \cos^3(c+dx) \sin(c+dx)}{4ad} \\
&\quad - \frac{\cos^4(c+dx) \sin(c+dx)}{d(a+a \cos(c+dx))} + \frac{4 \sin^3(c+dx)}{3ad} + \frac{15 \int 1 dx}{8a} \\
&= \frac{15x}{8a} - \frac{4 \sin(c+dx)}{ad} + \frac{15 \cos(c+dx) \sin(c+dx)}{8ad} \\
&\quad + \frac{5 \cos^3(c+dx) \sin(c+dx)}{4ad} - \frac{\cos^4(c+dx) \sin(c+dx)}{d(a+a \cos(c+dx))} + \frac{4 \sin^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.47

$$\int \frac{\cos^5(c+dx)}{a+a \cos(c+dx)} dx = \sec\left(\frac{c}{2}\right) \sec\left(\frac{1}{2}(c+dx)\right) \left(360dx \cos\left(\frac{dx}{2}\right) + 360dx \cos\left(c+\frac{dx}{2}\right) - 552 \sin\left(\frac{dx}{2}\right) - 168 \sin\left(c+\frac{dx}{2}\right) - 120 \sin\left(c+\frac{3dx}{2}\right) + 40 \sin\left[2c+\frac{5dx}{2}\right] + 40 \sin\left[3c+\frac{5dx}{2}\right] - 5 \sin\left[3c+\frac{7dx}{2}\right] - 5 \sin\left[4c+\frac{7dx}{2}\right] + 3 \sin\left[4c+\frac{9dx}{2}\right] + 3 \sin\left[5c+\frac{9dx}{2}\right]\right) / (384ad)$$

[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x]),x]

[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(360*d*x*Cos[(d*x)/2] + 360*d*x*Cos[c + (d*x)/2] - 552*Sin[(d*x)/2] - 168*Sin[c + (d*x)/2] - 120*Sin[c + (3*d*x)/2] - 120*Sin[2*c + (3*d*x)/2] + 40*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c + (5*d*x)/2] - 5*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + 3*Sin[4*c + (9*d*x)/2] + 3*Sin[5*c + (9*d*x)/2]))/(384*a*d)

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

method	result
parallelrisc	$\frac{180dx + \tan\left(\frac{dx}{2} + \frac{c}{2}\right) (-221 + 3 \cos(4dx+4c) - 2 \cos(3dx+3c) + 38 \cos(2dx+2c) - 82 \cos(dx+c))}{96ad}$
derivativedivides	$-\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-\frac{25 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4} - \frac{115 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12} - \frac{109 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12} - \frac{7 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4} + \frac{15 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^4 da}$
default	$-\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-\frac{25 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4} - \frac{115 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12} - \frac{109 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{12} - \frac{7 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4} + \frac{15 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^4 da}$
risc	$\frac{15x}{8a} + \frac{7ie^{i(dx+c)}}{8ad} - \frac{7ie^{-i(dx+c)}}{8ad} - \frac{2i}{da(e^{i(dx+c)}+1)} + \frac{\sin(4dx+4c)}{32ad} - \frac{\sin(3dx+3c)}{12ad} + \frac{\sin(2dx+2c)}{2ad}$
norman	$\frac{15x}{8a} - \frac{11 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4da} - \frac{95 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6da} - \frac{86 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3da} - \frac{155 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6da} - \frac{45 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4da} - \frac{\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} + \frac{75x}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^5}$

[In] `int(cos(d*x+c)^5/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)`

[Out] $1/96*(180*d*x+\tan(1/2*d*x+1/2*c))*(-221+3*\cos(4*d*x+4*c)-2*\cos(3*d*x+3*c)+38*\cos(2*d*x+2*c)-82*\cos(d*x+c))/a/d$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.67

$$\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)} dx = \frac{45 dx \cos(dx+c) + 45 dx + (6 \cos(dx+c)^4 - 2 \cos(dx+c)^3 + 13 \cos(dx+c)^2 - 19 \cos(dx+c) - 64)}{24(ad \cos(dx+c) + ad)}$$

[In] `integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="fricas")`

[Out] $1/24*(45*d*x*\cos(d*x+c) + 45*d*x + (6*\cos(d*x+c)^4 - 2*\cos(d*x+c)^3 + 13*\cos(d*x+c)^2 - 19*\cos(d*x+c) - 64)*\sin(d*x+c))/(a*d*\cos(d*x+c) + a*d)$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(102) = 204$.

Time = 1.93 (sec) , antiderivative size = 882, normalized size of antiderivative = 7.47

$$\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)} dx = \text{Too large to display}$$

[In] `integrate(cos(d*x+c)**5/(a+a*cos(d*x+c)),x)`

[Out] `Piecewise((45*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 270*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 180*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) + 45*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 24*tan(c/2 + d*x/2)**9/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 246*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d))`

6*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 374*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 314*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d) - 66*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d), Ne(d, 0)), (x*cos(c)**5/(a*cos(c) + a), True))

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.84

$$\int \frac{\cos^5(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{\frac{21 \sin(dx+c)}{\cos(dx+c)+1} + \frac{109 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{115 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{75 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a + \frac{4 a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6 a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{4 a \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{a \sin(dx+c)^8}{(\cos(dx+c)+1)^8}} - \frac{45 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{12 \sin(dx+c)}{a(\cos(dx+c)+1)}$$

12 d

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{\cos^5(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{\frac{45(dx+c)}{a} - \frac{24 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a} - \frac{2 \left(75 \tan(\frac{1}{2} dx + \frac{1}{2} c)^7 + 115 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 109 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 21 \tan(\frac{1}{2} dx + \frac{1}{2} c) \right)}{\left(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1 \right)^4 a}}{24 d}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] 1/24*(45*(d*x + c)/a - 24*tan(1/2*d*x + 1/2*c)/a - 2*(75*tan(1/2*d*x + 1/2*c)^7 + 115*tan(1/2*d*x + 1/2*c)^5 + 109*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d

Mupad [B] (verification not implemented)

Time = 16.48 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \frac{\cos^5(c + dx)}{a + a \cos(c + dx)} dx = \frac{15x}{8a} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad} - \frac{\frac{25 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{4} + \frac{115 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{12} + \frac{109 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{12} + \frac{7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4}}{ad \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)^4}$$

```
[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x)),x)
```

```
[Out] (15*x)/(8*a) - tan(c/2 + (d*x)/2)/(a*d) - ((7*tan(c/2 + (d*x)/2))/4 + (109*
tan(c/2 + (d*x)/2)^3)/12 + (115*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 + (d
*x)/2)^7)/4)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.44 $\int \frac{\cos^4(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	498
Rubi [A] (verified)	498
Mathematica [A] (verified)	500
Maple [A] (verified)	500
Fricas [A] (verification not implemented)	501
Sympy [B] (verification not implemented)	501
Maxima [A] (verification not implemented)	502
Giac [A] (verification not implemented)	502
Mupad [B] (verification not implemented)	503

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\cos^4(c+dx)}{a+a \cos(c+dx)} dx = -\frac{3x}{2a} + \frac{4 \sin(c+dx)}{ad} - \frac{3 \cos(c+dx) \sin(c+dx)}{2ad} - \frac{\cos^3(c+dx) \sin(c+dx)}{d(a+a \cos(c+dx))} - \frac{4 \sin^3(c+dx)}{3ad}$$

[Out] $-3/2*x/a+4*\sin(d*x+c)/a/d-3/2*\cos(d*x+c)*\sin(d*x+c)/a/d-\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))-4/3*\sin(d*x+c)^3/a/d$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2846, 2827, 2715, 8, 2713}

$$\int \frac{\cos^4(c+dx)}{a+a \cos(c+dx)} dx = -\frac{4 \sin^3(c+dx)}{3ad} + \frac{4 \sin(c+dx)}{ad} - \frac{\sin(c+dx) \cos^3(c+dx)}{d(a \cos(c+dx) + a)} - \frac{3 \sin(c+dx) \cos(c+dx)}{2ad} - \frac{3x}{2a}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^4/(a + a*\text{Cos}[c + d*x]),x]$

[Out] $(-3*x)/(2*a) + (4*\text{Sin}[c + d*x])/(a*d) - (3*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*a*d) - (\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(d*(a + a*\text{Cos}[c + d*x])) - (4*\text{Sin}[c + d*x]^3)/(3*a*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^3(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{\int \cos^2(c + dx)(3a - 4a \cos(c + dx)) dx}{a^2} \\
&= -\frac{\cos^3(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{3 \int \cos^2(c + dx) dx}{a} + \frac{4 \int \cos^3(c + dx) dx}{a} \\
&= -\frac{3 \cos(c + dx) \sin(c + dx)}{2ad} - \frac{\cos^3(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} \\
&\quad - \frac{3 \int 1 dx}{2a} - \frac{4 \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{ad} \\
&= -\frac{3x}{2a} + \frac{4 \sin(c + dx)}{ad} - \frac{3 \cos(c + dx) \sin(c + dx)}{2ad} - \frac{\cos^3(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{4 \sin^3(c + dx)}{3ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int \frac{\cos^4(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{\sec\left(\frac{c}{2}\right) \sec\left(\frac{1}{2}(c + dx)\right) \left(-36dx \cos\left(\frac{dx}{2}\right) - 36dx \cos\left(c + \frac{dx}{2}\right) + 69 \sin\left(\frac{dx}{2}\right) + 21 \sin\left(c + \frac{dx}{2}\right) + 18 \sin\left(c + \frac{3dx}{2}\right) - 2 \sin[2c + (5dx)/2] - 2 \sin[3c + (5dx)/2] + \sin[3c + (7dx)/2] + \sin[4c + (7dx)/2]\right)}{48ad}$$

`[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x]),x]`

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2]
+ 69*Sin[(d*x)/2] + 21*Sin[c + (d*x)/2] + 18*Sin[c + (3*d*x)/2] + 18*Sin[2*c
+ (3*d*x)/2] - 2*Sin[2*c + (5*d*x)/2] - 2*Sin[3*c + (5*d*x)/2] + Sin[3*c
+ (7*d*x)/2] + Sin[4*c + (7*d*x)/2]))/(48*a*d)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

method	result
parallelsch	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) (31 + \cos(3dx+3c) - \cos(2dx+2c) + 17 \cos(dx+c)) - 18dx}{12ad}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{8 \left(-\frac{5 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{8} - \frac{2 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{3} - \frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8} \right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3} - 3 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da}$
default	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{8 \left(-\frac{5 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{8} - \frac{2 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{3} - \frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8} \right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3} - 3 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da}$
risch	$-\frac{3x}{2a} - \frac{7ie^{i(dx+c)}}{8ad} + \frac{7ie^{-i(dx+c)}}{8ad} + \frac{2i}{da(e^{i(dx+c)}+1)} + \frac{\sin(3dx+3c)}{12ad} - \frac{\sin(2dx+2c)}{4ad}$
norman	$\frac{\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{3x}{2a} + \frac{4 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} + \frac{37 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{3da} + \frac{49 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{3da} + \frac{9 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{da} - \frac{6x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{a} - \frac{9x \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{a}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^4}$

`[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)`

```
[Out] 1/12*(tan(1/2*d*x+1/2*c)*(31+cos(3*d*x+3*c)-cos(2*d*x+2*c)+17*cos(d*x+c))-1
8*d*x)/a/d
```


Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\cos^4(c + dx)}{a + a \cos(c + dx)} dx = \frac{9 dx \cos(dx + c) + 9 dx - (2 \cos(dx + c)^3 - \cos(dx + c)^2 + 7 \cos(dx + c) + 16) \sin(dx + c)}{6(ad \cos(dx + c) + ad)}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] -1/6*(9*d*x*cos(d*x + c) + 9*d*x - (2*cos(d*x + c)^3 - cos(d*x + c)^2 + 7*cos(d*x + c) + 16)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(80) = 160.

Time = 1.22 (sec) , antiderivative size = 570, normalized size of antiderivative = 6.06

$$\int \frac{\cos^4(c + dx)}{a + a \cos(c + dx)} dx = \begin{cases} -\frac{9dx \tan^6\left(\frac{c}{2} + \frac{dx}{2}\right)}{6ad \tan^6\left(\frac{c}{2} + \frac{dx}{2}\right) + 18ad \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 18ad \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6ad} - \frac{27dx \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right)}{6ad \tan^6\left(\frac{c}{2} + \frac{dx}{2}\right) + 18ad \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 18ad \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6ad} \\ \frac{x \cos^4(c)}{a \cos(c) + a} \end{cases}$$

[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c)),x)

```
[Out] Piecewise((-9*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*d*x*tan(c/2 + d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 9*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 6*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 48*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 50*tan(c/2 + d*x/2)**3/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 24*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.87

$$\int \frac{\cos^4(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1} + \frac{16 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a + \frac{3 a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{9 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{3 \sin(dx+c)}{a(\cos(dx+c)+1)}$$

$$3d$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxima")

```
[Out] 1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{\cos^4(c + dx)}{a + a \cos(c + dx)} dx$$

$$= -\frac{\frac{9(dx+c)}{a} - \frac{6 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a} - \frac{2(15 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 16 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 9 \tan(\frac{1}{2} dx + \frac{1}{2} c))}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1)^3 a}}{6d}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac")

```
[Out] -1/6*(9*(d*x + c)/a - 6*tan(1/2*d*x + 1/2*c)/a - 2*(15*tan(1/2*d*x + 1/2*c)^5 + 16*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\cos^4(c + dx)}{a + a \cos(c + dx)} dx = \frac{\frac{15 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{8} + \frac{3 \sin\left(\frac{3c}{2} + \frac{3dx}{2}\right)}{4} - \frac{\sin\left(\frac{5c}{2} + \frac{5dx}{2}\right)}{12} + \frac{\sin\left(\frac{7c}{2} + \frac{7dx}{2}\right)}{24}}{a d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)} - \frac{3x}{2a}$$

[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x)),x)

```
[Out] ((15*sin(c/2 + (d*x)/2))/8 + (3*sin((3*c)/2 + (3*d*x)/2))/4 - sin((5*c)/2 +
(5*d*x)/2)/12 + sin((7*c)/2 + (7*d*x)/2)/24)/(a*d*cos(c/2 + (d*x)/2)) - (3
*x)/(2*a)
```

3.45 $\int \frac{\cos^3(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	504
Rubi [A] (verified)	504
Mathematica [A] (verified)	505
Maple [A] (verified)	505
Fricas [A] (verification not implemented)	506
Sympy [B] (verification not implemented)	506
Maxima [A] (verification not implemented)	507
Giac [A] (verification not implemented)	507
Mupad [B] (verification not implemented)	508

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{\cos^3(c+dx)}{a+a \cos(c+dx)} dx = \frac{3x}{2a} - \frac{2 \sin(c+dx)}{ad} + \frac{3 \cos(c+dx) \sin(c+dx)}{2ad} - \frac{\cos^2(c+dx) \sin(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] $3/2*x/a-2*\sin(d*x+c)/a/d+3/2*\cos(d*x+c)*\sin(d*x+c)/a/d-\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2846, 2813}

$$\int \frac{\cos^3(c+dx)}{a+a \cos(c+dx)} dx = -\frac{2 \sin(c+dx)}{ad} - \frac{\sin(c+dx) \cos^2(c+dx)}{d(a \cos(c+dx) + a)} + \frac{3 \sin(c+dx) \cos(c+dx)}{2ad} + \frac{3x}{2a}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^3/(a + a*\text{Cos}[c + d*x]),x]$

[Out] $(3*x)/(2*a) - (2*\text{Sin}[c + d*x])/(a*d) + (3*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*a*d) - (\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(d*(a + a*\text{Cos}[c + d*x]))$

Rule 2813

$\text{Int}[(a + b*\sin[e + f*x])*(c + d*\sin[e + f*x]), x_Symbol] \rightarrow \text{Simp}[(2*a*c + b*d)*(x/2), x] + (-\text{Simp}[(b*c + a*d)*(\text{Cos}[e + f*x]/f), x] - \text{Simp}[b*d*\text{Cos}[e + f*x]*(\text{Sin}[e + f*x]/(2*f)), x]) /;$ Free

$Q[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 2846

$\text{Int}[\left((c_{.}) + (d_{.})\sin[e_{.} + (f_{.})(x_{.})]\right)^{(n_{.})}/\left((a_{.}) + (b_{.})\sin[e_{.} + (f_{.})(x_{.})]\right), x_Symbol] \rightarrow \text{Simp}[(-b*c - a*d)*\text{Cos}[e + f*x]*\left((c + d*\text{Sin}[e + f*x])^{(n - 1)}/(a*f*(a + b*\text{Sin}[e + f*x]))\right), x] - \text{Dist}[d/(a*b), \text{Int}[(c + d*\text{Sin}[e + f*x])^{(n - 2)}*\text{Simp}[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*\text{Sin}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ (\text{IntegerQ}[2*n] \ || \ \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\cos^2(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{\int \cos(c + dx)(2a - 3a \cos(c + dx)) dx}{a^2} \\ &= \frac{3x}{2a} - \frac{2 \sin(c + dx)}{ad} + \frac{3 \cos(c + dx) \sin(c + dx)}{2ad} - \frac{\cos^2(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

$$\int \frac{\cos^3(c + dx)}{a + a \cos(c + dx)} dx = \frac{\sec\left(\frac{c}{2}\right) \sec\left(\frac{1}{2}(c + dx)\right) \left(12dx \cos\left(\frac{dx}{2}\right) + 12dx \cos\left(c + \frac{dx}{2}\right) - 20 \sin\left(\frac{dx}{2}\right) - 4 \sin\left(c + \frac{dx}{2}\right) - 3 \sin\left(c + \frac{3dx}{2}\right)\right)}{16ad}$$

[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x]),x]

[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(12*d*x*Cos[(d*x)/2] + 12*d*x*Cos[c + (d*x)/2] - 20*Sin[(d*x)/2] - 4*Sin[c + (d*x)/2] - 3*Sin[c + (3*d*x)/2] - 3*Sin[2*c + (3*d*x)/2] + Sin[2*c + (5*d*x)/2] + Sin[3*c + (5*d*x)/2])/(16*a*d)

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

method	result
parallelrisc	$\frac{6dx + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)(-7 + \cos(2dx + 2c) - 2\cos(dx + c))}{4ad}$
derivativedivides	$-\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-3\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 3\arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^2}$
default	$\frac{-\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-3\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 3\arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da}$
risc	$\frac{3x}{2a} + \frac{ie^{i(dx+c)}}{2ad} - \frac{ie^{-i(dx+c)}}{2ad} - \frac{2i}{da(e^{i(dx+c)}+1)} + \frac{\sin(2dx+2c)}{4ad}$
norman	$\frac{\frac{3x}{2a} - \frac{2\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} - \frac{7\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da} - \frac{6\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da} - \frac{\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} + \frac{9x\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2a} + \frac{9x\left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2a} + \frac{3x\left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2a}}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^3}$

[In] `int(cos(d*x+c)^3/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)`

[Out] $1/4*(6*d*x + \tan(1/2*d*x + 1/2*c)*(-7 + \cos(2*d*x + 2*c) - 2*\cos(d*x + c)))/a/d$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \frac{\cos^3(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{3 dx \cos(dx + c) + 3 dx + (\cos(dx + c))^2 - \cos(dx + c) - 4) \sin(dx + c)}{2(ad \cos(dx + c) + ad)}$$

[In] `integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fricas")`

[Out] $1/2*(3*d*x*\cos(d*x + c) + 3*d*x + (\cos(d*x + c)^2 - \cos(d*x + c) - 4)*\sin(d*x + c))/(a*d*\cos(d*x + c) + a*d)$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(65) = 130$.

Time = 0.82 (sec) , antiderivative size = 325, normalized size of antiderivative = 4.28

$$\int \frac{\cos^3(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \left\{ \begin{array}{l} \frac{3dx \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right)}{2ad \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 4ad \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 2ad} + \frac{6dx \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{2ad \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 4ad \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 2ad} + \frac{3dx}{2ad \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 4ad \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 2ad} - \frac{x \cos^3(c)}{a \cos(c) + a} \end{array} \right.$$

[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c)),x)

[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 6*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*d*x/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*tan(c/2 + d*x/2)**5/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 10*tan(c/2 + d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 4*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a), True))

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.75

$$\int \frac{\cos^3(c + dx)}{a + a \cos(c + dx)} dx = -\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1} + \frac{3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a + \frac{2a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} - \frac{3 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(c + dx)}{a + a \cos(c + dx)} dx = \frac{\frac{3(dx+c)}{a} - \frac{2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a} - \frac{2 \left(3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + \tan(\frac{1}{2} dx + \frac{1}{2} c)\right)}{\left(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1\right)^2 a}}{2d}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] 1/2*(3*(d*x + c)/a - 2*tan(1/2*d*x + 1/2*c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2*a)/d

Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int \frac{\cos^3(c + dx)}{a + a \cos(c + dx)} dx =$$

$$-\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) - \frac{3 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)(c+dx)}{2} + 3 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 2 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{a d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)}$$

[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x)),x)

[Out] -(sin(c/2 + (d*x)/2) - (3*cos(c/2 + (d*x)/2)*(c + d*x))/2 + 3*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(a*d*cos(c/2 + (d*x)/2))

3.46 $\int \frac{\cos^2(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	509
Rubi [A] (verified)	509
Mathematica [B] (verified)	510
Maple [A] (verified)	511
Fricas [A] (verification not implemented)	511
Sympy [B] (verification not implemented)	511
Maxima [B] (verification not implemented)	512
Giac [A] (verification not implemented)	512
Mupad [B] (verification not implemented)	513

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\cos^2(c+dx)}{a+a \cos(c+dx)} dx = -\frac{x}{a} + \frac{\sin(c+dx)}{ad} + \frac{\sin(c+dx)}{ad(1+\cos(c+dx))}$$

[Out] $-x/a + \sin(d*x+c)/a/d + \sin(d*x+c)/a/d/(1+\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2825, 12, 2814, 2727}

$$\int \frac{\cos^2(c+dx)}{a+a \cos(c+dx)} dx = \frac{\sin(c+dx)}{ad} + \frac{\sin(c+dx)}{ad(\cos(c+dx)+1)} - \frac{x}{a}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^2/(a + a*\text{Cos}[c + d*x]), x]$

[Out] $-(x/a) + \text{Sin}[c + d*x]/(a*d) + \text{Sin}[c + d*x]/(a*d*(1 + \text{Cos}[c + d*x]))$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match}[\text{Q}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]]$

Rule 2727

$\text{Int}[(a_*) + (b_*)*\text{sin}[(c_*) + (d_*)*(x_)]^{(-1)}, x_Symbol] \rightarrow \text{Simp}[-\text{Cos}[c + d*x]/(d*(b + a*\text{Sin}[c + d*x])), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)
)*(x_)], x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sin(c + dx)}{ad} - \frac{\int \frac{a \cos(c+dx)}{a+a \cos(c+dx)} dx}{a} \\
&= \frac{\sin(c + dx)}{ad} - \int \frac{\cos(c + dx)}{a + a \cos(c + dx)} dx \\
&= -\frac{x}{a} + \frac{\sin(c + dx)}{ad} + \int \frac{1}{a + a \cos(c + dx)} dx \\
&= -\frac{x}{a} + \frac{\sin(c + dx)}{ad} + \frac{\sin(c + dx)}{d(a + a \cos(c + dx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(43) = 86.

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.07

$$\begin{aligned}
&\int \frac{\cos^2(c + dx)}{a + a \cos(c + dx)} dx \\
&= \frac{\sec\left(\frac{c}{2}\right) \sec\left(\frac{1}{2}(c + dx)\right) \left(-2dx \cos\left(\frac{dx}{2}\right) - 2dx \cos\left(c + \frac{dx}{2}\right) + 5 \sin\left(\frac{dx}{2}\right) + \sin\left(c + \frac{dx}{2}\right) + \sin\left(c + \frac{3dx}{2}\right) + \sin\left(2c + \frac{3dx}{2}\right)\right)}{4ad}
\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-2*d*x*Cos[(d*x)/2] - 2*d*x*Cos[c + (d*x)/2] +
5*Sin[(d*x)/2] + Sin[c + (d*x)/2] + Sin[c + (3*d*x)/2] + Sin[2*c + (3*d*x)/
2]))/(4*a*d)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{-dx + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)(\cos(dx+c)+2)}{ad}$	31
derivativedivides	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} - 2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da}$	56
default	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} - 2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da}$	56
risch	$-\frac{x}{a} - \frac{ie^{i(dx+c)}}{2ad} + \frac{ie^{-i(dx+c)}}{2ad} + \frac{2i}{da(e^{i(dx+c)}+1)}$	66
norman	$\frac{\frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} - \frac{x}{a} + \frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} + \frac{4(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right))}{da} - \frac{2x(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))}{a} - \frac{x(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right))}{a}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}$	112

```
[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*x+tan(1/2*d*x+1/2*c)*(cos(d*x+c)+2))/a/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{\cos^2(c + dx)}{a + a \cos(c + dx)} dx = -\frac{dx \cos(dx + c) + dx - (\cos(dx + c) + 2) \sin(dx + c)}{ad \cos(dx + c) + ad}$$

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) + d*x - (cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(31) = 62.

Time = 0.56 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.00

$$\int \frac{\cos^2(c + dx)}{a + a \cos(c + dx)} dx = \begin{cases} -\frac{dx \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + ad} - \frac{dx}{ad \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + ad} + \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + ad} + \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + ad} & \text{for } d \neq 0 \\ \frac{x \cos^2(c)}{a \cos(c) + a} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c)),x)

[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) - d*x/(a*d*tan(c/2 + d*x/2)**2 + a*d) + tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**2 + a*d) + 3*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a), True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(43) = 86.

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \frac{\cos^2(c + dx)}{a + a \cos(c + dx)} dx = -\frac{\frac{2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{2 \sin(dx+c)}{\left(a + \frac{a \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{\cos^2(c + dx)}{a + a \cos(c + dx)} dx = -\frac{\frac{dx+c}{a} - \frac{\tan(\frac{1}{2} dx + \frac{1}{2} c)}{a} - \frac{2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1)a}}{d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] -((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d

Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{\cos^2(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + (-c - dx) \cos\left(\frac{c}{2} + \frac{dx}{2}\right) + \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{a d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)}$$

[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x)),x)

[Out] (sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)*(c + d*x) + 2*cos(c/2 + (d*x)/2)^2 *sin(c/2 + (d*x)/2))/(a*d*cos(c/2 + (d*x)/2))

3.47 $\int \frac{\cos(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	514
Rubi [A] (verified)	514
Mathematica [B] (verified)	515
Maple [A] (verified)	515
Fricas [A] (verification not implemented)	516
Sympy [A] (verification not implemented)	516
Maxima [A] (verification not implemented)	516
Giac [A] (verification not implemented)	517
Mupad [B] (verification not implemented)	517

Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{\cos(c+dx)}{a+a \cos(c+dx)} dx = \frac{x}{a} - \frac{\sin(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] x/a-sin(d*x+c)/d/(a+a*cos(d*x+c))

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2814, 2727}

$$\int \frac{\cos(c+dx)}{a+a \cos(c+dx)} dx = \frac{x}{a} - \frac{\sin(c+dx)}{d(a \cos(c+dx) + a)}$$

[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x]),x]

[Out] x/a - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{x}{a} - \int \frac{1}{a + a \cos(c + dx)} dx \\ &= \frac{x}{a} - \frac{\sin(c + dx)}{d(a + a \cos(c + dx))} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.38

$$\begin{aligned} &\int \frac{\cos(c + dx)}{a + a \cos(c + dx)} dx \\ &= -\frac{\sin(c + dx) \left(\arcsin(\cos(c + dx))(1 + \cos(c + dx)) + \sqrt{\sin^2(c + dx)} \right)}{ad\sqrt{1 - \cos(c + dx)}(1 + \cos(c + dx))^{3/2}} \end{aligned}$$

[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x]),x]

[Out] -((Sin[c + d*x]*(ArcSin[Cos[c + d*x]]*(1 + Cos[c + d*x]) + Sqrt[Sin[c + d*x]^2]))/(a*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(3/2)))

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
parallelrisc	$\frac{dx - \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad}$	23
risc	$\frac{x}{a} - \frac{2i}{da(e^{i(dx+c)}+1)}$	29
derivativedivides	$\frac{-\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da}$	32
default	$\frac{-\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da}$	32
norman	$\frac{\frac{x}{a} + \frac{x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a} - \frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} - \frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{da}}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}$	75

[In] int(cos(d*x+c)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] (d*x-tan(1/2*d*x+1/2*c))/a/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{\cos(c + dx)}{a + a \cos(c + dx)} dx = \frac{dx \cos(dx + c) + dx - \sin(dx + c)}{ad \cos(dx + c) + ad}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] (d*x*cos(d*x + c) + d*x - sin(d*x + c))/(a*d*cos(d*x + c) + a*d)

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\cos(c + dx)}{a + a \cos(c + dx)} dx = \begin{cases} \frac{x}{a} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad} & \text{for } d \neq 0 \\ \frac{x \cos(c)}{a \cos(c) + a} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x)

[Out] Piecewise((x/a - tan(c/2 + d*x/2)/(a*d), Ne(d, 0)), (x*cos(c)/(a*cos(c) + a), True))

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int \frac{\cos(c + dx)}{a + a \cos(c + dx)} dx = \frac{\frac{2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] (2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c + dx)}{a + a \cos(c + dx)} dx = \frac{\frac{dx+c}{a} - \frac{\tan(\frac{1}{2} dx + \frac{1}{2} c)}{a}}{d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] ((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a)/d

Mupad [B] (verification not implemented)

Time = 14.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\cos(c + dx)}{a + a \cos(c + dx)} dx = \frac{x}{a} - \frac{\tan(\frac{c}{2} + \frac{dx}{2})}{a d}$$

[In] int(cos(c + d*x)/(a + a*cos(c + d*x)),x)

[Out] x/a - tan(c/2 + (d*x)/2)/(a*d)

3.48 $\int \frac{1}{a+a \cos(c+dx)} dx$

Optimal result	518
Rubi [A] (verified)	518
Mathematica [A] (verified)	519
Maple [A] (verified)	519
Fricas [A] (verification not implemented)	519
Sympy [A] (verification not implemented)	520
Maxima [A] (verification not implemented)	520
Giac [A] (verification not implemented)	520
Mupad [B] (verification not implemented)	521

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{1}{a+a \cos(c+dx)} dx = \frac{\sin(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] `sin(d*x+c)/d/(a+a*cos(d*x+c))`

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2727}

$$\int \frac{1}{a+a \cos(c+dx)} dx = \frac{\sin(c+dx)}{d(a \cos(c+dx)+a)}$$

[In] `Int[(a + a*Cos[c + d*x])^(-1), x]`

[Out] `Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))`

Rule 2727

`Int[((a_) + (b_.)*sin[(c_) + (d_.)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rubi steps

$$\text{integral} = \frac{\sin(c+dx)}{d(a+a \cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{a + a \cos(c + dx)} dx = \frac{\tan\left(\frac{1}{2}(c + dx)\right)}{ad}$$

[In] Integrate[(a + a*Cos[c + d*x])^(-1),x]

[Out] Tan[(c + d*x)/2]/(a*d)

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da}$	17
default	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da}$	17
norman	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da}$	17
parallelrisc	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da}$	17
risch	$\frac{2i}{da(e^{i(dx+c)}+1)}$	23

[In] int(1/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] 1/d/a*tan(1/2*d*x+1/2*c)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + a \cos(c + dx)} dx = \frac{\sin(dx + c)}{ad \cos(dx + c) + ad}$$

[In] integrate(1/(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] sin(d*x + c)/(a*d*cos(d*x + c) + a*d)

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{a + a \cos(c + dx)} dx = \begin{cases} \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad} & \text{for } d \neq 0 \\ \frac{x}{a \cos(c) + a} & \text{otherwise} \end{cases}$$

[In] integrate(1/(a+a*cos(d*x+c)),x)

[Out] Piecewise((tan(c/2 + d*x/2)/(a*d), Ne(d, 0)), (x/(a*cos(c) + a), True))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{a + a \cos(c + dx)} dx = \frac{\sin(dx + c)}{ad(\cos(dx + c) + 1)}$$

[In] integrate(1/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] sin(d*x + c)/(a*d*(cos(d*x + c) + 1))

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{a + a \cos(c + dx)} dx = \frac{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{ad}$$

[In] integrate(1/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] tan(1/2*d*x + 1/2*c)/(a*d)

Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{a + a \cos(c + dx)} dx = \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{a d}$$

[In] int(1/(a + a*cos(c + d*x)),x)

[Out] tan(c/2 + (d*x)/2)/(a*d)

3.49 $\int \frac{\sec(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	522
Rubi [A] (verified)	522
Mathematica [B] (verified)	523
Maple [A] (verified)	523
Fricas [A] (verification not implemented)	524
Sympy [F]	524
Maxima [A] (verification not implemented)	525
Giac [A] (verification not implemented)	525
Mupad [B] (verification not implemented)	525

Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \frac{\sec(c+dx)}{a+a \cos(c+dx)} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] $\operatorname{arctanh}(\sin(d*x+c))/a/d - \sin(d*x+c)/d/(a+a*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2826, 3855, 2727}

$$\int \frac{\sec(c+dx)}{a+a \cos(c+dx)} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{d(a \cos(c+dx) + a)}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c + d*x]/(a + a*\operatorname{Cos}[c + d*x]), x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]]/(a*d) - \operatorname{Sin}[c + d*x]/(d*(a + a*\operatorname{Cos}[c + d*x]))$

Rule 2727

$\operatorname{Int}[(a + b*\sin[(c + d)*(x)])^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[-\operatorname{Cos}[c + d*x]/(d*(b + a*\operatorname{Sin}[c + d*x])), x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x$ && $\operatorname{EqQ}[a^2 - b^2, 0]$

Rule 2826

$\operatorname{Int}[1/((a + b*\sin[(e + f)*(x)])*((c + d)*\sin[(e + f)*(x)])), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[b/(b*c - a*d), \operatorname{Int}[1/(a + b*\operatorname{Sin}[e + f*x]),$

`x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \sec(c + dx) dx}{a} - \int \frac{1}{a + a \cos(c + dx)} dx \\ &= \frac{\operatorname{arctanh}(\sin(c + dx))}{ad} - \frac{\sin(c + dx)}{d(a + a \cos(c + dx))} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 103 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.71

$$\int \frac{\sec(c + dx)}{a + a \cos(c + dx)} dx = \frac{2 \cos\left(\frac{1}{2}(c + dx)\right) \left(\cos\left(\frac{1}{2}(c + dx)\right) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) + \sin\left(\frac{1}{2}(c + dx)\right)}{ad(1 + \cos(c + dx))}$$

`[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x]),x]`

`[Out] (-2*Cos[(c + d*x)/2]*(Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])) + Sec[c/2]*Sin[(d*x)/2]) / (a*d*(1 + Cos[c + d*x]))`

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$\frac{-\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da}$	46
default	$\frac{-\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da}$	46
parallelrisc	$\frac{-\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da}$	46
norman	$-\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{ad} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{ad}$	58
risc	$-\frac{2i}{da(e^{i(dx+c)}+1)} - \frac{\ln(e^{i(dx+c)}-i)}{da} + \frac{\ln(e^{i(dx+c)}+i)}{ad}$	65

[In] `int(sec(d*x+c)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)`

[Out] `1/d/a*(-tan(1/2*d*x+1/2*c)-ln(tan(1/2*d*x+1/2*c)-1)+ln(tan(1/2*d*x+1/2*c)+1))`

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{\sec(c+dx)}{a+a\cos(c+dx)} dx$$

$$= \frac{(\cos(dx+c)+1)\log(\sin(dx+c)+1) - (\cos(dx+c)+1)\log(-\sin(dx+c)+1) - 2\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

[In] `integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

[Out] `1/2*((cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - (cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)`

Sympy [F]

$$\int \frac{\sec(c+dx)}{a+a\cos(c+dx)} dx = \frac{\int \frac{\sec(c+dx)}{\cos(c+dx)+1} dx}{a}$$

[In] `integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x)`

[Out] `Integral(sec(c + d*x)/(cos(c + d*x) + 1), x)/a`

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

$$\int \frac{\sec(c + dx)}{a + a \cos(c + dx)} dx = \frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{\sec(c + dx)}{a + a \cos(c + dx)} dx = \frac{\frac{\log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a} - \frac{\log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a} - \frac{\tan(\frac{1}{2} dx + \frac{1}{2} c)}{a}}{d}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))/a - tan(1/2*d*x + 1/2*c)/a)/d

Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sec(c + dx)}{a + a \cos(c + dx)} dx = \frac{2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{a d}$$

[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))),x)

[Out] (2*atanh(tan(c/2 + (d*x)/2)) - tan(c/2 + (d*x)/2))/(a*d)

3.50 $\int \frac{\sec^2(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	526
Rubi [A] (verified)	526
Mathematica [B] (verified)	527
Maple [A] (verified)	528
Fricas [A] (verification not implemented)	528
Sympy [F]	529
Maxima [B] (verification not implemented)	529
Giac [A] (verification not implemented)	529
Mupad [B] (verification not implemented)	530

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\sec^2(c+dx)}{a+a \cos(c+dx)} dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{ad} + \frac{2 \tan(c+dx)}{ad} - \frac{\tan(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] $-\operatorname{arctanh}(\sin(d*x+c))/a/d+2*\tan(d*x+c)/a/d-\tan(d*x+c)/d/(a+a*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2847, 2827, 3852, 8, 3855}

$$\int \frac{\sec^2(c+dx)}{a+a \cos(c+dx)} dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{ad} + \frac{2 \tan(c+dx)}{ad} - \frac{\tan(c+dx)}{d(a \cos(c+dx) + a)}$$

[In] $\text{Int}[\text{Sec}[c + d*x]^2/(a + a*\text{Cos}[c + d*x]),x]$

[Out] $-(\text{ArcTanh}[\text{Sin}[c + d*x]]/(a*d)) + (2*\text{Tan}[c + d*x])/(a*d) - \text{Tan}[c + d*x]/(d*(a + a*\text{Cos}[c + d*x]))$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2827

$\text{Int}[(b_*\sin[(e_*) + (f_*)*(x_*)])^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)])], x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d)), Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\tan(c + dx)}{d(a + a \cos(c + dx))} - \frac{\int (-2a + a \cos(c + dx)) \sec^2(c + dx) dx}{a^2} \\
&= -\frac{\tan(c + dx)}{d(a + a \cos(c + dx))} - \frac{\int \sec(c + dx) dx}{a} + \frac{2 \int \sec^2(c + dx) dx}{a} \\
&= -\frac{\operatorname{arctanh}(\sin(c + dx))}{ad} - \frac{\tan(c + dx)}{d(a + a \cos(c + dx))} - \frac{2 \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{ad} \\
&= -\frac{\operatorname{arctanh}(\sin(c + dx))}{ad} + \frac{2 \tan(c + dx)}{ad} - \frac{\tan(c + dx)}{d(a + a \cos(c + dx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 188 vs. 2(53) = 106.

Time = 0.78 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.55

$$\begin{aligned}
&\int \frac{\sec^2(c + dx)}{a + a \cos(c + dx)} dx \\
&= \frac{2 \cos\left(\frac{1}{2}(c + dx)\right) \left(\sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) + \cos\left(\frac{1}{2}(c + dx)\right) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)\right)}{ad(1 + \cos(c + dx))}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x]), x]

```
[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1} + \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) - \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1} - \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da}$	74
default	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1} + \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) - \frac{1}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1} - \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da}$	74
parallelrisch	$\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \cos(dx+c) - \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) \cos(dx+c) + 2 \cos(dx+c) \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{ad \cos(dx+c)}$	82
norman	$\frac{\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} - \frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da}}{\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{ad} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{ad}$	93
risch	$\frac{2i(e^{2i(dx+c)} + e^{i(dx+c)} + 2)}{da(e^{i(dx+c)} + 1)(e^{2i(dx+c)} + 1)} + \frac{\ln(e^{i(dx+c)} - i)}{da} - \frac{\ln(e^{i(dx+c)} + i)}{ad}$	98

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(tan(1/2*d*x+1/2*c)-1/(tan(1/2*d*x+1/2*c)-1)+ln(tan(1/2*d*x+1/2*c)-1)-1/(tan(1/2*d*x+1/2*c)+1)-ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.83

$$\int \frac{\sec^2(c + dx)}{a + a \cos(c + dx)} dx = \frac{(\cos(dx + c))^2 + \cos(dx + c) \log(\sin(dx + c) + 1) - (\cos(dx + c))^2 + \cos(dx + c) \log(-\sin(dx + c) + 1)}{2(ad \cos(dx + c))^2 + ad \cos(dx + c)}$$

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - (cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) + 1)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

SymPy [F]

$$\int \frac{\sec^2(c + dx)}{a + a \cos(c + dx)} dx = \frac{\int \frac{\sec^2(c+dx)}{\cos(c+dx)+1} dx}{a}$$

[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c)),x)

[Out] Integral(sec(c + d*x)**2/(cos(c + d*x) + 1), x)/a

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(53) = 106.

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \frac{\sec^2(c + dx)}{a + a \cos(c + dx)} dx$$

$$= -\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{2 \sin(dx+c)}{\left(a - \frac{a \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] -(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int \frac{\sec^2(c + dx)}{a + a \cos(c + dx)} dx$$

$$= -\frac{\frac{\log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a} - \frac{\log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a} - \frac{\tan(\frac{1}{2} dx + \frac{1}{2} c)}{a} + \frac{2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)a}}{d}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))/a - tan(1/2*d*x + 1/2*c)/a + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d

Mupad [B] (verification not implemented)

Time = 14.80 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \frac{\sec^2(c + dx)}{a + a \cos(c + dx)} dx = \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a - a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2\right)} - \frac{2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{ad} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{ad}$$

[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))),x)

[Out] (2*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) - (2*atanh(tan(c/2 + (d*x)/2)))/(a*d) + tan(c/2 + (d*x)/2)/(a*d)

3.51 $\int \frac{\sec^3(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	531
Rubi [A] (verified)	531
Mathematica [B] (verified)	533
Maple [A] (verified)	533
Fricas [A] (verification not implemented)	534
Sympy [F]	534
Maxima [B] (verification not implemented)	534
Giac [A] (verification not implemented)	535
Mupad [B] (verification not implemented)	535

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\sec^3(c+dx)}{a+a \cos(c+dx)} dx = \frac{3 \operatorname{arctanh}(\sin(c+dx))}{2ad} - \frac{2 \tan(c+dx)}{ad} + \frac{3 \sec(c+dx) \tan(c+dx)}{2ad} - \frac{\sec(c+dx) \tan(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] $3/2*\operatorname{arctanh}(\sin(d*x+c))/a/d-2*\tan(d*x+c)/a/d+3/2*\sec(d*x+c)*\tan(d*x+c)/a/d-\sec(d*x+c)*\tan(d*x+c)/d/(a+a*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2847, 2827, 3853, 3855, 3852, 8}

$$\int \frac{\sec^3(c+dx)}{a+a \cos(c+dx)} dx = \frac{3 \operatorname{arctanh}(\sin(c+dx))}{2ad} - \frac{2 \tan(c+dx)}{ad} + \frac{3 \tan(c+dx) \sec(c+dx)}{2ad} - \frac{\tan(c+dx) \sec(c+dx)}{d(a \cos(c+dx) + a)}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]^3/(a+a*\operatorname{Cos}[c+d*x]),x]$

[Out] $(3*\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]])/(2*a*d) - (2*\operatorname{Tan}[c+d*x])/(a*d) + (3*\operatorname{Sec}[c+d*x]*\operatorname{Tan}[c+d*x])/(2*a*d) - (\operatorname{Sec}[c+d*x]*\operatorname{Tan}[c+d*x])/(d*(a+a*\operatorname{Cos}[c+d*x]))$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 2847

`Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d)), Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3853

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sec(c + dx) \tan(c + dx)}{d(a + a \cos(c + dx))} - \frac{\int (-3a + 2a \cos(c + dx)) \sec^3(c + dx) dx}{a^2} \\
 &= -\frac{\sec(c + dx) \tan(c + dx)}{d(a + a \cos(c + dx))} - \frac{2 \int \sec^2(c + dx) dx}{a} + \frac{3 \int \sec^3(c + dx) dx}{a} \\
 &= \frac{3 \sec(c + dx) \tan(c + dx)}{2ad} - \frac{\sec(c + dx) \tan(c + dx)}{d(a + a \cos(c + dx))} \\
 &\quad + \frac{3 \int \sec(c + dx) dx}{2a} + \frac{2 \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{ad}
 \end{aligned}$$

$$= \frac{3a \operatorname{arctanh}(\sin(c+dx))}{2ad} - \frac{2 \tan(c+dx)}{ad} + \frac{3 \sec(c+dx) \tan(c+dx)}{2ad} - \frac{\sec(c+dx) \tan(c+dx)}{d(a+a \cos(c+dx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(83) = 166.

Time = 1.17 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.94

$$\int \frac{\sec^3(c+dx)}{a+a \cos(c+dx)} dx$$

$$\cos\left(\frac{1}{2}(c+dx)\right) \left(-4 \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) + \cos\left(\frac{1}{2}(c+dx)\right) \left(-6 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 6 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)$$

[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x]), x]

[Out] (Cos[(c + d*x)/2]*(-4*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(-6*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2) - (4*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]))*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(2*a*d*(1 + Cos[c + d*x]))

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{(-3 \cos(2dx+2c)-3) \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(3 \cos(2dx+2c)+3) \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-2 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)(1+2 \cos(2dx+2c))}{2ad(1+\cos(2dx+2c))}$
derivativedivides	$-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$
default	$-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{3 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$
norman	$-\frac{2 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{3 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{3 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$
risch	$-\frac{i\left(3 e^{4i(dx+c)}+3 e^{3i(dx+c)}+5 e^{2i(dx+c)}+e^{i(dx+c)}+4\right)}{da\left(e^{2i(dx+c)}+1\right)^2\left(e^{i(dx+c)}+1\right)}+\frac{3 \ln\left(e^{i(dx+c)}+i\right)}{2ad}-\frac{3 \ln\left(e^{i(dx+c)}-i\right)}{2da}$

[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a), x, method=_RETURNVERBOSE)

[Out] $\frac{1}{2} * ((-3 * \cos(2 * d * x + 2 * c) - 3) * \ln(\tan(1/2 * d * x + 1/2 * c) - 1) + (3 * \cos(2 * d * x + 2 * c) + 3) * \ln(\tan(1/2 * d * x + 1/2 * c) + 1) - 2 * \tan(1/2 * d * x + 1/2 * c) * (1 + 2 * \cos(2 * d * x + 2 * c) + \cos(d * x + c))) / a / d / (1 + \cos(2 * d * x + 2 * c))$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{3 (\cos(dx + c)^3 + \cos(dx + c)^2) \log(\sin(dx + c) + 1) - 3 (\cos(dx + c)^3 + \cos(dx + c)^2) \log(-\sin(dx + c) + 1) - 2 * (4 * \cos(dx + c)^2 + \cos(dx + c) - 1) * \sin(dx + c)}{4 (ad \cos(dx + c)^3 + ad \cos(dx + c)^2)}$$

[In] `integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fricas")`

[Out] $\frac{1}{4} * (3 * (\cos(d * x + c)^3 + \cos(d * x + c)^2) * \log(\sin(d * x + c) + 1) - 3 * (\cos(d * x + c)^3 + \cos(d * x + c)^2) * \log(-\sin(d * x + c) + 1) - 2 * (4 * \cos(d * x + c)^2 + \cos(d * x + c) - 1) * \sin(d * x + c)) / (a * d * \cos(d * x + c)^3 + a * d * \cos(d * x + c)^2)$

Sympy [F]

$$\int \frac{\sec^3(c + dx)}{a + a \cos(c + dx)} dx = \frac{\int \frac{\sec^3(c + dx)}{\cos(c + dx) + 1} dx}{a}$$

[In] `integrate(sec(d*x+c)**3/(a+a*cos(d*x+c)),x)`

[Out] `Integral(sec(c + d*x)**3/(cos(c + d*x) + 1), x)/a`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(79) = 158$.

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

$$\int \frac{\sec^3(c + dx)}{a + a \cos(c + dx)} dx$$

$$= -\frac{2 \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - \frac{3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) - \frac{3 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a} + \frac{3 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a} + \frac{2 \sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

[In] `integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxima")`

[Out] $-1/2*(2*(\sin(dx + c)/(\cos(dx + c) + 1) - 3*\sin(dx + c)^3/(\cos(dx + c) + 1)^3)/(a - 2*a*\sin(dx + c)^2/(\cos(dx + c) + 1)^2 + a*\sin(dx + c)^4/(\cos(dx + c) + 1)^4) - 3*\log(\sin(dx + c)/(\cos(dx + c) + 1) + 1)/a + 3*\log(\sin(dx + c)/(\cos(dx + c) + 1) - 1)/a + 2*\sin(dx + c)/(a*(\cos(dx + c) + 1)))/d$

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

$$\int \frac{\sec^3(c + dx)}{a + a \cos(c + dx)} dx = \frac{3 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a} - \frac{3 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a} - \frac{2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a} + \frac{2 (3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - \tan(\frac{1}{2} dx + \frac{1}{2} c))}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^2 a}$$

= $\frac{\quad}{2d}$

[In] `integrate(sec(dx+c)^3/(a+a*cos(dx+c)),x, algorithm="giac")`

[Out] $1/2*(3*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)))/a - 3*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/a - 2*\tan(1/2*d*x + 1/2*c)/a + 2*(3*\tan(1/2*d*x + 1/2*c)^3 - \tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 - 1)^2*a)/d$

Mupad [B] (verification not implemented)

Time = 14.76 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{\sec^3(c + dx)}{a + a \cos(c + dx)} dx = \frac{3 \operatorname{atanh}(\tan(\frac{c}{2} + \frac{dx}{2}))}{a d} - \frac{\tan(\frac{c}{2} + \frac{dx}{2})}{a d} - \frac{\tan(\frac{c}{2} + \frac{dx}{2}) - 3 \tan(\frac{c}{2} + \frac{dx}{2})^3}{d (a \tan(\frac{c}{2} + \frac{dx}{2})^4 - 2 a \tan(\frac{c}{2} + \frac{dx}{2})^2 + a)}$$

[In] `int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))),x)`

[Out] $(3*\operatorname{atanh}(\tan(c/2 + (d*x)/2)))/(a*d) - \tan(c/2 + (d*x)/2)/(a*d) - (\tan(c/2 + (d*x)/2) - 3*\tan(c/2 + (d*x)/2)^3)/(d*(a - 2*a*\tan(c/2 + (d*x)/2)^2 + a*\tan(c/2 + (d*x)/2)^4)$

3.52 $\int \frac{\sec^4(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	536
Rubi [A] (verified)	536
Mathematica [B] (verified)	538
Maple [A] (verified)	538
Fricas [A] (verification not implemented)	539
Sympy [F]	539
Maxima [B] (verification not implemented)	539
Giac [A] (verification not implemented)	540
Mupad [B] (verification not implemented)	540

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\sec^4(c+dx)}{a+a \cos(c+dx)} dx = -\frac{3\arctanh(\sin(c+dx))}{2ad} + \frac{4 \tan(c+dx)}{ad} - \frac{3 \sec(c+dx) \tan(c+dx)}{2ad} - \frac{\sec^2(c+dx) \tan(c+dx)}{d(a+a \cos(c+dx))} + \frac{4 \tan^3(c+dx)}{3ad}$$

[Out] $-3/2*\arctanh(\sin(d*x+c))/a/d+4*\tan(d*x+c)/a/d-3/2*\sec(d*x+c)*\tan(d*x+c)/a/d - \sec(d*x+c)^2*\tan(d*x+c)/d/(a+a*\cos(d*x+c))+4/3*\tan(d*x+c)^3/a/d$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2847, 2827, 3852, 3853, 3855}

$$\int \frac{\sec^4(c+dx)}{a+a \cos(c+dx)} dx = -\frac{3\arctanh(\sin(c+dx))}{2ad} + \frac{4 \tan^3(c+dx)}{3ad} + \frac{4 \tan(c+dx)}{ad} - \frac{3 \tan(c+dx) \sec(c+dx)}{2ad} - \frac{\tan(c+dx) \sec^2(c+dx)}{d(a \cos(c+dx) + a)}$$

[In] $\text{Int}[\text{Sec}[c + d*x]^4/(a + a*\text{Cos}[c + d*x]),x]$

[Out] $(-3*\text{ArcTanh}[\text{Sin}[c + d*x]])/(2*a*d) + (4*\text{Tan}[c + d*x])/(a*d) - (3*\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/(2*a*d) - (\text{Sec}[c + d*x]^2*\text{Tan}[c + d*x])/(d*(a + a*\text{Cos}[c + d*x])) + (4*\text{Tan}[c + d*x]^3)/(3*a*d)$

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d)), Int[(c + d*Sin[e + f*x])^n*(a^n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sec^2(c + dx) \tan(c + dx)}{d(a + a \cos(c + dx))} - \frac{\int (-4a + 3a \cos(c + dx)) \sec^4(c + dx) dx}{a^2} \\
 &= -\frac{\sec^2(c + dx) \tan(c + dx)}{d(a + a \cos(c + dx))} - \frac{3 \int \sec^3(c + dx) dx}{a} + \frac{4 \int \sec^4(c + dx) dx}{a} \\
 &= -\frac{3 \sec(c + dx) \tan(c + dx)}{2ad} - \frac{\sec^2(c + dx) \tan(c + dx)}{d(a + a \cos(c + dx))} \\
 &\quad - \frac{3 \int \sec(c + dx) dx}{2a} - \frac{4 \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{ad}
 \end{aligned}$$

$$= -\frac{3a \operatorname{arctanh}(\sin(c+dx))}{2ad} + \frac{4 \tan(c+dx)}{ad} - \frac{3 \sec(c+dx) \tan(c+dx)}{2ad} \\ - \frac{\sec^2(c+dx) \tan(c+dx)}{d(a+a \cos(c+dx))} + \frac{4 \tan^3(c+dx)}{3ad}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 368 vs. $2(103) = 206$.

Time = 3.13 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.57

$$\int \frac{\sec^4(c+dx)}{a+a \cos(c+dx)} dx \\ = \frac{\cos\left(\frac{1}{2}(c+dx)\right) \left(6 \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) + \frac{1}{8} \cos\left(\frac{1}{2}(c+dx)\right) \sec(c) \sec^3(c+dx) \left(9 \cos(2c+3dx) \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{a+a \cos(c+dx)}$$

[In] Integrate[Sec[c + d*x]^4/(a + a*Cos[c + d*x]),x]

[Out] (Cos[(c + d*x)/2]*(6*Sec[c/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2]*Sec[c]*Sec[c + d*x]^3*(9*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 27*Cos[d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])) + 27*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 9*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 48*Sin[d*x] - 12*Sin[2*c + d*x] - 6*Sin[c + 2*d*x] - 6*Sin[3*c + 2*d*x] + 20*Sin[2*c + 3*d*x]))/8)/(3*a*d*(1 + Cos[c + d*x]))

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

method	result
norman	$\frac{\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} - \frac{4 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{da} + \frac{25 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3da} - \frac{8 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da} + \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2ad} - \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2ad}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} + \frac{1}{\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{5}{2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2} - \frac{1}{3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} - \frac{1}{\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{5}{2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2}}{da}$
default	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} + \frac{1}{\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{5}{2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2} - \frac{1}{3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^3} - \frac{1}{\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{5}{2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{3 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2}}{da}$
parallelsch	$\frac{(27 \cos(dx+c) + 9 \cos(3dx+3c)) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + (-27 \cos(dx+c) - 9 \cos(3dx+3c)) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + 44 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{6ad(\cos(3dx+3c) + 3 \cos(dx+c))}$
risch	$\frac{i(9 e^{6i(dx+c)} + 9 e^{5i(dx+c)} + 24 e^{4i(dx+c)} + 24 e^{3i(dx+c)} + 39 e^{2i(dx+c)} + 7 e^{i(dx+c)} + 16)}{3da(e^{2i(dx+c)} + 1)^3(e^{i(dx+c)} + 1)} - \frac{3 \ln(e^{i(dx+c)} + i)}{2ad} + \frac{3 \ln(e^{i(dx+c)} - i)}{2ad}$

[In] `int(sec(d*x+c)^4/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)`

[Out] $(1/d/a*\tan(1/2*d*x+1/2*c)^7-4/d/a*\tan(1/2*d*x+1/2*c)+25/3/d/a*\tan(1/2*d*x+1/2*c)^3-8/d/a*\tan(1/2*d*x+1/2*c)^5)/(\tan(1/2*d*x+1/2*c)^2-1)^3+3/2/a/d*\ln(\tan(1/2*d*x+1/2*c)-1)-3/2/a/d*\ln(\tan(1/2*d*x+1/2*c)+1)$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)} dx = \frac{9(\cos(dx+c)^4 + \cos(dx+c)^3) \log(\sin(dx+c)+1) - 9(\cos(dx+c)^4 + \cos(dx+c)^3) \log(-\sin(dx+c)+1)}{12(ad\cos(dx+c)^4 + ad\cos(dx+c)^3)}$$

[In] `integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fricas")`

[Out] $-1/12*(9*(\cos(d*x+c)^4 + \cos(d*x+c)^3)*\log(\sin(d*x+c)+1) - 9*(\cos(d*x+c)^4 + \cos(d*x+c)^3)*\log(-\sin(d*x+c)+1) - 2*(16*\cos(d*x+c)^3 + 7*\cos(d*x+c)^2 - \cos(d*x+c) + 2)*\sin(d*x+c))/(a*d*\cos(d*x+c)^4 + a*d*\cos(d*x+c)^3)$

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)} dx = \frac{\int \frac{\sec^4(c+dx)}{\cos(c+dx)+1} dx}{a}$$

[In] `integrate(sec(d*x+c)**4/(a+a*cos(d*x+c)),x)`

[Out] `Integral(sec(c+d*x)**4/(cos(c+d*x)+1),x)/a`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(97) = 194.

Time = 0.21 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)} dx = \frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1} - \frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right) - \frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a} + \frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a} + \frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}}{a - \frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}$$

$6d$

[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{6} * (2 * (9 * \sin(d * x + c) / (\cos(d * x + c) + 1) - 16 * \sin(d * x + c)^3 / (\cos(d * x + c) + 1)^3 + 15 * \sin(d * x + c)^5 / (\cos(d * x + c) + 1)^5) / (a - 3 * a * \sin(d * x + c)^2 / (\cos(d * x + c) + 1)^2 + 3 * a * \sin(d * x + c)^4 / (\cos(d * x + c) + 1)^4 - a * \sin(d * x + c)^6 / (\cos(d * x + c) + 1)^6) - 9 * \log(\sin(d * x + c) / (\cos(d * x + c) + 1) + 1) / a + 9 * \log(\sin(d * x + c) / (\cos(d * x + c) + 1) - 1) / a + 6 * \sin(d * x + c) / (a * (\cos(d * x + c) + 1))) / d$

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{\sec^4(c + dx)}{a + a \cos(c + dx)} dx = \frac{\frac{9 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a} - \frac{9 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{a} - \frac{6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a} + \frac{2 \left(15 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 16 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 9 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)^3 a}}{6 d}$$

[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] $-\frac{1}{6} * (9 * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) + 1)) / a - 9 * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) - 1)) / a - 6 * \tan(1/2 * d * x + 1/2 * c) / a + 2 * (15 * \tan(1/2 * d * x + 1/2 * c)^5 - 16 * \tan(1/2 * d * x + 1/2 * c)^3 + 9 * \tan(1/2 * d * x + 1/2 * c)) / ((\tan(1/2 * d * x + 1/2 * c)^2 - 1)^3 * a)) / d$

Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{\sec^4(c + dx)}{a + a \cos(c + dx)} dx = \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{a d} - \frac{3 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a d} - \frac{5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 - \frac{16 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3} + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{a d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)^3}$$

[In] int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))),x)

[Out] $\frac{\tan(c/2 + (d * x)/2)}{a * d} - (3 * \operatorname{atanh}(\tan(c/2 + (d * x)/2))) / (a * d) - (3 * \tan(c/2 + (d * x)/2) - (16 * \tan(c/2 + (d * x)/2)^3) / 3 + 5 * \tan(c/2 + (d * x)/2)^5) / (a * d * (\tan(c/2 + (d * x)/2)^2 - 1)^3)$

3.53 $\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^2} dx$

Optimal result	541
Rubi [A] (verified)	541
Mathematica [A] (verified)	543
Maple [A] (verified)	544
Fricas [A] (verification not implemented)	544
Sympy [B] (verification not implemented)	545
Maxima [A] (verification not implemented)	545
Giac [A] (verification not implemented)	546
Mupad [B] (verification not implemented)	546

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{5x}{a^2} + \frac{12 \sin(c+dx)}{a^2 d} - \frac{5 \cos(c+dx) \sin(c+dx)}{a^2 d} - \frac{10 \cos^3(c+dx) \sin(c+dx)}{3a^2 d(1+\cos(c+dx))} - \frac{\cos^4(c+dx) \sin(c+dx)}{3d(a+a \cos(c+dx))^2} - \frac{4 \sin^3(c+dx)}{a^2 d}$$

[Out] $-5*x/a^2+12*\sin(d*x+c)/a^2/d-5*\cos(d*x+c)*\sin(d*x+c)/a^2/d-10/3*\cos(d*x+c)^3*\sin(d*x+c)/a^2/d/(1+\cos(d*x+c))-1/3*\cos(d*x+c)^4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2-4*\sin(d*x+c)^3/a^2/d$

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2844, 3056, 2827, 2715, 8, 2713}

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{4 \sin^3(c+dx)}{a^2 d} + \frac{12 \sin(c+dx)}{a^2 d} - \frac{10 \sin(c+dx) \cos^3(c+dx)}{3a^2 d(\cos(c+dx)+1)} - \frac{5 \sin(c+dx) \cos(c+dx)}{a^2 d} - \frac{5x}{a^2} - \frac{\sin(c+dx) \cos^4(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] $\text{Int}[\text{Cos}[c+d*x]^5/(a+a*\text{Cos}[c+d*x])^2,x]$

[Out] $(-5*x)/a^2 + (12*\text{Sin}[c+d*x])/(a^2*d) - (5*\text{Cos}[c+d*x]*\text{Sin}[c+d*x])/(a^2*d) - (10*\text{Cos}[c+d*x]^3*\text{Sin}[c+d*x])/(3*a^2*d*(1+\text{Cos}[c+d*x])) - (\text{Cos}[$

$$c + d*x]^4*\sin[c + d*x]/(3*d*(a + a*\cos[c + d*x])^2) - (4*\sin[c + d*x]^3)/(a^2*d)$$

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sine[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sine[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sine[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sine[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sine[e + f*x])^m*((c + d*Sine[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sine[e + f*x])^(m + 1)*(c + d*Sine[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sine[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegerQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sine[e + f*x])^m*((c + d*Sine[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sine[e + f*x])^(m + 1)*(c + d*Sine[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sine[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

egerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^3(c+dx)(4a-6a\cos(c+dx))}{a+a\cos(c+dx)} dx}{3a^2} \\
 &= -\frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
 &\quad - \frac{\int \cos^2(c+dx)(30a^2-36a^2\cos(c+dx)) dx}{3a^4} \\
 &= -\frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
 &\quad - \frac{10\int \cos^2(c+dx) dx}{a^2} + \frac{12\int \cos^3(c+dx) dx}{a^2} \\
 &= -\frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} \\
 &\quad - \frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{5\int 1 dx}{a^2} - \frac{12\text{Subst}(\int (1-x^2) dx, x, -\sin(c+dx))}{a^2d} \\
 &= -\frac{5x}{a^2} + \frac{12\sin(c+dx)}{a^2d} - \frac{5\cos(c+dx)\sin(c+dx)}{a^2d} \\
 &\quad - \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{4\sin^3(c+dx)}{a^2d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2} dx$$

$$= \frac{\sin(c+dx) \left(60 \arcsin(\cos(c+dx)) \cos^4\left(\frac{1}{2}(c+dx)\right) + (24 + 33 \cos(c+dx) + 6 \cos^2(c+dx) - \cos^3(c+dx)) \right)}{3a^2d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{5/2}}$$

[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^2,x]

[Out] (Sin[c + d*x]*(60*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^4 + (24 + 33*Cos[c + d*x] + 6*Cos[c + d*x]^2 - Cos[c + d*x]^3 + Cos[c + d*x]^4)*Sqrt[Sin[c + d*x]^2]))/(3*a^2*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(5/2))

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{43\left(\cos(dx+c)+\frac{14\cos(2dx+2c)}{129}-\frac{\cos(3dx+3c)}{129}+\frac{\cos(4dx+4c)}{258}+\frac{73}{86}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sec^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40dx}{8a^2d}$
derivativedivides	$-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-20\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da^2}$
default	$-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(-\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-20\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da^2}$
risch	$-\frac{5x}{a^2}+\frac{ie^{2i(dx+c)}}{4da^2}-\frac{15ie^{i(dx+c)}}{8a^2d}+\frac{15ie^{-i(dx+c)}}{8a^2d}-\frac{ie^{-2i(dx+c)}}{4da^2}+\frac{2i(15e^{2i(dx+c)}+27e^{i(dx+c)}+14)}{3da^2(e^{i(dx+c)}+1)^3}+\frac{\sin(3dx+3c)}{12a^2d}$
norman	$-\frac{5x}{a}+\frac{21\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{143\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{521\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{230\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{185\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}+\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{11}{3da}\frac{1}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] 1/8*(43*(cos(d*x+c)+14/129*cos(2*d*x+2*c)-1/129*cos(3*d*x+3*c)+1/258*cos(4*d*x+4*c)+73/86)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2-40*d*x)/a^2/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2} dx = \frac{15dx\cos(dx+c)^2+30dx\cos(dx+c)+15dx-(\cos(dx+c)^4-\cos(dx+c)^3+6\cos(dx+c)^2+33\cos(dx+c)+24)\sin(dx+c)}{3(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d)}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] -1/3*(15*d*x*cos(d*x+c)^2+30*d*x*cos(d*x+c)+15*d*x-(cos(d*x+c)^4-cos(d*x+c)^3+6*cos(d*x+c)^2+33*cos(d*x+c)+24)*sin(d*x+c))/(a^2*d*cos(d*x+c)^2+2*a^2*d*cos(d*x+c)+a^2*d)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(117) = 234$.

Time = 2.88 (sec) , antiderivative size = 700, normalized size of antiderivative = 5.65

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \left\{ \begin{array}{l} -\frac{30dx \tan^6\left(\frac{c}{2} + \frac{dx}{2}\right)}{6a^2d \tan^6\left(\frac{c}{2} + \frac{dx}{2}\right) + 18a^2d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 18a^2d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6a^2d} - \frac{90dx \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right)}{6a^2d \tan^6\left(\frac{c}{2} + \frac{dx}{2}\right) + 18a^2d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 18a^2d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)} \\ \frac{x \cos^5(c)}{(a \cos(c) + a)^2} \end{array} \right.$$

[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**2,x)

[Out] Piecewise((-30*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 30*d*x/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 24*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 138*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 160*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 63*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*cos(c)**5/(a*cos(c) + a)**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.67

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{4 \left(\frac{9 \sin(dx+c)}{\cos(dx+c)+1} + \frac{20 \sin^3(dx+c)}{(\cos(dx+c)+1)^3} + \frac{15 \sin^5(dx+c)}{(\cos(dx+c)+1)^5} \right) + \frac{27 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin^3(dx+c)}{(\cos(dx+c)+1)^3} - \frac{60 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] $\frac{1}{6} * (4 * (9 * \sin(dx + c) / (\cos(dx + c) + 1) + 20 * \sin(dx + c)^3 / (\cos(dx + c) + 1)^3 + 15 * \sin(dx + c)^5 / (\cos(dx + c) + 1)^5) / (a^2 + 3 * a^2 * \sin(dx + c)^2 / (\cos(dx + c) + 1)^2 + 3 * a^2 * \sin(dx + c)^4 / (\cos(dx + c) + 1)^4 + a^2 * \sin(dx + c)^6 / (\cos(dx + c) + 1)^6) + (27 * \sin(dx + c) / (\cos(dx + c) + 1) - \sin(dx + c)^3 / (\cos(dx + c) + 1)^3) / a^2 - 60 * \arctan(\sin(dx + c) / (\cos(dx + c) + 1)) / a^2) / d$

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\frac{30(dx+c)}{a^2} - \frac{4 \left(15 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 20 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 9 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1 \right)^3 a^2} + \frac{a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 27 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^6}}{6d}$$

[In] `integrate(cos(dx+c)^5/(a+a*cos(dx+c))^2,x, algorithm="giac")`

[Out] $-1/6 * (30 * (dx + c) / a^2 - 4 * (15 * \tan(1/2 * dx + 1/2 * c)^5 + 20 * \tan(1/2 * dx + 1/2 * c)^3 + 9 * \tan(1/2 * dx + 1/2 * c)) / ((\tan(1/2 * dx + 1/2 * c)^2 + 1)^3 * a^2) + (a^4 * \tan(1/2 * dx + 1/2 * c)^3 - 27 * a^4 * \tan(1/2 * dx + 1/2 * c)) / a^6) / d$

Mupad [B] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 28 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 60 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 40 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{6 a^2 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^3}$$

[In] `int(cos(c + dx)^5/(a + a*cos(c + dx))^2,x)`

[Out] $-(\sin(c/2 + (dx)/2) - 28 * \cos(c/2 + (dx)/2)^2 * \sin(c/2 + (dx)/2) - 60 * \cos(c/2 + (dx)/2)^4 * \sin(c/2 + (dx)/2) + 40 * \cos(c/2 + (dx)/2)^6 * \sin(c/2 + (dx)/2) - 16 * \cos(c/2 + (dx)/2)^8 * \sin(c/2 + (dx)/2) + 30 * \cos(c/2 + (dx)/2)^3 * (c + dx)) / (6 * a^2 * d * \cos(c/2 + (dx)/2)^3)$

3.54 $\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^2} dx$

Optimal result	547
Rubi [A] (verified)	547
Mathematica [A] (verified)	549
Maple [A] (verified)	549
Fricas [A] (verification not implemented)	550
Sympy [B] (verification not implemented)	550
Maxima [A] (verification not implemented)	551
Giac [A] (verification not implemented)	551
Mupad [B] (verification not implemented)	552

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{7x}{2a^2} - \frac{16 \sin(c+dx)}{3a^2d} + \frac{7 \cos(c+dx) \sin(c+dx)}{2a^2d} - \frac{8 \cos^2(c+dx) \sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^3(c+dx) \sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] $7/2*x/a^2-16/3*\sin(d*x+c)/a^2/d+7/2*\cos(d*x+c)*\sin(d*x+c)/a^2/d-8/3*\cos(d*x+c)^2*\sin(d*x+c)/a^2/d/(1+\cos(d*x+c))-1/3*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2$

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2844, 3056, 2813}

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{16 \sin(c+dx)}{3a^2d} - \frac{8 \sin(c+dx) \cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)} + \frac{7 \sin(c+dx) \cos(c+dx)}{2a^2d} + \frac{7x}{2a^2} - \frac{\sin(c+dx) \cos^3(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] $\text{Int}[\text{Cos}[c+d*x]^4/(a+a*\text{Cos}[c+d*x])^2,x]$

[Out] $(7*x)/(2*a^2) - (16*\text{Sin}[c+d*x])/(3*a^2*d) + (7*\text{Cos}[c+d*x]*\text{Sin}[c+d*x])/(2*a^2*d) - (8*\text{Cos}[c+d*x]^2*\text{Sin}[c+d*x])/(3*a^2*d*(1+\text{Cos}[c+d*x])) - (\text{Cos}[c+d*x]^3*\text{Sin}[c+d*x])/(3*d*(a+a*\text{Cos}[c+d*x])^2)$

Rule 2813

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*
(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(3a-5a\cos(c+dx))}{a+a\cos(c+dx)} dx}{3a^2} \\
&= -\frac{8\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{\int \cos(c+dx)(16a^2-21a^2\cos(c+dx)) dx}{3a^4} \\
&= \frac{7x}{2a^2} - \frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} \\
&\quad - \frac{8\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{\sin(c + dx) \left(-84 \arcsin(\cos(c + dx)) \cos^4\left(\frac{1}{2}(c + dx)\right) + (-32 - 43 \cos(c + dx) - 6 \cos^2(c + dx) + 3 \cos^3(c + dx)) \right)}{6a^2 d \sqrt{1 - \cos(c + dx)} (1 + \cos(c + dx))^{5/2}}$$

`[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]`

```
[Out] (Sin[c + d*x]*(-84*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^4 + (-32 - 43*Cos[c + d*x] - 6*Cos[c + d*x]^2 + 3*Cos[c + d*x]^3)*Sqrt[Sin[c + d*x]^2]))/(6*a^2*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{-163 \left(\cos(dx+c) + \frac{12 \cos(2dx+2c)}{163} - \frac{3 \cos(3dx+3c)}{163} + \frac{140}{163} \right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sec^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + 168 dx}{48a^2 d}$
derivativedivides	$\frac{\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3} - 7 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-10 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 6 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + 14 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \right)}{2d a^2}$
default	$\frac{\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3} - 7 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-10 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 6 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + 14 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \right)}{2d a^2}$
risch	$\frac{7x}{2a^2} - \frac{ie^{2i(dx+c)}}{8da^2} + \frac{ie^{i(dx+c)}}{a^2 d} - \frac{ie^{-i(dx+c)}}{a^2 d} + \frac{ie^{-2i(dx+c)}}{8da^2} - \frac{2i(12e^{2i(dx+c)} + 21e^{i(dx+c)} + 11)}{3da^2(e^{i(dx+c)} + 1)^3}$
norman	$\frac{7x}{2a} - \frac{13 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da} - \frac{149 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{6da} - \frac{100 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{3da} - \frac{18 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{da} - \frac{17 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{6da} + \frac{\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right)}{6da} + \frac{14}{a \left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^4}$

`[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)`

```
[Out] 1/48*(-163*(cos(d*x+c)+12/163*cos(2*d*x+2*c)-3/163*cos(3*d*x+3*c)+140/163)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^2+168*d*x)/a^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{21 dx \cos(dx + c)^2 + 42 dx \cos(dx + c) + 21 dx + (3 \cos(dx + c)^3 - 6 \cos(dx + c)^2 - 43 \cos(dx + c) - 32) \sin(dx + c)}{6 (a^2 d \cos(dx + c)^2 + 2 a^2 d \cos(dx + c) + a^2 d)}$$

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(21*d*x*cos(d*x + c)^2 + 42*d*x*cos(d*x + c) + 21*d*x + (3*cos(d*x + c)^3 - 6*cos(d*x + c)^2 - 43*cos(d*x + c) - 32)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(107) = 214.

Time = 1.77 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.62

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \begin{cases} \frac{21dx \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right)}{6a^2d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 12a^2d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6a^2d} + \frac{42dx \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{6a^2d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 12a^2d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6a^2d} + \frac{21dx}{6a^2d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 12a^2d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6a^2d} \\ \frac{x \cos^4(c)}{(a \cos(c) + a)^2} \end{cases}$$

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((21*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*d*x/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 19*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 71*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 39*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.44

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= - \frac{6 \left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{5 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) + \frac{21 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{42 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{a^2 + \frac{2a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} \frac{1}{6d}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

```
[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{\frac{21(dx+c)}{a^2} - \frac{6 \left(5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1 \right)^2 a^2} + \frac{a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 21 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^6}}{6d}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="giac")

```
[Out] 1/6*(21*(d*x + c)/a^2 - 6*(5*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 21*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 22 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 30 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 12 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{6 a^2 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^3}$$

[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^2,x)

[Out] (sin(c/2 + (d*x)/2) - 22*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 30*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) + 21*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)

3.55 $\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^2} dx$

Optimal result	553
Rubi [A] (verified)	553
Mathematica [A] (verified)	555
Maple [A] (verified)	555
Fricas [A] (verification not implemented)	556
Sympy [B] (verification not implemented)	556
Maxima [A] (verification not implemented)	557
Giac [A] (verification not implemented)	557
Mupad [B] (verification not implemented)	557

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{2x}{a^2} + \frac{4 \sin(c+dx)}{3a^2d} + \frac{2 \sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^2(c+dx) \sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] $-2*x/a^2+4/3*\sin(d*x+c)/a^2/d+2*\sin(d*x+c)/a^2/d/(1+\cos(d*x+c))-1/3*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2844, 3047, 3102, 12, 2814, 2727}

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{4 \sin(c+dx)}{3a^2d} + \frac{2 \sin(c+dx)}{a^2d(\cos(c+dx)+1)} - \frac{2x}{a^2} - \frac{\sin(c+dx) \cos^2(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] $\text{Int}[\text{Cos}[c+d*x]^3/(a+a*\text{Cos}[c+d*x])^2,x]$

[Out] $(-2*x)/a^2 + (4*\text{Sin}[c+d*x])/(3*a^2*d) + (2*\text{Sin}[c+d*x])/(a^2*d*(1+\text{Cos}[c+d*x])) - (\text{Cos}[c+d*x]^2*\text{Sin}[c+d*x])/(3*d*(a+a*\text{Cos}[c+d*x])^2)$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!Match}[\text{Q}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]]$

Rule 2727

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c +
d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)(2a-4a\cos(c+dx))}{a+a\cos(c+dx)} dx}{3a^2} \\ &= -\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{2a\cos(c+dx)-4a\cos^2(c+dx)}{a+a\cos(c+dx)} dx}{3a^2} \end{aligned}$$

[In] `int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{3} * (7 * \tan(1/2 * d * x + 1/2 * c) * (\cos(d * x + c) + 3/28 * \cos(2 * d * x + 2 * c) + 23/28) * \sec(1/2 * d * x + 1/2 * c)^2 - 6 * d * x) / a^2 / d$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{6 dx \cos(dx + c)^2 + 12 dx \cos(dx + c) + 6 dx - (3 \cos(dx + c)^2 + 14 \cos(dx + c) + 10) \sin(dx + c)}{3 (a^2 d \cos(dx + c)^2 + 2 a^2 d \cos(dx + c) + a^2 d)}$$

[In] `integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

[Out] $-1/3 * (6 * d * x * \cos(d * x + c)^2 + 12 * d * x * \cos(d * x + c) + 6 * d * x - (3 * \cos(d * x + c)^2 + 14 * \cos(d * x + c) + 10) * \sin(d * x + c)) / (a^2 * d * \cos(d * x + c)^2 + 2 * a^2 * d * \cos(d * x + c) + a^2 * d)$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(73) = 146$.

Time = 1.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.51

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^2} dx = \left\{ \begin{array}{l} -\frac{12 dx \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{6 a^2 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6 a^2 d} - \frac{12 dx}{6 a^2 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6 a^2 d} - \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{6 a^2 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6 a^2 d} + \frac{14 \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{6 a^2 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6 a^2 d} + \frac{27 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{6 a^2 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 6 a^2 d} \\ \frac{x \cos^3(c)}{(a \cos(c) + a)^2} \end{array} \right.$$

[In] `integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**2,x)`

[Out] `Piecewise((-12*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 12*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 14*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 27*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a)**2, True))`

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\frac{15 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{24 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} + \frac{12 \sin(dx+c)}{\left(a^2 + \frac{a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^2} dx = -\frac{\frac{12(dx+c)}{a^2} - \frac{12 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1)a^2} + \frac{a^4 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 15 a^4 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a^6}}{6 d}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] -1/6*(12*(d*x + c)/a^2 - 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d

Mupad [B] (verification not implemented)

Time = 14.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 16 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 12 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 12 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (c + dx)}{6 a^2 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^3}$$

[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^2,x)

[Out] -(sin(c/2 + (d*x)/2) - 16*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 12*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^2*d*cos(c/2 + (d*x)/2)^3)

3.56 $\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^2} dx$

Optimal result	558
Rubi [A] (verified)	558
Mathematica [A] (verified)	559
Maple [A] (verified)	560
Fricas [A] (verification not implemented)	560
Sympy [A] (verification not implemented)	561
Maxima [A] (verification not implemented)	561
Giac [A] (verification not implemented)	561
Mupad [B] (verification not implemented)	562

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{x}{a^2} - \frac{5 \sin(c+dx)}{3a^2d(1+\cos(c+dx))} + \frac{\sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] $x/a^2-5/3*\sin(d*x+c)/a^2/d/(1+\cos(d*x+c))+1/3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2837, 2814, 2727}

$$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{5 \sin(c+dx)}{3a^2d(\cos(c+dx)+1)} + \frac{x}{a^2} + \frac{\sin(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] `Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]`

[Out] $x/a^2 - (5*\sin[c + d*x])/(3*a^2*d*(1 + \cos[c + d*x])) + \sin[c + d*x]/(3*d*(a + a*\cos[c + d*x])^2)$

Rule 2727

`Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2837

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] := Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
+ 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int \frac{-2a + 3a \cos(c + dx)}{a + a \cos(c + dx)} dx}{3a^2} \\ &= \frac{x}{a^2} + \frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2} - \frac{5 \int \frac{1}{a + a \cos(c + dx)} dx}{3a} \\ &= \frac{x}{a^2} + \frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2} - \frac{5 \sin(c + dx)}{3d(a^2 + a^2 \cos(c + dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\sin(c + dx) \left(12 \arcsin(\cos(c + dx)) \cos^4\left(\frac{1}{2}(c + dx)\right) + (4 + 5 \cos(c + dx)) \sqrt{\sin^2(c + dx)} \right)}{3a^2 d \sqrt{1 - \cos(c + dx)} (1 + \cos(c + dx))^{5/2}}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -1/3*(Sin[c + d*x]*(12*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^4 + (4 + 5*Cos
[c + d*x])*Sqrt[Sin[c + d*x]^2]))/(a^2*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c
+ d*x])^(5/2))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) + 6dx - 9 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{6a^2d}$	36
derivativedivides	$\frac{\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) - 3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 4 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2da^2}$	46
default	$\frac{\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) - 3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 4 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2da^2}$	46
risch	$\frac{x}{a^2} - \frac{2i(6e^{2i(dx+c)} + 9e^{i(dx+c)} + 5)}{3da^2(e^{i(dx+c)} + 1)^3}$	53
norman	$\frac{\frac{x}{a} + \frac{x(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right))}{a} - \frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da} - \frac{17(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right))}{6da} - \frac{7(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right))}{6da} + \frac{\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{6da} + \frac{2x(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))}{a}}{a\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}$	133

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] 1/6*(tan(1/2*d*x+1/2*c)^3+6*d*x-9*tan(1/2*d*x+1/2*c))/a^2/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{3 dx \cos(dx + c)^2 + 6 dx \cos(dx + c) + 3 dx - (5 \cos(dx + c) + 4) \sin(dx + c)}{3(a^2d \cos(dx + c)^2 + 2a^2d \cos(dx + c) + a^2d)}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (5*cos(d*x + c) + 4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^2} dx = \begin{cases} \frac{x}{a^2} + \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{6a^2d} - \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2a^2d} & \text{for } d \neq 0 \\ \frac{x \cos^2(c)}{(a \cos(c) + a)^2} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**2,x)

[Out] Piecewise((x/a**2 + tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*tan(c/2 + d*x/2)/(2*a**2*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^2} dx = -\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{12 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\frac{6(dx+c)}{a^2} + \frac{a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 9a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^6}}{6d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] 1/6*(6*(d*x + c)/a^2 + (a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d

Mupad [B] (verification not implemented)

Time = 14.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 - 9 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 6 dx}{6 a^2 d}$$

[In] `int(cos(c + d*x)^2/(a + a*cos(c + d*x))^2,x)`

[Out] `(tan(c/2 + (d*x)/2)^3 - 9*tan(c/2 + (d*x)/2) + 6*d*x)/(6*a^2*d)`

3.57 $\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^2} dx$

Optimal result	563
Rubi [A] (verified)	563
Mathematica [A] (verified)	564
Maple [A] (verified)	564
Fricas [A] (verification not implemented)	565
Sympy [A] (verification not implemented)	565
Maxima [A] (verification not implemented)	565
Giac [A] (verification not implemented)	566
Mupad [B] (verification not implemented)	566

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{\sin(c+dx)}{3d(a+a \cos(c+dx))^2} + \frac{2 \sin(c+dx)}{3d(a^2+a^2 \cos(c+dx))}$$

[Out] $-1/3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2+2/3*\sin(d*x+c)/d/(a^2+a^2*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2829, 2727}

$$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{2 \sin(c+dx)}{3d(a^2 \cos(c+dx) + a^2)} - \frac{\sin(c+dx)}{3d(a \cos(c+dx) + a^2)}$$

[In] $\text{Int}[\text{Cos}[c + d*x]/(a + a*\text{Cos}[c + d*x])^2, x]$

[Out] $-1/3*\text{Sin}[c + d*x]/(d*(a + a*\text{Cos}[c + d*x])^2) + (2*\text{Sin}[c + d*x])/(3*d*(a^2 + a^2*\text{Cos}[c + d*x]))$

Rule 2727

$\text{Int}[(a + (b_*)*\sin[(c_*) + (d_*)*(x_*)])^{-1}, x_Symbol] \rightarrow \text{Simp}[-\text{Cos}[c + d*x]/(d*(b + a*\sin[c + d*x])), x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 2829

$\text{Int}[(a + (b_*)*\sin[(e_*) + (f_*)*(x_*)])^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)]), x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*\text{Cos}[e + f*x]*((a + b*\sin[e + f*x])^{(m_*)} - a^2), x]$

$x)^m/(a*f*(2*m + 1)), x] + \text{Dist}[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m + 1}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}]$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{2 \int \frac{1}{a + a \cos(c + dx)} dx}{3a} \\ &= -\frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{2 \sin(c + dx)}{3d(a^2 + a^2 \cos(c + dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{(1 + 2 \cos(c + dx)) \sin(c + dx)}{3a^2 d (1 + \cos(c + dx))^2}$$

[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^2,x]

[Out] ((1 + 2*Cos[c + d*x])*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])^2)

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

method	result	size
parallelrisc	$-\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 3\right)}{6a^2 d}$	31
derivativedivides	$-\frac{\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da^2}$	32
default	$-\frac{\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da^2}$	32
risc	$\frac{2i(3e^{2i(dx+c)} + 3e^{i(dx+c)} + 2)}{3da^2(e^{i(dx+c)} + 1)^3}$	47
norman	$\frac{\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da} + \frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3da} - \frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{6da}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a}$	76

[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] -1/6*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^2-3)/a^2/d

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{(2 \cos(dx + c) + 1) \sin(dx + c)}{3 (a^2 d \cos(dx + c)^2 + 2 a^2 d \cos(dx + c) + a^2 d)}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/3*(2*cos(d*x + c) + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^2} dx = \begin{cases} -\frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{6a^2d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2a^2d} & \text{for } d \neq 0 \\ \frac{x \cos(c)}{(a \cos(c) + a)^2} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**2,x)

[Out] Piecewise((-tan(c/2 + d*x/2)**3/(6*a**2*d) + tan(c/2 + d*x/2)/(2*a**2*d), Ne(d, 0)), (x*cos(c)/(a*cos(c) + a)**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\frac{3 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6 a^2 d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^2*d)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^2} dx = -\frac{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{6 a^2 d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] -1/6*(tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(a^2*d)

Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^2} dx = -\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 3\right)}{6 a^2 d}$$

[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^2,x)

[Out] -(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 - 3))/(6*a^2*d)

$$3.58 \quad \int \frac{1}{(a+a \cos(c+dx))^2} dx$$

Optimal result	567
Rubi [A] (verified)	567
Mathematica [A] (verified)	568
Maple [A] (verified)	568
Fricas [A] (verification not implemented)	569
Sympy [A] (verification not implemented)	569
Maxima [A] (verification not implemented)	569
Giac [A] (verification not implemented)	570
Mupad [B] (verification not implemented)	570

Optimal result

Integrand size = 12, antiderivative size = 55

$$\int \frac{1}{(a+a \cos(c+dx))^2} dx = \frac{\sin(c+dx)}{3d(a+a \cos(c+dx))^2} + \frac{\sin(c+dx)}{3d(a^2+a^2 \cos(c+dx))}$$

[Out] 1/3*sin(d*x+c)/d/(a+a*cos(d*x+c))^2+1/3*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2729, 2727}

$$\int \frac{1}{(a+a \cos(c+dx))^2} dx = \frac{\sin(c+dx)}{3d(a^2 \cos(c+dx)+a^2)} + \frac{\sin(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] Int[(a + a*Cos[c + d*x])^(-2), x]

[Out] Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*d*(a^2 + a^2*Cos[c + d*x]))

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n

+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] & & EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int \frac{1}{a + a \cos(c + dx)} dx}{3a} \\ &= \frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\sin(c + dx)}{3d(a^2 + a^2 \cos(c + dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a + a \cos(c + dx))^2} dx = \frac{\cos\left(\frac{1}{2}(c + dx)\right) \left(3 \sin\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{3}{2}(c + dx)\right)\right)}{3a^2 d (1 + \cos(c + dx))^2}$$

[In] Integrate[(a + a*Cos[c + d*x])^(-2),x]

[Out] (Cos[(c + d*x)/2]*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

method	result	size
parallelrisc	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 3\right)}{6a^2 d}$	31
derivativedivides	$\frac{\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3} + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2}$	32
default	$\frac{\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3} + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2}$	32
risc	$\frac{2i(3e^{i(dx+c)} + 1)}{3d a^2 (e^{i(dx+c)} + 1)^3}$	36
norman	$\frac{\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da} + \frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{6da}}{a}$	42

[In] int(1/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] 1/6*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^2+3)/a^2/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a + a \cos(c + dx))^2} dx = \frac{(\cos(dx + c) + 2) \sin(dx + c)}{3(a^2 d \cos(dx + c)^2 + 2a^2 d \cos(dx + c) + a^2 d)}$$

[In] integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/3*(cos(d*x + c) + 2)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a + a \cos(c + dx))^2} dx = \begin{cases} \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{6a^2d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2a^2d} & \text{for } d \neq 0 \\ \frac{x}{(a \cos(c) + a)^2} & \text{otherwise} \end{cases}$$

[In] integrate(1/(a+a*cos(d*x+c))**2,x)

[Out] Piecewise((tan(c/2 + d*x/2)**3/(6*a**2*d) + tan(c/2 + d*x/2)/(2*a**2*d), Ne(d, 0)), (x/(a*cos(c) + a)**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a + a \cos(c + dx))^2} dx = \frac{\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6a^2d}$$

[In] integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^2*d)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a + a \cos(c + dx))^2} dx = \frac{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{6 a^2 d}$$

[In] integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] 1/6*(tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/(a^2*d)

Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a + a \cos(c + dx))^2} dx = \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 3\right)}{6 a^2 d}$$

[In] int(1/(a + a*cos(c + d*x))^2,x)

[Out] (tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 + 3))/(6*a^2*d)

3.59 $\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^2} dx$

Optimal result	571
Rubi [A] (verified)	571
Mathematica [B] (verified)	573
Maple [A] (verified)	573
Fricas [A] (verification not implemented)	574
Sympy [F]	574
Maxima [A] (verification not implemented)	574
Giac [A] (verification not implemented)	575
Mupad [B] (verification not implemented)	575

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2 d} - \frac{4 \sin(c+dx)}{3a^2 d(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] $\operatorname{arctanh}(\sin(d*x+c))/a^2/d-4/3*\sin(d*x+c)/a^2/d/(1+\cos(d*x+c))-1/3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2845, 3057, 12, 3855}

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2 d} - \frac{4 \sin(c+dx)}{3a^2 d(\cos(c+dx)+1)} - \frac{\sin(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]/(a+a*\operatorname{Cos}[c+d*x])^2,x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]]/(a^2*d) - (4*\operatorname{Sin}[c+d*x])/(3*a^2*d*(1+\operatorname{Cos}[c+d*x])) - \operatorname{Sin}[c+d*x]/(3*d*(a+a*\operatorname{Cos}[c+d*x])^2)$

Rule 12

$\operatorname{Int}[(a_*)*(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!Match} Q[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 2845

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int \frac{(3a - a \cos(c + dx)) \sec(c + dx)}{a + a \cos(c + dx)} dx}{3a^2} \\
&= -\frac{4 \sin(c + dx)}{3a^2 d (1 + \cos(c + dx))} - \frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int 3a^2 \sec(c + dx) dx}{3a^4} \\
&= -\frac{4 \sin(c + dx)}{3a^2 d (1 + \cos(c + dx))} - \frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int \sec(c + dx) dx}{a^2} \\
&= \frac{\operatorname{arctanh}(\sin(c + dx))}{a^2 d} - \frac{4 \sin(c + dx)}{3a^2 d (1 + \cos(c + dx))} - \frac{\sin(c + dx)}{3d(a + a \cos(c + dx))^2}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 152 vs. $2(66) = 132$.

Time = 0.44 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.30

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{2 \cos\left(\frac{1}{2}(c + dx)\right) \left(6 \cos^3\left(\frac{1}{2}(c + dx)\right) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{3a^2d(1 + \cos(c + dx))}$$

[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^2,x]

[Out] $(-2*\text{Cos}[(c + d*x)/2]*(6*\text{Cos}[(c + d*x)/2]^3*(\text{Log}[\text{Cos}[(c + d*x)/2] - \text{Sin}[(c + d*x)/2]] - \text{Log}[\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2]]) + \text{Sec}[c/2]*\text{Sin}[(d*x)/2] + 8*\text{Cos}[(c + d*x)/2]^2*\text{Sec}[c/2]*\text{Sin}[(d*x)/2] + \text{Cos}[(c + d*x)/2]*\text{Tan}[c/2])/((3*a^2*d*(1 + \text{Cos}[c + d*x])^2)$

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{-\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) - 3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + 2 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2da^2}$	62
default	$\frac{-\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) - 3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 2 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + 2 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2da^2}$	62
parallelrisc	$\frac{-\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 9 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 6 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + 6 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{6a^2d}$	62
norman	$\frac{-\frac{3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da} - \frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{6da}}{a} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{da^2} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^2d}$	82
risc	$-\frac{2i(3e^{2i(dx+c)} + 9e^{i(dx+c)} + 4)}{3da^2(e^{i(dx+c)} + 1)^3} + \frac{\ln(e^{i(dx+c)} + i)}{da^2} - \frac{\ln(e^{i(dx+c)} - i)}{a^2d}$	89

[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] $1/2/d/a^2*(-1/3*\tan(1/2*d*x+1/2*c)^3-3*\tan(1/2*d*x+1/2*c)-2*\ln(\tan(1/2*d*x+1/2*c)-1)+2*\ln(\tan(1/2*d*x+1/2*c)+1))$

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.73

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2} dx = \frac{3(\cos(dx+c)^2+2\cos(dx+c)+1)\log(\sin(dx+c)+1)-3(\cos(dx+c)^2+2\cos(dx+c)+1)\log(-\sin(dx+c)+1)-2(4\cos(dx+c)+5)\sin(dx+c)}{6(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d)}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2} dx = \int \frac{\sec(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1} \frac{dx}{a^2}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**2,x)

[Out] Integral(sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2} dx = -\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{\frac{6 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^2} - \frac{6 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{a^2} - \frac{a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 9 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^6}}{6 d}$$

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - (a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B] (verification not implemented)

Time = 14.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^2} dx = -\frac{9 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 12 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{6 a^2 d}$$

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^2),x)
```

```
[Out] -(9*tan(c/2 + (d*x)/2) - 12*atanh(tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)^3)/(6*a^2*d)
```

3.60 $\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^2} dx$

Optimal result	576
Rubi [A] (verified)	576
Mathematica [B] (verified)	578
Maple [A] (verified)	578
Fricas [A] (verification not implemented)	579
Sympy [F]	580
Maxima [A] (verification not implemented)	580
Giac [A] (verification not implemented)	580
Mupad [B] (verification not implemented)	581

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{2\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{10 \tan(c+dx)}{3a^2d} - \frac{2 \tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\tan(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] $-2*\operatorname{arctanh}(\sin(d*x+c))/a^2/d+10/3*\tan(d*x+c)/a^2/d-2*\tan(d*x+c)/a^2/d/(1+\cos(d*x+c))-1/3*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^2$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2845, 3057, 2827, 3852, 8, 3855}

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{2\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{10 \tan(c+dx)}{3a^2d} - \frac{2 \tan(c+dx)}{a^2d(\cos(c+dx)+1)} - \frac{\tan(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]^2/(a+a*\operatorname{Cos}[c+d*x])^2,x]$

[Out] $(-2*\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]])/(a^2*d) + (10*\operatorname{Tan}[c+d*x])/(3*a^2*d) - (2*\operatorname{Tan}[c+d*x])/(a^2*d*(1+\operatorname{Cos}[c+d*x])) - \operatorname{Tan}[c+d*x]/(3*d*(a+a*\operatorname{Cos}[c+d*x])^2)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sine[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sine[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2845

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sine[e + f*x])^m*((c + d*Sine[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sine[e + f*x])^(m + 1)*(c + d*Sine[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sine[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sine[e + f*x])^m*((c + d*Sine[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sine[e + f*x])^(m + 1)*(c + d*Sine[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sine[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\text{integral} = -\frac{\tan(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int \frac{(4a - 2a \cos(c + dx)) \sec^2(c + dx)}{a + a \cos(c + dx)} dx}{3a^2}$$

$$\begin{aligned}
&= -\frac{2 \tan(c + dx)}{a^2 d(1 + \cos(c + dx))} - \frac{\tan(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int (10a^2 - 6a^2 \cos(c + dx)) \sec^2(c + dx) dx}{3a^4} \\
&= -\frac{2 \tan(c + dx)}{a^2 d(1 + \cos(c + dx))} - \frac{\tan(c + dx)}{3d(a + a \cos(c + dx))^2} - \frac{2 \int \sec(c + dx) dx}{a^2} + \frac{10 \int \sec^2(c + dx) dx}{3a^2} \\
&= -\frac{2 \operatorname{arctanh}(\sin(c + dx))}{a^2 d} - \frac{2 \tan(c + dx)}{a^2 d(1 + \cos(c + dx))} \\
&\quad - \frac{\tan(c + dx)}{3d(a + a \cos(c + dx))^2} - \frac{10 \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{3a^2 d} \\
&= -\frac{2 \operatorname{arctanh}(\sin(c + dx))}{a^2 d} + \frac{10 \tan(c + dx)}{3a^2 d} - \frac{2 \tan(c + dx)}{a^2 d(1 + \cos(c + dx))} - \frac{\tan(c + dx)}{3d(a + a \cos(c + dx))^2}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 239 vs. $2(81) = 162$.

Time = 1.07 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.95

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$2 \cos\left(\frac{1}{2}(c + dx)\right) \left(\sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) + 14 \cos^2\left(\frac{1}{2}(c + dx)\right) \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) + 6 \cos^3\left(\frac{1}{2}(c + dx)\right) \left(2 \log\left(\cos\left(\frac{1}{2}\right)\right) \right) \right)$$

[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]

[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*(2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\left(\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{2da^2}$
default	$\frac{\left(\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{2da^2}$
parallelrisch	$\frac{6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+7\left(\sec^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos(dx+c)+\frac{5\cos(2dx+c)}{2}\right)}{3a^2d\cos(dx+c)}$
norman	$\frac{-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a}+\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}-\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da^2}$
risch	$\frac{4i\left(3e^{4i(dx+c)}+9e^{3i(dx+c)}+11e^{2i(dx+c)}+12e^{i(dx+c)}+5\right)}{3da^2\left(e^{i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}+1\right)}+\frac{2\ln\left(e^{i(dx+c)}-i\right)}{a^2d}-\frac{2\ln\left(e^{i(dx+c)}+i\right)}{da^2}$

[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+5*tan(1/2*d*x+1/2*c)-2/(tan(1/2*d*x+1/2*c)+1)-4*ln(tan(1/2*d*x+1/2*c)+1)-2/(tan(1/2*d*x+1/2*c)-1)+4*ln(tan(1/2*d*x+1/2*c)-1))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.80

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2} dx = \frac{3(\cos(dx+c))^3+2\cos(dx+c)^2+\cos(dx+c)}{3(a^2d\cos(dx+c))^3+2a^2d\cos(dx+c)} \log(\sin(dx+c)+1) - \frac{3(\cos(dx+c))^3+2\cos(dx+c)^2+\cos(dx+c)}{3(a^2d\cos(dx+c))^3+2a^2d\cos(dx+c)} \log(-\sin(dx+c)+1) - \frac{(10\cos(dx+c)^2+14\cos(dx+c)+3)\sin(dx+c)}{a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] -1/3*(3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - (10*cos(d*x + c)^2 + 14*cos(d*x + c) + 3)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))

SymPy [F]

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\int \frac{\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1} dx}{a^2}$$

[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**2,x)

[Out] Integral(sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.79

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{\frac{15 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{12 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^2} + \frac{12 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^2} + \frac{12 \sin(dx+c)}{\left(a^2 - \frac{a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6d}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.31

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^2} dx =$$

$$\frac{\frac{12 \log\left(|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)|}{a^2} - \frac{12 \log\left(|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right)|}{a^2} + \frac{12 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right) a^2} - \frac{a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 15 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^6}}{6d}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] -1/6*(12*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d

Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{6 a^2 d} - \frac{4 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^2 d} - \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - a^2\right)} + \frac{5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2 a^2 d}$$

[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^2),x)

[Out] $\frac{\tan(c/2 + (d*x)/2)^3}{6*a^2*d} - \frac{4*\operatorname{atanh}(\tan(c/2 + (d*x)/2))}{a^2*d} - \frac{2*\tan(c/2 + (d*x)/2)}{d*(a^2*\tan(c/2 + (d*x)/2)^2 - a^2)} + \frac{5*\tan(c/2 + (d*x)/2)}{2*a^2*d}$

3.61 $\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^2} dx$

Optimal result	582
Rubi [A] (verified)	582
Mathematica [B] (verified)	584
Maple [A] (verified)	585
Fricas [A] (verification not implemented)	585
Sympy [F]	586
Maxima [A] (verification not implemented)	586
Giac [A] (verification not implemented)	587
Mupad [B] (verification not implemented)	587

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{7 \operatorname{arctanh}(\sin(c+dx))}{2a^2d} - \frac{16 \tan(c+dx)}{3a^2d} + \frac{7 \sec(c+dx) \tan(c+dx)}{2a^2d} - \frac{8 \sec(c+dx) \tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec(c+dx) \tan(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] 7/2*arctanh(sin(d*x+c))/a^2/d-16/3*tan(d*x+c)/a^2/d+7/2*sec(d*x+c)*tan(d*x+c)/a^2/d-8/3*sec(d*x+c)*tan(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^2

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2845, 3057, 2827, 3853, 3855, 3852, 8}

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{7 \operatorname{arctanh}(\sin(c+dx))}{2a^2d} - \frac{16 \tan(c+dx)}{3a^2d} + \frac{7 \tan(c+dx) \sec(c+dx)}{2a^2d} - \frac{8 \tan(c+dx) \sec(c+dx)}{3a^2d(\cos(c+dx)+1)} - \frac{\tan(c+dx) \sec(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (16*Tan[c + d*x])/(3*a^2*d) + (7*Sec[
c + d*x]*Tan[c + d*x])/(2*a^2*d) - (8*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(
1 + Cos[c + d*x])) - (Sec[c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^
2)
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*SIN[e + f*x])^
m*((c + d*SIN[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*SIN[e + f*x],
x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*SIN[e + f*x])^m*((c + d*SIN[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*SIN[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)])*(b_.)^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*(n - 2)/(n - 1),
```

`Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sec(c + dx) \tan(c + dx)}{3d(a + a \cos(c + dx))^2} + \frac{\int \frac{(5a - 3a \cos(c + dx)) \sec^3(c + dx)}{a + a \cos(c + dx)} dx}{3a^2} \\
 &= -\frac{8 \sec(c + dx) \tan(c + dx)}{3a^2 d(1 + \cos(c + dx))} - \frac{\sec(c + dx) \tan(c + dx)}{3d(a + a \cos(c + dx))^2} \\
 &\quad + \frac{\int (21a^2 - 16a^2 \cos(c + dx)) \sec^3(c + dx) dx}{3a^4} \\
 &= -\frac{8 \sec(c + dx) \tan(c + dx)}{3a^2 d(1 + \cos(c + dx))} - \frac{\sec(c + dx) \tan(c + dx)}{3d(a + a \cos(c + dx))^2} \\
 &\quad - \frac{16 \int \sec^2(c + dx) dx}{3a^2} + \frac{7 \int \sec^3(c + dx) dx}{a^2} \\
 &= \frac{7 \sec(c + dx) \tan(c + dx)}{2a^2 d} - \frac{8 \sec(c + dx) \tan(c + dx)}{3a^2 d(1 + \cos(c + dx))} - \frac{\sec(c + dx) \tan(c + dx)}{3d(a + a \cos(c + dx))^2} \\
 &\quad + \frac{7 \int \sec(c + dx) dx}{2a^2} + \frac{16 \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{3a^2 d} \\
 &= \frac{7 \arctanh(\sin(c + dx))}{2a^2 d} - \frac{16 \tan(c + dx)}{3a^2 d} + \frac{7 \sec(c + dx) \tan(c + dx)}{2a^2 d} \\
 &\quad - \frac{8 \sec(c + dx) \tan(c + dx)}{3a^2 d(1 + \cos(c + dx))} - \frac{\sec(c + dx) \tan(c + dx)}{3d(a + a \cos(c + dx))^2}
 \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(119) = 238.

Time = 1.59 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.45

$$\begin{aligned}
 &\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^2} dx \\
 &= \frac{\cos\left(\frac{1}{2}(c + dx)\right) \left(-2 \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) - 40 \cos^2\left(\frac{1}{2}(c + dx)\right) \sec\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right) + 3 \cos^3\left(\frac{1}{2}(c + dx)\right) \right) \left(-14 \log\left(\frac{a + a \cos(c + dx)}{2}\right) \right)}{3a^2 d}
 \end{aligned}$$

`[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]`

```
[Out] (Cos[(c + d*x)/2]*(-2*Sec[c/2]*Sin[(d*x)/2] - 40*Cos[(c + d*x)/2]^2*Sec[c/2]
]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]^3*(-14*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2) - (8*
Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - 2*Cos[(c + d*x)
/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$
default	$-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parallelrisc	$\frac{(-42\cos(2dx+2c)-42)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(42\cos(2dx+2c)+42)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-60\left(\sec^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\cos(dx+c)\right)}{12a^2d(1+\cos(2dx+2c))}$
norman	$-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{71\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{19\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^2d}+\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da^2}$
risc	$-\frac{i(21e^{6i(dx+c)}+63e^{5i(dx+c)}+98e^{4i(dx+c)}+126e^{3i(dx+c)}+97e^{2i(dx+c)}+75e^{i(dx+c)}+32)}{3da^2(e^{2i(dx+c)}+1)^2(e^{i(dx+c)}+1)^3}+\frac{7\ln(e^{i(dx+c)}+i)}{2da^2}-\frac{7\ln(e^{i(dx+c)}-i)}{2da^2}$

```
[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(-1/(tan(1/2*d*x+1/2*c)+1)^2+5/(tan(1/2*d*x+1/2*c)+1)+7*ln(tan(1/
2*d*x+1/2*c)+1)-1/3*tan(1/2*d*x+1/2*c)^3-7*tan(1/2*d*x+1/2*c)+1/(tan(1/2*d*
x+1/2*c)-1)^2+5/(tan(1/2*d*x+1/2*c)-1)-7*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.36

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2} dx$$

$$= \frac{21(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2)\log(\sin(dx+c)+1)-21(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2)\log(\sin(dx+c)-1)+12(a^2d\cos(dx+c))^4}{12(a^2d\cos(dx+c))^4+21(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2)}$$

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

[Out] $\frac{1}{12} \cdot (21 \cdot (\cos(dx + c))^4 + 2 \cdot \cos(dx + c)^3 + \cos(dx + c)^2) \cdot \log(\sin(dx + c) + 1) - 21 \cdot (\cos(dx + c))^4 + 2 \cdot \cos(dx + c)^3 + \cos(dx + c)^2) \cdot \log(-\sin(dx + c) + 1) - 2 \cdot (32 \cdot \cos(dx + c)^3 + 43 \cdot \cos(dx + c)^2 + 6 \cdot \cos(dx + c) - 3) \cdot \sin(dx + c) / (a^2 \cdot d \cdot \cos(dx + c)^4 + 2 \cdot a^2 \cdot d \cdot \cos(dx + c)^3 + a^2 \cdot d \cdot \cos(dx + c)^2)$

Sympy [F]

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\int \frac{\sec^3(c + dx)}{\cos^2(c + dx) + 2 \cos(c + dx) + 1} dx}{a^2}$$

[In] integrate(sec(dx+c)**3/(a+a*cos(dx+c))**2,x)

[Out] Integral(sec(c + dx)**3/(cos(c + dx)**2 + 2*cos(c + dx) + 1), x)/a**2

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.60

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{6 \left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} - \frac{5 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) + \frac{21 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^2} + \frac{21 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^2}}{a^2 - \frac{2a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} \cdot d$$

[In] integrate(sec(dx+c)^3/(a+a*cos(dx+c))^2,x, algorithm="maxima")

[Out] $-\frac{1}{6} \cdot (6 \cdot (3 \cdot \sin(dx + c) / (\cos(dx + c) + 1) - 5 \cdot \sin(dx + c)^3 / (\cos(dx + c) + 1)^3) / (a^2 - 2 \cdot a^2 \cdot \sin(dx + c)^2 / (\cos(dx + c) + 1)^2 + a^2 \cdot \sin(dx + c)^4 / (\cos(dx + c) + 1)^4) + (21 \cdot \sin(dx + c) / (\cos(dx + c) + 1) + \sin(dx + c)^3 / (\cos(dx + c) + 1)^3) / a^2 - 21 \cdot \log(\sin(dx + c) / (\cos(dx + c) + 1) + 1) / a^2 + 21 \cdot \log(\sin(dx + c) / (\cos(dx + c) + 1) - 1) / a^2) / d$

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{21 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a^2} - \frac{21 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a^2} + \frac{6 \left(5 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 3 \tan(\frac{1}{2} dx + \frac{1}{2} c) \right)}{\left(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1 \right)^2 a^2} - \frac{a^4 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 21 a^4 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a^6}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] 1/6*(21*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 21*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(5*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 21*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d

Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{7 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^2 d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{6 a^2 d}$$

$$- \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{d \left(a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^2 \right)}$$

$$- \frac{7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2 a^2 d}$$

[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^2),x)

[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (3*tan(c/2 + (d*x)/2) - 5*tan(c/2 + (d*x)/2)^3)/(d*(a^2*tan(c/2 + (d*x)/2)^4 - 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (7*tan(c/2 + (d*x)/2))/(2*a^2*d)

3.62 $\int \frac{\sec^4(c+dx)}{(a+a \cos(c+dx))^2} dx$

Optimal result	588
Rubi [A] (verified)	588
Mathematica [B] (verified)	590
Maple [A] (verified)	591
Fricas [A] (verification not implemented)	591
Sympy [F]	592
Maxima [A] (verification not implemented)	592
Giac [A] (verification not implemented)	593
Mupad [B] (verification not implemented)	593

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{\sec^4(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{5 \operatorname{arctanh}(\sin(c+dx))}{a^2 d} + \frac{12 \tan(c+dx)}{a^2 d} - \frac{5 \sec(c+dx) \tan(c+dx)}{a^2 d} - \frac{10 \sec^2(c+dx) \tan(c+dx)}{3a^2 d(1+\cos(c+dx))} - \frac{\sec^2(c+dx) \tan(c+dx)}{3d(a+a \cos(c+dx))^2} + \frac{4 \tan^3(c+dx)}{a^2 d}$$

[Out] $-5 \operatorname{arctanh}(\sin(d*x+c))/a^2/d + 12 \tan(d*x+c)/a^2/d - 5 \sec(d*x+c) \tan(d*x+c)/a^2/d - 10/3 \sec(d*x+c)^2 \tan(d*x+c)/a^2/d / (1+\cos(d*x+c)) - 1/3 \sec(d*x+c)^2 \tan(d*x+c)/d / (a+a \cos(d*x+c))^2 + 4 \tan(d*x+c)^3/a^2/d$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2845, 3057, 2827, 3852, 3853, 3855}

$$\int \frac{\sec^4(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{5 \operatorname{arctanh}(\sin(c+dx))}{a^2 d} + \frac{4 \tan^3(c+dx)}{a^2 d} + \frac{12 \tan(c+dx)}{a^2 d} - \frac{5 \tan(c+dx) \sec(c+dx)}{a^2 d} - \frac{10 \tan(c+dx) \sec^2(c+dx)}{3a^2 d(\cos(c+dx)+1)} - \frac{\tan(c+dx) \sec^2(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] $\text{Int}[\text{Sec}[c+d*x]^4/(a+a*\text{Cos}[c+d*x])^2,x]$


```
[Out] (-5*ArcTanh[Sin[c + d*x]]/(a^2*d) + (12*Tan[c + d*x])/(a^2*d) - (5*Sec[c +
d*x]*Tan[c + d*x])/(a^2*d) - (10*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1
+ Cos[c + d*x])) - (Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^
2) + (4*Tan[c + d*x]^3)/(a^2*d)
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]^(n_)), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(6a-4a\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)} dx}{3a^2} \\
&= -\frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad + \frac{\int (36a^2-30a^2\cos(c+dx))\sec^4(c+dx) dx}{3a^4} \\
&= -\frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{10\int \sec^3(c+dx) dx}{a^2} + \frac{12\int \sec^4(c+dx) dx}{a^2} \\
&= -\frac{5\sec(c+dx)\tan(c+dx)}{a^2d} - \frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} \\
&\quad - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{5\int \sec(c+dx) dx}{a^2} \\
&\quad - \frac{12\text{Subst}(\int (1+x^2) dx, x, -\tan(c+dx))}{a^2d} \\
&= -\frac{5\text{arctanh}(\sin(c+dx))}{a^2d} + \frac{12\tan(c+dx)}{a^2d} - \frac{5\sec(c+dx)\tan(c+dx)}{a^2d} \\
&\quad - \frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{4\tan^3(c+dx)}{a^2d}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(133) = 266$.

Time = 2.91 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.58

$$\begin{aligned}
&\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2} dx \\
&= \frac{960\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{a^2d} + \dots
\end{aligned}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (960*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Sec[c
```

$$\begin{aligned} & + d*x]^3*(-3*\text{Sin}[(d*x)/2] + 155*\text{Sin}[(3*d*x)/2] - 153*\text{Sin}[c - (d*x)/2] + 21 \\ & * \text{Sin}[c + (d*x)/2] - 135*\text{Sin}[2*c + (d*x)/2] + 25*\text{Sin}[c + (3*d*x)/2] + 45*\text{Sin} \\ & [2*c + (3*d*x)/2] - 85*\text{Sin}[3*c + (3*d*x)/2] + 99*\text{Sin}[c + (5*d*x)/2] + 21*\text{Si} \\ & n[2*c + (5*d*x)/2] + 33*\text{Sin}[3*c + (5*d*x)/2] - 45*\text{Sin}[4*c + (5*d*x)/2] + 57 \\ & * \text{Sin}[2*c + (7*d*x)/2] + 18*\text{Sin}[3*c + (7*d*x)/2] + 24*\text{Sin}[4*c + (7*d*x)/2] - \\ & 15*\text{Sin}[5*c + (7*d*x)/2] + 24*\text{Sin}[3*c + (9*d*x)/2] + 11*\text{Sin}[4*c + (9*d*x)/2 \\ &] + 13*\text{Sin}[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + \text{Cos}[c + d*x])^2) \end{aligned}$$

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3} + 9 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{2}{3\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} + \frac{3}{\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{10}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1} - 10 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{3\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}\right)}{2da^2}$
default	$\frac{\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3} + 9 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{2}{3\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} + \frac{3}{\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{10}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1} - 10 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{3\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}\right)}{2da^2}$
norman	$-\frac{21 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da} + \frac{80 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3da} - \frac{23 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da} + \frac{4 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da} + \frac{\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)}{6da} + \frac{5 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^2 d} - \frac{1}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) a}$
parallelrisc	$\frac{(90 \cos(dx+c) + 30 \cos(3dx+3c)) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + (-90 \cos(dx+c) - 30 \cos(3dx+3c)) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + 95 \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \cos\left(\frac{3dx}{2} + \frac{3c}{2}\right)\right)}{6a^2 d (\cos(3dx+3c) + 3 \cos(dx+c))}$
risc	$\frac{2i(15 e^{8i(dx+c)} + 45 e^{7i(dx+c)} + 85 e^{6i(dx+c)} + 135 e^{5i(dx+c)} + 153 e^{4i(dx+c)} + 155 e^{3i(dx+c)} + 99 e^{2i(dx+c)} + 57 e^{i(dx+c)} + 24)}{3a^2 d (e^{2i(dx+c)} + 1)^3 (e^{i(dx+c)} + 1)^3}$

[In] int(sec(d*x+c)^4/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+9*tan(1/2*d*x+1/2*c)-2/3/(tan(1/2*d*x+1/2*c)+1)^3+3/(tan(1/2*d*x+1/2*c)+1)^2-10/(tan(1/2*d*x+1/2*c)+1)-10*ln(tan(1/2*d*x+1/2*c)+1)-2/3/(tan(1/2*d*x+1/2*c)-1)^3-3/(tan(1/2*d*x+1/2*c)-1)^2-10/(tan(1/2*d*x+1/2*c)-1)+10*ln(tan(1/2*d*x+1/2*c)-1))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

$$\int \frac{\sec^4(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{15 (\cos(dx + c))^5 + 2 \cos(dx + c)^4 + \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 15 (\cos(dx + c))^5 + 2 \cos(dx + c)^4 + \cos(dx + c)^3 \log(\sin(dx + c) - 1)}{6 (a^2 d \cos(dx + c) + a^2 d)}$$

[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] $-1/6*(15*(\cos(dx + c))^5 + 2*\cos(dx + c)^4 + \cos(dx + c)^3)*\log(\sin(dx + c) + 1) - 15*(\cos(dx + c))^5 + 2*\cos(dx + c)^4 + \cos(dx + c)^3)*\log(-\sin(dx + c) + 1) - 2*(24*\cos(dx + c)^4 + 33*\cos(dx + c)^3 + 6*\cos(dx + c)^2 - \cos(dx + c) + 1)*\sin(dx + c))/(a^2*d*\cos(dx + c)^5 + 2*a^2*d*\cos(dx + c)^4 + a^2*d*\cos(dx + c)^3)$

Sympy [F]

$$\int \frac{\sec^4(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\int \frac{\sec^4(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1} dx}{a^2}$$

[In] integrate(sec(dx+c)**4/(a+a*cos(dx+c))**2,x)

[Out] Integral(sec(c + dx)**4/(cos(c + dx)**2 + 2*cos(c + dx) + 1), x)/a**2

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.76

$$\int \frac{\sec^4(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{4 \left(\frac{9 \sin(dx+c)}{\cos(dx+c)+1} - \frac{20 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right) + \frac{27 \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{30 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^2} + \frac{30 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^2}}{a^2 - \frac{3 a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{6d}{6d}$$

[In] integrate(sec(dx+c)^4/(a+a*cos(dx+c))^2,x, algorithm="maxima")

[Out] $1/6*(4*(9*\sin(dx + c)/(\cos(dx + c) + 1) - 20*\sin(dx + c)^3/(\cos(dx + c) + 1)^3 + 15*\sin(dx + c)^5/(\cos(dx + c) + 1)^5)/(a^2 - 3*a^2*\sin(dx + c)^2/(\cos(dx + c) + 1)^2 + 3*a^2*\sin(dx + c)^4/(\cos(dx + c) + 1)^4 - a^2*\sin(dx + c)^6/(\cos(dx + c) + 1)^6) + (27*\sin(dx + c)/(\cos(dx + c) + 1) + \sin(dx + c)^3/(\cos(dx + c) + 1)^3)/a^2 - 30*\log(\sin(dx + c)/(\cos(dx + c) + 1) + 1)/a^2 + 30*\log(\sin(dx + c)/(\cos(dx + c) + 1) - 1)/a^2)/d$

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(c + dx)}{(a + a \cos(c + dx))^2} dx =$$

$$\frac{30 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a^2} - \frac{30 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a^2} + \frac{4 \left(15 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 20 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 9 \tan(\frac{1}{2} dx + \frac{1}{2} c) \right)}{\left(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1 \right)^3 a^2} - \frac{a^4 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{6 d}$$

[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] -1/6*(30*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 30*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 4*(15*tan(1/2*d*x + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d

Mupad [B] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.15

$$\int \frac{\sec^4(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{6 a^2 d} - \frac{10 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^2 d}$$

$$- \frac{10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 - \frac{40 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3} + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - a^2 \right)} + \frac{9 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2 a^2 d}$$

[In] int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))^2),x)

[Out] tan(c/2 + (d*x)/2)^3/(6*a^2*d) - (10*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (6*tan(c/2 + (d*x)/2) - (40*tan(c/2 + (d*x)/2)^3)/3 + 10*tan(c/2 + (d*x)/2)^5)/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 - a^2)) + (9*tan(c/2 + (d*x)/2))/(2*a^2*d)

3.63 $\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^3} dx$

Optimal result	594
Rubi [A] (verified)	594
Mathematica [A] (verified)	596
Maple [A] (verified)	596
Fricas [A] (verification not implemented)	597
Sympy [B] (verification not implemented)	597
Maxima [A] (verification not implemented)	598
Giac [A] (verification not implemented)	598
Mupad [B] (verification not implemented)	599

Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{13x}{2a^3} - \frac{152 \sin(c+dx)}{15a^3d} + \frac{13 \cos(c+dx) \sin(c+dx)}{2a^3d} - \frac{\cos^4(c+dx) \sin(c+dx)}{5d(a+a \cos(c+dx))^3} - \frac{11 \cos^3(c+dx) \sin(c+dx)}{15ad(a+a \cos(c+dx))^2} - \frac{76 \cos^2(c+dx) \sin(c+dx)}{15d(a^3+a^3 \cos(c+dx))}$$

[Out] $13/2*x/a^3-152/15*\sin(d*x+c)/a^3/d+13/2*\cos(d*x+c)*\sin(d*x+c)/a^3/d-1/5*\cos(d*x+c)^4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^3-11/15*\cos(d*x+c)^3*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^2-76/15*\cos(d*x+c)^2*\sin(d*x+c)/d/(a^3+a^3*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2844, 3056, 2813}

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{152 \sin(c+dx)}{15a^3d} - \frac{76 \sin(c+dx) \cos^2(c+dx)}{15d(a^3 \cos(c+dx) + a^3)} + \frac{13 \sin(c+dx) \cos(c+dx)}{2a^3d} + \frac{13x}{2a^3} - \frac{\sin(c+dx) \cos^4(c+dx)}{5d(a \cos(c+dx) + a)^3} - \frac{11 \sin(c+dx) \cos^3(c+dx)}{15ad(a \cos(c+dx) + a)^2}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^5/(a + a*\text{Cos}[c + d*x])^3, x]$

[Out] $(13x)/(2a^3) - (152\sin[c + dx])/(15a^3d) + (13\cos[c + dx]\sin[c + dx])/(2a^3d) - (\cos[c + dx]^4\sin[c + dx])/(5d(a + a\cos[c + dx])^3) - (11\cos[c + dx]^3\sin[c + dx])/(15ad(a + a\cos[c + dx])^2) - (76\cos[c + dx]^2\sin[c + dx])/(15d(a^3 + a^3\cos[c + dx]))$

Rule 2813

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)x], x_Symbol] \rightarrow \text{Simp}[(2ac + bd)(x/2), x] + (-\text{Simp}[(bc + ad)\cos[e + fx]/f], x) - \text{Simp}[bd\cos[e + fx](\sin[e + fx]/(2f)), x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[bc - ad, 0]$

Rule 2844

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)x]^m, x_Symbol] \rightarrow \text{Simp}[(bc - ad)\cos[e + fx](a + b\sin[e + fx])^m, x] + \text{Dist}[1/(ab(2m + 1)), \text{Int}[(a + b\sin[e + fx])^{m+1}(c + d\sin[e + fx])^{n-2}], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[bc - ad, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{GtQ}[n, 1] \&\& (\text{IntegersQ}[2m, 2n] \mid\mid (\text{IntegerQ}[m] \&\& \text{EqQ}[c, 0]))$

Rule 3056

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)x]^m(A_.) + (B_.)\sin[(e_.) + (f_.)x]^n, x_Symbol] \rightarrow \text{Simp}[(Aa - Ab)\cos[e + fx](a + b\sin[e + fx])^m, x] - \text{Dist}[1/(ab(2m + 1)), \text{Int}[(a + b\sin[e + fx])^{m+1}(c + d\sin[e + fx])^{n-1}], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, x\} \&\& \text{NeQ}[bc - ad, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2] \&\& \text{GtQ}[n, 0] \&\& \text{IntegerQ}[2m] \&\& (\text{IntegerQ}[2n] \mid\mid \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\cos^4(c + dx) \sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{\int \frac{\cos^3(c + dx)(4a - 7a \cos(c + dx))}{(a + a \cos(c + dx))^2} dx}{5a^2} \\ &= -\frac{\cos^4(c + dx) \sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{11 \cos^3(c + dx) \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} - \frac{\int \frac{\cos^2(c + dx)(33a^2 - 43a^2 \cos(c + dx))}{a + a \cos(c + dx)} dx}{15a^4} \\ &= -\frac{\cos^4(c + dx) \sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{11 \cos^3(c + dx) \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} \\ &\quad - \frac{76 \cos^2(c + dx) \sin(c + dx)}{15d(a^3 + a^3 \cos(c + dx))} - \frac{\int \cos(c + dx) (152a^3 - 195a^3 \cos(c + dx)) dx}{15a^6} \end{aligned}$$

$$= \frac{13x}{2a^3} - \frac{152 \sin(c+dx)}{15a^3d} + \frac{13 \cos(c+dx) \sin(c+dx)}{2a^3d} - \frac{\cos^4(c+dx) \sin(c+dx)}{5d(a+a \cos(c+dx))^3} \\ - \frac{11 \cos^3(c+dx) \sin(c+dx)}{15ad(a+a \cos(c+dx))^2} - \frac{76 \cos^2(c+dx) \sin(c+dx)}{15d(a^3+a^3 \cos(c+dx))}$$

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.83

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right) \csc^6(c+dx) \sin^7\left(\frac{1}{2}(c+dx)\right) \left(12480 \arcsin(\cos(c+dx)) \cos^6\left(\frac{1}{2}(c+dx)\right) + (4303 + 6000 \cos(c+dx)) \sqrt{\sin^2(c+dx)}\right)}{15a^3d}$$

[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^3,x]

[Out] -1/15*(Cos[(c + d*x)/2]*Csc[c + d*x]^6*Sin[(c + d*x)/2]^7*(12480*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^6 + (4303 + 6000*Cos[c + d*x] + 1856*Cos[2*(c + d*x)] + 90*Cos[3*(c + d*x)] - 15*Cos[4*(c + d*x)])*Sqrt[Sin[c + d*x]^2]))/(a^3*d*Sqrt[Sin[c + d*x]^2])

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.50

method	result
parallelrisch	$-\frac{1001 \left(\cos(dx+c) + \frac{928 \cos(2dx+2c)}{3003} + \frac{15 \cos(3dx+3c)}{1001} - \frac{5 \cos(4dx+4c)}{2002} + \frac{331}{462} \right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sec^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + \frac{13dx}{2}}{a^3d}$
derivativedivides	$-\frac{\left(\frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{5} + \frac{8 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{3} - 31 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-28 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 20 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + 52 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \right)}{4da^3}$
default	$-\frac{\left(\frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{5} + \frac{8 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{3} - 31 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-28 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 20 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + 52 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \right)}{4da^3}$
risch	$\frac{13x}{2a^3} - \frac{ie^{2i(dx+c)}}{8da^3} + \frac{3ie^{i(dx+c)}}{2da^3} - \frac{3ie^{-i(dx+c)}}{2da^3} + \frac{ie^{-2i(dx+c)}}{8da^3} - \frac{2i(150e^{4i(dx+c)} + 525e^{3i(dx+c)} + 745e^{2i(dx+c)} + 480e^{i(dx+c)} + 150)}{15da^3(e^{i(dx+c)} + 1)^5}$
norman	$\frac{13x}{2a} - \frac{51 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4da} - \frac{721 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{12da} - \frac{6613 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{60da} - \frac{1165 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{12da} - \frac{475 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{12da} - \frac{59 \left(\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{12da}$

[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] 13/160*(-77*(cos(d*x+c)+928/3003*cos(2*d*x+2*c)+15/1001*cos(3*d*x+3*c)-5/2002*cos(4*d*x+4*c)+331/462)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4+80*d*x)/a^3/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= \frac{195 dx \cos(dx + c)^3 + 585 dx \cos(dx + c)^2 + 585 dx \cos(dx + c) + 195 dx + (15 \cos(dx + c)^4 - 45 \cos(dx + c)^3 + 30 a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 + 3 a^3 d \cos(dx + c) + a^3 d)}{30 (a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 + 3 a^3 d \cos(dx + c) + a^3 d)}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/30*(195*d*x*cos(d*x + c)^3 + 585*d*x*cos(d*x + c)^2 + 585*d*x*cos(d*x + c) + 195*d*x + (15*cos(d*x + c)^4 - 45*cos(d*x + c)^3 - 479*cos(d*x + c)^2 - 717*cos(d*x + c) - 304)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(143) = 286.

Time = 3.89 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.09

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= \begin{cases} \frac{390dx \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right)}{60a^3d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 120a^3d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 60a^3d} + \frac{780dx \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{60a^3d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 120a^3d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 60a^3d} + \frac{390dx}{60a^3d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 120a^3d} \\ \frac{x \cos^5(c)}{(a \cos(c) + a)^3} \end{cases}$$

[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**3,x)

[Out] Piecewise((390*d*x*tan(c/2 + d*x/2)**4/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 780*d*x*tan(c/2 + d*x/2)**2/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 390*d*x/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 3*tan(c/2 + d*x/2)**9/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) + 34*tan(c/2 + d*x/2)**7/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 388*tan(c/2 + d*x/2)**5/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 1310*tan(c/2 + d*x/2)**3/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d) - 765*tan(c/2 + d*x/2)/(60*a**3*d*tan(c/2 + d*x/2)**4 + 120*a**3*d*tan(c/2 + d*x/2)**2 + 60*a**3*d), Ne(d, 0)), (x*cos(c)**5/(a*cos(c) + a)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.20

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= -\frac{60 \left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} + \frac{7 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) + \frac{465 \sin(dx+c)}{\cos(dx+c)+1} - \frac{40 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{780 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60 d}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= -\frac{\frac{390(dx+c)}{a^3} - \frac{60 \left(7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1 \right)^2 a^3} - \frac{3 a^{12} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 40 a^{12} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 465 a^{12} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^{15}}}{60 d}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="giac")

```
[Out] 1/60*(390*(d*x + c)/a^3 - 60*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5 - 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 14.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^3} dx =$$

$$\frac{3 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 46 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 508 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 420 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 120 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^8 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 390 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (c + dx)}{60 a^3 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^5}$$

[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x))^3,x)

```
[Out] -(3*sin(c/2 + (d*x)/2) - 46*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 508*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) - 120*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) - 390*cos(c/2 + (d*x)/2)^5*(c + d*x))/(60*a^3*d*cos(c/2 + (d*x)/2)^5)
```

3.64 $\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^3} dx$

Optimal result	600
Rubi [A] (verified)	600
Mathematica [A] (verified)	602
Maple [A] (verified)	603
Fricas [A] (verification not implemented)	603
Sympy [B] (verification not implemented)	604
Maxima [A] (verification not implemented)	604
Giac [A] (verification not implemented)	605
Mupad [B] (verification not implemented)	605

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{3x}{a^3} + \frac{9 \sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx) \sin(c+dx)}{5d(a+a \cos(c+dx))^3} - \frac{3 \cos^2(c+dx) \sin(c+dx)}{5ad(a+a \cos(c+dx))^2} + \frac{3 \sin(c+dx)}{d(a^3+a^3 \cos(c+dx))}$$

[Out] $-3*x/a^3+9/5*\sin(d*x+c)/a^3/d-1/5*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^3-3/5*\cos(d*x+c)^2*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^2+3*\sin(d*x+c)/d/(a^3+a^3*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2844, 3056, 3047, 3102, 12, 2814, 2727}

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{9 \sin(c+dx)}{5a^3d} + \frac{3 \sin(c+dx)}{d(a^3 \cos(c+dx) + a^3)} - \frac{3x}{a^3} - \frac{\sin(c+dx) \cos^3(c+dx)}{5d(a \cos(c+dx) + a)^3} - \frac{3 \sin(c+dx) \cos^2(c+dx)}{5ad(a \cos(c+dx) + a)^2}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^4/(a + a*\text{Cos}[c + d*x])^3, x]$

[Out] $(-3*x)/a^3 + (9*\text{Sin}[c + d*x])/(5*a^3*d) - (\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(5*d*(a + a*\text{Cos}[c + d*x])^3) - (3*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(5*a*d*(a + a*\text{Cos}[c + d*x])^2) + (3*\text{Sin}[c + d*x])/(d*(a^3 + a^3*\text{Cos}[c + d*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/(c_ + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3047

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_ + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_ + (d_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3102

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(3a-6a\cos(c+dx))}{(a+a\cos(c+dx))^2} dx}{5a^2} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)(18a^2-27a^2\cos(c+dx))}{a+a\cos(c+dx)} dx}{15a^4} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{18a^2\cos(c+dx)-27a^2\cos^2(c+dx)}{a+a\cos(c+dx)} dx}{15a^4} \\
&= \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{45a^3\cos(c+dx)}{a+a\cos(c+dx)} dx}{15a^5} \\
&= \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{3\int \frac{\cos(c+dx)}{a+a\cos(c+dx)} dx}{a^2} \\
&= -\frac{3x}{a^3} + \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
&\quad - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{3\int \frac{1}{a+a\cos(c+dx)} dx}{a^2} \\
&= -\frac{3x}{a^3} + \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
&\quad - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{3\sin(c+dx)}{d(a^3+a^3\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\begin{aligned}
&\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3} dx \\
&= \frac{\sin(c+dx) \left(120 \arcsin(\cos(c+dx)) \cos^6\left(\frac{1}{2}(c+dx)\right) + (24 + 57 \cos(c+dx) + 39 \cos^2(c+dx) + 5 \cos^3(c+dx)) \sqrt{1 - \cos(c+dx)} \right)}{5a^3d\sqrt{1 - \cos(c+dx)}(1 + \cos(c+dx))^{7/2}}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^3,x]

[Out] (Sin[c + d*x]*(120*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^6 + (24 + 57*Cos[c + d*x] + 39*Cos[c + d*x]^2 + 5*Cos[c + d*x]^3)*Sqrt[Sin[c + d*x]^2]))/(5*a^3*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(7/2))

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.55

method	result
parallelrisch	$\frac{243 \left(\cos(dx+c) + \frac{26 \cos(2dx+2c)}{81} + \frac{5 \cos(3dx+3c)}{243} + \frac{58}{81} \right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sec^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 3dx}{a^3 d}$
derivativedivides	$\frac{\left(\frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{5} \right) - 2 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + 17 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{8 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} - 24 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d a^3}$
default	$\frac{\left(\frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{5} \right) - 2 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + 17 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{8 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} - 24 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{4d a^3}$
risch	$-\frac{3x}{a^3} - \frac{ie^{i(dx+c)}}{2da^3} + \frac{ie^{-i(dx+c)}}{2da^3} + \frac{4i(15e^{4i(dx+c)} + 50e^{3i(dx+c)} + 70e^{2i(dx+c)} + 45e^{i(dx+c)} + 12)}{5da^3(e^{i(dx+c)} + 1)^5}$
norman	$-\frac{3x}{a} + \frac{25 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4da} + \frac{45 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{2da} + \frac{591 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{20da} + \frac{81 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{5da} + \frac{51 \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{20da} - \frac{3 \left(\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{10da} \frac{1}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^4 a^2}$

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] 3/80*(81*(cos(d*x+c)+26/81*cos(2*d*x+2*c))+5/243*cos(3*d*x+3*c)+58/81)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4-80*d*x/a^3/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{15 dx \cos(dx + c)^3 + 45 dx \cos(dx + c)^2 + 45 dx \cos(dx + c) + 15 dx - (5 \cos(dx + c)^3 + 39 \cos(dx + c) + 24) \sin(dx + c)}{5 (a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 + 3 a^3 d \cos(dx + c) + a^3 d)}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] -1/5*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) + 15*d*x - (5*cos(d*x + c)^3 + 39*cos(d*x + c)^2 + 57*cos(d*x + c) + 24)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(109) = 218.

Time = 2.38 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.02

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= \begin{cases} -\frac{60dx \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{20a^3 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 20a^3 d} - \frac{60dx}{20a^3 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 20a^3 d} + \frac{\tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{20a^3 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 20a^3 d} - \frac{9 \tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{20a^3 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 20a^3 d} + \frac{75 \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{20a^3 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 20a^3 d} \\ \frac{x \cos^4(c)}{(a \cos(c) + a)^3} \end{cases}$$

[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**3,x)

[Out] Piecewise((-60*d*x*tan(c/2 + d*x/2)**2/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) - 60*d*x/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + tan(c/2 + d*x/2)**7/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) - 9*tan(c/2 + d*x/2)**5/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + 75*tan(c/2 + d*x/2)**3/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + 125*tan(c/2 + d*x/2)/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= \frac{\frac{40 \sin(dx+c)}{\left(a^3 + \frac{a^3 \sin^2(dx+c)}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} + \frac{\frac{85 \sin(dx+c)}{\cos(dx+c)+1} - \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{120 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{20 d}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] 1/20*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= -\frac{\frac{60(dx+c)}{a^3} - \frac{40 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1)a^3} - \frac{a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 10 a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 85 a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a^{15}}}{20 d}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="giac")

```
[Out] -1/20*(60*(d*x + c)/a^3 - 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 - 10*a^12*tan(1/2*d*x + 1/2*c)^3 + 85*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= \frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 12 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 96 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 40 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{20 a^3 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^5}$$

[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^3,x)

```
[Out] (sin(c/2 + (d*x)/2) - 12*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 96*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 40*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) - 60*cos(c/2 + (d*x)/2)^5*(c + d*x))/(20*a^3*d*cos(c/2 + (d*x)/2)^5)
```

3.65 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3} dx$

Optimal result	606
Rubi [A] (verified)	606
Mathematica [A] (verified)	608
Maple [A] (verified)	608
Fricas [A] (verification not implemented)	609
Sympy [A] (verification not implemented)	609
Maxima [A] (verification not implemented)	609
Giac [A] (verification not implemented)	610
Mupad [B] (verification not implemented)	610

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{x}{a^3} - \frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{29\sin(c+dx)}{15d(a^3+a^3\cos(c+dx))}$$

[Out] $x/a^3 - 1/5*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^3 + 7/15*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^2 - 29/15*\sin(d*x+c)/d/(a^3+a^3*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2844, 3047, 3098, 2814, 2727}

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3} dx = -\frac{29\sin(c+dx)}{15d(a^3\cos(c+dx)+a^3)} + \frac{x}{a^3} - \frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3} + \frac{7\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

[In] $\text{Int}[\text{Cos}[c+d*x]^3/(a+a*\text{Cos}[c+d*x])^3, x]$

[Out] $x/a^3 - (\text{Cos}[c+d*x]^2*\text{Sin}[c+d*x])/(5*d*(a+a*\text{Cos}[c+d*x])^3) + (7*\text{Sin}[c+d*x])/(15*a*d*(a+a*\text{Cos}[c+d*x])^2) - (29*\text{Sin}[c+d*x])/(15*d*(a^3+a^3*\text{Cos}[c+d*x]))$

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3047

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3098

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(A*b - a*B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\cos^2(c + dx) \sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{\int \frac{\cos(c+dx)(2a-5a \cos(c+dx))}{(a+a \cos(c+dx))^2} dx}{5a^2} \\ &= -\frac{\cos^2(c + dx) \sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{\int \frac{2a \cos(c+dx)-5a \cos^2(c+dx)}{(a+a \cos(c+dx))^2} dx}{5a^2} \end{aligned}$$

[Out] $1/60*(-3*\tan(1/2*d*x+1/2*c)^5+20*\tan(1/2*d*x+1/2*c)^3+60*d*x-105*\tan(1/2*d*x+1/2*c))/a^3/d$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.21

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{15 dx \cos(dx+c)^3 + 45 dx \cos(dx+c)^2 + 45 dx \cos(dx+c) + 15 dx - (32 \cos(dx+c)^2 + 51 \cos(dx+c) + 22) \sin(dx+c)}{15 (a^3 d \cos(dx+c)^3 + 3 a^3 d \cos(dx+c)^2 + 3 a^3 d \cos(dx+c) + a^3 d)}$$

[In] `integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] $1/15*(15*d*x*\cos(d*x+c)^3 + 45*d*x*\cos(d*x+c)^2 + 45*d*x*\cos(d*x+c) + 15*d*x - (32*\cos(d*x+c)^2 + 51*\cos(d*x+c) + 22)*\sin(d*x+c))/(a^3*d*\cos(d*x+c)^3 + 3*a^3*d*\cos(d*x+c)^2 + 3*a^3*d*\cos(d*x+c) + a^3*d)$

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3} dx = \begin{cases} \frac{x}{a^3} - \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{20a^3d} + \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{3a^3d} - \frac{7\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x \cos^3(c)}{(a \cos(c)+a)^3} & \text{otherwise} \end{cases}$$

[In] `integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**3,x)`

[Out] `Piecewise((x/a**3 - tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)**3/(3*a**3*d) - 7*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a)**3, True))`

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3} dx = -\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{20 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60 d} - \frac{120 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}$$

[In] `integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="maxima")`

[Out] $-1/60*((105*\sin(d*x+c)/(\cos(d*x+c)+1) - 20*\sin(d*x+c)^3/(\cos(d*x+c)+1)^3 + 3*\sin(d*x+c)^5/(\cos(d*x+c)+1)^5)/a^3 - 120*\arctan(\sin(d*x+c)/(\cos(d*x+c)+1))/a^3/d$

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.71

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{\frac{60(dx+c)}{a^3} - \frac{3a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 20a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 105a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a^{15}}}{60d}$$

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*a^12*tan(1/2*d*x + 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{x}{a^3} - \frac{\frac{32 \sin(\frac{c}{2} + \frac{dx}{2}) \cos(\frac{c}{2} + \frac{dx}{2})^4}{15} - \frac{13 \sin(\frac{c}{2} + \frac{dx}{2}) \cos(\frac{c}{2} + \frac{dx}{2})^2}{30} + \frac{\sin(\frac{c}{2} + \frac{dx}{2})}{20}}{a^3 d \cos(\frac{c}{2} + \frac{dx}{2})^5}$$

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^3,x)
```

```
[Out] x/a^3 - (sin(c/2 + (d*x)/2)/20 - (13*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2))/30 + (32*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/15)/(a^3*d*cos(c/2 + (d*x)/2)^5)
```

3.66 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3} dx$

Optimal result	611
Rubi [A] (verified)	611
Mathematica [A] (verified)	612
Maple [A] (verified)	613
Fricas [A] (verification not implemented)	613
Sympy [A] (verification not implemented)	614
Maxima [A] (verification not implemented)	614
Giac [A] (verification not implemented)	614
Mupad [B] (verification not implemented)	615

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{7\sin(c+dx)}{15d(a^3+a^3\cos(c+dx))}$$

[Out] 1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-8/15*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+7/15*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2837, 2829, 2727}

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{7\sin(c+dx)}{15d(a^3\cos(c+dx)+a^3)} - \frac{8\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2} + \frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]

[Out] Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) - (8*Sin[c + d*x])/((15*a*d*(a + a*Cos[c + d*x])^2) + (7*Sin[c + d*x]))/(15*d*(a^3 + a^3*Cos[c + d*x]))

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b

$^2, 0]$

Rule 2829

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2837

```
Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
+ 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{\int \frac{-3a + 5a \cos(c + dx)}{(a + a \cos(c + dx))^2} dx}{5a^2} \\ &= \frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{8 \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} + \frac{7 \int \frac{1}{a + a \cos(c + dx)} dx}{15a^2} \\ &= \frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{8 \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} + \frac{7 \sin(c + dx)}{15d(a^3 + a^3 \cos(c + dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{(2 + 6 \cos(c + dx) + 7 \cos^2(c + dx)) \sin(c + dx)}{15a^3d(1 + \cos(c + dx))^3}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ((2 + 6*Cos[c + d*x] + 7*Cos[c + d*x]^2)*Sin[c + d*x])/(15*a^3*d*(1 + Cos[c
+ d*x])^3)
```


Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{\left(\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da^3}$	45
default	$\frac{\left(\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da^3}$	45
parallelrisc	$\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60a^3d}$	47
risc	$\frac{2i\left(15e^{4i(dx+c)}+30e^{3i(dx+c)}+40e^{2i(dx+c)}+20e^{i(dx+c)}+7\right)}{15da^3\left(e^{i(dx+c)}+1\right)^5}$	69
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3da}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{30da}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{15da}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}}{a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	114

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3} dx$$

$$= \frac{(7\cos(dx+c)^2+6\cos(dx+c)+2)\sin(dx+c)}{15(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d)}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/15*(7*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^3} dx = \begin{cases} \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{20a^3d} - \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{6a^3d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x \cos^2(c)}{(a \cos(c) + a)^3} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**3,x)

[Out] Piecewise((tan(c/2 + d*x/2)**5/(20*a**3*d) - tan(c/2 + d*x/2)**3/(6*a**3*d) + tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{\frac{15 \sin(dx+c)}{\cos(dx+c)+1} - \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60 a^3 d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 10 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 15 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{60 a^3 d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 - 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x + 1/2*c))/(a^3*d)

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 15\right)}{60 a^3 d}$$

[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^3,x)

[Out] (tan(c/2 + (d*x)/2)*(3*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 15)/(60*a^3*d)

3.67 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3} dx$

Optimal result	616
Rubi [A] (verified)	616
Mathematica [A] (verified)	617
Maple [A] (verified)	618
Fricas [A] (verification not implemented)	618
Sympy [A] (verification not implemented)	619
Maxima [A] (verification not implemented)	619
Giac [A] (verification not implemented)	619
Mupad [B] (verification not implemented)	620

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3} dx = -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{\sin(c+dx)}{5d(a^3+a^3\cos(c+dx))}$$

[Out] $-1/5*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^3+1/5*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^2+1/5*\sin(d*x+c)/d/(a^3+a^3*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2829, 2729, 2727}

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{\sin(c+dx)}{5d(a^3\cos(c+dx)+a^3)} + \frac{\sin(c+dx)}{5ad(a\cos(c+dx)+a)^2} - \frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^3,x]

[Out] $-1/5*\text{Sin}[c + d*x]/(d*(a + a*\text{Cos}[c + d*x])^3) + \text{Sin}[c + d*x]/(5*a*d*(a + a*\text{Cos}[c + d*x])^2) + \text{Sin}[c + d*x]/(5*d*(a^3 + a^3*\text{Cos}[c + d*x]))$

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b

$\wedge 2, 0]$

Rule 2729

`Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]`

Rule 2829

`Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]`

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{3 \int \frac{1}{(a + a \cos(c + dx))^2} dx}{5a} \\ &= -\frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{\sin(c + dx)}{5ad(a + a \cos(c + dx))^2} + \frac{\int \frac{1}{a + a \cos(c + dx)} dx}{5a^2} \\ &= -\frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{\sin(c + dx)}{5ad(a + a \cos(c + dx))^2} + \frac{\sin(c + dx)}{5d(a^3 + a^3 \cos(c + dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.53

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{(1 + 3 \cos(c + dx) + \cos^2(c + dx)) \sin(c + dx)}{5a^3 d (1 + \cos(c + dx))^3}$$

[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^3,x]

[Out] ((1 + 3*Cos[c + d*x] + Cos[c + d*x]^2)*Sin[c + d*x])/(5*a^3*d*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

method	result	size
parallelrisc	$-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)-5\right)}{20a^3d}$	31
derivativedivides	$-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da^3}$	32
default	$-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da^3}$	32
risc	$\frac{2i\left(5e^{3i(dx+c)}+5e^{2i(dx+c)}+5e^{i(dx+c)}+1\right)}{5da^3\left(e^{i(dx+c)}+1\right)^5}$	58
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}$	95

[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] -1/20*tan(1/2*d*x+1/2*c)*(tan(1/2*d*x+1/2*c)^4-5)/a^3/d

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3} dx$$

$$= \frac{(\cos(dx+c)^2+3\cos(dx+c)+1)\sin(dx+c)}{5(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d)}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/5*(cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.58

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^3} dx = \begin{cases} -\frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{20a^3d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x \cos(c)}{(a \cos(c) + a)^3} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**3,x)

[Out] Piecewise((-tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*cos(c)/(a*cos(c) + a)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{\frac{5 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{20 a^3 d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] 1/20*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^3} dx = -\frac{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{20 a^3 d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] -1/20*(tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c))/(a^3*d)

Mupad [B] (verification not implemented)

Time = 14.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.36

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^3} dx = -\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 5\right)}{20 a^3 d}$$

[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^3,x)

[Out] -(tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^4 - 5))/(20*a^3*d)

$$3.68 \quad \int \frac{1}{(a+a \cos(c+dx))^3} dx$$

Optimal result	621
Rubi [A] (verified)	621
Mathematica [A] (verified)	622
Maple [A] (verified)	622
Fricas [A] (verification not implemented)	623
Sympy [A] (verification not implemented)	623
Maxima [A] (verification not implemented)	624
Giac [A] (verification not implemented)	624
Mupad [B] (verification not implemented)	624

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \frac{1}{(a+a \cos(c+dx))^3} dx = \frac{\sin(c+dx)}{5d(a+a \cos(c+dx))^3} + \frac{2 \sin(c+dx)}{15ad(a+a \cos(c+dx))^2} + \frac{2 \sin(c+dx)}{15d(a^3+a^3 \cos(c+dx))}$$

[Out] 1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3+2/15*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2+2/15*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2729, 2727}

$$\int \frac{1}{(a+a \cos(c+dx))^3} dx = \frac{2 \sin(c+dx)}{15d(a^3 \cos(c+dx) + a^3)} + \frac{2 \sin(c+dx)}{15ad(a \cos(c+dx) + a)^2} + \frac{\sin(c+dx)}{5d(a \cos(c+dx) + a)^3}$$

[In] Int[(a + a*cos[c + d*x])^(-3), x]

[Out] Sin[c + d*x]/(5*d*(a + a*cos[c + d*x])^3) + (2*Sin[c + d*x])/(15*a*d*(a + a*cos[c + d*x])^2) + (2*Sin[c + d*x])/(15*d*(a^3 + a^3*cos[c + d*x]))

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b

$^2, 0]$

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{2 \int \frac{1}{(a + a \cos(c + dx))^2} dx}{5a} \\ &= \frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{2 \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} + \frac{2 \int \frac{1}{a + a \cos(c + dx)} dx}{15a^2} \\ &= \frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{2 \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} + \frac{2 \sin(c + dx)}{15d(a^3 + a^3 \cos(c + dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\begin{aligned} &\int \frac{1}{(a + a \cos(c + dx))^3} dx \\ &= \frac{\cos\left(\frac{1}{2}(c + dx)\right) \left(10 \sin\left(\frac{1}{2}(c + dx)\right) + 5 \sin\left(\frac{3}{2}(c + dx)\right) + \sin\left(\frac{5}{2}(c + dx)\right)\right)}{15a^3d(1 + \cos(c + dx))^3} \end{aligned}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(-3),x]
```

```
[Out] (Cos[(c + d*x)/2]*(10*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + Sin[(5*(c
+ d*x))/2]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{\left(\frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{5}\right) + \frac{2\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4da^3}}$	45
default	$\frac{\left(\frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{5}\right) + \frac{2\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4da^3}}$	45
risch	$\frac{4i(10e^{2i(dx+c)} + 5e^{i(dx+c)} + 1)}{15da^3(e^{i(dx+c)} + 1)^5}$	47
parallelrisch	$\frac{3\left(\frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) + 10\left(\frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) + 15\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{60a^3d}$	47
norman	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4da} + \frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{6da} + \frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{20da}$	61

[In] `int(1/(a+cos(d*x+c))*a^3,x,method=_RETURNVERBOSE)`

[Out] $1/4/d/a^3*(1/5*\tan(1/2*d*x+1/2*c)^5+2/3*\tan(1/2*d*x+1/2*c)^3+\tan(1/2*d*x+1/2*c))$

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a + a \cos(c + dx))^3} dx$$

$$= \frac{(2 \cos(dx + c)^2 + 6 \cos(dx + c) + 7) \sin(dx + c)}{15 (a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 + 3 a^3 d \cos(dx + c) + a^3 d)}$$

[In] `integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] $1/15*(2*\cos(d*x + c)^2 + 6*\cos(d*x + c) + 7)*\sin(d*x + c)/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d)$

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a + a \cos(c + dx))^3} dx = \begin{cases} \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{20a^3d} + \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{6a^3d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x}{(a \cos(c) + a)^3} & \text{otherwise} \end{cases}$$

[In] `integrate(1/(a+a*cos(d*x+c))**3,x)`

[Out] `Piecewise((tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)**3/(6*a**3*d) + tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x/(a*cos(c) + a)**3, True))`

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a + a \cos(c + dx))^3} dx = \frac{\frac{15 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60 a^3 d}$$

[In] integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a + a \cos(c + dx))^3} dx = \frac{3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 10 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 15 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{60 a^3 d}$$

[In] integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 + 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x + 1/2*c))/(a^3*d)

Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{1}{(a + a \cos(c + dx))^3} dx = \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 15\right)}{60 a^3 d}$$

[In] int(1/(a + a*cos(c + d*x))^3,x)

[Out] (tan(c/2 + (d*x)/2)*(10*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + 15))/(60*a^3*d)

3.69 $\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^3} dx$

Optimal result	625
Rubi [A] (verified)	625
Mathematica [B] (verified)	627
Maple [A] (verified)	627
Fricas [A] (verification not implemented)	628
Sympy [F]	628
Maxima [A] (verification not implemented)	628
Giac [A] (verification not implemented)	629
Mupad [B] (verification not implemented)	629

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^3 d} - \frac{\sin(c+dx)}{5d(a+a \cos(c+dx))^3} - \frac{7 \sin(c+dx)}{15ad(a+a \cos(c+dx))^2} - \frac{22 \sin(c+dx)}{15d(a^3+a^3 \cos(c+dx))}$$

[Out] $\operatorname{arctanh}(\sin(d*x+c))/a^3/d-1/5*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^3-7/15*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^2-22/15*\sin(d*x+c)/d/(a^3+a^3*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2845, 3057, 12, 3855}

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^3 d} - \frac{22 \sin(c+dx)}{15d(a^3 \cos(c+dx) + a^3)} - \frac{7 \sin(c+dx)}{15ad(a \cos(c+dx) + a)^2} - \frac{\sin(c+dx)}{5d(a \cos(c+dx) + a)^3}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]/(a+a*\operatorname{Cos}[c+d*x])^3,x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]]/(a^3*d) - \operatorname{Sin}[c+d*x]/(5*d*(a+a*\operatorname{Cos}[c+d*x])^3) - (7*\operatorname{Sin}[c+d*x])/(15*a*d*(a+a*\operatorname{Cos}[c+d*x])^2) - (22*\operatorname{Sin}[c+d*x])/(15*d*(a^3+a^3*\operatorname{Cos}[c+d*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3855

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{\int \frac{(5a - 2a \cos(c + dx)) \sec(c + dx)}{(a + a \cos(c + dx))^2} dx}{5a^2} \\
 &= -\frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{7 \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} + \frac{\int \frac{(15a^2 - 7a^2 \cos(c + dx)) \sec(c + dx)}{a + a \cos(c + dx)} dx}{15a^4} \\
 &= -\frac{\sin(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{7 \sin(c + dx)}{15ad(a + a \cos(c + dx))^2} \\
 &\quad - \frac{22 \sin(c + dx)}{15d(a^3 + a^3 \cos(c + dx))} + \frac{\int 15a^3 \sec(c + dx) dx}{15a^6}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
&\quad - \frac{22\sin(c+dx)}{15d(a^3+a^3\cos(c+dx))} + \frac{\int \sec(c+dx) dx}{a^3} \\
&= \frac{\operatorname{arctanh}(\sin(c+dx))}{a^3d} - \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
&\quad - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{22\sin(c+dx)}{15d(a^3+a^3\cos(c+dx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(97) = 194.

Time = 0.55 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.07

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(60\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{15a^3d(1+\cos(c+dx))^3}$$

[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^3,x]

[Out] (-2*Cos[(c + d*x)/2]*(60*Cos[(c + d*x)/2]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*Sec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 88*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]*Tan[c/2] + 14*Cos[(c + d*x)/2]^3*Tan[c/2))/(15*a^3*d*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{\left(\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}\right)-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4da^3}$	75
default	$-\frac{\left(\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}\right)-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{4da^3}$	75
parallelrisch	$\frac{-3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-20\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-60\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+60\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-105\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60a^3d}$	75
norman	$\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3da}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3d}$	101
risch	$-\frac{2i\left(15e^{4i(dx+c)}+75e^{3i(dx+c)}+145e^{2i(dx+c)}+95e^{i(dx+c)}+22\right)}{15da^3\left(e^{i(dx+c)}+1\right)^5}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{a^3d}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{da^3}$	111

[In] `int(sec(d*x+c)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4}d/a^3(-1/5*\tan(1/2*d*x+1/2*c)^5-4/3*\tan(1/2*d*x+1/2*c)^3-7*\tan(1/2*d*x+1/2*c)-4*\ln(\tan(1/2*d*x+1/2*c)-1)+4*\ln(\tan(1/2*d*x+1/2*c)+1))$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.63

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3} dx$$

$$= \frac{15(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1)\log(\sin(dx+c)+1)-15(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1)\log(-\sin(dx+c)+1)-2(22\cos(dx+c)^2+51\cos(dx+c)+32)\sin(dx+c)}{30(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d)}$$

[In] `integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

[Out] $\frac{1}{30}*(15*(\cos(dx+c)^3+3*\cos(dx+c)^2+3*\cos(dx+c)+1)*\log(\sin(dx+c)+1)-15*(\cos(dx+c)^3+3*\cos(dx+c)^2+3*\cos(dx+c)+1)*\log(-\sin(dx+c)+1)-2*(22*\cos(dx+c)^2+51*\cos(dx+c)+32)*\sin(dx+c))/(a^3*d*\cos(dx+c)^3+3*a^3*d*\cos(dx+c)^2+3*a^3*d*\cos(dx+c)+a^3*d)$

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3} dx = \int \frac{\frac{\sec(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1} dx}{a^3}$$

[In] `integrate(sec(d*x+c)/(a+a*cos(d*x+c))**3,x)`

[Out] `Integral(sec(c+d*x)/(cos(c+d*x)**3+3*cos(c+d*x)**2+3*cos(c+d*x)+1),x)/a**3`

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3} dx$$

$$= -\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1} + \frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} + \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}$$

60 d

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out]
$$-1/60*((105*\sin(d*x + c)/(\cos(d*x + c) + 1) + 20*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/a^3 - 60*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a^3 + 60*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a^3)/d$$

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= \frac{60 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a^3} - \frac{60 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a^3} - \frac{3 a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 20 a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 105 a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a^{15}}}{60 d}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out]
$$1/60*(60*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/a^3 - (3*a^{12}*\tan(1/2*d*x + 1/2*c)^5 + 20*a^{12}*\tan(1/2*d*x + 1/2*c)^3 + 105*a^{12}*\tan(1/2*d*x + 1/2*c))/a^{15})/d$$

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.60

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= -\frac{105 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 120 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) + 20 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{60 a^3 d}$$

[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^3),x)

[Out]
$$-(105*\tan(c/2 + (d*x)/2) - 120*\operatorname{atanh}(\tan(c/2 + (d*x)/2)) + 20*\tan(c/2 + (d*x)/2)^3 + 3*\tan(c/2 + (d*x)/2)^5)/(60*a^3*d)$$

3.70 $\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^3} dx$

Optimal result	630
Rubi [A] (verified)	630
Mathematica [B] (verified)	632
Maple [A] (verified)	633
Fricas [A] (verification not implemented)	633
Sympy [F]	634
Maxima [A] (verification not implemented)	634
Giac [A] (verification not implemented)	634
Mupad [B] (verification not implemented)	635

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{3\operatorname{arctanh}(\sin(c+dx))}{a^3d} + \frac{24 \tan(c+dx)}{5a^3d} - \frac{\tan(c+dx)}{5d(a+a \cos(c+dx))^3} - \frac{3 \tan(c+dx)}{5ad(a+a \cos(c+dx))^2} - \frac{3 \tan(c+dx)}{d(a^3+a^3 \cos(c+dx))}$$

[Out] $-3*\operatorname{arctanh}(\sin(d*x+c))/a^3/d+24/5*\tan(d*x+c)/a^3/d-1/5*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^3-3/5*\tan(d*x+c)/a/d/(a+a*\cos(d*x+c))^2-3*\tan(d*x+c)/d/(a^3+a^3*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2845, 3057, 2827, 3852, 8, 3855}

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{3\operatorname{arctanh}(\sin(c+dx))}{a^3d} + \frac{24 \tan(c+dx)}{5a^3d} - \frac{3 \tan(c+dx)}{d(a^3 \cos(c+dx) + a^3)} - \frac{3 \tan(c+dx)}{5ad(a \cos(c+dx) + a)^2} - \frac{\tan(c+dx)}{5d(a \cos(c+dx) + a)^3}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]^2/(a+a*\operatorname{Cos}[c+d*x])^3,x]$

[Out] $(-3 \operatorname{ArcTanh}[\sin[c + dx]])/(a^3 d) + (24 \tan[c + dx])/(5 a^3 d) - \tan[c + dx]/(5 d (a + a \cos[c + dx])^3) - (3 \tan[c + dx])/(5 a d (a + a \cos[c + dx])^2) - (3 \tan[c + dx])/(d (a^3 + a^3 \cos[c + dx]))$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2827

$\operatorname{Int}[(b_)\sin[e_ + f x] + (f_)(x_)]^{(m_)}((c_ + (d_)\sin[e_ + f x] + (f_)(x_))^{(n_)}], x_Symbol] \rightarrow \operatorname{Dist}[c, \operatorname{Int}[(b \sin[e + f x])^m, x], x] + \operatorname{Dist}[d/b, \operatorname{Int}[(b \sin[e + f x])^{m+1}, x], x] /; \operatorname{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2845

$\operatorname{Int}[(a_ + (b_)\sin[e_ + f x] + (f_)(x_)]^{(m_)}((c_ + (d_)\sin[e_ + f x] + (f_)(x_))^{(n_)}], x_Symbol] \rightarrow \operatorname{Simp}[b^2 \cos[e + f x] (a + b \sin[e + f x])^m (c + d \sin[e + f x])^{n+1} / (a f (2m+1) (b c - a d)), x] + \operatorname{Dist}[1 / (a (2m+1) (b c - a d)), \operatorname{Int}[(a + b \sin[e + f x])^{m+1} (c + d \sin[e + f x])^n \operatorname{Simp}[b c (m+1) - a d (2m+n+2) + b d (m+n+2) \sin[e + f x], x], x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \operatorname{NeQ}[b c - a d, 0] \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{NeQ}[c^2 - d^2, 0] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{!GtQ}[n, 0] \&\& (\operatorname{IntegerSqrt}[2m, 2n] \mid\mid (\operatorname{IntegerQ}[m] \&\& \operatorname{EqQ}[c, 0]))$

Rule 3057

$\operatorname{Int}[(a_ + (b_)\sin[e_ + f x] + (f_)(x_)]^{(m_)}((A_ + (B_)\sin[e_ + f x] + (f_)(x_))^{(n_)}], x_Symbol] \rightarrow \operatorname{Simp}[b (A b - a B) \cos[e + f x] (a + b \sin[e + f x])^m (c + d \sin[e + f x])^{n+1} / (a f (2m+1) (b c - a d)), x] + \operatorname{Dist}[1 / (a (2m+1) (b c - a d)), \operatorname{Int}[(a + b \sin[e + f x])^{m+1} (c + d \sin[e + f x])^n \operatorname{Simp}[B (a c m + b d (n+1)) + A (b c (m+1) - a d (2m+n+2)) + d (A b - a B) (m+n+2) \sin[e + f x], x], x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \operatorname{NeQ}[b c - a d, 0] \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{NeQ}[c^2 - d^2, 0] \&\& \operatorname{LtQ}[m, -2^{(-1)}] \&\& \operatorname{!GtQ}[n, 0] \&\& \operatorname{IntegerQ}[2m] \&\& (\operatorname{IntegerQ}[2n] \mid\mid \operatorname{EqQ}[c, 0])$

Rule 3852

$\operatorname{Int}[\operatorname{csc}[(c_ + (d_)(x_)]^{(n_)}], x_Symbol] \rightarrow \operatorname{Dist}[-d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}], x], x], x, \operatorname{Cot}[c + dx]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3855

$\operatorname{Int}[\operatorname{csc}[(c_ + (d_)(x_)]], x_Symbol] \rightarrow \operatorname{Simp}[-\operatorname{ArcTanh}[\cos[c + dx]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(6a-3a\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2} dx}{5a^2} \\
 &= -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{\int \frac{(27a^2-18a^2\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)} dx}{15a^4} \\
 &= -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2} \\
 &\quad - \frac{3\tan(c+dx)}{d(a^3+a^3\cos(c+dx))} + \frac{\int (72a^3-45a^3\cos(c+dx))\sec^2(c+dx) dx}{15a^6} \\
 &= -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2} \\
 &\quad - \frac{3\tan(c+dx)}{d(a^3+a^3\cos(c+dx))} - \frac{3\int \sec(c+dx) dx}{a^3} + \frac{24\int \sec^2(c+dx) dx}{5a^3} \\
 &= -\frac{3\text{arctanh}(\sin(c+dx))}{a^3d} - \frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2} \\
 &\quad - \frac{3\tan(c+dx)}{d(a^3+a^3\cos(c+dx))} - \frac{24\text{Subst}(\int 1 dx, x, -\tan(c+dx))}{5a^3d} \\
 &= -\frac{3\text{arctanh}(\sin(c+dx))}{a^3d} + \frac{24\tan(c+dx)}{5a^3d} - \frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3} \\
 &\quad - \frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{3\tan(c+dx)}{d(a^3+a^3\cos(c+dx))}
 \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 286 vs. 2(112) = 224.

Time = 1.13 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.55

$$\begin{aligned}
 &\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3} dx \\
 &= \frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right) + 8\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right) + 76\cos^4\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\right)}{\dots}
 \end{aligned}$$

[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]

[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + 8*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 76*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 20*Cos[(c + d*x)/2]^5*(3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2]))*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + Cos[(c + d*x)/2]*Tan[c/2] + 8*Cos[(c + d*x)/2]^3*Tan[c/2))/(5*a^3*d*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\left(\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}\right)+2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4da^3}$
default	$\frac{\left(\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}\right)+2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+17\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{4da^3}$
parallelrisc	$\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+\frac{57\left(\cos(dx+c)+\frac{\cos(2dx+2c)}{2}+\frac{2\cos(3dx+3c)}{19}+\frac{67}{114}\right)\cos(dx+c)}{20}}{a^3d\cos(dx+c)}$
norman	$-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{15\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}+\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20da}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3d}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3d}$
risc	$\frac{2i\left(15e^{6i(dx+c)}+75e^{5i(dx+c)}+160e^{4i(dx+c)}+200e^{3i(dx+c)}+189e^{2i(dx+c)}+105e^{i(dx+c)}+24\right)}{5da^3\left(e^{i(dx+c)}+1\right)^5\left(e^{2i(dx+c)}+1\right)}+\frac{3\ln\left(e^{i(dx+c)}-i\right)}{a^3d}$

[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^3+17*tan(1/2*d*x+1/2*c)-4/(tan(1/2*d*x+1/2*c)+1)-12*ln(tan(1/2*d*x+1/2*c)+1)-4/(tan(1/2*d*x+1/2*c)-1)+12*ln(tan(1/2*d*x+1/2*c)-1))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.70

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{15(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c))\log(\sin(dx+c)+1)-15(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c))\log(-\sin(dx+c)+1)-2(24\cos(dx+c)^3+57\cos(dx+c)^2+39\cos(dx+c)+5)\sin(dx+c)}{10(a^3d\cos(dx+c))^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c)}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] -1/10*(15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(24*cos(d*x + c)^3 + 57*cos(d*x + c)^2 + 39*cos(d*x + c) + 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))

SymPy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{\int \frac{\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1} dx}{a^3}$$

[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**3,x)

[Out] Integral(sec(c + d*x)**2/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x)/a**3

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.47

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3} dx$$

$$= \frac{\frac{40 \sin(dx+c)}{\left(a^3 - \frac{a^3 \sin^2(dx+c)}{(\cos(dx+c)+1)^2}\right) (\cos(dx+c)+1)} + \frac{\frac{85 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10 \sin^3(dx+c)}{(\cos(dx+c)+1)^3} + \frac{\sin^5(dx+c)}{(\cos(dx+c)+1)^5}}{a^3} - \frac{60 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^3} + \frac{60 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^3}}{20 d}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] 1/20*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3} dx =$$

$$\frac{\frac{60 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a^3} - \frac{60 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a^3} + \frac{40 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1) a^3} - \frac{a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 10 a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 85 a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a^{15}}}{20 d}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] -1/20*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 + 10*a^12*tan(1/2*d*x + 1/2*c)^3 + 85*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d

Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{2a^3 d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{20a^3 d} - \frac{6 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^3 d}$$

$$- \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - a^3\right)} + \frac{17 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4a^3 d}$$

[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^3),x)

[Out] tan(c/2 + (d*x)/2)^3/(2*a^3*d) + tan(c/2 + (d*x)/2)^5/(20*a^3*d) - (6*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x)/2)^2 - a^3)) + (17*tan(c/2 + (d*x)/2))/(4*a^3*d)

3.71 $\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^3} dx$

Optimal result	636
Rubi [A] (verified)	636
Mathematica [B] (verified)	639
Maple [A] (verified)	639
Fricas [A] (verification not implemented)	640
Sympy [F]	640
Maxima [A] (verification not implemented)	640
Giac [A] (verification not implemented)	641
Mupad [B] (verification not implemented)	641

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{13 \operatorname{arctanh}(\sin(c+dx))}{2a^3d} - \frac{152 \tan(c+dx)}{15a^3d} + \frac{13 \sec(c+dx) \tan(c+dx)}{2a^3d} - \frac{\sec(c+dx) \tan(c+dx)}{5d(a+a \cos(c+dx))^3} - \frac{11 \sec(c+dx) \tan(c+dx)}{15ad(a+a \cos(c+dx))^2} - \frac{76 \sec(c+dx) \tan(c+dx)}{15d(a^3+a^3 \cos(c+dx))}$$

[Out] 13/2*arctanh(sin(d*x+c))/a^3/d-152/15*tan(d*x+c)/a^3/d+13/2*sec(d*x+c)*tan(d*x+c)/a^3/d-1/5*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^3-11/15*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^2-76/15*sec(d*x+c)*tan(d*x+c)/d/(a^3+a^3*cos(d*x+c))

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2845, 3057, 2827, 3853, 3855, 3852, 8}

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{13 \operatorname{arctanh}(\sin(c+dx))}{2a^3d} - \frac{152 \tan(c+dx)}{15a^3d} + \frac{13 \tan(c+dx) \sec(c+dx)}{2a^3d} - \frac{76 \tan(c+dx) \sec(c+dx)}{15d(a^3 \cos(c+dx) + a^3)} - \frac{11 \tan(c+dx) \sec(c+dx)}{15ad(a \cos(c+dx) + a)^2} - \frac{\tan(c+dx) \sec(c+dx)}{5d(a \cos(c+dx) + a)^3}$$

[In] Int[Sec[c + d*x]^3/(a + a*cos[c + d*x])^3,x]

[Out] $(13 \cdot \text{ArcTanh}[\sin[c + dx]]) / (2a^3d) - (152 \cdot \tan[c + dx]) / (15a^3d) + (13 \cdot \sec[c + dx] \cdot \tan[c + dx]) / (2a^3d) - (\sec[c + dx] \cdot \tan[c + dx]) / (5d(a + a \cos[c + dx])^3) - (11 \cdot \sec[c + dx] \cdot \tan[c + dx]) / (15ad(a + a \cos[c + dx])^2) - (76 \cdot \sec[c + dx] \cdot \tan[c + dx]) / (15d(a^3 + a^3 \cos[c + dx]))$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a \cdot x, x] /; \text{FreeQ}[a, x]$

Rule 2827

$\text{Int}[(b \cdot \sin[e + fx] + f \cdot x)^m \cdot (c + d \cdot \sin[e + fx])], x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b \cdot \sin[e + fx])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b \cdot \sin[e + fx])^{m+1}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2845

$\text{Int}[(a + b \cdot \sin[e + fx])^m \cdot (c + d \cdot \sin[e + fx] + f \cdot x)^n], x_Symbol] \rightarrow \text{Simp}[b^2 \cos[e + fx] \cdot (a + b \cdot \sin[e + fx])^m \cdot (c + d \cdot \sin[e + fx])^{n+1} / (a \cdot f \cdot (2m+1) \cdot (b \cdot c - a \cdot d)), x] + \text{Dist}[1 / (a \cdot (2m+1) \cdot (b \cdot c - a \cdot d)), \text{Int}[(a + b \cdot \sin[e + fx])^{m+1} \cdot (c + d \cdot \sin[e + fx])^n \cdot \text{Simp}[b \cdot c \cdot (m+1) - a \cdot d \cdot (2m+n+2) + b \cdot d \cdot (m+n+2) \cdot \sin[e + fx], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{!GtQ}[n, 0] \ \&\& \ (\text{IntegerSQ}[2m, 2n] \ || \ (\text{IntegerQ}[m] \ \&\& \ \text{EqQ}[c, 0]))]$

Rule 3057

$\text{Int}[(a + b \cdot \sin[e + fx])^m \cdot (A + B \cdot \sin[e + fx] + f \cdot x)^n \cdot (c + d \cdot \sin[e + fx])], x_Symbol] \rightarrow \text{Simp}[b \cdot (A \cdot b - a \cdot B) \cdot \cos[e + fx] \cdot (a + b \cdot \sin[e + fx])^m \cdot (c + d \cdot \sin[e + fx])^{n+1} / (a \cdot f \cdot (2m+1) \cdot (b \cdot c - a \cdot d)), x] + \text{Dist}[1 / (a \cdot (2m+1) \cdot (b \cdot c - a \cdot d)), \text{Int}[(a + b \cdot \sin[e + fx])^{m+1} \cdot (c + d \cdot \sin[e + fx])^n \cdot \text{Simp}[B \cdot (a \cdot c \cdot m + b \cdot d \cdot (n+1)) + A \cdot (b \cdot c \cdot (m+1) - a \cdot d \cdot (2m+n+2)) + d \cdot (A \cdot b - a \cdot B) \cdot (m+n+2) \cdot \sin[e + fx], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ \text{!GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2m] \ \&\& \ (\text{IntegerQ}[2n] \ || \ \text{EqQ}[c, 0])]$

Rule 3852

$\text{Int}[\csc[c + d \cdot x]^n], x_Symbol] \rightarrow \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}], x], x], x, \text{Cot}[c + dx], x] /; \text{FreeQ}\{c, d\}, x \ \&\& \ \text{IGtQ}[n/2, 0]$

Rule 3853

$\text{Int}[(\csc[c + d \cdot x] + d \cdot x) \cdot (b \cdot \csc[c + d \cdot x])^n], x_Symbol] \rightarrow \text{Simp}[(-b) \cdot \cos[c + dx] \cdot (b \cdot \csc[c + dx])^{n-1} / (d \cdot (n-1)), x] + \text{Dist}[b^2 \cdot (n-2) / (n-1),$

Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sec(c + dx) \tan(c + dx)}{5d(a + a \cos(c + dx))^3} + \frac{\int \frac{(7a - 4a \cos(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^2} dx}{5a^2} \\
 &= -\frac{\sec(c + dx) \tan(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{11 \sec(c + dx) \tan(c + dx)}{15ad(a + a \cos(c + dx))^2} + \frac{\int \frac{(43a^2 - 33a^2 \cos(c + dx)) \sec^3(c + dx)}{a + a \cos(c + dx)} dx}{15a^4} \\
 &= -\frac{\sec(c + dx) \tan(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{11 \sec(c + dx) \tan(c + dx)}{15ad(a + a \cos(c + dx))^2} \\
 &\quad - \frac{76 \sec(c + dx) \tan(c + dx)}{15d(a^3 + a^3 \cos(c + dx))} + \frac{\int (195a^3 - 152a^3 \cos(c + dx)) \sec^3(c + dx) dx}{15a^6} \\
 &= -\frac{\sec(c + dx) \tan(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{11 \sec(c + dx) \tan(c + dx)}{15ad(a + a \cos(c + dx))^2} \\
 &\quad - \frac{76 \sec(c + dx) \tan(c + dx)}{15d(a^3 + a^3 \cos(c + dx))} - \frac{152 \int \sec^2(c + dx) dx}{15a^3} + \frac{13 \int \sec^3(c + dx) dx}{a^3} \\
 &= \frac{13 \sec(c + dx) \tan(c + dx)}{2a^3d} - \frac{\sec(c + dx) \tan(c + dx)}{5d(a + a \cos(c + dx))^3} - \frac{11 \sec(c + dx) \tan(c + dx)}{15ad(a + a \cos(c + dx))^2} \\
 &\quad - \frac{76 \sec(c + dx) \tan(c + dx)}{15d(a^3 + a^3 \cos(c + dx))} + \frac{13 \int \sec(c + dx) dx}{2a^3} + \frac{152 \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{15a^3d} \\
 &= \frac{13 \arctanh(\sin(c + dx))}{2a^3d} - \frac{152 \tan(c + dx)}{15a^3d} \\
 &\quad + \frac{13 \sec(c + dx) \tan(c + dx)}{2a^3d} - \frac{\sec(c + dx) \tan(c + dx)}{5d(a + a \cos(c + dx))^3} \\
 &\quad - \frac{11 \sec(c + dx) \tan(c + dx)}{15ad(a + a \cos(c + dx))^2} - \frac{76 \sec(c + dx) \tan(c + dx)}{15d(a^3 + a^3 \cos(c + dx))}
 \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. $2(156) = 312$.

Time = 2.91 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.20

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{24960 \cos^6\left(\frac{1}{2}(c + dx)\right) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)\right)}{\dots}$$

[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]

[Out] $-1/480*(24960*\text{Cos}[(c + d*x)/2]^6*(\text{Log}[\text{Cos}[(c + d*x)/2] - \text{Sin}[(c + d*x)/2]] - \text{Log}[\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2]]) + \text{Cos}[(c + d*x)/2]*\text{Sec}[c/2]*\text{Sec}[c]*\text{Sec}[c + d*x]^2*(-1235*\text{Sin}[(d*x)/2] + 3805*\text{Sin}[(3*d*x)/2] - 4329*\text{Sin}[c - (d*x)/2] + 1989*\text{Sin}[c + (d*x)/2] - 3575*\text{Sin}[2*c + (d*x)/2] - 475*\text{Sin}[c + (3*d*x)/2] + 2005*\text{Sin}[2*c + (3*d*x)/2] - 2275*\text{Sin}[3*c + (3*d*x)/2] + 2673*\text{Sin}[c + (5*d*x)/2] + 105*\text{Sin}[2*c + (5*d*x)/2] + 1593*\text{Sin}[3*c + (5*d*x)/2] - 975*\text{Sin}[4*c + (5*d*x)/2] + 1325*\text{Sin}[2*c + (7*d*x)/2] + 255*\text{Sin}[3*c + (7*d*x)/2] + 875*\text{Sin}[4*c + (7*d*x)/2] - 195*\text{Sin}[5*c + (7*d*x)/2] + 304*\text{Sin}[3*c + (9*d*x)/2] + 90*\text{Sin}[4*c + (9*d*x)/2] + 214*\text{Sin}[5*c + (9*d*x)/2])/(a^3*d*(1 + \text{Cos}[c + d*x])^3)$

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} - \frac{8\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} - 31 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{2}{\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} + \frac{14}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1} + 26 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + \dots$
default	$-\frac{\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} - \frac{8\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} - 31 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{2}{\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} + \frac{14}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1} + 26 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + \dots$
parallelrisch	$\frac{(-1560 \cos(2dx+2c) - 1560) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + (1560 \cos(2dx+2c) + 1560) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - 2331 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{240a^3 d(1 + \cos(2dx+2c))}$
norman	$-\frac{51 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4da} + \frac{131 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6da} - \frac{97 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15da} - \frac{17 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{30da} - \frac{\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)}{20da} - \frac{13 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2a^3 d}$
risch	$-\frac{i(195 e^{8i(dx+c)} + 975 e^{7i(dx+c)} + 2275 e^{6i(dx+c)} + 3575 e^{5i(dx+c)} + 4329 e^{4i(dx+c)} + 3805 e^{3i(dx+c)} + 2673 e^{2i(dx+c)} + 132)}{15d a^3 (e^{2i(dx+c)} + 1)^2 (e^{i(dx+c)} + 1)^5}$

[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] $1/4/d/a^3*(-1/5*\tan(1/2*d*x+1/2*c)^5 - 8/3*\tan(1/2*d*x+1/2*c)^3 - 31*\tan(1/2*d*x+1/2*c) - 2/(\tan(1/2*d*x+1/2*c)+1)^2 + 14/(\tan(1/2*d*x+1/2*c)+1) + 26*\ln(\tan(1/2$

$*d*x+1/2*c)+1)+2/(\tan(1/2*d*x+1/2*c)-1)^2+14/(\tan(1/2*d*x+1/2*c)-1)-26*\ln(\tan(1/2*d*x+1/2*c)-1))$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.32

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{195(\cos(dx+c))^5 + 3\cos(dx+c)^4 + 3\cos(dx+c)^3 + \cos(dx+c)^2 \log(\sin(dx+c)+1) - 195(\cos(dx+c))^2 \log(-\sin(dx+c)+1) - 2*(304*\cos(dx+c)^4 + 717*\cos(dx+c)^3 + 479*\cos(dx+c)^2 + 45*\cos(dx+c) - 15)*\sin(dx+c)}{60(a^3*d*\cos(dx+c)^5 + 3*a^3*d*\cos(dx+c)^4 + 3*a^3*d*\cos(dx+c)^3 + a^3*d*\cos(dx+c)^2)}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/60*(195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(304*cos(d*x + c)^4 + 717*cos(d*x + c)^3 + 479*cos(d*x + c)^2 + 45*cos(d*x + c) - 15)*sin(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3} dx = \int \frac{\sec^3(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1} dx$$

[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**3,x)

[Out] Integral(sec(c + d*x)**3/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x)/a**3

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1} - \frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3 - \frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1} + \frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} + \frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out]
$$-1/60*(60*(5*\sin(d*x + c)/(\cos(d*x + c) + 1) - 7*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/(a^3 - 2*a^3*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + a^3*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4) + (465*\sin(d*x + c)/(\cos(d*x + c) + 1) + 40*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)/a^3 - 390*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a^3 + 390*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a^3)/d$$

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= \frac{\frac{390 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a^3} - \frac{390 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a^3} + \frac{60 (7 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 5 \tan(\frac{1}{2} dx + \frac{1}{2} c))}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^2 a^3} - \frac{3 a^{12} \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 40 a^{12}}{60 d}}{60 d}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out]
$$1/60*(390*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^3 - 390*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(7*\tan(1/2*d*x + 1/2*c)^3 - 5*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*a^12*\tan(1/2*d*x + 1/2*c)^5 + 40*a^12*\tan(1/2*d*x + 1/2*c)^3 + 465*a^12*\tan(1/2*d*x + 1/2*c))/a^15)/d$$

Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{13 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^3 d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{20 a^3 d} - \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3 a^3 d} - \frac{5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{d \left(a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^3\right)} - \frac{31 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4 a^3 d}$$

[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^3),x)

[Out]
$$(13*\operatorname{atanh}(\tan(c/2 + (d*x)/2)))/(a^3*d) - \tan(c/2 + (d*x)/2)^5/(20*a^3*d) - (2*\tan(c/2 + (d*x)/2)^3)/(3*a^3*d) - (5*\tan(c/2 + (d*x)/2) - 7*\tan(c/2 + (d*x)/2)^3)/(d*(a^3*\tan(c/2 + (d*x)/2)^4 - 2*a^3*\tan(c/2 + (d*x)/2)^2 + a^3) - (31*\tan(c/2 + (d*x)/2))/(4*a^3*d)$$

3.72 $\int \frac{\cos^6(c+dx)}{(a+a \cos(c+dx))^4} dx$

Optimal result	642
Rubi [A] (verified)	642
Mathematica [A] (verified)	644
Maple [A] (verified)	644
Fricas [A] (verification not implemented)	645
Sympy [B] (verification not implemented)	646
Maxima [A] (verification not implemented)	646
Giac [A] (verification not implemented)	647
Mupad [B] (verification not implemented)	647

Optimal result

Integrand size = 21, antiderivative size = 184

$$\int \frac{\cos^6(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{21x}{2a^4} - \frac{576 \sin(c+dx)}{35a^4d} + \frac{21 \cos(c+dx) \sin(c+dx)}{2a^4d} - \frac{43 \cos^3(c+dx) \sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{288 \cos^2(c+dx) \sin(c+dx)}{35a^4d(1+\cos(c+dx))} - \frac{\cos^5(c+dx) \sin(c+dx)}{7d(a+a \cos(c+dx))^4} - \frac{2 \cos^4(c+dx) \sin(c+dx)}{5ad(a+a \cos(c+dx))^3}$$

[Out] 21/2*x/a^4-576/35*sin(d*x+c)/a^4/d+21/2*cos(d*x+c)*sin(d*x+c)/a^4/d-43/35*cos(d*x+c)^3*sin(d*x+c)/a^4/d/(1+cos(d*x+c))^2-288/35*cos(d*x+c)^2*sin(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*cos(d*x+c)^5*sin(d*x+c)/d/(a+a*cos(d*x+c))^4-2/5*cos(d*x+c)^4*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2844, 3056, 2813}

$$\int \frac{\cos^6(c+dx)}{(a+a \cos(c+dx))^4} dx = -\frac{576 \sin(c+dx)}{35a^4d} - \frac{43 \sin(c+dx) \cos^3(c+dx)}{35a^4d(\cos(c+dx)+1)^2} - \frac{288 \sin(c+dx) \cos^2(c+dx)}{35a^4d(\cos(c+dx)+1)} + \frac{21 \sin(c+dx) \cos(c+dx)}{2a^4d} + \frac{21x}{2a^4} - \frac{\sin(c+dx) \cos^5(c+dx)}{7d(a \cos(c+dx)+a)^4} - \frac{2 \sin(c+dx) \cos^4(c+dx)}{5ad(a \cos(c+dx)+a)^3}$$

[In] Int[Cos[c + d*x]^6/(a + a*cos[c + d*x])^4,x]

```
[Out] (21*x)/(2*a^4) - (576*Sin[c + d*x])/(35*a^4*d) + (21*Cos[c + d*x]*Sin[c + d
*x])/(2*a^4*d) - (43*Cos[c + d*x]^3*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*
x])^2) - (288*Cos[c + d*x]^2*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*x])) -
(Cos[c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*Cos[c + d*x
]^4*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2813

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)
*(x_)]), x_Symbol] := Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^4(c+dx)(5a-9a\cos(c+dx))}{(a+a\cos(c+dx))^3} dx}{7a^2} \\ &= -\frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^4(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^3(c+dx)(56a^2-73a^2\cos(c+dx))}{(a+a\cos(c+dx))^2} dx}{35a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{43 \cos^3(c+dx) \sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^5(c+dx) \sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\
&\quad - \frac{2 \cos^4(c+dx) \sin(c+dx)}{5ad(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(387a^3-477a^3\cos(c+dx))}{a+a\cos(c+dx)} dx}{105a^6} \\
&= -\frac{43 \cos^3(c+dx) \sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^5(c+dx) \sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2 \cos^4(c+dx) \sin(c+dx)}{5ad(a+a\cos(c+dx))^3} \\
&\quad - \frac{288 \cos^2(c+dx) \sin(c+dx)}{35d(a^4+a^4\cos(c+dx))} - \frac{\int \cos(c+dx)(1728a^4-2205a^4\cos(c+dx)) dx}{105a^8} \\
&= \frac{21x}{2a^4} - \frac{576 \sin(c+dx)}{35a^4d} + \frac{21 \cos(c+dx) \sin(c+dx)}{2a^4d} - \frac{43 \cos^3(c+dx) \sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} \\
&\quad - \frac{\cos^5(c+dx) \sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2 \cos^4(c+dx) \sin(c+dx)}{5ad(a+a\cos(c+dx))^3} - \frac{288 \cos^2(c+dx) \sin(c+dx)}{35d(a^4+a^4\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4} dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right) \csc^8(c+dx) \sin^9\left(\frac{1}{2}(c+dx)\right) \left(188160 \arcsin(\cos(c+dx)) \cos^8\left(\frac{1}{2}(c+dx)\right) + (55656 + 85762 \cos(c+dx) + 37504 \cos(2(c+dx)) + 7873 \cos(3(c+dx)) + 280 \cos(4(c+dx)) - 35 \cos(5(c+dx))) \sqrt{\sin^2(c+dx)}\right)}{a^4 d \sqrt{\sin^2(c+dx)}}$$

[In] Integrate[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^4,x]

[Out] -1/35*(Cos[(c + d*x)/2]*Csc[c + d*x]^8*Sin[(c + d*x)/2]^9*(188160*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^8 + (55656 + 85762*Cos[c + d*x] + 37504*Cos[2*(c + d*x)] + 7873*Cos[3*(c + d*x)] + 280*Cos[4*(c + d*x)] - 35*Cos[5*(c + d*x)])*Sqrt[Sin[c + d*x]^2])/(a^4*d*Sqrt[Sin[c + d*x]^2])

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

method	result
parallelrisch	$\frac{-85762 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\cos(dx+c) + \frac{18752 \cos(2dx+2c)}{42881} + \frac{7873 \cos(3dx+3c)}{85762} + \frac{140 \cos(4dx+4c)}{42881} - \frac{35 \cos(5dx+5c)}{85762} + \frac{27828}{42881} \right) \left(\sec^6\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{8960a^4d}$
derivativedivides	$\frac{\left(\frac{\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{7} - \frac{9 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{5} + 13 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 111 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-72 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 56 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + 168 \arctan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{8da^4}$
default	$\frac{\left(\frac{\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{7} - \frac{9 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{5} + 13 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 111 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-72 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 56 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + 168 \arctan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{8da^4}$
risch	$\frac{21x}{2a^4} - \frac{ie^{2i(dx+c)}}{8a^4d} + \frac{2ie^{i(dx+c)}}{da^4} - \frac{2ie^{-i(dx+c)}}{da^4} + \frac{ie^{-2i(dx+c)}}{8a^4d} - \frac{2i(700e^{6i(dx+c)} + 3675e^{5i(dx+c)} + 8505e^{4i(dx+c)} + 1575e^{3i(dx+c)} + 105e^{2i(dx+c)} + 7e^{i(dx+c)} + 1)}{35da^4}$

[In] int(cos(d*x+c)^6/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)

[Out] 1/8960*(-85762*tan(1/2*d*x+1/2*c)*(cos(d*x+c)+18752/42881*cos(2*d*x+2*c)+7873/85762*cos(3*d*x+3*c)+140/42881*cos(4*d*x+4*c)-35/85762*cos(5*d*x+5*c)+27828/42881)*sec(1/2*d*x+1/2*c)^6+94080*d*x)/a^4/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93

$$\int \frac{\cos^6(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{735 dx \cos(dx + c)^4 + 2940 dx \cos(dx + c)^3 + 4410 dx \cos(dx + c)^2 + 2940 dx \cos(dx + c) + 735 dx + (35 \cos(dx + c)^5 - 140 \cos(dx + c)^4 - 2012 \cos(dx + c)^3 - 4548 \cos(dx + c)^2 - 3873 \cos(dx + c) - 1152) \sin(dx + c)}{70 (a^4 d \cos(dx + c)^4 + 4 a^4 d \cos(dx + c)^3 + 4 a^4 d \cos(dx + c)^2 + 4 a^4 d \cos(dx + c) + a^4 d)}$$

[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="fricas")

[Out] 1/70*(735*d*x*cos(d*x + c)^4 + 2940*d*x*cos(d*x + c)^3 + 4410*d*x*cos(d*x + c)^2 + 2940*d*x*cos(d*x + c) + 735*d*x + (35*cos(d*x + c)^5 - 140*cos(d*x + c)^4 - 2012*cos(d*x + c)^3 - 4548*cos(d*x + c)^2 - 3873*cos(d*x + c) - 1152)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(175) = 350.

Time = 9.08 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.88

$$\int \frac{\cos^6(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \left\{ \begin{array}{l} \frac{2940dx \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right)}{280a^4d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 560a^4d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 280a^4d} + \frac{5880dx \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{280a^4d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 560a^4d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 280a^4d} + \frac{2940dx}{280a^4d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 560a^4d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 280a^4d} \\ \frac{x \cos^6(c)}{(a \cos(c) + a)^4} \end{array} \right.$$

[In] integrate(cos(d*x+c)**6/(a+a*cos(d*x+c))**4,x)

[Out] Piecewise((2940*d*x*tan(c/2 + d*x/2)**4/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) + 5880*d*x*tan(c/2 + d*x/2)**2/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) + 2940*d*x/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) + 5*tan(c/2 + d*x/2)**11/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) - 53*tan(c/2 + d*x/2)**9/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) + 334*tan(c/2 + d*x/2)**7/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) - 3038*tan(c/2 + d*x/2)**5/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) - 9835*tan(c/2 + d*x/2)**3/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d) - 5845*tan(c/2 + d*x/2)/(280*a**4*d*tan(c/2 + d*x/2)**4 + 560*a**4*d*tan(c/2 + d*x/2)**2 + 280*a**4*d), Ne(d, 0)), (x*cos(c)**6/(a*cos(c) + a)**4, True))

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.11

$$\int \frac{\cos^6(c + dx)}{(a + a \cos(c + dx))^4} dx =$$

$$\frac{280 \left(\frac{7 \sin(dx+c)}{\cos(dx+c)+1} + \frac{9 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^4 + \frac{2a^4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^4 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{3885 \sin(dx+c)}{\cos(dx+c)+1} - \frac{455 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{63 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{5880 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}$$

280 d

[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x

+ c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) - 455 *sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.70

$$\int \frac{\cos^6(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{\frac{2940(dx+c)}{a^4} - \frac{280\left(9 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1\right)^2 a^4} + \frac{5 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 63 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 455 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 3885 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^{28}}}{280 d}$$

[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/280*(2940*(d*x + c)/a^4 - 280*(9*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (5*a^24*tan(1/2*d*x + 1/2*c)^7 - 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 - 3885*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d

Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int \frac{\cos^6(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{5 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 78 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 596 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 4408 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 2520 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^8 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 560 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 2940 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^7 (c + dx)}{280 a^4 d}$$

[In] int(cos(c + d*x)^6/(a + a*cos(c + d*x))^4,x)

[Out] (5*sin(c/2 + (d*x)/2) - 78*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 596*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 4408*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) - 2520*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 560*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) + 2940*cos(c/2 + (d*x)/2)^7*(c + d*x))/(280*a^4*d*cos(c/2 + (d*x)/2)^7)

3.73 $\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^4} dx$

Optimal result	648
Rubi [A] (verified)	648
Mathematica [A] (verified)	651
Maple [A] (verified)	651
Fricas [A] (verification not implemented)	652
Sympy [A] (verification not implemented)	652
Maxima [A] (verification not implemented)	653
Giac [A] (verification not implemented)	653
Mupad [B] (verification not implemented)	654

Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^4} dx = -\frac{4x}{a^4} + \frac{244 \sin(c+dx)}{105a^4d} - \frac{88 \cos^2(c+dx) \sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} + \frac{4 \sin(c+dx)}{a^4d(1+\cos(c+dx))} - \frac{\cos^4(c+dx) \sin(c+dx)}{7d(a+a \cos(c+dx))^4} - \frac{12 \cos^3(c+dx) \sin(c+dx)}{35ad(a+a \cos(c+dx))^3}$$

[Out] $-4*x/a^4+244/105*\sin(d*x+c)/a^4/d-88/105*\cos(d*x+c)^2*\sin(d*x+c)/a^4/d/(1+\cos(d*x+c))^2+4*\sin(d*x+c)/a^4/d/(1+\cos(d*x+c))-1/7*\cos(d*x+c)^4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^4-12/35*\cos(d*x+c)^3*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^3$

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2844, 3056, 3047, 3102, 12, 2814, 2727}

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{244 \sin(c+dx)}{105a^4d} - \frac{88 \sin(c+dx) \cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2} + \frac{4 \sin(c+dx)}{a^4d(\cos(c+dx)+1)} - \frac{4x}{a^4} - \frac{\sin(c+dx) \cos^4(c+dx)}{7d(a \cos(c+dx)+a)^4} - \frac{12 \sin(c+dx) \cos^3(c+dx)}{35ad(a \cos(c+dx)+a)^3}$$

[In] $\text{Int}[\text{Cos}[c+d*x]^5/(a+a*\text{Cos}[c+d*x])^4,x]$

```
[Out] (-4*x)/a^4 + (244*Sin[c + d*x])/(105*a^4*d) - (88*Cos[c + d*x]^2*Sin[c + d*
x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*Sin[c + d*x])/(a^4*d*(1 + Cos[c +
d*x])) - (Cos[c + d*x]^4*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (12*
Cos[c + d*x]^3*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 2727

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c +
d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
```

$b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /;$ Free
 $Q[\{a, b, c, d, e, f, A, B\}, x] \&\& NeQ[b*c - a*d, 0] \&\& EqQ[a^2 - b^2, 0] \&\&$
 $NeQ[c^2 - d^2, 0] \&\& LtQ[m, -2^{(-1)}] \&\& GtQ[n, 0] \&\& IntegerQ[2*m] \&\& (Int$
 $egerQ[2*n] || EqQ[c, 0])$

Rule 3102

$Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*sin[(e_.)$
 $+ (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> Simp[(-C)*Co$
 $s[e + f*x]*((a + b*Sin[e + f*x])^{(m + 1)}/(b*f*(m + 2))), x] + Dist[1/(b*(m$
 $+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m$
 $+ 2) - a*C)*Sin[e + f*x], x], x] /;$ FreeQ[{a, b, e, f, A, B, C, m}, x]
 $\&\& !LtQ[m, -1]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^3(c+dx)(4a-8a\cos(c+dx))}{(a+a\cos(c+dx))^3} dx}{7a^2} \\
 &= -\frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(36a^2-52a^2\cos(c+dx))}{(a+a\cos(c+dx))^2} dx}{35a^4} \\
 &= -\frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\
 &\quad - \frac{12\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(176a^3-244a^3\cos(c+dx))}{a+a\cos(c+dx)} dx}{105a^6} \\
 &= -\frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\
 &\quad - \frac{12\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{\int \frac{176a^3\cos(c+dx)-244a^3\cos^2(c+dx)}{a+a\cos(c+dx)} dx}{105a^6} \\
 &= \frac{244\sin(c+dx)}{105a^4d} - \frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\
 &\quad - \frac{12\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{\int \frac{420a^4\cos(c+dx)}{a+a\cos(c+dx)} dx}{105a^7} \\
 &= \frac{244\sin(c+dx)}{105a^4d} - \frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\
 &\quad - \frac{12\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{4\int \frac{\cos(c+dx)}{a+a\cos(c+dx)} dx}{a^3}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{4x}{a^4} + \frac{244 \sin(c+dx)}{105a^4d} - \frac{88 \cos^2(c+dx) \sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} \\
&\quad - \frac{\cos^4(c+dx) \sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12 \cos^3(c+dx) \sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{4 \int \frac{1}{a+a\cos(c+dx)} dx}{a^3} \\
&= -\frac{4x}{a^4} + \frac{244 \sin(c+dx)}{105a^4d} - \frac{88 \cos^2(c+dx) \sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx) \sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\
&\quad - \frac{12 \cos^3(c+dx) \sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{4 \sin(c+dx)}{d(a^4+a^4\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4} dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right) \sin^5\left(\frac{1}{2}(c+dx)\right) \left(7350 + 6678 \cos(c+dx) - 5432 \cos(2(c+dx)) - 6333 \cos(3(c+dx))\right)}{210a^4d(-1+\cos(c+dx))}$$

[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^4,x]

[Out] -1/210*(Cos[(c + d*x)/2]*Sin[(c + d*x)/2]^5*(7350 + 6678*Cos[c + d*x] - 5432*Cos[2*(c + d*x)] - 6333*Cos[3*(c + d*x)] - 2158*Cos[4*(c + d*x)] - 105*Cos[5*(c + d*x)] + 53760*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^6*Sqrt[Sin[c + d*x]^2]))/(a^4*d*(-1 + Cos[c + d*x])^3*(1 + Cos[c + d*x])^4)

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.51

method	result
parallelrisch	$\frac{781 \left(\cos(dx+c) + \frac{2741 \cos(2dx+2c)}{6248} + \frac{74 \cos(3dx+3c)}{781} + \frac{105 \cos(4dx+4c)}{24992} + \frac{16171}{24992} \right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sec^6\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 840dx}{210a^4d}$
derivativedivides	$-\frac{\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{7} + \frac{7\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} - \frac{23\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} + 49 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{16 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} - 64 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8da^4}$
default	$-\frac{\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{7} + \frac{7\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} - \frac{23\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} + 49 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{16 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} - 64 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8da^4}$
risch	$-\frac{4x}{a^4} - \frac{ie^{i(dx+c)}}{2da^4} + \frac{ie^{-i(dx+c)}}{2da^4} + \frac{4i(525e^{6i(dx+c)}+2625e^{5i(dx+c)}+5950e^{4i(dx+c)}+7420e^{3i(dx+c)}+5397e^{2i(dx+c)}+105d a^4(e^{i(dx+c)}+1))^7}{105d a^4(e^{i(dx+c)}+1)^7}$
norman	$-\frac{4x}{a} - \frac{\tan^{17}\left(\frac{dx}{2} + \frac{c}{2}\right)}{56ad} + \frac{65 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da} + \frac{113\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3da} + \frac{2059\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{30da} + \frac{1271\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{21da} + \frac{2075\left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{84da}$

[In] `int(cos(d*x+c)^5/(a+cos(d*x+c))*a^4,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{210} * (781 * (\cos(dx+c) + 2741/6248 * \cos(2*dx+2*c)) + 74/781 * \cos(3*dx+3*c) + 105/2 * 4992 * \cos(4*dx+4*c) + 16171/24992) * \tan(1/2*dx+1/2*c) * \sec(1/2*dx+1/2*c)^6 - 840 * dx / a^4/d$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4} dx = \frac{420 dx \cos(dx+c)^4 + 1680 dx \cos(dx+c)^3 + 2520 dx \cos(dx+c)^2 + 1680 dx \cos(dx+c) + 420 dx - 105 (a^4 d \cos(dx+c)^4 + 4 a^4 d \cos(dx+c)^3 + 6 a^4 d \cos(dx+c)^2 + 4 a^4 d \cos(dx+c) + a^4 d)}{105 (a^4 d \cos(dx+c)^4 + 4 a^4 d \cos(dx+c)^3 + 6 a^4 d \cos(dx+c)^2 + 4 a^4 d \cos(dx+c) + a^4 d)}$$

[In] `integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x,algorithm="fricas")`

[Out] $-\frac{1}{105} * (420 * dx * \cos(dx+c)^4 + 1680 * dx * \cos(dx+c)^3 + 2520 * dx * \cos(dx+c)^2 + 1680 * dx * \cos(dx+c) + 420 * dx - (105 * \cos(dx+c)^4 + 1184 * \cos(dx+c)^3 + 2636 * \cos(dx+c)^2 + 2236 * \cos(dx+c) + 664) * \sin(dx+c)) / (a^4 * dx * \cos(dx+c)^4 + 4 * a^4 * dx * \cos(dx+c)^3 + 6 * a^4 * dx * \cos(dx+c)^2 + 4 * a^4 * dx * \cos(dx+c) + a^4 * dx)$

Sympy [A] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.87

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4} dx = \begin{cases} -\frac{3360 dx \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 840 a^4 d} - \frac{3360 dx}{840 a^4 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 840 a^4 d} - \frac{15 \tan^9\left(\frac{c}{2} + \frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 840 a^4 d} + \frac{132 \tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{840 a^4 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 840 a^4 d} \\ \frac{x \cos^5(c)}{(a \cos(c) + a)^4} \end{cases}$$

[In] `integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**4,x)`

[Out] `Piecewise((-3360*d*x*tan(c/2 + d*x/2)**2/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 3360*d*x/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 15*tan(c/2 + d*x/2)**9/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 132*tan(c/2 + d*x/2)**7/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) - 658*tan(c/2 + d*x/2)**5/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 4340*tan(c/2 + d*x/2)**3/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d) + 6825*tan(c/2 + d*x/2)/(840*a**4*d*tan(c/2 + d*x/2)**2 + 840*a**4*d), Ne(d, 0)), (x*cos(c)**5/(a*cos(c) + a)**4, True))`

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{\frac{1680 \sin(dx+c)}{\left(a^4 + \frac{a^4 \sin^2(dx+c)}{\cos(dx+c)+1}\right) (\cos(dx+c)+1)} + \frac{\frac{5145 \sin(dx+c)}{\cos(dx+c)+1} - \frac{805 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{147 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{6720 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{840 d}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] 1/840*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^4} dx =$$

$$\frac{\frac{3360(dx+c)}{a^4} - \frac{1680 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1\right) a^4} + \frac{15 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 147 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 805 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 5145 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^{28}}}{840 d}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] -1/840*(3360*(d*x + c)/a^4 - 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^4) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*a^24*tan(1/2*d*x + 1/2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d

Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^4} dx =$$

$$\frac{15 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 192 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 1144 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 6112 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 3360 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^8 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 1680 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 840 a^4 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^{12}}{840 a^4 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^7}$$

[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x))^4,x)

[Out] -(15*sin(c/2 + (d*x)/2) - 192*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 1144*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 6112*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) - 1680*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 3360*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 1680*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2) + 840*a^4*d*cos(c/2 + (d*x)/2)^12)/(840*a^4*d*cos(c/2 + (d*x)/2)^7)

3.74 $\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^4} dx$

Optimal result	655
Rubi [A] (verified)	655
Mathematica [A] (verified)	657
Maple [A] (verified)	658
Fricas [A] (verification not implemented)	658
Sympy [A] (verification not implemented)	659
Maxima [A] (verification not implemented)	659
Giac [A] (verification not implemented)	659
Mupad [B] (verification not implemented)	660

Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{x}{a^4} + \frac{11 \sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{43 \sin(c+dx)}{21a^4d(1+\cos(c+dx))} - \frac{\cos^3(c+dx) \sin(c+dx)}{7d(a+a \cos(c+dx))^4} - \frac{2 \cos^2(c+dx) \sin(c+dx)}{7ad(a+a \cos(c+dx))^3}$$

[Out] $x/a^4+11/21*\sin(d*x+c)/a^4/d/(1+\cos(d*x+c))^2-43/21*\sin(d*x+c)/a^4/d/(1+\cos(d*x+c))-1/7*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^4-2/7*\cos(d*x+c)^2*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^3$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2844, 3056, 3047, 3098, 2814, 2727}

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^4} dx = -\frac{43 \sin(c+dx)}{21a^4d(\cos(c+dx)+1)} + \frac{11 \sin(c+dx)}{21a^4d(\cos(c+dx)+1)^2} + \frac{x}{a^4} - \frac{\sin(c+dx) \cos^3(c+dx)}{7d(a \cos(c+dx)+a)^4} - \frac{2 \sin(c+dx) \cos^2(c+dx)}{7ad(a \cos(c+dx)+a)^3}$$

[In] $\text{Int}[\text{Cos}[c+d*x]^4/(a+a*\text{Cos}[c+d*x])^4,x]$

[Out] $x/a^4 + (11*\text{Sin}[c+d*x])/(21*a^4*d*(1+\text{Cos}[c+d*x])) - (43*\text{Sin}[c+d*x])/(21*a^4*d*(1+\text{Cos}[c+d*x])) - (\text{Cos}[c+d*x]^3*\text{Sin}[c+d*x])/(7*d*(a+a*\text{Cos}[c+d*x])^4) - (2*\text{Cos}[c+d*x]^2*\text{Sin}[c+d*x])/(7*a*d*(a+a*\text{Cos}[c+d*x])^3)$

Rule 2727

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c +
d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/(c_ + (d_)*sin[(e_) + (f_
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)])*(c_ + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*(c_ + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3098

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(A*b - a*
```

$B + bC) \cdot \text{Cos}[e + f \cdot x] \cdot ((a + b \cdot \text{Sin}[e + f \cdot x])^m / (a \cdot f \cdot (2 \cdot m + 1))), x] + \text{Dist}[1 / (a^2 \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Sin}[e + f \cdot x])^{m+1} \cdot \text{Simp}[a \cdot A \cdot (m+1) + m \cdot (b \cdot B - a \cdot C) + b \cdot C \cdot (2 \cdot m + 1) \cdot \text{Sin}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^3(c+dx) \sin(c+dx)}{7d(a+a \cos(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)(3a-7a \cos(c+dx))}{(a+a \cos(c+dx))^3} dx}{7a^2} \\
&= -\frac{\cos^3(c+dx) \sin(c+dx)}{7d(a+a \cos(c+dx))^4} - \frac{2 \cos^2(c+dx) \sin(c+dx)}{7ad(a+a \cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(20a^2-35a^2 \cos(c+dx))}{(a+a \cos(c+dx))^2} dx}{35a^4} \\
&= -\frac{\cos^3(c+dx) \sin(c+dx)}{7d(a+a \cos(c+dx))^4} - \frac{2 \cos^2(c+dx) \sin(c+dx)}{7ad(a+a \cos(c+dx))^3} - \frac{\int \frac{20a^2 \cos(c+dx)-35a^2 \cos^2(c+dx)}{(a+a \cos(c+dx))^2} dx}{35a^4} \\
&= \frac{11 \sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\cos^3(c+dx) \sin(c+dx)}{7d(a+a \cos(c+dx))^4} \\
&\quad - \frac{2 \cos^2(c+dx) \sin(c+dx)}{7ad(a+a \cos(c+dx))^3} + \frac{\int \frac{-110a^3+105a^3 \cos(c+dx)}{a+a \cos(c+dx)} dx}{105a^6} \\
&= \frac{x}{a^4} + \frac{11 \sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\cos^3(c+dx) \sin(c+dx)}{7d(a+a \cos(c+dx))^4} \\
&\quad - \frac{2 \cos^2(c+dx) \sin(c+dx)}{7ad(a+a \cos(c+dx))^3} - \frac{43 \int \frac{1}{a+a \cos(c+dx)} dx}{21a^3} \\
&= \frac{x}{a^4} + \frac{11 \sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\cos^3(c+dx) \sin(c+dx)}{7d(a+a \cos(c+dx))^4} \\
&\quad - \frac{2 \cos^2(c+dx) \sin(c+dx)}{7ad(a+a \cos(c+dx))^3} - \frac{43 \sin(c+dx)}{21d(a^4+a^4 \cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{32 \cos\left(\frac{1}{2}(c+dx)\right) \csc^8(c+dx) \sin^9\left(\frac{1}{2}(c+dx)\right) \left(336 \arcsin(\cos(c+dx)) \cos^8\left(\frac{1}{2}(c+dx)\right) + (94+146 \cos(c+dx)) \sqrt{\sin^2(c+dx)}\right)}{21a^4d}$$

[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^4,x]

[Out] (-32*Cos[(c + d*x)/2]*Csc[c + d*x]^8*Sin[(c + d*x)/2]^9*(336*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^8 + (94 + 146*Cos[c + d*x] + 62*Cos[2*(c + d*x)] + 13*Cos[3*(c + d*x)])*Sqrt[Sin[c + d*x]^2]))/(21*a^4*d*Sqrt[Sin[c + d*x]^2])

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.50

method	result
parallelrisch	$\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-21\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+77\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+168dx-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{168a^4d}$
derivativedivides	$\frac{\left(\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{7}\right)-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da^4}$
default	$\frac{\left(\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{7}\right)-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da^4}$
risch	$\frac{x}{a^4}-\frac{4i\left(42e^{6i(dx+c)}+189e^{5i(dx+c)}+413e^{4i(dx+c)}+497e^{3i(dx+c)}+357e^{2i(dx+c)}+140e^{i(dx+c)}+26\right)}{21da^4\left(e^{i(dx+c)}+1\right)^7}$
norman	$\frac{\frac{x}{a}+\frac{x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{169\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24da}-\frac{229\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24da}-\frac{293\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}-\frac{121\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{168da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

```
[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/168*(3*tan(1/2*d*x+1/2*c)^7-21*tan(1/2*d*x+1/2*c)^5+77*tan(1/2*d*x+1/2*c)^3+168*d*x-315*tan(1/2*d*x+1/2*c))/a^4/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.20

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4} dx$$

$$= \frac{21 dx \cos(dx+c)^4 + 84 dx \cos(dx+c)^3 + 126 dx \cos(dx+c)^2 + 84 dx \cos(dx+c) + 21 dx - (52 \cos(dx+c)^4 + 4a^4d \cos(dx+c)^3 + 6a^4d \cos(dx+c)^2 + 4a^4d \cos(dx+c) + a^4d)}{21(a^4d \cos(dx+c)^4 + 4a^4d \cos(dx+c)^3 + 6a^4d \cos(dx+c)^2 + 4a^4d \cos(dx+c) + a^4d)}$$

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/21*(21*d*x*cos(d*x+c)^4+84*d*x*cos(d*x+c)^3+126*d*x*cos(d*x+c)^2+84*d*x*cos(d*x+c)+21*d*x-(52*cos(d*x+c)^4+4*a^4*d*cos(d*x+c)^3+6*a^4*d*cos(d*x+c)^2+4*a^4*d*cos(d*x+c)+a^4*d))
```

Sympy [A] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \begin{cases} \frac{x}{a^4} + \frac{\tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{56a^4d} - \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{8a^4d} + \frac{11 \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{24a^4d} - \frac{15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x \cos^4(c)}{(a \cos(c) + a)^4} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**4,x)

[Out] Piecewise((x/a**4 + tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/(8*a**4*d) + 11*tan(c/2 + d*x/2)**3/(24*a**4*d) - 15*tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**4, True))

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= - \frac{\frac{315 \sin(dx+c)}{\cos(dx+c)+1} - \frac{77 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{336 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{168 d}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{\frac{168(dx+c)}{a^4} + \frac{3a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 21a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 77a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 315a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^{28}}}{168 d}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/168*(168*(d*x + c)/a^4 + (3*a^24*tan(1/2*d*x + 1/2*c)^7 - 21*a^24*tan(1/2*d*x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 - 315*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d

Mupad [B] (verification not implemented)

Time = 14.84 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{x}{a^4} + \frac{-\frac{52 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6}{21} + \frac{16 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4}{21} - \frac{5 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{28} + \frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{56}}{a^4 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^7}$$

[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^4,x)

[Out] x/a^4 + (sin(c/2 + (d*x)/2)/56 - (5*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2))/28 + (16*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/21 - (52*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2))/21)/(a^4*d*cos(c/2 + (d*x)/2)^7)

$$3.75 \quad \int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^4} dx$$

Optimal result	661
Rubi [A] (verified)	661
Mathematica [A] (verified)	663
Maple [A] (verified)	663
Fricas [A] (verification not implemented)	664
Sympy [A] (verification not implemented)	664
Maxima [A] (verification not implemented)	665
Giac [A] (verification not implemented)	665
Mupad [B] (verification not implemented)	665

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^4} dx = -\frac{18 \sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} + \frac{12 \sin(c+dx)}{35a^4d(1+\cos(c+dx))} - \frac{\cos^2(c+dx) \sin(c+dx)}{7d(a+a \cos(c+dx))^4} + \frac{8 \sin(c+dx)}{35ad(a+a \cos(c+dx))^3}$$

[Out] $-18/35*\sin(d*x+c)/a^4/d/(1+\cos(d*x+c))^2+12/35*\sin(d*x+c)/a^4/d/(1+\cos(d*x+c))-1/7*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^4+8/35*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^3$

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2844, 3047, 3098, 2829, 2727}

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{12 \sin(c+dx)}{35a^4d(\cos(c+dx)+1)} - \frac{18 \sin(c+dx)}{35a^4d(\cos(c+dx)+1)^2} - \frac{\sin(c+dx) \cos^2(c+dx)}{7d(a \cos(c+dx)+a)^4} + \frac{8 \sin(c+dx)}{35ad(a \cos(c+dx)+a)^3}$$

[In] $\text{Int}[\text{Cos}[c+d*x]^3/(a+a*\text{Cos}[c+d*x])^4,x]$

[Out] $(-18*\text{Sin}[c+d*x])/(35*a^4*d*(1+\text{Cos}[c+d*x])^2) + (12*\text{Sin}[c+d*x])/(35*a^4*d*(1+\text{Cos}[c+d*x])) - (\text{Cos}[c+d*x]^2*\text{Sin}[c+d*x])/(7*d*(a+a*\text{Cos}[c+d*x])^4) + (8*\text{Sin}[c+d*x])/(35*a*d*(a+a*\text{Cos}[c+d*x])^3)$

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2829

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3047

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3098

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(A*b - a*B + b*C)*Cos[e + f*x]*((a + b*sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\cos^2(c + dx) \sin(c + dx)}{7d(a + a \cos(c + dx))^4} - \frac{\int \frac{\cos(c + dx)(2a - 6a \cos(c + dx))}{(a + a \cos(c + dx))^3} dx}{7a^2} \\ &= -\frac{\cos^2(c + dx) \sin(c + dx)}{7d(a + a \cos(c + dx))^4} - \frac{\int \frac{2a \cos(c + dx) - 6a \cos^2(c + dx)}{(a + a \cos(c + dx))^3} dx}{7a^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{8\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{-24a^2+30a^2\cos(c+dx)}{(a+a\cos(c+dx))^2} dx}{35a^4} \\
&= -\frac{18\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\
&\quad + \frac{8\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{12\int \frac{1}{a+a\cos(c+dx)} dx}{35a^3} \\
&= -\frac{18\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} \\
&\quad + \frac{8\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{12\sin(c+dx)}{35d(a^4+a^4\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.49

$$\begin{aligned}
&\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4} dx \\
&= \frac{(2+8\cos(c+dx)+13\cos^2(c+dx)+12\cos^3(c+dx))\sin(c+dx)}{35a^4d(1+\cos(c+dx))^4}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]

[Out] ((2 + 8*Cos[c + d*x] + 13*Cos[c + d*x]^2 + 12*Cos[c + d*x]^3)*Sin[c + d*x]) / (35*a^4*d*(1 + Cos[c + d*x])^4)

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

method	result
parallelrisch	$-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{21\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{56a^4d}$
derivativedivides	$-\frac{\left(\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da^4}$
default	$-\frac{\left(\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da^4}$
risch	$\frac{2i\left(35e^{6i(dx+c)}+105e^{5i(dx+c)}+210e^{4i(dx+c)}+210e^{3i(dx+c)}+147e^{2i(dx+c)}+49e^{i(dx+c)}+12\right)}{35da^4\left(e^{i(dx+c)}+1\right)^7}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{4da}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40da}-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70da}+\frac{13\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280da}+\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140da}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3a^3}$

[In] `int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)`

[Out] $-1/56*(\tan(1/2*d*x+1/2*c)^6-21/5*\tan(1/2*d*x+1/2*c)^4+7*\tan(1/2*d*x+1/2*c)^2-7)*\tan(1/2*d*x+1/2*c)/a^4/d$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4} dx$$

$$= \frac{(12\cos(dx+c)^3 + 13\cos(dx+c)^2 + 8\cos(dx+c) + 2)\sin(dx+c)}{35(a^4d\cos(dx+c)^4 + 4a^4d\cos(dx+c)^3 + 6a^4d\cos(dx+c)^2 + 4a^4d\cos(dx+c) + a^4d)}$$

[In] `integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fricas")`

[Out] $1/35*(12*\cos(d*x+c)^3 + 13*\cos(d*x+c)^2 + 8*\cos(d*x+c) + 2)*\sin(d*x+c)/(a^4*d*\cos(d*x+c)^4 + 4*a^4*d*\cos(d*x+c)^3 + 6*a^4*d*\cos(d*x+c)^2 + 4*a^4*d*\cos(d*x+c) + a^4*d)$

Sympy [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4} dx$$

$$= \begin{cases} -\frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x\cos^3(c)}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

[In] `integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**4,x)`

[Out] `Piecewise((-tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*tan(c/2 + d*x/2)**5/(40*a**4*d) - tan(c/2 + d*x/2)**3/(8*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a)**4, True))`

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^4} dx = \frac{\frac{35 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280 a^4 d}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^4*d)

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^4} dx = \frac{5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 21 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{280 a^4 d}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] -1/280*(5*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d*x + 1/2*c)^3 - 35*tan(1/2*d*x + 1/2*c))/(a^4*d)

Mupad [B] (verification not implemented)

Time = 15.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^4} dx = \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 21 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 35 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 35\right)}{280 a^4 d}$$

[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^4,x)

[Out] -(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 5*tan(c/2 + (d*x)/2)^6 - 35))/(280*a^4*d)

3.76 $\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^4} dx$

Optimal result	666
Rubi [A] (verified)	666
Mathematica [A] (verified)	668
Maple [A] (verified)	668
Fricas [A] (verification not implemented)	669
Sympy [A] (verification not implemented)	669
Maxima [A] (verification not implemented)	669
Giac [A] (verification not implemented)	670
Mupad [B] (verification not implemented)	670

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{\sin(c+dx)}{7d(a+a \cos(c+dx))^4} - \frac{11 \sin(c+dx)}{35ad(a+a \cos(c+dx))^3} + \frac{13 \sin(c+dx)}{105d(a^2+a^2 \cos(c+dx))^2} + \frac{13 \sin(c+dx)}{105d(a^4+a^4 \cos(c+dx))}$$

[Out] 1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4-11/35*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3+13/105*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+13/105*sin(d*x+c)/d/(a^4+a^4*cos(d*x+c))

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2837, 2829, 2729, 2727}

$$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{13 \sin(c+dx)}{105d(a^4 \cos(c+dx) + a^4)} + \frac{13 \sin(c+dx)}{105d(a^2 \cos(c+dx) + a^2)^2} - \frac{11 \sin(c+dx)}{35ad(a \cos(c+dx) + a)^3} + \frac{\sin(c+dx)}{7d(a \cos(c+dx) + a)^4}$$

[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]

[Out] Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) - (11*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + (13*Sin[c + d*x])/(105*d*(a^2 + a^2*Cos[c + d*x])^2) + (13*Sin[c + d*x])/(105*d*(a^4 + a^4*Cos[c + d*x]))

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*cos[c + d*x]*((a + b*sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2829

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2837

Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[b*cos[e + f*x]*((a + b*sin[e + f*x])^m/(a*f*(2*m + 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(a*m - b*(2*m + 1)*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} + \frac{\int \frac{-4a + 7a \cos(c + dx)}{(a + a \cos(c + dx))^3} dx}{7a^2} \\
 &= \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} - \frac{11 \sin(c + dx)}{35ad(a + a \cos(c + dx))^3} + \frac{13 \int \frac{1}{(a + a \cos(c + dx))^2} dx}{35a^2} \\
 &= \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} - \frac{11 \sin(c + dx)}{35ad(a + a \cos(c + dx))^3} \\
 &\quad + \frac{13 \sin(c + dx)}{105d(a^2 + a^2 \cos(c + dx))^2} + \frac{13 \int \frac{1}{a + a \cos(c + dx)} dx}{105a^3} \\
 &= \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} - \frac{11 \sin(c + dx)}{35ad(a + a \cos(c + dx))^3} \\
 &\quad + \frac{13 \sin(c + dx)}{105d(a^2 + a^2 \cos(c + dx))^2} + \frac{13 \sin(c + dx)}{105d(a^4 + a^4 \cos(c + dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{(8 + 32 \cos(c + dx) + 52 \cos^2(c + dx) + 13 \cos^3(c + dx)) \sin(c + dx)}{105a^4d(1 + \cos(c + dx))^4}$$

[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]

[Out] ((8 + 32*Cos[c + d*x] + 52*Cos[c + d*x]^2 + 13*Cos[c + d*x]^3)*Sin[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^4)

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

method	result	size
derivativdivides	$\frac{\left(\frac{\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{7} - \frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{5} - \frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da^4}$	58
default	$\frac{\left(\frac{\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{7} - \frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{5} - \frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3}\right) + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da^4}$	58
parallelrisc	$\frac{15\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 21\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 35\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 105\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{840a^4d}$	60
risc	$\frac{2i(105e^{5i(dx+c)} + 175e^{4i(dx+c)} + 280e^{3i(dx+c)} + 168e^{2i(dx+c)} + 91e^{i(dx+c)} + 13)}{105da^4(e^{i(dx+c)} + 1)^7}$	80
norman	$\frac{\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da} + \frac{5\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24da} + \frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{60da} - \frac{31\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{420da} + \frac{3\left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{280da} + \frac{\tan^{11}\left(\frac{dx}{2} + \frac{c}{2}\right)}{56da}}{a^3\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}$	133

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)

[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{(13 \cos(dx + c)^3 + 52 \cos(dx + c)^2 + 32 \cos(dx + c) + 8) \sin(dx + c)}{105 (a^4 d \cos(dx + c)^4 + 4 a^4 d \cos(dx + c)^3 + 6 a^4 d \cos(dx + c)^2 + 4 a^4 d \cos(dx + c) + a^4 d)}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fricas")

[Out] 1/105*(13*cos(d*x + c)^3 + 52*cos(d*x + c)^2 + 32*cos(d*x + c) + 8)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \begin{cases} \frac{\tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{56a^4d} - \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{40a^4d} - \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{24a^4d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x \cos^2(c)}{(a \cos(c) + a)^4} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**4,x)

[Out] Piecewise((tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/(40*a**4*d) - tan(c/2 + d*x/2)**3/(24*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**4, True))

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^4} dx = \frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840 a^4 d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^4*d)

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{15 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 21 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 105 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{840 a^4 d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/840*(15*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*d*x + 1/2*c)^3 + 105*tan(1/2*d*x + 1/2*c))/(a^4*d)

Mupad [B] (verification not implemented)

Time = 14.94 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= -\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(-15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 21 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 35 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 105\right)}{840 a^4 d}$$

[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^4,x)

[Out] -(tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 - 15*tan(c/2 + (d*x)/2)^6 - 105))/(840*a^4*d)

$$3.77 \quad \int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^4} dx$$

Optimal result	671
Rubi [A] (verified)	671
Mathematica [A] (verified)	673
Maple [A] (verified)	673
Fricas [A] (verification not implemented)	674
Sympy [A] (verification not implemented)	674
Maxima [A] (verification not implemented)	674
Giac [A] (verification not implemented)	675
Mupad [B] (verification not implemented)	675

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^4} dx = -\frac{\sin(c+dx)}{7d(a+a \cos(c+dx))^4} + \frac{4 \sin(c+dx)}{35ad(a+a \cos(c+dx))^3} + \frac{8 \sin(c+dx)}{105d(a^2+a^2 \cos(c+dx))^2} + \frac{8 \sin(c+dx)}{105d(a^4+a^4 \cos(c+dx))}$$

[Out] -1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+4/35*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3+8/105*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+8/105*sin(d*x+c)/d/(a^4+a^4*cos(d*x+c))

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2829, 2729, 2727}

$$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{8 \sin(c+dx)}{105d(a^4 \cos(c+dx) + a^4)} + \frac{8 \sin(c+dx)}{105d(a^2 \cos(c+dx) + a^2)^2} + \frac{4 \sin(c+dx)}{35ad(a \cos(c+dx) + a)^3} - \frac{\sin(c+dx)}{7d(a \cos(c+dx) + a)^4}$$

[In] Int[Cos[c + d*x]/(a + a*cos[c + d*x])^4,x]

[Out] -1/7*Sin[c + d*x]/(d*(a + a*cos[c + d*x])^4) + (4*Sin[c + d*x])/(35*a*d*(a + a*cos[c + d*x])^3) + (8*Sin[c + d*x])/(105*d*(a^2 + a^2*cos[c + d*x])^2) + (8*Sin[c + d*x])/(105*d*(a^4 + a^4*cos[c + d*x]))

Rule 2727

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c +
d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4 \int \frac{1}{(a+a\cos(c+dx))^3} dx}{7a} \\
&= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{8 \int \frac{1}{(a+a\cos(c+dx))^2} dx}{35a^2} \\
&= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} \\
&\quad + \frac{8\sin(c+dx)}{105d(a^2+a^2\cos(c+dx))^2} + \frac{8 \int \frac{1}{a+a\cos(c+dx)} dx}{105a^3} \\
&= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} \\
&\quad + \frac{8\sin(c+dx)}{105d(a^2+a^2\cos(c+dx))^2} + \frac{8\sin(c+dx)}{105d(a^4+a^4\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{(13 + 52 \cos(c + dx) + 32 \cos^2(c + dx) + 8 \cos^3(c + dx)) \sin(c + dx)}{105a^4 d (1 + \cos(c + dx))^4}$$

`[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^4, x]`

```
[Out] ((13 + 52*Cos[c + d*x] + 32*Cos[c + d*x]^2 + 8*Cos[c + d*x]^3)*Sin[c + d*x]
)/(105*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.51

method	result	size
parallelrisch	$-\frac{\left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{7(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right))}{5} - \frac{7(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))}{3} - 7\right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{56a^4 d}$	57
derivativedivides	$-\frac{\frac{(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right))}{7} - \frac{(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right))}{5} + \frac{(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right))}{3} + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da^4}$	58
default	$-\frac{(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right))}{7} - \frac{(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right))}{5} + \frac{(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right))}{3} + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da^4}$	58
risch	$\frac{8i(35e^{4i(dx+c)} + 35e^{3i(dx+c)} + 42e^{2i(dx+c)} + 14e^{i(dx+c)} + 2)}{105da^4(e^{i(dx+c)} + 1)^7}$	69
norman	$\frac{\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da} + \frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{6da} + \frac{\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{60da} - \frac{3(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right))}{70da} - \frac{\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)}{56da}}{a^3\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$	114

`[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)`

```
[Out] -1/56*(tan(1/2*d*x+1/2*c)^6+7/5*tan(1/2*d*x+1/2*c)^4-7/3*tan(1/2*d*x+1/2*c)
^2-7)*tan(1/2*d*x+1/2*c)/a^4/d
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4} dx$$

$$= \frac{(8\cos(dx+c)^3 + 32\cos(dx+c)^2 + 52\cos(dx+c) + 13)\sin(dx+c)}{105(a^4d\cos(dx+c)^4 + 4a^4d\cos(dx+c)^3 + 6a^4d\cos(dx+c)^2 + 4a^4d\cos(dx+c) + a^4d)}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fricas")

[Out] 1/105*(8*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 52*cos(d*x + c) + 13)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.76

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4} dx$$

$$= \begin{cases} -\frac{\tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{56a^4d} - \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{40a^4d} + \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{24a^4d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**4,x)

[Out] Piecewise((-tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/(40*a**4*d) + tan(c/2 + d*x/2)**3/(24*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0)), (x*cos(c)/(a*cos(c) + a)**4, True))

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4} dx = \frac{105\sin(dx+c)}{\cos(dx+c)+1} + \frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7} \frac{1}{840a^4d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^4*d)

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^4} dx =$$

$$\frac{15 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 21 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 105 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{840 a^4 d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] -1/840*(15*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/(a^4*d)

Mupad [B] (verification not implemented)

Time = 14.94 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(-15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 21 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 35 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 105\right)}{840 a^4 d}$$

[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^4,x)

[Out] (tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 - 15*tan(c/2 + (d*x)/2)^6 + 105))/(840*a^4*d)

3.78 $\int \frac{1}{(a+a \cos(c+dx))^4} dx$

Optimal result	676
Rubi [A] (verified)	676
Mathematica [A] (verified)	677
Maple [A] (verified)	678
Fricas [A] (verification not implemented)	678
Sympy [A] (verification not implemented)	679
Maxima [A] (verification not implemented)	679
Giac [A] (verification not implemented)	679
Mupad [B] (verification not implemented)	680

Optimal result

Integrand size = 12, antiderivative size = 112

$$\int \frac{1}{(a+a \cos(c+dx))^4} dx = \frac{\sin(c+dx)}{7d(a+a \cos(c+dx))^4} + \frac{3 \sin(c+dx)}{35ad(a+a \cos(c+dx))^3} + \frac{2 \sin(c+dx)}{35d(a^2+a^2 \cos(c+dx))^2} + \frac{2 \sin(c+dx)}{35d(a^4+a^4 \cos(c+dx))}$$

[Out] 1/7*sin(d*x+c)/d/(a+a*cos(d*x+c))^4+3/35*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^3+2/35*sin(d*x+c)/d/(a^2+a^2*cos(d*x+c))^2+2/35*sin(d*x+c)/d/(a^4+a^4*cos(d*x+c))

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2729, 2727}

$$\int \frac{1}{(a+a \cos(c+dx))^4} dx = \frac{2 \sin(c+dx)}{35d(a^4 \cos(c+dx) + a^4)} + \frac{2 \sin(c+dx)}{35d(a^2 \cos(c+dx) + a^2)^2} + \frac{3 \sin(c+dx)}{35ad(a \cos(c+dx) + a)^3} + \frac{\sin(c+dx)}{7d(a \cos(c+dx) + a)^4}$$

[In] Int[(a + a*Cos[c + d*x])^(-4), x]

[Out] Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) + (3*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3) + (2*Sin[c + d*x])/(35*d*(a^2 + a^2*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(35*d*(a^4 + a^4*Cos[c + d*x]))

Rule 2727


```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c +
d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} + \frac{3 \int \frac{1}{(a + a \cos(c + dx))^3} dx}{7a} \\
&= \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} + \frac{3 \sin(c + dx)}{35ad(a + a \cos(c + dx))^3} + \frac{6 \int \frac{1}{(a + a \cos(c + dx))^2} dx}{35a^2} \\
&= \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} + \frac{3 \sin(c + dx)}{35ad(a + a \cos(c + dx))^3} \\
&\quad + \frac{2 \sin(c + dx)}{35d(a^2 + a^2 \cos(c + dx))^2} + \frac{2 \int \frac{1}{a + a \cos(c + dx)} dx}{35a^3} \\
&= \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} + \frac{3 \sin(c + dx)}{35ad(a + a \cos(c + dx))^3} \\
&\quad + \frac{2 \sin(c + dx)}{35d(a^2 + a^2 \cos(c + dx))^2} + \frac{2 \sin(c + dx)}{35d(a^4 + a^4 \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\begin{aligned}
&\int \frac{1}{(a + a \cos(c + dx))^4} dx \\
&= \frac{\cos\left(\frac{1}{2}(c + dx)\right) \left(35 \sin\left(\frac{1}{2}(c + dx)\right) + 21 \sin\left(\frac{3}{2}(c + dx)\right) + 7 \sin\left(\frac{5}{2}(c + dx)\right) + \sin\left(\frac{7}{2}(c + dx)\right)\right)}{70a^4d(1 + \cos(c + dx))^4}
\end{aligned}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(-4), x]
```

```
[Out] (Cos[(c + d*x)/2]*(35*Sin[(c + d*x)/2] + 21*Sin[(3*(c + d*x))/2] + 7*Sin[(5
*(c + d*x))/2] + Sin[(7*(c + d*x))/2]))/(70*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

method	result	size
derivativedivides	$\frac{\left(\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da^4}$	56
default	$\frac{\left(\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{7}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da^4}$	56
risch	$\frac{4i\left(35e^{3i(dx+c)}+21e^{2i(dx+c)}+7e^{i(dx+c)}+1\right)}{35da^4\left(e^{i(dx+c)}+1\right)^7}$	58
parallelrisc	$\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+35\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{280a^4d}$	60
norman	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40da}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}$ a^3	80

[In] int(1/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)

[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+a\cos(c+dx))^4} dx$$

$$= \frac{(2\cos(dx+c)^3+8\cos(dx+c)^2+13\cos(dx+c)+12)\sin(dx+c)}{35(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d)}$$

[In] integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="fricas")

[Out] 1/35*(2*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 13*cos(d*x + c) + 12)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a + a \cos(c + dx))^4} dx = \begin{cases} \frac{\tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{56a^4d} + \frac{3 \tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{40a^4d} + \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{8a^4d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x}{(a \cos(c) + a)^4} & \text{otherwise} \end{cases}$$

[In] integrate(1/(a+a*cos(d*x+c))**4,x)

[Out] Piecewise((tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*tan(c/2 + d*x/2)**5/(40*a**4*d) + tan(c/2 + d*x/2)**3/(8*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0)), (x/(a*cos(c) + a)**4, True))

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a + a \cos(c + dx))^4} dx = \frac{\frac{35 \sin(dx+c)}{\cos(dx+c)+1} + \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280 a^4 d}$$

[In] integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^4*d)

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{1}{(a + a \cos(c + dx))^4} dx = \frac{5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 21 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{280 a^4 d}$$

[In] integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/280*(5*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d*x + 1/2*c)^3 + 35*tan(1/2*d*x + 1/2*c))/(a^4*d)

Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 21 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 35 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 35\right)}{280 a^4 d}$$

[In] int(1/(a + a*cos(c + d*x))^4,x)

[Out] (tan(c/2 + (d*x)/2)*(35*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 5*tan(c/2 + (d*x)/2)^6 + 35))/(280*a^4*d)

3.79 $\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^4} dx$

Optimal result	681
Rubi [A] (verified)	681
Mathematica [A] (verified)	683
Maple [A] (verified)	684
Fricas [A] (verification not implemented)	684
Sympy [F]	685
Maxima [A] (verification not implemented)	685
Giac [A] (verification not implemented)	685
Mupad [B] (verification not implemented)	686

Optimal result

Integrand size = 19, antiderivative size = 120

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4 d} - \frac{11 \sin(c+dx)}{21 a^4 d (1+\cos(c+dx))^2} - \frac{32 \sin(c+dx)}{21 a^4 d (1+\cos(c+dx))} - \frac{\sin(c+dx)}{7 d (a+a \cos(c+dx))^4} - \frac{2 \sin(c+dx)}{7 a d (a+a \cos(c+dx))^3}$$

[Out] $\operatorname{arctanh}(\sin(d*x+c))/a^4/d - 11/21*\sin(d*x+c)/a^4/d/(1+\cos(d*x+c))^2 - 32/21*\sin(d*x+c)/a^4/d/(1+\cos(d*x+c)) - 1/7*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^4 - 2/7*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^3$

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2845, 3057, 12, 3855}

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4 d} - \frac{32 \sin(c+dx)}{21 a^4 d (\cos(c+dx)+1)} - \frac{11 \sin(c+dx)}{21 a^4 d (\cos(c+dx)+1)^2} - \frac{2 \sin(c+dx)}{7 a d (a \cos(c+dx)+a)^3} - \frac{\sin(c+dx)}{7 d (a \cos(c+dx)+a)^4}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]/(a+a*\operatorname{Cos}[c+d*x])^4, x]$

[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) - (11*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*x])^2) - (32*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*x])) - Sin[c + d*x]/(7*d*(a + a*cos[c + d*x])^4) - (2*Sin[c + d*x])/(7*a*d*(a + a*cos[c + d*x])^3)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3855

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} + \frac{\int \frac{(7a - 3a \cos(c + dx)) \sec(c + dx)}{(a + a \cos(c + dx))^3} dx}{7a^2} \\ &= -\frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} - \frac{2 \sin(c + dx)}{7ad(a + a \cos(c + dx))^3} + \frac{\int \frac{(35a^2 - 20a^2 \cos(c + dx)) \sec(c + dx)}{(a + a \cos(c + dx))^2} dx}{35a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{11 \sin(c + dx)}{21a^4d(1 + \cos(c + dx))^2} - \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} \\
&\quad - \frac{2 \sin(c + dx)}{7ad(a + a \cos(c + dx))^3} + \frac{\int \frac{(105a^3 - 55a^3 \cos(c + dx)) \sec(c + dx)}{a + a \cos(c + dx)} dx}{105a^6} \\
&= -\frac{11 \sin(c + dx)}{21a^4d(1 + \cos(c + dx))^2} - \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} - \frac{2 \sin(c + dx)}{7ad(a + a \cos(c + dx))^3} \\
&\quad - \frac{32 \sin(c + dx)}{21d(a^4 + a^4 \cos(c + dx))} + \frac{\int 105a^4 \sec(c + dx) dx}{105a^8} \\
&= -\frac{11 \sin(c + dx)}{21a^4d(1 + \cos(c + dx))^2} - \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} \\
&\quad - \frac{2 \sin(c + dx)}{7ad(a + a \cos(c + dx))^3} - \frac{32 \sin(c + dx)}{21d(a^4 + a^4 \cos(c + dx))} + \frac{\int \sec(c + dx) dx}{a^4} \\
&= \frac{\operatorname{arctanh}(\sin(c + dx))}{a^4d} - \frac{11 \sin(c + dx)}{21a^4d(1 + \cos(c + dx))^2} - \frac{\sin(c + dx)}{7d(a + a \cos(c + dx))^4} \\
&\quad - \frac{2 \sin(c + dx)}{7ad(a + a \cos(c + dx))^3} - \frac{32 \sin(c + dx)}{21d(a^4 + a^4 \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.54

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{-1344 \cos^8\left(\frac{1}{2}(c + dx)\right) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)\right)}{84a^4d(1 + \cos(c + dx))^4}$$

[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^4,x]

[Out] (-1344*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-686*Sin[(d*x)/2] + 434*Sin[c + (d*x)/2] - 525*Sin[c + (3*d*x)/2] + 147*Sin[2*c + (3*d*x)/2] - 203*Sin[2*c + (5*d*x)/2] + 21*Sin[3*c + (5*d*x)/2] - 32*Sin[3*c + (7*d*x)/2]))/(84*a^4*d*(1 + Cos[c + d*x])^4)

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8da^4}$
default	$-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8da^4}$
parallelrisc	$\frac{-3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-21\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-77\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-168\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+168\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{168a^4d}$
norman	$\frac{15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24da}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da^4}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^4d}$
risc	$-\frac{2i\left(21e^{6i(dx+c)}+147e^{5i(dx+c)}+434e^{4i(dx+c)}+686e^{3i(dx+c)}+525e^{2i(dx+c)}+203e^{i(dx+c)}+32\right)}{21da^4\left(e^{i(dx+c)}+1\right)^7}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{da^4}-$

[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)

[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-tan(1/2*d*x+1/2*c)^5-11/3*tan(1/2*d*x+1/2*c)^3-15*tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*c)-1)+8*ln(tan(1/2*d*x+1/2*c)+1))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.68

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4} dx$$

$$= \frac{21(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1)\log(\sin(dx+c)+1)-21(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1)\log(-\sin(dx+c)+1)-2*(32\cos(dx+c)^3+107\cos(dx+c)^2+124\cos(dx+c)+52)*\sin(dx+c)}{42(a^4d\cos(dx+c)+a^4d)}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fricas")

[Out] 1/42*(21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 + 107*cos(d*x + c)^2 + 124*cos(d*x + c) + 52)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

Sympy [F]

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^4} dx = \int \frac{\sec(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1} \frac{dx}{a^4}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**4,x)

[Out] Integral(sec(c + d*x)/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) + 1), x)/a**4

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.16

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^4} dx = \frac{\frac{315 \sin(dx+c)}{\cos(dx+c)+1} + \frac{77 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{168 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^4} + \frac{168 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^4}$$

168 d

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^4} dx = \frac{\frac{168 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a^4} - \frac{168 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a^4} - \frac{3 a^{24} \tan(\frac{1}{2} dx + \frac{1}{2} c)^7 + 21 a^{24} \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 77 a^{24} \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 315 a^{24} \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a^{28}}}{168 d}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/168*(168*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 168*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - (3*a^24*tan(1/2*d*x + 1/2*c)^7 + 21*a^24*tan(1/2*d*x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 + 315*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d

Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= -\frac{\frac{11 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{24 a^4} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{8 a^4} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{56 a^4} - \frac{2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^4} + \frac{15 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8 a^4}}{d}$$

[In] `int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^4),x)`

[Out] `-((11*tan(c/2 + (d*x)/2)^3)/(24*a^4) + tan(c/2 + (d*x)/2)^5/(8*a^4) + tan(c/2 + (d*x)/2)^7/(56*a^4) - (2*atanh(tan(c/2 + (d*x)/2)))/a^4 + (15*tan(c/2 + (d*x)/2))/(8*a^4))/d`

3.80 $\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^4} dx$

Optimal result	687
Rubi [A] (verified)	687
Mathematica [B] (verified)	690
Maple [A] (verified)	690
Fricas [A] (verification not implemented)	691
Sympy [F]	691
Maxima [A] (verification not implemented)	691
Giac [A] (verification not implemented)	692
Mupad [B] (verification not implemented)	692

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^4} dx = -\frac{4\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{664 \tan(c+dx)}{105a^4d} - \frac{88 \tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{4 \tan(c+dx)}{a^4d(1+\cos(c+dx))} - \frac{\tan(c+dx)}{7d(a+a \cos(c+dx))^4} - \frac{12 \tan(c+dx)}{35ad(a+a \cos(c+dx))^3}$$

[Out] $-4*\operatorname{arctanh}(\sin(d*x+c))/a^4/d+664/105*\tan(d*x+c)/a^4/d-88/105*\tan(d*x+c)/a^4/d/(1+\cos(d*x+c))^2-4*\tan(d*x+c)/a^4/d/(1+\cos(d*x+c))-1/7*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^4-12/35*\tan(d*x+c)/a/d/(a+a*\cos(d*x+c))^3$

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2845, 3057, 2827, 3852, 8, 3855}

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^4} dx = -\frac{4\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{664 \tan(c+dx)}{105a^4d} - \frac{4 \tan(c+dx)}{a^4d(\cos(c+dx)+1)} - \frac{88 \tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2} - \frac{12 \tan(c+dx)}{35ad(a \cos(c+dx)+a)^3} - \frac{\tan(c+dx)}{7d(a \cos(c+dx)+a)^4}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]^2/(a+a*\operatorname{Cos}[c+d*x])^4,x]$

[Out] $(-4 \operatorname{ArcTanh}[\sin[c + dx]])/(a^4 d) + (664 \operatorname{Tan}[c + dx])/(105 a^4 d) - (88 \operatorname{Tan}[c + dx])/(105 a^4 d (1 + \cos[c + dx])^2) - (4 \operatorname{Tan}[c + dx])/(a^4 d (1 + \cos[c + dx])) - \operatorname{Tan}[c + dx]/(7 d (a + a \cos[c + dx])^4) - (12 \operatorname{Tan}[c + dx])/(35 a d (a + a \cos[c + dx])^3)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] \text{ /; } \operatorname{FreeQ}[a, x]$

Rule 2827

$\operatorname{Int}[(b_.) \sin[(e_.) + (f_.) (x_)]^{(m_)} ((c_.) + (d_.) \sin[(e_.) + (f_.) (x_)]), x_Symbol] \rightarrow \operatorname{Dist}[c, \operatorname{Int}[(b \sin[e + fx])^m, x], x] + \operatorname{Dist}[d/b, \operatorname{Int}[(b \sin[e + fx])^{m+1}, x], x] \text{ /; } \operatorname{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2845

$\operatorname{Int}[(a_.) + (b_.) \sin[(e_.) + (f_.) (x_)]^{(m_)} ((c_.) + (d_.) \sin[(e_.) + (f_.) (x_)]^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[b^2 \cos[e + fx] (a + b \sin[e + fx])^m (c + d \sin[e + fx])^{n+1} / (a f (2m+1) (b c - a d)), x] + \operatorname{Dist}[1 / (a (2m+1) (b c - a d)), \operatorname{Int}[(a + b \sin[e + fx])^{m+1} (c + d \sin[e + fx])^n \operatorname{Simp}[b c (m+1) - a d (2m+n+2) + b d (m+n+2) \sin[e + fx], x], x], x] \text{ /; } \operatorname{FreeQ}[\{a, b, c, d, e, f, n\}, x] \ \&\& \operatorname{NeQ}[b c - a d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0] \ \&\& \operatorname{LtQ}[m, -1] \ \&\& \operatorname{!GtQ}[n, 0] \ \&\& (\operatorname{IntegerSQ}[2m, 2n] \ || \ (\operatorname{IntegerQ}[m] \ \&\& \operatorname{EqQ}[c, 0]))$

Rule 3057

$\operatorname{Int}[(a_.) + (b_.) \sin[(e_.) + (f_.) (x_)]^{(m_)} ((A_.) + (B_.) \sin[(e_.) + (f_.) (x_)]^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[b (A b - a B) \cos[e + fx] (a + b \sin[e + fx])^m (c + d \sin[e + fx])^{n+1} / (a f (2m+1) (b c - a d)), x] + \operatorname{Dist}[1 / (a (2m+1) (b c - a d)), \operatorname{Int}[(a + b \sin[e + fx])^{m+1} (c + d \sin[e + fx])^n \operatorname{Simp}[B (a c m + b d (n+1)) + A (b c (m+1) - a d (2m+n+2)) + d (A b - a B) (m+n+2) \sin[e + fx], x], x], x] \text{ /; } \operatorname{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \operatorname{NeQ}[b c - a d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0] \ \&\& \operatorname{LtQ}[m, -2^{(-1)}] \ \&\& \operatorname{!GtQ}[n, 0] \ \&\& \operatorname{IntegerQ}[2m] \ \&\& (\operatorname{IntegerQ}[2n] \ || \ \operatorname{EqQ}[c, 0])$

Rule 3852

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.) (x_)]^{(n_)}], x_Symbol] \rightarrow \operatorname{Dist}[-d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}], x], x], x, \operatorname{Cot}[c + dx]], x] \text{ /; } \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3855

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(8a-4a\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3} dx}{7a^2} \\
 &= -\frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{(52a^2-36a^2\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2} dx}{35a^4} \\
 &= -\frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} \\
 &\quad - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{(244a^3-176a^3\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)} dx}{105a^6} \\
 &= -\frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} \\
 &\quad - \frac{4\tan(c+dx)}{d(a^4+a^4\cos(c+dx))} + \frac{\int (664a^4-420a^4\cos(c+dx))\sec^2(c+dx) dx}{105a^8} \\
 &= -\frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} \\
 &\quad - \frac{4\tan(c+dx)}{d(a^4+a^4\cos(c+dx))} - \frac{4\int \sec(c+dx) dx}{a^4} + \frac{664\int \sec^2(c+dx) dx}{105a^4} \\
 &= -\frac{4\arctanh(\sin(c+dx))}{a^4d} - \frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} \\
 &\quad - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{4\tan(c+dx)}{d(a^4+a^4\cos(c+dx))} - \frac{664\text{Subst}(\int 1 dx, x, -\tan(c+dx))}{105a^4d} \\
 &= -\frac{4\arctanh(\sin(c+dx))}{a^4d} + \frac{664\tan(c+dx)}{105a^4d} - \frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} \\
 &\quad - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{4\tan(c+dx)}{d(a^4+a^4\cos(c+dx))}
 \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 341 vs. $2(135) = 270$.

Time = 3.38 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.53

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^4} dx$$

$$= \frac{107520 \cos^8\left(\frac{1}{2}(c + dx)\right) \left(\log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)\right)}{1680 a^4 d (1 + \cos(c + dx))^4}$$

[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]

[Out] (107520*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]*(-10780*Sin[(d*x)/2] + 18788*Sin[(3*d*x)/2] - 20524*Sin[c - (d*x)/2] + 14644*Sin[c + (d*x)/2] - 16660*Sin[2*c + (d*x)/2] - 4690*Sin[c + (3*d*x)/2] + 14378*Sin[2*c + (3*d*x)/2] - 9100*Sin[3*c + (3*d*x)/2] + 11668*Sin[c + (5*d*x)/2] - 630*Sin[2*c + (5*d*x)/2] + 9358*Sin[3*c + (5*d*x)/2] - 2940*Sin[4*c + (5*d*x)/2] + 4228*Sin[2*c + (7*d*x)/2] + 315*Sin[3*c + (7*d*x)/2] + 3493*Sin[4*c + (7*d*x)/2] - 420*Sin[5*c + (7*d*x)/2] + 664*Sin[3*c + (9*d*x)/2] + 105*Sin[4*c + (9*d*x)/2] + 559*Sin[5*c + (9*d*x)/2]))/(1680*a^4*d*(1 + Cos[c + d*x])^4)

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{7} + \frac{7\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{5} + \frac{23\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3} + 49\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{8}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1} - 32\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{8}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1}}{8da^4}$
default	$\frac{\frac{\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{7} + \frac{7\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{5} + \frac{23\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{3} + 49\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{8}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1} - 32\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{8}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1}}{8da^4}$
parallelrisc	$\frac{3360\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\cos(dx+c) - 3360\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)\cos(dx+c) + 2861\left(\cos(dx+c) + \frac{1650\cos(2dx+2c)}{2861} + \frac{559\cos(3dx+3c)}{2861}\right)}{840a^4d\cos(dx+c)}$
norman	$-\frac{65\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da} + \frac{31\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{6da} + \frac{47\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)}{60da} + \frac{11\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{70da} + \frac{\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)}{56da} + \frac{4\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^4d} - \frac{8}{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1}$
risc	$\frac{8i(105e^{8i(dx+c)} + 735e^{7i(dx+c)} + 2275e^{6i(dx+c)} + 4165e^{5i(dx+c)} + 5131e^{4i(dx+c)} + 4697e^{3i(dx+c)} + 2917e^{2i(dx+c)} + 1057e^{i(dx+c)})}{105da^4(e^{i(dx+c)} + 1)^7(e^{2i(dx+c)} + 1)}$

[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)

[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+7/5*tan(1/2*d*x+1/2*c)^5+23/3*tan(1/2*d*x+1/2*c)^3+49*tan(1/2*d*x+1/2*c)-8/(tan(1/2*d*x+1/2*c)+1)-32*ln(tan(1/2*d*x+1/2*c)+1)-8/(tan(1/2*d*x+1/2*c)-1))

$$x+1/2*c)+1)-8/(\tan(1/2*d*x+1/2*c)-1)+32*\ln(\tan(1/2*d*x+1/2*c)-1))$$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.73

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4} dx = \frac{210(\cos(dx+c))^5 + 4\cos(dx+c)^4 + 6\cos(dx+c)^3 + 4\cos(dx+c)^2 + \cos(dx+c)\log(\sin(dx+c)+1) - 210(\cos(dx+c))^5 + 4\cos(dx+c)^4 + 6\cos(dx+c)^3 + 4\cos(dx+c)^2 + \cos(dx+c)\log(-\sin(dx+c)+1) - (664\cos(dx+c)^4 + 2236\cos(dx+c)^3 + 2636\cos(dx+c)^2 + 1184\cos(dx+c) + 105)\sin(dx+c)}{a^4 d \cos(dx+c)^5 + 4a^4 d \cos(dx+c)^4 + 6a^4 d \cos(dx+c)^3 + 4a^4 d \cos(dx+c)^2 + a^4 d \cos(dx+c)}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fricas")

[Out] -1/105*(210*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - (664*cos(d*x + c)^4 + 2236*cos(d*x + c)^3 + 2636*cos(d*x + c)^2 + 1184*cos(d*x + c) + 105)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4} dx = \frac{\int \frac{\sec^2(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1} dx}{a^4}$$

[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**4,x)

[Out] Integral(sec(c + d*x)**2/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) + 1), x)/a**4

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.38

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4} dx = \frac{1680 \sin(dx+c)}{(a^4 - \frac{a^4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2})(\cos(dx+c)+1)} + \frac{\frac{5145 \sin(dx+c)}{\cos(dx+c)+1} + \frac{805 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{147 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4} - \frac{3360 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^4} + \dots$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] 1/840*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^4} dx = \frac{\frac{3360 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^4} - \frac{3360 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{a^4} + \frac{1680 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)a^4} - \frac{15 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 147 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 805 a^{24} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3}{840 d}}{840 d}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] -1/840*(3360*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^4) - (15*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*a^24*tan(1/2*d*x + 1/2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d

Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^4} dx = \frac{23 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{24 a^4 d} + \frac{7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{40 a^4 d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{56 a^4 d} - \frac{8 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^4 d} - \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - a^4\right)} + \frac{49 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8 a^4 d}$$

[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^4),x)

[Out] (23*tan(c/2 + (d*x)/2)^3)/(24*a^4*d) + (7*tan(c/2 + (d*x)/2)^5)/(40*a^4*d) + tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (8*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (2*tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2)^2 - a^4)) + (49*tan(c/2 + (d*x)/2))/(8*a^4*d)

3.81 $\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^4} dx$

Optimal result	693
Rubi [A] (verified)	693
Mathematica [B] (verified)	696
Maple [A] (verified)	697
Fricas [A] (verification not implemented)	697
Sympy [F]	698
Maxima [A] (verification not implemented)	698
Giac [A] (verification not implemented)	698
Mupad [B] (verification not implemented)	699

Optimal result

Integrand size = 21, antiderivative size = 185

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^4} dx = \frac{21 \operatorname{arctanh}(\sin(c+dx))}{2a^4d} - \frac{576 \tan(c+dx)}{35a^4d} + \frac{21 \sec(c+dx) \tan(c+dx)}{2a^4d} - \frac{43 \sec(c+dx) \tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{288 \sec(c+dx) \tan(c+dx)}{35a^4d(1+\cos(c+dx))} - \frac{\sec(c+dx) \tan(c+dx)}{7d(a+a \cos(c+dx))^4} - \frac{2 \sec(c+dx) \tan(c+dx)}{5ad(a+a \cos(c+dx))^3}$$

[Out] 21/2*arctanh(sin(d*x+c))/a^4/d-576/35*tan(d*x+c)/a^4/d+21/2*sec(d*x+c)*tan(d*x+c)/a^4/d-43/35*sec(d*x+c)*tan(d*x+c)/a^4/d/(1+cos(d*x+c))^2-288/35*sec(d*x+c)*tan(d*x+c)/a^4/d/(1+cos(d*x+c))-1/7*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^4-2/5*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^3

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {2845, 3057, 2827, 3853, 3855, 3852, 8}

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^4} dx = \frac{21 \operatorname{arctanh}(\sin(c + dx))}{2a^4 d} - \frac{576 \tan(c + dx)}{35a^4 d} + \frac{21 \tan(c + dx) \sec(c + dx)}{2a^4 d} - \frac{288 \tan(c + dx) \sec(c + dx)}{35a^4 d (\cos(c + dx) + 1)} - \frac{43 \tan(c + dx) \sec(c + dx)}{35a^4 d (\cos(c + dx) + 1)^2} - \frac{2 \tan(c + dx) \sec(c + dx)}{5ad(a \cos(c + dx) + a)^3} - \frac{\tan(c + dx) \sec(c + dx)}{7d(a \cos(c + dx) + a)^4}$$

[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]

[Out] (21*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (576*Tan[c + d*x])/(35*a^4*d) + (21*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - (43*Sec[c + d*x]*Tan[c + d*x])/(35*a^4*d*(1 + Cos[c + d*x])^2) - (288*Sec[c + d*x]*Tan[c + d*x])/(35*a^4*d*(1 + Cos[c + d*x])) - (Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*Sec[c + d*x]*Tan[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2845

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*

$d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2) * \text{Sin}[e + f*x], x], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3852

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d*x]*((b*\text{Csc}[c + d*x])^{(n - 1)}/(d*(n - 1))), x] + \text{Dist}[b^2*((n - 2)/(n - 1)), \text{Int}[(b*\text{Csc}[c + d*x])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sec(c + dx) \tan(c + dx)}{7d(a + a \cos(c + dx))^4} + \frac{\int \frac{(9a - 5a \cos(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^3} dx}{7a^2} \\
 &= -\frac{\sec(c + dx) \tan(c + dx)}{7d(a + a \cos(c + dx))^4} - \frac{2 \sec(c + dx) \tan(c + dx)}{5ad(a + a \cos(c + dx))^3} + \frac{\int \frac{(73a^2 - 56a^2 \cos(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^2} dx}{35a^4} \\
 &= -\frac{43 \sec(c + dx) \tan(c + dx)}{35a^4 d(1 + \cos(c + dx))^2} - \frac{\sec(c + dx) \tan(c + dx)}{7d(a + a \cos(c + dx))^4} \\
 &\quad - \frac{2 \sec(c + dx) \tan(c + dx)}{5ad(a + a \cos(c + dx))^3} + \frac{\int \frac{(477a^3 - 387a^3 \cos(c + dx)) \sec^3(c + dx)}{a + a \cos(c + dx)} dx}{105a^6} \\
 &= -\frac{43 \sec(c + dx) \tan(c + dx)}{35a^4 d(1 + \cos(c + dx))^2} - \frac{\sec(c + dx) \tan(c + dx)}{7d(a + a \cos(c + dx))^4} - \frac{2 \sec(c + dx) \tan(c + dx)}{5ad(a + a \cos(c + dx))^3} \\
 &\quad - \frac{288 \sec(c + dx) \tan(c + dx)}{35d(a^4 + a^4 \cos(c + dx))} + \frac{\int (2205a^4 - 1728a^4 \cos(c + dx)) \sec^3(c + dx) dx}{105a^8} \\
 &= -\frac{43 \sec(c + dx) \tan(c + dx)}{35a^4 d(1 + \cos(c + dx))^2} - \frac{\sec(c + dx) \tan(c + dx)}{7d(a + a \cos(c + dx))^4} - \frac{2 \sec(c + dx) \tan(c + dx)}{5ad(a + a \cos(c + dx))^3} \\
 &\quad - \frac{288 \sec(c + dx) \tan(c + dx)}{35d(a^4 + a^4 \cos(c + dx))} - \frac{576 \int \sec^2(c + dx) dx}{35a^4} + \frac{21 \int \sec^3(c + dx) dx}{a^4}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{21 \sec(c+dx) \tan(c+dx)}{2a^4d} - \frac{43 \sec(c+dx) \tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\sec(c+dx) \tan(c+dx)}{7d(a+a\cos(c+dx))^4} \\
&\quad - \frac{2 \sec(c+dx) \tan(c+dx)}{5ad(a+a\cos(c+dx))^3} - \frac{288 \sec(c+dx) \tan(c+dx)}{35d(a^4+a^4\cos(c+dx))} \\
&\quad + \frac{21 \int \sec(c+dx) dx}{2a^4} + \frac{576 \text{Subst}(\int 1 dx, x, -\tan(c+dx))}{35a^4d} \\
&= \frac{21 \arctanh(\sin(c+dx))}{2a^4d} - \frac{576 \tan(c+dx)}{35a^4d} + \frac{21 \sec(c+dx) \tan(c+dx)}{2a^4d} \\
&\quad - \frac{43 \sec(c+dx) \tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\sec(c+dx) \tan(c+dx)}{7d(a+a\cos(c+dx))^4} \\
&\quad - \frac{2 \sec(c+dx) \tan(c+dx)}{5ad(a+a\cos(c+dx))^3} - \frac{288 \sec(c+dx) \tan(c+dx)}{35d(a^4+a^4\cos(c+dx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 455 vs. 2(185) = 370.

Time = 6.78 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.46

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4} dx &= -\frac{168 \cos^8\left(\frac{c}{2} + \frac{dx}{2}\right) \log\left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) - \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d(a+a\cos(c+dx))^4} \\
&+ \frac{168 \cos^8\left(\frac{c}{2} + \frac{dx}{2}\right) \log\left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) + \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d(a+a\cos(c+dx))^4} \\
&+ \frac{\cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sec\left(\frac{c}{2}\right) \sec(c) \sec^2(c+dx) \left(24402 \sin\left(\frac{dx}{2}\right) - 55556 \sin\left(\frac{3dx}{2}\right) + 61054 \sin\left(c - \frac{dx}{2}\right) - 33614 \sin\left(c + \frac{dx}{2}\right)\right)}{d(a+a\cos(c+dx))^4}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]

[Out] (-168*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]/(d*(a + a*Cos[c + d*x])^4) + (168*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]/(d*(a + a*Cos[c + d*x])^4) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(24402*Sin[(d*x)/2] - 55556*Sin[(3*d*x)/2] + 61054*Sin[c - (d*x)/2] - 33614*Sin[c + (d*x)/2] + 51842*Sin[2*c + (d*x)/2] + 12460*Sin[c + (3*d*x)/2] - 33716*Sin[2*c + (3*d*x)/2] + 34300*Sin[3*c + (3*d*x)/2] - 39788*Sin[c + (5*d*x)/2] + 2940*Sin[2*c + (5*d*x)/2] - 26068*Sin[3*c + (5*d*x)/2] + 16660*Sin[4*c + (5*d*x)/2] - 21351*Sin[2*c + (7*d*x)/2] - 1295*Sin[3*c + (7*d*x)/2] - 14911*Sin[4*c + (7*d*x)/2] + 5145*Sin[5*c + (7*d*x)/2] - 7329*Sin[3*c + (9*d*x)/2] - 1225*Sin[4*c + (9*d*x)/2] - 5369*Sin[5*c + (9*d*x)/2] + 735*Sin[6*c + (9*d*x)/2] - 1152*Sin[4*c + (11*d*x)/2] - 280*Sin[5*c + (11*d*x)/2] - 872*Sin[6*c + (11*d*x)/2))/(2240*d*(a + a*Cos[c + d*x])^4)

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{7} - \frac{9(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{5} - 13(\tan^3(\frac{dx}{2} + \frac{c}{2})) - 111 \tan(\frac{dx}{2} + \frac{c}{2}) - \frac{4}{(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)^2} + \frac{36}{\tan(\frac{dx}{2} + \frac{c}{2}) + 1} + 84 \ln(\frac{4}{8da^4})$
default	$-\frac{(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{7} - \frac{9(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{5} - 13(\tan^3(\frac{dx}{2} + \frac{c}{2})) - 111 \tan(\frac{dx}{2} + \frac{c}{2}) - \frac{4}{(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)^2} + \frac{36}{\tan(\frac{dx}{2} + \frac{c}{2}) + 1} + 84 \ln(\frac{4}{8da^4})$
parallelrisch	$\frac{(-23520 \cos(2dx+2c) - 23520) \ln(\tan(\frac{dx}{2} + \frac{c}{2}) - 1) + (23520 \cos(2dx+2c) + 23520) \ln(\tan(\frac{dx}{2} + \frac{c}{2}) + 1) - 34168 \tan(\frac{dx}{2} + \frac{c}{2})}{2240a^4 d(1 + \cos(\frac{dx}{2} + \frac{c}{2}))}$
norman	$-\frac{167 \tan(\frac{dx}{2} + \frac{c}{2})}{8da} + \frac{281(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{8da} - \frac{217(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{20da} - \frac{167(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{140da} - \frac{53(\tan^9(\frac{dx}{2} + \frac{c}{2}))}{280da} - \frac{\tan^{11}(\frac{dx}{2} + \frac{c}{2})}{56da} - \dots$
risch	$\frac{i(735 e^{10i(dx+c)} + 5145 e^{9i(dx+c)} + 16660 e^{8i(dx+c)} + 34300 e^{7i(dx+c)} + 51842 e^{6i(dx+c)} + 61054 e^{5i(dx+c)} + 55556 e^{4i(dx+c)} + 35d^4 (e^{2i(dx+c)} + 1)^2 (e^{i(dx+c)} + 1)^7)}{(\tan^2(\frac{dx}{2} + \frac{c}{2}) - 1)^2 a^3}$

[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)

[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-9/5*tan(1/2*d*x+1/2*c)^5-13*tan(1/2*d*x+1/2*c)^3-111*tan(1/2*d*x+1/2*c)-4/(tan(1/2*d*x+1/2*c)+1)^2+36/(tan(1/2*d*x+1/2*c)+1)+84*ln(tan(1/2*d*x+1/2*c)+1)+4/(tan(1/2*d*x+1/2*c)-1)^2+36/(tan(1/2*d*x+1/2*c)-1)-84*ln(tan(1/2*d*x+1/2*c)-1))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4} dx$$

$$= \frac{735(\cos(dx+c)^6 + 4\cos(dx+c)^5 + 6\cos(dx+c)^4 + 4\cos(dx+c)^3 + \cos(dx+c)^2) \log(\sin(dx+c))}{\dots}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fricas")

[Out] 1/140*(735*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 735*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(1152*cos(d*x + c)^5 + 3873*cos(d*x + c)^4 + 4548*cos(d*x + c)^3 + 2012*cos(d*x + c)^2 + 140*cos(d*x + c) - 35)*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)

SymPy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4} dx = \int \frac{\sec^3(c+dx)}{\cos^4(c+dx)+4\cos^3(c+dx)+6\cos^2(c+dx)+4\cos(c+dx)+1} \frac{dx}{a^4}$$

[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**4,x)

[Out] Integral(sec(c + d*x)**3/(cos(c + d*x)**4 + 4*cos(c + d*x)**3 + 6*cos(c + d*x)**2 + 4*cos(c + d*x) + 1), x)/a**4

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.25

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4} dx = \frac{280 \left(\frac{7 \sin(dx+c)}{\cos(dx+c)+1} - \frac{9 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^4 - \frac{2a^4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^4 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{3885 \sin(dx+c)}{\cos(dx+c)+1} + \frac{455 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{63 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{2940 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^4} + \frac{2940 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^4}$$

280 d

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="maxima")

[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) + 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4} dx = \frac{2940 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{a^4} - \frac{2940 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{a^4} + \frac{280 \left(9 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 7 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2 - 1} - \frac{5a^{24} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 63a^{24} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5}{a^{24}}$$

280 d

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/280*(2940*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 2940*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 280*(9*tan(1/2*d*x + 1/2*c)^3 - 7*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (5*a^24*tan(1/2*d*x + 1/2*c)^7 + 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 + 3885*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d

Mupad [B] (verification not implemented)

Time = 15.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^4} dx = \frac{21 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^4 d} - \frac{9 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{40 a^4 d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{56 a^4 d} - \frac{13 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{8 a^4 d} - \frac{7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 9 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{d \left(a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2 a^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a^4 \right)} - \frac{111 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8 a^4 d}$$

[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^4),x)

[Out] (21*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (9*tan(c/2 + (d*x)/2)^5)/(40*a^4*d) - tan(c/2 + (d*x)/2)^7/(56*a^4*d) - (13*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) - (7*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^3)/(d*(a^4*tan(c/2 + (d*x)/2)^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4) - (111*tan(c/2 + (d*x)/2))/(8*a^4*d)

$$3.82 \quad \int \frac{\cos^7(c+dx)}{(a+a \cos(c+dx))^5} dx$$

Optimal result	700
Rubi [A] (verified)	700
Mathematica [A] (verified)	703
Maple [A] (verified)	703
Fricas [A] (verification not implemented)	704
Sympy [B] (verification not implemented)	704
Maxima [A] (verification not implemented)	705
Giac [A] (verification not implemented)	705
Mupad [B] (verification not implemented)	706

Optimal result

Integrand size = 21, antiderivative size = 225

$$\int \frac{\cos^7(c+dx)}{(a+a \cos(c+dx))^5} dx = \frac{31x}{2a^5} - \frac{7664 \sin(c+dx)}{315a^5d} + \frac{31 \cos(c+dx) \sin(c+dx)}{2a^5d} - \frac{\cos^6(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{17 \cos^5(c+dx) \sin(c+dx)}{63ad(a+a \cos(c+dx))^4} - \frac{28 \cos^4(c+dx) \sin(c+dx)}{45a^2d(a+a \cos(c+dx))^3} - \frac{577 \cos^3(c+dx) \sin(c+dx)}{315a^3d(a+a \cos(c+dx))^2} - \frac{3832 \cos^2(c+dx) \sin(c+dx)}{315d(a^5+a^5 \cos(c+dx))}$$

[Out] 31/2*x/a^5-7664/315*sin(d*x+c)/a^5/d+31/2*cos(d*x+c)*sin(d*x+c)/a^5/d-1/9*cos(d*x+c)^6*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-17/63*cos(d*x+c)^5*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4-28/45*cos(d*x+c)^4*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3-577/315*cos(d*x+c)^3*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-3832/315*cos(d*x+c)^2*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

= {2844, 3056, 2813}

$$\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5} dx = -\frac{7664\sin(c+dx)}{315a^5d} - \frac{3832\sin(c+dx)\cos^2(c+dx)}{315d(a^5\cos(c+dx)+a^5)} + \frac{31\sin(c+dx)\cos(c+dx)}{2a^5d} + \frac{31x}{2a^5} - \frac{577\sin(c+dx)\cos^3(c+dx)}{315a^3d(a\cos(c+dx)+a)^2} - \frac{28\sin(c+dx)\cos^4(c+dx)}{45a^2d(a\cos(c+dx)+a)^3} - \frac{\sin(c+dx)\cos^6(c+dx)}{9d(a\cos(c+dx)+a)^5} - \frac{17\sin(c+dx)\cos^5(c+dx)}{63ad(a\cos(c+dx)+a)^4}$$

[In] Int[Cos[c + d*x]^7/(a + a*Cos[c + d*x])^5,x]

[Out] (31*x)/(2*a^5) - (7664*Sin[c + d*x])/(315*a^5*d) + (31*Cos[c + d*x]*Sin[c + d*x])/(2*a^5*d) - (Cos[c + d*x]^6*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) - (17*Cos[c + d*x]^5*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (28*Cos[c + d*x]^4*Sin[c + d*x])/(45*a^2*d*(a + a*Cos[c + d*x])^3) - (577*Cos[c + d*x]^3*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[c + d*x])^2) - (3832*Cos[c + d*x]^2*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))

Rule 2813

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Cos[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n-1)/(a*f*(2*m+1))), x] + Dist[1/(a*b*(2*m+1)), Int[(a + b*Sin[e + f*x])^(m+1)*(c + d*Sin[e + f*x])^(n-2)*Simp[b*(c^2*(m+1) + d^2*(n-1)) + a*c*d*(m-n+1) + d*(a*d*(m-n+1) + b*c*(m+n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m+1))), x] - Dist[1/(a*b*(2*m+1)), Int[(a + b*Sin[e + f*x])^(m+1)*(c + d*Sin[e + f*x])^(n-1)*Simp[A*(a*d*n - b*c*(m+1)) - B*(a*c*m + b*d*n) - d*(a*B*(m-n) + A*b*(m+n+1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&

NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^5(c+dx)(6a-11a\cos(c+dx))}{(a+a\cos(c+dx))^4} dx}{9a^2} \\
&= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^4(c+dx)(85a^2-111a^2\cos(c+dx))}{(a+a\cos(c+dx))^3} dx}{63a^4} \\
&= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
&\quad - \frac{28\cos^4(c+dx)\sin(c+dx)}{45a^2d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^3(c+dx)(784a^3-947a^3\cos(c+dx))}{(a+a\cos(c+dx))^2} dx}{315a^6} \\
&= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
&\quad - \frac{28\cos^4(c+dx)\sin(c+dx)}{45a^2d(a+a\cos(c+dx))^3} - \frac{577\cos^3(c+dx)\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{\int \frac{\cos^2(c+dx)(5193a^4-6303a^4\cos(c+dx))}{a+a\cos(c+dx)} dx}{945a^8} \\
&= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
&\quad - \frac{28\cos^4(c+dx)\sin(c+dx)}{45a^2d(a+a\cos(c+dx))^3} - \frac{577\cos^3(c+dx)\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{3832\cos^2(c+dx)\sin(c+dx)}{315d(a^5+a^5\cos(c+dx))} \\
&\quad - \frac{\int \cos(c+dx)(22992a^5-29295a^5\cos(c+dx)) dx}{945a^{10}} \\
&= \frac{31x}{2a^5} - \frac{7664\sin(c+dx)}{315a^5d} + \frac{31\cos(c+dx)\sin(c+dx)}{2a^5d} - \frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} \\
&\quad - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\cos^4(c+dx)\sin(c+dx)}{45a^2d(a+a\cos(c+dx))^3} \\
&\quad - \frac{577\cos^3(c+dx)\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} - \frac{3832\cos^2(c+dx)\sin(c+dx)}{315d(a^5+a^5\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 8.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.64

$$\int \frac{\cos^7(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{\cos\left(\frac{1}{2}(c + dx)\right) \csc^{10}(c + dx) \sin^9\left(\frac{1}{2}(c + dx)\right) \left(984312 + 1035321 \cos(c + dx) - 484476 \cos(2(c + dx))\right)}{1024a^5d}$$

[In] Integrate[Cos[c + d*x]^7/(a + a*Cos[c + d*x])^5,x]

[Out] -1/1260*(Cos[(c + d*x)/2]*Csc[c + d*x]^10*Sin[(c + d*x)/2]^9*(984312 + 1035321*Cos[c + d*x] - 484476*Cos[2*(c + d*x)] - 933309*Cos[3*(c + d*x)] - 491576*Cos[4*(c + d*x)] - 106807*Cos[5*(c + d*x)] - 3780*Cos[6*(c + d*x)] + 315*Cos[7*(c + d*x)] + 9999360*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^8*sqrt[Si n[c + d*x]^2]))/(a^5*d)

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.44

method	result
parallelrisch	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\cos(6dx+6c) - \frac{854012 \cos(dx+c)}{63} - \frac{2250427 \cos(2dx+2c)}{315} - \frac{143054 \cos(3dx+3c)}{63} - \frac{113422 \cos(4dx+4c)}{315} - 10 \cos(5dx+5c) - 2627186/315\right)}{1024a^5d}$
derivativedivides	$-\frac{\left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{9} + \frac{10\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{7} - \frac{48\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} + 50\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 351 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-176\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1024}{(1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))}$
default	$-\frac{\left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{9} + \frac{10\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{7} - \frac{48\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} + 50\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 351 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{-176\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1024}{(1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))}$
risch	$\frac{31x}{2a^5} - \frac{ie^{2i(dx+c)}}{8a^5d} + \frac{5ie^{i(dx+c)}}{2a^5d} - \frac{5ie^{-i(dx+c)}}{2a^5d} + \frac{ie^{-2i(dx+c)}}{8a^5d} - \frac{2i(11025 e^{8i(dx+c)} + 77175 e^{7i(dx+c)} + 247695 e^{6i(dx+c)} + 57120 e^{5i(dx+c)} + 57120 e^{4i(dx+c)} + 247695 e^{3i(dx+c)} + 77175 e^{2i(dx+c)} + 11025 e^{i(dx+c)} + 1)}{16d a^5}$

[In] int(cos(d*x+c)^7/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)

[Out] 1/1024*(tan(1/2*d*x+1/2*c)*(cos(6*d*x+6*c)-854012/63*cos(d*x+c)-2250427/315*cos(2*d*x+2*c)-143054/63*cos(3*d*x+3*c)-113422/315*cos(4*d*x+4*c)-10*cos(5*d*x+5*c)-2627186/315)*sec(1/2*d*x+1/2*c)^8+15872*d*x)/a^5/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.92

$$\int \frac{\cos^7(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{9765 dx \cos(dx + c)^5 + 48825 dx \cos(dx + c)^4 + 97650 dx \cos(dx + c)^3 + 97650 dx \cos(dx + c)^2 + 48825 dx \cos(dx + c) + 9765}{630 (a^5 d \cos(dx + c))^5}$$

[In] integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="fricas")

[Out] 1/630*(9765*d*x*cos(d*x + c)^5 + 48825*d*x*cos(d*x + c)^4 + 97650*d*x*cos(d*x + c)^3 + 97650*d*x*cos(d*x + c)^2 + 48825*d*x*cos(d*x + c) + 9765*d*x + (315*cos(d*x + c)^6 - 1575*cos(d*x + c)^5 - 28828*cos(d*x + c)^4 - 87440*cos(d*x + c)^3 - 112119*cos(d*x + c)^2 - 66875*cos(d*x + c) - 15328)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(214) = 428.

Time = 19.70 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.61

$$\int \frac{\cos^7(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \left\{ \begin{array}{l} \frac{78120 dx \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right)}{5040 a^5 d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 10080 a^5 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 5040 a^5 d} + \frac{156240 dx \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{5040 a^5 d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 10080 a^5 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 5040 a^5 d} + \frac{9765 dx \cos^7(c)}{(a \cos(c) + a)^5} \end{array} \right.$$

[In] integrate(cos(d*x+c)**7/(a+a*cos(d*x+c))**5,x)

[Out] Piecewise((78120*d*x*tan(c/2 + d*x/2)**4/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) + 156240*d*x*tan(c/2 + d*x/2)**2/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) + 78120*d*x/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) - 35*tan(c/2 + d*x/2)**13/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) + 380*tan(c/2 + d*x/2)**11/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) - 2159*tan(c/2 + d*x/2)**9/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 + d*x/2)**2 + 5040*a**5*d) + 10152*tan(c/2 + d*x/2)**7/(5040*a**5*d*tan(c/2 + d*x/2)**4 + 10080*a**5*d*tan(c/2 +

$d*x/2)**2 + 5040*a**5*d) - 82089*\tan(c/2 + d*x/2)**5/(5040*a**5*d*\tan(c/2 + d*x/2)**4 + 10080*a**5*d*\tan(c/2 + d*x/2)**2 + 5040*a**5*d) - 260820*\tan(c/2 + d*x/2)**3/(5040*a**5*d*\tan(c/2 + d*x/2)**4 + 10080*a**5*d*\tan(c/2 + d*x/2)**2 + 5040*a**5*d) - 155925*\tan(c/2 + d*x/2)/(5040*a**5*d*\tan(c/2 + d*x/2)**4 + 10080*a**5*d*\tan(c/2 + d*x/2)**2 + 5040*a**5*d), Ne(d, 0)), (x*cos(c)**7/(a*cos(c) + a)**5, True))$

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00

$$\int \frac{\cos^7(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{5040 \left(\frac{9 \sin(dx+c)}{\cos(dx+c)+1} + \frac{11 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right) + \frac{110565 \sin(dx+c) - 15750 \sin(dx+c)^3 + 3024 \sin(dx+c)^5 - 450 \sin(dx+c)^7 + 35 \sin(dx+c)^9}{\cos(dx+c)+1} - \frac{156240 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040 d}$$

[In] integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="maxima")

[Out] $-1/5040*(5040*(9*\sin(d*x + c)/(\cos(d*x + c) + 1) + 11*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/(a^5 + 2*a^5*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + a^5*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4) + (110565*\sin(d*x + c)/(\cos(d*x + c) + 1) - 15750*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 3024*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 - 450*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7 + 35*\sin(d*x + c)^9/(\cos(d*x + c) + 1)^9)/a^5 - 156240*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^5)/d$

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.64

$$\int \frac{\cos^7(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{78120(dx+c)}{a^5} - \frac{5040 \left(11 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 9 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1 \right)^2 a^5} - \frac{35 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 - 450 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 3024 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 15750 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 110565 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^{45}}}{5040 d}$$

[In] integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="giac")

[Out] $1/5040*(78120*(d*x + c)/a^5 - 5040*(11*\tan(1/2*d*x + 1/2*c)^3 + 9*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5) - (35*a^40*\tan(1/2*d*x + 1/2*c)^9 - 450*a^40*\tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*\tan(1/2*d*x + 1/2*c)^5 - 15750*a^40*\tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*\tan(1/2*d*x + 1/2*c))/a^45)/d$

Mupad [B] (verification not implemented)

Time = 15.43 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.80

$$\int \frac{\cos^7(c + dx)}{(a + a \cos(c + dx))^5} dx =$$

$$\frac{35 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 590 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 4584 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 23288 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 129824 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^8 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 55440 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 10080 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 78120 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^9 (c + dx)}{(5040 a^5 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right))^9}$$

[In] int(cos(c + d*x)^7/(a + a*cos(c + d*x))^5,x)

[Out] -(35*sin(c/2 + (d*x)/2) - 590*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 4584*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 23288*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) + 129824*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) + 55440*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2) - 10080*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2) - 78120*cos(c/2 + (d*x)/2)^9*(c + d*x))/(5040*a^5*d*cos(c/2 + (d*x)/2)^9)

3.83 $\int \frac{\cos^6(c+dx)}{(a+a \cos(c+dx))^5} dx$

Optimal result	707
Rubi [A] (verified)	707
Mathematica [A] (verified)	710
Maple [A] (verified)	711
Fricas [A] (verification not implemented)	711
Sympy [A] (verification not implemented)	712
Maxima [A] (verification not implemented)	712
Giac [A] (verification not implemented)	713
Mupad [B] (verification not implemented)	713

Optimal result

Integrand size = 21, antiderivative size = 191

$$\int \frac{\cos^6(c+dx)}{(a+a \cos(c+dx))^5} dx = -\frac{5x}{a^5} + \frac{181 \sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} \\ - \frac{5 \cos^4(c+dx) \sin(c+dx)}{21ad(a+a \cos(c+dx))^4} - \frac{29 \cos^3(c+dx) \sin(c+dx)}{63a^2d(a+a \cos(c+dx))^3} \\ - \frac{67 \cos^2(c+dx) \sin(c+dx)}{63a^3d(a+a \cos(c+dx))^2} + \frac{5 \sin(c+dx)}{d(a^5+a^5 \cos(c+dx))}$$

[Out] $-5*x/a^5+181/63*\sin(d*x+c)/a^5/d-1/9*\cos(d*x+c)^5*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^5-5/21*\cos(d*x+c)^4*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^4-29/63*\cos(d*x+c)^3*\sin(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^3-67/63*\cos(d*x+c)^2*\sin(d*x+c)/a^3/d/(a+a*\cos(d*x+c))^2+5*\sin(d*x+c)/d/(a^5+a^5*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2844, 3056, 3047, 3102, 12, 2814, 2727}

$$\int \frac{\cos^6(c+dx)}{(a+a \cos(c+dx))^5} dx = \frac{181 \sin(c+dx)}{63a^5d} + \frac{5 \sin(c+dx)}{d(a^5 \cos(c+dx) + a^5)} - \frac{5x}{a^5} \\ - \frac{67 \sin(c+dx) \cos^2(c+dx)}{63a^3d(a \cos(c+dx) + a)^2} - \frac{29 \sin(c+dx) \cos^3(c+dx)}{63a^2d(a \cos(c+dx) + a)^3} \\ - \frac{\sin(c+dx) \cos^5(c+dx)}{9d(a \cos(c+dx) + a)^5} - \frac{5 \sin(c+dx) \cos^4(c+dx)}{21ad(a \cos(c+dx) + a)^4}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^6/(a + a*\text{Cos}[c + d*x])^5, x]$

```
[Out] (-5*x)/a^5 + (181*Sin[c + d*x])/(63*a^5*d) - (Cos[c + d*x]^5*Sin[c + d*x])/
(9*d*(a + a*Cos[c + d*x])^5) - (5*Cos[c + d*x]^4*Sin[c + d*x])/(21*a*d*(a +
a*Cos[c + d*x])^4) - (29*Cos[c + d*x]^3*Sin[c + d*x])/(63*a^2*d*(a + a*Cos
[c + d*x])^3) - (67*Cos[c + d*x]^2*Sin[c + d*x])/(63*a^3*d*(a + a*Cos[c + d
*x])^2) + (5*Sin[c + d*x])/(d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 2727

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c +
d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
```


1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3102

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^5(c + dx) \sin(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{\int \frac{\cos^4(c + dx)(5a - 10a \cos(c + dx))}{(a + a \cos(c + dx))^4} dx}{9a^2} \\
 &= -\frac{\cos^5(c + dx) \sin(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{5 \cos^4(c + dx) \sin(c + dx)}{21ad(a + a \cos(c + dx))^4} - \frac{\int \frac{\cos^3(c + dx)(60a^2 - 85a^2 \cos(c + dx))}{(a + a \cos(c + dx))^3} dx}{63a^4} \\
 &= -\frac{\cos^5(c + dx) \sin(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{5 \cos^4(c + dx) \sin(c + dx)}{21ad(a + a \cos(c + dx))^4} \\
 &\quad - \frac{29 \cos^3(c + dx) \sin(c + dx)}{63a^2d(a + a \cos(c + dx))^3} - \frac{\int \frac{\cos^2(c + dx)(435a^3 - 570a^3 \cos(c + dx))}{(a + a \cos(c + dx))^2} dx}{315a^6} \\
 &= -\frac{\cos^5(c + dx) \sin(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{5 \cos^4(c + dx) \sin(c + dx)}{21ad(a + a \cos(c + dx))^4} \\
 &\quad - \frac{29 \cos^3(c + dx) \sin(c + dx)}{63a^2d(a + a \cos(c + dx))^3} - \frac{67 \cos^2(c + dx) \sin(c + dx)}{63a^3d(a + a \cos(c + dx))^2} \\
 &\quad - \frac{\int \frac{\cos(c + dx)(2010a^4 - 2715a^4 \cos(c + dx))}{a + a \cos(c + dx)} dx}{945a^8} \\
 &= -\frac{\cos^5(c + dx) \sin(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{5 \cos^4(c + dx) \sin(c + dx)}{21ad(a + a \cos(c + dx))^4} \\
 &\quad - \frac{29 \cos^3(c + dx) \sin(c + dx)}{63a^2d(a + a \cos(c + dx))^3} - \frac{67 \cos^2(c + dx) \sin(c + dx)}{63a^3d(a + a \cos(c + dx))^2} \\
 &\quad - \frac{\int \frac{2010a^4 \cos(c + dx) - 2715a^4 \cos^2(c + dx)}{a + a \cos(c + dx)} dx}{945a^8}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{181 \sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{5 \cos^4(c+dx) \sin(c+dx)}{21ad(a+a \cos(c+dx))^4} \\
&\quad - \frac{29 \cos^3(c+dx) \sin(c+dx)}{63a^2d(a+a \cos(c+dx))^3} - \frac{67 \cos^2(c+dx) \sin(c+dx)}{63a^3d(a+a \cos(c+dx))^2} - \frac{\int \frac{4725a^5 \cos(c+dx)}{a+a \cos(c+dx)} dx}{945a^9} \\
&= \frac{181 \sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{5 \cos^4(c+dx) \sin(c+dx)}{21ad(a+a \cos(c+dx))^4} \\
&\quad - \frac{29 \cos^3(c+dx) \sin(c+dx)}{63a^2d(a+a \cos(c+dx))^3} - \frac{67 \cos^2(c+dx) \sin(c+dx)}{63a^3d(a+a \cos(c+dx))^2} - \frac{5 \int \frac{\cos(c+dx)}{a+a \cos(c+dx)} dx}{a^4} \\
&= -\frac{5x}{a^5} + \frac{181 \sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{5 \cos^4(c+dx) \sin(c+dx)}{21ad(a+a \cos(c+dx))^4} \\
&\quad - \frac{29 \cos^3(c+dx) \sin(c+dx)}{63a^2d(a+a \cos(c+dx))^3} - \frac{67 \cos^2(c+dx) \sin(c+dx)}{63a^3d(a+a \cos(c+dx))^2} + \frac{5 \int \frac{1}{a+a \cos(c+dx)} dx}{a^4} \\
&= -\frac{5x}{a^5} + \frac{181 \sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{5 \cos^4(c+dx) \sin(c+dx)}{21ad(a+a \cos(c+dx))^4} \\
&\quad - \frac{29 \cos^3(c+dx) \sin(c+dx)}{63a^2d(a+a \cos(c+dx))^3} - \frac{67 \cos^2(c+dx) \sin(c+dx)}{63a^3d(a+a \cos(c+dx))^2} + \frac{5 \sin(c+dx)}{d(a^5+a^5 \cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.98 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \frac{\cos^6(c+dx)}{(a+a \cos(c+dx))^5} dx$$

$$= \frac{2 \cos\left(\frac{1}{2}(c+dx)\right) \sin^9\left(\frac{1}{2}(c+dx)\right) \left(161280 \arcsin(\cos(c+dx)) \cos^{10}\left(\frac{1}{2}(c+dx)\right) + (42676 + 69350 \cos(c+dx))\right)}{63a^5d(-1 + \cos(c+dx))^4}$$

[In] Integrate[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^5,x]

[Out] (2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]^9*(161280*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^10 + (42676 + 69350*Cos[c + d*x] + 36632*Cos[2*(c + d*x)] + 11675*Cos[3*(c + d*x)] + 1892*Cos[4*(c + d*x)] + 63*Cos[5*(c + d*x)])*Sqrt[Sin[c + d*x]^2])/(63*a^5*d*(-1 + Cos[c + d*x])^4*(1 + Cos[c + d*x])^5*Sqrt[Sin[c + d*x]^2])

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.46

method	result
parallelrisc	$\frac{34675 \left(\cos(dx+c) + \frac{964 \cos(2dx+2c)}{1825} + \frac{467 \cos(3dx+3c)}{2774} + \frac{946 \cos(4dx+4c)}{34675} + \frac{63 \cos(5dx+5c)}{69350} + \frac{21338}{34675} \right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sec^8\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 8064}{a^5 d}$
derivativedivides	$\frac{\left(\frac{\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)}{9} - \frac{8 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{7} + 6 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 24 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + 129 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{32 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} - 160 \arcsin\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}\right) \right)}{16d a^5}$
default	$\frac{\left(\frac{\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)}{9} - \frac{8 \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{7} + 6 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 24 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + 129 \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{32 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} - 160 \arcsin\left(\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}\right) \right)}{16d a^5}$
risc	$-\frac{5x}{a^5} - \frac{ie^{i(dx+c)}}{2a^5d} + \frac{ie^{-i(dx+c)}}{2a^5d} + \frac{2i(945e^{8i(dx+c)} + 6300e^{7i(dx+c)} + 19740e^{6i(dx+c)} + 36414e^{5i(dx+c)} + 43092e^{4i(dx+c)} + 31500e^{3i(dx+c)} + 15750e^{2i(dx+c)} + 3150e^{i(dx+c)} + 315)}{63d a^5 (e^{i(dx+c)} + 1)^5}$

[In] int(cos(d*x+c)^6/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)

[Out] 5/8064*(6935*(cos(d*x+c)+964/1825*cos(2*d*x+2*c)+467/2774*cos(3*d*x+3*c)+946/34675*cos(4*d*x+4*c)+63/69350*cos(5*d*x+5*c)+21338/34675)*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^8-8064*d*x)/a^5/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.04

$$\int \frac{\cos^6(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{315 dx \cos(dx + c)^5 + 1575 dx \cos(dx + c)^4 + 3150 dx \cos(dx + c)^3 + 3150 dx \cos(dx + c)^2 + 1575 dx \cos(dx + c) + 315}{63 (a^5 d \cos(dx + c)^5 + 5 a^5 d \cos(dx + c)^4 + 15750 a^5 d \cos(dx + c)^3 + 15750 a^5 d \cos(dx + c)^2 + 3150 a^5 d \cos(dx + c) + 315)}$$

[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="fricas")

[Out] -1/63*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x + c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (63*cos(d*x + c)^5 + 946*cos(d*x + c)^4 + 2840*cos(d*x + c)^3 + 3633*cos(d*x + c)^2 + 2165*cos(d*x + c) + 496)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)

Sympy [A] (verification not implemented)

Time = 11.54 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.68

$$\int \frac{\cos^6(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \left\{ \begin{array}{l} -\frac{5040dx \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{1008a^5 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 1008a^5 d} - \frac{5040dx}{1008a^5 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 1008a^5 d} + \frac{7 \tan^{11}\left(\frac{c}{2} + \frac{dx}{2}\right)}{1008a^5 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 1008a^5 d} - \frac{65 \tan^9\left(\frac{c}{2} + \frac{dx}{2}\right)}{1008a^5 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 1008a^5 d} \\ \frac{x \cos^6(c)}{(a \cos(c) + a)^5} \end{array} \right.$$

`[In] integrate(cos(d*x+c)**6/(a+a*cos(d*x+c))**5,x)`

```
[Out] Piecewise((-5040*d*x*tan(c/2 + d*x/2)**2/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) - 5040*d*x/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) + 7*tan(c/2 + d*x/2)**11/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) - 65*tan(c/2 + d*x/2)**9/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) + 306*tan(c/2 + d*x/2)**7/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) - 1134*tan(c/2 + d*x/2)**5/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) + 6615*tan(c/2 + d*x/2)**3/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d) + 10143*tan(c/2 + d*x/2)/(1008*a**5*d*tan(c/2 + d*x/2)**2 + 1008*a**5*d), Ne(d, 0)), (x*cos(c)**6/(a*cos(c) + a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.93

$$\int \frac{\cos^6(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{\frac{2016 \sin(dx+c)}{\left(a^5 + \frac{a^5 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right) (\cos(dx+c)+1)}{1008 d} + \frac{\frac{8127 \sin(dx+c)}{\cos(dx+c)+1} - \frac{1512 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{72 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{7 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{10080 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}$$

`[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="maxima")`

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 + a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) - 1512*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 72*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.68

$$\int \frac{\cos^6(c + dx)}{(a + a \cos(c + dx))^5} dx =$$

$$\frac{\frac{5040(dx+c)}{a^5} - \frac{2016 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1)a^5} - \frac{7 a^{40} \tan(\frac{1}{2} dx + \frac{1}{2} c)^9 - 72 a^{40} \tan(\frac{1}{2} dx + \frac{1}{2} c)^7 + 378 a^{40} \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 1512 a^{40} \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 8127 a^{40} \tan(\frac{1}{2} dx + \frac{1}{2} c)}{a^{45}}}{1008 d}$$

[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="giac")

```
[Out] -1/1008*(5040*(d*x + c)/a^5 - 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 - 72*a^40*tan(1/2*d*x + 1/2*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 - 1512*a^40*tan(1/2*d*x + 1/2*c)^3 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

Mupad [B] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.83

$$\int \frac{\cos^6(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{7 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 100 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 636 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 2512 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 10096 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^8 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 5040 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^9 (c + dx)}{1008 a^5 d}$$

[In] int(cos(c + d*x)^6/(a + a*cos(c + d*x))^5,x)

```
[Out] (7*sin(c/2 + (d*x)/2) - 100*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) + 636*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) - 2512*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) + 10096*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2) - 5040*cos(c/2 + (d*x)/2)^9*(c + d*x))/(1008*a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.84 $\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^5} dx$

Optimal result	714
Rubi [A] (verified)	714
Mathematica [A] (verified)	717
Maple [A] (verified)	717
Fricas [A] (verification not implemented)	718
Sympy [A] (verification not implemented)	718
Maxima [A] (verification not implemented)	719
Giac [A] (verification not implemented)	719
Mupad [B] (verification not implemented)	719

Optimal result

Integrand size = 21, antiderivative size = 168

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^5} dx = \frac{x}{a^5} - \frac{\cos^4(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{13 \cos^3(c+dx) \sin(c+dx)}{63ad(a+a \cos(c+dx))^4} - \frac{34 \cos^2(c+dx) \sin(c+dx)}{105a^2d(a+a \cos(c+dx))^3} + \frac{173 \sin(c+dx)}{315a^3d(a+a \cos(c+dx))^2} - \frac{661 \sin(c+dx)}{315d(a^5+a^5 \cos(c+dx))}$$

[Out] x/a^5-1/9*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-13/63*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4-34/105*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+173/315*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-661/315*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2844, 3056, 3047, 3098, 2814, 2727}

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^5} dx = -\frac{661 \sin(c+dx)}{315d(a^5 \cos(c+dx) + a^5)} + \frac{x}{a^5} + \frac{173 \sin(c+dx)}{315a^3d(a \cos(c+dx) + a)^2} - \frac{34 \sin(c+dx) \cos^2(c+dx)}{105a^2d(a \cos(c+dx) + a)^3} - \frac{\sin(c+dx) \cos^4(c+dx)}{9d(a \cos(c+dx) + a)^5} - \frac{13 \sin(c+dx) \cos^3(c+dx)}{63ad(a \cos(c+dx) + a)^4}$$

[In] Int[Cos[c + d*x]^5/(a + a*cos[c + d*x])^5,x]

```
[Out] x/a^5 - (Cos[c + d*x]^4*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) - (13*Cos[c + d*x]^3*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (34*Cos[c + d*x]^2*Sin[c + d*x])/(105*a^2*d*(a + a*Cos[c + d*x])^3) + (173*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[c + d*x])^2) - (661*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

egerQ[2*n] || EqQ[c, 0])

Rule 3098

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^3(c+dx)(4a-9a\cos(c+dx))}{(a+a\cos(c+dx))^4} dx}{9a^2} \\
 &= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)(39a^2-63a^2\cos(c+dx))}{(a+a\cos(c+dx))^3} dx}{63a^4} \\
 &= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
 &\quad - \frac{34\cos^2(c+dx)\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(204a^3-315a^3\cos(c+dx))}{(a+a\cos(c+dx))^2} dx}{315a^6} \\
 &= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
 &\quad - \frac{34\cos^2(c+dx)\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} - \frac{\int \frac{204a^3\cos(c+dx)-315a^3\cos^2(c+dx)}{(a+a\cos(c+dx))^2} dx}{315a^6} \\
 &= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
 &\quad - \frac{34\cos^2(c+dx)\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} \\
 &\quad + \frac{173\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} + \frac{\int \frac{-1038a^4+945a^4\cos(c+dx)}{a+a\cos(c+dx)} dx}{945a^8} \\
 &= \frac{x}{a^5} - \frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
 &\quad - \frac{34\cos^2(c+dx)\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} + \frac{173\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} - \frac{661\int \frac{1}{a+a\cos(c+dx)} dx}{315a^4}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{x}{a^5} - \frac{\cos^4(c+dx) \sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13 \cos^3(c+dx) \sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
&\quad - \frac{34 \cos^2(c+dx) \sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} \\
&\quad + \frac{173 \sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} - \frac{661 \sin(c+dx)}{315d(a^5+a^5\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.76

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{8 \cos\left(\frac{1}{2}(c+dx)\right) \csc^{10}(c+dx) \sin^{11}\left(\frac{1}{2}(c+dx)\right) \left(80640 \arcsin(\cos(c+dx)) \cos^{10}\left(\frac{1}{2}(c+dx)\right) + (20689\right)}{315a^5d\sqrt{\dots}}$$

[In] Integrate[Cos[c + d*x]^5/(a + a*cos[c + d*x])^5,x]

[Out] (-8*cos[(c + d*x)/2]*Csc[c + d*x]^10*Sin[(c + d*x)/2]^11*(80640*ArcSin[Cos[c + d*x]]*Cos[(c + d*x)/2]^10 + (20689 + 33440*cos[c + d*x] + 17648*cos[2*(c + d*x)] + 5480*cos[3*(c + d*x)] + 863*cos[4*(c + d*x)])*Sqrt[Sin[c + d*x]^2]))/(315*a^5*d*Sqrt[Sin[c + d*x]^2])

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.46

method	result
parallelrisc	$\frac{-35\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+270\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1008\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2730\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5040dx-9765 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5040a^5d}$
derivativedivides	$-\frac{\left(\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{9}\right)+\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{26\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32 \arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da^5}$
default	$-\frac{\left(\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{9}\right)+\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{26\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-31 \tan\left(\frac{dx}{2}+\frac{c}{2}\right)+32 \arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da^5}$
risc	$\frac{x}{a^5} - \frac{2i(1575 e^{8i(dx+c)}+9450 e^{7i(dx+c)}+28350 e^{6i(dx+c)}+50400 e^{5i(dx+c)}+58338 e^{4i(dx+c)}+44142 e^{3i(dx+c)}+21618 e^{2i(dx+c)}+5400 e^{i(dx+c)}+540)}{315da^5(e^{i(dx+c)}+1)^9}$

[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)

[Out] 1/5040*(-35*tan(1/2*d*x+1/2*c)^9+270*tan(1/2*d*x+1/2*c)^7-1008*tan(1/2*d*x+1/2*c)^5+2730*tan(1/2*d*x+1/2*c)^3+5040*d*x-9765*tan(1/2*d*x+1/2*c))/a^5/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.12

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{315 dx \cos(dx + c)^5 + 1575 dx \cos(dx + c)^4 + 3150 dx \cos(dx + c)^3 + 3150 dx \cos(dx + c)^2 + 1575 dx \cos(dx + c) + 315}{315 (a^5 d \cos(dx + c)^5 + 5 a^5 d \cos(dx + c)^4 + 10 a^5 d \cos(dx + c)^3 + 10 a^5 d \cos(dx + c)^2 + 5 a^5 d \cos(dx + c) + a^5 d)}$$

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x + c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (863*cos(d*x + c)^4 + 2740*cos(d*x + c)^3 + 3549*cos(d*x + c)^2 + 2125*cos(d*x + c) + 488)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [A] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.69

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \begin{cases} \frac{x}{a^5} - \frac{\tan^9\left(\frac{c}{2} + \frac{dx}{2}\right)}{144a^5d} + \frac{3 \tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{56a^5d} - \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{5a^5d} + \frac{13 \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{24a^5d} - \frac{31 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x \cos^5(c)}{(a \cos(c) + a)^5} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((x/a**5 - tan(c/2 + d*x/2)**9/(144*a**5*d) + 3*tan(c/2 + d*x/2)**7/(56*a**5*d) - tan(c/2 + d*x/2)**5/(5*a**5*d) + 13*tan(c/2 + d*x/2)**3/(24*a**5*d) - 31*tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**5/(a*cos(c) + a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= - \frac{\frac{9765 \sin(dx+c)}{\cos(dx+c)+1} - \frac{2730 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1008 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{270 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{10080 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040 d}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="maxima")

[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) - 2730*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 270*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.60

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{5040(dx+c) - \frac{35 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 - 270 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 1008 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 2730 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 9765 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^{45}}}{5040 d}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="giac")

[Out] 1/5040*(5040*(d*x + c)/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 - 270*a^40*tan(1/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 - 2730*a^40*tan(1/2*d*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d

Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{x}{a^5}$$

$$- \frac{\frac{863 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^8}{315} - \frac{356 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6}{315} + \frac{169 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4}{420} - \frac{41 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{504}}{a^5 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^9}$$

```
[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x))^5,x)
```

```
[Out] x/a^5 - (sin(c/2 + (d*x)/2)/144 - (41*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2))/504 + (169*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/420 - (356*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2))/315 + (863*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2))/315)/(a^5*d*cos(c/2 + (d*x)/2)^9)
```

3.85 $\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^5} dx$

Optimal result	721
Rubi [A] (verified)	721
Mathematica [A] (verified)	724
Maple [A] (verified)	724
Fricas [A] (verification not implemented)	725
Sympy [A] (verification not implemented)	725
Maxima [A] (verification not implemented)	725
Giac [A] (verification not implemented)	726
Mupad [B] (verification not implemented)	726

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^5} dx = -\frac{\cos^3(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{11 \cos^2(c+dx) \sin(c+dx)}{63ad(a+a \cos(c+dx))^4} + \frac{67 \sin(c+dx)}{315a^2d(a+a \cos(c+dx))^3} - \frac{142 \sin(c+dx)}{315a^3d(a+a \cos(c+dx))^2} + \frac{83 \sin(c+dx)}{315d(a^5+a^5 \cos(c+dx))}$$

[Out] $-1/9*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^5-11/63*\cos(d*x+c)^2*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^4+67/315*\sin(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^3-142/315*\sin(d*x+c)/a^3/d/(a+a*\cos(d*x+c))^2+83/315*\sin(d*x+c)/d/(a^5+a^5*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2844, 3056, 3047, 3098, 2829, 2727}

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^5} dx = \frac{83 \sin(c+dx)}{315d(a^5 \cos(c+dx) + a^5)} - \frac{142 \sin(c+dx)}{315a^3d(a \cos(c+dx) + a)^2} + \frac{67 \sin(c+dx)}{315a^2d(a \cos(c+dx) + a)^3} - \frac{\sin(c+dx) \cos^3(c+dx)}{9d(a \cos(c+dx) + a)^5} - \frac{11 \sin(c+dx) \cos^2(c+dx)}{63ad(a \cos(c+dx) + a)^4}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^4/(a + a*\text{Cos}[c + d*x])^5, x]$

[Out] $-1/9*(\cos[c + dx]^3*\sin[c + dx])/(d*(a + a*\cos[c + dx])^5) - (11*\cos[c + dx]^2*\sin[c + dx])/(63*a*d*(a + a*\cos[c + dx])^4) + (67*\sin[c + dx])/(315*a^2*d*(a + a*\cos[c + dx])^3) - (142*\sin[c + dx])/(315*a^3*d*(a + a*\cos[c + dx])^2) + (83*\sin[c + dx])/(315*d*(a^5 + a^5*\cos[c + dx]))$

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + dx]/(d*(b + a*sin[c + dx])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2829

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3047

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int

egerQ[2*n] || EqQ[c, 0])

Rule 3098

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)^2], x_Symbol] :> Simp[(A*b - a*B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^2(c+dx)(3a-8a\cos(c+dx))}{(a+a\cos(c+dx))^4} dx}{9a^2} \\
 &= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos(c+dx)(22a^2-45a^2\cos(c+dx))}{(a+a\cos(c+dx))^3} dx}{63a^4} \\
 &= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{22a^2\cos(c+dx)-45a^2\cos^2(c+dx)}{(a+a\cos(c+dx))^3} dx}{63a^4} \\
 &= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
 &\quad + \frac{67\sin(c+dx)}{315a^2d(a+a\cos(c+dx))^3} + \frac{\int \frac{-201a^3+225a^3\cos(c+dx)}{(a+a\cos(c+dx))^2} dx}{315a^6} \\
 &= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
 &\quad + \frac{67\sin(c+dx)}{315a^2d(a+a\cos(c+dx))^3} - \frac{142\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} + \frac{83\int \frac{1}{a+a\cos(c+dx)} dx}{315a^4} \\
 &= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
 &\quad + \frac{67\sin(c+dx)}{315a^2d(a+a\cos(c+dx))^3} \\
 &\quad - \frac{142\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} + \frac{83\sin(c+dx)}{315d(a^5+a^5\cos(c+dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5} dx$$

$$= \frac{(8+40\cos(c+dx)+84\cos^2(c+dx)+100\cos^3(c+dx)+83\cos^4(c+dx))\sin(c+dx)}{315a^5d(1+\cos(c+dx))^5}$$

[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^5,x]

[Out] ((8 + 40*Cos[c + d*x] + 84*Cos[c + d*x]^2 + 100*Cos[c + d*x]^3 + 83*Cos[c + d*x]^4)*Sin[c + d*x])/(315*a^5*d*(1 + Cos[c + d*x])^5)

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

method	result
derivativdivides	$\frac{\frac{(\tan^9(\frac{dx}{2} + \frac{c}{2}))}{9} - \frac{4(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{7} + \frac{6(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{5} - \frac{4(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{3} + \tan(\frac{dx}{2} + \frac{c}{2})}{16da^5}$
default	$\frac{(\tan^9(\frac{dx}{2} + \frac{c}{2}))}{9} - \frac{4(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{7} + \frac{6(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{5} - \frac{4(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{3} + \tan(\frac{dx}{2} + \frac{c}{2})}{16da^5}$
parallelrisch	$\frac{35(\tan^9(\frac{dx}{2} + \frac{c}{2})) - 180(\tan^7(\frac{dx}{2} + \frac{c}{2})) + 378(\tan^5(\frac{dx}{2} + \frac{c}{2})) - 420(\tan^3(\frac{dx}{2} + \frac{c}{2})) + 315 \tan(\frac{dx}{2} + \frac{c}{2})}{5040a^5d}$
risch	$\frac{2i(315e^{8i(dx+c)} + 1260e^{7i(dx+c)} + 3360e^{6i(dx+c)} + 5040e^{5i(dx+c)} + 5418e^{4i(dx+c)} + 3612e^{3i(dx+c)} + 1728e^{2i(dx+c)} + 432e^{i(dx+c)})}{315da^5(e^{i(dx+c)} + 1)^9}$
norman	$\frac{\frac{\tan^{17}(\frac{dx}{2} + \frac{c}{2})}{144ad} + \frac{\tan(\frac{dx}{2} + \frac{c}{2})}{16da} + \frac{\tan^3(\frac{dx}{2} + \frac{c}{2})}{6da} + \frac{7(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{60da} + \frac{\tan^7(\frac{dx}{2} + \frac{c}{2})}{70da} + \frac{109(\tan^9(\frac{dx}{2} + \frac{c}{2}))}{2520da} + \frac{19(\tan^{11}(\frac{dx}{2} + \frac{c}{2}))}{630da} - \frac{11(\tan^{13}(\frac{dx}{2} + \frac{c}{2}))}{10080da} + \frac{11(\tan^{15}(\frac{dx}{2} + \frac{c}{2}))}{120960da}}{(1 + \tan^2(\frac{dx}{2} + \frac{c}{2}))^4} a^4$

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)

[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{(83 \cos(dx + c)^4 + 100 \cos(dx + c)^3 + 84 \cos(dx + c)^2 + 40 \cos(dx + c) + 8) \sin(dx + c)}{315 (a^5 d \cos(dx + c)^5 + 5 a^5 d \cos(dx + c)^4 + 10 a^5 d \cos(dx + c)^3 + 10 a^5 d \cos(dx + c)^2 + 5 a^5 d \cos(dx + c) + a^5 d)}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="fricas")

[Out] 1/315*(83*cos(d*x + c)^4 + 100*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 40*cos(d*x + c) + 8)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)

Sympy [A] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \begin{cases} \frac{\tan^9\left(\frac{c}{2} + \frac{dx}{2}\right)}{144a^5d} - \frac{\tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{28a^5d} + \frac{3 \tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{40a^5d} - \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{12a^5d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x \cos^4(c)}{(a \cos(c) + a)^5} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**5,x)

[Out] Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**7/(28*a**5*d) + 3*tan(c/2 + d*x/2)**5/(40*a**5*d) - tan(c/2 + d*x/2)**3/(12*a**5*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**5, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{315 \sin(dx+c)}{\cos(dx+c)+1} - \frac{420 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{180 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}$$

$$5040 a^5 d$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="maxima")

[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 420*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 180*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.46

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 - 180 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 378 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 420 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 315 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{5040 a^5 d}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="giac")

[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 - 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/2*d*x + 1/2*c)^5 - 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(a^5*d)

Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.82

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) \left(315 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 420 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 378 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 180 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 35 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^8\right)}{5040 a^5 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^9}$$

[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^5,x)

[Out] (sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 - 180*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 - 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(5040*a^5*d*cos(c/2 + (d*x)/2)^9)

3.86 $\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^5} dx$

Optimal result	727
Rubi [A] (verified)	727
Mathematica [A] (verified)	729
Maple [A] (verified)	730
Fricas [A] (verification not implemented)	730
Sympy [A] (verification not implemented)	731
Maxima [A] (verification not implemented)	731
Giac [A] (verification not implemented)	731
Mupad [B] (verification not implemented)	732

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^5} dx = -\frac{\cos^2(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a \cos(c+dx))^4}$$

$$-\frac{17 \sin(c+dx)}{63a^2d(a+a \cos(c+dx))^3} + \frac{5 \sin(c+dx)}{63a^3d(a+a \cos(c+dx))^2}$$

$$+\frac{5 \sin(c+dx)}{63d(a^5+a^5 \cos(c+dx))}$$

[Out] $-1/9*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^5+1/7*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^4-17/63*\sin(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^3+5/63*\sin(d*x+c)/a^3/d/(a+a*\cos(d*x+c))^2+5/63*\sin(d*x+c)/d/(a^5+a^5*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2844, 3047, 3098, 2829, 2729, 2727}

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^5} dx = \frac{5 \sin(c+dx)}{63d(a^5 \cos(c+dx) + a^5)} + \frac{5 \sin(c+dx)}{63a^3d(a \cos(c+dx) + a)^2}$$

$$-\frac{17 \sin(c+dx)}{63a^2d(a \cos(c+dx) + a)^3}$$

$$-\frac{\sin(c+dx) \cos^2(c+dx)}{9d(a \cos(c+dx) + a)^5} + \frac{\sin(c+dx)}{7ad(a \cos(c+dx) + a)^4}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^3/(a + a*\text{Cos}[c + d*x])^5, x]$

[Out] $-1/9*(\cos[c + dx]^2 \sin[c + dx]) / (d*(a + a \cos[c + dx])^5) + \sin[c + dx] / (7*a*d*(a + a \cos[c + dx])^4) - (17*\sin[c + dx]) / (63*a^2*d*(a + a \cos[c + dx])^3) + (5*\sin[c + dx]) / (63*a^3*d*(a + a \cos[c + dx])^2) + (5*\sin[c + dx]) / (63*d*(a^5 + a^5*\cos[c + dx]))$

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*cos[c + d*x]*((a + b*sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2829

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegerQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3047

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3098

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)^2], x_Symbol] := Simp[(A*b - a*

$B + bC) \cdot \text{Cos}[e + f \cdot x] \cdot ((a + b \cdot \text{Sin}[e + f \cdot x])^m / (a \cdot f \cdot (2 \cdot m + 1))), x] + \text{Dist}[1 / (a^2 \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Sin}[e + f \cdot x])^{m+1} \cdot \text{Simp}[a \cdot A \cdot (m+1) + m \cdot (b \cdot B - a \cdot C) + b \cdot C \cdot (2 \cdot m + 1) \cdot \text{Sin}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^2(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{\int \frac{\cos(c+dx)(2a-7a \cos(c+dx))}{(a+a \cos(c+dx))^4} dx}{9a^2} \\
 &= -\frac{\cos^2(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{\int \frac{2a \cos(c+dx)-7a \cos^2(c+dx)}{(a+a \cos(c+dx))^4} dx}{9a^2} \\
 &= -\frac{\cos^2(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a \cos(c+dx))^4} + \frac{\int \frac{-36a^2+49a^2 \cos(c+dx)}{(a+a \cos(c+dx))^3} dx}{63a^4} \\
 &= -\frac{\cos^2(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a \cos(c+dx))^4} \\
 &\quad - \frac{17 \sin(c+dx)}{63a^2d(a+a \cos(c+dx))^3} + \frac{5 \int \frac{1}{(a+a \cos(c+dx))^2} dx}{21a^3} \\
 &= -\frac{\cos^2(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a \cos(c+dx))^4} \\
 &\quad - \frac{17 \sin(c+dx)}{63a^2d(a+a \cos(c+dx))^3} + \frac{5 \sin(c+dx)}{63a^3d(a+a \cos(c+dx))^2} + \frac{5 \int \frac{1}{a+a \cos(c+dx)} dx}{63a^4} \\
 &= -\frac{\cos^2(c+dx) \sin(c+dx)}{9d(a+a \cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a \cos(c+dx))^4} - \frac{17 \sin(c+dx)}{63a^2d(a+a \cos(c+dx))^3} \\
 &\quad + \frac{5 \sin(c+dx)}{63a^3d(a+a \cos(c+dx))^2} + \frac{5 \sin(c+dx)}{63d(a^5+a^5 \cos(c+dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 2.69 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.45

$$\begin{aligned}
 &\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^5} dx \\
 &= \frac{(2+10 \cos(c+dx)+21 \cos^2(c+dx)+25 \cos^3(c+dx)+5 \cos^4(c+dx)) \sin(c+dx)}{63a^5d(1+\cos(c+dx))^5}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]

[Out] ((2 + 10*Cos[c + d*x] + 21*Cos[c + d*x]^2 + 25*Cos[c + d*x]^3 + 5*Cos[c + d*x]^4)*Sin[c + d*x])/(63*a^5*d*(1 + Cos[c + d*x])^5)

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

method	result
parallelrisc	$-\frac{\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{18\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{144a^5d}$
derivativedivides	$-\frac{\left(\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{9}+\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da^5}$
default	$-\frac{\left(\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{9}+\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da^5}$
risc	$\frac{2i\left(63e^{7i(dx+c)}+147e^{6i(dx+c)}+315e^{5i(dx+c)}+315e^{4i(dx+c)}+273e^{3i(dx+c)}+117e^{2i(dx+c)}+45e^{i(dx+c)}+5\right)}{63da^5\left(e^{i(dx+c)}+1\right)^9}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48da}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}-\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{112da}+\frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1008da}+\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{336da}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{336da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}a^4$

[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)

[Out] -1/144*(tan(1/2*d*x+1/2*c)^8-18/7*tan(1/2*d*x+1/2*c)^6+6*tan(1/2*d*x+1/2*c)^2-9)*tan(1/2*d*x+1/2*c)/a^5/d

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5} dx$$

$$= \frac{(5\cos(dx+c)^4+25\cos(dx+c)^3+21\cos(dx+c)^2+10\cos(dx+c)+2)\sin(dx+c)}{63(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+10a^5d\cos(dx+c)^2+5a^5d\cos(dx+c))}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="fricas")

[Out] 1/63*(5*cos(d*x + c)^4 + 25*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)

Sympy [A] (verification not implemented)

Time = 4.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \begin{cases} -\frac{\tan^9\left(\frac{c}{2} + \frac{dx}{2}\right)}{144a^5d} + \frac{\tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{56a^5d} - \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{24a^5d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x \cos^3(c)}{(a \cos(c) + a)^5} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**5,x)

[Out] Piecewise((-tan(c/2 + d*x/2)**9/(144*a**5*d) + tan(c/2 + d*x/2)**7/(56*a**5*d) - tan(c/2 + d*x/2)**3/(24*a**5*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a)**5, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{\frac{63 \sin(dx+c)}{\cos(dx+c)+1} - \frac{42 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{18 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{7 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008 a^5 d}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="maxima")

[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) - 42*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.40

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 - 18 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 42 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 63 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{1008 a^5 d}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="giac")

[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^7 + 42*tan(1/2*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.39

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= -\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 18 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 42 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 63\right)}{1008 a^5 d}$$

[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^5,x)

[Out] -(tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 + 7*tan(c/2 + (d*x)/2)^8 - 63))/(1008*a^5*d)

$$3.87 \quad \int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^5} dx$$

Optimal result	733
Rubi [A] (verified)	733
Mathematica [A] (verified)	735
Maple [A] (verified)	735
Fricas [A] (verification not implemented)	736
Sympy [A] (verification not implemented)	736
Maxima [A] (verification not implemented)	736
Giac [A] (verification not implemented)	737
Mupad [B] (verification not implemented)	737

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^5} dx = \frac{\sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{2 \sin(c+dx)}{9ad(a+a \cos(c+dx))^4} + \frac{\sin(c+dx)}{15a^2d(a+a \cos(c+dx))^3} + \frac{2 \sin(c+dx)}{45a^3d(a+a \cos(c+dx))^2} + \frac{2 \sin(c+dx)}{45d(a^5+a^5 \cos(c+dx))}$$

[Out] 1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5-2/9*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4+1/15*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+2/45*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2+2/45*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2837, 2829, 2729, 2727}

$$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^5} dx = \frac{2 \sin(c+dx)}{45d(a^5 \cos(c+dx) + a^5)} + \frac{2 \sin(c+dx)}{45a^3d(a \cos(c+dx) + a)^2} + \frac{\sin(c+dx)}{15a^2d(a \cos(c+dx) + a)^3} - \frac{2 \sin(c+dx)}{9ad(a \cos(c+dx) + a)^4} + \frac{\sin(c+dx)}{9d(a \cos(c+dx) + a)^5}$$

[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]

```
[Out] Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) - (2*Sin[c + d*x])/(9*a*d*(a + a*
Cos[c + d*x])^4) + Sin[c + d*x]/(15*a^2*d*(a + a*Cos[c + d*x])^3) + (2*Sin[
c + d*x])/(45*a^3*d*(a + a*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(45*d*(a^5 +
a^5*Cos[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c +
d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eEq[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2837

```
Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_),
x_Symbol] := Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
+ 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{\int \frac{-5a + 9a \cos(c + dx)}{(a + a \cos(c + dx))^4} dx}{9a^2} \\
&= \frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{2 \sin(c + dx)}{9ad(a + a \cos(c + dx))^4} + \frac{\int \frac{1}{(a + a \cos(c + dx))^3} dx}{3a^2} \\
&= \frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{2 \sin(c + dx)}{9ad(a + a \cos(c + dx))^4} \\
&\quad + \frac{\sin(c + dx)}{15a^2d(a + a \cos(c + dx))^3} + \frac{2 \int \frac{1}{(a + a \cos(c + dx))^2} dx}{15a^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{15a^2d(a+a\cos(c+dx))^3} \\
&\quad + \frac{2\sin(c+dx)}{45a^3d(a+a\cos(c+dx))^2} + \frac{2\int \frac{1}{a+a\cos(c+dx)} dx}{45a^4} \\
&= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{15a^2d(a+a\cos(c+dx))^3} \\
&\quad + \frac{2\sin(c+dx)}{45a^3d(a+a\cos(c+dx))^2} + \frac{2\sin(c+dx)}{45d(a^5+a^5\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.47

$$\begin{aligned}
&\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5} dx \\
&= \frac{(2+10\cos(c+dx)+21\cos^2(c+dx)+10\cos^3(c+dx)+2\cos^4(c+dx))\sin(c+dx)}{45a^5d(1+\cos(c+dx))^5}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]

[Out] ((2 + 10*Cos[c + d*x] + 21*Cos[c + d*x]^2 + 10*Cos[c + d*x]^3 + 2*Cos[c + d*x]^4)*Sin[c + d*x])/(45*a^5*d*(1 + Cos[c + d*x])^5)

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

method	result	size
derivativedivides	$\frac{\left(\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{9}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da^5}$	45
default	$\frac{\left(\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{9}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da^5}$	45
parallelrisch	$\frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-18\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+45\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{720a^5d}$	47
risch	$\frac{4i\left(30e^{6i(dx+c)}+45e^{5i(dx+c)}+81e^{4i(dx+c)}+54e^{3i(dx+c)}+36e^{2i(dx+c)}+9e^{i(dx+c)}+1\right)}{45da^5\left(e^{i(dx+c)}+1\right)^9}$	91
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80da}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{20da}-\frac{13\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{720da}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{72da}+\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^4}$	15

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)

[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-2/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{(2 \cos(dx + c))^4 + 10 \cos(dx + c)^3 + 21 \cos(dx + c)^2 + 10 \cos(dx + c) + 2) \sin(dx + c)}{45 (a^5 d \cos(dx + c))^5 + 5 a^5 d \cos(dx + c)^4 + 10 a^5 d \cos(dx + c)^3 + 10 a^5 d \cos(dx + c)^2 + 5 a^5 d \cos(dx + c)}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="fricas")

```
[Out] 1/45*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [A] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.49

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^5} dx = \begin{cases} \frac{\tan^9\left(\frac{c}{2} + \frac{dx}{2}\right)}{144a^5d} - \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{40a^5d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x \cos^2(c)}{(a \cos(c) + a)^5} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**5,x)

```
[Out] Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**5/(40*a**5*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{\frac{45 \sin(dx+c)}{\cos(dx+c)+1} - \frac{18 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{720 a^5 d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="maxima")

```
[Out] 1/720*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.33

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 - 18 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 45 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{720 a^5 d}$$

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/720*(5*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^5 + 45*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

Mupad [B] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 18 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 45\right)}{720 a^5 d}$$

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(5*tan(c/2 + (d*x)/2)^8 - 18*tan(c/2 + (d*x)/2)^4 + 45)/(720*a^5*d)
```

$$3.88 \quad \int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5} dx$$

Optimal result	738
Rubi [A] (verified)	738
Mathematica [A] (verified)	740
Maple [A] (verified)	740
Fricas [A] (verification not implemented)	741
Sympy [A] (verification not implemented)	741
Maxima [A] (verification not implemented)	741
Giac [A] (verification not implemented)	742
Mupad [B] (verification not implemented)	742

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5} dx = -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{5\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{21a^2d(a+a\cos(c+dx))^3} + \frac{2\sin(c+dx)}{63ad(a^2+a^2\cos(c+dx))^2} + \frac{2\sin(c+dx)}{63d(a^5+a^5\cos(c+dx))}$$

[Out] -1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5+5/63*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4+1/21*a^2*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+2/63*sin(d*x+c)/a/d/(a^2+a^2*cos(d*x+c))^2+2/63*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2829, 2729, 2727}

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{2\sin(c+dx)}{63d(a^5\cos(c+dx)+a^5)} + \frac{2\sin(c+dx)}{63ad(a^2\cos(c+dx)+a^2)^2} + \frac{\sin(c+dx)}{21a^2d(a\cos(c+dx)+a)^3} + \frac{5\sin(c+dx)}{63ad(a\cos(c+dx)+a)^4} - \frac{\sin(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^5,x]

[Out] $-1/9*\text{Sin}[c + d*x]/(d*(a + a*\text{Cos}[c + d*x])^5) + (5*\text{Sin}[c + d*x])/(63*a*d*(a + a*\text{Cos}[c + d*x])^4) + \text{Sin}[c + d*x]/(21*a^2*d*(a + a*\text{Cos}[c + d*x])^3) + (2*\text{Sin}[c + d*x])/(63*a*d*(a^2 + a^2*\text{Cos}[c + d*x])^2) + (2*\text{Sin}[c + d*x])/(63*d*(a^5 + a^5*\text{Cos}[c + d*x]))$

Rule 2727

$\text{Int}[(a + (b_*)\text{sin}[(c_*) + (d_*)(x_*)])^{-1}, x_Symbol] \rightarrow \text{Simp}[-\text{Cos}[c + d*x]/(d*(b + a*\text{Sin}[c + d*x])), x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 2729

$\text{Int}[(a + (b_*)\text{sin}[(c_*) + (d_*)(x_*)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[b*\text{Cos}[c + d*x]*((a + b*\text{Sin}[c + d*x])^n/(a*d*(2*n + 1))), x] + \text{Dist}[(n + 1)/(a*(2*n + 1)), \text{Int}[(a + b*\text{Sin}[c + d*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2829

$\text{Int}[(a + (b_*)\text{sin}[(e_*) + (f_*)(x_*)])^{(m_*)}*((c_*) + (d_*)\text{sin}[(e_*) + (f_*)(x_*)]), x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*\text{Cos}[e + f*x]*((a + b*\text{Sin}[e + f*x])^m/(a*f*(2*m + 1))), x] + \text{Dist}[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{5 \int \frac{1}{(a + a \cos(c + dx))^4} dx}{9a} \\
 &= -\frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{5 \sin(c + dx)}{63ad(a + a \cos(c + dx))^4} + \frac{5 \int \frac{1}{(a + a \cos(c + dx))^3} dx}{21a^2} \\
 &= -\frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{5 \sin(c + dx)}{63ad(a + a \cos(c + dx))^4} \\
 &\quad + \frac{\sin(c + dx)}{21a^2d(a + a \cos(c + dx))^3} + \frac{2 \int \frac{1}{(a + a \cos(c + dx))^2} dx}{21a^3} \\
 &= -\frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{5 \sin(c + dx)}{63ad(a + a \cos(c + dx))^4} \\
 &\quad + \frac{\sin(c + dx)}{21a^2d(a + a \cos(c + dx))^3} + \frac{2 \sin(c + dx)}{63a^3d(a + a \cos(c + dx))^2} + \frac{2 \int \frac{1}{a + a \cos(c + dx)} dx}{63a^4} \\
 &= -\frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{5 \sin(c + dx)}{63ad(a + a \cos(c + dx))^4} + \frac{\sin(c + dx)}{21a^2d(a + a \cos(c + dx))^3} \\
 &\quad + \frac{2 \sin(c + dx)}{63a^3d(a + a \cos(c + dx))^2} + \frac{2 \sin(c + dx)}{63d(a^5 + a^5 \cos(c + dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.46

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5} dx$$

$$= \frac{(5+25\cos(c+dx)+21\cos^2(c+dx)+10\cos^3(c+dx)+2\cos^4(c+dx))\sin(c+dx)}{63a^5d(1+\cos(c+dx))^5}$$

[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^5,x]

[Out] ((5 + 25*Cos[c + d*x] + 21*Cos[c + d*x]^2 + 10*Cos[c + d*x]^3 + 2*Cos[c + d*x]^4)*Sin[c + d*x])/(63*a^5*d*(1 + Cos[c + d*x])^5)

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.40

method	result	size
parallelrisch	$-\frac{\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{18\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{144a^5d}$	57
derivativedivides	$-\frac{\left(\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{9}-\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da^5}$	58
default	$-\frac{\left(\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{9}-\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da^5}$	58
risch	$\frac{4i\left(63e^{5i(dx+c)}+63e^{4i(dx+c)}+84e^{3i(dx+c)}+36e^{2i(dx+c)}+9e^{i(dx+c)}+1\right)}{63da^5\left(e^{i(dx+c)}+1\right)^9}$	80
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48da}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{56da}-\frac{25\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1008da}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144da}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}$	133

[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)

[Out] -1/144*(tan(1/2*d*x+1/2*c)^8+18/7*tan(1/2*d*x+1/2*c)^6-6*tan(1/2*d*x+1/2*c)^2-9)*tan(1/2*d*x+1/2*c)/a^5/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{(2 \cos(dx + c)^4 + 10 \cos(dx + c)^3 + 21 \cos(dx + c)^2 + 25 \cos(dx + c) + 5) \sin(dx + c)}{63 (a^5 d \cos(dx + c))^5 + 5 a^5 d \cos(dx + c)^4 + 10 a^5 d \cos(dx + c)^3 + 10 a^5 d \cos(dx + c)^2 + 5 a^5 d \cos(dx + c) + a^5 d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="fricas")

```
[Out] 1/63*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 25*cos(d*x + c) + 5)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [A] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \begin{cases} -\frac{\tan^9\left(\frac{c}{2} + \frac{dx}{2}\right)}{144a^5d} - \frac{\tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{56a^5d} + \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{24a^5d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x \cos(c)}{(a \cos(c) + a)^5} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**5,x)

```
[Out] Piecewise((-tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**7/(56*a**5*d) + tan(c/2 + d*x/2)**3/(24*a**5*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)/(a*cos(c) + a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{\frac{63 \sin(dx+c)}{\cos(dx+c)+1} + \frac{42 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{18 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{7 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008 a^5 d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="maxima")

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.41

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 + 18 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 42 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 63 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{1008 a^5 d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="giac")

[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 + 18*tan(1/2*d*x + 1/2*c)^7 - 42*tan(1/2*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)

Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(-7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 18 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 42 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 63\right)}{1008 a^5 d}$$

[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^5,x)

[Out] (tan(c/2 + (d*x)/2)*(42*tan(c/2 + (d*x)/2)^2 - 18*tan(c/2 + (d*x)/2)^6 - 7*tan(c/2 + (d*x)/2)^8 + 63))/(1008*a^5*d)

$$3.89 \quad \int \frac{1}{(a+a \cos(c+dx))^5} dx$$

Optimal result	743
Rubi [A] (verified)	743
Mathematica [A] (verified)	745
Maple [C] (verified)	745
Fricas [A] (verification not implemented)	746
Sympy [A] (verification not implemented)	746
Maxima [A] (verification not implemented)	746
Giac [A] (verification not implemented)	747
Mupad [B] (verification not implemented)	747

Optimal result

Integrand size = 12, antiderivative size = 143

$$\int \frac{1}{(a+a \cos(c+dx))^5} dx = \frac{\sin(c+dx)}{9d(a+a \cos(c+dx))^5} + \frac{4 \sin(c+dx)}{63ad(a+a \cos(c+dx))^4}$$

$$+ \frac{4 \sin(c+dx)}{105a^2d(a+a \cos(c+dx))^3}$$

$$+ \frac{8 \sin(c+dx)}{315ad(a^2+a^2 \cos(c+dx))^2} + \frac{8 \sin(c+dx)}{315d(a^5+a^5 \cos(c+dx))}$$

[Out] 1/9*sin(d*x+c)/d/(a+a*cos(d*x+c))^5+4/63*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^4+4/105*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3+8/315*sin(d*x+c)/a/d/(a^2+a^2*cos(d*x+c))^2+8/315*sin(d*x+c)/d/(a^5+a^5*cos(d*x+c))

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2729, 2727}

$$\int \frac{1}{(a+a \cos(c+dx))^5} dx = \frac{8 \sin(c+dx)}{315d(a^5 \cos(c+dx) + a^5)} + \frac{8 \sin(c+dx)}{315ad(a^2 \cos(c+dx) + a^2)^2}$$

$$+ \frac{4 \sin(c+dx)}{105a^2d(a \cos(c+dx) + a)^3}$$

$$+ \frac{4 \sin(c+dx)}{63ad(a \cos(c+dx) + a)^4} + \frac{\sin(c+dx)}{9d(a \cos(c+dx) + a)^5}$$

[In] Int[(a + a*cos[c + d*x])^(-5), x]

[Out] $\text{Sin}[c + d*x]/(9*d*(a + a*\text{Cos}[c + d*x])^5) + (4*\text{Sin}[c + d*x])/(63*a*d*(a + a*\text{Cos}[c + d*x])^4) + (4*\text{Sin}[c + d*x])/(105*a^2*d*(a + a*\text{Cos}[c + d*x])^3) + (8*\text{Sin}[c + d*x])/(315*a*d*(a^2 + a^2*\text{Cos}[c + d*x])^2) + (8*\text{Sin}[c + d*x])/(315*d*(a^5 + a^5*\text{Cos}[c + d*x]))$

Rule 2727

$\text{Int}[(a_) + (b_)*\text{sin}[(c_) + (d_)*(x_)]^{-1}, x_Symbol] \rightarrow \text{Simp}[-\text{Cos}[c + d*x]/(d*(b + a*\text{Sin}[c + d*x])), x] /;$ $\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2729

$\text{Int}[(a_) + (b_)*\text{sin}[(c_) + (d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[b*\text{Cos}[c + d*x]*((a + b*\text{Sin}[c + d*x])^n/(a*d*(2*n + 1))), x] + \text{Dist}[(n + 1)/(a*(2*n + 1)), \text{Int}[(a + b*\text{Sin}[c + d*x])^{(n + 1)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{4 \int \frac{1}{(a + a \cos(c + dx))^4} dx}{9a} \\
 &= \frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{4 \sin(c + dx)}{63ad(a + a \cos(c + dx))^4} + \frac{4 \int \frac{1}{(a + a \cos(c + dx))^3} dx}{21a^2} \\
 &= \frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{4 \sin(c + dx)}{63ad(a + a \cos(c + dx))^4} \\
 &\quad + \frac{4 \sin(c + dx)}{105a^2d(a + a \cos(c + dx))^3} + \frac{8 \int \frac{1}{(a + a \cos(c + dx))^2} dx}{105a^3} \\
 &= \frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{4 \sin(c + dx)}{63ad(a + a \cos(c + dx))^4} + \frac{4 \sin(c + dx)}{105a^2d(a + a \cos(c + dx))^3} \\
 &\quad + \frac{8 \sin(c + dx)}{315a^3d(a + a \cos(c + dx))^2} + \frac{8 \int \frac{1}{a + a \cos(c + dx)} dx}{315a^4} \\
 &= \frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{4 \sin(c + dx)}{63ad(a + a \cos(c + dx))^4} + \frac{4 \sin(c + dx)}{105a^2d(a + a \cos(c + dx))^3} \\
 &\quad + \frac{8 \sin(c + dx)}{315a^3d(a + a \cos(c + dx))^2} + \frac{8 \sin(c + dx)}{315d(a^5 + a^5 \cos(c + dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{\cos\left(\frac{1}{2}(c + dx)\right) \left(126 \sin\left(\frac{1}{2}(c + dx)\right) + 84 \sin\left(\frac{3}{2}(c + dx)\right) + 36 \sin\left(\frac{5}{2}(c + dx)\right) + 9 \sin\left(\frac{7}{2}(c + dx)\right) + \sin\left(\frac{9}{2}(c + dx)\right)\right)}{315a^5 d (1 + \cos(c + dx))^5}$$

[In] Integrate[(a + a*Cos[c + d*x])^(-5),x]

[Out] (Cos[(c + d*x)/2]*(126*Sin[(c + d*x)/2] + 84*Sin[(3*(c + d*x))/2] + 36*Sin[(5*(c + d*x))/2] + 9*Sin[(7*(c + d*x))/2] + Sin[(9*(c + d*x))/2]))/(315*a^5*d*(1 + Cos[c + d*x])^5)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{16i(126e^{4i(dx+c)} + 84e^{3i(dx+c)} + 36e^{2i(dx+c)} + 9e^{i(dx+c)} + 1)}{315da^5(e^{i(dx+c)} + 1)^9}$	69
derivativedivides	$\frac{\frac{\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)}{9} + \frac{4\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{7} + \frac{6\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} + \frac{4\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16da^5}$	71
default	$\frac{\frac{\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)}{9} + \frac{4\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{7} + \frac{6\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5} + \frac{4\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3} + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16da^5}$	71
parallelrisch	$\frac{35\left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 180\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 378\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 420\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 315\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{5040a^5d}$	73
norman	$\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{16da} + \frac{\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{12da} + \frac{3\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{40da} + \frac{\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)}{28da} + \frac{\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right)}{144da}$	99

[In] int(1/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)

[Out] 16/315*I*(126*exp(4*I*(d*x+c))+84*exp(3*I*(d*x+c))+36*exp(2*I*(d*x+c))+9*exp(I*(d*x+c))+1)/d/a^5/(exp(I*(d*x+c))+1)^9

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{(8 \cos(dx + c)^4 + 40 \cos(dx + c)^3 + 84 \cos(dx + c)^2 + 100 \cos(dx + c) + 83) \sin(dx + c)}{315 (a^5 d \cos(dx + c)^5 + 5 a^5 d \cos(dx + c)^4 + 10 a^5 d \cos(dx + c)^3 + 10 a^5 d \cos(dx + c)^2 + 5 a^5 d \cos(dx + c) + a^5 d)}$$

`[In] integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="fricas")`

```
[Out] 1/315*(8*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 100*cos(d*x + c) + 83)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [A] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a + a \cos(c + dx))^5} dx$$

$$= \begin{cases} \frac{\tan^9\left(\frac{c+dx}{2}\right)}{144a^5d} + \frac{\tan^7\left(\frac{c+dx}{2}\right)}{28a^5d} + \frac{3\tan^5\left(\frac{c+dx}{2}\right)}{40a^5d} + \frac{\tan^3\left(\frac{c+dx}{2}\right)}{12a^5d} + \frac{\tan\left(\frac{c+dx}{2}\right)}{16a^5d} & \text{for } d \neq 0 \\ \frac{x}{(a \cos(c) + a)^5} & \text{otherwise} \end{cases}$$

`[In] integrate(1/(a+a*cos(d*x+c))**5,x)`

```
[Out] Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) + tan(c/2 + d*x/2)**7/(28*a**5*d) + 3*tan(c/2 + d*x/2)**5/(40*a**5*d) + tan(c/2 + d*x/2)**3/(12*a**5*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x/(a*cos(c) + a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{\frac{315 \sin(dx+c)}{\cos(dx+c)+1} + \frac{420 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{180 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040 a^5 d}$$

[In] integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="maxima")

[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) + 420*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 180*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int \frac{1}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 + 180 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 378 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 420 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 315 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{5040 a^5 d}$$

[In] integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="giac")

[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 + 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/2*d*x + 1/2*c)^5 + 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(a^5*d)

Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) \left(315 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 420 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 378 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 180 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 35 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^8\right)}{5040 a^5 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^9}$$

[In] int(1/(a + a*cos(c + d*x))^5,x)

[Out] (sin(c/2 + (d*x)/2)*(315*cos(c/2 + (d*x)/2)^8 + 35*sin(c/2 + (d*x)/2)^8 + 180*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 378*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 + 420*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2))/(5040*a^5*d*cos(c/2 + (d*x)/2)^9)

3.90 $\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^5} dx$

Optimal result	748
Rubi [A] (verified)	748
Mathematica [A] (verified)	750
Maple [A] (verified)	751
Fricas [A] (verification not implemented)	751
Sympy [F]	752
Maxima [A] (verification not implemented)	752
Giac [A] (verification not implemented)	752
Mupad [B] (verification not implemented)	753

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^5} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^5 d} - \frac{\sin(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{13 \sin(c+dx)}{63ad(a+a \cos(c+dx))^4} - \frac{34 \sin(c+dx)}{105a^2 d(a+a \cos(c+dx))^3} - \frac{173 \sin(c+dx)}{315a^3 d(a+a \cos(c+dx))^2} - \frac{488 \sin(c+dx)}{315d(a^5+a^5 \cos(c+dx))}$$

[Out] $\operatorname{arctanh}(\sin(dx+c))/a^5/d-1/9*\sin(dx+c)/d/(a+a*\cos(dx+c))^5-13/63*\sin(dx+c)/a/d/(a+a*\cos(dx+c))^4-34/105*\sin(dx+c)/a^2/d/(a+a*\cos(dx+c))^3-173/315*\sin(dx+c)/a^3/d/(a+a*\cos(dx+c))^2-488/315*\sin(dx+c)/d/(a^5+a^5*\cos(dx+c))$

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2845, 3057, 12, 3855}

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^5} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^5 d} - \frac{488 \sin(c+dx)}{315d(a^5 \cos(c+dx) + a^5)} - \frac{173 \sin(c+dx)}{315a^3 d(a \cos(c+dx) + a)^2} - \frac{34 \sin(c+dx)}{105a^2 d(a \cos(c+dx) + a)^3} - \frac{13 \sin(c+dx)}{63ad(a \cos(c+dx) + a)^4} - \frac{\sin(c+dx)}{9d(a \cos(c+dx) + a)^5}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]/(a+a*\operatorname{Cos}[c+d*x])^5,x]$


```
[Out] ArcTanh[Sin[c + d*x]]/(a^5*d) - Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) -
(13*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (34*Sin[c + d*x])/(105
*a^2*d*(a + a*Cos[c + d*x])^3) - (173*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[c
+ d*x])^2) - (488*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 2845

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{\int \frac{(9a - 4a \cos(c + dx)) \sec(c + dx)}{(a + a \cos(c + dx))^4} dx}{9a^2} \\ &= -\frac{\sin(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{13 \sin(c + dx)}{63ad(a + a \cos(c + dx))^4} + \frac{\int \frac{(63a^2 - 39a^2 \cos(c + dx)) \sec(c + dx)}{(a + a \cos(c + dx))^3} dx}{63a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
&\quad - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} + \frac{\int \frac{(315a^3-204a^3\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2} dx}{315a^6} \\
&= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} \\
&\quad - \frac{173\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(945a^4-519a^4\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)} dx}{945a^8} \\
&= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} \\
&\quad - \frac{173\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} - \frac{488\sin(c+dx)}{315d(a^5+a^5\cos(c+dx))} + \frac{\int 945a^5\sec(c+dx) dx}{945a^{10}} \\
&= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} \\
&\quad - \frac{173\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} - \frac{488\sin(c+dx)}{315d(a^5+a^5\cos(c+dx))} + \frac{\int \sec(c+dx) dx}{a^5} \\
&= \frac{\operatorname{arctanh}(\sin(c+dx))}{a^5d} - \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
&\quad - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} - \frac{173\sin(c+dx)}{315a^3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{488\sin(c+dx)}{315d(a^5+a^5\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.38

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right) \left(80640 \cos^9\left(\frac{1}{2}(c+dx)\right) \left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{\dots}$$

[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^5,x]

[Out] -1/2520*(Cos[(c + d*x)/2]*(80640*Cos[(c + d*x)/2]^9*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*(35973*Sin[(d*x)/2] - 25515*Sin[c + (d*x)/2] + 29757*Sin[c + (3*d*x)/2] - 11235*Sin[2*c + (3*d*x)/2] + 14733*Sin[2*c + (5*d*x)/2] - 2835*Sin[3*c + (5*d*x)/2] + 4077*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c + (7*d*x)/2] + 488*Sin[4*c + (9*d*x)/2]))/(a^5*d*(1 + Cos[c + d*x])^5)

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-\frac{(\tan^9(\frac{dx}{2} + \frac{c}{2}))}{9} - \frac{6(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{7} - \frac{16(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{5} - \frac{26(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{3} - 31 \tan(\frac{dx}{2} + \frac{c}{2}) - 16 \ln(\tan(\frac{dx}{2} + \frac{c}{2}) - 1) + 16 \ln(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)}{16da^5}$
default	$\frac{-\frac{(\tan^9(\frac{dx}{2} + \frac{c}{2}))}{9} - \frac{6(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{7} - \frac{16(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{5} - \frac{26(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{3} - 31 \tan(\frac{dx}{2} + \frac{c}{2}) - 16 \ln(\tan(\frac{dx}{2} + \frac{c}{2}) - 1) + 16 \ln(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)}{16da^5}$
parallelrisch	$\frac{-35(\tan^9(\frac{dx}{2} + \frac{c}{2})) - 270(\tan^7(\frac{dx}{2} + \frac{c}{2})) - 1008(\tan^5(\frac{dx}{2} + \frac{c}{2})) - 2730(\tan^3(\frac{dx}{2} + \frac{c}{2})) - 9765 \tan(\frac{dx}{2} + \frac{c}{2}) - 5040 \ln(\tan(\frac{dx}{2} + \frac{c}{2}) - 1) + 5040 \ln(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)}{5040a^5d}$
norman	$\frac{\frac{31 \tan(\frac{dx}{2} + \frac{c}{2})}{16da} - \frac{13(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{24da} - \frac{\tan^5(\frac{dx}{2} + \frac{c}{2})}{5da} - \frac{3(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{56da} - \frac{\tan^9(\frac{dx}{2} + \frac{c}{2})}{144da}}{a^4} + \frac{\ln(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)}{a^5d} - \frac{\ln(\tan(\frac{dx}{2} + \frac{c}{2}) - 1)}{a^5d}$
risch	$\frac{2i(315 e^{8i(dx+c)} + 2835 e^{7i(dx+c)} + 11235 e^{6i(dx+c)} + 25515 e^{5i(dx+c)} + 35973 e^{4i(dx+c)} + 29757 e^{3i(dx+c)} + 14733 e^{2i(dx+c)} + 4725 e^{i(dx+c)} + 315)}{315d a^5 (e^{i(dx+c)} + 1)^9}$

[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)

[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9-6/7*tan(1/2*d*x+1/2*c)^7-16/5*tan(1/2*d*x+1/2*c)^5-26/3*tan(1/2*d*x+1/2*c)^3-31*tan(1/2*d*x+1/2*c)-16*ln(tan(1/2*d*x+1/2*c)-1)+16*ln(tan(1/2*d*x+1/2*c)+1))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.61

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$= \frac{315 (\cos(dx + c)^5 + 5 \cos(dx + c)^4 + 10 \cos(dx + c)^3 + 10 \cos(dx + c)^2 + 5 \cos(dx + c) + 1) \log(\sin(dx + c) + 1) - 315 (\cos(dx + c)^5 + 5 \cos(dx + c)^4 + 10 \cos(dx + c)^3 + 10 \cos(dx + c)^2 + 5 \cos(dx + c) + 1) \log(\sin(dx + c) - 1) - 2(488 \cos(dx + c)^4 + 2125 \cos(dx + c)^3 + 3549 \cos(dx + c)^2 + 2740 \cos(dx + c) + 863) \sin(dx + c)}{a^5 d \cos(dx + c)^5 + 5 a^5 d \cos(dx + c)^4 + 10 a^5 d \cos(dx + c)^3 + 10 a^5 d \cos(dx + c)^2 + 5 a^5 d \cos(dx + c) + a^5 d}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="fricas")

[Out] 1/630*(315*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 315*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(488*cos(d*x + c)^4 + 2125*cos(d*x + c)^3 + 3549*cos(d*x + c)^2 + 2740*cos(d*x + c) + 863)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{\int \frac{\sec(c+dx)}{\cos^5(c+dx)+5\cos^4(c+dx)+10\cos^3(c+dx)+10\cos^2(c+dx)+5\cos(c+dx)+1} dx}{a^5}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**5,x)

[Out] Integral(sec(c + d*x)/(cos(c + d*x)**5 + 5*cos(c + d*x)**4 + 10*cos(c + d*x)**3 + 10*cos(c + d*x)**2 + 5*cos(c + d*x) + 1), x)/a**5

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{\frac{9765 \sin(dx+c)}{\cos(dx+c)+1} + \frac{2730 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1008 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{270 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{5040 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{a^5} + \frac{5040 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 1\right)}{a^5}}{5040 d}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="maxima")

[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) + 2730*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 270*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{\frac{5040 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^5} - \frac{5040 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{a^5} - \frac{35 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 + 270 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 1008 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5}{a^{45}}}{5040 d}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="giac")

[Out] 1/5040*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 + 270*a^40*tan(1/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 + 2730*a^40*tan(1/2*d*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.65

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^5} dx =$$

$$-\frac{\frac{13 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{24 a^5} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{5 a^5} + \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{56 a^5} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{144 a^5} - \frac{2 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^5} + \frac{31 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16 a^5}}{d}$$

[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^5),x)

[Out] -((13*tan(c/2 + (d*x)/2)^3)/(24*a^5) + tan(c/2 + (d*x)/2)^5/(5*a^5) + (3*tan(c/2 + (d*x)/2)^7)/(56*a^5) + tan(c/2 + (d*x)/2)^9/(144*a^5) - (2*atanh(tan(c/2 + (d*x)/2)))/a^5 + (31*tan(c/2 + (d*x)/2))/(16*a^5))/d

3.91 $\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^5} dx$

Optimal result	754
Rubi [A] (verified)	754
Mathematica [B] (verified)	757
Maple [A] (verified)	758
Fricas [A] (verification not implemented)	758
Sympy [F]	759
Maxima [A] (verification not implemented)	759
Giac [A] (verification not implemented)	759
Mupad [B] (verification not implemented)	760

Optimal result

Integrand size = 21, antiderivative size = 168

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^5} dx = -\frac{5\operatorname{arctanh}(\sin(c+dx))}{a^5d} + \frac{496 \tan(c+dx)}{63a^5d} - \frac{\tan(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{5 \tan(c+dx)}{21ad(a+a \cos(c+dx))^4} - \frac{29 \tan(c+dx)}{63a^2d(a+a \cos(c+dx))^3} - \frac{67 \tan(c+dx)}{63a^3d(a+a \cos(c+dx))^2} - \frac{5 \tan(c+dx)}{d(a^5+a^5 \cos(c+dx))}$$

[Out] $-5*\operatorname{arctanh}(\sin(d*x+c))/a^5/d+496/63*\tan(d*x+c)/a^5/d-1/9*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^5-5/21*\tan(d*x+c)/a/d/(a+a*\cos(d*x+c))^4-29/63*\tan(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^3-67/63*\tan(d*x+c)/a^3/d/(a+a*\cos(d*x+c))^2-5*\tan(d*x+c)/d/(a^5+a^5*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {2845, 3057, 2827, 3852, 8, 3855}

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5} dx = -\frac{5\operatorname{arctanh}(\sin(c+dx))}{a^5d} + \frac{496\tan(c+dx)}{63a^5d} - \frac{5\tan(c+dx)}{d(a^5\cos(c+dx)+a^5)} - \frac{67\tan(c+dx)}{63a^3d(a\cos(c+dx)+a)^2} - \frac{29\tan(c+dx)}{63a^2d(a\cos(c+dx)+a)^3} - \frac{5\tan(c+dx)}{21ad(a\cos(c+dx)+a)^4} - \frac{\tan(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]

[Out] (-5*ArcTanh[Sin[c + d*x]]/(a^5*d) + (496*Tan[c + d*x])/(63*a^5*d) - Tan[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) - (5*Tan[c + d*x])/(21*a*d*(a + a*Cos[c + d*x])^4) - (29*Tan[c + d*x])/(63*a^2*d*(a + a*Cos[c + d*x])^3) - (67*Tan[c + d*x])/(63*a^3*d*(a + a*Cos[c + d*x])^2) - (5*Tan[c + d*x])/(d*(a^5 + a^5*Cos[c + d*x])))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2845

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b

$d*(n + 1) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2) * \text{Sin}[e + f*x], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3852

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] := \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3855

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] := \text{Simp}[-\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\tan(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{\int \frac{(10a - 5a \cos(c + dx)) \sec^2(c + dx)}{(a + a \cos(c + dx))^4} dx}{9a^2} \\
 &= -\frac{\tan(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{5 \tan(c + dx)}{21ad(a + a \cos(c + dx))^4} + \frac{\int \frac{(85a^2 - 60a^2 \cos(c + dx)) \sec^2(c + dx)}{(a + a \cos(c + dx))^3} dx}{63a^4} \\
 &= -\frac{\tan(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{5 \tan(c + dx)}{21ad(a + a \cos(c + dx))^4} \\
 &\quad - \frac{29 \tan(c + dx)}{63a^2d(a + a \cos(c + dx))^3} + \frac{\int \frac{(570a^3 - 435a^3 \cos(c + dx)) \sec^2(c + dx)}{(a + a \cos(c + dx))^2} dx}{315a^6} \\
 &= -\frac{\tan(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{5 \tan(c + dx)}{21ad(a + a \cos(c + dx))^4} - \frac{29 \tan(c + dx)}{63a^2d(a + a \cos(c + dx))^3} \\
 &\quad - \frac{67 \tan(c + dx)}{63a^3d(a + a \cos(c + dx))^2} + \frac{\int \frac{(2715a^4 - 2010a^4 \cos(c + dx)) \sec^2(c + dx)}{a + a \cos(c + dx)} dx}{945a^8} \\
 &= -\frac{\tan(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{5 \tan(c + dx)}{21ad(a + a \cos(c + dx))^4} \\
 &\quad - \frac{29 \tan(c + dx)}{63a^2d(a + a \cos(c + dx))^3} - \frac{67 \tan(c + dx)}{63a^3d(a + a \cos(c + dx))^2} \\
 &\quad - \frac{5 \tan(c + dx)}{d(a^5 + a^5 \cos(c + dx))} + \frac{\int (7440a^5 - 4725a^5 \cos(c + dx)) \sec^2(c + dx) dx}{945a^{10}}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4} \\
&\quad - \frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3} - \frac{67\tan(c+dx)}{63a^3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{5\tan(c+dx)}{d(a^5+a^5\cos(c+dx))} - \frac{5\int\sec(c+dx)dx}{a^5} + \frac{496\int\sec^2(c+dx)dx}{63a^5} \\
&= -\frac{5\arctanh(\sin(c+dx))}{a^5d} - \frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4} \\
&\quad - \frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3} - \frac{67\tan(c+dx)}{63a^3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{5\tan(c+dx)}{d(a^5+a^5\cos(c+dx))} - \frac{496\text{Subst}(\int 1 dx, x, -\tan(c+dx))}{63a^5d} \\
&= -\frac{5\arctanh(\sin(c+dx))}{a^5d} + \frac{496\tan(c+dx)}{63a^5d} - \frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5} \\
&\quad - \frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3} \\
&\quad - \frac{67\tan(c+dx)}{63a^3d(a+a\cos(c+dx))^2} - \frac{5\tan(c+dx)}{d(a^5+a^5\cos(c+dx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 393 vs. $2(168) = 336$.

Time = 5.52 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.34

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5} dx$$

$$\frac{322560 \cos^{10}\left(\frac{1}{2}(c+dx)\right) \left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{1}$$

[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]

[Out] (322560*Cos[(c + d*x)/2]^10*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Sec[c + d*x]*(-33978*Sin[(d*x)/2] + 52002*Sin[(3*d*x)/2] - 56952*Sin[c - (d*x)/2] + 43722*Sin[c + (d*x)/2] - 47208*Sin[2*c + (d*x)/2] - 18144*Sin[c + (3*d*x)/2] + 41796*Sin[2*c + (3*d*x)/2] - 28350*Sin[3*c + (3*d*x)/2] + 34578*Sin[c + (5*d*x)/2] - 5691*Sin[2*c + (5*d*x)/2] + 28719*Sin[3*c + (5*d*x)/2] - 11550*Sin[4*c + (5*d*x)/2] + 15517*Sin[2*c + (7*d*x)/2] - 504*Sin[3*c + (7*d*x)/2] + 13186*Sin[4*c + (7*d*x)/2] - 2835*Sin[5*c + (7*d*x)/2] + 4149*Sin[3*c + (9*d*x)/2] + 252*Sin[4*c + (9*d*x)/2] + 3582*Sin[5*c + (9*d*x)/2] - 315*Sin[6*c + (9*d*x)/2] + 496*Sin[4*c + (11*d*x)/2] + 63*Sin[5*c + (11*d*x)/2] + 433*Sin[6*c + (11*d*x)/2]))/(2016*a^5*d*(1 + Cos[c + d*x])^5)

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\left(\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{9}+\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+80\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{16da^5}$
default	$\frac{\left(\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{9}+\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+129\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+80\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)}{16da^5}$
parallelrisc	$\frac{40320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-40320\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+31846\left(\cos(dx+c)+\frac{10010\cos(2dx+2c)}{15923}+\frac{425}{8064a^5d}\cos(dx+c)\right)}{8064a^5d\cos(dx+c)}$
norman	$\frac{-\frac{161\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16da}+\frac{105\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16da}+\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{17\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56da}+\frac{65\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1008da}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{144da}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^4}+5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$
risc	$\frac{2i\left(315e^{10i(dx+c)}+2835e^{9i(dx+c)}+11550e^{8i(dx+c)}+28350e^{7i(dx+c)}+47208e^{6i(dx+c)}+56952e^{5i(dx+c)}+52002e^{4i(dx+c)}+31500e^{3i(dx+c)}+15750e^{2i(dx+c)}+3150e^{i(dx+c)}+315\right)}{63da^5\left(e^{i(dx+c)}+1\right)^9\left(e^{2i(dx+c)}+1\right)}$

[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^5,x,method=_RETURNVERBOSE)

[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7+6*tan(1/2*d*x+1/2*c)^5+24*tan(1/2*d*x+1/2*c)^3+129*tan(1/2*d*x+1/2*c)-16/(tan(1/2*d*x+1/2*c)-1)+80*ln(tan(1/2*d*x+1/2*c)-1)-16/(tan(1/2*d*x+1/2*c)+1)-80*ln(tan(1/2*d*x+1/2*c)+1))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.65

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{315(\cos(dx+c)^6+5\cos(dx+c)^5+10\cos(dx+c)^4+10\cos(dx+c)^3+5\cos(dx+c)^2+\cos(dx+c))}{(a+a\cos(c+dx))^5}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="fricas")

[Out] -1/126*(315*(cos(d*x+c)^6+5*cos(d*x+c)^5+10*cos(d*x+c)^4+10*cos(d*x+c)^3+5*cos(d*x+c)^2+cos(d*x+c))*log(sin(d*x+c)+1)-315*(cos(d*x+c)^6+5*cos(d*x+c)^5+10*cos(d*x+c)^4+10*cos(d*x+c)^3+5*cos(d*x+c)^2+cos(d*x+c))*log(-sin(d*x+c)+1)-2*(496*cos(d*x+c)^5+2165*cos(d*x+c)^4+3633*cos(d*x+c)^3+2840*cos(d*x+c)^2+946*cos(d*x+c)+63)*sin(d*x+c))/(a^5*d*cos(d*x+c)^6+5*a^5*d*cos(d*x+c)^5+10*a^5*d*cos(d*x+c)^4+10*a^5*d*cos(d*x+c)^3+5*a^5*d*cos(d*x+c)^2+a^5*d*cos(d*x+c))

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{\int \frac{\sec^2(c+dx)}{\cos^5(c+dx)+5\cos^4(c+dx)+10\cos^3(c+dx)+10\cos^2(c+dx)+5\cos(c+dx)+1} dx}{a^5}$$

[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**5,x)

[Out] Integral(sec(c + d*x)**2/(cos(c + d*x)**5 + 5*cos(c + d*x)**4 + 10*cos(c + d*x)**3 + 10*cos(c + d*x)**2 + 5*cos(c + d*x) + 1), x)/a**5

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.23

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{\frac{2016 \sin(dx+c)}{\left(a^5 - \frac{a^5 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right) (\cos(dx+c)+1)} + \frac{\frac{8127 \sin(dx+c)}{\cos(dx+c)+1} + \frac{1512 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{72 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{7 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{5040 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{1008 d}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="maxima")

[Out] 1/1008*(2016*sin(d*x + c)/((a^5 - a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) + 1512*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 72*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{\frac{5040 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^5} - \frac{5040 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right)}{a^5} + \frac{2016 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right) a^5} - \frac{7 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 + 72 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 378 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 1512 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 8127 a^{40} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(a^5 - \frac{a^5 \tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan^2\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1}\right) \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)}}{1008 d}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="giac")

[Out] $-1/1008*(5040*\log(\tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*\log(\tan(1/2*d*x + 1/2*c) - 1))/a^5 + 2016*\tan(1/2*d*x + 1/2*c)/((\tan(1/2*d*x + 1/2*c)^2 - 1)*a^5) - (7*a^{40}*\tan(1/2*d*x + 1/2*c)^9 + 72*a^{40}*\tan(1/2*d*x + 1/2*c)^7 + 378*a^{40}*\tan(1/2*d*x + 1/2*c)^5 + 1512*a^{40}*\tan(1/2*d*x + 1/2*c)^3 + 8127*a^{40}*\tan(1/2*d*x + 1/2*c))/a^{45}/d$

Mupad [B] (verification not implemented)

Time = 14.78 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{2 a^5 d} + \frac{3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{8 a^5 d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7}{14 a^5 d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{144 a^5 d} - \frac{10 \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^5 d} - \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(a^5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - a^5\right)} + \frac{129 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{16 a^5 d}$$

[In] `int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^5),x)`

[Out] $(3*\tan(c/2 + (d*x)/2)^3)/(2*a^5*d) + (3*\tan(c/2 + (d*x)/2)^5)/(8*a^5*d) + \tan(c/2 + (d*x)/2)^7/(14*a^5*d) + \tan(c/2 + (d*x)/2)^9/(144*a^5*d) - (10*\operatorname{atanh}(\tan(c/2 + (d*x)/2)))/(a^5*d) - (2*\tan(c/2 + (d*x)/2))/(d*(a^5*\tan(c/2 + (d*x)/2)^2 - a^5)) + (129*\tan(c/2 + (d*x)/2))/(16*a^5*d)$

3.92 $\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^5} dx$

Optimal result	761
Rubi [A] (verified)	761
Mathematica [B] (verified)	764
Maple [A] (verified)	765
Fricas [A] (verification not implemented)	766
Sympy [F]	766
Maxima [A] (verification not implemented)	766
Giac [A] (verification not implemented)	767
Mupad [B] (verification not implemented)	767

Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^5} dx = \frac{31 \operatorname{arctanh}(\sin(c+dx))}{2a^5d} - \frac{7664 \tan(c+dx)}{315a^5d} + \frac{31 \sec(c+dx) \tan(c+dx)}{2a^5d} - \frac{\sec(c+dx) \tan(c+dx)}{9d(a+a \cos(c+dx))^5} - \frac{17 \sec(c+dx) \tan(c+dx)}{63ad(a+a \cos(c+dx))^4} - \frac{28 \sec(c+dx) \tan(c+dx)}{45a^2d(a+a \cos(c+dx))^3} - \frac{577 \sec(c+dx) \tan(c+dx)}{315a^3d(a+a \cos(c+dx))^2} - \frac{3832 \sec(c+dx) \tan(c+dx)}{315d(a^5+a^5 \cos(c+dx))}$$

```
[Out] 31/2*arctanh(sin(d*x+c))/a^5/d-7664/315*tan(d*x+c)/a^5/d+31/2*sec(d*x+c)*tan(d*x+c)/a^5/d-1/9*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^5-17/63*sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^4-28/45*sec(d*x+c)*tan(d*x+c)/a^2/d/(a+a*cos(d*x+c))^3-577/315*sec(d*x+c)*tan(d*x+c)/a^3/d/(a+a*cos(d*x+c))^2-3832/315*sec(d*x+c)*tan(d*x+c)/d/(a^5+a^5*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {2845, 3057, 2827, 3853, 3855, 3852, 8}

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{31\operatorname{arctanh}(\sin(c+dx))}{2a^5d} - \frac{7664\tan(c+dx)}{315a^5d} + \frac{31\tan(c+dx)\sec(c+dx)}{2a^5d} - \frac{3832\tan(c+dx)\sec(c+dx)}{315d(a^5\cos(c+dx)+a^5)} - \frac{577\tan(c+dx)\sec(c+dx)}{315a^3d(a\cos(c+dx)+a)^2} - \frac{28\tan(c+dx)\sec(c+dx)}{45a^2d(a\cos(c+dx)+a)^3} - \frac{17\tan(c+dx)\sec(c+dx)}{63ad(a\cos(c+dx)+a)^4} - \frac{\tan(c+dx)\sec(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]

[Out] (31*ArcTanh[Sin[c + d*x]]/(2*a^5*d) - (7664*Tan[c + d*x])/(315*a^5*d) + (31*Sec[c + d*x]*Tan[c + d*x])/(2*a^5*d) - (Sec[c + d*x]*Tan[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) - (17*Sec[c + d*x]*Tan[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (28*Sec[c + d*x]*Tan[c + d*x])/(45*a^2*d*(a + a*Cos[c + d*x])^3) - (577*Sec[c + d*x]*Tan[c + d*x])/(315*a^3*d*(a + a*Cos[c + d*x])^2) - (3832*Sec[c + d*x]*Tan[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x])))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2845

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),

Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sec(c + dx) \tan(c + dx)}{9d(a + a \cos(c + dx))^5} + \frac{\int \frac{(11a - 6a \cos(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^4} dx}{9a^2} \\
 &= -\frac{\sec(c + dx) \tan(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{17 \sec(c + dx) \tan(c + dx)}{63ad(a + a \cos(c + dx))^4} + \frac{\int \frac{(111a^2 - 85a^2 \cos(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^3} dx}{63a^4} \\
 &= -\frac{\sec(c + dx) \tan(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{17 \sec(c + dx) \tan(c + dx)}{63ad(a + a \cos(c + dx))^4} \\
 &\quad - \frac{28 \sec(c + dx) \tan(c + dx)}{45a^2d(a + a \cos(c + dx))^3} + \frac{\int \frac{(947a^3 - 784a^3 \cos(c + dx)) \sec^3(c + dx)}{(a + a \cos(c + dx))^2} dx}{315a^6} \\
 &= -\frac{\sec(c + dx) \tan(c + dx)}{9d(a + a \cos(c + dx))^5} - \frac{17 \sec(c + dx) \tan(c + dx)}{63ad(a + a \cos(c + dx))^4} - \frac{28 \sec(c + dx) \tan(c + dx)}{45a^2d(a + a \cos(c + dx))^3} \\
 &\quad - \frac{577 \sec(c + dx) \tan(c + dx)}{315a^3d(a + a \cos(c + dx))^2} + \frac{\int \frac{(6303a^4 - 5193a^4 \cos(c + dx)) \sec^3(c + dx)}{a + a \cos(c + dx)} dx}{945a^8}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\sec(c+dx)\tan(c+dx)}{45a^2d(a+a\cos(c+dx))^3} \\
&\quad - \frac{577\sec(c+dx)\tan(c+dx)}{315a^3d(a+a\cos(c+dx))^2} - \frac{3832\sec(c+dx)\tan(c+dx)}{315d(a^5+a^5\cos(c+dx))} \\
&\quad + \frac{\int(29295a^5-22992a^5\cos(c+dx))\sec^3(c+dx)dx}{945a^{10}} \\
&= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4} \\
&\quad - \frac{28\sec(c+dx)\tan(c+dx)}{45a^2d(a+a\cos(c+dx))^3} - \frac{577\sec(c+dx)\tan(c+dx)}{315a^3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{3832\sec(c+dx)\tan(c+dx)}{315d(a^5+a^5\cos(c+dx))} - \frac{7664\int\sec^2(c+dx)dx}{315a^5} + \frac{31\int\sec^3(c+dx)dx}{a^5} \\
&= \frac{31\sec(c+dx)\tan(c+dx)}{2a^5d} - \frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} \\
&\quad - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\sec(c+dx)\tan(c+dx)}{45a^2d(a+a\cos(c+dx))^3} \\
&\quad - \frac{577\sec(c+dx)\tan(c+dx)}{315a^3d(a+a\cos(c+dx))^2} - \frac{3832\sec(c+dx)\tan(c+dx)}{315d(a^5+a^5\cos(c+dx))} \\
&\quad + \frac{31\int\sec(c+dx)dx}{2a^5} + \frac{7664\text{Subst}(\int 1dx, x, -\tan(c+dx))}{315a^5d} \\
&= \frac{31\text{arctanh}(\sin(c+dx))}{2a^5d} - \frac{7664\tan(c+dx)}{315a^5d} \\
&\quad + \frac{31\sec(c+dx)\tan(c+dx)}{2a^5d} - \frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5} \\
&\quad - \frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\sec(c+dx)\tan(c+dx)}{45a^2d(a+a\cos(c+dx))^3} \\
&\quad - \frac{577\sec(c+dx)\tan(c+dx)}{315a^3d(a+a\cos(c+dx))^2} - \frac{3832\sec(c+dx)\tan(c+dx)}{315d(a^5+a^5\cos(c+dx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 507 vs. $2(224) = 448$.

Time = 7.09 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.26

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5} dx &= -\frac{496\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a+a\cos(c+dx))^5} \\
&\quad + \frac{496\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a+a\cos(c+dx))^5} \\
&\quad + \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sec\left(\frac{c}{2}\right)\sec(c)\sec^2(c+dx)\left(1472562\sin\left(\frac{dx}{2}\right)-2822886\sin\left(\frac{3dx}{2}\right)+3057654\sin\left(c-\frac{dx}{2}\right)-1\right)}{d(a+a\cos(c+dx))^5}
\end{aligned}$$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.31

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5} dx$$

$$= \frac{9765 (\cos(dx+c))^7 + 5 \cos(dx+c)^6 + 10 \cos(dx+c)^5 + 10 \cos(dx+c)^4 + 5 \cos(dx+c)^3 + \cos(dx+c)^2}{a^5}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="fricas")

```
[Out] 1/1260*(9765*(cos(d*x + c)^7 + 5*cos(d*x + c)^6 + 10*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 5*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 9765*(cos(d*x + c)^7 + 5*cos(d*x + c)^6 + 10*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 5*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(15328*cos(d*x + c)^6 + 66875*cos(d*x + c)^5 + 112119*cos(d*x + c)^4 + 87440*cos(d*x + c)^3 + 28828*cos(d*x + c)^2 + 1575*cos(d*x + c) - 315)*sin(d*x + c))/(a^5*d*cos(d*x + c)^7 + 5*a^5*d*cos(d*x + c)^6 + 10*a^5*d*cos(d*x + c)^5 + 10*a^5*d*cos(d*x + c)^4 + 5*a^5*d*cos(d*x + c)^3 + a^5*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5} dx = \frac{\int \frac{\sec^3(c+dx)}{\cos^5(c+dx)+5\cos^4(c+dx)+10\cos^3(c+dx)+10\cos^2(c+dx)+5\cos(c+dx)+1} dx}{a^5}$$

[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**5,x)

```
[Out] Integral(sec(c + d*x)**3/(cos(c + d*x)**5 + 5*cos(c + d*x)**4 + 10*cos(c + d*x)**3 + 10*cos(c + d*x)**2 + 5*cos(c + d*x) + 1), x)/a**5
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.12

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5} dx =$$

$$\frac{5040 \left(\frac{9 \sin(dx+c)}{\cos(dx+c)+1} - \frac{11 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^5 - \frac{2 a^5 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^5 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{110565 \sin(dx+c)}{\cos(dx+c)+1} + \frac{15750 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3024 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{450 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5} - \frac{78120 \log(\sin(dx+c)+1)}{a^5}$$

5040 d

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="maxima")

[Out] $-1/5040*(5040*(9*\sin(d*x + c)/(\cos(d*x + c) + 1) - 11*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3)/(a^5 - 2*a^5*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + a^5*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4) + (110565*\sin(d*x + c)/(\cos(d*x + c) + 1) + 15750*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 3024*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 + 450*\sin(d*x + c)^7/(\cos(d*x + c) + 1)^7 + 35*\sin(d*x + c)^9/(\cos(d*x + c) + 1)^9)/a^5 - 78120*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)/a^5 + 78120*\log(\sin(d*x + c)/(\cos(d*x + c) + 1) - 1)/a^5)/d$

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.76

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^5} dx$$

$$\frac{78120 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a^5} - \frac{78120 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a^5} + \frac{5040 (11 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 9 \tan(\frac{1}{2} dx + \frac{1}{2} c))}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^2 a^5} - \frac{35 a^{40} \tan(\frac{1}{2} dx + \frac{1}{2} c)}{5040 d}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="giac")

[Out] $1/5040*(78120*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^5 - 78120*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/a^5 + 5040*(11*\tan(1/2*d*x + 1/2*c)^3 - 9*\tan(1/2*d*x + 1/2*c))/((\tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^5) - (35*a^{40}*\tan(1/2*d*x + 1/2*c)^9 + 450*a^{40}*\tan(1/2*d*x + 1/2*c)^7 + 3024*a^{40}*\tan(1/2*d*x + 1/2*c)^5 + 15750*a^{40}*\tan(1/2*d*x + 1/2*c)^3 + 110565*a^{40}*\tan(1/2*d*x + 1/2*c))/a^5)/d$

Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.80

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^5} dx = \frac{31 \operatorname{atanh}(\tan(\frac{c}{2} + \frac{dx}{2}))}{a^5 d} - \frac{3 \tan(\frac{c}{2} + \frac{dx}{2})^5}{5 a^5 d}$$

$$- \frac{5 \tan(\frac{c}{2} + \frac{dx}{2})^7}{56 a^5 d} - \frac{\tan(\frac{c}{2} + \frac{dx}{2})^9}{144 a^5 d} - \frac{25 \tan(\frac{c}{2} + \frac{dx}{2})^3}{8 a^5 d}$$

$$- \frac{9 \tan(\frac{c}{2} + \frac{dx}{2}) - 11 \tan(\frac{c}{2} + \frac{dx}{2})^3}{d (a^5 \tan(\frac{c}{2} + \frac{dx}{2})^4 - 2 a^5 \tan(\frac{c}{2} + \frac{dx}{2})^2 + a^5)}$$

$$- \frac{351 \tan(\frac{c}{2} + \frac{dx}{2})}{16 a^5 d}$$

[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^5),x)

[Out] (31*atanh(tan(c/2 + (d*x)/2)))/(a^5*d) - (3*tan(c/2 + (d*x)/2)^5)/(5*a^5*d)
 - (5*tan(c/2 + (d*x)/2)^7)/(56*a^5*d) - tan(c/2 + (d*x)/2)^9/(144*a^5*d) -
 (25*tan(c/2 + (d*x)/2)^3)/(8*a^5*d) - (9*tan(c/2 + (d*x)/2) - 11*tan(c/2 +
 (d*x)/2)^3)/(d*(a^5*tan(c/2 + (d*x)/2)^4 - 2*a^5*tan(c/2 + (d*x)/2)^2 + a^5)) - (351*tan(c/2 + (d*x)/2))/(16*a^5*d)

3.93 $\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^6} dx$

Optimal result	769
Rubi [A] (verified)	769
Mathematica [A] (verified)	772
Maple [A] (verified)	772
Fricas [A] (verification not implemented)	773
Sympy [A] (verification not implemented)	773
Maxima [A] (verification not implemented)	773
Giac [A] (verification not implemented)	774
Mupad [B] (verification not implemented)	774

Optimal result

Integrand size = 21, antiderivative size = 184

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^6} dx = \frac{130 \sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{268 \sin(c+dx)}{693a^6d(1+\cos(c+dx))^2}$$

$$+ \frac{146 \sin(c+dx)}{693a^6d(1+\cos(c+dx))} - \frac{\cos^4(c+dx) \sin(c+dx)}{11d(a+a \cos(c+dx))^6}$$

$$- \frac{14 \cos^3(c+dx) \sin(c+dx)}{99ad(a+a \cos(c+dx))^5} - \frac{118 \cos^2(c+dx) \sin(c+dx)}{693a^2d(a+a \cos(c+dx))^4}$$

[Out] 130/693*sin(d*x+c)/a^6/d/(1+cos(d*x+c))^3-268/693*sin(d*x+c)/a^6/d/(1+cos(d*x+c))^2+146/693*sin(d*x+c)/a^6/d/(1+cos(d*x+c))-1/11*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^6-14/99*cos(d*x+c)^3*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^5-118/693*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^4

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2844, 3056, 3047, 3098, 2829, 2727}

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^6} dx = \frac{146 \sin(c+dx)}{693a^6d(\cos(c+dx)+1)} - \frac{268 \sin(c+dx)}{693a^6d(\cos(c+dx)+1)^2}$$

$$+ \frac{130 \sin(c+dx)}{693a^6d(\cos(c+dx)+1)^3} - \frac{118 \sin(c+dx) \cos^2(c+dx)}{693a^2d(a \cos(c+dx)+a)^4}$$

$$- \frac{\sin(c+dx) \cos^4(c+dx)}{11d(a \cos(c+dx)+a)^6} - \frac{14 \sin(c+dx) \cos^3(c+dx)}{99ad(a \cos(c+dx)+a)^5}$$

[In] Int[Cos[c + d*x]^5/(a + a*cos[c + d*x])^6,x]

```
[Out] (130*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c + d*x])^3) - (268*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c + d*x])^2) + (146*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^4*Sin[c + d*x])/(11*d*(a + a*Cos[c + d*x])^6) - (14*Cos[c + d*x]^3*Sin[c + d*x])/(99*a*d*(a + a*Cos[c + d*x])^5) - (118*Cos[c + d*x]^2*Sin[c + d*x])/(693*a^2*d*(a + a*Cos[c + d*x])^4)
```

Rule 2727

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2844

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3098

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] :> Simp[(A*b - a*B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{\int \frac{\cos^3(c+dx)(4a-10a\cos(c+dx))}{(a+a\cos(c+dx))^5} dx}{11a^2} \\
 &= -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^2(c+dx)(42a^2-76a^2\cos(c+dx))}{(a+a\cos(c+dx))^4} dx}{99a^4} \\
 &= -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} \\
 &\quad - \frac{118\cos^2(c+dx)\sin(c+dx)}{693a^2d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos(c+dx)(236a^3-414a^3\cos(c+dx))}{(a+a\cos(c+dx))^3} dx}{693a^6} \\
 &= -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} \\
 &\quad - \frac{118\cos^2(c+dx)\sin(c+dx)}{693a^2d(a+a\cos(c+dx))^4} - \frac{\int \frac{236a^3\cos(c+dx)-414a^3\cos^2(c+dx)}{(a+a\cos(c+dx))^3} dx}{693a^6} \\
 &= \frac{130\sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} \\
 &\quad - \frac{118\cos^2(c+dx)\sin(c+dx)}{693a^2d(a+a\cos(c+dx))^4} + \frac{\int \frac{-1950a^4+2070a^4\cos(c+dx)}{(a+a\cos(c+dx))^2} dx}{3465a^8} \\
 &= \frac{130\sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} \\
 &\quad - \frac{118\cos^2(c+dx)\sin(c+dx)}{693a^2d(a+a\cos(c+dx))^4} - \frac{268\sin(c+dx)}{693d(a^3+a^3\cos(c+dx))^2} + \frac{146\int \frac{1}{a+a\cos(c+dx)} dx}{693a^5} \\
 &= \frac{130\sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} \\
 &\quad - \frac{118\cos^2(c+dx)\sin(c+dx)}{693a^2d(a+a\cos(c+dx))^4} - \frac{268\sin(c+dx)}{693d(a^3+a^3\cos(c+dx))^2} + \frac{146\sin(c+dx)}{693d(a^6+a^6\cos(c+dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 5.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.41

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^6} dx$$

$$= \frac{(8 + 48 \cos(c + dx) + 124 \cos^2(c + dx) + 184 \cos^3(c + dx) + 183 \cos^4(c + dx) + 146 \cos^5(c + dx)) \sin(c + dx)}{693a^6d(1 + \cos(c + dx))^6}$$

`[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^6,x]`

```
[Out] ((8 + 48*Cos[c + d*x] + 124*Cos[c + d*x]^2 + 184*Cos[c + d*x]^3 + 183*Cos[c + d*x]^4 + 146*Cos[c + d*x]^5)*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c + d*x])^6)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result
parallelrisc	$-\frac{\left(\tan^{10}\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{55(\tan^8(\frac{dx}{2} + \frac{c}{2}))}{9} + \frac{110(\tan^6(\frac{dx}{2} + \frac{c}{2}))}{7} - 22(\tan^4(\frac{dx}{2} + \frac{c}{2})) + \frac{55(\tan^2(\frac{dx}{2} + \frac{c}{2}))}{3} - 11\right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{352a^6d}$
derivativedivides	$-\frac{\frac{(\tan^{11}(\frac{dx}{2} + \frac{c}{2}))}{11} + \frac{5(\tan^9(\frac{dx}{2} + \frac{c}{2}))}{9} - \frac{10(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{7} + 2(\tan^5(\frac{dx}{2} + \frac{c}{2})) - \frac{5(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{3} + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{32da^6}$
default	$-\frac{(\tan^{11}(\frac{dx}{2} + \frac{c}{2}))}{11} + \frac{5(\tan^9(\frac{dx}{2} + \frac{c}{2}))}{9} - \frac{10(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{7} + 2(\tan^5(\frac{dx}{2} + \frac{c}{2})) - \frac{5(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{3} + \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{32da^6}$
risc	$\frac{2i(693e^{10i(dx+c)} + 3465e^{9i(dx+c)} + 11550e^{8i(dx+c)} + 23100e^{7i(dx+c)} + 33726e^{6i(dx+c)} + 33726e^{5i(dx+c)} + 25080e^{4i(dx+c)} + 15030e^{3i(dx+c)} + 4200e^{2i(dx+c)} + 630e^{i(dx+c)} + 1)}{693da^6(e^{i(dx+c)} + 1)^{11}}$

`[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^6,x,method=_RETURNVERBOSE)`

```
[Out] -1/352*(tan(1/2*d*x+1/2*c)^10-55/9*tan(1/2*d*x+1/2*c)^8+110/7*tan(1/2*d*x+1/2*c)^6-22*tan(1/2*d*x+1/2*c)^4+55/3*tan(1/2*d*x+1/2*c)^2-11)*tan(1/2*d*x+1/2*c)/a^6/d
```


Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.80

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^6} dx$$

$$= \frac{(146 \cos(dx + c)^5 + 183 \cos(dx + c)^4 + 184 \cos(dx + c)^3 + 124 \cos(dx + c)^2 + 48 \cos(dx + c) + 8) \sin(dx + c)}{693 (a^6 d \cos(dx + c)^6 + 6 a^6 d \cos(dx + c)^5 + 15 a^6 d \cos(dx + c)^4 + 20 a^6 d \cos(dx + c)^3 + 15 a^6 d \cos(dx + c)^2 + 6 a^6 d \cos(dx + c) + a^6 d)}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="fricas")

[Out] 1/693*(146*cos(d*x + c)^5 + 183*cos(d*x + c)^4 + 184*cos(d*x + c)^3 + 124*cos(d*x + c)^2 + 48*cos(d*x + c) + 8)*sin(d*x + c)/(a^6*d*cos(d*x + c)^6 + 6*a^6*d*cos(d*x + c)^5 + 15*a^6*d*cos(d*x + c)^4 + 20*a^6*d*cos(d*x + c)^3 + 15*a^6*d*cos(d*x + c)^2 + 6*a^6*d*cos(d*x + c) + a^6*d)

Sympy [A] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^6} dx$$

$$= \begin{cases} -\frac{\tan^{11}\left(\frac{c}{2} + \frac{dx}{2}\right)}{352a^6d} + \frac{5 \tan^9\left(\frac{c}{2} + \frac{dx}{2}\right)}{288a^6d} - \frac{5 \tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{112a^6d} + \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{16a^6d} - \frac{5 \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{96a^6d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{32a^6d} & \text{for } d \neq 0 \\ \frac{x \cos^5(c)}{(a \cos(c) + a)^6} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**6,x)

[Out] Piecewise((-tan(c/2 + d*x/2)**11/(352*a**6*d) + 5*tan(c/2 + d*x/2)**9/(288*a**6*d) - 5*tan(c/2 + d*x/2)**7/(112*a**6*d) + tan(c/2 + d*x/2)**5/(16*a**6*d) - 5*tan(c/2 + d*x/2)**3/(96*a**6*d) + tan(c/2 + d*x/2)/(32*a**6*d), Ne(d, 0)), (x*cos(c)**5/(a*cos(c) + a)**6, True))

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^6} dx$$

$$= \frac{\frac{693 \sin(dx+c)}{\cos(dx+c)+1} - \frac{1155 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1386 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{990 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{385 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{63 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{22176 a^6 d}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="maxima")

[Out] $\frac{1}{22176} * (693 * \sin(d*x + c) / (\cos(d*x + c) + 1) - 1155 * \sin(d*x + c)^3 / (\cos(d*x + c) + 1)^3 + 1386 * \sin(d*x + c)^5 / (\cos(d*x + c) + 1)^5 - 990 * \sin(d*x + c)^7 / (\cos(d*x + c) + 1)^7 + 385 * \sin(d*x + c)^9 / (\cos(d*x + c) + 1)^9 - 63 * \sin(d*x + c)^11 / (\cos(d*x + c) + 1)^11) / (a^6 * d)$

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^6} dx = \frac{63 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{11} - 385 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 + 990 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 1386 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 1155 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 63 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{22176 a^6 d}$$

[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="giac")

[Out] $-1/22176 * (63 * \tan(1/2 * d * x + 1/2 * c)^{11} - 385 * \tan(1/2 * d * x + 1/2 * c)^9 + 990 * \tan(1/2 * d * x + 1/2 * c)^7 - 1386 * \tan(1/2 * d * x + 1/2 * c)^5 + 1155 * \tan(1/2 * d * x + 1/2 * c)^3 - 693 * \tan(1/2 * d * x + 1/2 * c)) / (a^6 * d)$

Mupad [B] (verification not implemented)

Time = 15.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int \frac{\cos^5(c + dx)}{(a + a \cos(c + dx))^6} dx = \frac{\frac{495 \sin\left(\frac{3c}{2} + \frac{3dx}{2}\right)}{8} + \frac{495 \sin\left(\frac{5c}{2} + \frac{5dx}{2}\right)}{16} + \frac{275 \sin\left(\frac{7c}{2} + \frac{7dx}{2}\right)}{8} + \frac{55 \sin\left(\frac{9c}{2} + \frac{9dx}{2}\right)}{8} + \frac{73 \sin\left(\frac{11c}{2} + \frac{11dx}{2}\right)}{16}}{22176 a^6 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^{11}}$$

[In] int(cos(c + d*x)^5/(a + a*cos(c + d*x))^6,x)

[Out] $((495 * \sin((3 * c) / 2 + (3 * d * x) / 2)) / 8 + (495 * \sin((5 * c) / 2 + (5 * d * x) / 2)) / 16 + (275 * \sin((7 * c) / 2 + (7 * d * x) / 2)) / 8 + (55 * \sin((9 * c) / 2 + (9 * d * x) / 2)) / 8 + (73 * \sin((11 * c) / 2 + (11 * d * x) / 2)) / 16) / (22176 * a^6 * d * \cos(c / 2 + (d * x) / 2)^{11})$

3.94 $\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^6} dx$

Optimal result	775
Rubi [A] (verified)	775
Mathematica [A] (verified)	778
Maple [A] (verified)	778
Fricas [A] (verification not implemented)	779
Sympy [A] (verification not implemented)	779
Maxima [A] (verification not implemented)	779
Giac [A] (verification not implemented)	780
Mupad [B] (verification not implemented)	780

Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^6} dx = -\frac{241 \sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3} + \frac{61 \sin(c+dx)}{1155a^6d(1+\cos(c+dx))^2} + \frac{61 \sin(c+dx)}{1155a^6d(1+\cos(c+dx))} - \frac{\cos^3(c+dx) \sin(c+dx)}{11d(a+a \cos(c+dx))^6} - \frac{4 \cos^2(c+dx) \sin(c+dx)}{33ad(a+a \cos(c+dx))^5} + \frac{9 \sin(c+dx)}{77a^2d(a+a \cos(c+dx))^4}$$

[Out] -241/1155*sin(d*x+c)/a^6/d/(1+cos(d*x+c))^3+61/1155*sin(d*x+c)/a^6/d/(1+cos(d*x+c))^2+61/1155*sin(d*x+c)/a^6/d/(1+cos(d*x+c))-1/11*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^6-4/33*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^5+9/77*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^4

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2844, 3056, 3047, 3098, 2829, 2729, 2727}

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^6} dx = \frac{61 \sin(c+dx)}{1155a^6d(\cos(c+dx)+1)} + \frac{61 \sin(c+dx)}{1155a^6d(\cos(c+dx)+1)^2} - \frac{241 \sin(c+dx)}{1155a^6d(\cos(c+dx)+1)^3} + \frac{9 \sin(c+dx)}{77a^2d(a \cos(c+dx)+a)^4} - \frac{\sin(c+dx) \cos^3(c+dx)}{11d(a \cos(c+dx)+a)^6} - \frac{4 \sin(c+dx) \cos^2(c+dx)}{33ad(a \cos(c+dx)+a)^5}$$

[In] Int[Cos[c + d*x]^4/(a + a*cos[c + d*x])^6,x]

[Out] $(-241*\sin[c + d*x])/(1155*a^6*d*(1 + \cos[c + d*x])^3) + (61*\sin[c + d*x])/(1155*a^6*d*(1 + \cos[c + d*x])^2) + (61*\sin[c + d*x])/(1155*a^6*d*(1 + \cos[c + d*x])) - (\cos[c + d*x]^3*\sin[c + d*x])/(11*d*(a + a*\cos[c + d*x])^6) - (4*\cos[c + d*x]^2*\sin[c + d*x])/(33*a*d*(a + a*\cos[c + d*x])^5) + (9*\sin[c + d*x])/(77*a^2*d*(a + a*\cos[c + d*x])^4)$

Rule 2727

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := Simp[-Cos[c + d*x]/(d*(b + a*sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*cos[c + d*x]*((a + b*sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2829

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegerQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3047

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3056

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])

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Rule 3098

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Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{\int \frac{\cos^2(c+dx)(3a-9a\cos(c+dx))}{(a+a\cos(c+dx))^5} dx}{11a^2} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos(c+dx)(24a^2-57a^2\cos(c+dx))}{(a+a\cos(c+dx))^4} dx}{99a^4} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} - \frac{\int \frac{24a^2\cos(c+dx)-57a^2\cos^2(c+dx)}{(a+a\cos(c+dx))^4} dx}{99a^4} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} \\
&\quad + \frac{9\sin(c+dx)}{77a^2d(a+a\cos(c+dx))^4} + \frac{\int \frac{-324a^3+399a^3\cos(c+dx)}{(a+a\cos(c+dx))^3} dx}{693a^6} \\
&= -\frac{241\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3} - \frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} \\
&\quad - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} + \frac{9\sin(c+dx)}{77a^2d(a+a\cos(c+dx))^4} + \frac{61\int \frac{1}{(a+a\cos(c+dx))^2} dx}{385a^4} \\
&= -\frac{241\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3} - \frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} \\
&\quad + \frac{9\sin(c+dx)}{77a^2d(a+a\cos(c+dx))^4} + \frac{61\sin(c+dx)}{1155d(a^3+a^3\cos(c+dx))^2} + \frac{61\int \frac{1}{a+a\cos(c+dx)} dx}{1155a^5}
\end{aligned}$$

$$= -\frac{241 \sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3} - \frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5}$$

$$+ \frac{9\sin(c+dx)}{77a^2d(a+a\cos(c+dx))^4} + \frac{61\sin(c+dx)}{1155d(a^3+a^3\cos(c+dx))^2} + \frac{61\sin(c+dx)}{1155d(a^6+a^6\cos(c+dx))}$$

Mathematica [A] (verified)

Time = 5.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.43

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6} dx$$

$$= \frac{(16+96\cos(c+dx)+248\cos^2(c+dx)+368\cos^3(c+dx)+366\cos^4(c+dx)+61\cos^5(c+dx))\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^6}$$

[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^6,x]

[Out] ((16 + 96*Cos[c + d*x] + 248*Cos[c + d*x]^2 + 368*Cos[c + d*x]^3 + 366*Cos[c + d*x]^4 + 61*Cos[c + d*x]^5)*Sin[c + d*x])/(1155*a^6*d*(1 + Cos[c + d*x])^6)

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.48

method	result
derivativdivides	$\frac{(\tan^{11}(\frac{dx}{2} + \frac{c}{2}))}{11} - \frac{(\tan^9(\frac{dx}{2} + \frac{c}{2}))}{3} + \frac{2(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{7} + \frac{2(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{5} - (\tan^3(\frac{dx}{2} + \frac{c}{2})) + \tan(\frac{dx}{2} + \frac{c}{2})}{32da^6}$
default	$\frac{(\tan^{11}(\frac{dx}{2} + \frac{c}{2}))}{11} - \frac{(\tan^9(\frac{dx}{2} + \frac{c}{2}))}{3} + \frac{2(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{7} + \frac{2(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{5} - (\tan^3(\frac{dx}{2} + \frac{c}{2})) + \tan(\frac{dx}{2} + \frac{c}{2})}{32da^6}$
parallelrisc	$\frac{105(\tan^{11}(\frac{dx}{2} + \frac{c}{2})) - 385(\tan^9(\frac{dx}{2} + \frac{c}{2})) + 330(\tan^7(\frac{dx}{2} + \frac{c}{2})) + 462(\tan^5(\frac{dx}{2} + \frac{c}{2})) - 1155(\tan^3(\frac{dx}{2} + \frac{c}{2})) + 1155 \tan(\frac{dx}{2} + \frac{c}{2})}{36960a^6d}$
risc	$\frac{2i(1155e^{9i(dx+c)} + 3465e^{8i(dx+c)} + 9240e^{7i(dx+c)} + 12936e^{6i(dx+c)} + 15246e^{5i(dx+c)} + 10890e^{4i(dx+c)} + 6600e^{3i(dx+c)} + 2200e^{2i(dx+c)} + 220e^{i(dx+c)} + 1)}{1155da^6(e^{i(dx+c)} + 1)^{11}}$

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^6,x,method=_RETURNVERBOSE)

[Out] 1/32/d/a^6*(1/11*tan(1/2*d*x+1/2*c)^11-1/3*tan(1/2*d*x+1/2*c)^9+2/7*tan(1/2*d*x+1/2*c)^7+2/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^6} dx$$

$$= \frac{(61 \cos(dx + c)^5 + 366 \cos(dx + c)^4 + 368 \cos(dx + c)^3 + 248 \cos(dx + c)^2 + 96 \cos(dx + c) + 16) \sin(dx + c)}{1155 (a^6 d \cos(dx + c)^6 + 6 a^6 d \cos(dx + c)^5 + 15 a^6 d \cos(dx + c)^4 + 20 a^6 d \cos(dx + c)^3 + 15 a^6 d \cos(dx + c)^2 + 6 a^6 d \cos(dx + c) + a^6 d)}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="fricas")

[Out] 1/1155*(61*cos(d*x + c)^5 + 366*cos(d*x + c)^4 + 368*cos(d*x + c)^3 + 248*cos(d*x + c)^2 + 96*cos(d*x + c) + 16)*sin(d*x + c)/(a^6*d*cos(d*x + c)^6 + 6*a^6*d*cos(d*x + c)^5 + 15*a^6*d*cos(d*x + c)^4 + 20*a^6*d*cos(d*x + c)^3 + 15*a^6*d*cos(d*x + c)^2 + 6*a^6*d*cos(d*x + c) + a^6*d)

Sympy [A] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^6} dx$$

$$= \begin{cases} \frac{\tan^{11}\left(\frac{c}{2} + \frac{dx}{2}\right)}{352a^6d} - \frac{\tan^9\left(\frac{c}{2} + \frac{dx}{2}\right)}{96a^6d} + \frac{\tan^7\left(\frac{c}{2} + \frac{dx}{2}\right)}{112a^6d} + \frac{\tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{80a^6d} - \frac{\tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{32a^6d} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{32a^6d} & \text{for } d \neq 0 \\ \frac{x \cos^4(c)}{(a \cos(c) + a)^6} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**6,x)

[Out] Piecewise((tan(c/2 + d*x/2)**11/(352*a**6*d) - tan(c/2 + d*x/2)**9/(96*a**6*d) + tan(c/2 + d*x/2)**7/(112*a**6*d) + tan(c/2 + d*x/2)**5/(80*a**6*d) - tan(c/2 + d*x/2)**3/(32*a**6*d) + tan(c/2 + d*x/2)/(32*a**6*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**6, True))

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.72

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^6} dx$$

$$= \frac{1155 \sin(dx+c) - \frac{1155 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{462 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{330 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{385 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} + \frac{105 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{36960 a^6 d}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="maxima")

[Out] $\frac{1}{36960} * (1155 * \sin(d*x + c) / (\cos(d*x + c) + 1) - 1155 * \sin(d*x + c)^3 / (\cos(d*x + c) + 1)^3 + 462 * \sin(d*x + c)^5 / (\cos(d*x + c) + 1)^5 + 330 * \sin(d*x + c)^7 / (\cos(d*x + c) + 1)^7 - 385 * \sin(d*x + c)^9 / (\cos(d*x + c) + 1)^9 + 105 * \sin(d*x + c)^11 / (\cos(d*x + c) + 1)^11) / (a^6 * d)$

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.48

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^6} dx$$

$$= \frac{105 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{11} - 385 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 + 330 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 462 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 1155 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 1155}{36960 a^6 d}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="giac")

[Out] $\frac{1}{36960} * (105 * \tan(1/2*d*x + 1/2*c)^{11} - 385 * \tan(1/2*d*x + 1/2*c)^9 + 330 * \tan(1/2*d*x + 1/2*c)^7 + 462 * \tan(1/2*d*x + 1/2*c)^5 - 1155 * \tan(1/2*d*x + 1/2*c)^3 + 1155 * \tan(1/2*d*x + 1/2*c)) / (a^6 * d)$

Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.86

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^6} dx$$

$$= \frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) \left(1155 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 1155 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^8 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 462 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 1155 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 462 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 1155 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 1155}{36960 a^6 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)}$$

[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^6,x)

[Out] $(\sin(c/2 + (d*x)/2) * (1155 * \cos(c/2 + (d*x)/2)^{10} + 105 * \sin(c/2 + (d*x)/2)^{10} - 385 * \cos(c/2 + (d*x)/2)^2 * \sin(c/2 + (d*x)/2)^8 + 330 * \cos(c/2 + (d*x)/2)^4 * \sin(c/2 + (d*x)/2)^6 + 462 * \cos(c/2 + (d*x)/2)^6 * \sin(c/2 + (d*x)/2)^4 - 1155 * \cos(c/2 + (d*x)/2)^8 * \sin(c/2 + (d*x)/2)^2) / (36960 * a^6 * d * \cos(c/2 + (d*x)/2)^{11})$

3.95 $\int \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} dx$

Optimal result	781
Rubi [A] (verified)	781
Mathematica [A] (verified)	783
Maple [A] (verified)	784
Fricas [A] (verification not implemented)	784
Sympy [F(-1)]	784
Maxima [A] (verification not implemented)	785
Giac [A] (verification not implemented)	785
Mupad [F(-1)]	785

Optimal result

Integrand size = 23, antiderivative size = 158

$$\int \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{32a \sin(c + dx)}{45d \sqrt{a + a \cos(c + dx)}} + \frac{16a \cos^3(c + dx) \sin(c + dx)}{63d \sqrt{a + a \cos(c + dx)}} + \frac{2a \cos^4(c + dx) \sin(c + dx)}{9d \sqrt{a + a \cos(c + dx)}} - \frac{64 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{315d} + \frac{32(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105ad}$$

[Out] 32/105*(a+a*cos(d*x+c))^(3/2)*sin(d*x+c)/a/d+32/45*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/63*a*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/9*a*cos(d*x+c)^4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-64/315*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used

= {2849, 2838, 2830, 2725}

$$\int \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{2a \sin(c + dx) \cos^4(c + dx)}{9d \sqrt{a \cos(c + dx) + a}} + \frac{16a \sin(c + dx) \cos^3(c + dx)}{63d \sqrt{a \cos(c + dx) + a}} + \frac{32 \sin(c + dx) (a \cos(c + dx) + a)^{3/2}}{105ad} - \frac{64 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{315d} + \frac{32a \sin(c + dx)}{45d \sqrt{a \cos(c + dx) + a}}$$

[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]],x]

[Out] (32*a*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a*Cos[c + d*x]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Cos[c + d*x]^4*Sin[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (64*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (32*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*a*d)

Rule 2725

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2830

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2838

Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2849

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])

```

n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a \cos^4(c + dx) \sin(c + dx)}{9d \sqrt{a + a \cos(c + dx)}} + \frac{8}{9} \int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{16a \cos^3(c + dx) \sin(c + dx)}{63d \sqrt{a + a \cos(c + dx)}} + \frac{2a \cos^4(c + dx) \sin(c + dx)}{9d \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{16}{21} \int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{16a \cos^3(c + dx) \sin(c + dx)}{63d \sqrt{a + a \cos(c + dx)}} + \frac{2a \cos^4(c + dx) \sin(c + dx)}{9d \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{32(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105ad} \\
&\quad + \frac{32 \int \left(\frac{3a}{2} - a \cos(c + dx)\right) \sqrt{a + a \cos(c + dx)} dx}{105a} \\
&= \frac{16a \cos^3(c + dx) \sin(c + dx)}{63d \sqrt{a + a \cos(c + dx)}} + \frac{2a \cos^4(c + dx) \sin(c + dx)}{9d \sqrt{a + a \cos(c + dx)}} \\
&\quad - \frac{64 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{315d} \\
&\quad + \frac{32(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105ad} + \frac{16}{45} \int \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{32a \sin(c + dx)}{45d \sqrt{a + a \cos(c + dx)}} + \frac{16a \cos^3(c + dx) \sin(c + dx)}{63d \sqrt{a + a \cos(c + dx)}} + \frac{2a \cos^4(c + dx) \sin(c + dx)}{9d \sqrt{a + a \cos(c + dx)}} \\
&\quad - \frac{64 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{315d} + \frac{32(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.58

$$\begin{aligned}
&\int \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{\sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(1890 \sin\left(\frac{1}{2}(c + dx)\right) + 420 \sin\left(\frac{3}{2}(c + dx)\right) + 252 \sin\left(\frac{5}{2}(c + dx)\right)\right) +}{2520d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]],x]

[Out] $(\text{Sqrt}[a*(1 + \text{Cos}[c + d*x])]*\text{Sec}[(c + d*x)/2]*(1890*\text{Sin}[(c + d*x)/2] + 420*\text{Sin}[(3*(c + d*x))/2] + 252*\text{Sin}[(5*(c + d*x))/2] + 45*\text{Sin}[(7*(c + d*x))/2] + 35*\text{Sin}[(9*(c + d*x))/2]))/(2520*d)$

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(560 \left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 800 \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 552 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 104 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 107\right) \sqrt{2}}{315 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	97

[In] `int(cos(d*x+c)^4*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $2/315*\cos(1/2*d*x+1/2*c)*a*\sin(1/2*d*x+1/2*c)*(560*\cos(1/2*d*x+1/2*c)^8-800*\cos(1/2*d*x+1/2*c)^6+552*\cos(1/2*d*x+1/2*c)^4-104*\cos(1/2*d*x+1/2*c)^2+107)*2^(1/2)/(a*\cos(1/2*d*x+1/2*c)^2)^(1/2)/d$

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.46

$$\int \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{2 (35 \cos(dx + c)^4 + 40 \cos(dx + c)^3 + 48 \cos(dx + c)^2 + 64 \cos(dx + c) + 128) \sqrt{a \cos(dx + c) + a \sin(dx + c)}}{315 (d \cos(dx + c) + d)}$$

[In] `integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $2/315*(35*\cos(d*x + c)^4 + 40*\cos(d*x + c)^3 + 48*\cos(d*x + c)^2 + 64*\cos(d*x + c) + 128)*\text{sqrt}(a*\cos(d*x + c) + a)*\sin(d*x + c)/(d*\cos(d*x + c) + d)$

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**4*(a+a*cos(d*x+c))**(1/2),x)`

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.50

$$\int \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{(35 \sqrt{2} \sin(\frac{9}{2} dx + \frac{9}{2} c) + 45 \sqrt{2} \sin(\frac{7}{2} dx + \frac{7}{2} c) + 252 \sqrt{2} \sin(\frac{5}{2} dx + \frac{5}{2} c) + 420 \sqrt{2} \sin(\frac{3}{2} dx + \frac{3}{2} c) + 1890 \sqrt{2} \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{2520 d}$$

[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

```
[Out] 1/2520*(35*sqrt(2)*sin(9/2*d*x + 9/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/2*c) +
252*sqrt(2)*sin(5/2*d*x + 5/2*c) + 420*sqrt(2)*sin(3/2*d*x + 3/2*c) + 1890
*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.74

$$\int \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{\sqrt{2}(35 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{9}{2} dx + \frac{9}{2} c) + 45 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{7}{2} dx + \frac{7}{2} c) + 252 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{5}{2} dx + \frac{5}{2} c) + 420 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 1890 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{2520 d}$$

[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

```
[Out] 1/2520*sqrt(2)*(35*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 45*sgn(
cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 252*sgn(cos(1/2*d*x + 1/2*c))*
sin(5/2*d*x + 5/2*c) + 420*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) +
1890*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \cos(c + dx)^4 \sqrt{a + a \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2), x)

3.96 $\int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx$

Optimal result	786
Rubi [A] (verified)	786
Mathematica [A] (verified)	788
Maple [A] (verified)	788
Fricas [A] (verification not implemented)	789
Sympy [F(-1)]	789
Maxima [A] (verification not implemented)	789
Giac [A] (verification not implemented)	790
Mupad [F(-1)]	790

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{4a \sin(c + dx)}{5d \sqrt{a + a \cos(c + dx)}} + \frac{2a \cos^3(c + dx) \sin(c + dx)}{7d \sqrt{a + a \cos(c + dx)}} - \frac{8 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{35d} + \frac{12(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35ad}$$

[Out] $12/35*(a+a*\cos(d*x+c))^(3/2)*\sin(d*x+c)/a/d+4/5*a*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)+2/7*a*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)-8/35*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/d$

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2849, 2838, 2830, 2725}

$$\int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{2a \sin(c + dx) \cos^3(c + dx)}{7d \sqrt{a \cos(c + dx) + a}} + \frac{12 \sin(c + dx) (a \cos(c + dx) + a)^{3/2}}{35ad} - \frac{8 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{35d} + \frac{4a \sin(c + dx)}{5d \sqrt{a \cos(c + dx) + a}}$$

[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]],x]

[Out] (4*a*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) - (8*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(35*d) + (12*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*a*d)

Rule 2725

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2830

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m/(f*(m + 1)))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2838

Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(m_), x_Symbol] := Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2849

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2a \cos^3(c + dx) \sin(c + dx)}{7d\sqrt{a + a \cos(c + dx)}} + \frac{6}{7} \int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx \\ &= \frac{2a \cos^3(c + dx) \sin(c + dx)}{7d\sqrt{a + a \cos(c + dx)}} + \frac{12(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35ad} \\ &\quad + \frac{12 \int \left(\frac{3a}{2} - a \cos(c + dx)\right) \sqrt{a + a \cos(c + dx)} dx}{35a} \end{aligned}$$

$$\begin{aligned}
&= \frac{2a \cos^3(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{8\sqrt{a+a\cos(c+dx)} \sin(c+dx)}{35d} \\
&\quad + \frac{12(a+a\cos(c+dx))^{3/2} \sin(c+dx)}{35ad} + \frac{2}{5} \int \sqrt{a+a\cos(c+dx)} dx \\
&= \frac{4a \sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2a \cos^3(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{8\sqrt{a+a\cos(c+dx)} \sin(c+dx)}{35d} + \frac{12(a+a\cos(c+dx))^{3/2} \sin(c+dx)}{35ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\begin{aligned}
&\int \cos^3(c+dx) \sqrt{a+a\cos(c+dx)} dx \\
&= \frac{\sqrt{a(1+\cos(c+dx))} \sec\left(\frac{1}{2}(c+dx)\right) \left(105 \sin\left(\frac{1}{2}(c+dx)\right) + 35 \sin\left(\frac{3}{2}(c+dx)\right) + 7 \sin\left(\frac{5}{2}(c+dx)\right) + 5 \sin\left(\frac{7}{2}(c+dx)\right)\right)}{140d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(105*Sin[(c + d*x)/2] + 35*Sin[(3*(c + d*x))/2] + 7*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2]))/(140*d)

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(40 \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 36 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 22 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 9\right) \sqrt{2}}{35 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	84

[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/35*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(40*cos(1/2*d*x+1/2*c)^6-36*cos(1/2*d*x+1/2*c)^4+22*cos(1/2*d*x+1/2*c)^2+9)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

$$\int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{2(5 \cos(dx + c)^3 + 6 \cos(dx + c)^2 + 8 \cos(dx + c) + 16) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{35(d \cos(dx + c) + d)}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 2/35*(5*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.53

$$\int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{(5\sqrt{2} \sin(\frac{7}{2} dx + \frac{7}{2} c) + 7\sqrt{2} \sin(\frac{5}{2} dx + \frac{5}{2} c) + 35\sqrt{2} \sin(\frac{3}{2} dx + \frac{3}{2} c) + 105\sqrt{2} \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{140 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/140*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c) + 35*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{\sqrt{2} \left(5 \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{7}{2} dx + \frac{7}{2} c \right) + 7 \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{5}{2} dx + \frac{5}{2} c \right) + 35 \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{3}{2} dx + \frac{3}{2} c \right) + 105 \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sqrt{a}}{140 d}$$

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/140*sqrt(2)*(5*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 7*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 35*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 105*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \cos(c + dx)^3 \sqrt{a + a \cos(c + dx)} dx$$

```
[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2), x)
```

3.97 $\int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx$

Optimal result	791
Rubi [A] (verified)	791
Mathematica [A] (verified)	793
Maple [A] (verified)	793
Fricas [A] (verification not implemented)	793
Sympy [F]	794
Maxima [A] (verification not implemented)	794
Giac [A] (verification not implemented)	794
Mupad [F(-1)]	795

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{14a \sin(c + dx)}{15d \sqrt{a + a \cos(c + dx)}} - \frac{4 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5ad}$$

[Out] $2/5*(a+a*\cos(d*x+c))^{(3/2)}*\sin(d*x+c)/a/d+14/15*a*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}-4/15*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2838, 2830, 2725}

$$\int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{2 \sin(c + dx)(a \cos(c + dx) + a)^{3/2}}{5ad} - \frac{4 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{15d} + \frac{14a \sin(c + dx)}{15d \sqrt{a \cos(c + dx) + a}}$$

[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]],x]

[Out] $(14*a*\sin[c + d*x])/(15*d*\sqrt{a + a*\cos[c + d*x]}) - (4*\sqrt{a + a*\cos[c + d*x]}*\sin[c + d*x])/(15*d) + (2*(a + a*\cos[c + d*x])^{(3/2)}*\sin[c + d*x])/(5*a*d)$

Rule 2725

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_),
x_Symbol] := Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !L
tQ[m, -2^(-1)]
```

Rubi steps

integral

$$\begin{aligned}
&= \frac{2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5ad} + \frac{2 \int \left(\frac{3a}{2} - a \cos(c + dx)\right) \sqrt{a + a \cos(c + dx)} dx}{5a} \\
&= -\frac{4\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15d} \\
&\quad + \frac{2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5ad} + \frac{7}{15} \int \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{14a \sin(c + dx)}{15d\sqrt{a + a \cos(c + dx)}} - \frac{4\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15d} \\
&\quad + \frac{2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.79

$$\int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{\sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(30 \sin\left(\frac{1}{2}(c + dx)\right) + 5 \sin\left(\frac{3}{2}(c + dx)\right) + 3 \sin\left(\frac{5}{2}(c + dx)\right)\right)}{30d}$$

[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(30*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(30*d)

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(12 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 4 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 7\right) \sqrt{2}}{15 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	71

[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*cos(1/2*d*x+1/2*c)^4-4*cos(1/2*d*x+1/2*c)^2+7)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{2 \sqrt{a \cos(dx + c) + a} (3 \cos^2(dx + c) + 4 \cos(dx + c) + 8) \sin(dx + c)}{15 (d \cos(dx + c) + d)}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 2/15*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c)^2 + 4*cos(d*x + c) + 8)*sin(d*x + c)/(d*cos(d*x + c) + d)

Sympy [F]

$$\int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \sqrt{a(\cos(c + dx) + 1)} \cos^2(c + dx) dx$$

[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{(3\sqrt{2} \sin(\frac{5}{2} dx + \frac{5}{2} c) + 5\sqrt{2} \sin(\frac{3}{2} dx + \frac{3}{2} c) + 30\sqrt{2} \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{30 d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/30*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c) + 30*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{\sqrt{2}(3 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{5}{2} dx + \frac{5}{2} c) + 5 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 30 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{30 d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/30*sqrt(2)*(3*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 5*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 30*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \cos(c + dx)^2 \sqrt{a + a \cos(c + dx)} dx$$

```
[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2), x)
```

3.98 $\int \cos(c + dx) \sqrt{a + a \cos(c + dx)} dx$

Optimal result	796
Rubi [A] (verified)	796
Mathematica [A] (verified)	797
Maple [A] (verified)	797
Fricas [A] (verification not implemented)	798
Sympy [F]	798
Maxima [A] (verification not implemented)	798
Giac [A] (verification not implemented)	799
Mupad [F(-1)]	799

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \cos(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{2a \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} + \frac{2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d}$$

[Out] $2/3*a*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/3*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2830, 2725}

$$\int \cos(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{2 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{3d} + \frac{2a \sin(c + dx)}{3d \sqrt{a \cos(c + dx) + a}}$$

[In] `Int[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]],x]`

[Out] $(2*a*\sin[c + d*x])/(3*d*\sqrt{a + a*\cos[c + d*x]}) + (2*\sqrt{a + a*\cos[c + d*x]}*\sin[c + d*x])/(3*d)$

Rule 2725

`Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] -> Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq`

$Q[a^2 - b^2, 0]$

Rule 2830

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3} \int \sqrt{a + a \cos(c + dx)} dx \\ &= \frac{2a \sin(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} + \frac{2\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\begin{aligned} &\int \cos(c + dx) \sqrt{a + a \cos(c + dx)} dx \\ &= \frac{\sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(3 \sin\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{3}{2}(c + dx)\right)\right)}{3d} \end{aligned}$$

[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*d)

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(1 + 2 \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{2}}{3 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	58

[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/3*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(1+2*cos(1/2*d*x+1/2*c)^2)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

$$\int \cos(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{2 \sqrt{a \cos(dx + c) + a} (\cos(dx + c) + 2) \sin(dx + c)}{3 (d \cos(dx + c) + d)}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) + 2)*sin(d*x + c)/(d*cos(d*x + c) + d)

Sympy [F]

$$\int \cos(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \sqrt{a (\cos(c + dx) + 1)} \cos(c + dx) dx$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \cos(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{(\sqrt{2} \sin(\frac{3}{2} dx + \frac{3}{2} c) + 3 \sqrt{2} \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{3 d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/3*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \cos(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{\sqrt{2} \left(\operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{3}{2} dx + \frac{3}{2} c \right) + 3 \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sqrt{a}}{3d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/3*sqrt(2)*(sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 3*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \cos(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2), x)

3.99 $\int \sqrt{a + a \cos(c + dx)} dx$

Optimal result	800
Rubi [A] (verified)	800
Mathematica [A] (verified)	801
Maple [A] (verified)	801
Fricas [A] (verification not implemented)	801
Sympy [F]	802
Maxima [A] (verification not implemented)	802
Giac [A] (verification not implemented)	802
Mupad [B] (verification not implemented)	802

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \sqrt{a + a \cos(c + dx)} dx = \frac{2a \sin(c + dx)}{d\sqrt{a + a \cos(c + dx)}}$$

[Out] $2*a*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2725}

$$\int \sqrt{a + a \cos(c + dx)} dx = \frac{2a \sin(c + dx)}{d\sqrt{a \cos(c + dx) + a}}$$

[In] `Int[Sqrt[a + a*Cos[c + d*x]],x]`

[Out] `(2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])`

Rule 2725

`Int[Sqrt[(a_) + (b_.)*sin[(c_) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rubi steps

$$\text{integral} = \frac{2a \sin(c + dx)}{d\sqrt{a + a \cos(c + dx)}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sqrt{a + a \cos(c + dx)} dx = \frac{2\sqrt{a(1 + \cos(c + dx))} \tan\left(\frac{1}{2}(c + dx)\right)}{d}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]],x]

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/d

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{2a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2}}{\sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	43
risch	$-\frac{i\sqrt{2} \sqrt{a(e^{i(dx+c)}+1)^2 e^{-i(dx+c)} (e^{i(dx+c)}-1)}}{(e^{i(dx+c)}+1)d}$	60

[In] int((a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \sqrt{a + a \cos(c + dx)} dx = \frac{2\sqrt{a \cos(dx + c) + a \sin(dx + c)}}{d \cos(dx + c) + d}$$

[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)

Sympy [F]

$$\int \sqrt{a + a \cos(c + dx)} dx = \int \sqrt{a \cos(c + dx) + a} dx$$

[In] integrate((a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a*cos(c + d*x) + a), x)

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sqrt{a + a \cos(c + dx)} dx = \frac{2\sqrt{2}\sqrt{a} \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{d}$$

[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \sqrt{a + a \cos(c + dx)} dx = \frac{2\sqrt{2}\sqrt{a} \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{d}$$

[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 2*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d

Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \sqrt{a + a \cos(c + dx)} dx = \frac{2 \sin(c + dx) \sqrt{a (\cos(c + dx) + 1)}}{d (\cos(c + dx) + 1)}$$

[In] int((a + a*cos(c + d*x))^(1/2),x)

[Out] (2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*(cos(c + d*x) + 1))

3.100 $\int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx$

Optimal result	803
Rubi [A] (verified)	803
Mathematica [A] (verified)	804
Maple [B] (verified)	804
Fricas [A] (verification not implemented)	805
Sympy [F]	805
Maxima [F]	805
Giac [A] (verification not implemented)	806
Mupad [F(-1)]	806

Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx = \frac{2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{d}$$

[Out] $2*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*a^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2852, 212}

$$\int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx = \frac{2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]]*\operatorname{Sec}[c + d*x], x]$

[Out] $(2*\operatorname{Sqrt}[a]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])])/d$

Rule 212

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

Rule 2852

$\operatorname{Int}[\operatorname{Sqrt}[(a_ + (b_)*\sin[(e_.) + (f_)*(x_)])]/((c_.) + (d_)*\sin[(e_.) + (f_)*(x_)]), x_Symbol] \rightarrow \operatorname{Dist}[-2*(b/f), \operatorname{Subst}[\operatorname{Int}[1/(b*c + a*d - d*x^2), x$

], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \\ &= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\begin{aligned} &\int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx \\ &= \frac{\sqrt{2}\text{arctanh}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right) \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right)}{d} \end{aligned}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x],x]

[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2])/d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(31) = 62.

Time = 1.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.92

method	result
default	$\frac{\sqrt{a} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\ln\left(\frac{4\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a+8a}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}}\right) + \ln\left(-\frac{4\left(\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2}}\right) \right)}{\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$

[In] int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c),x,method=_RETURNVERBOSE)

[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.95

$$\int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx$$

$$= \left[\frac{\sqrt{a} \log \left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - 4\sqrt{a} \cos(dx+c) + a\sqrt{a}(\cos(dx+c)-2) \sin(dx+c) + 8a}{\cos(dx+c)^3 + \cos(dx+c)^2} \right)}{2d}, \frac{\sqrt{-a} \arctan \left(\frac{2\sqrt{a} \cos(dx+c) + a\sqrt{-a}}{a \cos(dx+c)^2 - a \cos(dx+c)} \right)}{d} \right]$$

[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="fricas")

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a)*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqrt(-a)*arctan(2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

Sympy [F]

$$\int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx = \int \sqrt{a (\cos(c + dx) + 1)} \sec(c + dx) dx$$

[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c),x)

[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x), x)

Maxima [F]

$$\int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx = \int \sqrt{a \cos(dx + c) + a} \sec(dx + c) dx$$

[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="maxima")

[Out] integrate(sqrt(a*cos(d*x + c) + a)*sec(d*x + c), x)

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx = -\frac{\sqrt{a} \log\left(\frac{|-2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}\right) \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{d}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] -sqrt(a)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx = \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos(c + dx)} dx$$

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.101 $\int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx$

Optimal result	807
Rubi [A] (verified)	807
Mathematica [A] (verified)	808
Maple [B] (verified)	809
Fricas [B] (verification not implemented)	809
Sympy [F]	810
Maxima [B] (verification not implemented)	810
Giac [A] (verification not implemented)	811
Mupad [F(-1)]	811

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx = \frac{\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{d} + \frac{a \tan(c + dx)}{d \sqrt{a + a \cos(c + dx)}}$$

[Out] $\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*a^{(1/2)}/d+a*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2851, 2852, 212}

$$\int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx = \frac{\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{d} + \frac{a \tan(c + dx)}{d \sqrt{a \cos(c + dx) + a}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]]*\operatorname{Sec}[c + d*x]^2, x]$

[Out] $(\operatorname{Sqrt}[a]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])])/d + (a*\operatorname{Tan}[c + d*x])/(\operatorname{d}*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_) + (
f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]))], x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/((c_) + (d_.)*sin[(e_) + (
f_.)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{a \tan(c + dx)}{d\sqrt{a + a \cos(c + dx)}} + \frac{1}{2} \int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx \\ &= \frac{a \tan(c + dx)}{d\sqrt{a + a \cos(c + dx)}} - \frac{a \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \\ &= \frac{\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} + \frac{a \tan(c + dx)}{d\sqrt{a + a \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\begin{aligned} &\int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx \\ &= \frac{\sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \sec(c + dx) \left(\sqrt{2} \operatorname{arctanh}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) \cos(c + dx) + 2 \sin\left(\frac{1}{2}(c + dx)\right)}{2d} \end{aligned}$$

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*ArcTanh[
Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(54) = 108.

Time = 1.22 (sec) , antiderivative size = 383, normalized size of antiderivative = 6.18

method	result
default	$\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-2a \left(\ln\left(\frac{4\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a+8a}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}} \right) + \ln\left(-\frac{4\left(\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2}} \right) \right)$

[In] `int((a+cos(d*x+c))*a^(1/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)`

[Out] $\cos(1/2*d*x+1/2*c)*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*a*(\ln(4/(2*\cos(1/2*d*x+1/2*c)+2^{(1/2)}))*(2^{(1/2)}*a*\cos(1/2*d*x+1/2*c)+2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)+2*a})+\ln(-4/(2*\cos(1/2*d*x+1/2*c)-2^{(1/2)}))*(2^{(1/2)}*a*\cos(1/2*d*x+1/2*c)-2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)-2*a}))*\sin(1/2*d*x+1/2*c)^2+\ln(4/(2*\cos(1/2*d*x+1/2*c)+2^{(1/2)}))*(2^{(1/2)}*a*\cos(1/2*d*x+1/2*c)+2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)+2*a})*a+\ln(-4/(2*\cos(1/2*d*x+1/2*c)-2^{(1/2)}))*(2^{(1/2)}*a*\cos(1/2*d*x+1/2*c)-2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)-2*a})*a+2*2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)})/a^{(1/2)}/(2*\cos(1/2*d*x+1/2*c)+2^{(1/2)})/(2*\cos(1/2*d*x+1/2*c)-2^{(1/2)})/\sin(1/2*d*x+1/2*c)/(a*\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/d$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(54) = 108.

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.26

$$\int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx$$

$$= \frac{(\cos(dx + c)^2 + \cos(dx + c)) \sqrt{a} \log\left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - 4\sqrt{a \cos(dx+c) + a} \sqrt{a} (\cos(dx+c) - 2) \sin(dx+c) + 8a}{\cos(dx+c)^3 + \cos(dx+c)^2}\right) + 4}{4(d \cos(dx + c)^2 + d \cos(dx + c))}$$

[In] `integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="fricas")`

[Out] $1/4*((\cos(d*x + c)^2 + \cos(d*x + c))*\sqrt{a}*\log((a*\cos(d*x + c)^3 - 7*a*\cos(d*x + c)^2 - 4*\sqrt{a*\cos(d*x + c) + a}*\sqrt{a}*(\cos(d*x + c) - 2)*\sin(d*x + c) + 8*a)/(\cos(d*x + c)^3 + \cos(d*x + c)^2)) + 4*\sqrt{a*\cos(d*x + c) + a}*\sin(d*x + c))/(d*\cos(d*x + c)^2 + d*\cos(d*x + c))$

Sympy [F]

$$\int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx = \int \sqrt{a (\cos(c + dx) + 1)} \sec^2(c + dx) dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. 2(54) = 108.

Time = 0.40 (sec) , antiderivative size = 1170, normalized size of antiderivative = 18.87

$$\int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((4*sqrt(2)*sin(1/2*d*x + 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (4*sqrt(2)*sin(1/2*d*x + 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 2*(2*sqrt(2)*sin(3/2*d*x + 3/2*c) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) - 4*(sqrt(2)*cos(2*d*x + 2*c
```

) + sqrt(2))*sin(5/2*d*x + 5/2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*sin(1/2*d*x + 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.68

$$\int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx = \frac{\sqrt{2} \left(\sqrt{2} \log \left(\frac{-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) + \frac{4 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)}{2 \sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1} \right) \sqrt{a}}{4d}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="giac")

[Out] -1/4*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/(2*sin(1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx = \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos(c + dx)^2} dx$$

[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)

[Out] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)

3.102 $\int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx$

Optimal result	812
Rubi [A] (verified)	812
Mathematica [A] (verified)	814
Maple [B] (verified)	814
Fricas [A] (verification not implemented)	815
Sympy [F]	815
Maxima [B] (verification not implemented)	815
Giac [A] (verification not implemented)	817
Mupad [F(-1)]	818

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx = \frac{3\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4d} + \frac{3a \tan(c + dx)}{4d \sqrt{a + a \cos(c + dx)}} + \frac{a \sec(c + dx) \tan(c + dx)}{2d \sqrt{a + a \cos(c + dx)}}$$

[Out] $3/4 * \operatorname{arctanh}(\sin(d*x+c) * a^{1/2} / (a+a*\cos(d*x+c))^{1/2}) * a^{1/2} / d + 3/4 * a * \tan(d*x+c) / d / (a+a*\cos(d*x+c))^{1/2} + 1/2 * a * \sec(d*x+c) * \tan(d*x+c) / d / (a+a*\cos(d*x+c))^{1/2}$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2851, 2852, 212}

$$\int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx = \frac{3\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4d} + \frac{3a \tan(c + dx)}{4d \sqrt{a \cos(c + dx) + a}} + \frac{a \tan(c + dx) \sec(c + dx)}{2d \sqrt{a \cos(c + dx) + a}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]] * \operatorname{Sec}[c + d*x]^3, x]$

[Out] $(3\sqrt{a}\operatorname{ArcTanh}[\sqrt{a}\sin[c+dx]]/\sqrt{a+a\cos[c+dx]])/(4d) + (3a\tan[c+dx])/(4d\sqrt{a+a\cos[c+dx]}) + (a\sec[c+dx]\tan[c+dx])/(2d\sqrt{a+a\cos[c+dx]})$

Rule 212

$\operatorname{Int}[(a_1 + (b_1)(x_1)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]\operatorname{Rt}[-b, 2]))\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2](x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{LtQ}[b, 0])$

Rule 2851

$\operatorname{Int}[\sqrt{(a_1 + (b_1)\sin[e_1] + (f_1)(x_1))}((c_1) + (d_1)\sin[e_1] + (f_1)(x_1))]^{(n_1)}, x_Symbol] \rightarrow \operatorname{Simp}[(b_1c_1 - a_1d_1)\cos[e_1 + f_1x]((c_1 + d_1\sin[e_1 + f_1x])^{(n_1 + 1)})/(f_1(n_1 + 1)(c_1^2 - d_1^2)\sqrt{a_1 + b_1\sin[e_1 + f_1x]}), x] + \operatorname{Dist}[(2n_1 + 3)((b_1c_1 - a_1d_1)/(2b_1(n_1 + 1)(c_1^2 - d_1^2))), \operatorname{Int}[\sqrt{a_1 + b_1\sin[e_1 + f_1x]}(c_1 + d_1\sin[e_1 + f_1x])^{(n_1 + 1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \&\& \operatorname{NeQ}[b_1c_1 - a_1d_1, 0] \&\& \operatorname{EqQ}[a_1^2 - b_1^2, 0] \&\& \operatorname{NeQ}[c_1^2 - d_1^2, 0] \&\& \operatorname{LtQ}[n_1, -1] \&\& \operatorname{NeQ}[2n_1 + 3, 0] \&\& \operatorname{IntegerQ}[2n_1]$

Rule 2852

$\operatorname{Int}[\sqrt{(a_1 + (b_1)\sin[e_1] + (f_1)(x_1))}/((c_1) + (d_1)\sin[e_1] + (f_1)(x_1)), x_Symbol] \rightarrow \operatorname{Dist}[-2(b_1/f_1), \operatorname{Subst}[\operatorname{Int}[1/(b_1c_1 + a_1d_1 - d_1x^2), x], x, b_1(\cos[e_1 + f_1x]/\sqrt{a_1 + b_1\sin[e_1 + f_1x]})], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \&\& \operatorname{NeQ}[b_1c_1 - a_1d_1, 0] \&\& \operatorname{EqQ}[a_1^2 - b_1^2, 0] \&\& \operatorname{NeQ}[c_1^2 - d_1^2, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{a \sec(c+dx) \tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{3}{4} \int \sqrt{a+a\cos(c+dx)} \sec^2(c+dx) dx \\ &= \frac{3a \tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a \sec(c+dx) \tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{3}{8} \int \sqrt{a+a\cos(c+dx)} \sec(c+dx) dx \\ &= \frac{3a \tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a \sec(c+dx) \tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} \\ &\quad - \frac{(3a) \operatorname{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} \\ &= \frac{3\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{3a \tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a \sec(c+dx) \tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx$$

$$= \frac{\sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \sec^2(c + dx) \left(3\sqrt{2} \operatorname{arctanh}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) \cos^2(c + dx) + \sin\left(\frac{1}{2}(c + dx)\right)}{8d}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3,x]

[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(3*Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + Sin[(c + d*x)/2] + 3*Sin[(3*(c + d*x))/2]))/(8*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(86) = 172.

Time = 1.56 (sec) , antiderivative size = 551, normalized size of antiderivative = 5.40

method	result
default	$\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(12a \left(\ln\left(\frac{4\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a+8a}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}}\right) + \ln\left(-\frac{4\left(\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2}}\right) \right)$

[In] int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*a*(ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^4+(-12*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-12*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a-12*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+10*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)/a^(1/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.52

$$\int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx$$

$$= \frac{3 (\cos(dx + c)^3 + \cos(dx + c)^2) \sqrt{a} \log\left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - 4\sqrt{a} \cos(dx+c) + a\sqrt{a}(\cos(dx+c)-2) \sin(dx+c) + 8a}{\cos(dx+c)^3 + \cos(dx+c)^2}\right)}{16 (d \cos(dx + c))^3 + d \cos(dx + c)^2}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="fricas")

```
[Out] 1/16*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*s
in(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x +
c) + a)*(3*cos(d*x + c) + 2)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x +
c)^2)
```

Sympy [F]

$$\int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx = \int \sqrt{a (\cos(c + dx) + 1)} \sec^3(c + dx) dx$$

[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**3,x)

[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**3, x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2642 vs. 2(86) = 172.

Time = 4.16 (sec) , antiderivative size = 2642, normalized size of antiderivative = 25.90

$$\int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="maxima")

```
[Out] 1/16*(3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
```

$$\begin{aligned}
& 2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2)) * c \\
& \cos(4*d*x + 4*c)^2 + 12*(\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2)) * \cos(2*d*x + 2*c)^2 + 3*(\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2)) * \sin(4*d*x + 4*c)^2 + 12*(\log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2)) * \sin(2*d*x + 2*c)^2 - 24*\sqrt{2} * \\
& \cos(7/2*d*x + 7/2*c) * \sin(2*d*x + 2*c) - 8*\sqrt{2} * \cos(5/2*d*x + 5/2*c) * \sin(\\
& 2*d*x + 2*c) + 2*(6*(\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c) + 2)) * \cos(2*d*x + 2*c) + 6*\sqrt{2} * \sin(7/2*d*x + 7/2*c) + 2*\sqrt{2} * \sin \\
& (5/2*d*x + 5/2*c) - 2*\sqrt{2} * \sin(3/2*d*x + 3/2*c) - 6*\sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c) + 3*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(2 \\
& *\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*\log(2*\cos(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c) + 2)) * \cos(4*d*x + 4*c) - 4*(2*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 6*\sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c) - 3*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2 \\
&) + 3*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} * c \\
& \cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(2*\cos(1/2
\end{aligned}$$

```

*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d*
x + 7/2*c) - sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) +
3*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(5/2*d*x + 5/2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*c
os(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) - 12
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
2*c) + 1)*d)

```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.28

$$\int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx =$$

$$\frac{\sqrt{2} \left(3 \sqrt{2} \log \left(\frac{-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)} \right) \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) + \frac{4 \left(6 \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 - 5 \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{\left(2 \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)^2} \right)}{16 d}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="giac")

[Out] -1/16*sqrt(2)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(6*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 5*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx = \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos(c + dx)^3} dx$$

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.103 $\int \sqrt{a + a \cos(c + dx)} \sec^4(c + dx) dx$

Optimal result	819
Rubi [A] (verified)	819
Mathematica [A] (verified)	821
Maple [B] (verified)	822
Fricas [A] (verification not implemented)	822
Sympy [F]	823
Maxima [B] (verification not implemented)	823
Giac [A] (verification not implemented)	827
Mupad [F(-1)]	827

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \sqrt{a + a \cos(c + dx)} \sec^4(c + dx) dx = \frac{5\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{8d} + \frac{5a \tan(c + dx)}{8d \sqrt{a + a \cos(c + dx)}} + \frac{5a \sec(c + dx) \tan(c + dx)}{12d \sqrt{a + a \cos(c + dx)}} + \frac{a \sec^2(c + dx) \tan(c + dx)}{3d \sqrt{a + a \cos(c + dx)}}$$

[Out] $5/8*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*a^{(1/2)}/d+5/8*a*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+5/12*a*\sec(d*x+c)*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+1/3*a*\sec(d*x+c)^2*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used

= {2851, 2852, 212}

$$\int \sqrt{a + a \cos(c + dx)} \sec^4(c + dx) dx = \frac{5\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{8d} + \frac{5a \tan(c + dx)}{8d\sqrt{a \cos(c + dx) + a}} + \frac{a \tan(c + dx) \sec^2(c + dx)}{3d\sqrt{a \cos(c + dx) + a}} + \frac{5a \tan(c + dx) \sec(c + dx)}{12d\sqrt{a \cos(c + dx) + a}}$$

[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^4,x]

[Out] (5*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/(8*d) + (5*a*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (5*a*Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2851

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]))], x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rule 2852

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a \sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{6} \int \sqrt{a+a\cos(c+dx)} \sec^3(c+dx) dx \\
 &= \frac{5a \sec(c+dx) \tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a \sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{5}{8} \int \sqrt{a+a\cos(c+dx)} \sec^2(c+dx) dx \\
 &= \frac{5a \tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a \sec(c+dx) \tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{a \sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{16} \int \sqrt{a+a\cos(c+dx)} \sec(c+dx) dx \\
 &= \frac{5a \tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a \sec(c+dx) \tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{a \sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{(5a) \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} \\
 &= \frac{5\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{5a \tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{5a \sec(c+dx) \tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a \sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.79

$$\begin{aligned}
 &\int \sqrt{a+a\cos(c+dx)} \sec^4(c+dx) dx \\
 &= \frac{\sqrt{a(1+\cos(c+dx))} \sec\left(\frac{1}{2}(c+dx)\right) \sec^3(c+dx) (30\sqrt{2} \operatorname{arctanh}(\sqrt{2} \sin\left(\frac{1}{2}(c+dx)\right))) \cos^3(c+dx) + 42 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{96d}
 \end{aligned}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^4,x]

[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(30*Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 42*Sin[(c + d*x)/2] + 5*(Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))) / (96*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. 2(118) = 236.

Time = 1.78 (sec) , antiderivative size = 717, normalized size of antiderivative = 5.20

method	result
default	$\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-120a \left(\ln\left(\frac{4\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a+8a}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}} \right) + \ln\left(-\frac{4 \left(\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2}} \right) \right)$

[In] int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-120*a*(ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a)))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*sin(1/2*d*x+1/2*c)^6+60*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-90*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a-90*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a-160*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+15*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+15*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+66*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.20

$$\int \sqrt{a + a \cos(c + dx)} \sec^4(c + dx) dx$$

$$= \frac{15 (\cos(dx + c)^4 + \cos(dx + c)^3) \sqrt{a} \log\left(\frac{a \cos(dx + c)^3 - 7a \cos(dx + c)^2 - 4\sqrt{a \cos(dx + c) + a} \sqrt{a} (\cos(dx + c) - 2) \sin(dx + c) + 8a}{\cos(dx + c)^3 + \cos(dx + c)^2}\right)}{96 (d \cos(dx + c))^4 + d \cos(dx + c)}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="fricas")

```
[Out] 1/96*(15*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*
sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x +
c) + a)*(15*cos(d*x + c)^2 + 10*cos(d*x + c) + 8)*sin(d*x + c))/(d*cos(d*x
+ c)^4 + d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sqrt{a + a \cos(c + dx)} \sec^4(c + dx) dx = \int \sqrt{a (\cos(c + dx) + 1)} \sec^4(c + dx) dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**4,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5115 vs. 2(118) = 236.

Time = 45.83 (sec) , antiderivative size = 5115, normalized size of antiderivative = 37.07

$$\int \sqrt{a + a \cos(c + dx)} \sec^4(c + dx) dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(15*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt
(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
r(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(6*d*x + 6*c)^2
+ 135*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(
2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
r(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4*c)^2 +
135*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)
```

```

*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c)^2 + 1
5*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*sin(6*d*x + 6*c)^2 + 135*(
sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(
1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c)^2 + 135*(s
qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c)^2 - 120*(sin
(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*cos(13/2*d*x + 13/
2*c) + 2*(45*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 8*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4*
c) + 45*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt
(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/

```

$$\begin{aligned}
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) - 8*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + \\
& 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2}(2) \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 15*\sqrt{2}*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*s \\
& qrt(2)*\sin(1/2*d*x + 1/2*c) + 2) + 60*\sin(11/2*d*x + 11/2*c) + 200*\sin(9/2* \\
& d*x + 9/2*c) + 168*\sin(7/2*d*x + 7/2*c) + 12*\sin(5/2*d*x + 5/2*c) - 20*\sin(\\
& 3/2*d*x + 3/2*c) - 120*\sin(1/2*d*x + 1/2*c))*\cos(6*d*x + 6*c) - 360*(\sin(4* \\
& d*x + 4*c) + \sin(2*d*x + 2*c))*\cos(11/2*d*x + 11/2*c) - 1200*(\sin(4*d*x + 4 \\
& *c) + \sin(2*d*x + 2*c))*\cos(9/2*d*x + 9/2*c) + 6*(45*(\sqrt{2})*\log(2*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}* \\
& \sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - 8*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 15*\sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*s \\
& qrt(2)*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) + 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) + 168*\sin(7/2*d*x + 7/2*c) + 12*\sin(5/2*d*x + 5/2*c) - 20*\sin(3/2*d*x + \\
& 3/2*c) - 120*\sin(1/2*d*x + 1/2*c))*\cos(4*d*x + 4*c) + 30*(3*\sqrt{2})*\log(2*c \\
& os(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2* \\
& sqrt(2)*\sin(1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - 4*\sin(3/2*d*x + 3/2*c) - 24*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + \\
& 2*c) + 120*(\cos(6*d*x + 6*c) + 3*\cos(4*d*x + 4*c) + 3*\cos(2*d*x + 2*c) + 1 \\
&)*\sin(13/2*d*x + 13/2*c) + 2*(45*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2
\end{aligned}$$

$$\begin{aligned}
& * \sqrt{2} \cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
& * \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 8*\sin(1/2*d*x + 1/ \\
& 2*c)) * \sin(4*d*x + 4*c) + 45*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 8*\sin(1/2*d*x + 1/2*c)) \\
& * \sin(2*d*x + 2*c) - 60*\cos(11/2*d*x + 11/2*c) - 200*\cos(9/2*d*x + 9/2*c) - \\
& 168*\cos(7/2*d*x + 7/2*c) - 12*\cos(5/2*d*x + 5/2*c) + 20*\cos(3/2*d*x + 3/2*c \\
&)) * \sin(6*d*x + 6*c) + 120*(3*\cos(4*d*x + 4*c) + 3*\cos(2*d*x + 2*c) + 1)*\sin \\
& (11/2*d*x + 11/2*c) + 400*(3*\cos(4*d*x + 4*c) + 3*\cos(2*d*x + 2*c) + 1)*\sin \\
& (9/2*d*x + 9/2*c) + 6*(45*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 8*\sin(1/2*d*x + 1/2*c)) * \sin \\
& (2*d*x + 2*c) - 168*\cos(7/2*d*x + 7/2*c) - 12*\cos(5/2*d*x + 5/2*c) + 20*\cos \\
& (3/2*d*x + 3/2*c)) * \sin(4*d*x + 4*c) + 336*(3*\cos(2*d*x + 2*c) + 1)*\sin(7/ \\
& 2*d*x + 7/2*c) + 24*(3*\cos(2*d*x + 2*c) + 1)*\sin(5/2*d*x + 5/2*c) - 1008*\cos \\
& (7/2*d*x + 7/2*c) * \sin(2*d*x + 2*c) - 72*\cos(5/2*d*x + 5/2*c) * \sin(2*d*x + 2 \\
& *c) + 120*\cos(3/2*d*x + 3/2*c) * \sin(2*d*x + 2*c) + 15*\sqrt{2}*\log(2*\cos(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) + 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c) + 2) - 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2) - 40*\sin(3/2*d*x + 3/2*c) - 120*\sin(1/2*d*x + 1/2*c)) * \sqrt{a} / ((\sqrt{2} \\
& *\cos(6*d*x + 6*c)^2 + 9*\sqrt{2}*\cos(4*d*x + 4*c)^2 + 9*\sqrt{2}*\cos(2*d* \\
& x + 2*c)^2 + \sqrt{2}*\sin(6*d*x + 6*c)^2 + 9*\sqrt{2}*\sin(4*d*x + 4*c)^2 + 18 \\
& *\sqrt{2}*\sin(4*d*x + 4*c) * \sin(2*d*x + 2*c) + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + \\
& 2*(3*\sqrt{2}*\cos(4*d*x + 4*c) + 3*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2}))*\cos(\\
& 6*d*x + 6*c) + 6*(3*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2}))*\cos(4*d*x + 4*c) + \\
& 6*(\sqrt{2}*\sin(4*d*x + 4*c) + \sqrt{2}*\sin(2*d*x + 2*c)) * \sin(6*d*x + 6*c) + \\
& 6*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2}))*d
\end{aligned}$$

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.12

$$\int \sqrt{a + a \cos(c + dx)} \sec^4(c + dx) dx =$$

$$\frac{\sqrt{2} \left(15 \sqrt{2} \log \left(\frac{-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) + \frac{4 \left(60 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^5 - 80 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 + 33 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) \right)}{(2 \sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^3} \sqrt{a} \right)}{96 d}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="giac")

```
[Out] -1/96*sqrt(2)*(15*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(60*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 80*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 33*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/((2*sin(1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a \cos(c + dx)} \sec^4(c + dx) dx = \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos(c + dx)^4} dx$$

[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)

[Out] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^4, x)

3.104 $\int \cos^3(c + dx)(a + a \cos(c + dx))^{3/2} dx$

Optimal result	828
Rubi [A] (verified)	828
Mathematica [A] (verified)	831
Maple [A] (verified)	831
Fricas [A] (verification not implemented)	831
Sympy [F(-1)]	832
Maxima [A] (verification not implemented)	832
Giac [A] (verification not implemented)	832
Mupad [F(-1)]	833

Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{68a^2 \sin(c + dx)}{45d\sqrt{a + a \cos(c + dx)}} + \frac{34a^2 \cos^3(c + dx) \sin(c + dx)}{63d\sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \cos^4(c + dx) \sin(c + dx)}{9d\sqrt{a + a \cos(c + dx)}} - \frac{136a\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{315d} + \frac{68(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105d}$$

[Out] $68/105*(a+a*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+68/45*a^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)+34/63*a^2*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)+2/9*a^2*\cos(d*x+c)^4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)-136/315*a*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/d$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2842, 21, 2849, 2838, 2830, 2725}

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{2a^2 \sin(c + dx) \cos^4(c + dx)}{9d\sqrt{a \cos(c + dx) + a}} + \frac{34a^2 \sin(c + dx) \cos^3(c + dx)}{63d\sqrt{a \cos(c + dx) + a}} + \frac{68a^2 \sin(c + dx)}{45d\sqrt{a \cos(c + dx) + a}} + \frac{68 \sin(c + dx)(a \cos(c + dx) + a)^{3/2}}{105d} - \frac{136a \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{315d}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^3*(a + a*\text{Cos}[c + d*x])^(3/2), x]$


```
[Out] (68*a^2*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (34*a^2*Cos[c + d*x]
]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Cos[c + d*x]^4*Sin
in[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (136*a*Sqrt[a + a*Cos[c + d*x
]])*Sin[c + d*x])/(315*d) + (68*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10
5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] := Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !L
tQ[m, -2^(-1)]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]^(n_.), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2 \cos^4(c + dx) \sin(c + dx)}{9d\sqrt{a + a \cos(c + dx)}} + \frac{2}{9} \int \frac{\cos^3(c + dx) \left(\frac{17a^2}{2} + \frac{17}{2}a^2 \cos(c + dx) \right)}{\sqrt{a + a \cos(c + dx)}} dx \\
&= \frac{2a^2 \cos^4(c + dx) \sin(c + dx)}{9d\sqrt{a + a \cos(c + dx)}} + \frac{1}{9}(17a) \int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{34a^2 \cos^3(c + dx) \sin(c + dx)}{63d\sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \cos^4(c + dx) \sin(c + dx)}{9d\sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{1}{21}(34a) \int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{34a^2 \cos^3(c + dx) \sin(c + dx)}{63d\sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \cos^4(c + dx) \sin(c + dx)}{9d\sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{68(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105d} \\
&\quad + \frac{68}{105} \int \left(\frac{3a}{2} - a \cos(c + dx) \right) \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{34a^2 \cos^3(c + dx) \sin(c + dx)}{63d\sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \cos^4(c + dx) \sin(c + dx)}{9d\sqrt{a + a \cos(c + dx)}} \\
&\quad - \frac{136a\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{315d} \\
&\quad + \frac{68(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105d} + \frac{1}{45}(34a) \int \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{68a^2 \sin(c + dx)}{45d\sqrt{a + a \cos(c + dx)}} + \frac{34a^2 \cos^3(c + dx) \sin(c + dx)}{63d\sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \cos^4(c + dx) \sin(c + dx)}{9d\sqrt{a + a \cos(c + dx)}} \\
&\quad - \frac{136a\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{315d} + \frac{68(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.57

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{a\sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) (3780 \sin\left(\frac{1}{2}(c + dx)\right) + 1050 \sin\left(\frac{3}{2}(c + dx)\right) + 378 \sin\left(\frac{5}{2}(c + dx)\right) + 105 \sin\left(\frac{7}{2}(c + dx)\right) + 35 \sin\left(\frac{9}{2}(c + dx)\right))}{2520d}$$

[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2),x]

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3780*Sin[(c + d*x)/2] + 1050*Sin[(3*(c + d*x))/2] + 378*Sin[(5*(c + d*x))/2] + 135*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*d)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(280 \left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 220 \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 114 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 47 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 94\right) \sqrt{2}}{315 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	99

[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)

```
[Out] 4/315*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(280*cos(1/2*d*x+1/2*c)^8-220*cos(1/2*d*x+1/2*c)^6+114*cos(1/2*d*x+1/2*c)^4+47*cos(1/2*d*x+1/2*c)^2+94)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.48

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{2(35a \cos(dx + c)^4 + 85a \cos(dx + c)^3 + 102a \cos(dx + c)^2 + 136a \cos(dx + c) + 272a) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{315(d \cos(dx + c) + d)}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

```
[Out] 2/315*(35*a*cos(d*x + c)^4 + 85*a*cos(d*x + c)^3 + 102*a*cos(d*x + c)^2 + 136*a*cos(d*x + c) + 272*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{3/2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.52

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{(35 \sqrt{2} a \sin(\frac{9}{2} dx + \frac{9}{2} c) + 135 \sqrt{2} a \sin(\frac{7}{2} dx + \frac{7}{2} c) + 378 \sqrt{2} a \sin(\frac{5}{2} dx + \frac{5}{2} c) + 1050 \sqrt{2} a \sin(\frac{3}{2} dx + \frac{3}{2} c) + 3780 \sqrt{2} a \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{2520 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/2520*(35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.75

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{\sqrt{2}(35 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{9}{2} dx + \frac{9}{2} c) + 135 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{7}{2} dx + \frac{7}{2} c) + 378 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{5}{2} dx + \frac{5}{2} c) + 1050 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 3780 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{2520 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/2520*sqrt(2)*(35*a*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 135*a*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 378*a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 1050*a*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 3780*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{3/2} dx = \int \cos(c + dx)^3 (a + a \cos(c + dx))^{3/2} dx$$

```
[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2), x)
```

```
[Out] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2), x)
```

3.105 $\int \cos^2(c + dx)(a + a \cos(c + dx))^{3/2} dx$

Optimal result	834
Rubi [A] (verified)	834
Mathematica [A] (verified)	836
Maple [A] (verified)	836
Fricas [A] (verification not implemented)	837
Sympy [F(-1)]	837
Maxima [A] (verification not implemented)	837
Giac [A] (verification not implemented)	838
Mupad [F(-1)]	838

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{152a^2 \sin(c + dx)}{105d\sqrt{a + a \cos(c + dx)}} + \frac{38a\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{105d} - \frac{4(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7ad}$$

[Out] $-4/35*(a+a*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+2/7*(a+a*\cos(d*x+c))^(5/2)*\sin(d*x+c)/a/d+152/105*a^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)+38/105*a*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/d$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2838, 2830, 2726, 2725}

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{152a^2 \sin(c + dx)}{105d\sqrt{a \cos(c + dx) + a}} + \frac{2 \sin(c + dx)(a \cos(c + dx) + a)^{5/2}}{7ad} - \frac{4 \sin(c + dx)(a \cos(c + dx) + a)^{3/2}}{35d} + \frac{38a \sin(c + dx)\sqrt{a \cos(c + dx) + a}}{105d}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^2*(a + a*\text{Cos}[c + d*x])^(3/2), x]$

```
[Out] (152*a^2*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (38*a*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(105*d) - (4*(a + a*Cos[c + d*x])^(3/2)*Sin[c +
d*x])/(35*d) + (2*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_),
x_Symbol] := Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !L
tQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7ad} \\ &+ \frac{2 \int \left(\frac{5a}{2} - a \cos(c + dx)\right) (a + a \cos(c + dx))^{3/2} dx}{7a} \\ &= -\frac{4(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35d} \\ &+ \frac{2(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7ad} + \frac{19}{35} \int (a + a \cos(c + dx))^{3/2} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{38a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d} - \frac{4(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} \\
&\quad + \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad} + \frac{1}{105}(76a) \int \sqrt{a+a\cos(c+dx)} dx \\
&= \frac{152a^2\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{38a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d} \\
&\quad - \frac{4(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2} dx = \frac{a\sqrt{a(1+\cos(c+dx))} \sec\left(\frac{1}{2}(c+dx)\right) (735 \sin\left(\frac{1}{2}(c+dx)\right) + 175 \sin\left(\frac{3}{2}(c+dx)\right) + 63 \sin\left(\frac{5}{2}(c+dx)\right))}{420d}$$

[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2), x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(735*Sin[(c + d*x)/2] + 175*Sin[(3*(c + d*x))/2] + 63*Sin[(5*(c + d*x))/2] + 15*Sin[(7*(c + d*x))/2]))/(420*d)

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(60 \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 12 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 19 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 38\right) \sqrt{2}}{105 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	86

[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(3/2), x, method=_RETURNVERBOSE)

[Out] 4/105*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(60*cos(1/2*d*x+1/2*c)^6-12*cos(1/2*d*x+1/2*c)^4+19*cos(1/2*d*x+1/2*c)^2+38)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{2(15a \cos(dx + c)^3 + 39a \cos(dx + c)^2 + 52a \cos(dx + c) + 104a) \sqrt{a \cos(dx + c) + a \sin(dx + c)}}{105(d \cos(dx + c) + d)}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 2/105*(15*a*cos(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 52*a*cos(d*x + c) + 104*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{3/2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{(15 \sqrt{2} a \sin(\frac{7}{2} dx + \frac{7}{2} c) + 63 \sqrt{2} a \sin(\frac{5}{2} dx + \frac{5}{2} c) + 175 \sqrt{2} a \sin(\frac{3}{2} dx + \frac{3}{2} c) + 735 \sqrt{2} a \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{420 d}$$

[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/420*(15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{\sqrt{2}(15 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{7}{2} dx + \frac{7}{2} c) + 63 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{5}{2} dx + \frac{5}{2} c) + 175 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 735 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{420 a d}$$

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/420*sqrt(2)*(15*a*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 63*a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 175*a*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 735*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{3/2} dx = \int \cos(c + dx)^2 (a + a \cos(c + dx))^{3/2} dx$$

```
[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2), x)
```

3.106 $\int \cos(c + dx)(a + a \cos(c + dx))^{3/2} dx$

Optimal result	839
Rubi [A] (verified)	839
Mathematica [A] (verified)	840
Maple [A] (verified)	841
Fricas [A] (verification not implemented)	841
Sympy [F]	841
Maxima [A] (verification not implemented)	842
Giac [A] (verification not implemented)	842
Mupad [F(-1)]	842

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \cos(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{8a^2 \sin(c + dx)}{5d\sqrt{a + a \cos(c + dx)}} + \frac{2a\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{5d} + \frac{2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d}$$

[Out] $2/5*(a+a*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+8/5*a^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)+2/5*a*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/d$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2830, 2726, 2725}

$$\int \cos(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{8a^2 \sin(c + dx)}{5d\sqrt{a \cos(c + dx) + a}} + \frac{2a \sin(c + dx)\sqrt{a \cos(c + dx) + a}}{5d} + \frac{2 \sin(c + dx)(a \cos(c + dx) + a)^{3/2}}{5d}$$

[In] $\text{Int}[\text{Cos}[c + d*x]*(a + a*\text{Cos}[c + d*x])^(3/2), x]$

[Out] $(8*a^2*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (2*a*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(5*d) + (2*(a + a*\text{Cos}[c + d*x])^(3/2)*\text{Sin}[c + d*x])/(5*d)$

Rule 2725

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[-2*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]])), x] /;$ $\text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{Eq}$

$Q[a^2 - b^2, 0]$

Rule 2726

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{3}{5} \int (a + a \cos(c + dx))^{3/2} dx \\
 &= \frac{2a\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{5d} + \frac{2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
 &\quad + \frac{1}{5}(4a) \int \sqrt{a + a \cos(c + dx)} dx \\
 &= \frac{8a^2 \sin(c + dx)}{5d\sqrt{a + a \cos(c + dx)}} + \frac{2a\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{5d} \\
 &\quad + \frac{2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.78

$$\int \cos(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{a\sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(20 \sin\left(\frac{1}{2}(c + dx)\right) + 5 \sin\left(\frac{3}{2}(c + dx)\right) + \sin\left(\frac{5}{2}(c + dx)\right)\right)}{10d}$$

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2), x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(20*Sin[(c + d*x)/2] + 5*Sin
[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(10*d)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(2 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right) \sqrt{2}}{5 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	71

[In] `int(cos(d*x+c)*(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{4}{5} \cos\left(\frac{1}{2} d x + \frac{1}{2} c\right) a^2 \sin\left(\frac{1}{2} d x + \frac{1}{2} c\right) \left(2 \cos\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 + \cos\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 + 2\right) 2^{1/2} / \left(a \cos\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2\right)^{1/2} / d$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.64

$$\int \cos(c + dx) (a + a \cos(c + dx))^{3/2} dx = \frac{2 \left(a \cos(dx + c)^2 + 3 a \cos(dx + c) + 6 a \right) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{5 (d \cos(dx + c) + d)}$$

[In] `integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] $\frac{2}{5} (a \cos(dx + c)^2 + 3 a \cos(dx + c) + 6 a) \sqrt{a \cos(dx + c) + a} \sin(dx + c) / (d \cos(dx + c) + d)$

Sympy [F]

$$\int \cos(c + dx) (a + a \cos(c + dx))^{3/2} dx = \int (a(\cos(c + dx) + 1))^{3/2} \cos(c + dx) dx$$

[In] `integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(3/2),x)`

[Out] `Integral((a*(cos(c + d*x) + 1))**(3/2)*cos(c + d*x), x)`

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int \cos(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{(\sqrt{2}a \sin(\frac{5}{2} dx + \frac{5}{2} c) + 5 \sqrt{2}a \sin(\frac{3}{2} dx + \frac{3}{2} c) + 20 \sqrt{2}a \sin(\frac{1}{2} dx + \frac{1}{2} c))\sqrt{a}}{10 d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/10*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \cos(c + dx)(a + a \cos(c + dx))^{3/2} dx = \frac{\sqrt{2}(a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{5}{2} dx + \frac{5}{2} c) + 5 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 20 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c))\sqrt{a}}{10 d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/10*sqrt(2)*(a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 5*a*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 20*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx)(a + a \cos(c + dx))^{3/2} dx = \int \cos(c + dx) (a + a \cos(c + dx))^{3/2} dx$$

[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2), x)

3.107 $\int (a + a \cos(c + dx))^{3/2} dx$

Optimal result	843
Rubi [A] (verified)	843
Mathematica [A] (verified)	844
Maple [A] (verified)	844
Fricas [A] (verification not implemented)	845
Sympy [F]	845
Maxima [A] (verification not implemented)	845
Giac [A] (verification not implemented)	846
Mupad [F(-1)]	846

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int (a + a \cos(c + dx))^{3/2} dx = \frac{8a^2 \sin(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} + \frac{2a\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d}$$

[Out] $8/3*a^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/3*a*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$\int (a + a \cos(c + dx))^{3/2} dx = \frac{8a^2 \sin(c + dx)}{3d\sqrt{a \cos(c + dx) + a}} + \frac{2a \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{3d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}, x]$

[Out] $(8*a^2*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (2*a*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*d)$

Rule 2725

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]] , x_Symbol] \rightarrow \text{Simp}[-2*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]])), x] /;$ $\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2726

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2a\sqrt{a + a\cos(c + dx)}\sin(c + dx)}{3d} + \frac{1}{3}(4a) \int \sqrt{a + a\cos(c + dx)} dx \\ &= \frac{8a^2\sin(c + dx)}{3d\sqrt{a + a\cos(c + dx)}} + \frac{2a\sqrt{a + a\cos(c + dx)}\sin(c + dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (a + a\cos(c + dx))^{3/2} dx = \frac{a\sqrt{a(1 + \cos(c + dx))}\sec\left(\frac{1}{2}(c + dx)\right)\left(9\sin\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{3}{2}(c + dx)\right)\right)}{3d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2),x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(9*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*d)

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{4a^2\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\right)\sqrt{2}}{3\sqrt{a\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}d}$	58

[In] int((a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 4/3*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(cos(1/2*d*x+1/2*c)^2+2)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int (a + a \cos(c + dx))^{3/2} dx = \frac{2(a \cos(dx + c) + 5a) \sqrt{a \cos(dx + c) + a \sin(dx + c)}}{3(d \cos(dx + c) + d)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 2/3*(a*cos(d*x + c) + 5*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)

Sympy [F]

$$\int (a + a \cos(c + dx))^{3/2} dx = \int (a \cos(c + dx) + a)^{\frac{3}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))**(3/2),x)

[Out] Integral((a*cos(c + d*x) + a)**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int (a + a \cos(c + dx))^{3/2} dx = \frac{(\sqrt{2}a \sin(\frac{3}{2} dx + \frac{3}{2} c) + 9 \sqrt{2}a \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{3d}$$

[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (a + a \cos(c + dx))^{3/2} dx = \frac{\sqrt{2}(\operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 9 \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{3d}$$

[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] 1/3*sqrt(2)*(a*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 9*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} dx = \int (a + a \cos(c + dx))^{3/2} dx$$

[In] int((a + a*cos(c + d*x))^(3/2),x)

[Out] int((a + a*cos(c + d*x))^(3/2), x)

3.108 $\int (a + a \cos(c + dx))^{3/2} \sec(c + dx) dx$

Optimal result	847
Rubi [A] (verified)	847
Mathematica [A] (verified)	849
Maple [B] (verified)	849
Fricas [B] (verification not implemented)	849
Sympy [F]	850
Maxima [F]	850
Giac [A] (verification not implemented)	850
Mupad [F(-1)]	851

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (a + a \cos(c + dx))^{3/2} \sec(c + dx) dx = \frac{2a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{d} + \frac{2a^2 \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}}$$

[Out] $2*a^{(3/2)}*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+2*a^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec), antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2842, 21, 2852, 212}

$$\int (a + a \cos(c + dx))^{3/2} \sec(c + dx) dx = \frac{2a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{d} + \frac{2a^2 \sin(c + dx)}{d \sqrt{a \cos(c + dx) + a}}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^{(3/2)}*\operatorname{Sec}[c + d*x], x]$

[Out] $(2*a^{(3/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])])/d + (2*a^2*\operatorname{Sin}[c + d*x])/(d*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])$

Rule 21

$\operatorname{Int}[(u_*)*((a_) + (b_)*(v_))^{(m_)}*((c_) + (d_)*(v_))^{(n_)}, x_Symbol] \rightarrow$
 $\operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m + n)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n, x\}$
 $\&\& \operatorname{EqQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[m] \&\& (!\operatorname{IntegerQ}[n] \mid\mid \operatorname{SimplerQ}[c + d*x,$
 $a + b*x])$

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2842

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2852

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a^2 \sin(c + dx)}{d\sqrt{a + a \cos(c + dx)}} + 2 \int \frac{\left(\frac{a^2}{2} + \frac{1}{2}a^2 \cos(c + dx)\right) \sec(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx \\
 &= \frac{2a^2 \sin(c + dx)}{d\sqrt{a + a \cos(c + dx)}} + a \int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx \\
 &= \frac{2a^2 \sin(c + dx)}{d\sqrt{a + a \cos(c + dx)}} - \frac{(2a^2) \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \\
 &= \frac{2a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} + \frac{2a^2 \sin(c + dx)}{d\sqrt{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int (a + a \cos(c + dx))^{3/2} \sec(c + dx) dx = \frac{a \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(\sqrt{2} \operatorname{arctanh}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right) + 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x], x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sin[(c + d*x)/2]))/d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(58) = 116.

Time = 1.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.17

method	result
default	$\frac{\sqrt{a} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(2\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a} + \ln\left(\frac{4\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a} + 8a}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}}\right) a + \ln\left(-\frac{4}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2}}\right) a + \ln\left(\frac{4}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}}\right) a\right)}{\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$

[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c), x, method=_RETURNVERBOSE)

[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a)*a+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.92

$$\int (a + a \cos(c + dx))^{3/2} \sec(c + dx) dx = \frac{(a \cos(dx + c) + a) \sqrt{a} \log\left(\frac{a \cos(dx + c)^3 - 7 a \cos(dx + c)^2 - 4 \sqrt{a \cos(dx + c) + a} \sqrt{a} (\cos(dx + c) - 2) \sin(dx + c) + 8 a}{\cos(dx + c)^3 + \cos(dx + c)^2}\right) + 4 \sqrt{a} \cos(dx + c)}{2 (d \cos(dx + c) + d)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")

[Out] $\frac{1}{2} * ((a * \cos(d * x + c) + a) * \sqrt{a} * \log((a * \cos(d * x + c))^3 - 7 * a * \cos(d * x + c)^2 - 4 * \sqrt{a * \cos(d * x + c) + a} * \sqrt{a} * (\cos(d * x + c) - 2) * \sin(d * x + c) + 8 * a) / (\cos(d * x + c)^3 + \cos(d * x + c)^2)) + 4 * \sqrt{a * \cos(d * x + c) + a} * a * \sin(d * x + c)) / (d * \cos(d * x + c) + d)$

Sympy [F]

$$\int (a + a \cos(c + dx))^{3/2} \sec(c + dx) dx = \int (a(\cos(c + dx) + 1))^{\frac{3}{2}} \sec(c + dx) dx$$

[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c),x)

[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*sec(c + d*x), x)

Maxima [F]

$$\int (a + a \cos(c + dx))^{3/2} \sec(c + dx) dx = \int (a \cos(dx + c) + a)^{\frac{3}{2}} \sec(dx + c) dx$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int (a + a \cos(c + dx))^{3/2} \sec(c + dx) dx = \frac{\sqrt{2} \left(\sqrt{2} a \log \left(\frac{|-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) - 4 a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) \right) \sqrt{a}}{2d}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")

[Out] $-\frac{1}{2} * \sqrt{2} * (\sqrt{2} * a * \log(\operatorname{abs}(-2 * \sqrt{2} + 4 * \sin(1/2 * d * x + 1/2 * c)) / \operatorname{abs}(2 * \sqrt{2} + 4 * \sin(1/2 * d * x + 1/2 * c)))) * \operatorname{sgn}(\cos(1/2 * d * x + 1/2 * c)) - 4 * a * \operatorname{sgn}(\cos(1/2 * d * x + 1/2 * c)) * \sin(1/2 * d * x + 1/2 * c)) * \sqrt{a} / d$

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec(c + dx) dx = \int \frac{(a + a \cos(c + dx))^{3/2}}{\cos(c + dx)} dx$$

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

```
[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

3.109 $\int (a + a \cos(c + dx))^{3/2} \sec^2(c + dx) dx$

Optimal result	852
Rubi [A] (verified)	852
Mathematica [A] (verified)	854
Maple [B] (verified)	854
Fricas [B] (verification not implemented)	855
Sympy [F(-1)]	855
Maxima [B] (verification not implemented)	855
Giac [A] (verification not implemented)	856
Mupad [F(-1)]	857

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int (a + a \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \frac{3a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{d} + \frac{a^2 \tan(c + dx)}{d \sqrt{a + a \cos(c + dx)}}$$

[Out] $3a^{3/2} \operatorname{arctanh}(\sin(dx+c) \cdot a^{1/2} / (a+a \cos(dx+c))^{1/2}) / d + a^2 \tan(dx+c) / d / (a+a \cos(dx+c))^{1/2}$

Rubi [A] (verified)

Time = 0.14 (sec), antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2841, 21, 2852, 212}

$$\int (a + a \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \frac{3a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{d} + \frac{a^2 \tan(c + dx)}{d \sqrt{a \cos(c + dx) + a}}$$

[In] $\text{Int}[(a + a \cos[c + dx])^{3/2} \sec^2[c + dx], x]$

[Out] $(3a^{3/2} \operatorname{ArcTanh}[\sqrt{a} \sin[c + dx] / \sqrt{a + a \cos[c + dx]}]) / d + (a^2 \tan[c + dx]) / (d \sqrt{a + a \cos[c + dx]})$

Rule 21

$\text{Int}[(u_*) \cdot ((a_*) + (b_*) \cdot (v_*))^{(m_*)} \cdot ((c_*) + (d_*) \cdot (v_*))^{(n_*)}, x_Symbol] :=$
 $\text{Dist}[(b/d)^m, \text{Int}[u \cdot (c + d \cdot v)^{(m+n)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, n\}, x$
 $\&\& \text{EqQ}[b \cdot c - a \cdot d, 0] \&\& \text{IntegerQ}[m] \&\& (!\text{IntegerQ}[n] \parallel \text{SimplerQ}[c + dx,$
 $a + b \cdot x])$

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2841

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2852

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a^2 \tan(c + dx)}{d\sqrt{a + a \cos(c + dx)}} - a \int \frac{\left(-\frac{3a}{2} - \frac{3}{2}a \cos(c + dx)\right) \sec(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx \\
 &= \frac{a^2 \tan(c + dx)}{d\sqrt{a + a \cos(c + dx)}} + \frac{1}{2}(3a) \int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx \\
 &= \frac{a^2 \tan(c + dx)}{d\sqrt{a + a \cos(c + dx)}} - \frac{(3a^2) \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \\
 &= \frac{3a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} + \frac{a^2 \tan(c + dx)}{d\sqrt{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int (a + a \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \frac{a \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \sec(c + dx) \left(3\sqrt{2} \operatorname{arctanh}\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right) \cos(c + dx) + 2d\right)}{2d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(3*Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*Sin[(c + d*x)/2]))/(2*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(57) = 114.

Time = 1.37 (sec) , antiderivative size = 385, normalized size of antiderivative = 5.92

method	result
default	$\sqrt{a} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-6a \left(\ln\left(\frac{4\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a+8a}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}} \right) + \ln\left(-\frac{4 \left(\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)} \right) \right)$

[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-6*a*(ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

$2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(2*d*x + 2*c)^2 + (2*\sqrt{2})*a*\sin(3/2*d*x + 3/2*c) + 6*\sqrt{2})*a*\sin(1/2*d*x + 1/2*c) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\sin(2*d*x + 2*c)^2 - 4*\sqrt{2})*a*\sin(3/2*d*x + 3/2*c) + 4*\sqrt{2})*a*\sin(1/2*d*x + 1/2*c) - 2*(\sqrt{2})*a*\sin(3/2*d*x + 3/2*c) - 5*\sqrt{2})*a*\sin(1/2*d*x + 1/2*c) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(2*d*x + 2*c) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 2*(\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a*\sin(7/2*d*x + 7/2*c) - 6*(\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a*\sin(5/2*d*x + 5/2*c) + 2*(3*\sqrt{2})*a*\cos(3/2*d*x + 3/2*c) + \sqrt{2})*a*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c))*\sqrt{a}/((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*d)$

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int (a + a \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \frac{\sqrt{2} \left(3 \sqrt{2} a \log \left(\frac{|-2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) + \frac{4 a \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)}{2 \sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1} \right) \sqrt{a}}{4 d}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")

[Out] $-1/4*\sqrt{2}*(3*\sqrt{2}*a*\log(\text{abs}(-2*\sqrt{2}) + 4*\sin(1/2*d*x + 1/2*c)))/\text{abs}(2*\sqrt{2} + 4*\sin(1/2*d*x + 1/2*c))*\text{sgn}(\cos(1/2*d*x + 1/2*c)) + 4*a*\text{sgn}(\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c)/(2*\sin(1/2*d*x + 1/2*c)^2 - 1))*\sqrt{a}/d$

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \int \frac{(a + a \cos(c + dx))^{3/2}}{\cos(c + dx)^2} dx$$

[In] $\text{int}((a + a*\cos(c + d*x))^{3/2}/\cos(c + d*x)^2, x)$

[Out] $\text{int}((a + a*\cos(c + d*x))^{3/2}/\cos(c + d*x)^2, x)$

3.110 $\int (a + a \cos(c + dx))^{3/2} \sec^3(c + dx) dx$

Optimal result	858
Rubi [A] (verified)	858
Mathematica [A] (verified)	860
Maple [B] (verified)	860
Fricas [A] (verification not implemented)	861
Sympy [F(-1)]	861
Maxima [B] (verification not implemented)	862
Giac [A] (verification not implemented)	864
Mupad [F(-1)]	864

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int (a + a \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \frac{7a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4d} + \frac{7a^2 \tan(c + dx)}{4d\sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d\sqrt{a + a \cos(c + dx)}}$$

[Out] $7/4*a^{(3/2)}*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+7/4*a^2*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+1/2*a^2*\sec(d*x+c)*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2841, 21, 2851, 2852, 212}

$$\int (a + a \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \frac{7a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4d} + \frac{7a^2 \tan(c + dx)}{4d\sqrt{a \cos(c + dx) + a}} + \frac{a^2 \tan(c + dx) \sec(c + dx)}{2d\sqrt{a \cos(c + dx) + a}}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^{(3/2)}*\operatorname{Sec}[c + d*x]^3, x]$

[Out] $(7*a^{(3/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])])/(4*d) + (7*a^2*\operatorname{Tan}[c + d*x])/(4*d*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]]) + (a^2*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])$

Rule 21

```
Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplifierQ[c + d*x,
  a + b*x])
```

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
  ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
  (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
  *Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
  d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
  2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
  *(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
  , e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
  && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
  (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (
  f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
  + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
  t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
  f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
  && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
  1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (
  f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
  ], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d,
  e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d\sqrt{a + a \cos(c + dx)}} - \frac{1}{2}a \int \frac{\left(-\frac{7a}{2} - \frac{7}{2}a \cos(c + dx)\right) \sec^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$\begin{aligned}
&= \frac{a^2 \sec(c+dx) \tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{4}(7a) \int \sqrt{a+a\cos(c+dx)} \sec^2(c+dx) dx \\
&= \frac{7a^2 \tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \sec(c+dx) \tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{1}{8}(7a) \int \sqrt{a+a\cos(c+dx)} \sec(c+dx) dx \\
&= \frac{7a^2 \tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \sec(c+dx) \tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{(7a^2) \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} \\
&= \frac{7a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{7a^2 \tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \sec(c+dx) \tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int (a+a\cos(c+dx))^{3/2} \sec^3(c+dx) dx = \frac{a\sqrt{a(1+\cos(c+dx))} \sec\left(\frac{1}{2}(c+dx)\right) \sec^2(c+dx) (7\sqrt{2}\operatorname{arctanh}(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right))) \cos^2(c+dx)}{8d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(7*Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - 3*Sin[(c + d*x)/2] + 7*Sin[(3*(c + d*x))/2]))/(8*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(90) = 180.

Time = 1.56 (sec) , antiderivative size = 551, normalized size of antiderivative = 5.20

method	result
default	$ \frac{\sqrt{a} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(28a \left(\ln \left(\frac{4\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a+8a}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}} \right) + \ln \left(-\frac{4 \left(\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)} \right) \right) $

[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)


```
[Out] 1/2*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(28*a*(ln(4/(
2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2))*
(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2
)-2*a)))*sin(1/2*d*x+1/2*c)^4+(-28*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(
1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*
a))*a-28*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)
-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a-28*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^2+7*ln(4/(2*cos(1/2*d*x+
1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2)*a^(1/2)+2*a))*a+7*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a
*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+
18*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(
1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x
+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.53

$$\int (a + a \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \frac{7(a \cos(dx + c)^3 + a \cos(dx + c)^2) \sqrt{a} \log\left(\frac{a \cos(dx + c)^3 - 7a \cos(dx + c)^2 - 4\sqrt{a \cos(dx + c) + a} \sqrt{a} (\cos(dx + c) - 2) \sin(dx + c)}{\cos(dx + c)^3 + \cos(dx + c)^2}\right) + 4(7a \cos(dx + c) + 2a) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{16(d \cos(dx + c))^3 + d \cos(dx + c)}$$

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3
- 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(7*a*cos(d*x
+ c) + 2*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*co
s(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```


$c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 7\sqrt{2}a\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + 7\sqrt{2}a\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 7\sqrt{2}a\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 12a\sin(3/2dx + 3/2c) - 56a\sin(1/2dx + 1/2c))\sin(2dx + 2c))\sin(4dx + 4c) + 80(2a\cos(2dx + 2c) + a)\sin(7/2dx + 7/2c) + 8(2a\cos(2dx + 2c)^2 + 2a\sin(2dx + 2c)^2 + 23a\cos(2dx + 2c) + 11a)\sin(5/2dx + 5/2c) + 24a\sin(3/2dx + 3/2c) - 56a\sin(1/2dx + 1/2c))\sqrt{a}/((\sqrt{2}\cos(4dx + 4c)^2 + 4\sqrt{2}\cos(2dx + 2c)^2 + \sqrt{2}\sin(4dx + 4c)^2 + 4\sqrt{2}\sin(4dx + 4c)\sin(2dx + 2c) + 4\sqrt{2}\sin(2dx + 2c)^2 + 2(2\sqrt{2}\cos(2dx + 2c) + \sqrt{2})\cos(4dx + 4c) + 4\sqrt{2}\cos(2dx + 2c) + \sqrt{2}))d$

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int (a + a \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \frac{\sqrt{2} \left(7\sqrt{2}a \log \left(\frac{|-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) + \frac{4 \left(14 \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) \right)^3 - 9 \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \left(2 \sin(\frac{1}{2} dx + \frac{1}{2} c) \right)^2 - 1 \right)^2}{16d}}{16d}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")

[Out] -1/16*sqrt(2)*(7*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(14*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 9*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \int \frac{(a + a \cos(c + dx))^{3/2}}{\cos(c + dx)^3} dx$$

[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)

[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)

3.111 $\int (a + a \cos(c + dx))^{3/2} \sec^4(c + dx) dx$

Optimal result	865
Rubi [A] (verified)	865
Mathematica [A] (verified)	867
Maple [B] (verified)	868
Fricas [A] (verification not implemented)	868
Sympy [F(-1)]	869
Maxima [B] (verification not implemented)	869
Giac [A] (verification not implemented)	873
Mupad [F(-1)]	873

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int (a + a \cos(c + dx))^{3/2} \sec^4(c + dx) dx = \frac{11a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{8d} + \frac{11a^2 \tan(c + dx)}{8d\sqrt{a + a \cos(c + dx)}} + \frac{11a^2 \sec(c + dx) \tan(c + dx)}{12d\sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sec^2(c + dx) \tan(c + dx)}{3d\sqrt{a + a \cos(c + dx)}}$$

[Out] $11/8*a^{(3/2)}*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+11/8*a^2*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+11/12*a^2*\sec(d*x+c)*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+1/3*a^2*\sec(d*x+c)^2*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2841, 21, 2851, 2852, 212}

$$\int (a + a \cos(c + dx))^{3/2} \sec^4(c + dx) dx = \frac{11a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{8d} + \frac{11a^2 \tan(c + dx)}{8d\sqrt{a \cos(c + dx) + a}} + \frac{a^2 \tan(c + dx) \sec^2(c + dx)}{3d\sqrt{a \cos(c + dx) + a}} + \frac{11a^2 \tan(c + dx) \sec(c + dx)}{12d\sqrt{a \cos(c + dx) + a}}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^{(3/2)}*\operatorname{Sec}[c + d*x]^4, x]$

[Out] $(11*a^{(3/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])])/(8*d) + (11*a^2*\operatorname{Tan}[c + d*x])/(8*d*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]]) + (11*a^2*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(12*d*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]]) + (a^2*\operatorname{Sec}[c + d*x]^2*\operatorname{Tan}[c + d*x])/(3*d*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])$

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
  ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
  (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
  *Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
  d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
  2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
  *(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
  , e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
  && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
  (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
  f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
  + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
  t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
  f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
  && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
  1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
  f_.)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
  ], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d,
  e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{a^2 \sec^2(c + dx) \tan(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} - \frac{1}{3}a \int \frac{\left(-\frac{11a}{2} - \frac{11}{2}a \cos(c + dx)\right) \sec^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$\begin{aligned}
&= \frac{a^2 \sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{6}(11a) \int \sqrt{a+a\cos(c+dx)} \sec^3(c+dx) dx \\
&= \frac{11a^2 \sec(c+dx) \tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{1}{8}(11a) \int \sqrt{a+a\cos(c+dx)} \sec^2(c+dx) dx \\
&= \frac{11a^2 \tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2 \sec(c+dx) \tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{a^2 \sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{16}(11a) \int \sqrt{a+a\cos(c+dx)} \sec(c+dx) dx \\
&= \frac{11a^2 \tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2 \sec(c+dx) \tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{a^2 \sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{(11a^2) \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} \\
&= \frac{11a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{11a^2 \tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{11a^2 \sec(c+dx) \tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int (a+a\cos(c+dx))^{3/2} \sec^4(c+dx) dx = \frac{a\sqrt{a(1+\cos(c+dx))} \sec\left(\frac{1}{2}(c+dx)\right) \sec^3(c+dx) (66\sqrt{2}\operatorname{arctanh}(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right))) \cos^3(c+dx) + 11a^2 \operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right) + 11a^2 \tan(c+dx)}{96d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(66*Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 54*Sin[(c + d*x)/2] + 11*(Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))) / (96*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(124) = 248.

Time = 1.90 (sec) , antiderivative size = 718, normalized size of antiderivative = 4.99

method	result
default	$\sqrt{a} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-264a \left(\ln\left(\frac{4\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a+8a}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}} \right) + \ln\left(-\frac{4 \left(\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)} \right) \right)$

[In] `int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{6} a^{1/2} \cos\left(\frac{1}{2} d x + \frac{1}{2} c\right) \left(a \sin\left(\frac{1}{2} d x + \frac{1}{2} c\right) \right)^2 \left(-264 a \left(\ln\left(\frac{4 \sqrt{2} a \cos\left(\frac{d x}{2} + \frac{c}{2}\right) + 4 \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{d x}{2} + \frac{c}{2}\right)\right)} \sqrt{a+8 a}}{2 \cos\left(\frac{d x}{2} + \frac{c}{2}\right) + \sqrt{2}} \right) + \ln\left(-\frac{4 \left(\sqrt{2} a \cos\left(\frac{d x}{2} + \frac{c}{2}\right) - \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{d x}{2} + \frac{c}{2}\right)\right)}\right)}{2 \cos\left(\frac{d x}{2} + \frac{c}{2}\right)} \right) \right) \right. \\ \left. + \frac{33 a \cos^4(dx+c) + a \cos^3(dx+c)}{96 (d \cos(dx+c))^4 + \dots} \right) \frac{1}{d}$$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.20

$$\int (a + a \cos(c + dx))^{3/2} \sec^4(c + dx) dx = \frac{33 (a \cos(dx+c)^4 + a \cos(dx+c)^3) \sqrt{a} \log\left(\frac{a \cos(dx+c)^3 - 7 a \cos(dx+c)^2 - 4 \sqrt{a \cos(dx+c)} + a \sqrt{a} (\cos(dx+c) - 2) \sin(dx+c)}{\cos(dx+c)^3 + \cos(dx+c)^2}\right)}{96 (d \cos(dx+c))^4 + \dots}$$

[In] `integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="fricas")`


```
[Out] 1/96*(33*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^4(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5542 vs. 2(124) = 248.

Time = 164.44 (sec) , antiderivative size = 5542, normalized size of antiderivative = 38.49

$$\int (a + a \cos(c + dx))^{3/2} \sec^4(c + dx) dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(774*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 162*sqrt(2)*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(6*d*x + 6*c)^2 + 9*(14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 33*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 + 9*(14*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 90*sqrt(2)*
```

$$\begin{aligned}
& a \sin(1/2*d*x + 1/2*c) - 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 33*a*lo \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(2*d*x + 2*c)^2 + (14*\sqrt{2}*a*\sin(3 \\
& /2*d*x + 3/2*c) + 90*\sqrt{2}*a*\sin(1/2*d*x + 1/2*c) - 33*a*\log(2*\cos(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) - 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{r \\
& t(2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\sin(6*d*x + \\
& 6*c)^2 + 9*(14*\sqrt{2}*a*\sin(3/2*d*x + 3/2*c) + 90*\sqrt{2}*a*\sin(1/2*d*x + \\
& 1/2*c) - 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*1 \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 33*a*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2))*\sin(4*d*x + 4*c)^2 + 9*(14*\sqrt{2}*a*\sin(3/2*d*x + 3/2* \\
& c) + 90*\sqrt{2}*a*\sin(1/2*d*x + 1/2*c) - 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) - 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*lo \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\sin(2*d*x + 2*c)^2 + 44* \\
& \sqrt{2}*a*\sin(3/2*d*x + 3/2*c) + 132*\sqrt{2}*a*\sin(1/2*d*x + 1/2*c) + 14*(s \\
& \sqrt{2}*a*\sin(6*d*x + 6*c) + 3*\sqrt{2}*a*\sin(4*d*x + 4*c) + 3*\sqrt{2}*a*\sin(\\
& 2*d*x + 2*c))*\cos(15/2*d*x + 15/2*c) + 90*(\sqrt{2}*a*\sin(6*d*x + 6*c) + 3*s \\
& \sqrt{2}*a*\sin(4*d*x + 4*c) + 3*\sqrt{2}*a*\sin(2*d*x + 2*c))*\cos(13/2*d*x + 13 \\
& /2*c) - 2*(87*\sqrt{2}*a*\sin(11/2*d*x + 11/2*c) + 157*\sqrt{2}*a*\sin(9/2*d*x \\
& + 9/2*c) + 129*\sqrt{2}*a*\sin(7/2*d*x + 7/2*c) + 27*\sqrt{2}*a*\sin(5/2*d*x + \\
& 5/2*c) - 29*\sqrt{2}*a*\sin(3/2*d*x + 3/2*c) - 111*\sqrt{2}*a*\sin(1/2*d*x + 1/ \\
& 2*c) - 3*(14*\sqrt{2}*a*\sin(3/2*d*x + 3/2*c) + 90*\sqrt{2}*a*\sin(1/2*d*x + 1/ \\
& 2*c) - 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 33*a*\log(2*\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*s \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c) + 2))*\cos(2*d*x + 2*c) - 33*a*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) - 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}(\ \\
& 2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 14*(\sqrt{2}) \\
& *a*\cos(6*d*x + 6*c) + 3*\sqrt{2})*a*\cos(4*d*x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x \\
& + 2*c) + \sqrt{2})*a*\sin(15/2*d*x + 15/2*c) - 90*(\sqrt{2})*a*\cos(6*d*x + 6*c) \\
& + 3*\sqrt{2})*a*\cos(4*d*x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a) \\
& *\sin(13/2*d*x + 13/2*c) + 2*(87*\sqrt{2})*a*\cos(11/2*d*x + 11/2*c) + 157*\sqrt{2} \\
& (2)*a*\cos(9/2*d*x + 9/2*c) + 129*\sqrt{2})*a*\cos(7/2*d*x + 7/2*c) + 27*\sqrt{2} \\
& (2)*a*\cos(5/2*d*x + 5/2*c) - 15*\sqrt{2})*a*\cos(3/2*d*x + 3/2*c) - 21*\sqrt{2})*a \\
& *\cos(1/2*d*x + 1/2*c) + 3*(14*\sqrt{2})*a*\sin(3/2*d*x + 3/2*c) + 90*\sqrt{2})*a \\
& *\sin(1/2*d*x + 1/2*c) - 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 33*a*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*s \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c) + 2))*\sin(4*d*x + 4*c) + 3*(14*\sqrt{2})*a*\sin(3/ \\
& 2*d*x + 3/2*c) + 90*\sqrt{2})*a*\sin(1/2*d*x + 1/2*c) - 33*a*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) - 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}(\ \\
& 2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\sin(2*d*x + \\
& 2*c))*\sin(6*d*x + 6*c) - 174*(3*\sqrt{2})*a*\cos(4*d*x + 4*c) + 3*\sqrt{2})*a*\cos \\
& (2*d*x + 2*c) + \sqrt{2})*a*\sin(11/2*d*x + 11/2*c) - 314*(3*\sqrt{2})*a*\cos(4 \\
& *d*x + 4*c) + 3*\sqrt{2})*a*\cos(2*d*x + 2*c) + \sqrt{2})*a*\sin(9/2*d*x + 9/2*c \\
&) + 18*(43*\sqrt{2})*a*\cos(7/2*d*x + 7/2*c) + 9*\sqrt{2})*a*\cos(5/2*d*x + 5/2*c \\
&) - 5*\sqrt{2})*a*\cos(3/2*d*x + 3/2*c) - 7*\sqrt{2})*a*\cos(1/2*d*x + 1/2*c) + (\\
& 14*\sqrt{2})*a*\sin(3/2*d*x + 3/2*c) + 90*\sqrt{2})*a*\sin(1/2*d*x + 1/2*c) - 33* \\
& a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*a*\log(2*\cos(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) + 33*a*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned} & \frac{1}{2}c)^2 - 2\sqrt{2}\cos(\frac{1}{2}dx + \frac{1}{2}c) - 2\sqrt{2}\sin(\frac{1}{2}dx + \frac{1}{2}c) \\ & + 2)\sin(2dx + 2c)\sin(4dx + 4c) - 258(3\sqrt{2})a\cos(2dx + 2c) \\ & + \sqrt{2}a\sin(\frac{7}{2}dx + \frac{7}{2}c) - 54(3\sqrt{2})a\cos(2dx + 2c) + \sqrt{2}a\sin(\frac{5}{2}dx + \frac{5}{2}c) \\ & - 18(5\sqrt{2})a\cos(\frac{3}{2}dx + \frac{3}{2}c) + 7\sqrt{2}a\cos(\frac{1}{2}dx + \frac{1}{2}c)\sin(2dx + 2c)\sqrt{a} \\ & / ((2(3\cos(4dx + 4c) + 3\cos(2dx + 2c) + 1)\cos(6dx + 6c) + \cos(6dx + 6c)^2 + 6(3\cos(2dx + 2c) + 1)\cos(4dx + 4c) + 9\cos(4dx + 4c)^2 + 9\cos(2dx + 2c)^2 + 6(\sin(4dx + 4c) + \sin(2dx + 2c))\sin(6dx + 6c) + \sin(6dx + 6c)^2 + 9\sin(4dx + 4c)^2 + 18\sin(4dx + 4c)\sin(2dx + 2c) + 9\sin(2dx + 2c)^2 + 6\cos(2dx + 2c) + 1)d \end{aligned}$$

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int (a + a \cos(c + dx))^{3/2} \sec^4(c + dx) dx = \sqrt{2} \left(33 \sqrt{2} a \log \left(\frac{|-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) + \frac{4 \left(132 a \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 176 a \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 63 a \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{(2 \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1)^3} \right) \sqrt{a} / d$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="giac")

[Out] -1/96*sqrt(2)*(33*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(132*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 176*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 63*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^4(c + dx) dx = \int \frac{(a + a \cos(c + dx))^{3/2}}{\cos(c + dx)^4} dx$$

[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)

[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^4, x)

3.112 $\int \cos^3(c + dx)(a + a \cos(c + dx))^{5/2} dx$

Optimal result	874
Rubi [A] (verified)	874
Mathematica [A] (verified)	877
Maple [A] (verified)	878
Fricas [A] (verification not implemented)	878
Sympy [F(-1)]	878
Maxima [A] (verification not implemented)	879
Giac [A] (verification not implemented)	879
Mupad [F(-1)]	879

Optimal result

Integrand size = 23, antiderivative size = 203

$$\begin{aligned} \int \cos^3(c + dx)(a + a \cos(c + dx))^{5/2} dx &= \frac{284a^3 \sin(c + dx)}{99d\sqrt{a + a \cos(c + dx)}} \\ &+ \frac{710a^3 \cos^3(c + dx) \sin(c + dx)}{693d\sqrt{a + a \cos(c + dx)}} + \frac{46a^3 \cos^4(c + dx) \sin(c + dx)}{99d\sqrt{a + a \cos(c + dx)}} \\ &- \frac{568a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{693d} \\ &+ \frac{2a^2 \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{11d} \\ &+ \frac{284a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{231d} \end{aligned}$$

[Out] $284/231*a*(a+a*\cos(d*x+c))^{(3/2)}*\sin(d*x+c)/d+284/99*a^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+710/693*a^3*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+46/99*a^3*\cos(d*x+c)^4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}-568/693*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d+2/11*a^2*\cos(d*x+c)^4*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {2842, 3060, 2849, 2838, 2830, 2725}

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2} dx = \frac{46a^3 \sin(c+dx) \cos^4(c+dx)}{99d\sqrt{a\cos(c+dx)+a}} + \frac{710a^3 \sin(c+dx) \cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}} + \frac{284a^3 \sin(c+dx)}{99d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2 \sin(c+dx) \cos^4(c+dx) \sqrt{a\cos(c+dx)+a}}{11d} - \frac{568a^2 \sin(c+dx) \sqrt{a\cos(c+dx)+a}}{693d} + \frac{284a \sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{231d}$$

[In] Int[Cos[c + d*x]^3*(a + a*cos[c + d*x])^(5/2), x]

[Out] (284*a^3*Sin[c + d*x])/(99*d*Sqrt[a + a*cos[c + d*x]]) + (710*a^3*cos[c + d*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*cos[c + d*x]]) + (46*a^3*cos[c + d*x]^4*Sin[c + d*x])/(99*d*Sqrt[a + a*cos[c + d*x]]) - (568*a^2*Sqrt[a + a*cos[c + d*x]]*Sin[c + d*x])/(693*d) + (2*a^2*cos[c + d*x]^4*Sqrt[a + a*cos[c + d*x]]*Sin[c + d*x])/(11*d) + (284*a*(a + a*cos[c + d*x])^(3/2)*Sin[c + d*x])/(231*d)

Rule 2725

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2830

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2838

Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2842

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^n_, x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x]

```

])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))

```

Rule 2849

```

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

```

Rule 3060

```

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2 \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{11d} \\
&+ \frac{2}{11} \int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} \left(\frac{19a^2}{2} + \frac{23}{2} a^2 \cos(c + dx) \right) dx \\
&= \frac{46a^3 \cos^4(c + dx) \sin(c + dx)}{99d \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{11d} \\
&+ \frac{1}{99} (355a^2) \int \cos^3(c + dx) \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{710a^3 \cos^3(c + dx) \sin(c + dx)}{693d \sqrt{a + a \cos(c + dx)}} + \frac{46a^3 \cos^4(c + dx) \sin(c + dx)}{99d \sqrt{a + a \cos(c + dx)}} \\
&+ \frac{2a^2 \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{11d} \\
&+ \frac{1}{231} (710a^2) \int \cos^2(c + dx) \sqrt{a + a \cos(c + dx)} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{710a^3 \cos^3(c + dx) \sin(c + dx)}{693d\sqrt{a + a \cos(c + dx)}} + \frac{46a^3 \cos^4(c + dx) \sin(c + dx)}{99d\sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{2a^2 \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{11d} \\
&\quad + \frac{284a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{231d} \\
&\quad + \frac{1}{231}(284a) \int \left(\frac{3a}{2} - a \cos(c + dx) \right) \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{710a^3 \cos^3(c + dx) \sin(c + dx)}{693d\sqrt{a + a \cos(c + dx)}} + \frac{46a^3 \cos^4(c + dx) \sin(c + dx)}{99d\sqrt{a + a \cos(c + dx)}} \\
&\quad - \frac{568a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{693d} \\
&\quad + \frac{2a^2 \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{11d} \\
&\quad + \frac{284a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{231d} + \frac{1}{99}(142a^2) \int \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{284a^3 \sin(c + dx)}{99d\sqrt{a + a \cos(c + dx)}} + \frac{710a^3 \cos^3(c + dx) \sin(c + dx)}{693d\sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{46a^3 \cos^4(c + dx) \sin(c + dx)}{99d\sqrt{a + a \cos(c + dx)}} - \frac{568a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{693d} \\
&\quad + \frac{2a^2 \cos^4(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{11d} \\
&\quad + \frac{284a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{231d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{a^2 \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) (31878 \sin\left(\frac{1}{2}(c + dx)\right) + 8778 \sin\left(\frac{3}{2}(c + dx)\right) + 3465 \sin\left(\frac{5}{2}(c + dx)\right) + 1287 \sin\left(\frac{7}{2}(c + dx)\right) + 385 \sin\left(\frac{9}{2}(c + dx)\right) + 63 \sin\left(\frac{11}{2}(c + dx)\right))}{11088d}$$

[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5/2),x]

[Out] (a^2*sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(31878*Sin[(c + d*x)/2] + 8778*Sin[(3*(c + d*x))/2] + 3465*Sin[(5*(c + d*x))/2] + 1287*Sin[(7*(c + d*x))/2] + 385*Sin[(9*(c + d*x))/2] + 63*Sin[(11*(c + d*x))/2]))/(11088*d)

Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.55

method	result
default	$\frac{8 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(504 \left(\cos^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 364 \left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 178 \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 75 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 100 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 100}{693 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} d$

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/693*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(504*cos(1/2*d*x+1/2*c)^10-
364*cos(1/2*d*x+1/2*c)^8+178*cos(1/2*d*x+1/2*c)^6+75*cos(1/2*d*x+1/2*c)^4+1
00*cos(1/2*d*x+1/2*c)^2+200)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{2(63a^2 \cos(dx + c)^5 + 224a^2 \cos(dx + c)^4 + 355a^2 \cos(dx + c)^3 + 426a^2 \cos(dx + c)^2 + 568a^2 \cos(dx + c) + 1136a^2) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{693(d \cos(dx + c) + d)}$$

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^2*cos(d*x + c)^5 + 224*a^2*cos(d*x + c)^4 + 355*a^2*cos(d*x + c)
)^3 + 426*a^2*cos(d*x + c)^2 + 568*a^2*cos(d*x + c) + 1136*a^2)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{5/2} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.55

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{(63 \sqrt{2} a^2 \sin(\frac{11}{2} dx + \frac{11}{2} c) + 385 \sqrt{2} a^2 \sin(\frac{9}{2} dx + \frac{9}{2} c) + 1287 \sqrt{2} a^2 \sin(\frac{7}{2} dx + \frac{7}{2} c) + 3465 \sqrt{2} a^2 \sin(\frac{5}{2} dx + \frac{5}{2} c) + 8778 \sqrt{2} a^2 \sin(\frac{3}{2} dx + \frac{3}{2} c) + 31878 \sqrt{2} a^2 \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{11088 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/11088*(63*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 3465*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 31878*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 1.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.77

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{\sqrt{2}(63 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{11}{2} dx + \frac{11}{2} c) + 385 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{9}{2} dx + \frac{9}{2} c) + 1287 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{7}{2} dx + \frac{7}{2} c) + 3465 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{5}{2} dx + \frac{5}{2} c) + 8778 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 31878 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{11088 d}$$

[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/11088*sqrt(2)*(63*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(11/2*d*x + 11/2*c) + 385*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 1287*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 3465*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 8778*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 31878*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c + dx)(a + a \cos(c + dx))^{5/2} dx = \int \cos(c + dx)^3 (a + a \cos(c + dx))^{5/2} dx$$

[In] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^3*(a + a*cos(c + d*x))^(5/2), x)

3.113 $\int \cos^2(c + dx)(a + a \cos(c + dx))^{5/2} dx$

Optimal result	880
Rubi [A] (verified)	880
Mathematica [A] (verified)	882
Maple [A] (verified)	882
Fricas [A] (verification not implemented)	883
Sympy [F(-1)]	883
Maxima [A] (verification not implemented)	883
Giac [A] (verification not implemented)	884
Mupad [F(-1)]	884

Optimal result

Integrand size = 23, antiderivative size = 146

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{832a^3 \sin(c + dx)}{315d\sqrt{a + a \cos(c + dx)}} + \frac{208a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{315d} + \frac{26a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105d} - \frac{4(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{63d} + \frac{2(a + a \cos(c + dx))^{7/2} \sin(c + dx)}{9ad}$$

[Out] $26/105*a*(a+a*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d-4/63*(a+a*\cos(d*x+c))^(5/2)*\sin(d*x+c)/d+2/9*(a+a*\cos(d*x+c))^(7/2)*\sin(d*x+c)/a/d+832/315*a^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)+208/315*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/d$

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2838, 2830, 2726, 2725}

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{832a^3 \sin(c + dx)}{315d\sqrt{a \cos(c + dx) + a}} + \frac{208a^2 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{315d} + \frac{2 \sin(c + dx)(a \cos(c + dx) + a)^{7/2}}{9ad} - \frac{4 \sin(c + dx)(a \cos(c + dx) + a)^{5/2}}{63d} + \frac{26a \sin(c + dx)(a \cos(c + dx) + a)^{3/2}}{105d}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^2*(a + a*\text{Cos}[c + d*x])^(5/2), x]$

```
[Out] (832*a^3*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (208*a^2*Sqrt[a +
a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (26*a*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(105*d) - (4*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) +
(2*(a + a*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_),
x_Symbol] := Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !L
tQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2(a + a \cos(c + dx))^{7/2} \sin(c + dx)}{9ad} \\ &+ \frac{2 \int \left(\frac{7a}{2} - a \cos(c + dx)\right) (a + a \cos(c + dx))^{5/2} dx}{9a} \\ &= -\frac{4(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{63d} \\ &+ \frac{2(a + a \cos(c + dx))^{7/2} \sin(c + dx)}{9ad} + \frac{13}{21} \int (a + a \cos(c + dx))^{5/2} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{26a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105d} - \frac{4(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{63d} \\
&\quad + \frac{2(a + a \cos(c + dx))^{7/2} \sin(c + dx)}{9ad} + \frac{1}{105}(104a) \int (a + a \cos(c + dx))^{3/2} dx \\
&= \frac{208a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{315d} + \frac{26a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105d} \\
&\quad - \frac{4(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{63d} + \frac{2(a + a \cos(c + dx))^{7/2} \sin(c + dx)}{9ad} \\
&\quad + \frac{1}{315}(416a^2) \int \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{832a^3 \sin(c + dx)}{315d \sqrt{a + a \cos(c + dx)}} + \frac{208a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{315d} \\
&\quad + \frac{26a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{105d} \\
&\quad - \frac{4(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{63d} + \frac{2(a + a \cos(c + dx))^{7/2} \sin(c + dx)}{9ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{a^2 \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(8190 \sin\left(\frac{1}{2}(c + dx)\right) + 2100 \sin\left(\frac{3}{2}(c + dx)\right) + 756 \sin\left(\frac{5}{2}(c + dx)\right) + 100 \sin\left(\frac{7}{2}(c + dx)\right) + 35 \sin\left(\frac{9}{2}(c + dx)\right)\right)}{2520d}$$

[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2), x]

[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(8190*Sin[(c + d*x)/2] + 2100*Sin[(3*(c + d*x))/2] + 756*Sin[(5*(c + d*x))/2] + 225*Sin[(7*(c + d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*d)

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{8 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(140 \left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 20 \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 39 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 52 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 104\right) \sqrt{2}}{315 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	99

[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(5/2), x, method=_RETURNVERBOSE)

[Out] $8/315 \cos(1/2 dx + 1/2 c) a^3 \sin(1/2 dx + 1/2 c) (140 \cos(1/2 dx + 1/2 c)^8 - 20 \cos(1/2 dx + 1/2 c)^6 + 39 \cos(1/2 dx + 1/2 c)^4 + 52 \cos(1/2 dx + 1/2 c)^2 + 104) 2^{1/2} / (a \cos(1/2 dx + 1/2 c)^2)^{1/2} / d$

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{2(35 a^2 \cos(dx + c)^4 + 130 a^2 \cos(dx + c)^3 + 219 a^2 \cos(dx + c)^2 + 292 a^2 \cos(dx + c) + 584 a^2) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{315 (d \cos(dx + c) + d)}$$

[In] `integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] $2/315(35 a^2 \cos(dx + c)^4 + 130 a^2 \cos(dx + c)^3 + 219 a^2 \cos(dx + c)^2 + 292 a^2 \cos(dx + c) + 584 a^2) \sqrt{a \cos(dx + c) + a} \sin(dx + c) / (d \cos(dx + c) + d)$

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{5/2} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(5/2),x)`

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{(35 \sqrt{2} a^2 \sin(\frac{9}{2} dx + \frac{9}{2} c) + 225 \sqrt{2} a^2 \sin(\frac{7}{2} dx + \frac{7}{2} c) + 756 \sqrt{2} a^2 \sin(\frac{5}{2} dx + \frac{5}{2} c) + 2100 \sqrt{2} a^2 \sin(\frac{3}{2} dx + \frac{3}{2} c) + 8190 \sqrt{2} a^2 \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{2520 d}$$

[In] `integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] $1/2520(35 \sqrt{2} a^2 \sin(9/2 dx + 9/2 c) + 225 \sqrt{2} a^2 \sin(7/2 dx + 7/2 c) + 756 \sqrt{2} a^2 \sin(5/2 dx + 5/2 c) + 2100 \sqrt{2} a^2 \sin(3/2 dx + 3/2 c) + 8190 \sqrt{2} a^2 \sin(1/2 dx + 1/2 c)) \sqrt{a} / d$

Giac [A] (verification not implemented)

none

Time = 0.81 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{\sqrt{2}(35 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{9}{2} dx + \frac{9}{2} c) + 225 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{7}{2} dx + \frac{7}{2} c) + 756 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{5}{2} dx + \frac{5}{2} c) + 2100 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 8190 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{d}$$

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/2520*sqrt(2)*(35*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(9/2*d*x + 9/2*c) + 225
*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 756*a^2*sgn(cos(1/2*d
*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 2100*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(
3/2*d*x + 3/2*c) + 8190*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))
*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(a + a \cos(c + dx))^{5/2} dx = \int \cos(c + dx)^2 (a + a \cos(c + dx))^{5/2} dx$$

```
[In] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2), x)
```


3.114 $\int \cos(c + dx)(a + a \cos(c + dx))^{5/2} dx$

Optimal result	885
Rubi [A] (verified)	885
Mathematica [A] (verified)	887
Maple [A] (verified)	887
Fricas [A] (verification not implemented)	887
Sympy [F(-1)]	888
Maxima [A] (verification not implemented)	888
Giac [A] (verification not implemented)	888
Mupad [F(-1)]	889

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \cos(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{64a^3 \sin(c + dx)}{21d\sqrt{a + a \cos(c + dx)}} + \frac{16a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{7d} + \frac{2(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d}$$

[Out] $2/7*a*(a+a*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+2/7*(a+a*\cos(d*x+c))^(5/2)*\sin(d*x+c)/d+64/21*a^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)+16/21*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/d$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2830, 2726, 2725}

$$\int \cos(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{64a^3 \sin(c + dx)}{21d\sqrt{a \cos(c + dx) + a}} + \frac{16a^2 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{21d} + \frac{2a \sin(c + dx)(a \cos(c + dx) + a)^{3/2}}{7d} + \frac{2 \sin(c + dx)(a \cos(c + dx) + a)^{5/2}}{7d}$$

[In] $\text{Int}[\text{Cos}[c + d*x]*(a + a*\text{Cos}[c + d*x])^(5/2), x]$

[Out] $(64a^3 \sin[c + dx]) / (21d \sqrt{a + a \cos[c + dx]}) + (16a^2 \sqrt{a + a \cos[c + dx]} \sin[c + dx]) / (21d) + (2a(a + a \cos[c + dx])^{3/2} \sin[c + dx]) / (7d) + (2(a + a \cos[c + dx])^{5/2} \sin[c + dx]) / (7d)$

Rule 2725

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2726

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n), Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]

Rule 2830

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d} + \frac{5}{7} \int (a + a \cos(c + dx))^{5/2} dx \\
 &= \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{7d} + \frac{2(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\
 &\quad + \frac{1}{7}(8a) \int (a + a \cos(c + dx))^{3/2} dx \\
 &= \frac{16a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{7d} \\
 &\quad + \frac{2(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d} + \frac{1}{21}(32a^2) \int \sqrt{a + a \cos(c + dx)} dx \\
 &= \frac{64a^3 \sin(c + dx)}{21d \sqrt{a + a \cos(c + dx)}} + \frac{16a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{21d} \\
 &\quad + \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{7d} + \frac{2(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int \cos(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{a^2 \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(315 \sin\left(\frac{1}{2}(c + dx)\right) + 77 \sin\left(\frac{3}{2}(c + dx)\right) + 3(7 \sin\left(\frac{5}{2}(c + dx)\right) + \sin\left(\frac{7}{2}(c + dx)\right))\right)}{84d}$$

[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2), x]

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(315*Sin[(c + d*x)/2] + 77*Sin[(3*(c + d*x))/2] + 3*(7*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]))) / (84*d)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{8 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^3 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(6 \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 3 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 8\right) \sqrt{2}}{21 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	86

[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^(5/2), x, method=_RETURNVERBOSE)

```
[Out] 8/21*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(6*cos(1/2*d*x+1/2*c)^6+3*cos(1/2*d*x+1/2*c)^4+4*cos(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int \cos(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{2 \left(3 a^2 \cos(dx + c)^3 + 12 a^2 \cos(dx + c)^2 + 23 a^2 \cos(dx + c) + 46 a^2\right) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{21 (d \cos(dx + c) + d)}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2), x, algorithm="fricas")

```
[Out] 2/21*(3*a^2*cos(d*x + c)^3 + 12*a^2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) + 46*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c + dx)(a + a \cos(c + dx))^{5/2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \cos(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{(3\sqrt{2}a^2 \sin(\frac{7}{2}dx + \frac{7}{2}c) + 21\sqrt{2}a^2 \sin(\frac{5}{2}dx + \frac{5}{2}c) + 77\sqrt{2}a^2 \sin(\frac{3}{2}dx + \frac{3}{2}c) + 315\sqrt{2}a^2 \sin(\frac{1}{2}dx + \frac{1}{2}c))\sqrt{a}}{84d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/84*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \cos(c + dx)(a + a \cos(c + dx))^{5/2} dx = \frac{\sqrt{2}(3a^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) \sin(\frac{7}{2}dx + \frac{7}{2}c) + 21a^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) \sin(\frac{5}{2}dx + \frac{5}{2}c) + 77a^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) \sin(\frac{3}{2}dx + \frac{3}{2}c) + 315a^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) \sin(\frac{1}{2}dx + \frac{1}{2}c))\sqrt{a}}{84d}$$

[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/84*sqrt(2)*(3*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 21*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 77*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 315*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx)(a + a \cos(c + dx))^{5/2} dx = \int \cos(c + dx) (a + a \cos(c + dx))^{5/2} dx$$

```
[In] int(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2), x)
```

```
[Out] int(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2), x)
```

3.115 $\int (a + a \cos(c + dx))^{5/2} dx$

Optimal result	890
Rubi [A] (verified)	890
Mathematica [A] (verified)	891
Maple [A] (verified)	892
Fricas [A] (verification not implemented)	892
Sympy [F]	892
Maxima [A] (verification not implemented)	893
Giac [A] (verification not implemented)	893
Mupad [F(-1)]	893

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int (a + a \cos(c + dx))^{5/2} dx = \frac{64a^3 \sin(c + dx)}{15d\sqrt{a + a \cos(c + dx)}} + \frac{16a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d}$$

[Out] $2/5*a*(a+a*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+64/15*a^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)+16/15*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/d$

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$\int (a + a \cos(c + dx))^{5/2} dx = \frac{64a^3 \sin(c + dx)}{15d\sqrt{a \cos(c + dx) + a}} + \frac{16a^2 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{15d} + \frac{2a \sin(c + dx) (a \cos(c + dx) + a)^{3/2}}{5d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^(5/2), x]$

[Out] $(64*a^3*\text{Sin}[c + d*x])/(15*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (16*a^2*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(15*d) + (2*a*(a + a*\text{Cos}[c + d*x])^(3/2)*\text{Sin}[c + d*x])/(5*d)$

Rule 2725

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[-2*b*(\text{Cos}[c + d*x]/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]])), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{Eq}$

Q[a^2 - b^2, 0]

Rule 2726

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n), Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{1}{5}(8a) \int (a + a \cos(c + dx))^{3/2} dx \\
 &= \frac{16a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
 &\quad + \frac{1}{15}(32a^2) \int \sqrt{a + a \cos(c + dx)} dx \\
 &= \frac{64a^3 \sin(c + dx)}{15d \sqrt{a + a \cos(c + dx)}} + \frac{16a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15d} \\
 &\quad + \frac{2a(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int (a + a \cos(c + dx))^{5/2} dx = \frac{a^2 \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(150 \sin\left(\frac{1}{2}(c + dx)\right) + 25 \sin\left(\frac{3}{2}(c + dx)\right) + 3 \sin\left(\frac{5}{2}(c + dx)\right)\right)}{30d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2), x]

[Out] (a^2*sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(150*Sin[(c + d*x)/2] + 25*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(30*d)

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{8a^3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(3 \cos^4\left(\frac{dx}{2} + \frac{c}{2}\right) + 4 \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 8\right) \sqrt{2}}{15 \sqrt{a \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)} d}$	73

[In] `int((a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{8}{15} a^3 \cos\left(\frac{1}{2} d x + \frac{1}{2} c\right) \sin\left(\frac{1}{2} d x + \frac{1}{2} c\right) \left(3 \cos\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 + 4 \cos\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 + 8\right) 2^{1/2} / \left(a \cos\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2\right)^{1/2} / d$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int (a + a \cos(c + dx))^{5/2} dx = \frac{2(3a^2 \cos(dx + c)^2 + 14a^2 \cos(dx + c) + 43a^2) \sqrt{a \cos(dx + c) + a \sin(dx + c)}}{15(d \cos(dx + c) + d)}$$

[In] `integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] $\frac{2}{15} (3a^2 \cos(dx + c)^2 + 14a^2 \cos(dx + c) + 43a^2) \sqrt{a \cos(dx + c) + a \sin(dx + c)} / (d \cos(dx + c) + d)$

Sympy [F]

$$\int (a + a \cos(c + dx))^{5/2} dx = \int (a \cos(c + dx) + a)^{\frac{5}{2}} dx$$

[In] `integrate((a+a*cos(d*x+c))**(5/2),x)`

[Out] `Integral((a*cos(c + d*x) + a)**(5/2), x)`

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int (a + a \cos(c + dx))^{5/2} dx = \frac{(3\sqrt{2}a^2 \sin(\frac{5}{2}dx + \frac{5}{2}c) + 25\sqrt{2}a^2 \sin(\frac{3}{2}dx + \frac{3}{2}c) + 150\sqrt{2}a^2 \sin(\frac{1}{2}dx + \frac{1}{2}c))\sqrt{a}}{30d}$$

[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int (a + a \cos(c + dx))^{5/2} dx = \frac{\sqrt{2}(3a^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) \sin(\frac{5}{2}dx + \frac{5}{2}c) + 25a^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) \sin(\frac{3}{2}dx + \frac{3}{2}c) + 150a^2 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c)) \sin(\frac{1}{2}dx + \frac{1}{2}c))\sqrt{a}}{30d}$$

[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/30*sqrt(2)*(3*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 25*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 150*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} dx = \int (a + a \cos(c + dx))^{5/2} dx$$

[In] int((a + a*cos(c + d*x))^(5/2),x)

[Out] int((a + a*cos(c + d*x))^(5/2), x)

3.116 $\int (a + a \cos(c + dx))^{5/2} \sec(c + dx) dx$

Optimal result	894
Rubi [A] (verified)	894
Mathematica [A] (verified)	896
Maple [B] (verified)	896
Fricas [A] (verification not implemented)	897
Sympy [F(-1)]	897
Maxima [F]	897
Giac [A] (verification not implemented)	898
Mupad [F(-1)]	898

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (a + a \cos(c + dx))^{5/2} \sec(c + dx) dx = \frac{2a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} + \frac{14a^3 \sin(c+dx)}{3d\sqrt{a+a \cos(c+dx)}} + \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{3d}$$

[Out] $2*a^{(5/2)}*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+14/3*a^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/3*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2842, 3060, 2852, 212}

$$\int (a + a \cos(c + dx))^{5/2} \sec(c + dx) dx = \frac{2a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d} + \frac{14a^3 \sin(c+dx)}{3d\sqrt{a \cos(c+dx)+a}} + \frac{2a^2 \sin(c+dx) \sqrt{a \cos(c+dx)+a}}{3d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^{(5/2)}*\operatorname{Sec}[c + d*x], x]$

[Out] $(2*a^{(5/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])]/d + (14*a^3*\operatorname{Sin}[c + d*x])/(3*d*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]]) + (2*a^2*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/(3*d)$

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2842

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Ssin[e + f*x])^(m - 2)*(c + d*Ssin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2852

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Ssin[e + f*x])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3060

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*B*Cos[e + f*x]*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a + b*Ssin[e + f*x]))], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Ssin[e + f*x]]*(c + d*Ssin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d} \\ &+ \frac{2}{3} \int \sqrt{a + a \cos(c + dx)} \left(\frac{3a^2}{2} + \frac{7}{2} a^2 \cos(c + dx) \right) \sec(c + dx) dx \\ &= \frac{14a^3 \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d} \\ &+ a^2 \int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{14a^3 \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{3d} \\
&\quad - \frac{(2a^3) \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&= \frac{2a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{14a^3 \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int (a+a\cos(c+dx))^{5/2} \sec(c+dx) dx = \frac{2a^2 \sqrt{a(1+\cos(c+dx))} \left(3\operatorname{arctanh}\left(\sqrt{1-\cos(c+dx)}\right) + \sqrt{1-\cos(c+dx)}(8+\cos(c+dx)) \right) \tan\left(\frac{c+dx}{2}\right)}{3d\sqrt{1-\cos(c+dx)}}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x], x]

[Out] (2*a^2*Sqrt[a*(1 + Cos[c + d*x])]*(3*ArcTanh[Sqrt[1 - Cos[c + d*x]]] + Sqrt[1 - Cos[c + d*x]]*(8 + Cos[c + d*x]))*Tan[(c + d*x)/2])/(3*d*Sqrt[1 - Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(84) = 168.

Time = 1.97 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.51

method	result
default	$ \frac{a^{\frac{3}{2}} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-4\sqrt{2} \sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} + 18\sqrt{2} \sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a} + 3 \ln\left(\frac{4\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right)}{3 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}\right) \right)}{3 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} $

[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c), x, method=_RETURNVERBOSE)

[Out] 1/3*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+18*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.50

$$\int (a + a \cos(c + dx))^{5/2} \sec(c + dx) dx = \frac{3(a^2 \cos(dx + c) + a^2) \sqrt{a} \log\left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - 4\sqrt{a \cos(dx+c) + a} \sqrt{a} (\cos(dx+c) - 2) \sin(dx+c) + 8a}{\cos(dx+c)^3 + \cos(dx+c)^2}\right) + 4(a^2 \cos(dx+c) + a) \sin(dx+c)}{6(d \cos(dx+c) + d)}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(a^2*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a \cos(c + dx))^{5/2} \sec(c + dx) dx = \int (a \cos(dx + c) + a)^{5/2} \sec(dx + c) dx$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.22

$$\int (a + a \cos(c + dx))^{5/2} \sec(c + dx) dx =$$

$$\frac{\sqrt{2} \left(8 a^2 \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 3 \sqrt{2} a^2 \log \left(\frac{|-2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) \right)}{6 d}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")

[Out] -1/6*sqrt(2)*(8*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 3*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) - 36*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec(c + dx) dx = \int \frac{(a + a \cos(c + dx))^{5/2}}{\cos(c + dx)} dx$$

[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x),x)

[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x), x)

3.117 $\int (a + a \cos(c + dx))^{5/2} \sec^2(c + dx) dx$

Optimal result	899
Rubi [A] (verified)	899
Mathematica [C] (warning: unable to verify)	901
Maple [B] (verified)	901
Fricas [A] (verification not implemented)	902
Sympy [F(-1)]	902
Maxima [B] (verification not implemented)	903
Giac [A] (verification not implemented)	910
Mupad [F(-1)]	910

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int (a + a \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \frac{5a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} + \frac{a^3 \sin(c+dx)}{d \sqrt{a+a \cos(c+dx)}} + \frac{a^2 \sqrt{a+a \cos(c+dx)} \tan(c+dx)}{d}$$

[Out] $5a^{5/2} \operatorname{arctanh}(\sin(dx+c) a^{1/2} / (a+a \cos(dx+c))^{1/2}) / d + a^3 \sin(dx+c) / d / (a+a \cos(dx+c))^{1/2} + a^2 (a+a \cos(dx+c))^{1/2} \tan(dx+c) / d$

Rubi [A] (verified)

Time = 0.24 (sec), antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2841, 3060, 2852, 212}

$$\int (a + a \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \frac{5a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d} + \frac{a^3 \sin(c+dx)}{d \sqrt{a \cos(c+dx)+a}} + \frac{a^2 \tan(c+dx) \sqrt{a \cos(c+dx)+a}}{d}$$

[In] $\operatorname{Int}[(a + a \operatorname{Cos}[c + d*x])^{5/2} \operatorname{Sec}[c + d*x]^2, x]$

[Out] $(5a^{5/2} \operatorname{ArcTanh}[(\operatorname{Sqrt}[a] \operatorname{Sin}[c + d*x]) / \operatorname{Sqrt}[a + a \operatorname{Cos}[c + d*x]]) / d + (a^3 \operatorname{Sin}[c + d*x]) / (d \operatorname{Sqrt}[a + a \operatorname{Cos}[c + d*x]]) + (a^2 \operatorname{Sqrt}[a + a \operatorname{Cos}[c + d*x]] \operatorname{Tan}[c + d*x]) / d$

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (
f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]))], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2 \sqrt{a + a \cos(c + dx)} \tan(c + dx)}{d} \\
&\quad - a \int \left(-\frac{5a}{2} - \frac{1}{2} a \cos(c + dx) \right) \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx \\
&= \frac{a^3 \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{a + a \cos(c + dx)} \tan(c + dx)}{d} \\
&\quad + \frac{1}{2} (5a^2) \int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{a^3 \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \sqrt{a+a\cos(c+dx)} \tan(c+dx)}{d} \\
&\quad - \frac{(5a^3) \operatorname{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&= \frac{5a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a^3 \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \sqrt{a+a\cos(c+dx)} \tan(c+dx)}{d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.58 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.36

$$\int (a+a\cos(c+dx))^{5/2} \sec^2(c+dx) dx = \frac{(a(1+\cos(c+dx)))^{5/2} \left(-\frac{35}{8}(2080+3131\cos(c+dx)+728\cos(2(c+dx))+61\cos(3(c+dx))) \right)}{d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]

[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*((-35*(2080 + 3131*Cos[c + d*x] + 728*Cos[2*(c + d*x)] + 61*Cos[3*(c + d*x)])*Csc[(c + d*x)/2]^6)/8 + (105*ArcTanh[Sqrt[1 - Cos[c + d*x]]]*(1767 + 1252*Cos[c + d*x] + 872*Cos[2*(c + d*x)] + 108*Cos[3*(c + d*x)] + Cos[4*(c + d*x)])*Csc[(c + d*x)/2]^6)/(16*Sqrt[1 - Cos[c + d*x]]) + 1024*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, 2*Sin[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^3)/(6720*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(82) = 164.

Time = 6.56 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.70

method	result
default	$ a^{\frac{3}{2}} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-8\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 10 \ln \left(-\frac{4 \left(\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2}} \right) \right) $

[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)

```
[Out] a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2-10*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*sin(1/2*d*x+1/2*c)^2*a-10*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*sin(1/2*d*x+1/2*c)^2*a+6*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+5*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+5*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.78

$$\int (a + a \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \frac{5 (a^2 \cos(dx + c)^2 + a^2 \cos(dx + c)) \sqrt{a} \log \left(\frac{a \cos(dx + c)^3 - 7 a \cos(dx + c)^2 - 4 \sqrt{a} \cos(dx + c) + a \sqrt{a} (\cos(dx + c) - 2) \sin(dx + c)}{\cos(dx + c)^3 + \cos(dx + c)^2} \right)}{4 (d \cos(dx + c))^2 + d \cos(dx + c)}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*a^2*cos(d*x + c) + a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10847 vs. 2(82) = 164.

Time = 0.67 (sec) , antiderivative size = 10847, normalized size of antiderivative = 117.90

$$\int (a + a \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] -1/252*(1449*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)^3*sin(2*d*x + 2*c) - 63*(sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 25*sqrt(2)*a^2*cos(2*d*x + 2*c) + 24*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c)^3 - 252*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 21*(5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 15*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 15*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 15*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 5*(5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 12*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 6*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 6*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 6*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 6*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*s

$$\begin{aligned}
& \sqrt{2} \cos(1/2 dx + 1/2 c) - 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) \cos(2 dx + 2c) + (25\sqrt{2} a^2 \cos(3/2 dx + 3/2 c) + 198\sqrt{2} a^2 \cos(1/2 dx + 1/2 c)) \sin(2 dx + 2c) \cos(5/2 dx + 5/2 c)^2 + 21(60\sqrt{2} a^2 \sin(1/2 dx + 1/2 c)^3 - 15(a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2\sqrt{2} \cos(1/2 dx + 1/2 c) + 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2\sqrt{2} \cos(1/2 dx + 1/2 c) - 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) + a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2\sqrt{2} \cos(1/2 dx + 1/2 c) + 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2\sqrt{2} \cos(1/2 dx + 1/2 c) - 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) \cos(1/2 dx + 1/2 c)^2 - 15(a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2\sqrt{2} \cos(1/2 dx + 1/2 c) + 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2\sqrt{2} \cos(1/2 dx + 1/2 c) - 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) + a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2\sqrt{2} \cos(1/2 dx + 1/2 c) + 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2\sqrt{2} \cos(1/2 dx + 1/2 c) - 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) \sin(1/2 dx + 1/2 c)^2 + 25(\sqrt{2} a^2 \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} a^2 \sin(1/2 dx + 1/2 c)^2) \sin(3/2 dx + 3/2 c) + 12(5\sqrt{2} a^2 \cos(1/2 dx + 1/2 c)^2 - \sqrt{2} a^2 \sin(1/2 dx + 1/2 c)) \cos(2 dx + 2c)^2 - 315(a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2\sqrt{2} \cos(1/2 dx + 1/2 c) + 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2\sqrt{2} \cos(1/2 dx + 1/2 c) - 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) + a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2\sqrt{2} \cos(1/2 dx + 1/2 c) + 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2\sqrt{2} \cos(1/2 dx + 1/2 c) - 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) \cos(1/2 dx + 1/2 c)^2 + 21(69\sqrt{2} a^2 \cos(5/2 dx + 5/2 c) \sin(2 dx + 2c) - 144\sqrt{2} a^2 \sin(1/2 dx + 1/2 c) + (25\sqrt{2} a^2 \sin(3/2 dx + 3/2 c) + 54\sqrt{2} a^2 \sin(1/2 dx + 1/2 c) - 15 a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2\sqrt{2} \cos(1/2 dx + 1/2 c) + 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) + 15 a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2\sqrt{2} \cos(1/2 dx + 1/2 c) - 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - 15 a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2\sqrt{2} \cos(1/2 dx + 1/2 c) + 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) + 15 a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2\sqrt{2} \cos(1/2 dx + 1/2 c) - 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - 15 a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2\sqrt{2} \cos(1/2 dx + 1/2 c) + 2\sqrt{2} \sin(1/2 dx + 1/2 c) + 2) + 15 a^2 \log(2 \cos(1/2 dx + 1/2 c))^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2\sqrt{2} \cos(1/2 dx + 1/2 c) -
\end{aligned}$$

$$\begin{aligned}
& 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + (25\sqrt{2}a^2\sin(3/2dx + 3/2c) \\
& + 54\sqrt{2}a^2\sin(1/2dx + 1/2c) - 15a^2\log(2\cos(1/2dx + 1/2c))^2 \\
& + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin \\
& \sin(1/2dx + 1/2c) + 2) + 15a^2\log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2d \\
& *x + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/ \\
& 2c) + 2) - 15a^2\log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 \\
& - 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + 15 \\
& *a^2\log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos \\
& \cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2))\sin(2dx + 2c)^ \\
& 2 + 5(5\sqrt{2}a^2\sin(3/2dx + 3/2c) - 18\sqrt{2}a^2\sin(1/2dx + 1/ \\
& 2c) - 6a^2\log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2} \\
& \sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + 6a^2\log \\
& \log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2d \\
& *x + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 6a^2\log(2\cos(1/2dx \\
& + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) + 2 \\
& *sqrt{2}\sin(1/2dx + 1/2c) + 2) + 6a^2\log(2\cos(1/2dx + 1/2c))^2 + 2 \\
& *sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/ \\
& 2dx + 1/2c) + 2))\cos(2dx + 2c) + 5(5\sqrt{2}a^2\cos(3/2dx + 3/2 \\
& c) + 12\sqrt{2}a^2\cos(1/2dx + 1/2c))\sin(2dx + 2c))\sin(5/2dx + 5 \\
& /2c)^2 + 21(60\sqrt{2}a^2\sin(1/2dx + 1/2c)^3 - 15(a^2\log(2\cos(1/2 \\
& *dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) \\
& + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - a^2\log(2\cos(1/2dx + 1/2c))^2 + \\
& 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(\\
& 1/2dx + 1/2c) + 2) + a^2\log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2dx + \\
& 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) \\
& + 2) - a^2\log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2} \\
& (2)\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2))\cos(1/2dx \\
& + 1/2c)^2 - 15(a^2\log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c) \\
& ^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - \\
& a^2\log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos \\
& \cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + a^2\log(2\cos(1/ \\
& 2dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c \\
&) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - a^2\log(2\cos(1/2dx + 1/2c))^2 \\
& + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin \\
& (1/2dx + 1/2c) + 2))\sin(1/2dx + 1/2c)^2 + 25(\sqrt{2}a^2\cos(1/2d \\
& *x + 1/2c)^2 + \sqrt{2}a^2\sin(1/2dx + 1/2c)^2)\sin(3/2dx + 3/2c) + 1 \\
& 2(5\sqrt{2}a^2\cos(1/2dx + 1/2c)^2 - \sqrt{2}a^2)\sin(1/2dx + 1/2c) \\
&)\sin(2dx + 2c)^2 - 315(a^2\log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2d \\
& *x + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2 \\
& *c) + 2) - a^2\log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 + 2 \\
& \sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + a^2\log \\
& \log(2\cos(1/2dx + 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2d \\
& *x + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - a^2\log(2\cos(1/2dx + \\
& 1/2c))^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2} \\
& \sqrt{2}\sin(1/2dx + 1/2c) + 2))\sin(1/2dx + 1/2c)^2 - 35(\sqrt{2}a^2*
\end{aligned}$$

$$\begin{aligned}
& \cos(5/2*d*x + 5/2*c)^2*\sin(2*d*x + 2*c) + 2*\sqrt{2}*a^2*\cos(5/2*d*x + 5/2*c) \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(5/2*d*x + 5/2*c)^2 \\
& *2*\sin(2*d*x + 2*c) + 2*\sqrt{2}*a^2*\sin(5/2*d*x + 5/2*c)*\sin(2*d*x + 2*c)*\sin \\
& (1/2*d*x + 1/2*c) + (\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*a^2*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\sin(2*d*x + 2*c))*\cos(13/2*d*x + 13/2*c) - 135*(\sqrt{2} \\
& *a^2*\cos(5/2*d*x + 5/2*c)^2*\sin(2*d*x + 2*c) + 2*\sqrt{2}*a^2*\cos(5/2*d*x + \\
& 5/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(5/2*d*x + 5/ \\
& 2*c)^2*\sin(2*d*x + 2*c) + 2*\sqrt{2}*a^2*\sin(5/2*d*x + 5/2*c)*\sin(2*d*x + 2* \\
& c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*a^2 \\
& *\sin(1/2*d*x + 1/2*c)^2)*\sin(2*d*x + 2*c))*\cos(11/2*d*x + 11/2*c) - 98*(\sqrt{2} \\
& *a^2*\cos(5/2*d*x + 5/2*c)^2*\sin(2*d*x + 2*c) + 2*\sqrt{2}*a^2*\cos(5/2*d*x \\
& + 5/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(5/2*d*x \\
& + 5/2*c)^2*\sin(2*d*x + 2*c) + 2*\sqrt{2}*a^2*\sin(5/2*d*x + 5/2*c)*\sin(2*d*x \\
& + 2*c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *a^2*\sin(1/2*d*x + 1/2*c)^2)*\sin(2*d*x + 2*c))*\cos(9/2*d*x + 9/2*c) + 390*(\\
& \sqrt{2}*a^2*\cos(5/2*d*x + 5/2*c)^2*\sin(2*d*x + 2*c) + 2*\sqrt{2}*a^2*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(5/2*d \\
& *x + 5/2*c)^2*\sin(2*d*x + 2*c) + 2*\sqrt{2}*a^2*\sin(5/2*d*x + 5/2*c)*\sin(2*d \\
& *x + 2*c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*a^2*\sin(1/2*d*x + 1/2*c)^2)*\sin(2*d*x + 2*c))*\cos(7/2*d*x + 7/2*c) + 21 \\
& *(10*(5*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)*\sin(3/2*d*x + 3/2*c) + 12*\sqrt{2}* \\
& a^2*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(a^2*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *a^2*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) + a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(1/2*d*x + 1/2* \\
& c))*\cos(2*d*x + 2*c)^2 + 10*(5*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + 12*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(a^2 \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2))*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c)^2 + 10*(5*\sqrt{2}*a^2*\cos(1/2*d \\
& *x + 1/2*c)*\sin(3/2*d*x + 3/2*c) + 12*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)*\sin(\\
& 1/2*d*x + 1/2*c) - 6*(a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + a^2*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1
\end{aligned}$$

$$\begin{aligned}
& /2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c) + 2)) * \cos(1/2*d*x + 1/2*c) * \cos(2*d*x + 2*c) - 30*(a^ \\
& 2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2)) * \cos(1/2*d*x + 1/2*c) + (50*\sqrt{2})*a^2*\cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x \\
& + 1/2*c) + 189*\sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)^2 + 69*\sqrt{2})*a^2*\sin(1/2 \\
& *d*x + 1/2*c)^2 * \sin(2*d*x + 2*c) * \cos(5/2*d*x + 5/2*c) + 21*(60*\sqrt{2})*a^ \\
& 2*\sin(1/2*d*x + 1/2*c)^3 - 30*(a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + a^2 \\
& * \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2)) * \cos(1/2*d*x + 1/2*c)^2 - 30*(a^2*\log(2 \\
& * \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& (2)* \sin(1/2*d*x + 1/2*c) + 2) + a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2)) * \sin \\
& (1/2*d*x + 1/2*c)^2 + 25*(\sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*a^2* \\
& \sin(1/2*d*x + 1/2*c)^2 * \sin(3/2*d*x + 3/2*c) + 12*(5*\sqrt{2})*a^2*\cos(1/2*d* \\
& x + 1/2*c)^2 - 2*\sqrt{2})*a^2 * \sin(1/2*d*x + 1/2*c) * \cos(2*d*x + 2*c) + 35*(\\
& \sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*a^2*\sin(1/2*d*x + 1/2*c)^2 + (\\
& \sqrt{2})*a^2*\cos(2*d*x + 2*c) + \sqrt{2})*a^2 * \cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2})* \\
& a^2*\cos(2*d*x + 2*c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c \\
&)) * \cos(5/2*d*x + 5/2*c) + (\sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*a^2 \\
& * \sin(1/2*d*x + 1/2*c)^2 * \cos(2*d*x + 2*c) + 2*(\sqrt{2})*a^2*\cos(2*d*x + 2*c) \\
& * \sin(1/2*d*x + 1/2*c) + \sqrt{2})*a^2*\sin(1/2*d*x + 1/2*c) * \sin(5/2*d*x + 5/2 \\
& *c) * \sin(13/2*d*x + 13/2*c) + 135*(\sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& a^2*\sin(1/2*d*x + 1/2*c)^2 + (\sqrt{2})*a^2*\cos(2*d*x + 2*c) + \sqrt{2})*a^ \\
& ^2 * \cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2})*a^2*\cos(2*d*x + 2*c) + \sqrt{2})*a^2 * \sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2})*a^2*\cos(2*d*x + 2*c) * \cos(1/2*d*x + 1/2*c \\
&) + \sqrt{2})*a^2*\cos(1/2*d*x + 1/2*c) * \cos(5/2*d*x + 5/2*c) + (\sqrt{2})*a^2*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*a^2*\sin(1/2*d*x + 1/2*c)^2 * \cos(2*d*x + 2*c \\
&) + 2*(\sqrt{2})*a^2*\cos(2*d*x + 2*c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2})*a^2*\sin(
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c)) * \sin(5/2*d*x + 5/2*c)) * \sin(11/2*d*x + 11/2*c) + 7*(9*\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 + 9*\sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c)^2 - (5*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c)^2 + 5*\sqrt{2}) * a^2 * \sin(2*d*x + 2*c)^2 - 4*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c) - 9*\sqrt{2}) * a^2 * \cos(5/2*d*x + 5/2*c)^2 - 5*(\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c)^2 * \cos(2*d*x + 2*c)^2 - (5*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c)^2 + 5*\sqrt{2}) * a^2 * \sin(2*d*x + 2*c)^2 - 4*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c) - 9*\sqrt{2}) * a^2 * \sin(5/2*d*x + 5/2*c)^2 - 5*(\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c)^2 * \sin(2*d*x + 2*c)^2 - 2*(5*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c)^2 * \cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c) * \sin(2*d*x + 2*c)^2 - 4*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c) * \cos(1/2*d*x + 1/2*c) - 9*\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c) * \cos(5/2*d*x + 5/2*c) + 4*(\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c)^2 * \cos(2*d*x + 2*c) - 2*(5*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c)^2 * \sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}) * a^2 * \sin(2*d*x + 2*c)^2 * \sin(1/2*d*x + 1/2*c) - 4*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c) * \sin(1/2*d*x + 1/2*c) - 9*\sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c) * \sin(5/2*d*x + 5/2*c)) * \sin(9/2*d*x + 9/2*c) - 15*(35*\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 + 35*\sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c)^2 + (9*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c)^2 + 9*\sqrt{2}) * a^2 * \sin(2*d*x + 2*c)^2 + 44*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c) + 35*\sqrt{2}) * a^2 * \cos(5/2*d*x + 5/2*c)^2 + 9*(\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c)^2 * \cos(2*d*x + 2*c)^2 + (9*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c)^2 + 9*\sqrt{2}) * a^2 * \sin(2*d*x + 2*c)^2 + 44*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c) + 35*\sqrt{2}) * a^2 * \sin(5/2*d*x + 5/2*c)^2 + 9*(\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c)^2 * \sin(2*d*x + 2*c)^2 + 2*(9*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c)^2 * \cos(1/2*d*x + 1/2*c) + 9*\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c) * \sin(2*d*x + 2*c)^2 + 44*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c) * \cos(1/2*d*x + 1/2*c) + 35*\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c) * \cos(5/2*d*x + 5/2*c) + 44*(\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c)^2 * \cos(2*d*x + 2*c) + 2*(9*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c)^2 * \sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}) * a^2 * \sin(2*d*x + 2*c)^2 * \sin(1/2*d*x + 1/2*c) + 44*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c) * \sin(1/2*d*x + 1/2*c) + 35*\sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c) * \sin(5/2*d*x + 5/2*c)) * \sin(7/2*d*x + 7/2*c) - 21*(72*\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 + 72*\sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c)^2 + 3*(\sqrt{2}) * a^2 * \cos(2*d*x + 2*c)^2 + \sqrt{2}) * a^2 * \sin(2*d*x + 2*c)^2 + 25*\sqrt{2}) * a^2 * \cos(2*d*x + 2*c) + 24*\sqrt{2}) * a^2 * \cos(5/2*d*x + 5/2*c)^2 + (3*\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 - 50*\sqrt{2}) * a^2 * \sin(3/2*d*x + 3/2*c) * \sin(1/2*d*x + 1/2*c) - 117*\sqrt{2}) * a^2 * \sin(1/2*d*x + 1/2*c)^2 + 12*\sqrt{2}) * a^2 + 30*(a^2 * \log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}) * \cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \sin(1/2*d*x + 1/2*c) + 2) - a^2 * \log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}) * \cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}) * \sin(1/2*d*x + 1/2*c) + 2) + a^2 * \log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}) * \cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \sin(1/2*d*x + 1/2*c) + 2) - a^2 * \log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}) * \cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}) * \sin(1/2*d*x + 1/2*c) + 2)) * \sin(1/2*d*x + 1/2*c) * \cos(2*d*x + 2*c)^2 + (3*\sqrt{2}) * a^2 * \cos(1/2*d*x + 1/2*c)^2 - 50*\sqrt{2}) * a^2 * \sin(3/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 3/2*c)*\sin(1/2*d*x + 1/2*c) - 117*\sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 12*\sqrt{2}*a^2 + 30*(a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + a^2*\log(2*c \\
& os(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c) + 2))*\sin(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c)^2 + 12* \\
& sqrt{2}*a^2 + 6*(\sqrt{2}*a^2*\cos(2*d*x + 2*c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{ \\
& (2)*a^2*\cos(1/2*d*x + 1/2*c)*\sin(2*d*x + 2*c)^2 + 25*\sqrt{2}*a^2*\cos(2*d*x \\
& + 2*c)*\cos(1/2*d*x + 1/2*c) - 23*\sqrt{2}*a^2*\sin(2*d*x + 2*c)*\sin(1/2*d*x + \\
& 1/2*c) + 24*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c) + (75*s \\
& qrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^2 - 50*\sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c)*\sin \\
& (1/2*d*x + 1/2*c) - 45*\sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c)^2 + 24*\sqrt{2}*a^2 \\
& + 60*(a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*c \\
& os(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + a^2*\log(2*\cos(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2))*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) - 10*(5*\sqrt{2}*a^2*\cos(\\
& 3/2*d*x + 3/2*c)*\sin(1/2*d*x + 1/2*c) + 12*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c) \\
& *\sin(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 30*(a^2*\log(2*\cos(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c) + 2) + a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - a^2*l \\
& og(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\sin(1/2*d*x + 1/2*c))*s \\
& in(5/2*d*x + 5/2*c) + 105*(12*\sqrt{2}*a^2*\cos(1/2*d*x + 1/2*c)^3 + 12*\sqrt{2} \\
& (2)*a^2*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c)^2 + 5*(\sqrt{2}*a^2*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c)^2)*\cos(3/2*d*x + 3/2*c) \\
&)*\sin(2*d*x + 2*c))*\sqrt{a}/(((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*c \\
& os(2*d*x + 2*c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + (\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(2*d*x + 2*c)^2 + (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + \\
& 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + (\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2)*\sin(2*d*x + 2*c)^2 + 2*(\cos(2*d*x + 2*c)^ \\
& 2*\cos(1/2*d*x + 1/2*c) + \cos(1/2*d*x + 1/2*c)*\sin(2*d*x + 2*c)^2 + 2*\cos(2* \\
& d*x + 2*c)*\cos(1/2*d*x + 1/2*c) + \cos(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c \\
&) + 2*(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2)*\cos(2*d*x + 2*c) + \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*(\cos(2*d*x + 2*c)^2*\sin(1/2*d*x + 1/2*c) + \sin(2 \\
& *d*x + 2*c)^2*\sin(1/2*d*x + 1/2*c) + 2*\cos(2*d*x + 2*c)*\sin(1/2*d*x + 1/2*c
\end{aligned}$$

) + sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + sin(1/2*d*x + 1/2*c)^2)*d)

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.47

$$\int (a + a \cos(c + dx))^{5/2} \sec^2(c + dx) dx =$$

$$\frac{\sqrt{2} \left(5 \sqrt{2} a^2 \log \left(\frac{|-2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) - 8 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) + \dots \right)}{4 d}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")

[Out] -1/4*sqrt(2)*(5*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) - 8*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c) + 4*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/(2*sin(1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \int \frac{(a + a \cos(c + dx))^{5/2}}{\cos(c + dx)^2} dx$$

[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)

[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)

3.118 $\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx$

Optimal result	911
Rubi [A] (verified)	911
Mathematica [C] (verified)	913
Maple [B] (verified)	913
Fricas [A] (verification not implemented)	914
Sympy [F(-1)]	914
Maxima [B] (verification not implemented)	915
Giac [A] (verification not implemented)	917
Mupad [F(-1)]	918

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \frac{19a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4d} + \frac{9a^3 \tan(c + dx)}{4d\sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d}$$

[Out] $19/4*a^{(5/2)}*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+9/4*a^3*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+1/2*a^2*\sec(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2841, 3059, 2852, 212}

$$\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \frac{19a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4d} + \frac{9a^3 \tan(c + dx)}{4d\sqrt{a \cos(c + dx) + a}} + \frac{a^2 \tan(c + dx) \sec(c + dx) \sqrt{a \cos(c + dx) + a}}{2d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^{(5/2)}*\operatorname{Sec}[c + d*x]^3, x]$

[Out] $(19*a^{(5/2)}*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])])/(4*d) + (9*a^3*\operatorname{Tan}[c + d*x])/(4*d*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]]) + (a^2*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]]*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d)$

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2841

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2852

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3059

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d} \\ &\quad - \frac{1}{2}a \int \left(-\frac{9a}{2} - \frac{5}{2}a \cos(c + dx) \right) \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx \\ &= \frac{9a^3 \tan(c + dx)}{4d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d} \\ &\quad + \frac{1}{8}(19a^2) \int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{9a^3 \tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad - \frac{(19a^3)\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} \\
&= \frac{19a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{9a^3 \tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{a^2\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.52

$$\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx) dx = \frac{a^2\sqrt{a(1+\cos(c+dx))}\sec^5\left(\frac{1}{2}(c+dx)\right)\left(19\sqrt{2}\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)+19\sqrt{2}\cos(2(c+dx))\right)}{1}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]

[Out] -1/32*(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^5*(19*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 19*Sqrt[2]*Cos[2*(c + d*x)])*(Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) - 19*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 28*Sin[(c + d*x)/2] - 44*Sin[(3*(c + d*x))/2]))/(d*(-1 + Tan[(c + d*x)/2]^2)^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(90) = 180.

Time = 27.53 (sec) , antiderivative size = 551, normalized size of antiderivative = 5.20

method	result
default	$a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(76a\left(\ln\left(\frac{4\sqrt{2}a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a+8a}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)\right)+\ln\left(-\frac{4\left(\sqrt{2}a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right)}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\right)$

[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)

```
[Out] 1/2*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(76*a*(ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^4+(-44*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-76*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a-76*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+26*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+19*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+19*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.60

$$\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \frac{19 (a^2 \cos(dx + c)^3 + a^2 \cos(dx + c)^2) \sqrt{a} \log\left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - 4\sqrt{a} \cos(dx+c) + a\sqrt{a}(\cos(dx+c)-2)}{\cos(dx+c)^3 + \cos(dx+c)^2}\right)}{16 (d \cos(dx + c))^3 + d \cos(dx + c)}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(19*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3667 vs. $2(90) = 180$.

Time = 3.25 (sec) , antiderivative size = 3667, normalized size of antiderivative = 34.59

$$\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")

[Out] $-1/16*(150*\sqrt{2}*a^2*\cos(7/2*d*x + 7/2*c)*\sin(2*d*x + 2*c) + 154*\sqrt{2}*a^2*\cos(5/2*d*x + 5/2*c)*\sin(2*d*x + 2*c) - 28*\sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c) + 44*\sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c) - (3*\sqrt{2}*a^2*\sin(7/2*d*x + 7/2*c) + 5*\sqrt{2}*a^2*\sin(5/2*d*x + 5/2*c) - 17*\sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c) - 55*\sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c) + 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(4*d*x + 4*c)^2 + 4*(17*\sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c) + 55*\sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c) - 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(2*d*x + 2*c)^2 - 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - (3*\sqrt{2}*a^2*\sin(7/2*d*x + 7/2*c) + 5*\sqrt{2}*a^2*\sin(5/2*d*x + 5/2*c) - 17*\sqrt{2}*a^2*\sin(3/2*d*x + 3/2*c) - 55*\sqrt{2}*a^2*\sin(1/2*d*x + 1/2*c) + 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*\log(2*\cos(1/2*d*x + 1/2*c))^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*($


```

t(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*s
in(13/2*d*x + 13/2*c) - 11*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*co
s(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/2*d*x + 11/2*c) - 45*(sqrt(2)*a^2*cos(
4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/2*d*x +
9/2*c) - (12*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 20*sqrt(2)
*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 75*sqrt(2)*a^2*cos(7/2*d*x + 7
/2*c) - 77*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) - 45*sqrt(2)*a^2*cos(3/2*d*x +
3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) - 4*(17*sqrt(2)*a^2*sin(3/2*d*
x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*
d*x + 2*c))*sin(4*d*x + 4*c) - 6*(2*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 2*sqrt
(2)*a^2*sin(2*d*x + 2*c)^2 + 27*sqrt(2)*a^2*cos(2*d*x + 2*c) + 13*sqrt(2)*a
^2)*sin(7/2*d*x + 7/2*c) - 2*(10*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 10*sqrt(2
)*a^2*sin(2*d*x + 2*c)^2 + 87*sqrt(2)*a^2*cos(2*d*x + 2*c) + 41*sqrt(2)*a^2
)*sin(5/2*d*x + 5/2*c) + 2*(45*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 11*sqrt(2
)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sqrt(a)/((2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
+ 4*cos(2*d*x + 2*c) + 1)*d)

```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32

$$\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx =$$

$$\frac{\sqrt{2} \left(19 \sqrt{2} a^2 \log \left(\frac{\left| -2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|}{\left| 2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right|} \right) \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{4 \left(22 a^2 \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 13 a^2 \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{\left(2 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1 \right)} \right)}{16 d}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")

```

[Out] -1/16*sqrt(2)*(19*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/
abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(22*
a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 - 13*a^2*sgn(cos(1/2*d
*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a
)/d

```

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \int \frac{(a + a \cos(c + dx))^{5/2}}{\cos(c + dx)^3} dx$$

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

3.119 $\int (a + a \cos(c + dx))^{5/2} \sec^4(c + dx) dx$

Optimal result	919
Rubi [A] (verified)	919
Mathematica [C] (verified)	922
Maple [B] (verified)	922
Fricas [A] (verification not implemented)	923
Sympy [F(-1)]	923
Maxima [B] (verification not implemented)	924
Giac [A] (verification not implemented)	928
Mupad [F(-1)]	929

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int (a + a \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \frac{25a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{8d} + \frac{25a^3 \tan(c + dx)}{8d \sqrt{a + a \cos(c + dx)}} + \frac{13a^3 \sec(c + dx) \tan(c + dx)}{12d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) \tan(c + dx)}{3d}$$

[Out] 25/8*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+25/8*a^3*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+13/12*a^3*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a^2*sec(d*x+c)^2*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2841, 3059, 2851, 2852, 212}

$$\int (a + a \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \frac{25a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{8d} + \frac{25a^3 \tan(c + dx)}{8d \sqrt{a \cos(c + dx) + a}} + \frac{13a^3 \tan(c + dx) \sec(c + dx)}{12d \sqrt{a \cos(c + dx) + a}} + \frac{a^2 \tan(c + dx) \sec^2(c + dx) \sqrt{a \cos(c + dx) + a}}{3d}$$

[In] Int[(a + a*cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]

```
[Out] (25*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/(8*d)
+ (25*a^3*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (13*a^3*Sec[c + d
*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c +
d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_) + (
f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]))], x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/((c_) + (d_.)*sin[(e_) + (
f_.)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_) + (
f_.)*(x_)])*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]))], x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
```

*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
 1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) \tan(c + dx)}{3d} \\
 &\quad - \frac{1}{3} a \int \left(-\frac{13a}{2} - \frac{9}{2} a \cos(c + dx) \right) \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx \\
 &= \frac{13a^3 \sec(c + dx) \tan(c + dx)}{12d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) \tan(c + dx)}{3d} \\
 &\quad + \frac{1}{8} (25a^2) \int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx \\
 &= \frac{25a^3 \tan(c + dx)}{8d \sqrt{a + a \cos(c + dx)}} + \frac{13a^3 \sec(c + dx) \tan(c + dx)}{12d \sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) \tan(c + dx)}{3d} \\
 &\quad + \frac{1}{16} (25a^2) \int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx \\
 &= \frac{25a^3 \tan(c + dx)}{8d \sqrt{a + a \cos(c + dx)}} + \frac{13a^3 \sec(c + dx) \tan(c + dx)}{12d \sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) \tan(c + dx)}{3d} \\
 &\quad - \frac{(25a^3) \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{8d} \\
 &= \frac{25a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{8d} + \frac{25a^3 \tan(c + dx)}{8d \sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{13a^3 \sec(c + dx) \tan(c + dx)}{12d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) \tan(c + dx)}{3d}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.97

$$\int (a + a \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \frac{a^2 \sqrt{a(1 + \cos(c + dx))} \sec^7\left(\frac{1}{2}(c + dx)\right) \left(225\sqrt{2} \cos(c + dx) \left(\log\left(i - \sqrt{2}e^{\frac{1}{2}i(c+dx)} - ie^{i(c+dx)}\right) - 1\right)\right)}{}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^7*(225*Sqrt[2]*Cos[c + d*x]*
(Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) + 75*Sqrt[2]*Cos[3*(c + d*x)]*
(Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) - 4*(114*Sin[(c + d*x)/2] - 7*Sin[(3*(c + d*x))/2] + 75*Sin[(5*(c + d*x))/2])))/(384*d*(-1 + Tan[(c + d*x)/2]^2)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. 2(124) = 248.

Time = 96.06 (sec) , antiderivative size = 717, normalized size of antiderivative = 4.98

method	result
default	$a^{\frac{3}{2}} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-600a \left(\ln\left(\frac{4\sqrt{2}a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a+8a}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}} \right) + \ln\left(-\frac{4 \left(\sqrt{2}a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \sqrt{2} \sqrt{a}\right)}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)} \right) \right)$

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-600*a*(ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^6+300*(2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+3*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-736*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-450*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))
```

$$-2*a)) * a - 450 * \ln(4 / (2 * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)} * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} + 2 * a)) * a * \sin(1/2 * d * x + 1/2 * c)^2 + 234 * 2^{(1/2)} * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} + 75 * \ln(-4 / (2 * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)} * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} - 2 * a)) * a + 75 * \ln(4 / (2 * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)} * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} + 2 * a)) * a / (2 * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)})^3 / (2 * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)})^3 / \sin(1/2 * d * x + 1/2 * c) / (a * \cos(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / d$$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.27

$$\int (a + a \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \frac{75 (a^2 \cos(dx + c)^4 + a^2 \cos(dx + c)^3) \sqrt{a} \log\left(\frac{a \cos(dx + c)^3 - 7a \cos(dx + c)^2 - 4\sqrt{a} \cos(dx + c) + a\sqrt{a}(\cos(dx + c) - 2)}{\cos(dx + c)^3 + \cos(dx + c)^2}\right) + 4 * (75 * a^2 * \cos(dx + c)^2 + 34 * a^2 * \cos(dx + c) + 8 * a^2) * \sqrt{a * \cos(dx + c) + a} * \sin(dx + c)}{96 (d \cos(dx + c))^4}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")

[Out] 1/96*(75*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)

[Out] Timed out

$$\begin{aligned}
& c) + 2) - 15\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2) - 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2 \\
&) - 34a^2\sin(3/2dx + 3/2c) - 120a^2\sin(1/2dx + 1/2c)\cos(2dx + 2c)^2 - (75\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 75\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + 7 \\
& 5\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 75\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 10a^2\sin(9/2dx + 9/2c) + 30a^2\sin(7/2dx + 7/2c) + 78a^2\sin(5/2dx + 5/2c) - 170a^2\sin(3/2dx + 3/2c) - 600a^2\sin(1/2dx + 1/2c)\sin(6dx + 6c)^2 - 9(75\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 75\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + 75\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 75\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + 30a^2\sin(7/2dx + 7/2c) + 78a^2\sin(5/2dx + 5/2c) - 170a^2\sin(3/2dx + 3/2c) - 600a^2\sin(1/2dx + 1/2c)\sin(4dx + 4c)^2 - 45(15\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 15\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + 15\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 15\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 34a^2\sin(3/2dx + 3/2c) - 120a^2\sin(1/2dx + 1/2c)\sin(2dx + 2c)^2 - 56a^2\sin(3/2dx + 3/2c) + 600a^2\sin(1/2dx + 1/2c) + 10(a^2\sin(6dx + 6c) + 3a^2\sin(4dx + 4c) + 3a^2\sin(2dx + 2c))\cos(21/2dx + 21/2c) - 30(a^2\sin(6dx + 6c) + 3a^2\sin(4dx + 4c) + 3a^2\sin(2dx + 2c))\cos(19/2dx + 19/2c) - 48(a^2\sin(6dx + 6c) + 3a^2\sin(4dx + 4c) + 3a^2\sin(2dx + 2c))\cos(17/2dx + 17/2c) + 80(a^2\sin(6dx + 6c) + 3a^2\sin(4dx + 4c) + 3a^2\sin(2dx + 2c))\cos(15/2dx + 15/2c) + 396(a^2\sin(6dx + 6c) + 3a^2\sin(4dx + 4c) + 3a^2\sin(2dx + 2c))\cos(13/2dx + 13/2c) - 6(25\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 25\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 + 2\sqrt{2}\cos(1/2dx + 1/2c) - 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) + 2 \\
& 5\sqrt{2}a^2\log(2\cos(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c)^2 - 2\sqrt{2}\cos(1/2dx + 1/2c) + 2\sqrt{2}\sin(1/2dx + 1/2c) + 2) - 25\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& (2)*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 170*a^2*\sin(1 \\
& 1/2*d*x + 11/2*c) - 19*a^2*\sin(3/2*d*x + 3/2*c) - 200*a^2*\sin(1/2*d*x + 1/2 \\
& *c) + (75*\sqrt{2})*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& 75*\sqrt{2})*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 75*\sqrt{2} \\
& *a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 75*\sqrt{2})* \\
& a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 30*a^2*\sin(7/2*d* \\
& x + 7/2*c) + 78*a^2*\sin(5/2*d*x + 5/2*c) - 170*a^2*\sin(3/2*d*x + 3/2*c) - 6 \\
& 00*a^2*\sin(1/2*d*x + 1/2*c))*\cos(4*d*x + 4*c) + 5*(15*\sqrt{2})*a^2*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2})*a^2*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 15*\sqrt{2})*a^2*\log(2*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2})*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - 34*a^2*\sin(3/2*d*x + 3/2*c) - 120*a^2*\sin(1/2*d* \\
& x + 1/2*c))*\cos(2*d*x + 2*c) - 10*(a^2*\cos(4*d*x + 4*c) + a^2*\cos(2*d*x + 2 \\
& *c) - 25*a^2)*\sin(9/2*d*x + 9/2*c) + 2*(15*a^2*\cos(2*d*x + 2*c) + 121*a^2)* \\
& \sin(7/2*d*x + 7/2*c) + (78*a^2*\cos(2*d*x + 2*c) + 161*a^2)*\sin(5/2*d*x + 5/ \\
& 2*c))*\cos(6*d*x + 6*c) + 3060*(a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c)) \\
& *\cos(11/2*d*x + 11/2*c) + 4560*(a^2*\sin(4*d*x + 4*c) + a^2*\sin(2*d*x + 2*c) \\
&)*\cos(9/2*d*x + 9/2*c) - 18*(25*\sqrt{2})*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c) + 2) - 25*\sqrt{2})*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) + 25*\sqrt{2})*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c) + 2) - 25*\sqrt{2})*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - 19*a^2*\sin(3/2*d*x + 3/2*c) - 200*a^2*\sin(1/2*d*x + 1/2*c) + 5*(15*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2})*a \\
& ^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 15*\sqrt{2})*a^2*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2})*a^2*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 34*a^2*\sin(3/2*d*x + 3/2*c) - \\
& 120*a^2*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 2*(15*a^2*\cos(2*d*x + 2*c) \\
& + 121*a^2)*\sin(7/2*d*x + 7/2*c) + (78*a^2*\cos(2*d*x + 2*c) + 161*a^2)*\sin(
\end{aligned}$$

$$\begin{aligned}
& 5/2*d*x + 5/2*c)) * \cos(4*d*x + 4*c) - 18*(25*\sqrt{2}*a^2*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*s \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 25*\sqrt{2}*a^2*\log(2*\cos(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) + 25*\sqrt{2}*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c) + 2) - 25*\sqrt{2}*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - 19*a^2*\sin(3/2*d*x + 3/2*c) - 200*a^2*\sin(1/2*d*x + 1/2*c) \\
&) * \cos(2*d*x + 2*c) - 10*(a^2*\cos(6*d*x + 6*c) + 3*a^2*\cos(4*d*x + 4*c) + 3* \\
& a^2*\cos(2*d*x + 2*c) + a^2)*\sin(21/2*d*x + 21/2*c) + 30*(a^2*\cos(6*d*x + 6* \\
& c) + 3*a^2*\cos(4*d*x + 4*c) + 3*a^2*\cos(2*d*x + 2*c) + a^2)*\sin(19/2*d*x + \\
& 19/2*c) + 48*(a^2*\cos(6*d*x + 6*c) + 3*a^2*\cos(4*d*x + 4*c) + 3*a^2*\cos(2*d \\
& *x + 2*c) + a^2)*\sin(17/2*d*x + 17/2*c) - 80*(a^2*\cos(6*d*x + 6*c) + 3*a^2* \\
& \cos(4*d*x + 4*c) + 3*a^2*\cos(2*d*x + 2*c) + a^2)*\sin(15/2*d*x + 15/2*c) - 3 \\
& 96*(a^2*\cos(6*d*x + 6*c) + 3*a^2*\cos(4*d*x + 4*c) + 3*a^2*\cos(2*d*x + 2*c) \\
& + a^2)*\sin(13/2*d*x + 13/2*c) - 2*(90*a^2*\sin(7/2*d*x + 7/2*c))*\sin(2*d*x + \\
& 2*c) + 234*a^2*\sin(5/2*d*x + 5/2*c))*\sin(2*d*x + 2*c) - 510*a^2*\cos(11/2*d*x \\
& + 11/2*c) - 760*a^2*\cos(9/2*d*x + 9/2*c) - 696*a^2*\cos(7/2*d*x + 7/2*c) - \\
& 405*a^2*\cos(5/2*d*x + 5/2*c) - 113*a^2*\cos(3/2*d*x + 3/2*c) - 30*(a^2*\sin(4 \\
& *d*x + 4*c) + a^2*\sin(2*d*x + 2*c))*\sin(9/2*d*x + 9/2*c) + 3*(75*\sqrt{2}*a^ \\
& 2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 75*\sqrt{2}*a^2*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 75*\sqrt{2}*a^2*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 75*\sqrt{2}*a^2*\log(2*\cos(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 30*a^2*\sin(7/2*d*x + 7/2*c) + 78*a^2* \\
& \sin(5/2*d*x + 5/2*c) - 170*a^2*\sin(3/2*d*x + 3/2*c) - 600*a^2*\sin(1/2*d*x + \\
& 1/2*c))*\sin(4*d*x + 4*c) + 15*(15*\sqrt{2}*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*si \\
& n(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2}*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c) + 2) + 15*\sqrt{2}*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - 15*\sqrt{2}*a^2*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) - 34*a^2*\sin(3/2*d*x + 3/2*c) - 120*a^2*\sin(1/2*d*x + 1/2*c))*\sin(2*d \\
& *x + 2*c))*\sin(6*d*x + 6*c) - 1020*(3*a^2*\cos(4*d*x + 4*c) + 3*a^2*\cos(2*d* \\
& x + 2*c) + a^2)*\sin(11/2*d*x + 11/2*c) + 10*(9*a^2*\cos(4*d*x + 4*c)^2 + 9*a \\
& ^2*\cos(2*d*x + 2*c)^2 + 9*a^2*\sin(4*d*x + 4*c)^2 + 18*a^2*\sin(4*d*x + 4*c)* \\
& \sin(2*d*x + 2*c) + 9*a^2*\sin(2*d*x + 2*c)^2 - 450*a^2*\cos(2*d*x + 2*c) - 15 \\
& 1*a^2 + 18*(a^2*\cos(2*d*x + 2*c) - 25*a^2)*\cos(4*d*x + 4*c))*\sin(9/2*d*x + \\
& 9/2*c) - 6*(90*a^2*\sin(7/2*d*x + 7/2*c))*\sin(2*d*x + 2*c) + 234*a^2*\sin(5/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 5/2*c)*\sin(2*d*x + 2*c) - 696*a^2*\cos(7/2*d*x + 7/2*c) - 405*a^2*\cos(\\
& 5/2*d*x + 5/2*c) - 113*a^2*\cos(3/2*d*x + 3/2*c) + 15*(15*\sqrt{2})*a^2*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2})*a^2*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 15*\sqrt{2})*a^2*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2})*a^2*\log(2*\cos(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c) + 2) - 34*a^2*\sin(3/2*d*x + 3/2*c) - 120*a^2*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(2*d*x + 2*c))*\sin(4*d*x + 4*c) - 18*(15*a^2*\cos(2*d*x + \\
& 2*c)^2 + 15*a^2*\sin(2*d*x + 2*c)^2 + 242*a^2*\cos(2*d*x + 2*c) + 79*a^2)*\sin \\
& (7/2*d*x + 7/2*c) - 6*(117*a^2*\cos(2*d*x + 2*c)^2 + 117*a^2*\sin(2*d*x + 2*c \\
&)^2 + 483*a^2*\cos(2*d*x + 2*c) + 148*a^2)*\sin(5/2*d*x + 5/2*c))*\sqrt{a}/((s \\
& \sqrt{2}*\cos(6*d*x + 6*c)^2 + 9*\sqrt{2}*\cos(4*d*x + 4*c)^2 + 9*\sqrt{2}*\cos(2* \\
& d*x + 2*c)^2 + \sqrt{2}*\sin(6*d*x + 6*c)^2 + 9*\sqrt{2}*\sin(4*d*x + 4*c)^2 + \\
& 18*\sqrt{2}*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 \\
& + 2*(3*\sqrt{2}*\cos(4*d*x + 4*c) + 3*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2}))*\co \\
& s(6*d*x + 6*c) + 6*(3*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2}))*\cos(4*d*x + 4*c) \\
& + 6*(\sqrt{2}*\sin(4*d*x + 4*c) + \sqrt{2}*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) \\
& + 6*\sqrt{2}*\cos(2*d*x + 2*c) + \sqrt{2}))*d)
\end{aligned}$$

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.15

$$\int (a + a \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \frac{\sqrt{2} \left(75 \sqrt{2} a^2 \log \left(\frac{-2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{2\sqrt{2} + 4 \sin(\frac{1}{2} dx + \frac{1}{2} c)} \right) \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) + \frac{4 \left(300 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) \right)^5 - 368 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 + 117 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)}{(2 \sin(\frac{1}{2} dx + \frac{1}{2} c))^2 - 1} \right)^3 \sqrt{a}}{96 d}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")

[Out] -1/96*sqrt(2)*(75*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(300*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 - 368*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^3 + 117*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^3*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \int \frac{(a + a \cos(c + dx))^{5/2}}{\cos(c + dx)^4} dx$$

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

3.120 $\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx$

Optimal result	930
Rubi [A] (verified)	930
Mathematica [C] (verified)	933
Maple [B] (verified)	933
Fricas [A] (verification not implemented)	934
Sympy [F(-1)]	935
Maxima [F(-1)]	935
Giac [A] (verification not implemented)	935
Mupad [F(-1)]	936

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx = \frac{163a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{64d}$$

$$+ \frac{163a^3 \tan(c + dx)}{64d \sqrt{a + a \cos(c + dx)}} + \frac{163a^3 \sec(c + dx) \tan(c + dx)}{96d \sqrt{a + a \cos(c + dx)}}$$

$$+ \frac{17a^3 \sec^2(c + dx) \tan(c + dx)}{24d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) \tan(c + dx)}{4d}$$

[Out] 163/64*a^(5/2)*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d+163/64*a^3*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+163/96*a^3*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+17/24*a^3*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/4*a^2*sec(d*x+c)^3*(a+a*cos(d*x+c))^(1/2)*tan(d*x+c)/d

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2841, 3059, 2851, 2852, 212}

$$\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx = \frac{163a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{64d}$$

$$+ \frac{163a^3 \tan(c + dx)}{64d \sqrt{a \cos(c + dx) + a}} + \frac{17a^3 \tan(c + dx) \sec^2(c + dx)}{24d \sqrt{a \cos(c + dx) + a}}$$

$$+ \frac{163a^3 \tan(c + dx) \sec(c + dx)}{96d \sqrt{a \cos(c + dx) + a}} + \frac{a^2 \tan(c + dx) \sec^3(c + dx) \sqrt{a \cos(c + dx) + a}}{4d}$$

[In] Int[(a + a*cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]

[Out] (163*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*cos[c + d*x]])/(64*d) + (163*a^3*Tan[c + d*x])/(64*d*Sqrt[a + a*cos[c + d*x]]) + (163*a^3*Sec[c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*cos[c + d*x]]) + (17*a^3*Sec[c + d*x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*cos[c + d*x]]) + (a^2*Sqrt[a + a*cos[c + d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2841

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2851

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rule 2852

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3059

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp

```
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) \tan(c + dx)}{4d} \\
&\quad - \frac{1}{4} a \int \left(-\frac{17a}{2} - \frac{13}{2} a \cos(c + dx) \right) \sqrt{a + a \cos(c + dx)} \sec^4(c + dx) dx \\
&= \frac{17a^3 \sec^2(c + dx) \tan(c + dx)}{24d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) \tan(c + dx)}{4d} \\
&\quad + \frac{1}{48} (163a^2) \int \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) dx \\
&= \frac{163a^3 \sec(c + dx) \tan(c + dx)}{96d \sqrt{a + a \cos(c + dx)}} + \frac{17a^3 \sec^2(c + dx) \tan(c + dx)}{24d \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) \tan(c + dx)}{4d} \\
&\quad + \frac{1}{64} (163a^2) \int \sqrt{a + a \cos(c + dx)} \sec^2(c + dx) dx \\
&= \frac{163a^3 \tan(c + dx)}{64d \sqrt{a + a \cos(c + dx)}} + \frac{163a^3 \sec(c + dx) \tan(c + dx)}{96d \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{17a^3 \sec^2(c + dx) \tan(c + dx)}{24d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) \tan(c + dx)}{4d} \\
&\quad + \frac{1}{128} (163a^2) \int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx \\
&= \frac{163a^3 \tan(c + dx)}{64d \sqrt{a + a \cos(c + dx)}} + \frac{163a^3 \sec(c + dx) \tan(c + dx)}{96d \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{17a^3 \sec^2(c + dx) \tan(c + dx)}{24d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{a + a \cos(c + dx)} \sec^3(c + dx) \tan(c + dx)}{4d} \\
&\quad - \frac{(163a^3) \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{64d}
\end{aligned}$$

$$= \frac{163a^{5/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{64d} + \frac{163a^3 \tan(c+dx)}{64d\sqrt{a+a \cos(c+dx)}} + \frac{163a^3 \sec(c+dx) \tan(c+dx)}{96d\sqrt{a+a \cos(c+dx)}} \\ + \frac{17a^3 \sec^2(c+dx) \tan(c+dx)}{24d\sqrt{a+a \cos(c+dx)}} + \frac{a^2 \sqrt{a+a \cos(c+dx)} \sec^3(c+dx) \tan(c+dx)}{4d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.69 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.13

$$\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx = \frac{a^2 \sqrt{a(1 + \cos(c + dx))} \sec^9\left(\frac{1}{2}(c + dx)\right) \left(1467\sqrt{2} \log\left(i - \sqrt{2}e^{\frac{1}{2}i(c+dx)} - ie^{i(c+dx)}\right) + 1956\sqrt{2} \cos(2(c + dx))\right)}{1}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]

[Out] -1/6144*(a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]^9*(1467*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 1956*Sqrt[2]*Cos[2*(c + d*x)]*(Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) + 489*Sqrt[2]*Cos[4*(c + d*x)]*(Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) - 1467*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 2060*Sin[(c + d*x)/2] - 6204*Sin[(3*(c + d*x))/2] - 652*Sin[(5*(c + d*x))/2] - 1956*Sin[(7*(c + d*x))/2]))/(d*(-1 + Tan[(c + d*x)/2]^2)^4)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 881 vs. 2(158) = 316.

Time = 299.26 (sec) , antiderivative size = 882, normalized size of antiderivative = 4.85

method	result	size
default	Expression too large to display	882

[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] 1/24*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7824*a*(ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*sin(1/2*d*x+1/2*c)^8-7824*(2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)

$$2) * a^{(1/2)} + 2 * \ln(4 / (2 * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)}) * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} + 2 * a) * a + 2 * \ln(-4 / (2 * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)}) * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} - 2 * a) * a) * \sin(1/2 * d * x + 1/2 * c)^6 + 1304 * (11 * 2^{(1/2)} * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} + 9 * \ln(4 / (2 * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)}) * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} + 2 * a) * a + 9 * \ln(-4 / (2 * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)}) * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} - 2 * a) * a) * \sin(1/2 * d * x + 1/2 * c)^4 + (-9212 * 2^{(1/2)} * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} - 3912 * \ln(4 / (2 * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)}) * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} + 2 * a) * a - 3912 * \ln(-4 / (2 * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)}) * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} - 2 * a) * a) * \sin(1/2 * d * x + 1/2 * c)^2 + 2094 * 2^{(1/2)} * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} + 489 * \ln(4 / (2 * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)}) * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} + 2 * a) * a + 489 * \ln(-4 / (2 * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)})) * (2^{(1/2)} * a * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)}) * (a * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a^{(1/2)} - 2 * a) * a) / (2 * \cos(1/2 * d * x + 1/2 * c) + 2^{(1/2)})^4 / (2 * \cos(1/2 * d * x + 1/2 * c) - 2^{(1/2)})^4 / \sin(1/2 * d * x + 1/2 * c) / (a * \cos(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / d$$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08

$$\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx = \frac{489 (a^2 \cos(dx + c)^5 + a^2 \cos(dx + c)^4) \sqrt{a} \log\left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - 4\sqrt{a \cos(dx+c)} + a\sqrt{a}(\cos(dx+c) - 2)}{\cos(dx+c)^3 + \cos(dx+c)^2}\right)}{768 (d$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="fricas")

[Out] 1/768*(489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**5,x)

[Out] Timed out

Maxima [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="maxima")

[Out] Timed out

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05

$$\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx =$$

$$\sqrt{2} \left(489 \sqrt{2} a^2 \log \left(\frac{|-2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|} \right) \operatorname{sgn} \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) + \frac{4 \left(3912 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c) \right)^7 - 7172 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^5 + 4606 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 1047 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)}{(2 \sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^4} \right) \sqrt{a} / d$$

768 d

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="giac")

```
[Out] -1/768*sqrt(2)*(489*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))
)/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))*sgn(cos(1/2*d*x + 1/2*c)) + 4*(3
912*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^7 - 7172*a^2*sgn(cos
(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)^5 + 4606*a^2*sgn(cos(1/2*d*x + 1/2*
c))*sin(1/2*d*x + 1/2*c)^3 - 1047*a^2*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x
+ 1/2*c))/(2*sin(1/2*d*x + 1/2*c)^2 - 1)^4)*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^5(c + dx) dx = \int \frac{(a + a \cos(c + dx))^{5/2}}{\cos(c + dx)^5} dx$$

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^5, x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^5, x)
```

3.121 $\int (a + a \cos(c + dx))^{7/2} dx$

Optimal result	937
Rubi [A] (verified)	937
Mathematica [A] (verified)	938
Maple [A] (verified)	939
Fricas [A] (verification not implemented)	939
Sympy [F(-1)]	939
Maxima [A] (verification not implemented)	940
Giac [A] (verification not implemented)	940
Mupad [F(-1)]	940

Optimal result

Integrand size = 14, antiderivative size = 119

$$\int (a + a \cos(c + dx))^{7/2} dx = \frac{256a^4 \sin(c + dx)}{35d\sqrt{a + a \cos(c + dx)}} + \frac{64a^3 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{35d} + \frac{24a^2 (a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2a(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d}$$

[Out] $24/35*a^2*(a+a*\cos(d*x+c))^{(3/2)}*\sin(d*x+c)/d+2/7*a*(a+a*\cos(d*x+c))^{(5/2)}*\sin(d*x+c)/d+256/35*a^4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+64/35*a^3*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2726, 2725}

$$\int (a + a \cos(c + dx))^{7/2} dx = \frac{256a^4 \sin(c + dx)}{35d\sqrt{a \cos(c + dx) + a}} + \frac{64a^3 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{35d} + \frac{24a^2 \sin(c + dx) (a \cos(c + dx) + a)^{3/2}}{35d} + \frac{2a \sin(c + dx) (a \cos(c + dx) + a)^{5/2}}{7d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(7/2)}, x]$

[Out] $(256a^4 \sin[c + dx]) / (35d \sqrt{a + a \cos[c + dx]}) + (64a^3 \sqrt{a + a \cos[c + dx]} \sin[c + dx]) / (35d) + (24a^2 (a + a \cos[c + dx])^{3/2} \sin[c + dx]) / (35d) + (2a (a + a \cos[c + dx])^{5/2} \sin[c + dx]) / (7d)$

Rule 2725

$\text{Int}[\sqrt{(a_) + (b_.) \sin[(c_) + (d_.) (x_)]}], x_Symbol] \rightarrow \text{Simp}[-2b(\cos[c + dx] / (d \sqrt{a + b \sin[c + dx]})), x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2726

$\text{Int}[(a_) + (b_.) \sin[(c_) + (d_.) (x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-b) \cos[c + dx] * ((a + b \sin[c + dx])^{(n-1)} / (d^n)), x] + \text{Dist}[a * ((2*n - 1) / n), \text{Int}[(a + b \sin[c + dx])^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[n - 1/2, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d} + \frac{1}{7}(12a) \int (a + a \cos(c + dx))^{5/2} dx \\
 &= \frac{24a^2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2a(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\
 &\quad + \frac{1}{35}(96a^2) \int (a + a \cos(c + dx))^{3/2} dx \\
 &= \frac{64a^3 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{35d} + \frac{24a^2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35d} \\
 &\quad + \frac{2a(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d} + \frac{1}{35}(128a^3) \int \sqrt{a + a \cos(c + dx)} dx \\
 &= \frac{256a^4 \sin(c + dx)}{35d \sqrt{a + a \cos(c + dx)}} + \frac{64a^3 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{35d} \\
 &\quad + \frac{24a^2(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2a(a + a \cos(c + dx))^{5/2} \sin(c + dx)}{7d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int (a + a \cos(c + dx))^{7/2} dx = \frac{a^3 \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(1225 \sin\left(\frac{1}{2}(c + dx)\right) + 245 \sin\left(\frac{3}{2}(c + dx)\right) + 49 \sin\left(\frac{5}{2}(c + dx)\right)\right)}{140d}$$

[In] $\text{Integrate}[(a + a \cos[c + dx])^{7/2}, x]$

[Out] $(a^3 \sqrt{a(1 + \cos[c + d*x])} \sec[(c + d*x)/2] * (1225 \sin[(c + d*x)/2] + 245 \sin[(3*(c + d*x))/2] + 49 \sin[(5*(c + d*x))/2] + 5 \sin[(7*(c + d*x))/2]) / (140*d)$

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{16a^4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(5 \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 6 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 8 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 16\right) \sqrt{2}}{35 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	86

[In] `int((a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)`

[Out] $16/35 * a^4 * \cos(1/2*d*x+1/2*c) * \sin(1/2*d*x+1/2*c) * (5*\cos(1/2*d*x+1/2*c)^6 + 6*\cos(1/2*d*x+1/2*c)^4 + 8*\cos(1/2*d*x+1/2*c)^2 + 16) * 2^{(1/2)} / (a*\cos(1/2*d*x+1/2*c)^2)^{(1/2)} / d$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int (a + a \cos(c + dx))^{7/2} dx = \frac{2(5a^3 \cos(dx + c)^3 + 27a^3 \cos(dx + c)^2 + 71a^3 \cos(dx + c) + 177a^3) \sqrt{a \cos(dx + c) + a \sin(dx + c)}}{35(d \cos(dx + c) + d)}$$

[In] `integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")`

[Out] $2/35 * (5*a^3 * \cos(d*x + c)^3 + 27*a^3 * \cos(d*x + c)^2 + 71*a^3 * \cos(d*x + c) + 177*a^3) * \sqrt{a * \cos(d*x + c) + a * \sin(d*x + c)} / (d * \cos(d*x + c) + d)$

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{7/2} dx = \text{Timed out}$$

[In] `integrate((a+a*cos(d*x+c))**(7/2),x)`

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.65

$$\int (a + a \cos(c + dx))^{7/2} dx = \frac{(5 \sqrt{2} a^3 \sin(\frac{7}{2} dx + \frac{7}{2} c) + 49 \sqrt{2} a^3 \sin(\frac{5}{2} dx + \frac{5}{2} c) + 245 \sqrt{2} a^3 \sin(\frac{3}{2} dx + \frac{3}{2} c) + 1225 \sqrt{2} a^3 \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{140 d}$$

[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")

```
[Out] 1/140*(5*sqrt(2)*a^3*sin(7/2*d*x + 7/2*c) + 49*sqrt(2)*a^3*sin(5/2*d*x + 5/2*c) + 245*sqrt(2)*a^3*sin(3/2*d*x + 3/2*c) + 1225*sqrt(2)*a^3*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int (a + a \cos(c + dx))^{7/2} dx = \frac{\sqrt{2}(5 a^3 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{7}{2} dx + \frac{7}{2} c) + 49 a^3 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{5}{2} dx + \frac{5}{2} c) + 245 a^3 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 1225 a^3 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{140 d}$$

[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="giac")

```
[Out] 1/140*sqrt(2)*(5*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(7/2*d*x + 7/2*c) + 49*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 245*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 1225*a^3*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{7/2} dx = \int (a + a \cos(c + dx))^{7/2} dx$$

[In] int((a + a*cos(c + d*x))^(7/2),x)

[Out] int((a + a*cos(c + d*x))^(7/2), x)

3.122 $\int \frac{\cos^4(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$

Optimal result	941
Rubi [A] (verified)	941
Mathematica [A] (verified)	944
Maple [A] (verified)	944
Fricas [A] (verification not implemented)	945
Sympy [F(-1)]	945
Maxima [B] (verification not implemented)	945
Giac [A] (verification not implemented)	1393
Mupad [F(-1)]	1394

Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{148 \sin(c+dx)}{105d\sqrt{a+a \cos(c+dx)}} - \frac{2 \cos^2(c+dx) \sin(c+dx)}{35d\sqrt{a+a \cos(c+dx)}} + \frac{2 \cos^3(c+dx) \sin(c+dx)}{7d\sqrt{a+a \cos(c+dx)}} + \frac{62\sqrt{a+a \cos(c+dx)} \sin(c+dx)}{105ad}$$

[Out] $\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}-148/105*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}-2/35*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/7*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+62/105*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/a/d$

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2857, 3062, 3047, 3102, 2830, 2728, 212}

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx) \cos^3(c+dx)}{7d\sqrt{a \cos(c+dx)+a}} - \frac{2 \sin(c+dx) \cos^2(c+dx)}{35d\sqrt{a \cos(c+dx)+a}} + \frac{62 \sin(c+dx) \sqrt{a \cos(c+dx)+a}}{105ad} - \frac{148 \sin(c+dx)}{105d\sqrt{a \cos(c+dx)+a}}$$

[In] Int[Cos[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) - (148*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*Cos[c + d*x]^2*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Cos[c + d*x]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (62*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(105*a*d)

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2830

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2857

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1)), Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3047

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3062

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])

```

Rule 3102

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2 \cos^3(c + dx) \sin(c + dx)}{7d\sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{\cos^2(c+dx)(-6a+a \cos(c+dx))}{\sqrt{a+a \cos(c+dx)}} dx}{7a} \\
&= -\frac{2 \cos^2(c + dx) \sin(c + dx)}{35d\sqrt{a + a \cos(c + dx)}} + \frac{2 \cos^3(c + dx) \sin(c + dx)}{7d\sqrt{a + a \cos(c + dx)}} - \frac{2 \int \frac{\cos(c+dx)(2a^2-\frac{31}{2}a^2 \cos(c+dx))}{\sqrt{a+a \cos(c+dx)}} dx}{35a^2} \\
&= -\frac{2 \cos^2(c + dx) \sin(c + dx)}{35d\sqrt{a + a \cos(c + dx)}} + \frac{2 \cos^3(c + dx) \sin(c + dx)}{7d\sqrt{a + a \cos(c + dx)}} - \frac{2 \int \frac{2a^2 \cos(c+dx)-\frac{31}{2}a^2 \cos^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx}{35a^2} \\
&= -\frac{2 \cos^2(c + dx) \sin(c + dx)}{35d\sqrt{a + a \cos(c + dx)}} + \frac{2 \cos^3(c + dx) \sin(c + dx)}{7d\sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{62\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{105ad} - \frac{4 \int \frac{-\frac{31a^3}{4} + \frac{37}{2}a^3 \cos(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx}{105a^3} \\
&= -\frac{148 \sin(c + dx)}{105d\sqrt{a + a \cos(c + dx)}} - \frac{2 \cos^2(c + dx) \sin(c + dx)}{35d\sqrt{a + a \cos(c + dx)}} + \frac{2 \cos^3(c + dx) \sin(c + dx)}{7d\sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{62\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{105ad} + \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx \\
&= -\frac{148 \sin(c + dx)}{105d\sqrt{a + a \cos(c + dx)}} - \frac{2 \cos^2(c + dx) \sin(c + dx)}{35d\sqrt{a + a \cos(c + dx)}} + \frac{2 \cos^3(c + dx) \sin(c + dx)}{7d\sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{62\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{105ad} - \frac{2 \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d}
\end{aligned}$$

$$= \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{148 \sin(c+dx)}{105d\sqrt{a+a \cos(c+dx)}} - \frac{2 \cos^2(c+dx) \sin(c+dx)}{35d\sqrt{a+a \cos(c+dx)}} + \frac{2 \cos^3(c+dx) \sin(c+dx)}{7d\sqrt{a+a \cos(c+dx)}} + \frac{62\sqrt{a+a \cos(c+dx)} \sin(c+dx)}{105ad}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.64

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

$$= \frac{\left(105\sqrt{2} \operatorname{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right) + 2\sqrt{1-\cos(c+dx)}(-43+31\cos(c+dx)-3\cos^2(c+dx)+15\cos^3(c+dx))\right) \sin(c+dx)}{105d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}$$

[In] Integrate[Cos[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]], x]

[Out] ((105*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]] + 2*Sqrt[1 - Cos[c + d*x]]*(-43 + 31*Cos[c + d*x] - 3*Cos[c + d*x]^2 + 15*Cos[c + d*x]^3))*Sin[c + d*x])/ (105*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.11

method	result
default	$\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2} \sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-240 \sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} + 336 \sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 280 \sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right) + 105a^{\frac{3}{2}} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}{105d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}$

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^(1/2), x, method=_RETURNVERBOSE)

[Out] 1/105*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-240*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+336*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4-280*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+105*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= \frac{4(15 \cos(dx + c)^3 - 3 \cos(dx + c)^2 + 31 \cos(dx + c) - 43) \sqrt{a \cos(dx + c) + a} \sin(dx + c) + \frac{105 \sqrt{2}(a \cos(dx + c) + a) \log(-\cos(dx + c)^2 - 2\sqrt{2}\sqrt{a \cos(dx + c) + a} \sin(dx + c) / \sqrt{a} - 2\cos(dx + c) - 3) / (\cos(dx + c)^2 + 2\cos(dx + c) + 1)) / \sqrt{a}}{210(ad \cos(dx + c) + ad)}$$

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/210*(4*(15*cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 31*cos(d*x + c) - 43)*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c) + 105*sqrt(2)*(a*cos(d*x + c) + a)*log(-(
cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) -
2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*co
s(d*x + c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696204 vs. 2(149) = 298.

Time = 17.57 (sec) , antiderivative size = 696204, normalized size of antiderivative = 4001.17

$$\int \frac{\cos^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5040*(180*(cos(5/2*d*x + 5/2*c)^2*sin(d*x + c) + 2*cos(5/2*d*x + 5/2*c)*
cos(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c)^2*sin(d*x + c) + s
in(5/2*d*x + 5/2*c)^2*sin(d*x + c) + 2*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3
/2*c)*sin(d*x + c) + sin(3/2*d*x + 3/2*c)^2*sin(d*x + c))*cos(9/2*d*x + 9/2
```

$$\begin{aligned}
& *c)^3 - 180*((\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c) + \\
& 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c) + 1)*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c) \\
& + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c) + 1)*\sin(3/ \\
& 2*dx + 3/2*c)^2)*\sin(9/2*dx + 9/2*c)^3 - 40*((\cos(dx + c))^2 + \sin(dx + \\
& c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c))^2 + \sin \\
& (dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) \\
& + (\cos(dx + c))^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2 \\
& *c)^2 + (\cos(dx + c))^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx \\
& + 5/2*c)^2 + 2*(\cos(dx + c))^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5 \\
& /2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c))^2 + \sin(dx + c)^2 + 2 \\
& *\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(7/2*dx + 7/2*c)^3 - 2*(840* \\
& (\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^3 + 336*(\cos(dx + c))^2 + \sin(dx + \\
& c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^3 - 840*\cos(5/2*dx + 5/2* \\
& c)^3*\sin(dx + c) + 21*(45*(\log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2* \\
& c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + 45*(\log(\cos(1 \\
& /2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \\
& \log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c \\
&) + 1))*\sin(dx + c)^2 + 90*(\log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2 \\
& *c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2* \\
& dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + 16*(\cos(dx + \\
& c))^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c) - 80*\cos(3 \\
& /2*dx + 3/2*c)*\sin(dx + c) + 45*\log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx \\
& + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - 45*\log(\cos(1/2*dx + 1/2*c))^2 + \\
& \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(5/2*dx + 5/2*c)^ \\
& 2 + 945*((\log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2*c)^2 - 2* \\
& \sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2*c))^2 + s \\
& \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2 \\
& *c))^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + c)^ \\
& 2 + 2*(\log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx \\
& + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin \\
& (1/2*dx + 1/2*c) + 1))*\cos(dx + c) + \log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2 \\
& *dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c))^2 \\
& + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(3/2*dx + 3/2*c \\
&)^2 + 21*(45*(\log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1 \\
& /2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2*c)^2 \\
& - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + 45*(\log(\cos(1/2*dx + 1/2*c \\
&)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx \\
& + c)^2 + 90*(\log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(\\
& 1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c))^2 + \sin(1/2*dx + 1/2*c)^2 \\
& - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + 16*(\cos(dx + c))^2 + \sin(dx \\
& + c)^2 + 7*\cos(dx + c) + 6)*\sin(3/2*dx + 3/2*c) - 40*\cos(5/2*dx + 5/2*c
\end{aligned}$$

$$\begin{aligned}
&) * \sin(d*x + c) + 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(5/2*d*x + 5/2*c)^2 + 945 * ((\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + \\
& 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2 \\
& *c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c)^2 - 180 * (\cos \\
& (5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2 \\
& *c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x + 5/2* \\
& c)^2 * \sin(d*x + c) + 2 * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c \\
&) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \cos(7/2*d*x + 7/2*c) + 42 * (16 * (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) * \sin \\
& (3/2*d*x + 3/2*c) - 20 * \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) - 20 * \sin(3/2*d*x \\
& + 3/2*c)^2 * \sin(d*x + c) + 45 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2* \\
& c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1 \\
&)) * \cos(3/2*d*x + 3/2*c) * \cos(5/2*d*x + 5/2*c) + 20 * ((\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2 \\
& *d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \\
& \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2) * \sin(7/2*d*x + 7/2*c) + 42 * (\\
& 20 * (\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 20 * (\cos(d*x + c) + 1) * \cos(3/ \\
& 2*d*x + 3/2*c)^2 + 4 * (4 * \cos(d*x + c)^2 + 4 * \sin(d*x + c)^2 + 13 * \cos(d*x + c) \\
& + 9) * \sin(3/2*d*x + 3/2*c)^2 + 40 * ((\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) \\
& - \sin(3/2*d*x + 3/2*c) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 12 * \cos(d*x + c) \\
& ^2 + 45 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \\
& \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2
\end{aligned}$$

$$\begin{aligned}
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 \\
& + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c \\
&) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)) * \sin(5/2*d*x + 5/2*c) + 168*(2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c \\
&)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3) * \sin(3/2*d*x \\
& + 3/2*c)) * \cos(9/2*d*x + 9/2*c)^2 - 42*(40*(\cos(d*x + c) + 1) * \sin(5/2*d*x + \\
& 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3 \\
& /2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3 * \sin(d*x + c) + (45*(\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1)) * \cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90*(\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1)) * \cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1) * \sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + 45*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1)) * \cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c \\
&)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + \\
& c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + \\
& c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6) * \sin(3/2*d*x \\
& + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c) * \sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * s
\end{aligned}$$

$$\begin{aligned}
& \ln(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*(90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)^3 - 90*((\cos(d*x + c) + 1)*\cos(5/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c) \\
& ^3 - 20*((\cos(d*x + c))^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c))^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c))^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c))^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c))^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c))^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2*\sin(7/2*d*x + 7/2*c)^3 - (840*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^ \\
& 3 + 336*(\cos(d*x + c))^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + 21*(45*(\log(\cos(1/2* \\
& d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2 \\
& *d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c) + 16*(\cos(d*x + c))^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos \\
& (1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 45*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c))^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^ \\
& 2 + (\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log \\
& (\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*d*x + 1/2*c))^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c))^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c) + 16*(\cos(d*x + c))^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d* \\
& x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(3/2*d*x + 3/2*c)^2 - 180*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^ \\
& 2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2* \\
& c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c)) \\
& * \cos(7/2*d*x + 7/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2 \\
& *c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5 \\
& /2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*c \\
& os(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/ \\
& 2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + \\
& 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(\\
& d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos \\
& (5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^ \\
& 2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1
\end{aligned}$$

$$\begin{aligned}
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^3 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^3 - (840*(\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin \\
& (d*x + c) + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945 \\
& *((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 2 \\
& 1*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 \\
& + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d \\
& *x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 180*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(
\end{aligned}$$

$$\begin{aligned}
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 - (840*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 4 \\
& 5*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d \\
& *x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& os(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\si \\
& n(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (1 \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) \\
& + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d* \\
& x + c))*\cos(9/2*d*x + 9/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c) \sin(3/2 dx + 3/2 c) - 20 \cos(3/2 dx \\
& x + 3/2 c)^2 \sin(dx + c) - 20 \sin(3/2 dx + 3/2 c)^2 \sin(dx + c) + 45((\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) \\
& + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx \\
& + 1/2 c) + 1)) \cos(dx + c)^2 + (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx \\
& + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin \\
& (1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \sin(dx + c)^2 + 2(\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + \\
& 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + \\
& 1/2 c) + 1)) \cos(dx + c) + \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 \\
& c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx \\
& * x + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \cos(3/2 dx + 3/2 c) \cos(5/2 \\
& dx + 5/2 c) + 20((\cos(dx + c)^2 + \sin(dx + c)^2 + 11 \cos(dx + c) + 10) \\
& * \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 11 \cos(dx + \\
& c) + 10) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 11 \cos(dx + c) + 10) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^ \\
& 2 + \sin(dx + c)^2 + 11 \cos(dx + c) + 10) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(\\
& dx + c)^2 + \sin(dx + c)^2 + 11 \cos(dx + c) + 10) \sin(5/2 dx + 5/2 c) \sin \\
& n(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 11 \cos(dx + c) + 1 \\
& 0) \sin(3/2 dx + 3/2 c)^2 \sin(7/2 dx + 7/2 c) + 42(20(\cos(dx + c) + 1) \\
& * \cos(5/2 dx + 5/2 c)^2 + 20(\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + 4 * \\
& (4 \cos(dx + c)^2 + 4 \sin(dx + c)^2 + 13 \cos(dx + c) + 9) \sin(3/2 dx + 3 \\
& /2 c)^2 + 40((\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c) - \sin(3/2 dx + 3/2 c) \\
&) \sin(dx + c)) \cos(5/2 dx + 5/2 c) + 12 \cos(dx + c)^2 + 45((\log(\cos(1/2 \\
& * dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log \\
& (\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) \\
& + 1)) \cos(dx + c)^2 + (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 \\
& + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + \\
& 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \sin(dx + c)^2 + 2(\log(\cos(1/2 dx \\
& x + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx \\
& + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \cos(dx + c) + \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin \\
& (1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c) \\
&)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \sin(3/2 dx + 3/2 c) + 12 \sin(dx + c)^2 \\
& + 24 \cos(dx + c) + 12) \sin(5/2 dx + 5/2 c) + 168(2(\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2 \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + 3 \cos(dx + c)^ \\
& 2 + 3 \sin(dx + c)^2 + 6 \cos(dx + c) + 3) \sin(3/2 dx + 3/2 c) \sin(9/2 dx \\
& x + 9/2 c)^2 - 21(40(\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^3 + 16(\cos(dx \\
& * x + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^3 - 4 \\
& 0 \cos(5/2 dx + 5/2 c)^3 \sin(dx + c) + (45(\log(\cos(1/2 dx + 1/2 c)^2 + \sin \\
& (1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 \\
& * c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \cos(dx + c)^ \\
& 2 + 45(\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx \\
& + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin \\
& (1/2 dx + 1/2 c) + 1)) \sin(dx + c)^2 + 90(\log(\cos(1/2 dx + 1/2 c)^2 +
\end{aligned}$$

$$\begin{aligned}
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45 \\
& *((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (\\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + \\
& 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x \\
& + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& os(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x \\
& + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c) \\
& ^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - \log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1))* \\
& \sin(d*x + c)^2 + 2*(\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 2 \\
& * \sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - \log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2} \\
& *c)^2 - 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1))*\cos(d*x + c) + \log(\cos(\frac{1}{2}d*x + \frac{1}{2}c \\
&)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - \log(\cos(\frac{1}{2}d* \\
& x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1))*\cos(\frac{3}{ \\
& 2*d*x + \frac{3}{2}c))*\cos(\frac{5}{2}d*x + \frac{5}{2}c) + 2*(20*(\cos(d*x + c) + 1)*\cos(\frac{5}{2}d*x \\
& + \frac{5}{2}c)^2 + 20*(\cos(d*x + c) + 1)*\cos(\frac{3}{2}d*x + \frac{3}{2}c)^2 + 4*(4*\cos(d*x + \\
& c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(\frac{3}{2}d*x + \frac{3}{2}c)^2 + 40 \\
& *((\cos(d*x + c) + 1)*\cos(\frac{3}{2}d*x + \frac{3}{2}c) - \sin(\frac{3}{2}d*x + \frac{3}{2}c)*\sin(d*x + \\
& c))*\cos(\frac{5}{2}d*x + \frac{5}{2}c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c \\
&)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - \log(\cos(\frac{1}{2}d* \\
& x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1))*\cos(d* \\
& x + c)^2 + (\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 2*\sin(\frac{1}{2} \\
& *d*x + \frac{1}{2}c) + 1) - \log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 - \\
& 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 \\
& + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - \log(\cos(\frac{1}{2}d*x + \\
& \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2*\sin(\frac{1}{2}d*x + \frac{1}{2}c) + 1))*\cos(d*x + \\
& c) + \log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 2*\sin(\frac{1}{2}d*x + \\
& \frac{1}{2}c) + 1) - \log(\cos(\frac{1}{2}d*x + \frac{1}{2}c)^2 + \sin(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2*\sin(\\
& \frac{1}{2}d*x + \frac{1}{2}c) + 1))*\sin(\frac{3}{2}d*x + \frac{3}{2}c) + 12*\sin(d*x + c)^2 + 24*\cos(d* \\
& x + c) + 12)*\sin(\frac{5}{2}d*x + \frac{5}{2}c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(\frac{3}{2}d*x + \frac{3}{2}c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x \\
& + c)^2 + 6*\cos(d*x + c) + 3)*\sin(\frac{3}{2}d*x + \frac{3}{2}c))*\cos(\frac{7}{2}d*x + \frac{7}{2}c))*\co \\
& s(\frac{9}{2}d*x + \frac{9}{2}c) - 930*((\cos(\frac{5}{2}d*x + \frac{5}{2}c)^2*\sin(d*x + c) + 2*\cos(\frac{5}{2} \\
& d*x + \frac{5}{2}c)*\cos(\frac{3}{2}d*x + \frac{3}{2}c)*\sin(d*x + c) + \cos(\frac{3}{2}d*x + \frac{3}{2}c)^2*\sin \\
& (d*x + c) + \sin(\frac{5}{2}d*x + \frac{5}{2}c)^2*\sin(d*x + c) + 2*\sin(\frac{5}{2}d*x + \frac{5}{2}c)*\si \\
& n(\frac{3}{2}d*x + \frac{3}{2}c)*\sin(d*x + c) + \sin(\frac{3}{2}d*x + \frac{3}{2}c)^2*\sin(d*x + c))*\cos(\\
& \frac{9}{2}d*x + \frac{9}{2}c)^2 + 2*(\cos(\frac{5}{2}d*x + \frac{5}{2}c)^2*\sin(d*x + c) + 2*\cos(\frac{5}{2}d*x \\
& + \frac{5}{2}c)*\cos(\frac{3}{2}d*x + \frac{3}{2}c)*\sin(d*x + c) + \cos(\frac{3}{2}d*x + \frac{3}{2}c)^2*\sin(d* \\
& x + c) + \sin(\frac{5}{2}d*x + \frac{5}{2}c)^2*\sin(d*x + c) + 2*\sin(\frac{5}{2}d*x + \frac{5}{2}c)*\sin(\frac{3} \\
& /2*d*x + \frac{3}{2}c)*\sin(d*x + c) + \sin(\frac{3}{2}d*x + \frac{3}{2}c)^2*\sin(d*x + c))*\cos(\frac{9}{2} \\
& *d*x + \frac{9}{2}c)*\cos(\frac{7}{2}d*x + \frac{7}{2}c) + (\cos(\frac{5}{2}d*x + \frac{5}{2}c)^2*\sin(d*x + c) + \\
& 2*\cos(\frac{5}{2}d*x + \frac{5}{2}c)*\cos(\frac{3}{2}d*x + \frac{3}{2}c)*\sin(d*x + c) + \cos(\frac{3}{2}d*x + \frac{3} \\
& /2*c)^2*\sin(d*x + c) + \sin(\frac{5}{2}d*x + \frac{5}{2}c)^2*\sin(d*x + c) + 2*\sin(\frac{5}{2}d*x \\
& + \frac{5}{2}c)*\sin(\frac{3}{2}d*x + \frac{3}{2}c)*\sin(d*x + c) + \sin(\frac{3}{2}d*x + \frac{3}{2}c)^2*\sin(d*x \\
& + c))*\cos(\frac{7}{2}d*x + \frac{7}{2}c)^2 + (\cos(\frac{5}{2}d*x + \frac{5}{2}c)^2*\sin(d*x + c) + 2*\co \\
& s(\frac{5}{2}d*x + \frac{5}{2}c)*\cos(\frac{3}{2}d*x + \frac{3}{2}c)*\sin(d*x + c) + \cos(\frac{3}{2}d*x + \frac{3}{2}c) \\
& ^2*\sin(d*x + c) + \sin(\frac{5}{2}d*x + \frac{5}{2}c)^2*\sin(d*x + c) + 2*\sin(\frac{5}{2}d*x + \frac{5}{2} \\
& *c)*\sin(\frac{3}{2}d*x + \frac{3}{2}c)*\sin(d*x + c) + \sin(\frac{3}{2}d*x + \frac{3}{2}c)^2*\sin(d*x + c) \\
&)*\sin(\frac{9}{2}d*x + \frac{9}{2}c)^2 + 2*(\cos(\frac{5}{2}d*x + \frac{5}{2}c)^2*\sin(d*x + c) + 2*\cos(\frac{5} \\
& /2*d*x + \frac{5}{2}c)*\cos(\frac{3}{2}d*x + \frac{3}{2}c)*\sin(d*x + c) + \cos(\frac{3}{2}d*x + \frac{3}{2}c)^2* \\
& \sin(d*x + c) + \sin(\frac{5}{2}d*x + \frac{5}{2}c)^2*\sin(d*x + c) + 2*\sin(\frac{5}{2}d*x + \frac{5}{2}c) \\
& *\sin(\frac{3}{2}d*x + \frac{3}{2}c)*\sin(d*x + c) + \sin(\frac{3}{2}d*x + \frac{3}{2}c)^2*\sin(d*x + c))*s
\end{aligned}$$

$$\begin{aligned}
& \sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x \\
& + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/ \\
& 2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*s \\
& \sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x \\
& + c))) - 2*(45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 45*((\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c) \\
& ^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 5*((4*\cos(d*x + c)^2 + 4*\sin \\
& (d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x \\
& + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + \\
& 13)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos \\
& (d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + \\
& c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (4 \\
& *\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(3/2*d*x + 3/ \\
& 2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 35*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 + \\
& 90*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\cos(7/2*d*x + 7/2*c) - (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2* \\
& c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5 \\
& /2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + \\
& c))*\sin(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) + 21*(40*(\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (\\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 90 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + 16 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c) - 80 * \cos(3/2*d*x + 3/2*c) * \sin(d*x \\
& + c) + 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(5/2*d*x + 5/2*c)^2 + 45 * ((\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \c \\
& os(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)^2 + (45 * (\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&)) * \cos(d*x + c)^2 + 45 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90 * (\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1)) * \cos(d*x + c) + 16 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6 \\
&) * \sin(3/2*d*x + 3/2*c) - 40 * \cos(5/2*d*x + 5/2*c) * \sin(d*x + c) + 45 * \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1)) * \sin(5/2*d*x + 5/2*c)^2 + 45 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c)^2 + 2 * (16 * (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) * \sin(3/2*d*x + 3/2*c) - 20 * \cos(\\
& 3/2*d*x + 3/2*c)^2 * \sin(d*x + c) - 20 * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \\
& 45 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 \\
& * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)) * \cos \\
& (5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 20*(\\
& \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x \\
& + c)^2 + 13*\cos(d*x + c) + 9) * \sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + \\
& 1) * \cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c) * \sin(d*x + c)) * \cos(5/2*d*x + \\
& 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1)) * \sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12) * \sin(5 \\
& /2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1) * \cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x \\
& + c) + 3) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c)) * \sin(9/2*d*x + 9/2*c) \\
& - 10*(2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos \\
& (5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2* \\
& c)^2) * \cos(7/2*d*x + 7/2*c)^2 + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + 7*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 7*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 \\
& + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + \\
& 5/2*c) * \sin(3/2*d*x + 3/2*c) + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2) * \sin(7/2*d*x + 7/2*c) + 2016*(((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x \\
& + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\si \\
& n(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 \\
& + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)* \\
& \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d* \\
& x + 7/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \\
& + 36*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/2*c)^2 \\
& 2) \cos(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2* \\
& \cos(dx + c) + 1) \cos(5/2*dx + 5/2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2*(\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) * \\
& \sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \sin(3/2*dx + 3/2*c)^2) \cos(9/2*dx + 9/2*c) \cos(7/2*dx + 7/2*c) + ((\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c)^2 \\
& + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/ \\
& 2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& *x + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*dx + 3/2*c) + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/2*c)^2) \cos(7/2 \\
& *dx + 7/2*c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) * \cos \\
& (5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1) \cos(5/2*dx + 5/2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*dx \\
& + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*d \\
& *x + 3/2*c)^2) \sin(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c) \cos(3/2*dx + 3/2*c) + (\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2*dx + 3/2*c)^2 \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/ \\
& 2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*d \\
& *x + 5/2*c) \sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(3/2*dx + 3/2*c)^2) \sin(9/2*dx + 9/2*c) \sin(7/2*dx + 7 \\
& /2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx \\
& + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(\\
& 5/2*dx + 5/2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& *x + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*dx + 3/2*c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/2*c \\
&)^2) \sin(7/2*dx + 7/2*c)^2) \sin(3/2*\arctan2(\sin(dx + c), \cos(dx + c))) + \\
& 30*(((74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\cos(\\
& 5/2*dx + 5/2*c)^2 + 2*(74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx \\
& + c) + 105)*\cos(5/2*dx + 5/2*c) \cos(3/2*dx + 3/2*c) + (74*\cos(dx + c)^2 \\
& + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\cos(3/2*dx + 3/2*c)^2 + (74 \\
& *\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\sin(5/2*dx + \\
& 5/2*c)^2 + 2*(74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 1
\end{aligned}$$

$$\begin{aligned}
& n(5/2*d*x + 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + 2 \\
& 1*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^ \\
& 2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d \\
& *x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + \\
& c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*l \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c \\
&) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/
\end{aligned}$$

$$\begin{aligned}
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*co \\
& s(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& os(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*1 \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45* \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) \\
& - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \si \\
& n(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x \\
& + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d* \\
& x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(
\end{aligned}$$

$$\begin{aligned}
& d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2* \\
& (20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c \\
&) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c \\
&)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) \\
& ^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2* \\
& c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x \\
& + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 - 504*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + \\
& 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) * \\
& \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + \\
& 1) \sin(3/2 dx + 3/2 c)^2) \cos(7/2 dx + 7/2 c)^2 + ((\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2 \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + \\
& 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(3/2 dx \\
& + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(5/ \\
& 2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \\
& * \sin(5/2 dx + 5/2 c) * \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2 \cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx + 9/2 c)^2 + 2 \\
& * ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(5/2 dx + 5/2 \\
& c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(5/2 dx \\
& + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx \\
& + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \\
& \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2 \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin \\
& \sin(9/2 dx + 9/2 c) * \sin(7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2 \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2 \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^ \\
& 2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(5/2 dx + 5/ \\
& 2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(5/2 dx \\
& + 5/2 c) * \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos \\
& (dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(7/2 dx + 7/2 c)^2) \cos(1/3 \arct \\
& \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) * \sin(2/3 \arctan 2(\sin(3/2 dx \\
& + 3/2 c), \cos(3/2 dx + 3/2 c))) + (90 * (\cos(5/2 dx + 5/2 c))^2 \sin(dx + \\
& c) + 2 \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) \sin(dx + c) + \cos(3/2 dx \\
& + 3/2 c)^2 \sin(dx + c) + \sin(5/2 dx + 5/2 c)^2 \sin(dx + c) + 2 \sin(5/2 \\
& dx + 5/2 c) \sin(3/2 dx + 3/2 c) \sin(dx + c) + \sin(3/2 dx + 3/2 c)^2 \sin \\
& (dx + c)) \cos(9/2 dx + 9/2 c)^3 - 90 * ((\cos(dx + c) + 1) \cos(5/2 dx + 5/ \\
& 2 c)^2 + 2 * (\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\\
& \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c) + 1) \sin(5/2 dx + \\
& 5/2 c)^2 + 2 * (\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) \\
& + (\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx + 9/2 c)^3 - 20 * ((\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^ \\
& 2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(5/2 dx + \\
& 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx \\
& + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos \\
& (dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2 \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(7
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2* \\
& d*x + 3/2*c)^2 - 180*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2 \\
& *d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d \\
& *x + 7/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(\\
& d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20 \\
& *((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin \\
& (7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20* \\
& (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x \\
& + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + \\
& 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(\\
& 5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(\\
& d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(
\end{aligned}$$

$$\begin{aligned}
& d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2 \\
& *c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(\\
& d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + \\
& 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*
\end{aligned}$$

$$\begin{aligned}
& c) - 20\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20\sin(3/2*d*x + 3/2*c)^2*\sin \\
& (d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d* \\
& x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x \\
& + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 \\
& + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\co \\
& s(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
& ^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) \\
& + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 \\
& + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 - (840* \\
& (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2* \\
& c)^3*\sin(d*x + c) + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3 \\
& /2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)* \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + \\
& c) + 10)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/ \\
& 2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*c \\
& \cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d* \\
& x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*c \\
& \cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\si \\
& n(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2* \\
& d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2* \\
& c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2* \\
& d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)* \\
& \sin(3/2*d*x + 3/2*c))*\sin(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\si \\
& n(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 9 \\
& 0*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)*\sin(7/2*d*x + 7/2*c) + 45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(c
\end{aligned}$$

$$\begin{aligned}
& \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 \\
& + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 \\
& + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) \\
& + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + \\
& 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) - 930*((\cos(5/2*d*x + 5/ \\
& 2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x \\
& + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c) \\
&)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) \\
&) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x \\
& + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + (\cos(5 \\
& /2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
&)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c) \\
& ^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + (\cos(5/2*d* \\
& x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin \\
& (d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + \\
& 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d* \\
& x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d \\
& *x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/ \\
& 2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + \\
& (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + \\
& 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2*\cos(1/ \\
& 2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 2*(45*((\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/ \\
& 2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 \\
& + 45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d* \\
& x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& n(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5 \\
& /2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c \\
&)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/ \\
& 2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + \\
& 1))*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + \\
& 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + \\
& 3/2*c)^2 + 40*((\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2 \\
& *c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c) \\
& ^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^ \\
& 2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d* \\
& x + 7/2*c))*\sin(9/2*d*x + 9/2*c) - 10*(2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c))*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 7*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 14*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 7*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/ \\
& 2*d*x + 7/2*c) + 2016*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d* \\
& x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)
\end{aligned}$$

$$\begin{aligned}
& * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2 * \cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2 * \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 * \sin(9/2*d*x + 9/2*c \\
&) * \sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2 * \sin(7/2*d*x + 7/2*c)^2 * \sin(1/3*\arctan2(\sin(3/2*d*x + \\
& 3/2*c), \cos(3/2*d*x + 3/2*c))) + 36*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
&)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 * \cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 * \cos(9/2*d*x + 9/ \\
& 2*c) * \cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos
\end{aligned}$$

$$\begin{aligned}
& s(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/ \\
& 2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*(\\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(\\
& 9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(3/2*\arctan \\
& 2(\sin(d*x + c), \cos(d*x + c))) + 30*(((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^ \\
& 2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + \\
& 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d \\
& *x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 1 \\
& 05)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos \\
& (d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d \\
& *x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2* \\
& c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3 \\
& /2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((74*\cos(d*x + c)^2 + 74*\sin(\\
& d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^ \\
& 2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + \\
& 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d \\
& *x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 1 \\
& 05)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((7
\end{aligned}$$

$$\begin{aligned}
& 4*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin \\
& (d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c \\
&)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 \\
& + ((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5 \\
& /2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 \\
& + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74* \\
& \cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 10 \\
& 5)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(\\
& d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + \\
& 9/2*c)^2 + 2*((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 1 \\
& 05)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179 \\
& *\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d* \\
& x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5 \\
& /2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 \\
& + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9 \\
& /2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((74*\cos(d*x + c)^2 + 74*\sin(d*x + c \\
&)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 \\
& + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2 \\
& *d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179 \\
& *\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin \\
& (d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/ \\
& 2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(1/2*\arctan2(\sin(d*x + c), \\
& \cos(d*x + c))))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)) \\
&)^2 + 2*(45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*c \\
& \cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin \\
& (5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/ \\
& 2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2* \\
& c)^2 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2
\end{aligned}$$

$$\begin{aligned}
& *c)^2) * \cos(7/2*d*x + 7/2*c) * \sin(7/2*d*x + 7/2*c) + 45 * (\cos(5/2*d*x + 5/2*c) \\
& ^2 * \sin(d*x + c) + 2 * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) \\
& + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) \\
& + 2 * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) + \sin(3/2*d*x + \\
& 3/2*c)^2 * \sin(d*x + c)) * \sin(7/2*d*x + 7/2*c)^2 - 21 * (40 * (\cos(d*x + c) + 1) * \\
& \sin(5/2*d*x + 5/2*c)^3 + 16 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + \\
& c) + 1) * \sin(3/2*d*x + 3/2*c)^3 - 40 * \cos(5/2*d*x + 5/2*c)^3 * \sin(d*x + c) + (\\
& 45 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/ \\
& 2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + 45 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + \\
& 90 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1 \\
& /2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + 16 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c) - 80 * \cos(3/2*d*x + 3/2*c) * \sin(d*x \\
& + c) + 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d \\
& *x + 1/2*c) + 1) - 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(5/2*d*x + 5/2*c)^2 + 45 * ((\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) \\
&) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos \\
& (d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)^2 + (45 * (\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) \\
&)) * \cos(d*x + c)^2 + 45 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90 * (\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + \\
& 1)) * \cos(d*x + c) + 16 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7 * \cos(d*x + c) + 6 \\
&) * \sin(3/2*d*x + 3/2*c) - 40 * \cos(5/2*d*x + 5/2*c) * \sin(d*x + c) + 45 * \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \\
& 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1 \\
& /2*c) + 1)) * \sin(5/2*d*x + 5/2*c)^2 + 45 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c)^2 + 2 * (16 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) * \sin(3/2*d*x + 3/2*c) - 20 * \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) - 20 * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 45 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c) + 2 * (20 * (\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 20 * (\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 4 * (4 * \cos(d*x + c)^2 + 4 * \sin(d*x + c)^2 + 13 * \cos(d*x + c) + 9) * \sin(3/2*d*x + 3/2*c)^2 + 40 * ((\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 12 * \cos(d*x + c)^2 + 45 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c) + 12 * \sin(d*x + c)^2 + 24 * \cos(d*x + c) + 12 * \sin(5/2*d*x + 5/2*c) + 8 * (2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 3 * \cos(d*x + c)^2 + 3 * \sin(d*x + c)^2 + 6 * \cos(d*x + c) + 3) * \sin(3/2*d*x + 3/2*c) * \cos(7/2*d*x + 7/2*c)) * \cos(9/2*d*x + 9/2*c) + 2 * (90 * (\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \cos(9/2*d*x + 9/2*c)^3 - 90 * ((\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2) * \sin(9/2*d*x + 9/2*c)^3 - 20 * ((\cos(d*x + c)^2 + \sin(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^3 - (840*(\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin \\
& (d*x + c) + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945 \\
& *((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 2 \\
& 1*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^ \\
& 2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d \\
& *x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& + 1)) \cdot \cos(dx + c)^2 + (\log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 \\
& + 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + \\
& 1/2 \cdot c)^2 - 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \sin(dx + c)^2 + 2 \cdot (\log(\cos(1/2 \cdot dx \\
& x + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \log(\cos(1/2 \cdot dx \\
& os(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1 \\
&)) \cdot \cos(dx + c) + \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2 \cdot \sin \\
& in(1/2 \cdot dx + 1/2 \cdot c) + 1) - \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c \\
&)^2 - 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \sin(3/2 \cdot dx + 3/2 \cdot c)^2 - 180 \cdot (\cos(5/2 \cdot dx \\
& x + 5/2 \cdot c)^2 \cdot \sin(dx + c) + 2 \cdot \cos(5/2 \cdot dx + 5/2 \cdot c) \cdot \cos(3/2 \cdot dx + 3/2 \cdot c) \cdot \sin \\
& (dx + c) + \cos(3/2 \cdot dx + 3/2 \cdot c)^2 \cdot \sin(dx + c) + \sin(5/2 \cdot dx + 5/2 \cdot c)^2 \cdot \sin \\
& n(dx + c) + 2 \cdot \sin(5/2 \cdot dx + 5/2 \cdot c) \cdot \sin(3/2 \cdot dx + 3/2 \cdot c) \cdot \sin(dx + c) + \sin \\
& (3/2 \cdot dx + 3/2 \cdot c)^2 \cdot \sin(dx + c)) \cdot \cos(7/2 \cdot dx + 7/2 \cdot c) + 42 \cdot (16 \cdot (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2 \cdot \cos(dx + c) + 1) \cdot \cos(3/2 \cdot dx + 3/2 \cdot c) \cdot \sin(3/2 \cdot dx \\
& x + 3/2 \cdot c) - 20 \cdot \cos(3/2 \cdot dx + 3/2 \cdot c)^2 \cdot \sin(dx + c) - 20 \cdot \sin(3/2 \cdot dx + 3/2 \cdot c \\
& c)^2 \cdot \sin(dx + c) + 45 \cdot ((\log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 \\
& + 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx \\
& + 1/2 \cdot c)^2 - 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \cos(dx + c)^2 + (\log(\cos(1/2 \cdot dx \\
& + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \log(\cos \\
& s(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1) \\
&)) \cdot \sin(dx + c)^2 + 2 \cdot (\log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + \\
& 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1 \\
& /2 \cdot c)^2 - 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \cos(dx + c) + \log(\cos(1/2 \cdot dx + 1/2 \\
& *c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \log(\cos(1/2 \cdot \\
& dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \cos(\\
& 3/2 \cdot dx + 3/2 \cdot c)) \cdot \cos(5/2 \cdot dx + 5/2 \cdot c) + 20 \cdot ((\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2 \cdot \cos(dx + c) + 1) \cdot \cos(5/2 \cdot dx + 5/2 \cdot c)^2 + 2 \cdot (\cos(dx + c)^2 + \sin(dx \\
& *x + c)^2 + 2 \cdot \cos(dx + c) + 1) \cdot \cos(5/2 \cdot dx + 5/2 \cdot c) \cdot \cos(3/2 \cdot dx + 3/2 \cdot c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cdot \cos(dx + c) + 1) \cdot \cos(3/2 \cdot dx + 3/2 \cdot c \\
&)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cdot \cos(dx + c) + 1) \cdot \sin(5/2 \cdot dx + \\
& 5/2 \cdot c)^2 + 2 \cdot (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cdot \cos(dx + c) + 1) \cdot \sin(5/2 \\
& *dx + 5/2 \cdot c) \cdot \sin(3/2 \cdot dx + 3/2 \cdot c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cdot \cos \\
& os(dx + c) + 1) \cdot \sin(3/2 \cdot dx + 3/2 \cdot c)^2) \cdot \sin(7/2 \cdot dx + 7/2 \cdot c) + 42 \cdot (20 \cdot (\cos \\
& (dx + c) + 1) \cdot \cos(5/2 \cdot dx + 5/2 \cdot c)^2 + 20 \cdot (\cos(dx + c) + 1) \cdot \cos(3/2 \cdot dx + \\
& 3/2 \cdot c)^2 + 4 \cdot (4 \cdot \cos(dx + c)^2 + 4 \cdot \sin(dx + c)^2 + 13 \cdot \cos(dx + c) + 9) \cdot \sin \\
& in(3/2 \cdot dx + 3/2 \cdot c)^2 + 40 \cdot ((\cos(dx + c) + 1) \cdot \cos(3/2 \cdot dx + 3/2 \cdot c) - \sin(3 \\
& /2 \cdot dx + 3/2 \cdot c) \cdot \sin(dx + c)) \cdot \cos(5/2 \cdot dx + 5/2 \cdot c) + 12 \cdot \cos(dx + c)^2 + 45 \\
& * ((\log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2 \cdot \sin(1/2 \cdot dx + 1/ \\
& 2 \cdot c) + 1) - \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2 \cdot \sin(1/2 \\
& *dx + 1/2 \cdot c) + 1)) \cdot \cos(dx + c)^2 + (\log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot \\
& dx + 1/2 \cdot c)^2 + 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \\
& \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \sin(dx + c)^2 + 2 \cdot (\\
& \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c \\
&) + 1) - \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2 \cdot \sin(1/2 \cdot dx \\
& x + 1/2 \cdot c) + 1)) \cdot \cos(dx + c) + \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + \\
& 1/2 \cdot c)^2 + 2 \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12* \\
& \sin(d*x + c)^2 + 24*\cos(d*x + c) + 12))*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 3 \\
& *\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3))*\sin(3/2*d*x + 3/2* \\
& c))*\cos(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c) \\
& ^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(3/2*d*x \\
& + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 4 \\
& 5*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + \\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 1 \\
& 6*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6))*\sin(3/2*d*x + 3/2* \\
& c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2* \\
& d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*s
\end{aligned}$$

$$\begin{aligned}
& \ln(dx + c)^2 + 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2* \\
& \sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2* \\
& c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + \log(\cos(1/2*dx + 1/2*c) \\
& ^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx \\
& + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(3/2 \\
& *dx + 3/2*c)^2 + 2*(16*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + \\
& 1)*\cos(3/2*dx + 3/2*c)*\sin(3/2*dx + 3/2*c) - 20*\cos(3/2*dx + 3/2*c)^2*s \\
& \sin(dx + c) - 20*\sin(3/2*dx + 3/2*c)^2*\sin(dx + c) + 45*((\log(\cos(1/2*dx \\
& + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(co \\
& s(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1) \\
&)*\cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2 \\
& *\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2 \\
& *c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + c)^2 + 2*(\log(\cos(1/2*dx + \\
& 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1 \\
& /2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*c \\
& \cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1 \\
& /2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 \\
& - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(3/2*dx + 3/2*c))*\cos(5/2*dx + 5/2*c) + \\
& 2*(20*(\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 20*(\cos(dx + c) + 1)*c \\
& \cos(3/2*dx + 3/2*c)^2 + 4*(4*\cos(dx + c)^2 + 4*\sin(dx + c)^2 + 13*\cos(dx \\
& + c) + 9)*\sin(3/2*dx + 3/2*c)^2 + 40*((\cos(dx + c) + 1)*\cos(3/2*dx + 3/2 \\
& *c) - \sin(3/2*dx + 3/2*c)*\sin(dx + c))*\cos(5/2*dx + 5/2*c) + 12*\cos(dx \\
& + c)^2 + 45*((\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1 \\
& /2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 \\
& - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2*c)^2 \\
& + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + \\
& 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + \\
& c)^2 + 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2* \\
& dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2 \\
& *\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 + \sin \\
& (1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c \\
&)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(3/2*dx + 3 \\
& /2*c) + 12*\sin(dx + c)^2 + 24*\cos(dx + c) + 12)*\sin(5/2*dx + 5/2*c) + 8* \\
& (2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2 \\
& *c)^2 + 3*\cos(dx + c)^2 + 3*\sin(dx + c)^2 + 6*\cos(dx + c) + 3)*\sin(3/2*d \\
& *x + 3/2*c))*\cos(7/2*dx + 7/2*c)^2 - (840*(\cos(dx + c) + 1)*\sin(5/2*dx + \\
& 5/2*c)^3 + 336*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(\\
& 3/2*dx + 3/2*c)^3 - 840*\cos(5/2*dx + 5/2*c)^3*\sin(dx + c) + 21*(45*(\log(\\
& \cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + \\
& 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + \\
& 1/2*c) + 1))*\cos(dx + c)^2 + 45*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx \\
& + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin \\
& (1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + c)^2 + 90*(\log \\
& (\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + \\
& 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 4 \\
& 5*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d \\
& *x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& \cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\si \\
& n(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (l \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) \\
& + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d* \\
& x + c))*\cos(9/2*d*x + 9/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d* \\
& x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((l \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)) * \cos(5/2* \\
& d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10) \\
& * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + \\
& c) + 10) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 11*\cos(d*x + c) + 10) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10) * \sin(5/2*d*x + 5/2*c) * \sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 1 \\
& 0) * \sin(3/2*d*x + 3/2*c)^2 * \sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1) \\
& * \cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 4* \\
& (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9) * \sin(3/2*d*x + 3 \\
& /2*c)^2 + 40*((\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c) \\
&) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 \\
& + 24*\cos(d*x + c) + 12) * \sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^ \\
& 2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3) * \sin(3/2*d*x + 3/2*c)) * \sin(9/2*d* \\
& x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^3 - 4 \\
& 0*\cos(5/2*d*x + 5/2*c)^3 * \sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^ \\
& 2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) \\
& + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + \\
& 3/2*c) - 80*\cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2* \\
&d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
&5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
&+ 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d* \\
&x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
&os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
&+ 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
&1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
&*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
&(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
&x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/ \\
&2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
&(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&+ 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
&c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
&*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/ \\
&2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
&1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
&in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 \\
&+ 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
&+ 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*si \\
&n(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 \\
&+ 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
&1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
&/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
&*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^ \\
&2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d* \\
&x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - \\
&20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
&^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
&1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
&1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c
\end{aligned}$$

$$\begin{aligned}
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(\\
& 3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2* \\
& d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin \\
& (d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(\\
& d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin \\
& (7/2*d*x + 7/2*c)^2 + 2*(45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))* \\
& \cos(7/2*d*x + 7/2*c)^2 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)*\sin(7/2*d*x + 7/2*c) + 45*(\cos \\
& (5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/ \\
& 2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2 \\
& *c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 - 21*(40*(\\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^ \\
& 3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2dx + 3/2c) - 80\cos(3/2dx \\
& + 3/2c) \sin(dx + c) + 45\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c \\
& c)^2 + 2\sin(1/2dx + 1/2c) + 1) - 45\log(\cos(1/2dx + 1/2c)^2 + \sin(1/ \\
& 2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1)) \cos(5/2dx + 5/2c)^2 + 45 \\
& *((\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/ \\
& 2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2 \\
& *dx + 1/2c) + 1)) \cos(dx + c)^2 + (\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2* \\
& dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \\
& \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1)) \sin(dx + c)^2 + 2*(\\
& \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c \\
&) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2*d \\
& x + 1/2c) + 1)) \cos(dx + c) + \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + \\
& 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1 \\
& /2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1)) \cos(3/2dx + 3/2c)^2 + (\\
& 45*(\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1 \\
& /2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/ \\
& 2dx + 1/2c) + 1)) \cos(dx + c)^2 + 45*(\log(\cos(1/2dx + 1/2c)^2 + \sin(\\
& 1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c) \\
& ^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1)) \sin(dx + c)^2 + \\
& 90*(\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + \\
& 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1 \\
& /2dx + 1/2c) + 1)) \cos(dx + c) + 16*(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 7\cos(dx + c) + 6) \sin(3/2dx + 3/2c) - 40\cos(5/2dx + 5/2c) \sin(dx \\
& + c) + 45\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2d \\
& *x + 1/2c) + 1) - 45\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - \\
& 2\sin(1/2dx + 1/2c) + 1)) \sin(5/2dx + 5/2c)^2 + 45*((\log(\cos(1/2dx \\
& + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(co \\
& s(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1) \\
&) \cos(dx + c)^2 + (\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2 \\
& *sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2 \\
& *c)^2 - 2\sin(1/2dx + 1/2c) + 1)) \sin(dx + c)^2 + 2*(\log(\cos(1/2dx + \\
& 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(1 \\
& /2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1)) *c \\
& os(dx + c) + \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1 \\
& /2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 \\
& - 2\sin(1/2dx + 1/2c) + 1)) \sin(3/2dx + 3/2c)^2 + 2*(16*(\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2dx + 3/2c) \sin(3/2dx \\
& + 3/2c) - 20\cos(3/2dx + 3/2c)^2 \sin(dx + c) - 20\sin(3/2dx + 3/2c) \\
& ^2 \sin(dx + c) + 45*((\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 \\
& + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + \\
& 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1)) \cos(dx + c)^2 + (\log(\cos(1/2dx + \\
& 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(\\
& 1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1)) * \\
& \sin(dx + c)^2 + 2*(\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2 \\
& *sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2
\end{aligned}$$

$$\begin{aligned}
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/ \\
& 2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + \\
& c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40 \\
& *((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d* \\
& x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d* \\
& x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x \\
& + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\co \\
& s(9/2*d*x + 9/2*c) - 930*((\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2* \\
& d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin \\
& (d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\si \\
& n(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(\\
& 9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d* \\
& x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3 \\
& /2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2 \\
& *d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + \\
& 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3 \\
& /2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c))*\cos(7/2*d*x + 7/2*c)^2 + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c) \\
& ^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2 \\
& *c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
&)*\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2* \\
& \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\s \\
& in(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x \\
& + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/ \\
& 2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*s
\end{aligned}$$

$$\begin{aligned}
& \text{in}(d*x + c)) * \sin(7/2*d*x + 7/2*c)^2 * \cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x \\
& + c))) - 2*(45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 45*((\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c) \\
& ^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 5*((4*\cos(d*x + c)^2 + 4*\sin \\
& (d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x \\
& + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + \\
& 13)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos \\
& (d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + \\
& c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (4 \\
& *\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(3/2*d*x + 3/ \\
& 2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 35*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 + \\
& 90*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\cos(7/2*d*x + 7/2*c) - (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2* \\
& c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5 \\
& /2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + \\
& c))*\sin(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) + 21*(40*(\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (\\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + \\
& 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\co \\
& s(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x \\
& + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + \\
& 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5 \\
& /2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x \\
& + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(9/2*d*x + 9/2*c) \\
& - 10*(2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2* \\
& c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 7*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 7*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^ \\
& 2 + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 2268*(((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x \\
& + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x +
\end{aligned}$$

$$\begin{aligned}
&^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
&+ c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\\
&\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)* \\
&\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
&1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos \\
&(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
&+ 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
&2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
&c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
&*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
&2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
&c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2 \\
&*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
&(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&+ 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
&+ c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
&(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
&^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
&+ 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d \\
&*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
&+ 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
&+ c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\\
&\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
&+ (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
&2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
&*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
&(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7 \\
&/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
&+ 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
&5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
&2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c) \\
&^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
&*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
&(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + \\
&30*(((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(\\
&5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
&+ c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 \\
&+ 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74 \\
&*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + \\
&5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 1 \\
&05)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin \\
&(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + \\
&9/2*c)^2 + 2*((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
&105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 17
\end{aligned}$$

$$\begin{aligned}
& n(dx + c) + 2\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c)\sin(dx + c) + \cos \\
& (3/2*dx + 3/2*c)^2\sin(dx + c) + \sin(5/2*dx + 5/2*c)^2\sin(dx + c) + 2* \\
& \sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c)\sin(dx + c) + \sin(3/2*dx + 3/2* \\
& c)^2\sin(dx + c))\cos(9/2*dx + 9/2*c)\cos(7/2*dx + 7/2*c) + (\cos(5/2*dx \\
& + 5/2*c)^2\sin(dx + c) + 2\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c)\sin(\\
& dx + c) + \cos(3/2*dx + 3/2*c)^2\sin(dx + c) + \sin(5/2*dx + 5/2*c)^2\sin \\
& (dx + c) + 2*\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c)\sin(dx + c) + \sin(\\
& 3/2*dx + 3/2*c)^2\sin(dx + c))\cos(7/2*dx + 7/2*c)^2 + (\cos(5/2*dx + 5/ \\
& 2*c)^2\sin(dx + c) + 2*\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c)\sin(dx + \\
& c) + \cos(3/2*dx + 3/2*c)^2\sin(dx + c) + \sin(5/2*dx + 5/2*c)^2\sin(dx \\
& + c) + 2*\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c)\sin(dx + c) + \sin(3/2*d \\
& *x + 3/2*c)^2\sin(dx + c))\sin(9/2*dx + 9/2*c)^2 + 2*(\cos(5/2*dx + 5/2*c \\
&)^2\sin(dx + c) + 2*\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c)\sin(dx + c) \\
& + \cos(3/2*dx + 3/2*c)^2\sin(dx + c) + \sin(5/2*dx + 5/2*c)^2\sin(dx + c \\
&) + 2*\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c)\sin(dx + c) + \sin(3/2*d \\
& *x + 3/2*c)^2\sin(dx + c))\sin(9/2*dx + 9/2*c)\sin(7/2*dx + 7/2*c) + (\cos(5 \\
& /2*dx + 5/2*c)^2\sin(dx + c) + 2*\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c \\
&)\sin(dx + c) + \cos(3/2*dx + 3/2*c)^2\sin(dx + c) + \sin(5/2*dx + 5/2*c) \\
& ^2\sin(dx + c) + 2*\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c)\sin(dx + c) \\
& + \sin(3/2*d*x + 3/2*c)^2\sin(dx + c))\sin(7/2*d*x + 7/2*c)^2*\cos(1/2*\arct \\
& an2(\sin(dx + c), \cos(dx + c))) - 2*(45*((\cos(dx + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(dx + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(dx + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(dx + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(dx + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c \\
&) + (\cos(dx + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 45* \\
& ((\cos(dx + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(dx + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(dx + c) + 1)*\cos(3/2*d*x + 3/2*c)^ \\
& 2 + (\cos(dx + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(dx + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(dx + c) + 1)*\sin(3/2*d*x + 3/2* \\
& c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 35*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(\\
& dx + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 70*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2*\cos(dx + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 35*(\cos(d \\
& *x + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 5 \\
& *((4*\cos(dx + c)^2 + 4*\sin(dx + c)^2 + 17*\cos(dx + c) + 13)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(4*\cos(dx + c)^2 + 4*\sin(dx + c)^2 + 17*\cos(dx + c) + 13) \\
& *\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (4*\cos(dx + c)^2 + 4*\sin(dx \\
& + c)^2 + 17*\cos(dx + c) + 13)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(dx + c)^2 + \\
& 4*\sin(dx + c)^2 + 17*\cos(dx + c) + 13)*\sin(5/2*d*x + 5/2*c)^2 + 2*(4*\cos \\
& (dx + c)^2 + 4*\sin(dx + c)^2 + 17*\cos(dx + c) + 13)*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c) + (4*\cos(dx + c)^2 + 4*\sin(dx + c)^2 + 17*\cos(dx + \\
& c) + 13)*\sin(3/2*d*x + 3/2*c)^2)\sin(7/2*d*x + 7/2*c)^2 + 35*(\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 70*(\cos(\\
& dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(\\
& 3/2*d*x + 3/2*c) + 35*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c)^2 + 90*((\cos(dx + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c) - (\cos(5/2*d*x + \\
& 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d* \\
& x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2 \\
& *d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) + \\
& 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + \\
& 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c \\
&)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) \\
& ^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\si \\
& n(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2* \\
& c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x \\
& + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)* \\
& \sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1)
\end{aligned}$$

$$\begin{aligned}
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 2*4*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(9/2*d*x + 9/2*c) - 10*(2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c))*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/
\end{aligned}$$

$$\begin{aligned}
& 2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + 7*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 14*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2 \\
& *d*x + 3/2*c) + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\co \\
& s(3/2*d*x + 3/2*c)^2 + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 7*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x \\
& + 7/2*c) + 560*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2 \\
& *c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^ \\
& 2)*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*s \\
& in(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d* \\
& x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c
\end{aligned}$$

$$\begin{aligned}
&)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(\\
&7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
&\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&+ 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
&x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + s \\
&\sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
&+ 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2* \\
&d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
&+ 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
&+ c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (c \\
&\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
&+ (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
&*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
&x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
&d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x \\
&+ c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(c \\
&\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*c \\
&\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&+ 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
&d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
&\sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + \\
&9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
&x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
&*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d* \\
&x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2* \\
&(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
&*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
&1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2 \\
&*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
&+ 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
&+ c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\\
&\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^ \\
&2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
&2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
&*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
&(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x \\
&+ c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
&\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
&\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
&1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
&c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
&(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx \\
& + 9/2 c) \sin(7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(d \\
& *x + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(d \\
& *x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 \\
& *(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c \\
&) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1) \sin(3/2 dx + 3/2 c)^2) \sin(7/2 dx + 7/2 c)^2 + (((\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx \\
& + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 \\
& dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin \\
& (5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c)^2 \\
& + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5 \\
& /2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 \\
& dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (co \\
& s(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \\
& * \cos(9/2 dx + 9/2 c) \cos(7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(\\
& dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 \\
& c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + \\
& 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/ \\
& 2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(7/2 dx + 7/2 c)^2 + ((\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2* \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \\
& * \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx \\
& + 9/2 c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& (5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(\\
& dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + \\
& 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx
\end{aligned}$$

$$\begin{aligned}
& + 1) \sin(3/2*d*x + 3/2*c)^2) \sin(9/2*d*x + 9/2*c) \sin(7/2*d*x + 7/2*c) + ((\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + \\
& 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \sin(7 \\
& /2*d*x + 7/2*c)^2) \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2* \\
& c))) \sin(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 2520*(\\
& ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x \\
& + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \cos \\
& (9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin \\
& (3/2*d*x + 3/2*c)^2) \cos(9/2*d*x + 9/2*c) \cos(7/2*d*x + 7/2*c) + ((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c) * \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \cos(7/2*d*x + \\
& 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \\
& \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3 \\
& /2*c)^2) \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5
\end{aligned}$$

$$\begin{aligned}
& /2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) \\
& + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d* \\
& x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*s \\
& \sin(7/2*d*x + 7/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + \\
& 3/2*c))) + 36*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2* \\
& c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 \\
&)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*
\end{aligned}$$

$$\begin{aligned}
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + \\
& c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 \\
& + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\\
& 74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 1 \\
& 79*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)*\sin(7/2 \\
& *d*x + 7/2*c) + ((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) \\
& + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + \\
& 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos \\
& (d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\si \\
& n(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d \\
& *x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c) \\
& ^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\si \\
& n(7/2*d*x + 7/2*c)^2)*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*\cos(2/7 \\
& *\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^2 + 2*(90*(\cos(5/2*d* \\
& x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\si \\
& n(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)^3 - 90*((\cos(d*x + c) \\
&) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\c \\
& os(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2 \\
& *d*x + 9/2*c)^3 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d \\
& *x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/ \\
& 2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^3 - (840*(\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + 21*(45* \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c \\
&) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c) + 1)) * \cos(5/2*d*x + 5/2*c)^2 + 945 * ((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \\
& \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos \\
& (d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)^2 + 21 * (45 * (\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1)) * \cos(d*x + c)^2 + 45 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90 * (\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1)) * \cos(d*x + c) + 16 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7 * \cos(d*x + c) + \\
& 6) * \sin(3/2*d*x + 3/2*c) - 40 * \cos(5/2*d*x + 5/2*c) * \sin(d*x + c) + 45 * \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \sin(5/2*d*x + 5/2*c)^2 + 945 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 \\
& + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c)^2 - 180 * (\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x \\
& + c) + 2 * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2* \\
& d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \sin(5 \\
& /2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \\
& \sin(d*x + c)) * \cos(7/2*d*x + 7/2*c) + 42 * (16 * (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) * \sin(3/2*d*x + 3/2*c) - 20 * \cos(\\
& 3/2*d*x + 3/2*c)^2 * \sin(d*x + c) - 20 * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \\
& 45 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 \\
& * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2
\end{aligned}$$

$$\begin{aligned}
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\co \\
& s(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos \\
& (d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^ \\
& 2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(\\
& d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24* \\
& \cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3* \\
& \sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(9/2*d*x + 9/ \\
& 2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(\\
& 5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) \\
& - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d* \\
& x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2
\end{aligned}$$

$$\begin{aligned}
& + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d \\
& *x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\si \\
& n(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d \\
& *x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2 \\
& *c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1))*\cos(5/2* \\
& d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d* \\
& x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + \\
& 40*((\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d* \\
& x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos \\
& (d*x + c) + 12))*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d \\
& *x + c)^2 + 6*\cos(d*x + c) + 3))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^ \\
& 2 + 2*(45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c))*\cos
\end{aligned}$$

$$\begin{aligned}
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)* \\
& \sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 4 \\
& 5*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/ \\
& 2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45 \\
& *((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(\\
& 5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\co \\
& s(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + \\
& c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/ \\
& 2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2 \\
& *d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x +
\end{aligned}$$

$$\begin{aligned}
& c) + 3) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c)) * \cos(9/2*d*x + 9/2*c) - \\
& 930 * ((\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \cos(5/2*d*x + 5/2*c) * \cos(3/2 \\
& *d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2* \\
& d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin \\
& \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \cos(9/2*d*x + 9/2*c)^2 + \\
& 2 * (\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d* \\
& x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x \\
& + 5/2*c)^2 * \sin(d*x + c) + 2 * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(\\
& d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \cos(9/2*d*x + 9/2*c) * \cos(7/ \\
& 2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \cos(5/2*d*x + 5/2 \\
& *c) * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) \\
& + \sin(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x \\
& + 3/2*c) * \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \cos(7/2*d*x + \\
& 7/2*c)^2 + (\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \cos(5/2*d*x + 5/2*c) * \cos \\
& \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin \\
& \sin(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/ \\
& 2*c) * \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \sin(9/2*d*x + 9/2* \\
& c)^2 + 2 * (\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \cos(5/2*d*x + 5/2*c) * \cos(\\
& 3/2*d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5 \\
& /2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c \\
&) * \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \sin(9/2*d*x + 9/2*c) * \\
& \sin(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \cos(5/2*d*x \\
& + 5/2*c) * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d* \\
& x + c) + \sin(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2 * \sin(5/2*d*x + 5/2*c) * \sin(3 \\
& /2*d*x + 3/2*c) * \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \sin(7/2 \\
& *d*x + 7/2*c)^2 * \cos(1/2 * \arctan2(\sin(d*x + c), \cos(d*x + c))) - 2 * (45 * ((\cos \\
& (d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c) + 1) * \cos(5/2*d*x + \\
& 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\\
& \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c) + 1) * \sin(5/2*d*x \\
& + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2) \\
& * \cos(9/2*d*x + 9/2*c)^2 + 45 * ((\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 \\
& * (\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 \\
& + 2 * (\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c) + 1) * \sin(3/2*d*x + 3/2*c)^2) * \cos(7/2*d*x + 7/2*c)^2 + 35 * (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 70 * (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos \\
& (3/2*d*x + 3/2*c) + 35 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + \\
& 1) * \cos(3/2*d*x + 3/2*c)^2 + 5 * ((4 * \cos(d*x + c)^2 + 4 * \sin(d*x + c)^2 + 17 * \cos \\
& \cos(d*x + c) + 13) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (4 * \cos(d*x + c)^2 + 4 * \sin(d*x + \\
& c)^2 + 17 * \cos(d*x + c) + 13) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\\
& 4 * \cos(d*x + c)^2 + 4 * \sin(d*x + c)^2 + 17 * \cos(d*x + c) + 13) * \cos(3/2*d*x + 3 \\
& /2*c)^2 + (4 * \cos(d*x + c)^2 + 4 * \sin(d*x + c)^2 + 17 * \cos(d*x + c) + 13) * \sin(\\
& 5/2*d*x + 5/2*c)^2 + 2 * (4 * \cos(d*x + c)^2 + 4 * \sin(d*x + c)^2 + 17 * \cos(d*x + \\
& c) + 13) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (4 * \cos(d*x + c)^2 + 4 *
\end{aligned}$$

$$\begin{aligned}
& \sin(dx + c)^2 + 17\cos(dx + c) + 13\sin(3/2dx + 3/2c)^2\sin(7/2dx \\
& + 7/2c)^2 + 35(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(\\
& 5/2dx + 5/2c)^2 + 70(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\sin(5/2dx + 5/2c)\sin(3/2dx + 3/2c) + 35(\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1)\sin(3/2dx + 3/2c)^2 + 90(((\cos(dx + c) \\
& + 1)\cos(5/2dx + 5/2c)^2 + 2(\cos(dx + c) + 1)\cos(5/2dx + 5/2c)\cos \\
& (3/2dx + 3/2c) + (\cos(dx + c) + 1)\cos(3/2dx + 3/2c)^2 + (\cos(dx + \\
& c) + 1)\sin(5/2dx + 5/2c)^2 + 2(\cos(dx + c) + 1)\sin(5/2dx + 5/2c)* \\
& \sin(3/2dx + 3/2c) + (\cos(dx + c) + 1)\sin(3/2dx + 3/2c)^2)\cos(7/2d \\
& *x + 7/2c) - (\cos(5/2dx + 5/2c)^2\sin(dx + c) + 2\cos(5/2dx + 5/2c) \\
& *\cos(3/2dx + 3/2c)\sin(dx + c) + \cos(3/2dx + 3/2c)^2\sin(dx + c) + \\
& \sin(5/2dx + 5/2c)^2\sin(dx + c) + 2\sin(5/2dx + 5/2c)\sin(3/2dx + \\
& 3/2c)\sin(dx + c) + \sin(3/2dx + 3/2c)^2\sin(dx + c))\sin(7/2dx + 7/ \\
& 2c))\cos(9/2dx + 9/2c) + 21(40(\cos(dx + c) + 1)\sin(5/2dx + 5/2c) \\
& ^3 + 16(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2dx \\
& + 3/2c)^3 - 40\cos(5/2dx + 5/2c)^3\sin(dx + c) + (45(\log(\cos(1/2dx \\
& + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos \\
& (1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1)) \\
& *\cos(dx + c)^2 + 45(\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + \\
& 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1 \\
& /2c)^2 - 2\sin(1/2dx + 1/2c) + 1))\sin(dx + c)^2 + 90(\log(\cos(1/2dx \\
& + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\co \\
& s(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1) \\
&)*\cos(dx + c) + 16(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)* \\
& \sin(3/2dx + 3/2c) - 80\cos(3/2dx + 3/2c)\sin(dx + c) + 45\log(\cos(1/ \\
& 2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - 4 \\
& 5\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2 \\
& *c) + 1))\cos(5/2dx + 5/2c)^2 + 45*((\log(\cos(1/2dx + 1/2c)^2 + \sin(1/ \\
& 2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 \\
& + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1))\cos(dx + c)^2 + (\\
& \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c \\
&) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2d \\
& *x + 1/2c) + 1))\sin(dx + c)^2 + 2*(\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2d \\
& *x + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \\
& \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1))\cos(dx + c) + \log(\co \\
& s(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) \\
& - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/ \\
& 2c) + 1))\cos(3/2dx + 3/2c)^2 + (45(\log(\cos(1/2dx + 1/2c)^2 + \sin(1 \\
& /2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c)^ \\
& 2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1))\cos(dx + c)^2 + \\
& 45(\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1 \\
& /2c) + 1) - \log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/ \\
& 2dx + 1/2c) + 1))\sin(dx + c)^2 + 90(\log(\cos(1/2dx + 1/2c)^2 + \sin(\\
& 1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \log(\cos(1/2dx + 1/2c) \\
& ^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1))\cos(dx + c) + 1
\end{aligned}$$

$$\begin{aligned}
& 6*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(9/2*d*x + 9/2*c) - 10*(2*((\cos(d*x +
\end{aligned}$$

$$\begin{aligned}
& 1) * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7 \\
& /2*c)^2*\sin(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 120*(((\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\co \\
& s(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2 \\
& *c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2 \\
& *c)^2*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + ((\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9 \\
& /2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) +
\end{aligned}$$

$$\begin{aligned}
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d \\
& *x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(3/2*d*x + 3/2*c)^2 - 90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^ \\
& 2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2* \\
& c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c)) \\
& *\cos(9/2*d*x + 9/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2 \\
& *c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5 \\
& /2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 1 \\
& 0)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 11*\cos(d*x + c) + 10)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d \\
& *x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(
\end{aligned}$$

$$\begin{aligned}
& 3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(\\
& d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 \\
& + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d \\
& *x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& os(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*c \\
& os(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*s \\
& in(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(9/2*d*x + 9/2 \\
& *c)^2 - 42*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5 \\
& /2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45* \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) \\
& - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x \\
& + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d \\
& x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + \\
& c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c \\
&)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2* \\
& d*x + 7/2*c)^2 - 140*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x \\
& + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3 \\
& /2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c \\
&)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c) \\
& ^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + \\
& 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)\sin(3/2*dx + 3/2*c)^2*\sin(7/2*dx + 7/2*c)^2 + (((\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3 \\
& /2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\co \\
& s(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2*\cos(9/2*dx + 9/2 \\
& *c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\co \\
& s(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\sin(3/2*dx + 3/2 \\
& *c)^2*\cos(9/2*dx + 9/2*c)*\cos(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(d \\
& *x + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3 \\
& /2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(3/2*dx \\
& + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\sin(5/2 \\
& *dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)* \\
& \sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2*\cos(7/2*dx + 7/2*c)^2 + ((\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^ \\
& 2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + \\
& 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2*\sin(9 \\
& /2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2* \\
& dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\sin(3 \\
& /2*dx + 3/2*c)^2*\sin(9/2*dx + 9/2*c)*\sin(7/2*dx + 7/2*c) + ((\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos \\
& (3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)* \\
& \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c)
\end{aligned}$$

$$\begin{aligned}
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) \\
& + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c \\
&)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 180*(\cos(5/2*d*x + 5/2*c)^2*\sin(\\
& d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3 \\
& /2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c) \\
& ^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 \\
& + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)) \\
& *\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c)*\sin
\end{aligned}$$

$$\begin{aligned}
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4* \\
& \cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2* \\
& c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
&)*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + \\
& 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + \\
& 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(9/2*d*x + \\
& 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos \\
& (5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + \\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \\
& 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2 \\
& *c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2 \\
& *d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d \\
& *x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + \\
& 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + \\
& 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20* \\
& \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 \\
& + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c \\
&)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2 \\
& *d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
&+ 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x \\
&+ c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 \\
&+ \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x \\
&+ c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(\\
&7/2*d*x + 7/2*c)^2 + (90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d \\
&*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(\\
&d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin \\
&(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9 \\
&/2*d*x + 9/2*c)^3 - 90*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
&d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1 \\
&))*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
&(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
&+ 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^3 - 20*((\cos(d*x + c)^2 + \\
&\sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
&c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d \\
&*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/ \\
&2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
&(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&+ 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
&+ c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^ \\
&3 - (840*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \\
&\sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d \\
&*x + 5/2*c)^3*\sin(d*x + c) + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
&*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&+ 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
&x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
&d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos \\
&(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - \\
&80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x \\
&+ 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
&\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
&c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2 \\
&*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
&d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
&d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
&(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
&2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 \\
&+ \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
&1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*
\end{aligned}$$

$$\begin{aligned}
& x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d* \\
& x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 9 \\
& 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - \\
& 180*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2* \\
& d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d \\
& *x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\si \\
& n(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) + 42 \\
& *(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3 \\
& /2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\si \\
& n(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(
\end{aligned}$$

$$\begin{aligned}
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2* \\
& c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos \\
& (d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos \\
& (d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d* \\
& x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) \\
& + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d* \\
& x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin \\
& (3/2*d*x + 3/2*c))*\cos(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90* \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d \\
& *x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c)^2 + 45 * (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 \\
& * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 \\
& * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \sin(dx + c)^2 + 90 * (\log(\cos(1/2 * dx + \\
& 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(\\
& 1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \\
& \cos(dx + c) + 16 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 7 * \cos(dx + c) + 6) * \sin \\
& (3/2 * dx + 3/2 * c) - 40 * \cos(5/2 * dx + 5/2 * c) * \sin(dx + c) + 45 * \log(\cos(1/2 * \\
& dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 45 * \\
& \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c \\
&) + 1)) * \sin(5/2 * dx + 5/2 * c)^2 + 45 * ((\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * \\
& dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \\
& \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(dx + c)^2 + (\log \\
& (\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) \\
& + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx \\
& + 1/2 * c) + 1)) * \sin(dx + c)^2 + 2 * (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx \\
& + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin \\
& (1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(dx + c) + \log(\cos(\\
& 1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \\
& \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * \\
& c) + 1)) * \sin(3/2 * dx + 3/2 * c)^2 + 2 * (16 * (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2 * \cos(dx + c) + 1) * \cos(3/2 * dx + 3/2 * c) * \sin(3/2 * dx + 3/2 * c) - 20 * \cos(3/2 * \\
& dx + 3/2 * c)^2 * \sin(dx + c) - 20 * \sin(3/2 * dx + 3/2 * c)^2 * \sin(dx + c) + 45 * (\\
& (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * \\
& c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx \\
& * dx + 1/2 * c) + 1)) * \cos(dx + c)^2 + (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx \\
& x + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin \\
& (1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \sin(dx + c)^2 + 2 * (\log \\
& (\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) \\
& + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx \\
& + 1/2 * c) + 1)) * \cos(dx + c) + \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/ \\
& 2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 \\
& * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(3/2 * dx + 3/2 * c)) * \cos(5/ \\
& 2 * dx + 5/2 * c) + 2 * (20 * (\cos(dx + c) + 1) * \cos(5/2 * dx + 5/2 * c)^2 + 20 * (\cos(\\
& dx + c) + 1) * \cos(3/2 * dx + 3/2 * c)^2 + 4 * (4 * \cos(dx + c)^2 + 4 * \sin(dx + c) \\
& ^2 + 13 * \cos(dx + c) + 9) * \sin(3/2 * dx + 3/2 * c)^2 + 40 * ((\cos(dx + c) + 1) * \cos \\
& (3/2 * dx + 3/2 * c) - \sin(3/2 * dx + 3/2 * c) * \sin(dx + c)) * \cos(5/2 * dx + 5/2 * \\
& c) + 12 * \cos(dx + c)^2 + 45 * ((\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/ \\
& 2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 \\
& * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(dx + c)^2 + (\log(\cos(1/ \\
& 2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log \\
& (\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) \\
& + 1)) * \sin(dx + c)^2 + 2 * (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c \\
&)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx \\
& x + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(dx + c) + \log(\cos(1/2 * dx \\
& + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& * \sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d \\
& *x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*c \\
& \cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c \\
&) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 - (840*(\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c \\
&) + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x \\
& + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)* \\
& \sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945*((\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(l \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(\\
& d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + \\
& 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d* \\
& x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 90*(\cos(5/2*d*x + 5/2*c) \\
& ^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) \\
& + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) \\
& - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d \\
& *x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x \\
& + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + \\
& 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos \\
& (d*x + c) + 10)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 11*\cos(d*x + c) + 10)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11* \\
& \cos(d*x + c) + 10)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9) \\
& *\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + \\
& 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 1 \\
& 2*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) * \sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9) * \sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12) * \sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c)^2 + 2*(45*(\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \cos(7/2*d*x + 7/2*c)^2 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2) * \cos(7/2*d*x + 7/2*c) * \sin(7/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(\\
& 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
& *c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(\\
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x \\
& + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3 \\
& /2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + \\
& c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c \\
&)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2* \\
& d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) - 930*((\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + \\
& c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d* \\
& x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*si \\
& n(d*x + c))*\cos(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) \\
& + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d* \\
& x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d \\
& *x + c))*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^ \\
& 2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) +
\end{aligned}$$

$$\begin{aligned}
& \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) \\
& + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + (\cos(5/2*d*x + 5/2*c)^2*\sin \\
& (d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(\\
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*s \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c \\
&)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d* \\
& x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2 \\
& *d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5 \\
& /2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x \\
& + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2* \\
& d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2)*\cos(1/2*\arctan2(\sin(d* \\
& x + c), \cos(d*x + c))) - 2*(45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 45*((\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(\\
& 7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 5*((4*\cos(d \\
& *x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 1 \\
& 7*\cos(d*x + c) + 13)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x \\
& + c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^ \\
& 2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d \\
& *x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2* \\
& d*x + 3/2*c)^2 + 90*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c) - (\cos(5/2*d*x + 5/2*c)^2*s \\
& \sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + co
\end{aligned}$$

$$\begin{aligned}
& s(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2 \\
& * \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2 \\
& *c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) + 21*(40*(\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3* \\
& \sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + \\
& 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*(\\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 9 \\
& 0*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7* \\
& \cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + \\
& c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + \\
& 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2* \\
& d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1))*\cos(5/2*d*x + \\
& 5/2*c)^2 + 20*(\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c \\
&)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + 40*(\\
& (\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
&)*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c \\
&) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x \\
& + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + \\
& c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(\\
& 9/2*d*x + 9/2*c) - 10*(2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c)*\sin(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin \\
& (3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + 7*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 14*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c) + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& n(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) * \sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9) * \sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12) * \sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c)^2 - 504*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 \\
& + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 \\
&)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d* \\
& x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d* \\
& x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) \\
& ^2)*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(2/3*\ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + (90*(\cos(5/2*d*x + 5/2 \\
& *c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + \\
& c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d* \\
& x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)^3 - 90*((\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + \\
& 9/2*c)^3 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/ \\
& 2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^3 - (840*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/ \\
& 2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + 21*(45*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(\\
& 3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log
\end{aligned}$$

$$\begin{aligned}
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 180*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + \\
& 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3 \\
& /2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c))*\cos(7/2*d*x + 7/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x \\
& + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d \\
& *x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d* \\
& x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + \\
& c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40* \\
& ((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c \\
&))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x \\
& + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x \\
& + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(9/2*d*x + 9/2*c)^2 \\
& - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x \\
& + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*c \\
& \cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2 \\
& *c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + \\
& c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d* \\
& x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c \\
&)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
& + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3 \\
& /2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5 \\
& /2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos \\
& (5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c \\
&)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c \\
&)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2 \\
& *c)^2 - 90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin \\
& (5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2 \\
& *c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c \\
&) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d
\end{aligned}$$

$$\begin{aligned}
& *x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - \\
& 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c \\
&)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(5/2*d*x + 5/2 \\
& *c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + \\
& c) + 10)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos \\
& (d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(3/2*d*x + 3/2*c)^2)* \\
& \sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& 0*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d \\
& *x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x \\
& + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\si \\
& n(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*co \\
& s(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\si \\
& n(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\si \\
& n(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*
\end{aligned}$$

$$\begin{aligned}
& d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d* \\
& x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^ \\
& 2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))* \\
& \cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
& + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + \\
& c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c) \\
& ^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(\\
& 45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x \\
& + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(\\
& d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 - 20 \\
& *((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos \\
& (7/2*d*x + 7/2*c)*\sin(7/2*d*x + 7/2*c) + 45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d* \\
& x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2 \\
& *d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(
\end{aligned}$$

$$\begin{aligned}
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x \\
& + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + \\
& 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/ \\
& 2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \\
& 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& 3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + \\
& 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3 \\
&)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) - 930*((\\
& \cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5 \\
& /2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)^2 + 2*(\cos \\
& (5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2* \\
& c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c \\
&) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + \\
& 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(\\
& 5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2* \\
& c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) \\
& ^2 + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2* \\
& d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d \\
& *x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\si \\
& n(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)^2 + \\
& 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x \\
& + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d \\
& *x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)*\sin(7/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2* \\
& c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + \\
& 7/2*c)^2)*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 2*(45*((\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/ \\
& 2*d*x + 9/2*c)^2 + 45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 3/2*d*x + 3/2*c)^2 + 5*((4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + \\
& c) + 13)*\cos(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + \\
& 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (4*\cos(d \\
& *x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13 \\
&)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x \\
& + c)^2 + 17*\cos(d*x + c) + 13)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c \\
&)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 + 90*(((\cos(d*x + c) + 1)*\co \\
& s(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/ \\
& 2*c) - (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/ \\
& 2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2 \\
& *d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)* \\
& \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c))*\c \\
& os(9/2*d*x + 9/2*c) + 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d* \\
& x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*
\end{aligned}$$

$$\begin{aligned}
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d \\
& *x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2 \\
& *d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40 \\
& *\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5 \\
& /2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x \\
& + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + \\
& 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos \\
& (3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d \\
& *x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 -
\end{aligned}$$

$$\begin{aligned}
& x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2) \cos(9/2*d*x + 9 \\
& /2*c) \cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*s \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c)^2) \cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2) \sin(9/2*d*x + 9/2*c)^2 + 2* \\
& ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2) \sin \\
& (9/2*d*x + 9/2*c) \sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2) \sin(7/2*d*x + 7/2*c)^2) \sin(1/3*\arctan \\
& 2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 36*(((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + \\
& 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)* \\
& \cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c \\
&)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x \\
& + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\si \\
& n(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 \\
&)*\sin(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 120*(((\cos(d*x + c)^2 + \si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + \\
& 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)* \\
& \cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c \\
&)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x \\
& + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\si \\
& n(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 \\
&)*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 30*(((74*\cos(d*x + c)^2 + \\
& 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74* \\
& \cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179* \\
& \cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x \\
& + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + \\
& c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\si
\end{aligned}$$

$$\begin{aligned}
& n(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + \\
& c) + 105)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)^2 + 2*((74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c) \\
& ^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + \\
& 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74* \\
& \cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179* \\
& \cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d* \\
& x + 7/2*c) + ((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 1 \\
& 05)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179 \\
& *\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d* \\
& x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5 \\
& /2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 \\
& + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2*\cos(7 \\
& /2*d*x + 7/2*c)^2 + ((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + \\
& c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^ \\
& 2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74 \\
& *\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105 \\
&)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos \\
& (d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2 \\
&)*\sin(9/2*d*x + 9/2*c)^2 + 2*((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179* \\
& \cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(\\
& d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + \\
& c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^ \\
& 2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74 \\
& *\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((74*\cos(d*x + c)^2 \\
& + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(7 \\
& 4*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 17 \\
& 9*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(\\
& d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)* \\
& \sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(1/2*\arctan \\
& 2(\sin(d*x + c), \cos(d*x + c))))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3 \\
& /2*d*x + 3/2*c)))^2 + 2*(45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/
\end{aligned}$$

$$\begin{aligned}
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12))*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 6\cos(dx + c) + 3)\sin(3/2*dx + 3/2*c))\cos(7/2*dx + 7/2*c))\cos(\\
& 9/2*dx + 9/2*c) + 2*(90*(\cos(5/2*dx + 5/2*c)^2*\sin(dx + c) + 2*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*dx + 3/2*c)*\sin(dx + c) + \cos(3/2*dx + 3/2*c)^2*\sin(\\
& dx + c) + \sin(5/2*dx + 5/2*c)^2*\sin(dx + c) + 2*\sin(5/2*dx + 5/2*c)*\sin \\
& (3/2*dx + 3/2*c)*\sin(dx + c) + \sin(3/2*dx + 3/2*c)^2*\sin(dx + c))\cos(9 \\
& /2*dx + 9/2*c)^3 - 90*((\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(\\
& dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c) + 1 \\
&)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(c \\
& os(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c) \\
& + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(9/2*dx + 9/2*c)^3 - 20*((\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*d \\
& *x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/ \\
& 2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*s \\
& in(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) \\
& + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(7/2*dx + 7/2*c)^ \\
& 3 - (840*(\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^3 + 336*(\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^3 - 840*\cos(5/2*d \\
& *x + 5/2*c)^3*\sin(dx + c) + 21*(45*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \\
& \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + 45*(\\
& \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c \\
&) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(dx + c)^2 + 90*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2* \\
& dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \\
& \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + 16*(c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c) - \\
& 80*\cos(3/2*dx + 3/2*c)*\sin(dx + c) + 45*\log(\cos(1/2*dx + 1/2*c)^2 + \sin \\
& (1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - 45*\log(\cos(1/2*dx + 1/ \\
& 2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(5/2*dx \\
& + 5/2*c)^2 + 945*((\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2* \\
& \sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2* \\
& c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2 \\
& *c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2* \\
& dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(\\
& dx + c)^2 + 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin \\
& (1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^ \\
& 2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 \\
& + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + \\
& 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(3/2*d \\
& *x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 \\
& + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + \\
& 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + 45*(\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(c
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d* \\
& x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 9 \\
& 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - \\
& 180*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2* \\
& d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d \\
& *x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\si \\
& n(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) + 42 \\
& *(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3 \\
& /2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\si \\
& n(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2* \\
& c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos \\
& (d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos \\
& (d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d* \\
& x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) \\
& + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d* \\
& x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\si \\
& n(3/2*d*x + 3/2*c))*\cos(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90* \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*co \\
& s(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d \\
& *x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& os(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin \\
& n(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& n(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2* \\
& d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*(\\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/ \\
& 2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(\\
& d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c) \\
& ^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2* \\
& c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d \\
& *x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c
\end{aligned}$$

$$\begin{aligned}
&) + 3) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c)^2 - (840 * (\cos(d*x + c) + \\
& 1) \sin(5/2*d*x + 5/2*c)^3 + 336 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d* \\
& x + c) + 1) \sin(3/2*d*x + 3/2*c)^3 - 840 * \cos(5/2*d*x + 5/2*c)^3 * \sin(d*x + c \\
&) + 21 * (45 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + 45 * (\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x \\
& + c)^2 + 90 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + 16 * (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2 * \cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c) - 80 * \cos(3/2*d*x + 3/2*c) * \\
& \sin(d*x + c) + 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * s \\
& \sin(1/2*d*x + 1/2*c) + 1) - 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(5/2*d*x + 5/2*c)^2 + 945 * ((\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/ \\
& 2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c \\
&) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)^2 + 21 * (45 * (lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x \\
& + 1/2*c) + 1)) * \cos(d*x + c)^2 + 45 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90 * (l \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x \\
& + 1/2*c) + 1)) * \cos(d*x + c) + 16 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7 * \cos(\\
& d*x + c) + 6) \sin(3/2*d*x + 3/2*c) - 40 * \cos(5/2*d*x + 5/2*c) * \sin(d*x + c) + \\
& 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1 \\
& /2*c) + 1) - 45 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin \\
& (1/2*d*x + 1/2*c) + 1)) * \sin(5/2*d*x + 5/2*c)^2 + 945 * ((\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos \\
& (d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d* \\
& x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*
\end{aligned}$$

$$\begin{aligned}
& *c)^2 - 21*(40*(\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^3 + 16*(\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^3 - 40*\cos(5 \\
& /2*dx + 5/2*c)^3*\sin(dx + c) + (45*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2* \\
& dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \\
& \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + 45* \\
& (\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2* \\
& c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(dx + c)^2 + 90*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2 \\
& *dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 \\
& + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + 16*(\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c) \\
& - 80*\cos(3/2*dx + 3/2*c)*\sin(dx + c) + 45*\log(\cos(1/2*dx + 1/2*c)^2 + \sin \\
& (1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - 45*\log(\cos(1/2*dx + 1 \\
& /2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(5/2*dx \\
& + 5/2*c)^2 + 45*((\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2* \\
& \sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2* \\
& c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2 \\
& *c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2* \\
& dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(\\
& dx + c)^2 + 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin \\
& (1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^ \\
& 2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 \\
& + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + \\
& 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(3/2*d \\
& x + 3/2*c)^2 + (45*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2 \\
& *sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2 \\
& *c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + 45*(\log(\cos(1/2*dx + \\
& 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(\\
& 1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))* \\
& \sin(dx + c)^2 + 90*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + \\
& 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/ \\
& 2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + 16*(\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 7*\cos(dx + c) + 6)*\sin(3/2*dx + 3/2*c) - 40*\cos(5/2*dx + \\
& 5/2*c)*\sin(dx + c) + 45*\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c) \\
& ^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - 45*\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2* \\
& dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(5/2*dx + 5/2*c)^2 + 45*(\\
& (\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2* \\
& c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*d \\
& x + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin \\
& (1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + c)^2 + 2*(\log \\
& (\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) \\
& + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*d \\
& x + 1/2*c) + 1))*\cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/ \\
& 2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(\\
& 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
& *c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(\\
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x \\
& + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3 \\
& /2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + \\
& c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c \\
&)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2* \\
& d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) - 930*((\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + \\
& c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d* \\
& x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin \\
& n(d*x + c))*\cos(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) \\
& + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d* \\
& x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d \\
& *x + c))*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^ \\
& 2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \\
& \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) \\
& + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + (\cos(5/2*d*x + 5/2*c)^2*\sin \\
& (d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(
\end{aligned}$$

$$\begin{aligned}
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 2*(45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 5*((4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 + 90*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c) - (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) + 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& 2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3* \\
& \sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + \\
& 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*(\\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 9 \\
& 0*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7* \\
& \cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + \\
& c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c) \sin(3/2 dx + \\
& 3/2 c) - 20\cos(3/2 dx + 3/2 c)^2 \sin(dx + c) - 20\sin(3/2 dx + 3/2 c)^2 \\
& * \sin(dx + c) + 45 * ((\log(\cos(1/2 dx + 1/2 c))^2 + \sin(1/2 dx + 1/2 c)^2 + \\
& 2\sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/ \\
& 2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(dx + c)^2 + (\log(\cos(1/2 dx + 1 \\
& /2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/ \\
& 2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \sin \\
& n(dx + c)^2 + 2 * (\log(\cos(1/2 dx + 1/2 c))^2 + \sin(1/2 dx + 1/2 c)^2 + 2 * \sin \\
& (1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c \\
&)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(dx + c) + \log(\cos(1/2 dx + 1/2 c)^ \\
& 2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx \\
& + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(3/2 \\
& dx + 3/2 c)) \cos(5/2 dx + 5/2 c) + 2 * (20 * (\cos(dx + c) + 1) \cos(5/2 dx + \\
& 5/2 c)^2 + 20 * (\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + 4 * (4 * \cos(dx + c \\
&)^2 + 4 * \sin(dx + c)^2 + 13 * \cos(dx + c) + 9) \sin(3/2 dx + 3/2 c)^2 + 40 * (\\
& (\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c) - \sin(3/2 dx + 3/2 c) \sin(dx + c) \\
&) \cos(5/2 dx + 5/2 c) + 12 * \cos(dx + c)^2 + 45 * ((\log(\cos(1/2 dx + 1/2 c))^ \\
& 2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx \\
& + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(dx \\
& + c)^2 + (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx \\
& * x + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 * \\
& \sin(1/2 dx + 1/2 c) + 1)) \sin(dx + c)^2 + 2 * (\log(\cos(1/2 dx + 1/2 c)^2 + \\
& \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1 \\
& /2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(dx + c \\
&) + \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1 \\
& /2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/ \\
& 2 dx + 1/2 c) + 1)) \sin(3/2 dx + 3/2 c) + 12 * \sin(dx + c)^2 + 24 * \cos(dx \\
& + c) + 12) * \sin(5/2 dx + 5/2 c) + 8 * (2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& * \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + 3 * \cos(dx + c)^2 + 3 * \sin(dx + \\
& c)^2 + 6 * \cos(dx + c) + 3) * \sin(3/2 dx + 3/2 c)) * \sin(7/2 dx + 7/2 c)) * \sin(\\
& 9/2 dx + 9/2 c) - 10 * (2 * ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \cos(dx + c) \\
& + 1) \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \cos(dx \\
& * x + c) + 1) \cos(5/2 dx + 5/2 c) * \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2 * \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2 * \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx \\
& * x + c)^2 + \sin(dx + c)^2 + 2 * \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) * \sin(3 \\
& /2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \cos(dx + c) + 1) \sin \\
& (3/2 dx + 3/2 c)^2) \cos(7/2 dx + 7/2 c)^2 + 7 * (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2 * \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 14 * (\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2 * \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) * \cos(3/2 dx + 3/2 \\
& * c) + 7 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \cos(dx + c) + 1) \cos(3/2 dx \\
& + 3/2 c)^2 + 7 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \cos(dx + c) + 1) \sin(5 \\
& /2 dx + 5/2 c)^2 + 14 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \cos(dx + c) + \\
& 1) \sin(5/2 dx + 5/2 c) * \sin(3/2 dx + 3/2 c) + 7 * (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2 * \cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) * \sin(7/2 dx + 7/2 c) +
\end{aligned}$$

$$\begin{aligned}
& 2268 * (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx \\
& + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5 \\
& /2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^ \\
& ^2 + 2*\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2*\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c) \\
& ^2)\cos(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& x + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2* \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c) \\
& *\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + \\
& 1)\sin(3/2*dx + 3/2*c)^2)\cos(9/2*dx + 9/2*c)\cos(7/2*dx + 7/2*c) + ((c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 \\
& + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\cos(5/2*dx + 5 \\
& /2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + \\
& c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(\\
& dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2*\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)\cos(7/ \\
& 2*dx + 7/2*c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)* \\
& \cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c \\
&) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2*\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2*\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx \\
& + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\sin(3/2* \\
& dx + 3/2*c)^2)\sin(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^ \\
& ^2 + 2*\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2*\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c) \\
& ^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\sin(5/2*dx + 5 \\
& /2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\sin(5/2* \\
& dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*co \\
& s(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)\sin(9/2*dx + 9/2*c)*\sin(7/2*dx + \\
& 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\cos(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\cos \\
& (5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2*\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2*\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(\\
& dx + c)^2 + 2*\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)\sin(3/2*dx + 3/2* \\
& c)^2)\sin(7/2*dx + 7/2*c)^2)\sin(1/3*\arctan2(\sin(3/2*dx + 3/2*c), \cos(3/2
\end{aligned}$$

$$\begin{aligned}
& \cos(d*x + c))) - 120*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x \\
& + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3 \\
& /2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c) \\
& *\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(3/2*\arctan2(\sin(d*x + c),
\end{aligned}$$

$$\begin{aligned}
& \cos(d*x + c))) + 30*((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c) \\
& ^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (7 \\
& 4*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 10 \\
& 5)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179* \\
& \cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^ \\
& 2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179 \\
& *\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin \\
& (d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/ \\
& 2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c) \\
& ^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (7 \\
& 4*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((74*\cos(d*x + c)^2 \\
& + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& 74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 1 \\
& 79*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin \\
& (d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d* \\
& x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d* \\
& x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c) \\
&)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + \\
& 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74 \\
& *\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179 \\
& *\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((7 \\
& 4*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*si \\
& n(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c) \\
&)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)* \\
& \sin(7/2*d*x + 7/2*c) + ((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d* \\
& x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + \\
& c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*
\end{aligned}$$

$$\begin{aligned}
& x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 17 \\
& 9*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d \\
& *x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c \\
&)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) * \\
& \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 930*((\cos(5/ \\
& 2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& *\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^ \\
& 2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \\
& \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d \\
& *x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\si \\
& n(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2* \\
& \sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \si \\
& n(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c \\
&) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d \\
& *x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d* \\
& x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + (\\
& \cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5 \\
& /2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos \\
& (5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2* \\
& c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c \\
&) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + \\
& 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(\\
& 5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2* \\
& c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c) \\
& ^2)*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 2*(45*((\cos(d*x + c) + 1 \\
&)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x \\
& + 9/2*c)^2 + 45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\si \\
& n(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/ \\
& 2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d* \\
& x + 3/2*c)^2 + 5*((4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + \\
& 13)*\cos(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*co
\end{aligned}$$

$$\begin{aligned}
& s(d*x + c) + 13) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (4*\cos(d*x + c) \\
&)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13) * \cos(3/2*d*x + 3/2*c)^2 + (4* \\
& \cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13) * \sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13) * \sin(\\
& 5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^ \\
& 2 + 17*\cos(d*x + c) + 13) * \sin(3/2*d*x + 3/2*c)^2 * \sin(7/2*d*x + 7/2*c)^2 + \\
& 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2 \\
& *c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d \\
& *x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 + 90*(((\cos(d*x + c) + 1) * \cos(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/ \\
& 2*c) + (\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1) * \sin(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2) * \cos(7/2*d*x + 7/2*c) - \\
& (\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x \\
& + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x + \\
& 5/2*c)^2 * \sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(d* \\
& x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \sin(7/2*d*x + 7/2*c)) * \cos(9/2 \\
& *d*x + 9/2*c) + 21*(40*(\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^3 - \\
& 40*\cos(5/2*d*x + 5/2*c)^3 * \sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) \\
& ^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c \\
&) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + \\
& 3/2*c) - 80*\cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos \\
& (5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \co \\
& s(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + 45*(\log(\cos(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5 \\
& /2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^ \\
& 2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 \\
& + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) \\
& ^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d \\
& *x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - \\
& 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c \\
&)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3 \\
& /2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin \\
& (3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2 \\
& *d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*(\\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2
\end{aligned}$$

$$\begin{aligned}
& c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + \\
& 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/ \\
& 2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2* \\
& \cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2*\cos(9/2*dx + 9/2*c)^2 + 2*((\cos(\\
& dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + \\
& 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2* \\
& c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2* \\
& \cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2*\cos(9/2*d \\
& *x + 9/2*c)*\cos(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*co \\
& s(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (co \\
& s(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 \\
& + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/ \\
& 2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1)\sin(3/2*dx + 3/2*c)^2*\cos(7/2*dx + 7/2*c)^2 + ((\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*d \\
& *x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(3/ \\
& 2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*s \\
& in(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2*\sin(9/2*dx + 9/2*c)^ \\
& 2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + \\
& 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/ \\
& 2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2* \\
& \cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^ \\
& 2*\sin(9/2*dx + 9/2*c)*\sin(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + si \\
& n(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c \\
&) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(3/2*dx + 3/ \\
& 2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx \\
& + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(\\
& 5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2*\sin(7/2*dx + 7/2*c)^2*\cos(2/3 \\
& *arctan2(\sin(3/2*dx + 3/2*c), \cos(3/2*dx + 3/2*c)))^2 + ((\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2* \\
& dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\cos(3
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c) \\
& ^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) \\
& ^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3 \\
& /2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + (((co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2 \\
& *d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2 \\
& *d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2 \\
& *c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c) \\
& ^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5 \\
& /2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2* \\
& dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1)\sin(3/2*dx + 3/2*c)^2*\sin(9/2*dx + 9/2*c)^2 + 2*((\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2* \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c) \\
& *\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2*\sin(9/2*dx \\
& + 9/2*c)*\sin(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + \\
& 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2* \\
& c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1)\sin(3/2*dx + 3/2*c)^2*\sin(7/2*dx + 7/2*c)^2*\sin(5/2*\arctan2(\sin(dx \\
& *x + c), \cos(dx + c))) - 120*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& x + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2* \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c) \\
& *\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\sin(3/2*dx + 3/2*c)^2*\cos(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)*\cos(3/2*dx \\
& + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2* \\
& dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin \\
& (5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2*\cos(9/2*dx + 9/2*c)*\cos \\
& (7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1)\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*d \\
& *x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/ \\
& 2*dx + 3/2*c)^2*\cos(7/2*dx + 7/2*c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) +
\end{aligned}$$

$$\begin{aligned}
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c) \\
& ^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 \\
& + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2* \\
& dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1)\sin(3/2*dx + 3/2*c)^2*\sin(9/2*dx + 9/2*c)^2 + 2*((\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2* \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c) \\
& *\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2*\sin(9/2*dx \\
& + 9/2*c)*\sin(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + \\
& 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2* \\
& c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1)\sin(3/2*dx + 3/2*c)^2*\sin(7/2*dx + 7/2*c)^2*\sin(3/2*\arctan2(\sin(dx \\
& + c), \cos(dx + c))) + 30*((74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179 \\
& *\cos(dx + c) + 105)*\cos(5/2*dx + 5/2*c)^2 + 2*(74*\cos(dx + c)^2 + 74*\sin \\
& (dx + c)^2 + 179*\cos(dx + c) + 105)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/ \\
& 2*c) + (74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\cos \\
& (3/2*dx + 3/2*c)^2 + (74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx \\
& + c) + 105)*\sin(5/2*dx + 5/2*c)^2 + 2*(74*\cos(dx + c)^2 + 74*\sin(dx + c) \\
& ^2 + 179*\cos(dx + c) + 105)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (7 \\
& 4*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\sin(3/2*dx \\
& + 3/2*c)^2*\cos(9/2*dx + 9/2*c)^2 + 2*((74*\cos(dx + c)^2 + 74*\sin(dx + c \\
&)^2 + 179*\cos(dx + c) + 105)*\cos(5/2*dx + 5/2*c)^2 + 2*(74*\cos(dx + c)^2 \\
& + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\cos(5/2*dx + 5/2*c)*\cos(3/2 \\
& *dx + 3/2*c) + (74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + \\
& 105)*\cos(3/2*dx + 3/2*c)^2 + (74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179 \\
& *\cos(dx + c) + 105)*\sin(5/2*dx + 5/2*c)^2 + 2*(74*\cos(dx + c)^2 + 74*\sin \\
& (dx + c)^2 + 179*\cos(dx + c) + 105)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/ \\
& 2*c) + (74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\sin \\
& (3/2*dx + 3/2*c)^2*\cos(9/2*dx + 9/2*c)*\cos(7/2*dx + 7/2*c) + ((74*\cos(dx \\
& + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\cos(5/2*dx + 5/2*c \\
&)^2 + 2*(74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\cos \\
& (5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (74*\cos(dx + c)^2 + 74*\sin(dx + \\
& c)^2 + 179*\cos(dx + c) + 105)*\cos(3/2*dx + 3/2*c)^2 + (74*\cos(dx + c)^2 \\
& + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\sin(5/2*dx + 5/2*c)^2 + 2*(\\
& 74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 179*\cos(dx + c) + 105)*\sin(5/2*dx \\
& + 5/2*c)*\sin(3/2*dx + 3/2*c) + (74*\cos(dx + c)^2 + 74*\sin(dx + c)^2 + 1 \\
& 79*\cos(dx + c) + 105)*\sin(3/2*dx + 3/2*c)^2*\cos(7/2*dx + 7/2*c)^2 + ((7
\end{aligned}$$

$$\begin{aligned}
& 4*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin \\
& n(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c \\
&)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^ \\
& 2 + 2*((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d* \\
& x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^ \\
& 2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (7 \\
& 4*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin \\
& n(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x \\
& + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 1 \\
& 79*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin \\
& in(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos \\
& os(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d* \\
& x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + \\
& c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d* \\
& x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x \\
& + c))))*\sin(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^2 - 1 \\
& 008*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& os(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 \\
&)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& os(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& in(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*
\end{aligned}$$

$$\begin{aligned}
& x + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \cos(7/2 \\
& dx + 7/2 c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& s(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^ \\
& 2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + \\
& 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx \\
& x + 3/2 c)^2 \sin(9/2 dx + 9/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 \\
& *c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx \\
& x + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(\\
& dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \sin(9/2 dx + 9/2 c) \sin(7/2 dx + 7/ \\
& 2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
& + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5 \\
& /2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c) \\
& ^2) \sin(7/2 dx + 7/2 c)^2 \cos(1/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx \\
& *x + 3/2 c))) \sin(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) \\
& + 2(90(\cos(5/2 dx + 5/2 c)^2 \sin(dx + c) + 2\cos(5/2 dx + 5/2 c) \cos(3 \\
& /2 dx + 3/2 c) \sin(dx + c) + \cos(3/2 dx + 3/2 c)^2 \sin(dx + c) + \sin(5/ \\
& 2 dx + 5/2 c)^2 \sin(dx + c) + 2\sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) \\
& * \sin(dx + c) + \sin(3/2 dx + 3/2 c)^2 \sin(dx + c)) \cos(9/2 dx + 9/2 c)^3 \\
& - 90((\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c) + 1) \cos \\
& (5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c) + 1) \cos(3/2 dx + 3 \\
& /2 c)^2 + (\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c) + 1) * \\
& \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c) + 1) \sin(3/2 dx \\
& + 3/2 c)^2) \sin(9/2 dx + 9/2 c)^3 - 20((\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (c \\
& s(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 * \\
& c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx \\
& + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(d \\
& *x + c) + 1) \sin(3/2 dx + 3/2 c)^2 \sin(7/2 dx + 7/2 c)^3 - (840(\cos(dx \\
& + c) + 1) \sin(5/2 dx + 5/2 c)^3 + 336(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^3 - 840 \cos(5/2 dx + 5/2 c)^3 \sin
\end{aligned}$$

$$\begin{aligned}
& (d*x + c) + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + \\
& 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945* \\
& ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21 \\
& *(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 \\
& + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d* \\
& x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 180*(\cos(5/2*d*x \\
& + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(
\end{aligned}$$

$$\begin{aligned}
& d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin \\
& (d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(\\
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) + 42*(16*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x \\
& + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c \\
&)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3 \\
& /2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\co \\
& s(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(\\
& d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\si \\
& n(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/ \\
& 2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45* \\
& ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12* \\
& \sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3* \\
& \cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c \\
&))*\cos(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^ \\
& 3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c)^2 + 45*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + \\
& 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/ \\
& 2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + c)^2 + 90*(\log(\cos(1/2*dx \\
& + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos \\
& (1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1)) \\
& *\cos(dx + c) + 16*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin \\
& (3/2*dx + 3/2*c) - 80*\cos(3/2*dx + 3/2*c)*\sin(dx + c) + 45*\log(\cos(1/2 \\
& *dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - 45 \\
& *\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2* \\
& c) + 1))*\cos(5/2*dx + 5/2*c)^2 + 45*((\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2 \\
& *dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 \\
& + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + (1 \\
& \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) \\
& + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx \\
& + 1/2*c) + 1))*\sin(dx + c)^2 + 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx \\
& x + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin \\
& (1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + \log(\cos \\
& (1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) \\
& - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2 \\
& *c) + 1))*\cos(3/2*dx + 3/2*c)^2 + (45*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/ \\
& 2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 \\
& + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + 4 \\
& 5*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2 \\
& *dx + 1/2*c) + 1))*\sin(dx + c)^2 + 90*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1 \\
& /2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^ \\
& 2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + 16 \\
& *(\cos(dx + c)^2 + \sin(dx + c)^2 + 7*\cos(dx + c) + 6)*\sin(3/2*dx + 3/2*c \\
&) - 40*\cos(5/2*dx + 5/2*c)*\sin(dx + c) + 45*\log(\cos(1/2*dx + 1/2*c)^2 + \\
& \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - 45*\log(\cos(1/2*dx + \\
& 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(5/2*dx \\
& *x + 5/2*c)^2 + 45*((\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + \\
& 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/ \\
& 2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + (\log(\cos(1/2*dx + 1 \\
& /2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin \\
& (dx + c)^2 + 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin \\
& (1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c) \\
&)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^ \\
& 2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx \\
& + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(3/2* \\
& dx + 3/2*c)^2 + 2*(16*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + \\
& 1))*\cos(3/2*dx + 3/2*c)*\sin(3/2*dx + 3/2*c) - 20*\cos(3/2*dx + 3/2*c)^2*\sin \\
& (dx + c) - 20*\sin(3/2*dx + 3/2*c)^2*\sin(dx + c) + 45*((\log(\cos(1/2*dx \\
& + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& s(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + \\
& 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + \\
& c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2* \\
& c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + \\
& c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + \\
& c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/ \\
& 2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(\\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2* \\
& c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d* \\
& x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 - (840*(\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + 21*(45*(\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45 \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d* \\
& x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 -
\end{aligned}$$

$$\begin{aligned}
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\co \\
& s(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& os(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin \\
& (3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*1 \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \si \\
& n(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + \\
& 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3 \\
& /2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c))*\cos(9/2*d*x + 9/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x \\
& + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d \\
& *x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*
\end{aligned}$$

$$\begin{aligned}
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + \\
& c) + 10)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10 \\
&)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(\\
& 4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/ \\
& 2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c) \\
& *\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\si \\
& n(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 \\
& + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 \\
& + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(9/2*d*x \\
& + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40 \\
& *\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \si \\
& n(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 \\
& + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
& + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3 \\
& /2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5 \\
& /2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1
\end{aligned}$$

$$\begin{aligned}
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2 \\
& *d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 \\
& + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + \\
& 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 \\
& + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x \\
& + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 2 \\
& 0*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^ \\
& 2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x \\
& + c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2 \\
& *c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3 \\
& /2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d \\
& *x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(\\
& d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d \\
& *x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin \\
& (7/2*d*x + 7/2*c)^2 + 2*(45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2* \\
& \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos \\
& (7/2*d*x + 7/2*c)^2 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)*\sin(7/2*d*x + 7/2*c) + 45*(\cos \\
& (5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2* \\
& c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c \\
&) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 - 21*(40*(\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3 \\
& *\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45* \\
& ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(1
\end{aligned}$$

$$\begin{aligned}
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (4 \\
& 5*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + \\
& 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7 \\
& * \cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + \\
& c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\co \\
& s(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + \\
& 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^ \\
& 2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\s \\
& in(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2 \\
& *d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1))*\cos(5/2*d*x \\
& + 5/2*c)^2 + 20*(\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + \\
& c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + 40* \\
& ((\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c \\
&))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x \\
& + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + \\
& c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos \\
& (9/2*d*x + 9/2*c) - 930*((\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(\\
& d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9 \\
& /2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/ \\
& 2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2* \\
& d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + \\
& 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/ \\
& 2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c))*\cos(7/2*d*x + 7/2*c)^2 + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^ \\
& 2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2* \\
& c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c)) \\
& *\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/ \\
& 2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*s \\
& \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)* \\
& \sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\si \\
& n(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + \\
& c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d* \\
& x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\si \\
& n(d*x + c))*\sin(7/2*d*x + 7/2*c)^2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + \\
& c))) - 2*(45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(\\
& 3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 45*((\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6) * \sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c) * \sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) * \sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9) * \sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos
\end{aligned}$$

$$\begin{aligned}
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 \\
& + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d* \\
& x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*s \\
& \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*s \\
& \sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& *\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x \\
& + 7/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \\
& 36*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 \\
&)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& os(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*s \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*
\end{aligned}$$

$$\begin{aligned}
& x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2) * \cos(7/2* \\
& d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos \\
& s(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d* \\
& x + 3/2*c)^2) * \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d* \\
& x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2) * \sin(9/2*d*x + 9/2*c) * \sin(7/2*d*x + 7/ \\
& 2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5 \\
& /2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c) \\
& ^2) * \sin(7/2*d*x + 7/2*c)^2) * \sin(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - \\
& 120*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2 \\
& *d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 \\
&) * \cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * s \\
& in(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&) * \sin(3/2*d*x + 3/2*c)^2) * \cos(9/2*d*x + 9/2*c) * \cos(7/2*d*x + 7/2*c) + ((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2 \\
& *c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*
\end{aligned}$$

$$\begin{aligned}
& x + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c) \\
&)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \cos(7/2 \\
& dx + 7/2 c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos \\
& (5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) \\
& + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2 \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2 \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + \\
& 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(3/2 dx \\
& x + 3/2 c)^2 \sin(9/2 dx + 9/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2 \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2 \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(5/2 dx + 5/2 \\
& c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(5/2 dx \\
& x + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx \\
& + c) + 1) \sin(3/2 dx + 3/2 c)^2 \sin(9/2 dx + 9/2 c) \sin(7/2 dx + 7/ \\
& 2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(5/2 dx \\
& + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \cos(5 \\
& /2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2 \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2 \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(3/2 dx + 3/2 c) \\
& ^2 \sin(7/2 dx + 7/2 c)^2 \sin(3/2 \arctan 2(\sin(dx + c), \cos(dx + c))) + \\
& 30(((74 \cos(dx + c)^2 + 74 \sin(dx + c)^2 + 179 \cos(dx + c) + 105) \cos(5 \\
& /2 dx + 5/2 c)^2 + 2(74 \cos(dx + c)^2 + 74 \sin(dx + c)^2 + 179 \cos(dx \\
& + c) + 105) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (74 \cos(dx + c)^2 \\
& + 74 \sin(dx + c)^2 + 179 \cos(dx + c) + 105) \cos(3/2 dx + 3/2 c)^2 + (74 \\
& \cos(dx + c)^2 + 74 \sin(dx + c)^2 + 179 \cos(dx + c) + 105) \sin(5/2 dx + \\
& 5/2 c)^2 + 2(74 \cos(dx + c)^2 + 74 \sin(dx + c)^2 + 179 \cos(dx + c) + 10 \\
& 5) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (74 \cos(dx + c)^2 + 74 \sin(dx \\
& + c)^2 + 179 \cos(dx + c) + 105) \sin(3/2 dx + 3/2 c)^2 \cos(9/2 dx + \\
& 9/2 c)^2 + 2(((74 \cos(dx + c)^2 + 74 \sin(dx + c)^2 + 179 \cos(dx + c) + 1 \\
& 05) \cos(5/2 dx + 5/2 c)^2 + 2(74 \cos(dx + c)^2 + 74 \sin(dx + c)^2 + 179 \\
& \cos(dx + c) + 105) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (74 \cos(dx \\
& x + c)^2 + 74 \sin(dx + c)^2 + 179 \cos(dx + c) + 105) \cos(3/2 dx + 3/2 c) \\
& ^2 + (74 \cos(dx + c)^2 + 74 \sin(dx + c)^2 + 179 \cos(dx + c) + 105) \sin(5 \\
& /2 dx + 5/2 c)^2 + 2(74 \cos(dx + c)^2 + 74 \sin(dx + c)^2 + 179 \cos(dx \\
& + c) + 105) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (74 \cos(dx + c)^2 \\
& + 74 \sin(dx + c)^2 + 179 \cos(dx + c) + 105) \sin(3/2 dx + 3/2 c)^2 \cos(9 \\
& /2 dx + 9/2 c) \cos(7/2 dx + 7/2 c) + ((74 \cos(dx + c)^2 + 74 \sin(dx + c) \\
&)^2 + 179 \cos(dx + c) + 105) \cos(5/2 dx + 5/2 c)^2 + 2(74 \cos(dx + c)^2 \\
& + 74 \sin(dx + c)^2 + 179 \cos(dx + c) + 105) \cos(5/2 dx + 5/2 c) \cos(3/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179 \\
& *\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin \\
& (d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/ \\
& 2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin \\
& (3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + ((74*\cos(d*x + c)^2 + 74*\sin(\\
& d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^ \\
& 2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + \\
& 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d \\
& *x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 1 \\
& 05)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2*((74*\cos(d*x + c)^2 \\
& + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(7 \\
& 4*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 17 \\
& 9*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(\\
& d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x \\
& + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)* \\
& \sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2 \\
& *c) + ((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d* \\
& x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^ \\
& 2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (7 \\
& 4*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*si \\
& n(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x \\
& + 7/2*c)^2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*\sin(2/3*\arctan2(s \\
& in(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 4*(45*(\cos(5/2*d*x + 5/2*c) \\
& ^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) \\
& + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 - 20*((\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)*\sin(7 \\
& /2*d*x + 7/2*c) + 45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x
\end{aligned}$$

$$\begin{aligned}
& + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2 \\
& *d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d \\
& *x + 7/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - \\
& 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
& ^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c \\
&) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*co \\
& s(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5 \\
& /2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^ \\
& 2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + si \\
& n(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 \\
& + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c) \\
& ^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d \\
& *x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - \\
& 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c \\
&)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3 \\
& /2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin \\
& (3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2 \\
& *d*x + 3/2*c)*\sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*(\\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c) + 12*\sin \\
& (d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos \\
& (d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c)) * \\
& \cos(7/2*d*x + 7/2*c)) * \cos(9/2*d*x + 9/2*c) + 4*(90*(\cos(5/2*d*x + 5/2*c)^2 * \\
& \sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + \\
& 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/ \\
& 2*c)^2*\sin(d*x + c)) * \cos(9/2*d*x + 9/2*c)^3 - 90*((\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c) \\
& ^3 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2)*\sin(7/2*d*x + 7/2*c)^3 - (840*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^ \\
& 3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + 21*(45*(\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^ \\
& 2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d* \\
& x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c
\end{aligned}$$

$$\begin{aligned}
& s(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + 3 \cos(dx + c)^2 + 3 \sin(dx + c)^2 \\
& + 6 \cos(dx + c) + 3) \sin(3/2 dx + 3/2 c) \cos(9/2 dx + 9/2 c)^2 - 21 (\\
& 40 (\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^3 + 16 (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2 \cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^3 - 40 \cos(5/2 dx + 5/2 \\
& *c)^3 \sin(dx + c) + (45 (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c) \\
& ^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx \\
& + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \cos(dx + c)^2 + 45 (\log(\cos(1/2 \\
& *dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log \\
& (\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) \\
& + 1)) \sin(dx + c)^2 + 90 (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c) \\
&)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx \\
& x + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \cos(dx + c) + 16 (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1) \sin(3/2 dx + 3/2 c) - 80 \cos(3/2 \\
& *dx + 3/2 c) \sin(dx + c) + 45 \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + \\
& 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - 45 \log(\cos(1/2 dx + 1/2 c)^2 + \sin \\
& (1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \cos(5/2 dx + 5/2 c)^2 \\
& + 45 ((\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx \\
& + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin \\
& (1/2 dx + 1/2 c) + 1)) \cos(dx + c)^2 + (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(\\
& 1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c) \\
& ^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \sin(dx + c)^2 + \\
& 2 (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1 \\
& /2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/ \\
& 2 dx + 1/2 c) + 1)) \cos(dx + c) + \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx \\
& x + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin \\
& (1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \cos(3/2 dx + 3/2 c)^2 \\
& + (45 (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx \\
& + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin \\
& (1/2 dx + 1/2 c) + 1)) \cos(dx + c)^2 + 45 (\log(\cos(1/2 dx + 1/2 c)^2 + \\
& \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/ \\
& 2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \sin(dx + c) \\
& ^2 + 90 (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx \\
& x + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin \\
& (1/2 dx + 1/2 c) + 1)) \cos(dx + c) + 16 (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 7 \cos(dx + c) + 6) \sin(3/2 dx + 3/2 c) - 40 \cos(5/2 dx + 5/2 c) \sin(dx \\
& + c) + 45 \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/ \\
& 2 dx + 1/2 c) + 1) - 45 \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c) \\
& ^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \sin(5/2 dx + 5/2 c)^2 + 45 ((\log(\cos(1/2 \\
& *dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log \\
& (\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) \\
& + 1)) \cos(dx + c)^2 + (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 \\
& + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + \\
& 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)) \sin(dx + c)^2 + 2 (\log(\cos(1/2 dx \\
& x + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) - \log(\cos \\
& (1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1)
\end{aligned}$$

$$\begin{aligned}
&)) * \cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1)) * \sin(3/2*dx + 3/2*c)^2 + 2*(16*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1) * \cos(3/2*dx + 3/2*c) * \sin(3/2*dx + 3/2*c) - 20*\cos(3/2*dx + 3/2*c)^2 * \sin(dx + c) - 20*\sin(3/2*dx + 3/2*c)^2 * \sin(dx + c) + 45*((\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1)) * \cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1)) * \sin(dx + c)^2 + 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1)) * \cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1)) * \cos(3/2*dx + 3/2*c) * \cos(5/2*dx + 5/2*c) + 2*(20*(\cos(dx + c) + 1) * \cos(5/2*dx + 5/2*c)^2 + 20*(\cos(dx + c) + 1) * \cos(3/2*dx + 3/2*c)^2 + 4*(4*\cos(dx + c)^2 + 4*\sin(dx + c)^2 + 13*\cos(dx + c) + 9) * \sin(3/2*dx + 3/2*c)^2 + 40*((\cos(dx + c) + 1) * \cos(3/2*dx + 3/2*c) - \sin(3/2*dx + 3/2*c) * \sin(dx + c)) * \cos(5/2*dx + 5/2*c) + 12*\cos(dx + c)^2 + 45*((\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1)) * \cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1)) * \sin(dx + c)^2 + 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1)) * \cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1)) * \sin(3/2*dx + 3/2*c) + 12*\sin(dx + c)^2 + 24*\cos(dx + c) + 12) * \sin(5/2*dx + 5/2*c) + 8*(2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1) * \cos(3/2*dx + 3/2*c)^2 + 3*\cos(dx + c)^2 + 3*\sin(dx + c)^2 + 6*\cos(dx + c) + 3) * \sin(3/2*dx + 3/2*c) * \cos(7/2*dx + 7/2*c)^2 + (90*(\cos(5/2*dx + 5/2*c)^2 * \sin(dx + c) + 2*\cos(5/2*dx + 5/2*c) * \cos(3/2*dx + 3/2*c) * \sin(dx + c) + \cos(3/2*dx + 3/2*c)^2 * \sin(dx + c) + \sin(5/2*dx + 5/2*c)^2 * \sin(dx + c) + 2*\sin(5/2*dx + 5/2*c) * \sin(3/2*dx + 3/2*c) * \sin(dx + c) + \sin(3/2*dx + 3/2*c)^2 * \sin(dx + c)) * \cos(9/2*dx + 9/2*c)^3 - 90*((\cos(dx + c) + 1) * \cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c) + 1) * \cos(5/2*dx + 5/2*c) * \cos(3/2*dx + 3/2*c) + (\cos(dx + c) + 1) * \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c) + 1) * \sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c) + 1) * \sin(5/2*dx + 5/2*c) * \sin(3/2*dx + 3/2*c) + (\cos(dx + c) + 1) * \sin(3/2*dx + 3/2*c)^2) * \sin(9/2*dx + 9/2*c)^3 - 20*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1) * \cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1) * \cos(5/2*dx + 5/2*c) * \cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1) * \cos(3/2*dx + 3/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(3/2*d*x + 3/2*c)^2 - 180*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12))*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 3
\end{aligned}$$

$$\begin{aligned}
& * \cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c) \\
&)*\cos(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& ^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 4 \\
& 5*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + \\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 1 \\
& 6*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2* \\
& c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2* \\
& d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*s \\
& \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
&+ 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2 \\
&*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
&1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*s \\
&\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x \\
&+ 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\co \\
&s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
&*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
&*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + \\
&1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
&/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
&\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
&/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
&- 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + \\
&2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*co \\
&s(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x \\
&+ c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
&*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x \\
&+ c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
&/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
&- 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 \\
&+ \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
&1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + \\
&c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
&d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
&*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3 \\
&/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8* \\
&(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
&*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d \\
&*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 - (840*(\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
&5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
&3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + 21*(45*(\log(\\
&\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
&1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
&1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
&+ 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log \\
&(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
&1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
&1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
&x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 4 \\
&5*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2* \\
& d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10) \\
& *\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + \\
& c) + 10)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)*\si \\
& n(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 1 \\
& 0)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4* \\
& (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3 \\
& /2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c) \\
&)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 \\
& + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^ \\
& 2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(9/2*d* \\
& x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 4 \\
& 0*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^ \\
& 2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\si \\
& n(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
& + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
&+ 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x \\
&+ 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
&+ 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
&+ 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
&1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
&*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
&(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
&+ 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/ \\
&2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
&(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&+ 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
&*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/ \\
&2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
&1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 \\
&+ 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
&+ 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
&(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 \\
&+ 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
&1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
&/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
&*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^ \\
&2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d \\
&x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - \\
&20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
&^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
&1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
&1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&+ 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
&x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d \\
&x + c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(\\
& 3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2* \\
& d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin \\
& (d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(\\
& d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin \\
& (7/2*d*x + 7/2*c)^2 + 2*(45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))* \\
& \cos(7/2*d*x + 7/2*c)^2 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)*\sin(7/2*d*x + 7/2*c) + 45*(\cos \\
& (5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/ \\
& 2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2 \\
& *c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 - 21*(40*(\\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^ \\
& 3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45 \\
& *((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (\\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + \\
& 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x \\
& + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& os(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x \\
& + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c) \\
& ^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + \\
& c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40 \\
& *((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d* \\
& x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d* \\
& x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x \\
& + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\co \\
& s(9/2*d*x + 9/2*c) - 930*((\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2* \\
& d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin \\
& (d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\si \\
& n(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(\\
& 9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d* \\
& x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3 \\
& /2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2 \\
& *d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + \\
& 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3 \\
& /2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c))*\cos(7/2*d*x + 7/2*c)^2 + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c) \\
& ^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2 \\
& *c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
&)*\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2* \\
& \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\s \\
& in(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x \\
& + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/ \\
& 2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*s \\
& in(d*x + c))*\sin(7/2*d*x + 7/2*c)^2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x \\
& + c))) - 2*(45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)^2 + 45*((\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c) \\
& ^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 5*((4*\cos(d*x + c)^2 + 4*\sin \\
& (d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x \\
& + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + \\
& 13)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos \\
& (d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + \\
& c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (4 \\
& *\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(3/2*d*x + 3/ \\
& 2*c)^2*\sin(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 35*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 + \\
& 90*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2*\cos(7/2*d*x + 7/2*c) - (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2* \\
& c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5 \\
& /2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + \\
& c))*\sin(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) + 21*(40*(\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (\\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + \\
& 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6 \\
&)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(\\
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \\
& 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*co \\
& s(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(
\end{aligned}$$

$$\begin{aligned}
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 \\
& + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)* \\
& \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d* \\
& x + 7/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \\
& + 36*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^ \\
& 2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*
\end{aligned}$$

$$\begin{aligned}
& 5/2*d*x + 5/2*c)^2 + 2*(74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d*x \\
& + c) + 105)*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3/2*c) + (74*cos(d*x + c)^2 \\
& + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*sin(3/2*d*x + 3/2*c)^2)*cos(\\
& 9/2*d*x + 9/2*c)*cos(7/2*d*x + 7/2*c) + ((74*cos(d*x + c)^2 + 74*sin(d*x + \\
& c)^2 + 179*cos(d*x + c) + 105)*cos(5/2*d*x + 5/2*c)^2 + 2*(74*cos(d*x + c)^ \\
& 2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*cos(5/2*d*x + 5/2*c)*cos(3/ \\
& 2*d*x + 3/2*c) + (74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) \\
& + 105)*cos(3/2*d*x + 3/2*c)^2 + (74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 17 \\
& 9*cos(d*x + c) + 105)*sin(5/2*d*x + 5/2*c)^2 + 2*(74*cos(d*x + c)^2 + 74*si \\
& n(d*x + c)^2 + 179*cos(d*x + c) + 105)*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3 \\
& /2*c) + (74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*si \\
& n(3/2*d*x + 3/2*c)^2)*cos(7/2*d*x + 7/2*c)^2 + ((74*cos(d*x + c)^2 + 74*sin \\
& (d*x + c)^2 + 179*cos(d*x + c) + 105)*cos(5/2*d*x + 5/2*c)^2 + 2*(74*cos(d* \\
& x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*cos(5/2*d*x + 5/2*c) \\
& *cos(3/2*d*x + 3/2*c) + (74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d* \\
& x + c) + 105)*cos(3/2*d*x + 3/2*c)^2 + (74*cos(d*x + c)^2 + 74*sin(d*x + c) \\
& ^2 + 179*cos(d*x + c) + 105)*sin(5/2*d*x + 5/2*c)^2 + 2*(74*cos(d*x + c)^2 \\
& + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*sin(5/2*d*x + 5/2*c)*sin(3/2* \\
& d*x + 3/2*c) + (74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + \\
& 105)*sin(3/2*d*x + 3/2*c)^2)*sin(9/2*d*x + 9/2*c)^2 + 2*((74*cos(d*x + c)^2 \\
& + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& 74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*cos(5/2*d*x \\
& + 5/2*c)*cos(3/2*d*x + 3/2*c) + (74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 1 \\
& 79*cos(d*x + c) + 105)*cos(3/2*d*x + 3/2*c)^2 + (74*cos(d*x + c)^2 + 74*sin \\
& (d*x + c)^2 + 179*cos(d*x + c) + 105)*sin(5/2*d*x + 5/2*c)^2 + 2*(74*cos(d* \\
& x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*sin(5/2*d*x + 5/2*c) \\
& *sin(3/2*d*x + 3/2*c) + (74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d* \\
& x + c) + 105)*sin(3/2*d*x + 3/2*c)^2)*sin(9/2*d*x + 9/2*c)*sin(7/2*d*x + 7/ \\
& 2*c) + ((74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*co \\
& s(5/2*d*x + 5/2*c)^2 + 2*(74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d \\
& *x + c) + 105)*cos(5/2*d*x + 5/2*c)*cos(3/2*d*x + 3/2*c) + (74*cos(d*x + c) \\
& ^2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*cos(3/2*d*x + 3/2*c)^2 + (\\
& 74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + 105)*sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(74*cos(d*x + c)^2 + 74*sin(d*x + c)^2 + 179*cos(d*x + c) + \\
& 105)*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3/2*c) + (74*cos(d*x + c)^2 + 74*s \\
& in(d*x + c)^2 + 179*cos(d*x + c) + 105)*sin(3/2*d*x + 3/2*c)^2)*sin(7/2*d*x \\
& + 7/2*c)^2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*cos(2/3*arctan2(\\
& sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - (840*(cos(d*x + c) + 1)*si \\
& n(5/2*d*x + 5/2*c)^3 + 336*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c \\
&) + 1)*sin(3/2*d*x + 3/2*c)^3 - 840*cos(5/2*d*x + 5/2*c)^3*sin(d*x + c) + 2 \\
& 1*(45*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x \\
& + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin \\
& (1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 45*(log(cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2 \\
& *c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^
\end{aligned}$$

$$\begin{aligned}
& 2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d \\
& *x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + \\
& c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c \\
&) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 90*(\cos(5/2*d*x + 5/2*c)^2*\sin \\
& (d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2* \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2* \\
& c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20 \\
& *\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& c) + 45 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c) \\
&)) * \cos(5/2*d*x + 5/2*c) + 20 * ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11 * \cos(d*x + c) + 10) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11 * \cos(d*x + c) + 10) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11 * \cos(d*x + c) + 10) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11 * \cos(d*x + c) + 10) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11 * \cos(d*x + c) + 10) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11 * \cos(d*x + c) + 10) * \sin(3/2*d*x + 3/2*c)^2) * \sin(7/2*d*x + 7/2*c) + 42 * (20 * (\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 20 * (\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 4 * (4 * \cos(d*x + c)^2 + 4 * \sin(d*x + c)^2 + 13 * \cos(d*x + c) + 9) * \sin(3/2*d*x + 3/2*c)^2 + 40 * ((\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 12 * \cos(d*x + c)^2 + 45 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c) + 12 * \sin(d*x + c)^2 + 24 * \cos(d*x + c) + 12) * \sin(5/2*d*x + 5/2*c) + 168 * (2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 3 * \cos(d*x + c)^2 + 3 * \sin(d*x + c)^2 + 6 * \cos(d*x + c) + 3) * \sin(3/2*d*x + 3/2*c) * \sin(9/2*d*x + 9/2*c)^2 - 21 * (40 * (\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^3 + 16 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^3 - 40 * \cos(5/2*d*x + 5/2*c)^3 * \sin(d*x + c) + (45 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + 45 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * c
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c) + 16(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin \\
& (3/2*dx + 3/2*c) - 80\cos(3/2*dx + 3/2*c)\sin(dx + c) + 45\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 45\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) \\
& + 1))\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(dx + c)^2 + (\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + \\
& 1/2*c) + 1))\sin(dx + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(dx + c) + \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c \\
&) + 1))\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(dx + c)^2 + 45* \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d \\
& *x + 1/2*c) + 1))\sin(dx + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(dx + c) + 16*(\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 7\cos(dx + c) + 6)\sin(3/2*d*x + 3/2*c) \\
& - 40\cos(5/2*d*x + 5/2*c)\sin(dx + c) + 45\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 45\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\sin(5/2*d*x \\
& + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(dx + c)^2 + (\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\sin(\\
& dx + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(dx + c) + \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\sin(3/2*d* \\
& x + 3/2*c)^2 + 2*(16*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(\\
& dx + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(dx + c) + 45*((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos \\
& (dx + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\sin(dx + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2* \\
& (20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3 \\
& /2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c \\
&) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c \\
&)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) \\
& ^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2* \\
& c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x \\
& + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 - 504*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + \\
& 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)* \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + \\
& 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx \\
& + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/ \\
& 2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \\
& * \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 * \sin(9/2 dx + 9/2 c)^2 + 2 \\
& * ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 \\
& c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
& + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 * \sin \\
& (9/2 dx + 9/2 c) \sin(7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/ \\
& 2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx \\
& + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 * \sin(7/2 dx + 7/2 c)^2 * \cos(1/3 \arctan \\
& 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) * \sin(2/3 \arctan 2(\sin(3/2 dx \\
& + 3/2 c), \cos(3/2 dx + 3/2 c))) + (90 * (\cos(5/2 dx + 5/2 c))^2 \sin(dx + \\
& c) + 2\cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) \sin(dx + c) + \cos(3/2 dx \\
& + 3/2 c)^2 \sin(dx + c) + \sin(5/2 dx + 5/2 c)^2 \sin(dx + c) + 2\sin(5/2 \\
& dx + 5/2 c) \sin(3/2 dx + 3/2 c) \sin(dx + c) + \sin(3/2 dx + 3/2 c)^2 \sin \\
& (dx + c)) \cos(9/2 dx + 9/2 c)^3 - 90 * ((\cos(dx + c) + 1) \cos(5/2 dx + 5/ \\
& 2 c)^2 + 2(\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\\
& \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c) + 1) \sin(5/2 dx + \\
& 5/2 c)^2 + 2(\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) \\
& + (\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) * \sin(9/2 dx + 9/2 c)^3 - 20 * ((\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 \\
& + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + \\
& 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) * \sin(7 \\
& /2 dx + 7/2 c)^3 - (840 * (\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^3 + 336 * (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^3 \\
& - 840 * \cos(5/2 dx + 5/2 c)^3 \sin(dx + c) + 21 * (45 * (\log(\cos(1/2 dx + 1/2 \\
& c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx \\
& + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(dx \\
& + c)^2 + 45 * (\log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin \\
& (1/2 dx + 1/2 c) + 1) - \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2
\end{aligned}$$

$$\begin{aligned}
& d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20 \\
& *((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin \\
& (7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20* \\
& (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x \\
& + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + \\
& 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(\\
& 5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(\\
& d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(\\
& d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d \\
&*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2 \\
&*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
&2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
&+ 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log \\
&(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
&1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
&1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
&1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
&/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\co \\
&s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&- \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
&2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
&+ 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(lo \\
&g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&+ 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
&+ 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
&x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + s \\
&\sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(l \\
&og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&+ 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
&+ 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(\\
&d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + \\
&45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
&/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
&(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2 \\
&*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
&d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
&d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
&/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
&- 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c) \\
&^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
&+ 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
&+ c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
&+ 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\si \\
&n(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + s \\
&\sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2* \\
&c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin \\
&(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*si \\
&n(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
&x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d* \\
&x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x \\
& + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 \\
& + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\co \\
& s(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
& ^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) \\
& + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 \\
& + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 - (840* \\
& (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2* \\
& c)^3*\sin(d*x + c) + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3 \\
& /2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^ \\
& 2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c) \\
&)^2 + 21*(45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d* \\
& x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 7*\cos(d*x + c) + 6) * \sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c \\
&) * \sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c)^2 - 90*(\cos(\\
& 5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2* \\
& c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x + 5/2*c \\
&)^2 * \sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) \\
& + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \cos(9/2*d*x + 9/2*c) + 42*(16*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) * \sin(\\
& 3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) - 20*\sin(3/2*d*x \\
& + 3/2*c)^2 * \sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) * \cos(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 11*\cos(d*x + c) + 10) * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10) * \cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10) * \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x +
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& * \cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)* \\
& \sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 4 \\
& 5*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/ \\
& 2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45 \\
& *((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(\\
& 5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + \\
& c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/ \\
& 2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2 \\
& *d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + \\
& c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(45*(\cos(5/2*d*x \\
& + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d \\
& *x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(\\
& d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3 \\
& /2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 - 20*((\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
&)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2* \\
& c)*\sin(7/2*d*x + 7/2*c) + 45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2* \\
& \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin \\
& (7/2*d*x + 7/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + \\
& 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2 \\
& *c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) \\
& + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + \\
& 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) - 930*((\cos(5/2*d*x + 5/ \\
& 2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x \\
& + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c \\
&)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c \\
&) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x \\
& + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + (\cos(5 \\
& /2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c \\
&)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c \\
&)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + (\cos(5/2*d* \\
& x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin \\
& n(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + \\
& 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d* \\
& x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d \\
& *x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/ \\
& 2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + \\
& (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + \\
& 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2)*\cos(1/ \\
& 2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 2*(45*((\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/ \\
& 2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 \\
& + 45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3 \\
& /2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 35* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)
\end{aligned}$$

$$\begin{aligned}
&^2 + 5*((4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/ \\
&2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) \\
&+ 13)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin \\
&n(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + \\
&c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)^2 + 2* \\
&(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + \\
&5/2*c)*\sin(3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos \\
&(d*x + c) + 13)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + 35*(\cos(d* \\
&x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 70 \\
&*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
&c) + 1)*\sin(3/2*d*x + 3/2*c)^2 + 90*(((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
&c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
&(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
&/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
&(\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c) - (\cos(5/2* \\
&d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin \\
&(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2* \\
&\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin \\
&(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2 \\
&*c) + 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 \\
&+ \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2 \\
&*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
&x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log \\
&(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&+ 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
&+ 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
&*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos \\
&(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - \\
&80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
&1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2 \\
&*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + \\
&5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
&n(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
&x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d* \\
&x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
&/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
&- 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x \\
&+ 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s
\end{aligned}$$

$$\begin{aligned}
& 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c) \\
& ^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 \\
& + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x \\
& + 7/2*c))*\sin(9/2*d*x + 9/2*c) - 10*(2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 7*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 14*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 7*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/ \\
& 2*d*x + 7/2*c) + 2016*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x \\
& + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c \\
&)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x + \\
& 3/2*c), \cos(3/2*d*x + 3/2*c))) + 36*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/ \\
& 2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
&)^2 + 179\cos(dx + c) + 105)\sin(3/2*dx + 3/2*c)^2)\cos(7/2*dx + 7/2*c)^2 \\
&+ ((74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(5/2*dx + 5/2*c)^2 \\
&+ 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(5/2*dx + 5/2*c) \\
&\cos(3/2*dx + 3/2*c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105) \\
&\cos(3/2*dx + 3/2*c)^2 + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105) \\
&\sin(5/2*dx + 5/2*c)^2 + 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105) \\
&\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) \\
&+ 105)\sin(3/2*dx + 3/2*c)^2)\sin(9/2*dx + 9/2*c)^2 + 2*((74\cos(dx + c)^2 + 74\sin(dx + c)^2 \\
&+ 179\cos(dx + c) + 105)\cos(5/2*dx + 5/2*c)^2 + 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 \\
&+ 179\cos(dx + c) + 105)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (74\cos(dx + c)^2 \\
&+ 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(3/2*dx + 3/2*c)^2 + (74\cos(dx + c)^2 \\
&+ 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\sin(5/2*dx + 5/2*c)^2 + 2*(74\cos(dx + c)^2 \\
&+ 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) \\
&+ (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\sin(3/2*dx + 3/2*c)^2) \\
&\sin(9/2*dx + 9/2*c)\sin(7/2*dx + 7/2*c) + ((74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) \\
&+ 105)\cos(5/2*dx + 5/2*c)^2 + 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) \\
&+ 105)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 \\
&+ 179\cos(dx + c) + 105)\cos(3/2*dx + 3/2*c)^2 + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179 \\
&\cos(dx + c) + 105)\sin(5/2*dx + 5/2*c)^2 + 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) \\
&+ 105)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 \\
&+ 179\cos(dx + c) + 105)\sin(3/2*dx + 3/2*c)^2)\sin(9/2*dx + 9/2*c)\sin(7/2*dx + 7/2*c) \\
&+ ((74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(5/2*dx + 5/2*c)^2 \\
&+ 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(5/2*dx + 5/2*c) \\
&\cos(3/2*dx + 3/2*c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105) \\
&\cos(3/2*dx + 3/2*c)^2 + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105) \\
&\sin(5/2*dx + 5/2*c)^2 + 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105) \\
&\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) \\
&+ 105)\sin(3/2*dx + 3/2*c)^2)\sin(7/2*dx + 7/2*c)^2)\sin(1/2*\arctan2(\sin(dx + c), \\
&\cos(dx + c)))\sin(2/3*\arctan2(\sin(3/2*dx + 3/2*c), \cos(3/2*dx + 3/2*c)))^2 \\
&+ 2*(45*(\cos(5/2*dx + 5/2*c)^2*\sin(dx + c) + 2*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) \\
&\sin(dx + c) + \cos(3/2*dx + 3/2*c)^2*\sin(dx + c) + \sin(5/2*dx + 5/2*c)^2*\sin(dx + c) \\
&+ 2*\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c)\sin(dx + c) + \sin(3/2*dx + 3/2*c)^2 \\
&\sin(dx + c) + \sin(3/2*dx + 3/2*c)^2*\sin(dx + c))*\cos(7/2*dx + 7/2*c)^2 - 20*((\cos(dx + c)^2 \\
&+ \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
&+ 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
&+ 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) \\
&+ 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1) \\
&\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) \\
&+ 1)*\sin(3/2*dx + 3/2*c)^2)\cos(7/2*dx + 7/2*c)\sin(7/2*dx + 7/2*c) + 45*(\cos(5/2*dx + 5/2*c) \\
&^2*\sin(dx + c) + 2*\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c)\sin(dx + c) + \cos(3/2*dx + 3/2*c) \\
&^2*\sin(dx + c) + \sin(5/2*dx + 5/2*c)^2*\sin(dx + c) + 2*\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) \\
&\sin(dx + c) + \sin(3/2*dx + 3/2*c)^2*\sin(dx + c))\sin(7/2*dx + 7/2*c)^2 - 21*(40*(\cos(dx + c) + 1) \\
&\sin(5/2*dx + 5/2*c)^3 + 16*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^3 \\
&- 40*\cos(5/2*dx + 5/2*c)^3*\sin(dx + c) + (
\end{aligned}$$

$$\begin{aligned}
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \\
& 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\co \\
& s(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x \\
& + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + \\
& 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5 \\
& /2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x \\
& + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) \\
& + 2*(90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5 \\
& /2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c \\
&)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)^ \\
& 3 - 90*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^3 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^3 - (840*(\cos(d*
\end{aligned}$$

$$\begin{aligned}
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 180*(\cos(5/2*d*x \\
&x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin \\
&(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin \\
&(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin \\
&(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) + 42*(16*(\cos(d*x + \\
&c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x \\
&x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2* \\
&c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^ \\
&2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
&+ 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x \\
&+ 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
&(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
&2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
&/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2 \\
&*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2* \\
&d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
&3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c) \\
&^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
&*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
&(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c \\
&)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
&5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
&*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
&(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos \\
&(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
&3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin \\
&(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3 \\
&/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45 \\
&*((\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
&2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
&*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
&d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
&x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
&1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
&/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12* \\
&\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d \\
&*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3 \\
&*cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2* \\
&c))*\cos(9/2*d*x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
&^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
&+ 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& * \cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 4 \\
& 5*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + \\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 1 \\
& 6*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2* \\
& c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2* \\
& d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2 \\
& *d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*s
\end{aligned}$$

$$\begin{aligned}
& \sin(dx + c) - 20\sin(3/2*dx + 3/2*c)^2\sin(dx + c) + 45*((\log(\cos(1/2*dx \\
& + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1) \\
&)*\cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2 \\
& *\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2 \\
& *c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + c)^2 + 2*(\log(\cos(1/2*dx + \\
& 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1 \\
& /2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos \\
& (dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1 \\
& /2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 \\
& - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(3/2*dx + 3/2*c))*\cos(5/2*dx + 5/2*c) + \\
& 2*(20*(\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 20*(\cos(dx + c) + 1)*\cos \\
& (3/2*dx + 3/2*c)^2 + 4*(4*\cos(dx + c)^2 + 4*\sin(dx + c)^2 + 13*\cos(dx \\
& + c) + 9)*\sin(3/2*dx + 3/2*c)^2 + 40*((\cos(dx + c) + 1)*\cos(3/2*dx + 3/2 \\
& *c) - \sin(3/2*dx + 3/2*c)*\sin(dx + c))*\cos(5/2*dx + 5/2*c) + 12*\cos(dx \\
& + c)^2 + 45*((\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1 \\
& /2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 \\
& - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2*c)^2 \\
& + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + \\
& 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + \\
& c)^2 + 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2* \\
& dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2 \\
& *\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 + \sin \\
& (1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c \\
&)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(3/2*dx + 3 \\
& /2*c) + 12*\sin(dx + c)^2 + 24*\cos(dx + c) + 12)*\sin(5/2*dx + 5/2*c) + 8* \\
& (2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2 \\
& *c)^2 + 3*\cos(dx + c)^2 + 3*\sin(dx + c)^2 + 6*\cos(dx + c) + 3)*\sin(3/2*d \\
& *x + 3/2*c))*\cos(7/2*dx + 7/2*c)^2 - (840*(\cos(dx + c) + 1)*\sin(5/2*dx + \\
& 5/2*c)^3 + 336*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(\\
& 3/2*dx + 3/2*c)^3 - 840*\cos(5/2*dx + 5/2*c)^3*\sin(dx + c) + 21*(45*(\log(\\
& \cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + \\
& 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + \\
& 1/2*c) + 1))*\cos(dx + c)^2 + 45*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx \\
& + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin \\
& (1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + c)^2 + 90*(\log \\
& (\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + \\
& 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + \\
& 1/2*c) + 1))*\cos(dx + c) + 16*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& + c) + 1)*\sin(3/2*dx + 3/2*c) - 80*\cos(3/2*dx + 3/2*c)*\sin(dx + c) + 4 \\
& 5*\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2 \\
& *c) + 1) - 45*\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1 \\
& /2*dx + 1/2*c) + 1))*\cos(5/2*dx + 5/2*c)^2 + 945*((\log(\cos(1/2*dx + 1/2* \\
& c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(d
\end{aligned}$$

$$\begin{aligned}
& *x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& \cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin \\
& (3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(3/2*d*x + 3/2*c)^2 - 90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) \\
& + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d* \\
& x + c))*\cos(9/2*d*x + 9/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d* \\
& x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)) * \cos(5/2* \\
& d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10) \\
& *\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + \\
& c) + 10)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)*\si \\
& n(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 1 \\
& 0)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4* \\
& (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3 \\
& /2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c \\
&)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 \\
& + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^ \\
& 2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(9/2*d* \\
& x + 9/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 4 \\
& 0*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^ \\
& 2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\si \\
& n(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
& + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)
\end{aligned}$$

$$\begin{aligned}
&)) * \sin(dx + c)^2 + 2 * (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 \\
&+ 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + \\
&1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(dx + c) + \log(\cos(1/2 * dx + 1/ \\
&2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 \\
&* dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos \\
&(3/2 * dx + 3/2 * c)^2 + (45 * (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c) \\
&)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx \\
&x + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(dx + c)^2 + 45 * (\log(\cos(1/ \\
&2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log \\
&(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) \\
&+ 1)) * \sin(dx + c)^2 + 90 * (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * \\
&c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx \\
&* x + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(dx + c) + 16 * (\cos(dx + c) \\
&)^2 + \sin(dx + c)^2 + 7 * \cos(dx + c) + 6) * \sin(3/2 * dx + 3/2 * c) - 40 * \cos(5/ \\
&2 * dx + 5/2 * c) * \sin(dx + c) + 45 * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + \\
&1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 45 * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin \\
&(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \sin(5/2 * dx + 5/2 * c)^2 \\
&+ 45 * ((\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx \\
&+ 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin \\
&(1/2 * dx + 1/2 * c) + 1)) * \cos(dx + c)^2 + (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin \\
&(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c) \\
&)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \sin(dx + c)^2 \\
&+ 2 * (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + \\
&1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1 \\
&/2 * dx + 1/2 * c) + 1)) * \cos(dx + c) + \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx \\
&* x + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \\
&\sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \sin(3/2 * dx + 3/2 * c) \\
&^2 + 2 * (16 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \cos(dx + c) + 1) * \cos(3/2 * dx \\
&x + 3/2 * c) * \sin(3/2 * dx + 3/2 * c) - 20 * \cos(3/2 * dx + 3/2 * c)^2 * \sin(dx + c) - \\
&20 * \sin(3/2 * dx + 3/2 * c)^2 * \sin(dx + c) + 45 * ((\log(\cos(1/2 * dx + 1/2 * c)^2 + \\
&\sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/ \\
&2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(dx + c) \\
&^2 + (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + \\
&1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(\\
&1/2 * dx + 1/2 * c) + 1)) * \sin(dx + c)^2 + 2 * (\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin \\
&(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \log(\cos(1/2 * dx + 1/2 * c) \\
&)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(dx + c) + \\
&\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) \\
&+ 1) - \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx \\
&x + 1/2 * c) + 1)) * \cos(3/2 * dx + 3/2 * c)) * \cos(5/2 * dx + 5/2 * c) + 2 * (20 * (\cos(dx \\
&x + c) + 1) * \cos(5/2 * dx + 5/2 * c)^2 + 20 * (\cos(dx + c) + 1) * \cos(3/2 * dx + 3/ \\
&2 * c)^2 + 4 * (4 * \cos(dx + c)^2 + 4 * \sin(dx + c)^2 + 13 * \cos(dx + c) + 9) * \sin(\\
&3/2 * dx + 3/2 * c)^2 + 40 * ((\cos(dx + c) + 1) * \cos(3/2 * dx + 3/2 * c) - \sin(3/2 * \\
&dx + 3/2 * c) * \sin(dx + c)) * \cos(5/2 * dx + 5/2 * c) + 12 * \cos(dx + c)^2 + 45 * ((\\
&\log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c)
\end{aligned}$$

$$\begin{aligned}
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin \\
& (d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(\\
& d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin \\
& (7/2*d*x + 7/2*c)^2 + 2*(45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))* \\
& \cos(7/2*d*x + 7/2*c)^2 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)*\sin(7/2*d*x + 7/2*c) + 45*(\cos \\
& (5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/ \\
& 2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2 \\
& *c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 - 21*(40*(\\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^ \\
& 3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45 \\
& *((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (\\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + \\
& 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x \\
& + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& os(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x \\
& + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c) \\
& ^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/ \\
& 2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1))*\cos(5/2*d*x \\
& + 5/2*c)^2 + 20*(\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + \\
& c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + 40 \\
& *((\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)) * \cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x \\
& x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x \\
& x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x \\
& + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos \\
& s(9/2*d*x + 9/2*c) - 930*((\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2* \\
& d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin \\
& (d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(\\
& 9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d* \\
& x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3 \\
& /2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2 \\
& *d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + \\
& 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3 \\
& /2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c))*\cos(7/2*d*x + 7/2*c)^2 + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c) \\
& ^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2 \\
& *c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
&)*\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2* \\
& \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin \\
& (9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x \\
& + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/ \\
& 2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin \\
& (d*x + c))*\sin(7/2*d*x + 7/2*c)^2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x \\
& + c))) - 2*(45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 45*((\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c) \\
& ^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 5*((4*\cos(d*x + c)^2 + 4*\sin \\
& (d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x \\
& + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + \\
& 13)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos \\
& (d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + \\
& c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (4 \\
& *\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(3/2*d*x + 3/ \\
& 2*c)^2*\sin(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 35*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 + \\
& 90*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\cos(7/2*d*x + 7/2*c) - (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2* \\
& c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5 \\
& /2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + \\
& c))*\sin(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) + 21*(40*(\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (\\
& 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + \\
& 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2
\end{aligned}$$

$$\begin{aligned}
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6 \\
&)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(\\
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \\
& 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\co \\
& s(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x \\
& + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + \\
& 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5 \\
& /2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x \\
& + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(9/2*d*x + 9/2*c) \\
& - 10*(2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2* \\
& c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 7*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 7*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^ \\
& 2 + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 2268*((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x \\
& + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*
\end{aligned}$$

$$\begin{aligned}
& 2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2 \\
& *d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*c \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d \\
& *x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^ \\
& 2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7 \\
& /2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2*\sin(7/2*d*x + 7/2*c)^2*\sin(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - \\
& 120*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^ \\
& 2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)* \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/
\end{aligned}$$

$$\begin{aligned}
& 2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2 \\
& *d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*c \\
& os(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d \\
& *x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^ \\
& 2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7 \\
& /2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + \\
& 30*(((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 \\
& + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74 \\
& *\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 1 \\
& 05)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin \\
& (d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + \\
& 9/2*c)^2 + 2*((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 17 \\
& 9*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d \\
& *x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c \\
&)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(\\
& 5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x \\
& + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 \\
& + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\cos(\\
& 9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((74*\cos(d*x + c)^2 + 74*\sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2*\cos(1/2*\arctan(2(\sin(d*x + c), \cos(d*x + c))) - 2*(45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 5*((4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 + 90*(((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c) - (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) + \\
& 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + \\
& 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c \\
&)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) \\
& ^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\si \\
& n(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2* \\
& c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x \\
& + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)* \\
& \sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c))*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 2*4*\cos(d*x + c) + 12))*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(9/2*d*x + 9/2*c) - 10*(2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c))*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c))*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(5/2*d*x + 5/2*c))*\cos(3/2*d*x + 3/2*c) + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + 7*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 14*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d
\end{aligned}$$

$$\begin{aligned}
& *x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + 7 * (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \sin(7/2*d*x \\
& + 7/2*c) + 595 * (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos \\
& (5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + \\
& 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x \\
& + 3/2*c)^2 * \cos(9/2*d*x + 9/2*c)^2 + 2 * ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2* \\
& c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x \\
& + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d \\
& *x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(9/2*d*x + 9/2*c) * \cos(7/2*d*x + 7/2 \\
& *c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + \\
& 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/ \\
& 2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \\
& \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^ \\
& 2) * \cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x \\
& + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos \\
& (d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) \\
&) * \sin(3/2*d*x + 3/2*c)^2 * \cos(9/2*d*x + 9/2*c)^2 + 2 * ((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d* \\
& x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5 \\
& /2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) \\
&) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
&)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(9/2*d*x + 9/2*c) * \cos(\\
& 7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \\
& \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) \\
&) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2* \\
& d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*c \\
& os(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + \\
& 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2 \\
& *d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x \\
& + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c
\end{aligned}$$

$$\begin{aligned}
&)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2* \\
& d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)^2 \\
& + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2) \\
& *\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c) \\
&)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2* \\
& c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& *\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x \\
& + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d \\
& *x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 \\
& 2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*((\cos \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2 \\
& *d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2 \\
& *d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\co \\
& s(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2 \\
& *c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x
\end{aligned}$$

$$\begin{aligned}
& 2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + \\
& 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx \\
& x + 3/2 c)^2) \sin(7/2 dx + 7/2 c)^2) \sin(5/2 \arctan2(\sin(dx + c), \cos(dx \\
& + c))) - 120 * (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(\\
& 5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx \\
& * x + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3 \\
& /2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx \\
& + 3/2 c)^2) \cos(9/2 dx + 9/2 c)^2 + 2 * ((\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c \\
&)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx \\
& + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& x + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c) \cos(7/2 dx + 7/2 \\
& c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + \\
& 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 \\
& * dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& os(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \\
&) \cos(7/2 dx + 7/2 c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin \\
& (3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) * \\
& \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx + 9/2 c)^2 + 2 * ((\cos(dx + c)^2 + \sin(dx \\
& * x + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3 \\
& /2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx \\
& + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 \\
& * dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) * \\
& \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx + 9/2 c) \sin(7/ \\
& 2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& s(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^
\end{aligned}$$

$$\begin{aligned}
& 2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2dx + 5/2c) \sin(3/2dx + \\
& 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2dx + \\
& x + 3/2c)^2) \sin(7/2dx + 7/2c)^2) \sin(3/2 \arctan 2(\sin(dx + c), \cos(dx + \\
& c))) + 30 * (((74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + \\
& 105)\cos(5/2dx + 5/2c)^2 + 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 17 \\
& 9\cos(dx + c) + 105)\cos(5/2dx + 5/2c)\cos(3/2dx + 3/2c) + (74\cos(dx + \\
& c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(3/2dx + 3/2c \\
&)^2 + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\sin(\\
& 5/2dx + 5/2c)^2 + 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + \\
& c) + 105)\sin(5/2dx + 5/2c)\sin(3/2dx + 3/2c) + (74\cos(dx + c)^2 \\
& + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\sin(3/2dx + 3/2c)^2) \cos(\\
& 9/2dx + 9/2c)^2 + 2*((74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + \\
& c) + 105)\cos(5/2dx + 5/2c)^2 + 2*(74\cos(dx + c)^2 + 74\sin(dx + \\
& c)^2 + 179\cos(dx + c) + 105)\cos(5/2dx + 5/2c)\cos(3/2dx + 3/2c) + \\
& (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(3/2dx \\
& x + 3/2c)^2 + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + \\
& 105)\sin(5/2dx + 5/2c)^2 + 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 17 \\
& 9\cos(dx + c) + 105)\sin(5/2dx + 5/2c)\sin(3/2dx + 3/2c) + (74\cos(dx \\
& c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\sin(3/2dx + 3/2c \\
&)^2) \cos(9/2dx + 9/2c) \cos(7/2dx + 7/2c) + ((74\cos(dx + c)^2 + 74\sin \\
& (dx + c)^2 + 179\cos(dx + c) + 105)\cos(5/2dx + 5/2c)^2 + 2*(74\cos(\\
& dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(5/2dx + 5/2 \\
& c)\cos(3/2dx + 3/2c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(\\
& dx + c) + 105)\cos(3/2dx + 3/2c)^2 + (74\cos(dx + c)^2 + 74\sin(dx + \\
& c)^2 + 179\cos(dx + c) + 105)\sin(5/2dx + 5/2c)^2 + 2*(74\cos(dx + c)^ \\
& 2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\sin(5/2dx + 5/2c)\sin(3/ \\
& 2dx + 3/2c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) \\
& + 105)\sin(3/2dx + 3/2c)^2) \cos(7/2dx + 7/2c)^2 + ((74\cos(dx + c)^2 \\
& + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(5/2dx + 5/2c)^2 + 2*(\\
& 74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(5/2dx \\
& + 5/2c)\cos(3/2dx + 3/2c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 1 \\
& 79\cos(dx + c) + 105)\cos(3/2dx + 3/2c)^2 + (74\cos(dx + c)^2 + 74\sin \\
& (dx + c)^2 + 179\cos(dx + c) + 105)\sin(5/2dx + 5/2c)^2 + 2*(74\cos(dx \\
& x + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\sin(5/2dx + 5/2c) \\
& * \sin(3/2dx + 3/2c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx \\
& x + c) + 105)\sin(3/2dx + 3/2c)^2) \sin(9/2dx + 9/2c)^2 + 2*((74\cos(dx \\
& x + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\cos(5/2dx + 5/2c \\
&)^2 + 2*(74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\co \\
& s(5/2dx + 5/2c)\cos(3/2dx + 3/2c) + (74\cos(dx + c)^2 + 74\sin(dx + \\
& c)^2 + 179\cos(dx + c) + 105)\cos(3/2dx + 3/2c)^2 + (74\cos(dx + c)^2 \\
& + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\sin(5/2dx + 5/2c)^2 + 2*(\\
& 74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c) + 105)\sin(5/2dx \\
& + 5/2c)\sin(3/2dx + 3/2c) + (74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 1 \\
& 79\cos(dx + c) + 105)\sin(3/2dx + 3/2c)^2) \sin(9/2dx + 9/2c) \sin(7/2 \\
& dx + 7/2c) + ((74\cos(dx + c)^2 + 74\sin(dx + c)^2 + 179\cos(dx + c)
\end{aligned}$$

$$\begin{aligned}
& + 105)\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + \\
& 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos \\
& (d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d \\
& *x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c) \\
& ^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\sin \\
& (7/2*d*x + 7/2*c)^2)*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))\cos(2/7 \\
& *\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 4*(90*(\cos(5/2*d*x \\
& + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d \\
& *x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(\\
& d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3 \\
& /2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)^3 - 90*((\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)* \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d \\
& *x + 9/2*c)^3 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2* \\
& d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^3 - (840*(\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)^3 + 336*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^3 - 840*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + 21*(45*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d \\
& *x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)^2 + 21*(45*(\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) * \cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 90*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) * \cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6) \\
& * \sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c) * \sin(d*x + c) + 45*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1)) * \sin(5/2*d*x + 5/2*c)^2 + 945*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c)^2 - 180*(\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + \\
& c) + 2*\cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d* \\
& x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2*\sin(5/2 \\
& *d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin \\
& (d*x + c)) * \cos(7/2*d*x + 7/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) * \sin(3/2*d*x + 3/2*c) - 20*\cos(3/ \\
& 2*d*x + 3/2*c)^2 * \sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 45 \\
& *((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)) * \cos(\\
& 5/2*d*x + 5/2*c) + 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1) * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin
\end{aligned}$$

$$\begin{aligned}
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d \\
& *x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 \\
& + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d* \\
& x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\co \\
& s(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d \\
& *x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\co \\
& s(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\si \\
& n(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\cos(9/2*d*x + 9/2* \\
& c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/ \\
& 2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - \\
& 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x \\
& + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d \\
& *x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1
\end{aligned}$$

$$\begin{aligned}
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*s \\
& \sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + \\
& 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(1 \\
& 6*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2* \\
& c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3 \\
& /2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + \\
& 1))*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + \\
& 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9))*\sin(3/2*d*x \\
& + 3/2*c)^2 + 40*((\cos(d*x + c) + 1))*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/ \\
& 2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2
\end{aligned}$$

$$\begin{aligned}
& n(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d* \\
& x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x \\
& + 3/2*c)^2 - 90*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2* \\
& c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + \\
& 9/2*c) + 42*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + \\
& c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d* \\
& x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 20*((\co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d \\
& *x + c) + 10)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 1 \\
& 1*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 11*\cos(d*x + c) + 10)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 11*\cos(d*x + c) + 10)*\sin(3/2*d*x + 3/2*c \\
&)^2*\sin(7/2*d*x + 7/2*c) + 42*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4* \\
& \sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/ \\
& 2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 1 \\
& 2)*\sin(5/2*d*x + 5/2*c) + 168*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + \\
& 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(9/2*d*x + 9/2*c)^2 - 21*(40* \\
& (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c) \\
& ^3*\sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d* \\
& x + 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 4 \\
& 5*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2* \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 \\
& + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x \\
& + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(45*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c))*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 - 20*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^ \\
& 2)*\cos(7/2*d*x + 7/2*c)*\sin(7/2*d*x + 7/2*c) + 45*(\cos(5/2*d*x + 5/2*c)^2*s \\
& \sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + co \\
& s(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2 \\
& *sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2 \\
& *c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 - 21*(40*(\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (45*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90* \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*co \\
& s(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d \\
& *x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& os(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*si \\
& n(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) * \sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c) + 2*(20*(\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9) * \sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2*c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12) * \sin(5/2*d*x + 5/2*c) + 8*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c)) * \cos(9/2*d*x + 9/2*c) - 930*((\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \cos(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2 * \sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d* \\
& x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(9/2*d*x + 9/2*c)*\cos(7/2* \\
& d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \\
& \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7 \\
& /2*c)^2 + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(\\
& 5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2* \\
& c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c) \\
& ^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/ \\
& 2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2 \\
& *d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)* \\
& \sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(9/2*d*x + 9/2*c)*\si \\
& n(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2 \\
& *d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\sin(7/2*d \\
& *x + 7/2*c)^2)*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 2*(45*((\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\c \\
& os(9/2*d*x + 9/2*c)^2 + 45*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + 35*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 70*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3 \\
& /2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)^2 + 5*((4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(\\
& d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c \\
&)^2 + 17*\cos(d*x + c) + 13)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (4* \\
& \cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 17*\cos(d*x + c) \\
& + 13)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (4*\cos(d*x + c)^2 + 4*si \\
& n(d*x + c)^2 + 17*\cos(d*x + c) + 13)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + \\
& 7/2*c)^2 + 35*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)^2 + 70*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + 35*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 + 90*((\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d \\
& *x + 3/2*c)^2 + 2*(16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 20*\cos(3/2*d*x + 3/2*c)^2*\sin \\
& (d*x + c) - 20*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 45*((\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 2 \\
& *(20*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 20*(\cos(d*x + c) + 1)*\cos(\\
& 3/2*d*x + 3/2*c)^2 + 4*(4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + \\
& c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 40*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
&) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*\cos(d*x + \\
& c)^2 + 45*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c \\
&)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2 \\
& *c) + 12*\sin(d*x + c)^2 + 24*\cos(d*x + c) + 12)*\sin(5/2*d*x + 5/2*c) + 8*(2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
&)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x \\
& + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(9/2*d*x + 9/2*c) - 10*(2*((\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^2)*\sin(1/3* \\
& \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 36*((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c \\
&)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\si \\
& n(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2 \\
& *d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2 \\
& *d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x \\
& + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2* \\
& d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((74*\cos(d*x + \\
& c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 \\
& + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74 \\
& *\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos \\
& s(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/ \\
& 2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos \\
& s(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(1/2* \\
& \arctan2(\sin(d*x + c), \cos(d*x + c))) * \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) - 1860*((\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^ \\
& 2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2* \\
& c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c)) \\
& *\cos(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/ \\
& 2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*s \\
& \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)* \\
& \sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos \\
& s(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + \\
& c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d* \\
& x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin \\
& n(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + (\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + \\
& 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3 \\
& /2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c))*\sin(9/2*d*x + 9/2*c)^2 + 2*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2* \\
& c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5 \\
& /2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + \\
& c))*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + (\cos(5/2*d*x + 5/2*c)^2*\sin \\
& (d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(\\
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*s \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c \\
&)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2)*\cos(1/2*\arctan2(\sin(d*x + c), \cos \\
& (d*x + c))) - 315*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d \\
& *x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d \\
& *x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*s \\
& in(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^ \\
& 2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^ \\
& 2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3 \\
& *arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)
\end{aligned}$$

$$\begin{aligned}
&^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx \\
&+ 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5 \\
&/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
&* \cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^ \\
&^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
&x + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + \\
&(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c) \\
&^2)*\sin(9/2*dx + 9/2*c)*\sin(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx \\
&+ c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + s \\
&\sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2* \\
&c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3 \\
&/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx \\
&x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin \\
&(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
&2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(7/2*dx + 7/2*c)^2 + (((co \\
&s(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 \\
&+ 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/ \\
&2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + \\
&c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(d \\
&>*x + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
&2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + \\
&c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\cos(9/2 \\
&*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1) \\
&*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + \\
&c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(d \\
&>*x + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \\
&\sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + \\
&c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx \\
&x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2 \\
&*dx + 3/2*c)^2)*\cos(9/2*dx + 9/2*c)*\cos(7/2*dx + 7/2*c) + ((\cos(dx + c) \\
&^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(d \\
&>*x + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3 \\
&/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*co \\
&s(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + \\
&1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
&+ c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin \\
&(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\cos(7/2*dx + 7/2 \\
&*c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx \\
&+ 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(\\
&5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
&2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c) \\
&^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(d \\
&>*x + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + \\
&(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c \\
&)^2)*\sin(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(d
\end{aligned}$$

$$\begin{aligned}
& *x + c) + 1) * \cos(5/2 * d * x + 5/2 * c)^2 + 2 * (\cos(d * x + c)^2 + \sin(d * x + c)^2 + \\
& 2 * \cos(d * x + c) + 1) * \cos(5/2 * d * x + 5/2 * c) * \cos(3/2 * d * x + 3/2 * c) + (\cos(d * x + \\
& c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \cos(3/2 * d * x + 3/2 * c)^2 + (\cos(d \\
& * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c)^2 + 2 \\
& * (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c \\
&) * \sin(3/2 * d * x + 3/2 * c) + (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) \\
& + 1) * \sin(3/2 * d * x + 3/2 * c)^2 * \sin(9/2 * d * x + 9/2 * c) * \sin(7/2 * d * x + 7/2 * c) + ((\\
& \cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \cos(5/2 * d * x + 5/2 * c)^2 \\
& + 2 * (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \cos(5/2 * d * x + \\
& 5/2 * c) * \cos(3/2 * d * x + 3/2 * c) + (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x \\
& + c) + 1) * \cos(3/2 * d * x + 3/2 * c)^2 + (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos \\
& (d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c)^2 + 2 * (\cos(d * x + c)^2 + \sin(d * x + c)^2 \\
& + 2 * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c) * \sin(3/2 * d * x + 3/2 * c) + (\cos(d * x \\
& + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(3/2 * d * x + 3/2 * c)^2 * \sin(7 \\
& /2 * d * x + 7/2 * c)^2 * \sin(2/3 * \arctan(2 * \sin(3/2 * d * x + 3/2 * c), \cos(3/2 * d * x + 3/2 * \\
& c)))^2 + 2 * (((\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \cos(5/2 \\
& * d * x + 5/2 * c)^2 + 2 * (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \\
& \cos(5/2 * d * x + 5/2 * c) * \cos(3/2 * d * x + 3/2 * c) + (\cos(d * x + c)^2 + \sin(d * x + c)^2 \\
& + 2 * \cos(d * x + c) + 1) * \cos(3/2 * d * x + 3/2 * c)^2 + (\cos(d * x + c)^2 + \sin(d * x \\
& + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c)^2 + 2 * (\cos(d * x + c)^2 + \sin \\
& (d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c) * \sin(3/2 * d * x + 3/2 * \\
& c) + (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(3/2 * d * x + 3 \\
& /2 * c)^2 * \cos(9/2 * d * x + 9/2 * c)^2 + 2 * ((\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos \\
& (d * x + c) + 1) * \cos(5/2 * d * x + 5/2 * c)^2 + 2 * (\cos(d * x + c)^2 + \sin(d * x + c)^2 \\
& + 2 * \cos(d * x + c) + 1) * \cos(5/2 * d * x + 5/2 * c) * \cos(3/2 * d * x + 3/2 * c) + (\cos(d * \\
& x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \cos(3/2 * d * x + 3/2 * c)^2 + (\cos \\
& (d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c)^2 \\
& + 2 * (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5 \\
& /2 * c) * \sin(3/2 * d * x + 3/2 * c) + (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + \\
& c) + 1) * \sin(3/2 * d * x + 3/2 * c)^2 * \cos(9/2 * d * x + 9/2 * c) * \cos(7/2 * d * x + 7/2 * c) \\
& + ((\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \cos(5/2 * d * x + 5/2 \\
& * c)^2 + 2 * (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \cos(5/2 * d * \\
& x + 5/2 * c) * \cos(3/2 * d * x + 3/2 * c) + (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(\\
& d * x + c) + 1) * \cos(3/2 * d * x + 3/2 * c)^2 + (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 \\
& * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c)^2 + 2 * (\cos(d * x + c)^2 + \sin(d * x + c \\
&)^2 + 2 * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c) * \sin(3/2 * d * x + 3/2 * c) + (\cos \\
& (d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(3/2 * d * x + 3/2 * c)^2 * \cos \\
& (7/2 * d * x + 7/2 * c)^2 + ((\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) \\
& + 1) * \cos(5/2 * d * x + 5/2 * c)^2 + 2 * (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * \\
& x + c) + 1) * \cos(5/2 * d * x + 5/2 * c) * \cos(3/2 * d * x + 3/2 * c) + (\cos(d * x + c)^2 + \sin \\
& (d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \cos(3/2 * d * x + 3/2 * c)^2 + (\cos(d * x + c)^2 \\
& + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c)^2 + 2 * (\cos(d * \\
& x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin(5/2 * d * x + 5/2 * c) * \sin(3/ \\
& 2 * d * x + 3/2 * c) + (\cos(d * x + c)^2 + \sin(d * x + c)^2 + 2 * \cos(d * x + c) + 1) * \sin \\
& (3/2 * d * x + 3/2 * c)^2 * \sin(9/2 * d * x + 9/2 * c)^2 + 2 * ((\cos(d * x + c)^2 + \sin(d * x
\end{aligned}$$

$$\begin{aligned}
& n(3/2*d*x + 3/2*c)^2 * \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 * \sin(9/2*d*x + 9/2*c) * \sin(7/2* \\
& d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2 * \sin(7/2*d*x + 7/2*c)^2 * \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos \\
& (3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2 * \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/ \\
& 2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 * \sin(9/2*d*x + 9/2*c) * \sin(7/2 \\
& *d*x + 7/2*c) + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2 * \sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*
\end{aligned}$$

$$\begin{aligned}
& 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)* \\
& \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + \\
& 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
&)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/ \\
& 2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2* \\
& d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 \\
& + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*
\end{aligned}$$

$$\begin{aligned}
& \cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d* \\
& x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2* \\
& c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 \\
&)*\sin(7/2*d*x + 7/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c)))^2 + 2*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*
\end{aligned}$$

$$\begin{aligned}
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2 \\
& *d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c \\
&)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + \\
& 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2 \\
& *c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*si \\
& n(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*s \\
& in(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5
\end{aligned}$$

$$\begin{aligned}
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2 \\
& *c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2 \\
& *c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos \\
& (2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\c \\
& os(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/ \\
& 2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\c \\
& os(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/ \\
& 2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + (\\
& ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*
\end{aligned}$$

$$\begin{aligned}
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos \\
& (9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*c \\
& os(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + \\
& 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3 \\
& /2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) \\
& + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d* \\
& x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*s \\
& in(7/2*d*x + 7/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + \\
& 3/2*c)))^2 + 2*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2 \\
& *c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^ \\
& 2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7 \\
& /2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d \\
& *x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c))))*\cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x \\
& + 7/2*c))) + 2*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^
\end{aligned}$$

$$\begin{aligned}
& 2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2dx + 5/2c) \sin(3/2dx + \\
& 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2dx \\
& x + 3/2c)^2) \cos(9/2dx + 9/2c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(5/2dx + 5/2c) \cos(3/2dx + 3/2c) + (c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2dx + 3/2c)^2 \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2dx + 5/2 \\
& *c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2dx \\
& x + 5/2c) \sin(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \sin(3/2dx + 3/2c)^2) \cos(9/2dx + 9/2c) \cos(7/2dx + 7/ \\
& 2c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2dx \\
& + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5 \\
& /2dx + 5/2c) \cos(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *cos(dx + c) + 1) \cos(3/2dx + 3/2c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \sin(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1) \sin(5/2dx + 5/2c) \sin(3/2dx + 3/2c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2dx + 3/2c) \\
& ^2) \cos(7/2dx + 7/2c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \cos(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2* \\
& cos(dx + c) + 1) \cos(5/2dx + 5/2c) \cos(3/2dx + 3/2c) + (\cos(dx + c)^ \\
& 2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2dx + 3/2c)^2 + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2dx + 5/2c)^2 + 2*(\\
& os(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2dx + 5/2c) *s \\
& in(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&) \sin(3/2dx + 3/2c)^2) \sin(9/2dx + 9/2c)^2 + 2*((\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^ \\
& 2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2dx + 5/2c) \cos(3/2dx + \\
& 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2dx \\
& x + 3/2c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5 \\
& /2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&) \sin(5/2dx + 5/2c) \sin(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1) \sin(3/2dx + 3/2c)^2) \sin(9/2dx + 9/2c) \sin(\\
& 7/2dx + 7/2c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) * \\
& cos(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c \\
&) + 1) \cos(5/2dx + 5/2c) \cos(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1) \cos(3/2dx + 3/2c)^2 + (\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2dx + 5/2c)^2 + 2*(\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2dx + 5/2c) \sin(3/2dx \\
& + 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 \\
& dx + 3/2c)^2) \sin(7/2dx + 7/2c)^2) \cos(2/3 \arctan2(\sin(3/2dx + 3/2c \\
&), \cos(3/2dx + 3/2c))) \log(\cos(1/7 \arctan2(\sin(7/2dx + 7/2c), \cos(7/ \\
& 2dx + 7/2c)))^2 + \sin(1/7 \arctan2(\sin(7/2dx + 7/2c), \cos(7/2dx + 7/ \\
& 2c)))^2 + 2\sin(1/7 \arctan2(\sin(7/2dx + 7/2c), \cos(7/2dx + 7/2c))) + \\
& 1) + 315*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 \\
& dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) *c
\end{aligned}$$

$$\begin{aligned}
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/ \\
& 2*c)^2*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + \\
& ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos \\
& (7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3 \\
& /2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d \\
& *x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/ \\
& 2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d
\end{aligned}$$

$$\begin{aligned}
& *x + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^ \\
& 2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + \\
& 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\cos(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3 \\
& /2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*co \\
& s(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + \\
& 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\cos(9/2*dx + 9/2 \\
& *c)*\cos(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + \\
& c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos \\
& (dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin \\
& (3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)* \\
& \sin(3/2*dx + 3/2*c)^2)*\cos(7/2*dx + 7/2*c)^2 + ((\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2 \\
& *c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + \\
& 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*si \\
& n(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(9/2*dx + 9/2*c)^2 + 2*((\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^ \\
& 2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + \\
& 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos \\
& (dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(9 \\
& /2*dx + 9/2*c)*\sin(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c \\
&)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx \\
& + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& x + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(7/2*dx + 7/2*c)^2)*\cos(2/3*\arctan2 \\
& (\sin(3/2*dx + 3/2*c), \cos(3/2*dx + 3/2*c)))^2 + ((\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/
\end{aligned}$$

$$\begin{aligned}
& 2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*s \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*(\\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(\\
& 9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9 \\
& /2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3 \\
& /2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d* \\
& x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + \\
& (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2 \\
& *c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^ \\
& 2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d* \\
& x + 3/2*c))))*\cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^ \\
& 2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 \\
&)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*s \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2* \\
& d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d* \\
& x + 3/2*c)^2) \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d* \\
& x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \sin(9/2*d*x + 9/2*c) \sin(7/2*d*x + 7/ \\
& 2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5 \\
& /2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c) \\
& ^2) \sin(7/2*d*x + 7/2*c)^2) \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d \\
& *x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) *c \\
& os(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d \\
& *x + 3/2*c)^2) \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^ \\
& 2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d \\
& *x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \sin(9/2*d*x + 9/2*c) \sin(7/2*d*x + 7 \\
& /2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(\\
& 5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c \\
&)^2) \sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2
\end{aligned}$$

$$\begin{aligned}
& * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) \\
& * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + \\
& 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(9/2*d*x + 9/2*c)^2 + 2 * ((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2* \\
& d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin \\
& (5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + \\
& 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(9/2*d*x + 9/2*c) * \cos \\
& (7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) \\
&) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + \\
& c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d \\
& *x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/ \\
& 2*d*x + 3/2*c)^2 * \cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c) \\
&)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c) \\
&)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + \\
& 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2 \\
& *d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos \\
& (d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(9/2*d*x + 9/2*c)^2 + 2 * ((\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 \\
& * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) \\
&) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) \\
& + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x \\
& + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos \\
& (d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(9/2*d* \\
& x + 9/2*c) * \cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos \\
& (d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + \\
& 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2 \\
& *c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) \\
&) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*
\end{aligned}$$

$$\begin{aligned}
& *c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(9/2* \\
& d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2* \\
& d*x + 3/2*c)^2 * \cos(9/2*d*x + 9/2*c) * \cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(7/2*d*x + 7/2* \\
& c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5 \\
& /2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c) \\
& ^2) * \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) \\
& * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1) * \sin(3/2*d*x + 3/2*c)^2 * \sin(9/2*d*x + 9/2*c) * \sin(7/2*d*x + 7/2*c) + ((c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5 \\
& /2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \sin(7/ \\
& 2*d*x + 7/2*c)^2) * \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c \\
&)))^2 + 2*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * c
\end{aligned}$$

$$\begin{aligned}
& 2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + \\
& 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx \\
& x + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5 \\
& /2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \sin(9/2 dx + 9/2 c)^2 + \\
& 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 \\
& *c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
& x + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(\\
& dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& * \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(\\
& dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin \\
& (9/2 dx + 9/2 c) \sin(7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c) \\
& ^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5 \\
& /2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 \\
& dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(7/2 dx + 7/2 c)^2) \cos(2/3 \arcc \\
& \tan^2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + ((\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx \\
& + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx \\
& *x + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(\\
& 5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx + 9/2 c)^2 + \\
& 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/ \\
& 2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
& *x + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin \\
& (9/2 dx + 9/2 c) \sin(7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
& *x + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c \\
&)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + \\
& 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 \\
& *dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(7/2 dx + 7/2 c)^2 + (((\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2*
\end{aligned}$$

$$\begin{aligned}
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2dx + 5/2c) \\
& * \cos(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\cos(3/2dx + 3/2c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1)\sin(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& s(dx + c) + 1)\sin(5/2dx + 5/2c)*\sin(3/2dx + 3/2c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2dx + 3/2c)^2*\cos(9/2dx \\
& + 9/2c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos \\
& (5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\cos(5/2dx + 5/2c)*\cos(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1)\cos(3/2dx + 3/2c)^2 + (\cos(dx + c)^2 + \sin \\
& dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2dx + 5/2c)*\sin(3/2dx + \\
& 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2dx \\
& + 3/2c)^2*\cos(9/2dx + 9/2c)*\cos(7/2dx + 7/2c) + ((\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2dx + 5/2c)^2 + 2*(\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2dx + 5/2c)*\cos(3/2d \\
& *x + 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/ \\
& 2dx + 3/2c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*s \\
& in(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1)\sin(5/2dx + 5/2c)*\sin(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1)\sin(3/2dx + 3/2c)^2*\cos(7/2dx + 7/2c)^ \\
& 2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2dx + 5 \\
& /2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2* \\
& dx + 5/2c)*\cos(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& s(dx + c) + 1)\cos(3/2dx + 3/2c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1)\sin(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1)\sin(5/2dx + 5/2c)*\sin(3/2dx + 3/2c) + (co \\
& s(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2dx + 3/2c)^2) \\
& * \sin(9/2dx + 9/2c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1)\cos(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& s(dx + c) + 1)\cos(5/2dx + 5/2c)*\cos(3/2dx + 3/2c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2dx + 3/2c)^2 + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2dx + 5/2c)^2 + 2*(co \\
& s(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2dx + 5/2c)*\si \\
& n(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \\
& * \sin(3/2dx + 3/2c)^2*\sin(9/2dx + 9/2c)*\sin(7/2dx + 7/2c) + ((\cos \\
& dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2dx + 5/2c)^2 + \\
& 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2dx + 5/2* \\
& c)*\cos(3/2dx + 3/2c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1)\cos(3/2dx + 3/2c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)\sin(5/2dx + 5/2c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2* \\
& \cos(dx + c) + 1)\sin(5/2dx + 5/2c)*\sin(3/2dx + 3/2c) + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2dx + 3/2c)^2*\sin(7/2d \\
& *x + 7/2c)^2*\sin(2/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) \\
& ^2 + 2*(((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2dx
\end{aligned}$$

$$\begin{aligned}
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7 \\
& /2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2 \\
& *d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c \\
&)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + \\
& 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2 \\
& *c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/ \\
& 2*d*x + 3/2*c))))*\cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 3/2*c))))*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + \\
& 7/2*c)))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 \\
& - 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) - 12 \\
& 60*(((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(5/2*d*x + 5 \\
& /2*c)^2 + 2*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(5/2* \\
& d*x + 5/2*c)*cos(3/2*d*x + 3/2*c) + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*cos(3/2*d*x + 3/2*c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + \\
& 2*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^2 + 2*(cos(d*x + c)^2 + sin(d*x + \\
& c)^2 + 2*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3/2*c) + (co \\
& s(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^2) \\
& *cos(9/2*d*x + 9/2*c)^2 + 2*((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + \\
& c) + 1)*cos(5/2*d*x + 5/2*c)^2 + 2*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*cos(5/2*d*x + 5/2*c)*cos(3/2*d*x + 3/2*c) + (cos(d*x + c)^2 \\
& + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(3/2*d*x + 3/2*c)^2 + (cos(d*x + \\
& c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^2 + 2*(co \\
& s(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)*si \\
& n(3/2*d*x + 3/2*c) + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1) \\
& *sin(3/2*d*x + 3/2*c)^2)*cos(9/2*d*x + 9/2*c)*cos(7/2*d*x + 7/2*c) + ((cos(\\
& d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2* \\
& c)*cos(3/2*d*x + 3/2*c) + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) \\
& + 1)*cos(3/2*d*x + 3/2*c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x \\
& + c) + 1)*sin(5/2*d*x + 5/2*c)^2 + 2*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2* \\
& cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3/2*c) + (cos(d*x + c) \\
& ^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^2)*cos(7/2*d \\
& *x + 7/2*c)^2 + (((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*co \\
& s(5/2*d*x + 5/2*c)^2 + 2*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) \\
& + 1)*cos(5/2*d*x + 5/2*c)*cos(3/2*d*x + 3/2*c) + (cos(d*x + c)^2 + sin(d*x \\
& + c)^2 + 2*cos(d*x + c) + 1)*cos(3/2*d*x + 3/2*c)^2 + (cos(d*x + c)^2 + sin \\
& (d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^2 + 2*(cos(d*x + c)^ \\
& 2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + \\
& 3/2*c) + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d* \\
& x + 3/2*c)^2)*cos(9/2*d*x + 9/2*c)^2 + 2*((cos(d*x + c)^2 + sin(d*x + c)^2 \\
& + 2*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c)^2 + 2*(cos(d*x + c)^2 + sin(d*x \\
& + c)^2 + 2*cos(d*x + c) + 1)*cos(5/2*d*x + 5/2*c)*cos(3/2*d*x + 3/2*c) + (c \\
& os(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(3/2*d*x + 3/2*c)^2 \\
& + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(5/2*d* \\
& x + 5/2*c)*sin(3/2*d*x + 3/2*c) + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(\\
& d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^2)*cos(9/2*d*x + 9/2*c)*cos(7/2*d*x + 7/ \\
& 2*c) + ((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(5 \\
& /2*d*x + 5/2*c)*cos(3/2*d*x + 3/2*c) + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2 \\
& *cos(d*x + c) + 1)*cos(3/2*d*x + 3/2*c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^ \\
& 2 + 2*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^2 + 2*(cos(d*x + c)^2 + sin(d*
\end{aligned}$$

$$\begin{aligned}
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) \\
& ^2)*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)* \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos \\
& (7/2*d*x + 7/2*c) + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2 \\
& *d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x \\
& + 9/2*c)*\sin(7/2*d*x + 7/2*c) + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c
\end{aligned}$$

$$\begin{aligned}
&)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^2*\cos(2/3*\arctan2(\sin(3/ \\
& 2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& *\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x \\
& + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*si \\
& n(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)^2 \\
& + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 \\
&)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d* \\
& x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*c \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d* \\
& x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c) \\
& ^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*((c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/ \\
& 2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d \\
& *x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/ \\
& 2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*c \\
& \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(d \\
& *x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin \\
& n(7/2 dx + 7/2 c)^2 + (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& x + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^ \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/ \\
& 2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin \\
& (3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 \\
& c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3 \\
& /2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx \\
& x + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin \\
& (5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c) \cos(7/2 dx \\
& *x + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5 \\
& /2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/ \\
& 2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + \\
& 3/2 c)^2) \cos(7/2 dx + 7/2 c)^2 + (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(d \\
& *x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^ \\
& ^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + \\
& 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c)^2 + 2((\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx \\
& *x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3 \\
& /2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \co \\
& s(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 \\
& *c) \cos(7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos
\end{aligned}$$

$$\begin{aligned}
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\si \\
& n(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9 \\
& /2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2 \\
& (\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/ \\
& 2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*s \\
& in(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*(\\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(\\
& 9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx \\
& + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(7/2*dx + 7/2*c)^2 + (((\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos \\
& (3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)* \\
& \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) \\
& + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\cos(9/2*dx + 9 \\
& /2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2 \\
& *dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)* \\
& \cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2* \\
& c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3 \\
& /2*c)^2)*\cos(9/2*dx + 9/2*c)*\cos(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^ \\
& 2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + \\
& 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx \\
& x + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5 \\
& /2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1 \\
&)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\cos(7/2*dx + 7/2*c)^2 + \\
& ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c \\
&)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx \\
& + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& x + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*c \\
& os(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin \\
& (9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) \\
& + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& x + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^ \\
& 2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/ \\
& 2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin \\
& (3/2*dx + 3/2*c)^2)*\sin(9/2*dx + 9/2*c)*\sin(7/2*dx + 7/2*c) + ((\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*c \\
& os(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1 \\
&)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c \\
&) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(
\end{aligned}$$

$$\begin{aligned}
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + \\
& 7/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \\
& 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2) \\
& *\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\si \\
& n(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d \\
& *x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2 \\
& *c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^ \\
& 2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d* \\
& x + 3/2*c))) * \sin(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^ \\
& 2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 \\
&)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\s \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2* \\
& d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\co \\
& s(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d* \\
& x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/ \\
& 2*c) + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^
\end{aligned}$$

$$\begin{aligned}
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2* \\
& d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\si \\
& n(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2 \\
& *d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 \\
& + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + \\
& 5/2*c) \sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \sin(3/2*dx + 3/2*c)^2 \sin(7/2*dx + 7/2*c)^2 + ((\cos(dx + c)^ \\
& 2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c) \cos(3/ \\
& 2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& (3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&) \sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(\\
& dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/2*c)^2 \cos(9/2*dx + 9/2* \\
& c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& (5/2*dx + 5/2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(\\
& dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*dx + 3/2*c) \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/2* \\
& c)^2 \cos(9/2*dx + 9/2*c) \cos(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c) \cos(3/2*dx + 3/ \\
& 2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2*dx + \\
& 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2* \\
& dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin \\
& in(5/2*dx + 5/2*c) \sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/2*c)^2 \cos(7/2*dx + 7/2*c)^2 + ((c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c)^2 \\
& + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5 \\
& /2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(\\
& dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*dx + 3/2*c) + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/2*c)^2 \sin(9/ \\
& 2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&) \cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \cos(5/2*dx + 5/2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(\\
& dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*d \\
& *x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/ \\
& 2*dx + 3/2*c)^2 \sin(9/2*dx + 9/2*c) \sin(7/2*dx + 7/2*c) + ((\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c)^2 + 2*(\cos(\\
& dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c) \cos(\\
& 3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& os(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) +
\end{aligned}$$

$$\begin{aligned}
& 1) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/ \\
& 2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2* \\
& (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos \\
& (9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x \\
& + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) \\
& + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& s(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2) \\
& * \sin(7/2*d*x + 7/2*c)^2) * \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))) * \log(\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)) \\
&)^2 + \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin \\
& (1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1) + 1260*((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/ \\
& 2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d \\
& *x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/ \\
& 2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/ \\
& 2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2 \\
& *c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) +
\end{aligned}$$

$$\begin{aligned}
& ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c) \\
&)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx \\
& + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& x + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& os(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)\cos \\
& (7/2*dx + 7/2*c)^2 + (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin \\
& n(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2 \\
& *dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(\\
& 3/2*dx + 3/2*c)^2)\cos(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin \\
& n(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c \\
&) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/ \\
& 2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx \\
& + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(\\
& 5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)\cos(9/2*dx + 9/2*c)\cos(7/2*d \\
& x + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/ \\
& 2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \\
& *\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2 \\
& *c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + \\
& 3/2*c)^2)\cos(7/2*dx + 7/2*c)^2 + (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& s(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 \\
& + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/ \\
& 2*c)\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1)\sin(3/2*dx + 3/2*c)^2)\sin(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2 \\
& *dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(\\
& 3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \\
& *\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx \\
& *x + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)\sin(9/2*dx + 9/2*c \\
&)\sin(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c)
\end{aligned}$$

$$\begin{aligned}
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2* \\
& d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 \\
& + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d* \\
& x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin \\
& (9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& *\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x \\
& + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \\
& \cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^2 + (((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) * \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x \\
& + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 \\
& + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)* \\
& \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d* \\
& x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^ \\
& 2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\co \\
& s(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2) \\
& *\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\co \\
& s(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\si \\
& n(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)
\end{aligned}$$

$$\begin{aligned}
& (7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2 \\
& *d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^ \\
& 2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x \\
& + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/ \\
& 2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x
\end{aligned}$$

$$\begin{aligned}
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c) \\
& ^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + \\
& 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& s(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)\sin(\\
& 7/2*dx + 7/2*c)^2)\cos(2/3*\arctan2(\sin(3/2*dx + 3/2*c), \cos(3/2*dx + 3/2 \\
& *c)))\sin(2/7*\arctan2(\sin(7/2*dx + 7/2*c), \cos(7/2*dx + 7/2*c)))^2 + (((\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^ \\
& 2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + \\
& 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)\cos(9 \\
& /2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2* \\
& dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3 \\
& /2*dx + 3/2*c)^2)\cos(9/2*dx + 9/2*c)\cos(7/2*dx + 7/2*c) + ((\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos \\
& (3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)* \\
& \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& x + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)\cos(7/2*dx + 7 \\
& /2*c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*co \\
& s(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2 \\
& *c)^2)\sin(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + \\
& 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2 \\
& *c)\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c
\end{aligned}$$

$$\begin{aligned}
&) + 1) \sin(3/2*d*x + 3/2*c)^2) \sin(9/2*d*x + 9/2*c) \sin(7/2*d*x + 7/2*c) + \\
& ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c) \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x \\
& + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \sin \\
& (7/2*d*x + 7/2*c)^2) \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/ \\
& 2*c)))^2 + 2*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
&) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/ \\
& 2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + \\
& 3/2*c)^2) \cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + \\
& 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \cos(9/2*d*x + 9/2*c) \cos(7/2*d*x + 7/2*c) \\
&) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2* \\
& d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \\
& * \cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \\
& \sin(3/2*d*x + 3/2*c)^2) \cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \\
& \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^
\end{aligned}$$

$$\begin{aligned}
& 2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/ \\
& 2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& s(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d* \\
& x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9 \\
& /2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
&)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d \\
& *x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx + \\
& 9/2 c) \sin(7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) * \\
& \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \sin(3/2 dx + 3/2 c)^2) \sin(7/2 dx + 7/2 c)^2 + (((\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^ \\
& 2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + \\
& 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx \\
& x + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5 \\
& /2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c)^2 + \\
& 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 \\
& *c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
& x + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(\\
& dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(\\
& dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) *c \\
& os(9/2 dx + 9/2 c) \cos(7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c) \\
& ^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5 \\
& /2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 * \\
& dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2co \\
& s(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(7/2 dx + 7/2 c)^2 + ((\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) *c \\
& os(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c \\
&) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(\\
& dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx + \\
& 9/2 c)^2 + 2(((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5 \\
& /2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/ \\
& 2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx +
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)*\sin(7 \\
& /2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*c \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d \\
& *x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*s \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d* \\
& x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)*\sin(\\
& 7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2* \\
& d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^ \\
& 2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) +
\end{aligned}$$

$$\begin{aligned}
& 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x \\
& + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + \\
& 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)* \\
& \sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c \\
&)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(2/3*ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*(((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c) \\
& ^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2
\end{aligned}$$

$$\begin{aligned}
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2 \\
& *d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2 \\
& *d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2 \\
& *c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/ \\
& 2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
&)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x +
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2* \\
& c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*s \\
& \sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*si \\
& n(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9 \\
& /2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2 \\
& *c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c) \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(\\
& dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(\\
& 3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin \\
& (3/2 dx + 3/2 c)^2 \sin(9/2 dx + 9/2 c) \sin(7/2 dx + 7/2 c) + ((\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2* \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \\
& * \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(7/2 dx \\
& + 7/2 c)^2) \cos(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) * \\
& \sin(2/7 \arctan 2(\sin(7/2 dx + 7/2 c), \cos(7/2 dx + 7/2 c)))^2 + (((\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2*(\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) * \\
& \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx \\
& + 9/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(\\
& 5/2 dx + 5/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2((\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3 \\
& /2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx \\
& + 3/2 c)^2) \cos(9/2 dx + 9/2 c) \cos(7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2((\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx \\
& x + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 \\
& * dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin \\
& (5/2 dx + 5/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(7/2 dx + 7/2 c)^2 \\
& + (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/ \\
& 2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
& * x + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) * \\
& \sin(9/2 dx + 9/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos
\end{aligned}$$

$$\begin{aligned}
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d* \\
& x + 7/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^ \\
& 2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) \\
& ^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/ \\
& 2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2 \\
& *d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d
\end{aligned}$$

$$\begin{aligned}
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c \\
&)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + \\
& 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2 \\
& *c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*si \\
& n(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*s \\
& in(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2* \\
& c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*d \\
& *x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/ \\
& 2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*s \\
& \sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) \\
& + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(9/2*dx + 9/2*c)* \\
& \sin(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + \\
& 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2 \\
& *dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(\\
& 3/2*dx + 3/2*c)^2)*\sin(7/2*dx + 7/2*c)^2 + (((\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2 \\
& *c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx \\
& + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5 \\
& /2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\cos(9/2*dx + 9/2*c)^2 + 2*((\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + \\
& 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2 \\
& *c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c \\
&) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& x + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\cos(9/2* \\
& dx + 9/2*c)*\cos(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos \\
& (dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx \\
& x + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 \\
& + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5 \\
& /2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + \\
& c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\cos(7/2*dx + 7/2*c)^2 + ((\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2* \\
& dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3 \\
& /2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)* \\
& \sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c \\
&) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(9/2*dx + 9/2*c) \\
& ^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx \\
& + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5
\end{aligned}$$

$$\begin{aligned}
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d* \\
& x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2* \\
& c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*s \\
& \sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3 \\
& /2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/ \\
& 2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*c \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/ \\
& 2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + (\\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(\\
& 9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*co \\
& s(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + \\
& 7/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \\
& 2*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos
\end{aligned}$$

$$\begin{aligned}
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)* \\
& \cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d* \\
& x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2* \\
& c) + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 \\
&)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))))*\cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^2 \\
& + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& 2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + \\
& 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx \\
& x + 3/2 c)^2) \sin(9/2 dx + 9/2 c)^2 + 2 * ((\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (c \\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 \\
& * c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx \\
& x + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(\\
& dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx + 9/2 c) \sin(7/2 dx + 7/ \\
& 2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
& + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5 \\
& /2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& * \cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c) \\
& ^2) \sin(7/2 dx + 7/2 c)^2 + (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \\
& \cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * (\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) * \\
& \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c)^2 + 2 * ((\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx \\
& + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx \\
& * x + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(\\
& 5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c) \cos \\
& (7/2 dx + 7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \\
& * \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx \\
& * x + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx \\
& x + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 \\
& * dx + 3/2 c)^2) \cos(7/2 dx + 7/2 c)^2 + (((\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c) \\
& ^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5 \\
& /2 c)^2 + 2 * (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 *
\end{aligned}$$

$$\begin{aligned}
& + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \sin(9/2*d*x + 9/2*c)^2 \\
& + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2 \\
& *d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2 \\
&) \sin(9/2*d*x + 9/2*c) \sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2 \\
& *c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5 \\
& /2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \sin(7/2*d*x + 7/2*c)^2 + (((\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2* \\
& c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \cos(9/2*d \\
& *x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) *c \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d \\
& *x + 3/2*c)^2) \cos(9/2*d*x + 9/2*c) \cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(\\
& 3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \cos(7/2*d*x + 7/2*c \\
&)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \cos(5/ \\
& 2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \\
& 2) \sin(9/2 dx + 9/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 \\
& + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \\
& \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx + 9/2 c) \sin(7/2 dx + 7/2 c) + ((\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 \\
& + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/ \\
& 2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(7/2 \\
& dx + 7/2 c)^2) \sin(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c) \\
&))^2 + 2(((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
& + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& (5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 \\
& c)^2) \cos(9/2 dx + 9/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + \\
& 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 \\
& c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
&) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c) \cos(7/2 dx + 7/2 c) + \\
& ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c \\
&)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
& + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos \\
& (7/2 dx + 7/2 c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2
\end{aligned}$$

$$\begin{aligned}
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x \\
& + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3 \\
& /2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(\\
& 3/2*d*x + 3/2*c))))*\sin(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2 \\
& *c))))^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*c \\
& os(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/ \\
& 2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + \\
& (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*co \\
& s(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\co \\
& s(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + \\
& 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)* \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2 \\
& * ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\si \\
& n(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^ \\
& 2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arct \\
& an2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d* \\
& x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
&)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + \\
& 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d* \\
& x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*s \\
& \sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)* \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x \\
& + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*si \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 \\
& + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)* \\
& \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d* \\
& x + 7/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^ \\
& 2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) \\
& ^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/ \\
& 2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c
\end{aligned}$$

$$\begin{aligned}
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx \\
&+ 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 \\
&dx + 3/2 c)^2 \sin(9/2 dx + 9/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 \\
&+ 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
&x + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + \\
&(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c) \\
&^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5 \\
&/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 \\
&dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
&(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \sin(9/2 dx + 9/2 c) \sin(7/2 dx + \\
&7/2 c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
&x + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
&(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
&2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
&+ c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) \\
&+ (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c \\
&c)^2 \sin(7/2 dx + 7/2 c)^2 \cos(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 \\
&* dx + 3/2 c))) \cos(2/7 \arctan 2(\sin(7/2 dx + 7/2 c), \cos(7/2 dx + 7/2 c) \\
&)) + 2(((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
&+ 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
&(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
&2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c) \\
&^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
&* x + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + \\
&(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c \\
&)^2 \cos(9/2 dx + 9/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
&* x + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
&2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + \\
&c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx \\
&* x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2 \\
&*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c \\
&)* \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
&+ 1) \sin(3/2 dx + 3/2 c)^2 \cos(9/2 dx + 9/2 c) \cos(7/2 dx + 7/2 c) + ((\\
&\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 \\
&+ 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + \\
&5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
&+ c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
&(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 \\
&+ 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx \\
&+ c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \cos(7 \\
&/2 dx + 7/2 c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \\
&* \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
&c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx \\
&* x + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& *x + 3/2*c)^2 + 90*((\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1)*\cos \\
& \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) + 1) \\
&)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c) - (\cos(5/2*d*x + 5/2*c)^2*\sin \\
& \sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2* \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2* \\
& c)^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c))*\cos(9/2*d*x + 9/2*c) + 21*(40*(\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 40*\cos(5/2*d*x + 5/2*c)^3*\sin \\
& \sin(d*x + c) + (45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& \sin(d*x + c)^2 + 90*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 80*\cos(3/2*d*x + \\
& 3/2*c)*\sin(d*x + c) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + 5/2*c)^2 + 45*((\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c)^2 + (45* \\
& (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(d*x + c)^2 + 45*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 90 \\
& *(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(d*x + c) + 16*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*c \\
& \cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 40*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c \\
&) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 45*((\log(\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& *x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7 \\
& /2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2)*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2* \\
& d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\co \\
& s(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d \\
& *x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/ \\
& 2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\co \\
& s(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& *\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7 \\
& /2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2 \\
& *c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + \\
& ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin \\
& (7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/ \\
& 2*c))))*\cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)))^2 + ((\\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(\\
& 9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 \\
& dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(\\
& 3/2 dx + 3/2 c)^2 \cos(9/2 dx + 9/2 c) \cos(7/2 dx + 7/2 c) + ((\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2*(\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos \\
& (3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \\
& \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1) \sin(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \cos(7/2 dx + \\
& 7/2 c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 \\
& dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& (5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) \\
&) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/ \\
& 2 c)^2 \sin(9/2 dx + 9/2 c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 \\
& + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/ \\
& 2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \sin(3/2 dx + 3/2 c)^2 \sin(9/2 dx + 9/2 c) \sin(7/2 dx + 7/2 c) + \\
& ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 \\
& c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx \\
& + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \sin \\
& (7/2 dx + 7/2 c)^2 \cos(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3 \\
& /2 c)))^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 \\
& dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& (5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 \\
& c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3 \\
& /2 c)^2 \sin(9/2 dx + 9/2 c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7 \\
& /2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c \\
&)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c)))^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3 \\
& /2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5 \\
& /2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x \\
& + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3 \\
& /2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)
\end{aligned}$$

$$\begin{aligned}
& * \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c \\
&)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
& 2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d \\
& *x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2 \\
& *d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(s \\
& \sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3 \\
& /2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(
\end{aligned}$$

$$\begin{aligned}
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/ \\
& 2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3 \\
& /2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\co \\
& s(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2 \\
& *c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\co \\
& s(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2 \\
& *c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9 \\
& /2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x
\end{aligned}$$

$$\begin{aligned}
&^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
&5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
&+ c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c \\
&) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
&/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
&d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
&s(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
&2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
&c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (co \\
&>s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2) \\
&*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
&+ 3/2*c))) * \cos(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + \\
&2*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/ \\
&2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d \\
&*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
&(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
&2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
&c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos \\
&(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)* \\
&\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
&c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
&(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
&+ \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
&c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
&(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
&(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
&\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d \\
&*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2 \\
&*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&+ 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
&+ c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
&>os(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
&2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d* \\
&x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(\\
&5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
&1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
&c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d \\
&*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
&+ \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3 \\
&/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
&+ 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
&2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
&c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
&(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c) \\
&)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx \\
& + 5/2*c)\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& x + c) + 1)\sin(3/2*dx + 3/2*c)^2)\sin(9/2*dx + 9/2*c)\sin(7/2*dx + 7/2* \\
& c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + \\
& 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2 \\
& *dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*c \\
& os(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2 \\
&)\sin(7/2*dx + 7/2*c)^2)\cos(2/3*\arctan2(\sin(3/2*dx + 3/2*c), \cos(3/2*dx \\
& + 3/2*c)))\sin(5/7*\arctan2(\sin(7/2*dx + 7/2*c), \cos(7/2*dx + 7/2*c))) - \\
& 280*(((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + \\
& 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/ \\
& 2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2* \\
& cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (\\
& cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^ \\
& 2)\cos(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2* \\
& cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\\
& cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)* \\
& sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\sin(3/2*dx + 3/2*c)^2)\cos(9/2*dx + 9/2*c)\cos(7/2*dx + 7/2*c) + ((c \\
& s(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 \\
& + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/ \\
& 2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& *x + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx + 3/2*c) + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)\cos(7/2 \\
& *dx + 7/2*c)^2 + (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)* \\
& cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c \\
&) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)\sin(3/2*dx \\
& + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2* \\
& dx + 3/2*c)^2)\cos(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& x + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) +
\end{aligned}$$

$$\begin{aligned}
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c) \\
& ^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5 \\
& /2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2* \\
& dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*co \\
& s(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2*\cos(9/2*dx + 9/2*c)*\cos(7/2*dx + \\
& 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1)\cos(3/2*d*x + 3/2*c)^2 + (\cos(dx + c) \\
&)^2 + 2\cos(dx + c) + 1)\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(\\
& dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*d*x + 3/2* \\
& c)^2*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(d \\
& *x + c) + 1)\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1)\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& *x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*d*x + 5/2*c) \\
&)*\sin(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1)\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*d*x + 5/2*c)*\cos(3/2*d* \\
& x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*si \\
& n(5/2*d*x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1)\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)*c \\
& os(7/2*d*x + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1)\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1)\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*d*x + 3/2*c)^2 + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*d*x + 5/2*c)*\sin(3/2* \\
& dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3 \\
& /2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1)\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(d \\
& *x + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*d*x + 3/2*c) \\
&)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2 \\
& *d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*c \\
& os(dx + c) + 1)\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d \\
& *x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*d*x + 5/2*c)^2 + 2 \\
& *(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*d*x + 5/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c)
\end{aligned}$$

$$\begin{aligned}
& + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d* \\
& x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\sin(\\
& 3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2* \\
& c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d \\
& *x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/ \\
& 2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c) \\
& ^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2
\end{aligned}$$

$$\begin{aligned}
& n(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2(\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2 \\
& *dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(\\
& 3/2*dx + 3/2*c)^2) \cos(9/2*dx + 9/2*c) \cos(7/2*dx + 7/2*c) + ((\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c)^2 + 2(\cos \\
& s(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c) \cos \\
& s(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \\
& * \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1) \sin(5/2*dx + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& *x + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \\
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/2*c)^2) \cos(7/2*dx + \\
& 7/2*c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2* \\
& dx + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& os(5/2*dx + 5/2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin \\
& n(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*dx + 3/2*c \\
&) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/ \\
& 2*c)^2) \sin(9/2*dx + 9/2*c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& s(dx + c) + 1) \cos(5/2*dx + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos \\
& s(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 \\
& + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/ \\
& 2*c) \sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \sin(3/2*dx + 3/2*c)^2) \sin(9/2*dx + 9/2*c) \sin(7/2*dx + 7/2*c) + \\
& ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2* \\
& c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx \\
& + 5/2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& *x + c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& os(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*dx + 3/2*c) + (\cos(dx \\
& *x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/2*c)^2) \sin \\
& n(7/2*dx + 7/2*c)^2) \cos(2/3 \arctan2(\sin(3/2*dx + 3/2*c), \cos(3/2*dx + 3 \\
& /2*c))) \cos(2/7 \arctan2(\sin(7/2*dx + 7/2*c), \cos(7/2*dx + 7/2*c)))^2 + (\\
& ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx + 5/2*c \\
&)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*dx \\
& + 5/2*c) \cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& *x + c) + 1) \cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& os(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c) \sin(3/2*dx + 3/2*c) + (\cos(dx \\
& *x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*dx + 3/2*c)^2) \cos \\
& (9/2*dx + 9/2*c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1) \cos(5/2*dx + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx
\end{aligned}$$

$$\begin{aligned}
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9 \\
& /2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*s \\
& \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3 \\
& /2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2* \\
& ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin \\
& (9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^2*\cos(2/3*arcta \\
& n2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2 \\
& *((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x
\end{aligned}$$

$$\begin{aligned}
& \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c) \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(\\
& dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(\\
& 3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin \\
& (3/2 dx + 3/2 c)^2 \sin(9/2 dx + 9/2 c) \sin(7/2 dx + 7/2 c) + ((\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2 \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \\
& * \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \sin(7/2 dx \\
& + 7/2 c)^2 \sin(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 \\
& + 2(((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + \\
& 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/ \\
& 2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2 \\
&) \cos(9/2 dx + 9/2 c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c) \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx \\
& + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) * \\
& \sin(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c) \cos(7/2 dx + 7/2 c) + ((\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 \\
& + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/ \\
& 2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
& + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx + 3/2 c)^2) \cos(7/2 \\
& dx + 7/2 c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos \\
& (5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
& + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(3/2 dx + 3/2 c)^2 + (\cos(dx + c)^2 + \sin \\
& (dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c) \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2 dx + 5/2 c) \sin(3/2 dx \\
& + 3/2 c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2 dx \\
& + 3/2 c)^2) \sin(9/2 dx + 9/2 c)^2 + 2(((\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c)^2 + 2(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2\cos(dx + c) + 1) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2*dx + 3/2*c)^2 \\
& + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*dx + 5/2*c)^2 \\
& + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*d \\
& *x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
& (dx + c) + 1) \sin(3/2*d*x + 3/2*c)^2 \sin(9/2*d*x + 9/2*c) \sin(7/2*d*x + 7 \\
& /2*c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*d*x \\
& + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(\\
& 5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c) \\
& ^2 + 2\cos(dx + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(d \\
& *x + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*d*x + 3/2*c \\
&)^2 \sin(7/2*d*x + 7/2*c)^2 \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c))) \sin(2/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c)) \\
&)^2 + (((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*d*x \\
& + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5 \\
& /2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *cos(dx + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(d \\
& x + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*d*x + 3/2*c) \\
& ^2) \cos(9/2*d*x + 9/2*c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(d \\
& x + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \\
& *cos(dx + c) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(d \\
& x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*d*x + 5/2*c) \\
& *sin(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + \\
& 1) \sin(3/2*d*x + 3/2*c)^2) \cos(9/2*d*x + 9/2*c) \cos(7/2*d*x + 7/2*c) + ((c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*d*x + 5/2*c)^2 \\
& + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \cos(5/2*d*x + 5 \\
& /2*c) \cos(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + \\
& c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(\\
& d*x + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2\cos(dx + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x + 3/2*c) + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2*d*x + 3/2*c)^2) \cos(7/ \\
& 2*d*x + 7/2*c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) * \\
& \cos(5/2*d*x + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c \\
&) + 1) \cos(5/2*d*x + 5/2*c) \cos(3/2*d*x + 3/2*c) + (\cos(dx + c)^2 + \sin(d \\
& x + c)^2 + 2\cos(dx + c) + 1) \cos(3/2*d*x + 3/2*c)^2 + (\cos(dx + c)^2 + s \\
& in(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*d*x + 5/2*c)^2 + 2(\cos(dx + c \\
&)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(5/2*d*x + 5/2*c) \sin(3/2*d*x \\
& + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1) \sin(3/2* \\
& d*x + 3/2*c)^2) \sin(9/2*d*x + 9/2*c)^2 + 2((\cos(dx + c)^2 + \sin(dx + c)^ \\
& 2 + 2\cos(dx + c) + 1) \cos(5/2*d*x + 5/2*c)^2 + 2(\cos(dx + c)^2 + \sin(d
\end{aligned}$$

$$\begin{aligned}
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + \\
& 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2* \\
& c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c)))^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(\\
& 5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d* \\
& x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + \\
& 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*c \\
& os(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*
\end{aligned}$$

$$\begin{aligned}
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2* \\
& c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*s \\
& \sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2* \\
& c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3 \\
& /2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/ \\
& 2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\cos(2/3*\arctan2(\\
& \sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2 \\
& *c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d \\
& *x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*si \\
& \sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^
\end{aligned}$$

$$\begin{aligned}
& 2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9 \\
& /2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c \\
&)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(\\
& 3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\c \\
& os(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/ \\
& 2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\c \\
& os(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \si \\
& n(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/ \\
& 2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/ \\
& 2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& *\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + (\\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& ^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(\\
& 9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos \\
& (dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + \\
& (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c \\
&)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx \\
& + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& x + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\sin(9/2*dx + 9/2*c)*\sin(7/2*dx + 7/2* \\
& c) + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + \\
& 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2 \\
& *dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*c \\
& os(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
& + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
& + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (c \\
& os(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2 \\
&)*\sin(7/2*dx + 7/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*dx + 3/2*c), \cos(3/2*dx \\
& + 3/2*c))))*\cos(2/7*\arctan2(\sin(7/2*dx + 7/2*c), \cos(7/2*dx + 7/2*c))) + \\
& 2*(((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5 \\
& /2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2* \\
& dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*co \\
& s(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
& 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (co \\
& s(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2) \\
& *\cos(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + \\
& c) + 1)*\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*co \\
& s(dx + c) + 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + \\
& c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(co \\
& s(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\si \\
& n(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1) \\
& *\sin(3/2*dx + 3/2*c)^2)*\cos(9/2*dx + 9/2*c)*\cos(7/2*dx + 7/2*c) + ((\cos(\\
& dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2*c)^2 + \\
& 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos(5/2*dx + 5/2* \\
& c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) \\
& + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx \\
& + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2* \\
& cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c) \\
& ^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^2)*\cos(7/2*d \\
& *x + 7/2*c)^2 + ((\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\cos \\
& (5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + \\
& 1)*\cos(5/2*dx + 5/2*c)*\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2*\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(\\
& dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 \\
& + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + \\
& 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx \\
& + 3/2*c)^2)*\sin(9/2*dx + 9/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 +
\end{aligned}$$

$$\begin{aligned}
&) + 1) \sin(3/2*d*x + 3/2*c)^2 * \sin(9/2*d*x + 9/2*c)^2 + 2 * ((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2* \\
& d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3 \\
& /2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \\
& \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c \\
&) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \sin(9/2*d*x + 9/2*c) \\
& * \sin(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) \\
& + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d* \\
& x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin \\
& (3/2*d*x + 3/2*c)^2 * \sin(7/2*d*x + 7/2*c)^2 + (((\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin \\
& (d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c \\
&) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x \\
& + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin \\
& (5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(9/2*d*x + 9/2*c)^2 + 2 * ((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 \\
& + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/ \\
& 2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + \\
& c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d \\
& *x + c) + 1) * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(9/2 \\
& *d*x + 9/2*c) * \cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \\
& \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c) \\
& ^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\cos(d \\
& *x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(3/2*d*x + 3/2*c)^2 + (\\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c)^ \\
& 2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(5/2*d*x + \\
& 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x \\
& + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \cos(7/2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) * \cos(3/2 \\
& *d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) * \cos(\\
& 3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + c) + 1) \\
& * \sin(5/2*d*x + 5/2*c)^2 + 2 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 * \cos(d*x + \\
& c) + 1) * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d \\
& *x + c)^2 + 2 * \cos(d*x + c) + 1) * \sin(3/2*d*x + 3/2*c)^2 * \sin(9/2*d*x + 9/2*c
\end{aligned}$$

$$\begin{aligned}
&)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx \\
&+ 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(\\
&5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + \\
&2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c) \\
&^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx \\
&*x + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + \\
&(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c \\
&)^2)*\sin(9/2*dx + 9/2*c)*\sin(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin(dx \\
&+ c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \\
&\sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2 \\
&*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + \\
&3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*d \\
&*x + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)*\sin \\
&(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 \\
&+ 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)*\sin(7/2*dx + 7/2*c)^2)*\sin(2 \\
&/3*\arctan2(\sin(3/2*dx + 3/2*c), \cos(3/2*dx + 3/2*c)))^2 + 2*((\cos(dx + \\
&c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos \\
&(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos \\
&(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)* \\
&\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) \\
&+ 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
&x + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin \\
&(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)*\cos(9/2*dx + 9 \\
&/2*c)^2 + 2*((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2 \\
&*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)* \\
&\cos(5/2*dx + 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^ \\
&^2 + 2\cos(dx + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx \\
&+ c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin \\
&(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2* \\
&c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3 \\
&/2*c)^2)*\cos(9/2*dx + 9/2*c)*\cos(7/2*dx + 7/2*c) + ((\cos(dx + c)^2 + \sin \\
&(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^ \\
&^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c)\cos(3/2*dx + \\
&3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(3/2*d \\
&x + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(5 \\
&/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1 \\
&)*\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c \\
&)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)*\cos(7/2*dx + 7/2*c)^2 + \\
&((\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx + 5/2*c \\
&)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\cos(5/2*dx \\
&+ 5/2*c)\cos(3/2*dx + 3/2*c) + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos(dx \\
&x + c) + 1)\cos(3/2*dx + 3/2*c)^2 + (\cos(dx + c)^2 + \sin(dx + c)^2 + 2\cos \\
&(dx + c) + 1)\sin(5/2*dx + 5/2*c)^2 + 2*(\cos(dx + c)^2 + \sin(dx + c)^ \\
&^2 + 2\cos(dx + c) + 1)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (\cos(dx \\
&x + c)^2 + \sin(dx + c)^2 + 2\cos(dx + c) + 1)\sin(3/2*dx + 3/2*c)^2)*\sin
\end{aligned}$$

$$\begin{aligned}
& (9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + s \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/ \\
& 2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin \\
& (3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(c \\
& \cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*c \\
& \cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1 \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d*x + \\
& 7/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) *si \\
& n(1/7*\arctan2(\sin(7/2*d*x + 7/2*c), \cos(7/2*d*x + 7/2*c))) + 5040*(((\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) \\
& *\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x \\
& + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos(d*x + c)^2 + \\
& \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d \\
& *x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*s \\
& \sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x \\
& + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^ \\
& 2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2* \\
& d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + \\
& c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (co
\end{aligned}$$

$$\begin{aligned}
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2) \\
& * \sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos \\
& (d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 \\
& + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin \\
& (3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) \\
& * \sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\cos(\\
& d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + \\
& 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2* \\
& c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
& + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x \\
& + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2* \\
& \cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
& ^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(7/2*d \\
& *x + 7/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \\
& + 72*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) \\
& ^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& * \cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& * \sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((\cos \\
& (d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/ \\
& 2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) \\
&) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c) \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2* \\
& d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + \\
& 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2* \\
& c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(5/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) \\
& - 240*(((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x \\
& + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) \\
& ^2)*\cos(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d* \\
& x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2 \\
& *\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d* \\
& x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2* \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& *\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + \\
& 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((c \\
& os(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5 \\
& /2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + \\
& c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(\\
& d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + \\
& c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(7/ \\
& 2*d*x + 7/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)* \\
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c \\
&) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + s \\
& in(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c \\
&)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2* \\
& d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^ \\
& 2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d* \\
& x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*co \\
& s(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + \\
& 7/2*c) + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d* \\
& x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos \\
& (5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + \\
& 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c \\
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
& + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2* \\
& c)^2)*\sin(7/2*d*x + 7/2*c)^2)*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) \\
& + 60*((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos \\
& (5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d* \\
& x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^ \\
& 2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2*c)^2 + (7 \\
& 4*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x \\
& + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*si \\
& n(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\cos(9/2*d*x \\
& + 9/2*c)^2 + 2*((74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + \\
& 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 1 \\
& 79*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (74*\cos(\\
& d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(3/2*d*x + 3/2* \\
& c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d* \\
& x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^ \\
& 2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(3/2*d*x + 3/2*c)^2)*\cos \\
& (9/2*d*x + 9/2*c)*\cos(7/2*d*x + 7/2*c) + ((74*\cos(d*x + c)^2 + 74*\sin(d*x + \\
& c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c) \\
& ^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)*\cos(3 \\
& /2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) \\
& + 105)*\cos(3/2*d*x + 3/2*c)^2 + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 1 \\
& 79*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d*x + c)^2 + 74*s \\
& in(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*s \\
& in(3/2*d*x + 3/2*c)^2)*\cos(7/2*d*x + 7/2*c)^2 + ((74*\cos(d*x + c)^2 + 74*si \\
& n(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c)^2 + 2*(74*\cos(d \\
& *x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d*x + c) + 105)*\cos(5/2*d*x + 5/2*c \\
&)*\cos(3/2*d*x + 3/2*c) + (74*\cos(d*x + c)^2 + 74*\sin(d*x + c)^2 + 179*\cos(d
\end{aligned}$$

$$\begin{aligned}
& *x + c) + 105) * \cos(3/2 * dx + 3/2 * c)^2 + (74 * \cos(dx + c)^2 + 74 * \sin(dx + c) \\
&)^2 + 179 * \cos(dx + c) + 105) * \sin(5/2 * dx + 5/2 * c)^2 + 2 * (74 * \cos(dx + c)^2 \\
& + 74 * \sin(dx + c)^2 + 179 * \cos(dx + c) + 105) * \sin(5/2 * dx + 5/2 * c) * \sin(3/2 \\
& * dx + 3/2 * c) + (74 * \cos(dx + c)^2 + 74 * \sin(dx + c)^2 + 179 * \cos(dx + c) + \\
& 105) * \sin(3/2 * dx + 3/2 * c)^2 * \sin(9/2 * dx + 9/2 * c)^2 + 2 * ((74 * \cos(dx + c)^2 \\
& + 74 * \sin(dx + c)^2 + 179 * \cos(dx + c) + 105) * \cos(5/2 * dx + 5/2 * c)^2 + 2 * \\
& (74 * \cos(dx + c)^2 + 74 * \sin(dx + c)^2 + 179 * \cos(dx + c) + 105) * \cos(5/2 * dx \\
& + 5/2 * c) * \cos(3/2 * dx + 3/2 * c) + (74 * \cos(dx + c)^2 + 74 * \sin(dx + c)^2 + \\
& 179 * \cos(dx + c) + 105) * \cos(3/2 * dx + 3/2 * c)^2 + (74 * \cos(dx + c)^2 + 74 * \sin \\
& (dx + c)^2 + 179 * \cos(dx + c) + 105) * \sin(5/2 * dx + 5/2 * c)^2 + 2 * (74 * \cos(dx \\
& + c)^2 + 74 * \sin(dx + c)^2 + 179 * \cos(dx + c) + 105) * \sin(5/2 * dx + 5/2 * c) \\
&) * \sin(3/2 * dx + 3/2 * c) + (74 * \cos(dx + c)^2 + 74 * \sin(dx + c)^2 + 179 * \cos(dx \\
& + c) + 105) * \sin(3/2 * dx + 3/2 * c)^2 * \sin(9/2 * dx + 9/2 * c) * \sin(7/2 * dx + 7 \\
& /2 * c) + ((74 * \cos(dx + c)^2 + 74 * \sin(dx + c)^2 + 179 * \cos(dx + c) + 105) * \cos \\
& (5/2 * dx + 5/2 * c)^2 + 2 * (74 * \cos(dx + c)^2 + 74 * \sin(dx + c)^2 + 179 * \cos(dx \\
& + c) + 105) * \cos(5/2 * dx + 5/2 * c) * \cos(3/2 * dx + 3/2 * c) + (74 * \cos(dx + c) \\
&)^2 + 74 * \sin(dx + c)^2 + 179 * \cos(dx + c) + 105) * \cos(3/2 * dx + 3/2 * c)^2 + \\
& (74 * \cos(dx + c)^2 + 74 * \sin(dx + c)^2 + 179 * \cos(dx + c) + 105) * \sin(5/2 * dx \\
& + 5/2 * c)^2 + 2 * (74 * \cos(dx + c)^2 + 74 * \sin(dx + c)^2 + 179 * \cos(dx + c) \\
& + 105) * \sin(5/2 * dx + 5/2 * c) * \sin(3/2 * dx + 3/2 * c) + (74 * \cos(dx + c)^2 + 74 * \\
& \sin(dx + c)^2 + 179 * \cos(dx + c) + 105) * \sin(3/2 * dx + 3/2 * c)^2 * \sin(7/2 * dx \\
& + 7/2 * c)^2 * \sin(1/2 * \arctan2(\sin(dx + c), \cos(dx + c)))) / (((\sqrt{2} * \cos \\
& (dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2})) * \cos \\
& (5/2 * dx + 5/2 * c)^2 + 2 * (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + \\
& 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2})) * \cos(5/2 * dx + 5/2 * c) * \cos(3/2 * dx + 3/2 * c) \\
&) + (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \cos(3/2 \\
& * dx + 3/2 * c)^2 + (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) \\
& + \sqrt{2}) * \cos(5/2 * dx + 5/2 * c)^2 + 2 * (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) \\
& + \sqrt{2}) * \sin(5/2 * dx + 5/2 * c) * \sin(3/2 * dx + 3/2 * c) + (\sqrt{2} * \cos(dx + c) \\
&)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \sin(3/2 * dx \\
& + 3/2 * c)^2 * \cos(9/2 * dx + 9/2 * c)^2 + 2 * ((\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \cos(5/2 * dx + 5/2 * c)^2 \\
& + 2 * (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \cos(5/2 * dx + 5/2 * c) * \cos(3/2 * dx + 3/2 * c) + (\sqrt{2} * \cos(dx + c) \\
& + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \cos(3/2 \\
& * dx + 3/2 * c)^2 + (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \sin(5/2 * dx + 5/2 * c)^2 + 2 * (\sqrt{2} * \cos(dx + c) \\
&)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \sin(5/2 * dx \\
& + 5/2 * c) * \sin(3/2 * dx + 3/2 * c) + (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c) \\
& + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \sin(3/2 * dx + 3/2 * c)^2 * \cos(9/2 \\
& * dx + 9/2 * c) * \cos(7/2 * dx + 7/2 * c) + ((\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin \\
& (dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2})) * \cos(5/2 * dx + 5/2 * c)^2 + 2 * \\
& (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \cos(5/2 * dx + 5/2 * c) * \cos(3/2 * dx + 3/2 * c) + (\sqrt{2} * \cos(dx + c)
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 9/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c))^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2 \\
& * \text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d* \\
& x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5 \\
& /2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\text{si} \\
& n(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\\
& \text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d \\
& *x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)*\sin(3/2* \\
& d*x + 3/2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2) \\
& *\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)*\cos(7 \\
& /2*d*x + 7/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqr} \\
& t(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + \\
& c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d* \\
& x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt} \\
& (2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt} \\
& (2))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + ((\text{sqr} \\
& t(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqr} \\
& t(2))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x \\
& + 3/2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\text{co} \\
& s(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqr} \\
& t(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2* \\
& c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d \\
& *x + c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos \\
& (d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\text{si} \\
& n(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \\
& \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5 \\
& /2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\text{co} \\
& s(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2)* \\
& \cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2)) \\
& *\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos \\
& (d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\text{si} \\
& n(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2) \\
& *\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2) \\
& *\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqr} \\
& t(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c \\
&)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d* \\
& x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(\\
& d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*
\end{aligned}$$

$$\begin{aligned}
& t(2)*\cos(d*x + c) + \text{sqrt}(2)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + \\
& c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d \\
& *x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d* \\
& x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt} \\
& (2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt} \\
& (2))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x \\
& + 3/2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d \\
& *x + 7/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2) \\
& *\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 \\
& + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + \\
& 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)* \\
& \cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2)) \\
& *\sin(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^ \\
& 2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/ \\
& 2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x \\
& + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + ((\text{sqrt}(2) \\
& *\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c) \\
& ^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3 \\
& /2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d* \\
& x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2) \\
&)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)^2 \\
& + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + \\
& c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/ \\
& 2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt} \\
& t(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c \\
&)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d* \\
& x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(\\
& d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2* \\
& \text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(5/ \\
& 2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin \\
& (d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2*\sin \\
& (9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2) \\
& *\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c)^2 \\
& + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + \\
& c) + \text{sqrt}(2))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt} \\
& (2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c
\end{aligned}$$

$$\begin{aligned}
&)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(5/2dx \\
&x + 5/2c) \sin(3/2dx + 3/2c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
&+ c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(3/2dx + 3/2c)^2) \sin(7/2 \\
&*dx + 7/2c)^2 \sin(2/3 \arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c) \\
&)) ^2 + 2 * (((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos \\
&(dx + c) + \sqrt{2}) \cos(5/2dx + 5/2c)^2 + 2 * (\sqrt{2} \cos(dx + c)^2 + s \\
&qrt(2) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(5/2dx + 5/2 \\
&*c) \cos(3/2dx + 3/2c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 \\
&+ 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2dx + 3/2c)^2 + (\sqrt{2} \cos(\\
&dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin \\
&(5/2dx + 5/2c)^2 + 2 * (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + \\
&2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(5/2dx + 5/2c) \sin(3/2dx + 3/2c) \\
&+ (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c \\
&+ \sqrt{2}) \sin(3/2dx + 3/2c)^2) \cos(9/2dx + 9/2c)^2 + 2 * ((\sqrt{2} *c \\
&os(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) *c \\
&os(5/2dx + 5/2c)^2 + 2 * (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 \\
&+ 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(5/2dx + 5/2c) \cos(3/2dx + 3/2 \\
&*c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx \\
&+ c) + \sqrt{2}) \cos(3/2dx + 3/2c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} * \\
&sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(5/2dx + 5/2c)^2 + \\
&2 * (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c \\
&+ \sqrt{2}) \sin(5/2dx + 5/2c) \sin(3/2dx + 3/2c) + (\sqrt{2} \cos(dx + \\
&c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 \\
&dx + 3/2c)^2) \cos(9/2dx + 9/2c) \cos(7/2dx + 7/2c) + ((\sqrt{2} \cos(d \\
&>*x + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(\\
&5/2dx + 5/2c)^2 + 2 * (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2 \\
&*sqrt(2) \cos(dx + c) + \sqrt{2}) \cos(5/2dx + 5/2c) \cos(3/2dx + 3/2c) \\
&+ (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \\
&+ \sqrt{2}) \cos(3/2dx + 3/2c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(\\
&dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(5/2dx + 5/2c)^2 + 2 * (\\
&sqrt(2) \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \\
&sqrt(2) \sin(5/2dx + 5/2c) \sin(3/2dx + 3/2c) + (\sqrt{2} \cos(dx + c)^ \\
&2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2dx \\
&+ 3/2c)^2) \cos(7/2dx + 7/2c)^2 + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin \\
&(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(5/2dx + 5/2c)^2 + 2 * \\
&(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \\
&sqrt(2) \cos(5/2dx + 5/2c) \cos(3/2dx + 3/2c) + (\sqrt{2} \cos(dx + c) \\
&^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2dx \\
&+ 3/2c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} * \\
&cos(dx + c) + \sqrt{2}) \sin(5/2dx + 5/2c)^2 + 2 * (\sqrt{2} \cos(dx + c)^2 \\
&+ \sqrt{2}) \sin(5/2dx + 5/2c) \sin(3/2dx + 3/2c) + (\sqrt{2} \cos(dx + c) \\
&)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2dx + 3/2c)^2) \sin(9/2dx \\
&+ 9/2c)^2 + 2 * ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} (\\
&2) \cos(dx + c) + \sqrt{2}) \cos(5/2dx + 5/2c)^2 + 2 * (\sqrt{2} \cos(dx + c)
\end{aligned}$$

$$\begin{aligned}
&^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(5/2 dx \\
&+ 5/2 c) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
&+ c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \\
&)\cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \\
&))\sin(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c \\
&)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(5/2 dx + 5/2 c) \sin(3/2 dx + \\
&3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx \\
&*x + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2) \sin(9/2 dx + 9/2 c) \sin(7/2 dx \\
&+ 7/2 c) + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos \\
&os(dx + c) + \sqrt{2} \cos(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 + \\
&\sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(5/2 dx + 5 \\
&/2 c) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c) \\
&^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos \\
&s(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin \\
&in(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 \\
&+ 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 * \\
&c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + \\
&c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2) \sin(7/2 dx + 7/2 c)^2) \cos(2/3 \arct \\
&an2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) \cos(2/7 \arctan2(\sin(7/2 dx \\
&*x + 7/2 c), \cos(7/2 dx + 7/2 c)))^2 + (((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \\
&)*\sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(5/2 dx + 5/2 c)^2 \\
&+ 2(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + \\
&c) + \sqrt{2} \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx \\
&+ c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(3/2 \\
&*dx + 3/2 c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \\
&(2) \cos(dx + c) + \sqrt{2} \sin(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c \\
&)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(5/2 dx \\
&x + 5/2 c) \sin(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
&+ c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2) \cos(9/2 \\
&*dx + 9/2 c)^2 + 2((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \\
&qrt(2) \cos(dx + c) + \sqrt{2} \cos(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx \\
&+ c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(5/2 \\
&*dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
&+ c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \\
&rt(2) \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \\
&rt(2) \sin(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
&+ c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(5/2 dx + 5/2 c) \sin(3/2 dx \\
&x + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos \\
&os(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2) \cos(9/2 dx + 9/2 c) \cos(7/2 \\
&*dx + 7/2 c) + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \\
&2) \cos(dx + c) + \sqrt{2} \cos(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c) \\
&^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(5/2 dx \\
&+ 5/2 c) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
&+ c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \\
&)\cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
&)) * \sin(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c) \\
&)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
&3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d \\
&*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\cos(7/2*d*x + 7/2*c)^2 + ((\sqrt{2} \\
&2)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&2))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
&c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + \\
&3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
&d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(5/2*d*x + 5/2*c) \\
&^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
&+ c) + \sqrt{2}))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d \\
&*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(\\
&3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + s \\
&qrt(2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2 \\
&*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
&d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*co \\
&s(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*c \\
&os(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + \\
&2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d \\
&*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(\\
&5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*s \\
&in(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2*s \\
&in(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)^ \\
&2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
&+ c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d* \\
&x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3 \\
&/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*s \\
&qrt(2)*\cos(d*x + c) + \sqrt{2}))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + \\
&c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(5/2* \\
&d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d \\
&*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2*\sin(7 \\
&/2*d*x + 7/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2* \\
&c)))^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
&d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + s \\
&qrt(2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2* \\
&c)*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
&+ 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d \\
&*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(\\
&5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2 \\
&*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
&+ (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
&+ \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)^2 + 2*((\sqrt{2})*co \\
&s(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*c
\end{aligned}$$

$$\begin{aligned}
& \cos(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2* \\
& c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(5/2*d*x + 5/2*c)^2 + \\
& 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2})*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + \\
& c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d \\
& *x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2*c) + ((\sqrt{2}*\cos(d* \\
& x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(5 \\
& /2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2* \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
& (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(5/2*d*x + 5/2*c)^2 + 2*(s \\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + s \\
& \sqrt{2})*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + \\
& 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^2 + (((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(5/2*d*x + 5/2*c)^2 + 2* \\
& (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2})*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c) \\
& ^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2})*\sin(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(5/2*d*x + \\
& 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c \\
&)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x \\
& + 9/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2)*\cos(d*x + c) + \sqrt{2})*\cos(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c) \\
& ^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(5/2*d*x \\
& + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c \\
&)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + \\
& 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d \\
& *x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\cos(9/2*d*x + 9/2*c)*\cos(7/2*d*x \\
& + 7/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2})*\cos(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(5/2*d*x + 5 \\
& /2*c)*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})* \\
& \cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin \\
& (5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
&), \cos(3/2*d*x + 3/2*c))^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
&c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2* \\
&c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
&+ c) + \sqrt{2}))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(5/2*d*x + 5/2*c)* \\
&\sin(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2 \\
&*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2* \\
&c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
&d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2* \\
&c)*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
&+ 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d \\
&>*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(\\
&5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2 \\
&*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) \\
&+ (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
&+ \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2*\sin(9/2*d*x + 9/2*c)*\sin(7/2*d*x + 7/2* \\
&c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
&+ c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)*\cos \\
&(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2* \\
&\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + \\
&c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(5/2* \\
&d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\sin(3/2*d*x + 3/2*c)^2*\sin(7/2*d*x + 7/2*c)^2 + (((\sqrt{2}*\cos(d*x \\
&+ c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(5/ \\
&2*d*x + 5/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + \\
&(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
&\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
&+ c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(5/2*d*x + 5/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c)^2 \\
&+ \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
&3/2*c)^2*\cos(9/2*d*x + 9/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin \\
&(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c)^2 + 2* \\
&(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
&\sqrt{2}))*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(d*x + c) \\
&)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x \\
&+ 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2})*
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c) + \sqrt{2}) \sin(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \\
& + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(5/2 dx + \\
& 5/2 c) \sin(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c \\
&)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 \cos(9/2 dx \\
& + 9/2 c) \cos(7/2 dx + 7/2 c) + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
& + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \\
& \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \\
& \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \\
& \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3 \\
& /2 c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx \\
& + c) + \sqrt{2}) \sin(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \\
& \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(5/2 dx + 5/2 \\
& c) \sin(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 \\
& + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 \cos(7/2 dx + 7 \\
& /2 c)^2 + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos \\
& (dx + c) + \sqrt{2}) \cos(5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \\
& \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(5/2 dx + 5/2 \\
& c) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 \\
& + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(dx \\
& + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin \\
& (5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + \\
& 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) \\
& + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c \\
&) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 \sin(9/2 dx + 9/2 c)^2 + 2((\sqrt{2} \cos \\
& (dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos \\
& (5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 \\
& + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 \\
& c) + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx \\
& + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin \\
& (dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(5/2 dx + 5/2 c)^2 + \\
& 2(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c \\
&) + \sqrt{2}) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + \\
& c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 \\
& dx + 3/2 c)^2 \sin(9/2 dx + 9/2 c) \sin(7/2 dx + 7/2 c) + ((\sqrt{2} \cos(dx \\
& + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(\\
& 5/2 dx + 5/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2 \\
& \sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(5/2 dx + 5/2 c) \cos(3/2 dx + 3/2 c) \\
& + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \\
& + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
& + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(5/2 dx + 5/2 c)^2 + 2(\\
& \sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \\
& \sqrt{2}) \sin(5/2 dx + 5/2 c) \sin(3/2 dx + 3/2 c) + (\sqrt{2} \cos(dx + c)^2 \\
& + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx \\
& + 3/2 c)^2 \sin(7/2 dx + 7/2 c)^2 \sin(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos \\
& (3/2 dx + 3/2 c)))^2 + 2(((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)
\end{aligned}$$

$$\begin{aligned}
&)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(5/2dx + 5/2c)^2 + 2(\sqrt{2}) \\
&*\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}) \\
&)\cos(5/2dx + 5/2c)\cos(3/2dx + 3/2c) + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}) \\
&)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c) \\
&)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx \\
&+ c) + \sqrt{2})\sin(5/2dx + 5/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2 + \sqrt{2}) \\
&)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(5/2dx + 5/2c)*\sin \\
&(3/2dx + 3/2c) + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2} \\
&\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2)\cos(9/2dx + 9/2c) \\
&)^2 + 2((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx \\
&+ c) + \sqrt{2})\cos(5/2dx + 5/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2 + \sqrt{2}) \\
&)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(5/2dx + 5/2c) \\
&)\cos(3/2dx + 3/2c) + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + \\
&2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(dx \\
&+ c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(5/2 \\
&dx + 5/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2} \\
&\cos(dx + c) + \sqrt{2})\sin(5/2dx + 5/2c)*\sin(3/2dx + 3/2c) + \\
&(\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) \\
&+ \sqrt{2})\sin(3/2dx + 3/2c)^2)\cos(9/2dx + 9/2c)\cos(7/2dx + 7/2c) \\
&)+ ((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + \\
&c) + \sqrt{2})\cos(5/2dx + 5/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2 + \sqrt{2}) \\
&)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(5/2dx + 5/2c)*\cos \\
&(3/2dx + 3/2c) + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2} \\
&\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(dx + \\
&c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(5/2dx \\
&+ 5/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2} \\
&\cos(dx + c) + \sqrt{2})\sin(5/2dx + 5/2c)*\sin(3/2dx + 3/2c) + (\sqrt{2} \\
&\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin \\
&(3/2dx + 3/2c)^2)\cos(7/2dx + 7/2c)^2 + ((\sqrt{2}\cos(dx + \\
&c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(5/2 \\
&dx + 5/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2} \\
&\cos(dx + c) + \sqrt{2})\cos(5/2dx + 5/2c)*\cos(3/2dx + 3/2c) + (\sqrt{2} \\
&\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}) \\
&)\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx \\
&+ c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(5/2dx + 5/2c)^2 + 2(\sqrt{2} \\
&\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}) \\
&)\sin(5/2dx + 5/2c)*\sin(3/2dx + 3/2c) + (\sqrt{2}\cos(dx + c)^2 + \\
&\sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/ \\
&2c)^2)\sin(9/2dx + 9/2c)^2 + 2((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx \\
&+ c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(5/2dx + 5/2c)^2 + 2(\sqrt{2} \\
&\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}) \\
&)\cos(5/2dx + 5/2c)*\cos(3/2dx + 3/2c) + (\sqrt{2}\cos(dx + c)^2 \\
&+ \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + \\
&3/2c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos \\
&(dx + c) + \sqrt{2})\sin(5/2dx + 5/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2 +
\end{aligned}$$


```

os(d*x + c) + sqrt(2))*cos(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2 +
sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(5/2*d*x + 5
/2*c)*cos(3/2*d*x + 3/2*c) + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)
^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*co
s(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*s
in(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2
+ 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3/2*
c) + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x +
c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2)*sin(9/2*d*x + 9/2*c)^2 + 2*((sqrt(2)
*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2)
)*cos(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)
^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(5/2*d*x + 5/2*c)*cos(3/2*d*x + 3
/2*c) + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*
x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)
)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(5/2*d*x + 5/2*c)^2
+ 2*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x +
c) + sqrt(2))*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3/2*c) + (sqrt(2)*cos(d*x
+ c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/
2*d*x + 3/2*c)^2)*sin(9/2*d*x + 9/2*c)*sin(7/2*d*x + 7/2*c) + ((sqrt(2)*cos
(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*co
s(5/2*d*x + 5/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 +
2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(5/2*d*x + 5/2*c)*cos(3/2*d*x + 3/2*c
) + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x +
c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*si
n(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(5/2*d*x + 5/2*c)^2 + 2
*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c)
+ sqrt(2))*sin(5/2*d*x + 5/2*c)*sin(3/2*d*x + 3/2*c) + (sqrt(2)*cos(d*x + c
)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*
x + 3/2*c)^2)*sin(7/2*d*x + 7/2*c)^2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))))*sqrt(a)*d

```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= \frac{105\sqrt{2} \log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{\sqrt{a} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{105\sqrt{2} \log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{\sqrt{a} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{16\sqrt{2} (30 a^{\frac{13}{2}} \sin(\frac{1}{2} dx + \frac{1}{2} c)^7 - 42 a^{\frac{13}{2}} \sin(\frac{1}{2} dx + \frac{1}{2} c)^5 + 35 a^{\frac{13}{2}} \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 7 a^{\frac{13}{2}} \sin(\frac{1}{2} dx + \frac{1}{2} c))}{a^7 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))}$$

210 d

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

```
[Out] 1/210*(105*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x +
1/2*c))) - 105*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2
*d*x + 1/2*c))) - 16*sqrt(2)*(30*a^(13/2)*sin(1/2*d*x + 1/2*c)^7 - 42*a^(13
/2)*sin(1/2*d*x + 1/2*c)^5 + 35*a^(13/2)*sin(1/2*d*x + 1/2*c)^3)/(a^7*sgn(c
os(1/2*d*x + 1/2*c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^4}{\sqrt{a + a \cos(c + dx)}} dx$$

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(1/2), x)
```

3.123 $\int \frac{\cos^3(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$

Optimal result	1395
Rubi [A] (verified)	1395
Mathematica [A] (verified)	1397
Maple [A] (verified)	1398
Fricas [A] (verification not implemented)	1398
Sympy [F(-1)]	1399
Maxima [B] (verification not implemented)	1399
Giac [A] (verification not implemented)	1988
Mupad [F(-1)]	1989

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{28 \sin(c+dx)}{15d\sqrt{a+a \cos(c+dx)}} + \frac{2 \cos^2(c+dx) \sin(c+dx)}{5d\sqrt{a+a \cos(c+dx)}} - \frac{2\sqrt{a+a \cos(c+dx)} \sin(c+dx)}{15ad}$$

[Out] $-\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+28/15*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/5*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}-2/15*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/a/d$

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2857, 3047, 3102, 2830, 2728, 212}

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx) \cos^2(c+dx)}{5d\sqrt{a \cos(c+dx)+a}} - \frac{2 \sin(c+dx) \sqrt{a \cos(c+dx)+a}}{15ad} + \frac{28 \sin(c+dx)}{15d\sqrt{a \cos(c+dx)+a}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^3/\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]],x]$

[Out] $-\left(\operatorname{Sqrt}[2]*\operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x]}{\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]]}\right]\right)/\left(\operatorname{Sqrt}[a]*d\right)+\left(28*\operatorname{Sin}[c+d*x]\right)/\left(15*d*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]]\right)+\left(2*\operatorname{Cos}[c+d*x]^2*\operatorname{Sin}[c+d*x]\right)/\left(5*d*\operatorname{Sqrt}[a*\operatorname{Cos}[c+d*x]+a]\right)-\left(2*\operatorname{Sin}[c+d*x]*\operatorname{Sqrt}[a*\operatorname{Cos}[c+d*x]+a]\right)/\left(15*a*d\right)$

$\cos[c + d*x]^2*\sin[c + d*x]/(5*d*\sqrt{a + a*\cos[c + d*x]}) - (2*\sqrt{a + a*\cos[c + d*x]}*\sin[c + d*x])/(15*a*d)$

Rule 212

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 2728

$\text{Int}[1/\sqrt{(a + (b \cdot \sin[c + d*x]) + (d \cdot x))}, x_Symbol] \rightarrow \text{Dist}[-2/d, \text{Subst}[\text{Int}[1/(2*a - x^2), x], x, b*(\cos[c + d*x]/\sqrt{a + b*\sin[c + d*x]})], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2830

$\text{Int}[(a + (b \cdot \sin[e + f*x]) + (d \cdot x))^m * ((c + d \cdot \sin[e + f*x]) + (f \cdot x)), x_Symbol] \rightarrow \text{Simp}[-(d)*\cos[e + f*x]*((a + b*\sin[e + f*x])^m / (f*(m + 1))), x] + \text{Dist}[(a*d*m + b*c*(m + 1))/(b*(m + 1)), \text{Int}[(a + b*\sin[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2^{(-1)}]$

Rule 2857

$\text{Int}[(c + d \cdot \sin[e + f*x])^n / \sqrt{(a + b \cdot \sin[e + f*x]) + (f \cdot x)}, x_Symbol] \rightarrow \text{Simp}[-2*d*\cos[e + f*x]*((c + d*\sin[e + f*x])^{n-1} / (f*(2*n - 1)*\sqrt{a + b*\sin[e + f*x]})), x] - \text{Dist}[1/(b*(2*n - 1)), \text{Int}[(c + d*\sin[e + f*x])^{n-2} / \sqrt{a + b*\sin[e + f*x]}]*\text{Simp}[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*\sin[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 3047

$\text{Int}[(a + (b \cdot \sin[e + f*x]) + (d \cdot x))^m * ((A + B \cdot \sin[e + f*x]) + (f \cdot x)) * ((c + d \cdot \sin[e + f*x]) + (f \cdot x)), x_Symbol] \rightarrow \text{Int}[(a + b*\sin[e + f*x])^m * (A*c + (B*c + A*d)*\sin[e + f*x] + B*d*\sin[e + f*x]^2), x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 3102

$\text{Int}[(a + (b \cdot \sin[e + f*x]) + (d \cdot x))^m * ((A + B \cdot \sin[e + f*x]) + (f \cdot x)) + (C \cdot \sin[e + f*x])^2, x_Symbol] \rightarrow \text{Simp}[(-C)*\cos[e + f*x]*((a + b*\sin[e + f*x])^{m+1} / (b*f*(m + 2))), x] + \text{Dist}[1/(b*(m + 2)), \text{Int}[(a + b*\sin[e + f*x])^m * \text{Simp}[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C, m\}, x]$

&& !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \cos^2(c + dx) \sin(c + dx)}{5d\sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{\cos(c+dx)(-4a+a \cos(c+dx))}{\sqrt{a+a \cos(c+dx)}} dx}{5a} \\
 &= \frac{2 \cos^2(c + dx) \sin(c + dx)}{5d\sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{-4a \cos(c+dx)+a \cos^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx}{5a} \\
 &= \frac{2 \cos^2(c + dx) \sin(c + dx)}{5d\sqrt{a + a \cos(c + dx)}} - \frac{2\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15ad} - \frac{2 \int \frac{\frac{a^2}{2} - 7a^2 \cos(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx}{15a^2} \\
 &= \frac{28 \sin(c + dx)}{15d\sqrt{a + a \cos(c + dx)}} + \frac{2 \cos^2(c + dx) \sin(c + dx)}{5d\sqrt{a + a \cos(c + dx)}} \\
 &\quad - \frac{2\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15ad} - \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx \\
 &= \frac{28 \sin(c + dx)}{15d\sqrt{a + a \cos(c + dx)}} + \frac{2 \cos^2(c + dx) \sin(c + dx)}{5d\sqrt{a + a \cos(c + dx)}} \\
 &\quad - \frac{2\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15ad} + \frac{2 \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \\
 &= -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{28 \sin(c + dx)}{15d\sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{2 \cos^2(c + dx) \sin(c + dx)}{5d\sqrt{a + a \cos(c + dx)}} - \frac{2\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{15ad}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\begin{aligned}
 &\int \frac{\cos^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx \\
 &= \frac{\left(-15\sqrt{2} \operatorname{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}\right) + \sqrt{1 - \cos(c + dx)}(29 - 2 \cos(c + dx) + 3 \cos(2(c + dx)))\right) \sin(c + dx)}{15d\sqrt{1 - \cos(c + dx)}\sqrt{a(1 + \cos(c + dx))}}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]], x]

[Out] ((-15*sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]] + Sqrt[1 - Cos[c + d*x]]*(29 - 2*Cos[c + d*x] + 3*Cos[2*(c + d*x)]))*Sin[c + d*x])/(15*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.31

method	result
default	$\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\left(-24\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sqrt{a\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} + 20\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} + 15\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}{15a^{\frac{3}{2}}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}d$

[In] `int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $-1/15*\cos(1/2*d*x+1/2*c)*2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-24*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}*\sin(1/2*d*x+1/2*c)^4+20*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}*\sin(1/2*d*x+1/2*c)^2+15*\ln(4*(a^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+a)/\cos(1/2*d*x+1/2*c))*a-30*a^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)})/a^{(3/2)}/\sin(1/2*d*x+1/2*c)/(a*\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/d$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= \frac{4\sqrt{a \cos(dx + c) + a}(3 \cos(dx + c)^2 - \cos(dx + c) + 13) \sin(dx + c) + 15\sqrt{2}(a \cos(dx + c) + a) \log\left(\frac{\cos(dx + c)^2 + 2\sqrt{a \cos(dx + c) + a}}{\cos(dx + c) + 1}\right)}{30(ad \cos(dx + c) + ad)}$$

[In] `integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $1/30*(4*\sqrt{a*\cos(d*x + c) + a}*(3*\cos(d*x + c)^2 - \cos(d*x + c) + 13)*\sin(d*x + c) + 15*\sqrt{2}*(a*\cos(d*x + c) + a)*\log(-(\cos(d*x + c)^2 + 2*\sqrt{a*\cos(d*x + c) + a})*\sqrt{a*\cos(d*x + c) + a}*\sin(d*x + c)/\sqrt{a} - 2*\cos(d*x + c) - 3)/(\cos(d*x + c)^2 + 2*\cos(d*x + c) + 1))/\sqrt{a})/(a*d*\cos(d*x + c) + a*d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908518 vs. 2(119) = 238.

Time = 17.66 (sec) , antiderivative size = 908518, normalized size of antiderivative = 6489.41

$$\int \frac{\cos^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

```
[Out] -1/1680*(84*(sqrt(2)*cos(3/2*d*x + 3/2*c)^2*sin(d*x + c) + 2*sqrt(2)*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + sqrt(2)*sin(3/2*d*x + 3/2*c)^2*sin(d*x + c) + 2*sqrt(2)*sin(3/2*d*x + 3/2*c)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c))*cos(7/2*d*x + 7/2*c)^3 - 84*((sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(7/2*d*x + 7/2*c)^3 - 24*((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c)^3 + 3*(420*sqrt(2)
```

$$\begin{aligned}
& * \cos(3/2*d*x + 3/2*c)^3 * \sin(d*x + c) - 420 * (\sqrt{2}) * \cos(d*x + c) + \sqrt{2}) \\
& * \sin(3/2*d*x + 3/2*c)^3 - 280 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 + 35 * ((3 * \sqrt{2}) \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) \\
& ^2 + (3 * \sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin \\
& (1/2*d*x + 1/2*c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) \\
& * \sin(d*x + c)^2 + 24 * \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + 2 * (3 * \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) \\
& + 3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \co \\
& s(3/2*d*x + 3/2*c)^2 - 35 * (8 * \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^3 - 3 * (\sqrt{2}) * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(\\
& 1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3 * (\sqrt{2}) * \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}) \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2* \\
& c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4 * (2 * \sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + 105 * (\sqrt{2}) * \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}) * \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) \\
& + 1)) * \cos(1/2*d*x + 1/2*c)^2 + 35 * ((3 * \sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \sqrt{2} * \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - \\
& 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (3 * \sqrt{2}) * \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + \\
& 1/2*c) + 1) - 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 12 * \sqrt{2} * \cos \\
& (3/2*d*x + 3/2*c) * \sin(d*x + c) + 2 * (3 * \sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \sqrt{2} * \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - \\
& 20 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 3 * \sqrt{2} * \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 32 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)^2 - 35 * (8 \\
& * \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^3 - 3 * (\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}) * \log(\cos(1/2*d* \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(1 \\
& /2*d*x + 1/2*c)^2 - 3 * (\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c \\
&)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + \\
& (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/ \\
& 2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))* \\
& \cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6 \\
& *(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))^3 - 3*(\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - sq \\
& rt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) \\
& + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x \\
& + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2* \\
& d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + \\
& 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - \\
& 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + (84*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& t(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3 \\
& /2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
&)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2* \\
& d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^3 - 84*((\sqrt{2})*\cos(d*x \\
& + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(d*x + c) + \sqrt{2}))* \\
& \sin(3/2*d*x + 3/2*c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(\\
& 1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)*\sin(\\
& 1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(\\
& 7/2*d*x + 7/2*c)^3 - 24*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + \\
& 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2} \\
&))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
&) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2 \\
& *\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^3 + 3*(
\end{aligned}$$

$$\begin{aligned}
& 420\sqrt{2}\cos(3/2*d*x + 3/2*c)^3\sin(d*x + c) - 420(\sqrt{2}\cos(d*x + c) \\
& + \sqrt{2})\sin(3/2*d*x + 3/2*c)^3 - 280\sqrt{2}\sin(1/2*d*x + 1/2*c)^3 + 3 \\
& 5*((3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}\sin(1/2*d*x + 1/2*c))\cos \\
& \cos(d*x + c)^2 + (3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}\sin(1/2*d* \\
& x + 1/2*c))\sin(d*x + c)^2 + 24\sqrt{2}\cos(1/2*d*x + 1/2*c)\sin(d*x + c) + \\
& 2*(3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}\sin(1/2*d*x + 1/2*c))\cos \\
& \cos(d*x + c) + 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}\sin(1/2*d*x + \\
& 1/2*c))\cos(3/2*d*x + 3/2*c)^2 - 35*(8\sqrt{2}\sin(1/2*d*x + 1/2*c)^3 - 3* \\
& (\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2 \\
& *d*x + 1/2*c) + 1))\sin(1/2*d*x + 1/2*c)^2 + 4*(2\sqrt{2}\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)\cos(d*x + c)^2 + 105*(\sqrt{2}\log(\cos \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2* \\
& d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c)^2 + 35*((3\sqrt{2}\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2} \\
&)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 8\sqrt{2}\sin(1/2*d*x + 1/2*c)\cos(d*x + c)^2 + (3\sqrt{2}\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin \\
& (1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}\sin(1/2*d*x + 1/2*c))\sin(d*x + c)^2 + 1 \\
& 2\sqrt{2}\cos(3/2*d*x + 3/2*c)\sin(d*x + c) + 2*(3\sqrt{2}\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2} \\
&)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 20\sqrt{2}\sin(1/2*d*x + 1/2*c)\cos(d*x + c) + 3\sqrt{2}\log(\cos \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 32\sqrt{2}\sin(1/2*d*x + 1/2*c)\sin(3/2*d*x + 3/2*c) \\
&)^2 - 35*(8\sqrt{2}\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) \\
& + 1))\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\sin(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x \\
& + 1/2*c))*\sin(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c)^2 + 56*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& *\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\si \\
& n(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \si \\
& n(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/ \\
& 2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d* \\
& x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(\\
& d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{ \\
& t(2)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(\\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c) - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d \\
& *x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2 \\
& *d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\c \\
& os(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1 \\
& /2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2})*\cos(d*
\end{aligned}$$

$$\begin{aligned}
& n(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 2*(6*(\sqrt{2})*\cos \\
& (d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)) * \cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \sin(d*x + c)^2 + 6*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2} \\
& *\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2 \\
& *c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \cos(d*x + c) - 3*(\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \sin(3/2*d*x \\
& + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c)) * \cos(5/2*d*x + 5/2*c)^2 + (84*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + \\
& c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2} \\
& (2)*\sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) * \\
& \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)) * \cos(7/2*d*x + 7/2*c)^3 - 84*((\sqrt{2} \\
& (2)*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(d*x + c) \\
& + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + (\sqrt{2})*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c) + 2*(\sqrt{2})*\cos(d* \\
& x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3 \\
& /2*c)) * \sin(7/2*d*x + 7/2*c)^3 - 24*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d \\
& *x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c)^ \\
& 2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + \\
& c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2 * \cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x \\
& + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/ \\
& 2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 * \\
& \cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin \\
& (d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c) * \sin(1/2*d*x + 1/2 \\
& *c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2
\end{aligned}$$

$$\begin{aligned}
& *c)^3 + 3*(420*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 420*(\sqrt{2}*c \\
& \cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^3 + 35*((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{ \\
& rt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{ \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 35*((3*\sqrt{2}*\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3* \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x \\
& + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{ \\
& rt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2* \\
& d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1
\end{aligned}$$

$$\begin{aligned}
&)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \\
&\sin(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos \\
&(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 56*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
&+ 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin \\
&(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
&(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) - 70*((8*\sqrt{2})* \\
&\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log \\
&(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
&1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c) \\
&*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
&+ 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1 \\
&/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) \\
&+ 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos \\
&(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
&1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
&2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos \\
&(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&- \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
&d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
&+ \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 70* \\
&(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2 \\
&*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
&(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
&1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^ \\
&2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2 \\
&*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c) \\
&)*\cos(d*x + c) - 8*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})* \\
&\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x \\
&+ 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos \\
&(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin \\
&(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d* \\
&x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
&(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d* \\
&x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2 \\
&*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(\\
&1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin \\
&(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2})*
\end{aligned}$$

$$\begin{aligned}
& t(2) \cdot \cos(dx + c) + \sqrt{2} \cdot \cos(3/2 \cdot dx + 3/2 \cdot c)^2 + (8\sqrt{2} \cdot \sin(1/2 \cdot dx \\
& x + 1/2 \cdot c)^2 - 3(\sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c) \\
& ^2 + 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin \\
& (1/2 \cdot dx + 1/2 \cdot c)^2 - 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 2 \\
& \cdot \sqrt{2} \cdot \cos(dx + c)^2 + (8\sqrt{2} \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 3(\sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c) \\
& ^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2\sin(1/2 \cdot dx + 1/2 \cdot c) \\
& + 1) - \sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2\sin \\
& (1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 2\sqrt{2} \cdot \sin(dx + c)^2 + \\
& 6\sqrt{2} \cdot \cos(1/2 \cdot dx + 1/2 \cdot c)^2 + 14\sqrt{2} \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 12 \cdot (\\
& \sqrt{2} \cdot \cos(dx + c) \cdot \cos(1/2 \cdot dx + 1/2 \cdot c) - \sqrt{2} \cdot \sin(dx + c) \cdot \sin(1/2 \cdot dx \\
& x + 1/2 \cdot c) + \sqrt{2} \cdot \cos(1/2 \cdot dx + 1/2 \cdot c) \cdot \cos(3/2 \cdot dx + 3/2 \cdot c) + 2 \cdot (3\sqrt{2} \\
& (2) \cdot \cos(1/2 \cdot dx + 1/2 \cdot c)^2 + 11\sqrt{2} \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 3(\sqrt{2} \\
& \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2\sin(1/2 \cdot dx + 1/2 \cdot c) \\
& c) + 1) - \sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2\sin \\
& in(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 2\sqrt{2} \cdot \cos(dx + c) - \\
& 3(\sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2\sin(1/2 \cdot dx \\
& dx + 1/2 \cdot c) + 1) - \sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c) \\
& c)^2 - 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \sin(1/2 \cdot dx + 1/2 \cdot c) + 2\sqrt{2} \cdot \sin(3 \\
& /2 \cdot dx + 3/2 \cdot c) - 140 \cdot (2\sqrt{2} \cdot \cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sqrt{2} \cdot \sin(1/2 \cdot dx \\
& dx + 1/2 \cdot c)) \cdot \cos(7/2 \cdot dx + 7/2 \cdot c)^2 + 105 \cdot (12\sqrt{2} \cdot \cos(3/2 \cdot dx + 3/2 \cdot c) \\
& ^3 \cdot \sin(dx + c) - 12(\sqrt{2} \cdot \cos(dx + c) + \sqrt{2} \cdot \sin(3/2 \cdot dx + 3/2 \cdot c))^3 \\
& - 8\sqrt{2} \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)^3 + ((3\sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c) \\
& ^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - 3\sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c) \\
& ^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - 8\sqrt{2} \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)) \cdot \cos(dx + c)^2 + (3\sqrt{2} \cdot \log(\cos(1/2 \cdot dx \\
& *dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - 3\sqrt{2} \\
& \sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2\sin(1/2 \cdot dx \\
& + 1/2 \cdot c) + 1) - 8\sqrt{2} \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)) \cdot \sin(dx + c)^2 + 24\sqrt{2} \cdot \cos(2 \\
&) \cdot \cos(1/2 \cdot dx + 1/2 \cdot c) \cdot \sin(dx + c) + 2 \cdot (3\sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c) \\
& ^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - 3\sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c) \\
& ^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - 8\sqrt{2} \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)) \cdot \cos(dx + c) + 3\sqrt{2} \cdot \log(\cos(1/2 \cdot dx \\
& x + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 + 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - 3\sqrt{2} \\
& t(2) \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2\sin(1/2 \cdot dx + \\
& 1/2 \cdot c) + 1) - 8\sqrt{2} \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)) \cdot \cos(3/2 \cdot dx + 3/2 \cdot c)^2 - (8\sqrt{2} \\
& \sqrt{2} \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)^3 - 3(\sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin \\
& (1/2 \cdot dx + 1/2 \cdot c)^2 + 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \sqrt{2} \cdot \log(\cos(1/2 \cdot dx \\
& + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \cos(1/2 \\
& *dx + 1/2 \cdot c)^2 - 3(\sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \\
& *c)^2 + 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \\
& \sin(1/2 \cdot dx + 1/2 \cdot c)^2 - 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)^2 \\
& + 4 \cdot (2\sqrt{2} \cdot \cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sqrt{2} \cdot \sin(1/2 \cdot dx + 1/2 \cdot c)) \cdot \cos \\
& s(dx + c)^2 + 3(\sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin(1/2 \cdot dx + 1/2 \cdot c) \\
& ^2 + 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1) - \sqrt{2} \cdot \log(\cos(1/2 \cdot dx + 1/2 \cdot c)^2 + \sin \\
& (1/2 \cdot dx + 1/2 \cdot c)^2 - 2\sin(1/2 \cdot dx + 1/2 \cdot c) + 1)) \cdot \cos(1/2 \cdot dx + 1/2 \cdot c)^2 +
\end{aligned}$$

$$\begin{aligned}
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6* \\
& (\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/ \\
& 2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^ \\
& 2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + \\
& 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3* \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c \\
&) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*s \\
& \sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/ \\
& 2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 3*(420*\sqrt{2})*\cos(3/2*d*x + 3/2*c \\
&)^3*\sin(d*x + c) - 420*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c \\
&)^3 - 280*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24 \\
& *\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 \\
& - 35*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x \\
& + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) - 70*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) -
\end{aligned}$$

$$\begin{aligned}
& 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sqrt{2}*\cos(d*x + c)^ \\
& 2 + \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2}))*\cos(3/2*d* \\
& x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9 \\
& *\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 8*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 8*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d* \\
& x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos \\
& (3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d \\
& *x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 1 \\
& 1*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(7/2*d*x + 7/ \\
& 2*c)^2 + 105*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^3 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/
\end{aligned}$$

$$\begin{aligned}
& /2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 6*(14*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(\\
& 1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 - 8*((\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& *\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + \\
& c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2 \\
& *\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))* \\
& \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos \\
& (5/2*d*x + 5/2*c)*\sin(5/2*d*x + 5/2*c) + 14*(\sqrt{2}*\cos(3/2*d*x + 3/2*c) \\
& ^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d \\
& *x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2 \\
& + 35*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 \\
& + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \\
& 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3* \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 12*\sqrt{2} \\
&) * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)^2 - (8 \\
& *\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1 \\
& /2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c \\
&)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)) * \\
& \sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) * \cos(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - \\
& 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)) * \cos(3/2*d \\
& *x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 84*((\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*
\end{aligned}$$

$$\begin{aligned}
& c) + (\sqrt{2})\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})\cos(3/2*d*x + \\
& 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})\sin(3/2*d*x + 3/2*c)^2* \\
& \sin(d*x + c) + 2*\sqrt{2})\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/ \\
& 2*c) + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + \\
& c))) - 6*(14*((\sqrt{2})\cos(d*x + c) + \sqrt{2}))\cos(3/2*d*x + 3/2*c)^2 + (s \\
& \sqrt{2})\cos(d*x + c) + \sqrt{2}))\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2})\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (s \\
& \sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\sqrt{2})\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(1/2*d*x + 1/2 \\
& *c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 14*((\sqrt{2})\cos(d*x + \\
& c) + \sqrt{2}))\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})\cos(d*x + c) + \sqrt{2}))\sin \\
& (3/2*d*x + 3/2*c)^2 + \sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\sqrt{2})\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2 \\
& *d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})\cos(d*x + c)*\sin(1/2 \\
& *d*x + 1/2*c) + \sqrt{2})\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2 \\
& *d*x + 5/2*c)^2 + 10*(\sqrt{2})\cos(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*s \\
& \sqrt{2})\cos(d*x + c) + \sqrt{2}))\cos(3/2*d*x + 3/2*c)^2 + 10*(\sqrt{2})\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + 2*((4*\sqrt{2}) \\
& \cos(d*x + c)^2 + 4*\sqrt{2})\sin(d*x + c)^2 + 15*\sqrt{2})\cos(d*x + c) + \\
& 11*\sqrt{2}))\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2})\cos(d*x + c)^2 + \\
& 4*\sqrt{2})\sin(d*x + c)^2 + 15*\sqrt{2})\cos(d*x + c) + 11*\sqrt{2}))\sin(3/2*d \\
& *x + 3/2*c)^2 + 4*(\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1 \\
& /2*c)^2)*\sin(d*x + c)^2 + 11*\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2})\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4 \\
& *\sqrt{2})\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 15*\sqrt{2})\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) + 11*\sqrt{2})\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 15*(\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c) + 2*(4*\sqrt{2})\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2})\sin(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + 15*\sqrt{2})\cos(d*x + c)*\sin(1/2*d*x + 1/2* \\
& c) + 11*\sqrt{2})\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5 \\
& /2*c)^2 + 10*(\sqrt{2})\cos(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*\sqrt{2})\cos \\
& (d*x + c) + \sqrt{2}))\sin(3/2*d*x + 3/2*c)^2 + 10*(\sqrt{2})\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 10*\sqrt{2})\cos(1 \\
& /2*d*x + 1/2*c)^2 + 10*\sqrt{2})\sin(1/2*d*x + 1/2*c)^2 + 28*(((\sqrt{2})\cos(d \\
& *x + c) + \sqrt{2}))\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})\cos(d*x + c) + \sqrt{2}) \\
&)*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*(\sqrt{2})\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos \\
& (1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2})\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos \\
& (5/2*d*x + 5/2*c) - (\sqrt{2})\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}* \\
& \sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{ \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d \\
& *x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2* \\
& d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\si \\
& n(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*s \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{ \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4* \\
& (2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x \\
& + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{ \\
& t(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*s \\
& \sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x
\end{aligned}$$

$$\begin{aligned}
& + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2} \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \cos(d*x + c) - 3*(\sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c) * \sin(5/2*d*x + 5/2*c) + 20*(\sqrt{2} \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c) - 12*(2*((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2 + 5*(\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + 5*(\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + 5*(\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 10*(\sqrt{2} \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) * \cos(3/2*d*x + 3/2*c) + 10*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 10*(\sqrt{2} \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c) + 56*(((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5 \\
& /2*c) + ((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*c \\
& \cos(d*x + c) + 5*\sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (4*\sqrt{2}*\cos(\\
& d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2})) \\
& * \sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2 * \cos(1/2*d*x + 1/ \\
& 2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + \\
& c) * \cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2 \\
& *c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * c \\
& \cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 4*\sqrt{2} * \\
& \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c) * \sin(1/2*d*x + \\
& 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x \\
& + 5/2*c)^2 * \sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) * \cos(2/3*\arctan2(\sin \\
& (3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 3*(420*\sqrt{2}*\cos(3/2*d*x \\
& + 3/2*c)^3 * \sin(d*x + c) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x \\
& + 3/2*c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& *\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \sin(d*x + c) \\
& ^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& *\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 3*\sqrt{2} * \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3 \\
& /2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(\\
& 1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2 \\
& *d*x + 1/2*c)) * \cos(d*x + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1 \\
& /2*d*x + 1/2*c)^2 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} * \sin \\
& (1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + \\
& 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2* \\
& c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^ \\
& 2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2} \\
& * \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + \\
& c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c) \\
&)*\cos(7/2*d*x + 7/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + \\
& 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x \\
& + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/ \\
& 2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)* \\
& \sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{ \\
& rt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& *\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c) \\
& ^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& *\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2* \\
& d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2* \\
& d*x + 1/2*c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + \\
& (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) -
\end{aligned}$$

$$\begin{aligned}
& \sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d* \\
& x + 1/2*c) * \sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2 \\
& *d*x + 1/2*c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos \\
& (d*x + c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6* \\
& (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + \\
& c)) * \cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 2*(6*(\sqrt{2} * \cos(d*x + c) + \\
& \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \cos(d*x + \\
& c)^2 + (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \sin(d*x + c)^2 + 6*\sqrt{2} * \cos(1/2*d \\
& *x + 1/2*c)^2 + 14*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2} * \cos(d*x + c) \\
&) * \cos(1/2*d*x + 1/2*c) - \sqrt{2} * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2} * \cos(1/2*d*x + 1 \\
& /2*c)^2 + 11*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \cos(d*x + c) - 3*(\sqrt{2} * \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \sin(3/2*d*x + 3/2*c) - 4 \\
& *(2*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \sin(5/2 \\
& *d*x + 5/2*c)^2 + 1260*((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + \\
& 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} \\
&) * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2} \\
&)) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1 \\
& /2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&) * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) * \cos(1
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2 \\
& *d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1 \\
& /2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2} \\
& *cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2 \\
& *c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3 \\
& /2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) \\
& *\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d \\
& *x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2*\cos(1/3*\arctan2(\sin(3/ \\
& 2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c))) + (84*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2} \\
& *sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^3 - 84*((\sqrt{2} \\
& *cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(d*x + c) + \sqrt{2} \\
& *sin(3/2*d*x + 3/2*c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& *cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\sin(7/2*d*x + 7/2*c)^3 - 24*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c
\end{aligned}$$

$$\begin{aligned}
&) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) \\
& ^3 + 3*(420*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 420*(\sqrt{2}*\cos(\\
& d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^3 + 35*((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\si \\
& n(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d* \\
& x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\si \\
& n(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 35*((3*\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + \\
& c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - sq
\end{aligned}$$

$$\begin{aligned}
& \text{rt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& \quad 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 \\
& \quad + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(\\
& \quad 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \\
& \quad \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)) * \sin \\
& \quad (1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 105*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 \\
& \quad + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1 \\
& \quad /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin \\
& \quad (1/2*d*x + 1/2*c)^2 + 56*(\text{sqrt}(2) * \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2 \\
& \quad * \text{sqrt}(2) * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \text{sqrt}(2) * \sin \\
& \quad (3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\text{sqrt}(2) * \sin(3/2*d*x + 3/2*c) * \sin(d*x \\
& \quad + c) * \sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(\\
& \quad 1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) - 70*((8*\text{sqrt}(2) * \cos \\
& \quad (1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c) \\
& \quad)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(\\
& \quad 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& \quad) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * \sin \\
& \quad (1/2*d*x + 1/2*c) - 3*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& \quad 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 \\
& \quad + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2* \\
& \quad c)) * \sin(d*x + c)^2 + 8*\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) + \\
& \quad 2*(8*\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\text{sqrt}(2) * \log(\cos \\
& \quad (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& \quad - \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& \quad *x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\text{sqrt}(2) * \log(\cos(1 \\
& \quad /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \quad \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& \quad + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \\
& \quad \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(3/2*d*x + 3/2*c) - 70*(8* \\
& \quad \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^3 - 3*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& \quad (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2*d* \\
& \quad x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/ \\
& \quad 2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& \quad 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \quad \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) \\
& \quad ^2 + 4*(2*\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)) * \sin(1/2*d*x + 1/2*c)) * \cos \\
& \quad (d*x + c) - 8*((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2*\text{sqrt}(\\
& \quad 2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + \\
& \quad 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \cos(d* \\
& \quad x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2*\text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3 \\
& \quad /2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + \\
& \quad 1/2*c)^2) * \sin(d*x + c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/ \\
& \quad 2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& \quad * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + \\
& \quad 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2) * \cos
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d \\
& *x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((\\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d \\
& *x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(\\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))^3 - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c \\
&))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + \\
& 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1 \\
& /2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + \\
& 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c))*\cos(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 35*((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(s \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2}*\cos(3/ \\
& 2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*sqr \\
& t(2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2* \\
& d*x + 7/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3 \\
& *(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (\\
& 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2* \\
& d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 8*((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) + 8*\sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) + 8*\sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 9*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \cos(d*x + c)^2 + (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \sin(d*x + c)^2 + 6*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) - \sqrt{2} * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \cos(d*x + c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(7/2*d*x + 7/2*c)
\end{aligned}$$

$$\begin{aligned}
&)^2 + 105*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos \\
&(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&)^3 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
&\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
&*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
&/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
&2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/ \\
&2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
&c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
&\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
&*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
&c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d \\
&*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3 \\
&*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
&*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\\
&\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
&1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
&2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos \\
&(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&- \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
&*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})* \\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&+ 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos \\
&(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&- 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
&*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*sq \\
&rt(2)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})* \\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&+ 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1 \\
&/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
&3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
&*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 \\
&- (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
&+ \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1 \\
&/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
&\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
&+ 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1
\end{aligned}$$

$$\begin{aligned}
& /2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2* \\
& c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2* \\
& c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& *\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2 \\
& *c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*si \\
& n(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c \\
&) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3 \\
& /2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))* \\
& \sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))* \\
& \cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2* \\
& d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\l \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2} \\
&) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \sin(5/2*d*x + 5/2 \\
& *c)^2 + 6*(14*(\sqrt{2} * \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \cos(\\
& 3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2} * \sin(3/2*d*x + \\
& 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \sin(1/2 \\
& *d*x + 1/2*c) + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2 \\
& *c)^2) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c)^2 - 8*((\sqrt{2} * \cos(d*x + c)^2 + \\
& \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \\
& \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} (\\
& 2) * \cos(d*x + c) + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^ \\
& 2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} \\
& * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos \\
& (3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \cos(d*x + c) * \sin(\\
& 1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(\\
& 5/2*d*x + 5/2*c) * \sin(5/2*d*x + 5/2*c) + 14*(\sqrt{2} * \cos(3/2*d*x + 3/2*c)^2 * \\
& \sin(d*x + c) + 2*\sqrt{2} * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x \\
& + c) + \sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \sin(3/2*d*x \\
& + 3/2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \sin(5/2*d*x + 5/2*c)^2 + \\
& 35*(12*\sqrt{2} * \cos(3/2*d*x + 3/2*c)^3 * \sin(d*x + c) - 12*(\sqrt{2} * \cos(d*x + \\
& c) + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 + (\\
& (3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(\\
& d*x + c)^2 + (3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)) * \sin(d*x + c)^2 + 24*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + 2* \\
& (3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(\\
& d*x + c) + 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} * \sin(1/2*d*x + 1/ \\
& 2*c)) * \cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2})*\cos \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 32*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2* \\
& d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin \\
& (d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3 \\
& * (\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x \\
& + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2 \\
& *d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 1 \\
& 4*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1 \\
& /2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1 \\
& /2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\
& x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c))*\co \\
& s(7/2*d*x + 7/2*c) + 84*((\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*s \\
& \sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2})*\cos(3/2 \\
& *d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& (2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d \\
& *x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + (\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + s \\
& \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) \\
& *\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1 \\
& /2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*s \\
& \sin(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c))\cos(3/2*d*x + 3/2*c) + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)\cos(d*x + c) + 2*(\sqrt{2})\cos(d*x + c)\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(1/2*d*x + 1/2*c))\sin(3/2*d*x + 3/2*c))\cos(5/2*d*x + 5/2*c) - (\sqrt{2})\cos(3/2*d*x + 3/2*c)^2\sin(d*x + c) + 2*\sqrt{2})\cos(3/2*d*x + 3/2*c)\cos(1/2*d*x + 1/2*c)\sin(d*x + c) + \sqrt{2})\sin(3/2*d*x + 3/2*c)^2\sin(d*x + c) + 2*\sqrt{2})\sin(3/2*d*x + 3/2*c)\sin(d*x + c)\sin(1/2*d*x + 1/2*c) + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)\sin(d*x + c))\sin(5/2*d*x + 5/2*c))\cos(7/2*d*x + 7/2*c) + 20*(\sqrt{2})\cos(d*x + c)^2\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c)\sin(d*x + c)^2 + 2*\sqrt{2})\cos(d*x + c)\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c))\cos(3/2*d*x + 3/2*c) + 20*(\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)\cos(d*x + c) - 35*(12*\sqrt{2})\cos(3/2*d*x + 3/2*c)^3\sin(d*x + c) - 12*(\sqrt{2})\cos(d*x + c) + \sqrt{2})\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2})\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})\sin(1/2*d*x + 1/2*c))\cos(d*x + c)^2 + (3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})\sin(1/2*d*x + 1/2*c))\sin(d*x + c)^2 + 24*\sqrt{2})\cos(1/2*d*x + 1/2*c)\sin(d*x + c) + 2*(3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})\sin(1/2*d*x + 1/2*c))\cos(d*x + c) + 3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})\sin(1/2*d*x + 1/2*c))\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))\sin(1/2*d*x + 1/2*c))\cos(d*x + c)^2 + 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})\sin(1/2*d*x + 1/2*c))\cos(d*x + c)^2 + (3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})\sin(1/2*d*x + 1/2*c))\sin(d*x + c)^2 + 12*\sqrt{2})\cos(3/2*d*x + 3/2*c)\sin(d*x + c) + 2*(3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(\\
& d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c \\
&) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*co \\
& s(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*s \\
& \sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&))*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c) + 20*(\sqrt{2})*\cos(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3 \\
& /2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c) - 12*(2*((\sqrt{2})*\cos(d*x + c)^2 + sq \\
& \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2* \\
& c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*co \\
& s(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2))*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2* \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3 \\
& /2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/ \\
& 2*d*x + 5/2*c)^2 + 5*(\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2})*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + 5*(\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2} \\
&))*\sin(3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 5* \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 10*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1 \\
& /2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c \\
&)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 10*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) + 10*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c \\
&) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c \\
&) + 56*(((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d \\
& *x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}* \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c \\
&)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin \\
& \sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2})* \\
& \cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))* \\
& *\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(\\
& d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}* \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d \\
& *x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x \\
& + c))) - 84*(((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2} \\
&)*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2} \\
&)*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x \\
& + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d \\
& *x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2} \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d \\
& *x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2 \\
& *d*x + 7/2*c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + \\
& 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*
\end{aligned}$$

$$\begin{aligned}
& \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 9(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(4\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + 4\sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 9\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + 5\sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c) \sin(7/2 dx + 7/2 c) \sin(5/2 dx + 5/2 c) + ((4\sqrt{2} \cos(dx + c)^2 + 4\sqrt{2} \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) + 5\sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (4\sqrt{2} \cos(dx + c)^2 + 4\sqrt{2} \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) + 5\sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + 5\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + 5\sqrt{2} (2) \sin(1/2 dx + 1/2 c)^2 + 2(4\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + 4\sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + 5\sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + 9(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(4\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + 4\sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 9\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + 5\sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c) \sin(5/2 dx + 5/2 c)^2 \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \sin(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 6(14(\sqrt{2} \cos(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2} \cos(3/2 dx + 3/2 c) \cos(1/2 dx + 1/2 c) \sin(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2} \sin(3/2 dx + 3/2 c) \sin(dx + c) \sin(1/2 dx + 1/2 c) + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)) \cos(5/2 dx + 5/2 c)^2 - 8((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c) \cos(5/2 dx + 5/2 c) \sin(5/2 dx + 5/2 c) + 14(\sqrt{2} \cos(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2} \cos(3/2 dx + 3/2 c) \cos(1/2 dx + 1/2 c) \sin(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2} \sin(3/2 dx + 3/2 c) \sin(dx + c) \sin(1/2 dx + 1/2 c) + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)) \sin(5/2 dx + 5/2 c)^2 + 35(12\sqrt{2} \cos(3/2 dx + 3/2 c)^3 \sin(dx + c) - 12(\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^3 - 8\sqrt{2} \sin(1/2 dx + 1/2 c)^3 + ((3\sqrt{2}) \log(\cos(1/2 dx + 1/2 c))^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*c \\
& os(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}(2 \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d* \\
& x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + \\
& 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d \\
& *x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((\\
& 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d \\
& *x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(\\
& 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& qrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1
\end{aligned}$$

$$\begin{aligned}
& /2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin \\
& (1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c \\
&))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - \\
& 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + \\
& 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2* \\
& d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1 \\
& /2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d \\
& *x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + \\
& 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12* \\
& (\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d \\
& *x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})* \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - \\
& 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(\\
& 3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d \\
& *x + 1/2*c))*\cos(5/2*d*x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 2*(84*(\sqrt{2})*\cos \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2 \\
& * \sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(\\
& 7/2*d*x + 7/2*c)^3 - 84*((\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x \\
& + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/ \\
& 2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^3 - 24*((\sqrt{2})* \\
& \cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2})* \\
& *\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2* \\
& d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})* \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d \\
& *x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^3 + 3*(420*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3 \\
& *\sin(d*x + c) - 420*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 \\
& - 280*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& *\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - \\
& 35*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + \\
& 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2
\end{aligned}$$

$$\begin{aligned}
& *c)) * \cos(dx + c)^2 + 105 * (\sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx \\
& + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c \\
&)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(1/2 * dx + 1 \\
& /2 * c)^2 + 35 * ((3 * \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^ \\
& 2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin \\
& (1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 8 * \sqrt{2} * \sin(1/2 * dx \\
& + 1/2 * c)) * \cos(dx + c)^2 + (3 * \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * \\
& dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2 * dx + \\
& 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 8 * \sqrt{2} \\
& * \sin(1/2 * dx + 1/2 * c)) * \sin(dx + c)^2 + 12 * \sqrt{2} * \cos(3/2 * dx + 3/2 * c) * \sin \\
& (dx + c) + 2 * (3 * \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^ \\
& 2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin \\
& (1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 20 * \sqrt{2} * \sin(1/2 * dx \\
& + 1/2 * c)) * \cos(dx + c) + 3 * \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx \\
& + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2 * dx + 1/ \\
& 2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 32 * \sqrt{2} * \\
& \sin(1/2 * dx + 1/2 * c)) * \sin(3/2 * dx + 3/2 * c)^2 - 35 * (8 * \sqrt{2} * \sin(1/2 * dx + \\
& 1/2 * c)^3 - 3 * (\sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + \\
& 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 \\
& * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(1/2 * dx + 1/2 * c)^2 - 3 * (\\
& \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx \\
& + 1/2 * c) + 1) - \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^ \\
& 2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \sin(1/2 * dx + 1/2 * c)^2 + 4 * (2 * \sqrt{2} * \cos(\\
& 1/2 * dx + 1/2 * c)^2 + \sqrt{2} * \sin(1/2 * dx + 1/2 * c)) * \sin(dx + c)^2 + 105 * (s \\
& \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx \\
& + 1/2 * c) + 1) - \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 \\
& - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \sin(1/2 * dx + 1/2 * c)^2 + 56 * (\sqrt{2} * \cos(3/ \\
& 2 * dx + 3/2 * c)^2 * \sin(dx + c) + 2 * \sqrt{2} * \cos(3/2 * dx + 3/2 * c) * \cos(1/2 * dx \\
& + 1/2 * c) * \sin(dx + c) + \sqrt{2} * \sin(3/2 * dx + 3/2 * c)^2 * \sin(dx + c) + 2 * \sqrt{2} \\
& * \sin(3/2 * dx + 3/2 * c) * \sin(dx + c) * \sin(1/2 * dx + 1/2 * c) + (\sqrt{2} * \cos(\\
& 1/2 * dx + 1/2 * c)^2 + \sqrt{2} * \sin(1/2 * dx + 1/2 * c)^2 * \sin(dx + c)) * \cos(5/2 * \\
& dx + 5/2 * c) - 70 * ((8 * \sqrt{2} * \cos(1/2 * dx + 1/2 * c) * \sin(1/2 * dx + 1/2 * c) - 3 \\
& * (\sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx \\
& * dx + 1/2 * c) + 1) - \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c \\
&)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(1/2 * dx + 1/2 * c)) * \cos(dx + c)^2 + (\\
& 8 * \sqrt{2} * \cos(1/2 * dx + 1/2 * c) * \sin(1/2 * dx + 1/2 * c) - 3 * (\sqrt{2} * \log(\cos(1/ \\
& 2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - s \\
& \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx \\
& + 1/2 * c) + 1)) * \cos(1/2 * dx + 1/2 * c)) * \sin(dx + c)^2 + 8 * \sqrt{2} * \cos(1/2 * dx \\
& + 1/2 * c) * \sin(1/2 * dx + 1/2 * c) + 2 * (8 * \sqrt{2} * \cos(1/2 * dx + 1/2 * c) * \sin(1/2 * \\
& dx + 1/2 * c) - 3 * (\sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c) \\
& ^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin \\
& (1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1)) * \cos(1/2 * dx + 1/2 * c)) * \co \\
& s(dx + c) - 3 * (\sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 \\
& + 2 * \sin(1/2 * dx + 1/2 * c) + 1) - \sqrt{2} * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\\
& \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + \\
& c) * \cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 8*((\sqrt{2} * \cos(d*x + c)^2 + \\
& \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3 \\
& /2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) \\
& * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2) * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c) \\
& ^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*s \\
& \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \co \\
& s(3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d* \\
& x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \cos(d*x + c) * \sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin \\
& (5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/ \\
& 2*c)^2 + (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \cos(d*x + c)^2 + (8*\sqrt{2} * \sin(1/2 \\
& *d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}) * \sin(d*x + c)^2 + 6*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) - s \\
& \sqrt{2} * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \co \\
& s(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2} * \sin(1 \\
& /2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) \\
&) + 2*\sqrt{2}) * \cos(d*x + c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}) * \sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2} * \cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(7/2*d*x + 7/2*c)^2 + 105*(1 \\
& 2*\sqrt{2} * \cos(3/2*d*x + 3/2*c)^3 * \sin(d*x + c) - 12*(\sqrt{2} * \cos(d*x + c) + \\
& \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 - 2*((8 \\
& * \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} * \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2} * \cos(1/2*d*x \\
& + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/ \\
& 2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x \\
& + 1/2*c) + 2*(8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(3/2*d*x + 3/2 \\
& *c) - 2*(8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d \\
& *x + 1/2*c)^2 + 4*(2*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x \\
& + 1/2*c)) * \cos(d*x + c) - 2*(6*(\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x \\
& + 3/2*c)^2 + (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \cos(d*x + c)^2 + (8*\sqrt{2} * \sin \\
& (1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2})) * \sin(d*x + c)^2 + 6*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) \\
& - \sqrt{2} * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
&) * \cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2} * \sin \\
& (1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1 \\
& /2*c) + 2*\sqrt{2})) * \cos(d*x + c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2})) * \sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2} * \cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \cos(5/2*d*x + 5/2*c)^2 + 3*(4 \\
& 20*\sqrt{2} * \cos(3/2*d*x + 3/2*c)^3 * \sin(d*x + c) - 420*(\sqrt{2} * \cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& + \sqrt{2}) \sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35 \\
& *((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x \\
& + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \\
& 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12 \\
& *\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 32*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
& ^2 - 35*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)) * \sin(d*x + c)^2 + 105*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2} * \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(7/2*d*x + 7/2*c) - 70*((8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 8*((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) + 8*\sqrt{2} * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) + 8*\sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 9*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2} * \cos(d*x + c) + \sqrt{2} * \cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2} * \sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2 \\
& * \text{sqrt}(2))*\cos(d*x + c)^2 + (8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\sin(d*x + c)^2 + \\
& 6*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 14*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 12*(\\
& \text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \text{sqrt}(2)*\sin(d*x + c)*\sin(1/2*d* \\
& x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + 11*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2) \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\cos(d*x + c) - \\
& 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\sin(3 \\
& /2*d*x + 3/2*c) - 140*(2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2* \\
& d*x + 1/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 105*(12*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c) \\
& ^3*\sin(d*x + c) - 12*(\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^ \\
& 3 - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^3 + ((3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\text{sqrt}(2)*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3* \\
& \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\text{sqrt}(2)*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{qr} \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*s \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2 \\
& *d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 4*(2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))*\co \\
& s(d*x + c)^2 + 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + \\
& ((3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + \\
& 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*co \\
& s(d*x + c)^2 + (3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x \\
& + 1/2*c))*sin(d*x + c)^2 + 12*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + \\
& 2*(3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + \\
& 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 20*sqrt(2)*sin(1/2*d*x + 1/2*c))*c \\
& os(d*x + c) + 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 \\
& + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin \\
& (1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 32*sqrt(2)*sin(1/2*d*x \\
& + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 - (8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 - 3*(s \\
& qrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x \\
& + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 \\
& - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2)*log(cos \\
& (1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) \\
& - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d \\
& *x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 + 4*(2*sqrt(2)*cos(1/2*d*x + 1/2*c \\
&)^2 + sqrt(2))*sin(1/2*d*x + 1/2*c))*sin(d*x + c)^2 + 3*(sqrt(2)*log(cos(1/ \\
& 2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - s \\
& qrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x \\
& + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 2*((8*sqrt(2)*cos(1/2*d*x + 1/2*c)* \\
& sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x \\
& + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c) \\
& ^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/ \\
& 2*c))*cos(d*x + c)^2 + (8*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) \\
& - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1 \\
& /2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c))*sin(d*x + c)^2 \\
& + 8*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 2*(8*sqrt(2)*cos(1 \\
& /2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^ \\
& 2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(\\
& 1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))* \\
& cos(1/2*d*x + 1/2*c))*cos(d*x + c) - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 \\
& + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/ \\
& 2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*co \\
& s(1/2*d*x + 1/2*c) - 6*(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d* \\
& x + 1/2*c)^2)*sin(d*x + c))*cos(3/2*d*x + 3/2*c) - 2*(8*sqrt(2)*sin(1/2*d*x \\
& + 1/2*c)^3 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 - \\
& 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2* \\
& d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2* \\
& c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 + 4*(2*sqrt(2)*c
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6* \\
& (\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/ \\
& 2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*(\\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^ \\
& 2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + \\
& 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3* \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) \\
&) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin \\
& (3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 6*(14*(\sqrt{2})*\cos(3/2*d*x + 3/2*c) \\
&)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(\\
& d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2* \\
& d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^ \\
& 2 - 8*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x \\
& + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \\
& \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3 \\
& /2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2) \\
& *\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d \\
& *x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1 \\
& /2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)*\sin(5/2*d*x + 5/2*c) + 14 \\
& *(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2 \\
& *c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(\\
& d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d* \\
& x + c))*\sin(5/2*d*x + 5/2*c)^2 + 35*(12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(\\
& d*x + c) - 12*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*s \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin
\end{aligned}$$

$$\begin{aligned}
& n(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*s \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + si \\
& n(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*s \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*lo \\
& \text{g}(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(\\
& 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*s \\
& \text{rt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*s \\
& \text{rt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x \\
& + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*si \\
& n(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*si \\
& n(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \text{qrt}(2))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*
\end{aligned}$$

$$\begin{aligned}
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2 \\
& *d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 84*((\sqrt{2})*\cos(3/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& (2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d* \\
& x + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*c \\
& os(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x \\
& + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + (\sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))* \\
& \cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*s \\
& \sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2 \\
& *d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& (2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(7/2*d \\
& *x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + s \\
& \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c) \\
& *\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(1/2*a \\
& rctan2(\sin(d*x + c), \cos(d*x + c))) - 6*(14*((\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2* \\
& c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2* \\
& c)^2 + 14*((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& (2)*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2 \\
& *(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&)*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 10*(\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + \\
& 3/2*c)^2 + 10*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&)^2)*\cos(d*x + c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c) \\
& ^2 + 15*\sqrt{2}*\cos(d*x + c) + 11*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c)) * \cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))^3 - 3 \\
& *(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} * \cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + 3*(\sqrt{2} * \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2} * \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (3*\sqrt{2} * \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 12*\sqrt{2} \\
& * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + 2*(3*\sqrt{2} * \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 20*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 3*\sqrt{2} * \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 32*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)^2 \\
& - (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c))^3 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos \\
& (1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1 \\
& /2*c)^2 + 4*(2*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2* \\
& c)) * \sin(d*x + c)^2 + 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*((8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1)) * \cos(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
& * \sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2 \\
& *c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) \\
&) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(3 \\
& /2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})* \\
& \cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \cos(d*x + c)^2 + (8 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2})) * \sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c) * \cos(1/2* \\
& d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})) * \sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2} \\
&) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(5/2*d*x + 5/2 \\
& *c) + 20*(\sqrt{2}*\cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c \\
&)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + sq \\
& rt(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c) - 12 \\
& *(2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + \\
& c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + s \\
&qrt(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + s \\
&qrt(2) * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \sin(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin \\
& (d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x \\
& + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + sq \\
& rt(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c))^2 + 2*(\text{sqrt}(2)*\cos(d*x + c))^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c))*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))^2*\cos(d*x + c) + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c))^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c))^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c))^2 + ((\text{sqrt}(2)*\cos(d*x \\
& + c))^2 + \text{sqrt}(2)*\sin(d*x + c))^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2 \\
& *d*x + 3/2*c))^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c))^2)*\cos(d*x + c))^2 + (\text{sqrt}(2)*\cos(d*x + c))^2 + \text{sqrt}(2)*\sin(d*x + c))^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c))^2 + (\text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))^2)*\sin(d*x + c))^2 + \text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))^2 + 2*(\text{sqrt}(2)*\cos(\\
& d*x + c))^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c) \\
& ^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c))^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c))^2*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c))^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\sin(7/2*d*x + 7/2*c))^2 + 2*((\text{sqrt}(2)*\cos(d*x + c))^2 + \text{sqrt}(2)*\sin(d*x \\
& + c))^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c))^2 + (\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))^2)*\cos(d*x + c))^2 + \\
& (\text{sqrt}(2)*\cos(d*x + c))^2 + \text{sqrt}(2)*\sin(d*x + c))^2 + 2*\text{sqrt}(2)*\cos(d*x + c) \\
& + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c))^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c))^2)*\sin(d*x + c))^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))^2 + 2*(\text{sqrt}(2)*\cos(d*x + c))^2*\cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c))^2 + 2*\text{sqrt}(2)*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
&) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))^2)*\cos \\
& (d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c))^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d* \\
& x + c))^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c) \\
&)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c))^2 + \text{sqrt}(2)*\sin(d*x + c))^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c))^2 + (\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))^2)*\cos(d*x + c))^2 + (\text{sqrt}(2) \\
&)*\cos(d*x + c))^2 + \text{sqrt}(2)*\sin(d*x + c))^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&)*\sin(3/2*d*x + 3/2*c))^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c))^2)*\sin(d*x + c))^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c))^2 + 2*(\text{sqrt}(2)*\cos(d*x + c))^2*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c))^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{s} \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c))^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))^2)*\cos(d*x + c) \\
&) + 2*(\text{sqrt}(2)*\cos(d*x + c))^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c))^2
\end{aligned}$$

$$\begin{aligned}
&*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2* \\
&\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2} \\
&(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2* \\
&((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + \\
&c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
&+ \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c) \\
&^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2 \\
&*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
&1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}* \\
&\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4 \\
&*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1 \\
&/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9* \\
&(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
&c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x \\
&+ c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
&5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c) \\
&*\cos(5/2*d*x + 5/2*c) + ((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c) \\
&^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
&(4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + \\
&c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
&+ \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + \\
&1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2* \\
&\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2} \\
&*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos \\
&(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
&*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2 \\
&*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c) \\
&)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2* \\
&c))*\cos(5/2*d*x + 5/2*c)^2 + ((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x \\
&+ c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2} \\
&*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
&^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) \\
&+ 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2 \\
&*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2* \\
&d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + \\
&c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
&9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2* \\
&c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
&1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x \\
&+ 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x \\
&+ c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
&3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin \\
&(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 \\
&+ 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(
\end{aligned}$$

$$\begin{aligned}
& d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}(\\
& 2)*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*c \\
& \cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2})*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}(\\
& 2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2})*\cos(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2} \\
& *\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/ \\
& 2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((4*\sqrt{2})*\cos \\
& (d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2})*\cos(d*x + c)^2 + 4*\sqrt{2}* \\
& \sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 \\
& + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(\\
& d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*(4*\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2} \\
& (2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(1/2*\text{arc} \\
& \text{tan2}(\sin(d*x + c), \cos(d*x + c))))*\cos(2/3*\text{arctan2}(\sin(3/2*d*x + 3/2*c), \cos \\
& (3/2*d*x + 3/2*c))) + 84*((\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2 \\
& *\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2})*\cos(3 \\
& /2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& *\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2 \\
& *d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + (\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d* \\
& x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \\
& \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2* \\
& c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2} \\
& (2)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c \\
&) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)) \\
& *\sin(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \\
& 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*
\end{aligned}$$

$$\begin{aligned}
& \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) \\
& + (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 6*(14*((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 14*((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 10*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 10*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 15*\sqrt{2}*\cos(d*x + c) + 11*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 15*\sqrt{2}*\cos(d*x + c) + 11*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 11*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 15*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 11*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 15*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 15*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 10*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 10*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 28*(((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*s \\
& \sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
& (\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1 \\
& /2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d \\
& *x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin \\
& (d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(\\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
& - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1
\end{aligned}$$

$$\begin{aligned}
& n(1/2*d*x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)* \\
& sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(7/2*d*x + 7/2*c))*sin(5/2*d* \\
& x + 5/2*c) + ((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)* \\
& cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2 \\
& *c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (sqrt(2)*cos(d*x + \\
& c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2* \\
& d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/ \\
& 2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*co \\
& s(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/ \\
& 2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(sqrt(2)*cos(\\
& 1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt \\
& (2)*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(d*x + c)^2*sin(1/2*d* \\
& x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2* \\
& d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c)^2 + (((sqrt(2)*cos \\
& (d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*co \\
& s(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d* \\
& x + 1/2*c)^2)*cos(d*x + c)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + \\
& c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + (sqrt(2)* \\
& cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + s \\
& sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2) \\
& *cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x \\
& + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(\\
& 2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)^2*sin(1/2 \\
& *d*x + 1/2*c) + sqrt(2)*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*cos \\
& (d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x \\
& + 3/2*c))*cos(7/2*d*x + 7/2*c)^2 + 2*((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin \\
& (d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (s \\
& sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c \\
&)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x \\
& + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + \\
& sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2*d*x + 1/2* \\
& c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2*d \\
& *x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d \\
& *x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + \\
& 3/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 \\
&)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*s \\
& in(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1 \\
& /2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*cos(7/2*d*x + 7 \\
& /2*c)*cos(5/2*d*x + 5/2*c) + ((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c \\
&)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*c \\
& os(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (s \\
& sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + s \\
& sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2* \\
& d*x + 5/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)) \\
&)^2 + 2*(((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(\\
& d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^ \\
& 2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x \\
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c))*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d* \\
& x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^ \\
& 2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\co \\
& s(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d* \\
& x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d* \\
& x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3 \\
& /2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 \\
& + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sq} \\
& \text{rt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + \\
& c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 2 \\
& *\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))* \\
& \cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\s \\
& \text{in}(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\c \\
& \text{os}(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2) \\
&)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2 \\
& *\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(\\
& (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d* \\
& x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))* \\
& \sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d* \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2* \\
& d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))))*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x \\
& + 5/2*c)))) - 1260*(((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos \\
& (d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin \\
& (3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d* \\
& x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2
\end{aligned}$$

$$\begin{aligned}
& *(\sqrt{2}\cos(dx + c)^2\sin(1/2*dx + 1/2*c) + \sqrt{2}\sin(dx + c)^2\sin(\\
& 1/2*dx + 1/2*c) + 2*\sqrt{2}\cos(dx + c)\sin(1/2*dx + 1/2*c) + \sqrt{2}\sin \\
& n(1/2*dx + 1/2*c))*\sin(3/2*dx + 3/2*c))*\cos(7/2*dx + 7/2*c)^2 + 2*((\sqrt{ \\
& 2)\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2*\sqrt{2}\cos(dx + c) + \sqrt{ \\
& 2))*\cos(3/2*dx + 3/2*c)^2 + (\sqrt{2}\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}\sin \\
& (1/2*dx + 1/2*c)^2)*\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin \\
& (dx + c)^2 + 2*\sqrt{2}\cos(dx + c) + \sqrt{2})*\sin(3/2*dx + 3/2*c)^2 + (s \\
& \sqrt{2}\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}\sin(1/2*dx + 1/2*c)^2)*\sin(dx + c \\
&)^2 + \sqrt{2}\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}\sin(1/2*dx + 1/2*c)^2 + 2*(\\
& \sqrt{2}\cos(dx + c)^2*\cos(1/2*dx + 1/2*c) + \sqrt{2}\cos(1/2*dx + 1/2*c)* \\
& \sin(dx + c)^2 + 2*\sqrt{2}\cos(dx + c)*\cos(1/2*dx + 1/2*c) + \sqrt{2}\cos(\\
& 1/2*dx + 1/2*c))*\cos(3/2*dx + 3/2*c) + 2*(\sqrt{2}\cos(1/2*dx + 1/2*c)^2 \\
& + \sqrt{2}\sin(1/2*dx + 1/2*c)^2)*\cos(dx + c) + 2*(\sqrt{2}\cos(dx + c)^2* \\
& \sin(1/2*dx + 1/2*c) + \sqrt{2}\sin(dx + c)^2*\sin(1/2*dx + 1/2*c) + 2*\sqrt{ \\
& 2)\cos(dx + c)*\sin(1/2*dx + 1/2*c) + \sqrt{2}\sin(1/2*dx + 1/2*c))*\sin(3 \\
& /2*dx + 3/2*c))*\cos(7/2*dx + 7/2*c)*\cos(5/2*dx + 5/2*c) + ((\sqrt{2}\cos(\\
& dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2*\sqrt{2}\cos(dx + c) + \sqrt{2}))*\cos \\
& (3/2*dx + 3/2*c)^2 + (\sqrt{2}\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}\sin(1/2*dx \\
& + 1/2*c)^2)*\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c \\
&)^2 + 2*\sqrt{2}\cos(dx + c) + \sqrt{2})*\sin(3/2*dx + 3/2*c)^2 + (\sqrt{2)*c \\
& os(1/2*dx + 1/2*c)^2 + \sqrt{2}\sin(1/2*dx + 1/2*c)^2)*\sin(dx + c)^2 + sq \\
& rt(2)*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}\sin(1/2*dx + 1/2*c)^2 + 2*(\sqrt{2)* \\
& cos(dx + c)^2*\cos(1/2*dx + 1/2*c) + \sqrt{2}\cos(1/2*dx + 1/2*c)*\sin(dx \\
& + c)^2 + 2*\sqrt{2}\cos(dx + c)*\cos(1/2*dx + 1/2*c) + \sqrt{2}\cos(1/2*dx \\
& + 1/2*c))*\cos(3/2*dx + 3/2*c) + 2*(\sqrt{2}\cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*dx + 1/2*c)^2)*\cos(dx + c) + 2*(\sqrt{2}\cos(dx + c)^2*\sin(1/2* \\
& dx + 1/2*c) + \sqrt{2}\sin(dx + c)^2*\sin(1/2*dx + 1/2*c) + 2*\sqrt{2}\cos(\\
& dx + c)*\sin(1/2*dx + 1/2*c) + \sqrt{2}\sin(1/2*dx + 1/2*c))*\sin(3/2*dx + \\
& 3/2*c))*\cos(5/2*dx + 5/2*c)^2 + ((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx \\
& x + c)^2 + 2*\sqrt{2}\cos(dx + c) + \sqrt{2}))*\cos(3/2*dx + 3/2*c)^2 + (\sqrt{ \\
& 2)\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}\sin(1/2*dx + 1/2*c)^2)*\cos(dx + c)^2 \\
& + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2*\sqrt{2}\cos(dx + c \\
&) + \sqrt{2})*\sin(3/2*dx + 3/2*c)^2 + (\sqrt{2}\cos(1/2*dx + 1/2*c)^2 + sqr \\
& t(2)*\sin(1/2*dx + 1/2*c)^2)*\sin(dx + c)^2 + \sqrt{2}\cos(1/2*dx + 1/2*c)^ \\
& 2 + \sqrt{2}\sin(1/2*dx + 1/2*c)^2 + 2*(\sqrt{2}\cos(dx + c)^2*\cos(1/2*dx \\
& + 1/2*c) + \sqrt{2}\cos(1/2*dx + 1/2*c)*\sin(dx + c)^2 + 2*\sqrt{2}\cos(dx \\
& + c)*\cos(1/2*dx + 1/2*c) + \sqrt{2}\cos(1/2*dx + 1/2*c))*\cos(3/2*dx + 3/2 \\
& *c) + 2*(\sqrt{2}\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}\sin(1/2*dx + 1/2*c)^2)*c \\
& os(dx + c) + 2*(\sqrt{2}\cos(dx + c)^2*\sin(1/2*dx + 1/2*c) + \sqrt{2}\sin(\\
& dx + c)^2*\sin(1/2*dx + 1/2*c) + 2*\sqrt{2}\cos(dx + c)*\sin(1/2*dx + 1/2* \\
& c) + \sqrt{2}\sin(1/2*dx + 1/2*c))*\sin(3/2*dx + 3/2*c))*\sin(7/2*dx + 7/2* \\
& c)^2 + 2*((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2*\sqrt{2}\cos(\\
& dx + c) + \sqrt{2}))*\cos(3/2*dx + 3/2*c)^2 + (\sqrt{2}\cos(1/2*dx + 1/2*c)^ \\
& 2 + \sqrt{2}\sin(1/2*dx + 1/2*c)^2)*\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^ \\
& 2 + \sqrt{2}\sin(dx + c)^2 + 2*\sqrt{2}\cos(dx + c) + \sqrt{2})*\sin(3/2*dx
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + \\
& ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) \\
& + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt} \\
& (2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c) \\
& ^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(\\
& d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(\\
& 2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
&)*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 56*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)* \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d \\
& *x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/ \\
& 2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\co \\
& s(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c \\
&)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x \\
& + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2) \\
&)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/ \\
& 2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\co \\
& s(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sq}
\end{aligned}$$

$$\begin{aligned}
& * \sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{s} \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2 \\
& *d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqr} \\
& \text{t}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(3/ \\
& 2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 84*((4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4 \\
& *\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\cos(3/2*d*x + \\
& 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
&)^2)*\cos(d*x + c)^2 + (4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5* \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\text{sq} \\
& \text{rt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)* \\
& \sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\text{sqrt}(2)*\cos(d*x + c) \\
&)^2*\sin(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 9*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
&))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((4*\text{sqrt}(2)*\cos(d*x + c) \\
&)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\text{sqr} \\
& \text{t}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c) \\
& ^2 + 5*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\text{sqr} \\
& \text{t}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\text{sqrt}(2)*\cos(\\
& d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + 9*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + \\
& ((4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x \\
& + c) + 5*\text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\text{sqrt}(2)*\cos(d*x + c) \\
&)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\sin(3/ \\
& 2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 5*\text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& 4*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9
\end{aligned}$$

$$\begin{aligned}
& *(\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\cos(d*x \\
& + c) + 2*(4*\sqrt{2}\cos(d*x + c)^2\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}\sin(d*x \\
& + c)^2\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}\cos(d*x + c)\sin(1/2*d*x + 1/2*c) \\
& + 5*\sqrt{2}\sin(1/2*d*x + 1/2*c))\sin(3/2*d*x + 3/2*c))\cos(5/2*d*x + 5/2*c \\
&)^2 + ((4*\sqrt{2}\cos(d*x + c)^2 + 4*\sqrt{2}\sin(d*x + c)^2 + 9*\sqrt{2}\cos \\
& (d*x + c) + 5*\sqrt{2}))\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\cos(d*x + c)^2 + (4*\sqrt{2}\cos(d* \\
& x + c)^2 + 4*\sqrt{2}\sin(d*x + c)^2 + 9*\sqrt{2}\cos(d*x + c) + 5*\sqrt{2}))\sin \\
& (3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2 \\
& *d*x + 1/2*c)^2)\sin(d*x + c)^2 + 5*\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{ \\
& 2)\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}\cos(d*x + c)^2\cos(1/2*d*x + 1/2* \\
& c) + 4*\sqrt{2}\cos(1/2*d*x + 1/2*c)\sin(d*x + c)^2 + 9*\sqrt{2}\cos(d*x + c) \\
& *\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}\cos(1/2*d*x + 1/2*c))\cos(3/2*d*x + 3/2*c \\
&) + 9*(\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\cos \\
& (d*x + c) + 2*(4*\sqrt{2}\cos(d*x + c)^2\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}\sin \\
& (d*x + c)^2\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}\cos(d*x + c)\sin(1/2*d*x + 1/ \\
& 2*c) + 5*\sqrt{2}\sin(1/2*d*x + 1/2*c))\sin(3/2*d*x + 3/2*c))\sin(7/2*d*x + \\
& 7/2*c)^2 + 2*((4*\sqrt{2}\cos(d*x + c)^2 + 4*\sqrt{2}\sin(d*x + c)^2 + 9*\sqrt{ \\
& 2)\cos(d*x + c) + 5*\sqrt{2}))\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\cos(d*x + c)^2 + (4*\sqrt{2} \\
& *\cos(d*x + c)^2 + 4*\sqrt{2}\sin(d*x + c)^2 + 9*\sqrt{2}\cos(d*x + c) + 5*\sqrt{ \\
& 2}))\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}* \\
& \sin(1/2*d*x + 1/2*c)^2)\sin(d*x + c)^2 + 5*\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \\
& 5*\sqrt{2}\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}\cos(d*x + c)^2\cos(1/2*d*x \\
& + 1/2*c) + 4*\sqrt{2}\cos(1/2*d*x + 1/2*c)\sin(d*x + c)^2 + 9*\sqrt{2}\cos(d \\
& *x + c)\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}\cos(1/2*d*x + 1/2*c))\cos(3/2*d*x \\
& + 3/2*c) + 9*(\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c) \\
& ^2)\cos(d*x + c) + 2*(4*\sqrt{2}\cos(d*x + c)^2\sin(1/2*d*x + 1/2*c) + 4*\sqrt{ \\
& 2}\sin(d*x + c)^2\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}\cos(d*x + c)\sin(1/2*d \\
& *x + 1/2*c) + 5*\sqrt{2}\sin(1/2*d*x + 1/2*c))\sin(3/2*d*x + 3/2*c))\sin(7/2 \\
& *d*x + 7/2*c)\sin(5/2*d*x + 5/2*c) + ((4*\sqrt{2}\cos(d*x + c)^2 + 4*\sqrt{2} \\
& *\sin(d*x + c)^2 + 9*\sqrt{2}\cos(d*x + c) + 5*\sqrt{2}))\cos(3/2*d*x + 3/2*c) \\
& ^2 + 4*(\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\cos \\
& (d*x + c)^2 + (4*\sqrt{2}\cos(d*x + c)^2 + 4*\sqrt{2}\sin(d*x + c)^2 + 9*\sqrt{ \\
& 2)\cos(d*x + c) + 5*\sqrt{2}))\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\sin(d*x + c)^2 + 5*\sqrt{2}* \\
& \cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*co \\
& s(d*x + c)^2\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}\cos(1/2*d*x + 1/2*c)\sin(d*x \\
& + c)^2 + 9*\sqrt{2}\cos(d*x + c)\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}\cos(1/2*d* \\
& x + 1/2*c))\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{ \\
& 2}\sin(1/2*d*x + 1/2*c)^2)\cos(d*x + c) + 2*(4*\sqrt{2}\cos(d*x + c)^2\sin(\\
& 1/2*d*x + 1/2*c) + 4*\sqrt{2}\sin(d*x + c)^2\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2} \\
&)\cos(d*x + c)\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}\sin(1/2*d*x + 1/2*c))\sin(3 \\
& /2*d*x + 3/2*c))\sin(5/2*d*x + 5/2*c)^2)\sin(1/2*\arctan2(\sin(d*x + c), \cos(\\
& d*x + c))))\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2
\end{aligned}$$

$$\begin{aligned}
& + (84*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x \\
& + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1 \\
& /2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c))*\cos(7/2*d*x + 7/2*c)^3 - 84*((\sqrt{2}*\cos(d*x + c) + \sqrt{2})* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2 \\
& *c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)) \\
& *\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^ \\
& 3 - 24*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) \\
& *\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^3 + 3*(420*\sqrt{2}*\cos(3 \\
& /2*d*x + 3/2*c)^3*\sin(d*x + c) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3 \\
& /2*d*x + 3/2*c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (\\
& 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d \\
& *x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2* \\
& d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1
\end{aligned}$$

$$\begin{aligned}
&)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
&\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos \\
&(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
&1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d* \\
&x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})* \\
&\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
&2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log \\
&(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&+ 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2})*\cos(3/2 \\
&*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
&1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d* \\
&x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2})* \\
&\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
&c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log \\
&(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
&1) - 32*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2})* \\
&2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
&*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1 \\
&/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x \\
&+ 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
&+ 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
&1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + \\
&4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d* \\
&x + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
&+ 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
&/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 5 \\
&6*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/ \\
&2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin \\
&(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c \\
&+ (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d \\
&*x + c))*\cos(5/2*d*x + 5/2*c) - 70*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2 \\
&*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos \\
&(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})* \\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
&+ 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
&- 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})* \\
&\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x \\
&+ 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x \\
&+ 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2 \\
&*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d \\
& *x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{ \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sqrt{2} \\
& *\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d* \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{ \\
& 2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{ \\
& t(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{ \\
& 2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2* \\
& d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (\\
& 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& 1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2 \\
& *d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\si \\
& n(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{ \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(7/2*d*x + \\
& 7/2*c)^2 + 105*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*s \\
& \sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^ \\
& 3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{ \\
& 2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*si \\
& \sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + \\
& 12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{ \\
& 2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2* \\
& c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\
& x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + \\
& 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d
\end{aligned}$$

$$\begin{aligned}
&) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(7/2*d*x + 7/2*c) - 70*((8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c) * \cos(d*x + c) - 8*((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) + 8*\sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) + 8*\sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 9*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/
\end{aligned}$$

$$\begin{aligned}
& 2*c) - 70*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}))*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}))*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}))*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 \\
& + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)) * \cos \\
& (d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c))*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2* \\
& c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 6*(14*(\sqrt{2})*\cos \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c))*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2 \\
& *\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(\\
& 5/2*d*x + 5/2*c)^2 - 8*((\sqrt{2}*\cos(d*x + c))^2 + \sqrt{2}*\sin(d*x + c))^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
& *\cos(d*x + c))^2 + \sqrt{2}*\sin(d*x + c))^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c))^2*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c))^2 + 2*\sqrt{2}*\cos(d*x + c))*\cos(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\sqrt{2}*\cos(d*x + c))^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c))^2* \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)*\sin(5/2* \\
& d*x + 5/2*c) + 14*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})* \\
& \cos(3/2*d*x + 3/2*c))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d* \\
& x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2 + 35*(12*\sqrt{2}*\cos(3/2*d* \\
& x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + \\
& 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c \\
&)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x \\
& + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d \\
& *x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c \\
&)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*
\end{aligned}$$

$$\begin{aligned}
& t(2) \sin(dx + c)^2 + 15\sqrt{2} \cos(dx + c) + 11\sqrt{2} \cos(3/2 dx + 3/2 c)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (4\sqrt{2} \cos(dx + c)^2 + 4\sqrt{2} \sin(dx + c)^2 + 15\sqrt{2} \cos(dx + c) + 11\sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + 11\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + 11\sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(4\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + 4\sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 15\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + 11\sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 15(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(4\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + 4\sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 15\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + 11\sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)) \sin(5/2 dx + 5/2 c)^2 + 10(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + 10(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + 10\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + 10\sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 28(((\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)) \cos(5/2 dx + 5/2 c) - (\sqrt{2} \cos(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2} \cos(3/2 dx + 3/2 c) \cos(1/2 dx + 1/2 c) \sin(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2} \sin(3/2 dx + 3/2 c) \sin(dx + c) \sin(1/2 dx + 1/2 c) + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)) \sin(5/2 dx + 5/2 c) + 20(\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 20(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) - 35(12\sqrt{2} \cos(3/2 dx + 3/2 c)^3 \sin(dx + c) - 12(\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^3 - 8\sqrt{2} \sin(1/2 dx + 1/2 c)^3 + ((3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(dx + c)^2 + (3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(dx + c)^2 + 24\sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c) + 2(3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(dx + c) + 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8* \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\text{sqrt}(2)*\sin(1/2*d* \\
& *x + 1/2*c)^3 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 \\
& - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3 \\
& *(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\text{sqrt}(2)*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (\\
& 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(d \\
& *x + c)^2 + 12*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\text{sqrt}(2)*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 20*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3* \\
& \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/ \\
& 2*d*x + 3/2*c)^2 - (8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\text{sqrt}(2)*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin \\
& (1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*((8*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/ \\
& 2*c) - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c)^2 + (8*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\text{sqrt}(2)*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)* \\
& \sin(1/2*d*x + 1/2*c) - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2* \\
& c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3* \\
& (\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
& - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\co \\
& s(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{ \\
& 2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\co \\
& s(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(\\
& d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{ \\
& 2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\l \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2 \\
& *c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))* \\
& \sin(5/2*d*x + 5/2*c) + 20*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{ \\
& 2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2* \\
& d*x + 7/2*c) - 12*(2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2* \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\c \\
& os(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))* \\
& \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\s \\
& in(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{ \\
& 2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
&)^2 \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \\
&)\sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx \\
&+ 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c) \sin(5/2 dx \\
&+ 5/2 c)^2 \sin(3/2 \arctan2(\sin(dx + c), \cos(dx + c))) - 84(((4\sqrt{2} * \\
&\cos(dx + c)^2 + 4\sqrt{2} \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) + 5\sqrt{2} \\
&(2)) \cos(3/2 dx + 3/2 c)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin \\
&(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (4\sqrt{2} \cos(dx + c)^2 + 4\sqrt{2} (\\
&2) \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) + 5\sqrt{2} (2)) \sin(3/2 dx + 3/2 c \\
&)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin \\
&(dx + c)^2 + 5\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + 5\sqrt{2} \sin(1/2 dx + \\
&1/2 c)^2 + 2(4\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + 4\sqrt{2} \cos \\
&(1/2 dx + 1/2 c) \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 \\
&c) + 5\sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 9(\sqrt{2} \cos \\
&(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(4\sqrt{2} \\
&\cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + 4\sqrt{2} \sin(dx + c)^2 \sin(1 \\
&/2 dx + 1/2 c) + 9\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + 5\sqrt{2} \sin \\
&(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c) \cos(7/2 dx + 7/2 c)^2 + 2((4\sqrt{2} \\
&\cos(dx + c)^2 + 4\sqrt{2} \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) + \\
&5\sqrt{2} (2)) \cos(3/2 dx + 3/2 c)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \\
&\sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (4\sqrt{2} \cos(dx + c)^2 + \\
&4\sqrt{2} \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) + 5\sqrt{2} (2)) \sin(3/2 dx \\
&+ 3/2 c)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 \\
&c)^2) \sin(dx + c)^2 + 5\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + 5\sqrt{2} \sin(1/2 \\
&>* dx + 1/2 c)^2 + 2(4\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + 4\sqrt{2} \\
&(2) \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) \cos(1/2 dx \\
&+ 1/2 c) + 5\sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 9(\sqrt{2} \\
&(2) \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + \\
&2(4\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + 4\sqrt{2} \sin(dx + c)^2 \\
&\sin(1/2 dx + 1/2 c) + 9\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + 5\sqrt{2} \\
&\sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c) \cos(7/2 dx + 7/2 c) \cos(\\
&5/2 dx + 5/2 c) + ((4\sqrt{2} \cos(dx + c)^2 + 4\sqrt{2} \sin(dx + c)^2 + \\
&9\sqrt{2} \cos(dx + c) + 5\sqrt{2} (2)) \cos(3/2 dx + 3/2 c)^2 + 4(\sqrt{2} \cos \\
&(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (4\sqrt{2} \\
&\cos(dx + c)^2 + 4\sqrt{2} \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) + \\
&5\sqrt{2} (2)) \sin(3/2 dx + 3/2 c)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \\
&\sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + 5\sqrt{2} \cos(1/2 dx + 1/2 \\
&c)^2 + 5\sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(4\sqrt{2} \cos(dx + c)^2 \cos(1 \\
&/2 dx + 1/2 c) + 4\sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 9\sqrt{2} \\
&\cos(dx + c) \cos(1/2 dx + 1/2 c) + 5\sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/ \\
&2 dx + 3/2 c) + 9(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + \\
&1/2 c)^2) \cos(dx + c) + 2(4\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \\
&4\sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 9\sqrt{2} \cos(dx + c) \sin \\
&(1/2 dx + 1/2 c) + 5\sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c) \cos \\
&(5/2 dx + 5/2 c)^2 + ((4\sqrt{2} \cos(dx + c)^2 + 4\sqrt{2} \sin(dx + c) \\
&^2 + 9\sqrt{2} \cos(dx + c) + 5\sqrt{2} (2)) \cos(3/2 dx + 3/2 c)^2 + 4(\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + \\
& c) + 5*\sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2* \\
& \cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2} \\
& \cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos \\
& \cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2 \\
& *c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c \\
&)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2* \\
& c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d \\
& *x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4* \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2})*\cos \\
& \cos(d*x + c) + 5*\sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d* \\
& x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(\\
& d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((4*\sqrt{2}*\cos(d*x \\
& + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos \\
& \cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d \\
& *x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4* \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5* \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2})*\cos \\
& \cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(1/2*\arctan2(\\
& \sin(d*x + c), \cos(d*x + c))))*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c)))^2 + 3*(420*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 42 \\
& 0*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^
\end{aligned}$$

$$\begin{aligned}
& c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2} \sin(3/2 dx + \\
& 3/2 c) \sin(dx + c) \sin(1/2 dx + 1/2 c) + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 \\
& + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c) \cos(7/2 dx + 7/2 c) - 70 * \\
& ((8\sqrt{2} \cos(1/2 dx + 1/2 c) \sin(1/2 dx + 1/2 c) - 3(\sqrt{2} \log(\cos(\\
& 1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \\
& \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx \\
& x + 1/2 c) + 1)) \cos(1/2 dx + 1/2 c)) \cos(dx + c)^2 + (8\sqrt{2} \cos(1/2 * \\
& dx + 1/2 c) \sin(1/2 dx + 1/2 c) - 3(\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \\
& \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 \\
& * dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos \\
& (1/2 dx + 1/2 c)) \sin(dx + c)^2 + 8\sqrt{2} \cos(1/2 dx + 1/2 c) \sin(1/2 * \\
& dx + 1/2 c) + 2(8\sqrt{2} \cos(1/2 dx + 1/2 c) \sin(1/2 dx + 1/2 c) - 3(\\
& \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx \\
& + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^ \\
& 2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(1/2 dx + 1/2 c)) \cos(dx + c) - 3*(sq \\
& rt(2) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + \\
& 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 \\
& - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(1/2 dx + 1/2 c) - 6(\sqrt{2} \cos(1/2 dx * \\
& x + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c) \cos(3/2 dx + \\
& 3/2 c) - 70(8\sqrt{2} \sin(1/2 dx + 1/2 c)^3 - 3(\sqrt{2} \log(\cos(1/2 dx \\
& + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \\
& * \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 * \\
& c) + 1)) \cos(1/2 dx + 1/2 c)^2 - 3(\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + s \\
& in(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx \\
& * x + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \sin(1 \\
& /2 dx + 1/2 c)^2 + 4(2\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2}) \sin(1/2 * \\
& dx + 1/2 c) \cos(dx + c) - 8((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + \\
& c)^2 + 9\sqrt{2} \cos(dx + c) + 8\sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} (\\
& 2) \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 \\
& + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 9\sqrt{2} \cos(dx + c) \\
& + 8\sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + sq \\
& rt(2) \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + 8\sqrt{2} \cos(1/2 dx + 1/2 * \\
& c)^2 + 8\sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2 \\
& * dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 9\sqrt{2} \cos \\
& (dx + c) \cos(1/2 dx + 1/2 c) + 8\sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx * \\
& x + 3/2 c) + 9(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 * \\
& c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} (\\
& 2) \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 9\sqrt{2} \cos(dx + c) \sin(1/2 dx \\
& + 1/2 c) + 8\sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c)) \sin(5/2 dx \\
& * x + 5/2 c) - 70(6(\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 \\
& + (8\sqrt{2} \sin(1/2 dx + 1/2 c)^2 - 3(\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^ \\
& 2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \log(\cos(\\
& 1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) * \\
& \sin(1/2 dx + 1/2 c) + 2\sqrt{2}) \cos(dx + c)^2 + (8\sqrt{2} \sin(1/2 dx + \\
& 1/2 c)^2 - 3(\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2
\end{aligned}$$

$$\begin{aligned}
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)
\end{aligned}$$

$$\begin{aligned}
& - 8\sqrt{2}\sin(1/2*d*x + 1/2*c))\sin(d*x + c)^2 + 12\sqrt{2}\cos(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) + 2*(3\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2})\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20\sqrt{2}) \\
& *\sin(1/2*d*x + 1/2*c))\cos(d*x + c) + 3\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2})\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 32\sqrt{2})\sin(1/2*d*x + 1/2*c))\sin(3/2*d*x + 3/2*c)^2 - (8\sqrt{2})\sin(1 \\
& /2*d*x + 1/2*c)^3 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c \\
&)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\si \\
& n(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))\sin(1/2*d*x + 1/2*c))\sin(d*x + c)^2 \\
& + 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))\sin(1/2*d*x + 1/2*c)^2 - 2*((8\sqrt{2} \\
&)\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}) * \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))\cos(1/2*d*x + 1/2*c))\cos(d*x + c)^2 + (8\sqrt{2})\cos(1/2*d*x + 1/2 \\
& *c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))\cos(1/2*d*x \\
& + 1/2*c))\sin(d*x + c)^2 + 8\sqrt{2})\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2 \\
& *c) + 2*(8\sqrt{2})\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}) * \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c))\cos(d*x + c) - 3*(\sqrt{2})\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})\cos(1/2*d*x + 1/2*c \\
&)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)\sin(d*x + c))\cos(3/2*d*x + 3/2*c) - \\
& 2*(8\sqrt{2})\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \\
& \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))\sin(1/2*d*x + 1 \\
& /2*c)^2 + 4*(2*\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))\sin(1/2*d*x + 1/2* \\
& c))\cos(d*x + c) - 2*(6*(\sqrt{2})\cos(d*x + c) + \sqrt{2}))\cos(3/2*d*x + 3/2* \\
& c)^2 + (8\sqrt{2})\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) +
\end{aligned}$$

$$\begin{aligned}
& 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \cos(d*x + c)^2 + (8*\sqrt{2}) * \sin(1/2*d \\
& *x + 1/2*c)^2 - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}) * \sin(d*x + c)^2 + 6*\sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}) * \sin \\
& (1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) - \sqrt{2}) * \sin \\
& (d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2}) * \cos(1/2*d*x + 1/2*c)) * \cos(\\
& 3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}) * \sin(1/2 \\
& *d*x + 1/2*c)^2 - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}) * \cos(d*x + c) - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}) * \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}) * \sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}) * \cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \sin(5/2*d*x + 5/2*c)^2 + 60*((\sqrt{2}) * \cos \\
& (d*x + c)^2 + \sqrt{2}) * \sin(d*x + c)^2 + 2*\sqrt{2}) * \cos(d*x + c) + \sqrt{2}) * \cos \\
& (3/2*d*x + 3/2*c)^2 + (\sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin \\
& (1/2*d*x + 1/2*c)^2 * \cos(d*x + c)^2 + (\sqrt{2}) * \cos(d*x + c)^2 + \sqrt{2}) * \sin \\
& (d*x + c)^2 + 2*\sqrt{2}) * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (s \\
& \sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c \\
&)^2 + \sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^2 + 2*(\\
& \sqrt{2}) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2}) * \cos(1/2*d*x + 1/2*c) * \\
& \sin(d*x + c)^2 + 2*\sqrt{2}) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2}) * \cos(\\
& 1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c) + 2*(\sqrt{2}) * \cos(d*x + c)^2 * \\
& \sin(1/2*d*x + 1/2*c) + \sqrt{2}) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \sin(3 \\
& /2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}) * \cos(d*x + c)^2 + \sqrt{2} \\
& (2) * \sin(d*x + c)^2 + 2*\sqrt{2}) * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c) \\
& ^2 + (\sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^2 * \cos(\\
& d*x + c)^2 + (\sqrt{2}) * \cos(d*x + c)^2 + \sqrt{2}) * \sin(d*x + c)^2 + 2*\sqrt{2}) * \cos \\
& (d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}) * \cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)^2 + \sqrt{2}) * \cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}) * \cos(d*x + c)^2 * \cos \\
& (1/2*d*x + 1/2*c) + \sqrt{2}) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2}) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2 \\
& *d*x + 3/2*c) + 2*(\sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1 \\
& /2*c)^2 * \cos(d*x + c) + 2*(\sqrt{2}) * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \cos(d*x + c) * \sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2* \\
& d*x + 7/2*c) * \cos(5/2*d*x + 5/2*c) + ((\sqrt{2}) * \cos(d*x + c)^2 + \sqrt{2}) * \sin(\\
& d*x + c)^2 + 2*\sqrt{2}) * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& (2) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c) \\
& ^2 + (\sqrt{2}) * \cos(d*x + c)^2 + \sqrt{2}) * \sin(d*x + c)^2 + 2*\sqrt{2}) * \cos(d*x +
\end{aligned}$$

$$\begin{aligned}
& *c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)* \\
& \sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))* \\
& \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
& *2)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
& (2))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2))*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2 \\
& *2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2))*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2* \\
& ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2))*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
& (2))*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d \\
& *x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2))*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c)) \\
& *\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
& (2))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& (2))*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
& (2))*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2))*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2 \\
& *d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c)))^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c) \\
&)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
& (2))*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + s
\end{aligned}$$

$$\begin{aligned}
& \text{qrt}(2) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \text{sqrt}(2) * \cos(d*x + c) * \sin(1/2 \\
& *d*x + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3/2*c) * \cos(7/2 \\
& *d*x + 7/2*c)^2 + 2 * ((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{s} \\
& \text{qrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d* \\
& x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \text{co} \\
& s(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \text{s} \\
& \text{in}(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d* \\
& *x + 1/2*c)^2) * \sin(d*x + c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \text{si} \\
& \text{n}(1/2*d*x + 1/2*c)^2 + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqr} \\
& \text{t}(2) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d \\
& *x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\text{sqrt}(\\
& 2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + \\
& 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(d*x + c)^2 * \sin \\
& (1/2*d*x + 1/2*c) + 2 * \text{sqrt}(2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \text{s} \\
& \text{in}(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c) * \cos(7/2*d*x + 7/2*c) * \cos(5/2*d*x \\
& + 5/2*c) + ((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \text{c} \\
& \text{os}(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2* \\
& c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \cos(d*x + \\
& c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3/2*d \\
& *x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2 \\
& *c)^2) * \sin(d*x + c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d* \\
& x + 1/2*c)^2 + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos \\
& (1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2 \\
& *c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\text{sqrt}(2) * \cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\text{sqrt}(\\
& 2) * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(d*x + c)^2 * \sin(1/2*d*x \\
& + 1/2*c) + 2 * \text{sqrt}(2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d \\
& *x + 1/2*c)) * \sin(3/2*d*x + 3/2*c) * \cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2) * \cos(d \\
& *x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(\\
& 3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x \\
& + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c) \\
& ^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \text{co} \\
& s(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \text{sqr} \\
& \text{t}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\text{sqrt}(2) * \text{c} \\
& \text{os}(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + \\
& c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + \\
& 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
& * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \sin(1/2*d \\
& *x + 1/2*c) + \text{sqrt}(2) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \text{sqrt}(2) * \cos(d \\
& *x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + \\
& 3/2*c) * \sin(7/2*d*x + 7/2*c)^2 + 2 * ((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d \\
& *x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqr} \\
& \text{t}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^ \\
& 2 + (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + \\
& c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqr}
\end{aligned}$$

$$\begin{aligned}
& \text{rt}(2) * \sin(1/2 * d * x + 1/2 * c)^2 * \sin(d * x + c)^2 + \text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c) \\
& ^2 + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)^2 + 2 * (\text{sqrt}(2) * \cos(d * x + c)^2 * \cos(1/2 * d * x \\
& + 1/2 * c) + \text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c) * \sin(d * x + c)^2 + 2 * \text{sqrt}(2) * \cos(d * x \\
& + c) * \cos(1/2 * d * x + 1/2 * c) + \text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)) * \cos(3/2 * d * x + 3/ \\
& 2 * c) + 2 * (\text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)^2 + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)^2) * \\
& \cos(d * x + c) + 2 * (\text{sqrt}(2) * \cos(d * x + c)^2 * \sin(1/2 * d * x + 1/2 * c) + \text{sqrt}(2) * \sin \\
& (d * x + c)^2 * \sin(1/2 * d * x + 1/2 * c) + 2 * \text{sqrt}(2) * \cos(d * x + c) * \sin(1/2 * d * x + 1/2 \\
& * c) + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)) * \sin(3/2 * d * x + 3/2 * c)) * \sin(7/2 * d * x + 7/2 \\
& * c) * \sin(5/2 * d * x + 5/2 * c) + ((\text{sqrt}(2) * \cos(d * x + c)^2 + \text{sqrt}(2) * \sin(d * x + c)^ \\
& 2 + 2 * \text{sqrt}(2) * \cos(d * x + c) + \text{sqrt}(2)) * \cos(3/2 * d * x + 3/2 * c)^2 + (\text{sqrt}(2) * \cos \\
& (1/2 * d * x + 1/2 * c)^2 + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)^2) * \cos(d * x + c)^2 + (\text{sqr} \\
& \text{t}(2) * \cos(d * x + c)^2 + \text{sqrt}(2) * \sin(d * x + c)^2 + 2 * \text{sqrt}(2) * \cos(d * x + c) + \text{sqr} \\
& \text{t}(2)) * \sin(3/2 * d * x + 3/2 * c)^2 + (\text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)^2 + \text{sqrt}(2) * \text{si} \\
& \text{n}(1/2 * d * x + 1/2 * c)^2) * \sin(d * x + c)^2 + \text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)^2 + \text{sqr} \\
& \text{t}(2) * \sin(1/2 * d * x + 1/2 * c)^2 + 2 * (\text{sqrt}(2) * \cos(d * x + c)^2 * \cos(1/2 * d * x + 1/2 * c) \\
&) + \text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c) * \sin(d * x + c)^2 + 2 * \text{sqrt}(2) * \cos(d * x + c) * \text{co} \\
& \text{s}(1/2 * d * x + 1/2 * c) + \text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)) * \cos(3/2 * d * x + 3/2 * c) + 2 \\
& * (\text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)^2 + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)^2) * \cos(d * x \\
& + c) + 2 * (\text{sqrt}(2) * \cos(d * x + c)^2 * \sin(1/2 * d * x + 1/2 * c) + \text{sqrt}(2) * \sin(d * x + c) \\
&)^2 * \sin(1/2 * d * x + 1/2 * c) + 2 * \text{sqrt}(2) * \cos(d * x + c) * \sin(1/2 * d * x + 1/2 * c) + \text{sqr} \\
& \text{t}(2) * \sin(1/2 * d * x + 1/2 * c) * \sin(3/2 * d * x + 3/2 * c)) * \sin(5/2 * d * x + 5/2 * c)^2 * \text{s} \\
& \text{in}(2/3 * \arctan2(\sin(3/2 * d * x + 3/2 * c), \cos(3/2 * d * x + 3/2 * c)))^2 + 2 * (((\text{sqrt}(2) \\
&) * \cos(d * x + c)^2 + \text{sqrt}(2) * \sin(d * x + c)^2 + 2 * \text{sqrt}(2) * \cos(d * x + c) + \text{sqrt}(2) \\
&)) * \cos(3/2 * d * x + 3/2 * c)^2 + (\text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)^2 + \text{sqrt}(2) * \sin(1 \\
& /2 * d * x + 1/2 * c)^2) * \cos(d * x + c)^2 + (\text{sqrt}(2) * \cos(d * x + c)^2 + \text{sqrt}(2) * \sin(d \\
& * x + c)^2 + 2 * \text{sqrt}(2) * \cos(d * x + c) + \text{sqrt}(2)) * \sin(3/2 * d * x + 3/2 * c)^2 + (\text{sqr} \\
& \text{t}(2) * \cos(1/2 * d * x + 1/2 * c)^2 + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)^2) * \sin(d * x + c)^ \\
& 2 + \text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)^2 + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)^2 + 2 * (\text{sq} \\
& \text{rt}(2) * \cos(d * x + c)^2 * \cos(1/2 * d * x + 1/2 * c) + \text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c) * \text{si} \\
& \text{n}(d * x + c)^2 + 2 * \text{sqrt}(2) * \cos(d * x + c) * \cos(1/2 * d * x + 1/2 * c) + \text{sqrt}(2) * \cos(1/ \\
& 2 * d * x + 1/2 * c)) * \cos(3/2 * d * x + 3/2 * c) + 2 * (\text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)^2 + \\
& \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)^2) * \cos(d * x + c) + 2 * (\text{sqrt}(2) * \cos(d * x + c)^2 * \text{si} \\
& \text{n}(1/2 * d * x + 1/2 * c) + \text{sqrt}(2) * \sin(d * x + c)^2 * \sin(1/2 * d * x + 1/2 * c) + 2 * \text{sqrt}(2) \\
&) * \cos(d * x + c) * \sin(1/2 * d * x + 1/2 * c) + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)) * \sin(3/2 \\
& * d * x + 3/2 * c)) * \cos(7/2 * d * x + 7/2 * c)^2 + 2 * ((\text{sqrt}(2) * \cos(d * x + c)^2 + \text{sqrt}(2) \\
&) * \sin(d * x + c)^2 + 2 * \text{sqrt}(2) * \cos(d * x + c) + \text{sqrt}(2)) * \cos(3/2 * d * x + 3/2 * c)^2 \\
& + (\text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)^2 + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)^2) * \cos(d * x \\
& + c)^2 + (\text{sqrt}(2) * \cos(d * x + c)^2 + \text{sqrt}(2) * \sin(d * x + c)^2 + 2 * \text{sqrt}(2) * \cos \\
& (d * x + c) + \text{sqrt}(2)) * \sin(3/2 * d * x + 3/2 * c)^2 + (\text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c) \\
& ^2 + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)^2) * \sin(d * x + c)^2 + \text{sqrt}(2) * \cos(1/2 * d * x + \\
& 1/2 * c)^2 + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 * c)^2 + 2 * (\text{sqrt}(2) * \cos(d * x + c)^2 * \cos(\\
& 1/2 * d * x + 1/2 * c) + \text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c) * \sin(d * x + c)^2 + 2 * \text{sqrt}(2) * \\
& \cos(d * x + c) * \cos(1/2 * d * x + 1/2 * c) + \text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)) * \cos(3/2 * d \\
& * x + 3/2 * c) + 2 * (\text{sqrt}(2) * \cos(1/2 * d * x + 1/2 * c)^2 + \text{sqrt}(2) * \sin(1/2 * d * x + 1/2 \\
& * c)^2) * \cos(d * x + c) + 2 * (\text{sqrt}(2) * \cos(d * x + c)^2 * \sin(1/2 * d * x + 1/2 * c) + \text{sqrt}
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c) \\
& ^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\co \\
& s(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin \\
& (5/2*d*x + 5/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/ \\
& 2*c))))*\cos(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))*\sin(2/ \\
& 5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + (84*(\sqrt{2}*\cos(3 \\
& /2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& *\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2 \\
& *d*x + 7/2*c)^3 - 84*((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
&) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d \\
& *x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^3 - 24*((\sqrt{2}*\cos \\
& (d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\co \\
& s(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
& *\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\sin(5/2*d*x + 5/2*c)^3 + 3*(420*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\si \\
& n(d*x + c) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - \\
& 280*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& *\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d* \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - 35* \\
& (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c) \\
&)*\cos(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2* \\
& c)^2 + 35*((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d* \\
& x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 56*(\sqrt{2}*\cos(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1 \\
& /2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x \\
& + 5/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2} \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x \\
& + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d \\
& *x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2} \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)) \\
& *\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& rt(2))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2} \\
& rt(2))*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2* \\
& c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& *\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2* \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3 \\
& /2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/ \\
& 2*d*x + 5/2*c) - 70*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c) \\
&)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d* \\
& x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2} \\
& (2))*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 3/2*c) + 2*(3*sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + 11*sqrt(2)*sin(1/2* \\
& d*x + 1/2*c)^2 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2* \\
& c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) + \\
& 2*sqrt(2))*cos(d*x + c) - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2* \\
& d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/ \\
& 2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x \\
& + 1/2*c) + 2*sqrt(2))*sin(3/2*d*x + 3/2*c) - 140*(2*sqrt(2)*cos(1/2*d*x + 1 \\
& /2*c)^2 + sqrt(2))*sin(1/2*d*x + 1/2*c))*cos(7/2*d*x + 7/2*c)^2 + 105*(12*s \\
& qrt(2)*cos(3/2*d*x + 3/2*c)^3*sin(d*x + c) - 12*(sqrt(2)*cos(d*x + c) + sqr \\
& t(2))*sin(3/2*d*x + 3/2*c)^3 - 8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 + ((3*sqrt(\\
& 2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - \\
& 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) \\
& ^2 + (3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin \\
& (1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x \\
& + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c)) \\
& *sin(d*x + c)^2 + 24*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 2*(3*sqrt(\\
& 2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - \\
& 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) \\
& + 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + \\
& 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*co \\
& s(3/2*d*x + 3/2*c)^2 - (8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 - 3*(sqrt(2)*log(c \\
& os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1 \\
&) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2 \\
& *d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2)*log(cos(1/2*d*x + 1 \\
& /2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*lo \\
& g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) \\
& + 1))*sin(1/2*d*x + 1/2*c)^2 + 4*(2*sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2 \\
&))*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + 3*(sqrt(2)*log(cos(1/2*d*x + 1/2* \\
& c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(c \\
& os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1 \\
&))*cos(1/2*d*x + 1/2*c)^2 + ((3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x \\
& + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(\\
& 2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + (3*sqrt(2)*log(cos(1/2*d*x + 1/2* \\
& c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log \\
& (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + \\
& 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(d*x + c)^2 + 12*sqrt(2)*cos(3/2*d \\
& *x + 3/2*c)*sin(d*x + c) + 2*(3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x \\
& + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 20*sqrt \\
& (2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&)*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \\
& \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2* \\
& c) + 2*\sqrt{2})*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (84*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)* \\
& \cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c))*\cos(7/2*d*x + 7/2*c)^3 - 84*((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
& *\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2* \\
& d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^3 - 24*((\\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
& *\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c \\
&)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin \\
& \sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^3 + 3*(420*\sqrt{2}*\cos(3/2*d*x + \\
& 3/2*c)^3*\sin(d*x + c) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*s \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 \\
& + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*s
\end{aligned}$$

$$\begin{aligned}
& \sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2 \\
& *c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d \\
& *x + 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 35*((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/ \\
& 2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) \\
& ^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 \\
& + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 56*(\sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(\\
& 1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))* \\
& \cos(5/2*d*x + 5/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/ \\
& 2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)* \\
& \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2* \\
& c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3 \\
& *(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2* \\
& d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{ \\
& 2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/ \\
& 2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/
\end{aligned}$$

$$\begin{aligned}
& 2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2} \\
& 2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2})*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(7/2*d*x + 7/2*c)^2 \\
& + 105*(12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(d*x \\
& + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 \\
& + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d* \\
& x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \\
& 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3* \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) - 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d
\end{aligned}$$

$$\begin{aligned}
& + 14\sqrt{2}\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}\cos(d*x + c)\cos(1/2*d*x \\
& + 1/2*c) - \sqrt{2}\sin(d*x + c)\sin(1/2*d*x + 1/2*c) + \sqrt{2}\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + 11*s \\
& \text{qrt}(2)\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2 \\
& *d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(c \\
& \text{os}(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*co \\
& \text{s}(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + 3*(420*\sqrt{2}\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 420*(\sqrt{2}\cos(d \\
& *x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2}\sin(1/2*d*x + 1/2*c \\
&)^3 + 35*((3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}\sin(1/2*d*x + 1/ \\
& 2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}\sin \\
& (1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c) + 2*(3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}\sin(1/2*d*x + 1/ \\
& 2*c))*\cos(d*x + c) + 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}\sin(1/ \\
& 2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}\sin(1/2*d*x + 1/2*c) \\
& ^3 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \text{in}(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 35*((3*\sqrt{2}\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 8*\sqrt{2}\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}\sin(1/2*d*x + 1/2*c))*\sin(d*x + c \\
&)^2 + 12*\sqrt{2}\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) -
\end{aligned}$$

$$\begin{aligned}
& 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2})*\cos(d*x + c) + 8*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d \\
& *x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2})*\sin \\
& n(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c)^2)*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/ \\
& 2*d*x + 1/2*c) + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^ \\
& 2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 8*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70 \\
& *(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin \\
& n(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + \\
& c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin \\
& n(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2 \\
& *(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3* \\
& (\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x \\
& + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2})*\cos(3/2*d*x \\
& + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 2 \\
& 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 \\
& - (8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c) \\
& ^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos \\
& (1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)) \\
& *cos(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d \\
& *x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2 \\
& *c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + \\
& 1/2*c)^2 + 4*(2*sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2))*sin(1/2*d*x + 1/ \\
& 2*c))*cos(d*x + c)^2 + 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x \\
& + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c) \\
& ^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/ \\
& 2*c)^2 + ((3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + \\
& 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/ \\
& 2*c))*cos(d*x + c)^2 + (3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x \\
& + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2* \\
& c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin \\
& (1/2*d*x + 1/2*c))*sin(d*x + c)^2 + 12*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(d*x \\
& + c) + 2*(3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + \\
& 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 20*sqrt(2)*sin(1/2*d*x + 1 \\
& /2*c))*cos(d*x + c) + 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + \\
& 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c) \\
& ^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 32*sqrt(2)*sin(\\
& 1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 - (8*sqrt(2)*sin(1/2*d*x + 1/2*c)^ \\
& 3 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(\\
& 1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + \\
& 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2) \\
& *log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2* \\
& c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 + 4*(2*sqrt(2)*cos(1/2*d*x \\
& + 1/2*c)^2 + sqrt(2))*sin(1/2*d*x + 1/2*c))*sin(d*x + c)^2 + 3*(sqrt(2)*lo \\
& g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) \\
& + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(\\
& 1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 2*((8*sqrt(2)*cos(1/2*d*x + \\
& 1/2*c)*sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x \\
& + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2* \\
& d*x + 1/2*c))*cos(d*x + c)^2 + (8*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x \\
& + 1/2*c) - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + \\
& 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c))*sin(d* \\
& x + c)^2 + 8*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 2*(8*sqrt(\\
& 2)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*log(cos(1/2*d*x + \\
& 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*
\end{aligned}$$

$$\begin{aligned}
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2 \\
& *c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) \\
& - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - \\
& 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(d \\
& *x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c \\
&)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos \\
& (d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& * \sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 6*(14*(\sqrt{2})*\cos(3/2*d*x \\
& + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2 \\
& *c))*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})* \\
& \sin(3/2*d*x + 3/2*c))*\sin(d*x + c))*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + \\
& 5/2*c)^2 - 8*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x \\
& + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2 \\
& *d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1
\end{aligned}$$

$$\begin{aligned}
& /2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)*\sin(5/2*d*x + 5/2 \\
& *c) + 14*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d \\
& *x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c \\
&)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2 + 35*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c \\
&)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c) \\
& ^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& *\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*(\\
& 2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/ \\
& 2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 \\
& + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \\
& 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d* \\
& x + 7/2*c)^2 + 14*((\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + \text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)* \\
& \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 10*(\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + 10*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + 2*((4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(\\
& d*x + c)^2 + 15*\text{sqrt}(2)*\cos(d*x + c) + 11*\text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + \\
& 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c)^2 + (4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 15*\text{sqrt}(2) \\
&)*\cos(d*x + c) + 11*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d* \\
& x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 11*\text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)^2 + 11*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\text{sqrt}(2)*c \\
& \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 15*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 11*\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 15*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 15*s \\
& \text{qrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 11*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)) \\
& *\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 10*(\text{sqrt}(2)*\cos(d*x + c)^2 \\
& + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + \\
& 3/2*c)^2 + 10*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c \\
&)^2)*\sin(d*x + c)^2 + 10*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 10*\text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 28*((\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2* \\
& c)^2 + (\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + \text{sqrt}(2)*co \\
& s(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x \\
& + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2 \\
& *c) + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) - (\text{sqrt}(2)*\cos(3/2 \\
& *d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt} \\
& (2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d \\
& *x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 20*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x \\
& + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2 \\
& *c) + 20*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)* \\
& \cos(d*x + c) - 35*(12*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\text{sqrt} \\
& t(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^3 - 8*\text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^3 + ((3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\
& x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3* \\
& (\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8 \\
& *\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d \\
& *x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c \\
&))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c) + 20*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2 \\
&)*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2 \\
& *d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d* \\
& x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3 \\
& /2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2})*\cos \\
& (d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d* \\
& x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2 \\
& *d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + \\
& 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c \\
&)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^ \\
& 2)*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 84*(((4*\text{sqrt}(2)*\cos(d*x + \\
& c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\cos(\\
& 3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d* \\
& x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d* \\
& x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c)^2 + 5*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\text{sqrt}(2)*\co \\
& s(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 9*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\sin(1/2*d* \\
& x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((4*\text{sqrt}(2)*\co \\
& s(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2) \\
&))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2) \\
&)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^ \\
& 2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c)^2 + 5*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c \\
&) + 5*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\text{sqr \\
& t}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + 9*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + \\
& 5/2*c) + ((4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2) \\
&)*\cos(d*x + c) + 5*\text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\text{sqrt}(2)*\co \\
& s(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2)
\end{aligned}$$

$$\begin{aligned}
&^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \cos(dx + c) + 2(4\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + 4\sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 9\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + 5\sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)) \sin(5/2 dx + 5/2 c)^2 \sin(1/2 \arctan 2(\sin(dx + c), \cos(dx + c))) \cos(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 3(420\sqrt{2} \cos(3/2 dx + 3/2 c)^3 \sin(dx + c) - 420(\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^3 - 280\sqrt{2} \sin(1/2 dx + 1/2 c)^3 + 35((3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(dx + c)^2 + (3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(dx + c)^2 + 24\sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c) + 2(3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(dx + c) + 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c)^2 - 35(8\sqrt{2} \sin(1/2 dx + 1/2 c)^3 - 3(\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(1/2 dx + 1/2 c)^2 - 3(\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \sin(1/2 dx + 1/2 c)^2 + 4(2\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2}) \sin(1/2 dx + 1/2 c) \cos(dx + c)^2 + 105(\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(1/2 dx + 1/2 c)^2 + 35((3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(dx + c)^2 + (3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(dx + c)^2 + 12\sqrt{2} \cos(3/2 dx + 3/2 c) \sin(dx + c) + 2(3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 20\sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(dx + c) + 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 32\sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)^2 - 35(8\sqrt{2} \sin(1/2 dx + 1/2 c)^3 - 3(\sqrt{2} \log(c
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + \\
& c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) - 70*((8*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x \\
& + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2 \\
& *c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - \\
& 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + \\
& 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2 \\
& *c))*\cos(d*x + c) - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9 \\
& *\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2} \\
& (2))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 8* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& * \cos(1/2*d*x + 1/2*c) + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
&) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d* \\
& x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2* \\
& c) - 70*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2} \\
& t(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
& - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin \\
& (d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2 \\
& *c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))* \\
& \cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& t(2))*\sin(1/2*d*x + 1/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2}*\cos(3 \\
& /2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/ \\
& 2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2} \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c \\
&)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2} \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d \\
& *x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d \\
& *x + 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d \\
& *x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)* \\
& \sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2})*\sin(1/2* \\
& d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\c \\
& \cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1 \\
& /2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)) \\
& *\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(\\
& 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \s \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d \\
& *x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d \\
& *x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + \\
& 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 \\
&) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin \\
& (d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7 \\
& /2*c) \cos(5/2*d*x + 5/2*c) + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c \\
&)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2} \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (s \\
& \sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + s \\
& \sqrt{2} \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin \\
& (1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + s \\
& \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos \\
& (1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + \\
& 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d* \\
& x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + \\
& c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(5/2*d*x + 5/2*c)^2 \\
& + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c \\
&) + \sqrt{2} \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin \\
& (1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin \\
& (d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2} \sin(3/2*d*x + 3/2*c \\
&)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin \\
& (d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + \\
& 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x \\
& + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c \\
&)) \sin(3/2*d*x + 3/2*c)) \sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2} \cos(d*x + c)^2 \\
& + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2} \cos(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c) \\
& ^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*s \\
& \sqrt{2} \cos(d*x + c) + \sqrt{2} \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + \\
& c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + \\
& 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \\
& \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2 \\
& *d*x + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2 \\
& *c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) *
\end{aligned}$$

$$\begin{aligned}
& \sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + (84*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^3 - 84*((\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^3 - 24*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^3 + 3*(420*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + \\
& 24\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2} \\
& (2)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& ^2 - 35*(8\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 35*((3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 56*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) +
\end{aligned}$$

$$\begin{aligned}
& 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos \\
& (5/2*d*x + 5/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c \\
&) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^ \\
& 2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c \\
&))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) \\
& - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d \\
& *x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sqrt{2}*\cos(d*x + c \\
&)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d* \\
& x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2 \\
& *\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c \\
&)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&)*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\
&2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
&) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c) \\
&))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})* \\
&\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
&x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
&c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + \\
&1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
&\sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2* \\
&d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
&1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2})*\cos(1/2 \\
&*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 1 \\
&05*(12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(d*x + \\
&c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + (\\
&(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
&d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
&2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(\\
&d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
&+ 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
&(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + \\
&1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2* \\
&(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
&d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
&2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(\\
&d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
&2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
&2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/ \\
&2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})* \\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
&2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
&*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2 \\
&*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
&1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
&+ \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d* \\
&x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
&2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
&+ \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(\\
&1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
&8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d* \\
&x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})* \\
&\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
&1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2})*\cos
\end{aligned}$$

$$\begin{aligned}
& \cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*(\\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2* \\
& d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin \\
& (d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3 \\
& *(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x \\
& + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2 \\
& *d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& c) + 1) - 3\sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2} \sin(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 \\
& + 12\sqrt{2} \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + 2*(3\sqrt{2} \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2} \\
& \sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 20\sqrt{2} \sin(1/2*d*x + 1/2*c) * \cos(d*x + c) + 3\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 3\sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32\sqrt{2} \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3 \\
& /2*c)^2 - 35*(8\sqrt{2} \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(\\
& 1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2 \\
& *d*x + 1/2*c) * \sin(d*x + c)^2 + 105*(\sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1 \\
& /2*d*x + 1/2*c)^2 + 28*(\sqrt{2} \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} \\
& \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2} * \sin(3 \\
& /2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} \sin(3/2*d*x + 3/2*c) * \sin(d*x + c \\
&) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2* \\
& d*x + 1/2*c)^2) * \sin(d*x + c) * \cos(7/2*d*x + 7/2*c) - 70*((8\sqrt{2} \cos(1/2 \\
& *d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) * \cos(d*x + c)^2 + (8\sqrt{2} \cos(1/2*d*x + 1/2*c) * \sin(1/ \\
& 2*d*x + 1/2*c) - 3*(\sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) * \\
& \sin(d*x + c)^2 + 8\sqrt{2} \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) + 2*(8 \\
& * \sqrt{2} \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) * \cos(d*x + c) - 3*(\sqrt{2} \log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c) * \cos(3/2*d*x + 3/2*c) - 70*(8\sqrt{2} \\
& \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d* \\
& x + 1/2*c)^2 - 3*(\sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + \\
& 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d \\
& *x + c) - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 9*\sqrt{2}*c \\
& \cos(d*x + c) + 8*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2}))*\sin(3 \\
& /2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2}*si \\
& n(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d \\
& *x + 1/2*c) + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*s \\
& in(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 8*\sqrt{2} \\
& *(\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6 \\
& *(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1 \\
& /2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
& *(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c) \\
& ^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + \\
& 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1 \\
& /2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3 \\
& *\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(sq \\
& rt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + \\
& c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))* \\
& \sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin \\
& (1/2*d*x + 1/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2})*\cos(3/2*d*x + 3 \\
& /2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/ \\
& 2*c)^3 - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*s \\
& qrt(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*
\end{aligned}$$

$$\begin{aligned}
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/ \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - \\
& (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\co \\
& s(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c) \\
&))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) \\
&)^2 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1 \\
& /2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + \\
& c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/ \\
& 2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) \\
& ^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - \\
& 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1 \\
& /2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x \\
& + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 12*(\sqrt{2})*\cos(d*x + c))*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c))*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3 \\
& *(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d* \\
& x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin \\
& (1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 6*(14*(\sqrt{2})*\cos(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c))*\cos(1/2*d*x + 1/2*c) \\
& *\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin \\
& (3/2*d*x + 3/2*c))*\sin(d*x + c))*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/ \\
& 2*c)^2 - 8*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos \\
& (d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c) \\
&)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*
\end{aligned}$$

$$\begin{aligned}
& x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2) \\
&)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d* \\
& x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)*\sin(5/2*d*x + 5/2*c) \\
& + 14*(\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x \\
& + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c) + 2*\text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1 \\
& /2*c) + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\text{s} \\
& \text{in}(d*x + c))*\sin(5/2*d*x + 5/2*c)^2 + 35*(12*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^3 \\
& *\sin(d*x + c) - 12*(\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^3 \\
& - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^3 + ((3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\text{sqrt}(2)*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{s} \\
& \text{qrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\text{sqrt}(2)*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(\\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\text{s} \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d \\
& *x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \text{s} \\
& \text{in}(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& + 4*(2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c)^2 + 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + (\\
& (3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c)^2 + (3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \text{s} \\
& \text{in}(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& (1/2*c)) * \sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2* \\
& (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2* \\
& c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + \\
& 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\co \\
& s(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3* \\
& (\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2} \\
& * \cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) +
\end{aligned}$$

$$\begin{aligned}
& 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 \\
& + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12 \\
& *(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin \\
& (3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2* \\
& d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 84*((\sqrt{2})*\cos \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d \\
& *x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2* \\
& \sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7 \\
& /2*d*x + 7/2*c)^2 + 2*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/ \\
& 2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& *\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d \\
& *x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + (s \\
& \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c) \\
& *\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c))*\cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2})*\cos \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2 \\
& *\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(\\
& 7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + (\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin \\
& (d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
&) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3 \\
& /2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(\\
& 1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 6*(14*((\sqrt{2})*\cos(d*x + c) + s \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d
\end{aligned}$$

$$\begin{aligned}
& *x + 3/2*c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\text{s} \\
& \text{in}(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + \\
& 7/2*c)^2 + 14*((\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\\
& \text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + \text{sqrt}(2)*\cos(1/2*d* \\
& x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)*\cos \\
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\text{s} \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&) + 2*(\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/ \\
& 2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 10*(\text{sqrt}(2)*\cos(d*x + \\
& c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d \\
& *x + 3/2*c)^2 + 10*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c)^2 + 2*((4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 15*\text{sqrt}(2)*\cos(d*x + c) + 11*\text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + 4* \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 15*\text{sqrt}(2)*\text{c} \\
& \text{os}(d*x + c) + 11*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 11*\text{sqrt}(2)*\text{c} \\
& \text{os}(1/2*d*x + 1/2*c)^2 + 11*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\text{sqrt}(2)*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 15*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 11*\text{sqrt}(2)*\cos(1/2*d* \\
& x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 15*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqr} \\
& \text{t}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 15*\text{sqrt} \\
& (2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 11*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\text{s} \\
& \text{in}(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 10*(\text{sqrt}(2)*\cos(d*x + c)^2 + \text{s} \\
& \text{qrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2 \\
& *c)^2 + 10*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\sin(d*x + c)^2 + 10*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 10*\text{sqrt}(2)*\sin(1/2*d \\
& *x + 1/2*c)^2 + 28*((\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^ \\
& 2 + (\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + \text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c \\
&)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) - (\text{sqrt}(2)*\cos(3/2*d* \\
& x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/ \\
& 2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2) \\
& *\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x \\
& + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 20*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c \\
&)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)
\end{aligned}$$

$$\begin{aligned}
& + 20*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) - 35*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^3 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^ \\
& 3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + \\
& 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2* \\
& c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\
& x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + \\
& 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + \\
& 1/2*c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x \\
& + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d* \\
& x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))* \\
& \cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
& *\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 \\
& + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c))*\cos \\
& (1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*c \\
& \cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2* \\
& d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin \\
& (d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& \cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3 \\
& * (\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x \\
& + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2 \\
& *d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& qrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 1 \\
& 4*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c))*\cos(1/2*d*x + 1 \\
& /2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\
& x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c))*\co \\
& s(7/2*d*x + 7/2*c) + 2*(84*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2 \\
& *\sqrt{2}*\cos(3/2*d*x + 3/2*c))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\s \\
& in(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^3 - 84*((\sqrt{2}*\cos \\
& (d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c))*\cos(1/2*d*x + 1/2*c) + \sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)* \\
& \sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))* \\
& \sin(7/2*d*x + 7/2*c)^3 - 24*((\sqrt{2}*\cos(d*x + c))^2 + \sqrt{2}*\sin(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 56*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*c
\end{aligned}$$

$$\begin{aligned}
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(\\
& 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2* \\
& c))*\sin(d*x + c)^2 + 12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x \\
& + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2})*\sin(1/2*d*x + 1/2*c) \\
&))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2* \\
& *d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2 \\
& *d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d \\
& *x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{ \\
& 2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sq \\
& rt(2))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*s \\
& qrt(2)*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{ \\
& t(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2* \\
& d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + \\
& 1/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 3*(420*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin \\
& (d*x + c) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 2 \\
& 80*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3* \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1
\end{aligned}$$

$$\begin{aligned}
&) - 8\sqrt{2}\sin(1/2*d*x + 1/2*c))\cos(d*x + c) + 3\sqrt{2}\log(\cos(1/2*d*x \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2} \\
& t(2)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8\sqrt{2}\sin(1/2*d*x + 1/2*c))\cos(3/2*d*x + 3/2*c)^2 - 35*(\\
& 8\sqrt{2}\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(\\
& 1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\sin(1/2*d*x + 1/2* \\
& c)^2 + 4*(2\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)) \\
& *\cos(d*x + c)^2 + 105*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c \\
&)^2 + 35*((3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}\sin(1/2*d*x + 1/ \\
& 2*c))\cos(d*x + c)^2 + (3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}\sin \\
& (1/2*d*x + 1/2*c))\sin(d*x + c)^2 + 12\sqrt{2}\cos(3/2*d*x + 3/2*c)\sin(d*x \\
& + c) + 2*(3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 20\sqrt{2}\sin(1/2*d*x + 1/ \\
& 2*c))\cos(d*x + c) + 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 32\sqrt{2}\sin(\\
& 1/2*d*x + 1/2*c))\sin(3/2*d*x + 3/2*c)^2 - 35*(8\sqrt{2}\sin(1/2*d*x + 1/2* \\
& c)^3 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
&)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2\sin(1/2*d*x + 1/2*c) + 1))\sin(1/2*d*x + 1/2*c)^2 + 4*(2\sqrt{2}\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c))\sin(d*x + c)^2 + 105*(\sqrt{2} \\
&)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))\sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2}\cos(3/2*d* \\
& x + 3/2*c)^2*\sin(d*x + c) + 2\sqrt{2}\cos(3/2*d*x + 3/2*c)\cos(1/2*d*x + 1/ \\
& 2*c)\sin(d*x + c) + \sqrt{2}\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2\sqrt{2} \\
& *\sin(3/2*d*x + 3/2*c)\sin(d*x + c)\sin(1/2*d*x + 1/2*c) + (\sqrt{2}\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\sin(d*x + c))\cos(7/2*d*x \\
& + 7/2*c) - 70*((8\sqrt{2}\cos(1/2*d*x + 1/2*c)\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\
&)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 8*((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) + 8*\sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 9*\sqrt{2}) * \cos(d*x + c) + 8*\sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 9*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 9*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \cos(d*x + c)^2 + (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \sin(d*x + c)^2 + 6*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) - \sqrt{2} * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2} *
\end{aligned}$$

$$\begin{aligned}
& \text{rt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& \quad 1/2*c) + 1) - 32*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8 \\
& * \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1 \\
& /2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c \\
&)^2 + 4*(2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))* \\
& \sin(d*x + c)^2 + 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*((8*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\text{sqrt}(2)*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin \\
& (1/2*d*x + 1/2*c) + 2*(8*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) \\
& - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - \\
& 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d \\
& *x + 3/2*c) - 2*(8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\text{sqrt}(2)*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqr \\
& t}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (1/2*d*x + 1/2*c)^2 + 4*(2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\cos(d*x + c)^2 + (8*\text{sqr \\
& t}(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c) + 2*\text{sqrt}(2))*\sin(d*x + c)^2 + 6*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + 14*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 12*(\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) - \text{sqrt}(2)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 11*\text{s} \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2 \\
& *d*x + 1/2*c) + 2*\text{sqrt}(2))*\cos(d*x + c) - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(c \\
& \text{os}(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c) - 4*(2*\text{sqrt}(2)*\text{co} \\
& \text{s}(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^ \\
& 2 + 6*(14*(\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\cos(3/2* \\
& d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2* \\
& c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 - 8*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt} \\
& (2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\text{c} \\
& \text{os}(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2* \\
& c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\text{co} \\
& \text{s}(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2) \\
&)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2 \\
& *d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{s} \\
& \text{qrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2* \\
& d*x + 5/2*c)*\sin(5/2*d*x + 5/2*c) + 14*(\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^2*\sin(\\
& d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\sin(3/2*d*x + 3/ \\
& 2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2 + 35*(\\
& 12*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^3 - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^3 + ((3*\text{s} \\
& \text{qrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x \\
& + c)^2 + (3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2 \\
& *c))*\sin(d*x + c)^2 + 24*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\text{s} \\
& \text{qrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x \\
& + c) + 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\text{si}
\end{aligned}$$

$$\begin{aligned}
& n(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d* \\
& x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c) \\
&)*cos(3/2*d*x + 3/2*c)^2 - (8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 - 3*(sqrt(2)*l \\
& og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) \\
& + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin \\
& (1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2)*log(cos(1/2*d*x \\
& + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2) \\
&)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2 \\
& *c) + 1))*sin(1/2*d*x + 1/2*c)^2 + 4*(2*sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sq \\
& rt(2))*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + 3*(sqrt(2)*log(cos(1/2*d*x + \\
& 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*l \\
& og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) \\
& + 1))*cos(1/2*d*x + 1/2*c)^2 + ((3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + si \\
& n(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2* \\
& d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*s \\
& qrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + (3*sqrt(2)*log(cos(1/2*d*x + \\
& 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2) \\
& *log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2* \\
& c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(d*x + c)^2 + 12*sqrt(2)*cos(3 \\
& /2*d*x + 3/2*c)*sin(d*x + c) + 2*(3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + si \\
& n(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2* \\
& d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 20* \\
& sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 3*sqrt(2)*log(cos(1/2*d*x + 1/ \\
& 2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*l \\
& og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) \\
& + 1) - 32*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 - (8*sqrt(2) \\
&)*sin(1/2*d*x + 1/2*c)^3 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2* \\
& d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/ \\
& 2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x \\
& + 1/2*c)^2 - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 \\
& + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 + 4 \\
& *(2*sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2))*sin(1/2*d*x + 1/2*c))*sin(d*x \\
& + c)^2 + 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + \\
& 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2* \\
& d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 2*((\\
& 8*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*log(cos(1/ \\
& 2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - s \\
& qrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x \\
& + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + (8*sqrt(2)*cos(1/2*d* \\
& x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d \\
& *x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1 \\
& /2*d*x + 1/2*c))*sin(d*x + c)^2 + 8*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(1/2*d* \\
& x + 1/2*c) + 2*(8*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 3*(sq
\end{aligned}$$

$$\begin{aligned}
& \text{rt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& \quad 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& \quad - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\text{sqrt} \\
& \quad (2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& \quad /2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& \quad 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\text{sqrt}(2) * \cos(1/2*d*x \\
& \quad + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(3/2*d*x + 3/ \\
& \quad 2*c) - 2*(8*\text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^3 - 3*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1 \\
& \quad /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log \\
& \quad (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& \quad + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& \quad 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2*d*x \\
& \quad + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2* \\
& \quad d*x + 1/2*c)^2 + 4*(2*\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)) * \sin(1/2*d*x \\
& \quad + 1/2*c)) * \cos(d*x + c) - 2*(6*(\text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x \\
& \quad + 3/2*c)^2 + (8*\text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2) * \log(\cos(1/2*d* \\
& \quad x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(\\
& \quad 2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& \quad 2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)) * \cos(d*x + c)^2 + (8*\text{sqrt}(2) * \sin \\
& \quad (1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& \quad + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c) \\
& \quad ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/ \\
& \quad 2*c) + 2*\text{sqrt}(2)) * \sin(d*x + c)^2 + 6*\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + 14*\text{sqrt} \\
& \quad (2) * \sin(1/2*d*x + 1/2*c)^2 + 12*(\text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c \\
& \quad) - \text{sqrt}(2) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c \\
& \quad)) * \cos(3/2*d*x + 3/2*c) + 2*(3*\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + 11*\text{sqrt}(2) * \\
& \quad \sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& \quad x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2* \\
& \quad c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + \\
& \quad 1/2*c) + 2*\text{sqrt}(2)) * \cos(d*x + c) - 3*(\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \quad \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2* \\
& \quad d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(\\
& \quad 1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c) - 4*(2*\text{sqrt}(2) * \cos(1/2*d \\
& \quad *x + 1/2*c)^2 + \text{sqrt}(2)) * \sin(1/2*d*x + 1/2*c)) * \cos(5/2*d*x + 5/2*c)) * \cos(7/ \\
& \quad 2*d*x + 7/2*c) + 84*((\text{sqrt}(2) * \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\text{sqrt}(\\
& \quad 2) * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \text{sqrt}(2) * \sin(3/2 \\
& \quad *d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\text{sqrt}(2) * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \\
& \quad \sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d* \\
& \quad x + 1/2*c)^2) * \sin(d*x + c)) * \cos(7/2*d*x + 7/2*c)^2 + 2*(\text{sqrt}(2) * \cos(3/2*d*x \\
& \quad + 3/2*c)^2 * \sin(d*x + c) + 2*\text{sqrt}(2) * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2 \\
& \quad *c) * \sin(d*x + c) + \text{sqrt}(2) * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\text{sqrt}(2) * \\
& \quad \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2) * \cos(1/2*d \\
& \quad *x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(7/2*d*x + \\
& \quad 7/2*c) * \cos(5/2*d*x + 5/2*c) + (\text{sqrt}(2) * \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) \\
& \quad + 2*\text{sqrt}(2) * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \text{sqrt}(
\end{aligned}$$

$$\begin{aligned}
& 2) * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2}*\cos \\
& (3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d \\
& *x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2* \\
& \sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)) * \sin(7 \\
& /2*d*x + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} \\
& (2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/ \\
& 2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& * \sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2 * \sin(d*x + c)) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5/2*c) + (s \\
& \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c) \\
& * \cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2 * \sin(d*x \\
& + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + \\
& c)) * \sin(5/2*d*x + 5/2*c)^2 * \cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - \\
& 6*(14*((\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c) + 2*(s \\
& \sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\si \\
& n(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 14*((\sqrt{2})*\cos(d*x + c) + sq \\
& rt(2))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d* \\
& x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&)^2 + 2*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\si \\
& n(1/2*d*x + 1/2*c)^2 * \cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + \\
& 5/2*c)^2 + 10*(\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 10*(\sqrt{2})*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c)^2 + 2*((4*\sqrt{2})*\cos \\
& (d*x + c)^2 + 4*\sqrt{2})*\sin(d*x + c)^2 + 15*\sqrt{2})*\cos(d*x + c) + 11*\sqrt{2} \\
& (2))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c)^2 + (4*\sqrt{2})*\cos(d*x + c)^2 + 4*\sqrt{2} \\
& (2))*\sin(d*x + c)^2 + 15*\sqrt{2})*\cos(d*x + c) + 11*\sqrt{2}))*\sin(3/2*d*x + 3/ \\
& 2*c)^2 + 4*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 \\
&) * \sin(d*x + c)^2 + 11*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(4*\sqrt{2})*\cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + 4*\sqrt{2} \\
& (2))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 15*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + 11*\sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 15*(\sqrt{2} \\
& (2))*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c) \\
& + 2*(4*\sqrt{2})*\cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 4*\sqrt{2})*\sin(d*x + c) \\
& ^2 * \sin(1/2*d*x + 1/2*c) + 15*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 11
\end{aligned}$$

$$\begin{aligned}
& * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c) * \sin(3/2 * d * x + 3/2 * c) * \sin(5/2 * d * x + 5/2 * c)^2 \\
& + 10 * (\sqrt{2} * \cos(d * x + c)^2 + \sqrt{2} * \sin(d * x + c)^2 + 2 * \sqrt{2} * \cos(d * x \\
& + c) + \sqrt{2}) * \sin(3/2 * d * x + 3/2 * c)^2 + 10 * (\sqrt{2} * \cos(1/2 * d * x + 1/2 * c)^2 \\
& + \sqrt{2} * \sin(1/2 * d * x + 1/2 * c)^2) * \sin(d * x + c)^2 + 10 * \sqrt{2} * \cos(1/2 * d * x \\
& + 1/2 * c)^2 + 10 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c)^2 + 28 * (((\sqrt{2} * \cos(d * x + c) \\
& + \sqrt{2}) * \cos(3/2 * d * x + 3/2 * c)^2 + (\sqrt{2} * \cos(d * x + c) + \sqrt{2}) * \sin(3 \\
& /2 * d * x + 3/2 * c)^2 + \sqrt{2} * \cos(1/2 * d * x + 1/2 * c)^2 + \sqrt{2} * \sin(1/2 * d * x + \\
& 1/2 * c)^2 + 2 * (\sqrt{2} * \cos(d * x + c) * \cos(1/2 * d * x + 1/2 * c) + \sqrt{2} * \cos(1/2 * d \\
& * x + 1/2 * c)) * \cos(3/2 * d * x + 3/2 * c) + (\sqrt{2} * \cos(1/2 * d * x + 1/2 * c)^2 + \sqrt{2} * \\
& \sin(1/2 * d * x + 1/2 * c)^2) * \cos(d * x + c) + 2 * (\sqrt{2} * \cos(d * x + c) * \sin(1/2 * d \\
& * x + 1/2 * c) + \sqrt{2} * \sin(1/2 * d * x + 1/2 * c)) * \sin(3/2 * d * x + 3/2 * c) * \cos(5/2 * d \\
& * x + 5/2 * c) - (\sqrt{2} * \cos(3/2 * d * x + 3/2 * c)^2 * \sin(d * x + c) + 2 * \sqrt{2} * \cos(\\
& 3/2 * d * x + 3/2 * c) * \cos(1/2 * d * x + 1/2 * c) * \sin(d * x + c) + \sqrt{2} * \sin(3/2 * d * x + \\
& 3/2 * c)^2 * \sin(d * x + c) + 2 * \sqrt{2} * \sin(3/2 * d * x + 3/2 * c) * \sin(d * x + c) * \sin(1/2 \\
& * d * x + 1/2 * c) + (\sqrt{2} * \cos(1/2 * d * x + 1/2 * c)^2 + \sqrt{2} * \sin(1/2 * d * x + 1/2 \\
& * c)^2) * \sin(d * x + c) * \sin(5/2 * d * x + 5/2 * c) * \cos(7/2 * d * x + 7/2 * c) + 20 * (\sqrt{2} * \\
& \cos(d * x + c)^2 * \cos(1/2 * d * x + 1/2 * c) + \sqrt{2} * \cos(1/2 * d * x + 1/2 * c) * \sin(d \\
& * x + c)^2 + 2 * \sqrt{2} * \cos(d * x + c) * \cos(1/2 * d * x + 1/2 * c) + \sqrt{2} * \cos(1/2 * d \\
& * x + 1/2 * c)) * \cos(3/2 * d * x + 3/2 * c) + 20 * (\sqrt{2} * \cos(1/2 * d * x + 1/2 * c)^2 + \sqrt{2} * \\
& \sin(1/2 * d * x + 1/2 * c)^2) * \cos(d * x + c) - 35 * (12 * \sqrt{2} * \cos(3/2 * d * x + 3 \\
& /2 * c)^3 * \sin(d * x + c) - 12 * (\sqrt{2} * \cos(d * x + c) + \sqrt{2}) * \sin(3/2 * d * x + 3/ \\
& 2 * c)^3 - 8 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c)^3 + ((3 * \sqrt{2} * \log(\cos(1/2 * d * x + 1 \\
& /2 * c)^2 + \sin(1/2 * d * x + 1/2 * c)^2 + 2 * \sin(1/2 * d * x + 1/2 * c) + 1) - 3 * \sqrt{2} * \\
& \log(\cos(1/2 * d * x + 1/2 * c)^2 + \sin(1/2 * d * x + 1/2 * c)^2 - 2 * \sin(1/2 * d * x + 1/2 * c \\
&) + 1) - 8 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c)) * \cos(d * x + c)^2 + (3 * \sqrt{2} * \log(\cos \\
& (1/2 * d * x + 1/2 * c)^2 + \sin(1/2 * d * x + 1/2 * c)^2 + 2 * \sin(1/2 * d * x + 1/2 * c) + 1) \\
& - 3 * \sqrt{2} * \log(\cos(1/2 * d * x + 1/2 * c)^2 + \sin(1/2 * d * x + 1/2 * c)^2 - 2 * \sin(1/ \\
& 2 * d * x + 1/2 * c) + 1) - 8 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c)) * \sin(d * x + c)^2 + 24 * \sqrt{2} * \\
& \cos(1/2 * d * x + 1/2 * c) * \sin(d * x + c) + 2 * (3 * \sqrt{2} * \log(\cos(1/2 * d * x + 1 \\
& /2 * c)^2 + \sin(1/2 * d * x + 1/2 * c)^2 + 2 * \sin(1/2 * d * x + 1/2 * c) + 1) - 3 * \sqrt{2} * \\
& \log(\cos(1/2 * d * x + 1/2 * c)^2 + \sin(1/2 * d * x + 1/2 * c)^2 - 2 * \sin(1/2 * d * x + 1/2 * c \\
&) + 1) - 8 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c)) * \cos(d * x + c) + 3 * \sqrt{2} * \log(\cos(1 \\
& /2 * d * x + 1/2 * c)^2 + \sin(1/2 * d * x + 1/2 * c)^2 + 2 * \sin(1/2 * d * x + 1/2 * c) + 1) - \\
& 3 * \sqrt{2} * \log(\cos(1/2 * d * x + 1/2 * c)^2 + \sin(1/2 * d * x + 1/2 * c)^2 - 2 * \sin(1/2 * d \\
& * x + 1/2 * c) + 1) - 8 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c)) * \cos(3/2 * d * x + 3/2 * c)^2 - \\
& (8 * \sqrt{2} * \sin(1/2 * d * x + 1/2 * c)^3 - 3 * (\sqrt{2} * \log(\cos(1/2 * d * x + 1/2 * c)^2 \\
& + \sin(1/2 * d * x + 1/2 * c)^2 + 2 * \sin(1/2 * d * x + 1/2 * c) + 1) - \sqrt{2} * \log(\cos(1/ \\
& 2 * d * x + 1/2 * c)^2 + \sin(1/2 * d * x + 1/2 * c)^2 - 2 * \sin(1/2 * d * x + 1/2 * c) + 1)) * \cos \\
& (1/2 * d * x + 1/2 * c)^2 - 3 * (\sqrt{2} * \log(\cos(1/2 * d * x + 1/2 * c)^2 + \sin(1/2 * d * x \\
& + 1/2 * c)^2 + 2 * \sin(1/2 * d * x + 1/2 * c) + 1) - \sqrt{2} * \log(\cos(1/2 * d * x + 1/2 * c) \\
& ^2 + \sin(1/2 * d * x + 1/2 * c)^2 - 2 * \sin(1/2 * d * x + 1/2 * c) + 1)) * \sin(1/2 * d * x + 1/ \\
& 2 * c)^2 + 4 * (2 * \sqrt{2} * \cos(1/2 * d * x + 1/2 * c)^2 + \sqrt{2}) * \sin(1/2 * d * x + 1/2 * c \\
&)) * \cos(d * x + c)^2 + 3 * (\sqrt{2} * \log(\cos(1/2 * d * x + 1/2 * c)^2 + \sin(1/2 * d * x + 1 \\
& /2 * c)^2 + 2 * \sin(1/2 * d * x + 1/2 * c) + 1) - \sqrt{2} * \log(\cos(1/2 * d * x + 1/2 * c)^2 \\
& + \sin(1/2 * d * x + 1/2 * c)^2 - 2 * \sin(1/2 * d * x + 1/2 * c) + 1)) * \cos(1/2 * d * x + 1/2 * c
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - \\
& 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin \\
& \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1 \\
& /2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x \\
& + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin \\
& \sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3 \\
& *(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d* \\
& x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin \\
& \sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c) + 20*(\sqrt{2})*\cos(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})* \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d \\
& *x + 3/2*c))*\sin(7/2*d*x + 7/2*c) - 12*(2*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos \\
& (d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 2*\sqrt{2})* \\
& \cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d \\
& *x + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2))*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d* \\
& x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d* \\
& x + 5/2*c)^2 + 5*(\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2))*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + 5*(\sqrt{2})*\cos \\
& s(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin \\
& \sin(3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2} \\
& (2))*\sin(1/2*d*x + 1/2*c)^2 + 10*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c \\
&) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 1 \\
& 0*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c) + 10*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - \\
& 630*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/ \\
& 2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)* \\
& \sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(\\
& d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + \\
& c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c \\
&))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + s \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*c \\
& \cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 \\
& *\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*sqr \\
& t(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(\\
& 3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5 \\
& /2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2* \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c) \\
& ^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *2*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2 \\
& *((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) \\
& + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt} \\
& (2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c) \\
& ^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(\\
& d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(\\
& 2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
&)*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(\\
& 2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(\\
& 2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(\\
& d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sq} \\
& rt(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c) \\
& ^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(s \\
& qrt(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(\\
& 2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/ \\
& 2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2) \\
& *\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(\\
& d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d* \\
& x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(\\
& 2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x \\
& + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(\\
& 2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 84*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^2 + 9\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + 5\sqrt{2}\cos(1/2d \\
& *x + 1/2c))\cos(3/2dx + 3/2c) + 9*(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{ \\
& t(2)\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2*(4\sqrt{2}\cos(dx + c)^2\sin \\
& (1/2dx + 1/2c) + 4\sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 9\sqrt{2} \\
& (2)\cos(dx + c)\sin(1/2dx + 1/2c) + 5\sqrt{2}\sin(1/2dx + 1/2c))\sin(\\
& 3/2dx + 3/2c))\cos(7/2dx + 7/2c)\cos(5/2dx + 5/2c) + ((4\sqrt{2})\cos \\
& os(dx + c)^2 + 4\sqrt{2}\sin(dx + c)^2 + 9\sqrt{2}\cos(dx + c) + 5\sqrt{2} \\
& (2))\cos(3/2dx + 3/2c)^2 + 4*(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin \\
& n(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (4\sqrt{2}\cos(dx + c)^2 + 4\sqrt{2} \\
&)\sin(dx + c)^2 + 9\sqrt{2}\cos(dx + c) + 5\sqrt{2}))\sin(3/2dx + 3/2c) \\
& ^2 + 4*(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin \\
& n(dx + c)^2 + 5\sqrt{2}\cos(1/2dx + 1/2c)^2 + 5\sqrt{2}\sin(1/2dx + 1 \\
& /2c)^2 + 2*(4\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + 4\sqrt{2}\cos(\\
& 1/2dx + 1/2c)\sin(dx + c)^2 + 9\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2 \\
& c) + 5\sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 9*(\sqrt{2}\cos(\\
& 1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2*(4\sqrt{ \\
& rt(2)\cos(dx + c)^2\sin(1/2dx + 1/2c) + 4\sqrt{2}\sin(dx + c)^2\sin(1/ \\
& 2dx + 1/2c) + 9\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + 5\sqrt{2}\sin \\
& n(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(5/2dx + 5/2c)^2 + ((4\sqrt{2} \\
& (2)\cos(dx + c)^2 + 4\sqrt{2}\sin(dx + c)^2 + 9\sqrt{2}\cos(dx + c) + 5 \\
& sqrt(2))\cos(3/2dx + 3/2c)^2 + 4*(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
& (2)\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (4\sqrt{2}\cos(dx + c)^2 + 4\sqrt{ \\
& rt(2)\sin(dx + c)^2 + 9\sqrt{2}\cos(dx + c) + 5\sqrt{2}))\sin(3/2dx + 3 \\
& /2c)^2 + 4*(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^ \\
& 2)\sin(dx + c)^2 + 5\sqrt{2}\cos(1/2dx + 1/2c)^2 + 5\sqrt{2}\sin(1/2d \\
& x + 1/2c)^2 + 2*(4\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + 4\sqrt{2} \\
&)\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 9\sqrt{2}\cos(dx + c)\cos(1/2dx + \\
& 1/2c) + 5\sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 9*(\sqrt{2} \\
&)\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2* \\
& (4\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + 4\sqrt{2}\sin(dx + c)^2\sin \\
& in(1/2dx + 1/2c) + 9\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + 5\sqrt{2} \\
& (2)\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\sin(7/2dx + 7/2c)^2 + 2* \\
& ((4\sqrt{2}\cos(dx + c)^2 + 4\sqrt{2}\sin(dx + c)^2 + 9\sqrt{2}\cos(dx + \\
& c) + 5\sqrt{2}))\cos(3/2dx + 3/2c)^2 + 4*(\sqrt{2}\cos(1/2dx + 1/2c)^2 \\
& + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (4\sqrt{2}\cos(dx + c)^ \\
& 2 + 4\sqrt{2}\sin(dx + c)^2 + 9\sqrt{2}\cos(dx + c) + 5\sqrt{2}))\sin(3/2 \\
& dx + 3/2c)^2 + 4*(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + \\
& 1/2c)^2)\sin(dx + c)^2 + 5\sqrt{2}\cos(1/2dx + 1/2c)^2 + 5\sqrt{2}\sin \\
& (1/2dx + 1/2c)^2 + 2*(4\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + 4\sqrt{ \\
& rt(2)\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 9\sqrt{2}\cos(dx + c)\cos(1/ \\
& 2dx + 1/2c) + 5\sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 9* \\
& (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + \\
& c) + 2*(4\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + 4\sqrt{2}\sin(dx + \\
& c)^2\sin(1/2dx + 1/2c) + 9\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \\
& 5\sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\sin(7/2dx + 7/2c)*
\end{aligned}$$

$$\begin{aligned}
& \sin(5/2*d*x + 5/2*c) + ((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 84*((\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 6*(14*((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \\
& * \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x \\
& + 7/2*c)^2 + 14*((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*c \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 10*(\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + 10*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d \\
& *x + c)^2 + 15*\sqrt{2}*\cos(d*x + c) + 11*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + \\
& 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 15*\sqrt{2}) \\
& *\cos(d*x + c) + 11*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 11*\sqrt{2}*c \\
& \cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2})*c \\
& \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 15*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 11*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 15*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*s \\
& \sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 15*s \\
& \sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c))* \\
& \sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 10*(\sqrt{2}*\cos(d*x + c)^2 + \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3 \\
& /2*c)^2 + 10*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 10*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 28*((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c \\
&)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2* \\
& c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) - (\sqrt{2}*\cos(3/2* \\
& d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& (\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d* \\
& x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 20*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2* \\
& c) + 20*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*c
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x \\
& + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8* \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d* \\
& x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2} \\
& *t(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c) \\
&)*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *t(2))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
& *t(2))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
& *t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) \\
& ^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c))*\cos \\
& (1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) * \sin(3/2*d*x + 3/2*c) - 4*(2 \\
& * \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \sin(5/2*d* \\
& x + 5/2*c) + 20*(\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(\\
& d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2* \\
& c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2* \\
& c) - 12*(2*((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos \\
& (d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c) \\
&)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d* \\
& x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2* \\
& c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(\\
& 1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2* \\
& c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} \\
&) * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d* \\
& x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2 + 5*(\sqrt{2} * \cos(d \\
& *x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(\\
& 3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d* \\
& x + 1/2*c)^2) * \cos(d*x + c)^2 + 5*(\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x \\
& + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2} \\
&) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 \\
& + 5*\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 10 \\
& * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
&) * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 10*(\sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 10*(\sqrt{2} * \cos(d*x + c) \\
&)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin \\
& (3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c) - 360*((\sqrt{2} * \cos(d*x + c)^2 + \\
& \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3 \\
& /2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) \\
& * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} \\
&) * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c) \\
&)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*s \\
& \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos \\
& (3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d* \\
& x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \cos(d*x + c) * \sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos \\
& (7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + \\
& 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c)^2 + (\text{sqrt}(2) \\
&)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{s} \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& *\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(\\
& 2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2 \\
& *d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(\\
& 2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d* \\
& x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3 \\
& /2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\c \\
& \text{os}(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{s} \\
& \text{qrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (((\text{sqrt}(2)* \\
& \text{cos}(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2)) \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(\\
& 2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt} \\
& (2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(\\
& d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{s} \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)* \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d \\
& *x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)* \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d \\
& *x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/ \\
& 2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\co \\
& \text{s}(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
&)^2*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x \\
& + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) \\
& + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
&) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) \\
& ^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x \\
& + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \\
& \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/ \\
& 2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)* \\
& \sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d* \\
& x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(\\
& d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/ \\
& 2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + \\
& c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d* \\
& x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + s \\
& \text{qrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*c
\end{aligned}$$

$$\begin{aligned}
& 3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7 \\
& /2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2 \\
& *\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)* \\
& \cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2)) \\
& *\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{s} \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2 \\
& *d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqr} \\
& \text{t}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\text{s} \\
& \text{in}(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d \\
& *x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2) \\
& *\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2 \\
& *d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\text{c} \\
& \text{os}(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqr} \\
& \text{t}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos \\
& (d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\text{c} \\
& \text{os}(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d* \\
& x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{s} \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2) \\
& *\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(\\
& 2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin \\
& (d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{s} \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c
\end{aligned}$$

$$\begin{aligned}
&)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx \\
&+ c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \\
&\sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2 \\
&c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2d \\
&x + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(d \\
&x + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + \\
&3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 \\
&)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin \\
&in(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1 \\
&/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\sin(7/2dx + 7 \\
&/2c)\sin(5/2dx + 5/2c) + ((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c \\
&)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos \\
&os(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (s \\
&qrt(2)\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + s \\
&qrt(2))\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin \\
&sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + s \\
&qrt(2)\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2 \\
&c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) \\
&\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + \\
&2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx \\
&x + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + \\
&c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \\
&\sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\sin(5/2dx + 5/2c)^2) \\
&\cos(2/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))))\sin(1/5\arct \\
&an2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) - 1260(((\sqrt{2}\cos(dx \\
&+ c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2 \\
&*dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1 \\
&/2c)^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 \\
&+ 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1 \\
&/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2} \\
&)\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx \\
&+ c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c) \\
&^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/ \\
&2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin \\
&n(1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx \\
&+ 1/2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx \\
&+ c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2 \\
&c))\cos(7/2dx + 7/2c)^2 + 2((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx \\
&+ c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2} \\
&)\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + \\
&(\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) \\
&+ \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
&2)\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 \\
&+ \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + \\
&1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx +
\end{aligned}$$

$$\begin{aligned}
& c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) \\
& + 2 * (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos \\
& (d*x + c) + 2 * (\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d* \\
& x + c)^2 \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c) \\
& * \cos(5/2*d*x + 5/2*c) + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + \\
& 2 * \sqrt{2} \cos(d*x + c) + \sqrt{2} \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \\
&) \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) + \sqrt{2} \\
&)) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1 \\
& /2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&) \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) \cos(1 \\
& /2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2 * (s \\
& \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c \\
&) + 2 * (\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \\
& * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&) \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(5/2*d*x + 5/2*c)^2 + ((s \\
& \sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) + s \\
& \sqrt{2} \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} * \\
& \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) + \sqrt{2} \sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x \\
& + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + \\
& 2 * (\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2* \\
& c) \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} \cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2 * (\sqrt{2} \cos(d*x + c) \\
& ^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2 * s \\
& \sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin \\
& (3/2*d*x + 3/2*c)) \sin(7/2*d*x + 7/2*c)^2 + 2 * ((\sqrt{2} \cos(d*x + c)^2 + s \\
& \sqrt{2} \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) + \sqrt{2} \cos(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos \\
& (d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2 * \sqrt{2} \\
&) \cos(d*x + c) + \sqrt{2} \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} \cos(d*x + c)^2 \\
& * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2 * \sqrt{2} \\
& \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(\\
& 3/2*d*x + 3/2*c) + 2 * (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x \\
& + 1/2*c)^2) \cos(d*x + c) + 2 * (\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} \cos(d*x + c) \sin(1 \\
& /2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \sin(7 \\
& /2*d*x + 7/2*c) \sin(5/2*d*x + 5/2*c) + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin \\
& (d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) + \sqrt{2} \cos(3/2*d*x + 3/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
&)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&)*\sin(5/2*d*x + 5/2*c)^3 + 3*(420*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + \\
& c) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2} \\
& (\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*s \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*s \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2} \\
& (\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x \\
& + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + \\
& 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d* \\
& x + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 3 \\
& 5*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d* \\
& x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \\
& 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))* \\
& \cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2})*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - \\
& 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + \\
& 5/2*c) - 70*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(\\
& d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2})*\cos \\
& (3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin \\
& (3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3* \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x \\
& + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2} \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d* \\
& x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1 \\
& /2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*c \\
& \cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 \\
& + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*s \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*sqr \\
& t(2))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c))*\cos(1/2*d*x + 1/2*c) - \sqrt{2}* \\
& \sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d \\
& *x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*s \\
& \sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 3*(420*\sqrt{2}* \\
& \cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))* \\
& \sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^ \\
& 2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))* \\
& \sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) \\
& + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(\\
& 1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3 \\
& *(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\co \\
& s(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sq \\
& rt(2)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8* \\
& \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
& *\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2})*\sin(3/2*d*x + 3/2*c)^ \\
& 2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d \\
& *x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 8* \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2 \\
& *c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2})*\cos(d* \\
& x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\c \\
& os(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqr \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\c \\
& os(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*si \\
& n(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/ \\
& 2*c) - 140*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c \\
&))*\sin(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x \\
& + c) - 12*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2} \\
& *(\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}(\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}(\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(\\
& 1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2* \\
& c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^ \\
& 2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^ \\
& 2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))* \\
& \sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) \\
& + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*s \\
& \sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}(\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x \\
& + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d \\
& *x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/ \\
& 2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*si \\
& n(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*co \\
& s(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2} \\
&)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2 \\
& *c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x \\
& + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2 \\
& *c))*\sin(5/2*d*x + 5/2*c)^2 + 6*(14*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x
\end{aligned}$$

$$\begin{aligned}
& + c) + 2\sqrt{2}\cos(3/2*d*x + 3/2*c)\cos(1/2*d*x + 1/2*c)\sin(d*x + c) + \\
& \sqrt{2}\sin(3/2*d*x + 3/2*c)^2\sin(d*x + c) + 2\sqrt{2}\sin(3/2*d*x + 3/2*c) \\
&)\sin(d*x + c)\sin(1/2*d*x + 1/2*c) + (\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2) \\
& \sin(d*x + c)\cos(5/2*d*x + 5/2*c)^2 - 8*((\sqrt{2}\cos(d*x + c)^2 + \sqrt{2}\sin(d*x + c)^2 + 2\sqrt{2}\cos(d*x + c) + \sqrt{2}\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\cos(d*x + c)^2 + (\sqrt{2}\cos(d*x + c)^2 + \sqrt{2}\sin(d*x + c)^2 + 2\sqrt{2}\cos(d*x + c) + \sqrt{2}\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\sin(d*x + c)^2 + \sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2 + 2(\sqrt{2}\cos(d*x + c)^2\cos(1/2*d*x + 1/2*c) + \sqrt{2}\cos(1/2*d*x + 1/2*c)\sin(d*x + c)^2 + 2\sqrt{2}\cos(d*x + c)\cos(1/2*d*x + 1/2*c) + \sqrt{2}\cos(1/2*d*x + 1/2*c)\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\cos(d*x + c) + 2*(\sqrt{2}\cos(d*x + c)^2\sin(1/2*d*x + 1/2*c) + \sqrt{2}\sin(d*x + c)^2\sin(1/2*d*x + 1/2*c) + 2\sqrt{2}\cos(d*x + c)\sin(1/2*d*x + 1/2*c) + \sqrt{2}\sin(1/2*d*x + 1/2*c)\sin(3/2*d*x + 3/2*c))\cos(5/2*d*x + 5/2*c)\sin(5/2*d*x + 5/2*c) + 14*(\sqrt{2}\cos(3/2*d*x + 3/2*c)^2\sin(d*x + c) + 2\sqrt{2}\cos(3/2*d*x + 3/2*c)\cos(1/2*d*x + 1/2*c)\sin(d*x + c) + \sqrt{2}\sin(3/2*d*x + 3/2*c)^2\sin(d*x + c) + 2\sqrt{2}\sin(3/2*d*x + 3/2*c)\sin(d*x + c)\sin(1/2*d*x + 1/2*c) + (\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)\sin(d*x + c))\sin(5/2*d*x + 5/2*c)^2 + 35*(12\sqrt{2}\cos(3/2*d*x + 3/2*c)^3\sin(d*x + c) - 12*(\sqrt{2}\cos(d*x + c) + \sqrt{2}\sin(3/2*d*x + 3/2*c))^3 - 8\sqrt{2}\sin(1/2*d*x + 1/2*c)^3 + ((3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}\sin(1/2*d*x + 1/2*c))\cos(d*x + c)^2 + (3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}\sin(1/2*d*x + 1/2*c))\sin(d*x + c)^2 + 24\sqrt{2}\cos(1/2*d*x + 1/2*c)\sin(d*x + c) + 2*(3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - 8\sqrt{2}\sin(1/2*d*x + 1/2*c))\cos(3/2*d*x + 3/2*c)^2 - (8\sqrt{2}\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\sin(1/2*d*x + 1/2*c)^2 + 4*(2\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c))\cos(d*x + c)^2 + 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& *x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(\\
& d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d* \\
& x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c \\
&) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\co \\
& s(5/2*d*x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 84*((\sqrt{2}*\cos(3/2*d*x + 3/2*c \\
&)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c))*\cos(1/2*d*x + 1/2*c)*\sin(\\
& d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2* \\
& d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^ \\
& 2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x \\
& + 3/2*c))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1 \\
& /2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\s \\
& in(d*x + c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + (\sqrt{2}*\cos(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c))*\cos(1/2*d*x + 1 \\
& /2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x \\
& + 5/2*c)^2 + (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(\\
& 3/2*d*x + 3/2*c))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2 \\
& *d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2* \\
& c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c))*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2 \\
& *d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c) \\
& * \sin(5/2*d*x + 5/2*c) + (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& * \cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(\\
& 3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2*\cos(1/2*\arctan2(\sin(\\
& d*x + c), \cos(d*x + c))) - 6*(14*((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2* \\
& d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2 \\
& *d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 14*(\\
& (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c \\
&) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*c \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d* \\
& x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 10*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*s \\
& \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + \\
& 10*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 15*\sqrt{2} \\
&)*\cos(d*x + c) + 11*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2} \\
&)*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 15*\sqrt{2}*\cos(d*x + c) + 11 \\
& *\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 11*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1 \\
& /2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 15*\sqrt{2} \\
&)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 11*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(\\
& 3/2*d*x + 3/2*c) + 15*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c \\
&) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 15*\sqrt{2}*\cos(d*x + c) \\
&)*\sin(1/2*d*x + 1/2*c) + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2* \\
& c))*\sin(5/2*d*x + 5/2*c)^2 + 10*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 10*(\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + \\
& 28*(((\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos \\
& (d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2})*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2} \\
&)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c) - (\text{sqrt}(2) * \cos(3/2*d*x + 3/2*c)^2 * \sin \\
& (d*x + c) + 2 * \text{sqrt}(2) * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c \\
&) + \text{sqrt}(2) * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2 * \text{sqrt}(2) * \sin(3/2*d*x + 3 \\
& /2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c) * \sin(5/2*d*x + 5/2*c)) * \cos(7/ \\
& 2*d*x + 7/2*c) + 20 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \\
& \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 20 * (\text{sqrt}(2) * \cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) - 35 * (\\
& 12 * \text{sqrt}(2) * \cos(3/2*d*x + 3/2*c)^3 * \sin(d*x + c) - 12 * (\text{sqrt}(2) * \cos(d*x + c) + \\
& \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^3 - 8 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^3 + ((3 * \text{sqrt} \\
& (2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x \\
& + 1/2*c) + 1) - 3 * \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \cos(d*x \\
& + c)^2 + (3 * \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \\
& \sin(1/2*d*x + 1/2*c) + 1) - 3 * \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2 \\
& *c)) * \sin(d*x + c)^2 + 24 * \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + 2 * (3 * \text{sqrt} \\
& (2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x \\
& + 1/2*c) + 1) - 3 * \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \cos(d*x \\
& + c) + 3 * \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin \\
& (1/2*d*x + 1/2*c) + 1) - 3 * \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c) \\
&) * \cos(3/2*d*x + 3/2*c)^2 - (8 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^3 - 3 * (\text{sqrt}(2) * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) \\
& + 1) - \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin \\
& (1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3 * (\text{sqrt}(2) * \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2 \\
& *c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4 * (2 * \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt} \\
& (2)) * \sin(1/2*d*x + 1/2*c) * \cos(d*x + c)^2 + 3 * (\text{sqrt}(2) * \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) \\
& + 1)) * \cos(1/2*d*x + 1/2*c)^2 + ((3 * \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \text{sqrt}(2) * \log(\cos(1/2 * \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \text{sqrt} \\
& (2) * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (3 * \text{sqrt}(2) * \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \text{sqrt}(2) \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2 * \\
& c) + 1) - 8 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 12 * \text{sqrt}(2) * \cos(3 \\
& /2*d*x + 3/2*c) * \sin(d*x + c) + 2 * (3 * \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \text{sqrt}(2) * \log(\cos(1/2 * \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 20 * \\
& \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 3 * \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}* \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x \\
& + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4 \\
& *(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x \\
& + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((\\
& 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1 \\
& /2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d* \\
& x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(sq \\
& rt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{ \\
& 2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/ \\
& 2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2* \\
& d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\si \\
& n(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*sq
\end{aligned}$$

$$\begin{aligned}
& \text{rt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 12 * (\text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) \\
&) - \text{sqrt}(2) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) \\
&) * \cos(3/2*d*x + 3/2*c) + 2 * (3 * \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + 11 * \text{sqrt}(2) * \\
& \sin(1/2*d*x + 1/2*c)^2 - 3 * (\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + \\
& 1/2*c) + 2 * \text{sqrt}(2) * \cos(d*x + c) - 3 * (\text{sqrt}(2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) * \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(\\
& 1/2*d*x + 1/2*c) + 2 * \text{sqrt}(2) * \sin(3/2*d*x + 3/2*c) - 4 * (2 * \text{sqrt}(2) * \cos(1/2*d \\
& *x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(5/2*d*x + 5/2*c) + 20 * (s \\
& \text{qrt}(2) * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(d*x + c)^2 * \sin(1/2 \\
& *d*x + 1/2*c) + 2 * \text{sqrt}(2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1 \\
& /2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c) * \sin(7/2*d*x + 7/2*c) - 12 * (2 * ((\text{sqrt}(\\
& 2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(\\
& 2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(\\
& 1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(\\
& d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2) * \sin(3/2*d*x + 3/2*c)^2 + (s \\
& \text{qrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c) \\
& ^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * (s \\
& \text{qrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * s \\
& \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1 \\
& /2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * s \\
& \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \text{sqrt}(\\
& 2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/ \\
& 2*d*x + 3/2*c) * \cos(5/2*d*x + 5/2*c)^2 + 5 * (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) \\
&) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2) * \cos(3/2*d*x + 3/2*c)^2 \\
& + 5 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(\\
& d*x + c)^2 + 5 * (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) \\
&) * \cos(d*x + c) + \text{sqrt}(2) * \sin(3/2*d*x + 3/2*c)^2 + 5 * (\text{sqrt}(2) * \cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + 5 * \text{sqrt}(2) * \cos(1 \\
& /2*d*x + 1/2*c)^2 + 5 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 10 * (\text{sqrt}(2) * \cos(d*x \\
& + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + \\
& 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) \\
&) * \cos(3/2*d*x + 3/2*c) + 10 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1 \\
& /2*d*x + 1/2*c)^2) * \cos(d*x + c) + 10 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \sin(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \text{sqrt}(2) * \cos(d*x + \\
& c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c) \\
&) * \sin(5/2*d*x + 5/2*c) + 56 * ((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + \\
& c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\\
& \text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \\
& \text{sqrt}(2) * \sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& *x + c)^2 * \sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
&) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2 \\
& *c)*\sin(5/2*d*x + 5/2*c) + ((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + \\
& c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{ \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^ \\
& 2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d* \\
& x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c) \\
& ^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9 \\
& *\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3 \\
& /2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) \\
&)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 6*(14*(\sqrt{ \\
& 2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c) \\
& *\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c))*\cos(5/2*d*x + 5/2*c)^2 - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c \\
&) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d* \\
& x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) \\
& *\sin(5/2*d*x + 5/2*c) + 14*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2 \\
& *\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2 + 35*(12*\sqrt{2}*\cos \\
& (3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin \\
& (3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3* \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x \\
& + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2} \\
& *t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d* \\
& x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))^3 - 3*(\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\si \\
& n(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1 \\
& /2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1 \\
& /2*d*x + 1/2*c)^2 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2 \\
& *c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c))^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3* \\
& (\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\co \\
& s(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(c
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 2*(84*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&^2) \sin(dx + c) \cos(7/2 dx + 7/2 c)^3 - 84 * ((\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2 * (\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 * (\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c)) \sin(7/2 dx + 7/2 c)^3 - 24 * ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2 * \sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2 * \sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2 * (\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2 * \sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + 2 * (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 * (\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 * \sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c)) \sin(5/2 dx + 5/2 c)^3 + 3 * (420 * \sqrt{2} \cos(3/2 dx + 3/2 c)^3 \sin(dx + c) - 420 * (\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^3 - 280 * \sqrt{2} \sin(1/2 dx + 1/2 c)^3 + 35 * ((3 * \sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 * \sin(1/2 dx + 1/2 c) + 1) - 3 * \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 * \sin(1/2 dx + 1/2 c) + 1) - 8 * \sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(dx + c)^2 + (3 * \sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 * \sin(1/2 dx + 1/2 c) + 1) - 3 * \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 * \sin(1/2 dx + 1/2 c) + 1) - 8 * \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(dx + c)^2 + 24 * \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c) + 2 * (3 * \sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 * \sin(1/2 dx + 1/2 c) + 1) - 3 * \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 * \sin(1/2 dx + 1/2 c) + 1) - 8 * \sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(dx + c) + 3 * \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 * \sin(1/2 dx + 1/2 c) + 1) - 3 * \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 * \sin(1/2 dx + 1/2 c) + 1) - 8 * \sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c)^2 - 35 * (8 * \sqrt{2} \sin(1/2 dx + 1/2 c)^3 - 3 * (\sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 * \sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 * \sin(1/2 dx + 1/2 c) + 1)) \cos(1/2 dx + 1/2 c)^2 - 3 * (\sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 * \sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 * \sin(1/2 dx + 1/2 c) + 1)) \sin(1/2 dx + 1/2 c)^2 + 4 * (2 * \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2}) \sin(1/2 dx + 1/2 c) \cos(dx + c)^2 + 105 * (\sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 * \sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 * \sin(1/2 dx + 1/2 c)
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^3 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3 \\
& *(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\text{sqrt}(2)*\text{co} \\
& s(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\text{sq} \\
& rt(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sq} \\
& rt(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\text{s} \\
& in(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\text{s} \\
& in(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c \\
&)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\text{co} \\
& s(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^ \\
& 2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sq} \\
& rt(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin \\
& (3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\text{sqrt}(2)*\cos(d*x + c) + \text{sq} \\
& rt(2))*\cos(3/2*d*x + 3/2*c)^2 + (8*\text{sqrt}(2))*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(\\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\cos(d*x + c)^ \\
& 2 + (8*\text{sqrt}(2))*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\sin(d*x + c)^2 + 6*\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + 14*\text{sqrt}(2))*\sin(1/2*d*x + 1/2*c)^2 + 12*(\text{sqrt}(2)*\cos(d*x + c)*\text{co} \\
& s(1/2*d*x + 1/2*c) - \text{sqrt}(2)*\sin(d*x + c))*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\text{co} \\
& s(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c \\
&)^2 + 11*\text{sqrt}(2))*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\text{c} \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\cos(d*x + c) - 3*(\text{sqrt}(2)*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sq} \\
& rt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c) - 140*(\\
& 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))*\cos(7/2*d \\
& *x + 7/2*c)^2 + 105*(12*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\text{s} \\
& qrt(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^3 - 8*\text{sqrt}(2)*\sin(1/2*d \\
& *x + 1/2*c)^3 + ((3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d \\
& *x + 1/2*c))*\cos(d*x + c)^2 + (3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)* \\
& \sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& *\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c) \\
& ^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& *\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2* \\
& d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2* \\
& d*x + 1/2*c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + \\
& (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + \\
& c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c) \\
&)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4 \\
& *(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(5/2 \\
& *d*x + 5/2*c)^2 + (84*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/ \\
& 2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& *\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c)^2 * \sin(d*x + c) * \cos(7/2*d*x + 7/2*c)^3 - 84 * ((\sqrt{2} * \cos(d*x \\
& + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c)^3 - 24 * ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c)^3 + 3 * (420 * \sqrt{2} * \cos(3/2*d*x + 3/2*c)^3 * \sin(d*x + c) - 420 * (\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^3 - 280 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 + 35 * ((3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 24 * \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + 2 * (3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - 3 * \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1) - 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c)^2 - 35 * (8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3 * (\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3 * (\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4 * (2 * \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + 105 * (\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 * \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 * \sin(1/2*d
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 + 35 * ((3*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (3*\sqrt{2})*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 12 \\
& * \sqrt{2} * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) + 2 * (3*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 20*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 3*\sqrt{2} * \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 32*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c) \\
& ^2 - 35 * (8*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3 * (\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3 * (\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d \\
& *x + 1/2*c)^2 + 4 * (2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x \\
& + 1/2*c)) * \sin(d*x + c)^2 + 105 * (\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d* \\
& x + 1/2*c)^2 + 56 * (\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \\
& \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2} * \sin(3/2*d* \\
& x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \sin \\
& (1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c)^2 * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) - 70 * ((8*\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3 * (\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2 \\
& *d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x \\
& + 1/2*c) - 3 * (\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \sin(d \\
& *x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) + 2 * (8*\sqrt{2} \\
&) * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3 * (\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3 * (\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1)) * \cos(1/2*d*x + 1/2*c) - 6 * (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c)^2 * \sin(d*x + c)) * \cos(3/2*d*x + 3/2*c) - 70 * (8*\sqrt{2})*\sin
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1 \\
& /2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2 \\
& *\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c) - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\co \\
& s(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
& - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\co \\
& s(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\co \\
& s(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(\\
& d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\l \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2 \\
& *c) - 140*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c) \\
&)*\cos(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + \\
& c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8 \\
& *sqrt(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*sqrt(2)*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(\\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 8*sqrt(2)*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*sqrt(2)*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8 \\
& *sqrt(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*sqrt(2)*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - 8*sqrt(2)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - 35*(8*sqrt \\
& t(2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d \\
& *x + 1/2*c)^2 - 3*(sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + si \\
& n(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& + 4*(2*sqrt(2)*\cos(1/2*d*x + 1/2*c)^2 + sqrt(2))*\sin(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c)^2 + 105*(sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + \\
& 35*((3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*\sin(1/2*d*x + 1/2*c)) \\
& *\cos(d*x + c)^2 + (3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*\sin(1/2* \\
& d*x + 1/2*c))*\sin(d*x + c)^2 + 12*sqrt(2)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& + 2*(3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*sqrt(2)*\sin(1/2*d*x + 1/2*c) \\
&)*\cos(d*x + c) + 3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*sqrt(2)*\sin(1/2*d \\
& *x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*sqrt(2)*\sin(1/2*d*x + 1/2*c)^3 \\
& - 3*(sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2)*l \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*sqrt(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + sqrt(2))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(sqrt(2)*lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(7/2*d*x + 7/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)) * \cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 8*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 9*\sqrt{2})*\cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 9*\sqrt{2})*\cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} * \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 2 \\
& 0*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d* \\
& x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + \\
& 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d \\
& *x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2* \\
& ((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2* \\
& d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/ \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2* \\
& d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + \\
& 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d \\
& *x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\cos(d*x + c)^2 + (8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\sin(d*x + c)^2 + 6*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 14*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 12*(\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \text{sqrt}(2)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 11*\text{sqrt}(2))*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\cos(d*x + c) - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c) - 4*(2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 6*(14*(\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 - 8*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2))*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)*\sin(5/2*d*x + 5/2*c) + 14*(\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2 + 35*(12*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^3 - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^3 + ((3*\text{sqrt}(2))*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d* \\
& x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1 \\
& /2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\l \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) \\
& - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + \\
& 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/ \\
& 2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c))*\cos(1/2*d*x + 1/2*c) - s \\
& \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\co \\
& s(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c))*\cos(7/2*d*x \\
& + 7/2*c) + 84*((\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\co \\
& s(3/2*d*x + 3/2*c))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x \\
& + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1
\end{aligned}$$

$$\begin{aligned}
& 2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c)^2 + 11*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 15*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/ \\
& 2*c) + 11*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 15*(\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
& 4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + 15*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 11*\sqrt{2} \\
& (\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 10 \\
& *(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + 10*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 10*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 28*((\sqrt{2}*\cos(d*x + c) + \sqrt{2})* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d* \\
& x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + \\
& 5/2*c) - (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d \\
& *x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c) \\
&)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) \\
& *\sin(d*x + c))*\sin(5/2*d*x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 20*(\sqrt{2})*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 20*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) - 35*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c) \\
& ^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^3 \\
& - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3* \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2 \\
& *d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos \\
& s(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + \\
& ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos \\
& s(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \\
& 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos \\
& os(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))^3 - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& *\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) \\
& - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 \\
& + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c))*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c))*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c) + 20*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c))*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c) - 12*(2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2))*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c))*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2))*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c))*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5
\end{aligned}$$

$$\begin{aligned}
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*s \\
& \text{qrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - 35 \\
& *(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& s(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c \\
&))*\cos(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 35*((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin \\
& in(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d \\
& *x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin \\
& in(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{ \\
& rt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(\sqrt{ \\
& t}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2}*\cos(3/2* \\
& d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& (2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d* \\
& x + 7/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(
\end{aligned}$$

$$\begin{aligned}
& \sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) * \cos(d*x + c)^2 + (8* \\
& \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3 * (\sqrt{2} * \log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 8 * \sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c) * \sin(1/2*d*x + 1/2*c) + 2 * (8 * \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d* \\
& x + 1/2*c) - 3 * (\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) * \cos(\\
& d*x + c) - 3 * (\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6 * (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c) \\
&) * \cos(3/2*d*x + 3/2*c) - 70 * (8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)^3 - 3 * (\sqrt{2} * \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3 * (\sqrt{2} * \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (2) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4 * (2 * \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c) * \cos(d*x + c) - 8 * ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 9 * \sqrt{2} * \cos(d*x + c) + 8 * \sqrt{2} * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 9 * \sqrt{2} * \cos(d*x + c) + 8 * \sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + 8 * \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 9 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + 8 * \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \cos(3/2*d*x + 3/2*c) + 9 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 9 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + 8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c) - 70 * (6 * (\sqrt{2} * \cos(d*x + c) + \sqrt{2} * \cos(3/2*d*x + 3/2*c))^2 + (8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3 * (\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c)^2 + (8 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3 * (\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \sin(d*x + c)^2 + 6 * \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + 14 * \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 12 * (\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2} \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(7/2*d*x + 7/2*c)^ \\
& 2 + 105*(12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(d \\
& *x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^ \\
& 3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)) \\
& *\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2* \\
& d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)) \\
& *\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2} \\
& (2)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) +
\end{aligned}$$

$$\begin{aligned}
& 1) - 20\sqrt{2}\sin(1/2*d*x + 1/2*c))\cos(d*x + c) + 3\sqrt{2}\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3* \\
& \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 32\sqrt{2}\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - \\
& (8*\sqrt{2}\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 4*(2*\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c) \\
&)*\sin(d*x + c)^2 + 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*((8*\sqrt{2}\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*l \\
& og(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*c \\
& os(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}\cos(1/2*d*x + 1/2*c)*s \\
& in(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) \\
&) - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) \\
& - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2 \\
& *d*x + 3/2*c) - 2*(8*\sqrt{2}\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\si \\
& n(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}\cos(d*x + c) + \sqrt{2})*co \\
& s(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*s \\
& \sqrt{2}\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c) + 2*\sqrt{2})*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
& + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11 \\
& *\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c \\
&)^2 + 1260*(((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos \\
& (d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + \\
& c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d \\
& *x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2})*\cos \\
& (d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + \\
& c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})* \\
& *\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2})*\cos(d*x + c \\
&)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2 \\
& *\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& x + 3/2*c))) + (84*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& * \cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^3 - 84*((\sqrt{2}*\cos(d*x + c \\
&) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(\\
& 3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2* \\
& d*x + 7/2*c)^3 - 24*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos \\
& (d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin \\
& (3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d \\
& *x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + \\
& 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^3 + 3*(420* \\
& \sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
& * \sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35*((\\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d \\
& *x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(\\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d \\
& *x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c))*\cos(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2})*\sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c) * \sin(d*x + c) \\
& + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 \\
& + 56*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x \\
& + 1/2*c) * \sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x \\
& + 3/2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 \\
& * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) - 70*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x \\
& + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x \\
& + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x \\
& + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2})*s
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin \\
& \sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin \\
& \sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2* \\
& d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + \\
& 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& \sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + \\
& (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin \\
& (d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& \sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3 \\
& *\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3 \\
& /2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
& \sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^
\end{aligned}$$

$$\begin{aligned}
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1 \\
& /2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c) \\
& *\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1 \\
& /2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2 \\
& *c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3 \\
& *(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d* \\
& x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos \\
& (d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{ \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos \\
& (d*x + c))*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c))*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{ \\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/ \\
& 2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))
\end{aligned}$$

$$\begin{aligned}
& * \cos(5/2*d*x + 5/2*c)^2 + 3*(420*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) \\
&) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x \\
& + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4 \\
& *(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x \\
& + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + 35 \\
& *((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\co \\
& s(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \\
& 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\c \\
& os(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3 \\
& *(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 28*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin
\end{aligned}$$

$$\begin{aligned}
& (5/2*d*x + 5/2*c) - 70*(6*(\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\cos(d*x + c)^2 + (8*\text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& + 2*\text{sqrt}(2))*\sin(d*x + c)^2 + 6*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 14*\text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c)^2 + 12*(\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \text{s} \\
& \text{qrt}(2)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) + 2*(3*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 11*\text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c \\
&) + 2*\text{sqrt}(2))*\cos(d*x + c) - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c) + 2*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c) - 140*(2*\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 105*(1 \\
& 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^3 - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^3 + ((3*\text{sq} \\
& \text{rt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c)^2 + (3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c))*\sin(d*x + c)^2 + 24*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\text{sq} \\
& \text{rt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c) + 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)) \\
& *\cos(3/2*d*x + 3/2*c)^2 - (8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\text{sqrt}(2)*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2) \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqr} \\
& \text{t}(2))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x \\
& + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2})*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2} \\
& *t(2)*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1 \\
& /2*c) + 2*\sqrt{2})*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 6*(1 \\
& 4*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/ \\
& 2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin \\
& (d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c \\
&) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d \\
& *x + c))*\cos(5/2*d*x + 5/2*c)^2 - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(\\
& d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& ^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d* \\
& x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3 \\
& /2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2) \\
& *\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/ \\
& 2*c)*\sin(5/2*d*x + 5/2*c) + 14*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2 + 35*(12*\sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})
\end{aligned}$$

$$\begin{aligned}
& *c)^2 + \sqrt{2}) * \sin(1/2*d*x + 1/2*c)) * \cos(5/2*d*x + 5/2*c)) * \cos(7/2*d*x + \\
& 7/2*c) + 84*((\sqrt{2} * \cos(3/2*d*x + 3/2*c))^2 * \sin(d*x + c) + 2*\sqrt{2} * \cos(3 \\
& /2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2} * \sin(3/2*d*x + 3 \\
& /2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \sin(1/2* \\
& d*x + 1/2*c) + (\sqrt{2} * \cos(1/2*d*x + 1/2*c))^2 + \sqrt{2} * \sin(1/2*d*x + 1/2* \\
& c))^2 * \sin(d*x + c)) * \cos(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2} * \cos(3/2*d*x + 3/2*c \\
&)^2 * \sin(d*x + c) + 2*\sqrt{2} * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin \\
& (d*x + c) + \sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \sin(3/2* \\
& d*x + 3/2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2} * \cos(1/2*d*x + 1/2 \\
& *c))^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c))^2 * \sin(d*x + c)) * \cos(7/2*d*x + 7/2*c) * \\
& \cos(5/2*d*x + 5/2*c) + (\sqrt{2} * \cos(3/2*d*x + 3/2*c))^2 * \sin(d*x + c) + 2*\sqrt{2} * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2} * \sin(3 \\
& /2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c \\
&) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2} * \cos(1/2*d*x + 1/2*c))^2 + \sqrt{2} * \sin(1/2* \\
& d*x + 1/2*c))^2 * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2} * \cos(3/2*d*x \\
& + 3/2*c))^2 * \sin(d*x + c) + 2*\sqrt{2} * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2 \\
& *c) * \sin(d*x + c) + \sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \\
& \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2} * \cos(1/2*d \\
& *x + 1/2*c))^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c))^2 * \sin(d*x + c)) * \sin(7/2*d*x + \\
& 7/2*c)^2 + 2*(\sqrt{2} * \cos(3/2*d*x + 3/2*c))^2 * \sin(d*x + c) + 2*\sqrt{2} * \cos(\\
& 3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2} * \sin(3/2*d*x + \\
& 3/2*c)^2 * \sin(d*x + c) + 2*\sqrt{2} * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \sin(1/2 \\
& *d*x + 1/2*c) + (\sqrt{2} * \cos(1/2*d*x + 1/2*c))^2 + \sqrt{2} * \sin(1/2*d*x + 1/2 \\
& *c))^2 * \sin(d*x + c)) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5/2*c) + (\sqrt{2} * \cos(3/2*d*x + 3/2*c))^2 * \sin(d*x + c) + 2*\sqrt{2} * \cos(3/2*d*x + 3/2*c) * \cos(1/2 \\
& *d*x + 1/2*c) * \sin(d*x + c) + \sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \\
& 2*\sqrt{2} * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c))^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c))^2 * \sin(d*x + c)) * \sin \\
& (5/2*d*x + 5/2*c)^2 * \cos(1/2 * \arctan2(\sin(d*x + c), \cos(d*x + c))) - 6*(14*(\\
& (\sqrt{2} * \cos(d*x + c) + \sqrt{2} * \cos(3/2*d*x + 3/2*c))^2 + (\sqrt{2} * \cos(d*x \\
& + c) + \sqrt{2} * \sin(3/2*d*x + 3/2*c))^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c))^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c))^2 + 2*(\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c \\
&) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + (\sqrt{2} * \cos(1/2*d \\
& *x + 1/2*c))^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c))^2 * \cos(d*x + c) + 2*(\sqrt{2} * \cos \\
& (d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d* \\
& x + 3/2*c)) * \cos(7/2*d*x + 7/2*c)^2 + 14*((\sqrt{2} * \cos(d*x + c) + \sqrt{2} * \cos(3/2*d*x + 3/2* \\
& c))^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c))^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c))^2 + 2* \\
& (\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \\
& \cos(3/2*d*x + 3/2*c) + (\sqrt{2} * \cos(1/2*d*x + 1/2*c))^2 + \sqrt{2} * \sin(1/2*d* \\
& x + 1/2*c))^2 * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2 \\
& + 10*(\sqrt{2} * \cos(d*x + c))^2 + \sqrt{2} * \sin(d*x + c))^2 + 2*\sqrt{2} * \cos(d*x \\
& + c) + \sqrt{2} * \cos(3/2*d*x + 3/2*c))^2 + 10*(\sqrt{2} * \cos(1/2*d*x + 1/2*c))^2 \\
& + \sqrt{2} * \sin(1/2*d*x + 1/2*c))^2 * \cos(d*x + c))^2 + 2*((4*\sqrt{2} * \cos(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 4\sqrt{2}\sin(dx + c)^2 + 15\sqrt{2}\cos(dx + c) + 11\sqrt{2})\cos(3/2dx + 3/2c)^2 + 4(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (4\sqrt{2}\cos(dx + c)^2 + 4\sqrt{2}\sin(dx + c)^2 + 15\sqrt{2}\cos(dx + c) + 11\sqrt{2})\sin(3/2dx + 3/2c)^2 + 4(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + 11\sqrt{2}\cos(1/2dx + 1/2c)^2 + 11\sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(4\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + 4\sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 15\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + 11\sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 15(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2(4\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + 4\sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 15\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + 11\sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\sin(5/2dx + 5/2c)^2 + 10(\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + 10(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + 10\sqrt{2}\cos(1/2dx + 1/2c)^2 + 10\sqrt{2}\sin(1/2dx + 1/2c)^2 + 28(((\sqrt{2}\cos(dx + c) + \sqrt{2}))\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(5/2dx + 5/2c) - (\sqrt{2}\cos(3/2dx + 3/2c)^2\sin(dx + c) + 2\sqrt{2}\cos(3/2dx + 3/2c)\cos(1/2dx + 1/2c)\sin(dx + c) + \sqrt{2}\sin(3/2dx + 3/2c)^2\sin(dx + c) + 2\sqrt{2}\sin(3/2dx + 3/2c)\sin(dx + c)\sin(1/2dx + 1/2c) + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c))\sin(5/2dx + 5/2c))\cos(7/2dx + 7/2c) + 20(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 20(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) - 35(12\sqrt{2}\cos(3/2dx + 3/2c)^3\sin(dx + c) - 12(\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^3 - 8\sqrt{2}\sin(1/2dx + 1/2c)^3 + ((3\sqrt{2})\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - 3\sqrt{2})\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1) - 8\sqrt{2}\sin(1/2dx + 1/2c))\cos(dx + c)^2 + (3\sqrt{2})\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - 3\sqrt{2})\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1) - 8\sqrt{2}\sin(1/2dx + 1/2c))\sin(dx + c)^2 + 24\sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c) + 2(3\sqrt{2})\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - 3\sqrt{2})\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1) - 8\sqrt{2}\sin(1/2dx + 1/2c))\cos(dx + c) + 3\sqrt{2})\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2c) + 1) - 3\sqrt{2})
\end{aligned}$$

$$\begin{aligned}
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + \\
& 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((\\
& 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(\\
& 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2})*\sin(1/2*d*x + 1 \\
& /2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin \\
& (1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) \\
&))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - \\
& 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + \\
& 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c) + 20*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c) - 12*(2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c) \\
& *\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)) \\
& *\sin(5/2*d*x + 5/2*c)^2)*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 84* \\
& (((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x \\
& + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& 2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2} \\
& (2)*\cos(d*x + c) + 5*\sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2})*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2} \\
& (2))*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3 \\
& /2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4* \\
& \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2})*\cos(3/2*d*x + \\
& 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + \\
& 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2} \\
& (2))*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c) \\
& ^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9 \\
& *\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&)*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((4*\sqrt{2} \\
& (2))*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5 \\
& *\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2))*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4* \\
& \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2} \\
& (2))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2} \\
& (2))*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2 \\
& *(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2} \\
& (2))*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin \\
& (1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c)))^2 + 6*(14*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(\\
& d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/ \\
& 2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 - 8*((\\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) +
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c \\
&)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})* \\
& *\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x \\
& + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d \\
& *x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*(2 \\
&))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2})*\cos \\
& (3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(d*x + c) + \sqrt{2})*\sin \\
& (3/2*d*x + 3/2*c)^3 - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*s \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x \\
& + c)^2 + 24*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*(2 \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/ \\
& 2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/
\end{aligned}$$

$$\begin{aligned}
& (8\sqrt{2}\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2}\sin(1/2*d*x + 1 \\
& /2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d* \\
& x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 3*(420*\sqrt{2}*\cos \\
& (3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 420*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin \\
& (3/2*d*x + 3/2*c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 \\
& + (3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin \\
& (d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + \\
& 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 3*\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(\\
& 3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{2}\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\cos(1/2*d*x + 1/2*c)^2 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 2 \\
& 0*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2 \\
& *d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin \\
& (d*x + c)^2 + 105*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& + 28*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x \\
& + 3/2*c))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c))*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c))*\cos(7/2*d*x + 7/2*c) - 70*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin \\
& (1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) \\
&))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c) - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) \\
&))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1 \\
& /2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 8*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 70*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}))*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}))*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}))*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2}))*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 105*(12*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}))*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}))*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2}))*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}))*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}))*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}))*\log(
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x \\
& x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1 \\
& /2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c \\
&)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 \\
& + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 \\
& + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin \\
& in(d*x + c)^2 + 12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) \\
& + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin \\
& in(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x \\
& + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d* \\
& x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2 \\
& *c))*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*\cos \\
& (d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2* \\
& c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})* \\
& \cos(d*x + c))*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c))*\sin(1/2*d*x + 1/2* \\
& c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2* \\
& c))*\sin(5/2*d*x + 5/2*c)^2 + 6*(14*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c) + s \\
& \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) \\
& *\sin(d*x + c))*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2))*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 - 8*((\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& (2) \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \\
& (2) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin \\
& (1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin \\
& (dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (s \\
& \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c \\
&)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2 * \\
& \sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) * \\
& \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(\\
& 1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2 * (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 \\
& + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 * (\sqrt{2} \cos(dx + c)^2 * \\
& \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \\
& (2) \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3 \\
& /2 dx + 3/2 c) \cos(5/2 dx + 5/2 c) \sin(5/2 dx + 5/2 c) + 14 * (\sqrt{2} \cos \\
& (3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2} \cos(3/2 dx + 3/2 c) \cos(1/2 * \\
& dx + 1/2 c) \sin(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2 \\
& * \sqrt{2} \sin(3/2 dx + 3/2 c) \sin(dx + c) \sin(1/2 dx + 1/2 c) + (\sqrt{2} * \\
& \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c) \sin(\\
& 5/2 dx + 5/2 c)^2 + 35 * (12\sqrt{2} \cos(3/2 dx + 3/2 c)^3 \sin(dx + c) - 1 \\
& 2 * (\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^3 - 8\sqrt{2} \sin(1 \\
& /2 dx + 1/2 c)^3 + ((3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + \\
& 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c) \\
& ^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1 \\
& /2 dx + 1/2 c)) \cos(dx + c)^2 + (3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + s \\
& in(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 \\
& * dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8 * \\
& \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(dx + c)^2 + 24\sqrt{2} \cos(1/2 dx + 1/2 \\
& * c) \sin(dx + c) + 2 * (3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + \\
& 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c) \\
& ^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \sin(1 \\
& /2 dx + 1/2 c)) \cos(dx + c) + 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(\\
& 1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - 3\sqrt{2} \log(\cos(1/2 dx \\
& x + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1) - 8\sqrt{2} \\
& \sqrt{2} \sin(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c)^2 - (8\sqrt{2} \sin(1/2 dx \\
& + 1/2 c)^3 - 3 * (\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 \\
& + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1 \\
& /2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(1/2 dx + 1/2 c)^2 - 3 \\
& * (\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx \\
& * x + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c \\
&)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \sin(1/2 dx + 1/2 c)^2 + 4 * (2\sqrt{2} \cos \\
& (1/2 dx + 1/2 c)^2 + \sqrt{2}) \sin(1/2 dx + 1/2 c) \cos(dx + c)^2 + 3 * (s \\
& \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx \\
& + 1/2 c) + 1) - \sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 \\
& - 2\sin(1/2 dx + 1/2 c) + 1)) \cos(1/2 dx + 1/2 c)^2 + ((3\sqrt{2} \log(\cos \\
& (1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) \\
& - 3\sqrt{2} \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/
\end{aligned}$$

$$\begin{aligned}
&) * \sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2* \\
& d*x + 1/2*c)^2) * \sin(d*x + c) * \sin(5/2*d*x + 5/2*c)^2 * \cos(1/2 * \arctan2(\sin(d \\
& *x + c), \cos(d*x + c))) - 6 * (14 * ((\text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d \\
& *x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^2 + \text{s} \\
& \text{qrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\text{sqrt}(2) \\
& * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2* \\
& d*x + 3/2*c) + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2* \\
& c)^2) * \cos(d*x + c) + 2 * (\text{sqrt}(2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c)^2 + 14 * ((\\
& \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(d*x + \\
& c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{s} \\
& \text{qrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2) * \cos(1/2*d* \\
& x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\text{sqrt}(2) * \text{co} \\
& \text{s}(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x \\
& + 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2 + 10 * (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \text{si} \\
& \text{n}(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + 1 \\
& 0 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x \\
& + c)^2 + 2 * ((4 * \text{sqrt}(2) * \cos(d*x + c)^2 + 4 * \text{sqrt}(2) * \sin(d*x + c)^2 + 15 * \text{sqrt} \\
& (2) * \cos(d*x + c) + 11 * \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + 4 * (\text{sqrt}(2) * \cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (4 * \text{sqrt}(2) \\
&) * \cos(d*x + c)^2 + 4 * \text{sqrt}(2) * \sin(d*x + c)^2 + 15 * \text{sqrt}(2) * \cos(d*x + c) + 11 * \\
& \text{sqrt}(2) * \sin(3/2*d*x + 3/2*c)^2 + 4 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(\\
& 2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + 11 * \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) \\
& ^2 + 11 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * (4 * \text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1/ \\
& 2*d*x + 1/2*c) + 4 * \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 15 * \text{sqrt}(2) \\
& * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + 11 * \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3 \\
& /2*d*x + 3/2*c) + 15 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x \\
& + 1/2*c)^2) * \cos(d*x + c) + 2 * (4 * \text{sqrt}(2) * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + 4 * \text{sqrt}(2) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 15 * \text{sqrt}(2) * \cos(d*x + c) * \\
& \sin(1/2*d*x + 1/2*c) + 11 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c) \\
&)) * \sin(5/2*d*x + 5/2*c)^2 + 10 * (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + \\
& c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^2 + 10 * (\text{sqrt}(\\
& 2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 \\
& + 10 * \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + 10 * \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& 8 * (((\text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(\\
& d*x + c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + (\text{sqrt}(2) * \cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\text{sqrt}(\\
& 2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/ \\
& 2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c) - (\text{sqrt}(2) * \cos(3/2*d*x + 3/2*c)^2 * \sin(\\
& d*x + c) + 2 * \text{sqrt}(2) * \cos(3/2*d*x + 3/2*c) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) \\
& + \text{sqrt}(2) * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 2 * \text{sqrt}(2) * \sin(3/2*d*x + 3/ \\
& 2*c) * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4* \\
& (2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x \\
& + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8 \\
& *\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - sq \\
& rt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + si \\
& n(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/ \\
& 2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x \\
& + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\
& *log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2} \\
& *log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2 \\
& *c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2} \\
& *sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c))*\cos(1/2*d*x + 1/2*c) \\
& - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin \\
& in(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1 \\
& /2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c) + 20*(sq \\
& rt(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}))*\sin(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}))*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c) - 12*(2*((\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}))*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2} \\
&))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}))*\sin(d \\
& *x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^ \\
& 2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c)^2 + 2*(sq \\
& rt(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}))*\cos(1/2*d*x + 1/2*c)*\si \\
& n(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}))*\cos(1/ \\
& 2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}))*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\si \\
& n(1/2*d*x + 1/2*c) + \sqrt{2}))*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(3/2 \\
& *d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 5*(\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 \\
& + 5*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c)^2)*\cos(d \\
& *x + c)^2 + 5*(\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}))*\sin(d*x + c)^2 + 2*\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2})*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2})*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 5*\sqrt{2}))*\sin(1/2*d*x + 1/2*c)^2 + 10*(\sqrt{2})*\cos(d*x + \\
& c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
& 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}))*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 10*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c) + 10*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2}))*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c \\
&)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&)*\sin(5/2*d*x + 5/2*c) - 630*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}))*\sin(d*x + \\
& c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\\
& \sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}))*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \\
& \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}))*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}))*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) \\
&)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}))*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c)^2)*\cos(d
\end{aligned}$$

$$\begin{aligned}
& *x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 \\
& + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/ \\
& 2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)* \\
& \sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(\\
& d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((s \\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + s \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}* \\
& \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\c \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c) \\
& ^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*s \\
& \sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\si \\
& \sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
& * \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c \\
&)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\c \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/ \\
& 2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + s \\
& \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2 \\
& *d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2 \\
& *d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\co \\
& s(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*s
\end{aligned}$$

$$\begin{aligned}
& c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c) \\
& ^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c) * \cos(5 \\
& /2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2}*\cos(\\
& d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin \\
& (3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2* \\
& (\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2} \\
& * \cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d* \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
& * \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& * \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \sin(3/2* \\
& d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
& * \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x \\
& + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\c \\
& os(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)) * \cos(3/2*d* \\
& x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2) * \cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& * \sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x \\
& + 7/2*c) * \sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 \\
& + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& + \sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2) \sin(1/2*d*x + 1/2*c)^2 \sin(d*x + c)^2 + \sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}) \cos(d*x + c)^2 \cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}) \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2}) \cos(d*x + \\
& c) \cos(1/2*d*x + 1/2*c) + \sqrt{2}) \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2* \\
& c) + 2*(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \cos \\
& (d*x + c) + 2*(\sqrt{2}) \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2}) \sin(d \\
& *x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) \cos(d*x + c) \sin(1/2*d*x + 1/2*c \\
&) + \sqrt{2}) \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \sin(5/2*d*x + 5/2*c \\
&)^2 \sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 84*(((4*\sqrt{2}) \cos(d*x \\
& + c)^2 + 4*\sqrt{2}) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) + 5*\sqrt{2}) \cos \\
& (3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(1/2* \\
& d*x + 1/2*c)^2) \cos(d*x + c)^2 + (4*\sqrt{2}) \cos(d*x + c)^2 + 4*\sqrt{2}) \sin(\\
& d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) + 5*\sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 + 4 \\
& *(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \sin(d*x \\
& + c)^2 + 5*\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}) \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*(4*\sqrt{2}) \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}) \cos(1/2*d* \\
& x + 1/2*c) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + 5 \\
& *\sqrt{2}) \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}) \cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(4*\sqrt{2}) * \\
& \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}) \sin(d*x + c)^2 \sin(1/2*d*x \\
& + 1/2*c) + 9*\sqrt{2}) \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}) \sin(1/2* \\
& d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c)^2 + 2*((4*\sqrt{2}) * \\
& \cos(d*x + c)^2 + 4*\sqrt{2}) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) + 5*\sqrt{2} \\
& (2)) \cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin \\
& (1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (4*\sqrt{2}) \cos(d*x + c)^2 + 4*\sqrt{2} \\
& (2)) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) + 5*\sqrt{2}) \sin(3/2*d*x + 3/2*c \\
&)^2 + 4*(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \sin \\
& (d*x + c)^2 + 5*\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}) \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(4*\sqrt{2}) \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}) \cos \\
& (1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) \cos(1/2*d*x + 1/2 \\
& *c) + 5*\sqrt{2}) \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}) \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(4*s \\
& \sqrt{2}) \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}) \sin(d*x + c)^2 \sin(1 \\
& /2*d*x + 1/2*c) + 9*\sqrt{2}) \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}) \sin \\
& (1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c) \cos(5/2*d*x \\
& + 5/2*c) + ((4*\sqrt{2}) \cos(d*x + c)^2 + 4*\sqrt{2}) \sin(d*x + c)^2 + 9*\sqrt{2} \\
& (2)) \cos(d*x + c) + 5*\sqrt{2}) \cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}) \cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (4*\sqrt{2}) * \\
& \cos(d*x + c)^2 + 4*\sqrt{2}) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) + 5*\sqrt{2} \\
& (2)) \sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin \\
& (1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + 5*\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \\
& 5*\sqrt{2}) \sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}) \cos(d*x + c)^2 \cos(1/2*d*x \\
& + 1/2*c) + 4*\sqrt{2}) \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d \\
& *x + c) \cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}) \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 3/2*c)) + 84*((\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos \\
& (3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + \\
& 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2 \\
& *c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\si \\
& n(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/ \\
& 2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c \\
&)*\cos(5/2*d*x + 5/2*c) + (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*s \\
& qrt(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2}*\cos(3/2*d \\
& *x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1 \\
& /2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(7/2*d*x \\
& + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\co \\
& s(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x \\
& + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1 \\
& /2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + (\sqrt{2} \\
& *\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1 \\
& /2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\s \\
& in(5/2*d*x + 5/2*c)^2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 6*(14 \\
& *((\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d* \\
& x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2} \\
& *\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2* \\
& d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 14*((\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/ \\
& 2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) \\
& ^2 + 10*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 10*(\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + 2*((4*\sqrt{2}*\cos(d*x \\
& + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 15*\sqrt{2}*\cos(d*x + c) + 11*\sqrt{2}))*
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c) + 20*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c) - 12*(2*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)
\end{aligned}$$

$$\begin{aligned}
& d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d* \\
& x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(\\
& 2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x \\
& + 5/2*c)^2 + (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2) \\
&)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/ \\
& 2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sq \\
& rt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)* \\
& \cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2)) \\
& *\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(\\
& 2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt} \\
& (2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(\\
& d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sq \\
& rt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)* \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d \\
& *x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& ^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& *\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d \\
& *x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 \\
& + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/ \\
& 2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)* \\
& \sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(\\
& d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((s \\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + s \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})* \\
& \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c) \\
& ^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*s \\
& \sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\si \\
& n(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3 \\
& /2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d* \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{ \\
& 2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 \\
& + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c \\
&)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&)*\sin(5/2*d*x + 5/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c))))*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - \\
& 1260*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*c \\
& \cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3 \\
& /2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) \\
& *\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c) \\
& ^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*c \\
& \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos \\
& (5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2 \\
& *\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}* \\
& \cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}) \\
& *\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2 \\
& *d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*s \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}) \\
& *\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((s \\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + s \\
& \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}* \\
& \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}* \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c) \\
& ^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*s \\
& \sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin \\
& (3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2})* \\
& \cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}* \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d \\
& *x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) + 56*(((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (s \\
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2*\sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 84*(((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 9*\sqrt{2}*\cos(d*x + c) + 5*\sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c) + 5*sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 9 \\
& *(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x \\
& + c) + 2*(4*sqrt(2)*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 4*sqrt(2)*sin(d*x \\
& + c)^2*sin(1/2*d*x + 1/2*c) + 9*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) \\
& + 5*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(7/2*d*x + 7/2*c \\
&)*sin(5/2*d*x + 5/2*c) + ((4*sqrt(2)*cos(d*x + c)^2 + 4*sqrt(2)*sin(d*x + c \\
&)^2 + 9*sqrt(2)*cos(d*x + c) + 5*sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + 4*(sqrt(\\
& 2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 \\
& + (4*sqrt(2)*cos(d*x + c)^2 + 4*sqrt(2)*sin(d*x + c)^2 + 9*sqrt(2)*cos(d*x \\
& + c) + 5*sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + 4*(sqrt(2)*cos(1/2*d*x + 1/2*c)^ \\
& 2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + 5*sqrt(2)*cos(1/2*d*x \\
& + 1/2*c)^2 + 5*sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(4*sqrt(2)*cos(d*x + c)^2 \\
& *cos(1/2*d*x + 1/2*c) + 4*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 9*s \\
& qrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + 5*sqrt(2)*cos(1/2*d*x + 1/2*c))* \\
& cos(3/2*d*x + 3/2*c) + 9*(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2* \\
& d*x + 1/2*c)^2)*cos(d*x + c) + 2*(4*sqrt(2)*cos(d*x + c)^2*sin(1/2*d*x + 1/ \\
& 2*c) + 4*sqrt(2)*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 9*sqrt(2)*cos(d*x + \\
& c)*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2 \\
& *c))*sin(5/2*d*x + 5/2*c)^2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))* \\
& cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*(84*(sqrt(\\
& 2)*cos(3/2*d*x + 3/2*c)^2*sin(d*x + c) + 2*sqrt(2)*cos(3/2*d*x + 3/2*c)*cos \\
& (1/2*d*x + 1/2*c)*sin(d*x + c) + sqrt(2)*sin(3/2*d*x + 3/2*c)^2*sin(d*x + c \\
&) + 2*sqrt(2)*sin(3/2*d*x + 3/2*c)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + (sqr \\
& t(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)) \\
& *cos(7/2*d*x + 7/2*c)^3 - 84*((sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x \\
& + 3/2*c)^2 + (sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + sqrt \\
& (2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*co \\
& s(d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x \\
& + 3/2*c) + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^ \\
& 2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*si \\
& n(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(7/2*d*x + 7/2*c)^3 - 24*((sqr \\
& t(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqr \\
& t(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*si \\
& n(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*si \\
& n(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + (\\
& sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + \\
& c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2* \\
& (sqrt(2)*cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c) \\
& *sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos \\
& (1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 \\
& + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)^2 \\
& *sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*sqr \\
& t(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(\\
& 3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c)^3 + 3*(420*sqrt(2)*cos(3/2*d*x + 3/2 \\
& *c)^3*sin(d*x + c) - 420*(sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2
\end{aligned}$$

$$\begin{aligned}
& *c)^3 - 280*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + \\
& 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& ^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d* \\
& x + 1/2*c)^2 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2} \\
& *sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c) \\
&)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2} \\
& *sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2} \\
& *cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 1
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2}*si \\
& n(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*sq \\
& rt(2)*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c \\
&) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c \\
&))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& 1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 140*(2*\sqrt{2})*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 1 \\
& 05*(12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(d*x + \\
& c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 + (\\
& (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2* \\
& (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - sq \\
& rt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& t(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2}*c \\
& os(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} \\
& rt(2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2* \\
& d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin \\
& (d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*c \\
& os(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3 \\
& *(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x \\
& + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c)) * \cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2})*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& \text{qrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 1 \\
& 4*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1 \\
& /2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1 \\
& /2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\
& x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c)^2 + \\
& 3*(420*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 420*(\sqrt{2})*\cos(d*x \\
& + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 280*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 \\
& + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \text{in}(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c \\
&))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \text{in}(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c \\
&))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - 35*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 \\
& - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*l \\
& \text{og}(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 105*(\sqrt{2})*lo \\
& \text{g}(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 + 35*((3*\sqrt{2})*\log(\cos(1/2*d*x \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 \\
& + 12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3 \\
& /2*c)^2 - 35*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(d*x + c)^2 + 105*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 28*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2} \\
& \sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3 \\
& /2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c \\
&)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2* \\
& d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c) - 70*((8*\sqrt{2})*\cos(1/2 \\
& *d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/ \\
& 2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))* \\
& \sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8 \\
& *\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 70*(8*\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c))*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 6*(14*(\sqrt{2})*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2})*\cos(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2*\sin(d*x + c))*\cos(5/2*d*x + 5/ \\
& 2*c)^2 - 8*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c \\
&)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c \\
&)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d* \\
& x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2) \\
&)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d* \\
& x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)*\sin(5/2*d*x + 5/2*c) \\
& + 14*(\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x \\
& + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2 \\
& *\sin(d*x + c) + 2*\text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1 \\
& /2*c) + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c))*\sin(5/2*d*x + 5/2*c)^2 + 35*(12*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^3 \\
& *\sin(d*x + c) - 12*(\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^3 \\
& - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^3 + ((3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\text{sqrt}(2)*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sq} \\
& \text{rt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(2)*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\text{sqrt}(2)*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\text{sqrt}(\\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\text{sq} \\
& \text{rt}(2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d \\
& *x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& + 4*(2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c)^2 + 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + (\\
& 3*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(s \\
& \text{qrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (8*\text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2 - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& \text{in}(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + \\
& 2*\text{sqrt}(2))*\cos(d*x + c)^2 + (8*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(2) \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \text{in}(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\sin(d*x + c)^2 \\
& + 6*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 14*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 12 \\
& *(\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \text{sqrt}(2)*\sin(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 11*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(\text{sqrt}(\\
& 2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\cos(d*x + c) \\
& - 3*(\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2))*\sin \\
& (3/2*d*x + 3/2*c) - 4*(2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2))*\sin(1/2* \\
& d*x + 1/2*c))*\cos(5/2*d*x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 84*((\text{sqrt}(2)*\cos \\
& (3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d \\
& *x + 1/2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2* \\
& \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2)*c \\
& \text{os}(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(7 \\
& /2*d*x + 7/2*c)^2 + 2*(\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt} \\
& (2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/ \\
& 2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) \\
& *\sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d \\
& *x + 1/2*c)^2)*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + (s \\
& \text{qrt}(2)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c) \\
& *\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x \\
& + c) + 2*\text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c))*\cos(5/2*d*x + 5/2*c)^2 + (\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) \\
& + 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \text{sqrt}(2) \\
&)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*s \\
& \text{in}(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(\text{sqrt}(2)*co \\
& \text{s}(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c) + \text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2 \\
& *\text{sqrt}(2)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\text{sqrt}(2)* \\
& \text{cos}(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\sin(\\
& 7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + (\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)^2*\sin \\
& (d*x + c) + 2*\text{sqrt}(2)*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c
\end{aligned}$$

$$\begin{aligned}
&) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2} \sin(3/2 dx + 3/2 c) \sin(dx + c) \sin(1/2 dx + 1/2 c) + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c) \sin(5/2 dx + 5/2 c)^2 \cos(1/2 \arctan2(\sin(dx + c), \cos(dx + c))) - 6(14((\sqrt{2} \cos(dx + c) + \sqrt{2})) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c)) \cos(7/2 dx + 7/2 c)^2 + 14((\sqrt{2} \cos(dx + c) + \sqrt{2})) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c)) \cos(5/2 dx + 5/2 c)^2 + 10(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + 10(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + 2((4\sqrt{2} \cos(dx + c))^2 + 4\sqrt{2} \sin(dx + c)^2 + 15\sqrt{2} \cos(dx + c) + 11\sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (4\sqrt{2} \cos(dx + c)^2 + 4\sqrt{2} \sin(dx + c)^2 + 15\sqrt{2} \cos(dx + c) + 11\sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + 4(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + 11\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + 11\sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(4\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + 4\sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 15\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + 11\sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + 15(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(4\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + 4\sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 15\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + 11\sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c) \sin(5/2 dx + 5/2 c)^2 + 10(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + 10(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + 10\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + 10\sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 28(((\sqrt{2} \cos(dx + c) + \sqrt{2})) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c)) \cos(5/2 dx + 5/2 c) - (\sqrt{2} \cos(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2} \cos(3/2 dx + 3/2 c) \cos(1/2 dx + 1/2 c) \sin(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 \sin(dx + c) + 2\sqrt{2})
\end{aligned}$$

$$\begin{aligned}
& c)^2 - (8\sqrt{2})\sin(1/2*d*x + 1/2*c)^3 - 3(\sqrt{2})\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c)^2 - 3(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\
& x + 1/2*c)^2 + 4*(2*\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + \\
& 1/2*c))*\sin(d*x + c)^2 + 3(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + \\
& 1/2*c)^2 - 2*((8\sqrt{2})\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3(\sqrt{2}) \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8\sqrt{2}) \\
& \cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3(\sqrt{2})\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}) \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8\sqrt{2})\cos(1/2*d*x + 1 \\
& /2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8\sqrt{2})\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x \\
& + 1/2*c) - 3(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d* \\
& x + c) - 3(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6(\sqrt{2}) \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))* \\
& \cos(3/2*d*x + 3/2*c) - 2*(8\sqrt{2})\sin(1/2*d*x + 1/2*c)^3 - 3(\sqrt{2})\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3(\sqrt{2})\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}) \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6(\sqrt{2})\cos(d*x + c) + \sqrt{2} \\
& (2))*\cos(3/2*d*x + 3/2*c)^2 + (8\sqrt{2})\sin(1/2*d*x + 1/2*c)^2 - 3(\sqrt{2}) \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)^2 \\
& + (8\sqrt{2})\sin(1/2*d*x + 1/2*c)^2 - 3(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6\sqrt{2})\cos(1/2*d*x + \\
& 1/2*c)^2 + 14\sqrt{2})\sin(1/2*d*x + 1/2*c)^2 + 12(\sqrt{2})\cos(d*x + c)*\cos \\
& (1/2*d*x + 1/2*c) - \sqrt{2})\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\cos \\
& (1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3\sqrt{2})\cos(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&^2 + 11\sqrt{2}\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c) - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c) + 20*(\sqrt{2})\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c) - 12*(2*((\sqrt{2})\cos(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})\cos(d*x + c)^2 + \sqrt{2} \\
&)\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}) \\
&)\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(1/2*d*x + 1/2*c) \\
&))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + 5*(\sqrt{2})\cos(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + 5*(\sqrt{2})\cos(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + 5*(\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2})\sin(1/2*d*x + 1/2*c)^2 + 10*(\sqrt{2})\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 10*(\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 10*(\sqrt{2})\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c) - 630*((\sqrt{2})\cos(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})\cos(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c))*\cos(
\end{aligned}$$

$$\begin{aligned}
& 3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7 \\
& /2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2 \\
& *\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)* \\
& \cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2)) \\
& *\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2 \\
& *d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{qr} \\
& \text{t}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*s \\
& \sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d \\
& *x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2) \\
& *\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2 \\
& *d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\text{c} \\
& \text{os}(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{qr} \\
& \text{t}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos \\
& (d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\text{c} \\
& \text{os}(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d* \\
& x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2) \\
& *\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(\\
& 2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin \\
& (d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c
\end{aligned}$$

$$\begin{aligned}
&)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx \\
&+ c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \\
&\sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2 \\
&c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2d \\
&*x + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(d \\
&*x + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + \\
&3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 \\
&)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin \\
&in(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1 \\
&/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\sin(7/2dx + 7 \\
&/2c)\sin(5/2dx + 5/2c) + ((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c \\
&)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}))\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos \\
&os(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (s \\
&qrt(2)\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + s \\
&qrt(2))\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin \\
&sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + s \\
&qrt(2)\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2 \\
&*c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) \\
&\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + \\
&2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx \\
&x + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + \\
&c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \\
&\sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\sin(5/2dx + 5/2c)^2) \\
&*\sin(1/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + 56(((\sqrt{2} \\
&2)\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2} \\
&2))\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(\\
&1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(\\
&dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}))\sin(3/2dx + 3/2c)^2 + (s \\
&rt(2)\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c) \\
&^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(s \\
&qrt(2)\cos(dx + c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)s \\
&in(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1 \\
&/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \\
&\sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2s \\
&in(1/2dx + 1/2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2} \\
&2)\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/ \\
&2dx + 3/2c))\cos(7/2dx + 7/2c)^2 + 2((\sqrt{2}\cos(dx + c)^2 + \sqrt{2} \\
&2)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}))\cos(3/2dx + 3/2c) \\
&^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(d \\
&*x + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos \\
&s(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c \\
&)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx \\
&+ 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos \\
&(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2} \\
&*\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2
\end{aligned}$$

$$\begin{aligned}
& 2*\arctan2(\sin(d*x + c), \cos(d*x + c))) * \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 84*((\sqrt{2}*\cos(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2*\sin(d*x + c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + (\sqrt{2}*\cos(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)^2 + (\sqrt{2}*\cos(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(\sqrt{2}*\cos(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2*\sin(d*x + c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + (\sqrt{2}*\cos(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c))^2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c)^2*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 105*(((\sqrt{2}*\cos(d*x + c))^2 + \sqrt{2}*\sin(d*x + c))^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c))^2 + \sqrt{2}*\sin(d*x + c))^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2 + 2*(\sqrt{2}*\cos(d*x + c))^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c))^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c))^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c))^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c))^2 + \sqrt{2}*\sin(d*x + c))^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c))^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))^2*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c))^2 + \sqrt{2}*\sin(d
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) \\
& + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \\
& + \sqrt{2} \sin(3/2 dx + 3/2 c) \cos(7/2 dx + 7/2 c) \cos(5/2 dx + 5/2 c) + \\
& ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \\
& + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \\
& 2) \sin(1/2 dx + 1/2 c)^2 \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \\
& 2) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 \\
& + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx \\
& + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \\
& + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \\
& \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \\
&) \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c) \\
&)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \cos(dx + c) + 2(\sqrt{2} \cos(dx + c) \\
&)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + \\
& 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \\
&) \sin(3/2 dx + 3/2 c) \cos(5/2 dx + 5/2 c)^2 + ((\sqrt{2} \cos(dx + c)^2 + \\
& \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c) \\
&)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \\
& \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \\
& \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c) \\
&)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c) \\
&)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c) \\
&)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \\
& \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos \\
& (3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx \\
& + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) \\
& + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin \\
& (1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)) \cos \\
& (7/2 dx + 7/2 c)^2 + 2((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + \\
& 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c) \\
&)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \\
&) \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \\
&)) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c) \\
&)^2 \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \\
&) \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \\
& \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) \\
& + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c) \\
&)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) \\
& + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) \\
& + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)) \cos(7/2 dx + 7/2 c) \cos(5/2 dx + 5/2 c) \\
& + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c) \\
&)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(s \\
& \text{qrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*c \\
& \cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(\\
& 2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqr \\
& t}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*c \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d* \\
& x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*s \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d* \\
& x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2 \\
& *d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + \\
& 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) \\
&)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 \\
& * \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\sqrt{2} \\
& * \cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
& * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(\\
& d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c) \\
& ^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
& * \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& * \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/ \\
& 2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
& * \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d \\
& *x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos \\
& (d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& * \cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2* \\
& d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& * \sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d \\
& *x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d \\
& *x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 \\
& + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/ \\
& 2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)* \\
& \cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2 \\
& *c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2 \\
& *c)^2 + (((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d* \\
& x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^ \\
& 2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\co \\
& s(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d* \\
& x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d* \\
& x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3 \\
& /2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 \\
& + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sq} \\
& \text{rt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + \\
& c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2 \\
& *\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))* \\
& \cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\s \\
& \text{in}(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\c \\
& \text{os}(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2) \\
&)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& /2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2) \\
& * \sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2 \\
& *c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\\
& \text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2) \\
& *\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 \\
& + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2 \\
& *c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c) \\
&)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2* \\
& \text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\s \\
& \text{in}(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sq} \\
& \text{rt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2* \\
& c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\co \\
& s(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2) \\
& *\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2* \\
& \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt} \\
& (2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3 \\
& /2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/ \\
& 2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2* \\
& \text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\c \\
& os(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))* \\
& \sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\s \\
& \text{in}(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sq} \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2* \\
& d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + \\
& 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\s
\end{aligned}$$

$$\begin{aligned}
& n(1/2*d*x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)* \\
& sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(7/2*d*x + 7/2*c)*sin(5/2*d* \\
& x + 5/2*c) + ((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)* \\
& cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2 \\
& *c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (sqrt(2)*cos(d*x + \\
& c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2* \\
& d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/ \\
& 2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*co \\
& s(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/ \\
& 2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(sqrt(2)*cos(\\
& 1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt \\
& (2)*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(d*x + c)^2*sin(1/2*d* \\
& x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2* \\
& d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c)^2)*cos(2/5*arctan2 \\
& (sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + (((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*si \\
& n(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (\\
& sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + \\
& c)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x \\
& + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + \\
& sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2*d*x + 1/2 \\
& *c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2* \\
& d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt(2)*cos(\\
& d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + \\
& 3/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^ \\
& 2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)* \\
& sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + \\
& 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*cos(7/2*d*x + \\
& 7/2*c)^2 + 2*((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)* \\
& cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2 \\
& *c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (sqrt(2)*cos(d*x + \\
& c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2* \\
& d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/ \\
& 2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*co \\
& s(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/ \\
& 2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(sqrt(2)*cos(\\
& 1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt \\
& (2)*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(d*x + c)^2*sin(1/2*d* \\
& x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2* \\
& d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*cos(7/2*d*x + 7/2*c)*cos(5/2*d*x + 5/2* \\
& c) + ((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x \\
& + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + \\
& sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (sqrt(2)*cos(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2*\cos(2/3* \\
& \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d \\
& *x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c \\
&) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c) \\
& *\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2 \\
& *\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + (((\\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(s \\
& \text{qrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*c \\
& \cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(\\
& 2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqr \\
& t}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*c \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d* \\
& x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*s \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d* \\
& x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2 \\
& *d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + \\
& 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) \\
&)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x
\end{aligned}$$

$$\begin{aligned}
& x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2) \\
&)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d* \\
& x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(\\
& d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c \\
&)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*c \\
& os(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + sq \\
& rt(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(\\
& d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c) \\
& ^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c \\
&)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2* \\
& \text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*co \\
& s(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c) \\
&)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&)*\sin(5/2*d*x + 5/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x \\
& + 3/2*c)))*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 \\
& + (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + \\
& c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + sq \\
& rt(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + sq \\
& rt(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2* \\
& c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\si \\
& n(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c \\
&)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c)) \\
& *\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^ \\
& 2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(sq \\
& rt(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*si \\
& n(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*si \\
& n(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2 \\
& *d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos \\
& os(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d* \\
& x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x \\
& + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d \\
& *x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3 \\
& /2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos \\
& os(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(s \\
& qrt(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5 \\
& /2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d \\
& *x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(2 \\
& /3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\sqrt{2}*\cos(d \\
& *x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d \\
& *x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 \\
& + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/ \\
& 2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)* \\
& \cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2 \\
& *c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2 \\
& *c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
&) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2 \\
& *(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c) \\
&)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + \\
& (((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& * \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 \\
& + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx \\
& + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + \\
& 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 \\
& c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} * \\
& \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c \\
&)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c \\
&)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2* \\
& \sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c)) * \sin \\
& (3/2 dx + 3/2 c) \cos(7/2 dx + 7/2 c) \cos(5/2 dx + 5/2 c) + ((\sqrt{2} * \\
& \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \\
& * \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 \\
& dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
& + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \\
& \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 \\
& + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \\
& \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx \\
& + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 \\
& dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \\
& \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 \\
& dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx \\
& + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx \\
& + 3/2 c) \cos(5/2 dx + 5/2 c)^2 + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
& + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \\
& \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + \\
& c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx \\
& + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \\
& \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 \\
& c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2 \\
& dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx \\
& + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + \\
& 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \\
& \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx \\
& + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + \\
& 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c) \sin(7/2 dx + \\
& 7/2 c)^2 + 2((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx \\
& + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 \\
& c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + \\
& c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 \\
& dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 \\
& c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx \\
& + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos \\
& (1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 \\
& c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx \\
& + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \\
& \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x \\
& + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + \\
& 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2* \\
& c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c) \\
& ^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \\
& \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin \\
& (3/2*d*x + 3/2*c) * \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c) \\
&)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos \\
& (d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos \\
& (d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos \\
& (1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * (\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/ \\
& 2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2 \\
& *d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2 \\
& *d*x + 7/2*c)^2 + 2 * ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos \\
& (d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin \\
& (3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d* \\
& *x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin \\
& (1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d* \\
& *x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + \\
& 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin \\
& (1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin \\
& (1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x \\
& + 5/2*c) + ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos \\
& (d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + \\
& c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2 \\
& *c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d* \\
& x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1
\end{aligned}$$

$$\begin{aligned}
& * \cos(dx + c)^2 * \cos(1/2*dx + 1/2*c) + \sqrt{2} * \cos(1/2*dx + 1/2*c) * \sin(dx \\
& + c)^2 + 2 * \sqrt{2} * \cos(dx + c) * \cos(1/2*dx + 1/2*c) + \sqrt{2} * \cos(1/2*dx \\
& + 1/2*c) * \cos(3/2*dx + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} * \\
& \sin(1/2*dx + 1/2*c)^2) * \cos(dx + c) + 2 * (\sqrt{2} * \cos(dx + c)^2 * \sin(1/2 \\
& * dx + 1/2*c) + \sqrt{2} * \sin(dx + c)^2 * \sin(1/2*dx + 1/2*c) + 2 * \sqrt{2} * \cos \\
& (dx + c) * \sin(1/2*dx + 1/2*c) + \sqrt{2} * \sin(1/2*dx + 1/2*c) * \sin(3/2*dx \\
& + 3/2*c)) * \cos(7/2*dx + 7/2*c)^2 + 2 * ((\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin \\
& (dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \cos(3/2*dx + 3/2*c)^2 + (s \\
& \sqrt{2} * \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} * \sin(1/2*dx + 1/2*c)^2) * \cos(dx + c \\
&)^2 + (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx \\
& + c) + \sqrt{2}) * \sin(3/2*dx + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*dx + 1/2*c)^2 + \\
& \sqrt{2} * \sin(1/2*dx + 1/2*c)^2) * \sin(dx + c)^2 + \sqrt{2} * \cos(1/2*dx + 1/2 \\
& * c)^2 + \sqrt{2} * \sin(1/2*dx + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(dx + c)^2 * \cos(1/2*d \\
& * x + 1/2*c) + \sqrt{2} * \cos(1/2*dx + 1/2*c) * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(d \\
& * x + c) * \cos(1/2*dx + 1/2*c) + \sqrt{2} * \cos(1/2*dx + 1/2*c)) * \cos(3/2*dx + \\
& 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} * \sin(1/2*dx + 1/2*c)^2 \\
&) * \cos(dx + c) + 2 * (\sqrt{2} * \cos(dx + c)^2 * \sin(1/2*dx + 1/2*c) + \sqrt{2} * s \\
& \sin(dx + c)^2 * \sin(1/2*dx + 1/2*c) + 2 * \sqrt{2} * \cos(dx + c) * \sin(1/2*dx + 1 \\
& /2*c) + \sqrt{2} * \sin(1/2*dx + 1/2*c) * \sin(3/2*dx + 3/2*c)) * \cos(7/2*dx + 7 \\
& /2*c) * \cos(5/2*dx + 5/2*c) + ((\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c \\
&)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \cos(3/2*dx + 3/2*c)^2 + (\sqrt{2} * c \\
& \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} * \sin(1/2*dx + 1/2*c)^2) * \cos(dx + c)^2 + (s \\
& \sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + s \\
& \sqrt{2}) * \sin(3/2*dx + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} * \\
& \sin(1/2*dx + 1/2*c)^2) * \sin(dx + c)^2 + \sqrt{2} * \cos(1/2*dx + 1/2*c)^2 + s \\
& \sqrt{2} * \sin(1/2*dx + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(dx + c)^2 * \cos(1/2*dx + 1/2 \\
& * c) + \sqrt{2} * \cos(1/2*dx + 1/2*c) * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) * \\
& \cos(1/2*dx + 1/2*c) + \sqrt{2} * \cos(1/2*dx + 1/2*c)) * \cos(3/2*dx + 3/2*c) + \\
& 2 * (\sqrt{2} * \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} * \sin(1/2*dx + 1/2*c)^2) * \cos(d \\
& * x + c) + 2 * (\sqrt{2} * \cos(dx + c)^2 * \sin(1/2*dx + 1/2*c) + \sqrt{2} * \sin(dx + \\
& c)^2 * \sin(1/2*dx + 1/2*c) + 2 * \sqrt{2} * \cos(dx + c) * \sin(1/2*dx + 1/2*c) + \\
& \sqrt{2} * \sin(1/2*dx + 1/2*c) * \sin(3/2*dx + 3/2*c)) * \cos(5/2*dx + 5/2*c)^2 \\
& + ((\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c \\
&) + \sqrt{2}) * \cos(3/2*dx + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*dx + 1/2*c)^2 + s \\
& \sqrt{2} * \sin(1/2*dx + 1/2*c)^2) * \cos(dx + c)^2 + (\sqrt{2} * \cos(dx + c)^2 + s \\
& \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \sin(3/2*dx + 3/2*c \\
&)^2 + (\sqrt{2} * \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} * \sin(1/2*dx + 1/2*c)^2) * \sin \\
& (dx + c)^2 + \sqrt{2} * \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} * \sin(1/2*dx + 1/2*c) \\
& ^2 + 2 * (\sqrt{2} * \cos(dx + c)^2 * \cos(1/2*dx + 1/2*c) + \sqrt{2} * \cos(1/2*dx + \\
& 1/2*c) * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) * \cos(1/2*dx + 1/2*c) + s \\
& \sqrt{2} * \cos(1/2*dx + 1/2*c)) * \cos(3/2*dx + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*dx + 1 \\
& /2*c)^2 + \sqrt{2} * \sin(1/2*dx + 1/2*c)^2) * \cos(dx + c) + 2 * (\sqrt{2} * \cos(dx \\
& + c)^2 * \sin(1/2*dx + 1/2*c) + \sqrt{2} * \sin(dx + c)^2 * \sin(1/2*dx + 1/2*c) \\
& + 2 * \sqrt{2} * \cos(dx + c) * \sin(1/2*dx + 1/2*c) + \sqrt{2} * \sin(1/2*dx + 1/2*c \\
&)) * \sin(3/2*dx + 3/2*c) * \sin(7/2*dx + 7/2*c)^2 + 2 * ((\sqrt{2} * \cos(dx + c)^
\end{aligned}$$

$$\begin{aligned}
& *x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 \\
& + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
& * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin \\
& (1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + \\
& c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2* \\
& c)) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5/2*c) + ((\sqrt{2} * \cos(d*x + c)^2 + \\
& \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \\
& \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \\
& \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 \\
& * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} \\
& * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos \\
& (3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(\\
& 1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(\\
& 5/2*d*x + 5/2*c)^2 + (((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 \\
& * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \\
& \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) \\
& * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2 \\
& *d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2 \\
& *d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) \\
& + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin \\
& (1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \\
& * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c)^2 + 2 * ((\sqrt{2} \\
& * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2} \\
& * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \\
& \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x \\
& + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + \\
& 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2* \\
& c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c) \\
& ^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} \\
& * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin \\
& (3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c) * \cos(5/2*d*x + 5/2*c) + ((\sqrt{2} * \cos
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + \\
& c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2 \\
& *\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))* \\
& \cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos \\
& (7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt} \\
& (2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt} \\
& (2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos \\
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2* \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c) \\
& ^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5 \\
& /2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt} \\
& (2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(\\
& d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin \\
& (3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2* \\
& (\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2) \\
& *\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d* \\
& x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt} \\
& (2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2) \\
& *\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}) \\
& * \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*c \\
& \cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d* \\
& x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x \\
& + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 \\
& + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2* \\
& c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*c \\
& \cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c \\
&) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c \\
&)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2* \\
& (\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2 \\
&)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(\\
& 3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c \\
&)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*
\end{aligned}$$

$$\begin{aligned}
& 3/2*c)) * \cos(7/2*d*x + 7/2*c) * \cos(5/2*d*x + 5/2*c) + ((\sqrt{2} * \cos(d*x + c) \\
& ^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c \\
&)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \\
& \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos \\
& s(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x \\
& + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + \\
& 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
&) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/ \\
& 2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) \\
& * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) \\
& * \cos(5/2*d*x + 5/2*c)^2 + (((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^ \\
& 2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{ \\
& t(2) * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{ \\
& t(2)) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin \\
& (1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{ \\
& t(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c \\
&) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos \\
& s(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 \\
& * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x \\
& + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c \\
&)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{ \\
& t(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c)^2 + \\
& 2 * ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c \\
&) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{ \\
& t(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{ \\
& t(2) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c \\
&)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin \\
& (d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{ \\
& t(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x \\
& + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c \\
&)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c) * \cos(5/2*d*x + 5/2*c) + ((\sqrt{ \\
& t(2) * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{ \\
& t(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin \\
& (1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin \\
& (d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{ \\
& t(2) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c \\
&)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (
\end{aligned}$$

$$\begin{aligned}
& \sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) * \\
& \sin(dx + c)^2 + 2 \sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(\\
& 1/2 dx + 1/2 c) * \cos(3/2 dx + 3/2 c) + 2 (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 \\
& + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 (\sqrt{2} \cos(dx + c)^2 * \\
& \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 \sqrt{2} \\
& (2) \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) * \sin(3 \\
& /2 dx + 3/2 c)) \cos(5/2 dx + 5/2 c)^2 + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \\
&) * \sin(dx + c)^2 + 2 \sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 \\
& + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx \\
& x + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2 \sqrt{2} \cos \\
& (dx + c) + \sqrt{2}) * \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c) \\
& ^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + \\
& 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2 (\sqrt{2} \cos(dx + c)^2 \cos(\\
& 1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2 \sqrt{2} * \\
& \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 d \\
& * x + 3/2 c) + 2 (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 \\
& * c)^2) \cos(dx + c) + 2 (\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \\
& (2) \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 \sqrt{2} \cos(dx + c) \sin(1/2 d * \\
& x + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) * \sin(3/2 dx + 3/2 c)) * \sin(7/2 d * \\
& x + 7/2 c)^2 + 2 ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2 \sqrt{2} \\
& (2) \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + \\
& 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx \\
& * x + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2 \sqrt{2} \cos(dx + c) + \sqrt{2}) * \sin(\\
& 3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx \\
& + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1 \\
& /2 dx + 1/2 c)^2 + 2 (\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \\
&) \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2 \sqrt{2} \cos(dx + c) \cos(1/2 dx \\
& + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2 (\sqrt{2} * \\
& \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 (\\
& \sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/ \\
& 2 dx + 1/2 c) + 2 \sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(\\
& 1/2 dx + 1/2 c) * \sin(3/2 dx + 3/2 c)) * \sin(7/2 dx + 7/2 c) * \sin(5/2 dx + \\
& 5/2 c) + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2 \sqrt{2} \cos(\\
& dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^ \\
& 2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^ \\
& ^2 + \sqrt{2} \sin(dx + c)^2 + 2 \sqrt{2} \cos(dx + c) + \sqrt{2}) * \sin(3/2 dx \\
& + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c) \\
& ^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + \\
& 1/2 c)^2 + 2 (\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/ \\
& 2 dx + 1/2 c) \sin(dx + c)^2 + 2 \sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) \\
& + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2 (\sqrt{2} \cos(1/2 * \\
& dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 (\sqrt{2} * \\
& \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + \\
& 1/2 c) + 2 \sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx \\
& + 1/2 c) * \sin(3/2 dx + 3/2 c)) * \sin(5/2 dx + 5/2 c)^2) \cos(2/3 \arctan 2(\sin
\end{aligned}$$

$$\begin{aligned}
& s(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d* \\
& x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d* \\
& x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3 \\
& /2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& \cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + \\
& c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2 \\
& *\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))* \\
& \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\s \\
& in(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\c \\
& os(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\s \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2 \\
& *\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*(\\
& (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d* \\
& x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/ \\
& 2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))* \\
& \sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& * \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \\
&) * \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/ \\
& 2 dx + 1/2 c)^2) * \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
& x + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) * \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \\
& (2) * \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) * \sin(dx + c)^2 \\
& + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2 * (\text{sqr} \\
& t(2) * \cos(dx + c)^2 * \cos(1/2 dx + 1/2 c) + \sqrt{2} * \cos(1/2 dx + 1/2 c) * \sin \\
& (dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) * \cos(1/2 dx + 1/2 c) + \sqrt{2} * \cos(1/2 \\
& * dx + 1/2 c)) * \cos(3/2 dx + 3/2 c) + 2 * (\sqrt{2} * \cos(1/2 dx + 1/2 c)^2 + \text{s} \\
& \text{qrt}(2) * \sin(1/2 dx + 1/2 c)^2) * \cos(dx + c) + 2 * (\sqrt{2} * \cos(dx + c)^2 * \sin \\
& (1/2 dx + 1/2 c) + \sqrt{2} * \sin(dx + c)^2 * \sin(1/2 dx + 1/2 c) + 2 * \sqrt{2} \\
& * \cos(dx + c) * \sin(1/2 dx + 1/2 c) + \sqrt{2} * \sin(1/2 dx + 1/2 c)) * \sin(3/2 \\
& dx + 3/2 c)) * \sin(5/2 dx + 5/2 c)^2 * \cos(2/3 * \arctan2(\sin(3/2 dx + 3/2 c), \\
& \cos(3/2 dx + 3/2 c))) * \sin(2/5 * \arctan2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx \\
& + 5/2 c)))^2 + (((\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} (\\
& 2) * \cos(dx + c) + \sqrt{2}) * \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} * \cos(1/2 dx + \\
& 1/2 c)^2 + \sqrt{2} * \sin(1/2 dx + 1/2 c)^2) * \cos(dx + c)^2 + (\sqrt{2} * \cos(dx \\
& x + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \sin(3 \\
& /2 dx + 3/2 c)^2 + (\sqrt{2} * \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} * \sin(1/2 dx + \\
& 1/2 c)^2) * \sin(dx + c)^2 + \sqrt{2} * \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} * \sin(1/ \\
& 2 dx + 1/2 c)^2 + 2 * (\sqrt{2} * \cos(dx + c)^2 * \cos(1/2 dx + 1/2 c) + \sqrt{2} \\
& * \cos(1/2 dx + 1/2 c) * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) * \cos(1/2 dx + \\
& 1/2 c) + \sqrt{2} * \cos(1/2 dx + 1/2 c)) * \cos(3/2 dx + 3/2 c) + 2 * (\sqrt{2} * \text{c} \\
& \text{os}(1/2 dx + 1/2 c)^2 + \sqrt{2} * \sin(1/2 dx + 1/2 c)^2) * \cos(dx + c) + 2 * (\text{s} \\
& \text{qrt}(2) * \cos(dx + c)^2 * \sin(1/2 dx + 1/2 c) + \sqrt{2} * \sin(dx + c)^2 * \sin(1/2 \\
& * dx + 1/2 c) + 2 * \sqrt{2} * \cos(dx + c) * \sin(1/2 dx + 1/2 c) + \sqrt{2} * \sin(1 \\
& /2 dx + 1/2 c)) * \sin(3/2 dx + 3/2 c)) * \cos(7/2 dx + 7/2 c)^2 + 2 * ((\sqrt{2} \\
& * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2} \\
&) * \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} * \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} * \sin(1/ \\
& 2 dx + 1/2 c)^2) * \cos(dx + c)^2 + (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx \\
& x + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \\
& (2) * \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} * \sin(1/2 dx + 1/2 c)^2) * \sin(dx + c)^2 \\
& + \sqrt{2} * \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} * \sin(1/2 dx + 1/2 c)^2 + 2 * (\text{sqr} \\
& t(2) * \cos(dx + c)^2 * \cos(1/2 dx + 1/2 c) + \sqrt{2} * \cos(1/2 dx + 1/2 c) * \sin \\
& (dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) * \cos(1/2 dx + 1/2 c) + \sqrt{2} * \cos(1/2 \\
& * dx + 1/2 c)) * \cos(3/2 dx + 3/2 c) + 2 * (\sqrt{2} * \cos(1/2 dx + 1/2 c)^2 + \text{s} \\
& \text{qrt}(2) * \sin(1/2 dx + 1/2 c)^2) * \cos(dx + c) + 2 * (\text{s} \\
& \text{qrt}(2) * \cos(dx + c)^2 * \sin(1/2 dx + 1/2 c) + \sqrt{2} * \sin(dx + c)^2 * \sin(1/2 \\
& * dx + 1/2 c) + 2 * \sqrt{2} * \cos(dx + c) * \sin(1/2 dx + 1/2 c) + \sqrt{2} * \sin(1 \\
& /2 dx + 1/2 c)) * \sin(3/2 dx + 3/2 c)) * \cos(7/2 dx + 7/2 c) * \cos(5/2 dx + 5/2 c) + ((\sqrt{2} * \cos(dx \\
& + c)^2 + \sqrt{2} * \sin(dx + c)^2 + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \cos(3/ \\
& 2 dx + 3/2 c)^2 + (\sqrt{2} * \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} * \sin(1/2 dx + \\
& 1/2 c)^2) * \cos(dx + c)^2 + (\sqrt{2} * \cos(dx + c)^2 + \sqrt{2} * \sin(dx + c)^2 \\
& + 2 * \sqrt{2} * \cos(dx + c) + \sqrt{2}) * \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} * \cos(\\
& 1/2 dx + 1/2 c)^2 + \sqrt{2} * \sin(1/2 dx + 1/2 c)^2) * \sin(dx + c)^2 + \sqrt{2} (
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c))^2 + \sqrt{2} \\
& (2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*c \\
& \cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*c \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2 \\
& *d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2* \\
& d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c))^2 + \sqrt{2}*\sin(\\
& d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& ^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d* \\
& x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3 \\
& /2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) \\
& *\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/ \\
& 2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\\
& \sqrt{2}*\cos(d*x + c))^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c) \\
&)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin \\
& (3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c))^2 + \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)* \\
& \cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2)*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 \\
& 2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(\\
& 1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(\\
& 7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)* \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d \\
& *x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/ \\
& 2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c \\
&)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x \\
& + 5/2*c)^2 + (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2) \\
& *\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2 \\
& *d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt} \\
& (2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos \\
& (d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(\\
& 2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*
\end{aligned}$$

$$\begin{aligned}
& c)) * \sin(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2} * \cos(d*x + c)^2 \\
& + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 \\
& * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + \\
& c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 \\
& * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \\
& \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2* \\
& d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2* \\
& c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c)^2 + 2 * ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} \\
& * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2} \\
& * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(\\
& 1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * \\
& (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + \\
& c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c) \\
& ^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c) * \sin(5 \\
& /2*d*x + 5/2*c) + ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(\\
& d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \sin(\\
& 3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * \\
& (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1 \\
& /2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(\\
& 1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c)^2 * \cos(2/5 * \arctan2(\sin(5/2 \\
& *d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 2 * (((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x \\
&+ 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} \cos(d*x + c)^2 \cos \\
&(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2 * \sqrt{2} \\
&* \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) * \cos(3/2* \\
&d*x + 3/2*c) + 2 * (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/ \\
&2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{ \\
&t(2) \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} \cos(d*x + c) \sin(1/2*d \\
&*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d \\
&*x + 7/2*c)^2 + 2 * ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2 * \sqrt{ \\
&t(2) \cos(d*x + c) + \sqrt{2} \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x \\
&+ 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} \cos(\\
&d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) + \sqrt{2}) * \sin \\
&(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x \\
&+ 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(\\
&1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&(2) \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) \cos(1/2*d*x \\
&+ 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} \\
&* \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * \\
&(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1 \\
&/2*d*x + 1/2*c) + 2 * \sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin \\
&(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c) * \cos(5/2*d*x + \\
&5/2*c) + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2 * \sqrt{2} \cos \\
&(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c) \\
&^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c) \\
&^2 + \sqrt{2} \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x \\
&+ 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c \\
&)^2) * \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x \\
&+ 1/2*c)^2 + 2 * (\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1 \\
&/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c \\
&) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} \cos(1/2 \\
&*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} \\
&* \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + \\
&1/2*c) + 2 * \sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x \\
&+ 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2} \cos(d*x \\
&+ c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2 * \sqrt{2} \cos(d*x + c) + \sqrt{2}) * \cos(3/ \\
&2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + \\
&1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 \\
&+ 2 * \sqrt{2} \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(\\
&1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} \\
&(2) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} \cos \\
&(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c \\
&)^2 + 2 * \sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1 \\
&/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin \\
&in(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x \\
&+ 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} \cos(d*x
\end{aligned}$$

$$\begin{aligned}
& n(1/2*d*x + 1/2*c)^2*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\sin(5/2*d*x + 5/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c)))^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sq} \\
& \text{rt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos \\
& (d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\text{si} \\
& \text{n}(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d* \\
& x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d* \\
& x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2 \\
& *(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\text{si} \\
& \text{n}(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt} \\
& (2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt} \\
& (2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin \\
& (d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c \\
&)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\\
& \text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)* \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt} \\
& (2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3 \\
& /2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(\\
& d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c \\
&)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*c \\
& \text{os}(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + sq \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(\\
& d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d \\
& *x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c)^2 * \cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
&) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin \\
& (7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^ \\
& 2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/ \\
& 2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d \\
& *x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(\\
& 3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
& \sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(2/3*\text{arc} \\
& \text{tan2}(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*((\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7 \\
& /2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2 \\
& *d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& *\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + \\
& 5/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\co \\
& s(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + (((\sqrt{2}*\c \\
& os(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\c \\
& os(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d* \\
& x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2 \\
& *d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& *\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + \\
& 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c)^2 + \\
& (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1 \\
& /2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
&)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^ \\
& 2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + s \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*s \\
& \sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + sq \\
& rt(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c \\
&)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d* \\
& x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2 \\
& *\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c) \\
&)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&)*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + sq \\
& rt(2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2* \\
& c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*c \\
& \cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2* \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/ \\
& 2*d*x + 5/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c \\
&)))^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d \\
& *x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 \\
& + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + \\
& 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1 \\
& /2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(\\
& 2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c \\
&)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/ \\
& 2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 \\
& + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt} \\
& (2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c \\
&)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2* \\
& \text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c \\
&) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin \\
& (5/2*d*x + 5/2*c)^2 + (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2) \\
&)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/ \\
& 2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& *\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(\\
& 2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*(\\
& (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2) \\
&)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 \\
& + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d* \\
& x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2* \\
& c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2 \\
& *\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))* \\
& \sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2) \\
& *\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d* \\
& x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{qr} \\
& t(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2) \\
& *\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2* \\
& d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)* \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d \\
& *x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/ \\
& 2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\text{co} \\
& s(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
&)^2*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x \\
& + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2) \\
&)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/ \\
& 2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\co \\
& s(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sq} \\
& rt(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/ \\
& 2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d* \\
& x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 \\
& + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + \\
& 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d* \\
& x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\co \\
& s(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos(d*x + \\
& c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d* \\
& x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (
\end{aligned}$$

$$\begin{aligned}
& n(1/2*d*x + 1/2*c)^2*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\sin(5/2*d*x + 5/2*c)^2*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c)))^2 + 2*(((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2 \\
& *\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)* \\
& \cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2)) \\
& *\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2 \\
& *d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqr} \\
& \text{t}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*s \\
& \sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((s \\
& \text{qrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + s \\
& \text{qrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)* \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2* \\
& c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*c \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c) \\
& ^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*s \\
& \text{qrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\si \\
& \sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*c \\
& \cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(\\
& 2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqr} \\
& \text{t}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*c \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d* \\
& x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin \\
& (d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c
\end{aligned}$$

$$\begin{aligned}
&)^2 + (\sqrt{2} \cos(dx + c))^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c))^2 + \\
&\sqrt{2} \sin(1/2 dx + 1/2 c)^2 \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c))^2 \cos(1/2 dx + 1/2 c) + \\
&\sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + \\
&2(\sqrt{2} \cos(1/2 dx + 1/2 c))^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \cos(dx + c) + 2(\sqrt{2} \cos(dx + c))^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + \\
&2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c) \sin(7/2 dx + 7/2 c)^2 + 2((\sqrt{2} \cos(dx + c))^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c))^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c))^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c))^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \sin(dx + c)^2 + 2(\sqrt{2} \cos(dx + c))^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c))^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \cos(dx + c) + 2(\sqrt{2} \cos(dx + c))^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c) \sin(7/2 dx + 7/2 c) \sin(5/2 dx + 5/2 c) + ((\sqrt{2} \cos(dx + c))^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c))^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c))^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c))^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \sin(dx + c)^2 + 2(\sqrt{2} \cos(dx + c))^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c))^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \cos(dx + c) + 2(\sqrt{2} \cos(dx + c))^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c) \sin(5/2 dx + 5/2 c)^2 \cos(2/3 \arctan^2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) \sin(2/5 \arctan^2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c)))^2 + (((\sqrt{2} \cos(dx + c))^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c))^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c))^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c))^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c))^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) *
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 \\
& + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + s \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*s \\
& \sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + sq \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + ((sq \\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + sq \\
& \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*s \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*s \\
& \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*co \\
& s(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^ \\
& 2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*sq \\
& \sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin \\
& (3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2})*c \\
& os(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2)*co \\
& s(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2 \\
& *d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + sq \\
& \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2* \\
& d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*sq \\
& \sqrt{2}))*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos \\
& (d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin \\
& (3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d* \\
& x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d* \\
& x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c) + 2 \\
& *(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x \\
& + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c \\
&)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d* \\
& x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2) \\
&)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d* \\
& x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2*\sin(2/3*\arctan2(\sin \\
& (3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*(((\text{sqrt}(2)*\cos(d*x + c)^2 \\
& + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2) \\
&)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt} \\
& (2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + \\
& c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2 \\
& *\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))* \\
& \cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos \\
& (7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt} \\
& (2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt} \\
& (2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos
\end{aligned}$$

$$\begin{aligned}
& s(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin \\
& (5/2*d*x + 5/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/ \\
& 2*c)))^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c) \\
&)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x \\
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2) \\
&)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(\\
& d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c) \\
&)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(\\
& d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c) \\
&)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
&)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2* \\
& \text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c) \\
&)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&)*\sin(5/2*d*x + 5/2*c)^2 + (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos
\end{aligned}$$

$$\begin{aligned}
& *x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& *\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d* \\
& x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d* \\
& x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& *\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d \\
& *x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(\\
& 3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
& \sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + \\
& 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^ \\
& 2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})* \\
& \cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(2/3*\arctan2(\sin \\
& (3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3 \\
& /2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) \\
& *\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& *\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c) \\
& ^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*co \\
& s(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos \\
& (7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2) \\
&)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& *\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(\\
& 2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2 \\
& *d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(\\
& 2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d* \\
& x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3 \\
& /2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(s \\
& \text{qrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*c \\
& \cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))* \\
& \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(\\
& 2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& *x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqr} \\
& \text{t}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*c \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d* \\
& x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*s \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d* \\
& x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2 \\
& *d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& *\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + \\
& 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c \\
&)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^ \\
& 2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\cos(2/5*\ar \\
& ctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 2*(((\text{sqrt}(2)*\cos(d*x + \\
& c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2* \\
& d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/ \\
& 2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d \\
& *x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^ \\
& 2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2 \\
& *c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2* \\
& c))*\cos(7/2*d*x + 7/2*c)^2 + 2*(((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c \\
&)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)* \\
& \cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2) \\
&)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*cos(5/2*d*x + 5/2*c)^2 + (((s \\
& sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + s \\
& sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)* \\
& sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)* \\
& sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + \\
& (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x \\
& + c)^2 + sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2* \\
& c)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*c \\
& os(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) \\
& ^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c) \\
& ^2*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*s \\
& sqrt(2)*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*si \\
& n(3/2*d*x + 3/2*c))*cos(7/2*d*x + 7/2*c)^2 + 2*((sqrt(2)*cos(d*x + c)^2 + s \\
& sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2 \\
& *c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*c \\
& os(d*x + c)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2) \\
&)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1 \\
& /2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2* \\
& d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2 \\
& *cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt \\
& t(2)*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(\\
& 3/2*d*x + 3/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x \\
& + 1/2*c)^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + \\
& sqrt(2)*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1 \\
& /2*d*x + 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*cos(7 \\
& /2*d*x + 7/2*c)*cos(5/2*d*x + 5/2*c) + ((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*s \\
& in(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + \\
& (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + \\
& c)^2 + (sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d* \\
& x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 \\
& + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + sqrt(2)*cos(1/2*d*x + 1/ \\
& 2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2 + 2*(sqrt(2)*cos(d*x + c)^2*cos(1/2 \\
& *d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*sqrt(2)*cos \\
& (d*x + c)*cos(1/2*d*x + 1/2*c) + sqrt(2)*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x \\
& + 3/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c) \\
& ^2)*cos(d*x + c) + 2*(sqrt(2)*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sqrt(2) \\
& *sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*sqrt(2)*cos(d*x + c)*sin(1/2*d*x + \\
& 1/2*c) + sqrt(2)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*cos(5/2*d*x + \\
& 5/2*c)^2 + (((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)* \\
& cos(d*x + c) + sqrt(2))*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2 \\
& *c)^2 + sqrt(2)*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (sqrt(2)*cos(d*x + \\
& c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2* \\
& d*x + 3/2*c)^2 + (sqrt(2)*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*sin(1/2*d*x + 1/
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\co \\
& s(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{ \\
& 2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d* \\
& x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2})*\co \\
& s(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\c \\
& os(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d* \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{ \\
& 2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\co \\
& s(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2})*\cos(d*x + \\
& c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\c \\
& os(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d* \\
& x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c \\
&))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\co \\
& s(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sq \\
& rt(2)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + sq \\
& rt(2))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\s \\
& in(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + sq \\
& rt(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2* \\
& c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\c \\
& os(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + s
\end{aligned}$$

$$\begin{aligned}
& /2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5 \\
& /2*c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c) \\
& ^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x \\
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
&) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2) \\
& *\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d \\
& *x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c) \\
& ^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt} \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d \\
& *x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^ \\
& 2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*s \\
& \text{qrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d* \\
& x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + \\
& c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2 \\
& *c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)* \\
& \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))* \\
& \sin(5/2*d*x + 5/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + \\
& 3/2*c))))*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + \\
& (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) \\
&) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt} \\
& t(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt} \\
& t(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c
\end{aligned}$$

$$\begin{aligned}
&)^2 + (\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)*\sin \\
&(d*x + c)^2 + \sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c) \\
&^2 + 2*(\sqrt{2}\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}\cos(1/2*d*x + \\
&1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&(2)\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}\cos(1/2*d*x + 1 \\
&/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}\cos(d*x \\
&+ c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
&+ 2*\sqrt{2}\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}\sin(1/2*d*x + 1/2*c) \\
&))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}\cos(d*x + c)^ \\
&2 + \sqrt{2}\sin(d*x + c)^2 + 2*\sqrt{2}\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x \\
&+ 3/2*c)^2 + (\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c) \\
&^2)*\cos(d*x + c)^2 + (\sqrt{2}\cos(d*x + c)^2 + \sqrt{2}\sin(d*x + c)^2 + 2*s \\
&\sqrt{2}\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}\cos(1/2*d* \\
&x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}\cos \\
&(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}\cos(d*x + \\
&c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
&2*\sqrt{2}\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}\cos(1/2*d*x + 1/2*c)) \\
&*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2 \\
&*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}\cos(d*x + c)^2*\sin(1/2*d*x + 1/2 \\
&*c) + \sqrt{2}\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}\cos(d*x + c)* \\
&\sin(1/2*d*x + 1/2*c) + \sqrt{2}\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))* \\
&\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2}\cos(d*x + c)^2 + \sqrt{2} \\
&(2)\sin(d*x + c)^2 + 2*\sqrt{2}\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c) \\
&^2 + (\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
&d*x + c)^2 + (\sqrt{2}\cos(d*x + c)^2 + \sqrt{2}\sin(d*x + c)^2 + 2*\sqrt{2}*c \\
&os(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}\cos(1/2*d*x + 1/2* \\
&c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}\cos(1/2*d*x \\
&+ 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}\cos(d*x + c)^2*c \\
&os(1/2*d*x + 1/2*c) + \sqrt{2}\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}\cos(1/2*d*x + 1/2*c))*\cos(3/2 \\
&*d*x + 3/2*c) + 2*(\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1 \\
&/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}\cos(d*x + c)*\sin(1/2* \\
&d*x + 1/2*c) + \sqrt{2}\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2* \\
&d*x + 5/2*c)^2 + ((\sqrt{2}\cos(d*x + c)^2 + \sqrt{2}\sin(d*x + c)^2 + 2*\sqrt{2} \\
&(2)\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}\cos(1/2*d*x + \\
&1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}\cos(d \\
&*x + c)^2 + \sqrt{2}\sin(d*x + c)^2 + 2*\sqrt{2}\cos(d*x + c) + \sqrt{2})*\sin(\\
&3/2*d*x + 3/2*c)^2 + (\sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x \\
&+ 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1 \\
&/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}\cos(d*x + c)*\cos(1/2*d*x \\
&+ 1/2*c) + \sqrt{2}\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}* \\
&\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
&\sqrt{2}\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}\sin(d*x + c)^2*\sin(1/
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c \\
&) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + s \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*s \\
& \sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + sq \\
& rt(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (((\sqrt{2}*\cos(d*x + c) \\
& ^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2* \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\co \\
& s(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c) \\
& *\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&)*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\co \\
& s(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (sq \\
& rt(2)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + sq \\
& rt(2))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\s \\
& \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + sq \\
& rt(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2* \\
& c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*c \\
& os(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos \\
& (5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\co \\
& s(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*s \\
& \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d
\end{aligned}$$

$$\begin{aligned}
& *c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2 \\
& *c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((s \\
& \text{qrt}(2)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + s \\
& \text{qrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} * \\
& \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2 * \\
& c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c) \\
& ^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*s \\
& \text{qrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin \\
& (3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + s \\
& \text{qrt}(2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2 * \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2 \\
& *\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{ \\
& t(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(\\
& 3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7 \\
& /2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} * \\
& \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d * \\
& x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2 \\
& *d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& *\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + \\
& 5/2*c)^2 + (((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} * \\
& \cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + \\
& c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2 * \\
& d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\co \\
& s(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{ \\
& 2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d* \\
& x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2})*\co \\
& s(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\c \\
& os(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d* \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{ \\
& 2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\co \\
& s(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + \\
& c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\c \\
& os(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d* \\
& x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c \\
&))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\co \\
& s(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sq \\
& rt(2)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + sq \\
& rt(2))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\s \\
& in(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + sq \\
& rt(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2* \\
& c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\c \\
& os(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + s
\end{aligned}$$

$$\begin{aligned}
& \text{qrt}(2) * \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(7/2*d*x + 7/2*c)^2 + \\
& 2 * ((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + \\
& c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sq} \\
& \text{rt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sq} \\
& \text{rt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2* \\
& c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \text{si} \\
& \text{n}(d*x + c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c \\
&)^2 + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x \\
& + 1/2*c) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \text{sq} \\
& \text{rt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\text{sqrt}(2) * \cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\text{sqrt}(2) * \cos(d* \\
& x + c)^2 * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + 2 * \text{sqrt}(2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2* \\
& c)) * \sin(3/2*d*x + 3/2*c) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5/2*c) + ((\text{sq} \\
& \text{rt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sq} \\
& \text{rt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \text{si} \\
& \text{n}(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \text{si} \\
& \text{n}(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^2 + (\\
& \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + \\
& c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * \\
& (\text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) \\
& * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos \\
& (1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 \\
& * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \text{sq} \\
& \text{rt}(2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(\\
& 3/2*d*x + 3/2*c) * \sin(5/2*d*x + 5/2*c)^2 * \sin(2/3 * \arctan2(\sin(3/2*d*x + 3/2 \\
& *c), \cos(3/2*d*x + 3/2*c)))^2 + 2 * (((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d \\
& *x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sq} \\
& \text{rt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^ \\
& 2 + (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + \\
& c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sq} \\
& \text{rt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) \\
& ^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x \\
& + c) * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/ \\
& 2*c) + 2 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \\
& \cos(d*x + c) + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin \\
& (d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \text{sqrt}(2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2 \\
& *c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c) * \cos(7/2*d*x + 7/2 \\
& *c)^2 + 2 * ((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos \\
& (d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) \\
& ^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \cos(d*x + c) \\
& ^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3/2*d*x \\
& + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&)^2 * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1 \\
& /2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c \\
&) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} \\
& * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + \\
& 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c) * \cos(5/2*d*x + 5/2*c) \\
& + ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c \\
&) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} \\
& * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c \\
&)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin \\
& (d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x \\
& + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c \\
&)) * \sin(3/2*d*x + 3/2*c) * \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2} * \cos(d*x + c)^2 \\
& + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) \\
& * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} \\
& * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c \\
&)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \\
& \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \cos \\
& (3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d \\
& *x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c \\
&) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3/2*c)) * \sin \\
& (7/2*d*x + 7/2*c)^2 + 2 * ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 \\
& + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} \\
& * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2} \\
&) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(\\
& 1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \cos(3/2*d*x + 3/2*c) + 2 * (\\
& \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + \\
& c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 \\
& * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& ^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) \\
& + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
&) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d* \\
& x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c) \\
& *\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2) \\
&)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{s} \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
&) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)* \\
& \sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d* \\
& x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(\\
& d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/ \\
& 2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + \\
& c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d* \\
& x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c \\
&))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + s \\
& \text{qrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*c \\
& \cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2 \\
&)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 \\
& *\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqr} \\
& \text{t}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(\\
& 3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5 \\
& /2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*s \\
& \text{qrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d* \\
& x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*co \\
& s(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*s \\
& \sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d \\
& *x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*si \\
& n(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqr} \\
& \text{t}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d \\
& *x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(\\
& 2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + \\
& 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/ \\
& 2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\co \\
& s(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + \\
& c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d* \\
& x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c \\
&))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& *\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d \\
& *x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\s \\
& in(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2 \\
& *\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}* \\
& \cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))* \\
& *\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c)^2 * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2 \\
& *d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) \\
& + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin \\
& (1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \\
& * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c)^2 * \sin(2/ \\
& 3 * \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2 * (((\sqrt{2} * \cos \\
& (d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2} * \cos \\
& (3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d* \\
& x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + \\
& c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \\
& * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x \\
& + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x \\
& + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2 \\
& *d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos \\
& (d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x \\
& + 3/2*c)) * \cos(7/2*d*x + 7/2*c)^2 + 2 * ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin \\
& (d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2} * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) \\
&)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x \\
& + c) + \sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d \\
& *x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d \\
& *x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + \\
& 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) \\
&) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin \\
& (d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7 \\
& /2*c) * \cos(5/2*d*x + 5/2*c) + ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c) \\
&)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2} * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \\
& \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2} * \\
& \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin \\
& (1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + \\
& 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d* \\
& x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 \\
& + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
&) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& t(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
& t(2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c \\
&)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c) \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + \\
& c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2 \\
& *c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)* \\
& \sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))* \\
& \sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
& (2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\c \\
& os(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\co \\
& s(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2))*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2 \\
& *d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& t(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2* \\
& d*x + 5/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)) \\
&))*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 2*((\sqrt{2} \\
& (2)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
& (2))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin
\end{aligned}$$

$$\begin{aligned}
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c \\
&)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\\
& \sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)* \\
& \sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3 \\
& /2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
& (2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c) \\
& ^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(\\
& os(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos \\
& s(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2))*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2 \\
& *d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2* \\
& d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(\\
& d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& ^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d* \\
& x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3 \\
& /2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) \\
& *\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^ \\
& 2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + \\
& 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx \\
& + 1/2 c) \sin(3/2 dx + 3/2 c) \sin(7/2 dx + 7/2 c)^2 + 2((\sqrt{2} \cos(dx \\
& + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3 \\
& /2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + \\
& 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 \\
& + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos \\
& (1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \\
& \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos \\
& (dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + \\
& c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + \\
& 1/2 c) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} * \\
& \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx \\
& + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx \\
& + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3 \\
& /2 c) \sin(7/2 dx + 7/2 c) \sin(5/2 dx + 5/2 c) + ((\sqrt{2} \cos(dx + c)^2 \\
& + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + \\
& 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \\
& \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \\
& \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx \\
& + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos \\
& (1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + \\
& c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2 \\
& * \sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) * \\
& \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 * \\
& dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 * \\
& c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) * \\
& \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c) * \\
& \sin(5/2 dx + 5/2 c)^2 \cos(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + \\
& 3/2 c)))) * \log(\cos(1/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 \\
& + \sin(1/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 - 2 \sin \\
& (1/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) + 1) + 105 * ((\sqrt{2} \\
& \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \\
& \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin \\
& (1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin \\
& (dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\\
& \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c \\
&)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2 * (\\
& \sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) * \\
& \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos \\
& (1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 \\
& + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 * \\
& \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \\
& \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3 \\
& /2 dx + 3/2 c) \cos(7/2 dx + 7/2 c)^2 + 2 * ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& (2) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(3/2 dx + 3/2 c) \\
& ^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(\\
& dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos \\
& (dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 \\
& c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx \\
& + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos \\
& (1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \\
&) \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 \\
& * dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1 \\
& /2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin \\
& (dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 \\
& dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)) \cos(7/2 \\
& dx + 7/2 c) \cos(5/2 dx + 5/2 c) + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin \\
& (dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \\
& \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) \\
& ^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + \\
& c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \\
& \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c) \\
&)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx \\
& x + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx \\
& + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3 \\
& /2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \\
& * \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin \\
& (dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/ \\
& 2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)) \cos(5/2 dx + 5/ \\
& 2 c)^2 + (((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos \\
& (dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c) \\
& ^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c) \\
& ^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx \\
& + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c) \\
&)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx \\
& + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1 \\
& /2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) \\
&) + \sqrt{2} \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 \\
& * dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \\
& * \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + \\
& 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx \\
& + 1/2 c) \sin(3/2 dx + 3/2 c)) \cos(7/2 dx + 7/2 c)^2 + 2((\sqrt{2} \cos(dx \\
& * x + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(\\
& 3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx \\
& + 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c) \\
& ^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos \\
& (1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \\
& \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos \\
& (dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx +
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + \\
& 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
& \sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2d \\
& x + 1/2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(d \\
& x + c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + \\
& 3/2c))\cos(7/2dx + 7/2c)\cos(5/2dx + 5/2c) + ((\sqrt{2}\cos(dx + c)^ \\
& 2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx \\
& + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c) \\
& ^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2s \\
& \sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2d \\
& x + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos \\
& (1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + \\
& c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + \\
& 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)) \\
& \cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2 \\
& dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2 \\
& c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c) \\
& \sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c)) \\
& \cos(5/2dx + 5/2c)^2 + (((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 \\
& + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(\\
& 1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2} \\
& \cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2} \\
&)\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin \\
& (1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
& \sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) \\
& + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos \\
& (1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2 \\
& (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + \\
& c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + c) \\
& ^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2} \\
& \sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(7/2dx + 7/2c)^2 + 2 \\
& *((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) \\
& + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
& \sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2} \\
& \sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c) \\
& ^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin \\
& (dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^ \\
& 2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + \\
& 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2} \\
& \cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/ \\
& 2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx \\
& + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + \\
& 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c) \\
&)\sin(3/2dx + 3/2c))\cos(7/2dx + 7/2c)\cos(5/2dx + 5/2c) + ((\sqrt{2} \\
& \cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& 2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(\\
& 1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(\\
& d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^2 + (\text{sq} \\
& \text{rt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c) \\
& ^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\text{s} \\
& \text{qrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * \text{s} \\
& \text{in}(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1 \\
& /2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \text{s} \\
& \text{in}(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \text{sqrt}(\\
& 2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/ \\
& 2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) \\
& * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 \\
& + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x \\
& + c)^2 + (\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(\\
& d*x + c) + \text{sqrt}(2)) * \sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^ \\
& 2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \text{c} \\
& \text{os}(d*x + c) * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d* \\
& x + 3/2*c) + 2 * (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2* \\
& c)^2) * \cos(d*x + c) + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(\\
& 2) * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \text{sqrt}(2) * \cos(d*x + c) * \sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x \\
& + 7/2*c)^2 + 2 * ((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(\\
& 2) * \cos(d*x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \cos(d* \\
& x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3 \\
& /2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + \\
& 1/2*c)^2) * \sin(d*x + c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
& * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\text{sqrt}(2) * \text{c} \\
& \text{os}(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\text{s} \\
& \text{qrt}(2) * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(d*x + c)^2 * \sin(1/2 \\
& *d*x + 1/2*c) + 2 * \text{sqrt}(2) * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \sin(1 \\
& /2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5 \\
& /2*c) + ((\text{sqrt}(2) * \cos(d*x + c)^2 + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d \\
& *x + c) + \text{sqrt}(2)) * \cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\text{sqrt}(2) * \cos(d*x + c)^2 \\
& + \text{sqrt}(2) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) + \text{sqrt}(2)) * \sin(3/2*d*x + \\
& 3/2*c)^2 + (\text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + 1/2*c)^ \\
& 2) * \sin(d*x + c)^2 + \text{sqrt}(2) * \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) * \sin(1/2*d*x + \\
& 1/2*c)^2 + 2 * (\text{sqrt}(2) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) * \cos(1/2 \\
& *d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \text{sqrt}(2) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& (3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c \\
&)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sq} \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(\\
& d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(\\
& d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sq} \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& ^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + \\
& c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{s} \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c \\
&)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d* \\
& x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d* \\
& x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3 \\
& /2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2) \\
& *\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/ \\
& 2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/ \\
& 2*c))*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c) \\
& ^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos \\
& (1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sq} \\
& \text{rt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sq} \\
& \text{rt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sq} \\
& \text{rt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos \\
& (1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{s} \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + \\
& ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) \\
& + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt} \\
& (2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c) \\
& ^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(\\
& d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(\\
& 2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c \\
&)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\text{si} \\
& \text{in}(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/ \\
& 2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 \\
& + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqr} \\
& \text{t}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c \\
&)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2* \\
& \text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\text{c} \\
& \text{os}(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c \\
&) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\text{si} \\
& \text{n}(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\text{co} \\
& \text{s}(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2) \\
& *\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/ \\
& 2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sq} \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2* \\
& \text{sin}(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\\
& \text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2) \\
& *\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 \\
& + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2 \\
& *c))*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c) \\
&)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c) * \sin \\
& (3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5/2*c) + ((\sqrt{2} * \\
& \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) \\
& * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2 \\
& *d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x \\
& + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)^2 \\
& + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin \\
& (d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2* \\
& d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&) * \sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c)^2 * \sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \\
& \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d \\
& *x + 3/2*c)) * \sin(5/2*d*x + 5/2*c)^2 * \cos(2/3 * \arctan 2(\sin(3/2*d*x + 3/2*c), \\
& \cos(3/2*d*x + 3/2*c))) * \cos(2/5 * \arctan 2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + \\
& 5/2*c)))^2 + (((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} \\
&) * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x \\
& + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/ \\
& 2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} \\
&) * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/ \\
& 2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2} * \\
& \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) \\
& * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2 \\
& *d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x \\
& + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)^2 \\
& + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&) * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin \\
& (d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2* \\
& d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&) * \sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + c)^2 * \sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \\
& \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d \\
& *x + 3/2*c)) * \cos(7/2*d*x + 7/2*c) * \cos(5/2*d*x + 5/2*c) + ((\sqrt{2} * \cos(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(3/2 \\
& *dx + 3/2*c)^2 + (\sqrt{2} \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \sin(1/2*dx + 1 \\
& /2*c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 \\
& + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(3/2*dx + 3/2*c)^2 + (\sqrt{2} \cos(1 \\
& /2*dx + 1/2*c)^2 + \sqrt{2} \sin(1/2*dx + 1/2*c)^2) \sin(dx + c)^2 + \sqrt{2} \\
&) \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \sin(1/2*dx + 1/2*c)^2 + 2(\sqrt{2} \cos(\\
& dx + c)^2 \cos(1/2*dx + 1/2*c) + \sqrt{2} \cos(1/2*dx + 1/2*c) \sin(dx + c) \\
& ^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2*dx + 1/2*c) + \sqrt{2} \cos(1/2*dx + 1/ \\
& 2*c)) \cos(3/2*dx + 3/2*c) + 2(\sqrt{2} \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \sin \\
& (1/2*dx + 1/2*c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2*dx \\
& + 1/2*c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2*dx + 1/2*c) + 2\sqrt{2} \cos(dx \\
& + c) \sin(1/2*dx + 1/2*c) + \sqrt{2} \sin(1/2*dx + 1/2*c)) \sin(3/2*dx + 3/2 \\
& *c)) \cos(5/2*dx + 5/2*c)^2 + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + \\
& c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(3/2*dx + 3/2*c)^2 + (\sqrt{2} * \\
& \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \sin(1/2*dx + 1/2*c)^2) \cos(dx + c)^2 + (\\
& \sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \\
& \sqrt{2} \sin(3/2*dx + 3/2*c)^2 + (\sqrt{2} \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*dx + 1/2*c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2*dx + 1/2*c)^2 + \\
& \sqrt{2} \sin(1/2*dx + 1/2*c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2*dx + 1/ \\
& 2*c) + \sqrt{2} \cos(1/2*dx + 1/2*c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \\
& * \cos(1/2*dx + 1/2*c) + \sqrt{2} \cos(1/2*dx + 1/2*c)) \cos(3/2*dx + 3/2*c) \\
& + 2(\sqrt{2} \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \sin(1/2*dx + 1/2*c)^2) \cos(d \\
& *x + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2*dx + 1/2*c) + \sqrt{2} \sin(dx \\
& + c)^2 \sin(1/2*dx + 1/2*c) + 2\sqrt{2} \cos(dx + c) \sin(1/2*dx + 1/2*c) + \\
& \sqrt{2} \sin(1/2*dx + 1/2*c)) \sin(3/2*dx + 3/2*c)) \sin(7/2*dx + 7/2*c)^2 \\
& + 2((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx \\
& + c) + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(3/2*dx + 3/2*c)^2 + (\sqrt{2} \cos(1/2*dx + 1/2*c)^2 + \\
& \sqrt{2} \sin(1/2*dx + 1/2*c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \\
& \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(3/2*dx + 3/ \\
& 2*c)^2 + (\sqrt{2} \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \sin(1/2*dx + 1/2*c)^2) * \\
& \sin(dx + c)^2 + \sqrt{2} \cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \sin(1/2*dx + 1/2 \\
& *c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2*dx + 1/2*c) + \sqrt{2} \cos(1/2*d \\
& x + 1/2*c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2*dx + 1/2*c) + s \\
& \sqrt{2} \cos(1/2*dx + 1/2*c)) \cos(3/2*dx + 3/2*c) + 2(\sqrt{2} \cos(1/2*d \\
& x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d \\
& x + 1/2*c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(\\
& dx + c)^2 \sin(1/2*d \\
& x + 1/2*c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2*d \\
& x + 1/2 \\
& c) + 2\sqrt{2} \cos(dx + c) \sin(1/2*d \\
& x + 1/2 \\
& c) + \sqrt{2} \sin(1/2*d \\
& x + 1/ \\
& 2*c)) \sin(3/2*d \\
& x + 3/2 \\
& *c)) \sin(7/2*d \\
& x + 7/2 \\
& *c) \sin(5/2*d \\
& x + 5/2 \\
& *c) + ((s \\
& \sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + s \\
& \sqrt{2} \cos(3/2*d \\
& x + 3/2 \\
& *c)^2 + (\sqrt{2} \cos(1/2*d \\
& x + 1/2 \\
& *c)^2 + \sqrt{2} * \\
& \sin(1/2*d \\
& x + 1/2 \\
& *c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} * \\
& \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \sin(3/2*d \\
& x + 3/2 \\
& *c)^2 + \\
& (\sqrt{2} \cos(1/2*d \\
& x + 1/2 \\
& *c)^2 + \sqrt{2} \sin(1/2*d \\
& x + 1/2 \\
& *c)^2) \sin(dx \\
& + c)^2 + \sqrt{2} \cos(1/2*d \\
& x + 1/2 \\
& *c)^2 + \sqrt{2} \sin(1/2*d \\
& x + 1/2 \\
& *c)^2 + \\
& 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2*d \\
& x + 1/2 \\
& *c) + \sqrt{2} \cos(1/2*d \\
& x + 1/2 \\
& *c)
\end{aligned}$$

$$\begin{aligned}
& d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/ \\
& 2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d \\
& *x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^ \\
& 2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2 \\
& *c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2* \\
& c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \\
& \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)* \\
& \cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(\\
& 2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^ \\
& 2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sq} \\
& \text{rt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(\\
& 1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(\\
& 5/2*d*x + 5/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2 \\
& *c)))^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c) \\
& ^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x \\
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
&)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
&) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2) \\
& *\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d \\
& *x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c) \\
& ^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\co \\
& s(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sq} \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\c \\
& os(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + \\
& 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
& \sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2d \\
& x + 1/2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(d \\
& x + c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + \\
& 3/2c))\sin(7/2dx + 7/2c)\sin(5/2dx + 5/2c) + ((\sqrt{2}\cos(dx + c)^ \\
& 2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx \\
& + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c) \\
& ^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2} \\
& \cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx \\
& x + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos \\
& (1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + \\
& c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + \\
& 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)) \\
& \cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2 \\
& dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2 \\
& c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c) \\
& \sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c)) \\
& \sin(5/2dx + 5/2c)^2 + (((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 \\
& + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(\\
& 1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2} \\
& \cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2} \\
&)\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin \\
& (1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
& \sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) \\
& + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos \\
& (1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2 \\
& (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + \\
& c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + c) \\
& ^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2} \\
& \sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(7/2dx + 7/2c)^2 + 2 \\
& ((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) \\
& + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
& \sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2} \\
& \sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c) \\
& ^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx \\
& + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 \\
& + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + \\
& 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2} \\
& \cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/ \\
& 2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx \\
& + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + \\
& 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c) \\
&)\sin(3/2dx + 3/2c))\cos(7/2dx + 7/2c)\cos(5/2dx + 5/2c) + ((\sqrt{2} \\
& \cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& t(2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c) \\
&)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}* \\
& \cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*c \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/ \\
& 2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + s \\
& \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2 \\
& *d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2 \\
& *d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d \\
& *x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d \\
& *x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*s \\
& \sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5 \\
& /2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2 \\
& *(((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c \\
&) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c \\
&)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^ \\
& 2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + \\
& 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \\
& * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2 \\
& *d*x + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2 \\
& *c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) * \\
& \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) * \\
& \cos(7/2*d*x + 7/2*c) \cos(5/2*d*x + 5/2*c) + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \\
& (2) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \cos(3/2*d*x + 3/2*c) \\
& ^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(\\
& d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} *c \\
& os(d*x + c) + \sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 *c \\
& s(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} (2 \\
&) \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2 \\
& *d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1 \\
& /2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + sq \\
& rt(2) \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(5/2* \\
& d*x + 5/2*c)^2 + (((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*sq \\
& rt(2) \cos(d*x + c) + \sqrt{2}) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(\\
& d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin \\
& (3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x \\
& + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} (\\
& 2) \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2* \\
& (\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin \\
& (1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2} \\
& (2) \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2} \\
& (2)) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(\\
& 1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(\\
& d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 + (sq \\
& rt(2) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + c) \\
& ^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(s \\
& qrt(2) \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) *s \\
& in(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1 \\
& /2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 *s \\
& in(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} (\\
& 2) \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/ \\
& 2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c) \cos(5/2*d*x + 5/2*c) + ((\sqrt{2} \cos(d
\end{aligned}$$

$$\begin{aligned}
& *x + c)^2 + \sqrt{2}*\sin(dx + c)^2 + 2*\sqrt{2}*\cos(dx + c) + \sqrt{2})*\cos(\\
& 3/2*dx + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}*\sin(1/2*dx \\
& + 1/2*c)^2)*\cos(dx + c)^2 + (\sqrt{2}*\cos(dx + c)^2 + \sqrt{2}*\sin(dx + c) \\
& ^2 + 2*\sqrt{2}*\cos(dx + c) + \sqrt{2}))*\sin(3/2*dx + 3/2*c)^2 + (\sqrt{2})*\cos \\
& s(1/2*dx + 1/2*c)^2 + \sqrt{2}*\sin(1/2*dx + 1/2*c)^2)*\sin(dx + c)^2 + \sqrt{2} \\
& t(2)*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}*\sin(1/2*dx + 1/2*c)^2 + 2*(\sqrt{2})*\cos \\
& os(dx + c)^2*\cos(1/2*dx + 1/2*c) + \sqrt{2}*\cos(1/2*dx + 1/2*c)*\sin(dx + \\
& c)^2 + 2*\sqrt{2}*\cos(dx + c)*\cos(1/2*dx + 1/2*c) + \sqrt{2}*\cos(1/2*dx + \\
& 1/2*c))*\cos(3/2*dx + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*dx + 1/2*c)^2)*\cos(dx + c) + 2*(\sqrt{2})*\cos(dx + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + \sqrt{2}*\sin(dx + c)^2*\sin(1/2*dx + 1/2*c) + 2*\sqrt{2}*\cos(d \\
& *x + c)*\sin(1/2*dx + 1/2*c) + \sqrt{2}*\sin(1/2*dx + 1/2*c))*\sin(3/2*dx + \\
& 3/2*c))*\cos(5/2*dx + 5/2*c)^2 + ((\sqrt{2})*\cos(dx + c)^2 + \sqrt{2}*\sin(dx \\
& + c)^2 + 2*\sqrt{2}*\cos(dx + c) + \sqrt{2}))*\cos(3/2*dx + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}*\sin(1/2*dx + 1/2*c)^2)*\cos(dx + c)^2 \\
& + (\sqrt{2})*\cos(dx + c)^2 + \sqrt{2}*\sin(dx + c)^2 + 2*\sqrt{2}*\cos(dx + c) \\
& + \sqrt{2}))*\sin(3/2*dx + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*dx + 1/2*c)^2)*\sin(dx + c)^2 + \sqrt{2}*\cos(1/2*dx + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*dx + 1/2*c)^2 + 2*(\sqrt{2})*\cos(dx + c)^2*\cos(1/2*dx + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*dx + 1/2*c)*\sin(dx + c)^2 + 2*\sqrt{2}*\cos(dx + \\
& c)*\cos(1/2*dx + 1/2*c) + \sqrt{2}*\cos(1/2*dx + 1/2*c))*\cos(3/2*dx + 3/2* \\
& c) + 2*(\sqrt{2})*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}*\sin(1/2*dx + 1/2*c)^2)*\cos \\
& s(dx + c) + 2*(\sqrt{2})*\cos(dx + c)^2*\sin(1/2*dx + 1/2*c) + \sqrt{2}*\sin(d \\
& *x + c)^2*\sin(1/2*dx + 1/2*c) + 2*\sqrt{2}*\cos(dx + c)*\sin(1/2*dx + 1/2*c \\
&) + \sqrt{2}*\sin(1/2*dx + 1/2*c))*\sin(3/2*dx + 3/2*c))*\sin(7/2*dx + 7/2*c \\
&)^2 + 2*((\sqrt{2})*\cos(dx + c)^2 + \sqrt{2}*\sin(dx + c)^2 + 2*\sqrt{2}*\cos(d \\
& *x + c) + \sqrt{2}))*\cos(3/2*dx + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*dx + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*dx + 1/2*c)^2)*\cos(dx + c)^2 + (\sqrt{2})*\cos(dx + c)^2 \\
& + \sqrt{2}*\sin(dx + c)^2 + 2*\sqrt{2}*\cos(dx + c) + \sqrt{2}))*\sin(3/2*dx + \\
& 3/2*c)^2 + (\sqrt{2})*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}*\sin(1/2*dx + 1/2*c)^ \\
& 2)*\sin(dx + c)^2 + \sqrt{2}*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}*\sin(1/2*dx + \\
& 1/2*c)^2 + 2*(\sqrt{2})*\cos(dx + c)^2*\cos(1/2*dx + 1/2*c) + \sqrt{2}*\cos(1/2 \\
& *dx + 1/2*c)*\sin(dx + c)^2 + 2*\sqrt{2}*\cos(dx + c)*\cos(1/2*dx + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*dx + 1/2*c))*\cos(3/2*dx + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*dx + 1/2*c)^2)*\cos(dx + c) + 2*(\sqrt{2})*\cos \\
& os(dx + c)^2*\sin(1/2*dx + 1/2*c) + \sqrt{2}*\sin(dx + c)^2*\sin(1/2*dx + 1 \\
& /2*c) + 2*\sqrt{2}*\cos(dx + c)*\sin(1/2*dx + 1/2*c) + \sqrt{2}*\sin(1/2*dx + \\
& 1/2*c))*\sin(3/2*dx + 3/2*c))*\sin(7/2*dx + 7/2*c)*\sin(5/2*dx + 5/2*c) + \\
& ((\sqrt{2})*\cos(dx + c)^2 + \sqrt{2}*\sin(dx + c)^2 + 2*\sqrt{2}*\cos(dx + c) \\
& + \sqrt{2}))*\cos(3/2*dx + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*dx + 1/2*c)^2)*\cos(dx + c)^2 + (\sqrt{2})*\cos(dx + c)^2 + \sqrt{2} \\
&)*\sin(dx + c)^2 + 2*\sqrt{2}*\cos(dx + c) + \sqrt{2}))*\sin(3/2*dx + 3/2*c)^ \\
& 2 + (\sqrt{2})*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}*\sin(1/2*dx + 1/2*c)^2)*\sin(d \\
& *x + c)^2 + \sqrt{2}*\cos(1/2*dx + 1/2*c)^2 + \sqrt{2}*\sin(1/2*dx + 1/2*c)^2 \\
& + 2*(\sqrt{2})*\cos(dx + c)^2*\cos(1/2*dx + 1/2*c) + \sqrt{2}*\cos(1/2*dx + 1
\end{aligned}$$

$$\begin{aligned}
& /2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c)) \\
& *\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d \\
& *x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d \\
& *x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7 \\
& /2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos \\
& (d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + \\
& c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c \\
&) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + s \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + sq \\
& rt(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + (((\sqrt{2}*\cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d \\
& *x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\sin(5/2*d*x + 5/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c))))*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)) \\
&) + 2*(((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) \\
&)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2
\end{aligned}$$

$$\begin{aligned}
& *c)) * \cos(7/2*d*x + 7/2*c) * \cos(5/2*d*x + 5/2*c) + ((\sqrt{2} * \cos(d*x + c)^2 + \\
& \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3 \\
& /2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) \\
& * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} \\
& (2) * \cos(d*x + c) + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c) \\
& ^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * s \\
& \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos \\
& s(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d* \\
& x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos \\
& (5/2*d*x + 5/2*c)^2 + ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 \\
& * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \\
& \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) \\
& * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2 \\
& *d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + s \\
& \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2 \\
& *d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} \\
& t(2) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) \\
& + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * s \\
& \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c)^2 + 2 * ((s \\
& \sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + s \\
& \sqrt{2} * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \\
& \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x \\
& + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + \\
& 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2* \\
& c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c) \\
& ^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * s \\
& \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin \\
& n(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5/2*c) + ((\sqrt{2} * \cos \\
& (d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \\
& \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2* \\
& d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x \\
& + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \\
& \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} *
\end{aligned}$$

$$\begin{aligned}
& + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&)*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (((\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c \\
&)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2* \\
& \sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c \\
&) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos \\
& (7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2 \\
& *2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/ \\
& 2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d \\
& *x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(\\
& 3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
& \sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (((\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d* \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&)*\sin(3/2*d*x + 3/2*c))^2 + \cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(\\
& d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (s \\
& rt(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c) \\
& ^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(s \\
& qrt(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*s \\
& in(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*s \\
& in(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/ \\
& 2*d*x + 3/2*c))^2 + \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1 \\
& /2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*c \\
& os(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d* \\
& x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))^*\sin(7/2*d*x \\
& + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d* \\
& x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3 \\
& /2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})*c \\
& os(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(s \\
& qrt(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))^*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5 \\
& /2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d \\
& *x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^
\end{aligned}$$

$$\begin{aligned}
& 2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos \\
& (d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)) * \sin(3/2*d*x + 3/2*c) * \sin(5/2*d*x + 5/2*c)^2 * \sin(2/3 * \arctan 2(\sin(\\
& 3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2 * ((\sqrt{2} * \cos(d*x + c)^2 + \\
& \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \\
& \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \\
& \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \\
& \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos \\
& (3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(\\
& 1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(\\
& 7/2*d*x + 7/2*c)^2 + 2 * ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + \\
& 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \\
& \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \\
& \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) \\
& + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \\
& \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c) * \cos(5/2*d*x + \\
& 5/2*c) + ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \\
& \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + \\
& c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \\
& \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2})*\co \\
& s(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*c \\
& os(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& *cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d \\
& x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\co \\
& s(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\si \\
& n(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x \\
& + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2 \\
& d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(\\
& d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})* \\
& \sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + \\
& 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + \\
& c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})* \\
& cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\\
& \sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \\
& \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) \\
& *cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d \\
& *x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 \\
&)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\cos(2/5*\arc \\
& tan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + (((\sqrt{2})*\cos(d*x + \\
& c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2 \\
& dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}) \\
& \cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx \\
& x + c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 \\
& + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2 \\
& c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(\\
& 1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + \\
& 1/2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + \\
& c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c \\
&))\cos(7/2dx + 7/2c)^2 + 2((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + \\
& c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}) \\
& \cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\\
& \sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \\
& \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}) \\
& \sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \\
& \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/ \\
& 2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) \\
& \cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) \\
& + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx \\
& x + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx \\
& + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \\
& \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(7/2dx + 7/2c)* \\
& \cos(5/2dx + 5/2c) + ((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2 \\
& \sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2 \\
& dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2}) \\
& \cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}) \\
& \sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2 \\
& dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}) \\
& \sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + s \\
& \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2 \\
& dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2} \\
& \cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) \\
& + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + c)^2s \\
& \sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2}) \\
& \sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(5/2dx + 5/2c)^2 + ((\sqrt{2} \\
& \cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2} \\
& \cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\si \\
& n(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\si \\
& n(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\\
& \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + \\
& c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2 \\
& (\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c) \\
& \sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos \\
& (1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 \\
& + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)* \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d \\
& *x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(d*x \\
& + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2 \\
& *d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2 \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& ^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\cos(5/2*d*x + 5/2*c)^2 + (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1 \\
& /2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c \\
&)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^ \\
& 2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x \\
& + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \\
& \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3 \\
& /2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2) \\
& *\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2 \\
& *c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\\
& \text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \\
& \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2) \\
& *\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + \sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\sqrt{2})\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2 \\
& *c)*\sin(d*x + c)^2 + 2*\sqrt{2})\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}) \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})\cos(1/2*d*x + 1/2*c \\
&)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})\cos(d*x + c \\
&)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2})\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(1/2*d*x + 1/2*c))*s \\
& \sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2})\cos(d*x + c)^2 + sq \\
& rt(2))*\sin(d*x + c)^2 + 2*\sqrt{2})\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2* \\
& c)^2 + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*co \\
& s(d*x + c)^2 + (\sqrt{2})\cos(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*\sqrt{2}) \\
& *\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})\cos(d*x + c)^2* \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2))*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c))*\cos(3 \\
& /2*d*x + 3/2*c) + 2*(\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2})\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})\cos(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2})\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/ \\
& 2*d*x + 7/2*c)^2 + 2*((\sqrt{2})\cos(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2* \\
& \sqrt{2})\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})\c \\
& os(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*\sqrt{2})\cos(d*x + c) + \sqrt{2}))* \\
& \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2* \\
& d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\s \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + sq \\
& rt(2))*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})\cos(d*x + c)*\cos(1/2* \\
& d*x + 1/2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
& (2))*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + \\
& 2*(\sqrt{2})\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(d*x + c)^2*\si \\
& n(1/2*d*x + 1/2*c) + 2*\sqrt{2})\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}) \\
& \sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d* \\
& x + 5/2*c) + ((\sqrt{2})\cos(d*x + c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*\sqrt{2}) \\
& \cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})\cos(d*x + \\
& c)^2 + \sqrt{2})\sin(d*x + c)^2 + 2*\sqrt{2})\cos(d*x + c) + \sqrt{2}))*\sin(3/2* \\
& d*x + 3/2*c)^2 + (\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/ \\
& 2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(\sqrt{2})\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})\co \\
& s(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})\cos(d*x + c)*\cos(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2})\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2} \\
& (2))*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})\sin(d*x + c)^2*\sin(1/2*d*
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(2/3*\arctan2 \\
& (\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& t(2)*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c \\
&)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2* \\
& \sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos \\
& os(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c \\
&) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin \\
& n(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin \\
& n(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
& (2))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2))*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2 \\
& *2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& (2))*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/ \\
& 2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d \\
& *x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(\\
& 3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})* \\
& cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
& \sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + (((\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d* \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2*\sin(d*x + c)^2 + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d* \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + \\
& c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d* \\
& x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c \\
&))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2* \\
& c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos \\
& (1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + \\
& 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2* \\
& c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d* \\
& x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&)*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d* \\
& x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos \\
& (d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}* \\
& \cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d \\
& *x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d* \\
& x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d* \\
& x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d \\
& *x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(\\
& 3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
& \sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + \\
& 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^ \\
& 2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}* \\
& \cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (((\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/ \\
& 2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2} \\
&)\cos(dx + c) + \sqrt{2}\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1 \\
& /2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2 \\
& dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2 \\
& \cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2} \\
& \cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(\\
& 3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx \\
& + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \\
& \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1 \\
& /2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(7 \\
& /2dx + 7/2c)^2 + 2((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2 \\
& \sqrt{2}\cos(dx + c) + \sqrt{2}\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2 \\
& dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2} \\
& \cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}) \\
& \sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2 \\
& dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
& \sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + \sqrt{2} \\
& \cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2 \\
& dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2} \\
& \cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) \\
& + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + c)^2\sin \\
& (1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2} \\
& \sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(7/2dx + 7/2c)\cos(5/2d \\
& x + 5/2c) + ((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2} \\
& \cos(dx + c) + \sqrt{2}\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/ \\
& 2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx \\
& + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2 \\
& dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1 \\
& /2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2 \\
& dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + \sqrt{2} \\
& \cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1 \\
& /2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos \\
& (1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2} \\
& \cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2d \\
& x + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2 \\
& dx + 1/2c))\sin(3/2dx + 3/2c))\cos(5/2dx + 5/2c)^2 + ((\sqrt{2}\cos \\
& (dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos \\
& (3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2d \\
& x + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + \\
& c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2} \\
& \cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2} \\
& \cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2} \\
& \cos(dx + c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx \\
& + c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx \\
& + 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& 2) * \sin(1/2*d*x + 1/2*c)^2 * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2 * \\
& *d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos \\
& (d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x \\
& + 3/2*c)) * \sin(7/2*d*x + 7/2*c)^2 + 2 * ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin \\
& (d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (s \\
& \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c \\
&)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x \\
& + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2 * \\
& c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d \\
& *x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d \\
& *x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + \\
& 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 \\
&) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin \\
& (d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7 \\
& /2*c) * \sin(5/2*d*x + 5/2*c) + ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c \\
&)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (s \\
& \sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + s \\
& \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2 * \\
& d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + s \\
& \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + \\
& 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d * \\
& x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + \\
& c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c)^2) \\
& * \cos(2/3 * \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \log(\cos(1/2 * \\
& \arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + \sin(1/2 * \arctan2(\sin(d*x + c), \cos(\\
& d*x + c)))^2 - 2 * \sin(1/2 * \arctan2(\sin(d*x + c), \cos(d*x + c))) + 1) - 6 * (14 * \\
& ((\sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(d*x \\
& + c) + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2 * \\
& c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + (\sqrt{2} * \cos(1/2 * \\
& d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \\
& \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d \\
& *x + 3/2*c)) * \cos(7/2*d*x + 7/2*c)^2 + 14 * ((\sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \\
& \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2 \\
& *c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * (\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) \\
& * \cos(3/2*d*x + 3/2*c) + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d \\
& *x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& 2 + 10*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + 10*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + 2*((4*\sqrt{2}*\cos(d*x \\
& + c)^2 + 4*\sqrt{2}*\sin(d*x + c)^2 + 15*\sqrt{2}*\cos(d*x + c) + 11*\sqrt{2})*\cos \\
& \cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\sqrt{2}*\cos(d*x + c)^2 + 4*\sqrt{2}*\sin \\
& (d*x + c)^2 + 15*\sqrt{2}*\cos(d*x + c) + 11*\sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 \\
& + 4*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d \\
& *x + c)^2 + 11*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*(4*\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)*\sin(d*x + c)^2 + 15*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2* \\
& c) + 11*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 15*(\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4* \\
& \sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}*\sin(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + 15*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 11*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + 10*(\\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + 10*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&)^2 + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 28*((\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x \\
& + 3/2*c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/ \\
& 2*c) - (\sqrt{2}*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 2*\sqrt{2}*\cos(3/2*d*x \\
& + 3/2*c)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c)^2 \\
& *2*\sin(d*x + c) + 2*\sqrt{2}*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)* \\
& \sin(d*x + c))*\sin(5/2*d*x + 5/2*c))*\cos(7/2*d*x + 7/2*c) + 20*(\sqrt{2}*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) \\
& ^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 20*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) - 35*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3 \\
& *sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 \\
& - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2} \\
& t(2)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d \\
& *x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 \\
& + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + (\\
& (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(\\
& d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2})*\cos(3/2*d*x + 3/2*c))*\sin(d*x + c) + 2* \\
& (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c))*\sin \\
& (1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2* \\
& c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c))*\sin(1/2*d*x + 1/2*c) - \\
& 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2
\end{aligned}$$

$$\begin{aligned}
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + \\
& 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2* \\
& *d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\co \\
& s(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3* \\
& (\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(s \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 \\
& + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12 \\
& *(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) \\
& - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin \\
& (3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2* \\
& d*x + 1/2*c))*\sin(5/2*d*x + 5/2*c) + 20*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/ \\
& 2*c))*\sin(7/2*d*x + 7/2*c) - 12*(2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 \\
& + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 \\
& + 2\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2* \\
& c)) \cos(3/2*d*x + 3/2*c) + 2(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(\\
& 1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2\sqrt{2} \cos(d*x + \\
& c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c) \\
&)) \cos(5/2*d*x + 5/2*c)^2 + (((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c) \\
&)^2 + 2\sqrt{2} \cos(d*x + c) + \sqrt{2})) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos \\
& os(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (s \\
& qrt(2) \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) + s \\
& qrt(2)) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin \\
& (1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + s \\
& qrt(2) \sin(1/2*d*x + 1/2*c)^2 + 2(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) \cos \\
& (1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + \\
& 2(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d* \\
& x + c) + 2(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + \\
& c)^2 \sin(1/2*d*x + 1/2*c) + 2\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c)^2 \\
& + 2((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + \\
& c) + \sqrt{2})) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + s \\
& qrt(2) \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + s \\
& qrt(2) \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin \\
& in(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2* \\
& c)^2 + 2(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x \\
& + 1/2*c) \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + s \\
& qrt(2) \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2(\sqrt{2} \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2(\sqrt{2} \cos(d \\
& *x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c \\
&) + 2\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2 \\
& *c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c) \cos(5/2*d*x + 5/2*c) + ((s \\
& qrt(2) \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) + s \\
& qrt(2)) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin \\
& in(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin \\
& in(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) + \sqrt{2})) \sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + \\
& c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \\
&) \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos \\
& s(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2(\sqrt{2} \cos(d*x + c)^2 \\
& 2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2\sqrt{2} \\
& qrt(2) \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin \\
& (3/2*d*x + 3/2*c)) \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& \sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + \\
& (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + \\
& c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx \\
& x + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 \\
& + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/ \\
& 2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2 \\
& *dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos \\
& (dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx \\
& + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c) \\
& ^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}) \\
& *\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + \\
& 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(7/2dx + \\
& 7/2c)\cos(5/2dx + 5/2c) + ((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + \\
& c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}) \\
& *\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + \\
& (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \\
& \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
&)*\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \\
& \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1 \\
& /2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) \\
&)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) \\
& + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx \\
& + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + \\
& c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) \\
& + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(5/2dx + 5/2c)^ \\
& 2 + ((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + \\
& c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
&)*\sin(1/2dx + 1/2c)^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2} \\
&)*\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2 \\
& *c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)*\sin \\
& (dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c) \\
& ^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx \\
& + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2} \\
&)\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + \\
& 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx \\
& + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) \\
&) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2 \\
& *c))\sin(3/2dx + 3/2c))\sin(7/2dx + 7/2c)^2 + 2((\sqrt{2}\cos(dx + c) \\
&)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx \\
& x + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c) \\
& ^2)\cos(dx + c)^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2 \\
& *\sqrt{2}\cos(dx + c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx \\
& + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos \\
& (1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + \\
& c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2
\end{aligned}$$

$$\begin{aligned}
& 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x \\
& + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 \\
& + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) \\
& + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + \\
& c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2* \\
& c) + 2*(\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos \\
& (d*x + c) + 2*(\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d \\
& *x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) \\
&) + \sqrt{2} * \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c \\
&)^2 + 2*((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d \\
& *x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 \\
& + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) \\
& * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2 \\
& *d*x + 1/2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} * \cos \\
& (d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1 \\
& /2*c) + 2*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)) * \sin(3/2*d*x + 3/2*c) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5/2*c) + \\
& ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) \\
& + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} \\
& * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d \\
& *x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1 \\
& /2*c) * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&) * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} * \cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(\sqrt{2} * \cos(d*x + \\
& c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) \\
& * \sin(3/2*d*x + 3/2*c) * \sin(5/2*d*x + 5/2*c)^2 * \cos(2/3 * \arctan^2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin \\
& (d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2})) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c \\
&)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x \\
& + c) + \sqrt{2})) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*
\end{aligned}$$

$$\begin{aligned}
& c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d \\
& *x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d \\
& *x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + \\
& 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 \\
&) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin \\
& (d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \sin(7/2*d*x + 7 \\
& /2*c)^2 + 2*((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos \\
& (d*x + c) + \sqrt{2}) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + \\
& c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2 \\
& *c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos \\
& (1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \\
& (2) \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d \\
& *x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \sin(7/2*d*x + 7/2*c) \sin(5/2*d*x + 5/2*c \\
&) + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + \\
& c) + \sqrt{2}) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \\
& \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin \\
& (d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x \\
& + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d \\
& *x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2 \\
& *c)) \sin(3/2*d*x + 3/2*c)) \sin(5/2*d*x + 5/2*c)^2 + (((\sqrt{2} \cos(d*x + c) \\
& ^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \cos(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c \\
&)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2* \\
& \sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x \\
& + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + \\
& 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \\
&) \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/ \\
& 2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \\
& * \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c))
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2 \\
& *d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& *\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2* \\
& d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(\\
& d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& ^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d* \\
& x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3 \\
& /2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) \\
& *\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/ \\
& 2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^ \\
& 2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})* \\
& \cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d* \\
& x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^ \\
& 2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d* \\
& x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d* \\
& x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3 \\
& /2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 \\
& + 2\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2* \\
& c)) \cos(3/2*d*x + 3/2*c) + 2(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(\\
& 1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2\sqrt{2} \cos(d*x + \\
& c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c) \\
&)) \sin(5/2*d*x + 5/2*c)^2 + (((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c) \\
&)^2 + 2\sqrt{2} \cos(d*x + c) + \sqrt{2}) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos \\
& os(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (s \\
& qrt(2) \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) + s \\
& qrt(2)) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin \\
& (1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + s \\
& qrt(2) \sin(1/2*d*x + 1/2*c)^2 + 2(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) \cos \\
& (1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + \\
& 2(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d* \\
& x + c) + 2(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + \\
& c)^2 \sin(1/2*d*x + 1/2*c) + 2\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c)^2 \\
& + 2(((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + \\
& c) + \sqrt{2}) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + s \\
& qrt(2) \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + s \\
& qrt(2) \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin(3/2*d*x + 3/2 \\
& *c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin \\
& in(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2* \\
& c)^2 + 2(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x \\
& + 1/2*c) \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + s \\
& qrt(2) \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2(\sqrt{2} \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2(\sqrt{2} \cos(d \\
& *x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c \\
&) + 2\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2 \\
& *c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c) \cos(5/2*d*x + 5/2*c) + ((s \\
& qrt(2) \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) + s \\
& qrt(2)) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin \\
& in(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin \\
& in(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + \\
& c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \\
&) \sin(d*x + c)^2 + 2\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos \\
& s(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2(\sqrt{2} \cos(d*x + c)^2 \\
& 2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2\sqrt{2} \\
& qrt(2) \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin \\
& (3/2*d*x + 3/2*c)) \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2}
\end{aligned}$$

$c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2$
 $*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) *$
 $\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*$
 $d*x + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*$
 $c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) *$
 $\sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c) \sin(3/2*d*x + 3/2*c) *$
 $\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2$
 $+ 2*\sqrt{2} \cos(d*x + c) + \sqrt{2})) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos($
 $1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{$
 $2) \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{$
 $2) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin$
 $(1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{$
 $2) \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c)$
 $+ \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos$
 $(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \cos(3/2*d*x + 3/2*c) + 2*$
 $(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x +$
 $c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)$
 $^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{$
 $2) \sin(1/2*d*x + 1/2*c) \sin(3/2*d*x + 3/2*c) \cos(7/2*d*x + 7/2*c) \cos(5$
 $/2*d*x + 5/2*c) + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{$
 $2) \cos(d*x + c) + \sqrt{2})) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x$
 $+ 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos($
 $d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin$
 $(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x$
 $+ 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin($
 $1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2}$
 $) \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x$
 $+ 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}$
 $* \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*$
 $(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1$
 $/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin$
 $(1/2*d*x + 1/2*c) \sin(3/2*d*x + 3/2*c) \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}$
 $* \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2})$
 $) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/$
 $2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*$
 $x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{$
 $2) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2$
 $+ \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{$
 $2) \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin$
 $(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2$
 $*d*x + 1/2*c) \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{$
 $2) \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin$
 $(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}$
 $* \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c) \sin(3/2*$
 $d*x + 3/2*c) \sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2}$

$$\begin{aligned}
& * \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2} \cos(3/2 dx + 3/2 c)^2 \\
& + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx \\
& + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx \\
& + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 \\
& + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + \\
& 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1 \\
& /2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos \\
& os(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 dx \\
& x + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 \\
& c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \\
& 2) \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx \\
& + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c) \sin(7/2 dx \\
& + 7/2 c) \sin(5/2 dx + 5/2 c) + ((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
& + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \\
& 2) \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 \\
& + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \\
& + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \\
& 2) \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2 \\
& + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos(dx + c)^2 \cos(1/2 dx + \\
& 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + \\
& c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 \\
& c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos \\
& s(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(dx \\
& * x + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx + c) \sin(1/2 dx + 1/2 c \\
&) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)) \sin(5/2 dx + 5/2 c \\
&)^2 \cos(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) \sin(2/5 \\
& \arctan2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c)))^2 + (((\sqrt{2} \cos(dx \\
& + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/ \\
& 2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + \\
& 1/2 c)^2) \cos(dx + c)^2 + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 \\
& + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(\\
& 1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \\
& 2) \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2 + 2(\sqrt{2} \cos \\
& (dx + c)^2 \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1/2 c) \sin(dx + c \\
&)^2 + 2\sqrt{2} \cos(dx + c) \cos(1/2 dx + 1/2 c) + \sqrt{2} \cos(1/2 dx + 1 \\
& /2 c)) \cos(3/2 dx + 3/2 c) + 2(\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin \\
& in(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2(\sqrt{2} \cos(dx + c)^2 \sin(1/2 dx \\
& + 1/2 c) + \sqrt{2} \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2\sqrt{2} \cos(dx \\
& + c) \sin(1/2 dx + 1/2 c) + \sqrt{2} \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/ \\
& 2 c)) \cos(7/2 dx + 7/2 c)^2 + 2((\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx \\
& + c)^2 + 2\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos(3/2 dx + 3/2 c)^2 + (\sqrt{2} \\
& 2) \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 \\
& + (\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \sin(dx + c)^2 + 2\sqrt{2} \cos(dx + c) \\
& + \sqrt{2}) \sin(3/2 dx + 3/2 c)^2 + (\sqrt{2} \cos(1/2 dx + 1/2 c)^2 + \sqrt{2} \\
& 2) \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \sqrt{2} \cos(1/2 dx + 1/2 c)^2
\end{aligned}$$

$$\begin{aligned}
& + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + \\
& c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2* \\
& c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos \\
& s(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d \\
& *x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c \\
&) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c) \cos(7/2*d*x + 7/2*c \\
&) \cos(5/2*d*x + 5/2*c) + ((\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 \\
& + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \\
&) \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2} \\
&) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(\\
& 1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&) \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2*(\\
& \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + \\
& c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^ \\
& 2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&) \sin(1/2*d*x + 1/2*c) \sin(3/2*d*x + 3/2*c) \cos(5/2*d*x + 5/2*c)^2 + ((\\
& \sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \\
& \sqrt{2}) \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&) \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \\
&) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x \\
& + c)^2 + \sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(\sqrt{2} \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2 \\
& *c) \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \\
& \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c \\
&)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c \\
&)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2} \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin \\
& in(3/2*d*x + 3/2*c) \sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2} \cos(d*x + c)^2 + \\
& \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \cos(d*x + c) + \sqrt{2}) \cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) * \\
& \cos(d*x + c)^2 + (\sqrt{2} \cos(d*x + c)^2 + \sqrt{2} \sin(d*x + c)^2 + 2*\sqrt{2} \\
&) \cos(d*x + c) + \sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2} \cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \cos(d*x + c)^ \\
& 2 \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2} \\
&) \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + \sqrt{2} \cos(1/2*d*x + 1/2*c)) \cos \\
& (3/2*d*x + 3/2*c) + 2*(\sqrt{2} \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \sin(1/2*d*x \\
& + 1/2*c)^2) \cos(d*x + c) + 2*(\sqrt{2} \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2} \sin(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \cos(d*x + c) \sin(\\
& 1/2*d*x + 1/2*c) + \sqrt{2} \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c) \sin(
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2 \\
& * \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt} \\
& \text{t}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(\\
& 3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1 \\
& /2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5 \\
& /2*d*x + 5/2*c)^2 + (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2* \\
& \text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\text{c} \\
& \text{os}(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))* \\
& \sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\text{s} \\
& \text{in}(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sq} \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2* \\
& d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + \\
& 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\text{s} \\
& \text{i}(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\text{sq} \\
& \text{rt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sq} \\
& \text{rt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\text{s} \\
& \text{in}(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\text{s} \\
& \text{in}(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + \\
& (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
&)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\text{c} \\
& \text{os}(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^ \\
& 2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sq} \\
& \text{rt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin \\
& (3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\text{c} \\
& \text{os}(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\text{c} \\
& \text{os}(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2) \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d* \\
& x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d* \\
& x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\text{c} \\
& \text{os}(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x
\end{aligned}$$

$$\begin{aligned}
& /2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\co \\
& s(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x \\
& + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/ \\
& 2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\co \\
& s(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sq \\
& rt(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/ \\
& 2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d* \\
& x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2})*\co \\
& s(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + (((\sqrt{2}*\cos(d*x + \\
& c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2* \\
& d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d \\
& *x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^ \\
& 2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*
\end{aligned}$$

$$\begin{aligned}
& c)) * \cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
&)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)* \\
& \cos(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
& *\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2} \\
& *\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin \\
& (d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + \\
& (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + \\
& c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2 \\
& *2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin \\
& (3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
& *\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2* \\
& c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& *\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3 \\
& /2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/ \\
& 2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\si \\
& n(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})* \\
& \sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + \\
& 5/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \\
& 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sq \\
& rt(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sq \\
& rt(2)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2* \\
& c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\si \\
& n(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sq \\
& rt(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d* \\
& x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c) \\
& ^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2* \\
& \sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\co \\
& s(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
& 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c) \\
& *\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))* \\
& \cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^ \\
& 2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2* \\
& ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^ \\
& 2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d \\
& *x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c)) \\
& *\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^ \\
& 2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\si \\
& n(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\si \\
& n(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2 \\
& *d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
& , \cos(3/2*d*x + 3/2*c))))*\sin(3/5*\arctan2(\sin(5/2*d*x + 5/2*c) , \cos(5/2*d*x \\
& + 5/2*c))) - 420*(((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos \\
& (d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\si
\end{aligned}$$

$$\begin{aligned}
& n(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d* \\
& x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2 \\
& *(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\text{si} \\
& n(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt} \\
& (2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt} \\
& (2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin \\
& (d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c \\
&)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\\
& \text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)* \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt} \\
& (2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3 \\
& /2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt}(2)*\cos(\\
& d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos \\
& (3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c \\
&)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + sq \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(\\
& d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d \\
& *x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{qr} \\
& \text{t}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^ \\
& 2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + \\
& c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + sq \\
& \text{rt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x \\
& + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/ \\
& 2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)* \\
& \cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& 2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
& + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^ \\
& 2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
& 2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/ \\
& 2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& 2)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d \\
& *x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(\\
& 3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& 2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
& \sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\cos(2/3*\text{arc} \\
& \tan^2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\sqrt{2}*\cos(d*x + c) \\
&)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d* \\
& x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2 \\
& *\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\c \\
& os(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x \\
& + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c) \\
&)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c) \\
&)*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c) \\
&)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\c \\
& os(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\s \\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \s \\
& \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sqrt{2}))*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2 \\
& *c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)* \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x +
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))* \\
& \sin(5/2*d*x + 5/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + \\
& 3/2*c)))^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
& (2)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d \\
& *x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(\\
& 3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
& \sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d \\
& *x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^ \\
& 2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(sq \\
& rt(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\si \\
& n(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\si \\
& n(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/ \\
& 2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d* \\
& x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^ \\
& 2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + sqrt \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\co \\
& s(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d* \\
& x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d* \\
& x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3 \\
& /2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x \\
& + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + \\
& (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& 2) * \sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + \\
& c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c \\
&) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos \\
& (d*x + c) + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d* \\
& x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) \\
& + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c) \\
& ^2 + 2 * ((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d* \\
& x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 \\
& + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 \\
& + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + \\
& 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 \\
&) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2* \\
& d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \\
& \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos \\
& (d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/ \\
& 2*c) + 2 * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + \\
& 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5/2*c) + (\\
& (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \\
& \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&) * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} \\
&) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \sin(3/2*d*x + 3/2*c)^2 \\
& + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \sin(d* \\
& x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 \\
& + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/ \\
& 2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + \sqrt{2} \\
& * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 * (\sqrt{2} * \cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2 * (\sqrt{2} * \cos(d*x + \\
& c)^2 * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 \\
& * \sqrt{2} * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \\
& \sin(3/2*d*x + 3/2*c)) * \sin(5/2*d*x + 5/2*c)^2 * \cos(2/3 * \arctan2(\sin(3/2*d*x + \\
& 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(1/5 * \arctan2(\sin(5/2*d*x + 5/2*c), \cos(\\
& 5/2*d*x + 5/2*c))) - 1260 * (((\sqrt{2} * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 \\
& + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos \\
& (1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + (\sqrt{2} \\
& * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) + \sqrt{2} \\
&) * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin \\
& (1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + \sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c)^2 + 2 * (\sqrt{2} * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) \\
&) + \sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2 * \sqrt{2} * \cos(d*x + c) * \cos \\
& (1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2 \\
& * (\sqrt{2} * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x
\end{aligned}$$

$$\begin{aligned}
& + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c) \\
&)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sq} \\
& \text{rt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + \\
& 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) \\
&) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sq} \\
& \text{rt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sq} \\
& \text{rt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c) \\
&)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
&))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((\text{sqrt} \\
& (2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt} \\
& (2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin \\
& (d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (s \\
& \text{qrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c) \\
&)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\\
& \text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)* \\
& \sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt} \\
& (2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3 \\
& /2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2) \\
&)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 \\
& + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(\\
& 1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)* \\
& \cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d \\
& *x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt} \\
& (2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d* \\
& x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d* \\
& x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt} \\
& (2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d \\
& *x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(
\end{aligned}$$

$$\begin{aligned}
& 3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
& \text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + \\
& 5/2*c) + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(\\
& d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^ \\
& 2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x \\
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)* \\
& \cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(1/3*\arctan2(\sin \\
& (3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 56*(((\text{sqrt}(2)*\cos(d*x + c)^2 + \\
& \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)* \\
& \cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(\\
& 2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^ \\
& 2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sq} \\
& \text{rt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(\\
& 1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(\\
& 7/2*d*x + 7/2*c)^2 + 2*((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + \\
& 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2) \\
& *\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2) \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/ \\
& 2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sq} \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2* \\
& d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x \\
& + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/ \\
& 2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}* \\
& \cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*co \\
& s(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(sq \\
& rt(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((\sqrt{2}*co \\
& s(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*c \\
& os(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2} \\
&)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d* \\
& x + c)^2 + 2*\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2}*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*co \\
& s(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*si \\
& n(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2* \\
& d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(\\
& d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2}* \\
& \sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + \\
& 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + \\
& c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}* \\
& \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\\
& \sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) +
\end{aligned}$$

$$\begin{aligned}
& \sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \\
& * \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + \sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}) \cos(d*x + c)^2 \cos(1/2*d*x + 1/ \\
& 2*c) + \sqrt{2}) \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 2*\sqrt{2}) \cos(d*x + c) \\
& * \cos(1/2*d*x + 1/2*c) + \sqrt{2}) \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) \\
& + 2*(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \cos(d \\
& *x + c) + 2*(\sqrt{2}) \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + \sqrt{2}) \sin(d*x \\
& + c)^2 \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}) \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}) \sin(1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \sin(5/2*d*x + 5/2*c)^2 \\
&) \sin(3/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) - 84*(((4*\sqrt{2}) \cos(d*x + \\
& c)^2 + 4*\sqrt{2}) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) + 5*\sqrt{2}) \cos(3 \\
& /2*d*x + 3/2*c)^2 + 4*(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x \\
& + 1/2*c)^2) \cos(d*x + c)^2 + (4*\sqrt{2}) \cos(d*x + c)^2 + 4*\sqrt{2}) \sin(d*x \\
& + c)^2 + 9*\sqrt{2}) \cos(d*x + c) + 5*\sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 + 4*(s \\
& \sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \sin(d*x + c \\
&)^2 + 5*\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}) \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*(4*\sqrt{2}) \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}) \cos(1/2*d*x + \\
& 1/2*c) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) \cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}) \\
& \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}) \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(4*\sqrt{2}) \cos \\
& (d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}) \sin(d*x + c)^2 \sin(1/2*d*x + 1 \\
& /2*c) + 9*\sqrt{2}) \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}) \sin(1/2*d*x \\
& + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c)^2 + 2*((4*\sqrt{2}) \cos \\
& (d*x + c)^2 + 4*\sqrt{2}) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) + 5*\sqrt{2}) \\
&) \cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(\\
& 1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (4*\sqrt{2}) \cos(d*x + c)^2 + 4*\sqrt{2}) \sin \\
& (d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) + 5*\sqrt{2}) \sin(3/2*d*x + 3/2*c)^2 \\
& + 4*(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \sin(\\
& d*x + c)^2 + 5*\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}) \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*(4*\sqrt{2}) \cos(d*x + c)^2 \cos(1/2*d*x + 1/2*c) + 4*\sqrt{2}) \cos(1/ \\
& 2*d*x + 1/2*c) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) \cos(1/2*d*x + 1/2*c) \\
& + 5*\sqrt{2}) \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/2*c) + 9*(\sqrt{2}) \cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c) + 2*(4*\sqrt{2}) \\
& \cos(d*x + c)^2 \sin(1/2*d*x + 1/2*c) + 4*\sqrt{2}) \sin(d*x + c)^2 \sin(1/2* \\
& d*x + 1/2*c) + 9*\sqrt{2}) \cos(d*x + c) \sin(1/2*d*x + 1/2*c) + 5*\sqrt{2}) \sin(\\
& 1/2*d*x + 1/2*c)) \sin(3/2*d*x + 3/2*c)) \cos(7/2*d*x + 7/2*c) \cos(5/2*d*x + \\
& 5/2*c) + ((4*\sqrt{2}) \cos(d*x + c)^2 + 4*\sqrt{2}) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos \\
& (d*x + c) + 5*\sqrt{2}) \cos(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}) \cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2}) \sin(1/2*d*x + 1/2*c)^2) \cos(d*x + c)^2 + (4*\sqrt{2}) \cos \\
& (d*x + c)^2 + 4*\sqrt{2}) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + c) + 5*\sqrt{2}) \\
&) \sin(3/2*d*x + 3/2*c)^2 + 4*(\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}) \sin(\\
& 1/2*d*x + 1/2*c)^2) \sin(d*x + c)^2 + 5*\sqrt{2}) \cos(1/2*d*x + 1/2*c)^2 + 5*\sqrt{2}) \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*(4*\sqrt{2}) \cos(d*x + c)^2 \cos(1/2*d*x + 1 \\
& /2*c) + 4*\sqrt{2}) \cos(1/2*d*x + 1/2*c) \sin(d*x + c)^2 + 9*\sqrt{2}) \cos(d*x + \\
& c) \cos(1/2*d*x + 1/2*c) + 5*\sqrt{2}) \cos(1/2*d*x + 1/2*c)) \cos(3/2*d*x + 3/
\end{aligned}$$

$$\begin{aligned}
& 2*c) + 9*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)* \\
& \cos(d*x + c) + 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2) \\
& *\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + \\
& 1/2*c) + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x \\
& + 5/2*c)^2 + ((4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt} \\
& t(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4*\text{sqrt}(2) \\
&)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt} \\
& rt(2))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 \\
& + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d* \\
& x + 1/2*c) + 4*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(\\
& d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 9*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c \\
&)^2)*\cos(d*x + c) + 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 4*\text{sqrt} \\
& rt(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/ \\
& 2*d*x + 7/2*c)^2 + 2*((4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\text{c} \\
& os(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (4 \\
& *\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) \\
& + 5*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5*\text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2*\cos \\
& (1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 9*\text{sqrt}(\\
& 2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(\\
& 3/2*d*x + 3/2*c) + 9*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + 4*\text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 9*\text{sqrt}(2)*\cos(d*x + c)*\text{s} \\
& in(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)) \\
& *\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4 \\
& *\text{sqrt}(2)*\sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\cos(3/2*d*x + \\
& 3/2*c)^2 + 4*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c \\
&)^2)*\cos(d*x + c)^2 + (4*\text{sqrt}(2)*\cos(d*x + c)^2 + 4*\text{sqrt}(2)*\sin(d*x + c)^2 \\
& + 9*\text{sqrt}(2)*\cos(d*x + c) + 5*\text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + 4*(\text{sqrt}(2)*\text{c} \\
& os(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + 5* \\
& \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(4*\text{sqrt} \\
& rt(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)* \\
& \sin(d*x + c)^2 + 9*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\text{c} \\
& os(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 9*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(4*\text{sqrt}(2)*\cos(d*x + c \\
&)^2*\sin(1/2*d*x + 1/2*c) + 4*\text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 9*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + 5*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c \\
&))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2)*\sin(1/2*\arctan2(\sin(d*x + \\
& c), \cos(d*x + c))))*\text{sqrt}(a)/((((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*c
\end{aligned}$$

$$\begin{aligned}
&^2 + (a\cos(dx + c)^2 + a\sin(dx + c)^2 + 2a\cos(dx + c) + a)\sin(3/2dx \\
&*x + 3/2c)^2 + (a\cos(1/2dx + 1/2c)^2 + a\sin(1/2dx + 1/2c)^2)\sin(dx \\
&*x + c)^2 + a\sin(1/2dx + 1/2c)^2 + 2*(a\cos(dx + c)^2*\cos(1/2dx + 1/ \\
&2c) + a\cos(1/2dx + 1/2c)*\sin(dx + c)^2 + 2*a\cos(dx + c)*\cos(1/2dx \\
&+ 1/2c) + a\cos(1/2dx + 1/2c))*\cos(3/2dx + 3/2c) + 2*(a\cos(1/2dx \\
&+ 1/2c)^2 + a\sin(1/2dx + 1/2c)^2)*\cos(dx + c) + 2*(a\cos(dx + c)^2* \\
&\sin(1/2dx + 1/2c) + a\sin(dx + c)^2*\sin(1/2dx + 1/2c) + 2*a\cos(dx \\
&+ c)*\sin(1/2dx + 1/2c) + a\sin(1/2dx + 1/2c))*\sin(3/2dx + 3/2c))*c \\
&os(7/2dx + 7/2c)*\cos(5/2dx + 5/2c) + ((a\cos(dx + c)^2 + a\sin(dx + \\
&c)^2 + 2a\cos(dx + c) + a)*\cos(3/2dx + 3/2c)^2 + (a\cos(1/2dx + 1/ \\
&2c)^2 + a\sin(1/2dx + 1/2c)^2)*\cos(dx + c)^2 + a\cos(1/2dx + 1/2c)^2 \\
&+ (a\cos(dx + c)^2 + a\sin(dx + c)^2 + 2a\cos(dx + c) + a)*\sin(3/2dx \\
&+ 3/2c)^2 + (a\cos(1/2dx + 1/2c)^2 + a\sin(1/2dx + 1/2c)^2)*\sin(dx \\
&+ c)^2 + a\sin(1/2dx + 1/2c)^2 + 2*(a\cos(dx + c)^2*\cos(1/2dx + 1/2* \\
&c) + a\cos(1/2dx + 1/2c)*\sin(dx + c)^2 + 2*a\cos(dx + c)*\cos(1/2dx + \\
&1/2c) + a\cos(1/2dx + 1/2c))*\cos(3/2dx + 3/2c) + 2*(a\cos(1/2dx + \\
&1/2c)^2 + a\sin(1/2dx + 1/2c)^2)*\cos(dx + c) + 2*(a\cos(dx + c)^2*si \\
&n(1/2dx + 1/2c) + a\sin(dx + c)^2*\sin(1/2dx + 1/2c) + 2*a\cos(dx + \\
&c)*\sin(1/2dx + 1/2c) + a\sin(1/2dx + 1/2c))*\sin(3/2dx + 3/2c))*cos \\
&(5/2dx + 5/2c)^2 + (((a\cos(dx + c)^2 + a\sin(dx + c)^2 + 2a\cos(dx \\
&+ c) + a)*\cos(3/2dx + 3/2c)^2 + (a\cos(1/2dx + 1/2c)^2 + a\sin(1/2dx \\
&x + 1/2c)^2)*\cos(dx + c)^2 + a\cos(1/2dx + 1/2c)^2 + (a\cos(dx + c)^2 \\
&+ a\sin(dx + c)^2 + 2a\cos(dx + c) + a)*\sin(3/2dx + 3/2c)^2 + (a\cos \\
&(1/2dx + 1/2c)^2 + a\sin(1/2dx + 1/2c)^2)*\sin(dx + c)^2 + a\sin(1/2* \\
&dx + 1/2c)^2 + 2*(a\cos(dx + c)^2*\cos(1/2dx + 1/2c) + a\cos(1/2dx + \\
&1/2c)*\sin(dx + c)^2 + 2*a\cos(dx + c)*\cos(1/2dx + 1/2c) + a\cos(1/2* \\
&dx + 1/2c))*\cos(3/2dx + 3/2c) + 2*(a\cos(1/2dx + 1/2c)^2 + a\sin(1/ \\
&2dx + 1/2c)^2)*\cos(dx + c) + 2*(a\cos(dx + c)^2*\sin(1/2dx + 1/2c) + \\
&a\sin(dx + c)^2*\sin(1/2dx + 1/2c) + 2*a\cos(dx + c)*\sin(1/2dx + 1/ \\
&2c) + a\sin(1/2dx + 1/2c))*\sin(3/2dx + 3/2c))*\cos(7/2dx + 7/2c)^2 \\
&+ 2*((a\cos(dx + c)^2 + a\sin(dx + c)^2 + 2a\cos(dx + c) + a)*\cos(3/2dx \\
&*x + 3/2c)^2 + (a\cos(1/2dx + 1/2c)^2 + a\sin(1/2dx + 1/2c)^2)*\cos(dx \\
&*x + c)^2 + a\cos(1/2dx + 1/2c)^2 + (a\cos(dx + c)^2 + a\sin(dx + c)^2 \\
&+ 2a\cos(dx + c) + a)*\sin(3/2dx + 3/2c)^2 + (a\cos(1/2dx + 1/2c)^2 \\
&+ a\sin(1/2dx + 1/2c)^2)*\sin(dx + c)^2 + a\sin(1/2dx + 1/2c)^2 + 2* \\
&(a\cos(dx + c)^2*\cos(1/2dx + 1/2c) + a\cos(1/2dx + 1/2c)*\sin(dx + c \\
&)^2 + 2*a\cos(dx + c)*\cos(1/2dx + 1/2c) + a\cos(1/2dx + 1/2c))*\cos(3 \\
&/2dx + 3/2c) + 2*(a\cos(1/2dx + 1/2c)^2 + a\sin(1/2dx + 1/2c)^2)*c \\
&os(dx + c) + 2*(a\cos(dx + c)^2*\sin(1/2dx + 1/2c) + a\sin(dx + c)^2*s \\
&in(1/2dx + 1/2c) + 2*a\cos(dx + c)*\sin(1/2dx + 1/2c) + a\sin(1/2dx \\
&+ 1/2c))*\sin(3/2dx + 3/2c))*\cos(7/2dx + 7/2c)*\cos(5/2dx + 5/2c) \\
&+ ((a\cos(dx + c)^2 + a\sin(dx + c)^2 + 2a\cos(dx + c) + a)*\cos(3/2dx \\
&+ 3/2c)^2 + (a\cos(1/2dx + 1/2c)^2 + a\sin(1/2dx + 1/2c)^2)*\cos(dx \\
&+ c)^2 + a\cos(1/2dx + 1/2c)^2 + (a\cos(dx + c)^2 + a\sin(dx + c)^2 + \\
&2a\cos(dx + c) + a)*\sin(3/2dx + 3/2c)^2 + (a\cos(1/2dx + 1/2c)^2 + \\
&a\cos(1/2dx + 1/2c)^2)^2
\end{aligned}$$

$$\begin{aligned}
& a \sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)^2 + a \sin(1/2*d*x + 1/2*c)^2 + 2*(a \\
& * \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + a \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 \\
& + 2*a * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + a \cos(1/2*d*x + 1/2*c) * \cos(3/2 \\
& *d*x + 3/2*c) + 2*(a \cos(1/2*d*x + 1/2*c)^2 + a \sin(1/2*d*x + 1/2*c)^2) * \cos \\
& (d*x + c) + 2*(a \cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + a \sin(d*x + c)^2 * \sin \\
& (1/2*d*x + 1/2*c) + 2*a * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + a \sin(1/2*d*x + \\
& 1/2*c) * \sin(3/2*d*x + 3/2*c) * \cos(5/2*d*x + 5/2*c)^2 + ((a \cos(d*x + c)^2 \\
& + a \sin(d*x + c)^2 + 2*a * \cos(d*x + c) + a) * \cos(3/2*d*x + 3/2*c)^2 + (a \cos(\\
& 1/2*d*x + 1/2*c)^2 + a \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + a \cos(1/2*d \\
& *x + 1/2*c)^2 + (a \cos(d*x + c)^2 + a \sin(d*x + c)^2 + 2*a * \cos(d*x + c) + a \\
&) * \sin(3/2*d*x + 3/2*c)^2 + (a \cos(1/2*d*x + 1/2*c)^2 + a \sin(1/2*d*x + 1/2* \\
& c)^2) * \sin(d*x + c)^2 + a \sin(1/2*d*x + 1/2*c)^2 + 2*(a \cos(d*x + c)^2 * \cos(1 \\
& /2*d*x + 1/2*c) + a \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*a * \cos(d*x + c) * \\
& \cos(1/2*d*x + 1/2*c) + a \cos(1/2*d*x + 1/2*c) * \cos(3/2*d*x + 3/2*c) + 2*(a \cos \\
& (1/2*d*x + 1/2*c)^2 + a \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(a \cos(\\
& d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + a \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2 \\
& * a * \cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + a \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x \\
& + 3/2*c) * \sin(7/2*d*x + 7/2*c)^2 + 2*((a \cos(d*x + c)^2 + a \sin(d*x + c)^2 \\
& + 2*a * \cos(d*x + c) + a) * \cos(3/2*d*x + 3/2*c)^2 + (a \cos(1/2*d*x + 1/2*c)^2 \\
& + a \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + a \cos(1/2*d*x + 1/2*c)^2 + (a \cos \\
& (d*x + c)^2 + a \sin(d*x + c)^2 + 2*a * \cos(d*x + c) + a) * \sin(3/2*d*x + 3/2 \\
& *c)^2 + (a \cos(1/2*d*x + 1/2*c)^2 + a \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 \\
& + a \sin(1/2*d*x + 1/2*c)^2 + 2*(a \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + a \\
& * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*a * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c \\
&) + a \cos(1/2*d*x + 1/2*c) * \cos(3/2*d*x + 3/2*c) + 2*(a \cos(1/2*d*x + 1/2*c \\
&)^2 + a \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(a \cos(d*x + c)^2 * \sin(1/2* \\
& d*x + 1/2*c) + a \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*a * \cos(d*x + c) * \sin \\
& (1/2*d*x + 1/2*c) + a \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(7/2*d \\
& *x + 7/2*c) * \sin(5/2*d*x + 5/2*c) + ((a \cos(d*x + c)^2 + a \sin(d*x + c)^2 + \\
& 2*a * \cos(d*x + c) + a) * \cos(3/2*d*x + 3/2*c)^2 + (a \cos(1/2*d*x + 1/2*c)^2 + \\
& a \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + a \cos(1/2*d*x + 1/2*c)^2 + (a \cos \\
& (d*x + c)^2 + a \sin(d*x + c)^2 + 2*a * \cos(d*x + c) + a) * \sin(3/2*d*x + 3/2*c \\
&)^2 + (a \cos(1/2*d*x + 1/2*c)^2 + a \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 \\
& + a \sin(1/2*d*x + 1/2*c)^2 + 2*(a \cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + a \cos \\
& (1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*a * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) \\
& + a \cos(1/2*d*x + 1/2*c) * \cos(3/2*d*x + 3/2*c) + 2*(a \cos(1/2*d*x + 1/2*c) \\
&)^2 + a \sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(a \cos(d*x + c)^2 * \sin(1/2*d* \\
& x + 1/2*c) + a \sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*a * \cos(d*x + c) * \sin(1 \\
& /2*d*x + 1/2*c) + a \sin(1/2*d*x + 1/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(5/2*d*x \\
& + 5/2*c)^2 * \cos(2/3 * \arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 \\
& + ((a \cos(d*x + c)^2 + a \sin(d*x + c)^2 + 2*a * \cos(d*x + c) + a) * \cos(3/2*d*x \\
& + 3/2*c)^2 + (a \cos(1/2*d*x + 1/2*c)^2 + a \sin(1/2*d*x + 1/2*c)^2) * \cos(d* \\
& x + c)^2 + a \cos(1/2*d*x + 1/2*c)^2 + (a \cos(d*x + c)^2 + a \sin(d*x + c)^2 \\
& + 2*a * \cos(d*x + c) + a) * \sin(3/2*d*x + 3/2*c)^2 + (a \cos(1/2*d*x + 1/2*c)^2 \\
& + a \sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + a \sin(1/2*d*x + 1/2*c)^2 + 2*(
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x \\
& + 1/2*c)^2) * \cos(d*x + c) + 2*(a*\cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + a*\sin \\
& (d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + \\
& a*\sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(7/2*d*x + 7/2*c) * \cos(5/2 \\
& *d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + \\
& a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2 \\
& *c)^2) * \cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin \\
& (d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d* \\
& x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + a*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*(a*\cos(d*x + c)^2 * \cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c) \\
& * \sin(d*x + c)^2 + 2*a*\cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1 \\
& /2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + \\
& 1/2*c)^2) * \cos(d*x + c) + 2*(a*\cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + a*\sin(\\
& d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + a \\
& * \sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2*c)) * \cos(5/2*d*x + 5/2*c)^2 + ((a*\cos \\
& (d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2* \\
& c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 \\
& + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos \\
& (d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(\\
& 1/2*d*x + 1/2*c)^2) * \sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d* \\
& x + c)^2 * \cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*a \\
& * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + \\
& 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + \\
& c) + 2*(a*\cos(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2 * \sin(1/2*d* \\
& x + 1/2*c) + 2*a*\cos(d*x + c) * \sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c) \\
&) * \sin(3/2*d*x + 3/2*c)) * \sin(7/2*d*x + 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + a*\sin \\
& (d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d \\
& *x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + a*\cos(1/2*d*x + \\
& 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin \\
& (3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2) \\
& * \sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2 * \cos(1/2*d* \\
& x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*a*\cos(d*x + c) * \cos(1 \\
& /2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1 \\
& /2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(a*\cos(d*x + \\
& c)^2 * \sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2 * \sin(1/2*d*x + 1/2*c) + 2*a*\cos \\
& (d*x + c) * \sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c)) * \sin(3/2*d*x + 3/2 \\
& *c)) * \sin(7/2*d*x + 7/2*c) * \sin(5/2*d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin \\
& (d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x \\
& + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3 \\
& /2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2) * \sin \\
& (d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2 * \cos(1/2*d*x \\
& + 1/2*c) + a*\cos(1/2*d*x + 1/2*c) * \sin(d*x + c)^2 + 2*a*\cos(d*x + c) * \cos(1/2 \\
& *d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2 \\
& *d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2) * \cos(d*x + c) + 2*(a*\cos(d*x + c \\
\end{aligned}$$

$$\begin{aligned}
&)^2 \sin(1/2 dx + 1/2 c) + a \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 a \cos(dx + c) \sin(1/2 dx + 1/2 c) + a \sin(1/2 dx + 1/2 c) \\
&)) \sin(5/2 dx + 5/2 c)^2 \sin(2/3 \arctan 2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c)))^2 + 2 * ((a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 a \cos(dx + c) \\
&)) + a) \cos(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + a \cos(1/2 dx + 1/2 c)^2 + (a \cos(dx + c)^2 + \\
&a \sin(dx + c)^2 + 2 a \cos(dx + c) + a) \sin(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + a \sin(1/2 dx \\
&+ 1/2 c)^2 + 2 (a \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + a \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2 a \cos(dx + c) \cos(1/2 dx + 1/2 c) + a \cos(1/2 dx \\
&+ 1/2 c)) \cos(3/2 dx + 3/2 c) + 2 (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 (a \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + a \\
&\sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 a \cos(dx + c) \sin(1/2 dx + 1/2 c) + a \sin(1/2 dx + 1/2 c)) \sin(3/2 dx + 3/2 c) \cos(7/2 dx + 7/2 c)^2 + 2 \\
&* ((a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 a \cos(dx + c) + a) \cos(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx \\
&+ c)^2 + a \cos(1/2 dx + 1/2 c)^2 + (a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 a \cos(dx + c) + a) \sin(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + \\
&a \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + a \sin(1/2 dx + 1/2 c)^2 + 2 (a \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + a \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 \\
&+ 2 a \cos(dx + c) \cos(1/2 dx + 1/2 c) + a \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + 2 (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx \\
&+ c) + 2 (a \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + a \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 a \cos(dx + c) \sin(1/2 dx + 1/2 c) + a \sin(1/2 dx + \\
&1/2 c)) \sin(3/2 dx + 3/2 c) \cos(7/2 dx + 7/2 c) \cos(5/2 dx + 5/2 c) + (\\
&(a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 a \cos(dx + c) + a) \cos(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx + \\
&c)^2 + a \cos(1/2 dx + 1/2 c)^2 + (a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 a \cos(dx + c) + a) \sin(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \\
&\sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + a \sin(1/2 dx + 1/2 c)^2 + 2 (a \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + a \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + \\
&2 a \cos(dx + c) \cos(1/2 dx + 1/2 c) + a \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + 2 (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx \\
&x + c) + 2 (a \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + a \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 a \cos(dx + c) \sin(1/2 dx + 1/2 c) + a \sin(1/2 dx + 1/ \\
&2 c)) \sin(3/2 dx + 3/2 c) \cos(5/2 dx + 5/2 c)^2 + ((a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 a \cos(dx + c) + a) \cos(3/2 dx + 3/2 c)^2 + (a \cos(1/2 \\
&* dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + a \cos(1/2 dx + 1/2 c)^2 + (a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 a \cos(dx + c) + a) \sin \\
&\sin(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + a \sin(1/2 dx + 1/2 c)^2 + 2 (a \cos(dx + c)^2 \cos(1/2 \\
&dx + 1/2 c) + a \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2 a \cos(dx + c) \cos(1/2 dx + 1/2 c) + a \cos(1/2 dx + 1/2 c)) \cos(3/2 dx + 3/2 c) + 2 (a \cos \\
&(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 (a \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + a \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 a \\
&+ c)^2 \sin(1/2 dx + 1/2 c) + a \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 a
\end{aligned}$$

$$\begin{aligned}
& \cos(dx + c) \sin(1/2 dx + 1/2 c) + a \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c) \\
& \sin(7/2 dx + 7/2 c)^2 + 2 * ((a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 \\
& * a \cos(dx + c) + a) \cos(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \\
& * \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + a \cos(1/2 dx + 1/2 c)^2 + (a \cos \\
& (dx + c)^2 + a \sin(dx + c)^2 + 2 * a \cos(dx + c) + a) \sin(3/2 dx + 3/2 c) \\
& ^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + \\
& a \sin(1/2 dx + 1/2 c)^2 + 2 * (a \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + a \cos \\
& (1/2 dx + 1/2 c) \sin(dx + c)^2 + 2 * a \cos(dx + c) \cos(1/2 dx + 1/2 c) + \\
& a \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2 * (a \cos(1/2 dx + 1/2 c)^2 \\
& + a \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 * (a \cos(dx + c)^2 \sin(1/2 dx \\
& + 1/2 c) + a \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 * a \cos(dx + c) \sin(1/ \\
& 2 dx + 1/2 c) + a \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)) \sin(7/2 dx \\
& + 7/2 c) \sin(5/2 dx + 5/2 c) + ((a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 * a \\
& * \cos(dx + c) + a) \cos(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \sin \\
& (1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + a \cos(1/2 dx + 1/2 c)^2 + (a \cos(dx \\
& + c)^2 + a \sin(dx + c)^2 + 2 * a \cos(dx + c) + a) \sin(3/2 dx + 3/2 c)^2 \\
& + (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \sin(dx + c)^2 + a \\
& * \sin(1/2 dx + 1/2 c)^2 + 2 * (a \cos(dx + c)^2 \cos(1/2 dx + 1/2 c) + a \cos(\\
& 1/2 dx + 1/2 c) \sin(dx + c)^2 + 2 * a \cos(dx + c) \cos(1/2 dx + 1/2 c) + a \\
& * \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2 * (a \cos(1/2 dx + 1/2 c)^2 + \\
& a \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 * (a \cos(dx + c)^2 \sin(1/2 dx + \\
& 1/2 c) + a \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 * a \cos(dx + c) \sin(1/2 \\
& dx + 1/2 c) + a \sin(1/2 dx + 1/2 c) \sin(3/2 dx + 3/2 c)) \sin(5/2 dx + \\
& 5/2 c)^2 \cos(2/3 \arctan2(\sin(3/2 dx + 3/2 c), \cos(3/2 dx + 3/2 c))) \cos \\
& (2/5 \arctan2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c)))^2 + (((a \cos(dx \\
& + c)^2 + a \sin(dx + c)^2 + 2 * a \cos(dx + c) + a) \cos(3/2 dx + 3/2 c)^2 + \\
& (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + a \cos \\
& (1/2 dx + 1/2 c)^2 + (a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 * a \cos(dx + \\
& c) + a) \sin(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx \\
& + 1/2 c)^2) \sin(dx + c)^2 + a \sin(1/2 dx + 1/2 c)^2 + 2 * (a \cos(dx + c) \\
& ^2 \cos(1/2 dx + 1/2 c) + a \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2 * a \cos(dx \\
& + c) \cos(1/2 dx + 1/2 c) + a \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) \\
& + 2 * (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 * \\
& (a \cos(dx + c)^2 \sin(1/2 dx + 1/2 c) + a \sin(dx + c)^2 \sin(1/2 dx + 1/2 \\
& c) + 2 * a \cos(dx + c) \sin(1/2 dx + 1/2 c) + a \sin(1/2 dx + 1/2 c) \sin(3 \\
& /2 dx + 3/2 c)) \cos(7/2 dx + 7/2 c)^2 + 2 * ((a \cos(dx + c)^2 + a \sin(dx \\
& + c)^2 + 2 * a \cos(dx + c) + a) \cos(3/2 dx + 3/2 c)^2 + (a \cos(1/2 dx + 1/ \\
& 2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx + c)^2 + a \cos(1/2 dx + 1/2 c)^2 \\
& + (a \cos(dx + c)^2 + a \sin(dx + c)^2 + 2 * a \cos(dx + c) + a) \sin(3/2 dx \\
& + 3/2 c)^2 + (a \cos(1/2 dx + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \sin(dx \\
& + c)^2 + a \sin(1/2 dx + 1/2 c)^2 + 2 * (a \cos(dx + c)^2 \cos(1/2 dx + 1/2 \\
& c) + a \cos(1/2 dx + 1/2 c) \sin(dx + c)^2 + 2 * a \cos(dx + c) \cos(1/2 dx \\
& + 1/2 c) + a \cos(1/2 dx + 1/2 c) \cos(3/2 dx + 3/2 c) + 2 * (a \cos(1/2 dx \\
& + 1/2 c)^2 + a \sin(1/2 dx + 1/2 c)^2) \cos(dx + c) + 2 * (a \cos(dx + c)^2 \sin \\
& (1/2 dx + 1/2 c) + a \sin(dx + c)^2 \sin(1/2 dx + 1/2 c) + 2 * a \cos(dx +
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((a*\cos(d*x + c)^2 + a*\sin(d*x + \\
& c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2 \\
& *c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 \\
& + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x \\
& + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2* \\
& c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + \\
& 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + \\
& 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*si \\
& n(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin \\
& (7/2*d*x + 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x \\
& + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d \\
& *x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^ \\
& 2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*co \\
& s(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x \\
& + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2 \\
& *d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) \\
& + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/ \\
& 2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)*s \\
& in(5/2*d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + \\
& c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 \\
& + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(\\
& 1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + \\
& 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d \\
& *x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2 \\
& *d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2* \\
& c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2 + \\
& (((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + \\
& 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + \\
& a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a \\
& *\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^ \\
& 2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2 \\
& *d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos \\
& (d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((a*\cos(d*x + c)^ \\
& 2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*co
\end{aligned}$$

$$\begin{aligned}
& s(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2* \\
& *d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + \\
& a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/ \\
& 2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos \\
& (1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c \\
&)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\\
& a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*co \\
& s(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d* \\
& x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((a*\cos(d*x + c)^2 \\
& + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(\\
& 1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d \\
& *x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a \\
&)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2* \\
& c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1 \\
& /2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)* \\
& \cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a* \\
& \cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(\\
& d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2 \\
& *a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + (((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + \\
& 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + \\
& a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*c \\
& os(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2* \\
& c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 \\
& + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a* \\
& \cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) \\
& + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c) \\
& ^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(\\
& 1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d* \\
& x + 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + \\
& a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/ \\
& 2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*s \\
& in(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d \\
& *x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c) \\
&)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin \\
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2* \\
& d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin
\end{aligned}$$

$$\begin{aligned}
& (d*x + c)^2 + 2*a*cos(d*x + c) + a)*sin(3/2*d*x + 3/2*c)^2 + (a*cos(1/2*d*x \\
& + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + a*sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*(a*cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + a*cos(1/2*d*x + 1/2*c)* \\
& sin(d*x + c)^2 + 2*a*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + a*cos(1/2*d*x + 1/ \\
& 2*c))*cos(3/2*d*x + 3/2*c) + 2*(a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + \\
& 1/2*c)^2)*cos(d*x + c) + 2*(a*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + a*sin(d \\
& *x + c)^2*sin(1/2*d*x + 1/2*c) + 2*a*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + a* \\
& sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*cos(5/2*d*x + 5/2*c)^2 + ((a*co \\
& s(d*x + c)^2 + a*sin(d*x + c)^2 + 2*a*cos(d*x + c) + a)*cos(3/2*d*x + 3/2*c \\
&)^2 + (a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 \\
& + a*cos(1/2*d*x + 1/2*c)^2 + (a*cos(d*x + c)^2 + a*sin(d*x + c)^2 + 2*a*cos \\
& (d*x + c) + a)*sin(3/2*d*x + 3/2*c)^2 + (a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1 \\
& /2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + a*sin(1/2*d*x + 1/2*c)^2 + 2*(a*cos(d*x \\
& + c)^2*cos(1/2*d*x + 1/2*c) + a*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*a* \\
& cos(d*x + c)*cos(1/2*d*x + 1/2*c) + a*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3 \\
& /2*c) + 2*(a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c \\
&) + 2*(a*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + a*sin(d*x + c)^2*sin(1/2*d*x \\
& + 1/2*c) + 2*a*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + a*sin(1/2*d*x + 1/2*c)) \\
& *sin(3/2*d*x + 3/2*c))*sin(7/2*d*x + 7/2*c)^2 + 2*((a*cos(d*x + c)^2 + a*si \\
& n(d*x + c)^2 + 2*a*cos(d*x + c) + a)*cos(3/2*d*x + 3/2*c)^2 + (a*cos(1/2*d* \\
& x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + a*cos(1/2*d*x + 1 \\
& /2*c)^2 + (a*cos(d*x + c)^2 + a*sin(d*x + c)^2 + 2*a*cos(d*x + c) + a)*sin(\\
& 3/2*d*x + 3/2*c)^2 + (a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)* \\
& sin(d*x + c)^2 + a*sin(1/2*d*x + 1/2*c)^2 + 2*(a*cos(d*x + c)^2*cos(1/2*d*x \\
& + 1/2*c) + a*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*a*cos(d*x + c)*cos(1/ \\
& 2*d*x + 1/2*c) + a*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(a*cos(1/ \\
& 2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(a*cos(d*x + \\
& c)^2*sin(1/2*d*x + 1/2*c) + a*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*a*cos \\
& (d*x + c)*sin(1/2*d*x + 1/2*c) + a*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2* \\
& c))*sin(7/2*d*x + 7/2*c)*sin(5/2*d*x + 5/2*c) + ((a*cos(d*x + c)^2 + a*sin(\\
& d*x + c)^2 + 2*a*cos(d*x + c) + a)*cos(3/2*d*x + 3/2*c)^2 + (a*cos(1/2*d*x \\
& + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + a*cos(1/2*d*x + 1/2 \\
& *c)^2 + (a*cos(d*x + c)^2 + a*sin(d*x + c)^2 + 2*a*cos(d*x + c) + a)*sin(3/ \\
& 2*d*x + 3/2*c)^2 + (a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*si \\
& n(d*x + c)^2 + a*sin(1/2*d*x + 1/2*c)^2 + 2*(a*cos(d*x + c)^2*cos(1/2*d*x + \\
& 1/2*c) + a*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*a*cos(d*x + c)*cos(1/2* \\
& d*x + 1/2*c) + a*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(a*cos(1/2* \\
& d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(a*cos(d*x + c) \\
& ^2*sin(1/2*d*x + 1/2*c) + a*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*a*cos(d \\
& *x + c)*sin(1/2*d*x + 1/2*c) + a*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) \\
&)*sin(5/2*d*x + 5/2*c)^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x \\
& + 3/2*c)))^2 + ((a*cos(d*x + c)^2 + a*sin(d*x + c)^2 + 2*a*cos(d*x + c) + \\
& a)*cos(3/2*d*x + 3/2*c)^2 + (a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2 \\
& *c)^2)*cos(d*x + c)^2 + a*cos(1/2*d*x + 1/2*c)^2 + (a*cos(d*x + c)^2 + a*si \\
& n(d*x + c)^2 + 2*a*cos(d*x + c) + a)*sin(3/2*d*x + 3/2*c)^2 + (a*cos(1/2*d*
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a* \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + \\
& a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d* \\
& *x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7 \\
& /2*c)*\cos(5/2*d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*co \\
& s(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(\\
& 1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x \\
& + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + \\
& (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*si \\
& n(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2 \\
& *d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*co \\
& s(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x \\
& + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2 \\
& *c)^2 + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3 \\
& /2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*c \\
& os(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + \\
& c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2* \\
& c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*c \\
& os(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c) \\
& ^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((a*\cos(d*x \\
& + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + \\
& (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*c \\
& os(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x \\
& + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d* \\
& x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c) \\
& ^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d \\
& *x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
& + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2 \\
& *(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/ \\
& 2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(\\
& 3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d*x + 5/2*c) + ((a*\cos(d*x + \\
& c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (\\
& a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos \\
& (1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + \\
& c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x \\
& + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2 \\
& *\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x \\
& + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) +
\end{aligned}$$

$$\begin{aligned}
& 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\\
& a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/ \\
& 2*d*x + 3/2*c))*\sin(5/2*d*x + 5/2*c)^2*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c) \\
&), \cos(3/2*d*x + 3/2*c)))^2 + 2*((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2* \\
& a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a* \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(\\
& d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^ \\
& 2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos \\
& (1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 \\
& + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2 \\
& *d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + \\
& 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a) \\
& *\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c \\
&)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(\\
& d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x \\
& + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*s \\
& in(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2 \\
& *c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1 \\
& /2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d* \\
& x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*s \\
& in(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x \\
& + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*c \\
& os(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d* \\
& x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + \\
& 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c \\
&)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin \\
& (d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c \\
&))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin \\
& (1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((a*\cos(d \\
& *x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a \\
& *\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d* \\
& x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2* \\
& d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + \\
& c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos \\
& (d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2* \\
& c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) +
\end{aligned}$$

$$\begin{aligned}
& 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin \\
& n(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + a*\sin(d \\
& *x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + \\
& 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2* \\
& c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2 \\
& *d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin \\
& (d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + \\
& 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d \\
& *x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d \\
& *x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^ \\
& 2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d* \\
& x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)) \\
& *\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x \\
& + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1 \\
& /2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c) \\
& ^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d \\
& *x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d \\
& *x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/ \\
& 2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x \\
& + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2* \\
& \sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin \\
& in(5/2*d*x + 5/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + \\
& 3/2*c)))*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + \\
& (((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x \\
& + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + \\
& 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + \\
& a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a* \\
& \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2* \\
& d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(\\
& d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(\\
& 1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + \\
& 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 \\
& + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos \\
& (1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2* \\
& d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + \\
& a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(\\
& 1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) \\
& *\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a \\
& *\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos
\end{aligned}$$

$$\begin{aligned}
& (d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x \\
& + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + \\
& a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1 \\
& /2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d* \\
& x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a) \\
& *\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c \\
&)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/ \\
& 2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*c \\
& os(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*c \\
& os(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2* \\
& a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2 \\
& *a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos \\
& (d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c) \\
& ^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \\
& a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*c \\
& os(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \\
& a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 \\
& + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x \\
& + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x \\
& + 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a) \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin \\
& (d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x \\
& + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)* \\
& sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a* \\
& sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c))*\sin(5/2*d* \\
& x + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)* \\
& cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d \\
& *x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + \\
& 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*si \\
& n(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/ \\
& 2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*si
\end{aligned}$$

$$\begin{aligned}
& (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2* \\
& a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a* \\
& \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
& 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d* \\
& x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/ \\
& 2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((a \\
& *\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& ^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a* \\
& \cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2 \\
& *a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2* \\
& c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((a*\cos(d*x + c)^2 + a*\sin \\
& (d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d* \\
& x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + \\
& 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin \\
& (3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2) \\
& *\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d* \\
& x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1 \\
& /2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1 \\
& /2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos \\
& (d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2 \\
& *c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a \\
& *\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin \\
& (1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d \\
& *x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 \\
& + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(\\
& 1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a \\
& *\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + \\
& a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + \\
& 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos \\
& (d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c)^2 * \cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x \\
& + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + \\
& (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*s \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*c \\
& \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a \\
& *sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1 \\
& /2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d* \\
& x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(5/2*d*x + 5/ \\
& 2*c)^2 * \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((\\
& a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3 \\
& /2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a \\
& *cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*s \\
& \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos \\
& (d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + \\
& 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x \\
& + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x \\
& + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2 \\
& *d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2 \\
& *c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + \\
& a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x \\
& + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)* \\
& \sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2 \\
& *d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*co \\
& s(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*co \\
& s(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d* \\
& x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a \\
& *cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c))*\sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + a* \\
& \sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2* \\
& d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + \\
& 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*si \\
& \sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d \\
& *x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(\\
& 1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*c \\
& \cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/ \\
& 2*c))*\sin(5/2*d*x + 5/2*c)^2 + (((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a \\
& *cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*s \\
& \sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d
\end{aligned}$$

$$\begin{aligned}
& *x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 \\
& + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a \\
& * \sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(\\
& 1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a \\
& * \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + \\
& a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2* \\
& d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + \\
& 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)* \\
& \cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c) \\
& ^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d \\
& *x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + \\
& 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\si \\
& n(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2* \\
& c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/ \\
& 2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x \\
& + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\si \\
& n(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x \\
& + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\co \\
& s(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x \\
& + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1 \\
& /2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(\\
& d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c) \\
&)*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c)^2 + ((a*\cos(d* \\
& x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 \\
& + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a* \\
& \cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x \\
& + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d \\
& *x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c) \\
&)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(\\
& d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) \\
&) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + \\
& 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1 \\
& /2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin \\
& (3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + a*\sin(d* \\
& x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + \\
& 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c) \\
&)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2* \\
& d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(
\end{aligned}$$

$$\begin{aligned}
& d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) \\
& + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x \\
& + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2 \\
& * \sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x \\
& + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))* \\
& \sin(7/2*d*x + 7/2*c)*\sin(5/2*d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x \\
& + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c) \\
& ^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 \\
& + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x \\
& + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2 \\
& *c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x \\
& + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x \\
& + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*s \\
& \sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + \\
& c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin \\
& (5/2*d*x + 5/2*c)^2*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3 \\
& /2*c)))^2 + 2*(((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a) \\
&)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2* \\
& c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin \\
& (d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x \\
& + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)* \\
& \sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/ \\
& 2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d \\
& *x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a* \\
& \sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((a* \\
& \cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2 \\
& *c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 \\
& + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*c \\
& \cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin \\
& (1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d \\
& *x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2* \\
& a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + \\
& c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c) \\
&))*\sin(3/2*d*x + 3/2*c))*\cos(7/2*d*x + 7/2*c)*\cos(5/2*d*x + 5/2*c) + ((a*co \\
& s(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c) \\
&)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 \\
& + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos \\
& (d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x
\end{aligned}$$

$$\begin{aligned}
& (1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) \\
&)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\\
& a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\co \\
& s(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \\
& 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d* \\
& x + 3/2*c))*\cos(7/2*d*x + 7/2*c)^2 + 2*((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^ \\
& 2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (\\
& a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3 \\
& /2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c \\
&)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \\
& a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2 \\
& *c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2 \\
& *c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/ \\
& 2*d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*s \\
& in(1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(7/2 \\
& *d*x + 7/2*c))*\cos(5/2*d*x + 5/2*c) + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 \\
& + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 \\
& + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a* \\
& cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2 \\
& *c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^ \\
& 2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a \\
& *cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c \\
&) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c \\
&)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2* \\
& d*x + 1/2*c) + a*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin \\
& (1/2*d*x + 1/2*c) + a*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(5/2*d \\
& *x + 5/2*c)^2 + ((a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + \\
& a)*\cos(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*si \\
& n(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d* \\
& x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*(a*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c) \\
&)*\sin(d*x + c)^2 + 2*a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1 \\
& /2*c))*\cos(3/2*d*x + 3/2*c) + 2*(a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + \\
& 1/2*c)^2)*\cos(d*x + c) + 2*(a*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + a*\sin(\\
& d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*a*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + a \\
& *sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(7/2*d*x + 7/2*c)^2 + 2*((a \\
& *cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a*\cos(d*x + c) + a)*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) \\
& ^2 + a*\cos(1/2*d*x + 1/2*c)^2 + (a*\cos(d*x + c)^2 + a*\sin(d*x + c)^2 + 2*a* \\
& cos(d*x + c) + a)*\sin(3/2*d*x + 3/2*c)^2 + (a*\cos(1/2*d*x + 1/2*c)^2 + a*si \\
& n(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + a*\sin(1/2*d*x + 1/2*c)^2 + 2*(a*\cos(\\
& d*x + c)^2*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2 \\
& *a*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + a*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x
\end{aligned}$$

+ 3/2*c) + 2*(a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(a*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + a*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*a*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + a*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(7/2*d*x + 7/2*c)*sin(5/2*d*x + 5/2*c) + ((a*cos(d*x + c)^2 + a*sin(d*x + c)^2 + 2*a*cos(d*x + c) + a)*cos(3/2*d*x + 3/2*c)^2 + (a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + a*cos(1/2*d*x + 1/2*c)^2 + (a*cos(d*x + c)^2 + a*sin(d*x + c)^2 + 2*a*cos(d*x + c) + a)*sin(3/2*d*x + 3/2*c)^2 + (a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + a*sin(1/2*d*x + 1/2*c)^2 + 2*(a*cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + a*cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*a*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + a*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(a*cos(1/2*d*x + 1/2*c)^2 + a*sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + 2*(a*cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + a*sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*a*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + a*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c)^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*d)

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int \frac{\cos^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{\frac{15\sqrt{2}\log(\sin(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{\sqrt{a}\operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} - \frac{15\sqrt{2}\log(-\sin(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{\sqrt{a}\operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} - \frac{4\sqrt{2}(12a^{\frac{9}{2}}\sin(\frac{1}{2}dx + \frac{1}{2}c)^5 - 10a^{\frac{9}{2}}\sin(\frac{1}{2}dx + \frac{1}{2}c)^3 + 15a^{\frac{9}{2}}\sin(\frac{1}{2}dx + \frac{1}{2}c))}{a^5\operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))}}{30d}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] -1/30*(15*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - 15*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - 4*sqrt(2)*(12*a^(9/2)*sin(1/2*d*x + 1/2*c)^5 - 10*a^(9/2)*sin(1/2*d*x + 1/2*c)^3 + 15*a^(9/2)*sin(1/2*d*x + 1/2*c))/(a^5*sgn(cos(1/2*d*x + 1/2*c))))/d

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^3}{\sqrt{a + a \cos(c + dx)}} dx$$

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(1/2), x)
```

3.124 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$

Optimal result	1990
Rubi [A] (verified)	1990
Mathematica [A] (verified)	1992
Maple [A] (verified)	1992
Fricas [A] (verification not implemented)	1992
Sympy [F]	1993
Maxima [B] (verification not implemented)	1993
Giac [A] (verification not implemented)	2006
Mupad [B] (verification not implemented)	2006

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{4 \sin(c+dx)}{3d\sqrt{a+a \cos(c+dx)}} + \frac{2\sqrt{a+a \cos(c+dx)} \sin(c+dx)}{3ad}$$

[Out] $\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}-4/3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/3*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/a/d$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2838, 2830, 2728, 212}

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx) \sqrt{a \cos(c+dx)+a}}{3ad} - \frac{4 \sin(c+dx)}{3d\sqrt{a \cos(c+dx)+a}}$$

[In] $\text{Int}[\text{Cos}[c+d*x]^2/\text{Sqrt}[a+a*\text{Cos}[c+d*x]],x]$

[Out] $(\text{Sqrt}[2]*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Sin}[c+d*x])/(\text{Sqrt}[2]*\text{Sqrt}[a+a*\text{Cos}[c+d*x]])])/(\text{Sqrt}[a]*d) - (4*\text{Sin}[c+d*x])/(3*d*\text{Sqrt}[a+a*\text{Cos}[c+d*x]]) + (2*\text{Sqrt}[a+a*\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(3*a*d)$

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2830

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2838

Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3ad} + \frac{2 \int \frac{\frac{a}{2} - a \cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx}{3a} \\
 &= -\frac{4 \sin(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} + \frac{2\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3ad} + \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx \\
 &= -\frac{4 \sin(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} + \frac{2\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3ad} \\
 &\quad - \frac{2 \text{Subst}\left(\int \frac{1}{2a - x^2} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{d} \\
 &= \frac{\sqrt{2} \arctanh\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2}\sqrt{a + a \cos(c + dx)}}\right)}{\sqrt{ad}} - \frac{4 \sin(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} + \frac{2\sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3ad}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{\left(\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{2}}\right) - \frac{2}{3}(1 - \cos(c + dx))^{3/2}\right) \sin(c + dx)}{d \sqrt{1 - \cos(c + dx)} \sqrt{a(1 + \cos(c + dx))}}$$

[In] Integrate[Cos[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]

[Out] ((Sqrt[2]*ArcTanh[Sqrt[1 - Cos[c + d*x]]/Sqrt[2]] - (2*(1 - Cos[c + d*x])^(3/2))/3)*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-4 \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} + 3 \ln\left(\frac{4 \sqrt{a} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} + 4a}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right) a\right)}{3a^{\frac{3}{2}} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$	132

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+3*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a/a^(3/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.26

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{4 \sqrt{a \cos(dx + c) + a} (\cos(dx + c) - 1) \sin(dx + c) + \frac{3 \sqrt{2} (a \cos(dx + c) + a) \log\left(-\frac{\cos(dx + c)^2 - 2 \sqrt{2} \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{\cos(dx + c)^2 + 2 \cos(dx + c) + 1}\right)}{\sqrt{a}}}{6(ad \cos(dx + c) + ad)}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] 1/6*(4*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sin(d*x + c) + 3*sqrt(2)
*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\cos^2(c + dx)}{\sqrt{a (\cos(c + dx) + 1)}} dx$$

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(1/2), x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19437 vs. 2(87) = 174.

Time = 0.66 (sec) , antiderivative size = 19437, normalized size of antiderivative = 186.89

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2), x, algorithm="maxima")
```

```
[Out] 1/60*(20*(cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)^3 + 8*(cos(d*x + c)^2 + si
n(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^3 - 20*cos(5/2*d*x
+ 5/2*c)^3*sin(d*x + c) + 2*(15*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 15*(log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1))*sin(d*x + c)^2 + 30*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 4*(cos(d*
x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - 20*c
os(3/2*d*x + 3/2*c)*sin(d*x + c) + 15*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 15*log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(5/2*d*x + 5/2
*c)^2 + 30*((log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x +
```

$$\begin{aligned}
& c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/ \\
& 2*c)^2 + 2*(10*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 4*(\cos(d*x + c))^ \\
& 2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 10*\cos(5/ \\
& 2*d*x + 5/2*c)^3*\sin(d*x + c) + (15*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 15*(\\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(d*x + c)^2 + 30*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 4*(\co \\
& s(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - \\
& 20*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 15*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(5/2*d*x + \\
& 5/2*c)^2 + 15*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*si \\
& n(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d* \\
& x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x \\
& + 3/2*c)^2 + (15*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 15*(\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*si \\
& n(d*x + c)^2 + 30*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 4*(\cos(d*x + c)^2 + \sin(\\
& d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 10*\cos(5/2*d*x + 5/ \\
& 2*c)*\sin(d*x + c) + 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 15*((lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 2*(4*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(3/2*d*x + 3/2*c) - 5*\cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) - 5*\sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + 15*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 10*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2*(5*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 5*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 10*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 3*\cos(d*x + c)^2 + 15*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(5/2*d*x + 5/2*c) + 2*(2*(\cos(d*x + c))^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c) - 2*4*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)
\end{aligned}$$

$$\begin{aligned}
&)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 10*((2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\cos(5/2*d*x + 5/2*c)^2 + 2*(2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\cos(3/2*d*x + 3/2*c)^2 + (2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\sin(5/2*d*x + 5/2*c)^2 + 2*(2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c)^2)*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))))*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*(15*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 15*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 30*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 4*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6)*\sin(3/2*d*x + 3/2*c) - 10*\cos(5/2*d*x + 5/2*c)*\sin(d*x + c) + 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(5/2*d*x + 5/2*c)^2 + 30*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c)^2 + 12*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 2*(10*(\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^3 + 4*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^3 - 10*\cos(5/2*d*x + 5/2*c)^3*\sin(d*x + c) + (15*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& /2*c)^2*\sin(d*x + c) + 15*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3/2*d*x + 3/2*c))*\cos(5/2*d*x + 5/2*c) + 10*(\cos(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)^2*\sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2*\sin(d*x + c) + 2*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \sin(3/2*d*x + 3/2*c)^2*\sin(d*x + c))*\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 2*(5*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 5*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 10*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c))*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 3*\cos(d*x + c)^2 + 15*((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(5/2*d*x + 5/2*c) + 2*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c) - 24*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 10*((2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\cos(5/2*d*x + 5/2*c)^2 + 2*(2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\cos(3/2*d*x + 3/2*c)^2 + (2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\sin(5/2*d*x + 5/2*c)^2 + 2*(2*\cos(d*x + c)^2 + 2*\sin(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^2 + 5\cos(dx + c) + 3)\sin(5/2*dx + 5/2*c)*\sin(3/2*dx + 3/2*c) + (2 \\
& * \cos(dx + c)^2 + 2\sin(dx + c)^2 + 5\cos(dx + c) + 3)\sin(3/2*dx + 3/2* \\
& c)^2)*\sin(1/2*\arctan2(\sin(dx + c), \cos(dx + c)))\sin(2/3*\arctan2(\sin(3/2 \\
& *dx + 3/2*c), \cos(3/2*dx + 3/2*c)))^2 + 4*(4*(\cos(dx + c)^2 + \sin(dx + \\
& c)^2 + 2\cos(dx + c) + 1)*\cos(3/2*dx + 3/2*c)*\sin(3/2*dx + 3/2*c) - 5*co \\
& s(3/2*dx + 3/2*c)^2*\sin(dx + c) - 5*\sin(3/2*dx + 3/2*c)^2*\sin(dx + c) + \\
& 15*((\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + \\
& 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(\\
& 1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + (\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1 \\
& /2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^ \\
& 2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + c)^2 + \\
& 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/ \\
& 2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2 \\
& *dx + 1/2*c) + 1))*\cos(dx + c) + \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx \\
& + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin \\
& (1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(3/2*dx + 3/2*c))*\cos \\
& (5/2*dx + 5/2*c) + 4*(10*(\cos(dx + c) + 1)*\sin(5/2*dx + 5/2*c)^3 + 4*(\\
& \cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*dx + 3/2*c)^ \\
& 3 - 10*\cos(5/2*dx + 5/2*c)^3*\sin(dx + c) + (15*(\log(\cos(1/2*dx + 1/2*c)^ \\
& 2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx \\
& + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx \\
& + c)^2 + 15*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/ \\
& 2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - \\
& 2*\sin(1/2*dx + 1/2*c) + 1))*\sin(dx + c)^2 + 30*(\log(\cos(1/2*dx + 1/2*c) \\
& ^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx \\
& + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx \\
& + c) + 4*(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\cos(dx + c) + 1)*\sin(3/2*d* \\
& x + 3/2*c) - 20*\cos(3/2*dx + 3/2*c)*\sin(dx + c) + 15*\log(\cos(1/2*dx + 1/ \\
& 2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - 15*\log(\cos(\\
& 1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))* \\
& \cos(5/2*dx + 5/2*c)^2 + 15*((\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/ \\
& 2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2 \\
& *dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c)^2 + (\log(\cos(1/ \\
& 2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log \\
& (\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) \\
& + 1))*\sin(dx + c)^2 + 2*(\log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*dx + 1/2*c) \\
&)^2 + 2*\sin(1/2*dx + 1/2*c) + 1) - \log(\cos(1/2*dx + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*dx + 1/2*c) + 1))*\cos(dx + c) + \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\cos(3/2*d*x + 3/2*c)^2 + (15*(\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(dx + c)^2 + 15*(\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/
\end{aligned}$$

$$\begin{aligned}
& 2*c) + 1)) * \sin(d*x + c)^2 + 30 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + 4 * (\cos(d*x \\
& + c)^2 + \sin(d*x + c)^2 + 7*\cos(d*x + c) + 6) * \sin(3/2*d*x + 3/2*c) - 10 * \cos \\
& (5/2*d*x + 5/2*c) * \sin(d*x + c) + 15 * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 15 * \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(5/2*d*x + 5/2*c \\
&)^2 + 15 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c) \\
& ^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3/2*d*x + 3/2* \\
& c)^2 + 2 * (4 * (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1) * \cos(3/2* \\
& d*x + 3/2*c) * \sin(3/2*d*x + 3/2*c) - 5 * \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) - \\
& 5 * \sin(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + 15 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) \\
& ^2 + (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1)) * \cos(3/2*d*x + 3/2*c) * \cos(5/2*d*x + 5/2*c) + 10 * (\cos(5/2*d \\
& *x + 5/2*c)^2 * \sin(d*x + c) + 2 * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) * \sin \\
& (d*x + c) + \cos(3/2*d*x + 3/2*c)^2 * \sin(d*x + c) + \sin(5/2*d*x + 5/2*c)^2 * \sin \\
& (d*x + c) + 2 * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) * \sin(d*x + c) + \sin \\
& (3/2*d*x + 3/2*c)^2 * \sin(d*x + c)) * \cos(1/2 * \arctan2(\sin(d*x + c), \cos(d*x + \\
& c))) + 2 * (5 * (\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c)^2 + 5 * (\cos(d*x + c) + 1 \\
&) * \cos(3/2*d*x + 3/2*c)^2 + (4 * \cos(d*x + c)^2 + 4 * \sin(d*x + c)^2 + 13 * \cos(d* \\
& x + c) + 9) * \sin(3/2*d*x + 3/2*c)^2 + 10 * ((\cos(d*x + c) + 1) * \cos(3/2*d*x + 3 \\
& /2*c) - \sin(3/2*d*x + 3/2*c) * \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 3 * \cos(d*x \\
& + c)^2 + 15 * ((\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x \\
& + c)^2 + 2 * (\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 -
\end{aligned}$$

$$\begin{aligned}
& + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2 + ((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\log(\cos(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 + \sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c)))^2 - 2*\sin(1/2*\arctan2(\sin(d*x + c), \cos(d*x + c))) + 1) + 4*(5*(\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 5*(\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (4*\cos(d*x + c)^2 + 4*\sin(d*x + c)^2 + 13*\cos(d*x + c) + 9)*\sin(3/2*d*x + 3/2*c)^2 + 10*((\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 3*\cos(d*x + c)^2 + 15*((\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + (\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(\log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(\cos(1/2*d*x + 1/2*c))^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3/2*d*x + 3/2*c) + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(5/2*d*x + 5/2*c) + 4*(2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c) - 60*((\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)^2 + (\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 5/2*c)^2 + 2*(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (\cos(d*x + c)^2 + \sin(d*x + c)^2 \\
& + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)^2)*\sin(1/3*\arctan2(\sin(3/2*d*x \\
& + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 20*((2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 \\
& + 5*\cos(d*x + c) + 3)*\cos(5/2*d*x + 5/2*c)^2 + 2*(2*\cos(d*x + c)^2 + 2*\sin \\
& (d*x + c)^2 + 5*\cos(d*x + c) + 3)*\cos(5/2*d*x + 5/2*c)*\cos(3/2*d*x + 3/2*c) \\
& + (2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\cos(3/2*d*x + \\
& 3/2*c)^2 + (2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c) + 3)*\sin(\\
& 5/2*d*x + 5/2*c)^2 + 2*(2*\cos(d*x + c)^2 + 2*\sin(d*x + c)^2 + 5*\cos(d*x + c \\
&) + 3)*\sin(5/2*d*x + 5/2*c)*\sin(3/2*d*x + 3/2*c) + (2*\cos(d*x + c)^2 + 2*\sin \\
& (d*x + c)^2 + 5*\cos(d*x + c) + 3)*\sin(3/2*d*x + 3/2*c)^2)*\sin(1/2*\arctan2(\\
& \sin(d*x + c), \cos(d*x + c))) * \sqrt{a} / (((\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) \\
& * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + c) + \sqrt{2}) * a * \cos(5/2*d*x + 5/2 \\
& *c)^2 + 2 * (\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * \\
& a * \cos(d*x + c) + \sqrt{2}) * a * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\sqrt{ \\
& 2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + c \\
&) + \sqrt{2}) * a * \cos(3/2*d*x + 3/2*c)^2 + ((\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) \\
& * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + c) + \sqrt{2}) * a * \cos(5/2*d*x + 5/ \\
& 2*c)^2 + 2 * (\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) \\
& * a * \cos(d*x + c) + \sqrt{2}) * a * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (s \\
& \sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + \\
& c) + \sqrt{2}) * a * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) \\
& * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + c) + \sqrt{2}) * a * \sin(5/2*d*x + 5/ \\
& 2*c)^2 + 2 * (\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) \\
& * a * \cos(d*x + c) + \sqrt{2}) * a * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (s \\
& \sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + \\
& c) + \sqrt{2}) * a * \sin(3/2*d*x + 3/2*c)^2) * \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c \\
&), \cos(3/2*d*x + 3/2*c)))^2 + (\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x \\
& + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + c) + \sqrt{2}) * a * \sin(5/2*d*x + 5/2*c)^2 + 2 * \\
& (\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x \\
& + c) + \sqrt{2}) * a * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\sqrt{2}) * a * \cos \\
& (d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + c) + \sqrt{2}) \\
& * a * \sin(3/2*d*x + 3/2*c)^2 + ((\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x \\
& + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + c) + \sqrt{2}) * a * \cos(5/2*d*x + 5/2*c)^2 + 2 \\
& * (\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x \\
& + c) + \sqrt{2}) * a * \cos(5/2*d*x + 5/2*c) * \cos(3/2*d*x + 3/2*c) + (\sqrt{2}) * a * \cos \\
& (d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + c) + \sqrt{2}) \\
& * a * \cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x \\
& + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + c) + \sqrt{2}) * a * \sin(5/2*d*x + 5/2*c)^2 + 2 \\
& * (\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x \\
& + c) + \sqrt{2}) * a * \sin(5/2*d*x + 5/2*c) * \sin(3/2*d*x + 3/2*c) + (\sqrt{2}) * a * \cos \\
& (d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 + 2 * \sqrt{2}) * a * \cos(d*x + c) + \sqrt{2}) \\
& * a * \sin(3/2*d*x + 3/2*c)^2) * \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c)))^2 + 2 * ((\sqrt{2}) * a * \cos(d*x + c)^2 + \sqrt{2}) * a * \sin(d*x + c)^2 \\
& + 2 * \sqrt{2}) * a * \cos(d*x + c) + \sqrt{2}) * a * \cos(5/2*d*x + 5/2*c)^2 + 2 * (\sqrt{2})
\end{aligned}$$

) $\cdot a \cdot \cos(dx + c)^2 + \sqrt{2} \cdot a \cdot \sin(dx + c)^2 + 2 \cdot \sqrt{2} \cdot a \cdot \cos(dx + c) + \sqrt{2} \cdot a \cdot \cos(5/2 \cdot dx + 5/2 \cdot c) \cdot \cos(3/2 \cdot dx + 3/2 \cdot c) + (\sqrt{2} \cdot a \cdot \cos(dx + c)^2 + \sqrt{2} \cdot a \cdot \sin(dx + c)^2 + 2 \cdot \sqrt{2} \cdot a \cdot \cos(dx + c) + \sqrt{2} \cdot a \cdot \cos(3/2 \cdot dx + 3/2 \cdot c)^2 + (\sqrt{2} \cdot a \cdot \cos(dx + c)^2 + \sqrt{2} \cdot a \cdot \sin(dx + c)^2 + 2 \cdot \sqrt{2} \cdot a \cdot \cos(dx + c) + \sqrt{2} \cdot a \cdot \sin(5/2 \cdot dx + 5/2 \cdot c)^2 + 2 \cdot (\sqrt{2} \cdot a \cdot \cos(dx + c)^2 + \sqrt{2} \cdot a \cdot \sin(dx + c)^2 + 2 \cdot \sqrt{2} \cdot a \cdot \cos(dx + c) + \sqrt{2} \cdot a \cdot \sin(5/2 \cdot dx + 5/2 \cdot c) \cdot \sin(3/2 \cdot dx + 3/2 \cdot c) + (\sqrt{2} \cdot a \cdot \cos(dx + c)^2 + \sqrt{2} \cdot a \cdot \sin(dx + c)^2 + 2 \cdot \sqrt{2} \cdot a \cdot \cos(dx + c) + \sqrt{2} \cdot a \cdot \sin(3/2 \cdot dx + 3/2 \cdot c)^2) \cdot \cos(2/3 \cdot \arctan2(\sin(3/2 \cdot dx + 3/2 \cdot c), \cos(3/2 \cdot dx + 3/2 \cdot c)))) \cdot d$

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= -\frac{\frac{8\sqrt{2}\sin(\frac{1}{2}dx + \frac{1}{2}c)^3}{\sqrt{a}\operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} - \frac{3\sqrt{2}\log(\sin(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{\sqrt{a}\operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} + \frac{3\sqrt{2}\log(-\sin(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{\sqrt{a}\operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))}}{6d}$$

[In] integrate(cos(dx+c)^2/(a+a*cos(dx+c))^(1/2),x, algorithm="giac")

[Out] -1/6*(8*sqrt(2)*sin(1/2*dx + 1/2*c)^3/(sqrt(a)*sgn(cos(1/2*dx + 1/2*c))) - 3*sqrt(2)*log(sin(1/2*dx + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*dx + 1/2*c))) + 3*sqrt(2)*log(-sin(1/2*dx + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*dx + 1/2*c))))/d

Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{2 \sin(c + dx) \sqrt{a + a \cos(c + dx)}}{3 a d}$$

$$- \frac{2 \left(4 a^2 E\left(\frac{c}{2} + \frac{dx}{2} \mid 1\right) - 3 a^2 F\left(\frac{c}{2} + \frac{dx}{2} \mid 1\right) \right) \sqrt{\frac{a + a \cos(c + dx)}{2 a}}}{3 a^2 d \sqrt{a + a \cos(c + dx)}}$$

[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(1/2),x)

[Out] (2*sin(c + d*x)*(a + a*cos(c + d*x))^(1/2))/(3*a*d) - (2*(4*a^2*ellipticE(c/2 + (d*x)/2, 1) - 3*a^2*ellipticF(c/2 + (d*x)/2, 1))*((a + a*cos(c + d*x))/(2*a))^(1/2))/(3*a^2*d*(a + a*cos(c + d*x))^(1/2))

$$3.125 \quad \int \frac{\cos(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	2007
Rubi [A] (verified)	2007
Mathematica [A] (verified)	2008
Maple [A] (verified)	2009
Fricas [A] (verification not implemented)	2009
Sympy [F]	2009
Maxima [B] (verification not implemented)	2010
Giac [A] (verification not implemented)	2022
Mupad [B] (verification not implemented)	2023

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\cos(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx)}{d \sqrt{a+a \cos(c+dx)}}$$

[Out] $-\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2830, 2728, 212}

$$\int \frac{\cos(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{2 \sin(c+dx)}{d \sqrt{a \cos(c+dx) + a}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a \cos(c+dx) + a}}\right)}{\sqrt{ad}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]/\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]], x]$

[Out] $-((\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])])/(\operatorname{Sqrt}[a]*d)) + (2*\operatorname{Sin}[c + d*x])/d*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]]$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{Gt} Q[a, 0] \parallel \operatorname{Lt} Q[b, 0])$

Rule 2728

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2 \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}} - \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx \\ &= \frac{2 \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}} + \frac{2 \text{Subst}\left(\int \frac{1}{2a - x^2} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{d} \\ &= -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2} \sqrt{a + a \cos(c + dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\begin{aligned} &\int \frac{\cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx \\ &= -\frac{2 \cos\left(\frac{1}{2}(c + dx)\right) \left(\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) - 2 \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d \sqrt{a(1 + \cos(c + dx))}} \end{aligned}$$

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(ArcTanh[Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2]))/(d*S
qrt[a*(1 + Cos[c + d*x])])
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\left(2\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - \ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} + 4a}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right)a\right)}{a^{\frac{3}{2}}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}d$	120

[In] `int(cos(d*x+c)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\cos(1/2*d*x+1/2*c)*2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*a^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} - \ln(4*(a^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+a)/\cos(1/2*d*x+1/2*c))*a/a^{(3/2)}/\sin(1/2*d*x+1/2*c)/(a*\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/d$

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2 + \frac{2\sqrt{2}\sqrt{a}\cos(dx+c)+a\sin(dx+c)}{\sqrt{a}} - 2\cos(dx+c) - 3}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right)}{\sqrt{a}} + 4\sqrt{a\cos(dx+c)+a\sin(dx+c)}$$

$$2(ad\cos(dx+c)+ad)$$

[In] `integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $1/2*(\sqrt{2}*(a*\cos(d*x+c)+a)*\log(-(\cos(d*x+c)^2 + 2*\sqrt{2}*\sqrt{a}*\cos(d*x+c)+a*\sin(d*x+c))/\sqrt{a} - 2*\cos(d*x+c) - 3)/(\cos(d*x+c)^2 + 2*\cos(d*x+c) + 1))/\sqrt{a} + 4*\sqrt{a*\cos(d*x+c)+a*\sin(d*x+c)}/(a*d*\cos(d*x+c)+a*d)$

Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx = \int \frac{\cos(c+dx)}{\sqrt{a(\cos(c+dx)+1)}} dx$$

[In] `integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)`[Out] `Integral(cos(c+d*x)/sqrt(a*(cos(c+d*x)+1)),x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18948 vs. $2(62) = 124$.

Time = 0.58 (sec) , antiderivative size = 18948, normalized size of antiderivative = 259.56

$$\int \frac{\cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] $-1/12*(12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + (12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 +$

$$\begin{aligned}
& *x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8 \\
& *sqrt(2)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*sqrt(2)*\cos(1/2*d \\
& *x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/ \\
& 2*d*x + 1/2*c) - 6*(sqrt(2)*\cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*\sin(1/2*d*x + \\
& 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*sqrt(2)*\sin(1/2*d*x + 1 \\
& /2*c)^3 - 3*(sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(s \\
& qrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*sqrt(2)*\cos(1 \\
& /2*d*x + 1/2*c)^2 + sqrt(2))*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 2*(6*(sqr \\
& t(2)*\cos(d*x + c) + sqrt(2))*\cos(3/2*d*x + 3/2*c)^2 + (8*sqrt(2)*\sin(1/2*d* \\
& x + 1/2*c)^2 - 3*(sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2 \\
& *sqrt(2))*\cos(d*x + c)^2 + (8*sqrt(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2)*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*sqrt(2))*\sin(d*x + c)^2 + \\
& 6*sqrt(2)*\cos(1/2*d*x + 1/2*c)^2 + 14*sqrt(2)*\sin(1/2*d*x + 1/2*c)^2 + 12*(\\
& sqrt(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - sqrt(2)*\sin(d*x + c)*\sin(1/2*d* \\
& x + 1/2*c) + sqrt(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*sqrt \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + 11*sqrt(2)*\sin(1/2*d*x + 1/2*c)^2 - 3*(sqrt(2) \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*sqrt(2))*\cos(d*x + c) - \\
& 3*(sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*sqrt(2))*\sin(3 \\
& /2*d*x + 3/2*c) - 4*(2*sqrt(2)*\cos(1/2*d*x + 1/2*c)^2 + sqrt(2))*\sin(1/2*d* \\
& x + 1/2*c))*\cos(2/3*arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 \\
& + ((3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*\sin(1/2*d*x + 1/2*c))*\cos \\
& (d*x + c)^2 + (3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*\sin(1/2*d* \\
& x + 1/2*c))*\sin(d*x + c)^2 + 12*sqrt(2)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) +
\end{aligned}$$

$$\begin{aligned}
& 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 12*((\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c)^2 + (\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2*\sin(d*x + c)^2 + \sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2}*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + (12*\sqrt{2}*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)^2) * \sin(d*x + c) * \cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(d*x + c)^2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2}*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)) * \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) -
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + ((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin \\
& (d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (s \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c \\
&)^2 + (\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x \\
& + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \\
& \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2* \\
& c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d \\
& *x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d \\
& *x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 \\
&)*\cos(d*x + c) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*s \\
& \sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1 \\
& /2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(2/3*\arctan2 \\
& (\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sqrt{2})*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2* \\
& d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(\\
& d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + \\
& 3/2*c) + 2*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^ \\
& 2)*\cos(d*x + c) + 2*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*s \\
& \sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d* \\
& x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2})*\co \\
& s(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*s \\
& \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d \\
& *x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\si \\
& n(1/2*d*x + 1/2*c)^2 + 2*(\sqrt{2})*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c)*\cos(1/2*d \\
& *x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + \\
& 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(d*x + c)^2*\sin \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*s \\
& \sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/ \\
& 2*c), \cos(3/2*d*x + 3/2*c))) + 2*(\sqrt{2})*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2* \\
& c) + \sqrt{2})*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)*s \\
& \sin(1/2*d*x + 1/2*c) + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*1 \\
& \log(\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(1/3 \\
& *\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin(1/3*\arctan2 \\
& (\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))) + 1) - 3*((\sqrt{2})*\cos(d*x + \\
& c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d \\
& *x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2 \\
& *c)^2)*\cos(d*x + c)^2 + ((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + \\
& 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\sqrt{2} \\
&)*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2})*\cos(d*x + c) + \sqrt{2} \\
&))*\sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1 \\
& /2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& \cos(3/2*d*x + 3/2*c))\wedge 2 - 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d \\
& *x + 3/2*c))) + 1) - 2*(6*(\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/ \\
& 2*c)\wedge 2 + (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)\wedge 2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)\wedge 2 + \sin(1/2*d*x + 1/2*c)\wedge 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)\wedge 2 + \sin(1/2*d*x + 1/2*c)\wedge 2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c)\wedge 2 + (8*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c)\wedge 2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)\wedge 2 + \sin(1/2*d*x + 1/2 \\
& *c)\wedge 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)\wedge 2 + \\
& \sin(1/2*d*x + 1/2*c)\wedge 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}))*\sin(d*x + c)\wedge 2 + 6*\sqrt{2}*\cos(1/2*d*x + 1/2*c)\wedge 2 + 14*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c)\wedge 2 + 12*(\sqrt{2}*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2} \\
& *\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos \\
& (3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}*\cos(1/2*d*x + 1/2*c)\wedge 2 + 11*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c)\wedge 2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)\wedge 2 + \sin(1/2*d*x + 1 \\
& /2*c)\wedge 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)\wedge 2 \\
& + \sin(1/2*d*x + 1/2*c)\wedge 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) \\
&) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)\wedge 2 + \sin(1 \\
& /2*d*x + 1/2*c)\wedge 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)\wedge 2 + \sin(1/2*d*x + 1/2*c)\wedge 2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)\wedge 2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c) - 12*((\sqrt{2}*\cos(d*x + c)\wedge 2 + \sqrt{2} \\
& *\sin(d*x + c)\wedge 2 + 2*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3/2*d*x + 3/2* \\
& c)\wedge 2 + (\sqrt{2}*\cos(1/2*d*x + 1/2*c)\wedge 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)\wedge 2)*\cos \\
& (d*x + c)\wedge 2 + (\sqrt{2}*\cos(d*x + c)\wedge 2 + \sqrt{2}*\sin(d*x + c)\wedge 2 + 2*\sqrt{2} \\
& *\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c)\wedge 2 + (\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c)\wedge 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)\wedge 2)*\sin(d*x + c)\wedge 2 + \sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c)\wedge 2 + \sqrt{2}*\sin(1/2*d*x + 1/2*c)\wedge 2 + 2*(\sqrt{2}*\cos(d*x + c)\wedge 2* \\
& \cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)\wedge 2 + 2*\sqrt{2} \\
& *\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \sqrt{2}*\cos(1/2*d*x + 1/2*c))*\cos(3 \\
& /2*d*x + 3/2*c) + 2*(\sqrt{2}*\cos(1/2*d*x + 1/2*c)\wedge 2 + \sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c)\wedge 2)*\cos(d*x + c) + 2*(\sqrt{2}*\cos(d*x + c)\wedge 2*\sin(1/2*d*x + 1/2*c) + \\
& \sqrt{2}*\sin(d*x + c)\wedge 2*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\cos(d*x + c)*\sin(1/ \\
& 2*d*x + 1/2*c) + \sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(1/ \\
& 3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))/(((\cos(d*x + c)\wedge 2 + \\
& \sin(d*x + c)\wedge 2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)\wedge 2 + (\cos(1/2*d*x \\
& + 1/2*c)\wedge 2 + \sin(1/2*d*x + 1/2*c)\wedge 2)*\cos(d*x + c)\wedge 2 + ((\cos(d*x + c)\wedge 2 + \sin \\
& (d*x + c)\wedge 2 + 2*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)\wedge 2 + (\cos(1/2*d*x + \\
& 1/2*c)\wedge 2 + \sin(1/2*d*x + 1/2*c)\wedge 2)*\cos(d*x + c)\wedge 2 + (\cos(d*x + c)\wedge 2 + \sin(\\
& d*x + c)\wedge 2 + 2*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)\wedge 2 + (\cos(1/2*d*x + 1/ \\
& 2*c)\wedge 2 + \sin(1/2*d*x + 1/2*c)\wedge 2)*\sin(d*x + c)\wedge 2 + 2*(\cos(d*x + c)\wedge 2*\cos(1/2 \\
& *d*x + 1/2*c) + \cos(1/2*d*x + 1/2*c))*\sin(d*x + c)\wedge 2 + 2*\cos(d*x + c)*\cos(1/ \\
& 2*d*x + 1/2*c) + \cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\cos(1/2*d* \\
& x + 1/2*c)\wedge 2 + \sin(1/2*d*x + 1/2*c)\wedge 2)*\cos(d*x + c) + \cos(1/2*d*x + 1/2*c)\wedge \\
& 2 + 2*(\cos(d*x + c)\wedge 2*\sin(1/2*d*x + 1/2*c) + \sin(d*x + c)\wedge 2*\sin(1/2*d*x + 1 \\
& /2*c) + 2*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sin(1/2*d*x + 1/2*c))*\sin(3/2
\end{aligned}$$

```

*d*x + 3/2*c) + sin(1/2*d*x + 1/2*c)^2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x
+ c) + 1)*sin(3/2*d*x + 3/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2)*sin(d*x + c)^2 + ((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x +
c) + 1)*cos(3/2*d*x + 3/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2)*cos(d*x + c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c)
+ 1)*sin(3/2*d*x + 3/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2)*sin(d*x + c)^2 + 2*(cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + cos(1/2*d*x
+ 1/2*c)*sin(d*x + c)^2 + 2*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + cos(1/2*d*x
+ 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2)*cos(d*x + c) + cos(1/2*d*x + 1/2*c)^2 + 2*(cos(d*x + c)^2*sin(1/
2*d*x + 1/2*c) + sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*cos(d*x + c)*sin(1
/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + sin(1/2*d*x
+ 1/2*c)^2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 2*(cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + cos(1/2*d*x + 1/2*c)*sin(d*x + c
)^2 + 2*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + cos(1/2*d*x + 1/2*c))*cos(3/2*d
*x + 3/2*c) + 2*(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*cos(d*x +
c) + cos(1/2*d*x + 1/2*c)^2 + 2*((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(
d*x + c) + 1)*cos(3/2*d*x + 3/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2)*cos(d*x + c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x
+ c) + 1)*sin(3/2*d*x + 3/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2)*sin(d*x + c)^2 + 2*(cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + cos(1/
2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + cos(1
/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2)*cos(d*x + c) + cos(1/2*d*x + 1/2*c)^2 + 2*(cos(d*x + c)^2*
sin(1/2*d*x + 1/2*c) + sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*cos(d*x + c)
*sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + sin(1/
2*d*x + 1/2*c)^2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 2*(cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sin(d*x + c)^2*sin(1/2*d*x +
1/2*c) + 2*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c))*sin(3
/2*d*x + 3/2*c) + sin(1/2*d*x + 1/2*c)^2)*sqrt(a)*d)

```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int \frac{\cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= - \frac{\frac{\sqrt{2} \log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{\sqrt{a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))}} - \frac{\sqrt{2} \log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{\sqrt{a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))}} - \frac{4 \sqrt{2} \sin(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))}}}{2d}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

```
[Out] -1/2*(sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c)/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) )/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int \frac{\cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{2 \left(2E\left(\frac{c}{2} + \frac{dx}{2} \mid 1\right) - F\left(\frac{c}{2} + \frac{dx}{2} \mid 1\right) \right) \sqrt{\frac{a + a \cos(c + dx)}{2a}}}{d \sqrt{a + a \cos(c + dx)}}$$

```
[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(2*ellipticE(c/2 + (d*x)/2, 1) - ellipticF(c/2 + (d*x)/2, 1))*((a + a*cos(c + d*x))/(2*a))^(1/2))/(d*(a + a*cos(c + d*x))^(1/2))
```

$$3.126 \quad \int \frac{1}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	2024
Rubi [A] (verified)	2024
Mathematica [A] (verified)	2025
Maple [C] (warning: unable to verify)	2025
Fricas [A] (verification not implemented)	2026
Sympy [F]	2026
Maxima [B] (verification not implemented)	2026
Giac [B] (verification not implemented)	2027
Mupad [B] (verification not implemented)	2027

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+a \cos(c+dx)}} dx = \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}}$$

[Out] $\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)}}*2^{(1/2)/d/a^{(1/2)}}$

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2728, 212}

$$\int \frac{1}{\sqrt{a+a \cos(c+dx)}} dx = \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}}$$

[In] $\operatorname{Int}[1/\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]],x]$

[Out] $(\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])]) / (\operatorname{Sqrt}[a]*d)$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]]),
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \\ &= \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{a}d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx = \frac{2\text{arctanh}\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) \cos\left(\frac{1}{2}(c + dx)\right)}{d\sqrt{a(1 + \cos(c + dx))}}$$

```
[In] Integrate[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])
])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\sqrt{2} \operatorname{am}^{-1}\left(\frac{dx}{2} + \frac{c}{2} 1\right)}{d \sec\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \operatorname{csgn}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$	56

```
[In] int(1/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)/sec(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/csgn(cos(1/2*
d*x+1/2*c))*InverseJacobiAM(1/2*d*x+1/2*c,1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx = \left[\frac{\sqrt{2} \log \left(-\frac{\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a \cos(dx+c)+a} \sin(dx+c) - 2 \cos(dx+c) - 3}{\sqrt{a} \cos(dx+c)^2 + 2 \cos(dx+c) + 1} \right)}{2\sqrt{ad}}, \right. \\ \left. - \frac{\sqrt{2}\sqrt{-\frac{1}{a}} \arctan \left(\frac{\sqrt{2}\sqrt{a \cos(dx+c)+a} \sqrt{-\frac{1}{a}}}{\sin(dx+c)} \right)}{d} \right]$$

[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)
*sqrt(-1/a)/sin(d*x + c))/d]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{1}{\sqrt{a \cos(c + dx) + a}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(1/sqrt(a*cos(c + d*x) + a), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(37) = 74.

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx \\ = \frac{\sqrt{2} \log \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 2 \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right) - \sqrt{2} \log \left(\cos \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 2 \sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right)}{2\sqrt{ad}}$$

[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(37) = 74$.

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= \frac{\frac{\sqrt{2} \log\left(\left|\frac{1}{\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)} + \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 2\right|\right)}{\sqrt{a} \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{\sqrt{2} \log\left(\left|\frac{1}{\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)} + \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 2\right|\right)}{\sqrt{a} \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}}{4d}$$

[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/4*(sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) + 2))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) - 2))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))))/d

Mupad [B] (verification not implemented)

Time = 14.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx = \frac{F\left(\frac{c}{2} + \frac{dx}{2} \mid 1\right) \sqrt{\frac{2(a+a \cos(c+dx))}{a}}}{d \sqrt{a + a \cos(c + dx)}}$$

[In] int(1/(a + a*cos(c + d*x))^(1/2),x)

[Out] (ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a*cos(c + d*x))^(1/2))

$$3.127 \quad \int \frac{\sec(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	2028
Rubi [A] (verified)	2028
Mathematica [A] (verified)	2029
Maple [B] (verified)	2030
Fricas [B] (verification not implemented)	2030
Sympy [F]	2031
Maxima [F]	2031
Giac [A] (verification not implemented)	2031
Mupad [F(-1)]	2032

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}}$$

[Out] $2*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d/a^{(1/2)}-\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2859, 2728, 212, 2852}

$$\int \frac{\sec(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}}$$

[In] `Int[Sec[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]`

[Out] $(2*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])]/(\operatorname{Sqrt}[a]*d) - (\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c + d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Cos}[c + d*x]])])]/(\operatorname{Sqrt}[a]*d)$

Rule 212

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2852

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2859

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \frac{\sqrt{a + a \cos(c + dx)} \sec(c + dx) dx}{a}}{a} - \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx \\ &= -\frac{2 \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} + \frac{2 \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \\ &= \frac{2 \text{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \text{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\begin{aligned} &\int \frac{\sec(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx \\ &= -\frac{2(\text{arctanh}(\sin(\frac{1}{2}(c + dx))) - \sqrt{2}\text{arctanh}(\sqrt{2}\sin(\frac{1}{2}(c + dx)))) \cos(\frac{1}{2}(c + dx))}{d\sqrt{a(1 + \cos(c + dx))}} \end{aligned}$$

[In] Integrate[Sec[c + d*x]/Sqrt[a + a*Cos[c + d*x]], x]

[Out] (-2*(ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]])*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(70) = 140.

Time = 1.64 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.66

method	result
default	$-\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\sqrt{2} \ln\left(\frac{4\sqrt{a} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} + 4a}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right) - \ln\left(\frac{4\sqrt{2} a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{a+8a}}{2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}}\right) - \ln\left(\frac{\sqrt{a} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{d}\right)}{\sqrt{a} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} d$

[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] $-\cos(1/2*d*x+1/2*c)*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2^{(1/2)}*\ln(4*(a^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+a)/\cos(1/2*d*x+1/2*c))- \ln(4/(2*\cos(1/2*d*x+1/2*c)+2^{(1/2)}))*(2^{(1/2)}*a*\cos(1/2*d*x+1/2*c)+2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)+2*a}) - \ln(-4/(2*\cos(1/2*d*x+1/2*c)-2^{(1/2)}))*(2^{(1/2)}*a*\cos(1/2*d*x+1/2*c)-2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)-2*a}))/a^{(1/2)}/\sin(1/2*d*x+1/2*c)/(a*\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/d$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.93

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{\sqrt{2}\sqrt{a} \log\left(-\frac{\cos(dx+c)^2 + 2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c) - 2\cos(dx+c) - 3}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right) + \sqrt{a} \log\left(\frac{a\cos(dx+c)^3 - 7a\cos(dx+c)^2 - 4\sqrt{a\cos(dx+c)+a}\cos(dx+c) + 8a}{\cos(dx+c)^3 + \cos(dx+c)}\right)}{2ad}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $1/2*(\sqrt{2}*\sqrt{a}*\log(-(\cos(d*x+c)^2 + 2*\sqrt{2}*\sqrt{a*\cos(d*x+c)+a}*\sin(d*x+c)/\sqrt{a} - 2*\cos(d*x+c) - 3)/(\cos(d*x+c)^2 + 2*\cos(d*x+c) + 1)) + \sqrt{a}*\log((a*\cos(d*x+c)^3 - 7*a*\cos(d*x+c)^2 - 4*\sqrt{a*\cos(d*x+c)+a}*\sqrt{a}*(\cos(d*x+c) - 2)*\sin(d*x+c) + 8*a)/(\cos(d*x+c)^3 + \cos(d*x+c)^2)))/(a*d)$

Sympy [F]

$$\int \frac{\sec(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\sec(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}} dx$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)

Maxima [F]

$$\int \frac{\sec(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\sec(dx + c)}{\sqrt{a \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)/sqrt(a*cos(d*x + c) + a), x)

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.42

$$\int \frac{\sec(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= \frac{\sqrt{2} \left(\frac{\sqrt{2} \log \left(\frac{-2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{2\sqrt{2} + 4 \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)} \right)}{\operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} + \frac{\log\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)}{\operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{\log\left(-\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)}{\operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} \right)}{2\sqrt{ad}}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))/sgn(cos(1/2*d*x + 1/2*c)) + log(sin(1/2*d*x + 1/2*c) + 1)/sgn(cos(1/2*d*x + 1/2*c)) - log(-sin(1/2*d*x + 1/2*c) + 1)/sgn(cos(1/2*d*x + 1/2*c)))/(sqrt(a)*d)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx) \sqrt{a + a \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(1/2)), x)
```

$$3.128 \quad \int \frac{\sec^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	2033
Rubi [A] (verified)	2033
Mathematica [C] (verified)	2035
Maple [B] (verified)	2035
Fricas [B] (verification not implemented)	2036
Sympy [F]	2036
Maxima [B] (verification not implemented)	2037
Giac [F(-2)]	2049
Mupad [F(-1)]	2049

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\tan(c+dx)}{d \sqrt{a+a \cos(c+dx)}}$$

[Out] $-\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d/a^{(1/2)}+\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2858, 3064, 2728, 212, 2852}

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} + \frac{\tan(c+dx)}{d \sqrt{a \cos(c+dx)+a}}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]^2/\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]],x]$

[Out] $-(\text{ArcTanh}[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{a+a\cos[c+dx]}}]/(\sqrt{a}d)) + (\sqrt{2}\text{ArcTanh}[\frac{\sqrt{a}\sin[c+dx]}{\sqrt{2}\sqrt{a+a\cos[c+dx]}}]) / (\sqrt{a}d) + \frac{\tan[c+dx]}{d\sqrt{a+a\cos[c+dx]}}$

Rule 212

$\text{Int}[(a_ + (b_)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]\text{Rt}[-b, 2]))\text{ArcTanh}[\text{Rt}[-b, 2](x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 2728

$\text{Int}[1/\sqrt{(a_ + (b_)\sin[(c_ + (d_)(x_)]), x_Symbol] \rightarrow \text{Dist}[-2/d, \text{Subst}[\text{Int}[1/(2a - x^2), x], x, b(\cos[c + dx]/\sqrt{a + b\sin[c + dx]})], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2852

$\text{Int}[\sqrt{(a_ + (b_)\sin[(e_ + (f_)(x_)])/((c_ + (d_)\sin[(e_ + (f_)(x_)]), x_Symbol] \rightarrow \text{Dist}[-2(b/f), \text{Subst}[\text{Int}[1/(b*c + a*d - dx^2), x], x, b(\cos[e + fx]/\sqrt{a + b\sin[e + fx]})], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 2858

$\text{Int}[(c_ + (d_)\sin[(e_ + (f_)(x_)])^{n_}/\sqrt{(a_ + (b_)\sin[(e_ + (f_)(x_)]), x_Symbol] \rightarrow \text{Simp}[-d*\cos[e + fx]*(c + d\sin[e + fx])^{n+1}/(f*(n+1)*(c^2 - d^2)*\sqrt{a + b\sin[e + fx]}), x] - \text{Dist}[1/(2*b*(n+1)*(c^2 - d^2)), \text{Int}[(c + d\sin[e + fx])^{n+1}*(\text{Simp}[a*d - 2*b*c*(n+1) + b*d*(2*n+3)*\sin[e + fx], x]/\sqrt{a + b\sin[e + fx]}), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 3064

$\text{Int}[(A_ + (B_)\sin[(e_ + (f_)(x_)])/(\sqrt{(a_ + (b_)\sin[(e_ + (f_)(x_)] + (f_)(x_)]*((c_ + (d_)\sin[(e_ + (f_)(x_)]), x_Symbol] \rightarrow \text{Dist}[(A*b - a*B)/(b*c - a*d), \text{Int}[1/\sqrt{a + b\sin[e + fx]}, x], x] + \text{Dist}[(B*c - A*d)/(b*c - a*d), \text{Int}[\sqrt{a + b\sin[e + fx]}/(c + d\sin[e + fx]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\text{integral} = \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{(a-a\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{2a}$$

$$\begin{aligned}
&= \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \sqrt{a+a\cos(c+dx)} \sec(c+dx) dx}{2a} + \int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx \\
&= \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&\quad - \frac{2\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.78

$$\begin{aligned}
&\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx \\
&= \frac{\cos\left(\frac{1}{2}(c+dx)\right) \left(\sqrt{2}\log\left(i - \sqrt{2}e^{\frac{1}{2}i(c+dx)} - ie^{i(c+dx)}\right) - \sqrt{2}\log\left(i + \sqrt{2}e^{\frac{1}{2}i(c+dx)} - ie^{i(c+dx)}\right) - 4\log(\cos\right)}{2d\sqrt{a(1+}}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Cos[(c + d*x)/2]*(Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - 4*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 4*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] + 4*Sec[c + d*x]*Sin[(c + d*x)/2]))/(2*d*Sqrt[a*(1 + Cos[c + d*x])])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(91) = 182.

Time = 1.75 (sec) , antiderivative size = 466, normalized size of antiderivative = 4.31

method	result
default	$\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\left(2a\left(-2\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+4a}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right)+\ln\left(\frac{4\sqrt{2}a\cos\left(\frac{dx}{2} + \frac{c}{2}\right)+4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sqrt{a+8a}}{2\cos\left(\frac{dx}{2} + \frac{c}{2}\right)+\sqrt{2}}\right)\right)}{2d\sqrt{a(1+\cos(dx+c))}}$

[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-2*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))+ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a)))*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a-ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(91) = 182.

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.19

$$\int \frac{\sec^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= \frac{(\cos(dx + c))^2 + \cos(dx + c))\sqrt{a} \log\left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 + 4\sqrt{a \cos(dx+c) + a}\sqrt{a}(\cos(dx+c) - 2) \sin(dx+c) + 8a}{\cos(dx+c)^3 + \cos(dx+c)^2}\right) + \frac{2\sqrt{a} \cos(dx+c)}{4(ad \cos(dx+c))^2}}{4(ad \cos(dx+c))^2}$$

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 2*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\sec^2(c + dx)}{\sqrt{a (\cos(c + dx) + 1)}} dx$$

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x)
```


$$\begin{aligned}
& 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \log \\
& (2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d* \\
& x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x + 1/2* \\
& c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2} \\
&)\sin(1/2*d*x + 1/2*c) + 2))\cos(d*x + c)^2 + (2\sqrt{2}\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 2\sqrt{2} \\
&)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2} \\
& (2)\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \log(2\cos(1 \\
& /2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2* \\
& c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \log(2\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/ \\
& 2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) \\
& ^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2))* \\
& \sin(d*x + c)^2 - 4\sqrt{2}\cos(1/2*d*x + 1/2*c)\sin(d*x + c) + 2*(2\sqrt{2} \\
&)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2* \\
& c) + 1) - 2\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) - \log(2\cos(1/2 \\
& *d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) \\
& + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x + 1/2*c)^2 + 2s \\
& in(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2* \\
& d*x + 1/2*c) + 2) - \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 \\
& - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + 1 \\
& og(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2* \\
& d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c) + 2\sqrt{2} \\
&)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 2\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2\sin(1/2*d*x + 1/2*c) + 1) + 4\sqrt{2}\sin(1/2*d*x + 1/2*c) - \log(2\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c \\
&) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2 \\
& *d*x + 1/2*c) + 2) - \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \\
& \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2 \\
& *d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2))*\cos(3*d*x + 3*c)^2 + (\\
& (2\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2* \\
& d*x + 1/2*c) + 1) - 2\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin \\
& (1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d* \\
& x + 1/2*c) + 2) + \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + \\
& 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \log \\
& (2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d* \\
& x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x + 1/2* \\
& c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2} \\
&)\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c)^2 + (2\sqrt{2}\log(\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))* \\
& \sin(d*x + c)^2 - 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(2*\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) - \log(2*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 1 \\
& og(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c) + 2*\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) + 4*\sqrt{2}*\sin(1/2*d*x + 1/2*c) - \log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(2*d*x + 2*c)^2 + 6 \\
& *(2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) - lo \\
& g(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c) \\
& ^2 + ((2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*si \\
& n(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*
\end{aligned}$$

$$\begin{aligned}
& c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) \\
& - \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(\\
& 1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x \\
& + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2* \\
& \sqrt{2}\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c)^2 + (2\sqrt{2})\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) - 2*s \\
& \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \log(2 \\
& *\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x \\
& + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \log(2\cos(1/2*d*x + 1/2*c) \\
& ^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}* \\
& \sin(1/2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + \\
& 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) \\
& + 2))*\sin(d*x + c)^2 - 4\sqrt{2}\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(2*s \\
& \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x \\
& + 1/2*c) + 1) - 2\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2\sin(1/2*d*x + 1/2*c) + 1) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) - \log(2*c \\
& \cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + \\
& 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x + 1/2*c)^2 \\
& + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin \\
& (1/2*d*x + 1/2*c) + 2) - \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + \\
& 2) + \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos \\
& (1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c) + 2* \\
& \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x \\
& + 1/2*c) + 1) - 2\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2\sin(1/2*d*x + 1/2*c) + 1) + 4\sqrt{2}\sin(1/2*d*x + 1/2*c) - \log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + \\
& 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x + 1/2*c)^ \\
& 2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin \\
& (1/2*d*x + 1/2*c) + 2) - \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1 \\
& /2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + \\
& 2) + \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos \\
& (1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2))*\sin(3*d*x + 3*c) \\
& ^2 + ((2\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin \\
& (1/2*d*x + 1/2*c) + 1) - 2\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) - \log(2\cos(1/2*d*x + 1/2*c)^2 + \\
& 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(\\
& 1/2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2* \\
& c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) \\
& - \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(\\
& 1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \log(2\cos(1/2*d*x \\
& + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2* \\
& \sqrt{2}\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c)^2 + (2\sqrt{2})\log(\cos(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*s \\
& \text{qrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *s\text{qrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \log(2 \\
& *\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& + 2))*\sin(d*x + c)^2 - 4*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(2*s \\
& \text{qrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 2*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) - \log(2*c \\
& \text{os}(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\text{si} \\
& \text{n}(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + \\
& 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\text{co} \\
& \text{s}(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c) + 2* \\
& \text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 2*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 4*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) - \log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\text{s} \\
& \text{in}(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + \\
& 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\text{c} \\
& \text{os}(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2))*\sin(2*d*x + 2*c) \\
& ^2 + 2*((2*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - 2*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\text{si} \\
& \text{n}(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + \\
& 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\text{co} \\
& \text{s}(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - \\
& 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c)^2 + 2*(2*\text{sqrt}(2)*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 2*\text{sqrt}(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) - \log(2*\cos(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c) + 2*\sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) - \log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\sin(d*x + c)^2 + 2*((2*\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos \\
& s(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2))*\cos(d*x + c)^3 + 3*(2*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 1 \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c \\
&)^2 + ((2*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2 \\
&) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c) + 2*\sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& t(2) \cdot \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 4*\sqrt{2}*\sin(1/2*d*x + 1/2*c) - \log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\sin(d*x + c)^2 + ((2*\sqrt{2})*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 2*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2))*\cos(d*x + c)^2 + (2*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2})*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\sin(d*x + c)^2 - 4*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(2*\sqrt{2})*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*sqr \\
& t(2)*\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x + c) + 2*\sqrt{2})*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2})* \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) + 4*\sqrt{2}*\sin(1/2*d*x + 1/2*c) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) -
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 4*\sqrt{2} \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c) + 2))*\sin(2*d*x + 2*c) + ((2*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\cos(d*x \\
& + c)^2 + 2*(2*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*\co \\
& s(d*x + c) + 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 4*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2))*s \\
& \sin(d*x + c) + 4*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x + 3*c) + 2*((2*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 2*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c) + 2) + \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}
\end{aligned}$$

+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))/((cos(d*x + c))^4 + sin(d*x + c)^4 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(3*d*x + 3*c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(2*d*x + 2*c)^2 + 4*cos(d*x + c)^3 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3*d*x + 3*c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 2*(cos(d*x + c)^3 + (cos(d*x + c) + 1)*sin(d*x + c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + 2*(cos(d*x + c)^3 + (cos(d*x + c) + 1)*sin(d*x + c)^2 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 6*cos(d*x + c)^2 + 2*(sin(d*x + c)^3 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(2*d*x + 2*c) + (cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(d*x + c))*sin(3*d*x + 3*c) + 2*(sin(d*x + c)^3 + (cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(d*x + c))*sin(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sqrt(a)*d

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Exception raised: TypeError}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^2 \sqrt{a + a \cos(c + dx)}} dx$$

[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(1/2)), x)

3.129 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$

Optimal result	2050
Rubi [A] (verified)	2050
Mathematica [C] (verified)	2052
Maple [B] (verified)	2053
Fricas [B] (verification not implemented)	2054
Sympy [F]	2054
Maxima [F(-1)]	2054
Giac [F(-2)]	2055
Mupad [F(-1)]	2055

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{7 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4\sqrt{ad}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\tan(c+dx)}{4d\sqrt{a+a \cos(c+dx)}} + \frac{\sec(c+dx) \tan(c+dx)}{2d\sqrt{a+a \cos(c+dx)}}$$

[Out] $7/4*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d/a^{(1/2)}-\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}-1/4*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+1/2*\sec(d*x+c)*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2858, 3063, 3064, 2728, 212, 2852}

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{7 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4\sqrt{ad}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} - \frac{\tan(c+dx)}{4d\sqrt{a \cos(c+dx)+a}} + \frac{\tan(c+dx) \sec(c+dx)}{2d\sqrt{a \cos(c+dx)+a}}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]^3/\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]],x]$

[Out] $(7*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])]/(4*\operatorname{Sqrt}[a]*d) - (\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])])$

)]/(Sqrt[a]*d) - Tan[c + d*x]/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (Sec[c + d*x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2852

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2858

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x])), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 3064

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[(A

*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{(a-3a\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
 &= -\frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{(-\frac{7a^2}{2} + \frac{1}{2}a^2\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{4a^2} \\
 &= -\frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{7\int \sqrt{a+a\cos(c+dx)}\sec(c+dx) dx}{8a} - \int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx \\
 &= -\frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} \\
 &\quad - \frac{7\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{2\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
 &= \frac{7\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{ad}} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{ad}} \\
 &\quad - \frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.45

$$\begin{aligned}
 &\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx \\
 &= \frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-7\sqrt{2}\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)+7\sqrt{2}\log\left(i+\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)\right)}{\sqrt{a+a\cos(c+dx)}}
 \end{aligned}$$

[In] Integrate[Sec[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]], x]


```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^2*(-7*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 7*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 16*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + Cos[2*(c + d*x)]*(-7*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 7*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 16*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 16*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]]) - 16*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] + 20*Sin[(c + d*x)/2] - 4*Sin[(3*(c + d*x))/2]))/(16*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(122) = 244$.

Time = 1.80 (sec) , antiderivative size = 677, normalized size of antiderivative = 4.61

method	result
default	$-\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\left(-4a\left(-8\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+4a}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right)+7\ln\left(-\frac{4\left(\sqrt{2}a\cos\left(\frac{dx}{2} + \frac{c}{2}\right)-\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}{2\cos\left(\frac{dx}{2} + \frac{c}{2}\right)-\sqrt{2}}\right)\right)}{\dots}$

```
[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*a*(-8*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))+7*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))+7*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*sin(1/2*d*x+1/2*c)^4+(-32*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-4*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+28*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a+28*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)*sin(1/2*d*x+1/2*c)^2+8*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-7*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a-7*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(122) = 244.

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.71

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= \frac{7 (\cos(dx + c)^3 + \cos(dx + c)^2) \sqrt{a} \log\left(\frac{a \cos(dx+c)^3 - 7a \cos(dx+c)^2 - 4\sqrt{a} \cos(dx+c) + a\sqrt{a}(\cos(dx+c)-2) \sin(dx+c) + 8a}{\cos(dx+c)^3 + \cos(dx+c)^2}\right) - 16 (ad \cos(dx+c)^3 + a^2 \cos(dx+c)^2)}{16 (ad \cos(dx+c)^3 + a^2 \cos(dx+c)^2)}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/16*(7*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)

Sympy [F]

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\sec^3(c + dx)}{\sqrt{a (\cos(c + dx) + 1)}} dx$$

[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)**3/sqrt(a*(cos(c + d*x) + 1)), x)

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Timed out

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^3 \sqrt{a + a \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(1/2)), x)
```

3.130 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$

Optimal result	2056
Rubi [A] (verified)	2056
Mathematica [C] (verified)	2059
Maple [B] (verified)	2059
Fricas [A] (verification not implemented)	2060
Sympy [F]	2061
Maxima [F(-1)]	2061
Giac [F(-2)]	2061
Mupad [F(-1)]	2061

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = -\frac{9 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{8\sqrt{ad}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{7 \tan(c+dx)}{8d\sqrt{a+a \cos(c+dx)}} - \frac{\sec(c+dx) \tan(c+dx)}{12d\sqrt{a+a \cos(c+dx)}} + \frac{\sec^2(c+dx) \tan(c+dx)}{3d\sqrt{a+a \cos(c+dx)}}$$

[Out] $-9/8*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d/a^{(1/2)}+\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+7/8*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}-1/12*\sec(d*x+c)*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+1/3*\sec(d*x+c)^2*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2858, 3063, 3064, 2728, 212, 2852}

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = -\frac{9 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{8\sqrt{ad}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} + \frac{7 \tan(c+dx)}{8d\sqrt{a \cos(c+dx)+a}} + \frac{\tan(c+dx) \sec^2(c+dx)}{3d\sqrt{a \cos(c+dx)+a}} - \frac{\tan(c+dx) \sec(c+dx)}{12d\sqrt{a \cos(c+dx)+a}}$$

[In] Int[Sec[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (-9*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/(8*Sqrt[a]*d + (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d) + (7*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) - (Sec[c + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (Sec[c + d*x]^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2852

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2858

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m

+ 1/2, 0])

Rule 3064

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sec^2(c + dx) \tan(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{(a-5a \cos(c+dx)) \sec^3(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx}{6a} \\
 &= -\frac{\sec(c + dx) \tan(c + dx)}{12d\sqrt{a + a \cos(c + dx)}} + \frac{\sec^2(c + dx) \tan(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{\left(-\frac{21a^2}{2} + \frac{3}{2}a^2 \cos(c+dx)\right) \sec^2(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx}{12a^2} \\
 &= \frac{7 \tan(c + dx)}{8d\sqrt{a + a \cos(c + dx)}} - \frac{\sec(c + dx) \tan(c + dx)}{12d\sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{\sec^2(c + dx) \tan(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{\left(\frac{27a^3}{4} - \frac{21}{4}a^3 \cos(c+dx)\right) \sec(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx}{12a^3} \\
 &= \frac{7 \tan(c + dx)}{8d\sqrt{a + a \cos(c + dx)}} - \frac{\sec(c + dx) \tan(c + dx)}{12d\sqrt{a + a \cos(c + dx)}} + \frac{\sec^2(c + dx) \tan(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} \\
 &\quad - \frac{9 \int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx}{16a} + \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx \\
 &= \frac{7 \tan(c + dx)}{8d\sqrt{a + a \cos(c + dx)}} - \frac{\sec(c + dx) \tan(c + dx)}{12d\sqrt{a + a \cos(c + dx)}} + \frac{\sec^2(c + dx) \tan(c + dx)}{3d\sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{9 \text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{8d} - \frac{2 \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \\
 &= -\frac{9 \arctanh\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{8\sqrt{ad}} + \frac{\sqrt{2} \arctanh\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} \\
 &\quad + \frac{7 \tan(c + dx)}{8d\sqrt{a + a \cos(c + dx)}} - \frac{\sec(c + dx) \tan(c + dx)}{12d\sqrt{a + a \cos(c + dx)}} + \frac{\sec^2(c + dx) \tan(c + dx)}{3d\sqrt{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.13

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right) \sec^3(c+dx) \left(9\cos(c+dx) \left(9\sqrt{2}\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)-9\sqrt{2}\log\left(i+\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)\right)\right)}{\sqrt{a+a\cos(c+dx)}}$$

[In] Integrate[Sec[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^3*(9*Cos[c + d*x]*(9*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - 9*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - 32*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 32*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]]) + 3*Cos[3*(c + d*x)]*(9*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - 9*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] - 32*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 32*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]]) + 4*(78*Sin[(c + d*x)/2] - 25*Sin[(3*(c + d*x))/2] + 21*Sin[(5*(c + d*x))/2]))/(192*d*Sqrt[a*(1 + Cos[c + d*x])])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. 2(152) = 304.

Time = 2.04 (sec) , antiderivative size = 883, normalized size of antiderivative = 4.88

method	result	size
default	Expression too large to display	883

[In] int(sec(d*x+c)^4/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(16*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))-9*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))-9*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*sin(1/2*d*x+1/2*c)^6+(576*2^(1/2)*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a-324*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*a-324*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*a+168*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))*sin(1/2*d*x+1/2*c)^4+(-288*2^(1/2)

) * ln(4 * (a^(1/2) * (a * sin(1/2 * d * x + 1/2 * c))^2)^(1/2) + a) / cos(1/2 * d * x + 1/2 * c) * a + 162 * ln(4 / (2 * cos(1/2 * d * x + 1/2 * c) + 2^(1/2))) * (2^(1/2) * a * cos(1/2 * d * x + 1/2 * c) + 2^(1/2) * (a * sin(1/2 * d * x + 1/2 * c))^2)^(1/2) * a^(1/2) + 2 * a) * a + 162 * ln(-4 / (2 * cos(1/2 * d * x + 1/2 * c) - 2^(1/2))) * (2^(1/2) * a * cos(1/2 * d * x + 1/2 * c) - 2^(1/2) * (a * sin(1/2 * d * x + 1/2 * c))^2)^(1/2) * a^(1/2) - 2 * a) * a - 160 * 2^(1/2) * (a * sin(1/2 * d * x + 1/2 * c))^2)^(1/2) * a^(1/2) * sin(1/2 * d * x + 1/2 * c)^2 + 48 * 2^(1/2) * ln(4 * (a^(1/2) * (a * sin(1/2 * d * x + 1/2 * c))^2)^(1/2) + a) / cos(1/2 * d * x + 1/2 * c) * a - 27 * ln(4 / (2 * cos(1/2 * d * x + 1/2 * c) + 2^(1/2))) * (2^(1/2) * a * cos(1/2 * d * x + 1/2 * c) + 2^(1/2) * (a * sin(1/2 * d * x + 1/2 * c))^2)^(1/2) * a^(1/2) + 2 * a) * a - 27 * ln(-4 / (2 * cos(1/2 * d * x + 1/2 * c) - 2^(1/2))) * (2^(1/2) * a * cos(1/2 * d * x + 1/2 * c) - 2^(1/2) * (a * sin(1/2 * d * x + 1/2 * c))^2)^(1/2) * a^(1/2) - 2 * a) * a + 54 * 2^(1/2) * (a * sin(1/2 * d * x + 1/2 * c))^2)^(1/2) * a^(1/2) / a^(3/2) / (2 * cos(1/2 * d * x + 1/2 * c) - 2^(1/2))^3 / (2 * cos(1/2 * d * x + 1/2 * c) + 2^(1/2))^3 / sin(1/2 * d * x + 1/2 * c) / (a * cos(1/2 * d * x + 1/2 * c))^2)^(1/2) / d

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.45

$$\int \frac{\sec^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= \frac{27 (\cos(dx + c)^4 + \cos(dx + c)^3) \sqrt{a} \log\left(\frac{a \cos(dx + c)^3 - 7a \cos(dx + c)^2 + 4\sqrt{a \cos(dx + c) + a} \sqrt{a} (\cos(dx + c) - 2) \sin(dx + c) + 8a}{\cos(dx + c)^3 + \cos(dx + c)^2}\right)}{1}$$

[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/96*(27*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*(21*cos(d*x + c)^2 - 2*cos(d*x + c) + 8)*sin(d*x + c) + 48*sqrt(2)*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)

Sympy [F]

$$\int \frac{\sec^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\sec^4(c + dx)}{\sqrt{a(\cos(c + dx) + 1)}} dx$$

[In] `integrate(sec(d*x+c)**4/(a+a*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sec(c + d*x)**4/sqrt(a*(cos(c + d*x) + 1)), x)`

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Timed out}$$

[In] `integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] Timed out

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Exception raised: TypeError}$$

[In] `integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")`

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^4 \sqrt{a + a \cos(c + dx)}} dx$$

[In] `int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2)),x)`

[Out] `int(1/(cos(c + d*x)^4*(a + a*cos(c + d*x))^(1/2)), x)`

$$3.131 \quad \int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	2062
Rubi [A] (verified)	2062
Mathematica [A] (verified)	2065
Maple [A] (verified)	2065
Fricas [A] (verification not implemented)	2066
Sympy [F(-1)]	2066
Maxima [F(-1)]	2066
Giac [A] (verification not implemented)	2067
Mupad [F(-1)]	2067

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = -\frac{15 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos^3(c+dx) \sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}} + \frac{31 \sin(c+dx)}{5ad\sqrt{a+a \cos(c+dx)}} + \frac{9 \cos^2(c+dx) \sin(c+dx)}{10ad\sqrt{a+a \cos(c+dx)}} - \frac{13\sqrt{a+a \cos(c+dx)} \sin(c+dx)}{10a^2d}$$

[Out] $-1/2*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}-15/4*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+31/5*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(1/2)}+9/10*\cos(d*x+c)^2*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(1/2)}-13/10*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/a^2/d$

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2844, 3062, 3047, 3102, 2830, 2728, 212}

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = -\frac{15 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{13 \sin(c+dx) \sqrt{a \cos(c+dx)+a}}{10a^2d} - \frac{\sin(c+dx) \cos^3(c+dx)}{2d(a \cos(c+dx)+a)^{3/2}} + \frac{9 \sin(c+dx) \cos^2(c+dx)}{10ad\sqrt{a \cos(c+dx)+a}} + \frac{31 \sin(c+dx)}{5ad\sqrt{a \cos(c+dx)+a}}$$

[In] Int[Cos[c + d*x]^4/(a + a*cos[c + d*x])^(3/2), x]

[Out] (-15*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*cos[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(2*d*(a + a*cos[c + d*x])^(3/2)) + (31*Sin[c + d*x])/(5*a*d*Sqrt[a + a*cos[c + d*x]]) + (9*cos[c + d*x]^2*Sin[c + d*x])/(10*a*d*Sqrt[a + a*cos[c + d*x]]) - (13*Sqrt[a + a*cos[c + d*x]]*Sin[c + d*x])/(10*a^2*d)

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2830

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^n, x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3047

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3062

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])

```

Rule 3102

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\cos^2(c+dx)(3a-\frac{9}{2}a\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\cos(c+dx)(-9a^2+\frac{39}{4}a^2\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}} dx}{5a^3} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-9a^2\cos(c+dx)+\frac{39}{4}a^2\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{5a^3} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{13\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{10a^2d} - \frac{2\int \frac{\frac{39a^3}{8}-\frac{93}{4}a^3\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{15a^4} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{31\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{13\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{10a^2d} - \frac{15\int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{31\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{13\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{10a^2d} + \frac{15\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad}
\end{aligned}$$

$$= -\frac{15\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{31\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} \\ + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{13\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{10a^2d}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.64

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{\left(-75\sqrt{2}\operatorname{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)(1+\cos(c+dx)) + 2\sqrt{1-\cos(c+dx)}\right)}{20d\sqrt{1-\cos(c+dx)}(a)}$$

[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(3/2), x]

[Out] ((-75*sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*(1 + Cos[c + d*x]) + 2*sqrt[1 - Cos[c + d*x]]*(47 + 39*Cos[c + d*x] - 2*Cos[2*(c + d*x)] + Cos[3*(c + d*x)]))*Sin[c + d*x])/(20*d*sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.45

method	result
default	$\frac{\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-32\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+32\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+75\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\right)}{20\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{5/2}}$

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/20/cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-32*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^6+32*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+75*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*sin(1/2*d*x+1/2*c)^2*a-80*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2-75*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a+85*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.01

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \frac{75 \sqrt{2} (\cos(dx + c)^2 + 2 \cos(dx + c) + 1) \sqrt{a} \log\left(-\frac{a \cos(dx+c)^2 + 2 \sqrt{2} \sqrt{a \cos(dx+c)} + a}{\cos(dx+c)^2 + 2 \cos(dx+c) + 1}\right) + 4 \sqrt{a} \sin(dx+c) - 2 a \cos(dx+c) - 3 a}{40 (a^2 d \cos(dx+c)^2 + 2 a^2 d \cos(dx+c) + a^2 d)}$$

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x + c) + 49)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 1.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx =$$

$$\frac{75 \sqrt{2} \log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^{\frac{3}{2}} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{75 \sqrt{2} \log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^{\frac{3}{2}} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} + \frac{10 \sqrt{2} \sin(\frac{1}{2} dx + \frac{1}{2} c)}{(\sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1) a^{\frac{3}{2}} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{32 \sqrt{2} (2 a^{\frac{17}{2}} \sin(\frac{1}{2} dx + \frac{1}{2} c))}{a^{10} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))}$$

$$40 d$$

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/40*(75*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))) - 75*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))) + 10*sqrt(2)*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))) - 32*sqrt(2)*(2*a^(17/2)*sin(1/2*d*x + 1/2*c)^5 + 5*a^(17/2)*sin(1/2*d*x + 1/2*c))/(a^10*sgn(cos(1/2*d*x + 1/2*c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^4}{(a + a \cos(c + dx))^{3/2}} dx$$

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(3/2), x)
```

$$3.132 \quad \int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	2068
Rubi [A] (verified)	2068
Mathematica [A] (verified)	2070
Maple [A] (verified)	2071
Fricas [A] (verification not implemented)	2071
Sympy [F(-1)]	2072
Maxima [F(-1)]	2072
Giac [A] (verification not implemented)	2072
Mupad [F(-1)]	2072

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{11 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos^2(c+dx) \sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}} - \frac{13 \sin(c+dx)}{3ad\sqrt{a+a \cos(c+dx)}} + \frac{7\sqrt{a+a \cos(c+dx)} \sin(c+dx)}{6a^2d}$$

[Out] $-1/2*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}+11/4*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}-13/3*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(1/2)}+7/6*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/a^2/d$

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2844, 3047, 3102, 2830, 2728, 212}

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{11 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{7 \sin(c+dx) \sqrt{a \cos(c+dx)+a}}{6a^2d} - \frac{\sin(c+dx) \cos^2(c+dx)}{2d(a \cos(c+dx)+a)^{3/2}} - \frac{13 \sin(c+dx)}{3ad\sqrt{a \cos(c+dx)+a}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^3/(a+a*\operatorname{Cos}[c+d*x])^{(3/2)},x]$

[Out] $(11*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*d) - (\operatorname{Cos}[c+d*x]^2*\operatorname{Sin}[c+d*x])/(2*d*(a+a*\operatorname{Cos}[c+d*x])$

$)^{(3/2)} - (13*\text{Sin}[c + d*x])/(3*a*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (7*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(6*a^2*d)$

Rule 212

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]))*\text{ArcTanh}[\text{Rt}[-b, 2]*(x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 2728

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*\text{sin}[(c_.) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Dist}[-2/d, \text{Subst}[\text{Int}[1/(2*a - x^2), x], x, b*(\text{Cos}[c + d*x]/\text{Sqrt}[a + b*\text{Sin}[c + d*x]])], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 2830

$\text{Int}[(a_ + (b_)*\text{sin}[(e_.) + (f_)*(x_)])^{(m_)*((c_.) + (d_)*\text{sin}[(e_.) + (f_)*(x_)])}, x_Symbol] \rightarrow \text{Simp}[(-d)*\text{Cos}[e + f*x]*((a + b*\text{Sin}[e + f*x])^{(m/(f*(m + 1)))}), x] + \text{Dist}[(a*d*m + b*c*(m + 1))/(b*(m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{LtQ}[m, -2^{(-1)}]$

Rule 2844

$\text{Int}[(a_ + (b_)*\text{sin}[(e_.) + (f_)*(x_)])^{(m_)*((c_.) + (d_)*\text{sin}[(e_.) + (f_)*(x_)])^{(n_)}}, x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*((c + d*\text{Sin}[e + f*x])^{(n - 1)/(a*f*(2*m + 1))}), x] + \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*(c + d*\text{Sin}[e + f*x])^{(n - 2)}*\text{Simp}[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{GtQ}[n, 1] \&\& (\text{IntegersQ}[2*m, 2*n] \parallel (\text{IntegerQ}[m] \&\& \text{EqQ}[c, 0]))$

Rule 3047

$\text{Int}[(a_ + (b_)*\text{sin}[(e_.) + (f_)*(x_)])^{(m_)*((A_.) + (B_)*\text{sin}[(e_.) + (f_)*(x_)])^{(c_.) + (d_)*\text{sin}[(e_.) + (f_)*(x_)]]}, x_Symbol] \rightarrow \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(A*c + (B*c + A*d)*\text{Sin}[e + f*x] + B*d*\text{Sin}[e + f*x]^2), x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

Rule 3102

$\text{Int}[(a_ + (b_)*\text{sin}[(e_.) + (f_)*(x_)])^{(m_)*((A_.) + (B_)*\text{sin}[(e_.) + (f_)*(x_)]) + (C_)*\text{sin}[(e_.) + (f_)*(x_)]^2}, x_Symbol] \rightarrow \text{Simp}[(-C)*\text{Cos}[e + f*x]*((a + b*\text{Sin}[e + f*x])^{(m + 1)})/(b*f*(m + 2)), x] + \text{Dist}[1/(b*(m + 2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*\text{Simp}[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m$

+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)(2a-\frac{7}{2}a\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
 &= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{2a\cos(c+dx)-\frac{7}{2}a\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
 &= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{6a^2d} - \frac{\int \frac{-\frac{7a^2}{4}+\frac{13}{2}a^2\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{3a^3} \\
 &= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{13\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{7\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{6a^2d} + \frac{11\int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
 &= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{13\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{7\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{6a^2d} - \frac{11\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad} \\
 &= \frac{11\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} \\
 &\quad - \frac{13\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} + \frac{7\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{6a^2d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{\left(33\sqrt{2}\text{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\right)(1+\cos(c+dx))+2\sqrt{1-\cos(c+dx)}}{12d\sqrt{1-\cos(c+dx)}(a(1+\cos(c+dx)))}$$

[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2), x]

[Out] ((33*sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*(1 + Cos[c + d*x]) + 2*sqrt[1 - Cos[c + d*x]]*(-19 - 12*Cos[c + d*x] + 4*Cos[c + d*x]^2))*Sin[c + d*x]) / (12*d*sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.61

method	result
default	$\frac{\sqrt{2} \sqrt{a \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \left(16 \sqrt{a \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \sqrt{a \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} + 8 \sqrt{a \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \sqrt{a \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} - 33 \ln \left(\frac{4 \sqrt{a} \sqrt{a \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right)} \right)}{12 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) a^{\frac{5}{2}} \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{a \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}}$

[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)

```
[Out] 1/12*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^4+8*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2-33*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*sin(1/2*d*x+1/2*c)^2*a-27*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+33*ln(4*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a)/cos(1/2*d*x+1/2*c))*a)/cos(1/2*d*x+1/2*c)/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.20

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{33\sqrt{2}(\cos(dx+c)^2+2\cos(dx+c)+1)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)}}{\cos(dx+c)^2}\right)}{24(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d)}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

```
[Out] 1/24*(33*sqrt(2)*(cos(d*x+c)^2+2*cos(d*x+c)+1)*sqrt(a)*log(-(a*cos(d*x+c)^2-2*sqrt(2)*sqrt(a*cos(d*x+c)+a))*sqrt(a)*sin(d*x+c)-2*a*cos(d*x+c)-3*a)/(cos(d*x+c)^2+2*cos(d*x+c)+1))+4*sqrt(a*cos(d*x+c)+a)*(4*cos(d*x+c)^2-12*cos(d*x+c)-19)*sin(d*x+c)/(a^2*d*cos(d*x+c)^2+2*a^2*d*cos(d*x+c)+a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.70

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \frac{\frac{3\sqrt{2}\sin(\frac{1}{2}dx + \frac{1}{2}c)}{(\sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)a^{\frac{3}{2}}\text{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} - \frac{8\sqrt{2}(2a^{\frac{9}{2}}\sin(\frac{1}{2}dx + \frac{1}{2}c)^3 + 3a^{\frac{9}{2}}\sin(\frac{1}{2}dx + \frac{1}{2}c))}{a^6\text{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))}}{12d}$$

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*(3*sqrt(2)*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a^(3/2)*
sgn(cos(1/2*d*x + 1/2*c))) - 8*sqrt(2)*(2*a^(9/2)*sin(1/2*d*x + 1/2*c)^3 +
3*a^(9/2)*sin(1/2*d*x + 1/2*c))/(a^6*sgn(cos(1/2*d*x + 1/2*c)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^3}{(a + a \cos(c + dx))^{3/2}} dx$$

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(3/2), x)
```

$$3.133 \quad \int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	2073
Rubi [A] (verified)	2073
Mathematica [A] (verified)	2075
Maple [A] (verified)	2075
Fricas [A] (verification not implemented)	2075
Sympy [F]	2076
Maxima [F(-1)]	2076
Giac [A] (verification not implemented)	2076
Mupad [F(-1)]	2077

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = -\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}} + \frac{2 \sin(c+dx)}{ad\sqrt{a+a \cos(c+dx)}}$$

[Out] $1/2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}-7/4*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)*2^{(1/2)}}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+2*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2837, 2830, 2728, 212}

$$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = -\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{2 \sin(c+dx)}{ad\sqrt{a \cos(c+dx)+a}} + \frac{\sin(c+dx)}{2d(a \cos(c+dx)+a)^{3/2}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^2/(a+a*\operatorname{Cos}[c+d*x])^{(3/2)},x]$

[Out] $(-7*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)*d} + \operatorname{Sin}[c+d*x]/(2*d*(a+a*\operatorname{Cos}[c+d*x])^{(3/2)}) + (2*\operatorname{Sin}[c+d*x])/(a*d*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])$

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2830

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2837

Int[sin[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(m_)), x_Symbol] := Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} + \frac{\int \frac{-\frac{3a}{2} + 2a \cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx}{2a^2} \\
 &= \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} + \frac{2 \sin(c + dx)}{ad \sqrt{a + a \cos(c + dx)}} - \frac{7 \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx}{4a} \\
 &= \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} + \frac{2 \sin(c + dx)}{ad \sqrt{a + a \cos(c + dx)}} + \frac{7 \text{Subst}\left(\int \frac{1}{2a - x^2} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{2ad} \\
 &= -\frac{7 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2} \sqrt{a + a \cos(c + dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} + \frac{2 \sin(c + dx)}{ad \sqrt{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \frac{\left(-7\sqrt{2}\operatorname{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}\right)\right) (1 + \cos(c + dx)) + 2\sqrt{1 - \cos(c + dx)}}{4d\sqrt{1 - \cos(c + dx)}(a(1 + \cos(c + dx)))^{3/2}}$$

[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2), x]

[Out] ((-7*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*(1 + Cos[c + d*x]) + 2*Sqrt[1 - Cos[c + d*x]]*(5 + 4*Cos[c + d*x]))*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.65

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-7\sqrt{2} \ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4a}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right) a\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{2}\sqrt{a} + \sqrt{2}}{4\cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^{\frac{5}{2}} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} d$

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-7*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*a^(1/2)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(5/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.56

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \frac{7\sqrt{2}(\cos(dx + c)^2 + 2\cos(dx + c) + 1)\sqrt{a} \log\left(-\frac{a\cos(dx+c)^2 + 2\sqrt{2}\sqrt{a\cos(dx+c)}}{\cos(dx+c)^2 + 1}\right)}{8(a^2d\cos(dx + c))^2}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2), x, algorithm="fricas")

[Out] 1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d*x

+ c) + a)*(4*cos(d*x + c) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F]

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\cos^2(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(3/2), x)

[Out] Integral(cos(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2), x)

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] Timed out

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.38

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \frac{\frac{7\sqrt{2}\log(\sin(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^{\frac{3}{2}}\text{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} - \frac{7\sqrt{2}\log(-\sin(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^{\frac{3}{2}}\text{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} - \frac{16\sqrt{2}\sin(\frac{1}{2}dx + \frac{1}{2}c)}{a^{\frac{3}{2}}\text{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} + \frac{2\sqrt{2}\sin(\frac{1}{2}dx + \frac{1}{2}c)}{(\sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)a^{\frac{3}{2}}\text{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))}}{8d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2), x, algorithm="giac")

[Out] -1/8*(7*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))) - 7*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))) - 16*sqrt(2)*sin(1/2*d*x + 1/2*c)/(a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a^(3/2)*sgn(cos(1/2*d*x + 1/2*c)))/d

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^2}{(a + a \cos(c + dx))^{3/2}} dx$$

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(3/2), x)
```

```
[Out] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(3/2), x)
```

3.134 $\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$

Optimal result	2078
Rubi [A] (verified)	2078
Mathematica [A] (verified)	2079
Maple [B] (verified)	2079
Fricas [B] (verification not implemented)	2080
Sympy [F]	2080
Maxima [B] (verification not implemented)	2081
Giac [A] (verification not implemented)	2111
Mupad [F(-1)]	2111

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}}$$

[Out] $-1/2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}+3/4*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2829, 2728, 212}

$$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a \cos(c+dx)+a)^{3/2}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]/(a+a*\operatorname{Cos}[c+d*x])^{(3/2)},x]$

[Out] $(3*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)*d} - \operatorname{Sin}[c+d*x]/(2*d*(a+a*\operatorname{Cos}[c+d*x])^{(3/2)})$

Rule 212

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]))*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 2728

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3 \int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx}{4a} \\ &= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{3 \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad} \\ &= \frac{3 \arctanh\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{3 \arctanh\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \cos^3\left(\frac{1}{2}(c+dx)\right) - \frac{1}{2} \sin(c+dx)}{d(a(1+\cos(c+dx)))^{3/2}}$$

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^(3/2), x]
```

```
[Out] (3*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 - Sin[c + d*x]/2)/(d*(a*(1
+ Cos[c + d*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(62) = 124.

Time = 1.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.82

method	result	size
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(3\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4a}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\right)}{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)a^{\frac{5}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}d}$	140

[In] `int(cos(d*x+c)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4}(a\sin(1/2dx+1/2c)^2)^{1/2}(3\sqrt{2}^{1/2}\ln(2(2a^{1/2}(a\sin(1/2dx+1/2c)^2)^{1/2}+2a)/\cos(1/2dx+1/2c))a\cos(1/2dx+1/2c)^2-2^{1/2}(a\sin(1/2dx+1/2c)^2)^{1/2}a^{1/2})/\cos(1/2dx+1/2c)/a^{5/2}/\sin(1/2dx+1/2c)/(a\cos(1/2dx+1/2c)^2)^{1/2}/d$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(62) = 124$.

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.00

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{3\sqrt{2}(\cos(dx+c)^2+2\cos(dx+c)+1)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{8(a^2d\cos(dx+c))^2+2a^2d\cos(dx+c)}$$

[In] `integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] $\frac{1}{8}(3\sqrt{2}(\cos(dx+c)^2+2\cos(dx+c)+1)\sqrt{a}\log(-(a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a})\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a)/(\cos(dx+c)^2+2\cos(dx+c)+1))-4\sqrt{2}a\cos(dx+c)\sqrt{a}\sin(dx+c))/(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d)$

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \int \frac{\cos(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}} dx$$

[In] `integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)`

[Out] `Integral(cos(c+d*x)/(a*(cos(c+d*x)+1))**(3/2),x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46532 vs. 2(62) = 124.

Time = 2.59 (sec) , antiderivative size = 46532, normalized size of antiderivative = 604.31

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/16*(3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 4*sqrt(2)*sin(5/2*d*x + 5/2*c))*cos(3*d*x + 3*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + (3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x + 3*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 3*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 27*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 - 96*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 60*sqrt(2)*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 2*(9*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 9*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sqrt(2)*sin(5/2*d*x + 5/2*c) + 16*

$$\begin{aligned}
& \sqrt{2} \sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + \\
& 3*c) + 60*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\cos(5/2*d*x + \\
& 5/2*c) + 6*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 6*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 2*(9*(\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d \\
& *x + c) + 10*\sqrt{2}*\cos(5/2*d*x + 5/2*c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) \\
& - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x + 3*c) - 20*(3*\sqrt{2}*\cos(2* \\
& d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(5/2*d*x + 5/2*c) + 6*(9* \\
& (\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 16*\sqrt{2}*\cos(3/2*d*x + 3 \\
& /2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 32*(3*\sqrt{2}*\cos \\
& (d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) + 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(6/5*\arctan2(\sin(5/2*d*x + 5/2 \\
& *c), \cos(5/2*d*x + 5/2*c)))^2 + 9*(3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 3*d*x + 3*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 + \\
& 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(3*d*x + 3*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (2*d*x + 2*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 - 96
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(2/ \\
& 5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 3*(\sqrt{2}*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) + 4*\sqrt{2}*\sin(5/2*d*x + 5/2*c))*\sin(3*d*x + 3*c)^2 + 2 \\
& 7*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c)^2 + 27*(\sqrt{2}*\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + (3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*co \\
& s(3*d*x + 3*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 \\
& + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(3*d*x + 3*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(2*d*x + 2*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 - \\
& 96*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 60*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)*\sin(d*x + c) + 2*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) \\
& + 9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*s \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 10*\sqrt{2}*\sin(5/2*d*x + 5/2*c) + 16*\sqrt{2}*\sin(3/2*d*x + \\
& 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 60*(\sqrt{2}*\si \\
& n(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1
\end{aligned}$$

$$\begin{aligned}
&) + 16\sqrt{2}\sin(3/2*d*x + 3/2*c) + 10\sqrt{2}\sin(1/2*d*x + 1/2*c))\cos(\\
& 2*d*x + 2*c) + 6*(3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 10\sqrt{2}\sin(1/2* \\
& d*x + 1/2*c))\cos(d*x + c) + 2*(9*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d* \\
& *x + 2*c) + 9*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 10\sqrt{2}*c \\
& os(5/2*d*x + 5/2*c) - 16\sqrt{2}\cos(3/2*d*x + 3/2*c) - 10\sqrt{2}\cos(1/2* \\
& d*x + 1/2*c))\sin(3*d*x + 3*c) - 20*(3\sqrt{2}\cos(2*d*x + 2*c) + 3\sqrt{2}) \\
& *\cos(d*x + c) + \sqrt{2})\sin(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2}\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\sin(d*x + c) - 16\sqrt{2}\cos(3/2*d*x + 3/2*c) - 10\sqrt{2}\cos \\
& (1/2*d*x + 1/2*c))\sin(2*d*x + 2*c) + 32*(3\sqrt{2}\cos(d*x + c) + \sqrt{2}) \\
& *\sin(3/2*d*x + 3/2*c) + 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 20\sqrt{2}\sin \\
& (1/2*d*x + 1/2*c))*\sin(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2 \\
& *c)))^2 + 9*(3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3*d*x + 3*c)^2 + 27*(s \\
& \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 + 27*(\sqrt{2}\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - s \\
& \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + 3*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3*d \\
& *x + 3*c)^2 + 27*(\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c)^2 + 27* \\
& (\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 - 96\sqrt{2}\cos(3/2*d*x + \\
& 3/2*c)\sin(d*x + c) - 60\sqrt{2}\cos(1/2*d*x + 1/2*c)\sin(d*x + c) + 2*(9* \\
& (\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 9*(\sqrt{2}\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - sq \\
& rt(2)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\cos(d*x + c) + 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 10*\sqrt{2}*(\\
& 2)*\sin(5/2*d*x + 5/2*c) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 60*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2} \\
& *\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sqrt{2}*\sin(3/2*d \\
& *x + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 6*(3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + \\
& c) + 2*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 10*\sqrt{2}*\cos(5/2*d*x + 5/2*c) - 16 \\
& *\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x \\
& + 3*c) - 20*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})* \\
& *\sin(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) \\
& - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(2 \\
& *d*x + 2*c) + 32*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c) + \\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(\\
& 4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 96*(\sqrt{2})*\cos \\
& (3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin \\
& (2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2 \\
& *d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2} \\
& (2)*\cos(d*x + c) + \sqrt{2}))*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) \\
& + \sqrt{2}*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
& (2))*\cos(3/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))\sin(2/5*\arctan \\
& 2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 9*(3*(\sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(3*d*x + 3*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (2*d*x + 2*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 3 \\
& *(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3*d*x + 3*c)^2 + 27*(\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(2*d*x + 2*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(d*x + c)^2 - 96*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 60*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\cos(2*d*x + 2*c) + 9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) - 10*\sqrt{2}*\sin(5/2*d*x + 5/2*c) + 16*\sqrt{2} \\
& *\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c \\
&) + 60*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\cos(5/2*d*x + 5/2* \\
& c) + 6*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c))*\cos(2*d*x + 2*c) + 6*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 1 \\
& 0*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 2*(9*(\sqrt{2}*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(2*d*x + 2*c) + 9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + \\
& c) + 10*\sqrt{2}*\cos(5/2*d*x + 5/2*c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 1 \\
& 0*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x + 3*c) - 20*(3*\sqrt{2}*\cos(2*d*x \\
& + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}*\sin(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c \\
&) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 32*(3*\sqrt{2}*\cos(d \\
& *x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c
\end{aligned}$$

$$\begin{aligned}
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 9* \\
& (\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 2*(9*\sqrt{2})*\cos(2*d*x + 2 \\
& *c) + 9*\sqrt{2}*\cos(d*x + c) - 2*\sqrt{2})*\sin(5/2*d*x + 5/2*c) + 3*\sqrt{2}* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*si \\
& n(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 60*(\sqrt{2})*\sin(2*d*x + 2*c) + \sqrt{2} \\
& (\sqrt{2})*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sqrt{2}*\sin(3/2 \\
& *d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 6*(3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x \\
& + c) + 3*(3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3*d*x + 3*c)^2 + 27*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 + 27*(\sqrt{2})*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3*d*x \\
& + 3*c)^2 + 27*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c)^2 + 27*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 - 96*\sqrt{2})*\cos(3/2*d*x + 3/ \\
& 2*c)*\sin(d*x + c) - 60*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(9*(\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 9*(\sqrt{2})*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 10*\sqrt{2}* \\
& \sin(5/2*d*x + 5/2*c) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c))*\cos(3*d*x + 3*c) + 60*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin \\
& (d*x + c))*\cos(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sqrt{2}*\sin(3/2*d*x \\
& + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 6*(3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) \\
& + 2*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 9*(\sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 10*\sqrt{2}*\cos(5/2*d*x + 5/2*c) - 16*\sqrt{2} \\
& *\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x + 3 \\
& *c) - 20*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin \\
& (5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - \\
& 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(2*d* \\
& x + 2*c) + 32*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c) + 3*\sqrt{2} \\
& *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(4/5 \\
& *\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 3*(3*(\sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(3*d*x + 3*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(2*d*x + 2*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 \\
& + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(3*d*x + 3*c)^2 + 27*(\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log
\end{aligned}$$

$$\begin{aligned}
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 - 96*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 60*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 10*\sqrt{2}*\sin(5/2*d*x + 5/2*c) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 60*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 6*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 2*(9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 10*\sqrt{2}*\cos(5/2*d*x + 5/2*c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x + 3*c) - 20*(3*\sqrt{2})*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}*\sin(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 32*(3*\sqrt{2})*\cos(d*x + c) + \sqrt{2}*\sin(3/2*d*x + 3/2*c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 2*(18*(\sqrt{2})*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + 9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 10*\sqrt{2} \\
& * \cos(5/2*d*x + 5/2*c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c))*\sin(3*d*x + 3*c) + 2*(27*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 27*\sqrt{2} \\
& * \cos(d*x + c)^2 + 27*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 54*\sqrt{2}*\sin(2*d*x \\
& + 2*c)*\sin(d*x + c) + 27*\sqrt{2}*\sin(d*x + c)^2 + 6*(9*\sqrt{2}*\cos(d*x + c \\
&) - 2*\sqrt{2})*\cos(2*d*x + 2*c) - 12*\sqrt{2}*\cos(d*x + c) - 7*\sqrt{2})*\sin(\\
& 5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 16 \\
& * \sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x \\
& + 2*c) + 32*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c) - 16*(\sqrt{2} \\
& * \cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d* \\
& x + c)^2 + \sqrt{2}*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2} \\
& * \sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2} \\
& * \cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3*d*x + 3*c) + \\
& 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d \\
& *x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) \\
& + \sqrt{2})*\sin(3/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - \\
& 6*(\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\co \\
& s(d*x + c)^2 + \sqrt{2}*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + \\
& 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3* \\
& \sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3*d*x + 3* \\
& c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin \\
& (2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2})*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)) \\
&) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))* \\
& \cos(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 6*(3*(\sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) + 2*\sqrt{2}*\sin(5/2*d*x + 5/2*c))*\cos(3*d*x + 3*c \\
&)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 + 27*(\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) + 2*\sqrt{2}*\sin(5/2*d*x + 5/2*c))*\sin(3*d*x + 3*c)^2 + 27*(\sqrt{2}*\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c)^2 + 27*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(c
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\sin(d*x + c)^2 - 96*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 60*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&))*\cos(2*d*x + 2*c) + 9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 2* \\
& (9*\sqrt{2})*\cos(2*d*x + 2*c) + 9*\sqrt{2}*\cos(d*x + c) - 2*\sqrt{2})*\sin(5/2*d \\
& *x + 5/2*c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sqrt{2}*\sin(3/2*d*x \\
& + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 60*(\sqrt{2})* \\
& \sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*co \\
& s(2*d*x + 2*c) + 6*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 10*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c))*\cos(d*x + c) + 3*(3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3 \\
& *d*x + 3*c)^2 + 27*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 + 2 \\
& 7*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - sq \\
& rt(2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(3*d*x + 3*c)^2 + 27*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (2*d*x + 2*c)^2 + 27*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 - 96* \\
& \sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 60*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& *\sin(d*x + c) + 2*(9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 9 \\
& *(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) - 10*\sqrt{2}*\sin(5/2*d*x + 5/2*c) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2 \\
& *c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 60*(\sqrt{2}*\sin(2 \\
& *d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + \\
& 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(2*d \\
& *x + 2*c) + 6*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 10*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c) + 2*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x \\
& + 2*c) + 9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 10*\sqrt{2}*\cos(\\
& 5/2*d*x + 5/2*c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c))*\sin(3*d*x + 3*c) - 20*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos \\
& (d*x + c) + \sqrt{2}*\sin(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2}*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\sin(d*x + c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 32*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin \\
& (3/2*d*x + 3/2*c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 20*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c))*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c) \\
&)) + 2*(18*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin(5/2*d*x + \\
& 5/2*c) + 9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 9*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 10*\sqrt{2}*\cos(5/2*d*x + 5/2*c) - 1 \\
& 6*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x \\
& + 3*c) + 2*(27*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 27*\sqrt{2}*\cos(d*x + c)^2 + 27 \\
& *\sqrt{2}*\sin(2*d*x + 2*c)^2 + 54*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + 27 \\
& *\sqrt{2}*\sin(d*x + c)^2 + 6*(9*\sqrt{2}*\cos(d*x + c) - 2*\sqrt{2})*\cos(2*d*x \\
& + 2*c) - 12*\sqrt{2}*\cos(d*x + c) - 7*\sqrt{2}))*\sin(5/2*d*x + 5/2*c) + 6*(9*(
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 2*(9*\sqrt{2}*\cos(2*d*x + 2*c) + 9*\sqrt{2}*\cos(d*x + c) - 2*\sqrt{2}))*\sin(5/2*d*x + 5/2*c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 60*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2})*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sqrt{2}*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 6*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 10*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 2*(18*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + 9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)))*\sin(2*d*x + 2*c) + 9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 10*\sqrt{2}*\cos(5/2*d*x + 5/2*c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x + 3*c) + 2*(27*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 27*\sqrt{2}*\cos(d*x + c)^2 + 27*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 54*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + 27*\sqrt{2}*\sin(d*x + c)^2 + 6*(9*\sqrt{2}*\cos(d*x + c) - 2*\sqrt{2}))*\cos(2*d*x + 2*c) - 12*\sqrt{2}*\cos(d*x + c) - 7*\sqrt{2}))*\sin(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 32*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 6*(\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 3*((\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
&)*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(3*d*x + 3*c)^2 \\
& + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) \\
& + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos \\
& (d*x + c) + \sqrt{2})*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin \\
& (3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(6/5*\arctan2(\sin(5/2* \\
& d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 9*(\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9* \\
& \sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(3*d*x + \\
& 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d* \\
& x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2})*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + \\
& c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(4/5*\arctan2(\sin \\
& (5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 9*(\sqrt{2}*\cos(3*d*x + 3*c)^2 \\
& + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(3 \\
& *d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)* \\
& \sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3 \\
& *\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2})*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x \\
& + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(2/5*\arctan2 \\
& (\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + (\sqrt{2}*\cos(3*d*x + 3 \\
& *c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin \\
& (3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2 \\
& *c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) \\
& + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x \\
& + c) + \sqrt{2})*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin \\
& (d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(6/5* \\
& \arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 9*(\sqrt{2}*\cos(3*d \\
& *x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d \\
& *x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x \\
& + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2})*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2} \\
&)*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin \\
& (4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 18*(\sqrt{2} \\
&)*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2}*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2} \\
&)*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2} \\
&)*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(3*d*x + 3*c) + 6*(3 \\
& *\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) \\
& + \sqrt{2}*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2} \\
&)*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 9*(\sqrt{2} \\
&)*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 \\
& + \sqrt{2})*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2 \\
& *c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos \\
& (d*x + c) + \sqrt{2}))*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2} \\
&)*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(2 \\
& /5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + \sqrt{2}*\cos(3*d \\
& *x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + sqr \\
& t(2)*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d \\
& *x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x \\
& + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2}))*\cos(2*d*x + 2*c) + 2*(\sqrt{2}*\cos(3*d*x + 3*c)^2 + \\
& 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(3*d*x \\
& + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(\\
& d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*sqr \\
& t(2)*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \\
& \sqrt{2}))*\cos(2*d*x + 2*c) + 3*(\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(\\
& 2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(3*d*x + 3*c)^2 + 9* \\
& \sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*s \\
& \sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + \\
& c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(\\
& 2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2})*\sin(d*x + c))*\sin(3*d* \\
& x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(4/5*\arctan2(\sin(5/2*d*x + \\
& 5/2*c), \cos(5/2*d*x + 5/2*c))) + 3*(\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2})* \\
& \cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(3*d*x + 3*c)^2 \\
& + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + \\
& 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d \\
& *x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))* \\
& \cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2})*\sin(d*x + c))*\sin(\\
& 3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(2/5*\arctan2(\sin(5/2*d* \\
& x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2})* \\
& \sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(6/5* \\
& \arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 6*(\sqrt{2}*\cos(3*d*x \\
& + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2} \\
&)*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x \\
& + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + \\
& 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2})* \\
& \cos(d*x + c) + \sqrt{2}))*\cos(2*d*x + 2*c) + 3*(\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9* \\
& \sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2})*\sin(3*d*x + \\
& 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d* \\
& x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2} \\
&)*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + s \\
& \sqrt{2}))*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2})*\sin(d*x + \\
& c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(2/5*\arctan2(si \\
& n(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \\
& \sqrt{2})*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))* \\
& \cos(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 6*(\sqrt{2})*c
\end{aligned}$$

$$\begin{aligned}
&) + \sqrt{2}) \cos(3dx + 3c) + 6(3\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos(2dx + 2c) + 6(\sqrt{2}) \sin(2dx + 2c) + \sqrt{2}) \sin(dx + c)) \sin(3dx + 3c) + 6\sqrt{2}) \cos(dx + c) + \sqrt{2}) \sin(2/5 \arctan 2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c))) \sin(6/5 \arctan 2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c))) + 6\sqrt{2}) \cos(dx + c) + \sqrt{2}) \log(\cos(1/5 \arctan 2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c)))^2 + \sin(1/5 \arctan 2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c)))^2 - 2\sin(1/5 \arctan 2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c)))) + 1) + 2(36(\sqrt{2}) \sin(2dx + 2c) + \sqrt{2}) \sin(dx + c)) \sin(5/2 dx + 5/2 c) + 9(\sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \sin(2dx + 2c) + 9(\sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \sin(dx + c) + 10\sqrt{2}) \cos(5/2 dx + 5/2 c) - 16\sqrt{2}) \cos(3/2 dx + 3/2 c) - 10\sqrt{2}) \cos(1/2 dx + 1/2 c)) \sin(3dx + 3c) + 4(27\sqrt{2}) \cos(2dx + 2c)^2 + 27\sqrt{2}) \cos(dx + c)^2 + 27\sqrt{2}) \sin(2dx + 2c)^2 + 54\sqrt{2}) \sin(2dx + 2c) \sin(dx + c) + 27\sqrt{2}) \sin(dx + c)^2 + 3(18\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos(2dx + 2c) + 3\sqrt{2}) \cos(dx + c) - 2\sqrt{2}) \sin(5/2 dx + 5/2 c) + 6(9(\sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2\sin(1/2 dx + 1/2 c) + 1) - \sqrt{2}) \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2\sin(1/2 dx + 1/2 c) + 1)) \sin(dx + c) - 16\sqrt{2}) \cos(3/2 dx + 3/2 c) - 10\sqrt{2}) \cos(1/2 dx + 1/2 c)) \sin(2dx + 2c) + 32(3\sqrt{2}) \cos(dx + c) + \sqrt{2}) \sin(3/2 dx + 3/2 c) - 2(6\sqrt{2}) \cos(3dx + 3c)^2 \cos(5/2 dx + 5/2 c) + 6\sqrt{2}) \cos(5/2 dx + 5/2 c) \sin(3dx + 3c)^2 + 12(3\sqrt{2}) \cos(2dx + 2c) + 3\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos(3dx + 3c) \cos(5/2 dx + 5/2 c) + 36(\sqrt{2}) \sin(2dx + 2c) + \sqrt{2}) \sin(dx + c)) \cos(5/2 dx + 5/2 c) \sin(3dx + 3c) + 6(9\sqrt{2}) \cos(2dx + 2c)^2 + 9\sqrt{2}) \cos(dx + c)^2 + 9\sqrt{2}) \sin(2dx + 2c)^2 + 18\sqrt{2}) \sin(2dx + 2c) \sin(dx + c) + 9\sqrt{2}) \sin(dx + c)^2 + 6(3\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos(2dx + 2c) + 6\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos(5/2 dx + 5/2 c) - 16(\sqrt{2}) \cos(3dx + 3c)^2 + 9\sqrt{2}) \cos(2dx + 2c)^2 + 9\sqrt{2}) \cos(dx + c)^2 + \sqrt{2}) \sin(3dx + 3c)^2 + 9\sqrt{2}) \sin(2dx + 2c)^2 + 18\sqrt{2}) \sin(2dx + 2c) \sin(dx + c) + 9\sqrt{2}) \sin(dx + c)^2 + 2(3\sqrt{2}) \cos(2dx + 2c) + 3\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos(3dx + 3c) + 6(3\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos(2dx + 2c) + 6(\sqrt{2}) \sin(2dx + 2c) + \sqrt{2}) \sin(dx + c)) \sin(3dx + 3c) + 6\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos(3/5 \arctan 2(\sin(5/2 dx + 5/2 c), \cos(5/2 dx + 5/2 c))) - 6(\sqrt{2}) \cos(3dx + 3c)^2 + 9\sqrt{2}) \cos(2dx + 2c)^2 + 9\sqrt{2}) \cos(dx + c)^2 + \sqrt{2}) \sin(3dx + 3c)^2 + 9\sqrt{2}) \sin(2dx + 2c)^2 + 18\sqrt{2}) \sin(2dx + 2c) \sin(dx + c) + 9\sqrt{2}) \sin(dx + c)^2 + 2(3\sqrt{2}) \cos(2dx + 2c) + 3\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos(3dx + 3c) + 6(3\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos(2dx + 2c) + 6(\sqrt{2}) \sin(2dx + 2c) + \sqrt{2}) \sin(dx + c)) \sin(3dx + 3c) + 6\sqrt{2}) \cos(dx + c) + \sqrt{2}) \cos
\end{aligned}$$

$$\begin{aligned}
& 2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x + 3*c) - 20*(3*\sqrt{2})*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(5/2*d*x + 5/2*c) + 6 \\
& *(9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 16*\sqrt{2}*\cos(3/2*d*x \\
& + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 32*(3*\sqrt{2}*(\\
& 2)*\cos(d*x + c) + \sqrt{2}))*\sin(3/2*d*x + 3/2*c) + 3*\sqrt{2})*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& (2)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) + 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(4/5*\arctan2(\sin(5/2*d*x + \\
& 5/2*c), \cos(5/2*d*x + 5/2*c))) - 3*(3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos \\
& (3*d*x + 3*c)^2 + 27*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 \\
& + 27*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\sin(3*d*x + 3*c)^2 + 27*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(2*d*x + 2*c)^2 + 27*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 - \\
& 96*\sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 60*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)*\sin(d*x + c) + 2*(9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) \\
& + 9*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*s \\
& \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) - 10*\sqrt{2})*\sin(5/2*d*x + 5/2*c) + 16*\sqrt{2})*\sin(3/2*d*x + \\
& 3/2*c) + 10*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 60*(\sqrt{2})*\sin \\
& (2*d*x + 2*c) + \sqrt{2})*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2})* \\
& *log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) + 16*\sqrt{2})*\sin(3/2*d*x + 3/2*c) + 10*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(
\end{aligned}$$

$$\begin{aligned}
& n(1/2*d*x + 1/2*c) + 1)) * \cos(3*d*x + 3*c)^2 + 27*(\sqrt{2} * \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1)) * \cos(2*d*x + 2*c)^2 + 27*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + \\
& c)^2 + 3*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(3*d*x + 3*c)^2 + 27*(\sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1)) * \sin(2*d*x + 2*c)^2 + 27*(\sqrt{2} * \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1)) * \sin(d*x + c)^2 - 96*\sqrt{2} * \cos(3/2*d*x + 3/2*c) * \sin(d*x + c) - 6 \\
& 0*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \sin(d*x + c) + 2*(9*(\sqrt{2} * \log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1)) * \cos(2*d*x + 2*c) + 9*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + \\
& c) + 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 10*\sqrt{2} * \sin(5/2*d*x + 5/2*c) \\
& + 16*\sqrt{2} * \sin(3/2*d*x + 3/2*c) + 10*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(3* \\
& d*x + 3*c) + 60*(\sqrt{2} * \sin(2*d*x + 2*c) + \sqrt{2} * \sin(d*x + c)) * \cos(5/2*d \\
& *x + 5/2*c) + 6*(9*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(d*x + c) + 3*\sqrt{2} \\
&) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 3*\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sqrt{2} * \sin(3/2*d*x + 3/2*c) + 10*\sqrt{2} \\
&) * \sin(1/2*d*x + 1/2*c)) * \cos(2*d*x + 2*c) + 6*(3*\sqrt{2} * \log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) + 10*\sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 2*(9*(\sqrt{2} * \log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1)) * \sin(2*d*x + 2*c) + 9*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \\
& \sin(d*x + c) + 10*\sqrt{2} * \cos(5/2*d*x + 5/2*c) - 16*\sqrt{2} * \cos(3/2*d*x + 3 \\
& /2*c) - 10*\sqrt{2} * \cos(1/2*d*x + 1/2*c)) * \sin(3*d*x + 3*c) - 20*(3*\sqrt{2} * \c \\
& os(2*d*x + 2*c) + 3*\sqrt{2} * \cos(d*x + c) + \sqrt{2} * \sin(5/2*d*x + 5/2*c) + \\
& 6*(9*(\sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 16*\sqrt{2}*\cos(3/2*d*x + 3/2*c) - 10*\sqrt{2}*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 32*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))))*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 32*(\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(2*d*x + 2*c) + 3*(\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(3/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 36*(\sqrt{2}*\cos(3*d*x + 3*c)^2*\cos(5/2*d*x + 5/2*c) + \sqrt{2}*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c)*\cos(5/2*d*x + 5/2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) + (9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(2*d*x + 2*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(5/2*d*x + 5/2*c) - (\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))))*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 12*(\sqrt{2}*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*\sin(d*x + c)^2 + 2*(3*\sqrt{2}*\cos(2*d*x + 2*c) + 3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*\sin(2*d*x + 2*c) + \sqrt{2}*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*\sqrt{2}*\cos(d*x + c) + \sqrt{2}))*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) +
\end{aligned}$$

$a^2 \cos(3dx + 3c)^2 + 9a^2 \cos(2dx + 2c)^2 + 9a^2 \cos(dx + c)^2 +$
 $a^2 \sin(3dx + 3c)^2 + 9a^2 \sin(2dx + 2c)^2 + 18a^2 \sin(2dx + 2c)$
 $\sin(dx + c) + 9a^2 \sin(dx + c)^2 + 6a^2 \cos(dx + c) + a^2 + 2(3a^2 \cos(2dx + 2c) +$
 $3a^2 \cos(dx + c) + a^2) \cos(3dx + 3c) + 6(3a^2 \cos(dx + c) + a^2) \cos(2dx + 2c) +$
 $6(a^2 \sin(2dx + 2c) + a^2 \sin(dx + c)) \sin(3dx + 3c) \sin(4/5 \arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx +$
 $5/2c))) + (a^2 \cos(3dx + 3c)^2 + 9a^2 \cos(2dx + 2c)^2 + 9a^2 \cos(dx + c)^2 + a^2 \sin(3dx + 3c)^2 +$
 $9a^2 \sin(2dx + 2c)^2 + 18a^2 \sin(2dx + 2c) \sin(dx + c) + 9a^2 \sin(dx + c)^2 + 6a^2 \cos(dx + c) + a^2$
 $+ 2(3a^2 \cos(2dx + 2c) + 3a^2 \cos(dx + c) + a^2) \cos(3dx + 3c) + 6(3a^2 \cos(dx + c) + a^2) \cos(2dx + 2c) +$
 $6(a^2 \sin(2dx + 2c) + a^2 \sin(dx + c)) \sin(3dx + 3c) \sin(2/5 \arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx +$
 $5/2c))) \sin(6/5 \arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c)))) * d$

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \frac{\frac{3\sqrt{2} \log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^{3/2} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{3\sqrt{2} \log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^{3/2} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} + \frac{2\sqrt{2} \sin(\frac{1}{2} dx + \frac{1}{2} c)}{(\sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1) a^{3/2} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))}}{8d}$$

[In] integrate(cos(dx+c)/(a+a*cos(dx+c))^(3/2),x, algorithm="giac")

[Out] 1/8*(3*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))) - 3*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(1/2*d*x + 1/2*c))) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a^(3/2)*sgn(cos(1/2*d*x + 1/2*c)))/d

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx$$

[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)/(a + a*cos(c + d*x))^(3/2), x)

3.135 $\int \frac{1}{(a+a \cos(c+dx))^{3/2}} dx$

Optimal result	2112
Rubi [A] (verified)	2112
Mathematica [A] (verified)	2113
Maple [B] (verified)	2113
Fricas [B] (verification not implemented)	2114
Sympy [F]	2114
Maxima [B] (verification not implemented)	2115
Giac [A] (verification not implemented)	2125
Mupad [F(-1)]	2125

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{1}{(a+a \cos(c+dx))^{3/2}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}}$$

[Out] 1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)+1/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2729, 2728, 212}

$$\int \frac{1}{(a+a \cos(c+dx))^{3/2}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c+dx)}{2d(a \cos(c+dx)+a)^{3/2}}$$

[In] Int[(a + a*Cos[c + d*x])^(-3/2), x]

[Out] ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a \cos(c+dx)}} dx}{4a} \\ &= \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{2ad} \\ &= \frac{\operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2}} dx = \frac{\cos^2\left(\frac{1}{2}(c + dx)\right) \left(\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c + dx)\right)\right)\right) \cos\left(\frac{1}{2}(c + dx)\right) + \tan\left(\frac{1}{2}(c + dx)\right)}{d(a(1 + \cos(c + dx)))^{3/2}}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(-3/2), x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + Tan[(c +
d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(62) = 124.

Time = 1.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.79

method	result	size
default	$\frac{\sqrt{a \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \left(\sqrt{2} \ln \left(\frac{4\sqrt{a} \sqrt{a \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 4a}}{\cos \left(\frac{dx}{2} + \frac{c}{2} \right)} \right) a \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sqrt{2} \sqrt{a \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \sqrt{a} \right)}{4a^{\frac{5}{2}} \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{a \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} d}$	138

[In] `int(1/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4} a^{-5/2} / \cos(1/2 d x + 1/2 c) * (a \sin(1/2 d x + 1/2 c)^2)^{1/2} * (2^{1/2} \ln(2 * (2 a^{1/2} * (a \sin(1/2 d x + 1/2 c)^2)^{1/2} + 2 a) / \cos(1/2 d x + 1/2 c)) * a \cos(1/2 d x + 1/2 c)^2 + 2^{1/2} * (a \sin(1/2 d x + 1/2 c)^2)^{1/2} * a^{1/2}) / \sin(1/2 d x + 1/2 c) / (a \cos(1/2 d x + 1/2 c)^2)^{1/2} / d$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(62) = 124$.

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2}} dx = \frac{\sqrt{2} (\cos(dx + c)^2 + 2 \cos(dx + c) + 1) \sqrt{a} \log \left(-\frac{a \cos(dx+c)^2 - 2\sqrt{2}\sqrt{a \cos(dx+c)+a}\sqrt{\cos(dx+c)^2+2 \cos}}{\cos(dx+c)^2+2 \cos} \right)}{8 (a^2 d \cos(dx + c))^2 + 2 a^2 d \cos(dx + c)}$$

[In] `integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] $\frac{1}{8} * (\sqrt{2} * (\cos(dx + c)^2 + 2 \cos(dx + c) + 1) * \sqrt{a} * \log(-a \cos(dx + c)^2 - 2 \sqrt{2} * \sqrt{a \cos(dx + c) + a} * \sqrt{a} * \sin(dx + c) - 2 * a * \cos(dx + c) - 3 * a) / (\cos(dx + c)^2 + 2 \cos(dx + c) + 1)) + 4 * \sqrt{a \cos(dx + c) + a} * \sin(dx + c)) / (a^2 * d * \cos(dx + c)^2 + 2 * a^2 * d * \cos(dx + c) + a^2 * d)$

Sympy [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{1}{(a \cos(c + dx) + a)^{\frac{3}{2}}} dx$$

[In] `integrate(1/(a+a*cos(d*x+c))**(3/2),x)`

[Out] `Integral((a*cos(c + d*x) + a)**(-3/2), x)`

$$\begin{aligned}
& 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*(\sin(2*d*x + 2*c) + \sin \\
& (d*x + c))*\sin(3*d*x + 3*c) + \sin(3*d*x + 3*c)^2 + 9*\sin(2*d*x + 2*c)^2 + 1 \\
& 8*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin \\
& (4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 3*(2*(3*\cos(2* \\
& d*x + 2*c) + 3*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + \cos(3*d*x + 3*c)^2 + 6* \\
& (3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + \\
& c)^2 + 6*(\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + \sin(3*d*x + 3* \\
& c)^2 + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x \\
& + c)^2 + 6*\cos(d*x + c) + 1)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2 \\
& *d*x + 3/2*c))))*\cos(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&)) - 4*(8*\cos(3*d*x + 3*c)^2*\sin(3/2*d*x + 3/2*c) - 72*\cos(2*d*x + 2*c)^2*\sin \\
& (3/2*d*x + 3/2*c) - 144*\cos(2*d*x + 2*c)*\cos(d*x + c)*\sin(3/2*d*x + 3/2*c) \\
&) - 8*\sin(3*d*x + 3*c)^2*\sin(3/2*d*x + 3/2*c) - 72*\sin(2*d*x + 2*c)^2*\sin(3 \\
& /2*d*x + 3/2*c) - 16*(3*\cos(3/2*d*x + 3/2*c)*\sin(2*d*x + 2*c) + 3*\cos(3/2*d \\
& *x + 3/2*c)*\sin(d*x + c) - \sin(3/2*d*x + 3/2*c))*\cos(3*d*x + 3*c) - 48*(\cos \\
& (3/2*d*x + 3/2*c)*\sin(3*d*x + 3*c) + 3*\cos(3/2*d*x + 3/2*c)*\sin(2*d*x + 2*c) \\
&) - (3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - \cos(3*d*x + 3*c)*\sin(3/2*d* \\
& x + 3/2*c) - 3*\cos(2*d*x + 2*c)*\sin(3/2*d*x + 3/2*c) + 3*\cos(3/2*d*x + 3/2* \\
& c)*\sin(d*x + c))*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c) \\
&)) - 16*(\cos(3*d*x + 3*c)*\cos(3/2*d*x + 3/2*c) + 3*\sin(2*d*x + 2*c)*\sin(3/2 \\
& *d*x + 3/2*c) + 3*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c)) \\
& *\sin(3*d*x + 3*c) - 48*(3*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + \\
& 3/2*c))*\sin(2*d*x + 2*c) - 8*(9*\cos(d*x + c)^2 + 9*\sin(d*x + c)^2 - 1)*\sin \\
& (3/2*d*x + 3/2*c) - 48*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 3*(2*(3*\cos(2*d* \\
& x + 2*c) + 3*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + \cos(3*d*x + 3*c)^2 + 6*(3 \\
& *\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c) \\
& ^2 + 6*(\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + \sin(3*d*x + 3*c) \\
& ^2 + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + \\
& c)^2 + 6*\cos(d*x + c) + 1)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d \\
& *x + 3/2*c))))*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) \\
& - 4*(8*\cos(3*d*x + 3*c)^2*\sin(3/2*d*x + 3/2*c) - 72*\cos(2*d*x + 2*c)^2*\sin \\
& (3/2*d*x + 3/2*c) - 144*\cos(2*d*x + 2*c)*\cos(d*x + c)*\sin(3/2*d*x + 3/2*c) \\
& - 8*\sin(3*d*x + 3*c)^2*\sin(3/2*d*x + 3/2*c) - 72*\sin(2*d*x + 2*c)^2*\sin(3/2 \\
& *d*x + 3/2*c) - 16*(3*\cos(3/2*d*x + 3/2*c)*\sin(2*d*x + 2*c) + 3*\cos(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) - \sin(3/2*d*x + 3/2*c))*\cos(3*d*x + 3*c) - 16*(\cos(3 \\
& *d*x + 3*c)*\cos(3/2*d*x + 3/2*c) + 3*\sin(2*d*x + 2*c)*\sin(3/2*d*x + 3/2*c) \\
& + 3*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c))*\sin(3*d*x + 3 \\
& *c) - 48*(3*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c))*\sin(2 \\
& *d*x + 2*c) - 8*(9*\cos(d*x + c)^2 + 9*\sin(d*x + c)^2 - 1)*\sin(3/2*d*x + 3/2 \\
& *c) - 48*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 3*(2*(3*\cos(2*d*x + 2*c) + 3*c \\
& \cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + \cos(3*d*x + 3*c)^2 + 6*(3*\cos(d*x + c) \\
& + 1)*\cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*(\sin(2* \\
& d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + \sin(3*d*x + 3*c)^2 + 9*\sin(2* \\
& d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(\\
& d*x + c) + 1)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))
\end{aligned}$$

$$\begin{aligned}
& x + c) + 1) \cos(3d*x + 3*c) + 6*(3*\cos(d*x + c) + 1) \cos(2*d*x + 2*c) + 9* \\
& \cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 3*(2*(3*\cos(2*d*x + 2*c) + 3*\cos(d* \\
& x + c) + 1) \cos(3*d*x + 3*c) + \cos(3*d*x + 3*c)^2 + 6*(3*\cos(d*x + c) + 1)* \\
& \cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*(\sin(2*d*x + \\
& 2*c) + \sin(d*x + c)) \sin(3*d*x + 3*c) + \sin(3*d*x + 3*c)^2 + 9*\sin(2*d*x + \\
& 2*c)^2 + 18*\sin(2*d*x + 2*c) \sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + \\
& c) + 1) \cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(\\
& (\sin(2*d*x + 2*c) + \sin(d*x + c)) \cos(3*d*x + 3*c) + \sin(2*d*x + 2*c) + \sin \\
& (d*x + c)) \sin(3*d*x + 3*c) + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c) \sin \\
& (d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1) \cos(4/3*\arctan2(\sin(3/2 \\
& *d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(3*(2*\cos(2*d*x + 2*c) + 2*\cos(d* \\
& x + c) + 1) \cos(3*d*x + 3*c)^2 + \cos(3*d*x + 3*c)^3 + (\cos(3*d*x + 3*c) + 1 \\
&) \sin(3*d*x + 3*c)^2 + 3*(2*(3*\cos(d*x + c) + 2)*\cos(2*d*x + 2*c) + 3*\cos(2 \\
& *d*x + 2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(2*d*x + 2*c)^2 + 6*\sin(2*d*x + 2*c \\
&) \sin(d*x + c) + 3*\sin(d*x + c)^2 + 4*\cos(d*x + c) + 1) \cos(3*d*x + 3*c) + \\
& 6*(3*\cos(d*x + c) + 1) \cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x \\
& + c)^2 + 6*((\sin(2*d*x + 2*c) + \sin(d*x + c)) \cos(3*d*x + 3*c) + \sin(2*d*x \\
& + 2*c) + \sin(d*x + c)) \sin(3*d*x + 3*c) + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d \\
& *x + 2*c) \sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1) \cos(2/3*\ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*((\sin(2*d*x + 2*c) + \\
& \sin(d*x + c)) \cos(3*d*x + 3*c)^2 + 2*(\sin(2*d*x + 2*c) + \sin(d*x + c)) \cos(\\
& 3*d*x + 3*c) + \sin(2*d*x + 2*c) + \sin(d*x + c)) \sin(3*d*x + 3*c) + 9*\sin(2* \\
& d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c) \sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*(6*(\\
& \sin(2*d*x + 2*c) + \sin(d*x + c)) \sin(3*d*x + 3*c)^2 + \sin(3*d*x + 3*c)^3 + \\
& (2*(3*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + 1) \cos(3*d*x + 3*c) + \cos(3*d*x + \\
& 3*c)^2 + 6*(3*\cos(d*x + c) + 1) \cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + \\
& 9*\cos(d*x + c)^2 + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c) \sin(d*x + c) \\
& + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1) \sin(3*d*x + 3*c) + 3*(2*(3*\cos(2*d \\
& *x + 2*c) + 3*\cos(d*x + c) + 1) \cos(3*d*x + 3*c) + \cos(3*d*x + 3*c)^2 + 6*(\\
& 3*\cos(d*x + c) + 1) \cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c \\
&)^2 + 6*(\sin(2*d*x + 2*c) + \sin(d*x + c)) \sin(3*d*x + 3*c) + \sin(3*d*x + 3* \\
& c)^2 + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c) \sin(d*x + c) + 9*\sin(d*x \\
& + c)^2 + 6*\cos(d*x + c) + 1) \sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2* \\
& d*x + 3/2*c)))) \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)) \\
&) + 6*(6*(\sin(2*d*x + 2*c) + \sin(d*x + c)) \sin(3*d*x + 3*c)^2 + \sin(3*d*x + \\
& 3*c)^3 + (2*(3*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + 1) \cos(3*d*x + 3*c) + \cos \\
& (3*d*x + 3*c)^2 + 6*(3*\cos(d*x + c) + 1) \cos(2*d*x + 2*c) + 9*\cos(2*d*x + \\
& 2*c)^2 + 9*\cos(d*x + c)^2 + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c) \sin \\
& (d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1) \sin(3*d*x + 3*c)) \sin(2/ \\
& 3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*\cos(d*x + c) + 1 \\
&) \log(\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(\\
& 1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 2*\sin(1/3*\arct \\
& an2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1) - (2*(3*\cos(2*d*x + 2 \\
& *c) + 3*\cos(d*x + c) + 2)*\cos(3*d*x + 3*c)^3 + \cos(3*d*x + 3*c)^4 + 6*(\sin(\\
& 2*d*x + 2*c) + \sin(d*x + c)) \sin(3*d*x + 3*c)^3 + \sin(3*d*x + 3*c)^4 + 3*(6
\end{aligned}$$

$(\cos(dx + c) + 1)\cos(2dx + 2c) + 3\cos(2dx + 2c)^2 + 3\cos(dx + c)^2 + 3\sin(2dx + 2c)^2 + 6\sin(2dx + 2c)\sin(dx + c) + 3\sin(dx + c)^2 + 6\cos(dx + c) + 2)\cos(3dx + 3c)^2 + 9(2(3\cos(2dx + 2c) + 3\cos(dx + c) + 1)\cos(3dx + 3c) + \cos(3dx + 3c)^2 + 6(3\cos(dx + c) + 1)\cos(2dx + 2c) + 9\cos(2dx + 2c)^2 + 9\cos(dx + c)^2 + 6(\sin(2dx + 2c) + \sin(dx + c))\sin(3dx + 3c) + \sin(3dx + 3c)^2 + 9\sin(2dx + 2c)^2 + 18\sin(2dx + 2c)\sin(dx + c) + 9\sin(dx + c)^2 + 6\cos(dx + c) + 1)\cos(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + 9(2(3\cos(2dx + 2c) + 3\cos(dx + c) + 1)\cos(3dx + 3c) + \cos(3dx + 3c)^2 + 6(3\cos(dx + c) + 1)\cos(2dx + 2c) + 9\cos(2dx + 2c)^2 + 9\cos(dx + c)^2 + 6(\sin(2dx + 2c) + \sin(dx + c))\sin(3dx + 3c) + \sin(3dx + 3c)^2 + 9\sin(2dx + 2c)^2 + 18\sin(2dx + 2c)\sin(dx + c) + 9\sin(dx + c)^2 + 6\cos(dx + c) + 1)\cos(2/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + (2(3\cos(2dx + 2c) + 3\cos(dx + c) + 2)\cos(3dx + 3c) + 2\cos(3dx + 3c)^2 + 6(3\cos(dx + c) + 1)\cos(2dx + 2c) + 9\cos(2dx + 2c)^2 + 9\cos(dx + c)^2 + 9\sin(2dx + 2c)^2 + 18\sin(2dx + 2c)\sin(dx + c) + 9\sin(dx + c)^2 + 6\cos(dx + c) + 2)\sin(3dx + 3c)^2 + 9(2(3\cos(2dx + 2c) + 3\cos(dx + c) + 1)\cos(3dx + 3c) + \cos(3dx + 3c)^2 + 6(3\cos(dx + c) + 1)\cos(2dx + 2c) + 9\cos(2dx + 2c)^2 + 9\cos(dx + c)^2 + 6(\sin(2dx + 2c) + \sin(dx + c))\sin(3dx + 3c) + \sin(3dx + 3c)^2 + 9\sin(2dx + 2c)^2 + 18\sin(2dx + 2c)\sin(dx + c) + 9\sin(dx + c)^2 + 6\cos(dx + c) + 1)\sin(4/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + 9(2(3\cos(2dx + 2c) + 3\cos(dx + c) + 1)\cos(3dx + 3c) + \cos(3dx + 3c)^2 + 6(3\cos(dx + c) + 1)\cos(2dx + 2c) + 9\cos(2dx + 2c)^2 + 9\cos(dx + c)^2 + 6(\sin(2dx + 2c) + \sin(dx + c))\sin(3dx + 3c) + \sin(3dx + 3c)^2 + 9\sin(2dx + 2c)^2 + 18\sin(2dx + 2c)\sin(dx + c) + 9\sin(dx + c)^2 + 6\cos(dx + c) + 1)\sin(2/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + 2(9(2\cos(dx + c) + 1)\cos(2dx + 2c) + 9\cos(2dx + 2c)^2 + 9\cos(dx + c)^2 + 9\sin(2dx + 2c)^2 + 18\sin(2dx + 2c)\sin(dx + c) + 9\sin(dx + c)^2 + 9\cos(dx + c) + 2)\cos(3dx + 3c) + 6(3\cos(dx + c) + 1)\cos(2dx + 2c) + 9\cos(2dx + 2c)^2 + 9\cos(dx + c)^2 + 6(3(2\cos(2dx + 2c) + 2\cos(dx + c) + 1)\cos(3dx + 3c)^2 + \cos(3dx + 3c)^3 + (\cos(3dx + 3c) + 1)\sin(3dx + 3c)^2 + 3(2(3\cos(dx + c) + 2)\cos(2dx + 2c) + 3\cos(2dx + 2c)^2 + 3\cos(dx + c)^2 + 3\sin(2dx + 2c)^2 + 6\sin(2dx + 2c)\sin(dx + c) + 3\sin(dx + c)^2 + 4\cos(dx + c) + 1)\cos(3dx + 3c) + 6(3\cos(dx + c) + 1)\cos(2dx + 2c) + 9\cos(2dx + 2c)^2 + 9\cos(dx + c)^2 + 3(2(3\cos(2dx + 2c) + 3\cos(dx + c) + 1)\cos(3dx + 3c) + \cos(3dx + 3c)^2 + 6(3\cos(dx + c) + 1)\cos(2dx + 2c) + 9\cos(2dx + 2c)^2 + 9\cos(dx + c)^2 + 6(\sin(2dx + 2c) + \sin(dx + c))\sin(3dx + 3c) + \sin(3dx + 3c)^2 + 9\sin(2dx + 2c)^2 + 18\sin(2dx + 2c)\sin(dx + c) + 9\sin(dx + c)^2 + 6\cos(dx + c) + 1)\cos(2/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + 6((\sin(2dx + 2c) + \sin(dx + c))\cos(3dx + 3c) + \sin(2dx + 2c) + \sin(dx + c))\sin(3dx + 3c) + 9\sin(2dx + 2c)^2 + 18\sin(2$

$$\begin{aligned}
& *d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(3*(2*\cos(2*d*x + 2*c) + 2*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c)^2 + \cos(3*d*x + 3*c)^3 + (\cos(3*d*x + 3*c) + 1)*\sin(3*d*x + 3*c)^2 + 3*(2*(3*\cos(d*x + c) + 2)*\cos(2*d*x + 2*c) + 3*\cos(2*d*x + 2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(2*d*x + 2*c)^2 + 6*\sin(2*d*x + 2*c)*\sin(d*x + c) + 3*\sin(d*x + c)^2 + 4*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 6*(3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*((\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(3*d*x + 3*c) + \sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*((\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(3*d*x + 3*c)^2 + 2*(\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(3*d*x + 3*c) + \sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*(6*(\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c)^2 + \sin(3*d*x + 3*c)^3 + (2*(3*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + \cos(3*d*x + 3*c)^2 + 6*(3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin(3*d*x + 3*c) + 3*(2*(3*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + \cos(3*d*x + 3*c)^2 + 6*(3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*(\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + \sin(3*d*x + 3*c)^2 + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))*\sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(6*(\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c)^2 + \sin(3*d*x + 3*c)^3 + (2*(3*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + \cos(3*d*x + 3*c)^2 + 6*(3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin(3*d*x + 3*c))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*\cos(d*x + c) + 1)*\log(\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + \sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 1) - 32*(2*(3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c)*\cos(3/2*d*x + 3/2*c) + 3*\cos(2*d*x + 2*c)^2*\cos(3/2*d*x + 3/2*c) + 3*\cos(3/2*d*x + 3/2*c)*\sin(2*d*x + 2*c)^2 + 2*(\cos(2*d*x + 2*c)*\cos(3/2*d*x + 3/2*c) + \cos(3/2*d*x + 3/2*c)*\cos(d*x + c) - \sin(2*d*x + 2*c)*\sin(3/2*d*x + 3/2*c) - \sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\cos(3*d*x + 3*c) + (3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + 2*\cos(d*x + c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - \sin(3/2*d*x + 3/2*c))*\sin(2*d*x + 2*c) - 2*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c))*\sin(3*d*x + 3*c) + 32*(6*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) + \cos(3/2*d*x + 3/2*c))*\sin(2*d*x + 2*c) + 32*(3*\cos(d*x + c)^2 + 3*\sin(d*x + c)^2 + \cos(d*x + c))*\sin(3/2*d*x + 3/2*c) + 32*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 4*(3*(2*\cos(2*d*x + 2*c) + 2*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c)^2 + \cos(3*d*x + 3*c)^2
\end{aligned}$$

$$\begin{aligned}
& 3 + (\cos(3d*x + 3*c) + 1)*\sin(3d*x + 3*c)^2 + 3*(2*(3*\cos(d*x + c) + 2)*\cos(2d*x + 2*c) + 3*\cos(2d*x + 2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(2d*x + 2*c)^2 + 6*\sin(2d*x + 2*c)*\sin(d*x + c) + 3*\sin(d*x + c)^2 + 4*\cos(d*x + c) + 1)*\cos(3d*x + 3*c) + 6*(3*\cos(d*x + c) + 1)*\cos(2d*x + 2*c) + 9*\cos(2d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 3*(2*(3*\cos(2d*x + 2*c) + 3*\cos(d*x + c) + 1)*\cos(3d*x + 3*c) + \cos(3d*x + 3*c)^2 + 6*(3*\cos(d*x + c) + 1)*\cos(2d*x + 2*c) + 9*\cos(2d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*(\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(3d*x + 3*c) + \sin(3d*x + 3*c)^2 + 9*\sin(2d*x + 2*c)^2 + 18*\sin(2d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 3*(2*(3*\cos(2d*x + 2*c) + 3*\cos(d*x + c) + 1)*\cos(3d*x + 3*c) + \cos(3d*x + 3*c)^2 + 6*(3*\cos(d*x + c) + 1)*\cos(2d*x + 2*c) + 9*\cos(2d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*(\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(3d*x + 3*c) + \sin(3d*x + 3*c)^2 + 9*\sin(2d*x + 2*c)^2 + 18*\sin(2d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*((\sin(2d*x + 2*c) + \sin(d*x + c))*\cos(3d*x + 3*c) + \sin(2d*x + 2*c) + \sin(d*x + c))*\sin(3d*x + 3*c) + 9*\sin(2d*x + 2*c)^2 + 18*\sin(2d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin(5/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 4*(8*\cos(3d*x + 3*c)^2*\cos(3/2*d*x + 3/2*c) + 48*(3*\cos(d*x + c) + 1)*\cos(2d*x + 2*c)*\cos(3/2*d*x + 3/2*c) + 72*\cos(2d*x + 2*c)^2*\cos(3/2*d*x + 3/2*c) - 8*\cos(3/2*d*x + 3/2*c)*\sin(3d*x + 3*c)^2 + 72*\cos(3/2*d*x + 3/2*c)*\sin(2d*x + 2*c)^2 + 144*\cos(3/2*d*x + 3/2*c)*\sin(2d*x + 2*c)*\sin(d*x + c) + 16*((3*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) + 3*\cos(2d*x + 2*c)*\cos(3/2*d*x + 3/2*c))*\cos(3d*x + 3*c) + 8*(9*\cos(d*x + c)^2 + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - 3*(2*(3*\cos(2d*x + 2*c) + 3*\cos(d*x + c) + 1)*\cos(3d*x + 3*c) + \cos(3d*x + 3*c)^2 + 6*(3*\cos(d*x + c) + 1)*\cos(2d*x + 2*c) + 9*\cos(2d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*(\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(3d*x + 3*c) + \sin(3d*x + 3*c)^2 + 9*\sin(2d*x + 2*c)^2 + 18*\sin(2d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*((3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + \cos(3d*x + 3*c))*\sin(3/2*d*x + 3/2*c) + 3*\cos(2d*x + 2*c)*\sin(3/2*d*x + 3/2*c))*\sin(3d*x + 3*c) - 48*(\cos(3/2*d*x + 3/2*c)*\sin(3d*x + 3*c) + 3*\cos(3/2*d*x + 3/2*c)*\sin(2d*x + 2*c) - (3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - \cos(3d*x + 3*c)*\sin(3/2*d*x + 3/2*c) - 3*\cos(2d*x + 2*c)*\sin(3/2*d*x + 3/2*c) + 3*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) * \sin(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 4*(8*\cos(3d*x + 3*c)^2*\cos(3/2*d*x + 3/2*c) + 48*(3*\cos(d*x + c) + 1)*\cos(2d*x + 2*c)*\cos(3/2*d*x + 3/2*c) + 72*\cos(2d*x + 2*c)^2*\cos(3/2*d*x + 3/2*c) - 8*\cos(3/2*d*x + 3/2*c)*\sin(3d*x + 3*c)^2 + 72*\cos(3/2*d*x + 3/2*c)*\sin(2d*x + 2*c)^2 + 144*\cos(3/2*d*x + 3/2*c)*\sin(2d*x + 2*c)*\sin(d*x + c) + 16*((3*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) + 3*\cos(2d*x + 2*c)*\cos(3/2*d*x + 3/2*c))*\cos(3d*x + 3*c) + 8*(9*\cos(d*x + c)^2 + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - 3*(2*(3*\cos(2d*x + 2*c) + 3*\cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& + 1) \cos(3d*x + 3*c) + \cos(3d*x + 3*c)^2 + 6*(3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*(\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + \sin(3*d*x + 3*c)^2 + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1) * \cos(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 16*((3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + \cos(3*d*x + 3*c)*\sin(3/2*d*x + 3/2*c) + 3*\cos(2*d*x + 2*c)*\sin(3/2*d*x + 3/2*c))*\sin(3*d*x + 3*c))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) - 4*(3*(2*\cos(2*d*x + 2*c) + 2*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c)^2 + \cos(3*d*x + 3*c)^3 + (\cos(3*d*x + 3*c) + 1)*\sin(3*d*x + 3*c)^2 + 3*(2*(3*\cos(d*x + c) + 2)*\cos(2*d*x + 2*c) + 3*\cos(2*d*x + 2*c)^2 + 3*\cos(d*x + c)^2 + 3*\sin(2*d*x + 2*c)^2 + 6*\sin(2*d*x + 2*c)*\sin(d*x + c) + 3*\sin(d*x + c)^2 + 4*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 6*(3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 9*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*((\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(3*d*x + 3*c) + \sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 9*\sin(2*d*x + 2*c)^2 + 18*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))))/((\sqrt{2})*a*\cos(3*d*x + 3*c)^4 + \sqrt{2})*a*\sin(3*d*x + 3*c)^4 + 2*(3*\sqrt{2})*a*\cos(2*d*x + 2*c) + 3*\sqrt{2})*a*\cos(d*x + c) + 2*\sqrt{2})*a*\cos(3*d*x + 3*c)^3 + 6*(\sqrt{2})*a*\sin(2*d*x + 2*c) + \sqrt{2})*a*\sin(d*x + c))*\sin(3*d*x + 3*c)^3 + 9*\sqrt{2})*a*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2})*a*\cos(d*x + c)^2 + 9*\sqrt{2})*a*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2})*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2})*a*\sin(d*x + c)^2 + 3*(3*\sqrt{2})*a*\cos(2*d*x + 2*c)^2 + 3*\sqrt{2})*a*\cos(d*x + c)^2 + 3*\sqrt{2})*a*\sin(2*d*x + 2*c)^2 + 6*\sqrt{2})*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 3*\sqrt{2})*a*\sin(d*x + c)^2 + 6*\sqrt{2})*a*\cos(d*x + c) + 6*(\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a*\cos(2*d*x + 2*c) + 2*\sqrt{2})*a*\cos(3*d*x + 3*c)^2 + 9*(\sqrt{2})*a*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2})*a*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2})*a*\cos(d*x + c)^2 + \sqrt{2})*a*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2})*a*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2})*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2})*a*\sin(d*x + c)^2 + 6*\sqrt{2})*a*\cos(d*x + c) + 2*(3*\sqrt{2})*a*\cos(2*d*x + 2*c) + 3*\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a*\cos(2*d*x + 2*c) + 6*(\sqrt{2})*a*\sin(2*d*x + 2*c) + \sqrt{2})*a*\sin(d*x + c))*\sin(3*d*x + 3*c) + \sqrt{2})*a*\cos(4/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + 9*(\sqrt{2})*a*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2})*a*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2})*a*\cos(d*x + c)^2 + \sqrt{2})*a*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2})*a*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2})*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2})*a*\sin(d*x + c)^2 + 6*\sqrt{2})*a*\cos(d*x + c) + 2*(3*\sqrt{2})*a*\cos(2*d*x + 2*c) + 3*\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a*\cos(2*d*x + 2*c) + 6*(\sqrt{2})*a*\sin(2*d*x + 2*c) + \sqrt{2})*a*\sin(d*x + c))*\sin(3*d*x + 3*c) + \sqrt{2})*a*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + (2*\sqrt{2})*a*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2})*a*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2})*a*\cos(d*x + c)^2 + 9*\sqrt{2})*a*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2})*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2})*a*\sin(d*x + c)^2 + 6*\sqrt{2})*a*\cos(d*x + c) + 2*(3*\sqrt{2})*a*\cos(2*d*x + 2*c) + 3*\sqrt{2})*a*\cos(d*x + c) + 2*
\end{aligned}$$

$$\begin{aligned}
& \sqrt{2} * a * \cos(3 * d * x + 3 * c) + 6 * (3 * \sqrt{2} * a * \cos(d * x + c) + \sqrt{2} * a * \cos(\\
& 2 * d * x + 2 * c) + 2 * \sqrt{2} * a * \sin(3 * d * x + 3 * c)^2 + 9 * (\sqrt{2} * a * \cos(3 * d * x + 3 \\
& * c)^2 + 9 * \sqrt{2} * a * \cos(2 * d * x + 2 * c)^2 + 9 * \sqrt{2} * a * \cos(d * x + c)^2 + \sqrt{2} (\\
& 2) * a * \sin(3 * d * x + 3 * c)^2 + 9 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)^2 + 18 * \sqrt{2} * a * \sin \\
& (2 * d * x + 2 * c) * \sin(d * x + c) + 9 * \sqrt{2} * a * \sin(d * x + c)^2 + 6 * \sqrt{2} * a * \cos(d \\
& * x + c) + 2 * (3 * \sqrt{2} * a * \cos(2 * d * x + 2 * c) + 3 * \sqrt{2} * a * \cos(d * x + c) + \sqrt{2} \\
& (2) * a * \cos(3 * d * x + 3 * c) + 6 * (3 * \sqrt{2} * a * \cos(d * x + c) + \sqrt{2} * a * \cos(2 * d * \\
& x + 2 * c) + 6 * (\sqrt{2} * a * \sin(2 * d * x + 2 * c) + \sqrt{2} * a * \sin(d * x + c)) * \sin(3 * d * \\
& x + 3 * c) + \sqrt{2} * a * \sin(4 / 3 * \arctan 2(\sin(3 / 2 * d * x + 3 / 2 * c), \cos(3 / 2 * d * x + 3 \\
& / 2 * c)))^2 + 9 * (\sqrt{2} * a * \cos(3 * d * x + 3 * c)^2 + 9 * \sqrt{2} * a * \cos(2 * d * x + 2 * c)^ \\
& 2 + 9 * \sqrt{2} * a * \cos(d * x + c)^2 + \sqrt{2} * a * \sin(3 * d * x + 3 * c)^2 + 9 * \sqrt{2} * a \\
& * \sin(2 * d * x + 2 * c)^2 + 18 * \sqrt{2} * a * \sin(2 * d * x + 2 * c) * \sin(d * x + c) + 9 * \sqrt{2} \\
&) * a * \sin(d * x + c)^2 + 6 * \sqrt{2} * a * \cos(d * x + c) + 2 * (3 * \sqrt{2} * a * \cos(2 * d * x + \\
& 2 * c) + 3 * \sqrt{2} * a * \cos(d * x + c) + \sqrt{2} * a * \cos(3 * d * x + 3 * c) + 6 * (3 * \sqrt{2} \\
&) * a * \cos(d * x + c) + \sqrt{2} * a * \cos(2 * d * x + 2 * c) + 6 * (\sqrt{2} * a * \sin(2 * d * x + 2 \\
& * c) + \sqrt{2} * a * \sin(d * x + c)) * \sin(3 * d * x + 3 * c) + \sqrt{2} * a * \sin(2 / 3 * \arctan 2 \\
& (\sin(3 / 2 * d * x + 3 / 2 * c), \cos(3 / 2 * d * x + 3 / 2 * c)))^2 + 6 * \sqrt{2} * a * \cos(d * x + c) \\
& + 2 * (9 * \sqrt{2} * a * \cos(2 * d * x + 2 * c)^2 + 9 * \sqrt{2} * a * \cos(d * x + c)^2 + 9 * \sqrt{2} \\
&) * a * \sin(2 * d * x + 2 * c)^2 + 18 * \sqrt{2} * a * \sin(2 * d * x + 2 * c) * \sin(d * x + c) + 9 * \sqrt{2} \\
& t(2) * a * \sin(d * x + c)^2 + 9 * \sqrt{2} * a * \cos(d * x + c) + 9 * (2 * \sqrt{2} * a * \cos(d * x + \\
& c) + \sqrt{2} * a * \cos(2 * d * x + 2 * c) + 2 * \sqrt{2} * a * \cos(3 * d * x + 3 * c) + 6 * (3 * \sqrt{2} \\
& rt(2) * a * \cos(d * x + c) + \sqrt{2} * a * \cos(2 * d * x + 2 * c) + 6 * (\sqrt{2} * a * \cos(3 * d * x \\
& + 3 * c)^3 + 9 * \sqrt{2} * a * \cos(2 * d * x + 2 * c)^2 + 9 * \sqrt{2} * a * \cos(d * x + c)^2 + 9 \\
& * \sqrt{2} * a * \sin(2 * d * x + 2 * c)^2 + 18 * \sqrt{2} * a * \sin(2 * d * x + 2 * c) * \sin(d * x + c) \\
& + 9 * \sqrt{2} * a * \sin(d * x + c)^2 + 3 * (2 * \sqrt{2} * a * \cos(2 * d * x + 2 * c) + 2 * \sqrt{2} * \\
& a * \cos(d * x + c) + \sqrt{2} * a * \cos(3 * d * x + 3 * c)^2 + (\sqrt{2} * a * \cos(3 * d * x + 3 * c) \\
&) + \sqrt{2} * a * \sin(3 * d * x + 3 * c)^2 + 6 * \sqrt{2} * a * \cos(d * x + c) + 3 * (3 * \sqrt{2} \\
&) * a * \cos(2 * d * x + 2 * c)^2 + 3 * \sqrt{2} * a * \cos(d * x + c)^2 + 3 * \sqrt{2} * a * \sin(2 * d * x \\
& + 2 * c)^2 + 6 * \sqrt{2} * a * \sin(2 * d * x + 2 * c) * \sin(d * x + c) + 3 * \sqrt{2} * a * \sin(d * x \\
& + c)^2 + 4 * \sqrt{2} * a * \cos(d * x + c) + 2 * (3 * \sqrt{2} * a * \cos(d * x + c) + 2 * \sqrt{2} \\
& (2) * a * \cos(2 * d * x + 2 * c) + \sqrt{2} * a * \cos(3 * d * x + 3 * c) + 6 * (3 * \sqrt{2} * a * \cos(d * x \\
& + c) + \sqrt{2} * a * \cos(2 * d * x + 2 * c) + 3 * (\sqrt{2} * a * \cos(3 * d * x + 3 * c)^2 + 9 * \sqrt{2} \\
&) * a * \cos(2 * d * x + 2 * c)^2 + 9 * \sqrt{2} * a * \cos(d * x + c)^2 + \sqrt{2} * a * \sin(3 * \\
& d * x + 3 * c)^2 + 9 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)^2 + 18 * \sqrt{2} * a * \sin(2 * d * x + 2 * \\
& c) * \sin(d * x + c) + 9 * \sqrt{2} * a * \sin(d * x + c)^2 + 6 * \sqrt{2} * a * \cos(d * x + c) + 2 \\
& * (3 * \sqrt{2} * a * \cos(2 * d * x + 2 * c) + 3 * \sqrt{2} * a * \cos(d * x + c) + \sqrt{2} * a * \cos(\\
& 3 * d * x + 3 * c) + 6 * (3 * \sqrt{2} * a * \cos(d * x + c) + \sqrt{2} * a * \cos(2 * d * x + 2 * c) + \\
& 6 * (\sqrt{2} * a * \sin(2 * d * x + 2 * c) + \sqrt{2} * a * \sin(d * x + c)) * \sin(3 * d * x + 3 * c) + \\
& \sqrt{2} * a * \cos(2 / 3 * \arctan 2(\sin(3 / 2 * d * x + 3 / 2 * c), \cos(3 / 2 * d * x + 3 / 2 * c))) + 6 \\
& * (\sqrt{2} * a * \sin(2 * d * x + 2 * c) + \sqrt{2} * a * \sin(d * x + c) + (\sqrt{2} * a * \sin(2 * d * \\
& x + 2 * c) + \sqrt{2} * a * \sin(d * x + c)) * \cos(3 * d * x + 3 * c)) * \sin(3 * d * x + 3 * c) + \sqrt{2} \\
& t(2) * a * \cos(4 / 3 * \arctan 2(\sin(3 / 2 * d * x + 3 / 2 * c), \cos(3 / 2 * d * x + 3 / 2 * c))) + 6 * (\sqrt{2} \\
&) * a * \cos(3 * d * x + 3 * c)^3 + 9 * \sqrt{2} * a * \cos(2 * d * x + 2 * c)^2 + 9 * \sqrt{2} * a * \\
& \cos(d * x + c)^2 + 9 * \sqrt{2} * a * \sin(2 * d * x + 2 * c)^2 + 18 * \sqrt{2} * a * \sin(2 * d * x + \\
& 2 * c) * \sin(d * x + c) + 9 * \sqrt{2} * a * \sin(d * x + c)^2 + 3 * (2 * \sqrt{2} * a * \cos(2 * d * x +
\end{aligned}$$

$$\begin{aligned}
& 2*c) + 2*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(3*d*x + 3*c)^2 + (\sqrt{2} \\
& *a*\cos(3*d*x + 3*c) + \sqrt{2}*a*\sin(3*d*x + 3*c)^2 + 6*\sqrt{2}*a*\cos(d*x + \\
& c) + 3*(3*\sqrt{2}*a*\cos(2*d*x + 2*c)^2 + 3*\sqrt{2}*a*\cos(d*x + c)^2 + 3*\sqrt{2} \\
& *a*\sin(2*d*x + 2*c)^2 + 6*\sqrt{2}*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 3* \\
& \sqrt{2}*a*\sin(d*x + c)^2 + 4*\sqrt{2}*a*\cos(d*x + c) + 2*(3*\sqrt{2}*a*\cos(d* \\
& x + c) + 2*\sqrt{2}*a*\cos(2*d*x + 2*c) + \sqrt{2}*a*\cos(3*d*x + 3*c) + 6*(3 \\
& *\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*a*\sin(2* \\
& d*x + 2*c) + \sqrt{2}*a*\sin(d*x + c) + (\sqrt{2}*a*\sin(2*d*x + 2*c) + \sqrt{2} \\
& *a*\sin(d*x + c))*\cos(3*d*x + 3*c))*\sin(3*d*x + 3*c) + \sqrt{2}*a*\cos(2/3*ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*((\sqrt{2}*a*\sin(2*d* \\
& x + 2*c) + \sqrt{2}*a*\sin(d*x + c))*\cos(3*d*x + 3*c)^2 + \sqrt{2}*a*\sin(2*d*x \\
& + 2*c) + \sqrt{2}*a*\sin(d*x + c) + 2*(\sqrt{2}*a*\sin(2*d*x + 2*c) + \sqrt{2}* \\
& a*\sin(d*x + c))*\cos(3*d*x + 3*c))*\sin(3*d*x + 3*c) + 6*(\sqrt{2}*a*\sin(3*d*x \\
& + 3*c)^3 + 6*(\sqrt{2}*a*\sin(2*d*x + 2*c) + \sqrt{2}*a*\sin(d*x + c))*\sin(3*d \\
& *x + 3*c)^2 + (\sqrt{2}*a*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*a*\cos(2*d*x + 2*c)^ \\
& 2 + 9*\sqrt{2}*a*\cos(d*x + c)^2 + 9*\sqrt{2}*a*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2} \\
&)*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*a*\sin(d*x + c)^2 + 6*\sqrt{2}* \\
& a*\cos(d*x + c) + 2*(3*\sqrt{2}*a*\cos(2*d*x + 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) \\
& + \sqrt{2}*a*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos \\
& (2*d*x + 2*c) + \sqrt{2}*a*\sin(3*d*x + 3*c) + 3*(\sqrt{2}*a*\cos(3*d*x + 3* \\
& c)^2 + 9*\sqrt{2}*a*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*a*\cos(d*x + c)^2 + \sqrt{2} \\
&)*a*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*a*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*a*\sin \\
& (2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*a*\sin(d*x + c)^2 + 6*\sqrt{2}*a*\cos(d* \\
& x + c) + 2*(3*\sqrt{2}*a*\cos(2*d*x + 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2} \\
&)*a*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(2*d*x \\
& + 2*c) + 6*(\sqrt{2}*a*\sin(2*d*x + 2*c) + \sqrt{2}*a*\sin(d*x + c))*\sin(3*d*x \\
& + 3*c) + \sqrt{2}*a*\sin(2/3*arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/ \\
& 2*c))))*\sin(4/3*arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 6*(\sqrt{2} \\
& *a*\sin(3*d*x + 3*c)^3 + 6*(\sqrt{2}*a*\sin(2*d*x + 2*c) + \sqrt{2}*a*\sin \\
& (d*x + c))*\sin(3*d*x + 3*c)^2 + (\sqrt{2}*a*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*a \\
& *\cos(2*d*x + 2*c)^2 + 9*\sqrt{2}*a*\cos(d*x + c)^2 + 9*\sqrt{2}*a*\sin(2*d*x + \\
& 2*c)^2 + 18*\sqrt{2}*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*a*\sin(d*x + \\
& c)^2 + 6*\sqrt{2}*a*\cos(d*x + c) + 2*(3*\sqrt{2}*a*\cos(2*d*x + 2*c) + 3*\sqrt{2} \\
&)*a*\cos(d*x + c) + \sqrt{2}*a*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*a*\cos(d*x + \\
& c) + \sqrt{2}*a*\cos(2*d*x + 2*c) + \sqrt{2}*a*\sin(3*d*x + 3*c))*\sin(2/3*ar \\
& ctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + \sqrt{2}*a*\sqrt{a}*d)
\end{aligned}$$

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2}} dx = \frac{\sqrt{2} \left(\frac{\log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{\operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{\log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{\operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{2 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{(\sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1) \operatorname{asgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} \right)}{8 \sqrt{ad}}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(log(sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) -
log(-sin(1/2*d*x + 1/2*c) + 1)/(a*sgn(cos(1/2*d*x + 1/2*c))) - 2*sin(1/2*d*
x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a*sgn(cos(1/2*d*x + 1/2*c)))/sqrt(a)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{1}{(a + a \cos(c + dx))^{3/2}} dx$$

```
[In] int(1/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int(1/(a + a*cos(c + d*x))^(3/2), x)
```

$$3.136 \quad \int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	2126
Rubi [A] (verified)	2126
Mathematica [C] (verified)	2128
Maple [B] (verified)	2128
Fricas [B] (verification not implemented)	2129
Sympy [F]	2129
Maxima [F]	2129
Giac [A] (verification not implemented)	2130
Mupad [F(-1)]	2130

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{3/2}d} - \frac{5 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}}$$

[Out] $2*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d-1/2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}-5/4*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2845, 3064, 2728, 212, 2852}

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{a^{3/2}d} - \frac{5 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a \cos(c+dx)+a)^{3/2}}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]/(a+a*\operatorname{Cos}[c+d*x])^{(3/2)},x]$

[Out] $(2*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])]/(a^{(3/2)}*d) - (5*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*d) - \operatorname{Sin}[c+d*x]/(2*d*(a+a*\operatorname{Cos}[c+d*x])^{(3/2)}))$

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2852

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3064

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} + \frac{\int \frac{(2a - \frac{1}{2}a \cos(c + dx)) \sec(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx}{2a^2} \\ &= -\frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2}} + \frac{\int \sqrt{a + a \cos(c + dx)} \sec(c + dx) dx}{a^2} - \frac{5 \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx}{4a} \end{aligned}$$

$$\begin{aligned}
 &= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} \\
 &\quad + \frac{5\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad} \\
 &= \frac{2\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{5\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.99

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{\cos^3\left(\frac{1}{2}(c+dx)\right) \left(-4\sqrt{2}\log\left(i - \sqrt{2}e^{\frac{1}{2}i(c+dx)} - ie^{i(c+dx)}\right) + 4\sqrt{2}\log\left(i + \sqrt{2}e^{i(c+dx)}\right)\right)}{(a+a\cos(c+dx))^{3/2}}$$

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^(3/2), x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(-4*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))] + 4*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]) + 10*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 10*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] - (Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^(-2) + (Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^(-2))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(93) = 186.

Time = 1.95 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.54

method	result
default	$ -\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(5\sqrt{2} \ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} + 4a}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right) a\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 4 \ln\left(\frac{4\sqrt{2}a\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sqrt{a+8a}}{2\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}}\right)}{4a^{\frac{5}{2}}\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sin\left(\frac{dx}{2} + \frac{c}{2}\right)} $

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^(3/2), x, method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^2-4*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)
```

$$x+1/2*c)^2*a-4*\ln(-4/(2*\cos(1/2*d*x+1/2*c)-2^(1/2))*(2^(1/2)*a*\cos(1/2*d*x+1/2*c)-2^(1/2)*(a*\sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*\cos(1/2*d*x+1/2*c)^2*a+2^(1/2)*(a*\sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/\sin(1/2*d*x+1/2*c)/(a*\cos(1/2*d*x+1/2*c)^2)^(1/2)/d$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(93) = 186.

Time = 0.26 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.23

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{5\sqrt{2}(\cos(dx+c)^2+2\cos(dx+c)+1)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)}}{\cos(dx+c)^2+1}\right)}{\cos(dx+c)^2+1}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \int \frac{\sec(c+dx)}{(a(\cos(c+dx)+1))^{3/2}} dx$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)

[Out] Integral(sec(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x)

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \int \frac{\sec(dx+c)}{(a\cos(dx+c)+a)^{3/2}} dx$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)/(a*cos(d*x + c) + a)^(3/2), x)

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.20

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx =$$

$$\frac{\sqrt{2} \left(\frac{4\sqrt{2} \log\left(\frac{|-2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}{|2\sqrt{2}+4 \sin(\frac{1}{2} dx + \frac{1}{2} c)|}\right)}{a} + \frac{5 \log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a} - \frac{5 \log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a} - \frac{2 \sin(\frac{1}{2} dx + \frac{1}{2} c)}{(\sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)a} \right)}{8\sqrt{a}d\operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}$$

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(4*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(1/2*d*x + 1/2*c)))/a + 5*log(sin(1/2*d*x + 1/2*c) + 1)/a - 5*log(-sin(1/2*d*x + 1/2*c) + 1)/a - 2*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)*a))/(sqrt(a)*d*sgn(cos(1/2*d*x + 1/2*c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{1}{\cos(c + dx) (a + a \cos(c + dx))^{3/2}} dx$$

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(3/2)), x)
```

$$3.137 \quad \int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	2131
Rubi [A] (verified)	2131
Mathematica [A] (verified)	2133
Maple [B] (verified)	2134
Fricas [B] (verification not implemented)	2134
Sympy [F]	2135
Maxima [B] (verification not implemented)	2135
Giac [F(-2)]	2227
Mupad [F(-1)]	2227

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = -\frac{3 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{3/2}d} + \frac{9 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2\sqrt{a+a \cos(c+dx)}}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\tan(c+dx)}{2d(a+a \cos(c+dx))^{3/2}} + \frac{3 \tan(c+dx)}{2ad\sqrt{a+a \cos(c+dx)}}$$

[Out] $-3*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d+9/4*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}-1/2*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}+3/2*\tan(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2845, 3063, 3064, 2728, 212, 2852}

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = -\frac{3 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{a^{3/2}d} + \frac{9 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2\sqrt{a \cos(c+dx)+a}}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{3 \tan(c+dx)}{2ad\sqrt{a \cos(c+dx)+a}} - \frac{\tan(c+dx)}{2d(a \cos(c+dx)+a)^{3/2}}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]^2/(a+a*\operatorname{Cos}[c+d*x])^{(3/2)},x]$

```
[Out] (-3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/(a^(3/2)*d) +
(9*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*
Sqrt[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (3*Tan
[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2845

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (
f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```


Rule 3064

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{(3a-\frac{3}{2}a\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
 &= -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{(-3a^2+\frac{3}{2}a^2\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{2a^3} \\
 &= -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} \\
 &\quad - \frac{3\int \sqrt{a+a\cos(c+dx)}\sec(c+dx) dx}{2a^2} + \frac{9\int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
 &= -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{3\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} - \frac{9\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad} \\
 &= -\frac{3\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{9\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} \\
 &\quad - \frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{9\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right) - 6\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2ad\sqrt{a(1+\cos(c+dx))}}$$

[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2), x]

[Out] (9*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] - 6*Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (3 + 2*Sec[c + d*x])*Tan[(c + d*x)/2])/ (2*a*d*Sqrt[a*(1 + Cos[c + d*x])])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(119) = 238.

Time = 1.68 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.94

method	result
default	$\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(18\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4a}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\ln\left(\frac{4\sqrt{2}a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a+8a}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)\right)$

[In] `int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{2}(a\sin(1/2dx+1/2c)^2)^{1/2}(18\sqrt{2}\ln(2(2a^{1/2}(a\sin(1/2dx+1/2c)^2)^{1/2}+2a)/\cos(1/2dx+1/2c))a\cos(1/2dx+1/2c)^4-12\ln(4/(2\cos(1/2dx+1/2c)+2^{1/2}))(2^{1/2}a\cos(1/2dx+1/2c)+2^{1/2}(a\sin(1/2dx+1/2c)^2)^{1/2}a^{1/2}+2a))\cos(1/2dx+1/2c)^4a-12\ln(-4/(2\cos(1/2dx+1/2c)-2^{1/2}))(2^{1/2}a\cos(1/2dx+1/2c)-2^{1/2}(a\sin(1/2dx+1/2c)^2)^{1/2}a^{1/2}-2a))\cos(1/2dx+1/2c)^4a-9\sqrt{2}\ln(2(2a^{1/2}(a\sin(1/2dx+1/2c)^2)^{1/2}+2a)/\cos(1/2dx+1/2c))a\cos(1/2dx+1/2c)^2+6\cos(1/2dx+1/2c)^2(a\sin(1/2dx+1/2c)^2)^{1/2}2^{1/2}a^{1/2}+6\ln(4/(2\cos(1/2dx+1/2c)+2^{1/2}))(2^{1/2}a\cos(1/2dx+1/2c)+2^{1/2}(a\sin(1/2dx+1/2c)^2)^{1/2}a^{1/2}+2a))\cos(1/2dx+1/2c)^2a+6\ln(-4/(2\cos(1/2dx+1/2c)-2^{1/2}))(2^{1/2}a\cos(1/2dx+1/2c)-2^{1/2}(a\sin(1/2dx+1/2c)^2)^{1/2}a^{1/2}-2a))\cos(1/2dx+1/2c)^2a-2^{1/2}(a\sin(1/2dx+1/2c)^2)^{1/2}a^{1/2}/a^{5/2}/\cos(1/2dx+1/2c)/(2\cos(1/2dx+1/2c)-2^{1/2})/(2\cos(1/2dx+1/2c)+2^{1/2})/\sin(1/2dx+1/2c)/(a\cos(1/2dx+1/2c)^2)^{1/2}/d$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(119) = 238.

Time = 0.31 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.99

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{9\sqrt{2}(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c))\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}}{\dots}\right)}{\dots}$$

[In] `integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{8}(9\sqrt{2})(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c))\sqrt{a}\log(-a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a})\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a)/(\cos(dx+c)^2+2\cos(dx+c)+1)+6(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c))\sqrt{a}\log((a\cos(dx+c)^3-7a\cos(dx+c)^2+4\sqrt{a\cos(dx+c)+a})\sqrt{a})(\cos(dx+c)$$

) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c) + 2)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))

Sympy [F]

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\sec^2(c + dx)}{(a(\cos(c + dx) + 1))^{3/2}} dx$$

[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)

[Out] Integral(sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139280 vs. 2(119) = 238.

Time = 5.93 (sec) , antiderivative size = 139280, normalized size of antiderivative = 967.22

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Too large to display}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] -1/4*(48*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2)*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x + 3*c)^4 + 432*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2)*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^4 + 243*(

$$\begin{aligned}
& + 1) + 16*\sin(3/2*d*x + 3/2*c) + 24*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c)^{\wedge} \\
& 3 + 324*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 + \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
& (\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^{\wedge}2 + \sin(1/2*d*x \\
& + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^{\wedge}2 + s \\
& \sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + \sin(1/2*d*x + 1/2*c)) \\
& *\cos(d*x + c)^{\wedge}3 + 8*(48*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^{\wedge}2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 \\
& + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + s \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^{\wedge} \\
& 2 + \sin(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d* \\
& x + 1/2*c)^{\wedge}2 + \sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2* \\
& d*x + 2*c) + 45*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2 \\
& *c)^{\wedge}2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2 \\
&) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sqrt{2} \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\sqrt{2} \\
& \log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2* \\
& d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^{\wedge}2 + \sin(\\
& 1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2* \\
& c)^{\wedge}2 + \sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + \\
& 24*\cos(5/2*d*x + 5/2*c) + 16*\cos(3/2*d*x + 3/2*c) - 24*\cos(1/2*d*x + 1/2*c \\
&))*\sin(3*d*x + 3*c)^{\wedge}3 + 24*(63*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^{\wedge}2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + \\
& 1/2*c)^{\wedge}2 + \sin(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos \\
& (1/2*d*x + 1/2*c)^{\wedge}2 + \sin(1/2*d*x + 1/2*c)^{\wedge}2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(d*x + c) - 16*\cos(3/2*d*x + 3/2*c) - 24*\cos(1/2*d*x + 1/2*c))*\sin(2*d* \\
& x + 2*c)^{\wedge}3 - 324*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^{\wedge}3 + (3*(\sqrt{2}*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sin(1/2*d*x + 1/2*c)^{\wedge}2 + 2*\sqrt{2}*\cos(1/2*d*x + 1
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin \\
& \sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& \sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2 \\
&) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sin(3/2*d*x + 3/2*c) + 6*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + \\
& 2*c) + 18*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
& *c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& *c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 2*\sin(1/2*d*x + 1/ \\
& 2*c))*\cos(d*x + c) + 2*(9*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& 2*d*x + 2*c) + 9*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& *c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) \\
& + 6*\cos(5/2*d*x + 5/2*c) + 8*\cos(3/2*d*x + 3/2*c) - 6*\cos(1/2*d*x + 1/2*c) \\
& *\sin(3*d*x + 3*c) - 12*(3*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + 1)*\sin(5/2*d* \\
& x + 5/2*c) + 6*(9*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2} \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) \\
& + 8*\cos(3/2*d*x + 3/2*c) - 6*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) - 16*(\\
& 3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + 48*\cos(3/2*d*x + 3/2*c)*\sin(d*x \\
& + c) - 36*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) + 12*\sin(1/2*d*x + 1/2*c))*\cos(5*d*x + 5*c)^2 + 9*(3* \\
& (\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(3*d*x + 3*c)^2 + 27*(\sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& s(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\sin(d*x + c) + 8*\cos(3/2*d*x + 3/2*c) - 6*\cos(1/2*d*x + 1/2*c))*\sin(2* \\
& d*x + 2*c) - 16*(3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + 48*\cos(3/2*d*x \\
& + 3/2*c)*\sin(d*x + c) - 36*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 3*\sqrt{2}*lo \\
& g(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\si \\
& n(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 12*\sin(1/2*d*x + 1/2*c))*\cos(4*d*x \\
& + 4*c)^2 + (1056*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\s \\
& qrt(2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2} \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \si \\
& n(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + \\
& 2*c)^2 + 891*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + \\
& 480*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\s \\
& qrt(2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\ \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\ \\
& \sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c)^2 + 459*(\\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}) \\
& *\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2 \\
& *\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 576*(\sin(2*d*x \\
& + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 24*(81*(\sqrt{2}*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 27*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - 27*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c) + 2) + 27*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) - 27*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 8 \\
& 1*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) + 81*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) - 16*\sin(3/2*d*x + 3/2*c) + 40*\sin(1/2*d*x + 1/2*c))*\cos \\
& (2*d*x + 2*c) + 54*(11*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c) + 2) - 11*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& + 11*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 11*\sqrt{2}*(\\
& 2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 33*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 33*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) + 16*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 96*(10*\cos(2*d*x + 2*c) + 9*\cos \\
& (d*x + c) + 3)*\sin(5/2*d*x + 5/2*c) + 8*(117*(\sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*s
\end{aligned}$$

$$\begin{aligned}
& \sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \\
& * \sin(1/2 dx + 1/2 c)^2 + 2 \sqrt{2} \cos(1/2 dx + 1/2 c) - 2 \sqrt{2} \sin(1/ \\
& 2 dx + 1/2 c) + 2) + \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx \\
& + 1/2 c)^2 - 2 \sqrt{2} \cos(1/2 dx + 1/2 c) + 2 \sqrt{2} \sin(1/2 dx + 1/2 c \\
&) + 2) - \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c)^2 - \\
& 2 \sqrt{2} \cos(1/2 dx + 1/2 c) - 2 \sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - 3 \log \\
& (\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) \\
& + 1) + 3 \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx \\
& x + 1/2 c) + 1)) * \sin(dx + c) + 80 \cos(3/2 dx + 3/2 c) - 72 \cos(1/2 dx + \\
& 1/2 c)) * \sin(2 dx + 2 c) - 96 (3 \cos(dx + c) + 1) * \sin(3/2 dx + 3/2 c) + 6 \\
& 72 \cos(3/2 dx + 3/2 c) * \sin(dx + c) - 576 \cos(1/2 dx + 1/2 c) * \sin(dx + c \\
&) + 99 \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2 \sqrt{2} \\
& \sqrt{2} \cos(1/2 dx + 1/2 c) + 2 \sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - 99 \sqrt{2} \\
& \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2 \sqrt{2} \cos \\
& (1/2 dx + 1/2 c) - 2 \sqrt{2} \sin(1/2 dx + 1/2 c) + 2) + 99 \sqrt{2} \log(2 \\
& * \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2 \sqrt{2} \cos(1/2 dx \\
& + 1/2 c) + 2 \sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - 99 \sqrt{2} \log(2 \cos(1/2 dx \\
& * x + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2 \sqrt{2} \cos(1/2 dx + 1/2 c) - \\
& 2 \sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - 297 \log(\cos(1/2 dx + 1/2 c)^2 + \sin \\
& (1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) + 297 \log(\cos(1/2 dx + 1 \\
& /2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + 1) + 288 \sin(1/ \\
& 2 dx + 1/2 c)) * \cos(3 dx + 3 c)^2 + (1971 * (\sqrt{2} \log(2 \cos(1/2 dx + 1/2 \\
& * c)^2 + 2 \sin(1/2 dx + 1/2 c)^2 + 2 \sqrt{2} \cos(1/2 dx + 1/2 c) + 2 \sqrt{2} \\
& (2) * \sin(1/2 dx + 1/2 c) + 2) - \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin \\
& (1/2 dx + 1/2 c)^2 + 2 \sqrt{2} \cos(1/2 dx + 1/2 c) - 2 \sqrt{2} \sin(1/2 dx \\
& x + 1/2 c) + 2) + \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/ \\
& 2 c)^2 - 2 \sqrt{2} \cos(1/2 dx + 1/2 c) + 2 \sqrt{2} \sin(1/2 dx + 1/2 c) + \\
& 2) - \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2 \sqrt{2} \\
& \sqrt{2} \cos(1/2 dx + 1/2 c) - 2 \sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - 3 \log(\cos \\
& (1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) \\
& + 3 \log(\cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + \\
& 1/2 c) + 1)) * \cos(dx + c)^2 + 675 (\sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \\
& * \sin(1/2 dx + 1/2 c)^2 + 2 \sqrt{2} \cos(1/2 dx + 1/2 c) + 2 \sqrt{2} \sin(1/ \\
& 2 dx + 1/2 c) + 2) - \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx \\
& + 1/2 c)^2 + 2 \sqrt{2} \cos(1/2 dx + 1/2 c) - 2 \sqrt{2} \sin(1/2 dx + 1/2 c \\
&) + 2) + \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c)^2 - \\
& 2 \sqrt{2} \cos(1/2 dx + 1/2 c) + 2 \sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - \sqrt{2} \\
& (2) \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c)^2 - 2 \sqrt{2} \cos \\
& (1/2 dx + 1/2 c) - 2 \sqrt{2} \sin(1/2 dx + 1/2 c) + 2) - 3 \log(\cos(1/2 dx \\
& + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/2 c) + 1) + 3 \log(\\
& \cos(1/2 dx + 1/2 c)^2 + \sin(1/2 dx + 1/2 c)^2 - 2 \sin(1/2 dx + 1/2 c) + \\
& 1)) * \sin(dx + c)^2 + 18 (73 \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/ \\
& 2 dx + 1/2 c)^2 + 2 \sqrt{2} \cos(1/2 dx + 1/2 c) + 2 \sqrt{2} \sin(1/2 dx + \\
& 1/2 c) + 2) - 73 \sqrt{2} \log(2 \cos(1/2 dx + 1/2 c)^2 + 2 \sin(1/2 dx + 1/ \\
& 2 c)^2 + 2 \sqrt{2} \cos(1/2 dx + 1/2 c) - 2 \sqrt{2} \sin(1/2 dx + 1/2 c) +
\end{aligned}$$

$$\begin{aligned}
& 2) + 73\sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - 73\sqrt{2} \\
& \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos \\
& (1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - 219\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) + 21 \\
& 9\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2 \\
& *c) + 1) + 80\sin(1/2*d*x + 1/2*c)\cos(d*x + c) + 416*(3\cos(d*x + c) + 1) \\
& *\sin(3/2*d*x + 3/2*c) - 96\cos(3/2*d*x + 3/2*c)\sin(d*x + c) - 576\cos(1/2* \\
& d*x + 1/2*c)\sin(d*x + c) + 219\sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin \\
& (1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d \\
& *x + 1/2*c) + 2) - 219\sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x \\
& + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2* \\
& c) + 2) + 219\sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) \\
& ^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \\
& 219\sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2} \\
& \cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - 657\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + \\
& 1) + 657\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d* \\
& x + 1/2*c) + 1) + 480\sin(1/2*d*x + 1/2*c)\cos(2*d*x + 2*c)^2 + 162*(\sqrt{2} \\
&)\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(\\
& 1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log(2\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2 \\
& *c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}\log(2\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2} \\
& (2)\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin \\
& (1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d* \\
& x + 1/2*c) + 2) - 3\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) + 3\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) + 2\sin(1/2*d*x + 1/2*c)\cos(d*x + c \\
&)^2 + (3*(\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + \\
& 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
& \log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos \\
& (1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}\log(2\cos \\
& (1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1 \\
& /2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log(2\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2} \\
& \sin(1/2*d*x + 1/2*c) + 2) - 3\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) + 3\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1)\cos(3*d*x + 3*c)^2 + \\
& 27*(\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2} \\
& \cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log(2\cos \\
& (1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2* \\
& c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}\log(2\cos(1/2*d*x + 1/2* \\
& d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) \\
& + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(d*x + c) + 8*\cos(3/2*d*x + 3/2*c) - 6*\cos(1/2*d*x + 1/2*c \\
&))*\sin(2*d*x + 2*c) - 16*(3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + 48*\cos \\
& (3/2*d*x + 3/2*c)*\sin(d*x + c) - 36*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 3*s \\
& \text{qrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\text{sqrt}(2)*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + 3*\text{sqrt}(2)*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + \\
& 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 12*\sin(1/2*d*x + 1/2*c))* \\
& \sin(5*d*x + 5*c)^2 + 9*(3*(\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \\
& \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2) \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x \\
& + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 3*d*x + 3*c)^2 + 27*(\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \text{sqrt}(\\
& 2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2 \\
& *c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x \\
& + 2*c)^2 + 27*(\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) \\
& - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(\\
& 2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \text{sqrt}(2)*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d* \\
& x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - \\
& 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) + 2*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 2*(9*(\sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 9*(\sqrt{2}*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& *sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 6*\cos(5/2*d*x + 5/2*c) + 8*\cos \\
& (3/2*d*x + 3/2*c) - 6*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x + 3*c) - 12*(3*\cos(2* \\
& d*x + 2*c) + 3*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) + 6*(9*(\sqrt{2})*\log(2 \\
& *\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 8*\cos(3/2*d*x + 3/2*c) - 6*co \\
& s(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) - 16*(3*\cos(d*x + c) + 1)*\sin(3/2*d*x \\
& + 3/2*c) + 48*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 36*\cos(1/2*d*x + 1/2*c)*s \\
& in(d*x + c) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*sqr \\
& t(2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 39*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 39*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 39*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 39*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 117*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 117*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 80*\sin(3/2*d*x + 3/2*c) + 72*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 18*(17*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 17*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 17*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 17*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 51*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 51*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 32*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 192*(3*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) + 24*(81*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 16*\cos(3/2*d*x + 3/2*c) - 40*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) - 224*(3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + 288*\cos(3/2*d*x + 3/2*c))*\sin(d*x + c) - 864*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c) + 51*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 51*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 51*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 51*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 153*\log(\cos(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 153*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) + 192*\sin(1/2*d*x + 1/2*c))*\sin(3*d*x + 3*c)^2 + (864*(\sqrt{2})*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 + 675*(\sqrt{2})*\log(2*\cos(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 1971*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(d*x + c)^2 + 24*(63*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1))*\cos(d*x + c) + 21*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) - 21*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2} \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2)*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& *sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sin(3/2*d*x + 3/2*c))*\sin(3* \\
& d*x + 3*c)^2 + 9*(12*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2} \\
& (2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x \\
& + 2*c) + 9*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2 \\
& *\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*s \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sin(3/2*d*x + 3/2*c))* \\
& \sin(2*d*x + 2*c)^2 + 48*(3*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c) + 27*(3*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) -
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) + 2*\sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) + 2*(9*(s \\
& \text{qrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \text{sqrt}(2)*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*s \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(2*d*x + 2*c) + 9*(\text{sqrt}(2)*\text{lo} \\
& \text{g}(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d \\
& *x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - \\
& 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\text{si} \\
& \text{n}(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c) + 6*\cos(5/2*d*x + 5/2*c) + 8 \\
& * \cos(3/2*d*x + 3/2*c) - 6*\cos(1/2*d*x + 1/2*c)) * \sin(3*d*x + 3*c) - 12*(3*\co \\
& s(2*d*x + 2*c) + 3*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) + 6*(9*(\text{sqrt}(2)*\text{l} \\
& \text{og}(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2* \\
& d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
& - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\text{s} \\
& \text{i}(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + \\
& 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(d*x + c) + 8*\cos(3/2*d*x + 3/2*c) - \\
& 6*\cos(1/2*d*x + 1/2*c)) * \sin(2*d*x + 2*c) - 16*(3*\cos(d*x + c) + 1) * \sin(3/2* \\
& d*x + 3/2*c) + 48*\cos(3/2*d*x + 3/2*c) * \sin(d*x + c) - 36*\cos(1/2*d*x + 1/2* \\
& c) * \sin(d*x + c) + 3*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& + 2) - 3*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + 3*\text{sq} \\
& \text{rt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\c \\
& \text{os}(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\text{sqrt}(2)*\log(2
\end{aligned}$$

$$\begin{aligned}
& * \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 12*s \\
& \sin(1/2*d*x + 1/2*c))*\cos(4*d*x + 4*c) + 2*(90*(\sqrt{2})*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 + 81*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + \\
& 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(d*x + c)^2 + 54*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))* \\
& \sin(2*d*x + 2*c)^2 + 54*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2} \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d* \\
& x + c)^2 + 72*(\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + (171
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - 9*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 9*\sqrt{2} \\
& (2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\sqrt{2}*\log(2*c \\
& os(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 27*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 27*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 30*s \\
& in(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 36*((4*\cos(2*d*x + 2*c) + 3*\cos(d*x \\
& + c) + 1)*\sin(2*d*x + 2*c) + (3*\cos(d*x + c) + 1)*\sin(d*x + c) + 4*\cos(2*d \\
& *x + 2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 2*(135*(\sqrt{2}*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 54*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1))*\sin(d*x + c)^2 + 18*(5*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) - 5*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) + 5*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - 5*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& t(2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& + 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) + 7*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 8*(3*\cos(d*x + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c) + 96*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 72*\cos(1/2* \\
& d*x + 1/2*c)*\sin(d*x + c) + 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*
\end{aligned}$$

$$\begin{aligned}
& x + 1/2*c) + 2) - 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) + 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15 \\
& *\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 45*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + \\
& 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) + 42*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 9*(3*\sqrt{2}*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 8*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + \\
& 2*(4*(9*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 9* \\
& (\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 6*\cos(5/2*d*x + \\
& 5/2*c) + 8*\cos(3/2*d*x + 3/2*c) - 6*\cos(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) \\
& + 6*(4*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) + 4*(9* \\
& (\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2 \\
& * \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 8*\cos(3/2*d*x + \\
& 3/2*c) - 6*\cos(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 8*(3*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c) - 18*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 3*(12*(\sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 9*(\sqrt{2}*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) + 16*\sin(3/2*d*x + 3/2*c))*\sin(2*d*x + 2*c) + 9*(3*(\sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\c \\
& \cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*c \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \sqrt{2}*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 48*\sin(3/2*d*x + 3/2*c)*\sin(d*x + \\
& c) - 6*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x + 3*c) - 12*(7*(3*\cos(d*x + c) + 1)* \\
& \cos(2*d*x + 2*c) + 12*\cos(2*d*x + 2*c)^2 + 9*\cos(d*x + c)^2 + 6*\cos(d*x + c \\
&) + 1)*\sin(5/2*d*x + 5/2*c) + 6*(4*(9*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*si \\
& n(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\sin(d*x + c) + 8*\cos(3/2*d*x + 3/2*c) - 6*\cos(1/2*d*x + 1/2*c))*\cos \\
& (2*d*x + 2*c) + 8*(3*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - 18*\cos(d*x + \\
& c)*\cos(1/2*d*x + 1/2*c) + 9*(3*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*si \\
& n(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\cos(d*x + c) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\lo \\
& g(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1
\end{aligned}$$

$$\begin{aligned}
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + \\
& 48*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) - 6*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + \\
& 2*c) - 36*(3*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \cos(1/2*d*x + 1/2*c))*\sin(\\
& d*x + c) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*1 \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 12*\sin(1/2* \\
& d*x + 1/2*c))*\cos(5*d*x + 5*c) + 6*(12*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& *d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& (2))*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3 \\
& *log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\
& c) + 1))*\cos(3*d*x + 3*c)^3 + 108*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
& (2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(2*d*x + 2*c)^3 + 81*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& (2))*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\lo \\
& g(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\co \\
& s(d*x + c)^3 + 3*(28*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x \\
& + 2*c) + 27*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^ \\
& 2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 9* \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\sqrt{2}*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 9*\sqrt{2}*\log(2*\cos(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - 27*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 27*\log(\cos(1/2*d*x + 1/2*c)^2 + si \\
& n(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 16*\sin(5/2*d*x + 5/2*c \\
&) - 16*\sin(3/2*d*x + 3/2*c) + 16*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c)^2 + \\
& 3*(99*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + si \\
& n(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 33*\sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 33*\sqrt{2}*\log(2*c \\
& os(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 33*\sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2
\end{aligned}$$

$$\begin{aligned}
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d \\
& *x + 1/2*c) + 1))*\sin(d*x + c)^2 + 144*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c)^2 \\
& + 2*(90*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c)^2 + 8 \\
& 1*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 54*(\sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*s
\end{aligned}$$

$$\begin{aligned}
& \ln(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c)^2 + 54*(\sqrt{2}*\log(2 \\
& * \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 72*(\sin(2*d*x + 2*c) + \sin(\\
& d*x + c))*\cos(5/2*d*x + 5/2*c) + (171*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\si \\
& n(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(d*x + c) + 57*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) - 57*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& + 57*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 57*\sqrt{2} \\
& (2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 171*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 171* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) - 80*\sin(3/2*d*x + 3/2*c) + 96*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c \\
&) + 18*(3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*s \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 5*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c) - 6*(16*\cos(2*d*x + 2*c) + 15*\cos(d*x + c) + 5)*\sin(\\
& 5/2*d*x + 5/2*c) + 12*(9*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 8*\cos(3/2*d*x + 3/2*c) - 6*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) - 24*(3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + 96*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 72*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 9*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 9*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 27*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 27*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 30*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 36*((4*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + 1)*\sin(2*d*x + 2*c) + (3*\cos(d*x + c) + 1)*\sin(d*x + c) + 4*\cos(2*d*x + 2*c)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 2*(135*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^2 + 54*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 18*(5*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*)
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 5*\sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 5*\sqrt{2}*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 5*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 7*\sin(1/2*d*x + 1/2*c)) \\
& *\cos(d*x + c) - 8*(3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + 96*\cos(3/2*d* \\
& x + 3/2*c)*\sin(d*x + c) - 72*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 15*\sqrt{2} \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2}*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/ \\
& 2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 15*\sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 42*\sin(1/2*d*x + 1/2*c))*\cos \\
& (2*d*x + 2*c) + 9*(3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3 \\
& *\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 8 \\
& *\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 2*(4*(9*(\sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 9*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/
\end{aligned}$$

$$\begin{aligned}
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sin(3/2*d*x \\
& + 3/2*c))*\sin(2*d*x + 2*c) + 9*(3*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
& (2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2} \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) \\
& + 48*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) - 6*\cos(1/2*d*x + 1/2*c))*\sin(3*d*x \\
& + 3*c) - 12*(7*(3*\cos(d*x + c) + 1))*\cos(2*d*x + 2*c) + 12*\cos(2*d*x + 2*c) \\
& ^2 + 9*\cos(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) + 6*(4*(9* \\
& (\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 8*\cos(3/2*d*x + \\
& 3/2*c) - 6*\cos(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 8*(3*\cos(d*x + c) + 1) \\
& *\cos(3/2*d*x + 3/2*c) - 18*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 9*(3*(\sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^3 + 4*(387*(\sqrt{2}) * \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \\
& \sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*si \\
& n(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 129*\sqrt{2}*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 129*\sqrt{2}*\log(2*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*sqr \\
& t(2)*\sin(1/2*d*x + 1/2*c) + 2) + 129*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) - 129*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - 387*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) + 387*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sin(3/2*d*x + 3/2*c) + 168*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c)^2 + 81*(5*\sqrt{2})*\log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) - 5*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) + 5*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2) - 5*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 1 \\
& 5*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) + 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\
& /2*d*x + 1/2*c) + 1) + 6*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 4*(144*(sqr \\
& t(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2})*co \\
& s(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*co \\
& s(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*sqr \\
& t(2)*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 117*(\sqrt{2})*lo \\
& g(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d
\end{aligned}$$

$$\begin{aligned}
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& n(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 39*\sqrt{2}*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 39*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) + 39*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& n(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c) + 2) - 39*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) - 117*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) + 117*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 112*\sin(3/2*d*x + 3/2*c) + 24*\sin(1/2*d* \\
& x + 1/2*c))*\sin(2*d*x + 2*c)^2 + 96*(3*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2* \\
& c)*\sin(d*x + c) + 27*(15*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2 \\
& *\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d \\
& *x + c) + 5*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 5 \\
& *\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 5*\sqrt{2}*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 5*\sqrt{2}*\log(2*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& n(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 15*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 2*\sin(1/2* \\
& d*x + 1/2*c))*\sin(d*x + c)^2 + 144*((4*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + \\
& 1)*\sin(2*d*x + 2*c) + (3*\cos(d*x + c) + 1)*\sin(d*x + c) + 4*\cos(2*d*x + 2*c \\
&)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 3*(459*(\sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& *\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c) + 2) + \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2 \\
& * \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c)^2 + 171*(\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin \\
& (1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/ \\
& 2*c) + 2) + \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - s \\
& \text{qrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)* \\
& \cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*1 \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1))*\sin(d*x + c)^2 + 6*(51*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c) + 2) - 51*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& + 2) + 51*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 51* \\
& \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 153*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + \\
& 153*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) + 64*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 32*(3*\cos(d*x + c) + 1 \\
&)*\sin(3/2*d*x + 3/2*c) + 160*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 192*\cos(1/ \\
& 2*d*x + 1/2*c)*\sin(d*x + c) + 51*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*s \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2* \\
& d*x + 1/2*c) + 2) - 51*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c) + 2) + 51*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \\
& 51*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt} \\
& (2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 153*\log(\co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& + 153*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) + 128*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 27*(5*\text{sqrt}(2)* \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 5*\text{sqrt}(2)*\log(2*\cos(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2* \\
& c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + 5*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) - 5*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 12*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 6*(64*(3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 112*\cos(2*d*x + 2*c)^2 + 81*\cos(d*x + c)^2 + 16*\sin(2*d*x + 2*c)^2 + 24*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sin(d*x + c)^2 + 54*\cos(d*x + c) + 9)*\sin(5/2*d*x + 5/2*c) + 8*(3*(45*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 16*\cos(3/2*d*x + 3/2*c) - 24*\cos(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 8*(3*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - 54*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + 18*(6*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 2*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 2*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 2*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 2*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 2*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 6*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 6*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + \sin(1/2*d*x + 1/2*c))*\sin(d*x + c) + 120*\sin(3/2*d*x + 3/2*c)*\sin(d*x + c) - 18*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + 24*(9*\cos(d*x + c)^2 + 21*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 144*(3*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \cos(1/2*d*x + 1/2*c))*\sin(d*x + c) + 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) + 15*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 15 \\
& *\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 45*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + \\
& 45*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) + 54*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 36*(16*\sin(2*d*x + \\
& 2*c)^3 + 8*(3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c)*\sin(d*x + c) + 16*\cos(2*d* \\
& x + 2*c)^2*\sin(d*x + c) + 40*\sin(2*d*x + 2*c)^2*\sin(d*x + c) + 9*\sin(d*x + \\
& c)^3 + (8*(3*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 16*\cos(2*d*x + 2*c)^2 + 9 \\
& *\cos(d*x + c)^2 + 33*\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin(2*d*x + 2*c) \\
& + (9*\cos(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c \\
&) + 6*(189*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - s \\
& qrt(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2})* \\
& \cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*s \\
& qrt(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c)^3 + 9* \\
& (21*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 21*\sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 21*\sqrt{2}*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 21*\sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 63*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 63*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 22*\sin(1/2*d*x \\
& + 1/2*c))*\cos(d*x + c)^2 - 32*(3*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c) + 9*(21*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& qrt(2)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/ \\
& 2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
&) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) \\
& + 7*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& \text{rt}(2) \cdot \cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 7*\text{sqrt}(2) \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + 7*\text{sqrt}(2)*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/ \\
& 2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 7*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sq} \\
& \text{rt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 21*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 21*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 6*\sin(1/2*d*x + 1 \\
& /2*c))*\sin(d*x + c)^2 + 3*(21*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c) + 2) - 21*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& + 2) + 21*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 21* \\
& \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2) \\
& *\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 63*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 6 \\
& 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1) + 44*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 24*(9*\cos(d*x + c)^2 + 5 \\
& *\sin(d*x + c)^2 + 6*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 48*(3*\cos(d*x \\
& + c)*\cos(1/2*d*x + 1/2*c) + \cos(1/2*d*x + 1/2*c))*\sin(d*x + c) + 7*\text{sqrt}(2)* \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 7*\text{sqrt}(2)*\log(2*\cos(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2* \\
& c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + 7*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) - 7*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2 \\
& *d*x + 1/2*c) + 2) - 21*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) + 21*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 22*\sin(1/2*d*x + 1/2*c))*\cos(2 \\
& *d*x + 2*c) - 48*(9*\sin(d*x + c)^3 + (9*\cos(d*x + c)^2 + 6*\cos(d*x + c) + 1 \\
&)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) + 36*(\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2) \\
&)*\sin(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c) + 2) + \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2 \\
&) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sq} \\
& \text{rt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\
& + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) + 3*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 2*(12*(\text{sqrt}(2)*\log(2*\co \\
& s(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1
\end{aligned}$$

$$\begin{aligned}
& d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1 \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*s \\
& \sin(d*x + c) - 16*\cos(3/2*d*x + 3/2*c))*\cos(3*d*x + 3*c)^2 + 9*(9*(\sqrt{2}*1 \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 16*\cos(3/2*d*x + 3/2*c))* \\
& \cos(2*d*x + 2*c)^2 + 3*(28*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*(\\
& 2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin \\
& (2*d*x + 2*c) + 27*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + s \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c \\
&) + 16*\cos(5/2*d*x + 5/2*c) + 16*\cos(3/2*d*x + 3/2*c) - 16*\cos(1/2*d*x + 1/ \\
& 2*c))*\sin(3*d*x + 3*c)^2 + 3*(99*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1
\end{aligned}$$

$$\begin{aligned}
&)) * \sin(dx + c) + 16 * \cos(3/2 * dx + 3/2 * c) - 48 * \cos(1/2 * dx + 1/2 * c) * \sin(2 * \\
&dx + 2 * c)^2 - 48 * (3 * \cos(dx + c) + 1) * \sin(3/2 * dx + 3/2 * c) * \sin(dx + c) - \\
&108 * \cos(1/2 * dx + 1/2 * c) * \sin(dx + c)^2 + 2 * (3 * (9 * (\sqrt{2}) * \log(2 * \cos(1/2 * dx \\
&+ 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) + \\
&2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * c) + 2) - \sqrt{2} * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 \\
&+ 2 * \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) - 2 * \sqrt{2}) * \sin \\
&(1/2 * dx + 1/2 * c) + 2) + \sqrt{2} * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx \\
&* dx + 1/2 * c)^2 - 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) + 2 * \sqrt{2}) * \sin(1/2 * dx + 1/ \\
&2 * c) + 2) - \sqrt{2} * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c)^2 \\
&- 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) - 2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * c) + 2) - 3 \\
&* \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * \\
&c) + 1) + 3 * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 \\
&* dx + 1/2 * c) + 1)) * \sin(dx + c) - 16 * \cos(3/2 * dx + 3/2 * c) * \cos(2 * dx + 2 * c \\
&) - 16 * (3 * \cos(dx + c) + 1) * \cos(3/2 * dx + 3/2 * c) - 6 * (4 * \sin(2 * dx + 2 * c) + \\
&3 * \sin(dx + c)) * \sin(5/2 * dx + 5/2 * c) + 4 * (9 * (\sqrt{2}) * \log(2 * \cos(1/2 * dx + 1/ \\
&2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) + 2 * \sqrt{2} \\
&(2) * \sin(1/2 * dx + 1/2 * c) + 2) - \sqrt{2} * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin \\
&n(1/2 * dx + 1/2 * c)^2 + 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) - 2 * \sqrt{2}) * \sin(1/2 * dx \\
&* dx + 1/2 * c) + 2) + \sqrt{2} * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1 \\
&/2 * c)^2 - 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) + 2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * c) + \\
&2) - \sqrt{2} * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sqrt{2} \\
& * \cos(1/2 * dx + 1/2 * c) - 2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * c) + 2) - 3 * \log(c \\
&os(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1 \\
&) + 3 * \log(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + \\
&1/2 * c) + 1)) * \cos(2 * dx + 2 * c) + 9 * (\sqrt{2}) * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + \\
&2 * \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) + 2 * \sqrt{2}) * \sin(1 \\
&/2 * dx + 1/2 * c) + 2) - \sqrt{2} * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx \\
&+ 1/2 * c)^2 + 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) - 2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * \\
&c) + 2) + \sqrt{2} * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c)^2 - \\
&2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) + 2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * c) + 2) - \sqrt{2} \\
&t(2) * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sqrt{2}) * \cos \\
&s(1/2 * dx + 1/2 * c) - 2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * c) + 2) - 3 * \log(\cos(1/2 * dx \\
&x + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) + 3 * \log \\
&(\cos(1/2 * dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + \\
&1)) * \cos(dx + c) + 3 * \sqrt{2}) * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx \\
&+ 1/2 * c)^2 + 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) + 2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * c \\
&) + 2) - 3 * \sqrt{2}) * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c)^2 \\
&+ 2 * \sqrt{2}) * \cos(1/2 * dx + 1/2 * c) - 2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * c) + 2) + 3 * \\
&\sqrt{2}) * \log(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sqrt{2}) \\
&* \cos(1/2 * dx + 1/2 * c) + 2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * c) + 2) - 3 * \sqrt{2}) * \log \\
&(2 * \cos(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sqrt{2}) * \cos(1/2 * dx \\
&x + 1/2 * c) - 2 * \sqrt{2}) * \sin(1/2 * dx + 1/2 * c) + 2) - 9 * \log(\cos(1/2 * dx + 1/2 * \\
&c)^2 + \sin(1/2 * dx + 1/2 * c)^2 + 2 * \sin(1/2 * dx + 1/2 * c) + 1) + 9 * \log(\cos(1/2 \\
&* dx + 1/2 * c)^2 + \sin(1/2 * dx + 1/2 * c)^2 - 2 * \sin(1/2 * dx + 1/2 * c) + 1) - 8 * \\
&\sin(3/2 * dx + 3/2 * c) + 6 * \sin(1/2 * dx + 1/2 * c)) * \sin(2 * dx + 2 * c) + 9 * (3 * (\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& *d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + \\
& 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + \\
& sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2) \\
&)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(\\
& 2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x \\
& + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c) \\
&)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2* \\
& d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(\\
& 3*d*x + 3*c)^2 + 27*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + \\
& 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) \\
& + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(\\
& 2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(\\
& 1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2 \\
& *c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c)^2 + \\
& sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2*d*x + \\
& 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x \\
& + 2*c)^2 + 27*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) \\
& - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(\\
& 2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log \\
& (2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d* \\
& x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d* \\
& x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - \\
& 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2*d*x + 1/2*c) \\
& ^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + \\
& 3*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt \\
& t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*l \\
& og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2* \\
& d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2* \\
& d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) \\
& + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^ \\
& 2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*s \\
& in(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 27*(sq \\
& rt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*c \\
& os(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*c \\
& os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + \\
& 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + \\
& 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq \\
& rt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
\end{aligned}$$

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c)^2 + 27*(\sqrt{2})* \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2 \\
& *d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})* \\
& \sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 2*(9*(\sqrt{2})*\log(2*\cos \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1 \\
& /2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 9*(\sqrt{2})*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1 \\
& /2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/ \\
& 2*c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + \\
& 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1) - 6*\sin(5/2*d*x + 5/2*c) - 8*\sin(3/2*d*x + 3/2*c) + 6*\sin(1/2*d*x \\
& + 1/2*c))*\cos(3*d*x + 3*c) + 36*(\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 5/2*c) + 6*(9*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2} \\
& * \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + \\
& c) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2} \\
& * \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*\log(2*c \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2* \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(3/2*d*x + 3 \\
& /2*c) + 6*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 18*(\sqrt{2})*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1 \\
& /2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2* \\
& c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) + 2*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 2*(9*(\sqrt{2} \\
& * \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(\\
& 1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& * \sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2*c) + 9*(\sqrt{2})*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2
\end{aligned}$$

$$\begin{aligned}
& 1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2 \\
& *c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(\\
& 2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + \\
& 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2*d*x + 1/2*c)^2 + si \\
& n(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c) \\
&)*cos(d*x + c) - 24*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*sin(5/2*d*x + \\
& 5/2*c) + (171*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2* \\
& c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) \\
& - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt \\
& (2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*lo \\
& g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d \\
& *x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d \\
& *x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - \\
& 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1 \\
& /2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2*d*x + 1/2*c \\
&)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + \\
& 80*cos(3/2*d*x + 3/2*c) - 96*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 32*(3 \\
& *cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 72*cos(3/2*d*x + 3/2*c)*sin(d*x + \\
& c) - 90*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 6*sqrt(2)*log(2*cos(1/2*d*x + \\
& 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq \\
& rt(2)*sin(1/2*d*x + 1/2*c) + 2) - 6*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + \\
& 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1 \\
& /2*d*x + 1/2*c) + 2) + 6*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d \\
& *x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/ \\
& 2*c) + 2) - 6*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - \\
& 18*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1 \\
& /2*c) + 1) + 18*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin \\
& (1/2*d*x + 1/2*c) + 1) + 24*sin(1/2*d*x + 1/2*c))*sin(3*d*x + 3*c) - 12*(4* \\
& (3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*sin(2*d*x + 2*c) + 3*(3*cos(d*x + \\
& c) + 1)*sin(d*x + c) + 9*cos(2*d*x + 2*c)*sin(d*x + c))*sin(5/2*d*x + 5/2* \\
& c) + 2*(54*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 \\
& + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - s \\
& qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)* \\
& cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2* \\
& cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + \\
& 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + \\
& 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s \\
& qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d \\
& *x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(cos(1/2*d*x + 1/2*c)^2 \\
& + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 \\
& + 54*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s \\
& qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)
\end{aligned}$$

$$\begin{aligned}
& 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 2*\sin(\\
& 1/2*d*x + 1/2*c))*\cos(d*x + c) - 32*(3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2* \\
& c) + 24*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 126*\cos(1/2*d*x + 1/2*c)*\sin(d* \\
& x + c) + 6*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 6* \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 6*\sqrt{2}*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 6*\sqrt{2}*\log(2*\cos(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 18*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 18*\log(\cos(1/2*d*x + 1/ \\
& 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 24*\sin(1/2* \\
& d*x + 1/2*c))*\sin(2*d*x + 2*c) + 9*(9*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1 \\
& /2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& *\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
&) + 1))*\cos(d*x + c)^2 + 6*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + \\
& 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) \\
& + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d* \\
& x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 2* \\
& \sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
& *\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) \\
& - \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2} \\
& (2)\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - 3\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) + \\
& 3\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/ \\
& 2*c) + 1))\sin(d*x + c) - 16\cos(3/2*d*x + 3/2*c))\cos(3*d*x + 3*c)^2 + 9*(\\
& 9*(\sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2} \\
& (2)\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log \\
& (2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d \\
& *x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}\log(2\cos(1/2*d \\
& *x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + \\
& 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 \\
& + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin \\
& (1/2*d*x + 1/2*c) + 2) - 3\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) + 3\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\sin(d*x + c) - 16\cos(3/2*d* \\
& x + 3/2*c))\cos(2*d*x + 2*c)^2 + 3*(28*(\sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^ \\
& 2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin \\
& (1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + \\
& 1/2*c) + 2) + \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) \\
& ^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \\
& \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2} \\
&)\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - 3\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) + 3 \\
& *log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2* \\
& c) + 1))\sin(2*d*x + 2*c) + 27*(\sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin \\
& (1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d \\
& *x + 1/2*c) + 2) - \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1 \\
& /2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + \\
& 2) + \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2} \\
& \cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
& *log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2}\cos(1/ \\
& 2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - 3\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) + 3\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(d*x + c) + 16\cos(5/2*d*x + 5/2*c) + 16\cos(3/2*d*x + 3/2*c) - 16\cos(\\
& 1/2*d*x + 1/2*c))\sin(3*d*x + 3*c)^2 + 3*(99*(\sqrt{2}\log(2\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2} \\
& \sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin \\
& (1/2*d*x + 1/2*c)^2 + 2\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d* \\
& x + 1/2*c) + 2) + \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + \\
& 1/2*c)^2 - 2\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) \\
& + 2) - \sqrt{2}\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2} \\
& \cos(1/2*d*x + 1/2*c) - 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - 3\log(
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\sin(d*x + c) + 16*\cos(3/2*d*x + 3/2*c) - 48*\cos(1/2*d*x + 1/ \\
& 2*c))*\sin(2*d*x + 2*c)^2 - 48*(3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c)*\sin \\
& (d*x + c) - 108*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*(3*(9*(\sqrt{2})*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x \\
& + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2 \\
& *\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/ \\
& 2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) - 16*\cos(3/2*d*x + 3/2*c))*\cos \\
& (2*d*x + 2*c) - 16*(3*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) - 6*(4*\sin(2*d \\
& *x + 2*c) + 3*\sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + 4*(9*(\sqrt{2})*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + \\
& 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*s \\
& in(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 9*(\sqrt{2})*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*s \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/ \\
& 2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3 \\
& *\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9
\end{aligned}$$

$$\begin{aligned}
& + 3/2*c)*\sin(d*x + c))*\cos(2*d*x + 2*c) - 16*(9*\cos(d*x + c)^2 + 6*\cos(d*x \\
& + c) + 1)*\cos(3/2*d*x + 3/2*c) + 2*(6*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin \\
& (1/2*d*x + 1/2*c) + 2) - \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1 \\
& /2*c) + 2) + \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^ \\
& 2 - 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - \\
& \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2})* \\
& *\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2 \\
& *d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\
&) + 1))*\cos(3*d*x + 3*c)^2 + 54*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2* \\
& d*x + 1/2*c) + 2) - \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) \\
& + 2) + \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2})* \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2})*\cos(1 \\
& /2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\cos(2*d*x + 2*c)^2 + 54*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c) + 2) - \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) + \\
& \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2})* \\
& *\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2})*\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2})*\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& d*x + c)^2 + 90*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2 \\
&) - \sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2})* \\
& *\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2})*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2})*\cos(1/2* \\
& d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2})*\log(2*\cos(1/2* \\
& d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) \\
& - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2* \\
& c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(2*d*x + 2* \\
& c)^2 + 81*(\sqrt{2})*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sqrt{2})*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2})* \\
& \log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2})*\cos \\
& (1/2*d*x + 1/2*c) - 2*\sqrt{2})*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2})*\log(2*c
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c)^2 + 4*(\\
& 9*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2} \\
& (2)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log \\
& (2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin \\
& (1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(2*d*x + 2*c) + 9*(\sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1 \\
& /2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1 \\
& /2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2* \\
& c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2* \\
& c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x \\
& + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\
& \sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(2*\cos(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c \\
&) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2 \\
& *c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c) + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c \\
&) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) - 6*\sin(5/2*d*x + 5/2*c) - 8*\sin(3/2*d*x + 3/2 \\
& *c) + 6*\sin(1/2*d*x + 1/2*c))*\cos(3*d*x + 3*c) + 6*(16*\sin(2*d*x + 2*c) + 1 \\
& 5*\sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 12*(9*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2} \\
& \sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2* \\
& d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(
\end{aligned}$$

$$\begin{aligned}
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x \\
& + 1/2*c) + 1))*\cos(d*x + c) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d \\
& *x + 1/2*c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
& + 2) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*s \\
& \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - \\
& 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sin(3/2*d*x + 3/2*c) + 6*\sin(1/2*d*x + 1/2*c))*\cos(2*d*x + 2*c) + 36*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 2*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) - 24*(3*\cos(2*d*x + 2*c) + 3*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) + (171*(\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(d*x + c) + 80*\cos(3/2*d*x + 3/2*c) - 96*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) - 32*(3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) + 72*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 90*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 6*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 6*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 6*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 6*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 18*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*
\end{aligned}$$

$$\begin{aligned}
& \text{qrt}(2) * \sin(1/2*d*x + 1/2*c) + 2) - 3*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(\\
& 1/2*d*x + 1/2*c) + 2) + 3*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2* \\
& d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1 \\
& /2*c) + 2) - 3*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c \\
&)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) \\
& - 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1) + 9*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1) - 8*\sin(3/2*d*x + 3/2*c) + 6*\sin(1/2*d*x + 1/2*c))*\text{co} \\
& \text{s}(2*d*x + 2*c) + 36*(\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) \\
& + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + \text{sqrt}(\\
& 2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(\\
& 1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2 \\
& *c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 2*\sin(1/2 \\
& *d*x + 1/2*c))*\cos(d*x + c) - 32*(3*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) \\
& + 24*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) - 126*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c) + 6*\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 6*\text{sq} \\
& \text{r}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\text{co} \\
& \text{s}(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + 6*\text{sqrt}(2)*\log(2* \\
& \cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 6*\text{sqrt}(2)*\log(2*\cos(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2 \\
& *\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 18*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 18*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 24*\sin(1/2*d*x \\
& + 1/2*c))*\sin(2*d*x + 2*c) + 9*(9*(\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + \\
& 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1 \\
& /2*d*x + 1/2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2* \\
& c) + 2) + \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - \text{sq} \\
& \text{r}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\text{sqrt}(2)*\text{co} \\
& \text{s}(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(d*x + c)^2 + 6*(\text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/ \\
& 2*c) + 2) - \text{sqrt}(2)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) - 2*\text{sqrt}(2)*\sin(1/2*d*x + 1/2*c) + 2) + s
\end{aligned}$$

$$\begin{aligned}
& 2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + \\
& 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) \\
&)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) \\
& - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1 \\
& /2*c) + 1) + 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(\\
& 1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 513*(sqrt(2)*log(2*cos(1/2*d*x \\
& + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2* \\
& sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + \\
& 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1 \\
& /2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x \\
& + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2* \\
& c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - \\
& 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*l \\
& og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) \\
& + 1) + 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d \\
& *x + 1/2*c) + 1))*cos(d*x + c)^2 + 1377*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c) \\
& ^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)* \\
& sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + \\
& 1/2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c \\
&)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) \\
& - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(\\
& 2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1 \\
& /2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + \\
& 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2 \\
& *c) + 1))*sin(d*x + c)^2 + 24*(45*(sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2 \\
& *sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/ \\
& 2*d*x + 1/2*c) + 2) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x \\
& + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c \\
&) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - \\
& 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt \\
& (2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos \\
& (1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(cos(1/2*d*x \\
& + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 3*log(\\
& cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + \\
& 1))*cos(d*x + c) + 15*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x \\
& + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c \\
&) + 2) - 15*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 \\
& + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 1 \\
& 5*sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(\\
& 2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 15*sqrt(2)* \\
& log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2 \\
& *d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 45*log(cos(1/2*d*x + \\
& 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) + 45*log(co \\
& s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
\end{aligned}$$

$$\begin{aligned}
& - 16\sin(3/2*d*x + 3/2*c) + 24\sin(1/2*d*x + 1/2*c))\cos(2*d*x + 2*c) + 18 \\
& *(19\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2})\cos(1/2*d*x + 1/2*c) + 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - 19\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2})\cos(1/2*d*x + 1/2*c) - 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) + 19\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2})\cos(1/2*d*x + 1/2*c) + 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - 19\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2})\cos(1/2*d*x + 1/2*c) - 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - 57\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) + 57\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) + 32\sin(1/2*d*x + 1/2*c))\cos(d*x + c) - 160*(3\cos(d*x + c) + 1)\sin(3/2*d*x + 3/2*c) - 2 \\
& 88\cos(3/2*d*x + 3/2*c)\sin(d*x + c) - 1152\cos(1/2*d*x + 1/2*c)\sin(d*x + c) + 57\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2})\cos(1/2*d*x + 1/2*c) + 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - 57\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2})\cos(1/2*d*x + 1/2*c) - 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) + 57\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2})\cos(1/2*d*x + 1/2*c) + 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - 57\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2})\cos(1/2*d*x + 1/2*c) - 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - 171\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) + 171\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) + 192\sin(1/2*d*x + 1/2*c))\sin(2*d*x + 2*c) + 9*(45*(\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2})\cos(1/2*d*x + 1/2*c) + 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2})\cos(1/2*d*x + 1/2*c) - 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2})\cos(1/2*d*x + 1/2*c) + 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2})\cos(1/2*d*x + 1/2*c) - 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - 3\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) + 3\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1))\cos(d*x + c)^2 + 6*(5\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2})\cos(1/2*d*x + 1/2*c) + 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - 5\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2})\cos(1/2*d*x + 1/2*c) - 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) + 5\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2})\cos(1/2*d*x + 1/2*c) + 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - 5\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 - 2\sqrt{2})\cos(1/2*d*x + 1/2*c) - 2\sqrt{2})\sin(1/2*d*x + 1/2*c) + 2) - 15\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c) + 1) + 15\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2\sin(1/2*d*x + 1/2*c) + 1) + 8\sin(1/2*d*x + 1/2*c))\cos(d*x + c) + 5\sqrt{2})\log(2\cos(1/2*d*x + 1/2*c)^2 + 2\sin(1/2*d*x + 1/2*c)^2 + 2\sqrt{2})\cos(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
& + 2\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - 5\sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c) \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2} \\
& * \sin(1/2*d*x + 1/2*c) + 2) + 5*\sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}\sin(1/2*d* \\
& x + 1/2*c) + 2) - 5*\sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 - 2*\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}\sin(1/2*d*x + 1/2*c) \\
& + 2) - 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d \\
& *x + 1/2*c) + 1) + 15*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - \\
& 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c) - 6*co \\
& s(1/2*d*x + 1/2*c))*\sin(3*d*x + 3*c) - 12*(4*(3*\cos(d*x + c) + 1)*\cos(2*d* \\
& x + 2*c)^2 + 48*\cos(2*d*x + 2*c)^3 + 27*\cos(d*x + c)^3 + 16*(3*\cos(2*d*x + \\
& 2*c) + 3*\cos(d*x + c) + 1)*\sin(2*d*x + 2*c)^2 + 9*(3*\cos(d*x + c) + 1)*\sin(\\
& d*x + c)^2 + (99*\cos(d*x + c)^2 + 27*\sin(d*x + c)^2 + 66*\cos(d*x + c) + 11) \\
& *\cos(2*d*x + 2*c) + 27*\cos(d*x + c)^2 + 24*((3*\cos(d*x + c) + 1)*\sin(d*x + \\
& c) + 3*\cos(2*d*x + 2*c)*\sin(d*x + c))*\sin(2*d*x + 2*c) + 9*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c) + 2*(567*(\sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*s \\
& in(1/2*d*x + 1/2*c)^2 + 2*\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}\sin(1/2* \\
& d*x + 1/2*c) + 2) - \sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}\sin(1/2*d*x + 1/2*c) \\
& + 2) + \sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2* \\
& \sqrt{2}\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}\cos(1 \\
& /2*d*x + 1/2*c) - 2*\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(co \\
& s(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&)*\sin(d*x + c)^3 + 12*(63*(\sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c)^2 + 2*\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}\sin(1/2*d*x + \\
& 1/2*c) + 2) - \sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \\
& \sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log(\\
& 2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}\cos(1/2*d*x \\
& + 1/2*c) - 2*\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - 3*\log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 3*\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(\\
& d*x + c) - 16*\cos(3/2*d*x + 3/2*c) - 24*\cos(1/2*d*x + 1/2*c))*\cos(2*d*x + 2 \\
& *c)^2 - 162*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + 96*(3*\cos(d*x + c) + 1)*s \\
& in(3/2*d*x + 3/2*c)*\sin(d*x + c) - 594*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 \\
& - 48*(4*(3*\cos(d*x + c) + 1)*\cos(3/2*d*x + 3/2*c) + 9*\cos(d*x + c)*\cos(1/2* \\
& d*x + 1/2*c) - 9*(3*(\sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sqrt{2}\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}\sin(1/2*d*x + 1/2*c) \\
& + 2) - \sqrt{2}\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sqrt{2}\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) + \sqrt{2} \\
&)*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}\cos(\\
& 1/2*d*x + 1/2*c) + 2*\sqrt{2}\sin(1/2*d*x + 1/2*c) + 2) - \sqrt{2}\log(2*\cos(
\end{aligned}$$

$$\begin{aligned}
& \ln(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 21*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 16*\sin(1/ \\
& 2*d*x + 1/2*c))*\sin(d*x + c) - 18*\cos(1/2*d*x + 1/2*c))*\sin(2*d*x + 2*c) + \\
& 16*(27*\cos(d*x + c)^3 + 9*(3*\cos(d*x + c) + 1)*\sin(d*x + c)^2 + 27*\cos(d*x \\
& + c)^2 + 9*\cos(d*x + c) + 1)*\sin(3/2*d*x + 3/2*c) - 36*(9*\cos(d*x + c)^2*\co \\
& s(1/2*d*x + 1/2*c) + 6*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \cos(1/2*d*x + 1/ \\
& 2*c))*\sin(d*x + c) + 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2* \\
& c) + 2) - 3*\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sqrt{2}*\cos(1/2*d*x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) + 3 \\
& *\sqrt{2}*\log(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c) + 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 3*\sqrt{2}*\lo \\
& g(2*\cos(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)^2 - 2*\sqrt{2}*\cos(1/2*d \\
& *x + 1/2*c) - 2*\sqrt{2}*\sin(1/2*d*x + 1/2*c) + 2) - 9*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) + 9*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) + 1 \\
& 2*\sin(1/2*d*x + 1/2*c))/((16*\sqrt{2})*a*\cos(3*d*x + 3*c)^4 + 144*\sqrt{2})*a*c \\
& os(2*d*x + 2*c)^4 + 81*\sqrt{2})*a*\cos(d*x + c)^4 + 16*\sqrt{2})*a*\sin(3*d*x + \\
& 3*c)^4 + 144*\sqrt{2})*a*\sin(2*d*x + 2*c)^4 + 504*\sqrt{2})*a*\sin(2*d*x + 2*c)^ \\
& 3*\sin(d*x + c) + 81*\sqrt{2})*a*\sin(d*x + c)^4 + 108*\sqrt{2})*a*\cos(d*x + c)^3 \\
& + 8*(16*\sqrt{2})*a*\cos(2*d*x + 2*c) + 15*\sqrt{2})*a*\cos(d*x + c) + 5*\sqrt{2} \\
& *a)*\cos(3*d*x + 3*c)^3 + 168*(3*\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a)*\cos(2*d \\
& *x + 2*c)^3 + 8*(16*\sqrt{2})*a*\sin(2*d*x + 2*c) + 15*\sqrt{2})*a*\sin(d*x + c) \\
&)*\sin(3*d*x + 3*c)^3 + 54*\sqrt{2})*a*\cos(d*x + c)^2 + (\sqrt{2})*a*\cos(3*d*x + \\
& 3*c)^2 + 9*\sqrt{2})*a*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2})*a*\cos(d*x + c)^2 + \sqrt{ \\
& 2})*a*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2})*a*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2})*a*si \\
& n(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2})*a*\sin(d*x + c)^2 + 6*\sqrt{2})*a*\cos(\\
& d*x + c) + 2*(3*\sqrt{2})*a*\cos(2*d*x + 2*c) + 3*\sqrt{2})*a*\cos(d*x + c) + sqr \\
& t(2)*a)*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a)*\cos(2*d \\
& *x + 2*c) + 6*(\sqrt{2})*a*\sin(2*d*x + 2*c) + \sqrt{2})*a*\sin(d*x + c))*\sin(3*d \\
& *x + 3*c) + \sqrt{2})*a)*\cos(5*d*x + 5*c)^2 + 9*(\sqrt{2})*a*\cos(3*d*x + 3*c)^2 \\
& + 9*\sqrt{2})*a*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2})*a*\cos(d*x + c)^2 + \sqrt{2})*a \\
& \sin(3*d*x + 3*c)^2 + 9*\sqrt{2})*a*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2})*a*\sin(2*d* \\
& x + 2*c)*\sin(d*x + c) + 9*\sqrt{2})*a*\sin(d*x + c)^2 + 6*\sqrt{2})*a*\cos(d*x + \\
& c) + 2*(3*\sqrt{2})*a*\cos(2*d*x + 2*c) + 3*\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a \\
&)*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a)*\cos(2*d*x + 2 \\
& *c) + 6*(\sqrt{2})*a*\sin(2*d*x + 2*c) + \sqrt{2})*a*\sin(d*x + c))*\sin(3*d*x + 3 \\
& *c) + \sqrt{2})*a)*\cos(4*d*x + 4*c)^2 + (352*\sqrt{2})*a*\cos(2*d*x + 2*c)^2 + 2 \\
& 97*\sqrt{2})*a*\cos(d*x + c)^2 + 160*\sqrt{2})*a*\sin(2*d*x + 2*c)^2 + 312*\sqrt{2} \\
&)*\sin(2*d*x + 2*c)*\sin(d*x + c) + 153*\sqrt{2})*a*\sin(d*x + c)^2 + 198*\sqrt{ \\
& 2})*a*\cos(d*x + c) + 216*(3*\sqrt{2})*a*\cos(d*x + c) + \sqrt{2})*a)*\cos(2*d*x + \\
& 2*c) + 33*\sqrt{2})*a)*\cos(3*d*x + 3*c)^2 + (657*\sqrt{2})*a*\cos(d*x + c)^2 + \\
& 225*\sqrt{2})*a*\sin(d*x + c)^2 + 438*\sqrt{2})*a*\cos(d*x + c) + 73*\sqrt{2})*a)*c \\
& os(2*d*x + 2*c)^2 + (\sqrt{2})*a*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2})*a*\cos(2*d*x + \\
& 2*c)^2 + 9*\sqrt{2})*a*\cos(d*x + c)^2 + \sqrt{2})*a*\sin(3*d*x + 3*c)^2 + 9*sqr
\end{aligned}$$

$$\begin{aligned}
& t(2)*a*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9* \\
& \sqrt{2}*a*\sin(d*x + c)^2 + 6*\sqrt{2}*a*\cos(d*x + c) + 2*(3*\sqrt{2}*a*\cos(2* \\
& d*x + 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a)*\cos(3*d*x + 3*c) + 6*(3* \\
& \sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a)*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*a*\sin(2*d \\
& *x + 2*c) + \sqrt{2}*a*\sin(d*x + c))*\sin(3*d*x + 3*c) + \sqrt{2}*a*\sin(5*d*x \\
& + 5*c)^2 + 9*(\sqrt{2}*a*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*a*\cos(2*d*x + 2*c)^ \\
& 2 + 9*\sqrt{2}*a*\cos(d*x + c)^2 + \sqrt{2}*a*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*a \\
& *\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2} \\
&)*a*\sin(d*x + c)^2 + 6*\sqrt{2}*a*\cos(d*x + c) + 2*(3*\sqrt{2}*a*\cos(2*d*x + \\
& 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a)*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2} \\
&)*a*\cos(d*x + c) + \sqrt{2}*a)*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*a*\sin(2*d*x + 2 \\
& *c) + \sqrt{2}*a*\sin(d*x + c))*\sin(3*d*x + 3*c) + \sqrt{2}*a*\sin(4*d*x + 4*c \\
&)^2 + (32*\sqrt{2}*a*\cos(3*d*x + 3*c)^2 + 160*\sqrt{2}*a*\cos(2*d*x + 2*c)^2 + \\
& 153*\sqrt{2}*a*\cos(d*x + c)^2 + 352*\sqrt{2}*a*\sin(2*d*x + 2*c)^2 + 648*\sqrt{2} \\
&)*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 297*\sqrt{2}*a*\sin(d*x + c)^2 + 102*\sqrt{2} \\
&)*a*\cos(d*x + c) + 8*(16*\sqrt{2}*a*\cos(2*d*x + 2*c) + 15*\sqrt{2}*a*\cos \\
& (d*x + c) + 5*\sqrt{2}*a)*\cos(3*d*x + 3*c) + 104*(3*\sqrt{2}*a*\cos(d*x + c) + \\
& \sqrt{2}*a)*\cos(2*d*x + 2*c) + 17*\sqrt{2}*a*\sin(3*d*x + 3*c)^2 + (288*\sqrt{2} \\
&)*a*\cos(2*d*x + 2*c)^2 + 225*\sqrt{2}*a*\cos(d*x + c)^2 + 657*\sqrt{2}*a*\sin \\
& (d*x + c)^2 + 150*\sqrt{2}*a*\cos(d*x + c) + 168*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2} \\
&)*a*\cos(2*d*x + 2*c) + 25*\sqrt{2}*a*\sin(2*d*x + 2*c)^2 + 18*(9*\sqrt{2} \\
&)*a*\cos(d*x + c)^2 + 6*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(d*x + c)^2 \\
& + 12*\sqrt{2}*a*\cos(d*x + c) + 2*(4*\sqrt{2}*a*\cos(3*d*x + 3*c)^3 + 36*\sqrt{2} \\
&)*a*\cos(2*d*x + 2*c)^3 + 27*\sqrt{2}*a*\cos(d*x + c)^3 + 27*\sqrt{2}*a*\cos(d*x \\
& + c)^2 + (28*\sqrt{2}*a*\cos(2*d*x + 2*c) + 27*\sqrt{2}*a*\cos(d*x + c) + 9*\sqrt{2} \\
&)*a*\cos(3*d*x + 3*c)^2 + 33*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a)*\cos \\
& (2*d*x + 2*c)^2 + (4*\sqrt{2}*a*\cos(3*d*x + 3*c) + 4*\sqrt{2}*a*\cos(2*d*x + 2 \\
& *c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a)*\sin(3*d*x + 3*c)^2 + 9*(4*\sqrt{2} \\
&)*a*\cos(2*d*x + 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(2*d*x + 2 \\
& *c)^2 + 9*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a)*\sin(d*x + c)^2 + 9*\sqrt{2} \\
&)*a*\cos(d*x + c) + 3*(\sqrt{2}*a*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*a*\cos(2*d*x + \\
& 2*c)^2 + 9*\sqrt{2}*a*\cos(d*x + c)^2 + \sqrt{2}*a*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2} \\
&)*a*\sin(2*d*x + 2*c)^2 + 18*\sqrt{2}*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9* \\
& \sqrt{2}*a*\sin(d*x + c)^2 + 6*\sqrt{2}*a*\cos(d*x + c) + 2*(3*\sqrt{2}*a*\cos(2* \\
& d*x + 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a)*\cos(3*d*x + 3*c) + 6*(3* \\
& \sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a)*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*a*\sin(2*d \\
& *x + 2*c) + \sqrt{2}*a*\sin(d*x + c))*\sin(3*d*x + 3*c) + \sqrt{2}*a*\cos(4*d*x \\
& + 4*c) + 2*(30*\sqrt{2}*a*\cos(2*d*x + 2*c)^2 + 27*\sqrt{2}*a*\cos(d*x + c)^2 \\
& + 18*\sqrt{2}*a*\sin(2*d*x + 2*c)^2 + 36*\sqrt{2}*a*\sin(2*d*x + 2*c)*\sin(d*x + \\
& c) + 18*\sqrt{2}*a*\sin(d*x + c)^2 + 18*\sqrt{2}*a*\cos(d*x + c) + 19*(3*\sqrt{2} \\
&)*a*\cos(d*x + c) + \sqrt{2}*a)*\cos(2*d*x + 2*c) + 3*\sqrt{2}*a*\cos(3*d*x + \\
& 3*c) + 2*(45*\sqrt{2}*a*\cos(d*x + c)^2 + 18*\sqrt{2}*a*\sin(d*x + c)^2 + 30*\sqrt{2} \\
&)*a*\cos(d*x + c) + 5*\sqrt{2}*a)*\cos(2*d*x + 2*c) + 6*(4*\sqrt{2}*a*\cos(2 \\
& *d*x + 2*c)*\sin(d*x + c) + 4*(\sqrt{2}*a*\sin(2*d*x + 2*c) + \sqrt{2}*a*\sin(d* \\
& x + c))*\cos(3*d*x + 3*c) + (4*\sqrt{2}*a*\cos(2*d*x + 2*c) + 3*\sqrt{2}*a*\cos(
\end{aligned}$$

$$\begin{aligned}
& d*x + c) + \sqrt{2}*a*\sin(2*d*x + 2*c) + (3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2} \\
&)*a*\sin(d*x + c))*\sin(3*d*x + 3*c) + 18*(4*\sqrt{2}*a*\cos(2*d*x + 2*c)*\sin(\\
& d*x + c) + (3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(d*x + c))*\sin(2*d*x + \\
& 2*c) + \sqrt{2}*a*\cos(5*d*x + 5*c) + 6*(4*\sqrt{2}*a*\cos(3*d*x + 3*c)^3 + 3 \\
& 6*\sqrt{2}*a*\cos(2*d*x + 2*c)^3 + 27*\sqrt{2}*a*\cos(d*x + c)^3 + 27*\sqrt{2}*a \\
& *\cos(d*x + c)^2 + (28*\sqrt{2}*a*\cos(2*d*x + 2*c) + 27*\sqrt{2}*a*\cos(d*x + c \\
&) + 9*\sqrt{2}*a*\cos(3*d*x + 3*c)^2 + 33*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2} \\
&)*a*\cos(2*d*x + 2*c)^2 + (4*\sqrt{2}*a*\cos(3*d*x + 3*c) + 4*\sqrt{2}*a*\cos(2 \\
& *d*x + 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(3*d*x + 3*c)^2 + 9* \\
& (4*\sqrt{2}*a*\cos(2*d*x + 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(2 \\
& *d*x + 2*c)^2 + 9*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(d*x + c)^2 + 9 \\
& *\sqrt{2}*a*\cos(d*x + c) + 2*(30*\sqrt{2}*a*\cos(2*d*x + 2*c)^2 + 27*\sqrt{2}*a \\
& *\cos(d*x + c)^2 + 18*\sqrt{2}*a*\sin(2*d*x + 2*c)^2 + 36*\sqrt{2}*a*\sin(2*d*x \\
& + 2*c)*\sin(d*x + c) + 18*\sqrt{2}*a*\sin(d*x + c)^2 + 18*\sqrt{2}*a*\cos(d*x + \\
& c) + 19*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(2*d*x + 2*c) + 3*\sqrt{2} \\
&)*a*\cos(3*d*x + 3*c) + 2*(45*\sqrt{2}*a*\cos(d*x + c)^2 + 18*\sqrt{2}*a*\sin(d* \\
& x + c)^2 + 30*\sqrt{2}*a*\cos(d*x + c) + 5*\sqrt{2}*a*\cos(2*d*x + 2*c) + 6*(4 \\
& *\sqrt{2}*a*\cos(2*d*x + 2*c)*\sin(d*x + c) + 4*(\sqrt{2}*a*\sin(2*d*x + 2*c) + \\
& \sqrt{2}*a*\sin(d*x + c))*\cos(3*d*x + 3*c) + (4*\sqrt{2}*a*\cos(2*d*x + 2*c) + \\
& 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(2*d*x + 2*c) + (3*\sqrt{2}*a*\cos(d \\
& *x + c) + \sqrt{2}*a*\sin(d*x + c))*\sin(3*d*x + 3*c) + 18*(4*\sqrt{2}*a*\cos(2 \\
& *d*x + 2*c)*\sin(d*x + c) + (3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(d*x + \\
& c))*\sin(2*d*x + 2*c) + \sqrt{2}*a*\cos(4*d*x + 4*c) + 2*(192*\sqrt{2}*a*\cos(\\
& 2*d*x + 2*c)^3 + 135*\sqrt{2}*a*\cos(d*x + c)^3 + 135*\sqrt{2}*a*\cos(d*x + c)^ \\
& 2 + 172*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(2*d*x + 2*c)^2 + 4*(48*s \\
& \sqrt{2}*a*\cos(2*d*x + 2*c) + 39*\sqrt{2}*a*\cos(d*x + c) + 13*\sqrt{2}*a*\sin(2 \\
& *d*x + 2*c)^2 + 45*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(d*x + c)^2 + \\
& 45*\sqrt{2}*a*\cos(d*x + c) + 3*(153*\sqrt{2}*a*\cos(d*x + c)^2 + 57*\sqrt{2}*a* \\
& \sin(d*x + c)^2 + 102*\sqrt{2}*a*\cos(d*x + c) + 17*\sqrt{2}*a*\cos(2*d*x + 2*c \\
&) + 24*(15*\sqrt{2}*a*\cos(2*d*x + 2*c)*\sin(d*x + c) + 4*(3*\sqrt{2}*a*\cos(d*x \\
& + c) + \sqrt{2}*a*\sin(d*x + c))*\sin(2*d*x + 2*c) + 5*\sqrt{2}*a*\cos(3*d*x \\
& + 3*c) + 14*(27*\sqrt{2}*a*\cos(d*x + c)^3 + 27*\sqrt{2}*a*\cos(d*x + c)^2 + 9* \\
& (3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(d*x + c)^2 + 9*\sqrt{2}*a*\cos(d*x \\
& + c) + \sqrt{2}*a*\cos(2*d*x + 2*c) + 2*(4*\sqrt{2}*a*\sin(3*d*x + 3*c)^3 + 3 \\
& 6*\sqrt{2}*a*\sin(2*d*x + 2*c)^3 + 27*\sqrt{2}*a*\cos(2*d*x + 2*c)^2*\sin(d*x + \\
& c) + 99*\sqrt{2}*a*\sin(2*d*x + 2*c)^2*\sin(d*x + c) + 27*\sqrt{2}*a*\sin(d*x + \\
& c)^3 + (4*\sqrt{2}*a*\sin(2*d*x + 2*c) + 3*\sqrt{2}*a*\sin(d*x + c))*\cos(3*d*x \\
& + 3*c)^2 + (28*\sqrt{2}*a*\sin(2*d*x + 2*c) + 27*\sqrt{2}*a*\sin(d*x + c))*\sin(\\
& 3*d*x + 3*c)^2 + 18*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(2*d*x + 2*c) \\
& *\sin(d*x + c) + 2*(9*\sqrt{2}*a*\cos(2*d*x + 2*c)*\sin(d*x + c) + 4*(3*\sqrt{2} \\
&)*a*\cos(2*d*x + 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(2*d*x + 2*c \\
&) + 3*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(d*x + c))*\cos(3*d*x + 3*c) \\
& + 3*(\sqrt{2}*a*\cos(3*d*x + 3*c)^2 + 9*\sqrt{2}*a*\cos(2*d*x + 2*c)^2 + 9*\sqrt{2} \\
&)*a*\cos(d*x + c)^2 + \sqrt{2}*a*\sin(3*d*x + 3*c)^2 + 9*\sqrt{2}*a*\sin(2*d* \\
& x + 2*c)^2 + 18*\sqrt{2}*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 9*\sqrt{2}*a*\sin(d
\end{aligned}$$

$$\begin{aligned}
& *x + c)^2 + 6*\sqrt{2}*a*\cos(d*x + c) + 2*(3*\sqrt{2}*a*\cos(2*d*x + 2*c) + 3* \\
& \sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(3*d*x + 3*c) + 6*(3*\sqrt{2}*a*\cos(d \\
& *x + c) + \sqrt{2}*a*\cos(2*d*x + 2*c) + 6*(\sqrt{2}*a*\sin(2*d*x + 2*c) + \sqrt{2} \\
& *a*\sin(d*x + c))*\sin(3*d*x + 3*c) + \sqrt{2}*a*\sin(4*d*x + 4*c) + 2*(2* \\
& \sqrt{2}*a*\cos(3*d*x + 3*c)^2 + 18*\sqrt{2}*a*\cos(2*d*x + 2*c)^2 + 18*\sqrt{2} \\
& *a*\cos(d*x + c)^2 + 30*\sqrt{2}*a*\sin(2*d*x + 2*c)^2 + 57*\sqrt{2}*a*\sin(2*d* \\
& x + 2*c)*\sin(d*x + c) + 27*\sqrt{2}*a*\sin(d*x + c)^2 + 12*\sqrt{2}*a*\cos(d*x \\
& + c) + 4*(3*\sqrt{2}*a*\cos(2*d*x + 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2} \\
& *a*\cos(3*d*x + 3*c) + 12*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(2*d*x \\
& + 2*c) + 2*\sqrt{2}*a*\sin(3*d*x + 3*c) + 2*(18*\sqrt{2}*a*\cos(2*d*x + 2*c)^2 \\
& + 18*\sqrt{2}*a*\cos(d*x + c)^2 + 45*\sqrt{2}*a*\sin(d*x + c)^2 + 12*\sqrt{2}*a \\
& *\cos(d*x + c) + 12*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(2*d*x + 2*c) \\
& + 2*\sqrt{2}*a*\sin(2*d*x + 2*c) + 3*(9*\sqrt{2}*a*\cos(d*x + c)^2 + 6*\sqrt{2} \\
& *a*\cos(d*x + c) + \sqrt{2}*a*\sin(d*x + c))*\sin(5*d*x + 5*c) + 6*(4*\sqrt{2}* \\
& a*\sin(3*d*x + 3*c)^3 + 36*\sqrt{2}*a*\sin(2*d*x + 2*c)^3 + 27*\sqrt{2}*a*\cos(2 \\
& *d*x + 2*c)^2*\sin(d*x + c) + 99*\sqrt{2}*a*\sin(2*d*x + 2*c)^2*\sin(d*x + c) + \\
& 27*\sqrt{2}*a*\sin(d*x + c)^3 + (4*\sqrt{2}*a*\sin(2*d*x + 2*c) + 3*\sqrt{2}*a* \\
& \sin(d*x + c))*\cos(3*d*x + 3*c)^2 + (28*\sqrt{2}*a*\sin(2*d*x + 2*c) + 27*\sqrt{2} \\
& (2)*a*\sin(d*x + c))*\sin(3*d*x + 3*c)^2 + 18*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2} \\
& *a*\cos(2*d*x + 2*c)*\sin(d*x + c) + 2*(9*\sqrt{2}*a*\cos(2*d*x + 2*c)*\sin \\
& (d*x + c) + 4*(3*\sqrt{2}*a*\cos(2*d*x + 2*c) + 3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2} \\
& *a*\sin(2*d*x + 2*c) + 3*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(d* \\
& x + c))*\cos(3*d*x + 3*c) + 2*(2*\sqrt{2}*a*\cos(3*d*x + 3*c)^2 + 18*\sqrt{2}*a \\
& *\cos(2*d*x + 2*c)^2 + 18*\sqrt{2}*a*\cos(d*x + c)^2 + 30*\sqrt{2}*a*\sin(2*d*x \\
& + 2*c)^2 + 57*\sqrt{2}*a*\sin(2*d*x + 2*c)*\sin(d*x + c) + 27*\sqrt{2}*a*\sin(d* \\
& x + c)^2 + 12*\sqrt{2}*a*\cos(d*x + c) + 4*(3*\sqrt{2}*a*\cos(2*d*x + 2*c) + 3* \\
& \sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(3*d*x + 3*c) + 12*(3*\sqrt{2}*a*\cos(\\
& d*x + c) + \sqrt{2}*a*\cos(2*d*x + 2*c) + 2*\sqrt{2}*a*\sin(3*d*x + 3*c) + 2* \\
& (18*\sqrt{2}*a*\cos(2*d*x + 2*c)^2 + 18*\sqrt{2}*a*\cos(d*x + c)^2 + 45*\sqrt{2} \\
& *a*\sin(d*x + c)^2 + 12*\sqrt{2}*a*\cos(d*x + c) + 12*(3*\sqrt{2}*a*\cos(d*x + c \\
&) + \sqrt{2}*a*\cos(2*d*x + 2*c) + 2*\sqrt{2}*a*\sin(2*d*x + 2*c) + 3*(9*\sqrt{2} \\
& (2)*a*\cos(d*x + c)^2 + 6*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin(d*x + c))* \\
& \sin(4*d*x + 4*c) + 2*(192*\sqrt{2}*a*\sin(2*d*x + 2*c)^3 + 156*\sqrt{2}*a*\cos(\\
& 2*d*x + 2*c)^2*\sin(d*x + c) + 516*\sqrt{2}*a*\sin(2*d*x + 2*c)^2*\sin(d*x + c) \\
& + 135*\sqrt{2}*a*\sin(d*x + c)^3 + 4*(16*\sqrt{2}*a*\sin(2*d*x + 2*c) + 15*\sqrt{2} \\
& (2)*a*\sin(d*x + c))*\cos(3*d*x + 3*c)^2 + 96*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2} \\
& *a*\cos(2*d*x + 2*c)*\sin(d*x + c) + 8*(21*\sqrt{2}*a*\cos(2*d*x + 2*c)*\sin \\
& (d*x + c) + (24*\sqrt{2}*a*\cos(2*d*x + 2*c) + 21*\sqrt{2}*a*\cos(d*x + c) + \\
& 7*\sqrt{2}*a*\sin(2*d*x + 2*c) + 6*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\sin \\
& (d*x + c))*\cos(3*d*x + 3*c) + (192*\sqrt{2}*a*\cos(2*d*x + 2*c)^2 + 171*\sqrt{2} \\
& (2)*a*\cos(d*x + c)^2 + 459*\sqrt{2}*a*\sin(d*x + c)^2 + 114*\sqrt{2}*a*\cos(d*x \\
& + c) + 120*(3*\sqrt{2}*a*\cos(d*x + c) + \sqrt{2}*a*\cos(2*d*x + 2*c) + 19*\sqrt{2} \\
& (2)*a*\sin(2*d*x + 2*c) + 15*(9*\sqrt{2}*a*\cos(d*x + c)^2 + 6*\sqrt{2}*a*\cos \\
& (d*x + c) + \sqrt{2}*a*\sin(d*x + c))*\sin(3*d*x + 3*c) + 6*(84*\sqrt{2}*a*\cos \\
& (2*d*x + 2*c)^2*\sin(d*x + c) + 63*\sqrt{2}*a*\sin(d*x + c)^3 + 48*(3*\sqrt{2}(2)
\end{aligned}$$

```
*a*cos(d*x + c) + sqrt(2)*a*cos(2*d*x + 2*c)*sin(d*x + c) + 7*(9*sqrt(2)*a
*cos(d*x + c)^2 + 6*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a*sin(d*x + c))*sin(2
*d*x + 2*c) + sqrt(2)*a)*sqrt(a)*d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{1}{\cos(c + dx)^2 (a + a \cos(c + dx))^{3/2}} dx$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(3/2)), x)
```

$$3.138 \quad \int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	2228
Rubi [A] (verified)	2228
Mathematica [C] (verified)	2231
Maple [B] (verified)	2232
Fricas [A] (verification not implemented)	2233
Sympy [F]	2233
Maxima [F(-2)]	2233
Giac [F(-2)]	2234
Mupad [F(-1)]	2234

Optimal result

Integrand size = 23, antiderivative size = 185

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{19 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{13 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{7 \tan(c+dx)}{4ad\sqrt{a+a \cos(c+dx)}} - \frac{\sec(c+dx) \tan(c+dx)}{2d(a+a \cos(c+dx))^{3/2}} + \frac{\sec(c+dx) \tan(c+dx)}{ad\sqrt{a+a \cos(c+dx)}}$$

[Out] 19/4*arctanh(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d-13/4*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*sec(d*x+c)*tan(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)-7/4*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)+sec(d*x+c)*tan(d*x+c)/a/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2845, 3063, 3064, 2728, 212, 2852}

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{19 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4a^{3/2}d} - \frac{13 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{7 \tan(c+dx)}{4ad\sqrt{a \cos(c+dx)+a}} + \frac{\tan(c+dx) \sec(c+dx)}{ad\sqrt{a \cos(c+dx)+a}} - \frac{\tan(c+dx) \sec(c+dx)}{2d(a \cos(c+dx)+a)^{3/2}}$$

[In] Int[Sec[c + d*x]^3/(a + a*cos[c + d*x])^(3/2), x]

[Out] (19*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*cos[c + d*x]])/(4*a^(3/2)*d) - (13*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*cos[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) - (7*Tan[c + d*x])/(4*a*d*Sqrt[a + a*cos[c + d*x]]) - (Sec[c + d*x]*Tan[c + d*x])/(2*d*(a + a*cos[c + d*x])^(3/2)) + (Sec[c + d*x]*Tan[c + d*x])/(a*d*Sqrt[a + a*cos[c + d*x]])

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*sin[c + d*x])]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2852

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*sin[e + f*x])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ

$[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{EqQ}[m + 1/2, 0])$

Rule 3064

$\text{Int}[\frac{(A_.) + (B_.) \sin[(e_.) + (f_.)(x_.)]}{(\text{Sqrt}[(a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)] * ((c_.) + (d_.) \sin[(e_.) + (f_.)(x_.)]))}, x_Symbol] \ :> \ \text{Dist}[(A * b - a * B) / (b * c - a * d), \text{Int}[1 / \text{Sqrt}[a + b * \text{Sin}[e + f * x]], x], x] + \text{Dist}[(B * c - A * d) / (b * c - a * d), \text{Int}[\text{Sqrt}[a + b * \text{Sin}[e + f * x]] / (c + d * \text{Sin}[e + f * x]), x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b * c - a * d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{(4a-\frac{5}{2}a\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
 &= -\frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{(-7a^2+6a^2\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{4a^3} \\
 &= -\frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} \\
 &\quad + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{(\frac{19a^3}{2}-\frac{7}{2}a^3\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{4a^4} \\
 &= -\frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{19\int \sqrt{a+a\cos(c+dx)}\sec(c+dx) dx}{8a^2} - \frac{13\int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
 &= -\frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} \\
 &\quad - \frac{19\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4ad} + \frac{13\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad} \\
 &= \frac{19\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{13\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} \\
 &\quad - \frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.93 (sec) , antiderivative size = 841, normalized size of antiderivative = 4.55

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{19 \cos^3\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{ie^{ic}x}{-1+e^{ic}} - \frac{\log(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)})}{d} \right)}{2\sqrt{2}(a(1+\cos(c+dx)))^{3/2}}$$

$$+ \frac{19 \cos^3\left(\frac{c}{2} + \frac{dx}{2}\right) \left(-\frac{ie^{ic}x}{-1+e^{ic}} + \frac{\log(i+\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)})}{d} \right)}{2\sqrt{2}(a(1+\cos(c+dx)))^{3/2}}$$

$$+ \frac{13 \cos^3\left(\frac{c}{2} + \frac{dx}{2}\right) \log\left(\cos\left(\frac{c}{4} + \frac{dx}{4}\right) - \sin\left(\frac{c}{4} + \frac{dx}{4}\right)\right)}{d(a(1+\cos(c+dx)))^{3/2}}$$

$$- \frac{13 \cos^3\left(\frac{c}{2} + \frac{dx}{2}\right) \log\left(\cos\left(\frac{c}{4} + \frac{dx}{4}\right) + \sin\left(\frac{c}{4} + \frac{dx}{4}\right)\right)}{d(a(1+\cos(c+dx)))^{3/2}}$$

$$- \frac{\cos^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{2d(a(1+\cos(c+dx)))^{3/2} \left(\cos\left(\frac{c}{4} + \frac{dx}{4}\right) - \sin\left(\frac{c}{4} + \frac{dx}{4}\right)\right)^2}$$

$$+ \frac{\cos^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{2d(a(1+\cos(c+dx)))^{3/2} \left(\cos\left(\frac{c}{4} + \frac{dx}{4}\right) + \sin\left(\frac{c}{4} + \frac{dx}{4}\right)\right)^2}$$

$$+ \frac{\cos^3\left(\frac{c}{2} + \frac{dx}{2}\right) \sin\left(\frac{dx}{2}\right)}{d(a(1+\cos(c+dx)))^{3/2} \left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right) \left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) - \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)^2}$$

$$+ \frac{\cos^3\left(\frac{c}{2} + \frac{dx}{2}\right) \left(-5 \cos\left(\frac{c}{2}\right) + 7 \sin\left(\frac{c}{2}\right)\right)}{2d(a(1+\cos(c+dx)))^{3/2} \left(\cos\left(\frac{c}{2}\right) - \sin\left(\frac{c}{2}\right)\right) \left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) - \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}$$

$$+ \frac{\cos^3\left(\frac{c}{2} + \frac{dx}{2}\right) \sin\left(\frac{dx}{2}\right)}{d(a(1+\cos(c+dx)))^{3/2} \left(\cos\left(\frac{c}{2}\right) + \sin\left(\frac{c}{2}\right)\right) \left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) + \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)^2}$$

$$+ \frac{\cos^3\left(\frac{c}{2} + \frac{dx}{2}\right) \left(5 \cos\left(\frac{c}{2}\right) + 7 \sin\left(\frac{c}{2}\right)\right)}{2d(a(1+\cos(c+dx)))^{3/2} \left(\cos\left(\frac{c}{2}\right) + \sin\left(\frac{c}{2}\right)\right) \left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) + \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}$$

[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]

[Out] (19*Cos[c/2 + (d*x)/2]^3*((I*E^(I*c)*x)/(-1 + E^(I*c)) - Log[I - Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]/d))/(2*Sqrt[2]*(a*(1 + Cos[c + d*x]))^(3/2)) + (19*Cos[c/2 + (d*x)/2]^3*((-I)*E^(I*c)*x)/(-1 + E^(I*c)) + Log[I + Sqrt[2]*E^((I/2)*(c + d*x)) - I*E^(I*(c + d*x))]/d))/(2*Sqrt[2]*(a*(1 + Cos[c + d*x]))^(3/2)) + (13*Cos[c/2 + (d*x)/2]^3*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]]/(d*(a*(1 + Cos[c + d*x]))^(3/2)) - (13*Cos[c/2 + (d*x)/2]^3*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4]]/(d*(a*(1 + Cos[c + d*x]))^(3/2)) - Cos[c/2 + (d*x)/2]^3/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^2) + Cos[c/2 + (d*x)/2]^3/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^2) + (Cos[c/2

$$+ (d*x)/2]^3*\text{Sin}[(d*x)/2]/(d*(a*(1 + \text{Cos}[c + d*x]))^{(3/2)}*(\text{Cos}[c/2] - \text{Sin}[c/2]))*(\text{Cos}[c/2 + (d*x)/2] - \text{Sin}[c/2 + (d*x)/2])^2) + (\text{Cos}[c/2 + (d*x)/2]^3*(-5*\text{Cos}[c/2] + 7*\text{Sin}[c/2]))/(2*d*(a*(1 + \text{Cos}[c + d*x]))^{(3/2)}*(\text{Cos}[c/2] - \text{Sin}[c/2]))*(\text{Cos}[c/2 + (d*x)/2] - \text{Sin}[c/2 + (d*x)/2])) + (\text{Cos}[c/2 + (d*x)/2]^3*\text{Sin}[(d*x)/2])/ (d*(a*(1 + \text{Cos}[c + d*x]))^{(3/2)}*(\text{Cos}[c/2] + \text{Sin}[c/2]))*(\text{Cos}[c/2 + (d*x)/2] + \text{Sin}[c/2 + (d*x)/2])^2) + (\text{Cos}[c/2 + (d*x)/2]^3*(5*\text{Cos}[c/2] + 7*\text{Sin}[c/2]))/(2*d*(a*(1 + \text{Cos}[c + d*x]))^{(3/2)}*(\text{Cos}[c/2] + \text{Sin}[c/2]))*(\text{Cos}[c/2 + (d*x)/2] + \text{Sin}[c/2 + (d*x)/2]))$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(156) = 312$.

Time = 1.81 (sec) , antiderivative size = 807, normalized size of antiderivative = 4.36

method	result	size
default	Expression too large to display	807

[In] `int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/2*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}*(104*2^{(1/2)}*\ln(2*(2*a^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}+2*a)/\text{cos}(1/2*d*x+1/2*c))*\text{cos}(1/2*d*x+1/2*c)^6*a-76*\ln(-4/(2*\text{cos}(1/2*d*x+1/2*c)-2^{(1/2)}))*(2^{(1/2)}*a*\text{cos}(1/2*d*x+1/2*c)-2^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}-2*a))*\text{cos}(1/2*d*x+1/2*c)^6*a-76*\ln(4/(2*\text{cos}(1/2*d*x+1/2*c)+2^{(1/2)}))*(2^{(1/2)}*a*\text{cos}(1/2*d*x+1/2*c)+2^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}+2*a))*\text{cos}(1/2*d*x+1/2*c)^6*a-104*2^{(1/2)}*\ln(2*(2*a^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}+2*a)/\text{cos}(1/2*d*x+1/2*c))*a*\text{cos}(1/2*d*x+1/2*c)^4+28*2^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}*\text{cos}(1/2*d*x+1/2*c)^4+76*\ln(-4/(2*\text{cos}(1/2*d*x+1/2*c)-2^{(1/2)}))*(2^{(1/2)}*a*\text{cos}(1/2*d*x+1/2*c)-2^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}-2*a))*\text{cos}(1/2*d*x+1/2*c)^4*a+76*\ln(4/(2*\text{cos}(1/2*d*x+1/2*c)+2^{(1/2)}))*(2^{(1/2)}*a*\text{cos}(1/2*d*x+1/2*c)+2^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}+2*a))*\text{cos}(1/2*d*x+1/2*c)^4*a+26*2^{(1/2)}*\ln(2*(2*a^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}+2*a)/\text{cos}(1/2*d*x+1/2*c))*a*\text{cos}(1/2*d*x+1/2*c)^2-22*\text{cos}(1/2*d*x+1/2*c)^2*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}*2^{(1/2)}*a^{(1/2)}-19*\ln(-4/(2*\text{cos}(1/2*d*x+1/2*c)-2^{(1/2)}))*(2^{(1/2)}*a*\text{cos}(1/2*d*x+1/2*c)-2^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}-2*a))*\text{cos}(1/2*d*x+1/2*c)^2*a-19*\ln(4/(2*\text{cos}(1/2*d*x+1/2*c)+2^{(1/2)}))*(2^{(1/2)}*a*\text{cos}(1/2*d*x+1/2*c)+2^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}+2*a))*\text{cos}(1/2*d*x+1/2*c)^2*a+2*2^{(1/2)}*(a*\text{sin}(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)})/a^{(5/2)}/\text{cos}(1/2*d*x+1/2*c)/(2*\text{cos}(1/2*d*x+1/2*c)-2^{(1/2)})^2/(2*\text{cos}(1/2*d*x+1/2*c)+2^{(1/2)})^2/\text{sin}(1/2*d*x+1/2*c)/(a*\text{cos}(1/2*d*x+1/2*c)^2)^{(1/2)}/d$$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.63

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \frac{26\sqrt{2}(\cos(dx + c)^4 + 2\cos(dx + c)^3 + \cos(dx + c)^2)\sqrt{a} \log\left(-\frac{a \cos(dx + c)^2 + 2a \cos(dx + c) + a}{a \cos(dx + c)^2 + 2a \cos(dx + c) + a}\right) + 19(\cos(dx + c)^4 + 2\cos(dx + c)^3 + \cos(dx + c)^2)\sqrt{a} \log\left(\frac{a \cos(dx + c)^3 - 7a \cos(dx + c)^2 - 4\sqrt{a} \cos(dx + c) + a}{a \cos(dx + c)^2 + 2a \cos(dx + c) + a}\right) - 4\sqrt{a} \log\left(\frac{a \cos(dx + c) + a}{a \cos(dx + c)^2 + 2a \cos(dx + c) + a}\right) + 8a \sin(dx + c)}{(a^2 d \cos(dx + c)^4 + 2a^2 d \cos(dx + c)^3 + a^2 d \cos(dx + c)^2)}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/16*(26*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 19*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt(a*cos(d*x + c) + a)*(7*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)*sin(d*x + c)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)

Sympy [F]

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\sec^3(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)

[Out] Integral(sec(c + d*x)**3/(a*(cos(c + d*x) + 1))**(3/2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Exception raised: RuntimeError}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Memory limit reached. Please jump to an outer pointer, quit program and enlarge thememory limits before executing the program again.

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Exception raised: TypeError}$$

[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
 UT:sage2:=int(sage0,sageVARx);OUTPUT:sym2poly/r2sym(const gen & e,const in
 dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{1}{\cos(c + dx)^3 (a + a \cos(c + dx))^{3/2}} dx$$

[In] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)),x)

[Out] int(1/(cos(c + d*x)^3*(a + a*cos(c + d*x))^(3/2)), x)

$$3.139 \quad \int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	2235
Rubi [A] (verified)	2235
Mathematica [A] (verified)	2238
Maple [A] (verified)	2238
Fricas [A] (verification not implemented)	2239
Sympy [F(-1)]	2239
Maxima [F(-1)]	2239
Giac [A] (verification not implemented)	2240
Mupad [F(-1)]	2240

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = \frac{163 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^3(c+dx) \sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} - \frac{17 \cos^2(c+dx) \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}} - \frac{197 \sin(c+dx)}{24a^2d\sqrt{a+a \cos(c+dx)}} + \frac{95\sqrt{a+a \cos(c+dx)} \sin(c+dx)}{48a^3d}$$

[Out] $-1/4*\cos(d*x+c)^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(5/2)-17/16*\cos(d*x+c)^2*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^(3/2)+163/32*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*\cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-197/24*\sin(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^(1/2)+95/48*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/a^3/d$

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2844, 3056, 3047, 3102, 2830, 2728, 212}

$$\int \frac{\cos^4(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = \frac{163 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{95 \sin(c+dx) \sqrt{a \cos(c+dx)+a}}{48a^3d} - \frac{197 \sin(c+dx)}{24a^2d\sqrt{a \cos(c+dx)+a}} - \frac{\sin(c+dx) \cos^3(c+dx)}{4d(a \cos(c+dx)+a)^{5/2}} - \frac{17 \sin(c+dx) \cos^2(c+dx)}{16ad(a \cos(c+dx)+a)^{3/2}}$$

[In] Int[Cos[c + d*x]^4/(a + a*cos[c + d*x])^(5/2),x]

[Out] (163*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*cos[c + d*x]])]/(16*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(4*d*(a + a*cos[c + d*x])^(5/2)) - (17*cos[c + d*x]^2*Sin[c + d*x])/(16*a*d*(a + a*cos[c + d*x])^(3/2)) - (197*Sin[c + d*x])/(24*a^2*d*Sqrt[a + a*cos[c + d*x]]) + (95*Sqrt[a + a*cos[c + d*x]]*Sin[c + d*x])/(48*a^3*d)

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2830

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2844

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3047

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3056


```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])

```

Rule 3102

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)^2], x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos^2(c+dx)(3a-\frac{11}{2}a\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)(17a^2-\frac{95}{4}a^2\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{17a^2\cos(c+dx)-\frac{95}{4}a^2\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{95\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{48a^3d} - \frac{\int \frac{-\frac{95a^3}{8}+\frac{197}{4}a^3\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{12a^5} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad - \frac{197\sin(c+dx)}{24a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{95\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{48a^3d} + \frac{163\int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx}{32a^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{197\sin(c+dx)}{24a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{95\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{48a^3d} - \frac{163\text{Subst}\left(\int\frac{1}{2a-x^2}dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{16a^2d} \\
&= \frac{163\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{197\sin(c+dx)}{24a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{95\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{48a^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.67

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{\left(-978\sqrt{2}\text{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right) + \sqrt{1-\cos(c+dx)}(379+479\cos(c+dx)+80\cos(2(c+dx)))\right)\sin(c+dx)}{48d\sqrt{1-\cos(c+dx)}(a(1+\cos(c+dx)))^{5/2}}$$

[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(5/2), x]

[Out] -1/48*((-978*sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4 + Sqrt[1 - Cos[c + d*x]]*(379 + 479*Cos[c + d*x] + 80*Cos[2*(c + d*x)] - 8*Cos[3*(c + d*x)])*Sin[c + d*x])/(d*sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.32

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(128\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+489\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4a}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-512\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+96\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}{48d\sqrt{1-\cos(c+dx)}(a(1+\cos(c+dx)))^{5/2}}$

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^(5/2), x, method=_RETURNVERBOSE)

[Out] 1/96/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(128*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+489*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)

$*x+1/2*c)^4-512*2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}*\cos(1/2*d*x+1/2*c)^4-87*\cos(1/2*d*x+1/2*c)^2*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*2^{(1/2)}*a^{(1/2)}+6*2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)})/a^{(7/2)}/\sin(1/2*d*x+1/2*c)/(a*\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/d$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.14

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{489\sqrt{2}(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1)\sqrt{a}\log\left(-\frac{a\cos(dx+c)}{a+\cos(dx+c)}\right) + \sqrt{a}\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1\right)}{(a+a\cos(c+dx))^{5/2}}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 - 160*cos(d*x + c)^2 - 503*cos(d*x + c) - 299)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

Giac [A] (verification not implemented)

none

Time = 3.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \frac{3\sqrt{2}\left(29\sqrt{a}\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 27\sqrt{a}\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)^2 a^3 \operatorname{sgn}\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)} - \frac{128\sqrt{2}\left(a^{13/2}\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 3a^{13/2}\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{a^9 \operatorname{sgn}\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)} \frac{1}{96d}$$

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*(3*sqrt(2)*(29*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 27*sqrt(a)*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(1/2*d*x + 1/2*c))) - 128*sqrt(2)*(a^(13/2)*sin(1/2*d*x + 1/2*c)^3 + 3*a^(13/2)*sin(1/2*d*x + 1/2*c))/(a^9*sgn(cos(1/2*d*x + 1/2*c)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^4}{(a + a \cos(c + dx))^{5/2}} dx$$

```
[In] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*cos(c + d*x))^(5/2), x)
```

$$3.140 \quad \int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	2241
Rubi [A] (verified)	2241
Mathematica [A] (verified)	2243
Maple [A] (verified)	2244
Fricas [A] (verification not implemented)	2244
Sympy [F(-1)]	2245
Maxima [F(-1)]	2245
Giac [A] (verification not implemented)	2245
Mupad [F(-1)]	2246

Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = -\frac{75 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^2(c+dx) \sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} + \frac{13 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}} + \frac{9 \sin(c+dx)}{4a^2d\sqrt{a+a \cos(c+dx)}}$$

[Out] $-1/4*\cos(d*x+c)^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(5/2)}+13/16*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(3/2)}-75/32*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)})/(a+a*\cos(d*x+c))^{(1/2)}/a^{(5/2)}/d*2^{(1/2)}+9/4*\sin(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2844, 3047, 3098, 2830, 2728, 212}

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = -\frac{75 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{9 \sin(c+dx)}{4a^2d\sqrt{a \cos(c+dx)+a}} - \frac{\sin(c+dx) \cos^2(c+dx)}{4d(a \cos(c+dx)+a)^{5/2}} + \frac{13 \sin(c+dx)}{16ad(a \cos(c+dx)+a)^{3/2}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^3/(a+a*\operatorname{Cos}[c+d*x])^{(5/2)},x]$

[Out] $(-75*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])])/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*d) - (\operatorname{Cos}[c+d*x]^2*\operatorname{Sin}[c+d*x])/(4*d*(a+a*\operatorname{Cos}[c+d*x]))$

$x)^{(5/2)} + (13*\sin[c + d*x])/(16*a*d*(a + a*\cos[c + d*x])^{(3/2)}) + (9*\sin[c + d*x])/(4*a^2*d*\sqrt{a + a*\cos[c + d*x]})$

Rule 212

$\text{Int}[(a + (b \cdot x^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2])) \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot (x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 2728

$\text{Int}[1/\sqrt{(a + (b \cdot \sin[c + d \cdot x])^2)}, x_Symbol] \rightarrow \text{Dist}[-2/d, \text{Subst}[\text{Int}[1/(2a - x^2), x], x, b \cdot (\cos[c + d \cdot x]/\sqrt{a + b \cdot \sin[c + d \cdot x]})], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2830

$\text{Int}[(a + (b \cdot \sin[e + f \cdot x])^m) \cdot ((c + d \cdot \sin[e + f \cdot x]) + (f \cdot x))], x_Symbol] \rightarrow \text{Simp}[-d \cdot \cos[e + f \cdot x] \cdot (a + b \cdot \sin[e + f \cdot x])^m / (f \cdot (m + 1)), x] + \text{Dist}[(a \cdot d \cdot m + b \cdot c \cdot (m + 1)) / (b \cdot (m + 1)), \text{Int}[(a + b \cdot \sin[e + f \cdot x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2^{(-1)}]$

Rule 2844

$\text{Int}[(a + (b \cdot \sin[e + f \cdot x])^m) \cdot ((c + d \cdot \sin[e + f \cdot x]) + (f \cdot x))^n], x_Symbol] \rightarrow \text{Simp}[(b \cdot c - a \cdot d) \cdot \cos[e + f \cdot x] \cdot (a + b \cdot \sin[e + f \cdot x])^m \cdot (c + d \cdot \sin[e + f \cdot x])^{n-1} / (a \cdot f \cdot (2 \cdot m + 1)), x] + \text{Dist}[1 / (a \cdot b \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \sin[e + f \cdot x])^{m+1} \cdot (c + d \cdot \sin[e + f \cdot x])^{n-2} \cdot \text{Simp}[b \cdot (c^2 \cdot (m + 1) + d^2 \cdot (n - 1)) + a \cdot c \cdot d \cdot (m - n + 1) + d \cdot (a \cdot d \cdot (m - n + 1) + b \cdot c \cdot (m + n)) \cdot \sin[e + f \cdot x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ (\text{IntegersQ}[2 \cdot m, 2 \cdot n] \ || \ (\text{IntegerQ}[m] \ \&\& \ \text{EqQ}[c, 0]))$

Rule 3047

$\text{Int}[(a + (b \cdot \sin[e + f \cdot x])^m) \cdot ((A + B \cdot \sin[e + f \cdot x]) + (f \cdot x)) \cdot ((c + d \cdot \sin[e + f \cdot x]) + (f \cdot x))], x_Symbol] \rightarrow \text{Int}[(a + b \cdot \sin[e + f \cdot x])^m \cdot (A \cdot c + (B \cdot c + A \cdot d) \cdot \sin[e + f \cdot x] + B \cdot d \cdot \sin[e + f \cdot x]^2), x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0]$

Rule 3098

$\text{Int}[(a + (b \cdot \sin[e + f \cdot x])^m) \cdot ((A + B \cdot \sin[e + f \cdot x]) + (f \cdot x)) + (C \cdot \sin[e + f \cdot x])^2], x_Symbol] \rightarrow \text{Simp}[(A \cdot b - a \cdot B + b \cdot C) \cdot \cos[e + f \cdot x] \cdot (a + b \cdot \sin[e + f \cdot x])^m / (a \cdot f \cdot (2 \cdot m + 1)), x] + \text{Dist}[1 / (a^2 \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \sin[e + f \cdot x])^{m+1} \cdot \text{Simp}[a \cdot A \cdot (m + 1) + m \cdot (b \cdot C - a \cdot B + b \cdot C) \cdot \sin[e + f \cdot x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C, m\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0]$

B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos(c+dx)(2a-\frac{9}{2}a\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
 &= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{2a\cos(c+dx)-\frac{9}{2}a\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
 &= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{-\frac{39a^2}{4}+9a^2\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
 &= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
 &\quad + \frac{9\sin(c+dx)}{4a^2d\sqrt{a+a\cos(c+dx)}} - \frac{75\int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
 &= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
 &\quad + \frac{9\sin(c+dx)}{4a^2d\sqrt{a+a\cos(c+dx)}} + \frac{75\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{16a^2d} \\
 &= -\frac{75\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} \\
 &\quad + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{4a^2d\sqrt{a+a\cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{\left(-150\sqrt{2}\text{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right) + \sqrt{1-\cos(c+dx)}\right)}{16d\sqrt{1-\cos(c+dx)}(a(1+\cos(c+dx)))}$$

[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(5/2), x]

[Out] ((-150*Sqrt[2]*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4 + Sqrt[1 - Cos[c + d*x]]*(49 + 85*Cos[c + d*x] + 32*Cos[c + d*x]^2))*Sin[c + d*x])/(16*d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-75\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4a}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+64\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a}\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+21\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}d$

```
[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-75*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^4+64*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*a^(1/2)-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.37

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{75\sqrt{2}(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1)\sqrt{a}\log\left(-\frac{a\cos(dx+c)}{a^3d\cos(dx+c)}\right)}{64(a^3d\cos(dx+c))^2}$$

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(75*sqrt(2)*(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)*sqrt(a)*log(-(a*cos(d*x+c)^2+2*sqrt(2)*sqrt(a*cos(d*x+c)+a)*sqrt(a)*sin(d*x+c)-2*a*cos(d*x+c)-3*a)/(cos(d*x+c)^2+2*cos(d*x+c)+1))+4*sqrt(a*cos(d*x+c)+a)*(32*cos(d*x+c)^2+85*cos(d*x+c)+49)*sin(d*x+c))/(a^3*d*cos(d*x+c)^3+3*a^3*d*cos(d*x+c)^2+3*a^3*d*cos(d*x+c)+a^3*d)
```


Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

Giac [A] (verification not implemented)

none

Time = 2.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx =$$

$$\frac{\frac{75 \sqrt{2} \log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^{\frac{5}{2}} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{75 \sqrt{2} \log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^{\frac{5}{2}} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{128 \sqrt{2} \sin(\frac{1}{2} dx + \frac{1}{2} c)}{a^{\frac{5}{2}} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} + \frac{2 \sqrt{2} (21 \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 19 \sin(\frac{1}{2} dx + \frac{1}{2} c))}{(\sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^2 a^{\frac{5}{2}} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))}}{64 d}$$

[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

```
[Out] -1/64*(75*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(1/2*d*x +
1/2*c))) - 75*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(1/2*d
*x + 1/2*c))) - 128*sqrt(2)*sin(1/2*d*x + 1/2*c)/(a^(5/2)*sgn(cos(1/2*d*x +
1/2*c))) + 2*sqrt(2)*(21*sin(1/2*d*x + 1/2*c)^3 - 19*sin(1/2*d*x + 1/2*c))
/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^(5/2)*sgn(cos(1/2*d*x + 1/2*c)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^3}{(a + a \cos(c + dx))^{5/2}} dx$$

```
[In] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(5/2), x)
```

```
[Out] int(cos(c + d*x)^3/(a + a*cos(c + d*x))^(5/2), x)
```

$$3.141 \quad \int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	2247
Rubi [A] (verified)	2247
Mathematica [A] (verified)	2249
Maple [A] (verified)	2249
Fricas [B] (verification not implemented)	2249
Sympy [F(-1)]	2250
Maxima [F(-1)]	2250
Giac [A] (verification not implemented)	2250
Mupad [F(-1)]	2251

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = \frac{19 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} - \frac{13 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}}$$

[Out] $1/4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(5/2)}-13/16*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(3/2)}+19/32*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)}})/a^{(5/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2837, 2829, 2728, 212}

$$\int \frac{\cos^2(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = \frac{19 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{13 \sin(c+dx)}{16ad(a \cos(c+dx)+a)^{3/2}} + \frac{\sin(c+dx)}{4d(a \cos(c+dx)+a)^{5/2}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^2/(a+a*\operatorname{Cos}[c+d*x])^{(5/2)},x]$

[Out] $(19*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])])/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*d) + \operatorname{Sin}[c+d*x]/(4*d*(a+a*\operatorname{Cos}[c+d*x])^{(5/2)}) - (13*\operatorname{Sin}[c+d*x])/(16*a*d*(a+a*\operatorname{Cos}[c+d*x])^{(3/2)})$

Rule 212

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2829

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2837

Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] := Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} + \frac{\int \frac{-\frac{5a}{2} + 4a \cos(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx}{4a^2} \\
 &= \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} - \frac{13 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{3/2}} + \frac{19 \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx}{32a^2} \\
 &= \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} - \frac{13 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{3/2}} \\
 &\quad - \frac{19 \text{Subst}\left(\int \frac{1}{2a - x^2} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{16a^2d} \\
 &= \frac{19 \arctanh\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2}\sqrt{a + a \cos(c + dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} - \frac{13 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \frac{\left(76\sqrt{2}\operatorname{arctanh}\left(\sqrt{\sin^2\left(\frac{1}{2}(c + dx)\right)}\right)\cos^4\left(\frac{1}{2}(c + dx)\right) - 2\sqrt{1 - \cos(c + dx)}\right)}{32d\sqrt{1 - \cos(c + dx)}(a(1 + \cos(c + dx)))^{5/2}}$$

[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2), x]

```
[Out] ((76*sqrt(2)*ArcTanh[Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4 - 2*sqrt[1 - Cos[c + d*x]])*(9 + 13*Cos[c + d*x])*Sin[c + d*x]/(32*d*sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(19\sqrt{2} \ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4a}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right) a\left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 13\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{2}\sqrt{a} + 2\sqrt{2}}{32 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)^3 a^{\frac{7}{2}} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} d$

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^(5/2), x, method=_RETURNVERBOSE)

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(19*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^4-13*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*a^(1/2)+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(88) = 176.

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.76

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \frac{19\sqrt{2}(\cos(dx + c)^3 + 3\cos(dx + c)^2 + 3\cos(dx + c) + 1)\sqrt{a} \log\left(-\frac{a\cos(dx + c)}{a + \cos(dx + c)}\right)}{64(a^3d\cos(dx + c))^3}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2), x, algorithm="fricas")

```
[Out] 1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
```

)) - 4*sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c) + 9)*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

Giac [A] (verification not implemented)

none

Time = 1.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \frac{\frac{19\sqrt{2}\log(\sin(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^{5/2}\operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} - \frac{19\sqrt{2}\log(-\sin(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{a^{5/2}\operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))} + \frac{2\sqrt{2}(13\sqrt{a}\sin(\frac{1}{2}dx + \frac{1}{2}c)^3 - 11\sqrt{a}\sin(\frac{1}{2}dx + \frac{1}{2}c))}{(\sin(\frac{1}{2}dx + \frac{1}{2}c)^2 - 1)^2 a^3 \operatorname{sgn}(\cos(\frac{1}{2}dx + \frac{1}{2}c))}}{64d}$$

[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] 1/64*(19*sqrt(2)*log(sin(1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(1/2*d*x + 1/2*c))) - 19*sqrt(2)*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(1/2*d*x + 1/2*c))) + 2*sqrt(2)*(13*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 11*sqrt(a)*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(1/2*d*x + 1/2*c))))/d

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^2}{(a + a \cos(c + dx))^{5/2}} dx$$

```
[In] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(5/2), x)
```

```
[Out] int(cos(c + d*x)^2/(a + a*cos(c + d*x))^(5/2), x)
```

$$3.142 \quad \int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	2252
Rubi [A] (verified)	2252
Mathematica [A] (verified)	2254
Maple [A] (verified)	2254
Fricas [B] (verification not implemented)	2254
Sympy [F]	2255
Maxima [F(-1)]	2255
Giac [A] (verification not implemented)	2255
Mupad [F(-1)]	2255

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = \frac{5 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} + \frac{5 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}}$$

[Out] $-1/4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(5/2)}+5/16*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(3/2)}+5/32*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2829, 2729, 2728, 212}

$$\int \frac{\cos(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = \frac{5 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{5 \sin(c+dx)}{16ad(a \cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx)}{4d(a \cos(c+dx)+a)^{5/2}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]/(a+a*\operatorname{Cos}[c+d*x])^{(5/2)},x]$

[Out] $(5*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])])/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*d) - \operatorname{Sin}[c+d*x]/(4*d*(a+a*\operatorname{Cos}[c+d*x])^{(5/2)}) + (5*\operatorname{Sin}[c+d*x])/(16*a*d*(a+a*\operatorname{Cos}[c+d*x])^{(3/2)})$

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2829

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5 \int \frac{1}{(a+a\cos(c+dx))^{3/2}} dx}{8a} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5 \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{5 \int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5 \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
 &\quad - \frac{5 \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{16a^2d} \\
 &= \frac{5 \arctanh\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5 \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{40\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)+2\sin(c+dx)+5\sin(2(c+dx))}{32d(a(1+\cos(c+dx)))^{5/2}}$$

[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^(5/2), x]

[Out] (40*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 2*Sin[c + d*x] + 5*Sin[2*(c + d*x)])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(5\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4a}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{2}\sqrt{a}-2\sqrt{2}\sqrt{a}\right)}{32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3a^{\frac{7}{2}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}d$

[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^(5/2), x, method=_RETURNVERBOSE)

[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^4+5*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*a^(1/2)-2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/a^(7/2)/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(88) = 176.

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.76

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{5\sqrt{2}(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1)\sqrt{a}\log\left(-\frac{a\cos(dx+c)}{a^2}\right)}{64(a^3d\cos(dx+c))^3+}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2), x, algorithm="fricas")

[Out] 1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d*x + c) + a)*(5*cos(d*x + c) + 1)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F]

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)}{(a(\cos(c + dx) + 1))^{5/2}} dx$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)

[Out] Integral(cos(c + d*x)/(a*(cos(c + d*x) + 1))**(5/2), x)

Maxima [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

Giac [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = -\frac{\sqrt{2} \left(5\sqrt{a} \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 3\sqrt{a} \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)}{32 \left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1 \right)^2 a^3 \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}$$

[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] -1/32*sqrt(2)*(5*sqrt(a)*sin(1/2*d*x + 1/2*c)^3 - 3*sqrt(a)*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^3*d*sgn(cos(1/2*d*x + 1/2*c)))

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx$$

[In] int(cos(c + d*x)/(a + a*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)/(a + a*cos(c + d*x))^(5/2), x)

3.143 $\int \frac{1}{(a+a \cos(c+dx))^{5/2}} dx$

Optimal result	2256
Rubi [A] (verified)	2256
Mathematica [A] (verified)	2257
Maple [A] (verified)	2258
Fricas [B] (verification not implemented)	2258
Sympy [F]	2258
Maxima [B] (verification not implemented)	2259
Giac [A] (verification not implemented)	2310
Mupad [F(-1)]	2310

Optimal result

Integrand size = 14, antiderivative size = 107

$$\int \frac{1}{(a+a \cos(c+dx))^{5/2}} dx = \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} + \frac{3 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}}$$

[Out] 1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)+3/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+3/32*arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2729, 2728, 212}

$$\int \frac{1}{(a+a \cos(c+dx))^{5/2}} dx = \frac{3 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{3 \sin(c+dx)}{16ad(a \cos(c+dx)+a)^{3/2}} + \frac{\sin(c+dx)}{4d(a \cos(c+dx)+a)^{5/2}}$$

[In] Int[(a + a*Cos[c + d*x])^(-5/2),x]

[Out] (3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (3*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2729

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[c + d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} + \frac{3 \int \frac{1}{(a + a \cos(c + dx))^{3/2}} dx}{8a} \\
 &= \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} + \frac{3 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{3/2}} + \frac{3 \int \frac{1}{\sqrt{a + a \cos(c + dx)}} dx}{32a^2} \\
 &= \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} + \frac{3 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{3/2}} \\
 &\quad - \frac{3 \text{Subst}\left(\int \frac{1}{2a - x^2} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{16a^2d} \\
 &= \frac{3 \arctanh\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2}\sqrt{a + a \cos(c + dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} + \frac{3 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2}} dx = \frac{24 \arctanh\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) \cos^5\left(\frac{1}{2}(c + dx)\right) + 14 \sin(c + dx) + 3 \sin(2(c + dx))}{32d(a(1 + \cos(c + dx)))^{5/2}}$$

[In] Integrate[(a + a*Cos[c + d*x])^(-5/2), x]

[Out] (24*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 14*Sin[c + d*x] + 3*Sin[2*(c + d*x)])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

method	result
default	$\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(3\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4a}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{2}\sqrt{a}+2\sqrt{2}\sqrt{a}\right)}{32a^{\frac{7}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}d$

[In] `int(1/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{32}a^{-7/2}/\cos(1/2*d*x+1/2*c)^3*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(3*2^{(1/2)}*\ln(2*(2*a^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+2*a)/\cos(1/2*d*x+1/2*c))*a*\cos(1/2*d*x+1/2*c)^4+3*\cos(1/2*d*x+1/2*c)^2*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*2^{(1/2)}*a^{(1/2)}+2*2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)})/\sin(1/2*d*x+1/2*c)/(a*\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/d$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(88) = 176.

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2}} dx = \frac{3\sqrt{2}(\cos(dx + c)^3 + 3\cos(dx + c)^2 + 3\cos(dx + c) + 1)\sqrt{a} \log\left(-\frac{a \cos(dx + c)}{a + a \cos(c + dx)}\right)}{64(a^3 d \cos(dx + c)^3 + \dots)}$$

[In] `integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] $\frac{1}{64}*(3*\sqrt{2}*(\cos(d*x + c)^3 + 3*\cos(d*x + c)^2 + 3*\cos(d*x + c) + 1)*\sqrt{a}*\log\left(-\frac{a*\cos(d*x + c)}{a + a*\cos(c + dx)}\right) + 4*\sqrt{a*\cos(d*x + c) + a}*(3*\cos(d*x + c) + 7)*\sin(d*x + c))/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d)$

Sympy [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{1}{(a \cos(c + dx) + a)^{5/2}} dx$$

[In] `integrate(1/(a+a*cos(d*x+c))**(5/2),x)`

[Out] `Integral((a*cos(c + d*x) + a)**(-5/2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84332 vs. 2(88) = 176.

Time = 14.94 (sec) , antiderivative size = 84332, normalized size of antiderivative = 788.15

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2}} dx = \text{Too large to display}$$

[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/32*(512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(5*d*x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 - 512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(5*d*x + 5*c)^2 + 2560*cos(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 1024*(20*cos(2*d*x + 2*c) + 10*cos(d*x + c) + 1)*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 10240*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*sin(4*d*x + 4*c)^2*sin(5/2

$$\begin{aligned}
& *d*x + 5/2*c) + 10240*\sin(3*d*x + 3*c)^2*\sin(5/2*d*x + 5/2*c) + 2560*(5*(2* \\
& \sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c) \\
&)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d \\
& *x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*s \\
& \sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + \\
& 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(\\
& 8/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*si \\
& \sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)* \\
& \sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x \\
& + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin \\
& (5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4 \\
& *c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(6/ \\
& 5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*\sin(\\
& 2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\si \\
& \sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + \\
& 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/ \\
& /2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c \\
&)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(4/5* \\
& \arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 2560*(5*(2*\sin(2*d \\
& *x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5 \\
& *d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/ \\
& 2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2* \\
& d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*s \\
& \sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(2/5*arc \\
& \tan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 512*(5*\cos(4*d*x + 4* \\
& c)^2*\sin(5/2*d*x + 5/2*c) + 4*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*co \\
& s(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c) + 20*\cos(3*d*x + 3*c)^2*\sin(5/2*d*x + 5 \\
& /2*c) + 5*\sin(4*d*x + 4*c)^2*\sin(5/2*d*x + 5/2*c) + 20*\sin(3*d*x + 3*c)^2*s \\
& \sin(5/2*d*x + 5/2*c) + 2*((10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2 \\
& *d*x + 5/2*c) + 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(4*d*x + 4*c) \\
& + 2*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 2*(5*(2*\sin(\\
& 2*d*x + 2*c) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + 10*\sin(3*d*x + 3*c)*\sin \\
& (5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(4*d*x + 4*c) + 4*(5*(2*\sin(2* \\
& d*x + 2*c) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin \\
& (3*d*x + 3*c) + (4*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 20*\cos(2*d*x + 2 \\
& *c)^2 + 5*\cos(d*x + c)^2 + 20*\sin(2*d*x + 2*c)^2 + 20*\sin(2*d*x + 2*c)*\sin(\\
& d*x + c) + 5*\sin(d*x + c)^2 + 2*\cos(d*x + c))*\sin(5/2*d*x + 5/2*c))*\cos(5*d \\
& *x + 5*c) + 512*((20*\cos(2*d*x + 2*c) + 10*\cos(d*x + c) + 1)*\sin(5/2*d*x + \\
& 5/2*c) + 20*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(4*d*x + 4*c) + 512*(\\
& 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) - 12*(10*(\sin(4*d*x \\
& + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x \\
& + 5*c)^2 + \sin(5*d*x + 5*c)^3 + (2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3* \\
& c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x \\
& + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) \\
& + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5
\end{aligned}$$

$$\begin{aligned}
& * \cos(dx + c) + 1) \cos(3dx + 3c) + 100 \cos(3dx + 3c)^2 + 20(5 \cos(dx + c) + 1) \cos(2dx + 2c) + 100 \cos(2dx + 2c)^2 + 25 \cos(dx + c)^2 + \\
& 50(2 \sin(3dx + 3c) + 2 \sin(2dx + 2c) + \sin(dx + c)) \sin(4dx + 4c) + 25 \sin(4dx + 4c)^2 + 100(2 \sin(2dx + 2c) + \sin(dx + c)) \sin(3dx + 3c) + 100 \sin(3dx + 3c)^2 + 100 \sin(2dx + 2c)^2 + 100 \sin(2dx + 2c) \sin(dx + c) + 25 \sin(dx + c)^2 + 10 \cos(dx + c) + 1) \sin(5dx + 5c) + 5(2(5 \cos(4dx + 4c) + 10 \cos(3dx + 3c) + 10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(5dx + 5c) + \cos(5dx + 5c)^2 + 10(10 \cos(3dx + 3c) + 10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(4dx + 4c) + 25 \cos(4dx + 4c)^2 + 20(10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(3dx + 3c) + 100 \cos(3dx + 3c)^2 + 20(5 \cos(dx + c) + 1) \cos(2dx + 2c) + 100 \cos(2dx + 2c)^2 + 25 \cos(dx + c)^2 + 10(\sin(4dx + 4c) + 2 \sin(3dx + 3c) + 2 \sin(2dx + 2c) + \sin(dx + c)) \sin(5dx + 5c) + \sin(5dx + 5c)^2 + 50(2 \sin(3dx + 3c) + 2 \sin(2dx + 2c) + \sin(dx + c)) \sin(4dx + 4c) + 25 \sin(4dx + 4c)^2 + 100(2 \sin(2dx + 2c) + \sin(dx + c)) \sin(3dx + 3c) + 100 \sin(3dx + 3c)^2 + 100 \sin(2dx + 2c)^2 + 100 \sin(2dx + 2c) \sin(dx + c) + 25 \sin(dx + c)^2 + 10 \cos(dx + c) + 1) \sin(8/5 \arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) + 10(2(5 \cos(4dx + 4c) + 10 \cos(3dx + 3c) + 10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(5dx + 5c) + \cos(5dx + 5c)^2 + 10(10 \cos(3dx + 3c) + 10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(4dx + 4c) + 25 \cos(4dx + 4c)^2 + 20(10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(3dx + 3c) + 100 \cos(3dx + 3c)^2 + 20(5 \cos(dx + c) + 1) \cos(2dx + 2c) + 100 \cos(2dx + 2c)^2 + 25 \cos(dx + c)^2 + 10(\sin(4dx + 4c) + 2 \sin(3dx + 3c) + 2 \sin(2dx + 2c) + \sin(dx + c)) \sin(5dx + 5c) + \sin(5dx + 5c)^2 + 50(2 \sin(3dx + 3c) + 2 \sin(2dx + 2c) + \sin(dx + c)) \sin(4dx + 4c) + 25 \sin(4dx + 4c)^2 + 100(2 \sin(2dx + 2c) + \sin(dx + c)) \sin(3dx + 3c) + 100 \sin(3dx + 3c)^2 + 100 \sin(2dx + 2c)^2 + 100 \sin(2dx + 2c) \sin(dx + c) + 25 \sin(dx + c)^2 + 10 \cos(dx + c) + 1) \sin(6/5 \arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) + 10(2(5 \cos(4dx + 4c) + 10 \cos(3dx + 3c) + 10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(5dx + 5c) + \cos(5dx + 5c)^2 + 10(10 \cos(3dx + 3c) + 10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(4dx + 4c) + 25 \cos(4dx + 4c)^2 + 20(10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(3dx + 3c) + 100 \cos(3dx + 3c)^2 + 20(5 \cos(dx + c) + 1) \cos(2dx + 2c) + 100 \cos(2dx + 2c)^2 + 25 \cos(dx + c)^2 + 10(\sin(4dx + 4c) + 2 \sin(3dx + 3c) + 2 \sin(2dx + 2c) + \sin(dx + c)) \sin(5dx + 5c) + \sin(5dx + 5c)^2 + 50(2 \sin(3dx + 3c) + 2 \sin(2dx + 2c) + \sin(dx + c)) \sin(4dx + 4c) + 25 \sin(4dx + 4c)^2 + 100(2 \sin(2dx + 2c) + \sin(dx + c)) \sin(3dx + 3c) + 100 \sin(3dx + 3c)^2 + 100 \sin(2dx + 2c)^2 + 100 \sin(2dx + 2c) \sin(dx + c) + 25 \sin(dx + c)^2 + 10 \cos(dx + c) + 1) \sin(4/5 \arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) + 5(2(5 \cos(4dx + 4c) + 10 \cos(3dx + 3c) + 10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(5dx + 5c) + \cos(5dx + 5c)^2 + 10(10 \cos(3dx + 3c) + 10 \cos(2dx + 2c) + 5 \cos(dx + c) + 1) \cos(4dx + 4c) + 25 \cos(4dx + 4c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) * \cos(9/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 4*(128*\cos(5*d*x + 5*c)^2*\sin(5/2*d*x + 5/2*c) - 3200*\cos(4*d*x + 4*c)^2*\sin(5/2*d*x + 5/2*c) - 12800*(2*\cos(2*d*x + 2*c) + \cos(d*x + c))*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c) - 12800*\cos(3*d*x + 3*c)^2*\sin(5/2*d*x + 5/2*c) - 128*\sin(5*d*x + 5*c)^2*\sin(5/2*d*x + 5/2*c) - 3200*\sin(4*d*x + 4*c)^2*\sin(5/2*d*x + 5/2*c) - 12800*\sin(3*d*x + 3*c)^2*\sin(5/2*d*x + 5/2*c) - 256*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - \sin(5/2*d*x + 5/2*c))*\cos(5*d*x + 5*c) - 6400*((2*\cos(2*d*x + 2*c) + \cos(d*x + c))*\sin(5/2*d*x + 5/2*c) + 2*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(4*d*x + 4*c) - 1280*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) - 2560*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 2560*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 1280*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 256*(\cos(5*d*x + 5*c)*\cos(5/2*d*x + 5/2*c) + 5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + 5*\sin(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) + 10*\sin(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(5*d*x + 5*c) - 1280*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + 10*\sin(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(4*d*x + 4*c) - 2560*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(3*d*x + 3*c) - 128*(100*\cos(2*d*x + 2*c)^2 + 100*\cos(2*d*x + 2*c)*\cos(d*x + c) + 25*\cos(d*x + c)^2 + 100
\end{aligned}$$

$$\begin{aligned}
& * \sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 \\
& - 1)*\sin(5/2*d*x + 5/2*c) - 70*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) \\
&) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x \\
& + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + \\
& 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5* \\
& \cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x \\
& + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + \\
& 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + \\
& c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(\\
& 2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100 \\
& *(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c) \\
&)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d \\
& *x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(7/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(\\
& 5/2*d*x + 5/2*c))) + 70*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*c \\
& \cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 \\
& + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(\\
& 4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x \\
& + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + \\
& 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(\\
& 4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(\\
& 5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + \\
& 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(\\
& 2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 10 \\
& 0*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^ \\
& 2 + 10*\cos(d*x + c) + 1)*\sin(3/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x \\
& + 5/2*c))) + 15*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x \\
& + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(1 \\
& 0*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + \\
& 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1 \\
&)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2 \\
& *d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + \\
& 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + \\
& 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + s \\
& \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + \\
& 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2* \\
& d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*c \\
& \cos(d*x + c) + 1)*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c) \\
&))) * \cos(8/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 56*(10*(\\
& \sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))* \\
& \sin(5*d*x + 5*c)^2 + \sin(5*d*x + 5*c)^3 + (2*(5*\cos(4*d*x + 4*c) + 10*\cos(3 \\
& *d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \\
& \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(\\
& d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + \\
& 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20* \\
& (5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4 \\
& *d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c \\
&))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100 \\
& *\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*s \\
& \sin(5*d*x + 5*c) + 10*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(\\
& 2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + \\
& 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d \\
& *x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c \\
&) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)* \\
& \cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d \\
& *x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d \\
& *x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c \\
&) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d \\
& *x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*s \\
& \sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + \\
& 10*\cos(d*x + c) + 1)*\sin(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5 \\
& /2*c))) + 10*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + \\
& 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*c \\
& os(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c \\
&) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*c \\
& os(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d* \\
& x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c \\
&) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c \\
&) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(\\
& d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c \\
&) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x \\
& + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(\\
& d*x + c) + 1)*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) \\
& + 5*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5* \\
& \cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x \\
& + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*co \\
& s(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x \\
& + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) \\
& + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin \\
& (3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5 \\
& *d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) \\
& *\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d \\
& *x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 \\
& + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) \\
& + 1)*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))))*\cos(7/5* \\
& \arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 8*(128*\cos(5*d*x + 5 \\
& *c)^2*\sin(5/2*d*x + 5/2*c) - 3200*\cos(4*d*x + 4*c)^2*\sin(5/2*d*x + 5/2*c) - \\
& 12800*(2*\cos(2*d*x + 2*c) + \cos(d*x + c))*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5 \\
& /2*c) - 12800*\cos(3*d*x + 3*c)^2*\sin(5/2*d*x + 5/2*c) - 128*\sin(5*d*x + 5*c \\
&)^2*\sin(5/2*d*x + 5/2*c) - 3200*\sin(4*d*x + 4*c)^2*\sin(5/2*d*x + 5/2*c) - 1
\end{aligned}$$

$$\begin{aligned}
& 2800*\sin(3*d*x + 3*c)^2*\sin(5/2*d*x + 5/2*c) - 256*(5*(2*\sin(2*d*x + 2*c) + \\
& \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4* \\
& c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - \sin(5/2*d*x + 5/2*c))*\cos(5 \\
& *d*x + 5*c) - 6400*((2*\cos(2*d*x + 2*c) + \cos(d*x + c))*\sin(5/2*d*x + 5/2*c \\
&) + 2*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(4*d*x + 4*c) - 1280*(2*\sin \\
& (2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) - 2560*(5*(2*\sin(2*d*x + \\
& 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x \\
& + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c) \\
& *\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x \\
& + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5 \\
& /2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(4/5*\arctan2 \\
& (\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 1280*(5*(2*\sin(2*d*x + 2*c) \\
& + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5* \\
& c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(\\
& 3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2 \\
& *c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d* \\
& x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(2/5*\arctan2(\sin(\\
& 5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 256*(\cos(5*d*x + 5*c)*\cos(5/2*d* \\
& x + 5/2*c) + 5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + 5 \\
& *\sin(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) + 10*\sin(3*d*x + 3*c)*\sin(5/2*d*x + \\
& 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(5*d*x + 5*c) - 1280*(5*(2*\sin(2*d*x + 2* \\
& c) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + 10*\sin(3*d*x + 3*c)*\sin(5/2*d*x + \\
& 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(4*d*x + 4*c) - 2560*(5*(2*\sin(2*d*x + 2 \\
& *c) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(3*d*x \\
& + 3*c) - 128*(100*\cos(2*d*x + 2*c)^2 + 100*\cos(2*d*x + 2*c)*\cos(d*x + c) + \\
& 25*\cos(d*x + c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + \\
& c) + 25*\sin(d*x + c)^2 - 1)*\sin(5/2*d*x + 5/2*c) + 70*(2*(5*\cos(4*d*x + 4* \\
& c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5* \\
& d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + \\
& 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(1 \\
& 0*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + \\
& 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 \\
& + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2* \\
& d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\si \\
& n(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\si \\
& n(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c \\
&) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)* \\
& \sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(3/5*\arctan2(\sin \\
& (5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 15*(2*(5*\cos(4*d*x + 4*c) + 10* \\
& \cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5* \\
& c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5 \\
& *\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2* \\
& d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 \\
& + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\co \\
& s(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*
\end{aligned}$$

$c) + \sin(d*x + c)) * \sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c) * \sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1) * \sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))), \cos(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))) - 8*(128*\cos(5*d*x + 5*c)^2 * \sin(5/2*d*x + 5/2*c) - 3200*\cos(4*d*x + 4*c)^2 * \sin(5/2*d*x + 5/2*c) - 12800*(2*\cos(2*d*x + 2*c) + \cos(d*x + c)) * \cos(3*d*x + 3*c) * \sin(5/2*d*x + 5/2*c) - 12800*\cos(3*d*x + 3*c)^2 * \sin(5/2*d*x + 5/2*c) - 128*\sin(5*d*x + 5*c)^2 * \sin(5/2*d*x + 5/2*c) - 3200*\sin(4*d*x + 4*c)^2 * \sin(5/2*d*x + 5/2*c) - 12800*\sin(3*d*x + 3*c)^2 * \sin(5/2*d*x + 5/2*c) - 256*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + 5*\cos(5/2*d*x + 5/2*c) * \sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c) * \sin(3*d*x + 3*c) - \sin(5/2*d*x + 5/2*c)) * \cos(5*d*x + 5*c) - 6400*((2*\cos(2*d*x + 2*c) + \cos(d*x + c)) * \sin(5/2*d*x + 5/2*c) + 2*\cos(3*d*x + 3*c) * \sin(5/2*d*x + 5/2*c)) * \cos(4*d*x + 4*c) - 1280*(2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) - 1280*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c) * \sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c) * \sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c) * \sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c) * \sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c) * \sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c) * \sin(5/2*d*x + 5/2*c)) * \cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))) - 256*(\cos(5*d*x + 5*c) * \cos(5/2*d*x + 5/2*c) + 5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(5/2*d*x + 5/2*c) + 5*\sin(4*d*x + 4*c) * \sin(5/2*d*x + 5/2*c) + 10*\sin(3*d*x + 3*c) * \sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c) * \sin(5*d*x + 5*c) - 1280*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(5/2*d*x + 5/2*c) + 10*\sin(3*d*x + 3*c) * \sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c) * \sin(4*d*x + 4*c) - 2560*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)) * \sin(3*d*x + 3*c) - 128*(100*\cos(2*d*x + 2*c)^2 + 100*\cos(2*d*x + 2*c) * \cos(d*x + c) + 25*\cos(d*x + c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c) * \sin(d*x + c) + 25*\sin(d*x + c)^2 - 1) * \sin(5/2*d*x + 5/2*c) + 70*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1) * \cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c) * \sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1) * \sin(3/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))) + 15*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 1$

$$\begin{aligned}
& 0*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 56*(10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c)^2 + \sin(5*d*x + 5*c)^3 + (2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(5*d*x + 5*c) + 5*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))))*\cos(3/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 4*(128*\cos(5*d*x + 5*c)^2*\sin(5/2*d*x + 5/2*c) - 3200*\cos(4*d*x + 4*c)^2*\sin(5/2*d*x + 5/2*c) - 12800*(2*\cos(2*d*x + 2*c) + \cos(d*x + c))*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c) - 12800*\cos(3*d*x + 3*c)^2*\sin(5/2*d*x + 5/2*c) - 128*\sin(5*d*x + 5*c)^2*\sin(5/2*d*x + 5/2*c) - 3200*\sin(4*d*x + 4*c)^2*\sin(5/2*d*x + 5/2*c) - 12800*\sin(3*d*x + 3*c)^2*\sin(5/2*d*x + 5/2*c) - 256*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 5*\cos(5/2*d*x + 5/2*c))*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - \sin(5/2*d*x + 5/2*c))*\cos(5*d*x + 5*c) - 6400*((2*\cos(2*d*x + 2*c) + \cos(d*x + c))*\sin(5/2*d*x + 5/2*c) + 2*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(4*d*x + 4*c) - 1280*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) - 25
\end{aligned}$$

$$\begin{aligned}
& 6*(\cos(5*d*x + 5*c)*\cos(5/2*d*x + 5/2*c) + 5*(2*\sin(2*d*x + 2*c) + \sin(d*x \\
& + c))*\sin(5/2*d*x + 5/2*c) + 5*\sin(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) + 10*\sin(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(5*d*x + 5*c) \\
& - 1280*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + 10*\sin(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(4*d*x + 4*c) \\
& - 2560*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(3*d*x + 3*c) - 128*(100*\cos(2*d*x + 2*c)^2 + 100*\cos(2*d*x + 2*c)*\cos(d*x + c) + 25*\cos(d*x + c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 - 1)*\sin(5/2*d*x + 5/2*c) + 15*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))))*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 12*(10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c)^2 + \sin(5*d*x + 5*c)^3 + (2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(5*d*x + 5*c))*\cos(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 3*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 2)*\cos(5*d*x + 5*c)^3 + \cos(5*d*x + 5*c)^4 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c)^3 + \sin(5*d*x + 5*c)^4 + (10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 3)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 3)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 3)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 30*\cos(d*x + c)
\end{aligned}$$

$$\begin{aligned}
&) + 6) \cos(5d*x + 5c)^2 + 25*(2*(5\cos(4d*x + 4c) + 10\cos(3d*x + 3c) \\
& + 10\cos(2d*x + 2c) + 5\cos(d*x + c) + 1)\cos(5d*x + 5c) + \cos(5d*x + \\
& 5c)^2 + 10*(10\cos(3d*x + 3c) + 10\cos(2d*x + 2c) + 5\cos(d*x + c) + \\
& 1)\cos(4d*x + 4c) + 25\cos(4d*x + 4c)^2 + 20*(10\cos(2d*x + 2c) + 5\cos \\
& \cos(d*x + c) + 1)\cos(3d*x + 3c) + 100\cos(3d*x + 3c)^2 + 20*(5\cos(d*x \\
& + c) + 1)\cos(2d*x + 2c) + 100\cos(2d*x + 2c)^2 + 25\cos(d*x + c)^2 + 1 \\
& 0*(\sin(4d*x + 4c) + 2\sin(3d*x + 3c) + 2\sin(2d*x + 2c) + \sin(d*x + c \\
&))\sin(5d*x + 5c) + \sin(5d*x + 5c)^2 + 50*(2\sin(3d*x + 3c) + 2\sin(2 \\
& *d*x + 2c) + \sin(d*x + c))\sin(4d*x + 4c) + 25\sin(4d*x + 4c)^2 + 100* \\
& (2\sin(2d*x + 2c) + \sin(d*x + c))\sin(3d*x + 3c) + 100\sin(3d*x + 3c) \\
& ^2 + 100\sin(2d*x + 2c)^2 + 100\sin(2d*x + 2c)\sin(d*x + c) + 25\sin(d* \\
& x + c)^2 + 10\cos(d*x + c) + 1)\cos(8/5\arctan2(\sin(5/2d*x + 5/2c), \cos(5 \\
& /2d*x + 5/2c)))^2 + 100*(2*(5\cos(4d*x + 4c) + 10\cos(3d*x + 3c) + 10 \\
& *cos(2d*x + 2c) + 5\cos(d*x + c) + 1)\cos(5d*x + 5c) + \cos(5d*x + 5c) \\
& ^2 + 10*(10\cos(3d*x + 3c) + 10\cos(2d*x + 2c) + 5\cos(d*x + c) + 1)*co \\
& s(4d*x + 4c) + 25\cos(4d*x + 4c)^2 + 20*(10\cos(2d*x + 2c) + 5\cos(d* \\
& x + c) + 1)\cos(3d*x + 3c) + 100\cos(3d*x + 3c)^2 + 20*(5\cos(d*x + c) \\
& + 1)\cos(2d*x + 2c) + 100\cos(2d*x + 2c)^2 + 25\cos(d*x + c)^2 + 10*(si \\
& n(4d*x + 4c) + 2\sin(3d*x + 3c) + 2\sin(2d*x + 2c) + \sin(d*x + c))*si \\
& n(5d*x + 5c) + \sin(5d*x + 5c)^2 + 50*(2\sin(3d*x + 3c) + 2\sin(2d*x \\
& + 2c) + \sin(d*x + c))\sin(4d*x + 4c) + 25\sin(4d*x + 4c)^2 + 100*(2*si \\
& n(2d*x + 2c) + \sin(d*x + c))\sin(3d*x + 3c) + 100\sin(3d*x + 3c)^2 + \\
& 100\sin(2d*x + 2c)^2 + 100\sin(2d*x + 2c)\sin(d*x + c) + 25\sin(d*x + c \\
&)^2 + 10\cos(d*x + c) + 1)\cos(6/5\arctan2(\sin(5/2d*x + 5/2c), \cos(5/2d* \\
& x + 5/2c)))^2 + 100*(2*(5\cos(4d*x + 4c) + 10\cos(3d*x + 3c) + 10\cos(\\
& 2d*x + 2c) + 5\cos(d*x + c) + 1)\cos(5d*x + 5c) + \cos(5d*x + 5c)^2 + \\
& 10*(10\cos(3d*x + 3c) + 10\cos(2d*x + 2c) + 5\cos(d*x + c) + 1)\cos(4d \\
& *x + 4c) + 25\cos(4d*x + 4c)^2 + 20*(10\cos(2d*x + 2c) + 5\cos(d*x + c \\
&) + 1)\cos(3d*x + 3c) + 100\cos(3d*x + 3c)^2 + 20*(5\cos(d*x + c) + 1)* \\
& \cos(2d*x + 2c) + 100\cos(2d*x + 2c)^2 + 25\cos(d*x + c)^2 + 10*(\sin(4d \\
& *x + 4c) + 2\sin(3d*x + 3c) + 2\sin(2d*x + 2c) + \sin(d*x + c))\sin(5d \\
& *x + 5c) + \sin(5d*x + 5c)^2 + 50*(2\sin(3d*x + 3c) + 2\sin(2d*x + 2c \\
&) + \sin(d*x + c))\sin(4d*x + 4c) + 25\sin(4d*x + 4c)^2 + 100*(2\sin(2d \\
& *x + 2c) + \sin(d*x + c))\sin(3d*x + 3c) + 100\sin(3d*x + 3c)^2 + 100*s \\
& in(2d*x + 2c)^2 + 100\sin(2d*x + 2c)\sin(d*x + c) + 25\sin(d*x + c)^2 + \\
& 10\cos(d*x + c) + 1)\cos(4/5\arctan2(\sin(5/2d*x + 5/2c), \cos(5/2d*x + 5 \\
& /2c)))^2 + 25*(2*(5\cos(4d*x + 4c) + 10\cos(3d*x + 3c) + 10\cos(2d*x \\
& + 2c) + 5\cos(d*x + c) + 1)\cos(5d*x + 5c) + \cos(5d*x + 5c)^2 + 10*(10 \\
& *cos(3d*x + 3c) + 10\cos(2d*x + 2c) + 5\cos(d*x + c) + 1)\cos(4d*x + 4 \\
& *c) + 25\cos(4d*x + 4c)^2 + 20*(10\cos(2d*x + 2c) + 5\cos(d*x + c) + 1) \\
& *cos(3d*x + 3c) + 100\cos(3d*x + 3c)^2 + 20*(5\cos(d*x + c) + 1)\cos(2* \\
& d*x + 2c) + 100\cos(2d*x + 2c)^2 + 25\cos(d*x + c)^2 + 10*(\sin(4d*x + 4 \\
& *c) + 2\sin(3d*x + 3c) + 2\sin(2d*x + 2c) + \sin(d*x + c))\sin(5d*x + 5 \\
& *c) + \sin(5d*x + 5c)^2 + 50*(2\sin(3d*x + 3c) + 2\sin(2d*x + 2c) + si \\
& n(d*x + c))\sin(4d*x + 4c) + 25\sin(4d*x + 4c)^2 + 100*(2\sin(2d*x + 2
\end{aligned}$$

$$\begin{aligned}
& *c) + \sin(dx + c)) * \sin(3dx + 3c) + 100 * \sin(3dx + 3c)^2 + 100 * \sin(2d \\
& *x + 2c)^2 + 100 * \sin(2dx + 2c) * \sin(dx + c) + 25 * \sin(dx + c)^2 + 10 * \cos \\
& s(dx + c) + 1) * \cos(2/5 * \arctan2(\sin(5/2 * dx + 5/2 * c), \cos(5/2 * dx + 5/2 * c)) \\
&)^2 + (2 * (5 * \cos(4 * dx + 4 * c) + 10 * \cos(3 * dx + 3 * c) + 10 * \cos(2 * dx + 2 * c) + \\
& 5 * \cos(dx + c) + 2) * \cos(5 * dx + 5 * c) + 2 * \cos(5 * dx + 5 * c)^2 + 10 * (10 * \cos(3 * \\
& dx + 3 * c) + 10 * \cos(2 * dx + 2 * c) + 5 * \cos(dx + c) + 1) * \cos(4 * dx + 4 * c) + 2 \\
& 5 * \cos(4 * dx + 4 * c)^2 + 20 * (10 * \cos(2 * dx + 2 * c) + 5 * \cos(dx + c) + 1) * \cos(3 * \\
& dx + 3 * c) + 100 * \cos(3 * dx + 3 * c)^2 + 20 * (5 * \cos(dx + c) + 1) * \cos(2 * dx + 2 \\
& * c) + 100 * \cos(2 * dx + 2 * c)^2 + 25 * \cos(dx + c)^2 + 50 * (2 * \sin(3 * dx + 3 * c) + \\
& 2 * \sin(2 * dx + 2 * c) + \sin(dx + c)) * \sin(4 * dx + 4 * c) + 25 * \sin(4 * dx + 4 * c)^ \\
& 2 + 100 * (2 * \sin(2 * dx + 2 * c) + \sin(dx + c)) * \sin(3 * dx + 3 * c) + 100 * \sin(3 * dx \\
& * x + 3 * c)^2 + 100 * \sin(2 * dx + 2 * c)^2 + 100 * \sin(2 * dx + 2 * c) * \sin(dx + c) + 2 \\
& 5 * \sin(dx + c)^2 + 10 * \cos(dx + c) + 2) * \sin(5 * dx + 5 * c)^2 + 25 * (2 * (5 * \cos(4 \\
& * dx + 4 * c) + 10 * \cos(3 * dx + 3 * c) + 10 * \cos(2 * dx + 2 * c) + 5 * \cos(dx + c) + \\
& 1) * \cos(5 * dx + 5 * c) + \cos(5 * dx + 5 * c)^2 + 10 * (10 * \cos(3 * dx + 3 * c) + 10 * \cos \\
& (2 * dx + 2 * c) + 5 * \cos(dx + c) + 1) * \cos(4 * dx + 4 * c) + 25 * \cos(4 * dx + 4 * c)^ \\
& 2 + 20 * (10 * \cos(2 * dx + 2 * c) + 5 * \cos(dx + c) + 1) * \cos(3 * dx + 3 * c) + 100 * \cos \\
& s(3 * dx + 3 * c)^2 + 20 * (5 * \cos(dx + c) + 1) * \cos(2 * dx + 2 * c) + 100 * \cos(2 * dx \\
& + 2 * c)^2 + 25 * \cos(dx + c)^2 + 10 * (\sin(4 * dx + 4 * c) + 2 * \sin(3 * dx + 3 * c) + \\
& 2 * \sin(2 * dx + 2 * c) + \sin(dx + c)) * \sin(5 * dx + 5 * c) + \sin(5 * dx + 5 * c)^2 + \\
& 50 * (2 * \sin(3 * dx + 3 * c) + 2 * \sin(2 * dx + 2 * c) + \sin(dx + c)) * \sin(4 * dx + 4 * \\
& c) + 25 * \sin(4 * dx + 4 * c)^2 + 100 * (2 * \sin(2 * dx + 2 * c) + \sin(dx + c)) * \sin(3 * \\
& dx + 3 * c) + 100 * \sin(3 * dx + 3 * c)^2 + 100 * \sin(2 * dx + 2 * c)^2 + 100 * \sin(2 * dx \\
& * x + 2 * c) * \sin(dx + c) + 25 * \sin(dx + c)^2 + 10 * \cos(dx + c) + 1) * \sin(8/5 * \ar \\
& ctan2(\sin(5/2 * dx + 5/2 * c), \cos(5/2 * dx + 5/2 * c)))^2 + 100 * (2 * (5 * \cos(4 * dx \\
& + 4 * c) + 10 * \cos(3 * dx + 3 * c) + 10 * \cos(2 * dx + 2 * c) + 5 * \cos(dx + c) + 1) * \cos \\
& s(5 * dx + 5 * c) + \cos(5 * dx + 5 * c)^2 + 10 * (10 * \cos(3 * dx + 3 * c) + 10 * \cos(2 * dx \\
& * x + 2 * c) + 5 * \cos(dx + c) + 1) * \cos(4 * dx + 4 * c) + 25 * \cos(4 * dx + 4 * c)^2 + 2 \\
& 0 * (10 * \cos(2 * dx + 2 * c) + 5 * \cos(dx + c) + 1) * \cos(3 * dx + 3 * c) + 100 * \cos(3 * dx \\
& * x + 3 * c)^2 + 20 * (5 * \cos(dx + c) + 1) * \cos(2 * dx + 2 * c) + 100 * \cos(2 * dx + 2 * \\
& c)^2 + 25 * \cos(dx + c)^2 + 10 * (\sin(4 * dx + 4 * c) + 2 * \sin(3 * dx + 3 * c) + 2 * \sin \\
& (2 * dx + 2 * c) + \sin(dx + c)) * \sin(5 * dx + 5 * c) + \sin(5 * dx + 5 * c)^2 + 50 * (\\
& 2 * \sin(3 * dx + 3 * c) + 2 * \sin(2 * dx + 2 * c) + \sin(dx + c)) * \sin(4 * dx + 4 * c) + \\
& 25 * \sin(4 * dx + 4 * c)^2 + 100 * (2 * \sin(2 * dx + 2 * c) + \sin(dx + c)) * \sin(3 * dx + \\
& 3 * c) + 100 * \sin(3 * dx + 3 * c)^2 + 100 * \sin(2 * dx + 2 * c)^2 + 100 * \sin(2 * dx + 2 \\
& * c) * \sin(dx + c) + 25 * \sin(dx + c)^2 + 10 * \cos(dx + c) + 1) * \sin(6/5 * \arctan2 \\
& (\sin(5/2 * dx + 5/2 * c), \cos(5/2 * dx + 5/2 * c)))^2 + 100 * (2 * (5 * \cos(4 * dx + 4 * c \\
&) + 10 * \cos(3 * dx + 3 * c) + 10 * \cos(2 * dx + 2 * c) + 5 * \cos(dx + c) + 1) * \cos(5 * dx \\
& * x + 5 * c) + \cos(5 * dx + 5 * c)^2 + 10 * (10 * \cos(3 * dx + 3 * c) + 10 * \cos(2 * dx + 2 \\
& * c) + 5 * \cos(dx + c) + 1) * \cos(4 * dx + 4 * c) + 25 * \cos(4 * dx + 4 * c)^2 + 20 * (10 \\
& * \cos(2 * dx + 2 * c) + 5 * \cos(dx + c) + 1) * \cos(3 * dx + 3 * c) + 100 * \cos(3 * dx + \\
& 3 * c)^2 + 20 * (5 * \cos(dx + c) + 1) * \cos(2 * dx + 2 * c) + 100 * \cos(2 * dx + 2 * c)^2 \\
& + 25 * \cos(dx + c)^2 + 10 * (\sin(4 * dx + 4 * c) + 2 * \sin(3 * dx + 3 * c) + 2 * \sin(2 * dx \\
& * x + 2 * c) + \sin(dx + c)) * \sin(5 * dx + 5 * c) + \sin(5 * dx + 5 * c)^2 + 50 * (2 * \sin \\
& (3 * dx + 3 * c) + 2 * \sin(2 * dx + 2 * c) + \sin(dx + c)) * \sin(4 * dx + 4 * c) + 25 * \sin
\end{aligned}$$

$$\begin{aligned}
& *d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 \\
& + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 5*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*((\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5*d*x + 5*c) + \sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(8/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 20*((10*\cos(4*d*x + 4*c) + 20*\cos(3*d*x + 3*c) + 20*\cos(2*d*x + 2*c) + 10*\cos(d*x + c) + 3)*\cos(5*d*x + 5*c)^2 + \cos(5*d*x + 5*c)^3 + (\cos(5*d*x + 5*c) + 1)*\sin(5*d*x + 5*c)^2 + (10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 2)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 2)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 2)*\cos(2*
\end{aligned}$$

$$\begin{aligned}
& d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3*d*x + \\
& 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x \\
& + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*s \\
& \sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + \\
& c) + 25*\sin(d*x + c)^2 + 20*\cos(d*x + c) + 3)*\cos(5*d*x + 5*c) + 10*(10*co \\
& s(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) \\
& + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*co \\
& s(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x \\
& + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(2*(5*\cos(4*d*x + \\
& 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos \\
& (5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x \\
& + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20 \\
& *(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d* \\
& x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c \\
&)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin \\
& (2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2 \\
& *sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 2 \\
& 5*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + \\
& 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2* \\
& c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(4/5*\arctan2(\\
& \sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 5*(2*(5*\cos(4*d*x + 4*c) + 1 \\
& 0*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + \\
& 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + \\
& 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(\\
& 2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^ \\
& 2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25* \\
& \cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + \\
& 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d* \\
& x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d \\
& *x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 10 \\
& 0*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d* \\
& x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(2/5*\arctan2(\sin(5/2*d \\
& *x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*((\sin(4*d*x + 4*c) + 2*\sin(3*d*x + \\
& 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5*d*x + 5*c) + \sin(4*d*x + 4 \\
& *c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5 \\
& *c) + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x \\
& + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*s \\
& \sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin \\
& (2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(6 \\
& /5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 20*((10*\cos(4*d*x \\
& + 4*c) + 20*\cos(3*d*x + 3*c) + 20*\cos(2*d*x + 2*c) + 10*\cos(d*x + c) + 3)* \\
& \cos(5*d*x + 5*c)^2 + \cos(5*d*x + 5*c)^3 + (\cos(5*d*x + 5*c) + 1)*\sin(5*d*x \\
& + 5*c)^2 + (10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) \\
& + 2)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5 \\
& *\cos(d*x + c) + 2)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*
\end{aligned}$$

$$\begin{aligned}
& x + c) + 2) * \cos(2*d*x + 2*c) + 100 * \cos(2*d*x + 2*c)^2 + 25 * \cos(d*x + c)^2 + \\
& 50 * (2 * \sin(3*d*x + 3*c) + 2 * \sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(4*d*x + 4* \\
& c) + 25 * \sin(4*d*x + 4*c)^2 + 100 * (2 * \sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(3* \\
& d*x + 3*c) + 100 * \sin(3*d*x + 3*c)^2 + 100 * \sin(2*d*x + 2*c)^2 + 100 * \sin(2*d* \\
& x + 2*c) * \sin(d*x + c) + 25 * \sin(d*x + c)^2 + 20 * \cos(d*x + c) + 3) * \cos(5*d*x \\
& + 5*c) + 10 * (10 * \cos(3*d*x + 3*c) + 10 * \cos(2*d*x + 2*c) + 5 * \cos(d*x + c) + 1 \\
&) * \cos(4*d*x + 4*c) + 25 * \cos(4*d*x + 4*c)^2 + 20 * (10 * \cos(2*d*x + 2*c) + 5 * \cos \\
& (d*x + c) + 1) * \cos(3*d*x + 3*c) + 100 * \cos(3*d*x + 3*c)^2 + 20 * (5 * \cos(d*x + \\
& c) + 1) * \cos(2*d*x + 2*c) + 100 * \cos(2*d*x + 2*c)^2 + 25 * \cos(d*x + c)^2 + 5 * \\
& (2 * (5 * \cos(4*d*x + 4*c) + 10 * \cos(3*d*x + 3*c) + 10 * \cos(2*d*x + 2*c) + 5 * \cos(\\
& d*x + c) + 1) * \cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10 * (10 * \cos(3*d*x + 3* \\
& c) + 10 * \cos(2*d*x + 2*c) + 5 * \cos(d*x + c) + 1) * \cos(4*d*x + 4*c) + 25 * \cos(4* \\
& d*x + 4*c)^2 + 20 * (10 * \cos(2*d*x + 2*c) + 5 * \cos(d*x + c) + 1) * \cos(3*d*x + 3* \\
& c) + 100 * \cos(3*d*x + 3*c)^2 + 20 * (5 * \cos(d*x + c) + 1) * \cos(2*d*x + 2*c) + 10 \\
& 0 * \cos(2*d*x + 2*c)^2 + 25 * \cos(d*x + c)^2 + 10 * (\sin(4*d*x + 4*c) + 2 * \sin(3*d \\
& *x + 3*c) + 2 * \sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(5*d*x + 5*c) + \sin(5*d*x \\
& + 5*c)^2 + 50 * (2 * \sin(3*d*x + 3*c) + 2 * \sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin \\
& (4*d*x + 4*c) + 25 * \sin(4*d*x + 4*c)^2 + 100 * (2 * \sin(2*d*x + 2*c) + \sin(d*x + \\
& c)) * \sin(3*d*x + 3*c) + 100 * \sin(3*d*x + 3*c)^2 + 100 * \sin(2*d*x + 2*c)^2 + 1 \\
& 00 * \sin(2*d*x + 2*c) * \sin(d*x + c) + 25 * \sin(d*x + c)^2 + 10 * \cos(d*x + c) + 1) \\
& * \cos(2/5 * \arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10 * ((\sin(4* \\
& d*x + 4*c) + 2 * \sin(3*d*x + 3*c) + 2 * \sin(2*d*x + 2*c) + \sin(d*x + c)) * \cos(5* \\
& d*x + 5*c) + \sin(4*d*x + 4*c) + 2 * \sin(3*d*x + 3*c) + 2 * \sin(2*d*x + 2*c) + \sin \\
& (d*x + c)) * \sin(5*d*x + 5*c) + 50 * (2 * \sin(3*d*x + 3*c) + 2 * \sin(2*d*x + 2*c) \\
& + \sin(d*x + c)) * \sin(4*d*x + 4*c) + 25 * \sin(4*d*x + 4*c)^2 + 100 * (2 * \sin(2*d* \\
& x + 2*c) + \sin(d*x + c)) * \sin(3*d*x + 3*c) + 100 * \sin(3*d*x + 3*c)^2 + 100 * \sin \\
& (2*d*x + 2*c)^2 + 100 * \sin(2*d*x + 2*c) * \sin(d*x + c) + 25 * \sin(d*x + c)^2 + \\
& 10 * \cos(d*x + c) + 1) * \cos(4/5 * \arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/ \\
& 2*c))) + 10 * ((10 * \cos(4*d*x + 4*c) + 20 * \cos(3*d*x + 3*c) + 20 * \cos(2*d*x + 2* \\
& c) + 10 * \cos(d*x + c) + 3) * \cos(5*d*x + 5*c)^2 + \cos(5*d*x + 5*c)^3 + (\cos(5* \\
& d*x + 5*c) + 1) * \sin(5*d*x + 5*c)^2 + (10 * (10 * \cos(3*d*x + 3*c) + 10 * \cos(2*d* \\
& x + 2*c) + 5 * \cos(d*x + c) + 2) * \cos(4*d*x + 4*c) + 25 * \cos(4*d*x + 4*c)^2 + 2 \\
& 0 * (10 * \cos(2*d*x + 2*c) + 5 * \cos(d*x + c) + 2) * \cos(3*d*x + 3*c) + 100 * \cos(3*d \\
& *x + 3*c)^2 + 20 * (5 * \cos(d*x + c) + 2) * \cos(2*d*x + 2*c) + 100 * \cos(2*d*x + 2* \\
& c)^2 + 25 * \cos(d*x + c)^2 + 50 * (2 * \sin(3*d*x + 3*c) + 2 * \sin(2*d*x + 2*c) + \sin \\
& (d*x + c)) * \sin(4*d*x + 4*c) + 25 * \sin(4*d*x + 4*c)^2 + 100 * (2 * \sin(2*d*x + 2 \\
& *c) + \sin(d*x + c)) * \sin(3*d*x + 3*c) + 100 * \sin(3*d*x + 3*c)^2 + 100 * \sin(2*d \\
& *x + 2*c)^2 + 100 * \sin(2*d*x + 2*c) * \sin(d*x + c) + 25 * \sin(d*x + c)^2 + 20 * \cos \\
& (d*x + c) + 3) * \cos(5*d*x + 5*c) + 10 * (10 * \cos(3*d*x + 3*c) + 10 * \cos(2*d*x + \\
& 2*c) + 5 * \cos(d*x + c) + 1) * \cos(4*d*x + 4*c) + 25 * \cos(4*d*x + 4*c)^2 + 20 * (\\
& 10 * \cos(2*d*x + 2*c) + 5 * \cos(d*x + c) + 1) * \cos(3*d*x + 3*c) + 100 * \cos(3*d*x \\
& + 3*c)^2 + 20 * (5 * \cos(d*x + c) + 1) * \cos(2*d*x + 2*c) + 100 * \cos(2*d*x + 2*c)^ \\
& 2 + 25 * \cos(d*x + c)^2 + 10 * ((\sin(4*d*x + 4*c) + 2 * \sin(3*d*x + 3*c) + 2 * \sin(\\
& 2*d*x + 2*c) + \sin(d*x + c)) * \cos(5*d*x + 5*c) + \sin(4*d*x + 4*c) + 2 * \sin(3* \\
& d*x + 3*c) + 2 * \sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(5*d*x + 5*c) + 50 * (2 * \sin
\end{aligned}$$

$$\begin{aligned}
& n(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) \\
&) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*((\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5*d*x + 5*c)^2 + 2*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5*d*x + 5*c) + \sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*(10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c)^2 + \sin(5*d*x + 5*c)^3 + (2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(5*d*x + 5*c) + 10*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 1
\end{aligned}$$

$$\begin{aligned}
& 00*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c) \\
& ^2 + 10*\cos(d*x + c) + 1)*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x \\
& + 5/2*c))) + 5*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x \\
& + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(1 \\
& 0*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + \\
& 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1 \\
&)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2 \\
& *d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + \\
& 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + \\
& 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + s \\
& in(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + \\
& 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2* \\
& d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*c \\
& os(d*x + c) + 1)*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c) \\
&))*\sin(8/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 20*(10*(\\
& \sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))* \\
& \sin(5*d*x + 5*c)^2 + \sin(5*d*x + 5*c)^3 + (2*(5*\cos(4*d*x + 4*c) + 10*\cos(3 \\
& *d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \\
& \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(\\
& d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + \\
& 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20* \\
& (5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x \\
& + c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4 \\
& *d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c \\
&))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100 \\
& *\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*s \\
& in(5*d*x + 5*c) + 10*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(\\
& 2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + \\
& 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d \\
& *x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c \\
&) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)* \\
& \cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d \\
& *x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d \\
& *x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c \\
&) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d \\
& *x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*s \\
& in(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + \\
& 10*\cos(d*x + c) + 1)*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5 \\
& /2*c))) + 5*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2 \\
& *c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*c \\
& os(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) \\
& + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*c \\
& os(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x \\
& + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) \\
& + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c)
\end{aligned}$$

$$\begin{aligned}
& + \sin(5d*x + 5*c)^2 + 50*(2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(4d*x + 4*c) + 25*\sin(4d*x + 4*c)^2 + 100*(2*\sin(2d*x + 2*c) \\
& + \sin(d*x + c))*\sin(3d*x + 3*c) + 100*\sin(3d*x + 3*c)^2 + 100*\sin(2d*x + 2*c)^2 + 100*\sin(2d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) * \\
& \sin(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 20*(10*(\sin(4d*x + 4*c) + 2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(5d*x + 5*c)^2 + \sin(5d*x + 5*c)^3 + (2*(5*\cos(4d*x + 4*c) + 10*\cos(3d*x + 3*c) + 10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5d*x + 5*c) + \cos(5d*x + 5*c)^2 + 10*(10*\cos(3d*x + 3*c) + 10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4d*x + 4*c) + 25*\cos(4d*x + 4*c)^2 + 20*(10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3d*x + 3*c) + 100*\cos(3d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2d*x + 2*c) + 100*\cos(2d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(4d*x + 4*c) + 25*\sin(4d*x + 4*c)^2 + 100*(2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(3d*x + 3*c) + 100*\sin(3d*x + 3*c)^2 + 100*\sin(2d*x + 2*c)^2 + 100*\sin(2d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(5d*x + 5*c) + 5*(2*(5*\cos(4d*x + 4*c) + 10*\cos(3d*x + 3*c) + 10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5d*x + 5*c) + \cos(5d*x + 5*c)^2 + 10*(10*\cos(3d*x + 3*c) + 10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4d*x + 4*c) + 25*\cos(4d*x + 4*c)^2 + 20*(10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3d*x + 3*c) + 100*\cos(3d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2d*x + 2*c) + 100*\cos(2d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4d*x + 4*c) + 2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(5d*x + 5*c) + \sin(5d*x + 5*c)^2 + 50*(2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(4d*x + 4*c) + 25*\sin(4d*x + 4*c)^2 + 100*(2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(3d*x + 3*c) + 100*\sin(3d*x + 3*c)^2 + 100*\sin(2d*x + 2*c)^2 + 100*\sin(2d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) * \\
& \sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*(10*(\sin(4d*x + 4*c) + 2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(5d*x + 5*c)^2 + \sin(5d*x + 5*c)^3 + (2*(5*\cos(4d*x + 4*c) + 10*\cos(3d*x + 3*c) + 10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5d*x + 5*c) + \cos(5d*x + 5*c)^2 + 10*(10*\cos(3d*x + 3*c) + 10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4d*x + 4*c) + 25*\cos(4d*x + 4*c)^2 + 20*(10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3d*x + 3*c) + 100*\cos(3d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2d*x + 2*c) + 100*\cos(2d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(4d*x + 4*c) + 25*\sin(4d*x + 4*c)^2 + 100*(2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(3d*x + 3*c) + 100*\sin(3d*x + 3*c)^2 + 100*\sin(2d*x + 2*c)^2 + 100*\sin(2d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(5d*x + 5*c))*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*\cos(d*x + c) + 1)*\log(\cos(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + \sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 2*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) +
\end{aligned}$$

$$\begin{aligned}
& 1) - 3*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + \\
& \quad 5*\cos(d*x + c) + 2)*\cos(5*d*x + 5*c)^3 + \cos(5*d*x + 5*c)^4 + 10*(\sin(4*d*x \\
& \quad x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x \\
& \quad x + 5*c)^3 + \sin(5*d*x + 5*c)^4 + (10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + \\
& \quad 2*c) + 5*\cos(d*x + c) + 3)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(\\
& \quad 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 3)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x \\
& \quad + 3*c)^2 + 20*(5*\cos(d*x + c) + 3)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 \\
& \quad + 25*\cos(d*x + c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x \\
& \quad + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) \\
& \quad + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x \\
& \quad + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 30*\cos(d \\
& \quad *x + c) + 6)*\cos(5*d*x + 5*c)^2 + 25*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x \\
& \quad + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5 \\
& \quad *d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + \\
& \quad c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) \\
& \quad + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos \\
& \quad (d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c) \\
& \quad ^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d \\
& \quad *x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2 \\
& \quad *\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 \\
& \quad + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x \\
& \quad + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25* \\
& \quad \sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(8/5*\arctan2(\sin(5/2*d*x + 5/2*c), \\
& \quad \cos(5/2*d*x + 5/2*c)))^2 + 100*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) \\
& \quad) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x \\
& \quad + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + \\
& \quad 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5* \\
& \quad \cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x \\
& \quad + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + \\
& \quad 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + \\
& \quad c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(\\
& \quad 2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100 \\
& \quad *(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c \\
& \quad)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d \\
& \quad *x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(\\
& \quad 5/2*d*x + 5/2*c)))^2 + 100*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 1 \\
& \quad 0*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c \\
& \quad)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos \\
& \quad (4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d \\
& \quad *x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) \\
& \quad + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin \\
& \quad (4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin \\
& \quad (5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x \\
& \quad + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin \\
& \quad (2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& *d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d \\
& *x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(6/5*a \\
& rctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 100*(2*(5*\cos(4*d*x \\
& + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*c \\
& os(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d \\
& *x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + \\
& 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3* \\
& d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2 \\
& *c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*s \\
& in(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50* \\
& (2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + \\
& 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x \\
& + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + \\
& 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(4/5*arctan \\
& 2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 25*(2*(5*\cos(4*d*x + 4*c \\
&) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d \\
& *x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2 \\
& *c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10 \\
& *cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + \\
& 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 \\
& + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d \\
& *x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin \\
& (3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*si \\
& n(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) \\
& + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*s \\
& in(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(2/5*arctan2(\sin(\\
& 5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c)))^2 + 2*(5*(20*\cos(3*d*x + 3*c) + 20 \\
& *cos(2*d*x + 2*c) + 10*\cos(d*x + c) + 3)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + \\
& 4*c)^2 + 10*(20*\cos(2*d*x + 2*c) + 10*\cos(d*x + c) + 3)*\cos(3*d*x + 3*c) + \\
& 100*\cos(3*d*x + 3*c)^2 + 10*(10*\cos(d*x + c) + 3)*\cos(2*d*x + 2*c) + 100*co \\
& s(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x \\
& + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*s \\
& in(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + \\
& 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + \\
& c)^2 + 15*\cos(d*x + c) + 2)*\cos(5*d*x + 5*c) + 10*(10*\cos(3*d*x + 3*c) + 10 \\
& *cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4 \\
& *c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 10 \\
& 0*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2 \\
& *d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*((10*\cos(4*d*x + 4*c) + 20*\cos(3*d*x \\
& + 3*c) + 20*\cos(2*d*x + 2*c) + 10*\cos(d*x + c) + 3)*\cos(5*d*x + 5*c)^2 + c \\
& os(5*d*x + 5*c)^3 + (\cos(5*d*x + 5*c) + 1)*\sin(5*d*x + 5*c)^2 + (10*(10*\cos \\
& (3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 2)*\cos(4*d*x + 4*c) \\
& + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 2)*\cos \\
& (3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 2)*\cos(2*d*x \\
& + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3*d*x + 3*c
\end{aligned}$$

$$\begin{aligned}
&) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) \\
& + 25*\sin(d*x + c)^2 + 20*\cos(d*x + c) + 3)*\cos(5*d*x + 5*c) + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 2 \\
& 5*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) \\
& + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) \\
& + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) \\
& + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 \\
& + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) \\
& + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) \\
& + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \\
& \cos(5/2*d*x + 5/2*c))) + 10*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) \\
&) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) \\
& + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 \\
& + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) \\
& + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 \\
& + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 5*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x \\
& + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) \\
& + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) \\
& + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) \\
& + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) \\
& + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \\
& \cos(5/2*d*x + 5/2*c))) + 10*((\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5*d*x + 5*c) + \sin(4*d*x + 4*c) + 2*\sin(
\end{aligned}$$

$$\begin{aligned}
& 3d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + 50*(2* \\
& \sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25 \\
& *\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3 \\
& *c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c \\
&)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(8/5*\arctan2(s \\
& \sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 20*((10*\cos(4*d*x + 4*c) + 20 \\
& *\cos(3*d*x + 3*c) + 20*\cos(2*d*x + 2*c) + 10*\cos(d*x + c) + 3)*\cos(5*d*x + \\
& 5*c)^2 + \cos(5*d*x + 5*c)^3 + (\cos(5*d*x + 5*c) + 1)*\sin(5*d*x + 5*c)^2 + (\\
& 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 2)*\cos(4*d \\
& *x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c \\
&) + 2)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 2)* \\
& \cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3 \\
& *d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(\\
& 4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + \\
& 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin \\
& (d*x + c) + 25*\sin(d*x + c)^2 + 20*\cos(d*x + c) + 3)*\cos(5*d*x + 5*c) + 10* \\
& (10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x \\
& + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + \\
& 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos \\
& (2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(2*(5*\cos(4 \\
& *d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + \\
& 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos \\
& (2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^ \\
& 2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\co \\
& s(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x \\
& + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + \\
& 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + \\
& 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4* \\
& c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3* \\
& d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d* \\
& x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(4/5*\ar \\
& ctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 5*(2*(5*\cos(4*d*x + 4* \\
& c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5* \\
& d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + \\
& 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(1 \\
& 0*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + \\
& 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 \\
& + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2* \\
& d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*si \\
& n(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*s \\
& in(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c \\
&) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)* \\
& \sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(2/5*\arctan2(\sin \\
& (5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*((\sin(4*d*x + 4*c) + 2*\sin(3 \\
& *d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5*d*x + 5*c) + \sin(4*d
\end{aligned}$$

$$\begin{aligned}
& x + 2c)^2 + 25\cos(dx + c)^2 + 50*(2*\sin(3*dx + 3*c) + 2*\sin(2*dx + 2*c) \\
&) + \sin(dx + c))*\sin(4*dx + 4*c) + 25*\sin(4*dx + 4*c)^2 + 100*(2*\sin(2*d \\
& *x + 2*c) + \sin(dx + c))*\sin(3*dx + 3*c) + 100*\sin(3*dx + 3*c)^2 + 100*s \\
& \sin(2*dx + 2*c)^2 + 100*\sin(2*dx + 2*c)*\sin(dx + c) + 25*\sin(dx + c)^2 + \\
& 20*\cos(dx + c) + 3)*\cos(5*dx + 5*c) + 10*(10*\cos(3*dx + 3*c) + 10*\cos(2 \\
& *dx + 2*c) + 5*\cos(dx + c) + 1)*\cos(4*dx + 4*c) + 25*\cos(4*dx + 4*c)^2 \\
& + 20*(10*\cos(2*dx + 2*c) + 5*\cos(dx + c) + 1)*\cos(3*dx + 3*c) + 100*\cos(\\
& 3*dx + 3*c)^2 + 20*(5*\cos(dx + c) + 1)*\cos(2*dx + 2*c) + 100*\cos(2*dx + \\
& 2*c)^2 + 25*\cos(dx + c)^2 + 10*((\sin(4*dx + 4*c) + 2*\sin(3*dx + 3*c) + \\
& 2*\sin(2*dx + 2*c) + \sin(dx + c))*\cos(5*dx + 5*c) + \sin(4*dx + 4*c) + 2* \\
& \sin(3*dx + 3*c) + 2*\sin(2*dx + 2*c) + \sin(dx + c))*\sin(5*dx + 5*c) + 50 \\
& *(2*\sin(3*dx + 3*c) + 2*\sin(2*dx + 2*c) + \sin(dx + c))*\sin(4*dx + 4*c) \\
& + 25*\sin(4*dx + 4*c)^2 + 100*(2*\sin(2*dx + 2*c) + \sin(dx + c))*\sin(3*dx \\
& + 3*c) + 100*\sin(3*dx + 3*c)^2 + 100*\sin(2*dx + 2*c)^2 + 100*\sin(2*dx + \\
& 2*c)*\sin(dx + c) + 25*\sin(dx + c)^2 + 10*\cos(dx + c) + 1)*\cos(2/5*\arcta \\
& n2(\sin(5/2*dx + 5/2*c), \cos(5/2*dx + 5/2*c))) + 10*((\sin(4*dx + 4*c) + 2 \\
& *sin(3*dx + 3*c) + 2*\sin(2*dx + 2*c) + \sin(dx + c))*\cos(5*dx + 5*c)^2 + \\
& 2*(\sin(4*dx + 4*c) + 2*\sin(3*dx + 3*c) + 2*\sin(2*dx + 2*c) + \sin(dx + \\
& c))*\cos(5*dx + 5*c) + \sin(4*dx + 4*c) + 2*\sin(3*dx + 3*c) + 2*\sin(2*dx \\
& + 2*c) + \sin(dx + c))*\sin(5*dx + 5*c) + 50*(2*\sin(3*dx + 3*c) + 2*\sin(2* \\
& dx + 2*c) + \sin(dx + c))*\sin(4*dx + 4*c) + 25*\sin(4*dx + 4*c)^2 + 100*(\\
& 2*\sin(2*dx + 2*c) + \sin(dx + c))*\sin(3*dx + 3*c) + 100*\sin(3*dx + 3*c)^ \\
& 2 + 100*\sin(2*dx + 2*c)^2 + 100*\sin(2*dx + 2*c)*\sin(dx + c) + 25*\sin(dx \\
& + c)^2 + 10*(10*(\sin(4*dx + 4*c) + 2*\sin(3*dx + 3*c) + 2*\sin(2*dx + 2*c) \\
&) + \sin(dx + c))*\sin(5*dx + 5*c)^2 + \sin(5*dx + 5*c)^3 + (2*(5*\cos(4*dx \\
& + 4*c) + 10*\cos(3*dx + 3*c) + 10*\cos(2*dx + 2*c) + 5*\cos(dx + c) + 1)*c \\
& os(5*dx + 5*c) + \cos(5*dx + 5*c)^2 + 10*(10*\cos(3*dx + 3*c) + 10*\cos(2*d \\
& *x + 2*c) + 5*\cos(dx + c) + 1)*\cos(4*dx + 4*c) + 25*\cos(4*dx + 4*c)^2 + \\
& 20*(10*\cos(2*dx + 2*c) + 5*\cos(dx + c) + 1)*\cos(3*dx + 3*c) + 100*\cos(3* \\
& dx + 3*c)^2 + 20*(5*\cos(dx + c) + 1)*\cos(2*dx + 2*c) + 100*\cos(2*dx + 2 \\
& *c)^2 + 25*\cos(dx + c)^2 + 50*(2*\sin(3*dx + 3*c) + 2*\sin(2*dx + 2*c) + s \\
& \sin(dx + c))*\sin(4*dx + 4*c) + 25*\sin(4*dx + 4*c)^2 + 100*(2*\sin(2*dx + \\
& 2*c) + \sin(dx + c))*\sin(3*dx + 3*c) + 100*\sin(3*dx + 3*c)^2 + 100*\sin(2* \\
& dx + 2*c)^2 + 100*\sin(2*dx + 2*c)*\sin(dx + c) + 25*\sin(dx + c)^2 + 10*c \\
& os(dx + c) + 1)*\sin(5*dx + 5*c) + 10*(2*(5*\cos(4*dx + 4*c) + 10*\cos(3*d \\
& x + 3*c) + 10*\cos(2*dx + 2*c) + 5*\cos(dx + c) + 1)*\cos(5*dx + 5*c) + \cos \\
& (5*dx + 5*c)^2 + 10*(10*\cos(3*dx + 3*c) + 10*\cos(2*dx + 2*c) + 5*\cos(dx \\
& + c) + 1)*\cos(4*dx + 4*c) + 25*\cos(4*dx + 4*c)^2 + 20*(10*\cos(2*dx + 2* \\
& c) + 5*\cos(dx + c) + 1)*\cos(3*dx + 3*c) + 100*\cos(3*dx + 3*c)^2 + 20*(5* \\
& \cos(dx + c) + 1)*\cos(2*dx + 2*c) + 100*\cos(2*dx + 2*c)^2 + 25*\cos(dx + \\
& c)^2 + 10*(\sin(4*dx + 4*c) + 2*\sin(3*dx + 3*c) + 2*\sin(2*dx + 2*c) + \sin \\
& (dx + c))*\sin(5*dx + 5*c) + \sin(5*dx + 5*c)^2 + 50*(2*\sin(3*dx + 3*c) + \\
& 2*\sin(2*dx + 2*c) + \sin(dx + c))*\sin(4*dx + 4*c) + 25*\sin(4*dx + 4*c)^ \\
& 2 + 100*(2*\sin(2*dx + 2*c) + \sin(dx + c))*\sin(3*dx + 3*c) + 100*\sin(3*d \\
& x + 3*c)^2 + 100*\sin(2*dx + 2*c)^2 + 100*\sin(2*dx + 2*c)*\sin(dx + c) + 2
\end{aligned}$$

$$\begin{aligned}
& 5*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \\
& \cos(5/2*d*x + 5/2*c))) + 10*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) \\
& + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + \\
& 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + \\
& 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*c \\
& \cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x \\
& + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 1 \\
& 0*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c \\
&))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2 \\
& *d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100* \\
& (2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c) \\
& ^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d* \\
& x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5 \\
& /2*d*x + 5/2*c))) + 5*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos \\
& (2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + \\
& 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4* \\
& d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + \\
& c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1) \\
& *\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4* \\
& d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5* \\
& d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2* \\
& c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2* \\
& d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100* \\
& \sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 \\
& + 10*\cos(d*x + c) + 1)*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + \\
& 5/2*c))))*\sin(8/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 20 \\
& *(10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x \\
& + c))*\sin(5*d*x + 5*c)^2 + \sin(5*d*x + 5*c)^3 + (2*(5*\cos(4*d*x + 4*c) + 10 \\
& *\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5 \\
& *c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + \\
& 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2 \\
& *d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 \\
& + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*c \\
& \cos(d*x + c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) \\
& *\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d \\
& *x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 \\
& + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) \\
& + 1)*\sin(5*d*x + 5*c) + 10*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 1 \\
& 0*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c \\
&)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*c \\
& \cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d \\
& *x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) \\
& + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(s \\
& in(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*s \\
& in(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x
\end{aligned}$$

$$\begin{aligned}
& 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2 \\
& *d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 \\
& + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*c \\
& \cos(d*x + c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) \\
& * \sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d \\
& *x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 \\
& + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) \\
& + 1)*\sin(5*d*x + 5*c))*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + \\
& 5/2*c))) + 10*\cos(d*x + c) + 1)*\log(\cos(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos \\
& (5/2*d*x + 5/2*c)))^2 + \sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x \\
& + 5/2*c)))^2 - 2*\sin(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c) \\
&))) + 1) - 512*(5*\cos(4*d*x + 4*c)^2*\cos(5/2*d*x + 5/2*c) + 4*(10*\cos(2*d*x \\
& + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c)*\cos(5/2*d*x + 5/2*c) + 20*co \\
& s(3*d*x + 3*c)^2*\cos(5/2*d*x + 5/2*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + \\
& 4*c)^2 + 20*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c)^2 + 2*((2*\cos(2*d*x + 2*c) \\
&) + \cos(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(4*d*x + 4*c)*\cos(5/2*d*x + 5/2 \\
& *c) + 2*\cos(3*d*x + 3*c)*\cos(5/2*d*x + 5/2*c) - (2*\sin(2*d*x + 2*c) + \sin(d \\
& *x + c))*\sin(5/2*d*x + 5/2*c) - \sin(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 2*s \\
& in(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\cos(5*d*x + 5*c) + 2*((10*\cos(2*d*x + \\
& 2*c) + 5*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) + 10*\cos(3*d*x + 3*c)*\cos(\\
& 5/2*d*x + 5/2*c))*\cos(4*d*x + 4*c) + (4*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2* \\
& c) + 20*\cos(2*d*x + 2*c)^2 + 5*\cos(d*x + c)^2 + 20*\sin(2*d*x + 2*c)^2 + 20* \\
& \sin(2*d*x + 2*c)*\sin(d*x + c) + 5*\sin(d*x + c)^2 + 2*\cos(d*x + c))*\cos(5/2* \\
& d*x + 5/2*c) + 2*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) \\
&) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - \sin(5/2*d*x + 5/2*c))*\sin(4* \\
& d*x + 4*c) + 4*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) \\
& - \sin(5/2*d*x + 5/2*c))*\sin(3*d*x + 3*c) - 2*(2*\sin(2*d*x + 2*c) + \sin(d*x \\
& + c))*\sin(5/2*d*x + 5/2*c))*\sin(5*d*x + 5*c) + 512*(10*(2*\sin(2*d*x + 2*c) \\
& + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + 20*\sin(3*d*x + 3*c)*\sin(5/2*d*x + 5/ \\
& 2*c) + \cos(5/2*d*x + 5/2*c))*\sin(4*d*x + 4*c) + 1024*(10*(2*\sin(2*d*x + 2*c) \\
&) + \sin(d*x + c))*\sin(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c))*\sin(3*d*x + \\
& 3*c) + 512*(2*(10*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 20*\cos(2*d*x + 2*c)^ \\
& 2 + 5*\cos(d*x + c)^2 + 20*\sin(2*d*x + 2*c)^2 + 20*\sin(2*d*x + 2*c)*\sin(d*x \\
& + c) + 5*\sin(d*x + c)^2 + \cos(d*x + c))*\sin(5/2*d*x + 5/2*c) + 12*((10*\cos(\\
& 4*d*x + 4*c) + 20*\cos(3*d*x + 3*c) + 20*\cos(2*d*x + 2*c) + 10*\cos(d*x + c) \\
& + 3)*\cos(5*d*x + 5*c)^2 + \cos(5*d*x + 5*c)^3 + (\cos(5*d*x + 5*c) + 1)*\sin(5 \\
& *d*x + 5*c)^2 + (10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x \\
& + c) + 2)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) \\
&) + 5*\cos(d*x + c) + 2)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*c \\
& \cos(d*x + c) + 2)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c) \\
&)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x \\
& + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*s \\
& in(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin \\
& (2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 20*\cos(d*x + c) + 3)*\cos(5 \\
& *d*x + 5*c) + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c
\end{aligned}$$

$$\begin{aligned}
& d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) \\
&) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*((\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5*d*x + 5*c) + \sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(9/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 4*(128*\cos(5*d*x + 5*c)^2*\cos(5/2*d*x + 5/2*c) + 3200*\cos(4*d*x + 4*c)^2*\cos(5/2*d*x + 5/2*c) + 2560*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c)*\cos(5/2*d*x + 5/2*c) + 12800*\cos(3*d*x + 3*c)^2*\cos(5/2*d*x + 5/2*c) - 128*\cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c)^2 + 3200*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c)^2 + 12800*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) + 12800*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c)^2 + 256*((10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) + 5*\cos(4*d*x + 4*c)*\cos(5/2*d*x + 5/2*c) + 10*\cos(3*d*x + 3*c)*\cos(5/2*d*x + 5/2*c))*\cos(5*d*x + 5*c) + 1280*((10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) + 10*\cos(3*d*x + 3*c)*\cos(5/2*d*x + 5/2*c))*\cos(4*d*x + 4*c) + 128*(20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) + 70*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(7/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 70*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 +
\end{aligned}$$

$$\begin{aligned}
& 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) \\
& + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) \\
& + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) \\
& + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(3/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) \\
& - 15*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) \\
& + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) \\
& + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) \\
& + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 \\
& + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) \\
& + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) \\
& + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) \\
& + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) \\
& + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) \\
& + 256*((10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) + \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) \\
& + 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) + 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(5*d*x + 5*c) \\
& + 6400*((2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 2*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c))*\sin(4*d*x + 4*c) \\
& - 2560*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) \\
& + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) \\
& - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) \\
& - 2560*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) \\
& + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) \\
& - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) \\
& - 1280*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c) \\
& + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) \\
& - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) \\
& + 56*((10*\cos(4*d*x + 4*c) + 20*\cos(3*d*x + 3*c) + 20*\cos(2*d*x + 2*c) + 10*\cos(d*x + c) + 3)*\cos(5*d*x + 5*c)^2 + \cos(5*d*x + 5*c)^3 + (\cos(5*d*x + 5*c) + 1)*\sin(5*d*x + 5*c)^2 \\
& + (10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 2)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 2)*\cos(3*d*x + 3*c) \\
& + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x
\end{aligned}$$

$$\begin{aligned}
& + c) + 2) \cdot \cos(2dx + 2c) + 100 \cdot \cos(2dx + 2c)^2 + 25 \cdot \cos(dx + c)^2 + 5 \\
& 0 \cdot (2 \cdot \sin(3dx + 3c) + 2 \cdot \sin(2dx + 2c) + \sin(dx + c)) \cdot \sin(4dx + 4c) \\
& + 25 \cdot \sin(4dx + 4c)^2 + 100 \cdot (2 \cdot \sin(2dx + 2c) + \sin(dx + c)) \cdot \sin(3dx \\
& x + 3c) + 100 \cdot \sin(3dx + 3c)^2 + 100 \cdot \sin(2dx + 2c)^2 + 100 \cdot \sin(2dx \\
& + 2c) \cdot \sin(dx + c) + 25 \cdot \sin(dx + c)^2 + 20 \cdot \cos(dx + c) + 3) \cdot \cos(5dx + \\
& 5c) + 10 \cdot (10 \cdot \cos(3dx + 3c) + 10 \cdot \cos(2dx + 2c) + 5 \cdot \cos(dx + c) + 1) \cdot \\
& \cos(4dx + 4c) + 25 \cdot \cos(4dx + 4c)^2 + 20 \cdot (10 \cdot \cos(2dx + 2c) + 5 \cdot \cos(\\
& dx + c) + 1) \cdot \cos(3dx + 3c) + 100 \cdot \cos(3dx + 3c)^2 + 20 \cdot (5 \cdot \cos(dx + c \\
&) + 1) \cdot \cos(2dx + 2c) + 100 \cdot \cos(2dx + 2c)^2 + 25 \cdot \cos(dx + c)^2 + 10 \cdot (\\
& 2 \cdot (5 \cdot \cos(4dx + 4c) + 10 \cdot \cos(3dx + 3c) + 10 \cdot \cos(2dx + 2c) + 5 \cdot \cos(d \\
& *x + c) + 1) \cdot \cos(5dx + 5c) + \cos(5dx + 5c)^2 + 10 \cdot (10 \cdot \cos(3dx + 3c \\
&) + 10 \cdot \cos(2dx + 2c) + 5 \cdot \cos(dx + c) + 1) \cdot \cos(4dx + 4c) + 25 \cdot \cos(4d \\
& *x + 4c)^2 + 20 \cdot (10 \cdot \cos(2dx + 2c) + 5 \cdot \cos(dx + c) + 1) \cdot \cos(3dx + 3c \\
&) + 100 \cdot \cos(3dx + 3c)^2 + 20 \cdot (5 \cdot \cos(dx + c) + 1) \cdot \cos(2dx + 2c) + 100 \\
& \cdot \cos(2dx + 2c)^2 + 25 \cdot \cos(dx + c)^2 + 10 \cdot (\sin(4dx + 4c) + 2 \cdot \sin(3dx \\
& x + 3c) + 2 \cdot \sin(2dx + 2c) + \sin(dx + c)) \cdot \sin(5dx + 5c) + \sin(5dx \\
& + 5c)^2 + 50 \cdot (2 \cdot \sin(3dx + 3c) + 2 \cdot \sin(2dx + 2c) + \sin(dx + c)) \cdot \sin(\\
& 4dx + 4c) + 25 \cdot \sin(4dx + 4c)^2 + 100 \cdot (2 \cdot \sin(2dx + 2c) + \sin(dx + \\
& c)) \cdot \sin(3dx + 3c) + 100 \cdot \sin(3dx + 3c)^2 + 100 \cdot \sin(2dx + 2c)^2 + 10 \\
& 0 \cdot \sin(2dx + 2c) \cdot \sin(dx + c) + 25 \cdot \sin(dx + c)^2 + 10 \cdot \cos(dx + c) + 1) \cdot \\
& \cos(6/5 \cdot \arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) + 10 \cdot (2 \cdot (5 \cdot \cos \\
& (4dx + 4c) + 10 \cdot \cos(3dx + 3c) + 10 \cdot \cos(2dx + 2c) + 5 \cdot \cos(dx + c) \\
& + 1) \cdot \cos(5dx + 5c) + \cos(5dx + 5c)^2 + 10 \cdot (10 \cdot \cos(3dx + 3c) + 10 \cdot c \\
& os(2dx + 2c) + 5 \cdot \cos(dx + c) + 1) \cdot \cos(4dx + 4c) + 25 \cdot \cos(4dx + 4c \\
&)^2 + 20 \cdot (10 \cdot \cos(2dx + 2c) + 5 \cdot \cos(dx + c) + 1) \cdot \cos(3dx + 3c) + 100 \cdot \\
& \cos(3dx + 3c)^2 + 20 \cdot (5 \cdot \cos(dx + c) + 1) \cdot \cos(2dx + 2c) + 100 \cdot \cos(2d \\
& *x + 2c)^2 + 25 \cdot \cos(dx + c)^2 + 10 \cdot (\sin(4dx + 4c) + 2 \cdot \sin(3dx + 3c) \\
& + 2 \cdot \sin(2dx + 2c) + \sin(dx + c)) \cdot \sin(5dx + 5c) + \sin(5dx + 5c)^2 \\
& + 50 \cdot (2 \cdot \sin(3dx + 3c) + 2 \cdot \sin(2dx + 2c) + \sin(dx + c)) \cdot \sin(4dx + \\
& 4c) + 25 \cdot \sin(4dx + 4c)^2 + 100 \cdot (2 \cdot \sin(2dx + 2c) + \sin(dx + c)) \cdot \sin(\\
& 3dx + 3c) + 100 \cdot \sin(3dx + 3c)^2 + 100 \cdot \sin(2dx + 2c)^2 + 100 \cdot \sin(2 \\
& dx + 2c) \cdot \sin(dx + c) + 25 \cdot \sin(dx + c)^2 + 10 \cdot \cos(dx + c) + 1) \cdot \cos(4/5 \\
& \arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) + 5 \cdot (2 \cdot (5 \cdot \cos(4dx + \\
& 4c) + 10 \cdot \cos(3dx + 3c) + 10 \cdot \cos(2dx + 2c) + 5 \cdot \cos(dx + c) + 1) \cdot \cos(\\
& 5dx + 5c) + \cos(5dx + 5c)^2 + 10 \cdot (10 \cdot \cos(3dx + 3c) + 10 \cdot \cos(2dx \\
& + 2c) + 5 \cdot \cos(dx + c) + 1) \cdot \cos(4dx + 4c) + 25 \cdot \cos(4dx + 4c)^2 + 20 \cdot \\
& (10 \cdot \cos(2dx + 2c) + 5 \cdot \cos(dx + c) + 1) \cdot \cos(3dx + 3c) + 100 \cdot \cos(3dx \\
& + 3c)^2 + 20 \cdot (5 \cdot \cos(dx + c) + 1) \cdot \cos(2dx + 2c) + 100 \cdot \cos(2dx + 2c) \\
& ^2 + 25 \cdot \cos(dx + c)^2 + 10 \cdot (\sin(4dx + 4c) + 2 \cdot \sin(3dx + 3c) + 2 \cdot \sin(\\
& 2dx + 2c) + \sin(dx + c)) \cdot \sin(5dx + 5c) + \sin(5dx + 5c)^2 + 50 \cdot (2 \cdot \\
& \sin(3dx + 3c) + 2 \cdot \sin(2dx + 2c) + \sin(dx + c)) \cdot \sin(4dx + 4c) + 25 \\
& \cdot \sin(4dx + 4c)^2 + 100 \cdot (2 \cdot \sin(2dx + 2c) + \sin(dx + c)) \cdot \sin(3dx + 3 \\
& *c) + 100 \cdot \sin(3dx + 3c)^2 + 100 \cdot \sin(2dx + 2c)^2 + 100 \cdot \sin(2dx + 2c \\
&) \cdot \sin(dx + c) + 25 \cdot \sin(dx + c)^2 + 10 \cdot \cos(dx + c) + 1) \cdot \cos(2/5 \cdot \arctan2(s \\
& in(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) + 10 \cdot ((\sin(4dx + 4c) + 2 \cdot \sin
\end{aligned}$$

$$\begin{aligned}
& (3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \cos(5*d*x + 5*c) + \sin(4 \\
& *d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(5 \\
& *d*x + 5*c) + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin \\
& \sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x \\
& + c)) * \sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + \\
& 100*\sin(2*d*x + 2*c) * \sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + \\
& 1) * \sin(7/5 * \arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 8*(128*\cos \\
& s(5*d*x + 5*c)^2 * \cos(5/2*d*x + 5/2*c) + 3200*\cos(4*d*x + 4*c)^2 * \cos(5/2*d*x \\
& + 5/2*c) + 2560*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(3*d*x + 3*c \\
&) * \cos(5/2*d*x + 5/2*c) + 12800*\cos(3*d*x + 3*c)^2 * \cos(5/2*d*x + 5/2*c) - 12 \\
& 8*\cos(5/2*d*x + 5/2*c) * \sin(5*d*x + 5*c)^2 + 3200*\cos(5/2*d*x + 5/2*c) * \sin(4 \\
& *d*x + 4*c)^2 + 12800*(2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \cos(5/2*d*x + 5/2 \\
& *c) * \sin(3*d*x + 3*c) + 12800*\cos(5/2*d*x + 5/2*c) * \sin(3*d*x + 3*c)^2 + 256* \\
& ((10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) + 5*\cos(4* \\
& d*x + 4*c) * \cos(5/2*d*x + 5/2*c) + 10*\cos(3*d*x + 3*c) * \cos(5/2*d*x + 5/2*c)) \\
& * \cos(5*d*x + 5*c) + 1280*((10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(5/ \\
& 2*d*x + 5/2*c) + 10*\cos(3*d*x + 3*c) * \cos(5/2*d*x + 5/2*c)) * \cos(4*d*x + 4*c) \\
& + 128*(20*(5*\cos(d*x + c) + 1) * \cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + \\
& 25*\cos(d*x + c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c) * \sin(d*x \\
& + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1) * \cos(5/2*d*x + 5/2*c) - 70*(\\
& 2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d \\
& *x + c) + 1) * \cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c \\
&) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(4*d*x + 4*c) + 25*\cos(4*d \\
& *x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(3*d*x + 3*c \\
&) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1) * \cos(2*d*x + 2*c) + 100 \\
& * \cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d* \\
& x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(5*d*x + 5*c) + \sin(5*d*x \\
& + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(\\
& 4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + \\
& c)) * \sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 10 \\
& 0*\sin(2*d*x + 2*c) * \sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1) * \\
& \cos(3/5 * \arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 15*(2*(5*\cos \\
& (4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) \\
& + 1) * \cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos \\
& os(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c \\
&)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1) * \cos(3*d*x + 3*c) + 100* \\
& \cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1) * \cos(2*d*x + 2*c) + 100*\cos(2*d \\
& *x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) \\
& + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 \\
& + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(4*d*x + \\
& 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c)) * \sin(\\
& 3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2* \\
& d*x + 2*c) * \sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1) * \cos(1/5* \\
& \arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 256*((10*\cos(2*d*x + \\
& 2*c) + 5*\cos(d*x + c) + 1) * \sin(5/2*d*x + 5/2*c) + \cos(5*d*x + 5*c) * \sin(5/2
\end{aligned}$$

$$\begin{aligned}
& *d*x + 5/2*c) + 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) + 10*\cos(3*d*x + 3*c) \\
& *\sin(5/2*d*x + 5/2*c))*\sin(5*d*x + 5*c) + 6400*((2*\sin(2*d*x + 2*c) + \sin \\
& (d*x + c))*\cos(5/2*d*x + 5/2*c) + 2*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c))* \\
& \sin(4*d*x + 4*c) - 2560*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x \\
& + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin \\
& in(4*d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x \\
& + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/ \\
& 2*d*x + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3 \\
& *c)*\sin(5/2*d*x + 5/2*c))*\sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x \\
& + 5/2*c))) - 1280*(5*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2 \\
& *c) + \cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4* \\
& d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) - (10*\cos(2*d*x + 2*c \\
&) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5*d*x + 5*c)*\sin(5/2*d*x \\
& + 5/2*c) - 5*\cos(4*d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3*d*x + 3*c)*\sin \\
& in(5/2*d*x + 5/2*c))*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/ \\
& 2*c))))*\sin(6/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 8*(1 \\
& 28*\cos(5*d*x + 5*c)^2*\cos(5/2*d*x + 5/2*c) + 3200*\cos(4*d*x + 4*c)^2*\cos(5/ \\
& 2*d*x + 5/2*c) + 2560*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x \\
& + 3*c)*\cos(5/2*d*x + 5/2*c) + 12800*\cos(3*d*x + 3*c)^2*\cos(5/2*d*x + 5/2*c) \\
& - 128*\cos(5/2*d*x + 5/2*c)*\sin(5*d*x + 5*c)^2 + 3200*\cos(5/2*d*x + 5/2*c)* \\
& \sin(4*d*x + 4*c)^2 + 12800*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x \\
& + 5/2*c)*\sin(3*d*x + 3*c) + 12800*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c)^2 + \\
& 256*((10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) + 5*c \\
& os(4*d*x + 4*c)*\cos(5/2*d*x + 5/2*c) + 10*\cos(3*d*x + 3*c)*\cos(5/2*d*x + 5/ \\
& 2*c))*\cos(5*d*x + 5*c) + 1280*((10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*c \\
& os(5/2*d*x + 5/2*c) + 10*\cos(3*d*x + 3*c)*\cos(5/2*d*x + 5/2*c))*\cos(4*d*x + \\
& 4*c) + 128*(20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c \\
&)^2 + 25*\cos(d*x + c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin \\
& (d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) - \\
& 70*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5* \\
& \cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x \\
& + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*co \\
& s(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x \\
& + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) \\
& + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin \\
& (3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + \sin(5 \\
& *d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c)) \\
& *\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d \\
& *x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 \\
& + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) \\
& + 1)*\cos(3/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 15*(2*(\\
& 5*\cos(4*d*x + 4*c) + 10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x \\
& + c) + 1)*\cos(5*d*x + 5*c) + \cos(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + \\
& 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x \\
& + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) +
\end{aligned}$$

$$\begin{aligned}
& 100\cos(3d*x + 3*c)^2 + 20*(5\cos(d*x + c) + 1)\cos(2d*x + 2*c) + 100\cos(2d*x + 2*c)^2 + 25\cos(d*x + c)^2 + 10*(\sin(4d*x + 4*c) + 2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(5d*x + 5*c) + \sin(5d*x + 5*c)^2 + 50*(2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(4d*x + 4*c) + 25*\sin(4d*x + 4*c)^2 + 100*(2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(3d*x + 3*c) + 100*\sin(3d*x + 3*c)^2 + 100*\sin(2d*x + 2*c)^2 + 100*\sin(2d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)\cos(1/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 256*((10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) + \cos(5d*x + 5*c)*\sin(5/2*d*x + 5/2*c) + 5*\cos(4d*x + 4*c)*\sin(5/2*d*x + 5/2*c) + 10*\cos(3d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(5d*x + 5*c) + 6400*((2*\sin(2d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + 2*\cos(5/2*d*x + 5/2*c)*\sin(3d*x + 3*c))*\sin(4d*x + 4*c) - 1280*(5*(2*\sin(2d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c) + \cos(5/2*d*x + 5/2*c)*\sin(5d*x + 5*c) + 5*\cos(5/2*d*x + 5/2*c)*\sin(4d*x + 4*c) + 10*\cos(5/2*d*x + 5/2*c)*\sin(3d*x + 3*c) - (10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\sin(5/2*d*x + 5/2*c) - \cos(5d*x + 5*c)*\sin(5/2*d*x + 5/2*c) - 5*\cos(4d*x + 4*c)*\sin(5/2*d*x + 5/2*c) - 10*\cos(3d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 56*((10*\cos(4d*x + 4*c) + 20*\cos(3d*x + 3*c) + 20*\cos(2d*x + 2*c) + 10*\cos(d*x + c) + 3)*\cos(5d*x + 5*c)^2 + \cos(5d*x + 5*c)^3 + (\cos(5d*x + 5*c) + 1)*\sin(5d*x + 5*c)^2 + (10*(10*\cos(3d*x + 3*c) + 10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 2)*\cos(4d*x + 4*c) + 25*\cos(4d*x + 4*c)^2 + 20*(10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 2)*\cos(3d*x + 3*c) + 100*\cos(3d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 2)*\cos(2d*x + 2*c) + 100*\cos(2d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50*(2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(4d*x + 4*c) + 25*\sin(4d*x + 4*c)^2 + 100*(2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(3d*x + 3*c) + 100*\sin(3d*x + 3*c)^2 + 100*\sin(2d*x + 2*c)^2 + 100*\sin(2d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 20*\cos(d*x + c) + 3)*\cos(5d*x + 5*c) + 10*(10*\cos(3d*x + 3*c) + 10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4d*x + 4*c) + 25*\cos(4d*x + 4*c)^2 + 20*(10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3d*x + 3*c) + 100*\cos(3d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2d*x + 2*c) + 100*\cos(2d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 5*(2*(5*\cos(4d*x + 4*c) + 10*\cos(3d*x + 3*c) + 10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5d*x + 5*c) + \cos(5d*x + 5*c)^2 + 10*(10*\cos(3d*x + 3*c) + 10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(4d*x + 4*c) + 25*\cos(4d*x + 4*c)^2 + 20*(10*\cos(2d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(3d*x + 3*c) + 100*\cos(3d*x + 3*c)^2 + 20*(5*\cos(d*x + c) + 1)*\cos(2d*x + 2*c) + 100*\cos(2d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 10*(\sin(4d*x + 4*c) + 2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(5d*x + 5*c) + \sin(5d*x + 5*c)^2 + 50*(2*\sin(3d*x + 3*c) + 2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(4d*x + 4*c) + 25*\sin(4d*x + 4*c)^2 + 100*(2*\sin(2d*x + 2*c) + \sin(d*x + c))*\sin(3d*x + 3*c) + 100*\sin(3d*x + 3*c)^2 + 100*\sin(2d*x + 2*c)^2 + 100*\sin(2d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x +
\end{aligned}$$

$$\begin{aligned}
& 5/2*c))) + 10*((\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) \\
& + \sin(d*x + c))*\cos(5*d*x + 5*c) + \sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + \\
& 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(5*d*x + 5*c) + 50*(2*\sin(3*d*x + 3* \\
& c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4 \\
& *c)^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(\\
& 3*d*x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) \\
& + 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\sin(3/5*\arctan2(\sin(5/2*d*x + 5 \\
& /2*c), \cos(5/2*d*x + 5/2*c))) - 4*(128*\cos(5*d*x + 5*c)^2*\cos(5/2*d*x + 5/2 \\
& *c) + 3200*\cos(4*d*x + 4*c)^2*\cos(5/2*d*x + 5/2*c) + 2560*(10*\cos(2*d*x + 2 \\
& *c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c)*\cos(5/2*d*x + 5/2*c) + 12800*\cos \\
& (3*d*x + 3*c)^2*\cos(5/2*d*x + 5/2*c) - 128*\cos(5/2*d*x + 5/2*c)*\sin(5*d*x + \\
& 5*c)^2 + 3200*\cos(5/2*d*x + 5/2*c)*\sin(4*d*x + 4*c)^2 + 12800*(2*\sin(2*d*x \\
& + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c) + 12800*\cos(5 \\
& /2*d*x + 5/2*c)*\sin(3*d*x + 3*c)^2 + 256*((10*\cos(2*d*x + 2*c) + 5*\cos(d*x \\
& + c) + 1)*\cos(5/2*d*x + 5/2*c) + 5*\cos(4*d*x + 4*c)*\cos(5/2*d*x + 5/2*c) + \\
& 10*\cos(3*d*x + 3*c)*\cos(5/2*d*x + 5/2*c))*\cos(5*d*x + 5*c) + 1280*((10*\cos(\\
& 2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) + 10*\cos(3*d*x + 3* \\
& c)*\cos(5/2*d*x + 5/2*c))*\cos(4*d*x + 4*c) + 128*(20*(5*\cos(d*x + c) + 1)*\co \\
& s(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 100*\sin(2*d*x \\
& + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sin(d*x + c)^2 + 10*\cos(\\
& d*x + c) + 1)*\cos(5/2*d*x + 5/2*c) - 15*(2*(5*\cos(4*d*x + 4*c) + 10*\cos(3*d \\
& *x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1)*\cos(5*d*x + 5*c) + \co \\
& s(5*d*x + 5*c)^2 + 10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d* \\
& x + c) + 1)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2 \\
& *c) + 5*\cos(d*x + c) + 1)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5 \\
& *\cos(d*x + c) + 1)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + \\
& c)^2 + 10*(\sin(4*d*x + 4*c) + 2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \si \\
& n(d*x + c))*\sin(5*d*x + 5*c) + \sin(5*d*x + 5*c)^2 + 50*(2*\sin(3*d*x + 3*c) \\
& + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c) + 25*\sin(4*d*x + 4*c) \\
& ^2 + 100*(2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(3*d*x + 3*c) + 100*\sin(3*d \\
& *x + 3*c)^2 + 100*\sin(2*d*x + 2*c)^2 + 100*\sin(2*d*x + 2*c)*\sin(d*x + c) + \\
& 25*\sin(d*x + c)^2 + 10*\cos(d*x + c) + 1)*\cos(1/5*\arctan2(\sin(5/2*d*x + 5/2* \\
& c), \cos(5/2*d*x + 5/2*c))) + 256*((10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 1 \\
&)*\sin(5/2*d*x + 5/2*c) + \cos(5*d*x + 5*c)*\sin(5/2*d*x + 5/2*c) + 5*\cos(4*d* \\
& x + 4*c)*\sin(5/2*d*x + 5/2*c) + 10*\cos(3*d*x + 3*c)*\sin(5/2*d*x + 5/2*c))*\s \\
& in(5*d*x + 5*c) + 6400*((2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\cos(5/2*d*x + 5 \\
& /2*c) + 2*\cos(5/2*d*x + 5/2*c)*\sin(3*d*x + 3*c))*\sin(4*d*x + 4*c))*\sin(2/5* \\
& arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) - 12*((10*\cos(4*d*x + \\
& 4*c) + 20*\cos(3*d*x + 3*c) + 20*\cos(2*d*x + 2*c) + 10*\cos(d*x + c) + 3)*\cos \\
& (5*d*x + 5*c)^2 + \cos(5*d*x + 5*c)^3 + (\cos(5*d*x + 5*c) + 1)*\sin(5*d*x + 5 \\
& *c)^2 + (10*(10*\cos(3*d*x + 3*c) + 10*\cos(2*d*x + 2*c) + 5*\cos(d*x + c) + 2 \\
&)*\cos(4*d*x + 4*c) + 25*\cos(4*d*x + 4*c)^2 + 20*(10*\cos(2*d*x + 2*c) + 5*\co \\
& s(d*x + c) + 2)*\cos(3*d*x + 3*c) + 100*\cos(3*d*x + 3*c)^2 + 20*(5*\cos(d*x + \\
& c) + 2)*\cos(2*d*x + 2*c) + 100*\cos(2*d*x + 2*c)^2 + 25*\cos(d*x + c)^2 + 50 \\
& *(2*\sin(3*d*x + 3*c) + 2*\sin(2*d*x + 2*c) + \sin(d*x + c))*\sin(4*d*x + 4*c)
\end{aligned}$$

$$\begin{aligned}
& \cos(3dx + 3c)^2 + 100\sqrt{2}a^2\cos(2dx + 2c)^2 + 25\sqrt{2}a^2\cos(dx + c)^2 + \sqrt{2}a^2\sin(5dx + 5c)^2 + 25\sqrt{2}a^2\sin(4dx + 4c)^2 + 100\sqrt{2}a^2\sin(3dx + 3c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)\sin(dx + c) + 25\sqrt{2}a^2\sin(dx + c)^2 + 10\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2 + 2(5\sqrt{2}a^2\cos(4dx + 4c) + 10\sqrt{2}a^2\cos(3dx + 3c) + 10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(5dx + 5c) + 10(10\sqrt{2}a^2\cos(3dx + 3c) + 10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(4dx + 4c) + 20(10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(3dx + 3c) + 20(5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(2dx + 2c) + 10(\sqrt{2}a^2\sin(4dx + 4c) + 2\sqrt{2}a^2\sin(3dx + 3c) + 2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(5dx + 5c) + 50(2\sqrt{2}a^2\sin(3dx + 3c) + 2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(4dx + 4c) + 100(2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(3dx + 3c)\cos(2/5\arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c)))^2 + (2\sqrt{2}a^2\cos(5dx + 5c))^2 + 25\sqrt{2}a^2\cos(4dx + 4c)^2 + 100\sqrt{2}a^2\cos(3dx + 3c)^2 + 100\sqrt{2}a^2\cos(2dx + 2c)^2 + 25\sqrt{2}a^2\cos(dx + c)^2 + 25\sqrt{2}a^2\sin(4dx + 4c)^2 + 100\sqrt{2}a^2\sin(3dx + 3c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)\sin(dx + c) + 25\sqrt{2}a^2\sin(dx + c)^2 + 10\sqrt{2}a^2\cos(dx + c) + 2\sqrt{2}a^2 + 2(5\sqrt{2}a^2\cos(4dx + 4c) + 10\sqrt{2}a^2\cos(3dx + 3c) + 10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + 2\sqrt{2}a^2)\cos(5dx + 5c) + 10(10\sqrt{2}a^2\cos(3dx + 3c) + 10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(4dx + 4c) + 20(10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(3dx + 3c) + 20(5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(2dx + 2c) + 50(2\sqrt{2}a^2\sin(3dx + 3c) + 2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(4dx + 4c) + 100(2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(3dx + 3c)\sin(5dx + 5c)^2 + 25(\sqrt{2}a^2\cos(5dx + 5c))^2 + 25\sqrt{2}a^2\cos(4dx + 4c)^2 + 100\sqrt{2}a^2\cos(3dx + 3c)^2 + 100\sqrt{2}a^2\cos(2dx + 2c)^2 + 25\sqrt{2}a^2\cos(dx + c)^2 + \sqrt{2}a^2\sin(5dx + 5c)^2 + 25\sqrt{2}a^2\sin(4dx + 4c)^2 + 100\sqrt{2}a^2\sin(3dx + 3c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)\sin(dx + c) + 25\sqrt{2}a^2\sin(dx + c)^2 + 10\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2 + 2(5\sqrt{2}a^2\cos(4dx + 4c) + 10\sqrt{2}a^2\cos(3dx + 3c) + 10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(5dx + 5c) + 10(10\sqrt{2}a^2\cos(3dx + 3c) + 10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(4dx + 4c) + 20(10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(3dx + 3c) + 20(5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(2dx + 2c) + 10(\sqrt{2}a^2\sin(4dx + 4c) + 2\sqrt{2}a^2\sin(3dx + 3c) + 2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(5dx + 5c)
\end{aligned}$$

$$\begin{aligned}
& c) + 10\sqrt{2}a^2\cos(3dx + 3c) + 10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2\cos(5dx + 5c) + 10(10\sqrt{2}a^2\cos(3dx + 3c) + 10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2\cos(4dx + 4c) + 20(10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2\cos(3dx + 3c) + 20(5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2\cos(2dx + 2c) + 10(\sqrt{2}a^2\sin(4dx + 4c) + 2\sqrt{2}a^2\sin(3dx + 3c) + 2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(5dx + 5c) + 50(2\sqrt{2}a^2\sin(3dx + 3c) + 2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(4dx + 4c) + 100(2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(3dx + 3c))\sin(2/5\arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c)))^2 + \sqrt{2}a^2 + 2(25\sqrt{2}a^2\cos(4dx + 4c))^2 + 100\sqrt{2}a^2\cos(3dx + 3c)^2 + 100\sqrt{2}a^2\cos(2dx + 2c)^2 + 25\sqrt{2}a^2\cos(dx + c)^2 + 25\sqrt{2}a^2\sin(4dx + 4c)^2 + 100\sqrt{2}a^2\sin(3dx + 3c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)\sin(dx + c) + 25\sqrt{2}a^2\sin(dx + c)^2 + 15\sqrt{2}a^2\cos(dx + c) + 2\sqrt{2}a^2 + 5(20\sqrt{2}a^2\cos(3dx + 3c) + 20\sqrt{2}a^2\cos(2dx + 2c) + 10\sqrt{2}a^2\cos(dx + c) + 3\sqrt{2}a^2)\cos(4dx + 4c) + 10(20\sqrt{2}a^2\cos(2dx + 2c) + 10\sqrt{2}a^2\cos(dx + c) + 3\sqrt{2}a^2)\cos(3dx + 3c) + 10(10\sqrt{2}a^2\cos(dx + c) + 3\sqrt{2}a^2)\cos(2dx + 2c) + 50(2\sqrt{2}a^2\sin(3dx + 3c) + 2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(4dx + 4c) + 100(2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(3dx + 3c))\cos(5dx + 5c) + 10(10\sqrt{2}a^2\cos(3dx + 3c) + 10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(4dx + 4c) + 20(10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(3dx + 3c) + 20(5\sqrt{2}a^2\cos(dx + c) + \sqrt{2}a^2)\cos(2dx + 2c) + 10(\sqrt{2}a^2\cos(5dx + 5c))^3 + 25\sqrt{2}a^2\cos(4dx + 4c)^2 + 100\sqrt{2}a^2\cos(3dx + 3c)^2 + 100\sqrt{2}a^2\cos(2dx + 2c)^2 + 25\sqrt{2}a^2\cos(dx + c)^2 + 25\sqrt{2}a^2\sin(4dx + 4c)^2 + 100\sqrt{2}a^2\sin(3dx + 3c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)\sin(dx + c) + 25\sqrt{2}a^2\sin(dx + c)^2 + 10\sqrt{2}a^2\cos(dx + c) + (10\sqrt{2}a^2\cos(4dx + 4c) + 20\sqrt{2}a^2\cos(3dx + 3c) + 20\sqrt{2}a^2\cos(2dx + 2c) + 10\sqrt{2}a^2\cos(dx + c) + 3\sqrt{2}a^2)\cos(5dx + 5c)^2 + (\sqrt{2}a^2\cos(5dx + 5c) + \sqrt{2}a^2)\sin(5dx + 5c)^2 + \sqrt{2}a^2 + (25\sqrt{2}a^2\cos(4dx + 4c))^2 + 100\sqrt{2}a^2\cos(3dx + 3c)^2 + 100\sqrt{2}a^2\cos(2dx + 2c)^2 + 25\sqrt{2}a^2\cos(dx + c)^2 + 25\sqrt{2}a^2\sin(4dx + 4c)^2 + 100\sqrt{2}a^2\sin(3dx + 3c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)^2 + 100\sqrt{2}a^2\sin(2dx + 2c)\sin(dx + c) + 25\sqrt{2}a^2\sin(dx + c)^2 + 20\sqrt{2}a^2\cos(dx + c) + 3\sqrt{2}a^2 + 10(10\sqrt{2}a^2\cos(3dx + 3c) + 10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + 2\sqrt{2}a^2)\cos(4dx + 4c) + 20(10\sqrt{2}a^2\cos(2dx + 2c) + 5\sqrt{2}a^2\cos(dx + c) + 2\sqrt{2}a^2)\cos(3dx + 3c) + 20(5\sqrt{2}a^2\cos(dx + c) + 2\sqrt{2}a^2)\cos(2dx +
\end{aligned}$$

$$\begin{aligned}
& 2*c) + 50*(2*\sqrt{2}*a^2*\sin(3*d*x + 3*c) + 2*\sqrt{2}*a^2*\sin(2*d*x + 2*c) \\
& + \sqrt{2}*a^2*\sin(d*x + c))*\sin(4*d*x + 4*c) + 100*(2*\sqrt{2}*a^2*\sin(2*d*x \\
& + 2*c) + \sqrt{2}*a^2*\sin(d*x + c))*\sin(3*d*x + 3*c))*\cos(5*d*x + 5*c) + 10 \\
& *(10*\sqrt{2}*a^2*\cos(3*d*x + 3*c) + 10*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2} \\
& *a^2*\cos(d*x + c) + \sqrt{2}*a^2*\cos(4*d*x + 4*c) + 20*(10*\sqrt{2}*a^2* \\
& \cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2}*a^2*\cos(3*d*x + 3* \\
& c) + 20*(5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2}*a^2*\cos(2*d*x + 2*c) + 10*(\sqrt{2} \\
& *a^2*\cos(5*d*x + 5*c))^2 + 25*\sqrt{2}*a^2*\cos(4*d*x + 4*c))^2 + 100*\sqrt{2} \\
& *a^2*\cos(3*d*x + 3*c))^2 + 100*\sqrt{2}*a^2*\cos(2*d*x + 2*c))^2 + 25*\sqrt{2} \\
& *a^2*\cos(d*x + c))^2 + \sqrt{2}*a^2*\sin(5*d*x + 5*c))^2 + 25*\sqrt{2}*a^2*\sin \\
& (4*d*x + 4*c))^2 + 100*\sqrt{2}*a^2*\sin(3*d*x + 3*c))^2 + 100*\sqrt{2}*a^2*\sin \\
& (2*d*x + 2*c))^2 + 100*\sqrt{2}*a^2*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\sqrt{2} \\
& *a^2*\sin(d*x + c))^2 + 10*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2}*a^2 + 2*(5*\sqrt{2} \\
& *a^2*\cos(4*d*x + 4*c) + 10*\sqrt{2}*a^2*\cos(3*d*x + 3*c) + 10*\sqrt{2}*a^2 \\
& *\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2}*a^2*\cos(5*d*x + 5 \\
& *c) + 10*(10*\sqrt{2}*a^2*\cos(3*d*x + 3*c) + 10*\sqrt{2}*a^2*\cos(2*d*x + 2*c) \\
& + 5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2}*a^2*\cos(4*d*x + 4*c) + 20*(10*\sqrt{2} \\
& *a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2}*a^2*\cos(3* \\
& d*x + 3*c) + 20*(5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2}*a^2*\cos(2*d*x + 2*c) \\
& + 10*(\sqrt{2}*a^2*\sin(4*d*x + 4*c) + 2*\sqrt{2}*a^2*\sin(3*d*x + 3*c) + 2*\sqrt{2} \\
& *a^2*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(d*x + c))*\sin(5*d*x + 5*c) + 5 \\
& 0*(2*\sqrt{2}*a^2*\sin(3*d*x + 3*c) + 2*\sqrt{2}*a^2*\sin(2*d*x + 2*c) + \sqrt{2} \\
&)*a^2*\sin(d*x + c))*\sin(4*d*x + 4*c) + 100*(2*\sqrt{2}*a^2*\sin(2*d*x + 2*c) \\
& + \sqrt{2}*a^2*\sin(d*x + c))*\sin(3*d*x + 3*c))*\cos(6/5*\arctan2(\sin(5/2*d*x + \\
& 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*(\sqrt{2}*a^2*\cos(5*d*x + 5*c))^2 + 25*\sqrt{2} \\
& *a^2*\cos(4*d*x + 4*c))^2 + 100*\sqrt{2}*a^2*\cos(3*d*x + 3*c))^2 + 100*\sqrt{2} \\
& *a^2*\cos(2*d*x + 2*c))^2 + 25*\sqrt{2}*a^2*\cos(d*x + c))^2 + \sqrt{2}*a^2* \\
& \sin(5*d*x + 5*c))^2 + 25*\sqrt{2}*a^2*\sin(4*d*x + 4*c))^2 + 100*\sqrt{2}*a^2*\sin \\
& (3*d*x + 3*c))^2 + 100*\sqrt{2}*a^2*\sin(2*d*x + 2*c))^2 + 100*\sqrt{2}*a^2*\sin \\
& (2*d*x + 2*c)*\sin(d*x + c) + 25*\sqrt{2}*a^2*\sin(d*x + c))^2 + 10*\sqrt{2}*a^2 \\
& *\cos(d*x + c) + \sqrt{2}*a^2 + 2*(5*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 10*\sqrt{2} \\
&)*a^2*\cos(3*d*x + 3*c) + 10*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos \\
& (d*x + c) + \sqrt{2}*a^2*\cos(5*d*x + 5*c) + 10*(10*\sqrt{2}*a^2*\cos(3*d*x + \\
& 3*c) + 10*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2} \\
& *a^2*\cos(4*d*x + 4*c) + 20*(10*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2} \\
& *a^2*\cos(d*x + c) + \sqrt{2}*a^2*\cos(3*d*x + 3*c) + 20*(5*\sqrt{2}*a^2*\cos(d \\
& *x + c) + \sqrt{2}*a^2*\cos(2*d*x + 2*c) + 10*(\sqrt{2}*a^2*\sin(4*d*x + 4*c) \\
& + 2*\sqrt{2}*a^2*\sin(3*d*x + 3*c) + 2*\sqrt{2}*a^2*\sin(2*d*x + 2*c) + \sqrt{2} \\
& *a^2*\sin(d*x + c))*\sin(5*d*x + 5*c) + 50*(2*\sqrt{2}*a^2*\sin(3*d*x + 3*c) + \\
& 2*\sqrt{2}*a^2*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(d*x + c))*\sin(4*d*x + 4*c) \\
& + 100*(2*\sqrt{2}*a^2*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(d*x + c))*\sin(3*d* \\
& x + 3*c))*\cos(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 5* \\
& (\sqrt{2}*a^2*\cos(5*d*x + 5*c))^2 + 25*\sqrt{2}*a^2*\cos(4*d*x + 4*c))^2 + 100*\sqrt{2} \\
& *a^2*\cos(3*d*x + 3*c))^2 + 100*\sqrt{2}*a^2*\cos(2*d*x + 2*c))^2 + 25*\sqrt{2} \\
& *a^2*\cos(d*x + c))^2 + \sqrt{2}*a^2*\sin(5*d*x + 5*c))^2 + 25*\sqrt{2}*a^2*s
\end{aligned}$$

$$\begin{aligned}
& c) + \sqrt{2}a^2 \cos(3dx + 3c) + 20(5\sqrt{2}a^2 \cos(dx + c) + \sqrt{2} \\
& (2)a^2 \cos(2dx + 2c) + 10(\sqrt{2}a^2 \cos(5dx + 5c)^2 + 25\sqrt{2} \\
& a^2 \cos(4dx + 4c)^2 + 100\sqrt{2}a^2 \cos(3dx + 3c)^2 + 100\sqrt{2} \\
& a^2 \cos(2dx + 2c)^2 + 25\sqrt{2}a^2 \cos(dx + c)^2 + \sqrt{2}a^2 \sin(5 \\
& dx + 5c)^2 + 25\sqrt{2}a^2 \sin(4dx + 4c)^2 + 100\sqrt{2}a^2 \sin(3dx \\
& x + 3c)^2 + 100\sqrt{2}a^2 \sin(2dx + 2c)^2 + 100\sqrt{2}a^2 \sin(2dx \\
& + 2c) \sin(dx + c) + 25\sqrt{2}a^2 \sin(dx + c)^2 + 10\sqrt{2}a^2 \cos(dx \\
& + c) + \sqrt{2}a^2 + 2(5\sqrt{2}a^2 \cos(4dx + 4c) + 10\sqrt{2}a^2 \\
& \cos(3dx + 3c) + 10\sqrt{2}a^2 \cos(2dx + 2c) + 5\sqrt{2}a^2 \cos(dx \\
& + c) + \sqrt{2}a^2) \cos(5dx + 5c) + 10(10\sqrt{2}a^2 \cos(3dx + 3c) \\
& + 10\sqrt{2}a^2 \cos(2dx + 2c) + 5\sqrt{2}a^2 \cos(dx + c) + \sqrt{2}a^2 \\
& 2) \cos(4dx + 4c) + 20(10\sqrt{2}a^2 \cos(2dx + 2c) + 5\sqrt{2}a^2 \cos \\
& (dx + c) + \sqrt{2}a^2) \cos(3dx + 3c) + 20(5\sqrt{2}a^2 \cos(dx + c) \\
&) + \sqrt{2}a^2) \cos(2dx + 2c) + 10(\sqrt{2}a^2 \sin(4dx + 4c) + 2\sqrt{2} \\
& \sqrt{2}a^2 \sin(3dx + 3c) + 2\sqrt{2}a^2 \sin(2dx + 2c) + \sqrt{2}a^2 \sin \\
& (dx + c)) \sin(5dx + 5c) + 50(2\sqrt{2}a^2 \sin(3dx + 3c) + 2\sqrt{2} \\
& (2)a^2 \sin(2dx + 2c) + \sqrt{2}a^2 \sin(dx + c)) \sin(4dx + 4c) + 100 \\
& (2\sqrt{2}a^2 \sin(2dx + 2c) + \sqrt{2}a^2 \sin(dx + c)) \sin(3dx + 3c) \\
&)) \cos(4/5 \arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) + 5(\sqrt{2} \\
& (2)a^2 \cos(5dx + 5c)^2 + 25\sqrt{2}a^2 \cos(4dx + 4c)^2 + 100\sqrt{2} \\
& a^2 \cos(3dx + 3c)^2 + 100\sqrt{2}a^2 \cos(2dx + 2c)^2 + 25\sqrt{2}a^2 \\
& a^2 \cos(dx + c)^2 + \sqrt{2}a^2 \sin(5dx + 5c)^2 + 25\sqrt{2}a^2 \sin(4dx \\
& *x + 4c)^2 + 100\sqrt{2}a^2 \sin(3dx + 3c)^2 + 100\sqrt{2}a^2 \sin(2dx \\
& x + 2c)^2 + 100\sqrt{2}a^2 \sin(2dx + 2c) \sin(dx + c) + 25\sqrt{2}a^2 \\
& * \sin(dx + c)^2 + 10\sqrt{2}a^2 \cos(dx + c) + \sqrt{2}a^2 + 2(5\sqrt{2}a^2 \\
& a^2 \cos(4dx + 4c) + 10\sqrt{2}a^2 \cos(3dx + 3c) + 10\sqrt{2}a^2 \cos \\
& (2dx + 2c) + 5\sqrt{2}a^2 \cos(dx + c) + \sqrt{2}a^2) \cos(5dx + 5c) \\
& + 10(10\sqrt{2}a^2 \cos(3dx + 3c) + 10\sqrt{2}a^2 \cos(2dx + 2c) + 5 \\
& * \sqrt{2}a^2 \cos(dx + c) + \sqrt{2}a^2) \cos(4dx + 4c) + 20(10\sqrt{2}a^2 \\
& a^2 \cos(2dx + 2c) + 5\sqrt{2}a^2 \cos(dx + c) + \sqrt{2}a^2) \cos(3dx \\
& + 3c) + 20(5\sqrt{2}a^2 \cos(dx + c) + \sqrt{2}a^2) \cos(2dx + 2c) + 1 \\
& 0(\sqrt{2}a^2 \sin(4dx + 4c) + 2\sqrt{2}a^2 \sin(3dx + 3c) + 2\sqrt{2} \\
& (2) \\
&)a^2 \sin(2dx + 2c) + \sqrt{2}a^2 \sin(dx + c)) \sin(5dx + 5c) + 50(2 \\
& * \sqrt{2}a^2 \sin(3dx + 3c) + 2\sqrt{2}a^2 \sin(2dx + 2c) + \sqrt{2}a^2 \\
& 2) \sin(dx + c)) \sin(4dx + 4c) + 100(2\sqrt{2}a^2 \sin(2dx + 2c) + \sqrt{2} \\
& (2) \\
&)a^2 \sin(dx + c)) \sin(3dx + 3c)) \cos(2/5 \arctan2(\sin(5/2dx + 5/2 \\
& *c), \cos(5/2dx + 5/2c))) + 10(\sqrt{2}a^2 \sin(4dx + 4c) + 2\sqrt{2} \\
& (2) \\
&)a^2 \sin(3dx + 3c) + 2\sqrt{2}a^2 \sin(2dx + 2c) + \sqrt{2}a^2 \sin(dx \\
& + c) + (\sqrt{2}a^2 \sin(4dx + 4c) + 2\sqrt{2}a^2 \sin(3dx + 3c) + 2 \\
& * \sqrt{2}a^2 \sin(2dx + 2c) + \sqrt{2}a^2 \sin(dx + c)) \cos(5dx + 5c)) \\
& \sin(5dx + 5c) + 50(2\sqrt{2}a^2 \sin(3dx + 3c) + 2\sqrt{2}a^2 \sin(2 \\
& *dx + 2c) + \sqrt{2}a^2 \sin(dx + c)) \sin(4dx + 4c) + 100(2\sqrt{2}a^2 \\
& \sin(2dx + 2c) + \sqrt{2}a^2 \sin(dx + c)) \sin(3dx + 3c)) \cos(6/5 \arctan2 \\
& (\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) + 20(\sqrt{2}a^2 \cos(5 \\
& *dx + 5c)^3 + 25\sqrt{2}a^2 \cos(4dx + 4c)^2 + 100\sqrt{2}a^2 \cos(3dx
\end{aligned}$$

$$\begin{aligned}
& *x + 3*c)^2 + 100*\sqrt{2}*a^2*\cos(2*d*x + 2*c)^2 + 25*\sqrt{2}*a^2*\cos(d*x + \\
& c)^2 + 25*\sqrt{2}*a^2*\sin(4*d*x + 4*c)^2 + 100*\sqrt{2}*a^2*\sin(3*d*x + 3*c \\
&)^2 + 100*\sqrt{2}*a^2*\sin(2*d*x + 2*c)^2 + 100*\sqrt{2}*a^2*\sin(2*d*x + 2*c) \\
& *\sin(d*x + c) + 25*\sqrt{2}*a^2*\sin(d*x + c)^2 + 10*\sqrt{2}*a^2*\cos(d*x + c) \\
& + (10*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 20*\sqrt{2}*a^2*\cos(3*d*x + 3*c) + 20* \\
& \sqrt{2}*a^2*\cos(2*d*x + 2*c) + 10*\sqrt{2}*a^2*\cos(d*x + c) + 3*\sqrt{2}*a^2) \\
& *\cos(5*d*x + 5*c)^2 + (\sqrt{2}*a^2*\cos(5*d*x + 5*c) + \sqrt{2}*a^2)*\sin(5*d* \\
& x + 5*c)^2 + \sqrt{2}*a^2 + (25*\sqrt{2}*a^2*\cos(4*d*x + 4*c)^2 + 100*\sqrt{2}) \\
& *a^2*\cos(3*d*x + 3*c)^2 + 100*\sqrt{2}*a^2*\cos(2*d*x + 2*c)^2 + 25*\sqrt{2}*a \\
& ^2*\cos(d*x + c)^2 + 25*\sqrt{2}*a^2*\sin(4*d*x + 4*c)^2 + 100*\sqrt{2}*a^2*\sin \\
& (3*d*x + 3*c)^2 + 100*\sqrt{2}*a^2*\sin(2*d*x + 2*c)^2 + 100*\sqrt{2}*a^2*\sin \\
& (2*d*x + 2*c)*\sin(d*x + c) + 25*\sqrt{2}*a^2*\sin(d*x + c)^2 + 20*\sqrt{2}*a^2* \\
& \cos(d*x + c) + 3*\sqrt{2}*a^2 + 10*(10*\sqrt{2}*a^2*\cos(3*d*x + 3*c) + 10*\sqrt{2} \\
& *a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x + c) + 2*\sqrt{2}*a^2)*\cos \\
& (4*d*x + 4*c) + 20*(10*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x \\
& + c) + 2*\sqrt{2}*a^2)*\cos(3*d*x + 3*c) + 20*(5*\sqrt{2}*a^2*\cos(d*x + c) + \\
& 2*\sqrt{2}*a^2)*\cos(2*d*x + 2*c) + 50*(2*\sqrt{2}*a^2*\sin(3*d*x + 3*c) + 2*\sqrt{2} \\
& *a^2*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(d*x + c))*\sin(4*d*x + 4*c) + 1 \\
& 00*(2*\sqrt{2}*a^2*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(d*x + c))*\sin(3*d*x + \\
& 3*c))*\cos(5*d*x + 5*c) + 10*(10*\sqrt{2}*a^2*\cos(3*d*x + 3*c) + 10*\sqrt{2}*a \\
& ^2*\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2}*a^2)*\cos(4*d*x + \\
& 4*c) + 20*(10*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x + c) + \\
& \sqrt{2}*a^2)*\cos(3*d*x + 3*c) + 20*(5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2}*a^2) \\
& *\cos(2*d*x + 2*c) + 5*(\sqrt{2}*a^2*\cos(5*d*x + 5*c)^2 + 25*\sqrt{2}*a^2*\cos \\
& (4*d*x + 4*c)^2 + 100*\sqrt{2}*a^2*\cos(3*d*x + 3*c)^2 + 100*\sqrt{2}*a^2*\cos \\
& (2*d*x + 2*c)^2 + 25*\sqrt{2}*a^2*\cos(d*x + c)^2 + \sqrt{2}*a^2*\sin(5*d*x + 5 \\
& *c)^2 + 25*\sqrt{2}*a^2*\sin(4*d*x + 4*c)^2 + 100*\sqrt{2}*a^2*\sin(3*d*x + 3*c \\
&)^2 + 100*\sqrt{2}*a^2*\sin(2*d*x + 2*c)^2 + 100*\sqrt{2}*a^2*\sin(2*d*x + 2*c) \\
& *\sin(d*x + c) + 25*\sqrt{2}*a^2*\sin(d*x + c)^2 + 10*\sqrt{2}*a^2*\cos(d*x + c) \\
& + \sqrt{2}*a^2 + 2*(5*\sqrt{2}*a^2*\cos(4*d*x + 4*c) + 10*\sqrt{2}*a^2*\cos(3*d \\
& *x + 3*c) + 10*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x + c) + \\
& \sqrt{2}*a^2)*\cos(5*d*x + 5*c) + 10*(10*\sqrt{2}*a^2*\cos(3*d*x + 3*c) + 10*\sqrt{2} \\
& *a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2}*a^2)*\cos(4 \\
& *d*x + 4*c) + 20*(10*\sqrt{2}*a^2*\cos(2*d*x + 2*c) + 5*\sqrt{2}*a^2*\cos(d*x \\
& + c) + \sqrt{2}*a^2)*\cos(3*d*x + 3*c) + 20*(5*\sqrt{2}*a^2*\cos(d*x + c) + \sqrt{2} \\
& *a^2)*\cos(2*d*x + 2*c) + 10*(\sqrt{2}*a^2*\sin(4*d*x + 4*c) + 2*\sqrt{2}*a^2 \\
& *\sin(3*d*x + 3*c) + 2*\sqrt{2}*a^2*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(d*x \\
& + c))*\sin(5*d*x + 5*c) + 50*(2*\sqrt{2}*a^2*\sin(3*d*x + 3*c) + 2*\sqrt{2}*a^2 \\
& *\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(d*x + c))*\sin(4*d*x + 4*c) + 100*(2*\sqrt{2} \\
& *a^2*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin(d*x + c))*\sin(3*d*x + 3*c))*\cos \\
& (2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*(\sqrt{2}*a^2 \\
& *\sin(4*d*x + 4*c) + 2*\sqrt{2}*a^2*\sin(3*d*x + 3*c) + 2*\sqrt{2}*a^2*\sin(2*d* \\
& x + 2*c) + \sqrt{2}*a^2*\sin(d*x + c) + (\sqrt{2}*a^2*\sin(4*d*x + 4*c) + 2*\sqrt{2} \\
& *a^2*\sin(3*d*x + 3*c) + 2*\sqrt{2}*a^2*\sin(2*d*x + 2*c) + \sqrt{2}*a^2*\sin \\
& (d*x + c))*\cos(5*d*x + 5*c))*\sin(5*d*x + 5*c) + 50*(2*\sqrt{2}*a^2*\sin(3*d*
\end{aligned}$$

$$\begin{aligned}
& x + 3c) + 2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c))\sin(4 \\
& *dx + 4c) + 100*(2\sqrt{2}a^2\sin(2dx + 2c) + \sqrt{2}a^2\sin(dx + c \\
&))*\sin(3dx + 3c))*\cos(4/5*\arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/ \\
& 2c))) + 10*(\sqrt{2}a^2\cos(5dx + 5c)^3 + 25\sqrt{2}a^2\cos(4dx + 4* \\
& c)^2 + 100\sqrt{2}a^2\cos(3dx + 3c)^2 + 100\sqrt{2}a^2\cos(2dx + 2*c \\
&)^2 + 25\sqrt{2}a^2\cos(dx + c)^2 + 25\sqrt{2}a^2\sin(4dx + 4*c)^2 + 1 \\
& 00\sqrt{2}a^2\sin(3dx + 3c)^2 + 100\sqrt{2}a^2\sin(2dx + 2*c)^2 + 10 \\
& 0\sqrt{2}a^2\sin(2dx + 2*c)*\sin(dx + c) + 25\sqrt{2}a^2\sin(dx + c)^2 \\
& + 10\sqrt{2}a^2\cos(dx + c) + (10\sqrt{2}a^2\cos(4dx + 4*c) + 20\sqrt{2} \\
& (2)a^2\cos(3dx + 3*c) + 20\sqrt{2}a^2\cos(2dx + 2*c) + 10\sqrt{2}a^2 \\
& *cos(dx + c) + 3\sqrt{2}a^2*\cos(5dx + 5*c)^2 + (\sqrt{2}a^2*\cos(5dx \\
& + 5*c) + \sqrt{2}a^2)*\sin(5dx + 5*c)^2 + \sqrt{2}a^2 + (25\sqrt{2}a^2*co \\
& s(4dx + 4*c)^2 + 100\sqrt{2}a^2*\cos(3dx + 3*c)^2 + 100\sqrt{2}a^2*\cos \\
& (2dx + 2*c)^2 + 25\sqrt{2}a^2*\cos(dx + c)^2 + 25\sqrt{2}a^2*\sin(4dx \\
& + 4*c)^2 + 100\sqrt{2}a^2*\sin(3dx + 3*c)^2 + 100\sqrt{2}a^2*\sin(2dx + \\
& 2*c)^2 + 100\sqrt{2}a^2*\sin(2dx + 2*c)*\sin(dx + c) + 25\sqrt{2}a^2*si \\
& n(dx + c)^2 + 20\sqrt{2}a^2*\cos(dx + c) + 3\sqrt{2}a^2 + 10*(10\sqrt{2}(2) \\
& *a^2*\cos(3dx + 3*c) + 10\sqrt{2}a^2*\cos(2dx + 2*c) + 5\sqrt{2}a^2*\cos \\
& (dx + c) + 2\sqrt{2}a^2)*\cos(4dx + 4*c) + 20*(10\sqrt{2}a^2*\cos(2dx \\
& + 2*c) + 5\sqrt{2}a^2*\cos(dx + c) + 2\sqrt{2}a^2)*\cos(3dx + 3*c) + 20* \\
& (5\sqrt{2}a^2*\cos(dx + c) + 2\sqrt{2}a^2)*\cos(2dx + 2*c) + 50*(2\sqrt{2}(2) \\
& *a^2*\sin(3dx + 3*c) + 2\sqrt{2}a^2*\sin(2dx + 2*c) + \sqrt{2}a^2*\sin(\\
& dx + c))*\sin(4dx + 4*c) + 100*(2\sqrt{2}a^2*\sin(2dx + 2*c) + \sqrt{2}a^2* \\
& \sin(dx + c))*\sin(3dx + 3*c))*\cos(5dx + 5*c) + 10*(10\sqrt{2}a^2*c \\
& os(3dx + 3*c) + 10\sqrt{2}a^2*\cos(2dx + 2*c) + 5\sqrt{2}a^2*\cos(dx + \\
& c) + \sqrt{2}a^2)*\cos(4dx + 4*c) + 20*(10\sqrt{2}a^2*\cos(2dx + 2*c) + \\
& 5\sqrt{2}a^2*\cos(dx + c) + \sqrt{2}a^2)*\cos(3dx + 3*c) + 20*(5\sqrt{2}(2) \\
& *a^2*\cos(dx + c) + \sqrt{2}a^2)*\cos(2dx + 2*c) + 10*(\sqrt{2}a^2*\sin(4d \\
& *x + 4*c) + 2\sqrt{2}a^2*\sin(3dx + 3*c) + 2\sqrt{2}a^2*\sin(2dx + 2*c) \\
& + \sqrt{2}a^2*\sin(dx + c) + (\sqrt{2}a^2*\sin(4dx + 4*c) + 2\sqrt{2}a^2 \\
& *sin(3dx + 3*c) + 2\sqrt{2}a^2*\sin(2dx + 2*c) + \sqrt{2}a^2*\sin(dx + \\
& c))*\cos(5dx + 5*c))*\sin(5dx + 5*c) + 50*(2\sqrt{2}a^2*\sin(3dx + 3*c) \\
& + 2\sqrt{2}a^2*\sin(2dx + 2*c) + \sqrt{2}a^2*\sin(dx + c))*\sin(4dx + 4 \\
& *c) + 100*(2\sqrt{2}a^2*\sin(2dx + 2*c) + \sqrt{2}a^2*\sin(dx + c))*\sin(3 \\
& *dx + 3*c))*\cos(2/5*\arctan2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) + \\
& 10*(\sqrt{2}a^2*\sin(4dx + 4*c) + 2\sqrt{2}a^2*\sin(3dx + 3*c) + 2\sqrt{2} \\
& (2)a^2*\sin(2dx + 2*c) + \sqrt{2}a^2*\sin(dx + c) + (\sqrt{2}a^2*\sin(4dx \\
& + 4*c) + 2\sqrt{2}a^2*\sin(3dx + 3*c) + 2\sqrt{2}a^2*\sin(2dx + 2*c) \\
& + \sqrt{2}a^2*\sin(dx + c))*\cos(5dx + 5*c)^2 + 2*(\sqrt{2}a^2*\sin(4dx + \\
& 4*c) + 2\sqrt{2}a^2*\sin(3dx + 3*c) + 2\sqrt{2}a^2*\sin(2dx + 2*c) + s \\
&qrt(2)a^2*\sin(dx + c))*\cos(5dx + 5*c))*\sin(5dx + 5*c) + 50*(2\sqrt{2}(2) \\
& *a^2*\sin(3dx + 3*c) + 2\sqrt{2}a^2*\sin(2dx + 2*c) + \sqrt{2}a^2*\sin(dx \\
& + c))*\sin(4dx + 4*c) + 100*(2\sqrt{2}a^2*\sin(2dx + 2*c) + \sqrt{2}a^2 \\
& *sin(dx + c))*\sin(3dx + 3*c) + 10*(\sqrt{2}a^2*\sin(5dx + 5*c)^3 + 10* \\
& (\sqrt{2}a^2*\sin(4dx + 4*c) + 2\sqrt{2}a^2*\sin(3dx + 3*c) + 2\sqrt{2}a^2*
\end{aligned}$$

$$\begin{aligned}
& a^2 \sin(2dx + 2c) + \sqrt{2} a^2 \sin(dx + c) \sin(5dx + 5c)^2 + (\sqrt{2} a^2 \cos(5dx + 5c)^2 + 25\sqrt{2} a^2 \cos(4dx + 4c)^2 + 100\sqrt{2} a^2 \cos(3dx + 3c)^2 + 100\sqrt{2} a^2 \cos(2dx + 2c)^2 + 25\sqrt{2} a^2 \cos(dx + c)^2 + 25\sqrt{2} a^2 \sin(4dx + 4c)^2 + 100\sqrt{2} a^2 \sin(3dx + 3c)^2 + 100\sqrt{2} a^2 \sin(2dx + 2c)^2 + 100\sqrt{2} a^2 \sin(2dx + 2c) \sin(dx + c) + 25\sqrt{2} a^2 \sin(dx + c)^2 + 10\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2 + 2(5\sqrt{2} a^2 \cos(4dx + 4c) + 10\sqrt{2} a^2 \cos(3dx + 3c) + 10\sqrt{2} a^2 \cos(2dx + 2c) + 5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(5dx + 5c) + 10(10\sqrt{2} a^2 \cos(3dx + 3c) + 10\sqrt{2} a^2 \cos(2dx + 2c) + 5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(4dx + 4c) + 20(10\sqrt{2} a^2 \cos(2dx + 2c) + 5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(3dx + 3c) + 20(5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(2dx + 2c) + 50(2\sqrt{2} a^2 \sin(3dx + 3c) + 2\sqrt{2} a^2 \sin(2dx + 2c) + \sqrt{2} a^2 \sin(dx + c)) \sin(4dx + 4c) + 100(2\sqrt{2} a^2 \sin(2dx + 2c) + \sqrt{2} a^2 \sin(dx + c)) \sin(3dx + 3c) \sin(5dx + 5c) + 10(\sqrt{2} a^2 \cos(5dx + 5c)^2 + 25\sqrt{2} a^2 \cos(4dx + 4c)^2 + 100\sqrt{2} a^2 \cos(3dx + 3c)^2 + 100\sqrt{2} a^2 \cos(2dx + 2c)^2 + 25\sqrt{2} a^2 \cos(dx + c)^2 + \sqrt{2} a^2 \sin(5dx + 5c)^2 + 25\sqrt{2} a^2 \sin(4dx + 4c)^2 + 100\sqrt{2} a^2 \sin(3dx + 3c)^2 + 100\sqrt{2} a^2 \sin(2dx + 2c)^2 + 100\sqrt{2} a^2 \sin(2dx + 2c) \sin(dx + c) + 25\sqrt{2} a^2 \sin(dx + c)^2 + 10\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2 + 2(5\sqrt{2} a^2 \cos(4dx + 4c) + 10\sqrt{2} a^2 \cos(3dx + 3c) + 10\sqrt{2} a^2 \cos(2dx + 2c) + 5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(5dx + 5c) + 10(10\sqrt{2} a^2 \cos(3dx + 3c) + 10\sqrt{2} a^2 \cos(2dx + 2c) + 5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(4dx + 4c) + 20(10\sqrt{2} a^2 \cos(2dx + 2c) + 5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(3dx + 3c) + 20(5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(2dx + 2c) + 10(\sqrt{2} a^2 \sin(4dx + 4c) + 2\sqrt{2} a^2 \sin(3dx + 3c) + 2\sqrt{2} a^2 \sin(2dx + 2c) + \sqrt{2} a^2 \sin(dx + c)) \sin(5dx + 5c) + 50(2\sqrt{2} a^2 \sin(3dx + 3c) + 2\sqrt{2} a^2 \sin(2dx + 2c) + \sqrt{2} a^2 \sin(dx + c)) \sin(4dx + 4c) + 100(2\sqrt{2} a^2 \sin(2dx + 2c) + \sqrt{2} a^2 \sin(dx + c)) \sin(3dx + 3c) \sin(6/5 \arctan 2(\sin(5/2dx + 5/2c), \cos(5/2dx + 5/2c))) + 10(\sqrt{2} a^2 \cos(5dx + 5c)^2 + 25\sqrt{2} a^2 \cos(4dx + 4c)^2 + 100\sqrt{2} a^2 \cos(3dx + 3c)^2 + 100\sqrt{2} a^2 \cos(2dx + 2c)^2 + 25\sqrt{2} a^2 \cos(dx + c)^2 + \sqrt{2} a^2 \sin(5dx + 5c)^2 + 25\sqrt{2} a^2 \sin(4dx + 4c)^2 + 100\sqrt{2} a^2 \sin(3dx + 3c)^2 + 100\sqrt{2} a^2 \sin(2dx + 2c)^2 + 100\sqrt{2} a^2 \sin(2dx + 2c) \sin(dx + c) + 25\sqrt{2} a^2 \sin(dx + c)^2 + 10\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2 + 2(5\sqrt{2} a^2 \cos(4dx + 4c) + 10\sqrt{2} a^2 \cos(3dx + 3c) + 10\sqrt{2} a^2 \cos(2dx + 2c) + 5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(5dx + 5c) + 10(10\sqrt{2} a^2 \cos(3dx + 3c) + 10\sqrt{2} a^2 \cos(2dx + 2c) + 5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(4dx + 4c) + 20(10\sqrt{2} a^2 \cos(2dx + 2c) + 5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(3dx + 3c) + 20(5\sqrt{2} a^2 \cos(dx + c) + \sqrt{2} a^2) \cos(2dx + 2c)
\end{aligned}$$

$$\begin{aligned}
& (2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2*\cos(2*d*x + 2*c) + 50*(2*\text{sqrt}(2)*a^2*\sin(3*d*x + 3*c) + 2*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c) + \text{sqrt}(2)*a^2*\sin(d*x + c)) \\
& * \sin(4*d*x + 4*c) + 100*(2*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c) + \text{sqrt}(2)*a^2*\sin(d*x + c)) * \sin(3*d*x + 3*c)) * \sin(5*d*x + 5*c) + 5*(\text{sqrt}(2)*a^2*\cos(5*d*x + 5*c)^2 + 25*\text{sqrt}(2)*a^2*\cos(4*d*x + 4*c)^2 + 100*\text{sqrt}(2)*a^2*\cos(3*d*x + 3*c)^2 + 100*\text{sqrt}(2)*a^2*\cos(2*d*x + 2*c)^2 + 25*\text{sqrt}(2)*a^2*\cos(d*x + c)^2 + \text{sqrt}(2)*a^2*\sin(5*d*x + 5*c)^2 + 25*\text{sqrt}(2)*a^2*\sin(4*d*x + 4*c)^2 + 100*\text{sqrt}(2)*a^2*\sin(3*d*x + 3*c)^2 + 100*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c)^2 + 100*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\text{sqrt}(2)*a^2*\sin(d*x + c)^2 + 10*\text{sqrt}(2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2 + 2*(5*\text{sqrt}(2)*a^2*\cos(4*d*x + 4*c) + 10*\text{sqrt}(2)*a^2*\cos(3*d*x + 3*c) + 10*\text{sqrt}(2)*a^2*\cos(2*d*x + 2*c) + 5*\text{sqrt}(2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2)*\cos(5*d*x + 5*c) + 10*(10*\text{sqrt}(2)*a^2*\cos(3*d*x + 3*c) + 10*\text{sqrt}(2)*a^2*\cos(2*d*x + 2*c) + 5*\text{sqrt}(2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2)*\cos(4*d*x + 4*c) + 20*(10*\text{sqrt}(2)*a^2*\cos(2*d*x + 2*c) + 5*\text{sqrt}(2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2)*\cos(3*d*x + 3*c) + 20*(5*\text{sqrt}(2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2)*\cos(2*d*x + 2*c) + 10*(\text{sqrt}(2)*a^2*\sin(4*d*x + 4*c) + 2*\text{sqrt}(2)*a^2*\sin(3*d*x + 3*c) + 2*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c) + \text{sqrt}(2)*a^2*\sin(d*x + c))*\sin(5*d*x + 5*c) + 50*(2*\text{sqrt}(2)*a^2*\sin(3*d*x + 3*c) + 2*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c) + \text{sqrt}(2)*a^2*\sin(d*x + c))*\sin(4*d*x + 4*c) + 100*(2*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c) + \text{sqrt}(2)*a^2*\sin(d*x + c))*\sin(3*d*x + 3*c)) * \sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) * \sin(4/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) + 10*(\text{sqrt}(2)*a^2*\sin(5*d*x + 5*c)^3 + 10*(\text{sqrt}(2)*a^2*\sin(4*d*x + 4*c) + 2*\text{sqrt}(2)*a^2*\sin(3*d*x + 3*c) + 2*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c) + \text{sqrt}(2)*a^2*\sin(d*x + c))*\sin(5*d*x + 5*c)^2 + (\text{sqrt}(2)*a^2*\cos(5*d*x + 5*c)^2 + 25*\text{sqrt}(2)*a^2*\cos(4*d*x + 4*c)^2 + 100*\text{sqrt}(2)*a^2*\cos(3*d*x + 3*c)^2 + 100*\text{sqrt}(2)*a^2*\cos(2*d*x + 2*c)^2 + 25*\text{sqrt}(2)*a^2*\cos(d*x + c)^2 + 25*\text{sqrt}(2)*a^2*\sin(4*d*x + 4*c)^2 + 100*\text{sqrt}(2)*a^2*\sin(3*d*x + 3*c)^2 + 100*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c)^2 + 100*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c)*\sin(d*x + c) + 25*\text{sqrt}(2)*a^2*\sin(d*x + c)^2 + 10*\text{sqrt}(2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2 + 2*(5*\text{sqrt}(2)*a^2*\cos(4*d*x + 4*c) + 10*\text{sqrt}(2)*a^2*\cos(3*d*x + 3*c) + 10*\text{sqrt}(2)*a^2*\cos(2*d*x + 2*c) + 5*\text{sqrt}(2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2)*\cos(5*d*x + 5*c) + 10*(10*\text{sqrt}(2)*a^2*\cos(3*d*x + 3*c) + 10*\text{sqrt}(2)*a^2*\cos(2*d*x + 2*c) + 5*\text{sqrt}(2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2)*\cos(4*d*x + 4*c) + 20*(10*\text{sqrt}(2)*a^2*\cos(2*d*x + 2*c) + 5*\text{sqrt}(2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2)*\cos(3*d*x + 3*c) + 20*(5*\text{sqrt}(2)*a^2*\cos(d*x + c) + \text{sqrt}(2)*a^2)*\cos(2*d*x + 2*c) + 50*(2*\text{sqrt}(2)*a^2*\sin(3*d*x + 3*c) + 2*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c) + \text{sqrt}(2)*a^2*\sin(d*x + c))*\sin(4*d*x + 4*c) + 100*(2*\text{sqrt}(2)*a^2*\sin(2*d*x + 2*c) + \text{sqrt}(2)*a^2*\sin(d*x + c))*\sin(3*d*x + 3*c)) * \sin(5*d*x + 5*c)) * \sin(2/5*\arctan2(\sin(5/2*d*x + 5/2*c), \cos(5/2*d*x + 5/2*c))) * \text{sqrt}(a)*d)
\end{aligned}$$

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2}} dx = \frac{\sqrt{2} \left(\frac{3 \log(\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{3 \log(-\sin(\frac{1}{2} dx + \frac{1}{2} c) + 1)}{a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} - \frac{2 (3 \sin(\frac{1}{2} dx + \frac{1}{2} c)^3 - 5 \sin(\frac{1}{2} dx + \frac{1}{2} c))}{(\sin(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^2 a^2 \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c))} \right)}{64 \sqrt{ad}}$$

[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

```
[Out] 1/64*sqrt(2)*(3*log(sin(1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(1/2*d*x + 1/2*c))
) - 3*log(-sin(1/2*d*x + 1/2*c) + 1)/(a^2*sgn(cos(1/2*d*x + 1/2*c))) - 2*(
3*sin(1/2*d*x + 1/2*c)^3 - 5*sin(1/2*d*x + 1/2*c))/((sin(1/2*d*x + 1/2*c)^2
- 1)^2*a^2*sgn(cos(1/2*d*x + 1/2*c)))/(sqrt(a)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{1}{(a + a \cos(c + dx))^{5/2}} dx$$

[In] int(1/(a + a*cos(c + d*x))^(5/2),x)

[Out] int(1/(a + a*cos(c + d*x))^(5/2), x)

$$3.144 \quad \int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	2311
Rubi [A] (verified)	2311
Mathematica [C] (verified)	2313
Maple [B] (verified)	2314
Fricas [B] (verification not implemented)	2314
Sympy [F]	2315
Maxima [F]	2315
Giac [F(-2)]	2315
Mupad [F(-1)]	2316

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{5/2} d} - \frac{43 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2} d} - \frac{\sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} - \frac{11 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}}$$

[Out] $2*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(5/2)}/d-1/4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(5/2)}-11/16*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(3/2)}-43/32*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2845, 3057, 3064, 2728, 212, 2852}

$$\int \frac{\sec(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{a^{5/2} d} - \frac{43 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2} d} - \frac{11 \sin(c+dx)}{16ad(a \cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx)}{4d(a \cos(c+dx)+a)^{5/2}}$$

[In] $\operatorname{Int}[\operatorname{Sec}[c+d*x]/(a+a*\operatorname{Cos}[c+d*x])^{(5/2)},x]$

[Out] $(2*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])]/(a^{(5/2)}*d) - (43*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Cos}[c+d*x]])])/(16$

$\sqrt{2} a^{5/2} d - \sin[c + dx] / (4d(a + a\cos[c + dx])^{5/2}) - (11\sin[c + dx]) / (16ad(a + a\cos[c + dx])^{3/2})$

Rule 212

$\text{Int}[(a_ + (b_)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1/(\text{Rt}[a, 2]\text{Rt}[-b, 2])) \text{ArcTanh}[\text{Rt}[-b, 2](x/\text{Rt}[a, 2])], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 2728

$\text{Int}[1/\sqrt{(a_ + (b_)\sin[(c_ + (d_)(x_)]), x_Symbol] \rightarrow \text{Dist}[-2/d, \text{Subst}[\text{Int}[1/(2a - x^2), x], x, b(\cos[c + dx]/\sqrt{a + b\sin[c + dx]])], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2845

$\text{Int}[(a_ + (b_)\sin[(e_ + (f_)(x_)])^{(m_)}((c_ + (d_)\sin[(e_ + (f_)(x_)])^{(n_)}), x_Symbol] \rightarrow \text{Simp}[b^2\cos[e + fx](a + b\sin[e + fx])^m(c + d\sin[e + fx])^{n+1}/(af(2m+1)(bc - ad)), x] + \text{Dist}[1/(a(2m+1)(bc - ad)), \text{Int}[(a + b\sin[e + fx])^{m+1}(c + d\sin[e + fx])^n \text{Simp}[b^2c(m+1) - ad(2m+n+2) + b^2d(m+n+2)\sin[e + fx], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{NeQ}[b^2c - ad, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ (\text{IntegerSqrt}[2m, 2n] \ || \ (\text{IntegerQ}[m] \ \&\& \ \text{EqQ}[c, 0]))$

Rule 2852

$\text{Int}[\sqrt{(a_ + (b_)\sin[(e_ + (f_)(x_)])/((c_ + (d_)\sin[(e_ + (f_)(x_)]), x_Symbol] \rightarrow \text{Dist}[-2(b/f), \text{Subst}[\text{Int}[1/(b^2c + a^2d - d^2x^2), x], x, b(\cos[e + fx]/\sqrt{a + b\sin[e + fx]])], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b^2c - a^2d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3057

$\text{Int}[(a_ + (b_)\sin[(e_ + (f_)(x_)])^{(m_)}((A_ + (B_)\sin[(e_ + (f_)(x_)])^{(n_)}), x_Symbol] \rightarrow \text{Simp}[b(Ab - aB)\cos[e + fx](a + b\sin[e + fx])^m(c + d\sin[e + fx])^{n+1}/(af(2m+1)(bc - ad)), x] + \text{Dist}[1/(a(2m+1)(bc - ad)), \text{Int}[(a + b\sin[e + fx])^{m+1}(c + d\sin[e + fx])^n \text{Simp}[B(ac^m + b^2d(n+1)) + A(bc^m(m+1) - ad(2m+n+2)) + d(Ab - aB)(m+n+2)\sin[e + fx], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x \ \&\& \ \text{NeQ}[b^2c - a^2d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{-1}] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2m] \ \&\& \ (\text{IntegerQ}[2n] \ || \ \text{EqQ}[c, 0])$

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{(4a-\frac{3}{2}a\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
&= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{(8a^2-\frac{11}{4}a^2\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
&= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{\int \sqrt{a+a\cos(c+dx)}\sec(c+dx) dx}{a^3} - \frac{43\int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
&= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad - \frac{2\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^2d} + \frac{43\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{16a^2d} \\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{43\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\
&\quad - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.76 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.94

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{\cos^5\left(\frac{1}{2}(c+dx)\right) \left(-32\sqrt{2}\log\left(i - \sqrt{2}e^{\frac{1}{2}i(c+dx)} - ie^{i(c+dx)}\right) + 32\sqrt{2}\log\left(i + \sqrt{2}e^{\frac{1}{2}i(c+dx)} + ie^{i(c+dx)}\right)\right)}{(a+a\cos(c+dx))^{5/2}}$$

[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^(5/2), x]

[Out] (Cos[(c + d*x)/2]^5*(-32*Sqrt[2]*Log[I - Sqrt[2]*E^((I/2)*(c + d*x))] - I*E^(I*(c + d*x))) + 32*Sqrt[2]*Log[I + Sqrt[2]*E^((I/2)*(c + d*x))] - I*E^(I*(c + d*x))) / (a + a*Cos[c + d*x])^(5/2)

+ d*x))] + 86*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 86*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] - (Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^(-4) - 1/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 + (Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^(-4) + 11/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(119) = 238.

Time = 1.75 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.26

method	result
default	$-\frac{\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(43\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4a}}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-32\ln\left(\frac{4\sqrt{2}a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{a+1}}{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{2}}\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$

[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*a*cos(1/2*d*x+1/2*c)^4-32*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)+2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)+2*a))*cos(1/2*d*x+1/2*c)^4*a-32*ln(-4/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*(2^(1/2)*a*cos(1/2*d*x+1/2*c)-2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)-2*a))*cos(1/2*d*x+1/2*c)^4*a+11*cos(1/2*d*x+1/2*c)^2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*2^(1/2)*a^(1/2)+2*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(119) = 238.

Time = 0.28 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.07

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{43\sqrt{2}(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1)\sqrt{a}\log\left(-\frac{a\cos(dx+c)}{\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a\right)}{(a+a\cos(c+dx))^{5/2}}$$

[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/sin(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

$a) * (\cos(dx + c) - 2) * \sin(dx + c) + 8 * a) / (\cos(dx + c)^3 + \cos(dx + c)^2)$
 $) - 4 * \sqrt{a * \cos(dx + c) + a} * (11 * \cos(dx + c) + 15) * \sin(dx + c) / (a^3 * d * \cos(dx + c)^3 + 3 * a^3 * d * \cos(dx + c)^2 + 3 * a^3 * d * \cos(dx + c) + a^3 * d)$

Sympy [F]

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{\sec(c + dx)}{(a(\cos(c + dx) + 1))^{5/2}} dx$$

[In] integrate(sec(dx+c)/(a+a*cos(dx+c))**(5/2),x)

[Out] Integral(sec(c + dx)/(a*(cos(c + dx) + 1))**(5/2), x)

Maxima [F]

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{\sec(dx + c)}{(a \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(sec(dx+c)/(a+a*cos(dx+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(dx + c)/(a*cos(dx + c) + a)^(5/2), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \text{Exception raised: TypeError}$$

[In] integrate(sec(dx+c)/(a+a*cos(dx+c))^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
 UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
 dex_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{1}{\cos(c + dx) (a + a \cos(c + dx))^{5/2}} dx$$

```
[In] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)), x)
```

```
[Out] int(1/(cos(c + d*x)*(a + a*cos(c + d*x))^(5/2)), x)
```


$$3.145 \quad \int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	2317
Rubi [A] (verified)	2317
Mathematica [C] (verified)	2320
Maple [B] (verified)	2321
Fricas [B] (verification not implemented)	2322
Sympy [F]	2322
Maxima [F(-1)]	2322
Giac [F(-2)]	2323
Mupad [F(-1)]	2323

Optimal result

Integrand size = 23, antiderivative size = 174

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = -\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{5/2}d} + \frac{115 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} - \frac{15 \tan(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}} + \frac{35 \tan(c+dx)}{16a^2d\sqrt{a+a \cos(c+dx)}}$$

[Out] $-5*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(5/2)}/d+115/32*\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}-1/4*\tan(d*x+c)/d/(a+a*\cos(d*x+c))^{(5/2)}-15/16*\tan(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(3/2)}+35/16*\tan(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2845, 3057, 3063, 3064, 2728, 212, 2852}

$$\int \frac{\sec^2(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = -\frac{5 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{a^{5/2}d} + \frac{115 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{35 \tan(c+dx)}{16a^2d\sqrt{a \cos(c+dx)+a}} - \frac{15 \tan(c+dx)}{16ad(a \cos(c+dx)+a)^{3/2}} - \frac{\tan(c+dx)}{4d(a \cos(c+dx)+a)^{5/2}}$$

[In] Int[Sec[c + d*x]^2/(a + a*cos[c + d*x])^(5/2), x]

[Out] (-5*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*cos[c + d*x]])/(a^(5/2)*d) + (115*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*cos[c + d*x])^(5/2)) - (15*Tan[c + d*x])/(16*a*d*(a + a*cos[c + d*x])^(3/2)) + (35*Tan[c + d*x])/(16*a^2*d*Sqrt[a + a*cos[c + d*x]])

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2728

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*sin[c + d*x])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2852

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x], x, b*(Cos[e + f*x]/Sqrt[a + b*sin[e + f*x])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[

$b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}]$
 $\&\& !\text{GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] \mid\mid \text{EqQ}[c, 0])$

Rule 3063

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)x]]^{(m_.)}((A_.) + (B_.)\sin[(e_.) + (f_.)x])^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(B*c - A*d)\cos[e + f*x](a + b\sin[e + f*x])^m((c + d\sin[e + f*x])^{n+1})/(f*(n+1)*(c^2 - d^2)), x] + \text{Dist}[1/(b*(n+1)*(c^2 - d^2)), \text{Int}[(a + b\sin[e + f*x])^m(c + d\sin[e + f*x])^{n+1}]\text{Simp}[A*(a*d*m + b*c*(n+1)) - B*(a*c*m + b*d*(n+1)) + b*(B*c - A*d)*(m+n+2)*\sin[e + f*x], x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[n, -1] \&\& (\text{IntegerQ}[n] \mid\mid \text{EqQ}[m + 1/2, 0])$

Rule 3064

$\text{Int}[(A_.) + (B_.)\sin[(e_.) + (f_.)x]]/(\text{Sqrt}[(a_.) + (b_.)\sin[(e_.) + (f_.)x]]*(c_.) + (d_.)\sin[(e_.) + (f_.)x]), x_Symbol] \rightarrow \text{Dist}[(A*b - a*B)/(b*c - a*d), \text{Int}[1/\text{Sqrt}[a + b\sin[e + f*x]], x], x] + \text{Dist}[(B*c - A*d)/(b*c - a*d), \text{Int}[\text{Sqrt}[a + b\sin[e + f*x]]/(c + d\sin[e + f*x]), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{(5a-\frac{5}{2}a\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\ &= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{(\frac{35a^2}{2}-\frac{45}{4}a^2\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\ &= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\ &\quad + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{(-20a^3+\frac{35}{4}a^3\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx}{8a^5} \\ &= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} \\ &\quad - \frac{5\int \sqrt{a+a\cos(c+dx)}\sec(c+dx) dx}{2a^3} + \frac{115\int \frac{1}{\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{5\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^2d} - \frac{115\text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{16a^2d} \\
&= -\frac{5\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{115\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\
&\quad - \frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.97 (sec) , antiderivative size = 731, normalized size of antiderivative = 4.20

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx &= \frac{10\sqrt{2}\cos^5\left(\frac{c}{2} + \frac{dx}{2}\right) \left(-\frac{ie^{ic}x}{-1+e^{ic}} + \frac{\log\left(i-\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)}{d}\right)}{(a(1+\cos(c+dx)))^{5/2}} \\
&+ \frac{10\sqrt{2}\cos^5\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{ie^{ic}x}{-1+e^{ic}} - \frac{\log\left(i+\sqrt{2}e^{\frac{1}{2}i(c+dx)}-ie^{i(c+dx)}\right)}{d}\right)}{(a(1+\cos(c+dx)))^{5/2}} \\
&- \frac{115\cos^5\left(\frac{c}{2} + \frac{dx}{2}\right) \log\left(\cos\left(\frac{c}{4} + \frac{dx}{4}\right) - \sin\left(\frac{c}{4} + \frac{dx}{4}\right)\right)}{4d(a(1+\cos(c+dx)))^{5/2}} \\
&+ \frac{115\cos^5\left(\frac{c}{2} + \frac{dx}{2}\right) \log\left(\cos\left(\frac{c}{4} + \frac{dx}{4}\right) + \sin\left(\frac{c}{4} + \frac{dx}{4}\right)\right)}{4d(a(1+\cos(c+dx)))^{5/2}} \\
&+ \frac{\cos^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{8d(a(1+\cos(c+dx)))^{5/2} \left(\cos\left(\frac{c}{4} + \frac{dx}{4}\right) - \sin\left(\frac{c}{4} + \frac{dx}{4}\right)\right)^4} \\
&+ \frac{19\cos^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{8d(a(1+\cos(c+dx)))^{5/2} \left(\cos\left(\frac{c}{4} + \frac{dx}{4}\right) - \sin\left(\frac{c}{4} + \frac{dx}{4}\right)\right)^2} \\
&- \frac{\cos^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{8d(a(1+\cos(c+dx)))^{5/2} \left(\cos\left(\frac{c}{4} + \frac{dx}{4}\right) + \sin\left(\frac{c}{4} + \frac{dx}{4}\right)\right)^4} \\
&- \frac{19\cos^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{8d(a(1+\cos(c+dx)))^{5/2} \left(\cos\left(\frac{c}{4} + \frac{dx}{4}\right) + \sin\left(\frac{c}{4} + \frac{dx}{4}\right)\right)^2} \\
&+ \frac{4\cos^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{d(a(1+\cos(c+dx)))^{5/2} \left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) - \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)} \\
&- \frac{4\cos^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{d(a(1+\cos(c+dx)))^{5/2} \left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) + \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]

[Out] $(10\sqrt{2}\cos[c/2 + (d*x)/2]^5 * (((-1)*E^{(I*c)*x})/(-1 + E^{(I*c)}) + \text{Log}[I - \sqrt{2}*E^{((I/2)*(c + d*x))} - I*E^{(I*(c + d*x))}]/d))/(a*(1 + \cos[c + d*x])^{5/2}) + (10\sqrt{2}\cos[c/2 + (d*x)/2]^5 * ((I*E^{(I*c)*x})/(-1 + E^{(I*c)}) - \text{Log}[I + \sqrt{2}*E^{((I/2)*(c + d*x))} - I*E^{(I*(c + d*x))}]/d))/(a*(1 + \cos[c + d*x])^{5/2}) - (115*\cos[c/2 + (d*x)/2]^5 * \text{Log}[\cos[c/4 + (d*x)/4] - \sin[c/4 + (d*x)/4]])/(4*d*(a*(1 + \cos[c + d*x])^{5/2})) + (115*\cos[c/2 + (d*x)/2]^5 * \text{Log}[\cos[c/4 + (d*x)/4] + \sin[c/4 + (d*x)/4]])/(4*d*(a*(1 + \cos[c + d*x])^{5/2})) + \cos[c/2 + (d*x)/2]^5/(8*d*(a*(1 + \cos[c + d*x])^{5/2})*(\cos[c/4 + (d*x)/4] - \sin[c/4 + (d*x)/4])^4) + (19*\cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + \cos[c + d*x])^{5/2})*(\cos[c/4 + (d*x)/4] - \sin[c/4 + (d*x)/4])^2) - \cos[c/2 + (d*x)/2]^5/(8*d*(a*(1 + \cos[c + d*x])^{5/2})*(\cos[c/4 + (d*x)/4] + \sin[c/4 + (d*x)/4])^4) - (19*\cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + \cos[c + d*x])^{5/2})*(\cos[c/4 + (d*x)/4] + \sin[c/4 + (d*x)/4])^2) + (4*\cos[c/2 + (d*x)/2]^5)/(d*(a*(1 + \cos[c + d*x])^{5/2})*(\cos[c/2 + (d*x)/2] - \sin[c/2 + (d*x)/2])) - (4*\cos[c/2 + (d*x)/2]^5)/(d*(a*(1 + \cos[c + d*x])^{5/2})*(\cos[c/2 + (d*x)/2] + \sin[c/2 + (d*x)/2]))$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(145) = 290$.

Time = 1.69 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.45

method	result
default	$\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(230\sqrt{2} \ln\left(\frac{4\sqrt{a}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4a}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right) \left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^a - 160 \ln\left(\frac{4\sqrt{2}a\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 4\sqrt{2}\sqrt{a\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sqrt{a}}{2\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{2}}\right) \sqrt{a}\right)$

[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)

[Out] $1/16*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(230*2^{(1/2)}*\ln(2*(2*a^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+2*a)/\cos(1/2*d*x+1/2*c))*\cos(1/2*d*x+1/2*c)^6*a-160*\ln(4/(2*\cos(1/2*d*x+1/2*c)+2^{(1/2)}))*(2^{(1/2)}*a*\cos(1/2*d*x+1/2*c)+2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}+2*a))*\cos(1/2*d*x+1/2*c)^6*a-160*\ln(-4/(2*\cos(1/2*d*x+1/2*c)-2^{(1/2)}))*(2^{(1/2)}*a*\cos(1/2*d*x+1/2*c)-2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}-2*a))*\cos(1/2*d*x+1/2*c)^6*a-115*2^{(1/2)}*\ln(2*(2*a^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+2*a)/\cos(1/2*d*x+1/2*c))*a*\cos(1/2*d*x+1/2*c)^4+70*2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}*\cos(1/2*d*x+1/2*c)^4+80*\ln(4/(2*\cos(1/2*d*x+1/2*c)+2^{(1/2)}))*(2^{(1/2)}*a*\cos(1/2*d*x+1/2*c)+2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}+2*a))*\cos(1/2*d*x+1/2*c)^4*a+80*\ln(-4/(2*\cos(1/2*d*x+1/2*c)-2^{(1/2)}))*(2^{(1/2)}*a*\cos(1/2*d*x+1/2*c)-2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}-2*a))*\cos(1/2*d*x+1/2*c)^4*a-15*\cos(1/2*d*x+1/2*c)^2*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*2^{(1/2)}*a^{(1/2)}$

$$)-2*2^{(1/2)}*(a*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^{(1/2)}/a^{(7/2)}/\cos(1/2*d*x+1/2*c)^3/(2*\cos(1/2*d*x+1/2*c)+2^{(1/2)})/(2*\cos(1/2*d*x+1/2*c)-2^{(1/2)})/\sin(1/2*d*x+1/2*c)/(a*\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/d$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(145) = 290.

Time = 0.27 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.90

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{115\sqrt{2}(\cos(dx+c)^4 + 3\cos(dx+c)^3 + 3\cos(dx+c)^2 + \cos(dx+c))\sqrt{a}\log\left(\frac{\cos(dx+c)^2 + 2\cos(dx+c) + 1}{\cos(dx+c)^2 + 2\cos(dx+c) + 1}\right) + 80(\cos(dx+c)^4 + 3\cos(dx+c)^3 + 3\cos(dx+c)^2 + \cos(dx+c))\sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7a\cos(dx+c)^2 + 4\sqrt{a}\cos(dx+c) + a}{a\cos(dx+c)^3 + \cos(dx+c)^2}\right) + 4\sqrt{a}\cos(dx+c) + a(35\cos(dx+c)^2 + 55\cos(dx+c) + 16)\sin(dx+c)}{a^3d\cos(dx+c)^4 + 3a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2 + a^3d\cos(dx+c)}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/64*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \int \frac{\sec^2(c+dx)}{(a(\cos(c+dx)+1))^{5/2}} dx$$

[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)

[Out] Integral(sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(5/2), x)

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{1}{\cos(c + dx)^2 (a + a \cos(c + dx))^{5/2}} dx$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + a*cos(c + d*x))^(5/2)), x)
```

3.146 $\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx)) dx$

Optimal result	2324
Rubi [A] (verified)	2324
Mathematica [C] (warning: unable to verify)	2326
Maple [A] (verified)	2326
Fricas [C] (verification not implemented)	2327
Sympy [F(-1)]	2327
Maxima [F]	2328
Giac [F]	2328
Mupad [B] (verification not implemented)	2328

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx)) dx = \frac{6aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{10a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d}$$

$$+ \frac{10a \sqrt{\cos(c + dx)} \sin(c + dx)}{21d}$$

$$+ \frac{2a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}$$

$$+ \frac{2a \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d}$$

[Out] $6/5*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+10/21*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/7*a*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d+10/21*a*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

= {2827, 2715, 2719, 2720}

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))dx = \frac{10a \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} + \frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{5d} \\ + \frac{2a \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{7d} \\ + \frac{2a \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{5d} \\ + \frac{10a \sin(c+dx) \sqrt{\cos(c+dx)}}{21d}$$

[In] Int[Cos[c + d*x]^(5/2)*(a + a*cos[c + d*x]),x]

[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(21*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\text{integral} = a \int \cos^{\frac{5}{2}}(c+dx) dx + a \int \cos^{\frac{7}{2}}(c+dx) dx \\ = \frac{2a \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{2a \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} \\ + \frac{1}{5}(3a) \int \sqrt{\cos(c+dx)} dx + \frac{1}{7}(5a) \int \cos^{\frac{3}{2}}(c+dx) dx$$


```
[In] int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-122*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))dx$$

$$= \frac{2(15a\cos(dx+c)^2+21a\cos(dx+c)+25a)\sqrt{\cos(dx+c)}\sin(dx+c)-25i\sqrt{2}a\text{weierstrassPInverse}(\dots)}{\dots}$$

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
[Out] 1/105*(2*(15*a*cos(d*x + c)^2 + 21*a*cos(d*x + c) + 25*a)*sqrt(cos(d*x + c))*sin(d*x + c) - 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))dx = \int (a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}dx$$

[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))dx = \int (a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}dx$$

[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 15.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\begin{aligned} & \int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))dx \\ &= -\frac{2a\cos(c+dx)^{7/2}\sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c+dx)^2\right)}{7d\sqrt{\sin(c+dx)^2}} \\ & \quad -\frac{2a\cos(c+dx)^{9/2}\sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c+dx)^2\right)}{9d\sqrt{\sin(c+dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x)),x)

[Out] - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))

3.147 $\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx)) dx$

Optimal result	2329
Rubi [A] (verified)	2329
Mathematica [C] (warning: unable to verify)	2331
Maple [A] (verified)	2331
Fricas [C] (verification not implemented)	2332
Sympy [F(-1)]	2332
Maxima [F]	2332
Giac [F]	2333
Mupad [B] (verification not implemented)	2333

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx)) dx = \frac{6aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\ + \frac{2a\sqrt{\cos(c + dx)} \sin(c + dx)}{3d} \\ + \frac{2a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}$$

[Out] $6/5*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/3*a*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2827, 2715, 2720, 2719}

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx)) dx = \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{6aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} \\ + \frac{2a \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d} \\ + \frac{2a \sin(c + dx) \sqrt{\cos(c + dx)}}{3d}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]^{(3/2)}*(a + a*\operatorname{Cos}[c + d*x]), x]$

[Out] $(6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)$

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \cos^{\frac{3}{2}}(c + dx) dx + a \int \cos^{\frac{5}{2}}(c + dx) dx \\
 &= \frac{2a\sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} \\
 &\quad + \frac{1}{3}a \int \frac{1}{\sqrt{\cos(c + dx)}} dx + \frac{1}{5}(3a) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{6aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2a \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\
 &\quad + \frac{2a\sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}
 \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.56 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.67

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))dx$$

$$= \frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{9(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c))))\csc(c)\sec(c)}{\sqrt{\sec^2(c)}}-20\cos(c+dx)\right)}{\sqrt{\sec^2(c)}}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]),x]

[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((9*(3*Cos[c - d*x - ArcTan[Tan[c]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + 2*Cos[c + d*x]*(-18*Cot[c] + 10*Sin[c + d*x] + 3*Sin[2*(c + d*x)]) - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 6.66 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.52

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}a\left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-28\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\right)}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$\frac{2a\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d$

[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx)) dx$$

$$= \frac{2(3a \cos(dx + c) + 5a)\sqrt{\cos(dx + c)} \sin(dx + c) - 5i\sqrt{2a} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{d}$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/15*(2*(3*a*cos(d*x + c) + 5*a)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx)) dx = \int (a \cos(dx + c) + a) \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx)) dx = \int (a \cos(dx + c) + a) \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\begin{aligned} & \int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx)) dx \\ &= \frac{2a F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{3d} + \frac{2a \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} \\ & \quad - \frac{2a \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7d \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x)),x)

[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))

3.148 $\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx)) dx$

Optimal result	2334
Rubi [A] (verified)	2334
Mathematica [C] (warning: unable to verify)	2335
Maple [B] (verified)	2336
Fricas [C] (verification not implemented)	2336
Sympy [F]	2337
Maxima [F]	2337
Giac [F]	2337
Mupad [B] (verification not implemented)	2337

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx)) dx = \frac{2aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a\sqrt{\cos(c + dx)} \sin(c + dx)}{3d}$$

[Out] $2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2827, 2719, 2715, 2720}

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx)) dx = \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a \sin(c + dx)\sqrt{\cos(c + dx)}}{3d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*(a + a*\operatorname{Cos}[c + d*x]), x]$

[Out] $(2*a*\operatorname{EllipticE}[(c + d*x)/2, 2])/d + (2*a*\operatorname{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/(3*d)$

Rule 2715

$\operatorname{Int}[(b*\sin[(c_.) + (d_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(-b)*\operatorname{Cos}[c + d*x]*(b*\operatorname{Sin}[c + d*x])^{(n-1)}/(d*n), x] + \operatorname{Dist}[b^2*((n-1)/n), \operatorname{Int}[(b*\operatorname{Sin}[$

$c + d*x]^{(n - 2), x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2 * n]$

Rule 2719

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticF}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2827

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \sqrt{\cos(c + dx)} dx + a \int \cos^{\frac{3}{2}}(c + dx) dx \\ &= \frac{2aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a\sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3}a \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\ &= \frac{2aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a\sqrt{\cos(c + dx)} \sin(c + dx)}{3d} \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.19 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.64

$$\begin{aligned} &\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx)) dx \\ &= \frac{a(1 + \cos(c + dx)) \sec^2\left(\frac{1}{2}(c + dx)\right) \left(\frac{3(3 \cos(c - dx - \arctan(\tan(c))) + \cos(c + dx + \arctan(\tan(c)))) \csc(c) \sec(c)}{\sqrt{\sec^2(c)}} - 4 \cos(c + dx)\right)}{\sqrt{\sec^2(c)}} \end{aligned}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]),x]

[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((3*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 4*Cos[c

+ d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] - 4*Cos[c + d*x]*(3*Cot[c] - Sin[c + d*x]) - 6*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(107) = 214.

Time = 5.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.69

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} a\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$
parts	$\frac{2a\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$

[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out]
$$-2/3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)-2*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(\cos(1/2*d*x+1/2*c),2^(1/2))-3*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(\cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^(1/2)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))dx$$

$$= \frac{2a\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}a\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{d}$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out]
$$1/3*(2*a*\sqrt{\cos(dx+c)}*\sin(dx+c) - I*\sqrt{2}*a*\text{weierstrassPInverse}(-4,0,\cos(dx+c)+I*\sin(dx+c)) + I*\sqrt{2}*a*\text{weierstrassPInverse}(-4,0,\cos(dx+c)-I*\sin(dx+c)))/d$$

0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a*weierstrassZeta(-4, 0, w
 eierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*a*w
 eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))))/d

Sympy [F]

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx)) dx = a \left(\int \sqrt{\cos(c + dx)} dx + \int \cos^{\frac{3}{2}}(c + dx) dx \right)$$

[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c)),x)

[Out] a*(Integral(sqrt(cos(c + d*x)), x) + Integral(cos(c + d*x)**(3/2), x))

Maxima [F]

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx)) dx = \int (a \cos(dx + c) + a) \sqrt{\cos(dx + c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx)) dx = \int (a \cos(dx + c) + a) \sqrt{\cos(dx + c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx)) dx = \frac{2a E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2a F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{3d} + \frac{2a \sqrt{\cos(c + dx)} \sin(c + dx)}{3d}$$

[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x)),x)

[Out] (2*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*a*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)

3.149 $\int \frac{a+a \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx$

Optimal result	2338
Rubi [A] (verified)	2338
Mathematica [C] (verified)	2339
Maple [A] (verified)	2340
Fricas [C] (verification not implemented)	2340
Sympy [F]	2341
Maxima [F]	2341
Giac [F]	2341
Mupad [B] (verification not implemented)	2341

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \frac{2aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

[Out] $2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d$

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2827, 2720, 2719}

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])/Sqrt[\operatorname{Cos}[c + d*x]], x]$

[Out] $(2*a*\operatorname{EllipticE}[(c + d*x)/2, 2])/d + (2*a*\operatorname{EllipticF}[(c + d*x)/2, 2])/d$

Rule 2719

$\operatorname{Int}[Sqrt[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \operatorname{Simp}[(2/d)*\operatorname{EllipticE}[(1/2)*(c - \operatorname{Pi}/2 + d*x), 2], x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \frac{1}{\sqrt{\cos(c + dx)}} dx + a \int \sqrt{\cos(c + dx)} dx \\ &= \frac{2aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.43

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \frac{a \sqrt{\cos(c + dx)} (1 + \cos(c + dx)) \sec^2\left(\frac{1}{2}(c + dx)\right) \left(-2 \sqrt{\cos^2(dx - \arctan(\cot(c)))} \sqrt{\csc^2(c)} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \sin^2\left(\frac{1}{2}(c + dx)\right)\right)\right)}{1}$$

[In] Integrate[(a + a*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]

[Out] (a*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-2*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Tan[d*x + ArcTan[Tan[c]]] - (HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Tan[d*x + ArcTan[Tan[c]]])/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(2*d)

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.29

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$
parts	$\frac{2a \operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2} \sqrt{2}\right)}{d} + \frac{2a\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)a\sqrt{2}e^{-i(dx+c)}}{d\sqrt{\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}} - \frac{i\left(\frac{\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)}+e^{i(dx+c)}}}\right)}{\sqrt{\left(e^{2i(dx+c)}+1\right)e^{i(dx+c)}}}$

[In] int((a+cos(d*x+c)*a)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.06

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{-i\sqrt{2}a\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i\sqrt{2}a\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{d}$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")

```
[Out] (-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```


Sympy [F]

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = a \left(\int \frac{1}{\sqrt{\cos(c + dx)}} dx + \int \sqrt{\cos(c + dx)} dx \right)$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(1/2),x)

[Out] a*(Integral(1/sqrt(cos(c + d*x)), x) + Integral(sqrt(cos(c + d*x)), x))

Maxima [F]

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \int \frac{a \cos(dx + c) + a}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \int \frac{a \cos(dx + c) + a}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 14.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \frac{2a \left(E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) + F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) \right)}{d}$$

[In] int((a + a*cos(c + d*x))/cos(c + d*x)^(1/2),x)

[Out] (2*a*(ellipticE(c/2 + (d*x)/2, 2) + ellipticF(c/2 + (d*x)/2, 2)))/d

$$3.150 \quad \int \frac{a+a \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	2342
Rubi [A] (verified)	2342
Mathematica [C] (warning: unable to verify)	2343
Maple [A] (verified)	2344
Fricas [C] (verification not implemented)	2344
Sympy [F]	2345
Maxima [F]	2345
Giac [F]	2345
Mupad [B] (verification not implemented)	2345

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = -\frac{2aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a \sin(c + dx)}{d\sqrt{\cos(c + dx)}}$$

[Out] $-2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*a*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2827, 2716, 2719, 2720}

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} - \frac{2aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a \sin(c + dx)}{d\sqrt{\cos(c + dx)}}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])/(\operatorname{Cos}[c + d*x])^{(3/2)}, x]$

[Out] $(-2*a*\operatorname{EllipticE}[(c + d*x)/2, 2])/d + (2*a*\operatorname{EllipticF}[(c + d*x)/2, 2])/d + (2*a*\operatorname{Sin}[c + d*x])/(d*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]])$

Rule 2716

$\operatorname{Int}[(b*\sin[(c_.) + (d_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Cos}[c + d*x]*((b*\operatorname{Sin}[c + d*x])^{(n + 1)}/(b*d*(n + 1))), x] + \operatorname{Dist}[(n + 2)/(b^2*(n + 1)), \operatorname{In}$

`t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]`

Rule 2719

`Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2720

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + a \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a \sin(c + dx)}{d\sqrt{\cos(c + dx)}} - a \int \sqrt{\cos(c + dx)} dx \\
 &= -\frac{2aE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a \sin(c + dx)}{d\sqrt{\cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.67

$$\begin{aligned}
 &\int \frac{a + a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{a(1 + \cos(c + dx)) \operatorname{sec}^2\left(\frac{1}{2}(c + dx)\right) \left(4 \cos(dx) \operatorname{csc}(c) - \frac{(3 \cos(c - dx - \arctan(\tan(c))) + \cos(c + dx + \arctan(\tan(c)))) \operatorname{csc}(c) \operatorname{sec}(c)}{\sqrt{\operatorname{sec}^2(c)}}\right)}{\sqrt{\operatorname{sec}^2(c)}}
 \end{aligned}$$

[In] `Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(3/2), x]`

[Out] `(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(4*Cos[d*x]*Csc[c] - ((3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[S`

$$\frac{e^{c^2} - 4 \cos[c + dx] \sqrt{\cos[dx - \text{ArcTan}[\text{Cot}[c]]]^2} \sqrt{\csc[c]^2} \text{HypergeometricPFQ}\left[\left\{\frac{1}{4}, \frac{1}{2}\right\}, \left\{\frac{5}{4}\right\}, \sin[dx - \text{ArcTan}[\text{Cot}[c]]]^2\right] \sec[dx - \text{ArcTan}[\text{Cot}[c]]] \sin[c] + 2 \cos[c] \csc[dx + \text{ArcTan}[\text{Tan}[c]]] \text{HypergeometricPFQ}\left[\left\{-\frac{1}{2}, -\frac{1}{4}\right\}, \left\{\frac{3}{4}\right\}, \cos[dx + \text{ArcTan}[\text{Tan}[c]]]^2\right] \sqrt{\sec[c]^2} \sqrt{\sin[dx + \text{ArcTan}[\text{Tan}[c]]]^2}}{4 dx \sqrt{\cos[c + dx]}}$$

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.60

method	result
default	$\frac{2a \left(2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \cos \left(\frac{dx}{2} + \frac{c}{2} \right) - \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} F \left(\cos \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) - \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \right)}{\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} d}$
parts	$-\frac{2a \left(-2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \right)}{\sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} d}$

[In] int((a+cos(d*x+c)*a)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] $2*a*(2*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)-(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.74

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{-i \sqrt{2} a \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{2 \cos(dx + c)}$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] $(-I*\text{sqrt}(2)*a*\cos(d*x + c)*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + I*\text{sqrt}(2)*a*\cos(d*x + c)*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - I*\text{sqrt}(2)*a*\cos(d*x + c)*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) + I*\text{sqrt}(2)*a*\cos(d*x + c)*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) + 2*a*\text{sqrt}(\cos(d*x + c))*\sin(d*x + c))/(d*\cos(d*x + c))$

Sympy [F]

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = a \left(\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + \int \frac{1}{\sqrt{\cos(c + dx)}} dx \right)$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(3/2), x)

[Out] a*(Integral(cos(c + d*x)**(-3/2), x) + Integral(1/sqrt(cos(c + d*x)), x))

Maxima [F]

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{a \cos(dx + c) + a}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{a \cos(dx + c) + a}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 15.55 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2a F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2a \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

[In] int((a + a*cos(c + d*x))/cos(c + d*x)^(3/2), x)

[Out] (2*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

$$3.151 \quad \int \frac{a+a \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	2346
Rubi [A] (verified)	2346
Mathematica [C] (warning: unable to verify)	2347
Maple [B] (verified)	2348
Fricas [C] (verification not implemented)	2349
Sympy [F(-1)]	2349
Maxima [F]	2349
Giac [F]	2350
Mupad [B] (verification not implemented)	2350

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{a+a \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} dx = -\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{d} + \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2a \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[Out] $-2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+2*a*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2827, 2716, 2720, 2719}

$$\int \frac{a+a \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} - \frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{d} \\ + \frac{2a \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])/ \operatorname{Cos}[c + d*x]^{(5/2)}, x]$

[Out] $(-2*a*\operatorname{EllipticE}[(c + d*x)/2, 2])/d + (2*a*\operatorname{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a*\operatorname{Sin}[c + d*x])/(3*d*\operatorname{Cos}[c + d*x]^{(3/2)}) + (2*a*\operatorname{Sin}[c + d*x])/(d*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]])$

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx + a \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2a \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{1}{3} a \int \frac{1}{\sqrt{\cos(c + dx)}} dx - a \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{2a E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2a \sin(c + dx)}{d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.51 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.98

$$\begin{aligned}
&\int \frac{a + a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{a(1 + \cos(c + dx)) \sec^2\left(\frac{1}{2}(c + dx)\right) \left(2(3 \cos(c) + \cos(dx)) - \cos(2c + dx) + 3 \cos(c + 2dx)\right) \csc(c) - 4 \cos}{}
\end{aligned}$$

```
[In] Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(5/2), x]
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(2*(3*Cos[c] + Cos[d*x] - Cos[2*c + d*x] + 3*Cos[c + 2*d*x])*Csc[c] - 4*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] - (3*Cos[c + d*x]*Sec[c]*(-2*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Sin[d*x + ArcTan[Tan[c]]) + (3*Cos[c - d*x - ArcTan[Tan[c]]) + Cos[c + d*x + ArcTan[Tan[c]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Cos[c + d*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(127) = 254.

Time = 4.27 (sec) , antiderivative size = 368, normalized size of antiderivative = 4.43

method	result
default	$-\frac{2\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} a \left(12(\sin^4(\frac{dx}{2} + \frac{c}{2})) \cos(\frac{dx}{2} + \frac{c}{2}) - 2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2}))} - 1 F(\cos(\frac{dx}{2} + \frac{c}{2})) \right)}{3\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})} (2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1)$
parts	$-\frac{2a \left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2}))} - 1 F(\cos(\frac{dx}{2} + \frac{c}{2}), \sqrt{2}) (\sin^2(\frac{dx}{2} + \frac{c}{2})) - 2(\sin^2(\frac{dx}{2} + \frac{c}{2})) \cos(\frac{dx}{2} + \frac{c}{2}) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right)}{3\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})} (2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1)}$

```
[In] int((a+cos(d*x+c)*a)/cos(d*x+c)^(5/2), x, method=_RETURNVERBOSE)
[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*sin(1/2*d*x+1/2*c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*sin(1/2*d*x+1/2*c)^2-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```


Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.11

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{-i \sqrt{2} a \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3i \sqrt{2} a \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3i \sqrt{2} a \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2(3a \cos(dx + c) + a) \sqrt{\cos(dx + c)} \sin(dx + c)}{(d \cos(dx + c))^2}$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] 1/3*(-I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{a \cos(dx + c) + a}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{a \cos(dx + c) + a}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2 a \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}} + \frac{2 a \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{3 d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}}$$

[In] int((a + a*cos(c + d*x))/cos(c + d*x)^(5/2),x)

[Out] (2*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))

$$3.152 \quad \int \frac{a+a \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	2351
Rubi [A] (verified)	2351
Mathematica [C] (warning: unable to verify)	2353
Maple [B] (verified)	2354
Fricas [C] (verification not implemented)	2354
Sympy [F(-1)]	2355
Maxima [F]	2355
Giac [F]	2355
Mupad [B] (verification not implemented)	2355

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{a+a \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)} dx = -\frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{5d} + \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2a \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{6a \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[Out] $-6/5*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+2/3*a*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+6/5*a*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2827, 2716, 2719, 2720}

$$\int \frac{a+a \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} - \frac{6aE\left(\frac{1}{2}(c+dx)|2\right)}{5d} \\ + \frac{2a \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{6a \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])/(\operatorname{Cos}[c + d*x]^{(7/2)}), x]$

[Out] $(-6*a*\operatorname{EllipticE}[(c + d*x)/2, 2])/(5*d) + (2*a*\operatorname{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a*\operatorname{Sin}[c + d*x])/(5*d*\operatorname{Cos}[c + d*x]^{(5/2)}) + (2*a*\operatorname{Sin}[c + d*x])/(3*d*\operatorname{Cos}[c + d*x]^{(3/2)}) + (6*a*\operatorname{Sin}[c + d*x])/(5*d*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]])$

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a \int \frac{1}{\cos^{\frac{7}{2}}(c + dx)} dx + a \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{1}{3}a \int \frac{1}{\sqrt{\cos(c + dx)}} dx + \frac{1}{5}(3a) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} \\
&\quad + \frac{6a \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} - \frac{1}{5}(3a) \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{6a E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\
&\quad + \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{6a \sin(c + dx)}{5d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.32 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.30

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx = a \left(\sqrt{\cos(c + dx)(1 + \cos(c + dx))} \sec^2\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{3 \csc(c) \sec(c)}{5d} \right. \right. \\ \left. \left. + \frac{\sec(c) \sec^3(c + dx) \sin(dx)}{5d} + \frac{\sec(c) \sec^2(c + dx)(3 \sin(c) + 5 \sin(dx))}{15d} \right. \right. \\ \left. \left. + \frac{\sec(c) \sec(c + dx)(5 \sin(c) + 9 \sin(dx))}{15d} \right) \right. \\ \left. \frac{(1 + \cos(c + dx)) \csc(c) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \sin^2(dx - \arctan(\cot(c)))\right) \sec^2\left(\frac{c}{2} + \frac{dx}{2}\right) \sec(dx - \arctan(\cot(c)))}{3d\sqrt{\cos(c) \cos(dx + \arctan(\tan(c)))}} \right. \\ \left. + \frac{3(1 + \cos(c + dx)) \csc(c) \sec^2\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{{}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; \cos^2(dx + \arctan(\tan(c)))\right) \sin(dx + \arctan(\tan(c)))}{\sqrt{1 - \cos(dx + \arctan(\tan(c)))} \sqrt{1 + \cos(dx + \arctan(\tan(c)))} \sqrt{\cos(c) \cos(dx + \arctan(\tan(c)))}} \right)}{10d} \right)$$

[In] Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(7/2), x]

[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((3*Csc[c]*Sec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*x]^2*(3*Sin[c] + 5*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 9*Sin[d*x]))/(15*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (3*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*(HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Tan[c]^2])*Sqrt[1 + Tan[c]^2) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(10*d))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(147) = 294.

Time = 6.21 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.46

method	result
default	$4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} a \left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{12\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{2}} + \frac{7\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}} \right)$
parts	$2a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} \left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-1\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}} E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right) \right)$

```
[In] int((a+cos(d*x+c)*a)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/40*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.69

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{-5i \sqrt{2} a \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} a \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 9i \sqrt{2} a \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 9i \sqrt{2} a \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{15}$$

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
```

`I*sqrt(2)*a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*a*cos(d*x + c)^2 + 5*a*cos(d*x + c) + 3*a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3)`

Sympy [F(-1)]

Timed out.

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] `integrate((a+a*cos(d*x+c))/cos(d*x+c)**(7/2),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{a \cos(dx + c) + a}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] `integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")`

[Out] `integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)`

Giac [F]

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{a \cos(dx + c) + a}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] `integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")`

[Out] `integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)`

Mupad [B] (verification not implemented)

Time = 15.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{a + a \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx = \frac{2a \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{3d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}} + \frac{2a \sin(c + dx) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c + dx)^2\right)}{5d \cos(c + dx)^{5/2} \sqrt{\sin(c + dx)^2}}$$

```
[In] int((a + a*cos(c + d*x))/cos(c + d*x)^(7/2),x)
```

```
[Out] (2*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c +  
d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a*sin(c + d*x)*hypergeom([-5/4, 1/  
2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2))
```


3.153 $\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2 dx$

Optimal result	2357
Rubi [A] (verified)	2358
Mathematica [C] (verified)	2359
Maple [A] (verified)	2360
Fricas [C] (verification not implemented)	2360
Sympy [F(-1)]	2361
Maxima [F]	2361
Giac [F]	2361
Mupad [B] (verification not implemented)	2362

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2 dx = \frac{32a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{20a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{20a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{32a^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} + \frac{4a^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{2a^2 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d}$$

```
[Out] 32/15*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/21*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/45*a^2*cos(d*x+c)^(3/2)*sin(d*x+c)/d+4/7*a^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^2*cos(d*x+c)^(7/2)*sin(d*x+c)/d+20/21*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2715, 2719, 2720}

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2 dx = \frac{20a^2 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{32a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{2a^2 \sin(c + dx) \cos^{\frac{7}{2}}(c + dx)}{9d} + \frac{4a^2 \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{7d} + \frac{32a^2 \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{45d} + \frac{20a^2 \sin(c + dx) \sqrt{\cos(c + dx)}}{21d}$$

[In] Int[Cos[c + d*x]^(5/2)*(a + a*cos[c + d*x])^2,x]

[Out] (32*a^2*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a^2*EllipticF[(c + d*x)/2, 2])/(21*d) + (20*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (32*a^2*cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a^2*cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^2*cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)

Rule 2715

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_)*sin[(e_.) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_.) + (f_)*(x_)])^(m_), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +

$f*x])^n, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{RationalQ}[n]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(a^2 \cos^{\frac{5}{2}}(c+dx) + 2a^2 \cos^{\frac{7}{2}}(c+dx) + a^2 \cos^{\frac{9}{2}}(c+dx) \right) dx \\
 &= a^2 \int \cos^{\frac{5}{2}}(c+dx) dx + a^2 \int \cos^{\frac{9}{2}}(c+dx) dx + (2a^2) \int \cos^{\frac{7}{2}}(c+dx) dx \\
 &= \frac{2a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{4a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + \frac{2a^2 \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{9d} \\
 &\quad + \frac{1}{5}(3a^2) \int \sqrt{\cos(c+dx)} dx + \frac{1}{9}(7a^2) \int \cos^{\frac{5}{2}}(c+dx) dx + \frac{1}{7}(10a^2) \int \cos^{\frac{3}{2}}(c+dx) dx \\
 &= \frac{6a^2 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{20a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{21d} \\
 &\quad + \frac{32a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{45d} + \frac{4a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} \\
 &\quad + \frac{2a^2 \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{9d} + \frac{1}{15}(7a^2) \int \sqrt{\cos(c+dx)} dx \\
 &\quad + \frac{1}{21}(10a^2) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
 &= \frac{32a^2 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{15d} + \frac{20a^2 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} \\
 &\quad + \frac{20a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{21d} + \frac{32a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{45d} \\
 &\quad + \frac{4a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + \frac{2a^2 \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{9d}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.28 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.73

$$\begin{aligned}
 &\int \cos^{\frac{5}{2}}(c+dx)(a + a \cos(c+dx))^2 dx \\
 &= \frac{a^2(1 + \cos(c+dx))^2 \sec^4\left(\frac{1}{2}(c+dx)\right) \left(\frac{672(3 \cos(c-dx - \arctan(\tan(c))) + \cos(c+dx + \arctan(\tan(c)))) \csc(c) \sec(c)}{\sqrt{\sec^2(c)}} - 1200 \cos \right)}{1}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2,x]

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((672*(3*Cos[c - d*x - ArcTan[
Tan[c]]) + Cos[c + d*x + ArcTan[Tan[c]])]*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 1
200*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeom
etricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[C
ot[c]])*Sin[c] + Cos[c + d*x]*(-2688*Cot[c] + 1380*Sin[c + d*x] + 518*Sin[2
*(c + d*x)] + 180*Sin[3*(c + d*x)] + 35*Sin[4*(c + d*x)]) - 1344*Cos[c]*Csc
[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Arc
Tan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(5040*d*
Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 12.15 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.77

method	result
default	$\frac{4\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}a^2\left(560\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)-960\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)+608\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)-96\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{315\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$
parts	$\frac{2a^2\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

```
[In] int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(560*cos
(1/2*d*x+1/2*c)^11-960*cos(1/2*d*x+1/2*c)^9+608*cos(1/2*d*x+1/2*c)^7-96*cos
(1/2*d*x+1/2*c)^5-205*cos(1/2*d*x+1/2*c)^3+75*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+93*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2 dx =$$

$$2 \left(75i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 75i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/315*(75*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 75*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 168*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 168*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*a^2*cos(d*x + c)^3 + 90*a^2*cos(d*x + c)^2 + 112*a^2*cos(d*x + c) + 150*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2 dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2 dx = \int (a \cos(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}} dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^2 dx = \int (a \cos(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}} dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 15.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93

$$\begin{aligned}
& \int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2 dx \\
&= -\frac{2a^2 \cos(c+dx)^{7/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c+dx)^2\right)}{7d \sqrt{\sin(c+dx)^2}} \\
&\quad - \frac{4a^2 \cos(c+dx)^{9/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c+dx)^2\right)}{9d \sqrt{\sin(c+dx)^2}} \\
&\quad - \frac{2a^2 \cos(c+dx)^{11/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{4}; \frac{15}{4}; \cos(c+dx)^2\right)}{11d \sqrt{\sin(c+dx)^2}}
\end{aligned}$$

```
[In] int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] - (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c
+ d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*a^2*cos(c + d*x)^(9/2)*sin(c +
d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1
/2)) - (2*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4,
cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2))
```

3.154 $\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2 dx$

Optimal result	2363
Rubi [A] (verified)	2363
Mathematica [C] (verified)	2365
Maple [A] (verified)	2366
Fricas [C] (verification not implemented)	2366
Sympy [F(-1)]	2367
Maxima [F]	2367
Giac [F]	2367
Mupad [B] (verification not implemented)	2367

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2 dx = \frac{12a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{8a^2 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{7d} \\ + \frac{8a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{7d} \\ + \frac{4a^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} \\ + \frac{2a^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d}$$

[Out] $12/5*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+8/7*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4/5*a^2*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/7*a^2*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d+8/7*a^2*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used

= {2836, 2715, 2720, 2719}

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2 dx = \frac{8a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{7d} + \frac{12a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} \\ + \frac{2a^2 \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{7d} \\ + \frac{4a^2 \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d} \\ + \frac{8a^2 \sin(c + dx) \sqrt{\cos(c + dx)}}{7d}$$

[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2,x]

[Out] (12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2])/(7*d) + (8*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (4*a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^2 \cos^{\frac{3}{2}}(c + dx) + 2a^2 \cos^{\frac{5}{2}}(c + dx) + a^2 \cos^{\frac{7}{2}}(c + dx) \right) dx \\
&= a^2 \int \cos^{\frac{3}{2}}(c + dx) dx + a^2 \int \cos^{\frac{7}{2}}(c + dx) dx + (2a^2) \int \cos^{\frac{5}{2}}(c + dx) dx \\
&= \frac{2a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{4a^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2a^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} \\
&\quad + \frac{1}{3} a^2 \int \frac{1}{\sqrt{\cos(c + dx)}} dx + \frac{1}{7} (5a^2) \int \cos^{\frac{3}{2}}(c + dx) dx + \frac{1}{5} (6a^2) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{12a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2a^2 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\
&\quad + \frac{8a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{7d} + \frac{4a^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad + \frac{2a^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{1}{21} (5a^2) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{12a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{8a^2 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{7d} + \frac{8a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{7d} \\
&\quad + \frac{4a^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2a^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.74 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \cos^{\frac{3}{2}}(c + dx) (a + a \cos(c + dx))^2 dx$$

$$\frac{a^2 (1 + \cos(c + dx))^2 \sec^4\left(\frac{1}{2}(c + dx)\right) \left(\frac{42(3 \cos(c - dx - \arctan(\tan(c))) + \cos(c + dx + \arctan(\tan(c)))) \csc(c) \sec(c)}{\sqrt{\sec^2(c)}} - 80 \cos(c - \dots \right)}{280 d \sqrt{\cos(c + dx)}}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*cos[c + d*x])^2,x]

[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((42*(3*Cos[c - d*x - ArcTan[Tan[c]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 80 *Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-168*Cot[c] + 85*Sin[c + d*x] + 28*Sin[2*(c + d*x)] + 5*Sin[3*(c + d*x)]) - 84*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(280*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 9.44 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.25

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}a^2\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-116\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+126\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{35\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$
parts	$-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$

[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c))*a^2,x,method=_RETURNVERBOSE)

[Out]
$$-4/35*\left(\left(2*\cos\left(1/2*d*x+1/2*c\right)\right)^2-1\right)*\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*a^2*\left(40*\cos\left(1/2*d*x+1/2*c\right)*\sin\left(1/2*d*x+1/2*c\right)^8-116*\cos\left(1/2*d*x+1/2*c\right)*\sin\left(1/2*d*x+1/2*c\right)^6+126*\sin\left(1/2*d*x+1/2*c\right)^4*\cos\left(1/2*d*x+1/2*c\right)-39*\sin\left(1/2*d*x+1/2*c\right)^2*\cos\left(1/2*d*x+1/2*c\right)+10*\left(\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*\left(2*\sin\left(1/2*d*x+1/2*c\right)^2-1\right)^{(1/2)}*\text{EllipticF}\left(\cos\left(1/2*d*x+1/2*c\right),2^{(1/2)}\right)-21*\left(\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*\left(2*\sin\left(1/2*d*x+1/2*c\right)^2-1\right)^{(1/2)}*\text{EllipticE}\left(\cos\left(1/2*d*x+1/2*c\right),2^{(1/2)}\right)\right)/\left(-2*\sin\left(1/2*d*x+1/2*c\right)^4+\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}/\sin\left(1/2*d*x+1/2*c\right)/(2*\cos\left(1/2*d*x+1/2*c\right)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2 dx =$$

$$2\left(10i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-10i\sqrt{2}a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))\right)/d$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out]
$$-2/35*\left(10*I*\sqrt{2}*a^2*\text{weierstrassPInverse}(-4,0,\cos(dx+c)+I*\sin(dx+c))-10*I*\sqrt{2}*a^2*\text{weierstrassPInverse}(-4,0,\cos(dx+c)-I*\sin(dx+c))-21*I*\sqrt{2}*a^2*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+I*\sin(dx+c)))+21*I*\sqrt{2}*a^2*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-I*\sin(dx+c)))\right)-\left(5*a^2*\cos(dx+c)^2+14*a^2*\cos(dx+c)+20*a^2\right)*\sqrt{\cos(dx+c)}*\sin(dx+c)/d$$

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2 dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2 dx = \int (a \cos(dx + c) + a)^2 \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2 dx = \int (a \cos(dx + c) + a)^2 \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.07

$$\begin{aligned} & \int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2 dx \\ &= \frac{2 \left(a^2 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + a^2 \sqrt{\cos(c + dx)} \sin(c + dx) \right)}{3d} \\ & \quad - \frac{4a^2 \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7d \sqrt{\sin(c + dx)}^2} \\ & \quad - \frac{2a^2 \cos(c + dx)^{9/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c + dx)^2\right)}{9d \sqrt{\sin(c + dx)}^2} \end{aligned}$$

```
[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2,x)
```

```
[Out] (2*(a^2*ellipticF(c/2 + (d*x)/2, 2) + a^2*cos(c + d*x)^(1/2)*sin(c + d*x))  
/(3*d) - (4*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4,  
cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^2*cos(c + d*x)^(9/2)*  
sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)  
)^2)^(1/2))
```

3.155 $\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^2 dx$

Optimal result	2369
Rubi [A] (verified)	2369
Mathematica [C] (warning: unable to verify)	2371
Maple [A] (verified)	2371
Fricas [C] (verification not implemented)	2372
Sympy [F(-1)]	2372
Maxima [F]	2372
Giac [F]	2373
Mupad [B] (verification not implemented)	2373

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^2 dx = \frac{16a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{4a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{4a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2a^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}$$

[Out] $16/5*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4/3*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a^2*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+4/3*a^2*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.13 (sec), antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2719, 2715, 2720}

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^2 dx = \frac{4a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{16a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2a^2 \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d} + \frac{4a^2 \sin(c + dx) \sqrt{\cos(c + dx)}}{3d}$$

[In] Int[Sqrt[Cos[c + d*x]]*(a + a*cos[c + d*x])^2,x]

[Out] (16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2])/(3*d) + (4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)

Rule 2715

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_)*sin[(e_.) + (f_)*(x_)])^(n_)*((a_.) + (b_)*sin[(e_.) + (f_)*(x_)])^(m_), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(a^2 \sqrt{\cos(c + dx)} + 2a^2 \cos^{\frac{3}{2}}(c + dx) + a^2 \cos^{\frac{5}{2}}(c + dx) \right) dx \\
 &= a^2 \int \sqrt{\cos(c + dx)} dx + a^2 \int \cos^{\frac{5}{2}}(c + dx) dx + (2a^2) \int \cos^{\frac{3}{2}}(c + dx) dx \\
 &= \frac{2a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{4a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2a^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} \\
 &\quad + \frac{1}{5} (3a^2) \int \sqrt{\cos(c + dx)} dx + \frac{1}{3} (2a^2) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{16a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{4a^2 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\
 &\quad + \frac{4a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2a^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}
 \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.87 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.47

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2 dx$$

$$= \frac{a^2(1+\cos(c+dx))^2 \sec^4\left(\frac{1}{2}(c+dx)\right) \left(\frac{12(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c)))) \csc(c) \sec(c)}{\sqrt{\sec^2(c)}} - 20 \cos(c - \dots)\right)}{\dots}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2,x]

[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((12*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-48*Cot[c] + 20*Sin[c + d*x] + 3*Sin[2*(c + d*x)]) - 24*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]]))

Maple [A] (verified)

Time = 7.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.63

method	result
default	$\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} a^2 \left(-12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-13\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$
parts	$\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} \left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)} \sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-13*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.57

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^2 dx =$$

$$2 \left(5i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right) / d$$

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^2*cos(d*x + c) + 10*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^2 dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^2 dx = \int (a \cos(dx + c) + a)^2 \sqrt{\cos(dx + c)} dx$$

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```


Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2 dx = \int (a\cos(dx+c)+a)^2 \sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\begin{aligned} & \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2 dx \\ &= \frac{2a^2 E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{4a^2 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{3d} + \frac{4a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d} \\ & \quad - \frac{2a^2 \cos(c+dx)^{7/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c+dx)^2\right)}{7d \sqrt{\sin(c+dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2,x)

[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (4*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))

$$3.156 \quad \int \frac{(a+a \cos(c+dx))^2}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	2374
Rubi [A] (verified)	2374
Mathematica [C] (warning: unable to verify)	2376
Maple [B] (verified)	2376
Fricas [C] (verification not implemented)	2377
Sympy [F]	2377
Maxima [F]	2377
Giac [F]	2378
Mupad [B] (verification not implemented)	2378

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = \frac{4a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{8a^2 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d}$$

[Out] $4a^2(\cos(1/2dx+1/2c)^2)^{1/2}/\cos(1/2dx+1/2c)*\text{EllipticE}(\sin(1/2dx+1/2c), 2^{1/2})/d+8/3a^2(\cos(1/2dx+1/2c)^2)^{1/2}/\cos(1/2dx+1/2c)*\text{EllipticF}(\sin(1/2dx+1/2c), 2^{1/2})/d+2/3a^2\sin(dx+c)*\cos(dx+c)^{1/2}/d$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2720, 2719, 2715}

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = \frac{8a^2 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{4a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a^2 \sin(c + dx) \sqrt{\cos(c + dx)}}{3d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^2/\text{Sqrt}[\text{Cos}[c + d*x]], x]$

[Out] $(4a^2*\text{EllipticE}[(c + d*x)/2, 2])/d + (8a^2*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2a^2*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*d)$

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*
x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{\sqrt{\cos(c+dx)}} + 2a^2 \sqrt{\cos(c+dx)} + a^2 \cos^{\frac{3}{2}}(c+dx) \right) dx \\
&= a^2 \int \frac{1}{\sqrt{\cos(c+dx)}} dx + a^2 \int \cos^{\frac{3}{2}}(c+dx) dx + (2a^2) \int \sqrt{\cos(c+dx)} dx \\
&= \frac{4a^2 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2a^2 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} \\
&\quad + \frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d} + \frac{1}{3} a^2 \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{4a^2 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{8a^2 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
 Time = 3.86 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.34

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = \frac{a^2(1 + \cos(c + dx))^2 \sec^4\left(\frac{1}{2}(c + dx)\right) \left(\frac{3(3 \cos(c - dx - \arctan(\tan(c))) + \cos(c + dx + \arctan(\tan(c)))) \csc(c) \sec(c)}{\sqrt{\sec^2(c)}} - 8 \cos(c + dx)\right)}{\dots}$$

[In] Integrate[(a + a*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]

[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((3*(3*Cos[c - d*x - ArcTan[Tan[c]]) + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 8*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]])*Sin[c] + 2*Cos[c + d*x]*(-6*Cot[c] + Sin[c + d*x]) - 6*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[c + d*x]]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(113) = 226.
 Time = 3.94 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.40

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a^2} \left(2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\right) + \frac{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right)}{d}$
parts	$\frac{2a^2 \operatorname{am}^{-1}\left(\frac{dx}{2} + \frac{c}{2} \sqrt{2}\right)}{d} - \frac{2a^2 \sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} \left(4\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}$

[In] int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.00

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{2 \left(a^2 \sqrt{\cos(dx + c)} \sin(dx + c) - 2i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 3I \sqrt{2} a^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I \sin(dx + c))) - 3I \sqrt{2} a^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I \sin(dx + c))) \right)}{d}$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 2/3*(a^2*sqrt(cos(d*x + c))*sin(d*x + c) - 2*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d

Sympy [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = a^2 \left(\int \frac{1}{\sqrt{\cos(c + dx)}} dx + \int 2\sqrt{\cos(c + dx)} dx + \int \cos^{\frac{3}{2}}(c + dx) dx \right)$$

[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)

[Out] a**2*(Integral(1/sqrt(cos(c + d*x)), x) + Integral(2*sqrt(cos(c + d*x)), x) + Integral(cos(c + d*x)**(3/2), x))

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^2}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^2}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 14.89 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = \frac{4a^2 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{8a^2 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{3d} + \frac{2a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d}$$

[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^(1/2),x)

[Out] (4*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (8*a^2*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)

$$3.157 \quad \int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	2379
Rubi [A] (verified)	2379
Mathematica [C] (verified)	2380
Maple [B] (verified)	2381
Fricas [C] (verification not implemented)	2381
Sympy [F(-1)]	2382
Maxima [F]	2382
Giac [F]	2382
Mupad [B] (verification not implemented)	2382

Optimal result

Integrand size = 23, antiderivative size = 44

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{4a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a^2 \sin(c + dx)}{d\sqrt{\cos(c + dx)}}$$

[Out] $4*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2716, 2719, 2720}

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{4a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a^2 \sin(c + dx)}{d\sqrt{\cos(c + dx)}}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^2/\operatorname{Cos}[c + d*x]^{(3/2)}, x]$

[Out] $(4*a^2*\operatorname{EllipticF}[(c + d*x)/2, 2])/d + (2*a^2*\operatorname{Sin}[c + d*x])/(d*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]])$

Rule 2716

$\operatorname{Int}[(b* \sin[(c + d*x)])^n, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Cos}[c + d*x]*((b*\operatorname{Sin}[c + d*x])^{n+1}/(b*d*(n+1))), x] + \operatorname{Dist}[(n+2)/(b^2*(n+1)), \operatorname{Int}[(b*\operatorname{Sin}[c + d*x])^{n+2}, x], x] /; \operatorname{FreeQ}\{b, c, d, x\} \ \&\& \operatorname{LtQ}[n, -1] \ \&\& \operatorname{IntegerQ}[2*n]$

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{\cos^{\frac{3}{2}}(c+dx)} + \frac{2a^2}{\sqrt{\cos(c+dx)}} + a^2 \sqrt{\cos(c+dx)} \right) dx \\
&= a^2 \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)} dx + a^2 \int \sqrt{\cos(c+dx)} dx + (2a^2) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{2a^2 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{4a^2 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} + \frac{2a^2 \sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a^2 \int \sqrt{\cos(c+dx)} dx \\
&= \frac{4a^2 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} + \frac{2a^2 \sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.55

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2a^2 \csc(c + dx) \left(-3 \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cos^2(c + dx)\right) + \cos(c + dx) \left(6 \text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \cos^2(c + dx)\right) + \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \cos^2(c + dx)\right)\right)\right)}{3d\sqrt{\cos(c + dx)}}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a^2*Csc[c + d*x]*(-3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]
+ Cos[c + d*x]*(6*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2] + Cos[c
+ d*x]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]
^2])/(3*d*Sqrt[Cos[c + d*x]])
```


Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(68) = 136.

Time = 4.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.20

method	result
default	$\frac{4a^2 \left(-\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}} d$
parts	$\frac{2a^2 \left(-2\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}} d$

[In] `int((a+cos(d*x+c))*a^2/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{-4a^2(-\cos(1/2dx+1/2c))*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{(1/2)*\sin(1/2dx+1/2c)^2+(\sin(1/2dx+1/2c)^2)^{(1/2)}*(2*\sin(1/2dx+1/2c)^2-1)^{(1/2)}*EllipticF(\cos(1/2dx+1/2c),2^{(1/2)})*(-2*\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{(1/2)}}{(-2*\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{(1/2)}/\sin(1/2dx+1/2c)/(2*\cos(1/2dx+1/2c)^2-1)^{(1/2)}/d}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.20

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2 \left(i \sqrt{2} a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} a^2 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{d \cos(dx + c)}$$

[In] `integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")`

[Out]
$$-2*(I*\sqrt{2})*a^2*\cos(d*x + c)*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) - I*\sqrt{2}*a^2*\cos(d*x + c)*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - a^2*\sqrt{\cos(d*x + c)}*\sin(d*x + c)/(d*\cos(d*x + c))$$

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 14.90 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2 a^2 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{4 a^2 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{2 a^2 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^(3/2),x)

[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

$$3.158 \quad \int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	2383
Rubi [A] (verified)	2383
Mathematica [C] (verified)	2385
Maple [B] (verified)	2385
Fricas [C] (verification not implemented)	2386
Sympy [F(-1)]	2386
Maxima [F]	2386
Giac [F]	2387
Mupad [B] (verification not implemented)	2387

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)} dx = -\frac{4a^2 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{8a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d}$$

$$+ \frac{2a^2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2 \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

[Out] $-4*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+8/3*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+4*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2716, 2720, 2719}

$$\int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{8a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} - \frac{4a^2 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d}$$

$$+ \frac{2a^2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2 \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^2/\operatorname{Cos}[c + d*x]^{(5/2)}, x]$

[Out] $(-4*a^2*\operatorname{EllipticE}[(c + d*x)/2, 2])/d + (8*a^2*\operatorname{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a^2*\operatorname{Sin}[c + d*x])/(3*d*\operatorname{Cos}[c + d*x]^{(3/2)}) + (4*a^2*\operatorname{Sin}[c + d*x])/(d*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]])$

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2}{\cos^{\frac{5}{2}}(c + dx)} + \frac{2a^2}{\cos^{\frac{3}{2}}(c + dx)} + \frac{a^2}{\sqrt{\cos(c + dx)}} \right) dx \\
&= a^2 \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx + a^2 \int \frac{1}{\sqrt{\cos(c + dx)}} dx + (2a^2) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a^2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{4a^2 \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
&\quad + \frac{1}{3} a^2 \int \frac{1}{\sqrt{\cos(c + dx)}} dx - (2a^2) \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{4a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{8a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a^2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{4a^2 \sin(c + dx)}{d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{2a^2 \csc(c + dx) \left(\text{Hypergeometric2F1} \left(-\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \cos^2(c + dx) \right) + 6 \cos(c + dx) \text{Hypergeometric2F1} \left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cos^2(c + dx) \right) \right)}{3d \cos^{\frac{3}{2}}(c + dx)}$$

[In] Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]

[Out] (2*a^2*Csc[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 6*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - 3*Cos[c + d*x]^2*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(3*d*Cos[c + d*x]^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(135) = 270.

Time = 4.83 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.08

method	result
default	$-\frac{4\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a^2 \left(12\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 6\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}$
parts	$-\frac{2a^2 \left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1 F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}$

[In] int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.05

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2 \left(2i \sqrt{2} a^2 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 2i \sqrt{2} a^2 \cos(dx + c) \right)}{\dots}$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] -2/3*(2*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 2*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (6*a^2*cos(d*x + c) + a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 14.96 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\begin{aligned} \int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx &= \frac{2 a^2 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} \\ &+ \frac{4 a^2 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}} \\ &+ \frac{2 a^2 \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{3 d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^(5/2),x)

[Out] (2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (4*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))

$$3.159 \quad \int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	2388
Rubi [A] (verified)	2388
Mathematica [C] (verified)	2390
Maple [B] (verified)	2390
Fricas [C] (verification not implemented)	2391
Sympy [F(-1)]	2391
Maxima [F]	2392
Giac [F]	2392
Mupad [B] (verification not implemented)	2392

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)} dx = -\frac{16a^2 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{4a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2a^2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[Out] $-16/5*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4/3*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+4/3*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+16/5*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2716, 2719, 2720}

$$\int \frac{(a+a \cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{4a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} - \frac{16a^2 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} \\ + \frac{4a^2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2a^2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{16a^2 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[In] $\operatorname{Int}[(a+a \cos[c+dx])^2/\cos[c+dx]^{(7/2)}, x]$

[Out] $(-16*a^2*\operatorname{EllipticE}[(c+dx)/2, 2])/(5*d) + (4*a^2*\operatorname{EllipticF}[(c+dx)/2, 2])/(3*d) + (2*a^2*\sin[c+dx])/(5*d*\cos[c+dx]^{(5/2)}) + (4*a^2*\sin[c+dx])/(5*d*\sqrt{\cos[c+dx]})$

*x]]/(3*d*Cos[c + d*x]^(3/2)) + (16*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{a^2}{\cos^{\frac{7}{2}}(c + dx)} + \frac{2a^2}{\cos^{\frac{5}{2}}(c + dx)} + \frac{a^2}{\cos^{\frac{3}{2}}(c + dx)} \right) dx \\
 &= a^2 \int \frac{1}{\cos^{\frac{7}{2}}(c + dx)} dx + a^2 \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + (2a^2) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2a^2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{4a^2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2a^2 \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{1}{5} (3a^2) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &\quad + \frac{1}{3} (2a^2) \int \frac{1}{\sqrt{\cos(c + dx)}} dx - a^2 \int \sqrt{\cos(c + dx)} dx \\
 &= -\frac{2a^2 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{4a^2 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a^2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\
 &\quad + \frac{4a^2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{16a^2 \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} - \frac{1}{5} (3a^2) \int \sqrt{\cos(c + dx)} dx
 \end{aligned}$$

$$= -\frac{16a^2 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{4a^2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2a^2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx \\ = \frac{2a^2 \csc(c + dx) \left(3 \operatorname{Hypergeometric2F1}\left(-\frac{5}{4}, \frac{1}{2}, -\frac{1}{4}, \cos^2(c + dx)\right) + 5 \cos(c + dx) \left(2 \operatorname{Hypergeometric2F1}\left(-\frac{5}{4}, \frac{1}{2}, -\frac{1}{4}, \cos^2(c + dx)\right) + 5 \cos(c + dx)\right)\right)}{15d \cos^{\frac{5}{2}}(c + dx)}$$

[In] Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(7/2), x]

[Out] (2*a^2*Csc[c + d*x]*(3*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] + 5*Cos[c + d*x]*(2*Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 3*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2])/(15*d*Cos[c + d*x]^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(157) = 314.

Time = 7.27 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

method	result
default	$8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^2 \left(-\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}{12\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2}\right)^2} + \frac{17\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{30\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} \right)$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^2)^(1/2)

$4 + \sin(1/2*d*x + 1/2*c)^2)^{1/2} * (\text{EllipticF}(\cos(1/2*d*x + 1/2*c), 2^{1/2}) - \text{EllipticE}(\cos(1/2*d*x + 1/2*c), 2^{1/2})) - 1/80 * \cos(1/2*d*x + 1/2*c) * (-2 * \sin(1/2*d*x + 1/2*c)^4 + \sin(1/2*d*x + 1/2*c)^2)^{1/2} / (\cos(1/2*d*x + 1/2*c)^{2-1/2})^3 / \sin(1/2*d*x + 1/2*c) / (2 * \cos(1/2*d*x + 1/2*c)^{2-1})^{1/2} / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{7/2}(c + dx)} dx =$$

$$2 \left(5i \sqrt{2} a^2 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^2 \cos(dx + c) \right)$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] -2/15*(5*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 12*I*sqrt(2)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (24*a^2*cos(d*x + c)^2 + 10*a^2*cos(d*x + c) + 3*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)

Mupad [B] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(a + a \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx = \frac{6a^2 \sin(c + dx) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c + dx)^2\right) + 20a^2 \cos(c + dx) \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{15d \cos(c + dx)^{5/2} \sqrt{1 - \cos(c + dx)}}$$

[In] int((a + a*cos(c + d*x))^2/cos(c + d*x)^(7/2),x)

[Out] (6*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*a^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 30*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))

3.160 $\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3 dx$

Optimal result	2393
Rubi [A] (verified)	2394
Mathematica [C] (verified)	2396
Maple [A] (verified)	2396
Fricas [C] (verification not implemented)	2397
Sympy [F(-1)]	2397
Maxima [F]	2397
Giac [F]	2398
Mupad [B] (verification not implemented)	2398

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3 dx = \frac{68a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{44a^3 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{44a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{68a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} + \frac{6a^3 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{2a^3 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d}$$

```
[Out] 68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+44/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+68/45*a^3*cos(d*x+c)^(3/2)*sin(d*x+c)/d+6/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^3*cos(d*x+c)^(7/2)*sin(d*x+c)/d+44/21*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2715, 2720, 2719}

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3 dx = \frac{44a^3 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{68a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{2a^3 \sin(c + dx) \cos^{\frac{7}{2}}(c + dx)}{9d} + \frac{6a^3 \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{7d} + \frac{68a^3 \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{45d} + \frac{44a^3 \sin(c + dx) \sqrt{\cos(c + dx)}}{21d}$$

[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3,x]

[Out] (68*a^3*EllipticE[(c + d*x)/2, 2])/(15*d) + (44*a^3*EllipticF[(c + d*x)/2, 2])/(21*d) + (44*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (68*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (6*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^3*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +

f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3 \cos^{\frac{3}{2}}(c + dx) + 3a^3 \cos^{\frac{5}{2}}(c + dx) + 3a^3 \cos^{\frac{7}{2}}(c + dx) + a^3 \cos^{\frac{9}{2}}(c + dx) \right) dx \\
&= a^3 \int \cos^{\frac{3}{2}}(c + dx) dx + a^3 \int \cos^{\frac{9}{2}}(c + dx) dx \\
&\quad + (3a^3) \int \cos^{\frac{5}{2}}(c + dx) dx + (3a^3) \int \cos^{\frac{7}{2}}(c + dx) dx \\
&= \frac{2a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{6a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad + \frac{6a^3 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{2a^3 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d} \\
&\quad + \frac{1}{3} a^3 \int \frac{1}{\sqrt{\cos(c + dx)}} dx + \frac{1}{9} (7a^3) \int \cos^{\frac{5}{2}}(c + dx) dx \\
&\quad + \frac{1}{5} (9a^3) \int \sqrt{\cos(c + dx)} dx + \frac{1}{7} (15a^3) \int \cos^{\frac{3}{2}}(c + dx) dx \\
&= \frac{18a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2a^3 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\
&\quad + \frac{44a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{68a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} \\
&\quad + \frac{6a^3 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{2a^3 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d} \\
&\quad + \frac{1}{15} (7a^3) \int \sqrt{\cos(c + dx)} dx + \frac{1}{7} (5a^3) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{68a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{44a^3 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} \\
&\quad + \frac{44a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{68a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} \\
&\quad + \frac{6a^3 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{2a^3 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.71 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.73

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3 dx$$

$$= \frac{a^3(1+\cos(c+dx))^3 \sec^6\left(\frac{1}{2}(c+dx)\right) \left(\frac{1428(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c)))) \csc(c) \sec(c)}{\sqrt{\sec^2(c)}} - 2640 \cos\right)}{}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3,x]

[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((1428*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 2640*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-5712*Cot[c] + 2910*Sin[c + d*x] + 1022*Sin[2*(c + d*x)] + 270*Sin[3*(c + d*x)] + 35*Sin[4*(c + d*x)]) - 2856*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(10080*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 12.95 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.77

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-600\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+212\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+66\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{315\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$
parts	Expression too large to display

[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3 dx =$$

$$2 \left(165i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 165i \sqrt{2} a^3 \text{weierstrassPInverse} \right.$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] -2/315*(165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x + c)^2 + 238*a^3*cos(d*x + c) + 330*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3 dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3 dx = \int (a \cos(dx + c) + a)^3 \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3 dx = \int (a \cos(dx + c) + a)^3 \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 15.00 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.40

$$\begin{aligned} & \int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3 dx \\ &= \frac{2 \left(a^3 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + a^3 \sqrt{\cos(c + dx)} \sin(c + dx) \right)}{3d} \\ & - \frac{2 \left(\frac{33a^3 \cos(c+dx)^{7/2} \sin(c+dx)}{\sqrt{\sin(c+dx)^2}} - \frac{5a^3 \cos(c+dx)^{11/2} \sin(c+dx)}{\sqrt{\sin(c+dx)^2}} \right) {}_2F_1\left(\frac{1}{2}, \frac{11}{4}; \frac{15}{4}; \cos(c + dx)^2\right)}{77d} \\ & - \frac{2a^3 \cos(c + dx)^{9/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c + dx)^2\right)}{3d \sqrt{\sin(c + dx)^2}} \\ & - \frac{104a^3 \cos(c + dx)^{11/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{4}; \frac{19}{4}; \cos(c + dx)^2\right)}{385d \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3,x)

[Out] (2*(a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3*cos(c + d*x)^(1/2)*sin(c + d*x)))/(3*d) - (2*((33*a^3*cos(c + d*x)^(7/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2) - (5*a^3*cos(c + d*x)^(11/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2))*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(77*d) - (2*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2)) - (104*a^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 19/4, cos(c + d*x)^2))/(385*d*(sin(c + d*x)^2)^(1/2))

3.161 $\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^3 dx$

Optimal result	2399
Rubi [A] (verified)	2399
Mathematica [C] (warning: unable to verify)	2401
Maple [A] (verified)	2402
Fricas [C] (verification not implemented)	2402
Sympy [F(-1)]	2403
Maxima [F]	2403
Giac [F]	2403
Mupad [B] (verification not implemented)	2403

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^3 dx = \frac{28a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{52a^3 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{52a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{6a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2a^3 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d}$$

```
[Out] 28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+6/5*a^3*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a^3*cos(d*x+c)^(5/2)*sin(d*x+c)/d+52/21*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used

= {2836, 2719, 2715, 2720}

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3 dx = \frac{52a^3 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} + \frac{28a^3 E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d} + \frac{2a^3 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{7d} + \frac{6a^3 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{5d} + \frac{52a^3 \sin(c+dx) \sqrt{\cos(c+dx)}}{21d}$$

[In] Int[Sqrt[Cos[c + d*x]]*(a + a*cos[c + d*x])^3,x]

[Out] (28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2, 2])/(21*d) + (52*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (6*a^3*cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^3*cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^3 \sqrt{\cos(c+dx)} + 3a^3 \cos^{\frac{3}{2}}(c+dx) + 3a^3 \cos^{\frac{5}{2}}(c+dx) + a^3 \cos^{\frac{7}{2}}(c+dx) \right) dx \\
&= a^3 \int \sqrt{\cos(c+dx)} dx + a^3 \int \cos^{\frac{7}{2}}(c+dx) dx \\
&\quad + (3a^3) \int \cos^{\frac{3}{2}}(c+dx) dx + (3a^3) \int \cos^{\frac{5}{2}}(c+dx) dx \\
&= \frac{2a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{d} + \frac{6a^3 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{2a^3 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + \frac{1}{7} (5a^3) \int \cos^{\frac{3}{2}}(c+dx) dx \\
&\quad + a^3 \int \frac{1}{\sqrt{\cos(c+dx)}} dx + \frac{1}{5} (9a^3) \int \sqrt{\cos(c+dx)} dx \\
&= \frac{28a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{2a^3 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} \\
&\quad + \frac{52a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{21d} + \frac{6a^3 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{2a^3 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + \frac{1}{21} (5a^3) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{28a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{52a^3 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} \\
&\quad + \frac{52a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{21d} \\
&\quad + \frac{6a^3 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{2a^3 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.46 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\begin{aligned}
&\int \sqrt{\cos(c+dx)} (a + a \cos(c+dx))^3 dx \\
&= \frac{a^3 (1 + \cos(c+dx))^3 \sec^6\left(\frac{1}{2}(c+dx)\right) \left(\frac{294(3 \cos(c-dx - \arctan(\tan(c))) + \cos(c+dx + \arctan(\tan(c)))) \csc(c) \sec(c)}{\sqrt{\sec^2(c)}} - 520 \cos(c) \right)}{5}
\end{aligned}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3,x]

[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((294*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 520 Cos[c])

20*cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-1176*Cot[c] + 535*Sin[c + d*x] + 126*Sin[2*(c + d*x)] + 15*Sin[3*(c + d*x)]) - 588*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])/(1680*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 10.53 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.25

method	result
default	$\frac{4\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}a^3\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+602\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105\sqrt{-2\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$
parts	Expression too large to display

[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out]
$$-4/105*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^3*(120*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^8-432*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+602*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)-208*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+65*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-147*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3 dx = \frac{2\left(65i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-65i\sqrt{2}a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))\right)}{105}$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out]
$$-2/105*(65*I*\sqrt{2})*a^3*\text{weierstrassPInverse}(-4,0,\cos(d*x+c)+I*\sin(d*x+c))-65*I*\sqrt{2}*a^3*\text{weierstrassPInverse}(-4,0,\cos(d*x+c)-I*\sin(d*x+c))-147*I*\sqrt{2}*a^3*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d*x+c)+I*\sin(d*x+c)))+147*I*\sqrt{2}*a^3*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d*x+c)-I*\sin(d*x+c)))$$

, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 130*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^3 dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^3 dx = \int (a \cos(dx + c) + a)^3 \sqrt{\cos(dx + c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^3 dx = \int (a \cos(dx + c) + a)^3 \sqrt{\cos(dx + c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\begin{aligned} & \int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^3 dx \\ &= \frac{2 \left(a^3 E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) + a^3 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right) + a^3 \sqrt{\cos(c + dx)} \sin(c + dx) \right)}{d} \\ & \quad - \frac{6 a^3 \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7 d \sqrt{\sin(c + dx)^2}} \\ & \quad - \frac{2 a^3 \cos(c + dx)^{9/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c + dx)^2\right)}{9 d \sqrt{\sin(c + dx)^2}} \end{aligned}$$

```
[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3,x)
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```
[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + a^3*ellipticF(c/2 + (d*x)/2, 2) + a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (6*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```


$$3.162 \quad \int \frac{(a+a \cos(c+dx))^3}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	2405
Rubi [A] (verified)	2405
Mathematica [C] (warning: unable to verify)	2407
Maple [A] (verified)	2407
Fricas [C] (verification not implemented)	2408
Sympy [F(-1)]	2408
Maxima [F]	2408
Giac [F]	2409
Mupad [B] (verification not implemented)	2409

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \frac{36a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{4a^3 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} \\ + \frac{2a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{d} + \frac{2a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}$$

[Out] $36/5*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a^3*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2*a^3*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2720, 2719, 2715}

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \frac{4a^3 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{36a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} \\ + \frac{2a^3 \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d} + \frac{2a^3 \sin(c + dx) \sqrt{\cos(c + dx)}}{d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^3/\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]], x]$

[Out] $(36*a^3*\operatorname{EllipticE}[(c + d*x)/2, 2])/(5*d) + (4*a^3*\operatorname{EllipticF}[(c + d*x)/2, 2])/d + (2*a^3*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/d + (2*a^3*\operatorname{Cos}[c + d*x]^{(3/2)}*\operatorname{Sin}[c + d*x])/(5*d)$

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^3}{\sqrt{\cos(c + dx)}} + 3a^3 \sqrt{\cos(c + dx)} + 3a^3 \cos^{\frac{3}{2}}(c + dx) + a^3 \cos^{\frac{5}{2}}(c + dx) \right) dx \\
&= a^3 \int \frac{1}{\sqrt{\cos(c + dx)}} dx + a^3 \int \cos^{\frac{5}{2}}(c + dx) dx \\
&\quad + (3a^3) \int \sqrt{\cos(c + dx)} dx + (3a^3) \int \cos^{\frac{3}{2}}(c + dx) dx \\
&= \frac{6a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a^3 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{d} \\
&\quad + \frac{2a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{1}{5} (3a^3) \int \sqrt{\cos(c + dx)} dx + a^3 \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{36a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{4a^3 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} \\
&\quad + \frac{2a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{d} + \frac{2a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.01 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.56

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{a^3(1 + \cos(c + dx))^3 \sec^6\left(\frac{1}{2}(c + dx)\right) \left(\frac{9(3 \cos(c - dx - \arctan(\tan(c))) + \cos(c + dx + \arctan(\tan(c)))) \csc(c) \sec(c)}{\sqrt{\sec^2(c)}} - 20 \cos(c + dx)\right)}{40 d \sqrt{\cos(c + dx)}}$$

[In] Integrate[(a + a*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]

[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((9*(3*Cos[c - d*x - ArcTan[Tan[c]]) + Cos[c + d*x + ArcTan[Tan[c]])]*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]^2)*Sin[c] + Cos[c + d*x]*(-36*Cot[c] + 10*Sin[c + d*x] + Sin[2*(c + d*x)]) - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(40*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 5.94 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.75

method	result
default	$-\frac{4\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a^3\left(-4\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 14\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 6\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}$
parts	$\frac{2a^3 \operatorname{am}^{-1}\left(\frac{dx}{2} + \frac{c}{2}, \sqrt{2}\right)}{d} - \frac{2a^3\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(-8\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 6\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}$

[In] int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx =$$

$$2 \left(5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right) / d$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -2/5*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c) + 5*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/d

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^3}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^3}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 13.92 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \frac{6a^3 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{4a^3 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{2a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{d} - \frac{2a^3 \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7d \sqrt{\sin(c + dx)^2}}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^(1/2),x)

[Out] (6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (2*a^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))

$$3.163 \quad \int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	2410
Rubi [A] (verified)	2410
Mathematica [C] (verified)	2412
Maple [A] (verified)	2412
Fricas [C] (verification not implemented)	2413
Sympy [F(-1)]	2413
Maxima [F]	2413
Giac [F]	2414
Mupad [B] (verification not implemented)	2414

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{4a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{20a^3 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2a^3 \sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d}$$

[Out] 4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/d+20/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/d+2*a^3*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a^3*sin(d*x+c)*cos(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2836, 2716, 2719, 2720, 2715}

$$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{20a^3 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{4a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} \\ + \frac{2a^3 \sin(c+dx) \sqrt{\cos(c+dx)}}{3d} + \frac{2a^3 \sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[In] Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(3/2), x]

[Out] (4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^3*Sin[c + d*x])/(d*sqrt[Cos[c + d*x]]) + (2*a^3*sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{a^3}{\cos^{\frac{3}{2}}(c + dx)} + \frac{3a^3}{\sqrt{\cos(c + dx)}} + 3a^3 \sqrt{\cos(c + dx)} + a^3 \cos^{\frac{3}{2}}(c + dx) \right) dx \\
 &= a^3 \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + a^3 \int \cos^{\frac{3}{2}}(c + dx) dx \\
 &\quad + (3a^3) \int \frac{1}{\sqrt{\cos(c + dx)}} dx + (3a^3) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{6a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{6a^3 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a^3 \sin(c + dx)}{d\sqrt{\cos(c + dx)}} \\
 &\quad + \frac{2a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3}a^3 \int \frac{1}{\sqrt{\cos(c + dx)}} dx - a^3 \int \sqrt{\cos(c + dx)} dx
 \end{aligned}$$

$$= \frac{4a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{20a^3 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2a^3 \sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx = \\ \frac{2a^3 \csc(c + dx) \left(-3 \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cos^2(c + dx)\right) \sqrt{\sin^2(c + dx) + \cos(c + dx)} \left(-1 + \cos(c + dx)\right) \right)}{\cos^{\frac{3}{2}}(c + dx)}$$

[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(3/2), x]

[Out] (-2*a^3*Csc[c + d*x]*(-3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + Cos[c + d*x]*(-1 + Cos[c + d*x]^2 + 10*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 3*Cos[c + d*x]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(3*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 5.97 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.89

method	result
default	$\frac{4a^3 \left(2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \cos \left(\frac{dx}{2} + \frac{c}{2} \right) - 4 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \cos \left(\frac{dx}{2} + \frac{c}{2} \right) + 5 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} F \left(\cos \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) - 3 \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \right)}{3d}$
parts	$\frac{2a^3 \left(-2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \right)}{\sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \right)} d$

[In] int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.98

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx =$$

$$2 \left(5i \sqrt{2} a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c) \right)$$

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I
*sqrt(2)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c) + 3*a^3)*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 14.64 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\begin{aligned} \int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx &= \frac{6 a^3 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{20 a^3 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{3 d} \\ &+ \frac{2 a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{3 d} \\ &+ \frac{2 a^3 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^(3/2),x)

[Out] (6*a^3*ellipticE(c/2 + (d*x)/2, 2))/d + (20*a^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

$$3.164 \quad \int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	2415
Rubi [A] (verified)	2415
Mathematica [C] (verified)	2417
Maple [B] (verified)	2417
Fricas [C] (verification not implemented)	2418
Sympy [F(-1)]	2418
Maxima [F]	2418
Giac [F]	2419
Mupad [B] (verification not implemented)	2419

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)} dx = -\frac{4a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{20a^3 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2a^3 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3 \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

[Out] $-4*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+20/3*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+6*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2716, 2720, 2719}

$$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{20a^3 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} - \frac{4a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} \\ + \frac{2a^3 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3 \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^3/\text{Cos}[c + d*x]^{(5/2)}, x]$

[Out] $(-4*a^3*\text{EllipticE}[(c + d*x)/2, 2])/d + (20*a^3*\text{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*a^3*\text{Sin}[c + d*x])/(3*d*\text{Cos}[c + d*x]^{(3/2)}) + (6*a^3*\text{Sin}[c + d*x])/(d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^3}{\cos^{\frac{5}{2}}(c + dx)} + \frac{3a^3}{\cos^{\frac{3}{2}}(c + dx)} + \frac{3a^3}{\sqrt{\cos(c + dx)}} + a^3 \sqrt{\cos(c + dx)} \right) dx \\
&= a^3 \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx + a^3 \int \sqrt{\cos(c + dx)} dx \\
&\quad + (3a^3) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + (3a^3) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{6a^3 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a^3 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} \\
&\quad + \frac{6a^3 \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{1}{3} a^3 \int \frac{1}{\sqrt{\cos(c + dx)}} dx - (3a^3) \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{4a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{20a^3 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a^3 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.53

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{2a^3 \csc(c + dx) \left(\text{Hypergeometric2F1} \left(-\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \cos^2(c + dx) \right) + 9 \cos(c + dx) \text{Hypergeometric2F1} \left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cos^2(c + dx) \right) \right)}{\sqrt{\sin(c + dx)}}$$

[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]

[Out] (2*a^3*Csc[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 9*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - Cos[c + d*x]^2*(9*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2] + Cos[c + d*x]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2])/(3*d*Cos[c + d*x]^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(135) = 270.

Time = 6.45 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.08

method	result
default	$4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} a^3 \left(18\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 10\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1 F\left(\dots\right) \right)$
parts	$2a^3 \left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1 F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right) / \left(3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \right) \right)$

[In] int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))

$$)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.05

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2 \left(5i \sqrt{2} a^3 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c) \right)}{\dots}$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out]
$$-2/3*(5*I*\sqrt{2}*a^3*\cos(d*x + c)^2*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) - 5*I*\sqrt{2}*a^3*\cos(d*x + c)^2*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + 3*I*\sqrt{2}*a^3*\cos(d*x + c)^2*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) - 3*I*\sqrt{2}*a^3*\cos(d*x + c)^2*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) - (9*a^3*\cos(d*x + c) + a^3)*\sqrt{\cos(d*x + c)}*\sin(d*x + c))/(d*\cos(d*x + c)^2)$$

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.38

$$\begin{aligned} \int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx &= \frac{2(a^3 E(\frac{c}{2} + \frac{dx}{2} | 2) + 3a^3 F(\frac{c}{2} + \frac{dx}{2} | 2))}{d} \\ &+ \frac{6a^3 \sin(c + dx) {}_2F_1(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}} \\ &+ \frac{2a^3 \sin(c + dx) {}_2F_1(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2)}{3d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^(5/2),x)

[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a^3*ellipticF(c/2 + (d*x)/2, 2)))/d + (6*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))

$$3.165 \quad \int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	2420
Rubi [A] (verified)	2420
Mathematica [C] (verified)	2422
Maple [B] (verified)	2422
Fricas [C] (verification not implemented)	2423
Sympy [F(-1)]	2423
Maxima [F]	2424
Giac [F]	2424
Mupad [B] (verification not implemented)	2424

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)} dx = -\frac{36a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{4a^3 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d}$$

$$+ \frac{2a^3 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3 \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{36a^3 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[Out] $-36/5*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+2*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+36/5*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2716, 2719, 2720}

$$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{4a^3 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} - \frac{36a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d}$$

$$+ \frac{2a^3 \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{2a^3 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{36a^3 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^3/\text{Cos}[c + d*x]^{(7/2)}, x]$

[Out] $(-36a^3 \text{EllipticE}[(c + dx)/2, 2]) / (5d) + (4a^3 \text{EllipticF}[(c + dx)/2, 2]) / d + (2a^3 \sin[c + dx]) / (5d \cos[c + dx]^{5/2}) + (2a^3 \sin[c + dx]) / (d \cos[c + dx]^{3/2}) + (36a^3 \sin[c + dx]) / (5d \sqrt{\cos[c + dx]})$

Rule 2716

$\text{Int}[(b \sin[c + dx] + d(x))^{n-1}, x_Symbol] \rightarrow \text{Simp}[\cos[c + dx] * ((b \sin[c + dx])^{n+1} / (b*d*(n+1))), x] + \text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(b \sin[c + dx])^{n+2}, x], x] /;$ FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

$\text{Int}[\sqrt{\sin[c + dx]}, x_Symbol] \rightarrow \text{Simp}[(2/d) \text{EllipticE}[(1/2)*(c - \pi/2 + dx), 2], x] /;$ FreeQ[{c, d}, x]

Rule 2720

$\text{Int}[1/\sqrt{\sin[c + dx]}, x_Symbol] \rightarrow \text{Simp}[(2/d) \text{EllipticF}[(1/2)*(c - \pi/2 + dx), 2], x] /;$ FreeQ[{c, d}, x]

Rule 2836

$\text{Int}[(d \sin[e + fx] + f(x))^{n-1} * ((a + b \sin[e + fx])^m * (d \sin[e + fx])^n), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[(a + b \sin[e + fx])^m * (d \sin[e + fx])^n, x], x] /;$ FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{a^3}{\cos^{\frac{7}{2}}(c + dx)} + \frac{3a^3}{\cos^{\frac{5}{2}}(c + dx)} + \frac{3a^3}{\cos^{\frac{3}{2}}(c + dx)} + \frac{a^3}{\sqrt{\cos(c + dx)}} \right) dx \\
 &= a^3 \int \frac{1}{\cos^{\frac{7}{2}}(c + dx)} dx + a^3 \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad + (3a^3) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx + (3a^3) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2a^3 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a^3 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
 &\quad + \frac{1}{5}(3a^3) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + a^3 \int \frac{1}{\sqrt{\cos(c + dx)}} dx - (3a^3) \int \sqrt{\cos(c + dx)} dx \\
 &= -\frac{6a^3 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{4a^3 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a^3 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\
 &\quad + \frac{2a^3 \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)} + \frac{36a^3 \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} - \frac{1}{5}(3a^3) \int \sqrt{\cos(c + dx)} dx
 \end{aligned}$$

$$= -\frac{36a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{4a^3 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} \\ + \frac{2a^3 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3 \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{36a^3 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx \\ = \frac{2a^3 \csc(c + dx) \left(\operatorname{Hypergeometric2F1}\left(-\frac{5}{4}, \frac{1}{2}, -\frac{1}{4}, \cos^2(c + dx)\right) + 5 \cos(c + dx) \operatorname{Hypergeometric2F1}\left(-\frac{3}{4}, \frac{1}{2}, -\frac{1}{4}, \cos^2(c + dx)\right)\right)}{\cos^{\frac{7}{2}}(c + dx)}$$

[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(7/2), x]

[Out] (2*a^3*Csc[c + d*x]*(Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] + 5*Cos[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + Cos[c + d*x]*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - Cos[c + d*x]*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]))) * Sqrt[Sin[c + d*x]^2]) / (5*d*Cos[c + d*x]^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(157) = 314.

Time = 7.48 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.30

method	result
default	$16 \sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} a^3 \left(\frac{7 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2(\cos^2(\frac{dx}{2} + \frac{c}{2}))} + 1 F(\cos(\frac{dx}{2} + \frac{c}{2}), \sqrt{2})}{10 \sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}} - \frac{\cos(\frac{dx}{2} + \frac{c}{2}) \sqrt{-2(\sin^2(\frac{dx}{2} + \frac{c}{2}))}}{16(\cos^2(\frac{dx}{2} + \frac{c}{2}))} \right)$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-1/16*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(sin(1/2*d*x+1/2*c)^2)^(1/2))

$\sqrt{x+1/2c} \sqrt{-2\cos(1/2dx+1/2c)+1} / (-2\sin(1/2dx+1/2c) \sqrt{4+\sin(1/2dx+1/2c)^2} \sqrt{\text{EllipticF}(\cos(1/2dx+1/2c), 2)} - \text{EllipticE}(\cos(1/2dx+1/2c), 2)) - 1/160 \cos(1/2dx+1/2c) \sqrt{-2\sin(1/2dx+1/2c)^4 + \sin(1/2dx+1/2c)^2} / (\cos(1/2dx+1/2c)^2 - 1/2) \sqrt{3} / \sin(1/2dx+1/2c) / (2\cos(1/2dx+1/2c)^2 - 1) \sqrt{1/2} / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.71

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx = \frac{2 \left(5i \sqrt{2} a^3 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c) \right)}{\dots}$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] $-2/5*(5*I*\sqrt{2})*a^3*\cos(d*x + c)^3*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) - 5*I*\sqrt{2})*a^3*\cos(d*x + c)^3*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + 9*I*\sqrt{2})*a^3*\cos(d*x + c)^3*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) - 9*I*\sqrt{2})*a^3*\cos(d*x + c)^3*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) - (18*a^3*\cos(d*x + c)^2 + 5*a^3*\cos(d*x + c) + a^3)*\sqrt{\cos(d*x + c)*\sin(d*x + c)}/(d*\cos(d*x + c)^3)$

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)

Mupad [B] (verification not implemented)

Time = 15.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\begin{aligned} \int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx &= \frac{2 a^3 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} \\ &+ \frac{6 a^3 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}} \\ &+ \frac{2 a^3 \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}} \\ &+ \frac{2 a^3 \sin(c + dx) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c + dx)^2\right)}{5 d \cos(c + dx)^{5/2} \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^(7/2),x)

[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^3*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2))

$$3.166 \quad \int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	2425
Rubi [A] (verified)	2425
Mathematica [C] (verified)	2427
Maple [B] (verified)	2427
Fricas [C] (verification not implemented)	2428
Sympy [F(-1)]	2429
Maxima [F]	2429
Giac [F]	2429
Mupad [B] (verification not implemented)	2429

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)} dx = -\frac{28a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{52a^3 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d}$$

$$+ \frac{2a^3 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{52a^3 \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{28a^3 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[Out] $-28/5*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+52/21*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/7*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(7/2)}+6/5*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+52/21*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+28/5*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2716, 2720, 2719}

$$\int \frac{(a+a \cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{52a^3 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} - \frac{28a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d}$$

$$+ \frac{52a^3 \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{2a^3 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} + \frac{28a^3 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[In] Int[(a + a*cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]

[Out] (-28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*cos[c + d*x]^(7/2)) + (6*a^3*Sin[c + d*x])/(5*d*cos[c + d*x]^(5/2)) + (52*a^3*Sin[c + d*x])/(21*d*cos[c + d*x]^(3/2)) + (28*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{a^3}{\cos^{\frac{9}{2}}(c + dx)} + \frac{3a^3}{\cos^{\frac{7}{2}}(c + dx)} + \frac{3a^3}{\cos^{\frac{5}{2}}(c + dx)} + \frac{a^3}{\cos^{\frac{3}{2}}(c + dx)} \right) dx \\
 &= a^3 \int \frac{1}{\cos^{\frac{9}{2}}(c + dx)} dx + a^3 \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &\quad + (3a^3) \int \frac{1}{\cos^{\frac{7}{2}}(c + dx)} dx + (3a^3) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2a^3 \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
 &\quad + \frac{1}{7}(5a^3) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx + a^3 \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad - a^3 \int \sqrt{\cos(c + dx)} dx + \frac{1}{5}(9a^3) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{2a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2a^3 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} + \frac{2a^3 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{6a^3 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{52a^3 \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{28a^3 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}} \\
&\quad + \frac{1}{21}(5a^3) \int \frac{1}{\sqrt{\cos(c+dx)}} dx - \frac{1}{5}(9a^3) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{28a^3 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{52a^3 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} + \frac{2a^3 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{6a^3 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{52a^3 \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{28a^3 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx = \frac{2a^3 \csc(c + dx) \left(5 \operatorname{Hypergeometric2F1}\left(-\frac{7}{4}, \frac{1}{2}, -\frac{3}{4}, \cos^2(c + dx)\right) + 7 \cos(c + dx) \left(3 \operatorname{Hypergeometric2F1}\left(-\frac{5}{4}, \frac{1}{2}, -\frac{1}{4}, \cos^2(c + dx)\right) + 5 \operatorname{Cos}[c + dx] \operatorname{Hypergeometric2F1}\left[-\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \cos^2(c + dx)\right] + \operatorname{Cos}[c + dx] \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cos^2(c + dx)\right]\right)\right) \sqrt{\sin^2(c + dx)}}{(35d \cos^{\frac{7}{2}}(c + dx))}$$

[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(9/2), x]

[Out] (2*a^3*Csc[c + d*x]*(5*Hypergeometric2F1[-7/4, 1/2, -3/4, Cos[c + d*x]^2] + 7*Cos[c + d*x]*(3*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] + 5*Cos[c + d*x]*(Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]))) * Sqrt[Sin[c + d*x]^2]) / (35*d*Cos[c + d*x]^(7/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(179) = 358.

Time = 10.41 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.99

method	result
default	$ -\frac{16 \sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1) (\sin^2(\frac{dx}{2} + \frac{c}{2}))} a^3 \left(-\frac{13 \cos(\frac{dx}{2} + \frac{c}{2}) \sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}}{168(\cos^2(\frac{dx}{2} + \frac{c}{2}) - \frac{1}{2})^2} + \frac{53 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2(\cos^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}}{105 \sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}} \right)}{35d \cos^{\frac{7}{2}}(c + dx)} $
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] $-16*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^3*(-13/168*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^2+53/105*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1/448*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^4-7/10*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)/(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-7/20*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-3/160*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^3)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.46

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx = \frac{2 \left(65i \sqrt{2} a^3 \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 65i \sqrt{2} a^3 \cos(dx + c) \right)}{\dots}$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="fricas")

[Out] $-2/105*(65*I*\sqrt{2})*a^3*\cos(d*x + c)^4*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) - 65*I*\sqrt{2})*a^3*\cos(d*x + c)^4*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + 147*I*\sqrt{2})*a^3*\cos(d*x + c)^4*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) - 147*I*\sqrt{2})*a^3*\cos(d*x + c)^4*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) - (294*a^3*\cos(d*x + c)^3 + 130*a^3*\cos(d*x + c)^2 + 63*a^3*\cos(d*x + c) + 15*a^3)*\sqrt{\cos(d*x + c)}*\sin(d*x + c))/(d*\cos(d*x + c)^4)$

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{9}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{9}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)

Mupad [B] (verification not implemented)

Time = 15.57 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{(a + a \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx$$

$$= \frac{2a^3 \sin(c+dx) {}_2F_1\left(-\frac{7}{4}, \frac{1}{2}; -\frac{3}{4}; \cos(c+dx)^2\right)}{7} + \frac{6a^3 \cos(c+dx) \sin(c+dx) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c+dx)^2\right)}{5} + \frac{2a^3 \cos(c + dx)^2 \sin}{d \cos(c + dx)^{7/2} \sqrt{}}$$

[In] int((a + a*cos(c + d*x))^3/cos(c + d*x)^(9/2),x)

[Out] ((2*a^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + (6*a^3*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/5 + 2*a^3*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 2*a^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))

3.167 $\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^4 dx$

Optimal result	2430
Rubi [A] (verified)	2431
Mathematica [C] (warning: unable to verify)	2433
Maple [A] (verified)	2434
Fricas [C] (verification not implemented)	2434
Sympy [F(-1)]	2435
Maxima [F]	2435
Giac [F]	2435
Mupad [B] (verification not implemented)	2436

Optimal result

Integrand size = 23, antiderivative size = 173

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^4 dx = \frac{128a^4 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{904a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{231d} + \frac{904a^4 \sqrt{\cos(c + dx)} \sin(c + dx)}{231d} + \frac{128a^4 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} + \frac{150a^4 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{77d} + \frac{8a^4 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d} + \frac{2a^4 \cos^{\frac{9}{2}}(c + dx) \sin(c + dx)}{11d}$$

```
[Out] 128/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+904/231*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+128/45*a^4*cos(d*x+c)^(3/2)*sin(d*x+c)/d+150/77*a^4*cos(d*x+c)^(5/2)*sin(d*x+c)/d+8/9*a^4*cos(d*x+c)^(7/2)*sin(d*x+c)/d+2/11*a^4*cos(d*x+c)^(9/2)*sin(d*x+c)/d+904/231*a^4*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2715, 2720, 2719}

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4 dx = \frac{904a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{231d} + \frac{128a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{15d} + \frac{2a^4 \sin(c+dx) \cos^{\frac{9}{2}}(c+dx)}{11d} + \frac{8a^4 \sin(c+dx) \cos^{\frac{7}{2}}(c+dx)}{9d} + \frac{150a^4 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{77d} + \frac{128a^4 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{45d} + \frac{904a^4 \sin(c+dx) \sqrt{\cos(c+dx)}}{231d}$$

[In] Int[Cos[c + d*x]^(3/2)*(a + a*cos[c + d*x])^4,x]

[Out] (128*a^4*EllipticE[(c + d*x)/2, 2])/(15*d) + (904*a^4*EllipticF[(c + d*x)/2, 2])/(231*d) + (904*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (128*a^4*cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (150*a^4*cos[c + d*x]^(5/2)*Sin[c + d*x])/(77*d) + (8*a^4*cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (2*a^4*cos[c + d*x]^(9/2)*Sin[c + d*x])/(11*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^4 \cos^{\frac{3}{2}}(c + dx) + 4a^4 \cos^{\frac{5}{2}}(c + dx) + 6a^4 \cos^{\frac{7}{2}}(c + dx) + 4a^4 \cos^{\frac{9}{2}}(c + dx) \right. \\
&\quad \left. + a^4 \cos^{\frac{11}{2}}(c + dx) \right) dx \\
&= a^4 \int \cos^{\frac{3}{2}}(c + dx) dx + a^4 \int \cos^{\frac{11}{2}}(c + dx) dx + (4a^4) \int \cos^{\frac{5}{2}}(c + dx) dx \\
&\quad + (4a^4) \int \cos^{\frac{9}{2}}(c + dx) dx + (6a^4) \int \cos^{\frac{7}{2}}(c + dx) dx \\
&= \frac{2a^4 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{8a^4 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad + \frac{12a^4 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{8a^4 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d} \\
&\quad + \frac{2a^4 \cos^{\frac{9}{2}}(c + dx) \sin(c + dx)}{11d} + \frac{1}{3} a^4 \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad + \frac{1}{11} (9a^4) \int \cos^{\frac{7}{2}}(c + dx) dx + \frac{1}{5} (12a^4) \int \sqrt{\cos(c + dx)} dx \\
&\quad + \frac{1}{9} (28a^4) \int \cos^{\frac{5}{2}}(c + dx) dx + \frac{1}{7} (30a^4) \int \cos^{\frac{3}{2}}(c + dx) dx \\
&= \frac{24a^4 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2a^4 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\
&\quad + \frac{74a^4 \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{128a^4 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} \\
&\quad + \frac{150a^4 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{77d} + \frac{8a^4 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d} \\
&\quad + \frac{2a^4 \cos^{\frac{9}{2}}(c + dx) \sin(c + dx)}{11d} + \frac{1}{77} (45a^4) \int \cos^{\frac{3}{2}}(c + dx) dx \\
&\quad + \frac{1}{7} (10a^4) \int \frac{1}{\sqrt{\cos(c + dx)}} dx + \frac{1}{15} (28a^4) \int \sqrt{\cos(c + dx)} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{128a^4 E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{15d} + \frac{74a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} \\
&\quad + \frac{904a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{231d} + \frac{128a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{45d} \\
&\quad + \frac{150a^4 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{77d} + \frac{8a^4 \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{9d} \\
&\quad + \frac{2a^4 \cos^{\frac{9}{2}}(c+dx) \sin(c+dx)}{11d} + \frac{1}{77} (15a^4) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{128a^4 E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{15d} + \frac{904a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{231d} \\
&\quad + \frac{904a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{231d} \\
&\quad + \frac{128a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{45d} + \frac{150a^4 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{77d} \\
&\quad + \frac{8a^4 \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{9d} + \frac{2a^4 \cos^{\frac{9}{2}}(c+dx) \sin(c+dx)}{11d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4 dx$$

$$\begin{aligned}
&a^4(1+\cos(c+dx))^4 \sec^8\left(\frac{1}{2}(c+dx)\right) \left(-108480 \cos(c+dx) \sqrt{\cos^2(dx - \arctan(\cot(c)))} \sqrt{\csc^2(c)} {}_2F_1\left(\frac{1}{4}, \right.\right. \\
&= \left.\left. \right.\right)
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^4,x]

[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(-108480*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-236544*Cot[c] + 122610*Sin[c + d*x] + 45584*Sin[2*(c + d*x)] + 14445*Sin[3*(c + d*x)] + 3080*Sin[4*(c + d*x)] + 315*Sin[5*(c + d*x)]) + (59136*Sec[c]*(-2*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(443520*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 15.95 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.58

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4\left(5040\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5320\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1740\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+326\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+326\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1740\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5320\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5040}\right)$
parts	Expression too large to display

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -8/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(5040*cos(1/2*d*x+1/2*c)^13-5320*cos(1/2*d*x+1/2*c)^11+1740*cos(1/2*d*x+1/2*c)^9+326*cos(1/2*d*x+1/2*c)^7+678*cos(1/2*d*x+1/2*c)^5-4465*cos(1/2*d*x+1/2*c)^3+1695*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3696*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2001*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.09

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4 dx = \frac{2\left(3390i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-3390i\sqrt{2}a^4\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))\right)}{\dots}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/3465*(3390*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 3390*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 7392*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 7392*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (315*a^4*cos(d*x + c)^4 + 1540*a^4*cos(d*x + c)^3 + 3375*a^4*cos(d*x + c)^2 + 4928*a^4*cos(d*x + c) + 6780*a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^4 dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^4 dx = \int (a \cos(dx + c) + a)^4 \cos(dx + c)^{\frac{3}{2}} dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^4 dx = \int (a \cos(dx + c) + a)^4 \cos(dx + c)^{\frac{3}{2}} dx$$

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 15.03 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\begin{aligned}
& \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4 dx \\
&= \frac{2a^4 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{3d} + \frac{2a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d} \\
&\quad - \frac{8a^4 \cos(c+dx)^{7/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c+dx)^2\right)}{7d \sqrt{\sin(c+dx)^2}} \\
&\quad - \frac{4a^4 \cos(c+dx)^{9/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c+dx)^2\right)}{3d \sqrt{\sin(c+dx)^2}} \\
&\quad - \frac{8a^4 \cos(c+dx)^{11/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{4}; \frac{15}{4}; \cos(c+dx)^2\right)}{11d \sqrt{\sin(c+dx)^2}} \\
&\quad - \frac{2a^4 \cos(c+dx)^{13/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{13}{4}; \frac{17}{4}; \cos(c+dx)^2\right)}{13d \sqrt{\sin(c+dx)^2}}
\end{aligned}$$

[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^4,x)

```
[Out] (2*a^4*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a^4*cos(c + d*x)^(1/2)*sin(c
+ d*x))/(3*d) - (8*a^4*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4
], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*a^4*cos(c + d*x)
)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin
(c + d*x)^2)^(1/2)) - (8*a^4*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/
2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (2*a^4*cos
(c + d*x)^(13/2)*sin(c + d*x)*hypergeom([1/2, 13/4], 17/4, cos(c + d*x)^2)
)/(13*d*(sin(c + d*x)^2)^(1/2))
```


3.168 $\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^4 dx$

Optimal result	2437
Rubi [A] (verified)	2438
Mathematica [C] (verified)	2440
Maple [A] (verified)	2440
Fricas [C] (verification not implemented)	2441
Sympy [F(-1)]	2441
Maxima [F]	2441
Giac [F]	2442
Mupad [B] (verification not implemented)	2442

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^4 dx = \frac{152a^4 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{32a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{7d} + \frac{32a^4 \sqrt{\cos(c + dx)} \sin(c + dx)}{7d} + \frac{122a^4 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} + \frac{8a^4 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{2a^4 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d}$$

```
[Out] 152/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+122/45*a^4*cos(d*x+c)^(3/2)*sin(d*x+c)/d+8/7*a^4*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a^4*cos(d*x+c)^(7/2)*sin(d*x+c)/d+32/7*a^4*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2719, 2715, 2720}

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4 dx = \frac{32a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{7d} + \frac{152a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{15d} + \frac{2a^4 \sin(c+dx) \cos^{\frac{7}{2}}(c+dx)}{9d} + \frac{8a^4 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{7d} + \frac{122a^4 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{45d} + \frac{32a^4 \sin(c+dx) \sqrt{\cos(c+dx)}}{7d}$$

[In] Int[Sqrt[Cos[c + d*x]]*(a + a*cos[c + d*x])^4,x]

[Out] (152*a^4*EllipticE[(c + d*x)/2, 2])/(15*d) + (32*a^4*EllipticF[(c + d*x)/2, 2])/(7*d) + (32*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (122*a^4*cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (8*a^4*cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^4*cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)

Rule 2715

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_)*sin[(e_.) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_.) + (f_)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +

f*x))^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a^4 \sqrt{\cos(c+dx)} + 4a^4 \cos^{\frac{3}{2}}(c+dx) + 6a^4 \cos^{\frac{5}{2}}(c+dx) + 4a^4 \cos^{\frac{7}{2}}(c+dx) \right. \\
&\quad \left. + a^4 \cos^{\frac{9}{2}}(c+dx) \right) dx \\
&= a^4 \int \sqrt{\cos(c+dx)} dx + a^4 \int \cos^{\frac{3}{2}}(c+dx) dx + (4a^4) \int \cos^{\frac{5}{2}}(c+dx) dx \\
&\quad + (4a^4) \int \cos^{\frac{7}{2}}(c+dx) dx + (6a^4) \int \cos^{\frac{9}{2}}(c+dx) dx \\
&= \frac{2a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{8a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d} + \frac{12a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{8a^4 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + \frac{2a^4 \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{9d} + \frac{1}{9}(7a^4) \int \cos^{\frac{5}{2}}(c \\
&\quad + dx) dx + \frac{1}{3}(4a^4) \int \frac{1}{\sqrt{\cos(c+dx)}} dx + \frac{1}{7}(20a^4) \int \cos^{\frac{3}{2}}(c \\
&\quad + dx) dx + \frac{1}{5}(18a^4) \int \sqrt{\cos(c+dx)} dx \\
&= \frac{46a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{8a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\
&\quad + \frac{32a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{7d} + \frac{122a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{45d} \\
&\quad + \frac{8a^4 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + \frac{2a^4 \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{9d} \\
&\quad + \frac{1}{15}(7a^4) \int \sqrt{\cos(c+dx)} dx + \frac{1}{21}(20a^4) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{152a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{15d} + \frac{32a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{7d} \\
&\quad + \frac{32a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{7d} + \frac{122a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{45d} \\
&\quad + \frac{8a^4 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + \frac{2a^4 \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{9d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.93 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.73

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4 dx$$

$$= \frac{a^4(1+\cos(c+dx))^4 \sec^8\left(\frac{1}{2}(c+dx)\right) \left(\frac{3192(3\cos(c-dx-\arctan(\tan(c)))+\cos(c+dx+\arctan(\tan(c)))) \csc(c) \sec(c)}{\sqrt{\sec^2(c)}} - 5760 \cos \right)}{\dots}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4,x]

[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*((3192*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 5760*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-12768*Cot[c] + 6120*Sin[c + d*x] + 1778*Sin[2*(c + d*x)] + 360*Sin[3*(c + d*x)] + 35*Sin[4*(c + d*x)]) - 6384*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(20160*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 12.70 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.77

method	result
default	$\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} a^4 \left(280\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+34\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+72\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{315\sqrt{-2}\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
parts	Expression too large to display

[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^4,x,method=_RETURNVERBOSE)

[Out] -8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*cos(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*cos(1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-399*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^4 dx =$$

$$2 \left(360i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 360i \sqrt{2} a^4 \text{weierstrassPInverse} \right.$$

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/315*(360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 798*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 798*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*a^4*cos(d*x + c)^3 + 180*a^4*cos(d*x + c)^2 + 427*a^4*cos(d*x + c) + 720*a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^4 dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{\cos(c + dx)}(a + a \cos(c + dx))^4 dx = \int (a \cos(dx + c) + a)^4 \sqrt{\cos(dx + c)} dx$$

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4 dx = \int (a\cos(dx+c)+a)^4 \sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 15.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.52

$$\begin{aligned} & \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4 dx \\ &= \frac{2 \left(3a^4 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + 4a^4 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + 4a^4 \sqrt{\cos(c+dx)} \sin(c+dx) \right)}{3d} \\ & \quad - \frac{2 \left(\frac{66a^4 \cos(c+dx)^{7/2} \sin(c+dx)}{\sqrt{\sin(c+dx)^2}} - \frac{17a^4 \cos(c+dx)^{11/2} \sin(c+dx)}{\sqrt{\sin(c+dx)^2}} \right) {}_2F_1\left(\frac{1}{2}, \frac{11}{4}; \frac{15}{4}; \cos(c+dx)^2\right)}{77d} \\ & \quad - \frac{8a^4 \cos(c+dx)^{9/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c+dx)^2\right)}{9d \sqrt{\sin(c+dx)^2}} \\ & \quad - \frac{208a^4 \cos(c+dx)^{11/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{4}; \frac{19}{4}; \cos(c+dx)^2\right)}{385d \sqrt{\sin(c+dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^4,x)

[Out] (2*(3*a^4*ellipticE(c/2 + (d*x)/2, 2) + 4*a^4*ellipticF(c/2 + (d*x)/2, 2) + 4*a^4*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*((66*a^4*cos(c + d*x)^(7/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2) - (17*a^4*cos(c + d*x)^(11/2)*sin(c + d*x))/(sin(c + d*x)^2)^(1/2))*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(77*d) - (8*a^4*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (208*a^4*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 19/4, cos(c + d*x)^2))/(385*d*(sin(c + d*x)^2)^(1/2))

$$3.169 \quad \int \frac{(a+a \cos(c+dx))^4}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	2443
Rubi [A] (verified)	2443
Mathematica [C] (warning: unable to verify)	2445
Maple [A] (verified)	2446
Fricas [C] (verification not implemented)	2446
Sympy [F(-1)]	2447
Maxima [F]	2447
Giac [F]	2447
Mupad [B] (verification not implemented)	2447

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(a+a \cos(c+dx))^4}{\sqrt{\cos(c+dx)}} dx = \frac{64a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{136a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d}$$

$$+ \frac{94a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{21d}$$

$$+ \frac{8a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{2a^4 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d}$$

[Out] 64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+136/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a^4*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a^4*cos(d*x+c)^(5/2)*sin(d*x+c)/d+94/21*a^4*sin(d*x+c)*cos(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2720, 2719, 2715}

$$\int \frac{(a+a \cos(c+dx))^4}{\sqrt{\cos(c+dx)}} dx = \frac{136a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} + \frac{64a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d}$$

$$+ \frac{2a^4 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{7d} + \frac{8a^4 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$+ \frac{94a^4 \sin(c+dx) \sqrt{\cos(c+dx)}}{21d}$$

[In] Int[(a + a*cos[c + d*x])^4/Sqrt[Cos[c + d*x]],x]

[Out] (64*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (136*a^4*EllipticF[(c + d*x)/2, 2])/(21*d) + (94*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (8*a^4*cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^4*cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a^4}{\sqrt{\cos(c + dx)}} + 4a^4 \sqrt{\cos(c + dx)} + 6a^4 \cos^{\frac{3}{2}}(c + dx) + 4a^4 \cos^{\frac{5}{2}}(c + dx) \right. \\ &\quad \left. + a^4 \cos^{\frac{7}{2}}(c + dx) \right) dx \\ &= a^4 \int \frac{1}{\sqrt{\cos(c + dx)}} dx + a^4 \int \cos^{\frac{7}{2}}(c + dx) dx + (4a^4) \int \sqrt{\cos(c + dx)} dx \\ &\quad + (4a^4) \int \cos^{\frac{5}{2}}(c + dx) dx + (6a^4) \int \cos^{\frac{3}{2}}(c + dx) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{8a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} \\
&\quad + \frac{4a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{d} + \frac{8a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{2a^4 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + \frac{1}{7}(5a^4) \int \cos^{\frac{3}{2}}(c+dx) dx \\
&\quad + (2a^4) \int \frac{1}{\sqrt{\cos(c+dx)}} dx + \frac{1}{5}(12a^4) \int \sqrt{\cos(c+dx)} dx \\
&= \frac{64a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{6a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} \\
&\quad + \frac{94a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{21d} + \frac{8a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{2a^4 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d} + \frac{1}{21}(5a^4) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{64a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{136a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} \\
&\quad + \frac{94a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{21d} \\
&\quad + \frac{8a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{2a^4 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{7d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.78 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{a^4(1 + \cos(c + dx))^4 \sec^8\left(\frac{1}{2}(c + dx)\right) \left(\frac{672(3 \cos(c - dx - \arctan(\tan(c))) + \cos(c + dx + \arctan(\tan(c)))) \csc(c) \sec(c)}{\sqrt{\sec^2(c)}} - 1360 \cos \right)}{1}$$

[In] Integrate[(a + a*Cos[c + d*x])^4/Sqrt[Cos[c + d*x]],x]

[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*((672*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 1360*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c + d*x]*(-2688*Cot[c] + 955*Sin[c + d*x] + 168*Sin[2*(c + d*x)] + 15*Sin[3*(c + d*x)]) - 1344*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(3360*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 9.15 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.25

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}} a^4 \left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-258\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+448\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] -8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+85*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.34

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\cos(c + dx)}} dx =$$

$$\frac{2 \left(170i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 170i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{\dots}$$

[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="fricas")

```
[Out] -2/105*(170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 336*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 336*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*a^4*cos(d*x + c)^2 + 84*a^4*cos(d*x + c) + 235*a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^4}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^4}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 15.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.21

$$\begin{aligned} & \int \frac{(a + a \cos(c + dx))^4}{\sqrt{\cos(c + dx)}} dx \\ &= \frac{2 \left(4 a^4 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + 3 a^4 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + 2 a^4 \sqrt{\cos(c + dx)} \sin(c + dx) \right)}{d} \\ & \quad - \frac{8 a^4 \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7 d \sqrt{\sin(c + dx)^2}} \\ & \quad - \frac{2 a^4 \cos(c + dx)^{9/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c + dx)^2\right)}{9 d \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^(1/2),x)

[Out] (2*(4*a^4*ellipticE(c/2 + (d*x)/2, 2) + 3*a^4*ellipticF(c/2 + (d*x)/2, 2) + 2*a^4*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (8*a^4*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a^4*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))

$$3.170 \quad \int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	2449
Rubi [A] (verified)	2449
Mathematica [C] (verified)	2451
Maple [A] (verified)	2451
Fricas [C] (verification not implemented)	2452
Sympy [F(-1)]	2452
Maxima [F]	2453
Giac [F]	2453
Mupad [B] (verification not implemented)	2453

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{56a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{32a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d}$$

$$+ \frac{2a^4 \sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{8a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d}$$

$$+ \frac{2a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d}$$

[Out] 56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/d+2/5*a^4*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2*a^4*sin(d*x+c)/d/cos(d*x+c)^(1/2)+8/3*a^4*sin(d*x+c)*cos(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2836, 2716, 2719, 2720, 2715}

$$\int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{32a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{56a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d}$$

$$+ \frac{2a^4 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{5d}$$

$$+ \frac{8a^4 \sin(c+dx) \sqrt{\cos(c+dx)}}{3d} + \frac{2a^4 \sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(3/2),x]

[Out] (56*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (32*a^4*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (8*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])^n], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a^4}{\cos^{\frac{3}{2}}(c + dx)} + \frac{4a^4}{\sqrt{\cos(c + dx)}} + 6a^4\sqrt{\cos(c + dx)} + 4a^4\cos^{\frac{3}{2}}(c + dx) \right. \\ &\quad \left. + a^4\cos^{\frac{5}{2}}(c + dx) \right) dx \\ &= a^4 \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + a^4 \int \cos^{\frac{5}{2}}(c + dx) dx + (4a^4) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\ &\quad + (4a^4) \int \cos^{\frac{3}{2}}(c + dx) dx + (6a^4) \int \sqrt{\cos(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{12a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{8a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} \\
&+ \frac{2a^4 \sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{8a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d} \\
&+ \frac{2a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{1}{5}(3a^4) \int \sqrt{\cos(c+dx)} dx \\
&- a^4 \int \sqrt{\cos(c+dx)} dx + \frac{1}{3}(4a^4) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{56a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{32a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{2a^4 \sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\
&+ \frac{8a^4 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d} + \frac{2a^4 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.94 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.32

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2a^4 \csc(c + dx) \left(-15 \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cos^2(c + dx)\right) \sqrt{\sin^2(c + dx)} + \cos(c + dx) \left(-((20 + 3\cos(c + dx)) \sin^2(c + dx) + 80 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \cos^2(c + dx)\right] \sqrt{\sin^2(c + dx)} + 33 \cos(c + dx) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \cos^2(c + dx)\right] \sqrt{\sin^2(c + dx)} \right) \right)}{(15d \sqrt{\cos(c + dx)})}$$

[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(3/2),x]

[Out] (-2*a^4*Csc[c + d*x]*(-15*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + Cos[c + d*x]*(-(20 + 3*Cos[c + d*x])*Sin[c + d*x]^2) + 80*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 33*Cos[c + d*x]*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(15*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 8.78 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.63

method	result
default	$8a^4 \left(6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 26 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 19 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 20 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)} - 15 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)} \right)$
parts	Expression too large to display

```
[In] int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/15*a^4*(6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-26*sin(1/2*d*x+1/2*c)^4*
*cos(1/2*d*x+1/2*c)+19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-20*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.63

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{3}{2}}(c + dx)} dx =$$

$$2 \left(40i \sqrt{2} a^4 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 40i \sqrt{2} a^4 \cos(dx + c) \right)$$

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(40*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
) + I*sin(d*x + c)) - 40*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 42*I*sqrt(2)*a^4*cos(d*x + c)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
42*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^4*cos(d*x + c)^2 + 20*a^4*cos(
d*x + c) + 15*a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```


Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^4}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^4}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 15.02 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.25

$$\begin{aligned} \int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{3}{2}}(c + dx)} dx &= \frac{12 a^4 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{32 a^4 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{3 d} \\ &+ \frac{8 a^4 \sqrt{\cos(c + dx)} \sin(c + dx)}{3 d} \\ &+ \frac{2 a^4 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)}^2} \\ &- \frac{2 a^4 \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7 d \sqrt{\sin(c + dx)}^2} \end{aligned}$$

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^(3/2),x)

[Out] (12*a^4*ellipticE(c/2 + (d*x)/2, 2))/d + (32*a^4*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (8*a^4*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^4*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) - (2*a^4*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))

$$3.171 \quad \int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	2454
Rubi [A] (verified)	2454
Mathematica [C] (verified)	2456
Maple [B] (verified)	2456
Fricas [C] (verification not implemented)	2457
Sympy [F(-1)]	2457
Maxima [F]	2458
Giac [F]	2458
Mupad [B] (verification not implemented)	2458

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{40a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a^4 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{8a^4 \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + \frac{2a^4 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d}$$

[Out] 40/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/d+2/3*a^4*sin(d*x+c)/d/cos(d*x+c)^(3/2)+8*a^4*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a^4*sin(d*x+c)*cos(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2836, 2716, 2720, 2719, 2715}

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{40a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a^4 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2a^4 \sin(c + dx) \sqrt{\cos(c + dx)}}{3d} + \frac{8a^4 \sin(c + dx)}{d\sqrt{\cos(c + dx)}}$$

[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(5/2), x]

[Out] (40*a^4*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^4*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (8*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a^4}{\cos^{\frac{5}{2}}(c + dx)} + \frac{4a^4}{\cos^{\frac{3}{2}}(c + dx)} + \frac{6a^4}{\sqrt{\cos(c + dx)}} + 4a^4\sqrt{\cos(c + dx)} \right. \\ &\quad \left. + a^4 \cos^{\frac{3}{2}}(c + dx) \right) dx \\ &= a^4 \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx + a^4 \int \cos^{\frac{3}{2}}(c + dx) dx + (4a^4) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\ &\quad + (4a^4) \int \sqrt{\cos(c + dx)} dx + (6a^4) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \end{aligned}$$


```
[Out] 8/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2 \left(10i \sqrt{2} a^4 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} a^4 \cos(dx + c) \right)}{\dots}$$

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(10*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 10*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (a^4*cos(d*x + c)^2 + 12*a^4*cos(d*x + c) + a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^4}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^4}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 15.02 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.48

$$\begin{aligned} & \int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{5}{2}}(c + dx)} dx \\ &= \frac{2 \left(12 a^4 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + 19 a^4 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + a^4 \sqrt{\cos(c + dx)} \sin(c + dx) \right)}{3d} \\ &+ \frac{8 a^4 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}} \\ &+ \frac{2 a^4 \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{3d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^(5/2),x)

[Out] (2*(12*a^4*ellipticE(c/2 + (d*x)/2, 2) + 19*a^4*ellipticF(c/2 + (d*x)/2, 2) + a^4*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (8*a^4*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^4*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))

$$3.172 \quad \int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	2459
Rubi [A] (verified)	2459
Mathematica [C] (verified)	2461
Maple [B] (verified)	2461
Fricas [C] (verification not implemented)	2462
Sympy [F(-1)]	2462
Maxima [F]	2463
Giac [F]	2463
Mupad [B] (verification not implemented)	2463

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)} dx = -\frac{56a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{32a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2a^4 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{8a^4 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{66a^4 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[Out] $-56/5*a^4*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+32/3*a^4*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*a^4*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+8/3*a^4*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+66/5*a^4*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2716, 2719, 2720}

$$\int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{32a^4 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} - \frac{56a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} \\ + \frac{8a^4 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2a^4 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{66a^4 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[In] $\text{Int}[(a+a*\text{Cos}[c+d*x])^4/\text{Cos}[c+d*x]^{(7/2)}, x]$

[Out] $(-56*a^4*\text{EllipticE}[(c+d*x)/2, 2])/(5*d) + (32*a^4*\text{EllipticF}[(c+d*x)/2, 2])/(3*d) + (2*a^4*\text{Sin}[c+d*x])/(5*d*\text{Cos}[c+d*x]^{(5/2)}) + (8*a^4*\text{Sin}[c +$

$d*x]/(3*d*\text{Cos}[c + d*x]^{(3/2)}) + (66*a^4*\text{Sin}[c + d*x]/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]))$

Rule 2716

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[\text{Cos}[c + d*x]*((b*\text{Sin}[c + d*x])^{(n + 1)}/(b*d*(n + 1))), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n + 2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2719

$\text{Int}[\text{Sqrt}[\text{sin}[(c_*) + (d_*)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_*) + (d_*)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticF}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2836

$\text{Int}[(d_*)*\text{sin}[(e_*) + (f_*)*(x_)]^{(n_*)}*((a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)]^{(m_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[(a + b*\text{sin}[e + f*x])^m*(d*\text{sin}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0] \&\& \text{RationalQ}[n]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a^4}{\cos^{\frac{7}{2}}(c + dx)} + \frac{4a^4}{\cos^{\frac{5}{2}}(c + dx)} + \frac{6a^4}{\cos^{\frac{3}{2}}(c + dx)} + \frac{4a^4}{\sqrt{\cos(c + dx)}} \right. \\ &\quad \left. + a^4 \sqrt{\cos(c + dx)} \right) dx \\ &= a^4 \int \frac{1}{\cos^{\frac{7}{2}}(c + dx)} dx + a^4 \int \sqrt{\cos(c + dx)} dx + (4a^4) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx \\ &\quad + (4a^4) \int \frac{1}{\sqrt{\cos(c + dx)}} dx + (6a^4) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2a^4 E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{8a^4 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2a^4 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\ &\quad + \frac{8a^4 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{12a^4 \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{1}{5} (3a^4) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\ &\quad + \frac{1}{3} (4a^4) \int \frac{1}{\sqrt{\cos(c + dx)}} dx - (6a^4) \int \sqrt{\cos(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= -\frac{10a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{32a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{2a^4 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{8a^4 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{66a^4 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}} - \frac{1}{5}(3a^4) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{56a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{32a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\
&\quad + \frac{2a^4 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{8a^4 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{66a^4 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.40

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{2a^4 \csc(c + dx) \left(3 \operatorname{Hypergeometric2F1}\left(-\frac{5}{4}, \frac{1}{2}, -\frac{1}{4}, \cos^2(c + dx)\right) - 5 \cos(c + dx) \left(-4 \operatorname{Hypergeometric2F1}\right)}{\dots}$$

[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(7/2),x]

[Out] (2*a^4*Csc[c + d*x]*(3*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] - 5*Cos[c + d*x]*(-4*Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + Cos[c + d*x]*(-18*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] + 12*Cos[c + d*x]*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2] + Cos[c + d*x]^2*Hypergeometric2F1[1/2, 3/4, 7/4, Cos[c + d*x]^2]))) * Sqrt[Sin[c + d*x]^2]) / (15*d*Cos[c + d*x]^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(157) = 314.

Time = 8.79 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

method	result
default	$ -\frac{32 \sqrt{-\left(-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a^4} \left(\frac{41 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{60 \sqrt{-2 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} - 7 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right) $
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

```
[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(41/60*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-7/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/24*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^2-33/40*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*
c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-1/320*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{7}{2}}(c + dx)} dx =$$

$$2 \left(40i \sqrt{2} a^4 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 40i \sqrt{2} a^4 \cos(dx + c) \right)$$

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(40*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 40*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 42*I*sqrt(2)*a^4*cos(d*x + c)^3*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 42*I*sqrt(2)*a^4*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (99*a^4*cos(d*x + c)^2 + 20
*a^4*cos(d*x + c) + 3*a^4)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)
^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^4}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(7/2), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^4}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(7/2), x)

Mupad [B] (verification not implemented)

Time = 15.64 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\begin{aligned} & \int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{7}{2}}(c + dx)} dx \\ &= \frac{2 \left(a^4 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + 4 a^4 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) \right)}{d} \\ &+ \frac{2 \left(\frac{34 a^4 \sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{\sin(c+dx)^2}} + \frac{a^4 \sin(c+dx)}{\cos(c+dx)^{5/2} \sqrt{\sin(c+dx)^2}} \right) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c+dx)^2\right)}{5d} \\ &+ \frac{8 a^4 \sin(c+dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c+dx)^2\right)}{3d \cos(c+dx)^{3/2} \sqrt{\sin(c+dx)^2}} \\ &- \frac{8 a^4 \sin(c+dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{7}{4}; \cos(c+dx)^2\right)}{15d \sqrt{\cos(c+dx)} \sqrt{\sin(c+dx)^2}} \end{aligned}$$

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^(7/2),x)

[Out] (2*(a^4*ellipticE(c/2 + (d*x)/2, 2) + 4*a^4*ellipticF(c/2 + (d*x)/2, 2)))/d + (2*((34*a^4*sin(c + d*x))/(cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (a^4*sin(c + d*x))/(cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2))))*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(5*d) + (8*a^4*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) - (8*a^4*sin(c + d*x)*hypergeom([-1/4, 1/2], 7/4, cos(c + d*x)^2))/(15*d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

$$3.173 \quad \int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	2464
Rubi [A] (verified)	2464
Mathematica [C] (verified)	2466
Maple [B] (verified)	2467
Fricas [C] (verification not implemented)	2467
Sympy [F(-1)]	2468
Maxima [F]	2468
Giac [F]	2468
Mupad [B] (verification not implemented)	2469

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)} dx = -\frac{64a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{136a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d}$$

$$+ \frac{2a^4 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} + \frac{8a^4 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{94a^4 \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{64a^4 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[Out] $-64/5*a^4*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+136/21*a^4*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/7*a^4*\sin(d*x+c)/d/\cos(d*x+c)^{(7/2)}+8/5*a^4*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+94/21*a^4*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+64/5*a^4*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2836, 2716, 2720, 2719}

$$\int \frac{(a+a \cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{136a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} - \frac{64a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d}$$

$$+ \frac{94a^4 \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{8a^4 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{2a^4 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} + \frac{64a^4 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(9/2), x]

[Out] (-64*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (136*a^4*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (8*a^4*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (94*a^4*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (64*a^4*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2836

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{a^4}{\cos^{\frac{9}{2}}(c + dx)} + \frac{4a^4}{\cos^{\frac{7}{2}}(c + dx)} + \frac{6a^4}{\cos^{\frac{5}{2}}(c + dx)} + \frac{4a^4}{\cos^{\frac{3}{2}}(c + dx)} \right. \\ &\quad \left. + \frac{a^4}{\sqrt{\cos(c + dx)}} \right) dx \\ &= a^4 \int \frac{1}{\cos^{\frac{9}{2}}(c + dx)} dx + a^4 \int \frac{1}{\sqrt{\cos(c + dx)}} dx + (4a^4) \int \frac{1}{\cos^{\frac{7}{2}}(c + dx)} dx \\ &\quad + (4a^4) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + (6a^4) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} + \frac{2a^4 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} + \frac{8a^4 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{4a^4 \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{8a^4 \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \frac{1}{7}(5a^4) \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)} dx + (2a^4) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&\quad + \frac{1}{5}(12a^4) \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)} dx - (4a^4) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{8a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{6a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} + \frac{2a^4 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{8a^4 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{94a^4 \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{64a^4 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}} \\
&\quad + \frac{1}{21}(5a^4) \int \frac{1}{\sqrt{\cos(c+dx)}} dx - \frac{1}{5}(12a^4) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{64a^4 E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{136a^4 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} + \frac{2a^4 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{8a^4 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{94a^4 \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{64a^4 \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\begin{aligned}
&\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{9}{2}}(c + dx)} dx \\
&= \frac{2a^4 \csc(c + dx) \left(5 \operatorname{Hypergeometric2F1}\left(-\frac{7}{4}, \frac{1}{2}, -\frac{3}{4}, \cos^2(c + dx)\right) + 7 \cos(c + dx) \left(4 \operatorname{Hypergeometric2F1}\left(-\frac{5}{4}, \frac{1}{2}, -\frac{1}{4}, \cos^2(c + dx)\right) + 5 \cos(c + dx) \left(2 \operatorname{Hypergeometric2F1}\left(-\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \cos^2(c + dx)\right) + 4 \cos(c + dx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \cos^2(c + dx)\right) - \cos(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \cos^2(c + dx)\right) \right) \right) \sqrt{\sin(c + dx)^2}}{35d \cos^{\frac{7}{2}}(c + dx)}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(9/2),x]

[Out] (2*a^4*Csc[c + d*x]*(5*Hypergeometric2F1[-7/4, 1/2, -3/4, Cos[c + d*x]^2] + 7*Cos[c + d*x]*(4*Hypergeometric2F1[-5/4, 1/2, -1/4, Cos[c + d*x]^2] + 5*Cos[c + d*x]*(2*Hypergeometric2F1[-3/4, 1/2, 1/4, Cos[c + d*x]^2] + 4*Cos[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Cos[c + d*x]^2] - Cos[c + d*x]^2*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]))) * Sqrt[Sin[c + d*x]^2]) / (35*d*Cos[c + d*x]^(7/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(179) = 358.

Time = 10.46 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.99

method	result
default	$32\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} a^4 \left(\frac{253\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{420\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}} - \frac{47\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{672} \right)$
parts	Expression too large to display

[In] `int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)`

[Out]
$$-32*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^4*(253/420*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-47/672*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^2-4/5*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)/(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-2/5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-1/80*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^3-1/896*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^4)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.46

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{9}{2}}(c + dx)} dx = \frac{2 \left(170i \sqrt{2} a^4 \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 170i \sqrt{2} a^4 \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 336 i \sqrt{2} a^4 \cos(dx + c)^4 \right)}{d}$$

[In] `integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="fricas")`

[Out]
$$-2/105*(170*I*\sqrt{2})*a^4*\cos(d*x + c)^4*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) - 170*I*\sqrt{2})*a^4*\cos(d*x + c)^4*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + 336*I*\sqrt{2})*a^4*\cos(d*x + c)^4$$

```
4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 336*I*sqrt(2)*a^4*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (672*a^4*cos(d*x + c)^3 + 235*a^4*cos(d*x + c)^2 + 84*a^4*cos(d*x + c) + 15*a^4)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{9}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^4}{\cos(dx + c)^{\frac{9}{2}}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^4}{\cos(dx + c)^{\frac{9}{2}}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(9/2), x)
```


Mupad [B] (verification not implemented)

Time = 15.67 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.35

$$\begin{aligned}
\int \frac{(a + a \cos(c + dx))^4}{\cos^{\frac{9}{2}}(c + dx)} dx &= \frac{2 a^4 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} \\
&+ \frac{8 a^4 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)}^2} \\
&+ \frac{4 a^4 \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)}^2} \\
&+ \frac{8 a^4 \sin(c + dx) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c + dx)^2\right)}{5 d \cos(c + dx)^{5/2} \sqrt{\sin(c + dx)}^2} \\
&+ \frac{2 a^4 \sin(c + dx) {}_2F_1\left(-\frac{7}{4}, \frac{1}{2}; -\frac{3}{4}; \cos(c + dx)^2\right)}{7 d \cos(c + dx)^{7/2} \sqrt{\sin(c + dx)}^2}
\end{aligned}$$

[In] int((a + a*cos(c + d*x))^4/cos(c + d*x)^(9/2),x)

```

[Out] (2*a^4*ellipticF(c/2 + (d*x)/2, 2))/d + (8*a^4*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
+ (4*a^4*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
+ (8*a^4*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2))
+ (2*a^4*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/(7*d*cos(c + d*x)^(7/2)*(sin(c + d*x)^2)^(1/2))

```

3.174 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	2470
Rubi [A] (verified)	2470
Mathematica [C] (verified)	2472
Maple [A] (verified)	2472
Fricas [C] (verification not implemented)	2473
Sympy [F(-1)]	2473
Maxima [F]	2474
Giac [F]	2474
Mupad [F(-1)]	2474

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a \cos(c+dx)} dx = \frac{21E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5ad} - \frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3ad}$$

$$- \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{3ad}$$

$$+ \frac{7 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5ad} - \frac{\cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] 21/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a/d+7/5*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d-cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))-5/3*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2846, 2827, 2715, 2720, 2719}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a \cos(c+dx)} dx = -\frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3ad} + \frac{21E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5ad}$$

$$- \frac{\sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{d(a \cos(c+dx) + a)} + \frac{7 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{5ad}$$

$$- \frac{5 \sin(c+dx) \sqrt{\cos(c+dx)}}{3ad}$$

[In] Int[Cos[c + d*x]^(7/2)/(a + a*cos[c + d*x]),x]

[Out] (21*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) + (7*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*cos[c + d*x]))

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2846

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{\int \cos^{\frac{3}{2}}(c + dx) \left(\frac{5a}{2} - \frac{7}{2}a \cos(c + dx)\right) dx}{a^2} \\ &= -\frac{\cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{5 \int \cos^{\frac{3}{2}}(c + dx) dx}{2a} + \frac{7 \int \cos^{\frac{5}{2}}(c + dx) dx}{2a} \end{aligned}$$

$$\begin{aligned}
&= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} \\
&\quad - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{5\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{6a} + \frac{21\int\sqrt{\cos(c+dx)}dx}{10a} \\
&= \frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} \\
&\quad + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.86 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.46

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)} dx = \frac{\cos^2\left(\frac{1}{2}(c+dx)\right) \left(\frac{2i\sqrt{2}e^{-i(c+dx)} \left(63(1+e^{2i(c+dx)}) + 63(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) + 25e^{i(c+dx)} \right)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{1}$$

[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*cos[c + d*x]), x]

[Out] (Cos[(c + d*x)/2]^2*((2*I)*Sqrt[2]*(63*(1 + E^((2*I)*(c + d*x)))) + 63*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])/((d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) - (2*Sqrt[Cos[c + d*x]]*Csc[c]*(15 + 10*Cos[d*x]*Sin[c]^2 - 6*Cos[c]*(-8 + Cos[2*d*x]*Sin[c]^2) + 30*Sec[(c + d*x)/2]*Sin[c/2]*Sin[(d*x)/2] + 5*Sin[2*c]*Sin[d*x] - 3*Cos[2*c]*Sin[c]*Sin[2*d*x]))/d)/(15*a*(1 + Cos[c + d*x]))

Maple [A] (verified)

Time = 5.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(25F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+63E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{15a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}\right)}$

[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a), x, method=_RETURNVERBOSE)

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)^4+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.62

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)} dx$$

$$= \frac{2(6\cos(dx+c)^2 - 4\cos(dx+c) - 25)\sqrt{\cos(dx+c)}\sin(dx+c) - 25(-i\sqrt{2}\cos(dx+c) - i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + I\sin(dx+c)) - 25(I\sqrt{2}\cos(dx+c) + I\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) - I\sin(dx+c)) - 63(-I\sqrt{2}\cos(dx+c) - I\sqrt{2})\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + I\sin(dx+c))) - 63(I\sqrt{2}\cos(dx+c) + I\sqrt{2})\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - I\sin(dx+c)))}}{(a*d*\cos(d*x + c) + a*d)}$$

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(2*(6*cos(d*x + c)^2 - 4*cos(d*x + c) - 25)*sqrt(cos(d*x + c))*sin(d*x + c) - 25*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 25*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{a \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{a \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\cos(c + dx)^{7/2}}{a + a \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x)), x)

3.175 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	2475
Rubi [A] (verified)	2475
Mathematica [C] (verified)	2477
Maple [A] (verified)	2477
Fricas [C] (verification not implemented)	2478
Sympy [F(-1)]	2478
Maxima [F]	2478
Giac [F]	2479
Mupad [F(-1)]	2479

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a \cos(c+dx)} dx = -\frac{3E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} + \frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3ad} \\ + \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{3ad} - \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] $-3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d+5/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d-\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))+5/3*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2846, 2827, 2719, 2715, 2720}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a \cos(c+dx)} dx = \frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3ad} - \frac{3E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} \\ - \frac{\sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{d(a \cos(c+dx) + a)} + \frac{5 \sin(c+dx) \sqrt{\cos(c+dx)}}{3ad}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^{(5/2)}/(a+a*\operatorname{Cos}[c+d*x]), x]$

[Out] $(-3*\operatorname{EllipticE}[(c+d*x)/2, 2])/(a*d) + (5*\operatorname{EllipticF}[(c+d*x)/2, 2])/(3*a*d) + (5*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(3*a*d) - (\operatorname{Cos}[c+d*x]^{(3/2)}*\operatorname{Sin}[c+d*x])/(d*(a+a*\operatorname{Cos}[c+d*x]))$

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2846

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{\int \sqrt{\cos(c + dx)} \left(\frac{3a}{2} - \frac{5}{2}a \cos(c + dx)\right) dx}{a^2} \\
 &= -\frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} - \frac{3 \int \sqrt{\cos(c + dx)} dx}{2a} + \frac{5 \int \cos^{\frac{3}{2}}(c + dx) dx}{2a} \\
 &= -\frac{3E\left(\frac{1}{2}(c + dx) \mid 2\right)}{ad} + \frac{5\sqrt{\cos(c + dx)} \sin(c + dx)}{3ad} \\
 &\quad - \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d(a + a \cos(c + dx))} + \frac{5 \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{6a}
 \end{aligned}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} + \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.35 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.89

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)} dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right) \left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\left(9(1+e^{2i(c+dx)})+9(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right)\operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+5e^{i(c+dx)}\right)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{1}$$

[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]

[Out] (Cos[(c + d*x)/2]^2*((-2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) + (2*Sqrt[Cos[c + d*x]]*Csc[c]*(3 + 6*Cos[c] + 2*Cos[d*x]*Sin[c]^2 + 6*Sec[(c + d*x)/2]*Sin[c/2]*Sin[(d*x)/2] + Sin[2*c]*Sin[d*x]))/d)/(3*a*(1 + Cos[c + d*x]))

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.15

method	result
default	$-\frac{\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(5F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+9E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{3a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/a*cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.98

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{2(2 \cos(dx + c) + 5)\sqrt{\cos(dx + c)} \sin(dx + c) - 5(i\sqrt{2} \cos(dx + c) + i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx + c)) - 5(-i\sqrt{2} \cos(dx + c) - i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i\sin(dx + c)) - 9(i\sqrt{2} \cos(dx + c) + i\sqrt{2})\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))) - 9(-i\sqrt{2} \cos(dx + c) - i\sqrt{2})\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i\sin(dx + c)))}{a*d*\cos(d*x + c) + a*d}$$

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{a \cos(dx + c) + a} dx$$

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)} dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)} dx = \int \frac{\cos(c+dx)^{5/2}}{a+a\cos(c+dx)} dx$$

[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x)), x)

3.176 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	2480
Rubi [A] (verified)	2480
Mathematica [C] (verified)	2482
Maple [A] (verified)	2482
Fricas [C] (verification not implemented)	2483
Sympy [F]	2483
Maxima [F]	2483
Giac [F]	2484
Mupad [F(-1)]	2484

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a \cos(c+dx)} dx = \frac{3E\left(\frac{1}{2}(c+dx)|2\right)}{ad} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] 3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a/d-sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2846, 2827, 2720, 2719}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a \cos(c+dx)} dx = -\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} + \frac{3E\left(\frac{1}{2}(c+dx)|2\right)}{ad} - \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a \cos(c+dx) + a)}$$

[In] Int[Cos[c + d*x]^(3/2)/(a + a*cos[c + d*x]),x]

[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2846

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \frac{\frac{a}{2} - \frac{3}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{a^2} \\
 &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a} + \frac{3 \int \sqrt{\cos(c+dx)} dx}{2a} \\
 &= \frac{3E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a\cos(c+dx))}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
 Time = 2.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.67

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c + dx)\right) \left(\frac{2i\sqrt{2}e^{-i(c+dx)}\left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right)\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+e^{i(c+dx)}(-1+e^{2i(c+dx)})}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{a(1 + \cos(c + dx))}$$

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x)))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d * E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) - (2*Sqrt[Cos[c + d*x]]*(2*Cot[c] + Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]))/d)/(a*(1 + Cos[c + d*x]))
```

Maple [A] (verified)

Time = 3.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.76

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) + 3E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) + a\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\right)}{d}$

```
[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.58

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{(i\sqrt{2} \cos(dx + c) + i\sqrt{2}) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + (-i\sqrt{2} \cos(dx + c) + i\sqrt{2}) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3(-i\sqrt{2} \cos(dx + c) - i\sqrt{2}) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 3(i\sqrt{2} \cos(dx + c) + i\sqrt{2}) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) - 2 \operatorname{sqrt}(\cos(dx + c)) \sin(dx + c)}{(a*d*\cos(dx + c) + a*d)}$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*sqrt(cos(d*x + c))*sin(d*x + c)/(a*d*cos(d*x + c) + a*d)

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\cos(c+dx)+1} dx}{a}$$

[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)

[Out] Integral(cos(c + d*x)**(3/2)/(cos(c + d*x) + 1), x)/a

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{a \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{a \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\cos(c + dx)^{3/2}}{a + a \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x)), x)

$$3.177 \quad \int \frac{\sqrt{\cos(c+dx)}}{a+a \cos(c+dx)} dx$$

Optimal result	2485
Rubi [A] (verified)	2485
Mathematica [C] (verified)	2486
Maple [A] (verified)	2487
Fricas [C] (verification not implemented)	2487
Sympy [F]	2488
Maxima [F]	2488
Giac [F]	2488
Mupad [F(-1)]	2488

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a \cos(c+dx)} dx = -\frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] $-(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d+(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d+\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2848, 2827, 2720, 2719}

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a \cos(c+dx)} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} - \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} + \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a \cos(c+dx) + a)}$$

[In] $\text{Int}[\text{Sqrt}[\text{Cos}[c + d*x]]/(a + a*\text{Cos}[c + d*x]), x]$

[Out] $-(\text{EllipticE}[(c + d*x)/2, 2]/(a*d)) + \text{EllipticF}[(c + d*x)/2, 2]/(a*d) + (\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(d*(a + a*\text{Cos}[c + d*x]))$

Rule 2719

`Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2720

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 2848

`Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_))/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(-b)*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(a*f*(a + b*Sin[e + f*x]))), x] + Dist[d*(n/(a*b)), Int[(c + d*Sin[e + f*x])^(n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2*n] || EqQ[c, 0])`

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{a-a\cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{2a^2} \\ &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a} - \frac{\int \sqrt{\cos(c+dx)} dx}{2a} \\ &= -\frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a\cos(c+dx))} \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.00 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.66

$$\begin{aligned} &\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)} dx \\ &= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right) \left(-\frac{2i\sqrt{2}e^{-i(c+dx)}(1+e^{2i(c+dx)}+(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}})\text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)+e^{i(c+dx)}(-1+e^{2ic})}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{a(1+\cos(c+dx))} \end{aligned}$$

[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*cos[c + d*x]),x]

[Out] $(\cos[(c + d*x)/2]^2 * ((-2*I)*\sqrt{2}*(1 + E^{((2*I)*(c + d*x))}) + (-1 + E^{((2*I)*c)})*\sqrt{1 + E^{((2*I)*(c + d*x))}}] * \text{Hypergeometric2F1}[-1/4, 1/2, 3/4, -E^{((2*I)*(c + d*x))}] + E^{I*(c + d*x)}*(-1 + E^{((2*I)*c)})*\sqrt{1 + E^{((2*I)*(c + d*x))}}] * \text{Hypergeometric2F1}[1/4, 1/2, 5/4, -E^{((2*I)*(c + d*x))}]))/(d * E^{I*(c + d*x)}*(-1 + E^{((2*I)*c)})*\sqrt{(1 + E^{((2*I)*(c + d*x))})/E^{I*(c + d*x)}}) + (2*\sqrt{\cos[c + d*x]}*(\text{Csc}[c] + \text{Sec}[c/2]*\text{Sec}[(c + d*x)/2]*\text{Sin}[(d*x)/2]))/d)/(a*(1 + \cos[c + d*x]))$

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.83

method	result
default	$-\frac{\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) + E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}{a \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}$

[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] $-((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\cos(1/2*d*x+1/2*c)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}))+2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2)/a/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{\cos(c + dx)}}{a + a \cos(c + dx)} dx$$

$$= \frac{(-i\sqrt{2}\cos(dx + c) - i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + (i\sqrt{2}\cos(dx + c) + i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i\sin(dx + c))}{2} + \frac{(-i\sqrt{2}\cos(dx + c) - i\sqrt{2})\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c))) + (i\sqrt{2}\cos(dx + c) + i\sqrt{2})\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i\sin(dx + c)))}{2}$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] $1/2*((-I*\sqrt{2})*\cos(d*x + c) - I*\sqrt{2})*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + (I*\sqrt{2})*\cos(d*x + c) + I*\sqrt{2})*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + (-I*\sqrt{2})*\cos(d*x + c) - I*\sqrt{2})*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) + (I*\sqrt{2})*\cos(d*x + c) + I*\sqrt{2})*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)))$

-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(cos(d*x + c))*sin(d*x + c)/(a*d*cos(d*x + c) + a*d)

Sympy [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{a + a \cos(c + dx)} dx = \frac{\int \frac{\sqrt{\cos(c+dx)}}{\cos(c+dx)+1} dx}{a}$$

[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)

[Out] Integral(sqrt(cos(c + d*x))/(cos(c + d*x) + 1), x)/a

Maxima [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{a + a \cos(c + dx)} dx = \int \frac{\sqrt{\cos(dx + c)}}{a \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{a + a \cos(c + dx)} dx = \int \frac{\sqrt{\cos(dx + c)}}{a \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{a + a \cos(c + dx)} dx = \int \frac{\sqrt{\cos(c + dx)}}{a + a \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x)), x)

$$3.178 \quad \int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))} dx$$

Optimal result	2489
Rubi [A] (verified)	2489
Mathematica [C] (verified)	2490
Maple [A] (verified)	2491
Fricas [C] (verification not implemented)	2491
Sympy [F]	2492
Maxima [F]	2492
Giac [F]	2492
Mupad [F(-1)]	2492

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))} dx = \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a \cos(c+dx))}$$

[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a/d+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a/d-sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2847, 2827, 2720, 2719}

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} + \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} - \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a \cos(c+dx) + a)}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]

[Out] EllipticE[(c + d*x)/2, 2]/(a*d) + EllipticF[(c + d*x)/2, 2]/(a*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))

Rule 2719

`Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2720

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 2847

`Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))], x] + Dist[d/(a*(b*c - a*d)), Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])`

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \frac{-\frac{a}{2} - \frac{1}{2}a \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{a^2} \\ &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a} + \frac{\int \sqrt{\cos(c+dx)} dx}{2a} \\ &= \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d(a+a\cos(c+dx))} \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.07 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.67

$$\begin{aligned} &\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} dx \\ &= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right) \left(\frac{2i\sqrt{2}e^{-i(c+dx)} \left(1+e^{2i(c+dx)} + (-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right) \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) - e^{i(c+dx)}(-1+e^{2ic})}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{a(1+\cos(c+dx))} \end{aligned}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]

[Out] $(\cos[(c + dx)/2]^2 * ((2I) * \sqrt{2} * (1 + E^{((2I)(c + dx))}) + (-1 + E^{((2I)c)}) * \sqrt{1 + E^{((2I)(c + dx))}}] * \text{Hypergeometric2F1}[-1/4, 1/2, 3/4, -E^{((2I)(c + dx))}] - E^{(I(c + dx))} * (-1 + E^{((2I)c)}) * \sqrt{1 + E^{((2I)(c + dx))}}] * \text{Hypergeometric2F1}[1/4, 1/2, 5/4, -E^{((2I)(c + dx))}])) / (d * E^{(I(c + dx))} * (-1 + E^{((2I)c)}) * \sqrt{(1 + E^{((2I)(c + dx))}) / E^{(I(c + dx))}}]) - (2 * \sqrt{\cos[c + dx]} * (\csc[c] + \sec[c/2] * \sec[(c + dx)/2] * \sin[(dx)/2])) / d) / (a * (1 + \cos[c + dx]))$

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.86

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\left(F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) + \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\right)}{a \cos\left(\frac{dx}{2} + \frac{c}{2}\right)}$

[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] $((2 * \cos(1/2 * dx + 1/2 * c) ^ 2 - 1) * \sin(1/2 * dx + 1/2 * c) ^ 2) ^ (1/2) * (-\cos(1/2 * dx + 1/2 * c) * (2 * \sin(1/2 * dx + 1/2 * c) ^ 2 - 1) ^ (1/2) * (\sin(1/2 * dx + 1/2 * c) ^ 2) ^ (1/2) * (\text{EllipticF}(\cos(1/2 * dx + 1/2 * c), 2 ^ (1/2)) - \text{EllipticE}(\cos(1/2 * dx + 1/2 * c), 2 ^ (1/2)))) + 2 * \sin(1/2 * dx + 1/2 * c) ^ 4 - \sin(1/2 * dx + 1/2 * c) ^ 2) / a / \cos(1/2 * dx + 1/2 * c) / (-2 * \sin(1/2 * dx + 1/2 * c) ^ 4 + \sin(1/2 * dx + 1/2 * c) ^ 2) ^ (1/2) / \sin(1/2 * dx + 1/2 * c) / (2 * \cos(1/2 * dx + 1/2 * c) ^ 2 - 1) ^ (1/2) / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.63

$$\int \frac{1}{\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} dx$$

$$= \frac{(-i \sqrt{2} \cos(dx + c) - i \sqrt{2}) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + (i \sqrt{2} \cos(dx + c) + i \sqrt{2}) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + (i \sqrt{2} \cos(dx + c) + i \sqrt{2}) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + (-i \sqrt{2} \cos(dx + c) - i \sqrt{2}) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{(a + a \cos(c + dx))}$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] $1/2 * ((-I * \sqrt{2}) * \cos(dx + c) - I * \sqrt{2}) * \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I * \sin(dx + c)) + (I * \sqrt{2}) * \cos(dx + c) + I * \sqrt{2}) * \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I * \sin(dx + c)) + (I * \sqrt{2}) * \cos(dx + c) + I * \sqrt{2}) * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I * \sin(dx + c))) + (-I * \sqrt{2}) * \cos(dx + c) - I * \sqrt{2}) * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I * \sin(dx + c)))$

`-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*sqrt(cos(d*x + c))*sin(d*x + c)/(a*d*cos(d*x + c) + a*d)`

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} dx = \frac{\int \frac{1}{\cos^{\frac{3}{2}}(c+dx) + \sqrt{\cos(c+dx)}} dx}{a}$$

`[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)`

`[Out] Integral(1/(cos(c + d*x)**(3/2) + sqrt(cos(c + d*x))), x)/a`

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} dx = \int \frac{1}{(a \cos(dx + c) + a)\sqrt{\cos(dx + c)}} dx$$

`[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")`

`[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)`

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} dx = \int \frac{1}{(a \cos(dx + c) + a)\sqrt{\cos(dx + c)}} dx$$

`[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")`

`[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} dx = \int \frac{1}{\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} dx$$

`[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))),x)`

`[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))), x)`

$$3.179 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))} dx$$

Optimal result	2493
Rubi [A] (verified)	2493
Mathematica [C] (verified)	2495
Maple [A] (verified)	2495
Fricas [C] (verification not implemented)	2496
Sympy [F]	2496
Maxima [F]	2497
Giac [F]	2497
Mupad [F(-1)]	2497

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))} dx = -\frac{3E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} + \frac{3 \sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a \cos(c+dx))}$$

[Out] $-3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d - (\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d + 3*\sin(d*x+c)/a/d/\cos(d*x+c)^{(1/2)} - \sin(d*x+c)/d/(a+a*\cos(d*x+c))/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2847, 2827, 2716, 2719, 2720}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))} dx = -\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} - \frac{3E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} + \frac{3 \sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a \cos(c+dx) + a)}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*cos[c + d*x])),x]

[Out] (-3*EllipticE[(c + d*x)/2, 2]/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) + (3*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]])*(a + a*cos[c + d*x]))

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2847

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d)), Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c + dx)}{d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))} - \frac{\int \frac{-\frac{3a}{2} + \frac{1}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx}{a^2} \\ &= -\frac{\sin(c + dx)}{d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a} + \frac{3 \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)} dx}{2a} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} \\
&\quad - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} - \frac{3\int\sqrt{\cos(c+dx)}dx}{2a} \\
&= -\frac{3E\left(\frac{1}{2}(c+dx)\mid 2\right)}{ad} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} \\
&\quad + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.79 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.09

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right) \left(-\frac{2i\sqrt{2}e^{-i(c+dx)}(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}})\text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)-e^{i(c+dx)}(-\dots)}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{a(1+$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*cos[c + d*x])),x]

[Out] (Cos[(c + d*x)/2]^2*((-2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) + ((2*Cos[(c - d*x)/2] + Cos[(3*c + d*x)/2] + 3*Cos[(c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(2*d*Sqrt[Cos[c + d*x]]))/(a*(1 + Cos[c + d*x]))

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.64

method	result
default	$ -\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \left(F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - 3E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \right) $

[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

```
[Out] -(-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.46

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} dx$$

$$= \frac{2(3\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c) + (i\sqrt{2}\cos(dx+c)^2 + i\sqrt{2}\cos(dx+c))\text{weierstrassPInverse}(\dots)}{\dots}$$

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + (I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} dx = \frac{\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)+\cos^{\frac{3}{2}}(c+dx)} dx}{a}$$

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(1/(cos(c + d*x)**(5/2) + cos(c + d*x)**(3/2)), x)/a
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} dx = \int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} dx = \int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} dx = \int \frac{1}{\cos(c+dx)^{3/2}(a+a\cos(c+dx))} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))),x)

[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))), x)

$$3.180 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))} dx$$

Optimal result	2498
Rubi [A] (verified)	2498
Mathematica [C] (verified)	2500
Maple [B] (verified)	2500
Fricas [C] (verification not implemented)	2501
Sympy [F]	2502
Maxima [F]	2502
Giac [F]	2502
Mupad [F(-1)]	2502

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))} dx = \frac{3E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} + \frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3ad} \\ + \frac{5 \sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)} - \frac{3 \sin(c+dx)}{ad \sqrt{\cos(c+dx)}} \\ - \frac{\sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))}$$

[Out] $3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d+5/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d+5/3*\sin(d*x+c)/a/d/\cos(d*x+c)^{(3/2)}-\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))-3*\sin(d*x+c)/a/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2847, 2827, 2716, 2720, 2719}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))} dx = \frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3ad} + \frac{3E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} \\ - \frac{\sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)(a \cos(c+dx) + a)} \\ + \frac{5 \sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)} - \frac{3 \sin(c+dx)}{ad \sqrt{\cos(c+dx)}}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]

[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d) + (5*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (3*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2847

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d)), Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} - \frac{\int \frac{-\frac{5a}{2} + \frac{3}{2}a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx}{a^2} \\ &= -\frac{\sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} - \frac{3 \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx}{2a} + \frac{5 \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx}{2a} \end{aligned}$$

$$\begin{aligned}
&= \frac{5 \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)} - \frac{3 \sin(c + dx)}{ad \sqrt{\cos(c + dx)}} - \frac{\sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))} \\
&\quad + \frac{5 \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{6a} + \frac{3 \int \sqrt{\cos(c + dx)} dx}{2a} \\
&= \frac{3E\left(\frac{1}{2}(c + dx) \mid 2\right)}{ad} + \frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3ad} + \frac{5 \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)} \\
&\quad - \frac{3 \sin(c + dx)}{ad \sqrt{\cos(c + dx)}} - \frac{\sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.91 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.68

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))} dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c + dx)\right) \left(\frac{2i\sqrt{2}e^{-i(c+dx)} \left(9(1+e^{2i(c+dx)})+9(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right) \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) - 5e^{i(c+dx)}(-1+e^{2i(c+dx)})}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{1}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]

[Out] (Cos[(c + d*x)/2]^2*((2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x)))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) - ((10*Cos[(c - d*x)/2] + 8*Cos[(3*c + d*x)/2] + 4*Cos[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 9*Cos[(3*c + 5*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(4*d*Cos[c + d*x]^(3/2)))/(3*a*(1 + Cos[c + d*x]))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(166) = 332.

Time = 3.73 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.33

method	result
default	$ \sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \left(10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+x)}{2}}\right) $

[In] `int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{3} \frac{(-(-2 \cos(\frac{1}{2} d x + \frac{1}{2} c)^2 + 1) \sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{1/2}}{a \cos(\frac{1}{2} d x + \frac{1}{2} c) \sin(\frac{1}{2} d x + \frac{1}{2} c)^3} \frac{4 \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 - 4 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 + 1}{(2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 + \sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{1/2}} \frac{10 \cos(\frac{1}{2} d x + \frac{1}{2} c) \operatorname{EllipticF}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{1/2}) (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{1/2} (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{1/2} \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 18 \cos(\frac{1}{2} d x + \frac{1}{2} c) \operatorname{EllipticE}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{1/2}) (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{1/2} (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{1/2} \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 36 \sin(\frac{1}{2} d x + \frac{1}{2} c)^6 - 5 \cos(\frac{1}{2} d x + \frac{1}{2} c) (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{1/2} (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{1/2} \operatorname{EllipticF}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{1/2}) + 9 \cos(\frac{1}{2} d x + \frac{1}{2} c) (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{1/2} (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{1/2} \operatorname{EllipticE}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{1/2}) + 44 \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 - 11 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2}{(2 \cos(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{1/2}} \frac{1}{d}$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.08

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))} dx = \frac{2(9 \cos(dx + c)^2 + 4 \cos(dx + c) - 2) \sqrt{\cos(dx + c)} \sin(dx + c) + 5(i \sqrt{2} \cos(dx + c)^3 + i \sqrt{2} \cos(dx + c))}{\dots}$$

[In] `integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")`

[Out]
$$\frac{-1/6(2(9 \cos(dx + c)^2 + 4 \cos(dx + c) - 2) \sqrt{\cos(dx + c)} \sin(dx + c) + 5(I \sqrt{2} \cos(dx + c)^3 + I \sqrt{2} \cos(dx + c)^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + I \sin(dx + c)) + 5(-I \sqrt{2} \cos(dx + c)^3 - I \sqrt{2} \cos(dx + c)^2) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - I \sin(dx + c)) + 9(-I \sqrt{2} \cos(dx + c)^3 - I \sqrt{2} \cos(dx + c)^2) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + I \sin(dx + c))) + 9(I \sqrt{2} \cos(dx + c)^3 + I \sqrt{2} \cos(dx + c)^2) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - I \sin(dx + c)))}{(a \cos(dx + c)^3 + a d \cos(dx + c)^2)}$$

Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} dx = \frac{\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)+\cos^{\frac{5}{2}}(c+dx)} dx}{a}$$

[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)

[Out] Integral(1/(cos(c + d*x)**(7/2) + cos(c + d*x)**(5/2)), x)/a

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} dx = \int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} dx = \int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))} dx = \int \frac{1}{\cos(c+dx)^{5/2}(a+a\cos(c+dx))} dx$$

[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))),x)

[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))), x)

$$3.181 \quad \int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$$

Optimal result	2503
Rubi [A] (verified)	2503
Mathematica [C] (verified)	2505
Maple [A] (verified)	2506
Fricas [C] (verification not implemented)	2506
Sympy [F(-1)]	2507
Maxima [F]	2507
Giac [F]	2507
Mupad [F(-1)]	2508

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{56E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5a^2d} - \frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{a^2d}$$

$$- \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{a^2d} + \frac{56 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15a^2d}$$

$$- \frac{3 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] 56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a^2/d+56/15*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d-3*cos(d*x+c)^(5/2)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2-5*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2844, 3056, 2827, 2715, 2720, 2719}

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{a^2d} + \frac{56E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5a^2d}$$

$$- \frac{3 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)} + \frac{56 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{15a^2d}$$

$$- \frac{5 \sin(c+dx) \sqrt{\cos(c+dx)}}{a^2d} - \frac{\sin(c+dx) \cos^{\frac{7}{2}}(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] Int[Cos[c + d*x]^(9/2)/(a + a*cos[c + d*x])^2,x]

[Out] (56*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(a^2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d) + (56*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d) - (3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*cos[c + d*x])^2)

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m +

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1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7a}{2} - \frac{11}{2}a\cos(c+dx)\right)}{a+a\cos(c+dx)} dx}{3a^2} \\
&= -\frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{45a^2}{2} - 28a^2\cos(c+dx)\right) dx}{3a^4} \\
&= -\frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{15\int \cos^{\frac{3}{2}}(c+dx) dx}{2a^2} + \frac{28\int \cos^{\frac{5}{2}}(c+dx) dx}{3a^2} \\
&= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} \\
&\quad - \frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a^2} + \frac{28\int \sqrt{\cos(c+dx)} dx}{5a^2} \\
&= \frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} \\
&\quad + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.94 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx \\
&= \frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)(-240 - 1186\cos(c+dx) + 340\cos(2(c+dx)) + 207\cos(3(c+dx)) - 20\cos(4(c+dx)))}{(a+a\cos(c+dx))^2}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(9/2)/(a + a*cos[c + d*x])^2,x]

[Out] (Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-240 - 1186*Cos[c + d*x] + 340*Cos[2*(c + d*x)] + 207*Cos[3*(c + d*x)] - 20*Cos[4*(c + d*x)] + 3*Cos[5*(c + d*x)] + 600*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2) + 1792*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)))/(120*a^2*d)

Maple [A] (verified)

Time = 6.38 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.77

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(96\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-150\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{1}{2}+\frac{\cos(dx+c)}{2}}\right)}{30a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}$

[In] int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.80

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx$$

$$= \frac{2(6 \cos(dx + c)^3 - 8 \cos(dx + c)^2 - 94 \cos(dx + c) - 75) \sqrt{\cos(dx + c)} \sin(dx + c) - 75(-i\sqrt{2} \cos(dx + c) + i\sqrt{2} \sin(dx + c))}{(a + a \cos(c + dx))^2}$$

[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/30*(2*(6*cos(d*x + c)^3 - 8*cos(d*x + c)^2 - 94*cos(d*x + c) - 75)*sqrt(cos(d*x + c))*sin(d*x + c) - 75*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 75*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 168*(-I*sqrt(2)*cos(d*x + c) + I*sqrt(2)*sin(d*x + c))

2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 168*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{9}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{9}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(c + dx)^{9/2}}{(a + a \cos(c + dx))^2} dx$$

```
[In] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^2, x)
```

```
[Out] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^2, x)
```


$$3.182 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$$

Optimal result	2509
Rubi [A] (verified)	2509
Mathematica [C] (verified)	2511
Maple [A] (verified)	2512
Fricas [C] (verification not implemented)	2512
Sympy [F(-1)]	2513
Maxima [F]	2513
Giac [F]	2513
Mupad [F(-1)]	2513

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{7E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2 d} + \frac{10 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2 d} \\ + \frac{10\sqrt{\cos(c+dx)} \sin(c+dx)}{3a^2 d} \\ - \frac{7 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2 d(1+\cos(c+dx))} - \frac{\cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] $-7*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d+10/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d-7/3*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a^2/d/(1+\cos(d*x+c))-1/3*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2+10/3*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2844, 3056, 2827, 2719, 2715, 2720}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{10 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2 d} - \frac{7E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2 d} \\ - \frac{7 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{3a^2 d(\cos(c+dx)+1)} + \frac{10 \sin(c+dx) \sqrt{\cos(c+dx)}}{3a^2 d} \\ - \frac{\sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] Int[Cos[c + d*x]^(7/2)/(a + a*cos[c + d*x])^2,x]

[Out] (-7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) + (10*sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) - (7*cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*cos[c + d*x])^2)

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*SIN[e + f*x])^m*((c + d*SIN[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*SIN[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*SIN[e + f*x])^m*((c + d*SIN[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +

$b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /;$ Free
 $Q[\{a, b, c, d, e, f, A, B\}, x] \&\& NeQ[b*c - a*d, 0] \&\& EqQ[a^2 - b^2, 0] \&\&$
 $NeQ[c^2 - d^2, 0] \&\& LtQ[m, -2^{(-1)}] \&\& GtQ[n, 0] \&\& IntegerQ[2*m] \&\& (Int$
 $egerQ[2*n] || EqQ[c, 0])$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-\frac{9}{2}a\cos(c+dx)\right)}{a+a\cos(c+dx)} dx}{3a^2} \\
 &= -\frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
 &\quad - \frac{\int \sqrt{\cos(c+dx)}\left(\frac{21a^2}{2}-15a^2\cos(c+dx)\right) dx}{3a^4} \\
 &= -\frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
 &\quad - \frac{7\int \sqrt{\cos(c+dx)} dx}{2a^2} + \frac{5\int \cos^{\frac{3}{2}}(c+dx) dx}{a^2} \\
 &= -\frac{7E\left(\frac{1}{2}(c+dx)|2\right)}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} \\
 &\quad - \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3a^2} \\
 &= -\frac{7E\left(\frac{1}{2}(c+dx)|2\right)}{a^2d} + \frac{10\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} \\
 &\quad - \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.61 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\begin{aligned}
 &\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx \\
 &= \frac{\sqrt{\cos(c+dx)}\csc^3(c+dx)(15+76\cos(c+dx)-24\cos(2(c+dx))-12\cos(3(c+dx))+\cos(4(c+dx)))}{(12a^2d)}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^2,x]

[Out] (Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(15 + 76*Cos[c + d*x] - 24*Cos[2*(c + d*x)] - 12*Cos[3*(c + d*x)] + Cos[4*(c + d*x)] - 40*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2) - 112*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)))/(12*a^2*d)

Maple [A] (verified)

Time = 6.20 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.96

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(16\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\right)\right)}{6a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.01

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx$$

$$= \frac{2(2\cos(dx+c)^2+13\cos(dx+c)+10)\sqrt{\cos(dx+c)}\sin(dx+c)-10(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c))}{(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d)}$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

```
[Out] 1/6*(2*(2*cos(d*x+c)^2+13*cos(d*x+c)+10)*sqrt(cos(d*x+c))*sin(d*x+c)-10*(I*sqrt(2)*cos(d*x+c)^2+2*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c))-10*(-I*sqrt(2)*cos(d*x+c)^2-2*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))-21*(I*sqrt(2)*cos(d*x+c)^2+2*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c)))-21*(-I*sqrt(2)*cos(d*x+c)^2-2*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))))/(a^2*d*cos(d*x+c)^2+2*a^2*d*cos(d*x+c)+a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(c + dx)^{7/2}}{(a + a \cos(c + dx))^2} dx$$

[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^2,x)

[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^2, x)

$$3.183 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$$

Optimal result	2514
Rubi [A] (verified)	2514
Mathematica [C] (verified)	2516
Maple [A] (verified)	2516
Fricas [C] (verification not implemented)	2517
Sympy [F(-1)]	2517
Maxima [F]	2518
Giac [F]	2518
Mupad [F(-1)]	2518

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{4E\left(\frac{1}{2}(c+dx)|2\right)}{a^2d} - \frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d} \\ - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] $4*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d - 5/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d - 1/3*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2 - 5/3*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d/(1+\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2844, 3056, 2827, 2720, 2719}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d} + \frac{4E\left(\frac{1}{2}(c+dx)|2\right)}{a^2d} \\ - \frac{5 \sin(c+dx) \sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)} - \frac{\sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{3d(a \cos(c+dx)+a)^2}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^{(5/2)}/(a+a*\operatorname{Cos}[c+d*x])^2, x]$

[Out] $(4*\operatorname{EllipticE}[(c+d*x)/2, 2])/ (a^2*d) - (5*\operatorname{EllipticF}[(c+d*x)/2, 2]) / (3*a^2*d) - (5*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x]) / (3*a^2*d*(1+\operatorname{Cos}[c+d*x])) - (\operatorname{Cos}[c+d*x]^{(3/2)}*\operatorname{Sin}[c+d*x]) / (3*d*(a+a*\operatorname{Cos}[c+d*x])^2)$

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2844

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3056

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \cos(c + dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)} \left(\frac{3a}{2} - \frac{7}{2}a \cos(c+dx)\right) dx}{a + a \cos(c+dx)}}{3a^2} \\ &= -\frac{5\sqrt{\cos(c + dx)} \sin(c + dx)}{3a^2d(1 + \cos(c + dx))} - \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \cos(c + dx))^2} - \frac{\int \frac{\frac{5a^2}{2} - 6a^2 \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{3a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{5\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{6a^2} + \frac{2\int\sqrt{\cos(c+dx)}dx}{a^2} \\
&= \frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} \\
&\quad - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.50 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\begin{aligned}
&\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx \\
&= \sqrt{\cos(c+dx)} \operatorname{csc}^3(c+dx) (-6 - 46\cos(c+dx) + 14\cos(2(c+dx)) + 6\cos(3(c+dx)) + 20\operatorname{Hypergeometric}
\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-6 - 46*Cos[c + d*x] + 14*Cos[2*(c + d*
x)] + 6*Cos[3*(c + d*x)] + 20*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]
^2]*(Sin[c + d*x]^2)^(3/2) + 64*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2, 7/
4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)))/(12*a^2*d)
```

Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.29

method	result
default	$ \frac{\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+64\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}} $

```
[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*x
+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d*x+1
```


$/2*c)^{2-1}/a^2/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)^3/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^{2-1})^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.39

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx = \frac{2(6\cos(dx+c)+5)\sqrt{\cos(dx+c)}\sin(dx+c) + 5(-i\sqrt{2}\cos(dx+c)^2 - 2i\sqrt{2}\cos(dx+c) - i\sqrt{2})}{\dots}$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] $-1/6*(2*(6*\cos(d*x + c) + 5)*\sqrt{\cos(d*x + c)}*\sin(d*x + c) + 5*(-I*\sqrt{2})*\cos(d*x + c)^2 - 2*I*\sqrt{2}*\cos(d*x + c) - I*\sqrt{2})*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + 5*(I*\sqrt{2})*\cos(d*x + c)^2 + 2*I*\sqrt{2}*\cos(d*x + c) + I*\sqrt{2})*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + 12*(-I*\sqrt{2})*\cos(d*x + c)^2 - 2*I*\sqrt{2}*\cos(d*x + c) - I*\sqrt{2})*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) + 12*(I*\sqrt{2})*\cos(d*x + c)^2 + 2*I*\sqrt{2}*\cos(d*x + c) + I*\sqrt{2})*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))))/(a^2*d*\cos(d*x + c)^2 + 2*a^2*d*\cos(d*x + c) + a^2*d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(c + dx)^{5/2}}{(a + a \cos(c + dx))^2} dx$$

[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^2,x)

[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^2, x)

$$3.184 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$$

Optimal result	2519
Rubi [A] (verified)	2519
Mathematica [C] (verified)	2521
Maple [A] (verified)	2521
Fricas [C] (verification not implemented)	2522
Sympy [F(-1)]	2522
Maxima [F]	2523
Giac [F]	2523
Mupad [F(-1)]	2523

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2 d} + \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2 d} \\ + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{a^2 d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] $-(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d+2/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d+\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d/(1+\cos(d*x+c))-1/3*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^2$

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2844, 3057, 2827, 2720, 2719}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2 d} - \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2 d} \\ + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{a^2 d(\cos(c+dx)+1)} - \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{3d(a \cos(c+dx)+a)^2}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^{(3/2)}/(a+a*\operatorname{Cos}[c+d*x])^2, x]$

[Out] $-(\operatorname{EllipticE}[(c+d*x)/2, 2]/(a^2*d)) + (2*\operatorname{EllipticF}[(c+d*x)/2, 2])/(3*a^2*d) + (\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(a^2*d*(1+\operatorname{Cos}[c+d*x])) - (\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(3*d*(a+a*\operatorname{Cos}[c+d*x])^2)$

Rule 2719

$\text{Int}[\sqrt{\sin[(c_.) + (d_.)*(x_)]}], x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\sqrt{\sin[(c_.) + (d_.)*(x_)]}], x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticF}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2827

$\text{Int}[(b_.*\sin[(e_.) + (f_.)*(x_)])^{(m_)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2844

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])^{(m_)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^{(n - 1)}/(a*f*(2*m + 1))), x] + \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^{(n - 2)}*\text{Simp}[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{GtQ}[n, 1] \&\& (\text{IntegersQ}[2*m, 2*n] \parallel (\text{IntegerQ}[m] \&\& \text{EqQ}[c, 0]))]$

Rule 3057

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])^{(m_)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[b*(A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^{(n + 1)}/(a*f*(2*m + 1)*(b*c - a*d))), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^n*\text{Simp}[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] \parallel \text{EqQ}[c, 0])]$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{3d(a + a \cos(c + dx))^2} - \frac{\int \frac{\frac{a}{2} - \frac{5}{2}a \cos(c + dx)}{\sqrt{\cos(c + dx)}(a + a \cos(c + dx))} dx}{3a^2} \\ &= \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{a^2 d(1 + \cos(c + dx))} - \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{3d(a + a \cos(c + dx))^2} - \frac{\int \frac{-a^2 + \frac{3}{2}a^2 \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx}{3a^4} \end{aligned}$$

$$\begin{aligned}
&= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{a^2 d (1 + \cos(c+dx))} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a + a \cos(c+dx))^2} \\
&\quad + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3a^2} - \frac{\int \sqrt{\cos(c+dx)} dx}{2a^2} \\
&= -\frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2 d} + \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2 d} \\
&\quad + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{a^2 d (1 + \cos(c+dx))} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a + a \cos(c+dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx = \frac{2\sqrt{\cos(c+dx)} \operatorname{csc}^3(c+dx) (-2\cos(c+dx) + \cos(2(c+dx))) + \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \cos^2(c+dx)\right)}{3a^2 d}$$

[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]

[Out] (-2*sqrt[Cos[c + d*x]]*Csc[c + d*x]^3*(-2*Cos[c + d*x] + Cos[2*(c + d*x)] + Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2) + 2*Cos[c + d*x]*Hypergeometric2F1[3/4, 5/2, 7/4, Cos[c + d*x]^2]*(Sin[c + d*x]^2)^(3/2)))/(3*a^2*d)

Maple [A] (verified)

Time = 3.26 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.36

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{6a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x+1/2*c)^2)

$\frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.46

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx$$

$$= \frac{2(3\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c) - 2(i\sqrt{2}\cos(dx+c)^2 + 2i\sqrt{2}\cos(dx+c) + i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + I\sin(dx+c)) - 2(-I\sqrt{2}\cos(dx+c)^2 - 2I\sqrt{2}\cos(dx+c) - I\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) - I\sin(dx+c)) - 3(I\sqrt{2}\cos(dx+c)^2 + 2I\sqrt{2}\cos(dx+c) + I\sqrt{2})\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + I\sin(dx+c))) - 3(-I\sqrt{2}\cos(dx+c)^2 - 2I\sqrt{2}\cos(dx+c) - I\sqrt{2})\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - I\sin(dx+c)))}{(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c) + a^2d)}$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(2*(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 2*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(c + dx)^{3/2}}{(a + a \cos(c + dx))^2} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^2,x)

[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^2, x)

3.185 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^2} dx$

Optimal result	2524
Rubi [A] (verified)	2524
Mathematica [C] (verified)	2525
Maple [B] (verified)	2526
Fricas [C] (verification not implemented)	2526
Sympy [F]	2527
Maxima [F]	2527
Giac [F]	2527
Mupad [F(-1)]	2527

Optimal result

Integrand size = 23, antiderivative size = 57

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^2} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] 1/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a^2/d+1/3*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^2

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2843, 21, 2720}

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^2} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d} + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{3d(a \cos(c+dx) + a)^2}$$

[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^2,x]

[Out] EllipticF[(c + d*x)/2, 2]/(3*a^2*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)

Rule 21

```
Int[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x_Symbol] :>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```


Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2843

```
Int[((a_) + (b_)*sin[(e_) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{a}{2} + \frac{1}{2}a \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} dx}{3a^2} \\ &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{6a^2} \\ &= \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a+a\cos(c+dx))^2} \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\begin{aligned} &\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2} dx \\ &= \frac{\sqrt{\cos(c+dx)} \csc(c+dx) \left(-\text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \cos^2(c+dx)\right) \sqrt{\sin^2(c+dx)} + \tan^2\left(\frac{1}{2}(c+dx)\right) \right)}{3a^2d} \end{aligned}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]) + Tan[(c + d*x)/2]^2)/(3*a^2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(77) = 154$.

Time = 2.90 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.30

method	result
default	$-\frac{\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d$

[In] `int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)`

[Out]
$$-1/6*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*\cos(1/2*d*x+1/2*c)^2+1)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^(1/2))*\cos(1/2*d*x+1/2*c)^3+2*\cos(1/2*d*x+1/2*c)^4-3*\cos(1/2*d*x+1/2*c)^2+1)/a^2/\cos(1/2*d*x+1/2*c)^3/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^(1/2)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2} dx$$

$$= \frac{(-i\sqrt{2}\cos(dx+c)^2 - 2i\sqrt{2}\cos(dx+c) - i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))}{6(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c) + a^2d)}$$

[In] `integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

[Out]
$$1/6*((-I*\sqrt{2}*\cos(d*x+c)^2 - 2*I*\sqrt{2}*\cos(d*x+c) - I*\sqrt{2})*\text{weierstrassPInverse}(-4, 0, \cos(d*x+c) + I*\sin(d*x+c)) + (I*\sqrt{2}*\cos(d*x+c)^2 + 2*I*\sqrt{2}*\cos(d*x+c) + I*\sqrt{2})*\text{weierstrassPInverse}(-4, 0, \cos(d*x+c) - I*\sin(d*x+c)) + 2*\sqrt{\cos(d*x+c)}*\sin(d*x+c))/(a^2*d*\cos(d*x+c)^2 + 2*a^2*d*\cos(d*x+c) + a^2*d)$$

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2} dx = \int \frac{\sqrt{\cos(c+dx)}}{\cos^2(c+dx)+2\cos(c+dx)+1} \frac{dx}{a^2}$$

[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)

[Out] Integral(sqrt(cos(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2} dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^2} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2} dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^2} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^2,x)

[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^2, x)

$$3.186 \quad \int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^2} dx$$

Optimal result	2528
Rubi [A] (verified)	2528
Mathematica [C] (verified)	2530
Maple [A] (verified)	2531
Fricas [C] (verification not implemented)	2531
Sympy [F]	2532
Maxima [F]	2532
Giac [F]	2532
Mupad [F(-1)]	2532

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^2} dx = \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2 d} + \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2 d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{a^2 d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

[Out] $(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d+2/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d-\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d/(1+\cos(d*x+c))-1/3*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^2$

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2845, 3057, 2827, 2720, 2719}

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^2} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2 d} + \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2 d} - \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2 d(\cos(c+dx)+1)} - \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a \cos(c+dx)+a)^2}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*cos[c + d*x])^2),x]

[Out] EllipticE[(c + d*x)/2, 2]/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*cos[c + d*x])^2)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2845

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{5a}{2} - \frac{1}{2}a \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} dx}{3a^2} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{a^2 + \frac{3}{2}a^2 \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{3a^4} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3a^2} + \frac{\int \sqrt{\cos(c+dx)} dx}{2a^2} \\
&= \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2d} + \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d} \\
&\quad - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{3d(a+a\cos(c+dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.41 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.79

$$\begin{aligned}
&\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx \\
&\cos^4\left(\frac{1}{2}(c+dx)\right) \left(\frac{4i\sqrt{2}e^{-i(c+dx)}\left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right) \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) - 2e^{i(c+dx)}(-1+e^{2ic})}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right) \\
&= \frac{\hspace{15em}}{3a^2}
\end{aligned}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2), x]

[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x)))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) - (Sqrt[Cos[c + d*x]]*(7*Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 3*Cos[(c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(2*d))/(3*a^2*(1 + Cos[c + d*x])^2)

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.36

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

[In] `int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)`

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.46

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx =$$

$$\frac{2(3\cos(dx+c)+4)\sqrt{\cos(dx+c)}\sin(dx+c)+2(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2})\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d)}$$

[In] `integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")`

```
[Out] -1/6*(2*(3*cos(d*x + c) + 4)*sqrt(cos(d*x + c))*sin(d*x + c) + 2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx = \frac{\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)+2\cos^{\frac{3}{2}}(c+dx)+\sqrt{\cos(c+dx)}} dx}{a^2}$$

[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)

[Out] Integral(1/(cos(c + d*x)**(5/2) + 2*cos(c + d*x)**(3/2) + sqrt(cos(c + d*x))), x)/a**2

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx = \int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx = \int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx = \int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2), x)

[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^2), x)

$$3.187 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^2} dx$$

Optimal result	2533
Rubi [A] (verified)	2533
Mathematica [C] (verified)	2536
Maple [B] (verified)	2536
Fricas [C] (verification not implemented)	2537
Sympy [F]	2537
Maxima [F]	2538
Giac [F]	2538
Mupad [F(-1)]	2538

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^2} dx = -\frac{4E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2 d} - \frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2 d}$$

$$+ \frac{4 \sin(c+dx)}{a^2 d \sqrt{\cos(c+dx)}}$$

$$- \frac{5 \sin(c+dx)}{3a^2 d \sqrt{\cos(c+dx)}(1+\cos(c+dx))}$$

$$- \frac{\sin(c+dx)}{3d \sqrt{\cos(c+dx)}(a+a \cos(c+dx))^2}$$

[Out] $-4*(\cos(1/2*d*x+1/2*c))^2^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d-5/3*(\cos(1/2*d*x+1/2*c))^2^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d+4*\sin(d*x+c)/a^2/d/\cos(d*x+c)^{(1/2)}-5/3*\sin(d*x+c)/a^2/d/(1+\cos(d*x+c))/\cos(d*x+c)^{(1/2)}-1/3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {2845, 3057, 2827, 2716, 2719, 2720}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = -\frac{5 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d} - \frac{4E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2d} + \frac{4 \sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5 \sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^2}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2), x]

[Out] (-4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) + (4*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - (5*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2)

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(

$a*(2*m + 1)*(b*c - a*d)$, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sin(c + dx)}{3d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^2} + \frac{\int \frac{\frac{7a}{2} - \frac{3}{2}a\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))} dx}{3a^2} \\
 &= -\frac{5\sin(c + dx)}{3a^2d\sqrt{\cos(c + dx)}(1 + \cos(c + dx))} \\
 &\quad - \frac{\sin(c + dx)}{3d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^2} + \frac{\int \frac{6a^2 - \frac{5}{2}a^2\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx}{3a^4} \\
 &= -\frac{5\sin(c + dx)}{3a^2d\sqrt{\cos(c + dx)}(1 + \cos(c + dx))} - \frac{\sin(c + dx)}{3d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^2} \\
 &\quad - \frac{5\int \frac{1}{\sqrt{\cos(c + dx)}} dx}{6a^2} + \frac{2\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx}{a^2} \\
 &= -\frac{5\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3a^2d} + \frac{4\sin(c + dx)}{a^2d\sqrt{\cos(c + dx)}} \\
 &\quad - \frac{5\sin(c + dx)}{3a^2d\sqrt{\cos(c + dx)}(1 + \cos(c + dx))} \\
 &\quad - \frac{\sin(c + dx)}{3d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^2} - \frac{2\int \sqrt{\cos(c + dx)} dx}{a^2}
 \end{aligned}$$

$$= -\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.37 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.46

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} dx$$

$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right) \left(-\frac{4i\sqrt{2}e^{-i(c+dx)}(12(1+e^{2i(c+dx)})+12(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}})\operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-5e^{i(c+dx)}}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{\dots}$$

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(12*(1 + E^((2*I)*(c + d*x)))) + 12*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) + ((29*Cos[(c - d*x)/2] + 19*Cos[(3*c + d*x)/2] + 31*Cos[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 12*Cos[(3*c + 5*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(4*d*Sqrt[Cos[c + d*x]])))/(3*a^2*(1 + Cos[c + d*x])^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(176) = 352.

Time = 3.29 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.98

method	result
default	$-\frac{2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(5F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-12E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{\dots}$

```
[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*
```

$$\frac{\sin(1/2*d*x+1/2*c)^2 - 2*(2*\sin(1/2*d*x+1/2*c)^2 - 1)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (5*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2)^{(1/2)}) - 12*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2)^{(1/2)}) * \cos(1/2*d*x+1/2*c) - 48*(-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \sin(1/2*d*x+1/2*c)^6 + 86*(-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \sin(1/2*d*x+1/2*c)^4 - 37*(-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \sin(1/2*d*x+1/2*c)^2}{a^2 \cos(1/2*d*x+1/2*c)^3 (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2 - 1)^{(1/2)}} / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.34

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} dx$$

$$= \frac{2(12\cos(dx+c)^2 + 19\cos(dx+c) + 6)\sqrt{\cos(dx+c)}\sin(dx+c) - 5(-i\sqrt{2}\cos(dx+c)^3 - 2i\sqrt{2}\cos(dx+c))}{a^2}$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(2*(12*cos(d*x + c)^2 + 19*cos(d*x + c) + 6)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 12*(I*sqrt(2)*cos(d*x + c)^3 + 2*I*sqrt(2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 12*(-I*sqrt(2)*cos(d*x + c)^3 - 2*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = \frac{\int \frac{1}{\cos^{\frac{7}{2}}(c+dx) + 2\cos^{\frac{5}{2}}(c+dx) + \cos^{\frac{3}{2}}(c+dx)} dx}{a^2}$$

[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)

[Out] Integral(1/(cos(c + d*x)**(7/2) + 2*cos(c + d*x)**(5/2) + cos(c + d*x)**(3/2)), x)/a**2

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = \int \frac{1}{(a\cos(dx+c)+a)^2 \cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = \int \frac{1}{(a\cos(dx+c)+a)^2 \cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = \int \frac{1}{\cos(c+dx)^{3/2} (a+a\cos(c+dx))^2} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2), x)

[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^2), x)

$$3.188 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^2} dx$$

Optimal result	2539
Rubi [A] (verified)	2539
Mathematica [C] (verified)	2542
Maple [B] (verified)	2542
Fricas [C] (verification not implemented)	2543
Sympy [F]	2543
Maxima [F]	2544
Giac [F]	2544
Mupad [F(-1)]	2544

Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^2} dx = \frac{7E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2 d} + \frac{10 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2 d}$$

$$+ \frac{10 \sin(c+dx)}{3a^2 d \cos^{\frac{3}{2}}(c+dx)} - \frac{7 \sin(c+dx)}{a^2 d \sqrt{\cos(c+dx)}}$$

$$- \frac{7 \sin(c+dx)}{3a^2 d \cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))}$$

$$- \frac{\sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^2}$$

```
[Out] 7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+10/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)-7/3*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/(1+cos(d*x+c))-1/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2-7*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {2845, 3057, 2827, 2716, 2720, 2719}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = \frac{10 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d} + \frac{7E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2d}$$

$$- \frac{7 \sin(c+dx)}{3a^2d \cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}$$

$$+ \frac{10 \sin(c+dx)}{3a^2d \cos^{\frac{3}{2}}(c+dx)} - \frac{7 \sin(c+dx)}{a^2d \sqrt{\cos(c+dx)}}$$

$$- \frac{\sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)(a \cos(c+dx)+a)^2}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2), x]

[Out] (7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*a^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)) - (7*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - (7*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2845

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^

$m*((c + d*\sin[e + f*x])^{(n + 1)/(a*f*(2*m + 1)*(b*c - a*d)}), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)*(c + d*\sin[e + f*x])^n} \text{Simp}[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{!GtQ}[n, 0] \&\& (\text{IntegerSQ}[2*m, 2*n] || (\text{IntegerQ}[m] \&\& \text{EqQ}[c, 0]))$

Rule 3057

$\text{Int}[(a + b*\sin[e + f*x])^{(m)}*((A + B*\sin[e + f*x]) + (f*(x))^{(n)}), x_Symbol] := \text{Simp}[b*(A*b - a*B)*\cos[e + f*x]*(a + b*\sin[e + f*x])^{(m)}*((c + d*\sin[e + f*x])^{(n + 1)/(a*f*(2*m + 1)*(b*c - a*d)}), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)*(c + d*\sin[e + f*x])^n} \text{Simp}[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{!GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] || \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2} + \frac{\int \frac{\frac{9a}{2} - \frac{5}{2}a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))} dx}{3a^2} \\
 &= -\frac{7 \sin(c + dx)}{3a^2 d \cos^{\frac{3}{2}}(c + dx)(1 + \cos(c + dx))} \\
 &\quad - \frac{\sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2} + \frac{\int \frac{15a^2 - \frac{21}{2}a^2 \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx}{3a^4} \\
 &= -\frac{7 \sin(c + dx)}{3a^2 d \cos^{\frac{3}{2}}(c + dx)(1 + \cos(c + dx))} - \frac{\sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2} \\
 &\quad - \frac{7 \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx}{2a^2} + \frac{5 \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx}{a^2} \\
 &= \frac{10 \sin(c + dx)}{3a^2 d \cos^{\frac{3}{2}}(c + dx)} - \frac{7 \sin(c + dx)}{a^2 d \sqrt{\cos(c + dx)}} - \frac{7 \sin(c + dx)}{3a^2 d \cos^{\frac{3}{2}}(c + dx)(1 + \cos(c + dx))} \\
 &\quad - \frac{\sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2} + \frac{5 \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3a^2} + \frac{7 \int \sqrt{\cos(c + dx)} dx}{2a^2} \\
 &= \frac{7E\left(\frac{1}{2}(c + dx) \mid 2\right)}{a^2 d} + \frac{10 \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3a^2 d} + \frac{10 \sin(c + dx)}{3a^2 d \cos^{\frac{3}{2}}(c + dx)} \\
 &\quad - \frac{7 \sin(c + dx)}{a^2 d \sqrt{\cos(c + dx)}} - \frac{7 \sin(c + dx)}{3a^2 d \cos^{\frac{3}{2}}(c + dx)(1 + \cos(c + dx))} - \frac{\sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
 Time = 4.96 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.25

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2} dx$$

$$= \frac{\cos^4\left(\frac{1}{2}(c+dx)\right) \left(\frac{4i\sqrt{2}e^{-i(c+dx)} \left(21(1+e^{2i(c+dx)}) + 21(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \right) \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) - 10e^{i(c+dx)} \left(-\frac{21(1+e^{2i(c+dx)}) + 21(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{\dots} \right)}{\dots}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2), x]

[Out] (Cos[(c + d*x)/2]^4*((4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c + d*x)))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 10*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])/((d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) - ((82*Cos[(c - d*x)/2] + 65*Cos[(3*c + d*x)/2] + 68*Cos[(c + 3*d*x)/2] + 37*Cos[(5*c + 3*d*x)/2] + 53*Cos[(3*c + 5*d*x)/2] + 10*Cos[(7*c + 5*d*x)/2] + 21*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(8*d*Cos[c + d*x]^(3/2)))/(3*a^2*(1 + Cos[c + d*x])^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(198) = 396.
 Time = 4.55 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.55

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\dots} \left(\frac{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}{3\cos\left(\frac{dx}{2} + \frac{c}{2}\right)^3} + \frac{6\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)} - \frac{22\sqrt{\frac{1}{2} - \cos\left(\frac{dx}{2} + \frac{c}{2}\right)}}{\dots} \right)$

[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+1/2*c)*(

$$\frac{-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}/(\cos(1/2dx+1/2c)^2-1/2)^2+16\sin(1/2dx+1/2c)^2\cos(1/2dx+1/2c)/(-(-2\cos(1/2dx+1/2c)^2+1)\sin(1/2dx+1/2c)^2)^{1/2}}{\sin(1/2dx+1/2c)/(2\cos(1/2dx+1/2c)^2-1)^{1/2}}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.09

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = \frac{2(21\cos(dx+c)^3+32\cos(dx+c)^2+8\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)+10(i\sqrt{2}\cos(dx+c))}{(a^2d\cos(dx+c))^4+2a^2d\cos(dx+c)^3+a^2d\cos(dx+c)^2}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out]
$$\frac{-1/6*(2*(21*\cos(d*x+c)^3+32*\cos(d*x+c)^2+8*\cos(d*x+c)-2)*\sqrt{\cos(d*x+c)}*\sin(d*x+c)+10*(I*\sqrt{2}*\cos(d*x+c)^4+2*I*\sqrt{2}*\cos(d*x+c)^3+I*\sqrt{2}*\cos(d*x+c)^2)*\text{weierstrassPInverse}(-4,0,\cos(d*x+c)+I*\sin(d*x+c))+10*(-I*\sqrt{2}*\cos(d*x+c)^4-2*I*\sqrt{2}*\cos(d*x+c)^3-I*\sqrt{2}*\cos(d*x+c)^2)*\text{weierstrassPInverse}(-4,0,\cos(d*x+c)-I*\sin(d*x+c))+21*(-I*\sqrt{2}*\cos(d*x+c)^4-2*I*\sqrt{2}*\cos(d*x+c)^3-I*\sqrt{2}*\cos(d*x+c)^2)*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d*x+c)+I*\sin(d*x+c)))+21*(I*\sqrt{2}*\cos(d*x+c)^4+2*I*\sqrt{2}*\cos(d*x+c)^3+I*\sqrt{2}*\cos(d*x+c)^2)*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d*x+c)-I*\sin(d*x+c)))}{(a^2*d*\cos(d*x+c))^4+2*a^2*d*\cos(d*x+c)^3+a^2*d*\cos(d*x+c)^2}$$

Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = \frac{\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)+2\cos^{\frac{7}{2}}(c+dx)+\cos^{\frac{5}{2}}(c+dx)} dx}{a^2}$$

[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)

[Out] Integral(1/(cos(c+d*x)**(9/2)+2*cos(c+d*x)**(7/2)+cos(c+d*x)**(5/2)), x)/a**2

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = \int \frac{1}{(a\cos(dx+c)+a)^2 \cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = \int \frac{1}{(a\cos(dx+c)+a)^2 \cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2} dx = \int \frac{1}{\cos(c+dx)^{\frac{5}{2}}(a+a\cos(c+dx))^2} dx$$

[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2), x)

[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^2), x)

$$3.189 \quad \int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx$$

Optimal result	2545
Rubi [A] (verified)	2545
Mathematica [C] (verified)	2548
Maple [A] (verified)	2548
Fricas [C] (verification not implemented)	2549
Sympy [F(-1)]	2549
Maxima [F]	2550
Giac [F]	2550
Mupad [F(-1)]	2550

Optimal result

Integrand size = 23, antiderivative size = 207

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{231E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} - \frac{21 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d}$$

$$- \frac{21\sqrt{\cos(c+dx)} \sin(c+dx)}{2a^3d} + \frac{77 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{10a^3d}$$

$$- \frac{\cos^{\frac{9}{2}}(c+dx) \sin(c+dx)}{5d(a+a \cos(c+dx))^3} - \frac{4 \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{5ad(a+a \cos(c+dx))^2}$$

$$- \frac{63 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{10d(a^3+a^3 \cos(c+dx))}$$

```
[Out] 231/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-21/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+77/10*cos(d*x+c)^(3/2)*sin(d*x+c)/a^3/d-1/5*cos(d*x+c)^(9/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-4/5*cos(d*x+c)^(7/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-63/10*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))-21/2*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {2844, 3056, 2827, 2715, 2720, 2719}

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx = -\frac{21 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d} + \frac{231E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d}$$

$$- \frac{63 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{10d(a^3 \cos(c+dx) + a^3)} + \frac{77 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{10a^3d}$$

$$- \frac{21 \sin(c+dx) \sqrt{\cos(c+dx)}}{2a^3d}$$

$$- \frac{\sin(c+dx) \cos^{\frac{9}{2}}(c+dx)}{5d(a \cos(c+dx) + a)^3} - \frac{4 \sin(c+dx) \cos^{\frac{7}{2}}(c+dx)}{5ad(a \cos(c+dx) + a)^2}$$

[In] Int[Cos[c + d*x]^(11/2)/(a + a*cos[c + d*x])^3,x]

[Out] (231*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (21*EllipticF[(c + d*x)/2, 2])/(2*a^3*d) - (21*sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) + (77*cos[c + d*x]^(3/2)*Sin[c + d*x])/(10*a^3*d) - (Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*cos[c + d*x])^3) - (4*cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*a*d*(a + a*cos[c + d*x])^2) - (63*cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*cos[c + d*x]))

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*sin[c + d*x])^(n-1)/(d*n)), x] + Dist[b^2*((n-1)/n), Int[(b*sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2844

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*sin[e

+ f*x]]^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(\frac{9a}{2}-\frac{15}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2} dx}{5a^2} \\
 &= -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(42a^2-\frac{105}{2}a^2\cos(c+dx)\right)}{a+a\cos(c+dx)} dx}{15a^4} \\
 &= -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} \\
 &\quad - \frac{63\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} - \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{945a^3}{4}-\frac{1155}{4}a^3\cos(c+dx)\right) dx}{15a^6} \\
 &= -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} \\
 &\quad - \frac{63\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} - \frac{63\int \cos^{\frac{3}{2}}(c+dx) dx}{4a^3} + \frac{77\int \cos^{\frac{5}{2}}(c+dx) dx}{4a^3} \\
 &= -\frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} \\
 &\quad - \frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} \\
 &\quad - \frac{63\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} - \frac{21\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{4a^3} + \frac{231\int \sqrt{\cos(c+dx)} dx}{20a^3}
 \end{aligned}$$

$$= \frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{21\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} - \frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d}$$

$$+ \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} - \frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}$$

$$- \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{63\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10d(a^3+a^3\cos(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.85

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$$

$$= \frac{\sqrt{\cos(c+dx)}\operatorname{csc}(c+dx)\left(-((614+2995\cos(c+dx))-766\cos(2(c+dx))-1139\cos(3(c+dx))+290\cos(4(c+dx)))-10\cos(5(c+dx))+\cos(6(c+dx))\right)+127\cos(7(c+dx))+7040\cos(8(c+dx))+1680\operatorname{Hypergeometric2F1}\left[\frac{1}{4},\frac{1}{2},\frac{5}{4},\cos^2(c+dx)\right]\sqrt{\sin^2(c+dx)}+7040\cos(c+dx)\operatorname{Hypergeometric2F1}\left[\frac{3}{4},\frac{7}{2},\frac{7}{4},\cos^2(c+dx)\right]\sqrt{\sin^2(c+dx)}\right)}{160a^3d}$$

```
[In] Integrate[Cos[c + d*x]^(11/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-((614 + 2995*Cos[c + d*x] - 766*Cos[2*(c + d*x)] - 1139*Cos[3*(c + d*x)] + 290*Cos[4*(c + d*x)] + 127*Cos[5*(c + d*x)] - 10*Cos[6*(c + d*x)] + Cos[7*(c + d*x)])*Csc[c + d*x]^4 + 1680*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 7040*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(160*a^3*d)
```

Maple [A] (verified)

Time = 11.21 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(64\left(\cos^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-288\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-76\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-210\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)}{20a^3}$

```
[In] int(cos(d*x+c)^(11/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1/2*d*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*cos(1/2*d*x+1/2*c)^3)
```


$\frac{d^{2x+1/2}c}{a^3(-2\sin(1/2d^{2x+1/2}c)^4+\sin(1/2d^{2x+1/2}c)^2)^{1/2}}/c$
 $\frac{\cos(1/2d^{2x+1/2}c)^5/\sin(1/2d^{2x+1/2}c)}{(2\cos(1/2d^{2x+1/2}c)^2-1)^{1/2}}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.76

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$$

$$= \frac{2(4\cos(dx+c)^4 - 8\cos(dx+c)^3 - 147\cos(dx+c)^2 - 238\cos(dx+c) - 105)\sqrt{\cos(dx+c)}\sin(dx+c) - 105(-I\sqrt{2})\cos(dx+c)^3 - 3I\sqrt{2}\cos(dx+c)^2 - 3I\sqrt{2}\cos(dx+c) - I\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + I\sin(dx+c)) - 105(I\sqrt{2})\cos(dx+c)^3 + 3I\sqrt{2}\cos(dx+c)^2 + 3I\sqrt{2}\cos(dx+c) + I\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) - I\sin(dx+c)) - 231(-I\sqrt{2})\cos(dx+c)^3 - 3I\sqrt{2}\cos(dx+c)^2 - 3I\sqrt{2}\cos(dx+c) - I\sqrt{2})\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + I\sin(dx+c))) - 231(I\sqrt{2})\cos(dx+c)^3 + 3I\sqrt{2}\cos(dx+c)^2 + 3I\sqrt{2}\cos(dx+c) + I\sqrt{2})\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - I\sin(dx+c)))}{(a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2 + 3a^3d\cos(dx+c) + a^3d)}$$

[In] integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/20*(2*(4*cos(d*x + c)^4 - 8*cos(d*x + c)^3 - 147*cos(d*x + c)^2 - 238*cos(d*x + c) - 105)*sqrt(cos(d*x + c))*sin(d*x + c) - 105*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 105*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 231*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 231*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(11/2)/(a+a*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{11}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{11}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(11/2)/(a*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\cos^{\frac{11}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{11}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(11/2)/(a*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{11}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(c + dx)^{11/2}}{(a + a \cos(c + dx))^3} dx$$

[In] int(cos(c + d*x)^(11/2)/(a + a*cos(c + d*x))^3,x)

[Out] int(cos(c + d*x)^(11/2)/(a + a*cos(c + d*x))^3, x)

$$3.190 \quad \int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx$$

Optimal result	2551
Rubi [A] (verified)	2551
Mathematica [C] (verified)	2554
Maple [A] (verified)	2554
Fricas [C] (verification not implemented)	2555
Sympy [F(-1)]	2555
Maxima [F]	2555
Giac [F]	2556
Mupad [F(-1)]	2556

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{119E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} + \frac{11 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d}$$

$$+ \frac{11\sqrt{\cos(c+dx)} \sin(c+dx)}{2a^3d} - \frac{\cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{5d(a+a \cos(c+dx))^3}$$

$$- \frac{2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3ad(a+a \cos(c+dx))^2} - \frac{119 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{30d(a^3+a^3 \cos(c+dx))}$$

[Out] $-119/10*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^3/d+11/2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^3/d-1/5*\cos(d*x+c)^{(7/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^3-2/3*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^2-119/30*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a^3+a^3*\cos(d*x+c))+11/2*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^3/d$

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2844, 3056, 2827, 2719, 2715, 2720}

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{11 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d} - \frac{119E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d}$$

$$- \frac{119 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{30d(a^3 \cos(c+dx) + a^3)} + \frac{11 \sin(c+dx) \sqrt{\cos(c+dx)}}{2a^3d}$$

$$- \frac{\sin(c+dx) \cos^{\frac{7}{2}}(c+dx)}{5d(a \cos(c+dx) + a)^3} - \frac{2 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{3ad(a \cos(c+dx) + a)^2}$$

[In] Int[Cos[c + d*x]^(9/2)/(a + a*cos[c + d*x])^3,x]

[Out] (-119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2])/(2*a^3*d) + (11*sqrt[Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - (Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*cos[c + d*x])^3) - (2*cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a + a*cos[c + d*x])^2) - (119*cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*cos[c + d*x]))

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m +

1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7a}{2} - \frac{13}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2} dx}{5a^2} \\
&= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(25a^2 - \frac{69}{2}a^2\cos(c+dx)\right)}{a+a\cos(c+dx)} dx}{15a^4} \\
&= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} \\
&\quad - \frac{119\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30d(a^3+a^3\cos(c+dx))} - \frac{\int \sqrt{\cos(c+dx)}\left(\frac{357a^3}{4} - \frac{495}{4}a^3\cos(c+dx)\right) dx}{15a^6} \\
&= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} \\
&\quad - \frac{119\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30d(a^3+a^3\cos(c+dx))} - \frac{119\int \sqrt{\cos(c+dx)} dx}{20a^3} + \frac{33\int \cos^{\frac{3}{2}}(c+dx) dx}{4a^3} \\
&= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
&\quad - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{119\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30d(a^3+a^3\cos(c+dx))} + \frac{11\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{4a^3} \\
&= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} + \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} \\
&\quad - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{119\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30d(a^3+a^3\cos(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$$

$$= \frac{\sqrt{\cos(c+dx)} \csc(c+dx) \left((511 + 2260 \cos(c+dx) - 559 \cos(2(c+dx)) - 910 \cos(3(c+dx)) + 245 \cos(4(c+dx))) \right)}{(240 a^3 d)}$$

[In] Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^3,x]

[Out] (Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((511 + 2260*Cos[c + d*x] - 559*Cos[2*(c + d*x)] - 910*Cos[3*(c + d*x)] + 245*Cos[4*(c + d*x)] + 90*Cos[5*(c + d*x)] - 5*Cos[6*(c + d*x)])*Csc[c + d*x]^4 - 1320*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 5440*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(240*a^3*d)

Maple [A] (verified)

Time = 10.45 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(160\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+468\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60a^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}\right)}{60a^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

[In] int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.96

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$2 (20 \cos(dx + c)^3 + 237 \cos(dx + c)^2 + 376 \cos(dx + c) + 165) \sqrt{\cos(dx + c)} \sin(dx + c) - 165 (i \sqrt{2})$$

[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/60*(2*(20*cos(d*x + c)^3 + 237*cos(d*x + c)^2 + 376*cos(d*x + c) + 165)*sqrt(cos(d*x + c))*sin(d*x + c) - 165*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 165*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 357*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 357*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{9}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{9}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(c + dx)^{9/2}}{(a + a \cos(c + dx))^3} dx$$

[In] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^3,x)

[Out] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^3, x)

$$3.191 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx$$

Optimal result	2557
Rubi [A] (verified)	2557
Mathematica [C] (verified)	2559
Maple [A] (verified)	2560
Fricas [C] (verification not implemented)	2560
Sympy [F(-1)]	2561
Maxima [F]	2561
Giac [F]	2561
Mupad [F(-1)]	2561

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{49E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} - \frac{13 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d}$$

$$- \frac{\cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d(a+a \cos(c+dx))^3} - \frac{8 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15ad(a+a \cos(c+dx))^2}$$

$$- \frac{13 \sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a^3+a^3 \cos(c+dx))}$$

[Out] 49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a^3/d-1/5*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-8/15*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-13/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2844, 3056, 2827, 2720, 2719}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{13 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d} + \frac{49E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d}$$

$$- \frac{13 \sin(c+dx) \sqrt{\cos(c+dx)}}{6d(a^3 \cos(c+dx) + a^3)}$$

$$- \frac{\sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{5d(a \cos(c+dx) + a)^3} - \frac{8 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{15ad(a \cos(c+dx) + a)^2}$$

[In] Int[Cos[c + d*x]^(7/2)/(a + a*cos[c + d*x])^3,x]

[Out] (49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*cos[c + d*x])^3) - (8*cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d*(a + a*cos[c + d*x])^2) - (13*sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2844

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3056

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)(\frac{5a}{2}-\frac{11}{2}a\cos(c+dx))}{(a+a\cos(c+dx))^2} dx}{5a^2} \\
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)}(12a^2-\frac{41}{2}a^2\cos(c+dx))}{a+a\cos(c+dx)} dx}{15a^4} \\
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
&\quad - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a^3+a^3\cos(c+dx))} - \frac{\int \frac{\frac{65a^3}{4}-\frac{147}{4}a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{15a^6} \\
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
&\quad - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a^3+a^3\cos(c+dx))} - \frac{13\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{12a^3} + \frac{49\int \sqrt{\cos(c+dx)} dx}{20a^3} \\
&= \frac{49E(\frac{1}{2}(c+dx)|2)}{10a^3d} - \frac{13\text{EllipticF}(\frac{1}{2}(c+dx),2)}{6a^3d} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
&\quad - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a^3+a^3\cos(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$$

$$\sqrt{\cos(c+dx)}\csc(c+dx)\left(-((241+860\cos(c+dx))-164\cos(2(c+dx))-410\cos(3(c+dx))+115\cos(4(c+dx)))\right)$$

[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^3,x]

[Out] (Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(-((241 + 860*Cos[c + d*x] - 164*Cos[2*(c + d*x)] - 410*Cos[3*(c + d*x)] + 115*Cos[4*(c + d*x)] + 30*Cos[5*(c + d*x)])*Csc[c + d*x]^4) + 520*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 2240*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(240*a^3*d)

Maple [A] (verified)

Time = 10.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(348\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{60a^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

```
[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{2(87\cos(dx+c)^2+146\cos(dx+c)+65)\sqrt{\cos(dx+c)}\sin(dx+c)+65(-i\sqrt{2}\cos(dx+c))^3-3i\sqrt{2}\cos(dx+c)}{(a+a\cos(c+dx))^3}$$

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(2*(87*cos(d*x+c)^2+146*cos(d*x+c)+65)*sqrt(cos(d*x+c))*sin(d*x+c)+65*(-I*sqrt(2)*cos(d*x+c)^3-3*I*sqrt(2)*cos(d*x+c)^2-3*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c))+65*(I*sqrt(2)*cos(d*x+c)^3+3*I*sqrt(2)*cos(d*x+c)^2+3*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))+147*(-I*sqrt(2)*cos(d*x+c)^3-3*I*sqrt(2)*cos(d*x+c)^2-3*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c)))+147*(I*sqrt(2)*cos(d*x+c)^3+3*I*sqrt(2)*cos(d*x+c)^2+3*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))))/(a^3*d*cos(d*x+c)^3+3*a^3*d*cos(d*x+c)^2+3*a^3*d*cos(d*x+c)+a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(c + dx)^{7/2}}{(a + a \cos(c + dx))^3} dx$$

[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^3,x)

[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^3, x)

$$3.192 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx$$

Optimal result	2562
Rubi [A] (verified)	2562
Mathematica [C] (verified)	2564
Maple [A] (verified)	2565
Fricas [C] (verification not implemented)	2565
Sympy [F(-1)]	2566
Maxima [F]	2566
Giac [F]	2566
Mupad [F(-1)]	2567

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{9E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d}$$

$$-\frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d(a+a \cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)} \sin(c+dx)}{5ad(a+a \cos(c+dx))^2}$$

$$+ \frac{9\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3 \cos(c+dx))}$$

[Out] -9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a^3/d-1/5*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-2/5*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^2+9/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2844, 3056, 3057, 2827, 2720, 2719}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d} - \frac{9E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d}$$

$$+ \frac{9 \sin(c+dx) \sqrt{\cos(c+dx)}}{10d(a^3 \cos(c+dx) + a^3)} - \frac{\sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{5d(a \cos(c+dx) + a)^3}$$

$$- \frac{2 \sin(c+dx) \sqrt{\cos(c+dx)}}{5ad(a \cos(c+dx) + a)^2}$$

[In] Int[Cos[c + d*x]^(5/2)/(a + a*cos[c + d*x])^3,x]

[Out] (-9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^3*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*cos[c + d*x])^3) - (2* Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*cos[c + d*x])^2) + (9*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2844

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3056

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3057

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2} - \frac{9}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2} dx}{5a^2} \\
&= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{3a^2 - \frac{21}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} dx}{15a^4} \\
&= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} \\
&\quad + \frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} - \frac{\int \frac{-\frac{15a^3}{4} + \frac{27}{4}a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{15a^6} \\
&= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} \\
&\quad + \frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{4a^3} - \frac{9\int \sqrt{\cos(c+dx)} dx}{20a^3} \\
&= -\frac{9E\left(\frac{1}{2}(c+dx)|2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
&\quad - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{10d(a^3+a^3\cos(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.02 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx \\
&= \frac{\sqrt{\cos(c+dx)}\csc(c+dx)\left((259+120\cos(c+dx))+84\cos(2(c+dx))-280\cos(3(c+dx))+105\cos(4(c+dx))\right)}{(a+a\cos(c+dx))^3}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*cos[c + d*x])^3,x]

[Out] (Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((259 + 120*Cos[c + d*x] + 84*Cos[2*(c + d*x)] - 280*Cos[3*(c + d*x)] + 105*Cos[4*(c + d*x)])*Csc[c + d*x]^4 - 280*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 960*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(560*a^3*d)

Maple [A] (verified)

Time = 9.68 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{20a^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$2 \left(9 \cos(dx + c)^2 + 12 \cos(dx + c) + 5 \right) \sqrt{\cos(dx + c)} \sin(dx + c) - 5 \left(i \sqrt{2} \cos(dx + c)^3 + 3i \sqrt{2} \cos(dx + c) \right)$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/20*(2*(9*cos(d*x + c)^2 + 12*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))

$c) - I \sin(dx + c) - 9(I\sqrt{2}\cos(dx + c)^3 + 3I\sqrt{2}\cos(dx + c)^2 + 3I\sqrt{2}\cos(dx + c) + I\sqrt{2}) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + I\sin(dx + c))) - 9(-I\sqrt{2}\cos(dx + c)^3 - 3I\sqrt{2}\cos(dx + c)^2 - 3I\sqrt{2}\cos(dx + c) - I\sqrt{2}) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - I\sin(dx + c))) / (a^3 d \cos(dx + c)^3 + 3a^3 d \cos(dx + c)^2 + 3a^3 d \cos(dx + c) + a^3 d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(c + dx)^{5/2}}{(a + a \cos(c + dx))^3} dx$$

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^3, x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^3, x)
```

$$3.193 \quad \int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^3} dx$$

Optimal result	2568
Rubi [A] (verified)	2568
Mathematica [C] (verified)	2570
Maple [A] (verified)	2571
Fricas [C] (verification not implemented)	2571
Sympy [F(-1)]	2572
Maxima [F]	2572
Giac [F]	2572
Mupad [F(-1)]	2572

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d} \\ - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a \cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)} \sin(c+dx)}{15ad(a+a \cos(c+dx))^2} \\ + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3 \cos(c+dx))}$$

[Out] -1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a^3/d-1/5*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^3+4/15*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^2+1/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2844, 3057, 2827, 2720, 2719}

$$\int \frac{\cos^3(c+dx)}{(a+a \cos(c+dx))^3} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d} - \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} \\ + \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{10d(a^3 \cos(c+dx) + a^3)} + \frac{4 \sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a \cos(c+dx) + a)^2} \\ - \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a \cos(c+dx) + a)^3}$$

[In] Int[Cos[c + d*x]^(3/2)/(a + a*cos[c + d*x])^3,x]

[Out] $-1/10 \text{EllipticE}[(c + dx)/2, 2]/(a^3 d) + \text{EllipticF}[(c + dx)/2, 2]/(6 a^3 d) - (\text{Sqrt}[\text{Cos}[c + dx]] \text{Sin}[c + dx])/(5 d (a + a \text{Cos}[c + dx])^3) + (4 \text{Sqrt}[\text{Cos}[c + dx]] \text{Sin}[c + dx])/(15 a d (a + a \text{Cos}[c + dx])^2) + (\text{Sqrt}[\text{Cos}[c + dx]] \text{Sin}[c + dx])/(10 d (a^3 + a^3 \text{Cos}[c + dx]))$

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2844

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\frac{a}{2} - \frac{7}{2}a \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx}{5a^2} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)} \sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\int \frac{-\frac{a^2}{2} - 2a^2 \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} dx}{15a^4} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)} \sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
&\quad + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} - \frac{\int \frac{-\frac{5a^3}{4} + \frac{3}{4}a^3 \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{15a^6} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)} \sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
&\quad + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} - \frac{\int \sqrt{\cos(c+dx)} dx}{20a^3} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{12a^3} \\
&= -\frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
&\quad + \frac{4\sqrt{\cos(c+dx)} \sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3\cos(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx = \frac{\sqrt{\cos(c+dx)} \csc(c+dx) \left((497 - 1160 \cos(c+dx) + 812 \cos(2(c+dx)) - 280 \cos(3(c+dx))) + 35 \cos(4(c+dx)) \right)}{(a+a\cos(c+dx))^3}$$

[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]

[Out] -1/1680*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*((497 - 1160*Cos[c + d*x] + 812*Cos[2*(c + d*x)] - 280*Cos[3*(c + d*x)] + 35*Cos[4*(c + d*x)])*Csc[c + d*x]^4 + 280*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + 320*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(a^3*d)

Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{60a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\left(\sin^4\right)}}$

[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$$

$$= \frac{2(3\cos(dx+c)^2+14\cos(dx+c)+5)\sqrt{\cos(dx+c)}\sin(dx+c)-5(i\sqrt{2}\cos(dx+c))^3+3i\sqrt{2}\cos(dx+c)}}{(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d)}$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

```
[Out] 1/60*(2*(3*cos(d*x+c)^2+14*cos(d*x+c)+5)*sqrt(cos(d*x+c))*sin(d*x+c)-5*(I*sqrt(2)*cos(d*x+c)^3+3*I*sqrt(2)*cos(d*x+c)^2+3*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c))-5*(-I*sqrt(2)*cos(d*x+c)^3-3*I*sqrt(2)*cos(d*x+c)^2-3*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))-3*(I*sqrt(2)*cos(d*x+c)^3+3*I*sqrt(2)*cos(d*x+c)^2+3*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c)))-3*(-I*sqrt(2)*cos(d*x+c)^3-3*I*sqrt(2)*cos(d*x+c)^2-3*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))))/(a^3*d*cos(d*x+c)^3+3*a^3*d*cos(d*x+c)^2+3*a^3*d*cos(d*x+c)+a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\cos(c + dx)^{\frac{3}{2}}}{(a + a \cos(c + dx))^3} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^3,x)

[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^3, x)

3.194 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^3} dx$

Optimal result	2573
Rubi [A] (verified)	2573
Mathematica [C] (verified)	2575
Maple [A] (verified)	2576
Fricas [C] (verification not implemented)	2576
Sympy [F]	2577
Maxima [F]	2577
Giac [F]	2577
Mupad [F(-1)]	2577

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^3} dx = \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a \cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{15ad(a+a \cos(c+dx))^2} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3 \cos(c+dx))}$$

```
[Out] 1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^3+1/15*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^2-1/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2843, 3057, 2827, 2720, 2719}

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^3} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d} + \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} - \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{10d(a^3 \cos(c+dx) + a^3)} + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{15ad(a \cos(c+dx) + a)^2} + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{5d(a \cos(c+dx) + a)^3}$$

[In] Int[Sqrt[Cos[c + d*x]]/(a + a*cos[c + d*x])^3,x]

[Out] EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*cos[c + d*x])^3) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*cos[c + d*x])^2) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2843

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*(m + 1) - b*d*(m + n + 1)*sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{a}{2} + \frac{3}{2}a \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx}{5a^2} \\
&= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{2a^2 + \frac{1}{2}a^2 \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} dx}{15a^4} \\
&= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
&\quad - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} + \frac{\int \frac{\frac{5a^3}{4} + \frac{3}{4}a^3 \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{15a^6} \\
&= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{15ad(a+a\cos(c+dx))^2} \\
&\quad - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)} dx}{20a^3} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{12a^3} \\
&= \frac{E(\frac{1}{2}(c+dx)|2)}{10a^3d} + \frac{\text{EllipticF}(\frac{1}{2}(c+dx), 2)}{6a^3d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
&\quad + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3\cos(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3} dx = \frac{\sqrt{\cos(c+dx)} \csc(c+dx) \left(\frac{1}{8}(-847 + 1440 \cos(c+dx) - 532 \cos(2(c+dx)) + 35 \cos(4(c+dx))) \right) \csc^4(c+dx)}{15a^3d}$$

[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^3,x]

[Out] -1/210*(Sqrt[Cos[c + d*x]]*Csc[c + d*x]*(((-847 + 1440*Cos[c + d*x] - 532*Cos[2*(c + d*x)] + 35*Cos[4*(c + d*x)])*Csc[c + d*x]^4)/8 + 35*Hypergeometric2F1[1/4, 1/2, 5/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] - 40*Cos[c + d*x]*Hypergeometric2F1[3/4, 7/2, 7/4, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(a^3*d)

Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.74

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{60a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

```
[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2))*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3} dx = \frac{2(3\cos(dx+c)^2+4\cos(dx+c)-5)\sqrt{\cos(dx+c)}\sin(dx+c)+5(i\sqrt{2}\cos(dx+c))^3+3i\sqrt{2}\cos(dx+c)}}{(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d)}$$

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(2*(3*cos(d*x+c)^2+4*cos(d*x+c)-5)*sqrt(cos(d*x+c))*sin(d*x+c)+5*(I*sqrt(2)*cos(d*x+c)^3+3*I*sqrt(2)*cos(d*x+c)^2+3*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c))+5*(-I*sqrt(2)*cos(d*x+c)^3-3*I*sqrt(2)*cos(d*x+c)^2-3*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))+3*(-I*sqrt(2)*cos(d*x+c)^3-3*I*sqrt(2)*cos(d*x+c)^2-3*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c)))+3*(I*sqrt(2)*cos(d*x+c)^3+3*I*sqrt(2)*cos(d*x+c)^2+3*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))))/(a^3*d*cos(d*x+c)^3+3*a^3*d*cos(d*x+c)^2+3*a^3*d*cos(d*x+c)+a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3} dx = \frac{\int \frac{\sqrt{\cos(c+dx)}}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1} dx}{a^3}$$

[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)

[Out] Integral(sqrt(cos(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x)/a**3

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3} dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3} dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^3} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^3,x)

[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^3, x)

$$3.195 \quad \int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^3} dx$$

Optimal result	2578
Rubi [A] (verified)	2578
Mathematica [C] (warning: unable to verify)	2580
Maple [A] (verified)	2581
Fricas [C] (verification not implemented)	2581
Sympy [F]	2582
Maxima [F]	2582
Giac [F]	2582
Mupad [F(-1)]	2583

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^3} dx = \frac{9E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d}$$

$$- \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a \cos(c+dx))^3}$$

$$- \frac{2\sqrt{\cos(c+dx)} \sin(c+dx)}{5ad(a+a \cos(c+dx))^2}$$

$$- \frac{9\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3 \cos(c+dx))}$$

[Out] 9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^3-2/5*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^2-9/10*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+a^3*cos(d*x+c))

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

= {2845, 3057, 2827, 2720, 2719}

$$\int \frac{1}{\sqrt{\cos(c+dx)(a+a\cos(c+dx))^3}} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d} + \frac{9E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} - \frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{10d(a^3\cos(c+dx)+a^3)} - \frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{5ad(a\cos(c+dx)+a)^2} - \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*cos[c + d*x])^3), x]

[Out] (9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^3*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*cos[c + d*x])^3) - (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*cos[c + d*x])^2) - (9*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{9a}{2} - \frac{3}{2}a \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} dx}{5a^2} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)} \sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{21a^2}{2} - 3a^2 \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} dx}{15a^4} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)} \sin(c+dx)}{5ad(a+a\cos(c+dx))^2} \\
&\quad - \frac{9\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} + \frac{\int \frac{\frac{15a^3}{4} + \frac{27}{4}a^3 \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx}{15a^6} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)} \sin(c+dx)}{5ad(a+a\cos(c+dx))^2} \\
&\quad - \frac{9\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3\cos(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{4a^3} + \frac{9 \int \sqrt{\cos(c+dx)} dx}{20a^3} \\
&= \frac{9E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{5d(a+a\cos(c+dx))^3} \\
&\quad - \frac{2\sqrt{\cos(c+dx)} \sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{9\sqrt{\cos(c+dx)} \sin(c+dx)}{10d(a^3+a^3\cos(c+dx))}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.94 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.96

$$\begin{aligned}
&\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} dx \\
&= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right) \csc\left(\frac{c}{2}\right) \sec\left(\frac{c}{2}\right) \left(\frac{36(3\cos(c-dx-\arctan(\tan(c))) + \cos(c+dx+\arctan(\tan(c)))) \sec(c)}{\sqrt{\sec^2(c)}} - \cos(c+dx)\right) (62 \cos\left(\frac{c}{2}\right) \sec\left(\frac{c}{2}\right) \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3)}{\dots}
\end{aligned}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*cos[c + d*x])^3),x]

[Out] (Cos[(c + d*x)/2]^6*Csc[c/2]*Sec[c/2]*((36*(3*cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sec[c])/Sqrt[Sec[c]^2 - Cos[c + d*x]*(62*cos[(c - d*x)/2] + 28*cos[(3*c + d*x)/2] + 40*cos[(c + 3*d*x)/2] + 5*cos[(5*c + 3*d*x)/2] + 9*cos[(3*c + 5*d*x)/2])*Sec[(c + d*x)/2]^5 - (80*cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]])/Sqrt[Csc[c]^2 - (72*HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2)*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(40*a^3*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.73

method	result
default	$\frac{\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{20a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2))*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1/2*c)^4+cos(1/2*d*x+1/2*c)^2+1/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} dx = \frac{2(9\cos(dx+c)^2+22\cos(dx+c)+15)\sqrt{\cos(dx+c)}\sin(dx+c)+5(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c))}{\dots}$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

```
[Out] -1/20*(2*(9*cos(d*x + c)^2 + 22*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(d
*x + c) + 5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sq
rt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2
- 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c)) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x
+ c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(I*sqrt(2)*cos(d
*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(
2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*
x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} dx = \frac{\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)+3\cos^{\frac{5}{2}}(c+dx)+3\cos^{\frac{3}{2}}(c+dx)+\sqrt{\cos(c+dx)}} dx}{a^3}$$

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(1/(cos(c + d*x)**(7/2) + 3*cos(c + d*x)**(5/2) + 3*cos(c + d*x)**(
3/2) + sqrt(cos(c + d*x))), x)/a**3
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} dx = \int \frac{1}{(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}} dx$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} dx = \int \frac{1}{(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}} dx$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} dx = \int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} dx$$

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3), x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^3), x)
```

$$3.196 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^3} dx$$

Optimal result	2584
Rubi [A] (verified)	2585
Mathematica [C] (verified)	2587
Maple [B] (verified)	2588
Fricas [C] (verification not implemented)	2588
Sympy [F]	2589
Maxima [F(-2)]	2589
Giac [F]	2589
Mupad [F(-1)]	2590

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^3} dx = -\frac{49E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} - \frac{13 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d}$$

$$+ \frac{49 \sin(c+dx)}{10a^3d \sqrt{\cos(c+dx)}}$$

$$- \frac{\sin(c+dx)}{5d \sqrt{\cos(c+dx)}(a+a \cos(c+dx))^3}$$

$$- \frac{8 \sin(c+dx)}{15ad \sqrt{\cos(c+dx)}(a+a \cos(c+dx))^2}$$

$$- \frac{13 \sin(c+dx)}{6d \sqrt{\cos(c+dx)}(a^3+a^3 \cos(c+dx))}$$

```
[Out] -49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+49/10*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)-1/5*sin(d*x+c)/d/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2)-8/15*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)-13/6*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2845, 3057, 2827, 2716, 2719, 2720}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} dx = -\frac{13 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d} - \frac{49E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} + \frac{49 \sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{13 \sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a^3 \cos(c+dx) + a^3)} - \frac{8 \sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a \cos(c+dx) + a)^2} - \frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a \cos(c+dx) + a)^3}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3), x]

[Out] (-49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) + (49*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) - (13*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x]))

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b*\sin[e + f*x]^{(m + 1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2845

$\text{Int}[(a + (b_*)*\sin[(e_*) + (f_*)*(x_*)])^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)])^{(n_*)}, x_Symbol] := \text{Simp}[b^2*\cos[e + f*x]*(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^{(n + 1)}/(a*f*(2*m + 1)*(b*c - a*d))), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^n*\text{Simp}[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{!GtQ}[n, 0] \&\& (\text{IntegerSqrt}[2*m, 2*n] || (\text{IntegerQ}[m] \&\& \text{EqQ}[c, 0]))$

Rule 3057

$\text{Int}[(a + (b_*)*\sin[(e_*) + (f_*)*(x_*)])^{(m_*)}*((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_*)])^{(n_*)}, x_Symbol] := \text{Simp}[b*(A*b - a*B)*\cos[e + f*x]*(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^{(n + 1)}/(a*f*(2*m + 1)*(b*c - a*d))), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^n*\text{Simp}[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{!GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] || \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c + dx)}{5d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^3} + \frac{\int \frac{\frac{11a}{2} - \frac{5}{2}a\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^2} dx}{5a^2} \\ &= -\frac{\sin(c + dx)}{5d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^3} \\ &\quad - \frac{8\sin(c + dx)}{15ad\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^2} + \frac{\int \frac{\frac{41a^2}{2} - 12a^2\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))} dx}{15a^4} \\ &= -\frac{\sin(c + dx)}{5d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^3} - \frac{8\sin(c + dx)}{15ad\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^2} \\ &\quad - \frac{13\sin(c + dx)}{6d\sqrt{\cos(c + dx)}(a^3 + a^3\cos(c + dx))} + \frac{\int \frac{\frac{147a^3}{4} - \frac{65}{4}a^3\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx}{15a^6} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} \\
&\quad - \frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a^3+a^3\cos(c+dx))} - \frac{13\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{12a^3} + \frac{49\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)}dx}{20a^3} \\
&= -\frac{13\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} \\
&\quad - \frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} \\
&\quad - \frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a^3+a^3\cos(c+dx))} - \frac{49\int\sqrt{\cos(c+dx)}dx}{20a^3} \\
&= -\frac{49E\left(\frac{1}{2}(c+dx)\mid 2\right)}{10a^3d} - \frac{13\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d} \\
&\quad + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} \\
&\quad - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} - \frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a^3+a^3\cos(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.44 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.01

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} dx$$

$$= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right) \left(-\frac{4i\sqrt{2}e^{-i(c+dx)} \left(147(1+e^{2i(c+dx)}) + 147(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}} \right) \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) - 65e^{i(c+dx)}}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{\dots}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3), x]

[Out] (Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(147*(1 + E^((2*I)*(c + d*x)))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 1243*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2] + 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(16*d*Sqrt[Cos[c + d*x]])))/(15*a^3*(1 + Cos[c + d*x])^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(213) = 426$.

Time = 4.15 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.07

method	result
default	$-\frac{-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(65F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-147E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{\dots}$

[In] `int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -1/60*(-2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(-2 \\ & * \sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(65*\text{EllipticF}(\cos(1/2*d*x \\ & +1/2*c),2^{(1/2)})-147*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*\cos(1/2*d*x+1/2 \\ & *c)*\sin(1/2*d*x+1/2*c)^4+4*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2* \\ & c)^2-1)^{(1/2)}*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(65*\text{Elli \\ & pticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-147*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})) \\ &)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)-2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2 \\ & * \sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^ \\ & 2)^{(1/2)}*(65*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-147*\text{EllipticE}(\cos(1/2*d* \\ & x+1/2*c),2^{(1/2)}))*\cos(1/2*d*x+1/2*c)+588*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2* \\ & d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^8-1634*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(\\ & 1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^6+1488*(-2*\sin(1/2*d*x+1/2*c)^4+ \\ & \sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^4-439*(-2*\sin(1/2*d*x+1/2*c) \\ & ^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2)/a^3/\cos(1/2*d*x+1/2*c) \\ & ^5/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/ \\ & (2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.18

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} dx$$

$$= \frac{2(147\cos(dx+c)^3+376\cos(dx+c)^2+295\cos(dx+c)+60)\sqrt{\cos(dx+c)}\sin(dx+c)-65(-i\sqrt{2}c$$

[In] `integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")`

[Out]
$$\begin{aligned} & 1/60*(2*(147*\cos(dx+c)^3+376*\cos(dx+c)^2+295*\cos(dx+c)+60)*s \\ & \text{qrt}(\cos(dx+c))*\sin(dx+c)-65*(-I*\text{sqrt}(2)*\cos(dx+c)^4-3*I*\text{sqrt}(2) \\ &)*\cos(dx+c)^3-3*I*\text{sqrt}(2)*\cos(dx+c)^2-I*\text{sqrt}(2)*\cos(dx+c))*\text{wei} \end{aligned}$$


```
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 65*(I*sqrt(2)*cos(
d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + I*sq
rt(2)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
)) - 147*(I*sqrt(2)*cos(d*x + c)^4 + 3*I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(
2)*cos(d*x + c)^2 + I*sqrt(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 147*(-I*sqrt(2)*cos(d*
x + c)^4 - 3*I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - I*sqrt
(2)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} dx = \frac{\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)+3\cos^{\frac{7}{2}}(c+dx)+3\cos^{\frac{5}{2}}(c+dx)+\cos^{\frac{3}{2}}(c+dx)}{a^3} dx$$

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(1/(cos(c + d*x)**(9/2) + 3*cos(c + d*x)**(7/2) + 3*cos(c + d*x)**(
5/2) + cos(c + d*x)**(3/2)), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} dx = \int \frac{1}{(a\cos(dx+c)+a)^3 \cos(dx+c)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3} dx = \int \frac{1}{\cos(c + dx)^{\frac{3}{2}}(a + a \cos(c + dx))^3} dx$$

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3), x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^3), x)
```

$$3.197 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^3} dx$$

Optimal result	2591
Rubi [A] (verified)	2592
Mathematica [C] (verified)	2594
Maple [A] (verified)	2595
Fricas [C] (verification not implemented)	2595
Sympy [F]	2596
Maxima [F]	2596
Giac [F]	2596
Mupad [F(-1)]	2597

Optimal result

Integrand size = 23, antiderivative size = 207

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^3} dx = \frac{119E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} + \frac{11 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d}$$

$$+ \frac{11 \sin(c+dx)}{2a^3d \cos^{\frac{3}{2}}(c+dx)} - \frac{119 \sin(c+dx)}{10a^3d \sqrt{\cos(c+dx)}}$$

$$- \frac{\sin(c+dx)}{5d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^3}$$

$$- \frac{2 \sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^2}$$

$$- \frac{119 \sin(c+dx)}{30d \cos^{\frac{3}{2}}(c+dx)(a^3+a^3 \cos(c+dx))}$$

```
[Out] 119/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/a^3/d+11/2*sin(d*x+c)/a^3/d/cos(d*x+c)^(3/2)-1/5*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3-2/3*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2-119/30*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a^3+a^3*cos(d*x+c))-119/10*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2845, 3057, 2827, 2716, 2720, 2719}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3} dx = \frac{11 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d} + \frac{119E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{10a^3d} - \frac{119 \sin(c+dx)}{30d \cos^{\frac{3}{2}}(c+dx)(a^3 \cos(c+dx) + a^3)} + \frac{11 \sin(c+dx)}{2a^3d \cos^{\frac{3}{2}}(c+dx)} - \frac{119 \sin(c+dx)}{10a^3d \sqrt{\cos(c+dx)}} - \frac{2 \sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)(a \cos(c+dx) + a)^2} - \frac{\sin(c+dx)}{5d \cos^{\frac{3}{2}}(c+dx)(a \cos(c+dx) + a)^3}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3), x]

[Out] (119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2])/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Cos[c + d*x]^(3/2)) - (119*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*Cos[c + d*x]))

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sine[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sine[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sine[e + f*x])^m*((c + d*Sine[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sine[e + f*x])^(m + 1)*(c + d*Sine[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sine[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sine[e + f*x])^m*((c + d*Sine[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sine[e + f*x])^(m + 1)*(c + d*Sine[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sine[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sin(c + dx)}{5d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3} + \frac{\int \frac{\frac{13a}{2} - \frac{7}{2}a \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^2} dx}{5a^2} \\
 &= -\frac{\sin(c + dx)}{5d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3} \\
 &\quad - \frac{2 \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2} + \frac{\int \frac{\frac{69a^2}{2} - 25a^2 \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))} dx}{15a^4} \\
 &= -\frac{\sin(c + dx)}{5d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^3} - \frac{2 \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^2} \\
 &\quad - \frac{119 \sin(c + dx)}{30d \cos^{\frac{3}{2}}(c + dx)(a^3 + a^3 \cos(c + dx))} + \frac{\int \frac{\frac{495a^3}{4} - \frac{357}{4}a^3 \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} dx}{15a^6}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{5d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^3} - \frac{2 \sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^2} \\
&\quad - \frac{119 \sin(c+dx)}{30d \cos^{\frac{3}{2}}(c+dx)(a^3+a^3 \cos(c+dx))} - \frac{119 \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)} dx}{20a^3} + \frac{33 \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)} dx}{4a^3} \\
&= \frac{11 \sin(c+dx)}{2a^3 d \cos^{\frac{3}{2}}(c+dx)} - \frac{119 \sin(c+dx)}{10a^3 d \sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^3} \\
&\quad - \frac{2 \sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^2} - \frac{119 \sin(c+dx)}{30d \cos^{\frac{3}{2}}(c+dx)(a^3+a^3 \cos(c+dx))} \\
&\quad + \frac{11 \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{4a^3} + \frac{119 \int \sqrt{\cos(c+dx)} dx}{20a^3} \\
&= \frac{119E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3 d} + \frac{11 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3 d} + \frac{11 \sin(c+dx)}{2a^3 d \cos^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{119 \sin(c+dx)}{10a^3 d \sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^3} \\
&\quad - \frac{2 \sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^2} - \frac{119 \sin(c+dx)}{30d \cos^{\frac{3}{2}}(c+dx)(a^3+a^3 \cos(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.83 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.90

$$\begin{aligned}
&\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^3} dx \\
&= \frac{\cos^6\left(\frac{1}{2}(c+dx)\right) \left(\frac{4i\sqrt{2}e^{-i(c+dx)} \left(119(1+e^{2i(c+dx)})+119(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right) \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) - 55e^{i(c+dx)}}{d(-1+e^{2ic})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}} \right)}{\cos^6\left(\frac{1}{2}(c+dx)\right)}
\end{aligned}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3), x]

[Out] (Cos[(c + d*x)/2]^6*(((4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x)))) + 119*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) - 55*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) - ((5134*Cos[(c - d*x)/2] + 4148*Cos[(3*c + d*x)/2] + 4664*Cos[(c + 3*d*x)/2] + 2476*Cos[(5*c + 3*d*x)/2] + 3340*Cos[(3*c + 5*d*x)/2] + 944*Cos[(7*c + 5*d*x)/2] + 1620*Cos[(5*c + 7*d*x)/2] + 165*Cos[(9*c + 7*d*x)/2] + 357*Cos[(7*c + 9*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(96*d*Cos[c + d*x]^(3/2)))/(5*a^3*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2}))) + 1}(\sin^2(\frac{dx}{2} + \frac{c}{2}))}{\left(\frac{\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}}{5 \cos(\frac{dx}{2} + \frac{c}{2})^5} + \frac{32\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}}{15 \cos(\frac{dx}{2} + \frac{c}{2})^3} + \frac{118\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}}{\dots} \right)}$

[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out]
$$-1/4*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/a^3*(1/5*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)^5+32/15*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)^3+118/5*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)-128/5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+238/5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-4/3*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^2+48*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)/(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.00

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3} dx = \frac{2(357\cos(dx+c)^4 + 906\cos(dx+c)^3 + 695\cos(dx+c)^2 + 120\cos(dx+c) - 20)\sqrt{\cos(dx+c)}\sin(dx+c)}{\dots}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out]
$$-1/60*(2*(357*\cos(d*x+c)^4 + 906*\cos(d*x+c)^3 + 695*\cos(d*x+c)^2 + 120*\cos(d*x+c) - 20)*\sqrt{\cos(d*x+c)}*\sin(d*x+c) + 165*(I*\sqrt{2}*\cos(d*x+c)^5 + 3*I*\sqrt{2}*\cos(d*x+c)^4 + 3*I*\sqrt{2}*\cos(d*x+c)^3 + I*\sqrt{2}*\cos(d*x+c)^2)*\text{weierstrassPInverse}(-4, 0, \cos(d*x+c) + I*\sin(d*x+c)) + 165*(-I*\sqrt{2}*\cos(d*x+c)^5 - 3*I*\sqrt{2}*\cos(d*x+c)^4 - 3*I*\sqrt{2}*\cos(d*x+c)^3 - I*\sqrt{2}*\cos(d*x+c)^2)*\text{weierstrassPInverse}(-4, 0,$$

$\cos(dx + c) - I\sin(dx + c)) + 357*(-I\sqrt{2})\cos(dx + c)^5 - 3*I\sqrt{2}\cos(dx + c)^4 - 3*I\sqrt{2}\cos(dx + c)^3 - I\sqrt{2}\cos(dx + c)^2)*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I\sin(dx + c))) + 357*(I\sqrt{2})\cos(dx + c)^5 + 3*I\sqrt{2}\cos(dx + c)^4 + 3*I\sqrt{2}\cos(dx + c)^3 + I\sqrt{2}\cos(dx + c)^2)*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I\sin(dx + c))) / (a^3*d*\cos(dx + c)^5 + 3*a^3*d*\cos(dx + c)^4 + 3*a^3*d*\cos(dx + c)^3 + a^3*d*\cos(dx + c)^2)$

Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^3} dx = \frac{\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)+3\cos^{\frac{9}{2}}(c+dx)+3\cos^{\frac{7}{2}}(c+dx)+\cos^{\frac{5}{2}}(c+dx)} dx}{a^3}$$

[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)

[Out] Integral(1/(cos(c + d*x)**(11/2) + 3*cos(c + d*x)**(9/2) + 3*cos(c + d*x)**(7/2) + cos(c + d*x)**(5/2)), x)/a**3

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^3} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^3} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^3} dx = \int \frac{1}{\cos(c + dx)^{\frac{5}{2}}(a + a \cos(c + dx))^3} dx$$

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3), x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^3), x)
```

3.198 $\int \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx$

Optimal result	2598
Rubi [A] (verified)	2598
Mathematica [A] (verified)	2600
Maple [A] (verified)	2601
Fricas [A] (verification not implemented)	2601
Sympy [F(-1)]	2601
Maxima [B] (verification not implemented)	2602
Giac [F]	2603
Mupad [F(-1)]	2604

Optimal result

Integrand size = 25, antiderivative size = 154

$$\int \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{5\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{8d} + \frac{5a\sqrt{\cos(c+dx)} \sin(c+dx)}{8d\sqrt{a+a \cos(c+dx)}} + \frac{5a \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d\sqrt{a+a \cos(c+dx)}} + \frac{a \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a \cos(c+dx)}}$$

[Out] 5/8*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+5/12*a*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+1/3*a*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+5/8*a*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used

= {2849, 2853, 222}

$$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} dx = \frac{5\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{8d} + \frac{a \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a \cos(c+dx)+a}} + \frac{5a \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a \cos(c+dx)+a}} + \frac{5a \sin(c+dx) \sqrt{\cos(c+dx)}}{8d\sqrt{a \cos(c+dx)+a}}$$

[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]],x]

[Out] (5*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/(8*d) + (5*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (5*a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2849

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rubi steps

$$\text{integral} = \frac{a \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a \cos(c+dx)}} + \frac{5}{6} \int \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} dx$$

$$\begin{aligned}
&= \frac{5a \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{5}{8} \int \sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)} dx \\
&= \frac{5a\sqrt{\cos(c+dx)} \sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{a \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{16} \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{5a\sqrt{\cos(c+dx)} \sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{a \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{5 \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} \\
&= \frac{5\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{5a\sqrt{\cos(c+dx)} \sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{5a \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\begin{aligned}
&\int \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a\cos(c+dx)} dx \\
&= \frac{\sqrt{a(1+\cos(c+dx))} \sec\left(\frac{1}{2}(c+dx)\right) \left(15\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\sqrt{\cos(c+dx)}(14 \sin\left(\frac{1}{2}(c+dx)\right) + 3 \sin\left(\frac{3}{2}(c+dx)\right) + 2 \sin\left(\frac{5}{2}(c+dx)\right))\right)}{48d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(14*Sin[(c + d*x)/2] + 3*Sin[(3*(c + d*x))/2] + 2*Sin[(5*(c + d*x))/2]))) / (48*d)

Maple [A] (verified)

Time = 11.51 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17

method	result
default	$\frac{\left(8 \sin(dx+c) \cos^2(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 10 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 15 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 15 \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)}{24d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] `int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{24} \frac{d \left(8 \sin(dx+c) \cos^2(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 10 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 15 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 15 \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) \right)}{(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.70

$$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} dx = \frac{\sqrt{a \cos(dx+c)+a} (8 \cos^2(dx+c) + 10 \cos(dx+c) + 15) \sqrt{\cos(dx+c)} \sin(dx+c) - 15 \sqrt{a} (\cos(dx+c) + 1) \arctan\left(\sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}}\right)}{24(d \cos(dx+c) + d)}$$

[In] `integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{24} \frac{\left(\sqrt{a \cos(dx+c)+a} (8 \cos^2(dx+c) + 10 \cos(dx+c) + 15) \sqrt{\cos(dx+c)} \sin(dx+c) - 15 \sqrt{a} (\cos(dx+c) + 1) \arctan\left(\sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}}\right) \right)}{d \cos(dx+c) + d}$$

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)`

[Out] Timed out


```

+ 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))/d

```

Giac [F]

$$\int \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \sqrt{a \cos(dx + c) + a \cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \cos(c + dx)^{5/2} \sqrt{a + a \cos(c + dx)} dx$$

```
[In] int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2), x)
```


3.199 $\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx$

Optimal result	2605
Rubi [A] (verified)	2605
Mathematica [A] (verified)	2607
Maple [A] (verified)	2607
Fricas [A] (verification not implemented)	2607
Sympy [F]	2608
Maxima [B] (verification not implemented)	2608
Giac [F]	2609
Mupad [F(-1)]	2609

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{3\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4d} + \frac{3a \sqrt{\cos(c + dx)} \sin(c + dx)}{4d \sqrt{a + a \cos(c + dx)}} + \frac{a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d \sqrt{a + a \cos(c + dx)}}$$

[Out] $\frac{3}{4} \arcsin(\sin(dx+c) \cdot a^{1/2} / (a+a \cos(dx+c))^{1/2}) \cdot a^{1/2} / d + \frac{1}{2} a \cos(dx+c)^{3/2} \sin(dx+c) / d / (a+a \cos(dx+c))^{1/2} + \frac{3}{4} a \sin(dx+c) \cos(dx+c)^{1/2} / d / (a+a \cos(dx+c))^{1/2}$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2849, 2853, 222}

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx = \frac{3\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4d} + \frac{a \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{2d \sqrt{a \cos(c + dx) + a}} + \frac{3a \sin(c + dx) \sqrt{\cos(c + dx)}}{4d \sqrt{a \cos(c + dx) + a}}$$

[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]],x]

[Out] (3*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/(4*d) + (3*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2849

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \cos(c + dx)}} + \frac{3}{4} \int \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)} dx \\
 &= \frac{3a\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a + a \cos(c + dx)}} + \frac{a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \cos(c + dx)}} + \frac{3}{8} \int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{3a\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a + a \cos(c + dx)}} + \frac{a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \cos(c + dx)}} \\
 &\quad - \frac{3 \text{Subst}\left(\int \frac{1}{\sqrt{1 - \frac{x^2}{a}}} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{4d} \\
 &= \frac{3\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{4d} + \frac{3a\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a + a \cos(c + dx)}} + \frac{a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{\sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(3\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right) + 2\sqrt{\cos(c + dx)}(2 \sin\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{3}{2}(c + dx)\right))\right)}{8d}$$

[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))) / (8*d)

Maple [A] (verified)

Time = 11.60 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.25

method	result
default	$\frac{\left(2 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 3 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 3 \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right) (\sqrt{\cos(dx+c)} \sqrt{a(1+\cos(dx+c))})}{4d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4/d*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx$$

$$= \frac{\sqrt{a \cos(dx + c) + a} (2 \cos(dx + c) + 3) \sqrt{\cos(dx + c)} \sin(dx + c) - 3 \sqrt{a} (\cos(dx + c) + 1) \arctan\left(\frac{\sqrt{a \cos(dx + c) + a}}{\sqrt{a} \sin(dx + c)}\right)}{4(d \cos(dx + c) + d)}$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/4*(sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)

Sympy [F]

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \sqrt{a(\cos(c + dx) + 1)} \cos^{\frac{3}{2}}(c + dx) dx$$

[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3/2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(96) = 192.

Time = 0.47 (sec) , antiderivative size = 1059, normalized size of antiderivative = 9.13

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct

$\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 1 + \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - 1)) / d$

Giac [F]

$$\int \cos^{3/2}(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \sqrt{a \cos(dx + c) + a \cos(dx + c)^{3/2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \cos^{3/2}(c + dx) \sqrt{a + a \cos(c + dx)} dx = \int \cos(c + dx)^{3/2} \sqrt{a + a \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2), x)

3.200 $\int \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)} dx$

Optimal result	2610
Rubi [A] (verified)	2610
Mathematica [A] (verified)	2611
Maple [A] (verified)	2612
Fricas [A] (verification not implemented)	2612
Sympy [F]	2612
Maxima [B] (verification not implemented)	2613
Giac [F]	2613
Mupad [F(-1)]	2614

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)} dx = \frac{\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{d} + \frac{a \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}}$$

[Out] arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d+a*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2849, 2853, 222}

$$\int \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)} dx = \frac{\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{d} + \frac{a \sin(c + dx) \sqrt{\cos(c + dx)}}{d \sqrt{a \cos(c + dx) + a}}$$

[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/d + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2849

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*SIN[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*SIN[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*SIN[e + f*x]]*(c + d*SIN[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*SIN[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2} \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\ &= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\ &= \frac{\sqrt{a} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\begin{aligned} &\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)} dx \\ &= \frac{\sqrt{a(1+\cos(c+dx))} \sec\left(\frac{1}{2}(c+dx)\right) \left(\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\sqrt{\cos(c+dx)} \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} \end{aligned}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]], x]

[Out] (Sqrt[a*(1 + Cos[c + d*x])] * Sec[(c + d*x)/2] * (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)

Maple [A] (verified)

Time = 11.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\left(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\cos(dx+c)}\sqrt{a(1+\cos(dx+c))}\right)}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	108

[In] `int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `1/d*(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)`

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}dx = \frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - \sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)+d}$$

[In] `integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x,algorithm="fricas")`

[Out] `-(sqrt(a)*(cos(d*x+c)+1)*arctan(sqrt(a*cos(d*x+c)+a)*sqrt(cos(d*x+c)))/(sqrt(a)*sin(d*x+c)) - sqrt(a*cos(d*x+c)+a)*sqrt(cos(d*x+c))*sin(d*x+c))/(d*cos(d*x+c)+d)`

Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}dx = \int \sqrt{a(\cos(c+dx)+1)}\sqrt{\cos(c+dx)}dx$$

[In] `integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(a*(cos(c+d*x)+1))*sqrt(cos(c+d*x)),x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(62) = 124$.

Time = 0.44 (sec) , antiderivative size = 791, normalized size of antiderivative = 10.99

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{4}*(2*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^(1/4)*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(d*x + c) - (\cos(d*x + c) - 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))) * \sqrt{a} + \sqrt{a}*(\arctan2(-(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^(1/4)*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(d*x + c) - \cos(d*x + c)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^(1/4)*(\cos(d*x + c)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) + \sin(d*x + c)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))) + 1) - \arctan2(-(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^(1/4)*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(d*x + c) - \cos(d*x + c)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^(1/4)*(\cos(d*x + c)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) + \sin(d*x + c)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))) - 1) - \arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^(1/4)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^(1/4)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) + 1) + \arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^(1/4)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^(1/4)*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - 1))) / d$

Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}dx = \int \sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)} dx = \int \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)} dx$$

```
[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2), x)
```

$$3.201 \quad \int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	2615
Rubi [A] (verified)	2615
Mathematica [A] (verified)	2616
Maple [B] (verified)	2616
Fricas [A] (verification not implemented)	2617
Sympy [F]	2617
Maxima [B] (verification not implemented)	2617
Giac [F]	2618
Mupad [F(-1)]	2618

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx = \frac{2\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d}$$

[Out] 2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)/d

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2853, 222}

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx = \frac{2\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d}$$

[In] Int[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]

[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/d

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos

$[e + f*x]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]]], x] /; \text{FreeQ}\{a, b, d, e, f\}, x\} \&\& \text{Eq} Q[a^2 - b^2, 0] \&\& \text{Eq} Q[d, a/b]$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \\ &= \frac{2\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\begin{aligned} &\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\ &= \frac{\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right) \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right)}{d} \end{aligned}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]

[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2])/d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(31) = 62.

Time = 4.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{a(1+\cos(dx+c))} \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}{d\sqrt{\cos(dx+c)}}$	72

[In] int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

$$= \left[\frac{\sqrt{-a} \log \left(\frac{2a \cos(dx+c)^2 - 2\sqrt{a \cos(dx+c)+a} \sqrt{-a} \sqrt{\cos(dx+c)} \sin(dx+c) + a \cos(dx+c) - a}{\cos(dx+c)+1} \right)}{d}, \right. \\ \left. - \frac{2\sqrt{a} \arctan \left(\frac{\sqrt{a \cos(dx+c)+a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)} \right)}{d} \right]$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

```
[Out] [sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sqrt(cos(d*x + c))*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))/d, - 2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/d]
```

Sympy [F]

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \int \frac{\sqrt{a (\cos(c + dx) + 1)}}{\sqrt{\cos(c + dx)}} dx$$

[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)

[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/sqrt(cos(c + d*x)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(31) = 62.

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.95

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{\sqrt{a} \arctan \left((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{\frac{1}{4}} \sin \left(\frac{1}{2} \arctan(\sin(2dx + 2c)) \right) \right)}{1}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] $\sqrt{a} \arctan 2((\cos(2dx + 2c))^2 + \sin(2dx + 2c))^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + \sin(dx + c), (\cos(2dx + 2c))^2 + \sin(2dx + 2c))^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + \cos(dx + c))/d$

Giac [F]

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \int \frac{\sqrt{a \cos(dx + c) + a}}{\sqrt{\cos(dx + c)}} dx$$

[In] `integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

[In] `int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)`

[Out] `int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)`

$$3.202 \quad \int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	2619
Rubi [A] (verified)	2619
Mathematica [A] (verified)	2620
Maple [A] (verified)	2620
Fricas [A] (verification not implemented)	2620
Sympy [F]	2621
Maxima [B] (verification not implemented)	2621
Giac [A] (verification not implemented)	2621
Mupad [B] (verification not implemented)	2622

Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{2a \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}$$

[Out] $2*a*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2850}

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{2a \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}$$

[In] $\text{Int}[\text{Sqrt}[a + a*\text{Cos}[c + d*x]]/\text{Cos}[c + d*x]^{(3/2)}, x]$

[Out] $(2*a*\text{Sin}[c + d*x])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 2850

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[-2*b^2*(\text{Cos}[e + f*x]/(f*(b*c + a*d)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\text{integral} = \frac{2a \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2\sqrt{a(1 + \cos(c + dx))} \tan\left(\frac{1}{2}(c + dx)\right)}{d\sqrt{\cos(c + dx)}}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(3/2), x]

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{2\sqrt{a(1+\cos(dx+c))} \sin(dx+c)}{d(1+\cos(dx+c))\sqrt{\cos(dx+c)}}$	42

[In] int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] 2/d*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/(1+cos(d*x+c))/cos(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2\sqrt{a \cos(dx + c) + a}\sqrt{\cos(dx + c)} \sin(dx + c)}{d \cos(dx + c)^2 + d \cos(dx + c)}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="fricas")

[Out] 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))

Sympy [F]

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{a (\cos(c + dx) + 1)}}{\cos^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2), x)

[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/cos(c + d*x)**(3/2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(32) = 64$.

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2 \left(\frac{\sqrt{2}\sqrt{a} \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sqrt{2}\sqrt{a} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{3}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{3}{2}}}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] 2*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{4 \sqrt{2} \sqrt{a} \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \tan(\frac{1}{4} dx + \frac{1}{4} c)}{\sqrt{\tan(\frac{1}{4} dx + \frac{1}{4} c)^4 - 6 \tan(\frac{1}{4} dx + \frac{1}{4} c)^2 + 1} d}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="giac")

[Out] 4*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/(sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)

Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2 \sin(c + dx) \sqrt{a (\cos(c + dx) + 1)}}{d \sqrt{\cos(c + dx)} (\cos(c + dx) + 1)}$$

[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)

[Out] (2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*cos(c + d*x)^(1/2)*(cos(c + d*x) + 1))

$$3.203 \quad \int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	2623
Rubi [A] (verified)	2623
Mathematica [A] (verified)	2624
Maple [A] (verified)	2624
Fricas [A] (verification not implemented)	2625
Sympy [F]	2625
Maxima [B] (verification not implemented)	2625
Giac [A] (verification not implemented)	2626
Mupad [B] (verification not implemented)	2626

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2a \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{4a \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}$$

[Out] $2/3*a*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(1/2)}+4/3*a*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2851, 2850}

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2a \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} + \frac{4a \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx) + a}}$$

[In] $\text{Int}[\text{Sqrt}[a + a*\text{Cos}[c + d*x]]/\text{Cos}[c + d*x]^{(5/2)}, x]$

[Out] $(2*a*\text{Sin}[c + d*x])/(3*d*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (4*a*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 2850

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{2}{3} \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{4a \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2\sqrt{a(1 + \cos(c + dx))}(1 + 2 \cos(c + dx)) \tan\left(\frac{1}{2}(c + dx)\right)}{3d \cos^{\frac{3}{2}}(c + dx)}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(5/2), x]

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 2*Cos[c + d*x])*Tan[(c + d*x)/2])/(3*d*Cos[c + d*x]^(3/2))

Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{2 \sin(dx+c)(2 \cos(dx+c)+1) \sqrt{a(1+\cos(dx+c))}}{3d(1+\cos(dx+c)) \cos(dx+c)^{\frac{3}{2}}}$	52

[In] `int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{2}{3} \frac{d \sin(dx+c) (2 \cos(dx+c)+1) (a(1+\cos(dx+c)))^{1/2}}{(1+\cos(dx+c)) \cos(dx+c)^{3/2}}$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{2 \sqrt{a \cos(dx + c) + a} (2 \cos(dx + c) + 1) \sqrt{\cos(dx + c)} \sin(dx + c)}{3 (d \cos(dx + c))^3 + d \cos(dx + c)^2}$$

[In] `integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] $\frac{2}{3} \frac{\sqrt{a \cos(dx + c) + a} (2 \cos(dx + c) + 1) \sqrt{\cos(dx + c)} \sin(dx + c)}{(d \cos(dx + c))^3 + d \cos(dx + c)^2}$

Sympy [F]

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{\sqrt{a (\cos(c + dx) + 1)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

[In] `integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)`

[Out] `Integral(sqrt(a*(cos(c + d*x) + 1))/cos(c + d*x)**(5/2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(65) = 130$.

Time = 0.37 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{2 \left(\frac{3 \sqrt{2} \sqrt{a} \sin(dx+c)}{\cos(dx+c)+1} - \frac{4 \sqrt{2} \sqrt{a} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sqrt{2} \sqrt{a} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^2}{3 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{5}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{5}{2}} \left(\frac{2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 1 \right)}$$

[In] `integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")`

```
[Out] 2/3*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2
))*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{4\sqrt{2} \left(\left(3 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 10 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 3 \right) \sqrt{a} \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)}{3 \left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1 \right)^{\frac{3}{2}} d}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*((3*tan(1/4*d*x + 1/4*c)^2 - 10)*tan(1/4*d*x + 1/4*c)^2 + 3)*sq
rt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)
^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{4 \sqrt{a} (\cos(c + dx) + 1) (\sin(c + dx) + \sin(2c + 2dx) + \sin(3c + 3dx))}{3d \sqrt{\cos(c + dx)} (3 \cos(c + dx) + 2 \cos(2c + 2dx) + \cos(3c + 3dx) + 2)}$$

```
[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] (4*(a*(cos(c + d*x) + 1))^(1/2)*(sin(c + d*x) + sin(2*c + 2*d*x) + sin(3*c
+ 3*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + c
os(3*c + 3*d*x) + 2))
```

$$3.204 \quad \int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	2627
Rubi [A] (verified)	2627
Mathematica [A] (verified)	2629
Maple [A] (verified)	2629
Fricas [A] (verification not implemented)	2629
Sympy [F(-1)]	2630
Maxima [B] (verification not implemented)	2630
Giac [A] (verification not implemented)	2630
Mupad [B] (verification not implemented)	2631

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{2a \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{8a \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{16a \sin(c+dx)}{15d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}$$

[Out] 2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+8/15*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+16/15*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2851, 2850}

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{8a \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} + \frac{2a \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} + \frac{16a \sin(c+dx)}{15d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx) + a}}$$

[In] Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]

[Out] (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])

Rule 2850

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{4}{5} \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{8a \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{8}{15} \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{8a \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{16a \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{2\sqrt{a(1 + \cos(c + dx))(7 + 4 \cos(c + dx) + 4 \cos(2(c + dx)))} \tan\left(\frac{1}{2}(c + dx)\right)}{15d \cos^{\frac{5}{2}}(c + dx)}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(7/2), x]

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(7 + 4*Cos[c + d*x] + 4*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2 \sin(dx+c)(8 \cos^2(dx+c)+4 \cos(dx+c)+3) \sqrt{a(1+\cos(dx+c))}}{15d(1+\cos(dx+c)) \cos(dx+c)^{\frac{5}{2}}}$	62

[In] int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] 2/15/d*sin(d*x+c)*(8*cos(d*x+c)^2+4*cos(d*x+c)+3)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(5/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{2\sqrt{a \cos(dx + c) + a}(8 \cos(dx + c)^2 + 4 \cos(dx + c) + 3)\sqrt{\cos(dx + c)} \sin(dx + c)}{15(d \cos(dx + c)^4 + d \cos(dx + c)^3)}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2), x, algorithm="fricas")

[Out] 2/15*sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2), x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(97) = 194.

Time = 0.34 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{2 \left(\frac{15 \sqrt{2} \sqrt{a} \sin(dx+c)}{\cos(dx+c)+1} - \frac{25 \sqrt{2} \sqrt{a} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{17 \sqrt{2} \sqrt{a} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{7 \sqrt{2} \sqrt{a} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^3}{15 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{7}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{7}{2}} \left(\frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 1 \right)}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2), x, algorithm="maxima")

[Out] 2/15*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{4 \sqrt{2} \left(\left(\left(5 \left(3 \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 - 20 \right) \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 + 282 \right) \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 - 100 \right) \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2}{15 \left(\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^4 - 6 \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 + 1 \right)^{\frac{5}{2}}} d$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] $\frac{4}{15}\sqrt{2} * (((5*(3*\tan(1/4*d*x + 1/4*c))^2 - 20)*\tan(1/4*d*x + 1/4*c))^2 + 282)*\tan(1/4*d*x + 1/4*c)^2 - 100)*\tan(1/4*d*x + 1/4*c)^2 + 15)*\sqrt{a}*\operatorname{sgn}(\cos(1/2*d*x + 1/2*c))*\tan(1/4*d*x + 1/4*c)/((\tan(1/4*d*x + 1/4*c))^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1)^{(5/2)*d}$

Mupad [B] (verification not implemented)

Time = 16.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{8 \sqrt{a} (\cos(c + dx) + 1) (7 \sin(c + dx) + 4 \sin(2c + 2dx) + 9 \sin(3c + 3dx) + 2 \sin(4c + 4dx) + 2 \sin(5c + 5dx))}{15 d \sqrt{\cos(c + dx)} (10 \cos(c + dx) + 8 \cos(2c + 2dx) + 5 \cos(3c + 3dx) + 2 \cos(4c + 4dx) + \cos(5c + 5dx) + 6)}$$

[In] int((a + a*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)

[Out] $(8*(a*(\cos(c + d*x) + 1))^{(1/2)}*(7*\sin(c + d*x) + 4*\sin(2*c + 2*d*x) + 9*\sin(3*c + 3*d*x) + 2*\sin(4*c + 4*d*x) + 2*\sin(5*c + 5*d*x)))/(15*d*\cos(c + d*x)^{(1/2)}*(10*\cos(c + d*x) + 8*\cos(2*c + 2*d*x) + 5*\cos(3*c + 3*d*x) + 2*\cos(4*c + 4*d*x) + \cos(5*c + 5*d*x) + 6))$

$$3.205 \quad \int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	2632
Rubi [A] (verified)	2632
Mathematica [A] (verified)	2634
Maple [A] (verified)	2634
Fricas [A] (verification not implemented)	2635
Sympy [F(-1)]	2635
Maxima [B] (verification not implemented)	2635
Giac [A] (verification not implemented)	2636
Mupad [B] (verification not implemented)	2636

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{2a \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{12a \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{16a \sin(c+dx)}{35d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{32a \sin(c+dx)}{35d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}$$

```
[Out] 2/7*a*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+12/35*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+16/35*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+32/35*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used

= {2851, 2850}

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx = \frac{16a \sin(c + dx)}{35d \cos^{\frac{3}{2}}(c + dx) \sqrt{a \cos(c + dx) + a}} + \frac{12a \sin(c + dx)}{35d \cos^{\frac{5}{2}}(c + dx) \sqrt{a \cos(c + dx) + a}} + \frac{2a \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx) \sqrt{a \cos(c + dx) + a}} + \frac{32a \sin(c + dx)}{35d \sqrt{\cos(c + dx)} \sqrt{a \cos(c + dx) + a}}$$

[In] Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(9/2), x]

[Out] (2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (12*a*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a*Sin[c + d*x])/(35*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (32*a*Sin[c + d*x])/(35*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])

Rule 2850

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2a \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{6}{7} \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx \\ &= \frac{2a \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\ &\quad + \frac{12a \sin(c + dx)}{35d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{24}{35} \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2a \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{12a \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} \\
&\quad + \frac{16a \sin(c+dx)}{35d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{16}{35} \int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2a \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{12a \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} \\
&\quad + \frac{16a \sin(c+dx)}{35d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{32a \sin(c+dx)}{35d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\begin{aligned}
&\int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)} dx \\
&= \frac{2\sqrt{a(1+\cos(c+dx))}(9+18\cos(c+dx)+4\cos(2(c+dx))+4\cos(3(c+dx)))\tan\left(\frac{1}{2}(c+dx)\right)}{35d \cos^{\frac{7}{2}}(c+dx)}
\end{aligned}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(9/2), x]

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(9 + 18*Cos[c + d*x] + 4*Cos[2*(c + d*x)] + 4*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(35*d*Cos[c + d*x]^(7/2))

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{2 \sin(dx+c)(16(\cos^3(dx+c))+8(\cos^2(dx+c))+6 \cos(dx+c)+5) \sqrt{a(1+\cos(dx+c))}}{35d(1+\cos(dx+c)) \cos(dx+c)^{\frac{7}{2}}}$	72

[In] int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(9/2), x, method=_RETURNVERBOSE)

[Out] 2/35/d*sin(d*x+c)*(16*cos(d*x+c)^3+8*cos(d*x+c)^2+6*cos(d*x+c)+5)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx$$

$$= \frac{2(16 \cos(dx + c)^3 + 8 \cos(dx + c)^2 + 6 \cos(dx + c) + 5) \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c)}{35(d \cos(dx + c)^5 + d \cos(dx + c)^4)}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")

```
[Out] 2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*
x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(129) = 258.

Time = 0.37 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx$$

$$= \frac{2 \left(\frac{35 \sqrt{2} \sqrt{a} \sin(dx+c)}{\cos(dx+c)+1} - \frac{70 \sqrt{2} \sqrt{a} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{84 \sqrt{2} \sqrt{a} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{58 \sqrt{2} \sqrt{a} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{9 \sqrt{2} \sqrt{a} \sin(dx+c)^9}{(\cos(dx+c)+1)^9} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)} \right)}{35 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{9}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{9}{2}} \left(\frac{4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{4 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8} \right)}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")

```
[Out] 2/35*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)
```


$$\begin{aligned} & 2)^{(1/2)} + 3*\exp(c*3i + d*x*3i)*(\exp(- c*1i - d*x*1i)/2 + \exp(c*1i + d*x*1i) \\ &)/2)^{(1/2)} + 3*\exp(c*4i + d*x*4i)*(\exp(- c*1i - d*x*1i)/2 + \exp(c*1i + d*x* \\ & 1i)/2)^{(1/2)} + 3*\exp(c*5i + d*x*5i)*(\exp(- c*1i - d*x*1i)/2 + \exp(c*1i + d* \\ & x*1i)/2)^{(1/2)} + \exp(c*6i + d*x*6i)*(\exp(- c*1i - d*x*1i)/2 + \exp(c*1i + d* \\ & x*1i)/2)^{(1/2)} + \exp(c*7i + d*x*7i)*(\exp(- c*1i - d*x*1i)/2 + \exp(c*1i + d* \\ & x*1i)/2)^{(1/2)}) \end{aligned}$$

3.206 $\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{3}{2}} dx$

Optimal result	2638
Rubi [A] (verified)	2638
Mathematica [A] (verified)	2640
Maple [A] (verified)	2641
Fricas [A] (verification not implemented)	2641
Sympy [F(-1)]	2641
Maxima [B] (verification not implemented)	2642
Giac [F]	2643
Mupad [F(-1)]	2644

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{3}{2}} dx = \frac{11a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{8d} + \frac{11a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{8d \sqrt{a+a \cos(c+dx)}} + \frac{11a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)}} + \frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d \sqrt{a+a \cos(c+dx)}}$$

[Out] $11/8*a^{(3/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+11/12*a^2*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+1/3*a^2*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+11/8*a^2*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2842, 21, 2849, 2853, 222}

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{3}{2}} dx = \frac{11a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{8d} + \frac{a^2 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{3d \sqrt{a \cos(c+dx)+a}} + \frac{11a^2 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{12d \sqrt{a \cos(c+dx)+a}} + \frac{11a^2 \sin(c+dx) \sqrt{\cos(c+dx)}}{8d \sqrt{a \cos(c+dx)+a}}$$

[In] Int[Cos[c + d*x]^(3/2)*(a + a*cos[c + d*x])^(3/2),x]

[Out] (11*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*cos[c + d*x]])/(8*d) + (11*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*cos[c + d*x]]) + (11*a^2*cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*cos[c + d*x]]) + (a^2*cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*cos[c + d*x]])

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplifierQ[c + d*x, a + b*x])

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2842

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegerQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2849

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{3} \int \frac{\cos^{\frac{3}{2}}(c+dx) \left(\frac{11a^2}{2} + \frac{11}{2}a^2 \cos(c+dx) \right)}{\sqrt{a+a\cos(c+dx)}} dx \\
&= \frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{6} (11a) \int \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a\cos(c+dx)} dx \\
&= \frac{11a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{1}{8} (11a) \int \sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)} dx \\
&= \frac{11a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{16} (11a) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{11a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{(11a) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right)}{8d} \\
&= \frac{11a^{3/2} \arcsin \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right)}{8d} + \frac{11a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{11a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{3}{2}} dx = \frac{a\sqrt{a(1+\cos(c+dx))} \sec\left(\frac{1}{2}(c+dx)\right) \left(33\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\sqrt{\cos(c+dx)}(26\sin\left(\frac{1}{2}(c+dx)\right) + 9\sin\left(\frac{3}{2}(c+dx)\right)) \right)}{48d}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2), x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(33*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(26*Sin[(c + d*x)/2] + 9*Sin[(3*(c + d*x))/2]) + 2*Sin[(5*(c + d*x))/2]))/(48*d)

Maple [A] (verified)

Time = 11.91 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.13

method	result
default	$\frac{\left(8 \sin(dx+c) \cos^2(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 22 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 33 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 33 \arctan\left(\frac{\tan(dx+c)}{1+\cos(dx+c)}\right)\right) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{24d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] `int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)`

```
[Out] 1/24/d*(8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+22*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*a*(1+cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2} dx = \frac{(8a\cos(dx+c)^2 + 22a\cos(dx+c) + 33a)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c) - 33a\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{24(d\cos(dx+c)+d)}$$

[In] `integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")`

```
[Out] 1/24*((8*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 33*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 33*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(3/2),x)`

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1942 vs. 2(134) = 268.

Time = 0.59 (sec) , antiderivative size = 1942, normalized size of antiderivative = 12.14

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{3/2} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/96*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1))*sin(3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1))*((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1 + a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1) - (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1))*sqrt(a) + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1)) + 1) - a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))

$3*d*x + 3*c)) + 1)) * \sin(1/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) -$
 $\cos(1/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) * \sin(1/2 * \arctan2(\sin(2$
 $/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \arctan2(\sin(3*d*x$
 $+ 3*c), \cos(3*d*x + 3*c))) + 1))), (\cos(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3$
 $*d*x + 3*c)))^2 + \sin(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 +$
 $2 * \cos(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)^{(1/4)} * (\cos(1/3 * \arctan2$
 $(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) * \cos(1/2 * \arctan2(\sin(2/3 * \arctan$
 $2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \arctan2(\sin(3*d*x + 3*c), c$
 $os(3*d*x + 3*c))) + 1)) + \sin(1/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c$
 $))) * \sin(1/2 * \arctan2(\sin(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), c$
 $os(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1))) - 1) - a * \arctan2$
 $((\cos(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + \sin(2/3 * \arctan2($
 $\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + 2 * \cos(2/3 * \arctan2(\sin(3*d*x + 3*c)$
 $, \cos(3*d*x + 3*c))) + 1)^{(1/4)} * \sin(1/2 * \arctan2(\sin(2/3 * \arctan2(\sin(3*d*x +$
 $3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*$
 $c))) + 1)), (\cos(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + \sin(2$
 $/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2 + 2 * \cos(2/3 * \arctan2(\sin(3$
 $*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)^{(1/4)} * \cos(1/2 * \arctan2(\sin(2/3 * \arctan2($
 $\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos$
 $(3*d*x + 3*c))) + 1)) + 1) + a * \arctan2((\cos(2/3 * \arctan2(\sin(3*d*x + 3*c), c$
 $os(3*d*x + 3*c)))^2 + \sin(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^2$
 $+ 2 * \cos(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)^{(1/4)} * \sin(1$
 $/2 * \arctan2(\sin(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3 * \ar$
 $ctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)), (\cos(2/3 * \arctan2(\sin(3*d*$
 $x + 3*c), \cos(3*d*x + 3*c)))^2 + \sin(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*$
 $x + 3*c)))^2 + 2 * \cos(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)^$
 $(1/4) * \cos(1/2 * \arctan2(\sin(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))),$
 $\cos(2/3 * \arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)) - 1)) * \sqrt{a}) /$
 d

Giac [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{3/2} dx = \int (a \cos(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{3/2} dx = \int \cos(c + dx)^{3/2} (a + a \cos(c + dx))^{3/2} dx$$

```
[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2), x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2), x)
```


3.207 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2} dx$

Optimal result	2645
Rubi [A] (verified)	2645
Mathematica [A] (verified)	2647
Maple [A] (verified)	2647
Fricas [A] (verification not implemented)	2648
Sympy [F]	2648
Maxima [B] (verification not implemented)	2648
Giac [F]	2649
Mupad [F(-1)]	2649

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2} dx = \frac{7a^{3/2} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{7a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \cos^{3/2}(c+dx) \sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

[Out] $7/4*a^{(3/2)*\arcsin(\sin(d*x+c)*a^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)})/d+1/2*a^2*\cos(d*x+c)^{(3/2)*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+7/4*a^2*\sin(d*x+c)*\cos(d*x+c)^{(1/2)/d/(a+a*\cos(d*x+c))^{(1/2)}}$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2842, 21, 2849, 2853, 222}

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2} dx = \frac{7a^{3/2} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d} + \frac{a^2 \sin(c+dx) \cos^{3/2}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}} + \frac{7a^2 \sin(c+dx) \sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

[In] $\text{Int}[\text{Sqrt}[\text{Cos}[c+d*x]]*(a+a*\text{Cos}[c+d*x])^{(3/2)},x]$

[Out] $(7*a^{(3/2)*\text{ArcSin}[(\text{Sqrt}[a]*\text{Sin}[c+d*x])/(\text{Sqrt}[a+a*\text{Cos}[c+d*x]])])/(4*d) + (7*a^2*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(4*d*\text{Sqrt}[a+a*\text{Cos}[c+d*x]]) + (a^2*\text{Cos}[c+d*x]^{(3/2)*\text{Sin}[c+d*x]})/(2*d*\text{Sqrt}[a+a*\text{Cos}[c+d*x]])$

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x]
)^(n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\text{integral} = \frac{a^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \cos(c + dx)}} + \frac{1}{2} \int \frac{\sqrt{\cos(c + dx)} \left(\frac{7a^2}{2} + \frac{7}{2}a^2 \cos(c + dx) \right)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$\begin{aligned}
&= \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{4}(7a) \int \sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)} dx \\
&= \frac{7a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(7a) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{7a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{(7a) \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} \\
&= \frac{7a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{7a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.77

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2} dx = \frac{a\sqrt{a(1+\cos(c+dx))} \sec\left(\frac{1}{2}(c+dx)\right) \left(7\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c+dx)\right)\right) + 2\sqrt{\cos(c+dx)}\right) + 2\sqrt{\cos(c+dx)}(6\sqrt{a(1+\cos(c+dx))})}{8d}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2), x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(7*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(6*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))) / (8*d)

Maple [A] (verified)

Time = 11.74 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22

method	result
default	$\frac{\left(2 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 7 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 7 \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right) \sqrt{a(1+\cos(dx+c))} (\sqrt{\cos(dx+c)})}{4d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/4/d*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2} dx = \frac{(2a\cos(dx+c)+7a)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-7(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}{4(d\cos(dx+c)+d)}$$

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*cos(d*x + c) + 7*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2} dx = \int (a(\cos(c+dx)+1))^{3/2} \sqrt{\cos(c+dx)} dx$$

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. 2(100) = 200.

Time = 0.47 (sec) , antiderivative size = 1080, normalized size of antiderivative = 9.00

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2} dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c) + 1)^(1/4), sin(2*d*x + 2*c))))/d
```

$$2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))), \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) * \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) * \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) * \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) * \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1) - a \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) * \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) * \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) * \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) * \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) - 1) - a \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 1) + a \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - 1)) * \sqrt{a}) / d$$

Giac [F]

$$\int \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^{3/2} dx = \int (a \cos(dx + c) + a)^{3/2} \sqrt{\cos(dx + c)} dx$$

[In] integrate(cos(dx+c)^(1/2)*(a+a*cos(dx+c))^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(dx + c) + a)^(3/2)*sqrt(cos(dx + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^{3/2} dx = \int \sqrt{\cos(c + dx)} (a + a \cos(c + dx))^{3/2} dx$$

[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2), x)

$$3.208 \quad \int \frac{(a+a \cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	2650
Rubi [A] (verified)	2650
Mathematica [A] (verified)	2652
Maple [B] (verified)	2652
Fricas [A] (verification not implemented)	2652
Sympy [F]	2653
Maxima [B] (verification not implemented)	2653
Giac [F(-1)]	2654
Mupad [F(-1)]	2654

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \frac{3a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} + \frac{a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}}$$

[Out] $3a^{3/2} \arcsin(\sin(dx+c) a^{1/2} / (a+a \cos(dx+c))^{1/2}) / d + a^2 \sin(dx+c) \cos(dx+c)^{1/2} / d / (a+a \cos(dx+c))^{1/2}$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2842, 21, 2853, 222}

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \frac{3a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d} + \frac{a^2 \sin(c + dx) \sqrt{\cos(c + dx)}}{d \sqrt{a \cos(c + dx) + a}}$$

[In] $\text{Int}[(a + a \cos[c + dx])^{3/2} / \text{Sqrt}[\cos[c + dx]], x]$

[Out] $(3a^{3/2} \text{ArcSin}[(\text{Sqrt}[a] \sin[c + dx]) / \text{Sqrt}[a + a \cos[c + dx]]) / d + (a^2 \text{Sqrt}[\cos[c + dx]] \sin[c + dx]) / (d \text{Sqrt}[a + a \cos[c + dx]])$

Rule 21

$\text{Int}[(u_.) * ((a_.) + (b_.) * (v_.))^{(m_.)} * ((c_.) + (d_.) * (v_.))^{(n_.)}, x_Symbol] :=$
 $\text{Dist}[(b/d)^m, \text{Int}[u * (c + d*v)^{(m+n)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, n\}, x$
 $\&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& (!\text{IntegerQ}[n] || \text{SimplerQ}[c + dx,$
 $a + b*x])$

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2842

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}} + \int \frac{\frac{3a^2}{2} + \frac{3}{2}a^2 \cos(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} dx \\
 &= \frac{a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}} + \frac{1}{2}(3a) \int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}} - \frac{(3a) \text{Subst}\left(\int \frac{1}{\sqrt{1 - \frac{x^2}{a}}} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{d} \\
 &= \frac{3a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}}\right)}{d} + \frac{a^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \frac{a \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(3\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right) + 2\sqrt{\cos\left(\frac{1}{2}(c + dx)\right)}\right)}{2d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(65) = 130.

Time = 13.85 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.03

method	result
default	$\frac{\left(3 \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) + \cos(dx+c) \sin(dx+c) + 3 \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)}{d \sqrt{\cos(dx+c)} (1+\cos(dx+c))}$

[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(3*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+cos(d*x+c)*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/(1+cos(d*x+c))*a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \frac{\sqrt{a \cos(dx + c) + a} a \sqrt{\cos(dx + c)} \sin(dx + c) - 3(a \cos(dx + c) + a) \sqrt{a} \arctan\left(\frac{\sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)}}{\sqrt{a} \sin(dx + c)}\right)}{d \cos(dx + c) + d}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] (sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - 3*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)

SymPy [F]

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a(\cos(c + dx) + 1))^{3/2}}{\sqrt{\cos(c + dx)}} dx$$

[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)

[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/sqrt(cos(c + d*x)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. 2(65) = 130.

Time = 0.45 (sec) , antiderivative size = 803, normalized size of antiderivative = 10.71

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx$$

[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)

[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)

$$3.209 \quad \int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	2655
Rubi [A] (verified)	2655
Mathematica [A] (verified)	2657
Maple [B] (verified)	2657
Fricas [A] (verification not implemented)	2657
Sympy [F]	2658
Maxima [B] (verification not implemented)	2658
Giac [F]	2659
Mupad [F(-1)]	2659

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} + \frac{2a^2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}$$

[Out] $2*a^{(3/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+2*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2841, 21, 2853, 222}

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d} + \frac{2a^2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a \cos(c + dx) + a}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}/\text{Cos}[c + d*x]^{(3/2)}, x]$

[Out] $(2*a^{(3/2)}*\text{ArcSin}[(\text{Sqrt}[a]*\text{Sin}[c + d*x])/(\text{Sqrt}[a + a*\text{Cos}[c + d*x]])])/d + (2*a^2*\text{Sin}[c + d*x])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a^2 \sin(c + dx)}{d\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} - (2a) \int \frac{-\frac{a}{2} - \frac{1}{2}a \cos(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} dx \\
 &= \frac{2a^2 \sin(c + dx)}{d\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} + a \int \frac{\sqrt{a + a\cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{2a^2 \sin(c + dx)}{d\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} - \frac{(2a) \text{Subst}\left(\int \frac{1}{\sqrt{1 - \frac{x^2}{a}}} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a\cos(c + dx)}}\right)}{d} \\
 &= \frac{2a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a + a\cos(c + dx)}}\right)}{d} + \frac{2a^2 \sin(c + dx)}{d\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \frac{a \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \left(\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) \sqrt{\cos(c + dx)}}{d \sqrt{\cos(c + dx)}}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2), x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/(d*Sqrt[Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(66) = 132.

Time = 5.44 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.89

method	result
default	$\frac{2\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) + \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) + \sin(dx+c)\right)\sqrt{a}}{d(1+\cos(dx+c))\sqrt{\cos(dx+c)}}$

[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] 2/d*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2))/(1+cos(d*x+c))/cos(d*x+c)^(1/2)*a

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \frac{2\left(\sqrt{a \cos(dx + c) + aa} \sqrt{\cos(dx + c)} \sin(dx + c) - (a \cos(dx + c))^2 + a \cos(dx + c)\right)}{d \cos(dx + c)^2 + d \cos(dx + c)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2), x, algorithm="fricas")

[Out] 2*(sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - (a*cos(d*x + c))^2 + a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)))/(sqrt(a)*sin(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))

SymPy [F]

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(a(\cos(c + dx) + 1))^{3/2}}{\cos^{3/2}(c + dx)} dx$$

[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)

[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/cos(c + d*x)**(3/2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. 2(66) = 132.

Time = 0.47 (sec) , antiderivative size = 997, normalized size of antiderivative = 13.12

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*cos(2*d*x + 2*c)

$c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sqrt{a} + 4*(a \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - (a \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) - a) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) \sqrt{a}) / ((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * d)$

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{3/2}} dx$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(a + a \cos(c + dx))^{3/2}}{\cos(c + dx)^{3/2}} dx$$

[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)

[Out] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)

$$3.210 \quad \int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	2660
Rubi [A] (verified)	2660
Mathematica [A] (verified)	2661
Maple [A] (verified)	2662
Fricas [A] (verification not implemented)	2662
Sympy [F]	2662
Maxima [A] (verification not implemented)	2663
Giac [F(-1)]	2663
Mupad [B] (verification not implemented)	2663

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2a^2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{10a^2 \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}$$

[Out] $2/3*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(1/2)}+10/3*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2841, 21, 2850}

$$\int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2a^2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} + \frac{10a^2 \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx) + a}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}/\text{Cos}[c + d*x]^{(5/2)}, x]$

[Out] $(2*a^2*\text{Sin}[c + d*x])/(3*d*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (10*a^2*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 21


```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{1}{3}(2a) \int \frac{-\frac{5a}{2} - \frac{5}{2}a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx \\
&= \frac{2a^2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{1}{3}(5a) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a^2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{10a^2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2a \sqrt{a(1 + \cos(c + dx))} (1 + 5 \cos(c + dx)) \tan\left(\frac{1}{2}(c + dx)\right)}{3d \cos^{\frac{3}{2}}(c + dx)}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2), x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 5*Cos[c + d*x])*Tan[(c + d*x)/2])/(3*d
*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 5.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{2 \sin(dx+c)(5 \cos(dx+c)+1) \sqrt{a(1+\cos(dx+c))} a}{3d(1+\cos(dx+c)) \cos(dx+c)^{\frac{3}{2}}}$	53

[In] `int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{2}{3} \frac{d \sin(dx+c) (5 \cos(dx+c) + 1) (a(1 + \cos(dx+c)))^{1/2}}{(1 + \cos(dx+c))^{3/2} c}$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \frac{2(5a \cos(dx + c) + a) \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c)}{3(d \cos(dx + c))^3 + d \cos(dx + c)^2}$$

[In] `integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] $\frac{2}{3} \frac{(5a \cos(dx + c) + a) \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c)}{(d \cos(dx + c))^3 + d \cos(dx + c)^2}$

Sympy [F]

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \int \frac{(a(\cos(c + dx) + 1))^{3/2}}{\cos^{5/2}(c + dx)} dx$$

[In] `integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)`

[Out] `Integral((a*(cos(c + d*x) + 1))**(3/2)/cos(c + d*x)**(5/2), x)`

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \frac{4 \left(\frac{3\sqrt{2}a^{3/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{5\sqrt{2}a^{3/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{2\sqrt{2}a^{3/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{3d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{5/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{5/2}}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] 4/3*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 15.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \frac{2a \sqrt{a (\cos(c + dx) + 1)} (5 \sin(c + dx) + 2 \sin(2c + 2dx) + 5 \sin(3c + 3dx) + 3 \cos(c + dx) + 2 \cos(2c + 2dx) + \cos(3c + 3dx) + 2)}{3d \sqrt{\cos(c + dx)} (3 \cos(c + dx) + 2 \cos(2c + 2dx) + \cos(3c + 3dx) + 2)}$$

[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)

[Out] (2*a*(a*(cos(c + d*x) + 1))^(1/2)*(5*sin(c + d*x) + 2*sin(2*c + 2*d*x) + 5*sin(3*c + 3*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x) + 2))

$$3.211 \quad \int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	2664
Rubi [A] (verified)	2664
Mathematica [A] (verified)	2666
Maple [A] (verified)	2666
Fricas [A] (verification not implemented)	2667
Sympy [F(-1)]	2667
Maxima [B] (verification not implemented)	2667
Giac [F(-1)]	2668
Mupad [B] (verification not implemented)	2668

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{2a^2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{6a^2 \sin(c+dx)}{5d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{12a^2 \sin(c+dx)}{5d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}$$

[Out] $2/5*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}/(a+a*\cos(d*x+c))^{(1/2)}+6/5*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(1/2)}+12/5*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2841, 21, 2851, 2850}

$$\int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{6a^2 \sin(c+dx)}{5d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} + \frac{2a^2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} + \frac{12a^2 \sin(c+dx)}{5d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx) + a}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}/\text{Cos}[c + d*x]^{(7/2)}, x]$

[Out] $(2*a^2*\text{Sin}[c + d*x])/(5*d*\text{Cos}[c + d*x]^{(5/2)}*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (6*a^2*\text{Sin}[c + d*x])/(5*d*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (12*a^2*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 21

```
Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplifierQ[c + d*x,
  a + b*x])
```

Rule 2841

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2a^2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{1}{5}(2a) \int \frac{-\frac{9a}{2} - \frac{9}{2}a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx \\ &= \frac{2a^2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{1}{5}(9a) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2a^2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{6a^2 \sin(c + dx)}{5d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{1}{5}(6a) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a^2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{6a^2 \sin(c + dx)}{5d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{12a^2 \sin(c + dx)}{5d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

$$\int \frac{(a + a \cos(c + dx))^{\frac{3}{2}}}{\cos^{\frac{7}{2}}(c + dx)} dx = \frac{2a \sqrt{a(1 + \cos(c + dx))} (4 + 3 \cos(c + dx) + 3 \cos(2(c + dx))) \tan\left(\frac{1}{2}(c + dx)\right)}{5d \cos^{\frac{5}{2}}(c + dx)}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2), x]

[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(5*d*Cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{2 \sin(dx+c) (6 \cos^2(dx+c) + 3 \cos(dx+c) + 1) \sqrt{a(1+\cos(dx+c))} a}{5d(1+\cos(dx+c)) \cos(dx+c)^{\frac{5}{2}}}$	63

[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] 2/5/d*sin(d*x+c)*(6*cos(d*x+c)^2+3*cos(d*x+c)+1)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(5/2)*a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \frac{2(6a \cos(dx + c)^2 + 3a \cos(dx + c) + a) \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)}}{5(d \cos(dx + c)^4 + d \cos(dx + c)^3)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(103) = 206.

Time = 0.35 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \frac{4 \left(\frac{5\sqrt{2}a^{3/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{10\sqrt{2}a^{3/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{7\sqrt{2}a^{3/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{2\sqrt{2}a^{3/2} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right) \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{7/2}}{5d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{7/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{7/2} \left(\frac{2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} \right)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 4/5*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 16.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \frac{4a \sqrt{a (\cos(c + dx) + 1)} (8 \sin(c + dx) + 6 \sin(2c + 2dx) + 11 \sin(3c + 3dx) + 3 \sin(4c + 4dx) + 3 \sin(5c + 5dx))}{5d \sqrt{\cos(c + dx)} (10 \cos(c + dx) + 8 \cos(2c + 2dx) + 5 \cos(3c + 3dx) + 2 \cos(4c + 4dx) + \cos(5c + 5dx) + 6)}$$

```
[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (4*a*(a*(cos(c + d*x) + 1))^(1/2)*(8*sin(c + d*x) + 6*sin(2*c + 2*d*x) + 11
*sin(3*c + 3*d*x) + 3*sin(4*c + 4*d*x) + 3*sin(5*c + 5*d*x)))/(5*d*cos(c +
d*x)^(1/2)*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*c
os(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```


$$3.212 \quad \int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	2669
Rubi [A] (verified)	2669
Mathematica [A] (verified)	2671
Maple [A] (verified)	2671
Fricas [A] (verification not implemented)	2672
Sympy [F(-1)]	2672
Maxima [A] (verification not implemented)	2672
Giac [C] (verification not implemented)	2673
Mupad [B] (verification not implemented)	3115

Optimal result

Integrand size = 25, antiderivative size = 161

$$\begin{aligned} \int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)} dx &= \frac{2a^2 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} \\ &+ \frac{26a^2 \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{104a^2 \sin(c+dx)}{105d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} \\ &+ \frac{208a^2 \sin(c+dx)}{105d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}} \end{aligned}$$

[Out] $2/7*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(7/2)}/(a+a*\cos(d*x+c))^{(1/2)}+26/35*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}/(a+a*\cos(d*x+c))^{(1/2)}+104/105*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(1/2)}+208/105*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2841, 21, 2851, 2850}

$$\begin{aligned} \int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)} dx &= \frac{104a^2 \sin(c+dx)}{105d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} \\ &+ \frac{26a^2 \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} + \frac{2a^2 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} \\ &+ \frac{208a^2 \sin(c+dx)}{105d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx) + a}} \end{aligned}$$

[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2), x]

[Out] (2*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (2*6*a^2*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (104*a^2*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (208*a^2*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])

Rule 21

Int[(u_.)*((a_.) + (b_.)*(v_.))^(m_.)*((c_.) + (d_.)*(v_.))^(n_.), x_Symbol] :> Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 2841

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2850

Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rubi steps

$$\text{integral} = \frac{2a^2 \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{1}{7}(2a) \int \frac{-\frac{13a}{2} - \frac{13}{2}a \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx$$

$$\begin{aligned}
&= \frac{2a^2 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{1}{7}(13a) \int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx \\
&= \frac{2a^2 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{26a^2 \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} \\
&\quad + \frac{1}{35}(52a) \int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2a^2 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{26a^2 \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} \\
&\quad + \frac{104a^2 \sin(c+dx)}{105d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{1}{105}(104a) \int \frac{\sqrt{a+a \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2a^2 \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{26a^2 \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} \\
&\quad + \frac{104a^2 \sin(c+dx)}{105d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{208a^2 \sin(c+dx)}{105d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int \frac{(a+a \cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{2a \sqrt{a(1+\cos(c+dx))}(41+117 \cos(c+dx)+26 \cos(2(c+dx))+26 \cos(3(c+dx)))}{105d \cos^{\frac{7}{2}}(c+dx)}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2), x]

[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(41 + 117*Cos[c + d*x] + 26*Cos[2*(c + d*x)] + 26*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))

Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{2 \sin(dx+c)(104(\cos^3(dx+c))+52(\cos^2(dx+c))+39 \cos(dx+c)+15) \sqrt{a(1+\cos(dx+c))} a}{105d(1+\cos(dx+c)) \cos(dx+c)^{\frac{7}{2}}}$	73

[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(9/2), x, method=_RETURNVERBOSE)

[Out] 2/105/d*sin(d*x+c)*(104*cos(d*x+c)^3+52*cos(d*x+c)^2+39*cos(d*x+c)+15)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(7/2)*a

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.53

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \frac{2(104a \cos(dx + c)^3 + 52a \cos(dx + c)^2 + 39a \cos(dx + c) + 15a) \sqrt{a \cos(dx + c)}}{105(d \cos(dx + c)^5 + d \cos(dx + c)^4)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")

[Out] 2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.63

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \frac{4 \left(\frac{105 \sqrt{2} a^{3/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{245 \sqrt{2} a^{3/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{273 \sqrt{2} a^{3/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{171 \sqrt{2} a^{3/2} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{38 \sqrt{2} a^{3/2} \sin(dx+c)^9}{(\cos(dx+c)+1)^9} \right)}{105 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{9/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{9/2} \left(\frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 1 \right)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")

[Out] 4/105*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 233.17 (sec) , antiderivative size = 87931, normalized size of antiderivative = 546.16

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="giac")

[Out] 134217728/105*sqrt(2)*sqrt(-tan(1/4*d*x + c)^4*tan(1/2*c)^8 + 14*tan(1/4*d*x + c)^4*tan(1/2*c)^6 - 24*tan(1/4*d*x + c)^3*tan(1/2*c)^7 + 6*tan(1/4*d*x + c)^2*tan(1/2*c)^8 + 56*tan(1/4*d*x + c)^3*tan(1/2*c)^5 - 84*tan(1/4*d*x + c)^2*tan(1/2*c)^6 + 24*tan(1/4*d*x + c)*tan(1/2*c)^7 - tan(1/2*c)^8 - 14*tan(1/4*d*x + c)^4*tan(1/2*c)^2 + 56*tan(1/4*d*x + c)^3*tan(1/2*c)^3 - 56*tan(1/4*d*x + c)*tan(1/2*c)^5 + 14*tan(1/2*c)^6 + tan(1/4*d*x + c)^4 - 24*tan(1/4*d*x + c)^3*tan(1/2*c) + 84*tan(1/4*d*x + c)^2*tan(1/2*c)^2 - 56*tan(1/4*d*x + c)*tan(1/2*c)^3 - 6*tan(1/4*d*x + c)^2 + 24*tan(1/4*d*x + c)*tan(1/2*c) - 14*tan(1/2*c)^2 + 1)*(((((((((((((((((-26*I*a*e^(1055/2*I*c) - 10504*I*a*e^(1053/2*I*c) - 2116556*I*a*e^(1051/2*I*c) - 283618504*I*a*e^(1049/2*I*c) - 28432755026*I*a*e^(1047/2*I*c) - 2274620402080*I*a*e^(1045/2*I*c) - 151262256738489*I*a*e^(1043/2*I*c) - 8600339740332756*I*a*e^(1041/2*I*c) - 426791859624382434*I*a*e^(1039/2*I*c) - 18778841824711012321*I*a*e^(1037/2*I*c) - 741764252188078830689*I*a*e^(1035/2*I*c) - 26568646859265646058950*I*a*e^(1033/2*I*c) - 870123185139944470654786*I*a*e^(1031/2*I*c) - 26237560685859258651673169*I*a*e^(1029/2*I*c) - 732777588939374143249503406*I*a*e^(1027/2*I*c) - 19052217362358251291769232228*I*a*e^(1025/2*I*c) - 463207036476152149238073238832*I*a*e^(1023/2*I*c) - 10572019483402811301975746281474*I*a*e^(1021/2*I*c) - 227298420835979818145918471098570*I*a*e^(1019/2*I*c) - 4617746921046245570620730707402520*I*a*e^(1017/2*I*c) - 88891629710203223556668933908311334*I*a*e^(1015/2*I*c) - 1625446980015332177755918014501608982*I*a*e^(1013/2*I*c) - 28297555097893393021020676691580988233*I*a*e^(1011/2*I*c) - 469985499033039677874623273935334240076*I*a*e^(1009/2*I*c) - 7461020183678979972532247003767832497030*I*a*e^(1007/2*I*c) - 113407514349957436967142625811402104148561*I*a*e^(1005/2*I*c) - 1653132753283300154895933847581652791644965*I*a*e^(1003/2*I*c) - 23143861022126290935795147446967794330788764*I*a*e^(1001/2*I*c) - 311615599009998457055428626562611218440152906*I*a*e^(999/2*I*c) - 4040258091712579222349027082245857300045844265*I*a*e^(997/2*I*c) - 50503236456021701342291384690765044913121286031*I*a*e^(995/2*I*c) - 609297262840711915743663065579736975124435730278*I*a*e^(993/2*I*c) - 7102123372769126439065816004234917371096063247516*I*a*e^(991/2*I*c) - 8006032914110785684222094424093216574599312586835*I*a*e^(989/2*I*c) - 873599857808953513616291826808640309239785388357815*I*a*e^(987/2*I*c) - 9235203345455625183635292584058316806336315160604010*I*a*e^(985/2*I*c) - 94660893371344177720163398371692357473361249542281324*I*a*e^(983/2*I*c) - 9414928240218725385714

07241795810657020217893835828847*I*a*e^(981/2*I*c) - 9092846502857218261462
182868864364624762389989516884454*I*a*e^(979/2*I*c) - 853329543261206742723
11677629126536894715822244726480090*I*a*e^(977/2*I*c) - 7786641378440594926
87126944618806483697276408481076541824*I*a*e^(975/2*I*c) - 6913027812800192
462704129875030929367293369791365329368306*I*a*e^(973/2*I*c) - 597484082603
36333152741949120162468054098096806527992694330*I*a*e^(971/2*I*c) - 5029991
67177513108237013793834321163983016591115064062192504*I*a*e^(969/2*I*c) - 4
126888421983249887687507689195856719564200342340690922960020*I*a*e^(967/2*I
*c) - 33015189183442105731730952102686078554556811480087134425669934*I*a*e^
(965/2*I*c) - 2576627521764579828166282791344882800098730222812489282691092
47*I*a*e^(963/2*I*c) - 1962629045379290087751577541491949170474015064554905
158361368638*I*a*e^(961/2*I*c) - 145971076634287904419124491510555697826555
51054988172538106007306*I*a*e^(959/2*I*c) - 1060529033630690131355782203401
14498079609654804470793711887113055*I*a*e^(957/2*I*c) - 7529792233112361356
73287737101570895147168873416246633317325038975*I*a*e^(955/2*I*c) - 5226590
054479657822483780963008017596290103718605086715148151861646*I*a*e^(953/2*I
c) - 35480723438231712844684785105847570910860568249280080967679923825908
I*a*e^(951/2*I*c) - 2356471921691593101191613443407382001865279341517292517
38259408174007*I*a*e^(949/2*I*c) - 1531718700483241796992497361844851327831
244614221060296959000799843324*I*a*e^(947/2*I*c) - 974738630828190218232984
6941952263067390253415184368339278176002138446*I*a*e^(945/2*I*c) - 60747701
724909506776211674773541506177265806300575091658980807046860704*I*a*e^(943/
2*I*c) - 370884792983254000550721425873326574910124282599731660698821060848
882320*I*a*e^(941/2*I*c) - 221894153908280997998580146226159783968435134891
2275577377603658269150756*I*a*e^(939/2*I*c) - 13012954846383921360783439001
134235337905843280455893166840706360597524134*I*a*e^(937/2*I*c) - 748256379
26340979926222451933992320121926362312065461272342106878301448536*I*a*e^(93
5/2*I*c) - 4219747349096187780741342759182006525742147279404438445345023730
85580328604*I*a*e^(933/2*I*c) - 2334517419885966874350965488026816679466208
310259855732507435105175313457396*I*a*e^(931/2*I*c) - 126733614191388349406
17178471273655660069856526406967342118703762672110480200*I*a*e^(929/2*I*c)
- 6752682980371408820784121748009856094383803689375257216147308962311713997
1604*I*a*e^(927/2*I*c) - 35322638927283492431418931705836395420317184204941
0810171865734368883820818996*I*a*e^(925/2*I*c) - 18143510046388305354509746
49080451232909962840518095183307668250502738979794444*I*a*e^(923/2*I*c) - 9
153283321740997122951578693088645450336613285754578953646231535187679288519
816*I*a*e^(921/2*I*c) - 453641734231472475081355495620115384286835604208391
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38736238943107985690499222615150129577567013343972814900*I*a*e^(917/2*I*c)
- 1057267590272325636639773903752531171523549537193854805338038288437194124
266978930*I*a*e^(915/2*I*c) - 497385563358030362012603263786947750010767618
2499873076094518413397303603804906240*I*a*e^(913/2*I*c) - 23005253792558757
618650193250261667174892446545331020900285683620126145948425151860*I*a*e^(9
11/2*I*c) - 104632479285330168704567624173205774202975142869561651398142015
702621321511418637970*I*a*e^(909/2*I*c) - 468046912422543827579364617290976

131668618292605267581426808880930942056836225740850*I*a*e^(907/2*I*c) - 205
954451320666914305466797161323566250098209629523881816279242995442881462955
8400860*I*a*e^(905/2*I*c) - 89163138809086160619416729929999844829860757859
70885880652620253394990773311152181860*I*a*e^(903/2*I*c) - 3798422694278446
6728066963318356113137634905014288785261844980339121431692865121528770*I*a*
e^(901/2*I*c) - 1592555129036669659473920504826277765285552562864759807148
7710356765619338164447678480*I*a*e^(899/2*I*c) - 65724378186986858433214274
5237797306576908038181174879375861022306506494725134312464240*I*a*e^(897/2*
I*c) - 26703310804381367392516994211884771528169838826465183136183000551904
98774861721956084120*I*a*e^(895/2*I*c) - 1068253687813551381159868667815830
8084092593434324791253827853349664088304211712490029720*I*a*e^(893/2*I*c) -
42083973507831971365951079505656822321370151622842020026197681285864461893
407471640353960*I*a*e^(891/2*I*c) - 163287484325599994722817042441220130107
697316248983794524997603902832643977337412661872760*I*a*e^(889/2*I*c) - 624
082643961029037312709284672126976697873212538908890262985442647874324392878
156690829740*I*a*e^(887/2*I*c) - 234986055260470027437041195245292043865000
6836481438508181807279875096925888687475349639360*I*a*e^(885/2*I*c) - 87178
451215385439650006555089275761564545099651245782276661118615995304704731654
94741121930*I*a*e^(883/2*I*c) - 3187118311465183573627822548071998513136074
2804792653167920265913904648857223319205558481280*I*a*e^(881/2*I*c) - 11483
195709771919270611169705763287240925693313421210595325532758731557146619122
5892722458940*I*a*e^(879/2*I*c) - 40780746171419707117887262008218441136589
6862940021562248885259817851441393751779677614254090*I*a*e^(877/2*I*c) - 14
276653476597153178712146836003013566298180781570118096313232006836650152070
96525546411654850*I*a*e^(875/2*I*c) - 4927496729283083450012099767840203352
055720285319781857700963595150774626713503066668245663200*I*a*e^(873/2*I*c)
- 167688628137425704973593276857456712573650589092714891128297609865083833
52816248603242238251460*I*a*e^(871/2*I*c) - 5627373794228121721583257810007
3235858273628493027174707116905455498600552994477017353335693130*I*a*e^(869
/2*I*c) - 18624270141530015026036766190979622193201973253702463401097664654
2647178366376613371237978468950*I*a*e^(867/2*I*c) - 60795232506538055223176
5878979364337922618562001789681944692673308704528340024671175049604847740*I
*a*e^(865/2*I*c) - 19575864696310598451382153078727701034973417419760501462
98563390984316998623891058083233354950720*I*a*e^(863/2*I*c) - 6218376119870
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708312030*I*a*e^(861/2*I*c) - 194885761759582598684920866636691359938916703
26935036733424079659395654947395041671226077968039510*I*a*e^(859/2*I*c) - 6
026593628490055787427805827112924362891485407231105426369995644449795378662
2754638704751947767300*I*a*e^(857/2*I*c) - 18390480722449191984653021156617
2865275646148609389197477057237389158397210607704335080668307676320*I*a*e^(
855/2*I*c) - 55383894059703720980142009333903146592834534246696813432717035
6436277614050695088234925854972461590*I*a*e^(853/2*I*c) - 16461983593387429
705294567317256050448984847501474521523234085808828867857964631377742508443
90402000*I*a*e^(851/2*I*c) - 4829780128388681541713419953174190238612578362
193415201128746557290101817249355474415799153209807700*I*a*e^(849/2*I*c) -

139880302353639240750491684435137322249209761735967189805879375809728582596
76577959002425056421304360*I*a*e^(847/2*I*c) - 3999507572369123703819527876
8780599832619352981502600583944474510975840688388265315582010871954211560*I
*a*e^(845/2*I*c) - 11290502937326806460867891716679720547384498704113418483
5386265805093053876406216627623784532569846520*I*a*e^(843/2*I*c) - 31471121
452937142499278871384574742584918973031441639870412126582700441797744347223
4197727267055736520*I*a*e^(841/2*I*c) - 86623957642181530750728127464111311
5501900542064258358681945153517620235306644376865761912243559851440*I*a*e^(
839/2*I*c) - 23546413672838444747557085543861571414604520196309663641049065
30187054447060414009835524070257156040960*I*a*e^(837/2*I*c) - 6321293101377
705534598501434563578869433768111389255715203589457357528663969729197548539
465525802001790*I*a*e^(835/2*I*c) - 167615577354420865588650781906925881608
19052629045007656502709531547992735484033270393241163681859856540*I*a*e^(83
3/2*I*c) - 4390181120150456850569009393359897486332167308618300385026364806
2593209407109355666758585680043907834980*I*a*e^(831/2*I*c) - 11359046846756
325605798652651416795478177684709910176760999437070925021737758657289652605
0715752951812430*I*a*e^(829/2*I*c) - 29035145179318989889949274155669704377
7221617969524731942883589465588614512062844825790429284872923161710*I*a*e^(
827/2*I*c) - 73326261594425025070029099259126214159993758041023262339626619
3728726497598828508117264547235381193884620*I*a*e^(825/2*I*c) - 18296954886
447219366136705626726114716446217590419862915929740000765974011103480568764
43996820050899846880*I*a*e^(823/2*I*c) - 4511403508274736689214557264253227
256924552084399115779078878703890873777350251660642170480830307883930190*I*
a*e^(821/2*I*c) - 109922736874110426596082787940287968274450210553298017218
49776762014980912735003773441507133726549972274140*I*a*e^(819/2*I*c) - 2646
887067379749675909233214252546196368008033957904180984157401017492392301796
0550488705783118611025231900*I*a*e^(817/2*I*c) - 62991788302738889334378246
177490929980502834395575555703321916458754517369753884106316753560824690112
805080*I*a*e^(815/2*I*c) - 148170372481752681412605856390708157556929105970
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*c) - 344505081824778437645788999066343041845046916298803112223744796287266
870120059274579365813673652734652520860*I*a*e^(811/2*I*c) - 791797833199627
836948743904893083516778740662517693894919607742925640625105771158237179198
488030147594263020*I*a*e^(809/2*I*c) - 179905602323577179718681091771937925
4648844020291348114199995277306725921780651883266662884996146999653568600*I
*a*e^(807/2*I*c) - 40412318710901090466871534282586219688752552627638209851
44556783792101620081675023154167930533684581024874268*I*a*e^(805/2*I*c) - 8
975285853701358274702317193638300392915140185413802501007132352939624150862
231577879738561005464285728950292*I*a*e^(803/2*I*c) - 197094604905649653855
618030405974776279198936239645498502512476341445027319617021654807432872983
91566662175368*I*a*e^(801/2*I*c) - 4279759582983167298530610807902207660664
8329018648897202993638057186135621171784408511776555724216303427661062*I*a*
e^(799/2*I*c) - 91898464215137332126596549205674856438317557197351584304760
723812330051881370624842695522772767931492290924348*I*a*e^(797/2*I*c) - 195
149410499491059972295971340541623205014478485259218183564112297195495432447

981697623978433168097927188489160*I*a*e^(795/2*I*c) - 409847262333561884793
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832293291122032*I*a*e^(793/2*I*c) - 851330476367280675931975863605407903147
652289552972334476403837141273395148705506900158560019626231978044203758*I*
a*e^(791/2*I*c) - 174912593443959921315268323941947738188912530507648037732
6824972897585677418916594095435586707281927023930690932*I*a*e^(789/2*I*c) -
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07964145674445025619936991568879491783*I*a*e^(787/2*I*c) - 7146743752116835
696646457888685953635190050205364442589239754498947790079457620288669958662
869113747109252011972*I*a*e^(785/2*I*c) - 142142105345772785134655638543704
332319345315321319820871878648165896494954210260296992075052433965603872064
39630*I*a*e^(783/2*I*c) - 2796949758150298987804402115989629470528185352041
4801800856230671667421994293925781447451022557190042490393634543*I*a*e<sup>(781
/2*I*c)</sup> - 54452642914272730607209059956928793329411850728280306564009762144
470353759860633440913049919720954752768197793167*I*a*e^(779/2*I*c) - 104893
986154804101296916258335583569488825031337940879705887720891017009057737287
233126607468682692414623104616138*I*a*e^(777/2*I*c) - 199942201925298499861
324281146334926533714538945870178594046781580871260287770985229020625257582
980635730225596574*I*a*e^(775/2*I*c) - 377142540527901948948638950467507037
490336862439884119528845344255236737968714764289755834224007993516760655927
631*I*a*e^(773/2*I*c) - 704009729937813921690706496710889899788434054131439
662107181131420712162600554109916450181425438322392204166968902*I*a*e<sup>(771/
2*I*c)</sup> - 130061517427101052448113664695315832473694446405293294683683020301
5550268173675607727307084706325098884799728006340*I*a*e^(769/2*I*c) - 23781
592730321211106042463797053005815111645937589678722392818969981732015589876
39246325006007724766172710488108360*I*a*e^(767/2*I*c) - 4304076708196868623
182033776806377443199259967769349066155975950428294505280637590876357992866
456385230699336439762*I*a*e^(765/2*I*c) - 771065067832387504303985863253313
53206682438658369247383646577680905625502860759679978431131374349108868965
4262026*I*a*e^(763/2*I*c) - 13674110058458123094352164694691456954602488602
500355510203822923728942351461390874202410489991829018224022172756144*I*a*e
^(761/2*I*c) - 240065213305186194101177870502624116083503068842597365983963
70271394691940783119373097697849962988467299495366349818*I*a*e^(759/2*I*c)
- 4172599134954602125985964954763710995733952767863225515182161583206688887
4425029726202125176968278133786685507865790*I*a*e^(757/2*I*c) - 71805309593
088734534182328800742749493154891157519618233998054566080846925267026007081
349149718882656641899769699343*I*a*e^(755/2*I*c) - 122350052518531164014106
438492390774762115503639398988470214202703839744840033391227823927738136823
467053920486003100*I*a*e^(753/2*I*c) - 206430920167470779129078232414834138
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774122*I*a*e^(751/2*I*c) - 344900353549768914526411942374597368779545029739
787329071887057800090344123223642583873423162351462269773310642073463*I*a*e
^(749/2*I*c) - 570671487070895844624376668699871118872005188740963481132840
070150675796254780502758777604671643556111867698875355315*I*a*e^(747/2*I*c)
- 935142309058644648996306800248685199060934241266676135556650246097289205

034092387373949739321723050371978110125781708*I*a*e^(745/2*I*c) - 151772675
772323885245833852336431135734569046757961595531551453905021540080154769175
8586164717780698532261211210676134*I*a*e^(743/2*I*c) - 24398277418473876406
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90253696743286780667183*I*a*e^(741/2*I*c) - 3885077898135324755861645497801
928788835406962443757329897190841925861451659303725355539278317874676713417
912602151785*I*a*e^(739/2*I*c) - 612830727974489757665370637383400835846202
652485602861930650162676637289161428815346565423045111826183623444214492769
0*I*a*e^(737/2*I*c) - 95764982457308878595876534947130519638685080921098936
19828631489328621274896472800096828116619256422203001427328938060*I*a*e^(73
5/2*I*c) - 1482597913528423746226719438636827970156048051160166462384448383
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797137379235963979112570181508105761299864241*I*a*e^(731/2*I*c) - 345626886
583356492112358151163225379977564500741633872843777815593138933788809596699
59729851744028098328487235184728902*I*a*e^(729/2*I*c) - 5205042453660871068
507693566203030807490703686374070277936847572552205529179649231030372150241
8829383396474602416972540*I*a*e^(727/2*I*c) - 77676555142903920918742869451
767200086404378777218239528959225606553643216179633134107155344214086969807
897418659931017*I*a*e^(725/2*I*c) - 114875803385901547035330733037349925264
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633198*I*a*e^(723/2*I*c) - 168369630997561227559333636030680505097931214578
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a*e^(721/2*I*c) - 244578820412941222413395448190925169184058675448514568342
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2*I*c) - 352141077922736730546898110712009539372175536582481680338223417210
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824411098205470727732618356886370847708708442*I*a*e^(715/2*I*c) - 710934676
236178199315293629653949575146187423228339879602954615038356642192952138658
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330*I*a*e^(705/2*I*c) - 353676461326065445325819276169145836984896738655835
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e^(703/2*I*c) - 47511697379014573962071767119679331793636774785811816128460
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*I*c) - 6328710189082709801343939529235301914241975683533849906310574474924
327748271043712210167104784210984705855467961916306161*I*a*e^(699/2*I*c) -
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9214592564997399354344495040435849157657695842*I*a*e^(697/2*I*c) - 10948781
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655762254832610159791452992357965388879209915336157123282457319545983523340
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136322171921666098*I*a*e^(689/2*I*c) - 296418870982327739739856543305793794
838212447693239802111754473132194627045318804008018024008069296463793430151
30723883672*I*a*e^(687/2*I*c) - 3724112127270207428677853672812764225881806
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4788*I*a*e^(685/2*I*c) - 46402932547295212381676170516766288251068082667686
289662170986814423296955583080165076111831556746240823052636895914088352*I*
a*e^(683/2*I*c) - 573425601373144520885417340865419166341645485251082885818
23624632355858872350966292386326440523022152879291694690278241978*I*a*e<sup>(68
1/2*I*c)</sup> - 7027789085614364628701759332010686918276096400284793099171435196
8190586465418479358035827449094009745978784506403119811120*I*a*e<sup>(679/2*I*c
)</sup> - 85421852279646559159940111327081238178956474950230875118810643314639931
537207781953980988720021516214669798986825724375032*I*a*e^(677/2*I*c) - 102
973123976091249841486572077221888501123401210539358250850710575960244576128
548550903800052937848163824888063710773611688*I*a*e^(675/2*I*c) - 123105242
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706049223826034210909894133154305711632*I*a*e^(673/2*I*c) - 145954292246109
699719912222123781738335390625966506127269098354886794694991875083928883625
876737543812581750368360348693528*I*a*e^(671/2*I*c) - 171605486957459662750
676535846047739137214911399784738960265764988070463454234816351437243248376
883372505719428491381980392*I*a*e^(669/2*I*c) - 200079105947663271955632509
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087111914188372627836780990986990309150303501014036898617069788744248382411
616930060590160*I*a*e^(665/2*I*c) - 265165814533423959664985597822349856950
188516226578299292400272539208068973637723841893399770487082840158358266555
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244218355398012742691434651542769453942299897409407388624108299633805109409
320*I*a*e^(661/2*I*c) - 339562583656041705384495512461799490148091494844161
116673933307762871742676972723303982143114815055600538598674333818006140*I*
a*e^(659/2*I*c) - 379247649666166689932538744439842684315020161725910584926
467825646640883618440200155492765803014212451201418879737084175200*I*a*e<sup>(6
57/2*I*c)</sup> - 419811308608661332767758302246251767306850733653256595228909948
287744169176955320554102884264686683175846620329681501389720*I*a*e<sup>(655/2*I
*c)</sup> - 460516895758104311450607127871317475876181781180615971932371325200463
116468437759451218270512614846167126411504887103639420*I*a*e^(653/2*I*c) -
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401801645044637308666501943921790160591726142140*I*a*e^(651/2*I*c) - 538852

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 653608342888243843496016154319713358492360*I*a*e^(649/2*I*c) - 574502411709
 033654260208328538715135728998551050899066786172225938409580464129402448537
 468149317355804595816458074515452280*I*a*e^(647/2*I*c) - 606377755149130644
 685483641133783894074517633951559003505177038260609373787904347095041198589
 577627025869566789105127854940*I*a*e^(645/2*I*c) - 633369970466231151639593
 498633052500816992588222289708061904408614478071012324018301393989621376114
 495936107929638109352240*I*a*e^(643/2*I*c) - 654382036818887115292417789242
 500921067425262240728690563625955515997936064156892229191882650976483089469
 657316496979497920*I*a*e^(641/2*I*c) - 668367696536351754028217381429124369
 355105991706596728061963354863236879295944678255966593383757885721698666786
 195936988720*I*a*e^(639/2*I*c) - 674372239032754395378205038535525134413991
 821067808642593147982202493843652677927082541781810369810638842234476194328
 917280*I*a*e^(637/2*I*c) - 671573416376600344255476890840084107068748497197
 932624701404034109653368013046774679359731432461763294766813316740570321920
 *I*a*e^(635/2*I*c) - 659320582553919912075191619269245382876605084236933842
 646778081417120118613397607349078853158222767507670564574204234442160*I*a*e
 ^ (633/2*I*c) - 637170080700014936844214998715409219682319023159649705213690
 092322129022799756117985062386207907676879268835925347571814760*I*a*e^(631/
 2*I*c) - 60491494623493828221086113012403617143413939275335944189009476407
 229605663075886937693800533384123491506489579681833036880*I*a*e^(629/2*I*c)
 - 562607154534249611414703842333518335752632730873777259062662265415265197
 318757342578842160394845870139211686607258163271980*I*a*e^(627/2*I*c) - 510
 570917219603298930927086412148133828632499451055506210033280538039998585805
 570450486684217149260136863823464698025514400*I*a*e^(625/2*I*c) - 449405910
 836006748962393603823185186535716033707509621536220013769001652160266321868
 773593038071461031867255150268333080200*I*a*e^(623/2*I*c) - 379979787117931
 179687380662746570800769009286292103738378140902975891504813248606746718458
 290904172947709059780815722352620*I*a*e^(621/2*I*c) - 303409839775241340348
 800685971884147465797827674549775052181699893605576672194645212093459076166
 299568495347142098625376060*I*a*e^(619/2*I*c) - 221034257892392428506776513
 729559457247579927221728582941832739190163959979535188907184118401352847011
 402341171183153330400*I*a*e^(617/2*I*c) - 134373946463544483939193158440319
 190954779730332932854832052915711911612344383418766608011863070070271134159
 756438724511480*I*a*e^(615/2*I*c) - 450864052506273528333921347261618114674
 346898319541826591071586007968674493473147224142350757139877682529058579431
 68960940*I*a*e^(613/2*I*c) + 4508640525062735283339213472616181146743468983
 195418265910715860079686744934731472241423507571398776825290585794316896094
 0*I*a*e^(611/2*I*c) + 13437394646354448393919315844031919095477973033293285
 4832052915711911612344383418766608011863070070271134159756438724511480*I*a*
 e^(609/2*I*c) + 22103425789239242850677651372955945724757992722172858294183
 2739190163959979535188907184118401352847011402341171183153330400*I*a*e^(607
 /2*I*c) + 30340983977524134034880068597188414746579782767454977505218169989
 3605576672194645212093459076166299568495347142098625376060*I*a*e^(605/2*I*c
) + 37997978711793117968738066274657080076900928629210373837814090297589150

4813248606746718458290904172947709059780815722352620*I*a*e^(603/2*I*c) + 44
940591083600674896239360382318518653571603370750962153622001376900165216026
6321868773593038071461031867255150268333080200*I*a*e^(601/2*I*c) + 51057091
721960329893092708641214813382863249945105550621003328053803999858580557045
0486684217149260136863823464698025514400*I*a*e^(599/2*I*c) + 56260715453424
961141470384233351833575263273087377725906266226541526519731875734257884216
0394845870139211686607258163271980*I*a*e^(597/2*I*c) + 60491494623493828222
108611301240361714341393927533594418900947640722960566307588693769380053338
4123491506489579681833036880*I*a*e^(595/2*I*c) + 63717008070001493684421499
871540921968231902315964970521369009232212902279975611798506238620790767687
9268835925347571814760*I*a*e^(593/2*I*c) + 65932058255391991207519161926924
538287660508423693384264677808141712011861339760734907885315822276750767056
4574204234442160*I*a*e^(591/2*I*c) + 67157341637660034425547689084008410706
874849719793262470140403410965336801304677467935973143246176329476681331674
0570321920*I*a*e^(589/2*I*c) + 67437223903275439537820503853552513441399182
106780864259314798220249384365267792708254178181036981063884223447619432891
7280*I*a*e^(587/2*I*c) + 66836769653635175402821738142912436935510599170659
6728061963354863236879295944678255966593383757885721698666786195936988720*I
*a*e^(585/2*I*c) + 65438203681888711529241778924250092106742526224072869056
3625955515997936064156892229191882650976483089469657316496979497920*I*a*e^(
583/2*I*c) + 63336997046623115163959349863305250081699258822228970806190440
8614478071012324018301393989621376114495936107929638109352240*I*a*e^(581/2*
I*c) + 60637775514913064468548364113378389407451763395155900350517703826060
9373787904347095041198589577627025869566789105127854940*I*a*e^(579/2*I*c) +
57450241170903365426020832853871513572899855105089906678617222593840958046
4129402448537468149317355804595816458074515452280*I*a*e^(577/2*I*c) + 53885
281323820259810451411439759853728315471784068284835012155363183588104204929
9653608342888243843496016154319713358492360*I*a*e^(575/2*I*c) + 50051414111
687569446796470145331706246370984263721939905498360654812297696040180164504
4637308666501943921790160591726142140*I*a*e^(573/2*I*c) + 46051689575810431
145060712787131747587618178118061597193237132520046311646843775945121827051
2614846167126411504887103639420*I*a*e^(571/2*I*c) + 41981130860866133276775
830224625176730685073365325659522890994828774416917695532055410288426468668
3175846620329681501389720*I*a*e^(569/2*I*c) + 37924764966616668993253874443
984268431502016172591058492646782564664088361844020015549276580301421245120
1418879737084175200*I*a*e^(567/2*I*c) + 33956258365604170538449551246179949
014809149484416111667393330776287174267697272330398214311481505560053859867
4333818006140*I*a*e^(565/2*I*c) + 30137141726354276247494280303342811319844
751824421835539801274269143465154276945394229989740940738862410829963380510
9409320*I*a*e^(563/2*I*c) + 26516581453342395966498559782234985695018851622
657829929240027253920806897363772384189339977048708284015835826655548698188
0*I*a*e^(561/2*I*c) + 23131634811694770274393784785855608711191418837262783
6780990986990309150303501014036898617069788744248382411616930060590160*I*a*
e^(559/2*I*c) + 20007910594766327195563250951598099940618344458553311779784
9122654482023351610633374676069055506407150129677270455507223800*I*a*e^(557

$/2*I*c) + 17160548695745966275067653584604773913721491139978473896026576498$
 $8070463454234816351437243248376883372505719428491381980392*I*a*e^{(555/2*I*c}$
 $) + 14595429224610969971991222212378173833539062596650612726909835488679469$
 $4991875083928883625876737543812581750368360348693528*I*a*e^{(553/2*I*c) + 12}$
 $310524297035092393865167811728644375491035940742076769967817399860391398128$
 $5412874706049223826034210909894133154305711632*I*a*e^{(551/2*I*c) + 10297312}$
 $397609124984148657207722188850112340121053935825085071057596024457612854855$
 $0903800052937848163824888063710773611688*I*a*e^{(549/2*I*c) + 85421852279646}$
 $559159940111327081238178956474950230875118810643314639931537207781953980988$
 $720021516214669798986825724375032*I*a*e^{(547/2*I*c) + 702778908561436462870}$
 $175933201068691827609640028479309917143519681905864654184793580358274490940$
 $09745978784506403119811120*I*a*e^{(545/2*I*c) + 5734256013731445208854173408}$
 $654191663416454852510828858182362463235585887235096629238632644052302215287$
 $9291694690278241978*I*a*e^{(543/2*I*c) + 46402932547295212381676170516766288}$
 $251068082667686289662170986814423296955583080165076111831556746240823052636$
 $895914088352*I*a*e^{(541/2*I*c) + 372411212727020742867785367281276422588180}$
 $680725597659094510647668238594209452592717403098304203348491058356450383541$
 $34788*I*a*e^{(539/2*I*c) + 2964188709823277397398565433057937948382124476932}$
 $3980211175447313219462704531880400801802400806929646379343015130723883672*I$
 $*a*e^{(537/2*I*c) + 23398582340521584219692739918602629576737063867809667736}$
 $348081290947722283666066820099655484755971820828136322171921666098*I*a*e^{(5}$
 $35/2*I*c) + 183175263910895824505565576225483261015979145299235796538887920$
 $99153361571232824573195459835233402581137624623986434143640*I*a*e^{(533/2*I*$
 $c) + 1422096255936507476331430849012547478681676010516903806540846148028804}$
 $7031021663614568854283660670738808692795002194734697*I*a*e^{(531/2*I*c) + 10}$
 $948781640409755536653618878020227586096531153252918259760911769700775752607$
 $091211059932659598053974790570736751499078948*I*a*e^{(529/2*I*c) + 835921585}$
 $525270059274636632763640027045000249563752770351589923039914836784921459256$
 $4997399354344495040435849157657695842*I*a*e^{(527/2*I*c) + 63287101890827098}$
 $013439395292353019142419756835338499063105744749243277482710437122101671047$
 $84210984705855467961916306161*I*a*e^{(525/2*I*c) + 4751169737901457396207176}$
 $711967933179363677478581181612846070387281280000757129577641826784358070259$
 $730216727235818690609*I*a*e^{(523/2*I*c) + 353676461326065445325819276169145}$
 $836984896738655835870701106999280831997418448425789700221226148531872501461$
 $0659806587718*I*a*e^{(521/2*I*c) + 26104512439239812601045343301245482286721}$
 $738771000303049982303620541264736593591686034766432115959581784629113174285}$
 $91330*I*a*e^{(519/2*I*c) + 1910343223430788602372199148905316683910021942415}$
 $008389270429889705177139255081101273218946424866081924816129975411078465*I*$
 $a*e^{(517/2*I*c) + 138603841629113966677521737046806457587286373607779128668}$
 $2394282745847550566082001450826563620627885230711507370056591878*I*a*e^{(515}$
 $/2*I*c) + 99698359441354323795166381340478739419369614575348281904494353290}$
 $4006038513638159184208380334024402798975173818169325844*I*a*e^{(513/2*I*c) +}$
 $71093467623617819931529362965394957514618742322833987960295461503835664219}$
 $2952138658369772402012030511506524366087182336*I*a*e^{(511/2*I*c) + 50255047}$
 $099219587862641183807985000665977919581454351822399978129428805689682441109$

8205470727732618356886370847708708442*I*a*e^(509/2*I*c) + 35214107792273673
054689811071200953937217553658248168033822341721022234930736468426372761186
0803222344128198352327695010*I*a*e^(507/2*I*c) + 24457882041294122241339544
819092516918405867544851456834270443556042755361490528304811218700574696409
0261842716041958472*I*a*e^(505/2*I*c) + 16836963099756122755933363603068050
509793121457804118094145046106347725583677774110650313941478387593461840350
6189272070*I*a*e^(503/2*I*c) + 11487580338590154703533073303734992526459079
117533429461015509605659384806546748484528708374651502442061828925151563319
8*I*a*e^(501/2*I*c) + 77676555142903920918742869451767200086404378777218239
528959225606553643216179633134107155344214086969807897418659931017*I*a*e^(4
99/2*I*c) + 520504245366087106850769356620303080749070368637407027793684757
25522055291796492310303721502418829383396474602416972540*I*a*e^(497/2*I*c)
+ 3456268865833564921123581511632253799775645007416338728437778155931389337
8880959669959729851744028098328487235184728902*I*a*e^(495/2*I*c) + 22741305
878635147307038153488672825947987978019457328121790385481218602329797137379
235963979112570181508105761299864241*I*a*e^(493/2*I*c) + 148259791352842374
622671943863682797015604805116016646238444838349315151229328570577209911885
92107170607681823985077157*I*a*e^(491/2*I*c) + 9576498245730887859587653494
713051963868508092109893619828631489328621274896472800096828116619256422203
001427328938060*I*a*e^(489/2*I*c) + 612830727974489757665370637383400835846
202652485602861930650162676637289161428815346565423045111826183623444214492
7690*I*a*e^(487/2*I*c) + 38850778981353247558616454978019287888354069624437
57329897190841925861451659303725355539278317874676713417912602151785*I*a*e^
(485/2*I*c) + 2439827741847387640648859734577926475542979210917703206702312
668313095518685617607372855661697190253696743286780667183*I*a*e^(483/2*I*c)
+ 151772675772323885245833852336431135734569046757961595531551453905021540
0801547691758586164717780698532261211210676134*I*a*e^(481/2*I*c) + 93514230
905864464899630680024868519906093424126667613555665024609728920503409238737
3949739321723050371978110125781708*I*a*e^(479/2*I*c) + 57067148707089584462
437666869987111887200518874096348113284007015067579625478050275877760467164
3556111867698875355315*I*a*e^(477/2*I*c) + 34490035354976891452641194237459
736877954502973978732907188705780009034412322364258387342316235146226977331
0642073463*I*a*e^(475/2*I*c) + 20643092016747077912907823241483413891537685
7125870091984391255439368652648269437789361363756117101262608074750774122*I
*a*e^(473/2*I*c) + 12235005251853116401410643849239077476211550363939898847
0214202703839744840033391227823927738136823467053920486003100*I*a*e^(471/2*
I*c) + 71805309593088734534182328800742749493154891157519618233998054566080
846925267026007081349149718882656641899769699343*I*a*e^(469/2*I*c) + 417259
913495460212598596495476371099573395276786322551518216158320668888744250297
26202125176968278133786685507865790*I*a*e^(467/2*I*c) + 2400652133051861941
011778705026241160835030688425973659839637027139469194078311937309769784996
2988467299495366349818*I*a*e^(465/2*I*c) + 13674110058458123094352164694691
456954602488602500355510203822923728942351461390874202410489991829018224022
172756144*I*a*e^(463/2*I*c) + 771065067832387504303985863253313532066682438
6583692473836465776809056255028607596799784311313743491088689654262026*I*a*

$e^{(461/2*I*c)} + 43040767081968686231820337768063774431992599677693490661559$
 $75950428294505280637590876357992866456385230699336439762*I*a*e^{(459/2*I*c)}$
 $+ 2378159273032121110604246379705300581511164593758967872239281896998173201$
 $558987639246325006007724766172710488108360*I*a*e^{(457/2*I*c)} + 130061517427$
 $101052448113664695315832473694446405293294683683020301555026817367560772730$
 $7084706325098884799728006340*I*a*e^{(455/2*I*c)} + 70400972993781392169070649$
 $671088989978843405413143966210718113142071216260055410991645018142543832239$
 $2204166968902*I*a*e^{(453/2*I*c)} + 37714254052790194894863895046750703749033$
 $6862439884119528845344255236737968714764289755834224007993516760655927631*I$
 $*a*e^{(451/2*I*c)} + 19994220192529849986132428114633492653371453894587017859$
 $4046781580871260287770985229020625257582980635730225596574*I*a*e^{(449/2*I*c)}$
 $) + 10489398615480410129691625833558356948882503133794087970588772089101700$
 $9057737287233126607468682692414623104616138*I*a*e^{(447/2*I*c)} + 54452642914$
 $272730607209059956928793329411850728280306564009762144470353759860633440913$
 $049919720954752768197793167*I*a*e^{(445/2*I*c)} + 279694975815029898780440211$
 $598962947052818535204148018008562306716674219942939257814474510225571900424$
 $90393634543*I*a*e^{(443/2*I*c)} + 1421421053457727851346556385437043323193453$
 $1532131982087187864816589649495421026029699207505243396560387206439630*I*a*$
 $e^{(441/2*I*c)} + 71467437521168356966464578886859536351900502053644425892397$
 $54498947790079457620288669958662869113747109252011972*I*a*e^{(439/2*I*c)} + 3$
 $554806987990197776445869286315539003513648984994079123806662496505394242907$
 $964145674445025619936991568879491783*I*a*e^{(437/2*I*c)} + 174912593443959921$
 $315268323941947738188912530507648037732682497289758567741891659409543558670$
 $7281927023930690932*I*a*e^{(435/2*I*c)} + 85133047636728067593197586360540790$
 $314765228955297233447640383714127339514870550690015856001962623197804420375$
 $8*I*a*e^{(433/2*I*c)} + 40984726233356188479317607991100077494323790789672505$
 $0192873946316017656875993526712458367191600832293291122032*I*a*e^{(431/2*I*c)}$
 $) + 19514941049949105997229597134054162320501447848525921818356411229719549$
 $5432447981697623978433168097927188489160*I*a*e^{(429/2*I*c)} + 91898464215137$
 $332126596549205674856438317557197351584304760723812330051881370624842695522$
 $772767931492290924348*I*a*e^{(427/2*I*c)} + 427975958298316729853061080790220$
 $766066483290186488972029936380571861356211717844085117765557242163034276610$
 $62*I*a*e^{(425/2*I*c)} + 1970946049056496538556180304059747762791989362396454$
 $9850251247634144502731961702165480743287298391566662175368*I*a*e^{(423/2*I*c)}$
 $) + 89752858537013582747023171936383003929151401854138025010071323529396241$
 $50862231577879738561005464285728950292*I*a*e^{(421/2*I*c)} + 4041231871090109$
 $046687153428258621968875255262763820985144556783792101620081675023154167930$
 $533684581024874268*I*a*e^{(419/2*I*c)} + 179905602323577179718681091771937925$
 $4648844020291348114199995277306725921780651883266662884996146999653568600*I$
 $*a*e^{(417/2*I*c)} + 79179783319962783694874390489308351677874066251769389491$
 $9607742925640625105771158237179198488030147594263020*I*a*e^{(415/2*I*c)} + 34$
 $450508182477843764578899906634304184504691629880311222374479628726687012005$
 $9274579365813673652734652520860*I*a*e^{(413/2*I*c)} + 14817037248175268141260$
 $585639070815755692910597075048405825208338340280724047627516470799400628423$
 $1733473540*I*a*e^{(411/2*I*c)} + 62991788302738889334378246177490929980502834$

395575555703321916458754517369753884106316753560824690112805080*I*a*e^(409/
2*I*c) + 264688706737974967590923321425254619636800803395790418098415740101
74923923017960550488705783118611025231900*I*a*e^(407/2*I*c) + 1099227368741
104265960827879402879682744502105532980172184977676201498091273500377344150
7133726549972274140*I*a*e^(405/2*I*c) + 45114035082747366892145572642532272
56924552084399115779078878703890873777350251660642170480830307883930190*I*a
*e^(403/2*I*c) + 1829695488644721936613670562672611471644621759041986291592
974000076597401110348056876443996820050899846880*I*a*e^(401/2*I*c) + 733262
615944250250700290992591262141599937580410232623396266193728726497598828508
117264547235381193884620*I*a*e^(399/2*I*c) + 290351451793189898899492741556
697043777221617969524731942883589465588614512062844825790429284872923161710
*I*a*e^(397/2*I*c) + 113590468467563256057986526514167954781776847099101767
609994370709250217377586572896526050715752951812430*I*a*e^(395/2*I*c) + 439
018112015045685056900939335989748633216730861830038502636480625932094071093
55666758585680043907834980*I*a*e^(393/2*I*c) + 1676155773544208655886507819
069258816081905262904500765650270953154799273548403327039324116368185985654
0*I*a*e^(391/2*I*c) + 63212931013777055345985014345635788694337681113892557
15203589457357528663969729197548539465525802001790*I*a*e^(389/2*I*c) + 2354
641367283844474755708554386157141460452019630966364104906530187054447060414
009835524070257156040960*I*a*e^(387/2*I*c) + 866239576421815307507281274641
113115501900542064258358681945153517620235306644376865761912243559851440*I*
a*e^(385/2*I*c) + 314711214529371424992788713845747425849189730314416398704
121265827004417977443472234197727267055736520*I*a*e^(383/2*I*c) + 112905029
373268064608678917166797205473844987041134184835386265805093053876406216627
623784532569846520*I*a*e^(381/2*I*c) + 399950757236912370381952787687805998
32619352981502600583944474510975840688388265315582010871954211560*I*a*e^(37
9/2*I*c) + 1398803023536392407504916844351373222492097617359671898058793758
0972858259676577959002425056421304360*I*a*e^(377/2*I*c) + 48297801283886815
417134199531741902386125783621934152011287465572901018172493554744157991532
09807700*I*a*e^(375/2*I*c) + 1646198359338742970529456731725605044898484750
147452152323408580882886785796463137774250844390402000*I*a*e^(373/2*I*c) +
553838940597037209801420093339031465928345342466968134327170356436277614050
695088234925854972461590*I*a*e^(371/2*I*c) + 183904807224491919846530211566
172865275646148609389197477057237389158397210607704335080668307676320*I*a*e
^(369/2*I*c) + 602659362849005578742780582711292436289148540723110542636999
56444497953786622754638704751947767300*I*a*e^(367/2*I*c) + 1948857617595825
986849208666366913599389167032693503673342407965939565494739504167122607796
8039510*I*a*e^(365/2*I*c) + 62183761198708529277566209666440895464202440075
71780243876369631090631048335601455624785708312030*I*a*e^(363/2*I*c) + 1957
586469631059845138215307872770103497341741976050146298563390984316998623891
058083233354950720*I*a*e^(361/2*I*c) + 607952325065380552231765878979364337
922618562001789681944692673308704528340024671175049604847740*I*a*e^(359/2*I
*c) + 186242701415300150260367661909796221932019732537024634010976646542647
178366376613371237978468950*I*a*e^(357/2*I*c) + 562737379422812172158325781
00073235858273628493027174707116905455498600552994477017353335693130*I*a*e^

(355/2*I*c) + 1676886281374257049735932768574567125736505890927148911282976
0986508383352816248603242238251460*I*a*e^(353/2*I*c) + 49274967292830834500
12099767840203352055720285319781857700963595150774626713503066668245663200*
I*a*e^(351/2*I*c) + 1427665347659715317871214683600301356629818078157011809
631323200683665015207096525546411654850*I*a*e^(349/2*I*c) + 407807461714197
071178872620082184411365896862940021562248885259817851441393751779677614254
090*I*a*e^(347/2*I*c) + 114831957097719192706111697057632872409256933134212
105953255327587315571466191225892722458940*I*a*e^(345/2*I*c) + 318711831146
518357362782254807199851313607428047926531679202659139046488572233192055584
81280*I*a*e^(343/2*I*c) + 8717845121538543965000655508927576156454509965124
578227666111861599530470473165494741121930*I*a*e^(341/2*I*c) + 234986055260
470027437041195245292043865000683648143850818180727987509692588868747534963
9360*I*a*e^(339/2*I*c) + 62408264396102903731270928467212697669787321253890
8890262985442647874324392878156690829740*I*a*e^(337/2*I*c) + 16328748432559
999472281704244122013010769731624898379452499760390283264397733741266187276
0*I*a*e^(335/2*I*c) + 42083973507831971365951079505656822321370151622842020
026197681285864461893407471640353960*I*a*e^(333/2*I*c) + 106825368781355138
11598686678158308084092593434324791253827853349664088304211712490029720*I*a
*e^(331/2*I*c) + 2670331080438136739251699421188477152816983882646518313618
300055190498774861721956084120*I*a*e^(329/2*I*c) + 657243781869868584332142
745237797306576908038181174879375861022306506494725134312464240*I*a*e^(327/
2*I*c) + 159255512903666965947392050482627777652855525628647598071487710356
765619338164447678480*I*a*e^(325/2*I*c) + 379842269427844667280669633183561
13137634905014288785261844980339121431692865121528770*I*a*e^(323/2*I*c) + 8
916313880908616061941672992999984482986075785970885880652620253394990773311
152181860*I*a*e^(321/2*I*c) + 205954451320666914305466797161323566250098209
6295238818162792429954428814629558400860*I*a*e^(319/2*I*c) + 46804691242254
3827579364617290976131668618292605267581426808880930942056836225740850*I*a*
e^(317/2*I*c) + 10463247928533016870456762417320577420297514286956165139814
2015702621321511418637970*I*a*e^(315/2*I*c) + 23005253792558757618650193250
261667174892446545331020900285683620126145948425151860*I*a*e^(313/2*I*c) +
497385563358030362012603263786947750010767618249987307609451841339730360380
4906240*I*a*e^(311/2*I*c) + 10572675902723256366397739037525311715235495371
93854805338038288437194124266978930*I*a*e^(309/2*I*c) + 2209122678371764024
57239938736238943107985690499222615150129577567013343972814900*I*a*e^(307/2
*I*c) + 4536417342314724750813554956201153842868356042083917260831474702678
0336705078884*I*a*e^(305/2*I*c) + 91532833217409971229515786930886454503366
13285754578953646231535187679288519816*I*a*e^(303/2*I*c) + 1814351004638830
535450974649080451232909962840518095183307668250502738979794444*I*a*e^(301/
2*I*c) + 353226389272834924314189317058363954203171842049410810171865734368
883820818996*I*a*e^(299/2*I*c) + 675268298037140882078412174800985609438380
36893752572161473089623117139971604*I*a*e^(297/2*I*c) + 1267336141913883494
0617178471273655660069856526406967342118703762672110480200*I*a*e^(295/2*I*c
) + 23345174198859668743509654880268166794662083102598557325074351051753134
57396*I*a*e^(293/2*I*c) + 4219747349096187780741342759182006525742147279404

43844534502373085580328604*I*a*e^(291/2*I*c) + 7482563792634097992622245193
 3992320121926362312065461272342106878301448536*I*a*e^(289/2*I*c) + 13012954
 846383921360783439001134235337905843280455893166840706360597524134*I*a*e^(2
 87/2*I*c) + 221894153908280997998580146226159783968435134891227557737760365
 8269150756*I*a*e^(285/2*I*c) + 37088479298325400055072142587332657491012428
 2599731660698821060848882320*I*a*e^(283/2*I*c) + 60747701724909506776211674
 773541506177265806300575091658980807046860704*I*a*e^(281/2*I*c) + 974738630
 8281902182329846941952263067390253415184368339278176002138446*I*a*e^(279/2*
 I*c) + 15317187004832417969924973618448513278312446142210602969590007998433
 24*I*a*e^(277/2*I*c) + 2356471921691593101191613443407382001865279341517292
 51738259408174007*I*a*e^(275/2*I*c) + 3548072343823171284468478510584757091
 0860568249280080967679923825908*I*a*e^(273/2*I*c) + 52265900544796578224837
 80963008017596290103718605086715148151861646*I*a*e^(271/2*I*c) + 7529792233
 11236135673287737101570895147168873416246633317325038975*I*a*e^(269/2*I*c)
 + 106052903363069013135578220340114498079609654804470793711887113055*I*a*e^
 (267/2*I*c) + 1459710766342879044191244915105556978265555105498817253810600
 7306*I*a*e^(265/2*I*c) + 19626290453792900877515775414919491704740150645549
 05158361368638*I*a*e^(263/2*I*c) + 2576627521764579828166282791344882800098
 73022281248928269109247*I*a*e^(261/2*I*c) + 3301518918344210573173095210268
 6078554556811480087134425669934*I*a*e^(259/2*I*c) + 41268884219832498876875
 07689195856719564200342340690922960020*I*a*e^(257/2*I*c) + 5029991671775131
 08237013793834321163983016591115064062192504*I*a*e^(255/2*I*c) + 5974840826
 0336333152741949120162468054098096806527992694330*I*a*e^(253/2*I*c) + 69130
 27812800192462704129875030929367293369791365329368306*I*a*e^(251/2*I*c) + 7
 78664137844059492687126944618806483697276408481076541824*I*a*e^(249/2*I*c)
 + 85332954326120674272311677629126536894715822244726480090*I*a*e^(247/2*I*c
) + 9092846502857218261462182868864364624762389989516884454*I*a*e^(245/2*I*
 c) + 941492824021872538571407241795810657020217893835828847*I*a*e^(243/2*I*
 c) + 94660893371344177720163398371692357473361249542281324*I*a*e^(241/2*I*c
) + 9235203345455625183635292584058316806336315160604010*I*a*e^(239/2*I*c)
 + 873599857808953513616291826808640309239785388357815*I*a*e^(237/2*I*c) + 8
 0060329141110785684222094424093216574599312586835*I*a*e^(235/2*I*c) + 71021
 23372769126439065816004234917371096063247516*I*a*e^(233/2*I*c) + 6092972628
 40711915743663065579736975124435730278*I*a*e^(231/2*I*c) + 5050323645602170
 1342291384690765044913121286031*I*a*e^(229/2*I*c) + 40402580917125792223490
 27082245857300045844265*I*a*e^(227/2*I*c) + 3116155990099984570554286265626
 11218440152906*I*a*e^(225/2*I*c) + 2314386102212629093579514744696779433078
 8764*I*a*e^(223/2*I*c) + 1653132753283300154895933847581652791644965*I*a*e^
 (221/2*I*c) + 113407514349957436967142625811402104148561*I*a*e^(219/2*I*c)
 + 7461020183678979972532247003767832497030*I*a*e^(217/2*I*c) + 469985499033
 039677874623273935334240076*I*a*e^(215/2*I*c) + 282975550978933930210206766
 91580988233*I*a*e^(213/2*I*c) + 1625446980015332177755918014501608982*I*a*e
 ^ (211/2*I*c) + 88891629710203223556668933908311334*I*a*e^(209/2*I*c) + 4617
 746921046245570620730707402520*I*a*e^(207/2*I*c) + 227298420835979818145918
 471098570*I*a*e^(205/2*I*c) + 10572019483402811301975746281474*I*a*e^(203/2

$*I*c) + 463207036476152149238073238832*I*a*e^{(201/2*I*c)} + 1905221736235825$
 $1291769232228*I*a*e^{(199/2*I*c)} + 732777588939374143249503406*I*a*e^{(197/2*$
 $I*c)} + 26237560685859258651673169*I*a*e^{(195/2*I*c)} + 870123185139944470654$
 $786*I*a*e^{(193/2*I*c)} + 26568646859265646058950*I*a*e^{(191/2*I*c)} + 7417642$
 $52188078830689*I*a*e^{(189/2*I*c)} + 18778841824711012321*I*a*e^{(187/2*I*c)} +$
 $426791859624382434*I*a*e^{(185/2*I*c)} + 8600339740332756*I*a*e^{(183/2*I*c)}$
 $+ 151262256738489*I*a*e^{(181/2*I*c)} + 2274620402080*I*a*e^{(179/2*I*c)} + 284$
 $32755026*I*a*e^{(177/2*I*c)} + 283618504*I*a*e^{(175/2*I*c)} + 2116556*I*a*e^{(1$
 $73/2*I*c)} + 10504*I*a*e^{(171/2*I*c)} + 26*I*a*e^{(169/2*I*c)})*\tan(1/4*d*x + c$
 $)/(e^{(531*I*c)} + 432*e^{(530*I*c)} + 93096*e^{(529*I*c)} + 13343760*e^{(528*I*c)}$
 $+ 1431118260*e^{(527*I*c)} + 122503723056*e^{(526*I*c)} + 8718181624155*e^{(525$
 $*I*c)} + 530563624556832*e^{(524*I*c)} + 28186192554792138*e^{(523*I*c)} + 13278$
 $82849274858880*e^{(522*I*c)} + 56169444526926562260*e^{(521*I*c)} + 21548641447$
 $81257856128*e^{(520*I*c)} + 75599817092670157806639*e^{(519*I*c)} + 24424556298$
 $94502983849104*e^{(518*I*c)} + 73099207817335597247098038*e^{(517*I*c)} + 20370$
 $31259470368160131922320*e^{(516*I*c)} + 53090127264630963470039804475*e^{(515*$
 $I*c)} + 1299146645993240318167826532288*e^{(514*I*c)} + 2995254774926549967525$
 $7842032197*e^{(513*I*c)} + 652650253343206047453620559993840*e^{(512*I*c)} + 13$
 $477227799524701956579274210395326*e^{(511*I*c)} + 264410375780310742518099326$
 $419685040*e^{(510*I*c)} + 4939666610818025798809586352543471345*e^{(509*I*c)} +$
 $88054927598941411145869950813388040256*e^{(508*I*c)} + 150060274793739728640$
 $5577818722691539392*e^{(507*I*c)} + 24489837337812338687718622491865013839488$
 $*e^{(506*I*c)} + 383360155801054824529764688213114368047154*e^{(505*I*c)} + 576$
 $4601046563151304213854710715346838447392*e^{(504*I*c)} + 83380839911837894453$
 $136303673785039051506805*e^{(503*I*c)} + 116158141373397175153362251190904691$
 $7188768400*e^{(502*I*c)} + 15603911277687607099721623771744933086920587272*e^{($
 $501*I*c)} + 202347509724462171313966643580234078508179838320*e^{(500*I*c)} +$
 $2535667460650279776834561566186591213109251642859*e^{(499*I*c)} + 30735366512$
 $830562160991166338490057308062762518496*e^{(498*I*c)} + 360688613036389349413$
 $809780004559963548775423325255*e^{(497*I*c)} + 410154543993719579395995670844$
 $2496709433800261224880*e^{(496*I*c)} + 45230940039830738332025694784646206844$
 $854827698075736*e^{(495*I*c)} + 484093410240488718655917025303662581091659126$
 $182344528*e^{(494*I*c)} + 503202490340145182407421394376601192202650700631198$
 $2753*e^{(493*I*c)} + 50836369508171099437019348610847391946736185108017183136$
 $*e^{(492*I*c)} + 499467506558531733671585862910572702811545035730398749530*e^{($
 $491*I*c)} + 4775398607100853263534207733818266777478693412738731031680*e^{(4$
 $90*I*c)} + 44456708175258821024400946210535004523775722190977468484496*e^{(48$
 $9*I*c)} + 403212225957798188840846139960995624144491271694336796459584*e^{(48$
 $8*I*c)} + 3564764890628724017088487996688178929195787613958545474804845*e^{(4$
 $87*I*c)} + 30736217404321009965231037419663053962881035281709221697785072*e^{($
 $486*I*c)} + 258585348715977270155829115684193411072034541491364393985491350$
 $*e^{(485*I*c)} + 212370296918887131826671878122392706783994901572729388406538$
 $8080*e^{(484*I*c)} + 17033886027390615741040977721655541665612162275485028584$
 $310890417*e^{(483*I*c)} + 133490210052026183779673313868332303530332906163247$
 $194627808410304*e^{(482*I*c)} + 102253643746829673729306586270524644969368741$

5559865844306888705423*e^(481*I*c) + 76590105201875496517771183576768719270
81898989131125755798204236112*e^(480*I*c) + 5611708107634117538408757018518
8538660375932013674735519055227368366*e^(479*I*c) + 40234969226612115893400
3582839428785116904903936409545602519219664720*e^(478*I*c) + 28239051519365
86678382525706564457280290098698638597987628380245881715*e^(477*I*c) + 1940
7979215594566593535008103303255257745408070082431338945184797463936*e^(476*
I*c) + 13065766022656041933512143438993896188459543406998482430714933213174
7540*e^(475*I*c) + 86188485109499190876424680547467242860375731548445397471
3612812215428992*e^(474*I*c) + 55725511573286711210162164163075961618619559
69011697222340926210112854418*e^(473*I*c) + 3532444720677901811537805282078
9411687581004582367431006205879633729015200*e^(472*I*c) + 21960128133951556
1500261478844190024870555261281946058839614044697037963695*e^(471*I*c) + 13
39214374254245553564884406801945353385000254030655765953770237607180089968*
e^(470*I*c) + 8013729580790752434361964945761543761469520791210746972675870
481058674277844*e^(469*I*c) + 470650446111351581084873533674842431026982488
38312635876283099427442745866704*e^(468*I*c) + 2713612075032665707344865170
77181014801775322183181055638619257836143271472358*e^(467*I*c) + 1536333238
444927583532734556016494671674916578907116984548489078241693926940560*e^(46
6*I*c) + 854301344112621233483354066506962147247908583804136056455072203672
3654297540205*e^(465*I*c) + 46668223548266017806854592468100570289355960869
613650856575756758180182223308768*e^(464*I*c) + 250501028608928332469340456
829902067712233644464602753159945727868485722395506952*e^(463*I*c) + 132149
805527130085142999386663161987442453442518818359204972768757103215643507728
0*e^(462*I*c) + 68529932231457366873288853116177954355929408414398663510796
55652312894721972796266*e^(461*I*c) + 3494107161327670464947794304333945020
1504075335160361865916029213860778606230624960*e^(460*I*c) + 17519317050061
8300241515632381912285157790097816049220671217212220015297133400636060*e^(4
59*I*c) + 86397993362233034955629682002839551319870806494050570212606865293
6800794826651264256*e^(458*I*c) + 41915425006568261480933394145441591439644
78472492315931809171859902114109005939942952*e^(457*I*c) + 2000800680303004
7137293278250321597113540716201983333126349281186679153199068045257216*e^(4
56*I*c) + 93986915313068179149083606065681482780836060510530154618486949839
467131378859885998210*e^(455*I*c) + 434546676780280045346344498763892540797
175105756827515509297024187660299345484920192480*e^(454*I*c) + 197779298066
581813565130009432623915860544887080697086057732538502860998303453467231850
0*e^(453*I*c) + 88627521427569572856813408857649045979353495693553218156477
21172537159186491471311666400*e^(452*I*c) + 3910803125560180947653753536961
1844440844903751605645023514572352045248104262933598850730*e^(451*I*c) + 16
99563279699297677390209665262925328370450547712754455653441737668654093670
6073847337600*e^(450*I*c) + 72752101071839422929177407384469425579873866706
7535379759732795567942578751384250780476310*e^(449*I*c) + 30679742964317473
64198159623962463671617006419626851426148418602934852907379021659761911840*
e^(448*I*c) + 1274721961650332054135634306256284736860162214085678602544581
4532037904111523242298235713300*e^(447*I*c) + 52190912207661824215812271854
269748071292843243227894769229690720010547141334131610989636000*e^(446*I*c)

+ 210594301385648471184329078880317504953361839954159427434009884661777259
 752542647709150036990*e^(445*I*c) + 837579206923411932458786486765373533946
 545239708990769488724813982189165104589895518909256320*e^(444*I*c) + 328387
 476055581867672630948030673442015509858394807446901416817187444217010964852
 1627538755920*e^(443*I*c) + 12693496932964920565073673637181280088548682508
 880255337280065006566138696041797353216584528640*e^(442*I*c) + 483794897564
 340998438577918165893794068150426093403787475864371457816462454220451012304
 17309900*e^(441*I*c) + 1818346614061779013153301296771453811664491884131941
 41169344354754920969034952610378945282257600*e^(440*I*c) + 6740255305431330
 088948457752366252374507431144735445378181704471346071025756766760566753289
 61590*e^(439*I*c) + 2464382190807439609079774226855679629367885709776435876
 630851716253962696192341706239192878728160*e^(438*I*c) + 888829502875102466
 704420383760797610148005313441861447462076752282486891195988435266644491740
 4000*e^(437*I*c) + 31626644674725547731176795687527653571305969985923688392
 112164915553242573269490908989570248533280*e^(436*I*c) + 111034148797008819
 443143895644469242295049867464313710969257619338899133799285616020069872611
 710850*e^(435*I*c) + 384655842080666274454063078784837174998949052500975322
 162003392549953413592461519365177908682078400*e^(434*I*c) + 131505212093069
 212210229710532762284233587074342853089107298353586228009444660772347380047
 7453914130*e^(433*I*c) + 44372109178431823477643495444439046990200565950694
 70847193617092114714077633077234972825351226979360*e^(432*I*c) + 1477795509
 661712899871274518207149536217650697318308165023360527405167762497046434024
 2755840025673760*e^(431*I*c) + 48584258153140280447314836868772131390195412
 419046732778458706015096881437076337910793584122475073760*e^(430*I*c) + 157
 685845528850918721462877864435090257583149415561323427386562894447598277935
 629800939237175625149830*e^(429*I*c) + 505293663123015258878483025738812813
 203397766845340065381261016353419722382620393032535960660921950400*e^(428*I
 *c) + 159877110105819269227052899967744474268563100623245618584492522014400
 2305878120380828483988663574829100*e^(427*I*c) + 49952419562791381802051867
 444016880243882721139212556637349569469275713055331467768987878780596851084
 80*e^(426*I*c) + 1541311121148602393729497082079737671608134478816338654352
 2421939737507962125854981881879168348260330000*e^(425*I*c) + 46970224727117
 281826454045018070670522559756627580347784535320014963482632359729444541885
 102274546002560*e^(424*I*c) + 141379938253556843280565505807403304130606130
 725434751745794079833141361748917639986145377066437210546190*e^(423*I*c) +
 420358024835146798583611210145942154684437949365647899088372524802156222884
 839580011688655664280691773600*e^(422*I*c) + 123466804189240997878001808175
 544021601258247639694193796589963195307920397422213879460432849897214476690
 0*e^(421*I*c) + 35827180021632960614145367037151098971071982527392845461493
 43102348456124089657428594946438660859773886240*e^(420*I*c) + 1027160253020
 288900249781351684945259097151280952906066519730109705221006457608834802323
 4671975463677418470*e^(419*I*c) + 29097651061247453406647569781836910062165
 559852359052804259154165687125428752562385492373749486351714453120*e^(418*I
 *c) + 814520814138291118288754175642500548460376933128114804929091607581959
 89155768107022568350953861815940704090*e^(417*I*c) + 2253205325932206577679

411092895162489997945210155641349828272417100196754866944996893124665619072
12627820000*e^(416*I*c) + 6160031160229795849371257017578872129983543009899
91362628038861093914561332071191909714949426587936910303300*e^(415*I*c) + 1
664475034387211809394917743502937638978574937754764763987835872410449930690
131572904279995484581013965001440*e^(414*I*c) + 444541225929547462506765951
419831296601541629996893039334563034591410972074057361888498052001002845149
6996210*e^(413*I*c) + 11735856926245118493113091002501604032341876985999082
823520672241530200188223826392982302194084667538488665600*e^(412*I*c) + 306
275810542219573783905472892776091295728139310827335202473872260000200435382
79468776707958420892547870128680*e^(411*I*c) + 7901914955876656925478398848
723238835290914498274717185677246322380899336709150340287646727017612434269
9654400*e^(410*I*c) + 20155794742479409802677247846204088339586751256293086
2943753568690084015585598010154781548625239409581907397500*e^(409*I*c) + 50
832459930108546016697862968303266142765447408293904809793963839156729879578
8389433842285751054665210868287680*e^(408*I*c) + 12675970172948129134001462
760421269299864802928701903991075543110799642272801965224753701087384778563
11765699610*e^(407*I*c) + 3125683493178701743479704750307490178666292150720
179363604335113528623329684606343185540756019935662148267863968*e^(406*I*c)
+ 762178879191204706203884091779937460042889225819436763668294435609668140
0246312138001769285020661445991073249416*e^(405*I*c) + 18379807084003359766
027649217621144116091735572216620788861535803449702273802588359076704241840
733513439114113248*e^(404*I*c) + 438349721429193776853786922330210637445540
33100928502737480438978976746989895784070951905237783490374305934542955*e^(
403*I*c) + 1033997554672574364898478376407537547182043944730557950014676043
2641987655556873829531737211096115196005647730480*e^(402*I*c) + 2412460212
824400617929083177830328761948015971332060520912869970437291453457558057100
81489006741839439573984832678*e^(401*I*c) + 5567563887111823403410261927342
195461136517683173805390058936790493947140170636985652727288136690547790772
08977840*e^(400*I*c) + 1271033082938048950201360554831270342662343991277504
612342300366025046741742856580445289401786656311685859023084716*e^(399*I*c)
+ 287049613141231445183467471535358943955329443080853193346608628854370924
6230769151392180699413405623017247753532944*e^(398*I*c) + 64133818958559251
847582314510625563803285949385110065770152181197865363902130572840182022016
31094434819584025113465*e^(397*I*c) + 1417648365287570495720201334324111790
436997779665384995990252442198063518901163481565327960549778338288893276673
0080*e^(396*I*c) + 31004319206069417077069363141423487431828009098184744635
678652284177439464941651812564519144918003174108077634846014*e^(395*I*c) +
670917061305296691250198992100215765802378434622295353862950870761892978499
95931360645605292130961496106707521506432*e^(394*I*c) + 1436576871380447969
429471197042595384588181994235168246745862936910561192098663581236377722454
09530799230553767222252*e^(393*I*c) + 3043844711068133360102841601239063704
338888284906274226525512369667909161745208577591439301401871734923949819082
58944*e^(392*I*c) + 6382188929145337415058063996624885560667836004960918764
08374974877448971778036074996245581124283460438065182071976085*e^(391*I*c)
+ 1324311324984027428355222938147682378672860708817161741448749689593588020

860847508703702325320304649883120684987556400*e^(390*I*c) + 271958928348374
392604080510108034192124453031125460725092919277390933152322663503581567286
2569296693711643521070331394*e^(389*I*c) + 55274988490311783558612300093266
682839262900821584671180006985027193799390459183442221927421457112570280409
74074674736*e^(388*I*c) + 1111949964536320108088106282488633849242537565844
8977935535846349290425821570383090425411418521516670371372045206568345*e^(3
87*I*c) + 22140735001708603270915180769241391662035578755903979148909213603
822554792749183517160255571915875356439553717130797888*e^(386*I*c) + 436383
000758151710259462114644656896189657734886609858579456574798540851088518579
11222911989837615452608512356008400295*e^(385*I*c) + 8513953323478645577958
995946490063776073572970562122138000583720836915779467304967542879981787543
1430246332625899630160*e^(384*I*c) + 16443750067690689274132326015439427850
395456193602013359658180644935724027734944792733451710580999530009354927993
1273178*e^(383*I*c) + 31440903580822586156559543693835494544547399104312972
2046747228813030925204968503418566818838611866709807040793495364496*e^(382*
I*c) + 59515761550043151494747928233654705382792608791642526349718775702941
3385471835434198246807096214536895441388306027237899*e^(381*I*c) + 11153982
855456015505333280456001843179938993592179967298183407042218011956674104858
46996179056733558512452238583160792512*e^(380*I*c) + 2069698289500860643461
665762373807957513019424041178871904960551829412449344722432125679417958403
007551179298315947373776*e^(379*I*c) + 380260499670589110696462063384896480
703709885451018226324303059729563076035359753197432475226638919318576087827
4188013440*e^(378*I*c) + 69178389452148442784933304593613949233723338536198
79637372673184942859712431066345726870422099893124890777678037369988150*e^(
377*I*c) + 1246214044053725808492859670987206685775707094312486855450094815
4756863454308032925408340311237850017814707896986969086816*e^(376*I*c) + 22
231341131801535345406399037721686840208397941952580135584645966746736656716
271554826476282991066076564921432614339399735*e^(375*I*c) + 392742004143298
611693979445162250010812274333985850072063992312119071577953597196482415987
54266579840244551491476467899952*e^(374*I*c) + 6871246601598564151246858617
365974773487959171009835465278612493602307394314104957360664856300535941171
2764895683903806088*e^(373*I*c) + 11906059184966054683476569322767644906758
414824888267844750482607723633344451345409512666875005729581119164335690897
2191440*e^(372*I*c) + 20432555726518600076740271023084789645976158392276369
8235433212833313077783041040074669379017394836761539649081690630811665*e^(3
71*I*c) + 34731005381093529041945556055595731412956921073574598323436965997
6413374774078000173070075248654524917179128950507443058208*e^(370*I*c) + 58
474957368230458617938462884488332758149896988654038037889676799907561496400
7174600811092945356635118795824799369716742109*e^(369*I*c) + 97521033944404
931875728231176351778667322317559445794638327926463508504100491730029590427
5433144848532459919875479817581584*e^(368*I*c) + 16110925414000605259548593
752641941783476434718370782014462624356151429445873378335135860227298495233
58436493586042252995608*e^(367*I*c) + 2636662410430799340447522284778244283
740751068658140726576446671207798325606832295937705061686297930296382338892
574900819440*e^(366*I*c) + 427482690772059175252671133682087150084434564792

238547153435933360618957183244464136413289310866357620513387067215626416411
5*e^(365*I*c) + 68664253375186682626626937509089569657329245781421816306221
57802899880874681551031136314064199948604001529894566235238597088*e^(364*I*
c) + 1092721060347354481027979234784453607458889680623004111008954473160586
3146104181739039426674855453466097402330688331845602302*e^(363*I*c) + 17229
502824367647334400721998417596703948657394738805209391636597370380572398964
715080095366818322029152193635869784095333760*e^(362*I*c) + 269177947010866
150978901202368905011051467999960217751957108662262286389847034568326941532
30611607263444183501026198563419616*e^(361*I*c) + 4167044037539054364341821
934227174804003507149011908058528152249818881837590690036870123453130463316
3446319945130196476913600*e^(360*I*c) + 63923019433761989090614801288635098
123199445102303122616544648208998767803944455777042886552738499747183713136
069104651812215*e^(359*I*c) + 971730550247426800586167224613688926611412955
402634930130327460835361573242683333904003089583183702191548871697022574447
56176*e^(358*I*c) + 1463904484563511812182373827403741241916648199977469880
76598391862733629670142241546375533903130605297580105675355629160198162*e^(
357*I*c) + 2185631666596493122474836409562721492124991153838287710296542833
63972585118090479413696638108156385244646591328454425745117584*e^(356*I*c)
+ 3234131780148410037141511382460791525763609760350584578904099377387231715
37036573043681997163745313602400139153046673668433091*e^(355*I*c) + 4743230
435631005423773386299319662481291759769823324460180560093911540204388959031
40822967769494019446166779954024655344116288*e^(354*I*c) + 6895184493287935
599032604181499741902535783400588950355896064682446805915561181703040050375
63669880057908765898949268614772285*e^(353*I*c) + 9935556536495211272264439
608202336493864885100818925458667004440966615827904412418308556095770620395
55625090943332264901780720*e^(352*I*c) + 1419164481422176573858234013898999
62882232233309573730716310743838935832201454293617293175086458389621425307
051750612129761498*e^(351*I*c) + 200949611009268773815278208568373722272796
882429905873921544608349935146762533444967075776406669065615094901494404399
4822823920*e^(350*I*c) + 28208192985622159591075298072896284496213867989894
363693931160698940187812010002756331044983989593466317955680225199744001302
81*e^(349*I*c) + 3925697658415778352768103942856011840211642769621717217996
614398473887186074391482638547212826538270453912634540299792270321024*e^(34
8*I*c) + 541666280405243634958559598281835795386625846164435401820515891774
2576425344364964596750653177677803492186817305171175032011500*e^(347*I*c) +
74103726128911522243646332961280439716571932803277543043822356727737814030
23814127610355562505271969045177704726054907145784960*e^(346*I*c) + 1005220
952436958182758815498534554967803144574449998520825938543560974027240030145
4246872041775159838468077381562338745636398374*e^(345*I*c) + 13521230411945
436915558854706851796541567399811656870021567352975326815467817846533289123
871696056195231696146162720992221760992*e^(344*I*c) + 180353273381774554711
775685948516829779783464497771935720876885103924268845192729915608513263938
52241961470040819793627127923997*e^(343*I*c) + 2385639856556280203069527817
421264083328215417400645945829264406094792490731373592156169015393990601751
8647182491616573724049744*e^(342*I*c) + 31295263688189838313772775873307260

334117227258629501992358695636092662866062819845689064235813622974150120668
921391878398978380*e^(341*I*c) + 407159889637019189500203483367364234205133
113590104852469190746528833939708053744708301562297056473122654775842560272
12762941040*e^(340*I*c) + 5253922334674077114258709237025706953606031964443
950166761048276795580027605289243215279881460797511036622494508142812188847
3324*e^(339*I*c) + 67244087969080703823703257199663047606890610482090494619
492802935130215979819469966383336788693900139115594646893784095418472336*e^
(338*I*c) + 853681184302153128482312917396737358877462018516662996003921994
18764750086828198719872744047767783667325326289221881974987582215*e^(337*I*
c) + 1075047374065769161234803991697596333283214074194005100174988498305986
21565428266546315933920821527544726380201659114903834605888*e^(336*I*c) + 1
342977420234794299046296161045596100960747587210680227044689380630170596880
23363436458971534964665036319889119229809973806909680*e^(335*I*c) + 1664323
329225891951305583292663987533898239557375985275560960930624735597695457723
21978969318904192572733997888230986469005970880*e^(334*I*c) + 2046222955357
291095198299167898672253194297051625600826488403949654231128093365919212903
09392396263834674368977840527147037426908*e^(333*I*c) + 2495930722825658663
983899515096192026826344554871287146314618917708232013675276457937702037887
84677343934971424317987895255031936*e^(332*I*c) + 3020606380308684634611394
422793604997189069174825248944833562201961383770508289113830568604253701611
57201493696073712322595776808*e^(331*I*c) + 3627063075638432311356991574185
107324524206140136241688793121876452334501539279757933268347807413912034301
53093712635355523960320*e^(330*I*c) + 4321478564640869380238115618086785895
947020479046742822979596588001709844567990677518780448066190124526368913507
31618278545690160*e^(329*I*c) + 5109076151111345074526238461471371414502757
224443163855316484292308516866358277174884645003316233857774007449505384106
37735936000*e^(328*I*c) + 5993784847717334748093761424015548502070649721181
375729492575036514445419393090252768960496225156302631621845263943172854573
68300*e^(327*I*c) + 6977891069259246148167137476846827850836598190279522444
47043355869741368500452561164024636073401929693105801105522738405349028160*
e^(326*I*c) + 8061696713276255324245753400897757336819949915766744469223540
99714615192085443245663852257001115644286660304979476023966071898200*e^(325
*I*c) + 9243200528675225840357774957610723512225344207848619600018210205094
68146356756433795246446491396141583854513687104334429566707520*e^(324*I*c)
+ 1051782100428834371944508170051219187116816349766953322637182149610875004
223784784183284961906494422955462431208645690802526770780*e^(323*I*c) + 118
781794307939031610880232479811012902082278266008724859936764348120020604682
2166144285425922229375413676535071141005286431481600*e^(322*I*c) + 13313961
146267230358024621235315820503397499960141524528353059563674253702227586217
53922458727524856072950880960657564720475838500*e^(321*I*c) + 1481187117089
246662466955694965677855524730313260552690821602657176218737426245522795329
891464091005878304304075953693546767206080*e^(320*I*c) + 163556974464142190
065788638128907665322758017205667758745140206923435528368748965961391376195
9140773339736014790081814516625224440*e^(319*I*c) + 17926490780896322989369
457284819693349643915975062850884883506229372525334209808031443164317014521

90522716124797875257437516360640*e^(318*I*c) + 1950286550780181919244992961
204487010056460362845218501674423766266321558791436917317878702232679213868
287926294665202769722927380*e^(317*I*c) + 210614190346834430711254976120248
454340279425235248219981741042486967726271509828843764651868348794546277422
3656471345899082156800*e^(316*I*c) + 22577262191038562868128330126815737654
962622414206129320761431511719608545541241446990230098420805151579235293571
89869943515991200*e^(315*I*c) + 2402464595569686086120001803034211056739445
588621946141384106162886246161815149763025030834875234067267774023433418269
982431265280*e^(314*I*c) + 253776641546503033081547174669298859606991189469
722505292832045254217558715484809648333120980743011394301539836266967333795
7755720*e^(313*I*c) + 26611006479757835838282351392014419301783966433834239
035838625472558807723820492010155372149008327456015197371418498025066852640
00*e^(312*I*c) + 2770073207150768645597507281382065497924968466054527414122
339827333783770068305883487309979315983718403740872884345746380680204260*e^
(311*I*c) + 286250312632046179777066778072564418499125562317462617567905067
2100848988119391841466573417019247590580735265143427289340450811200*e^(310*
I*c) + 29364942143518684987032394554267711043448273062675589165508774672324
55153286140521089582733932202553130712723836983468866230908800*e^(309*I*c)
+ 2990498949622543608538129380283866335190087115124858818143787619186957111
903765723974899651518555144924290346242595167274383008960*e^(308*I*c) + 302
337164350822502717560317521295321902248504585073184530751900827738515473146
1213388035579159917590062343527464977286601165100620*e^(307*I*c) + 30344083
555309570757877317453225679816846165501628454732576796742802169473567857838
43205604202307836897073595410412575660465787520*e^(306*I*c) + 3023371643508
225027175603175212953219022485045850731845307519008277385154731461213388035
579159917590062343527464977286601165100620*e^(305*I*c) + 299049894962254360
853812938028386633519008711512485881814378761918695711190376572397489965151
8555144924290346242595167274383008960*e^(304*I*c) + 29364942143518684987032
394554267711043448273062675589165508774672324551532861405210895827339322025
53130712723836983468866230908800*e^(303*I*c) + 2862503126320461797770667780
725644184991255623174626175679050672100848988119391841466573417019247590580
735265143427289340450811200*e^(302*I*c) + 277007320715076864559750728138206
549792496846605452741412233982733378377006830588348730997931598371840374087
2884345746380680204260*e^(301*I*c) + 26611006479757835838282351392014419301
783966433834239035838625472558807723820492010155372149008327456015197371418
49802506685264000*e^(300*I*c) + 2537766415465030330815471746692988596069911
894697225052928320452542175587154848096483331209807430113943015398362669673
337957755720*e^(299*I*c) + 240246459556968608612000180303421105673944558862
194614138410616288624616181514976302503083487523406726777402343341826998243
1265280*e^(298*I*c) + 22577262191038562868128330126815737654962622414206129
320761431511719608545541241446990230098420805151579235293571898699435159912
00*e^(297*I*c) + 2106141903468344307112549761202484543402794252352482199817
410424869677262715098288437646518683487945462774223656471345899082156800*e^
(296*I*c) + 195028655078018191924499296120448701005646036284521850167442376
6266321558791436917317878702232679213868287926294665202769722927380*e^(295*

$I*c) + 17926490780896322989369457284819693349643915975062850884883506229372$
 $52533420980803144316431701452190522716124797875257437516360640*e^(294*I*c)$
 $+ 1635569744641421900657886381289076653227580172056677587451402069234355283$
 $687489659613913761959140773339736014790081814516625224440*e^(293*I*c) + 148$
 $118711708924666246695569496567785552473031326055269082160265717621873742624$
 $5522795329891464091005878304304075953693546767206080*e^(292*I*c) + 13313961$
 $146267230358024621235315820503397499960141524528353059563674253702227586217$
 $53922458727524856072950880960657564720475838500*e^(291*I*c) + 1187817943079$
 $390316108802324798110129020822782660087248599367643481200206046822166144285$
 $425922229375413676535071141005286431481600*e^(290*I*c) + 105178210042883437$
 $194450817005121918711681634976695332263718214961087500422378478418328496190$
 $6494422955462431208645690802526770780*e^(289*I*c) + 92432005286752258403577$
 $749576107235122253442078486196000182102050946814635675643379524644649139614$
 $1583854513687104334429566707520*e^(288*I*c) + 80616967132762553242457534008$
 $977573368199499157667444692235409971461519208544324566385225700111564428666$
 $0304979476023966071898200*e^(287*I*c) + 69778910692592461481671374768468278$
 $508365981902795224444704335586974136850045256116402463607340192969310580110$
 $5522738405349028160*e^(286*I*c) + 59937848477173347480937614240155485020706$
 $497211813757294925750365144454193930902527689604962251563026316218452639431$
 $7285457368300*e^(285*I*c) + 51090761511113450745262384614713714145027572244$
 $431638553164842923085168663582771748846450033162338577740074495053841063773$
 $5936000*e^(284*I*c) + 43214785646408693802381156180867858959470204790467428$
 $229795965880017098445679906775187804480661901245263689135073161827854569016$
 $0*e^(283*I*c) + 36270630756384323113569915741851073245242061401362416887931$
 $2187645233450153927975793326834780741391203430153093712635355523960320*e^(2$
 $82*I*c) + 30206063803086846346113944227936049971890691748252489448335622019$
 $6138377050828911383056860425370161157201493696073712322595776808*e^(281*I*c$
 $) + 24959307228256586639838995150961920268263445548712871463146189177082320$
 $1367527645793770203788784677343934971424317987895255031936*e^(280*I*c) + 20$
 $462229553572910951982991678986722531942970516256008264884039496542311280933$
 $6591921290309392396263834674368977840527147037426908*e^(279*I*c) + 16643233$
 $292258919513055832926639875338982395573759852755609609306247355976954577232$
 $1978969318904192572733997888230986469005970880*e^(278*I*c) + 13429774202347$
 $942990462961610455961009607475872106802270446893806301705968802336343645897$
 $1534964665036319889119229809973806909680*e^(277*I*c) + 10750473740657691612$
 $348039916975963332832140741940051001749884983059862156542826654631593392082$
 $1527544726380201659114903834605888*e^(276*I*c) + 85368118430215312848231291$
 $739673735887746201851666299600392199418764750086828198719872744047767783667$
 $325326289221881974987582215*e^(275*I*c) + 672440879690807038237032571996630$
 $476068906104820904946194928029351302159798194699663833367886939001391155946$
 $46893784095418472336*e^(274*I*c) + 5253922334674077114258709237025706953606$
 $031964443950166761048276795580027605289243215279881460797511036622494508142$
 $8121888473324*e^(273*I*c) + 40715988963701918950020348336736423420513311359$
 $010485246919074652883393970805374470830156229705647312265477584256027212762$
 $941040*e^(272*I*c) + 312952636881898383137727758733072603341172272586295019$

92358695636092662866062819845689064235813622974150120668921391878398978380*
e^(271*I*c) + 2385639856556280203069527817421264083328215417400645945829264
4060947924907313735921561690153939906017518647182491616573724049744*e^(270*
I*c) + 18035327338177455471177568594851682977978346449777193572087688510392
426884519272991560851326393852241961470040819793627127923997*e^(269*I*c) +
135212304119454369155588547068517965415673998116568700215673529753268154678
17846533289123871696056195231696146162720992221760992*e^(268*I*c) + 1005220
952436958182758815498534554967803144574449998520825938543560974027240030145
4246872041775159838468077381562338745636398374*e^(267*I*c) + 74103726128911
522243646332961280439716571932803277543043822356727737814030238141276103555
62505271969045177704726054907145784960*e^(266*I*c) + 5416662804052436349585
595982818357953866258461644354018205158917742576425344364964596750653177677
803492186817305171175032011500*e^(265*I*c) + 392569765841577835276810394285
601184021164276962171721799661439847388718607439148263854721282653827045391
2634540299792270321024*e^(264*I*c) + 28208192985622159591075298072896284496
213867989894363693931160698940187812010002756331044983989593466317955680225
19974400130281*e^(263*I*c) + 2009496110092687738152782085683737222727968824
299058739215446083499351467625334449670757764066690656150949014944043994822
823920*e^(262*I*c) + 14191644814221765738582340138989996288223223330957373
0716310743838935832201454293617293175086458389621425307051750612129761498*e
^(261*I*c) + 99355565364952112722644396082023364938648851008189254586670044
4096661582790441241830855609577062039555625090943332264901780720*e^(260*I*c
) + 68951844932879355990326041814997419025357834005889503558960646824468059
1556118170304005037563669880057908765898949268614772285*e^(259*I*c) + 47432
304356310054237733862993196624812917597698233244601805600939115402043889590
3140822967769494019446166779954024655344116288*e^(258*I*c) + 32341317801484
100371415113824607915257636097603505845789040993773872317153703657304368199
7163745313602400139153046673668433091*e^(257*I*c) + 21856316665964931224748
364095627214921249911538382877102965428336397258511809047941369663810815638
5244646591328454425745117584*e^(256*I*c) + 14639044845635118121823738274037
412419166481999774698807659839186273362967014224154637553390313060529758010
5675355629160198162*e^(255*I*c) + 97173055024742680058616722461368892661141
295540263493013032746083536157324268333390400308958318370219154887169702257
444756176*e^(254*I*c) + 639230194337619890906148012886350981231994451023031
22616544648208998767803944455777042886552738499747183713136069104651812215*
e^(253*I*c) + 4167044037539054364341821934227174804003507149011908058528152
2498188818375906900368701234531304633163446319945130196476913600*e^(252*I*c
) + 26917794701086615097890120236890501105146799996021775195710866226228638
984703456832694153230611607263444183501026198563419616*e^(251*I*c) + 172295
028243676473344007219984175967039486573947388052093916365973703805723989647
15080095366818322029152193635869784095333760*e^(250*I*c) + 1092721060347354
481027979234784453607458889680623004111008954473160586314610418173903942667
4855453466097402330688331845602302*e^(249*I*c) + 68664253375186682626626937
509089569657329245781421816306221578028998808746815510311363140641999486040
01529894566235238597088*e^(248*I*c) + 4274826907720591752526711336820871500

844345647922385471534359333606189571832444641364132893108663576205133870672
 156264164115*e^(247*I*c) + 263666241043079934044752228477824428374075106865
 814072657644667120779832560683229593770506168629793029638233889257490081944
 0*e^(246*I*c) + 16110925414000605259548593752641941783476434718370782014462
 62435615142944587337833513586022729849523358436493586042252995608*e^(245*I*
 c) + 9752103394440493187572823117635177866732231755944579463832792646350850
 41004917300295904275433144848532459919875479817581584*e^(244*I*c) + 5847495
 736823045861793846288448833275814989698865403803788967679990756149640071746
 00811092945356635118795824799369716742109*e^(243*I*c) + 3473100538109352904
 194555605559573141295692107357459832343696599764133747740780001730700752486
 54524917179128950507443058208*e^(242*I*c) + 2043255572651860007674027102308
 478964597615839227636982354332128333130777830410400746693790173948367615396
 49081690630811665*e^(241*I*c) + 1190605918496605468347656932276764490675841
 482488826784475048260772363334445134540951266687500572958111916433569089721
 91440*e^(240*I*c) + 6871246601598564151246858617365974773487959171009835465
 2786124936023073943141049573606648563005359411712764895683903806088*e^(239*
 I*c) + 39274200414329861169397944516225001081227433398585007206399231211907
 157795359719648241598754266579840244551491476467899952*e^(238*I*c) + 222313
 411318015353454063990377216868402083979419525801355846459667467366567162715
 54826476282991066076564921432614339399735*e^(237*I*c) + 1246214044053725808
 492859670987206685775707094312486855450094815475686345430803292540834031123
 7850017814707896986969086816*e^(236*I*c) + 69178389452148442784933304593613
 949233723338536198796373726731849428597124310663457268704220998931248907776
 78037369988150*e^(235*I*c) + 3802604996705891106964620633848964807037098854
 510182263243030597295630760353597531974324752266389193185760878274188013440
 *e^(234*I*c) + 206969828950086064346166576237380795751301942404117887190496
 0551829412449344722432125679417958403007551179298315947373776*e^(233*I*c) +
 11153982855456015505333280456001843179938993592179967298183407042218011956
 67410485846996179056733558512452238583160792512*e^(232*I*c) + 5951576155004
 315149474792823365470538279260879164252634971877570294133854718354341982468
 07096214536895441388306027237899*e^(231*I*c) + 3144090358082258615655954369
 383549454454739910431297220467472288130309252049685034185668188386118667098
 07040793495364496*e^(230*I*c) + 1644375006769068927413232601543942785039545
 619360201335965818064493572402773494479273345171058099953000935492799312731
 78*e^(229*I*c) + 8513953323478645577958995946490063776073572970562122138000
 5837208369157794673049675428799817875431430246332625899630160*e^(228*I*c) +
 43638300075815171025946211464465689618965773488660985857945657479854085108
 851857911222911989837615452608512356008400295*e^(227*I*c) + 221407350017086
 032709151807692413916620355787559039791489092136038225547927491835171602555
 71915875356439553717130797888*e^(226*I*c) + 1111949964536320108088106282488
 633849242537565844897793553584634929042582157038309042541141852151667037137
 2045206568345*e^(225*I*c) + 55274988490311783558612300093266682839262900821
 58467118000698502719379939045918344222192742145711257028040974074674736*e^(
 224*I*c) + 2719589283483743926040805101080341921244530311254607250929192773
 909331523226635035815672862569296693711643521070331394*e^(223*I*c) + 132431

132498402742835522293814768237867286070881716174144874968959358802086084750
8703702325320304649883120684987556400*e^(222*I*c) + 63821889291453374150580
639966248855606678360049609187640837497487744897177803607499624558112428346
0438065182071976085*e^(221*I*c) + 30438447110681333601028416012390637043388
882849062742265255123696679091617452085775914393014018717349239498190825894
4*e^(220*I*c) + 14365768713804479694294711970425953845881819942351682467458
6293691056119209866358123637772245409530799230553767222252*e^(219*I*c) + 67
091706130529669125019899210021576580237843462229535386295087076189297849995
931360645605292130961496106707521506432*e^(218*I*c) + 310043192060694170770
693631414234874318280090981847446356786522841774394649416518125645191449180
03174108077634846014*e^(217*I*c) + 1417648365287570495720201334324111790436
997779665384995990252442198063518901163481565327960549778338288893276673008
0*e^(216*I*c) + 64133818958559251847582314510625563803285949385110065770152
18119786536390213057284018202201631094434819584025113465*e^(215*I*c) + 2870
496131412314451834674715353589439553294430808531933466086288543709246230769
151392180699413405623017247753532944*e^(214*I*c) + 127103308293804895020136
055483127034266234399127750461234230036602504674174285658044528940178665631
1685859023084716*e^(213*I*c) + 55675638871118234034102619273421954611365176
8317380539005893679049394714017063698565272728813669054779077208977840*e<sup>(2
12*I*c)</sup> + 24124602128244006179290831778303287619480159713320605209128699704
3729145345755805710081489006741839439573984832678*e^(211*I*c) + 10339975546
725743648984783764075375471820439447305579500146760432641987655555687382953
1737211096115196005647730480*e^(210*I*c) + 43834972142919377685378692233021
063744554033100928502737480438978976746989895784070951905237783490374305934
542955*e^(209*I*c) + 183798070840033597660276492176211441160917355722166207
88861535803449702273802588359076704241840733513439114113248*e^(208*I*c) + 7
621788791912047062038840917799374600428892258194367636682944356096681400246
312138001769285020661445991073249416*e^(207*I*c) + 312568349317870174347970
475030749017866629215072017936360433511352862332968460634318554075601993566
2148267863968*e^(206*I*c) + 12675970172948129134001462760421269299864802928
70190399107554311079964227280196522475370108738477856311765699610*e<sup>(205*I*
c)</sup> + 5083245993010854601669786296830326614276544740829390480979396383915672
98795788389433842285751054665210868287680*e^(204*I*c) + 2015579474247940980
267724784620408833958675125629308629437535686900840155855980101547815486252
39409581907397500*e^(203*I*c) + 7901914955876656925478398848723238835290914
4982747171856772463223808993367091503402876467270176124342699654400*e<sup>(202*
I*c)</sup> + 30627581054221957378390547289277609129572813931082733520247387226000
020043538279468776707958420892547870128680*e^(201*I*c) + 117358569262451184
931130910025016040323418769859990828235206722415302001882238263929823021940
84667538488665600*e^(200*I*c) + 4445412259295474625067659514198312966015416
299968930393345630345914109720740573618884980520010028451496996210*e<sup>(199*I
*c)</sup> + 166447503438721180939491774350293763897857493775476476398783587241044
9930690131572904279995484581013965001440*e^(198*I*c) + 61600311602297958493
712570175788721299835430098999136262803886109391456133207119190971494942658
7936910303300*e^(197*I*c) + 22532053259322065776794110928951624899979452101

5564134982827241710019675486694499689312466561907212627820000*e^(196*I*c) +
81452081413829111828875417564250054846037693312811480492909160758195989155
768107022568350953861815940704090*e^(195*I*c) + 290976510612474534066475697
818369100621655598523590528042591541656871254287525623854923737494863517144
53120*e^(194*I*c) + 1027160253020288900249781351684945259097151280952906066
5197301097052210064576088348023234671975463677418470*e^(193*I*c) + 35827180
021632960614145367037151098971071982527392845461493431023484561240896574285
94946438660859773886240*e^(192*I*c) + 1234668041892409978780018081755440216
012582476396941937965899631953079203974222138794604328498972144766900*e^(19
1*I*c) + 420358024835146798583611210145942154684437949365647899088372524802
156222884839580011688655664280691773600*e^(190*I*c) + 141379938253556843280
565505807403304130606130725434751745794079833141361748917639986145377066437
210546190*e^(189*I*c) + 469702247271172818264540450180706705225597566275803
47784535320014963482632359729444541885102274546002560*e^(188*I*c) + 1541311
121148602393729497082079737671608134478816338654352242193973750796212585498
1881879168348260330000*e^(187*I*c) + 49952419562791381802051867444016880243
88272113921255663734956946927571305533146776898787878059685108480*e^(186*I*
c) + 1598771101058192692270528999677444742685631006232456185844925220144002
305878120380828483988663574829100*e^(185*I*c) + 505293663123015258878483025
738812813203397766845340065381261016353419722382620393032535960660921950400
*e^(184*I*c) + 157685845528850918721462877864435090257583149415561323427386
562894447598277935629800939237175625149830*e^(183*I*c) + 485842581531402804
473148368687721313901954124190467327784587060150968814370763379107935841224
75073760*e^(182*I*c) + 1477795509661712899871274518207149536217650697318308
1650233605274051677624970464340242755840025673760*e^(181*I*c) + 44372109178
431823477643495444439046990200565950694708471936170921147140776330772349728
25351226979360*e^(180*I*c) + 1315052120930692122102297105327622842335870743
428530891072983535862280094446607723473800477453914130*e^(179*I*c) + 384655
842080666274454063078784837174998949052500975322162003392549953413592461519
365177908682078400*e^(178*I*c) + 111034148797008819443143895644469242295049
867464313710969257619338899133799285616020069872611710850*e^(177*I*c) + 316
266446747255477311767956875276535713059699859236883921121649155532425732694
90908989570248533280*e^(176*I*c) + 8888295028751024667044203837607976101480
053134418614474620767522824868911959884352666444917404000*e^(175*I*c) + 246
438219080743960907977422685567962936788570977643587663085171625396269619234
1706239192878728160*e^(174*I*c) + 67402553054313300889484577523662523745074
3114473544537818170447134607102575676676056675328961590*e^(173*I*c) + 18183
466140617790131533012967714538116644918841319414116934435475492096903495261
0378945282257600*e^(172*I*c) + 48379489756434099843857791816589379406815042
609340378747586437145781646245422045101230417309900*e^(171*I*c) + 126934969
329649205650736736371812800885486825088802553372800650065661386960417973532
16584528640*e^(170*I*c) + 3283874760555818676726309480306734420155098583948
074469014168171874442170109648521627538755920*e^(169*I*c) + 837579206923411
932458786486765373533946545239708990769488724813982189165104589895518909256
320*e^(168*I*c) + 210594301385648471184329078880317504953361839954159427434

009884661777259752542647709150036990*e^(167*I*c) + 521909122076618242158122
71854269748071292843243227894769229690720010547141334131610989636000*e^(166
*I*c) + 1274721961650332054135634306256284736860162214085678602544581453203
7904111523242298235713300*e^(165*I*c) + 30679742964317473641981596239624636
71617006419626851426148418602934852907379021659761911840*e^(164*I*c) + 7275
210107183942292917740738446942557987386670675353797597327955679425787513842
50780476310*e^(163*I*c) + 1699563279699297677739020966526292532837045054771
27544556534417376686540936706073847337600*e^(162*I*c) + 3910803125560180947
6537535369611844440844903751605645023514572352045248104262933598850730*e^(1
61*I*c) + 88627521427569572856813408857649045979353495693553218156477211725
37159186491471311666400*e^(160*I*c) + 1977792980665818135651300094326239158
605448870806970860577325385028609983034534672318500*e^(159*I*c) + 434546676
780280045346344498763892540797175105756827515509297024187660299345484920192
480*e^(158*I*c) + 939869153130681791490836060656814827808360605105301546184
86949839467131378859885998210*e^(157*I*c) + 2000800680303004713729327825032
1597113540716201983333126349281186679153199068045257216*e^(156*I*c) + 41915
425006568261480933394145441591439644784724923159318091718599021141090059399
42952*e^(155*I*c) + 8639799336223303495562968200283955131987080649405057021
26068652936800794826651264256*e^(154*I*c) + 1751931705006183002415156323819
12285157790097816049220671217212220015297133400636060*e^(153*I*c) + 3494107
161327670464947794304333945020150407533516036186591602921386077860623062496
0*e^(152*I*c) + 68529932231457366873288853116177954355929408414398663510796
55652312894721972796266*e^(151*I*c) + 1321498055271300851429993866631619874
424534425188183592049727687571032156435077280*e^(150*I*c) + 250501028608928
332469340456829902067712233644464602753159945727868485722395506952*e^(149*I
*c) + 466682235482660178068545924681005702893559608696136508565757567581801
82223308768*e^(148*I*c) + 8543013441126212334833540665069621472479085838041
360564550722036723654297540205*e^(147*I*c) + 153633323844492758353273455601
6494671674916578907116984548489078241693926940560*e^(146*I*c) + 27136120750
3266570734486517077181014801775322183181055638619257836143271472358*e^(145*
I*c) + 47065044611135158108487353367484243102698248838312635876283099427442
745866704*e^(144*I*c) + 801372958079075243436196494576154376146952079121074
6972675870481058674277844*e^(143*I*c) + 13392143742542455535648844068019453
53385000254030655765953770237607180089968*e^(142*I*c) + 2196012813395155615
00261478844190024870555261281946058839614044697037963695*e^(141*I*c) + 3532
4447206779018115378052820789411687581004582367431006205879633729015200*e^(1
40*I*c) + 55725511573286711210162164163075961618619559690116972223409262101
12854418*e^(139*I*c) + 8618848510949919087642468054746724286037573154844539
74713612812215428992*e^(138*I*c) + 1306576602265604193351214343899389618845
95434069984824307149332131747540*e^(137*I*c) + 1940797921559456659353500810
3303255257745408070082431338945184797463936*e^(136*I*c) + 28239051519365866
78382525706564457280290098698638597987628380245881715*e^(135*I*c) + 4023496
92266121158934003582839428785116904903936409545602519219664720*e^(134*I*c)
+ 56117081076341175384087570185188538660375932013674735519055227368366*e^(1
33*I*c) + 76590105201875496517771183576768719270818989891311257557982042361

$12 * e^{(132 * I * c)} + 1022536437468296737293065862705246449693687415559865844306$
 $888705423 * e^{(131 * I * c)} + 133490210052026183779673313868332303530332906163247$
 $194627808410304 * e^{(130 * I * c)} + 170338860273906157410409777216555416656121622$
 $75485028584310890417 * e^{(129 * I * c)} + 2123702969188871318266718781223927067839$
 $949015727293884065388080 * e^{(128 * I * c)} + 258585348715977270155829115684193411$
 $072034541491364393985491350 * e^{(127 * I * c)} + 307362174043210099652310374196630$
 $53962881035281709221697785072 * e^{(126 * I * c)} + 3564764890628724017088487996688$
 $178929195787613958545474804845 * e^{(125 * I * c)} + 403212225957798188840846139960$
 $995624144491271694336796459584 * e^{(124 * I * c)} + 444567081752588210244009462105$
 $35004523775722190977468484496 * e^{(123 * I * c)} + 4775398607100853263534207733818$
 $266777478693412738731031680 * e^{(122 * I * c)} + 499467506558531733671585862910572$
 $702811545035730398749530 * e^{(121 * I * c)} + 508363695081710994370193486108473919$
 $46736185108017183136 * e^{(120 * I * c)} + 5032024903401451824074213943766011922026$
 $507006311982753 * e^{(119 * I * c)} + 484093410240488718655917025303662581091659126$
 $182344528 * e^{(118 * I * c)} + 452309400398307383320256947846462068448548276980757$
 $36 * e^{(117 * I * c)} + 4101545439937195793959956708442496709433800261224880 * e^{(11$
 $6 * I * c)} + 360688613036389349413809780004559963548775423325255 * e^{(115 * I * c)} +$
 $30735366512830562160991166338490057308062762518496 * e^{(114 * I * c)} + 2535667460$
 $650279776834561566186591213109251642859 * e^{(113 * I * c)} + 202347509724462171313$
 $966643580234078508179838320 * e^{(112 * I * c)} + 156039112776876070997216237717449$
 $33086920587272 * e^{(111 * I * c)} + 1161581413733971751533622511909046917188768400$
 $* e^{(110 * I * c)} + 83380839911837894453136303673785039051506805 * e^{(109 * I * c)} + 5$
 $764601046563151304213854710715346838447392 * e^{(108 * I * c)} + 383360155801054824$
 $529764688213114368047154 * e^{(107 * I * c)} + 244898373378123386877186224918650138$
 $39488 * e^{(106 * I * c)} + 1500602747937397286405577818722691539392 * e^{(105 * I * c)} +$
 $88054927598941411145869950813388040256 * e^{(104 * I * c)} + 4939666610818025798809$
 $586352543471345 * e^{(103 * I * c)} + 264410375780310742518099326419685040 * e^{(102 * I$
 $* c)} + 13477227799524701956579274210395326 * e^{(101 * I * c)} + 6526502533432060474$
 $53620559993840 * e^{(100 * I * c)} + 29952547749265499675257842032197 * e^{(99 * I * c)} +$
 $1299146645993240318167826532288 * e^{(98 * I * c)} + 53090127264630963470039804475 *$
 $e^{(97 * I * c)} + 2037031259470368160131922320 * e^{(96 * I * c)} + 73099207817335597247$
 $098038 * e^{(95 * I * c)} + 2442455629894502983849104 * e^{(94 * I * c)} + 7559981709267015$
 $7806639 * e^{(93 * I * c)} + 2154864144781257856128 * e^{(92 * I * c)} + 561694445269265622$
 $60 * e^{(91 * I * c)} + 1327882849274858880 * e^{(90 * I * c)} + 28186192554792138 * e^{(89 * I * c)}$
 $+ 530563624556832 * e^{(88 * I * c)} + 8718181624155 * e^{(87 * I * c)} + 122503723056 * e^{(86 * I * c)}$
 $+ 1431118260 * e^{(85 * I * c)} + 13343760 * e^{(84 * I * c)} + 93096 * e^{(83 * I * c)}$
 $+ 432 * e^{(82 * I * c)} + e^{(81 * I * c)} + 14 * (26 * a * e^{(1055/2 * I * c)} + 10504 * a * e^{(1053/$
 $2 * I * c)} + 2116556 * a * e^{(1051/2 * I * c)} + 283618504 * a * e^{(1049/2 * I * c)} + 2843275502$
 $6 * a * e^{(1047/2 * I * c)} + 2274620402080 * a * e^{(1045/2 * I * c)} + 151262256738437 * a * e^{(1043/2 * I * c)}$
 $+ 8600339740311748 * a * e^{(1041/2 * I * c)} + 426791859620149322 * a * e^{(1039/2 * I * c)}$
 $+ 18778841824143775343 * a * e^{(1037/2 * I * c)} + 741764252131213332757 * a * e^{(1035/2 * I * c)}$
 $+ 26568646854716407696970 * a * e^{(1033/2 * I * c)} + 8701231848374$
 $20284430070 * a * e^{(1031/2 * I * c)} + 26237560668658611978090055 * a * e^{(1029/2 * I * c)}$
 $+ 732777588085793048574300590 * a * e^{(1027/2 * I * c)} + 19052217324800742177269360$
 $704 * a * e^{(1025/2 * I * c)} + 463207034992633568486133297596 * a * e^{(1023/2 * I * c)} + 10$

572019430266010048013097752934*a*e^(1021/2*I*c) + 2272984190957551165865667
99957934*a*e^(1019/2*I*c) + 4617746868571980127702698250753676*a*e^(1017/2*
I*c) + 88891628244678704139848327305754038*a*e^(1015/2*I*c) + 1625446941911
901543891066605266178710*a*e^(1013/2*I*c) + 2829755417150959822835916906873
7992545*a*e^(1011/2*I*c) + 469985477889846368776571005635121060828*a*e^(100
9/2*I*c) + 7461019729104126380477146521488232920426*a*e^(1007/2*I*c) + 1134
07505114998207828286839959356818331515*a*e^(1005/2*I*c) + 16531325755122432
20555792873383292580108953*a*e^(1003/2*I*c) + 23143857771494702366699332162
923093803819912*a*e^(1001/2*I*c) + 3116155424202189605705498912105541315411
98338*a*e^(999/2*I*c) + 4040257151844206159542266588079905870752602511*a*e^
(997/2*I*c) + 50503221535858091096582689376221786704600600455*a*e^(995/2*I*
c) + 609297036058359460142063362206138699889504487122*a*e^(993/2*I*c) + 710
2120067046396299634193292688723536713017300128*a*e^(991/2*I*c) + 8006028286
2006487921860873353861357672073071040769*a*e^(989/2*I*c) + 8735992347087648
58055721340390736019595548391597091*a*e^(987/2*I*c) + 923519526684946931752
1124217042763676973766253104794*a*e^(985/2*I*c) + 9466079239161646653433315
1235841916067978700197242708*a*e^(983/2*I*c) + 9414916057875226479999780186
15368308830544085253128617*a*e^(981/2*I*c) + 909283230328133681623358022746
2577871197813963276075330*a*e^(979/2*I*c) + 8533279426386174534195194156863
3801044299209189190469846*a*e^(977/2*I*c) + 7786623913490808819340683704304
76103178748088552087631588*a*e^(975/2*I*c) + 691300935061032244604742862740
8811159158338885747490127762*a*e^(973/2*I*c) + 5974821903120267852023564378
9006093143612681508950204493778*a*e^(971/2*I*c) + 5029972852031920598400132
44325631734915939121404020646790260*a*e^(969/2*I*c) + 412687024698502953601
8107086136692505798915934457479856887816*a*e^(967/2*I*c) + 3301501862719472
6039854786048619784314920578570513381395990898*a*e^(965/2*I*c) + 2576611959
39290308606607761298064029010980446277923637727940695*a*e^(963/2*I*c) + 196
2615229865836518270067732662567794203130442183114304875477510*a*e^(961/2*I*
c) + 14596988265590225707951774554192673478049322662209596418132188458*a*e^
(959/2*I*c) + 1060518982683337569105648975586886380808069152381085818548682
57941*a*e^(957/2*I*c) + 752970977560607563719398585207010863604321557616289
473063991767039*a*e^(955/2*I*c) + 52265240935076461936828392759689436086267
86700142487822871933708874*a*e^(953/2*I*c) + 354802086973620760088737097767
02294541967395718297832569250047180556*a*e^(951/2*I*c) + 235643271710602776
985976812503397918723360707843190238666454384153669*a*e^(949/2*I*c) + 15316
89544688533128267835220826660575517763556541402057407422275055260*a*e^(947/
2*I*c) + 974717450252413924844218873977501756256764954078688783935818704091
2530*a*e^(945/2*I*c) + 6074619805356191958352949482025200643288016840259290
6594363059656788960*a*e^(943/2*I*c) + 3708743568261706225612110315792885225
36503760739536119117445103487930880*a*e^(941/2*I*c) + 221887070141102104978
9421436850301976335757207676690062759443158094615652*a*e^(939/2*I*c) + 1301
2484431533492378746242079203193435247631682976658777500945950758604298*a*e^
(937/2*I*c) + 7482258059928548093324070479619691298514226456115682634871717
4043201630392*a*e^(935/2*I*c) + 4219552816550570709386119371835225871920863
13169152717515130361078410110468*a*e^(933/2*I*c) + 233439620049209021698057

7767310744650580417606589598674389591775624513302292*a*e^(931/2*I*c) + 1267
 2621445440067623486707294305789706782209558457586283948374216939117542680*a
 *e^(929/2*I*c) + 6752240336121207913629144799891108270384536184910825436843
 8686584600219974916*a*e^(927/2*I*c) + 3532004347423079615370447529901245286
 53649931040810723777175782628615964698844*a*e^(925/2*I*c) + 181420178966324
 7503683990869126253542318071421878870447076671569392523282426716*a*e^(923/2
 *I*c) + 9152441985761079445443716091984800770861766098755365001578215027814
 025637822344*a*e^(921/2*I*c) + 45359519734612250739996296106148376154884994
 292039155402760649106566139277798596*a*e^(919/2*I*c) + 22088700947448554841
 0918212534797139518487225151752041095525819264180469636552620*a*e^(917/2*I*
 c) + 1057133035608080334517467842102247001229173401193395800195028598793946
 241985524410*a*e^(915/2*I*c) + 49731519454378183332079785198122292839906876
 81266209867827402557241427416724759600*a*e^(913/2*I*c) + 230016401216572533
 16651693705684837372513278520836331991407615544187744222242128500*a*e^(911/
 2*I*c) + 104614252934905612199658590379588736548577356327149679159709209471
 984279805545840510*a*e^(909/2*I*c) + 46795660437124083156463767484415712775
 0964967487710258048711564849492193413275081370*a*e^(907/2*I*c) + 2059104851
 233963888842379525772936034391701199184168338071977072713080793446996296340
 *a*e^(905/2*I*c) + 89142102752459377470130588902599904067720257310317223264
 39774933310018480720586176220*a*e^(903/2*I*c) + 379743334997202596581506260
 55074672602084485890782692205932372642164257255828275277230*a*e^(901/2*I*c)
 + 159209767180955530325047227583177604233946016419587011038408978676950649
 660142022326320*a*e^(899/2*I*c) + 65703578670331707387227618346665553683329
 4806445692036603437064145864115975619273310760*a*e^(897/2*I*c) + 2669400972
 789686981296836875076250459931673689078037577064328933999433324588323107010
 000*a*e^(895/2*I*c) + 10678445536825411376746680463152055034872867036734705
 052278088324574859010216795309754920*a*e^(893/2*I*c) + 42066267367318216193
 842191416909738948525619885259401159592359828689296015791863825798480*a*e^(
 891/2*I*c) + 16321208300699726872915924654990666086737306928579994350965202
 2515969021706052860626138960*a*e^(889/2*I*c) + 6237666345675784717891574646
 27305169392418716892107406574086770229679140188972330140850540*a*e^(887/2*I
 *c) + 234855692258423706200419810716707127388721793315640118242397689383442
 5480017846564439100360*a*e^(885/2*I*c) + 8712550834307422843463603955913018
 153198551629326082886421600824607707881453826119225268410*a*e^(883/2*I*c) +
 31850013002449494191788674279414994671607865224244945000200665576261090031
 451234274836807440*a*e^(881/2*I*c) + 11474859600895439947056082052328280297
 2560754779166281576532046193721730617184676146208454500*a*e^(879/2*I*c) + 4
 074841744653947362116914863644020728687905524304051798805074217510942824078
 35858091343026670*a*e^(877/2*I*c) + 142643037425091059133790826183498942347
 1882981709863583159775371537718016932012468351480138570*a*e^(875/2*I*c) + 4
 922849139190761971912744518550936831750167555165468970439911091242197041305
 307652588826163080*a*e^(873/2*I*c) + 16751630043585648045466097105076233663
 413980413177353774456822305187643841236848558981932298420*a*e^(871/2*I*c) +
 56210773708404693912307787410989229580969216199887991549658097359306716028
 570932883897745531110*a*e^(869/2*I*c) + 18601597745255025509215791744032430

0818135019254799070536796872549263250091086090579550703698310*a*e^(867/2*I*c) + 6071476558117846180525934068675294373925386810781848667804435379321003
 36168022295156652479524980*a*e^(865/2*I*c) + 195477130098723468429915723650
 2544878646869538838413631422449979548647041435372549308338323983400*a*e^(86
 3/2*I*c) + 6208666376133327662787020371327223738306905395229403481688606039
 769774018193877054980125219300650*a*e^(861/2*I*c) + 19455556288210142197792
 399701441683457177993888604394571936075981048677773457499402098681541345230
 *a*e^(859/2*I*c) + 60155208943075208721970485637459885199389358781088969898
 509767201637525536232949225931564636017060*a*e^(857/2*I*c) + 18353862986603
 064551692928939851624258351084780879082478093337125350402392953629920710068
 1758720560*a*e^(855/2*I*c) + 5526445943943309814046835890851146422809065536
 79728598394985334382381190569025961084792025986226970*a*e^(853/2*I*c) + 164
 235585052369408178165987548747970648759912590938897517892628070386544161757
 8142680863389145680280*a*e^(851/2*I*c) + 4817584949915740964852355271716409
 235848785641998844676677492209743884414375678304126912661883888620*a*e^(849
 /2*I*c) + 13949845316042293786528514094535620274951482636695793685790781893
 931237847288558891416636782386235120*a*e^(847/2*I*c) + 39877107005816808842
 474290443185467296349332073686639497504608960180007893671571725169560655925
 320000*a*e^(845/2*I*c) + 11254540176202178596753890614510262974982280482013
 1541462936025973893544847020246173693665097678958840*a*e^(843/2*I*c) + 3136
 293088020715462733207570165074527613810476767113929416368676209987438529454
 22691370541694930187760*a*e^(841/2*I*c) + 863027290376938136286146320587382
 966922778739244634235519200955649593051035611434988433819531197898760*a*e^(
 839/2*I*c) + 23452275720630232633764343055786061690911043360812978713567227
 44639904016570907272826407742463307924640*a*e^(837/2*I*c) + 629406122506786
 447764803532819614352598375975595100354714074485443115126708614333203407245
 7898773808110*a*e^(835/2*I*c) + 1668379197470897218586915631182682117015529
 7607333534880763775663291322286056634684196941640082885613980*a*e^(833/2*I*
 c) + 4368256487624130087749635757075188322826448323418589792049592125168459
 6213376975639862720078190049382420*a*e^(831/2*I*c) + 1129801682493579003864
 946113022334253254841486610959361670814713456006638529110286397566551348292
 21364250*a*e^(829/2*I*c) + 288673978616124351576066193803621410940049987233
 564354069965128114229512608260120320153840315109513240510*a*e^(827/2*I*c) +
 72870957701304930685145583004578625839578559495705560420492538784203999462
 5652269962227400682354338997620*a*e^(825/2*I*c) + 1817491183507803277646434
 154563247687916502767732579734578654547931596925444779173074576528904820222
 193680*a*e^(823/2*I*c) + 44790946053124559401311878991986757364901787900997
 45194145235324134619699772544528969129449757743039635770*a*e^(821/2*I*c) +
 10907792551328601262236194966417553379047773698515875937376972639114184123
 61493183977361964833748319598140*a*e^(819/2*I*c) + 262506700598237870871324
 313040849655957123767320824379561378686494524463490057790192498722891461579
 88186820*a*e^(817/2*I*c) + 624350634603888862944275086050183454453771965931
 51678464550029733503722062639199339939077739315562684600360*a*e^(815/2*I*c)
 + 146767097463970982808607791792512370924531634905313051233506563629059084
 334858680054056774768242705313471500*a*e^(813/2*I*c) + 34101053275050840101

051257257425643355058819225226351490993566391141835465779948294033981751718
 1660633925100*a*e^(811/2*I*c) + 7831995110286048303052651712417830735588716
 41563886451764214478656600184324031802959178677178450742159624260*a*e^(809/
 2*I*c) + 177815160679273569318457630517100655459173088980146347083457156964
 9456632925027058049883289543294859615339320*a*e^(807/2*I*c) + 3991010196204
 332937212586851998834375311877876034549525880121307974429565165949974007981
 491960685666506428548*a*e^(805/2*I*c) + 88560519659530348796623406573386029
 65083737597696868985656979387880769494954864304287309406159689433401344052*
 a*e^(803/2*I*c) + 194296978761766218912342065517275740088550706895032287006
 62784554532194008982832223012645019617538312082871128*a*e^(801/2*I*c) + 421
 488318183379605264415423924469270354566939628166561985944026101410544615947
 49815537318993350672298883213302*a*e^(799/2*I*c) + 904114465590476241075636
 388127901365070921139042817273413125290723804538206691540873902278005764014
 18085164068*a*e^(797/2*I*c) + 191780398312540108164969569134383375747089477
 08500852416545282627700304209125929751214708830470427357620162840*a*e^(795/
 2*I*c) + 40230206688627611091580537050825344058292667366383467679120937981
 7073708402847882776363322804524102236998117616*a*e^(793/2*I*c) + 8346255778
 624730272866697000320711631326880473631426951620586027757135789410782767919
 42737285481618912831112014*a*e^(791/2*I*c) + 171256256379489298835175825371
 058083911930675117962488080466572725027821268194300172115925962699426835365
 3902636*a*e^(789/2*I*c) + 3475684327725554198246437246074163598962204579122
 918009051534296803648946904541960047776709789822578169105209259*a*e^(787/2*
 I*c) + 69774539909700958938705179903395862097896076997538737829421306949459
 46528220744372046715941677441907103193732516*a*e^(785/2*I*c) + 138560661023
 300269915703514452458130971519949149654481078997705650476054273369480158720
 27856400037860001309803670*a*e^(783/2*I*c) + 272202840140473281110545726489
 135154639891847008248762990130210512503186300606320662319185591740786585673
 45880625*a*e^(781/2*I*c) + 529027743018574367549566488112897453499435435611
 47991932957809598337563243029219771808687832301138643124607095515*a*e^(779/
 2*I*c) + 101723341642534027856325802326625923100834568123573784283611965294
 829026168598516825508102699558265012155505286390*a*e^(777/2*I*c) + 19352737
 493776155106975449584051738189540453017146222106374993722910424723376312660
 6164713214930951706536170129402*a*e^(775/2*I*c) + 3643065882595299330012120
 113496787036688688258721282165017591615610211270169829625182642712312661274
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 719647983387318924115049192814394269280038737908609793825310009937379942*a*
 e^(771/2*I*c) + 12508856412562750179042237925459335912703372679853519227298
 92360270373029610277920725182173946163411522304315334632*a*e^(769/2*I*c) +
 228186549100043532565208157119295931459955274509410039814758576890137193267
 2555405076513220299715006918296079490268*a*e^(767/2*I*c) + 4119630675993972
 883163200200627180864136924478788743836907012989394926215330996549026090671
 132192170289614995953334*a*e^(765/2*I*c) + 73611505472290883580787692740396
 109097532014264372074343776475669425303448966351717988617006043301044168346
 68519430*a*e^(763/2*I*c) + 130189433375914762178356958680531819179445499007
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61/2*I*c) + 227914492230573423183794384642924067618367063004688929366445938
50736816226075816614000005128630118631272047930585354*a*e^(759/2*I*c) + 394
964421757197854804800626261040141950619117362572826461311558661205389432919
98823092670646932544497465667715800518*a*e^(757/2*I*c) + 677575411667699551
794684595940634031110056452951432934169471446823452063745500517009076235710
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852271484*a*e^(753/2*I*c) + 19350570483018105418915972164288713013847558252
7934017732420563157275710384038780519016341083221717538160410071629286*a*e^
(751/2*I*c) + 3221652051451423635701184039210812231721036704288274349032173
67898408752791686796216445582208987707216661270752348189*a*e^(749/2*I*c) +
531097086226002776844822503685026440629965503462454662271767664538175076793
501990496089430124065992327885079121429343*a*e^(747/2*I*c) + 86697045893774
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3573129277135387631125832240*a*e^(745/2*I*c) + 1401505022864821759990514911
774358655464920980642918627032979165269601299928150161830021936830813736871
417390373928686*a*e^(743/2*I*c) + 22437274442601993561690753071153304749782
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89*a*e^(741/2*I*c) + 355759052364744337578248508929766292922882754286156673
4110641737329597784391653357883472975676999315605445384252967857*a*e^(739/2
*I*c) + 5586988939674645914325363550716282684830193122934675954566650601501
265101993368547501997769261446740200272717454879998*a*e^(737/2*I*c) + 86908
344287462640820635666870922769031917378894184093446251988265759445886970611
99305932440621485102366745761048030632*a*e^(735/2*I*c) + 133916171433813415
617973822475038584786946514869762411301721921550415535983934943380724683336
18785797627855047879535719*a*e^(733/2*I*c) + 204417835019961259801359073735
729598558718872777900564562643674240367953383314829824410987395778323946443
81899556974181*a*e^(731/2*I*c) + 309132980920478453984687001663630696239627
341410546638371233948289257131780166068100757728741033115735840681003114039
90*a*e^(729/2*I*c) + 463169064421787428773107530128748877142514289777958793
15358012090947020067282331495803931605151341983289375819303497428*a*e^(727/
2*I*c) + 687588340824290493941730231472874639260477373072581343050011943174
95857981578577369701491983169829988876464082023881439*a*e^(725/2*I*c) + 101
143876200907701458145589433650452267328419069640282011909099693348827229087
774814701746425164097849836694542125988386*a*e^(723/2*I*c) + 14743495097122
257892001302754560741045764055468918914770666990802436741262512060095882016
9170652789316898683463434699434*a*e^(721/2*I*c) + 2129800728872132484276957
411841611559855081082818965937685889049569831974956079164659144035890164426
99445092549476690660*a*e^(719/2*I*c) + 304918178534066785990007348109251411
831147410206280250705996608101203138903042698601813697013463781101658444684
909249658*a*e^(717/2*I*c) + 43267510031147823268192037944542330700966435986
8176731174023649559638171040661252577006509195005627113792068208970590674*a
*e^(715/2*I*c) + 6085600003789944462454360725994938388425387255926195914009
72596394327368638438650114993937818761497030717896138241363060*a*e^(713/2*I
*c) + 848470498946001634567581956874393834016146789764834275392720475291429

146661138402672991032856375125997801887991514934480*a*e^(711/2*I*c) + 11727
113126584939536498388369849154117904014614045634840053843288777020371629671
20070585970799957845874604153354692701754*a*e^(709/2*I*c) + 160692624306626
054554481901005007128427431243975148175968419123604564153261042308353022715
1796510775305152151669953445033*a*e^(707/2*I*c) + 2183136663673238874306231
186152025274688130245012528032646406775075995457404272096460063109834340128
997113442263319468362*a*e^(705/2*I*c) + 29408711824717505312385503328846026
959178235822249108880611909943048931093638636029968254017904185889712749007
67917143382*a*e^(703/2*I*c) + 392836097149201063465198049040617150440014024
676010743447318264062075917380066973102851810619473316703925248564260153788
3*a*e^(701/2*I*c) + 5203763588007696555866441944538215327243563612139283244
175384029625341239582454293417845888737748393246675398200066703617*a*e^(699
/2*I*c) + 68363653907343935490083740060526889623052085887268921617240025435
45109906070875455109463119424154633158689756128755804310*a*e^(697/2*I*c) +
890769951839117842912980641402497841569803028507622901399151629297555758814
3549142778717881866319712550105543451994921740*a*e^(695/2*I*c) + 1151250370
118722932321754624292796931844868480402772030853088627243410611741428798087
6927354781721804664473558899761393515*a*e^(693/2*I*c) + 1475943096641637266
026391584135364038980566865549449523589098309458194274936068085056080535365
5895626485394213426659661080*a*e^(691/2*I*c) + 1877141777593854101016581170
965589125417404752437554585584113289037225874320065787936848261828833468461
5818699392418342286*a*e^(689/2*I*c) + 2368560957440153387573223776855359788
750496757000209731449894900700078096335324585104460952194609728104516073162
8195279464*a*e^(687/2*I*c) + 2965274439563358013315895980670116385427021037
517126720851863333216136474940528326162682504535882839974231851656862748495
6*a*e^(685/2*I*c) + 3683590220184064905914922714927576872979791050338928752
0100466864775463578466769724598377043872510103817701793820475775984*a*e^(68
3/2*I*c) + 4540854188536614387278871721876377305403940060205798303633337681
6522774495334926169325754390780348094316862266665965991846*a*e^(681/2*I*c)
+ 5555176982051654281141921884692657977308709975397899169675671615736895025
3678159527340671056699387281634743181637981363440*a*e^(679/2*I*c) + 6745081
347112670693549051856820239084922164027549460520665066790705151832929495199
8879978783463264636089137056491006523208*a*e^(677/2*I*c) + 8129071018204356
616701379173280267172954967390383067644635798771483933905417558570172260688
4268548187402687095553510812392*a*e^(675/2*I*c) + 9725126358288451269473326
158366739119204994534604768047558453841096717393945458322040125713163484526
9872208901982900506288*a*e^(673/2*I*c) + 1155013660100496806832702398001686
029727691282930709002561705121779982424918713245705795383463814149293141687
62720397958392*a*e^(671/2*I*c) + 136192832394833900835393035860717631858240
644738806126199965701046979626441751102715242872161933462105757019809834502
589368*a*e^(669/2*I*c) + 15945393607422544108744443544559732263801034995914
0938824355029415044951858795370137083023423725977662050522985425476876440*a
*e^(667/2*I*c) + 1853828765700121741147743144859216195375688381584788222948
99038884738131663676924882800088555461036440639784323290982435920*a*e^(665/
2*I*c) + 214041440126824842352708511077845618878866223151781361633546874774

038204026567692011971405221478790305862726998272063222200*a*e^(663/2*I*c) +
24544913252943878657146028256914731662384082738982168509152746364888136352
8910037224877635235250886963822468275620946735000*a*e^(661/2*I*c) + 2795780
678217740015444298371203092765322605167049308307176674216212482251758423025
86295274927274086096056979473472729880620*a*e^(659/2*I*c) + 316348884263616
362042842543901080546281501211987656004780654651440103299682244533088533089
753616985689239381646035489221440*a*e^(657/2*I*c) + 35562792952360209042496
295254069656463922322436052266745359625063593797852563838274166718529799876
5569584449936989927995480*a*e^(655/2*I*c) + 3972259113259586183629325664274
897110088063706955062184972890026109827136705312561541269284123968426450371
63118148106920740*a*e^(653/2*I*c) + 440898149491503523444778198806793839878
322252525780377816191849083993038457480195337323463748937132292723543647234
218039660*a*e^(651/2*I*c) + 48634649377888960220864954194100002855346932926
715456497046092005153453977747649119560232954097527602192211291936153687644
0*a*e^(649/2*I*c) + 5332228934934298858594093264598308973341326947698611499
75266872019015402485363293405873999908400003099563986230923614600520*a*e^(6
47/2*I*c) + 581134524659849570516520835583290845326108734568414348508022104
061332738062063508414909055483914854232496613990505843010500*a*e^(645/2*I*c
) + 62965030424428278136286812761655590978380494878441543704862783087307572
0560012090018464221573758434571727274802802603945840*a*e^(643/2*I*c) + 6783
085540050749008177339337287136946439110689948051691473515175034050265104208
44066523647882858701963834358113089172938960*a*e^(641/2*I*c) + 726625524054
874396729548904429989754634649183846135021779065652886206176807006743446378
198756010809903052637351826187568320*a*e^(639/2*I*c) + 77410445211553094712
953191086231935146942887913541796258981191251639696468099085798266657081471
2400614658358562654106747680*a*e^(637/2*I*c) + 8202448213103733279998304866
566586510148506276324277415446009091322998816007490343410817448694018867024
49976206139706099120*a*e^(635/2*I*c) + 864551488026486190923176963138034067
155722686853321421184941164808358133963280958662401384573838526420349485337
999344825920*a*e^(633/2*I*c) + 90654338091137046511284880039597581167135684
78957413165441736279423496908180987797774108295321067258447012481189965253
0280*a*e^(631/2*I*c) + 9457615196590840744070207016357652066287072685109508
59918127044094577511351195111798902113793353834963879647040364097292000*a*e
^(629/2*I*c) + 981776163519459256143964209232624766016543826754463955803919
524083126774434994582010832915047435963630135877730601620142540*a*e^(627/2*
I*c) + 10141929688536566257914620127857582334930245710109349535376353605121
77986386038527036220871061980615293450139380863514373440*a*e^(625/2*I*c) +
104265810628987872856563640672132080919144434896624853275387262442621354456
4508176781308633408517935011981967399688638658360*a*e^(623/2*I*c) + 1066862
354734951629341085781295112245624204034159509029552678063480622244236687699
044241201275152181775644021605917221039140*a*e^(621/2*I*c) + 10865442458195
107126822246415653572134714564509872887323281441630636076100532214509469992
70202022551820770028778526533989420*a*e^(619/2*I*c) + 110149237354129975153
461547990969736040946334905572381036460229516993161090718787392685524564368
7343585906730161451031127120*a*e^(617/2*I*c) + 1111547006738804576763416827

720669435301619875769248333073060137308247506189143702187094023228399816342
 788014313884847207960*a*e^(615/2*I*c) + 11166011452861623220858153208126762
 061145208586965493301675500631812147692556046807180719300626426382965790959
 12747329485300*a*e^(613/2*I*c) + 111660114528616232208581532081267620611452
 085869654933016755006318121476925560468071807193006264263829657909591274732
 9485300*a*e^(611/2*I*c) + 1111547006738804576763416827720669435301619875769
 248333073060137308247506189143702187094023228399816342788014313884847207960
 *a*e^(609/2*I*c) + 11014923735412997515346154799096973604094633490557238103
 64602295169931610907187873926855245643687343585906730161451031127120*a*e<sup>(6
 07/2*I*c)</sup> + 108654424581951071268222464156535721347145645098728873232814416
 3063607610053221450946999270202022551820770028778526533989420*a*e<sup>(605/2*I*
 c)</sup> + 1066862354734951629341085781295112245624204034159509029552678063480622
 244236687699044241201275152181775644021605917221039140*a*e^(603/2*I*c) + 10
 426581062898787285656364067213208091914443489662485327538726244262135445645
 08176781308633408517935011981967399688638658360*a*e^(601/2*I*c) + 101419296
 885365662579146201278575823349302457101093495353763536051217798638603852703
 6220871061980615293450139380863514373440*a*e^(599/2*I*c) + 9817761635194592
 561439642092326247660165438267544639558039195240831267744349945820108329150
 47435963630135877730601620142540*a*e^(597/2*I*c) + 945761519659084074407020
 701635765206628707268510950859918127044094577511351195111798902113793353834
 963879647040364097292000*a*e^(595/2*I*c) + 90654338091137046511284880039597
 58116713568478957413165441736279423496908180987797774108295321067258447012
 4811899652530280*a*e^(593/2*I*c) + 8645514880264861909231769631380340671557
 226868533214211849411648083581339632809586624013845738385264203494853379993
 44825920*a*e^(591/2*I*c) + 820244821310373327999830486656658651014850627632
 427741544600909132299881600749034341081744869401886702449976206139706099120
 *a*e^(589/2*I*c) + 77410445211553094712953191086231935146942887913541796258
 9811912516396964680990857982666570814712400614658358562654106747680*a*e<sup>(58
 7/2*I*c)</sup> + 7266255240548743967295489044299897546346491838461350217790656528
 86206176807006743446378198756010809903052637351826187568320*a*e^(585/2*I*c)
 + 678308554005074900817733933728713694643911068994805169147351517503405026
 510420844066523647882858701963834358113089172938960*a*e^(583/2*I*c) + 62965
 030424428278136286812761655590978380494878441543704862783087307572056001209
 0018464221573758434571727274802802603945840*a*e^(581/2*I*c) + 5811345246598
 495705165208355832908453261087345684143485080221040613327380620635084149090
 55483914854232496613990505843010500*a*e^(579/2*I*c) + 533222893493429885859
 409326459830897334132694769861149975266872019015402485363293405873999908400
 003099563986230923614600520*a*e^(577/2*I*c) + 48634649377888960220864954194
 100002855346932926715456497046092005153453977747649119560232954097527602192
 2112919361536876440*a*e^(575/2*I*c) + 4408981494915035234447781988067938398
 783222525257803778161918490839930384574801953373234637489371322927235436472
 34218039660*a*e^(573/2*I*c) + 397225911325958618362932566427489711008806370
 695506218497289002610982713670531256154126928412396842645037163118148106920
 740*a*e^(571/2*I*c) + 35562792952360209042496295254069656463922322436052266
 7453596250635937978525638382741667185297998765569584449936989927995480*a*e

(569/2*I*c) + 3163488842636163620428425439010805462815012119876560047806546
51440103299682244533088533089753616985689239381646035489221440*a*e^(567/2*I
*c) + 279578067821774001544429837120309276532260516704930830717667421621248
225175842302586295274927274086096056979473472729880620*a*e^(565/2*I*c) + 24
544913252943878657146028256914731662384082738982168509152746364888136352891
0037224877635235250886963822468275620946735000*a*e^(563/2*I*c) + 2140414401
268248423527085110778456188788662231517813616335468747740382040265676920119
71405221478790305862726998272063222200*a*e^(561/2*I*c) + 185382876570012174
114774314485921619537568838158478822294899038884738131663676924882800088555
461036440639784323290982435920*a*e^(559/2*I*c) + 15945393607422544108744443
544559732263801034995914093882435502941504495185879537013708302342372597766
2050522985425476876440*a*e^(557/2*I*c) + 1361928323948339008353930358607176
318582406447388061261999657010469796264417511027152428721619334621057570198
09834502589368*a*e^(555/2*I*c) + 115501366010049680683270239800168602972769
128293070900256170512177998242491871324570579538346381414929314168762720397
958392*a*e^(553/2*I*c) + 97251263582884512694733261583667391192049945346047
680475584538410967173939454583220401257131634845269872208901982900506288*a*
e^(551/2*I*c) + 81290710182043566167013791732802671729549673903830676446357
987714839339054175585701722606884268548187402687095553510812392*a*e^(549/2*
I*c) + 67450813471126706935490518568202390849221640275494605206650667907051
518329294951998879978783463264636089137056491006523208*a*e^(547/2*I*c) + 55
551769820516542811419218846926579773087099753978991696756716157368950253678
159527340671056699387281634743181637981363440*a*e^(545/2*I*c) + 45408541885
366143872788717218763773054039400602057983036333376816522774495334926169325
754390780348094316862266665965991846*a*e^(543/2*I*c) + 36835902201840649059
149227149275768729797910503389287520100466864775463578466769724598377043872
510103817701793820475775984*a*e^(541/2*I*c) + 29652744395633580133158959806
701163854270210375171267208518633332161364749405283261626825045358828399742
318516568627484956*a*e^(539/2*I*c) + 23685609574401533875732237768553597887
504967570002097314498949007000780963353245851044609521946097281045160731628
195279464*a*e^(537/2*I*c) + 18771417775938541010165811709655891254174047524
375545855841132890372258743200657879368482618288334684615818699392418342286
*a*e^(535/2*I*c) + 14759430966416372660263915841353640389805668655494495235
890983094581942749360680850560805353655895626485394213426659661080*a*e^(533
/2*I*c) + 11512503701187229323217546242927969318448684804027720308530886272
434106117414287980876927354781721804664473558899761393515*a*e^(531/2*I*c) +
89076995183911784291298064140249784156980302850762290139915162929755575881
43549142778717881866319712550105543451994921740*a*e^(529/2*I*c) + 683636539
073439354900837400605268896230520858872689216172400254354510990607087545510
9463119424154633158689756128755804310*a*e^(527/2*I*c) + 5203763588007696555
866441944538215327243563612139283244175384029625341239582454293417845888737
748393246675398200066703617*a*e^(525/2*I*c) + 39283609714920106346519804904
061715044001402467601074344731826406207591738006697310285181061947331670392
52485642601537883*a*e^(523/2*I*c) + 294087118247175053123855033288460269591
782358222491088806119099430489310936386360299682540179041858897127490076791

7143382*a*e^(521/2*I*c) + 2183136663673238874306231186152025274688130245012
528032646406775075995457404272096460063109834340128997113442263319468362*a*
e^(519/2*I*c) + 16069262430662605455448190100500712842743124397514817596841
91236045641532610423083530227151796510775305152151669953445033*a*e^(517/2*I
*c) + 117271131265849395364983883698491541179040146140456348400538432887770
2037162967120070585970799957845874604153354692701754*a*e^(515/2*I*c) + 8484
704989460016345675819568743938340161467897648342753927204752914291466611384
02672991032856375125997801887991514934480*a*e^(513/2*I*c) + 608560000378994
446245436072599493838842538725592619591400972596394327368638438650114993937
818761497030717896138241363060*a*e^(511/2*I*c) + 43267510031147823268192037
944542330700966435986817673117402364955963817104066125257700650919500562711
3792068208970590674*a*e^(509/2*I*c) + 3049181785340667859900073481092514118
311474102062802507059966081012031389030426986018136970134637811016584446849
09249658*a*e^(507/2*I*c) + 212980072887213248427695741184161155985508108281
896593768588904956983197495607916465914403589016442699445092549476690660*a*
e^(505/2*I*c) + 14743495097122257892001302754560741045764055468918914770666
9908024367412625120600958820169170652789316898683463434699434*a*e^(503/2*I*
c) + 1011438762009077014581455894336504522673284190696402820119090996933488
27229087774814701746425164097849836694542125988386*a*e^(501/2*I*c) + 687588
340824290493941730231472874639260477373072581343050011943174958579815785773
69701491983169829988876464082023881439*a*e^(499/2*I*c) + 463169064421787428
773107530128748877142514289777958793153580120909470200672823314958039316051
51341983289375819303497428*a*e^(497/2*I*c) + 309132980920478453984687001663
630696239627341410546638371233948289257131780166068100757728741033115735840
68100311403990*a*e^(495/2*I*c) + 204417835019961259801359073735729598558718
872777900564562643674240367953383314829824410987395778323946443818995569741
81*a*e^(493/2*I*c) + 133916171433813415617973822475038584786946514869762411
30172192155041553598393494338072468333618785797627855047879535719*a*e^(491/
2*I*c) + 869083442874626408206356668709227690319173788941840934462519882657
5944588697061199305932440621485102366745761048030632*a*e^(489/2*I*c) + 5586
988939674645914325363550716282684830193122934675954566650601501265101993368
547501997769261446740200272717454879998*a*e^(487/2*I*c) + 35575905236474433
757824850892976629292288275428615667341106417373295977843916533578834729756
76999315605445384252967857*a*e^(485/2*I*c) + 224372744426019935616907530711
533047497820991345500293367004318952790277086251364680160400675824242631357
2029851100889*a*e^(483/2*I*c) + 1401505022864821759990514911774358655464920
980642918627032979165269601299928150161830021936830813736871417390373928686
*a*e^(481/2*I*c) + 86697045893774080059910011005227233408633616681348521582
3374714091680381820801848615843383573129277135387631125832240*a*e^(479/2*I*
c) + 5310970862260027768448225036850264406299655034624546622717676645381750
76793501990496089430124065992327885079121429343*a*e^(477/2*I*c) + 322165205
145142363570118403921081223172103670428827434903217367898408752791686796216
445582208987707216661270752348189*a*e^(475/2*I*c) + 19350570483018105418915
972164288713013847558252793401773242056315727571038403878051901634108322171
7538160410071629286*a*e^(473/2*I*c) + 1150786790741569210038962115067440407

574481297555180726109745976924991164767807359592015878667774273738541398522
 71484*a*e^(471/2*I*c) + 677575411667699551794684595940634031110056452951432
 93416947144682345206374550051700907623571000874184381488493815415*a*e<sup>(469/
 2*I*c)</sup> + 394964421757197854804800626261040141950619117362572826461311558661
 20538943291998823092670646932544497465667715800518*a*e^(467/2*I*c) + 227914
 492230573423183794384642924067618367063004688929366445938507368162260758166
 14000005128630118631272047930585354*a*e^(465/2*I*c) + 130189433375914762178
 356958680531819179445499007194845922504447026408380059377508923151909569968
 62238788704893903660*a*e^(463/2*I*c) + 736115054722908835807876927403961090
 975320142643720743437764756694253034489663517179886170060433010441683466851
 9430*a*e^(461/2*I*c) + 4119630675993972883163200200627180864136924478788743
 836907012989394926215330996549026090671132192170289614995953334*a*e<sup>(459/2*
 I*c)</sup> + 22818654910004353256520815711929593145995527450941003981475857689013
 71932672555405076513220299715006918296079490268*a*e^(457/2*I*c) + 125088564
 125627501790422379254593359127033726798535192272989236027037302961027792072
 5182173946163411522304315334632*a*e^(455/2*I*c) + 6786059851087274217018825
 233023932842112497196479833873189241150491928143942692800387379086097938253
 10009937379942*a*e^(453/2*I*c) + 364306588259529933001212011349678703668868
 825872128216501759161561021127016982962518264271231266127473606437283673*a*
 e^(451/2*I*c) + 19352737493776155106975449584051738189540453017146222106374
 9937229104247233763126606164713214930951706536170129402*a*e^(449/2*I*c) + 1
 017233416425340278563258023266259231008345681235737842836119652948290261685
 98516825508102699558265012155505286390*a*e^(447/2*I*c) + 529027743018574367
 549566488112897453499435435611479919329578095983375632430292197718086878323
 01138643124607095515*a*e^(445/2*I*c) + 272202840140473281110545726489135154
 639891847008248762990130210512503186300606320662319185591740786585673458806
 25*a*e^(443/2*I*c) + 138560661023300269915703514452458130971519949149654481
 07899770565047605427336948015872027856400037860001309803670*a*e^(441/2*I*c)
 + 697745399097009589387051799033958620978960769975387378294213069494594652
 8220744372046715941677441907103193732516*a*e^(439/2*I*c) + 3475684327725554
 198246437246074163598962204579122918009051534296803648946904541960047776709
 789822578169105209259*a*e^(437/2*I*c) + 17125625637948929883517582537105808
 391193067511796248808046657272502782126819430017211592596269942683536539026
 36*a*e^(435/2*I*c) + 834625577862473027286669700032071163132688047363142695
 162058602775713578941078276791942737285481618912831112014*a*e^(433/2*I*c) +
 40230206688627611091580537050825344058292667366383467679120937981707370840
 2847882776363322804524102236998117616*a*e^(431/2*I*c) + 1917803983125401081
 649695691343833757470894770850008524165452826277003042091259297512147088304
 70427357620162840*a*e^(429/2*I*c) + 904114465590476241075636388127901365070
 92113904281727341312529072380453820669154087390227800576401418085164068*a*e
^(427/2*I*c) + 421488318183379605264415423924469270354566939628166561985944
 02610141054461594749815537318993350672298883213302*a*e^(425/2*I*c) + 194296
 978761766218912342065517275740088550706895032287006627845545321940089828322
 23012645019617538312082871128*a*e^(423/2*I*c) + 885605196595303487966234065
 733860296508373759769686898565697938788076949495486430428730940615968943340

1344052*a*e^(421/2*I*c) + 3991010196204332937212586851998834375311877876034
 549525880121307974429565165949974007981491960685666506428548*a*e^{(419/2*I*c}
) + 17781516067927356931845763051710065545917308898014634708345715696494566
 32925027058049883289543294859615339320*a*e^(417/2*I*c) + 783199511028604830
 305265171241783073558871641563886451764214478656600184324031802959178677178
 450742159624260*a*e^(415/2*I*c) + 34101053275050840101051257257425643355058
 8192252263514909935663911418354657799482940339817517181660633925100*a*e⁽⁴¹
 3/2*I*c) + 1467670974639709828086077917925123709245316349053130512335065636
 29059084334858680054056774768242705313471500*a*e^(411/2*I*c) + 624350634603
 888862944275086050183454453771965931516784645500297335037220626391993399390
 77739315562684600360*a*e^(409/2*I*c) + 262506700598237870871324313040849655
 95712376732082437956137868649452446349005779019249872289146157988186820*a*e
^(407/2*I*c) + 109077925513286012622236194966417553379047773698515875937376
 97263911418412361493183977361964833748319598140*a*e^(405/2*I*c) + 447909460
 531245594013118789919867573649017879009974519414523532413461969977254452896
 9129449757743039635770*a*e^(403/2*I*c) + 1817491183507803277646434154563247
 687916502767732579734578654547931596925444779173074576528904820222193680*a*
 e^(401/2*I*c) + 72870957701304930685145583004578625839578559495705560420492
 5387842039994625652269962227400682354338997620*a*e^(399/2*I*c) + 2886739786
 161243515760661938036214109400499872335643540699651281142295126082601203201
 53840315109513240510*a*e^(397/2*I*c) + 112980168249357900386494611302233425
 325484148661095936167081471345600663852911028639756655134829221364250*a*e⁽
 395/2*I*c) + 43682564876241300877496357570751883228264483234185897920495921
 251684596213376975639862720078190049382420*a*e^(393/2*I*c) + 16683791974708
 972185869156311826821170155297607333534880763775663291322286056634684196941
 640082885613980*a*e^(391/2*I*c) + 62940612250678644776480353281961435259837
 59755951003547140744854431151267086143332034072457898773808110*a*e^{(389/2*I}
 *c) + 234522757206302326337643430557860616909110433608129787135672274463990
 4016570907272826407742463307924640*a*e^(387/2*I*c) + 8630272903769381362861
 463205873829669227787392446342355192009556495930510356114349884338195311978
 98760*a*e^(385/2*I*c) + 313629308802071546273320757016507452761381047676711
 392941636867620998743852945422691370541694930187760*a*e^(383/2*I*c) + 11254
 540176202178596753890614510262974982280482013154146293602597389354484702024
 6173693665097678958840*a*e^(381/2*I*c) + 3987710700581680884247429044318546
 7296349332073686639497504608960180007893671571725169560655925320000*a*e⁽³⁷
 9/2*I*c) + 1394984531604229378652851409453562027495148263669579368579078189
 3931237847288558891416636782386235120*a*e^(377/2*I*c) + 4817584949915740964
 852355271716409235848785641998844676677492209743884414375678304126912661883
 888620*a*e^(375/2*I*c) + 16423558505236940817816598754874797064875991259093
 88975178926280703865441617578142680863389145680280*a*e^(373/2*I*c) + 552644
 594394330981404683589085114642280906553679728598394985334382381190569025961
 084792025986226970*a*e^(371/2*I*c) + 18353862986603064551692928939851624258
 3510847808790824780933371253504023929536299207100681758720560*a*e^{(369/2*I*}
 c) + 6015520894307520872197048563745988519938935878108896989850976720163752
 5536232949225931564636017060*a*e^(367/2*I*c) + 1945555628821014219779239970

1441683457177993888604394571936075981048677773457499402098681541345230*a*e^{-(365/2*I*c)} + 6208666376133327662787020371327223738306905395229403481688606
039769774018193877054980125219300650*a*e^{-(363/2*I*c)} + 19547713009872346842
991572365025448786468695388384136314224499795486470414353725493083383239834
00*a*e^{-(361/2*I*c)} + 607147655811784618052593406867529437392538681078184866
780443537932100336168022295156652479524980*a*e^{-(359/2*I*c)} + 18601597745255
025509215791744032430081813501925479907053679687254926325009108609057955070
3698310*a*e^{-(357/2*I*c)} + 5621077370840469391230778741098922958096921619988
7991549658097359306716028570932883897745531110*a*e^{-(355/2*I*c)} + 1675163004
358564804546609710507623366341398041317735377445682230518764384123684855898
1932298420*a*e^{-(353/2*I*c)} + 4922849139190761971912744518550936831750167555
165468970439911091242197041305307652588826163080*a*e^{-(351/2*I*c)} + 14264303
742509105913379082618349894234718829817098635831597753715377180169320124683
51480138570*a*e^{-(349/2*I*c)} + 407484174465394736211691486364402072868790552
430405179880507421751094282407835858091343026670*a*e^{-(347/2*I*c)} + 11474859
600895439947056082052328280297256075477916628157653204619372173061718467614
6208454500*a*e^{-(345/2*I*c)} + 3185001300244949419178867427941499467160786522
4244945000200665576261090031451234274836807440*a*e^{-(343/2*I*c)} + 8712550834
307422843463603955913018153198551629326082886421600824607707881453826119225
268410*a*e^{-(341/2*I*c)} + 23485569225842370620041981071670712738872179331564
01182423976893834425480017846564439100360*a*e^{-(339/2*I*c)} + 623766634567578
471789157464627305169392418716892107406574086770229679140188972330140850540
*a*e^{-(337/2*I*c)} + 16321208300699726872915924654990666086737306928579994350
9652022515969021706052860626138960*a*e^{-(335/2*I*c)} + 4206626736731821619384
2191416909738948525619885259401159592359828689296015791863825798480*a*e<sup>-(33
3/2*I*c)</sup> + 1067844553682541137674668046315205503487286703673470505227808832
4574859010216795309754920*a*e^{-(331/2*I*c)} + 2669400972789686981296836875076
250459931673689078037577064328933999433324588323107010000*a*e^{-(329/2*I*c)} +
65703578670331707387227618346665553683329480644569203660343706414586411597
5619273310760*a*e^{-(327/2*I*c)} + 1592097671809555303250472275831776042339460
16419587011038408978676950649660142022326320*a*e^{-(325/2*I*c)} + 379743334997
20259658150626055074672602084485890782692205932372642164257255828275277230*
a*e^{-(323/2*I*c)} + 891421027524593774701305889025999040677202573103172232643
9774933310018480720586176220*a*e^{-(321/2*I*c)} + 2059104851233963888842379525
772936034391701199184168338071977072713080793446996296340*a*e^{-(319/2*I*c)} +
46795660437124083156463767484415712775096496748771025804871156484949219341
3275081370*a*e^{-(317/2*I*c)} + 1046142529349056121996585903795887365485773563
27149679159709209471984279805545840510*a*e^{-(315/2*I*c)} + 230016401216572533
16651693705684837372513278520836331991407615544187744222242128500*a*e<sup>-(313/
2*I*c)</sup> + 497315194543781833320797851981222928399068768126620986782740255724
1427416724759600*a*e^{-(311/2*I*c)} + 1057133035608080334517467842102247001229
173401193395800195028598793946241985524410*a*e^{-(309/2*I*c)} + 22088700947448
5548410918212534797139518487225151752041095525819264180469636552620*a*e<sup>-(30
7/2*I*c)</sup> + 4535951973461225073999629610614837615488499429203915540276064910
6566139277798596*a*e^{-(305/2*I*c)} + 9152441985761079445443716091984800770861

766098755365001578215027814025637822344*a*e^(303/2*I*c) + 18142017896632475
 03683990869126253542318071421878870447076671569392523282426716*a*e^(301/2*I
 *c) + 353200434742307961537044752990124528653649931040810723777175782628615
 964698844*a*e^(299/2*I*c) + 67522403361212079136291447998911082703845361849
 108254368438686584600219974916*a*e^(297/2*I*c) + 12672621445440067623486707
 294305789706782209558457586283948374216939117542680*a*e^(295/2*I*c) + 23343
 96200492090216980577767310744650580417606589598674389591775624513302292*a*e
 ^((293/2*I*c) + 421955281655057070938611937183522587192086313169152717515130
 361078410110468*a*e^(291/2*I*c) + 74822580599285480933240704796196912985142
 264561156826348717174043201630392*a*e^(289/2*I*c) + 13012484431533492378746
 242079203193435247631682976658777500945950758604298*a*e^(287/2*I*c) + 22188
 70701411021049789421436850301976335757207676690062759443158094615652*a*e^(2
 85/2*I*c) + 370874356826170622561211031579288522536503760739536119117445103
 487930880*a*e^(283/2*I*c) + 60746198053561919583529494820252006432880168402
 592906594363059656788960*a*e^(281/2*I*c) + 97471745025241392484421887397750
 17562567649540786887839358187040912530*a*e^(279/2*I*c) + 153168954468853312
 8267835220826660575517763556541402057407422275055260*a*e^(277/2*I*c) + 2356
 43271710602776985976812503397918723360707843190238666454384153669*a*e^(275/
 2*I*c) + 354802086973620760088737097767022945419673957182978325692500471805
 56*a*e^(273/2*I*c) + 522652409350764619368283927596894360862678670014248782
 2871933708874*a*e^(271/2*I*c) + 7529709775606075637193985852070108636043215
 57616289473063991767039*a*e^(269/2*I*c) + 106051898268333756910564897558688
 638080806915238108581854868257941*a*e^(267/2*I*c) + 14596988265590225707951
 774554192673478049322662209596418132188458*a*e^(265/2*I*c) + 19626152298658
 36518270067732662567794203130442183114304875477510*a*e^(263/2*I*c) + 257661
 195939290308606607761298064029010980446277923637727940695*a*e^(261/2*I*c) +
 33015018627194726039854786048619784314920578570513381395990898*a*e^(259/2*
 I*c) + 4126870246985029536018107086136692505798915934457479856887816*a*e^(2
 57/2*I*c) + 502997285203192059840013244325631734915939121404020646790260*a*
 e^(255/2*I*c) + 59748219031202678520235643789006093143612681508950204493778
 *a*e^(253/2*I*c) + 69130093506103224460474286274088111591583388857474901277
 62*a*e^(251/2*I*c) + 778662391349080881934068370430476103178748088552087631
 588*a*e^(249/2*I*c) + 85332794263861745341951941568633801044299209189190469
 846*a*e^(247/2*I*c) + 90928323032813368162335802274625778711978139632760753
 30*a*e^(245/2*I*c) + 941491605787522647999978018615368308830544085253128617
 *a*e^(243/2*I*c) + 94660792391616466534333151235841916067978700197242708*a*
 e^(241/2*I*c) + 9235195266849469317521124217042763676973766253104794*a*e^(2
 39/2*I*c) + 873599234708764858055721340390736019595548391597091*a*e^(237/2*
 I*c) + 80060282862006487921860873353861357672073071040769*a*e^(235/2*I*c) +
 7102120067046396299634193292688723536713017300128*a*e^(233/2*I*c) + 609297
 036058359460142063362206138699889504487122*a*e^(231/2*I*c) + 50503221535858
 091096582689376221786704600600455*a*e^(229/2*I*c) + 40402571518442061595422
 66588079905870752602511*a*e^(227/2*I*c) + 311615542420218960570549891210554
 131541198338*a*e^(225/2*I*c) + 23143857771494702366699332162923093803819912
 *a*e^(223/2*I*c) + 1653132575512243220555792873383292580108953*a*e^(221/2*I

$$\begin{aligned}
& *c) + 113407505114998207828286839959356818331515*a*e^{(219/2*I*c)} + 74610197 \\
& 29104126380477146521488232920426*a*e^{(217/2*I*c)} + 469985477889846368776571 \\
& 005635121060828*a*e^{(215/2*I*c)} + 28297554171509598228359169068737992545*a* \\
& e^{(213/2*I*c)} + 1625446941911901543891066605266178710*a*e^{(211/2*I*c)} + 888 \\
& 91628244678704139848327305754038*a*e^{(209/2*I*c)} + 461774686857198012770269 \\
& 8250753676*a*e^{(207/2*I*c)} + 227298419095755116586566799957934*a*e^{(205/2*I* \\
& *c)} + 10572019430266010048013097752934*a*e^{(203/2*I*c)} + 463207034992633568 \\
& 486133297596*a*e^{(201/2*I*c)} + 19052217324800742177269360704*a*e^{(199/2*I*c \\
&)} + 732777588085793048574300590*a*e^{(197/2*I*c)} + 2623756066865861197809005 \\
& 5*a*e^{(195/2*I*c)} + 870123184837420284430070*a*e^{(193/2*I*c)} + 265686468547 \\
& 16407696970*a*e^{(191/2*I*c)} + 741764252131213332757*a*e^{(189/2*I*c)} + 18778 \\
& 841824143775343*a*e^{(187/2*I*c)} + 426791859620149322*a*e^{(185/2*I*c)} + 8600 \\
& 339740311748*a*e^{(183/2*I*c)} + 151262256738437*a*e^{(181/2*I*c)} + 2274620402 \\
& 080*a*e^{(179/2*I*c)} + 28432755026*a*e^{(177/2*I*c)} + 283618504*a*e^{(175/2*I* \\
& c)} + 2116556*a*e^{(173/2*I*c)} + 10504*a*e^{(171/2*I*c)} + 26*a*e^{(169/2*I*c)})/ \\
& (e^{(531*I*c)} + 432*e^{(530*I*c)} + 93096*e^{(529*I*c)} + 13343760*e^{(528*I*c)} + \\
& 1431118260*e^{(527*I*c)} + 122503723056*e^{(526*I*c)} + 8718181624155*e^{(525*I \\
& *c)} + 530563624556832*e^{(524*I*c)} + 28186192554792138*e^{(523*I*c)} + 1327882 \\
& 849274858880*e^{(522*I*c)} + 56169444526926562260*e^{(521*I*c)} + 2154864144781 \\
& 257856128*e^{(520*I*c)} + 75599817092670157806639*e^{(519*I*c)} + 2442455629894 \\
& 502983849104*e^{(518*I*c)} + 73099207817335597247098038*e^{(517*I*c)} + 2037031 \\
& 259470368160131922320*e^{(516*I*c)} + 53090127264630963470039804475*e^{(515*I* \\
& c)} + 1299146645993240318167826532288*e^{(514*I*c)} + 299525477492654996752578 \\
& 42032197*e^{(513*I*c)} + 652650253343206047453620559993840*e^{(512*I*c)} + 1347 \\
& 7227799524701956579274210395326*e^{(511*I*c)} + 26441037578031074251809932641 \\
& 9685040*e^{(510*I*c)} + 4939666610818025798809586352543471345*e^{(509*I*c)} + 8 \\
& 8054927598941411145869950813388040256*e^{(508*I*c)} + 15006027479373972864055 \\
& 77818722691539392*e^{(507*I*c)} + 24489837337812338687718622491865013839488*e \\
& ^{(506*I*c)} + 383360155801054824529764688213114368047154*e^{(505*I*c)} + 57646 \\
& 01046563151304213854710715346838447392*e^{(504*I*c)} + 8338083991183789445313 \\
& 6303673785039051506805*e^{(503*I*c)} + 11615814137339717515336225119090469171 \\
& 88768400*e^{(502*I*c)} + 15603911277687607099721623771744933086920587272*e^{(5 \\
& 01*I*c)} + 202347509724462171313966643580234078508179838320*e^{(500*I*c)} + 25 \\
& 35667460650279776834561566186591213109251642859*e^{(499*I*c)} + 3073536651283 \\
& 0562160991166338490057308062762518496*e^{(498*I*c)} + 36068861303638934941380 \\
& 9780004559963548775423325255*e^{(497*I*c)} + 41015454399371957939599567084424 \\
& 96709433800261224880*e^{(496*I*c)} + 4523094003983073833202569478464620684485 \\
& 4827698075736*e^{(495*I*c)} + 48409341024048871865591702530366258109165912618 \\
& 2344528*e^{(494*I*c)} + 50320249034014518240742139437660119220265070063119827 \\
& 53*e^{(493*I*c)} + 50836369508171099437019348610847391946736185108017183136*e \\
& ^{(492*I*c)} + 499467506558531733671585862910572702811545035730398749530*e^{(4 \\
& 91*I*c)} + 4775398607100853263534207733818266777478693412738731031680*e^{(490 \\
& *I*c)} + 44456708175258821024400946210535004523775722190977468484496*e^{(489* \\
& I*c)} + 403212225957798188840846139960995624144491271694336796459584*e^{(488* \\
& I*c)} + 3564764890628724017088487996688178929195787613958545474804845*e^{(487
\end{aligned}$$

$*I*c) + 30736217404321009965231037419663053962881035281709221697785072*e^(4$
 $86*I*c) + 258585348715977270155829115684193411072034541491364393985491350*e$
 $^(485*I*c) + 21237029691888713182667187812239270678399490157272938840653880$
 $80*e^(484*I*c) + 1703388602739061574104097772165554166561216227548502858431$
 $0890417*e^(483*I*c) + 13349021005202618377967331386833230353033290616324719$
 $4627808410304*e^(482*I*c) + 10225364374682967372930658627052464496936874155$
 $59865844306888705423*e^(481*I*c) + 7659010520187549651777118357676871927081$
 $898989131125755798204236112*e^(480*I*c) + 561170810763411753840875701851885$
 $38660375932013674735519055227368366*e^(479*I*c) + 4023496922661211589340035$
 $82839428785116904903936409545602519219664720*e^(478*I*c) + 2823905151936586$
 $678382525706564457280290098698638597987628380245881715*e^(477*I*c) + 194079$
 $79215594566593535008103303255257745408070082431338945184797463936*e^(476*I*$
 $c) + 1306576602265604193351214343899389618845954340699848243071493321317475$
 $40*e^(475*I*c) + 8618848510949919087642468054746724286037573154844539747136$
 $12812215428992*e^(474*I*c) + 5572551157328671121016216416307596161861955969$
 $011697222340926210112854418*e^(473*I*c) + 353244472067790181153780528207894$
 $11687581004582367431006205879633729015200*e^(472*I*c) + 2196012813395155615$
 $00261478844190024870555261281946058839614044697037963695*e^(471*I*c) + 1339$
 $214374254245553564884406801945353385000254030655765953770237607180089968*e$
 $^(470*I*c) + 801372958079075243436196494576154376146952079121074697267587048$
 $1058674277844*e^(469*I*c) + 47065044611135158108487353367484243102698248838$
 $312635876283099427442745866704*e^(468*I*c) + 271361207503266570734486517077$
 $181014801775322183181055638619257836143271472358*e^(467*I*c) + 153633323844$
 $4927583532734556016494671674916578907116984548489078241693926940560*e^(466*$
 $I*c) + 85430134411262123348335406650696214724790858380413605645507220367236$
 $54297540205*e^(465*I*c) + 4666822354826601780685459246810057028935596086961$
 $3650856575756758180182223308768*e^(464*I*c) + 25050102860892833246934045682$
 $9902067712233644464602753159945727868485722395506952*e^(463*I*c) + 13214980$
 $55271300851429993866631619874424534425188183592049727687571032156435077280*$
 $e^(462*I*c) + 6852993223145736687328885311617795435592940841439866351079655$
 $652312894721972796266*e^(461*I*c) + 349410716132767046494779430433394502015$
 $04075335160361865916029213860778606230624960*e^(460*I*c) + 1751931705006183$
 $00241515632381912285157790097816049220671217212220015297133400636060*e^(459$
 $*I*c) + 8639799336223303495562968200283955131987080649405057021260686529368$
 $00794826651264256*e^(458*I*c) + 4191542500656826148093339414544159143964478$
 $472492315931809171859902114109005939942952*e^(457*I*c) + 200080068030300471$
 $37293278250321597113540716201983333126349281186679153199068045257216*e^(456$
 $*I*c) + 9398691531306817914908360606568148278083606051053015461848694983946$
 $7131378859885998210*e^(455*I*c) + 43454667678028004534634449876389254079717$
 $5105756827515509297024187660299345484920192480*e^(454*I*c) + 19777929806658$
 $18135651300094326239158605448870806970860577325385028609983034534672318500*$
 $e^(453*I*c) + 8862752142756957285681340885764904597935349569355321815647721$
 $172537159186491471311666400*e^(452*I*c) + 391080312556018094765375353696118$
 $44440844903751605645023514572352045248104262933598850730*e^(451*I*c) + 1699$
 $563279699297677739020966526292532837045054771275445565344173766865409367060$

73847337600*e^(450*I*c) + 7275210107183942292917740738446942557987386670675
 35379759732795567942578751384250780476310*e^(449*I*c) + 3067974296431747364
 198159623962463671617006419626851426148418602934852907379021659761911840*e^
 (448*I*c) + 127472196165033205413563430625628473686016221408567860254458145
 32037904111523242298235713300*e^(447*I*c) + 5219091220766182421581227185426
 9748071292843243227894769229690720010547141334131610989636000*e^(446*I*c) +
 21059430138564847118432907888031750495336183995415942743400988466177725975
 2542647709150036990*e^(445*I*c) + 83757920692341193245878648676537353394654
 5239708990769488724813982189165104589895518909256320*e^(444*I*c) + 32838747
 605558186767263094803067344201550985839480744690141681718744421701096485216
 27538755920*e^(443*I*c) + 1269349693296492056507367363718128008854868250888
 0255337280065006566138696041797353216584528640*e^(442*I*c) + 48379489756434
 099843857791816589379406815042609340378747586437145781646245422045101230417
 309900*e^(441*I*c) + 181834661406177901315330129677145381166449188413194141
 169344354754920969034952610378945282257600*e^(440*I*c) + 674025530543133008
 894845775236625237450743114473544537818170447134607102575676676056675328961
 590*e^(439*I*c) + 246438219080743960907977422685567962936788570977643587663
 0851716253962696192341706239192878728160*e^(438*I*c) + 88882950287510246670
 442038376079761014800531344186144746207675228248689119598843526664449174040
 00*e^(437*I*c) + 3162664467472554773117679568752765357130596998592368839211
 2164915553242573269490908989570248533280*e^(436*I*c) + 11103414879700881944
 314389564446924229504986746431371096925761933889913379928561602006987261171
 0850*e^(435*I*c) + 38465584208066627445406307878483717499894905250097532216
 2003392549953413592461519365177908682078400*e^(434*I*c) + 13150521209306921
 221022971053276228423358707434285308910729835358622800944466077234738004774
 53914130*e^(433*I*c) + 4437210917843182347764349544443904699020056595069470
 847193617092114714077633077234972825351226979360*e^(432*I*c) + 147779550966
 171289987127451820714953621765069731830816502336052740516776249704643402427
 55840025673760*e^(431*I*c) + 4858425815314028044731483686877213139019541241
 9046732778458706015096881437076337910793584122475073760*e^(430*I*c) + 15768
 584552885091872146287786443509025758314941556132342738656289444759827793562
 9800939237175625149830*e^(429*I*c) + 50529366312301525887848302573881281320
 3397766845340065381261016353419722382620393032535960660921950400*e^(428*I*c
) + 15987711010581926922705289996774447426856310062324561858449252201440023
 05878120380828483988663574829100*e^(427*I*c) + 4995241956279138180205186744
 401688024388272113921255663734956946927571305533146776898787878059685108480
 *e^(426*I*c) + 154131112114860239372949708207973767160813447881633865435224
 21939737507962125854981881879168348260330000*e^(425*I*c) + 4697022472711728
 182645404501807067052255975662758034778453532001496348263235972944454188510
 2274546002560*e^(424*I*c) + 14137993825355684328056550580740330413060613072
 5434751745794079833141361748917639986145377066437210546190*e^(423*I*c) + 42
 035802483514679858361121014594215468443794936564789908837252480215622288483
 9580011688655664280691773600*e^(422*I*c) + 12346680418924099787800180817554
 40216012582476396941937965899631953079203974222138794604328498972144766900*
 e^(421*I*c) + 3582718002163296061414536703715109897107198252739284546149343

102348456124089657428594946438660859773886240*e^(420*I*c) + 102716025302028
890024978135168494525909715128095290606651973010970522100645760883480232346
71975463677418470*e^(419*I*c) + 2909765106124745340664756978183691006216555
9852359052804259154165687125428752562385492373749486351714453120*e^(418*I*c
) + 81452081413829111828875417564250054846037693312811480492909160758195989
155768107022568350953861815940704090*e^(417*I*c) + 225320532593220657767941
109289516248999794521015564134982827241710019675486694499689312466561907212
627820000*e^(416*I*c) + 616003116022979584937125701757887212998354300989991
362628038861093914561332071191909714949426587936910303300*e^(415*I*c) + 166
447503438721180939491774350293763897857493775476476398783587241044993069013
1572904279995484581013965001440*e^(414*I*c) + 44454122592954746250676595141
983129660154162999689303933456303459141097207405736188849805200100284514969
96210*e^(413*I*c) + 1173585692624511849311309100250160403234187698599908282
3520672241530200188223826392982302194084667538488665600*e^(412*I*c) + 30627
581054221957378390547289277609129572813931082733520247387226000020043538279
468776707958420892547870128680*e^(411*I*c) + 790191495587665692547839884872
323883529091449827471718567724632238089933670915034028764672701761243426996
54400*e^(410*I*c) + 2015579474247940980267724784620408833958675125629308629
43753568690084015585598010154781548625239409581907397500*e^(409*I*c) + 5083
245993010854601669786296830326614276544740829390480979396383915672987957883
89433842285751054665210868287680*e^(408*I*c) + 1267597017294812913400146276
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765699610*e^(407*I*c) + 312568349317870174347970475030749017866629215072017
9363604335113528623329684606343185540756019935662148267863968*e^(406*I*c) +
76217887919120470620388409177993746004288922581943676366829443560966814002
46312138001769285020661445991073249416*e^(405*I*c) + 1837980708400335976602
764921762114411609173557221662078886153580344970227380258835907670424184073
3513439114113248*e^(404*I*c) + 43834972142919377685378692233021063744554033
100928502737480438978976746989895784070951905237783490374305934542955*e^(40
3*I*c) + 103399755467257436489847837640753754718204394473055795001467604326
41987655556873829531737211096115196005647730480*e^(402*I*c) + 241246021282
440061792908317783032876194801597133206052091286997043729145345755805710081
489006741839439573984832678*e^(401*I*c) + 556756388711182340341026192734219
546113651768317380539005893679049394714017063698565272728813669054779077208
977840*e^(400*I*c) + 127103308293804895020136055483127034266234399127750461
2342300366025046741742856580445289401786656311685859023084716*e^(399*I*c) +
28704961314123144518346747153535894395532944308085319334660862885437092462
30769151392180699413405623017247753532944*e^(398*I*c) + 6413381895855925184
758231451062556380328594938511006577015218119786536390213057284018202201631
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699777966538499599025244219806351890116348156532796054977833828889327667300
80*e^(396*I*c) + 3100431920606941707706936314142348743182800909818474463567
8652284177439464941651812564519144918003174108077634846014*e^(395*I*c) + 67
091706130529669125019899210021576580237843462229535386295087076189297849995
931360645605292130961496106707521506432*e^(394*I*c) + 143657687138044796942

947119704259538458818199423516824674586293691056119209866358123637772245409
530799230553767222252*e^(393*I*c) + 304384471106813336010284160123906370433
888828490627422652551236966790916174520857759143930140187173492394981908258
944*e^(392*I*c) + 638218892914533741505806399662488556066783600496091876408
374974877448971778036074996245581124283460438065182071976085*e^(391*I*c) +
132431132498402742835522293814768237867286070881716174144874968959358802086
0847508703702325320304649883120684987556400*e^(390*I*c) + 27195892834837439
260408051010803419212445303112546072509291927739093315232266350358156728625
69296693711643521070331394*e^(389*I*c) + 5527498849031178355861230009326668
283926290082158467118000698502719379939045918344222192742145711257028040974
074674736*e^(388*I*c) + 111194996453632010808810628248863384924253756584489
77935535846349290425821570383090425411418521516670371372045206568345*e^(387
*I*c) + 2214073500170860327091518076924139166203557875590397914890921360382
2554792749183517160255571915875356439553717130797888*e^(386*I*c) + 43638300
075815171025946211464465689618965773488660985857945657479854085108851857911
222911989837615452608512356008400295*e^(385*I*c) + 851395332347864557795899
594649006377607357297056212213800058372083691577946730496754287998178754314
30246332625899630160*e^(384*I*c) + 1644375006769068927413232601543942785039
545619360201335965818064493572402773494479273345171058099953000935492799312
73178*e^(383*I*c) + 3144090358082258615655954369383549454454739910431297220
46747228813030925204968503418566818838611866709807040793495364496*e^(382*I*
c) + 5951576155004315149474792823365470538279260879164252634971877570294133
85471835434198246807096214536895441388306027237899*e^(381*I*c) + 1115398285
545601550533328045600184317993899359217996729818340704221801195667410485846
996179056733558512452238583160792512*e^(380*I*c) + 206969828950086064346166
576237380795751301942404117887190496055182941244934472243212567941795840300
7551179298315947373776*e^(379*I*c) + 38026049967058911069646206338489648070
370988545101822632430305972956307603535975319743247522663891931857608782741
88013440*e^(378*I*c) + 6917838945214844278493330459361394923372333853619879
637372673184942859712431066345726870422099893124890777678037369988150*e^(37
7*I*c) + 124621404405372580849285967098720668577570709431248685545009481547
56863454308032925408340311237850017814707896986969086816*e^(376*I*c) + 2223
134113180153534540639903772168684020839794195258013558464596674673665671627
1554826476282991066076564921432614339399735*e^(375*I*c) + 39274200414329861
169397944516225001081227433398585007206399231211907157795359719648241598754
266579840244551491476467899952*e^(374*I*c) + 687124660159856415124685861736
597477348795917100983546527861249360230739431410495736066485630053594117127
64895683903806088*e^(373*I*c) + 1190605918496605468347656932276764490675841
482488826784475048260772363334445134540951266687500572958111916433569089721
91440*e^(372*I*c) + 2043255572651860007674027102308478964597615839227636982
35433212833313077783041040074669379017394836761539649081690630811665*e^(371
*I*c) + 3473100538109352904194555605559573141295692107357459832343696599764
13374774078000173070075248654524917179128950507443058208*e^(370*I*c) + 5847
495736823045861793846288448833275814989698865403803788967679990756149640071
74600811092945356635118795824799369716742109*e^(369*I*c) + 9752103394440493

187572823117635177866732231755944579463832792646350850410049173002959042754
33144848532459919875479817581584*e^(368*I*c) + 1611092541400060525954859375
264194178347643471837078201446262435615142944587337833513586022729849523358
436493586042252995608*e^(367*I*c) + 263666241043079934044752228477824428374
075106865814072657644667120779832560683229593770506168629793029638233889257
4900819440*e^(366*I*c) + 42748269077205917525267113368208715008443456479223
85471534359333606189571832444641364132893108663576205133870672156264164115*
e^(365*I*c) + 6866425337518668262662693750908956965732924578142181630622157
802899880874681551031136314064199948604001529894566235238597088*e^(364*I*c)
+ 109272106034735448102797923478445360745888968062300411100895447316058631
46104181739039426674855453466097402330688331845602302*e^(363*I*c) + 1722950
282436764733440072199841759670394865739473880520939163659737038057239896471
5080095366818322029152193635869784095333760*e^(362*I*c) + 26917794701086615
097890120236890501105146799996021775195710866226228638984703456832694153230
611607263444183501026198563419616*e^(361*I*c) + 416704403753905436434182193
422717480400350714901190805852815224981888183759069003687012345313046331634
46319945130196476913600*e^(360*I*c) + 6392301943376198909061480128863509812
319944510230312261654464820899876780394445577704288655273849974718371313606
9104651812215*e^(359*I*c) + 97173055024742680058616722461368892661141295540
26349301303274608353615732426833390400308958318370219154887169702257444756
176*e^(358*I*c) + 146390448456351181218237382740374124191664819997746988076
598391862733629670142241546375533903130605297580105675355629160198162*e^(35
7*I*c) + 218563166659649312247483640956272149212499115383828771029654283363
972585118090479413696638108156385244646591328454425745117584*e^(356*I*c) +
323413178014841003714151138246079152576360976035058457890409937738723171537
036573043681997163745313602400139153046673668433091*e^(355*I*c) + 474323043
563100542377338629931966248129175976982332446018056009391154020438895903140
822967769494019446166779954024655344116288*e^(354*I*c) + 689518449328793559
903260418149974190253578340058895035589606468244680591556118170304005037563
669880057908765898949268614772285*e^(353*I*c) + 993555653649521127226443960
820233649386488510081892545866700444096661582790441241830855609577062039555
625090943332264901780720*e^(352*I*c) + 141916448142217657385823401389899962
88223223330957373071631074383893583220145429361729317508645838962142530705
1750612129761498*e^(351*I*c) + 20094961100926877381527820856837372227279688
242990587392154460834993514676253344496707577640666906561509490149440439948
22823920*e^(350*I*c) + 2820819298562215959107529807289628449621386798989436
369393116069894018781201000275633104498398959346631795568022519974400130281
*e^(349*I*c) + 392569765841577835276810394285601184021164276962171721799661
4398473887186074391482638547212826538270453912634540299792270321024*e^(348*
I*c) + 54166628040524363495855959828183579538662584616443540182051589177425
76425344364964596750653177677803492186817305171175032011500*e^(347*I*c) + 7
410372612891152224364633296128043971657193280327754304382235672773781403023
814127610355562505271969045177704726054907145784960*e^(346*I*c) + 100522095
243695818275881549853455496780314457444999852082593854356097402724003014542
46872041775159838468077381562338745636398374*e^(345*I*c) + 1352123041194543

691555885470685179654156739981165687002156735297532681546781784653328912387
1696056195231696146162720992221760992*e^(344*I*c) + 18035327338177455471177
568594851682977978346449777193572087688510392426884519272991560851326393852
241961470040819793627127923997*e^(343*I*c) + 238563985655628020306952781742
126408332821541740064594582926440609479249073137359215616901539399060175186
47182491616573724049744*e^(342*I*c) + 3129526368818983831377277587330726033
411722725862950199235869563609266286606281984568906423581362297415012066892
1391878398978380*e^(341*I*c) + 40715988963701918950020348336736423420513311
359010485246919074652883393970805374470830156229705647312265477584256027212
762941040*e^(340*I*c) + 525392233467407711425870923702570695360603196444395
016676104827679558002760528924321527988146079751103662249450814281218884733
24*e^(339*I*c) + 6724408796908070382370325719966304760689061048209049461949
2802935130215979819469966383336788693900139115594646893784095418472336*e^(3
38*I*c) + 85368118430215312848231291739673735887746201851666299600392199418
764750086828198719872744047767783667325326289221881974987582215*e^(337*I*c)
+ 107504737406576916123480399169759633328321407419400510017498849830598621
565428266546315933920821527544726380201659114903834605888*e^(336*I*c) + 134
297742023479429904629616104559610096074758721068022704468938063017059688023
363436458971534964665036319889119229809973806909680*e^(335*I*c) + 166432332
922589195130558329266398753389823955737598527556096093062473559769545772321
978969318904192572733997888230986469005970880*e^(334*I*c) + 204622295535729
109519829916789867225319429705162560082648840394965423112809336591921290309
392396263834674368977840527147037426908*e^(333*I*c) + 249593072282565866398
389951509619202682634455487128714631461891770823201367527645793770203788784
677343934971424317987895255031936*e^(332*I*c) + 302060638030868463461139442
279360499718906917482524894483356220196138377050828911383056860425370161157
201493696073712322595776808*e^(331*I*c) + 362706307563843231135699157418510
732452420614013624168879312187645233450153927975793326834780741391203430153
093712635355523960320*e^(330*I*c) + 432147856464086938023811561808678589594
702047904674282297959658800170984456799067751878044806619012452636891350731
618278545690160*e^(329*I*c) + 510907615111134507452623846147137141450275722
444316385531648429230851686635827717488464500331623385777400744950538410637
735936000*e^(328*I*c) + 599378484771733474809376142401554850207064972118137
572949257503651444541939309025276896049622515630263162184526394317285457368
300*e^(327*I*c) + 697789106925924614816713747684682785083659819027952244447
043355869741368500452561164024636073401929693105801105522738405349028160*e^(
326*I*c) + 806169671327625532424575340089775733681994991576674446922354099
714615192085443245663852257001115644286660304979476023966071898200*e^(325*I
*c) + 924320052867522584035777495761072351222534420784861960001821020509468
146356756433795246446491396141583854513687104334429566707520*e^(324*I*c) +
105178210042883437194450817005121918711681634976695332263718214961087500422
3784784183284961906494422955462431208645690802526770780*e^(323*I*c) + 11878
179430793903161088023247981101290208227826600872485993676434812002060468221
6614428542592229375413676535071141005286431481600*e^(322*I*c) + 1331396114
626723035802462123531582050339749996014152452835305956367425370222758621753

922458727524856072950880960657564720475838500*e^(321*I*c) + 148118711708924
666246695569496567785552473031326055269082160265717621873742624552279532989
1464091005878304304075953693546767206080*e^(320*I*c) + 16355697446414219006
578863812890766532275801720566775874514020692343552836874896596139137619591
40773339736014790081814516625224440*e^(319*I*c) + 1792649078089632298936945
728481969334964391597506285088488350622937252533420980803144316431701452190
522716124797875257437516360640*e^(318*I*c) + 195028655078018191924499296120
448701005646036284521850167442376626632155879143691731787870223267921386828
7926294665202769722927380*e^(317*I*c) + 21061419034683443071125497612024845
434027942523524821998174104248696772627150982884376465186834879454627742236
56471345899082156800*e^(316*I*c) + 2257726219103856286812833012681573765496
262241420612932076143151171960854554124144699023009842080515157923529357189
869943515991200*e^(315*I*c) + 240246459556968608612000180303421105673944558
862194614138410616288624616181514976302503083487523406726777402343341826998
2431265280*e^(314*I*c) + 25377664154650303308154717466929885960699118946972
250529283204525421755871548480964833312098074301139430153983626696733379577
55720*e^(313*I*c) + 2661100647975783583828235139201441930178396643383423903
583862547255880772382049201015537214900832745601519737141849802506685264000
*e^(312*I*c) + 277007320715076864559750728138206549792496846605452741412233
9827333783770068305883487309979315983718403740872884345746380680204260*e^(3
11*I*c) + 28625031263204617977706677807256441849912556231746261756790506721
00848988119391841466573417019247590580735265143427289340450811200*e^(310*I*
c) + 2936494214351868498703239455426771104344827306267558916550877467232455
153286140521089582733932202553130712723836983468866230908800*e^(309*I*c) +
299049894962254360853812938028386633519008711512485881814378761918695711190
3765723974899651518555144924290346242595167274383008960*e^(308*I*c) + 30233
716435082250271756031752129532190224850458507318453075190082773851547314612
13388035579159917590062343527464977286601165100620*e^(307*I*c) + 3034408355
530957075787731745322567981684616550162845473257679674280216947356785783843
205604202307836897073595410412575660465787520*e^(306*I*c) + 302337164350822
502717560317521295321902248504585073184530751900827738515473146121338803557
9159917590062343527464977286601165100620*e^(305*I*c) + 29904989496225436085
381293802838663351900871151248588181437876191869571119037657239748996515185
55144924290346242595167274383008960*e^(304*I*c) + 2936494214351868498703239
455426771104344827306267558916550877467232455153286140521089582733932202553
130712723836983468866230908800*e^(303*I*c) + 286250312632046179777066778072
564418499125562317462617567905067210084898811939184146657341701924759058073
5265143427289340450811200*e^(302*I*c) + 27700732071507686455975072813820654
979249684660545274141223398273337837700683058834873099793159837184037408728
84345746380680204260*e^(301*I*c) + 2661100647975783583828235139201441930178
396643383423903583862547255880772382049201015537214900832745601519737141849
802506685264000*e^(300*I*c) + 253776641546503033081547174669298859606991189
469722505292832045254217558715484809648333120980743011394301539836266967333
7957755720*e^(299*I*c) + 24024645955696860861200018030342110567394455886219
461413841061628862461618151497630250308348752340672677740234334182699824312

65280*e^(298*I*c) + 2257726219103856286812833012681573765496262241420612932
076143151171960854554124144699023009842080515157923529357189869943515991200
*e^(297*I*c) + 210614190346834430711254976120248454340279425235248219981741
0424869677262715098288437646518683487945462774223656471345899082156800*e^(2
96*I*c) + 19502865507801819192449929612044870100564603628452185016744237662
66321558791436917317878702232679213868287926294665202769722927380*e^(295*I*
c) + 1792649078089632298936945728481969334964391597506285088488350622937252
533420980803144316431701452190522716124797875257437516360640*e^(294*I*c) +
163556974464142190065788638128907665322758017205667758745140206923435528368
7489659613913761959140773339736014790081814516625224440*e^(293*I*c) + 14811
871170892466624669556949656778555247303132605526908216026571762187374262455
22795329891464091005878304304075953693546767206080*e^(292*I*c) + 1331396114
626723035802462123531582050339749996014152452835305956367425370222758621753
922458727524856072950880960657564720475838500*e^(291*I*c) + 118781794307939
031610880232479811012902082278266008724859936764348120020604682216614428542
5922229375413676535071141005286431481600*e^(290*I*c) + 10517821004288343719
445081700512191871168163497669533226371821496108750042237847841832849619064
94422955462431208645690802526770780*e^(289*I*c) + 9243200528675225840357774
957610723512225344207848619600018210205094681463567564337952464464913961415
83854513687104334429566707520*e^(288*I*c) + 8061696713276255324245753400897
757336819949915766744469223540997146151920854432456638522570011156442866603
04979476023966071898200*e^(287*I*c) + 6977891069259246148167137476846827850
836598190279522444470433558697413685004525611640246360734019296931058011055
22738405349028160*e^(286*I*c) + 5993784847717334748093761424015548502070649
721181375729492575036514445419393090252768960496225156302631621845263943172
85457368300*e^(285*I*c) + 5109076151111345074526238461471371414502757224443
163855316484292308516866358277174884645003316233857774007449505384106377359
36000*e^(284*I*c) + 4321478564640869380238115618086785895947020479046742822
97959658800170984456799067751878044806619012452636891350731618278545690160*
e^(283*I*c) + 3627063075638432311356991574185107324524206140136241688793121
87645233450153927975793326834780741391203430153093712635355523960320*e^(282
*I*c) + 3020606380308684634611394422793604997189069174825248944833562201961
38377050828911383056860425370161157201493696073712322595776808*e^(281*I*c)
+ 2495930722825658663983899515096192026826344554871287146314618917708232013
67527645793770203788784677343934971424317987895255031936*e^(280*I*c) + 2046
222955357291095198299167898672253194297051625600826488403949654231128093365
91921290309392396263834674368977840527147037426908*e^(279*I*c) + 1664323329
225891951305583292663987533898239557375985275560960930624735597695457723219
78969318904192572733997888230986469005970880*e^(278*I*c) + 1342977420234794
299046296161045596100960747587210680227044689380630170596880233634364589715
34964665036319889119229809973806909680*e^(277*I*c) + 1075047374065769161234
803991697596333283214074194005100174988498305986215654282665463159339208215
27544726380201659114903834605888*e^(276*I*c) + 8536811843021531284823129173
967373588774620185166629960039219941876475008682819871987274404776778366732
5326289221881974987582215*e^(275*I*c) + 67244087969080703823703257199663047

606890610482090494619492802935130215979819469966383336788693900139115594646
 893784095418472336*e^(274*I*c) + 525392233467407711425870923702570695360603
 196444395016676104827679558002760528924321527988146079751103662249450814281
 21888473324*e^(273*I*c) + 4071598896370191895002034833673642342051331135901
 048524691907465288339397080537447083015622970564731226547758425602721276294
 1040*e^(272*I*c) + 31295263688189838313772775873307260334117227258629501992
 358695636092662866062819845689064235813622974150120668921391878398978380*e^(271*I*c) + 238563985655628020306952781742126408332821541740064594582926440
 60947924907313735921561690153939906017518647182491616573724049744*e^(270*I*c) + 1803532733817745547117756859485168297797834644977719357208768851039242
 6884519272991560851326393852241961470040819793627127923997*e^(269*I*c) + 13
 521230411945436915558854706851796541567399811656870021567352975326815467817
 846533289123871696056195231696146162720992221760992*e^(268*I*c) + 100522095
 243695818275881549853455496780314457444999852082593854356097402724003014542
 46872041775159838468077381562338745636398374*e^(267*I*c) + 7410372612891152
 224364633296128043971657193280327754304382235672773781403023814127610355562
 505271969045177704726054907145784960*e^(266*I*c) + 541666280405243634958559
 598281835795386625846164435401820515891774257642534436496459675065317767780
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 34540299792270321024*e^(264*I*c) + 2820819298562215959107529807289628449621
 386798989436369393116069894018781201000275633104498398959346631795568022519
 974400130281*e^(263*I*c) + 200949611009268773815278208568373722272796882429
 905873921544608349935146762533444967075776406669065615094901494404399482282
 3920*e^(262*I*c) + 1419164481422176573858234013898999628822322333095737307
 16310743838935832201454293617293175086458389621425307051750612129761498*e^(261*I*c) + 9935556536495211272264439608202336493864885100818925458667004440
 96661582790441241830855609577062039555625090943332264901780720*e^(260*I*c)
 + 6895184493287935599032604181499741902535783400588950355896064682446805915
 56118170304005037563669880057908765898949268614772285*e^(259*I*c) + 4743230
 435631005423773386299319662481291759769823324460180560093911540204388959031
 40822967769494019446166779954024655344116288*e^(258*I*c) + 3234131780148410
 037141511382460791525763609760350584578904099377387231715370365730436819971
 63745313602400139153046673668433091*e^(257*I*c) + 2185631666596493122474836
 409562721492124991153838287710296542833639725851180904794136966381081563852
 44646591328454425745117584*e^(256*I*c) + 1463904484563511812182373827403741
 241916648199977469880765983918627336296701422415463755339031306052975801056
 75355629160198162*e^(255*I*c) + 9717305502474268005861672246136889266114129
 55402634930130327460835361573242683339040030895831837021915488716970225744
 4756176*e^(254*I*c) + 63923019433761989090614801288635098123199445102303122
 616544648208998767803944455777042886552738499747183713136069104651812215*e^(253*I*c) + 416704403753905436434182193422717480400350714901190805852815224
 98188818375906900368701234531304633163446319945130196476913600*e^(252*I*c)
 + 2691779470108661509789012023689050110514679999602177519571086622622863898
 4703456832694153230611607263444183501026198563419616*e^(251*I*c) + 17229502

824367647334400721998417596703948657394738805209391636597370380572398964715
080095366818322029152193635869784095333760*e^(250*I*c) + 109272106034735448
102797923478445360745888968062300411100895447316058631461041817390394266748
55453466097402330688331845602302*e^(249*I*c) + 6866425337518668262662693750
908956965732924578142181630622157802899880874681551031136314064199948604001
529894566235238597088*e^(248*I*c) + 427482690772059175252671133682087150084
434564792238547153435933360618957183244464136413289310866357620513387067215
6264164115*e^(247*I*c) + 26366624104307993404475222847782442837407510686581
40726576446671207798325606832295937705061686297930296382338892574900819440*
e^(246*I*c) + 1611092541400060525954859375264194178347643471837078201446262
435615142944587337833513586022729849523358436493586042252995608*e^(245*I*c)
+ 975210339444049318757282311763517786673223175594457946383279264635085041
004917300295904275433144848532459919875479817581584*e^(244*I*c) + 584749573
682304586179384628844883327581498969886540380378896767999075614964007174600
811092945356635118795824799369716742109*e^(243*I*c) + 347310053810935290419
455560555957314129569210735745983234369659976413374774078000173070075248654
524917179128950507443058208*e^(242*I*c) + 204325557265186000767402710230847
896459761583922763698235433212833313077783041040074669379017394836761539649
081690630811665*e^(241*I*c) + 119060591849660546834765693227676449067584148
248882678447504826077236333444513454095126668750057295811191643356908972191
440*e^(240*I*c) + 687124660159856415124685861736597477348795917100983546527
86124936023073943141049573606648563005359411712764895683903806088*e^(239*I*
c) + 3927420041432986116939794451622500108122743339858500720639923121190715
7795359719648241598754266579840244551491476467899952*e^(238*I*c) + 22231341
131801535345406399037721686840208397941952580135584645966746736656716271554
826476282991066076564921432614339399735*e^(237*I*c) + 124621404405372580849
285967098720668577570709431248685545009481547568634543080329254083403112378
50017814707896986969086816*e^(236*I*c) + 6917838945214844278493330459361394
923372333853619879637372673184942859712431066345726870422099893124890777678
037369988150*e^(235*I*c) + 380260499670589110696462063384896480703709885451
0182263243030597295630760353597531974324752266389193185760878274188013440*e
^(234*I*c) + 20696982895008606434616657623738079575130194240411788719049605
51829412449344722432125679417958403007551179298315947373776*e^(233*I*c) + 1
115398285545601550533328045600184317993899359217996729818340704221801195667
410485846996179056733558512452238583160792512*e^(232*I*c) + 595157615500431
514947479282336547053827926087916425263497187757029413385471835434198246807
096214536895441388306027237899*e^(231*I*c) + 314409035808225861565595436938
354945445473991043129722046747228813030925204968503418566818838611866709807
040793495364496*e^(230*I*c) + 164437500676906892741323260154394278503954561
936020133596581806449357240277349447927334517105809995300093549279931273178
*e^(229*I*c) + 851395332347864557795899594649006377607357297056212213800058
37208369157794673049675428799817875431430246332625899630160*e^(228*I*c) + 4
363830007581517102594621146446568961896577348866098585794565747985408510885
1857911222911989837615452608512356008400295*e^(227*I*c) + 22140735001708603
270915180769241391662035578755903979148909213603822554792749183517160255571

915875356439553717130797888*e^(226*I*c) + 111194996453632010808810628248863
384924253756584489779355358463492904258215703830904254114185215166703713720
45206568345*e^(225*I*c) + 5527498849031178355861230009326668283926290082158
467118000698502719379939045918344222192742145711257028040974074674736*e^(22
4*I*c) + 271958928348374392604080510108034192124453031125460725092919277390
9331523226635035815672862569296693711643521070331394*e^(223*I*c) + 13243113
249840274283552229381476823786728607088171617414487496895935880208608475087
03702325320304649883120684987556400*e^(222*I*c) + 6382188929145337415058063
996624885560667836004960918764083749748774489717780360749962455811242834604
38065182071976085*e^(221*I*c) + 3043844711068133360102841601239063704338888
28490627422652551236966790916174520857759143930140187173492394981908258944*
e^(220*I*c) + 1436576871380447969429471197042595384588181994235168246745862
93691056119209866358123637772245409530799230553767222252*e^(219*I*c) + 6709
170613052966912501989921002157658023784346222953538629508707618929784999593
1360645605292130961496106707521506432*e^(218*I*c) + 31004319206069417077069
363141423487431828009098184744635678652284177439464941651812564519144918003
174108077634846014*e^(217*I*c) + 141764836528757049572020133432411179043699
77796653849959902524421980635189011634815653279605497783382888932766730080*
e^(216*I*c) + 6413381895855925184758231451062556380328594938511006577015218
119786536390213057284018202201631094434819584025113465*e^(215*I*c) + 287049
613141231445183467471535358943955329443080853193346608628854370924623076915
1392180699413405623017247753532944*e^(214*I*c) + 12710330829380489502013605
548312703426623439912775046123423003660250467417428565804452894017866563116
85859023084716*e^(213*I*c) + 5567563887111823403410261927342195461136517683
17380539005893679049394714017063698565272728813669054779077208977840*e^(212
*I*c) + 2412460212824400617929083177830328761948015971332060520912869970437
29145345755805710081489006741839439573984832678*e^(211*I*c) + 1033997554672
57436489847837640753754718204394473055795001467604326419876555568738295317
37211096115196005647730480*e^(210*I*c) + 4383497214291937768537869223302106
374455403310092850273748043897897674698989578407095190523778349037430593454
2955*e^(209*I*c) + 18379807084003359766027649217621144116091735572216620788
861535803449702273802588359076704241840733513439114113248*e^(208*I*c) + 762
178879191204706203884091779937460042889225819436763668294435609668140024631
2138001769285020661445991073249416*e^(207*I*c) + 31256834931787017434797047
503074901786662921507201793636043351135286233296846063431855407560199356621
48267863968*e^(206*I*c) + 1267597017294812913400146276042126929986480292870
190399107554311079964227280196522475370108738477856311765699610*e^(205*I*c)
+ 508324599301085460166978629683032661427654474082939048097939638391567298
795788389433842285751054665210868287680*e^(204*I*c) + 201557947424794098026
772478462040883395867512562930862943753568690084015585598010154781548625239
409581907397500*e^(203*I*c) + 790191495587665692547839884872323883529091449
82747171856772463223808993367091503402876467270176124342699654400*e^(202*I*
c) + 3062758105422195737839054728927760912957281393108273352024738722600002
0043538279468776707958420892547870128680*e^(201*I*c) + 11735856926245118493
113091002501604032341876985999082823520672241530200188223826392982302194084

667538488665600*e^(200*I*c) + 444541225929547462506765951419831296601541629
9968930393345630345914109720740573618884980520010028451496996210*e^(199*I*c
) + 16644750343872118093949177435029376389785749377547647639878358724104499
30690131572904279995484581013965001440*e^(198*I*c) + 6160031160229795849371
257017578872129983543009899913626280388610939145613320711919097149494265879
36910303300*e^(197*I*c) + 2253205325932206577679411092895162489997945210155
64134982827241710019675486694499689312466561907212627820000*e^(196*I*c) + 8
145208141382911182887541756425005484603769331281148049290916075819598915576
8107022568350953861815940704090*e^(195*I*c) + 29097651061247453406647569781
836910062165559852359052804259154165687125428752562385492373749486351714453
120*e^(194*I*c) + 102716025302028890024978135168494525909715128095290606651
97301097052210064576088348023234671975463677418470*e^(193*I*c) + 3582718002
163296061414536703715109897107198252739284546149343102348456124089657428594
946438660859773886240*e^(192*I*c) + 123466804189240997878001808175544021601
2582476396941937965899631953079203974222138794604328498972144766900*e^(191*
I*c) + 42035802483514679858361121014594215468443794936564789908837252480215
6222884839580011688655664280691773600*e^(190*I*c) + 14137993825355684328056
550580740330413060613072543475174579407983314136174891763998614537706643721
0546190*e^(189*I*c) + 46970224727117281826454045018070670522559756627580347
784535320014963482632359729444541885102274546002560*e^(188*I*c) + 154131112
114860239372949708207973767160813447881633865435224219397375079621258549818
81879168348260330000*e^(187*I*c) + 4995241956279138180205186744401688024388
272113921255663734956946927571305533146776898787878059685108480*e^(186*I*c)
+ 159877110105819269227052899967744474268563100623245618584492522014400230
5878120380828483988663574829100*e^(185*I*c) + 50529366312301525887848302573
8812813203397766845340065381261016353419722382620393032535960660921950400*e
^(184*I*c) + 15768584552885091872146287786443509025758314941556132342738656
2894447598277935629800939237175625149830*e^(183*I*c) + 48584258153140280447
314836868772131390195412419046732778458706015096881437076337910793584122475
073760*e^(182*I*c) + 147779550966171289987127451820714953621765069731830816
50233605274051677624970464340242755840025673760*e^(181*I*c) + 4437210917843
18234776434954443904699020056595069470847193617092114714077633077234972825
351226979360*e^(180*I*c) + 131505212093069212210229710532762284233587074342
8530891072983535862280094446607723473800477453914130*e^(179*I*c) + 38465584
208066627445406307878483717499894905250097532216200339254995341359246151936
5177908682078400*e^(178*I*c) + 11103414879700881944314389564446924229504986
7464313710969257619338899133799285616020069872611710850*e^(177*I*c) + 31626
644674725547731176795687527653571305969985923688392112164915553242573269490
908989570248533280*e^(176*I*c) + 888829502875102466704420383760797610148005
3134418614474620767522824868911959884352666444917404000*e^(175*I*c) + 24643
821908074396090797742268556796293678857097764358766308517162539626961923417
06239192878728160*e^(174*I*c) + 6740255305431330088948457752366252374507431
14473544537818170447134607102575676676056675328961590*e^(173*I*c) + 1818346
614061779013153301296771453811664491884131941411693443547549209690349526103
78945282257600*e^(172*I*c) + 4837948975643409984385779181658937940681504260

9340378747586437145781646245422045101230417309900*e^(171*I*c) + 12693496932
 964920565073673637181280088548682508880255337280065006566138696041797353216
 584528640*e^(170*I*c) + 328387476055581867672630948030673442015509858394807
 4469014168171874442170109648521627538755920*e^(169*I*c) + 83757920692341193
 245878648676537353394654523970899076948872481398218916510458989551890925632
 0*e^(168*I*c) + 21059430138564847118432907888031750495336183995415942743400
 9884661777259752542647709150036990*e^(167*I*c) + 52190912207661824215812271
 854269748071292843243227894769229690720010547141334131610989636000*e<sup>(166*I
 *c)</sup> + 127472196165033205413563430625628473686016221408567860254458145320379
 04111523242298235713300*e^(165*I*c) + 3067974296431747364198159623962463671
 617006419626851426148418602934852907379021659761911840*e^(164*I*c) + 727521
 010718394229291774073844694255798738667067535379759732795567942578751384250
 780476310*e^(163*I*c) + 169956327969929767773902096652629253283704505477127
 544556534417376686540936706073847337600*e^(162*I*c) + 391080312556018094765
 37535369611844440844903751605645023514572352045248104262933598850730*e<sup>(161
 *I*c)</sup> + 8862752142756957285681340885764904597935349569355321815647721172537
 159186491471311666400*e^(160*I*c) + 197779298066581813565130009432623915860
 5448870806970860577325385028609983034534672318500*e^(159*I*c) + 43454667678
 028004534634449876389254079717510575682751550929702418766029934548492019248
 0*e^(158*I*c) + 93986915313068179149083606065681482780836060510530154618486
 949839467131378859885998210*e^(157*I*c) + 200080068030300471372932782503215
 97113540716201983333126349281186679153199068045257216*e^(156*I*c) + 4191542
 500656826148093339414544159143964478472492315931809171859902114109005939942
 952*e^(155*I*c) + 863979933622330349556296820028395513198708064940505702126
 068652936800794826651264256*e^(154*I*c) + 175193170500618300241515632381912
 285157790097816049220671217212220015297133400636060*e^(153*I*c) + 349410716
 13276704649477943043339450201504075335160361865916029213860778606230624960*
 e^(152*I*c) + 6852993223145736687328885311617795435592940841439866351079655
 652312894721972796266*e^(151*I*c) + 132149805527130085142999386663161987442
 4534425188183592049727687571032156435077280*e^(150*I*c) + 25050102860892833
 2469340456829902067712233644464602753159945727868485722395506952*e<sup>(149*I*c
)</sup> + 46668223548266017806854592468100570289355960869613650856575756758180182
 223308768*e^(148*I*c) + 854301344112621233483354066506962147247908583804136
 0564550722036723654297540205*e^(147*I*c) + 15363332384449275835327345560164
 94671674916578907116984548489078241693926940560*e^(146*I*c) + 2713612075032
 66570734486517077181014801775322183181055638619257836143271472358*e<sup>(145*I*
 c)</sup> + 4706504461113515810848735336748424310269824883831263587628309942744274
 5866704*e^(144*I*c) + 80137295807907524343619649457615437614695207912107469
 72675870481058674277844*e^(143*I*c) + 1339214374254245553564884406801945353
 385000254030655765953770237607180089968*e^(142*I*c) + 219601281339515561500
 261478844190024870555261281946058839614044697037963695*e^(141*I*c) + 353244
 47206779018115378052820789411687581004582367431006205879633729015200*e<sup>(140
 *I*c)</sup> + 5572551157328671121016216416307596161861955969011697222340926210112
 854418*e^(139*I*c) + 861884851094991908764246805474672428603757315484453974
 713612812215428992*e^(138*I*c) + 130657660226560419335121434389938961884595

434069984824307149332131747540*e^(137*I*c) + 194079792155945665935350081033
 03255257745408070082431338945184797463936*e^(136*I*c) + 2823905151936586678
 382525706564457280290098698638597987628380245881715*e^(135*I*c) + 402349692
 266121158934003582839428785116904903936409545602519219664720*e^(134*I*c) +
 56117081076341175384087570185188538660375932013674735519055227368366*e^(133
 *I*c) + 7659010520187549651777118357676871927081898989131125755798204236112
 *e^(132*I*c) + 102253643746829673729306586270524644969368741555986584430688
 8705423*e^(131*I*c) + 13349021005202618377967331386833230353033290616324719
 4627808410304*e^(130*I*c) + 17033886027390615741040977721655541665612162275
 485028584310890417*e^(129*I*c) + 212370296918887131826671878122392706783994
 9015727293884065388080*e^(128*I*c) + 25858534871597727015582911568419341107
 2034541491364393985491350*e^(127*I*c) + 30736217404321009965231037419663053
 962881035281709221697785072*e^(126*I*c) + 356476489062872401708848799668817
 8929195787613958545474804845*e^(125*I*c) + 40321222595779818884084613996099
 5624144491271694336796459584*e^(124*I*c) + 44456708175258821024400946210535
 004523775722190977468484496*e^(123*I*c) + 477539860710085326353420773381826
 6777478693412738731031680*e^(122*I*c) + 49946750655853173367158586291057270
 2811545035730398749530*e^(121*I*c) + 50836369508171099437019348610847391946
 736185108017183136*e^(120*I*c) + 503202490340145182407421394376601192202650
 7006311982753*e^(119*I*c) + 48409341024048871865591702530366258109165912618
 2344528*e^(118*I*c) + 45230940039830738332025694784646206844854827698075736
 *e^(117*I*c) + 4101545439937195793959956708442496709433800261224880*e^(116*
 I*c) + 360688613036389349413809780004559963548775423325255*e^(115*I*c) + 30
 735366512830562160991166338490057308062762518496*e^(114*I*c) + 253566746065
 0279776834561566186591213109251642859*e^(113*I*c) + 20234750972446217131396
 6643580234078508179838320*e^(112*I*c) + 15603911277687607099721623771744933
 086920587272*e^(111*I*c) + 1161581413733971751533622511909046917188768400*e
 ^((110*I*c) + 83380839911837894453136303673785039051506805*e^(109*I*c) + 576
 4601046563151304213854710715346838447392*e^(108*I*c) + 38336015580105482452
 9764688213114368047154*e^(107*I*c) + 24489837337812338687718622491865013839
 488*e^(106*I*c) + 1500602747937397286405577818722691539392*e^(105*I*c) + 88
 054927598941411145869950813388040256*e^(104*I*c) + 493966661081802579880958
 6352543471345*e^(103*I*c) + 264410375780310742518099326419685040*e^(102*I*c
) + 13477227799524701956579274210395326*e^(101*I*c) + 652650253343206047453
 620559993840*e^(100*I*c) + 29952547749265499675257842032197*e^(99*I*c) + 12
 99146645993240318167826532288*e^(98*I*c) + 53090127264630963470039804475*e^
 (97*I*c) + 2037031259470368160131922320*e^(96*I*c) + 7309920781733559724709
 8038*e^(95*I*c) + 2442455629894502983849104*e^(94*I*c) + 755998170926701578
 06639*e^(93*I*c) + 2154864144781257856128*e^(92*I*c) + 56169444526926562260
 *e^(91*I*c) + 1327882849274858880*e^(90*I*c) + 28186192554792138*e^(89*I*c)
 + 530563624556832*e^(88*I*c) + 8718181624155*e^(87*I*c) + 122503723056*e^(
 86*I*c) + 1431118260*e^(85*I*c) + 13343760*e^(84*I*c) + 93096*e^(83*I*c) +
 432*e^(82*I*c) + e^(81*I*c))*tan(1/4*d*x + c) + 7*(338*I*a*e^(1055/2*I*c)
 + 136552*I*a*e^(1053/2*I*c) + 27515228*I*a*e^(1051/2*I*c) + 3687040552*I*a*
 e^(1049/2*I*c) + 369625815338*I*a*e^(1047/2*I*c) + 29570065227040*I*a*e^(10

$45/2 * I * c) + 1966409337599317 * I * a * e^{(1043/2 * I * c)} + 111804416623905668 * I * a * e^{(1041/2 * I * c)} + 5548294175032309402 * I * a * e^{(1039/2 * I * c)} + 2441249437098984204$
 $93 * I * a * e^{(1037/2 * I * c)} + 9642935277307714791837 * I * a * e^{(1035/2 * I * c)} + 3453924$
 $09079468621758030 * I * a * e^{(1033/2 * I * c)} + 11311601400768793085009738 * I * a * e^{(10$
 $31/2 * I * c)} + 341088288572157297772118077 * I * a * e^{(1029/2 * I * c)} + 95261086391402$
 $31470517528278 * I * a * e^{(1027/2 * I * c)} + 247678824959506386373000637444 * I * a * e^{(1$
 $025/2 * I * c)} + 6021691444519566631530369807136 * I * a * e^{(1023/2 * I * c)} + 137436252$
 $221498552065844084546042 * I * a * e^{(1021/2 * I * c)} + 29548794360631569349939310672$
 $26770 * I * a * e^{(1019/2 * I * c)} + 60030708924112460138240147528356200 * I * a * e^{(1017/$
 $2 * I * c)} + 1155591156922028898629041321537843182 * I * a * e^{(1015/2 * I * c)} + 2113080$
 $9978126689911485296435923679086 * I * a * e^{(1013/2 * I * c)} + 3678681977448171261151$
 $32997049285770709 * I * a * e^{(1011/2 * I * c)} + 610981106456226790697858505795658035$
 $5548 * I * a * e^{(1009/2 * I * c)} + 96993253296241745199299054522607239475550 * I * a * e^{($
 $1007/2 * I * c)} + 1474297501848124542057148378045565788918733 * I * a * e^{(1005/2 * I * c$
 $) + 21490722237212978408403217414663518345362065 * I * a * e^{(1003/2 * I * c)} + 30087$
 $0128273961175599469606519345892299669212 * I * a * e^{(1001/2 * I * c)} + 4051001655313$
 $083033785087944018198318088989698 * I * a * e^{(999/2 * I * c)} + 525233363944859296592$
 $33288472451790119792705845 * I * a * e^{(997/2 * I * c)} + 6565417755175106462345442804$
 $31980818352772409123 * I * a * e^{(995/2 * I * c)} + 7920859881151658505680929024974848$
 $479434840259054 * I * a * e^{(993/2 * I * c)} + 923275377293759706647808962879230340914$
 $45656319868 * I * a * e^{(991/2 * I * c)} + 1040783353217938776411028748013653576556213$
 $918915335 * I * a * e^{(989/2 * I * c)} + 113567856889895513439085155111988872331623017$
 $12666395 * I * a * e^{(987/2 * I * c)} + 1200574819111759471271173912783354250387275251$
 $57415250 * I * a * e^{(985/2 * I * c)} + 1230589594126194023203320811726992132021299197$
 $471177852 * I * a * e^{(983/2 * I * c)} + 122393823461605138922406012384550471250110286$
 $91886256771 * I * a * e^{(981/2 * I * c)} + 1182067205269991890935975421673724568536642$
 $93435038820382 * I * a * e^{(979/2 * I * c)} + 1109325204761582846669749366309189735990$
 $268133663930730770 * I * a * e^{(977/2 * I * c)} + 101225988592648902717939158690840034$
 $20710826310616818118992 * I * a * e^{(975/2 * I * c)} + 8986899228987572447754752957763$
 $0120615631448882291724117498 * I * a * e^{(973/2 * I * c)} + 77672552243281059007302851$
 $7365937240359454002830981290341970 * I * a * e^{(971/2 * I * c)} + 65389515297967354699$
 $11010316324282233950224197184638863745512 * I * a * e^{(969/2 * I * c)} + 5364918594328$
 $1059583991832533828279475735386275724477961967700 * I * a * e^{(967/2 * I * c)} + 42919$
 $4047823886492115016704404554514518112725730803903112532182 * I * a * e^{(965/2 * I * c$
 $) + 3349584649215974064494562288169726260989776567520122206199301331 * I * a * e^{($
 $963/2 * I * c)} + 2551390123780806833326128892534213199648383667021549525296876$
 $5174 * I * a * e^{(961/2 * I * c)} + 18976001127512012080908245217116426083644447991720$
 $0503452767697538 * I * a * e^{(959/2 * I * c)} + 13786676382429068339094872244483678916$
 $82486392320460023203024208675 * I * a * e^{(957/2 * I * c)} + 9788564956238722791049897$
 $452775743960339323004999489860937041472915 * I * a * e^{(955/2 * I * c)} + 679443512140$
 $79532127966887502579811300811007213403186988581793087318 * I * a * e^{(953/2 * I * c)}$
 $+ 461239107567627147326076654057456658457043425511345276082020328203684 * I * a$
 $* e^{(951/2 * I * c)} + 3063335070065447315308133852848066992101021248688567743307$
 $391113472411 * I * a * e^{(949/2 * I * c)} + 199117598387417926031189003205662174209347$
 $86172730376127765302242075212 * I * a * e^{(947/2 * I * c)} + 1267117847094533386056607$

78068542544022643349256399659726946153656233718*I*a*e^(945/2*I*c) + 7896900
39989072858764397361560374790810333801284467329771776351321048032*I*a*e^(94
3/2*I*c) + 4821293518673085167363493338203848847844190253193965493680746373
955080080*I*a*e^(941/2*I*c) + 288448227683149535792126719085007079507251784
23613479919214905815452892948*I*a*e^(939/2*I*c) + 1691590012546678217913607
84785389073333265377052642698186735860494211567022*I*a*e^(937/2*I*c) + 9726
72122556805618091233812449540815508255737026698123751589823682626604408*I*a
*e^(935/2*I*c) + 5485282326270569168343744931160098443379763576662271067319
928989360513500012*I*a*e^(933/2*I*c) + 303463009922187912110612328959183307
55427695817262812270205461788254462121188*I*a*e^(931/2*I*c) + 1647388919690
44198589212814142825245654731022138637322268898435719103132844200*I*a*e^(92
9/2*I*c) + 8777602140423293412419858009732851598736550581903364235112068015
39777723334852*I*a*e^(927/2*I*c) + 4591423692429468515885016139926071595050
527848377998359965229708397246546408548*I*a*e^(925/2*I*c) + 235835770676392
10318661600390456214302440996107182761770233626717895694882382172*I*a*e^(92
3/2*I*c) + 1189758463401729131592487255558182069289626151174463048430443261
62984826050727208*I*a*e^(921/2*I*c) + 5896411214077176351589511972323511292
97384981427873806303623085470095427193276692*I*a*e^(919/2*I*c) + 2871353973
777698342367234312486286687762934618612668642380811677650394711966142500*I*
a*e^(917/2*I*c) + 137417856595605575898866999448538754167996491063998205665
14814463893361892904950890*I*a*e^(915/2*I*c) + 6464603885648283613014530104
1965583999891241698291024005815352807403014487811899520*I*a*e^(913/2*I*c) +
29899596829739071545656839188275994608759453512170090365350364591685949769
0395095780*I*a*e^(911/2*I*c) + 13598573971342772024370692045568229995410159
16103929234362752517223588368000249304810*I*a*e^(909/2*I*c) + 6082802098350
359025768710715675956495702650305299754400572492569343098453009652566250*I*
a*e^(907/2*I*c) + 267652772109858246881160789136273609283582202235984950948
33068710253849363218567711980*I*a*e^(905/2*I*c) + 1158699669051248530097127
43230581680050040635976036640419536918816038916044404058390580*I*a*e^(903/2
*I*c) + 4935968762662148107626418822454818734573678869054029943502438078406
69121360440299203610*I*a*e^(901/2*I*c) + 2069405755469517768606845303743959
442907534110949655104608338576642070816206712850594640*I*a*e^(899/2*I*c) +
854000449095519783654305856856490873812705535148165262053330176201136460459
5026952327120*I*a*e^(897/2*I*c) + 34695679479197391254672720198579891366138
778568280423896803407887950191615247559195575960*I*a*e^(895/2*I*c) + 138791
049050218664398373326905547232792143053299682659302792667147805101052262490
681909560*I*a*e^(893/2*I*c) + 546737062480322552904613119794649473739260578
277683675349293374134581393610464155972682280*I*a*e^(891/2*I*c) + 212122716
885477530932922055684330195745925621532769525019636715611216148495682866483
2181880*I*a*e^(889/2*I*c) + 81067449514020403556506738911560136863838278415
81501510916125303851411890302126855054047420*I*a*e^(887/2*I*c) + 3052207467
221923878854205859343853485258636427341837120005521482648212740331282598694
3237280*I*a*e^(885/2*I*c) + 11322593106386724431761639455345559749498779183
9142975472506570258163576992032763272923073090*I*a*e^(883/2*I*c) + 41390126
754989055823435182991213105576849191069489002127889405055369599620111544891

$8378784640 * I * a * e^{(881/2 * I * c)} + 14911452921628698997872885605312811427061256$
 $23861982068118427515045600599195560543505352431020 * I * a * e^{(879/2 * I * c)} + 5295$
 $019379440696397366776161064889279568712285275229242010057123976569166715041$
 $171797137891170 * I * a * e^{(877/2 * I * c)} + 185349026160449005232573409986289689481$
 $49584677829515375456881018487654174288958798896933969850 * I * a * e^{(875/2 * I * c)}$
 $+ 6396431913830536683852424390350391246093878486472172857619258907137976008$
 $6707383054171805732800 * I * a * e^{(873/2 * I * c)} + 21764983871873411985186978485868$
 $9378603106679847986805041872690065735347538753124146680974449780 * I * a * e^{(871$
 $/2 * I * c)} + 73029655258696344225597087065783914623049345083825117804758766586$
 $6638789410736040062955533068290 * I * a * e^{(869/2 * I * c)} + 24166102823474984668967$
 $41078363790625389200690273045307234207106791714360472853141666795331774750 * I * a * e^{(867/2 * I * c)} + 7887248495370051935609113701125546897394431652357414444$
 $786810665029425502851513884434989024419020 * I * a * e^{(865/2 * I * c)} + 253921808396$
 $207175587073434951207934973174564120010599738564064332994760182863564542152$
 $10996398560 * I * a * e^{(863/2 * I * c)} + 8064419173488996776559216166904176121469433$
 $8439517250809770511678016755719669133078696159918617190 * I * a * e^{(861/2 * I * c)} +$
 $25268931035482758388494149738730795207903213724380640051561109730300983802$
 $5277886805474018066255230 * I * a * e^{(859/2 * I * c)} + 78123640003492233583452583227$
 $3074822606607778875807184513559794733690991968890185038246494481956500 * I * a * e^{(857/2 * I * c)} + 23834175127565223830229822759473644638224519265209696409146$
 $74528214045561036716921751876861448214560 * I * a * e^{(855/2 * I * c)} + 7175946538198$
 $935299155529026996842352667522857865938994004026182478124225655917226024463$
 $989568859870 * I * a * e^{(853/2 * I * c)} + 213234849147017215169538860194659781513497$
 $85426322129525596551026913910149503567591658927381185274800 * I * a * e^{(851/2 * I * c)}$
 $+ 6254243401902741848509484824848454113338082680345171591207671628116975$
 $3247585230468246410100921183300 * I * a * e^{(849/2 * I * c)} + 18107807683744905331264$
 $387812935563056139984311930680824984749925337461704160453834409464953801340$
 $8680 * I * a * e^{(847/2 * I * c)} + 51756820355312954410804386109705453232980813714151$
 $2218550020340601981346633003923694388693344112445480 * I * a * e^{(845/2 * I * c)} + 14$
 $605462002593652074486957173582431842768148914859289752956162795465010155732$
 $51800164340914796390068760 * I * a * e^{(843/2 * I * c)} + 4069524455400128914928462086$
 $271400007173410848833155506269926856142013580290716696933427177861645554760$
 $* I * a * e^{(841/2 * I * c)} + 111966121821141964442037838123563295873180241684479076$
 $57035795800817072585681312382705199633772907655120 * I * a * e^{(839/2 * I * c)} + 3042$
 $128121962949457071547645652476485619314620152367555522897979398723961297037$
 $4604657916876884833815680 * I * a * e^{(837/2 * I * c)} + 81629829044671572879258630168$
 $605666151289808623903865249335836814408606538176579973692550178994519263270$
 $* I * a * e^{(835/2 * I * c)} + 216337990832542916463946367030741381088306321383016439$
 $555020828318352316877090017854969031272563599670220 * I * a * e^{(833/2 * I * c)} + 566$
 $318310112973422190074463206605332641947751448347307430314678972924597750157$
 $502827727169011647198755540 * I * a * e^{(831/2 * I * c)} + 146441145990259158305146533$
 $104978388890275336672209564306551623036491156422842953732683402693987815861$
 $5190 * I * a * e^{(829/2 * I * c)} + 37408523468650487964094117821532204335247599947800$
 $65112544112920800396613415405338362337137549877683070230 * I * a * e^{(827/2 * I * c)}$
 $+ 9440883227662672550065366923781042631437653007247333510037128999390205650$

430820323780125609508653387501660*I*a*e^(825/2*I*c) + 235406497364212308866
095824188703992052061298469617125235817947664250271910002525674642295882105
47056463840*I*a*e^(823/2*I*c) + 5799848688724343199572068100416317544857349
4680504990319637888579588763738426026942298616787064620503722070*I*a*e^(821
/2*I*c) + 14120023726763338224103829629480899357391515954966181764655361743
7359133700881157615051697163328436455695020*I*a*e^(819/2*I*c) + 33970536672
655873836914010596386451773527523414340051373935767448448366505976532073655
7902256832515882267500*I*a*e^(817/2*I*c) + 80769022503565948483170779513278
182902665161792021013282953291691617090536488162475060828002976266044874284
0*I*a*e^(815/2*I*c) + 18979705353287041219813221325693084005669224783346327
82784394455220770810883560414441976268438153636677378420*I*a*e^(813/2*I*c)
+ 4408214207480768029716973149859780404324438947607836425499375750084309188
687239341140052519808074207080308780*I*a*e^(811/2*I*c) + 101202318762952166
047217781137436284619629964897559405277691717525951814998806078062909622822
36974162704806460*I*a*e^(809/2*I*c) + 2296668787724545702579410207337767735
7995629076173854095854698682796300148885688222697251615914395078685755000*I
*a*e^(807/2*I*c) + 51524243379903114703452475096762441385775808238928447266
375386475837054738084592021134353327686946496437282284*I*a*e^(805/2*I*c) +
114276017965510704536422999678624463235888712139759411494933394918036613892
504103005096855020042456078555128196*I*a*e^(803/2*I*c) + 250583999994178547
973769655475783365474416927100959735219628296998411286972584917312607492527
753131218181041384*I*a*e^(801/2*I*c) + 543288572546872620415741148755828577
467632034006400828150383133866837214270664111437963716798350612156148456206
*I*a*e^(799/2*I*c) + 116469103885404056506882596475507370970791963843625556
6567555148408120876309968864222625458155464316902364952524*I*a*e^(797/2*I*c
) + 24689795321202892272688406168200044504581174237694806443432279718895668
76733150004369035355668553948933308821480*I*a*e^(795/2*I*c) + 5175761305072
823696040196698965533450997327744793152831047341015798393926092260966184527
256512322695767910061296*I*a*e^(793/2*I*c) + 107301092713177499323083813829
105498651961301555042478503350150672469877026772333775321254702023915671511
17668374*I*a*e^(791/2*I*c) + 2200037804428534402873482573286667665876293487
1965249674527380850469340847187834791650053923537769840230493764196*I*a*e^(
789/2*I*c) + 44614390963628429378874329909878550605230830963183562020504214
280108729796984369361375912054446637154294300258539*I*a*e^(787/2*I*c) + 894
872505724728370624201553901098068070796612063573577599972562878494813596597
99894879391911743427849611697524276*I*a*e^(785/2*I*c) + 1775460876760394876
532070304768547726662435411567457405751271227211038129174484594994757961542
10708959328249028630*I*a*e^(783/2*I*c) + 3484552538386882013605057839411137
322515274454773630574061883426726340639345544052866635925010406288601835439
05219*I*a*e^(781/2*I*c) + 6765361123754535335353223274353047625071160083629
88186332586271393372786322039392090789530327993045045404821029011*I*a*e^(77
9/2*I*c) + 1299466170334480341543135560982800973046388591632081102321927761
098390304943531108062897890291528379823914088825794*I*a*e^(777/2*I*c) + 246
939675213444454420928792091871436788268416425362146048269276656983377978163
3857461623947418830676182596868630102*I*a*e^(775/2*I*c) + 46429120789156830

103580860070870170239739657131314593283519782257106530869458014982756831946
47474013409696663828963*I*a*e^(773/2*I*c) + 8637448429412603145232295486970
579164573288359043608779825334105587873485306782005310584032334923334469052
992471566*I*a*e^(771/2*I*c) + 159000166293990983770633365148321190576578131
48770829120355232070502630384804444889767486179756113401575843696792900*I*a
*e^(769/2*I*c) + 2896332700674538255726052339252100276616242443147314330601
5072868867675529419511652966969349719029403862600773620600*I*a*e^(767/2*I*c
) + 52210724333347160801501773165819812944993848127565638741792799335959055
388789921819771479296204943724701127192070026*I*a*e^(765/2*I*c) + 931436044
993931263445166225811972204036547143672127723996445360856060800463349334285
45626738637712217067955072450098*I*a*e^(763/2*I*c) + 1644561376434840230278
392207278101839798107429845574439865731064657301602431360510964884269705632
21576065210083954112*I*a*e^(761/2*I*c) + 2873906059262755425755484216228398
712102349721261608857322496442215933427846303190237392024424792026905463808
83489714*I*a*e^(759/2*I*c) + 4970982705744169777939999010222689126124145532
53938578661999730743751218491838334678754461337804861175391987745632150*I*a
*e^(757/2*I*c) + 8511007554959481673119673847993823253939084737081335367440
04690162732128381564701822378684740339904953764853311657379*I*a*e^(755/2*I*
c) + 1442482860527021727025998678990411107321021387835542377307580287512051
573989107700940225360216432991175352627674951660*I*a*e^(753/2*I*c) + 242021
155405915375047953074533158882160450744844571122745730374460166840055721317
8896517376949590952343768990797772626*I*a*e^(751/2*I*c) + 40200468440323258
066066869587785394442990764582328053362087483228106281545807646529066353454
66720611170521299087695579*I*a*e^(749/2*I*c) + 6610986490722064658800908996
594970089148031698056312546893974727551725296838192813109416462543781169819
141205979640295*I*a*e^(747/2*I*c) + 107641858801753142474554137806023873166
880496795733541361846183879500257001189713176124123961781803519004352221120
93164*I*a*e^(745/2*I*c) + 1735395255611804409186625004390427952487663973701
3840121333225227870238823195868916660184536540709110237107927786882862*I*a*
e^(743/2*I*c) + 27703880658797418014141798369767308004525850429299648596969
106478869158139812082333717388082946821508766049481077973059*I*a*e^(741/2*I
*c) + 437956160097493579765068474053492147508352811690398993662144001844139
44991292881347390284533191154910866650017714941045*I*a*e^(739/2*I*c) + 6856
326772480152583553009118563234215337021944196809319949343958539927990099302
7594546478734905346362251571732607984770*I*a*e^(737/2*I*c) + 10630321158236
029079093688060762090286918969908906507228226986583823766209641521560821346
5309152317417335803254777646540*I*a*e^(735/2*I*c) + 16323700153006415236755
606122558009743798044040551757218392703576723018130996281464206087944437728
5948362185495154086161*I*a*e^(733/2*I*c) + 24827424893256161794570369766352
520061007857443406140981388900138235391030810939089972770658040659664048257
4074089879213*I*a*e^(731/2*I*c) + 37403241875119881172022846385732487428813
284882744325815756034266411597497467396971732361062007898502816726694679824
7966*I*a*e^(729/2*I*c) + 55818109155457679534086754423714982536666008344943
2838329374039537797461902854863923357577080240107340008718422024591820*I*a*
e^(727/2*I*c) + 82518864671237129917668538300374154515196623739664827393661

4407065191999961220292971482894130114743521964516001714179861*I*a*e^(725/2*I*c) + 12085585125205618679814100562904687888330622454974050388723038643523
 72871609518678342756379906875785368808430218531927334*I*a*e^(723/2*I*c) + 1
 753648721621948344665323035344500928169246731037156030237049745174658096990
 360960149750629287727493716142281998954108750*I*a*e^(721/2*I*c) + 252117074
 822840814497403707614675960285810411663855353744002021745697040909473065379
 3237536846366906505696581378882054616*I*a*e^(719/2*I*c) + 35914593751591787
 152728742998311655187833617405919229803455910474415178178055595362360826739
 85676829650893577170263379850*I*a*e^(717/2*I*c) + 5069592243524225405652688
 460326152875966709816050275878460357717440876793202354867243322880737700534
 668203200200223745266*I*a*e^(715/2*I*c) + 709141321503593510840471686830456
 660355600325275476682344989682912218724940528811036840580613971249581289084
 1064231210928*I*a*e^(713/2*I*c) + 98304751991270654066101070302328422717822
 031819476675378378907425848514987139484100775572274654750919377102497693237
 67412*I*a*e^(711/2*I*c) + 1350586343554588426504002605984859829589996048371
 6033585986818438381609952126332420911813536022155582505474802194829519694*I
 *a*e^(709/2*I*c) + 1839078777852711402811233434183425152417220026075511506
 882808089209223235028530568167961786178831641276993498874053500525*I*a*e^(7
 07/2*I*c) + 248217417034951034650827387237655682083567507014810112474664187
 97200402450920850968639227625439549849474276227435039858410*I*a*e^(705/2*I*
 c) + 3320791740547982497177231684382745550440305039953544294261353941150935
 0547092434970208286776394915006932929394522500297294*I*a*e^(703/2*I*c) + 44
 040443029368273279921818885276545283392557505519036132927785267603972753156
 572011647468523788004999404542569517581517357*I*a*e^(701/2*I*c) + 579008746
 767270585768451007780664586553146539732308445153172147345172454032245121379
 19416910585252662901629186232759644253*I*a*e^(699/2*I*c) + 7546823932870119
 439529542933759974862513581047508822037598365484305167320538287023700169066
 7166243440408007001704577025946*I*a*e^(697/2*I*c) + 97523794881693437070748
 447421304334356560534454944874598301077916325487914962129843509415087293500
 937741240715003092219284*I*a*e^(695/2*I*c) + 124952561789572611845340672901
 98496356888471113299103324124065991407644096671531128521516242907079372487
 240401894820911301*I*a*e^(693/2*I*c) + 158740600917927756806842461346197698
 018633387914866001093265977618455672590296399398391407324627011561459934928
 460625495480*I*a*e^(691/2*I*c) + 199966978737582534787918030254773650478417
 629776542960095080035676386188635182585291285252871357795555019530030846146
 346634*I*a*e^(689/2*I*c) + 249789388564135872584605676300302394509382132318
 737927877105275729996941134754667820904263042980484891534320044406713265976
 *I*a*e^(687/2*I*c) + 309422498620400775071454638476849117557897927017418041
 379379645423910232537399353417216157449077614118952637848517622461364*I*a*e
 ^ (685/2*I*c) + 380108285039915468450978858646485727016104746077018138051055
 881056469177249022540749317142917534972043461283677156449777056*I*a*e^(683/
 2*I*c) + 463077887159345985240066365048769653961899496722147648006706160892
 430828599260536532398498029152984249659429757073514254834*I*a*e^(681/2*I*c)
 + 559504894566289361813285134367579689512812244042262151586772699084107339
 658616004499973602767883158496957453596573417243760*I*a*e^(679/2*I*c) + 670

450434518367673310598723205770260142877601963255249446103714768964388684117
 340565887082959657303903689573624209578443416*I*a*e^(677/2*I*c) + 796800960
 938061420567296985439828502401638873253959462072849963044169306655915733067
 106371194259686348644202335486139211144*I*a*e^(675/2*I*c) + 939200230533694
 163569508797061901464826518892456171897778418746558378982592001120989002153
 289888436624007732395176649784016*I*a*e^(673/2*I*c) + 109797755820963888178
 127205348435487247165418262903769157172660389764200902221523637457788073902
 6045741294959273615557371064*I*a*e^(671/2*I*c) + 12730750365056582893357722
 256858724777153745047266128618306504565373058620598129538692444814409538758
 71577522924627878599496*I*a*e^(669/2*I*c) + 1463976940963912447651365515467
 185405762501924914077047324754706711366589407997958028777358245267180524254
 879552356822975000*I*a*e^(667/2*I*c) + 166964498101759185688009959290938726
 432806535901211729968766539207882830804280636546810086048766576965125518215
 6660820289680*I*a*e^(665/2*I*c) + 18884633505011777972779840776652500676250
 657886706129959370168287364866134703413117858923348162704534361325290917160
 95385240*I*a*e^(663/2*I*c) + 2118197643147292177460018767745386996600286988
 408731141330032157425554534393958723652997267660637525977591682505254490180
 360*I*a*e^(661/2*I*c) + 235597159356506010403841878445160808428982313839723
 0756698258679170071745498684434157111905965169387211600490743639011934220*I
 *a*e^(659/2*I*c) + 25982652607794097753203519002343915395098479161698383899
 84592795272199305469446132274424760298191811242353465634710031180000*I*a*e^
 (657/2*I*c) + 2840937680391253376694793235386260264165181954461073860260663
 943093784238240140243587013416430815228684501592017944484715960*I*a*e^(655/
 2*I*c) + 307927617921356210953777107862724068817115384009612166108122831524
 4104452786839486078569491845014868170250665115287493949260*I*a*e^(653/2*I*c
) + 33080734958809439532983859033062834196656613098294098946198541637745434
 45959563910317004475455576865647862947887778962083020*I*a*e^(651/2*I*c) + 3
 521732622041061038550984940990187934660572168825766556068910780855010177940
 257565117450855025856247255801996413769987213480*I*a*e^(649/2*I*c) + 371439
 792624457786523536688662927317155788405019136216329247132549822732080321052
 0277222404279215283455652034473210863303640*I*a*e^(647/2*I*c) + 38801097142
 781304168851412266740882386770409188825219490326196287424645516944761604588
 75162535150778871755003462307751583020*I*a*e^(645/2*I*c) + 4012977992311501
 006598317779033127050054097994290386865552989735233629291236389911293983802
 554742032304564034581648071441520*I*a*e^(643/2*I*c) + 410736991435895460891
 381503873310576186892582307966048646911286224748129369356598267601802184398
 2158540517444046989929778560*I*a*e^(641/2*I*c) + 41581042941815303182691599
 910307796054226321620259136051809827170067553972826431184119541582131734856
 27554911230413551069360*I*a*e^(639/2*I*c) + 4160645719004275565304960215369
 141503063578927293698222851613642937196324199454397834076656748547427159039
 358357564304622240*I*a*e^(637/2*I*c) + 411129028232729378919209515264935437
 007088137901238016084937413844813831101022171430989588860911791187017594280
 7299965844160*I*a*e^(635/2*I*c) + 40073348039561720648476456245739750447529
 022059146357971496852115844588791691449653159891664422568049504033358903448
 43916080*I*a*e^(633/2*I*c) + 3847221655924066177393579828048385255032885389

661433978074662073291344120217371070464927892827695267540237246884342651895
880*I*a*e^(631/2*I*c) + 363065196986830578863605993260490446530758652609376
9642523526793124670415020840772141816616075958474486988941581617999689040*I
*a*e^(629/2*I*c) + 33586610479866735734417779268260981728826287891982343116
69728226199222850309825295635814951021622934550781683257670586071740*I*a*e^(627/2*I*c) + 3033651196230917471784130478514016494934585854447176100608031
487731329011389156939224211578883484516187572097853270424000800*I*a*e^(625/2*I*c) + 265937888393631608837786315928610665306860335528607818507061214359
0704786554387476399696078202766125630847092154086813465000*I*a*e^(623/2*I*c)
) + 22408950295594289363714204936872577855087935094800690649179661997933772
05061951097939009853477230036485584868189563257145660*I*a*e^(621/2*I*c) + 1
784439224726138591870127611501980562247256627957670212130377044528036983916
903547976062125338219081770039268290168415803980*I*a*e^(619/2*I*c) + 129729
073691761268934956170566184481228086508778832627261684651445176762560315771
8949168500603037628131561607285054911012000*I*a*e^(617/2*I*c) + 78758107057
402302344708628907689331282129505846540059599829360414446802659436547122088
6365550607609800151696278587848326040*I*a*e^(615/2*I*c) + 26407461608063870
256740547485279735699023562580769891006453063959288214072008619586399697905
0655375410841792287731053730620*I*a*e^(613/2*I*c) - 26407461608063870256740
547485279735699023562580769891006453063959288214072008619586399697905065537
5410841792287731053730620*I*a*e^(611/2*I*c) - 78758107057402302344708628907
68933128212950584654005959982936041444680265943654712208863655060760980015
1696278587848326040*I*a*e^(609/2*I*c) - 12972907369176126893495617056618448
122808650877883262726168465144517676256031577189491685006030376281315616072
85054911012000*I*a*e^(607/2*I*c) - 1784439224726138591870127611501980562247
256627957670212130377044528036983916903547976062125338219081770039268290168
415803980*I*a*e^(605/2*I*c) - 224089502955942893637142049368725778550879350
948006906491796619979337720506195109793900985347723003648558486818956325714
5660*I*a*e^(603/2*I*c) - 26593788839363160883778631592861066530686033552860
78185070612143590704786554387476399696078202766125630847092154086813465000*
I*a*e^(601/2*I*c) - 3033651196230917471784130478514016494934585854447176100
608031487731329011389156939224211578883484516187572097853270424000800*I*a*e^(599/2*I*c) - 335866104798667357344177792682609817288262878919823431166972
8226199222850309825295635814951021622934550781683257670586071740*I*a*e^(597/2*I*c) - 36306519698683057886360599326049044653075865260937696425235267931
24670415020840772141816616075958474486988941581617999689040*I*a*e^(595/2*I*c) - 3847221655924066177393579828048385255032885389661433978074662073291344
120217371070464927892827695267540237246884342651895880*I*a*e^(593/2*I*c) -
400733480395617206484764562457397504475290220591463579714968521158445887916
9144965315989166442256804950403335890344843916080*I*a*e^(591/2*I*c) - 41112
902823272937891920951526493543700708813790123801608493741384481383110102217
14309895888609117911870175942807299965844160*I*a*e^(589/2*I*c) - 4160645719
004275565304960215369141503063578927293698222851613642937196324199454397834
076656748547427159039358357564304622240*I*a*e^(587/2*I*c) - 415810429418153
031826915999103077960542263216202591360518098271700675539728264311841195415

8213173485627554911230413551069360*I*a*e^(585/2*I*c) - 41073699143589546089
138150387331057618689258230796604864691128622474812936935659826760180218439
82158540517444046989929778560*I*a*e^(583/2*I*c) - 4012977992311501006598317
779033127050054097994290386865552989735233629291236389911293983802554742032
304564034581648071441520*I*a*e^(581/2*I*c) - 388010971427813041688514122667
408823867704091888252194903261962874246455169447616045887516253515077887175
5003462307751583020*I*a*e^(579/2*I*c) - 37143979262445778652353668866292731
715578840501913621632924713254982273208032105202772224042792152834556520344
73210863303640*I*a*e^(577/2*I*c) - 3521732622041061038550984940990187934660
572168825766556068910780855010177940257565117450855025856247255801996413769
987213480*I*a*e^(575/2*I*c) - 330807349588094395329838590330628341966566130
982940989461985416377454344595956391031700447545557686564786294788777896208
3020*I*a*e^(573/2*I*c) - 30792761792135621095377710786272406881711538400961
21661081228315244104452786839486078569491845014868170250665115287493949260*
I*a*e^(571/2*I*c) - 2840937680391253376694793235386260264165181954461073860
260663943093784238240140243587013416430815228684501592017944484715960*I*a*e
^(569/2*I*c) - 259826526077940977532035190023439153950984791616983838998459
2795272199305469446132274424760298191811242353465634710031180000*I*a*e^(567
/2*I*c) - 23559715935650601040384187844516080842898231383972307566982586791
70071745498684434157111905965169387211600490743639011934220*I*a*e^(565/2*I*
c) - 2118197643147292177460018767745386996600286988408731141330032157425554
534393958723652997267660637525977591682505254490180360*I*a*e^(563/2*I*c) -
188846335050117779727798407766525006762506578867061299593701682873648661347
0341311785892334816270453436132529091716095385240*I*a*e^(561/2*I*c) - 16696
449810175918568800995929093872643280653590121172996876653920788283080428063
65468100860487665769651255182156660820289680*I*a*e^(559/2*I*c) - 1463976940
963912447651365515467185405762501924914077047324754706711366589407997958028
777358245267180524254879552356822975000*I*a*e^(557/2*I*c) - 127307503650565
828933577222568587247771537450472661286183065045653730586205981295386924448
1440953875871577522924627878599496*I*a*e^(555/2*I*c) - 10979775582096388817
812720534843548724716541826290376915717266038976420090222152363745778807390
26045741294959273615557371064*I*a*e^(553/2*I*c) - 9392002305336941635695087
970619014648265188924561718977784187465583789825920011209890021532898884366
24007732395176649784016*I*a*e^(551/2*I*c) - 7968009609380614205672969854398
285024016388732539594620728499630441693066559157330671063711942596863486442
02335486139211144*I*a*e^(549/2*I*c) - 6704504345183676733105987232057702601
428776019632552494461037147689643886841173405658870829596573039036895736242
09578443416*I*a*e^(547/2*I*c) - 5595048945662893618132851343675796895128122
440422621515867726990841073396586160044999736027678831584969574535965734172
43760*I*a*e^(545/2*I*c) - 4630778871593459852400663650487696539618994967221
47648006706160892430828599260536532398498029152984249659429757073514254834*
I*a*e^(543/2*I*c) - 3801082850399154684509788586464857270161047460770181380
51055881056469177249022540749317142917534972043461283677156449777056*I*a*e^
(541/2*I*c) - 3094224986204007750714546384768491175578979270174180413793796
45423910232537399353417216157449077614118952637848517622461364*I*a*e^(539/2

$*I*c)$ - 2497893885641358725846056763003023945093821323187379278771052757299
 96941134754667820904263042980484891534320044406713265976 $*I*a*e^{(537/2*I*c)}$
 - 1999669787375825347879180302547736504784176297765429600950800356763861886
 35182585291285252871357795555019530030846146346634 $*I*a*e^{(535/2*I*c)}$ - 1587
 406009179277568068424613461976980186333879148660010932659776184556725902963
 99398391407324627011561459934928460625495480 $*I*a*e^{(533/2*I*c)}$ - 1249525617
 895726118453406729019849635688847111329910333241240659914076440966715311285
 21516242907079372487240401894820911301 $*I*a*e^{(531/2*I*c)}$ - 9752379488169343
 707074844742130433435656053445494487459830107791632548791496212984350941508
 7293500937741240715003092219284 $*I*a*e^{(529/2*I*c)}$ - 75468239328701194395295
 429337599748625135810475088220375983654843051673205382870237001690667166243
 440408007001704577025946 $*I*a*e^{(527/2*I*c)}$ - 579008746767270585768451007780
 664586553146539732308445153172147345172454032245121379194169105852526629016
 29186232759644253 $*I*a*e^{(525/2*I*c)}$ - 4404044302936827327992181888527654528
 339255750551903613292778526760397275315657201164746852378800499940454256951
 7581517357 $*I*a*e^{(523/2*I*c)}$ - 33207917405479824971772316843827455504403050
 399535442942613539411509350547092434970208286776394915006932929394522500297
 294 $*I*a*e^{(521/2*I*c)}$ - 248217417034951034650827387237655682083567507014810
 11247466418797200402450920850968639227625439549849474276227435039858410 $*I*a$
 $*e^{(519/2*I*c)}$ - 183907877785271140281123343418342515241722002607551150688
 2808089209223235028530568167961786178831641276993498874053500525 $*I*a*e^{(517$
 $/2*I*c)}$ - 13505863435545884265040026059848598295899960483716033585986818438
 381609952126332420911813536022155582505474802194829519694 $*I*a*e^{(515/2*I*c)}$
 - 983047519912706540661010703023284227178220318194766753783789074258485149
 8713948410077557227465475091937710249769323767412 $*I*a*e^{(513/2*I*c)}$ - 70914
 132150359351084047168683045666035560032527547668234498968291221872494052881
 10368405806139712495812890841064231210928 $*I*a*e^{(511/2*I*c)}$ - 5069592243524
 225405652688460326152875966709816050275878460357717440876793202354867243322
 880737700534668203200200223745266 $*I*a*e^{(509/2*I*c)}$ - 359145937515917871527
 287429983116551878336174059192298034559104744151781780555953623608267398567
 6829650893577170263379850 $*I*a*e^{(507/2*I*c)}$ - 25211707482284081449740370761
 467596028581041166385535374400202174569704090947306537932375368463669065056
 96581378882054616 $*I*a*e^{(505/2*I*c)}$ - 1753648721621948344665323035344500928
 169246731037156030237049745174658096990360960149750629287727493716142281998
 954108750 $*I*a*e^{(503/2*I*c)}$ - 120855851252056186798141005629046878883306224
 549740503887230386435237287160951867834275637990687578536880843021853192733
 4 $*I*a*e^{(501/2*I*c)}$ - 82518864671237129917668538300374154515196623739664827
 3936614407065191999961220292971482894130114743521964516001714179861 $*I*a*e^{($
 $499/2*I*c)}$ - 55818109155457679534086754423714982536666008344943283832937403
 9537797461902854863923357577080240107340008718422024591820 $*I*a*e^{(497/2*I*c$
 $)}$ - 37403241875119881172022846385732487428813284882744325815756034266411597
 4974673969717323610620078985028167266946798247966 $*I*a*e^{(495/2*I*c)}$ - 24827
 424893256161794570369766352520061007857443406140981388900138235391030810939
 0899727706580406596640482574074089879213 $*I*a*e^{(493/2*I*c)}$ - 16323700153006
 415236755606122558009743798044040551757218392703576723018130996281464206087

9444377285948362185495154086161*I*a*e^(491/2*I*c) - 10630321158236029079093
 688060762090286918969908906507228226986583823766209641521560821346530915231
 7417335803254777646540*I*a*e^(489/2*I*c) - 68563267724801525835530091185632
 342153370219441968093199493439585399279900993027594546478734905346362251571
 732607984770*I*a*e^(487/2*I*c) - 437956160097493579765068474053492147508352
 811690398993662144001844139449912928813473902845331911549108666500177149410
 45*I*a*e^(485/2*I*c) - 2770388065879741801414179836976730800452585042929964
 8596969106478869158139812082333717388082946821508766049481077973059*I*a*e^(
 483/2*I*c) - 17353952556118044091866250043904279524876639737013840121333225
 227870238823195868916660184536540709110237107927786882862*I*a*e^(481/2*I*c)
 - 107641858801753142474554137806023873166880496795733541361846183879500257
 00118971317612412396178180351900435222112093164*I*a*e^(479/2*I*c) - 6610986
 490722064658800908996594970089148031698056312546893974727551725296838192813
 109416462543781169819141205979640295*I*a*e^(477/2*I*c) - 402004684403232580
 66068695877853944429907645823280533620874832281062815458076465290663534546
 6720611170521299087695579*I*a*e^(475/2*I*c) - 24202115540591537504795307453
 315888216045074484457112274573037446016684005572131788965173769495909523437
 68990797772626*I*a*e^(473/2*I*c) - 1442482860527021727025998678990411107321
 021387835542377307580287512051573989107700940225360216432991175352627674951
 660*I*a*e^(471/2*I*c) - 851100755495948167311967384799382325393908473708133
 536744004690162732128381564701822378684740339904953764853311657379*I*a*e^(4
 69/2*I*c) - 497098270574416977793999901022268912612414553253938578661999730
 743751218491838334678754461337804861175391987745632150*I*a*e^(467/2*I*c) -
 287390605926275542575548421622839871210234972126160885732249644221593342784
 630319023739202442479202690546380883489714*I*a*e^(465/2*I*c) - 164456137643
 484023027839220727810183979810742984557443986573106465730160243136051096488
 426970563221576065210083954112*I*a*e^(463/2*I*c) - 931436044993931263445166
 225811972204036547143672127723996445360856060800463349334285456267386377122
 17067955072450098*I*a*e^(461/2*I*c) - 5221072433334716080150177316581981294
 499384812756563874179279933595905538878992181977147929620494372470112719207
 0026*I*a*e^(459/2*I*c) - 28963327006745382557260523392521002766162424431473
 143306015072868867675529419511652966969349719029403862600773620600*I*a*e^(4
 57/2*I*c) - 159000166293990983770633365148321190576578131487708291203552320
 70502630384804444889767486179756113401575843696792900*I*a*e^(455/2*I*c) - 8
 637448429412603145232295486970579164573288359043608779825334105587873485306
 782005310584032334923334469052992471566*I*a*e^(453/2*I*c) - 464291207891568
 301035808600708701702397396571313145932835197822571065308694580149827568319
 4647474013409696663828963*I*a*e^(451/2*I*c) - 24693967521344445442092879209
 187143678826841642536214604826927665698337797816338574616239474188306761825
 96868630102*I*a*e^(449/2*I*c) - 1299466170334480341543135560982800973046388
 591632081102321927761098390304943531108062897890291528379823914088825794*I*
 a*e^(447/2*I*c) - 676536112375453533535322327435304762507116008362988186332
 586271393372786322039392090789530327993045045404821029011*I*a*e^(445/2*I*c)
 - 348455253838688201360505783941113732251527445477363057406188342672634063
 934554405286663592501040628860183543905219*I*a*e^(443/2*I*c) - 177546087676

039487653207030476854772666243541156745740575127122721103812917448459499475
796154210708959328249028630*I*a*e^(441/2*I*c) - 894872505724728370624201553
901098068070796612063573577599972562878494813596597998948793919117434278496
11697524276*I*a*e^(439/2*I*c) - 4461439096362842937887432990987855060523083
0963183562020504214280108729796984369361375912054446637154294300258539*I*a*
e^(437/2*I*c) - 22000378044285344028734825732866676658762934871965249674527
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1480*I*a*e^(429/2*I*c) - 11646910388540405650688259647550737097079196384362
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*I*c) - 5432885725468726204157411487558285774676320340064008281503831338668
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/2*I*c) - 22966687877245457025794102073377677357995629076173854095854698682
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I*c) - 80769022503565948483170779513278182902665161792021013282953291691617
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*a*e^(405/2*I*c) - 57998486887243431995720681004163175448573494680504990319
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) - 14644114599025915830514653310497838889027533667220956430655162303649115
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 $) - 18107807683744905331264387812935563056139984311930680824984749925337461$
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*I*a*e^(323/2*I*c) - 115869966905124853009712743230581680050040635976036640
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9/2*I*c) - 6082802098350359025768710715675956495702650305299754400572492569
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 ^ (263/2*I*c) - 334958464921597406449456228816972626098977656752012220619930
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 19/2*I*c) - 96993253296241745199299054522607239475550*I*a*e^(217/2*I*c) - 6
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) - 247678824959506386373000637444*I*a*e^(199/2*I*c) - 95261086391402314705
 17528278*I*a*e^(197/2*I*c) - 341088288572157297772118077*I*a*e^(195/2*I*c)
 - 11311601400768793085009738*I*a*e^(193/2*I*c) - 345392409079468621758030*I
 *a*e^(191/2*I*c) - 9642935277307714791837*I*a*e^(189/2*I*c) - 2441249437098
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 065227040*I*a*e^(179/2*I*c) - 369625815338*I*a*e^(177/2*I*c) - 3687040552*I
 *a*e^(175/2*I*c) - 27515228*I*a*e^(173/2*I*c) - 136552*I*a*e^(171/2*I*c) -
 338*I*a*e^(169/2*I*c))/(e^(531*I*c) + 432*e^(530*I*c) + 93096*e^(529*I*c) +
 13343760*e^(528*I*c) + 1431118260*e^(527*I*c) + 122503723056*e^(526*I*c) +
 8718181624155*e^(525*I*c) + 530563624556832*e^(524*I*c) + 2818619255479213
 8*e^(523*I*c) + 1327882849274858880*e^(522*I*c) + 56169444526926562260*e^(5
 21*I*c) + 2154864144781257856128*e^(520*I*c) + 75599817092670157806639*e^(5
 19*I*c) + 2442455629894502983849104*e^(518*I*c) + 7309920781733559724709803

$8 * e^{(517 * I * c)} + 2037031259470368160131922320 * e^{(516 * I * c)} + 5309012726463096$
 $3470039804475 * e^{(515 * I * c)} + 1299146645993240318167826532288 * e^{(514 * I * c)} + 2$
 $9952547749265499675257842032197 * e^{(513 * I * c)} + 65265025334320604745362055999$
 $3840 * e^{(512 * I * c)} + 13477227799524701956579274210395326 * e^{(511 * I * c)} + 264410$
 $375780310742518099326419685040 * e^{(510 * I * c)} + 493966661081802579880958635254$
 $3471345 * e^{(509 * I * c)} + 88054927598941411145869950813388040256 * e^{(508 * I * c)} +$
 $1500602747937397286405577818722691539392 * e^{(507 * I * c)} + 24489837337812338687$
 $718622491865013839488 * e^{(506 * I * c)} + 383360155801054824529764688213114368047$
 $154 * e^{(505 * I * c)} + 5764601046563151304213854710715346838447392 * e^{(504 * I * c)} +$
 $83380839911837894453136303673785039051506805 * e^{(503 * I * c)} + 116158141373397$
 $1751533622511909046917188768400 * e^{(502 * I * c)} + 15603911277687607099721623771$
 $744933086920587272 * e^{(501 * I * c)} + 202347509724462171313966643580234078508179$
 $838320 * e^{(500 * I * c)} + 2535667460650279776834561566186591213109251642859 * e^{(4$
 $99 * I * c)} + 30735366512830562160991166338490057308062762518496 * e^{(498 * I * c)} +$
 $360688613036389349413809780004559963548775423325255 * e^{(497 * I * c)} + 410154543$
 $9937195793959956708442496709433800261224880 * e^{(496 * I * c)} + 45230940039830738$
 $332025694784646206844854827698075736 * e^{(495 * I * c)} + 484093410240488718655917$
 $025303662581091659126182344528 * e^{(494 * I * c)} + 503202490340145182407421394376$
 $6011922026507006311982753 * e^{(493 * I * c)} + 50836369508171099437019348610847391$
 $946736185108017183136 * e^{(492 * I * c)} + 499467506558531733671585862910572702811$
 $545035730398749530 * e^{(491 * I * c)} + 477539860710085326353420773381826677747869$
 $3412738731031680 * e^{(490 * I * c)} + 44456708175258821024400946210535004523775722$
 $190977468484496 * e^{(489 * I * c)} + 403212225957798188840846139960995624144491271$
 $694336796459584 * e^{(488 * I * c)} + 356476489062872401708848799668817892919578761$
 $3958545474804845 * e^{(487 * I * c)} + 30736217404321009965231037419663053962881035$
 $281709221697785072 * e^{(486 * I * c)} + 258585348715977270155829115684193411072034$
 $541491364393985491350 * e^{(485 * I * c)} + 212370296918887131826671878122392706783$
 $9949015727293884065388080 * e^{(484 * I * c)} + 17033886027390615741040977721655541$
 $665612162275485028584310890417 * e^{(483 * I * c)} + 133490210052026183779673313868$
 $332303530332906163247194627808410304 * e^{(482 * I * c)} + 102253643746829673729306$
 $5862705246449693687415559865844306888705423 * e^{(481 * I * c)} + 76590105201875496$
 $51777118357676871927081898989131125755798204236112 * e^{(480 * I * c)} + 5611708107$
 $6341175384087570185188538660375932013674735519055227368366 * e^{(479 * I * c)} + 40$
 $2349692266121158934003582839428785116904903936409545602519219664720 * e^{(478 *$
 $I * c)} + 28239051519365866783825257065644572802900986986385979876283802458817$
 $15 * e^{(477 * I * c)} + 1940797921559456659353500810330325525774540807008243133894$
 $5184797463936 * e^{(476 * I * c)} + 13065766022656041933512143438993896188459543406$
 $9984824307149332131747540 * e^{(475 * I * c)} + 86188485109499190876424680547467242$
 $8603757315484453974713612812215428992 * e^{(474 * I * c)} + 55725511573286711210162$
 $16416307596161861955969011697222340926210112854418 * e^{(473 * I * c)} + 3532444720$
 $6779018115378052820789411687581004582367431006205879633729015200 * e^{(472 * I * c$
 $) + 21960128133951556150026147884419002487055526128194605883961404469703796$
 $3695 * e^{(471 * I * c)} + 13392143742542455535648844068019453533850002540306557659$
 $53770237607180089968 * e^{(470 * I * c)} + 8013729580790752434361964945761543761469$
 $520791210746972675870481058674277844 * e^{(469 * I * c)} + 470650446111351581084873$

53367484243102698248838312635876283099427442745866704*e^(468*I*c) + 2713612
 07503266570734486517077181014801775322183181055638619257836143271472358*e^(
 467*I*c) + 1536333238444927583532734556016494671674916578907116984548489078
 241693926940560*e^(466*I*c) + 854301344112621233483354066506962147247908583
 8041360564550722036723654297540205*e^(465*I*c) + 46668223548266017806854592
 468100570289355960869613650856575756758180182223308768*e^(464*I*c) + 250501
 028608928332469340456829902067712233644464602753159945727868485722395506952
 *e^(463*I*c) + 132149805527130085142999386663161987442453442518818359204972
 7687571032156435077280*e^(462*I*c) + 68529932231457366873288853116177954355
 92940841439866351079655652312894721972796266*e^(461*I*c) + 3494107161327670
 4649477943043339450201504075335160361865916029213860778606230624960*e^(460*
 I*c) + 17519317050061830024151563238191228515779009781604922067121721222001
 5297133400636060*e^(459*I*c) + 86397993362233034955629682002839551319870806
 4940505702126068652936800794826651264256*e^(458*I*c) + 41915425006568261480
 93339414544159143964478472492315931809171859902114109005939942952*e^(457*I*
 c) + 2000800680303004713729327825032159711354071620198333312634928118667915
 3199068045257216*e^(456*I*c) + 93986915313068179149083606065681482780836060
 510530154618486949839467131378859885998210*e^(455*I*c) + 434546676780280045
 346344498763892540797175105756827515509297024187660299345484920192480*e^(45
 4*I*c) + 197779298066581813565130009432623915860544887080697086057732538502
 8609983034534672318500*e^(453*I*c) + 88627521427569572856813408857649045979
 35349569355321815647721172537159186491471311666400*e^(452*I*c) + 3910803125
 560180947653753536961184444084490375160564502351457235204524810426293359885
 0730*e^(451*I*c) + 16995632796992976777390209665262925328370450547712754455
 6534417376686540936706073847337600*e^(450*I*c) + 72752101071839422929177407
 3844694255798738667067535379759732795567942578751384250780476310*e^(449*I*c
) + 30679742964317473641981596239624636716170064196268514261484186029348529
 07379021659761911840*e^(448*I*c) + 1274721961650332054135634306256284736860
 1622140856786025445814532037904111523242298235713300*e^(447*I*c) + 52190912
 207661824215812271854269748071292843243227894769229690720010547141334131610
 989636000*e^(446*I*c) + 210594301385648471184329078880317504953361839954159
 427434009884661777259752542647709150036990*e^(445*I*c) + 837579206923411932
 458786486765373533946545239708990769488724813982189165104589895518909256320
 *e^(444*I*c) + 328387476055581867672630948030673442015509858394807446901416
 8171874442170109648521627538755920*e^(443*I*c) + 12693496932964920565073673
 637181280088548682508880255337280065006566138696041797353216584528640*e^(44
 2*I*c) + 483794897564340998438577918165893794068150426093403787475864371457
 81646245422045101230417309900*e^(441*I*c) + 1818346614061779013153301296771
 45381166449188413194141169344354754920969034952610378945282257600*e^(440*I*
 c) + 6740255305431330088948457752366252374507431144735445378181704471346071
 02575676676056675328961590*e^(439*I*c) + 2464382190807439609079774226855679
 629367885709776435876630851716253962696192341706239192878728160*e^(438*I*c)
 + 888829502875102466704420383760797610148005313441861447462076752282486891
 1959884352666444917404000*e^(437*I*c) + 31626644674725547731176795687527653
 571305969985923688392112164915553242573269490908989570248533280*e^(436*I*c)

+ 111034148797008819443143895644469242295049867464313710969257619338899133
799285616020069872611710850*e^(435*I*c) + 384655842080666274454063078784837
174998949052500975322162003392549953413592461519365177908682078400*e^(434*I
*c) + 131505212093069212210229710532762284233587074342853089107298353586228
0094446607723473800477453914130*e^(433*I*c) + 44372109178431823477643495444
43904699020056595069470847193617092114714077633077234972825351226979360*e^(
432*I*c) + 1477795509661712899871274518207149536217650697318308165023360527
4051677624970464340242755840025673760*e^(431*I*c) + 48584258153140280447314
836868772131390195412419046732778458706015096881437076337910793584122475073
760*e^(430*I*c) + 157685845528850918721462877864435090257583149415561323427
386562894447598277935629800939237175625149830*e^(429*I*c) + 505293663123015
258878483025738812813203397766845340065381261016353419722382620393032535960
660921950400*e^(428*I*c) + 159877110105819269227052899967744474268563100623
2456185844925220144002305878120380828483988663574829100*e^(427*I*c) + 49952
419562791381802051867444016880243882721139212556637349569469275713055331467
76898787878059685108480*e^(426*I*c) + 1541311121148602393729497082079737671
6081344788163386543522421939737507962125854981881879168348260330000*e^(425*
I*c) + 46970224727117281826454045018070670522559756627580347784535320014963
482632359729444541885102274546002560*e^(424*I*c) + 141379938253556843280565
505807403304130606130725434751745794079833141361748917639986145377066437210
546190*e^(423*I*c) + 420358024835146798583611210145942154684437949365647899
08837252480215622884839580011688655664280691773600*e^(422*I*c) + 123466804
189240997878001808175544021601258247639694193796589963195307920397422213879
4604328498972144766900*e^(421*I*c) + 35827180021632960614145367037151098971
07198252739284546149343102348456124089657428594946438660859773886240*e^(420
*I*c) + 1027160253020288900249781351684945259097151280952906066519730109705
2210064576088348023234671975463677418470*e^(419*I*c) + 29097651061247453406
647569781836910062165559852359052804259154165687125428752562385492373749486
351714453120*e^(418*I*c) + 814520814138291118288754175642500548460376933128
11480492909160758195989155768107022568350953861815940704090*e^(417*I*c) + 2
253205325932206577679411092895162489997945210155641349828272417100196754866
94499689312466561907212627820000*e^(416*I*c) + 6160031160229795849371257017
578872129983543009899913626280388610939145613320711919097149494265879369103
03300*e^(415*I*c) + 1664475034387211809394917743502937638978574937754764763
987835872410449930690131572904279995484581013965001440*e^(414*I*c) + 444541
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8884980520010028451496996210*e^(413*I*c) + 11735856926245118493113091002501
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600*e^(412*I*c) + 306275810542219573783905472892776091295728139310827335202
47387226000020043538279468776707958420892547870128680*e^(411*I*c) + 7901914
955876656925478398848723238835290914498274717185677246322380899336709150340
2876467270176124342699654400*e^(410*I*c) + 20155794742479409802677247846204
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7500*e^(409*I*c) + 50832459930108546016697862968303266142765447408293904809
7939638391567298795788389433842285751054665210868287680*e^(408*I*c) + 12675

970172948129134001462760421269299864802928701903991075543110799642272801965
22475370108738477856311765699610*e^(407*I*c) + 3125683493178701743479704750
307490178666292150720179363604335113528623329684606343185540756019935662148
267863968*e^(406*I*c) + 762178879191204706203884091779937460042889225819436
7636682944356096681400246312138001769285020661445991073249416*e^(405*I*c) +
18379807084003359766027649217621144116091735572216620788861535803449702273
802588359076704241840733513439114113248*e^(404*I*c) + 438349721429193776853
786922330210637445540331009285027374804389789767469898957840709519052377834
90374305934542955*e^(403*I*c) + 1033997554672574364898478376407537547182043
9447305579500146760432641987655556873829531737211096115196005647730480*e^(
402*I*c) + 2412460212824400617929083177830328761948015971332060520912869970
43729145345755805710081489006741839439573984832678*e^(401*I*c) + 5567563887
111823403410261927342195461136517683173805390058936790493947140170636985652
72728813669054779077208977840*e^(400*I*c) + 1271033082938048950201360554831
270342662343991277504612342300366025046741742856580445289401786656311685859
023084716*e^(399*I*c) + 287049613141231445183467471535358943955329443080853
1933466086288543709246230769151392180699413405623017247753532944*e^(398*I*c
) + 64133818958559251847582314510625563803285949385110065770152181197865363
90213057284018202201631094434819584025113465*e^(397*I*c) + 1417648365287570
49572020133432411179043699779665384995990252442198063518901163481565327960
5497783382888932766730080*e^(396*I*c) + 31004319206069417077069363141423487
431828009098184744635678652284177439464941651812564519144918003174108077634
846014*e^(395*I*c) + 670917061305296691250198992100215765802378434622295353
86295087076189297849995931360645605292130961496106707521506432*e^(394*I*c)
+ 1436576871380447969429471197042595384588181994235168246745862936910561192
09866358123637772245409530799230553767222252*e^(393*I*c) + 3043844711068133
360102841601239063704338888284906274226525512369667909161745208577591439301
40187173492394981908258944*e^(392*I*c) + 6382188929145337415058063996624885
560667836004960918764083749748774489717780360749962455811242834604380651820
71976085*e^(391*I*c) + 1324311324984027428355222938147682378672860708817161
741448749689593588020860847508703702325320304649883120684987556400*e^(390*I
*c) + 271958928348374392604080510108034192124453031125460725092919277390933
1523226635035815672862569296693711643521070331394*e^(389*I*c) + 55274988490
311783558612300093266682839262900821584671180006985027193799390459183442221
92742145711257028040974074674736*e^(388*I*c) + 1111949964536320108088106282
488633849242537565844897793553584634929042582157038309042541141852151667037
1372045206568345*e^(387*I*c) + 22140735001708603270915180769241391662035578
755903979148909213603822554792749183517160255571915875356439553717130797888
*e^(386*I*c) + 436383000758151710259462114644656896189657734886609858579456
57479854085108851857911222911989837615452608512356008400295*e^(385*I*c) + 8
513953323478645577958995946490063776073572970562122138000583720836915779467
3049675428799817875431430246332625899630160*e^(384*I*c) + 16443750067690689
274132326015439427850395456193602013359658180644935724027734944792733451710
5809995300093549279931273178*e^(383*I*c) + 31440903580822586156559543693835
494544547399104312972204674722881303092520496850341856681883861186670980704

0793495364496*e^(382*I*c) + 59515761550043151494747928233654705382792608791
6425263497187757029413385471835434198246807096214536895441388306027237899*e
^(381*I*c) + 11153982855456015505333280456001843179938993592179967298183407
04221801195667410485846996179056733558512452238583160792512*e^(380*I*c) + 2
069698289500860643461665762373807957513019424041178871904960551829412449344
722432125679417958403007551179298315947373776*e^(379*I*c) + 380260499670589
110696462063384896480703709885451018226324303059729563076035359753197432475
2266389193185760878274188013440*e^(378*I*c) + 69178389452148442784933304593
613949233723338536198796373726731849428597124310663457268704220998931248907
77678037369988150*e^(377*I*c) + 1246214044053725808492859670987206685775707
094312486855450094815475686345430803292540834031123785001781470789698696908
6816*e^(376*I*c) + 22231341131801535345406399037721686840208397941952580135
584645966746736656716271554826476282991066076564921432614339399735*e^(375*I
*c) + 392742004143298611693979445162250010812274333985850072063992312119071
57795359719648241598754266579840244551491476467899952*e^(374*I*c) + 6871246
601598564151246858617365974773487959171009835465278612493602307394314104957
3606648563005359411712764895683903806088*e^(373*I*c) + 11906059184966054683
476569322767644906758414824888267844750482607723633344451345409512666875005
7295811191643356908972191440*e^(372*I*c) + 20432555726518600076740271023084
789645976158392276369823543321283331307778304104007466937901739483676153964
9081690630811665*e^(371*I*c) + 34731005381093529041945556055595731412956921
073574598323436965997641337477407800017307007524865452491717912895050744305
8208*e^(370*I*c) + 58474957368230458617938462884488332758149896988654038037
8896767999075614964007174600811092945356635118795824799369716742109*e^(369*
I*c) + 97521033944404931875728231176351778667322317559445794638327926463508
5041004917300295904275433144848532459919875479817581584*e^(368*I*c) + 16110
925414000605259548593752641941783476434718370782014462624356151429445873378
33513586022729849523358436493586042252995608*e^(367*I*c) + 2636662410430799
340447522284778244283740751068658140726576446671207798325606832295937705061
686297930296382338892574900819440*e^(366*I*c) + 427482690772059175252671133
682087150084434564792238547153435933360618957183244464136413289310866357620
5133870672156264164115*e^(365*I*c) + 68664253375186682626626937509089569657
329245781421816306221578028998808746815510311363140641999486040015298945662
35238597088*e^(364*I*c) + 1092721060347354481027979234784453607458889680623
004111008954473160586314610418173903942667485545346609740233068833184560230
2*e^(363*I*c) + 17229502824367647334400721998417596703948657394738805209391
636597370380572398964715080095366818322029152193635869784095333760*e^(362*I
*c) + 269177947010866150978901202368905011051467999960217751957108662262286
38984703456832694153230611607263444183501026198563419616*e^(361*I*c) + 4167
044037539054364341821934227174804003507149011908058528152249818881837590690
0368701234531304633163446319945130196476913600*e^(360*I*c) + 63923019433761
989090614801288635098123199445102303122616544648208998767803944455777042886
552738499747183713136069104651812215*e^(359*I*c) + 971730550247426800586167
224613688926611412955402634930130327460835361573242683333904003089583183702
19154887169702257444756176*e^(358*I*c) + 1463904484563511812182373827403741

241916648199977469880765983918627336296701422415463755339031306052975801056
 75355629160198162*e^(357*I*c) + 2185631666596493122474836409562721492124991
 153838287710296542833639725851180904794136966381081563852446465913284544257
 45117584*e^(356*I*c) + 3234131780148410037141511382460791525763609760350584
 57890409937738723171537036573043681997163745313602400139153046673668433091*
 e^(355*I*c) + 4743230435631005423773386299319662481291759769823324460180560
 09391154020438895903140822967769494019446166779954024655344116288*e^(354*I*
 c) + 6895184493287935599032604181499741902535783400588950355896064682446805
 91556118170304005037563669880057908765898949268614772285*e^(353*I*c) + 9935
 556536495211272264439608202336493864885100818925458667004440966615827904412
 41830855609577062039555625090943332264901780720*e^(352*I*c) + 1419164481422
 17657385823401389899962882232233309573730716310743838935832201454293617293
 175086458389621425307051750612129761498*e^(351*I*c) + 200949611009268773815
 278208568373722272796882429905873921544608349935146762533444967075776406669
 0656150949014944043994822823920*e^(350*I*c) + 28208192985622159591075298072
 896284496213867989894363693931160698940187812010002756331044983989593466317
 95568022519974400130281*e^(349*I*c) + 3925697658415778352768103942856011840
 211642769621717217996614398473887186074391482638547212826538270453912634540
 299792270321024*e^(348*I*c) + 541666280405243634958559598281835795386625846
 164435401820515891774257642534436496459675065317767780349218681730517117503
 2011500*e^(347*I*c) + 74103726128911522243646332961280439716571932803277543
 04382235672773781403023814127610355562505271969045177704726054907145784960*
 e^(346*I*c) + 1005220952436958182758815498534554967803144574449998520825938
 5435609740272400301454246872041775159838468077381562338745636398374*e^(345*
 I*c) + 13521230411945436915558854706851796541567399811656870021567352975326
 815467817846533289123871696056195231696146162720992221760992*e^(344*I*c) +
 180353273381774554711775685948516829779783464497771935720876885103924268845
 19272991560851326393852241961470040819793627127923997*e^(343*I*c) + 2385639
 856556280203069527817421264083328215417400645945829264406094792490731373592
 1561690153939906017518647182491616573724049744*e^(342*I*c) + 31295263688189
 838313772775873307260334117227258629501992358695636092662866062819845689064
 235813622974150120668921391878398978380*e^(341*I*c) + 407159889637019189500
 203483367364234205133113590104852469190746528833939708053744708301562297056
 47312265477584256027212762941040*e^(340*I*c) + 5253922334674077114258709237
 025706953606031964443950166761048276795580027605289243215279881460797511036
 6224945081428121888473324*e^(339*I*c) + 67244087969080703823703257199663047
 606890610482090494619492802935130215979819469966383336788693900139115594646
 893784095418472336*e^(338*I*c) + 853681184302153128482312917396737358877462
 018516662996003921994187647500868281987198727440477677836673253262892218819
 74987582215*e^(337*I*c) + 1075047374065769161234803991697596333283214074194
 005100174988498305986215654282665463159339208215275447263802016591149038346
 05888*e^(336*I*c) + 1342977420234794299046296161045596100960747587210680227
 04468938063017059688023363436458971534964665036319889119229809973806909680*
 e^(335*I*c) + 1664323329225891951305583292663987533898239557375985275560960
 93062473559769545772321978969318904192572733997888230986469005970880*e^(334

*I*c) + 2046222955357291095198299167898672253194297051625600826488403949654
23112809336591921290309392396263834674368977840527147037426908*e^(333*I*c)
+ 2495930722825658663983899515096192026826344554871287146314618917708232013
67527645793770203788784677343934971424317987895255031936*e^(332*I*c) + 3020
606380308684634611394422793604997189069174825248944833562201961383770508289
11383056860425370161157201493696073712322595776808*e^(331*I*c) + 3627063075
638432311356991574185107324524206140136241688793121876452334501539279757933
26834780741391203430153093712635355523960320*e^(330*I*c) + 4321478564640869
380238115618086785895947020479046742822979596588001709844567990677518780448
06619012452636891350731618278545690160*e^(329*I*c) + 5109076151111345074526
238461471371414502757224443163855316484292308516866358277174884645003316233
85777400744950538410637735936000*e^(328*I*c) + 5993784847717334748093761424
015548502070649721181375729492575036514445419393090252768960496225156302631
62184526394317285457368300*e^(327*I*c) + 6977891069259246148167137476846827
850836598190279522444470433558697413685004525611640246360734019296931058011
05522738405349028160*e^(326*I*c) + 8061696713276255324245753400897757336819
949915766744469223540997146151920854432456638522570011156442866603049794760
23966071898200*e^(325*I*c) + 9243200528675225840357774957610723512225344207
848619600018210205094681463567564337952464464913961415838545136871043344295
66707520*e^(324*I*c) + 1051782100428834371944508170051219187116816349766953
322637182149610875004223784784183284961906494422955462431208645690802526770
780*e^(323*I*c) + 118781794307939031610880232479811012902082278266008724859
9367643481200206046822166144285425922229375413676535071141005286431481600*e
^(322*I*c) + 13313961146267230358024621235315820503397499960141524528353059
56367425370222758621753922458727524856072950880960657564720475838500*e^(321
*I*c) + 1481187117089246662466955694965677855524730313260552690821602657176
218737426245522795329891464091005878304304075953693546767206080*e^(320*I*c)
+ 163556974464142190065788638128907665322758017205667758745140206923435528
3687489659613913761959140773339736014790081814516625224440*e^(319*I*c) + 17
926490780896322989369457284819693349643915975062850884883506229372525334209
80803144316431701452190522716124797875257437516360640*e^(318*I*c) + 1950286
550780181919244992961204487010056460362845218501674423766266321558791436917
317878702232679213868287926294665202769722927380*e^(317*I*c) + 210614190346
834430711254976120248454340279425235248219981741042486967726271509828843764
6518683487945462774223656471345899082156800*e^(316*I*c) + 22577262191038562
868128330126815737654962622414206129320761431511719608545541241446990230098
42080515157923529357189869943515991200*e^(315*I*c) + 2402464595569686086120
001803034211056739445588621946141384106162886246161815149763025030834875234
067267774023433418269982431265280*e^(314*I*c) + 253776641546503033081547174
669298859606991189469722505292832045254217558715484809648333120980743011394
3015398362669673337957755720*e^(313*I*c) + 26611006479757835838282351392014
419301783966433834239035838625472558807723820492010155372149008327456015197
37141849802506685264000*e^(312*I*c) + 2770073207150768645597507281382065497
924968466054527414122339827333783770068305883487309979315983718403740872884
345746380680204260*e^(311*I*c) + 286250312632046179777066778072564418499125

562317462617567905067210084898811939184146657341701924759058073526514342728
 9340450811200*e^(310*I*c) + 29364942143518684987032394554267711043448273062
 675589165508774672324551532861405210895827339322025531307127238369834688662
 30908800*e^(309*I*c) + 2990498949622543608538129380283866335190087115124858
 818143787619186957111903765723974899651518555144924290346242595167274383008
 960*e^(308*I*c) + 302337164350822502717560317521295321902248504585073184530
 7519008277385154731461213388035579159917590062343527464977286601165100620*e
 ^ (307*I*c) + 30344083555309570757877317453225679816846165501628454732576796
 74280216947356785783843205604202307836897073595410412575660465787520*e^(306
 *I*c) + 3023371643508225027175603175212953219022485045850731845307519008277
 385154731461213388035579159917590062343527464977286601165100620*e^(305*I*c)
 + 299049894962254360853812938028386633519008711512485881814378761918695711
 1903765723974899651518555144924290346242595167274383008960*e^(304*I*c) + 29
 364942143518684987032394554267711043448273062675589165508774672324551532861
 40521089582733932202553130712723836983468866230908800*e^(303*I*c) + 2862503
 126320461797770667780725644184991255623174626175679050672100848988119391841
 466573417019247590580735265143427289340450811200*e^(302*I*c) + 277007320715
 076864559750728138206549792496846605452741412233982733378377006830588348730
 9979315983718403740872884345746380680204260*e^(301*I*c) + 26611006479757835
 838282351392014419301783966433834239035838625472558807723820492010155372149
 00832745601519737141849802506685264000*e^(300*I*c) + 2537766415465030330815
 471746692988596069911894697225052928320452542175587154848096483331209807430
 113943015398362669673337957755720*e^(299*I*c) + 240246459556968608612000180
 303421105673944558862194614138410616288624616181514976302503083487523406726
 7774023433418269982431265280*e^(298*I*c) + 22577262191038562868128330126815
 737654962622414206129320761431511719608545541241446990230098420805151579235
 29357189869943515991200*e^(297*I*c) + 2106141903468344307112549761202484543
 402794252352482199817410424869677262715098288437646518683487945462774223656
 471345899082156800*e^(296*I*c) + 195028655078018191924499296120448701005646
 036284521850167442376626632155879143691731787870223267921386828792629466520
 2769722927380*e^(295*I*c) + 17926490780896322989369457284819693349643915975
 062850884883506229372525334209808031443164317014521905227161247978752574375
 16360640*e^(294*I*c) + 1635569744641421900657886381289076653227580172056677
 587451402069234355283687489659613913761959140773339736014790081814516625224
 440*e^(293*I*c) + 148118711708924666246695569496567785552473031326055269082
 1602657176218737426245522795329891464091005878304304075953693546767206080*e
 ^ (292*I*c) + 13313961146267230358024621235315820503397499960141524528353059
 56367425370222758621753922458727524856072950880960657564720475838500*e^(291
 *I*c) + 1187817943079390316108802324798110129020822782660087248599367643481
 20020604682216614428542592229375413676535071141005286431481600*e^(290*I*c)
 + 105178210042883437194450817005121918711681634976695332263718214961087500
 4223784784183284961906494422955462431208645690802526770780*e^(289*I*c) + 92
 432005286752258403577749576107235122253442078486196000182102050946814635675
 6433795246446491396141583854513687104334429566707520*e^(288*I*c) + 80616967
 132762553242457534008977573368199499157667444692235409971461519208544324566

3852257001115644286660304979476023966071898200*e^(287*I*c) + 69778910692592
461481671374768468278508365981902795224444704335586974136850045256116402463
6073401929693105801105522738405349028160*e^(286*I*c) + 59937848477173347480
937614240155485020706497211813757294925750365144454193930902527689604962251
5630263162184526394317285457368300*e^(285*I*c) + 51090761511113450745262384
614713714145027572244431638553164842923085168663582771748846450033162338577
7400744950538410637735936000*e^(284*I*c) + 43214785646408693802381156180867
858959470204790467428229795965880017098445679906775187804480661901245263689
1350731618278545690160*e^(283*I*c) + 36270630756384323113569915741851073245
242061401362416887931218764523345015392797579332683478074139120343015309371
2635355523960320*e^(282*I*c) + 30206063803086846346113944227936049971890691
748252489448335622019613837705082891138305686042537016115720149369607371232
2595776808*e^(281*I*c) + 24959307228256586639838995150961920268263445548712
871463146189177082320136752764579377020378878467734393497142431798789525503
1936*e^(280*I*c) + 20462229553572910951982991678986722531942970516256008264
8840394965423112809336591921290309392396263834674368977840527147037426908*e
^(279*I*c) + 16643233292258919513055832926639875338982395573759852755609609
3062473559769545772321978969318904192572733997888230986469005970880*e^(278*
I*c) + 13429774202347942990462961610455961009607475872106802270446893806301
7059688023363436458971534964665036319889119229809973806909680*e^(277*I*c) +
10750473740657691612348039916975963332832140741940051001749884983059862156
5428266546315933920821527544726380201659114903834605888*e^(276*I*c) + 85368
118430215312848231291739673735887746201851666299600392199418764750086828198
719872744047767783667325326289221881974987582215*e^(275*I*c) + 672440879690
807038237032571996630476068906104820904946194928029351302159798194699663833
36788693900139115594646893784095418472336*e^(274*I*c) + 5253922334674077114
258709237025706953606031964443950166761048276795580027605289243215279881460
7975110366224945081428121888473324*e^(273*I*c) + 40715988963701918950020348
336736423420513311359010485246919074652883393970805374470830156229705647312
265477584256027212762941040*e^(272*I*c) + 312952636881898383137727758733072
603341172272586295019923586956360926628660628198456890642358136229741501206
68921391878398978380*e^(271*I*c) + 2385639856556280203069527817421264083328
215417400645945829264406094792490731373592156169015393990601751864718249161
6573724049744*e^(270*I*c) + 18035327338177455471177568594851682977978346449
777193572087688510392426884519272991560851326393852241961470040819793627127
923997*e^(269*I*c) + 135212304119454369155588547068517965415673998116568700
21567352975326815467817846533289123871696056195231696146162720992221760992*
e^(268*I*c) + 1005220952436958182758815498534554967803144574449998520825938
5435609740272400301454246872041775159838468077381562338745636398374*e^(267*
I*c) + 74103726128911522243646332961280439716571932803277543043822356727737
81403023814127610355562505271969045177704726054907145784960*e^(266*I*c) + 5
416662804052436349585595982818357953866258461644354018205158917742576425344
364964596750653177677803492186817305171175032011500*e^(265*I*c) + 392569765
841577835276810394285601184021164276962171721799661439847388718607439148263
8547212826538270453912634540299792270321024*e^(264*I*c) + 28208192985622159

591075298072896284496213867989894363693931160698940187812010002756331044983
98959346631795568022519974400130281*e^(263*I*c) + 2009496110092687738152782
085683737222727968824299058739215446083499351467625334449670757764066690656
150949014944043994822823920*e^(262*I*c) + 141916448142217657385823401389899
96288223223330957373071631074383893583220145429361729317508645838962142530
7051750612129761498*e^(261*I*c) + 99355565364952112722644396082023364938648
851008189254586670044409666158279044124183085560957706203955562509094333226
4901780720*e^(260*I*c) + 68951844932879355990326041814997419025357834005889
503558960646824468059155611817030400503756366988005790876589894926861477228
5*e^(259*I*c) + 47432304356310054237733862993196624812917597698233244601805
6009391154020438895903140822967769494019446166779954024655344116288*e^(258*
I*c) + 32341317801484100371415113824607915257636097603505845789040993773872
3171537036573043681997163745313602400139153046673668433091*e^(257*I*c) + 21
856316665964931224748364095627214921249911538382877102965428336397258511809
0479413696638108156385244646591328454425745117584*e^(256*I*c) + 14639044845
635118121823738274037412419166481999774698807659839186273362967014224154637
5533903130605297580105675355629160198162*e^(255*I*c) + 97173055024742680058
616722461368892661141295540263493013032746083536157324268333390400308958318
370219154887169702257444756176*e^(254*I*c) + 639230194337619890906148012886
350981231994451023031226165446482089987678039444557770428865527384997471837
13136069104651812215*e^(253*I*c) + 4167044037539054364341821934227174804003
507149011908058528152249818881837590690036870123453130463316344631994513019
6476913600*e^(252*I*c) + 26917794701086615097890120236890501105146799996021
775195710866226228638984703456832694153230611607263444183501026198563419616
*e^(251*I*c) + 172295028243676473344007219984175967039486573947388052093916
36597370380572398964715080095366818322029152193635869784095333760*e^(250*I*
c) + 1092721060347354481027979234784453607458889680623004111008954473160586
3146104181739039426674855453466097402330688331845602302*e^(249*I*c) + 68664
253375186682626626937509089569657329245781421816306221578028998808746815510
31136314064199948604001529894566235238597088*e^(248*I*c) + 4274826907720591
752526711336820871500844345647922385471534359333606189571832444641364132893
108663576205133870672156264164115*e^(247*I*c) + 26366241043079934044752228
477824428374075106865814072657644667120779832560683229593770506168629793029
6382338892574900819440*e^(246*I*c) + 16110925414000605259548593752641941783
476434718370782014462624356151429445873378335135860227298495233584364935860
42252995608*e^(245*I*c) + 9752103394440493187572823117635177866732231755944
57946383279264635085041004917300295904275433144848532459919875479817581584*
e^(244*I*c) + 5847495736823045861793846288448833275814989698865403803788967
67999075614964007174600811092945356635118795824799369716742109*e^(243*I*c)
+ 3473100538109352904194555605559573141295692107357459832343696599764133747
74078000173070075248654524917179128950507443058208*e^(242*I*c) + 2043255572
651860007674027102308478964597615839227636982354332128333130777830410400746
69379017394836761539649081690630811665*e^(241*I*c) + 1190605918496605468347
656932276764490675841482488826784475048260772363334445134540951266687500572
95811191643356908972191440*e^(240*I*c) + 6871246601598564151246858617365974

773487959171009835465278612493602307394314104957360664856300535941171276489
5683903806088*e^(239*I*c) + 39274200414329861169397944516225001081227433398
585007206399231211907157795359719648241598754266579840244551491476467899952
*e^(238*I*c) + 222313411318015353454063990377216868402083979419525801355846
45966746736656716271554826476282991066076564921432614339399735*e^(237*I*c)
+ 1246214044053725808492859670987206685775707094312486855450094815475686345
4308032925408340311237850017814707896986969086816*e^(236*I*c) + 69178389452
148442784933304593613949233723338536198796373726731849428597124310663457268
70422099893124890777678037369988150*e^(235*I*c) + 3802604996705891106964620
633848964807037098854510182263243030597295630760353597531974324752266389193
185760878274188013440*e^(234*I*c) + 206969828950086064346166576237380795751
301942404117887190496055182941244934472243212567941795840300755117929831594
7373776*e^(233*I*c) + 11153982855456015505333280456001843179938993592179967
29818340704221801195667410485846996179056733558512452238583160792512*e^(232
*I*c) + 5951576155004315149474792823365470538279260879164252634971877570294
13385471835434198246807096214536895441388306027237899*e^(231*I*c) + 3144090
358082258615655954369383549454454739910431297220467472288130309252049685034
18566818838611866709807040793495364496*e^(230*I*c) + 1644375006769068927413
232601543942785039545619360201335965818064493572402773494479273345171058099
95300093549279931273178*e^(229*I*c) + 8513953323478645577958995946490063776
073572970562122138000583720836915779467304967542879981787543143024633262589
9630160*e^(228*I*c) + 43638300075815171025946211464465689618965773488660985
857945657479854085108851857911222911989837615452608512356008400295*e^(227*I
*c) + 221407350017086032709151807692413916620355787559039791489092136038225
54792749183517160255571915875356439553717130797888*e^(226*I*c) + 1111949964
536320108088106282488633849242537565844897793553584634929042582157038309042
5411418521516670371372045206568345*e^(225*I*c) + 55274988490311783558612300
093266682839262900821584671180006985027193799390459183442221927421457112570
28040974074674736*e^(224*I*c) + 2719589283483743926040805101080341921244530
311254607250929192773909331523226635035815672862569296693711643521070331394
*e^(223*I*c) + 132431132498402742835522293814768237867286070881716174144874
9689593588020860847508703702325320304649883120684987556400*e^(222*I*c) + 63
821889291453374150580639966248855606678360049609187640837497487744897177803
6074996245581124283460438065182071976085*e^(221*I*c) + 30438447110681333601
028416012390637043388882849062742265255123696679091617452085775914393014018
7173492394981908258944*e^(220*I*c) + 14365768713804479694294711970425953845
881819942351682467458629369105611920986635812363777224540953079923055376722
2252*e^(219*I*c) + 67091706130529669125019899210021576580237843462229535386
295087076189297849995931360645605292130961496106707521506432*e^(218*I*c) +
310043192060694170770693631414234874318280090981847446356786522841774394649
41651812564519144918003174108077634846014*e^(217*I*c) + 1417648365287570495
720201334324111790436997779665384995990252442198063518901163481565327960549
7783382888932766730080*e^(216*I*c) + 64133818958559251847582314510625563803
285949385110065770152181197865363902130572840182022016310944348195840251134
65*e^(215*I*c) + 2870496131412314451834674715353589439553294430808531933466

086288543709246230769151392180699413405623017247753532944*e^(214*I*c) + 127
 103308293804895020136055483127034266234399127750461234230036602504674174285
 6580445289401786656311685859023084716*e^(213*I*c) + 55675638871118234034102
 619273421954611365176831738053900589367904939471401706369856527272881366905
 4779077208977840*e^(212*I*c) + 24124602128244006179290831778303287619480159
 7133206052091286997043729145345755805710081489006741839439573984832678*e^(2
 11*I*c) + 10339975546725743648984783764075375471820439447305579500146760432
 641987655556873829531737211096115196005647730480*e^(210*I*c) + 43834972142
 919377685378692233021063744554033100928502737480438978976746989895784070951
 905237783490374305934542955*e^(209*I*c) + 183798070840033597660276492176211
 441160917355722166207888615358034497022738025883590767042418407335134391141
 13248*e^(208*I*c) + 7621788791912047062038840917799374600428892258194367636
 682944356096681400246312138001769285020661445991073249416*e^(207*I*c) + 312
 568349317870174347970475030749017866629215072017936360433511352862332968460
 6343185540756019935662148267863968*e^(206*I*c) + 12675970172948129134001462
 760421269299864802928701903991075543110799642272801965224753701087384778563
 11765699610*e^(205*I*c) + 5083245993010854601669786296830326614276544740829
 39048097939638391567298795788389433842285751054665210868287680*e^(204*I*c)
 + 2015579474247940980267724784620408833958675125629308629437535686900840155
 85598010154781548625239409581907397500*e^(203*I*c) + 7901914955876656925478
 398848723238835290914498274717185677246322380899336709150340287646727017612
 4342699654400*e^(202*I*c) + 30627581054221957378390547289277609129572813931
 082733520247387226000020043538279468776707958420892547870128680*e^(201*I*c)
 + 117358569262451184931130910025016040323418769859990828235206722415302001
 88223826392982302194084667538488665600*e^(200*I*c) + 4445412259295474625067
 659514198312966015416299968930393345630345914109720740573618884980520010028
 451496996210*e^(199*I*c) + 166447503438721180939491774350293763897857493775
 4764763987835872410449930690131572904279995484581013965001440*e^(198*I*c) +
 61600311602297958493712570175788721299835430098999136262803886109391456133
 2071191909714949426587936910303300*e^(197*I*c) + 22532053259322065776794110
 928951624899979452101556413498282724171001967548669449968931246656190721262
 7820000*e^(196*I*c) + 81452081413829111828875417564250054846037693312811480
 492909160758195989155768107022568350953861815940704090*e^(195*I*c) + 290976
 510612474534066475697818369100621655598523590528042591541656871254287525623
 85492373749486351714453120*e^(194*I*c) + 1027160253020288900249781351684945
 2590971512809529060665197301097052210064576088348023234671975463677418470*e
 ^^(193*I*c) + 35827180021632960614145367037151098971071982527392845461493431
 02348456124089657428594946438660859773886240*e^(192*I*c) + 1234668041892409
 978780018081755440216012582476396941937965899631953079203974222138794604328
 498972144766900*e^(191*I*c) + 420358024835146798583611210145942154684437949
 365647899088372524802156222884839580011688655664280691773600*e^(190*I*c) +
 141379938253556843280565505807403304130606130725434751745794079833141361748
 917639986145377066437210546190*e^(189*I*c) + 469702247271172818264540450180
 70670522559756627580347784535320014963482632359729444541885102274546002560*
 e^(188*I*c) + 1541311121148602393729497082079737671608134478816338654352242

1939737507962125854981881879168348260330000*e^(187*I*c) + 49952419562791381
802051867444016880243882721139212556637349569469275713055331467768987878780
59685108480*e^(186*I*c) + 1598771101058192692270528999677444742685631006232
456185844925220144002305878120380828483988663574829100*e^(185*I*c) + 505293
663123015258878483025738812813203397766845340065381261016353419722382620393
032535960660921950400*e^(184*I*c) + 157685845528850918721462877864435090257
583149415561323427386562894447598277935629800939237175625149830*e^(183*I*c)
+ 485842581531402804473148368687721313901954124190467327784587060150968814
37076337910793584122475073760*e^(182*I*c) + 1477795509661712899871274518207
1495362176506973183081650233605274051677624970464340242755840025673760*e^(1
81*I*c) + 44372109178431823477643495444439046990200565950694708471936170921
14714077633077234972825351226979360*e^(180*I*c) + 1315052120930692122102297
105327622842335870743428530891072983535862280094446607723473800477453914130
*e^(179*I*c) + 384655842080666274454063078784837174998949052500975322162003
392549953413592461519365177908682078400*e^(178*I*c) + 111034148797008819443
143895644469242295049867464313710969257619338899133799285616020069872611710
850*e^(177*I*c) + 316266446747255477311767956875276535713059699859236883921
12164915553242573269490908989570248533280*e^(176*I*c) + 8888295028751024667
044203837607976101480053134418614474620767522824868911959884352666444917404
000*e^(175*I*c) + 246438219080743960907977422685567962936788570977643587663
0851716253962696192341706239192878728160*e^(174*I*c) + 67402553054313300889
484577523662523745074311447354453781817044713460710257567667605667532896159
0*e^(173*I*c) + 18183466140617790131533012967714538116644918841319414116934
4354754920969034952610378945282257600*e^(172*I*c) + 48379489756434099843857
791816589379406815042609340378747586437145781646245422045101230417309900*e^
(171*I*c) + 126934969329649205650736736371812800885486825088802553372800650
06566138696041797353216584528640*e^(170*I*c) + 3283874760555818676726309480
306734420155098583948074469014168171874442170109648521627538755920*e^(169*I
*c) + 837579206923411932458786486765373533946545239708990769488724813982189
165104589895518909256320*e^(168*I*c) + 210594301385648471184329078880317504
953361839954159427434009884661777259752542647709150036990*e^(167*I*c) + 521
909122076618242158122718542697480712928432432278947692296907200105471413341
31610989636000*e^(166*I*c) + 1274721961650332054135634306256284736860162214
0856786025445814532037904111523242298235713300*e^(165*I*c) + 30679742964317
473641981596239624636716170064196268514261484186029348529073790216597619118
40*e^(164*I*c) + 7275210107183942292917740738446942557987386670675353797597
32795567942578751384250780476310*e^(163*I*c) + 1699563279699297677739020966
52629253283704505477127544556534417376686540936706073847337600*e^(162*I*c)
+ 3910803125560180947653753536961184444084490375160564502351457235204524810
4262933598850730*e^(161*I*c) + 88627521427569572856813408857649045979353495
69355321815647721172537159186491471311666400*e^(160*I*c) + 1977792980665818
135651300094326239158605448870806970860577325385028609983034534672318500*e^
(159*I*c) + 434546676780280045346344498763892540797175105756827515509297024
187660299345484920192480*e^(158*I*c) + 939869153130681791490836060656814827
80836060510530154618486949839467131378859885998210*e^(157*I*c) + 2000800680

303004713729327825032159711354071620198333312634928118667915319906804525721
 6*e^(156*I*c) + 41915425006568261480933394145441591439644784724923159318091
 71859902114109005939942952*e^(155*I*c) + 8639799336223303495562968200283955
 13198708064940505702126068652936800794826651264256*e^(154*I*c) + 1751931705
 00618300241515632381912285157790097816049220671217212220015297133400636060*
 e^(153*I*c) + 3494107161327670464947794304333945020150407533516036186591602
 9213860778606230624960*e^(152*I*c) + 68529932231457366873288853116177954355
 92940841439866351079655652312894721972796266*e^(151*I*c) + 1321498055271300
 851429993866631619874424534425188183592049727687571032156435077280*e<sup>(150*I
 *c)</sup> + 250501028608928332469340456829902067712233644464602753159945727868485
 722395506952*e^(149*I*c) + 466682235482660178068545924681005702893559608696
 13650856575756758180182223308768*e^(148*I*c) + 8543013441126212334833540665
 069621472479085838041360564550722036723654297540205*e^(147*I*c) + 153633323
 8444927583532734556016494671674916578907116984548489078241693926940560*e<sup>(1
 46*I*c)</sup> + 27136120750326657073448651707718101480177532218318105563861925783
 6143271472358*e^(145*I*c) + 47065044611135158108487353367484243102698248838
 312635876283099427442745866704*e^(144*I*c) + 801372958079075243436196494576
 1543761469520791210746972675870481058674277844*e^(143*I*c) + 13392143742542
 45553564884406801945353385000254030655765953770237607180089968*e^(142*I*c)
 + 2196012813395155615002614788441900248705552612819460588396140446970379636
 95*e^(141*I*c) + 3532444720677901811537805282078941168758100458236743100620
 5879633729015200*e^(140*I*c) + 55725511573286711210162164163075961618619559
 69011697222340926210112854418*e^(139*I*c) + 8618848510949919087642468054746
 72428603757315484453974713612812215428992*e^(138*I*c) + 1306576602265604193
 35121434389938961884595434069984824307149332131747540*e^(137*I*c) + 1940797
 9215594566593535008103303255257745408070082431338945184797463936*e<sup>(136*I*c
)</sup> + 2823905151936586678382525706564457280290098698638597987628380245881715*
 e^(135*I*c) + 4023496922661211589340035828394287851169049039364095456025192
 19664720*e^(134*I*c) + 5611708107634117538408757018518853866037593201367473
 5519055227368366*e^(133*I*c) + 76590105201875496517771183576768719270818989
 89131125755798204236112*e^(132*I*c) + 1022536437468296737293065862705246449
 693687415559865844306888705423*e^(131*I*c) + 133490210052026183779673313868
 332303530332906163247194627808410304*e^(130*I*c) + 170338860273906157410409
 77721655541665612162275485028584310890417*e^(129*I*c) + 2123702969188871318
 266718781223927067839949015727293884065388080*e^(128*I*c) + 258585348715977
 270155829115684193411072034541491364393985491350*e^(127*I*c) + 307362174043
 21009965231037419663053962881035281709221697785072*e^(126*I*c) + 3564764890
 628724017088487996688178929195787613958545474804845*e^(125*I*c) + 403212225
 957798188840846139960995624144491271694336796459584*e^(124*I*c) + 444567081
 75258821024400946210535004523775722190977468484496*e^(123*I*c) + 4775398607
 100853263534207733818266777478693412738731031680*e^(122*I*c) + 499467506558
 531733671585862910572702811545035730398749530*e^(121*I*c) + 508363695081710
 99437019348610847391946736185108017183136*e^(120*I*c) + 5032024903401451824
 074213943766011922026507006311982753*e^(119*I*c) + 484093410240488718655917
 025303662581091659126182344528*e^(118*I*c) + 452309400398307383320256947846

46206844854827698075736*e^(117*I*c) + 4101545439937195793959956708442496709
 433800261224880*e^(116*I*c) + 360688613036389349413809780004559963548775423
 325255*e^(115*I*c) + 30735366512830562160991166338490057308062762518496*e^(
 114*I*c) + 2535667460650279776834561566186591213109251642859*e^(113*I*c) +
 202347509724462171313966643580234078508179838320*e^(112*I*c) + 156039112776
 87607099721623771744933086920587272*e^(111*I*c) + 1161581413733971751533622
 511909046917188768400*e^(110*I*c) + 833808399118378944531363036737850390515
 06805*e^(109*I*c) + 5764601046563151304213854710715346838447392*e^(108*I*c)
 + 383360155801054824529764688213114368047154*e^(107*I*c) + 244898373378123
 38687718622491865013839488*e^(106*I*c) + 1500602747937397286405577818722691
 539392*e^(105*I*c) + 88054927598941411145869950813388040256*e^(104*I*c) + 4
 939666610818025798809586352543471345*e^(103*I*c) + 264410375780310742518099
 326419685040*e^(102*I*c) + 13477227799524701956579274210395326*e^(101*I*c)
 + 652650253343206047453620559993840*e^(100*I*c) + 2995254774926549967525784
 2032197*e^(99*I*c) + 1299146645993240318167826532288*e^(98*I*c) + 530901272
 64630963470039804475*e^(97*I*c) + 2037031259470368160131922320*e^(96*I*c) +
 73099207817335597247098038*e^(95*I*c) + 2442455629894502983849104*e^(94*I*
 c) + 75599817092670157806639*e^(93*I*c) + 2154864144781257856128*e^(92*I*c)
 + 56169444526926562260*e^(91*I*c) + 1327882849274858880*e^(90*I*c) + 28186
 192554792138*e^(89*I*c) + 530563624556832*e^(88*I*c) + 8718181624155*e^(87*
 I*c) + 122503723056*e^(86*I*c) + 1431118260*e^(85*I*c) + 13343760*e^(84*I*c
) + 93096*e^(83*I*c) + 432*e^(82*I*c) + e^(81*I*c)))*tan(1/4*d*x + c) - 28*
 (338*a*e^(1055/2*I*c) + 136552*a*e^(1053/2*I*c) + 27515228*a*e^(1051/2*I*c)
 + 3687040552*a*e^(1049/2*I*c) + 369625815338*a*e^(1047/2*I*c) + 2957006522
 7040*a*e^(1045/2*I*c) + 1966409337599161*a*e^(1043/2*I*c) + 111804416623842
 644*a*e^(1041/2*I*c) + 5548294175019610066*a*e^(1039/2*I*c) + 2441249437081
 96709459*a*e^(1037/2*I*c) + 9642935277137118257641*a*e^(1035/2*I*c) + 34539
 2409065820898531490*a*e^(1033/2*I*c) + 11311601399861219435495350*a*e^(1031
 /2*I*c) + 341088288520555248394683275*a*e^(1029/2*I*c) + 952610863657947943
 7964936790*a*e^(1027/2*I*c) + 247678824846833277253335834952*a*e^(1025/2*I*
 c) + 6021691440068977811219750309348*a*e^(1023/2*I*c) + 1374362520620865068
 10801408828862*a*e^(1021/2*I*c) + 2954879430842410604932011265442302*a*e^(1
 019/2*I*c) + 60030708766686810922979823601133828*a*e^(1017/2*I*c) + 1155591
 152525353155901693951332528574*a*e^(1015/2*I*c) + 2113080986381305149781821
 5704863434230*a*e^(1013/2*I*c) + 367868194965564832606935721682843410685*a*
 e^(1011/2*I*c) + 6109811001129869767843196320577458616044*a*e^(1009/2*I*c)
 + 96993251932443912018716975689508023006378*a*e^(1007/2*I*c) + 147429747414
 1465450580152071694890996247655*a*e^(1005/2*I*c) + 214907217038591505014775
 95681763435720778909*a*e^(1003/2*I*c) + 30087011852119230244408516213566272
 3458505456*a*e^(1001/2*I*c) + 405100148552598682984514700153591381830206719
 4*a*e^(999/2*I*c) + 52523333574538984648429489695519406526756976563*a*e^(99
 7/2*I*c) + 656541730750769495168952644914474242735369097675*a*e^(995/2*I*c)
 + 7920859200695792807006454406332207329276812132106*a*e^(993/2*I*c) + 9232
 7527810400690773733044860754764248910029932264*a*e^(991/2*I*c) + 1040783214
 351939655185196194449040653397856679303077*a*e^(989/2*I*c) + 11356783819252

976784499704281554895724049545296747183*a*e^(987/2*I*c) + 12005745766900216
 9157406380951976541404817260989994202*a*e^(985/2*I*c) + 1230589291098041959
 151565198792286611164919449523155204*a*e^(983/2*I*c) + 12239378690259640676
 098190698651937625584285361664235941*a*e^(981/2*I*c) + 11820667791274226530
 8151068632779853612619007457778914450*a*e^(979/2*I*c) + 1109324724380705292
 283449093231952034907213153230066378718*a*e^(977/2*I*c) + 10122593617438421
 454872945774471611256945529490238801736444*a*e^(975/2*I*c) + 89868936876015
 124138088939755918047910164857950059626671506*a*e^(973/2*I*c) + 77672495443
 7796152386576526960981118865651225484076140617434*a*e^(971/2*I*c) + 6538945
 880517523468866323420234290611269386227470050003903740*a*e^(969/2*I*c) + 53
 649131382810104977802029620946554089637208561026255904068048*a*e^(967/2*I*c
) + 429193535791556847068138327994764831727769099957108368643958154*a*e^(96
 5/2*I*c) + 3349579976888722895534725449848627543256623010923459353099036635
 *a*e^(963/2*I*c) + 25513859756352077572322437078487111620320963476059001013
 262542630*a*e^(961/2*I*c) + 18975965275398356047987369720091612753396352814
 3615261433871315234*a*e^(959/2*I*c) + 1378664619969635806325626967277078749
 677173011656393858195336896873*a*e^(957/2*I*c) + 97885401924542563212551710
 83155760919251578109432314011159159160067*a*e^(955/2*I*c) + 679441531018873
 78416790537745291711484709226979885783161630835768882*a*e^(953/2*I*c) + 461
 23756141520107036405797767537275375289232929441981287451225816828*a*e^(951
 /2*I*c) + 30633232928597028773294909300288637116798055874454197751858096994
 25497*a*e^(949/2*I*c) + 199116722447441451059815872075200132414974384236728
 24426122213941696300*a*e^(947/2*I*c) + 126711148304267514241908091838693049
 594473410683838555696089694914431850*a*e^(945/2*I*c) + 78968552145187676915
 5582812528911180433623630278166458083659977181181120*a*e^(943/2*I*c) + 4821
 262154262488189638345987905919984432326565079712079669628450138845920*a*e^(
 941/2*I*c) + 28844609848991141927798576089491975808162650525713844636694817
 073554205876*a*e^(939/2*I*c) + 16915758712613441712465074792030898222500107
 1457475688802526063335881435874*a*e^(937/2*I*c) + 9726629305633107245966511
 92822395090437846904768028338179628461156644761176*a*e^(935/2*I*c) + 548522
 3830696074870232670430705003547243336234497802574847555967196592940404*a*e^(
 933/2*I*c) + 3034593643234220864981688011795852870078325964381028101501180
 5920593868190276*a*e^(931/2*I*c) + 1647366661889354043514224268663063314483
 37931278840039090867647515115028970040*a*e^(929/2*I*c) + 877746897443579121
 236075377490355068404009709042755817010349573425737869900308*a*e^(927/2*I*c
) + 45913455966455716820928177167561427183292380224569005106080054438360338
 79600972*a*e^(925/2*I*c) + 235831280056761522204069876752068777200271990892
 52365783381244222564261342679308*a*e^(923/2*I*c) + 118973313857946531056557
 15724616432599593579578912435085803330684419827752028872*a*e^(921/2*I*c) +
 58962711066691395067675754627703313240993647585074960288651941051528148983
 5432948*a*e^(919/2*I*c) + 2871277913186241419584133533827623298277835710354
 997128730316529679482826595460860*a*e^(917/2*I*c) + 13741380386275892220936
 955428884365279657568364963495869127376859464108093967124930*a*e^(915/2*I*c
) + 64643918893425950177388154847859391478782846138908883463204853647802214
 592593258800*a*e^(913/2*I*c) + 29898507900269692408923793667457065368702389

2479672655281622093032615608331810198500*a*e^(911/2*I*c) + 1359802460968616
270231868299814338076025075555715544920009211265679248387475802712230*a*e^(
909/2*I*c) + 60825298300431024154835691072077923487801633182127113659132393
32777970405599581316610*a*e^(907/2*I*c) + 267639513250336669548004139167656
68459687382409352116237121465028288206531963685595620*a*e^(905/2*I*c) + 115
863621300152298439647525945111489920370403992774304311355052277945386826708
814291260*a*e^(903/2*I*c) + 49356702364086764029997288950294474894310688112
5111509805464276652017586793148209575990*a*e^(901/2*I*c) + 2069267679845930
130381175334056457688285267649989023134787201410929393396836695689578160*a*
e^(899/2*I*c) + 85393764955921532889458592844632368727921627023914493546129
81234209920197053609719548280*a*e^(897/2*I*c) + 346928703063628310309900516
75345388059171005150015300151358457747567034555085057721123200*a*e^(895/2*I
*c) + 138778687909946600865367388761087962678079037327882508538788609272464
232291741120175647560*a*e^(893/2*I*c) + 54668354816232031875560754306354085
9142453668714252535105543063481098088223075742274691040*a*e^(891/2*I*c) + 2
120999195474520962343810357637122282201408039497316115675157834511715525841
111903287821280*a*e^(889/2*I*c) + 81057891441839307976610991290380915545455
87802074817715094441746173533262684806046690657020*a*e^(887/2*I*c) + 305181
301359518097999904378987712298812491727618069833461963108699953378023766097
17118084280*a*e^(885/2*I*c) + 113209905005294985203964615738175308537606762
720615035742593838863752559149966030961364432530*a*e^(883/2*I*c) + 41383715
743997151395747534710612209041809610312109350338458642259708854838700764277
8689392720*a*e^(881/2*I*c) + 1490892736239643826349298194172347730859393184
729348390955768602845045895181329808155623088100*a*e^(879/2*I*c) + 52940394
824274988423621015252224339762845321369798999653176734581985547286008981823
20842047910*a*e^(877/2*I*c) + 185311575956915491903936356712375479862040958
35636771457399022595077629357340612025141454239410*a*e^(875/2*I*c) + 639502
185792610192892198743153344773370298585574856438019092724321074768545481194
03002345066040*a*e^(873/2*I*c) + 217597528931373271755655817928113868631916
330150550078003801892262646350629271469008473415746660*a*e^(871/2*I*c) + 73
010532580291508847306886642344653438470518818729882934266406849662113709558
9328874374034838030*a*e^(869/2*I*c) + 2415921333551524990268389104618889650
799105367832328831867199400393567997657263303511921645584430*a*e^(867/2*I*c
) + 78848019705208119554410088376458628352127125163445206326896900553986229
20399678675555557890805540*a*e^(865/2*I*c) + 253836166254933485118820982113
67472769175014354807241300095082666759379081238794081850941236922200*a*e^(8
63/2*I*c) + 806146354312988706244395040330930438241580527688457923866164254
30054670214293015106952021225106450*a*e^(861/2*I*c) + 252588736344355055032
432845569683371071833282115166243901085698005080677674847942191033976516438
790*a*e^(859/2*I*c) + 78089892499768235098611025781775998802237733577449444
7021128338922394168739338059483402091817158180*a*e^(857/2*I*c) + 2382300742
453358079505817090919571011220019979440937399265243279675473364667112224683
580742416880880*a*e^(855/2*I*c) + 71723015394236205179352863839024286597402
54815627164523216102592820676307371873206856996395387197810*a*e^(853/2*I*c)
+ 213117498303262122069866741394786678912571477458440628814790901335345210

94852002361976434826499389640*a*e^(851/2*I*c) + 625051628242471442925710487
33415581477053088822642095333462366631829558281303110976783578410692040060*
a*e^(849/2*I*c) + 180961288176734209022437313266581102621868947359969028289
341259075645977162094115793342533495420437760*a*e^(847/2*I*c) + 51720711862
527423933715170812385400189629059534876927495124302268825779661057231498734
0186199489059600*a*e^(845/2*I*c) + 1459444560745512684204677392868982903207
097170274091924589689734580305663102531694189766409440027416920*a*e^(843/2*
I*c) + 40662075707504936772057335424718975259655520778602911959930973823174
71471408387205631440905418660098080*a*e^(841/2*I*c) + 111867557359821196291
988306484377261013122785735064571683491382063725291990370949000144430217058
86779480*a*e^(839/2*I*c) + 303923712903854797854537577563382489444180178336
93287676089824301076398097218660304024089324314169620320*a*e^(837/2*I*c) +
815461248614793975396145017238741603804067318787033837641689949980554328569
93829770850278568809909050230*a*e^(835/2*I*c) + 216098738191195691073884838
885130051606941660632814593281807643078453044461425859841133487398485483388
540*a*e^(833/2*I*c) + 56564314344488004859133467053941411771442658443299199
6919574779333959694062618454004998070039855100793060*a*e^(831/2*I*c) + 1462
530218978002981214579686179196234895642459370576507150726645426795532204892
523476949341669543643800450*a*e^(829/2*I*c) + 37356763881922178059298875437
491269152714003934322610550534396246143578332587567355865516316424683950690
30*a*e^(827/2*I*c) + 942682005728444778196310755751381759067350544069564584
5734753585643633077515924434771991502983134459465060*a*e^(825/2*I*c) + 2350
291388414583187433121250509539022598335395261617304489120785691712272664473
4230198671436996652682109840*a*e^(823/2*I*c) + 5789847874548404866954317068
245529292730188272321513952449528244196894384174589566247623093618880995136
9410*a*e^(821/2*I*c) + 1409384442983762699168937114336324967714380857089225
41655406018353943269164564745597706818400543286501171020*a*e^(819/2*I*c) +
339028422987833397855870661410580272783241944520304992916961272142323793865
878264830598039819203382521385460*a*e^(817/2*I*c) + 80596100957913925945111
236549855602442049297404176384842308042398321863605332592288792671695920641
6040146280*a*e^(815/2*I*c) + 1893606613431364445653113349865458640784337196
419968099348407060708046548568397551017687935610548803578917500*a*e^(813/2*
I*c) + 43973333030386009091653033607968159002906500495742365171786730492219
55238982869664008596667433434500227270300*a*e^(811/2*I*c) + 100934252910422
529258020368638012421632116064743382522556636322387459122205834683440665777
03311911110184928980*a*e^(809/2*I*c) + 229014298029247469899833989909783486
36420948713161325757783624831891169692007204195378391317610973058655935960*
a*e^(807/2*I*c) + 513672538957285383949905581668971446228052831255870390485
41605029080310508962829559065760790760349012514574324*a*e^(805/2*I*c) + 113
902787380345137253175540176895466156435942786320399177793549184461969982254
888132235316813167766293611215076*a*e^(803/2*I*c) + 24970703838266908715955
524234509458143854697844302578334009195027744760044974013600717240698439882
1206740076664*a*e^(801/2*I*c) + 5412519688777523321188136602492159282836024
35291415731537931205691521494543428945687343868761091899299763448926*a*e^(7
99/2*I*c) + 116001605090320766784189852473116448704256191850911800787750661

2429202374013486500542675475746584871022464343284*a*e^(797/2*I*c) + 2458371
678013599792486645299638407502874417936997854986824910964593814714891899418
206416846940147483817202804120*a*e^(795/2*I*c) + 51519670409967739886114822
874423495999554963399688711960157152409688729297264125600396680370886804417
79084914448*a*e^(793/2*I*c) + 106773451206115010316915177025157556222038526
33360648841958791107152970141386266895230517166551174392300030788342*a*e<sup>(7
91/2*I*c)</sup> + 218846998697445948227976513999219963532135383045671481597307612
56766296942857944789316336187135948566011636746108*a*e^(789/2*I*c) + 443636
454312537928664444903673060748123098651704911177085122325992861735120251940
47460621336633494994931616185367*a*e^(787/2*I*c) + 889498384731512101262028
680973754500267348876403858613232524870352317449834499616651977412760371967
52019899934708*a*e^(785/2*I*c) + 176407157870248188675039822837973453779541
220475971003861656126215795163365626566036433061912593074816945660675950*a*
e^(783/2*I*c) + 34606840858170005542670863991251388916277002843247539209630
3434765417571235354958450150764552565844163750312201645*a*e^(781/2*I*c) + 6
715894581490223423381875873103135599401069792029954889309548169394108055424
31176677931049253422876784424817016775*a*e^(779/2*I*c) + 128932750625588726
632450392400553280714817181084005573147574293437509654798000623860010169713
7751135171334427911230*a*e^(777/2*I*c) + 2448844783514115940204994952758843
903903066137200218152757612724023451060388905375768675328804787228900093398
681626*a*e^(775/2*I*c) + 46017069769727428834674047753392005152720786506232
12895074567249940816451512616367674551127607081333270181526998549*a*e<sup>(773/
2*I*c)</sup> + 855573479483030694276062658642039017301381734938615418725528935144
2131157588880818086547678120350930967986819636206*a*e^(771/2*I*c) + 1573972
719789718418643197826017636319226366234443559992072034335787378276063519772
3475350184525299985472022233368096*a*e^(769/2*I*c) + 2865229670974996507625
784570864717780384181865418129149779789649856611348772690428196044307022518
1805904476964093204*a*e^(767/2*I*c) + 5161367757526037511405735505421276062
482895606687503232118780055778547642671930573936597865733021532003186231832
8782*a*e^(765/2*I*c) + 9200978996533686300086017071701952083217979747204988
7078430610217525483442688425464817079510726308915387248081319590*a*e<sup>(763/2
*I*c)</sup> + 1623259213671149201253425026233024146083260448733520018556420724261
78071258568253753649777482260604487791662279095380*a*e^(761/2*I*c) + 283430
806741231720767506648452381579979167064691921935266522465999039151913754937
570379832176219683475585745980384642*a*e^(759/2*I*c) + 48981527576421563052
339169497851992367594557903028751693728789261368473412858996478196365109587
2597973866512819688854*a*e^(757/2*I*c) + 8378465523833960829378545780771133
691849743259713967787534422362640065376947479240555279396070574248092948564
90727675*a*e^(755/2*I*c) + 141861446347474593193958069062579643562092740232
6725707128978611183225665658663115625141407985113241273578748581984652*a*e<sup>(
753/2*I*c)</sup> + 2377677385281450780276535003632106957838315432294980074044721
444697000658850433913097456261486621128187350479818244518*a*e^(751/2*I*c) +
39450372888274194780186776805941821459331688317845552766846120920946111584
71077576449964287503346809348891265398259857*a*e^(749/2*I*c) + 648007496619
664487295966394891574067999964358342900357460871681820105936044613941126232

9726012628871843958339023337019*a*e^(747/2*I*c) + 1053806516425389968093408
092491104937584830698685055708458055195564245125177119141858169242447850124
6323800746129573800*a*e^(745/2*I*c) + 1696738771149622447782344394159137878
323962029350474049231489872889689774553299313340966785701350373451261766548
0477878*a*e^(743/2*I*c) + 2704978381176806334395584332893138892256030096597
2200104848675102101401505754650642355872221358066002741301627025812757*a*e^(
741/2*I*c) + 427001033010337786505355770869414806680330299009873584071186
5716884587219045914023413725085698021434756439677135383181*a*e^(739/2*I*c)
+ 6674705712707426861902397340424444672885789658469134008604771883281723793
7379017743449379623461239962563329874824019974*a*e^(737/2*I*c) + 1033225961
822791393446925833729217924813400623274124576045730552780866028877481850485
31214610304639645383482596676893056*a*e^(735/2*I*c) + 158394676850912040582
941140342441453463440676565557261331557154602069504766857910520692027298938
218954032119203110414947*a*e^(733/2*I*c) + 24048622815686739096475523786252
682807080504230546515678836444262776197236498239628989965376105158600417615
6357053438713*a*e^(731/2*I*c) + 3616318324123533619791585984364395733623932
070386522961325116375286608547568899933388743889737647373105591087219618049
50*a*e^(729/2*I*c) + 538632494001899503823161396844566466894862982480795241
656663858431115927193564638280643271545140293504620018648806033924*a*e^(727
/2*I*c) + 79467717159055783443196598168336866386492622066483733727957858721
9324929588856910779705968796220532402758676260405879107*a*e^(725/2*I*c) + 1
161406469681514686930231057169282810901036258055622532359913997313204117553
887393212328822273602211299474414691345126818*a*e^(723/2*I*c) + 16814976956
710006362151445829636071570171785214616874983091326371239089327772448953675
07216314619532694155538887238368802*a*e^(721/2*I*c) + 241184975049333266478
700555010611783273056698304791710723156537037797315349049446529989109255128
3476181625151984127753260*a*e^(719/2*I*c) + 3427439132936962218965045556690
424755097418918332324019145162160654311058062898282097500536856553443553377
411829820283274*a*e^(717/2*I*c) + 48259026274667626988238077733212032316012
634932178285321894505374255289467801481143830273687521592051509482054081474
95002*a*e^(715/2*I*c) + 673287592148262938582698346865379444871928897581859
7029221935080325837839653180167035072543738369936072307403835829117580*a*e^(
713/2*I*c) + 9308078563784043781017671130610248182087046913252916996880386
765608626993343281123190939712394958769614572944446169239240*a*e^(711/2*I*c
) + 12752082903980424859973782251580364392177685341537086604305318227869656
657064262679580181162371557742791081894355841587442*a*e^(709/2*I*c) + 17313
637307222450012881686695186108423735807528718156899578111200509974666183011
894235219699372581227561012842803845507749*a*e^(707/2*I*c) + 23297332527241
334865760084343467850152477287148995908606680514595755613320785430623562079
774389532374413873170123179910986*a*e^(705/2*I*c) + 31071306638867887145486
464243102830719563075151549726084666050681757993075426708570569040294724757
843388933295162320833726*a*e^(703/2*I*c) + 41074582160388455294632132080196
811133391850599884224560084096210500036772683324177263871865645028380575643
276712484382279*a*e^(701/2*I*c) + 53823524702485318601656281997663882683788
476647986193555036044978138774992020063718500148457278526659285673723602694

203421*a*e^(699/2*I*c) + 69916813461334650415678596824358129491815956630827
853757961542905961932986850395133931124724066002703731115391919663302990*a*
e^(697/2*I*c) + 90038229799029117818251903422428983408657431542110202767693
858296039149513675064655386150204671625355804206371140871319100*a*e^(695/2*
I*c) + 11495651216480428881356632049176888176830290754603443012317990989343
8427077959770152895978203628978930419723138966501119415*a*e^(693/2*I*c) + 1
455215056428878225474028796306562457100557747936373122749107932013003212812
57473947648611370199879573400230644017226525880*a*e^(691/2*I*c) + 182655864
263565676185586987570691832452212014065070429107609486721955832263268162000
180393266176385464480617543310025867318*a*e^(689/2*I*c) + 22734165436953896
383651350084459811904542387980282194079961590608083280046930635800881807383
5544585361649070452644644673032*a*e^(687/2*I*c) + 2806013656366115729105352
583294567568504401498926246662118118539858260307469619478112589657783088973
72914648649596408103468*a*e^(685/2*I*c) + 343473066630504003957461711079255
275849509437970035160833870481102720034535490851137644957145445343032779474
377183842954352*a*e^(683/2*I*c) + 41697974254250439905326704655692781831581
925610162955494026538762567812695289478714234210243727085600233712896612536
7035438*a*e^(681/2*I*c) + 5020932161084111007513087369465492553260793218252
50649068631607319287111256994298784601423217071295643346748150667815747120*
a*e^(679/2*I*c) + 599693463769285182016789414524002866479190256121281643609
130877951772604734279684159983894089320154006036997114520381278504*a*e^(677
/2*I*c) + 71052457584707104425174599313029575654897575295247587901665678008
8912410122013994517928255364300113796241185885146892353096*a*e^(675/2*I*c)
+ 8351490433163621046318995930487503058680371856689458161676107203582068277
17921729336701823612277340955328663616780414165744*a*e^(673/2*I*c) + 973902
45089215896557782693042189945216644054620610300129604309450410588695304883
563741799322908586007062103356708940279896*a*e^(671/2*I*c) + 11268509797513
965651407826291173959444409650356633676640287812794283033557922557964365952
32774704614843366308657035222636184*a*e^(669/2*I*c) + 129375433604327610763
551982574380675237083215776590283681600987892159353110715973908198332664853
4990963952447104637063475320*a*e^(667/2*I*c) + 1474036789113145125511586027
654519429767847115418748780301619677569124644817162925556814071528650814220
826475841711970398160*a*e^(665/2*I*c) + 16667688981919873229991442357374388
708795575677577514828113208026480816927001456076331784426018383079408299295
42219394816600*a*e^(663/2*I*c) + 187066221020888088529998809098955509325203
817865681910497590287277487884734303575272263127913613463619019462002877706
2771000*a*e^(661/2*I*c) + 2084078719336775409178999404560963834546779142378
693747318081619972210644062098160313254276418251294113576487118901711135260
*a*e^(659/2*I*c) + 23050562010856457734032115034958154060535070869056200726
15040978473923133843650811925101283679618986151289094096579669424320*a*e^(6
57/2*I*c) + 253134969683085098902842312671793025142106793828413761786635271
9881528467762070615224279447864998980982885764701335750299640*a*e^(655/2*I*
c) + 2760488471465536079001651407420406186773605926108195582908063805737766
190178484006749788753656431318199675121290619375614420*a*e^(653/2*I*c) + 29
898467547956308845174261169932700256750688732569707609559749165731697456555

50814846206617487155647589839236464701796279580*a*e^(651/2*I*c) + 321672557
507420652882892410005858474866554248955010598380892000356880845989548491941
7655857108624163274212339927440740825720*a*e^(649/2*I*c) + 3438442076114507
565828227629767657465219062203634063167744669216259140625679987917913352910
725024380172392492408078597847560*a*e^(647/2*I*c) + 36524219541131202253108
411756791413989630436235507972042053047897587472615246652915141798196494378
44381518984788554724814900*a*e^(645/2*I*c) + 385629012761269004523983508968
714311032763185387401712825777974712386427509471961921712939294748427543524
8946627983054371120*a*e^(643/2*I*c) + 4047954522936635863015958727879610537
748720406147393754728504285325965759343128650059937388360412866431049119645
619636844080*a*e^(641/2*I*c) + 42256779589263274939937634722559658068282936
758816331671495101524737232818312661778458989626625470882831534108115407639
37760*a*e^(639/2*I*c) + 438813356691874296623589709553772672914984549164820
8346990732139184405000815590708092488420647208468573326448979993238919840*a
*e^(637/2*I*c) + 4534439976333459310511482504990119650672839429847590278620
126153982188162070604003003067465034271078676682298452574885223760*a*e^(635
/2*I*c) + 46641735971564011756333525390837410179024153544719843471640392086
49991585553846020213682464029641764522414226537622903667360*a*e^(633/2*I*c)
+ 477735667572291477242788068085209794733382474511349117627392160272442581
9692043282810479708939732376220492567215846763821640*a*e^(631/2*I*c) + 4874
421304259617175276500599203042973459961166296204262993568026484810127299386
149073158190948249765374087088863694435755200*a*e^(629/2*I*c) + 49561511251
423650396692349838879762351218808782072049798900247151722813278366768316201
80909262717497111396726001076096795420*a*e^(627/2*I*c) + 502360397556685133
411332704356697506265365584465606240891406574370079536006792629619460725611
0274262048502871524134363430720*a*e^(625/2*I*c) + 5078020054192365588204904
825615019548624349688158033347603995255820206564757369117044034349799048501
173752875181190997850680*a*e^(623/2*I*c) + 51207212535516830195028848207874
961190989552457514397490474922793191033378444567666894164233085038287229411
22090578157783220*a*e^(621/2*I*c) + 515300800317335719578906194573631791526
712765914602757793744688268081645370584734147385693445188944784266441896079
8801389660*a*e^(619/2*I*c) + 5176060246095054646746769762022552048272184715
565679198279004129835648608965961963638205672345664098844084476717934767765
360*a*e^(617/2*I*c) + 51908489944204076650057446454521036150556279903379990
53339203475748827740596926190664008371199178877128179934687123252625080*a*e
^(615/2*I*c) + 519806428086287421165688613951756416147995641412176872720892
0751520508406216499432758187510936276181271178602086681965044900*a*e^(613/2
*I*c) + 5198064280862874211656886139517564161479956414121768727208920751520
508406216499432758187510936276181271178602086681965044900*a*e^(611/2*I*c) +
51908489944204076650057446454521036150556279903379990533392034757488277405
96926190664008371199178877128179934687123252625080*a*e^(609/2*I*c) + 517606
024609505464674676976202255204827218471556567919827900412983564860896596196
3638205672345664098844084476717934767765360*a*e^(607/2*I*c) + 5153008003173
357195789061945736317915267127659146027577937446882680816453705847341473856
934451889447842664418960798801389660*a*e^(605/2*I*c) + 51207212535516830195

028848207874961190989552457514397490474922793191033378444567666894164233085
03828722941122090578157783220*a*e^(603/2*I*c) + 507802005419236558820490482
561501954862434968815803334760399525582020656475736911704403434979904850117
3752875181190997850680*a*e^(601/2*I*c) + 5023603975566851334113327043566975
062653655844656062408914065743700795360067926296194607256110274262048502871
524134363430720*a*e^(599/2*I*c) + 49561511251423650396692349838879762351218
808782072049798900247151722813278366768316201809092627174971113967260010760
96795420*a*e^(597/2*I*c) + 487442130425961717527650059920304297345996116629
620426299356802648481012729938614907315819094824976537408708886369443575520
0*a*e^(595/2*I*c) + 4777356675722914772427880680852097947333824745113491176
273921602724425819692043282810479708939732376220492567215846763821640*a*e^(
593/2*I*c) + 46641735971564011756333525390837410179024153544719843471640392
08649991585553846020213682464029641764522414226537622903667360*a*e^(591/2*I
*c) + 453443997633345931051148250499011965067283942984759027862012615398218
8162070604003003067465034271078676682298452574885223760*a*e^(589/2*I*c) + 4
388133566918742966235897095537726729149845491648208346990732139184405000815
590708092488420647208468573326448979993238919840*a*e^(587/2*I*c) + 42256779
589263274939937634722559658068282936758816331671495101524737232818312661778
45898962662547088283153410811540763937760*a*e^(585/2*I*c) + 404795452293663
586301595872787961053774872040614739375472850428532596575934312865005993738
8360412866431049119645619636844080*a*e^(583/2*I*c) + 3856290127612690045239
835089687143110327631853874017128257779747123864275094719619217129392947484
275435248946627983054371120*a*e^(581/2*I*c) + 36524219541131202253108411756
791413989630436235507972042053047897587472615246652915141798196494378443815
18984788554724814900*a*e^(579/2*I*c) + 343844207611450756582822762976765746
521906220363406316774466921625914062567998791791335291072502438017239249240
8078597847560*a*e^(577/2*I*c) + 3216725575074206528828924100058584748665542
489550105983808920003568808459895484919417655857108624163274212339927440740
825720*a*e^(575/2*I*c) + 29898467547956308845174261169932700256750688732569
7076095597491657316974565550814846206617487155647589839236464701796279580*
a*e^(573/2*I*c) + 276048847146553607900165140742040618677360592610819558290
8063805737766190178484006749788753656431318199675121290619375614420*a*e^(57
1/2*I*c) + 2531349696830850989028423126717930251421067938284137617866352719
881528467762070615224279447864998980982885764701335750299640*a*e^(569/2*I*c
) + 23050562010856457734032115034958154060535070869056200726150409784739231
33843650811925101283679618986151289094096579669424320*a*e^(567/2*I*c) + 208
407871933677540917899940456096383454677914237869374731808161997221064406209
8160313254276418251294113576487118901711135260*a*e^(565/2*I*c) + 1870662210
208880885299988090989555093252038178656819104975902872774878847343035752722
631279136134636190194620028777062771000*a*e^(563/2*I*c) + 16667688981919873
229991442357374388708795575677577514828113208026480816927001456076331784426
01838307940829929542219394816600*a*e^(561/2*I*c) + 147403678911314512551158
602765451942976784711541874878030161967756912464481716292555681407152865081
4220826475841711970398160*a*e^(559/2*I*c) + 1293754336043276107635519825743
806752370832157765902836816009878921593531107159739081983326648534990963952

$447104637063475320 * a * e^{(557/2 * I * c)} + 11268509797513965651407826291173959444$
 $409650356633676640287812794283033557922557964365952327747046148433663086570$
 $35222636184 * a * e^{(555/2 * I * c)} + 973902450892158965557782693042189945216644054$
 $620610300129604309450410588695304883563741799322908586007062103356708940279$
 $896 * a * e^{(553/2 * I * c)} + 83514904331636210463189959304875030586803718566894581$
 $6167610720358206827717921729336701823612277340955328663616780414165744 * a * e^{(551/2 * I * c)}$
 $+ 7105245758470710442517459931302957565489757529524758790166567$
 $80088912410122013994517928255364300113796241185885146892353096 * a * e^{(549/2 * I * c)}$
 $+ 599693463769285182016789414524002866479190256121281643609130877951772$
 $604734279684159983894089320154006036997114520381278504 * a * e^{(547/2 * I * c)} + 50$
 $209321610841110075130873694654925532607932182525064906863160731928711125699$
 $4298784601423217071295643346748150667815747120 * a * e^{(545/2 * I * c)} + 4169797425$
 $425043990532670465569278183158192561016295549402653876256781269528947871423$
 $42102437270856002337128966125367035438 * a * e^{(543/2 * I * c)} + 343473066630504003$
 $957461711079255275849509437970035160833870481102720034535490851137644957145$
 $445343032779474377183842954352 * a * e^{(541/2 * I * c)} + 28060136563661157291053525$
 $832945675685044014989262466621181185398582603074696194781125896577830889737$
 $2914648649596408103468 * a * e^{(539/2 * I * c)} + 2273416543695389638365135008445981$
 $190454238798028219407996159060808328004693063580088180738355445853616490704$
 $52644644673032 * a * e^{(537/2 * I * c)} + 182655864263565676185586987570691832452212$
 $014065070429107609486721955832263268162000180393266176385464480617543310025$
 $867318 * a * e^{(535/2 * I * c)} + 14552150564288782254740287963065624571005577479363$
 $7312274910793201300321281257473947648611370199879573400230644017226525880 * a$
 $* e^{(533/2 * I * c)} + 1149565121648042888135663204917688817683029075460344301231$
 $79909893438427077959770152895978203628978930419723138966501119415 * a * e^{(531/2 * I * c)}$
 $+ 900382297990291178182519034224289834086574315421102027676938582960$
 $39149513675064655386150204671625355804206371140871319100 * a * e^{(529/2 * I * c)} +$
 $699168134613346504156785968243581294918159566308278537579615429059619329868$
 $50395133931124724066002703731115391919663302990 * a * e^{(527/2 * I * c)} + 538235247$
 $024853186016562819976638826837884766479861935550360449781387749920200637185$
 $00148457278526659285673723602694203421 * a * e^{(525/2 * I * c)} + 410745821603884552$
 $946321320801968111333918505998842245600840962105000367726833241772638718656$
 $45028380575643276712484382279 * a * e^{(523/2 * I * c)} + 310713066388678871454864642$
 $431028307195630751515497260846660506817579930754267085705690402947247578433$
 $88933295162320833726 * a * e^{(521/2 * I * c)} + 232973325272413348657600843434678501$
 $524772871489959086066805145957556133207854306235620797743895323744138731701$
 $23179910986 * a * e^{(519/2 * I * c)} + 173136373072224500128816866951861084237358075$
 $287181568995781112005099746661830118942352196993725812275610128428038455077$
 $49 * a * e^{(517/2 * I * c)} + 127520829039804248599737822515803643921776853415370866$
 $04305318227869656657064262679580181162371557742791081894355841587442 * a * e^{(515/2 * I * c)}$
 $+ 930807856378404378101767113061024818208704691325291699688038676$
 $5608626993343281123190939712394958769614572944446169239240 * a * e^{(513/2 * I * c)}$
 $+ 6732875921482629385826983468653794448719288975818597029221935080325837839$
 $653180167035072543738369936072307403835829117580 * a * e^{(511/2 * I * c)} + 48259026$
 $274667626988238077733212032316012634932178285321894505374255289467801481143$

83027368752159205150948205408147495002*a*e^(509/2*I*c) + 342743913293696221
 896504555669042475509741891833232401914516216065431105806289828209750053685
 6553443553377411829820283274*a*e^(507/2*I*c) + 2411849750493332664787005550
 106117832730566983047917107231565370377973153490494465299891092551283476181
 625151984127753260*a*e^(505/2*I*c) + 16814976956710006362151445829636071570
 171785214616874983091326371239089327772448953675072163146195326941555388872
 38368802*a*e^(503/2*I*c) + 116140646968151468693023105716928281090103625805
 5622532359913997313204117553887393212328822273602211299474414691345126818*a
 *e^(501/2*I*c) + 7946771715905578344319659816833686638649262206648373372795
 78587219324929588856910779705968796220532402758676260405879107*a*e^(499/2*I
 *c) + 538632494001899503823161396844566466894862982480795241656663858431115
 927193564638280643271545140293504620018648806033924*a*e^(497/2*I*c) + 36163
 183241235336197915859843643957336239320703865229613251163752866085475688999
 3338874388973764737310559108721961804950*a*e^(495/2*I*c) + 2404862281568673
 909647552378625268280708050423054651567883644426277619723649823962898996537
 61051586004176156357053438713*a*e^(493/2*I*c) + 158394676850912040582941140
 342441453463440676565557261331557154602069504766857910520692027298938218954
 032119203110414947*a*e^(491/2*I*c) + 10332259618227913934469258337292179248
 134006232741245760457305527808660288774818504853121461030463964538348259667
 6893056*a*e^(489/2*I*c) + 6674705712707426861902397340424444672885789658469
 1340086047718832817237937379017743449379623461239962563329874824019974*a*e
 ^ (487/2*I*c) + 427001033010337786505355770869414806680330299009873584071186
 5716884587219045914023413725085698021434756439677135383181*a*e^(485/2*I*c)
 + 2704978381176806334395584332893138892256030096597220010484867510210140150
 5754650642355872221358066002741301627025812757*a*e^(483/2*I*c) + 1696738771
 149622447782344394159137878323962029350474049231489872889689774553299313340
 9667857013503734512617665480477878*a*e^(481/2*I*c) + 1053806516425389968093
 408092491104937584830698685055708458055195564245125177119141858169242447850
 1246323800746129573800*a*e^(479/2*I*c) + 6480074966196644872959663948915740
 679999643583429003574608716818201059360446139411262329726012628871843958339
 023337019*a*e^(477/2*I*c) + 39450372888274194780186776805941821459331688317
 84555276684612092094611158471077576449964287503346809348891265398259857*a*e
 ^ (475/2*I*c) + 237767738528145078027653500363210695783831543229498007404472
 1444697000658850433913097456261486621128187350479818244518*a*e^(473/2*I*c)
 + 1418614463474745931939580690625796435620927402326725707128978611183225665
 658663115625141407985113241273578748581984652*a*e^(471/2*I*c) + 8378465238
 339608293785457807711336918497432597139677875344223626400653769474792405552
 7939607057424809294856490727675*a*e^(469/2*I*c) + 4898152757642156305233916
 949785199236759455790302875169372878926136847341285899647819636510958725979
 73866512819688854*a*e^(467/2*I*c) + 283430806741231720767506648452381579979
 167064691921935266522465999039151913754937570379832176219683475585745980384
 642*a*e^(465/2*I*c) + 16232592136711492012534250262330241460832604487335200
 1855642072426178071258568253753649777482260604487791662279095380*a*e^(463/2
 *I*c) + 9200978996533686300086017071701952083217979747204988707843061021752
 5483442688425464817079510726308915387248081319590*a*e^(461/2*I*c) + 5161367

757526037511405735505421276062482895606687503232118780055778547642671930573
9365978657330215320031862318328782*a*e^(459/2*I*c) + 2865229670974996507625
784570864717780384181865418129149779789649856611348772690428196044307022518
1805904476964093204*a*e^(457/2*I*c) + 1573972719789718418643197826017636319
226366234443559992072034335787378276063519772347535018452529998547202223336
8096*a*e^(455/2*I*c) + 8555734794830306942760626586420390173013817349386154
187255289351442131157588880818086547678120350930967986819636206*a*e^(453/2*
I*c) + 46017069769727428834674047753392005152720786506232128950745672499408
16451512616367674551127607081333270181526998549*a*e^(451/2*I*c) + 244884478
351411594020499495275884390390306613720021815275761272402345106038890537576
8675328804787228900093398681626*a*e^(449/2*I*c) + 1289327506255887266324503
924005532807148171810840055731475742934375096547980006238600101697137751135
171334427911230*a*e^(447/2*I*c) + 67158945814902234233818758731031355994010
6979202995488930954816939410805542431176677931049253422876784424817016775*a
*e^(445/2*I*c) + 3460684085817000554267086399125138891627700284324753920963
03434765417571235354958450150764552565844163750312201645*a*e^(443/2*I*c) +
176407157870248188675039822837973453779541220475971003861656126215795163365
626566036433061912593074816945660675950*a*e^(441/2*I*c) + 88949838473151210
126202868097375450026734887640385861323252487035231744983449961665197741276
037196752019899934708*a*e^(439/2*I*c) + 44363645431253792866444490367306074
812309865170491117708512232599286173512025194047460621336633494994931616185
367*a*e^(437/2*I*c) + 21884699869744594822797651399921996353213538304567148
159730761256766296942857944789316336187135948566011636746108*a*e^(435/2*I*c
) + 10677345120611501031691517702515755622203852633360648841958791107152970
141386266895230517166551174392300030788342*a*e^(433/2*I*c) + 51519670409967
739886114822874423495999554963399688711960157152409688729297264125600396680
37088680441779084914448*a*e^(431/2*I*c) + 245837167801359979248664529963840
750287441793699785498682491096459381471489189941820641684694014748381720280
4120*a*e^(429/2*I*c) + 1160016050903207667841898524731164487042561918509118
007877506612429202374013486500542675475746584871022464343284*a*e^(427/2*I*c
) + 54125196887775233211881366024921592828360243529141573153793120569152149
4543428945687343868761091899299763448926*a*e^(425/2*I*c) + 2497070383826690
871595552423450945814385469784430257833400919502774476004497401360071724069
84398821206740076664*a*e^(423/2*I*c) + 113902787380345137253175540176895466
156435942786320399177793549184461969982254888132235316813167766293611215076
*a*e^(421/2*I*c) + 51367253895728538394990558166897144622805283125587039048
541605029080310508962829559065760790760349012514574324*a*e^(419/2*I*c) + 22
901429802924746989983398990978348636420948713161325757783624831891169692007
204195378391317610973058655935960*a*e^(417/2*I*c) + 10093425291042252925802
036863801242163211606474338252255663632238745912220583468344066577703311911
110184928980*a*e^(415/2*I*c) + 43973333030386009091653033607968159002906500
49574236517178673049221955238982869664008596667433434500227270300*a*e^(413/
2*I*c) + 189360661343136444565311334986545864078433719641996809934840706070
8046548568397551017687935610548803578917500*a*e^(411/2*I*c) + 8059610095791
392594511123654985560244204929740417638484230804239832186360533259228879267

16959206416040146280*a*e^(409/2*I*c) + 339028422987833397855870661410580272
783241944520304992916961272142323793865878264830598039819203382521385460*a*
e^(407/2*I*c) + 14093844429837626991689371143363249677143808570892254165540
6018353943269164564745597706818400543286501171020*a*e^(405/2*I*c) + 5789847
874548404866954317068245529292730188272321513952449528244196894384174589566
2476230936188809951369410*a*e^(403/2*I*c) + 2350291388414583187433121250509
539022598335395261617304489120785691712272664473423019867143699665268210984
0*a*e^(401/2*I*c) + 9426820057284447781963107557513817590673505440695645845
734753585643633077515924434771991502983134459465060*a*e^(399/2*I*c) + 37356
763881922178059298875437491269152714003934322610550534396246143578332587567
35586551631642468395069030*a*e^(397/2*I*c) + 146253021897800298121457968617
919623489564245937057650715072664542679553220489252347694934166954364380045
0*a*e^(395/2*I*c) + 5656431434448800485913346705394141177144265844329919969
19574779333959694062618454004998070039855100793060*a*e^(393/2*I*c) + 216098
738191195691073884838885130051606941660632814593281807643078453044461425859
841133487398485483388540*a*e^(391/2*I*c) + 81546124861479397539614501723874
160380406731878703383764168994998055432856993829770850278568809909050230*a*
e^(389/2*I*c) + 30392371290385479785453757756338248944418017833693287676089
824301076398097218660304024089324314169620320*a*e^(387/2*I*c) + 11186755735
982119629198830648437726101312278573506457168349138206372529199037094900014
443021705886779480*a*e^(385/2*I*c) + 40662075707504936772057335424718975259
65552077860291195993097382317471471408387205631440905418660098080*a*e^(383/
2*I*c) + 145944456074551268420467739286898290320709717027409192458968973458
0305663102531694189766409440027416920*a*e^(381/2*I*c) + 5172071186252742393
371517081238540018962905953487692749512430226882577966105723149873401861994
89059600*a*e^(379/2*I*c) + 180961288176734209022437313266581102621868947359
969028289341259075645977162094115793342533495420437760*a*e^(377/2*I*c) + 62
505162824247144292571048733415581477053088822642095333462366631829558281303
110976783578410692040060*a*e^(375/2*I*c) + 21311749830326212206986674139478
667891257147745844062881479090133534521094852002361976434826499389640*a*e^(
373/2*I*c) + 71723015394236205179352863839024286597402548156271645232161025
92820676307371873206856996395387197810*a*e^(371/2*I*c) + 238230074245335807
950581709091957101122001997944093739926524327967547336466711222468358074241
6880880*a*e^(369/2*I*c) + 7808989249976823509861102578177599880223773357744
94447021128338922394168739338059483402091817158180*a*e^(367/2*I*c) + 252588
736344355055032432845569683371071833282115166243901085698005080677674847942
191033976516438790*a*e^(365/2*I*c) + 80614635431298870624439504033093043824
158052768845792386616425430054670214293015106952021225106450*a*e^(363/2*I*c
) + 25383616625493348511882098211367472769175014354807241300095082666759379
081238794081850941236922200*a*e^(361/2*I*c) + 78848019705208119554410088376
4586283521271251634452063268969005539862292039967867555557890805540*a*e^(3
59/2*I*c) + 241592133355152499026838910461888965079910536783232883186719940
0393567997657263303511921645584430*a*e^(357/2*I*c) + 7301053258029150884730
68866423446534384705188187298829342664068496621137095589328874374034838030*
a*e^(355/2*I*c) + 217597528931373271755655817928113868631916330150550078003

801892262646350629271469008473415746660*a*e^(353/2*I*c) + 63950218579261019
 289219874315334477337029858557485643801909272432107476854548119403002345066
 040*a*e^(351/2*I*c) + 18531157595691549190393635671237547986204095835636771
 457399022595077629357340612025141454239410*a*e^(349/2*I*c) + 52940394824274
 988423621015252224339762845321369798999653176734581985547286008981823208420
 47910*a*e^(347/2*I*c) + 149089273623964382634929819417234773085939318472934
 8390955768602845045895181329808155623088100*a*e^(345/2*I*c) + 4138371574399
 715139574753471061220904180961031210935033845864225970885483870076427786893
 92720*a*e^(343/2*I*c) + 113209905005294985203964615738175308537606762720615
 035742593838863752559149966030961364432530*a*e^(341/2*I*c) + 30518130135951
 809799990437898771229881249172761806983346196310869995337802376609717118084
 280*a*e^(339/2*I*c) + 81057891441839307976610991290380915545455878020748177
 15094441746173533262684806046690657020*a*e^(337/2*I*c) + 212099919547452096
 2343810357637122282201408039497316115675157834511715525841111903287821280*a
 *e^(335/2*I*c) + 5466835481623203187556075430635408591424536687142525351055
 43063481098088223075742274691040*a*e^(333/2*I*c) + 138778687909946600865367
 388761087962678079037327882508538788609272464232291741120175647560*a*e^(331
 /2*I*c) + 34692870306362831030990051675345388059171005150015300151358457747
 567034555085057721123200*a*e^(329/2*I*c) + 85393764955921532889458592844632
 36872792162702391449354612981234209920197053609719548280*a*e^(327/2*I*c) +
 206926767984593013038117533405645768828526764998902313478720141092939339683
 6695689578160*a*e^(325/2*I*c) + 4935670236408676402999728895029447489431068
 81125111509805464276652017586793148209575990*a*e^(323/2*I*c) + 115863621300
 152298439647525945111489920370403992774304311355052277945386826708814291260
 *a*e^(321/2*I*c) + 26763951325033666954800413916765668459687382409352116237
 121465028288206531963685595620*a*e^(319/2*I*c) + 60825298300431024154835691
 07207792348780163318212711365913239332777970405599581316610*a*e^(317/2*I*c)
 + 135980246096861627023186829981433807602507555571554492000921126567924838
 7475802712230*a*e^(315/2*I*c) + 2989850790026969240892379366745706536870238
 92479672655281622093032615608331810198500*a*e^(313/2*I*c) + 646439188934259
 50177388154847859391478782846138908883463204853647802214592593258800*a*e^(3
 11/2*I*c) + 137413803862758922209369554288843652796575683649634958691273768
 59464108093967124930*a*e^(309/2*I*c) + 287127791318624141958413353382762329
 8277835710354997128730316529679482826595460860*a*e^(307/2*I*c) + 5896271106
 66913950676757546277033132409936475850749602886519410515281489835432948*a*e
 ^ (305/2*I*c) + 118973313857946531056557157246164325995935795789124350858033
 330684419827752028872*a*e^(303/2*I*c) + 23583128005676152220406987675206877
 720027199089252365783381244222564261342679308*a*e^(301/2*I*c) + 45913455966
 45571682092817716756142718329238022456900510608005443836033879600972*a*e^(2
 99/2*I*c) + 877746897443579121236075377490355068404009709042755817010349573
 425737869900308*a*e^(297/2*I*c) + 16473666618893540435142242686630633144833
 7931278840039090867647515115028970040*a*e^(295/2*I*c) + 3034593643234220864
 9816880117958528700783259643810281015011805920593868190276*a*e^(293/2*I*c)
 + 5485223830696074870232670430705003547243336234497802574847555967196592940
 404*a*e^(291/2*I*c) + 97266293056331072459665119282239509043784690476802833

8179628461156644761176*a*e^(289/2*I*c) + 1691575871261344171246507479203089
 82225001071457475688802526063335881435874*a*e^(287/2*I*c) + 288446098489911
 41927798576089491975808162650525713844636694817073554205876*a*e^(285/2*I*c)
 + 482126215426248818963834598790591998443232656507971207966962845013884592
 0*a*e^(283/2*I*c) + 7896855214518767691555828125289111804336236302781664580
 83659977181181120*a*e^(281/2*I*c) + 126711148304267514241908091838693049594
 473410683838555696089694914431850*a*e^(279/2*I*c) + 19911672244744145105981
 587207520013241497438423672824426122213941696300*a*e^(277/2*I*c) + 30633232
 92859702877329490930028863711679805587445419775185809699425497*a*e^(275/2*I
 *c) + 461237561415201070364057977767537275375289232929441981287451225816828
 *a*e^(273/2*I*c) + 67944153101887378416790537745291711484709226979885783161
 630835768882*a*e^(271/2*I*c) + 97885401924542563212551710831557609192515781
 09432314011159159160067*a*e^(269/2*I*c) + 137866461996963580632562696727707
 8749677173011656393858195336896873*a*e^(267/2*I*c) + 1897596527539835604798
 73697200916127533963528143615261433871315234*a*e^(265/2*I*c) + 255138597563
 52077572322437078487111620320963476059001013262542630*a*e^(263/2*I*c) + 334
 9579976888722895534725449848627543256623010923459353099036635*a*e^(261/2*I*
 c) + 429193535791556847068138327994764831727769099957108368643958154*a*e^(2
 59/2*I*c) + 53649131382810104977802029620946554089637208561026255904068048*
 a*e^(257/2*I*c) + 653894588051752346886632342023429061126938622747005000390
 3740*a*e^(255/2*I*c) + 7767249544377961523865765269609811188656512254840761
 40617434*a*e^(253/2*I*c) + 898689368760151241380889397559180479101648579500
 59626671506*a*e^(251/2*I*c) + 101225936174384214548729457744716112569455294
 90238801736444*a*e^(249/2*I*c) + 110932472438070529228344909323195203490721
 3153230066378718*a*e^(247/2*I*c) + 1182066779127422653081510686327798536126
 19007457778914450*a*e^(245/2*I*c) + 122393786902596406760981906986519376255
 84285361664235941*a*e^(243/2*I*c) + 123058929109804195915156519879228661116
 4919449523155204*a*e^(241/2*I*c) + 1200574576690021691574063809519765414048
 17260989994202*a*e^(239/2*I*c) + 113567838192529767844997042815548957240495
 45296747183*a*e^(237/2*I*c) + 104078321435193965518519619444904065339785667
 9303077*a*e^(235/2*I*c) + 9232752781040069077373304486075476424891002993226
 4*a*e^(233/2*I*c) + 7920859200695792807006454406332207329276812132106*a*e^(
 231/2*I*c) + 656541730750769495168952644914474242735369097675*a*e^(229/2*I*
 c) + 52523333574538984648429489695519406526756976563*a*e^(227/2*I*c) + 4051
 001485525986829845147001535913818302067194*a*e^(225/2*I*c) + 30087011852119
 2302444085162135662723458505456*a*e^(223/2*I*c) + 2149072170385915050147759
 5681763435720778909*a*e^(221/2*I*c) + 1474297474141465450580152071694890996
 247655*a*e^(219/2*I*c) + 96993251932443912018716975689508023006378*a*e^(217
 /2*I*c) + 6109811001129869767843196320577458616044*a*e^(215/2*I*c) + 367868
 194965564832606935721682843410685*a*e^(213/2*I*c) + 21130809863813051497818
 215704863434230*a*e^(211/2*I*c) + 1155591152525353155901693951332528574*a*e
 ^((209/2*I*c) + 60030708766686810922979823601133828*a*e^(207/2*I*c) + 295487
 9430842410604932011265442302*a*e^(205/2*I*c) + 1374362520620865068108014088
 28862*a*e^(203/2*I*c) + 6021691440068977811219750309348*a*e^(201/2*I*c) + 2
 47678824846833277253335834952*a*e^(199/2*I*c) + 952610863657947943796493679

$0*a*e^{(197/2*I*c)} + 341088288520555248394683275*a*e^{(195/2*I*c)} + 113116013$
 $99861219435495350*a*e^{(193/2*I*c)} + 345392409065820898531490*a*e^{(191/2*I*c)}$
 $) + 9642935277137118257641*a*e^{(189/2*I*c)} + 244124943708196709459*a*e^{(187/2*I*c)}$
 $+ 5548294175019610066*a*e^{(185/2*I*c)} + 111804416623842644*a*e^{(183/2*I*c)}$
 $+ 1966409337599161*a*e^{(181/2*I*c)} + 29570065227040*a*e^{(179/2*I*c)}$
 $+ 369625815338*a*e^{(177/2*I*c)} + 3687040552*a*e^{(175/2*I*c)} + 27515228*a*e^{(173/2*I*c)}$
 $+ 136552*a*e^{(171/2*I*c)} + 338*a*e^{(169/2*I*c)})/(e^{(531*I*c)} + 432*e^{(530*I*c)}$
 $+ 93096*e^{(529*I*c)} + 13343760*e^{(528*I*c)} + 1431118260*e^{(527*I*c)}$
 $+ 122503723056*e^{(526*I*c)} + 8718181624155*e^{(525*I*c)} + 53056362$
 $4556832*e^{(524*I*c)} + 28186192554792138*e^{(523*I*c)} + 1327882849274858880*e^{(522*I*c)}$
 $+ 56169444526926562260*e^{(521*I*c)} + 2154864144781257856128*e^{(520*I*c)}$
 $+ 75599817092670157806639*e^{(519*I*c)} + 2442455629894502983849104*e^{(518*I*c)}$
 $+ 73099207817335597247098038*e^{(517*I*c)} + 203703125947036816013$
 $1922320*e^{(516*I*c)} + 53090127264630963470039804475*e^{(515*I*c)} + 129914664$
 $5993240318167826532288*e^{(514*I*c)} + 29952547749265499675257842032197*e^{(513*I*c)}$
 $+ 652650253343206047453620559993840*e^{(512*I*c)} + 134772277995247019$
 $56579274210395326*e^{(511*I*c)} + 264410375780310742518099326419685040*e^{(510*I*c)}$
 $+ 4939666610818025798809586352543471345*e^{(509*I*c)} + 880549275989414$
 $11145869950813388040256*e^{(508*I*c)} + 1500602747937397286405577818722691539$
 $392*e^{(507*I*c)} + 24489837337812338687718622491865013839488*e^{(506*I*c)} + 3$
 $83360155801054824529764688213114368047154*e^{(505*I*c)} + 5764601046563151304$
 $213854710715346838447392*e^{(504*I*c)} + 833808399118378944531363036737850390$
 $51506805*e^{(503*I*c)} + 1161581413733971751533622511909046917188768400*e^{(502*I*c)}$
 $+ 15603911277687607099721623771744933086920587272*e^{(501*I*c)} + 2023$
 $47509724462171313966643580234078508179838320*e^{(500*I*c)} + 2535667460650279$
 $776834561566186591213109251642859*e^{(499*I*c)} + 307353665128305621609911663$
 $38490057308062762518496*e^{(498*I*c)} + 3606886130363893494138097800045599635$
 $48775423325255*e^{(497*I*c)} + 4101545439937195793959956708442496709433800261$
 $224880*e^{(496*I*c)} + 45230940039830738332025694784646206844854827698075736*$
 $e^{(495*I*c)} + 484093410240488718655917025303662581091659126182344528*e^{(494*I*c)}$
 $+ 5032024903401451824074213943766011922026507006311982753*e^{(493*I*c)}$
 $+ 50836369508171099437019348610847391946736185108017183136*e^{(492*I*c)} + 4$
 $99467506558531733671585862910572702811545035730398749530*e^{(491*I*c)} + 4775$
 $398607100853263534207733818266777478693412738731031680*e^{(490*I*c)} + 444567$
 $08175258821024400946210535004523775722190977468484496*e^{(489*I*c)} + 4032122$
 $25957798188840846139960995624144491271694336796459584*e^{(488*I*c)} + 3564764$
 $890628724017088487996688178929195787613958545474804845*e^{(487*I*c)} + 307362$
 $17404321009965231037419663053962881035281709221697785072*e^{(486*I*c)} + 2585$
 $85348715977270155829115684193411072034541491364393985491350*e^{(485*I*c)} + 2$
 $123702969188871318266718781223927067839949015727293884065388080*e^{(484*I*c)}$
 $+ 17033886027390615741040977721655541665612162275485028584310890417*e^{(483*I*c)}$
 $+ 133490210052026183779673313868332303530332906163247194627808410304*$
 $e^{(482*I*c)} + 1022536437468296737293065862705246449693687415559865844306888$
 $705423*e^{(481*I*c)} + 765901052018754965177711835767687192708189898913112575$
 $5798204236112*e^{(480*I*c)} + 56117081076341175384087570185188538660375932013$

674735519055227368366*e^(479*I*c) + 402349692266121158934003582839428785116
904903936409545602519219664720*e^(478*I*c) + 282390515193658667838252570656
4457280290098698638597987628380245881715*e^(477*I*c) + 19407979215594566593
535008103303255257745408070082431338945184797463936*e^(476*I*c) + 130657660
226560419335121434389938961884595434069984824307149332131747540*e^(475*I*c)
+ 861884851094991908764246805474672428603757315484453974713612812215428992
*e^(474*I*c) + 557255115732867112101621641630759616186195596901169722234092
6210112854418*e^(473*I*c) + 35324447206779018115378052820789411687581004582
367431006205879633729015200*e^(472*I*c) + 219601281339515561500261478844190
024870555261281946058839614044697037963695*e^(471*I*c) + 133921437425424555
3564884406801945353385000254030655765953770237607180089968*e^(470*I*c) + 80
13729580790752434361964945761543761469520791210746972675870481058674277844*
e^(469*I*c) + 4706504461113515810848735336748424310269824883831263587628309
9427442745866704*e^(468*I*c) + 27136120750326657073448651707718101480177532
2183181055638619257836143271472358*e^(467*I*c) + 15363332384449275835327345
56016494671674916578907116984548489078241693926940560*e^(466*I*c) + 8543013
441126212334833540665069621472479085838041360564550722036723654297540205*e^
(465*I*c) + 466682235482660178068545924681005702893559608696136508565757567
58180182223308768*e^(464*I*c) + 2505010286089283324693404568299020677122336
44464602753159945727868485722395506952*e^(463*I*c) + 1321498055271300851429
993866631619874424534425188183592049727687571032156435077280*e^(462*I*c) +
685299322314573668732888531161779543559294084143986635107965565231289472197
2796266*e^(461*I*c) + 34941071613276704649477943043339450201504075335160361
865916029213860778606230624960*e^(460*I*c) + 175193170500618300241515632381
912285157790097816049220671217212220015297133400636060*e^(459*I*c) + 863979
933622330349556296820028395513198708064940505702126068652936800794826651264
256*e^(458*I*c) + 419154250065682614809333941454415914396447847249231593180
9171859902114109005939942952*e^(457*I*c) + 20008006803030047137293278250321
597113540716201983333126349281186679153199068045257216*e^(456*I*c) + 939869
153130681791490836060656814827808360605105301546184869498394671313788598859
98210*e^(455*I*c) + 4345466767802800453463444987638925407971751057568275155
09297024187660299345484920192480*e^(454*I*c) + 1977792980665818135651300094
326239158605448870806970860577325385028609983034534672318500*e^(453*I*c) +
886275214275695728568134088576490459793534956935532181564772117253715918649
1471311666400*e^(452*I*c) + 39108031255601809476537535369611844440844903751
605645023514572352045248104262933598850730*e^(451*I*c) + 169956327969929767
773902096652629253283704505477127544556534417376686540936706073847337600*e^
(450*I*c) + 727521010718394229291774073844694255798738667067535379759732795
567942578751384250780476310*e^(449*I*c) + 306797429643174736419815962396246
3671617006419626851426148418602934852907379021659761911840*e^(448*I*c) + 12
747219616503320541356343062562847368601622140856786025445814532037904111523
242298235713300*e^(447*I*c) + 521909122076618242158122718542697480712928432
43227894769229690720010547141334131610989636000*e^(446*I*c) + 2105943013856
484711843290788803175049533618399541594274340098846617772597525426477091500
36990*e^(445*I*c) + 8375792069234119324587864867653735339465452397089907694

88724813982189165104589895518909256320*e^(444*I*c) + 3283874760555818676726
 309480306734420155098583948074469014168171874442170109648521627538755920*e^
 (443*I*c) + 126934969329649205650736736371812800885486825088802553372800650
 06566138696041797353216584528640*e^(442*I*c) + 4837948975643409984385779181
 6589379406815042609340378747586437145781646245422045101230417309900*e^(441*
 I*c) + 18183466140617790131533012967714538116644918841319414116934435475492
 0969034952610378945282257600*e^(440*I*c) + 67402553054313300889484577523662
 5237450743114473544537818170447134607102575676676056675328961590*e^(439*I*c
) + 24643821908074396090797742268556796293678857097764358766308517162539626
 96192341706239192878728160*e^(438*I*c) + 8888295028751024667044203837607976
 101480053134418614474620767522824868911959884352666444917404000*e^(437*I*c)
 + 316266446747255477311767956875276535713059699859236883921121649155532425
 73269490908989570248533280*e^(436*I*c) + 1110341487970088194431438956444692
 42295049867464313710969257619338899133799285616020069872611710850*e^(435*I*
 c) + 3846558420806662744540630787848371749989490525009753221620033925499534
 13592461519365177908682078400*e^(434*I*c) + 1315052120930692122102297105327
 622842335870743428530891072983535862280094446607723473800477453914130*e^(43
 3*I*c) + 443721091784318234776434954444390469902005659506947084719361709211
 4714077633077234972825351226979360*e^(432*I*c) + 14777955096617128998712745
 182071495362176506973183081650233605274051677624970464340242755840025673760
 *e^(431*I*c) + 485842581531402804473148368687721313901954124190467327784587
 06015096881437076337910793584122475073760*e^(430*I*c) + 1576858455288509187
 214628778644350902575831494155613234273865628944475982779356298009392371756
 25149830*e^(429*I*c) + 5052936631230152588784830257388128132033977668453400
 65381261016353419722382620393032535960660921950400*e^(428*I*c) + 1598771101
 058192692270528999677444742685631006232456185844925220144002305878120380828
 483988663574829100*e^(427*I*c) + 499524195627913818020518674440168802438827
 2113921255663734956946927571305533146776898787878059685108480*e^(426*I*c) +
 15413111211486023937294970820797376716081344788163386543522421939737507962
 125854981881879168348260330000*e^(425*I*c) + 469702247271172818264540450180
 70670522559756627580347784535320014963482632359729444541885102274546002560*
 e^(424*I*c) + 1413799382535568432805655058074033041306061307254347517457940
 79833141361748917639986145377066437210546190*e^(423*I*c) + 4203580248351467
 985836112101459421546844379493656478990883725248021562228848395800116886556
 64280691773600*e^(422*I*c) + 1234668041892409978780018081755440216012582476
 396941937965899631953079203974222138794604328498972144766900*e^(421*I*c) +
 358271800216329606141453670371510989710719825273928454614934310234845612408
 9657428594946438660859773886240*e^(420*I*c) + 10271602530202889002497813516
 849452590971512809529060665197301097052210064576088348023234671975463677418
 470*e^(419*I*c) + 290976510612474534066475697818369100621655598523590528042
 59154165687125428752562385492373749486351714453120*e^(418*I*c) + 8145208141
 382911182887541756425005484603769331281148049290916075819598915576810702256
 8350953861815940704090*e^(417*I*c) + 22532053259322065776794110928951624899
 9794521015564134982827241710019675486694499689312466561907212627820000*e^(4
 16*I*c) + 61600311602297958493712570175788721299835430098999136262803886109

3914561332071191909714949426587936910303300*e^(415*I*c) + 16644750343872118
093949177435029376389785749377547647639878358724104499306901315729042799954
84581013965001440*e^(414*I*c) + 4445412259295474625067659514198312966015416
299968930393345630345914109720740573618884980520010028451496996210*e^(413*I
*c) + 117358569262451184931130910025016040323418769859990828235206722415302
00188223826392982302194084667538488665600*e^(412*I*c) + 3062758105422195737
839054728927760912957281393108273352024738722600002004353827946877670795842
0892547870128680*e^(411*I*c) + 79019149558766569254783988487232388352909144
982747171856772463223808993367091503402876467270176124342699654400*e^(410*I
*c) + 201557947424794098026772478462040883395867512562930862943753568690084
015585598010154781548625239409581907397500*e^(409*I*c) + 508324599301085460
166978629683032661427654474082939048097939638391567298795788389433842285751
054665210868287680*e^(408*I*c) + 126759701729481291340014627604212692998648
0292870190399107554311079964227280196522475370108738477856311765699610*e^(4
07*I*c) + 31256834931787017434797047503074901786662921507201793636043351135
28623329684606343185540756019935662148267863968*e^(406*I*c) + 7621788791912
047062038840917799374600428892258194367636682944356096681400246312138001769
285020661445991073249416*e^(405*I*c) + 183798070840033597660276492176211441
160917355722166207888615358034497022738025883590767042418407335134391141132
48*e^(404*I*c) + 4383497214291937768537869223302106374455403310092850273748
0438978976746989895784070951905237783490374305934542955*e^(403*I*c) + 10339
975546725743648984783764075375471820439447305579500146760432641987655555687
3829531737211096115196005647730480*e^(402*I*c) + 24124602128244006179290831
778303287619480159713320605209128699704372914534575580571008148900674183943
9573984832678*e^(401*I*c) + 55675638871118234034102619273421954611365176831
7380539005893679049394714017063698565272728813669054779077208977840*e^(400*
I*c) + 12710330829380489502013605548312703426623439912775046123423003660250
46741742856580445289401786656311685859023084716*e^(399*I*c) + 2870496131412
314451834674715353589439553294430808531933466086288543709246230769151392180
699413405623017247753532944*e^(398*I*c) + 641338189585592518475823145106255
638032859493851100657701521811978653639021305728401820220163109443481958402
5113465*e^(397*I*c) + 14176483652875704957202013343241117904369977796653849
959902524421980635189011634815653279605497783382888932766730080*e^(396*I*c)
+ 310043192060694170770693631414234874318280090981847446356786522841774394
64941651812564519144918003174108077634846014*e^(395*I*c) + 6709170613052966
912501989921002157658023784346222953538629508707618929784999593136064560529
2130961496106707521506432*e^(394*I*c) + 14365768713804479694294711970425953
845881819942351682467458629369105611920986635812363777224540953079923055376
7222252*e^(393*I*c) + 30438447110681333601028416012390637043388882849062742
2652551236966790916174520857759143930140187173492394981908258944*e^(392*I*c
) + 63821889291453374150580639966248855606678360049609187640837497487744897
1778036074996245581124283460438065182071976085*e^(391*I*c) + 13243113249840
274283552229381476823786728607088171617414487496895935880208608475087037023
25320304649883120684987556400*e^(390*I*c) + 2719589283483743926040805101080
341921244530311254607250929192773909331523226635035815672862569296693711643

$521070331394 * e^{(389 * I * c)} + 552749884903117835586123000932666828392629008215$
 $8467118000698502719379939045918344222192742145711257028040974074674736 * e^{(388 * I * c)}$
 $+ 11119499645363201080881062824886338492425375658448977935535846349$
 $290425821570383090425411418521516670371372045206568345 * e^{(387 * I * c)} + 221407$
 $350017086032709151807692413916620355787559039791489092136038225547927491835$
 $17160255571915875356439553717130797888 * e^{(386 * I * c)} + 4363830007581517102594$
 $621146446568961896577348866098585794565747985408510885185791122291198983761$
 $5452608512356008400295 * e^{(385 * I * c)} + 85139533234786455779589959464900637760$
 $735729705621221380005837208369157794673049675428799817875431430246332625899$
 $630160 * e^{(384 * I * c)} + 164437500676906892741323260154394278503954561936020133$
 $596581806449357240277349447927334517105809995300093549279931273178 * e^{(383 * I$
 $* c)} + 314409035808225861565595436938354945445473991043129722046747228813030$
 $925204968503418566818838611866709807040793495364496 * e^{(382 * I * c)} + 595157615$
 $500431514947479282336547053827926087916425263497187757029413385471835434198$
 $246807096214536895441388306027237899 * e^{(381 * I * c)} + 111539828554560155053332$
 $804560018431799389935921799672981834070422180119566741048584699617905673355$
 $8512452238583160792512 * e^{(380 * I * c)} + 20696982895008606434616657623738079575$
 $130194240411788719049605518294124493447224321256794179584030075511792983159$
 $47373776 * e^{(379 * I * c)} + 3802604996705891106964620633848964807037098854510182$
 $263243030597295630760353597531974324752266389193185760878274188013440 * e^{(37$
 $8 * I * c)} + 691783894521484427849333045936139492337233385361987963737267318494$
 $2859712431066345726870422099893124890777678037369988150 * e^{(377 * I * c)} + 12462$
 $140440537258084928596709872066857757070943124868554500948154756863454308032$
 $925408340311237850017814707896986969086816 * e^{(376 * I * c)} + 222313411318015353$
 $454063990377216868402083979419525801355846459667467366567162715548264762829$
 $91066076564921432614339399735 * e^{(375 * I * c)} + 3927420041432986116939794451622$
 $500108122743339858500720639923121190715779535971964824159875426657984024455$
 $1491476467899952 * e^{(374 * I * c)} + 68712466015985641512468586173659747734879591$
 $710098354652786124936023073943141049573606648563005359411712764895683903806$
 $088 * e^{(373 * I * c)} + 119060591849660546834765693227676449067584148248882678447$
 $504826077236333444513454095126668750057295811191643356908972191440 * e^{(372 * I$
 $* c)} + 204325557265186000767402710230847896459761583922763698235433212833313$
 $077783041040074669379017394836761539649081690630811665 * e^{(371 * I * c)} + 347310$
 $053810935290419455560555957314129569210735745983234369659976413374774078000$
 $173070075248654524917179128950507443058208 * e^{(370 * I * c)} + 584749573682304586$
 $179384628844883327581498969886540380378896767999075614964007174600811092945$
 $356635118795824799369716742109 * e^{(369 * I * c)} + 975210339444049318757282311763$
 $517786673223175594457946383279264635085041004917300295904275433144848532459$
 $919875479817581584 * e^{(368 * I * c)} + 161109254140006052595485937526419417834764$
 $347183707820144626243561514294458733783351358602272984952335843649358604225$
 $2995608 * e^{(367 * I * c)} + 26366624104307993404475222847782442837407510686581407$
 $26576446671207798325606832295937705061686297930296382338892574900819440 * e^{($
 $366 * I * c)} + 4274826907720591752526711336820871500844345647922385471534359333$
 $606189571832444641364132893108663576205133870672156264164115 * e^{(365 * I * c)} +$
 $686642533751866826266269375090895696573292457814218163062215780289988087468$

1551031136314064199948604001529894566235238597088*e^(364*I*c) + 10927210603
473544810279792347844536074588896806230041110089544731605863146104181739039
426674855453466097402330688331845602302*e^(363*I*c) + 172295028243676473344
007219984175967039486573947388052093916365973703805723989647150800953668183
22029152193635869784095333760*e^(362*I*c) + 2691779470108661509789012023689
050110514679999602177519571086622622863898470345683269415323061160726344418
3501026198563419616*e^(361*I*c) + 41670440375390543643418219342271748040035
071490119080585281522498188818375906900368701234531304633163446319945130196
476913600*e^(360*I*c) + 639230194337619890906148012886350981231994451023031
22616544648208998767803944455777042886552738499747183713136069104651812215*
e^(359*I*c) + 9717305502474268005861672246136889266114129554026349301303274
6083536157324268333390400308958318370219154887169702257444756176*e^(358*I*c
) + 14639044845635118121823738274037412419166481999774698807659839186273362
9670142241546375533903130605297580105675355629160198162*e^(357*I*c) + 21856
316665964931224748364095627214921249911538382877102965428336397258511809047
9413696638108156385244646591328454425745117584*e^(356*I*c) + 32341317801484
100371415113824607915257636097603505845789040993773872317153703657304368199
7163745313602400139153046673668433091*e^(355*I*c) + 47432304356310054237733
862993196624812917597698233244601805600939115402043889590314082296776949401
9446166779954024655344116288*e^(354*I*c) + 68951844932879355990326041814997
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5898949268614772285*e^(353*I*c) + 99355565364952112722644396082023364938648
851008189254586670044409666158279044124183085560957706203955562509094333226
4901780720*e^(352*I*c) + 14191644814221765738582340138989996288223223333095
737307163107438389358322014542936172931750864583896214253070517506121297614
98*e^(351*I*c) + 2009496110092687738152782085683737222727968824299058739215
446083499351467625334449670757764066690656150949014944043994822823920*e^(35
0*I*c) + 282081929856221595910752980728962844962138679898943636939311606989
4018781201000275633104498398959346631795568022519974400130281*e^(349*I*c) +
39256976584157783527681039428560118402116427696217172179966143984738871860
74391482638547212826538270453912634540299792270321024*e^(348*I*c) + 5416662
804052436349585595982818357953866258461644354018205158917742576425344364964
596750653177677803492186817305171175032011500*e^(347*I*c) + 741037261289115
222436463329612804397165719328032775430438223567277378140302381412761035556
2505271969045177704726054907145784960*e^(346*I*c) + 10052209524369581827588
154985345549678031445744499985208259385435609740272400301454246872041775159
838468077381562338745636398374*e^(345*I*c) + 135212304119454369155588547068
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96146162720992221760992*e^(344*I*c) + 1803532733817745547117756859485168297
797834644977719357208768851039242688451927299156085132639385224196147004081
9793627127923997*e^(343*I*c) + 23856398565562802030695278174212640833282154
174006459458292644060947924907313735921561690153939906017518647182491616573
724049744*e^(342*I*c) + 312952636881898383137727758733072603341172272586295
019923586956360926628660628198456890642358136229741501206689213918783989783
80*e^(341*I*c) + 4071598896370191895002034833673642342051331135901048524691

9074652883393970805374470830156229705647312265477584256027212762941040*e^(340*I*c) + 52539223346740771142587092370257069536060319644439501667610482767955800276052892432152798814607975110366224945081428121888473324*e^(339*I*c) + 67244087969080703823703257199663047606890610482090494619492802935130215979819469966383336788693900139115594646893784095418472336*e^(338*I*c) + 85368118430215312848231291739673735887746201851666299600392199418764750086828198719872744047767783667325326289221881974987582215*e^(337*I*c) + 107504737406576916123480399169759633328321407419400510017498849830598621565428266546315933920821527544726380201659114903834605888*e^(336*I*c) + 134297742023479429904629616104559610096074758721068022704468938063017059688023363436458971534964665036319889119229809973806909680*e^(335*I*c) + 166432332922589195130558329266398753389823955737598527556096093062473559769545772321978969318904192572733997888230986469005970880*e^(334*I*c) + 204622295535729109519829916789867225319429705162560082648840394965423112809336591921290309392396263834674368977840527147037426908*e^(333*I*c) + 249593072282565866398389951509619202682634455487128714631461891770823201367527645793770203788784677343934971424317987895255031936*e^(332*I*c) + 302060638030868463461139442279360499718906917482524894483356220196138377050828911383056860425370161157201493696073712322595776808*e^(331*I*c) + 362706307563843231135699157418510732452420614013624168879312187645233450153927975793326834780741391203430153093712635355523960320*e^(330*I*c) + 432147856464086938023811561808678589594702047904674282297959658800170984456799067751878044806619012452636891350731618278545690160*e^(329*I*c) + 51090761511134507452623846147137141450275722444316385531648429230851686635827717488464500331623385777400744950538410637735936000*e^(328*I*c) + 599378484771733474809376142401554850207064972118137572949257503651444541939309025276896049622515630263162184526394317285457368300*e^(327*I*c) + 697789106925924614816713747684682785083659819027952244447043355869741368500452561164024636073401929693105801105522738405349028160*e^(326*I*c) + 806169671327625532424575340089775733681994991576674446922354099714615192085443245663852257001115644286660304979476023966071898200*e^(325*I*c) + 924320052867522584035777495761072351222534420784861960001821020509468146356756433795246446491396141583854513687104334429566707520*e^(324*I*c) + 1051782100428834371944508170051219187116816349766953322637182149610875004223784784183284961906494422955462431208645690802526770780*e^(323*I*c) + 1187817943079390316108802324798110129020822782660087248599367643481200206046822166144285425922229375413676535071141005286431481600*e^(322*I*c) + 1331396114626723035802462123531582050339749996014152452835305956367425370222758621753922458727524856072950880960657564720475838500*e^(321*I*c) + 1481187117089246662466955694965677855524730313260552690821602657176218737426245522795329891464091005878304304075953693546767206080*e^(320*I*c) + 1635569744641421900657886381289076653227580172056677587451402069234355283687489659613913761959140773339736014790081814516625224440*e^(319*I*c) + 1792649078089632298936945728481969334964391597506285088488350622937252533420980803144316431701452190522716124797875257437516360640*e^(318*I*c) + 19502865507801819192449929612044870100564603628452185016744237662663215587914369173178787022326792138682879262946652027

69722927380*e^(317*I*c) + 2106141903468344307112549761202484543402794252352
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156800*e^(316*I*c) + 225772621910385628681283301268157376549626224142061293
207614315117196085455412414469902300984208051515792352935718986994351599120
0*e^(315*I*c) + 24024645955696860861200018030342110567394455886219461413841
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314*I*c) + 2537766415465030330815471746692988596069911894697225052928320452
542175587154848096483331209807430113943015398362669673337957755720*e^(313*I
*c) + 266110064797578358382823513920144193017839664338342390358386254725588
0772382049201015537214900832745601519737141849802506685264000*e^(312*I*c) +
27700732071507686455975072813820654979249684660545274141223398273337837700
68305883487309979315983718403740872884345746380680204260*e^(311*I*c) + 2862
503126320461797770667780725644184991255623174626175679050672100848988119391
841466573417019247590580735265143427289340450811200*e^(310*I*c) + 293649421
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9582733932202553130712723836983468866230908800*e^(309*I*c) + 29904989496225
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51518555144924290346242595167274383008960*e^(308*I*c) + 3023371643508225027
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917590062343527464977286601165100620*e^(307*I*c) + 303440835553095707578773
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3468866230908800*e^(303*I*c) + 28625031263204617977706677807256441849912556
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527414122339827333783770068305883487309979315983718403740872884345746380680
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358386254725588077238204920101553721490083274560151973714184980250668526400
0*e^(300*I*c) + 25377664154650303308154717466929885960699118946972250529283
20452542175587154848096483331209807430113943015398362669673337957755720*e^(
299*I*c) + 2402464595569686086120001803034211056739445588621946141384106162
886246161815149763025030834875234067267774023433418269982431265280*e^(298*I
*c) + 225772621910385628681283301268157376549626224142061293207614315117196
0854554124144699023009842080515157923529357189869943515991200*e^(297*I*c) +
21061419034683443071125497612024845434027942523524821998174104248696772627
15098288437646518683487945462774223656471345899082156800*e^(296*I*c) + 1950
286550780181919244992961204487010056460362845218501674423766266321558791436
917317878702232679213868287926294665202769722927380*e^(295*I*c) + 179264907
808963229893694572848196933496439159750628508848835062293725253342098080314
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219006578863812890766532275801720566775874514020692343552836874896596139137
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 466955694965677855524730313260552690821602657176218737426245522795329891464
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 212353158205033974999601415245283530595636742537022275862175392245872752485
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 534420784861960001821020509468146356756433795246446491396141583854513687104
 334429566707520*e^(288*I*c) + 806169671327625532424575340089775733681994991
 576674446922354099714615192085443245663852257001115644286660304979476023966
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 160*e^(286*I*c) + 599378484771733474809376142401554850207064972118137572949
 257503651444541939309025276896049622515630263162184526394317285457368300*e^
 (285*I*c) + 51090761511134507452623846147137141450275722444316385531648429
 230851686635827717488464500331623385777400744950538410637735936000*e^(284*I
 *c) + 432147856464086938023811561808678589594702047904674282297959658800170
 984456799067751878044806619012452636891350731618278545690160*e^(283*I*c) +
 362706307563843231135699157418510732452420614013624168879312187645233450153
 927975793326834780741391203430153093712635355523960320*e^(282*I*c) + 302060
 638030868463461139442279360499718906917482524894483356220196138377050828911
 383056860425370161157201493696073712322595776808*e^(281*I*c) + 249593072282
 565866398389951509619202682634455487128714631461891770823201367527645793770
 203788784677343934971424317987895255031936*e^(280*I*c) + 204622295535729109
 519829916789867225319429705162560082648840394965423112809336591921290309392
 396263834674368977840527147037426908*e^(279*I*c) + 166432332922589195130558
 329266398753389823955737598527556096093062473559769545772321978969318904192
 572733997888230986469005970880*e^(278*I*c) + 134297742023479429904629616104
 559610096074758721068022704468938063017059688023363436458971534964665036319
 889119229809973806909680*e^(277*I*c) + 107504737406576916123480399169759633
 328321407419400510017498849830598621565428266546315933920821527544726380201
 659114903834605888*e^(276*I*c) + 853681184302153128482312917396737358877462
 018516662996003921994187647500868281987198727440477677836673253262892218819
 74987582215*e^(275*I*c) + 6724408796908070382370325719966304760689061048209
 049461949280293513021597981946996638333678869390013911559464689378409541847
 2336*e^(274*I*c) + 52539223346740771142587092370257069536060319644439501667
 610482767955800276052892432152798814607975110366224945081428121888473324*e^
 (273*I*c) + 407159889637019189500203483367364234205133113590104852469190746
 52883393970805374470830156229705647312265477584256027212762941040*e^(272*I*
 c) + 3129526368818983831377277587330726033411722725862950199235869563609266
 2866062819845689064235813622974150120668921391878398978380*e^(271*I*c) + 23
 856398565562802030695278174212640833282154174006459458292644060947924907313

735921561690153939906017518647182491616573724049744*e^(270*I*c) + 180353273
381774554711775685948516829779783464497771935720876885103924268845192729915
60851326393852241961470040819793627127923997*e^(269*I*c) + 1352123041194543
691555885470685179654156739981165687002156735297532681546781784653328912387
1696056195231696146162720992221760992*e^(268*I*c) + 10052209524369581827588
154985345549678031445744499985208259385435609740272400301454246872041775159
838468077381562338745636398374*e^(267*I*c) + 741037261289115222436463329612
804397165719328032775430438223567277378140302381412761035556250527196904517
7704726054907145784960*e^(266*I*c) + 54166628040524363495855959828183579538
662584616443540182051589177425764253443649645967506531776778034921868173051
71175032011500*e^(265*I*c) + 3925697658415778352768103942856011840211642769
621717217996614398473887186074391482638547212826538270453912634540299792270
321024*e^(264*I*c) + 282081929856221595910752980728962844962138679898943636
9393116069894018781201000275633104498398959346631795568022519974400130281*e
^(263*I*c) + 20094961100926877381527820856837372227279688242990587392154460
83499351467625334449670757764066690656150949014944043994822823920*e^(262*I*
c) + 1419164481422176573858234013898999628822322333309573730716310743838935
832201454293617293175086458389621425307051750612129761498*e^(261*I*c) + 993
555653649521127226443960820233649386488510081892545866700444096661582790441
241830855609577062039555625090943332264901780720*e^(260*I*c) + 689518449328
793559903260418149974190253578340058895035589606468244680591556118170304005
037563669880057908765898949268614772285*e^(259*I*c) + 474323043563100542377
338629931966248129175976982332446018056009391154020438895903140822967769494
019446166779954024655344116288*e^(258*I*c) + 323413178014841003714151138246
079152576360976035058457890409937738723171537036573043681997163745313602400
139153046673668433091*e^(257*I*c) + 218563166659649312247483640956272149212
499115383828771029654283363972585118090479413696638108156385244646591328454
425745117584*e^(256*I*c) + 146390448456351181218237382740374124191664819997
746988076598391862733629670142241546375533903130605297580105675355629160198
162*e^(255*I*c) + 971730550247426800586167224613688926611412955402634930130
32746083536157324268333390400308958318370219154887169702257444756176*e^(254
*I*c) + 6392301943376198909061480128863509812319944510230312261654464820899
8767803944455777042886552738499747183713136069104651812215*e^(253*I*c) + 41
670440375390543643418219342271748040035071490119080585281522498188818375906
900368701234531304633163446319945130196476913600*e^(252*I*c) + 269177947010
866150978901202368905011051467999960217751957108662262286389847034568326941
53230611607263444183501026198563419616*e^(251*I*c) + 1722950282436764733440
072199841759670394865739473880520939163659737038057239896471508009536681832
2029152193635869784095333760*e^(250*I*c) + 10927210603473544810279792347844
536074588896806230041110089544731605863146104181739039426674855453466097402
330688331845602302*e^(249*I*c) + 686642533751866826266269375090895696573292
457814218163062215780289988087468155103113631406419994860400152989456623523
8597088*e^(248*I*c) + 42748269077205917525267113368208715008443456479223854
71534359333606189571832444641364132893108663576205133870672156264164115*e^(
247*I*c) + 2636662410430799340447522284778244283740751068658140726576446671

207798325606832295937705061686297930296382338892574900819440*e^(246*I*c) +
161109254140006052595485937526419417834764347183707820144626243561514294458
7337833513586022729849523358436493586042252995608*e^(245*I*c) + 97521033944
404931875728231176351778667322317559445794638327926463508504100491730029590
4275433144848532459919875479817581584*e^(244*I*c) + 58474957368230458617938
462884488332758149896988654038037889676799907561496400717460081109294535663
5118795824799369716742109*e^(243*I*c) + 34731005381093529041945556055595731
412956921073574598323436965997641337477407800017307007524865452491717912895
0507443058208*e^(242*I*c) + 20432555726518600076740271023084789645976158392
276369823543321283331307778304104007466937901739483676153964908169063081166
5*e^(241*I*c) + 11906059184966054683476569322767644906758414824888267844750
4826077236333444513454095126668750057295811191643356908972191440*e^(240*I*c
) + 68712466015985641512468586173659747734879591710098354652786124936023073
943141049573606648563005359411712764895683903806088*e^(239*I*c) + 392742004
143298611693979445162250010812274333985850072063992312119071577953597196482
41598754266579840244551491476467899952*e^(238*I*c) + 2223134113180153534540
639903772168684020839794195258013558464596674673665671627155482647628299106
6076564921432614339399735*e^(237*I*c) + 12462140440537258084928596709872066
857757070943124868554500948154756863454308032925408340311237850017814707896
986969086816*e^(236*I*c) + 691783894521484427849333045936139492337233385361
9879637372673184942859712431066345726870422099893124890777678037369988150*e
^(235*I*c) + 38026049967058911069646206338489648070370988545101822632430305
97295630760353597531974324752266389193185760878274188013440*e^(234*I*c) + 2
069698289500860643461665762373807957513019424041178871904960551829412449344
722432125679417958403007551179298315947373776*e^(233*I*c) + 111539828554560
155053332804560018431799389935921799672981834070422180119566741048584699617
9056733558512452238583160792512*e^(232*I*c) + 59515761550043151494747928233
654705382792608791642526349718775702941338547183543419824680709621453689544
1388306027237899*e^(231*I*c) + 31440903580822586156559543693835494544547399
104312972204674722881303092520496850341856681883861186670980704079349536449
6*e^(230*I*c) + 16443750067690689274132326015439427850395456193602013359658
1806449357240277349447927334517105809995300093549279931273178*e^(229*I*c) +
85139533234786455779589959464900637760735729705621221380005837208369157794
673049675428799817875431430246332625899630160*e^(228*I*c) + 436383000758151
710259462114644656896189657734886609858579456574798540851088518579112229119
89837615452608512356008400295*e^(227*I*c) + 2214073500170860327091518076924
139166203557875590397914890921360382255479274918351716025557191587535643955
3717130797888*e^(226*I*c) + 11119499645363201080881062824886338492425375658
448977935535846349290425821570383090425411418521516670371372045206568345*e^
(225*I*c) + 552749884903117835586123000932666828392629008215846711800069850
2719379939045918344222192742145711257028040974074674736*e^(224*I*c) + 27195
892834837439260408051010803419212445303112546072509291927739093315232266350
35815672862569296693711643521070331394*e^(223*I*c) + 1324311324984027428355
222938147682378672860708817161741448749689593588020860847508703702325320304
649883120684987556400*e^(222*I*c) + 638218892914533741505806399662488556066

783600496091876408374974877448971778036074996245581124283460438065182071976
085*e^(221*I*c) + 304384471106813336010284160123906370433888828490627422652
551236966790916174520857759143930140187173492394981908258944*e^(220*I*c) +
143657687138044796942947119704259538458818199423516824674586293691056119209
866358123637772245409530799230553767222252*e^(219*I*c) + 670917061305296691
250198992100215765802378434622295353862950870761892978499959313606456052921
30961496106707521506432*e^(218*I*c) + 3100431920606941707706936314142348743
182800909818474463567865228417743946494165181256451914491800317410807763484
6014*e^(217*I*c) + 14176483652875704957202013343241117904369977796653849959
902524421980635189011634815653279605497783382888932766730080*e^(216*I*c) +
641338189585592518475823145106255638032859493851100657701521811978653639021
3057284018202201631094434819584025113465*e^(215*I*c) + 28704961314123144518
346747153535894395532944308085319334660862885437092462307691513921806994134
05623017247753532944*e^(214*I*c) + 1271033082938048950201360554831270342662
343991277504612342300366025046741742856580445289401786656311685859023084716
*e^(213*I*c) + 556756388711182340341026192734219546113651768317380539005893
679049394714017063698565272728813669054779077208977840*e^(212*I*c) + 241246
021282440061792908317783032876194801597133206052091286997043729145345755805
710081489006741839439573984832678*e^(211*I*c) + 103399755467257436489847837
640753754718204394473055795001467604326419876555556873829531737211096115196
005647730480*e^(210*I*c) + 438349721429193776853786922330210637445540331009
28502737480438978976746989895784070951905237783490374305934542955*e^(209*I*
c) + 1837980708400335976602764921762114411609173557221662078886153580344970
2273802588359076704241840733513439114113248*e^(208*I*c) + 76217887919120470
620388409177993746004288922581943676366829443560966814002463121380017692850
20661445991073249416*e^(207*I*c) + 3125683493178701743479704750307490178666
292150720179363604335113528623329684606343185540756019935662148267863968*e^
(206*I*c) + 126759701729481291340014627604212692998648029287019039910755431
1079964227280196522475370108738477856311765699610*e^(205*I*c) + 50832459930
108546016697862968303266142765447408293904809793963839156729879578838943384
2285751054665210868287680*e^(204*I*c) + 20155794742479409802677247846204088
339586751256293086294375356869008401558559801015478154862523940958190739750
0*e^(203*I*c) + 79019149558766569254783988487232388352909144982747171856772
463223808993367091503402876467270176124342699654400*e^(202*I*c) + 306275810
542219573783905472892776091295728139310827335202473872260000200435382794687
76707958420892547870128680*e^(201*I*c) + 1173585692624511849311309100250160
403234187698599908282352067224153020018822382639298230219408466753848866560
0*e^(200*I*c) + 44454122592954746250676595141983129660154162999689303933456
30345914109720740573618884980520010028451496996210*e^(199*I*c) + 1664475034
387211809394917743502937638978574937754764763987835872410449930690131572904
279995484581013965001440*e^(198*I*c) + 616003116022979584937125701757887212
998354300989991362628038861093914561332071191909714949426587936910303300*e^
(197*I*c) + 225320532593220657767941109289516248999794521015564134982827241
710019675486694499689312466561907212627820000*e^(196*I*c) + 814520814138291
118288754175642500548460376933128114804929091607581959891557681070225683509

53861815940704090*e^(195*I*c) + 2909765106124745340664756978183691006216555
9852359052804259154165687125428752562385492373749486351714453120*e^(194*I*c
) + 10271602530202889002497813516849452590971512809529060665197301097052210
064576088348023234671975463677418470*e^(193*I*c) + 358271800216329606141453
670371510989710719825273928454614934310234845612408965742859494643866085977
3886240*e^(192*I*c) + 12346680418924099787800180817554402160125824763969419
37965899631953079203974222138794604328498972144766900*e^(191*I*c) + 4203580
248351467985836112101459421546844379493656478990883725248021562228848395800
11688655664280691773600*e^(190*I*c) + 1413799382535568432805655058074033041
30606130725434751745794079833141361748917639986145377066437210546190*e^(189
*I*c) + 4697022472711728182645404501807067052255975662758034778453532001496
3482632359729444541885102274546002560*e^(188*I*c) + 15413111211486023937294
970820797376716081344788163386543522421939737507962125854981881879168348260
330000*e^(187*I*c) + 499524195627913818020518674440168802438827211392125566
3734956946927571305533146776898787878059685108480*e^(186*I*c) + 15987711010
581926922705289996774447426856310062324561858449252201440023058781203808284
83988663574829100*e^(185*I*c) + 5052936631230152588784830257388128132033977
66845340065381261016353419722382620393032535960660921950400*e^(184*I*c) + 1
576858455288509187214628778644350902575831494155613234273865628944475982779
35629800939237175625149830*e^(183*I*c) + 4858425815314028044731483686877213
1390195412419046732778458706015096881437076337910793584122475073760*e^(182*
I*c) + 14777955096617128998712745182071495362176506973183081650233605274051
677624970464340242755840025673760*e^(181*I*c) + 443721091784318234776434954
4443904699020056595069470847193617092114714077633077234972825351226979360*e
^(180*I*c) + 13150521209306921221022971053276228423358707434285308910729835
35862280094446607723473800477453914130*e^(179*I*c) + 3846558420806662744540
630787848371749989490525009753221620033925499534135924615193651779086820784
00*e^(178*I*c) + 1110341487970088194431438956444692422950498674643137109692
57619338899133799285616020069872611710850*e^(177*I*c) + 3162664467472554773
117679568752765357130596998592368839211216491555324257326949090898957024853
3280*e^(176*I*c) + 88882950287510246670442038376079761014800531344186144746
20767522824868911959884352666444917404000*e^(175*I*c) + 2464382190807439609
079774226855679629367885709776435876630851716253962696192341706239192878728
160*e^(174*I*c) + 674025530543133008894845775236625237450743114473544537818
170447134607102575676676056675328961590*e^(173*I*c) + 181834661406177901315
330129677145381166449188413194141169344354754920969034952610378945282257600
*e^(172*I*c) + 483794897564340998438577918165893794068150426093403787475864
37145781646245422045101230417309900*e^(171*I*c) + 1269349693296492056507367
3637181280088548682508880255337280065006566138696041797353216584528640*e^(1
70*I*c) + 32838747605558186767263094803067344201550985839480744690141681718
74442170109648521627538755920*e^(169*I*c) + 8375792069234119324587864867653
73533946545239708990769488724813982189165104589895518909256320*e^(168*I*c)
+ 2105943013856484711843290788803175049533618399541594274340098846617772597
52542647709150036990*e^(167*I*c) + 5219091220766182421581227185426974807129
2843243227894769229690720010547141334131610989636000*e^(166*I*c) + 12747219

616503320541356343062562847368601622140856786025445814532037904111523242298
235713300*e^(165*I*c) + 306797429643174736419815962396246367161700641962685
1426148418602934852907379021659761911840*e^(164*I*c) + 72752101071839422929
1774073844694255798738667067535379759732795567942578751384250780476310*e^(1
63*I*c) + 16995632796992976777390209665262925328370450547712754455653441737
6686540936706073847337600*e^(162*I*c) + 39108031255601809476537535369611844
440844903751605645023514572352045248104262933598850730*e^(161*I*c) + 886275
214275695728568134088576490459793534956935532181564772117253715918649147131
1666400*e^(160*I*c) + 19777929806658181356513000943262391586054488708069708
60577325385028609983034534672318500*e^(159*I*c) + 4345466767802800453463444
98763892540797175105756827515509297024187660299345484920192480*e^(158*I*c)
+ 9398691531306817914908360606568148278083606051053015461848694983946713137
8859885998210*e^(157*I*c) + 20008006803030047137293278250321597113540716201
983333126349281186679153199068045257216*e^(156*I*c) + 419154250065682614809
3339414544159143964478472492315931809171859902114109005939942952*e^(155*I*c
) + 86397993362233034955629682002839551319870806494050570212606865293680079
4826651264256*e^(154*I*c) + 17519317050061830024151563238191228515779009781
6049220671217212220015297133400636060*e^(153*I*c) + 34941071613276704649477
943043339450201504075335160361865916029213860778606230624960*e^(152*I*c) +
685299322314573668732888531161779543559294084143986635107965565231289472197
2796266*e^(151*I*c) + 13214980552713008514299938666316198744245344251881835
92049727687571032156435077280*e^(150*I*c) + 2505010286089283324693404568299
02067712233644464602753159945727868485722395506952*e^(149*I*c) + 4666822354
8266017806854592468100570289355960869613650856575756758180182223308768*e^(1
48*I*c) + 85430134411262123348335406650696214724790858380413605645507220367
23654297540205*e^(147*I*c) + 1536333238444927583532734556016494671674916578
907116984548489078241693926940560*e^(146*I*c) + 271361207503266570734486517
077181014801775322183181055638619257836143271472358*e^(145*I*c) + 470650446
11135158108487353367484243102698248838312635876283099427442745866704*e^(144
*I*c) + 8013729580790752434361964945761543761469520791210746972675870481058
674277844*e^(143*I*c) + 133921437425424555356488440680194535338500025403065
5765953770237607180089968*e^(142*I*c) + 21960128133951556150026147884419002
4870555261281946058839614044697037963695*e^(141*I*c) + 35324447206779018115
378052820789411687581004582367431006205879633729015200*e^(140*I*c) + 557255
1157328671121016216416307596161861955969011697222340926210112854418*e^(139*
I*c) + 86188485109499190876424680547467242860375731548445397471361281221542
8992*e^(138*I*c) + 13065766022656041933512143438993896188459543406998482430
7149332131747540*e^(137*I*c) + 19407979215594566593535008103303255257745408
070082431338945184797463936*e^(136*I*c) + 282390515193658667838252570656445
7280290098698638597987628380245881715*e^(135*I*c) + 40234969226612115893400
3582839428785116904903936409545602519219664720*e^(134*I*c) + 56117081076341
175384087570185188538660375932013674735519055227368366*e^(133*I*c) + 765901
0520187549651777118357676871927081898989131125755798204236112*e^(132*I*c) +
1022536437468296737293065862705246449693687415559865844306888705423*e^(131
*I*c) + 133490210052026183779673313868332303530332906163247194627808410304*

$$\begin{aligned}
& e^{(130*I*c)} + 1703388602739061574104097772165554166561216227548502858431089 \\
& 0417*e^{(129*I*c)} + 21237029691888713182667187812239270678399490157272938840 \\
& 65388080*e^{(128*I*c)} + 2585853487159772701558291156841934110720345414913643 \\
& 93985491350*e^{(127*I*c)} + 3073621740432100996523103741966305396288103528170 \\
& 9221697785072*e^{(126*I*c)} + 35647648906287240170884879966881789291957876139 \\
& 58545474804845*e^{(125*I*c)} + 4032122259577981888408461399609956241444912716 \\
& 94336796459584*e^{(124*I*c)} + 4445670817525882102440094621053500452377572219 \\
& 0977468484496*e^{(123*I*c)} + 47753986071008532635342077338182667774786934127 \\
& 38731031680*e^{(122*I*c)} + 4994675065585317336715858629105727028115450357303 \\
& 98749530*e^{(121*I*c)} + 5083636950817109943701934861084739194673618510801718 \\
& 3136*e^{(120*I*c)} + 5032024903401451824074213943766011922026507006311982753* \\
& e^{(119*I*c)} + 484093410240488718655917025303662581091659126182344528*e^{(118 \\
& *I*c)} + 45230940039830738332025694784646206844854827698075736*e^{(117*I*c)} + \\
& 4101545439937195793959956708442496709433800261224880*e^{(116*I*c)} + 3606886 \\
& 13036389349413809780004559963548775423325255*e^{(115*I*c)} + 3073536651283056 \\
& 2160991166338490057308062762518496*e^{(114*I*c)} + 25356674606502797768345615 \\
& 66186591213109251642859*e^{(113*I*c)} + 2023475097244621713139666435802340785 \\
& 08179838320*e^{(112*I*c)} + 15603911277687607099721623771744933086920587272*e \\
& ^{(111*I*c)} + 1161581413733971751533622511909046917188768400*e^{(110*I*c)} + 8 \\
& 3380839911837894453136303673785039051506805*e^{(109*I*c)} + 57646010465631513 \\
& 04213854710715346838447392*e^{(108*I*c)} + 3833601558010548245297646882131143 \\
& 68047154*e^{(107*I*c)} + 24489837337812338687718622491865013839488*e^{(106*I*c \\
&)} + 1500602747937397286405577818722691539392*e^{(105*I*c)} + 8805492759894141 \\
& 1145869950813388040256*e^{(104*I*c)} + 4939666610818025798809586352543471345* \\
& e^{(103*I*c)} + 264410375780310742518099326419685040*e^{(102*I*c)} + 1347722779 \\
& 9524701956579274210395326*e^{(101*I*c)} + 652650253343206047453620559993840*e \\
& ^{(100*I*c)} + 29952547749265499675257842032197*e^{(99*I*c)} + 1299146645993240 \\
& 318167826532288*e^{(98*I*c)} + 53090127264630963470039804475*e^{(97*I*c)} + 203 \\
& 7031259470368160131922320*e^{(96*I*c)} + 73099207817335597247098038*e^{(95*I*c \\
&)} + 2442455629894502983849104*e^{(94*I*c)} + 75599817092670157806639*e^{(93*I* \\
& c)} + 2154864144781257856128*e^{(92*I*c)} + 56169444526926562260*e^{(91*I*c)} + \\
& 1327882849274858880*e^{(90*I*c)} + 28186192554792138*e^{(89*I*c)} + 53056362455 \\
& 6832*e^{(88*I*c)} + 8718181624155*e^{(87*I*c)} + 122503723056*e^{(86*I*c)} + 1431 \\
& 118260*e^{(85*I*c)} + 13343760*e^{(84*I*c)} + 93096*e^{(83*I*c)} + 432*e^{(82*I*c)} \\
& + e^{(81*I*c)}) * \tan(1/4*d*x + c) + 7*(-3718*I*a*e^{(1055/2*I*c)} - 1502072*I* \\
& a*e^{(1053/2*I*c)} - 302667508*I*a*e^{(1051/2*I*c)} - 40557446072*I*a*e^{(1049/2 \\
& *I*c)} - 4065883968718*I*a*e^{(1047/2*I*c)} - 325270717497440*I*a*e^{(1045/2*I* \\
& c)} - 21630502713590407*I*a*e^{(1043/2*I*c)} - 1229848582862122028*I*a*e^{(1041 \\
& /2*I*c)} - 61031235925186078942*I*a*e^{(1039/2*I*c)} - 2685374380786193144783* \\
& I*a*e^{(1037/2*I*c)} - 106072288048110242178847*I*a*e^{(1035/2*I*c)} - 37993164 \\
& 99692185181122010*I*a*e^{(1033/2*I*c)} - 124427615396355739905346318*I*a*e^{(1 \\
& 031/2*I*c)} - 3751971173605702746328148447*I*a*e^{(1029/2*I*c)} - 104787194996 \\
& 399169411032876338*I*a*e^{(1027/2*I*c)} - 2724467073052261042519716192524*I*a \\
& *e^{(1025/2*I*c)} - 66238605830373986929763608958656*I*a*e^{(1023/2*I*c)} - 151 \\
& 1798772310987071825034633753982*I*a*e^{(1021/2*I*c)} - 3250367372708464039374
\end{aligned}$$

5736284624870*I*a*e^(1019/2*I*c) - 660337796066223079759666054299242200*I*a
 *e^(1017/2*I*c) - 12711502667519783895793526074473028922*I*a*e^(1015/2*I*c)
 - 232438908235205496318447910669140546906*I*a*e^(1013/2*I*c) - 40465501381
 36102762173994148249330760039*I*a*e^(1011/2*I*c) - 672079208644143772696713
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 *a*e^(1005/2*I*c) - 236397937497882482710239205369322834941776795*I*a*e^(10
 03/2*I*c) - 3309571280975022580464977398526908985926399572*I*a*e^(1001/2*I*
 c) - 44561015944582807131288351508188541688368025238*I*a*e^(999/2*I*c) - 57
 7756662739414343075056408130898651215560673415*I*a*e^(997/2*I*c) - 72219589
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 I*c) - 8543973172918851719756469222929408444037078724153454872120390*I*a*e^
 (971/2*I*c) - 7192839149776427713448991220281034099272124959254612166822543
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*a*e^(933/2*I*c) - 33380444841812354407431172174362889642129947060455169568
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7/2*I*c) - 2944003577142698726124009086133102779114235240644160758492639805
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*e^(861/2*I*c) - 2778238434030150389912004289978892368939537453486552521474
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762504577571205434262057786505982168778594588998843843064160*I*a*e<sup>(855/2*I
*c)</sup> - 788866885356930177297637238699696087295784775621781001862517078036545
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)</sup> - 16083338628450132035920487995624082525516714816814575817900515648576615
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380*I*a*e^(811/2*I*c) - 110962940683089939119784926809098350196267205293097
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*c) - 251757924710457652984043156887340987777499765844068808649192255015133
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1/2*I*c) - 5948818874517551546477323998145990545303752046100553906871359525
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*I*a*e^(793/2*I*c) - 117321649413688805541489535769494623870692198518938820
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*c) - 240448066444268276150657582600282806736882299475006929773867640278082
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*I*c) - 7375237914096075753162757165114932907142763715319826841589592004329
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*e^(771/2*I*c) - 1727356648844560330636060166928567494261286446941231181267
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c) - 3143945599456919694267301189142992437773709005581330832402664217947159

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94*I*a*e^(759/2*I*c) - 5369433703713650579657543421925621273779896095493124
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57/2*I*c) - 918247884464415279278999087462958677682781850387787426747433314
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a*e^(743/2*I*c) - 295836191977555337516502759047146340088063683853123342002
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6863545628611035983085337507604532900*I*a*e^(735/2*I*c) - 17294477059214263
760088636654386762729080567200728007203192061357574554974641444591901300987
66051624362205245758670207771*I*a*e^(733/2*I*c) - 2624509576503561590920940
859305490105529291719205556945813050948090098978078963007543424661010613712
700954897972362495183*I*a*e^(731/2*I*c) - 394459845579235584131438630869091
688616597117378012582908079320650647984452263022600937980691694387762001248
5276018469226*I*a*e^(729/2*I*c) - 58720968143946275683812282183031367249802
560207599870758237349031412604961806367857641856683487210687551197919251693
49060*I*a*e^(727/2*I*c) - 8658474320748768850176377360129493160884016043350
179069387994509339240513007513825644052150379909781184428932642325725991*I*
a*e^(725/2*I*c) - 126464728874003373368658042722413436818510114448218997812
24539688451075809003555323364506814180545559579259228045710226754*I*a*e^(72
3/2*I*c) - 1829782956439305606836649875274920512367273492919051367157299055
4943995105221464517636015940848250377045895323290805291610*I*a*e^(721/2*I*c
) - 26227337368181122629870644431505178710075760072638475630349956011587927

854248611384364726503701818181005210900229213259176*I*a*e^(719/2*I*c) - 372
439498037343743798976414117862306566829323429902993434882550972030299097519
64165221448107634040170424408865199063339150*I*a*e^(717/2*I*c) - 5239944480
970766646493505752839643070619071693319360872460041164232215889290410408316
8727544999290987935418056417609518886*I*a*e^(715/2*I*c) - 73044855444748059
042443305340338046658849426364699492056128256421114683299164503187828529907
449597255183049531017656578288*I*a*e^(713/2*I*c) - 100894427492465881820210
044031354389458531264949909860927179527670590665562380069647321683456897981
727347188444555138501852*I*a*e^(711/2*I*c) - 138096263303296549257873301446
946161929200832100730252243863138612640918600095174736177924532345923756979
943811207087709274*I*a*e^(709/2*I*c) - 187308050497359467244546032059914316
902021455289947469407219847664575495361423597848611293739354236387332381220
310027105775*I*a*e^(707/2*I*c) - 251775360161376084897377488801495137606634
320723225849432463892924567486986972945320933780599953869938023457422052033
507310*I*a*e^(705/2*I*c) - 335408954754216210698116053661748898877578992418
661064723049500934091086950204858818941891841228452826384396254403564858074
*I*a*e^(703/2*I*c) - 442856413396521685634819129417520745967270283601088862
463141874230502485118099550417429454047383787479022883737738141901647*I*a*e
^(701/2*I*c) - 579562225293042134407748296189730817948676123423392101712466
430684150787138154370408270217791597537521052690534681094777663*I*a*e^(699/
2*I*c) - 751809406182173515834474930999752929931175527190042528290750068342
500184719150415256891020872036863319566794006361395795166*I*a*e^(697/2*I*c)
- 966734763200362088911450996294781727509924930263355610929302899030544704
282021262539760395229496521149023170036904352367324*I*a*e^(695/2*I*c) - 123
230929094885364247877935620085560816583582993928065911306556885596383211403
9518667176894361073144152565737831771354567511*I*a*e^(693/2*I*c) - 15572749
543636884974080928262296288151177990110096816240387416437531704841535169212
46034331278292249614057235994834309112040*I*a*e^(691/2*I*c) - 1951029428625
521159148702272207389699970324070295476096514707196995854917643770502268941
301284137902720651956083459514140654*I*a*e^(689/2*I*c) - 242345133480565914
733311284208599537821591350818552536451750600207637452850170881830271955344
6763078559382901836261131889256*I*a*e^(687/2*I*c) - 29846602160714443511932
004794183637479600618995280780624142655542950335078365386874190346265916083
51476175359584103001344444*I*a*e^(685/2*I*c) - 3644707984726321844885671363
210810979066210140813589811957097568654154365500556310960222110462594190863
697966659357844337376*I*a*e^(683/2*I*c) - 441320182176666215112684059021209
690173516855092646004758040837527592249384072393506741138735899841939688015
8733328711475814*I*a*e^(681/2*I*c) - 52988624495437595976650698117807905194
004016968785308752474523246470410378584341878153254556892207528898367029126
11111473360*I*a*e^(679/2*I*c) - 6309026175326811309123202300481088539484880
748756551060941678675574426326200109728754025077089532288910657077894543705
008776*I*a*e^(677/2*I*c) - 744910389926207712507533930739305232743825202785
942817072257714932602632217359931814815704828633351982935665889434059470818
4*I*a*e^(675/2*I*c) - 87220151109116905494649970042987671375105193524769158
11744669892179440284768413559220038431542413088622966991423825777102576*I*a

$e^{(673/2 \cdot I \cdot c)}$ - 1012761943112901113256883104445803082834216896465939566891
 1086917559825727413834246532010534829987050611067023384522066422504 $\cdot I \cdot a \cdot e^{(671/2 \cdot I \cdot c)}$ - 11662172128711377562308127546490438756126563569171539827972836
 511627219040565051420359980715312969450565938144063714770162456 $\cdot I \cdot a \cdot e^{(669/2 \cdot I \cdot c)}$ - 133178328902411595371902019461628998827468083267619624403201865933
 53473930246629933250050973279739565560426676777999389973000 $\cdot I \cdot a \cdot e^{(667/2 \cdot I \cdot c)}$ - 1508225861129425019938185854583130248301854593304849431308315269384735
 5205441385741162728852832003622428437181937942160447280 $\cdot I \cdot a \cdot e^{(665/2 \cdot I \cdot c)}$ -
 16938310832178831532822490891199231821109053460381935396940095961501487041
 734822855376465829835686556578407011793714960124040 $\cdot I \cdot a \cdot e^{(663/2 \cdot I \cdot c)}$ - 188
 639064756184990429860215837094004932466910713714827299980195356240710745621
 93650053426627399425629073637844277644750153560 $\cdot I \cdot a \cdot e^{(661/2 \cdot I \cdot c)}$ - 2083203
 668573329093655423228453425612501162709973030231841925570029701600228514043
 9445233578939463049867452727621615078639620 $\cdot I \cdot a \cdot e^{(659/2 \cdot I \cdot c)}$ - 22810972876
 883211286122694720045280171700534757884748490927484917240200147999923814567
 292507805999860803053865524582364804000 $\cdot I \cdot a \cdot e^{(657/2 \cdot I \cdot c)}$ - 247646717746537
 943274318671614025835099364502512825202355254688131160922508624627196838810
 23355338660499916659181180349795560 $\cdot I \cdot a \cdot e^{(655/2 \cdot I \cdot c)}$ - 2665338259907263972
 393674728320510756840957791783340693766849913374442906530005524057150842503
 6429118321680964755361170389060 $\cdot I \cdot a \cdot e^{(653/2 \cdot I \cdot c)}$ - 28434450046604756977423
 509538203693350662281539803802036385444751890833462571962860896974749954872
 457999903437614234319146820 $\cdot I \cdot a \cdot e^{(651/2 \cdot I \cdot c)}$ - 300632969035469927158522289
 443751171965033991608993398279627377566183933967808068189244111452769884960
 80480471422501896218680 $\cdot I \cdot a \cdot e^{(649/2 \cdot I \cdot c)}$ - 3149456054896788840456941192433
 960339464541906672760549598880732775198496323243194020723726538869474617993
 1762688836878800840 $\cdot I \cdot a \cdot e^{(647/2 \cdot I \cdot c)}$ - 32683348865778325383130633237123959
 605735110544382086527702946726198258184497868972918896266641810727368242389
 813054030133220 $\cdot I \cdot a \cdot e^{(645/2 \cdot I \cdot c)}$ - 335865737199706210494973739308999143800
 578848660449831488821544440472239763144499350791725442971653021501355619964
 18042983120 $\cdot I \cdot a \cdot e^{(643/2 \cdot I \cdot c)}$ - 3416431465489687648882423403358597201233661
 495861463644434120783025409767327514222749227479543034254076241258278000096
 0770560 $\cdot I \cdot a \cdot e^{(641/2 \cdot I \cdot c)}$ - 34381162126696627622627472821856495333522147075
 887509654873323568525944772491810111430218649338400622167547434461523829279
 760 $\cdot I \cdot a \cdot e^{(639/2 \cdot I \cdot c)}$ - 342074886829748944745010462946006975972275383016531
 94797160002682681665876148402637025420428006347009046317545272979001609440 $\cdot I \cdot a \cdot e^{(637/2 \cdot I \cdot c)}$ - 3362059800860642320065405835203109381204271641886196132
 5485316234413640793571327779830986471142061602134350650899849651277760 $\cdot I \cdot a \cdot e^{(635/2 \cdot I \cdot c)}$ - 32605705624939843682495195253232828524053826715848416651645
 889035921458935555487028933418123373614771794931330464399194235280 $\cdot I \cdot a \cdot e^{(633/2 \cdot I \cdot c)}$ - 311567109867585086547893089495950066396872100672757996618560872
 69851472589759467787944734060267442519580376947726659750489880 $\cdot I \cdot a \cdot e^{(631/2 \cdot I \cdot c)}$ - 2927672841618892055776840724607529365182033239622975352122060537267
 9227701104569238138367698160209738275122467287662716467440 $\cdot I \cdot a \cdot e^{(629/2 \cdot I \cdot c)}$
) - 26978353305400752041322382411052653974591789746746144606734240460942260
 089842260207957764573547124064623860447428875078078740 $\cdot I \cdot a \cdot e^{(627/2 \cdot I \cdot c)}$ -

242836498297182145731846228199333575666406674652245845379832169499539908069
 98169501947418479233595645585727784516401446933600*I*a*e^{-(625/2*I*c)} - 2122
 385655536659350820276514891261412730358387740296483604700583776671928033155
 7500411875109254434779078342142082375611830200*I*a*e^{-(623/2*I*c)} - 17838816
 347634183995008789781209806397083166041005840838361297522754230827274431117
 012331744863482652746434258554498280368660*I*a*e^{-(621/2*I*c)} - 141761464888
 017502067395597904384668082073933772652224452247447509405105716529542169443
 07486288863884549289668823221101795780*I*a*e^{-(619/2*I*c)} - 1029017357760796
 231586474548765393704099340718822618948182547718412282895541449947739012472
 0061044684730713865172442389148000*I*a*e^{-(617/2*I*c)} - 62406653602092301735
 393769820976385659315056333071113170888600517586672027890825069028429381527
 20706278974386615876935528840*I*a*e^{-(615/2*I*c)} - 2091397970086053758812619
 314132074314633768915002826637164459868374857836229715353712487792869819863
 186463104917716621200020*I*a*e^{-(613/2*I*c)} + 209139797008605375881261931413
 207431463376891500282663716445986837485783622971535371248779286981986318646
 3104917716621200020*I*a*e^{-(611/2*I*c)} + 62406653602092301735393769820976385
 659315056333071113170888600517586672027890825069028429381527207062789743866
 15876935528840*I*a*e^{-(609/2*I*c)} + 1029017357760796231586474548765393704099
 340718822618948182547718412282895541449947739012472006104468473071386517244
 2389148000*I*a*e^{-(607/2*I*c)} + 14176146488801750206739559790438466808207393
 377265222445224744750940510571652954216944307486288863884549289668823221101
 795780*I*a*e^{-(605/2*I*c)} + 178388163476341839950087897812098063970831660410
 058408383612975227542308272744311170123317448634826527464342585544982803686
 60*I*a*e^{-(603/2*I*c)} + 212238565536659350820276514891261412730358387740296
 4836047005837766719280331557500411875109254434779078342142082375611830200*I
 *a*e^{-(601/2*I*c)} + 24283649829718214573184622819933357566640667465224584537
 983216949953990806998169501947418479233595645585727784516401446933600*I*a*e
^{-(599/2*I*c)} + 269783533054007520413223824110526539745917897467461446067342
 40460942260089842260207957764573547124064623860447428875078078740*I*a*e⁻⁽⁵⁹
^{7/2*I*c)} + 2927672841618892055776840724607529365182033239622975352122060537
 2679227701104569238138367698160209738275122467287662716467440*I*a*e^{-(595/2*}
^{I*c)} + 31156710986758508654789308949595006639687210067275799661856087269851
 472589759467787944734060267442519580376947726659750489880*I*a*e^{-(593/2*I*c)}
 + 326057056249398436824951952532328285240538267158484166516458890359214589
 3555487028933418123373614771794931330464399194235280*I*a*e^{-(591/2*I*c)} + 3
 362059800860642320065405835203109381204271641886196132548531623441364079357
 1327779830986471142061602134350650899849651277760*I*a*e^{-(589/2*I*c)} + 34207
 488682974894474501046294600697597227538301653194797160002682681665876148402
 637025420428006347009046317545272979001609440*I*a*e^{-(587/2*I*c)} + 343811621
 266966276226274728218564953335221470758875096548733235685259447724918101114
 30218649338400622167547434461523829279760*I*a*e^{-(585/2*I*c)} + 3416431465489
 687648882423403358597201233661495861463644434120783025409767327514222749227
 4795430342540762412582780000960770560*I*a*e^{-(583/2*I*c)} + 33586573719970621
 049497373930899914380057884866044983148882154444047223976314449935079172544
 297165302150135561996418042983120*I*a*e^{-(581/2*I*c)} + 326833488657783253831

306332371239596057351105443820865277029467261982581844978689729188962666418
10727368242389813054030133220*I*a*e^(579/2*I*c) + 3149456054896788840456941
192433960339464541906672760549598880732775198496323243194020723726538869474
6179931762688836878800840*I*a*e^(577/2*I*c) + 30063296903546992715852228944
375117196503399160899339827962737756618393396780806818924411145276988496080
480471422501896218680*I*a*e^(575/2*I*c) + 284344500466047569774235095382036
933506622815398038020363854447518908334625719628608969747499548724579999034
37614234319146820*I*a*e^(573/2*I*c) + 2665338259907263972393674728320510756
840957791783340693766849913374442906530005524057150842503642911832168096475
5361170389060*I*a*e^(571/2*I*c) + 24764671774653794327431867161402583509936
450251282520235525468813116092250862462719683881023355338660499916659181180
349795560*I*a*e^(569/2*I*c) + 228109728768832112861226947200452801717005347
578847484909274849172402001479999238145672925078059998608030538655245823648
04000*I*a*e^(567/2*I*c) + 2083203668573329093655423228453425612501162709973
030231841925570029701600228514043944523357893946304986745272762161507863962
0*I*a*e^(565/2*I*c) + 18863906475618499042986021583709400493246691071371482
729998019535624071074562193650053426627399425629073637844277644750153560*I*
a*e^(563/2*I*c) + 169383108321788315328224908911992318211090534603819353969
40095961501487041734822855376465829835686556578407011793714960124040*I*a*e^(561/2*I*c)
+ 1508225861129425019938185854583130248301854593304849431308315
2693847355205441385741162728852832003622428437181937942160447280*I*a*e^(559/2*I*c)
+ 13317832890241159537190201946162899882746808326761962440320186593
353473930246629933250050973279739565560426676777999389973000*I*a*e^(557/2*I*c)
+ 116621721287113775623081275464904387561265635691715398279728365116272
19040565051420359980715312969450565938144063714770162456*I*a*e^(555/2*I*c)
+ 1012761943112901113256883104445803082834216896465939566891108691755982572
7413834246532010534829987050611067023384522066422504*I*a*e^(553/2*I*c) + 87
220151109116905494649970042987671375105193524769158117446698921794402847684
13559220038431542413088622966991423825777102576*I*a*e^(551/2*I*c) + 7449103
899262077125075339307393052327438252027859428170722577149326026322173599318
148157048286333519829356658894340594708184*I*a*e^(549/2*I*c) + 630902617532
681130912320230048108853948488074875655106094167867557442632620010972875402
5077089532288910657077894543705008776*I*a*e^(547/2*I*c) + 52988624495437595
976650698117807905194004016968785308752474523246470410378584341878153254556
8922075288983670291261111473360*I*a*e^(545/2*I*c) + 4413201821766662151126
840590212096901735168550926460047580408375275922493840723935067411387358998
419396880158733328711475814*I*a*e^(543/2*I*c) + 364470798472632184488567136
321081097906621014081358981195709756865415436550055631096022211046259419086
369796659357844337376*I*a*e^(541/2*I*c) + 29846602160714443511932004794183
637479600618995280780624142655542950335078365386874190346265916083514761753
59584103001344444*I*a*e^(539/2*I*c) + 2423451334805659147333112842085995378
215913508185525364517506002076374528501708818302719553446763078559382901836
261131889256*I*a*e^(537/2*I*c) + 195102942862552115914870227220738969997032
407029547609651470719699585491764377050226894130128413790272065195608345951
4140654*I*a*e^(535/2*I*c) + 15572749543636884974080928262296288151177990110

096816240387416437531704841535169212460343312782922496140572359948343091120
 40*I*a*e^(533/2*I*c) + 1232309290948853642478779356200855608165835829939280
 659113065568855963832114039518667176894361073144152565737831771354567511*I*
 a*e^(531/2*I*c) + 966734763200362088911450996294781727509924930263355610929
 302899030544704282021262539760395229496521149023170036904352367324*I*a*e^(5
 29/2*I*c) + 751809406182173515834474930999752929931175527190042528290750068
 342500184719150415256891020872036863319566794006361395795166*I*a*e^(527/2*I
 *c) + 579562225293042134407748296189730817948676123423392101712466430684150
 787138154370408270217791597537521052690534681094777663*I*a*e^(525/2*I*c) +
 442856413396521685634819129417520745967270283601088862463141874230502485118
 099550417429454047383787479022883737738141901647*I*a*e^(523/2*I*c) + 335408
 954754216210698116053661748898877578992418661064723049500934091086950204858
 818941891841228452826384396254403564858074*I*a*e^(521/2*I*c) + 251775360161
 376084897377488801495137606634320723225849432463892924567486986972945320933
 780599953869938023457422052033507310*I*a*e^(519/2*I*c) + 187308050497359467
 244546032059914316902021455289947469407219847664575495361423597848611293739
 354236387332381220310027105775*I*a*e^(517/2*I*c) + 138096263303296549257873
 301446946161929200832100730252243863138612640918600095174736177924532345923
 756979943811207087709274*I*a*e^(515/2*I*c) + 100894427492465881820210044031
 354389458531264949909860927179527670590665562380069647321683456897981727347
 188444555138501852*I*a*e^(513/2*I*c) + 730448554447480590424433053403380466
 588494263646994920561282564211146832991645031878285299074495972551830495310
 17656578288*I*a*e^(511/2*I*c) + 5239944480970766646493505752839643070619071
 693319360872460041164232215889290410408316872754499929098793541805641760951
 8886*I*a*e^(509/2*I*c) + 37243949803734374379897641411786230656682932342990
 299343488255097203029909751964165221448107634040170424408865199063339150*I*
 a*e^(507/2*I*c) + 262273373681811226298706444315051787100757600726384756303
 49956011587927854248611384364726503701818181005210900229213259176*I*a*e^(50
 5/2*I*c) + 1829782956439305606836649875274920512367273492919051367157299055
 4943995105221464517636015940848250377045895323290805291610*I*a*e^(503/2*I*c
) + 12646472887400337336865804272241343681851011444821899781224539688451075
 809003555323364506814180545559579259228045710226754*I*a*e^(501/2*I*c) + 865
 847432074876885017637736012949316088401604335017906938799450933924051300751
 3825644052150379909781184428932642325725991*I*a*e^(499/2*I*c) + 58720968143
 946275683812282183031367249802560207599870758237349031412604961806367857641
 85668348721068755119791925169349060*I*a*e^(497/2*I*c) + 3944598455792355841
 314386308690916886165971173780125829080793206506479844522630226009379806916
 943877620012485276018469226*I*a*e^(495/2*I*c) + 262450957650356159092094085
 930549010552929171920555694581305094809009897807896300754342466101061371270
 0954897972362495183*I*a*e^(493/2*I*c) + 17294477059214263760088636654386762
 729080567200728007203192061357574554974641444591901300987660516243622052457
 58670207771*I*a*e^(491/2*I*c) + 1128648994213736556251579671179535173610891
 367629572521988885688708464002698593853556863545628611035983085337507604532
 900*I*a*e^(489/2*I*c) + 729425552221874327486638200118763924277143204037696
 136287769063944888833152008172511657111399079703659693540539407822870*I*a*e

$\wedge(487/2*I*c) + 466822963677192661817331692307767055928258847749833780466192$
 $188737545805803679028894427817375583837205119809040687012935*I*a*e^{(485/2*I$
 $*c) + 295836191977555337516502759047146340088063683853123342002123301118180$
 $522953314880146816183888113061523938713872940614049*I*a*e^{(483/2*I*c) + 185$
 $633805219350815197506350903977557107252277227219361709459901437719214218026$
 $047853241796305910447871730808241890424442*I*a*e^{(481/2*I*c) + 115331633399$
 $545294784116715872366046039996439419217339810253656785321781700967045714777$
 $723718363358804096119332434747204*I*a*e^{(479/2*I*c) + 709421728625611640581$
 $601798688187324197887347397520234919328507749231563329446224925984401573067$
 $13277485913625566923405*I*a*e^{(477/2*I*c) + 4320205035785148116648701448006$
 $406297244718446957010989878166683915016192151982303287631165400297472693539$
 $1777697053449*I*a*e^{(475/2*I*c) + 26045177287912241176067222920901853851313$
 $554636809138208225364272427146197041803852090395533633595587669906282640335$
 $206*I*a*e^{(473/2*I*c) + 155436391595087510190621822992589087995698198761329$
 $85160704675357166344312257501069237830360698174149396040851872785700*I*a*e^{$
 $(471/2*I*c) + 9182478844644152792789990874629586776827818503877874267474333$
 $149197309164002316988444567545080601139568783596754328769*I*a*e^{(469/2*I*c)$
 $+ 536943370371365057965754342192562127377989609549312409405836889707519217$
 $9929265569591470701092735447407195008016032850*I*a*e^{(467/2*I*c) + 31076912$
 $890922450865942239142884964053899123899109838360878281617044308077919620448$
 $64438637368112933607786892490134694*I*a*e^{(465/2*I*c) + 1780195287370357324$
 $046533363473674962784595946909397017548451036643167283461157097955927272971$
 $697034280841666570691552*I*a*e^{(463/2*I*c) + 100924680821544677378647035106$
 $550416847900766313145845163296252762133611609519406119274163659697074508176$
 $3545649524358*I*a*e^{(461/2*I*c) + 56624796774753920331645228162081727228166$
 $630708266001409626784945725116717748737473906897305871977817503871634006084$
 $6*I*a*e^{(459/2*I*c) + 31439455994569196942673011891429924377737090055813308$
 $3240266421794715966982810304291470328806778764207633221621542600*I*a*e^{(457$
 $/2*I*c) + 17273566488445603306360601669285674942612864469412311812674186989$
 $4500269515702878867401279635331386515415677529225100*I*a*e^{(455/2*I*c) + 93$
 $908980208956081110791543951772985370600517586262249095832986389557258513757$
 $169577939649993369495960643164181810586*I*a*e^{(453/2*I*c) + 505160955717702$
 $736706806587300544605277632593563199783177893743454009384401250420614488090$
 $87158872073418586148121473*I*a*e^{(451/2*I*c) + 2688619372091907337098892954$
 $674504531907813287375704264122527194698302978274331378642387095176502732686$
 $1300318178642*I*a*e^{(449/2*I*c) + 14157449655540836037828005969786800176631$
 $864151856876634713547605341520538812026850026666968584541575064351905970774$
 $*I*a*e^{(447/2*I*c) + 737523791409607575316275716511493290714276371531982684$
 $1589592004329773561580320189977859708165438687906157904775921*I*a*e^{(445/2*$
 $I*c) + 38008556433995055200694535944109720280988684103462485758377464761067$
 $55638552361174314174233599317030879593032072609*I*a*e^{(443/2*I*c) + 1937670$
 $525270251509451678468483387984129872967538542952639048449569636581336018975$
 $500784323892713878389595738475570*I*a*e^{(441/2*I*c) + 977125696367158781879$
 $935803084928183110588096954532024018846643158415313905036398953539486071073$
 $924163775202836156*I*a*e^{(439/2*I*c) + 487384184545517828048292377274889764$

924631864066054597518560594306296038854621408587615306610365269386295410913
 209*I*a*e^(437/2*I*c) + 240448066444268276150657582600282806736882299475006
 929773867640278082209476385198832816186325534733945931054736716*I*a*e<sup>(435/
 2*I*c)</sup> + 117321649413688805541489535769494623870692198518938820841919455374
 946516030743273351488754702468032406699077952754*I*a*e^(433/2*I*c) + 566134
 964506713070924629978112007703878668613634184664482274913284090412872330960
 94607670158591842995212315780816*I*a*e^(431/2*I*c) + 2701621072515344146131
 402425368020110690494127021479232055469033618750854315350522823645075427649
 5512692743900280*I*a*e^(429/2*I*c) + 12748790059454570094944575356891183306
 355672515080113302539504189301580044931131951492677371440091923947401124164
 *I*a*e^(427/2*I*c) + 594881887451755154647732399814599054530375204610055390
 6871359525091394365290552015395275569402480394903253981466*I*a*e<sup>(425/2*I*c
)</sup> + 27446479221039674506653515289124453385084317345329773392589243546324184
 13755714066126762594816764534577943660024*I*a*e^(423/2*I*c) + 1252025659809
 341407225419006492626474816512541093036664002973216176119522637183550660057
 342417535420038395747756*I*a*e^(421/2*I*c) + 564659658143817076204432213944
 725285346043737598644238824183882546233010176533216691973768066147098418777
 561124*I*a*e^(419/2*I*c) + 25175792471045765298404315688734098777499765844
 068808649192255015133673742362290431699054972850328850024201000*I*a*e<sup>(417/
 2*I*c)</sup> + 110962940683089939119784926809098350196267205293097933934506361993
 436671367092877439898462338113225623209168660*I*a*e^(415/2*I*c) + 483444222
 719571672115283780376316317858047715234526783830382249238165252080018836178
 90956028571517412300257380*I*a*e^(413/2*I*c) + 2081916009657441298952351526
 902094940934436381403124863224816590166089265933322581676907490783064713110
 6973820*I*a*e^(411/2*I*c) + 88614103628935485133785436616669229792128791625
 46855368943021122572517216952715322579460798697748541783361640*I*a*e<sup>(409/2
 *I*c)</sup> + 3727685690743909795581760930725419308750587813165730468793831046647
 131796121280952242102308634280498612910500*I*a*e^(407/2*I*c) + 154969439176
 876112859296706362143095821677931109206307000795601115272064011371678113908
 3032984676774287045220*I*a*e^(405/2*I*c) + 63664343006830970779682307871047
 283430709842994913413062564401278355421062181433392867579320727100311006117
 0*I*a*e^(403/2*I*c) + 25844164954447508105108617067262460235318095770556449
 4685767284029136308908583837797261793042111767829441440*I*a*e^(401/2*I*c) +
 10366136343863029786110851000214496639470958282076427097998237731443762220
 5754841630360152661302764514627060*I*a*e^(399/2*I*c) + 41080064709877283734
 887584286870615187909186938494335754863409747746758750561170745510692753019
 453256230130*I*a*e^(397/2*I*c) + 160833386284501320359204879956240825255167
 14816814575817900515648576615466807762107000685827676876615047090*I*a*e<sup>(39
 5/2*I*c)</sup> + 6220463245033442486970965431823733752822029473563015032522697711
 385753292200817503166820643981171972227740*I*a*e^(393/2*I*c) + 237651562626
 525439152900401728989412156753196374638996570170926255037566888287901191239
 3122776372223417220*I*a*e^(391/2*I*c) + 89680795103511447814058492212920166
 6414256892054024424779321295278270156631852705762871914272291903841570*I*a*
 e^(389/2*I*c) + 33424726352953278678796796394401815339089529908062565016262
 5033301387415089670209519217032467321603326080*I*a*e^(387/2*I*c) + 12303086

815692272305679075836631831047214060472197947425229713527282252993132636810
8994384850189269222320*I*a*e^(385/2*I*c) + 44720399622948196823873997189864
500160905696730195693098411725777718941421992979705222237696090447011160*I*
a*e^(383/2*I*c) + 160512736936774650149178398393766191845774391311876711661
39084979089858430096894352960115226308965770760*I*a*e^(381/2*I*c) + 5688421
312395675282558972732461105838586797834128682182058119235866037766500737528
636435864135629276280*I*a*e^(379/2*I*c) + 199029717646465954833239130033434
7842069908578423832679534203876367723433092440766099450922294077090680*I*a*
e^(377/2*I*c) + 68746845182880446138347516110722617254137847246934010854845
3628584356757564117582602376376619756273900*I*a*e^(375/2*I*c) + 23440145177
183638490588541151419900277989260986134788228744397382430271359037999413957
1196743269525200*I*a*e^(373/2*I*c) + 78886688535693017729763723869969608729
578477562178100186251707803654595227995531245155841610652335370*I*a*e^(371/
2*I*c) + 262026661413661375338099973457430738047247625045775712054342620577
86505982168778594588998843843064160*I*a*e^(369/2*I*c) + 8589090245938569057
205496894537381121736556805993399422069994412586113650715896554879518565545
033500*I*a*e^(367/2*I*c) + 277823843403015038991200428997889236893953745348
6552521474417159987795798634573042959267684684956330*I*a*e^(365/2*I*c) + 88
669118294726958970476865059787128995449404848479690905773574060752253415308
6994088857099910307490*I*a*e^(363/2*I*c) + 27919956618760773479939843263385
5170873911810421163724268010453094050454030082867477501063799585760*I*a*e^(
361/2*I*c) + 86727049291498131194800264955595551093397528897054551562337232
229710003919599925462196648416340420*I*a*e^(359/2*I*c) + 265735099311760349
618670131544115731745198269463306465727402756230582911184472753753824374476
50250*I*a*e^(357/2*I*c) + 8030707825911039750901750056608809314825712783262
646850788620295561719866426138714811018171607190*I*a*e^(355/2*I*c) + 239344
956931123638090578121707109045258427789993815442329626445069531132916073064
4154303696630780*I*a*e^(353/2*I*c) + 70341919587773492725310001749344921533
0761703688880548654605473075438085070856781449196376998400*I*a*e^(351/2*I*c
) + 20383391724947901034896921116234923504212194691197220886731858105412239
4696504660030320280130750*I*a*e^(349/2*I*c) + 58232128415139821241222373924
893979763195025859874710580258166074043709621951021510992110247670*I*a*e^(3
47/2*I*c) + 163992260136891349729894299620340638416069451379678962703167909
95858119979554550799257281805220*I*a*e^(345/2*I*c) + 4552057982199619540279
669891611168972664512469922273304350295630649068449947661654859851520640*I*
a*e^(343/2*I*c) + 124527128456591689098616311211092805496806563010157154297
6439354848212020139249744172386191190*I*a*e^(341/2*I*c) + 33569016274523440
689722952306357800187992714315613796580609367280150561989873665145003393808
0*I*a*e^(339/2*I*c) + 89161435402471574683310567083005500079794884684361421
770824844212210441358440351257958080020*I*a*e^(337/2*I*c) + 233304558129349
191097882697145464428390837228147809505297967644587828053363202905160927830
80*I*a*e^(335/2*I*c) + 6013393403514090825503439756338023248243113472621004
894860736435704687187297040948536876280*I*a*e^(333/2*I*c) + 152653655744022
772632359258630324792448641870940805391309744522667013779988145370988028836
0*I*a*e^(331/2*I*c) + 38161498256504938298548772399547510632592125444794475

8076508528236388846570076902279935560*I*a*e^(329/2*I*c) + 93931668465861097
 295767669397193978348122793779758072033532762411099355052744964746076720*I*
 a*e^(327/2*I*c) + 227616207014150998824942862057363907088639144103903695744
 31103252189239994156364638800240*I*a*e^(325/2*I*c) + 5429167275423972414370
 118138151426061135560961696250085913601894114770264832855561732510*I*a*e^(3
 23/2*I*c) + 127448496163639347102604757977951048480001088144145292270612281
 1798463818843113785401980*I*a*e^(321/2*I*c) + 29440035771426987261240090861
 3310277911423524064416075849263980596572460467893101551780*I*a*e^(319/2*I*c
) + 66907190283119388459517143574812431686022847054878088870420694345689435
 187958917420750*I*a*e^(317/2*I*c) + 149576983980833836478728363167385041163
 57444050461844622220215541229121889145127946510*I*a*e^(315/2*I*c) + 3288810
 369087482330767112497943810956913958488942225831844466822949439789604958354
 380*I*a*e^(313/2*I*c) + 711078144382149111952346120629500861576171892034190
 598453444253505004747647977121920*I*a*e^(311/2*I*c) + 151154235563977220379
 337165749623108778682727639486653007034770480329113314879178190*I*a*e^(309/
 2*I*c) + 315838790301413954589785078831270108407496056038527631443724669058
 13381411392911500*I*a*e^(307/2*I*c) + 6485865431691803112160646709240463402
 571992800370597824291437072623168391271067612*I*a*e^(305/2*I*c) + 130870052
 7302742603332830336012830403130678747327429168347765689867244859352037688*I
 *a*e^(303/2*I*c) + 25941335757606125914021211615794333861772719757999942694
 5148843639870850445567092*I*a*e^(301/2*I*c) + 50504618901563664225694989020
 362043666401757813261032636384435135020270858362828*I*a*e^(299/2*I*c) + 965
 518472954734452614517906025929335216122691652350610206580605701146897250097
 2*I*a*e^(297/2*I*c) + 18120981235520913412285931908637755228208408122955628
 14006136853589089587942200*I*a*e^(295/2*I*c) + 3338044484181235440743117217
 43628896421299470604551695682547055334526562374348*I*a*e^(293/2*I*c) + 6033
 7325392389346145064651977945597823054838103742220792252383582775469810852*I
 *a*e^(291/2*I*c) + 10699270750568860036215311048748328462315940410814407912
 532812583183629052968*I*a*e^(289/2*I*c) + 186073015333228452816690112663564
 4029479060130343273430382098603478677010682*I*a*e^(287/2*I*c) + 31729021076
 3646745231429975366933483435752750035349554674608214217860330588*I*a*e^(285
 /2*I*c) + 53033810408194302213396652823533471045170883965946504040578005764
 694599600*I*a*e^(283/2*I*c) + 868653017859292125478617485640182306852250101
 1656089237471028876240107552*I*a*e^(281/2*I*c) + 13938211445475254340803901
 53712822738926746059871372659758491954013233298*I*a*e^(279/2*I*c) + 2190281
 90068671998320108302544300443107574826644022160996792142204598852*I*a*e^(27
 7/2*I*c) + 3369652871162467972977990009382872460065922983659119513319988804
 6174281*I*a*e^(275/2*I*c) + 50736095642605887109164530167025640325718296185
 50956017565632914253964*I*a*e^(273/2*I*c) + 7473852214293200139663566711060
 61647673665642790216192401997745384178*I*a*e^(271/2*I*c) + 1076738842851268
 17246373518618741221711008376494758493794059082539665*I*a*e^(269/2*I*c) + 1
 5165303771430260819236270425814926625403183922898905657641898587425*I*a*e^(
 267/2*I*c) + 20873553431310800483666932530451017660181981017813066706220177
 62198*I*a*e^(265/2*I*c) + 2806523604644513971683714343040105858307712417439
 56318553252853554*I*a*e^(263/2*I*c) + 3684536883691245223855750548794910894

6286792376090329160029006801*I*a*e^(261/2*I*c) + 47211276982847511798865587
 53661206453110344569457416745986959922*I*a*e^(259/2*I*c) + 5901403178367834
 51582400841140547729023595019680165231818378300*I*a*e^(257/2*I*c) + 7192839
 1497764277134489912202810340992721249592546121668225432*I*a*e^(255/2*I*c) +
 8543973172918851719756469222929408444037078724153454872120390*I*a*e^(253/2
 *I*c) + 988558176286135607113625950318361683964799348363993276036638*I*a*e^
 (251/2*I*c) + 111348517556517214393482430347520024901285561594013324999952*
 I*a*e^(249/2*I*c) + 1220257084693626697153761559909100395266739507352891270
 8550*I*a*e^(247/2*I*c) + 13002733575778782356308899839708718988433842410829
 17039402*I*a*e^(245/2*I*c) + 1346331570601825069666545683305221522471183987
 56303435361*I*a*e^(243/2*I*c) + 1353648149484655119161529698375387133690010
 8337339193172*I*a*e^(241/2*I*c) + 13206319777820792138551258325108244381849
 06553225344870*I*a*e^(239/2*I*c) + 1249246176482496308547568203279235126359
 52856544140745*I*a*e^(237/2*I*c) + 1144861503379709196383067840727158198722
 0664774793965*I*a*e^(235/2*I*c) + 10156027827667569616065111955295064346923
 26161772148*I*a*e^(233/2*I*c) + 8712944961972016777473838211417420612533807
 9715514*I*a*e^(231/2*I*c) + 72219589337910633543382527582412609270099586491
 53*I*a*e^(229/2*I*c) + 577756662739414343075056408130898651215560673415*I*a
 *e^(227/2*I*c) + 44561015944582807131288351508188541688368025238*I*a*e^(225
 /2*I*c) + 3309571280975022580464977398526908985926399572*I*a*e^(223/2*I*c)
 + 236397937497882482710239205369322834941776795*I*a*e^(221/2*I*c) + 1621727
 2150903922938701499630023289700137263*I*a*e^(219/2*I*c) + 10669257680745512
 95828818005563441210737450*I*a*e^(217/2*I*c) + 6720792086441437726967130810
 7198146913908*I*a*e^(215/2*I*c) + 4046550138136102762173994148249330760039*
 I*a*e^(213/2*I*c) + 232438908235205496318447910669140546906*I*a*e^(211/2*I*
 c) + 12711502667519783895793526074473028922*I*a*e^(209/2*I*c) + 66033779606
 6223079759666054299242200*I*a*e^(207/2*I*c) + 32503673727084640393745736284
 624870*I*a*e^(205/2*I*c) + 1511798772310987071825034633753982*I*a*e^(203/2*
 I*c) + 66238605830373986929763608958656*I*a*e^(201/2*I*c) + 272446707305226
 1042519716192524*I*a*e^(199/2*I*c) + 104787194996399169411032876338*I*a*e^(
 197/2*I*c) + 3751971173605702746328148447*I*a*e^(195/2*I*c) + 1244276153963
 55739905346318*I*a*e^(193/2*I*c) + 3799316499692185181122010*I*a*e^(191/2*I
 *c) + 106072288048110242178847*I*a*e^(189/2*I*c) + 2685374380786193144783*I
 *a*e^(187/2*I*c) + 61031235925186078942*I*a*e^(185/2*I*c) + 122984858286212
 2028*I*a*e^(183/2*I*c) + 21630502713590407*I*a*e^(181/2*I*c) + 325270717497
 440*I*a*e^(179/2*I*c) + 4065883968718*I*a*e^(177/2*I*c) + 40557446072*I*a*e
 ^((175/2*I*c) + 302667508*I*a*e^(173/2*I*c) + 1502072*I*a*e^(171/2*I*c) + 37
 18*I*a*e^(169/2*I*c))/(e^(531*I*c) + 432*e^(530*I*c) + 93096*e^(529*I*c) +
 13343760*e^(528*I*c) + 1431118260*e^(527*I*c) + 122503723056*e^(526*I*c) +
 8718181624155*e^(525*I*c) + 530563624556832*e^(524*I*c) + 28186192554792138
 *e^(523*I*c) + 1327882849274858880*e^(522*I*c) + 56169444526926562260*e^(52
 1*I*c) + 2154864144781257856128*e^(520*I*c) + 75599817092670157806639*e^(51
 9*I*c) + 2442455629894502983849104*e^(518*I*c) + 73099207817335597247098038
 *e^(517*I*c) + 2037031259470368160131922320*e^(516*I*c) + 53090127264630963
 470039804475*e^(515*I*c) + 1299146645993240318167826532288*e^(514*I*c) + 29

952547749265499675257842032197*e^(513*I*c) + 652650253343206047453620559993
 840*e^(512*I*c) + 13477227799524701956579274210395326*e^(511*I*c) + 2644103
 75780310742518099326419685040*e^(510*I*c) + 4939666610818025798809586352543
 471345*e^(509*I*c) + 88054927598941411145869950813388040256*e^(508*I*c) + 1
 500602747937397286405577818722691539392*e^(507*I*c) + 244898373378123386877
 18622491865013839488*e^(506*I*c) + 3833601558010548245297646882131143680471
 54*e^(505*I*c) + 5764601046563151304213854710715346838447392*e^(504*I*c) +
 83380839911837894453136303673785039051506805*e^(503*I*c) + 1161581413733971
 751533622511909046917188768400*e^(502*I*c) + 156039112776876070997216237717
 44933086920587272*e^(501*I*c) + 2023475097244621713139666435802340785081798
 38320*e^(500*I*c) + 2535667460650279776834561566186591213109251642859*e^(49
 9*I*c) + 30735366512830562160991166338490057308062762518496*e^(498*I*c) + 3
 60688613036389349413809780004559963548775423325255*e^(497*I*c) + 4101545439
 937195793959956708442496709433800261224880*e^(496*I*c) + 452309400398307383
 32025694784646206844854827698075736*e^(495*I*c) + 4840934102404887186559170
 25303662581091659126182344528*e^(494*I*c) + 5032024903401451824074213943766
 011922026507006311982753*e^(493*I*c) + 508363695081710994370193486108473919
 46736185108017183136*e^(492*I*c) + 4994675065585317336715858629105727028115
 45035730398749530*e^(491*I*c) + 4775398607100853263534207733818266777478693
 412738731031680*e^(490*I*c) + 444567081752588210244009462105350045237757221
 90977468484496*e^(489*I*c) + 4032122259577981888408461399609956241444912716
 94336796459584*e^(488*I*c) + 3564764890628724017088487996688178929195787613
 958545474804845*e^(487*I*c) + 307362174043210099652310374196630539628810352
 81709221697785072*e^(486*I*c) + 2585853487159772701558291156841934110720345
 41491364393985491350*e^(485*I*c) + 2123702969188871318266718781223927067839
 949015727293884065388080*e^(484*I*c) + 170338860273906157410409777216555416
 65612162275485028584310890417*e^(483*I*c) + 1334902100520261837796733138683
 32303530332906163247194627808410304*e^(482*I*c) + 1022536437468296737293065
 862705246449693687415559865844306888705423*e^(481*I*c) + 765901052018754965
 1777118357676871927081898989131125755798204236112*e^(480*I*c) + 56117081076
 341175384087570185188538660375932013674735519055227368366*e^(479*I*c) + 402
 349692266121158934003582839428785116904903936409545602519219664720*e^(478*I
 *c) + 282390515193658667838252570656445728029009869863859798762838024588171
 5*e^(477*I*c) + 19407979215594566593535008103303255257745408070082431338945
 184797463936*e^(476*I*c) + 130657660226560419335121434389938961884595434069
 984824307149332131747540*e^(475*I*c) + 861884851094991908764246805474672428
 603757315484453974713612812215428992*e^(474*I*c) + 557255115732867112101621
 6416307596161861955969011697222340926210112854418*e^(473*I*c) + 35324447206
 779018115378052820789411687581004582367431006205879633729015200*e^(472*I*c)
 + 219601281339515561500261478844190024870555261281946058839614044697037963
 695*e^(471*I*c) + 133921437425424555356488440680194535338500025403065576595
 3770237607180089968*e^(470*I*c) + 80137295807907524343619649457615437614695
 20791210746972675870481058674277844*e^(469*I*c) + 4706504461113515810848735
 3367484243102698248838312635876283099427442745866704*e^(468*I*c) + 27136120
 7503266570734486517077181014801775322183181055638619257836143271472358*e^(4

$67 * I * c) + 15363332384449275835327345560164946716749165789071169845484890782$
 $41693926940560 * e^{(466 * I * c)} + 8543013441126212334833540665069621472479085838$
 $041360564550722036723654297540205 * e^{(465 * I * c)} + 466682235482660178068545924$
 $68100570289355960869613650856575756758180182223308768 * e^{(464 * I * c)} + 2505010$
 $28608928332469340456829902067712233644464602753159945727868485722395506952 * e^{(463 * I * c)}$
 $+ 1321498055271300851429993866631619874424534425188183592049727$
 $687571032156435077280 * e^{(462 * I * c)} + 685299322314573668732888531161779543559$
 $2940841439866351079655652312894721972796266 * e^{(461 * I * c)} + 34941071613276704$
 $649477943043339450201504075335160361865916029213860778606230624960 * e^{(460 * I * c)}$
 $+ 175193170500618300241515632381912285157790097816049220671217212220015$
 $297133400636060 * e^{(459 * I * c)} + 863979933622330349556296820028395513198708064$
 $940505702126068652936800794826651264256 * e^{(458 * I * c)} + 419154250065682614809$
 $3339414544159143964478472492315931809171859902114109005939942952 * e^{(457 * I * c)}$
 $+ 20008006803030047137293278250321597113540716201983333126349281186679153$
 $199068045257216 * e^{(456 * I * c)} + 939869153130681791490836060656814827808360605$
 $10530154618486949839467131378859885998210 * e^{(455 * I * c)} + 4345466767802800453$
 $46344498763892540797175105756827515509297024187660299345484920192480 * e^{(454 * I * c)}$
 $+ 1977792980665818135651300094326239158605448870806970860577325385028$
 $609983034534672318500 * e^{(453 * I * c)} + 886275214275695728568134088576490459793$
 $5349569355321815647721172537159186491471311666400 * e^{(452 * I * c)} + 39108031255$
 $601809476537535369611844440844903751605645023514572352045248104262933598850$
 $730 * e^{(451 * I * c)} + 169956327969929767773902096652629253283704505477127544556$
 $534417376686540936706073847337600 * e^{(450 * I * c)} + 727521010718394229291774073$
 $844694255798738667067535379759732795567942578751384250780476310 * e^{(449 * I * c)}$
 $+ 306797429643174736419815962396246367161700641962685142614841860293485290$
 $7379021659761911840 * e^{(448 * I * c)} + 12747219616503320541356343062562847368601$
 $622140856786025445814532037904111523242298235713300 * e^{(447 * I * c)} + 521909122$
 $076618242158122718542697480712928432432278947692296907200105471413341316109$
 $89636000 * e^{(446 * I * c)} + 2105943013856484711843290788803175049533618399541594$
 $27434009884661777259752542647709150036990 * e^{(445 * I * c)} + 8375792069234119324$
 $58786486765373533946545239708990769488724813982189165104589895518909256320 * e^{(444 * I * c)}$
 $+ 3283874760555818676726309480306734420155098583948074469014168$
 $171874442170109648521627538755920 * e^{(443 * I * c)} + 126934969329649205650736736$
 $37181280088548682508880255337280065006566138696041797353216584528640 * e^{(442 * I * c)}$
 $+ 4837948975643409984385779181658937940681504260934037874758643714578$
 $1646245422045101230417309900 * e^{(441 * I * c)} + 18183466140617790131533012967714$
 $5381166449188413194141169344354754920969034952610378945282257600 * e^{(440 * I * c)}$
 $+ 67402553054313300889484577523662523745074311447354453781817044713460710$
 $2575676676056675328961590 * e^{(439 * I * c)} + 24643821908074396090797742268556796$
 $29367885709776435876630851716253962696192341706239192878728160 * e^{(438 * I * c)}$
 $+ 8888295028751024667044203837607976101480053134418614474620767522824868911$
 $959884352666444917404000 * e^{(437 * I * c)} + 316266446747255477311767956875276535$
 $71305969985923688392112164915553242573269490908989570248533280 * e^{(436 * I * c)}$
 $+ 1110341487970088194431438956444692422950498674643137109692576193388991337$
 $99285616020069872611710850 * e^{(435 * I * c)} + 3846558420806662744540630787848371$

74998949052500975322162003392549953413592461519365177908682078400*e^(434*I*c) + 1315052120930692122102297105327622842335870743428530891072983535862280
 094446607723473800477453914130*e^(433*I*c) + 443721091784318234776434954444
 3904699020056595069470847193617092114714077633077234972825351226979360*e^(4
 32*I*c) + 14777955096617128998712745182071495362176506973183081650233605274
 051677624970464340242755840025673760*e^(431*I*c) + 485842581531402804473148
 368687721313901954124190467327784587060150968814370763379107935841224750737
 60*e^(430*I*c) + 1576858455288509187214628778644350902575831494155613234273
 86562894447598277935629800939237175625149830*e^(429*I*c) + 5052936631230152
 588784830257388128132033977668453400653812610163534197223826203930325359606
 60921950400*e^(428*I*c) + 1598771101058192692270528999677444742685631006232
 456185844925220144002305878120380828483988663574829100*e^(427*I*c) + 499524
 195627913818020518674440168802438827211392125566373495694692757130553314677
 6898787878059685108480*e^(426*I*c) + 15413111211486023937294970820797376716
 081344788163386543522421939737507962125854981881879168348260330000*e^(425*I
 *c) + 469702247271172818264540450180706705225597566275803477845353200149634
 82632359729444541885102274546002560*e^(424*I*c) + 1413799382535568432805655
 058074033041306061307254347517457940798331413617489176399861453770664372105
 46190*e^(423*I*c) + 4203580248351467985836112101459421546844379493656478990
 88372524802156222884839580011688655664280691773600*e^(422*I*c) + 1234668041
 892409978780018081755440216012582476396941937965899631953079203974222138794
 604328498972144766900*e^(421*I*c) + 358271800216329606141453670371510989710
 7198252739284546149343102348456124089657428594946438660859773886240*e^(420*
 I*c) + 10271602530202889002497813516849452590971512809529060665197301097052
 210064576088348023234671975463677418470*e^(419*I*c) + 290976510612474534066
 475697818369100621655598523590528042591541656871254287525623854923737494863
 51714453120*e^(418*I*c) + 8145208141382911182887541756425005484603769331281
 1480492909160758195989155768107022568350953861815940704090*e^(417*I*c) + 22
 532053259322065776794110928951624899979452101556413498282724171001967548669
 4499689312466561907212627820000*e^(416*I*c) + 61600311602297958493712570175
 788721299835430098999136262803886109391456133207119190971494942658793691030
 3300*e^(415*I*c) + 16644750343872118093949177435029376389785749377547647639
 87835872410449930690131572904279995484581013965001440*e^(414*I*c) + 4445412
 259295474625067659514198312966015416299968930393345630345914109720740573618
 884980520010028451496996210*e^(413*I*c) + 117358569262451184931130910025016
 040323418769859990828235206722415302001882238263929823021940846675384886656
 00*e^(412*I*c) + 3062758105422195737839054728927760912957281393108273352024
 7387226000020043538279468776707958420892547870128680*e^(411*I*c) + 79019149
 558766569254783988487232388352909144982747171856772463223808993367091503402
 876467270176124342699654400*e^(410*I*c) + 201557947424794098026772478462040
 883395867512562930862943753568690084015585598010154781548625239409581907397
 500*e^(409*I*c) + 508324599301085460166978629683032661427654474082939048097
 939638391567298795788389433842285751054665210868287680*e^(408*I*c) + 126759
 701729481291340014627604212692998648029287019039910755431107996422728019652
 2475370108738477856311765699610*e^(407*I*c) + 31256834931787017434797047503

074901786662921507201793636043351135286233296846063431855407560199356621482
67863968*e^(406*I*c) + 7621788791912047062038840917799374600428892258194367
636682944356096681400246312138001769285020661445991073249416*e^(405*I*c) +
183798070840033597660276492176211441160917355722166207888615358034497022738
02588359076704241840733513439114113248*e^(404*I*c) + 4383497214291937768537
869223302106374455403310092850273748043897897674698989578407095190523778349
0374305934542955*e^(403*I*c) + 10339975546725743648984783764075375471820439
4473055795001467604326419876555556873829531737211096115196005647730480*e^(4
02*I*c) + 24124602128244006179290831778303287619480159713320605209128699704
3729145345755805710081489006741839439573984832678*e^(401*I*c) + 55675638871
118234034102619273421954611365176831738053900589367904939471401706369856527
2728813669054779077208977840*e^(400*I*c) + 12710330829380489502013605548312
703426623439912775046123423003660250467417428565804452894017866563116858590
23084716*e^(399*I*c) + 2870496131412314451834674715353589439553294430808531
933466086288543709246230769151392180699413405623017247753532944*e^(398*I*c)
+ 641338189585592518475823145106255638032859493851100657701521811978653639
0213057284018202201631094434819584025113465*e^(397*I*c) + 14176483652875704
95720201334324111790436997796653849959902524421980635189011634815653279605
497783382888932766730080*e^(396*I*c) + 310043192060694170770693631414234874
318280090981847446356786522841774394649416518125645191449180031741080776348
46014*e^(395*I*c) + 6709170613052966912501989921002157658023784346222953538
6295087076189297849995931360645605292130961496106707521506432*e^(394*I*c) +
14365768713804479694294711970425953845881819942351682467458629369105611920
9866358123637772245409530799230553767222252*e^(393*I*c) + 30438447110681333
601028416012390637043388882849062742265255123696679091617452085775914393014
0187173492394981908258944*e^(392*I*c) + 63821889291453374150580639966248855
606678360049609187640837497487744897177803607499624558112428346043806518207
1976085*e^(391*I*c) + 13243113249840274283552229381476823786728607088171617
41448749689593588020860847508703702325320304649883120684987556400*e^(390*I*
c) + 2719589283483743926040805101080341921244530311254607250929192773909331
523226635035815672862569296693711643521070331394*e^(389*I*c) + 552749884903
117835586123000932666828392629008215846711800069850271937993904591834422219
2742145711257028040974074674736*e^(388*I*c) + 11119499645363201080881062824
886338492425375658448977935535846349290425821570383090425411418521516670371
372045206568345*e^(387*I*c) + 221407350017086032709151807692413916620355787
55903979148909213603822554792749183517160255571915875356439553717130797888*
e^(386*I*c) + 4363830007581517102594621146446568961896577348866098585794565
7479854085108851857911222911989837615452608512356008400295*e^(385*I*c) + 85
139533234786455779589959464900637760735729705621221380005837208369157794673
049675428799817875431430246332625899630160*e^(384*I*c) + 164437500676906892
741323260154394278503954561936020133596581806449357240277349447927334517105
809995300093549279931273178*e^(383*I*c) + 314409035808225861565595436938354
945445473991043129722046747228813030925204968503418566818838611866709807040
793495364496*e^(382*I*c) + 595157615500431514947479282336547053827926087916
425263497187757029413385471835434198246807096214536895441388306027237899*e^

(381*I*c) + 111539828554560155053332804560018431799389935921799672981834070
4221801195667410485846996179056733558512452238583160792512*e^(380*I*c) + 20
696982895008606434616657623738079575130194240411788719049605518294124493447
22432125679417958403007551179298315947373776*e^(379*I*c) + 3802604996705891
106964620633848964807037098854510182263243030597295630760353597531974324752
266389193185760878274188013440*e^(378*I*c) + 691783894521484427849333045936
139492337233385361987963737267318494285971243106634572687042209989312489077
7678037369988150*e^(377*I*c) + 12462140440537258084928596709872066857757070
943124868554500948154756863454308032925408340311237850017814707896986969086
816*e^(376*I*c) + 222313411318015353454063990377216868402083979419525801355
84645966746736656716271554826476282991066076564921432614339399735*e^(375*I*
c) + 3927420041432986116939794451622500108122743339858500720639923121190715
7795359719648241598754266579840244551491476467899952*e^(374*I*c) + 68712466
015985641512468586173659747734879591710098354652786124936023073943141049573
606648563005359411712764895683903806088*e^(373*I*c) + 119060591849660546834
765693227676449067584148248882678447504826077236333444513454095126668750057
295811191643356908972191440*e^(372*I*c) + 204325557265186000767402710230847
896459761583922763698235433212833313077783041040074669379017394836761539649
081690630811665*e^(371*I*c) + 347310053810935290419455560555957314129569210
735745983234369659976413374774078000173070075248654524917179128950507443058
208*e^(370*I*c) + 584749573682304586179384628844883327581498969886540380378
896767999075614964007174600811092945356635118795824799369716742109*e^(369*I
*c) + 975210339444049318757282311763517786673223175594457946383279264635085
041004917300295904275433144848532459919875479817581584*e^(368*I*c) + 161109
254140006052595485937526419417834764347183707820144626243561514294458733783
3513586022729849523358436493586042252995608*e^(367*I*c) + 26366624104307993
404475222847782442837407510686581407265764466712077983256068322959377050616
86297930296382338892574900819440*e^(366*I*c) + 4274826907720591752526711336
820871500844345647922385471534359333606189571832444641364132893108663576205
133870672156264164115*e^(365*I*c) + 686642533751866826266269375090895696573
292457814218163062215780289988087468155103113631406419994860400152989456623
5238597088*e^(364*I*c) + 10927210603473544810279792347844536074588896806230
041110089544731605863146104181739039426674855453466097402330688331845602302
*e^(363*I*c) + 172295028243676473344007219984175967039486573947388052093916
36597370380572398964715080095366818322029152193635869784095333760*e^(362*I*
c) + 2691779470108661509789012023689050110514679999602177519571086622622863
8984703456832694153230611607263444183501026198563419616*e^(361*I*c) + 41670
440375390543643418219342271748040035071490119080585281522498188818375906900
368701234531304633163446319945130196476913600*e^(360*I*c) + 639230194337619
890906148012886350981231994451023031226165446482089987678039444557770428865
52738499747183713136069104651812215*e^(359*I*c) + 9717305502474268005861672
246136889266114129554026349301303274608353615732426833339040030895831837021
9154887169702257444756176*e^(358*I*c) + 14639044845635118121823738274037412
419166481999774698807659839186273362967014224154637553390313060529758010567
5355629160198162*e^(357*I*c) + 21856316665964931224748364095627214921249911

538382877102965428336397258511809047941369663810815638524464659132845442574
5117584*e^(356*I*c) + 32341317801484100371415113824607915257636097603505845
7890409937738723171537036573043681997163745313602400139153046673668433091*e
(355*I*c) + 47432304356310054237733862993196624812917597698233244601805600
9391154020438895903140822967769494019446166779954024655344116288*e^(354*I*c
) + 68951844932879355990326041814997419025357834005889503558960646824468059
1556118170304005037563669880057908765898949268614772285*e^(353*I*c) + 99355
565364952112722644396082023364938648851008189254586670044409666158279044124
1830855609577062039555625090943332264901780720*e^(352*I*c) + 14191644814221
76573858234013898999628822322333095737307163107438389358322014542936172931
75086458389621425307051750612129761498*e^(351*I*c) + 2009496110092687738152
782085683737222727968824299058739215446083499351467625334449670757764066690
656150949014944043994822823920*e^(350*I*c) + 282081929856221595910752980728
962844962138679898943636939311606989401878120100027563310449839895934663179
5568022519974400130281*e^(349*I*c) + 39256976584157783527681039428560118402
116427696217172179966143984738871860743914826385472128265382704539126345402
99792270321024*e^(348*I*c) + 5416662804052436349585595982818357953866258461
644354018205158917742576425344364964596750653177677803492186817305171175032
011500*e^(347*I*c) + 741037261289115222436463329612804397165719328032775430
4382235672773781403023814127610355562505271969045177704726054907145784960*e
(346*I*c) + 10052209524369581827588154985345549678031445744499985208259385
435609740272400301454246872041775159838468077381562338745636398374*e^(345*I
*c) + 135212304119454369155588547068517965415673998116568700215673529753268
15467817846533289123871696056195231696146162720992221760992*e^(344*I*c) + 1
803532733817745547117756859485168297797834644977719357208768851039242688451
9272991560851326393852241961470040819793627127923997*e^(343*I*c) + 23856398
565562802030695278174212640833282154174006459458292644060947924907313735921
561690153939906017518647182491616573724049744*e^(342*I*c) + 312952636881898
383137727758733072603341172272586295019923586956360926628660628198456890642
35813622974150120668921391878398978380*e^(341*I*c) + 4071598896370191895002
034833673642342051331135901048524691907465288339397080537447083015622970564
7312265477584256027212762941040*e^(340*I*c) + 52539223346740771142587092370
257069536060319644439501667610482767955800276052892432152798814607975110366
224945081428121888473324*e^(339*I*c) + 672440879690807038237032571996630476
068906104820904946194928029351302159798194699663833367886939001391155946468
93784095418472336*e^(338*I*c) + 8536811843021531284823129173967373588774620
185166629960039219941876475008682819871987274404776778366732532628922188197
4987582215*e^(337*I*c) + 10750473740657691612348039916975963332832140741940
051001749884983059862156542826654631593392082152754472638020165911490383460
5888*e^(336*I*c) + 13429774202347942990462961610455961009607475872106802270
4468938063017059688023363436458971534964665036319889119229809973806909680*e
(335*I*c) + 16643233292258919513055832926639875338982395573759852755609609
3062473559769545772321978969318904192572733997888230986469005970880*e^(334*
I*c) + 20462229553572910951982991678986722531942970516256008264884039496542
3112809336591921290309392396263834674368977840527147037426908*e^(333*I*c) +

24959307228256586639838995150961920268263445548712871463146189177082320136
 7527645793770203788784677343934971424317987895255031936*e^(332*I*c) + 30206
 063803086846346113944227936049971890691748252489448335622019613837705082891
 1383056860425370161157201493696073712322595776808*e^(331*I*c) + 36270630756
 384323113569915741851073245242061401362416887931218764523345015392797579332
 6834780741391203430153093712635355523960320*e^(330*I*c) + 43214785646408693
 802381156180867858959470204790467428229795965880017098445679906775187804480
 6619012452636891350731618278545690160*e^(329*I*c) + 51090761511113450745262
 384614713714145027572244431638553164842923085168663582771748846450033162338
 5777400744950538410637735936000*e^(328*I*c) + 59937848477173347480937614240
 155485020706497211813757294925750365144454193930902527689604962251563026316
 2184526394317285457368300*e^(327*I*c) + 69778910692592461481671374768468278
 508365981902795224444704335586974136850045256116402463607340192969310580110
 5522738405349028160*e^(326*I*c) + 80616967132762553242457534008977573368199
 499157667444692235409971461519208544324566385225700111564428666030497947602
 3966071898200*e^(325*I*c) + 92432005286752258403577749576107235122253442078
 486196000182102050946814635675643379524644649139614158385451368710433442956
 6707520*e^(324*I*c) + 10517821004288343719445081700512191871168163497669533
 226371821496108750042237847841832849619064944229554624312086456908025267707
 80*e^(323*I*c) + 1187817943079390316108802324798110129020822782660087248599
 367643481200206046822166144285425922229375413676535071141005286431481600*e^
 (322*I*c) + 133139611462672303580246212353158205033974999601415245283530595
 6367425370222758621753922458727524856072950880960657564720475838500*e^(321*
 I*c) + 14811871170892466624669556949656778555247303132605526908216026571762
 18737426245522795329891464091005878304304075953693546767206080*e^(320*I*c)
 + 1635569744641421900657886381289076653227580172056677587451402069234355283
 687489659613913761959140773339736014790081814516625224440*e^(319*I*c) + 179
 264907808963229893694572848196933496439159750628508848835062293725253342098
 0803144316431701452190522716124797875257437516360640*e^(318*I*c) + 19502865
 507801819192449929612044870100564603628452185016744237662663215587914369173
 17878702232679213868287926294665202769722927380*e^(317*I*c) + 2106141903468
 344307112549761202484543402794252352482199817410424869677262715098288437646
 518683487945462774223656471345899082156800*e^(316*I*c) + 225772621910385628
 681283301268157376549626224142061293207614315117196085455412414469902300984
 2080515157923529357189869943515991200*e^(315*I*c) + 24024645955696860861200
 018030342110567394455886219461413841061628862461618151497630250308348752340
 67267774023433418269982431265280*e^(314*I*c) + 2537766415465030330815471746
 692988596069911894697225052928320452542175587154848096483331209807430113943
 015398362669673337957755720*e^(313*I*c) + 266110064797578358382823513920144
 193017839664338342390358386254725588077238204920101553721490083274560151973
 7141849802506685264000*e^(312*I*c) + 27700732071507686455975072813820654979
 249684660545274141223398273337837700683058834873099793159837184037408728843
 45746380680204260*e^(311*I*c) + 2862503126320461797770667780725644184991255
 623174626175679050672100848988119391841466573417019247590580735265143427289
 340450811200*e^(310*I*c) + 293649421435186849870323945542677110434482730626

755891655087746723245515328614052108958273393220255313071272383698346886623
 0908800*e^(309*I*c) + 29904989496225436085381293802838663351900871151248588
 181437876191869571119037657239748996515185551449242903462425951672743830089
 60*e^(308*I*c) + 3023371643508225027175603175212953219022485045850731845307
 519008277385154731461213388035579159917590062343527464977286601165100620*e^
 (307*I*c) + 303440835553095707578773174532256798168461655016284547325767967
 4280216947356785783843205604202307836897073595410412575660465787520*e^(306*
 I*c) + 30233716435082250271756031752129532190224850458507318453075190082773
 85154731461213388035579159917590062343527464977286601165100620*e^(305*I*c)
 + 2990498949622543608538129380283866335190087115124858818143787619186957111
 903765723974899651518555144924290346242595167274383008960*e^(304*I*c) + 293
 649421435186849870323945542677110434482730626755891655087746723245515328614
 0521089582733932202553130712723836983468866230908800*e^(303*I*c) + 28625031
 263204617977706677807256441849912556231746261756790506721008489881193918414
 66573417019247590580735265143427289340450811200*e^(302*I*c) + 2770073207150
 768645597507281382065497924968466054527414122339827333783770068305883487309
 979315983718403740872884345746380680204260*e^(301*I*c) + 266110064797578358
 382823513920144193017839664338342390358386254725588077238204920101553721490
 0832745601519737141849802506685264000*e^(300*I*c) + 25377664154650303308154
 717466929885960699118946972250529283204525421755871548480964833312098074301
 13943015398362669673337957755720*e^(299*I*c) + 2402464595569686086120001803
 034211056739445588621946141384106162886246161815149763025030834875234067267
 774023433418269982431265280*e^(298*I*c) + 225772621910385628681283301268157
 376549626224142061293207614315117196085455412414469902300984208051515792352
 9357189869943515991200*e^(297*I*c) + 21061419034683443071125497612024845434
 027942523524821998174104248696772627150982884376465186834879454627742236564
 71345899082156800*e^(296*I*c) + 1950286550780181919244992961204487010056460
 362845218501674423766266321558791436917317878702232679213868287926294665202
 769722927380*e^(295*I*c) + 179264907808963229893694572848196933496439159750
 628508848835062293725253342098080314431643170145219052271612479787525743751
 6360640*e^(294*I*c) + 16355697446414219006578863812890766532275801720566775
 874514020692343552836874896596139137619591407733397360147900818145166252244
 40*e^(293*I*c) + 1481187117089246662466955694965677855524730313260552690821
 602657176218737426245522795329891464091005878304304075953693546767206080*e^
 (292*I*c) + 133139611462672303580246212353158205033974999601415245283530595
 6367425370222758621753922458727524856072950880960657564720475838500*e^(291*
 I*c) + 11878179430793903161088023247981101290208227826600872485993676434812
 0020604682216614428542592229375413676535071141005286431481600*e^(290*I*c)
 + 1051782100428834371944508170051219187116816349766953322637182149610875004
 223784784183284961906494422955462431208645690802526770780*e^(289*I*c) + 924
 320052867522584035777495761072351222534420784861960001821020509468146356756
 433795246446491396141583854513687104334429566707520*e^(288*I*c) + 806169671
 327625532424575340089775733681994991576674446922354099714615192085443245663
 852257001115644286660304979476023966071898200*e^(287*I*c) + 697789106925924
 614816713747684682785083659819027952244447043355869741368500452561164024636

073401929693105801105522738405349028160*e^(286*I*c) + 599378484771733474809
 376142401554850207064972118137572949257503651444541939309025276896049622515
 630263162184526394317285457368300*e^(285*I*c) + 510907615111134507452623846
 147137141450275722444316385531648429230851686635827717488464500331623385777
 400744950538410637735936000*e^(284*I*c) + 432147856464086938023811561808678
 589594702047904674282297959658800170984456799067751878044806619012452636891
 350731618278545690160*e^(283*I*c) + 362706307563843231135699157418510732452
 420614013624168879312187645233450153927975793326834780741391203430153093712
 635355523960320*e^(282*I*c) + 302060638030868463461139442279360499718906917
 482524894483356220196138377050828911383056860425370161157201493696073712322
 595776808*e^(281*I*c) + 249593072282565866398389951509619202682634455487128
 714631461891770823201367527645793770203788784677343934971424317987895255031
 936*e^(280*I*c) + 204622295535729109519829916789867225319429705162560082648
 840394965423112809336591921290309392396263834674368977840527147037426908*e^(
 279*I*c) + 166432332922589195130558329266398753389823955737598527556096093
 062473559769545772321978969318904192572733997888230986469005970880*e^(278*I
 *c) + 134297742023479429904629616104559610096074758721068022704468938063017
 059688023363436458971534964665036319889119229809973806909680*e^(277*I*c) +
 107504737406576916123480399169759633328321407419400510017498849830598621565
 428266546315933920821527544726380201659114903834605888*e^(276*I*c) + 853681
 184302153128482312917396737358877462018516662996003921994187647500868281987
 19872744047767783667325326289221881974987582215*e^(275*I*c) + 6724408796908
 070382370325719966304760689061048209049461949280293513021597981946996638333
 6788693900139115594646893784095418472336*e^(274*I*c) + 52539223346740771142
 587092370257069536060319644439501667610482767955800276052892432152798814607
 975110366224945081428121888473324*e^(273*I*c) + 407159889637019189500203483
 367364234205133113590104852469190746528833939708053744708301562297056473122
 65477584256027212762941040*e^(272*I*c) + 3129526368818983831377277587330726
 033411722725862950199235869563609266286606281984568906423581362297415012066
 8921391878398978380*e^(271*I*c) + 23856398565562802030695278174212640833282
 154174006459458292644060947924907313735921561690153939906017518647182491616
 573724049744*e^(270*I*c) + 180353273381774554711775685948516829779783464497
 771935720876885103924268845192729915608513263938522419614700408197936271279
 23997*e^(269*I*c) + 1352123041194543691555885470685179654156739981165687002
 1567352975326815467817846533289123871696056195231696146162720992221760992*e
 ^ (268*I*c) + 10052209524369581827588154985345549678031445744499985208259385
 435609740272400301454246872041775159838468077381562338745636398374*e^(267*I
 *c) + 741037261289115222436463329612804397165719328032775430438223567277378
 1403023814127610355562505271969045177704726054907145784960*e^(266*I*c) + 54
 166628040524363495855959828183579538662584616443540182051589177425764253443
 64964596750653177677803492186817305171175032011500*e^(265*I*c) + 3925697658
 415778352768103942856011840211642769621717217996614398473887186074391482638
 547212826538270453912634540299792270321024*e^(264*I*c) + 282081929856221595
 910752980728962844962138679898943636939311606989401878120100027563310449839
 8959346631795568022519974400130281*e^(263*I*c) + 20094961100926877381527820

856837372227279688242990587392154460834993514676253344496707577640666906561
50949014944043994822823920*e^(262*I*c) + 1419164481422176573858234013898999
62882232233309573730716310743838935832201454293617293175086458389621425307
051750612129761498*e^(261*I*c) + 993555653649521127226443960820233649386488
510081892545866700444096661582790441241830855609577062039555625090943332264
901780720*e^(260*I*c) + 689518449328793559903260418149974190253578340058895
035589606468244680591556118170304005037563669880057908765898949268614772285
*e^(259*I*c) + 474323043563100542377338629931966248129175976982332446018056
009391154020438895903140822967769494019446166779954024655344116288*e^(258*I
*c) + 323413178014841003714151138246079152576360976035058457890409937738723
171537036573043681997163745313602400139153046673668433091*e^(257*I*c) + 218
563166659649312247483640956272149212499115383828771029654283363972585118090
479413696638108156385244646591328454425745117584*e^(256*I*c) + 146390448456
351181218237382740374124191664819997746988076598391862733629670142241546375
533903130605297580105675355629160198162*e^(255*I*c) + 971730550247426800586
167224613688926611412955402634930130327460835361573242683333904003089583183
70219154887169702257444756176*e^(254*I*c) + 6392301943376198909061480128863
509812319944510230312261654464820899876780394445577704288655273849974718371
3136069104651812215*e^(253*I*c) + 41670440375390543643418219342271748040035
071490119080585281522498188818375906900368701234531304633163446319945130196
476913600*e^(252*I*c) + 269177947010866150978901202368905011051467999960217
75195710866226228638984703456832694153230611607263444183501026198563419616*
e^(251*I*c) + 1722950282436764733440072199841759670394865739473880520939163
6597370380572398964715080095366818322029152193635869784095333760*e^(250*I*c
) + 10927210603473544810279792347844536074588896806230041110089544731605863
146104181739039426674855453466097402330688331845602302*e^(249*I*c) + 686642
533751866826266269375090895696573292457814218163062215780289988087468155103
1136314064199948604001529894566235238597088*e^(248*I*c) + 42748269077205917
525267113368208715008443456479223854715343593336061895718324446413641328931
08663576205133870672156264164115*e^(247*I*c) + 2636662410430799340447522284
778244283740751068658140726576446671207798325606832295937705061686297930296
382338892574900819440*e^(246*I*c) + 161109254140006052595485937526419417834
764347183707820144626243561514294458733783351358602272984952335843649358604
2252995608*e^(245*I*c) + 97521033944404931875728231176351778667322317559445
7946383279264635085041004917300295904275433144848532459919875479817581584*e
^(244*I*c) + 58474957368230458617938462884488332758149896988654038037889676
7999075614964007174600811092945356635118795824799369716742109*e^(243*I*c) +
34731005381093529041945556055595731412956921073574598323436965997641337477
4078000173070075248654524917179128950507443058208*e^(242*I*c) + 20432555726
518600076740271023084789645976158392276369823543321283331307778304104007466
9379017394836761539649081690630811665*e^(241*I*c) + 11906059184966054683476
569322767644906758414824888267844750482607723633344451345409512666875005729
5811191643356908972191440*e^(240*I*c) + 68712466015985641512468586173659747
734879591710098354652786124936023073943141049573606648563005359411712764895
683903806088*e^(239*I*c) + 392742004143298611693979445162250010812274333985

85007206399231211907157795359719648241598754266579840244551491476467899952*
 $e^{(238*I*c)}$ + 2223134113180153534540639903772168684020839794195258013558464
 5966746736656716271554826476282991066076564921432614339399735* $e^{(237*I*c)}$ +
 12462140440537258084928596709872066857757070943124868554500948154756863454
 308032925408340311237850017814707896986969086816* $e^{(236*I*c)}$ + 691783894521
 484427849333045936139492337233385361987963737267318494285971243106634572687
 0422099893124890777678037369988150* $e^{(235*I*c)}$ + 38026049967058911069646206
 338489648070370988545101822632430305972956307603535975319743247522663891931
 85760878274188013440* $e^{(234*I*c)}$ + 2069698289500860643461665762373807957513
 019424041178871904960551829412449344722432125679417958403007551179298315947
 373776* $e^{(233*I*c)}$ + 111539828554560155053332804560018431799389935921799672
 9818340704221801195667410485846996179056733558512452238583160792512* $e^{(232*I*c)}$ +
 59515761550043151494747928233654705382792608791642526349718775702941
 3385471835434198246807096214536895441388306027237899* $e^{(231*I*c)}$ + 31440903
 580822586156559543693835494544547399104312972204674722881303092520496850341
 8566818838611866709807040793495364496* $e^{(230*I*c)}$ + 16443750067690689274132
 326015439427850395456193602013359658180644935724027734944792733451710580999
 5300093549279931273178* $e^{(229*I*c)}$ + 85139533234786455779589959464900637760
 735729705621221380005837208369157794673049675428799817875431430246332625899
 630160* $e^{(228*I*c)}$ + 436383000758151710259462114644656896189657734886609858
 57945657479854085108851857911222911989837615452608512356008400295* $e^{(227*I*c)}$ +
 2214073500170860327091518076924139166203557875590397914890921360382255
 4792749183517160255571915875356439553717130797888* $e^{(226*I*c)}$ + 11119499645
 363201080881062824886338492425375658448977935535846349290425821570383090425
 411418521516670371372045206568345* $e^{(225*I*c)}$ + 552749884903117835586123000
 932666828392629008215846711800069850271937993904591834422219274214571125702
 8040974074674736* $e^{(224*I*c)}$ + 27195892834837439260408051010803419212445303
 11254607250929192773909331523226635035815672862569296693711643521070331394*
 $e^{(223*I*c)}$ + 1324311324984027428355222938147682378672860708817161741448749
 689593588020860847508703702325320304649883120684987556400* $e^{(222*I*c)}$ + 638
 218892914533741505806399662488556066783600496091876408374974877448971778036
 074996245581124283460438065182071976085* $e^{(221*I*c)}$ + 304384471106813336010
 284160123906370433888828490627422652551236966790916174520857759143930140187
 173492394981908258944* $e^{(220*I*c)}$ + 143657687138044796942947119704259538458
 818199423516824674586293691056119209866358123637772245409530799230553767222
 252* $e^{(219*I*c)}$ + 670917061305296691250198992100215765802378434622295353862
 95087076189297849995931360645605292130961496106707521506432* $e^{(218*I*c)}$ + 3
 100431920606941707706936314142348743182800909818474463567865228417743946494
 1651812564519144918003174108077634846014* $e^{(217*I*c)}$ + 14176483652875704957
 20201334324111790436997796653849959902524421980635189011634815653279605497
 783382888932766730080* $e^{(216*I*c)}$ + 641338189585592518475823145106255638032
 859493851100657701521811978653639021305728401820220163109443481958402511346
 5* $e^{(215*I*c)}$ + 28704961314123144518346747153535894395532944308085319334660
 86288543709246230769151392180699413405623017247753532944* $e^{(214*I*c)}$ + 1271
 033082938048950201360554831270342662343991277504612342300366025046741742856

580445289401786656311685859023084716*e^(213*I*c) + 556756388711182340341026
192734219546113651768317380539005893679049394714017063698565272728813669054
779077208977840*e^(212*I*c) + 241246021282440061792908317783032876194801597
133206052091286997043729145345755805710081489006741839439573984832678*e^(21
1*I*c) + 103399755467257436489847837640753754718204394473055795001467604326
41987655556873829531737211096115196005647730480*e^(210*I*c) + 438349721429
193776853786922330210637445540331009285027374804389789767469898957840709519
05237783490374305934542955*e^(209*I*c) + 1837980708400335976602764921762114
411609173557221662078886153580344970227380258835907670424184073351343911411
3248*e^(208*I*c) + 76217887919120470620388409177993746004288922581943676366
82944356096681400246312138001769285020661445991073249416*e^(207*I*c) + 3125
683493178701743479704750307490178666292150720179363604335113528623329684606
343185540756019935662148267863968*e^(206*I*c) + 126759701729481291340014627
604212692998648029287019039910755431107996422728019652247537010873847785631
1765699610*e^(205*I*c) + 50832459930108546016697862968303266142765447408293
9048097939638391567298795788389433842285751054665210868287680*e^(204*I*c) +
20155794742479409802677247846204088339586751256293086294375356869008401558
5598010154781548625239409581907397500*e^(203*I*c) + 79019149558766569254783
988487232388352909144982747171856772463223808993367091503402876467270176124
342699654400*e^(202*I*c) + 306275810542219573783905472892776091295728139310
82733520247387226000020043538279468776707958420892547870128680*e^(201*I*c)
+ 1173585692624511849311309100250160403234187698599908282352067224153020018
8223826392982302194084667538488665600*e^(200*I*c) + 44454122592954746250676
595141983129660154162999689303933456303459141097207405736188849805200100284
51496996210*e^(199*I*c) + 1664475034387211809394917743502937638978574937754
764763987835872410449930690131572904279995484581013965001440*e^(198*I*c) +
616003116022979584937125701757887212998354300989991362628038861093914561332
071191909714949426587936910303300*e^(197*I*c) + 225320532593220657767941109
289516248999794521015564134982827241710019675486694499689312466561907212627
820000*e^(196*I*c) + 814520814138291118288754175642500548460376933128114804
92909160758195989155768107022568350953861815940704090*e^(195*I*c) + 2909765
106124745340664756978183691006216555985235905280425915416568712542875256238
5492373749486351714453120*e^(194*I*c) + 10271602530202889002497813516849452
590971512809529060665197301097052210064576088348023234671975463677418470*e^
(193*I*c) + 358271800216329606141453670371510989710719825273928454614934310
2348456124089657428594946438660859773886240*e^(192*I*c) + 12346680418924099
787800180817554402160125824763969419379658996319530792039742221387946043284
98972144766900*e^(191*I*c) + 4203580248351467985836112101459421546844379493
65647899088372524802156222884839580011688655664280691773600*e^(190*I*c) + 1
413799382535568432805655058074033041306061307254347517457940798331413617489
17639986145377066437210546190*e^(189*I*c) + 4697022472711728182645404501807
0670522559756627580347784535320014963482632359729444541885102274546002560*e
^(188*I*c) + 15413111211486023937294970820797376716081344788163386543522421
939737507962125854981881879168348260330000*e^(187*I*c) + 499524195627913818
020518674440168802438827211392125566373495694692757130553314677689878787805

9685108480*e^(186*I*c) + 15987711010581926922705289996774447426856310062324
 56185844925220144002305878120380828483988663574829100*e^(185*I*c) + 5052936
 631230152588784830257388128132033977668453400653812610163534197223826203930
 32535960660921950400*e^(184*I*c) + 1576858455288509187214628778644350902575
 83149415561323427386562894447598277935629800939237175625149830*e^(183*I*c)
 + 4858425815314028044731483686877213139019541241904673277845870601509688143
 7076337910793584122475073760*e^(182*I*c) + 14777955096617128998712745182071
 495362176506973183081650233605274051677624970464340242755840025673760*e<sup>(18
 1*I*c)</sup> + 443721091784318234776434954444390469902005659506947084719361709211
 4714077633077234972825351226979360*e^(180*I*c) + 13150521209306921221022971
 05327622842335870743428530891072983535862280094446607723473800477453914130*
 e^(179*I*c) + 3846558420806662744540630787848371749989490525009753221620033
 92549953413592461519365177908682078400*e^(178*I*c) + 1110341487970088194431
 438956444692422950498674643137109692576193388991337992856160200698726117108
 50*e^(177*I*c) + 3162664467472554773117679568752765357130596998592368839211
 2164915553242573269490908989570248533280*e^(176*I*c) + 88882950287510246670
 442038376079761014800531344186144746207675228248689119598843526664449174040
 00*e^(175*I*c) + 2464382190807439609079774226855679629367885709776435876630
 851716253962696192341706239192878728160*e^(174*I*c) + 674025530543133008894
 845775236625237450743114473544537818170447134607102575676676056675328961590
 *e^(173*I*c) + 181834661406177901315330129677145381166449188413194141169344
 354754920969034952610378945282257600*e^(172*I*c) + 483794897564340998438577
 91816589379406815042609340378747586437145781646245422045101230417309900*e<sup>(
 171*I*c)</sup> + 1269349693296492056507367363718128008854868250888025533728006500
 6566138696041797353216584528640*e^(170*I*c) + 32838747605558186767263094803
 06734420155098583948074469014168171874442170109648521627538755920*e<sup>(169*I*
 c)</sup> + 8375792069234119324587864867653735339465452397089907694887248139821891
 65104589895518909256320*e^(168*I*c) + 2105943013856484711843290788803175049
 53361839954159427434009884661777259752542647709150036990*e^(167*I*c) + 5219
 091220766182421581227185426974807129284324322789476922969072001054714133413
 1610989636000*e^(166*I*c) + 12747219616503320541356343062562847368601622140
 856786025445814532037904111523242298235713300*e^(165*I*c) + 306797429643174
 736419815962396246367161700641962685142614841860293485290737902165976191184
 0*e^(164*I*c) + 72752101071839422929177407384469425579873866706753537975973
 2795567942578751384250780476310*e^(163*I*c) + 16995632796992976777390209665
 2629253283704505477127544556534417376686540936706073847337600*e^(162*I*c) +
 39108031255601809476537535369611844440844903751605645023514572352045248104
 262933598850730*e^(161*I*c) + 886275214275695728568134088576490459793534956
 9355321815647721172537159186491471311666400*e^(160*I*c) + 19777929806658181
 35651300094326239158605448870806970860577325385028609983034534672318500*e<sup>(
 159*I*c)</sup> + 4345466767802800453463444987638925407971751057568275155092970241
 87660299345484920192480*e^(158*I*c) + 9398691531306817914908360606568148278
 0836060510530154618486949839467131378859885998210*e^(157*I*c) + 20008006803
 030047137293278250321597113540716201983333126349281186679153199068045257216
 *e^(156*I*c) + 419154250065682614809333941454415914396447847249231593180917

1859902114109005939942952*e^(155*I*c) + 86397993362233034955629682002839551
3198708064940505702126068652936800794826651264256*e^(154*I*c) + 17519317050
0618300241515632381912285157790097816049220671217212220015297133400636060*e
^(153*I*c) + 34941071613276704649477943043339450201504075335160361865916029
213860778606230624960*e^(152*I*c) + 685299322314573668732888531161779543559
2940841439866351079655652312894721972796266*e^(151*I*c) + 13214980552713008
51429993866631619874424534425188183592049727687571032156435077280*e^(150*I*
c) + 2505010286089283324693404568299020677122336444646027531599457278684857
22395506952*e^(149*I*c) + 4666822354826601780685459246810057028935596086961
3650856575756758180182223308768*e^(148*I*c) + 85430134411262123348335406650
69621472479085838041360564550722036723654297540205*e^(147*I*c) + 1536333238
444927583532734556016494671674916578907116984548489078241693926940560*e^(14
6*I*c) + 271361207503266570734486517077181014801775322183181055638619257836
143271472358*e^(145*I*c) + 470650446111351581084873533674842431026982488383
12635876283099427442745866704*e^(144*I*c) + 8013729580790752434361964945761
543761469520791210746972675870481058674277844*e^(143*I*c) + 133921437425424
5553564884406801945353385000254030655765953770237607180089968*e^(142*I*c) +
21960128133951556150026147884419002487055526128194605883961404469703796369
5*e^(141*I*c) + 35324447206779018115378052820789411687581004582367431006205
879633729015200*e^(140*I*c) + 557255115732867112101621641630759616186195596
9011697222340926210112854418*e^(139*I*c) + 86188485109499190876424680547467
2428603757315484453974713612812215428992*e^(138*I*c) + 13065766022656041933
5121434389938961884595434069984824307149332131747540*e^(137*I*c) + 19407979
215594566593535008103303255257745408070082431338945184797463936*e^(136*I*c)
+ 2823905151936586678382525706564457280290098698638597987628380245881715*e
^(135*I*c) + 40234969226612115893400358283942878511690490393640954560251921
9664720*e^(134*I*c) + 56117081076341175384087570185188538660375932013674735
519055227368366*e^(133*I*c) + 765901052018754965177711835767687192708189898
9131125755798204236112*e^(132*I*c) + 10225364374682967372930658627052464496
93687415559865844306888705423*e^(131*I*c) + 1334902100520261837796733138683
32303530332906163247194627808410304*e^(130*I*c) + 1703388602739061574104097
7721655541665612162275485028584310890417*e^(129*I*c) + 21237029691888713182
66718781223927067839949015727293884065388080*e^(128*I*c) + 2585853487159772
70155829115684193411072034541491364393985491350*e^(127*I*c) + 3073621740432
1009965231037419663053962881035281709221697785072*e^(126*I*c) + 35647648906
28724017088487996688178929195787613958545474804845*e^(125*I*c) + 4032122259
57798188840846139960995624144491271694336796459584*e^(124*I*c) + 4445670817
5258821024400946210535004523775722190977468484496*e^(123*I*c) + 47753986071
00853263534207733818266777478693412738731031680*e^(122*I*c) + 4994675065585
31733671585862910572702811545035730398749530*e^(121*I*c) + 5083636950817109
9437019348610847391946736185108017183136*e^(120*I*c) + 50320249034014518240
74213943766011922026507006311982753*e^(119*I*c) + 4840934102404887186559170
25303662581091659126182344528*e^(118*I*c) + 4523094003983073833202569478464
6206844854827698075736*e^(117*I*c) + 41015454399371957939599567084424967094
33800261224880*e^(116*I*c) + 3606886130363893494138097800045599635487754233

$25255e^{(115*I*c)} + 30735366512830562160991166338490057308062762518496e^{(14*I*c)} + 2535667460650279776834561566186591213109251642859e^{(113*I*c)} + 202347509724462171313966643580234078508179838320e^{(112*I*c)} + 15603911277687607099721623771744933086920587272e^{(111*I*c)} + 1161581413733971751533622511909046917188768400e^{(110*I*c)} + 83380839911837894453136303673785039051506805e^{(109*I*c)} + 5764601046563151304213854710715346838447392e^{(108*I*c)} + 383360155801054824529764688213114368047154e^{(107*I*c)} + 24489837337812338687718622491865013839488e^{(106*I*c)} + 1500602747937397286405577818722691539392e^{(105*I*c)} + 88054927598941411145869950813388040256e^{(104*I*c)} + 4939666610818025798809586352543471345e^{(103*I*c)} + 264410375780310742518099326419685040e^{(102*I*c)} + 13477227799524701956579274210395326e^{(101*I*c)} + 652650253343206047453620559993840e^{(100*I*c)} + 29952547749265499675257842032197e^{(99*I*c)} + 1299146645993240318167826532288e^{(98*I*c)} + 53090127264630963470039804475e^{(97*I*c)} + 2037031259470368160131922320e^{(96*I*c)} + 73099207817335597247098038e^{(95*I*c)} + 2442455629894502983849104e^{(94*I*c)} + 75599817092670157806639e^{(93*I*c)} + 2154864144781257856128e^{(92*I*c)} + 56169444526926562260e^{(91*I*c)} + 1327882849274858880e^{(90*I*c)} + 28186192554792138e^{(89*I*c)} + 530563624556832e^{(88*I*c)} + 8718181624155e^{(87*I*c)} + 122503723056e^{(86*I*c)} + 1431118260e^{(85*I*c)} + 13343760e^{(84*I*c)} + 93096e^{(83*I*c)} + 432e^{(82*I*c)} + e^{(81*I*c)}) * \tan(1/4*d*x + c) + 14*(3718*a*e^{(1055/2*I*c)} + 1502072*a*e^{(1053/2*I*c)} + 302667508*a*e^{(1051/2*I*c)} + 40557446072*a*e^{(1049/2*I*c)} + 4065883968718*a*e^{(1047/2*I*c)} + 325270717497440*a*e^{(1045/2*I*c)} + 21630502713590667*a*e^{(1043/2*I*c)} + 1229848582862227068*a*e^{(1041/2*I*c)} + 61031235925207244502*a*e^{(1039/2*I*c)} + 2685374380789029329793*a*e^{(1037/2*I*c)} + 106072288048394569716987*a*e^{(1035/2*I*c)} + 3799316499714931382700630*a*e^{(1033/2*I*c)} + 124427615397868362145478058*a*e^{(1031/2*I*c)} + 3751971173691706110924034857*a*e^{(1029/2*I*c)} + 104787195000667085382630850098*a*e^{(1027/2*I*c)} + 2724467073240049286222217499520*a*e^{(1025/2*I*c)} + 66238605837791619527049091465540*a*e^{(1023/2*I*c)} + 1511798772576673047875435387479418*a*e^{(1021/2*I*c)} + 32503673735785850571258452122985714*a*e^{(1019/2*I*c)} + 660337796328597830395723099421064436*a*e^{(1017/2*I*c)} + 12711502674847529112148726218999140522*a*e^{(1015/2*I*c)} + 232438908425726665209807096498775079690*a*e^{(1013/2*I*c)} + 4046550142768142823441656912104576646575*a*e^{(1011/2*I*c)} + 67207920970133725538564726906882769461732*a*e^{(1009/2*I*c)} + 1066925770347513486390409808978976432165366*a*e^{(1007/2*I*c)} + 16217272197080856640086895687618882654237557*a*e^{(1005/2*I*c)} + 236397938386786552298680619962344215977635255*a*e^{(1003/2*I*c)} + 3309571297229229358438719595781123324700153528*a*e^{(1001/2*I*c)} + 44561016227553011590271450799436600978561400862*a*e^{(999/2*I*c)} + 577756667439166347317361298827893686038031023201*a*e^{(997/2*I*c)} + 7221959008399380670648524146893979780310767551977*a*e^{(995/2*I*c)} + 87129450753762477310055827412704326695373797670990*a*e^{(993/2*I*c)} + 1015602799297538681881502332438578214644838744212768*a*e^{(991/2*I*c)} + 11448615265227028920332020054872464737892837624091599*a*e^{(989/2*I*c)} + 124924620764273645058915750915973878220098322691297101*a*e^{(987/2*I*c)} + 132063201818273405043449076959796054660794302120955437$

$4*a*e^{(985/2*I*c)} + 1353648199985191570695804906782920339703084559180097994$
 $8*a*e^{(983/2*I*c)} + 1346331631527910859062275189596447146295754269413452748$
 $55*a*e^{(981/2*I*c)} + 130027342859438463217880491586934807649249604052802694$
 $1982*a*e^{(979/2*I*c)} + 1220257164748036847894733934719754255437366012976630$
 $5670026*a*e^{(977/2*I*c)} + 1113485262918004111072465842269528005053268774794$
 $52354210332*a*e^{(975/2*I*c)} + 988558268629814228600928047340864800588772545$
 $018200620775758*a*e^{(973/2*I*c)} + 85439741194334071599751662430130262799909$
 $85549262801022565070*a*e^{(971/2*I*c)} + 719284009116603764023203187516727101$
 $60518473805095916511887372*a*e^{(969/2*I*c)} + 590140408754311076687013315279$
 $638319827311377295800031121209656*a*e^{(967/2*I*c)} + 47211285515018981347023$
 $49899679082580779514892160753741261926062*a*e^{(965/2*I*c)} + 368453766224328$
 $77842558414417003887278048975820601095126064678425*a*e^{(963/2*I*c)} + 280652$
 $429583871841898555612150376650333075795788342413441360641690*a*e^{(961/2*I*c)}$
 $) + 2087355940512950608443105897698920410302162080067836820821771280502*a*e$
 $^{(959/2*I*c)} + 151653088004861664024727661229744378319301207468364712918197$
 $80637051*a*e^{(957/2*I*c)} + 107673925545674144588693096021112687136443322698$
 $905146090803710183409*a*e^{(955/2*I*c)} + 74738555150888980082572859246533346$
 $9870510137536130275665450524715478*a*e^{(953/2*I*c)} + 5073612140276869022785$
 $810697789013952147928175584559508062569934520500*a*e^{(951/2*I*c)} + 33696548$
 $332879373510255581378537814235558502314864144954037409976543435*a*e^{(949/2*$
 $I*c)} + 21902833599928625777636963414661371310650964786170467741048845447104$
 $1828*a*e^{(947/2*I*c)} + 1393822204759320545751155434910006366369348464487459$
 $381028813178904280462*a*e^{(945/2*I*c)} + 86865377059570157468745566475036119$
 $78485575286577500733201504080157038368*a*e^{(943/2*I*c)} + 530338626559434445$
 $30651471417311659093748967671410808199772545872566250240*a*e^{(941/2*I*c)} +$
 $317290565438299295944227981890447073652423258243355469512774650547831808732$
 $*a*e^{(939/2*I*c)} + 18607325088575871328399195233663058049388744823152060882$
 $51791750125084597398*a*e^{(937/2*I*c)} + 106992860611465417411140377189131221$
 $66739129443783960920451139637868310953672*a*e^{(935/2*I*c)} + 603374228211089$
 $0544218880523770538090215653857306777647643655256342216048188*a*e^{(933/2*I$
 $*c)} + 333805055593388567427808201926960940156572529198077839859476641115104$
 $007423532*a*e^{(931/2*I*c)} + 18121018304254363008917205726286783630423541985$
 $55147726701275150075546643583080*a*e^{(929/2*I*c)} + 965520690630898105108766$
 $1502772471729668550920396172616931546671734191430604348*a*e^{(927/2*I*c)} + 5$
 $050474895167495659424171916502050073964613004151143876733776676640401872179$
 $9652*a*e^{(925/2*I*c)} + 2594141053437586143844290455743529031819611343826803$
 $34119395735344643117364969828*a*e^{(923/2*I*c)} + 130870474410323324494283271$
 $1379475161653066000371260361064765494153774929304542712*a*e^{(921/2*I*c)} + 6$
 $485888759441894264481925452577319190375803293665331722248553541369888292250$
 $431868*a*e^{(919/2*I*c)} + 31584005662708604544701764762131792650967524020242$
 $941992643782185239861413590496020*a*e^{(917/2*I*c)} + 15115491025736943469259$
 $6735808879851584404239307790047602041275902443495292459666790*a*e^{(915/2*I*$
 $c)} + 7110816734361854519240407936691412989696267227966143957428023612660936$
 $33641264085200*a*e^{(913/2*I*c)} + 328882849500609878052514256929680012854775$
 $1347678538502742954742586652543592821521100*a*e^{(911/2*I*c)} + 1495778983626$

0035053720288689121945297194278508615355005723695241121674541179126990690*a
*e^(909/2*I*c) + 6690764342465983818454577392490887995470152568741037622719
9214504402274227244630158470*a*e^(907/2*I*c) + 2944025642405920618949693968
68156932057924803903695357306824322577258371768427288287660*a*e^(905/2*I*c)
+ 127449552107113874458321583563395128080302542623822937083806236749728719
5705487702727780*a*e^(903/2*I*c) + 5429216947618612587547123012480165658464
605296922357528675368754249200100593150293185330*a*e^(901/2*I*c) + 22761850
427041663890460996661405723193407183947233163896700853295630681434965577357
299920*a*e^(899/2*I*c) + 93932713207395265486897880366664895741093700695924
344009200350068922566762007052749646040*a*e^(897/2*I*c) + 38161965549422077
4667949833382475342723385533974486856243098750370767001096283055945502640*a
*e^(895/2*I*c) + 1526557117570036622571918876778252127468437541774623614619
302679658497516840009278849352280*a*e^(893/2*I*c) + 60134824039422697813758
95136192720321233266638905496214866132974340762298443025559536997680*a*e^(8
91/2*I*c) + 233308349176317150934664814200838330299476688805134989663781375
98401377993510086729958534960*a*e^(889/2*I*c) + 891630246675288443652575569
46977068950919875040439960342188615542362550926345388079651264340*a*e^(887/
2*I*c) + 335696720738936570706713403614929944912226507516523422878616118942
760950780230341688669154360*a*e^(885/2*I*c) + 12452979254554264676850036797
83202930885617600557716500473909654045354638798448652726086548710*a*e^(883/
2*I*c) + 455216454198570115836444083320219736044501483971095697077034024097
3634689769418208918160590320*a*e^(881/2*I*c) + 1639964574077305109297539230
9744789183304727620673733635277139746257521706440274686262794198620*a*e^(87
9/2*I*c) + 5823375669931839000990917236761189568577386782678355064920050086
9687498091353927199571718405490*a*e^(877/2*I*c) + 2038401394201733506465998
63519636205648295919156240767205893957321010048179259062568208312088790*a*e
^(875/2*I*c) + 703442619798587327256518771953962641129807006145552665167689
345204662084900828221826842429079160*a*e^(873/2*I*c) + 23935364531901942975
725216426094858411453720475694518714819436739240842285781199927263643140430
20*a*e^(871/2*I*c) + 803102539288425206417745098787909283292860806299086446
1726329576684085419875495358549722887030650*a*e^(869/2*I*c) + 2657465386577
270594551688425896691792857739954492665110111547126128141566277464592080556
2286190810*a*e^(867/2*I*c) + 8673111081249706024196404770369878197649324163
5614918587219564911585800789497248326704733021564620*a*e^(865/2*I*c) + 2792
137812408373062631492392777412785041807648142163207685231265451122669432308
69925045964255370200*a*e^(863/2*I*c) + 886740231925532664522771055553453849
969744152256079998032209234008019622116815785290576730828104950*a*e^(861/2*
I*c) + 27784053052143436342726424781153687621685136571720170670300841434525
83457313885656093029057631444370*a*e^(859/2*I*c) + 858965006761491323319609
660407459749256654782245376571508197972507715623929328837727749947754206006
0*a*e^(857/2*I*c) + 2620451831207455850865238131984749663471988374403165108
8258490824465738576863175200241054655253538000*a*e^(855/2*I*c) + 7889273247
555062920716015061964951885967133305942536387072760442653882061519707285975
3978543934017670*a*e^(853/2*I*c) + 2344209058616675016735149514402754718218
28319962595741737151538794823588356166465622241503448244518760*a*e^(851/2*I

$*c) + 687530224472620092791173281345497074926271430076019616322085361144879$
 $429550058409565871621454060233940*a*e^{(849/2*I*c)} + 19904906928755272586672$
 $364990174607843663079232802966549317359979549411477932072990417750800461458$
 $53840*a*e^{(847/2*I*c)} + 568901947141022024606690860348400435132998267377735$
 $4247661562524221692115006358198880775065147099437120*a*e^{(845/2*I*c)} + 1605$
 $309814686536657061422698593617514802846668574542867033494369104831413462797$
 $7747488782883668338627720*a*e^{(843/2*I*c)} + 4472589130147944961802165361302$
 $1840925820831052165035724946865219883816662123055202896018457333687900560*a$
 $*e^{(841/2*I*c)} + 1230471825913272392910598378031329902365609515414260915025$
 $92133079889930820736735976930062642304445760760*a*e^{(839/2*I*c)} + 334295101$
 $291598394405385926974923341821234439369994003970291001771339818244857969207$
 $842071054369602329440*a*e^{(837/2*I*c)} + 89694641544949502177330565213252148$
 $9143683388084681377490890952178768366949153161001917497973558984634610*a*e^{$
 $(835/2*I*c)} + 2376911274978475170003732136235788181859385653298280842814105$
 $754624463620356183384021592556492451101686180*a*e^{(833/2*I*c)} + 62215793900$
 $366012619065913586228573658637652713363638132589281020472312697012877444357$
 $51925676619466254380*a*e^{(831/2*I*c)} + 160864475160840631632436890502158797$
 $61267599809808958182816872155074721707033313657170678606565171911318790*a*e$
 $^{(829/2*I*c)} + 410886153084858983661724972688741143514787801131605768393365$
 $45240589947982864125921136564058023293113177890*a*e^{(827/2*I*c)} + 103684586$
 $953158052661761002194856220016078202175436674022417030738316048059257865332$
 $989827933134737432425420*a*e^{(825/2*I*c)} + 25850394142741877598344935825722$
 $712932718703102540535230494615859661794773224169018013860902417961989654808$
 $0*a*e^{(823/2*I*c)} + 6368084508200525084133949033736136994089851317933782253$
 $22630926816010278188664354873365849405050807559590630*a*e^{(821/2*I*c)} + 155$
 $012618925132209145656629332104130466133237360530197770368114959324223703231$
 $0585758452049979836332182019460*a*e^{(819/2*I*c)} + 3728801747608926549551319$
 $037961429327447404631788357054035813617665417388029848305480269822322115047$
 $563031420*a*e^{(817/2*I*c)} + 88642599821438870660522413394953025195926517437$
 $06395609918105114241642344626154900178145218983856632762282840*a*e^{(815/2*I$
 $*c)} + 208263481397657281455023002502306335286253688110379262837891828378175$
 $72337323284403311770372970302291071410100*a*e^{(813/2*I*c)} + 483623360137755$
 $033433196931546008721802088936653299722550074646690367455218437107619016886$
 $65256224358994271700*a*e^{(811/2*I*c)} + 111007051148402125345220283714271173$
 $982629035385223122646805636583654935893527705499942693423715121989424818940$
 $*a*e^{(809/2*I*c)} + 25186525048428154589423818001709359561079691188227199925$
 $0070181159518716093352404254337261360592129286243867720*a*e^{(807/2*I*c)} + 5$
 $649177060380370278859200074337640760516480614093901081266350280919135724146$
 $47347178039923578299379859935849404*a*e^{(805/2*I*c)} + 125263879584746691376$
 $612500666962587390732663784320819917382780840045989023229609436217023987331$
 $8861004896614156*a*e^{(803/2*I*c)} + 2746087713045966492759819747057534537295$
 $794851142521901350884068686972703385462885090913563610742000426952431144*a$
 $e^{(801/2*I*c)} + 59521604667866061656535562465898355618400210490063676342925$
 $96565935280818195141374695236251836967708599780176746*a*e^{(799/2*I*c)} + 127$
 $564556178413915139143847288988400427256611136687918466219278080687859904900$

77238948492719552936499900344230684*a*e^(797/2*I*c) + 270335925308800022267
374653213591384648474841539826876975946990447382574548191382107376417426615
88856574025344360*a*e^(795/2*I*c) + 566524575090192173328098322533937654900
26446523156131066467177640614737878543683747754276689674112796652581318416*
a*e^(793/2*I*c) + 117407982036395964616775103428880300577811655638673060994
605539426451393694076705057517258363475234790358732617874*a*e^(791/2*I*c) +
24063719217692599555476976382219654969515010641276784024680601558878500146
6549798116731687549539743160206690512276*a*e^(789/2*I*c) + 4877938040040138
25009296221609997616820891599158990112266724901298572450074338737871626296
26332383304760538813509*a*e^(787/2*I*c) + 978002873261192283457582949969456
052834946311148564091945480083912375742750095496723188070578970297191691801
222556*a*e^(785/2*I*c) + 19395278696913220572173470105895471941193523887881
93290366880726140422435916649514401171793242696849555174393643850*a*e^(783/
2*I*c) + 380474446496892135901546187311383678115984042854196339778614189930
8945274545366930937702520484566257179138022478559*a*e^(781/2*I*c) + 7383289
569915905966227444696110983492150005009839693268039573457762597244641291406
843792278663524351127277610068501*a*e^(779/2*I*c) + 14173935643325598453202
412396621317173285893916076073572849970829958267591737907286619219624773200
857453140746381674*a*e^(777/2*I*c) + 26919576922579260675214576944282615653
058708260861805615759498421237241977159288165357847803942410262265158465467
750*a*e^(775/2*I*c) + 50582952218167250740970644422917614462631967442654432
142347921323029515220528364018397135511063228424091734992672055*a*e^(773/2*
I*c) + 94041409739585576088931966327005555323971108074136721579643463089364
171018456699048513026812223168440576516628683514*a*e^(771/2*I*c) + 17299512
375655010663737813235701030648756604997888820508826549533226330316550378006
6305408668687698920139970094538712*a*e^(769/2*I*c) + 3148973829894315833326
800968271629195316733024716804606400211477651853882990052078834882293334476
15352727288983946148*a*e^(767/2*I*c) + 567211894570524721264934335244788478
905608726318730689979939994574065235928861643029786499059347466854077954904
040746*a*e^(765/2*I*c) + 10110748001149593767195366249770947389642856046430
62799111778942015387325706624278964496575270768601912488833296828570*a*e^(7
63/2*I*c) + 178362473864714504760191755645106188793385377435414139252892685
5412119981497793799270417127134990753415746380839743700*a*e^(761/2*I*c) + 3
114056515501297399026968002850854928138954664552478667168351586612797176618
252981308448320828915776307805739513112726*a*e^(759/2*I*c) + 53811222485671
624211573835697566302203282933979388379820118345988236397087116939169806464
41642210742072277442692795738*a*e^(757/2*I*c) + 920371533810205201395399465
527232770116950697613530256141498159932967169113207475202283224142491191645
9710600587437561*a*e^(755/2*I*c) + 1558181593426048285354507841921955212887
518287711054373068632026342500146156781141126319224452754535552267981870752
0580*a*e^(753/2*I*c) + 2611308638642497861940627208748348431168677835346981
0280045602451813616101635095685826358744734323495284346010214813114*a*e^(75
1/2*I*c) + 4332158318999982001182551353596115363226170993620425085994442615
2513525897996934481741824822705710882414313119272790131*a*e^(749/2*I*c) + 7
115037980318233062499822718689669240619785943405408448288067061432154242494

6546285172306349522048789631550138215923153*a*e^(747/2*I*c) + 1156905307506
944415897628755570187804393124142353468057244506458568792123744039674834976
77636041575439277920548968229136*a*e^(745/2*I*c) + 186246058051488407504035
604030386554718066896118691201386319926538759246009834321804025886034942775
247389666416855412210*a*e^(743/2*I*c) + 29686989431234102588261475688184880
118254809571823190101744270051097013220747369708584337037406840973428721573
0709535159*a*e^(741/2*I*c) + 4685503252933359349996733690917671179857892633
80711414814986160196520127079260704512548045596385215149194149787362058847*
a*e^(739/2*I*c) + 732282573074945415241191984024036255668279273758317972537
870914592731850184818289172732027744954335864633099347774743778*a*e^(737/2*
I*c) + 11333264072490566772651823572830413026805965657533923303560137333862
67578802249101009054033933358184582360056565943982872*a*e^(735/2*I*c) + 173
702790473065474636132103848495836822591755750686241516886200436892400895652
9402849910157768607834197363252609518902089*a*e^(733/2*I*c) + 2636670293606
111780587780364179448299223239045229764093938477626209881636013364392058085
322915002420199553549738131324075*a*e^(731/2*I*c) + 39639120705364146651009
937160909528466211708624286633319606811297764001906634621181840423762056572
05476237354164748568266*a*e^(729/2*I*c) + 590246486578182874643375962611573
129936862803877686353791928675712191961984875667858576596127229260923575004
3165001541228*a*e^(727/2*I*c) + 8705750656639962805019357547403610399215124
329872130490106158502891057402404433904178771049787603257660231907442902723
601*a*e^(725/2*I*c) + 12719346510829980167691769356005135310633717185977594
866088135331067145022553996096352557277491441427340072720038453020094*a*e^(
723/2*I*c) + 18409060258976502497717330256964155108641749728335469201164427
871721771346762241189212154587072288919159051458074105345270*a*e^(721/2*I*c
) + 26395464183288948474053286863120161200727884088708580750264073545101364
060793692377715714056166397802814429763618417822876*a*e^(719/2*I*c) + 37495
624966289314777017990976471297474782544902192217644185887694081271123513263
069199075270691251282255239872596210514278*a*e^(717/2*I*c) + 52772584002591
202607152756706889895749307537002629823015890630924021431243053135308382138
154153897041915995771633984281806*a*e^(715/2*I*c) + 73592839927564405504042
127592762819931998928642896919811011301927328743837627845050154224299851252
425467281991751035964300*a*e^(713/2*I*c) + 10169164442095007002502401105745
299738150028346657473063428389325495918846662740294688296269207633447157874
3932496642334320*a*e^(711/2*I*c) + 1392453280183338807743649688008531425847
282555130434164378701124696315506658696053430278963530231765576944002004994
61146726*a*e^(709/2*I*c) + 188949118902042756365887767234643113787797409285
875028353410293464440403604554265358653936413844588923365518512812155564263
*a*e^(707/2*I*c) + 25409801152668679501782688899786288110769817827980307837
0347724142537822230167040460676758302527381200018338794267668213782*a*e^(70
5/2*I*c) + 3386672073660640286700358693759763701110349823161844797535928303
55581796704600824509215308211821300197041784861670006680074*a*e^(703/2*I*c)
+ 447387531042524414883193986439269112293394102569954189918196551016064656
984949980121125552746165717302725948575549803471765*a*e^(701/2*I*c) + 58581
00878400906389593525931152074933304093883043336485865087262549666627261727

1460912163699959246766169974995865625894415*a*e^(699/2*I*c) + 7603533619508
 856466413519768608931920221645629023321616921264819927797879971183785022008
 34924391895215671091534427467182154*a*e^(697/2*I*c) + 978325068291196614130
 480547884149187418948183584483275851528436468279773867175472874823772301653
 470715495906310752113906356*a*e^(695/2*I*c) + 12479104898517248121534775616
 840297489853271667611766490178937644627425409558103593603457524987374393381
 73328941693214996869*a*e^(693/2*I*c) + 157811889103642503017552977572861318
 482559349051118394199347762880076205789091813709818058882334843395130982221
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 130238883005535552359464916952802872195794689967236683993035275155224868084
 857426*a*e^(689/2*I*c) + 24598838797251652924524255220681788806897094208434
 92989019614310088740969485770831889636928119055954588263689191708179939864*
 a*e^(687/2*I*c) + 303235865118925529668391399505846717618104455596520952274
 9027694372314500932020478507945032419489336181643123979995151020196*a*e^(68
 5/2*I*c) + 3706784707748169939476939927418306117137807661216185577529307951
 968534891054102022599875800521028818591703507213651354601744*a*e^(683/2*I*c
) + 44935456833121271857607041861967869268635584109539353473633873054553903
 18278646691517592351147942162829757283672732654718906*a*e^(681/2*I*c) + 540
 232208427635848639217080716045249230373661054399375678239756345118322037171
 9416455539109849685147683838533792416380394000*a*e^(679/2*I*c) + 6441637428
 661563057501000533820791389212722899725158982575438650338723590004865678079
 355102867423082969123638559458360395064*a*e^(677/2*I*c) + 76183718752941105
 641961216543842531245115661575434720951365646384764643713912959262094351323
 61790499815979231439230353607576*a*e^(675/2*I*c) + 893726314795215763676240
 123736328390606017542376255907971017911134989189214771319978309964087646551
 3972490960354416509072464*a*e^(673/2*I*c) + 1040041893956882969269067276155
 936052619321420158220168021731056876014136218320434031791899929050060158731
 2837404209184604296*a*e^(671/2*I*c) + 1200686847036478596522350752575807406
 469533716196491478100076629062585153133503282262264135067153407165146097530
 2014909903304*a*e^(669/2*I*c) + 1375218304672657470338894314471659653008968
 125070292383699730867185051197745096339649145804854866887117801543267700931
 4283240*a*e^(667/2*I*c) + 1562819593656972303993395151637678670901706644270
 936369302808183275576843663926339199860583111614265415631132444804399401848
 0*a*e^(665/2*I*c) + 1762285033050908883568393767823870507177042348428327355
 2938412847052745751689386549302132045239654150557454790469375312979400*a*e^
 (663/2*I*c) + 1972020040177886458475718370782644717819555915506351150427313
 8952837613637477930413174805644097852673575370034436363705076200*a*e^(661/2
 *I*c) + 2190058446852995951675880083747992895130646084645872903937594321885
 3680130886800630510267887849815785681169229247800599653780*a*e^(659/2*I*c)
 + 2414098113448315778195563275126534688906203426418723305867304386086256924
 5974192234790942766740180569345117561729698386920640*a*e^(657/2*I*c) + 2641
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 9961354340519916453720031563673191039715534120*a*e^(655/2*I*c) + 2869634223
 995152667938703217258208832021982349861252001550039646159536474283494228504
 0710651207447747254997240669017588581660*a*e^(653/2*I*c) + 3095417237589497

876397294420077045455933742686055117788039367854141746562086078917937839946
7769874414988489807528551280728660*a*e^(651/2*I*c) + 3315959531533147356972
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5167083319916344381519908840*a*e^(649/2*I*c) + 3528396681263064114934115979
196139129202552397407753688852390329126523164795493945472750687369015565309
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3976853710509180*a*e^(645/2*I*c) + 3918541339920238362207362760432918908988
417940727975797151686773589816813803571996282173607691297122540475978028583
3325366160*a*e^(643/2*I*c) + 4091866128725715758651661611002142378118037703
292223884980027067357955333807395068123723386687793615259101311579333532260
5360*a*e^(641/2*I*c) + 4248493411020465375586079150675238474650067449607667
3967800901085937786065818126999716652181441925069428597465940270175008320*a
*e^(639/2*I*c) + 4387412945552773996857990988634756853286259349716761883768
1590989053474358012316788203258490840630555174376169174989395181280*a*e^(63
7/2*I*c) + 4508169844824075318026237569200492792750653834504367393946898174
1472441127727726396712184152830435376131283599596954371095120*a*e^(635/2*I*
c) + 4610868869849396905397932192065957729063974918507803832843791155932457
5990868098570200478146083617100445871110853633600291520*a*e^(633/2*I*c) + 4
696149188100867407357195325873944929763110725688718438336243163643469188472
2589930145727930608372648302523392227498002326040*a*e^(631/2*I*c) + 4765130
082013758403198130130623713278481811240638734049548215491661694777530918141
0534964011766375876504387207320222180574240*a*e^(629/2*I*c) + 4819329965953
023636896562916217235337720840743097068652259424626292339759049078074577607
3857916729329041495496560962652110580*a*e^(627/2*I*c) + 4860562864748522153
044803656843936522208397768105783646754526463486349861683531174982743205749
7076186357697241560068425420480*a*e^(625/2*I*c) + 4890818092533230025107915
677645484028717431286470578855522595573185675060135698807015026604282570904
0021440687898855883758920*a*e^(623/2*I*c) + 4912130122613277836827274921961
873872320155547846238824199438934372686877163666449245704006018847791874396
5347339416057414940*a*e^(621/2*I*c) + 4926446454215570587003863454233878821
243879853070543498183705855935512352332142045329005492441236069408635648626
7112390442900*a*e^(619/2*I*c) + 4935501586955109511259058056365233382148713
502940320429787113638575117708266882248311489582398937580644150072571054673
2275120*a*e^(617/2*I*c) + 4940704972449785792364641812280241807866341597430
108985830932523362871461431390261043765495141712346720821707039061058425124
0*a*e^(615/2*I*c) + 4943050029347945929542918221066724141015407481608712897
8164893559373471444466906250076413859410320538676562475573896009453900*a*e^
(613/2*I*c) + 4943050029347945929542918221066724141015407481608712897816489
3559373471444466906250076413859410320538676562475573896009453900*a*e^(611/2
*I*c) + 4940704972449785792364641812280241807866341597430108985830932523362
8714614313902610437654951417123467208217070390610584251240*a*e^(609/2*I*c)
+ 4935501586955109511259058056365233382148713502940320429787113638575117708
2668822483114895823989375806441500725710546732275120*a*e^(607/2*I*c) + 4926
446454215570587003863454233878821243879853070543498183705855935512352332142

0453290054924412360694086356486267112390442900*a*e^(605/2*I*c) + 4912130122
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 7040060188477918743965347339416057414940*a*e^(603/2*I*c) + 4890818092533230
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 2825709040021440687898855883758920*a*e^(601/2*I*c) + 4860562864748522153044
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 6186357697241560068425420480*a*e^(599/2*I*c) + 4819329965953023636896562916
 217235337720840743097068652259424626292339759049078074577607385791672932904
 1495496560962652110580*a*e^(597/2*I*c) + 4765130082013758403198130130623713
 278481811240638734049548215491661694777530918141053496401176637587650438720
 7320222180574240*a*e^(595/2*I*c) + 4696149188100867407357195325873944929763
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 8002326040*a*e^(593/2*I*c) + 4610868869849396905397932192065957729063974918
 507803832843791155932457599086809857020047814608361710044587111085363360029
 1520*a*e^(591/2*I*c) + 4508169844824075318026237569200492792750653834504367
 3939468981741472441127727726396712184152830435376131283599596954371095120*a
 *e^(589/2*I*c) + 4387412945552773996857990988634756853286259349716761883768
 1590989053474358012316788203258490840630555174376169174989395181280*a*e^(58
 7/2*I*c) + 4248493411020465375586079150675238474650067449607667396780090108
 5937786065818126999716652181441925069428597465940270175008320*a*e^(585/2*I*
 c) + 4091866128725715758651661611002142378118037703292223884980027067357955
 3338073950681237233866877936152591013115793335322605360*a*e^(583/2*I*c) + 3
 918541339920238362207362760432918908988417940727975797151686773589816813803
 5719962821736076912971225404759780285833325366160*a*e^(581/2*I*c) + 3730052
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 0252310284690807173977430553976853710509180*a*e^(579/2*I*c) + 3528396681263
 064114934115979196139129202552397407753688852390329126523164795493945472750
 6873690155653093593169057488857882680*a*e^(577/2*I*c) + 3315959531533147356
 972129975249200797729277076478250842859830047423405667239637471651007463754
 7675167083319916344381519908840*a*e^(575/2*I*c) + 3095417237589497876397294
 420077045455933742686055117788039367854141746562086078917937839946776987441
 4988489807528551280728660*a*e^(573/2*I*c) + 2869634223995152667938703217258
 208832021982349861252001550039646159536474283494228504071065120744774725499
 7240669017588581660*a*e^(571/2*I*c) + 2641554936314980668503966145439267020
 612265815370183362673368419575522391115846996135434051991645372003156367319
 1039715534120*a*e^(569/2*I*c) + 2414098113448315778195563275126534688906203
 426418723305867304386086256924597419223479094276674018056934511756172969838
 6920640*a*e^(567/2*I*c) + 2190058446852995951675880083747992895130646084645
 872903937594321885368013088680063051026788784981578568116922924780059965378
 0*a*e^(565/2*I*c) + 1972020040177886458475718370782644717819555915506351150
 4273138952837613637477930413174805644097852673575370034436363705076200*a*e^
 (563/2*I*c) + 1762285033050908883568393767823870507177042348428327355293841
 2847052745751689386549302132045239654150557454790469375312979400*a*e^(561/2
 *I*c) + 1562819593656972303993395151637678670901706644270936369302808183275
 5768436639263391998605831116142654156311324448043994018480*a*e^(559/2*I*c)

+ 1375218304672657470338894314471659653008968125070292383699730867185051197
 7450963396491458048548668871178015432677009314283240*a*e^(557/2*I*c) + 1200
 686847036478596522350752575807406469533716196491478100076629062585153133503
 2822622641350671534071651460975302014909903304*a*e^(555/2*I*c) + 1040041893
 956882969269067276155936052619321420158220168021731056876014136218320434031
 7918999290500601587312837404209184604296*a*e^(553/2*I*c) + 8937263147952157
 636762401237363283906060175423762559079710179111349891892147713199783099640
 876465513972490960354416509072464*a*e^(551/2*I*c) + 76183718752941105641961
 216543842531245115661575434720951365646384764643713912959262094351323617904
 99815979231439230353607576*a*e^(549/2*I*c) + 644163742866156305750100053382
 079138921272289972515898257543865033872359000486567807935510286742308296912
 3638559458360395064*a*e^(547/2*I*c) + 5402322084276358486392170807160452492
 303736610543993756782397563451183220371719416455539109849685147683838533792
 416380394000*a*e^(545/2*I*c) + 44935456833121271857607041861967869268635584
 109539353473633873054553903182786466915175923511479421628297572836727326547
 18906*a*e^(543/2*I*c) + 370678470774816993947693992741830611713780766121618
 5577529307951968534891054102022599875800521028818591703507213651354601744*a
 *e^(541/2*I*c) + 3032358651189255296683913995058467176181044555965209522749
 027694372314500932020478507945032419489336181643123979995151020196*a*e<sup>(539
 /2*I*c)</sup> + 24598838797251652924524255220681788806897094208434929890196143100
 88740969485770831889636928119055954588263689191708179939864*a*e^(537/2*I*c)
 + 197868002648028501464643875406112310549856313023888300553555235946491695
 2802872195794689967236683993035275155224868084857426*a*e^(535/2*I*c) + 1578
 118891036425030175529775728613184825593490511183941993477628800762057890918
 137098180588823348433951309822211104517241960*a*e^(533/2*I*c) + 12479104898
 517248121534775616840297489853271667611766490178937644627425409558103593603
 45752498737439338173328941693214996869*a*e^(531/2*I*c) + 978325068291196614
 130480547884149187418948183584483275851528436468279773867175472874823772301
 653470715495906310752113906356*a*e^(529/2*I*c) + 76035336195088564664135197
 686089319202216456290233216169212648199277978799711837850220083492439189521
 5671091534427467182154*a*e^(527/2*I*c) + 5858100878400906389593525931152074
 93330409388304333648586508726254966662726172714609121636999592467661699749
 95865625894415*a*e^(525/2*I*c) + 447387531042524414883193986439269112293394
 102569954189918196551016064656984949980121125552746165717302725948575549803
 471765*a*e^(523/2*I*c) + 33866720736606402867003586937597637011103498231618
 4479753592830355581796704600824509215308211821300197041784861670006680074*a
 *e^(521/2*I*c) + 2540980115266867950178268889978628811076981782798030783703
 47724142537822230167040460676758302527381200018338794267668213782*a*e<sup>(519/
 2*I*c)</sup> + 188949118902042756365887767234643113787797409285875028353410293464
 440403604554265358653936413844588923365518512812155564263*a*e^(517/2*I*c) +
 13924532801833388077436496880085314258472825551304341643787011246963155066
 5869605343027896353023176557694400200499461146726*a*e^(515/2*I*c) + 1016916
 444209500700250240110574529973815002834665747306342838932549591884666274029
 46882962692076334471578743932496642334320*a*e^(513/2*I*c) + 735928399275644
 055040421275927628199319989286428969198110113019273287438376278450501542242

99851252425467281991751035964300*a*e^(511/2*I*c) + 527725840025912026071527
 567068898957493075370026298230158906309240214312430531353083821381541538970
 41915995771633984281806*a*e^(509/2*I*c) + 374956249662893147770179909764712
 974747825449021922176441858876940812711235132630691990752706912512822552398
 72596210514278*a*e^(507/2*I*c) + 263954641832889484740532868631201612007278
 840887085807502640735451013640607936923777157140561663978028144297636184178
 22876*a*e^(505/2*I*c) + 184090602589765024977173302569641551086417497283354
 69201164427871721771346762241189212154587072288919159051458074105345270*a*e
^(503/2*I*c) + 127193465108299801676917693560051353106337171859775948660881
 35331067145022553996096352557277491441427340072720038453020094*a*e^{(501/2*I}
^{*c)} + 870575065663996280501935754740361039921512432987213049010615850289105
 7402404433904178771049787603257660231907442902723601*a*e^(499/2*I*c) + 5902
 464865781828746433759626115731299368628038776863537919286757121919619848756
 678585765961272292609235750043165001541228*a*e^(497/2*I*c) + 39639120705364
 146651009937160909528466211708624286633319606811297764001906634621181840423
 76205657205476237354164748568266*a*e^(495/2*I*c) + 263667029360611178058778
 036417944829922323904522976409393847762620988163601336439205808532291500242
 0199553549738131324075*a*e^(493/2*I*c) + 1737027904730654746361321038484958
 368225917557506862415168862004368924008956529402849910157768607834197363252
 609518902089*a*e^(491/2*I*c) + 11333264072490566772651823572830413026805965
 657533923303560137333862675788022491010090540339333581845823600565659439828
 72*a*e^(489/2*I*c) + 732282573074945415241191984024036255668279273758317972
 537870914592731850184818289172732027744954335864633099347774743778*a*e⁽⁴⁸⁷
^{/2*I*c)} + 46855032529333593499967336909176711798578926338071141481498616019
 6520127079260704512548045596385215149194149787362058847*a*e^(485/2*I*c) + 2
 968698943123410258826147568818488011825480957182319010174427005109701322074
 73697085843370374068409734287215730709535159*a*e^(483/2*I*c) + 186246058051
 488407504035604030386554718066896118691201386319926538759246009834321804025
 886034942775247389666416855412210*a*e^(481/2*I*c) + 11569053075069444158976
 287555701878043931241423534680572445064585687921237440396748349767763604157
 5439277920548968229136*a*e^(479/2*I*c) + 7115037980318233062499822718689669
 240619785943405408448288067061432154242494654628517230634952204878963155013
 8215923153*a*e^(477/2*I*c) + 4332158318999982001182551353596115363226170993
 6204250859944426152513525897996934481741824822705710882414313119272790131*a
^{*e}(475/2*I*c) + 2611308638642497861940627208748348431168677835346981028004
 5602451813616101635095685826358744734323495284346010214813114*a*e^{(473/2*I*}
^{c)} + 1558181593426048285354507841921955212887518287711054373068632026342500
 1461567811411263192244527545355522679818707520580*a*e^(471/2*I*c) + 9203715
 338102052013953994655272327701169506976135302561414981599329671691132074752
 022832241424911916459710600587437561*a*e^(469/2*I*c) + 53811222485671624211
 573835697566302203282933979388379820118345988236397087116939169806464416422
 10742072277442692795738*a*e^(467/2*I*c) + 311405651550129739902696800285085
 492813895466455247866716835158661279717661825298130844832082891577630780573
 9513112726*a*e^(465/2*I*c) + 1783624738647145047601917556451061887933853774
 354141392528926855412119981497793799270417127134990753415746380839743700*a*

$e^{(463/2*I*c)} + 10110748001149593767195366249770947389642856046430627991117$
 $78942015387325706624278964496575270768601912488833296828570*a*e^{(461/2*I*c)}$
 $+ 567211894570524721264934335244788478905608726318730689979939994574065235$
 $928861643029786499059347466854077954904040746*a*e^{(459/2*I*c)} + 31489738298$
 $943158333268009682716291953167330247168046064002114776518538829900520788348$
 $8229333447615352727288983946148*a*e^{(457/2*I*c)} + 1729951237565501066373781$
 $323570103064875660499788882050882654953322633031655037800663054086686876989$
 $20139970094538712*a*e^{(455/2*I*c)} + 940414097395855760889319663270055553239$
 $711080741367215796434630893641710184566990485130268122231684405765166286835$
 $14*a*e^{(453/2*I*c)} + 505829522181672507409706444229176144626319674426544321$
 $42347921323029515220528364018397135511063228424091734992672055*a*e^{(451/2*I$
 $*c)} + 269195769225792606752145769442826156530587082608618056157594984212372$
 $41977159288165357847803942410262265158465467750*a*e^{(449/2*I*c)} + 141739356$
 $433255984532024123966213171732858939160760735728499708299582675917379072866$
 $19219624773200857453140746381674*a*e^{(447/2*I*c)} + 738328956991590596622744$
 $469611098349215000500983969326803957345776259724464129140684379227866352435$
 $1127277610068501*a*e^{(445/2*I*c)} + 3804744464968921359015461873113836781159$
 $840428541963397786141899308945274545366930937702520484566257179138022478559$
 $*a*e^{(443/2*I*c)} + 19395278696913220572173470105895471941193523887881932903$
 $66880726140422435916649514401171793242696849555174393643850*a*e^{(441/2*I*c)}$
 $+ 978002873261192283457582949969456052834946311148564091945480083912375742$
 $750095496723188070578970297191691801222556*a*e^{(439/2*I*c)} + 48779380400401$
 $382500929622160999761682089159915899901122667249012985724500743387378716262$
 $9626332383304760538813509*a*e^{(437/2*I*c)} + 2406371921769259955547697638221$
 $965496951501064127678402468060155887850014665497981167316875495397431602066$
 $90512276*a*e^{(435/2*I*c)} + 117407982036395964616775103428880300577811655638$
 $673060994605539426451393694076705057517258363475234790358732617874*a*e^{(433$
 $/2*I*c)} + 56652457509019217332809832253393765490026446523156131066467177640$
 $614737878543683747754276689674112796652581318416*a*e^{(431/2*I*c)} + 27033592$
 $530880002226737465321359138464847484153982687697594699044738257454819138210$
 $737641742661588856574025344360*a*e^{(429/2*I*c)} + 12756455617841391513914384$
 $728898840042725661113668791846621927808068785990490077238948492719552936499$
 $900344230684*a*e^{(427/2*I*c)} + 59521604667866061656535562465898355618400210$
 $49006367634292596565935280818195141374695236251836967708599780176746*a*e^{(4$
 $25/2*I*c)} + 274608771304596649275981974705753453729579485114252190135088406$
 $8686972703385462885090913563610742000426952431144*a*e^{(423/2*I*c)} + 1252638$
 $795847466913766125006669625873907326637843208199173827808400459890232296094$
 $362170239873318861004896614156*a*e^{(421/2*I*c)} + 56491770603803702788592000$
 $743376407605164806140939010812663502809191357241464734717803992357829937985$
 $9935849404*a*e^{(419/2*I*c)} + 2518652504842815458942381800170935956107969118$
 $82271999250070181159518716093352404254337261360592129286243867720*a*e^{(417/$
 $2*I*c)} + 111007051148402125345220283714271173982629035385223122646805636583$
 $654935893527705499942693423715121989424818940*a*e^{(415/2*I*c)} + 48362336013$
 $775503343319693154600872180208893665329972255007464669036745521843710761901$
 $688665256224358994271700*a*e^{(413/2*I*c)} + 20826348139765728145502300250230$

633528625368811037926283789182837817572337323284403311770372970302291071410
 100*a*e^(411/2*I*c) + 88642599821438870660522413394953025195926517437063956
 09918105114241642344626154900178145218983856632762282840*a*e^(409/2*I*c) +
 372880174760892654955131903796142932744740463178835705403581361766541738802
 9848305480269822322115047563031420*a*e^(407/2*I*c) + 1550126189251322091456
 566293321041304661332373605301977703681149593242237032310585758452049979836
 332182019460*a*e^(405/2*I*c) + 63680845082005250841339490337361369940898513
 1793378225322630926816010278188664354873365849405050807559590630*a*e^(403/2
 *I*c) + 2585039414274187759834493582572271293271870310254053523049461585966
 17947732241690180138609024179619896548080*a*e^(401/2*I*c) + 103684586953158
 052661761002194856220016078202175436674022417030738316048059257865332989827
 933134737432425420*a*e^(399/2*I*c) + 41088615308485898366172497268874114351
 478780113160576839336545240589947982864125921136564058023293113177890*a*e^(
 397/2*I*c) + 16086447516084063163243689050215879761267599809808958182816872
 155074721707033313657170678606565171911318790*a*e^(395/2*I*c) + 62215793900
 366012619065913586228573658637652713363638132589281020472312697012877444357
 51925676619466254380*a*e^(393/2*I*c) + 237691127497847517000373213623578818
 1859385653298280842814105754624463620356183384021592556492451101686180*a*e^(
 391/2*I*c) + 8969464154494950217733056521325214891436833880846813774908909
 52178768366949153161001917497973558984634610*a*e^(389/2*I*c) + 334295101291
 598394405385926974923341821234439369994003970291001771339818244857969207842
 071054369602329440*a*e^(387/2*I*c) + 12304718259132723929105983780313299023
 6560951541426091502592133079889930820736735976930062642304445760760*a*e^(38
 5/2*I*c) + 4472589130147944961802165361302184092582083105216503572494686521
 9883816662123055202896018457333687900560*a*e^(383/2*I*c) + 1605309814686536
 657061422698593617514802846668574542867033494369104831413462797774748878288
 3668338627720*a*e^(381/2*I*c) + 5689019471410220246066908603484004351329982
 673777354247661562524221692115006358198880775065147099437120*a*e^(379/2*I*c
) + 19904906928755272586672364990174607843663079232802966549317359979549411
 47793207299041775080046145853840*a*e^(377/2*I*c) + 687530224472620092791173
 281345497074926271430076019616322085361144879429550058409565871621454060233
 940*a*e^(375/2*I*c) + 23442090586166750167351495144027547182182831996259574
 1737151538794823588356166465622241503448244518760*a*e^(373/2*I*c) + 7889273
 247555062920716015061964951885967133305942536387072760442653882061519707285
 9753978543934017670*a*e^(371/2*I*c) + 2620451831207455850865238131984749663
 4719883744031651088258490824465738576863175200241054655253538000*a*e^(369/2
 *I*c) + 8589650067614913233196096604074597492566547822453765715081979725077
 156239293288377277499477542060060*a*e^(367/2*I*c) + 27784053052143436342726
 424781153687621685136571720170670300841434525834573138856560930290576314443
 70*a*e^(365/2*I*c) + 886740231925532664522771055553453849969744152256079998
 032209234008019622116815785290576730828104950*a*e^(363/2*I*c) + 27921378124
 083730626314923927774127850418076481421632076852312654511226694323086992504
 5964255370200*a*e^(361/2*I*c) + 8673111081249706024196404770369878197649324
 1635614918587219564911585800789497248326704733021564620*a*e^(359/2*I*c) + 2
 657465386577270594551688425896691792857739954492665110111547126128141566277

4645920805562286190810*a*e^(357/2*I*c) + 8031025392884252064177450987879092
 832928608062990864461726329576684085419875495358549722887030650*a*e^(355/2*
 I*c) + 23935364531901942975725216426094858411453720475694518714819436739240
 84228578119992726364314043020*a*e^(353/2*I*c) + 703442619798587327256518771
 953962641129807006145552665167689345204662084900828221826842429079160*a*e^(
 351/2*I*c) + 20384013942017335064659986351963620564829591915624076720589395
 7321010048179259062568208312088790*a*e^(349/2*I*c) + 5823375669931839000990
 9172367611895685773867826783550649200500869687498091353927199571718405490*a
 *e^(347/2*I*c) + 1639964574077305109297539230974478918330472762067373363527
 7139746257521706440274686262794198620*a*e^(345/2*I*c) + 4552164541985701158
 364440833202197360445014839710956970770340240973634689769418208918160590320
 *a*e^(343/2*I*c) + 12452979254554264676850036797832029308856176005577165004
 73909654045354638798448652726086548710*a*e^(341/2*I*c) + 335696720738936570
 706713403614929944912226507516523422878616118942760950780230341688669154360
 *a*e^(339/2*I*c) + 89163024667528844365257556946977068950919875040439960342
 188615542362550926345388079651264340*a*e^(337/2*I*c) + 23330834917631715093
 466481420083833029947668880513498966378137598401377993510086729958534960*a*
 e^(335/2*I*c) + 60134824039422697813758951361927203212332666389054962148661
 32974340762298443025559536997680*a*e^(333/2*I*c) + 152655711757003662257191
 8876778252127468437541774623614619302679658497516840009278849352280*a*e^(33
 1/2*I*c) + 3816196554942207746679498333824753427233855339744868562430987503
 70767001096283055945502640*a*e^(329/2*I*c) + 939327132073952654868978803666
 64895741093700695924344009200350068922566762007052749646040*a*e^(327/2*I*c)
 + 227618504270416638904609966614057231934071839472331638967008532956306814
 34965577357299920*a*e^(325/2*I*c) + 542921694761861258754712301248016565846
 4605296922357528675368754249200100593150293185330*a*e^(323/2*I*c) + 1274495
 521071138744583215835633951280803025426238229370838062367497287195705487702
 727780*a*e^(321/2*I*c) + 29440256424059206189496939686815693205792480390369
 5357306824322577258371768427288287660*a*e^(319/2*I*c) + 6690764342465983818
 4545773924908879954701525687410376227199214504402274227244630158470*a*e^(31
 7/2*I*c) + 1495778983626003505372028868912194529719427850861535500572369524
 1121674541179126990690*a*e^(315/2*I*c) + 3288828495006098780525142569296800
 128547751347678538502742954742586652543592821521100*a*e^(313/2*I*c) + 71108
 167343618545192404079366914129896962672279661439574280236126609363364126408
 5200*a*e^(311/2*I*c) + 1511549102573694346925967358088798515844042393077900
 47602041275902443495292459666790*a*e^(309/2*I*c) + 315840056627086045447017
 64762131792650967524020242941992643782185239861413590496020*a*e^(307/2*I*c)
 + 648588875944189426448192545257731919037580329366533172224855354136988829
 2250431868*a*e^(305/2*I*c) + 1308704744103233244942832711379475161653066000
 371260361064765494153774929304542712*a*e^(303/2*I*c) + 25941410534375861438
 4429045574352903181961134382680334119395735344643117364969828*a*e^(301/2*I*
 c) + 5050474895167495659424171916502050073964613004151143876733776676640401
 8721799652*a*e^(299/2*I*c) + 9655206906308981051087661502772471729668550920
 396172616931546671734191430604348*a*e^(297/2*I*c) + 18121018304254363008917
 20572628678363042354198555147726701275150075546643583080*a*e^(295/2*I*c) +

333805055593388567427808201926960940156572529198077839859476641115104007423
 532*a*e^(293/2*I*c) + 60337422821108905442188805237705380902156538573067777
 647643655256342216048188*a*e^(291/2*I*c) + 10699286061146541741114037718913
 122166739129443783960920451139637868310953672*a*e^(289/2*I*c) + 18607325088
 57587132839919523366305804938874482315206088251791750125084597398*a*e<sup>(287/
 2*I*c)</sup> + 317290565438299295944227981890447073652423258243355469512774650547
 831808732*a*e^(285/2*I*c) + 53033862655943444530651471417311659093748967671
 410808199772545872566250240*a*e^(283/2*I*c) + 86865377059570157468745566475
 03611978485575286577500733201504080157038368*a*e^(281/2*I*c) + 139382220475
 9320545751155434910006366369348464487459381028813178904280462*a*e<sup>(279/2*I*
 c)</sup> + 2190283359992862577763696341466137131065096478617046774104884544710418
 28*a*e^(277/2*I*c) + 336965483328793735102555813785378142355585023148641449
 54037409976543435*a*e^(275/2*I*c) + 507361214027686902278581069778901395214
 7928175584559508062569934520500*a*e^(273/2*I*c) + 7473855515088898008257285
 92465333469870510137536130275665450524715478*a*e^(271/2*I*c) + 107673925545
 674144588693096021112687136443322698905146090803710183409*a*e^(269/2*I*c) +
 15165308800486166402472766122974437831930120746836471291819780637051*a*e<sup>(
 267/2*I*c)</sup> + 20873559405129506084431058976989204103021620800678368208217712
 80502*a*e^(265/2*I*c) + 280652429583871841898555612150376650333075795788342
 413441360641690*a*e^(263/2*I*c) + 36845376622432877842558414417003887278048
 975820601095126064678425*a*e^(261/2*I*c) + 47211285515018981347023498996790
 82580779514892160753741261926062*a*e^(259/2*I*c) + 590140408754311076687013
 315279638319827311377295800031121209656*a*e^(257/2*I*c) + 71928400911660376
 402320318751672710160518473805095916511887372*a*e^(255/2*I*c) + 85439741194
 33407159975166243013026279990985549262801022565070*a*e^(253/2*I*c) + 988558
 268629814228600928047340864800588772545018200620775758*a*e^(251/2*I*c) + 11
 1348526291800411107246584226952800505326877479452354210332*a*e^(249/2*I*c)
 + 12202571647480368478947339347197542554373660129766305670026*a*e<sup>(247/2*I*
 c)</sup> + 1300273428594384632178804915869348076492496040528026941982*a*e<sup>(245/2*
 I*c)</sup> + 134633163152791085906227518959644714629575426941345274855*a*e<sup>(243/2
 *I*c)</sup> + 13536481999851915706958049067829203397030845591800979948*a*e<sup>(241/2
 *I*c)</sup> + 1320632018182734050434490769597960546607943021209554374*a*e<sup>(239/2*
 I*c)</sup> + 124924620764273645058915750915973878220098322691297101*a*e<sup>(237/2*I*
 c)</sup> + 11448615265227028920332020054872464737892837624091599*a*e^(235/2*I*c)
 + 1015602799297538681881502332438578214644838744212768*a*e^(233/2*I*c) + 87
 129450753762477310055827412704326695373797670990*a*e^(231/2*I*c) + 72219590
 08399380670648524146893979780310767551977*a*e^(229/2*I*c) + 577756667439166
 347317361298827893686038031023201*a*e^(227/2*I*c) + 44561016227553011590271
 450799436600978561400862*a*e^(225/2*I*c) + 33095712972292293584387195957811
 23324700153528*a*e^(223/2*I*c) + 236397938386786552298680619962344215977635
 255*a*e^(221/2*I*c) + 16217272197080856640086895687618882654237557*a*e<sup>(219
 /2*I*c)</sup> + 1066925770347513486390409808978976432165366*a*e^(217/2*I*c) + 672
 07920970133725538564726906882769461732*a*e^(215/2*I*c) + 404655014276814282
 3441656912104576646575*a*e^(213/2*I*c) + 2324389084257266652098070964987750
 79690*a*e^(211/2*I*c) + 12711502674847529112148726218999140522*a*e^{(209/2*I}

$*c) + 660337796328597830395723099421064436*a*e^{(207/2*I*c)} + 32503673735785$
 $850571258452122985714*a*e^{(205/2*I*c)} + 1511798772576673047875435387479418*$
 $a*e^{(203/2*I*c)} + 66238605837791619527049091465540*a*e^{(201/2*I*c)} + 272446$
 $7073240049286222217499520*a*e^{(199/2*I*c)} + 104787195000667085382630850098*$
 $a*e^{(197/2*I*c)} + 3751971173691706110924034857*a*e^{(195/2*I*c)} + 1244276153$
 $97868362145478058*a*e^{(193/2*I*c)} + 3799316499714931382700630*a*e^{(191/2*I*$
 $c)} + 106072288048394569716987*a*e^{(189/2*I*c)} + 2685374380789029329793*a*e^{(187/2*I*c)}$
 $+ 61031235925207244502*a*e^{(185/2*I*c)} + 1229848582862227068*a*$
 $e^{(183/2*I*c)} + 21630502713590667*a*e^{(181/2*I*c)} + 325270717497440*a*e^{(17$
 $9/2*I*c)} + 4065883968718*a*e^{(177/2*I*c)} + 40557446072*a*e^{(175/2*I*c)} + 30$
 $2667508*a*e^{(173/2*I*c)} + 1502072*a*e^{(171/2*I*c)} + 3718*a*e^{(169/2*I*c)})/($
 $e^{(531*I*c)} + 432*e^{(530*I*c)} + 93096*e^{(529*I*c)} + 13343760*e^{(528*I*c)} +$
 $1431118260*e^{(527*I*c)} + 122503723056*e^{(526*I*c)} + 8718181624155*e^{(525*I*$
 $c)} + 530563624556832*e^{(524*I*c)} + 28186192554792138*e^{(523*I*c)} + 13278828$
 $49274858880*e^{(522*I*c)} + 56169444526926562260*e^{(521*I*c)} + 21548641447812$
 $57856128*e^{(520*I*c)} + 75599817092670157806639*e^{(519*I*c)} + 2442456298945$
 $02983849104*e^{(518*I*c)} + 73099207817335597247098038*e^{(517*I*c)} + 20370312$
 $59470368160131922320*e^{(516*I*c)} + 53090127264630963470039804475*e^{(515*I*c)}$
 $) + 1299146645993240318167826532288*e^{(514*I*c)} + 2995254774926549967525784$
 $2032197*e^{(513*I*c)} + 652650253343206047453620559993840*e^{(512*I*c)} + 13477$
 $227799524701956579274210395326*e^{(511*I*c)} + 264410375780310742518099326419$
 $685040*e^{(510*I*c)} + 4939666610818025798809586352543471345*e^{(509*I*c)} + 88$
 $054927598941411145869950813388040256*e^{(508*I*c)} + 150060274793739728640557$
 $7818722691539392*e^{(507*I*c)} + 24489837337812338687718622491865013839488*e^{(506*I*c)}$
 $+ 383360155801054824529764688213114368047154*e^{(505*I*c)} + 576460$
 $1046563151304213854710715346838447392*e^{(504*I*c)} + 83380839911837894453136$
 $303673785039051506805*e^{(503*I*c)} + 116158141373397175153362251190904691718$
 $8768400*e^{(502*I*c)} + 15603911277687607099721623771744933086920587272*e^{(50$
 $1*I*c)} + 202347509724462171313966643580234078508179838320*e^{(500*I*c)} + 253$
 $5667460650279776834561566186591213109251642859*e^{(499*I*c)} + 30735366512830$
 $562160991166338490057308062762518496*e^{(498*I*c)} + 360688613036389349413809$
 $780004559963548775423325255*e^{(497*I*c)} + 410154543993719579395995670844249$
 $6709433800261224880*e^{(496*I*c)} + 45230940039830738332025694784646206844854$
 $827698075736*e^{(495*I*c)} + 484093410240488718655917025303662581091659126182$
 $344528*e^{(494*I*c)} + 503202490340145182407421394376601192202650700631198275$
 $3*e^{(493*I*c)} + 50836369508171099437019348610847391946736185108017183136*e^{(492*I*c)}$
 $+ 499467506558531733671585862910572702811545035730398749530*e^{(49$
 $1*I*c)} + 4775398607100853263534207733818266777478693412738731031680*e^{(490*$
 $I*c)} + 44456708175258821024400946210535004523775722190977468484496*e^{(489*I$
 $*c)} + 403212225957798188840846139960995624144491271694336796459584*e^{(488*I$
 $*c)} + 3564764890628724017088487996688178929195787613958545474804845*e^{(487*$
 $I*c)} + 30736217404321009965231037419663053962881035281709221697785072*e^{(48$
 $6*I*c)} + 258585348715977270155829115684193411072034541491364393985491350*e^{(485*I*c)}$
 $+ 212370296918887131826671878122392706783994901572729388406538808$
 $0*e^{(484*I*c)} + 17033886027390615741040977721655541665612162275485028584310$

$890417 \cdot e^{(483 \cdot I \cdot c)} + 133490210052026183779673313868332303530332906163247194$
 $627808410304 \cdot e^{(482 \cdot I \cdot c)} + 102253643746829673729306586270524644969368741555$
 $9865844306888705423 \cdot e^{(481 \cdot I \cdot c)} + 76590105201875496517771183576768719270818$
 $98989131125755798204236112 \cdot e^{(480 \cdot I \cdot c)} + 5611708107634117538408757018518853$
 $8660375932013674735519055227368366 \cdot e^{(479 \cdot I \cdot c)} + 40234969226612115893400358$
 $2839428785116904903936409545602519219664720 \cdot e^{(478 \cdot I \cdot c)} + 28239051519365866$
 $78382525706564457280290098698638597987628380245881715 \cdot e^{(477 \cdot I \cdot c)} + 1940797$
 $9215594566593535008103303255257745408070082431338945184797463936 \cdot e^{(476 \cdot I \cdot c)}$
 $) + 13065766022656041933512143438993896188459543406998482430714933213174754$
 $0 \cdot e^{(475 \cdot I \cdot c)} + 86188485109499190876424680547467242860375731548445397471361$
 $2812215428992 \cdot e^{(474 \cdot I \cdot c)} + 55725511573286711210162164163075961618619559690$
 $11697222340926210112854418 \cdot e^{(473 \cdot I \cdot c)} + 3532444720677901811537805282078941$
 $1687581004582367431006205879633729015200 \cdot e^{(472 \cdot I \cdot c)} + 21960128133951556150$
 $0261478844190024870555261281946058839614044697037963695 \cdot e^{(471 \cdot I \cdot c)} + 13392$
 $14374254245553564884406801945353385000254030655765953770237607180089968 \cdot e^{(470 \cdot I \cdot c)}$
 $+ 8013729580790752434361964945761543761469520791210746972675870481$
 $058674277844 \cdot e^{(469 \cdot I \cdot c)} + 470650446111351581084873533674842431026982488383$
 $12635876283099427442745866704 \cdot e^{(468 \cdot I \cdot c)} + 2713612075032665707344865170771$
 $81014801775322183181055638619257836143271472358 \cdot e^{(467 \cdot I \cdot c)} + 1536333238444$
 $927583532734556016494671674916578907116984548489078241693926940560 \cdot e^{(466 \cdot I \cdot c)}$
 $+ 854301344112621233483354066506962147247908583804136056455072203672365$
 $4297540205 \cdot e^{(465 \cdot I \cdot c)} + 46668223548266017806854592468100570289355960869613$
 $650856575756758180182223308768 \cdot e^{(464 \cdot I \cdot c)} + 250501028608928332469340456829$
 $902067712233644464602753159945727868485722395506952 \cdot e^{(463 \cdot I \cdot c)} + 132149805$
 $5271300851429993866631619874424534425188183592049727687571032156435077280 \cdot e^{(462 \cdot I \cdot c)}$
 $+ 68529932231457366873288853116177954355929408414398663510796556$
 $52312894721972796266 \cdot e^{(461 \cdot I \cdot c)} + 3494107161327670464947794304333945020150$
 $4075335160361865916029213860778606230624960 \cdot e^{(460 \cdot I \cdot c)} + 17519317050061830$
 $0241515632381912285157790097816049220671217212220015297133400636060 \cdot e^{(459 \cdot I \cdot c)}$
 $+ 86397993362233034955629682002839551319870806494050570212606865293680$
 $0794826651264256 \cdot e^{(458 \cdot I \cdot c)} + 41915425006568261480933394145441591439644784$
 $72492315931809171859902114109005939942952 \cdot e^{(457 \cdot I \cdot c)} + 2000800680303004713$
 $7293278250321597113540716201983333126349281186679153199068045257216 \cdot e^{(456 \cdot I \cdot c)}$
 $+ 93986915313068179149083606065681482780836060510530154618486949839467$
 $131378859885998210 \cdot e^{(455 \cdot I \cdot c)} + 434546676780280045346344498763892540797175$
 $105756827515509297024187660299345484920192480 \cdot e^{(454 \cdot I \cdot c)} + 197779298066581$
 $8135651300094326239158605448870806970860577325385028609983034534672318500 \cdot e^{(453 \cdot I \cdot c)}$
 $+ 88627521427569572856813408857649045979353495693553218156477211$
 $72537159186491471311666400 \cdot e^{(452 \cdot I \cdot c)} + 3910803125560180947653753536961184$
 $4440844903751605645023514572352045248104262933598850730 \cdot e^{(451 \cdot I \cdot c)} + 16995$
 $632796992976777390209665262925328370450547712754455653441737668654093670607$
 $3847337600 \cdot e^{(450 \cdot I \cdot c)} + 72752101071839422929177407384469425579873866706753$
 $5379759732795567942578751384250780476310 \cdot e^{(449 \cdot I \cdot c)} + 30679742964317473641$
 $98159623962463671617006419626851426148418602934852907379021659761911840 \cdot e^{(448 \cdot I \cdot c)}$
 $+ 1274721961650332054135634306256284736860162214085678602544581453$

2037904111523242298235713300*e^(447*I*c) + 52190912207661824215812271854269
748071292843243227894769229690720010547141334131610989636000*e^(446*I*c) +
210594301385648471184329078880317504953361839954159427434009884661777259752
542647709150036990*e^(445*I*c) + 837579206923411932458786486765373533946545
239708990769488724813982189165104589895518909256320*e^(444*I*c) + 328387476
055581867672630948030673442015509858394807446901416817187444217010964852162
7538755920*e^(443*I*c) + 12693496932964920565073673637181280088548682508880
255337280065006566138696041797353216584528640*e^(442*I*c) + 483794897564340
998438577918165893794068150426093403787475864371457816462454220451012304173
09900*e^(441*I*c) + 1818346614061779013153301296771453811664491884131941411
69344354754920969034952610378945282257600*e^(440*I*c) + 6740255305431330088
948457752366252374507431144735445378181704471346071025756766760566753289615
90*e^(439*I*c) + 2464382190807439609079774226855679629367885709776435876630
851716253962696192341706239192878728160*e^(438*I*c) + 888829502875102466704
420383760797610148005313441861447462076752282486891195988435266644491740400
0*e^(437*I*c) + 31626644674725547731176795687527653571305969985923688392112
164915553242573269490908989570248533280*e^(436*I*c) + 111034148797008819443
143895644469242295049867464313710969257619338899133799285616020069872611710
850*e^(435*I*c) + 384655842080666274454063078784837174998949052500975322162
003392549953413592461519365177908682078400*e^(434*I*c) + 131505212093069212
210229710532762284233587074342853089107298353586228009444660772347380047745
3914130*e^(433*I*c) + 44372109178431823477643495444439046990200565950694708
47193617092114714077633077234972825351226979360*e^(432*I*c) + 1477795509661
712899871274518207149536217650697318308165023360527405167762497046434024275
5840025673760*e^(431*I*c) + 48584258153140280447314836868772131390195412419
046732778458706015096881437076337910793584122475073760*e^(430*I*c) + 157685
845528850918721462877864435090257583149415561323427386562894447598277935629
800939237175625149830*e^(429*I*c) + 505293663123015258878483025738812813203
397766845340065381261016353419722382620393032535960660921950400*e^(428*I*c)
+ 159877110105819269227052899967744474268563100623245618584492522014400230
5878120380828483988663574829100*e^(427*I*c) + 49952419562791381802051867444
01688024388272113921255663734956946927571305533146776898787878059685108480*
e^(426*I*c) + 1541311121148602393729497082079737671608134478816338654352242
1939737507962125854981881879168348260330000*e^(425*I*c) + 46970224727117281
826454045018070670522559756627580347784535320014963482632359729444541885102
274546002560*e^(424*I*c) + 141379938253556843280565505807403304130606130725
434751745794079833141361748917639986145377066437210546190*e^(423*I*c) + 420
358024835146798583611210145942154684437949365647899088372524802156222884839
580011688655664280691773600*e^(422*I*c) + 123466804189240997878001808175544
0216012582476396941937965899631953079203974222138794604328498972144766900*e
^(421*I*c) + 35827180021632960614145367037151098971071982527392845461493431
02348456124089657428594946438660859773886240*e^(420*I*c) + 1027160253020288
900249781351684945259097151280952906066519730109705221006457608834802323467
1975463677418470*e^(419*I*c) + 29097651061247453406647569781836910062165559
852359052804259154165687125428752562385492373749486351714453120*e^(418*I*c)

+ 814520814138291118288754175642500548460376933128114804929091607581959891
55768107022568350953861815940704090*e^(417*I*c) + 2253205325932206577679411
092895162489997945210155641349828272417100196754866944996893124665619072126
27820000*e^(416*I*c) + 6160031160229795849371257017578872129983543009899913
62628038861093914561332071191909714949426587936910303300*e^(415*I*c) + 1664
475034387211809394917743502937638978574937754764763987835872410449930690131
572904279995484581013965001440*e^(414*I*c) + 444541225929547462506765951419
831296601541629996893039334563034591410972074057361888498052001002845149699
6210*e^(413*I*c) + 11735856926245118493113091002501604032341876985999082823
520672241530200188223826392982302194084667538488665600*e^(412*I*c) + 306275
810542219573783905472892776091295728139310827335202473872260000200435382794
68776707958420892547870128680*e^(411*I*c) + 7901914955876656925478398848723
238835290914498274717185677246322380899336709150340287646727017612434269965
4400*e^(410*I*c) + 20155794742479409802677247846204088339586751256293086294
3753568690084015585598010154781548625239409581907397500*e^(409*I*c) + 50832
459930108546016697862968303266142765447408293904809793963839156729879578838
9433842285751054665210868287680*e^(408*I*c) + 12675970172948129134001462760
421269299864802928701903991075543110799642272801965224753701087384778563117
65699610*e^(407*I*c) + 3125683493178701743479704750307490178666292150720179
363604335113528623329684606343185540756019935662148267863968*e^(406*I*c) +
762178879191204706203884091779937460042889225819436763668294435609668140024
6312138001769285020661445991073249416*e^(405*I*c) + 18379807084003359766027
649217621144116091735572216620788861535803449702273802588359076704241840733
513439114113248*e^(404*I*c) + 438349721429193776853786922330210637445540331
00928502737480438978976746989895784070951905237783490374305934542955*e^(403
*I*c) + 1033997554672574364898478376407537547182043944730557950014676043264
1987655556873829531737211096115196005647730480*e^(402*I*c) + 2412460212824
400617929083177830328761948015971332060520912869970437291453457558057100814
89006741839439573984832678*e^(401*I*c) + 5567563887111823403410261927342195
461136517683173805390058936790493947140170636985652727288136690547790772089
77840*e^(400*I*c) + 1271033082938048950201360554831270342662343991277504612
342300366025046741742856580445289401786656311685859023084716*e^(399*I*c) +
287049613141231445183467471535358943955329443080853193346608628854370924623
0769151392180699413405623017247753532944*e^(398*I*c) + 64133818958559251847
582314510625563803285949385110065770152181197865363902130572840182022016310
94434819584025113465*e^(397*I*c) + 1417648365287570495720201334324111790436
997779665384995990252442198063518901163481565327960549778338288893276673008
0*e^(396*I*c) + 31004319206069417077069363141423487431828009098184744635678
652284177439464941651812564519144918003174108077634846014*e^(395*I*c) + 670
917061305296691250198992100215765802378434622295353862950870761892978499959
31360645605292130961496106707521506432*e^(394*I*c) + 1436576871380447969429
471197042595384588181994235168246745862936910561192098663581236377722454095
30799230553767222252*e^(393*I*c) + 3043844711068133360102841601239063704338
888284906274226525512369667909161745208577591439301401871734923949819082589
44*e^(392*I*c) + 6382188929145337415058063996624885560667836004960918764083

74974877448971778036074996245581124283460438065182071976085*e^(391*I*c) + 1
324311324984027428355222938147682378672860708817161741448749689593588020860
847508703702325320304649883120684987556400*e^(390*I*c) + 271958928348374392
604080510108034192124453031125460725092919277390933152322663503581567286256
9296693711643521070331394*e^(389*I*c) + 55274988490311783558612300093266682
839262900821584671180006985027193799390459183442221927421457112570280409740
74674736*e^(388*I*c) + 1111949964536320108088106282488633849242537565844897
7935535846349290425821570383090425411418521516670371372045206568345*e^(387*
I*c) + 22140735001708603270915180769241391662035578755903979148909213603822
554792749183517160255571915875356439553717130797888*e^(386*I*c) + 436383000
758151710259462114644656896189657734886609858579456574798540851088518579112
22911989837615452608512356008400295*e^(385*I*c) + 8513953323478645577958995
946490063776073572970562122138000583720836915779467304967542879981787543143
0246332625899630160*e^(384*I*c) + 16443750067690689274132326015439427850395
456193602013359658180644935724027734944792733451710580999530009354927993127
3178*e^(383*I*c) + 31440903580822586156559543693835494544547399104312972204
6747228813030925204968503418566818838611866709807040793495364496*e^(382*I*c
) + 59515761550043151494747928233654705382792608791642526349718775702941338
5471835434198246807096214536895441388306027237899*e^(381*I*c) + 11153982855
456015505333280456001843179938993592179967298183407042218011956674104858469
96179056733558512452238583160792512*e^(380*I*c) + 2069698289500860643461665
762373807957513019424041178871904960551829412449344722432125679417958403007
551179298315947373776*e^(379*I*c) + 380260499670589110696462063384896480703
709885451018226324303059729563076035359753197432475226638919318576087827418
8013440*e^(378*I*c) + 69178389452148442784933304593613949233723338536198796
37372673184942859712431066345726870422099893124890777678037369988150*e^(377
*I*c) + 1246214044053725808492859670987206685775707094312486855450094815475
6863454308032925408340311237850017814707896986969086816*e^(376*I*c) + 22231
341131801535345406399037721686840208397941952580135584645966746736656716271
554826476282991066076564921432614339399735*e^(375*I*c) + 392742004143298611
693979445162250010812274333985850072063992312119071577953597196482415987542
66579840244551491476467899952*e^(374*I*c) + 6871246601598564151246858617365
974773487959171009835465278612493602307394314104957360664856300535941171276
4895683903806088*e^(373*I*c) + 11906059184966054683476569322767644906758414
824888267844750482607723633344451345409512666875005729581119164335690897219
1440*e^(372*I*c) + 20432555726518600076740271023084789645976158392276369823
5433212833313077783041040074669379017394836761539649081690630811665*e^(371*
I*c) + 34731005381093529041945556055595731412956921073574598323436965997641
3374774078000173070075248654524917179128950507443058208*e^(370*I*c) + 58474
957368230458617938462884488332758149896988654038037889676799907561496400717
4600811092945356635118795824799369716742109*e^(369*I*c) + 97521033944404931
875728231176351778667322317559445794638327926463508504100491730029590427543
3144848532459919875479817581584*e^(368*I*c) + 16110925414000605259548593752
641941783476434718370782014462624356151429445873378335135860227298495233584
36493586042252995608*e^(367*I*c) + 2636662410430799340447522284778244283740

751068658140726576446671207798325606832295937705061686297930296382338892574
 900819440*e^(366*I*c) + 427482690772059175252671133682087150084434564792238
 5471534359333606189571832444641364132893108663576205133870672156264164115*e
^(365*I*c) + 68664253375186682626626937509089569657329245781421816306221578
 02899880874681551031136314064199948604001529894566235238597088*e^(364*I*c)
 + 1092721060347354481027979234784453607458889680623004111008954473160586314
 6104181739039426674855453466097402330688331845602302*e^(363*I*c) + 17229502
 824367647334400721998417596703948657394738805209391636597370380572398964715
 080095366818322029152193635869784095333760*e^(362*I*c) + 269177947010866150
 978901202368905011051467999960217751957108662262286389847034568326941532306
 11607263444183501026198563419616*e^(361*I*c) + 4167044037539054364341821934
 227174804003507149011908058528152249818881837590690036870123453130463316344
 6319945130196476913600*e^(360*I*c) + 63923019433761989090614801288635098123
 199445102303122616544648208998767803944455777042886552738499747183713136069
 104651812215*e^(359*I*c) + 971730550247426800586167224613688926611412955402
 634930130327460835361573242683333904003089583183702191548871697022574447561
 76*e^(358*I*c) + 1463904484563511812182373827403741241916648199977469880765
 98391862733629670142241546375533903130605297580105675355629160198162*e<sup>(357
 *I*c)</sup> + 2185631666596493122474836409562721492124991153838287710296542833639
 72585118090479413696638108156385244646591328454425745117584*e^(356*I*c) + 3
 234131780148410037141511382460791525763609760350584578904099377387231715370
 36573043681997163745313602400139153046673668433091*e^(355*I*c) + 4743230435
 631005423773386299319662481291759769823324460180560093911540204388959031408
 22967769494019446166779954024655344116288*e^(354*I*c) + 6895184493287935599
 032604181499741902535783400588950355896064682446805915561181703040050375636
 69880057908765898949268614772285*e^(353*I*c) + 9935556536495211272264439608
 202336493864885100818925458667004440966615827904412418308556095770620395556
 25090943332264901780720*e^(352*I*c) + 1419164481422176573858234013898999628
 82232233309573730716310743838935832201454293617293175086458389621425307051
 750612129761498*e^(351*I*c) + 200949611009268773815278208568373722272796882
 429905873921544608349935146762533444967075776406669065615094901494404399482
 2823920*e^(350*I*c) + 28208192985622159591075298072896284496213867989894363
 69393116069894018781201000275633104498398959346631795568022519974400130281*
 e^(349*I*c) + 3925697658415778352768103942856011840211642769621717217996614
 398473887186074391482638547212826538270453912634540299792270321024*e<sup>(348*I
 *c)</sup> + 541666280405243634958559598281835795386625846164435401820515891774257
 6425344364964596750653177677803492186817305171175032011500*e^(347*I*c) + 74
 103726128911522243646332961280439716571932803277543043822356727737814030238
 14127610355562505271969045177704726054907145784960*e^(346*I*c) + 1005220952
 436958182758815498534554967803144574449998520825938543560974027240030145424
 6872041775159838468077381562338745636398374*e^(345*I*c) + 13521230411945436
 915558854706851796541567399811656870021567352975326815467817846533289123871
 696056195231696146162720992221760992*e^(344*I*c) + 180353273381774554711775
 685948516829779783464497771935720876885103924268845192729915608513263938522
 41961470040819793627127923997*e^(343*I*c) + 2385639856556280203069527817421

264083328215417400645945829264406094792490731373592156169015393990601751864
7182491616573724049744*e^(342*I*c) + 31295263688189838313772775873307260334
117227258629501992358695636092662866062819845689064235813622974150120668921
391878398978380*e^(341*I*c) + 407159889637019189500203483367364234205133113
590104852469190746528833939708053744708301562297056473122654775842560272127
62941040*e^(340*I*c) + 5253922334674077114258709237025706953606031964443950
166761048276795580027605289243215279881460797511036622494508142812188847332
4*e^(339*I*c) + 67244087969080703823703257199663047606890610482090494619492
802935130215979819469966383336788693900139115594646893784095418472336*e^(33
8*I*c) + 853681184302153128482312917396737358877462018516662996003921994187
64750086828198719872744047767783667325326289221881974987582215*e^(337*I*c)
+ 1075047374065769161234803991697596333283214074194005100174988498305986215
65428266546315933920821527544726380201659114903834605888*e^(336*I*c) + 1342
977420234794299046296161045596100960747587210680227044689380630170596880233
63436458971534964665036319889119229809973806909680*e^(335*I*c) + 1664323329
225891951305583292663987533898239557375985275560960930624735597695457723219
78969318904192572733997888230986469005970880*e^(334*I*c) + 2046222955357291
095198299167898672253194297051625600826488403949654231128093365919212903093
92396263834674368977840527147037426908*e^(333*I*c) + 2495930722825658663983
899515096192026826344554871287146314618917708232013675276457937702037887846
77343934971424317987895255031936*e^(332*I*c) + 3020606380308684634611394422
793604997189069174825248944833562201961383770508289113830568604253701611572
01493696073712322595776808*e^(331*I*c) + 3627063075638432311356991574185107
324524206140136241688793121876452334501539279757933268347807413912034301530
93712635355523960320*e^(330*I*c) + 4321478564640869380238115618086785895947
020479046742822979596588001709844567990677518780448066190124526368913507316
18278545690160*e^(329*I*c) + 5109076151111345074526238461471371414502757224
443163855316484292308516866358277174884645003316233857774007449505384106377
35936000*e^(328*I*c) + 5993784847717334748093761424015548502070649721181375
729492575036514445419393090252768960496225156302631621845263943172854573683
00*e^(327*I*c) + 6977891069259246148167137476846827850836598190279522444470
43355869741368500452561164024636073401929693105801105522738405349028160*e^(
326*I*c) + 8061696713276255324245753400897757336819949915766744469223540997
14615192085443245663852257001115644286660304979476023966071898200*e^(325*I*
c) + 9243200528675225840357774957610723512225344207848619600018210205094681
46356756433795246446491396141583854513687104334429566707520*e^(324*I*c) + 1
051782100428834371944508170051219187116816349766953322637182149610875004223
784784183284961906494422955462431208645690802526770780*e^(323*I*c) + 118781
794307939031610880232479811012902082278266008724859936764348120020604682216
6144285425922229375413676535071141005286431481600*e^(322*I*c) + 13313961146
267230358024621235315820503397499960141524528353059563674253702227586217539
22458727524856072950880960657564720475838500*e^(321*I*c) + 1481187117089246
662466955694965677855524730313260552690821602657176218737426245522795329891
464091005878304304075953693546767206080*e^(320*I*c) + 163556974464142190065
788638128907665322758017205667758745140206923435528368748965961391376195914

0773339736014790081814516625224440*e^(319*I*c) + 17926490780896322989369457
 284819693349643915975062850884883506229372525334209808031443164317014521905
 22716124797875257437516360640*e^(318*I*c) + 1950286550780181919244992961204
 487010056460362845218501674423766266321558791436917317878702232679213868287
 926294665202769722927380*e^(317*I*c) + 210614190346834430711254976120248454
 340279425235248219981741042486967726271509828843764651868348794546277422365
 6471345899082156800*e^(316*I*c) + 22577262191038562868128330126815737654962
 622414206129320761431511719608545541241446990230098420805151579235293571898
 69943515991200*e^(315*I*c) + 2402464595569686086120001803034211056739445588
 621946141384106162886246161815149763025030834875234067267774023433418269982
 431265280*e^(314*I*c) + 253776641546503033081547174669298859606991189469722
 505292832045254217558715484809648333120980743011394301539836266967333795775
 5720*e^(313*I*c) + 26611006479757835838282351392014419301783966433834239035
 83862547255880772382049201015537214900832745601519737141849802506685264000*
 e^(312*I*c) + 2770073207150768645597507281382065497924968466054527414122339
 827333783770068305883487309979315983718403740872884345746380680204260*e^(31
 1*I*c) + 286250312632046179777066778072564418499125562317462617567905067210
 0848988119391841466573417019247590580735265143427289340450811200*e^(310*I*c
) + 29364942143518684987032394554267711043448273062675589165508774672324551
 53286140521089582733932202553130712723836983468866230908800*e^(309*I*c) + 2
 990498949622543608538129380283866335190087115124858818143787619186957111903
 765723974899651518555144924290346242595167274383008960*e^(308*I*c) + 302337
 164350822502717560317521295321902248504585073184530751900827738515473146121
 3388035579159917590062343527464977286601165100620*e^(307*I*c) + 30344083555
 309570757877317453225679816846165501628454732576796742802169473567857838432
 05604202307836897073595410412575660465787520*e^(306*I*c) + 3023371643508225
 027175603175212953219022485045850731845307519008277385154731461213388035579
 159917590062343527464977286601165100620*e^(305*I*c) + 299049894962254360853
 812938028386633519008711512485881814378761918695711190376572397489965151855
 5144924290346242595167274383008960*e^(304*I*c) + 29364942143518684987032394
 554267711043448273062675589165508774672324551532861405210895827339322025531
 30712723836983468866230908800*e^(303*I*c) + 2862503126320461797770667780725
 644184991255623174626175679050672100848988119391841466573417019247590580735
 265143427289340450811200*e^(302*I*c) + 277007320715076864559750728138206549
 792496846605452741412233982733378377006830588348730997931598371840374087288
 4345746380680204260*e^(301*I*c) + 26611006479757835838282351392014419301783
 966433834239035838625472558807723820492010155372149008327456015197371418498
 02506685264000*e^(300*I*c) + 2537766415465030330815471746692988596069911894
 697225052928320452542175587154848096483331209807430113943015398362669673337
 957755720*e^(299*I*c) + 240246459556968608612000180303421105673944558862194
 614138410616288624616181514976302503083487523406726777402343341826998243126
 5280*e^(298*I*c) + 22577262191038562868128330126815737654962622414206129320
 76143151171960854554124144699023009842080515157923529357189869943515991200*
 e^(297*I*c) + 2106141903468344307112549761202484543402794252352482199817410
 424869677262715098288437646518683487945462774223656471345899082156800*e^(29

6*I*c) + 195028655078018191924499296120448701005646036284521850167442376626
6321558791436917317878702232679213868287926294665202769722927380*e^(295*I*c
) + 17926490780896322989369457284819693349643915975062850884883506229372525
33420980803144316431701452190522716124797875257437516360640*e^(294*I*c) + 1
635569744641421900657886381289076653227580172056677587451402069234355283687
489659613913761959140773339736014790081814516625224440*e^(293*I*c) + 148118
711708924666246695569496567785552473031326055269082160265717621873742624552
2795329891464091005878304304075953693546767206080*e^(292*I*c) + 13313961146
267230358024621235315820503397499960141524528353059563674253702227586217539
22458727524856072950880960657564720475838500*e^(291*I*c) + 1187817943079390
316108802324798110129020822782660087248599367643481200206046822166144285425
922229375413676535071141005286431481600*e^(290*I*c) + 105178210042883437194
450817005121918711681634976695332263718214961087500422378478418328496190649
4422955462431208645690802526770780*e^(289*I*c) + 92432005286752258403577749
576107235122253442078486196000182102050946814635675643379524644649139614158
3854513687104334429566707520*e^(288*I*c) + 80616967132762553242457534008977
573368199499157667444692235409971461519208544324566385225700111564428666030
4979476023966071898200*e^(287*I*c) + 69778910692592461481671374768468278508
365981902795224444704335586974136850045256116402463607340192969310580110552
2738405349028160*e^(286*I*c) + 59937848477173347480937614240155485020706497
211813757294925750365144454193930902527689604962251563026316218452639431728
5457368300*e^(285*I*c) + 51090761511113450745262384614713714145027572244431
638553164842923085168663582771748846450033162338577740074495053841063773593
6000*e^(284*I*c) + 43214785646408693802381156180867858959470204790467428229
7959658800170984456799067751878044806619012452636891350731618278545690160*e
^(283*I*c) + 36270630756384323113569915741851073245242061401362416887931218
7645233450153927975793326834780741391203430153093712635355523960320*e^(282*
I*c) + 30206063803086846346113944227936049971890691748252489448335622019613
8377050828911383056860425370161157201493696073712322595776808*e^(281*I*c) +
24959307228256586639838995150961920268263445548712871463146189177082320136
7527645793770203788784677343934971424317987895255031936*e^(280*I*c) + 20462
229553572910951982991678986722531942970516256008264884039496542311280933659
1921290309392396263834674368977840527147037426908*e^(279*I*c) + 16643233292
258919513055832926639875338982395573759852755609609306247355976954577232197
8969318904192572733997888230986469005970880*e^(278*I*c) + 13429774202347942
990462961610455961009607475872106802270446893806301705968802336343645897153
4964665036319889119229809973806909680*e^(277*I*c) + 10750473740657691612348
039916975963332832140741940051001749884983059862156542826654631593392082152
7544726380201659114903834605888*e^(276*I*c) + 85368118430215312848231291739
673735887746201851666299600392199418764750086828198719872744047767783667325
326289221881974987582215*e^(275*I*c) + 672440879690807038237032571996630476
068906104820904946194928029351302159798194699663833367886939001391155946468
93784095418472336*e^(274*I*c) + 5253922334674077114258709237025706953606031
964443950166761048276795580027605289243215279881460797511036622494508142812
1888473324*e^(273*I*c) + 40715988963701918950020348336736423420513311359010

485246919074652883393970805374470830156229705647312265477584256027212762941
 040*e^(272*I*c) + 312952636881898383137727758733072603341172272586295019923
 58695636092662866062819845689064235813622974150120668921391878398978380*e^(
 271*I*c) + 2385639856556280203069527817421264083328215417400645945829264406
 0947924907313735921561690153939906017518647182491616573724049744*e^(270*I*c
) + 18035327338177455471177568594851682977978346449777193572087688510392426
 884519272991560851326393852241961470040819793627127923997*e^(269*I*c) + 135
 212304119454369155588547068517965415673998116568700215673529753268154678178
 46533289123871696056195231696146162720992221760992*e^(268*I*c) + 1005220952
 436958182758815498534554967803144574449998520825938543560974027240030145424
 6872041775159838468077381562338745636398374*e^(267*I*c) + 74103726128911522
 243646332961280439716571932803277543043822356727737814030238141276103555625
 05271969045177704726054907145784960*e^(266*I*c) + 5416662804052436349585595
 982818357953866258461644354018205158917742576425344364964596750653177677803
 492186817305171175032011500*e^(265*I*c) + 392569765841577835276810394285601
 184021164276962171721799661439847388718607439148263854721282653827045391263
 4540299792270321024*e^(264*I*c) + 28208192985622159591075298072896284496213
 867989894363693931160698940187812010002756331044983989593466317955680225199
 74400130281*e^(263*I*c) + 2009496110092687738152782085683737222727968824299
 058739215446083499351467625334449670757764066690656150949014944043994822823
 920*e^(262*I*c) + 14191644814221765738582340138989996288223223330957373071
 6310743838935832201454293617293175086458389621425307051750612129761498*e^(2
 61*I*c) + 99355565364952112722644396082023364938648851008189254586670044409
 6661582790441241830855609577062039555625090943332264901780720*e^(260*I*c) +
 68951844932879355990326041814997419025357834005889503558960646824468059155
 6118170304005037563669880057908765898949268614772285*e^(259*I*c) + 47432304
 356310054237733862993196624812917597698233244601805600939115402043889590314
 0822967769494019446166779954024655344116288*e^(258*I*c) + 32341317801484100
 371415113824607915257636097603505845789040993773872317153703657304368199716
 3745313602400139153046673668433091*e^(257*I*c) + 21856316665964931224748364
 095627214921249911538382877102965428336397258511809047941369663810815638524
 4646591328454425745117584*e^(256*I*c) + 14639044845635118121823738274037412
 419166481999774698807659839186273362967014224154637553390313060529758010567
 5355629160198162*e^(255*I*c) + 97173055024742680058616722461368892661141295
 540263493013032746083536157324268333390400308958318370219154887169702257444
 756176*e^(254*I*c) + 639230194337619890906148012886350981231994451023031226
 16544648208998767803944455777042886552738499747183713136069104651812215*e^(
 253*I*c) + 4167044037539054364341821934227174804003507149011908058528152249
 8188818375906900368701234531304633163446319945130196476913600*e^(252*I*c) +
 26917794701086615097890120236890501105146799996021775195710866226228638984
 703456832694153230611607263444183501026198563419616*e^(251*I*c) + 172295028
 243676473344007219984175967039486573947388052093916365973703805723989647150
 80095366818322029152193635869784095333760*e^(250*I*c) + 1092721060347354481
 027979234784453607458889680623004111008954473160586314610418173903942667485
 5453466097402330688331845602302*e^(249*I*c) + 68664253375186682626626937509

089569657329245781421816306221578028998808746815510311363140641999486040015
29894566235238597088*e^(248*I*c) + 4274826907720591752526711336820871500844
345647922385471534359333606189571832444641364132893108663576205133870672156
264164115*e^(247*I*c) + 263666241043079934044752228477824428374075106865814
0726576446671207798325606832295937705061686297930296382338892574900819440*e^(246*I*c) + 16110925414000605259548593752641941783476434718370782014462624
35615142944587337833513586022729849523358436493586042252995608*e^(245*I*c)
+ 9752103394440493187572823117635177866732231755944579463832792646350850410
04917300295904275433144848532459919875479817581584*e^(244*I*c) + 5847495736
823045861793846288448833275814989698865403803788967679990756149640071746008
11092945356635118795824799369716742109*e^(243*I*c) + 3473100538109352904194
555605559573141295692107357459832343696599764133747740780001730700752486545
24917179128950507443058208*e^(242*I*c) + 2043255572651860007674027102308478
964597615839227636982354332128333130777830410400746693790173948367615396490
81690630811665*e^(241*I*c) + 1190605918496605468347656932276764490675841482
488826784475048260772363334445134540951266687500572958111916433569089721914
40*e^(240*I*c) + 6871246601598564151246858617365974773487959171009835465278
6124936023073943141049573606648563005359411712764895683903806088*e^(239*I*c)
) + 39274200414329861169397944516225001081227433398585007206399231211907157
795359719648241598754266579840244551491476467899952*e^(238*I*c) + 222313411
318015353454063990377216868402083979419525801355846459667467366567162715548
26476282991066076564921432614339399735*e^(237*I*c) + 1246214044053725808492
859670987206685775707094312486855450094815475686345430803292540834031123785
0017814707896986969086816*e^(236*I*c) + 69178389452148442784933304593613949
233723338536198796373726731849428597124310663457268704220998931248907776780
37369988150*e^(235*I*c) + 3802604996705891106964620633848964807037098854510
182263243030597295630760353597531974324752266389193185760878274188013440*e^(234*I*c) + 206969828950086064346166576237380795751301942404117887190496055
1829412449344722432125679417958403007551179298315947373776*e^(233*I*c) + 11
153982855456015505333280456001843179938993592179967298183407042218011956674
10485846996179056733558512452238583160792512*e^(232*I*c) + 5951576155004315
149474792823365470538279260879164252634971877570294133854718354341982468070
96214536895441388306027237899*e^(231*I*c) + 3144090358082258615655954369383
549454454739910431297220467472288130309252049685034185668188386118667098070
40793495364496*e^(230*I*c) + 1644375006769068927413232601543942785039545619
36020133596581806449357240277349447927334517105809995300093549279931273178*
e^(229*I*c) + 8513953323478645577958995946490063776073572970562122138000583
7208369157794673049675428799817875431430246332625899630160*e^(228*I*c) + 43
638300075815171025946211464465689618965773488660985857945657479854085108851
857911222911989837615452608512356008400295*e^(227*I*c) + 221407350017086032
709151807692413916620355787559039791489092136038225547927491835171602555719
15875356439553717130797888*e^(226*I*c) + 1111949964536320108088106282488633
849242537565844897793553584634929042582157038309042541141852151667037137204
5206568345*e^(225*I*c) + 55274988490311783558612300093266682839262900821584
67118000698502719379939045918344222192742145711257028040974074674736*e⁽²²⁴

$*I*c) + 2719589283483743926040805101080341921244530311254607250929192773909$
 $331523226635035815672862569296693711643521070331394*e^{(223*I*c)} + 132431132$
 $498402742835522293814768237867286070881716174144874968959358802086084750870$
 $3702325320304649883120684987556400*e^{(222*I*c)} + 63821889291453374150580639$
 $966248855606678360049609187640837497487744897177803607499624558112428346043$
 $8065182071976085*e^{(221*I*c)} + 30438447110681333601028416012390637043388882$
 $8490627422652551236966790916174520857759143930140187173492394981908258944*e$
 $^{(220*I*c)} + 14365768713804479694294711970425953845881819942351682467458629$
 $3691056119209866358123637772245409530799230553767222252*e^{(219*I*c)} + 67091$
 $706130529669125019899210021576580237843462229535386295087076189297849995931$
 $360645605292130961496106707521506432*e^{(218*I*c)} + 310043192060694170770693$
 $631414234874318280090981847446356786522841774394649416518125645191449180031$
 $74108077634846014*e^{(217*I*c)} + 1417648365287570495720201334324111790436997$
 $7796653849959902524421980635189011634815653279605497783382888932766730080*e$
 $^{(216*I*c)} + 64133818958559251847582314510625563803285949385110065770152181$
 $19786536390213057284018202201631094434819584025113465*e^{(215*I*c)} + 2870496$
 $131412314451834674715353589439553294430808531933466086288543709246230769151$
 $392180699413405623017247753532944*e^{(214*I*c)} + 127103308293804895020136055$
 $483127034266234399127750461234230036602504674174285658044528940178665631168$
 $5859023084716*e^{(213*I*c)} + 55675638871118234034102619273421954611365176831$
 $7380539005893679049394714017063698565272728813669054779077208977840*e^{(212*$
 $I*c)} + 24124602128244006179290831778303287619480159713320605209128699704372$
 $9145345755805710081489006741839439573984832678*e^{(211*I*c)} + 10339975546725$
 $74364898478376407537547182043944730557950014676043264198765555687382953173$
 $7211096115196005647730480*e^{(210*I*c)} + 43834972142919377685378692233021063$
 $744554033100928502737480438978976746989895784070951905237783490374305934542$
 $955*e^{(209*I*c)} + 183798070840033597660276492176211441160917355722166207888$
 $61535803449702273802588359076704241840733513439114113248*e^{(208*I*c)} + 7621$
 $788791912047062038840917799374600428892258194367636682944356096681400246312$
 $138001769285020661445991073249416*e^{(207*I*c)} + 312568349317870174347970475$
 $030749017866629215072017936360433511352862332968460634318554075601993566214$
 $8267863968*e^{(206*I*c)} + 12675970172948129134001462760421269299864802928701$
 $90399107554311079964227280196522475370108738477856311765699610*e^{(205*I*c)}$
 $+ 5083245993010854601669786296830326614276544740829390480979396383915672987$
 $95788389433842285751054665210868287680*e^{(204*I*c)} + 2015579474247940980267$
 $724784620408833958675125629308629437535686900840155855980101547815486252394$
 $09581907397500*e^{(203*I*c)} + 7901914955876656925478398848723238835290914498$
 $2747171856772463223808993367091503402876467270176124342699654400*e^{(202*I*c}$
 $) + 30627581054221957378390547289277609129572813931082733520247387226000020$
 $043538279468776707958420892547870128680*e^{(201*I*c)} + 117358569262451184931$
 $130910025016040323418769859990828235206722415302001882238263929823021940846$
 $67538488665600*e^{(200*I*c)} + 4445412259295474625067659514198312966015416299$
 $968930393345630345914109720740573618884980520010028451496996210*e^{(199*I*c)}$
 $+ 166447503438721180939491774350293763897857493775476476398783587241044993$
 $0690131572904279995484581013965001440*e^{(198*I*c)} + 61600311602297958493712$

570175788721299835430098999136262803886109391456133207119190971494942658793
6910303300*e^(197*I*c) + 22532053259322065776794110928951624899979452101556
4134982827241710019675486694499689312466561907212627820000*e^(196*I*c) + 81
452081413829111828875417564250054846037693312811480492909160758195989155768
107022568350953861815940704090*e^(195*I*c) + 290976510612474534066475697818
369100621655598523590528042591541656871254287525623854923737494863517144531
20*e^(194*I*c) + 1027160253020288900249781351684945259097151280952906066519
7301097052210064576088348023234671975463677418470*e^(193*I*c) + 35827180021
632960614145367037151098971071982527392845461493431023484561240896574285949
46438660859773886240*e^(192*I*c) + 1234668041892409978780018081755440216012
582476396941937965899631953079203974222138794604328498972144766900*e^(191*I
*c) + 420358024835146798583611210145942154684437949365647899088372524802156
222884839580011688655664280691773600*e^(190*I*c) + 141379938253556843280565
505807403304130606130725434751745794079833141361748917639986145377066437210
546190*e^(189*I*c) + 469702247271172818264540450180706705225597566275803477
84535320014963482632359729444541885102274546002560*e^(188*I*c) + 1541311121
148602393729497082079737671608134478816338654352242193973750796212585498188
1879168348260330000*e^(187*I*c) + 49952419562791381802051867444016880243882
72113921255663734956946927571305533146776898787878059685108480*e^(186*I*c)
+ 1598771101058192692270528999677444742685631006232456185844925220144002305
878120380828483988663574829100*e^(185*I*c) + 505293663123015258878483025738
812813203397766845340065381261016353419722382620393032535960660921950400*e^
(184*I*c) + 157685845528850918721462877864435090257583149415561323427386562
894447598277935629800939237175625149830*e^(183*I*c) + 485842581531402804473
148368687721313901954124190467327784587060150968814370763379107935841224750
73760*e^(182*I*c) + 1477795509661712899871274518207149536217650697318308165
0233605274051677624970464340242755840025673760*e^(181*I*c) + 44372109178431
823477643495444439046990200565950694708471936170921147140776330772349728253
51226979360*e^(180*I*c) + 1315052120930692122102297105327622842335870743428
530891072983535862280094446607723473800477453914130*e^(179*I*c) + 384655842
080666274454063078784837174998949052500975322162003392549953413592461519365
177908682078400*e^(178*I*c) + 111034148797008819443143895644469242295049867
464313710969257619338899133799285616020069872611710850*e^(177*I*c) + 316266
446747255477311767956875276535713059699859236883921121649155532425732694909
08989570248533280*e^(176*I*c) + 8888295028751024667044203837607976101480053
134418614474620767522824868911959884352666444917404000*e^(175*I*c) + 246438
219080743960907977422685567962936788570977643587663085171625396269619234170
6239192878728160*e^(174*I*c) + 67402553054313300889484577523662523745074311
4473544537818170447134607102575676676056675328961590*e^(173*I*c) + 18183466
140617790131533012967714538116644918841319414116934435475492096903495261037
8945282257600*e^(172*I*c) + 48379489756434099843857791816589379406815042609
340378747586437145781646245422045101230417309900*e^(171*I*c) + 126934969329
649205650736736371812800885486825088802553372800650065661386960417973532165
84528640*e^(170*I*c) + 3283874760555818676726309480306734420155098583948074
469014168171874442170109648521627538755920*e^(169*I*c) + 837579206923411932

458786486765373533946545239708990769488724813982189165104589895518909256320
*e^(168*I*c) + 210594301385648471184329078880317504953361839954159427434009
884661777259752542647709150036990*e^(167*I*c) + 521909122076618242158122718
54269748071292843243227894769229690720010547141334131610989636000*e<sup>(166*I*
c)</sup> + 1274721961650332054135634306256284736860162214085678602544581453203790
4111523242298235713300*e^(165*I*c) + 30679742964317473641981596239624636716
17006419626851426148418602934852907379021659761911840*e^(164*I*c) + 7275210
107183942292917740738446942557987386670675353797597327955679425787513842507
80476310*e^(163*I*c) + 1699563279699297677739020966526292532837045054771275
44556534417376686540936706073847337600*e^(162*I*c) + 3910803125560180947653
7535369611844440844903751605645023514572352045248104262933598850730*e<sup>(161*
I*c)</sup> + 88627521427569572856813408857649045979353495693553218156477211725371
59186491471311666400*e^(160*I*c) + 1977792980665818135651300094326239158605
448870806970860577325385028609983034534672318500*e^(159*I*c) + 434546676780
280045346344498763892540797175105756827515509297024187660299345484920192480
*e^(158*I*c) + 939869153130681791490836060656814827808360605105301546184869
49839467131378859885998210*e^(157*I*c) + 2000800680303004713729327825032159
7113540716201983333126349281186679153199068045257216*e^(156*I*c) + 41915425
006568261480933394145441591439644784724923159318091718599021141090059399429
52*e^(155*I*c) + 8639799336223303495562968200283955131987080649405057021260
68652936800794826651264256*e^(154*I*c) + 1751931705006183002415156323819122
85157790097816049220671217212220015297133400636060*e^(153*I*c) + 3494107161
3276704649477943043339450201504075335160361865916029213860778606230624960*e
^ (152*I*c) + 68529932231457366873288853116177954355929408414398663510796556
52312894721972796266*e^(151*I*c) + 1321498055271300851429993866631619874424
534425188183592049727687571032156435077280*e^(150*I*c) + 250501028608928332
469340456829902067712233644464602753159945727868485722395506952*e^(149*I*c)
+ 466682235482660178068545924681005702893559608696136508565757567581801822
23308768*e^(148*I*c) + 8543013441126212334833540665069621472479085838041360
564550722036723654297540205*e^(147*I*c) + 153633323844492758353273455601649
4671674916578907116984548489078241693926940560*e^(146*I*c) + 27136120750326
6570734486517077181014801775322183181055638619257836143271472358*e<sup>(145*I*c
)</sup> + 47065044611135158108487353367484243102698248838312635876283099427442745
866704*e^(144*I*c) + 801372958079075243436196494576154376146952079121074697
2675870481058674277844*e^(143*I*c) + 13392143742542455535648844068019453533
85000254030655765953770237607180089968*e^(142*I*c) + 2196012813395155615002
61478844190024870555261281946058839614044697037963695*e^(141*I*c) + 3532444
7206779018115378052820789411687581004582367431006205879633729015200*e<sup>(140*
I*c)</sup> + 55725511573286711210162164163075961618619559690116972223409262101128
54418*e^(139*I*c) + 8618848510949919087642468054746724286037573154844539747
13612812215428992*e^(138*I*c) + 1306576602265604193351214343899389618845954
34069984824307149332131747540*e^(137*I*c) + 1940797921559456659353500810330
3255257745408070082431338945184797463936*e^(136*I*c) + 28239051519365866783
82525706564457280290098698638597987628380245881715*e^(135*I*c) + 4023496922
66121158934003582839428785116904903936409545602519219664720*e^(134*I*c) + 5

6117081076341175384087570185188538660375932013674735519055227368366*e^(133*I*c) + 7659010520187549651777118357676871927081898989131125755798204236112*e^(132*I*c) + 1022536437468296737293065862705246449693687415559865844306888705423*e^(131*I*c) + 133490210052026183779673313868332303530332906163247194627808410304*e^(130*I*c) + 17033886027390615741040977721655541665612162275485028584310890417*e^(129*I*c) + 2123702969188871318266718781223927067839949015727293884065388080*e^(128*I*c) + 258585348715977270155829115684193411072034541491364393985491350*e^(127*I*c) + 30736217404321009965231037419663053962881035281709221697785072*e^(126*I*c) + 3564764890628724017088487996688178929195787613958545474804845*e^(125*I*c) + 403212225957798188840846139960995624144491271694336796459584*e^(124*I*c) + 44456708175258821024400946210535004523775722190977468484496*e^(123*I*c) + 4775398607100853263534207733818266777478693412738731031680*e^(122*I*c) + 499467506558531733671585862910572702811545035730398749530*e^(121*I*c) + 50836369508171099437019348610847391946736185108017183136*e^(120*I*c) + 5032024903401451824074213943766011922026507006311982753*e^(119*I*c) + 484093410240488718655917025303662581091659126182344528*e^(118*I*c) + 45230940039830738332025694784646206844854827698075736*e^(117*I*c) + 4101545439937195793959956708442496709433800261224880*e^(116*I*c) + 360688613036389349413809780004559963548775423325255*e^(115*I*c) + 30735366512830562160991166338490057308062762518496*e^(114*I*c) + 2535667460650279776834561566186591213109251642859*e^(113*I*c) + 202347509724462171313966643580234078508179838320*e^(112*I*c) + 15603911277687607099721623771744933086920587272*e^(111*I*c) + 1161581413733971751533622511909046917188768400*e^(110*I*c) + 83380839911837894453136303673785039051506805*e^(109*I*c) + 5764601046563151304213854710715346838447392*e^(108*I*c) + 383360155801054824529764688213114368047154*e^(107*I*c) + 24489837337812338687718622491865013839488*e^(106*I*c) + 1500602747937397286405577818722691539392*e^(105*I*c) + 88054927598941411145869950813388040256*e^(104*I*c) + 4939666610818025798809586352543471345*e^(103*I*c) + 264410375780310742518099326419685040*e^(102*I*c) + 13477227799524701956579274210395326*e^(101*I*c) + 652650253343206047453620559993840*e^(100*I*c) + 29952547749265499675257842032197*e^(99*I*c) + 1299146645993240318167826532288*e^(98*I*c) + 53090127264630963470039804475*e^(97*I*c) + 2037031259470368160131922320*e^(96*I*c) + 73099207817335597247098038*e^(95*I*c) + 2442455629894502983849104*e^(94*I*c) + 75599817092670157806639*e^(93*I*c) + 2154864144781257856128*e^(92*I*c) + 56169444526926562260*e^(91*I*c) + 1327882849274858880*e^(90*I*c) + 28186192554792138*e^(89*I*c) + 530563624556832*e^(88*I*c) + 8718181624155*e^(87*I*c) + 122503723056*e^(86*I*c) + 1431118260*e^(85*I*c) + 13343760*e^(84*I*c) + 93096*e^(83*I*c) + 432*e^(82*I*c) + e^(81*I*c))*tan(1/4*d*x + c) + 7*(11154*I*a*e^(1055/2*I*c) + 4506216*I*a*e^(1053/2*I*c) + 908002524*I*a*e^(1051/2*I*c) + 121672338216*I*a*e^(1049/2*I*c) + 12197651906154*I*a*e^(1047/2*I*c) + 975812152492320*I*a*e^(1045/2*I*c) + 64891508140773093*I*a*e^(1043/2*I*c) + 3689545748587122372*I*a*e^(1041/2*I*c) + 183093707775710628858*I*a*e^(1039/2*I*c) + 8056123142378999966797*I*a*e^(1037/2*I*c) + 318216864146377884963053*I*a*e^(1035/2*I*c) + 11397949499240328225340750*I*a*e^(1033/2*I*c) + 3732828461999581039

46548090*I*a*e^(1031/2*I*c) + 11255913521436332875256492285*I*a*e^(1029/2*I*c) + 314361585019926536123171779990*I*a*e^(1027/2*I*c) + 81734012205088606
 69709781109364*I*a*e^(1025/2*I*c) + 198715817544529039864660110471376*I*a*e
 ^ (1023/2*I*c) + 4535396318845906415166472261686554*I*a*e^(1021/2*I*c) + 975
 11021243902906034367815029181218*I*a*e^(1019/2*I*c) + 198101339008777817114
 3744488041442712*I*a*e^(1017/2*I*c) + 3813450805531950148020285195722772446
 2*I*a*e^(1015/2*I*c) + 697316726077381488801524339658757026686*I*a*e^(1013/
 2*I*c) + 12139650447759376183227584660170495869589*I*a*e^(1011/2*I*c) + 201
 623763354433037155766729846494383292508*I*a*e^(1009/2*I*c) + 32007773205892
 57207465955323343789287604398*I*a*e^(1007/2*I*c) + 486518167851923908629493
 46064352748595789341*I*a*e^(1005/2*I*c) + 709193818893909656677523349582328
 791900299617*I*a*e^(1003/2*I*c) + 99287139599586441436114562761876907080752
 85068*I*a*e^(1001/2*I*c) + 133683049871200685767730244065634086417919364482
 *I*a*e^(999/2*I*c) + 1733270022057743255812637849164901656684569946869*I*a*
 e^(997/2*I*c) + 21665877338576587360651194480041798194029867090275*I*a*e^(9
 95/2*I*c) + 261388357024674483234213954236474266534206934500718*I*a*e^(993/
 2*I*c) + 3046808467328698391993011459852063034674399964693132*I*a*e^(991/2*
 I*c) + 34345846767794763840341317174417543493362860419944375*I*a*e^(989/2*I
 *c) + 374773875381762608573480596867444622808315521135770955*I*a*e^(987/2*I
 *c) + 3961896224254872041251938157236050043036325552867100290*I*a*e^(985/2*
 I*c) + 40609448120913176083174637180877049405994150689208285948*I*a*e^(983/
 2*I*c) + 403899515051838803400194879806531247927007890538062521027*I*a*e^(9
 81/2*I*c) + 3900820584110953912072698502834368654421340467896495241550*I*a*
 e^(979/2*I*c) + 36607718305457313571850609230094969421310836249109701091666
 *I*a*e^(977/2*I*c) + 334045615572410482684075553294675381058554260440184749
 831968*I*a*e^(975/2*I*c) + 296567519383571006655991658274309515837195178824
 6043891048682*I*a*e^(973/2*I*c) + 25631926334820519324289406607504181260913
 532669181012596611058*I*a*e^(971/2*I*c) + 215785242285994868818528731430540
 812422314520336288528460007864*I*a*e^(969/2*I*c) + 177042160825029715561159
 9130292037434814100286157679816873050532*I*a*e^(967/2*I*c) + 14163389239388
 825902842073391400806825940375203939456957445830422*I*a*e^(965/2*I*c) + 110
 536162580123562634802981277497355054368691566003022338877490451*I*a*e^(963/
 2*I*c) + 841957579184928615664618341834644691840690166368399913404427216774
 *I*a*e^(961/2*I*c) + 626207033177932400051380661316398100986836278394976825
 3661316894658*I*a*e^(959/2*I*c) + 45495947534779552008093888439933100139363
 372706825172710009091580883*I*a*e^(957/2*I*c) + 323021950031541227897801407
 171566170150519245202346540397613583355667*I*a*e^(955/2*I*c) + 224215804172
 7235117364957739459342847819393345842116256845583655236406*I*a*e^(953/2*I*c
) + 15220847247394291084416636302914971818756836097657800823465735035857764
 *I*a*e^(951/2*I*c) + 101089727467776956523475550836202489574004061008016807
 940002791738698571*I*a*e^(949/2*I*c) + 657085621385514241363241301495127058
 109065015699304745663817714211556940*I*a*e^(947/2*I*c) + 418147107090738761
 2659598631606751788208788794201536282802542477878283190*I*a*e^(945/2*I*c) +
 26059644761295218063521552347914441867779264013071310596786890247411087520
 *I*a*e^(943/2*I*c) + 159101807620372457710230554627597385485602304037309714

452742966256548901840*I*a*e^(941/2*I*c) + 951873187489146746238338008022239
906318071898028016864090568030803687398612*I*a*e^(939/2*I*c) + 558220743072
1767424910922169155711907398569721937114073480331164440862462638*I*a*e^(937
/2*I*c) + 32097922563848506943066473469340753662783076784232671471143949644
558156480632*I*a*e^(935/2*I*c) + 181012678179948465189891189876844483520479
568880173570917696339360628958538348*I*a*e^(933/2*I*c) + 100141772034465691
5251924901009750939990149160965149540929889824388402626455012*I*a*e^(931/2*
I*c) + 54363210824381420294157398855539147675527182044111772774155362548632
31523181480*I*a*e^(929/2*I*c) + 2896571400325784692154085898638895400812211
1612712363354360981851687424724108036*I*a*e^(927/2*I*c) + 15151479395044286
1336357006872512458064446653474553525802193583698620014121641124*I*a*e^(925
/2*I*c) + 77824546206109293824112895299300796116555830458311753797993658323
2925968316199836*I*a*e^(923/2*I*c) + 39261319752280473730214398836102939446
47446928673344297294745924701096160760888424*I*a*e^(921/2*I*c) + 1945776444
4728521352641555485388469197530472258189211796433938437083420010091733396*I
*a*e^(919/2*I*c) + 94752549937175934400499756270975710616491132843843008041
484744522257068322302549540*I*a*e^(917/2*I*c) + 453467570647678726762830796
324670748943540211918613115981382798996143666294810914890*I*a*e^(915/2*I*c)
+ 213325987645378922642673873920426655936692279915502179304543513972950935
8790624760320*I*a*e^(913/2*I*c) + 98665617992106740504226261681187999762187
42038737385255818701142786087196734211598180*I*a*e^(911/2*I*c) + 4487375453
728008640491091087886372186496684916097070987803931507965374355685111396241
0*I*a*e^(909/2*I*c) + 20072483864097435618425497602435711351463146370708337
9071558666747287853457166817321610*I*a*e^(907/2*I*c) + 88321698672020915183
7022414332672328111515000315030045611168408538524032471081357579180*I*a*e^(
905/2*I*c) + 38235310470745017739018826170797254746461372056086358408511845
00433382055447334225716180*I*a*e^(903/2*I*c) + 1628786013187342629766595419
5872007340389448696155366782102817863363964748733518124937370*I*a*e^(901/2*
I*c) + 68286519373700606662969518596072942990402829423660330718366006960865
727616800960284438480*I*a*e^(899/2*I*c) + 281802543079970773764222747501103
240510862783237481025696735711115713233601841503443789040*I*a*e^(897/2*I*c)
+ 114487866597334536288031280499292234026314441356757106089623565009992240
7159938671183630200*I*a*e^(895/2*I*c) + 45797580440835338476480708866180632
36901192162004233936078348854854553658379606034075133880*I*a*e^(893/2*I*c)
+ 1804082255618213689663871310089234435832805300717367962876565714745095939
8026341254411115720*I*a*e^(891/2*I*c) + 69994103899854449688044814471201743
418413806666583628183006633122034125384874442925881756120*I*a*e^(889/2*I*c)
+ 267495779341793686858892087186701348017864136095797204004377097030869586
488588546928368741180*I*a*e^(887/2*I*c) + 100711783765737336946872972571982
0597008572465469753257215153626598733911453021524113695566720*I*a*e^(885/2*
I*c) + 37360062305102756756100615441269715573934211219494108870036549757264
31985661061431612714966210*I*a*e^(883/2*I*c) + 1365694353786631656234502288
5038770780812495797097516350805731913565888244243229131324182944640*I*a*e^(
881/2*I*c) + 49200709831170847979783317949393803747072027694891369055993880
672007049593913929619833129271500*I*a*e^(879/2*I*c) + 174708148576954508133

108566432138482752871314388735390819701558304621069749665356028799825469570
 $*I*a*e^{(877/2*I*c)}$ + 611546710350755227094210555322738233573992238438880922
 442071495399840623270733748703638918730970 $*I*a*e^{(875/2*I*c)}$ + 211042686701
 657901173815999545210644960709417069397550562541081212722173793102352142744
 6997422880 $*I*a*e^{(873/2*I*c)}$ + 71809767087815641520119330793240663969412779
 79720732126305244304659930709704206531169354332309620 $*I*a*e^{(871/2*I*c)}$ + 2
 409441928790210422014636431525698756343032323446865385718272253087317365650
 7620757370776234725250 $*I*a*e^{(869/2*I*c)}$ + 79728801300455544122096358134599
 530539313996522871876150671326770147866624403122038941020982112670 $*I*a*e^{(867/2*I*c)}$ + 260210521546648324710601552017336263001665484044998430835364683
 575217699721326919568028661098556620 $*I*a*e^{(865/2*I*c)}$ + 837701525654515943
 160919065652989402268629196598907039545164356949845716819867090707769561650
 988160 $*I*a*e^{(863/2*I*c)}$ + 266042842931571103774675083618235061716695085166
 6088185525123590808329315051618495256897037944395590 $*I*a*e^{(861/2*I*c)}$ + 83
 359229219753395548749857408099513531307771208171816871848757958934640668766
 77629382653264393601630 $*I*a*e^{(859/2*I*c)}$ + 2577132301570592546450213827703
 5617975672430492105429256456221829154887242352647174124493051808094580 $*I*a*$
 $e^{(857/2*I*c)}$ + 78621408619495274059980620283854146208984244064535407477261
 926286982916818434658466433148202509344800 $*I*a*e^{(855/2*I*c)}$ + 236703836252
 587779323701434388787465249427045104853012907439025264086299638762864112564
 049076655732830 $*I*a*e^{(853/2*I*c)}$ + 703345279898755583562127912601131791334
 107956711719030548547943555221102827637472553536460508801363920 $*I*a*e^{(851/2*I*c)}$ + 206285295492081618602391594125111752795850045368393587517339248395
 2599009295139987985628233229731478980 $*I*a*e^{(849/2*I*c)}$ + 59722941276362590
 674151260506011593791666320614098219152734656592985426908363278178714123202
 20580305480 $*I*a*e^{(847/2*I*c)}$ + 1706960063470899691725822494164793138249211
 5532252012641576292881810859853135265796592414892124832389000 $*I*a*e^{(845/2*I*c)}$ + 48167052524691460784860596000543126325082971121698814633620291249190
 839220221789221157407204913360887960 $*I*a*e^{(843/2*I*c)}$ + 134201038644703507
 385888818415634883722891759405213144807057882964953199405626071972925310751
 903480110440 $*I*a*e^{(841/2*I*c)}$ + 369210997610108799944611140506231611589949
 279646373920938134598886382202697574571710903344492696653736880 $*I*a*e^{(839/2*I*c)}$ + 100308906580872377987604746044435692379102789056152055267941705646
 9828915431176758967907919151985703838720 $*I*a*e^{(837/2*I*c)}$ + 26914293841250
 094130608671518641177000729853137740390405183243788052379353447158363657003
 45548762139320230 $*I*a*e^{(835/2*I*c)}$ + 7132421149055578168006796797797943674
 026134220971516324623846545930681177797566466263020966077638684308780 $*I*a*e$
 $^{(833/2*I*c)}$ + 186695013151690567802855585778163774515738518543184957254613
 26689178551127951637356019031393026858408569620 $*I*a*e^{(831/2*I*c)}$ + 4827261
 875576824255901844040633795968313307135325975392553055937185084404452329574
 5943249184433038391906550 $*I*a*e^{(829/2*I*c)}$ + 12330238631943173760301230788
 568046145553392342651686293163036989653681052433842494658667855571241483833
 0710 $*I*a*e^{(827/2*I*c)}$ + 31115307812434068987682163412997317426688473577707
 5069163110831590213226340122660492406303919452492168793820 $*I*a*e^{(825/2*I*c)}$
) + 77577842500154404097242752774500834816804684020962178036963319225555426

6684373520182047073542988726249962080*I*a*e^(823/2*I*c) + 19111321825020891
 877081664786859019352013524355598757051509077545383989033395660206645244926
 74910743672362870*I*a*e^(821/2*I*c) + 4652229613980190281092438412966980324
 563941911273487870254673754078512908450865883806375186718602486878389420*I*
 a*e^(819/2*I*c) + 111911937372368801918281190224557235328802284114807569582
 10492395616704273180646550698293673440285122201506540*I*a*e^(817/2*I*c) + 2
 660501738325959941125486653387063070184684544114538539883726653793136940859
 5315781206468524649149866075877240*I*a*e^(815/2*I*c) + 62509941784743120997
 663879158557982155081149380070540181443212715349440122956954676521201490211
 883966367828020*I*a*e^(813/2*I*c) + 145164084395384796858807452169156193314
 757794283400173793760462753468308342626556407070906048758273598047775980*I*
 a*e^(811/2*I*c) + 333211138005255761240200864910933592348294347179262547321
 519853589485903531852562513295558687155952554066590460*I*a*e^(809/2*I*c) +
 756058495767730773659409931369268584451348889364804029349493357426339750910
 243652561427608338124503345850013560*I*a*e^(807/2*I*c) + 169586694384839863
 762082875542402504079804383725498790646540871293177075734811292316429856498
 3018936301335935212*I*a*e^(805/2*I*c) + 37605659493035666864233856362234082
 094465498792886874861655868160002208520695808993984464267495572535029400183
 08*I*a*e^(803/2*I*c) + 8244492425854132814040653088604670129631127112463111
 698869090178226433285329901659997552076882534722621154663912*I*a*e^(801/2*I
 *c) + 178709569969992802875872825562267623650671756493247528499380693654045
 51598981972431698224836196155095718947401358*I*a*e^(799/2*I*c) + 3830261707
 190416013199912010998899180421021407244116125726026137468537377000578615018
 9477516514391979924577468172*I*a*e^(797/2*I*c) + 81176276038870735325330852
 188233888929463718282951750613849345989835502222967166292107718062084091143
 361182517160*I*a*e^(795/2*I*c) + 170126842297170420289826544603513884546736
 532165273225663609085918787724504183838886776200509606191937033678162224*I*
 a*e^(793/2*I*c) + 352600027682002951366552195831292961904393452031839665434
 742581379773740338046042737991951645221266975380498373846*I*a*e^(791/2*I*c)
 + 722736724787787124132763159445766235279414024537115241684494028855350914
 895151999785034206184040168317292786589284*I*a*e^(789/2*I*c) + 146517146645
 611822858332172737413341944831073458968335363906687277370038676579338217934
 0075277656508354859972911131*I*a*e^(787/2*I*c) + 29378484206865194916839528
 936523698208356730302085912803094623807212834844907441695442915323208158084
 09450100179444*I*a*e^(785/2*I*c) + 5826728456998393838746283558785767736418
 444764516351018178688292977826358717393165474435299003815460564340303035510
 *I*a*e^(783/2*I*c) + 114313182922035384495360621046134469254143793530201476
 45218488194165850651837526675988253582086095570243158171594435*I*a*e^(781/2
 *I*c) + 2218531084901737614494058133508448821593174865751884625102376972508
 4130717621880836878455248575979783141226165618755*I*a*e^(779/2*I*c) + 42594
 522671068860883864561229655566037459988103599672363573691551514146031450943
 546205718366811710531905255429039490*I*a*e^(777/2*I*c) + 809062879793839605
 960149642537205140747506339361059008351912777079735013049055460658979356673
 54719683918092356762982*I*a*e^(775/2*I*c) + 1520450247419420210318434501382
 836778647301515591782012337386341960572966347276438075116206347002158599878

$58540208163 * I * a * e^{(773/2 * I * c)} + 2827122382504900816891349747732958623701013$
 $47118293726638809114306775532326665539002921023135199371428068547769357902 * I * a * e^{(771/2 * I * c)} + 5201402052117401461463288860339192842776562670130376159$
 $75662868102063644941116116140374194763868070844567225649530644 * I * a * e^{(769/2 * I * c)} + 9469358582484161376188178033743727416111418592823351320281378875528$
 $32387239243837518791104798117018522060501489612616 * I * a * e^{(767/2 * I * c)} + 1705$
 $948357752683688609383556611544971698931045395116831763212880566314009847795$
 $209812264993457998814619114688074679466 * I * a * e^{(765/2 * I * c)} + 304142566853240$
 $586400498135898699141795898114585235603666143538212285529007098174186120342$
 $9675035697323494807153105698 * I * a * e^{(763/2 * I * c)} + 53663051021007398309871063$
 $261074502884344499432343683812176509358219426518193152712057667397585559364$
 $19394930826692752 * I * a * e^{(761/2 * I * c)} + 9370897002428257267123315283721797254$
 $447538815988223371846399458761076724136173352675976157625073367090808727028$
 $817394 * I * a * e^{(759/2 * I * c)} + 161962868526188369395506297658517359366716193400$
 $33188646816345459125057185947699008626649435369257313552045494568159814 * I * a$
 $* e^{(757/2 * I * c)} + 2770761376042432109280977348834171005609101629943082281185$
 $036820550538666892248077868916795733514784560970590740995043 * I * a * e^{(755/2 * I * c)} + 46919468887999820120797066437767350896395887972340669168418033185399$
 $584359856915397601507103516462593793662665388320876 * I * a * e^{(753/2 * I * c)} + 786$
 $499297907510768672074582401143110998685640861404828533880308670564283751695$
 $02351361259161526296946312988137021594754 * I * a * e^{(751/2 * I * c)} + 1305136545651$
 $806596808576107518103309191826804511798147243018089725819932514585681580430$
 $20510042455457225755690757090571 * I * a * e^{(749/2 * I * c)} + 2144110099842286461896$
 $635924207376189992755025214652446364858385441675385555490569836303716960975$
 $60516804480364748061847 * I * a * e^{(747/2 * I * c)} + 3487329620520598271405637311130$
 $324137290678229939023511061930537039310926962392941111500081097682203263223$
 $48527847477500 * I * a * e^{(745/2 * I * c)} + 5615844530352673936409554025270746390789$
 $929068539086222880978579241968852012258019589818629155193134820307854602738$
 $84334 * I * a * e^{(743/2 * I * c)} + 8954365481450827453236125711936234058690741641172$
 $34358676031591115885958848259250172134566787604094987664728603369653891 * I * a$
 $* e^{(741/2 * I * c)} + 1413754020308820178961328889057375784750324941205731220632$
 $799352997567939467526832841222047335354848244171656614625516085 * I * a * e^{(739/2 * I * c)} + 221031391624875337635546246102968719766070020504848277575893242534$
 $2781809601155166037756063197334234247463515636702691330 * I * a * e^{(737/2 * I * c)} +$
 $34221343023174192329904603489067710817250505419905048934146238032397191152$
 $80808161335987355144864275649603454218872971580 * I * a * e^{(735/2 * I * c)} + 5247170$
 $71193742875498237689590963416558841382097777033506021664235403593006423444$
 $490084357597297711396346308669760798689 * I * a * e^{(733/2 * I * c)} + 796820700441231$
 $679735742088944894430487588626523574245830481249993762974648093193551920603$
 $7160539581522171987555750097661 * I * a * e^{(731/2 * I * c)} + 11984658394515165319634$
 $827787595449667237245852425113651072146233279869165517073131931513924164490$
 $275061920030161185859950 * I * a * e^{(729/2 * I * c)} + 178543014828327237381513629934$
 $276680941045982591828198036711915638221886077206236384288733091635402307821$
 $69354631068186188 * I * a * e^{(727/2 * I * c)} + 2634722491074523483769625594669165696$
 $247639632180233653128376580788401980064509198212034622424974493901613902271$

6374364629*I*a*e^(725/2*I*c) + 38514520648760918228036755788926097489724496
546941732018982203797769917480619897805747366276137271048739730353633877720
246*I*a*e^(723/2*I*c) + 557743650882071383061823428136907285675901598890415
79007946978228143889730779948322963998644219045352889801185546869084174*I*a
*e^(721/2*I*c) + 8001811557783642701076696476946993118662492011906234468644
5108628080498436960382404149332811237697500795135371424363931912*I*a*e^(719
/2*I*c) + 11373880699718367608654346517232895570860701851878364855552562510
6590297142781281354203188623909797488026117582818256419866*I*a*e^(717/2*I*c
) + 16018390310024491399589891810028467230424870758017612489626238141446543
8933861024310119111597234769579332371810044707792786*I*a*e^(715/2*I*c) + 22
353326578596883514387560025326324433543045327642356089211895885600284087274
9060322902817137435872704723487164743059560288*I*a*e^(713/2*I*c) + 30910198
454139291491998334324383454757099934023351069019471767444707960282631156876
4653637550247109038105458177964729351012*I*a*e^(711/2*I*c) + 42356590469530
397618028105246119646948625093649536670112135976865303107013996765644320517
9446365202776502809602693273968654*I*a*e^(709/2*I*c) + 57520537836464419636
834559585294277687340135091241285346717893214249887017844048901649968684842
1923980117904712934988541229*I*a*e^(707/2*I*c) + 77415994423928762785021381
764599701472101121370429364427262716503131266128077413187472071417102879206
0394385453881130835866*I*a*e^(705/2*I*c) + 10326833781408798598715685440931
614866301473319621108843354141229752824379283252892159956340795787271063539
02540296842372558*I*a*e^(703/2*I*c) + 1365384960843458647834068386583040556
831516179914170280355490859615430960938934067120381777848152158124027973811
688241828477*I*a*e^(701/2*I*c) + 178943906435616044494140836108479112889982
263247843589635556168785678329431717513293604789904498670400750515439460112
3847453*I*a*e^(699/2*I*c) + 23247415130512690281765260929546689247279241363
925098295022509200027825817048315555440437788944320885776609402364643431816
90*I*a*e^(697/2*I*c) + 2993987947288425518808928248754589667712106423890365
853065234018548819562445899531390409664170571394926881128145770065899220*I*
a*e^(695/2*I*c) + 382264697413215166970786157267327925910297091788357983560
6273218857975092162918434170953228420879543659937491953198146195925*I*a*e^(
693/2*I*c) + 48387996361111263172080964351011092440835595380669484893874298
83233492589499491605457749827375642095290439183595761588081080*I*a*e^(691/2
*I*c) + 6072817074204712468274441182835677570948620936784622005264549151564
266119016397112228983141779145954492755416086534786022346*I*a*e^(689/2*I*c)
+ 755685058467942384069281977003129681151363129234954433023067473886036825
9494432432567844365994608959978423828261012106341304*I*a*e^(687/2*I*c) + 93
241128788016496313851751411144345263829853283002222845386219384884437515564
77225816459938122698725278253649259377876875956*I*a*e^(685/2*I*c) + 1140793
644236414863961925322676715375573694014098823792666633979384227510052733374
298581396287055519700042503130062102254624*I*a*e^(683/2*I*c) + 13840604493
814049780973828546511041191138534310013963427847312833326059845471345783687
136583985872044817693603530350111267506*I*a*e^(681/2*I*c) + 166519619984923
331068036514841112023323662475383036270497690976414641089291627408982243821
37687725244771019687054923945046640*I*a*e^(679/2*I*c) + 1986782813850402792

435803837007843024336077185901900775461709270589489548232039076520583465372
 2079791333317381901945409007768*I*a*e^(677/2*I*c) + 23508246964781735141632
 365638448017151121169614422195680868391258194610581920728030535410178638205
 451448869498618378116964232*I*a*e^(675/2*I*c) + 275856288583184014177612634
 272945850537939719850315871027518382355931980531330615442637387956682970388
 85359604477137092618448*I*a*e^(673/2*I*c) + 3210285090892660409424213973529
 495224528101332037331055002089573485700408488036159340689037826111311863219
 6652156779246123832*I*a*e^(671/2*I*c) + 37051398263203087048517280736377192
 693587419362465698554580525240769695507938515050581196946526516020509145484
 606049843462088*I*a*e^(669/2*I*c) + 424096397262841225387048322393177248300
 565718290597660819717087019970485761655559389717552016690414553318678139650
 51654307480*I*a*e^(667/2*I*c) + 4814133828670270443494318370428184176235314
 002997972864172743486451690869192828601007960728080578432338915063701477396
 5009680*I*a*e^(665/2*I*c) + 54194499762890460723929105582897570216085973537
 596633216486796551038997346993003645175821744461702739407646907627975598738
 840*I*a*e^(663/2*I*c) + 605006596623126218031602601746579223502211520047673
 17545468752663230455453836071633783294815424678317791973306003900350353160*
 I*a*e^(661/2*I*c) + 6697469913077846538914940090465987167430309649621766851
 1479613781699518747276124181121029837065794584289560577716116602457420*I*a*
 e^(659/2*I*c) + 73515265460978022291049555461658974596744920966894424354716
 781203950620650045013650012377646825486222824081997699903889572320*I*a*e<sup>(6
 57/2*I*c)</sup> + 800058513579856460251215011652477765886785672686852468639929071
 65010082240622707911593521131774941396043731554410091241720760*I*a*e<sup>(655/2
 *I*c)</sup> + 8631656078816751274445725383171093200605106707512974307519342705085
 8246340444879284106183668280070526606382512365973492282060*I*a*e<sup>(653/2*I*c
)</sup> + 92306558934010460154314547976260787142987352242958060011321681639646026
 993007612606976698995821213357225605569003753064006540*I*a*e^(651/2*I*c) +
 978271710517758356614851108675746302673246010722659154094209718193563262939
 13318240743163564341172410866596207519180793797160*I*a*e^(649/2*I*c) + 1027
 255616438762116321534031127595360965131210111787842691346773077789867872256
 97189206232077844605004406406497176048297949080*I*a*e^(647/2*I*c) + 1068488
 932108919771042818651380177081098335409483043325317404719042989475223163580
 22815005300747182128774487161016238862329900*I*a*e^(645/2*I*c) + 1100488348
 410264759599026986441106407687167701855742432122574029623107476371567855418
 86064672440123040238982814411352193275760*I*a*e^(643/2*I*c) + 1121862667999
 892444118324104590243503739886206692631749566504553262289486407367882712926
 58432697539009158378809943007656485440*I*a*e^(641/2*I*c) + 1131360096196882
 560673685813226647177007066357388125297382388679396595908022821692269547999
 31975543848041993126041018399782640*I*a*e^(639/2*I*c) + 1127913961074666991
 335082928004885169897237417731391357410180679848860373091387562332435928223
 58432801130626891113712078476960*I*a*e^(637/2*I*c) + 1110685029295280185235
 551866034370726474823020737174091235959637950490979773787494728596101802753
 50151345478254745974421907840*I*a*e^(635/2*I*c) + 1079098652035309276305746
 801385484461332725509647484124799898717103380683274715519444786317210367402
 16042261083571574894545520*I*a*e^(633/2*I*c) + 1032875126216357270019162252

069851917717687059593101807010810800640163345906195862462379570150471128038
 08968581609901944100680*I*a*e^(631/2*I*c) + 9720518831669389454313543525560
 007384022285163822775012727908687942526348633328027036203571559250302587129
 1198541945539450000*I*a*e^(629/2*I*c) + 89699640886356096212186761465087430
 703849390130218154088737913649465978223346239090680080257616229441671165845
 538363436497340*I*a*e^(627/2*I*c) + 808409142977369308955701027437455136510
 013512703125447649382346137871132696658170647838056116764653358575671084824
 23695998240*I*a*e^(625/2*I*c) + 7073159819183244162539792986740854978145073
 507338632004690484931376624787671385123480232285480253099308049256849707060
 7833960*I*a*e^(623/2*I*c) + 59505040665512914709000743487513075110958013616
 100913522472530216019669696050974277244082570160190999482228790680261106380
 540*I*a*e^(621/2*I*c) + 473225651914242698298170107814085585967937467823732
 82618951353454733615818669575867423053741342217028108862881275991225842700*
 I*a*e^(619/2*I*c) + 3436977127874920653742628967722738146464464337985371712
 0169694951249851578045846724379802197163939310568646413075921689487840*I*a*
 e^(617/2*I*c) + 20852033260202120041673473658620917231794877194943715445744
 154690362463066942921456479058038515193180799997053917525437497240*I*a*e^(6
 15/2*I*c) + 698934188613444222883426912511264147212564818851551010323045382
 3285545505266418080876124175346096295740915799724115868021820*I*a*e^(613/2*
 I*c) - 69893418861344422288342691251126414721256481885155101032304538232855
 45505266418080876124175346096295740915799724115868021820*I*a*e^(611/2*I*c)
 - 2085203326020212004167347365862091723179487719494371544574415469036246306
 6942921456479058038515193180799997053917525437497240*I*a*e^(609/2*I*c) - 34
 369771278749206537426289677227381464644643379853717120169694951249851578045
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489/2*I*c) - 22103139162487533763554624610296871976607002050484827757589324
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c) - 1413754020308820178961328889057375784750324941205731220632799352997567
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*c) - 520140205211740146146328886033919284277656267013037615975662868102063
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*a*e^(447/2*I*c) - 22185310849017376144940581335084488215931748657518846251
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*c) - 114313182922035384495360621046134469254143793530201476452184881941658
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5/2*I*c) - 3526000276820029513665521958312929619043934520318396654347425813
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*I*c) - 8244492425854132814040653088604670129631127112463111698869090178226
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417/2*I*c) - 3332111380052557612402008649109335923482943471792625473215198
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^(409/2*I*c) - 111911937372368801918281190224557235328802284114807569582104
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/2*I*c) - 31115307812434068987682163412997317426688473577707506916311083159
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*I*a*e^(395/2*I*c) - 186695013151690567802855585778163774515738518543184957

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- 17069600634708996917258224941647931382492115532252012641576292881810
859853135265796592414892124832389000*I*a*e^(379/2*I*c) - 597229412763625906
741512605060115937916663206140982191527346565929854269083632781787141232022
0580305480*I*a*e^(377/2*I*c) - 20628529549208161860239159412511175279585004
53683935875173392483952599009295139987985628233229731478980*I*a*e^(375/2*I*c)
- 7033452798987555835621279126011317913341079567117190305485479435552211
02827637472553536460508801363920*I*a*e^(373/2*I*c) - 2367038362525877793237
014343887874652494270451048530129074390252640862996387628641125640490766557
32830*I*a*e^(371/2*I*c) - 7862140861949527405998062028385414620898424406453
5407477261926286982916818434658466433148202509344800*I*a*e^(369/2*I*c) - 25
771323015705925464502138277035617975672430492105429256456221829154887242352
647174124493051808094580*I*a*e^(367/2*I*c) - 833592292197533955487498574080
9951353130777120817181687184875795893464066876677629382653264393601630*I*a*
e^(365/2*I*c) - 26604284293157110377467508361823506171669508516660881855251
23590808329315051618495256897037944395590*I*a*e^(363/2*I*c) - 8377015256545
159431609190656529894022686291965989070395451643569498457168198670907077695
61650988160*I*a*e^(361/2*I*c) - 2602105215466483247106015520173362630016654
84044998430835364683575217699721326919568028661098556620*I*a*e^(359/2*I*c)
- 7972880130045554412209635813459953053931399652287187615067132677014786662
4403122038941020982112670*I*a*e^(357/2*I*c) - 24094419287902104220146364315
256987563430323234468653857182722530873173656507620757370776234725250*I*a*e
^(355/2*I*c) - 718097670878156415201193307932406639694127797972073212630524
4304659930709704206531169354332309620*I*a*e^(353/2*I*c) - 21104268670165790
117381599954521064496070941706939755056254108121272217379310235214274469974
22880*I*a*e^(351/2*I*c) - 6115467103507552270942105553227382335739922384388
80922442071495399840623270733748703638918730970*I*a*e^(349/2*I*c) - 1747081
485769545081331085664321384827528713143887353908197015583046210697496653560
28799825469570*I*a*e^(347/2*I*c) - 4920070983117084797978331794939380374707
2027694891369055993880672007049593913929619833129271500*I*a*e^(345/2*I*c) -
13656943537866316562345022885038770780812495797097516350805731913565888244
243229131324182944640*I*a*e^(343/2*I*c) - 373600623051027567561006154412697
1557393421121949410887003654975726431985661061431612714966210*I*a*e^(341/2*I*c)
- 10071178376573733694687297257198205970085724654697532572151536265987
33911453021524113695566720*I*a*e^(339/2*I*c) - 2674957793417936868588920871

86701348017864136095797204004377097030869586488588546928368741180*I*a*e^(33
 7/2*I*c) - 6999410389985444968804481447120174341841380666658362818300663312
 2034125384874442925881756120*I*a*e^(335/2*I*c) - 18040822556182136896638713
 100892344358328053007173679628765657147450959398026341254411115720*I*a*e^(3
 33/2*I*c) - 457975804408353384764807088661806323690119216200423393607834885
 4854553658379606034075133880*I*a*e^(331/2*I*c) - 11448786659733453628803128
 04992922340263144413567571060896235650099922407159938671183630200*I*a*e^(32
 9/2*I*c) - 2818025430799707737642227475011032405108627832374810256967357111
 15713233601841503443789040*I*a*e^(327/2*I*c) - 6828651937370060666296951859
 6072942990402829423660330718366006960865727616800960284438480*I*a*e^(325/2*
 I*c) - 16287860131873426297665954195872007340389448696155366782102817863363
 964748733518124937370*I*a*e^(323/2*I*c) - 382353104707450177390188261707972
 5474646137205608635840851184500433382055447334225716180*I*a*e^(321/2*I*c) -
 88321698672020915183702241433267232811151500031503004561116840853852403247
 1081357579180*I*a*e^(319/2*I*c) - 20072483864097435618425497602435711351463
 1463707083379071558666747287853457166817321610*I*a*e^(317/2*I*c) - 44873754
 537280086404910910878863721864966849160970709878039315079653743556851113962
 410*I*a*e^(315/2*I*c) - 986656179921067405042262616811879997621874203873738
 5255818701142786087196734211598180*I*a*e^(313/2*I*c) - 21332598764537892264
 26738739204266559366922799155021793045435139729509358790624760320*I*a*e^(31
 1/2*I*c) - 4534675706476787267628307963246707489435402119186131159813827989
 96143666294810914890*I*a*e^(309/2*I*c) - 9475254993717593440049975627097571
 0616491132843843008041484744522257068322302549540*I*a*e^(307/2*I*c) - 19457
 764444728521352641555485388469197530472258189211796433938437083420010091733
 396*I*a*e^(305/2*I*c) - 392613197522804737302143988361029394464744692867334
 4297294745924701096160760888424*I*a*e^(303/2*I*c) - 77824546206109293824112
 8952993007961165558304583117537979936583232925968316199836*I*a*e^(301/2*I*c
) - 15151479395044286133635700687251245806444665347455352580219358369862001
 4121641124*I*a*e^(299/2*I*c) - 28965714003257846921540858986388954008122111
 612712363354360981851687424724108036*I*a*e^(297/2*I*c) - 543632108243814202
 9415739885553914767552718204411177277415536254863231523181480*I*a*e^(295/2*
 I*c) - 10014177203446569152519249010097509399901491609651495409298898243884
 02626455012*I*a*e^(293/2*I*c) - 1810126781799484651898911898768444835204795
 68880173570917696339360628958538348*I*a*e^(291/2*I*c) - 3209792256384850694
 3066473469340753662783076784232671471143949644558156480632*I*a*e^(289/2*I*c
) - 55822074307217674249109221691557119073985697219371140734803311644408624
 62638*I*a*e^(287/2*I*c) - 9518731874891467462383380080222399063180718980280
 16864090568030803687398612*I*a*e^(285/2*I*c) - 1591018076203724577102305546
 27597385485602304037309714452742966256548901840*I*a*e^(283/2*I*c) - 2605964
 4761295218063521552347914441867779264013071310596786890247411087520*I*a*e^(
 281/2*I*c) - 41814710709073876126595986316067517882087887942015362828025424
 77878283190*I*a*e^(279/2*I*c) - 6570856213855142413632413014951270581090650
 15699304745663817714211556940*I*a*e^(277/2*I*c) - 1010897274677769565234755
 50836202489574004061008016807940002791738698571*I*a*e^(275/2*I*c) - 1522084
 7247394291084416636302914971818756836097657800823465735035857764*I*a*e^(273

$$\begin{aligned}
& /2*I*c) - 22421580417272351173649577394593428478193933458421162568455836552 \\
& 36406*I*a*e^{(271/2*I*c)} - 3230219500315412278978014071715661701505192452023 \\
& 46540397613583355667*I*a*e^{(269/2*I*c)} - 4549594753477955200809388843993310 \\
& 0139363372706825172710009091580883*I*a*e^{(267/2*I*c)} - 62620703317793240005 \\
& 13806613163981009868362783949768253661316894658*I*a*e^{(265/2*I*c)} - 8419575 \\
& 79184928615664618341834644691840690166368399913404427216774*I*a*e^{(263/2*I* \\
& c)} - 110536162580123562634802981277497355054368691566003022338877490451*I*a \\
& *e^{(261/2*I*c)} - 1416338923938882590284207339140080682594037520393945695744 \\
& 5830422*I*a*e^{(259/2*I*c)} - 17704216082502971556115991302920374348141002861 \\
& 57679816873050532*I*a*e^{(257/2*I*c)} - 2157852422859948688185287314305408124 \\
& 22314520336288528460007864*I*a*e^{(255/2*I*c)} - 2563192633482051932428940660 \\
& 7504181260913532669181012596611058*I*a*e^{(253/2*I*c)} - 29656751938357100665 \\
& 59916582743095158371951788246043891048682*I*a*e^{(251/2*I*c)} - 3340456155724 \\
& 10482684075553294675381058554260440184749831968*I*a*e^{(249/2*I*c)} - 3660771 \\
& 8305457313571850609230094969421310836249109701091666*I*a*e^{(247/2*I*c)} - 39 \\
& 00820584110953912072698502834368654421340467896495241550*I*a*e^{(245/2*I*c)} \\
& - 403899515051838803400194879806531247927007890538062521027*I*a*e^{(243/2*I* \\
& c)} - 40609448120913176083174637180877049405994150689208285948*I*a*e^{(241/2* \\
& I*c)} - 3961896224254872041251938157236050043036325552867100290*I*a*e^{(239/2 \\
& *I*c)} - 374773875381762608573480596867444622808315521135770955*I*a*e^{(237/2 \\
& *I*c)} - 34345846767794763840341317174417543493362860419944375*I*a*e^{(235/2* \\
& I*c)} - 3046808467328698391993011459852063034674399964693132*I*a*e^{(233/2*I* \\
& c)} - 261388357024674483234213954236474266534206934500718*I*a*e^{(231/2*I*c)} \\
& - 21665877338576587360651194480041798194029867090275*I*a*e^{(229/2*I*c)} - 17 \\
& 33270022057743255812637849164901656684569946869*I*a*e^{(227/2*I*c)} - 1336830 \\
& 49871200685767730244065634086417919364482*I*a*e^{(225/2*I*c)} - 9928713959958 \\
& 644143611456276187690708075285068*I*a*e^{(223/2*I*c)} - 709193818893909656677 \\
& 523349582328791900299617*I*a*e^{(221/2*I*c)} - 486518167851923908629493460643 \\
& 52748595789341*I*a*e^{(219/2*I*c)} - 3200777320589257207465955323343789287604 \\
& 398*I*a*e^{(217/2*I*c)} - 201623763354433037155766729846494383292508*I*a*e^{(2 \\
& 15/2*I*c)} - 12139650447759376183227584660170495869589*I*a*e^{(213/2*I*c)} - 6 \\
& 97316726077381488801524339658757026686*I*a*e^{(211/2*I*c)} - 3813450805531950 \\
& 1480202851957227724462*I*a*e^{(209/2*I*c)} - 19810133900877781711437444880414 \\
& 42712*I*a*e^{(207/2*I*c)} - 97511021243902906034367815029181218*I*a*e^{(205/2* \\
& I*c)} - 4535396318845906415166472261686554*I*a*e^{(203/2*I*c)} - 1987158175445 \\
& 29039864660110471376*I*a*e^{(201/2*I*c)} - 8173401220508860669709781109364*I* \\
& a*e^{(199/2*I*c)} - 314361585019926536123171779990*I*a*e^{(197/2*I*c)} - 112559 \\
& 13521436332875256492285*I*a*e^{(195/2*I*c)} - 373282846199958103946548090*I*a \\
& *e^{(193/2*I*c)} - 11397949499240328225340750*I*a*e^{(191/2*I*c)} - 31821686414 \\
& 6377884963053*I*a*e^{(189/2*I*c)} - 8056123142378999966797*I*a*e^{(187/2*I*c)} \\
& - 183093707775710628858*I*a*e^{(185/2*I*c)} - 3689545748587122372*I*a*e^{(183/ \\
& 2*I*c)} - 64891508140773093*I*a*e^{(181/2*I*c)} - 975812152492320*I*a*e^{(179/2 \\
& *I*c)} - 12197651906154*I*a*e^{(177/2*I*c)} - 121672338216*I*a*e^{(175/2*I*c)} - \\
& 908002524*I*a*e^{(173/2*I*c)} - 4506216*I*a*e^{(171/2*I*c)} - 11154*I*a*e^{(169 \\
& /2*I*c)))/(e^{(531*I*c)} + 432*e^{(530*I*c)} + 93096*e^{(529*I*c)} + 13343760*e^{(5
\end{aligned}$$

$28 * I^c) + 1431118260 * e^{(527 * I^c)} + 122503723056 * e^{(526 * I^c)} + 8718181624155$
 $* e^{(525 * I^c)} + 530563624556832 * e^{(524 * I^c)} + 28186192554792138 * e^{(523 * I^c)}$
 $+ 1327882849274858880 * e^{(522 * I^c)} + 56169444526926562260 * e^{(521 * I^c)} + 2154$
 $864144781257856128 * e^{(520 * I^c)} + 75599817092670157806639 * e^{(519 * I^c)} + 2442$
 $455629894502983849104 * e^{(518 * I^c)} + 73099207817335597247098038 * e^{(517 * I^c)}$
 $+ 2037031259470368160131922320 * e^{(516 * I^c)} + 53090127264630963470039804475 *$
 $e^{(515 * I^c)} + 1299146645993240318167826532288 * e^{(514 * I^c)} + 299525477492654$
 $99675257842032197 * e^{(513 * I^c)} + 652650253343206047453620559993840 * e^{(512 * I^c)}$
 $+ 13477227799524701956579274210395326 * e^{(511 * I^c)} + 26441037578031074251$
 $8099326419685040 * e^{(510 * I^c)} + 4939666610818025798809586352543471345 * e^{(509$
 $* I^c)} + 88054927598941411145869950813388040256 * e^{(508 * I^c)} + 15006027479373$
 $97286405577818722691539392 * e^{(507 * I^c)} + 2448983733781233868771862249186501$
 $3839488 * e^{(506 * I^c)} + 383360155801054824529764688213114368047154 * e^{(505 * I^c)}$
 $) + 5764601046563151304213854710715346838447392 * e^{(504 * I^c)} + 8338083991183$
 $7894453136303673785039051506805 * e^{(503 * I^c)} + 11615814137339717515336225119$
 $09046917188768400 * e^{(502 * I^c)} + 1560391127768760709972162377174493308692058$
 $7272 * e^{(501 * I^c)} + 202347509724462171313966643580234078508179838320 * e^{(500 *}$
 $I^c)} + 2535667460650279776834561566186591213109251642859 * e^{(499 * I^c)} + 3073$
 $5366512830562160991166338490057308062762518496 * e^{(498 * I^c)} + 36068861303638$
 $9349413809780004559963548775423325255 * e^{(497 * I^c)} + 41015454399371957939599$
 $56708442496709433800261224880 * e^{(496 * I^c)} + 4523094003983073833202569478464$
 $6206844854827698075736 * e^{(495 * I^c)} + 48409341024048871865591702530366258109$
 $1659126182344528 * e^{(494 * I^c)} + 50320249034014518240742139437660119220265070$
 $06311982753 * e^{(493 * I^c)} + 5083636950817109943701934861084739194673618510801$
 $7183136 * e^{(492 * I^c)} + 49946750655853173367158586291057270281154503573039874$
 $9530 * e^{(491 * I^c)} + 47753986071008532635342077338182667774786934127387310316$
 $80 * e^{(490 * I^c)} + 4445670817525882102440094621053500452377572219097746848449$
 $6 * e^{(489 * I^c)} + 40321222595779818884084613996099562414449127169433679645958$
 $4 * e^{(488 * I^c)} + 35647648906287240170884879966881789291957876139585454748048$
 $45 * e^{(487 * I^c)} + 3073621740432100996523103741966305396288103528170922169778$
 $5072 * e^{(486 * I^c)} + 25858534871597727015582911568419341107203454149136439398$
 $5491350 * e^{(485 * I^c)} + 21237029691888713182667187812239270678399490157272938$
 $84065388080 * e^{(484 * I^c)} + 1703388602739061574104097772165554166561216227548$
 $5028584310890417 * e^{(483 * I^c)} + 13349021005202618377967331386833230353033290$
 $6163247194627808410304 * e^{(482 * I^c)} + 10225364374682967372930658627052464496$
 $93687415559865844306888705423 * e^{(481 * I^c)} + 7659010520187549651777118357676$
 $871927081898989131125755798204236112 * e^{(480 * I^c)} + 561170810763411753840875$
 $70185188538660375932013674735519055227368366 * e^{(479 * I^c)} + 4023496922661211$
 $58934003582839428785116904903936409545602519219664720 * e^{(478 * I^c)} + 2823905$
 $151936586678382525706564457280290098698638597987628380245881715 * e^{(477 * I^c)}$
 $+ 19407979215594566593535008103303255257745408070082431338945184797463936 *$
 $e^{(476 * I^c)} + 1306576602265604193351214343899389618845954340699848243071493$
 $32131747540 * e^{(475 * I^c)} + 8618848510949919087642468054746724286037573154844$
 $53974713612812215428992 * e^{(474 * I^c)} + 5572551157328671121016216416307596161$
 $861955969011697222340926210112854418 * e^{(473 * I^c)} + 353244472067790181153780$

52820789411687581004582367431006205879633729015200*e^(472*I*c) + 2196012813
39515561500261478844190024870555261281946058839614044697037963695*e^(471*I*
c) + 1339214374254245553564884406801945353385000254030655765953770237607180
089968*e^(470*I*c) + 801372958079075243436196494576154376146952079121074697
2675870481058674277844*e^(469*I*c) + 47065044611135158108487353367484243102
698248838312635876283099427442745866704*e^(468*I*c) + 271361207503266570734
486517077181014801775322183181055638619257836143271472358*e^(467*I*c) + 153
633323844492758353273455601649467167491657890711698454848907824169392694056
0*e^(466*I*c) + 85430134411262123348335406650696214724790858380413605645507
22036723654297540205*e^(465*I*c) + 4666822354826601780685459246810057028935
5960869613650856575756758180182223308768*e^(464*I*c) + 25050102860892833246
9340456829902067712233644464602753159945727868485722395506952*e^(463*I*c) +
13214980552713008514299938666316198744245344251881835920497276875710321564
35077280*e^(462*I*c) + 6852993223145736687328885311617795435592940841439866
351079655652312894721972796266*e^(461*I*c) + 349410716132767046494779430433
39450201504075335160361865916029213860778606230624960*e^(460*I*c) + 1751931
705006183002415156323819122851577900978160492206712172122200152971334006360
60*e^(459*I*c) + 8639799336223303495562968200283955131987080649405057021260
68652936800794826651264256*e^(458*I*c) + 4191542500656826148093339414544159
143964478472492315931809171859902114109005939942952*e^(457*I*c) + 200080068
030300471372932782503215971135407162019833331263492811866791531990680452572
16*e^(456*I*c) + 9398691531306817914908360606568148278083606051053015461848
6949839467131378859885998210*e^(455*I*c) + 43454667678028004534634449876389
2540797175105756827515509297024187660299345484920192480*e^(454*I*c) + 19777
929806658181356513000943262391586054488708069708605773253850286099830345346
72318500*e^(453*I*c) + 8862752142756957285681340885764904597935349569355321
815647721172537159186491471311666400*e^(452*I*c) + 391080312556018094765375
35369611844440844903751605645023514572352045248104262933598850730*e^(451*I*
c) + 1699563279699297677739020966526292532837045054771275445565344173766865
40936706073847337600*e^(450*I*c) + 7275210107183942292917740738446942557987
38667067535379759732795567942578751384250780476310*e^(449*I*c) + 3067974296
431747364198159623962463671617006419626851426148418602934852907379021659761
911840*e^(448*I*c) + 127472196165033205413563430625628473686016221408567860
25445814532037904111523242298235713300*e^(447*I*c) + 5219091220766182421581
2271854269748071292843243227894769229690720010547141334131610989636000*e^(4
46*I*c) + 21059430138564847118432907888031750495336183995415942743400988466
1777259752542647709150036990*e^(445*I*c) + 83757920692341193245878648676537
3533946545239708990769488724813982189165104589895518909256320*e^(444*I*c) +
32838747605558186767263094803067344201550985839480744690141681718744421701
09648521627538755920*e^(443*I*c) + 1269349693296492056507367363718128008854
8682508880255337280065006566138696041797353216584528640*e^(442*I*c) + 48379
489756434099843857791816589379406815042609340378747586437145781646245422045
101230417309900*e^(441*I*c) + 181834661406177901315330129677145381166449188
413194141169344354754920969034952610378945282257600*e^(440*I*c) + 674025530
543133008894845775236625237450743114473544537818170447134607102575676676056

675328961590*e^(439*I*c) + 246438219080743960907977422685567962936788570977
6435876630851716253962696192341706239192878728160*e^(438*I*c) + 88882950287
510246670442038376079761014800531344186144746207675228248689119598843526664
44917404000*e^(437*I*c) + 3162664467472554773117679568752765357130596998592
3688392112164915553242573269490908989570248533280*e^(436*I*c) + 11103414879
700881944314389564446924229504986746431371096925761933889913379928561602006
9872611710850*e^(435*I*c) + 38465584208066627445406307878483717499894905250
0975322162003392549953413592461519365177908682078400*e^(434*I*c) + 13150521
209306921221022971053276228423358707434285308910729835358622800944466077234
73800477453914130*e^(433*I*c) + 4437210917843182347764349544443904699020056
595069470847193617092114714077633077234972825351226979360*e^(432*I*c) + 147
779550966171289987127451820714953621765069731830816502336052740516776249704
64340242755840025673760*e^(431*I*c) + 4858425815314028044731483686877213139
0195412419046732778458706015096881437076337910793584122475073760*e^(430*I*c
) + 15768584552885091872146287786443509025758314941556132342738656289444759
8277935629800939237175625149830*e^(429*I*c) + 50529366312301525887848302573
8812813203397766845340065381261016353419722382620393032535960660921950400*e
^(428*I*c) + 15987711010581926922705289996774447426856310062324561858449252
20144002305878120380828483988663574829100*e^(427*I*c) + 4995241956279138180
205186744401688024388272113921255663734956946927571305533146776898787878059
685108480*e^(426*I*c) + 154131112114860239372949708207973767160813447881633
86543522421939737507962125854981881879168348260330000*e^(425*I*c) + 4697022
472711728182645404501807067052255975662758034778453532001496348263235972944
4541885102274546002560*e^(424*I*c) + 14137993825355684328056550580740330413
0606130725434751745794079833141361748917639986145377066437210546190*e^(423*
I*c) + 42035802483514679858361121014594215468443794936564789908837252480215
6222884839580011688655664280691773600*e^(422*I*c) + 12346680418924099787800
180817554402160125824763969419379658996319530792039742221387946043284989721
44766900*e^(421*I*c) + 3582718002163296061414536703715109897107198252739284
546149343102348456124089657428594946438660859773886240*e^(420*I*c) + 102716
025302028890024978135168494525909715128095290606651973010970522100645760883
48023234671975463677418470*e^(419*I*c) + 2909765106124745340664756978183691
0062165559852359052804259154165687125428752562385492373749486351714453120*e
^(418*I*c) + 81452081413829111828875417564250054846037693312811480492909160
758195989155768107022568350953861815940704090*e^(417*I*c) + 225320532593220
657767941109289516248999794521015564134982827241710019675486694499689312466
561907212627820000*e^(416*I*c) + 616003116022979584937125701757887212998354
300989991362628038861093914561332071191909714949426587936910303300*e^(415*I
*c) + 166447503438721180939491774350293763897857493775476476398783587241044
9930690131572904279995484581013965001440*e^(414*I*c) + 44454122592954746250
676595141983129660154162999689303933456303459141097207405736188849805200100
28451496996210*e^(413*I*c) + 1173585692624511849311309100250160403234187698
5999082823520672241530200188223826392982302194084667538488665600*e^(412*I*c
) + 30627581054221957378390547289277609129572813931082733520247387226000020
043538279468776707958420892547870128680*e^(411*I*c) + 790191495587665692547

839884872323883529091449827471718567724632238089933670915034028764672701761
24342699654400*e^(410*I*c) + 2015579474247940980267724784620408833958675125
62930862943753568690084015585598010154781548625239409581907397500*e^(409*I*
c) + 5083245993010854601669786296830326614276544740829390480979396383915672
98795788389433842285751054665210868287680*e^(408*I*c) + 1267597017294812913
400146276042126929986480292870190399107554311079964227280196522475370108738
477856311765699610*e^(407*I*c) + 312568349317870174347970475030749017866629
2150720179363604335113528623329684606343185540756019935662148267863968*e^(4
06*I*c) + 76217887919120470620388409177993746004288922581943676366829443560
96681400246312138001769285020661445991073249416*e^(405*I*c) + 1837980708400
335976602764921762114411609173557221662078886153580344970227380258835907670
4241840733513439114113248*e^(404*I*c) + 43834972142919377685378692233021063
744554033100928502737480438978976746989895784070951905237783490374305934542
955*e^(403*I*c) + 103399755467257436489847837640753754718204394473055795001
467604326419876555556873829531737211096115196005647730480*e^(402*I*c) + 241
246021282440061792908317783032876194801597133206052091286997043729145345755
805710081489006741839439573984832678*e^(401*I*c) + 556756388711182340341026
192734219546113651768317380539005893679049394714017063698565272728813669054
779077208977840*e^(400*I*c) + 127103308293804895020136055483127034266234399
1277504612342300366025046741742856580445289401786656311685859023084716*e^(3
99*I*c) + 28704961314123144518346747153535894395532944308085319334660862885
43709246230769151392180699413405623017247753532944*e^(398*I*c) + 6413381895
855925184758231451062556380328594938511006577015218119786536390213057284018
202201631094434819584025113465*e^(397*I*c) + 141764836528757049572020133432
411179043699777966538499599025244219806351890116348156532796054977833828889
32766730080*e^(396*I*c) + 3100431920606941707706936314142348743182800909818
4744635678652284177439464941651812564519144918003174108077634846014*e^(395*
I*c) + 67091706130529669125019899210021576580237843462229535386295087076189
297849995931360645605292130961496106707521506432*e^(394*I*c) + 143657687138
044796942947119704259538458818199423516824674586293691056119209866358123637
772245409530799230553767222252*e^(393*I*c) + 304384471106813336010284160123
906370433888828490627422652551236966790916174520857759143930140187173492394
981908258944*e^(392*I*c) + 638218892914533741505806399662488556066783600496
091876408374974877448971778036074996245581124283460438065182071976085*e^(39
1*I*c) + 132431132498402742835522293814768237867286070881716174144874968959
3588020860847508703702325320304649883120684987556400*e^(390*I*c) + 27195892
834837439260408051010803419212445303112546072509291927739093315232266350358
15672862569296693711643521070331394*e^(389*I*c) + 5527498849031178355861230
009326668283926290082158467118000698502719379939045918344222192742145711257
028040974074674736*e^(388*I*c) + 111194996453632010808810628248863384924253
756584489779355358463492904258215703830904254114185215166703713720452065683
45*e^(387*I*c) + 2214073500170860327091518076924139166203557875590397914890
9213603822554792749183517160255571915875356439553717130797888*e^(386*I*c) +
43638300075815171025946211464465689618965773488660985857945657479854085108
851857911222911989837615452608512356008400295*e^(385*I*c) + 851395332347864

557795899594649006377607357297056212213800058372083691577946730496754287998
17875431430246332625899630160*e^(384*I*c) + 1644375006769068927413232601543
942785039545619360201335965818064493572402773494479273345171058099953000935
49279931273178*e^(383*I*c) + 3144090358082258615655954369383549454454739910
43129722046747228813030925204968503418566818838611866709807040793495364496*
e^(382*I*c) + 5951576155004315149474792823365470538279260879164252634971877
57029413385471835434198246807096214536895441388306027237899*e^(381*I*c) + 1
115398285545601550533328045600184317993899359217996729818340704221801195667
410485846996179056733558512452238583160792512*e^(380*I*c) + 206969828950086
064346166576237380795751301942404117887190496055182941244934472243212567941
7958403007551179298315947373776*e^(379*I*c) + 38026049967058911069646206338
489648070370988545101822632430305972956307603535975319743247522663891931857
60878274188013440*e^(378*I*c) + 6917838945214844278493330459361394923372333
853619879637372673184942859712431066345726870422099893124890777678037369988
150*e^(377*I*c) + 124621404405372580849285967098720668577570709431248685545
00948154756863454308032925408340311237850017814707896986969086816*e^(376*I*
c) + 2223134113180153534540639903772168684020839794195258013558464596674673
6656716271554826476282991066076564921432614339399735*e^(375*I*c) + 39274200
414329861169397944516225001081227433398585007206399231211907157795359719648
241598754266579840244551491476467899952*e^(374*I*c) + 687124660159856415124
685861736597477348795917100983546527861249360230739431410495736066485630053
59411712764895683903806088*e^(373*I*c) + 1190605918496605468347656932276764
490675841482488826784475048260772363334445134540951266687500572958111916433
56908972191440*e^(372*I*c) + 2043255572651860007674027102308478964597615839
227636982354332128333130777830410400746693790173948367615396490816906308116
65*e^(371*I*c) + 3473100538109352904194555605559573141295692107357459832343
69659976413374774078000173070075248654524917179128950507443058208*e^(370*I*
c) + 5847495736823045861793846288448833275814989698865403803788967679990756
14964007174600811092945356635118795824799369716742109*e^(369*I*c) + 9752103
394440493187572823117635177866732231755944579463832792646350850410049173002
95904275433144848532459919875479817581584*e^(368*I*c) + 1611092541400060525
954859375264194178347643471837078201446262435615142944587337833513586022729
849523358436493586042252995608*e^(367*I*c) + 263666241043079934044752228477
824428374075106865814072657644667120779832560683229593770506168629793029638
2338892574900819440*e^(366*I*c) + 42748269077205917525267113368208715008443
456479223854715343593336061895718324446413641328931086635762051338706721562
64164115*e^(365*I*c) + 6866425337518668262662693750908956965732924578142181
630622157802899880874681551031136314064199948604001529894566235238597088*e^
(364*I*c) + 109272106034735448102797923478445360745888968062300411100895447
31605863146104181739039426674855453466097402330688331845602302*e^(363*I*c)
+ 1722950282436764733440072199841759670394865739473880520939163659737038057
2398964715080095366818322029152193635869784095333760*e^(362*I*c) + 26917794
701086615097890120236890501105146799996021775195710866226228638984703456832
694153230611607263444183501026198563419616*e^(361*I*c) + 416704403753905436
434182193422717480400350714901190805852815224981888183759069003687012345313

04633163446319945130196476913600*e^(360*I*c) + 6392301943376198909061480128
863509812319944510230312261654464820899876780394445577704288655273849974718
3713136069104651812215*e^(359*I*c) + 97173055024742680058616722461368892661
141295540263493013032746083536157324268333390400308958318370219154887169702
257444756176*e^(358*I*c) + 146390448456351181218237382740374124191664819997
746988076598391862733629670142241546375533903130605297580105675355629160198
162*e^(357*I*c) + 218563166659649312247483640956272149212499115383828771029
654283363972585118090479413696638108156385244646591328454425745117584*e^(35
6*I*c) + 323413178014841003714151138246079152576360976035058457890409937738
723171537036573043681997163745313602400139153046673668433091*e^(355*I*c) +
474323043563100542377338629931966248129175976982332446018056009391154020438
895903140822967769494019446166779954024655344116288*e^(354*I*c) + 689518449
328793559903260418149974190253578340058895035589606468244680591556118170304
005037563669880057908765898949268614772285*e^(353*I*c) + 993555653649521127
226443960820233649386488510081892545866700444096661582790441241830855609577
062039555625090943332264901780720*e^(352*I*c) + 141916448142217657385823401
38989996288223223330957373071631074383893583220145429361729317508645838962
1425307051750612129761498*e^(351*I*c) + 20094961100926877381527820856837372
227279688242990587392154460834993514676253344496707577640666906561509490149
44043994822823920*e^(350*I*c) + 2820819298562215959107529807289628449621386
798989436369393116069894018781201000275633104498398959346631795568022519974
400130281*e^(349*I*c) + 392569765841577835276810394285601184021164276962171
721799661439847388718607439148263854721282653827045391263454029979227032102
4*e^(348*I*c) + 54166628040524363495855959828183579538662584616443540182051
58917742576425344364964596750653177677803492186817305171175032011500*e^(347
*I*c) + 7410372612891152224364633296128043971657193280327754304382235672773
781403023814127610355562505271969045177704726054907145784960*e^(346*I*c) +
100522095243695818275881549853455496780314457444999852082593854356097402724
00301454246872041775159838468077381562338745636398374*e^(345*I*c) + 1352123
041194543691555885470685179654156739981165687002156735297532681546781784653
3289123871696056195231696146162720992221760992*e^(344*I*c) + 18035327338177
455471177568594851682977978346449777193572087688510392426884519272991560851
326393852241961470040819793627127923997*e^(343*I*c) + 238563985655628020306
952781742126408332821541740064594582926440609479249073137359215616901539399
06017518647182491616573724049744*e^(342*I*c) + 3129526368818983831377277587
330726033411722725862950199235869563609266286606281984568906423581362297415
0120668921391878398978380*e^(341*I*c) + 40715988963701918950020348336736423
420513311359010485246919074652883393970805374470830156229705647312265477584
256027212762941040*e^(340*I*c) + 525392233467407711425870923702570695360603
196444395016676104827679558002760528924321527988146079751103662249450814281
21888473324*e^(339*I*c) + 6724408796908070382370325719966304760689061048209
049461949280293513021597981946996638333678869390013911559464689378409541847
2336*e^(338*I*c) + 85368118430215312848231291739673735887746201851666299600
392199418764750086828198719872744047767783667325326289221881974987582215*e^
(337*I*c) + 107504737406576916123480399169759633328321407419400510017498849

830598621565428266546315933920821527544726380201659114903834605888*e^(336*I*c) + 134297742023479429904629616104559610096074758721068022704468938063017
 059688023363436458971534964665036319889119229809973806909680*e^(335*I*c) + 166432332922589195130558329266398753389823955737598527556096093062473559769
 545772321978969318904192572733997888230986469005970880*e^(334*I*c) + 204622
 295535729109519829916789867225319429705162560082648840394965423112809336591
 921290309392396263834674368977840527147037426908*e^(333*I*c) + 249593072282
 565866398389951509619202682634455487128714631461891770823201367527645793770
 203788784677343934971424317987895255031936*e^(332*I*c) + 302060638030868463
 461139442279360499718906917482524894483356220196138377050828911383056860425
 370161157201493696073712322595776808*e^(331*I*c) + 362706307563843231135699
 157418510732452420614013624168879312187645233450153927975793326834780741391
 203430153093712635355523960320*e^(330*I*c) + 432147856464086938023811561808
 678589594702047904674282297959658800170984456799067751878044806619012452636
 891350731618278545690160*e^(329*I*c) + 51090761511134507452623846147137141
 450275722444316385531648429230851686635827717488464500331623385777400744950
 538410637735936000*e^(328*I*c) + 599378484771733474809376142401554850207064
 972118137572949257503651444541939309025276896049622515630263162184526394317
 285457368300*e^(327*I*c) + 697789106925924614816713747684682785083659819027
 952244447043355869741368500452561164024636073401929693105801105522738405349
 028160*e^(326*I*c) + 806169671327625532424575340089775733681994991576674446
 922354099714615192085443245663852257001115644286660304979476023966071898200
 *e^(325*I*c) + 924320052867522584035777495761072351222534420784861960001821
 020509468146356756433795246446491396141583854513687104334429566707520*e^(32
 4*I*c) + 105178210042883437194450817005121918711681634976695332263718214961
 0875004223784784183284961906494422955462431208645690802526770780*e^(323*I*c
) + 11878179430793903161088023247981101290208227826600872485993676434812002
 06046822166144285425922229375413676535071141005286431481600*e^(322*I*c) + 1
 331396114626723035802462123531582050339749996014152452835305956367425370222
 758621753922458727524856072950880960657564720475838500*e^(321*I*c) + 148118
 711708924666246695569496567785552473031326055269082160265717621873742624552
 2795329891464091005878304304075953693546767206080*e^(320*I*c) + 16355697446
 414219006578863812890766532275801720566775874514020692343552836874896596139
 13761959140773339736014790081814516625224440*e^(319*I*c) + 1792649078089632
 298936945728481969334964391597506285088488350622937252533420980803144316431
 701452190522716124797875257437516360640*e^(318*I*c) + 195028655078018191924
 499296120448701005646036284521850167442376626632155879143691731787870223267
 9213868287926294665202769722927380*e^(317*I*c) + 21061419034683443071125497
 612024845434027942523524821998174104248696772627150982884376465186834879454
 62774223656471345899082156800*e^(316*I*c) + 2257726219103856286812833012681
 573765496262241420612932076143151171960854554124144699023009842080515157923
 529357189869943515991200*e^(315*I*c) + 240246459556968608612000180303421105
 673944558862194614138410616288624616181514976302503083487523406726777402343
 3418269982431265280*e^(314*I*c) + 25377664154650303308154717466929885960699
 118946972250529283204525421755871548480964833312098074301139430153983626696

$73337957755720 \cdot e^{(313 \cdot I \cdot c)} + 2661100647975783583828235139201441930178396643$
 $383423903583862547255880772382049201015537214900832745601519737141849802506$
 $685264000 \cdot e^{(312 \cdot I \cdot c)} + 277007320715076864559750728138206549792496846605452$
 $741412233982733378377006830588348730997931598371840374087288434574638068020$
 $4260 \cdot e^{(311 \cdot I \cdot c)} + 28625031263204617977706677807256441849912556231746261756$
 $79050672100848988119391841466573417019247590580735265143427289340450811200 \cdot$
 $e^{(310 \cdot I \cdot c)} + 2936494214351868498703239455426771104344827306267558916550877$
 $467232455153286140521089582733932202553130712723836983468866230908800 \cdot e^{(30$
 $9 \cdot I \cdot c)} + 299049894962254360853812938028386633519008711512485881814378761918$
 $6957111903765723974899651518555144924290346242595167274383008960 \cdot e^{(308 \cdot I \cdot c$
 $) + 30233716435082250271756031752129532190224850458507318453075190082773851$
 $54731461213388035579159917590062343527464977286601165100620 \cdot e^{(307 \cdot I \cdot c)} + 3$
 $034408355530957075787731745322567981684616550162845473257679674280216947356$
 $785783843205604202307836897073595410412575660465787520 \cdot e^{(306 \cdot I \cdot c)} + 302337$
 $164350822502717560317521295321902248504585073184530751900827738515473146121$
 $3388035579159917590062343527464977286601165100620 \cdot e^{(305 \cdot I \cdot c)} + 29904989496$
 $225436085381293802838663351900871151248588181437876191869571119037657239748$
 $99651518555144924290346242595167274383008960 \cdot e^{(304 \cdot I \cdot c)} + 2936494214351868$
 $498703239455426771104344827306267558916550877467232455153286140521089582733$
 $932202553130712723836983468866230908800 \cdot e^{(303 \cdot I \cdot c)} + 286250312632046179777$
 $066778072564418499125562317462617567905067210084898811939184146657341701924$
 $7590580735265143427289340450811200 \cdot e^{(302 \cdot I \cdot c)} + 27700732071507686455975072$
 $813820654979249684660545274141223398273337837700683058834873099793159837184$
 $03740872884345746380680204260 \cdot e^{(301 \cdot I \cdot c)} + 2661100647975783583828235139201$
 $441930178396643383423903583862547255880772382049201015537214900832745601519$
 $737141849802506685264000 \cdot e^{(300 \cdot I \cdot c)} + 253776641546503033081547174669298859$
 $606991189469722505292832045254217558715484809648333120980743011394301539836$
 $2669673337957755720 \cdot e^{(299 \cdot I \cdot c)} + 24024645955696860861200018030342110567394$
 $455886219461413841061628862461618151497630250308348752340672677740234334182$
 $69982431265280 \cdot e^{(298 \cdot I \cdot c)} + 2257726219103856286812833012681573765496262241$
 $420612932076143151171960854554124144699023009842080515157923529357189869943$
 $515991200 \cdot e^{(297 \cdot I \cdot c)} + 210614190346834430711254976120248454340279425235248$
 $219981741042486967726271509828843764651868348794546277422365647134589908215$
 $6800 \cdot e^{(296 \cdot I \cdot c)} + 19502865507801819192449929612044870100564603628452185016$
 $74423766266321558791436917317878702232679213868287926294665202769722927380 \cdot$
 $e^{(295 \cdot I \cdot c)} + 1792649078089632298936945728481969334964391597506285088488350$
 $622937252533420980803144316431701452190522716124797875257437516360640 \cdot e^{(29$
 $4 \cdot I \cdot c)} + 163556974464142190065788638128907665322758017205667758745140206923$
 $4355283687489659613913761959140773339736014790081814516625224440 \cdot e^{(293 \cdot I \cdot c$
 $) + 14811871170892466624669556949656778555247303132605526908216026571762187$
 $37426245522795329891464091005878304304075953693546767206080 \cdot e^{(292 \cdot I \cdot c)} + 1$
 $331396114626723035802462123531582050339749996014152452835305956367425370222$
 $758621753922458727524856072950880960657564720475838500 \cdot e^{(291 \cdot I \cdot c)} + 118781$
 $794307939031610880232479811012902082278266008724859936764348120020604682216$
 $6144285425922229375413676535071141005286431481600 \cdot e^{(290 \cdot I \cdot c)} + 10517821004$

288343719445081700512191871168163497669533226371821496108750042237847841832
84961906494422955462431208645690802526770780*e^(289*I*c) + 9243200528675225
840357774957610723512225344207848619600018210205094681463567564337952464464
91396141583854513687104334429566707520*e^(288*I*c) + 8061696713276255324245
753400897757336819949915766744469223540997146151920854432456638522570011156
44286660304979476023966071898200*e^(287*I*c) + 6977891069259246148167137476
846827850836598190279522444470433558697413685004525611640246360734019296931
05801105522738405349028160*e^(286*I*c) + 5993784847717334748093761424015548
502070649721181375729492575036514445419393090252768960496225156302631621845
26394317285457368300*e^(285*I*c) + 5109076151111345074526238461471371414502
757224443163855316484292308516866358277174884645003316233857774007449505384
10637735936000*e^(284*I*c) + 4321478564640869380238115618086785895947020479
046742822979596588001709844567990677518780448066190124526368913507316182785
45690160*e^(283*I*c) + 3627063075638432311356991574185107324524206140136241
688793121876452334501539279757933268347807413912034301530937126353555239603
20*e^(282*I*c) + 3020606380308684634611394422793604997189069174825248944833
56220196138377050828911383056860425370161157201493696073712322595776808*e^(
281*I*c) + 2495930722825658663983899515096192026826344554871287146314618917
70823201367527645793770203788784677343934971424317987895255031936*e^(280*I*
c) + 2046222955357291095198299167898672253194297051625600826488403949654231
12809336591921290309392396263834674368977840527147037426908*e^(279*I*c) + 1
664323329225891951305583292663987533898239557375985275560960930624735597695
45772321978969318904192572733997888230986469005970880*e^(278*I*c) + 1342977
420234794299046296161045596100960747587210680227044689380630170596880233634
36458971534964665036319889119229809973806909680*e^(277*I*c) + 1075047374065
769161234803991697596333283214074194005100174988498305986215654282665463159
33920821527544726380201659114903834605888*e^(276*I*c) + 8536811843021531284
823129173967373588774620185166629960039219941876475008682819871987274404776
7783667325326289221881974987582215*e^(275*I*c) + 67244087969080703823703257
199663047606890610482090494619492802935130215979819469966383336788693900139
115594646893784095418472336*e^(274*I*c) + 525392233467407711425870923702570
695360603196444395016676104827679558002760528924321527988146079751103662249
45081428121888473324*e^(273*I*c) + 4071598896370191895002034833673642342051
331135901048524691907465288339397080537447083015622970564731226547758425602
7212762941040*e^(272*I*c) + 31295263688189838313772775873307260334117227258
629501992358695636092662866062819845689064235813622974150120668921391878398
978380*e^(271*I*c) + 238563985655628020306952781742126408332821541740064594
58292644060947924907313735921561690153939906017518647182491616573724049744*
e^(270*I*c) + 1803532733817745547117756859485168297797834644977719357208768
8510392426884519272991560851326393852241961470040819793627127923997*e^(269*
I*c) + 13521230411945436915558854706851796541567399811656870021567352975326
815467817846533289123871696056195231696146162720992221760992*e^(268*I*c) +
100522095243695818275881549853455496780314457444999852082593854356097402724
00301454246872041775159838468077381562338745636398374*e^(267*I*c) + 7410372
612891152224364633296128043971657193280327754304382235672773781403023814127

610355562505271969045177704726054907145784960*e^(266*I*c) + 541666280405243
 634958559598281835795386625846164435401820515891774257642534436496459675065
 3177677803492186817305171175032011500*e^(265*I*c) + 39256976584157783527681
 039428560118402116427696217172179966143984738871860743914826385472128265382
 70453912634540299792270321024*e^(264*I*c) + 2820819298562215959107529807289
 628449621386798989436369393116069894018781201000275633104498398959346631795
 568022519974400130281*e^(263*I*c) + 200949611009268773815278208568373722272
 796882429905873921544608349935146762533444967075776406669065615094901494404
 3994822823920*e^(262*I*c) + 14191644814221765738582340138989996288223223333
 095737307163107438389358322014542936172931750864583896214253070517506121297
 61498*e^(261*I*c) + 9935556536495211272264439608202336493864885100818925458
 66700444096661582790441241830855609577062039555625090943332264901780720*e^(
 260*I*c) + 6895184493287935599032604181499741902535783400588950355896064682
 44680591556118170304005037563669880057908765898949268614772285*e^(259*I*c)
 + 4743230435631005423773386299319662481291759769823324460180560093911540204
 38895903140822967769494019446166779954024655344116288*e^(258*I*c) + 3234131
 780148410037141511382460791525763609760350584578904099377387231715370365730
 43681997163745313602400139153046673668433091*e^(257*I*c) + 2185631666596493
 122474836409562721492124991153838287710296542833639725851180904794136966381
 08156385244646591328454425745117584*e^(256*I*c) + 1463904484563511812182373
 827403741241916648199977469880765983918627336296701422415463755339031306052
 97580105675355629160198162*e^(255*I*c) + 9717305502474268005861672246136889
 266114129554026349301303274608353615732426833339040030895831837021915488716
 9702257444756176*e^(254*I*c) + 63923019433761989090614801288635098123199445
 102303122616544648208998767803944455777042886552738499747183713136069104651
 812215*e^(253*I*c) + 416704403753905436434182193422717480400350714901190805
 85281522498188818375906900368701234531304633163446319945130196476913600*e^(
 252*I*c) + 2691779470108661509789012023689050110514679999602177519571086622
 6228638984703456832694153230611607263444183501026198563419616*e^(251*I*c) +
 17229502824367647334400721998417596703948657394738805209391636597370380572
 398964715080095366818322029152193635869784095333760*e^(250*I*c) + 109272106
 034735448102797923478445360745888968062300411100895447316058631461041817390
 39426674855453466097402330688331845602302*e^(249*I*c) + 6866425337518668262
 662693750908956965732924578142181630622157802899880874681551031136314064199
 948604001529894566235238597088*e^(248*I*c) + 427482690772059175252671133682
 087150084434564792238547153435933360618957183244464136413289310866357620513
 3870672156264164115*e^(247*I*c) + 26366624104307993404475222847782442837407
 510686581407265764466712077983256068322959377050616862979302963823388925749
 00819440*e^(246*I*c) + 1611092541400060525954859375264194178347643471837078
 201446262435615142944587337833513586022729849523358436493586042252995608*e^(
 245*I*c) + 975210339444049318757282311763517786673223175594457946383279264
 635085041004917300295904275433144848532459919875479817581584*e^(244*I*c) +
 584749573682304586179384628844883327581498969886540380378896767999075614964
 007174600811092945356635118795824799369716742109*e^(243*I*c) + 347310053810
 935290419455560555957314129569210735745983234369659976413374774078000173070

075248654524917179128950507443058208*e^(242*I*c) + 204325557265186000767402
710230847896459761583922763698235433212833313077783041040074669379017394836
761539649081690630811665*e^(241*I*c) + 119060591849660546834765693227676449
067584148248882678447504826077236333444513454095126668750057295811191643356
908972191440*e^(240*I*c) + 687124660159856415124685861736597477348795917100
98354652786124936023073943141049573606648563005359411712764895683903806088*
e^(239*I*c) + 3927420041432986116939794451622500108122743339858500720639923
1211907157795359719648241598754266579840244551491476467899952*e^(238*I*c) +
22231341131801535345406399037721686840208397941952580135584645966746736656
716271554826476282991066076564921432614339399735*e^(237*I*c) + 124621404405
372580849285967098720668577570709431248685545009481547568634543080329254083
40311237850017814707896986969086816*e^(236*I*c) + 6917838945214844278493330
459361394923372333853619879637372673184942859712431066345726870422099893124
890777678037369988150*e^(235*I*c) + 380260499670589110696462063384896480703
709885451018226324303059729563076035359753197432475226638919318576087827418
8013440*e^(234*I*c) + 20696982895008606434616657623738079575130194240411788
71904960551829412449344722432125679417958403007551179298315947373776*e^(233
*I*c) + 1115398285545601550533328045600184317993899359217996729818340704221
801195667410485846996179056733558512452238583160792512*e^(232*I*c) + 595157
615500431514947479282336547053827926087916425263497187757029413385471835434
198246807096214536895441388306027237899*e^(231*I*c) + 314409035808225861565
595436938354945445473991043129722046747228813030925204968503418566818838611
866709807040793495364496*e^(230*I*c) + 164437500676906892741323260154394278
503954561936020133596581806449357240277349447927334517105809995300093549279
931273178*e^(229*I*c) + 851395332347864557795899594649006377607357297056212
21380005837208369157794673049675428799817875431430246332625899630160*e^(228
*I*c) + 4363830007581517102594621146446568961896577348866098585794565747985
4085108851857911222911989837615452608512356008400295*e^(227*I*c) + 22140735
001708603270915180769241391662035578755903979148909213603822554792749183517
160255571915875356439553717130797888*e^(226*I*c) + 111194996453632010808810
628248863384924253756584489779355358463492904258215703830904254114185215166
70371372045206568345*e^(225*I*c) + 5527498849031178355861230009326668283926
290082158467118000698502719379939045918344222192742145711257028040974074674
736*e^(224*I*c) + 271958928348374392604080510108034192124453031125460725092
9192773909331523226635035815672862569296693711643521070331394*e^(223*I*c) +
13243113249840274283552229381476823786728607088171617414487496895935880208
60847508703702325320304649883120684987556400*e^(222*I*c) + 6382188929145337
415058063996624885560667836004960918764083749748774489717780360749962455811
24283460438065182071976085*e^(221*I*c) + 3043844711068133360102841601239063
704338888284906274226525512369667909161745208577591439301401871734923949819
08258944*e^(220*I*c) + 1436576871380447969429471197042595384588181994235168
24674586293691056119209866358123637772245409530799230553767222252*e^(219*I*
c) + 6709170613052966912501989921002157658023784346222953538629508707618929
7849995931360645605292130961496106707521506432*e^(218*I*c) + 31004319206069
417077069363141423487431828009098184744635678652284177439464941651812564519

144918003174108077634846014*e^(217*I*c) + 141764836528757049572020133432411
179043699777966538499599025244219806351890116348156532796054977833828889327
66730080*e^(216*I*c) + 6413381895855925184758231451062556380328594938511006
577015218119786536390213057284018202201631094434819584025113465*e^(215*I*c)
+ 287049613141231445183467471535358943955329443080853193346608628854370924
6230769151392180699413405623017247753532944*e^(214*I*c) + 12710330829380489
502013605548312703426623439912775046123423003660250467417428565804452894017
86656311685859023084716*e^(213*I*c) + 5567563887111823403410261927342195461
136517683173805390058936790493947140170636985652727288136690547790772089778
40*e^(212*I*c) + 2412460212824400617929083177830328761948015971332060520912
86997043729145345755805710081489006741839439573984832678*e^(211*I*c) + 1033
997554672574364898478376407537547182043944730557950014676043264198765555568
73829531737211096115196005647730480*e^(210*I*c) + 4383497214291937768537869
223302106374455403310092850273748043897897674698989578407095190523778349037
4305934542955*e^(209*I*c) + 18379807084003359766027649217621144116091735572
216620788861535803449702273802588359076704241840733513439114113248*e^(208*I
*c) + 762178879191204706203884091779937460042889225819436763668294435609668
1400246312138001769285020661445991073249416*e^(207*I*c) + 31256834931787017
434797047503074901786662921507201793636043351135286233296846063431855407560
19935662148267863968*e^(206*I*c) + 1267597017294812913400146276042126929986
480292870190399107554311079964227280196522475370108738477856311765699610*e^
(205*I*c) + 508324599301085460166978629683032661427654474082939048097939638
391567298795788389433842285751054665210868287680*e^(204*I*c) + 201557947424
794098026772478462040883395867512562930862943753568690084015585598010154781
548625239409581907397500*e^(203*I*c) + 790191495587665692547839884872323883
52909144982747171856772463223808993367091503402876467270176124342699654400*
e^(202*I*c) + 3062758105422195737839054728927760912957281393108273352024738
7226000020043538279468776707958420892547870128680*e^(201*I*c) + 11735856926
245118493113091002501604032341876985999082823520672241530200188223826392982
302194084667538488665600*e^(200*I*c) + 444541225929547462506765951419831296
6015416299968930393345630345914109720740573618884980520010028451496996210*e
^(199*I*c) + 16644750343872118093949177435029376389785749377547647639878358
72410449930690131572904279995484581013965001440*e^(198*I*c) + 6160031160229
795849371257017578872129983543009899913626280388610939145613320711919097149
49426587936910303300*e^(197*I*c) + 2253205325932206577679411092895162489997
94521015564134982827241710019675486694499689312466561907212627820000*e^(196
*I*c) + 8145208141382911182887541756425005484603769331281148049290916075819
5989155768107022568350953861815940704090*e^(195*I*c) + 29097651061247453406
647569781836910062165559852359052804259154165687125428752562385492373749486
351714453120*e^(194*I*c) + 102716025302028890024978135168494525909715128095
29060665197301097052210064576088348023234671975463677418470*e^(193*I*c) + 3
582718002163296061414536703715109897107198252739284546149343102348456124089
657428594946438660859773886240*e^(192*I*c) + 123466804189240997878001808175
544021601258247639694193796589963195307920397422213879460432849897214476690
0*e^(191*I*c) + 42035802483514679858361121014594215468443794936564789908837

2524802156222884839580011688655664280691773600*e^(190*I*c) + 14137993825355
684328056550580740330413060613072543475174579407983314136174891763998614537
7066437210546190*e^(189*I*c) + 46970224727117281826454045018070670522559756
627580347784535320014963482632359729444541885102274546002560*e^(188*I*c) +
154131112114860239372949708207973767160813447881633865435224219397375079621
25854981881879168348260330000*e^(187*I*c) + 4995241956279138180205186744401
688024388272113921255663734956946927571305533146776898787878059685108480*e^
(186*I*c) + 159877110105819269227052899967744474268563100623245618584492522
0144002305878120380828483988663574829100*e^(185*I*c) + 50529366312301525887
848302573881281320339776684534006538126101635341972238262039303253596066092
1950400*e^(184*I*c) + 15768584552885091872146287786443509025758314941556132
3427386562894447598277935629800939237175625149830*e^(183*I*c) + 48584258153
140280447314836868772131390195412419046732778458706015096881437076337910793
584122475073760*e^(182*I*c) + 147779550966171289987127451820714953621765069
73183081650233605274051677624970464340242755840025673760*e^(181*I*c) + 4437
210917843182347764349544443904699020056595069470847193617092114714077633077
234972825351226979360*e^(180*I*c) + 131505212093069212210229710532762284233
5870743428530891072983535862280094446607723473800477453914130*e^(179*I*c) +
38465584208066627445406307878483717499894905250097532216200339254995341359
2461519365177908682078400*e^(178*I*c) + 11103414879700881944314389564446924
2295049867464313710969257619338899133799285616020069872611710850*e^(177*I*c
) + 31626644674725547731176795687527653571305969985923688392112164915553242
573269490908989570248533280*e^(176*I*c) + 888829502875102466704420383760797
6101480053134418614474620767522824868911959884352666444917404000*e^(175*I*c
) + 24643821908074396090797742268556796293678857097764358766308517162539626
96192341706239192878728160*e^(174*I*c) + 6740255305431330088948457752366252
37450743114473544537818170447134607102575676676056675328961590*e^(173*I*c)
+ 1818346614061779013153301296771453811664491884131941411693443547549209690
34952610378945282257600*e^(172*I*c) + 4837948975643409984385779181658937940
6815042609340378747586437145781646245422045101230417309900*e^(171*I*c) + 12
693496932964920565073673637181280088548682508880255337280065006566138696041
797353216584528640*e^(170*I*c) + 328387476055581867672630948030673442015509
8583948074469014168171874442170109648521627538755920*e^(169*I*c) + 83757920
692341193245878648676537353394654523970899076948872481398218916510458989551
8909256320*e^(168*I*c) + 21059430138564847118432907888031750495336183995415
9427434009884661777259752542647709150036990*e^(167*I*c) + 52190912207661824
215812271854269748071292843243227894769229690720010547141334131610989636000
*e^(166*I*c) + 127472196165033205413563430625628473686016221408567860254458
14532037904111523242298235713300*e^(165*I*c) + 3067974296431747364198159623
962463671617006419626851426148418602934852907379021659761911840*e^(164*I*c)
+ 727521010718394229291774073844694255798738667067535379759732795567942578
751384250780476310*e^(163*I*c) + 169956327969929767773902096652629253283704
505477127544556534417376686540936706073847337600*e^(162*I*c) + 391080312556
018094765375353696118444408449037516056450235145723520452481042629335988507
30*e^(161*I*c) + 8862752142756957285681340885764904597935349569355321815647

721172537159186491471311666400*e^(160*I*c) + 197779298066581813565130009432
 6239158605448870806970860577325385028609983034534672318500*e^(159*I*c) + 43
 454667678028004534634449876389254079717510575682751550929702418766029934548
 4920192480*e^(158*I*c) + 93986915313068179149083606065681482780836060510530
 154618486949839467131378859885998210*e^(157*I*c) + 200080068030300471372932
 78250321597113540716201983333126349281186679153199068045257216*e^(156*I*c)
 + 4191542500656826148093339414544159143964478472492315931809171859902114109
 005939942952*e^(155*I*c) + 863979933622330349556296820028395513198708064940
 505702126068652936800794826651264256*e^(154*I*c) + 175193170500618300241515
 632381912285157790097816049220671217212220015297133400636060*e^(153*I*c) +
 349410716132767046494779430433394502015040753351603618659160292138607786062
 30624960*e^(152*I*c) + 6852993223145736687328885311617795435592940841439866
 351079655652312894721972796266*e^(151*I*c) + 132149805527130085142999386663
 1619874424534425188183592049727687571032156435077280*e^(150*I*c) + 25050102
 8608928332469340456829902067712233644464602753159945727868485722395506952*e
 ^ (149*I*c) + 46668223548266017806854592468100570289355960869613650856575756
 758180182223308768*e^(148*I*c) + 854301344112621233483354066506962147247908
 5838041360564550722036723654297540205*e^(147*I*c) + 15363332384449275835327
 34556016494671674916578907116984548489078241693926940560*e^(146*I*c) + 2713
 61207503266570734486517077181014801775322183181055638619257836143271472358*
 e^(145*I*c) + 4706504461113515810848735336748424310269824883831263587628309
 9427442745866704*e^(144*I*c) + 80137295807907524343619649457615437614695207
 91210746972675870481058674277844*e^(143*I*c) + 1339214374254245553564884406
 801945353385000254030655765953770237607180089968*e^(142*I*c) + 219601281339
 515561500261478844190024870555261281946058839614044697037963695*e^(141*I*c)
 + 353244472067790181153780528207894116875810045823674310062058796337290152
 00*e^(140*I*c) + 5572551157328671121016216416307596161861955969011697222340
 926210112854418*e^(139*I*c) + 861884851094991908764246805474672428603757315
 484453974713612812215428992*e^(138*I*c) + 130657660226560419335121434389938
 961884595434069984824307149332131747540*e^(137*I*c) + 194079792155945665935
 35008103303255257745408070082431338945184797463936*e^(136*I*c) + 2823905151
 936586678382525706564457280290098698638597987628380245881715*e^(135*I*c) +
 402349692266121158934003582839428785116904903936409545602519219664720*e^(13
 4*I*c) + 561170810763411753840875701851885386603759320136747355190552273683
 66*e^(133*I*c) + 7659010520187549651777118357676871927081898989131125755798
 204236112*e^(132*I*c) + 102253643746829673729306586270524644969368741555986
 5844306888705423*e^(131*I*c) + 13349021005202618377967331386833230353033290
 6163247194627808410304*e^(130*I*c) + 17033886027390615741040977721655541665
 612162275485028584310890417*e^(129*I*c) + 212370296918887131826671878122392
 7067839949015727293884065388080*e^(128*I*c) + 25858534871597727015582911568
 4193411072034541491364393985491350*e^(127*I*c) + 30736217404321009965231037
 419663053962881035281709221697785072*e^(126*I*c) + 356476489062872401708848
 7996688178929195787613958545474804845*e^(125*I*c) + 40321222595779818884084
 6139960995624144491271694336796459584*e^(124*I*c) + 44456708175258821024400
 946210535004523775722190977468484496*e^(123*I*c) + 477539860710085326353420

7733818266777478693412738731031680*e^(122*I*c) + 49946750655853173367158586
 2910572702811545035730398749530*e^(121*I*c) + 50836369508171099437019348610
 847391946736185108017183136*e^(120*I*c) + 503202490340145182407421394376601
 1922026507006311982753*e^(119*I*c) + 48409341024048871865591702530366258109
 1659126182344528*e^(118*I*c) + 45230940039830738332025694784646206844854827
 698075736*e^(117*I*c) + 410154543993719579395995670844249670943380026122488
 0*e^(116*I*c) + 360688613036389349413809780004559963548775423325255*e^(115*
 I*c) + 30735366512830562160991166338490057308062762518496*e^(114*I*c) + 253
 5667460650279776834561566186591213109251642859*e^(113*I*c) + 20234750972446
 2171313966643580234078508179838320*e^(112*I*c) + 15603911277687607099721623
 771744933086920587272*e^(111*I*c) + 116158141373397175153362251190904691718
 8768400*e^(110*I*c) + 83380839911837894453136303673785039051506805*e^(109*I
 *c) + 5764601046563151304213854710715346838447392*e^(108*I*c) + 38336015580
 1054824529764688213114368047154*e^(107*I*c) + 24489837337812338687718622491
 865013839488*e^(106*I*c) + 1500602747937397286405577818722691539392*e^(105*
 I*c) + 88054927598941411145869950813388040256*e^(104*I*c) + 493966661081802
 5798809586352543471345*e^(103*I*c) + 264410375780310742518099326419685040*e
 ^^(102*I*c) + 13477227799524701956579274210395326*e^(101*I*c) + 652650253343
 206047453620559993840*e^(100*I*c) + 29952547749265499675257842032197*e^(99*
 I*c) + 1299146645993240318167826532288*e^(98*I*c) + 53090127264630963470039
 804475*e^(97*I*c) + 2037031259470368160131922320*e^(96*I*c) + 7309920781733
 5597247098038*e^(95*I*c) + 2442455629894502983849104*e^(94*I*c) + 755998170
 92670157806639*e^(93*I*c) + 2154864144781257856128*e^(92*I*c) + 56169444526
 926562260*e^(91*I*c) + 1327882849274858880*e^(90*I*c) + 28186192554792138*e
 ^^(89*I*c) + 530563624556832*e^(88*I*c) + 8718181624155*e^(87*I*c) + 1225037
 23056*e^(86*I*c) + 1431118260*e^(85*I*c) + 13343760*e^(84*I*c) + 93096*e^(8
 3*I*c) + 432*e^(82*I*c) + e^(81*I*c)))*tan(1/4*d*x + c) - 8*(11154*a*e^(105
 5/2*I*c) + 4506216*a*e^(1053/2*I*c) + 908002524*a*e^(1051/2*I*c) + 12167233
 8216*a*e^(1049/2*I*c) + 12197651906154*a*e^(1047/2*I*c) + 975812152492320*a
 *e^(1045/2*I*c) + 64891508140773561*a*e^(1043/2*I*c) + 3689545748587311444*
 a*e^(1041/2*I*c) + 183093707775748726866*a*e^(1039/2*I*c) + 805612314238410
 5100019*a*e^(1037/2*I*c) + 318216864146889674614121*a*e^(1035/2*I*c) + 1139
 7949499281271404789090*a*e^(1033/2*I*c) + 373282846202680826204100054*a*e^(
 1031/2*I*c) + 11255913521591139154617026091*a*e^(1029/2*I*c) + 314361585027
 608802719103613974*a*e^(1027/2*I*c) + 8173401220846880695205758846440*a*e^(
 1025/2*I*c) + 198715817557880846019817635638340*a*e^(1023/2*I*c) + 45353963
 19324144520768178591431614*a*e^(1021/2*I*c) + 97511021259565231697992928696
 183262*a*e^(1019/2*I*c) + 1981013390560058542422694832888372388*a*e^(1017/2
 *I*c) + 38134508068509651338161782530249790846*a*e^(1015/2*I*c) + 697316726
 420326420226818605542124203670*a*e^(1013/2*I*c) + 1213965045609725416931589
 4492857358348125*a*e^(1011/2*I*c) + 201623763544733613950600829151019284608
 876*a*e^(1009/2*I*c) + 3200777324680738651028636558675556361155978*a*e^(100
 7/2*I*c) + 48651816868314506338966571934860812829084391*a*e^(1005/2*I*c) +
 709193820494019943354343470410215009861831325*a*e^(1003/2*I*c) + 9928713989
 218000067239217146577130297436860304*a*e^(1001/2*I*c) + 1336830503805832927

59807271902459155221921326266*a*e^(999/2*I*c) + 173327003051799449005454155
 5914528946987399507283*a*e^(997/2*I*c) + 2166587747288431567518701560103605
 4026439130732651*a*e^(995/2*I*c) + 2613883590661727412848063331363568543260
 16097969610*a*e^(993/2*I*c) + 304680849708779449002218856975054172848834318
 2667144*a*e^(991/2*I*c) + 3434584718442721669725639327266274282823521060344
 7717*a*e^(989/2*I*c) + 3747738809914961000287614260595582528481431164912335
 83*a*e^(987/2*I*c) + 396189629698902900697184144565228362792605043858053804
 2*a*e^(985/2*I*c) + 4060944903010454199463398191888662924984771679038975808
 4*a*e^(983/2*I*c) + 4038995260209810459155678125511202679913875302830052160
 05*a*e^(981/2*I*c) + 390082071197239239532077053114670415937565216051495878
 0466*a*e^(979/2*I*c) + 3660771974683332327017333510537817068380688319428350
 2572958*a*e^(977/2*I*c) + 3340456313007058705883041585142571550296109888736
 39068542876*a*e^(975/2*I*c) + 296567536011012069724922894795916996072115373
 5047384406327794*a*e^(973/2*I*c) + 2563192803917569444155965471867373913880
 4185560774185672766170*a*e^(971/2*I*c) + 2157852592378720168676186753222766
 51746761914034929879374220636*a*e^(969/2*I*c) + 177042177197442162270914587
 8462925518593050670543424508511160048*a*e^(967/2*I*c) + 1416339077592369884
 0349913696288805073818903748549770982099621066*a*e^(965/2*I*c) + 1105361766
 01461446339362095696904713163296927687419549616666327675*a*e^(963/2*I*c) +
 841957703671377547101115934348899568986163724166597446960696905670*a*e^(961
 /2*I*c) + 62620714077377466517268280518836252815367722301461118298469646459
 86*a*e^(959/2*I*c) + 454959565932048254589704164140715683955206104100666110
 11076531123433*a*e^(957/2*I*c) + 323022024354913226146606279108678219616651
 564707789831198960566522147*a*e^(955/2*I*c) + 22421586363406278621273200163
 33950189333356311019840571116834002118194*a*e^(953/2*I*c) + 152208518881827
 28740782750360825763433070135271873008844718192146049020*a*e^(951/2*I*c) +
 101089762818526848940409310346350932268283613848471249613323714958903385*a*
 e^(949/2*I*c) + 65708588432062408320642914193817365629501810011026851622404
 0799717224364*a*e^(947/2*I*c) + 4181472981318382654497651147465017493829958
 109415267437695790041142593706*a*e^(945/2*I*c) + 26059658326016189488791557
 218632775709151983873327181283009050908453688384*a*e^(943/2*I*c) + 15910190
 1781384472239523743932630706140119020186733129962943950053112026720*a*e^(94
 1/2*I*c) + 9518738267397863802852345898801577462611900015140162777134396547
 50846059956*a*e^(939/2*I*c) + 558221167660699249684548132726498124474001205
 7291236674457828201859232835234*a*e^(937/2*I*c) + 3209795016413281711580362
 1585892838016332860939330285771049589504254327493976*a*e^(935/2*I*c) + 1810
 12853831745679282416051344942263215904480326976416514252463543134260124404*
 a*e^(933/2*I*c) + 100141881512124243201119488135004288874173132790845790222
 2695201280195390793156*a*e^(931/2*I*c) + 5436327766912905456994738945036935
 633200047191898446394850650924966170366179640*a*e^(929/2*I*c) + 28965753998
 48336112837518724860972276280949244636300000698769985510049611994644*a*e^(
 927/2*I*c) + 15151502852110177909939525697359286550929763834100458067162723
 2414532875203636556*a*e^(925/2*I*c) + 7782468109778386565721189179139516347
 16997865633712601174220799241685098928335884*a*e^(923/2*I*c) + 392613958303
 7514980862266059833351755777935293123288314534508002591338991682219336*a*e^

$(921/2 * I * c) + 1945780653776869404820978100666431013047007585759267848387698$
 $3129530380918539494004 * a * e^{(919/2 * I * c)} + 9475277846893111808530906263413738$
 $5816780489586428340550184393460692866634869729660 * a * e^{(917/2 * I * c)} + 4534687$
 $884428446337415399584948720853235894643167677840355434888986277028599374999$
 $70 * a * e^{(915/2 * I * c)} + 213326624728047057622598356992495288818408186451218110$
 $5155913104938448572679018199600 * a * e^{(913/2 * I * c)} + 9866594526524333225284699$
 $746947423621056154537758266423132777202294818167314534899300 * a * e^{(911/2 * I * c)}$
 $) + 44873919662729856679145756027256058487052792227828665075782678570381536$
 $051164871117670 * a * e^{(909/2 * I * c)} + 20072565710547280650521283907508435534876$
 $1207242373555648013716429258893539543288139010 * a * e^{(907/2 * I * c)} + 8832209729$
 $112415312048723809408405986388772511591360555687880418590094749703331636653$
 $80 * a * e^{(905/2 * I * c)} + 382355012698486981006072048037821257663086364919700326$
 $8812200176029853869143883259372540 * a * e^{(903/2 * I * c)} + 1628794990357190574016$
 $2779090603628412377720468039404006167093404295378080939699332502390 * a * e^{(90$
 $1/2 * I * c)} + 6828693464305013994032391657856757594143852875697873817677875372$
 $8051071752354796643709360 * a * e^{(899/2 * I * c)} + 2818044320613592698679443982575$
 $79674637432999117233397059536410474449872076551395746129720 * a * e^{(897/2 * I * c)}$
 $+ 114488711702184599000087340701485051278874729063266215316010150922954094$
 $7474388471978656320 * a * e^{(895/2 * I * c)} + 4579795236480509202664411272468766919$
 $319562882778252434978404442189537483545578491382900040 * a * e^{(893/2 * I * c)} + 18$
 $040983595466984885878007365734448155005057683579992022758394768630235475311$
 $641910974225440 * a * e^{(891/2 * I * c)} + 69994790043406292839026175163841940048198$
 $660179478150539131705393762570348229558917524315680 * a * e^{(889/2 * I * c)} + 26749$
 $865656180899245730387429982824134435324087108391376211317145798417908643444$
 $5936669700220 * a * e^{(887/2 * I * c)} + 1007129713752251234868254087759675535888677$
 $155631019333522995080270882748550110427116557445880 * a * e^{(885/2 * I * c)} + 37360$
 $544899750791063541843619131572688619307962473318376105028254162418042956244$
 $49271975310930 * a * e^{(883/2 * I * c)} + 136571366295717469893522360083351382675021$
 $65617670776782506352316642516480582065287858511354960 * a * e^{(881/2 * I * c)} + 492$
 $014706466686391880133776761877789597454179588934807427700764455508644778118$
 $84521773749073060 * a * e^{(879/2 * I * c)} + 174711101080471994990887097621209864618$
 $235930291877442803658578096718181438954083545775558767270 * a * e^{(877/2 * I * c)} +$
 $61155799676552999930162284205039395874208655237461023649061524311242159357$
 $3435237134063616533170 * a * e^{(875/2 * I * c)} + 2110469371404518909945920440713248$
 $701022530412552795169170873593870389367656539594697616691490680 * a * e^{(873/2 * I * c)}$
 $+ 71811344262959625041553044254225672304315486409767418835058658684467$
 $99304861044299741930350386660 * a * e^{(871/2 * I * c)} + 240949959870381557960654502$
 $45382859737463433262674262689793654664935039408256036219860889044227150 * a * e^{(869/2 * I * c)}$
 $+ 797308795388823610199633020617662400108557220940525161396836$
 $18998330963400779203492036220152901230 * a * e^{(867/2 * I * c)} + 260217903483151829$
 $093716530516190668583821940861461483281216448823551734330201315644832519365$
 $482660 * a * e^{(865/2 * I * c)} + 83772737353983749316834968510939016703748900128953$
 $7635907868254276871639349234894861317508580200600 * a * e^{(863/2 * I * c)} + 2660517$
 $658969571246912951290672187792083797820694140421836394949047894222007034274$
 $605942965668984850 * a * e^{(861/2 * I * c)} + 83362266405828778143753953685966233702$

38373682844807454630106991237390698165510113390571091820411910*a*e^(859/2*I*c) + 257723424493994596747148455306987447158228614659746355547707657696516
 51918057666525566577884870538980*a*e^(857/2*I*c) + 786247831880172272212718
 198383648014818245439118249623311406619698585841828193229414408180452072092
 00*a*e^(855/2*I*c) + 236714854041181584447614451552651701797471499914982974
 463603390401213706685388710336557640001199330610*a*e^(853/2*I*c) + 70338076
 383100911366932827516044752365318982521945458459886812685865044840195458149
 7659682533841609480*a*e^(851/2*I*c) + 2062965693714750235067564405338921374
 554548468969024643489620343184854915274816524086649932505557498620*a*e^(849
 /2*I*c) + 59726475236274934029567277892456656166658008313319873294875998874
 21385232985834708325491572836136512320*a*e^(847/2*I*c) + 170706936790260123
 938426614442418799102036095130027247812807566205231412293118130551919029088
 78349108560*a*e^(845/2*I*c) + 481703886493362084343690627644084956210609037
 56379287547455136054183338852405439229133595793509420899160*a*e^(843/2*I*c)
 + 134211087455569391993037089971344850826794101522082918032904065365561154
 291491933979703574469218577504480*a*e^(841/2*I*c) + 36924087163246709980480
 044053159959407607774913382508761439855275786996954495311212595020599856332
 4077080*a*e^(839/2*I*c) + 1003176728997484447898771022327222595016528556449
 971374706484597613376742091449978571381520265096549223520*a*e^(837/2*I*c) +
 26916833190764380449005765173294894186764821830783328981317445588097824210
 79721983063323068088737367336630*a*e^(835/2*I*c) + 713314733144717145788973
 462038051706913328151470784513707706666333052589601057881527553406651105641
 8010940*a*e^(833/2*I*c) + 1867155163961814412425529406384621783071114295160
 4023512034950975052357363350856930638366116375863857882340*a*e^(831/2*I*c)
 + 4827833470059623787454741154691043207610309622065450969560675003193927073
 3858578346301164136903126745008770*a*e^(829/2*I*c) + 1233181216640724357575
 555885267829259130760244774776563032781298993663789826694352606370726303144
 15613631270*a*e^(827/2*I*c) + 311195856161614323352630504511761603172739768
 941213790291237430115115150792860742309567092063322246220261860*a*e^(825/2*
 I*c) + 77589328128135879137351410177930387017046370427941584364855954852901
 3942109194523969959827626412784222314640*a*e^(823/2*I*c) + 1911436768719782
 288999540325472146134322970444655309319583417672491603875380587266190085405
 224372301352193090*a*e^(821/2*I*c) + 46530274600032473346728935661421901200
 98523594045891127033466194722323946032022461301160748860597149007224780*a*e
 ^ (819/2*I*c) + 111932582254445713015042300716279828651654031726944825835164
 19343498008047004568591630706756038392221432223860*a*e^(817/2*I*c) + 266102
 947930130638238728238368746480972834091121747235656339391230401580406819995
 00835260559999844504634310120*a*e^(815/2*I*c) + 625232700732460321553587371
 050084348389683924504408397906937312545543701237544880788549287567167591576
 96466300*a*e^(813/2*I*c) + 145197342897244297433634089205884167687764814320
 219463291543467642188350261872080176726805087772378655781459100*a*e^(811/2*
 I*c) + 33329314195868704280258218151752459096226800790189417739733059481681
 4779451881594947809096302954987869523422420*a*e^(809/2*I*c) + 7562582975360
 233577252027947010632267401534276887167063717492873906459409169196863982496
 20530727474047018860760*a*e^(807/2*I*c) + 169634803178887279427713303057927

277079359213981549388986796158228213350979928778672092338004688069940429121
0612*a*e^(805/2*I*c) + 3761710771325835521801477079120704187229783360504762
321065604463917904203258553340376305595966043494428272365668*a*e^(803/2*I*c
) + 82471849970654506368339026450872779598993616840557604669492812893019726
68228176889972677919552911525524671443832*a*e^(801/2*I*c) + 178772164894330
951029064697572798411166624139605801184320488965100852620426851236985361777
12229678186507680743838*a*e^(799/2*I*c) + 383170010946699352365004737441442
116339073722214515022588412864096168170947539486408813156569598230715995485
29652*a*e^(797/2*I*c) + 812089511609375494408963761485072165160645985072002
46222957845322614756486503647277443720081824770412839738067480*a*e^(795/2*I
*c) + 170200221953232931986455647854273859562845748654874472429340774688584
087705524712065100647263201100765505852342928*a*e^(793/2*I*c) + 35276295031
401588148749225952837635364581450519265488913903945198226064169728006890525
9697307295129817347147157942*a*e^(791/2*I*c) + 7230943761554118904687583101
547420609492909802339004327428236753491839662168866663472119243910458994518
88015744508*a*e^(789/2*I*c) + 146594777843443224560876958048506099568446407
7663272283620316924693201551078744401781649591297172800316166593341847*a*e^
(787/2*I*c) + 2939514653165904511434655012105150828904007596825929476999671
465422724758128470982083055835294931929377197195285748*a*e^(785/2*I*c) + 58
302650299275916227107054982145988672056465410297488052315860618291374784874
39718627253349769176525651271282339950*a*e^(783/2*I*c) + 114387416774835002
287847205454594548722288194278596749810185800029555472736335279688751577272
29586789086765557400717*a*e^(781/2*I*c) + 222007214034938782361045031171802
474684508421507320435951147859643125395662798747629539547252171211010758089
61057863*a*e^(779/2*I*c) + 426261640814423367900443928740643212661243656930
55578062535947646438490845848561405141165578566022420904984992779262*a*e^(7
77/2*I*c) + 809705477346197792524991834059292203357066547910088788113919402
71150785533399580091596160301576141784097339728980090*a*e^(775/2*I*c) + 152
174114651836920279709831963803377982356186827751705484345314660768355526131
537482905934760724796686692614883019125*a*e^(773/2*I*c) + 28296876925658410
281879282983134031372419660366028167555906899906983413091676412157767918738
8938656817063251770095022*a*e^(771/2*I*c) + 5206445251665335960613757959406
570747666605448098580380508628333217633440609658655436048796007266599713454
31129847296*a*e^(769/2*I*c) + 947916737695979066656478842193532082586367581
805930780704754523836040119154533261655741871082094090548820191177736884*a*
e^(767/2*I*c) + 17078358836365758571867050504980966331579783103918753277191
42933387004630630670852842390662774725361962248022778723278*a*e^(765/2*I*c)
+ 304501954197584663501883258182032274485361678548198781813392827237146298
5129663419186138280673182752234472745408285510*a*e^(763/2*I*c) + 5373076061
973168504014181459519819721508275880784253991903654691196339007406080147476
696706726408511663297656449125300*a*e^(761/2*I*c) + 93835205260308053440484
819628244701833083469043395267356881867434313239135004639698687201315004288
30559888077010196418*a*e^(759/2*I*c) + 162195773890998650017790861946896936
634022983061549686004373771804028557232275860397899262942432280165504357426
02136054*a*e^(757/2*I*c) + 277501415665559516237560867163710586813167951094

44727785594328908441218495107612157679112376946419989454625088610911643*a*e
 $\wedge(755/2*I*c)$ + 469963266239089896578886833492238682795778407137777519034242
 99836299363657745857514934182927561870099251068169102199820*a*e $\wedge(753/2*I*c)$
 + 787874132322296430777642020879382478313017314162672625674445056459432851
 09559656037217616994737395772316148327891471302*a*e $\wedge(751/2*I*c)$ + 130757092
 982902523772408546910270270217553445051652898981442079835320326376505227529
 154501020728626159470929494044207633*a*e $\wedge(749/2*I*c)$ + 21483771935885237738
 394744368610522252356389816528190813049070485967794521941557311526730280410
 8890941570998616137840699*a*e $\wedge(747/2*I*c)$ + 3494734343971712888377042144779
 844754736698072103755994909988000274804998906578292863060593961100203846340
 76939313007368*a*e $\wedge(745/2*I*c)$ + 562856635056256522897295635187644403242633
 887267858968379604254363064108890906869496730352389810427434831340032592335
 350*a*e $\wedge(743/2*I*c)$ + 89760068438882857561569840914545736409207339419570854
 8474214631467676943800634808122870620940628733790867239511160865397*a*e $\wedge(74$
 $1/2*I*c)$ + 1417399437191054323839106392665411305862914101585825022283885579
 483586672008506936972830021650145621200685041810287064301*a*e $\wedge(739/2*I*c)$ +
 22163948552537554153117696425164084833125683279918194569435445726655879893
 41669065966415431960005186052908659861771471174*a*e $\wedge(737/2*I*c)$ + 343218022
 711576040491697168029506446272753132246810590899265221050901706072748366569
 5250186544493349252686330433955313376*a*e $\wedge(735/2*I*c)$ + 5263608430798021988
 512603950847390921256551866219090577603956112275050115597501030429610982200
 369929372389895438604735587*a*e $\wedge(733/2*I*c)$ + 79948488145396147587671881182
 940160466975819167796878015069418200801859756270153002058697145200704130301
 23095531003290745*a*e $\wedge(731/2*I*c)$ + 120274340335685963574796945743632655201
 492044667649156439996418350448071425395206138913435308553046616435786083915
 40037558*a*e $\wedge(729/2*I*c)$ + 179223442423092111123085142070904611727690600231
 86678888125375232294636141950170069922592546432072256690280201930015285124*
 a*e $\wedge(727/2*I*c)$ + 264544667257628713686194515680728643738272691440428880419
 55082747247267325324943486097333234252334413537567942897374486243*a*e $\wedge(725/$
 $2*I*c)$ + 386820109739823575544913927638282776265309386641426699074289633383
 97512335069535559962305954403609176276589299921949303042*a*e $\wedge(723/2*I*c)$ +
 560336084278280009691377030978044973196236559281085693587773018747505499031
 28662509955413199447848429732621413779639624290*a*e $\wedge(721/2*I*c)$ + 804158282
 228122214306018937726982639881571301397138573534003104431019647862664651064
 97352458508320193679809189270395189388*a*e $\wedge(719/2*I*c)$ + 114343631608925883
 407310373219046938499339036241108317448666887626687514027677976291595996209
 697034615765954684303162952874*a*e $\wedge(717/2*I*c)$ + 16109579844808771133633398
 501845128614249944752747636857583734750833717984985176122251946484471861153
 0929945237812223712858*a*e $\wedge(715/2*I*c)$ + 2248965180131652569006723228722098
 209463641972986735223565856373546259573908926828296912287224946232680396577
 16264090221100*a*e $\wedge(713/2*I*c)$ + 311123077520821182661400197392400502848451
 085231967404189415031214666716333382727640087182723744328071082256937126787
 518760*a*e $\wedge(711/2*I*c)$ + 42653785427007861538014680289194543197999430879059
 7186556743002620082507967337801063312521286799161839548882443173283066418*a
 *e $\wedge(709/2*I*c)$ + 5795405950312360962351926688236015842262090471528145938558

94141916452918266691180249939843124623425348125525255775702439429*a*e^(707/
2*I*c) + 780434260513757032567868066635568515793145500827706415413194947155
129096500358128954342424580618054152762185523771214603306*a*e^(705/2*I*c) +
10416945830850610852374324265843085406619236363691449958554363895366497512
92293693616230487032218200324810659084850331343102*a*e^(703/2*I*c) + 137823
005110446393543358966116275540837449787430178620058157911659070792095807986
0750586411465537396094529761988971561274055*a*e^(701/2*I*c) + 1807615478092
428422586742709696692811895332427746348368604855468543776298223239993229039
784665919612645743781029502335900285*a*e^(699/2*I*c) + 23502788927076768740
554130642361122245814004763554864200511318963106542906171225198904360869451
31040778770847639389879805902*a*e^(697/2*I*c) + 302961851346903955146825656
795861097743609586460997415007283495515682424639012969996237397242827275991
0731542596087674887228*a*e^(695/2*I*c) + 3872024419334554579420467046031395
921549818528162349987615698433421440396242863792881072470370499151581175133
609251380773047*a*e^(693/2*I*c) + 49067785929989401902314141434747060478328
753464000958730359655950068022003311180448835317526604878141714359891423493
60753080*a*e^(691/2*I*c) + 616580769754728699479328294686167382981523846128
840747077285016671845269213716152898133050036712251037966712392543828523595
8*a*e^(689/2*I*c) + 7683265610269106593100348009627034347824363151517363177
221420649792715877784524856324220119411390772969326329689886553378312*a*e^(
687/2*I*c) + 94949276593680083254330046494834403346153078624912876009466657
80127052373495730640538318474641629025405313130018388171357868*a*e^(685/2*I
*c) + 116373858411965061630973626230419974385250718712191236540146811766574
70875214492305790509082822504563489413921335587213879152*a*e^(683/2*I*c) +
141470440164930732450776615666314115455835411275845909958691630899066385040
60078124820435754206103794538604889656144397980398*a*e^(681/2*I*c) + 170589
23964015244403349928439769071471929182092907580335251582239822602748552573
62481376944694355051578717124066137978186800*a*e^(679/2*I*c) + 204053067463
745290545736774834770450591858211618481952507321379809428512590615108830028
74214129271492308383701478696424164392*a*e^(677/2*I*c) + 242142484147402472
888196588867656073071905413044581300074913364221933209580067043613589086740
27125910904606593453776587033928*a*e^(675/2*I*c) + 285080237233288780967575
214463695898134726405773514745595018550629865497920541737879721362675232987
52140308859911162875134192*a*e^(673/2*I*c) + 333015651374150036074961865003
399084901277051358311626016743046664252782794600423077963394789753609361729
19487408241032947288*a*e^(671/2*I*c) + 386009757817994088882815931562872586
787646154265778480927188004150473475198308221604715948399662199253469913478
96509355010712*a*e^(669/2*I*c) + 444022028468575168779244113679353800600509
809431711133506904397649540552018153247047505104517588034837044608909807592
42508920*a*e^(667/2*I*c) + 506899611846388853164867916703438317336756983469
739630786387516193599896178182048936106575818268253669196105487737781861306
40*a*e^(665/2*I*c) + 574369944241760348692836013425853069214373031885183180
99842499919884656824211783455869597757363845458740014261234014581626200*a*e
^(663/2*I*c) + 646037522633182149884507533121493887106735609341214381420087
83864379422317004435234461707945597946067866780736895289190700600*a*e^(661/

$2 * I * c) + 721385474218920200380483552391741118266211757756069602009642033114$
 $22673485210729157141087289517598043801848625484306799328540 * a * e^{(659/2 * I * c)}$
 $+ 799782343329502549924504866398131763500174836943329385000838225640827202$
 $08751313740900328892353116973355066036775494328765120 * a * e^{(657/2 * I * c)} + 880$
 $494248118632460070417048527172297393627823256713439522675382405838079517142$
 $47800388339978768161265599420577081086275111160 * a * e^{(655/2 * I * c)} + 962702250$
 $247078204115301096806946144372362669496151016816136859590987753201718371362$
 $74199340417404214583084453939806659575380 * a * e^{(653/2 * I * c)} + 104552444916312$
 $126006877416486437777717590908468787720243122447280267750201687271309307377$
 $742904285196753740983916932633642780 * a * e^{(651/2 * I * c)} + 11280419808233481207$
 $736486249079312363327270082232642717106213022801211237635329630755350945657$
 $1138136109437126237459229081720 * a * e^{(649/2 * I * c)} + 1209327793477144810108070$
 $737501166364398486296579108350120748553927291320467514747108884812019936874$
 $31592438447235726849647240 * a * e^{(647/2 * I * c)} + 128847681569002824540410082248$
 $936892936967491817732890598454086249078923694647500741465194936808582459082$
 $829851393734873468340 * a * e^{(645/2 * I * c)} + 13646359477785763333632590519188123$
 $937110654688598380889628333204942065186458624338743919006329074162216473998$
 $6933577650200880 * a * e^{(643/2 * I * c)} + 1437032217222094096452336290981841704102$
 $353084124771312904016667887995840142038690242790701986421975508963511877125$
 $56702920880 * a * e^{(641/2 * I * c)} + 150499745560043932121484821953750004569583777$
 $620078554370457838073613675824918437108013091616854991063642344717193005281$
 $918560 * a * e^{(639/2 * I * c)} + 15679879859448719992189716943464233141793387261169$
 $010187777798613091976355152812898085050331307868001189674596237358413988624$
 $0 * a * e^{(637/2 * I * c)} + 1625598053250439925129127412870968613885569003167654955$
 $35437174267090670767653306889517935410527555972812323487812389764362960 * a * e$
 $^{(635/2 * I * c)} + 167756607639017815122826725739409421275441245777071574031830$
 $523852183716988086830946970345672432604771944649492352368098672160 * a * e^{(633$
 $/2 * I * c)} + 17237732271319674646365057966354138721148026981289246438621489879$
 $4454356386250587378111441884776842588658854932896656803938120 * a * e^{(631/2 * I * c)}$
 $+ 1764234324054794538161313384363570099443410715872046797699194070535024$
 $96189053085565560755477873431708930459354064305146157120 * a * e^{(629/2 * I * c)} +$
 $179908153272706670000892525978345887462188900040185546322175125543470352160$
 $299635078744629399567970910285081924551936400573340 * a * e^{(627/2 * I * c)} + 18285$
 $418520065773445627970978893120176590668351478586444659742512659741786766206$
 $5408456075741019764641428549938761457515615040 * a * e^{(625/2 * I * c)} + 1852909802$
 $718963485090453822470458351579178399777068052822137195552854796179592377065$
 $68522352165899937243956163969083644543160 * a * e^{(623/2 * I * c)} + 187251708480744$
 $392625293783021587920321861289993201231307646538717961240769622101632401769$
 $654249280585153941428126471657060020 * a * e^{(621/2 * I * c)} + 18877012033954588099$
 $191816676545145142571992540109029074001938664626522043103659091926892710506$
 $9149477284814652455638584619100 * a * e^{(619/2 * I * c)} + 1898775162272441556498395$
 $374294551323308557442481546059427602460317895811199670662351353852040561290$
 $40864831011050801071902960 * a * e^{(617/2 * I * c)} + 190600027441770269331777872098$
 $625556063025053708101092087509203973053094773227276392899592510611027692949$
 $217367574644142011320 * a * e^{(615/2 * I * c)} + 19095639439011186040692574251968470$

144079787566872563589975084262108892587845309106733023243014795054575656690
7118285339121700*a*e^(613/2*I*c) + 1909563943901118604069257425196847014407
978756687256358997508426210889258784530910673302324301479505457565669071182
85339121700*a*e^(611/2*I*c) + 190600027441770269331777872098625556063025053
708101092087509203973053094773227276392899592510611027692949217367574644142
011320*a*e^(609/2*I*c) + 18987751622724415564983953742945513233085574424815
460594276024603178958111996706623513538520405612904086483101105080107190296
0*a*e^(607/2*I*c) + 1887701203395458809919181667654514514257199254010902907
40019386646265220431036590919268927105069149477284814652455638584619100*a*e
^(605/2*I*c) + 187251708480744392625293783021587920321861289993201231307646
538717961240769622101632401769654249280585153941428126471657060020*a*e^(603
/2*I*c) + 18529098027189634850904538224704583515791783997770680528221371955
5285479617959237706568522352165899937243956163969083644543160*a*e^(601/2*I*
c) + 1828541852006577344562797097889312017659066835147858644465974251265974
17867662065408456075741019764641428549938761457515615040*a*e^(599/2*I*c) +
179908153272706670000892525978345887462188900040185546322175125543470352160
299635078744629399567970910285081924551936400573340*a*e^(597/2*I*c) + 17642
343240547945381613133843635700994434107158720467976991940705350249618905308
5565560755477873431708930459354064305146157120*a*e^(595/2*I*c) + 1723773227
131967464636505796635413872114802698128924643862148987944543563862505873781
11441884776842588658854932896656803938120*a*e^(593/2*I*c) + 167756607639017
815122826725739409421275441245777071574031830523852183716988086830946970345
672432604771944649492352368098672160*a*e^(591/2*I*c) + 16255980532504399251
291274128709686138855690031676549553543717426709067076765330688951793541052
7555972812323487812389764362960*a*e^(589/2*I*c) + 1567987985944871999218971
69434642331417933872611690101877779861309197635515281289808505033130786800
11896745962373584139886240*a*e^(587/2*I*c) + 150499745560043932121484821953
750004569583777620078554370457838073613675824918437108013091616854991063642
344717193005281918560*a*e^(585/2*I*c) + 14370322172220940964523362909818417
041023530841247713129040166678879958401420386902427907019864219755089635118
7712556702920880*a*e^(583/2*I*c) + 1364635947778576333363259051918812393711
065468859838088962833320494206518645862433874391900632907416221647399869335
77650200880*a*e^(581/2*I*c) + 128847681569002824540410082248936892936967491
817732890598454086249078923694647500741465194936808582459082829851393734873
468340*a*e^(579/2*I*c) + 12093277934771448101080707375011663643984862965791
083501207485539272913204675147471088848120199368743159243844723572684964724
0*a*e^(577/2*I*c) + 1128041980823348120773648624907931236332727008223264271
71062130228012112376353296307553509456571138136109437126237459229081720*a*e
^(575/2*I*c) + 104552444916312126006877416486437777717590908468787720243122
447280267750201687271309307377742904285196753740983916932633642780*a*e^(573
/2*I*c) + 96270225024707820411530109680694614437236266949615101681613685959
098775320171837136274199340417404214583084453939806659575380*a*e^(571/2*I*c
) + 88049424811863246007041704852717229739362782325671343952267538240583807
951714247800388339978768161265599420577081086275111160*a*e^(569/2*I*c) + 79
978234332950254992450486639813176350017483694332938500083822564082720208751

313740900328892353116973355066036775494328765120*a*e^(567/2*I*c) + 72138547
 421892020038048355239174111826621175775606960200964203311422673485210729157
 141087289517598043801848625484306799328540*a*e^(565/2*I*c) + 64603752263318
 214988450753312149388710673560934121438142008783864379422317004435234461707
 945597946067866780736895289190700600*a*e^(563/2*I*c) + 57436994424176034869
 283601342585306921437303188518318099842499919884656824211783455869597757363
 845458740014261234014581626200*a*e^(561/2*I*c) + 50689961184638885316486791
 670343831733675698346973963078638751619359989617818204893610657581826825366
 919610548773778186130640*a*e^(559/2*I*c) + 44402202846857516877924411367935
 380060050980943171113350690439764954055201815324704750510451758803483704460
 890980759242508920*a*e^(557/2*I*c) + 38600975781799408888281593156287258678
 764615426577848092718800415047347519830822160471594839966219925346991347896
 509355010712*a*e^(555/2*I*c) + 33301565137415003607496186500339908490127705
 135831162601674304666425278279460042307796339478975360936172919487408241032
 947288*a*e^(553/2*I*c) + 28508023723328878096757521446369589813472640577351
 474559501855062986549792054173787972136267523298752140308859911162875134192
 *a*e^(551/2*I*c) + 24214248414740247288819658886765607307190541304458130007
 491336422193320958006704361358908674027125910904606593453776587033928*a*e⁽
 549/2*I*c) + 20405306746374529054573677483477045059185821161848195250732137
 980942851259061510883002874214129271492308383701478696424164392*a*e^{(547/2*}
 I*c) + 17058923964015244403349928439769071471929182092907580335251582239822
 260274855257362481376944694355051578717124066137978186800*a*e^(545/2*I*c) +
 14147044016493073245077661566631411545583541127584590995869163089906638504
 060078124820435754206103794538604889656144397980398*a*e^(543/2*I*c) + 11637
 385841196506163097362623041997438525071871219123654014681176657470875214492
 305790509082822504563489413921335587213879152*a*e^(541/2*I*c) + 94949276593
 680083254330046494834403346153078624912876009466657801270523734957306405383
 18474641629025405313130018388171357868*a*e^(539/2*I*c) + 768326561026910659
 310034800962703434782436315151736317722142064979271587778452485632422011941
 1390772969326329689886553378312*a*e^(537/2*I*c) + 6165807697547286994793282
 946861673829815238461288407470772850166718452692137161528981330500367122510
 379667123925438285235958*a*e^(535/2*I*c) + 49067785929989401902314141434747
 060478328753464000958730359655950068022003311180448835317526604878141714359
 89142349360753080*a*e^(533/2*I*c) + 387202441933455457942046704603139592154
 981852816234998761569843342144039624286379288107247037049915158117513360925
 1380773047*a*e^(531/2*I*c) + 3029618513469039551468256567958610977436095864
 609974150072834955156824246390129699962373972428272759910731542596087674887
 228*a*e^(529/2*I*c) + 23502788927076768740554130642361122245814004763554864
 20051131896310654290617122519890436086945131040778770847639389879805902*a*e
^(527/2*I*c) + 180761547809242842258674270969669281189533242774634836860485
 5468543776298223239993229039784665919612645743781029502335900285*a*e^{(525/2}
 *I*c) + 1378230051104463935433589661162755408374497874301786200581579116590
 707920958079860750586411465537396094529761988971561274055*a*e^(523/2*I*c) +
 10416945830850610852374324265843085406619236363691449958554363895366497512
 92293693616230487032218200324810659084850331343102*a*e^(521/2*I*c) + 780434

260513757032567868066635568515793145500827706415413194947155129096500358128
954342424580618054152762185523771214603306*a*e^(519/2*I*c) + 57954059503123
609623519266882360158422620904715281459385589414191645291826669118024993984
3124623425348125525255775702439429*a*e^(517/2*I*c) + 4265378542700786153801
468028919454319799943087905971865567430026200825079673378010633125212867991
61839548882443173283066418*a*e^(515/2*I*c) + 311123077520821182661400197392
400502848451085231967404189415031214666716333382727640087182723744328071082
256937126787518760*a*e^(513/2*I*c) + 22489651801316525690067232287220982094
636419729867352235658563735462595739089268282969122872249462326803965771626
4090221100*a*e^(511/2*I*c) + 1610957984480877113363339850184512861424994475
274763685758373475083371798498517612225194648447186115309299452378122237128
58*a*e^(509/2*I*c) + 114343631608925883407310373219046938499339036241108317
448666887626687514027677976291595996209697034615765954684303162952874*a*e^(
507/2*I*c) + 80415828222812221430601893772698263988157130139713857353400310
443101964786266465106497352458508320193679809189270395189388*a*e^(505/2*I*c
) + 56033608427828000969137703097804497319623655928108569358777301874750549
903128662509955413199447848429732621413779639624290*a*e^(503/2*I*c) + 38682
010973982357554491392763828277626530938664142669907428963338397512335069535
559962305954403609176276589299921949303042*a*e^(501/2*I*c) + 26454466725762
871368619451568072864373827269144042888041955082747247267325324943486097333
234252334413537567942897374486243*a*e^(499/2*I*c) + 17922344242309211112308
514207090461172769060023186678888125375232294636141950170069922592546432072
256690280201930015285124*a*e^(497/2*I*c) + 12027434033568596357479694574363
265520149204466764915643999641835044807142539520613891343530855304661643578
608391540037558*a*e^(495/2*I*c) + 79948488145396147587671881182940160466975
819167796878015069418200801859756270153002058697145200704130301230955310032
90745*a*e^(493/2*I*c) + 526360843079802198851260395084739092125655186621909
0577603956112275050115597501030429610982200369929372389895438604735587*a*e^(
491/2*I*c) + 3432180227115760404916971680295064462727531322468105908992652
210509017060727483665695250186544493349252686330433955313376*a*e^(489/2*I*c
) + 22163948552537554153117696425164084833125683279918194569435445726655879
89341669065966415431960005186052908659861771471174*a*e^(487/2*I*c) + 141739
943719105432383910639266541130586291410158582502228388557948358667200850693
6972830021650145621200685041810287064301*a*e^(485/2*I*c) + 8976006843888285
756156984091454573640920733941957085484742146314676769438006348081228706209
40628733790867239511160865397*a*e^(483/2*I*c) + 562856635056256522897295635
187644403242633887267858968379604254363064108890906869496730352389810427434
831340032592335350*a*e^(481/2*I*c) + 34947343439717128883770421447798447547
366980721037559949099880002748049989065782928630605939611002038463407693931
3007368*a*e^(479/2*I*c) + 2148377193588523773839474436861052225235638981652
81908130490704859677945219415573115267302804108890941570998616137840699*a*e
^(477/2*I*c) + 130757092982902523772408546910270270217553445051652898981442
079835320326376505227529154501020728626159470929494044207633*a*e^(475/2*I*c
) + 7878741323229643077764202087938247831301731416267262567444505645943285
109559656037217616994737395772316148327891471302*a*e^(473/2*I*c) + 46996326

623908989657888683349223868279577840713777751903424299836299363657745857514
934182927561870099251068169102199820*a*e^(471/2*I*c) + 27750141566555951623
756086716371058681316795109444727785594328908441218495107612157679112376946
419989454625088610911643*a*e^(469/2*I*c) + 16219577389099865001779086194689
693663402298306154968600437377180402855723227586039789926294243228016550435
742602136054*a*e^(467/2*I*c) + 93835205260308053440484819628244701833083469
04339526735688186743431323913500463969868720131500428830559888077010196418*
a*e^(465/2*I*c) + 537307606197316850401418145951981972150827588078425399190
3654691196339007406080147476696706726408511663297656449125300*a*e^(463/2*I*
c) + 3045019541975846635018832581820322744853616785481987818133928272371462
985129663419186138280673182752234472745408285510*a*e^(461/2*I*c) + 17078358
836365758571867050504980966331579783103918753277191429333870046306306708528
42390662774725361962248022778723278*a*e^(459/2*I*c) + 947916737695979066656
478842193532082586367581805930780704754523836040119154533261655741871082094
090548820191177736884*a*e^(457/2*I*c) + 52064452516653359606137579594065707
476666054480985803805086283332176334406096586554360487960072665997134543112
9847296*a*e^(455/2*I*c) + 2829687692565841028187928298313403137241966036602
81675559068999069834130916764121577679187388938656817063251770095022*a*e^(4
53/2*I*c) + 152174114651836920279709831963803377982356186827751705484345314
660768355526131537482905934760724796686692614883019125*a*e^(451/2*I*c) + 80
970547734619779252499183405929220335706654791008878811391940271150785533399
580091596160301576141784097339728980090*a*e^(449/2*I*c) + 42626164081442336
790044392874064321266124365693055578062535947646438490845848561405141165578
566022420904984992779262*a*e^(447/2*I*c) + 22200721403493878236104503117180
247468450842150732043595114785964312539566279874762953954725217121101075808
961057863*a*e^(445/2*I*c) + 11438741677483500228784720545459454872228819427
859674981018580002955547273633527968875157727229586789086765557400717*a*e^(
443/2*I*c) + 58302650299275916227107054982145988672056465410297488052315860
61829137478487439718627253349769176525651271282339950*a*e^(441/2*I*c) + 293
951465316590451143465501210515082890400759682592947699967146542272475812847
0982083055835294931929377197195285748*a*e^(439/2*I*c) + 1465947778434432245
608769580485060995684464077663272283620316924693201551078744401781649591297
172800316166593341847*a*e^(437/2*I*c) + 72309437615541189046875831015474206
094929098023390043274282367534918396621688666634721192439104589945188801574
4508*a*e^(435/2*I*c) + 3527629503140158814874922595283763536458145051926548
89139039451982260641697280068905259697307295129817347147157942*a*e^(433/2*I
*c) + 170200221953232931986455647854273859562845748654874472429340774688584
087705524712065100647263201100765505852342928*a*e^(431/2*I*c) + 81208951160
937549440896376148507216516064598507200246222957845322614756486503647277443
720081824770412839738067480*a*e^(429/2*I*c) + 38317001094669935236500473744
144211633907372221451502258841286409616817094753948640881315656959823071599
548529652*a*e^(427/2*I*c) + 17877216489433095102906469757279841116662413960
580118432048896510085262042685123698536177712229678186507680743838*a*e^(425
/2*I*c) + 82471849970654506368339026450872779598993616840557604669492812893
01972668228176889972677919552911525524671443832*a*e^(423/2*I*c) + 376171077

132583552180147707912070418722978336050476232106560446391790420325855334037
6305595966043494428272365668*a*e^(421/2*I*c) + 1696348031788872794277133030
579272770793592139815493889867961582282133509799287786720923380046880699404
291210612*a*e^(419/2*I*c) + 75625829753602335772520279470106322674015342768
8716706371749287390645940916919686398249620530727474047018860760*a*e^(417/2
*I*c) + 3332931419586870428025821815175245909622680079018941773973305948168
14779451881594947809096302954987869523422420*a*e^(415/2*I*c) + 145197342897
244297433634089205884167687764814320219463291543467642188350261872080176726
805087772378655781459100*a*e^(413/2*I*c) + 62523270073246032155358737105008
434838968392450440839790693731254554370123754488078854928756716759157696466
300*a*e^(411/2*I*c) + 26610294793013063823872823836874648097283409112174723
56563393912304015804068199950083526055999844504634310120*a*e^(409/2*I*c) +
11193258225444571301504230071627982865165403172694482583516419343498008047
004568591630706756038392221432223860*a*e^(407/2*I*c) + 46530274600032473346
728935661421901200985235940458911270334661947223239460320224613011607488605
97149007224780*a*e^(405/2*I*c) + 191143676871978228899954032547214613432297
0444655309319583417672491603875380587266190085405224372301352193090*a*e^(40
3/2*I*c) + 7758932812813587913735141017793038701704637042794158436485595485
29013942109194523969959827626412784222314640*a*e^(401/2*I*c) + 311195856161
614323352630504511761603172739768941213790291237430115115150792860742309567
092063322246220261860*a*e^(399/2*I*c) + 12331812166407243575755558852678292
5913076024477477656303278129899366378982669435260637072630314415613631270*a
*e^(397/2*I*c) + 4827833470059623787454741154691043207610309622065450969560
6750031939270733858578346301164136903126745008770*a*e^(395/2*I*c) + 1867155
163961814412425529406384621783071114295160402351203495097505235736335085693
0638366116375863857882340*a*e^(393/2*I*c) + 7133147331447171457889734620380
517069133281514707845137077066663330525896010578815275534066511056418010940
*a*e^(391/2*I*c) + 26916833190764380449005765173294894186764821830783328981
31744558809782421079721983063323068088737367336630*a*e^(389/2*I*c) + 100317
672899748444789877102232722259501652855644997137470648459761337674209144997
8571381520265096549223520*a*e^(387/2*I*c) + 3692408716324670998048004405315
99594076077749133825087614398552757869969544953112125950205998563324077080*
a*e^(385/2*I*c) + 134211087455569391993037089971344850826794101522082918032
904065365561154291491933979703574469218577504480*a*e^(383/2*I*c) + 48170388
649336208434369062764408495621060903756379287547455136054183338852405439229
133595793509420899160*a*e^(381/2*I*c) + 17070693679026012393842661444241879
910203609513002724781280756620523141229311813055191902908878349108560*a*e^(
379/2*I*c) + 59726475236274934029567277892456656166658008313319873294875998
87421385232985834708325491572836136512320*a*e^(377/2*I*c) + 206296569371475
023506756440533892137455454846896902464348962034318485491527481652408664993
2505557498620*a*e^(375/2*I*c) + 7033807638310091136693282751604475236531898
25219454584598868126858650448401954581497659682533841609480*a*e^(373/2*I*c)
+ 236714854041181584447614451552651701797471499914982974463603390401213706
685388710336557640001199330610*a*e^(371/2*I*c) + 78624783188017227221271819
838364801481824543911824962331140661969858584182819322941440818045207209200

$a^{369/2} + 25772342449399459674714845530698744715822861465974635554$
 $770765769651651918057666525566577884870538980 a^{367/2} + 83362266405$
 $828778143753953685966233702383736828448074546301069912373906981655101133905$
 $71091820411910 a^{365/2} + 266051765896957124691295129067218779208379$
 $7820694140421836394949047894222007034274605942965668984850 a^{363/2}$
 $+ 8377273735398374931683496851093901670374890012895376359078682542768716393$
 $49234894861317508580200600 a^{361/2} + 260217903483151829093716530516$
 $190668583821940861461483281216448823551734330201315644832519365482660 a^{359/2}$
 $+ 79730879538882361019963302061766240010855722094052516139683618$
 $998330963400779203492036220152901230 a^{357/2} + 24094995987038155796$
 $065450245382859737463433262674262689793654664935039408256036219860889044227$
 $150 a^{355/2} + 71811344262959625041553044254225672304315486409767418$
 $83505865868446799304861044299741930350386660 a^{353/2} + 211046937140$
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 $6691490680 a^{351/2} + 6115579967655299993016228420503939587420865523$
 $74610236490615243112421593573435237134063616533170 a^{349/2} + 174711$
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 $545775558767270 a^{347/2} + 4920147064668639188013377676187778959745$
 $417958893480742770076445550864477811884521773749073060 a^{345/2} + 13$
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 $065287858511354960 a^{343/2} + 37360544899750791063541843619131572688$
 $61930796247331837610502825416241804295624449271975310930 a^{341/2} +$
 $100712971375225123486825408775967553588867715563101933352299508027088274855$
 $0110427116557445880 a^{339/2} + 2674986565618089924573038742998282413$
 $44353240871083913762113171457984179086434445936669700220 a^{337/2} +$
 $699947900434062928390261751638419400481986601794781505391317053937625703482$
 $29558917524315680 a^{335/2} + 180409835954669848858780073657344481550$
 $05057683579992022758394768630235475311641910974225440 a^{333/2} + 457$
 $979523648050920266441127246876691931956288277825243497840444218953748354557$
 $8491382900040 a^{331/2} + 1144887117021845990000873407014850512788747$
 $290632662153160101509229540947474388471978656320 a^{329/2} + 28180443$
 $206135926986794439825757967463743299911723339705953641047444987207655139574$
 $6129720 a^{327/2} + 6828693464305013994032391657856757594143852875697$
 $8738176778753728051071752354796643709360 a^{325/2} + 1628794990357190$
 $5740162779090603628412377720468039404006167093404295378080939699332502390 a$
 $^{323/2} + 3823550126984869810060720480378212576630863649197003268812$
 $200176029853869143883259372540 a^{321/2} + 88322097291124153120487238$
 $0940840598638877251159136055568788041859009474970333163665380 a^{319/2}$
 $+ 2007256571054728065052128390750843553487612072423735556480137164292588$
 $93539543288139010 a^{317/2} + 448739196627298566791457560272560584870$
 $52792227828665075782678570381536051164871117670 a^{315/2} + 986659452$
 $652433322528469974694742362105615453775826642313277720229481816731453489930$
 $0 a^{313/2} + 2133266247280470576225983569924952888184081864512181105$
 $155913104938448572679018199600 a^{311/2} + 45346878844284463374153995$
 $8494872085323589464316767784035543488898627702859937499970 a^{309/2}$

+ 9475277846893111808530906263413738581678048958642834055018439346069286663
4869729660*a*e^(307/2*I*c) + 1945780653776869404820978100666431013047007585
7592678483876983129530380918539494004*a*e^(305/2*I*c) + 3926139583037514980
862266059833351755777935293123288314534508002591338991682219336*a*e^(303/2*
I*c) + 77824681097783865657211891791395163471699786563371260117422079924168
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38341004580671627232414532875203636556*a*e^(299/2*I*c) + 289657539984833611
2837518724860972276280949244636300000698769985510049611994644*a*e^(297/2*I
*c) + 543632776691290545699473894503693563320004719189844639485065092496617
0366179640*a*e^(295/2*I*c) + 1001418815121242432011194881350042888741731327
908457902222695201280195390793156*a*e^(293/2*I*c) + 18101285383174567928241
6051344942263215904480326976416514252463543134260124404*a*e^(291/2*I*c) + 3
209795016413281711580362158589283801633286093933028577104958950425432749397
6*a*e^(289/2*I*c) + 5582211676606992496845481327264981244740012057291236674
457828201859232835234*a*e^(287/2*I*c) + 95187382673978638028523458988015774
6261190001514016277713439654750846059956*a*e^(285/2*I*c) + 1591019017813844
72239523743932630706140119020186733129962943950053112026720*a*e^(283/2*I*c)
+ 260596583260161894887915572186327757091519838733271812830090509084536883
84*a*e^(281/2*I*c) + 418147298131838265449765114746501749382995810941526743
7695790041142593706*a*e^(279/2*I*c) + 6570858843206240832064291419381736562
95018100110268516224040799717224364*a*e^(277/2*I*c) + 101089762818526848940
409310346350932268283613848471249613323714958903385*a*e^(275/2*I*c) + 15220
851888182728740782750360825763433070135271873008844718192146049020*a*e^(273
/2*I*c) + 22421586363406278621273200163339501893333563110198405711168340021
18194*a*e^(271/2*I*c) + 323022024354913226146606279108678219616651564707789
831198960566522147*a*e^(269/2*I*c) + 45495956593204825458970416414071568395
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51883625281536772230146111829846964645986*a*e^(265/2*I*c) + 841957703671377
547101115934348899568986163724166597446960696905670*a*e^(263/2*I*c) + 11053
6176601461446339362095696904713163296927687419549616666327675*a*e^(261/2*I*
c) + 14163390775923698840349913696288805073818903748549770982099621066*a*e^
(259/2*I*c) + 1770421771974421622709145878462925518593050670543424508511160
048*a*e^(257/2*I*c) + 21578525923787201686761867532227665174676191403492987
9374220636*a*e^(255/2*I*c) + 2563192803917569444155965471867373913880418556
0774185672766170*a*e^(253/2*I*c) + 2965675360110120697249228947959169960721
153735047384406327794*a*e^(251/2*I*c) + 33404563130070587058830415851425715
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146704159375652160514958780466*a*e^(245/2*I*c) + 40389952602098104591556781
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1918886629249847716790389758084*a*e^(241/2*I*c) + 3961896296989029006971841
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2662742828235210603447717*a*e^(235/2*I*c) + 3046808497087794490022188569750
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4326016097969610*a*e^(231/2*I*c) + 2166587747288431567518701560103605402643
 9130732651*a*e^(229/2*I*c) + 1733270030517994490054541555914528946987399507
 283*a*e^(227/2*I*c) + 133683050380583292759807271902459155221921326266*a*e^(225/2*I*c) + 9928713989218000067239217146577130297436860304*a*e^(223/2*I*c)
) + 709193820494019943354343470410215009861831325*a*e^(221/2*I*c) + 4865181
 6868314506338966571934860812829084391*a*e^(219/2*I*c) + 3200777324680738651
 028636558675556361155978*a*e^(217/2*I*c) + 20162376354473361395060082915101
 9284608876*a*e^(215/2*I*c) + 12139650456097254169315894492857358348125*a*e^(213/2*I*c) + 697316726420326420226818605542124203670*a*e^(211/2*I*c) + 381
 34508068509651338161782530249790846*a*e^(209/2*I*c) + 198101339056005854242
 2694832888372388*a*e^(207/2*I*c) + 97511021259565231697992928696183262*a*e^(205/2*I*c) + 4535396319324144520768178591431614*a*e^(203/2*I*c) + 19871581
 7557880846019817635638340*a*e^(201/2*I*c) + 8173401220846880695205758846440
 *a*e^(199/2*I*c) + 314361585027608802719103613974*a*e^(197/2*I*c) + 1125591
 3521591139154617026091*a*e^(195/2*I*c) + 373282846202680826204100054*a*e^(193/2*I*c) + 11397949499281271404789090*a*e^(191/2*I*c) + 318216864146889674
 614121*a*e^(189/2*I*c) + 8056123142384105100019*a*e^(187/2*I*c) + 183093707
 775748726866*a*e^(185/2*I*c) + 3689545748587311444*a*e^(183/2*I*c) + 648915
 08140773561*a*e^(181/2*I*c) + 975812152492320*a*e^(179/2*I*c) + 12197651906
 154*a*e^(177/2*I*c) + 121672338216*a*e^(175/2*I*c) + 908002524*a*e^(173/2*I*c)
 *c) + 4506216*a*e^(171/2*I*c) + 11154*a*e^(169/2*I*c))/(e^(531*I*c) + 432*e^(530*I*c) + 93096*e^(529*I*c) + 13343760*e^(528*I*c) + 1431118260*e^(527*I*c) + 122503723056*e^(526*I*c) + 8718181624155*e^(525*I*c) + 53056362455683
 2*e^(524*I*c) + 28186192554792138*e^(523*I*c) + 1327882849274858880*e^(522*I*c) + 56169444526926562260*e^(521*I*c) + 2154864144781257856128*e^(520*I*c)
) + 75599817092670157806639*e^(519*I*c) + 2442455629894502983849104*e^(518*I*c) + 73099207817335597247098038*e^(517*I*c) + 203703125947036816013192232
 0*e^(516*I*c) + 53090127264630963470039804475*e^(515*I*c) + 129914664599324
 0318167826532288*e^(514*I*c) + 29952547749265499675257842032197*e^(513*I*c)
 + 652650253343206047453620559993840*e^(512*I*c) + 134772277995247019565792
 74210395326*e^(511*I*c) + 264410375780310742518099326419685040*e^(510*I*c)
 + 4939666610818025798809586352543471345*e^(509*I*c) + 880549275989414111458
 69950813388040256*e^(508*I*c) + 1500602747937397286405577818722691539392*e^(507*I*c) + 24489837337812338687718622491865013839488*e^(506*I*c) + 3833601
 55801054824529764688213114368047154*e^(505*I*c) + 5764601046563151304213854
 710715346838447392*e^(504*I*c) + 833808399118378944531363036737850390515068
 05*e^(503*I*c) + 1161581413733971751533622511909046917188768400*e^(502*I*c)
 + 15603911277687607099721623771744933086920587272*e^(501*I*c) + 2023475097
 24462171313966643580234078508179838320*e^(500*I*c) + 2535667460650279776834
 561566186591213109251642859*e^(499*I*c) + 307353665128305621609911663384900
 57308062762518496*e^(498*I*c) + 3606886130363893494138097800045599635487754
 23325255*e^(497*I*c) + 4101545439937195793959956708442496709433800261224880
 *e^(496*I*c) + 45230940039830738332025694784646206844854827698075736*e^(495*I*c) + 484093410240488718655917025303662581091659126182344528*e^(494*I*c)
 + 5032024903401451824074213943766011922026507006311982753*e^(493*I*c) + 508

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15977270155829115684193411072034541491364393985491350*e^(485*I*c) + 2123702
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33886027390615741040977721655541665612162275485028584310890417*e^(483*I*c)
+ 133490210052026183779673313868332303530332906163247194627808410304*e^(482
*I*c) + 1022536437468296737293065862705246449693687415559865844306888705423
*e^(481*I*c) + 765901052018754965177711835767687192708189898913112575579820
4236112*e^(480*I*c) + 56117081076341175384087570185188538660375932013674735
519055227368366*e^(479*I*c) + 402349692266121158934003582839428785116904903
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884851094991908764246805474672428603757315484453974713612812215428992*e^(47
4*I*c) + 557255115732867112101621641630759616186195596901169722234092621011
2854418*e^(473*I*c) + 35324447206779018115378052820789411687581004582367431
006205879633729015200*e^(472*I*c) + 219601281339515561500261478844190024870
555261281946058839614044697037963695*e^(471*I*c) + 133921437425424555356488
4406801945353385000254030655765953770237607180089968*e^(470*I*c) + 80137295
80790752434361964945761543761469520791210746972675870481058674277844*e^(469
*I*c) + 4706504461113515810848735336748424310269824883831263587628309942744
2745866704*e^(468*I*c) + 27136120750326657073448651707718101480177532218318
1055638619257836143271472358*e^(467*I*c) + 15363332384449275835327345560164
94671674916578907116984548489078241693926940560*e^(466*I*c) + 8543013441126
212334833540665069621472479085838041360564550722036723654297540205*e^(465*I
*c) + 466682235482660178068545924681005702893559608696136508565757567581801
82223308768*e^(464*I*c) + 2505010286089283324693404568299020677122336444646
02753159945727868485722395506952*e^(463*I*c) + 1321498055271300851429993866
631619874424534425188183592049727687571032156435077280*e^(462*I*c) + 685299
322314573668732888531161779543559294084143986635107965565231289472197279626
6*e^(461*I*c) + 34941071613276704649477943043339450201504075335160361865916
029213860778606230624960*e^(460*I*c) + 175193170500618300241515632381912285
157790097816049220671217212220015297133400636060*e^(459*I*c) + 863979933622
330349556296820028395513198708064940505702126068652936800794826651264256*e^
(458*I*c) + 419154250065682614809333941454415914396447847249231593180917185
9902114109005939942952*e^(457*I*c) + 20008006803030047137293278250321597113
540716201983333126349281186679153199068045257216*e^(456*I*c) + 939869153130
68179149083606065681482780836060510530154618486949839467131378859885998210*
e^(455*I*c) + 4345466767802800453463444987638925407971751057568275155092970

24187660299345484920192480*e^(454*I*c) + 1977792980665818135651300094326239
 158605448870806970860577325385028609983034534672318500*e^(453*I*c) + 886275
 214275695728568134088576490459793534956935532181564772117253715918649147131
 1666400*e^(452*I*c) + 39108031255601809476537535369611844440844903751605645
 023514572352045248104262933598850730*e^(451*I*c) + 169956327969929767773902
 096652629253283704505477127544556534417376686540936706073847337600*e^(450*I
 *c) + 727521010718394229291774073844694255798738667067535379759732795567942
 578751384250780476310*e^(449*I*c) + 306797429643174736419815962396246367161
 7006419626851426148418602934852907379021659761911840*e^(448*I*c) + 12747219
 616503320541356343062562847368601622140856786025445814532037904111523242298
 235713300*e^(447*I*c) + 521909122076618242158122718542697480712928432432278
 94769229690720010547141334131610989636000*e^(446*I*c) + 2105943013856484711
 84329078880317504953361839954159427434009884661777259752542647709150036990*
 e^(445*I*c) + 8375792069234119324587864867653735339465452397089907694887248
 13982189165104589895518909256320*e^(444*I*c) + 3283874760555818676726309480
 306734420155098583948074469014168171874442170109648521627538755920*e^(443*I
 *c) + 126934969329649205650736736371812800885486825088802553372800650065661
 38696041797353216584528640*e^(442*I*c) + 4837948975643409984385779181658937
 9406815042609340378747586437145781646245422045101230417309900*e^(441*I*c) +
 18183466140617790131533012967714538116644918841319414116934435475492096903
 4952610378945282257600*e^(440*I*c) + 67402553054313300889484577523662523745
 0743114473544537818170447134607102575676676056675328961590*e^(439*I*c) + 24
 643821908074396090797742268556796293678857097764358766308517162539626961923
 41706239192878728160*e^(438*I*c) + 8888295028751024667044203837607976101480
 053134418614474620767522824868911959884352666444917404000*e^(437*I*c) + 316
 266446747255477311767956875276535713059699859236883921121649155532425732694
 90908989570248533280*e^(436*I*c) + 1110341487970088194431438956444692422950
 49867464313710969257619338899133799285616020069872611710850*e^(435*I*c) + 3
 846558420806662744540630787848371749989490525009753221620033925499534135924
 61519365177908682078400*e^(434*I*c) + 1315052120930692122102297105327622842
 335870743428530891072983535862280094446607723473800477453914130*e^(433*I*c)
 + 443721091784318234776434954444390469902005659506947084719361709211471407
 7633077234972825351226979360*e^(432*I*c) + 14777955096617128998712745182071
 495362176506973183081650233605274051677624970464340242755840025673760*e^(43
 1*I*c) + 485842581531402804473148368687721313901954124190467327784587060150
 96881437076337910793584122475073760*e^(430*I*c) + 1576858455288509187214628
 778644350902575831494155613234273865628944475982779356298009392371756251498
 30*e^(429*I*c) + 5052936631230152588784830257388128132033977668453400653812
 61016353419722382620393032535960660921950400*e^(428*I*c) + 1598771101058192
 692270528999677444742685631006232456185844925220144002305878120380828483988
 663574829100*e^(427*I*c) + 499524195627913818020518674440168802438827211392
 1255663734956946927571305533146776898787878059685108480*e^(426*I*c) + 15413
 111211486023937294970820797376716081344788163386543522421939737507962125854
 981881879168348260330000*e^(425*I*c) + 469702247271172818264540450180706705
 22559756627580347784535320014963482632359729444541885102274546002560*e^(424

*I*c) + 1413799382535568432805655058074033041306061307254347517457940798331
41361748917639986145377066437210546190*e^(423*I*c) + 4203580248351467985836
112101459421546844379493656478990883725248021562228848395800116886556642806
91773600*e^(422*I*c) + 1234668041892409978780018081755440216012582476396941
937965899631953079203974222138794604328498972144766900*e^(421*I*c) + 358271
800216329606141453670371510989710719825273928454614934310234845612408965742
8594946438660859773886240*e^(420*I*c) + 10271602530202889002497813516849452
590971512809529060665197301097052210064576088348023234671975463677418470*e^
(419*I*c) + 290976510612474534066475697818369100621655598523590528042591541
65687125428752562385492373749486351714453120*e^(418*I*c) + 8145208141382911
182887541756425005484603769331281148049290916075819598915576810702256835095
3861815940704090*e^(417*I*c) + 22532053259322065776794110928951624899979452
1015564134982827241710019675486694499689312466561907212627820000*e^(416*I*c
) + 61600311602297958493712570175788721299835430098999136262803886109391456
1332071191909714949426587936910303300*e^(415*I*c) + 16644750343872118093949
177435029376389785749377547647639878358724104499306901315729042799954845810
13965001440*e^(414*I*c) + 4445412259295474625067659514198312966015416299968
930393345630345914109720740573618884980520010028451496996210*e^(413*I*c) +
117358569262451184931130910025016040323418769859990828235206722415302001882
23826392982302194084667538488665600*e^(412*I*c) + 3062758105422195737839054
728927760912957281393108273352024738722600002004353827946877670795842089254
7870128680*e^(411*I*c) + 79019149558766569254783988487232388352909144982747
171856772463223808993367091503402876467270176124342699654400*e^(410*I*c) +
201557947424794098026772478462040883395867512562930862943753568690084015585
598010154781548625239409581907397500*e^(409*I*c) + 508324599301085460166978
629683032661427654474082939048097939638391567298795788389433842285751054665
210868287680*e^(408*I*c) + 126759701729481291340014627604212692998648029287
0190399107554311079964227280196522475370108738477856311765699610*e^(407*I*c
) + 31256834931787017434797047503074901786662921507201793636043351135286233
29684606343185540756019935662148267863968*e^(406*I*c) + 7621788791912047062
038840917799374600428892258194367636682944356096681400246312138001769285020
661445991073249416*e^(405*I*c) + 183798070840033597660276492176211441160917
35572216620788861535803449702273802588359076704241840733513439114113248*e^(
404*I*c) + 4383497214291937768537869223302106374455403310092850273748043897
8976746989895784070951905237783490374305934542955*e^(403*I*c) + 10339975546
725743648984783764075375471820439447305579500146760432641987655555687382953
1737211096115196005647730480*e^(402*I*c) + 24124602128244006179290831778303
287619480159713320605209128699704372914534575580571008148900674183943957398
4832678*e^(401*I*c) + 55675638871118234034102619273421954611365176831738053
9005893679049394714017063698565272728813669054779077208977840*e^(400*I*c) +
12710330829380489502013605548312703426623439912775046123423003660250467417
42856580445289401786656311685859023084716*e^(399*I*c) + 2870496131412314451
834674715353589439553294430808531933466086288543709246230769151392180699413
405623017247753532944*e^(398*I*c) + 641338189585592518475823145106255638032
859493851100657701521811978653639021305728401820220163109443481958402511346

$5 * e^{(397 * I * c)} + 14176483652875704957202013343241117904369977796653849959902$
 $524421980635189011634815653279605497783382888932766730080 * e^{(396 * I * c)} + 310$
 $043192060694170770693631414234874318280090981847446356786522841774394649416$
 $51812564519144918003174108077634846014 * e^{(395 * I * c)} + 6709170613052966912501$
 $989921002157658023784346222953538629508707618929784999593136064560529213096$
 $1496106707521506432 * e^{(394 * I * c)} + 14365768713804479694294711970425953845881$
 $819942351682467458629369105611920986635812363777224540953079923055376722225$
 $2 * e^{(393 * I * c)} + 30438447110681333601028416012390637043388882849062742265255$
 $1236966790916174520857759143930140187173492394981908258944 * e^{(392 * I * c)} + 63$
 $821889291453374150580639966248855606678360049609187640837497487744897177803$
 $6074996245581124283460438065182071976085 * e^{(391 * I * c)} + 13243113249840274283$
 $552229381476823786728607088171617414487496895935880208608475087037023253203$
 $04649883120684987556400 * e^{(390 * I * c)} + 2719589283483743926040805101080341921$
 $244530311254607250929192773909331523226635035815672862569296693711643521070$
 $331394 * e^{(389 * I * c)} + 552749884903117835586123000932666828392629008215846711$
 $8000698502719379939045918344222192742145711257028040974074674736 * e^{(388 * I * c)}$
 $) + 11119499645363201080881062824886338492425375658448977935535846349290425$
 $821570383090425411418521516670371372045206568345 * e^{(387 * I * c)} + 221407350017$
 $086032709151807692413916620355787559039791489092136038225547927491835171602$
 $55571915875356439553717130797888 * e^{(386 * I * c)} + 4363830007581517102594621146$
 $446568961896577348866098585794565747985408510885185791122291198983761545260$
 $8512356008400295 * e^{(385 * I * c)} + 85139533234786455779589959464900637760735729$
 $705621221380005837208369157794673049675428799817875431430246332625899630160$
 $* e^{(384 * I * c)} + 164437500676906892741323260154394278503954561936020133596581$
 $806449357240277349447927334517105809995300093549279931273178 * e^{(383 * I * c)} +$
 $314409035808225861565595436938354945445473991043129722046747228813030925204$
 $968503418566818838611866709807040793495364496 * e^{(382 * I * c)} + 595157615500431$
 $514947479282336547053827926087916425263497187757029413385471835434198246807$
 $096214536895441388306027237899 * e^{(381 * I * c)} + 111539828554560155053332804560$
 $018431799389935921799672981834070422180119566741048584699617905673355851245$
 $2238583160792512 * e^{(380 * I * c)} + 20696982895008606434616657623738079575130194$
 $240411788719049605518294124493447224321256794179584030075511792983159473737$
 $76 * e^{(379 * I * c)} + 3802604996705891106964620633848964807037098854510182263243$
 $030597295630760353597531974324752266389193185760878274188013440 * e^{(378 * I * c)}$
 $+ 691783894521484427849333045936139492337233385361987963737267318494285971$
 $2431066345726870422099893124890777678037369988150 * e^{(377 * I * c)} + 12462140440$
 $537258084928596709872066857757070943124868554500948154756863454308032925408$
 $340311237850017814707896986969086816 * e^{(376 * I * c)} + 222313411318015353454063$
 $990377216868402083979419525801355846459667467366567162715548264762829910660$
 $76564921432614339399735 * e^{(375 * I * c)} + 3927420041432986116939794451622500108$
 $122743339858500720639923121190715779535971964824159875426657984024455149147$
 $6467899952 * e^{(374 * I * c)} + 68712466015985641512468586173659747734879591710098$
 $354652786124936023073943141049573606648563005359411712764895683903806088 * e^{(373 * I * c)}$
 $+ 119060591849660546834765693227676449067584148248882678447504826$
 $077236333444513454095126668750057295811191643356908972191440 * e^{(372 * I * c)} +$

204325557265186000767402710230847896459761583922763698235433212833313077783
041040074669379017394836761539649081690630811665*e^(371*I*c) + 347310053810
935290419455560555957314129569210735745983234369659976413374774078000173070
075248654524917179128950507443058208*e^(370*I*c) + 584749573682304586179384
628844883327581498969886540380378896767999075614964007174600811092945356635
118795824799369716742109*e^(369*I*c) + 975210339444049318757282311763517786
673223175594457946383279264635085041004917300295904275433144848532459919875
479817581584*e^(368*I*c) + 161109254140006052595485937526419417834764347183
707820144626243561514294458733783351358602272984952335843649358604225299560
8*e^(367*I*c) + 26366624104307993404475222847782442837407510686581407265764
46671207798325606832295937705061686297930296382338892574900819440*e^(366*I*
c) + 4274826907720591752526711336820871500844345647922385471534359333606189
571832444641364132893108663576205133870672156264164115*e^(365*I*c) + 686642
533751866826266269375090895696573292457814218163062215780289988087468155103
1136314064199948604001529894566235238597088*e^(364*I*c) + 10927210603473544
810279792347844536074588896806230041110089544731605863146104181739039426674
855453466097402330688331845602302*e^(363*I*c) + 172295028243676473344007219
984175967039486573947388052093916365973703805723989647150800953668183220291
52193635869784095333760*e^(362*I*c) + 2691779470108661509789012023689050110
514679999602177519571086622622863898470345683269415323061160726344418350102
6198563419616*e^(361*I*c) + 41670440375390543643418219342271748040035071490
119080585281522498188818375906900368701234531304633163446319945130196476913
600*e^(360*I*c) + 639230194337619890906148012886350981231994451023031226165
4464820899876780394445577042886552738499747183713136069104651812215*e^(359
*I*c) + 9717305502474268005861672246136889266114129554026349301303274608353
6157324268333390400308958318370219154887169702257444756176*e^(358*I*c) + 14
639044845635118121823738274037412419166481999774698807659839186273362967014
2241546375533903130605297580105675355629160198162*e^(357*I*c) + 21856316665
964931224748364095627214921249911538382877102965428336397258511809047941369
6638108156385244646591328454425745117584*e^(356*I*c) + 32341317801484100371
415113824607915257636097603505845789040993773872317153703657304368199716374
5313602400139153046673668433091*e^(355*I*c) + 47432304356310054237733862993
196624812917597698233244601805600939115402043889590314082296776949401944616
6779954024655344116288*e^(354*I*c) + 68951844932879355990326041814997419025
357834005889503558960646824468059155611817030400503756366988005790876589894
9268614772285*e^(353*I*c) + 99355565364952112722644396082023364938648851008
189254586670044409666158279044124183085560957706203955562509094333226490178
0720*e^(352*I*c) + 1419164481422176573858234013898999628822322333095737307
16310743838935832201454293617293175086458389621425307051750612129761498*e^(
351*I*c) + 2009496110092687738152782085683737222727968824299058739215446083
499351467625334449670757764066690656150949014944043994822823920*e^(350*I*c)
+ 282081929856221595910752980728962844962138679898943636939311606989401878
1201000275633104498398959346631795568022519974400130281*e^(349*I*c) + 39256
976584157783527681039428560118402116427696217172179966143984738871860743914
82638547212826538270453912634540299792270321024*e^(348*I*c) + 5416662804052

436349585595982818357953866258461644354018205158917742576425344364964596750
 653177677803492186817305171175032011500*e^(347*I*c) + 741037261289115222436
 463329612804397165719328032775430438223567277378140302381412761035556250527
 1969045177704726054907145784960*e^(346*I*c) + 10052209524369581827588154985
 345549678031445744499985208259385435609740272400301454246872041775159838468
 077381562338745636398374*e^(345*I*c) + 135212304119454369155588547068517965
 415673998116568700215673529753268154678178465332891238716960561952316961461
 62720992221760992*e^(344*I*c) + 1803532733817745547117756859485168297797834
 644977719357208768851039242688451927299156085132639385224196147004081979362
 7127923997*e^(343*I*c) + 23856398565562802030695278174212640833282154174006
 459458292644060947924907313735921561690153939906017518647182491616573724049
 744*e^(342*I*c) + 312952636881898383137727758733072603341172272586295019923
 58695636092662866062819845689064235813622974150120668921391878398978380*e^(
 341*I*c) + 4071598896370191895002034833673642342051331135901048524691907465
 2883393970805374470830156229705647312265477584256027212762941040*e^(340*I*c
) + 52539223346740771142587092370257069536060319644439501667610482767955800
 276052892432152798814607975110366224945081428121888473324*e^(339*I*c) + 672
 440879690807038237032571996630476068906104820904946194928029351302159798194
 69966383336788693900139115594646893784095418472336*e^(338*I*c) + 8536811843
 021531284823129173967373588774620185166629960039219941876475008682819871987
 2744047767783667325326289221881974987582215*e^(337*I*c) + 10750473740657691
 612348039916975963332832140741940051001749884983059862156542826654631593392
 0821527544726380201659114903834605888*e^(336*I*c) + 13429774202347942990462
 961610455961009607475872106802270446893806301705968802336343645897153496466
 5036319889119229809973806909680*e^(335*I*c) + 16643233292258919513055832926
 639875338982395573759852755609609306247355976954577232197896931890419257273
 3997888230986469005970880*e^(334*I*c) + 20462229553572910951982991678986722
 531942970516256008264884039496542311280933659192129030939239626383467436897
 7840527147037426908*e^(333*I*c) + 24959307228256586639838995150961920268263
 445548712871463146189177082320136752764579377020378878467734393497142431798
 7895255031936*e^(332*I*c) + 30206063803086846346113944227936049971890691748
 252489448335622019613837705082891138305686042537016115720149369607371232259
 5776808*e^(331*I*c) + 36270630756384323113569915741851073245242061401362416
 887931218764523345015392797579332683478074139120343015309371263535552396032
 0*e^(330*I*c) + 43214785646408693802381156180867858959470204790467428229795
 9658800170984456799067751878044806619012452636891350731618278545690160*e^(3
 29*I*c) + 51090761511113450745262384614713714145027572244431638553164842923
 0851686635827717488464500331623385777400744950538410637735936000*e^(328*I*c
) + 59937848477173347480937614240155485020706497211813757294925750365144454
 1939309025276896049622515630263162184526394317285457368300*e^(327*I*c) + 69
 778910692592461481671374768468278508365981902795224444704335586974136850045
 2561164024636073401929693105801105522738405349028160*e^(326*I*c) + 80616967
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 3852257001115644286660304979476023966071898200*e^(325*I*c) + 92432005286752
 258403577749576107235122253442078486196000182102050946814635675643379524644

6491396141583854513687104334429566707520*e^(324*I*c) + 10517821004288343719
445081700512191871168163497669533226371821496108750042237847841832849619064
94422955462431208645690802526770780*e^(323*I*c) + 1187817943079390316108802
324798110129020822782660087248599367643481200206046822166144285425922229375
413676535071141005286431481600*e^(322*I*c) + 133139611462672303580246212353
158205033974999601415245283530595636742537022275862175392245872752485607295
0880960657564720475838500*e^(321*I*c) + 14811871170892466624669556949656778
555247303132605526908216026571762187374262455227953298914640910058783043040
75953693546767206080*e^(320*I*c) + 1635569744641421900657886381289076653227
580172056677587451402069234355283687489659613913761959140773339736014790081
814516625224440*e^(319*I*c) + 179264907808963229893694572848196933496439159
750628508848835062293725253342098080314431643170145219052271612479787525743
7516360640*e^(318*I*c) + 19502865507801819192449929612044870100564603628452
185016744237662663215587914369173178787022326792138682879262946652027697229
27380*e^(317*I*c) + 2106141903468344307112549761202484543402794252352482199
817410424869677262715098288437646518683487945462774223656471345899082156800
*e^(316*I*c) + 225772621910385628681283301268157376549626224142061293207614
3151171960854554124144699023009842080515157923529357189869943515991200*e^(3
15*I*c) + 24024645955696860861200018030342110567394455886219461413841061628
86246161815149763025030834875234067267774023433418269982431265280*e^(314*I*
c) + 2537766415465030330815471746692988596069911894697225052928320452542175
587154848096483331209807430113943015398362669673337957755720*e^(313*I*c) +
266110064797578358382823513920144193017839664338342390358386254725588077238
2049201015537214900832745601519737141849802506685264000*e^(312*I*c) + 27700
732071507686455975072813820654979249684660545274141223398273337837700683058
83487309979315983718403740872884345746380680204260*e^(311*I*c) + 2862503126
320461797770667780725644184991255623174626175679050672100848988119391841466
573417019247590580735265143427289340450811200*e^(310*I*c) + 293649421435186
849870323945542677110434482730626755891655087746723245515328614052108958273
3932202553130712723836983468866230908800*e^(309*I*c) + 29904989496225436085
381293802838663351900871151248588181437876191869571119037657239748996515185
55144924290346242595167274383008960*e^(308*I*c) + 3023371643508225027175603
175212953219022485045850731845307519008277385154731461213388035579159917590
062343527464977286601165100620*e^(307*I*c) + 303440835553095707578773174532
256798168461655016284547325767967428021694735678578384320560420230783689707
3595410412575660465787520*e^(306*I*c) + 30233716435082250271756031752129532
190224850458507318453075190082773851547314612133880355791599175900623435274
64977286601165100620*e^(305*I*c) + 2990498949622543608538129380283866335190
087115124858818143787619186957111903765723974899651518555144924290346242595
167274383008960*e^(304*I*c) + 293649421435186849870323945542677110434482730
626755891655087746723245515328614052108958273393220255313071272383698346886
6230908800*e^(303*I*c) + 28625031263204617977706677807256441849912556231746
261756790506721008489881193918414665734170192475905807352651434272893404508
11200*e^(302*I*c) + 2770073207150768645597507281382065497924968466054527414
122339827333783770068305883487309979315983718403740872884345746380680204260

$*e^{(301*I*c)} + 266110064797578358382823513920144193017839664338342390358386$
 $2547255880772382049201015537214900832745601519737141849802506685264000*e^{(3$
 $00*I*c)} + 25377664154650303308154717466929885960699118946972250529283204525$
 $42175587154848096483331209807430113943015398362669673337957755720*e^{(299*I*}$
 $c)} + 2402464595569686086120001803034211056739445588621946141384106162886246$
 $161815149763025030834875234067267774023433418269982431265280*e^{(298*I*c)} +$
 $225772621910385628681283301268157376549626224142061293207614315117196085455$
 $4124144699023009842080515157923529357189869943515991200*e^{(297*I*c)} + 21061$
 $419034683443071125497612024845434027942523524821998174104248696772627150982$
 $88437646518683487945462774223656471345899082156800*e^{(296*I*c)} + 1950286550$
 $780181919244992961204487010056460362845218501674423766266321558791436917317$
 $878702232679213868287926294665202769722927380*e^{(295*I*c)} + 179264907808963$
 $229893694572848196933496439159750628508848835062293725253342098080314431643$
 $1701452190522716124797875257437516360640*e^{(294*I*c)} + 16355697446414219006$
 $578863812890766532275801720566775874514020692343552836874896596139137619591$
 $40773339736014790081814516625224440*e^{(293*I*c)} + 1481187117089246662466955$
 $694965677855524730313260552690821602657176218737426245522795329891464091005$
 $878304304075953693546767206080*e^{(292*I*c)} + 133139611462672303580246212353$
 $158205033974999601415245283530595636742537022275862175392245872752485607295$
 $0880960657564720475838500*e^{(291*I*c)} + 11878179430793903161088023247981101$
 $290208227826600872485993676434812002060468221661442854259222293754136765350$
 $71141005286431481600*e^{(290*I*c)} + 1051782100428834371944508170051219187116$
 $816349766953322637182149610875004223784784183284961906494422955462431208645$
 $690802526770780*e^{(289*I*c)} + 924320052867522584035777495761072351222534420$
 $784861960001821020509468146356756433795246446491396141583854513687104334429$
 $566707520*e^{(288*I*c)} + 806169671327625532424575340089775733681994991576674$
 $446922354099714615192085443245663852257001115644286660304979476023966071898$
 $200*e^{(287*I*c)} + 697789106925924614816713747684682785083659819027952244447$
 $043355869741368500452561164024636073401929693105801105522738405349028160*e^{(}$
 $286*I*c)} + 599378484771733474809376142401554850207064972118137572949257503$
 $651444541939309025276896049622515630263162184526394317285457368300*e^{(285*I}$
 $*c)} + 510907615111134507452623846147137141450275722444316385531648429230851$
 $686635827717488464500331623385777400744950538410637735936000*e^{(284*I*c)} +$
 $432147856464086938023811561808678589594702047904674282297959658800170984456$
 $799067751878044806619012452636891350731618278545690160*e^{(283*I*c)} + 362706$
 $307563843231135699157418510732452420614013624168879312187645233450153927975$
 $793326834780741391203430153093712635355523960320*e^{(282*I*c)} + 302060638030$
 $868463461139442279360499718906917482524894483356220196138377050828911383056$
 $860425370161157201493696073712322595776808*e^{(281*I*c)} + 249593072282565866$
 $398389951509619202682634455487128714631461891770823201367527645793770203788$
 $784677343934971424317987895255031936*e^{(280*I*c)} + 204622295535729109519829$
 $916789867225319429705162560082648840394965423112809336591921290309392396263$
 $834674368977840527147037426908*e^{(279*I*c)} + 166432332922589195130558329266$
 $398753389823955737598527556096093062473559769545772321978969318904192572733$
 $997888230986469005970880*e^{(278*I*c)} + 134297742023479429904629616104559610$

096074758721068022704468938063017059688023363436458971534964665036319889119
229809973806909680*e^(277*I*c) + 107504737406576916123480399169759633328321
407419400510017498849830598621565428266546315933920821527544726380201659114
903834605888*e^(276*I*c) + 853681184302153128482312917396737358877462018516
662996003921994187647500868281987198727440477677836673253262892218819749875
82215*e^(275*I*c) + 6724408796908070382370325719966304760689061048209049461
9492802935130215979819469966383336788693900139115594646893784095418472336*e
^(274*I*c) + 52539223346740771142587092370257069536060319644439501667610482
767955800276052892432152798814607975110366224945081428121888473324*e^(273*I
*c) + 407159889637019189500203483367364234205133113590104852469190746528833
93970805374470830156229705647312265477584256027212762941040*e^(272*I*c) + 3
129526368818983831377277587330726033411722725862950199235869563609266286606
2819845689064235813622974150120668921391878398978380*e^(271*I*c) + 23856398
565562802030695278174212640833282154174006459458292644060947924907313735921
561690153939906017518647182491616573724049744*e^(270*I*c) + 180353273381774
554711775685948516829779783464497771935720876885103924268845192729915608513
26393852241961470040819793627127923997*e^(269*I*c) + 1352123041194543691555
885470685179654156739981165687002156735297532681546781784653328912387169605
6195231696146162720992221760992*e^(268*I*c) + 10052209524369581827588154985
345549678031445744499985208259385435609740272400301454246872041775159838468
077381562338745636398374*e^(267*I*c) + 741037261289115222436463329612804397
165719328032775430438223567277378140302381412761035556250527196904517770472
6054907145784960*e^(266*I*c) + 54166628040524363495855959828183579538662584
616443540182051589177425764253443649645967506531776778034921868173051711750
32011500*e^(265*I*c) + 3925697658415778352768103942856011840211642769621717
217996614398473887186074391482638547212826538270453912634540299792270321024
*e^(264*I*c) + 282081929856221595910752980728962844962138679898943636939311
6069894018781201000275633104498398959346631795568022519974400130281*e^(263*
I*c) + 20094961100926877381527820856837372227279688242990587392154460834993
51467625334449670757764066690656150949014944043994822823920*e^(262*I*c) + 1
41916448142217657385823401389899962882232233309573730716310743838935832201
454293617293175086458389621425307051750612129761498*e^(261*I*c) + 993555653
649521127226443960820233649386488510081892545866700444096661582790441241830
855609577062039555625090943332264901780720*e^(260*I*c) + 689518449328793559
903260418149974190253578340058895035589606468244680591556118170304005037563
669880057908765898949268614772285*e^(259*I*c) + 474323043563100542377338629
931966248129175976982332446018056009391154020438895903140822967769494019446
166779954024655344116288*e^(258*I*c) + 323413178014841003714151138246079152
576360976035058457890409937738723171537036573043681997163745313602400139153
046673668433091*e^(257*I*c) + 218563166659649312247483640956272149212499115
383828771029654283363972585118090479413696638108156385244646591328454425745
117584*e^(256*I*c) + 146390448456351181218237382740374124191664819997746988
076598391862733629670142241546375533903130605297580105675355629160198162*e^
(255*I*c) + 971730550247426800586167224613688926611412955402634930130327460
83536157324268333390400308958318370219154887169702257444756176*e^(254*I*c)

+ 6392301943376198909061480128863509812319944510230312261654464820899876780
3944455777042886552738499747183713136069104651812215*e^(253*I*c) + 41670440
375390543643418219342271748040035071490119080585281522498188818375906900368
701234531304633163446319945130196476913600*e^(252*I*c) + 269177947010866150
978901202368905011051467999960217751957108662262286389847034568326941532306
11607263444183501026198563419616*e^(251*I*c) + 1722950282436764733440072199
841759670394865739473880520939163659737038057239896471508009536681832202915
2193635869784095333760*e^(250*I*c) + 10927210603473544810279792347844536074
588896806230041110089544731605863146104181739039426674855453466097402330688
331845602302*e^(249*I*c) + 686642533751866826266269375090895696573292457814
218163062215780289988087468155103113631406419994860400152989456623523859708
8*e^(248*I*c) + 42748269077205917525267113368208715008443456479223854715343
59333606189571832444641364132893108663576205133870672156264164115*e^(247*I*
c) + 2636662410430799340447522284778244283740751068658140726576446671207798
325606832295937705061686297930296382338892574900819440*e^(246*I*c) + 161109
254140006052595485937526419417834764347183707820144626243561514294458733783
3513586022729849523358436493586042252995608*e^(245*I*c) + 97521033944404931
875728231176351778667322317559445794638327926463508504100491730029590427543
3144848532459919875479817581584*e^(244*I*c) + 58474957368230458617938462884
488332758149896988654038037889676799907561496400717460081109294535663511879
5824799369716742109*e^(243*I*c) + 3473100538109352904194555605595731412956
921073574598323436965997641337477407800017307007524865452491717912895050744
3058208*e^(242*I*c) + 20432555726518600076740271023084789645976158392276369
8235433212833313077783041040074669379017394836761539649081690630811665*e^(2
41*I*c) + 11906059184966054683476569322767644906758414824888267844750482607
7236333444513454095126668750057295811191643356908972191440*e^(240*I*c) + 68
712466015985641512468586173659747734879591710098354652786124936023073943141
049573606648563005359411712764895683903806088*e^(239*I*c) + 392742004143298
611693979445162250010812274333985850072063992312119071577953597196482415987
54266579840244551491476467899952*e^(238*I*c) + 2223134113180153534540639903
772168684020839794195258013558464596674673665671627155482647628299106607656
4921432614339399735*e^(237*I*c) + 12462140440537258084928596709872066857757
070943124868554500948154756863454308032925408340311237850017814707896986969
086816*e^(236*I*c) + 691783894521484427849333045936139492337233385361987963
7372673184942859712431066345726870422099893124890777678037369988150*e^(235*
I*c) + 38026049967058911069646206338489648070370988545101822632430305972956
30760353597531974324752266389193185760878274188013440*e^(234*I*c) + 2069698
289500860643461665762373807957513019424041178871904960551829412449344722432
125679417958403007551179298315947373776*e^(233*I*c) + 111539828554560155053
332804560018431799389935921799672981834070422180119566741048584699617905673
3558512452238583160792512*e^(232*I*c) + 59515761550043151494747928233654705
382792608791642526349718775702941338547183543419824680709621453689544138830
6027237899*e^(231*I*c) + 31440903580822586156559543693835494544547399104312
9722046747228813030925204968503418566818838611866709807040793495364496*e^(2
30*I*c) + 16443750067690689274132326015439427850395456193602013359658180644

9357240277349447927334517105809995300093549279931273178*e^(229*I*c) + 85139
533234786455779589959464900637760735729705621221380005837208369157794673049
675428799817875431430246332625899630160*e^(228*I*c) + 436383000758151710259
462114644656896189657734886609858579456574798540851088518579112229119898376
15452608512356008400295*e^(227*I*c) + 2214073500170860327091518076924139166
203557875590397914890921360382255479274918351716025557191587535643955371713
0797888*e^(226*I*c) + 11119499645363201080881062824886338492425375658448977
935535846349290425821570383090425411418521516670371372045206568345*e^(225*I
*c) + 552749884903117835586123000932666828392629008215846711800069850271937
9939045918344222192742145711257028040974074674736*e^(224*I*c) + 27195892834
837439260408051010803419212445303112546072509291927739093315232266350358156
72862569296693711643521070331394*e^(223*I*c) + 1324311324984027428355222938
147682378672860708817161741448749689593588020860847508703702325320304649883
120684987556400*e^(222*I*c) + 638218892914533741505806399662488556066783600
496091876408374974877448971778036074996245581124283460438065182071976085*e^
(221*I*c) + 304384471106813336010284160123906370433888828490627422652551236
966790916174520857759143930140187173492394981908258944*e^(220*I*c) + 143657
687138044796942947119704259538458818199423516824674586293691056119209866358
123637772245409530799230553767222252*e^(219*I*c) + 670917061305296691250198
992100215765802378434622295353862950870761892978499959313606456052921309614
96106707521506432*e^(218*I*c) + 3100431920606941707706936314142348743182800
9098184744635678652284177439464941651812564519144918003174108077634846014*e
^(217*I*c) + 14176483652875704957202013343241117904369977796653849959902524
421980635189011634815653279605497783382888932766730080*e^(216*I*c) + 641338
189585592518475823145106255638032859493851100657701521811978653639021305728
4018202201631094434819584025113465*e^(215*I*c) + 28704961314123144518346747
153535894395532944308085319334660862885437092462307691513921806994134056230
17247753532944*e^(214*I*c) + 1271033082938048950201360554831270342662343991
277504612342300366025046741742856580445289401786656311685859023084716*e^(21
3*I*c) + 556756388711182340341026192734219546113651768317380539005893679049
394714017063698565272728813669054779077208977840*e^(212*I*c) + 241246021282
440061792908317783032876194801597133206052091286997043729145345755805710081
489006741839439573984832678*e^(211*I*c) + 103399755467257436489847837640753
754718204394473055795001467604326419876555556873829531737211096115196005647
730480*e^(210*I*c) + 438349721429193776853786922330210637445540331009285027
37480438978976746989895784070951905237783490374305934542955*e^(209*I*c) + 1
837980708400335976602764921762114411609173557221662078886153580344970227380
2588359076704241840733513439114113248*e^(208*I*c) + 76217887919120470620388
409177993746004288922581943676366829443560966814002463121380017692850206614
45991073249416*e^(207*I*c) + 3125683493178701743479704750307490178666292150
720179363604335113528623329684606343185540756019935662148267863968*e^(206*I
*c) + 126759701729481291340014627604212692998648029287019039910755431107996
4227280196522475370108738477856311765699610*e^(205*I*c) + 50832459930108546
016697862968303266142765447408293904809793963839156729879578838943384228575
1054665210868287680*e^(204*I*c) + 20155794742479409802677247846204088339586

7512562930862943753568690084015585598010154781548625239409581907397500*e^(203*I*c) + 79019149558766569254783988487232388352909144982747171856772463223
808993367091503402876467270176124342699654400*e^(202*I*c) + 306275810542219
573783905472892776091295728139310827335202473872260000200435382794687767079
58420892547870128680*e^(201*I*c) + 1173585692624511849311309100250160403234
1876985999082823520672241530200188223826392982302194084667538488665600*e^(200*I*c) + 44454122592954746250676595141983129660154162999689303933456303459
14109720740573618884980520010028451496996210*e^(199*I*c) + 1664475034387211
809394917743502937638978574937754764763987835872410449930690131572904279995
484581013965001440*e^(198*I*c) + 616003116022979584937125701757887212998354
300989991362628038861093914561332071191909714949426587936910303300*e^(197*I*c) + 225320532593220657767941109289516248999794521015564134982827241710019
675486694499689312466561907212627820000*e^(196*I*c) + 814520814138291118288
754175642500548460376933128114804929091607581959891557681070225683509538618
15940704090*e^(195*I*c) + 2909765106124745340664756978183691006216555985235
9052804259154165687125428752562385492373749486351714453120*e^(194*I*c) + 10
271602530202889002497813516849452590971512809529060665197301097052210064576
088348023234671975463677418470*e^(193*I*c) + 358271800216329606141453670371
510989710719825273928454614934310234845612408965742859494643866085977388624
0*e^(192*I*c) + 12346680418924099787800180817554402160125824763969419379658
99631953079203974222138794604328498972144766900*e^(191*I*c) + 4203580248351
467985836112101459421546844379493656478990883725248021562228848395800116886
55664280691773600*e^(190*I*c) + 1413799382535568432805655058074033041306061
30725434751745794079833141361748917639986145377066437210546190*e^(189*I*c)
+ 4697022472711728182645404501807067052255975662758034778453532001496348263
2359729444541885102274546002560*e^(188*I*c) + 15413111211486023937294970820
797376716081344788163386543522421939737507962125854981881879168348260330000
*e^(187*I*c) + 499524195627913818020518674440168802438827211392125566373495
6946927571305533146776898787878059685108480*e^(186*I*c) + 15987711010581926
922705289996774447426856310062324561858449252201440023058781203808284839886
63574829100*e^(185*I*c) + 5052936631230152588784830257388128132033977668453
40065381261016353419722382620393032535960660921950400*e^(184*I*c) + 1576858
455288509187214628778644350902575831494155613234273865628944475982779356298
00939237175625149830*e^(183*I*c) + 4858425815314028044731483686877213139019
5412419046732778458706015096881437076337910793584122475073760*e^(182*I*c) +
14777955096617128998712745182071495362176506973183081650233605274051677624
970464340242755840025673760*e^(181*I*c) + 443721091784318234776434954444390
4699020056595069470847193617092114714077633077234972825351226979360*e^(180*I*c) + 13150521209306921221022971053276228423358707434285308910729835358622
80094446607723473800477453914130*e^(179*I*c) + 3846558420806662744540630787
84837174998949052500975322162003392549953413592461519365177908682078400*e^(178*I*c) + 1110341487970088194431438956444692422950498674643137109692576193
38899133799285616020069872611710850*e^(177*I*c) + 3162664467472554773117679
5687527653571305969985923688392112164915553242573269490908989570248533280*e
^(176*I*c) + 88882950287510246670442038376079761014800531344186144746207675

22824868911959884352666444917404000*e^(175*I*c) + 2464382190807439609079774
226855679629367885709776435876630851716253962696192341706239192878728160*e^(
(174*I*c) + 674025530543133008894845775236625237450743114473544537818170447
134607102575676676056675328961590*e^(173*I*c) + 181834661406177901315330129
677145381166449188413194141169344354754920969034952610378945282257600*e^(17
2*I*c) + 483794897564340998438577918165893794068150426093403787475864371457
81646245422045101230417309900*e^(171*I*c) + 1269349693296492056507367363718
1280088548682508880255337280065006566138696041797353216584528640*e^(170*I*c
) + 32838747605558186767263094803067344201550985839480744690141681718744421
70109648521627538755920*e^(169*I*c) + 8375792069234119324587864867653735339
46545239708990769488724813982189165104589895518909256320*e^(168*I*c) + 2105
943013856484711843290788803175049533618399541594274340098846617772597525426
47709150036990*e^(167*I*c) + 5219091220766182421581227185426974807129284324
3227894769229690720010547141334131610989636000*e^(166*I*c) + 12747219616503
320541356343062562847368601622140856786025445814532037904111523242298235713
300*e^(165*I*c) + 306797429643174736419815962396246367161700641962685142614
8418602934852907379021659761911840*e^(164*I*c) + 72752101071839422929177407
3844694255798738667067535379759732795567942578751384250780476310*e^(163*I*c
) + 16995632796992976777390209665262925328370450547712754455653441737668654
0936706073847337600*e^(162*I*c) + 39108031255601809476537535369611844440844
903751605645023514572352045248104262933598850730*e^(161*I*c) + 886275214275
695728568134088576490459793534956935532181564772117253715918649147131166640
0*e^(160*I*c) + 19777929806658181356513000943262391586054488708069708605773
25385028609983034534672318500*e^(159*I*c) + 4345466767802800453463444987638
92540797175105756827515509297024187660299345484920192480*e^(158*I*c) + 9398
691531306817914908360606568148278083606051053015461848694983946713137885988
5998210*e^(157*I*c) + 20008006803030047137293278250321597113540716201983333
126349281186679153199068045257216*e^(156*I*c) + 419154250065682614809333941
4544159143964478472492315931809171859902114109005939942952*e^(155*I*c) + 86
397993362233034955629682002839551319870806494050570212606865293680079482665
1264256*e^(154*I*c) + 17519317050061830024151563238191228515779009781604922
0671217212220015297133400636060*e^(153*I*c) + 34941071613276704649477943043
339450201504075335160361865916029213860778606230624960*e^(152*I*c) + 685299
322314573668732888531161779543559294084143986635107965565231289472197279626
6*e^(151*I*c) + 13214980552713008514299938666316198744245344251881835920497
27687571032156435077280*e^(150*I*c) + 2505010286089283324693404568299020677
12233644464602753159945727868485722395506952*e^(149*I*c) + 4666822354826601
7806854592468100570289355960869613650856575756758180182223308768*e^(148*I*c
) + 85430134411262123348335406650696214724790858380413605645507220367236542
97540205*e^(147*I*c) + 1536333238444927583532734556016494671674916578907116
984548489078241693926940560*e^(146*I*c) + 271361207503266570734486517077181
014801775322183181055638619257836143271472358*e^(145*I*c) + 470650446111351
58108487353367484243102698248838312635876283099427442745866704*e^(144*I*c)
+ 8013729580790752434361964945761543761469520791210746972675870481058674277
844*e^(143*I*c) + 133921437425424555356488440680194535338500025403065576595

3770237607180089968*e^(142*I*c) + 21960128133951556150026147884419002487055
 5261281946058839614044697037963695*e^(141*I*c) + 35324447206779018115378052
 820789411687581004582367431006205879633729015200*e^(140*I*c) + 557255115732
 8671121016216416307596161861955969011697222340926210112854418*e^(139*I*c) +
 861884851094991908764246805474672428603757315484453974713612812215428992*e^(138*I*c) +
 13065766022656041933512143438993896188459543406998482430714933
 2131747540*e^(137*I*c) + 19407979215594566593535008103303255257745408070082
 431338945184797463936*e^(136*I*c) + 282390515193658667838252570656445728029
 0098698638597987628380245881715*e^(135*I*c) + 40234969226612115893400358283
 9428785116904903936409545602519219664720*e^(134*I*c) + 56117081076341175384
 087570185188538660375932013674735519055227368366*e^(133*I*c) + 765901052018
 7549651777118357676871927081898989131125755798204236112*e^(132*I*c) + 10225
 36437468296737293065862705246449693687415559865844306888705423*e^(131*I*c)
 + 133490210052026183779673313868332303530332906163247194627808410304*e<sup>(130
 *I*c)</sup> + 17033886027390615741040977721655541665612162275485028584310890417*e^(129*I*c) +
 21237029691888713182667187812239270678399490157272938840653880
 80*e^(128*I*c) + 2585853487159772701558291156841934110720345414913643939854
 91350*e^(127*I*c) + 3073621740432100996523103741966305396288103528170922169
 7785072*e^(126*I*c) + 35647648906287240170884879966881789291957876139585454
 74804845*e^(125*I*c) + 4032122259577981888408461399609956241444912716943367
 96459584*e^(124*I*c) + 4445670817525882102440094621053500452377572219097746
 8484496*e^(123*I*c) + 47753986071008532635342077338182667774786934127387310
 31680*e^(122*I*c) + 4994675065585317336715858629105727028115450357303987495
 30*e^(121*I*c) + 50836369508171099437019348610847391946736185108017183136*e^(120*I*c) +
 5032024903401451824074213943766011922026507006311982753*e<sup>(119
 *I*c)</sup> + 484093410240488718655917025303662581091659126182344528*e^(118*I*c)
 + 45230940039830738332025694784646206844854827698075736*e^(117*I*c) + 41015
 45439937195793959956708442496709433800261224880*e^(116*I*c) + 3606886130363
 89349413809780004559963548775423325255*e^(115*I*c) + 3073536651283056216099
 1166338490057308062762518496*e^(114*I*c) + 25356674606502797768345615661865
 91213109251642859*e^(113*I*c) + 2023475097244621713139666435802340785081798
 38320*e^(112*I*c) + 15603911277687607099721623771744933086920587272*e<sup>(111*
 I*c)</sup> + 1161581413733971751533622511909046917188768400*e^(110*I*c) + 8338083
 9911837894453136303673785039051506805*e^(109*I*c) + 57646010465631513042138
 54710715346838447392*e^(108*I*c) + 3833601558010548245297646882131143680471
 54*e^(107*I*c) + 24489837337812338687718622491865013839488*e^(106*I*c) + 15
 00602747937397286405577818722691539392*e^(105*I*c) + 8805492759894141114586
 9950813388040256*e^(104*I*c) + 4939666610818025798809586352543471345*e<sup>(103
 *I*c)</sup> + 264410375780310742518099326419685040*e^(102*I*c) + 1347722779952470
 1956579274210395326*e^(101*I*c) + 652650253343206047453620559993840*e<sup>(100*
 I*c)</sup> + 29952547749265499675257842032197*e^(99*I*c) + 1299146645993240318167
 826532288*e^(98*I*c) + 53090127264630963470039804475*e^(97*I*c) + 203703125
 9470368160131922320*e^(96*I*c) + 73099207817335597247098038*e^(95*I*c) + 24
 42455629894502983849104*e^(94*I*c) + 75599817092670157806639*e^(93*I*c) + 2
 154864144781257856128*e^(92*I*c) + 56169444526926562260*e^(91*I*c) + 132788

$$\begin{aligned}
& 2849274858880 * e^{(90 * I * c)} + 28186192554792138 * e^{(89 * I * c)} + 530563624556832 * e^{(88 * I * c)} \\
& + 8718181624155 * e^{(87 * I * c)} + 122503723056 * e^{(86 * I * c)} + 1431118260 * e^{(85 * I * c)} \\
& + 13343760 * e^{(84 * I * c)} + 93096 * e^{(83 * I * c)} + 432 * e^{(82 * I * c)} + e^{(81 * I * c)} \\
& \left. \right) * \tan(1/4 * d * x + c) + 7 * (-11154 * I * a * e^{(1055/2 * I * c)} - 4506216 * I * a * e^{(1053/2 * I * c)} \\
& - 908002524 * I * a * e^{(1051/2 * I * c)} - 121672338216 * I * a * e^{(1049/2 * I * c)} \\
& - 12197651906154 * I * a * e^{(1047/2 * I * c)} - 975812152492320 * I * a * e^{(1045/2 * I * c)} \\
& - 64891508140773093 * I * a * e^{(1043/2 * I * c)} - 3689545748587122372 * I * a * e^{(1041/2 * I * c)} \\
& - 183093707775710628858 * I * a * e^{(1039/2 * I * c)} - 8056123142378999966797 * I * a * e^{(1037/2 * I * c)} \\
& - 318216864146377884963053 * I * a * e^{(1035/2 * I * c)} - 11397949499240328225340750 * I * a * e^{(1033/2 * I * c)} \\
& - 373282846199958103946548090 * I * a * e^{(1031/2 * I * c)} - 11255913521436332875256492285 * I * a * e^{(1029/2 * I * c)} \\
& - 314361585019926536123171779990 * I * a * e^{(1027/2 * I * c)} - 8173401220508860669709781109364 * I * a * e^{(1025/2 * I * c)} \\
& - 198715817544529039864660110471376 * I * a * e^{(1023/2 * I * c)} - 4535396318845906415166472261686554 * I * a * e^{(1021/2 * I * c)} \\
& - 97511021243902906034367815029181218 * I * a * e^{(1019/2 * I * c)} - 1981013390087778171143744488041442712 * I * a * e^{(1017/2 * I * c)} \\
& - 38134508055319501480202851957227724462 * I * a * e^{(1015/2 * I * c)} - 697316726077381488801524339658757026686 * I * a * e^{(1013/2 * I * c)} \\
& - 12139650447759376183227584660170495869589 * I * a * e^{(1011/2 * I * c)} - 201623763354433037155766729846494383292508 * I * a * e^{(1009/2 * I * c)} \\
& - 3200777320589257207465955323343789287604398 * I * a * e^{(1007/2 * I * c)} - 48651816785192390862949346064352748595789341 * I * a * e^{(1005/2 * I * c)} \\
& - 709193818893909656677523349582328791900299617 * I * a * e^{(1003/2 * I * c)} - 9928713959958644143611456276187690708075285068 * I * a * e^{(1001/2 * I * c)} \\
& - 133683049871200685767730244065634086417919364482 * I * a * e^{(999/2 * I * c)} - 1733270022057743255812637849164901656684569946869 * I * a * e^{(997/2 * I * c)} \\
& - 21665877338576587360651194480041798194029867090275 * I * a * e^{(995/2 * I * c)} - 261388357024674483234213954236474266534206934500718 * I * a * e^{(993/2 * I * c)} \\
& - 3046808467328698391993011459852063034674399964693132 * I * a * e^{(991/2 * I * c)} - 34345846767794763840341317174417543493362860419944375 * I * a * e^{(989/2 * I * c)} \\
& - 374773875381762608573480596867444622808315521135770955 * I * a * e^{(987/2 * I * c)} - 3961896224254872041251938157236050043036325552867100290 * I * a * e^{(985/2 * I * c)} \\
& - 40609448120913176083174637180877049405994150689208285948 * I * a * e^{(983/2 * I * c)} - 403899515051838803400194879806531247927007890538062521027 * I * a * e^{(981/2 * I * c)} \\
& - 3900820584110953912072698502834368654421340467896495241550 * I * a * e^{(979/2 * I * c)} - 36607718305457313571850609230094969421310836249109701091666 * I * a * e^{(977/2 * I * c)} \\
& - 334045615572410482684075553294675381058554260440184749831968 * I * a * e^{(975/2 * I * c)} - 2965675193835710066559916582743095158371951788246043891048682 * I * a * e^{(973/2 * I * c)} \\
& - 25631926334820519324289406607504181260913532669181012596611058 * I * a * e^{(971/2 * I * c)} - 215785242285994868818528731430540812422314520336288528460007864 * I * a * e^{(969/2 * I * c)} \\
& - 1770421608250297155611599130292037434814100286157679816873050532 * I * a * e^{(967/2 * I * c)} - 14163389239388825902842073391400806825940375203939456957445830422 * I * a * e^{(965/2 * I * c)} \\
& - 110536162580123562634802981277497355054368691566003022338877490451 * I * a * e^{(963/2 * I * c)} - 841957579184928615664618341834644691840690166368399913404427216774 * I * a * e^{(961/2 * I * c)} \\
& - 6262070331779324000513806613163981009868362783949768253661316894658 * I * a * e^{(959/2 * I * c)} - 45495947534779552008093888439933100139363372706825172710009
\end{aligned}$$

091580883*I*a*e^(957/2*I*c) - 323021950031541227897801407171566170150519245
 202346540397613583355667*I*a*e^(955/2*I*c) - 224215804172723511736495773945
 9342847819393345842116256845583655236406*I*a*e^(953/2*I*c) - 15220847247394
 291084416636302914971818756836097657800823465735035857764*I*a*e^(951/2*I*c)
 - 101089727467776956523475550836202489574004061008016807940002791738698571
 *I*a*e^(949/2*I*c) - 657085621385514241363241301495127058109065015699304745
 663817714211556940*I*a*e^(947/2*I*c) - 418147107090738761265959863160675178
 8208788794201536282802542477878283190*I*a*e^(945/2*I*c) - 26059644761295218
 063521552347914441867779264013071310596786890247411087520*I*a*e^(943/2*I*c)
 - 159101807620372457710230554627597385485602304037309714452742966256548901
 840*I*a*e^(941/2*I*c) - 951873187489146746238338008022239906318071898028016
 864090568030803687398612*I*a*e^(939/2*I*c) - 558220743072176742491092216915
 5711907398569721937114073480331164440862462638*I*a*e^(937/2*I*c) - 32097922
 563848506943066473469340753662783076784232671471143949644558156480632*I*a*e
 ^ (935/2*I*c) - 181012678179948465189891189876844483520479568880173570917696
 339360628958538348*I*a*e^(933/2*I*c) - 100141772034465691525192490100975093
 9990149160965149540929889824388402626455012*I*a*e^(931/2*I*c) - 54363210824
 38142029415739885553914767552718204411177277415536254863231523181480*I*a*e^
 (929/2*I*c) - 2896571400325784692154085898638895400812211161271236335436098
 1851687424724108036*I*a*e^(927/2*I*c) - 15151479395044286133635700687251245
 8064446653474553525802193583698620014121641124*I*a*e^(925/2*I*c) - 77824546
 2061092938241128952993007961165558304583117537979936583232925968316199836*I
 *a*e^(923/2*I*c) - 39261319752280473730214398836102939446474469286733442972
 94745924701096160760888424*I*a*e^(921/2*I*c) - 1945776444472852135264155548
 5388469197530472258189211796433938437083420010091733396*I*a*e^(919/2*I*c) -
 94752549937175934400499756270975710616491132843843008041484744522257068322
 302549540*I*a*e^(917/2*I*c) - 453467570647678726762830796324670748943540211
 918613115981382798996143666294810914890*I*a*e^(915/2*I*c) - 213325987645378
 9226426738739204266559366922799155021793045435139729509358790624760320*I*a*
 e^(913/2*I*c) - 98665617992106740504226261681187999762187420387373852558187
 01142786087196734211598180*I*a*e^(911/2*I*c) - 4487375453728008640491091087
 8863721864966849160970709878039315079653743556851113962410*I*a*e^(909/2*I*c
) - 20072483864097435618425497602435711351463146370708337907155866674728785
 3457166817321610*I*a*e^(907/2*I*c) - 88321698672020915183702241433267232811
 1515000315030045611168408538524032471081357579180*I*a*e^(905/2*I*c) - 38235
 310470745017739018826170797254746461372056086358408511845004333820554473342
 25716180*I*a*e^(903/2*I*c) - 1628786013187342629766595419587200734038944869
 6155366782102817863363964748733518124937370*I*a*e^(901/2*I*c) - 68286519373
 700606662969518596072942990402829423660330718366006960865727616800960284438
 480*I*a*e^(899/2*I*c) - 281802543079970773764222747501103240510862783237481
 02569673571115713233601841503443789040*I*a*e^(897/2*I*c) - 114487866597334
 536288031280499292234026314441356757106089623565009992240715993867118363020
 0*I*a*e^(895/2*I*c) - 45797580440835338476480708866180632369011921620042339
 36078348854854553658379606034075133880*I*a*e^(893/2*I*c) - 1804082255618213
 68966387131008923443583280530071736796287656571474509593980263412544111572

$0 \cdot I \cdot a \cdot e^{(891/2 \cdot I \cdot c)}$ - 69994103899854449688044814471201743418413806666583628
183006633122034125384874442925881756120 $\cdot I \cdot a \cdot e^{(889/2 \cdot I \cdot c)}$ - 267495779341793
686858892087186701348017864136095797204004377097030869586488588546928368741
180 $\cdot I \cdot a \cdot e^{(887/2 \cdot I \cdot c)}$ - 100711783765737336946872972571982059700857246546975
3257215153626598733911453021524113695566720 $\cdot I \cdot a \cdot e^{(885/2 \cdot I \cdot c)}$ - 37360062305
102756756100615441269715573934211219494108870036549757264319856610614316127
14966210 $\cdot I \cdot a \cdot e^{(883/2 \cdot I \cdot c)}$ - 1365694353786631656234502288503877078081249579
7097516350805731913565888244243229131324182944640 $\cdot I \cdot a \cdot e^{(881/2 \cdot I \cdot c)}$ - 49200
709831170847979783317949393803747072027694891369055993880672007049593913929
619833129271500 $\cdot I \cdot a \cdot e^{(879/2 \cdot I \cdot c)}$ - 174708148576954508133108566432138482752
871314388735390819701558304621069749665356028799825469570 $\cdot I \cdot a \cdot e^{(877/2 \cdot I \cdot c)}$
- 611546710350755227094210555322738233573992238438880922442071495399840623
270733748703638918730970 $\cdot I \cdot a \cdot e^{(875/2 \cdot I \cdot c)}$ - 211042686701657901173815999545
2106449607094170693975505625410812127221737931023521427446997422880 $\cdot I \cdot a \cdot e^{(}$
 $873/2 \cdot I \cdot c)$ - 71809767087815641520119330793240663969412779797207321263052443
04659930709704206531169354332309620 $\cdot I \cdot a \cdot e^{(871/2 \cdot I \cdot c)}$ - 2409441928790210422
014636431525698756343032323446865385718272253087317365650762075737077623472
5250 $\cdot I \cdot a \cdot e^{(869/2 \cdot I \cdot c)}$ - 79728801300455544122096358134599530539313996522871
876150671326770147866624403122038941020982112670 $\cdot I \cdot a \cdot e^{(867/2 \cdot I \cdot c)}$ - 260210
521546648324710601552017336263001665484044998430835364683575217699721326919
568028661098556620 $\cdot I \cdot a \cdot e^{(865/2 \cdot I \cdot c)}$ - 837701525654515943160919065652989402
268629196598907039545164356949845716819867090707769561650988160 $\cdot I \cdot a \cdot e^{(863/}$
 $2 \cdot I \cdot c)$ - 266042842931571103774675083618235061716695085166608818552512359080
8329315051618495256897037944395590 $\cdot I \cdot a \cdot e^{(861/2 \cdot I \cdot c)}$ - 83359229219753395548
749857408099513531307771208171816871848757958934640668766776293826532643936
01630 $\cdot I \cdot a \cdot e^{(859/2 \cdot I \cdot c)}$ - 2577132301570592546450213827703561797567243049210
5429256456221829154887242352647174124493051808094580 $\cdot I \cdot a \cdot e^{(857/2 \cdot I \cdot c)}$ - 78
621408619495274059980620283854146208984244064535407477261926286982916818434
658466433148202509344800 $\cdot I \cdot a \cdot e^{(855/2 \cdot I \cdot c)}$ - 236703836252587779323701434388
787465249427045104853012907439025264086299638762864112564049076655732830 $\cdot I \cdot$
 $a \cdot e^{(853/2 \cdot I \cdot c)}$ - 703345279898755583562127912601131791334107956711719030548
547943555221102827637472553536460508801363920 $\cdot I \cdot a \cdot e^{(851/2 \cdot I \cdot c)}$ - 206285295
492081618602391594125111752795850045368393587517339248395259900929513998798
5628233229731478980 $\cdot I \cdot a \cdot e^{(849/2 \cdot I \cdot c)}$ - 59722941276362590674151260506011593
79166632061409821915273465659298542690836327817871412320220580305480 $\cdot I \cdot a \cdot e^{(}$
 $847/2 \cdot I \cdot c)$ - 1706960063470899691725822494164793138249211553225201264157629
2881810859853135265796592414892124832389000 $\cdot I \cdot a \cdot e^{(845/2 \cdot I \cdot c)}$ - 48167052524
691460784860596000543126325082971121698814633620291249190839220221789221157
407204913360887960 $\cdot I \cdot a \cdot e^{(843/2 \cdot I \cdot c)}$ - 134201038644703507385888818415634883
722891759405213144807057882964953199405626071972925310751903480110440 $\cdot I \cdot a \cdot e^{(}$
 $841/2 \cdot I \cdot c)$ - 369210997610108799944611140506231611589949279646373920938134
598886382202697574571710903344492696653736880 $\cdot I \cdot a \cdot e^{(839/2 \cdot I \cdot c)}$ - 100308906
580872377987604746044435692379102789056152055267941705646982891543117675896
7907919151985703838720 $\cdot I \cdot a \cdot e^{(837/2 \cdot I \cdot c)}$ - 26914293841250094130608671518641
17700072985313774039040518324378805237935344715836365700345548762139320230*

$I*a*e^{(835/2*I*c)} - 7132421149055578168006796797797943674026134220971516324$
 $623846545930681177797566466263020966077638684308780*I*a*e^{(833/2*I*c)} - 186$
 $695013151690567802855585778163774515738518543184957254613266891785511279516$
 $37356019031393026858408569620*I*a*e^{(831/2*I*c)} - 4827261875576824255901844$
 $040633795968313307135325975392553055937185084404452329574594324918443303839$
 $1906550*I*a*e^{(829/2*I*c)} - 12330238631943173760301230788568046145553392342$
 $6516862931630369896536810524338424946586678555712414838330710*I*a*e^{(827/2*$
 $I*c)} - 31115307812434068987682163412997317426688473577707506916311083159021$
 $3226340122660492406303919452492168793820*I*a*e^{(825/2*I*c)} - 77577842500154$
 $40409724275277450083481680468402096217803696331922555426668437352018204707$
 $3542988726249962080*I*a*e^{(823/2*I*c)} - 19111321825020891877081664786859019$
 $35201352435559875705150907754538398903339566020664524492674910743672362870*$
 $I*a*e^{(821/2*I*c)} - 4652229613980190281092438412966980324563941911273487870$
 $254673754078512908450865883806375186718602486878389420*I*a*e^{(819/2*I*c)} -$
 $111911937372368801918281190224557235328802284114807569582104923956167042731$
 $80646550698293673440285122201506540*I*a*e^{(817/2*I*c)} - 2660501738325959941$
 $125486653387063070184684544114538539883726653793136940859531578120646852464$
 $9149866075877240*I*a*e^{(815/2*I*c)} - 62509941784743120997663879158557982155$
 $081149380070540181443212715349440122956954676521201490211883966367828020*I*$
 $a*e^{(813/2*I*c)} - 145164084395384796858807452169156193314757794283400173793$
 $760462753468308342626556407070906048758273598047775980*I*a*e^{(811/2*I*c)} -$
 $333211138005255761240200864910933592348294347179262547321519853589485903531$
 $852562513295558687155952554066590460*I*a*e^{(809/2*I*c)} - 756058495767730773$
 $659409931369268584451348889364804029349493357426339750910243652561427608338$
 $124503345850013560*I*a*e^{(807/2*I*c)} - 169586694384839863762082875542402504$
 $079804383725498790646540871293177075734811292316429856498301893630133593521$
 $2*I*a*e^{(805/2*I*c)} - 37605659493035666864233856362234082094465498792886874$
 $86165586816000220852069580899398446426749557253502940018308*I*a*e^{(803/2*I*$
 $c)} - 8244492425854132814040653088604670129631127112463111698869090178226433$
 $285329901659997552076882534722621154663912*I*a*e^{(801/2*I*c)} - 178709569969$
 $992802875872825562267623650671756493247528499380693654045515989819724316982$
 $24836196155095718947401358*I*a*e^{(799/2*I*c)} - 3830261707190416013199912010$
 $998899180421021407244116125726026137468537377000578615018947751651439197992$
 $4577468172*I*a*e^{(797/2*I*c)} - 81176276038870735325330852188233888929463718$
 $282951750613849345989835502222967166292107718062084091143361182517160*I*a*e$
 $^{(795/2*I*c)} - 170126842297170420289826544603513884546736532165273225663609$
 $085918787724504183838886776200509606191937033678162224*I*a*e^{(793/2*I*c)} -$
 $352600027682002951366552195831292961904393452031839665434742581379773740338$
 $046042737991951645221266975380498373846*I*a*e^{(791/2*I*c)} - 722736724787787$
 $124132763159445766235279414024537115241684494028855350914895151999785034206$
 $184040168317292786589284*I*a*e^{(789/2*I*c)} - 146517146645611822858332172737$
 $413341944831073458968335363906687277370038676579338217934007527765650835485$
 $9972911131*I*a*e^{(787/2*I*c)} - 29378484206865194916839528936523698208356730$
 $30208591280309462380721283484490744169544291532320815808409450100179444*I*a$
 $*e^{(785/2*I*c)} - 5826728456998393838746283558785767736418444764516351018178$

688292977826358717393165474435299003815460564340303035510*I*a*e^(783/2*I*c)
- 114313182922035384495360621046134469254143793530201476452184881941658506
51837526675988253582086095570243158171594435*I*a*e^(781/2*I*c) - 2218531084
901737614494058133508448821593174865751884625102376972508413071762188083687
8455248575979783141226165618755*I*a*e^(779/2*I*c) - 42594522671068860883864
561229655566037459988103599672363573691551514146031450943546205718366811710
531905255429039490*I*a*e^(777/2*I*c) - 809062879793839605960149642537205140
747506339361059008351912777079735013049055460658979356673547196839180923567
62982*I*a*e^(775/2*I*c) - 1520450247419420210318434501382836778647301515591
78201233738634196057296634727643807511620634700215859987858540208163*I*a*e^(
773/2*I*c) - 2827122382504900816891349747732958623701013471182937266388091
14306775532326665539002921023135199371428068547769357902*I*a*e^(771/2*I*c)
- 5201402052117401461463288860339192842776562670130376159756628681020636449
41116116140374194763868070844567225649530644*I*a*e^(769/2*I*c) - 9469358582
484161376188178033743727416111418592823351320281378875528323872392438375187
91104798117018522060501489612616*I*a*e^(767/2*I*c) - 1705948357752683688609
383556611544971698931045395116831763212880566314009847795209812264993457998
814619114688074679466*I*a*e^(765/2*I*c) - 304142566853240586400498135898699
141795898114585235603666143538212285529007098174186120342967503569732349480
7153105698*I*a*e^(763/2*I*c) - 53663051021007398309871063261074502884344499
43234368381217650935821942651819315271205766739758555936419394930826692752*
I*a*e^(761/2*I*c) - 9370897002428257267123315283721797254447538815988223371
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2*I*c) - 161962868526188369395506297658517359366716193400331886468163454591
25057185947699008626649435369257313552045494568159814*I*a*e^(757/2*I*c) - 2
770761376042432109280977348834171005609101629943082281185036820550538666689
2248077868916795733514784560970590740995043*I*a*e^(755/2*I*c) - 46919468887
999820120797066437767350896395887972340669168418033185399584359856915397601
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074582401143110998685640861404828533880308670564283751695023513612591615262
96946312988137021594754*I*a*e^(751/2*I*c) - 1305136545651806596808576107518
103309191826804511798147243018089725819932514585681580430205100424554572257
55690757090571*I*a*e^(749/2*I*c) - 2144110099842286461896635924207376189992
75502521465244636485838544167538555490569836303716960975605168044803647480
61847*I*a*e^(747/2*I*c) - 3487329620520598271405637311130324137290678229939
02351106193053703931092696239294111150008109768220326322348527847477500*I*a
*e^(745/2*I*c) - 5615844530352673936409554025270746390789929068539086222880
97857924196885201225801958981862915519313482030785460273884334*I*a*e^(743/2
*I*c) - 8954365481450827453236125711936234058690741641172343586760315911158
85958848259250172134566787604094987664728603369653891*I*a*e^(741/2*I*c) - 1
413754020308820178961328889057375784750324941205731220632799352997567939467
526832841222047335354848244171656614625516085*I*a*e^(739/2*I*c) - 221031391
624875337635546246102968719766070020504848277575893242534278180960115516603
7756063197334234247463515636702691330*I*a*e^(737/2*I*c) - 34221343023174192
329904603489067710817250505419905048934146238032397191152808081613359873551

44864275649603454218872971580*I*a*e^(735/2*I*c) - 5247170711937428754982376
895909634165588413820977777033506021664235403593006423444490084357597297711
396346308669760798689*I*a*e^(733/2*I*c) - 796820700441231679735742088944894
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245852425113651072146233279869165517073131931513924164490275061920030161185
859950*I*a*e^(729/2*I*c) - 178543014828327237381513629934276680941045982591
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I*a*e^(727/2*I*c) - 2634722491074523483769625594669165696247639632180233653
1283765807884019800645091982120346224249744939016139022716374364629*I*a*e^(
725/2*I*c) - 38514520648760918228036755788926097489724496546941732018982203
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*c) - 557743650882071383061823428136907285675901598890415790079469782281438
89730779948322963998644219045352889801185546869084174*I*a*e^(721/2*I*c) - 8
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0382404149332811237697500795135371424363931912*I*a*e^(719/2*I*c) - 11373880
699718367608654346517232895570860701851878364855552562510659029714278128135
4203188623909797488026117582818256419866*I*a*e^(717/2*I*c) - 16018390310024
491399589891810028467230424870758017612489626238141446543893386102431011911
1597234769579332371810044707792786*I*a*e^(715/2*I*c) - 22353326578596883514
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5872704723487164743059560288*I*a*e^(713/2*I*c) - 30910198454139291491998334
324383454757099934023351069019471767444707960282631156876465363755024710903
8105458177964729351012*I*a*e^(711/2*I*c) - 42356590469530397618028105246119
646948625093649536670112135976865303107013996765644320517944636520277650280
9602693273968654*I*a*e^(709/2*I*c) - 57520537836464419636834559585294277687
340135091241285346717893214249887017844048901649968684842192398011790471293
4988541229*I*a*e^(707/2*I*c) - 77415994423928762785021381764599701472101121
370429364427262716503131266128077413187472071417102879206039438545388113083
5866*I*a*e^(705/2*I*c) - 10326833781408798598715685440931614866301473319621
10884335414122975282437928325289215995634079578727106353902540296842372558*
I*a*e^(703/2*I*c) - 1365384960843458647834068386583040556831516179914170280
355490859615430960938934067120381777848152158124027973811688241828477*I*a*e
^(701/2*I*c) - 178943906435616044494140836108479112889982263247843589635556
1687856783294317175132936047899044986704007505154394601123847453*I*a*e^(699
/2*I*c) - 23247415130512690281765260929546689247279241363925098295022509200
02782581704831555544043778894432088577660940236464343181690*I*a*e^(697/2*I*
c) - 2993987947288425518808928248754589667712106423890365853065234018548819
562445899531390409664170571394926881128145770065899220*I*a*e^(695/2*I*c) -
382264697413215166970786157267327925910297091788357983560627321885797509216
2918434170953228420879543659937491953198146195925*I*a*e^(693/2*I*c) - 48387
996361111263172080964351011092440835595380669484893874298832334925894994916
05457749827375642095290439183595761588081080*I*a*e^(691/2*I*c) - 6072817074
204712468274441182835677570948620936784622005264549151564266119016397112228
983141779145954492755416086534786022346*I*a*e^(689/2*I*c) - 755685058467942

384069281977003129681151363129234954433023067473886036825949443243256784436
5994608959978423828261012106341304*I*a*e^(687/2*I*c) - 93241128788016496313
851751411144345263829853283002222845386219384884437515564772258164599381226
98725278253649259377876875956*I*a*e^(685/2*I*c) - 1140793644236414863961925
322676715375573694014098823792666633979384227510052733374298581396287055551
9700042503130062102254624*I*a*e^(683/2*I*c) - 13840604493814049780973828546
511041191138534310013963427847312833326059845471345783687136583985872044817
693603530350111267506*I*a*e^(681/2*I*c) - 166519619984923331068036514841112
023323662475383036270497690976414641089291627408982243821376877252447710196
87054923945046640*I*a*e^(679/2*I*c) - 1986782813850402792435803837007843024
336077185901900775461709270589489548232039076520583465372207979133331738190
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116964232*I*a*e^(675/2*I*c) - 275856288583184014177612634272945850537939719
850315871027518382355931980531330615442637387956682970388853596044771370926
18448*I*a*e^(673/2*I*c) - 3210285090892660409424213973529495224528101332037
331055002089573485700408488036159340689037826111311863219665215677924612383
2*I*a*e^(671/2*I*c) - 37051398263203087048517280736377192693587419362465698
554580525240769695507938515050581196946526516020509145484606049843462088*I*
a*e^(669/2*I*c) - 424096397262841225387048322393177248300565718290597660819
71708701997048576165555938971755201669041455331867813965051654307480*I*a*e^
(667/2*I*c) - 4814133828670270443494318370428184176235314002997972864172743
4864516908691928286010079607280805784323389150637014773965009680*I*a*e^(665
/2*I*c) - 54194499762890460723929105582897570216085973537596633216486796551
038997346993003645175821744461702739407646907627975598738840*I*a*e^(663/2*I
*c) - 605006596623126218031602601746579223502211520047673175454687526632304
55453836071633783294815424678317791973306003900350353160*I*a*e^(661/2*I*c)
- 6697469913077846538914940090465987167430309649621766851147961378169951874
7276124181121029837065794584289560577716116602457420*I*a*e^(659/2*I*c) - 73
515265460978022291049555461658974596744920966894424354716781203950620650045
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11593521131774941396043731554410091241720760*I*a*e^(655/2*I*c) - 8631656078
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460154314547976260787142987352242958060011321681639646026993007612606976698
995821213357225605569003753064006540*I*a*e^(651/2*I*c) - 978271710517758356
614851108675746302673246010722659154094209718193563262939133182407431635643
41172410866596207519180793797160*I*a*e^(649/2*I*c) - 1027255616438762116321
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28774487161016238862329900*I*a*e^(645/2*I*c) - 1100488348410264759599026986
441106407687167701855742432122574029623107476371567855418860646724401230402
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 78809943007656485440*I*a*e^(641/2*I*c) - 1131360096196882560673685813226647
 177007066357388125297382388679396595908022821692269547999319755438480419931
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 897237417731391357410180679848860373091387562332435928223584328011306268911
 13712078476960*I*a*e^(637/2*I*c) - 1110685029295280185235551866034370726474
 823020737174091235959637950490979773787494728596101802753501513454782547459
 74421907840*I*a*e^(635/2*I*c) - 1079098652035309276305746801385484461332725
 509647484124799898717103380683274715519444786317210367402160422610835715748
 94545520*I*a*e^(633/2*I*c) - 1032875126216357270019162252069851917717687059
 593101807010810800640163345906195862462379570150471128038089685816099019441
 00680*I*a*e^(631/2*I*c) - 9720518831669389454313543525560007384022285163822
 775012727908687942526348633328027036203571559250302587129119854194553945000
 0*I*a*e^(629/2*I*c) - 89699640886356096212186761465087430703849390130218154
 088737913649465978223346239090680080257616229441671165845538363436497340*I*
 a*e^(627/2*I*c) - 808409142977369308955701027437455136510013512703125447649
 38234613787113269665817064783805611676465335857567108482423695998240*I*a*e^(625/2*I*c) - 7073159819183244162539792986740854978145073507338632004690484
 9313766247876713851234802322854802530993080492568497070607833960*I*a*e^(623/2*I*c) - 59505040665512914709000743487513075110958013616100913522472530216
 019669696050974277244082570160190999482228790680261106380540*I*a*e^(621/2*I*c) - 473225651914242698298170107814085585967937467823732826189513534547336
 15818669575867423053741342217028108862881275991225842700*I*a*e^(619/2*I*c) - 3436977127874920653742628967722738146464464337985371712016969495124985157
 8045846724379802197163939310568646413075921689487840*I*a*e^(617/2*I*c) - 20
 852033260202120041673473658620917231794877194943715445744154690362463066942
 921456479058038515193180799997053917525437497240*I*a*e^(615/2*I*c) - 698934
 188613444222883426912511264147212564818851551010323045382328554550526641808
 0876124175346096295740915799724115868021820*I*a*e^(613/2*I*c) + 69893418861
 344422288342691251126414721256481885155101032304538232855455052664180808761
 24175346096295740915799724115868021820*I*a*e^(611/2*I*c) + 2085203326020212
 004167347365862091723179487719494371544574415469036246306694292145647905803
 8515193180799997053917525437497240*I*a*e^(609/2*I*c) + 34369771278749206537
 426289677227381464644643379853717120169694951249851578045846724379802197163
 939310568646413075921689487840*I*a*e^(607/2*I*c) + 473225651914242698298170
 107814085585967937467823732826189513534547336158186695758674230537413422170
 28108862881275991225842700*I*a*e^(605/2*I*c) + 5950504066551291470900074348
 751307511095801361610091352247253021601966969605097427724408257016019099948
 2228790680261106380540*I*a*e^(603/2*I*c) + 70731598191832441625397929867408
 549781450735073386320046904849313766247876713851234802322854802530993080492
 568497070607833960*I*a*e^(601/2*I*c) + 808409142977369308955701027437455136
 510013512703125447649382346137871132696658170647838056116764653358575671084
 82423695998240*I*a*e^(599/2*I*c) + 8969964088635609621218676146508743070384
 939013021815408873791364946597822334623909068008025761622944167116584553836
 3436497340*I*a*e^(597/2*I*c) + 97205188316693894543135435255600073840222851

638227750127279086879425263486333280270362035715592503025871291198541945539
450000*I*a*e^(595/2*I*c) + 103287512621635727001916225206985191771768705959
310180701081080064016334590619586246237957015047112803808968581609901944100
680*I*a*e^(593/2*I*c) + 107909865203530927630574680138548446133272550964748
412479989871710338068327471551944478631721036740216042261083571574894545520
*I*a*e^(591/2*I*c) + 111068502929528018523555186603437072647482302073717409
123595963795049097977378749472859610180275350151345478254745974421907840*I*
a*e^(589/2*I*c) + 112791396107466699133508292800488516989723741773139135741
018067984886037309138756233243592822358432801130626891113712078476960*I*a*e
^(587/2*I*c) + 113136009619688256067368581322664717700706635738812529738238
867939659590802282169226954799931975543848041993126041018399782640*I*a*e^(5
85/2*I*c) + 112186266799989244411832410459024350373988620669263174956650455
326228948640736788271292658432697539009158378809943007656485440*I*a*e^(583/
2*I*c) + 110048834841026475959902698644110640768716770185574243212257402962
310747637156785541886064672440123040238982814411352193275760*I*a*e^(581/2*I
*c) + 106848893210891977104281865138017708109833540948304332531740471904298
947522316358022815005300747182128774487161016238862329900*I*a*e^(579/2*I*c)
+ 102725561643876211632153403112759536096513121011178784269134677307778986
787225697189206232077844605004406406497176048297949080*I*a*e^(577/2*I*c) +
978271710517758356614851108675746302673246010722659154094209718193563262939
13318240743163564341172410866596207519180793797160*I*a*e^(575/2*I*c) + 9230
655893401046015431454797626078714298735224295806001132168163964602699300761
2606976698995821213357225605569003753064006540*I*a*e^(573/2*I*c) + 86316560
788167512744457253831710932006051067075129743075193427050858246340444879284
106183668280070526606382512365973492282060*I*a*e^(571/2*I*c) + 800058513579
856460251215011652477765886785672686852468639929071650100822406227079115935
21131774941396043731554410091241720760*I*a*e^(569/2*I*c) + 7351526546097802
229104955546165897459674492096689442435471678120395062065004501365001237764
6825486222824081997699903889572320*I*a*e^(567/2*I*c) + 66974699130778465389
149400904659871674303096496217668511479613781699518747276124181121029837065
794584289560577716116602457420*I*a*e^(565/2*I*c) + 605006596623126218031602
601746579223502211520047673175454687526632304554538360716337832948154246783
17791973306003900350353160*I*a*e^(563/2*I*c) + 5419449976289046072392910558
289757021608597353759663321648679655103899734699300364517582174446170273940
7646907627975598738840*I*a*e^(561/2*I*c) + 48141338286702704434943183704281
841762353140029979728641727434864516908691928286010079607280805784323389150
637014773965009680*I*a*e^(559/2*I*c) + 424096397262841225387048322393177248
300565718290597660819717087019970485761655559389717552016690414553318678139
65051654307480*I*a*e^(557/2*I*c) + 3705139826320308704851728073637719269358
741936246569855458052524076969550793851505058119694652651602050914548460604
9843462088*I*a*e^(555/2*I*c) + 32102850908926604094242139735294952245281013
320373310550020895734857004084880361593406890378261113118632196652156779246
123832*I*a*e^(553/2*I*c) + 275856288583184014177612634272945850537939719850
315871027518382355931980531330615442637387956682970388853596044771370926184
48*I*a*e^(551/2*I*c) + 2350824696478173514163236563844801715112116961442219

5680868391258194610581920728030535410178638205451448869498618378116964232*I
 *a*e^(549/2*I*c) + 19867828138504027924358038370078430243360771859019007754
 617092705894895482320390765205834653722079791333317381901945409007768*I*a*e
^(547/2*I*c) + 166519619984923331068036514841112023323662475383036270497690
 97641464108929162740898224382137687725244771019687054923945046640*I*a*e⁽⁵⁴
^{5/2*I*c)} + 1384060449381404978097382854651104119113853431001396342784731283
 3326059845471345783687136583985872044817693603530350111267506*I*a*e^{(543/2*}
^{I*c)} + 11407936442364148639619253226767153755736940140988237926666339793842
 275100527333742985813962870555519700042503130062102254624*I*a*e^(541/2*I*c)
 + 932411287880164963138517514111443452638298532830022228453862193848844375
 1556477225816459938122698725278253649259377876875956*I*a*e^(539/2*I*c) + 75
 568505846794238406928197700312968115136312923495443302306747388603682594944
 32432567844365994608959978423828261012106341304*I*a*e^(537/2*I*c) + 6072817
 074204712468274441182835677570948620936784622005264549151564266119016397112
 228983141779145954492755416086534786022346*I*a*e^(535/2*I*c) + 483879963611
 112631720809643510110924408355953806694848938742988323349258949949160545774
 9827375642095290439183595761588081080*I*a*e^(533/2*I*c) + 38226469741321516
 697078615726732792591029709178835798356062732188579750921629184341709532284
 20879543659937491953198146195925*I*a*e^(531/2*I*c) + 2993987947288425518808
 928248754589667712106423890365853065234018548819562445899531390409664170571
 394926881128145770065899220*I*a*e^(529/2*I*c) + 232474151305126902817652609
 295466892472792413639250982950225092000278258170483155554404377889443208857
 7660940236464343181690*I*a*e^(527/2*I*c) + 17894390643561604449414083610847
 911288998226324784358963555616878567832943171751329360478990449867040075051
 54394601123847453*I*a*e^(525/2*I*c) + 1365384960843458647834068386583040556
 831516179914170280355490859615430960938934067120381777848152158124027973811
 688241828477*I*a*e^(523/2*I*c) + 103268337814087985987156854409316148663014
 733196211088433541412297528243792832528921599563407957872710635390254029684
 2372558*I*a*e^(521/2*I*c) + 77415994423928762785021381764599701472101121370
 429364427262716503131266128077413187472071417102879206039438545388113083586
 6*I*a*e^(519/2*I*c) + 57520537836464419636834559585294277687340135091241285
 3467178932142498870178440489016499686848421923980117904712934988541229*I*a*
 e^(517/2*I*c) + 42356590469530397618028105246119646948625093649536670112135
 9768653031070139967656443205179446365202776502809602693273968654*I*a*e⁽⁵¹⁵
^{/2*I*c)} + 30910198454139291491998334324383454757099934023351069019471767444
 7079602826311568764653637550247109038105458177964729351012*I*a*e^{(513/2*I*c}
⁾ + 22353326578596883514387560025326324433543045327642356089211895885600284
 0872749060322902817137435872704723487164743059560288*I*a*e^(511/2*I*c) + 16
 018390310024491399589891810028467230424870758017612489626238141446543893386
 1024310119111597234769579332371810044707792786*I*a*e^(509/2*I*c) + 11373880
 699718367608654346517232895570860701851878364855552562510659029714278128135
 4203188623909797488026117582818256419866*I*a*e^(507/2*I*c) + 80018115577836
 427010766964769469931186624920119062344686445108628080498436960382404149332
 811237697500795135371424363931912*I*a*e^(505/2*I*c) + 557743650882071383061
 823428136907285675901598890415790079469782281438897307799483229639986442190

45352889801185546869084174*I*a*e^(503/2*I*c) + 3851452064876091822803675578
892609748972449654694173201898220379776991748061989780574736627613727104873
9730353633877720246*I*a*e^(501/2*I*c) + 26347224910745234837696255946691656
962476396321802336531283765807884019800645091982120346224249744939016139022
716374364629*I*a*e^(499/2*I*c) + 178543014828327237381513629934276680941045
982591828198036711915638221886077206236384288733091635402307821693546310681
86188*I*a*e^(497/2*I*c) + 1198465839451516531963482778759544966723724585242
5113651072146233279869165517073131931513924164490275061920030161185859950*I
*a*e^(495/2*I*c) + 79682070044123167973574208894489443048758862652357424583
04812499937629746480931935519206037160539581522171987555750097661*I*a*e^(49
3/2*I*c) + 5247170711937428754982376895909634165588413820977777033506021664
235403593006423444490084357597297711396346308669760798689*I*a*e^(491/2*I*c)
+ 342213430231741923299046034890677108172505054199050489341462380323971911
5280808161335987355144864275649603454218872971580*I*a*e^(489/2*I*c) + 22103
139162487533763554624610296871976607002050484827757589324253427818096011551
66037756063197334234247463515636702691330*I*a*e^(487/2*I*c) + 1413754020308
820178961328889057375784750324941205731220632799352997567939467526832841222
047335354848244171656614625516085*I*a*e^(485/2*I*c) + 895436548145082745323
612571193623405869074164117234358676031591115885958848259250172134566787604
094987664728603369653891*I*a*e^(483/2*I*c) + 561584453035267393640955402527
074639078992906853908622288097857924196885201225801958981862915519313482030
785460273884334*I*a*e^(481/2*I*c) + 348732962052059827140563731113032413729
067822993902351106193053703931092696239294111150008109768220326322348527847
477500*I*a*e^(479/2*I*c) + 214411009984228646189663592420737618999275502521
465244636485838544167538555549056983630371696097560516804480364748061847*I
a*e^(477/2*I*c) + 130513654565180659680857610751810330919182680451179814724
301808972581993251458568158043020510042455457225755690757090571*I*a*e^(475/
2*I*c) + 786499297907510768672074582401143110998685640861404828533880308670
56428375169502351361259161526296946312988137021594754*I*a*e^(473/2*I*c) + 4
691946888799982012079706643776735089639588797234066916841803318539958435985
6915397601507103516462593793662665388320876*I*a*e^(471/2*I*c) + 27707613760
424321092809773488341710056091016299430822811850368205505386666892248077868
916795733514784560970590740995043*I*a*e^(469/2*I*c) + 161962868526188369395
506297658517359366716193400331886468163454591250571859476990086266494353692
57313552045494568159814*I*a*e^(467/2*I*c) + 9370897002428257267123315283721
797254447538815988223371846399458761076724136173352675976157625073367090808
727028817394*I*a*e^(465/2*I*c) + 536630510210073983098710632610745028843444
994323436838121765093582194265181931527120576673975855593641939493082669275
2*I*a*e^(463/2*I*c) + 30414256685324058640049813589869914179589811458523560
36661435382122855290070981741861203429675035697323494807153105698*I*a*e^(46
1/2*I*c) + 1705948357752683688609383556611544971698931045395116831763212880
566314009847795209812264993457998814619114688074679466*I*a*e^(459/2*I*c) +
946935858248416137618817803374372741611141859282335132028137887552832387239
243837518791104798117018522060501489612616*I*a*e^(457/2*I*c) + 520140205211
740146146328886033919284277656267013037615975662868102063644941116116140374

194763868070844567225649530644*I*a*e^(455/2*I*c) + 282712238250490081689134
 974773295862370101347118293726638809114306775532326665539002921023135199371
 428068547769357902*I*a*e^(453/2*I*c) + 152045024741942021031843450138283677
 864730151559178201233738634196057296634727643807511620634700215859987858540
 208163*I*a*e^(451/2*I*c) + 809062879793839605960149642537205140747506339361
 05900835191277707973501304905546065897935667354719683918092356762982*I*a*e^
 (449/2*I*c) + 4259452267106886088386456122965556603745998810359967236357369
 1551514146031450943546205718366811710531905255429039490*I*a*e^(447/2*I*c) +
 22185310849017376144940581335084488215931748657518846251023769725084130717
 621880836878455248575979783141226165618755*I*a*e^(445/2*I*c) + 114313182922
 035384495360621046134469254143793530201476452184881941658506518375266759882
 53582086095570243158171594435*I*a*e^(443/2*I*c) + 5826728456998393838746283
 558785767736418444764516351018178688292977826358717393165474435299003815460
 564340303035510*I*a*e^(441/2*I*c) + 293784842068651949168395289365236982083
 567303020859128030946238072128348449074416954429153232081580840945010017944
 4*I*a*e^(439/2*I*c) + 14651714664561182285833217273741334194483107345896833
 53639066872773700386765793382179340075277656508354859972911131*I*a*e^(437/2
 *I*c) + 7227367247877871241327631594457662352794140245371152416844940288553
 50914895151999785034206184040168317292786589284*I*a*e^(435/2*I*c) + 3526000
 276820029513665521958312929619043934520318396654347425813797737403380460427
 37991951645221266975380498373846*I*a*e^(433/2*I*c) + 1701268422971704202898
 265446035138845467365321652732256636090859187877245041838388867762005096061
 91937033678162224*I*a*e^(431/2*I*c) + 8117627603887073532533085218823388892
 94637182829517506138493459898355022296716629210771806208409114336118251716
 0*I*a*e^(429/2*I*c) + 38302617071904160131999120109988991804210214072441161
 257260261374685373770005786150189477516514391979924577468172*I*a*e^(427/2*I
 *c) + 178709569969992802875872825562267623650671756493247528499380693654045
 51598981972431698224836196155095718947401358*I*a*e^(425/2*I*c) + 8244492425
 854132814040653088604670129631127112463111698869090178226433285329901659997
 552076882534722621154663912*I*a*e^(423/2*I*c) + 376056594930356668642338563
 622340820944654987928868748616558681600022085206958089939844642674955725350
 2940018308*I*a*e^(421/2*I*c) + 16958669438483986376208287554240250407980438
 37254987906465408712931770757348112923164298564983018936301335935212*I*a*e^
 (419/2*I*c) + 7560584957677307736594099313692685844513488893648040293494933
 57426339750910243652561427608338124503345850013560*I*a*e^(417/2*I*c) + 3332
 111380052557612402008649109335923482943471792625473215198535894859035318525
 62513295558687155952554066590460*I*a*e^(415/2*I*c) + 1451640843953847968588
 074521691561933147577942834001737937604627534683083426265564070709060487582
 73598047775980*I*a*e^(413/2*I*c) + 6250994178474312099766387915855798215508
 1149380070540181443212715349440122956954676521201490211883966367828020*I*a*
 e^(411/2*I*c) + 26605017383259599411254866533870630701846845441145385398837
 266537931369408595315781206468524649149866075877240*I*a*e^(409/2*I*c) + 111
 911937372368801918281190224557235328802284114807569582104923956167042731806
 46550698293673440285122201506540*I*a*e^(407/2*I*c) + 4652229613980190281092
 438412966980324563941911273487870254673754078512908450865883806375186718602

486878389420*I*a*e^(405/2*I*c) + 191113218250208918770816647868590193520135
 2435559875705150907754538398903339566020664524492674910743672362870*I*a*e^(
 403/2*I*c) + 77577842500154404097242752774500834816804684020962178036963319
 2255554266684373520182047073542988726249962080*I*a*e^(401/2*I*c) + 31115307
 812434068987682163412997317426688473577707506916311083159021322634012266049
 2406303919452492168793820*I*a*e^(399/2*I*c) + 12330238631943173760301230788
 568046145553392342651686293163036989653681052433842494658667855571241483833
 0710*I*a*e^(397/2*I*c) + 48272618755768242559018440406337959683133071353259
 753925530559371850844044523295745943249184433038391906550*I*a*e^(395/2*I*c)
 + 186695013151690567802855585778163774515738518543184957254613266891785511
 27951637356019031393026858408569620*I*a*e^(393/2*I*c) + 7132421149055578168
 006796797797943674026134220971516324623846545930681177797566466263020966077
 638684308780*I*a*e^(391/2*I*c) + 269142938412500941306086715186411770007298
 5313774039040518324378805237935344715836365700345548762139320230*I*a*e^(389
 /2*I*c) + 10030890658087237798760474604443569237910278905615205526794170564
 69828915431176758967907919151985703838720*I*a*e^(387/2*I*c) + 3692109976101
 087999446111405062316115899492796463739209381345988863822026975745717109033
 44492696653736880*I*a*e^(385/2*I*c) + 1342010386447035073858888184156348837
 22891759405213144807057882964953199405626071972925310751903480110440*I*a*e^
 (383/2*I*c) + 4816705252469146078486059600054312632508297112169881463362029
 1249190839220221789221157407204913360887960*I*a*e^(381/2*I*c) + 17069600634
 708996917258224941647931382492115532252012641576292881810859853135265796592
 414892124832389000*I*a*e^(379/2*I*c) + 597229412763625906741512605060115937
 9166632061409821915273465659298542690836327817871412320220580305480*I*a*e^(
 377/2*I*c) + 20628529549208161860239159412511175279585004536839358751733924
 83952599009295139987985628233229731478980*I*a*e^(375/2*I*c) + 7033452798987
 555835621279126011317913341079567117190305485479435552211028276374725535364
 60508801363920*I*a*e^(373/2*I*c) + 2367038362525877793237014343887874652494
 27045104853012907439025264086299638762864112564049076655732830*I*a*e^(371/2
 *I*c) + 7862140861949527405998062028385414620898424406453540747726192628698
 2916818434658466433148202509344800*I*a*e^(369/2*I*c) + 25771323015705925464
 502138277035617975672430492105429256456221829154887242352647174124493051808
 094580*I*a*e^(367/2*I*c) + 833592292197533955487498574080995135313077712081
 7181687184875795893464066876677629382653264393601630*I*a*e^(365/2*I*c) + 26
 604284293157110377467508361823506171669508516660881855251235908083293150516
 18495256897037944395590*I*a*e^(363/2*I*c) + 8377015256545159431609190656529
 89402268629196598907039545164356949845716819867090707769561650988160*I*a*e^
 (361/2*I*c) + 2602105215466483247106015520173362630016654840449984308353646
 83575217699721326919568028661098556620*I*a*e^(359/2*I*c) + 7972880130045554
 412209635813459953053931399652287187615067132677014786662440312203894102098
 2112670*I*a*e^(357/2*I*c) + 24094419287902104220146364315256987563430323234
 468653857182722530873173656507620757370776234725250*I*a*e^(355/2*I*c) + 718
 097670878156415201193307932406639694127797972073212630524430465993070970420
 6531169354332309620*I*a*e^(353/2*I*c) + 21104268670165790117381599954521064
 49607094170693975505625410812127221737931023521427446997422880*I*a*e^(351/2

$*I*c) + 6115467103507552270942105553227382335739922384388809224420714953998$
 $40623270733748703638918730970*I*a*e^{(349/2*I*c)} + 1747081485769545081331085$
 $66432138482752871314388735390819701558304621069749665356028799825469570*I*a$
 $*e^{(347/2*I*c)} + 4920070983117084797978331794939380374707202769489136905599$
 $3880672007049593913929619833129271500*I*a*e^{(345/2*I*c)} + 13656943537866316$
 $562345022885038770780812495797097516350805731913565888244243229131324182944$
 $640*I*a*e^{(343/2*I*c)} + 373600623051027567561006154412697155739342112194941$
 $0887003654975726431985661061431612714966210*I*a*e^{(341/2*I*c)} + 10071178376$
 $573733694687297257198205970085724654697532572151536265987339114530215241136$
 $95566720*I*a*e^{(339/2*I*c)} + 2674957793417936868588920871867013480178641360$
 $95797204004377097030869586488588546928368741180*I*a*e^{(337/2*I*c)} + 6999410$
 $389985444968804481447120174341841380666658362818300663312203412538487444292$
 $5881756120*I*a*e^{(335/2*I*c)} + 18040822556182136896638713100892344358328053$
 $007173679628765657147450959398026341254411115720*I*a*e^{(333/2*I*c)} + 457975$
 $804408353384764807088661806323690119216200423393607834885485455365837960603$
 $4075133880*I*a*e^{(331/2*I*c)} + 11448786659733453628803128049929223402631444$
 $13567571060896235650099922407159938671183630200*I*a*e^{(329/2*I*c)} + 2818025$
 $430799707737642227475011032405108627832374810256967357111157132336018415034$
 $43789040*I*a*e^{(327/2*I*c)} + 6828651937370060666296951859607294299040282942$
 $3660330718366006960865727616800960284438480*I*a*e^{(325/2*I*c)} + 16287860131$
 $873426297665954195872007340389448696155366782102817863363964748733518124937$
 $370*I*a*e^{(323/2*I*c)} + 382353104707450177390188261707972547464613720560863$
 $5840851184500433382055447334225716180*I*a*e^{(321/2*I*c)} + 88321698672020915$
 $1837022414332672328111515000315030045611168408538524032471081357579180*I*a*$
 $e^{(319/2*I*c)} + 20072483864097435618425497602435711351463146370708337907155$
 $8666747287853457166817321610*I*a*e^{(317/2*I*c)} + 44873754537280086404910910$
 $878863721864966849160970709878039315079653743556851113962410*I*a*e^{(315/2*I$
 $*c)} + 986656179921067405042262616811879997621874203873738525581870114278608$
 $7196734211598180*I*a*e^{(313/2*I*c)} + 21332598764537892264267387392042665593$
 $66922799155021793045435139729509358790624760320*I*a*e^{(311/2*I*c)} + 4534675$
 $706476787267628307963246707489435402119186131159813827989961436662948109148$
 $90*I*a*e^{(309/2*I*c)} + 9475254993717593440049975627097571061649113284384300$
 $8041484744522257068322302549540*I*a*e^{(307/2*I*c)} + 19457764444728521352641$
 $555485388469197530472258189211796433938437083420010091733396*I*a*e^{(305/2*I$
 $*c)} + 392613197522804737302143988361029394464744692867334429729474592470109$
 $6160760888424*I*a*e^{(303/2*I*c)} + 77824546206109293824112895299300796116555$
 $8304583117537979936583232925968316199836*I*a*e^{(301/2*I*c)} + 15151479395044$
 $2861336357006872512458064446653474553525802193583698620014121641124*I*a*e^{($
 $299/2*I*c)} + 28965714003257846921540858986388954008122111612712363354360981$
 $851687424724108036*I*a*e^{(297/2*I*c)} + 543632108243814202941573988555391476$
 $7552718204411177277415536254863231523181480*I*a*e^{(295/2*I*c)} + 10014177203$
 $44656915251924901009750939990149160965149540929889824388402626455012*I*a*e^{$
 $(293/2*I*c)} + 1810126781799484651898911898768444835204795688801735709176963$
 $39360628958538348*I*a*e^{(291/2*I*c)} + 3209792256384850694306647346934075366$
 $2783076784232671471143949644558156480632*I*a*e^{(289/2*I*c)} + 55822074307217$

67424910922169155711907398569721937114073480331164440862462638*I*a*e^(287/2
*I*c) + 9518731874891467462383380080222399063180718980280168640905680308036
87398612*I*a*e^(285/2*I*c) + 1591018076203724577102305546275973854856023040
37309714452742966256548901840*I*a*e^(283/2*I*c) + 2605964476129521806352155
2347914441867779264013071310596786890247411087520*I*a*e^(281/2*I*c) + 41814
71070907387612659598631606751788208788794201536282802542477878283190*I*a*e^(
279/2*I*c) + 6570856213855142413632413014951270581090650156993047456638177
14211556940*I*a*e^(277/2*I*c) + 1010897274677769565234755508362024895740040
61008016807940002791738698571*I*a*e^(275/2*I*c) + 1522084724739429108441663
6302914971818756836097657800823465735035857764*I*a*e^(273/2*I*c) + 22421580
41727235117364957739459342847819393345842116256845583655236406*I*a*e^(271/2
*I*c) + 3230219500315412278978014071715661701505192452023465403976135833556
67*I*a*e^(269/2*I*c) + 4549594753477955200809388843993310013936337270682517
2710009091580883*I*a*e^(267/2*I*c) + 62620703317793240005138066131639810098
68362783949768253661316894658*I*a*e^(265/2*I*c) + 8419575791849286156646183
41834644691840690166368399913404427216774*I*a*e^(263/2*I*c) + 1105361625801
23562634802981277497355054368691566003022338877490451*I*a*e^(261/2*I*c) + 1
4163389239388825902842073391400806825940375203939456957445830422*I*a*e^(259
/2*I*c) + 1770421608250297155611599130292037434814100286157679816873050532*
I*a*e^(257/2*I*c) + 2157852422859948688185287314305408124223145203362885284
60007864*I*a*e^(255/2*I*c) + 2563192633482051932428940660750418126091353266
9181012596611058*I*a*e^(253/2*I*c) + 29656751938357100665599165827430951583
71951788246043891048682*I*a*e^(251/2*I*c) + 3340456155724104826840755532946
75381058554260440184749831968*I*a*e^(249/2*I*c) + 3660771830545731357185060
9230094969421310836249109701091666*I*a*e^(247/2*I*c) + 39008205841109539120
72698502834368654421340467896495241550*I*a*e^(245/2*I*c) + 4038995150518388
03400194879806531247927007890538062521027*I*a*e^(243/2*I*c) + 4060944812091
3176083174637180877049405994150689208285948*I*a*e^(241/2*I*c) + 39618962242
54872041251938157236050043036325552867100290*I*a*e^(239/2*I*c) + 3747738753
81762608573480596867444622808315521135770955*I*a*e^(237/2*I*c) + 3434584676
7794763840341317174417543493362860419944375*I*a*e^(235/2*I*c) + 30468084673
28698391993011459852063034674399964693132*I*a*e^(233/2*I*c) + 2613883570246
74483234213954236474266534206934500718*I*a*e^(231/2*I*c) + 2166587733857658
7360651194480041798194029867090275*I*a*e^(229/2*I*c) + 17332700220577432558
12637849164901656684569946869*I*a*e^(227/2*I*c) + 1336830498712006857677302
44065634086417919364482*I*a*e^(225/2*I*c) + 9928713959958644143611456276187
690708075285068*I*a*e^(223/2*I*c) + 709193818893909656677523349582328791900
299617*I*a*e^(221/2*I*c) + 48651816785192390862949346064352748595789341*I*a
*e^(219/2*I*c) + 3200777320589257207465955323343789287604398*I*a*e^(217/2*I
*c) + 201623763354433037155766729846494383292508*I*a*e^(215/2*I*c) + 121396
50447759376183227584660170495869589*I*a*e^(213/2*I*c) + 6973167260773814888
01524339658757026686*I*a*e^(211/2*I*c) + 3813450805531950148020285195722772
4462*I*a*e^(209/2*I*c) + 1981013390087778171143744488041442712*I*a*e^(207/2
*I*c) + 97511021243902906034367815029181218*I*a*e^(205/2*I*c) + 45353963188
45906415166472261686554*I*a*e^(203/2*I*c) + 1987158175445290398646601104713

$76 \cdot I \cdot a \cdot e^{(201/2 \cdot I \cdot c)} + 8173401220508860669709781109364 \cdot I \cdot a \cdot e^{(199/2 \cdot I \cdot c)} +$
 $314361585019926536123171779990 \cdot I \cdot a \cdot e^{(197/2 \cdot I \cdot c)} + 112559135214363328752564$
 $92285 \cdot I \cdot a \cdot e^{(195/2 \cdot I \cdot c)} + 373282846199958103946548090 \cdot I \cdot a \cdot e^{(193/2 \cdot I \cdot c)} + 1$
 $1397949499240328225340750 \cdot I \cdot a \cdot e^{(191/2 \cdot I \cdot c)} + 318216864146377884963053 \cdot I \cdot a \cdot$
 $e^{(189/2 \cdot I \cdot c)} + 8056123142378999966797 \cdot I \cdot a \cdot e^{(187/2 \cdot I \cdot c)} + 1830937077757106$
 $28858 \cdot I \cdot a \cdot e^{(185/2 \cdot I \cdot c)} + 3689545748587122372 \cdot I \cdot a \cdot e^{(183/2 \cdot I \cdot c)} + 648915081$
 $40773093 \cdot I \cdot a \cdot e^{(181/2 \cdot I \cdot c)} + 975812152492320 \cdot I \cdot a \cdot e^{(179/2 \cdot I \cdot c)} + 1219765190$
 $6154 \cdot I \cdot a \cdot e^{(177/2 \cdot I \cdot c)} + 121672338216 \cdot I \cdot a \cdot e^{(175/2 \cdot I \cdot c)} + 908002524 \cdot I \cdot a \cdot e^{($
 $173/2 \cdot I \cdot c)} + 4506216 \cdot I \cdot a \cdot e^{(171/2 \cdot I \cdot c)} + 11154 \cdot I \cdot a \cdot e^{(169/2 \cdot I \cdot c)}) / (e^{(531 \cdot I$
 $\cdot c)} + 432 \cdot e^{(530 \cdot I \cdot c)} + 93096 \cdot e^{(529 \cdot I \cdot c)} + 13343760 \cdot e^{(528 \cdot I \cdot c)} + 14311182$
 $60 \cdot e^{(527 \cdot I \cdot c)} + 122503723056 \cdot e^{(526 \cdot I \cdot c)} + 8718181624155 \cdot e^{(525 \cdot I \cdot c)} + 530$
 $563624556832 \cdot e^{(524 \cdot I \cdot c)} + 28186192554792138 \cdot e^{(523 \cdot I \cdot c)} + 1327882849274858$
 $880 \cdot e^{(522 \cdot I \cdot c)} + 56169444526926562260 \cdot e^{(521 \cdot I \cdot c)} + 2154864144781257856128$
 $\cdot e^{(520 \cdot I \cdot c)} + 75599817092670157806639 \cdot e^{(519 \cdot I \cdot c)} + 2442455629894502983849$
 $104 \cdot e^{(518 \cdot I \cdot c)} + 73099207817335597247098038 \cdot e^{(517 \cdot I \cdot c)} + 2037031259470368$
 $160131922320 \cdot e^{(516 \cdot I \cdot c)} + 53090127264630963470039804475 \cdot e^{(515 \cdot I \cdot c)} + 1299$
 $146645993240318167826532288 \cdot e^{(514 \cdot I \cdot c)} + 29952547749265499675257842032197 \cdot$
 $e^{(513 \cdot I \cdot c)} + 652650253343206047453620559993840 \cdot e^{(512 \cdot I \cdot c)} + 1347722779952$
 $4701956579274210395326 \cdot e^{(511 \cdot I \cdot c)} + 264410375780310742518099326419685040 \cdot e$
 $^{(510 \cdot I \cdot c)} + 4939666610818025798809586352543471345 \cdot e^{(509 \cdot I \cdot c)} + 8805492759$
 $8941411145869950813388040256 \cdot e^{(508 \cdot I \cdot c)} + 15006027479373972864055778187226$
 $91539392 \cdot e^{(507 \cdot I \cdot c)} + 24489837337812338687718622491865013839488 \cdot e^{(506 \cdot I \cdot c$
 $) + 383360155801054824529764688213114368047154 \cdot e^{(505 \cdot I \cdot c)} + 57646010465631$
 $51304213854710715346838447392 \cdot e^{(504 \cdot I \cdot c)} + 8338083991183789445313630367378$
 $5039051506805 \cdot e^{(503 \cdot I \cdot c)} + 1161581413733971751533622511909046917188768400 \cdot$
 $e^{(502 \cdot I \cdot c)} + 15603911277687607099721623771744933086920587272 \cdot e^{(501 \cdot I \cdot c)} +$
 $202347509724462171313966643580234078508179838320 \cdot e^{(500 \cdot I \cdot c)} + 25356674606$
 $50279776834561566186591213109251642859 \cdot e^{(499 \cdot I \cdot c)} + 3073536651283056216099$
 $1166338490057308062762518496 \cdot e^{(498 \cdot I \cdot c)} + 36068861303638934941380978000455$
 $9963548775423325255 \cdot e^{(497 \cdot I \cdot c)} + 41015454399371957939599567084424967094338$
 $00261224880 \cdot e^{(496 \cdot I \cdot c)} + 4523094003983073833202569478464620684485482769807$
 $5736 \cdot e^{(495 \cdot I \cdot c)} + 484093410240488718655917025303662581091659126182344528 \cdot e$
 $^{(494 \cdot I \cdot c)} + 5032024903401451824074213943766011922026507006311982753 \cdot e^{(493$
 $\cdot I \cdot c)} + 50836369508171099437019348610847391946736185108017183136 \cdot e^{(492 \cdot I \cdot c$
 $) + 499467506558531733671585862910572702811545035730398749530 \cdot e^{(491 \cdot I \cdot c)} +$
 $4775398607100853263534207733818266777478693412738731031680 \cdot e^{(490 \cdot I \cdot c)} + 4$
 $4456708175258821024400946210535004523775722190977468484496 \cdot e^{(489 \cdot I \cdot c)} + 40$
 $3212225957798188840846139960995624144491271694336796459584 \cdot e^{(488 \cdot I \cdot c)} + 35$
 $64764890628724017088487996688178929195787613958545474804845 \cdot e^{(487 \cdot I \cdot c)} + 3$
 $0736217404321009965231037419663053962881035281709221697785072 \cdot e^{(486 \cdot I \cdot c)} +$
 $258585348715977270155829115684193411072034541491364393985491350 \cdot e^{(485 \cdot I \cdot c$
 $) + 2123702969188871318266718781223927067839949015727293884065388080 \cdot e^{(484$
 $\cdot I \cdot c)} + 17033886027390615741040977721655541665612162275485028584310890417 \cdot e$
 $^{(483 \cdot I \cdot c)} + 13349021005202618377967331386833230353033290616324719462780841$
 $0304 \cdot e^{(482 \cdot I \cdot c)} + 10225364374682967372930658627052464496936874155598658443$

06888705423*e^(481*I*c) + 7659010520187549651777118357676871927081898989131
125755798204236112*e^(480*I*c) + 561170810763411753840875701851885386603759
32013674735519055227368366*e^(479*I*c) + 4023496922661211589340035828394287
85116904903936409545602519219664720*e^(478*I*c) + 2823905151936586678382525
706564457280290098698638597987628380245881715*e^(477*I*c) + 194079792155945
66593535008103303255257745408070082431338945184797463936*e^(476*I*c) + 1306
57660226560419335121434389938961884595434069984824307149332131747540*e^(475
*I*c) + 8618848510949919087642468054746724286037573154844539747136128122154
28992*e^(474*I*c) + 5572551157328671121016216416307596161861955969011697222
340926210112854418*e^(473*I*c) + 353244472067790181153780528207894116875810
04582367431006205879633729015200*e^(472*I*c) + 2196012813395155615002614788
44190024870555261281946058839614044697037963695*e^(471*I*c) + 1339214374254
245553564884406801945353385000254030655765953770237607180089968*e^(470*I*c)
+ 801372958079075243436196494576154376146952079121074697267587048105867427
7844*e^(469*I*c) + 47065044611135158108487353367484243102698248838312635876
283099427442745866704*e^(468*I*c) + 271361207503266570734486517077181014801
775322183181055638619257836143271472358*e^(467*I*c) + 153633323844492758353
2734556016494671674916578907116984548489078241693926940560*e^(466*I*c) + 85
430134411262123348335406650696214724790858380413605645507220367236542975402
05*e^(465*I*c) + 4666822354826601780685459246810057028935596086961365085657
5756758180182223308768*e^(464*I*c) + 25050102860892833246934045682990206771
2233644464602753159945727868485722395506952*e^(463*I*c) + 13214980552713008
51429993866631619874424534425188183592049727687571032156435077280*e^(462*I*
c) + 6852993223145736687328885311617795435592940841439866351079655652312894
721972796266*e^(461*I*c) + 349410716132767046494779430433394502015040753351
60361865916029213860778606230624960*e^(460*I*c) + 1751931705006183002415156
32381912285157790097816049220671217212220015297133400636060*e^(459*I*c) + 8
639799336223303495562968200283955131987080649405057021260686529368007948266
51264256*e^(458*I*c) + 4191542500656826148093339414544159143964478472492315
931809171859902114109005939942952*e^(457*I*c) + 200080068030300471372932782
50321597113540716201983333126349281186679153199068045257216*e^(456*I*c) + 9
398691531306817914908360606568148278083606051053015461848694983946713137885
9885998210*e^(455*I*c) + 43454667678028004534634449876389254079717510575682
7515509297024187660299345484920192480*e^(454*I*c) + 19777929806658181356513
00094326239158605448870806970860577325385028609983034534672318500*e^(453*I*
c) + 8862752142756957285681340885764904597935349569355321815647721172537159
186491471311666400*e^(452*I*c) + 391080312556018094765375353696118444408449
03751605645023514572352045248104262933598850730*e^(451*I*c) + 1699563279699
297677739020966526292532837045054771275445565344173766865409367060738473376
00*e^(450*I*c) + 7275210107183942292917740738446942557987386670675353797597
32795567942578751384250780476310*e^(449*I*c) + 3067974296431747364198159623
962463671617006419626851426148418602934852907379021659761911840*e^(448*I*c)
+ 127472196165033205413563430625628473686016221408567860254458145320379041
11523242298235713300*e^(447*I*c) + 5219091220766182421581227185426974807129
2843243227894769229690720010547141334131610989636000*e^(446*I*c) + 21059430

138564847118432907888031750495336183995415942743400988466177725975254264770
9150036990*e^(445*I*c) + 83757920692341193245878648676537353394654523970899
0769488724813982189165104589895518909256320*e^(444*I*c) + 32838747605558186
767263094803067344201550985839480744690141681718744421701096485216275387559
20*e^(443*I*c) + 1269349693296492056507367363718128008854868250888025533728
0065006566138696041797353216584528640*e^(442*I*c) + 48379489756434099843857
791816589379406815042609340378747586437145781646245422045101230417309900*e^
(441*I*c) + 181834661406177901315330129677145381166449188413194141169344354
754920969034952610378945282257600*e^(440*I*c) + 674025530543133008894845775
236625237450743114473544537818170447134607102575676676056675328961590*e^(43
9*I*c) + 246438219080743960907977422685567962936788570977643587663085171625
3962696192341706239192878728160*e^(438*I*c) + 88882950287510246670442038376
07976101480053134418614474620767522824868911959884352666444917404000*e^(437
*I*c) + 3162664467472554773117679568752765357130596998592368839211216491555
3242573269490908989570248533280*e^(436*I*c) + 11103414879700881944314389564
4469242295049867464313710969257619338899133799285616020069872611710850*e^(4
35*I*c) + 38465584208066627445406307878483717499894905250097532216200339254
9953413592461519365177908682078400*e^(434*I*c) + 13150521209306921221022971
05327622842335870743428530891072983535862280094446607723473800477453914130*
e^(433*I*c) + 4437210917843182347764349544443904699020056595069470847193617
092114714077633077234972825351226979360*e^(432*I*c) + 147779550966171289987
127451820714953621765069731830816502336052740516776249704643402427558400256
73760*e^(431*I*c) + 4858425815314028044731483686877213139019541241904673277
8458706015096881437076337910793584122475073760*e^(430*I*c) + 15768584552885
091872146287786443509025758314941556132342738656289444759827793562980093923
7175625149830*e^(429*I*c) + 50529366312301525887848302573881281320339776684
5340065381261016353419722382620393032535960660921950400*e^(428*I*c) + 15987
711010581926922705289996774447426856310062324561858449252201440023058781203
80828483988663574829100*e^(427*I*c) + 4995241956279138180205186744401688024
388272113921255663734956946927571305533146776898787878059685108480*e^(426*I
*c) + 154131112114860239372949708207973767160813447881633865435224219397375
07962125854981881879168348260330000*e^(425*I*c) + 4697022472711728182645404
501807067052255975662758034778453532001496348263235972944454188510227454600
2560*e^(424*I*c) + 14137993825355684328056550580740330413060613072543475174
5794079833141361748917639986145377066437210546190*e^(423*I*c) + 42035802483
514679858361121014594215468443794936564789908837252480215622288483958001168
8655664280691773600*e^(422*I*c) + 12346680418924099787800180817554402160125
82476396941937965899631953079203974222138794604328498972144766900*e^(421*I*
c) + 3582718002163296061414536703715109897107198252739284546149343102348456
124089657428594946438660859773886240*e^(420*I*c) + 102716025302028890024978
135168494525909715128095290606651973010970522100645760883480232346719754636
77418470*e^(419*I*c) + 2909765106124745340664756978183691006216555985235905
2804259154165687125428752562385492373749486351714453120*e^(418*I*c) + 81452
081413829111828875417564250054846037693312811480492909160758195989155768107
022568350953861815940704090*e^(417*I*c) + 225320532593220657767941109289516

24899979452101556413498282724171001967548669449968931246656190721262782000
*e^(416*I*c) + 616003116022979584937125701757887212998354300989991362628038
861093914561332071191909714949426587936910303300*e^(415*I*c) + 166447503438
721180939491774350293763897857493775476476398783587241044993069013157290427
9995484581013965001440*e^(414*I*c) + 44454122592954746250676595141983129660
15416299968930393345630345914109720740573618884980520010028451496996210*e^(
413*I*c) + 1173585692624511849311309100250160403234187698599908282352067224
1530200188223826392982302194084667538488665600*e^(412*I*c) + 30627581054221
957378390547289277609129572813931082733520247387226000020043538279468776707
958420892547870128680*e^(411*I*c) + 790191495587665692547839884872323883529
09144982747171856772463223808993367091503402876467270176124342699654400*e^(
410*I*c) + 2015579474247940980267724784620408833958675125629308629437535686
90084015585598010154781548625239409581907397500*e^(409*I*c) + 5083245993010
854601669786296830326614276544740829390480979396383915672987957883894338422
85751054665210868287680*e^(408*I*c) + 1267597017294812913400146276042126929
986480292870190399107554311079964227280196522475370108738477856311765699610
*e^(407*I*c) + 312568349317870174347970475030749017866629215072017936360433
5113528623329684606343185540756019935662148267863968*e^(406*I*c) + 76217887
919120470620388409177993746004288922581943676366829443560966814002463121380
01769285020661445991073249416*e^(405*I*c) + 1837980708400335976602764921762
114411609173557221662078886153580344970227380258835907670424184073351343911
4113248*e^(404*I*c) + 43834972142919377685378692233021063744554033100928502
737480438978976746989895784070951905237783490374305934542955*e^(403*I*c) +
103399755467257436489847837640753754718204394473055795001467604326419876555
556873829531737211096115196005647730480*e^(402*I*c) + 241246021282440061792
908317783032876194801597133206052091286997043729145345755805710081489006741
839439573984832678*e^(401*I*c) + 556756388711182340341026192734219546113651
768317380539005893679049394714017063698565272728813669054779077208977840*e^(
400*I*c) + 127103308293804895020136055483127034266234399127750461234230036
6025046741742856580445289401786656311685859023084716*e^(399*I*c) + 28704961
314123144518346747153535894395532944308085319334660862885437092462307691513
92180699413405623017247753532944*e^(398*I*c) + 6413381895855925184758231451
062556380328594938511006577015218119786536390213057284018202201631094434819
584025113465*e^(397*I*c) + 141764836528757049572020133432411179043699777966
53849959902524421980635189011634815653279605497783382888932766730080*e^(396
*I*c) + 3100431920606941707706936314142348743182800909818474463567865228417
7439464941651812564519144918003174108077634846014*e^(395*I*c) + 67091706130
529669125019899210021576580237843462229535386295087076189297849995931360645
605292130961496106707521506432*e^(394*I*c) + 143657687138044796942947119704
259538458818199423516824674586293691056119209866358123637772245409530799230
553767222252*e^(393*I*c) + 304384471106813336010284160123906370433888828490
627422652551236966790916174520857759143930140187173492394981908258944*e^(39
2*I*c) + 638218892914533741505806399662488556066783600496091876408374974877
448971778036074996245581124283460438065182071976085*e^(391*I*c) + 132431132
498402742835522293814768237867286070881716174144874968959358802086084750870

3702325320304649883120684987556400*e^(390*I*c) + 27195892834837439260408051
010803419212445303112546072509291927739093315232266350358156728625692966937
11643521070331394*e^(389*I*c) + 5527498849031178355861230009326668283926290
082158467118000698502719379939045918344222192742145711257028040974074674736
*e^(388*I*c) + 111194996453632010808810628248863384924253756584489779355358
46349290425821570383090425411418521516670371372045206568345*e^(387*I*c) + 2
214073500170860327091518076924139166203557875590397914890921360382255479274
9183517160255571915875356439553717130797888*e^(386*I*c) + 43638300075815171
025946211464465689618965773488660985857945657479854085108851857911222911989
837615452608512356008400295*e^(385*I*c) + 851395332347864557795899594649006
377607357297056212213800058372083691577946730496754287998178754314302463326
25899630160*e^(384*I*c) + 1644375006769068927413232601543942785039545619360
20133596581806449357240277349447927334517105809995300093549279931273178*e^(
383*I*c) + 3144090358082258615655954369383549454454739910431297220467472288
13030925204968503418566818838611866709807040793495364496*e^(382*I*c) + 5951
576155004315149474792823365470538279260879164252634971877570294133854718354
34198246807096214536895441388306027237899*e^(381*I*c) + 1115398285545601550
533328045600184317993899359217996729818340704221801195667410485846996179056
733558512452238583160792512*e^(380*I*c) + 206969828950086064346166576237380
795751301942404117887190496055182941244934472243212567941795840300755117929
8315947373776*e^(379*I*c) + 38026049967058911069646206338489648070370988545
10182263243030597295630760353597531974324752266389193185760878274188013440*
e^(378*I*c) + 6917838945214844278493330459361394923372333853619879637372673
184942859712431066345726870422099893124890777678037369988150*e^(377*I*c) +
124621404405372580849285967098720668577570709431248685545009481547568634543
08032925408340311237850017814707896986969086816*e^(376*I*c) + 2223134113180
153534540639903772168684020839794195258013558464596674673665671627155482647
6282991066076564921432614339399735*e^(375*I*c) + 39274200414329861169397944
516225001081227433398585007206399231211907157795359719648241598754266579840
244551491476467899952*e^(374*I*c) + 687124660159856415124685861736597477348
795917100983546527861249360230739431410495736066485630053594117127648956839
03806088*e^(373*I*c) + 1190605918496605468347656932276764490675841482488826
78447504826077236333444513454095126668750057295811191643356908972191440*e^(
372*I*c) + 2043255572651860007674027102308478964597615839227636982354332128
33313077783041040074669379017394836761539649081690630811665*e^(371*I*c) + 3
473100538109352904194555605559573141295692107357459832343696599764133747740
78000173070075248654524917179128950507443058208*e^(370*I*c) + 5847495736823
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92945356635118795824799369716742109*e^(369*I*c) + 9752103394440493187572823
117635177866732231755944579463832792646350850410049173002959042754331448485
32459919875479817581584*e^(368*I*c) + 1611092541400060525954859375264194178
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814072657644667120779832560683229593770506168629793029638233889257490081944
0*e^(366*I*c) + 42748269077205917525267113368208715008443456479223854715343

59333606189571832444641364132893108663576205133870672156264164115*e^(365*I*c) + 6866425337518668262662693750908956965732924578142181630622157802899880
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733440072199841759670394865739473880520939163659737038057239896471508009536
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444183501026198563419616*e^(361*I*c) + 416704403753905436434182193422717480
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30196476913600*e^(360*I*c) + 6392301943376198909061480128863509812319944510
230312261654464820899876780394445577704288655273849974718371313606910465181
2215*e^(359*I*c) + 97173055024742680058616722461368892661141295540263493013
032746083536157324268333390400308958318370219154887169702257444756176*e^(35
8*I*c) + 146390448456351181218237382740374124191664819997746988076598391862
733629670142241546375533903130605297580105675355629160198162*e^(357*I*c) +
218563166659649312247483640956272149212499115383828771029654283363972585118
090479413696638108156385244646591328454425745117584*e^(356*I*c) + 323413178
014841003714151138246079152576360976035058457890409937738723171537036573043
681997163745313602400139153046673668433091*e^(355*I*c) + 474323043563100542
377338629931966248129175976982332446018056009391154020438895903140822967769
494019446166779954024655344116288*e^(354*I*c) + 689518449328793559903260418
149974190253578340058895035589606468244680591556118170304005037563669880057
908765898949268614772285*e^(353*I*c) + 993555653649521127226443960820233649
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332264901780720*e^(352*I*c) + 141916448142217657385823401389899962882232233
330957373071631074383893583220145429361729317508645838962142530705175061212
9761498*e^(351*I*c) + 20094961100926877381527820856837372227279688242990587
39215446083499351467625334449670757764066690656150949014944043994822823920*
e^(350*I*c) + 2820819298562215959107529807289628449621386798989436369393116
069894018781201000275633104498398959346631795568022519974400130281*e^(349*I
*c) + 392569765841577835276810394285601184021164276962171721799661439847388
7186074391482638547212826538270453912634540299792270321024*e^(348*I*c) + 54
166628040524363495855959828183579538662584616443540182051589177425764253443
64964596750653177677803492186817305171175032011500*e^(347*I*c) + 7410372612
891152224364633296128043971657193280327754304382235672773781403023814127610
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040819793627127923997*e^(343*I*c) + 238563985655628020306952781742126408332
821541740064594582926440609479249073137359215616901539399060175186471824916
16573724049744*e^(342*I*c) + 3129526368818983831377277587330726033411722725

862950199235869563609266286606281984568906423581362297415012066892139187839
 8978380*e^(341*I*c) + 40715988963701918950020348336736423420513311359010485
 246919074652883393970805374470830156229705647312265477584256027212762941040
 *e^(340*I*c) + 525392233467407711425870923702570695360603196444395016676104
 82767955800276052892432152798814607975110366224945081428121888473324*e^(339
 *I*c) + 6724408796908070382370325719966304760689061048209049461949280293513
 0215979819469966383336788693900139115594646893784095418472336*e^(338*I*c) +
 85368118430215312848231291739673735887746201851666299600392199418764750086
 828198719872744047767783667325326289221881974987582215*e^(337*I*c) + 107504
 737406576916123480399169759633328321407419400510017498849830598621565428266
 546315933920821527544726380201659114903834605888*e^(336*I*c) + 134297742023
 479429904629616104559610096074758721068022704468938063017059688023363436458
 971534964665036319889119229809973806909680*e^(335*I*c) + 166432332922589195
 130558329266398753389823955737598527556096093062473559769545772321978969318
 904192572733997888230986469005970880*e^(334*I*c) + 204622295535729109519829
 916789867225319429705162560082648840394965423112809336591921290309392396263
 834674368977840527147037426908*e^(333*I*c) + 249593072282565866398389951509
 619202682634455487128714631461891770823201367527645793770203788784677343934
 971424317987895255031936*e^(332*I*c) + 302060638030868463461139442279360499
 718906917482524894483356220196138377050828911383056860425370161157201493696
 073712322595776808*e^(331*I*c) + 362706307563843231135699157418510732452420
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 355523960320*e^(330*I*c) + 432147856464086938023811561808678589594702047904
 674282297959658800170984456799067751878044806619012452636891350731618278545
 690160*e^(329*I*c) + 510907615111134507452623846147137141450275722444316385
 531648429230851686635827717488464500331623385777400744950538410637735936000
 *e^(328*I*c) + 599378484771733474809376142401554850207064972118137572949257
 503651444541939309025276896049622515630263162184526394317285457368300*e^(32
 7*I*c) + 697789106925924614816713747684682785083659819027952244447043355869
 741368500452561164024636073401929693105801105522738405349028160*e^(326*I*c)
 + 806169671327625532424575340089775733681994991576674446922354099714615192
 085443245663852257001115644286660304979476023966071898200*e^(325*I*c) + 924
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 433795246446491396141583854513687104334429566707520*e^(324*I*c) + 105178210
 042883437194450817005121918711681634976695332263718214961087500422378478418
 3284961906494422955462431208645690802526770780*e^(323*I*c) + 11878179430793
 903161088023247981101290208227826600872485993676434812002060468221661442854
 25922229375413676535071141005286431481600*e^(322*I*c) + 1331396114626723035
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 5878304304075953693546767206080*e^(320*I*c) + 16355697446414219006578863812
 890766532275801720566775874514020692343552836874896596139137619591407733397
 36014790081814516625224440*e^(319*I*c) + 1792649078089632298936945728481969
 334964391597506285088488350622937252533420980803144316431701452190522716124

797875257437516360640*e^(318*I*c) + 195028655078018191924499296120448701005
646036284521850167442376626632155879143691731787870223267921386828792629466
5202769722927380*e^(317*I*c) + 21061419034683443071125497612024845434027942
523524821998174104248696772627150982884376465186834879454627742236564713458
99082156800*e^(316*I*c) + 2257726219103856286812833012681573765496262241420
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991200*e^(315*I*c) + 240246459556968608612000180303421105673944558862194614
138410616288624616181514976302503083487523406726777402343341826998243126528
0*e^(314*I*c) + 25377664154650303308154717466929885960699118946972250529283
20452542175587154848096483331209807430113943015398362669673337957755720*e^(
313*I*c) + 2661100647975783583828235139201441930178396643383423903583862547
255880772382049201015537214900832745601519737141849802506685264000*e^(312*I
*c) + 277007320715076864559750728138206549792496846605452741412233982733378
3770068305883487309979315983718403740872884345746380680204260*e^(311*I*c) +
28625031263204617977706677807256441849912556231746261756790506721008489881
19391841466573417019247590580735265143427289340450811200*e^(310*I*c) + 2936
494214351868498703239455426771104344827306267558916550877467232455153286140
521089582733932202553130712723836983468866230908800*e^(309*I*c) + 299049894
962254360853812938028386633519008711512485881814378761918695711190376572397
4899651518555144924290346242595167274383008960*e^(308*I*c) + 30233716435082
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79159917590062343527464977286601165100620*e^(307*I*c) + 3034408355530957075
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90346242595167274383008960*e^(304*I*c) + 2936494214351868498703239455426771
104344827306267558916550877467232455153286140521089582733932202553130712723
836983468866230908800*e^(303*I*c) + 286250312632046179777066778072564418499
125562317462617567905067210084898811939184146657341701924759058073526514342
7289340450811200*e^(302*I*c) + 27700732071507686455975072813820654979249684
660545274141223398273337837700683058834873099793159837184037408728843457463
80680204260*e^(301*I*c) + 2661100647975783583828235139201441930178396643383
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264000*e^(300*I*c) + 253776641546503033081547174669298859606991189469722505
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0*e^(299*I*c) + 24024645955696860861200018030342110567394455886219461413841
06162886246161815149763025030834875234067267774023433418269982431265280*e^(
298*I*c) + 2257726219103856286812833012681573765496262241420612932076143151
171960854554124144699023009842080515157923529357189869943515991200*e^(297*I
*c) + 210614190346834430711254976120248454340279425235248219981741042486967
7262715098288437646518683487945462774223656471345899082156800*e^(296*I*c) +
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91436917317878702232679213868287926294665202769722927380*e^(295*I*c) + 1792

649078089632298936945728481969334964391597506285088488350622937252533420980
803144316431701452190522716124797875257437516360640*e^(294*I*c) + 163556974
464142190065788638128907665322758017205667758745140206923435528368748965961
3913761959140773339736014790081814516625224440*e^(293*I*c) + 14811871170892
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91464091005878304304075953693546767206080*e^(292*I*c) + 1331396114626723035
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524856072950880960657564720475838500*e^(291*I*c) + 118781794307939031610880
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62431208645690802526770780*e^(289*I*c) + 9243200528675225840357774957610723
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87104334429566707520*e^(288*I*c) + 8061696713276255324245753400897757336819
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279522444470433558697413685004525611640246360734019296931058011055227384053
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729492575036514445419393090252768960496225156302631621845263943172854573683
00*e^(285*I*c) + 5109076151111345074526238461471371414502757224443163855316
48429230851686635827717488464500331623385777400744950538410637735936000*e^(
284*I*c) + 4321478564640869380238115618086785895947020479046742822979596588
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c) + 3627063075638432311356991574185107324524206140136241688793121876452334
50153927975793326834780741391203430153093712635355523960320*e^(282*I*c) + 3
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722825658663983899515096192026826344554871287146314618917708232013675276457
93770203788784677343934971424317987895255031936*e^(280*I*c) + 2046222955357
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09392396263834674368977840527147037426908*e^(279*I*c) + 1664323329225891951
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36319889119229809973806909680*e^(277*I*c) + 1075047374065769161234803991697
596333283214074194005100174988498305986215654282665463159339208215275447263
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774620185166629960039219941876475008682819871987274404776778366732532628922
1881974987582215*e^(275*I*c) + 67244087969080703823703257199663047606890610
482090494619492802935130215979819469966383336788693900139115594646893784095
418472336*e^(274*I*c) + 525392233467407711425870923702570695360603196444395
016676104827679558002760528924321527988146079751103662249450814281218884733
24*e^(273*I*c) + 4071598896370191895002034833673642342051331135901048524691
9074652883393970805374470830156229705647312265477584256027212762941040*e^(2
72*I*c) + 31295263688189838313772775873307260334117227258629501992358695636

092662866062819845689064235813622974150120668921391878398978380*e^(271*I*c)
+ 238563985655628020306952781742126408332821541740064594582926440609479249
07313735921561690153939906017518647182491616573724049744*e^(270*I*c) + 1803
532733817745547117756859485168297797834644977719357208768851039242688451927
2991560851326393852241961470040819793627127923997*e^(269*I*c) + 13521230411
945436915558854706851796541567399811656870021567352975326815467817846533289
123871696056195231696146162720992221760992*e^(268*I*c) + 100522095243695818
275881549853455496780314457444999852082593854356097402724003014542468720417
75159838468077381562338745636398374*e^(267*I*c) + 7410372612891152224364633
296128043971657193280327754304382235672773781403023814127610355562505271969
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795386625846164435401820515891774257642534436496459675065317767780349218681
7305171175032011500*e^(265*I*c) + 39256976584157783527681039428560118402116
427696217172179966143984738871860743914826385472128265382704539126345402997
92270321024*e^(264*I*c) + 2820819298562215959107529807289628449621386798989
436369393116069894018781201000275633104498398959346631795568022519974400130
281*e^(263*I*c) + 200949611009268773815278208568373722272796882429905873921
5446083499351467625334449670757764066690656150949014944043994822823920*e^(2
62*I*c) + 1419164481422176573858234013898999628822322333095737307163107438
38935832201454293617293175086458389621425307051750612129761498*e^(261*I*c)
+ 9935556536495211272264439608202336493864885100818925458667004440966615827
90441241830855609577062039555625090943332264901780720*e^(260*I*c) + 6895184
493287935599032604181499741902535783400588950355896064682446805915561181703
04005037563669880057908765898949268614772285*e^(259*I*c) + 4743230435631005
423773386299319662481291759769823324460180560093911540204388959031408229677
69494019446166779954024655344116288*e^(258*I*c) + 3234131780148410037141511
382460791525763609760350584578904099377387231715370365730436819971637453136
02400139153046673668433091*e^(257*I*c) + 2185631666596493122474836409562721
492124991153838287710296542833639725851180904794136966381081563852446465913
28454425745117584*e^(256*I*c) + 1463904484563511812182373827403741241916648
199977469880765983918627336296701422415463755339031306052975801056753556291
60198162*e^(255*I*c) + 9717305502474268005861672246136889266114129554026349
3013032746083536157324268333390400308958318370219154887169702257444756176*e
^(254*I*c) + 63923019433761989090614801288635098123199445102303122616544648
208998767803944455777042886552738499747183713136069104651812215*e^(253*I*c)
+ 416704403753905436434182193422717480400350714901190805852815224981888183
75906900368701234531304633163446319945130196476913600*e^(252*I*c) + 2691779
470108661509789012023689050110514679999602177519571086622622863898470345683
2694153230611607263444183501026198563419616*e^(251*I*c) + 17229502824367647
334400721998417596703948657394738805209391636597370380572398964715080095366
818322029152193635869784095333760*e^(250*I*c) + 109272106034735448102797923
478445360745888968062300411100895447316058631461041817390394266748554534660
97402330688331845602302*e^(249*I*c) + 6866425337518668262662693750908956965
732924578142181630622157802899880874681551031136314064199948604001529894566
235238597088*e^(248*I*c) + 427482690772059175252671133682087150084434564792

238547153435933360618957183244464136413289310866357620513387067215626416411
 5*e^(247*I*c) + 26366624104307993404475222847782442837407510686581407265764
 46671207798325606832295937705061686297930296382338892574900819440*e<sup>(246*I*
 c)</sup> + 1611092541400060525954859375264194178347643471837078201446262435615142
 944587337833513586022729849523358436493586042252995608*e^(245*I*c) + 975210
 339444049318757282311763517786673223175594457946383279264635085041004917300
 295904275433144848532459919875479817581584*e^(244*I*c) + 584749573682304586
 179384628844883327581498969886540380378896767999075614964007174600811092945
 356635118795824799369716742109*e^(243*I*c) + 347310053810935290419455560555
 957314129569210735745983234369659976413374774078000173070075248654524917179
 128950507443058208*e^(242*I*c) + 204325557265186000767402710230847896459761
 583922763698235433212833313077783041040074669379017394836761539649081690630
 811665*e^(241*I*c) + 119060591849660546834765693227676449067584148248882678
 447504826077236333444513454095126668750057295811191643356908972191440*e<sup>(24
 0*I*c)</sup> + 687124660159856415124685861736597477348795917100983546527861249360
 23073943141049573606648563005359411712764895683903806088*e^(239*I*c) + 3927
 420041432986116939794451622500108122743339858500720639923121190715779535971
 9648241598754266579840244551491476467899952*e^(238*I*c) + 22231341131801535
 345406399037721686840208397941952580135584645966746736656716271554826476282
 991066076564921432614339399735*e^(237*I*c) + 124621404405372580849285967098
 720668577570709431248685545009481547568634543080329254083403112378500178147
 07896986969086816*e^(236*I*c) + 6917838945214844278493330459361394923372333
 853619879637372673184942859712431066345726870422099893124890777678037369988
 150*e^(235*I*c) + 380260499670589110696462063384896480703709885451018226324
 3030597295630760353597531974324752266389193185760878274188013440*e<sup>(234*I*c
)</sup> + 20696982895008606434616657623738079575130194240411788719049605518294124
 49344722432125679417958403007551179298315947373776*e^(233*I*c) + 1115398285
 545601550533328045600184317993899359217996729818340704221801195667410485846
 996179056733558512452238583160792512*e^(232*I*c) + 595157615500431514947479
 282336547053827926087916425263497187757029413385471835434198246807096214536
 895441388306027237899*e^(231*I*c) + 314409035808225861565595436938354945445
 473991043129722046747228813030925204968503418566818838611866709807040793495
 364496*e^(230*I*c) + 164437500676906892741323260154394278503954561936020133
 596581806449357240277349447927334517105809995300093549279931273178*e<sup>(229*I
 *c)</sup> + 851395332347864557795899594649006377607357297056212213800058372083691
 57794673049675428799817875431430246332625899630160*e^(228*I*c) + 4363830007
 581517102594621146446568961896577348866098585794565747985408510885185791122
 2911989837615452608512356008400295*e^(227*I*c) + 22140735001708603270915180
 769241391662035578755903979148909213603822554792749183517160255571915875356
 439553717130797888*e^(226*I*c) + 111194996453632010808810628248863384924253
 756584489779355358463492904258215703830904254114185215166703713720452065683
 45*e^(225*I*c) + 5527498849031178355861230009326668283926290082158467118000
 698502719379939045918344222192742145711257028040974074674736*e^(224*I*c) +
 271958928348374392604080510108034192124453031125460725092919277390933152322
 6635035815672862569296693711643521070331394*e^(223*I*c) + 13243113249840274

283552229381476823786728607088171617414487496895935880208608475087037023253
20304649883120684987556400*e^(222*I*c) + 6382188929145337415058063996624885
560667836004960918764083749748774489717780360749962455811242834604380651820
71976085*e^(221*I*c) + 3043844711068133360102841601239063704338888284906274
22652551236966790916174520857759143930140187173492394981908258944*e^(220*I*
c) + 1436576871380447969429471197042595384588181994235168246745862936910561
19209866358123637772245409530799230553767222252*e^(219*I*c) + 6709170613052
966912501989921002157658023784346222953538629508707618929784999593136064560
5292130961496106707521506432*e^(218*I*c) + 31004319206069417077069363141423
487431828009098184744635678652284177439464941651812564519144918003174108077
634846014*e^(217*I*c) + 141764836528757049572020133432411179043699777966538
49959902524421980635189011634815653279605497783382888932766730080*e^(216*I*
c) + 6413381895855925184758231451062556380328594938511006577015218119786536
390213057284018202201631094434819584025113465*e^(215*I*c) + 287049613141231
445183467471535358943955329443080853193346608628854370924623076915139218069
9413405623017247753532944*e^(214*I*c) + 12710330829380489502013605548312703
426623439912775046123423003660250467417428565804452894017866563116858590230
84716*e^(213*I*c) + 5567563887111823403410261927342195461136517683173805390
05893679049394714017063698565272728813669054779077208977840*e^(212*I*c) + 2
412460212824400617929083177830328761948015971332060520912869970437291453457
55805710081489006741839439573984832678*e^(211*I*c) + 1033997554672574364898
478376407537547182043944730557950014676043264198765555568738295317372110961
15196005647730480*e^(210*I*c) + 4383497214291937768537869223302106374455403
3100928502737480438978976746989895784070951905237783490374305934542955*e^(2
09*I*c) + 18379807084003359766027649217621144116091735572216620788861535803
449702273802588359076704241840733513439114113248*e^(208*I*c) + 762178879191
204706203884091779937460042889225819436763668294435609668140024631213800176
9285020661445991073249416*e^(207*I*c) + 31256834931787017434797047503074901
786662921507201793636043351135286233296846063431855407560199356621482678639
68*e^(206*I*c) + 1267597017294812913400146276042126929986480292870190399107
554311079964227280196522475370108738477856311765699610*e^(205*I*c) + 508324
599301085460166978629683032661427654474082939048097939638391567298795788389
433842285751054665210868287680*e^(204*I*c) + 201557947424794098026772478462
040883395867512562930862943753568690084015585598010154781548625239409581907
397500*e^(203*I*c) + 790191495587665692547839884872323883529091449827471718
56772463223808993367091503402876467270176124342699654400*e^(202*I*c) + 3062
758105422195737839054728927760912957281393108273352024738722600002004353827
9468776707958420892547870128680*e^(201*I*c) + 11735856926245118493113091002
501604032341876985999082823520672241530200188223826392982302194084667538488
665600*e^(200*I*c) + 444541225929547462506765951419831296601541629996893039
3345630345914109720740573618884980520010028451496996210*e^(199*I*c) + 16644
750343872118093949177435029376389785749377547647639878358724104499306901315
72904279995484581013965001440*e^(198*I*c) + 6160031160229795849371257017578
872129983543009899913626280388610939145613320711919097149494265879369103033
00*e^(197*I*c) + 2253205325932206577679411092895162489997945210155641349828

27241710019675486694499689312466561907212627820000*e^(196*I*c) + 8145208141
 382911182887541756425005484603769331281148049290916075819598915576810702256
 8350953861815940704090*e^(195*I*c) + 29097651061247453406647569781836910062
 165559852359052804259154165687125428752562385492373749486351714453120*e^(19
 4*I*c) + 102716025302028890024978135168494525909715128095290606651973010970
 52210064576088348023234671975463677418470*e^(193*I*c) + 3582718002163296061
 414536703715109897107198252739284546149343102348456124089657428594946438660
 859773886240*e^(192*I*c) + 123466804189240997878001808175544021601258247639
 6941937965899631953079203974222138794604328498972144766900*e^(191*I*c) + 42
 035802483514679858361121014594215468443794936564789908837252480215622288483
 9580011688655664280691773600*e^(190*I*c) + 14137993825355684328056550580740
 3304130606130725434751745794079833141361748917639986145377066437210546190*e
 ^ (189*I*c) + 46970224727117281826454045018070670522559756627580347784535320
 014963482632359729444541885102274546002560*e^(188*I*c) + 154131112114860239
 372949708207973767160813447881633865435224219397375079621258549818818791683
 48260330000*e^(187*I*c) + 4995241956279138180205186744401688024388272113921
 255663734956946927571305533146776898787878059685108480*e^(186*I*c) + 159877
 110105819269227052899967744474268563100623245618584492522014400230587812038
 0828483988663574829100*e^(185*I*c) + 50529366312301525887848302573881281320
 3397766845340065381261016353419722382620393032535960660921950400*e^(184*I*c
) + 15768584552885091872146287786443509025758314941556132342738656289444759
 8277935629800939237175625149830*e^(183*I*c) + 48584258153140280447314836868
 772131390195412419046732778458706015096881437076337910793584122475073760*e^
 (182*I*c) + 147779550966171289987127451820714953621765069731830816502336052
 74051677624970464340242755840025673760*e^(181*I*c) + 4437210917843182347764
 349544443904699020056595069470847193617092114714077633077234972825351226979
 360*e^(180*I*c) + 131505212093069212210229710532762284233587074342853089107
 2983535862280094446607723473800477453914130*e^(179*I*c) + 38465584208066627
 445406307878483717499894905250097532216200339254995341359246151936517790868
 2078400*e^(178*I*c) + 11103414879700881944314389564446924229504986746431371
 0969257619338899133799285616020069872611710850*e^(177*I*c) + 31626644674725
 547731176795687527653571305969985923688392112164915553242573269490908989570
 248533280*e^(176*I*c) + 888829502875102466704420383760797610148005313441861
 4474620767522824868911959884352666444917404000*e^(175*I*c) + 24643821908074
 396090797742268556796293678857097764358766308517162539626961923417062391928
 78728160*e^(174*I*c) + 6740255305431330088948457752366252374507431144735445
 37818170447134607102575676676056675328961590*e^(173*I*c) + 1818346614061779
 013153301296771453811664491884131941411693443547549209690349526103789452822
 57600*e^(172*I*c) + 4837948975643409984385779181658937940681504260934037874
 7586437145781646245422045101230417309900*e^(171*I*c) + 12693496932964920565
 073673637181280088548682508880255337280065006566138696041797353216584528640
 *e^(170*I*c) + 328387476055581867672630948030673442015509858394807446901416
 8171874442170109648521627538755920*e^(169*I*c) + 83757920692341193245878648
 6765373533946545239708990769488724813982189165104589895518909256320*e^(168*
 I*c) + 21059430138564847118432907888031750495336183995415942743400988466177

7259752542647709150036990*e^(167*I*c) + 52190912207661824215812271854269748
071292843243227894769229690720010547141334131610989636000*e^(166*I*c) + 127
472196165033205413563430625628473686016221408567860254458145320379041115232
42298235713300*e^(165*I*c) + 3067974296431747364198159623962463671617006419
626851426148418602934852907379021659761911840*e^(164*I*c) + 727521010718394
229291774073844694255798738667067535379759732795567942578751384250780476310
*e^(163*I*c) + 169956327969929767773902096652629253283704505477127544556534
417376686540936706073847337600*e^(162*I*c) + 391080312556018094765375353696
11844440844903751605645023514572352045248104262933598850730*e^(161*I*c) + 8
862752142756957285681340885764904597935349569355321815647721172537159186491
471311666400*e^(160*I*c) + 197779298066581813565130009432623915860544887080
6970860577325385028609983034534672318500*e^(159*I*c) + 43454667678028004534
6344498763892540797175105756827515509297024187660299345484920192480*e^(158*
I*c) + 93986915313068179149083606065681482780836060510530154618486949839467
131378859885998210*e^(157*I*c) + 200080068030300471372932782503215971135407
16201983333126349281186679153199068045257216*e^(156*I*c) + 4191542500656826
148093339414544159143964478472492315931809171859902114109005939942952*e^(15
5*I*c) + 863979933622330349556296820028395513198708064940505702126068652936
800794826651264256*e^(154*I*c) + 175193170500618300241515632381912285157790
097816049220671217212220015297133400636060*e^(153*I*c) + 349410716132767046
49477943043339450201504075335160361865916029213860778606230624960*e^(152*I*
c) + 6852993223145736687328885311617795435592940841439866351079655652312894
721972796266*e^(151*I*c) + 132149805527130085142999386663161987442453442518
8183592049727687571032156435077280*e^(150*I*c) + 25050102860892833246934045
6829902067712233644464602753159945727868485722395506952*e^(149*I*c) + 46668
223548266017806854592468100570289355960869613650856575756758180182223308768
*e^(148*I*c) + 854301344112621233483354066506962147247908583804136056455072
2036723654297540205*e^(147*I*c) + 15363332384449275835327345560164946716749
16578907116984548489078241693926940560*e^(146*I*c) + 2713612075032665707344
86517077181014801775322183181055638619257836143271472358*e^(145*I*c) + 4706
5044611135158108487353367484243102698248838312635876283099427442745866704*e
^(144*I*c) + 80137295807907524343619649457615437614695207912107469726758704
81058674277844*e^(143*I*c) + 1339214374254245553564884406801945353385000254
030655765953770237607180089968*e^(142*I*c) + 219601281339515561500261478844
190024870555261281946058839614044697037963695*e^(141*I*c) + 353244472067790
18115378052820789411687581004582367431006205879633729015200*e^(140*I*c) + 5
572551157328671121016216416307596161861955969011697222340926210112854418*e
^(139*I*c) + 861884851094991908764246805474672428603757315484453974713612812
215428992*e^(138*I*c) + 130657660226560419335121434389938961884595434069984
824307149332131747540*e^(137*I*c) + 194079792155945665935350081033032552577
45408070082431338945184797463936*e^(136*I*c) + 2823905151936586678382525706
564457280290098698638597987628380245881715*e^(135*I*c) + 402349692266121158
934003582839428785116904903936409545602519219664720*e^(134*I*c) + 561170810
76341175384087570185188538660375932013674735519055227368366*e^(133*I*c) + 7
659010520187549651777118357676871927081898989131125755798204236112*e^(132*I

$*c) + 1022536437468296737293065862705246449693687415559865844306888705423*e$
 $^{(131*I*c)} + 13349021005202618377967331386833230353033290616324719462780841$
 $0304*e^{(130*I*c)} + 17033886027390615741040977721655541665612162275485028584$
 $310890417*e^{(129*I*c)} + 212370296918887131826671878122392706783994901572729$
 $3884065388080*e^{(128*I*c)} + 25858534871597727015582911568419341107203454149$
 $1364393985491350*e^{(127*I*c)} + 30736217404321009965231037419663053962881035$
 $281709221697785072*e^{(126*I*c)} + 356476489062872401708848799668817892919578$
 $7613958545474804845*e^{(125*I*c)} + 40321222595779818884084613996099562414449$
 $1271694336796459584*e^{(124*I*c)} + 44456708175258821024400946210535004523775$
 $722190977468484496*e^{(123*I*c)} + 477539860710085326353420773381826677747869$
 $3412738731031680*e^{(122*I*c)} + 49946750655853173367158586291057270281154503$
 $5730398749530*e^{(121*I*c)} + 50836369508171099437019348610847391946736185108$
 $017183136*e^{(120*I*c)} + 503202490340145182407421394376601192202650700631198$
 $2753*e^{(119*I*c)} + 484093410240488718655917025303662581091659126182344528*e$
 $^{(118*I*c)} + 45230940039830738332025694784646206844854827698075736*e^{(117*I$
 $*c)} + 4101545439937195793959956708442496709433800261224880*e^{(116*I*c)} + 36$
 $0688613036389349413809780004559963548775423325255*e^{(115*I*c)} + 30735366512$
 $830562160991166338490057308062762518496*e^{(114*I*c)} + 253566746065027977683$
 $4561566186591213109251642859*e^{(113*I*c)} + 20234750972446217131396664358023$
 $4078508179838320*e^{(112*I*c)} + 15603911277687607099721623771744933086920587$
 $272*e^{(111*I*c)} + 1161581413733971751533622511909046917188768400*e^{(110*I*c$
 $) + 83380839911837894453136303673785039051506805*e^{(109*I*c)} + 576460104656$
 $3151304213854710715346838447392*e^{(108*I*c)} + 38336015580105482452976468821$
 $3114368047154*e^{(107*I*c)} + 24489837337812338687718622491865013839488*e^{(10$
 $6*I*c)} + 1500602747937397286405577818722691539392*e^{(105*I*c)} + 88054927598$
 $941411145869950813388040256*e^{(104*I*c)} + 493966661081802579880958635254347$
 $1345*e^{(103*I*c)} + 264410375780310742518099326419685040*e^{(102*I*c)} + 13477$
 $227799524701956579274210395326*e^{(101*I*c)} + 652650253343206047453620559993$
 $840*e^{(100*I*c)} + 29952547749265499675257842032197*e^{(99*I*c)} + 12991466459$
 $93240318167826532288*e^{(98*I*c)} + 53090127264630963470039804475*e^{(97*I*c)}$
 $+ 2037031259470368160131922320*e^{(96*I*c)} + 73099207817335597247098038*e^{(9$
 $5*I*c)} + 2442455629894502983849104*e^{(94*I*c)} + 75599817092670157806639*e^{($
 $93*I*c)} + 2154864144781257856128*e^{(92*I*c)} + 56169444526926562260*e^{(91*I*$
 $c)} + 1327882849274858880*e^{(90*I*c)} + 28186192554792138*e^{(89*I*c)} + 530563$
 $624556832*e^{(88*I*c)} + 8718181624155*e^{(87*I*c)} + 122503723056*e^{(86*I*c)} +$
 $1431118260*e^{(85*I*c)} + 13343760*e^{(84*I*c)} + 93096*e^{(83*I*c)} + 432*e^{(82$
 $*I*c)} + e^{(81*I*c)}) * \tan(1/4*d*x + c) + 14*(3718*a*e^{(1055/2*I*c)} + 1502072$
 $*a*e^{(1053/2*I*c)} + 302667508*a*e^{(1051/2*I*c)} + 40557446072*a*e^{(1049/2*I*$
 $c)} + 4065883968718*a*e^{(1047/2*I*c)} + 325270717497440*a*e^{(1045/2*I*c)} + 21$
 $630502713590667*a*e^{(1043/2*I*c)} + 1229848582862227068*a*e^{(1041/2*I*c)} + 6$
 $1031235925207244502*a*e^{(1039/2*I*c)} + 2685374380789029329793*a*e^{(1037/2*I$
 $*c)} + 106072288048394569716987*a*e^{(1035/2*I*c)} + 3799316499714931382700630$
 $*a*e^{(1033/2*I*c)} + 124427615397868362145478058*a*e^{(1031/2*I*c)} + 37519711$
 $73691706110924034857*a*e^{(1029/2*I*c)} + 104787195000667085382630850098*a*e^{$
 $(1027/2*I*c)} + 2724467073240049286222217499520*a*e^{(1025/2*I*c)} + 662386058$

37791619527049091465540*a*e^(1023/2*I*c) + 15117987725766730478754353874794
18*a*e^(1021/2*I*c) + 32503673735785850571258452122985714*a*e^(1019/2*I*c)
+ 660337796328597830395723099421064436*a*e^(1017/2*I*c) + 12711502674847529
112148726218999140522*a*e^(1015/2*I*c) + 2324389084257266652098070964987750
79690*a*e^(1013/2*I*c) + 4046550142768142823441656912104576646575*a*e^(1011
/2*I*c) + 67207920970133725538564726906882769461732*a*e^(1009/2*I*c) + 1066
925770347513486390409808978976432165366*a*e^(1007/2*I*c) + 1621727219708085
6640086895687618882654237557*a*e^(1005/2*I*c) + 236397938386786552298680619
962344215977635255*a*e^(1003/2*I*c) + 3309571297229229358438719595781123324
700153528*a*e^(1001/2*I*c) + 4456101622755301159027145079943660097856140086
2*a*e^(999/2*I*c) + 577756667439166347317361298827893686038031023201*a*e^(9
97/2*I*c) + 7221959008399380670648524146893979780310767551977*a*e^(995/2*I*
c) + 87129450753762477310055827412704326695373797670990*a*e^(993/2*I*c) + 1
015602799297538681881502332438578214644838744212768*a*e^(991/2*I*c) + 11448
615265227028920332020054872464737892837624091599*a*e^(989/2*I*c) + 12492462
0764273645058915750915973878220098322691297101*a*e^(987/2*I*c) + 1320632018
182734050434490769597960546607943021209554374*a*e^(985/2*I*c) + 13536481999
851915706958049067829203397030845591800979948*a*e^(983/2*I*c) + 13463316315
2791085906227518959644714629575426941345274855*a*e^(981/2*I*c) + 1300273428
594384632178804915869348076492496040528026941982*a*e^(979/2*I*c) + 12202571
647480368478947339347197542554373660129766305670026*a*e^(977/2*I*c) + 11134
8526291800411107246584226952800505326877479452354210332*a*e^(975/2*I*c) + 9
88558268629814228600928047340864800588772545018200620775758*a*e^(973/2*I*c)
+ 8543974119433407159975166243013026279990985549262801022565070*a*e^(971/2
*I*c) + 71928400911660376402320318751672710160518473805095916511887372*a*e^(
969/2*I*c) + 5901404087543110766870133152796383198273113772958000311212096
56*a*e^(967/2*I*c) + 472112855150189813470234989967908258077951489216075374
1261926062*a*e^(965/2*I*c) + 3684537662243287784255841441700388727804897582
0601095126064678425*a*e^(963/2*I*c) + 2806524295838718418985556121503766503
33075795788342413441360641690*a*e^(961/2*I*c) + 208735594051295060844310589
7698920410302162080067836820821771280502*a*e^(959/2*I*c) + 1516530880048616
6402472766122974437831930120746836471291819780637051*a*e^(957/2*I*c) + 1076
73925545674144588693096021112687136443322698905146090803710183409*a*e^(955/
2*I*c) + 747385551508889800825728592465333469870510137536130275665450524715
478*a*e^(953/2*I*c) + 50736121402768690227858106977890139521479281755845595
08062569934520500*a*e^(951/2*I*c) + 336965483328793735102555813785378142355
58502314864144954037409976543435*a*e^(949/2*I*c) + 219028335999286257776369
634146613713106509647861704677410488454471041828*a*e^(947/2*I*c) + 13938222
04759320545751155434910006366369348464487459381028813178904280462*a*e^(945/
2*I*c) + 868653770595701574687455664750361197848557528657750073320150408015
7038368*a*e^(943/2*I*c) + 5303386265594344453065147141731165909374896767141
0808199772545872566250240*a*e^(941/2*I*c) + 3172905654382992959442279818904
47073652423258243355469512774650547831808732*a*e^(939/2*I*c) + 186073250885
7587132839919523366305804938874482315206088251791750125084597398*a*e^(937/2
*I*c) + 1069928606114654174111403771891312216673912944378396092045113963786

$8310953672*a*e^{(935/2*I*c)} + 6033742282110890544218880523770538090215653857$
 $3067777647643655256342216048188*a*e^{(933/2*I*c)} + 3338050555933885674278082$
 $01926960940156572529198077839859476641115104007423532*a*e^{(931/2*I*c)} + 181$
 $210183042543630089172057262867836304235419855514772670127515007554664358308$
 $0*a*e^{(929/2*I*c)} + 9655206906308981051087661502772471729668550920396172616$
 $931546671734191430604348*a*e^{(927/2*I*c)} + 50504748951674956594241719165020$
 $500739646130041511438767337766766404018721799652*a*e^{(925/2*I*c)} + 25941410$
 $5343758614384429045574352903181961134382680334119395735344643117364969828*a$
 $*e^{(923/2*I*c)} + 1308704744103233244942832711379475161653066000371260361064$
 $765494153774929304542712*a*e^{(921/2*I*c)} + 64858887594418942644819254525773$
 $19190375803293665331722248553541369888292250431868*a*e^{(919/2*I*c)} + 315840$
 $056627086045447017647621317926509675240202429419926437821852398614135904960$
 $20*a*e^{(917/2*I*c)} + 151154910257369434692596735808879851584404239307790047$
 $602041275902443495292459666790*a*e^{(915/2*I*c)} + 71108167343618545192404079$
 $3669141298969626722796614395742802361266093633641264085200*a*e^{(913/2*I*c)}$
 $+ 3288828495006098780525142569296800128547751347678538502742954742586652543$
 $592821521100*a*e^{(911/2*I*c)} + 14957789836260035053720288689121945297194278$
 $508615355005723695241121674541179126990690*a*e^{(909/2*I*c)} + 66907643424659$
 $838184545773924908879954701525687410376227199214504402274227244630158470*a*$
 $e^{(907/2*I*c)} + 29440256424059206189496939686815693205792480390369535730682$
 $4322577258371768427288287660*a*e^{(905/2*I*c)} + 1274495521071138744583215835$
 $633951280803025426238229370838062367497287195705487702727780*a*e^{(903/2*I*c}$
 $) + 54292169476186125875471230124801656584646052969223575286753687542492001$
 $00593150293185330*a*e^{(901/2*I*c)} + 227618504270416638904609966614057231934$
 $07183947233163896700853295630681434965577357299920*a*e^{(899/2*I*c)} + 939327$
 $132073952654868978803666648957410937006959243440092003500689225667620070527$
 $49646040*a*e^{(897/2*I*c)} + 381619655494220774667949833382475342723385533974$
 $486856243098750370767001096283055945502640*a*e^{(895/2*I*c)} + 15265571175700$
 $366225719188767782521274684375417746236146193026796584975168400092788493522$
 $80*a*e^{(893/2*I*c)} + 601348240394226978137589513619272032123326663890549621$
 $4866132974340762298443025559536997680*a*e^{(891/2*I*c)} + 2333083491763171509$
 $3466481420083833029947668880513498966378137598401377993510086729958534960*a$
 $*e^{(889/2*I*c)} + 8916302466752884436525755694697706895091987504043996034218$
 $8615542362550926345388079651264340*a*e^{(887/2*I*c)} + 3356967207389365707067$
 $13403614929944912226507516523422878616118942760950780230341688669154360*a*e$
 $^{(885/2*I*c)} + 124529792545542646768500367978320293088561760055771650047390$
 $9654045354638798448652726086548710*a*e^{(883/2*I*c)} + 4552164541985701158364$
 $440833202197360445014839710956970770340240973634689769418208918160590320*a*$
 $e^{(881/2*I*c)} + 16399645740773051092975392309744789183304727620673733635277$
 $139746257521706440274686262794198620*a*e^{(879/2*I*c)} + 58233756699318390009$
 $909172367611895685773867826783550649200500869687498091353927199571718405490$
 $*a*e^{(877/2*I*c)} + 20384013942017335064659986351963620564829591915624076720$
 $5893957321010048179259062568208312088790*a*e^{(875/2*I*c)} + 7034426197985873$
 $272565187719539626411298070061455526651676893452046620849008282218268424290$
 $79160*a*e^{(873/2*I*c)} + 239353645319019429757252164260948584114537204756945$

1871481943673924084228578119992726364314043020*a*e^(871/2*I*c) + 8031025392
884252064177450987879092832928608062990864461726329576684085419875495358549
722887030650*a*e^(869/2*I*c) + 26574653865772705945516884258966917928577399
544926651101115471261281415662774645920805562286190810*a*e^(867/2*I*c) + 86
731110812497060241964047703698781976493241635614918587219564911585800789497
248326704733021564620*a*e^(865/2*I*c) + 27921378124083730626314923927774127
8504180764814216320768523126545112266943230869925045964255370200*a*e^(863/2
*I*c) + 8867402319255326645227710555534538499697441522560799980322092340080
19622116815785290576730828104950*a*e^(861/2*I*c) + 277840530521434363427264
247811536876216851365717201706703008414345258345731388565609302905763144437
0*a*e^(859/2*I*c) + 8589650067614913233196096604074597492566547822453765715
081979725077156239293288377277499477542060060*a*e^(857/2*I*c) + 26204518312
074558508652381319847496634719883744031651088258490824465738576863175200241
054655253538000*a*e^(855/2*I*c) + 78892732475550629207160150619649518859671
333059425363870727604426538820615197072859753978543934017670*a*e^(853/2*I*c
) + 23442090586166750167351495144027547182182831996259574173715153879482358
8356166465622241503448244518760*a*e^(851/2*I*c) + 6875302244726200927911732
813454970749262714300760196163220853611448794295500584095658716214540602339
40*a*e^(849/2*I*c) + 199049069287552725866723649901746078436630792328029665
4931735997954941147793207299041775080046145853840*a*e^(847/2*I*c) + 5689019
471410220246066908603484004351329982673777354247661562524221692115006358198
880775065147099437120*a*e^(845/2*I*c) + 16053098146865366570614226985936175
148028466685745428670334943691048314134627977747488782883668338627720*a*e^(
843/2*I*c) + 44725891301479449618021653613021840925820831052165035724946865
219883816662123055202896018457333687900560*a*e^(841/2*I*c) + 12304718259132
723929105983780313299023656095154142609150259213307988993082073673597693006
2642304445760760*a*e^(839/2*I*c) + 3342951012915983944053859269749233418212
34439369994003970291001771339818244857969207842071054369602329440*a*e^(837/
2*I*c) + 896946415449495021773305652132521489143683388084681377490890952178
768366949153161001917497973558984634610*a*e^(835/2*I*c) + 23769112749784751
700037321362357881818593856532982808428141057546244636203561833840215925564
92451101686180*a*e^(833/2*I*c) + 622157939003660126190659135862285736586376
5271336363813258928102047231269701287744435751925676619466254380*a*e^(831/2
*I*c) + 1608644751608406316324368905021587976126759980980895818281687215507
4721707033313657170678606565171911318790*a*e^(829/2*I*c) + 4108861530848589
836617249726887411435147878011316057683933654524058994798286412592113656405
8023293113177890*a*e^(827/2*I*c) + 1036845869531580526617610021948562200160
78202175436674022417030738316048059257865332989827933134737432425420*a*e^(8
25/2*I*c) + 258503941427418775983449358257227129327187031025405352304946158
596617947732241690180138609024179619896548080*a*e^(823/2*I*c) + 63680845082
005250841339490337361369940898513179337822532263092681601027818866435487336
5849405050807559590630*a*e^(821/2*I*c) + 1550126189251322091456566293321041
304661332373605301977703681149593242237032310585758452049979836332182019460
*a*e^(819/2*I*c) + 37288017476089265495513190379614293274474046317883570540
35813617665417388029848305480269822322115047563031420*a*e^(817/2*I*c) + 886

425998214388706605224133949530251959265174370639560991810511424164234462615
4900178145218983856632762282840*a*e^(815/2*I*c) + 2082634813976572814550230
025023063352862536881103792628378918283781757233732328440331177037297030229
1071410100*a*e^(813/2*I*c) + 4836233601377550334331969315460087218020889366
5329972255007464669036745521843710761901688665256224358994271700*a*e^(811/2
*I*c) + 1110070511484021253452202837142711739826290353852231226468056365836
54935893527705499942693423715121989424818940*a*e^(809/2*I*c) + 251865250484
281545894238180017093595610796911882271999250070181159518716093352404254337
261360592129286243867720*a*e^(807/2*I*c) + 56491770603803702788592000743376
407605164806140939010812663502809191357241464734717803992357829937985993584
9404*a*e^(805/2*I*c) + 1252638795847466913766125006669625873907326637843208
199173827808400459890232296094362170239873318861004896614156*a*e^(803/2*I*c
) + 27460877130459664927598197470575345372957948511425219013508840686869727
03385462885090913563610742000426952431144*a*e^(801/2*I*c) + 595216046678660
616565355624658983556184002104900636763429259656593528081819514137469523625
1836967708599780176746*a*e^(799/2*I*c) + 1275645561784139151391438472889884
004272566111366879184662192780806878599049007723894849271955293649990034423
0684*a*e^(797/2*I*c) + 2703359253088000222673746532135913846484748415398268
7697594699044738257454819138210737641742661588856574025344360*a*e^(795/2*I*
c) + 5665245750901921733280983225339376549002644652315613106646717764061473
7878543683747754276689674112796652581318416*a*e^(793/2*I*c) + 1174079820363
959646167751034288803005778116556386730609946055394264513936940767050575172
58363475234790358732617874*a*e^(791/2*I*c) + 240637192176925995554769763822
196549695150106412767840246806015588785001466549798116731687549539743160206
690512276*a*e^(789/2*I*c) + 48779380400401382500929622160999761682089159915
8999011226672490129857245007433873787162629626332383304760538813509*a*e^(78
7/2*I*c) + 9780028732611922834575829499694560528349463111485640919454800839
12375742750095496723188070578970297191691801222556*a*e^(785/2*I*c) + 193952
786969132205721734701058954719411935238878819329036688072614042243591664951
4401171793242696849555174393643850*a*e^(783/2*I*c) + 3804744464968921359015
461873113836781159840428541963397786141899308945274545366930937702520484566
257179138022478559*a*e^(781/2*I*c) + 73832895699159059662274446961109834921
500050098396932680395734577625972446412914068437922786635243511272776100685
01*a*e^(779/2*I*c) + 141739356433255984532024123966213171732858939160760735
72849970829958267591737907286619219624773200857453140746381674*a*e^(777/2*I
*c) + 269195769225792606752145769442826156530587082608618056157594984212372
41977159288165357847803942410262265158465467750*a*e^(775/2*I*c) + 505829522
181672507409706444229176144626319674426544321423479213230295152205283640183
97135511063228424091734992672055*a*e^(773/2*I*c) + 940414097395855760889319
663270055553239711080741367215796434630893641710184566990485130268122231684
40576516628683514*a*e^(771/2*I*c) + 172995123756550106637378132357010306487
566049978888205088265495332263303165503780066305408668687698920139970094538
712*a*e^(769/2*I*c) + 31489738298943158333268009682716291953167330247168046
0640021147765185388299005207883488229333447615352727288983946148*a*e^(767/2
*I*c) + 5672118945705247212649343352447884789056087263187306899799399945740

65235928861643029786499059347466854077954904040746*a*e^(765/2*I*c) + 101107
480011495937671953662497709473896428560464306279911177894201538732570662427
8964496575270768601912488833296828570*a*e^(763/2*I*c) + 1783624738647145047
601917556451061887933853774354141392528926855412119981497793799270417127134
990753415746380839743700*a*e^(761/2*I*c) + 31140565155012973990269680028508
549281389546645524786671683515866127971766182529813084483208289157763078057
39513112726*a*e^(759/2*I*c) + 538112224856716242115738356975663022032829339
7938837982011834598823639708711693916980646441642210742072277442692795738*a
*e^(757/2*I*c) + 9203715338102052013953994655272327701169506976135302561414
981599329671691132074752022832241424911916459710600587437561*a*e^(755/2*I*c
) + 15581815934260482853545078419219552128875182877110543730686320263425001
461567811411263192244527545355522679818707520580*a*e^(753/2*I*c) + 26113086
386424978619406272087483484311686778353469810280045602451813616101635095685
826358744734323495284346010214813114*a*e^(751/2*I*c) + 43321583189999820011
825513535961153632261709936204250859944426152513525897996934481741824822705
710882414313119272790131*a*e^(749/2*I*c) + 71150379803182330624998227186896
692406197859434054084482880670614321542424946546285172306349522048789631550
138215923153*a*e^(747/2*I*c) + 11569053075069444158976287555701878043931241
423534680572445064585687921237440396748349767763604157543927792054896822913
6*a*e^(745/2*I*c) + 1862460580514884075040356040303865547180668961186912013
86319926538759246009834321804025886034942775247389666416855412210*a*e^(743/
2*I*c) + 296869894312341025882614756881848801182548095718231901017442700510
970132207473697085843370374068409734287215730709535159*a*e^(741/2*I*c) + 46
855032529333593499967336909176711798578926338071141481498616019652012707926
0704512548045596385215149194149787362058847*a*e^(739/2*I*c) + 7322825730749
454152411919840240362556682792737583179725378709145927318501848182891727320
27744954335864633099347774743778*a*e^(737/2*I*c) + 113332640724905667726518
235728304130268059656575339233035601373338626757880224910100905403393335818
4582360056565943982872*a*e^(735/2*I*c) + 1737027904730654746361321038484958
368225917557506862415168862004368924008956529402849910157768607834197363252
609518902089*a*e^(733/2*I*c) + 26366702936061117805877803641794482992232390
452297640939384776262098816360133643920580853229150024201995535497381313240
75*a*e^(731/2*I*c) + 396391207053641466510099371609095284662117086242866333
1960681129776400190663462118184042376205657205476237354164748568266*a*e^(72
9/2*I*c) + 5902464865781828746433759626115731299368628038776863537919286757
121919619848756678585765961272292609235750043165001541228*a*e^(727/2*I*c) +
87057506566399628050193575474036103992151243298721304901061585028910574024
04433904178771049787603257660231907442902723601*a*e^(725/2*I*c) + 127193465
108299801676917693560051353106337171859775948660881353310671450225539960963
52557277491441427340072720038453020094*a*e^(723/2*I*c) + 184090602589765024
977173302569641551086417497283354692011644278717217713467622411892121545870
72288919159051458074105345270*a*e^(721/2*I*c) + 263954641832889484740532868
631201612007278840887085807502640735451013640607936923777157140561663978028
14429763618417822876*a*e^(719/2*I*c) + 374956249662893147770179909764712974
747825449021922176441858876940812711235132630691990752706912512822552398725

96210514278*a*e^(717/2*I*c) + 527725840025912026071527567068898957493075370
026298230158906309240214312430531353083821381541538970419159957716339842818
06*a*e^(715/2*I*c) + 735928399275644055040421275927628199319989286428969198
11011301927328743837627845050154224299851252425467281991751035964300*a*e^(7
13/2*I*c) + 101691644420950070025024011057452997381500283466574730634283893
254959188466627402946882962692076334471578743932496642334320*a*e^(711/2*I*c
) + 13924532801833388077436496880085314258472825551304341643787011246963155
0665869605343027896353023176557694400200499461146726*a*e^(709/2*I*c) + 1889
491189020427563658877672346431137877974092858750283534102934644404036045542
65358653936413844588923365518512812155564263*a*e^(707/2*I*c) + 254098011526
686795017826888997862881107698178279803078370347724142537822230167040460676
758302527381200018338794267668213782*a*e^(705/2*I*c) + 33866720736606402867
003586937597637011103498231618447975359283035558179670460082450921530821182
1300197041784861670006680074*a*e^(703/2*I*c) + 4473875310425244148831939864
392691122933941025699541899181965510160646569849499801211255527461657173027
25948575549803471765*a*e^(701/2*I*c) + 585810087840090638959352593115207493
330409388304333648586508726254966666272617271460912163699959246766169974995
865625894415*a*e^(699/2*I*c) + 76035336195088564664135197686089319202216456
290233216169212648199277978799711837850220083492439189521567109153442746718
2154*a*e^(697/2*I*c) + 9783250682911966141304805478841491874189481835844832
75851528436468279773867175472874823772301653470715495906310752113906356*a*e
^(695/2*I*c) + 124791048985172481215347756168402974898532716676117664901789
3764462742540955810359360345752498737439338173328941693214996869*a*e^(693/2
*I*c) + 1578118891036425030175529775728613184825593490511183941993477628800
762057890918137098180588823348433951309822211104517241960*a*e^(691/2*I*c) +
19786800264802850146464387540611231054985631302388830055355523594649169528
02872195794689967236683993035275155224868084857426*a*e^(689/2*I*c) + 245988
387972516529245242552206817888068970942084349298901961431008874096948577083
1889636928119055954588263689191708179939864*a*e^(687/2*I*c) + 3032358651189
255296683913995058467176181044555965209522749027694372314500932020478507945
032419489336181643123979995151020196*a*e^(685/2*I*c) + 37067847077481699394
769399274183061171378076612161855775293079519685348910541020225998758005210
28818591703507213651354601744*a*e^(683/2*I*c) + 449354568331212718576070418
619678692686355841095393534736338730545539031827864669151759235114794216282
9757283672732654718906*a*e^(681/2*I*c) + 5402322084276358486392170807160452
492303736610543993756782397563451183220371719416455539109849685147683838533
792416380394000*a*e^(679/2*I*c) + 64416374286615630575010005338207913892127
228997251589825754386503387235900048656780793551028674230829691236385594583
60395064*a*e^(677/2*I*c) + 761837187529411056419612165438425312451156615754
347209513656463847646437139129592620943513236179049981597923143923035360757
6*a*e^(675/2*I*c) + 8937263147952157636762401237363283906060175423762559079
710179111349891892147713199783099640876465513972490960354416509072464*a*e^(
673/2*I*c) + 10400418939568829692690672761559360526193214201582201680217310
568760141362183204340317918999290500601587312837404209184604296*a*e^(671/2*
I*c) + 12006868470364785965223507525758074064695337161964914781000766290625

851531335032822622641350671534071651460975302014909903304*a*e^(669/2*I*c) +
 13752183046726574703388943144716596530089681250702923836997308671850511977
 450963396491458048548668871178015432677009314283240*a*e^(667/2*I*c) + 15628
 195936569723039933951516376786709017066442709363693028081832755768436639263
 391998605831116142654156311324448043994018480*a*e^(665/2*I*c) + 17622850330
 509088835683937678238705071770423484283273552938412847052745751689386549302
 132045239654150557454790469375312979400*a*e^(663/2*I*c) + 19720200401778864
 584757183707826447178195559155063511504273138952837613637477930413174805644
 097852673575370034436363705076200*a*e^(661/2*I*c) + 21900584468529959516758
 800837479928951306460846458729039375943218853680130886800630510267887849815
 785681169229247800599653780*a*e^(659/2*I*c) + 24140981134483157781955632751
 265346889062034264187233058673043860862569245974192234790942766740180569345
 117561729698386920640*a*e^(657/2*I*c) + 26415549363149806685039661454392670
 206122658153701833626733684195755223911158469961354340519916453720031563673
 191039715534120*a*e^(655/2*I*c) + 28696342239951526679387032172582088320219
 823498612520015500396461595364742834942285040710651207447747254997240669017
 588581660*a*e^(653/2*I*c) + 30954172375894978763972944200770454559337426860
 551177880393678541417465620860789179378399467769874414988489807528551280728
 660*a*e^(651/2*I*c) + 33159595315331473569721299752492007977292770764782508
 428598300474234056672396374716510074637547675167083319916344381519908840*a*
 e^(649/2*I*c) + 35283966812630641149341159791961391292025523974077536888523
 903291265231647954939454727506873690155653093593169057488857882680*a*e^(647
 /2*I*c) + 37300521329459415731704251401002810979585734710978988333621496246
 233856339027608860252310284690807173977430553976853710509180*a*e^(645/2*I*c
) + 39185413399202383622073627604329189089884179407279757971516867735898168
 138035719962821736076912971225404759780285833325366160*a*e^(643/2*I*c) + 40
 918661287257157586516616110021423781180377032922238849800270673579553338073
 950681237233866877936152591013115793335322605360*a*e^(641/2*I*c) + 42484934
 110204653755860791506752384746500674496076673967800901085937786065818126999
 716652181441925069428597465940270175008320*a*e^(639/2*I*c) + 43874129455527
 739968579909886347568532862593497167618837681590989053474358012316788203258
 490840630555174376169174989395181280*a*e^(637/2*I*c) + 45081698448240753180
 262375692004927927506538345043673939468981741472441127727726396712184152830
 435376131283599596954371095120*a*e^(635/2*I*c) + 46108688698493969053979321
 920659577290639749185078038328437911559324575990868098570200478146083617100
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 449297631107256887184383362431636434691884722589930145727930608372648302523
 392227498002326040*a*e^(631/2*I*c) + 47651300820137584031981301306237132784
 818112406387340495482154916616947775309181410534964011766375876504387207320
 222180574240*a*e^(629/2*I*c) + 48193299659530236368965629162172353377208407
 430970686522594246262923397590490780745776073857916729329041495496560962652
 110580*a*e^(627/2*I*c) + 48605628647485221530448036568439365222083977681057
 836467545264634863498616835311749827432057497076186357697241560068425420480
 *a*e^(625/2*I*c) + 48908180925332300251079156776454840287174312864705788555
 225955731856750601356988070150266042825709040021440687898855883758920*a*e^(

623/2*I*c) + 49121301226132778368272749219618738723201555478462388241994389
343726868771636664492457040060188477918743965347339416057414940*a*e^(621/2*
I*c) + 49264464542155705870038634542338788212438798530705434981837058559355
123523321420453290054924412360694086356486267112390442900*a*e^(619/2*I*c) +
49355015869551095112590580563652333821487135029403204297871136385751177082
668822483114895823989375806441500725710546732275120*a*e^(617/2*I*c) + 49407
049724497857923646418122802418078663415974301089858309325233628714614313902
610437654951417123467208217070390610584251240*a*e^(615/2*I*c) + 49430500293
479459295429182210667241410154074816087128978164893559373471444466906250076
413859410320538676562475573896009453900*a*e^(613/2*I*c) + 49430500293479459
295429182210667241410154074816087128978164893559373471444466906250076413859
410320538676562475573896009453900*a*e^(611/2*I*c) + 49407049724497857923646
418122802418078663415974301089858309325233628714614313902610437654951417123
467208217070390610584251240*a*e^(609/2*I*c) + 49355015869551095112590580563
652333821487135029403204297871136385751177082668822483114895823989375806441
500725710546732275120*a*e^(607/2*I*c) + 49264464542155705870038634542338788
212438798530705434981837058559355123523321420453290054924412360694086356486
267112390442900*a*e^(605/2*I*c) + 49121301226132778368272749219618738723201
555478462388241994389343726868771636664492457040060188477918743965347339416
057414940*a*e^(603/2*I*c) + 48908180925332300251079156776454840287174312864
70578855225955731856750601356988070150266042825709040021440687898855883758
920*a*e^(601/2*I*c) + 48605628647485221530448036568439365222083977681057836
467545264634863498616835311749827432057497076186357697241560068425420480*a*
e^(599/2*I*c) + 48193299659530236368965629162172353377208407430970686522594
246262923397590490780745776073857916729329041495496560962652110580*a*e^(597
/2*I*c) + 47651300820137584031981301306237132784818112406387340495482154916
616947775309181410534964011766375876504387207320222180574240*a*e^(595/2*I*c
) + 46961491881008674073571953258739449297631107256887184383362431636434691
884722589930145727930608372648302523392227498002326040*a*e^(593/2*I*c) + 46
108688698493969053979321920659577290639749185078038328437911559324575990868
098570200478146083617100445871110853633600291520*a*e^(591/2*I*c) + 45081698
448240753180262375692004927927506538345043673939468981741472441127727726396
712184152830435376131283599596954371095120*a*e^(589/2*I*c) + 43874129455527
739968579909886347568532862593497167618837681590989053474358012316788203258
490840630555174376169174989395181280*a*e^(587/2*I*c) + 42484934110204653755
860791506752384746500674496076673967800901085937786065818126999716652181441
925069428597465940270175008320*a*e^(585/2*I*c) + 40918661287257157586516616
110021423781180377032922238849800270673579553338073950681237233866877936152
591013115793335322605360*a*e^(583/2*I*c) + 39185413399202383622073627604329
189089884179407279757971516867735898168138035719962821736076912971225404759
780285833325366160*a*e^(581/2*I*c) + 37300521329459415731704251401002810979
585734710978988333621496246233856339027608860252310284690807173977430553976
853710509180*a*e^(579/2*I*c) + 35283966812630641149341159791961391292025523
974077536888523903291265231647954939454727506873690155653093593169057488857
882680*a*e^(577/2*I*c) + 33159595315331473569721299752492007977292770764782

508428598300474234056672396374716510074637547675167083319916344381519908840
*a*e^(575/2*I*c) + 30954172375894978763972944200770454559337426860551177880
393678541417465620860789179378399467769874414988489807528551280728660*a*e⁽
573/2*I*c) + 28696342239951526679387032172582088320219823498612520015500396
461595364742834942285040710651207447747254997240669017588581660*a*e^{(571/2*}
I*c) + 26415549363149806685039661454392670206122658153701833626733684195755
223911158469961354340519916453720031563673191039715534120*a*e^(569/2*I*c) +
24140981134483157781955632751265346889062034264187233058673043860862569245
974192234790942766740180569345117561729698386920640*a*e^(567/2*I*c) + 21900
584468529959516758800837479928951306460846458729039375943218853680130886800
630510267887849815785681169229247800599653780*a*e^(565/2*I*c) + 19720200401
778864584757183707826447178195559155063511504273138952837613637477930413174
805644097852673575370034436363705076200*a*e^(563/2*I*c) + 17622850330509088
835683937678238705071770423484283273552938412847052745751689386549302132045
239654150557454790469375312979400*a*e^(561/2*I*c) + 15628195936569723039933
951516376786709017066442709363693028081832755768436639263391998605831116142
654156311324448043994018480*a*e^(559/2*I*c) + 13752183046726574703388943144
716596530089681250702923836997308671850511977450963396491458048548668871178
015432677009314283240*a*e^(557/2*I*c) + 12006868470364785965223507525758074
064695337161964914781000766290625851531335032822622641350671534071651460975
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214201582201680217310568760141362183204340317918999290500601587312837404209
184604296*a*e^(553/2*I*c) + 89372631479521576367624012373632839060601754237
625590797101791113498918921477131997830996408764655139724909603544165090724
64*a*e^(551/2*I*c) + 761837187529411056419612165438425312451156615754347209
5136564638476464371391295926209435132361790499815979231439230353607576*a*e⁽
549/2*I*c) + 6441637428661563057501000533820791389212722899725158982575438
650338723590004865678079355102867423082969123638559458360395064*a*e^{(547/2*}
I*c) + 54023220842763584863921708071604524923037366105439937567823975634511
83220371719416455539109849685147683838533792416380394000*a*e^(545/2*I*c) +
449354568331212718576070418619678692686355841095393534736338730545539031827
8646691517592351147942162829757283672732654718906*a*e^(543/2*I*c) + 3706784
707748169939476939927418306117137807661216185577529307951968534891054102022
599875800521028818591703507213651354601744*a*e^(541/2*I*c) + 30323586511892
552966839139950584671761810445559652095227490276943723145009320204785079450
32419489336181643123979995151020196*a*e^(539/2*I*c) + 245988387972516529245
242552206817888068970942084349298901961431008874096948577083188963692811905
5954588263689191708179939864*a*e^(537/2*I*c) + 1978680026480285014646438754
061123105498563130238883005535552359464916952802872195794689967236683993035
275155224868084857426*a*e^(535/2*I*c) + 15781188910364250301755297757286131
848255934905111839419934776288007620578909181370981805888233484339513098222
11104517241960*a*e^(533/2*I*c) + 124791048985172481215347756168402974898532
716676117664901789376446274254095581035936034575249873743933817332894169321
4996869*a*e^(531/2*I*c) + 9783250682911966141304805478841491874189481835844
83275851528436468279773867175472874823772301653470715495906310752113906356*

$a \cdot e^{(529/2 \cdot I \cdot c)}$ + 760353361950885646641351976860893192022164562902332161692
 126481992779787997118378502200834924391895215671091534427467182154 $\cdot a \cdot e^{(527/2 \cdot I \cdot c)}$ + 58581008784009063895935259311520749333040938830433364858650872625
 4966666272617271460912163699959246766169974995865625894415 $\cdot a \cdot e^{(525/2 \cdot I \cdot c)}$
 + 4473875310425244148831939864392691122933941025699541899181965510160646569
 84949980121125552746165717302725948575549803471765 $\cdot a \cdot e^{(523/2 \cdot I \cdot c)}$ + 338667
 207366064028670035869375976370111034982316184479753592830355581796704600824
 509215308211821300197041784861670006680074 $\cdot a \cdot e^{(521/2 \cdot I \cdot c)}$ + 25409801152668
 679501782688899786288110769817827980307837034772414253782223016704046067675
 8302527381200018338794267668213782 $\cdot a \cdot e^{(519/2 \cdot I \cdot c)}$ + 1889491189020427563658
 877672346431137877974092858750283534102934644404036045542653586539364138445
 88923365518512812155564263 $\cdot a \cdot e^{(517/2 \cdot I \cdot c)}$ + 139245328018333880774364968800
 853142584728255513043416437870112469631550665869605343027896353023176557694
 400200499461146726 $\cdot a \cdot e^{(515/2 \cdot I \cdot c)}$ + 10169164442095007002502401105745299738
 150028346657473063428389325495918846662740294688296269207633447157874393249
 6642334320 $\cdot a \cdot e^{(513/2 \cdot I \cdot c)}$ + 7359283992756440550404212759276281993199892864
 289691981101130192732874383762784505015422429985125242546728199175103596430
 0 $\cdot a \cdot e^{(511/2 \cdot I \cdot c)}$ + 5277258400259120260715275670688989574930753700262982301
 5890630924021431243053135308382138154153897041915995771633984281806 $\cdot a \cdot e^{(509/2 \cdot I \cdot c)}$ + 3749562496628931477701799097647129747478254490219221764418588769
 4081271123513263069199075270691251282255239872596210514278 $\cdot a \cdot e^{(507/2 \cdot I \cdot c)}$
 + 2639546418328894847405328686312016120072788408870858075026407354510136406
 0793692377715714056166397802814429763618417822876 $\cdot a \cdot e^{(505/2 \cdot I \cdot c)}$ + 1840906
 025897650249771733025696415510864174972833546920116442787172177134676224118
 9212154587072288919159051458074105345270 $\cdot a \cdot e^{(503/2 \cdot I \cdot c)}$ + 1271934651082998
 016769176935600513531063371718597759486608813533106714502255399609635255727
 7491441427340072720038453020094 $\cdot a \cdot e^{(501/2 \cdot I \cdot c)}$ + 8705750656639962805019357
 547403610399215124329872130490106158502891057402404433904178771049787603257
 660231907442902723601 $\cdot a \cdot e^{(499/2 \cdot I \cdot c)}$ + 59024648657818287464337596261157312
 993686280387768635379192867571219196198487566785857659612722926092357500431
 65001541228 $\cdot a \cdot e^{(497/2 \cdot I \cdot c)}$ + 396391207053641466510099371609095284662117086
 242866333196068112977640019066346211818404237620565720547623735416474856826
 6 $\cdot a \cdot e^{(495/2 \cdot I \cdot c)}$ + 2636670293606111780587780364179448299223239045229764093
 938477626209881636013364392058085322915002420199553549738131324075 $\cdot a \cdot e^{(493/2 \cdot I \cdot c)}$ + 17370279047306547463613210384849583682259175575068624151688620043
 68924008956529402849910157768607834197363252609518902089 $\cdot a \cdot e^{(491/2 \cdot I \cdot c)}$ +
 113332640724905667726518235728304130268059656575339233035601373338626757880
 2249101009054033933358184582360056565943982872 $\cdot a \cdot e^{(489/2 \cdot I \cdot c)}$ + 7322825730
 749454152411919840240362556682792737583179725378709145927318501848182891727
 32027744954335864633099347774743778 $\cdot a \cdot e^{(487/2 \cdot I \cdot c)}$ + 468550325293335934999
 673369091767117985789263380711414814986160196520127079260704512548045596385
 215149194149787362058847 $\cdot a \cdot e^{(485/2 \cdot I \cdot c)}$ + 29686989431234102588261475688184
 880118254809571823190101744270051097013220747369708584337037406840973428721
 5730709535159 $\cdot a \cdot e^{(483/2 \cdot I \cdot c)}$ + 1862460580514884075040356040303865547180668
 961186912013863199265387592460098343218040258860349427752473896664168554122

$10 * a * e^{(481/2 * I * c)} + 115690530750694441589762875557018780439312414235346805$
 $724450645856879212374403967483497677636041575439277920548968229136 * a * e^{(479$
 $/2 * I * c)} + 71150379803182330624998227186896692406197859434054084482880670614$
 $321542424946546285172306349522048789631550138215923153 * a * e^{(477/2 * I * c)} + 43$
 $321583189999820011825513535961153632261709936204250859944426152513525897996$
 $934481741824822705710882414313119272790131 * a * e^{(475/2 * I * c)} + 26113086386424$
 $978619406272087483484311686778353469810280045602451813616101635095685826358$
 $744734323495284346010214813114 * a * e^{(473/2 * I * c)} + 15581815934260482853545078$
 $419219552128875182877110543730686320263425001461567811411263192244527545355$
 $522679818707520580 * a * e^{(471/2 * I * c)} + 92037153381020520139539946552723277011$
 $695069761353025614149815993296716911320747520228322414249119164597106005874$
 $37561 * a * e^{(469/2 * I * c)} + 538112224856716242115738356975663022032829339793883$
 $7982011834598823639708711693916980646441642210742072277442692795738 * a * e^{(46$
 $7/2 * I * c)} + 3114056515501297399026968002850854928138954664552478667168351586$
 $612797176618252981308448320828915776307805739513112726 * a * e^{(465/2 * I * c)} + 17$
 $836247386471450476019175564510618879338537743541413925289268554121199814977$
 $93799270417127134990753415746380839743700 * a * e^{(463/2 * I * c)} + 101107480011495$
 $937671953662497709473896428560464306279911177894201538732570662427896449657$
 $5270768601912488833296828570 * a * e^{(461/2 * I * c)} + 5672118945705247212649343352$
 $447884789056087263187306899799399945740652359288616430297864990593474668540$
 $77954904040746 * a * e^{(459/2 * I * c)} + 314897382989431583332680096827162919531673$
 $302471680460640021147765185388299005207883488229333447615352727288983946148$
 $* a * e^{(457/2 * I * c)} + 17299512375655010663737813235701030648756604997888820508$
 $8265495332263303165503780066305408668687698920139970094538712 * a * e^{(455/2 * I * c)}$
 $+ 940414097395855760889319663270055532397110807413672157964346308936417$
 $1018456699048513026812223168440576516628683514 * a * e^{(453/2 * I * c)} + 5058295221$
 $816725074097064442291761446263196744265443214234792132302951522052836401839$
 $7135511063228424091734992672055 * a * e^{(451/2 * I * c)} + 2691957692257926067521457$
 $694428261565305870826086180561575949842123724197715928816535784780394241026$
 $2265158465467750 * a * e^{(449/2 * I * c)} + 1417393564332559845320241239662131717328$
 $589391607607357284997082995826759173790728661921962477320085745314074638167$
 $4 * a * e^{(447/2 * I * c)} + 7383289569915905966227444696110983492150005009839693268$
 $039573457762597244641291406843792278663524351127277610068501 * a * e^{(445/2 * I * c)}$
 $+ 38047444649689213590154618731138367811598404285419633977861418993089452$
 $74545366930937702520484566257179138022478559 * a * e^{(443/2 * I * c)} + 193952786969$
 $132205721734701058954719411935238878819329036688072614042243591664951440117$
 $1793242696849555174393643850 * a * e^{(441/2 * I * c)} + 9780028732611922834575829499$
 $694560528349463111485640919454800839123757427500954967231880705789702971916$
 $91801222556 * a * e^{(439/2 * I * c)} + 487793804004013825009296221609997616820891599$
 $158999011226672490129857245007433873787162629626332383304760538813509 * a * e^{($
 $437/2 * I * c)} + 24063719217692599555476976382219654969515010641276784024680601$
 $5588785001466549798116731687549539743160206690512276 * a * e^{(435/2 * I * c)} + 1174$
 $079820363959646167751034288803005778116556386730609946055394264513936940767$
 $05057517258363475234790358732617874 * a * e^{(433/2 * I * c)} + 566524575090192173328$
 $098322533937654900264465231561310664671776406147378785436837477542766896741$

12796652581318416*a*e^(431/2*I*c) + 270335925308800022267374653213591384648
 47484153982687697594699044738257454819138210737641742661588856574025344360*
 a*e^(429/2*I*c) + 127564556178413915139143847288988400427256611136687918466
 21927808068785990490077238948492719552936499900344230684*a*e^(427/2*I*c) +
 595216046678660616565355624658983556184002104900636763429259656593528081819
 5141374695236251836967708599780176746*a*e^(425/2*I*c) + 2746087713045966492
 759819747057534537295794851142521901350884068686972703385462885090913563610
 742000426952431144*a*e^(423/2*I*c) + 12526387958474669137661250066696258739
 07326637843208199173827808400459890232296094362170239873318861004896614156*
 a*e^(421/2*I*c) + 564917706038037027885920007433764076051648061409390108126
 635028091913572414647347178039923578299379859935849404*a*e^(419/2*I*c) + 25
 186525048428154589423818001709359561079691188227199925007018115951871609335
 2404254337261360592129286243867720*a*e^(417/2*I*c) + 1110070511484021253452
 202837142711739826290353852231226468056365836549358935277054999426934237151
 21989424818940*a*e^(415/2*I*c) + 483623360137755033433196931546008721802088
 93665329972255007464669036745521843710761901688665256224358994271700*a*e<sup>(4
 13/2*I*c)</sup> + 208263481397657281455023002502306335286253688110379262837891828
 37817572337323284403311770372970302291071410100*a*e^(411/2*I*c) + 886425998
 214388706605224133949530251959265174370639560991810511424164234462615490017
 8145218983856632762282840*a*e^(409/2*I*c) + 3728801747608926549551319037961
 429327447404631788357054035813617665417388029848305480269822322115047563031
 420*a*e^(407/2*I*c) + 15501261892513220914565662933210413046613323736053019
 77703681149593242237032310585758452049979836332182019460*a*e^(405/2*I*c) +
 636808450820052508413394903373613699408985131793378225322630926816010278188
 664354873365849405050807559590630*a*e^(403/2*I*c) + 25850394142741877598344
 935825722712932718703102540535230494615859661794773224169018013860902417961
 9896548080*a*e^(401/2*I*c) + 1036845869531580526617610021948562200160782021
 75436674022417030738316048059257865332989827933134737432425420*a*e<sup>(399/2*I
 *c)</sup> + 410886153084858983661724972688741143514787801131605768393365452405899
 47982864125921136564058023293113177890*a*e^(397/2*I*c) + 160864475160840631
 632436890502158797612675998098089581828168721550747217070333136571706786065
 65171911318790*a*e^(395/2*I*c) + 622157939003660126190659135862285736586376
 5271336363813258928102047231269701287744435751925676619466254380*a*e<sup>(393/2
 *I*c)</sup> + 2376911274978475170003732136235788181859385653298280842814105754624
 463620356183384021592556492451101686180*a*e^(391/2*I*c) + 89694641544949502
 177330565213252148914368338808468137749089095217876836694915316100191749797
 3558984634610*a*e^(389/2*I*c) + 3342951012915983944053859269749233418212344
 39369994003970291001771339818244857969207842071054369602329440*a*e<sup>(387/2*I
 *c)</sup> + 123047182591327239291059837803132990236560951541426091502592133079889
 930820736735976930062642304445760760*a*e^(385/2*I*c) + 44725891301479449618
 021653613021840925820831052165035724946865219883816662123055202896018457333
 687900560*a*e^(383/2*I*c) + 16053098146865366570614226985936175148028466685
 745428670334943691048314134627977747488782883668338627720*a*e^(381/2*I*c) +
 56890194714102202460669086034840043513299826737773542476615625242216921150
 06358198880775065147099437120*a*e^(379/2*I*c) + 199049069287552725866723649

901746078436630792328029665493173599795494114779320729904177508004614585384
0*a*e^(377/2*I*c) + 6875302244726200927911732813454970749262714300760196163
22085361144879429550058409565871621454060233940*a*e^(375/2*I*c) + 234420905
861667501673514951440275471821828319962595741737151538794823588356166465622
241503448244518760*a*e^(373/2*I*c) + 78892732475550629207160150619649518859
671333059425363870727604426538820615197072859753978543934017670*a*e^(371/2*
I*c) + 26204518312074558508652381319847496634719883744031651088258490824465
738576863175200241054655253538000*a*e^(369/2*I*c) + 85896500676149132331960
966040745974925665478224537657150819797250771562392932883772774994775420600
60*a*e^(367/2*I*c) + 277840530521434363427264247811536876216851365717201706
7030084143452583457313885656093029057631444370*a*e^(365/2*I*c) + 8867402319
255326645227710555534538499697441522560799980322092340080196221168157852905
76730828104950*a*e^(363/2*I*c) + 279213781240837306263149239277741278504180
764814216320768523126545112266943230869925045964255370200*a*e^(361/2*I*c) +
86731110812497060241964047703698781976493241635614918587219564911585800789
497248326704733021564620*a*e^(359/2*I*c) + 26574653865772705945516884258966
917928577399544926651101115471261281415662774645920805562286190810*a*e^(357
/2*I*c) + 80310253928842520641774509878790928329286080629908644617263295766
84085419875495358549722887030650*a*e^(355/2*I*c) + 239353645319019429757252
1642609485841145372047569451871481943673924084228578119992726364314043020*a
*e^(353/2*I*c) + 7034426197985873272565187719539626411298070061455526651676
89345204662084900828221826842429079160*a*e^(351/2*I*c) + 203840139420173350
646599863519636205648295919156240767205893957321010048179259062568208312088
790*a*e^(349/2*I*c) + 58233756699318390009909172367611895685773867826783550
649200500869687498091353927199571718405490*a*e^(347/2*I*c) + 16399645740773
051092975392309744789183304727620673733635277139746257521706440274686262794
198620*a*e^(345/2*I*c) + 45521645419857011583644408332021973604450148397109
56970770340240973634689769418208918160590320*a*e^(343/2*I*c) + 124529792545
542646768500367978320293088561760055771650047390965404535463879844865272608
6548710*a*e^(341/2*I*c) + 3356967207389365707067134036149299449122265075165
23422878616118942760950780230341688669154360*a*e^(339/2*I*c) + 891630246675
288443652575569469770689509198750404399603421886155423625509263453880796512
64340*a*e^(337/2*I*c) + 233308349176317150934664814200838330299476688805134
98966378137598401377993510086729958534960*a*e^(335/2*I*c) + 601348240394226
978137589513619272032123326663890549621486613297434076229844302555953699768
0*a*e^(333/2*I*c) + 1526557117570036622571918876778252127468437541774623614
619302679658497516840009278849352280*a*e^(331/2*I*c) + 38161965549422077466
7949833382475342723385533974486856243098750370767001096283055945502640*a*e^
(329/2*I*c) + 9393271320739526548689788036666489574109370069592434400920035
0068922566762007052749646040*a*e^(327/2*I*c) + 2276185042704166389046099666
1405723193407183947233163896700853295630681434965577357299920*a*e^(325/2*I*
c) + 5429216947618612587547123012480165658464605296922357528675368754249200
100593150293185330*a*e^(323/2*I*c) + 12744955210711387445832158356339512808
03025426238229370838062367497287195705487702727780*a*e^(321/2*I*c) + 294402
564240592061894969396868156932057924803903695357306824322577258371768427288

287660*a*e^(319/2*I*c) + 66907643424659838184545773924908879954701525687410
 376227199214504402274227244630158470*a*e^(317/2*I*c) + 14957789836260035053
 720288689121945297194278508615355005723695241121674541179126990690*a*e^(315
 /2*I*c) + 32888284950060987805251425692968001285477513476785385027429547425
 86652543592821521100*a*e^(313/2*I*c) + 711081673436185451924040793669141298
 969626722796614395742802361266093633641264085200*a*e^(311/2*I*c) + 15115491
 025736943469259673580887985158440423930779004760204127590244349529245966679
 0*a*e^(309/2*I*c) + 3158400566270860454470176476213179265096752402024294199
 2643782185239861413590496020*a*e^(307/2*I*c) + 6485888759441894264481925452
 577319190375803293665331722248553541369888292250431868*a*e^(305/2*I*c) + 13
 087047441032332449428327113794751616530660003712603610647654941537749293045
 42712*a*e^(303/2*I*c) + 259414105343758614384429045574352903181961134382680
 334119395735344643117364969828*a*e^(301/2*I*c) + 50504748951674956594241719
 165020500739646130041511438767337766766404018721799652*a*e^(299/2*I*c) + 96
 552069063089810510876615027724717296685509203961726169315466717341914306043
 48*a*e^(297/2*I*c) + 181210183042543630089172057262867836304235419855514772
 6701275150075546643583080*a*e^(295/2*I*c) + 3338050555933885674278082019269
 60940156572529198077839859476641115104007423532*a*e^(293/2*I*c) + 603374228
 21108905442188805237705380902156538573067777647643655256342216048188*a*e^(2
 91/2*I*c) + 106992860611465417411140377189131221667391294437839609204511396
 37868310953672*a*e^(289/2*I*c) + 186073250885758713283991952336630580493887
 4482315206088251791750125084597398*a*e^(287/2*I*c) + 3172905654382992959442
 27981890447073652423258243355469512774650547831808732*a*e^(285/2*I*c) + 530
 33862655943444530651471417311659093748967671410808199772545872566250240*a*e
 ^ (283/2*I*c) + 868653770595701574687455664750361197848557528657750073320150
 4080157038368*a*e^(281/2*I*c) + 1393822204759320545751155434910006366369348
 464487459381028813178904280462*a*e^(279/2*I*c) + 21902833599928625777636963
 4146613713106509647861704677410488454471041828*a*e^(277/2*I*c) + 3369654833
 2879373510255581378537814235558502314864144954037409976543435*a*e^(275/2*I*
 c) + 5073612140276869022785810697789013952147928175584559508062569934520500
 *a*e^(273/2*I*c) + 74738555150888980082572859246533346987051013753613027566
 5450524715478*a*e^(271/2*I*c) + 1076739255456741445886930960211126871364433
 22698905146090803710183409*a*e^(269/2*I*c) + 151653088004861664024727661229
 74437831930120746836471291819780637051*a*e^(267/2*I*c) + 208735594051295060
 8443105897698920410302162080067836820821771280502*a*e^(265/2*I*c) + 2806524
 29583871841898555612150376650333075795788342413441360641690*a*e^(263/2*I*c)
 + 36845376622432877842558414417003887278048975820601095126064678425*a*e^(2
 61/2*I*c) + 472112855150189813470234989967908258077951489216075374126192606
 2*a*e^(259/2*I*c) + 5901404087543110766870133152796383198273113772958000311
 21209656*a*e^(257/2*I*c) + 719284009116603764023203187516727101605184738050
 95916511887372*a*e^(255/2*I*c) + 854397411943340715997516624301302627999098
 5549262801022565070*a*e^(253/2*I*c) + 9885582686298142286009280473408648005
 88772545018200620775758*a*e^(251/2*I*c) + 111348526291800411107246584226952
 800505326877479452354210332*a*e^(249/2*I*c) + 12202571647480368478947339347
 197542554373660129766305670026*a*e^(247/2*I*c) + 13002734285943846321788049

15869348076492496040528026941982*a*e^(245/2*I*c) + 134633163152791085906227
 518959644714629575426941345274855*a*e^(243/2*I*c) + 13536481999851915706958
 049067829203397030845591800979948*a*e^(241/2*I*c) + 13206320181827340504344
 90769597960546607943021209554374*a*e^(239/2*I*c) + 124924620764273645058915
 750915973878220098322691297101*a*e^(237/2*I*c) + 11448615265227028920332020
 054872464737892837624091599*a*e^(235/2*I*c) + 10156027992975386818815023324
 38578214644838744212768*a*e^(233/2*I*c) + 871294507537624773100558274127043
 26695373797670990*a*e^(231/2*I*c) + 722195900839938067064852414689397978031
 0767551977*a*e^(229/2*I*c) + 5777566674391663473173612988278936860380310232
 01*a*e^(227/2*I*c) + 44561016227553011590271450799436600978561400862*a*e^(2
 25/2*I*c) + 3309571297229229358438719595781123324700153528*a*e^(223/2*I*c)
 + 236397938386786552298680619962344215977635255*a*e^(221/2*I*c) + 162172721
 97080856640086895687618882654237557*a*e^(219/2*I*c) + 106692577034751348639
 0409808978976432165366*a*e^(217/2*I*c) + 6720792097013372553856472690688276
 9461732*a*e^(215/2*I*c) + 4046550142768142823441656912104576646575*a*e^(213
 /2*I*c) + 232438908425726665209807096498775079690*a*e^(211/2*I*c) + 1271150
 2674847529112148726218999140522*a*e^(209/2*I*c) + 6603377963285978303957230
 99421064436*a*e^(207/2*I*c) + 32503673735785850571258452122985714*a*e^(205/
 2*I*c) + 1511798772576673047875435387479418*a*e^(203/2*I*c) + 6623860583779
 1619527049091465540*a*e^(201/2*I*c) + 2724467073240049286222217499520*a*e^(
 199/2*I*c) + 104787195000667085382630850098*a*e^(197/2*I*c) + 3751971173691
 706110924034857*a*e^(195/2*I*c) + 124427615397868362145478058*a*e^(193/2*I*
 c) + 3799316499714931382700630*a*e^(191/2*I*c) + 106072288048394569716987*a
 *e^(189/2*I*c) + 2685374380789029329793*a*e^(187/2*I*c) + 61031235925207244
 502*a*e^(185/2*I*c) + 1229848582862227068*a*e^(183/2*I*c) + 216305027135906
 67*a*e^(181/2*I*c) + 325270717497440*a*e^(179/2*I*c) + 4065883968718*a*e^(1
 77/2*I*c) + 40557446072*a*e^(175/2*I*c) + 302667508*a*e^(173/2*I*c) + 15020
 72*a*e^(171/2*I*c) + 3718*a*e^(169/2*I*c))/(e^(531*I*c) + 432*e^(530*I*c) +
 93096*e^(529*I*c) + 13343760*e^(528*I*c) + 1431118260*e^(527*I*c) + 122503
 723056*e^(526*I*c) + 8718181624155*e^(525*I*c) + 530563624556832*e^(524*I*c
) + 28186192554792138*e^(523*I*c) + 1327882849274858880*e^(522*I*c) + 56169
 444526926562260*e^(521*I*c) + 2154864144781257856128*e^(520*I*c) + 75599817
 092670157806639*e^(519*I*c) + 2442455629894502983849104*e^(518*I*c) + 73099
 207817335597247098038*e^(517*I*c) + 2037031259470368160131922320*e^(516*I*c
) + 53090127264630963470039804475*e^(515*I*c) + 129914664599324031816782653
 2288*e^(514*I*c) + 29952547749265499675257842032197*e^(513*I*c) + 652650253
 343206047453620559993840*e^(512*I*c) + 13477227799524701956579274210395326*
 e^(511*I*c) + 264410375780310742518099326419685040*e^(510*I*c) + 4939666610
 818025798809586352543471345*e^(509*I*c) + 880549275989414111458699508133880
 40256*e^(508*I*c) + 1500602747937397286405577818722691539392*e^(507*I*c) +
 24489837337812338687718622491865013839488*e^(506*I*c) + 3833601558010548245
 29764688213114368047154*e^(505*I*c) + 5764601046563151304213854710715346838
 447392*e^(504*I*c) + 83380839911837894453136303673785039051506805*e^(503*I*
 c) + 1161581413733971751533622511909046917188768400*e^(502*I*c) + 156039112
 77687607099721623771744933086920587272*e^(501*I*c) + 2023475097244621713139

66643580234078508179838320*e^(500*I*c) + 2535667460650279776834561566186591
 213109251642859*e^(499*I*c) + 307353665128305621609911663384900573080627625
 18496*e^(498*I*c) + 360688613036389349413809780004559963548775423325255*e^(
 497*I*c) + 4101545439937195793959956708442496709433800261224880*e^(496*I*c)
 + 45230940039830738332025694784646206844854827698075736*e^(495*I*c) + 4840
 93410240488718655917025303662581091659126182344528*e^(494*I*c) + 5032024903
 401451824074213943766011922026507006311982753*e^(493*I*c) + 508363695081710
 99437019348610847391946736185108017183136*e^(492*I*c) + 4994675065585317336
 71585862910572702811545035730398749530*e^(491*I*c) + 4775398607100853263534
 207733818266777478693412738731031680*e^(490*I*c) + 444567081752588210244009
 46210535004523775722190977468484496*e^(489*I*c) + 4032122259577981888408461
 39960995624144491271694336796459584*e^(488*I*c) + 3564764890628724017088487
 996688178929195787613958545474804845*e^(487*I*c) + 307362174043210099652310
 37419663053962881035281709221697785072*e^(486*I*c) + 2585853487159772701558
 29115684193411072034541491364393985491350*e^(485*I*c) + 2123702969188871318
 266718781223927067839949015727293884065388080*e^(484*I*c) + 170338860273906
 15741040977721655541665612162275485028584310890417*e^(483*I*c) + 1334902100
 52026183779673313868332303530332906163247194627808410304*e^(482*I*c) + 1022
 536437468296737293065862705246449693687415559865844306888705423*e^(481*I*c)
 + 7659010520187549651777118357676871927081898989131125755798204236112*e^(4
 80*I*c) + 56117081076341175384087570185188538660375932013674735519055227368
 366*e^(479*I*c) + 402349692266121158934003582839428785116904903936409545602
 519219664720*e^(478*I*c) + 282390515193658667838252570656445728029009869863
 8597987628380245881715*e^(477*I*c) + 19407979215594566593535008103303255257
 745408070082431338945184797463936*e^(476*I*c) + 130657660226560419335121434
 389938961884595434069984824307149332131747540*e^(475*I*c) + 861884851094991
 908764246805474672428603757315484453974713612812215428992*e^(474*I*c) + 557
 2551157328671121016216416307596161861955969011697222340926210112854418*e^(4
 73*I*c) + 35324447206779018115378052820789411687581004582367431006205879633
 729015200*e^(472*I*c) + 219601281339515561500261478844190024870555261281946
 058839614044697037963695*e^(471*I*c) + 133921437425424555356488440680194535
 3385000254030655765953770237607180089968*e^(470*I*c) + 80137295807907524343
 61964945761543761469520791210746972675870481058674277844*e^(469*I*c) + 4706
 5044611135158108487353367484243102698248838312635876283099427442745866704*e
 ^ (468*I*c) + 27136120750326657073448651707718101480177532218318105563861925
 7836143271472358*e^(467*I*c) + 15363332384449275835327345560164946716749165
 78907116984548489078241693926940560*e^(466*I*c) + 8543013441126212334833540
 665069621472479085838041360564550722036723654297540205*e^(465*I*c) + 466682
 23548266017806854592468100570289355960869613650856575756758180182223308768*
 e^(464*I*c) + 2505010286089283324693404568299020677122336444646027531599457
 27868485722395506952*e^(463*I*c) + 1321498055271300851429993866631619874424
 534425188183592049727687571032156435077280*e^(462*I*c) + 685299322314573668
 7328885311617795435592940841439866351079655652312894721972796266*e^(461*I*c
) + 34941071613276704649477943043339450201504075335160361865916029213860778
 606230624960*e^(460*I*c) + 175193170500618300241515632381912285157790097816

049220671217212220015297133400636060*e^(459*I*c) + 863979933622330349556296
820028395513198708064940505702126068652936800794826651264256*e^(458*I*c) +
419154250065682614809333941454415914396447847249231593180917185990211410900
5939942952*e^(457*I*c) + 20008006803030047137293278250321597113540716201983
333126349281186679153199068045257216*e^(456*I*c) + 939869153130681791490836
06065681482780836060510530154618486949839467131378859885998210*e^(455*I*c)
+ 4345466767802800453463444987638925407971751057568275155092970241876602993
45484920192480*e^(454*I*c) + 1977792980665818135651300094326239158605448870
806970860577325385028609983034534672318500*e^(453*I*c) + 886275214275695728
5681340885764904597935349569355321815647721172537159186491471311666400*e^(4
52*I*c) + 39108031255601809476537535369611844440844903751605645023514572352
045248104262933598850730*e^(451*I*c) + 169956327969929767773902096652629253
283704505477127544556534417376686540936706073847337600*e^(450*I*c) + 727521
010718394229291774073844694255798738667067535379759732795567942578751384250
780476310*e^(449*I*c) + 306797429643174736419815962396246367161700641962685
1426148418602934852907379021659761911840*e^(448*I*c) + 12747219616503320541
356343062562847368601622140856786025445814532037904111523242298235713300*e^(
447*I*c) + 521909122076618242158122718542697480712928432432278947692296907
20010547141334131610989636000*e^(446*I*c) + 2105943013856484711843290788803
17504953361839954159427434009884661777259752542647709150036990*e^(445*I*c)
+ 8375792069234119324587864867653735339465452397089907694887248139821891651
04589895518909256320*e^(444*I*c) + 3283874760555818676726309480306734420155
098583948074469014168171874442170109648521627538755920*e^(443*I*c) + 126934
969329649205650736736371812800885486825088802553372800650065661386960417973
53216584528640*e^(442*I*c) + 4837948975643409984385779181658937940681504260
9340378747586437145781646245422045101230417309900*e^(441*I*c) + 18183466140
617790131533012967714538116644918841319414116934435475492096903495261037894
5282257600*e^(440*I*c) + 67402553054313300889484577523662523745074311447354
4537818170447134607102575676676056675328961590*e^(439*I*c) + 24643821908074
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78728160*e^(438*I*c) + 888295028751024667044203837607976101480053134418614
474620767522824868911959884352666444917404000*e^(437*I*c) + 316266446747255
477311767956875276535713059699859236883921121649155532425732694909089895702
48533280*e^(436*I*c) + 1110341487970088194431438956444692422950498674643137
10969257619338899133799285616020069872611710850*e^(435*I*c) + 3846558420806
662744540630787848371749989490525009753221620033925499534135924615193651779
08682078400*e^(434*I*c) + 1315052120930692122102297105327622842335870743428
530891072983535862280094446607723473800477453914130*e^(433*I*c) + 443721091
784318234776434954444390469902005659506947084719361709211471407763307723497
2825351226979360*e^(432*I*c) + 14777955096617128998712745182071495362176506
973183081650233605274051677624970464340242755840025673760*e^(431*I*c) + 485
842581531402804473148368687721313901954124190467327784587060150968814370763
37910793584122475073760*e^(430*I*c) + 1576858455288509187214628778644350902
57583149415561323427386562894447598277935629800939237175625149830*e^(429*I*
c) + 5052936631230152588784830257388128132033977668453400653812610163534197

22382620393032535960660921950400*e^(428*I*c) + 1598771101058192692270528999
 677444742685631006232456185844925220144002305878120380828483988663574829100
 *e^(427*I*c) + 499524195627913818020518674440168802438827211392125566373495
 6946927571305533146776898787878059685108480*e^(426*I*c) + 15413111211486023
 937294970820797376716081344788163386543522421939737507962125854981881879168
 348260330000*e^(425*I*c) + 469702247271172818264540450180706705225597566275
 80347784535320014963482632359729444541885102274546002560*e^(424*I*c) + 1413
 799382535568432805655058074033041306061307254347517457940798331413617489176
 39986145377066437210546190*e^(423*I*c) + 4203580248351467985836112101459421
 54684437949365647899088372524802156222884839580011688655664280691773600*e^(
 422*I*c) + 1234668041892409978780018081755440216012582476396941937965899631
 953079203974222138794604328498972144766900*e^(421*I*c) + 358271800216329606
 141453670371510989710719825273928454614934310234845612408965742859494643866
 0859773886240*e^(420*I*c) + 10271602530202889002497813516849452590971512809
 529060665197301097052210064576088348023234671975463677418470*e^(419*I*c) +
 290976510612474534066475697818369100621655598523590528042591541656871254287
 52562385492373749486351714453120*e^(418*I*c) + 8145208141382911182887541756
 425005484603769331281148049290916075819598915576810702256835095386181594070
 4090*e^(417*I*c) + 22532053259322065776794110928951624899979452101556413498
 2827241710019675486694499689312466561907212627820000*e^(416*I*c) + 61600311
 602297958493712570175788721299835430098999136262803886109391456133207119190
 9714949426587936910303300*e^(415*I*c) + 16644750343872118093949177435029376
 38978574937754764763987835872410449930690131572904279995484581013965001440*
 e^(414*I*c) + 4445412259295474625067659514198312966015416299968930393345630
 345914109720740573618884980520010028451496996210*e^(413*I*c) + 117358569262
 451184931130910025016040323418769859990828235206722415302001882238263929823
 02194084667538488665600*e^(412*I*c) + 3062758105422195737839054728927760912
 9572813931082733520247387226000020043538279468776707958420892547870128680*e
 ^ (411*I*c) + 79019149558766569254783988487232388352909144982747171856772463
 223808993367091503402876467270176124342699654400*e^(410*I*c) + 201557947424
 794098026772478462040883395867512562930862943753568690084015585598010154781
 548625239409581907397500*e^(409*I*c) + 508324599301085460166978629683032661
 427654474082939048097939638391567298795788389433842285751054665210868287680
 *e^(408*I*c) + 126759701729481291340014627604212692998648029287019039910755
 4311079964227280196522475370108738477856311765699610*e^(407*I*c) + 31256834
 931787017434797047503074901786662921507201793636043351135286233296846063431
 85540756019935662148267863968*e^(406*I*c) + 7621788791912047062038840917799
 374600428892258194367636682944356096681400246312138001769285020661445991073
 249416*e^(405*I*c) + 183798070840033597660276492176211441160917355722166207
 88861535803449702273802588359076704241840733513439114113248*e^(404*I*c) + 4
 383497214291937768537869223302106374455403310092850273748043897897674698989
 5784070951905237783490374305934542955*e^(403*I*c) + 10339975546725743648984
 783764075375471820439447305579500146760432641987655555687382953173721109611
 5196005647730480*e^(402*I*c) + 24124602128244006179290831778303287619480159
 7133206052091286997043729145345755805710081489006741839439573984832678*e^(4

01*I*c) + 55675638871118234034102619273421954611365176831738053900589367904
9394714017063698565272728813669054779077208977840*e^(400*I*c) + 12710330829
380489502013605548312703426623439912775046123423003660250467417428565804452
89401786656311685859023084716*e^(399*I*c) + 2870496131412314451834674715353
589439553294430808531933466086288543709246230769151392180699413405623017247
753532944*e^(398*I*c) + 641338189585592518475823145106255638032859493851100
6577015218119786536390213057284018202201631094434819584025113465*e^(397*I*c
) + 14176483652875704957202013343241117904369977796653849959902524421980635
189011634815653279605497783382888932766730080*e^(396*I*c) + 310043192060694
170770693631414234874318280090981847446356786522841774394649416518125645191
44918003174108077634846014*e^(395*I*c) + 6709170613052966912501989921002157
658023784346222953538629508707618929784999593136064560529213096149610670752
1506432*e^(394*I*c) + 14365768713804479694294711970425953845881819942351682
4674586293691056119209866358123637772245409530799230553767222252*e^(393*I*c
) + 30438447110681333601028416012390637043388882849062742265255123696679091
6174520857759143930140187173492394981908258944*e^(392*I*c) + 63821889291453
374150580639966248855606678360049609187640837497487744897177803607499624558
1124283460438065182071976085*e^(391*I*c) + 13243113249840274283552229381476
823786728607088171617414487496895935880208608475087037023253203046498831206
84987556400*e^(390*I*c) + 2719589283483743926040805101080341921244530311254
607250929192773909331523226635035815672862569296693711643521070331394*e^(38
9*I*c) + 552749884903117835586123000932666828392629008215846711800069850271
9379939045918344222192742145711257028040974074674736*e^(388*I*c) + 11119499
645363201080881062824886338492425375658448977935535846349290425821570383090
425411418521516670371372045206568345*e^(387*I*c) + 221407350017086032709151
807692413916620355787559039791489092136038225547927491835171602555719158753
56439553717130797888*e^(386*I*c) + 4363830007581517102594621146446568961896
577348866098585794565747985408510885185791122291198983761545260851235600840
0295*e^(385*I*c) + 85139533234786455779589959464900637760735729705621221380
005837208369157794673049675428799817875431430246332625899630160*e^(384*I*c)
+ 164437500676906892741323260154394278503954561936020133596581806449357240
277349447927334517105809995300093549279931273178*e^(383*I*c) + 314409035808
225861565595436938354945445473991043129722046747228813030925204968503418566
818838611866709807040793495364496*e^(382*I*c) + 595157615500431514947479282
336547053827926087916425263497187757029413385471835434198246807096214536895
441388306027237899*e^(381*I*c) + 111539828554560155053332804560018431799389
935921799672981834070422180119566741048584699617905673355851245223858316079
2512*e^(380*I*c) + 20696982895008606434616657623738079575130194240411788719
04960551829412449344722432125679417958403007551179298315947373776*e^(379*I*
c) + 3802604996705891106964620633848964807037098854510182263243030597295630
760353597531974324752266389193185760878274188013440*e^(378*I*c) + 691783894
521484427849333045936139492337233385361987963737267318494285971243106634572
6870422099893124890777678037369988150*e^(377*I*c) + 12462140440537258084928
596709872066857757070943124868554500948154756863454308032925408340311237850
017814707896986969086816*e^(376*I*c) + 222313411318015353454063990377216868

402083979419525801355846459667467366567162715548264762829910660765649214326
14339399735*e^(375*I*c) + 3927420041432986116939794451622500108122743339858
5007206399231211907157795359719648241598754266579840244551491476467899952*e
^(374*I*c) + 68712466015985641512468586173659747734879591710098354652786124
936023073943141049573606648563005359411712764895683903806088*e^(373*I*c) +
119060591849660546834765693227676449067584148248882678447504826077236333444
513454095126668750057295811191643356908972191440*e^(372*I*c) + 204325557265
186000767402710230847896459761583922763698235433212833313077783041040074669
379017394836761539649081690630811665*e^(371*I*c) + 347310053810935290419455
560555957314129569210735745983234369659976413374774078000173070075248654524
917179128950507443058208*e^(370*I*c) + 584749573682304586179384628844883327
581498969886540380378896767999075614964007174600811092945356635118795824799
369716742109*e^(369*I*c) + 975210339444049318757282311763517786673223175594
457946383279264635085041004917300295904275433144848532459919875479817581584
*e^(368*I*c) + 161109254140006052595485937526419417834764347183707820144626
2435615142944587337833513586022729849523358436493586042252995608*e^(367*I*c
) + 26366624104307993404475222847782442837407510686581407265764466712077983
25606832295937705061686297930296382338892574900819440*e^(366*I*c) + 4274826
907720591752526711336820871500844345647922385471534359333606189571832444641
364132893108663576205133870672156264164115*e^(365*I*c) + 686642533751866826
266269375090895696573292457814218163062215780289988087468155103113631406419
9948604001529894566235238597088*e^(364*I*c) + 10927210603473544810279792347
844536074588896806230041110089544731605863146104181739039426674855453466097
402330688331845602302*e^(363*I*c) + 172295028243676473344007219984175967039
486573947388052093916365973703805723989647150800953668183220291521936358697
84095333760*e^(362*I*c) + 2691779470108661509789012023689050110514679999602
177519571086622622863898470345683269415323061160726344418350102619856341961
6*e^(361*I*c) + 41670440375390543643418219342271748040035071490119080585281
522498188818375906900368701234531304633163446319945130196476913600*e^(360*I
*c) + 639230194337619890906148012886350981231994451023031226165446482089987
67803944455777042886552738499747183713136069104651812215*e^(359*I*c) + 9717
305502474268005861672246136889266114129554026349301303274608353615732426833
3390400308958318370219154887169702257444756176*e^(358*I*c) + 14639044845635
118121823738274037412419166481999774698807659839186273362967014224154637553
3903130605297580105675355629160198162*e^(357*I*c) + 21856316665964931224748
364095627214921249911538382877102965428336397258511809047941369663810815638
5244646591328454425745117584*e^(356*I*c) + 32341317801484100371415113824607
915257636097603505845789040993773872317153703657304368199716374531360240013
9153046673668433091*e^(355*I*c) + 47432304356310054237733862993196624812917
597698233244601805600939115402043889590314082296776949401944616677995402465
5344116288*e^(354*I*c) + 68951844932879355990326041814997419025357834005889
503558960646824468059155611817030400503756366988005790876589894926861477228
5*e^(353*I*c) + 99355565364952112722644396082023364938648851008189254586670
0444096661582790441241830855609577062039555625090943332264901780720*e^(352*
I*c) + 14191644814221765738582340138989996288223223333095737307163107438389

35832201454293617293175086458389621425307051750612129761498*e^(351*I*c) + 2
009496110092687738152782085683737222727968824299058739215446083499351467625
334449670757764066690656150949014944043994822823920*e^(350*I*c) + 282081929
856221595910752980728962844962138679898943636939311606989401878120100027563
3104498398959346631795568022519974400130281*e^(349*I*c) + 39256976584157783
527681039428560118402116427696217172179966143984738871860743914826385472128
26538270453912634540299792270321024*e^(348*I*c) + 5416662804052436349585595
982818357953866258461644354018205158917742576425344364964596750653177677803
492186817305171175032011500*e^(347*I*c) + 741037261289115222436463329612804
397165719328032775430438223567277378140302381412761035556250527196904517770
4726054907145784960*e^(346*I*c) + 10052209524369581827588154985345549678031
445744499985208259385435609740272400301454246872041775159838468077381562338
745636398374*e^(345*I*c) + 135212304119454369155588547068517965415673998116
568700215673529753268154678178465332891238716960561952316961461627209922217
60992*e^(344*I*c) + 1803532733817745547117756859485168297797834644977719357
2087688510392426884519272991560851326393852241961470040819793627127923997*e
^(343*I*c) + 23856398565562802030695278174212640833282154174006459458292644
060947924907313735921561690153939906017518647182491616573724049744*e^(342*I
*c) + 312952636881898383137727758733072603341172272586295019923586956360926
62866062819845689064235813622974150120668921391878398978380*e^(341*I*c) + 4
071598896370191895002034833673642342051331135901048524691907465288339397080
5374470830156229705647312265477584256027212762941040*e^(340*I*c) + 52539223
346740771142587092370257069536060319644439501667610482767955800276052892432
152798814607975110366224945081428121888473324*e^(339*I*c) + 672440879690807
038237032571996630476068906104820904946194928029351302159798194699663833367
88693900139115594646893784095418472336*e^(338*I*c) + 8536811843021531284823
129173967373588774620185166629960039219941876475008682819871987274404776778
3667325326289221881974987582215*e^(337*I*c) + 10750473740657691612348039916
975963332832140741940051001749884983059862156542826654631593392082152754472
6380201659114903834605888*e^(336*I*c) + 13429774202347942990462961610455961
009607475872106802270446893806301705968802336343645897153496466503631988911
9229809973806909680*e^(335*I*c) + 16643233292258919513055832926639875338982
395573759852755609609306247355976954577232197896931890419257273399788823098
6469005970880*e^(334*I*c) + 20462229553572910951982991678986722531942970516
256008264884039496542311280933659192129030939239626383467436897784052714703
7426908*e^(333*I*c) + 24959307228256586639838995150961920268263445548712871
463146189177082320136752764579377020378878467734393497142431798789525503193
6*e^(332*I*c) + 30206063803086846346113944227936049971890691748252489448335
6220196138377050828911383056860425370161157201493696073712322595776808*e^(3
31*I*c) + 36270630756384323113569915741851073245242061401362416887931218764
5233450153927975793326834780741391203430153093712635355523960320*e^(330*I*c
) + 43214785646408693802381156180867858959470204790467428229795965880017098
4456799067751878044806619012452636891350731618278545690160*e^(329*I*c) + 51
09076151113450745262384614713714145027572244431638553164842923085168663582
7717488464500331623385777400744950538410637735936000*e^(328*I*c) + 59937848

477173347480937614240155485020706497211813757294925750365144454193930902527
6896049622515630263162184526394317285457368300*e^(327*I*c) + 69778910692592
461481671374768468278508365981902795224444704335586974136850045256116402463
6073401929693105801105522738405349028160*e^(326*I*c) + 80616967132762553242
457534008977573368199499157667444692235409971461519208544324566385225700111
5644286660304979476023966071898200*e^(325*I*c) + 92432005286752258403577749
576107235122253442078486196000182102050946814635675643379524644649139614158
3854513687104334429566707520*e^(324*I*c) + 10517821004288343719445081700512
191871168163497669533226371821496108750042237847841832849619064944229554624
31208645690802526770780*e^(323*I*c) + 1187817943079390316108802324798110129
020822782660087248599367643481200206046822166144285425922229375413676535071
141005286431481600*e^(322*I*c) + 133139611462672303580246212353158205033974
999601415245283530595636742537022275862175392245872752485607295088096065756
4720475838500*e^(321*I*c) + 14811871170892466624669556949656778555247303132
605526908216026571762187374262455227953298914640910058783043040759536935467
67206080*e^(320*I*c) + 1635569744641421900657886381289076653227580172056677
587451402069234355283687489659613913761959140773339736014790081814516625224
440*e^(319*I*c) + 179264907808963229893694572848196933496439159750628508848
8350622937252533420980803144316431701452190522716124797875257437516360640*e
^(318*I*c) + 19502865507801819192449929612044870100564603628452185016744237
66266321558791436917317878702232679213868287926294665202769722927380*e^(317
*I*c) + 2106141903468344307112549761202484543402794252352482199817410424869
677262715098288437646518683487945462774223656471345899082156800*e^(316*I*c)
+ 225772621910385628681283301268157376549626224142061293207614315117196085
4554124144699023009842080515157923529357189869943515991200*e^(315*I*c) + 24
024645955696860861200018030342110567394455886219461413841061628862461618151
49763025030834875234067267774023433418269982431265280*e^(314*I*c) + 2537766
415465030330815471746692988596069911894697225052928320452542175587154848096
483331209807430113943015398362669673337957755720*e^(313*I*c) + 266110064797
578358382823513920144193017839664338342390358386254725588077238204920101553
7214900832745601519737141849802506685264000*e^(312*I*c) + 27700732071507686
455975072813820654979249684660545274141223398273337837700683058834873099793
15983718403740872884345746380680204260*e^(311*I*c) + 2862503126320461797770
667780725644184991255623174626175679050672100848988119391841466573417019247
590580735265143427289340450811200*e^(310*I*c) + 293649421435186849870323945
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0712723836983468866230908800*e^(309*I*c) + 29904989496225436085381293802838
663351900871151248588181437876191869571119037657239748996515185551449242903
46242595167274383008960*e^(308*I*c) + 3023371643508225027175603175212953219
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977286601165100620*e^(307*I*c) + 303440835553095707578773174532256798168461
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5660465787520*e^(306*I*c) + 30233716435082250271756031752129532190224850458
507318453075190082773851547314612133880355791599175900623435274649772866011
65100620*e^(305*I*c) + 2990498949622543608538129380283866335190087115124858

818143787619186957111903765723974899651518555144924290346242595167274383008
960*e^(304*I*c) + 293649421435186849870323945542677110434482730626755891655
0877467232455153286140521089582733932202553130712723836983468866230908800*e
^(303*I*c) + 28625031263204617977706677807256441849912556231746261756790506
72100848988119391841466573417019247590580735265143427289340450811200*e^(302
*I*c) + 2770073207150768645597507281382065497924968466054527414122339827333
783770068305883487309979315983718403740872884345746380680204260*e^(301*I*c)
+ 266110064797578358382823513920144193017839664338342390358386254725588077
2382049201015537214900832745601519737141849802506685264000*e^(300*I*c) + 25
377664154650303308154717466929885960699118946972250529283204525421755871548
48096483331209807430113943015398362669673337957755720*e^(299*I*c) + 2402464
595569686086120001803034211056739445588621946141384106162886246161815149763
025030834875234067267774023433418269982431265280*e^(298*I*c) + 225772621910
385628681283301268157376549626224142061293207614315117196085455412414469902
3009842080515157923529357189869943515991200*e^(297*I*c) + 21061419034683443
071125497612024845434027942523524821998174104248696772627150982884376465186
83487945462774223656471345899082156800*e^(296*I*c) + 1950286550780181919244
992961204487010056460362845218501674423766266321558791436917317878702232679
213868287926294665202769722927380*e^(295*I*c) + 179264907808963229893694572
848196933496439159750628508848835062293725253342098080314431643170145219052
2716124797875257437516360640*e^(294*I*c) + 16355697446414219006578863812890
766532275801720566775874514020692343552836874896596139137619591407733397360
14790081814516625224440*e^(293*I*c) + 1481187117089246662466955694965677855
524730313260552690821602657176218737426245522795329891464091005878304304075
953693546767206080*e^(292*I*c) + 133139611462672303580246212353158205033974
999601415245283530595636742537022275862175392245872752485607295088096065756
4720475838500*e^(291*I*c) + 11878179430793903161088023247981101290208227826
600872485993676434812002060468221661442854259222293754136765350711410052864
31481600*e^(290*I*c) + 1051782100428834371944508170051219187116816349766953
322637182149610875004223784784183284961906494422955462431208645690802526770
780*e^(289*I*c) + 924320052867522584035777495761072351222534420784861960001
821020509468146356756433795246446491396141583854513687104334429566707520*e
^(288*I*c) + 806169671327625532424575340089775733681994991576674446922354099
714615192085443245663852257001115644286660304979476023966071898200*e^(287*I
*c) + 697789106925924614816713747684682785083659819027952244447043355869741
368500452561164024636073401929693105801105522738405349028160*e^(286*I*c) +
599378484771733474809376142401554850207064972118137572949257503651444541939
309025276896049622515630263162184526394317285457368300*e^(285*I*c) + 510907
615111134507452623846147137141450275722444316385531648429230851686635827717
488464500331623385777400744950538410637735936000*e^(284*I*c) + 432147856464
086938023811561808678589594702047904674282297959658800170984456799067751878
044806619012452636891350731618278545690160*e^(283*I*c) + 362706307563843231
135699157418510732452420614013624168879312187645233450153927975793326834780
741391203430153093712635355523960320*e^(282*I*c) + 302060638030868463461139
442279360499718906917482524894483356220196138377050828911383056860425370161

157201493696073712322595776808*e^(281*I*c) + 249593072282565866398389951509
619202682634455487128714631461891770823201367527645793770203788784677343934
971424317987895255031936*e^(280*I*c) + 204622295535729109519829916789867225
319429705162560082648840394965423112809336591921290309392396263834674368977
840527147037426908*e^(279*I*c) + 166432332922589195130558329266398753389823
955737598527556096093062473559769545772321978969318904192572733997888230986
469005970880*e^(278*I*c) + 134297742023479429904629616104559610096074758721
068022704468938063017059688023363436458971534964665036319889119229809973806
909680*e^(277*I*c) + 107504737406576916123480399169759633328321407419400510
017498849830598621565428266546315933920821527544726380201659114903834605888
*e^(276*I*c) + 853681184302153128482312917396737358877462018516662996003921
99418764750086828198719872744047767783667325326289221881974987582215*e^(275
*I*c) + 6724408796908070382370325719966304760689061048209049461949280293513
0215979819469966383336788693900139115594646893784095418472336*e^(274*I*c) +
52539223346740771142587092370257069536060319644439501667610482767955800276
052892432152798814607975110366224945081428121888473324*e^(273*I*c) + 407159
889637019189500203483367364234205133113590104852469190746528833939708053744
70830156229705647312265477584256027212762941040*e^(272*I*c) + 3129526368818
983831377277587330726033411722725862950199235869563609266286606281984568906
4235813622974150120668921391878398978380*e^(271*I*c) + 23856398565562802030
695278174212640833282154174006459458292644060947924907313735921561690153939
906017518647182491616573724049744*e^(270*I*c) + 180353273381774554711775685
948516829779783464497771935720876885103924268845192729915608513263938522419
61470040819793627127923997*e^(269*I*c) + 1352123041194543691555885470685179
654156739981165687002156735297532681546781784653328912387169605619523169614
6162720992221760992*e^(268*I*c) + 10052209524369581827588154985345549678031
445744499985208259385435609740272400301454246872041775159838468077381562338
745636398374*e^(267*I*c) + 741037261289115222436463329612804397165719328032
775430438223567277378140302381412761035556250527196904517770472605490714578
4960*e^(266*I*c) + 54166628040524363495855959828183579538662584616443540182
05158917742576425344364964596750653177677803492186817305171175032011500*e^(
265*I*c) + 3925697658415778352768103942856011840211642769621717217996614398
473887186074391482638547212826538270453912634540299792270321024*e^(264*I*c)
+ 282081929856221595910752980728962844962138679898943636939311606989401878
1201000275633104498398959346631795568022519974400130281*e^(263*I*c) + 20094
961100926877381527820856837372227279688242990587392154460834993514676253344
49670757764066690656150949014944043994822823920*e^(262*I*c) + 1419164481422
17657385823401389899962882232233309573730716310743838935832201454293617293
175086458389621425307051750612129761498*e^(261*I*c) + 993555653649521127226
443960820233649386488510081892545866700444096661582790441241830855609577062
039555625090943332264901780720*e^(260*I*c) + 689518449328793559903260418149
974190253578340058895035589606468244680591556118170304005037563669880057908
765898949268614772285*e^(259*I*c) + 474323043563100542377338629931966248129
175976982332446018056009391154020438895903140822967769494019446166779954024
655344116288*e^(258*I*c) + 323413178014841003714151138246079152576360976035

058457890409937738723171537036573043681997163745313602400139153046673668433
091*e^(257*I*c) + 218563166659649312247483640956272149212499115383828771029
654283363972585118090479413696638108156385244646591328454425745117584*e^(25
6*I*c) + 146390448456351181218237382740374124191664819997746988076598391862
733629670142241546375533903130605297580105675355629160198162*e^(255*I*c) +
971730550247426800586167224613688926611412955402634930130327460835361573242
68333390400308958318370219154887169702257444756176*e^(254*I*c) + 6392301943
376198909061480128863509812319944510230312261654464820899876780394445577704
2886552738499747183713136069104651812215*e^(253*I*c) + 41670440375390543643
418219342271748040035071490119080585281522498188818375906900368701234531304
633163446319945130196476913600*e^(252*I*c) + 269177947010866150978901202368
905011051467999960217751957108662262286389847034568326941532306116072634441
83501026198563419616*e^(251*I*c) + 1722950282436764733440072199841759670394
865739473880520939163659737038057239896471508009536681832202915219363586978
4095333760*e^(250*I*c) + 10927210603473544810279792347844536074588896806230
041110089544731605863146104181739039426674855453466097402330688331845602302
*e^(249*I*c) + 686642533751866826266269375090895696573292457814218163062215
7802899880874681551031136314064199948604001529894566235238597088*e^(248*I*c
) + 42748269077205917525267113368208715008443456479223854715343593336061895
71832444641364132893108663576205133870672156264164115*e^(247*I*c) + 2636662
410430799340447522284778244283740751068658140726576446671207798325606832295
937705061686297930296382338892574900819440*e^(246*I*c) + 161109254140006052
595485937526419417834764347183707820144626243561514294458733783351358602272
9849523358436493586042252995608*e^(245*I*c) + 97521033944404931875728231176
351778667322317559445794638327926463508504100491730029590427543314484853245
9919875479817581584*e^(244*I*c) + 58474957368230458617938462884488332758149
896988654038037889676799907561496400717460081109294535663511879582479936971
6742109*e^(243*I*c) + 34731005381093529041945556055595731412956921073574598
3234369659976413374774078000173070075248654524917179128950507443058208*e^(2
42*I*c) + 20432555726518600076740271023084789645976158392276369823543321283
3313077783041040074669379017394836761539649081690630811665*e^(241*I*c) + 11
906059184966054683476569322767644906758414824888267844750482607723633344451
3454095126668750057295811191643356908972191440*e^(240*I*c) + 68712466015985
641512468586173659747734879591710098354652786124936023073943141049573606648
563005359411712764895683903806088*e^(239*I*c) + 392742004143298611693979445
162250010812274333985850072063992312119071577953597196482415987542665798402
44551491476467899952*e^(238*I*c) + 2223134113180153534540639903772168684020
839794195258013558464596674673665671627155482647628299106607656492143261433
9399735*e^(237*I*c) + 12462140440537258084928596709872066857757070943124868
554500948154756863454308032925408340311237850017814707896986969086816*e^(23
6*I*c) + 691783894521484427849333045936139492337233385361987963737267318494
2859712431066345726870422099893124890777678037369988150*e^(235*I*c) + 38026
049967058911069646206338489648070370988545101822632430305972956307603535975
31974324752266389193185760878274188013440*e^(234*I*c) + 2069698289500860643
461665762373807957513019424041178871904960551829412449344722432125679417958

403007551179298315947373776*e^(233*I*c) + 111539828554560155053332804560018
431799389935921799672981834070422180119566741048584699617905673355851245223
8583160792512*e^(232*I*c) + 59515761550043151494747928233654705382792608791
6425263497187757029413385471835434198246807096214536895441388306027237899*e
^(231*I*c) + 31440903580822586156559543693835494544547399104312972204674722
8813030925204968503418566818838611866709807040793495364496*e^(230*I*c) + 16
443750067690689274132326015439427850395456193602013359658180644935724027734
9447927334517105809995300093549279931273178*e^(229*I*c) + 85139533234786455
779589959464900637760735729705621221380005837208369157794673049675428799817
875431430246332625899630160*e^(228*I*c) + 436383000758151710259462114644656
896189657734886609858579456574798540851088518579112229119898376154526085123
56008400295*e^(227*I*c) + 2214073500170860327091518076924139166203557875590
3979148909213603822554792749183517160255571915875356439553717130797888*e^(2
26*I*c) + 11119499645363201080881062824886338492425375658448977935535846349
290425821570383090425411418521516670371372045206568345*e^(225*I*c) + 552749
884903117835586123000932666828392629008215846711800069850271937993904591834
4222192742145711257028040974074674736*e^(224*I*c) + 27195892834837439260408
051010803419212445303112546072509291927739093315232266350358156728625692966
93711643521070331394*e^(223*I*c) + 1324311324984027428355222938147682378672
860708817161741448749689593588020860847508703702325320304649883120684987556
400*e^(222*I*c) + 638218892914533741505806399662488556066783600496091876408
374974877448971778036074996245581124283460438065182071976085*e^(221*I*c) +
304384471106813336010284160123906370433888828490627422652551236966790916174
520857759143930140187173492394981908258944*e^(220*I*c) + 143657687138044796
942947119704259538458818199423516824674586293691056119209866358123637772245
409530799230553767222252*e^(219*I*c) + 670917061305296691250198992100215765
802378434622295353862950870761892978499959313606456052921309614961067075215
06432*e^(218*I*c) + 3100431920606941707706936314142348743182800909818474463
5678652284177439464941651812564519144918003174108077634846014*e^(217*I*c) +
14176483652875704957202013343241117904369977796653849959902524421980635189
011634815653279605497783382888932766730080*e^(216*I*c) + 641338189585592518
475823145106255638032859493851100657701521811978653639021305728401820220163
1094434819584025113465*e^(215*I*c) + 28704961314123144518346747153535894395
532944308085319334660862885437092462307691513921806994134056230172477535329
44*e^(214*I*c) + 1271033082938048950201360554831270342662343991277504612342
300366025046741742856580445289401786656311685859023084716*e^(213*I*c) + 556
756388711182340341026192734219546113651768317380539005893679049394714017063
698565272728813669054779077208977840*e^(212*I*c) + 241246021282440061792908
317783032876194801597133206052091286997043729145345755805710081489006741839
439573984832678*e^(211*I*c) + 103399755467257436489847837640753754718204394
473055795001467604326419876555556873829531737211096115196005647730480*e^(21
0*I*c) + 438349721429193776853786922330210637445540331009285027374804389789
76746989895784070951905237783490374305934542955*e^(209*I*c) + 1837980708400
335976602764921762114411609173557221662078886153580344970227380258835907670
4241840733513439114113248*e^(208*I*c) + 76217887919120470620388409177993746

004288922581943676366829443560966814002463121380017692850206614459910732494
16*e^(207*I*c) + 3125683493178701743479704750307490178666292150720179363604
335113528623329684606343185540756019935662148267863968*e^(206*I*c) + 126759
701729481291340014627604212692998648029287019039910755431107996422728019652
2475370108738477856311765699610*e^(205*I*c) + 50832459930108546016697862968
303266142765447408293904809793963839156729879578838943384228575105466521086
8287680*e^(204*I*c) + 20155794742479409802677247846204088339586751256293086
2943753568690084015585598010154781548625239409581907397500*e^(203*I*c) + 79
019149558766569254783988487232388352909144982747171856772463223808993367091
503402876467270176124342699654400*e^(202*I*c) + 306275810542219573783905472
892776091295728139310827335202473872260000200435382794687767079584208925478
70128680*e^(201*I*c) + 1173585692624511849311309100250160403234187698599908
2823520672241530200188223826392982302194084667538488665600*e^(200*I*c) + 44
454122592954746250676595141983129660154162999689303933456303459141097207405
73618884980520010028451496996210*e^(199*I*c) + 1664475034387211809394917743
502937638978574937754764763987835872410449930690131572904279995484581013965
001440*e^(198*I*c) + 616003116022979584937125701757887212998354300989991362
628038861093914561332071191909714949426587936910303300*e^(197*I*c) + 225320
532593220657767941109289516248999794521015564134982827241710019675486694499
689312466561907212627820000*e^(196*I*c) + 814520814138291118288754175642500
54846037693312811480492909160758195989155768107022568350953861815940704090*
e^(195*I*c) + 2909765106124745340664756978183691006216555985235905280425915
4165687125428752562385492373749486351714453120*e^(194*I*c) + 10271602530202
889002497813516849452590971512809529060665197301097052210064576088348023234
671975463677418470*e^(193*I*c) + 358271800216329606141453670371510989710719
8252739284546149343102348456124089657428594946438660859773886240*e^(192*I*c
) + 12346680418924099787800180817554402160125824763969419379658996319530792
03974222138794604328498972144766900*e^(191*I*c) + 4203580248351467985836112
101459421546844379493656478990883725248021562228848395800116886556642806917
73600*e^(190*I*c) + 1413799382535568432805655058074033041306061307254347517
45794079833141361748917639986145377066437210546190*e^(189*I*c) + 4697022472
711728182645404501807067052255975662758034778453532001496348263235972944454
1885102274546002560*e^(188*I*c) + 15413111211486023937294970820797376716081
344788163386543522421939737507962125854981881879168348260330000*e^(187*I*c)
+ 499524195627913818020518674440168802438827211392125566373495694692757130
55331467768987878059685108480*e^(186*I*c) + 15987711010581926922705289996
77444742685631006232456185844925220144002305878120380828483988663574829100*
e^(185*I*c) + 5052936631230152588784830257388128132033977668453400653812610
16353419722382620393032535960660921950400*e^(184*I*c) + 1576858455288509187
214628778644350902575831494155613234273865628944475982779356298009392371756
25149830*e^(183*I*c) + 4858425815314028044731483686877213139019541241904673
2778458706015096881437076337910793584122475073760*e^(182*I*c) + 14777955096
617128998712745182071495362176506973183081650233605274051677624970464340242
755840025673760*e^(181*I*c) + 443721091784318234776434954444390469902005659
5069470847193617092114714077633077234972825351226979360*e^(180*I*c) + 13150

521209306921221022971053276228423358707434285308910729835358622800944466077
23473800477453914130*e^(179*I*c) + 3846558420806662744540630787848371749989
49052500975322162003392549953413592461519365177908682078400*e^(178*I*c) + 1
110341487970088194431438956444692422950498674643137109692576193388991337992
85616020069872611710850*e^(177*I*c) + 3162664467472554773117679568752765357
1305969985923688392112164915553242573269490908989570248533280*e^(176*I*c) +
88882950287510246670442038376079761014800531344186144746207675228248689119
59884352666444917404000*e^(175*I*c) + 2464382190807439609079774226855679629
367885709776435876630851716253962696192341706239192878728160*e^(174*I*c) +
674025530543133008894845775236625237450743114473544537818170447134607102575
676676056675328961590*e^(173*I*c) + 181834661406177901315330129677145381166
449188413194141169344354754920969034952610378945282257600*e^(172*I*c) + 483
794897564340998438577918165893794068150426093403787475864371457816462454220
45101230417309900*e^(171*I*c) + 1269349693296492056507367363718128008854868
2508880255337280065006566138696041797353216584528640*e^(170*I*c) + 32838747
605558186767263094803067344201550985839480744690141681718744421701096485216
27538755920*e^(169*I*c) + 8375792069234119324587864867653735339465452397089
90769488724813982189165104589895518909256320*e^(168*I*c) + 2105943013856484
711843290788803175049533618399541594274340098846617772597525426477091500369
90*e^(167*I*c) + 5219091220766182421581227185426974807129284324322789476922
9690720010547141334131610989636000*e^(166*I*c) + 12747219616503320541356343
062562847368601622140856786025445814532037904111523242298235713300*e^(165*I
*c) + 306797429643174736419815962396246367161700641962685142614841860293485
2907379021659761911840*e^(164*I*c) + 72752101071839422929177407384469425579
8738667067535379759732795567942578751384250780476310*e^(163*I*c) + 16995632
796992976777390209665262925328370450547712754455653441737668654093670607384
7337600*e^(162*I*c) + 39108031255601809476537535369611844440844903751605645
023514572352045248104262933598850730*e^(161*I*c) + 886275214275695728568134
0885764904597935349569355321815647721172537159186491471311666400*e^(160*I*c
) + 19777929806658181356513000943262391586054488708069708605773253850286099
83034534672318500*e^(159*I*c) + 4345466767802800453463444987638925407971751
05756827515509297024187660299345484920192480*e^(158*I*c) + 9398691531306817
9149083606065681482780836060510530154618486949839467131378859885998210*e^(1
57*I*c) + 20008006803030047137293278250321597113540716201983333126349281186
679153199068045257216*e^(156*I*c) + 419154250065682614809333941454415914396
4478472492315931809171859902114109005939942952*e^(155*I*c) + 86397993362233
0349556296820028395513198708064940505702126068652936800794826651264256*e^(1
54*I*c) + 17519317050061830024151563238191228515779009781604922067121721222
0015297133400636060*e^(153*I*c) + 34941071613276704649477943043339450201504
075335160361865916029213860778606230624960*e^(152*I*c) + 685299322314573668
7328885311617795435592940841439866351079655652312894721972796266*e^(151*I*c
) + 13214980552713008514299938666316198744245344251881835920497276875710321
56435077280*e^(150*I*c) + 2505010286089283324693404568299020677122336444646
02753159945727868485722395506952*e^(149*I*c) + 4666822354826601780685459246
8100570289355960869613650856575756758180182223308768*e^(148*I*c) + 85430134

41126212334833540665069621472479085838041360564550722036723654297540205*e^(147*I*c) + 1536333238444927583532734556016494671674916578907116984548489078
 241693926940560*e^(146*I*c) + 271361207503266570734486517077181014801775322
 183181055638619257836143271472358*e^(145*I*c) + 470650446111351581084873533
 67484243102698248838312635876283099427442745866704*e^(144*I*c) + 8013729580
 790752434361964945761543761469520791210746972675870481058674277844*e^(143*I
 *c) + 133921437425424555356488440680194535338500025403065576595377023760718
 0089968*e^(142*I*c) + 21960128133951556150026147884419002487055526128194605
 8839614044697037963695*e^(141*I*c) + 35324447206779018115378052820789411687
 581004582367431006205879633729015200*e^(140*I*c) + 557255115732867112101621
 6416307596161861955969011697222340926210112854418*e^(139*I*c) + 86188485109
 4991908764246805474672428603757315484453974713612812215428992*e^(138*I*c) +
 130657660226560419335121434389938961884595434069984824307149332131747540*e
 ^ (137*I*c) + 19407979215594566593535008103303255257745408070082431338945184
 797463936*e^(136*I*c) + 282390515193658667838252570656445728029009869863859
 7987628380245881715*e^(135*I*c) + 40234969226612115893400358283942878511690
 4903936409545602519219664720*e^(134*I*c) + 56117081076341175384087570185188
 538660375932013674735519055227368366*e^(133*I*c) + 765901052018754965177711
 8357676871927081898989131125755798204236112*e^(132*I*c) + 10225364374682967
 37293065862705246449693687415559865844306888705423*e^(131*I*c) + 1334902100
 52026183779673313868332303530332906163247194627808410304*e^(130*I*c) + 1703
 3886027390615741040977721655541665612162275485028584310890417*e^(129*I*c) +
 2123702969188871318266718781223927067839949015727293884065388080*e^(128*I*
 c) + 258585348715977270155829115684193411072034541491364393985491350*e^(127
 *I*c) + 30736217404321009965231037419663053962881035281709221697785072*e^(1
 26*I*c) + 3564764890628724017088487996688178929195787613958545474804845*e^(
 125*I*c) + 403212225957798188840846139960995624144491271694336796459584*e^(
 124*I*c) + 44456708175258821024400946210535004523775722190977468484496*e^(1
 23*I*c) + 4775398607100853263534207733818266777478693412738731031680*e^(122
 *I*c) + 499467506558531733671585862910572702811545035730398749530*e^(121*I*
 c) + 50836369508171099437019348610847391946736185108017183136*e^(120*I*c) +
 5032024903401451824074213943766011922026507006311982753*e^(119*I*c) + 4840
 93410240488718655917025303662581091659126182344528*e^(118*I*c) + 4523094003
 9830738332025694784646206844854827698075736*e^(117*I*c) + 41015454399371957
 93959956708442496709433800261224880*e^(116*I*c) + 3606886130363893494138097
 80004559963548775423325255*e^(115*I*c) + 3073536651283056216099116633849005
 7308062762518496*e^(114*I*c) + 25356674606502797768345615661865912131092516
 42859*e^(113*I*c) + 202347509724462171313966643580234078508179838320*e^(112
 *I*c) + 15603911277687607099721623771744933086920587272*e^(111*I*c) + 11615
 81413733971751533622511909046917188768400*e^(110*I*c) + 8338083991183789445
 3136303673785039051506805*e^(109*I*c) + 57646010465631513042138547107153468
 38447392*e^(108*I*c) + 383360155801054824529764688213114368047154*e^(107*I*
 c) + 24489837337812338687718622491865013839488*e^(106*I*c) + 15006027479373
 97286405577818722691539392*e^(105*I*c) + 8805492759894141114586995081338804
 0256*e^(104*I*c) + 4939666610818025798809586352543471345*e^(103*I*c) + 2644

$10375780310742518099326419685040 * e^{(102 * I * c)} + 1347722779952470195657927421$
 $0395326 * e^{(101 * I * c)} + 652650253343206047453620559993840 * e^{(100 * I * c)} + 29952$
 $547749265499675257842032197 * e^{(99 * I * c)} + 1299146645993240318167826532288 * e^{(98 * I * c)}$
 $+ 53090127264630963470039804475 * e^{(97 * I * c)} + 203703125947036816013$
 $1922320 * e^{(96 * I * c)} + 73099207817335597247098038 * e^{(95 * I * c)} + 24424556298945$
 $02983849104 * e^{(94 * I * c)} + 75599817092670157806639 * e^{(93 * I * c)} + 2154864144781$
 $257856128 * e^{(92 * I * c)} + 56169444526926562260 * e^{(91 * I * c)} + 132788284927485888$
 $0 * e^{(90 * I * c)} + 28186192554792138 * e^{(89 * I * c)} + 530563624556832 * e^{(88 * I * c)} +$
 $8718181624155 * e^{(87 * I * c)} + 122503723056 * e^{(86 * I * c)} + 1431118260 * e^{(85 * I * c)}$
 $+ 13343760 * e^{(84 * I * c)} + 93096 * e^{(83 * I * c)} + 432 * e^{(82 * I * c)} + e^{(81 * I * c)})) * \tan$
 $n(1/4 * d * x + c) + 7 * (3718 * I * a * e^{(1055/2 * I * c)} + 1502072 * I * a * e^{(1053/2 * I * c)} +$
 $302667508 * I * a * e^{(1051/2 * I * c)} + 40557446072 * I * a * e^{(1049/2 * I * c)} + 40658839687$
 $18 * I * a * e^{(1047/2 * I * c)} + 325270717497440 * I * a * e^{(1045/2 * I * c)} + 21630502713590$
 $407 * I * a * e^{(1043/2 * I * c)} + 1229848582862122028 * I * a * e^{(1041/2 * I * c)} + 610312359$
 $25186078942 * I * a * e^{(1039/2 * I * c)} + 2685374380786193144783 * I * a * e^{(1037/2 * I * c)}$
 $+ 106072288048110242178847 * I * a * e^{(1035/2 * I * c)} + 3799316499692185181122010 * I$
 $* a * e^{(1033/2 * I * c)} + 124427615396355739905346318 * I * a * e^{(1031/2 * I * c)} + 375197$
 $1173605702746328148447 * I * a * e^{(1029/2 * I * c)} + 104787194996399169411032876338 * I$
 $* a * e^{(1027/2 * I * c)} + 2724467073052261042519716192524 * I * a * e^{(1025/2 * I * c)} + 6$
 $6238605830373986929763608958656 * I * a * e^{(1023/2 * I * c)} + 1511798772310987071825$
 $034633753982 * I * a * e^{(1021/2 * I * c)} + 32503673727084640393745736284624870 * I * a * e^{(1019/2 * I * c)}$
 $+ 660337796066223079759666054299242200 * I * a * e^{(1017/2 * I * c)} + 1$
 $2711502667519783895793526074473028922 * I * a * e^{(1015/2 * I * c)} + 2324389082352054$
 $96318447910669140546906 * I * a * e^{(1013/2 * I * c)} + 404655013813610276217399414824$
 $9330760039 * I * a * e^{(1011/2 * I * c)} + 67207920864414377269671308107198146913908 * I$
 $* a * e^{(1009/2 * I * c)} + 1066925768074551295828818005563441210737450 * I * a * e^{(1007/2 * I * c)}$
 $+ 16217272150903922938701499630023289700137263 * I * a * e^{(1005/2 * I * c)} +$
 $236397937497882482710239205369322834941776795 * I * a * e^{(1003/2 * I * c)} + 3309571$
 $280975022580464977398526908985926399572 * I * a * e^{(1001/2 * I * c)} + 44561015944582$
 $807131288351508188541688368025238 * I * a * e^{(999/2 * I * c)} + 577756662739414343075$
 $056408130898651215560673415 * I * a * e^{(997/2 * I * c)} + 722195893379106335433825275$
 $8241260927009958649153 * I * a * e^{(995/2 * I * c)} + 87129449619720167774738382114174$
 $206125338079715514 * I * a * e^{(993/2 * I * c)} + 101560278276675696160651119552950643$
 $4692326161772148 * I * a * e^{(991/2 * I * c)} + 11448615033797091963830678407271581987$
 $220664774793965 * I * a * e^{(989/2 * I * c)} + 124924617648249630854756820327923512635$
 $952856544140745 * I * a * e^{(987/2 * I * c)} + 132063197778207921385512583251082443818$
 $4906553225344870 * I * a * e^{(985/2 * I * c)} + 13536481494846551191615296983753871336$
 $900108337339193172 * I * a * e^{(983/2 * I * c)} + 134633157060182506966654568330522152$
 $247118398756303435361 * I * a * e^{(981/2 * I * c)} + 130027335757787823563088998397087$
 $1898843384241082917039402 * I * a * e^{(979/2 * I * c)} + 12202570846936266971537615599$
 $091003952667395073528912708550 * I * a * e^{(977/2 * I * c)} + 111348517556517214393482$
 $430347520024901285561594013324999952 * I * a * e^{(975/2 * I * c)} + 988558176286135607$
 $113625950318361683964799348363993276036638 * I * a * e^{(973/2 * I * c)} + 854397317291$
 $8851719756469222929408444037078724153454872120390 * I * a * e^{(971/2 * I * c)} + 71928$
 $391497764277134489912202810340992721249592546121668225432 * I * a * e^{(969/2 * I * c)}$

+ 590140317836783451582400841140547729023595019680165231818378300*I*a*e^(9
 67/2*I*c) + 472112769828475117988655875366120645311034456945741674598695992
 2*I*a*e^(965/2*I*c) + 36845368836912452238557505487949108946286792376090329
 160029006801*I*a*e^(963/2*I*c) + 280652360464451397168371434304010585830771
 241743956318553252853554*I*a*e^(961/2*I*c) + 208735534313108004836669325304
 5101766018198101781306670622017762198*I*a*e^(959/2*I*c) + 15165303771430260
 819236270425814926625403183922898905657641898587425*I*a*e^(957/2*I*c) + 107
 673884285126817246373518618741221711008376494758493794059082539665*I*a*e^(9
 55/2*I*c) + 747385221429320013966356671106061647673665642790216192401997745
 384178*I*a*e^(953/2*I*c) + 507360956426058871091645301670256403257182961855
 0956017565632914253964*I*a*e^(951/2*I*c) + 33696528711624679729779900093828
 724600659229836591195133199888046174281*I*a*e^(949/2*I*c) + 219028190068671
 998320108302544300443107574826644022160996792142204598852*I*a*e^(947/2*I*c)
 + 139382114454752543408039015371282273892674605987137265975849195401323329
 8*I*a*e^(945/2*I*c) + 86865301785929212547861748564018230685225010116560892
 37471028876240107552*I*a*e^(943/2*I*c) + 5303381040819430221339665282353347
 1045170883965946504040578005764694599600*I*a*e^(941/2*I*c) + 31729021076364
 6745231429975366933483435752750035349554674608214217860330588*I*a*e^(939/2*
 I*c) + 18607301533322845281669011266356440294790601303432734303820986034786
 77010682*I*a*e^(937/2*I*c) + 1069927075056886003621531104874832846231594041
 0814407912532812583183629052968*I*a*e^(935/2*I*c) + 60337325392389346145064
 651977945597823054838103742220792252383582775469810852*I*a*e^(933/2*I*c) +
 333804448418123544074311721743628896421299470604551695682547055334526562374
 348*I*a*e^(931/2*I*c) + 181209812355209134122859319086377552282084081229556
 2814006136853589089587942200*I*a*e^(929/2*I*c) + 96551847295473445261451790
 60259293352161226916523506102065806057011468972500972*I*a*e^(927/2*I*c) + 5
 050461890156366422569498902036204366640175781326103263638443513502027085836
 2828*I*a*e^(925/2*I*c) + 25941335757606125914021211615794333861772719757999
 9426945148843639870850445567092*I*a*e^(923/2*I*c) + 13087005273027426033328
 30336012830403130678747327429168347765689867244859352037688*I*a*e^(921/2*I*
 c) + 6485865431691803112160646709240463402571992800370597824291437072623168
 391271067612*I*a*e^(919/2*I*c) + 315838790301413954589785078831270108407496
 05603852763144372466905813381411392911500*I*a*e^(917/2*I*c) + 1511542355639
 77220379337165749623108778682727639486653007034770480329113314879178190*I*a
 *e^(915/2*I*c) + 7110781443821491119523461206295008615761718920341905984534
 44253505004747647977121920*I*a*e^(913/2*I*c) + 3288810369087482330767112497
 943810956913958488942225831844466822949439789604958354380*I*a*e^(911/2*I*c)
 + 149576983980833836478728363167385041163574440504618446222202155412291218
 89145127946510*I*a*e^(909/2*I*c) + 6690719028311938845951714357481243168602
 2847054878088870420694345689435187958917420750*I*a*e^(907/2*I*c) + 29440035
 771426987261240090861331027791142352406441607584926398059657246046789310155
 1780*I*a*e^(905/2*I*c) + 12744849616363934710260475797795104848000108814414
 52922706122811798463818843113785401980*I*a*e^(903/2*I*c) + 5429167275423972
 414370118138151426061135560961696250085913601894114770264832855561732510*I*
 a*e^(901/2*I*c) + 227616207014150998824942862057363907088639144103903695744

$31103252189239994156364638800240 * I * a * e^{(899/2 * I * c)} + 9393166846586109729576$
 $7669397193978348122793779758072033532762411099355052744964746076720 * I * a * e^{($
 $897/2 * I * c)} + 38161498256504938298548772399547510632592125444794475807650852$
 $8236388846570076902279935560 * I * a * e^{(895/2 * I * c)} + 15265365574402277263235925$
 $86303247924486418709408053913097445226670137799881453709880288360 * I * a * e^{(89$
 $3/2 * I * c)} + 6013393403514090825503439756338023248243113472621004894860736435$
 $704687187297040948536876280 * I * a * e^{(891/2 * I * c)} + 233304558129349191097882697$
 $14546442839083722814780950529796764458782805336320290516092783080 * I * a * e^{(88$
 $9/2 * I * c)} + 8916143540247157468331056708300550007979488468436142177082484421$
 $2210441358440351257958080020 * I * a * e^{(887/2 * I * c)} + 33569016274523440689722952$
 $3063578001879927143156137965806093672801505619898736651450033938080 * I * a * e^{($
 $885/2 * I * c)} + 12452712845659168909861631121109280549680656301015715429764393$
 $54848212020139249744172386191190 * I * a * e^{(883/2 * I * c)} + 4552057982199619540279$
 $669891611168972664512469922273304350295630649068449947661654859851520640 * I *$
 $a * e^{(881/2 * I * c)} + 163992260136891349729894299620340638416069451379678962703$
 $16790995858119979554550799257281805220 * I * a * e^{(879/2 * I * c)} + 5823212841513982$
 $124122237392489397976319502585987471058025816607404370962195102151099211024$
 $7670 * I * a * e^{(877/2 * I * c)} + 20383391724947901034896921116234923504212194691197$
 $2208867318581054122394696504660030320280130750 * I * a * e^{(875/2 * I * c)} + 70341919$
 $587773492725310001749344921533076170368888054865460547307543808507085678144$
 $9196376998400 * I * a * e^{(873/2 * I * c)} + 23934495693112363809057812170710904525842$
 $77899938154423296264450695311329160730644154303696630780 * I * a * e^{(871/2 * I * c)}$
 $+ 8030707825911039750901750056608809314825712783262646850788620295561719866$
 $426138714811018171607190 * I * a * e^{(869/2 * I * c)} + 265735099311760349618670131544$
 $11573174519826946330646572740275623058291118447275375382437447650250 * I * a * e^{($
 $867/2 * I * c)} + 8672704929149813119480026495559555109339752889705455156233723$
 $2229710003919599925462196648416340420 * I * a * e^{(865/2 * I * c)} + 27919956618760773$
 $479939843263385517087391181042116372426801045309405045403008286747750106379$
 $9585760 * I * a * e^{(863/2 * I * c)} + 88669118294726958970476865059787128995449404848$
 $4796909057735740607522534153086994088857099910307490 * I * a * e^{(861/2 * I * c)} + 27$
 $782384340301503899120042899788923689395374534865525214744171599877957986345$
 $73042959267684684956330 * I * a * e^{(859/2 * I * c)} + 8589090245938569057205496894537$
 $381121736556805993399422069994412586113650715896554879518565545033500 * I * a * e$
 $^{(857/2 * I * c)} + 262026661413661375338099973457430738047247625045775712054342$
 $62057786505982168778594588998843843064160 * I * a * e^{(855/2 * I * c)} + 7888668853569$
 $301772976372386996960872957847756217810018625170780365459522799553124515584$
 $1610652335370 * I * a * e^{(853/2 * I * c)} + 23440145177183638490588541151419900277989$
 $2609861347882287443973824302713590379994139571196743269525200 * I * a * e^{(851/2 *$
 $I * c)} + 68746845182880446138347516110722617254137847246934010854845362858435$
 $6757564117582602376376619756273900 * I * a * e^{(849/2 * I * c)} + 19902971764646595483$
 $323913003343478420699085784238326795342038763677234330924407660994509222940$
 $77090680 * I * a * e^{(847/2 * I * c)} + 5688421312395675282558972732461105838586797834$
 $128682182058119235866037766500737528636435864135629276280 * I * a * e^{(845/2 * I * c)}$
 $+ 160512736936774650149178398393766191845774391311876711661390849790898584$
 $30096894352960115226308965770760 * I * a * e^{(843/2 * I * c)} + 4472039962294819682387$

399718986450016090569673019569309841172577771894142199297970522223769609044
 7011160*I*a*e^(841/2*I*c) + 12303086815692272305679075836631831047214060472
 1979474252297135272822529931326368108994384850189269222320*I*a*e^(839/2*I*c
) + 33424726352953278678796796394401815339089529908062565016262503330138741
 5089670209519217032467321603326080*I*a*e^(837/2*I*c) + 89680795103511447814
 058492212920166641425689205402442477932129527827015663185270576287191427229
 1903841570*I*a*e^(835/2*I*c) + 23765156262652543915290040172898941215675319
 63746389965701709262550375668882879011912393122776372223417220*I*a*e^(833/2
 *I*c) + 6220463245033442486970965431823733752822029473563015032522697711385
 753292200817503166820643981171972227740*I*a*e^(831/2*I*c) + 160833386284501
 320359204879956240825255167148168145758179005156485766154668077621070006858
 27676876615047090*I*a*e^(829/2*I*c) + 4108006470987728373488758428687061518
 7909186938494335754863409747746758750561170745510692753019453256230130*I*a*
 e^(827/2*I*c) + 10366136343863029786110851000214496639470958282076427097998
 237731443762205754841630360152661302764514627060*I*a*e^(825/2*I*c) + 25844
 164954447508105108617067262460235318095770556449468576728402913630890858383
 7797261793042111767829441440*I*a*e^(823/2*I*c) + 63664343006830970779682307
 871047283430709842994913413062564401278355421062181433392867579320727100311
 0061170*I*a*e^(821/2*I*c) + 15496943917687611285929670636214309582167793110
 92063070007956011152720640113716781139083032984676774287045220*I*a*e^(819/2
 *I*c) + 3727685690743909795581760930725419308750587813165730468793831046647
 131796121280952242102308634280498612910500*I*a*e^(817/2*I*c) + 886141036289
 354851337854366166692297921287916254685536894302112257251721695271532257946
 0798697748541783361640*I*a*e^(815/2*I*c) + 20819160096574412989523515269020
 949409344363814031248632248165901660892659333225816769074907830647131106973
 820*I*a*e^(813/2*I*c) + 483444222719571672115283780376316317858047715234526
 78383038224923816525208001883617890956028571517412300257380*I*a*e^(811/2*I*
 c) + 1109629406830899391197849268090983501962672052930979339345063619934366
 71367092877439898462338113225623209168660*I*a*e^(809/2*I*c) + 2517579247104
 57652984043156887340987774997658440688086491922550151336737423622904316990
 54972850328850024201000*I*a*e^(807/2*I*c) + 5646596581438170762044322139447
 252853460437375986442388241838825462330101765332166919737680661470984187775
 61124*I*a*e^(805/2*I*c) + 1252025659809341407225419006492626474816512541093
 036664002973216176119522637183550660057342417535420038395747756*I*a*e^(803/
 2*I*c) + 274464792210396745066535152891244533850843173453297733925892435463
 2418413755714066126762594816764534577943660024*I*a*e^(801/2*I*c) + 59488188
 745175515464773239981459905453037520461005539068713595250913943652905520153
 95275569402480394903253981466*I*a*e^(799/2*I*c) + 1274879005945457009494457
 535689118330635567251508011330253950418930158004493113195149267737144009192
 3947401124164*I*a*e^(797/2*I*c) + 27016210725153441461314024253680201106904
 941270214792320554690336187508543153505228236450754276495512692743900280*I*
 a*e^(795/2*I*c) + 566134964506713070924629978112007703878668613634184664482
 27491328409041287233096094607670158591842995212315780816*I*a*e^(793/2*I*c)
 + 1173216494136888055414895357694946238706921985189388208419194553749465160
 30743273351488754702468032406699077952754*I*a*e^(791/2*I*c) + 2404480664442

682761506575826002828067368822994750069297738676402780822094763851988328161
86325534733945931054736716*I*a*e^(789/2*I*c) + 4873841845455178280482923772
748897649246318640660545975185605943062960388546214085876153066103652693862
95410913209*I*a*e^(787/2*I*c) + 9771256963671587818799358030849281831105880
96954532024018846643158415313905036398953539486071073924163775202836156*I*a
*e^(785/2*I*c) + 1937670525270251509451678468483387984129872967538542952639
048449569636581336018975500784323892713878389595738475570*I*a*e^(783/2*I*c)
+ 380085564339950552006945359441097202809886841034624857583774647610675563
8552361174314174233599317030879593032072609*I*a*e^(781/2*I*c) + 73752379140
960757531627571651149329071427637153198268415895920043297735615803201899778
59708165438687906157904775921*I*a*e^(779/2*I*c) + 1415744965554083603782800
596978680017663186415185687663471354760534152053881202685002666696858454157
5064351905970774*I*a*e^(777/2*I*c) + 26886193720919073370988929546745045319
078132873757042641225271946983029782743313786423870951765027326861300318178
642*I*a*e^(775/2*I*c) + 505160955717702736706806587300544605277632593563199
78317789374345400938440125042061448809087158872073418586148121473*I*a*e^(77
3/2*I*c) + 9390898020895608111079154395177298537060051758626224909583298638
9557258513757169577939649993369495960643164181810586*I*a*e^(771/2*I*c) + 17
273566488445603306360601669285674942612864469412311812674186989450026951570
2878867401279635331386515415677529225100*I*a*e^(769/2*I*c) + 31439455994569
196942673011891429924377737090055813308324026642179471596698281030429147032
8806778764207633221621542600*I*a*e^(767/2*I*c) + 56624796774753920331645228
162081727228166630708266001409626784945725116717748737473906897305871977817
5038716340060846*I*a*e^(765/2*I*c) + 10092468082154467737864703510655041684
790076631314584516329625276213361160951940611927416365969707450817635456495
24358*I*a*e^(763/2*I*c) + 1780195287370357324046533363473674962784595946909
397017548451036643167283461157097955927272971697034280841666570691552*I*a*e
^(761/2*I*c) + 310769128909224508659422391428849640538991238991098383608782
8161704430807791962044864438637368112933607786892490134694*I*a*e^(759/2*I*c
) + 53694337037136505796575434219256212737798960954931240940583688970751921
79929265569591470701092735447407195008016032850*I*a*e^(757/2*I*c) + 9182478
844644152792789990874629586776827818503877874267474333149197309164002316988
444567545080601139568783596754328769*I*a*e^(755/2*I*c) + 155436391595087510
190621822992589087995698198761329851607046753571663443122575010692378303606
98174149396040851872785700*I*a*e^(753/2*I*c) + 2604517728791224117606722292
090185385131355463680913820822536427242714619704180385209039553363359558766
9906282640335206*I*a*e^(751/2*I*c) + 43202050357851481166487014480064062972
447184469570109898781666839150161921519823032876311654002974726935391777697
053449*I*a*e^(749/2*I*c) + 709421728625611640581601798688187324197887347397
52023491932850774923156332944622492598440157306713277485913625566923405*I*a
*e^(747/2*I*c) + 1153316333995452947841167158723660460399964394192173398102
5365678532178170096704571477723718363358804096119332434747204*I*a*e^(745/2
*I*c) + 1856338052193508151975063509039775571072522772272193617094599014377
19214218026047853241796305910447871730808241890424442*I*a*e^(743/2*I*c) + 2
958361919775553375165027590471463400880636838531233420021233011181805229533

14880146816183888113061523938713872940614049*I*a*e^(741/2*I*c) + 4668229636
771926618173316923077670559282588477498337804661921887375458058036790288944
27817375583837205119809040687012935*I*a*e^(739/2*I*c) + 7294255522218743274
866382001187639242771432040376961362877690639448888331520081725116571113990
79703659693540539407822870*I*a*e^(737/2*I*c) + 1128648994213736556251579671
179535173610891367629572521988885688708464002698593853556863545628611035983
085337507604532900*I*a*e^(735/2*I*c) + 172944770592142637600886366543867627
290805672007280072031920613575745549746414445919013009876605162436220524575
8670207771*I*a*e^(733/2*I*c) + 26245095765035615909209408593054901055292917
192055569458130509480900989780789630075434246610106137127009548979723624951
83*I*a*e^(731/2*I*c) + 3944598455792355841314386308690916886165971173780125
829080793206506479844522630226009379806916943877620012485276018469226*I*a*e
^(729/2*I*c) + 587209681439462756838122821830313672498025602075998707582373
4903141260496180636785764185668348721068755119791925169349060*I*a*e^(727/2*
I*c) + 86584743207487688501763773601294931608840160433501790693879945093392
40513007513825644052150379909781184428932642325725991*I*a*e^(725/2*I*c) + 1
264647288740033733686580427224134368185101144482189978122453968845107580900
3555323364506814180545559579259228045710226754*I*a*e^(723/2*I*c) + 18297829
564393056068366498752749205123672734929190513671572990554943995105221464517
636015940848250377045895323290805291610*I*a*e^(721/2*I*c) + 262273373681811
226298706444315051787100757600726384756303499560115879278542486113843647265
03701818181005210900229213259176*I*a*e^(719/2*I*c) + 3724394980373437437989
764141178623065668293234299029934348825509720302990975196416522144810763404
0170424408865199063339150*I*a*e^(717/2*I*c) + 52399444809707666464935057528
396430706190716933193608724600411642322158892904104083168727544999290987935
418056417609518886*I*a*e^(715/2*I*c) + 730448554447480590424433053403380466
588494263646994920561282564211146832991645031878285299074495972551830495310
17656578288*I*a*e^(713/2*I*c) + 1008944274924658818202100440313543894585312
649499098609271795276705906655623800696473216834568979817273471884445551385
01852*I*a*e^(711/2*I*c) + 1380962633032965492578733014469461619292008321007
30252243863138612640918600095174736177924532345923756979943811207087709274*
I*a*e^(709/2*I*c) + 1873080504973594672445460320599143169020214552899474694
07219847664575495361423597848611293739354236387332381220310027105775*I*a*e^
(707/2*I*c) + 2517753601613760848973774888014951376066343207232258494324638
92924567486986972945320933780599953869938023457422052033507310*I*a*e^(705/2
*I*c) + 3354089547542162106981160536617488988775789924186610647230495009340
91086950204858818941891841228452826384396254403564858074*I*a*e^(703/2*I*c)
+ 4428564133965216856348191294175207459672702836010888624631418742305024851
18099550417429454047383787479022883737738141901647*I*a*e^(701/2*I*c) + 5795
622252930421344077482961897308179486761234233921017124664306841507871381543
70408270217791597537521052690534681094777663*I*a*e^(699/2*I*c) + 7518094061
821735158344749309997529299311755271900425282907500683425001847191504152568
91020872036863319566794006361395795166*I*a*e^(697/2*I*c) + 9667347632003620
889114509962947817275099249302633556109293028990305447042820212625397603952
29496521149023170036904352367324*I*a*e^(695/2*I*c) + 1232309290948853642478

779356200855608165835829939280659113065568855963832114039518667176894361073
 144152565737831771354567511*I*a*e^(693/2*I*c) + 155727495436368849740809282
 622962881511779901100968162403874164375317048415351692124603433127829224961
 4057235994834309112040*I*a*e^(691/2*I*c) + 19510294286255211591487022722073
 896999703240702954760965147071969958549176437705022689413012841379027206519
 56083459514140654*I*a*e^(689/2*I*c) + 2423451334805659147333112842085995378
 215913508185525364517506002076374528501708818302719553446763078559382901836
 261131889256*I*a*e^(687/2*I*c) + 298466021607144435119320047941836374796006
 18995280780624142655429503350783653868741903462659160835147617535958410300
 1344444*I*a*e^(685/2*I*c) + 36447079847263218448856713632108109790662101408
 135898119570975686541543655005563109602221104625941908636979666593578443373
 76*I*a*e^(683/2*I*c) + 4413201821766662151126840590212096901735168550926460
 047580408375275922493840723935067411387358998419396880158733328711475814*I*
 a*e^(681/2*I*c) + 529886244954375959766506981178079051940040169687853087524
 7452324647041037858434187815325455689220752889836702912611111473360*I*a*e⁽
 679/2*I*c) + 63090261753268113091232023004810885394848807487565510609416786
 75574426326200109728754025077089532288910657077894543705008776*I*a*e^{(677/2}
 *I*c) + 7449103899262077125075339307393052327438252027859428170722577149326
 026322173599318148157048286333519829356658894340594708184*I*a*e^(675/2*I*c)
 + 872201511091169054946499700429876713751051935247691581174466989217944028
 4768413559220038431542413088622966991423825777102576*I*a*e^(673/2*I*c) + 10
 127619431129011132568831044458030828342168964659395668911086917559825727413
 834246532010534829987050611067023384522066422504*I*a*e^(671/2*I*c) + 116621
 721287113775623081275464904387561265635691715398279728365116272190405650514
 20359980715312969450565938144063714770162456*I*a*e^(669/2*I*c) + 1331783289
 024115953719020194616289988274680832676196244032018659335347393024662993325
 0050973279739565560426676777999389973000*I*a*e^(667/2*I*c) + 15082258611294
 250199381858545831302483018545933048494313083152693847355205441385741162728
 852832003622428437181937942160447280*I*a*e^(665/2*I*c) + 169383108321788315
 328224908911992318211090534603819353969400959615014870417348228553764658298
 35686556578407011793714960124040*I*a*e^(663/2*I*c) + 1886390647561849904298
 602158370940049324669107137148272999801953562407107456219365005342662739942
 5629073637844277644750153560*I*a*e^(661/2*I*c) + 20832036685733290936554232
 284534256125011627099730302318419255700297016002285140439445233578939463049
 867452727621615078639620*I*a*e^(659/2*I*c) + 228109728768832112861226947200
 452801717005347578847484909274849172402001479999238145672925078059998608030
 53865524582364804000*I*a*e^(657/2*I*c) + 2476467177465379432743186716140258
 350993645025128252023552546881311609225086246271968388102335533866049991665
 9181180349795560*I*a*e^(655/2*I*c) + 26653382599072639723936747283205107568
 409577917833406937668499133744429065300055240571508425036429118321680964755
 361170389060*I*a*e^(653/2*I*c) + 284344500466047569774235095382036933506622
 815398038020363854447518908334625719628608969747499548724579999034376142343
 19146820*I*a*e^(651/2*I*c) + 3006329690354699271585222894437511719650339916
 089933982796273775661839339678080681892441114527698849608048047142250189621
 8680*I*a*e^(649/2*I*c) + 31494560548967888404569411924339603394645419066727

605495988807327751984963232431940207237265388694746179931762688836878800840
 $*I*a*e^{(647/2*I*c)}$ + 326833488657783253831306332371239596057351105443820865
 27702946726198258184497868972918896266641810727368242389813054030133220*I*a
 $*e^{(645/2*I*c)}$ + 3358657371997062104949737393089991438005788486604498314888
 2154444047223976314449935079172544297165302150135561996418042983120*I*a*e^(
 $643/2*I*c$) + 34164314654896876488824234033585972012336614958614636444341207
 830254097673275142227492274795430342540762412582780000960770560*I*a*e^(641/
 $2*I*c$) + 343811621266966276226274728218564953335221470758875096548733235685
 25944772491810111430218649338400622167547434461523829279760*I*a*e^(639/2*I*
 c) + 3420748868297489447450104629460069759722753830165319479716000268268166
 5876148402637025420428006347009046317545272979001609440*I*a*e^(637/2*I*c) +
 33620598008606423200654058352031093812042716418861961325485316234413640793
 571327779830986471142061602134350650899849651277760*I*a*e^(635/2*I*c) + 326
 057056249398436824951952532328285240538267158484166516458890359214589355554
 87028933418123373614771794931330464399194235280*I*a*e^(633/2*I*c) + 3115671
 098675850865478930894959500663968721006727579966185608726985147258975946778
 7944734060267442519580376947726659750489880*I*a*e^(631/2*I*c) + 29276728416
 188920557768407246075293651820332396229753521220605372679227701104569238138
 367698160209738275122467287662716467440*I*a*e^(629/2*I*c) + 269783533054007
 520413223824110526539745917897467461446067342404609422600898422602079577645
 73547124064623860447428875078078740*I*a*e^(627/2*I*c) + 2428364982971821457
 318462281993335756664066746522458453798321694995399080699816950194741847923
 3595645585727784516401446933600*I*a*e^(625/2*I*c) + 21223856555366593508202
 765148912614127303583877402964836047005837766719280331557500411875109254434
 779078342142082375611830200*I*a*e^(623/2*I*c) + 178388163476341839950087897
 812098063970831660410058408383612975227542308272744311170123317448634826527
 46434258554498280368660*I*a*e^(621/2*I*c) + 1417614648880175020673955979043
 846680820739337726522244522474475094051057165295421694430748628886388454928
 9668823221101795780*I*a*e^(619/2*I*c) + 10290173577607962315864745487653937
 040993407188226189481825477184122828955414499477390124720061044684730713865
 172442389148000*I*a*e^(617/2*I*c) + 624066536020923017353937698209763856593
 150563330711131708886005175866720278908250690284293815272070627897438661587
 6935528840*I*a*e^(615/2*I*c) + 20913979700860537588126193141320743146337689
 150028266371644598683748578362297153537124877928698198631864631049177166212
 00020*I*a*e^(613/2*I*c) - 2091397970086053758812619314132074314633768915002
 826637164459868374857836229715353712487792869819863186463104917716621200020
 $*I*a*e^{(611/2*I*c)}$ - 624066536020923017353937698209763856593150563330711131
 7088860051758667202789082506902842938152720706278974386615876935528840*I*a*
 $e^{(609/2*I*c)}$ - 10290173577607962315864745487653937040993407188226189481825
 477184122828955414499477390124720061044684730713865172442389148000*I*a*e^(6
 $07/2*I*c)$ - 141761464888017502067395597904384668082073933772652224452247447
 50940510571652954216944307486288863884549289668823221101795780*I*a*e^(605/2
 $*I*c)$ - 1783881634763418399500878978120980639708316604100584083836129752275
 4230827274431117012331744863482652746434258554498280368660*I*a*e^(603/2*I*c
 $)$ - 21223856555366593508202765148912614127303583877402964836047005837766719

280331557500411875109254434779078342142082375611830200*I*a*e^(601/2*I*c) -
242836498297182145731846228199333575666406674652245845379832169499539908069
98169501947418479233595645585727784516401446933600*I*a*e^(599/2*I*c) - 2697
835330540075204132238241105265397459178974674614460673424046094226008984226
0207957764573547124064623860447428875078078740*I*a*e^(597/2*I*c) - 29276728
416188920557768407246075293651820332396229753521220605372679227701104569238
138367698160209738275122467287662716467440*I*a*e^(595/2*I*c) - 311567109867
585086547893089495950066396872100672757996618560872698514725897594677879447
34060267442519580376947726659750489880*I*a*e^(593/2*I*c) - 3260570562493984
368249519525323282852405382671584841665164588903592145893555548702893341812
3373614771794931330464399194235280*I*a*e^(591/2*I*c) - 33620598008606423200
654058352031093812042716418861961325485316234413640793571327779830986471142
061602134350650899849651277760*I*a*e^(589/2*I*c) - 342074886829748944745010
462946006975972275383016531947971600026826816658761484026370254204280063470
09046317545272979001609440*I*a*e^(587/2*I*c) - 3438116212669662762262747282
185649533352214707588750965487332356852594477249181011143021864933840062216
7547434461523829279760*I*a*e^(585/2*I*c) - 34164314654896876488824234033585
972012336614958614636444341207830254097673275142227492274795430342540762412
582780000960770560*I*a*e^(583/2*I*c) - 335865737199706210494973739308999143
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a*e^(353/2*I*c) - 703419195877734927253100017493449215330761703688880548654
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010348969211162349235042121946911972208867318581054122394696504660030320280
130750*I*a*e^(349/2*I*c) - 582321284151398212412223739248939797631950258598
74710580258166074043709621951021510992110247670*I*a*e^(347/2*I*c) - 1639922
601368913497298942996203406384160694513796789627031679099585811997955455079
9257281805220*I*a*e^(345/2*I*c) - 45520579821996195402796698916111689726645
12469922273304350295630649068449947661654859851520640*I*a*e^(343/2*I*c) - 1
245271284565916890986163112110928054968065630101571542976439354848212020139
249744172386191190*I*a*e^(341/2*I*c) - 335690162745234406897229523063578001
879927143156137965806093672801505619898736651450033938080*I*a*e^(339/2*I*c)
- 891614354024715746833105670830055000797948846843614217708248442122104413
58440351257958080020*I*a*e^(337/2*I*c) - 2333045581293491910978826971454644
2839083722814780950529796764458782805336320290516092783080*I*a*e^(335/2*I*c
) - 60133934035140908255034397563380232482431134726210048948607364357046871
87297040948536876280*I*a*e^(333/2*I*c) - 1526536557440227726323592586303247
924486418709408053913097445226670137799881453709880288360*I*a*e^(331/2*I*c)
- 381614982565049382985487723995475106325921254447944758076508528236388846
570076902279935560*I*a*e^(329/2*I*c) - 939316684658610972957676693971939783
48122793779758072033532762411099355052744964746076720*I*a*e^(327/2*I*c) - 2
276162070141509988249428620573639070886391441039036957443110325218923999415
6364638800240*I*a*e^(325/2*I*c) - 54291672754239724143701181381514260611355
60961696250085913601894114770264832855561732510*I*a*e^(323/2*I*c) - 1274484
961636393471026047579779510484800010881441452922706122811798463818843113785
401980*I*a*e^(321/2*I*c) - 294400357714269872612400908613310277911423524064
416075849263980596572460467893101551780*I*a*e^(319/2*I*c) - 669071902831193
88459517143574812431686022847054878088870420694345689435187958917420750*I*a
*e^(317/2*I*c) - 1495769839808338364787283631673850411635744405046184462222
0215541229121889145127946510*I*a*e^(315/2*I*c) - 32888103690874823307671124
97943810956913958488942225831844466822949439789604958354380*I*a*e^(313/2*I*
c) - 7110781443821491119523461206295008615761718920341905984534442535050047
47647977121920*I*a*e^(311/2*I*c) - 1511542355639772203793371657496231087786
82727639486653007034770480329113314879178190*I*a*e^(309/2*I*c) - 3158387903
0141395458978507883127010840749605603852763144372466905813381411392911500*I
*a*e^(307/2*I*c) - 64858654316918031121606467092404634025719928003705978242
91437072623168391271067612*I*a*e^(305/2*I*c) - 1308700527302742603332830336
012830403130678747327429168347765689867244859352037688*I*a*e^(303/2*I*c) -
259413357576061259140212116157943338617727197579999426945148843639870850445
567092*I*a*e^(301/2*I*c) - 505046189015636642256949890203620436664017578132
61032636384435135020270858362828*I*a*e^(299/2*I*c) - 9655184729547344526145
179060259293352161226916523506102065806057011468972500972*I*a*e^(297/2*I*c)
- 181209812355209134122859319086377552282084081229556281400613685358908958

7942200*I*a*e^(295/2*I*c) - 33380444841812354407431172174362889642129947060
 4551695682547055334526562374348*I*a*e^(293/2*I*c) - 60337325392389346145064
 651977945597823054838103742220792252383582775469810852*I*a*e^(291/2*I*c) -
 106992707505688600362153110487483284623159404108144079125328125831836290529
 68*I*a*e^(289/2*I*c) - 1860730153332284528166901126635644029479060130343273
 430382098603478677010682*I*a*e^(287/2*I*c) - 317290210763646745231429975366
 933483435752750035349554674608214217860330588*I*a*e^(285/2*I*c) - 530338104
 08194302213396652823533471045170883965946504040578005764694599600*I*a*e^(28
 3/2*I*c) - 8686530178592921254786174856401823068522501011656089237471028876
 240107552*I*a*e^(281/2*I*c) - 139382114454752543408039015371282273892674605
 9871372659758491954013233298*I*a*e^(279/2*I*c) - 21902819006867199832010830
 2544300443107574826644022160996792142204598852*I*a*e^(277/2*I*c) - 33696528
 711624679729779900093828724600659229836591195133199888046174281*I*a*e^(275/
 2*I*c) - 507360956426058871091645301670256403257182961855095601756563291425
 3964*I*a*e^(273/2*I*c) - 74738522142932001396635667110606164767366564279021
 6192401997745384178*I*a*e^(271/2*I*c) - 10767388428512681724637351861874122
 1711008376494758493794059082539665*I*a*e^(269/2*I*c) - 15165303771430260819
 236270425814926625403183922898905657641898587425*I*a*e^(267/2*I*c) - 208735
 5343131080048366693253045101766018198101781306670622017762198*I*a*e^(265/2*
 I*c) - 280652360464451397168371434304010585830771241743956318553252853554*I
 *a*e^(263/2*I*c) - 36845368836912452238557505487949108946286792376090329160
 029006801*I*a*e^(261/2*I*c) - 472112769828475117988655875366120645311034456
 9457416745986959922*I*a*e^(259/2*I*c) - 59014031783678345158240084114054772
 9023595019680165231818378300*I*a*e^(257/2*I*c) - 71928391497764277134489912
 202810340992721249592546121668225432*I*a*e^(255/2*I*c) - 854397317291885171
 9756469222929408444037078724153454872120390*I*a*e^(253/2*I*c) - 98855817628
 6135607113625950318361683964799348363993276036638*I*a*e^(251/2*I*c) - 11134
 8517556517214393482430347520024901285561594013324999952*I*a*e^(249/2*I*c) -
 12202570846936266971537615599091003952667395073528912708550*I*a*e^(247/2*I
 *c) - 1300273357577878235630889983970871898843384241082917039402*I*a*e^(245
 /2*I*c) - 134633157060182506966654568330522152247118398756303435361*I*a*e^(
 243/2*I*c) - 13536481494846551191615296983753871336900108337339193172*I*a*e
 ^ (241/2*I*c) - 1320631977782079213855125832510824438184906553225344870*I*a*
 e^(239/2*I*c) - 124924617648249630854756820327923512635952856544140745*I*a*
 e^(237/2*I*c) - 11448615033797091963830678407271581987220664774793965*I*a*e
 ^ (235/2*I*c) - 1015602782766756961606511195529506434692326161772148*I*a*e^(
 233/2*I*c) - 87129449619720167774738382114174206125338079715514*I*a*e^(231/
 2*I*c) - 7221958933791063354338252758241260927009958649153*I*a*e^(229/2*I*c
) - 577756662739414343075056408130898651215560673415*I*a*e^(227/2*I*c) - 44
 561015944582807131288351508188541688368025238*I*a*e^(225/2*I*c) - 330957128
 0975022580464977398526908985926399572*I*a*e^(223/2*I*c) - 23639793749788248
 2710239205369322834941776795*I*a*e^(221/2*I*c) - 16217272150903922938701499
 630023289700137263*I*a*e^(219/2*I*c) - 106692576807455129582881800556344121
 0737450*I*a*e^(217/2*I*c) - 67207920864414377269671308107198146913908*I*a*e
 ^ (215/2*I*c) - 4046550138136102762173994148249330760039*I*a*e^(213/2*I*c) -

232438908235205496318447910669140546906*I*a*e^(211/2*I*c) - 12711502667519
 783895793526074473028922*I*a*e^(209/2*I*c) - 660337796066223079759666054299
 242200*I*a*e^(207/2*I*c) - 32503673727084640393745736284624870*I*a*e^(205/2
 *I*c) - 1511798772310987071825034633753982*I*a*e^(203/2*I*c) - 662386058303
 73986929763608958656*I*a*e^(201/2*I*c) - 2724467073052261042519716192524*I*
 a*e^(199/2*I*c) - 104787194996399169411032876338*I*a*e^(197/2*I*c) - 375197
 1173605702746328148447*I*a*e^(195/2*I*c) - 124427615396355739905346318*I*a*
 e^(193/2*I*c) - 3799316499692185181122010*I*a*e^(191/2*I*c) - 1060722880481
 10242178847*I*a*e^(189/2*I*c) - 2685374380786193144783*I*a*e^(187/2*I*c) -
 61031235925186078942*I*a*e^(185/2*I*c) - 1229848582862122028*I*a*e^(183/2*I
 *c) - 21630502713590407*I*a*e^(181/2*I*c) - 325270717497440*I*a*e^(179/2*I*
 c) - 4065883968718*I*a*e^(177/2*I*c) - 40557446072*I*a*e^(175/2*I*c) - 3026
 67508*I*a*e^(173/2*I*c) - 1502072*I*a*e^(171/2*I*c) - 3718*I*a*e^(169/2*I*c
))/(e^(531*I*c) + 432*e^(530*I*c) + 93096*e^(529*I*c) + 13343760*e^(528*I*c
) + 1431118260*e^(527*I*c) + 122503723056*e^(526*I*c) + 8718181624155*e^(52
 5*I*c) + 530563624556832*e^(524*I*c) + 28186192554792138*e^(523*I*c) + 1327
 882849274858880*e^(522*I*c) + 56169444526926562260*e^(521*I*c) + 2154864144
 781257856128*e^(520*I*c) + 75599817092670157806639*e^(519*I*c) + 2442455629
 894502983849104*e^(518*I*c) + 73099207817335597247098038*e^(517*I*c) + 2037
 031259470368160131922320*e^(516*I*c) + 53090127264630963470039804475*e^(515
 *I*c) + 1299146645993240318167826532288*e^(514*I*c) + 299525477492654996752
 57842032197*e^(513*I*c) + 652650253343206047453620559993840*e^(512*I*c) + 1
 3477227799524701956579274210395326*e^(511*I*c) + 26441037578031074251809932
 6419685040*e^(510*I*c) + 4939666610818025798809586352543471345*e^(509*I*c)
 + 88054927598941411145869950813388040256*e^(508*I*c) + 15006027479373972864
 05577818722691539392*e^(507*I*c) + 2448983733781233868771862249186501383948
 8*e^(506*I*c) + 383360155801054824529764688213114368047154*e^(505*I*c) + 57
 64601046563151304213854710715346838447392*e^(504*I*c) + 8338083991183789445
 3136303673785039051506805*e^(503*I*c) + 11615814137339717515336225119090469
 17188768400*e^(502*I*c) + 15603911277687607099721623771744933086920587272*e
 ^ (501*I*c) + 202347509724462171313966643580234078508179838320*e^(500*I*c) +
 2535667460650279776834561566186591213109251642859*e^(499*I*c) + 3073536651
 2830562160991166338490057308062762518496*e^(498*I*c) + 36068861303638934941
 3809780004559963548775423325255*e^(497*I*c) + 41015454399371957939599567084
 42496709433800261224880*e^(496*I*c) + 4523094003983073833202569478464620684
 4854827698075736*e^(495*I*c) + 48409341024048871865591702530366258109165912
 6182344528*e^(494*I*c) + 50320249034014518240742139437660119220265070063119
 82753*e^(493*I*c) + 5083636950817109943701934861084739194673618510801718313
 6*e^(492*I*c) + 499467506558531733671585862910572702811545035730398749530*e
 ^ (491*I*c) + 4775398607100853263534207733818266777478693412738731031680*e^(
 490*I*c) + 44456708175258821024400946210535004523775722190977468484496*e^(4
 89*I*c) + 403212225957798188840846139960995624144491271694336796459584*e^(4
 88*I*c) + 3564764890628724017088487996688178929195787613958545474804845*e^(
 487*I*c) + 30736217404321009965231037419663053962881035281709221697785072*e
 ^ (486*I*c) + 25858534871597727015582911568419341107203454149136439398549135

$0 \cdot e^{(485 \cdot I \cdot c)} + 21237029691888713182667187812239270678399490157272938840653$
 $88080 \cdot e^{(484 \cdot I \cdot c)} + 1703388602739061574104097772165554166561216227548502858$
 $4310890417 \cdot e^{(483 \cdot I \cdot c)} + 13349021005202618377967331386833230353033290616324$
 $7194627808410304 \cdot e^{(482 \cdot I \cdot c)} + 10225364374682967372930658627052464496936874$
 $15559865844306888705423 \cdot e^{(481 \cdot I \cdot c)} + 7659010520187549651777118357676871927$
 $081898989131125755798204236112 \cdot e^{(480 \cdot I \cdot c)} + 561170810763411753840875701851$
 $88538660375932013674735519055227368366 \cdot e^{(479 \cdot I \cdot c)} + 4023496922661211589340$
 $03582839428785116904903936409545602519219664720 \cdot e^{(478 \cdot I \cdot c)} + 2823905151936$
 $586678382525706564457280290098698638597987628380245881715 \cdot e^{(477 \cdot I \cdot c)} + 194$
 $07979215594566593535008103303255257745408070082431338945184797463936 \cdot e^{(476$
 $\cdot I \cdot c)} + 1306576602265604193351214343899389618845954340699848243071493321317$
 $47540 \cdot e^{(475 \cdot I \cdot c)} + 8618848510949919087642468054746724286037573154844539747$
 $13612812215428992 \cdot e^{(474 \cdot I \cdot c)} + 5572551157328671121016216416307596161861955$
 $969011697222340926210112854418 \cdot e^{(473 \cdot I \cdot c)} + 353244472067790181153780528207$
 $89411687581004582367431006205879633729015200 \cdot e^{(472 \cdot I \cdot c)} + 2196012813395155$
 $61500261478844190024870555261281946058839614044697037963695 \cdot e^{(471 \cdot I \cdot c)} + 1$
 $339214374254245553564884406801945353385000254030655765953770237607180089968$
 $\cdot e^{(470 \cdot I \cdot c)} + 801372958079075243436196494576154376146952079121074697267587$
 $0481058674277844 \cdot e^{(469 \cdot I \cdot c)} + 47065044611135158108487353367484243102698248$
 $838312635876283099427442745866704 \cdot e^{(468 \cdot I \cdot c)} + 271361207503266570734486517$
 $077181014801775322183181055638619257836143271472358 \cdot e^{(467 \cdot I \cdot c)} + 153633323$
 $8444927583532734556016494671674916578907116984548489078241693926940560 \cdot e^{(4$
 $66 \cdot I \cdot c)} + 85430134411262123348335406650696214724790858380413605645507220367$
 $23654297540205 \cdot e^{(465 \cdot I \cdot c)} + 4666822354826601780685459246810057028935596086$
 $9613650856575756758180182223308768 \cdot e^{(464 \cdot I \cdot c)} + 25050102860892833246934045$
 $6829902067712233644464602753159945727868485722395506952 \cdot e^{(463 \cdot I \cdot c)} + 13214$
 $980552713008514299938666316198744245344251881835920497276875710321564350772$
 $80 \cdot e^{(462 \cdot I \cdot c)} + 6852993223145736687328885311617795435592940841439866351079$
 $655652312894721972796266 \cdot e^{(461 \cdot I \cdot c)} + 349410716132767046494779430433394502$
 $01504075335160361865916029213860778606230624960 \cdot e^{(460 \cdot I \cdot c)} + 1751931705006$
 $18300241515632381912285157790097816049220671217212220015297133400636060 \cdot e^{($
 $459 \cdot I \cdot c)} + 8639799336223303495562968200283955131987080649405057021260686529$
 $36800794826651264256 \cdot e^{(458 \cdot I \cdot c)} + 4191542500656826148093339414544159143964$
 $478472492315931809171859902114109005939942952 \cdot e^{(457 \cdot I \cdot c)} + 200080068030300$
 $47137293278250321597113540716201983333126349281186679153199068045257216 \cdot e^{($
 $456 \cdot I \cdot c)} + 9398691531306817914908360606568148278083606051053015461848694983$
 $9467131378859885998210 \cdot e^{(455 \cdot I \cdot c)} + 43454667678028004534634449876389254079$
 $7175105756827515509297024187660299345484920192480 \cdot e^{(454 \cdot I \cdot c)} + 19777929806$
 $658181356513000943262391586054488708069708605773253850286099830345346723185$
 $00 \cdot e^{(453 \cdot I \cdot c)} + 8862752142756957285681340885764904597935349569355321815647$
 $721172537159186491471311666400 \cdot e^{(452 \cdot I \cdot c)} + 391080312556018094765375353696$
 $11844440844903751605645023514572352045248104262933598850730 \cdot e^{(451 \cdot I \cdot c)} + 1$
 $699563279699297677739020966526292532837045054771275445565344173766865409367$
 $06073847337600 \cdot e^{(450 \cdot I \cdot c)} + 7275210107183942292917740738446942557987386670$
 $67535379759732795567942578751384250780476310 \cdot e^{(449 \cdot I \cdot c)} + 3067974296431747$

364198159623962463671617006419626851426148418602934852907379021659761911840
*e^(448*I*c) + 127472196165033205413563430625628473686016221408567860254458
14532037904111523242298235713300*e^(447*I*c) + 5219091220766182421581227185
4269748071292843243227894769229690720010547141334131610989636000*e^{(446*I*c}
) + 21059430138564847118432907888031750495336183995415942743400988466177725
9752542647709150036990*e^(445*I*c) + 83757920692341193245878648676537353394
6545239708990769488724813982189165104589895518909256320*e^(444*I*c) + 32838
747605558186767263094803067344201550985839480744690141681718744421701096485
21627538755920*e^(443*I*c) + 1269349693296492056507367363718128008854868250
8880255337280065006566138696041797353216584528640*e^(442*I*c) + 48379489756
434099843857791816589379406815042609340378747586437145781646245422045101230
417309900*e^(441*I*c) + 181834661406177901315330129677145381166449188413194
141169344354754920969034952610378945282257600*e^(440*I*c) + 674025530543133
008894845775236625237450743114473544537818170447134607102575676676056675328
961590*e^(439*I*c) + 246438219080743960907977422685567962936788570977643587
6630851716253962696192341706239192878728160*e^(438*I*c) + 88882950287510246
670442038376079761014800531344186144746207675228248689119598843526664449174
04000*e^(437*I*c) + 3162664467472554773117679568752765357130596998592368839
2112164915553242573269490908989570248533280*e^(436*I*c) + 11103414879700881
944314389564446924229504986746431371096925761933889913379928561602006987261
1710850*e^(435*I*c) + 38465584208066627445406307878483717499894905250097532
2162003392549953413592461519365177908682078400*e^(434*I*c) + 13150521209306
921221022971053276228423358707434285308910729835358622800944466077234738004
77453914130*e^(433*I*c) + 4437210917843182347764349544443904699020056595069
470847193617092114714077633077234972825351226979360*e^(432*I*c) + 147779550
966171289987127451820714953621765069731830816502336052740516776249704643402
42755840025673760*e^(431*I*c) + 4858425815314028044731483686877213139019541
2419046732778458706015096881437076337910793584122475073760*e^(430*I*c) + 15
768584552885091872146287786443509025758314941556132342738656289444759827793
5629800939237175625149830*e^(429*I*c) + 50529366312301525887848302573881281
3203397766845340065381261016353419722382620393032535960660921950400*e<sup>(428*
I*c)</sup> + 15987711010581926922705289996774447426856310062324561858449252201440
02305878120380828483988663574829100*e^(427*I*c) + 4995241956279138180205186
744401688024388272113921255663734956946927571305533146776898787878059685108
480*e^(426*I*c) + 154131112114860239372949708207973767160813447881633865435
22421939737507962125854981881879168348260330000*e^(425*I*c) + 4697022472711
728182645404501807067052255975662758034778453532001496348263235972944454188
5102274546002560*e^(424*I*c) + 14137993825355684328056550580740330413060613
0725434751745794079833141361748917639986145377066437210546190*e^(423*I*c) +
42035802483514679858361121014594215468443794936564789908837252480215622288
4839580011688655664280691773600*e^(422*I*c) + 12346680418924099787800180817
554402160125824763969419379658996319530792039742221387946043284989721447669
00*e^(421*I*c) + 3582718002163296061414536703715109897107198252739284546149
343102348456124089657428594946438660859773886240*e^(420*I*c) + 102716025302
028890024978135168494525909715128095290606651973010970522100645760883480232

$34671975463677418470 \cdot e^{(419 \cdot I \cdot c)} + 2909765106124745340664756978183691006216$
 $5559852359052804259154165687125428752562385492373749486351714453120 \cdot e^{(418 \cdot I \cdot c)} +$
 $81452081413829111828875417564250054846037693312811480492909160758195$
 $989155768107022568350953861815940704090 \cdot e^{(417 \cdot I \cdot c)} + 225320532593220657767$
 $941109289516248999794521015564134982827241710019675486694499689312466561907$
 $212627820000 \cdot e^{(416 \cdot I \cdot c)} + 616003116022979584937125701757887212998354300989$
 $991362628038861093914561332071191909714949426587936910303300 \cdot e^{(415 \cdot I \cdot c)} +$
 $166447503438721180939491774350293763897857493775476476398783587241044993069$
 $0131572904279995484581013965001440 \cdot e^{(414 \cdot I \cdot c)} + 44454122592954746250676595$
 $141983129660154162999689303933456303459141097207405736188849805200100284514$
 $96996210 \cdot e^{(413 \cdot I \cdot c)} + 1173585692624511849311309100250160403234187698599908$
 $2823520672241530200188223826392982302194084667538488665600 \cdot e^{(412 \cdot I \cdot c)} + 30$
 $627581054221957378390547289277609129572813931082733520247387226000020043538$
 $279468776707958420892547870128680 \cdot e^{(411 \cdot I \cdot c)} + 790191495587665692547839884$
 $872323883529091449827471718567724632238089933670915034028764672701761243426$
 $99654400 \cdot e^{(410 \cdot I \cdot c)} + 2015579474247940980267724784620408833958675125629308$
 $62943753568690084015585598010154781548625239409581907397500 \cdot e^{(409 \cdot I \cdot c)} + 5$
 $083245993010854601669786296830326614276544740829390480979396383915672987957$
 $88389433842285751054665210868287680 \cdot e^{(408 \cdot I \cdot c)} + 1267597017294812913400146$
 $276042126929986480292870190399107554311079964227280196522475370108738477856$
 $311765699610 \cdot e^{(407 \cdot I \cdot c)} + 312568349317870174347970475030749017866629215072$
 $0179363604335113528623329684606343185540756019935662148267863968 \cdot e^{(406 \cdot I \cdot c)}$
 $) + 76217887919120470620388409177993746004288922581943676366829443560966814$
 $00246312138001769285020661445991073249416 \cdot e^{(405 \cdot I \cdot c)} + 1837980708400335976$
 $602764921762114411609173557221662078886153580344970227380258835907670424184$
 $0733513439114113248 \cdot e^{(404 \cdot I \cdot c)} + 43834972142919377685378692233021063744554$
 $033100928502737480438978976746989895784070951905237783490374305934542955 \cdot e^{(403 \cdot I \cdot c)}$
 $+ 103399755467257436489847837640753754718204394473055795001467604$
 $32641987655556873829531737211096115196005647730480 \cdot e^{(402 \cdot I \cdot c)} + 241246021$
 $282440061792908317783032876194801597133206052091286997043729145345755805710$
 $081489006741839439573984832678 \cdot e^{(401 \cdot I \cdot c)} + 556756388711182340341026192734$
 $219546113651768317380539005893679049394714017063698565272728813669054779077$
 $208977840 \cdot e^{(400 \cdot I \cdot c)} + 127103308293804895020136055483127034266234399127750$
 $4612342300366025046741742856580445289401786656311685859023084716 \cdot e^{(399 \cdot I \cdot c)}$
 $) + 28704961314123144518346747153535894395532944308085319334660862885437092$
 $46230769151392180699413405623017247753532944 \cdot e^{(398 \cdot I \cdot c)} + 6413381895855925$
 $184758231451062556380328594938511006577015218119786536390213057284018202201$
 $631094434819584025113465 \cdot e^{(397 \cdot I \cdot c)} + 141764836528757049572020133432411179$
 $043699777966538499599025244219806351890116348156532796054977833828889327667$
 $30080 \cdot e^{(396 \cdot I \cdot c)} + 3100431920606941707706936314142348743182800909818474463$
 $5678652284177439464941651812564519144918003174108077634846014 \cdot e^{(395 \cdot I \cdot c)} +$
 $67091706130529669125019899210021576580237843462229535386295087076189297849$
 $995931360645605292130961496106707521506432 \cdot e^{(394 \cdot I \cdot c)} + 143657687138044796$
 $942947119704259538458818199423516824674586293691056119209866358123637772245$
 $409530799230553767222252 \cdot e^{(393 \cdot I \cdot c)} + 304384471106813336010284160123906370$

433888828490627422652551236966790916174520857759143930140187173492394981908
258944*e^(392*I*c) + 638218892914533741505806399662488556066783600496091876
408374974877448971778036074996245581124283460438065182071976085*e^(391*I*c)
+ 132431132498402742835522293814768237867286070881716174144874968959358802
0860847508703702325320304649883120684987556400*e^(390*I*c) + 27195892834837
439260408051010803419212445303112546072509291927739093315232266350358156728
62569296693711643521070331394*e^(389*I*c) + 5527498849031178355861230009326
668283926290082158467118000698502719379939045918344222192742145711257028040
974074674736*e^(388*I*c) + 111194996453632010808810628248863384924253756584
48977935535846349290425821570383090425411418521516670371372045206568345*e^(
387*I*c) + 2214073500170860327091518076924139166203557875590397914890921360
3822554792749183517160255571915875356439553717130797888*e^(386*I*c) + 43638
300075815171025946211464465689618965773488660985857945657479854085108851857
911222911989837615452608512356008400295*e^(385*I*c) + 851395332347864557795
899594649006377607357297056212213800058372083691577946730496754287998178754
31430246332625899630160*e^(384*I*c) + 1644375006769068927413232601543942785
039545619360201335965818064493572402773494479273345171058099953000935492799
31273178*e^(383*I*c) + 3144090358082258615655954369383549454454739910431297
22046747228813030925204968503418566818838611866709807040793495364496*e^(382
*I*c) + 5951576155004315149474792823365470538279260879164252634971877570294
13385471835434198246807096214536895441388306027237899*e^(381*I*c) + 1115398
285545601550533328045600184317993899359217996729818340704221801195667410485
846996179056733558512452238583160792512*e^(380*I*c) + 206969828950086064346
166576237380795751301942404117887190496055182941244934472243212567941795840
3007551179298315947373776*e^(379*I*c) + 38026049967058911069646206338489648
070370988545101822632430305972956307603535975319743247522663891931857608782
74188013440*e^(378*I*c) + 6917838945214844278493330459361394923372333853619
879637372673184942859712431066345726870422099893124890777678037369988150*e^(
377*I*c) + 124621404405372580849285967098720668577570709431248685545009481
54756863454308032925408340311237850017814707896986969086816*e^(376*I*c) + 2
223134113180153534540639903772168684020839794195258013558464596674673665671
6271554826476282991066076564921432614339399735*e^(375*I*c) + 39274200414329
861169397944516225001081227433398585007206399231211907157795359719648241598
754266579840244551491476467899952*e^(374*I*c) + 687124660159856415124685861
736597477348795917100983546527861249360230739431410495736066485630053594117
12764895683903806088*e^(373*I*c) + 1190605918496605468347656932276764490675
841482488826784475048260772363334445134540951266687500572958111916433569089
72191440*e^(372*I*c) + 2043255572651860007674027102308478964597615839227636
98235433212833313077783041040074669379017394836761539649081690630811665*e^(
371*I*c) + 3473100538109352904194555605559573141295692107357459832343696599
76413374774078000173070075248654524917179128950507443058208*e^(370*I*c) + 5
847495736823045861793846288448833275814989698865403803788967679990756149640
07174600811092945356635118795824799369716742109*e^(369*I*c) + 9752103394440
493187572823117635177866732231755944579463832792646350850410049173002959042
75433144848532459919875479817581584*e^(368*I*c) + 1611092541400060525954859

375264194178347643471837078201446262435615142944587337833513586022729849523
358436493586042252995608*e^(367*I*c) + 263666241043079934044752228477824428
374075106865814072657644667120779832560683229593770506168629793029638233889
2574900819440*e^(366*I*c) + 42748269077205917525267113368208715008443456479
223854715343593336061895718324446413641328931086635762051338706721562641641
15*e^(365*I*c) + 6866425337518668262662693750908956965732924578142181630622
157802899880874681551031136314064199948604001529894566235238597088*e^(364*I
*c) + 109272106034735448102797923478445360745888968062300411100895447316058
63146104181739039426674855453466097402330688331845602302*e^(363*I*c) + 1722
950282436764733440072199841759670394865739473880520939163659737038057239896
4715080095366818322029152193635869784095333760*e^(362*I*c) + 26917794701086
615097890120236890501105146799996021775195710866226228638984703456832694153
230611607263444183501026198563419616*e^(361*I*c) + 416704403753905436434182
193422717480400350714901190805852815224981888183759069003687012345313046331
63446319945130196476913600*e^(360*I*c) + 6392301943376198909061480128863509
812319944510230312261654464820899876780394445577704288655273849974718371313
6069104651812215*e^(359*I*c) + 97173055024742680058616722461368892661141295
540263493013032746083536157324268333390400308958318370219154887169702257444
756176*e^(358*I*c) + 146390448456351181218237382740374124191664819997746988
076598391862733629670142241546375533903130605297580105675355629160198162*e^
(357*I*c) + 218563166659649312247483640956272149212499115383828771029654283
363972585118090479413696638108156385244646591328454425745117584*e^(356*I*c)
+ 323413178014841003714151138246079152576360976035058457890409937738723171
537036573043681997163745313602400139153046673668433091*e^(355*I*c) + 474323
043563100542377338629931966248129175976982332446018056009391154020438895903
140822967769494019446166779954024655344116288*e^(354*I*c) + 689518449328793
559903260418149974190253578340058895035589606468244680591556118170304005037
563669880057908765898949268614772285*e^(353*I*c) + 993555653649521127226443
960820233649386488510081892545866700444096661582790441241830855609577062039
555625090943332264901780720*e^(352*I*c) + 141916448142217657385823401389899
96288223223330957373071631074383893583220145429361729317508645838962142530
7051750612129761498*e^(351*I*c) + 20094961100926877381527820856837372227279
688242990587392154460834993514676253344496707577640666906561509490149440439
94822823920*e^(350*I*c) + 2820819298562215959107529807289628449621386798989
436369393116069894018781201000275633104498398959346631795568022519974400130
281*e^(349*I*c) + 392569765841577835276810394285601184021164276962171721799
6614398473887186074391482638547212826538270453912634540299792270321024*e^(3
48*I*c) + 54166628040524363495855959828183579538662584616443540182051589177
42576425344364964596750653177677803492186817305171175032011500*e^(347*I*c)
+ 7410372612891152224364633296128043971657193280327754304382235672773781403
023814127610355562505271969045177704726054907145784960*e^(346*I*c) + 100522
095243695818275881549853455496780314457444999852082593854356097402724003014
54246872041775159838468077381562338745636398374*e^(345*I*c) + 1352123041194
543691555885470685179654156739981165687002156735297532681546781784653328912
3871696056195231696146162720992221760992*e^(344*I*c) + 18035327338177455471

177568594851682977978346449777193572087688510392426884519272991560851326393
852241961470040819793627127923997*e^(343*I*c) + 238563985655628020306952781
742126408332821541740064594582926440609479249073137359215616901539399060175
18647182491616573724049744*e^(342*I*c) + 3129526368818983831377277587330726
033411722725862950199235869563609266286606281984568906423581362297415012066
8921391878398978380*e^(341*I*c) + 40715988963701918950020348336736423420513
311359010485246919074652883393970805374470830156229705647312265477584256027
212762941040*e^(340*I*c) + 525392233467407711425870923702570695360603196444
395016676104827679558002760528924321527988146079751103662249450814281218884
73324*e^(339*I*c) + 6724408796908070382370325719966304760689061048209049461
9492802935130215979819469966383336788693900139115594646893784095418472336*e
^(338*I*c) + 85368118430215312848231291739673735887746201851666299600392199
418764750086828198719872744047767783667325326289221881974987582215*e^(337*I
*c) + 107504737406576916123480399169759633328321407419400510017498849830598
621565428266546315933920821527544726380201659114903834605888*e^(336*I*c) +
134297742023479429904629616104559610096074758721068022704468938063017059688
023363436458971534964665036319889119229809973806909680*e^(335*I*c) + 166432
332922589195130558329266398753389823955737598527556096093062473559769545772
321978969318904192572733997888230986469005970880*e^(334*I*c) + 204622295535
729109519829916789867225319429705162560082648840394965423112809336591921290
309392396263834674368977840527147037426908*e^(333*I*c) + 249593072282565866
398389951509619202682634455487128714631461891770823201367527645793770203788
784677343934971424317987895255031936*e^(332*I*c) + 302060638030868463461139
442279360499718906917482524894483356220196138377050828911383056860425370161
157201493696073712322595776808*e^(331*I*c) + 362706307563843231135699157418
510732452420614013624168879312187645233450153927975793326834780741391203430
153093712635355523960320*e^(330*I*c) + 432147856464086938023811561808678589
594702047904674282297959658800170984456799067751878044806619012452636891350
731618278545690160*e^(329*I*c) + 510907615111134507452623846147137141450275
722444316385531648429230851686635827717488464500331623385777400744950538410
637735936000*e^(328*I*c) + 599378484771733474809376142401554850207064972118
137572949257503651444541939309025276896049622515630263162184526394317285457
368300*e^(327*I*c) + 697789106925924614816713747684682785083659819027952244
447043355869741368500452561164024636073401929693105801105522738405349028160
*e^(326*I*c) + 806169671327625532424575340089775733681994991576674446922354
099714615192085443245663852257001115644286660304979476023966071898200*e^(32
5*I*c) + 924320052867522584035777495761072351222534420784861960001821020509
468146356756433795246446491396141583854513687104334429566707520*e^(324*I*c)
+ 105178210042883437194450817005121918711681634976695332263718214961087500
4223784784183284961906494422955462431208645690802526770780*e^(323*I*c) + 11
878179430793903161088023247981101290208227826600872485993676434812002060468
22166144285425922229375413676535071141005286431481600*e^(322*I*c) + 1331396
114626723035802462123531582050339749996014152452835305956367425370222758621
753922458727524856072950880960657564720475838500*e^(321*I*c) + 148118711708
924666246695569496567785552473031326055269082160265717621873742624552279532

9891464091005878304304075953693546767206080*e^(320*I*c) + 16355697446414219
006578863812890766532275801720566775874514020692343552836874896596139137619
59140773339736014790081814516625224440*e^(319*I*c) + 1792649078089632298936
945728481969334964391597506285088488350622937252533420980803144316431701452
190522716124797875257437516360640*e^(318*I*c) + 195028655078018191924499296
120448701005646036284521850167442376626632155879143691731787870223267921386
8287926294665202769722927380*e^(317*I*c) + 21061419034683443071125497612024
845434027942523524821998174104248696772627150982884376465186834879454627742
23656471345899082156800*e^(316*I*c) + 2257726219103856286812833012681573765
496262241420612932076143151171960854554124144699023009842080515157923529357
189869943515991200*e^(315*I*c) + 240246459556968608612000180303421105673944
558862194614138410616288624616181514976302503083487523406726777402343341826
9982431265280*e^(314*I*c) + 25377664154650303308154717466929885960699118946
972250529283204525421755871548480964833312098074301139430153983626696733379
57755720*e^(313*I*c) + 2661100647975783583828235139201441930178396643383423
903583862547255880772382049201015537214900832745601519737141849802506685264
000*e^(312*I*c) + 277007320715076864559750728138206549792496846605452741412
2339827333783770068305883487309979315983718403740872884345746380680204260*e
^(311*I*c) + 28625031263204617977706677807256441849912556231746261756790506
72100848988119391841466573417019247590580735265143427289340450811200*e^(310
*I*c) + 2936494214351868498703239455426771104344827306267558916550877467232
455153286140521089582733932202553130712723836983468866230908800*e^(309*I*c)
+ 299049894962254360853812938028386633519008711512485881814378761918695711
1903765723974899651518555144924290346242595167274383008960*e^(308*I*c) + 30
233716435082250271756031752129532190224850458507318453075190082773851547314
61213388035579159917590062343527464977286601165100620*e^(307*I*c) + 3034408
355530957075787731745322567981684616550162845473257679674280216947356785783
843205604202307836897073595410412575660465787520*e^(306*I*c) + 302337164350
822502717560317521295321902248504585073184530751900827738515473146121338803
5579159917590062343527464977286601165100620*e^(305*I*c) + 29904989496225436
085381293802838663351900871151248588181437876191869571119037657239748996515
18555144924290346242595167274383008960*e^(304*I*c) + 2936494214351868498703
239455426771104344827306267558916550877467232455153286140521089582733932202
553130712723836983468866230908800*e^(303*I*c) + 286250312632046179777066778
072564418499125562317462617567905067210084898811939184146657341701924759058
0735265143427289340450811200*e^(302*I*c) + 27700732071507686455975072813820
654979249684660545274141223398273337837700683058834873099793159837184037408
72884345746380680204260*e^(301*I*c) + 2661100647975783583828235139201441930
178396643383423903583862547255880772382049201015537214900832745601519737141
849802506685264000*e^(300*I*c) + 253776641546503033081547174669298859606991
189469722505292832045254217558715484809648333120980743011394301539836266967
3337957755720*e^(299*I*c) + 24024645955696860861200018030342110567394455886
219461413841061628862461618151497630250308348752340672677740234334182699824
31265280*e^(298*I*c) + 2257726219103856286812833012681573765496262241420612
932076143151171960854554124144699023009842080515157923529357189869943515991

200*e^(297*I*c) + 210614190346834430711254976120248454340279425235248219981
7410424869677262715098288437646518683487945462774223656471345899082156800*e
^(296*I*c) + 19502865507801819192449929612044870100564603628452185016744237
66266321558791436917317878702232679213868287926294665202769722927380*e^(295
*I*c) + 1792649078089632298936945728481969334964391597506285088488350622937
252533420980803144316431701452190522716124797875257437516360640*e^(294*I*c)
+ 163556974464142190065788638128907665322758017205667758745140206923435528
3687489659613913761959140773339736014790081814516625224440*e^(293*I*c) + 14
811871170892466624669556949656778555247303132605526908216026571762187374262
45522795329891464091005878304304075953693546767206080*e^(292*I*c) + 1331396
114626723035802462123531582050339749996014152452835305956367425370222758621
753922458727524856072950880960657564720475838500*e^(291*I*c) + 118781794307
939031610880232479811012902082278266008724859936764348120020604682216614428
5425922229375413676535071141005286431481600*e^(290*I*c) + 10517821004288343
719445081700512191871168163497669533226371821496108750042237847841832849619
06494422955462431208645690802526770780*e^(289*I*c) + 9243200528675225840357
774957610723512225344207848619600018210205094681463567564337952464464913961
41583854513687104334429566707520*e^(288*I*c) + 8061696713276255324245753400
897757336819949915766744469223540997146151920854432456638522570011156442866
60304979476023966071898200*e^(287*I*c) + 6977891069259246148167137476846827
850836598190279522444470433558697413685004525611640246360734019296931058011
05522738405349028160*e^(286*I*c) + 5993784847717334748093761424015548502070
649721181375729492575036514445419393090252768960496225156302631621845263943
17285457368300*e^(285*I*c) + 5109076151111345074526238461471371414502757224
443163855316484292308516866358277174884645003316233857774007449505384106377
35936000*e^(284*I*c) + 4321478564640869380238115618086785895947020479046742
822979596588001709844567990677518780448066190124526368913507316182785456901
60*e^(283*I*c) + 3627063075638432311356991574185107324524206140136241688793
12187645233450153927975793326834780741391203430153093712635355523960320*e^(
282*I*c) + 3020606380308684634611394422793604997189069174825248944833562201
96138377050828911383056860425370161157201493696073712322595776808*e^(281*I*
c) + 2495930722825658663983899515096192026826344554871287146314618917708232
01367527645793770203788784677343934971424317987895255031936*e^(280*I*c) + 2
046222955357291095198299167898672253194297051625600826488403949654231128093
36591921290309392396263834674368977840527147037426908*e^(279*I*c) + 1664323
329225891951305583292663987533898239557375985275560960930624735597695457723
21978969318904192572733997888230986469005970880*e^(278*I*c) + 1342977420234
794299046296161045596100960747587210680227044689380630170596880233634364589
71534964665036319889119229809973806909680*e^(277*I*c) + 1075047374065769161
234803991697596333283214074194005100174988498305986215654282665463159339208
21527544726380201659114903834605888*e^(276*I*c) + 8536811843021531284823129
173967373588774620185166629960039219941876475008682819871987274404776778366
7325326289221881974987582215*e^(275*I*c) + 67244087969080703823703257199663
047606890610482090494619492802935130215979819469966383336788693900139115594
646893784095418472336*e^(274*I*c) + 525392233467407711425870923702570695360

603196444395016676104827679558002760528924321527988146079751103662249450814
28121888473324*e^(273*I*c) + 4071598896370191895002034833673642342051331135
901048524691907465288339397080537447083015622970564731226547758425602721276
2941040*e^(272*I*c) + 31295263688189838313772775873307260334117227258629501
992358695636092662866062819845689064235813622974150120668921391878398978380
*e^(271*I*c) + 238563985655628020306952781742126408332821541740064594582926
44060947924907313735921561690153939906017518647182491616573724049744*e^(270
*I*c) + 1803532733817745547117756859485168297797834644977719357208768851039
2426884519272991560851326393852241961470040819793627127923997*e^(269*I*c) +
13521230411945436915558854706851796541567399811656870021567352975326815467
817846533289123871696056195231696146162720992221760992*e^(268*I*c) + 100522
095243695818275881549853455496780314457444999852082593854356097402724003014
54246872041775159838468077381562338745636398374*e^(267*I*c) + 7410372612891
152224364633296128043971657193280327754304382235672773781403023814127610355
562505271969045177704726054907145784960*e^(266*I*c) + 541666280405243634958
559598281835795386625846164435401820515891774257642534436496459675065317767
7803492186817305171175032011500*e^(265*I*c) + 39256976584157783527681039428
560118402116427696217172179966143984738871860743914826385472128265382704539
12634540299792270321024*e^(264*I*c) + 2820819298562215959107529807289628449
621386798989436369393116069894018781201000275633104498398959346631795568022
519974400130281*e^(263*I*c) + 200949611009268773815278208568373722272796882
429905873921544608349935146762533444967075776406669065615094901494404399482
2823920*e^(262*I*c) + 14191644814221765738582340138989996288223223333095737
30716310743838935832201454293617293175086458389621425307051750612129761498*
e^(261*I*c) + 9935556536495211272264439608202336493864885100818925458667004
44096661582790441241830855609577062039555625090943332264901780720*e^(260*I*
c) + 6895184493287935599032604181499741902535783400588950355896064682446805
91556118170304005037563669880057908765898949268614772285*e^(259*I*c) + 4743
230435631005423773386299319662481291759769823324460180560093911540204388959
03140822967769494019446166779954024655344116288*e^(258*I*c) + 3234131780148
410037141511382460791525763609760350584578904099377387231715370365730436819
97163745313602400139153046673668433091*e^(257*I*c) + 2185631666596493122474
836409562721492124991153838287710296542833639725851180904794136966381081563
85244646591328454425745117584*e^(256*I*c) + 1463904484563511812182373827403
741241916648199977469880765983918627336296701422415463755339031306052975801
05675355629160198162*e^(255*I*c) + 9717305502474268005861672246136889266114
129554026349301303274608353615732426833339040030895831837021915488716970225
7444756176*e^(254*I*c) + 63923019433761989090614801288635098123199445102303
12261654464820899876780394445577042886552738499747183713136069104651812215
*e^(253*I*c) + 416704403753905436434182193422717480400350714901190805852815
22498188818375906900368701234531304633163446319945130196476913600*e^(252*I*
c) + 2691779470108661509789012023689050110514679999602177519571086622622863
8984703456832694153230611607263444183501026198563419616*e^(251*I*c) + 17229
502824367647334400721998417596703948657394738805209391636597370380572398964
715080095366818322029152193635869784095333760*e^(250*I*c) + 109272106034735

448102797923478445360745888968062300411100895447316058631461041817390394266
74855453466097402330688331845602302*e^(249*I*c) + 6866425337518668262662693
750908956965732924578142181630622157802899880874681551031136314064199948604
001529894566235238597088*e^(248*I*c) + 427482690772059175252671133682087150
084434564792238547153435933360618957183244464136413289310866357620513387067
2156264164115*e^(247*I*c) + 26366624104307993404475222847782442837407510686
581407265764466712077983256068322959377050616862979302963823388925749008194
40*e^(246*I*c) + 1611092541400060525954859375264194178347643471837078201446
262435615142944587337833513586022729849523358436493586042252995608*e^(245*I
*c) + 975210339444049318757282311763517786673223175594457946383279264635085
041004917300295904275433144848532459919875479817581584*e^(244*I*c) + 584749
573682304586179384628844883327581498969886540380378896767999075614964007174
600811092945356635118795824799369716742109*e^(243*I*c) + 347310053810935290
419455560555957314129569210735745983234369659976413374774078000173070075248
654524917179128950507443058208*e^(242*I*c) + 204325557265186000767402710230
847896459761583922763698235433212833313077783041040074669379017394836761539
649081690630811665*e^(241*I*c) + 119060591849660546834765693227676449067584
148248882678447504826077236333444513454095126668750057295811191643356908972
191440*e^(240*I*c) + 687124660159856415124685861736597477348795917100983546
52786124936023073943141049573606648563005359411712764895683903806088*e^(239
*I*c) + 3927420041432986116939794451622500108122743339858500720639923121190
7157795359719648241598754266579840244551491476467899952*e^(238*I*c) + 22231
341131801535345406399037721686840208397941952580135584645966746736656716271
554826476282991066076564921432614339399735*e^(237*I*c) + 124621404405372580
849285967098720668577570709431248685545009481547568634543080329254083403112
37850017814707896986969086816*e^(236*I*c) + 6917838945214844278493330459361
394923372333853619879637372673184942859712431066345726870422099893124890777
678037369988150*e^(235*I*c) + 380260499670589110696462063384896480703709885
451018226324303059729563076035359753197432475226638919318576087827418801344
0*e^(234*I*c) + 20696982895008606434616657623738079575130194240411788719049
60551829412449344722432125679417958403007551179298315947373776*e^(233*I*c)
+ 1115398285545601550533328045600184317993899359217996729818340704221801195
667410485846996179056733558512452238583160792512*e^(232*I*c) + 595157615500
431514947479282336547053827926087916425263497187757029413385471835434198246
807096214536895441388306027237899*e^(231*I*c) + 314409035808225861565595436
938354945445473991043129722046747228813030925204968503418566818838611866709
807040793495364496*e^(230*I*c) + 164437500676906892741323260154394278503954
561936020133596581806449357240277349447927334517105809995300093549279931273
178*e^(229*I*c) + 851395332347864557795899594649006377607357297056212213800
05837208369157794673049675428799817875431430246332625899630160*e^(228*I*c)
+ 4363830007581517102594621146446568961896577348866098585794565747985408510
8851857911222911989837615452608512356008400295*e^(227*I*c) + 22140735001708
603270915180769241391662035578755903979148909213603822554792749183517160255
571915875356439553717130797888*e^(226*I*c) + 111194996453632010808810628248
863384924253756584489779355358463492904258215703830904254114185215166703713

$72045206568345 \cdot e^{(225 \cdot I \cdot c)} + 5527498849031178355861230009326668283926290082$
 $158467118000698502719379939045918344222192742145711257028040974074674736 \cdot e^{(224 \cdot I \cdot c)} + 271958928348374392604080510108034192124453031125460725092919277$
 $3909331523226635035815672862569296693711643521070331394 \cdot e^{(223 \cdot I \cdot c)} + 13243$
 $113249840274283552229381476823786728607088171617414487496895935880208608475$
 $08703702325320304649883120684987556400 \cdot e^{(222 \cdot I \cdot c)} + 6382188929145337415058$
 $063996624885560667836004960918764083749748774489717780360749962455811242834$
 $60438065182071976085 \cdot e^{(221 \cdot I \cdot c)} + 3043844711068133360102841601239063704338$
 $888284906274226525512369667909161745208577591439301401871734923949819082589$
 $44 \cdot e^{(220 \cdot I \cdot c)} + 1436576871380447969429471197042595384588181994235168246745$
 $86293691056119209866358123637772245409530799230553767222252 \cdot e^{(219 \cdot I \cdot c)} + 6$
 $709170613052966912501989921002157658023784346222953538629508707618929784999$
 $5931360645605292130961496106707521506432 \cdot e^{(218 \cdot I \cdot c)} + 31004319206069417077$
 $069363141423487431828009098184744635678652284177439464941651812564519144918$
 $003174108077634846014 \cdot e^{(217 \cdot I \cdot c)} + 141764836528757049572020133432411179043$
 $699777966538499599025244219806351890116348156532796054977833828889327667300$
 $80 \cdot e^{(216 \cdot I \cdot c)} + 6413381895855925184758231451062556380328594938511006577015$
 $218119786536390213057284018202201631094434819584025113465 \cdot e^{(215 \cdot I \cdot c)} + 287$
 $049613141231445183467471535358943955329443080853193346608628854370924623076$
 $9151392180699413405623017247753532944 \cdot e^{(214 \cdot I \cdot c)} + 12710330829380489502013$
 $605548312703426623439912775046123423003660250467417428565804452894017866563$
 $11685859023084716 \cdot e^{(213 \cdot I \cdot c)} + 5567563887111823403410261927342195461136517$
 $68317380539005893679049394714017063698565272728813669054779077208977840 \cdot e^{(212 \cdot I \cdot c)} + 2412460212824400617929083177830328761948015971332060520912869970$
 $43729145345755805710081489006741839439573984832678 \cdot e^{(211 \cdot I \cdot c)} + 1033997554$
 $67257436489847837640753754718204394473055795001467604326419876555568738295$
 $31737211096115196005647730480 \cdot e^{(210 \cdot I \cdot c)} + 4383497214291937768537869223302$
 $106374455403310092850273748043897897674698989578407095190523778349037430593$
 $4542955 \cdot e^{(209 \cdot I \cdot c)} + 18379807084003359766027649217621144116091735572216620$
 $788861535803449702273802588359076704241840733513439114113248 \cdot e^{(208 \cdot I \cdot c)} +$
 $762178879191204706203884091779937460042889225819436763668294435609668140024$
 $6312138001769285020661445991073249416 \cdot e^{(207 \cdot I \cdot c)} + 31256834931787017434797$
 $047503074901786662921507201793636043351135286233296846063431855407560199356$
 $62148267863968 \cdot e^{(206 \cdot I \cdot c)} + 1267597017294812913400146276042126929986480292$
 $870190399107554311079964227280196522475370108738477856311765699610 \cdot e^{(205 \cdot I \cdot c)} + 508324599301085460166978629683032661427654474082939048097939638391567$
 $298795788389433842285751054665210868287680 \cdot e^{(204 \cdot I \cdot c)} + 201557947424794098$
 $026772478462040883395867512562930862943753568690084015585598010154781548625$
 $239409581907397500 \cdot e^{(203 \cdot I \cdot c)} + 790191495587665692547839884872323883529091$
 $44982747171856772463223808993367091503402876467270176124342699654400 \cdot e^{(202 \cdot I \cdot c)} + 3062758105422195737839054728927760912957281393108273352024738722600$
 $0020043538279468776707958420892547870128680 \cdot e^{(201 \cdot I \cdot c)} + 11735856926245118$
 $493113091002501604032341876985999082823520672241530200188223826392982302194$
 $084667538488665600 \cdot e^{(200 \cdot I \cdot c)} + 444541225929547462506765951419831296601541$
 $6299968930393345630345914109720740573618884980520010028451496996210 \cdot e^{(199 \cdot I \cdot c)}$

$I*c) + 16644750343872118093949177435029376389785749377547647639878358724104$
 $49930690131572904279995484581013965001440*e^{(198*I*c)} + 6160031160229795849$
 $371257017578872129983543009899913626280388610939145613320711919097149494265$
 $87936910303300*e^{(197*I*c)} + 2253205325932206577679411092895162489997945210$
 $15564134982827241710019675486694499689312466561907212627820000*e^{(196*I*c)}$
 $+ 8145208141382911182887541756425005484603769331281148049290916075819598915$
 $5768107022568350953861815940704090*e^{(195*I*c)} + 29097651061247453406647569$
 $781836910062165559852359052804259154165687125428752562385492373749486351714$
 $453120*e^{(194*I*c)} + 102716025302028890024978135168494525909715128095290606$
 $65197301097052210064576088348023234671975463677418470*e^{(193*I*c)} + 3582718$
 $002163296061414536703715109897107198252739284546149343102348456124089657428$
 $594946438660859773886240*e^{(192*I*c)} + 123466804189240997878001808175544021$
 $6012582476396941937965899631953079203974222138794604328498972144766900*e^{(1$
 $91*I*c)} + 42035802483514679858361121014594215468443794936564789908837252480$
 $2156222884839580011688655664280691773600*e^{(190*I*c)} + 14137993825355684328$
 $056550580740330413060613072543475174579407983314136174891763998614537706643$
 $7210546190*e^{(189*I*c)} + 46970224727117281826454045018070670522559756627580$
 $347784535320014963482632359729444541885102274546002560*e^{(188*I*c)} + 154131$
 $112114860239372949708207973767160813447881633865435224219397375079621258549$
 $81881879168348260330000*e^{(187*I*c)} + 4995241956279138180205186744401688024$
 $388272113921255663734956946927571305533146776898787878059685108480*e^{(186*I$
 $*c)} + 159877110105819269227052899967744474268563100623245618584492522014400$
 $2305878120380828483988663574829100*e^{(185*I*c)} + 50529366312301525887848302$
 $573881281320339776684534006538126101635341972238262039303253596066092195040$
 $0*e^{(184*I*c)} + 15768584552885091872146287786443509025758314941556132342738$
 $6562894447598277935629800939237175625149830*e^{(183*I*c)} + 48584258153140280$
 $447314836868772131390195412419046732778458706015096881437076337910793584122$
 $475073760*e^{(182*I*c)} + 147779550966171289987127451820714953621765069731830$
 $81650233605274051677624970464340242755840025673760*e^{(181*I*c)} + 4437210917$
 $843182347764349544443904699020056595069470847193617092114714077633077234972$
 $825351226979360*e^{(180*I*c)} + 131505212093069212210229710532762284233587074$
 $3428530891072983535862280094446607723473800477453914130*e^{(179*I*c)} + 38465$
 $584208066627445406307878483717499894905250097532216200339254995341359246151$
 $9365177908682078400*e^{(178*I*c)} + 11103414879700881944314389564446924229504$
 $9867464313710969257619338899133799285616020069872611710850*e^{(177*I*c)} + 31$
 $626644674725547731176795687527653571305969985923688392112164915553242573269$
 $490908989570248533280*e^{(176*I*c)} + 888829502875102466704420383760797610148$
 $0053134418614474620767522824868911959884352666444917404000*e^{(175*I*c)} + 24$
 $643821908074396090797742268556796293678857097764358766308517162539626961923$
 $41706239192878728160*e^{(174*I*c)} + 6740255305431330088948457752366252374507$
 $43114473544537818170447134607102575676676056675328961590*e^{(173*I*c)} + 1818$
 $346614061779013153301296771453811664491884131941411693443547549209690349526$
 $10378945282257600*e^{(172*I*c)} + 4837948975643409984385779181658937940681504$
 $2609340378747586437145781646245422045101230417309900*e^{(171*I*c)} + 12693496$
 $932964920565073673637181280088548682508880255337280065006566138696041797353$

216584528640*e^(170*I*c) + 328387476055581867672630948030673442015509858394
 8074469014168171874442170109648521627538755920*e^(169*I*c) + 83757920692341
 193245878648676537353394654523970899076948872481398218916510458989551890925
 6320*e^(168*I*c) + 21059430138564847118432907888031750495336183995415942743
 4009884661777259752542647709150036990*e^(167*I*c) + 52190912207661824215812
 271854269748071292843243227894769229690720010547141334131610989636000*e^(16
 6*I*c) + 127472196165033205413563430625628473686016221408567860254458145320
 37904111523242298235713300*e^(165*I*c) + 3067974296431747364198159623962463
 671617006419626851426148418602934852907379021659761911840*e^(164*I*c) + 727
 521010718394229291774073844694255798738667067535379759732795567942578751384
 250780476310*e^(163*I*c) + 169956327969929767773902096652629253283704505477
 127544556534417376686540936706073847337600*e^(162*I*c) + 391080312556018094
 76537535369611844440844903751605645023514572352045248104262933598850730*e^(
 161*I*c) + 8862752142756957285681340885764904597935349569355321815647721172
 537159186491471311666400*e^(160*I*c) + 197779298066581813565130009432623915
 8605448870806970860577325385028609983034534672318500*e^(159*I*c) + 43454667
 678028004534634449876389254079717510575682751550929702418766029934548492019
 2480*e^(158*I*c) + 93986915313068179149083606065681482780836060510530154618
 486949839467131378859885998210*e^(157*I*c) + 200080068030300471372932782503
 21597113540716201983333126349281186679153199068045257216*e^(156*I*c) + 4191
 542500656826148093339414544159143964478472492315931809171859902114109005939
 942952*e^(155*I*c) + 863979933622330349556296820028395513198708064940505702
 126068652936800794826651264256*e^(154*I*c) + 175193170500618300241515632381
 912285157790097816049220671217212220015297133400636060*e^(153*I*c) + 349410
 716132767046494779430433394502015040753351603618659160292138607786062306249
 60*e^(152*I*c) + 6852993223145736687328885311617795435592940841439866351079
 655652312894721972796266*e^(151*I*c) + 132149805527130085142999386663161987
 4424534425188183592049727687571032156435077280*e^(150*I*c) + 25050102860892
 8332469340456829902067712233644464602753159945727868485722395506952*e^(149*
 I*c) + 46668223548266017806854592468100570289355960869613650856575756758180
 182223308768*e^(148*I*c) + 854301344112621233483354066506962147247908583804
 1360564550722036723654297540205*e^(147*I*c) + 15363332384449275835327345560
 16494671674916578907116984548489078241693926940560*e^(146*I*c) + 2713612075
 03266570734486517077181014801775322183181055638619257836143271472358*e^(145
 *I*c) + 4706504461113515810848735336748424310269824883831263587628309942744
 2745866704*e^(144*I*c) + 80137295807907524343619649457615437614695207912107
 46972675870481058674277844*e^(143*I*c) + 1339214374254245553564884406801945
 353385000254030655765953770237607180089968*e^(142*I*c) + 219601281339515561
 500261478844190024870555261281946058839614044697037963695*e^(141*I*c) + 353
 24447206779018115378052820789411687581004582367431006205879633729015200*e^(
 140*I*c) + 5572551157328671121016216416307596161861955969011697222340926210
 112854418*e^(139*I*c) + 861884851094991908764246805474672428603757315484453
 974713612812215428992*e^(138*I*c) + 130657660226560419335121434389938961884
 595434069984824307149332131747540*e^(137*I*c) + 194079792155945665935350081
 03303255257745408070082431338945184797463936*e^(136*I*c) + 2823905151936586

678382525706564457280290098698638597987628380245881715*e^(135*I*c) + 402349
 692266121158934003582839428785116904903936409545602519219664720*e^(134*I*c)
 + 56117081076341175384087570185188538660375932013674735519055227368366*e^(
 133*I*c) + 7659010520187549651777118357676871927081898989131125755798204236
 112*e^(132*I*c) + 102253643746829673729306586270524644969368741555986584430
 6888705423*e^(131*I*c) + 13349021005202618377967331386833230353033290616324
 7194627808410304*e^(130*I*c) + 17033886027390615741040977721655541665612162
 275485028584310890417*e^(129*I*c) + 212370296918887131826671878122392706783
 9949015727293884065388080*e^(128*I*c) + 25858534871597727015582911568419341
 1072034541491364393985491350*e^(127*I*c) + 30736217404321009965231037419663
 053962881035281709221697785072*e^(126*I*c) + 356476489062872401708848799668
 8178929195787613958545474804845*e^(125*I*c) + 40321222595779818884084613996
 0995624144491271694336796459584*e^(124*I*c) + 44456708175258821024400946210
 535004523775722190977468484496*e^(123*I*c) + 477539860710085326353420773381
 8266777478693412738731031680*e^(122*I*c) + 49946750655853173367158586291057
 2702811545035730398749530*e^(121*I*c) + 50836369508171099437019348610847391
 946736185108017183136*e^(120*I*c) + 503202490340145182407421394376601192202
 6507006311982753*e^(119*I*c) + 48409341024048871865591702530366258109165912
 6182344528*e^(118*I*c) + 45230940039830738332025694784646206844854827698075
 736*e^(117*I*c) + 4101545439937195793959956708442496709433800261224880*e^(1
 16*I*c) + 360688613036389349413809780004559963548775423325255*e^(115*I*c) +
 30735366512830562160991166338490057308062762518496*e^(114*I*c) + 253566746
 0650279776834561566186591213109251642859*e^(113*I*c) + 20234750972446217131
 3966643580234078508179838320*e^(112*I*c) + 15603911277687607099721623771744
 933086920587272*e^(111*I*c) + 116158141373397175153362251190904691718876840
 0*e^(110*I*c) + 83380839911837894453136303673785039051506805*e^(109*I*c) +
 5764601046563151304213854710715346838447392*e^(108*I*c) + 38336015580105482
 4529764688213114368047154*e^(107*I*c) + 24489837337812338687718622491865013
 839488*e^(106*I*c) + 1500602747937397286405577818722691539392*e^(105*I*c) +
 88054927598941411145869950813388040256*e^(104*I*c) + 493966661081802579880
 9586352543471345*e^(103*I*c) + 264410375780310742518099326419685040*e^(102*
 I*c) + 13477227799524701956579274210395326*e^(101*I*c) + 652650253343206047
 453620559993840*e^(100*I*c) + 29952547749265499675257842032197*e^(99*I*c) +
 1299146645993240318167826532288*e^(98*I*c) + 53090127264630963470039804475
 *e^(97*I*c) + 2037031259470368160131922320*e^(96*I*c) + 7309920781733559724
 7098038*e^(95*I*c) + 2442455629894502983849104*e^(94*I*c) + 755998170926701
 57806639*e^(93*I*c) + 2154864144781257856128*e^(92*I*c) + 56169444526926562
 260*e^(91*I*c) + 1327882849274858880*e^(90*I*c) + 28186192554792138*e^(89*I
 *c) + 530563624556832*e^(88*I*c) + 8718181624155*e^(87*I*c) + 122503723056*
 e^(86*I*c) + 143118260*e^(85*I*c) + 13343760*e^(84*I*c) + 93096*e^(83*I*c)
 + 432*e^(82*I*c) + e^(81*I*c))*tan(1/4*d*x + c) - 28*(338*a*e^(1055/2*I*c
) + 136552*a*e^(1053/2*I*c) + 27515228*a*e^(1051/2*I*c) + 3687040552*a*e^(1
 049/2*I*c) + 369625815338*a*e^(1047/2*I*c) + 29570065227040*a*e^(1045/2*I*c
) + 1966409337599161*a*e^(1043/2*I*c) + 111804416623842644*a*e^(1041/2*I*c)
 + 5548294175019610066*a*e^(1039/2*I*c) + 244124943708196709459*a*e^(1037/2

$*I*c) + 9642935277137118257641*a*e^{(1035/2*I*c)} + 345392409065820898531490*$
 $a*e^{(1033/2*I*c)} + 11311601399861219435495350*a*e^{(1031/2*I*c)} + 3410882885$
 $20555248394683275*a*e^{(1029/2*I*c)} + 9526108636579479437964936790*a*e^{(1027$
 $/2*I*c)} + 247678824846833277253335834952*a*e^{(1025/2*I*c)} + 602169144006897$
 $7811219750309348*a*e^{(1023/2*I*c)} + 137436252062086506810801408828862*a*e^{($
 $1021/2*I*c)} + 2954879430842410604932011265442302*a*e^{(1019/2*I*c)} + 6003070$
 $8766686810922979823601133828*a*e^{(1017/2*I*c)} + 115559115252535315590169395$
 $1332528574*a*e^{(1015/2*I*c)} + 21130809863813051497818215704863434230*a*e^{(1$
 $013/2*I*c)} + 367868194965564832606935721682843410685*a*e^{(1011/2*I*c)} + 610$
 $9811001129869767843196320577458616044*a*e^{(1009/2*I*c)} + 969932519324439120$
 $18716975689508023006378*a*e^{(1007/2*I*c)} + 14742974741414654505801520716948$
 $90996247655*a*e^{(1005/2*I*c)} + 21490721703859150501477595681763435720778909$
 $*a*e^{(1003/2*I*c)} + 300870118521192302444085162135662723458505456*a*e^{(1001$
 $/2*I*c)} + 4051001485525986829845147001535913818302067194*a*e^{(999/2*I*c)} +$
 $52523333574538984648429489695519406526756976563*a*e^{(997/2*I*c)} + 656541730$
 $750769495168952644914474242735369097675*a*e^{(995/2*I*c)} + 79208592006957928$
 $07006454406332207329276812132106*a*e^{(993/2*I*c)} + 923275278104006907737330$
 $44860754764248910029932264*a*e^{(991/2*I*c)} + 104078321435193965518519619444$
 $9040653397856679303077*a*e^{(989/2*I*c)} + 1135678381925297678449970428155489$
 $5724049545296747183*a*e^{(987/2*I*c)} + 1200574576690021691574063809519765414$
 $04817260989994202*a*e^{(985/2*I*c)} + 123058929109804195915156519879228661116$
 $4919449523155204*a*e^{(983/2*I*c)} + 1223937869025964067609819069865193762558$
 $4285361664235941*a*e^{(981/2*I*c)} + 1182066779127422653081510686327798536126$
 $19007457778914450*a*e^{(979/2*I*c)} + 110932472438070529228344909323195203490$
 $7213153230066378718*a*e^{(977/2*I*c)} + 1012259361743842145487294577447161125$
 $6945529490238801736444*a*e^{(975/2*I*c)} + 8986893687601512413808893975591804$
 $7910164857950059626671506*a*e^{(973/2*I*c)} + 7767249544377961523865765269609$
 $81118865651225484076140617434*a*e^{(971/2*I*c)} + 653894588051752346886632342$
 $0234290611269386227470050003903740*a*e^{(969/2*I*c)} + 5364913138281010497780$
 $2029620946554089637208561026255904068048*a*e^{(967/2*I*c)} + 4291935357915568$
 $47068138327994764831727769099957108368643958154*a*e^{(965/2*I*c)} + 334957997$
 $6888722895534725449848627543256623010923459353099036635*a*e^{(963/2*I*c)} + 2$
 $5513859756352077572322437078487111620320963476059001013262542630*a*e^{(961/2$
 $*I*c)} + 189759652753983560479873697200916127533963528143615261433871315234*$
 $a*e^{(959/2*I*c)} + 137866461996963580632562696727707874967717301165639385819$
 $5336896873*a*e^{(957/2*I*c)} + 9788540192454256321255171083155760919251578109$
 $432314011159159160067*a*e^{(955/2*I*c)} + 67944153101887378416790537745291711$
 $484709226979885783161630835768882*a*e^{(953/2*I*c)} + 46123756141520107036405$
 $7977767537275375289232929441981287451225816828*a*e^{(951/2*I*c)} + 3063323292$
 $859702877329490930028863711679805587445419775185809699425497*a*e^{(949/2*I*c$
 $) + 19911672244744145105981587207520013241497438423672824426122213941696300$
 $*a*e^{(947/2*I*c)} + 12671114830426751424190809183869304959447341068383855569$
 $6089694914431850*a*e^{(945/2*I*c)} + 7896855214518767691555828125289111804336$
 $23630278166458083659977181181120*a*e^{(943/2*I*c)} + 482126215426248818963834$
 $5987905919984432326565079712079669628450138845920*a*e^{(941/2*I*c)} + 2884460$

9848991141927798576089491975808162650525713844636694817073554205876*a*e^(93
9/2*I*c) + 1691575871261344171246507479203089822250010714574756888025260633
35881435874*a*e^(937/2*I*c) + 972662930563310724596651192822395090437846904
768028338179628461156644761176*a*e^(935/2*I*c) + 54852238306960748702326704
30705003547243336234497802574847555967196592940404*a*e^(933/2*I*c) + 303459
36432342208649816880117958528700783259643810281015011805920593868190276*a*e
^(931/2*I*c) + 164736666188935404351422426866306331448337931278840039090867
647515115028970040*a*e^(929/2*I*c) + 87774689744357912123607537749035506840
4009709042755817010349573425737869900308*a*e^(927/2*I*c) + 4591345596645571
682092817716756142718329238022456900510608005443836033879600972*a*e^(925/2*
I*c) + 23583128005676152220406987675206877720027199089252365783381244222564
261342679308*a*e^(923/2*I*c) + 11897331385794653105655715724616432599593579
578912435085803330684419827752028872*a*e^(921/2*I*c) + 5896271106669139506
76757546277033132409936475850749602886519410515281489835432948*a*e^(919/2*I
*c) + 287127791318624141958413353382762329827783571035499712873031652967948
2826595460860*a*e^(917/2*I*c) + 1374138038627589222093695542888436527965756
8364963495869127376859464108093967124930*a*e^(915/2*I*c) + 6464391889342595
0177388154847859391478782846138908883463204853647802214592593258800*a*e^(91
3/2*I*c) + 2989850790026969240892379366745706536870238924796726552816220930
32615608331810198500*a*e^(911/2*I*c) + 135980246096861627023186829981433807
6025075555715544920009211265679248387475802712230*a*e^(909/2*I*c) + 6082529
830043102415483569107207792348780163318212711365913239332777970405599581316
610*a*e^(907/2*I*c) + 26763951325033666954800413916765668459687382409352116
237121465028288206531963685595620*a*e^(905/2*I*c) + 11586362130015229843964
7525945111489920370403992774304311355052277945386826708814291260*a*e^(903/2
*I*c) + 4935670236408676402999728895029447489431068811251115098054642766520
17586793148209575990*a*e^(901/2*I*c) + 206926767984593013038117533405645768
8285267649989023134787201410929393396836695689578160*a*e^(899/2*I*c) + 8539
376495592153288945859284463236872792162702391449354612981234209920197053609
719548280*a*e^(897/2*I*c) + 34692870306362831030990051675345388059171005150
015300151358457747567034555085057721123200*a*e^(895/2*I*c) + 13877868790994
660086536738876108796267807903732788250853878860927246423229174112017564756
0*a*e^(893/2*I*c) + 5466835481623203187556075430635408591424536687142525351
05543063481098088223075742274691040*a*e^(891/2*I*c) + 212099919547452096234
3810357637122282201408039497316115675157834511715525841111903287821280*a*e
(889/2*I*c) + 8105789144183930797661099129038091554545587802074817715094441
746173533262684806046690657020*a*e^(887/2*I*c) + 30518130135951809799990437
898771229881249172761806983346196310869995337802376609717118084280*a*e^(885
/2*I*c) + 11320990500529498520396461573817530853760676272061503574259383886
3752559149966030961364432530*a*e^(883/2*I*c) + 4138371574399715139574753471
06122090418096103121093503384586422597088548387007642778689392720*a*e^(881/
2*I*c) + 149089273623964382634929819417234773085939318472934839095576860284
5045895181329808155623088100*a*e^(879/2*I*c) + 5294039482427498842362101525
222433976284532136979899965317673458198554728600898182320842047910*a*e^(877
/2*I*c) + 18531157595691549190393635671237547986204095835636771457399022595

077629357340612025141454239410*a*e^(875/2*I*c) + 63950218579261019289219874
 315334477337029858557485643801909272432107476854548119403002345066040*a*e^(
 873/2*I*c) + 21759752893137327175565581792811386863191633015055007800380189
 2262646350629271469008473415746660*a*e^(871/2*I*c) + 7301053258029150884730
 68866423446534384705188187298829342664068496621137095589328874374034838030*
 a*e^(869/2*I*c) + 241592133355152499026838910461888965079910536783232883186
 7199400393567997657263303511921645584430*a*e^(867/2*I*c) + 7884801970520811
 955441008837645862835212712516344520632689690055398622920399678675555557890
 805540*a*e^(865/2*I*c) + 25383616625493348511882098211367472769175014354807
 241300095082666759379081238794081850941236922200*a*e^(863/2*I*c) + 80614635
 431298870624439504033093043824158052768845792386616425430054670214293015106
 952021225106450*a*e^(861/2*I*c) + 25258873634435505503243284556968337107183
 3282115166243901085698005080677674847942191033976516438790*a*e^(859/2*I*c)
 + 7808989249976823509861102578177599880223773357744944470211283389223941687
 39338059483402091817158180*a*e^(857/2*I*c) + 238230074245335807950581709091
 9571011220019979440937399265243279675473364667112224683580742416880880*a*e^(
 855/2*I*c) + 7172301539423620517935286383902428659740254815627164523216102
 592820676307371873206856996395387197810*a*e^(853/2*I*c) + 21311749830326212
 206986674139478667891257147745844062881479090133534521094852002361976434826
 499389640*a*e^(851/2*I*c) + 62505162824247144292571048733415581477053088822
 642095333462366631829558281303110976783578410692040060*a*e^(849/2*I*c) + 18
 096128817673420902243731326658110262186894735996902828934125907564597716209
 4115793342533495420437760*a*e^(847/2*I*c) + 5172071186252742393371517081238
 54001896290595348769274951243022688257796610572314987340186199489059600*a*e
 ^ (845/2*I*c) + 145944456074551268420467739286898290320709717027409192458968
 9734580305663102531694189766409440027416920*a*e^(843/2*I*c) + 4066207570750
 493677205733542471897525965552077860291195993097382317471471408387205631440
 905418660098080*a*e^(841/2*I*c) + 11186755735982119629198830648437726101312
 278573506457168349138206372529199037094900014443021705886779480*a*e^(839/2*
 I*c) + 30392371290385479785453757756338248944418017833693287676089824301076
 398097218660304024089324314169620320*a*e^(837/2*I*c) + 81546124861479397539
 614501723874160380406731878703383764168994998055432856993829770850278568809
 909050230*a*e^(835/2*I*c) + 21609873819119569107388483888513005160694166063
 2814593281807643078453044461425859841133487398485483388540*a*e^(833/2*I*c)
 + 5656431434448800485913346705394141177144265844329919969195747793339596940
 62618454004998070039855100793060*a*e^(831/2*I*c) + 146253021897800298121457
 968617919623489564245937057650715072664542679553220489252347694934166954364
 3800450*a*e^(829/2*I*c) + 3735676388192217805929887543749126915271400393432
 261055053439624614357833258756735586551631642468395069030*a*e^(827/2*I*c) +
 94268200572844477819631075575138175906735054406956458457347535856436330775
 15924434771991502983134459465060*a*e^(825/2*I*c) + 235029138841458318743312
 125050953902259833539526161730448912078569171227266447342301986714369966526
 82109840*a*e^(823/2*I*c) + 578984787454840486695431706824552929273018827232
 15139524495282441968943841745895662476230936188809951369410*a*e^(821/2*I*c)
 + 140938444298376269916893711433632496771438085708922541655406018353943269

164564745597706818400543286501171020*a*e^(819/2*I*c) + 33902842298783339785
587066141058027278324194452030499291696127214232379386587826483059803981920
3382521385460*a*e^(817/2*I*c) + 8059610095791392594511123654985560244204929
74041763848423080423983218636053325922887926716959206416040146280*a*e^(815/
2*I*c) + 189360661343136444565311334986545864078433719641996809934840706070
8046548568397551017687935610548803578917500*a*e^(813/2*I*c) + 4397333303038
600909165303360796815900290650049574236517178673049221955238982869664008596
667433434500227270300*a*e^(811/2*I*c) + 10093425291042252925802036863801242
163211606474338252255663632238745912220583468344066577703311911110184928980
*a*e^(809/2*I*c) + 22901429802924746989983398990978348636420948713161325757
783624831891169692007204195378391317610973058655935960*a*e^(807/2*I*c) + 51
367253895728538394990558166897144622805283125587039048541605029080310508962
829559065760790760349012514574324*a*e^(805/2*I*c) + 11390278738034513725317
554017689546615643594278632039917779354918446196998225488813223531681316776
6293611215076*a*e^(803/2*I*c) + 2497070383826690871595552423450945814385469
78443025783340091950277447600449740136007172406984398821206740076664*a*e^(8
01/2*I*c) + 541251968877752332118813660249215928283602435291415731537931205
691521494543428945687343868761091899299763448926*a*e^(799/2*I*c) + 11600160
509032076678418985247311644870425619185091180078775066124292023740134865005
42675475746584871022464343284*a*e^(797/2*I*c) + 245837167801359979248664529
963840750287441793699785498682491096459381471489189941820641684694014748381
7202804120*a*e^(795/2*I*c) + 5151967040996773988611482287442349599955496339
968871196015715240968872929726412560039668037088680441779084914448*a*e^(793
/2*I*c) + 10677345120611501031691517702515755622203852633360648841958791107
152970141386266895230517166551174392300030788342*a*e^(791/2*I*c) + 21884699
869744594822797651399921996353213538304567148159730761256766296942857944789
316336187135948566011636746108*a*e^(789/2*I*c) + 44363645431253792866444490
367306074812309865170491117708512232599286173512025194047460621336633494994
931616185367*a*e^(787/2*I*c) + 88949838473151210126202868097375450026734887
640385861323252487035231744983449961665197741276037196752019899934708*a*e^(
785/2*I*c) + 17640715787024818867503982283797345377954122047597100386165612
6215795163365626566036433061912593074816945660675950*a*e^(783/2*I*c) + 3460
684085817000554267086399125138891627700284324753920963034347654175712353549
58450150764552565844163750312201645*a*e^(781/2*I*c) + 671589458149022342338
187587310313559940106979202995488930954816939410805542431176677931049253422
876784424817016775*a*e^(779/2*I*c) + 12893275062558872663245039240055328071
481718108400557314757429343750965479800062386001016971377511351713344279112
30*a*e^(777/2*I*c) + 244884478351411594020499495275884390390306613720021815
2757612724023451060388905375768675328804787228900093398681626*a*e^(775/2*I*
c) + 4601706976972742883467404775339200515272078650623212895074567249940816
451512616367674551127607081333270181526998549*a*e^(773/2*I*c) + 85557347948
303069427606265864203901730138173493861541872552893514421311575888808180865
47678120350930967986819636206*a*e^(771/2*I*c) + 157397271978971841864319782
601763631922636623444355999207203433578737827606351977234753501845252999854
72022233368096*a*e^(769/2*I*c) + 286522967097499650762578457086471778038418

18654181291497797896498566113487726904281960443070225181805904476964093204*
 $a * e^{(767/2 * I * c)}$ + 516136775752603751140573550542127606248289560668750323211
 87800557785476426719305739365978657330215320031862318328782*a*e^(765/2*I*c)
 + 920097899653368630008601707170195208321797974720498870784306102175254834
 42688425464817079510726308915387248081319590*a*e^(763/2*I*c) + 162325921367
 114920125342502623302414608326044873352001855642072426178071258568253753649
 777482260604487791662279095380*a*e^(761/2*I*c) + 28343080674123172076750664
 845238157997916706469192193526652246599903915191375493757037983217621968347
 5585745980384642*a*e^(759/2*I*c) + 4898152757642156305233916949785199236759
 455790302875169372878926136847341285899647819636510958725979738665128196888
 54*a*e^(757/2*I*c) + 837846552383396082937854578077113369184974325971396778
 753442236264006537694747924055527939607057424809294856490727675*a*e^(755/2*
 $I * c)$ + 14186144634747459319395806906257964356209274023267257071289786111832
 25665658663115625141407985113241273578748581984652*a*e^(753/2*I*c) + 237767
 738528145078027653500363210695783831543229498007404472144469700065885043391
 3097456261486621128187350479818244518*a*e^(751/2*I*c) + 3945037288827419478
 018677680594182145933168831784555276684612092094611158471077576449964287503
 346809348891265398259857*a*e^(749/2*I*c) + 64800749661966448729596639489157
 406799996435834290035746087168182010593604461394112623297260126288718439583
 39023337019*a*e^(747/2*I*c) + 105380651642538996809340809249110493758483069
 86850557084580551955642451251771191418581692424478501246323800746129573800*
 $a * e^{(745/2 * I * c)}$ + 169673877114962244778234439415913787832396202935047404923
 14898728896897745532993133409667857013503734512617665480477878*a*e^(743/2*I
 $* c)$ + 270497838117680633439558433289313889225603009659722001048486751021014
 01505754650642355872221358066002741301627025812757*a*e^(741/2*I*c) + 427001
 033010337778650535577086941480668033029900987358407118657168845872190459140
 23413725085698021434756439677135383181*a*e^(739/2*I*c) + 667470571270742686
 190239734042444467288578965846913400860477188328172379373790177434493796234
 61239962563329874824019974*a*e^(737/2*I*c) + 103322596182279139344692583372
 921792481340062327412457604573055278086602887748185048531214610304639645383
 482596676893056*a*e^(735/2*I*c) + 15839467685091204058294114034244145346344
 06765655726133155715460206950476685791052069202729893821895403211920311041
 4947*a*e^(733/2*I*c) + 2404862281568673909647552378625268280708050423054651
 56788364442627761972364982396289899653761051586004176156357053438713*a*e^(7
 $31/2 * I * c)$ + 361631832412353361979158598436439573362393207038652296132511637
 528660854756889993338874388973764737310559108721961804950*a*e^(729/2*I*c) +
 53863249400189950382316139684456646689486298248079524165666385843111592719
 3564638280643271545140293504620018648806033924*a*e^(727/2*I*c) + 7946771715
 905578344319659816833686638649262206648373372795785872193249295888569107797
 05968796220532402758676260405879107*a*e^(725/2*I*c) + 116140646968151468693
 023105716928281090103625805562253235991399731320411755388739321232882227360
 2211299474414691345126818*a*e^(723/2*I*c) + 1681497695671000636215144582963
 607157017178521461687498309132637123908932777244895367507216314619532694155
 538887238368802*a*e^(721/2*I*c) + 24118497504933326647870055501061178327305
 669830479171072315653703779731534904944652998910925512834761816251519841277

53260*a*e^(719/2*I*c) + 342743913293696221896504555669042475509741891833232
4019145162160654311058062898282097500536856553443553377411829820283274*a*e^(717/2*I*c) + 4825902627466762698823807773321203231601263493217828532189450
537425528946780148114383027368752159205150948205408147495002*a*e^(715/2*I*c)
) + 67328759214826293858269834686537944487192889758185970292219350803258378
39653180167035072543738369936072307403835829117580*a*e^(713/2*I*c) + 930807
856378404378101767113061024818208704691325291699688038676560862699334328112
3190939712394958769614572944446169239240*a*e^(711/2*I*c) + 1275208290398042
485997378225158036439217768534153708660430531822786965665706426267958018116
2371557742791081894355841587442*a*e^(709/2*I*c) + 1731363730722245001288168
669518610842373580752871815689957811120050997466618301189423521969937258122
7561012842803845507749*a*e^(707/2*I*c) + 2329733252724133486576008434346785
015247728714899590860668051459575561332078543062356207977438953237441387317
0123179910986*a*e^(705/2*I*c) + 3107130663886788714548646424310283071956307
515154972608466605068175799307542670857056904029472475784338893329516232083
3726*a*e^(703/2*I*c) + 4107458216038845529463213208019681113339185059988422
4560084096210500036772683324177263871865645028380575643276712484382279*a*e^(701/2*I*c) + 5382352470248531860165628199766388268378847664798619355503604
4978138774992020063718500148457278526659285673723602694203421*a*e^(699/2*I*c)
c) + 6991681346133465041567859682435812949181595663082785375796154290596193
2986850395133931124724066002703731115391919663302990*a*e^(697/2*I*c) + 9003
822979902911781825190342242898340865743154211020276769385829603914951367506
4655386150204671625355804206371140871319100*a*e^(695/2*I*c) + 1149565121648
042888135663204917688817683029075460344301231799098934384270779597701528959
78203628978930419723138966501119415*a*e^(693/2*I*c) + 145521505642887822547
402879630656245710055774793637312274910793201300321281257473947648611370199
879573400230644017226525880*a*e^(691/2*I*c) + 18265586426356567618558698757
069183245221201406507042910760948672195583226326816200018039326617638546448
0617543310025867318*a*e^(689/2*I*c) + 2273416543695389638365135008445981190
454238798028219407996159060808328004693063580088180738355445853616490704526
44644673032*a*e^(687/2*I*c) + 280601365636611572910535258329456756850440149
892624666211811853985826030746961947811258965778308897372914648649596408103
468*a*e^(685/2*I*c) + 34347306663050400395746171107925527584950943797003516
0833870481102720034535490851137644957145445343032779474377183842954352*a*e^(683/2*I*c) + 4169797425425043990532670465569278183158192561016295549402653
87625678126952894787142342102437270856002337128966125367035438*a*e^(681/2*I*c)
*c) + 502093216108411100751308736946549255326079321825250649068631607319287
111256994298784601423217071295643346748150667815747120*a*e^(679/2*I*c) + 59
969346376928518201678941452400286647919025612128164360913087795177260473427
9684159983894089320154006036997114520381278504*a*e^(677/2*I*c) + 7105245758
470710442517459931302957565489757529524758790166567800889124101220139945179
28255364300113796241185885146892353096*a*e^(675/2*I*c) + 835149043316362104
631899593048750305868037185668945816167610720358206827717921729336701823612
277340955328663616780414165744*a*e^(673/2*I*c) + 97390245089215896555778269
304218994521664405462061030012960430945041058869530488356374179932290858600

7062103356708940279896*a*e^(671/2*I*c) + 1126850979751396565140782629117395
944440965035663367664028781279428303355792255796436595232774704614843366308
657035222636184*a*e^(669/2*I*c) + 12937543360432761076355198257438067523708
321577659028368160098789215935311071597390819833266485349909639524471046370
63475320*a*e^(667/2*I*c) + 147403678911314512551158602765451942976784711541
874878030161967756912464481716292555681407152865081422082647584171197039816
0*a*e^(665/2*I*c) + 1666768898191987322999144235737438870879557567757751482
811320802648081692700145607633178442601838307940829929542219394816600*a*e^(
663/2*I*c) + 18706622102088808852999880909895550932520381786568191049759028
72774878847343035752722631279136134636190194620028777062771000*a*e^(661/2*I
*c) + 208407871933677540917899940456096383454677914237869374731808161997221
0644062098160313254276418251294113576487118901711135260*a*e^(659/2*I*c) + 2
305056201085645773403211503495815406053507086905620072615040978473923133843
650811925101283679618986151289094096579669424320*a*e^(657/2*I*c) + 25313496
968308509890284231267179302514210679382841376178663527198815284677620706152
24279447864998980982885764701335750299640*a*e^(655/2*I*c) + 276048847146553
607900165140742040618677360592610819558290806380573776619017848400674978875
3656431318199675121290619375614420*a*e^(653/2*I*c) + 2989846754795630884517
42611699327002567506887325697076095597491657316974565550814846206617487155
647589839236464701796279580*a*e^(651/2*I*c) + 32167255750742065288289241000
585847486655424895501059838089200035688084598954849194176558571086241632742
12339927440740825720*a*e^(649/2*I*c) + 343844207611450756582822762976765746
521906220363406316774466921625914062567998791791335291072502438017239249240
8078597847560*a*e^(647/2*I*c) + 3652421954113120225310841175679141398963043
623550797204205304789758747261524665291514179819649437844381518984788554724
814900*a*e^(645/2*I*c) + 38562901276126900452398350896871431103276318538740
17128257779747123864275094719619217129392947484275435248946627983054371120*
a*e^(643/2*I*c) + 404795452293663586301595872787961053774872040614739375472
8504285325965759343128650059937388360412866431049119645619636844080*a*e^(64
1/2*I*c) + 4225677958926327493993763472255965806828293675881633167149510152
473723281831266177845898962662547088283153410811540763937760*a*e^(639/2*I*c
) + 43881335669187429662358970955377267291498454916482083469907321391844050
00815590708092488420647208468573326448979993238919840*a*e^(637/2*I*c) + 453
443997633345931051148250499011965067283942984759027862012615398218816207060
4003003067465034271078676682298452574885223760*a*e^(635/2*I*c) + 4664173597
156401175633352539083741017902415354471984347164039208649991585553846020213
682464029641764522414226537622903667360*a*e^(633/2*I*c) + 47773566757229147
724278806808520979473338247451134911762739216027244258196920432828104797089
39732376220492567215846763821640*a*e^(631/2*I*c) + 487442130425961717527650
059920304297345996116629620426299356802648481012729938614907315819094824976
5374087088863694435755200*a*e^(629/2*I*c) + 4956151125142365039669234983887
976235121880878207204979890024715172281327836676831620180909262717497111396
726001076096795420*a*e^(627/2*I*c) + 50236039755668513341133270435669750626
536558446560624089140657437007953600679262961946072561102742620485028715241
34363430720*a*e^(625/2*I*c) + 507802005419236558820490482561501954862434968

815803334760399525582020656475736911704403434979904850117375287518119099785
0680*a*e^(623/2*I*c) + 5120721253551683019502884820787496119098955245751439
749047492279319103337844456766689416423308503828722941122090578157783220*a*
e^(621/2*I*c) + 51530080031733571957890619457363179152671276591460275779374
46882680816453705847341473856934451889447842664418960798801389660*a*e^(619/
2*I*c) + 517606024609505464674676976202255204827218471556567919827900412983
5648608965961963638205672345664098844084476717934767765360*a*e^(617/2*I*c)
+ 5190848994420407665005744645452103615055627990337999053339203475748827740
596926190664008371199178877128179934687123252625080*a*e^(615/2*I*c) + 51980
642808628742116568861395175641614799564141217687272089207515205084062164994
32758187510936276181271178602086681965044900*a*e^(613/2*I*c) + 519806428086
287421165688613951756416147995641412176872720892075152050840621649943275818
7510936276181271178602086681965044900*a*e^(611/2*I*c) + 5190848994420407665
005744645452103615055627990337999053339203475748827740596926190664008371199
178877128179934687123252625080*a*e^(609/2*I*c) + 51760602460950546467467697
620225520482721847155656791982790041298356486089659619636382056723456640988
44084476717934767765360*a*e^(607/2*I*c) + 515300800317335719578906194573631
791526712765914602757793744688268081645370584734147385693445188944784266441
8960798801389660*a*e^(605/2*I*c) + 5120721253551683019502884820787496119098
955245751439749047492279319103337844456766689416423308503828722941122090578
157783220*a*e^(603/2*I*c) + 50780200541923655882049048256150195486243496881
580333476039952558202065647573691170440343497990485011737528751811909978506
80*a*e^(601/2*I*c) + 502360397556685133411332704356697506265365584465606240
8914065743700795360067926296194607256110274262048502871524134363430720*a*e^
(599/2*I*c) + 4956151125142365039669234983887976235121880878207204979890024
715172281327836676831620180909262717497111396726001076096795420*a*e^(597/2*
I*c) + 48744213042596171752765005992030429734599611662962042629935680264848
10127299386149073158190948249765374087088863694435755200*a*e^(595/2*I*c) +
477735667572291477242788068085209794733382474511349117627392160272442581969
2043282810479708939732376220492567215846763821640*a*e^(593/2*I*c) + 4664173
597156401175633352539083741017902415354471984347164039208649991585553846020
213682464029641764522414226537622903667360*a*e^(591/2*I*c) + 45344399763334
593105114825049901196506728394298475902786201261539821881620706040030030674
65034271078676682298452574885223760*a*e^(589/2*I*c) + 438813356691874296623
589709553772672914984549164820834699073213918440500081559070809248842064720
8468573326448979993238919840*a*e^(587/2*I*c) + 4225677958926327493993763472
255965806828293675881633167149510152473723281831266177845898962662547088283
153410811540763937760*a*e^(585/2*I*c) + 40479545229366358630159587278796105
377487204061473937547285042853259657593431286500599373883604128664310491196
45619636844080*a*e^(583/2*I*c) + 385629012761269004523983508968714311032763
185387401712825777974712386427509471961921712939294748427543524894662798305
4371120*a*e^(581/2*I*c) + 3652421954113120225310841175679141398963043623550
797204205304789758747261524665291514179819649437844381518984788554724814900
*a*e^(579/2*I*c) + 34384420761145075658282276297676574652190622036340631677
44669216259140625679987917913352910725024380172392492408078597847560*a*e^(5

$77/2 * I * c) + 321672557507420652882892410005858474866554248955010598380892000$
 $3568808459895484919417655857108624163274212339927440740825720 * a * e^{(575/2 * I * c)}$
 $+ 2989846754795630884517426116993270025675068873256970760955974916573169$
 $74565550814846206617487155647589839236464701796279580 * a * e^{(573/2 * I * c)} + 27$
 $604884714655360790016514074204061867736059261081955829080638057377661901784$
 $84006749788753656431318199675121290619375614420 * a * e^{(571/2 * I * c)} + 253134969$
 $683085098902842312671793025142106793828413761786635271988152846776207061522$
 $4279447864998980982885764701335750299640 * a * e^{(569/2 * I * c)} + 2305056201085645$
 $773403211503495815406053507086905620072615040978473923133843650811925101283$
 $679618986151289094096579669424320 * a * e^{(567/2 * I * c)} + 20840787193367754091789$
 $994045609638345467791423786937473180816199722106440620981603132542764182512$
 $94113576487118901711135260 * a * e^{(565/2 * I * c)} + 187066221020888088529998809098$
 $955509325203817865681910497590287277487884734303575272263127913613463619019$
 $4620028777062771000 * a * e^{(563/2 * I * c)} + 1666768898191987322999144235737438870$
 $879557567757751482811320802648081692700145607633178442601838307940829929542$
 $219394816600 * a * e^{(561/2 * I * c)} + 14740367891131451255115860276545194297678471$
 $154187487803016196775691246448171629255568140715286508142208264758417119703$
 $98160 * a * e^{(559/2 * I * c)} + 129375433604327610763551982574380675237083215776590$
 $2836816009878921593531107159739081983326648534990963952447104637063475320 * a$
 $* e^{(557/2 * I * c)} + 1126850979751396565140782629117395944440965035663367664028$
 $781279428303355792255796436595232774704614843366308657035222636184 * a * e^{(555$
 $/2 * I * c)} + 97390245089215896555778269304218994521664405462061030012960430945$
 $0410588695304883563741799322908586007062103356708940279896 * a * e^{(553/2 * I * c)}$
 $+ 8351490433163621046318995930487503058680371856689458161676107203582068277$
 $17921729336701823612277340955328663616780414165744 * a * e^{(551/2 * I * c)} + 710524$
 $575847071044251745993130295756548975752952475879016656780088912410122013994$
 $517928255364300113796241185885146892353096 * a * e^{(549/2 * I * c)} + 59969346376928$
 $518201678941452400286647919025612128164360913087795177260473427968415998389$
 $4089320154006036997114520381278504 * a * e^{(547/2 * I * c)} + 5020932161084111007513$
 $087369465492553260793218252506490686316073192871112569942987846014232170712$
 $95643346748150667815747120 * a * e^{(545/2 * I * c)} + 416979742542504399053267046556$
 $927818315819256101629554940265387625678126952894787142342102437270856002337$
 $128966125367035438 * a * e^{(543/2 * I * c)} + 34347306663050400395746171107925527584$
 $950943797003516083387048110272003453549085113764495714544534303277947437718$
 $3842954352 * a * e^{(541/2 * I * c)} + 2806013656366115729105352583294567568504401498$
 $926246662118118539858260307469619478112589657783088973729146486495964081034$
 $68 * a * e^{(539/2 * I * c)} + 227341654369538963836513500844598119045423879802821940$
 $799615906080832800469306358008818073835544585361649070452644644673032 * a * e^{($
 $537/2 * I * c)} + 18265586426356567618558698757069183245221201406507042910760948$
 $6721955832263268162000180393266176385464480617543310025867318 * a * e^{(535/2 * I * c)}$
 $+ 1455215056428878225474028796306562457100557747936373122749107932013003$
 $21281257473947648611370199879573400230644017226525880 * a * e^{(533/2 * I * c)} + 114$
 $956512164804288813566320491768881768302907546034430123179909893438427077959$
 $770152895978203628978930419723138966501119415 * a * e^{(531/2 * I * c)} + 90038229799$
 $029117818251903422428983408657431542110202767693858296039149513675064655386$

150204671625355804206371140871319100*a*e^(529/2*I*c) + 69916813461334650415
678596824358129491815956630827853757961542905961932986850395133931124724066
002703731115391919663302990*a*e^(527/2*I*c) + 53823524702485318601656281997
663882683788476647986193555036044978138774992020063718500148457278526659285
673723602694203421*a*e^(525/2*I*c) + 41074582160388455294632132080196811133
391850599884224560084096210500036772683324177263871865645028380575643276712
484382279*a*e^(523/2*I*c) + 31071306638867887145486464243102830719563075151
549726084666050681757993075426708570569040294724757843388933295162320833726
*a*e^(521/2*I*c) + 23297332527241334865760084343467850152477287148995908606
680514595755613320785430623562079774389532374413873170123179910986*a*e^(519
/2*I*c) + 17313637307222450012881686695186108423735807528718156899578111200
509974666183011894235219699372581227561012842803845507749*a*e^(517/2*I*c) +
12752082903980424859973782251580364392177685341537086604305318227869656657
064262679580181162371557742791081894355841587442*a*e^(515/2*I*c) + 93080785
637840437810176711306102481820870469132529169968803867656086269933432811231
90939712394958769614572944446169239240*a*e^(513/2*I*c) + 673287592148262938
582698346865379444871928897581859702922193508032583783965318016703507254373
8369936072307403835829117580*a*e^(511/2*I*c) + 4825902627466762698823807773
321203231601263493217828532189450537425528946780148114383027368752159205150
948205408147495002*a*e^(509/2*I*c) + 34274391329369622189650455566904247550
974189183323240191451621606543110580628982820975005368565534435533774118298
20283274*a*e^(507/2*I*c) + 241184975049333266478700555010611783273056698304
7917107231565370377973153490494465299891092551283476181625151984127753260*a
*e^(505/2*I*c) + 1681497695671000636215144582963607157017178521461687498309
132637123908932777244895367507216314619532694155538887238368802*a*e^(503/2*
I*c) + 11614064696815146869302310571692828109010362580556225323599139973132
04117553887393212328822273602211299474414691345126818*a*e^(501/2*I*c) + 794
677171590557834431965981683368663864926220664837337279578587219324929588856
910779705968796220532402758676260405879107*a*e^(499/2*I*c) + 53863249400189
950382316139684456646689486298248079524165666385843111592719356463828064327
1545140293504620018648806033924*a*e^(497/2*I*c) + 3616318324123533619791585
984364395733623932070386522961325116375286608547568899933388743889737647373
10559108721961804950*a*e^(495/2*I*c) + 240486228156867390964755237862526828
070805042305465156788364442627761972364982396289899653761051586004176156357
053438713*a*e^(493/2*I*c) + 15839467685091204058294114034244145346344067656
5557261331557154602069504766857910520692027298938218954032119203110414947*a
*e^(491/2*I*c) + 1033225961822791393446925833729217924813400623274124576045
73055278086602887748185048531214610304639645383482596676893056*a*e^(489/2*I
*c) + 667470571270742686190239734042444467288578965846913400860477188328172
37937379017743449379623461239962563329874824019974*a*e^(487/2*I*c) + 427001
033010337778650535577086941480668033029900987358407118657168845872190459140
23413725085698021434756439677135383181*a*e^(485/2*I*c) + 270497838117680633
439558433289313889225603009659722001048486751021014015057546506423558722213
58066002741301627025812757*a*e^(483/2*I*c) + 169673877114962244778234439415
913787832396202935047404923148987288968977455329931334096678570135037345126

$17665480477878*a*e^{(481/2*I*c)} + 105380651642538996809340809249110493758483$
 $069868505570845805519556424512517711914185816924244785012463238007461295738$
 $00*a*e^{(479/2*I*c)} + 648007496619664487295966394891574067999964358342900357$
 $4608716818201059360446139411262329726012628871843958339023337019*a*e^{(477/2$
 $*I*c)} + 3945037288827419478018677680594182145933168831784555276684612092094$
 $611158471077576449964287503346809348891265398259857*a*e^{(475/2*I*c)} + 23776$
 $773852814507802765350036321069578383154322949800740447214446970006588504339$
 $13097456261486621128187350479818244518*a*e^{(473/2*I*c)} + 141861446347474593$
 $193958069062579643562092740232672570712897861118322566565866311562514140798$
 $5113241273578748581984652*a*e^{(471/2*I*c)} + 8378465523833960829378545780771$
 $133691849743259713967787534422362640065376947479240555279396070574248092948$
 $56490727675*a*e^{(469/2*I*c)} + 489815275764215630523391694978519923675945579$
 $030287516937287892613684734128589964781963651095872597973866512819688854*a*$
 $e^{(467/2*I*c)} + 28343080674123172076750664845238157997916706469192193526652$
 $2465999039151913754937570379832176219683475585745980384642*a*e^{(465/2*I*c)}$
 $+ 1623259213671149201253425026233024146083260448733520018556420724261780712$
 $58568253753649777482260604487791662279095380*a*e^{(463/2*I*c)} + 920097899653$
 $368630008601707170195208321797974720498870784306102175254834426884254648170$
 $79510726308915387248081319590*a*e^{(461/2*I*c)} + 516136775752603751140573550$
 $542127606248289560668750323211878005577854764267193057393659786573302153200$
 $31862318328782*a*e^{(459/2*I*c)} + 286522967097499650762578457086471778038418$
 $18654181291497797896498566113487726904281960443070225181805904476964093204*$
 $a*e^{(457/2*I*c)} + 157397271978971841864319782601763631922636623444355999207$
 $20343357873782760635197723475350184525299985472022233368096*a*e^{(455/2*I*c)}$
 $+ 855573479483030694276062658642039017301381734938615418725528935144213115$
 $7588880818086547678120350930967986819636206*a*e^{(453/2*I*c)} + 4601706976972$
 $742883467404775339200515272078650623212895074567249940816451512616367674551$
 $127607081333270181526998549*a*e^{(451/2*I*c)} + 24488447835141159402049949527$
 $588439039030661372002181527576127240234510603889053757686753288047872289000$
 $93398681626*a*e^{(449/2*I*c)} + 128932750625588726632450392400553280714817181$
 $0840055731475742934375096547980006238600101697137751135171334427911230*a*e^{($
 $447/2*I*c)} + 6715894581490223423381875873103135599401069792029954889309548$
 $16939410805542431176677931049253422876784424817016775*a*e^{(445/2*I*c)} + 346$
 $068408581700055426708639912513889162770028432475392096303434765417571235354$
 $958450150764552565844163750312201645*a*e^{(443/2*I*c)} + 17640715787024818867$
 $503982283797345377954122047597100386165612621579516336562656603643306191259$
 $3074816945660675950*a*e^{(441/2*I*c)} + 8894983847315121012620286809737545002$
 $673488764038586132325248703523174498344996166519774127603719675201989993470$
 $8*a*e^{(439/2*I*c)} + 4436364543125379286644449036730607481230986517049111770$
 $8512232599286173512025194047460621336633494994931616185367*a*e^{(437/2*I*c)}$
 $+ 2188469986974459482279765139992199635321353830456714815973076125676629694$
 $2857944789316336187135948566011636746108*a*e^{(435/2*I*c)} + 1067734512061150$
 $103169151770251575562220385263336064884195879110715297014138626689523051716$
 $6551174392300030788342*a*e^{(433/2*I*c)} + 5151967040996773988611482287442349$
 $599955496339968871196015715240968872929726412560039668037088680441779084914$

448*a*e^(431/2*I*c) + 24583716780135997924866452996384075028744179369978549
86824910964593814714891899418206416846940147483817202804120*a*e^(429/2*I*c)
+ 116001605090320766784189852473116448704256191850911800787750661242920237
4013486500542675475746584871022464343284*a*e^(427/2*I*c) + 5412519688777523
321188136602492159282836024352914157315379312056915214945434289456873438687
61091899299763448926*a*e^(425/2*I*c) + 249707038382669087159555242345094581
438546978443025783340091950277447600449740136007172406984398821206740076664
*a*e^(423/2*I*c) + 11390278738034513725317554017689546615643594278632039917
7793549184461969982254888132235316813167766293611215076*a*e^(421/2*I*c) + 5
136725389572853839499055816689714462280528312558703904854160502908031050896
2829559065760790760349012514574324*a*e^(419/2*I*c) + 2290142980292474698998
339899097834863642094871316132575778362483189116969200720419537839131761097
3058655935960*a*e^(417/2*I*c) + 1009342529104225292580203686380124216321160
6474338252255663632238745912220583468344066577703311911110184928980*a*e^(41
5/2*I*c) + 4397333303038600909165303360796815900290650049574236517178673049
221955238982869664008596667433434500227270300*a*e^(413/2*I*c) + 18936066134
313644456531133498654586407843371964199680993484070607080465485683975510176
87935610548803578917500*a*e^(411/2*I*c) + 805961009579139259451112365498556
024420492974041763848423080423983218636053325922887926716959206416040146280
*a*e^(409/2*I*c) + 33902842298783339785587066141058027278324194452030499291
6961272142323793865878264830598039819203382521385460*a*e^(407/2*I*c) + 1409
384442983762699168937114336324967714380857089225416554060183539432691645647
45597706818400543286501171020*a*e^(405/2*I*c) + 578984787454840486695431706
824552929273018827232151395244952824419689438417458956624762309361888099513
69410*a*e^(403/2*I*c) + 235029138841458318743312125050953902259833539526161
73044891207856917122726644734230198671436996652682109840*a*e^(401/2*I*c) +
942682005728444778196310755751381759067350544069564584573475358564363307751
5924434771991502983134459465060*a*e^(399/2*I*c) + 3735676388192217805929887
543749126915271400393432261055053439624614357833258756735586551631642468395
069030*a*e^(397/2*I*c) + 14625302189780029812145796861791962348956424593705
76507150726645426795532204892523476949341669543643800450*a*e^(395/2*I*c) +
565643143444880048591334670539414117714426584432991996919574779333959694062
618454004998070039855100793060*a*e^(393/2*I*c) + 21609873819119569107388483
888513005160694166063281459328180764307845304446142585984113348739848548338
8540*a*e^(391/2*I*c) + 8154612486147939753961450172387416038040673187870338
3764168994998055432856993829770850278568809909050230*a*e^(389/2*I*c) + 3039
237129038547978545375775633824894441801783369328767608982430107639809721866
0304024089324314169620320*a*e^(387/2*I*c) + 1118675573598211962919883064843
7726101312278573506457168349138206372529199037094900014443021705886779480*a
*e^(385/2*I*c) + 4066207570750493677205733542471897525965552077860291195993
097382317471471408387205631440905418660098080*a*e^(383/2*I*c) + 14594445607
455126842046773928689829032070971702740919245896897345803056631025316941897
66409440027416920*a*e^(381/2*I*c) + 517207118625274239337151708123854001896
290595348769274951243022688257796610572314987340186199489059600*a*e^(379/2*
I*c) + 18096128817673420902243731326658110262186894735996902828934125907564

5977162094115793342533495420437760*a*e^(377/2*I*c) + 6250516282424714429257
 104873341558147705308882264209533346236663182955828130311097678357841069204
 0060*a*e^(375/2*I*c) + 2131174983032621220698667413947866789125714774584406
 2881479090133534521094852002361976434826499389640*a*e^(373/2*I*c) + 7172301
 539423620517935286383902428659740254815627164523216102592820676307371873206
 856996395387197810*a*e^(371/2*I*c) + 23823007424533580795058170909195710112
 20019979440937399265243279675473364667112224683580742416880880*a*e^(369/2*I
 *c) + 780898924997682350986110257817759988022377335774494447021128338922394
 168739338059483402091817158180*a*e^(367/2*I*c) + 25258873634435505503243284
 5569683371071833282115166243901085698005080677674847942191033976516438790*a
 *e^(365/2*I*c) + 8061463543129887062443950403309304382415805276884579238661
 6425430054670214293015106952021225106450*a*e^(363/2*I*c) + 2538361662549334
 851188209821136747276917501435480724130009508266675937908123879408185094123
 6922200*a*e^(361/2*I*c) + 7884801970520811955441008837645862835212712516344
 520632689690055398622920399678675555557890805540*a*e^(359/2*I*c) + 24159213
 335515249902683891046188896507991053678323288318671994003935679976572633035
 11921645584430*a*e^(357/2*I*c) + 730105325802915088473068866423446534384705
 188187298829342664068496621137095589328874374034838030*a*e^(355/2*I*c) + 21
 759752893137327175565581792811386863191633015055007800380189226264635062927
 1469008473415746660*a*e^(353/2*I*c) + 6395021857926101928921987431533447733
 7029858557485643801909272432107476854548119403002345066040*a*e^(351/2*I*c)
 + 1853115759569154919039363567123754798620409583563677145739902259507762935
 7340612025141454239410*a*e^(349/2*I*c) + 5294039482427498842362101525222433
 976284532136979899965317673458198554728600898182320842047910*a*e^(347/2*I*c
) + 14908927362396438263492981941723477308593931847293483909557686028450458
 95181329808155623088100*a*e^(345/2*I*c) + 413837157439971513957475347106122
 090418096103121093503384586422597088548387007642778689392720*a*e^(343/2*I*c
) + 11320990500529498520396461573817530853760676272061503574259383886375255
 9149966030961364432530*a*e^(341/2*I*c) + 3051813013595180979999043789877122
 9881249172761806983346196310869995337802376609717118084280*a*e^(339/2*I*c)
 + 8105789144183930797661099129038091554545587802074817715094441746173533262
 684806046690657020*a*e^(337/2*I*c) + 21209991954745209623438103576371222822
 01408039497316115675157834511715525841111903287821280*a*e^(335/2*I*c) + 546
 683548162320318755607543063540859142453668714252535105543063481098088223075
 742274691040*a*e^(333/2*I*c) + 13877868790994660086536738876108796267807903
 7327882508538788609272464232291741120175647560*a*e^(331/2*I*c) + 3469287030
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 3200*a*e^(329/2*I*c) + 8539376495592153288945859284463236872792162702391449
 354612981234209920197053609719548280*a*e^(327/2*I*c) + 20692676798459301303
 81175334056457688285267649989023134787201410929393396836695689578160*a*e^(3
 25/2*I*c) + 493567023640867640299972889502944748943106881125111509805464276
 652017586793148209575990*a*e^(323/2*I*c) + 11586362130015229843964752594511
 1489920370403992774304311355052277945386826708814291260*a*e^(321/2*I*c) + 2
 676395132503366695480041391676566845968738240935211623712146502828820653196
 3685595620*a*e^(319/2*I*c) + 6082529830043102415483569107207792348780163318

212711365913239332777970405599581316610*a*e^(317/2*I*c) + 13598024609686162
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 15/2*I*c) + 298985079002696924089237936674570653687023892479672655281622093
 032615608331810198500*a*e^(313/2*I*c) + 64643918893425950177388154847859391
 478782846138908883463204853647802214592593258800*a*e^(311/2*I*c) + 13741380
 386275892220936955428884365279657568364963495869127376859464108093967124930
 *a*e^(309/2*I*c) + 28712779131862414195841335338276232982778357103549971287
 30316529679482826595460860*a*e^(307/2*I*c) + 589627110666913950676757546277
 033132409936475850749602886519410515281489835432948*a*e^(305/2*I*c) + 11897
 33138579465310565571572461643259959357957891243508580333068441982775202887
 2*a*e^(303/2*I*c) + 2358312800567615222040698767520687772002719908925236578
 3381244222564261342679308*a*e^(301/2*I*c) + 4591345596645571682092817716756
 142718329238022456900510608005443836033879600972*a*e^(299/2*I*c) + 87774689
 7443579121236075377490355068404009709042755817010349573425737869900308*a*e
 (297/2*I*c) + 1647366661889354043514224268663063314483379312788400390908676
 47515115028970040*a*e^(295/2*I*c) + 303459364323422086498168801179585287007
 83259643810281015011805920593868190276*a*e^(293/2*I*c) + 548522383069607487
 0232670430705003547243336234497802574847555967196592940404*a*e^(291/2*I*c)
 + 9726629305633107245966511928223950904378469047680283381796284611566447611
 76*a*e^(289/2*I*c) + 169157587126134417124650747920308982225001071457475688
 802526063335881435874*a*e^(287/2*I*c) + 28844609848991141927798576089491975
 808162650525713844636694817073554205876*a*e^(285/2*I*c) + 48212621542624881
 89638345987905919984432326565079712079669628450138845920*a*e^(283/2*I*c) +
 789685521451876769155582812528911180433623630278166458083659977181181120*a*
 e^(281/2*I*c) + 12671114830426751424190809183869304959447341068383855569608
 9694914431850*a*e^(279/2*I*c) + 1991167224474414510598158720752001324149743
 8423672824426122213941696300*a*e^(277/2*I*c) + 3063323292859702877329490930
 028863711679805587445419775185809699425497*a*e^(275/2*I*c) + 46123756141520
 1070364057977767537275375289232929441981287451225816828*a*e^(273/2*I*c) + 6
 7944153101887378416790537745291711484709226979885783161630835768882*a*e^(27
 1/2*I*c) + 9788540192454256321255171083155760919251578109432314011159159160
 067*a*e^(269/2*I*c) + 13786646199696358063256269672770787496771730116563938
 58195336896873*a*e^(267/2*I*c) + 189759652753983560479873697200916127533963
 528143615261433871315234*a*e^(265/2*I*c) + 25513859756352077572322437078487
 111620320963476059001013262542630*a*e^(263/2*I*c) + 33495799768887228955347
 25449848627543256623010923459353099036635*a*e^(261/2*I*c) + 429193535791556
 847068138327994764831727769099957108368643958154*a*e^(259/2*I*c) + 53649131
 382810104977802029620946554089637208561026255904068048*a*e^(257/2*I*c) + 65
 38945880517523468866323420234290611269386227470050003903740*a*e^(255/2*I*c)
 + 776724954437796152386576526960981118865651225484076140617434*a*e^(253/2*
 I*c) + 89868936876015124138088939755918047910164857950059626671506*a*e^(251
 /2*I*c) + 10122593617438421454872945774471611256945529490238801736444*a*e^(
 249/2*I*c) + 1109324724380705292283449093231952034907213153230066378718*a*e
 ^((247/2*I*c) + 118206677912742265308151068632779853612619007457778914450*a*
 e^(245/2*I*c) + 12239378690259640676098190698651937625584285361664235941*a*

$e^{(243/2*I*c)} + 1230589291098041959151565198792286611164919449523155204*a*e^{(241/2*I*c)} + 120057457669002169157406380951976541404817260989994202*a*e^{(239/2*I*c)} + 11356783819252976784499704281554895724049545296747183*a*e^{(237/2*I*c)} + 1040783214351939655185196194449040653397856679303077*a*e^{(235/2*I*c)} + 92327527810400690773733044860754764248910029932264*a*e^{(233/2*I*c)} + 7920859200695792807006454406332207329276812132106*a*e^{(231/2*I*c)} + 656541730750769495168952644914474242735369097675*a*e^{(229/2*I*c)} + 52523333574538984648429489695519406526756976563*a*e^{(227/2*I*c)} + 4051001485525986829845147001535913818302067194*a*e^{(225/2*I*c)} + 300870118521192302444085162135662723458505456*a*e^{(223/2*I*c)} + 21490721703859150501477595681763435720778909*a*e^{(221/2*I*c)} + 1474297474141465450580152071694890996247655*a*e^{(219/2*I*c)} + 96993251932443912018716975689508023006378*a*e^{(217/2*I*c)} + 6109811001129869767843196320577458616044*a*e^{(215/2*I*c)} + 367868194965564832606935721682843410685*a*e^{(213/2*I*c)} + 21130809863813051497818215704863434230*a*e^{(211/2*I*c)} + 1155591152525353155901693951332528574*a*e^{(209/2*I*c)} + 60030708766686810922979823601133828*a*e^{(207/2*I*c)} + 2954879430842410604932011265442302*a*e^{(205/2*I*c)} + 137436252062086506810801408828862*a*e^{(203/2*I*c)} + 6021691440068977811219750309348*a*e^{(201/2*I*c)} + 24767882484683327725335834952*a*e^{(199/2*I*c)} + 9526108636579479437964936790*a*e^{(197/2*I*c)} + 341088288520555248394683275*a*e^{(195/2*I*c)} + 11311601399861219435495350*a*e^{(193/2*I*c)} + 345392409065820898531490*a*e^{(191/2*I*c)} + 9642935277137118257641*a*e^{(189/2*I*c)} + 244124943708196709459*a*e^{(187/2*I*c)} + 5548294175019610066*a*e^{(185/2*I*c)} + 111804416623842644*a*e^{(183/2*I*c)} + 1966409337599161*a*e^{(181/2*I*c)} + 29570065227040*a*e^{(179/2*I*c)} + 369625815338*a*e^{(177/2*I*c)} + 3687040552*a*e^{(175/2*I*c)} + 27515228*a*e^{(173/2*I*c)} + 136552*a*e^{(171/2*I*c)} + 338*a*e^{(169/2*I*c)})/(e^{(531*I*c)} + 432*e^{(530*I*c)} + 93096*e^{(529*I*c)} + 13343760*e^{(528*I*c)} + 1431118260*e^{(527*I*c)} + 122503723056*e^{(526*I*c)} + 8718181624155*e^{(525*I*c)} + 530563624556832*e^{(524*I*c)} + 28186192554792138*e^{(523*I*c)} + 1327882849274858880*e^{(522*I*c)} + 56169444526926562260*e^{(521*I*c)} + 2154864144781257856128*e^{(520*I*c)} + 75599817092670157806639*e^{(519*I*c)} + 2442455629894502983849104*e^{(518*I*c)} + 73099207817335597247098038*e^{(517*I*c)} + 2037031259470368160131922320*e^{(516*I*c)} + 53090127264630963470039804475*e^{(515*I*c)} + 1299146645993240318167826532288*e^{(514*I*c)} + 29952547749265499675257842032197*e^{(513*I*c)} + 652650253343206047453620559993840*e^{(512*I*c)} + 13477227799524701956579274210395326*e^{(511*I*c)} + 264410375780310742518099326419685040*e^{(510*I*c)} + 4939666610818025798809586352543471345*e^{(509*I*c)} + 88054927598941411145869950813388040256*e^{(508*I*c)} + 1500602747937397286405577818722691539392*e^{(507*I*c)} + 24489837337812338687718622491865013839488*e^{(506*I*c)} + 383360155801054824529764688213114368047154*e^{(505*I*c)} + 5764601046563151304213854710715346838447392*e^{(504*I*c)} + 83380839911837894453136303673785039051506805*e^{(503*I*c)} + 1161581413733971751533622511909046917188768400*e^{(502*I*c)} + 15603911277687607099721623771744933086920587272*e^{(501*I*c)} + 202347509724462171313966643580234078508179838320*e^{(500*I*c)} + 2535667460650279776834561566186591213109251642859*e^{(499*I*c)} + 30735366512830562160991166338490057308062762518$

496*e^(498*I*c) + 360688613036389349413809780004559963548775423325255*e^(497*I*c) + 4101545439937195793959956708442496709433800261224880*e^(496*I*c) + 45230940039830738332025694784646206844854827698075736*e^(495*I*c) + 484093410240488718655917025303662581091659126182344528*e^(494*I*c) + 5032024903401451824074213943766011922026507006311982753*e^(493*I*c) + 50836369508171099437019348610847391946736185108017183136*e^(492*I*c) + 499467506558531733671585862910572702811545035730398749530*e^(491*I*c) + 4775398607100853263534207733818266777478693412738731031680*e^(490*I*c) + 44456708175258821024400946210535004523775722190977468484496*e^(489*I*c) + 403212225957798188840846139960995624144491271694336796459584*e^(488*I*c) + 3564764890628724017088487996688178929195787613958545474804845*e^(487*I*c) + 30736217404321009965231037419663053962881035281709221697785072*e^(486*I*c) + 258585348715977270155829115684193411072034541491364393985491350*e^(485*I*c) + 2123702969188871318266718781223927067839949015727293884065388080*e^(484*I*c) + 17033886027390615741040977721655541665612162275485028584310890417*e^(483*I*c) + 133490210052026183779673313868332303530332906163247194627808410304*e^(482*I*c) + 1022536437468296737293065862705246449693687415559865844306888705423*e^(481*I*c) + 7659010520187549651777118357676871927081898989131125755798204236112*e^(480*I*c) + 56117081076341175384087570185188538660375932013674735519055227368366*e^(479*I*c) + 402349692266121158934003582839428785116904903936409545602519219664720*e^(478*I*c) + 2823905151936586678382525706564457280290098698638597987628380245881715*e^(477*I*c) + 19407979215594566593535008103303255257745408070082431338945184797463936*e^(476*I*c) + 130657660226560419335121434389938961884595434069984824307149332131747540*e^(475*I*c) + 861884851094991908764246805474672428603757315484453974713612812215428992*e^(474*I*c) + 5572551157328671121016216416307596161861955969011697222340926210112854418*e^(473*I*c) + 35324447206779018115378052820789411687581004582367431006205879633729015200*e^(472*I*c) + 219601281339515561500261478844190024870555261281946058839614044697037963695*e^(471*I*c) + 1339214374254245553564884406801945353385000254030655765953770237607180089968*e^(470*I*c) + 8013729580790752434361964945761543761469520791210746972675870481058674277844*e^(469*I*c) + 47065044611135158108487353367484243102698248838312635876283099427442745866704*e^(468*I*c) + 271361207503266570734486517077181014801775322183181055638619257836143271472358*e^(467*I*c) + 1536333238444927583532734556016494671674916578907116984548489078241693926940560*e^(466*I*c) + 8543013441126212334833540665069621472479085838041360564550722036723654297540205*e^(465*I*c) + 46668223548266017806854592468100570289355960869613650856575756758180182223308768*e^(464*I*c) + 250501028608928332469340456829902067712233644464602753159945727868485722395506952*e^(463*I*c) + 1321498055271300851429993866631619874424534425188183592049727687571032156435077280*e^(462*I*c) + 685299322314573668732885311617795435592940841439866351079655652312894721972796266*e^(461*I*c) + 34941071613276704649477943043339450201504075335160361865916029213860778606230624960*e^(460*I*c) + 175193170500618300241515632381912285157790097816049220671217212220015297133400636060*e^(459*I*c) + 863979933622330349556296820028395513198708064940505702126068652936800794826651264256*e^(458*I*c) + 41

915425006568261480933394145441591439644784724923159318091718599021141090059
39942952*e^(457*I*c) + 2000800680303004713729327825032159711354071620198333
3126349281186679153199068045257216*e^(456*I*c) + 93986915313068179149083606
065681482780836060510530154618486949839467131378859885998210*e^(455*I*c) +
434546676780280045346344498763892540797175105756827515509297024187660299345
484920192480*e^(454*I*c) + 197779298066581813565130009432623915860544887080
6970860577325385028609983034534672318500*e^(453*I*c) + 88627521427569572856
81340885764904597935349569355321815647721172537159186491471311666400*e^(452
*I*c) + 3910803125560180947653753536961184444084490375160564502351457235204
5248104262933598850730*e^(451*I*c) + 16995632796992976777390209665262925328
3704505477127544556534417376686540936706073847337600*e^(450*I*c) + 72752101
071839422929177407384469425579873866706753537975973279556794257875138425078
0476310*e^(449*I*c) + 30679742964317473641981596239624636716170064196268514
26148418602934852907379021659761911840*e^(448*I*c) + 1274721961650332054135
6343062562847368601622140856786025445814532037904111523242298235713300*e^(4
47*I*c) + 52190912207661824215812271854269748071292843243227894769229690720
010547141334131610989636000*e^(446*I*c) + 210594301385648471184329078880317
504953361839954159427434009884661777259752542647709150036990*e^(445*I*c) +
837579206923411932458786486765373533946545239708990769488724813982189165104
589895518909256320*e^(444*I*c) + 328387476055581867672630948030673442015509
8583948074469014168171874442170109648521627538755920*e^(443*I*c) + 12693496
932964920565073673637181280088548682508880255337280065006566138696041797353
216584528640*e^(442*I*c) + 483794897564340998438577918165893794068150426093
40378747586437145781646245422045101230417309900*e^(441*I*c) + 1818346614061
779013153301296771453811664491884131941411693443547549209690349526103789452
82257600*e^(440*I*c) + 6740255305431330088948457752366252374507431144735445
37818170447134607102575676676056675328961590*e^(439*I*c) + 2464382190807439
609079774226855679629367885709776435876630851716253962696192341706239192878
728160*e^(438*I*c) + 888829502875102466704420383760797610148005313441861447
4620767522824868911959884352666444917404000*e^(437*I*c) + 31626644674725547
731176795687527653571305969985923688392112164915553242573269490908989570248
533280*e^(436*I*c) + 111034148797008819443143895644469242295049867464313710
969257619338899133799285616020069872611710850*e^(435*I*c) + 384655842080666
274454063078784837174998949052500975322162003392549953413592461519365177908
682078400*e^(434*I*c) + 131505212093069212210229710532762284233587074342853
0891072983535862280094446607723473800477453914130*e^(433*I*c) + 44372109178
431823477643495444439046990200565950694708471936170921147140776330772349728
25351226979360*e^(432*I*c) + 1477795509661712899871274518207149536217650697
3183081650233605274051677624970464340242755840025673760*e^(431*I*c) + 48584
258153140280447314836868772131390195412419046732778458706015096881437076337
910793584122475073760*e^(430*I*c) + 157685845528850918721462877864435090257
583149415561323427386562894447598277935629800939237175625149830*e^(429*I*c)
+ 505293663123015258878483025738812813203397766845340065381261016353419722
382620393032535960660921950400*e^(428*I*c) + 159877110105819269227052899967
7444742685631006232456185844925220144002305878120380828483988663574829100*e

$\wedge(427*I*c) + 49952419562791381802051867444016880243882721139212556637349569$
46927571305533146776898787878059685108480*e $\wedge(426*I*c) + 1541311121148602393$
729497082079737671608134478816338654352242193973750796212585498188187916834

8260330000*e $\wedge(425*I*c) + 46970224727117281826454045018070670522559756627580$
347784535320014963482632359729444541885102274546002560*e $\wedge(424*I*c) + 141379$
938253556843280565505807403304130606130725434751745794079833141361748917639

986145377066437210546190*e $\wedge(423*I*c) + 420358024835146798583611210145942154$
684437949365647899088372524802156222884839580011688655664280691773600*e $\wedge(42$
2*I*c) + 123466804189240997878001808175544021601258247639694193796589963195

3079203974222138794604328498972144766900*e $\wedge(421*I*c) + 35827180021632960614$
145367037151098971071982527392845461493431023484561240896574285949464386608

59773886240*e $\wedge(420*I*c) + 1027160253020288900249781351684945259097151280952$
9060665197301097052210064576088348023234671975463677418470*e $\wedge(419*I*c) + 29$
097651061247453406647569781836910062165559852359052804259154165687125428752

562385492373749486351714453120*e $\wedge(418*I*c) + 814520814138291118288754175642$
500548460376933128114804929091607581959891557681070225683509538618159407040

90*e $\wedge(417*I*c) + 2253205325932206577679411092895162489997945210155641349828$
27241710019675486694499689312466561907212627820000*e $\wedge(416*I*c) + 6160031160$
229795849371257017578872129983543009899913626280388610939145613320711919097

14949426587936910303300*e $\wedge(415*I*c) + 1664475034387211809394917743502937638$
978574937754764763987835872410449930690131572904279995484581013965001440*e \wedge
(414*I*c) + 444541225929547462506765951419831296601541629996893039334563034

5914109720740573618884980520010028451496996210*e $\wedge(413*I*c) + 11735856926245$
118493113091002501604032341876985999082823520672241530200188223826392982302

194084667538488665600*e $\wedge(412*I*c) + 306275810542219573783905472892776091295$
72813931082733520247387226000020043538279468776707958420892547870128680*e \wedge
(411*I*c) + 7901914955876656925478398848723238835290914498274717185677246322

3808993367091503402876467270176124342699654400*e $\wedge(410*I*c) + 20155794742479$
409802677247846204088339586751256293086294375356869008401558559801015478154

8625239409581907397500*e $\wedge(409*I*c) + 50832459930108546016697862968303266142$
7654474082939048097939638391567298795788389433842285751054665210868287680*e \wedge
(408*I*c) + 12675970172948129134001462760421269299864802928701903991075543

11079964227280196522475370108738477856311765699610*e $\wedge(407*I*c) + 3125683493$
178701743479704750307490178666292150720179363604335113528623329684606343185

540756019935662148267863968*e $\wedge(406*I*c) + 762178879191204706203884091779937$
460042889225819436763668294435609668140024631213800176928502066144599107324

9416*e $\wedge(405*I*c) + 18379807084003359766027649217621144116091735572216620788$
861535803449702273802588359076704241840733513439114113248*e $\wedge(404*I*c) + 438$
349721429193776853786922330210637445540331009285027374804389789767469898957

84070951905237783490374305934542955*e $\wedge(403*I*c) + 1033997554672574364898478$
37640753754718204394473055795001467604326419876555568738295317372110961151

96005647730480*e $\wedge(402*I*c) + 2412460212824400617929083177830328761948015971$
33206052091286997043729145345755805710081489006741839439573984832678*e $\wedge(401$
*I*c) + 5567563887111823403410261927342195461136517683173805390058936790493

94714017063698565272728813669054779077208977840*e $\wedge(400*I*c) + 1271033082938$

048950201360554831270342662343991277504612342300366025046741742856580445289
401786656311685859023084716*e^(399*I*c) + 287049613141231445183467471535358
943955329443080853193346608628854370924623076915139218069941340562301724775
3532944*e^(398*I*c) + 64133818958559251847582314510625563803285949385110065
77015218119786536390213057284018202201631094434819584025113465*e^(397*I*c)
+ 1417648365287570495720201334324111790436997779665384995990252442198063518
9011634815653279605497783382888932766730080*e^(396*I*c) + 31004319206069417
077069363141423487431828009098184744635678652284177439464941651812564519144
918003174108077634846014*e^(395*I*c) + 670917061305296691250198992100215765
802378434622295353862950870761892978499959313606456052921309614961067075215
06432*e^(394*I*c) + 1436576871380447969429471197042595384588181994235168246
74586293691056119209866358123637772245409530799230553767222252*e^(393*I*c)
+ 3043844711068133360102841601239063704338888284906274226525512369667909161
74520857759143930140187173492394981908258944*e^(392*I*c) + 6382188929145337
415058063996624885560667836004960918764083749748774489717780360749962455811
24283460438065182071976085*e^(391*I*c) + 1324311324984027428355222938147682
378672860708817161741448749689593588020860847508703702325320304649883120684
987556400*e^(390*I*c) + 271958928348374392604080510108034192124453031125460
7250929192773909331523226635035815672862569296693711643521070331394*e^(389*
I*c) + 55274988490311783558612300093266682839262900821584671180006985027193
79939045918344222192742145711257028040974074674736*e^(388*I*c) + 1111949964
536320108088106282488633849242537565844897793553584634929042582157038309042
5411418521516670371372045206568345*e^(387*I*c) + 22140735001708603270915180
769241391662035578755903979148909213603822554792749183517160255571915875356
439553717130797888*e^(386*I*c) + 436383000758151710259462114644656896189657
734886609858579456574798540851088518579112229119898376154526085123560084002
95*e^(385*I*c) + 8513953323478645577958995946490063776073572970562122138000
5837208369157794673049675428799817875431430246332625899630160*e^(384*I*c) +
16443750067690689274132326015439427850395456193602013359658180644935724027
7349447927334517105809995300093549279931273178*e^(383*I*c) + 31440903580822
586156559543693835494544547399104312972204674722881303092520496850341856681
8838611866709807040793495364496*e^(382*I*c) + 59515761550043151494747928233
654705382792608791642526349718775702941338547183543419824680709621453689544
1388306027237899*e^(381*I*c) + 11153982855456015505333280456001843179938993
592179967298183407042218011956674104858469961790567335585124522385831607925
12*e^(380*I*c) + 2069698289500860643461665762373807957513019424041178871904
960551829412449344722432125679417958403007551179298315947373776*e^(379*I*c)
+ 380260499670589110696462063384896480703709885451018226324303059729563076
0353597531974324752266389193185760878274188013440*e^(378*I*c) + 69178389452
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70422099893124890777678037369988150*e^(377*I*c) + 1246214044053725808492859
670987206685775707094312486855450094815475686345430803292540834031123785001
7814707896986969086816*e^(376*I*c) + 22231341131801535345406399037721686840
208397941952580135584645966746736656716271554826476282991066076564921432614
339399735*e^(375*I*c) + 392742004143298611693979445162250010812274333985850

07206399231211907157795359719648241598754266579840244551491476467899952*e^(374*I*c) + 68712466015985641512468586173659747734879591710098354652786124936023073943141049573606648563005359411712764895683903806088*e^(373*I*c) + 119060591849660546834765693227676449067584148248882678447504826077236333444513454095126668750057295811191643356908972191440*e^(372*I*c) + 204325557265186000767402710230847896459761583922763698235433212833313077783041040074669379017394836761539649081690630811665*e^(371*I*c) + 347310053810935290419455560555957314129569210735745983234369659976413374774078000173070075248654524917179128950507443058208*e^(370*I*c) + 584749573682304586179384628844883327581498969886540380378896767999075614964007174600811092945356635118795824799369716742109*e^(369*I*c) + 975210339444049318757282311763517786673223175594457946383279264635085041004917300295904275433144848532459919875479817581584*e^(368*I*c) + 1611092541400060525954859375264194178347643471837078201446262435615142944587337833513586022729849523358436493586042252995608*e^(367*I*c) + 2636662410430799340447522284778244283740751068658140726576446671207798325606832295937705061686297930296382338892574900819440*e^(366*I*c) + 4274826907720591752526711336820871500844345647922385471534359333606189571832444641364132893108663576205133870672156264164115*e^(365*I*c) + 6866425337518668262662693750908956965732924578142181630622157802899880874681551031136314064199948604001529894566235238597088*e^(364*I*c) + 10927210603473544810279792347844536074588896806230041110089544731605863146104181739039426674855453466097402330688331845602302*e^(363*I*c) + 17229502824367647334400721998417596703948657394738805209391636597370380572398964715080095366818322029152193635869784095333760*e^(362*I*c) + 26917794701086615097890120236890501105146799996021775195710866226228638984703456832694153230611607263444183501026198563419616*e^(361*I*c) + 41670440375390543643418219342271748040035071490119080585281522498188818375906900368701234531304633163446319945130196476913600*e^(360*I*c) + 63923019433761989090614801288635098123199445102303122616544648208998767803944455777042886552738499747183713136069104651812215*e^(359*I*c) + 97173055024742680058616722461368892661141295540263493013032746083536157324268333390400308958318370219154887169702257444756176*e^(358*I*c) + 146390448456351181218237382740374124191664819997746988076598391862733629670142241546375533903130605297580105675355629160198162*e^(357*I*c) + 218563166659649312247483640956272149212499115383828771029654283363972585118090479413696638108156385244646591328454425745117584*e^(356*I*c) + 323413178014841003714151138246079152576360976035058457890409937738723171537036573043681997163745313602400139153046673668433091*e^(355*I*c) + 474323043563100542377338629931966248129175976982332446018056009391154020438895903140822967769494019446166779954024655344116288*e^(354*I*c) + 689518449328793559903260418149974190253578340058895035589606468244680591556118170304005037563669880057908765898949268614772285*e^(353*I*c) + 993555653649521127226443960820233649386488510081892545866700444096661582790441241830855609577062039555625090943332264901780720*e^(352*I*c) + 141916448142217657385823401389899962882232233309573730716310743838935832201454293617293175086458389621425307051750612129761498*e^(351*I*c) + 200949611009268773815278208568373722272796882429905873921544608349935146762533

4449670757764066690656150949014944043994822823920*e^(350*I*c) + 28208192985
 622159591075298072896284496213867989894363693931160698940187812010002756331
 04498398959346631795568022519974400130281*e^(349*I*c) + 3925697658415778352
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 5636398374*e^(345*I*c) + 13521230411945436915558854706851796541567399811656
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 992*e^(344*I*c) + 180353273381774554711775685948516829779783464497771935720
 87688510392426884519272991560851326393852241961470040819793627127923997*e^(
 343*I*c) + 2385639856556280203069527817421264083328215417400645945829264406
 0947924907313735921561690153939906017518647182491616573724049744*e^(342*I*c
) + 31295263688189838313772775873307260334117227258629501992358695636092662
 866062819845689064235813622974150120668921391878398978380*e^(341*I*c) + 407
 159889637019189500203483367364234205133113590104852469190746528833939708053
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 2798814607975110366224945081428121888473324*e^(339*I*c) + 67244087969080703
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 693900139115594646893784095418472336*e^(338*I*c) + 853681184302153128482312
 917396737358877462018516662996003921994187647500868281987198727440477677836
 67325326289221881974987582215*e^(337*I*c) + 1075047374065769161234803991697
 596333283214074194005100174988498305986215654282665463159339208215275447263
 80201659114903834605888*e^(336*I*c) + 1342977420234794299046296161045596100
 960747587210680227044689380630170596880233634364589715349646650363198891192
 29809973806909680*e^(335*I*c) + 1664323329225891951305583292663987533898239
 557375985275560960930624735597695457723219789693189041925727339978882309864
 69005970880*e^(334*I*c) + 2046222955357291095198299167898672253194297051625
 600826488403949654231128093365919212903093923962638346743689778405271470374
 26908*e^(333*I*c) + 2495930722825658663983899515096192026826344554871287146
 31461891770823201367527645793770203788784677343934971424317987895255031936*
 e^(332*I*c) + 3020606380308684634611394422793604997189069174825248944833562
 20196138377050828911383056860425370161157201493696073712322595776808*e^(331
 *I*c) + 3627063075638432311356991574185107324524206140136241688793121876452
 33450153927975793326834780741391203430153093712635355523960320*e^(330*I*c)
 + 4321478564640869380238115618086785895947020479046742822979596588001709844
 56799067751878044806619012452636891350731618278545690160*e^(329*I*c) + 5109
 076151111345074526238461471371414502757224443163855316484292308516866358277
 17488464500331623385777400744950538410637735936000*e^(328*I*c) + 5993784847
 717334748093761424015548502070649721181375729492575036514445419393090252768
 96049622515630263162184526394317285457368300*e^(327*I*c) + 6977891069259246

148167137476846827850836598190279522444470433558697413685004525611640246360
73401929693105801105522738405349028160*e^(326*I*c) + 8061696713276255324245
753400897757336819949915766744469223540997146151920854432456638522570011156
44286660304979476023966071898200*e^(325*I*c) + 9243200528675225840357774957
610723512225344207848619600018210205094681463567564337952464464913961415838
54513687104334429566707520*e^(324*I*c) + 1051782100428834371944508170051219
187116816349766953322637182149610875004223784784183284961906494422955462431
208645690802526770780*e^(323*I*c) + 118781794307939031610880232479811012902
082278266008724859936764348120020604682216614428542592222937541367653507114
1005286431481600*e^(322*I*c) + 13313961146267230358024621235315820503397499
960141524528353059563674253702227586217539224587275248560729508809606575647
20475838500*e^(321*I*c) + 1481187117089246662466955694965677855524730313260
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206080*e^(320*I*c) + 163556974464142190065788638128907665322758017205667758
745140206923435528368748965961391376195914077333973601479008181451662522444
0*e^(319*I*c) + 17926490780896322989369457284819693349643915975062850884883
50622937252533420980803144316431701452190522716124797875257437516360640*e^(
318*I*c) + 1950286550780181919244992961204487010056460362845218501674423766
266321558791436917317878702232679213868287926294665202769722927380*e^(317*I
*c) + 210614190346834430711254976120248454340279425235248219981741042486967
7262715098288437646518683487945462774223656471345899082156800*e^(316*I*c) +
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54124144699023009842080515157923529357189869943515991200*e^(315*I*c) + 2402
464595569686086120001803034211056739445588621946141384106162886246161815149
763025030834875234067267774023433418269982431265280*e^(314*I*c) + 253776641
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267711043448273062675589165508774672324551532861405210895827339322025531307
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242595167274383008960*e^(308*I*c) + 302337164350822502717560317521295321902
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100620*e^(305*I*c) + 299049894962254360853812938028386633519008711512485881
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0*e^(304*I*c) + 29364942143518684987032394554267711043448273062675589165508

77467232455153286140521089582733932202553130712723836983468866230908800*e^(
 303*I*c) + 2862503126320461797770667780725644184991255623174626175679050672
 100848988119391841466573417019247590580735265143427289340450811200*e^(302*I
 *c) + 277007320715076864559750728138206549792496846605452741412233982733378
 3770068305883487309979315983718403740872884345746380680204260*e^(301*I*c) +
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 766415465030330815471746692988596069911894697225052928320452542175587154848
 096483331209807430113943015398362669673337957755720*e^(299*I*c) + 240246459
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 5030834875234067267774023433418269982431265280*e^(298*I*c) + 22577262191038
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 653227580172056677587451402069234355283687489659613913761959140773339736014
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 0*e^(289*I*c) + 92432005286752258403577749576107235122253442078486196000182
 1020509468146356756433795246446491396141583854513687104334429566707520*e^(2
 88*I*c) + 80616967132762553242457534008977573368199499157667444692235409971
 4615192085443245663852257001115644286660304979476023966071898200*e^(287*I*c
) + 69778910692592461481671374768468278508365981902795224444704335586974136
 8500452561164024636073401929693105801105522738405349028160*e^(286*I*c) + 59
 937848477173347480937614240155485020706497211813757294925750365144454193930
 9025276896049622515630263162184526394317285457368300*e^(285*I*c) + 51090761
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 4806619012452636891350731618278545690160*e^(283*I*c) + 36270630756384323113
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 1391203430153093712635355523960320*e^(282*I*c) + 30206063803086846346113944
 227936049971890691748252489448335622019613837705082891138305686042537016115
 7201493696073712322595776808*e^(281*I*c) + 24959307228256586639838995150961
 920268263445548712871463146189177082320136752764579377020378878467734393497

1424317987895255031936*e^(280*I*c) + 20462229553572910951982991678986722531
942970516256008264884039496542311280933659192129030939239626383467436897784
0527147037426908*e^(279*I*c) + 16643233292258919513055832926639875338982395
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9005970880*e^(278*I*c) + 13429774202347942990462961610455961009607475872106
802270446893806301705968802336343645897153496466503631988911922980997380690
9680*e^(277*I*c) + 10750473740657691612348039916975963332832140741940051001
7498849830598621565428266546315933920821527544726380201659114903834605888*e
^(276*I*c) + 85368118430215312848231291739673735887746201851666299600392199
418764750086828198719872744047767783667325326289221881974987582215*e^(275*I
*c) + 672440879690807038237032571996630476068906104820904946194928029351302
15979819469966383336788693900139115594646893784095418472336*e^(274*I*c) + 5
253922334674077114258709237025706953606031964443950166761048276795580027605
2892432152798814607975110366224945081428121888473324*e^(273*I*c) + 40715988
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830156229705647312265477584256027212762941040*e^(272*I*c) + 312952636881898
383137727758733072603341172272586295019923586956360926628660628198456890642
35813622974150120668921391878398978380*e^(271*I*c) + 2385639856556280203069
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6017518647182491616573724049744*e^(270*I*c) + 18035327338177455471177568594
851682977978346449777193572087688510392426884519272991560851326393852241961
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415673998116568700215673529753268154678178465332891238716960561952316961461
62720992221760992*e^(268*I*c) + 1005220952436958182758815498534554967803144
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60*e^(266*I*c) + 5416662804052436349585595982818357953866258461644354018205
158917742576425344364964596750653177677803492186817305171175032011500*e^(26
5*I*c) + 392569765841577835276810394285601184021164276962171721799661439847
3887186074391482638547212826538270453912634540299792270321024*e^(264*I*c) +
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110092687738152782085683737222727968824299058739215446083499351467625334449
670757764066690656150949014944043994822823920*e^(262*I*c) + 141916448142217
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597698233244601805600939115402043889590314082296776949401944616677995402465
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845789040993773872317153703657304368199716374531360240013915304667366843309
1*e^(257*I*c) + 21856316665964931224748364095627214921249911538382877102965

4283363972585118090479413696638108156385244646591328454425745117584*e^(256*I*c) + 146390448456351181218237382740374124191664819997746988076598391862733629670142241546375533903130605297580105675355629160198162*e^(255*I*c) + 97173055024742680058616722461368892661141295540263493013032746083536157324268333390400308958318370219154887169702257444756176*e^(254*I*c) + 63923019433761989090614801288635098123199445102303122616544648208998767803944455777042886552738499747183713136069104651812215*e^(253*I*c) + 41670440375390543643418219342271748040035071490119080585281522498188818375906900368701234531304633163446319945130196476913600*e^(252*I*c) + 26917794701086615097890120236890501105146799996021775195710866226228638984703456832694153230611607263444183501026198563419616*e^(251*I*c) + 17229502824367647334400721998417596703948657394738805209391636597370380572398964715080095366818322029152193635869784095333760*e^(250*I*c) + 10927210603473544810279792347844536074588896806230041110089544731605863146104181739039426674855453466097402330688331845602302*e^(249*I*c) + 6866425337518668262662693750908956965732924578142181630622157802899880874681551031136314064199948604001529894566235238597088*e^(248*I*c) + 4274826907720591752526711336820871500844345647922385471534359333606189571832444641364132893108663576205133870672156264164115*e^(247*I*c) + 2636662410430799340447522284778244283740751068658140726576446671207798325606832295937705061686297930296382338892574900819440*e^(246*I*c) + 1611092541400060525954859375264194178347643471837078201446262435615142944587337833513586022729849523358436493586042252995608*e^(245*I*c) + 975210339444049318757282311763517786673223175594457946383279264635085041004917300295904275433144848532459919875479817581584*e^(244*I*c) + 584749573682304586179384628844883327581498969886540380378896767999075614964007174600811092945356635118795824799369716742109*e^(243*I*c) + 347310053810935290419455560555957314129569210735745983234369659976413374774078000173070075248654524917179128950507443058208*e^(242*I*c) + 204325557265186000767402710230847896459761583922763698235433212833313077783041040074669379017394836761539649081690630811665*e^(241*I*c) + 119060591849660546834765693227676449067584148248882678447504826077236333444513454095126668750057295811191643356908972191440*e^(240*I*c) + 68712466015985641512468586173659747734879591710098354652786124936023073943141049573606648563005359411712764895683903806088*e^(239*I*c) + 39274200414329861169397944516225001081227433398585007206399231211907157795359719648241598754266579840244551491476467899952*e^(238*I*c) + 22231341131801535345406399037721686840208397941952580135584645966746736656716271554826476282991066076564921432614339399735*e^(237*I*c) + 12462140440537258084928596709872066857757070943124868554500948154756863454308032925408340311237850017814707896986969086816*e^(236*I*c) + 6917838945214844278493330459361394923372333853619879637372673184942859712431066345726870422099893124890777678037369988150*e^(235*I*c) + 3802604996705891106964620633848964807037098854510182263243030597295630760353597531974324752266389193185760878274188013440*e^(234*I*c) + 2069698289500860643461665762373807957513019424041178871904960551829412449344722432125679417958403007551179298315947373776*e^(233*I*c) + 11153982855456015505333280456001843179938993592179967298183407042218011956674104858469961790567335585124522385

83160792512*e^(232*I*c) + 5951576155004315149474792823365470538279260879164
25263497187757029413385471835434198246807096214536895441388306027237899*e^(
231*I*c) + 3144090358082258615655954369383549454454739910431297220467472288
13030925204968503418566818838611866709807040793495364496*e^(230*I*c) + 1644
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47927334517105809995300093549279931273178*e^(229*I*c) + 8513953323478645577
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008400295*e^(227*I*c) + 221407350017086032709151807692413916620355787559039
79148909213603822554792749183517160255571915875356439553717130797888*e^(226
*I*c) + 1111949964536320108088106282488633849242537565844897793553584634929
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22192742145711257028040974074674736*e^(224*I*c) + 2719589283483743926040805
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0*e^(222*I*c) + 63821889291453374150580639966248855606678360049609187640837
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432*e^(218*I*c) + 310043192060694170770693631414234874318280090981847446356
78652284177439464941651812564519144918003174108077634846014*e^(217*I*c) + 1
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582314510625563803285949385110065770152181197865363902130572840182022016310
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294430808531933466086288543709246230769151392180699413405623017247753532944
*e^(214*I*c) + 127103308293804895020136055483127034266234399127750461234230
0366025046741742856580445289401786656311685859023084716*e^(213*I*c) + 55675
638871118234034102619273421954611365176831738053900589367904939471401706369
8565272728813669054779077208977840*e^(212*I*c) + 24124602128244006179290831
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9573984832678*e^(211*I*c) + 10339975546725743648984783764075375471820439447
3055795001467604326419876555556873829531737211096115196005647730480*e^(210*
I*c) + 43834972142919377685378692233021063744554033100928502737480438978976
746989895784070951905237783490374305934542955*e^(209*I*c) + 183798070840033
597660276492176211441160917355722166207888615358034497022738025883590767042
41840733513439114113248*e^(208*I*c) + 7621788791912047062038840917799374600
428892258194367636682944356096681400246312138001769285020661445991073249416
*e^(207*I*c) + 312568349317870174347970475030749017866629215072017936360433

5113528623329684606343185540756019935662148267863968*e^(206*I*c) + 12675970
 172948129134001462760421269299864802928701903991075543110799642272801965224
 75370108738477856311765699610*e^(205*I*c) + 5083245993010854601669786296830
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 87680*e^(204*I*c) + 2015579474247940980267724784620408833958675125629308629
 43753568690084015585598010154781548625239409581907397500*e^(203*I*c) + 7901
 914955876656925478398848723238835290914498274717185677246322380899336709150
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 128680*e^(201*I*c) + 117358569262451184931130910025016040323418769859990828
 23520672241530200188223826392982302194084667538488665600*e^(200*I*c) + 4445
 412259295474625067659514198312966015416299968930393345630345914109720740573
 618884980520010028451496996210*e^(199*I*c) + 166447503438721180939491774350
 293763897857493775476476398783587241044993069013157290427999548458101396500
 1440*e^(198*I*c) + 61600311602297958493712570175788721299835430098999136262
 8038861093914561332071191909714949426587936910303300*e^(197*I*c) + 22532053
 259322065776794110928951624899979452101556413498282724171001967548669449968
 9312466561907212627820000*e^(196*I*c) + 81452081413829111828875417564250054
 846037693312811480492909160758195989155768107022568350953861815940704090*e^
 (195*I*c) + 290976510612474534066475697818369100621655598523590528042591541
 65687125428752562385492373749486351714453120*e^(194*I*c) + 1027160253020288
 900249781351684945259097151280952906066519730109705221006457608834802323467
 1975463677418470*e^(193*I*c) + 35827180021632960614145367037151098971071982
 52739284546149343102348456124089657428594946438660859773886240*e^(192*I*c)
 + 1234668041892409978780018081755440216012582476396941937965899631953079203
 974222138794604328498972144766900*e^(191*I*c) + 420358024835146798583611210
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 600*e^(190*I*c) + 141379938253556843280565505807403304130606130725434751745
 794079833141361748917639986145377066437210546190*e^(189*I*c) + 469702247271
 172818264540450180706705225597566275803477845353200149634826323597294445418
 85102274546002560*e^(188*I*c) + 1541311121148602393729497082079737671608134
 4788163386543522421939737507962125854981881879168348260330000*e^(187*I*c) +
 49952419562791381802051867444016880243882721139212556637349569469275713055
 33146776898787878059685108480*e^(186*I*c) + 1598771101058192692270528999677
 444742685631006232456185844925220144002305878120380828483988663574829100*e^
 (185*I*c) + 505293663123015258878483025738812813203397766845340065381261016
 353419722382620393032535960660921950400*e^(184*I*c) + 157685845528850918721
 462877864435090257583149415561323427386562894447598277935629800939237175625
 149830*e^(183*I*c) + 485842581531402804473148368687721313901954124190467327
 78458706015096881437076337910793584122475073760*e^(182*I*c) + 1477795509661
 712899871274518207149536217650697318308165023360527405167762497046434024275
 5840025673760*e^(181*I*c) + 44372109178431823477643495444439046990200565950
 69470847193617092114714077633077234972825351226979360*e^(180*I*c) + 1315052
 120930692122102297105327622842335870743428530891072983535862280094446607723
 473800477453914130*e^(179*I*c) + 384655842080666274454063078784837174998949

052500975322162003392549953413592461519365177908682078400*e^(178*I*c) + 111
034148797008819443143895644469242295049867464313710969257619338899133799285
616020069872611710850*e^(177*I*c) + 316266446747255477311767956875276535713
05969985923688392112164915553242573269490908989570248533280*e^(176*I*c) + 8
888295028751024667044203837607976101480053134418614474620767522824868911959
884352666444917404000*e^(175*I*c) + 246438219080743960907977422685567962936
7885709776435876630851716253962696192341706239192878728160*e^(174*I*c) + 67
402553054313300889484577523662523745074311447354453781817044713460710257567
6676056675328961590*e^(173*I*c) + 18183466140617790131533012967714538116644
9188413194141169344354754920969034952610378945282257600*e^(172*I*c) + 48379
489756434099843857791816589379406815042609340378747586437145781646245422045
101230417309900*e^(171*I*c) + 126934969329649205650736736371812800885486825
08880255337280065006566138696041797353216584528640*e^(170*I*c) + 3283874760
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538755920*e^(169*I*c) + 837579206923411932458786486765373533946545239708990
769488724813982189165104589895518909256320*e^(168*I*c) + 210594301385648471
184329078880317504953361839954159427434009884661777259752542647709150036990
*e^(167*I*c) + 521909122076618242158122718542697480712928432432278947692296
90720010547141334131610989636000*e^(166*I*c) + 1274721961650332054135634306
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) + 30679742964317473641981596239624636716170064196268514261484186029348529
07379021659761911840*e^(164*I*c) + 7275210107183942292917740738446942557987
38667067535379759732795567942578751384250780476310*e^(163*I*c) + 1699563279
699297677739020966526292532837045054771275445565344173766865409367060738473
37600*e^(162*I*c) + 3910803125560180947653753536961184444084490375160564502
3514572352045248104262933598850730*e^(161*I*c) + 88627521427569572856813408
85764904597935349569355321815647721172537159186491471311666400*e^(160*I*c)
+ 1977792980665818135651300094326239158605448870806970860577325385028609983
034534672318500*e^(159*I*c) + 434546676780280045346344498763892540797175105
756827515509297024187660299345484920192480*e^(158*I*c) + 939869153130681791
49083606065681482780836060510530154618486949839467131378859885998210*e^(157
*I*c) + 2000800680303004713729327825032159711354071620198333312634928118667
9153199068045257216*e^(156*I*c) + 41915425006568261480933394145441591439644
78472492315931809171859902114109005939942952*e^(155*I*c) + 8639799336223303
49556296820028395513198708064940505702126068652936800794826651264256*e^(154
*I*c) + 1751931705006183002415156323819122851577900978160492206712172122200
15297133400636060*e^(153*I*c) + 3494107161327670464947794304333945020150407
5335160361865916029213860778606230624960*e^(152*I*c) + 68529932231457366873
28885311617795435592940841439866351079655652312894721972796266*e^(151*I*c)
+ 1321498055271300851429993866631619874424534425188183592049727687571032156
435077280*e^(150*I*c) + 250501028608928332469340456829902067712233644464602
753159945727868485722395506952*e^(149*I*c) + 466682235482660178068545924681
00570289355960869613650856575756758180182223308768*e^(148*I*c) + 8543013441
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7*I*c) + 153633323844492758353273455601649467167491657890711698454848907824

1693926940560*e^(146*I*c) + 27136120750326657073448651707718101480177532218
 3181055638619257836143271472358*e^(145*I*c) + 47065044611135158108487353367
 484243102698248838312635876283099427442745866704*e^(144*I*c) + 801372958079
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) + 13392143742542455535648844068019453533850002540306557659537702376071800
 89968*e^(142*I*c) + 2196012813395155615002614788441900248705552612819460588
 39614044697037963695*e^(141*I*c) + 3532444720677901811537805282078941168758
 1004582367431006205879633729015200*e^(140*I*c) + 55725511573286711210162164
 16307596161861955969011697222340926210112854418*e^(139*I*c) + 8618848510949
 91908764246805474672428603757315484453974713612812215428992*e^(138*I*c) + 1
 30657660226560419335121434389938961884595434069984824307149332131747540*e^(
 137*I*c) + 1940797921559456659353500810330325525774540807008243133894518479
 7463936*e^(136*I*c) + 28239051519365866783825257065644572802900986986385979
 87628380245881715*e^(135*I*c) + 4023496922661211589340035828394287851169049
 03936409545602519219664720*e^(134*I*c) + 5611708107634117538408757018518853
 8660375932013674735519055227368366*e^(133*I*c) + 76590105201875496517771183
 57676871927081898989131125755798204236112*e^(132*I*c) + 1022536437468296737
 293065862705246449693687415559865844306888705423*e^(131*I*c) + 133490210052
 026183779673313868332303530332906163247194627808410304*e^(130*I*c) + 170338
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 123702969188871318266718781223927067839949015727293884065388080*e^(128*I*c)
 + 258585348715977270155829115684193411072034541491364393985491350*e^(127*I
 *c) + 30736217404321009965231037419663053962881035281709221697785072*e^(126
 *I*c) + 3564764890628724017088487996688178929195787613958545474804845*e^(12
 5*I*c) + 403212225957798188840846139960995624144491271694336796459584*e^(12
 4*I*c) + 44456708175258821024400946210535004523775722190977468484496*e^(123
 *I*c) + 4775398607100853263534207733818266777478693412738731031680*e^(122*I
 *c) + 499467506558531733671585862910572702811545035730398749530*e^(121*I*c)
 + 50836369508171099437019348610847391946736185108017183136*e^(120*I*c) + 5
 032024903401451824074213943766011922026507006311982753*e^(119*I*c) + 484093
 410240488718655917025303662581091659126182344528*e^(118*I*c) + 452309400398
 30738332025694784646206844854827698075736*e^(117*I*c) + 4101545439937195793
 959956708442496709433800261224880*e^(116*I*c) + 360688613036389349413809780
 004559963548775423325255*e^(115*I*c) + 307353665128305621609911663384900573
 08062762518496*e^(114*I*c) + 2535667460650279776834561566186591213109251642
 859*e^(113*I*c) + 202347509724462171313966643580234078508179838320*e^(112*I
 *c) + 15603911277687607099721623771744933086920587272*e^(111*I*c) + 1161581
 413733971751533622511909046917188768400*e^(110*I*c) + 833808399118378944531
 36303673785039051506805*e^(109*I*c) + 5764601046563151304213854710715346838
 447392*e^(108*I*c) + 383360155801054824529764688213114368047154*e^(107*I*c)
 + 24489837337812338687718622491865013839488*e^(106*I*c) + 1500602747937397
 286405577818722691539392*e^(105*I*c) + 880549275989414111458699508133880402
 56*e^(104*I*c) + 4939666610818025798809586352543471345*e^(103*I*c) + 264410
 375780310742518099326419685040*e^(102*I*c) + 134772277995247019565792742103
 95326*e^(101*I*c) + 652650253343206047453620559993840*e^(100*I*c) + 2995254

$7749265499675257842032197 * e^{(99 * I * c)} + 1299146645993240318167826532288 * e^{(98 * I * c)} + 53090127264630963470039804475 * e^{(97 * I * c)} + 2037031259470368160131922320 * e^{(96 * I * c)} + 73099207817335597247098038 * e^{(95 * I * c)} + 2442455629894502983849104 * e^{(94 * I * c)} + 75599817092670157806639 * e^{(93 * I * c)} + 2154864144781257856128 * e^{(92 * I * c)} + 56169444526926562260 * e^{(91 * I * c)} + 1327882849274858880 * e^{(90 * I * c)} + 28186192554792138 * e^{(89 * I * c)} + 530563624556832 * e^{(88 * I * c)} + 8718181624155 * e^{(87 * I * c)} + 122503723056 * e^{(86 * I * c)} + 1431118260 * e^{(85 * I * c)} + 13343760 * e^{(84 * I * c)} + 93096 * e^{(83 * I * c)} + 432 * e^{(82 * I * c)} + e^{(81 * I * c)}) * \tan(1/4 * d * x + c) + 7 * (-338 * I * a * e^{(1055/2 * I * c)} - 136552 * I * a * e^{(1053/2 * I * c)} - 27515228 * I * a * e^{(1051/2 * I * c)} - 3687040552 * I * a * e^{(1049/2 * I * c)} - 369625815338 * I * a * e^{(1047/2 * I * c)} - 29570065227040 * I * a * e^{(1045/2 * I * c)} - 1966409337599317 * I * a * e^{(1043/2 * I * c)} - 111804416623905668 * I * a * e^{(1041/2 * I * c)} - 5548294175032309402 * I * a * e^{(1039/2 * I * c)} - 244124943709898420493 * I * a * e^{(1037/2 * I * c)} - 9642935277307714791837 * I * a * e^{(1035/2 * I * c)} - 345392409079468621758030 * I * a * e^{(1033/2 * I * c)} - 11311601400768793085009738 * I * a * e^{(1031/2 * I * c)} - 341088288572157297772118077 * I * a * e^{(1029/2 * I * c)} - 9526108639140231470517528278 * I * a * e^{(1027/2 * I * c)} - 247678824959506386373000637444 * I * a * e^{(1025/2 * I * c)} - 6021691444519566631530369807136 * I * a * e^{(1023/2 * I * c)} - 137436252221498552065844084546042 * I * a * e^{(1021/2 * I * c)} - 2954879436063156934993931067226770 * I * a * e^{(1019/2 * I * c)} - 60030708924112460138240147528356200 * I * a * e^{(1017/2 * I * c)} - 1155591156922028898629041321537843182 * I * a * e^{(1015/2 * I * c)} - 21130809978126689911485296435923679086 * I * a * e^{(1013/2 * I * c)} - 367868197744817126115132997049285770709 * I * a * e^{(1011/2 * I * c)} - 6109811064562267906978585057956580355548 * I * a * e^{(1009/2 * I * c)} - 96993253296241745199299054522607239475550 * I * a * e^{(1007/2 * I * c)} - 1474297501848124542057148378045565788918733 * I * a * e^{(1005/2 * I * c)} - 21490722237212978408403217414663518345362065 * I * a * e^{(1003/2 * I * c)} - 300870128273961175599469606519345892299669212 * I * a * e^{(1001/2 * I * c)} - 4051001655313083033785087944018198318088989698 * I * a * e^{(999/2 * I * c)} - 52523336394485929659233288472451790119792705845 * I * a * e^{(997/2 * I * c)} - 656541775517510646234544280431980818352772409123 * I * a * e^{(995/2 * I * c)} - 7920859881151658505680929024974848479434840259054 * I * a * e^{(993/2 * I * c)} - 92327537729375970664780896287923034091445656319868 * I * a * e^{(991/2 * I * c)} - 1040783353217938776411028748013653576556213918915335 * I * a * e^{(989/2 * I * c)} - 11356785688989551343908515511198887233162301712666395 * I * a * e^{(987/2 * I * c)} - 120057481911175947127117391278335425038727525157415250 * I * a * e^{(985/2 * I * c)} - 1230589594126194023203320811726992132021299197471177852 * I * a * e^{(983/2 * I * c)} - 12239382346160513892240601238455047125011028691886256771 * I * a * e^{(981/2 * I * c)} - 118206720526999189093597542167372456853664293435038820382 * I * a * e^{(979/2 * I * c)} - 1109325204761582846669749366309189735990268133663930730770 * I * a * e^{(977/2 * I * c)} - 10122598859264890271793915869084003420710826310616818118992 * I * a * e^{(975/2 * I * c)} - 89868992289875724477547529577630120615631448882291724117498 * I * a * e^{(973/2 * I * c)} - 776725522432810590073028517365937240359454002830981290341970 * I * a * e^{(971/2 * I * c)} - 6538951529796735469911010316324282233950224197184638863745512 * I * a * e^{(969/2 * I * c)} - 53649185943281059583991832533828279475735386275724477961967700 * I * a * e^{(967/2 * I * c)} - 429194047823886492115016704404554514518112725730803903112532182 * I * a * e^{(965/2 * I * c)} - 33495846492159740644945622881$

69726260989776567520122206199301331*I*a*e^(963/2*I*c) - 2551390123780806833
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) - 1378667638242906833909487224448367891682486392320460023203024208675*I*a
 *e^(957/2*I*c) - 9788564956238722791049897452775743960339323004999489860937
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 *c) - 303463009922187912110612328959183307554276958172628122702054617882544
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 - 4591423692429468515885016139926071595050527848377998359965229708397246546
 408548*I*a*e^(925/2*I*c) - 235835770676392103186616003904562143024409961071
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 c) - 5896411214077176351589511972323511292973849814278738063036230854700954
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 618612668642380811677650394711966142500*I*a*e^(917/2*I*c) - 137417856595605
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 (915/2*I*c) - 6464603885648283613014530104196558399989124169829102400581535
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 24688116078913627360928358220223598495094833068710253849363218567711980*I*a
 *e^(905/2*I*c) - 1158699669051248530097127432305816800500406359760366404195
 36918816038916044404058390580*I*a*e^(903/2*I*c) - 4935968762662148107626418
 82245481873457367886905402994350243807840669121360440299203610*I*a*e^(901/2
 *I*c) - 2069405755469517768606845303743959442907534110949655104608338576642
 070816206712850594640*I*a*e^(899/2*I*c) - 854000449095519783654305856856490
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/2*I*c)</sup> - 21764983871873411985186978485868937860310667984798680504187269006
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*a*e^(869/2*I*c) - 24166102823474984668967410783637906253892006902730453072
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859/2*I*c)</sup> - 78123640003492233583452583227307482260660777887580718451355979
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*a*e^(849/2*I*c) - 18107807683744905331264387812935563056139984311930680824
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843/2*I*c)</sup> - 4069524455400128914928462086271400007173410848833155506269926
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 83020*I*a*e^(645/2*I*c) - 4012977992311501006598317779033127050054097994290
 386865552989735233629291236389911293983802554742032304564034581648071441520

$*I*a*e^{(643/2*I*c)} - 410736991435895460891381503873310576186892582307966048$
 $6469112862247481293693565982676018021843982158540517444046989929778560*I*a*$
 $e^{(641/2*I*c)} - 41581042941815303182691599910307796054226321620259136051809$
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 $*c)} - 411129028232729378919209515264935437007088137901238016084937413844813$
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 $8910064530639592882140720086195863996979050655375410841792287731053730620*I$
 $*a*e^{(613/2*I*c)} + 26407461608063870256740547485279735699023562580769891006$
 $4530639592882140720086195863996979050655375410841792287731053730620*I*a*e^{($
 $611/2*I*c)} + 78758107057402302344708628907689331282129505846540059599829360$
 $4144468026594365471220886365550607609800151696278587848326040*I*a*e^{(609/2*$
 $I*c)} + 12972907369176126893495617056618448122808650877883262726168465144517$
 $67625603157718949168500603037628131561607285054911012000*I*a*e^{(607/2*I*c)}$
 $+ 1784439224726138591870127611501980562247256627957670212130377044528036983$
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 $1097939009853477230036485584868189563257145660*I*a*e^{(603/2*I*c)} + 26593788$
 $839363160883778631592861066530686033552860781850706121435907047865543874763$
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99965844160*I*a*e^(589/2*I*c) + 4160645719004275565304960215369141503063578
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622240*I*a*e^(587/2*I*c) + 415810429418153031826915999103077960542263216202
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0*I*a*e^(585/2*I*c) + 41073699143589546089138150387331057618689258230796604
86469112862247481293693565982676018021843982158540517444046989929778560*I*a
*e^(583/2*I*c) + 4012977992311501006598317779033127050054097994290386865552
989735233629291236389911293983802554742032304564034581648071441520*I*a*e^(5
81/2*I*c) + 388010971427813041688514122667408823867704091888252194903261962
8742464551694476160458875162535150778871755003462307751583020*I*a*e^(579/2*
I*c) + 37143979262445778652353668866292731715578840501913621632924713254982
27320803210520277222404279215283455652034473210863303640*I*a*e^(577/2*I*c)
+ 3521732622041061038550984940990187934660572168825766556068910780855010177
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807349588094395329838590330628341966566130982940989461985416377454344595956
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9091716095385240*I*a*e^(561/2*I*c) + 16696449810175918568800995929093872643
280653590121172996876653920788283080428063654681008604876657696512551821566
60820289680*I*a*e^(559/2*I*c) + 1463976940963912447651365515467185405762501
924914077047324754706711366589407997958028777358245267180524254879552356822
975000*I*a*e^(557/2*I*c) + 127307503650565828933577222568587247771537450472
661286183065045653730586205981295386924448144095387587157752292462787859949
6*I*a*e^(555/2*I*c) + 10979775582096388817812720534843548724716541826290376
91571726603897642009022215236374577880739026045741294959273615557371064*I*a
*e^(553/2*I*c) + 9392002305336941635695087970619014648265188924561718977784
18746558378982592001120989002153289888436624007732395176649784016*I*a*e^(55
1/2*I*c) + 7968009609380614205672969854398285024016388732539594620728499630

44169306655915733067106371194259686348644202335486139211144*I*a*e^(549/2*I*c) + 6704504345183676733105987232057702601428776019632552494461037147689643
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 871593459852400663650487696539618994967221476480067061608924308285992605365
 32398498029152984249659429757073514254834*I*a*e^(543/2*I*c) + 3801082850399
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 42917534972043461283677156449777056*I*a*e^(541/2*I*c) + 3094224986204007750
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 77614118952637848517622461364*I*a*e^(539/2*I*c) + 2497893885641358725846056
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 91534320044406713265976*I*a*e^(537/2*I*c) + 1999669787375825347879180302547
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 11301*I*a*e^(531/2*I*c) + 9752379488169343707074844742130433435656053445494
 4874598301077916325487914962129843509415087293500937741240715003092219284*I
 *a*e^(529/2*I*c) + 75468239328701194395295429337599748625135810475088220375
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 27/2*I*c) + 579008746767270585768451007780664586553146539732308445153172147
 34517245403224512137919416910585252662901629186232759644253*I*a*e^(525/2*I*
 c) + 4404044302936827327992181888527654528339255750551903613292778526760397
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 207917405479824971772316843827455504403050399535442942613539411509350547092
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 6178831641276993498874053500525*I*a*e^(517/2*I*c) + 13505863435545884265040
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 582505474802194829519694*I*a*e^(515/2*I*c) + 983047519912706540661010703023
 284227178220318194766753783789074258485149871394841007755722746547509193771
 0249769323767412*I*a*e^(513/2*I*c) + 70914132150359351084047168683045666035
 560032527547668234498968291221872494052881103684058061397124958128908410642
 31210928*I*a*e^(511/2*I*c) + 5069592243524225405652688460326152875966709816
 050275878460357717440876793202354867243322880737700534668203200200223745266
 *I*a*e^(509/2*I*c) + 359145937515917871527287429983116551878336174059192298
 0345591047441517817805559536236082673985676829650893577170263379850*I*a*e^(
 507/2*I*c) + 25211707482284081449740370761467596028581041166385535374400202
 17456970409094730653793237536846366906505696581378882054616*I*a*e^(505/2*I*
 c) + 1753648721621948344665323035344500928169246731037156030237049745174658
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855851252056186798141005629046878883306224549740503887230386435237287160951
8678342756379906875785368808430218531927334*I*a*e^(501/2*I*c) + 82518864671
237129917668538300374154515196623739664827393661440706519199996122029297148
2894130114743521964516001714179861*I*a*e^(499/2*I*c) + 55818109155457679534
086754423714982536666008344943283832937403953779746190285486392335757708024
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732487428813284882744325815756034266411597497467396971732361062007898502816
7266946798247966*I*a*e^(495/2*I*c) + 24827424893256161794570369766352520061
007857443406140981388900138235391030810939089972770658040659664048257407408
9879213*I*a*e^(493/2*I*c) + 16323700153006415236755606122558009743798044040
5517572183927035767230181309962814642060879444377285948362185495154086161*I
*a*e^(491/2*I*c) + 10630321158236029079093688060762090286918969908906507228
2269865838237662096415215608213465309152317417335803254777646540*I*a*e^(489
/2*I*c) + 68563267724801525835530091185632342153370219441968093199493439585
399279900993027594546478734905346362251571732607984770*I*a*e^(487/2*I*c) +
437956160097493579765068474053492147508352811690398993662144001844139449912
92881347390284533191154910866650017714941045*I*a*e^(485/2*I*c) + 2770388065
879741801414179836976730800452585042929964859696910647886915813981208233371
7388082946821508766049481077973059*I*a*e^(483/2*I*c) + 17353952556118044091
866250043904279524876639737013840121333225227870238823195868916660184536540
709110237107927786882862*I*a*e^(481/2*I*c) + 107641858801753142474554137806
023873166880496795733541361846183879500257001189713176124123961781803519004
35222112093164*I*a*e^(479/2*I*c) + 6610986490722064658800908996594970089148
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295*I*a*e^(477/2*I*c) + 402004684403232580660668695877853944429907645823280
5336208748322810628154580764652906635345466720611170521299087695579*I*a*e^(
475/2*I*c) + 24202115540591537504795307453315888216045074484457112274573037
44601668400557213178896517376949590952343768990797772626*I*a*e^(473/2*I*c)
+ 1442482860527021727025998678990411107321021387835542377307580287512051573
989107700940225360216432991175352627674951660*I*a*e^(471/2*I*c) + 851100755
495948167311967384799382325393908473708133536744004690162732128381564701822
378684740339904953764853311657379*I*a*e^(469/2*I*c) + 497098270574416977793
999901022268912612414553253938578661999730743751218491838334678754461337804
861175391987745632150*I*a*e^(467/2*I*c) + 287390605926275542575548421622839
871210234972126160885732249644221593342784630319023739202442479202690546380
883489714*I*a*e^(465/2*I*c) + 164456137643484023027839220727810183979810742
984557443986573106465730160243136051096488426970563221576065210083954112*I*
a*e^(463/2*I*c) + 931436044993931263445166225811972204036547143672127723996
44536085606080046334933428545626738637712217067955072450098*I*a*e^(461/2*I*
c) + 5221072433334716080150177316581981294499384812756563874179279933595905
5388789921819771479296204943724701127192070026*I*a*e^(459/2*I*c) + 28963327
006745382557260523392521002766162424431473143306015072868867675529419511652
966969349719029403862600773620600*I*a*e^(457/2*I*c) + 159000166293990983770
633365148321190576578131487708291203552320705026303848044448897674861797561
13401575843696792900*I*a*e^(455/2*I*c) + 8637448429412603145232295486970579

164573288359043608779825334105587873485306782005310584032334923334469052992
471566*I*a*e^(453/2*I*c) + 464291207891568301035808600708701702397396571313
1459328351978225710653086945801498275683194647474013409696663828963*I*a*e^(
451/2*I*c) + 24693967521344445442092879209187143678826841642536214604826927
66569833779781633857461623947418830676182596868630102*I*a*e^(449/2*I*c) + 1
299466170334480341543135560982800973046388591632081102321927761098390304943
531108062897890291528379823914088825794*I*a*e^(447/2*I*c) + 676536112375453
533535322327435304762507116008362988186332586271393372786322039392090789530
327993045045404821029011*I*a*e^(445/2*I*c) + 348455253838688201360505783941
113732251527445477363057406188342672634063934554405286663592501040628860183
543905219*I*a*e^(443/2*I*c) + 177546087676039487653207030476854772666243541
156745740575127122721103812917448459499475796154210708959328249028630*I*a*e
^(441/2*I*c) + 894872505724728370624201553901098068070796612063573577599972
56287849481359659799894879391911743427849611697524276*I*a*e^(439/2*I*c) + 4
461439096362842937887432990987855060523083096318356202050421428010872979698
4369361375912054446637154294300258539*I*a*e^(437/2*I*c) + 22000378044285344
028734825732866676658762934871965249674527380850469340847187834791650053923
537769840230493764196*I*a*e^(435/2*I*c) + 107301092713177499323083813829105
498651961301555042478503350150672469877026772333775321254702023915671511176
68374*I*a*e^(433/2*I*c) + 5175761305072823696040196698965533450997327744793
152831047341015798393926092260966184527256512322695767910061296*I*a*e^(431/
2*I*c) + 246897953212028922726884061682000445045811742376948064434322797188
9566876733150004369035355668553948933308821480*I*a*e^(429/2*I*c) + 11646910
388540405650688259647550737097079196384362555665675551484081208763099688642
22625458155464316902364952524*I*a*e^(427/2*I*c) + 5432885725468726204157411
487558285774676320340064008281503831338668372142706641114379637167983506121
56148456206*I*a*e^(425/2*I*c) + 2505839999941785479737696554757833654744169
27100959735219628296998411286972584917312607492527753131218181041384*I*a*e
(423/2*I*c) + 1142760179655107045364229996786244632358887121397594114949333
94918036613892504103005096855020042456078555128196*I*a*e^(421/2*I*c) + 5152
424337990311470345247509676244138577580823892844726637538647583705473808459
2021134353327686946496437282284*I*a*e^(419/2*I*c) + 22966687877245457025794
102073377677357995629076173854095854698682796300148885688222697251615914395
078685755000*I*a*e^(417/2*I*c) + 101202318762952166047217781137436284619629
96489755940527769171752595181499880607806290962282236974162704806460*I*a*e
(415/2*I*c) + 4408214207480768029716973149859780404324438947607836425499375
750084309188687239341140052519808074207080308780*I*a*e^(413/2*I*c) + 189797
053532870412198132213256930840056692247833463278278439445522077081088356041
4441976268438153636677378420*I*a*e^(411/2*I*c) + 80769022503565948483170779
513278182902665161792021013282953291691617090536488162475060828002976266044
8742840*I*a*e^(409/2*I*c) + 33970536672655873836914010596386451773527523414
3400513739357674484483665059765320736557902256832515882267500*I*a*e^(407/2*
I*c) + 14120023726763338224103829629480899357391515954966181764655361743735
9133700881157615051697163328436455695020*I*a*e^(405/2*I*c) + 57998486887243
431995720681004163175448573494680504990319637888579588763738426026942298616

787064620503722070*I*a*e^(403/2*I*c) + 235406497364212308866095824188703992
05206129846961712523581794766425027191000252567464229588210547056463840*I*a
*e^(401/2*I*c) + 9440883227662672550065366923781042631437653007247333510037
128999390205650430820323780125609508653387501660*I*a*e^(399/2*I*c) + 374085
234686504879640941178215322043352475999478006511254411292080039661341540533
8362337137549877683070230*I*a*e^(397/2*I*c) + 14644114599025915830514653310
497838889027533667220956430655162303649115642284295373268340269398781586151
90*I*a*e^(395/2*I*c) + 5663183101129734221900744632066053326419477514483473
07430314678972924597750157502827727169011647198755540*I*a*e^(393/2*I*c) + 2
163379908325429164639463670307413810883063213830164395550208283183523168770
90017854969031272563599670220*I*a*e^(391/2*I*c) + 8162982904467157287925863
016860566615128980862390386524933583681440860653817657997369255017899451926
3270*I*a*e^(389/2*I*c) + 30421281219629494570715476456524764856193146201523
675555228979793987239612970374604657916876884833815680*I*a*e^(387/2*I*c) +
111966121821141964442037838123563295873180241684479076570357958008170725856
81312382705199633772907655120*I*a*e^(385/2*I*c) + 4069524455400128914928462
086271400007173410848833155506269926856142013580290716696933427177861645554
760*I*a*e^(383/2*I*c) + 146054620025936520744869571735824318427681489148592
8975295616279546501015573251800164340914796390068760*I*a*e^(381/2*I*c) + 51
756820355312954410804386109705453232980813714151221855002034060198134663300
3923694388693344112445480*I*a*e^(379/2*I*c) + 18107807683744905331264387812
9355630561399843119306808249847499253374617041604538344094649538013408680*I
*a*e^(377/2*I*c) + 62542434019027418485094848248484541133380826803451715912
076716281169753247585230468246410100921183300*I*a*e^(375/2*I*c) + 213234849
147017215169538860194659781513497854263221295255965510269139101495035675916
58927381185274800*I*a*e^(373/2*I*c) + 7175946538198935299155529026996842352
667522857865938994004026182478124225655917226024463989568859870*I*a*e^(371/
2*I*c) + 238341751275652238302298227594736446382245192652096964091467452821
4045561036716921751876861448214560*I*a*e^(369/2*I*c) + 78123640003492233583
452583227307482260660777887580718451355979473369099196889018503824649448195
6500*I*a*e^(367/2*I*c) + 25268931035482758388494149738730795207903213724380
6400515611097303009838025277886805474018066255230*I*a*e^(365/2*I*c) + 80644
191734889967765592161669041761214694338439517250809770511678016755719669133
078696159918617190*I*a*e^(363/2*I*c) + 253921808396207175587073434951207934
97317456412001059973856406433299476018286356454215210996398560*I*a*e^(361/2
*I*c) + 7887248495370051935609113701125546897394431652357414444786810665029
425502851513884434989024419020*I*a*e^(359/2*I*c) + 241661028234749846689674
1078363790625389200690273045307234207106791714360472853141666795331774750*I
*a*e^(357/2*I*c) + 73029655258696344225597087065783914623049345083825117804
7587665866638789410736040062955533068290*I*a*e^(355/2*I*c) + 21764983871873
411985186978485868937860310667984798680504187269006573534753875312414668097
4449780*I*a*e^(353/2*I*c) + 63964319138305366838524243903503912460938784864
721728576192589071379760086707383054171805732800*I*a*e^(351/2*I*c) + 185349
026160449005232573409986289689481495846778295153754568810184876541742889587
98896933969850*I*a*e^(349/2*I*c) + 5295019379440696397366776161064889279568

$712285275229242010057123976569166715041171797137891170 * I * a * e^{(347/2 * I * c)} +$
 $149114529216286989978728856053128114270612562386198206811842751504560059919$
 $5560543505352431020 * I * a * e^{(345/2 * I * c)} + 41390126754989055823435182991213105$
 $5768491910694890021278894050553695996201115448918378784640 * I * a * e^{(343/2 * I * c)}$
 $) + 11322593106386724431761639455345559749498779183914297547250657025816357$
 $6992032763272923073090 * I * a * e^{(341/2 * I * c)} + 30522074672219238788542058593438$
 $534852586364273418371200055214826482127403312825986943237280 * I * a * e^{(339/2 * I * c)}$
 $+ 810674495140204035565067389115601368638382784158150151091612530385141$
 $1890302126855054047420 * I * a * e^{(337/2 * I * c)} + 21212271688547753093292205568433$
 $01957459256215327695250196367156112161484956828664832181880 * I * a * e^{(335/2 * I * c)}$
 $+ 5467370624803225529046131197946494737392605782776836753492933741345813$
 $93610464155972682280 * I * a * e^{(333/2 * I * c)} + 1387910490502186643983733269055472$
 $32792143053299682659302792667147805101052262490681909560 * I * a * e^{(331/2 * I * c)}$
 $+ 3469567947919739125467272019857989136613877856828042389680340788795019161$
 $5247559195575960 * I * a * e^{(329/2 * I * c)} + 85400044909551978365430585685649087381$
 $27055351481652620533301762011364604595026952327120 * I * a * e^{(327/2 * I * c)} + 2069$
 $405755469517768606845303743959442907534110949655104608338576642070816206712$
 $850594640 * I * a * e^{(325/2 * I * c)} + 493596876266214810762641882245481873457367886$
 $905402994350243807840669121360440299203610 * I * a * e^{(323/2 * I * c)} + 115869966905$
 $124853009712743230581680050040635976036640419536918816038916044404058390580$
 $* I * a * e^{(321/2 * I * c)} + 267652772109858246881160789136273609283582202235984950$
 $94833068710253849363218567711980 * I * a * e^{(319/2 * I * c)} + 6082802098350359025768$
 $710715675956495702650305299754400572492569343098453009652566250 * I * a * e^{(317/2 * I * c)}$
 $+ 135985739713427720243706920455682299954101591610392923436275251722$
 $3588368000249304810 * I * a * e^{(315/2 * I * c)} + 29899596829739071545656839188275994$
 $6087594535121700903653503645916859497690395095780 * I * a * e^{(313/2 * I * c)} + 64646$
 $038856482836130145301041965583999891241698291024005815352807403014487811899$
 $520 * I * a * e^{(311/2 * I * c)} + 137417856595605575898866999448538754167996491063998$
 $20566514814463893361892904950890 * I * a * e^{(309/2 * I * c)} + 2871353973777698342367$
 $234312486286687762934618612668642380811677650394711966142500 * I * a * e^{(307/2 * I * c)}$
 $+ 589641121407717635158951197232351129297384981427873806303623085470095$
 $427193276692 * I * a * e^{(305/2 * I * c)} + 118975846340172913159248725555818206928962$
 $615117446304843044326162984826050727208 * I * a * e^{(303/2 * I * c)} + 235835770676392$
 $10318661600390456214302440996107182761770233626717895694882382172 * I * a * e^{(301/2 * I * c)}$
 $+ 4591423692429468515885016139926071595050527848377998359965229708$
 $397246546408548 * I * a * e^{(299/2 * I * c)} + 877760214042329341241985800973285159873$
 $65505819033642351120680153977723334852 * I * a * e^{(297/2 * I * c)} + 164738891969044$
 $198589212814142825245654731022138637322268898435719103132844200 * I * a * e^{(295/2 * I * c)}$
 $+ 303463009922187912110612328959183307554276958172628122702054617882$
 $54462121188 * I * a * e^{(293/2 * I * c)} + 5485282326270569168343744931160098443379763$
 $576662271067319928989360513500012 * I * a * e^{(291/2 * I * c)} + 972672122556805618091$
 $233812449540815508255737026698123751589823682626604408 * I * a * e^{(289/2 * I * c)} +$
 $169159001254667821791360784785389073333265377052642698186735860494211567022$
 $* I * a * e^{(287/2 * I * c)} + 288448227683149535792126719085007079507251784236134799$
 $19214905815452892948 * I * a * e^{(285/2 * I * c)} + 4821293518673085167363493338203848$

847844190253193965493680746373955080080*I*a*e^(283/2*I*c) + 789690039989072
 858764397361560374790810333801284467329771776351321048032*I*a*e^(281/2*I*c)
 + 126711784709453338605660778068542544022643349256399659726946153656233718
 *I*a*e^(279/2*I*c) + 199117598387417926031189003205662174209347861727303761
 27765302242075212*I*a*e^(277/2*I*c) + 3063335070065447315308133852848066992
 101021248688567743307391113472411*I*a*e^(275/2*I*c) + 461239107567627147326
 076654057456658457043425511345276082020328203684*I*a*e^(273/2*I*c) + 679443
 51214079532127966887502579811300811007213403186988581793087318*I*a*e^(271/2
 *I*c) + 9788564956238722791049897452775743960339323004999489860937041472915
 *I*a*e^(269/2*I*c) + 137866763824290683390948722444836789168248639232046002
 3203024208675*I*a*e^(267/2*I*c) + 18976001127512012080908245217116426083644
 4479917200503452767697538*I*a*e^(265/2*I*c) + 25513901237808068333261288925
 342131996483836670215495252968765174*I*a*e^(263/2*I*c) + 334958464921597406
 4494562288169726260989776567520122206199301331*I*a*e^(261/2*I*c) + 42919404
 7823886492115016704404554514518112725730803903112532182*I*a*e^(259/2*I*c) +
 53649185943281059583991832533828279475735386275724477961967700*I*a*e^(257/
 2*I*c) + 6538951529796735469911010316324282233950224197184638863745512*I*a*
 e^(255/2*I*c) + 77672552243281059007302851736593724035945400283098129034197
 0*I*a*e^(253/2*I*c) + 89868992289875724477547529577630120615631448882291724
 117498*I*a*e^(251/2*I*c) + 101225988592648902717939158690840034207108263106
 16818118992*I*a*e^(249/2*I*c) + 1109325204761582846669749366309189735990268
 133663930730770*I*a*e^(247/2*I*c) + 118206720526999189093597542167372456853
 664293435038820382*I*a*e^(245/2*I*c) + 122393823461605138922406012384550471
 25011028691886256771*I*a*e^(243/2*I*c) + 1230589594126194023203320811726992
 132021299197471177852*I*a*e^(241/2*I*c) + 120057481911175947127117391278335
 425038727525157415250*I*a*e^(239/2*I*c) + 113567856889895513439085155111988
 87233162301712666395*I*a*e^(237/2*I*c) + 1040783353217938776411028748013653
 576556213918915335*I*a*e^(235/2*I*c) + 923275377293759706647808962879230340
 91445656319868*I*a*e^(233/2*I*c) + 7920859881151658505680929024974848479434
 840259054*I*a*e^(231/2*I*c) + 656541775517510646234544280431980818352772409
 123*I*a*e^(229/2*I*c) + 52523336394485929659233288472451790119792705845*I*a*
 e^(227/2*I*c) + 4051001655313083033785087944018198318088989698*I*a*e^(225/
 2*I*c) + 300870128273961175599469606519345892299669212*I*a*e^(223/2*I*c) +
 21490722237212978408403217414663518345362065*I*a*e^(221/2*I*c) + 1474297501
 848124542057148378045565788918733*I*a*e^(219/2*I*c) + 969932532962417451992
 99054522607239475550*I*a*e^(217/2*I*c) + 6109811064562267906978585057956580
 355548*I*a*e^(215/2*I*c) + 367868197744817126115132997049285770709*I*a*e^(2
 13/2*I*c) + 21130809978126689911485296435923679086*I*a*e^(211/2*I*c) + 1155
 591156922028898629041321537843182*I*a*e^(209/2*I*c) + 600307089241124601382
 40147528356200*I*a*e^(207/2*I*c) + 2954879436063156934993931067226770*I*a*e
 ^((205/2*I*c) + 137436252221498552065844084546042*I*a*e^(203/2*I*c) + 602169
 1444519566631530369807136*I*a*e^(201/2*I*c) + 24767882495950638637300063744
 4*I*a*e^(199/2*I*c) + 9526108639140231470517528278*I*a*e^(197/2*I*c) + 3410
 88288572157297772118077*I*a*e^(195/2*I*c) + 11311601400768793085009738*I*a*
 e^(193/2*I*c) + 345392409079468621758030*I*a*e^(191/2*I*c) + 96429352773077

$14791837 * I * a * e^{(189/2 * I * c)} + 244124943709898420493 * I * a * e^{(187/2 * I * c)} + 5548$
 $294175032309402 * I * a * e^{(185/2 * I * c)} + 111804416623905668 * I * a * e^{(183/2 * I * c)} +$
 $1966409337599317 * I * a * e^{(181/2 * I * c)} + 29570065227040 * I * a * e^{(179/2 * I * c)} + 369$
 $625815338 * I * a * e^{(177/2 * I * c)} + 3687040552 * I * a * e^{(175/2 * I * c)} + 27515228 * I * a * e$
 $^{(173/2 * I * c)} + 136552 * I * a * e^{(171/2 * I * c)} + 338 * I * a * e^{(169/2 * I * c)}) / (e^{(531 * I * c)}$
 $+ 432 * e^{(530 * I * c)} + 93096 * e^{(529 * I * c)} + 13343760 * e^{(528 * I * c)} + 143111826$
 $0 * e^{(527 * I * c)} + 122503723056 * e^{(526 * I * c)} + 8718181624155 * e^{(525 * I * c)} + 5305$
 $63624556832 * e^{(524 * I * c)} + 28186192554792138 * e^{(523 * I * c)} + 13278828492748588$
 $80 * e^{(522 * I * c)} + 56169444526926562260 * e^{(521 * I * c)} + 2154864144781257856128 * e$
 $^{(520 * I * c)} + 75599817092670157806639 * e^{(519 * I * c)} + 24424556298945029838491$
 $04 * e^{(518 * I * c)} + 73099207817335597247098038 * e^{(517 * I * c)} + 20370312594703681$
 $60131922320 * e^{(516 * I * c)} + 53090127264630963470039804475 * e^{(515 * I * c)} + 12991$
 $46645993240318167826532288 * e^{(514 * I * c)} + 29952547749265499675257842032197 * e$
 $^{(513 * I * c)} + 652650253343206047453620559993840 * e^{(512 * I * c)} + 13477227799524$
 $701956579274210395326 * e^{(511 * I * c)} + 264410375780310742518099326419685040 * e^{(510 * I * c)}$
 $+ 4939666610818025798809586352543471345 * e^{(509 * I * c)} + 88054927598$
 $941411145869950813388040256 * e^{(508 * I * c)} + 150060274793739728640557781872269$
 $1539392 * e^{(507 * I * c)} + 24489837337812338687718622491865013839488 * e^{(506 * I * c)}$
 $+ 383360155801054824529764688213114368047154 * e^{(505 * I * c)} + 576460104656315$
 $1304213854710715346838447392 * e^{(504 * I * c)} + 83380839911837894453136303673785$
 $039051506805 * e^{(503 * I * c)} + 1161581413733971751533622511909046917188768400 * e$
 $^{(502 * I * c)} + 15603911277687607099721623771744933086920587272 * e^{(501 * I * c)} +$
 $202347509724462171313966643580234078508179838320 * e^{(500 * I * c)} + 253566746065$
 $0279776834561566186591213109251642859 * e^{(499 * I * c)} + 30735366512830562160991$
 $166338490057308062762518496 * e^{(498 * I * c)} + 360688613036389349413809780004559$
 $963548775423325255 * e^{(497 * I * c)} + 410154543993719579395995670844249670943380$
 $0261224880 * e^{(496 * I * c)} + 45230940039830738332025694784646206844854827698075$
 $736 * e^{(495 * I * c)} + 484093410240488718655917025303662581091659126182344528 * e^{(494 * I * c)}$
 $+ 5032024903401451824074213943766011922026507006311982753 * e^{(493 * I * c)}$
 $+ 50836369508171099437019348610847391946736185108017183136 * e^{(492 * I * c)}$
 $+ 499467506558531733671585862910572702811545035730398749530 * e^{(491 * I * c)} +$
 $4775398607100853263534207733818266777478693412738731031680 * e^{(490 * I * c)} + 44$
 $456708175258821024400946210535004523775722190977468484496 * e^{(489 * I * c)} + 403$
 $212225957798188840846139960995624144491271694336796459584 * e^{(488 * I * c)} + 356$
 $4764890628724017088487996688178929195787613958545474804845 * e^{(487 * I * c)} + 30$
 $736217404321009965231037419663053962881035281709221697785072 * e^{(486 * I * c)} +$
 $258585348715977270155829115684193411072034541491364393985491350 * e^{(485 * I * c)}$
 $+ 2123702969188871318266718781223927067839949015727293884065388080 * e^{(484 * I * c)}$
 $+ 17033886027390615741040977721655541665612162275485028584310890417 * e^{(483 * I * c)}$
 $+ 133490210052026183779673313868332303530332906163247194627808410$
 $304 * e^{(482 * I * c)} + 102253643746829673729306586270524644969368741555986584430$
 $6888705423 * e^{(481 * I * c)} + 76590105201875496517771183576768719270818989891311$
 $25755798204236112 * e^{(480 * I * c)} + 5611708107634117538408757018518853866037593$
 $2013674735519055227368366 * e^{(479 * I * c)} + 40234969226612115893400358283942878$
 $5116904903936409545602519219664720 * e^{(478 * I * c)} + 28239051519365866783825257$

06564457280290098698638597987628380245881715*e^(477*I*c) + 1940797921559456
6593535008103303255257745408070082431338945184797463936*e^(476*I*c) + 13065
7660226560419335121434389938961884595434069984824307149332131747540*e^(475*
I*c) + 86188485109499190876424680547467242860375731548445397471361281221542
8992*e^(474*I*c) + 55725511573286711210162164163075961618619559690116972223
40926210112854418*e^(473*I*c) + 3532444720677901811537805282078941168758100
4582367431006205879633729015200*e^(472*I*c) + 21960128133951556150026147884
4190024870555261281946058839614044697037963695*e^(471*I*c) + 13392143742542
45553564884406801945353385000254030655765953770237607180089968*e^(470*I*c)
+ 8013729580790752434361964945761543761469520791210746972675870481058674277
844*e^(469*I*c) + 470650446111351581084873533674842431026982488383126358762
83099427442745866704*e^(468*I*c) + 2713612075032665707344865170771810148017
75322183181055638619257836143271472358*e^(467*I*c) + 1536333238444927583532
734556016494671674916578907116984548489078241693926940560*e^(466*I*c) + 854
301344112621233483354066506962147247908583804136056455072203672365429754020
5*e^(465*I*c) + 46668223548266017806854592468100570289355960869613650856575
756758180182223308768*e^(464*I*c) + 250501028608928332469340456829902067712
233644464602753159945727868485722395506952*e^(463*I*c) + 132149805527130085
1429993866631619874424534425188183592049727687571032156435077280*e^(462*I*c
) + 68529932231457366873288853116177954355929408414398663510796556523128947
21972796266*e^(461*I*c) + 3494107161327670464947794304333945020150407533516
0361865916029213860778606230624960*e^(460*I*c) + 17519317050061830024151563
2381912285157790097816049220671217212220015297133400636060*e^(459*I*c) + 86
397993362233034955629682002839551319870806494050570212606865293680079482665
1264256*e^(458*I*c) + 41915425006568261480933394145441591439644784724923159
31809171859902114109005939942952*e^(457*I*c) + 200800680303004713729327825
0321597113540716201983333126349281186679153199068045257216*e^(456*I*c) + 93
986915313068179149083606065681482780836060510530154618486949839467131378859
885998210*e^(455*I*c) + 434546676780280045346344498763892540797175105756827
515509297024187660299345484920192480*e^(454*I*c) + 197779298066581813565130
0094326239158605448870806970860577325385028609983034534672318500*e^(453*I*c
) + 88627521427569572856813408857649045979353495693553218156477211725371591
86491471311666400*e^(452*I*c) + 3910803125560180947653753536961184444084490
3751605645023514572352045248104262933598850730*e^(451*I*c) + 16995632796992
976777390209665262925328370450547712754455653441737668654093670607384733760
0*e^(450*I*c) + 72752101071839422929177407384469425579873866706753537975973
2795567942578751384250780476310*e^(449*I*c) + 30679742964317473641981596239
62463671617006419626851426148418602934852907379021659761911840*e^(448*I*c)
+ 1274721961650332054135634306256284736860162214085678602544581453203790411
1523242298235713300*e^(447*I*c) + 52190912207661824215812271854269748071292
843243227894769229690720010547141334131610989636000*e^(446*I*c) + 210594301
385648471184329078880317504953361839954159427434009884661777259752542647709
150036990*e^(445*I*c) + 837579206923411932458786486765373533946545239708990
769488724813982189165104589895518909256320*e^(444*I*c) + 328387476055581867
672630948030673442015509858394807446901416817187444217010964852162753875592

$0 \cdot e^{(443 \cdot I \cdot c)} + 12693496932964920565073673637181280088548682508880255337280$
 $065006566138696041797353216584528640 \cdot e^{(442 \cdot I \cdot c)} + 483794897564340998438577$
 $91816589379406815042609340378747586437145781646245422045101230417309900 \cdot e^{($
 $441 \cdot I \cdot c)} + 1818346614061779013153301296771453811664491884131941411693443547$
 $54920969034952610378945282257600 \cdot e^{(440 \cdot I \cdot c)} + 6740255305431330088948457752$
 $36625237450743114473544537818170447134607102575676676056675328961590 \cdot e^{(439$
 $\cdot I \cdot c)} + 2464382190807439609079774226855679629367885709776435876630851716253$
 $962696192341706239192878728160 \cdot e^{(438 \cdot I \cdot c)} + 888829502875102466704420383760$
 $7976101480053134418614474620767522824868911959884352666444917404000 \cdot e^{(437 \cdot$
 $I \cdot c)} + 31626644674725547731176795687527653571305969985923688392112164915553$
 $242573269490908989570248533280 \cdot e^{(436 \cdot I \cdot c)} + 111034148797008819443143895644$
 $469242295049867464313710969257619338899133799285616020069872611710850 \cdot e^{(43$
 $5 \cdot I \cdot c)} + 384655842080666274454063078784837174998949052500975322162003392549$
 $953413592461519365177908682078400 \cdot e^{(434 \cdot I \cdot c)} + 131505212093069212210229710$
 $5327622842335870743428530891072983535862280094446607723473800477453914130 \cdot e$
 $^{(433 \cdot I \cdot c)} + 44372109178431823477643495444439046990200565950694708471936170$
 $92114714077633077234972825351226979360 \cdot e^{(432 \cdot I \cdot c)} + 1477795509661712899871$
 $274518207149536217650697318308165023360527405167762497046434024275584002567$
 $3760 \cdot e^{(431 \cdot I \cdot c)} + 48584258153140280447314836868772131390195412419046732778$
 $458706015096881437076337910793584122475073760 \cdot e^{(430 \cdot I \cdot c)} + 157685845528850$
 $918721462877864435090257583149415561323427386562894447598277935629800939237$
 $175625149830 \cdot e^{(429 \cdot I \cdot c)} + 505293663123015258878483025738812813203397766845$
 $340065381261016353419722382620393032535960660921950400 \cdot e^{(428 \cdot I \cdot c)} + 159877$
 $110105819269227052899967744474268563100623245618584492522014400230587812038$
 $0828483988663574829100 \cdot e^{(427 \cdot I \cdot c)} + 49952419562791381802051867444016880243$
 $88272113921255663734956946927571305533146776898787878059685108480 \cdot e^{(426 \cdot I \cdot$
 $c)} + 1541311121148602393729497082079737671608134478816338654352242193973750$
 $7962125854981881879168348260330000 \cdot e^{(425 \cdot I \cdot c)} + 46970224727117281826454045$
 $018070670522559756627580347784535320014963482632359729444541885102274546002$
 $560 \cdot e^{(424 \cdot I \cdot c)} + 141379938253556843280565505807403304130606130725434751745$
 $794079833141361748917639986145377066437210546190 \cdot e^{(423 \cdot I \cdot c)} + 420358024835$
 $146798583611210145942154684437949365647899088372524802156222884839580011688$
 $655664280691773600 \cdot e^{(422 \cdot I \cdot c)} + 123466804189240997878001808175544021601258$
 $2476396941937965899631953079203974222138794604328498972144766900 \cdot e^{(421 \cdot I \cdot c$
 $) + 35827180021632960614145367037151098971071982527392845461493431023484561$
 $24089657428594946438660859773886240 \cdot e^{(420 \cdot I \cdot c)} + 1027160253020288900249781$
 $351684945259097151280952906066519730109705221006457608834802323467197546367$
 $7418470 \cdot e^{(419 \cdot I \cdot c)} + 29097651061247453406647569781836910062165559852359052$
 $804259154165687125428752562385492373749486351714453120 \cdot e^{(418 \cdot I \cdot c)} + 814520$
 $814138291118288754175642500548460376933128114804929091607581959891557681070$
 $22568350953861815940704090 \cdot e^{(417 \cdot I \cdot c)} + 2253205325932206577679411092895162$
 $48999794521015564134982827241710019675486694499689312466561907212627820000 \cdot$
 $e^{(416 \cdot I \cdot c)} + 6160031160229795849371257017578872129983543009899913626280388$
 $61093914561332071191909714949426587936910303300 \cdot e^{(415 \cdot I \cdot c)} + 1664475034387$
 $211809394917743502937638978574937754764763987835872410449930690131572904279$

995484581013965001440*e^(414*I*c) + 444541225929547462506765951419831296601
5416299968930393345630345914109720740573618884980520010028451496996210*e^(4
13*I*c) + 11735856926245118493113091002501604032341876985999082823520672241
530200188223826392982302194084667538488665600*e^(412*I*c) + 306275810542219
573783905472892776091295728139310827335202473872260000200435382794687767079
58420892547870128680*e^(411*I*c) + 7901914955876656925478398848723238835290
9144982747171856772463223808993367091503402876467270176124342699654400*e^(4
10*I*c) + 20155794742479409802677247846204088339586751256293086294375356869
0084015585598010154781548625239409581907397500*e^(409*I*c) + 50832459930108
546016697862968303266142765447408293904809793963839156729879578838943384228
5751054665210868287680*e^(408*I*c) + 12675970172948129134001462760421269299
86480292870190399107554311079964227280196522475370108738477856311765699610*
e^(407*I*c) + 3125683493178701743479704750307490178666292150720179363604335
113528623329684606343185540756019935662148267863968*e^(406*I*c) + 762178879
191204706203884091779937460042889225819436763668294435609668140024631213800
1769285020661445991073249416*e^(405*I*c) + 18379807084003359766027649217621
144116091735572216620788861535803449702273802588359076704241840733513439114
113248*e^(404*I*c) + 438349721429193776853786922330210637445540331009285027
37480438978976746989895784070951905237783490374305934542955*e^(403*I*c) + 1
033997554672574364898478376407537547182043944730557950014676043264198765555
56873829531737211096115196005647730480*e^(402*I*c) + 2412460212824400617929
083177830328761948015971332060520912869970437291453457558057100814890067418
39439573984832678*e^(401*I*c) + 5567563887111823403410261927342195461136517
68317380539005893679049394714017063698565272728813669054779077208977840*e^(
400*I*c) + 1271033082938048950201360554831270342662343991277504612342300366
025046741742856580445289401786656311685859023084716*e^(399*I*c) + 287049613
141231445183467471535358943955329443080853193346608628854370924623076915139
2180699413405623017247753532944*e^(398*I*c) + 64133818958559251847582314510
625563803285949385110065770152181197865363902130572840182022016310944348195
84025113465*e^(397*I*c) + 1417648365287570495720201334324111790436997779665
3849959902524421980635189011634815653279605497783382888932766730080*e^(396*
I*c) + 31004319206069417077069363141423487431828009098184744635678652284177
439464941651812564519144918003174108077634846014*e^(395*I*c) + 670917061305
296691250198992100215765802378434622295353862950870761892978499959313606456
05292130961496106707521506432*e^(394*I*c) + 1436576871380447969429471197042
595384588181994235168246745862936910561192098663581236377722454095307992305
53767222252*e^(393*I*c) + 3043844711068133360102841601239063704338888284906
27422652551236966790916174520857759143930140187173492394981908258944*e^(392
*I*c) + 6382188929145337415058063996624885560667836004960918764083749748774
48971778036074996245581124283460438065182071976085*e^(391*I*c) + 1324311324
984027428355222938147682378672860708817161741448749689593588020860847508703
702325320304649883120684987556400*e^(390*I*c) + 271958928348374392604080510
108034192124453031125460725092919277390933152322663503581567286256929669371
1643521070331394*e^(389*I*c) + 55274988490311783558612300093266682839262900
82158467118000698502719379939045918344222192742145711257028040974074674736*

$e^{(388*I*c)} + 1111949964536320108088106282488633849242537565844897793553584$
 $6349290425821570383090425411418521516670371372045206568345*e^{(387*I*c)} + 22$
 $140735001708603270915180769241391662035578755903979148909213603822554792749$
 $183517160255571915875356439553717130797888*e^{(386*I*c)} + 436383000758151710$
 $259462114644656896189657734886609858579456574798540851088518579112229119898$
 $37615452608512356008400295*e^{(385*I*c)} + 8513953323478645577958995946490063$
 $776073572970562122138000583720836915779467304967542879981787543143024633262$
 $5899630160*e^{(384*I*c)} + 16443750067690689274132326015439427850395456193602$
 $0133596581806449357240277349447927334517105809995300093549279931273178*e^{(3$
 $83*I*c)} + 31440903580822586156559543693835494544547399104312972204674722881$
 $3030925204968503418566818838611866709807040793495364496*e^{(382*I*c)} + 59515$
 $761550043151494747928233654705382792608791642526349718775702941338547183543$
 $4198246807096214536895441388306027237899*e^{(381*I*c)} + 11153982855456015505$
 $333280456001843179938993592179967298183407042218011956674104858469961790567$
 $33558512452238583160792512*e^{(380*I*c)} + 2069698289500860643461665762373807$
 $957513019424041178871904960551829412449344722432125679417958403007551179298$
 $315947373776*e^{(379*I*c)} + 380260499670589110696462063384896480703709885451$
 $0182263243030597295630760353597531974324752266389193185760878274188013440*e$
 $^{(378*I*c)} + 69178389452148442784933304593613949233723338536198796373726731$
 $84942859712431066345726870422099893124890777678037369988150*e^{(377*I*c)} + 1$
 $246214044053725808492859670987206685775707094312486855450094815475686345430$
 $8032925408340311237850017814707896986969086816*e^{(376*I*c)} + 22231341131801$
 $535345406399037721686840208397941952580135584645966746736656716271554826476$
 $282991066076564921432614339399735*e^{(375*I*c)} + 392742004143298611693979445$
 $162250010812274333985850072063992312119071577953597196482415987542665798402$
 $44551491476467899952*e^{(374*I*c)} + 6871246601598564151246858617365974773487$
 $959171009835465278612493602307394314104957360664856300535941171276489568390$
 $3806088*e^{(373*I*c)} + 11906059184966054683476569322767644906758414824888267$
 $8447504826077236333444513454095126668750057295811191643356908972191440*e^{(3$
 $72*I*c)} + 20432555726518600076740271023084789645976158392276369823543321283$
 $3313077783041040074669379017394836761539649081690630811665*e^{(371*I*c)} + 34$
 $731005381093529041945556055595731412956921073574598323436965997641337477407$
 $8000173070075248654524917179128950507443058208*e^{(370*I*c)} + 58474957368230$
 $458617938462884488332758149896988654038037889676799907561496400717460081109$
 $2945356635118795824799369716742109*e^{(369*I*c)} + 97521033944404931875728231$
 $176351778667322317559445794638327926463508504100491730029590427543314484853$
 $2459919875479817581584*e^{(368*I*c)} + 16110925414000605259548593752641941783$
 $476434718370782014462624356151429445873378335135860227298495233584364935860$
 $42252995608*e^{(367*I*c)} + 2636662410430799340447522284778244283740751068658$
 $140726576446671207798325606832295937705061686297930296382338892574900819440$
 $*e^{(366*I*c)} + 427482690772059175252671133682087150084434564792238547153435$
 $9333606189571832444641364132893108663576205133870672156264164115*e^{(365*I*c$
 $) + 68664253375186682626626937509089569657329245781421816306221578028998808$
 $74681551031136314064199948604001529894566235238597088*e^{(364*I*c)} + 1092721$
 $060347354481027979234784453607458889680623004111008954473160586314610418173$

9039426674855453466097402330688331845602302*e^(363*I*c) + 17229502824367647
334400721998417596703948657394738805209391636597370380572398964715080095366
818322029152193635869784095333760*e^(362*I*c) + 269177947010866150978901202
368905011051467999960217751957108662262286389847034568326941532306116072634
44183501026198563419616*e^(361*I*c) + 4167044037539054364341821934227174804
003507149011908058528152249818881837590690036870123453130463316344631994513
0196476913600*e^(360*I*c) + 63923019433761989090614801288635098123199445102
303122616544648208998767803944455777042886552738499747183713136069104651812
215*e^(359*I*c) + 971730550247426800586167224613688926611412955402634930130
32746083536157324268333390400308958318370219154887169702257444756176*e^(358
*I*c) + 1463904484563511812182373827403741241916648199977469880765983918627
33629670142241546375533903130605297580105675355629160198162*e^(357*I*c) + 2
185631666596493122474836409562721492124991153838287710296542833639725851180
90479413696638108156385244646591328454425745117584*e^(356*I*c) + 3234131780
148410037141511382460791525763609760350584578904099377387231715370365730436
81997163745313602400139153046673668433091*e^(355*I*c) + 4743230435631005423
773386299319662481291759769823324460180560093911540204388959031408229677694
94019446166779954024655344116288*e^(354*I*c) + 6895184493287935599032604181
499741902535783400588950355896064682446805915561181703040050375636698800579
08765898949268614772285*e^(353*I*c) + 9935556536495211272264439608202336493
864885100818925458667004440966615827904412418308556095770620395556250909433
32264901780720*e^(352*I*c) + 1419164481422176573858234013898999628822322333
309573730716310743838935832201454293617293175086458389621425307051750612129
761498*e^(351*I*c) + 200949611009268773815278208568373722272796882429905873
9215446083499351467625334449670757764066690656150949014944043994822823920*e
^(350*I*c) + 28208192985622159591075298072896284496213867989894363693931160
69894018781201000275633104498398959346631795568022519974400130281*e^(349*I*
c) + 3925697658415778352768103942856011840211642769621717217996614398473887
186074391482638547212826538270453912634540299792270321024*e^(348*I*c) + 541
666280405243634958559598281835795386625846164435401820515891774257642534436
4964596750653177677803492186817305171175032011500*e^(347*I*c) + 74103726128
911522243646332961280439716571932803277543043822356727737814030238141276103
55562505271969045177704726054907145784960*e^(346*I*c) + 1005220952436958182
758815498534554967803144574449998520825938543560974027240030145424687204177
5159838468077381562338745636398374*e^(345*I*c) + 13521230411945436915558854
706851796541567399811656870021567352975326815467817846533289123871696056195
231696146162720992221760992*e^(344*I*c) + 180353273381774554711775685948516
829779783464497771935720876885103924268845192729915608513263938522419614700
40819793627127923997*e^(343*I*c) + 2385639856556280203069527817421264083328
215417400645945829264406094792490731373592156169015393990601751864718249161
6573724049744*e^(342*I*c) + 31295263688189838313772775873307260334117227258
629501992358695636092662866062819845689064235813622974150120668921391878398
978380*e^(341*I*c) + 407159889637019189500203483367364234205133113590104852
46919074652883393970805374470830156229705647312265477584256027212762941040*
e^(340*I*c) + 5253922334674077114258709237025706953606031964443950166761048

2767955800276052892432152798814607975110366224945081428121888473324*e^(339*I*c) + 67244087969080703823703257199663047606890610482090494619492802935130
 215979819469966383336788693900139115594646893784095418472336*e^(338*I*c) +
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 28198719872744047767783667325326289221881974987582215*e^(337*I*c) + 1075047
 374065769161234803991697596333283214074194005100174988498305986215654282665
 46315933920821527544726380201659114903834605888*e^(336*I*c) + 1342977420234
 794299046296161045596100960747587210680227044689380630170596880233634364589
 71534964665036319889119229809973806909680*e^(335*I*c) + 1664323329225891951
 305583292663987533898239557375985275560960930624735597695457723219789693189
 04192572733997888230986469005970880*e^(334*I*c) + 2046222955357291095198299
 167898672253194297051625600826488403949654231128093365919212903093923962638
 34674368977840527147037426908*e^(333*I*c) + 2495930722825658663983899515096
 192026826344554871287146314618917708232013675276457937702037887846773439349
 71424317987895255031936*e^(332*I*c) + 3020606380308684634611394422793604997
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 73712322595776808*e^(331*I*c) + 3627063075638432311356991574185107324524206
 140136241688793121876452334501539279757933268347807413912034301530937126353
 55523960320*e^(330*I*c) + 4321478564640869380238115618086785895947020479046
 742822979596588001709844567990677518780448066190124526368913507316182785456
 90160*e^(329*I*c) + 510907615111345074526238461471371414502757224443163855
 31648429230851686635827717488464500331623385777400744950538410637735936000*
 e^(328*I*c) + 5993784847717334748093761424015548502070649721181375729492575
 03651444541939309025276896049622515630263162184526394317285457368300*e^(327
 *I*c) + 6977891069259246148167137476846827850836598190279522444470433558697
 41368500452561164024636073401929693105801105522738405349028160*e^(326*I*c)
 + 8061696713276255324245753400897757336819949915766744469223540997146151920
 85443245663852257001115644286660304979476023966071898200*e^(325*I*c) + 9243
 200528675225840357774957610723512225344207848619600018210205094681463567564
 33795246446491396141583854513687104334429566707520*e^(324*I*c) + 1051782100
 428834371944508170051219187116816349766953322637182149610875004223784784183
 284961906494422955462431208645690802526770780*e^(323*I*c) + 118781794307939
 031610880232479811012902082278266008724859936764348120020604682216614428542
 5922229375413676535071141005286431481600*e^(322*I*c) + 13313961146267230358
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 907665322758017205667758745140206923435528368748965961391376195914077333973
 6014790081814516625224440*e^(319*I*c) + 17926490780896322989369457284819693
 349643915975062850884883506229372525334209808031443164317014521905227161247
 97875257437516360640*e^(318*I*c) + 1950286550780181919244992961204487010056
 460362845218501674423766266321558791436917317878702232679213868287926294665
 202769722927380*e^(317*I*c) + 210614190346834430711254976120248454340279425
 235248219981741042486967726271509828843764651868348794546277422365647134589

9082156800*e^(316*I*c) + 22577262191038562868128330126815737654962622414206
129320761431511719608545541241446990230098420805151579235293571898699435159
91200*e^(315*I*c) + 2402464595569686086120001803034211056739445588621946141
384106162886246161815149763025030834875234067267774023433418269982431265280
*e^(314*I*c) + 253776641546503033081547174669298859606991189469722505292832
0452542175587154848096483331209807430113943015398362669673337957755720*e^(3
13*I*c) + 26611006479757835838282351392014419301783966433834239035838625472
55880772382049201015537214900832745601519737141849802506685264000*e^(312*I*
c) + 2770073207150768645597507281382065497924968466054527414122339827333783
770068305883487309979315983718403740872884345746380680204260*e^(311*I*c) +
286250312632046179777066778072564418499125562317462617567905067210084898811
9391841466573417019247590580735265143427289340450811200*e^(310*I*c) + 29364
942143518684987032394554267711043448273062675589165508774672324551532861405
21089582733932202553130712723836983468866230908800*e^(309*I*c) + 2990498949
622543608538129380283866335190087115124858818143787619186957111903765723974
899651518555144924290346242595167274383008960*e^(308*I*c) + 302337164350822
502717560317521295321902248504585073184530751900827738515473146121338803557
9159917590062343527464977286601165100620*e^(307*I*c) + 30344083555309570757
877317453225679816846165501628454732576796742802169473567857838432056042023
07836897073595410412575660465787520*e^(306*I*c) + 3023371643508225027175603
175212953219022485045850731845307519008277385154731461213388035579159917590
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386633519008711512485881814378761918695711190376572397489965151855514492429
0346242595167274383008960*e^(304*I*c) + 29364942143518684987032394554267711
043448273062675589165508774672324551532861405210895827339322025531307127238
36983468866230908800*e^(303*I*c) + 2862503126320461797770667780725644184991
255623174626175679050672100848988119391841466573417019247590580735265143427
289340450811200*e^(302*I*c) + 277007320715076864559750728138206549792496846
605452741412233982733378377006830588348730997931598371840374087288434574638
0680204260*e^(301*I*c) + 26611006479757835838282351392014419301783966433834
239035838625472558807723820492010155372149008327456015197371418498025066852
64000*e^(300*I*c) + 2537766415465030330815471746692988596069911894697225052
928320452542175587154848096483331209807430113943015398362669673337957755720
*e^(299*I*c) + 240246459556968608612000180303421105673944558862194614138410
6162886246161815149763025030834875234067267774023433418269982431265280*e^(2
98*I*c) + 22577262191038562868128330126815737654962622414206129320761431511
71960854554124144699023009842080515157923529357189869943515991200*e^(297*I*
c) + 2106141903468344307112549761202484543402794252352482199817410424869677
262715098288437646518683487945462774223656471345899082156800*e^(296*I*c) +
195028655078018191924499296120448701005646036284521850167442376626632155879
1436917317878702232679213868287926294665202769722927380*e^(295*I*c) + 17926
490780896322989369457284819693349643915975062850884883506229372525334209808
03144316431701452190522716124797875257437516360640*e^(294*I*c) + 1635569744
641421900657886381289076653227580172056677587451402069234355283687489659613
913761959140773339736014790081814516625224440*e^(293*I*c) + 148118711708924

666246695569496567785552473031326055269082160265717621873742624552279532989
1464091005878304304075953693546767206080*e^(292*I*c) + 13313961146267230358
024621235315820503397499960141524528353059563674253702227586217539224587275
24856072950880960657564720475838500*e^(291*I*c) + 1187817943079390316108802
324798110129020822782660087248599367643481200206046822166144285425922229375
413676535071141005286431481600*e^(290*I*c) + 105178210042883437194450817005
121918711681634976695332263718214961087500422378478418328496190649442295546
2431208645690802526770780*e^(289*I*c) + 92432005286752258403577749576107235
122253442078486196000182102050946814635675643379524644649139614158385451368
7104334429566707520*e^(288*I*c) + 80616967132762553242457534008977573368199
499157667444692235409971461519208544324566385225700111564428666030497947602
3966071898200*e^(287*I*c) + 69778910692592461481671374768468278508365981902
795224444704335586974136850045256116402463607340192969310580110552273840534
9028160*e^(286*I*c) + 59937848477173347480937614240155485020706497211813757
294925750365144454193930902527689604962251563026316218452639431728545736830
0*e^(285*I*c) + 5109076151113450745262384614713714145027572244431638553164
8429230851686635827717488464500331623385777400744950538410637735936000*e^(2
84*I*c) + 43214785646408693802381156180867858959470204790467428229795965880
0170984456799067751878044806619012452636891350731618278545690160*e^(283*I*c
) + 36270630756384323113569915741851073245242061401362416887931218764523345
0153927975793326834780741391203430153093712635355523960320*e^(282*I*c) + 30
206063803086846346113944227936049971890691748252489448335622019613837705082
8911383056860425370161157201493696073712322595776808*e^(281*I*c) + 24959307
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3770203788784677343934971424317987895255031936*e^(280*I*c) + 20462229553572
910951982991678986722531942970516256008264884039496542311280933659192129030
9392396263834674368977840527147037426908*e^(279*I*c) + 16643233292258919513
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4192572733997888230986469005970880*e^(278*I*c) + 13429774202347942990462961
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746201851666299600392199418764750086828198719872744047767783667325326289221
881974987582215*e^(275*I*c) + 672440879690807038237032571996630476068906104
820904946194928029351302159798194699663833367886939001391155946468937840954
18472336*e^(274*I*c) + 5253922334674077114258709237025706953606031964443950
166761048276795580027605289243215279881460797511036622494508142812188847332
4*e^(273*I*c) + 40715988963701918950020348336736423420513311359010485246919
074652883393970805374470830156229705647312265477584256027212762941040*e^(27
2*I*c) + 312952636881898383137727758733072603341172272586295019923586956360
92662866062819845689064235813622974150120668921391878398978380*e^(271*I*c)
+ 2385639856556280203069527817421264083328215417400645945829264406094792490
7313735921561690153939906017518647182491616573724049744*e^(270*I*c) + 18035
327338177455471177568594851682977978346449777193572087688510392426884519272

991560851326393852241961470040819793627127923997*e^(269*I*c) + 135212304119
454369155588547068517965415673998116568700215673529753268154678178465332891
23871696056195231696146162720992221760992*e^(268*I*c) + 1005220952436958182
758815498534554967803144574449998520825938543560974027240030145424687204177
5159838468077381562338745636398374*e^(267*I*c) + 74103726128911522243646332
961280439716571932803277543043822356727737814030238141276103555625052719690
45177704726054907145784960*e^(266*I*c) + 5416662804052436349585595982818357
953866258461644354018205158917742576425344364964596750653177677803492186817
305171175032011500*e^(265*I*c) + 392569765841577835276810394285601184021164
276962171721799661439847388718607439148263854721282653827045391263454029979
2270321024*e^(264*I*c) + 28208192985622159591075298072896284496213867989894
363693931160698940187812010002756331044983989593466317955680225199744001302
81*e^(263*I*c) + 2009496110092687738152782085683737222727968824299058739215
446083499351467625334449670757764066690656150949014944043994822823920*e^(26
2*I*c) + 14191644814221765738582340138989996288223223330957373071631074383
8935832201454293617293175086458389621425307051750612129761498*e^(261*I*c) +
99355565364952112722644396082023364938648851008189254586670044409666158279
0441241830855609577062039555625090943332264901780720*e^(260*I*c) + 68951844
932879355990326041814997419025357834005889503558960646824468059155611817030
4005037563669880057908765898949268614772285*e^(259*I*c) + 47432304356310054
237733862993196624812917597698233244601805600939115402043889590314082296776
9494019446166779954024655344116288*e^(258*I*c) + 32341317801484100371415113
824607915257636097603505845789040993773872317153703657304368199716374531360
2400139153046673668433091*e^(257*I*c) + 21856316665964931224748364095627214
921249911538382877102965428336397258511809047941369663810815638524464659132
8454425745117584*e^(256*I*c) + 14639044845635118121823738274037412419166481
999774698807659839186273362967014224154637553390313060529758010567535562916
0198162*e^(255*I*c) + 97173055024742680058616722461368892661141295540263493
013032746083536157324268333390400308958318370219154887169702257444756176*e^
(254*I*c) + 639230194337619890906148012886350981231994451023031226165446482
0899876780394445777042886552738499747183713136069104651812215*e^(253*I*c)
+ 4167044037539054364341821934227174804003507149011908058528152249818881837
5906900368701234531304633163446319945130196476913600*e^(252*I*c) + 26917794
701086615097890120236890501105146799996021775195710866226228638984703456832
694153230611607263444183501026198563419616*e^(251*I*c) + 172295028243676473
344007219984175967039486573947388052093916365973703805723989647150800953668
18322029152193635869784095333760*e^(250*I*c) + 1092721060347354481027979234
784453607458889680623004111008954473160586314610418173903942667485545346609
7402330688331845602302*e^(249*I*c) + 68664253375186682626626937509089569657
329245781421816306221578028998808746815510311363140641999486040015298945662
35238597088*e^(248*I*c) + 4274826907720591752526711336820871500844345647922
385471534359333606189571832444641364132893108663576205133870672156264164115
*e^(247*I*c) + 263666241043079934044752228477824428374075106865814072657644
6671207798325606832295937705061686297930296382338892574900819440*e^(246*I*c
) + 16110925414000605259548593752641941783476434718370782014462624356151429

44587337833513586022729849523358436493586042252995608*e^(245*I*c) + 9752103
394440493187572823117635177866732231755944579463832792646350850410049173002
95904275433144848532459919875479817581584*e^(244*I*c) + 5847495736823045861
793846288448833275814989698865403803788967679990756149640071746008110929453
56635118795824799369716742109*e^(243*I*c) + 3473100538109352904194555605559
573141295692107357459832343696599764133747740780001730700752486545249171791
28950507443058208*e^(242*I*c) + 2043255572651860007674027102308478964597615
839227636982354332128333130777830410400746693790173948367615396490816906308
11665*e^(241*I*c) + 1190605918496605468347656932276764490675841482488826784
47504826077236333444513454095126668750057295811191643356908972191440*e^(240
*I*c) + 6871246601598564151246858617365974773487959171009835465278612493602
3073943141049573606648563005359411712764895683903806088*e^(239*I*c) + 39274
200414329861169397944516225001081227433398585007206399231211907157795359719
648241598754266579840244551491476467899952*e^(238*I*c) + 222313411318015353
454063990377216868402083979419525801355846459667467366567162715548264762829
91066076564921432614339399735*e^(237*I*c) + 1246214044053725808492859670987
206685775707094312486855450094815475686345430803292540834031123785001781470
7896986969086816*e^(236*I*c) + 69178389452148442784933304593613949233723338
536198796373726731849428597124310663457268704220998931248907776780373699881
50*e^(235*I*c) + 3802604996705891106964620633848964807037098854510182263243
030597295630760353597531974324752266389193185760878274188013440*e^(234*I*c)
+ 206969828950086064346166576237380795751301942404117887190496055182941244
9344722432125679417958403007551179298315947373776*e^(233*I*c) + 11153982855
456015505333280456001843179938993592179967298183407042218011956674104858469
96179056733558512452238583160792512*e^(232*I*c) + 5951576155004315149474792
823365470538279260879164252634971877570294133854718354341982468070962145368
95441388306027237899*e^(231*I*c) + 3144090358082258615655954369383549454454
739910431297220467472288130309252049685034185668188386118667098070407934953
64496*e^(230*I*c) + 1644375006769068927413232601543942785039545619360201335
96581806449357240277349447927334517105809995300093549279931273178*e^(229*I*
c) + 8513953323478645577958995946490063776073572970562122138000583720836915
7794673049675428799817875431430246332625899630160*e^(228*I*c) + 43638300075
815171025946211464465689618965773488660985857945657479854085108851857911222
911989837615452608512356008400295*e^(227*I*c) + 221407350017086032709151807
692413916620355787559039791489092136038225547927491835171602555719158753564
39553717130797888*e^(226*I*c) + 1111949964536320108088106282488633849242537
565844897793553584634929042582157038309042541141852151667037137204520656834
5*e^(225*I*c) + 55274988490311783558612300093266682839262900821584671180006
98502719379939045918344222192742145711257028040974074674736*e^(224*I*c) + 2
719589283483743926040805101080341921244530311254607250929192773909331523226
635035815672862569296693711643521070331394*e^(223*I*c) + 132431132498402742
835522293814768237867286070881716174144874968959358802086084750870370232532
0304649883120684987556400*e^(222*I*c) + 63821889291453374150580639966248855
606678360049609187640837497487744897177803607499624558112428346043806518207
1976085*e^(221*I*c) + 30438447110681333601028416012390637043388882849062742

2652551236966790916174520857759143930140187173492394981908258944*e^(220*I*c)
) + 14365768713804479694294711970425953845881819942351682467458629369105611
9209866358123637772245409530799230553767222252*e^(219*I*c) + 67091706130529
669125019899210021576580237843462229535386295087076189297849995931360645605
292130961496106707521506432*e^(218*I*c) + 310043192060694170770693631414234
874318280090981847446356786522841774394649416518125645191449180031741080776
34846014*e^(217*I*c) + 1417648365287570495720201334324111790436997779665384
9959902524421980635189011634815653279605497783382888932766730080*e^(216*I*c
) + 64133818958559251847582314510625563803285949385110065770152181197865363
90213057284018202201631094434819584025113465*e^(215*I*c) + 2870496131412314
451834674715353589439553294430808531933466086288543709246230769151392180699
413405623017247753532944*e^(214*I*c) + 127103308293804895020136055483127034
266234399127750461234230036602504674174285658044528940178665631168585902308
4716*e^(213*I*c) + 55675638871118234034102619273421954611365176831738053900
5893679049394714017063698565272728813669054779077208977840*e^(212*I*c) + 24
124602128244006179290831778303287619480159713320605209128699704372914534575
5805710081489006741839439573984832678*e^(211*I*c) + 10339975546725743648984
783764075375471820439447305579500146760432641987655555687382953173721109611
5196005647730480*e^(210*I*c) + 43834972142919377685378692233021063744554033
100928502737480438978976746989895784070951905237783490374305934542955*e^(20
9*I*c) + 183798070840033597660276492176211441160917355722166207888615358034
49702273802588359076704241840733513439114113248*e^(208*I*c) + 7621788791912
047062038840917799374600428892258194367636682944356096681400246312138001769
285020661445991073249416*e^(207*I*c) + 312568349317870174347970475030749017
866629215072017936360433511352862332968460634318554075601993566214826786396
8*e^(206*I*c) + 12675970172948129134001462760421269299864802928701903991075
54311079964227280196522475370108738477856311765699610*e^(205*I*c) + 5083245
993010854601669786296830326614276544740829390480979396383915672987957883894
33842285751054665210868287680*e^(204*I*c) + 2015579474247940980267724784620
408833958675125629308629437535686900840155855980101547815486252394095819073
97500*e^(203*I*c) + 7901914955876656925478398848723238835290914498274717185
6772463223808993367091503402876467270176124342699654400*e^(202*I*c) + 30627
581054221957378390547289277609129572813931082733520247387226000020043538279
468776707958420892547870128680*e^(201*I*c) + 117358569262451184931130910025
016040323418769859990828235206722415302001882238263929823021940846675384886
65600*e^(200*I*c) + 4445412259295474625067659514198312966015416299968930393
345630345914109720740573618884980520010028451496996210*e^(199*I*c) + 166447
503438721180939491774350293763897857493775476476398783587241044993069013157
2904279995484581013965001440*e^(198*I*c) + 61600311602297958493712570175788
721299835430098999136262803886109391456133207119190971494942658793691030330
0*e^(197*I*c) + 22532053259322065776794110928951624899979452101556413498282
7241710019675486694499689312466561907212627820000*e^(196*I*c) + 81452081413
829111828875417564250054846037693312811480492909160758195989155768107022568
350953861815940704090*e^(195*I*c) + 290976510612474534066475697818369100621
65559852359052804259154165687125428752562385492373749486351714453120*e^(194

$*I*c) + 1027160253020288900249781351684945259097151280952906066519730109705$
 $2210064576088348023234671975463677418470*e^{(193*I*c)} + 35827180021632960614$
 $145367037151098971071982527392845461493431023484561240896574285949464386608$
 $59773886240*e^{(192*I*c)} + 1234668041892409978780018081755440216012582476396$
 $941937965899631953079203974222138794604328498972144766900*e^{(191*I*c)} + 420$
 $358024835146798583611210145942154684437949365647899088372524802156222884839$
 $580011688655664280691773600*e^{(190*I*c)} + 141379938253556843280565505807403$
 $304130606130725434751745794079833141361748917639986145377066437210546190*e^{(189*I*c)}$
 $+ 469702247271172818264540450180706705225597566275803477845353200$
 $14963482632359729444541885102274546002560*e^{(188*I*c)} + 1541311121148602393$
 $729497082079737671608134478816338654352242193973750796212585498188187916834$
 $8260330000*e^{(187*I*c)} + 49952419562791381802051867444016880243882721139212$
 $55663734956946927571305533146776898787878059685108480*e^{(186*I*c)} + 1598771$
 $101058192692270528999677444742685631006232456185844925220144002305878120380$
 $828483988663574829100*e^{(185*I*c)} + 505293663123015258878483025738812813203$
 $397766845340065381261016353419722382620393032535960660921950400*e^{(184*I*c)}$
 $+ 157685845528850918721462877864435090257583149415561323427386562894447598$
 $277935629800939237175625149830*e^{(183*I*c)} + 485842581531402804473148368687$
 $72131390195412419046732778458706015096881437076337910793584122475073760*e^{(182*I*c)}$
 $+ 1477795509661712899871274518207149536217650697318308165023360527$
 $4051677624970464340242755840025673760*e^{(181*I*c)} + 44372109178431823477643$
 $495444439046990200565950694708471936170921147140776330772349728253512269793$
 $60*e^{(180*I*c)} + 1315052120930692122102297105327622842335870743428530891072$
 $983535862280094446607723473800477453914130*e^{(179*I*c)} + 384655842080666274$
 $454063078784837174998949052500975322162003392549953413592461519365177908682$
 $078400*e^{(178*I*c)} + 111034148797008819443143895644469242295049867464313710$
 $969257619338899133799285616020069872611710850*e^{(177*I*c)} + 316266446747255$
 $477311767956875276535713059699859236883921121649155532425732694909089895702$
 $48533280*e^{(176*I*c)} + 8888295028751024667044203837607976101480053134418614$
 $474620767522824868911959884352666444917404000*e^{(175*I*c)} + 246438219080743$
 $960907977422685567962936788570977643587663085171625396269619234170623919287$
 $8728160*e^{(174*I*c)} + 67402553054313300889484577523662523745074311447354453$
 $7818170447134607102575676676056675328961590*e^{(173*I*c)} + 18183466140617790$
 $131533012967714538116644918841319414116934435475492096903495261037894528225$
 $7600*e^{(172*I*c)} + 48379489756434099843857791816589379406815042609340378747$
 $586437145781646245422045101230417309900*e^{(171*I*c)} + 126934969329649205650$
 $73673637181280088548682508880255337280065006566138696041797353216584528640*$
 $e^{(170*I*c)} + 3283874760555818676726309480306734420155098583948074469014168$
 $171874442170109648521627538755920*e^{(169*I*c)} + 837579206923411932458786486$
 $765373533946545239708990769488724813982189165104589895518909256320*e^{(168*I$
 $*c)} + 210594301385648471184329078880317504953361839954159427434009884661777$
 $259752542647709150036990*e^{(167*I*c)} + 521909122076618242158122718542697480$
 $71292843243227894769229690720010547141334131610989636000*e^{(166*I*c)} + 1274$
 $721961650332054135634306256284736860162214085678602544581453203790411152324$
 $2298235713300*e^{(165*I*c)} + 30679742964317473641981596239624636716170064196$

26851426148418602934852907379021659761911840*e^(164*I*c) + 7275210107183942
29291774073844694255798738667067535379759732795567942578751384250780476310*
e^(163*I*c) + 1699563279699297677739020966526292532837045054771275445565344
17376686540936706073847337600*e^(162*I*c) + 3910803125560180947653753536961
1844440844903751605645023514572352045248104262933598850730*e^(161*I*c) + 88
627521427569572856813408857649045979353495693553218156477211725371591864914
71311666400*e^(160*I*c) + 1977792980665818135651300094326239158605448870806
970860577325385028609983034534672318500*e^(159*I*c) + 434546676780280045346
344498763892540797175105756827515509297024187660299345484920192480*e^(158*I
*c) + 939869153130681791490836060656814827808360605105301546184869498394671
31378859885998210*e^(157*I*c) + 2000800680303004713729327825032159711354071
6201983333126349281186679153199068045257216*e^(156*I*c) + 41915425006568261
48093339414544159143964478472492315931809171859902114109005939942952*e^(155
*I*c) + 8639799336223303495562968200283955131987080649405057021260686529368
00794826651264256*e^(154*I*c) + 1751931705006183002415156323819122851577900
97816049220671217212220015297133400636060*e^(153*I*c) + 3494107161327670464
9477943043339450201504075335160361865916029213860778606230624960*e^(152*I*c
) + 68529932231457366873288853116177954355929408414398663510796556523128947
21972796266*e^(151*I*c) + 1321498055271300851429993866631619874424534425188
183592049727687571032156435077280*e^(150*I*c) + 250501028608928332469340456
829902067712233644464602753159945727868485722395506952*e^(149*I*c) + 466682
23548266017806854592468100570289355960869613650856575756758180182223308768*
e^(148*I*c) + 8543013441126212334833540665069621472479085838041360564550722
036723654297540205*e^(147*I*c) + 153633323844492758353273455601649467167491
6578907116984548489078241693926940560*e^(146*I*c) + 27136120750326657073448
6517077181014801775322183181055638619257836143271472358*e^(145*I*c) + 47065
044611135158108487353367484243102698248838312635876283099427442745866704*e^
(144*I*c) + 801372958079075243436196494576154376146952079121074697267587048
1058674277844*e^(143*I*c) + 13392143742542455535648844068019453533850002540
30655765953770237607180089968*e^(142*I*c) + 2196012813395155615002614788441
90024870555261281946058839614044697037963695*e^(141*I*c) + 3532444720677901
8115378052820789411687581004582367431006205879633729015200*e^(140*I*c) + 55
72551157328671121016216416307596161861955969011697222340926210112854418*e^(
139*I*c) + 8618848510949919087642468054746724286037573154844539747136128122
15428992*e^(138*I*c) + 1306576602265604193351214343899389618845954340699848
24307149332131747540*e^(137*I*c) + 1940797921559456659353500810330325525774
5408070082431338945184797463936*e^(136*I*c) + 28239051519365866783825257065
64457280290098698638597987628380245881715*e^(135*I*c) + 4023496922661211589
34003582839428785116904903936409545602519219664720*e^(134*I*c) + 5611708107
6341175384087570185188538660375932013674735519055227368366*e^(133*I*c) + 76
59010520187549651777118357676871927081898989131125755798204236112*e^(132*I*
c) + 1022536437468296737293065862705246449693687415559865844306888705423*e^
(131*I*c) + 133490210052026183779673313868332303530332906163247194627808410
304*e^(130*I*c) + 170338860273906157410409777216555416656121622754850285843
10890417*e^(129*I*c) + 2123702969188871318266718781223927067839949015727293

$884065388080 \cdot e^{(128 \cdot I \cdot c)} + 258585348715977270155829115684193411072034541491$
 $364393985491350 \cdot e^{(127 \cdot I \cdot c)} + 307362174043210099652310374196630539628810352$
 $81709221697785072 \cdot e^{(126 \cdot I \cdot c)} + 3564764890628724017088487996688178929195787$
 $613958545474804845 \cdot e^{(125 \cdot I \cdot c)} + 403212225957798188840846139960995624144491$
 $271694336796459584 \cdot e^{(124 \cdot I \cdot c)} + 444567081752588210244009462105350045237757$
 $22190977468484496 \cdot e^{(123 \cdot I \cdot c)} + 4775398607100853263534207733818266777478693$
 $412738731031680 \cdot e^{(122 \cdot I \cdot c)} + 499467506558531733671585862910572702811545035$
 $730398749530 \cdot e^{(121 \cdot I \cdot c)} + 508363695081710994370193486108473919467361851080$
 $17183136 \cdot e^{(120 \cdot I \cdot c)} + 5032024903401451824074213943766011922026507006311982$
 $753 \cdot e^{(119 \cdot I \cdot c)} + 484093410240488718655917025303662581091659126182344528 \cdot e^{(118 \cdot I \cdot c)}$
 $+ 45230940039830738332025694784646206844854827698075736 \cdot e^{(117 \cdot I \cdot c)}$
 $+ 4101545439937195793959956708442496709433800261224880 \cdot e^{(116 \cdot I \cdot c)} + 360$
 $688613036389349413809780004559963548775423325255 \cdot e^{(115 \cdot I \cdot c)} + 307353665128$
 $30562160991166338490057308062762518496 \cdot e^{(114 \cdot I \cdot c)} + 2535667460650279776834$
 $561566186591213109251642859 \cdot e^{(113 \cdot I \cdot c)} + 202347509724462171313966643580234$
 $078508179838320 \cdot e^{(112 \cdot I \cdot c)} + 156039112776876070997216237717449330869205872$
 $72 \cdot e^{(111 \cdot I \cdot c)} + 1161581413733971751533622511909046917188768400 \cdot e^{(110 \cdot I \cdot c)}$
 $+ 83380839911837894453136303673785039051506805 \cdot e^{(109 \cdot I \cdot c)} + 5764601046563$
 $151304213854710715346838447392 \cdot e^{(108 \cdot I \cdot c)} + 383360155801054824529764688213$
 $114368047154 \cdot e^{(107 \cdot I \cdot c)} + 24489837337812338687718622491865013839488 \cdot e^{(106 \cdot I \cdot c)}$
 $+ 1500602747937397286405577818722691539392 \cdot e^{(105 \cdot I \cdot c)} + 880549275989$
 $41411145869950813388040256 \cdot e^{(104 \cdot I \cdot c)} + 4939666610818025798809586352543471$
 $345 \cdot e^{(103 \cdot I \cdot c)} + 264410375780310742518099326419685040 \cdot e^{(102 \cdot I \cdot c)} + 134772$
 $27799524701956579274210395326 \cdot e^{(101 \cdot I \cdot c)} + 6526502533432060474536205599938$
 $40 \cdot e^{(100 \cdot I \cdot c)} + 29952547749265499675257842032197 \cdot e^{(99 \cdot I \cdot c)} + 129914664599$
 $3240318167826532288 \cdot e^{(98 \cdot I \cdot c)} + 53090127264630963470039804475 \cdot e^{(97 \cdot I \cdot c)} +$
 $2037031259470368160131922320 \cdot e^{(96 \cdot I \cdot c)} + 73099207817335597247098038 \cdot e^{(95 \cdot I \cdot c)}$
 $+ 2442455629894502983849104 \cdot e^{(94 \cdot I \cdot c)} + 75599817092670157806639 \cdot e^{(93 \cdot I \cdot c)}$
 $+ 2154864144781257856128 \cdot e^{(92 \cdot I \cdot c)} + 56169444526926562260 \cdot e^{(91 \cdot I \cdot c)}$
 $+ 1327882849274858880 \cdot e^{(90 \cdot I \cdot c)} + 28186192554792138 \cdot e^{(89 \cdot I \cdot c)} + 5305636$
 $24556832 \cdot e^{(88 \cdot I \cdot c)} + 8718181624155 \cdot e^{(87 \cdot I \cdot c)} + 122503723056 \cdot e^{(86 \cdot I \cdot c)} +$
 $1431118260 \cdot e^{(85 \cdot I \cdot c)} + 13343760 \cdot e^{(84 \cdot I \cdot c)} + 93096 \cdot e^{(83 \cdot I \cdot c)} + 432 \cdot e^{(82 \cdot I \cdot c)}$
 $+ e^{(81 \cdot I \cdot c)}) \cdot \tan(1/4 \cdot d \cdot x + c) + 14 \cdot (26 \cdot a \cdot e^{(1055/2 \cdot I \cdot c)} + 10504 \cdot a \cdot e^{(1053/2 \cdot I \cdot c)}$
 $+ 2116556 \cdot a \cdot e^{(1051/2 \cdot I \cdot c)} + 283618504 \cdot a \cdot e^{(1049/2 \cdot I \cdot c)} + 2843$
 $2755026 \cdot a \cdot e^{(1047/2 \cdot I \cdot c)} + 2274620402080 \cdot a \cdot e^{(1045/2 \cdot I \cdot c)} + 151262256738437$
 $\cdot a \cdot e^{(1043/2 \cdot I \cdot c)} + 8600339740311748 \cdot a \cdot e^{(1041/2 \cdot I \cdot c)} + 426791859620149322 \cdot$
 $a \cdot e^{(1039/2 \cdot I \cdot c)} + 18778841824143775343 \cdot a \cdot e^{(1037/2 \cdot I \cdot c)} + 7417642521312133$
 $32757 \cdot a \cdot e^{(1035/2 \cdot I \cdot c)} + 26568646854716407696970 \cdot a \cdot e^{(1033/2 \cdot I \cdot c)} + 8701231$
 $84837420284430070 \cdot a \cdot e^{(1031/2 \cdot I \cdot c)} + 26237560668658611978090055 \cdot a \cdot e^{(1029/2 \cdot I \cdot c)}$
 $\cdot I \cdot c) + 732777588085793048574300590 \cdot a \cdot e^{(1027/2 \cdot I \cdot c)} + 19052217324800742177$
 $269360704 \cdot a \cdot e^{(1025/2 \cdot I \cdot c)} + 463207034992633568486133297596 \cdot a \cdot e^{(1023/2 \cdot I \cdot c)}$
 $) + 10572019430266010048013097752934 \cdot a \cdot e^{(1021/2 \cdot I \cdot c)} + 2272984190957551165$
 $86566799957934 \cdot a \cdot e^{(1019/2 \cdot I \cdot c)} + 4617746868571980127702698250753676 \cdot a \cdot e^{(1017/2 \cdot I \cdot c)}$
 $+ 88891628244678704139848327305754038 \cdot a \cdot e^{(1015/2 \cdot I \cdot c)} + 1625446$
 $941911901543891066605266178710 \cdot a \cdot e^{(1013/2 \cdot I \cdot c)} + 2829755417150959822835916$

9068737992545*a*e^(1011/2*I*c) + 469985477889846368776571005635121060828*a*
e^(1009/2*I*c) + 7461019729104126380477146521488232920426*a*e^(1007/2*I*c)
+ 113407505114998207828286839959356818331515*a*e^(1005/2*I*c) + 16531325755
12243220555792873383292580108953*a*e^(1003/2*I*c) + 23143857771494702366699
332162923093803819912*a*e^(1001/2*I*c) + 3116155424202189605705498912105541
31541198338*a*e^(999/2*I*c) + 404025715184420615954226658807990587075260251
1*a*e^(997/2*I*c) + 50503221535858091096582689376221786704600600455*a*e^(99
5/2*I*c) + 609297036058359460142063362206138699889504487122*a*e^(993/2*I*c)
+ 7102120067046396299634193292688723536713017300128*a*e^(991/2*I*c) + 8006
0282862006487921860873353861357672073071040769*a*e^(989/2*I*c) + 8735992347
08764858055721340390736019595548391597091*a*e^(987/2*I*c) + 923519526684946
9317521124217042763676973766253104794*a*e^(985/2*I*c) + 9466079239161646653
4333151235841916067978700197242708*a*e^(983/2*I*c) + 9414916057875226479999
78018615368308830544085253128617*a*e^(981/2*I*c) + 909283230328133681623358
0227462577871197813963276075330*a*e^(979/2*I*c) + 8533279426386174534195194
1568633801044299209189190469846*a*e^(977/2*I*c) + 7786623913490808819340683
70430476103178748088552087631588*a*e^(975/2*I*c) + 691300935061032244604742
8627408811159158338885747490127762*a*e^(973/2*I*c) + 5974821903120267852023
5643789006093143612681508950204493778*a*e^(971/2*I*c) + 5029972852031920598
40013244325631734915939121404020646790260*a*e^(969/2*I*c) + 412687024698502
9536018107086136692505798915934457479856887816*a*e^(967/2*I*c) + 3301501862
7194726039854786048619784314920578570513381395990898*a*e^(965/2*I*c) + 2576
61195939290308606607761298064029010980446277923637727940695*a*e^(963/2*I*c)
+ 1962615229865836518270067732662567794203130442183114304875477510*a*e^(96
1/2*I*c) + 1459698826559022570795177455419267347804932266220959641813218845
8*a*e^(959/2*I*c) + 1060518982683337569105648975586886380808069152381085818
54868257941*a*e^(957/2*I*c) + 752970977560607563719398585207010863604321557
616289473063991767039*a*e^(955/2*I*c) + 52265240935076461936828392759689436
08626786700142487822871933708874*a*e^(953/2*I*c) + 354802086973620760088737
09776702294541967395718297832569250047180556*a*e^(951/2*I*c) + 235643271710
602776985976812503397918723360707843190238666454384153669*a*e^(949/2*I*c) +
1531689544688533128267835220826660575517763556541402057407422275055260*a*e
^(947/2*I*c) + 974717450252413924844218873977501756256764954078688783935818
7040912530*a*e^(945/2*I*c) + 6074619805356191958352949482025200643288016840
2592906594363059656788960*a*e^(943/2*I*c) + 3708743568261706225612110315792
88522536503760739536119117445103487930880*a*e^(941/2*I*c) + 221887070141102
1049789421436850301976335757207676690062759443158094615652*a*e^(939/2*I*c)
+ 1301248443153349237874624207920319343524763168297665877750094595075860429
8*a*e^(937/2*I*c) + 7482258059928548093324070479619691298514226456115682634
8717174043201630392*a*e^(935/2*I*c) + 4219552816550570709386119371835225871
92086313169152717515130361078410110468*a*e^(933/2*I*c) + 233439620049209021
6980577767310744650580417606589598674389591775624513302292*a*e^(931/2*I*c)
+ 1267262144544006762348670729430578970678220955845758628394837421693911754
2680*a*e^(929/2*I*c) + 6752240336121207913629144799891108270384536184910825
4368438686584600219974916*a*e^(927/2*I*c) + 3532004347423079615370447529901

24528653649931040810723777175782628615964698844*a*e^(925/2*I*c) + 181420178
 9663247503683990869126253542318071421878870447076671569392523282426716*a*e^(
 923/2*I*c) + 9152441985761079445443716091984800770861766098755365001578215
 027814025637822344*a*e^(921/2*I*c) + 45359519734612250739996296106148376154
 884994292039155402760649106566139277798596*a*e^(919/2*I*c) + 22088700947448
 5548410918212534797139518487225151752041095525819264180469636552620*a*e^(91
 7/2*I*c) + 1057133035608080334517467842102247001229173401193395800195028598
 793946241985524410*a*e^(915/2*I*c) + 49731519454378183332079785198122292839
 90687681266209867827402557241427416724759600*a*e^(913/2*I*c) + 230016401216
 5725331665169370568483737251327852083633199140761554418774422242128500*a*e
 ^ (911/2*I*c) + 104614252934905612199658590379588736548577356327149679159709
 209471984279805545840510*a*e^(909/2*I*c) + 46795660437124083156463767484415
 7127750964967487710258048711564849492193413275081370*a*e^(907/2*I*c) + 2059
 104851233963888842379525772936034391701199184168338071977072713080793446996
 296340*a*e^(905/2*I*c) + 89142102752459377470130588902599904067720257310317
 22326439774933310018480720586176220*a*e^(903/2*I*c) + 379743334997202596581
 50626055074672602084485890782692205932372642164257255828275277230*a*e^(901/
 2*I*c) + 159209767180955530325047227583177604233946016419587011038408978676
 950649660142022326320*a*e^(899/2*I*c) + 65703578670331707387227618346665553
 6833294806445692036603437064145864115975619273310760*a*e^(897/2*I*c) + 2669
 400972789686981296836875076250459931673689078037577064328933999433324588323
 107010000*a*e^(895/2*I*c) + 10678445536825411376746680463152055034872867036
 734705052278088324574859010216795309754920*a*e^(893/2*I*c) + 42066267367318
 216193842191416909738948525619885259401159592359828689296015791863825798480
 *a*e^(891/2*I*c) + 16321208300699726872915924654990666086737306928579994350
 9652022515969021706052860626138960*a*e^(889/2*I*c) + 6237666345675784717891
 57464627305169392418716892107406574086770229679140188972330140850540*a*e^(8
 87/2*I*c) + 234855692258423706200419810716707127388721793315640118242397689
 3834425480017846564439100360*a*e^(885/2*I*c) + 8712550834307422843463603955
 913018153198551629326082886421600824607707881453826119225268410*a*e^(883/2*
 I*c) + 31850013002449494191788674279414994671607865224244945000200665576261
 090031451234274836807440*a*e^(881/2*I*c) + 11474859600895439947056082052328
 2802972560754779166281576532046193721730617184676146208454500*a*e^(879/2*I*
 c) + 4074841744653947362116914863644020728687905524304051798805074217510942
 82407835858091343026670*a*e^(877/2*I*c) + 142643037425091059133790826183498
 9423471882981709863583159775371537718016932012468351480138570*a*e^(875/2*I*
 c) + 4922849139190761971912744518550936831750167555165468970439911091242197
 041305307652588826163080*a*e^(873/2*I*c) + 16751630043585648045466097105076
 233663413980413177353774456822305187643841236848558981932298420*a*e^(871/2*
 I*c) + 56210773708404693912307787410989229580969216199887991549658097359306
 716028570932883897745531110*a*e^(869/2*I*c) + 18601597745255025509215791744
 0324300818135019254799070536796872549263250091086090579550703698310*a*e^(86
 7/2*I*c) + 6071476558117846180525934068675294373925386810781848667804435379
 32100336168022295156652479524980*a*e^(865/2*I*c) + 195477130098723468429915
 7236502544878646869538838413631422449979548647041435372549308338323983400*a

$e^{(863/2*I*c)}$ + 6208666376133327662787020371327223738306905395229403481688
 606039769774018193877054980125219300650*a*e^(861/2*I*c) + 19455556288210142
 197792399701441683457177993888604394571936075981048677773457499402098681541
 345230*a*e^(859/2*I*c) + 60155208943075208721970485637459885199389358781088
 969898509767201637525536232949225931564636017060*a*e^(857/2*I*c) + 18353862
 986603064551692928939851624258351084780879082478093337125350402392953629920
 7100681758720560*a*e^(855/2*I*c) + 5526445943943309814046835890851146422809
 06553679728598394985334382381190569025961084792025986226970*a*e^(853/2*I*c)
 + 164235585052369408178165987548747970648759912590938897517892628070386544
 1617578142680863389145680280*a*e^(851/2*I*c) + 4817584949915740964852355271
 716409235848785641998844676677492209743884414375678304126912661883888620*a*
 e^(849/2*I*c) + 13949845316042293786528514094535620274951482636695793685790
 781893931237847288558891416636782386235120*a*e^(847/2*I*c) + 39877107005816
 808842474290443185467296349332073686639497504608960180007893671571725169560
 655925320000*a*e^(845/2*I*c) + 11254540176202178596753890614510262974982280
 4820131541462936025973893544847020246173693665097678958840*a*e^(843/2*I*c)
 + 3136293088020715462733207570165074527613810476767113929416368676209987438
 52945422691370541694930187760*a*e^(841/2*I*c) + 863027290376938136286146320
 587382966922778739244634235519200955649593051035611434988433819531197898760
 *a*e^(839/2*I*c) + 23452275720630232633764343055786061690911043360812978713
 56722744639904016570907272826407742463307924640*a*e^(837/2*I*c) + 629406122
 506786447764803532819614352598375975595100354714074485443115126708614333203
 4072457898773808110*a*e^(835/2*I*c) + 1668379197470897218586915631182682117
 0155297607333534880763775663291322286056634684196941640082885613980*a*e⁽⁸³
^{3/2*I*c)} + 4368256487624130087749635757075188322826448323418589792049592125
 1684596213376975639862720078190049382420*a*e^(831/2*I*c) + 1129801682493579
 003864946113022334253254841486610959361670814713456006638529110286397566551
 34829221364250*a*e^(829/2*I*c) + 288673978616124351576066193803621410940049
 987233564354069965128114229512608260120320153840315109513240510*a*e^{(827/2*}
^{I*c)} + 72870957701304930685145583004578625839578559495705560420492538784203
 9994625652269962227400682354338997620*a*e^(825/2*I*c) + 1817491183507803277
 646434154563247687916502767732579734578654547931596925444779173074576528904
 820222193680*a*e^(823/2*I*c) + 44790946053124559401311878991986757364901787
 90099745194145235324134619699772544528969129449757743039635770*a*e^{(821/2*I}
^{*c)} + 109077925513286012622236194966417553379047773698515875937376972639114
 18412361493183977361964833748319598140*a*e^(819/2*I*c) + 262506700598237870
 871324313040849655957123767320824379561378686494524463490057790192498722891
 46157988186820*a*e^(817/2*I*c) + 624350634603888862944275086050183454453771
 96593151678464550029733503722062639199339939077739315562684600360*a*e^{(815/}
^{2*I*c)} + 146767097463970982808607791792512370924531634905313051233506563629
 059084334858680054056774768242705313471500*a*e^(813/2*I*c) + 34101053275050
 840101051257257425643355058819225226351490993566391141835465779948294033981
 7517181660633925100*a*e^(811/2*I*c) + 7831995110286048303052651712417830735
 58871641563886451764214478656600184324031802959178677178450742159624260*a*e
^(809/2*I*c) + 177815160679273569318457630517100655459173088980146347083457

1569649456632925027058049883289543294859615339320*a*e^(807/2*I*c) + 3991010
 196204332937212586851998834375311877876034549525880121307974429565165949974
 007981491960685666506428548*a*e^(805/2*I*c) + 88560519659530348796623406573
 386029650837375976968689856569793878807694949548643042873094061596894334013
 44052*a*e^(803/2*I*c) + 194296978761766218912342065517275740088550706895032
 28700662784554532194008982832223012645019617538312082871128*a*e^(801/2*I*c)
 + 421488318183379605264415423924469270354566939628166561985944026101410544
 61594749815537318993350672298883213302*a*e^(799/2*I*c) + 904114465590476241
 075636388127901365070921139042817273413125290723804538206691540873902278005
 76401418085164068*a*e^(797/2*I*c) + 191780398312540108164969569134383375747
 089477085000852416545282627700304209125929751214708830470427357620162840*a*
 e^(795/2*I*c) + 40230206688627611091580537050825344058292667366383467679120
 9379817073708402847882776363322804524102236998117616*a*e^(793/2*I*c) + 8346
 255778624730272866697000320711631326880473631426951620586027757135789410782
 76791942737285481618912831112014*a*e^(791/2*I*c) + 171256256379489298835175
 825371058083911930675117962488080466572725027821268194300172115925962699426
 8353653902636*a*e^(789/2*I*c) + 3475684327725554198246437246074163598962204
 579122918009051534296803648946904541960047776709789822578169105209259*a*e^(
 787/2*I*c) + 69774539909700958938705179903395862097896076997538737829421306
 94945946528220744372046715941677441907103193732516*a*e^(785/2*I*c) + 138560
 661023300269915703514452458130971519949149654481078997705650476054273369480
 15872027856400037860001309803670*a*e^(783/2*I*c) + 272202840140473281110545
 726489135154639891847008248762990130210512503186300606320662319185591740786
 58567345880625*a*e^(781/2*I*c) + 529027743018574367549566488112897453499435
 43561147991932957809598337563243029219771808687832301138643124607095515*a*e
 ^((779/2*I*c) + 101723341642534027856325802326625923100834568123573784283611
 965294829026168598516825508102699558265012155505286390*a*e^(777/2*I*c) + 19
 352737493776155106975449584051738189540453017146222106374993722910424723376
 3126606164713214930951706536170129402*a*e^(775/2*I*c) + 3643065882595299330
 012120113496787036688688258721282165017591615610211270169829625182642712312
 66127473606437283673*a*e^(773/2*I*c) + 678605985108727421701882523302393284
 211249719647983387318924115049192814394269280038737908609793825310009937379
 942*a*e^(771/2*I*c) + 12508856412562750179042237925459335912703372679853519
 22729892360270373029610277920725182173946163411522304315334632*a*e^(769/2*I
 *c) + 228186549100043532565208157119295931459955274509410039814758576890137
 1932672555405076513220299715006918296079490268*a*e^(767/2*I*c) + 4119630675
 993972883163200200627180864136924478788743836907012989394926215330996549026
 090671132192170289614995953334*a*e^(765/2*I*c) + 73611505472290883580787692
 740396109097532014264372074343776475669425303448966351717988617006043301044
 16834668519430*a*e^(763/2*I*c) + 130189433375914762178356958680531819179445
 49900719484592250444702640838005937750892315190956996862238788704893903660*
 a*e^(761/2*I*c) + 227914492230573423183794384642924067618367063004688929366
 44593850736816226075816614000005128630118631272047930585354*a*e^(759/2*I*c)
 + 394964421757197854804800626261040141950619117362572826461311558661205389
 43291998823092670646932544497465667715800518*a*e^(757/2*I*c) + 677575411667

699551794684595940634031110056452951432934169471446823452063745500517009076
23571000874184381488493815415*a*e^(755/2*I*c) + 115078679074156921003896211
506744040757448129755518072610974597692499116476780735959201587866777427373
854139852271484*a*e^(753/2*I*c) + 19350570483018105418915972164288713013847
558252793401773242056315727571038403878051901634108322171753816041007162928
6*a*e^(751/2*I*c) + 3221652051451423635701184039210812231721036704288274349
03217367898408752791686796216445582208987707216661270752348189*a*e^(749/2*I
*c) + 531097086226002776844822503685026440629965503462454662271767664538175
076793501990496089430124065992327885079121429343*a*e^(747/2*I*c) + 86697045
893774080059910011005227233408633616681348521582337471409168038182080184861
5843383573129277135387631125832240*a*e^(745/2*I*c) + 1401505022864821759990
514911774358655464920980642918627032979165269601299928150161830021936830813
736871417390373928686*a*e^(743/2*I*c) + 22437274442601993561690753071153304
749782099134550029336700431895279027708625136468016040067582424263135720298
51100889*a*e^(741/2*I*c) + 355759052364744337578248508929766292922882754286
1566734110641737329597784391653357883472975676999315605445384252967857*a*e^
(739/2*I*c) + 5586988939674645914325363550716282684830193122934675954566650
601501265101993368547501997769261446740200272717454879998*a*e^(737/2*I*c) +
86908344287462640820635666870922769031917378894184093446251988265759445886
97061199305932440621485102366745761048030632*a*e^(735/2*I*c) + 133916171433
813415617973822475038584786946514869762411301721921550415535983934943380724
68333618785797627855047879535719*a*e^(733/2*I*c) + 204417835019961259801359
073735729598558718872777900564562643674240367953383314829824410987395778323
94644381899556974181*a*e^(731/2*I*c) + 309132980920478453984687001663630696
239627341410546638371233948289257131780166068100757728741033115735840681003
11403990*a*e^(729/2*I*c) + 463169064421787428773107530128748877142514289777
95879315358012090947020067282331495803931605151341983289375819303497428*a*e
(727/2*I*c) + 687588340824290493941730231472874639260477373072581343050011
94317495857981578577369701491983169829988876464082023881439*a*e^(725/2*I*c)
+ 101143876200907701458145589433650452267328419069640282011909099693348827
229087774814701746425164097849836694542125988386*a*e^(723/2*I*c) + 14743495
097122257892001302754560741045764055468918914770666990802436741262512060095
8820169170652789316898683463434699434*a*e^(721/2*I*c) + 2129800728872132484
276957411841611559855081082818965937685889049569831974956079164659144035890
16442699445092549476690660*a*e^(719/2*I*c) + 304918178534066785990007348109
251411831147410206280250705996608101203138903042698601813697013463781101658
444684909249658*a*e^(717/2*I*c) + 43267510031147823268192037944542330700966
435986817673117402364955963817104066125257700650919500562711379206820897059
0674*a*e^(715/2*I*c) + 6085600003789944462454360725994938388425387255926195
91400972596394327368638438650114993937818761497030717896138241363060*a*e^(7
13/2*I*c) + 848470498946001634567581956874393834016146789764834275392720475
291429146661138402672991032856375125997801887991514934480*a*e^(711/2*I*c) +
11727113126584939536498388369849154117904014614045634840053843288777020371
62967120070585970799957845874604153354692701754*a*e^(709/2*I*c) + 160692624
306626054554481901005007128427431243975148175968419123604564153261042308353

0227151796510775305152151669953445033*a*e^(707/2*I*c) + 2183136663673238874
306231186152025274688130245012528032646406775075995457404272096460063109834
340128997113442263319468362*a*e^(705/2*I*c) + 29408711824717505312385503328
846026959178235822249108880611909943048931093638636029968254017904185889712
74900767917143382*a*e^(703/2*I*c) + 392836097149201063465198049040617150440
014024676010743447318264062075917380066973102851810619473316703925248564260
1537883*a*e^(701/2*I*c) + 5203763588007696555866441944538215327243563612139
283244175384029625341239582454293417845888737748393246675398200066703617*a*
e^(699/2*I*c) + 68363653907343935490083740060526889623052085887268921617240
02543545109906070875455109463119424154633158689756128755804310*a*e^(697/2*I
*c) + 890769951839117842912980641402497841569803028507622901399151629297555
7588143549142778717881866319712550105543451994921740*a*e^(695/2*I*c) + 1151
250370118722932321754624292796931844868480402772030853088627243410611741428
7980876927354781721804664473558899761393515*a*e^(693/2*I*c) + 1475943096641
637266026391584135364038980566865549449523589098309458194274936068085056080
5353655895626485394213426659661080*a*e^(691/2*I*c) + 1877141777593854101016
581170965589125417404752437554585584113289037225874320065787936848261828833
4684615818699392418342286*a*e^(689/2*I*c) + 2368560957440153387573223776855
359788750496757000209731449894900700078096335324585104460952194609728104516
0731628195279464*a*e^(687/2*I*c) + 2965274439563358013315895980670116385427
02103751712672085186333216136474940528326162682504535882839974231851656862
7484956*a*e^(685/2*I*c) + 3683590220184064905914922714927576872979791050338
9287520100466864775463578466769724598377043872510103817701793820475775984*a
*e^(683/2*I*c) + 4540854188536614387278871721876377305403940060205798303633
3376816522774495334926169325754390780348094316862266665965991846*a*e^(681/2
*I*c) + 5555176982051654281141921884692657977308709975397899169675671615736
8950253678159527340671056699387281634743181637981363440*a*e^(679/2*I*c) + 6
745081347112670693549051856820239084922164027549460520665066790705151832929
4951998879978783463264636089137056491006523208*a*e^(677/2*I*c) + 8129071018
204356616701379173280267172954967390383067644635798771483933905417558570172
2606884268548187402687095553510812392*a*e^(675/2*I*c) + 9725126358288451269
473326158366739119204994534604768047558453841096717393945458322040125713163
4845269872208901982900506288*a*e^(673/2*I*c) + 1155013660100496806832702398
001686029727691282930709002561705121779982424918713245705795383463814149293
14168762720397958392*a*e^(671/2*I*c) + 136192832394833900835393035860717631
858240644738806126199965701046979626441751102715242872161933462105757019809
834502589368*a*e^(669/2*I*c) + 15945393607422544108744443544559732263801034
995914093882435502941504495185879537013708302342372597766205052298542547687
6440*a*e^(667/2*I*c) + 1853828765700121741147743144859216195375688381584788
22294899038884738131663676924882800088555461036440639784323290982435920*a*e
^(665/2*I*c) + 214041440126824842352708511077845618878866223151781361633546
874774038204026567692011971405221478790305862726998272063222200*a*e^(663/2*
I*c) + 24544913252943878657146028256914731662384082738982168509152746364888
1363528910037224877635235250886963822468275620946735000*a*e^(661/2*I*c) + 2
795780678217740015444298371203092765322605167049308307176674216212482251758

42302586295274927274086096056979473472729880620*a*e^(659/2*I*c) + 316348884
263616362042842543901080546281501211987656004780654651440103299682244533088
533089753616985689239381646035489221440*a*e^(657/2*I*c) + 35562792952360209
042496295254069656463922322436052266745359625063593797852563838274166718529
7998765569584449936989927995480*a*e^(655/2*I*c) + 3972259113259586183629325
664274897110088063706955062184972890026109827136705312561541269284123968426
45037163118148106920740*a*e^(653/2*I*c) + 440898149491503523444778198806793
839878322252525780377816191849083993038457480195337323463748937132292723543
647234218039660*a*e^(651/2*I*c) + 48634649377888960220864954194100002855346
932926715456497046092005153453977747649119560232954097527602192211291936153
6876440*a*e^(649/2*I*c) + 5332228934934298858594093264598308973341326947698
61149975266872019015402485363293405873999908400003099563986230923614600520*
a*e^(647/2*I*c) + 581134524659849570516520835583290845326108734568414348508
022104061332738062063508414909055483914854232496613990505843010500*a*e^(645
/2*I*c) + 62965030424428278136286812761655590978380494878441543704862783087
3075720560012090018464221573758434571727274802802603945840*a*e^(643/2*I*c)
+ 6783085540050749008177339337287136946439110689948051691473515175034050265
10420844066523647882858701963834358113089172938960*a*e^(641/2*I*c) + 726625
524054874396729548904429989754634649183846135021779065652886206176807006743
446378198756010809903052637351826187568320*a*e^(639/2*I*c) + 77410445211553
094712953191086231935146942887913541796258981191251639696468099085798266657
0814712400614658358562654106747680*a*e^(637/2*I*c) + 8202448213103733279998
304866566586510148506276324277415446009091322998816007490343410817448694018
86702449976206139706099120*a*e^(635/2*I*c) + 864551488026486190923176963138
034067155722686853321421184941164808358133963280958662401384573838526420349
485337999344825920*a*e^(633/2*I*c) + 90654338091137046511284880039597581167
13568478957413165441736279423496908180987797774108295321067258447012481189
9652530280*a*e^(631/2*I*c) + 9457615196590840744070207016357652066287072685
109508599181270440945775113511951117989021137933538349638796470403640972920
00*a*e^(629/2*I*c) + 981776163519459256143964209232624766016543826754463955
803919524083126774434994582010832915047435963630135877730601620142540*a*e^(
627/2*I*c) + 10141929688536566257914620127857582334930245710109349535376353
60512177986386038527036220871061980615293450139380863514373440*a*e^(625/2*I
*c) + 104265810628987872856563640672132080919144434896624853275387262442621
3544564508176781308633408517935011981967399688638658360*a*e^(623/2*I*c) + 1
066862354734951629341085781295112245624204034159509029552678063480622244236
687699044241201275152181775644021605917221039140*a*e^(621/2*I*c) + 10865442
458195107126822246415653572134714564509872887323281441630636076100532214509
46999270202022551820770028778526533989420*a*e^(619/2*I*c) + 110149237354129
975153461547990969736040946334905572381036460229516993161090718787392685524
5643687343585906730161451031127120*a*e^(617/2*I*c) + 1111547006738804576763
416827720669435301619875769248333073060137308247506189143702187094023228399
816342788014313884847207960*a*e^(615/2*I*c) + 11166011452861623220858153208
126762061145208586965493301675500631812147692556046807180719300626426382965
79095912747329485300*a*e^(613/2*I*c) + 111660114528616232208581532081267620

611452085869654933016755006318121476925560468071807193006264263829657909591
 2747329485300*a*e^(611/2*I*c) + 1111547006738804576763416827720669435301619
 875769248333073060137308247506189143702187094023228399816342788014313884847
 207960*a*e^(609/2*I*c) + 11014923735412997515346154799096973604094633490557
 23810364602295169931610907187873926855245643687343585906730161451031127120*
 a*e^(607/2*I*c) + 108654424581951071268222464156535721347145645098728873232
 8144163063607610053221450946999270202022551820770028778526533989420*a*e^(60
 5/2*I*c) + 1066862354734951629341085781295112245624204034159509029552678063
 480622244236687699044241201275152181775644021605917221039140*a*e^(603/2*I*c
) + 10426581062898787285656364067213208091914443489662485327538726244262135
 44564508176781308633408517935011981967399688638658360*a*e^(601/2*I*c) + 101
 419296885365662579146201278575823349302457101093495353763536051217798638603
 8527036220871061980615293450139380863514373440*a*e^(599/2*I*c) + 9817761635
 194592561439642092326247660165438267544639558039195240831267744349945820108
 32915047435963630135877730601620142540*a*e^(597/2*I*c) + 945761519659084074
 407020701635765206628707268510950859918127044094577511351195111798902113793
 353834963879647040364097292000*a*e^(595/2*I*c) + 90654338091137046511284880
 039597581167135684789574131654417362794234969081809877977774108295321067258
 4470124811899652530280*a*e^(593/2*I*c) + 8645514880264861909231769631380340
 671557226868533214211849411648083581339632809586624013845738385264203494853
 37999344825920*a*e^(591/2*I*c) + 820244821310373327999830486656658651014850
 627632427741544600909132299881600749034341081744869401886702449976206139706
 099120*a*e^(589/2*I*c) + 77410445211553094712953191086231935146942887913541
 7962589811912516396964680990857982666570814712400614658358562654106747680*a
 *e^(587/2*I*c) + 7266255240548743967295489044299897546346491838461350217790
 65652886206176807006743446378198756010809903052637351826187568320*a*e^(585/
 2*I*c) + 678308554005074900817733933728713694643911068994805169147351517503
 405026510420844066523647882858701963834358113089172938960*a*e^(583/2*I*c) +
 62965030424428278136286812761655590978380494878441543704862783087307572056
 0012090018464221573758434571727274802802603945840*a*e^(581/2*I*c) + 5811345
 246598495705165208355832908453261087345684143485080221040613327380620635084
 14909055483914854232496613990505843010500*a*e^(579/2*I*c) + 533222893493429
 885859409326459830897334132694769861149975266872019015402485363293405873999
 908400003099563986230923614600520*a*e^(577/2*I*c) + 48634649377888960220864
 954194100002855346932926715456497046092005153453977747649119560232954097527
 6021922112919361536876440*a*e^(575/2*I*c) + 4408981494915035234447781988067
 938398783222525257803778161918490839930384574801953373234637489371322927235
 43647234218039660*a*e^(573/2*I*c) + 397225911325958618362932566427489711008
 806370695506218497289002610982713670531256154126928412396842645037163118148
 106920740*a*e^(571/2*I*c) + 35562792952360209042496295254069656463922322436
 052266745359625063593797852563838274166718529799876556958444993698992799548
 0*a*e^(569/2*I*c) + 3163488842636163620428425439010805462815012119876560047
 80654651440103299682244533088533089753616985689239381646035489221440*a*e^(5
 67/2*I*c) + 279578067821774001544429837120309276532260516704930830717667421
 621248225175842302586295274927274086096056979473472729880620*a*e^(565/2*I*c

) + 24544913252943878657146028256914731662384082738982168509152746364888136
3528910037224877635235250886963822468275620946735000*a*e^(563/2*I*c) + 2140
414401268248423527085110778456188788662231517813616335468747740382040265676
92011971405221478790305862726998272063222200*a*e^(561/2*I*c) + 185382876570
012174114774314485921619537568838158478822294899038884738131663676924882800
088555461036440639784323290982435920*a*e^(559/2*I*c) + 15945393607422544108
744443544559732263801034995914093882435502941504495185879537013708302342372
5977662050522985425476876440*a*e^(557/2*I*c) + 1361928323948339008353930358
607176318582406447388061261999657010469796264417511027152428721619334621057
57019809834502589368*a*e^(555/2*I*c) + 115501366010049680683270239800168602
972769128293070900256170512177998242491871324570579538346381414929314168762
720397958392*a*e^(553/2*I*c) + 97251263582884512694733261583667391192049945
346047680475584538410967173939454583220401257131634845269872208901982900506
288*a*e^(551/2*I*c) + 81290710182043566167013791732802671729549673903830676
446357987714839339054175585701722606884268548187402687095553510812392*a*e^(
549/2*I*c) + 67450813471126706935490518568202390849221640275494605206650667
907051518329294951998879978783463264636089137056491006523208*a*e^(547/2*I*c
) + 55551769820516542811419218846926579773087099753978991696756716157368950
253678159527340671056699387281634743181637981363440*a*e^(545/2*I*c) + 45408
541885366143872788717218763773054039400602057983036333376816522774495334926
169325754390780348094316862266665965991846*a*e^(543/2*I*c) + 36835902201840
649059149227149275768729797910503389287520100466864775463578466769724598377
043872510103817701793820475775984*a*e^(541/2*I*c) + 29652744395633580133158
959806701163854270210375171267208518633332161364749405283261626825045358828
399742318516568627484956*a*e^(539/2*I*c) + 23685609574401533875732237768553
597887504967570002097314498949007000780963353245851044609521946097281045160
731628195279464*a*e^(537/2*I*c) + 18771417775938541010165811709655891254174
047524375545855841132890372258743200657879368482618288334684615818699392418
342286*a*e^(535/2*I*c) + 14759430966416372660263915841353640389805668655494
495235890983094581942749360680850560805353655895626485394213426659661080*a*
e^(533/2*I*c) + 11512503701187229323217546242927969318448684804027720308530
886272434106117414287980876927354781721804664473558899761393515*a*e^(531/2*
I*c) + 89076995183911784291298064140249784156980302850762290139915162929755
57588143549142778717881866319712550105543451994921740*a*e^(529/2*I*c) + 683
636539073439354900837400605268896230520858872689216172400254354510990607087
5455109463119424154633158689756128755804310*a*e^(527/2*I*c) + 5203763588007
696555866441944538215327243563612139283244175384029625341239582454293417845
888737748393246675398200066703617*a*e^(525/2*I*c) + 39283609714920106346519
804904061715044001402467601074344731826406207591738006697310285181061947331
67039252485642601537883*a*e^(523/2*I*c) + 294087118247175053123855033288460
269591782358222491088806119099430489310936386360299682540179041858897127490
0767917143382*a*e^(521/2*I*c) + 2183136663673238874306231186152025274688130
245012528032646406775075995457404272096460063109834340128997113442263319468
362*a*e^(519/2*I*c) + 16069262430662605455448190100500712842743124397514817
59684191236045641532610423083530227151796510775305152151669953445033*a*e^(5

17/2*I*c) + 117271131265849395364983883698491541179040146140456348400538432
8877702037162967120070585970799957845874604153354692701754*a*e^(515/2*I*c)
+ 8484704989460016345675819568743938340161467897648342753927204752914291466
61138402672991032856375125997801887991514934480*a*e^(513/2*I*c) + 608560000
378994446245436072599493838842538725592619591400972596394327368638438650114
993937818761497030717896138241363060*a*e^(511/2*I*c) + 43267510031147823268
192037944542330700966435986817673117402364955963817104066125257700650919500
5627113792068208970590674*a*e^(509/2*I*c) + 3049181785340667859900073481092
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44684909249658*a*e^(507/2*I*c) + 212980072887213248427695741184161155985508
108281896593768588904956983197495607916465914403589016442699445092549476690
660*a*e^(505/2*I*c) + 14743495097122257892001302754560741045764055468918914
7706669908024367412625120600958820169170652789316898683463434699434*a*e^(50
3/2*I*c) + 1011438762009077014581455894336504522673284190696402820119090996
93348827229087774814701746425164097849836694542125988386*a*e^(501/2*I*c) +
687588340824290493941730231472874639260477373072581343050011943174958579815
78577369701491983169829988876464082023881439*a*e^(499/2*I*c) + 463169064421
787428773107530128748877142514289777958793153580120909470200672823314958039
31605151341983289375819303497428*a*e^(497/2*I*c) + 309132980920478453984687
001663630696239627341410546638371233948289257131780166068100757728741033115
73584068100311403990*a*e^(495/2*I*c) + 204417835019961259801359073735729598
558718872777900564562643674240367953383314829824410987395778323946443818995
56974181*a*e^(493/2*I*c) + 133916171433813415617973822475038584786946514869
76241130172192155041553598393494338072468333618785797627855047879535719*a*e
^(491/2*I*c) + 869083442874626408206356668709227690319173788941840934462519
8826575944588697061199305932440621485102366745761048030632*a*e^(489/2*I*c)
+ 5586988939674645914325363550716282684830193122934675954566650601501265101
993368547501997769261446740200272717454879998*a*e^(487/2*I*c) + 35575905236
4744337578248508929766292288275428615667341106417373295977843916533578834
72975676999315605445384252967857*a*e^(485/2*I*c) + 224372744426019935616907
530711533047497820991345500293367004318952790277086251364680160400675824242
6313572029851100889*a*e^(483/2*I*c) + 1401505022864821759990514911774358655
464920980642918627032979165269601299928150161830021936830813736871417390373
928686*a*e^(481/2*I*c) + 86697045893774080059910011005227233408633616681348
5215823374714091680381820801848615843383573129277135387631125832240*a*e^(47
9/2*I*c) + 5310970862260027768448225036850264406299655034624546622717676645
38175076793501990496089430124065992327885079121429343*a*e^(477/2*I*c) + 322
165205145142363570118403921081223172103670428827434903217367898408752791686
796216445582208987707216661270752348189*a*e^(475/2*I*c) + 19350570483018105
418915972164288713013847558252793401773242056315727571038403878051901634108
3221717538160410071629286*a*e^(473/2*I*c) + 1150786790741569210038962115067
440407574481297555180726109745976924991164767807359592015878667774273738541
39852271484*a*e^(471/2*I*c) + 677575411667699551794684595940634031110056452
95143293416947144682345206374550051700907623571000874184381488493815415*a*e
^(469/2*I*c) + 394964421757197854804800626261040141950619117362572826461311

55866120538943291998823092670646932544497465667715800518*a*e^(467/2*I*c) +
227914492230573423183794384642924067618367063004688929366445938507368162260
75816614000005128630118631272047930585354*a*e^(465/2*I*c) + 130189433375914
762178356958680531819179445499007194845922504447026408380059377508923151909
56996862238788704893903660*a*e^(463/2*I*c) + 736115054722908835807876927403
961090975320142643720743437764756694253034489663517179886170060433010441683
4668519430*a*e^(461/2*I*c) + 4119630675993972883163200200627180864136924478
788743836907012989394926215330996549026090671132192170289614995953334*a*e^(
459/2*I*c) + 22818654910004353256520815711929593145995527450941003981475857
68901371932672555405076513220299715006918296079490268*a*e^(457/2*I*c) + 125
088564125627501790422379254593359127033726798535192272989236027037302961027
7920725182173946163411522304315334632*a*e^(455/2*I*c) + 6786059851087274217
018825233023932842112497196479833873189241150491928143942692800387379086097
93825310009937379942*a*e^(453/2*I*c) + 364306588259529933001212011349678703
668868825872128216501759161561021127016982962518264271231266127473606437283
673*a*e^(451/2*I*c) + 19352737493776155106975449584051738189540453017146222
1063749937229104247233763126606164713214930951706536170129402*a*e^(449/2*I*
c) + 1017233416425340278563258023266259231008345681235737842836119652948290
26168598516825508102699558265012155505286390*a*e^(447/2*I*c) + 529027743018
574367549566488112897453499435435611479919329578095983375632430292197718086
87832301138643124607095515*a*e^(445/2*I*c) + 272202840140473281110545726489
135154639891847008248762990130210512503186300606320662319185591740786585673
45880625*a*e^(443/2*I*c) + 138560661023300269915703514452458130971519949149
65448107899770565047605427336948015872027856400037860001309803670*a*e^(441/
2*I*c) + 697745399097009589387051799033958620978960769975387378294213069494
5946528220744372046715941677441907103193732516*a*e^(439/2*I*c) + 3475684327
725554198246437246074163598962204579122918009051534296803648946904541960047
776709789822578169105209259*a*e^(437/2*I*c) + 17125625637948929883517582537
105808391193067511796248808046657272502782126819430017211592596269942683536
53902636*a*e^(435/2*I*c) + 834625577862473027286669700032071163132688047363
142695162058602775713578941078276791942737285481618912831112014*a*e^(433/2*
I*c) + 40230206688627611091580537050825344058292667366383467679120937981707
3708402847882776363322804524102236998117616*a*e^(431/2*I*c) + 1917803983125
401081649695691343833757470894770850008524165452826277003042091259297512147
08830470427357620162840*a*e^(429/2*I*c) + 904114465590476241075636388127901
365070921139042817273413125290723804538206691540873902278005764014180851640
68*a*e^(427/2*I*c) + 421488318183379605264415423924469270354566939628166561
98594402610141054461594749815537318993350672298883213302*a*e^(425/2*I*c) +
194296978761766218912342065517275740088550706895032287006627845545321940089
82832223012645019617538312082871128*a*e^(423/2*I*c) + 885605196595303487966
234065733860296508373759769686898565697938788076949495486430428730940615968
9433401344052*a*e^(421/2*I*c) + 3991010196204332937212586851998834375311877
876034549525880121307974429565165949974007981491960685666506428548*a*e^(419
/2*I*c) + 17781516067927356931845763051710065545917308898014634708345715696
49456632925027058049883289543294859615339320*a*e^(417/2*I*c) + 783199511028

604830305265171241783073558871641563886451764214478656600184324031802959178
 677178450742159624260*a*e^(415/2*I*c) + 34101053275050840101051257257425643
 3550588192252263514909935663911418354657799482940339817517181660633925100*a
 *e^(413/2*I*c) + 1467670974639709828086077917925123709245316349053130512335
 06563629059084334858680054056774768242705313471500*a*e^(411/2*I*c) + 624350
 634603888862944275086050183454453771965931516784645500297335037220626391993
 39939077739315562684600360*a*e^(409/2*I*c) + 262506700598237870871324313040
 849655957123767320824379561378686494524463490057790192498722891461579881868
 20*a*e^(407/2*I*c) + 109077925513286012622236194966417553379047773698515875
 93737697263911418412361493183977361964833748319598140*a*e^(405/2*I*c) + 447
 909460531245594013118789919867573649017879009974519414523532413461969977254
 4528969129449757743039635770*a*e^(403/2*I*c) + 1817491183507803277646434154
 563247687916502767732579734578654547931596925444779173074576528904820222193
 680*a*e^(401/2*I*c) + 72870957701304930685145583004578625839578559495705560
 4204925387842039994625652269962227400682354338997620*a*e^(399/2*I*c) + 2886
 739786161243515760661938036214109400499872335643540699651281142295126082601
 20320153840315109513240510*a*e^(397/2*I*c) + 112980168249357900386494611302
 233425325484148661095936167081471345600663852911028639756655134829221364250
 *a*e^(395/2*I*c) + 43682564876241300877496357570751883228264483234185897920
 495921251684596213376975639862720078190049382420*a*e^(393/2*I*c) + 16683791
 974708972185869156311826821170155297607333534880763775663291322286056634684
 196941640082885613980*a*e^(391/2*I*c) + 62940612250678644776480353281961435
 25983759755951003547140744854431151267086143332034072457898773808110*a*e^(3
 89/2*I*c) + 234522757206302326337643430557860616909110433608129787135672274
 4639904016570907272826407742463307924640*a*e^(387/2*I*c) + 8630272903769381
 362861463205873829669227787392446342355192009556495930510356114349884338195
 31197898760*a*e^(385/2*I*c) + 313629308802071546273320757016507452761381047
 676711392941636867620998743852945422691370541694930187760*a*e^(383/2*I*c) +
 11254540176202178596753890614510262974982280482013154146293602597389354484
 7020246173693665097678958840*a*e^(381/2*I*c) + 3987710700581680884247429044
 3185467296349332073686639497504608960180007893671571725169560655925320000*a
 *e^(379/2*I*c) + 1394984531604229378652851409453562027495148263669579368579
 0781893931237847288558891416636782386235120*a*e^(377/2*I*c) + 4817584949915
 740964852355271716409235848785641998844676677492209743884414375678304126912
 661883888620*a*e^(375/2*I*c) + 16423558505236940817816598754874797064875991
 25909388975178926280703865441617578142680863389145680280*a*e^(373/2*I*c) +
 552644594394330981404683589085114642280906553679728598394985334382381190569
 025961084792025986226970*a*e^(371/2*I*c) + 18353862986603064551692928939851
 6242583510847808790824780933371253504023929536299207100681758720560*a*e^(36
 9/2*I*c) + 6015520894307520872197048563745988519938935878108896989850976720
 1637525536232949225931564636017060*a*e^(367/2*I*c) + 1945555628821014219779
 239970144168345717799388860439457193607598104867777345749940209868154134523
 0*a*e^(365/2*I*c) + 6208666376133327662787020371327223738306905395229403481
 688606039769774018193877054980125219300650*a*e^(363/2*I*c) + 19547713009872
 346842991572365025448786468695388384136314224499795486470414353725493083383

23983400*a*e^(361/2*I*c) + 607147655811784618052593406867529437392538681078
184866780443537932100336168022295156652479524980*a*e^(359/2*I*c) + 18601597
745255025509215791744032430081813501925479907053679687254926325009108609057
9550703698310*a*e^(357/2*I*c) + 5621077370840469391230778741098922958096921
6199887991549658097359306716028570932883897745531110*a*e^(355/2*I*c) + 1675
163004358564804546609710507623366341398041317735377445682230518764384123684
8558981932298420*a*e^(353/2*I*c) + 4922849139190761971912744518550936831750
167555165468970439911091242197041305307652588826163080*a*e^(351/2*I*c) + 14
264303742509105913379082618349894234718829817098635831597753715377180169320
12468351480138570*a*e^(349/2*I*c) + 407484174465394736211691486364402072868
790552430405179880507421751094282407835858091343026670*a*e^(347/2*I*c) + 11
474859600895439947056082052328280297256075477916628157653204619372173061718
4676146208454500*a*e^(345/2*I*c) + 3185001300244949419178867427941499467160
7865224244945000200665576261090031451234274836807440*a*e^(343/2*I*c) + 8712
550834307422843463603955913018153198551629326082886421600824607707881453826
119225268410*a*e^(341/2*I*c) + 23485569225842370620041981071670712738872179
33156401182423976893834425480017846564439100360*a*e^(339/2*I*c) + 623766634
567578471789157464627305169392418716892107406574086770229679140188972330140
850540*a*e^(337/2*I*c) + 16321208300699726872915924654990666086737306928579
9943509652022515969021706052860626138960*a*e^(335/2*I*c) + 4206626736731821
6193842191416909738948525619885259401159592359828689296015791863825798480*a
*e^(333/2*I*c) + 1067844553682541137674668046315205503487286703673470505227
8088324574859010216795309754920*a*e^(331/2*I*c) + 2669400972789686981296836
875076250459931673689078037577064328933999433324588323107010000*a*e^(329/2*
I*c) + 6570357867033170738722761834666553683329480644569203660343706414586
4115975619273310760*a*e^(327/2*I*c) + 1592097671809555303250472275831776042
33946016419587011038408978676950649660142022326320*a*e^(325/2*I*c) + 379743
334997202596581506260550746726020844858907826922059323726421642572558282752
77230*a*e^(323/2*I*c) + 891421027524593774701305889025999040677202573103172
2326439774933310018480720586176220*a*e^(321/2*I*c) + 2059104851233963888842
379525772936034391701199184168338071977072713080793446996296340*a*e^(319/2*
I*c) + 46795660437124083156463767484415712775096496748771025804871156484949
2193413275081370*a*e^(317/2*I*c) + 1046142529349056121996585903795887365485
77356327149679159709209471984279805545840510*a*e^(315/2*I*c) + 230016401216
57253316651693705684837372513278520836331991407615544187744222242128500*a*e
^(313/2*I*c) + 497315194543781833320797851981222928399068768126620986782740
2557241427416724759600*a*e^(311/2*I*c) + 1057133035608080334517467842102247
001229173401193395800195028598793946241985524410*a*e^(309/2*I*c) + 22088700
9474485548410918212534797139518487225151752041095525819264180469636552620*a
*e^(307/2*I*c) + 4535951973461225073999629610614837615488499429203915540276
0649106566139277798596*a*e^(305/2*I*c) + 9152441985761079445443716091984800
770861766098755365001578215027814025637822344*a*e^(303/2*I*c) + 18142017896
63247503683990869126253542318071421878870447076671569392523282426716*a*e^(3
01/2*I*c) + 353200434742307961537044752990124528653649931040810723777175782
628615964698844*a*e^(299/2*I*c) + 67522403361212079136291447998911082703845

361849108254368438686584600219974916*a*e^(297/2*I*c) + 12672621445440067623
 486707294305789706782209558457586283948374216939117542680*a*e^(295/2*I*c) +
 23343962004920902169805777673107446505804176065895986743895917756245133022
 92*a*e^(293/2*I*c) + 421955281655057070938611937183522587192086313169152717
 515130361078410110468*a*e^(291/2*I*c) + 74822580599285480933240704796196912
 985142264561156826348717174043201630392*a*e^(289/2*I*c) + 13012484431533492
 378746242079203193435247631682976658777500945950758604298*a*e^(287/2*I*c) +
 2218870701411021049789421436850301976335757207676690062759443158094615652*
 a*e^(285/2*I*c) + 370874356826170622561211031579288522536503760739536119117
 445103487930880*a*e^(283/2*I*c) + 60746198053561919583529494820252006432880
 168402592906594363059656788960*a*e^(281/2*I*c) + 97471745025241392484421887
 39775017562567649540786887839358187040912530*a*e^(279/2*I*c) + 153168954468
 8533128267835220826660575517763556541402057407422275055260*a*e^(277/2*I*c)
 + 235643271710602776985976812503397918723360707843190238666454384153669*a*e
 ^ (275/2*I*c) + 354802086973620760088737097767022945419673957182978325692500
 47180556*a*e^(273/2*I*c) + 522652409350764619368283927596894360862678670014
 2487822871933708874*a*e^(271/2*I*c) + 7529709775606075637193985852070108636
 04321557616289473063991767039*a*e^(269/2*I*c) + 106051898268333756910564897
 558688638080806915238108581854868257941*a*e^(267/2*I*c) + 14596988265590225
 707951774554192673478049322662209596418132188458*a*e^(265/2*I*c) + 19626152
 29865836518270067732662567794203130442183114304875477510*a*e^(263/2*I*c) +
 257661195939290308606607761298064029010980446277923637727940695*a*e^(261/2*
 I*c) + 33015018627194726039854786048619784314920578570513381395990898*a*e^(
 259/2*I*c) + 4126870246985029536018107086136692505798915934457479856887816*
 a*e^(257/2*I*c) + 502997285203192059840013244325631734915939121404020646790
 260*a*e^(255/2*I*c) + 59748219031202678520235643789006093143612681508950204
 493778*a*e^(253/2*I*c) + 69130093506103224460474286274088111591583388857474
 90127762*a*e^(251/2*I*c) + 778662391349080881934068370430476103178748088552
 087631588*a*e^(249/2*I*c) + 85332794263861745341951941568633801044299209189
 190469846*a*e^(247/2*I*c) + 90928323032813368162335802274625778711978139632
 76075330*a*e^(245/2*I*c) + 941491605787522647999978018615368308830544085253
 128617*a*e^(243/2*I*c) + 94660792391616466534333151235841916067978700197242
 708*a*e^(241/2*I*c) + 9235195266849469317521124217042763676973766253104794*
 a*e^(239/2*I*c) + 873599234708764858055721340390736019595548391597091*a*e^(
 237/2*I*c) + 80060282862006487921860873353861357672073071040769*a*e^(235/2*
 I*c) + 7102120067046396299634193292688723536713017300128*a*e^(233/2*I*c) +
 609297036058359460142063362206138699889504487122*a*e^(231/2*I*c) + 50503221
 535858091096582689376221786704600600455*a*e^(229/2*I*c) + 40402571518442061
 59542266588079905870752602511*a*e^(227/2*I*c) + 311615542420218960570549891
 210554131541198338*a*e^(225/2*I*c) + 23143857771494702366699332162923093803
 819912*a*e^(223/2*I*c) + 1653132575512243220555792873383292580108953*a*e^(2
 21/2*I*c) + 113407505114998207828286839959356818331515*a*e^(219/2*I*c) + 74
 61019729104126380477146521488232920426*a*e^(217/2*I*c) + 469985477889846368
 776571005635121060828*a*e^(215/2*I*c) + 28297554171509598228359169068737992
 545*a*e^(213/2*I*c) + 1625446941911901543891066605266178710*a*e^(211/2*I*c)

+ 88891628244678704139848327305754038*a*e^(209/2*I*c) + 461774686857198012
 7702698250753676*a*e^(207/2*I*c) + 227298419095755116586566799957934*a*e^(2
 05/2*I*c) + 10572019430266010048013097752934*a*e^(203/2*I*c) + 463207034992
 633568486133297596*a*e^(201/2*I*c) + 19052217324800742177269360704*a*e^(199
 /2*I*c) + 73277588085793048574300590*a*e^(197/2*I*c) + 2623756066865861197
 8090055*a*e^(195/2*I*c) + 870123184837420284430070*a*e^(193/2*I*c) + 265686
 46854716407696970*a*e^(191/2*I*c) + 741764252131213332757*a*e^(189/2*I*c) +
 18778841824143775343*a*e^(187/2*I*c) + 426791859620149322*a*e^(185/2*I*c)
 + 8600339740311748*a*e^(183/2*I*c) + 151262256738437*a*e^(181/2*I*c) + 2274
 620402080*a*e^(179/2*I*c) + 28432755026*a*e^(177/2*I*c) + 283618504*a*e^(17
 5/2*I*c) + 2116556*a*e^(173/2*I*c) + 10504*a*e^(171/2*I*c) + 26*a*e^(169/2*
 I*c))/(e^(531*I*c) + 432*e^(530*I*c) + 93096*e^(529*I*c) + 13343760*e^(528*
 I*c) + 1431118260*e^(527*I*c) + 122503723056*e^(526*I*c) + 8718181624155*e^
 (525*I*c) + 530563624556832*e^(524*I*c) + 28186192554792138*e^(523*I*c) + 1
 327882849274858880*e^(522*I*c) + 56169444526926562260*e^(521*I*c) + 2154864
 144781257856128*e^(520*I*c) + 75599817092670157806639*e^(519*I*c) + 2442455
 629894502983849104*e^(518*I*c) + 73099207817335597247098038*e^(517*I*c) + 2
 037031259470368160131922320*e^(516*I*c) + 53090127264630963470039804475*e^(
 515*I*c) + 1299146645993240318167826532288*e^(514*I*c) + 299525477492654996
 75257842032197*e^(513*I*c) + 652650253343206047453620559993840*e^(512*I*c)
 + 13477227799524701956579274210395326*e^(511*I*c) + 26441037578031074251809
 9326419685040*e^(510*I*c) + 4939666610818025798809586352543471345*e^(509*I*
 c) + 88054927598941411145869950813388040256*e^(508*I*c) + 15006027479373972
 86405577818722691539392*e^(507*I*c) + 2448983733781233868771862249186501383
 9488*e^(506*I*c) + 383360155801054824529764688213114368047154*e^(505*I*c) +
 5764601046563151304213854710715346838447392*e^(504*I*c) + 8338083991183789
 4453136303673785039051506805*e^(503*I*c) + 11615814137339717515336225119090
 46917188768400*e^(502*I*c) + 1560391127768760709972162377174493308692058727
 2*e^(501*I*c) + 202347509724462171313966643580234078508179838320*e^(500*I*c
) + 2535667460650279776834561566186591213109251642859*e^(499*I*c) + 3073536
 6512830562160991166338490057308062762518496*e^(498*I*c) + 36068861303638934
 9413809780004559963548775423325255*e^(497*I*c) + 41015454399371957939599567
 08442496709433800261224880*e^(496*I*c) + 4523094003983073833202569478464620
 6844854827698075736*e^(495*I*c) + 48409341024048871865591702530366258109165
 9126182344528*e^(494*I*c) + 50320249034014518240742139437660119220265070063
 11982753*e^(493*I*c) + 5083636950817109943701934861084739194673618510801718
 3136*e^(492*I*c) + 49946750655853173367158586291057270281154503573039874953
 0*e^(491*I*c) + 4775398607100853263534207733818266777478693412738731031680*
 e^(490*I*c) + 44456708175258821024400946210535004523775722190977468484496*e
 ^ (489*I*c) + 403212225957798188840846139960995624144491271694336796459584*e
 ^ (488*I*c) + 3564764890628724017088487996688178929195787613958545474804845*
 e^(487*I*c) + 3073621740432100996523103741966305396288103528170922169778507
 2*e^(486*I*c) + 25858534871597727015582911568419341107203454149136439398549
 1350*e^(485*I*c) + 21237029691888713182667187812239270678399490157272938840
 65388080*e^(484*I*c) + 1703388602739061574104097772165554166561216227548502

8584310890417*e^(483*I*c) + 13349021005202618377967331386833230353033290616
 3247194627808410304*e^(482*I*c) + 10225364374682967372930658627052464496936
 87415559865844306888705423*e^(481*I*c) + 7659010520187549651777118357676871
 927081898989131125755798204236112*e^(480*I*c) + 561170810763411753840875701
 85188538660375932013674735519055227368366*e^(479*I*c) + 4023496922661211589
 34003582839428785116904903936409545602519219664720*e^(478*I*c) + 2823905151
 936586678382525706564457280290098698638597987628380245881715*e^(477*I*c) +
 19407979215594566593535008103303255257745408070082431338945184797463936*e^(
 476*I*c) + 1306576602265604193351214343899389618845954340699848243071493321
 31747540*e^(475*I*c) + 8618848510949919087642468054746724286037573154844539
 74713612812215428992*e^(474*I*c) + 5572551157328671121016216416307596161861
 955969011697222340926210112854418*e^(473*I*c) + 353244472067790181153780528
 20789411687581004582367431006205879633729015200*e^(472*I*c) + 2196012813395
 15561500261478844190024870555261281946058839614044697037963695*e^(471*I*c)
 + 1339214374254245553564884406801945353385000254030655765953770237607180089
 968*e^(470*I*c) + 801372958079075243436196494576154376146952079121074697267
 5870481058674277844*e^(469*I*c) + 47065044611135158108487353367484243102698
 248838312635876283099427442745866704*e^(468*I*c) + 271361207503266570734486
 517077181014801775322183181055638619257836143271472358*e^(467*I*c) + 153633
 3238444927583532734556016494671674916578907116984548489078241693926940560*e
 ^ (466*I*c) + 85430134411262123348335406650696214724790858380413605645507220
 36723654297540205*e^(465*I*c) + 4666822354826601780685459246810057028935596
 0869613650856575756758180182223308768*e^(464*I*c) + 25050102860892833246934
 0456829902067712233644464602753159945727868485722395506952*e^(463*I*c) + 13
 214980552713008514299938666316198744245344251881835920497276875710321564350
 77280*e^(462*I*c) + 6852993223145736687328885311617795435592940841439866351
 079655652312894721972796266*e^(461*I*c) + 349410716132767046494779430433394
 50201504075335160361865916029213860778606230624960*e^(460*I*c) + 1751931705
 00618300241515632381912285157790097816049220671217212220015297133400636060*
 e^(459*I*c) + 8639799336223303495562968200283955131987080649405057021260686
 52936800794826651264256*e^(458*I*c) + 4191542500656826148093339414544159143
 964478472492315931809171859902114109005939942952*e^(457*I*c) + 200080068030
 30047137293278250321597113540716201983333126349281186679153199068045257216*
 e^(456*I*c) + 9398691531306817914908360606568148278083606051053015461848694
 9839467131378859885998210*e^(455*I*c) + 43454667678028004534634449876389254
 0797175105756827515509297024187660299345484920192480*e^(454*I*c) + 19777929
 806658181356513000943262391586054488708069708605773253850286099830345346723
 18500*e^(453*I*c) + 8862752142756957285681340885764904597935349569355321815
 647721172537159186491471311666400*e^(452*I*c) + 391080312556018094765375353
 69611844440844903751605645023514572352045248104262933598850730*e^(451*I*c)
 + 1699563279699297677739020966526292532837045054771275445565344173766865409
 36706073847337600*e^(450*I*c) + 7275210107183942292917740738446942557987386
 67067535379759732795567942578751384250780476310*e^(449*I*c) + 3067974296431
 747364198159623962463671617006419626851426148418602934852907379021659761911
 840*e^(448*I*c) + 127472196165033205413563430625628473686016221408567860254

45814532037904111523242298235713300*e^(447*I*c) + 5219091220766182421581227
1854269748071292843243227894769229690720010547141334131610989636000*e^(446*
I*c) + 21059430138564847118432907888031750495336183995415942743400988466177
7259752542647709150036990*e^(445*I*c) + 83757920692341193245878648676537353
3946545239708990769488724813982189165104589895518909256320*e^(444*I*c) + 32
838747605558186767263094803067344201550985839480744690141681718744421701096
48521627538755920*e^(443*I*c) + 1269349693296492056507367363718128008854868
2508880255337280065006566138696041797353216584528640*e^(442*I*c) + 48379489
756434099843857791816589379406815042609340378747586437145781646245422045101
230417309900*e^(441*I*c) + 181834661406177901315330129677145381166449188413
194141169344354754920969034952610378945282257600*e^(440*I*c) + 674025530543
133008894845775236625237450743114473544537818170447134607102575676676056675
328961590*e^(439*I*c) + 246438219080743960907977422685567962936788570977643
5876630851716253962696192341706239192878728160*e^(438*I*c) + 88882950287510
246670442038376079761014800531344186144746207675228248689119598843526664449
17404000*e^(437*I*c) + 3162664467472554773117679568752765357130596998592368
8392112164915553242573269490908989570248533280*e^(436*I*c) + 11103414879700
881944314389564446924229504986746431371096925761933889913379928561602006987
2611710850*e^(435*I*c) + 38465584208066627445406307878483717499894905250097
5322162003392549953413592461519365177908682078400*e^(434*I*c) + 13150521209
306921221022971053276228423358707434285308910729835358622800944466077234738
00477453914130*e^(433*I*c) + 4437210917843182347764349544443904699020056595
069470847193617092114714077633077234972825351226979360*e^(432*I*c) + 147779
550966171289987127451820714953621765069731830816502336052740516776249704643
40242755840025673760*e^(431*I*c) + 4858425815314028044731483686877213139019
5412419046732778458706015096881437076337910793584122475073760*e^(430*I*c) +
15768584552885091872146287786443509025758314941556132342738656289444759827
7935629800939237175625149830*e^(429*I*c) + 50529366312301525887848302573881
2813203397766845340065381261016353419722382620393032535960660921950400*e^(4
28*I*c) + 15987711010581926922705289996774447426856310062324561858449252201
44002305878120380828483988663574829100*e^(427*I*c) + 4995241956279138180205
186744401688024388272113921255663734956946927571305533146776898787878059685
108480*e^(426*I*c) + 154131112114860239372949708207973767160813447881633865
43522421939737507962125854981881879168348260330000*e^(425*I*c) + 4697022472
711728182645404501807067052255975662758034778453532001496348263235972944454
1885102274546002560*e^(424*I*c) + 14137993825355684328056550580740330413060
6130725434751745794079833141361748917639986145377066437210546190*e^(423*I*c
) + 42035802483514679858361121014594215468443794936564789908837252480215622
2884839580011688655664280691773600*e^(422*I*c) + 12346680418924099787800180
817554402160125824763969419379658996319530792039742221387946043284989721447
66900*e^(421*I*c) + 3582718002163296061414536703715109897107198252739284546
149343102348456124089657428594946438660859773886240*e^(420*I*c) + 102716025
302028890024978135168494525909715128095290606651973010970522100645760883480
23234671975463677418470*e^(419*I*c) + 2909765106124745340664756978183691006
2165559852359052804259154165687125428752562385492373749486351714453120*e^(4

$18 \cdot I \cdot c) + 81452081413829111828875417564250054846037693312811480492909160758$
 $195989155768107022568350953861815940704090 \cdot e^{(417 \cdot I \cdot c)} + 225320532593220657$
 $767941109289516248999794521015564134982827241710019675486694499689312466561$
 $907212627820000 \cdot e^{(416 \cdot I \cdot c)} + 616003116022979584937125701757887212998354300$
 $989991362628038861093914561332071191909714949426587936910303300 \cdot e^{(415 \cdot I \cdot c)}$
 $+ 166447503438721180939491774350293763897857493775476476398783587241044993$
 $0690131572904279995484581013965001440 \cdot e^{(414 \cdot I \cdot c)} + 44454122592954746250676$
 $595141983129660154162999689303933456303459141097207405736188849805200100284$
 $51496996210 \cdot e^{(413 \cdot I \cdot c)} + 1173585692624511849311309100250160403234187698599$
 $9082823520672241530200188223826392982302194084667538488665600 \cdot e^{(412 \cdot I \cdot c)} +$
 $30627581054221957378390547289277609129572813931082733520247387226000020043$
 $538279468776707958420892547870128680 \cdot e^{(411 \cdot I \cdot c)} + 790191495587665692547839$
 $884872323883529091449827471718567724632238089933670915034028764672701761243$
 $42699654400 \cdot e^{(410 \cdot I \cdot c)} + 2015579474247940980267724784620408833958675125629$
 $30862943753568690084015585598010154781548625239409581907397500 \cdot e^{(409 \cdot I \cdot c)}$
 $+ 5083245993010854601669786296830326614276544740829390480979396383915672987$
 $95788389433842285751054665210868287680 \cdot e^{(408 \cdot I \cdot c)} + 1267597017294812913400$
 $146276042126929986480292870190399107554311079964227280196522475370108738477$
 $856311765699610 \cdot e^{(407 \cdot I \cdot c)} + 312568349317870174347970475030749017866629215$
 $0720179363604335113528623329684606343185540756019935662148267863968 \cdot e^{(406 \cdot$
 $I \cdot c)} + 76217887919120470620388409177993746004288922581943676366829443560966$
 $81400246312138001769285020661445991073249416 \cdot e^{(405 \cdot I \cdot c)} + 1837980708400335$
 $976602764921762114411609173557221662078886153580344970227380258835907670424$
 $1840733513439114113248 \cdot e^{(404 \cdot I \cdot c)} + 43834972142919377685378692233021063744$
 $554033100928502737480438978976746989895784070951905237783490374305934542955$
 $\cdot e^{(403 \cdot I \cdot c)} + 103399755467257436489847837640753754718204394473055795001467$
 $604326419876555556873829531737211096115196005647730480 \cdot e^{(402 \cdot I \cdot c)} + 241246$
 $021282440061792908317783032876194801597133206052091286997043729145345755805$
 $710081489006741839439573984832678 \cdot e^{(401 \cdot I \cdot c)} + 556756388711182340341026192$
 $734219546113651768317380539005893679049394714017063698565272728813669054779$
 $077208977840 \cdot e^{(400 \cdot I \cdot c)} + 127103308293804895020136055483127034266234399127$
 $7504612342300366025046741742856580445289401786656311685859023084716 \cdot e^{(399 \cdot$
 $I \cdot c)} + 28704961314123144518346747153535894395532944308085319334660862885437$
 $09246230769151392180699413405623017247753532944 \cdot e^{(398 \cdot I \cdot c)} + 6413381895855$
 $925184758231451062556380328594938511006577015218119786536390213057284018202$
 $201631094434819584025113465 \cdot e^{(397 \cdot I \cdot c)} + 141764836528757049572020133432411$
 $179043699777966538499599025244219806351890116348156532796054977833828889327$
 $66730080 \cdot e^{(396 \cdot I \cdot c)} + 3100431920606941707706936314142348743182800909818474$
 $4635678652284177439464941651812564519144918003174108077634846014 \cdot e^{(395 \cdot I \cdot c}$
 $) + 67091706130529669125019899210021576580237843462229535386295087076189297$
 $849995931360645605292130961496106707521506432 \cdot e^{(394 \cdot I \cdot c)} + 143657687138044$
 $796942947119704259538458818199423516824674586293691056119209866358123637772$
 $245409530799230553767222252 \cdot e^{(393 \cdot I \cdot c)} + 304384471106813336010284160123906$
 $370433888828490627422652551236966790916174520857759143930140187173492394981$
 $908258944 \cdot e^{(392 \cdot I \cdot c)} + 638218892914533741505806399662488556066783600496091$

876408374974877448971778036074996245581124283460438065182071976085*e^(391*I*c) + 132431132498402742835522293814768237867286070881716174144874968959358
8020860847508703702325320304649883120684987556400*e^(390*I*c) + 27195892834
837439260408051010803419212445303112546072509291927739093315232266350358156
72862569296693711643521070331394*e^(389*I*c) + 5527498849031178355861230009
326668283926290082158467118000698502719379939045918344222192742145711257028
040974074674736*e^(388*I*c) + 111194996453632010808810628248863384924253756
58448977935535846349290425821570383090425411418521516670371372045206568345*
e^(387*I*c) + 2214073500170860327091518076924139166203557875590397914890921
3603822554792749183517160255571915875356439553717130797888*e^(386*I*c) + 43
638300075815171025946211464465689618965773488660985857945657479854085108851
857911222911989837615452608512356008400295*e^(385*I*c) + 851395332347864557
795899594649006377607357297056212213800058372083691577946730496754287998178
75431430246332625899630160*e^(384*I*c) + 1644375006769068927413232601543942
785039545619360201335965818064493572402773494479273345171058099953000935492
79931273178*e^(383*I*c) + 3144090358082258615655954369383549454454739910431
29722046747228813030925204968503418566818838611866709807040793495364496*e^(
382*I*c) + 5951576155004315149474792823365470538279260879164252634971877570
29413385471835434198246807096214536895441388306027237899*e^(381*I*c) + 1115
398285545601550533328045600184317993899359217996729818340704221801195667410
485846996179056733558512452238583160792512*e^(380*I*c) + 206969828950086064
346166576237380795751301942404117887190496055182941244934472243212567941795
8403007551179298315947373776*e^(379*I*c) + 38026049967058911069646206338489
648070370988545101822632430305972956307603535975319743247522663891931857608
78274188013440*e^(378*I*c) + 6917838945214844278493330459361394923372333853
619879637372673184942859712431066345726870422099893124890777678037369988150
*e^(377*I*c) + 124621404405372580849285967098720668577570709431248685545009
48154756863454308032925408340311237850017814707896986969086816*e^(376*I*c)
+ 2223134113180153534540639903772168684020839794195258013558464596674673665
6716271554826476282991066076564921432614339399735*e^(375*I*c) + 39274200414
329861169397944516225001081227433398585007206399231211907157795359719648241
598754266579840244551491476467899952*e^(374*I*c) + 687124660159856415124685
861736597477348795917100983546527861249360230739431410495736066485630053594
11712764895683903806088*e^(373*I*c) + 1190605918496605468347656932276764490
675841482488826784475048260772363334445134540951266687500572958111916433569
08972191440*e^(372*I*c) + 2043255572651860007674027102308478964597615839227
63698235433212833313077783041040074669379017394836761539649081690630811665*
e^(371*I*c) + 3473100538109352904194555605559573141295692107357459832343696
59976413374774078000173070075248654524917179128950507443058208*e^(370*I*c)
+ 5847495736823045861793846288448833275814989698865403803788967679990756149
64007174600811092945356635118795824799369716742109*e^(369*I*c) + 9752103394
440493187572823117635177866732231755944579463832792646350850410049173002959
04275433144848532459919875479817581584*e^(368*I*c) + 1611092541400060525954
859375264194178347643471837078201446262435615142944587337833513586022729849
523358436493586042252995608*e^(367*I*c) + 263666241043079934044752228477824

428374075106865814072657644667120779832560683229593770506168629793029638233
8892574900819440*e^(366*I*c) + 42748269077205917525267113368208715008443456
479223854715343593336061895718324446413641328931086635762051338706721562641
64115*e^(365*I*c) + 6866425337518668262662693750908956965732924578142181630
622157802899880874681551031136314064199948604001529894566235238597088*e^(36
4*I*c) + 109272106034735448102797923478445360745888968062300411100895447316
05863146104181739039426674855453466097402330688331845602302*e^(363*I*c) + 1
722950282436764733440072199841759670394865739473880520939163659737038057239
8964715080095366818322029152193635869784095333760*e^(362*I*c) + 26917794701
086615097890120236890501105146799996021775195710866226228638984703456832694
153230611607263444183501026198563419616*e^(361*I*c) + 416704403753905436434
182193422717480400350714901190805852815224981888183759069003687012345313046
33163446319945130196476913600*e^(360*I*c) + 6392301943376198909061480128863
509812319944510230312261654464820899876780394445577704288655273849974718371
3136069104651812215*e^(359*I*c) + 97173055024742680058616722461368892661141
295540263493013032746083536157324268333390400308958318370219154887169702257
444756176*e^(358*I*c) + 146390448456351181218237382740374124191664819997746
988076598391862733629670142241546375533903130605297580105675355629160198162
*e^(357*I*c) + 218563166659649312247483640956272149212499115383828771029654
283363972585118090479413696638108156385244646591328454425745117584*e^(356*I
*c) + 323413178014841003714151138246079152576360976035058457890409937738723
171537036573043681997163745313602400139153046673668433091*e^(355*I*c) + 474
323043563100542377338629931966248129175976982332446018056009391154020438895
903140822967769494019446166779954024655344116288*e^(354*I*c) + 689518449328
793559903260418149974190253578340058895035589606468244680591556118170304005
037563669880057908765898949268614772285*e^(353*I*c) + 993555653649521127226
443960820233649386488510081892545866700444096661582790441241830855609577062
039555625090943332264901780720*e^(352*I*c) + 141916448142217657385823401389
89996288223223330957373071631074383893583220145429361729317508645838962142
5307051750612129761498*e^(351*I*c) + 20094961100926877381527820856837372227
279688242990587392154460834993514676253344496707577640666906561509490149440
43994822823920*e^(350*I*c) + 2820819298562215959107529807289628449621386798
989436369393116069894018781201000275633104498398959346631795568022519974400
130281*e^(349*I*c) + 392569765841577835276810394285601184021164276962171721
7996614398473887186074391482638547212826538270453912634540299792270321024*e
^(348*I*c) + 54166628040524363495855959828183579538662584616443540182051589
17742576425344364964596750653177677803492186817305171175032011500*e^(347*I*
c) + 7410372612891152224364633296128043971657193280327754304382235672773781
403023814127610355562505271969045177704726054907145784960*e^(346*I*c) + 100
522095243695818275881549853455496780314457444999852082593854356097402724003
01454246872041775159838468077381562338745636398374*e^(345*I*c) + 1352123041
194543691555885470685179654156739981165687002156735297532681546781784653328
9123871696056195231696146162720992221760992*e^(344*I*c) + 18035327338177455
471177568594851682977978346449777193572087688510392426884519272991560851326
393852241961470040819793627127923997*e^(343*I*c) + 238563985655628020306952

781742126408332821541740064594582926440609479249073137359215616901539399060
17518647182491616573724049744*e^(342*I*c) + 3129526368818983831377277587330
726033411722725862950199235869563609266286606281984568906423581362297415012
0668921391878398978380*e^(341*I*c) + 40715988963701918950020348336736423420
513311359010485246919074652883393970805374470830156229705647312265477584256
027212762941040*e^(340*I*c) + 525392233467407711425870923702570695360603196
444395016676104827679558002760528924321527988146079751103662249450814281218
88473324*e^(339*I*c) + 6724408796908070382370325719966304760689061048209049
461949280293513021597981946996638333678869390013911559464689378409541847233
6*e^(338*I*c) + 85368118430215312848231291739673735887746201851666299600392
199418764750086828198719872744047767783667325326289221881974987582215*e^(33
7*I*c) + 107504737406576916123480399169759633328321407419400510017498849830
598621565428266546315933920821527544726380201659114903834605888*e^(336*I*c)
+ 134297742023479429904629616104559610096074758721068022704468938063017059
688023363436458971534964665036319889119229809973806909680*e^(335*I*c) + 166
432332922589195130558329266398753389823955737598527556096093062473559769545
772321978969318904192572733997888230986469005970880*e^(334*I*c) + 204622295
535729109519829916789867225319429705162560082648840394965423112809336591921
290309392396263834674368977840527147037426908*e^(333*I*c) + 249593072282565
866398389951509619202682634455487128714631461891770823201367527645793770203
788784677343934971424317987895255031936*e^(332*I*c) + 302060638030868463461
139442279360499718906917482524894483356220196138377050828911383056860425370
161157201493696073712322595776808*e^(331*I*c) + 362706307563843231135699157
418510732452420614013624168879312187645233450153927975793326834780741391203
430153093712635355523960320*e^(330*I*c) + 432147856464086938023811561808678
589594702047904674282297959658800170984456799067751878044806619012452636891
350731618278545690160*e^(329*I*c) + 510907615111134507452623846147137141450
275722444316385531648429230851686635827717488464500331623385777400744950538
410637735936000*e^(328*I*c) + 599378484771733474809376142401554850207064972
118137572949257503651444541939309025276896049622515630263162184526394317285
457368300*e^(327*I*c) + 697789106925924614816713747684682785083659819027952
244447043355869741368500452561164024636073401929693105801105522738405349028
160*e^(326*I*c) + 806169671327625532424575340089775733681994991576674446922
354099714615192085443245663852257001115644286660304979476023966071898200*e^
(325*I*c) + 924320052867522584035777495761072351222534420784861960001821020
509468146356756433795246446491396141583854513687104334429566707520*e^(324*I
*c) + 105178210042883437194450817005121918711681634976695332263718214961087
5004223784784183284961906494422955462431208645690802526770780*e^(323*I*c) +
11878179430793903161088023247981101290208227826600872485993676434812002060
46822166144285425922229375413676535071141005286431481600*e^(322*I*c) + 1331
396114626723035802462123531582050339749996014152452835305956367425370222758
621753922458727524856072950880960657564720475838500*e^(321*I*c) + 148118711
708924666246695569496567785552473031326055269082160265717621873742624552279
5329891464091005878304304075953693546767206080*e^(320*I*c) + 16355697446414
219006578863812890766532275801720566775874514020692343552836874896596139137

61959140773339736014790081814516625224440*e^(319*I*c) + 1792649078089632298
936945728481969334964391597506285088488350622937252533420980803144316431701
452190522716124797875257437516360640*e^(318*I*c) + 195028655078018191924499
296120448701005646036284521850167442376626632155879143691731787870223267921
3868287926294665202769722927380*e^(317*I*c) + 21061419034683443071125497612
024845434027942523524821998174104248696772627150982884376465186834879454627
74223656471345899082156800*e^(316*I*c) + 2257726219103856286812833012681573
765496262241420612932076143151171960854554124144699023009842080515157923529
357189869943515991200*e^(315*I*c) + 240246459556968608612000180303421105673
944558862194614138410616288624616181514976302503083487523406726777402343341
8269982431265280*e^(314*I*c) + 25377664154650303308154717466929885960699118
946972250529283204525421755871548480964833312098074301139430153983626696733
37957755720*e^(313*I*c) + 2661100647975783583828235139201441930178396643383
423903583862547255880772382049201015537214900832745601519737141849802506685
264000*e^(312*I*c) + 277007320715076864559750728138206549792496846605452741
412233982733378377006830588348730997931598371840374087288434574638068020426
0*e^(311*I*c) + 28625031263204617977706677807256441849912556231746261756790
50672100848988119391841466573417019247590580735265143427289340450811200*e^(
310*I*c) + 2936494214351868498703239455426771104344827306267558916550877467
232455153286140521089582733932202553130712723836983468866230908800*e^(309*I
*c) + 299049894962254360853812938028386633519008711512485881814378761918695
7111903765723974899651518555144924290346242595167274383008960*e^(308*I*c) +
30233716435082250271756031752129532190224850458507318453075190082773851547
31461213388035579159917590062343527464977286601165100620*e^(307*I*c) + 3034
408355530957075787731745322567981684616550162845473257679674280216947356785
783843205604202307836897073595410412575660465787520*e^(306*I*c) + 302337164
350822502717560317521295321902248504585073184530751900827738515473146121338
8035579159917590062343527464977286601165100620*e^(305*I*c) + 29904989496225
436085381293802838663351900871151248588181437876191869571119037657239748996
51518555144924290346242595167274383008960*e^(304*I*c) + 2936494214351868498
703239455426771104344827306267558916550877467232455153286140521089582733932
202553130712723836983468866230908800*e^(303*I*c) + 286250312632046179777066
778072564418499125562317462617567905067210084898811939184146657341701924759
0580735265143427289340450811200*e^(302*I*c) + 27700732071507686455975072813
820654979249684660545274141223398273337837700683058834873099793159837184037
40872884345746380680204260*e^(301*I*c) + 2661100647975783583828235139201441
930178396643383423903583862547255880772382049201015537214900832745601519737
141849802506685264000*e^(300*I*c) + 253776641546503033081547174669298859606
991189469722505292832045254217558715484809648333120980743011394301539836266
9673337957755720*e^(299*I*c) + 24024645955696860861200018030342110567394455
886219461413841061628862461618151497630250308348752340672677740234334182699
82431265280*e^(298*I*c) + 2257726219103856286812833012681573765496262241420
612932076143151171960854554124144699023009842080515157923529357189869943515
991200*e^(297*I*c) + 210614190346834430711254976120248454340279425235248219
981741042486967726271509828843764651868348794546277422365647134589908215680

$0 * e^{(296 * I * c)} + 19502865507801819192449929612044870100564603628452185016744$
 $23766266321558791436917317878702232679213868287926294665202769722927380 * e^{($
 $295 * I * c)} + 1792649078089632298936945728481969334964391597506285088488350622$
 $937252533420980803144316431701452190522716124797875257437516360640 * e^{(294 * I$
 $* c)} + 163556974464142190065788638128907665322758017205667758745140206923435$
 $5283687489659613913761959140773339736014790081814516625224440 * e^{(293 * I * c)} +$
 $14811871170892466624669556949656778555247303132605526908216026571762187374$
 $26245522795329891464091005878304304075953693546767206080 * e^{(292 * I * c)} + 1331$
 $396114626723035802462123531582050339749996014152452835305956367425370222758$
 $621753922458727524856072950880960657564720475838500 * e^{(291 * I * c)} + 118781794$
 $307939031610880232479811012902082278266008724859936764348120020604682216614$
 $4285425922229375413676535071141005286431481600 * e^{(290 * I * c)} + 10517821004288$
 $343719445081700512191871168163497669533226371821496108750042237847841832849$
 $61906494422955462431208645690802526770780 * e^{(289 * I * c)} + 9243200528675225840$
 $357774957610723512225344207848619600018210205094681463567564337952464464913$
 $96141583854513687104334429566707520 * e^{(288 * I * c)} + 8061696713276255324245753$
 $400897757336819949915766744469223540997146151920854432456638522570011156442$
 $86660304979476023966071898200 * e^{(287 * I * c)} + 6977891069259246148167137476846$
 $827850836598190279522444470433558697413685004525611640246360734019296931058$
 $01105522738405349028160 * e^{(286 * I * c)} + 5993784847717334748093761424015548502$
 $070649721181375729492575036514445419393090252768960496225156302631621845263$
 $94317285457368300 * e^{(285 * I * c)} + 5109076151111345074526238461471371414502757$
 $224443163855316484292308516866358277174884645003316233857774007449505384106$
 $37735936000 * e^{(284 * I * c)} + 4321478564640869380238115618086785895947020479046$
 $742822979596588001709844567990677518780448066190124526368913507316182785456$
 $90160 * e^{(283 * I * c)} + 3627063075638432311356991574185107324524206140136241688$
 $79312187645233450153927975793326834780741391203430153093712635355523960320 *$
 $e^{(282 * I * c)} + 3020606380308684634611394422793604997189069174825248944833562$
 $20196138377050828911383056860425370161157201493696073712322595776808 * e^{(281$
 $* I * c)} + 2495930722825658663983899515096192026826344554871287146314618917708$
 $23201367527645793770203788784677343934971424317987895255031936 * e^{(280 * I * c)}$
 $+ 2046222955357291095198299167898672253194297051625600826488403949654231128$
 $09336591921290309392396263834674368977840527147037426908 * e^{(279 * I * c)} + 1664$
 $323329225891951305583292663987533898239557375985275560960930624735597695457$
 $72321978969318904192572733997888230986469005970880 * e^{(278 * I * c)} + 1342977420$
 $234794299046296161045596100960747587210680227044689380630170596880233634364$
 $58971534964665036319889119229809973806909680 * e^{(277 * I * c)} + 1075047374065769$
 $161234803991697596333283214074194005100174988498305986215654282665463159339$
 $20821527544726380201659114903834605888 * e^{(276 * I * c)} + 8536811843021531284823$
 $129173967373588774620185166629960039219941876475008682819871987274404776778$
 $3667325326289221881974987582215 * e^{(275 * I * c)} + 67244087969080703823703257199$
 $663047606890610482090494619492802935130215979819469966383336788693900139115$
 $594646893784095418472336 * e^{(274 * I * c)} + 525392233467407711425870923702570695$
 $360603196444395016676104827679558002760528924321527988146079751103662249450$
 $81428121888473324 * e^{(273 * I * c)} + 4071598896370191895002034833673642342051331$

135901048524691907465288339397080537447083015622970564731226547758425602721
2762941040*e^(272*I*c) + 31295263688189838313772775873307260334117227258629
501992358695636092662866062819845689064235813622974150120668921391878398978
380*e^(271*I*c) + 238563985655628020306952781742126408332821541740064594582
92644060947924907313735921561690153939906017518647182491616573724049744*e^(
270*I*c) + 1803532733817745547117756859485168297797834644977719357208768851
0392426884519272991560851326393852241961470040819793627127923997*e^(269*I*c
) + 13521230411945436915558854706851796541567399811656870021567352975326815
467817846533289123871696056195231696146162720992221760992*e^(268*I*c) + 100
522095243695818275881549853455496780314457444999852082593854356097402724003
01454246872041775159838468077381562338745636398374*e^(267*I*c) + 7410372612
891152224364633296128043971657193280327754304382235672773781403023814127610
355562505271969045177704726054907145784960*e^(266*I*c) + 541666280405243634
958559598281835795386625846164435401820515891774257642534436496459675065317
7677803492186817305171175032011500*e^(265*I*c) + 39256976584157783527681039
428560118402116427696217172179966143984738871860743914826385472128265382704
53912634540299792270321024*e^(264*I*c) + 2820819298562215959107529807289628
449621386798989436369393116069894018781201000275633104498398959346631795568
022519974400130281*e^(263*I*c) + 200949611009268773815278208568373722272796
882429905873921544608349935146762533444967075776406669065615094901494404399
4822823920*e^(262*I*c) + 14191644814221765738582340138989996288223223333095
737307163107438389358322014542936172931750864583896214253070517506121297614
98*e^(261*I*c) + 9935556536495211272264439608202336493864885100818925458667
00444096661582790441241830855609577062039555625090943332264901780720*e^(260
*I*c) + 6895184493287935599032604181499741902535783400588950355896064682446
80591556118170304005037563669880057908765898949268614772285*e^(259*I*c) + 4
743230435631005423773386299319662481291759769823324460180560093911540204388
95903140822967769494019446166779954024655344116288*e^(258*I*c) + 3234131780
148410037141511382460791525763609760350584578904099377387231715370365730436
81997163745313602400139153046673668433091*e^(257*I*c) + 2185631666596493122
474836409562721492124991153838287710296542833639725851180904794136966381081
56385244646591328454425745117584*e^(256*I*c) + 1463904484563511812182373827
403741241916648199977469880765983918627336296701422415463755339031306052975
80105675355629160198162*e^(255*I*c) + 9717305502474268005861672246136889266
114129554026349301303274608353615732426833339040030895831837021915488716970
2257444756176*e^(254*I*c) + 63923019433761989090614801288635098123199445102
303122616544648208998767803944455777042886552738499747183713136069104651812
215*e^(253*I*c) + 416704403753905436434182193422717480400350714901190805852
81522498188818375906900368701234531304633163446319945130196476913600*e^(252
*I*c) + 2691779470108661509789012023689050110514679999602177519571086622622
8638984703456832694153230611607263444183501026198563419616*e^(251*I*c) + 17
229502824367647334400721998417596703948657394738805209391636597370380572398
964715080095366818322029152193635869784095333760*e^(250*I*c) + 109272106034
735448102797923478445360745888968062300411100895447316058631461041817390394
26674855453466097402330688331845602302*e^(249*I*c) + 6866425337518668262662

693750908956965732924578142181630622157802899880874681551031136314064199948
604001529894566235238597088*e^(248*I*c) + 427482690772059175252671133682087
150084434564792238547153435933360618957183244464136413289310866357620513387
0672156264164115*e^(247*I*c) + 26366624104307993404475222847782442837407510
686581407265764466712077983256068322959377050616862979302963823388925749008
19440*e^(246*I*c) + 1611092541400060525954859375264194178347643471837078201
446262435615142944587337833513586022729849523358436493586042252995608*e^(24
5*I*c) + 975210339444049318757282311763517786673223175594457946383279264635
085041004917300295904275433144848532459919875479817581584*e^(244*I*c) + 584
749573682304586179384628844883327581498969886540380378896767999075614964007
174600811092945356635118795824799369716742109*e^(243*I*c) + 347310053810935
290419455560555957314129569210735745983234369659976413374774078000173070075
248654524917179128950507443058208*e^(242*I*c) + 204325557265186000767402710
230847896459761583922763698235433212833313077783041040074669379017394836761
539649081690630811665*e^(241*I*c) + 119060591849660546834765693227676449067
584148248882678447504826077236333444513454095126668750057295811191643356908
972191440*e^(240*I*c) + 687124660159856415124685861736597477348795917100983
54652786124936023073943141049573606648563005359411712764895683903806088*e^(
239*I*c) + 3927420041432986116939794451622500108122743339858500720639923121
1907157795359719648241598754266579840244551491476467899952*e^(238*I*c) + 22
231341131801535345406399037721686840208397941952580135584645966746736656716
271554826476282991066076564921432614339399735*e^(237*I*c) + 124621404405372
580849285967098720668577570709431248685545009481547568634543080329254083403
11237850017814707896986969086816*e^(236*I*c) + 6917838945214844278493330459
361394923372333853619879637372673184942859712431066345726870422099893124890
777678037369988150*e^(235*I*c) + 380260499670589110696462063384896480703709
885451018226324303059729563076035359753197432475226638919318576087827418801
3440*e^(234*I*c) + 20696982895008606434616657623738079575130194240411788719
04960551829412449344722432125679417958403007551179298315947373776*e^(233*I*
c) + 1115398285545601550533328045600184317993899359217996729818340704221801
195667410485846996179056733558512452238583160792512*e^(232*I*c) + 595157615
500431514947479282336547053827926087916425263497187757029413385471835434198
246807096214536895441388306027237899*e^(231*I*c) + 314409035808225861565595
436938354945445473991043129722046747228813030925204968503418566818838611866
709807040793495364496*e^(230*I*c) + 164437500676906892741323260154394278503
954561936020133596581806449357240277349447927334517105809995300093549279931
273178*e^(229*I*c) + 851395332347864557795899594649006377607357297056212213
80005837208369157794673049675428799817875431430246332625899630160*e^(228*I*
c) + 4363830007581517102594621146446568961896577348866098585794565747985408
5108851857911222911989837615452608512356008400295*e^(227*I*c) + 22140735001
708603270915180769241391662035578755903979148909213603822554792749183517160
255571915875356439553717130797888*e^(226*I*c) + 111194996453632010808810628
248863384924253756584489779355358463492904258215703830904254114185215166703
71372045206568345*e^(225*I*c) + 5527498849031178355861230009326668283926290
082158467118000698502719379939045918344222192742145711257028040974074674736

$e^{(224*I*c)} + 271958928348374392604080510108034192124453031125460725092919$
 $2773909331523226635035815672862569296693711643521070331394*e^{(223*I*c)} + 13$
 $243113249840274283552229381476823786728607088171617414487496895935880208608$
 $47508703702325320304649883120684987556400*e^{(222*I*c)} + 6382188929145337415$
 $058063996624885560667836004960918764083749748774489717780360749962455811242$
 $83460438065182071976085*e^{(221*I*c)} + 3043844711068133360102841601239063704$
 $338888284906274226525512369667909161745208577591439301401871734923949819082$
 $58944*e^{(220*I*c)} + 1436576871380447969429471197042595384588181994235168246$
 $74586293691056119209866358123637772245409530799230553767222252*e^{(219*I*c)}$
 $+ 6709170613052966912501989921002157658023784346222953538629508707618929784$
 $9995931360645605292130961496106707521506432*e^{(218*I*c)} + 31004319206069417$
 $077069363141423487431828009098184744635678652284177439464941651812564519144$
 $918003174108077634846014*e^{(217*I*c)} + 141764836528757049572020133432411179$
 $043699777966538499599025244219806351890116348156532796054977833828889327667$
 $30080*e^{(216*I*c)} + 6413381895855925184758231451062556380328594938511006577$
 $015218119786536390213057284018202201631094434819584025113465*e^{(215*I*c)} +$
 $287049613141231445183467471535358943955329443080853193346608628854370924623$
 $0769151392180699413405623017247753532944*e^{(214*I*c)} + 12710330829380489502$
 $013605548312703426623439912775046123423003660250467417428565804452894017866$
 $56311685859023084716*e^{(213*I*c)} + 5567563887111823403410261927342195461136$
 $51768317380539005893679049394714017063698565272728813669054779077208977840*$
 $e^{(212*I*c)} + 2412460212824400617929083177830328761948015971332060520912869$
 $97043729145345755805710081489006741839439573984832678*e^{(211*I*c)} + 1033997$
 $554672574364898478376407537547182043944730557950014676043264198765555568738$
 $29531737211096115196005647730480*e^{(210*I*c)} + 4383497214291937768537869223$
 $302106374455403310092850273748043897897674698989578407095190523778349037430$
 $5934542955*e^{(209*I*c)} + 18379807084003359766027649217621144116091735572216$
 $620788861535803449702273802588359076704241840733513439114113248*e^{(208*I*c)}$
 $+ 762178879191204706203884091779937460042889225819436763668294435609668140$
 $0246312138001769285020661445991073249416*e^{(207*I*c)} + 31256834931787017434$
 $797047503074901786662921507201793636043351135286233296846063431855407560199$
 $35662148267863968*e^{(206*I*c)} + 1267597017294812913400146276042126929986480$
 $292870190399107554311079964227280196522475370108738477856311765699610*e^{(20$
 $5*I*c)} + 508324599301085460166978629683032661427654474082939048097939638391$
 $567298795788389433842285751054665210868287680*e^{(204*I*c)} + 201557947424794$
 $098026772478462040883395867512562930862943753568690084015585598010154781548$
 $625239409581907397500*e^{(203*I*c)} + 790191495587665692547839884872323883529$
 $09144982747171856772463223808993367091503402876467270176124342699654400*e^{($
 $202*I*c)} + 3062758105422195737839054728927760912957281393108273352024738722$
 $6000020043538279468776707958420892547870128680*e^{(201*I*c)} + 11735856926245$
 $118493113091002501604032341876985999082823520672241530200188223826392982302$
 $194084667538488665600*e^{(200*I*c)} + 444541225929547462506765951419831296601$
 $5416299968930393345630345914109720740573618884980520010028451496996210*e^{(1$
 $99*I*c)} + 16644750343872118093949177435029376389785749377547647639878358724$
 $10449930690131572904279995484581013965001440*e^{(198*I*c)} + 6160031160229795$

849371257017578872129983543009899913626280388610939145613320711919097149494
26587936910303300*e^(197*I*c) + 2253205325932206577679411092895162489997945
21015564134982827241710019675486694499689312466561907212627820000*e^(196*I*
c) + 8145208141382911182887541756425005484603769331281148049290916075819598
9155768107022568350953861815940704090*e^(195*I*c) + 29097651061247453406647
569781836910062165559852359052804259154165687125428752562385492373749486351
714453120*e^(194*I*c) + 102716025302028890024978135168494525909715128095290
60665197301097052210064576088348023234671975463677418470*e^(193*I*c) + 3582
718002163296061414536703715109897107198252739284546149343102348456124089657
428594946438660859773886240*e^(192*I*c) + 123466804189240997878001808175544
0216012582476396941937965899631953079203974222138794604328498972144766900*e
^(191*I*c) + 42035802483514679858361121014594215468443794936564789908837252
4802156222884839580011688655664280691773600*e^(190*I*c) + 14137993825355684
328056550580740330413060613072543475174579407983314136174891763998614537706
6437210546190*e^(189*I*c) + 46970224727117281826454045018070670522559756627
580347784535320014963482632359729444541885102274546002560*e^(188*I*c) + 154
131112114860239372949708207973767160813447881633865435224219397375079621258
54981881879168348260330000*e^(187*I*c) + 4995241956279138180205186744401688
024388272113921255663734956946927571305533146776898787878059685108480*e^(18
6*I*c) + 159877110105819269227052899967744474268563100623245618584492522014
4002305878120380828483988663574829100*e^(185*I*c) + 50529366312301525887848
302573881281320339776684534006538126101635341972238262039303253596066092195
0400*e^(184*I*c) + 15768584552885091872146287786443509025758314941556132342
7386562894447598277935629800939237175625149830*e^(183*I*c) + 48584258153140
280447314836868772131390195412419046732778458706015096881437076337910793584
122475073760*e^(182*I*c) + 147779550966171289987127451820714953621765069731
83081650233605274051677624970464340242755840025673760*e^(181*I*c) + 4437210
917843182347764349544443904699020056595069470847193617092114714077633077234
972825351226979360*e^(180*I*c) + 131505212093069212210229710532762284233587
0743428530891072983535862280094446607723473800477453914130*e^(179*I*c) + 38
465584208066627445406307878483717499894905250097532216200339254995341359246
1519365177908682078400*e^(178*I*c) + 11103414879700881944314389564446924229
5049867464313710969257619338899133799285616020069872611710850*e^(177*I*c) +
31626644674725547731176795687527653571305969985923688392112164915553242573
269490908989570248533280*e^(176*I*c) + 888829502875102466704420383760797610
1480053134418614474620767522824868911959884352666444917404000*e^(175*I*c) +
24643821908074396090797742268556796293678857097764358766308517162539626961
92341706239192878728160*e^(174*I*c) + 6740255305431330088948457752366252374
50743114473544537818170447134607102575676676056675328961590*e^(173*I*c) + 1
818346614061779013153301296771453811664491884131941411693443547549209690349
52610378945282257600*e^(172*I*c) + 4837948975643409984385779181658937940681
5042609340378747586437145781646245422045101230417309900*e^(171*I*c) + 12693
496932964920565073673637181280088548682508880255337280065006566138696041797
353216584528640*e^(170*I*c) + 328387476055581867672630948030673442015509858
3948074469014168171874442170109648521627538755920*e^(169*I*c) + 83757920692

341193245878648676537353394654523970899076948872481398218916510458989551890
 9256320*e^(168*I*c) + 21059430138564847118432907888031750495336183995415942
 7434009884661777259752542647709150036990*e^(167*I*c) + 52190912207661824215
 812271854269748071292843243227894769229690720010547141334131610989636000*e^(166*I*c) + 127472196165033205413563430625628473686016221408567860254458145
 32037904111523242298235713300*e^(165*I*c) + 3067974296431747364198159623962
 463671617006419626851426148418602934852907379021659761911840*e^(164*I*c) +
 727521010718394229291774073844694255798738667067535379759732795567942578751
 384250780476310*e^(163*I*c) + 169956327969929767773902096652629253283704505
 477127544556534417376686540936706073847337600*e^(162*I*c) + 391080312556018
 09476537535369611844440844903751605645023514572352045248104262933598850730*
 e^(161*I*c) + 8862752142756957285681340885764904597935349569355321815647721
 172537159186491471311666400*e^(160*I*c) + 197779298066581813565130009432623
 9158605448870806970860577325385028609983034534672318500*e^(159*I*c) + 43454
 667678028004534634449876389254079717510575682751550929702418766029934548492
 0192480*e^(158*I*c) + 93986915313068179149083606065681482780836060510530154
 618486949839467131378859885998210*e^(157*I*c) + 200080068030300471372932782
 50321597113540716201983333126349281186679153199068045257216*e^(156*I*c) + 4
 191542500656826148093339414544159143964478472492315931809171859902114109005
 939942952*e^(155*I*c) + 863979933622330349556296820028395513198708064940505
 702126068652936800794826651264256*e^(154*I*c) + 175193170500618300241515632
 381912285157790097816049220671217212220015297133400636060*e^(153*I*c) + 349
 410716132767046494779430433394502015040753351603618659160292138607786062306
 24960*e^(152*I*c) + 6852993223145736687328885311617795435592940841439866351
 079655652312894721972796266*e^(151*I*c) + 132149805527130085142999386663161
 9874424534425188183592049727687571032156435077280*e^(150*I*c) + 25050102860
 8928332469340456829902067712233644464602753159945727868485722395506952*e^(149*I*c) + 46668223548266017806854592468100570289355960869613650856575756758
 180182223308768*e^(148*I*c) + 854301344112621233483354066506962147247908583
 8041360564550722036723654297540205*e^(147*I*c) + 15363332384449275835327345
 56016494671674916578907116984548489078241693926940560*e^(146*I*c) + 2713612
 07503266570734486517077181014801775322183181055638619257836143271472358*e^(145*I*c) + 4706504461113515810848735336748424310269824883831263587628309942
 7442745866704*e^(144*I*c) + 80137295807907524343619649457615437614695207912
 10746972675870481058674277844*e^(143*I*c) + 1339214374254245553564884406801
 945353385000254030655765953770237607180089968*e^(142*I*c) + 219601281339515
 561500261478844190024870555261281946058839614044697037963695*e^(141*I*c) +
 35324447206779018115378052820789411687581004582367431006205879633729015200*
 e^(140*I*c) + 5572551157328671121016216416307596161861955969011697222340926
 210112854418*e^(139*I*c) + 861884851094991908764246805474672428603757315484
 453974713612812215428992*e^(138*I*c) + 130657660226560419335121434389938961
 884595434069984824307149332131747540*e^(137*I*c) + 194079792155945665935350
 08103303255257745408070082431338945184797463936*e^(136*I*c) + 2823905151936
 586678382525706564457280290098698638597987628380245881715*e^(135*I*c) + 402
 349692266121158934003582839428785116904903936409545602519219664720*e^{(134*I}

$\ast c) + 56117081076341175384087570185188538660375932013674735519055227368366\ast$
 $e^{(133\ast I\ast c)} + 7659010520187549651777118357676871927081898989131125755798204$
 $236112\ast e^{(132\ast I\ast c)} + 102253643746829673729306586270524644969368741555986584$
 $4306888705423\ast e^{(131\ast I\ast c)} + 13349021005202618377967331386833230353033290616$
 $3247194627808410304\ast e^{(130\ast I\ast c)} + 17033886027390615741040977721655541665612$
 $162275485028584310890417\ast e^{(129\ast I\ast c)} + 212370296918887131826671878122392706$
 $7839949015727293884065388080\ast e^{(128\ast I\ast c)} + 25858534871597727015582911568419$
 $3411072034541491364393985491350\ast e^{(127\ast I\ast c)} + 30736217404321009965231037419$
 $663053962881035281709221697785072\ast e^{(126\ast I\ast c)} + 356476489062872401708848799$
 $6688178929195787613958545474804845\ast e^{(125\ast I\ast c)} + 40321222595779818884084613$
 $9960995624144491271694336796459584\ast e^{(124\ast I\ast c)} + 44456708175258821024400946$
 $210535004523775722190977468484496\ast e^{(123\ast I\ast c)} + 477539860710085326353420773$
 $3818266777478693412738731031680\ast e^{(122\ast I\ast c)} + 49946750655853173367158586291$
 $0572702811545035730398749530\ast e^{(121\ast I\ast c)} + 50836369508171099437019348610847$
 $391946736185108017183136\ast e^{(120\ast I\ast c)} + 503202490340145182407421394376601192$
 $2026507006311982753\ast e^{(119\ast I\ast c)} + 48409341024048871865591702530366258109165$
 $9126182344528\ast e^{(118\ast I\ast c)} + 45230940039830738332025694784646206844854827698$
 $075736\ast e^{(117\ast I\ast c)} + 4101545439937195793959956708442496709433800261224880\ast e$
 $^{(116\ast I\ast c)} + 360688613036389349413809780004559963548775423325255\ast e^{(115\ast I\ast c)}$
 $) + 30735366512830562160991166338490057308062762518496\ast e^{(114\ast I\ast c)} + 253566$
 $7460650279776834561566186591213109251642859\ast e^{(113\ast I\ast c)} + 20234750972446217$
 $1313966643580234078508179838320\ast e^{(112\ast I\ast c)} + 15603911277687607099721623771$
 $744933086920587272\ast e^{(111\ast I\ast c)} + 116158141373397175153362251190904691718876$
 $8400\ast e^{(110\ast I\ast c)} + 83380839911837894453136303673785039051506805\ast e^{(109\ast I\ast c)}$
 $+ 5764601046563151304213854710715346838447392\ast e^{(108\ast I\ast c)} + 38336015580105$
 $4824529764688213114368047154\ast e^{(107\ast I\ast c)} + 24489837337812338687718622491865$
 $013839488\ast e^{(106\ast I\ast c)} + 1500602747937397286405577818722691539392\ast e^{(105\ast I\ast c)}$
 $) + 88054927598941411145869950813388040256\ast e^{(104\ast I\ast c)} + 493966661081802579$
 $8809586352543471345\ast e^{(103\ast I\ast c)} + 264410375780310742518099326419685040\ast e^{(102\ast I\ast c)}$
 $+ 13477227799524701956579274210395326\ast e^{(101\ast I\ast c)} + 652650253343206$
 $047453620559993840\ast e^{(100\ast I\ast c)} + 29952547749265499675257842032197\ast e^{(99\ast I\ast c)}$
 $) + 1299146645993240318167826532288\ast e^{(98\ast I\ast c)} + 53090127264630963470039804$
 $475\ast e^{(97\ast I\ast c)} + 2037031259470368160131922320\ast e^{(96\ast I\ast c)} + 7309920781733559$
 $7247098038\ast e^{(95\ast I\ast c)} + 2442455629894502983849104\ast e^{(94\ast I\ast c)} + 755998170926$
 $70157806639\ast e^{(93\ast I\ast c)} + 2154864144781257856128\ast e^{(92\ast I\ast c)} + 56169444526926$
 $562260\ast e^{(91\ast I\ast c)} + 1327882849274858880\ast e^{(90\ast I\ast c)} + 28186192554792138\ast e^{(89\ast I\ast c)}$
 $+ 530563624556832\ast e^{(88\ast I\ast c)} + 8718181624155\ast e^{(87\ast I\ast c)} + 1225037230$
 $56\ast e^{(86\ast I\ast c)} + 1431118260\ast e^{(85\ast I\ast c)} + 13343760\ast e^{(84\ast I\ast c)} + 93096\ast e^{(83\ast I\ast c)}$
 $+ 432\ast e^{(82\ast I\ast c)} + e^{(81\ast I\ast c)})\ast \tan(1/4\ast d\ast x + c) + (26\ast I\ast a\ast e^{(1055/2\ast I\ast c)}$
 $+ 10504\ast I\ast a\ast e^{(1053/2\ast I\ast c)} + 2116556\ast I\ast a\ast e^{(1051/2\ast I\ast c)} + 283618504\ast I\ast a\ast$
 $e^{(1049/2\ast I\ast c)} + 28432755026\ast I\ast a\ast e^{(1047/2\ast I\ast c)} + 2274620402080\ast I\ast a\ast e^{(1045/2\ast I\ast c)}$
 $+ 151262256738489\ast I\ast a\ast e^{(1043/2\ast I\ast c)} + 8600339740332756\ast I\ast a\ast e^{(1041/2\ast I\ast c)}$
 $+ 426791859624382434\ast I\ast a\ast e^{(1039/2\ast I\ast c)} + 18778841824711012321\ast I\ast a\ast$
 $e^{(1037/2\ast I\ast c)} + 741764252188078830689\ast I\ast a\ast e^{(1035/2\ast I\ast c)} + 265686468592656$
 $46058950\ast I\ast a\ast e^{(1033/2\ast I\ast c)} + 870123185139944470654786\ast I\ast a\ast e^{(1031/2\ast I\ast c)} +$

26237560685859258651673169*I*a*e^(1029/2*I*c) + 73277758893937414324950340
 6*I*a*e^(1027/2*I*c) + 19052217362358251291769232228*I*a*e^(1025/2*I*c) + 4
 63207036476152149238073238832*I*a*e^(1023/2*I*c) + 105720194834028113019757
 46281474*I*a*e^(1021/2*I*c) + 227298420835979818145918471098570*I*a*e^(1019
 /2*I*c) + 4617746921046245570620730707402520*I*a*e^(1017/2*I*c) + 888916297
 10203223556668933908311334*I*a*e^(1015/2*I*c) + 162544698001533217775591801
 4501608982*I*a*e^(1013/2*I*c) + 28297555097893393021020676691580988233*I*a*
 e^(1011/2*I*c) + 469985499033039677874623273935334240076*I*a*e^(1009/2*I*c)
 + 7461020183678979972532247003767832497030*I*a*e^(1007/2*I*c) + 1134075143
 49957436967142625811402104148561*I*a*e^(1005/2*I*c) + 165313275328330015489
 5933847581652791644965*I*a*e^(1003/2*I*c) + 2314386102212629093579514744696
 7794330788764*I*a*e^(1001/2*I*c) + 3116155990099984570554286265626112184401
 52906*I*a*e^(999/2*I*c) + 4040258091712579222349027082245857300045844265*I*
 a*e^(997/2*I*c) + 50503236456021701342291384690765044913121286031*I*a*e^(99
 5/2*I*c) + 609297262840711915743663065579736975124435730278*I*a*e^(993/2*I*
 c) + 7102123372769126439065816004234917371096063247516*I*a*e^(991/2*I*c) +
 80060329141110785684222094424093216574599312586835*I*a*e^(989/2*I*c) + 8735
 99857808953513616291826808640309239785388357815*I*a*e^(987/2*I*c) + 9235203
 345455625183635292584058316806336315160604010*I*a*e^(985/2*I*c) + 946608933
 71344177720163398371692357473361249542281324*I*a*e^(983/2*I*c) + 9414928240
 21872538571407241795810657020217893835828847*I*a*e^(981/2*I*c) + 9092846502
 857218261462182868864364624762389989516884454*I*a*e^(979/2*I*c) + 853329543
 26120674272311677629126536894715822244726480090*I*a*e^(977/2*I*c) + 7786641
 37844059492687126944618806483697276408481076541824*I*a*e^(975/2*I*c) + 6913
 027812800192462704129875030929367293369791365329368306*I*a*e^(973/2*I*c) +
 59748408260336333152741949120162468054098096806527992694330*I*a*e^(971/2*I*
 c) + 502999167177513108237013793834321163983016591115064062192504*I*a*e^(96
 9/2*I*c) + 4126888421983249887687507689195856719564200342340690922960020*I*
 a*e^(967/2*I*c) + 330151891834421057317309521026860785545568114800871344256
 69934*I*a*e^(965/2*I*c) + 2576627521764579828166282791344882800098730222812
 48928269109247*I*a*e^(963/2*I*c) + 1962629045379290087751577541491949170474
 015064554905158361368638*I*a*e^(961/2*I*c) + 145971076634287904419124491510
 55569782655551054988172538106007306*I*a*e^(959/2*I*c) + 1060529033630690131
 35578220340114498079609654804470793711887113055*I*a*e^(957/2*I*c) + 7529792
 23311236135673287737101570895147168873416246633317325038975*I*a*e^(955/2*I*
 c) + 5226590054479657822483780963008017596290103718605086715148151861646*I*
 a*e^(953/2*I*c) + 354807234382317128446847851058475709108605682492800809676
 79923825908*I*a*e^(951/2*I*c) + 2356471921691593101191613443407382001865279
 34151729251738259408174007*I*a*e^(949/2*I*c) + 1531718700483241796992497361
 844851327831244614221060296959000799843324*I*a*e^(947/2*I*c) + 974738630828
 1902182329846941952263067390253415184368339278176002138446*I*a*e^(945/2*I*c
) + 60747701724909506776211674773541506177265806300575091658980807046860704
 *I*a*e^(943/2*I*c) + 370884792983254000550721425873326574910124282599731660
 698821060848882320*I*a*e^(941/2*I*c) + 221894153908280997998580146226159783
 9684351348912275577377603658269150756*I*a*e^(939/2*I*c) + 13012954846383921

360783439001134235337905843280455893166840706360597524134*I*a*e^(937/2*I*c)
+ 748256379263409799262224519339923201219263623120654612723421068783014485
36*I*a*e^(935/2*I*c) + 4219747349096187780741342759182006525742147279404438
44534502373085580328604*I*a*e^(933/2*I*c) + 2334517419885966874350965488026
816679466208310259855732507435105175313457396*I*a*e^(931/2*I*c) + 126733614
19138834940617178471273655660069856526406967342118703762672110480200*I*a*e^(
929/2*I*c) + 6752682980371408820784121748009856094383803689375257216147308
9623117139971604*I*a*e^(927/2*I*c) + 35322638927283492431418931705836395420
3171842049410810171865734368883820818996*I*a*e^(925/2*I*c) + 18143510046388
30535450974649080451232909962840518095183307668250502738979794444*I*a*e^(92
3/2*I*c) + 9153283321740997122951578693088645450336613285754578953646231535
187679288519816*I*a*e^(921/2*I*c) + 453641734231472475081355495620115384286
83560420839172608314747026780336705078884*I*a*e^(919/2*I*c) + 2209122678371
76402457239938736238943107985690499222615150129577567013343972814900*I*a*e^(
917/2*I*c) + 1057267590272325636639773903752531171523549537193854805338038
288437194124266978930*I*a*e^(915/2*I*c) + 497385563358030362012603263786947
7500107676182499873076094518413397303603804906240*I*a*e^(913/2*I*c) + 23005
253792558757618650193250261667174892446545331020900285683620126145948425151
860*I*a*e^(911/2*I*c) + 104632479285330168704567624173205774202975142869561
651398142015702621321511418637970*I*a*e^(909/2*I*c) + 468046912422543827579
364617290976131668618292605267581426808880930942056836225740850*I*a*e^(907/
2*I*c) + 205954451320666914305466797161323566250098209629523881816279242995
4428814629558400860*I*a*e^(905/2*I*c) + 89163138809086160619416729929999844
82986075785970885880652620253394990773311152181860*I*a*e^(903/2*I*c) + 3798
422694278446672806696331835611313763490501428878526184498033912143169286512
1528770*I*a*e^(901/2*I*c) + 1592555129036669659473920504826277765285552562
8647598071487710356765619338164447678480*I*a*e^(899/2*I*c) + 65724378186986
8584332142745237797306576908038181174879375861022306506494725134312464240*I
*a*e^(897/2*I*c) + 26703310804381367392516994211884771528169838826465183136
18300055190498774861721956084120*I*a*e^(895/2*I*c) + 1068253687813551381159
8686678158308084092593434324791253827853349664088304211712490029720*I*a*e^(
893/2*I*c) + 42083973507831971365951079505656822321370151622842020026197681
285864461893407471640353960*I*a*e^(891/2*I*c) + 163287484325599994722817042
441220130107697316248983794524997603902832643977337412661872760*I*a*e^(889/
2*I*c) + 624082643961029037312709284672126976697873212538908890262985442647
874324392878156690829740*I*a*e^(887/2*I*c) + 234986055260470027437041195245
2920438650006836481438508181807279875096925888687475349639360*I*a*e^(885/2*
I*c) + 87178451215385439650006555089275761564545099651245782276661118615995
30470473165494741121930*I*a*e^(883/2*I*c) + 3187118311465183573627822548071
9985131360742804792653167920265913904648857223319205558481280*I*a*e^(881/2*
I*c) + 11483195709771919270611169705763287240925693313421210595325532758731
5571466191225892722458940*I*a*e^(879/2*I*c) + 40780746171419707117887262008
2184411365896862940021562248885259817851441393751779677614254090*I*a*e^(877
/2*I*c) + 14276653476597153178712146836003013566298180781570118096313232006
83665015207096525546411654850*I*a*e^(875/2*I*c) + 4927496729283083450012099

767840203352055720285319781857700963595150774626713503066668245663200*I*a*e
^(873/2*I*c) + 167688628137425704973593276857456712573650589092714891128297
60986508383352816248603242238251460*I*a*e^(871/2*I*c) + 5627373794228121721
583257810007323585827362849302717470711690545549860055299447701735333569313
0*I*a*e^(869/2*I*c) + 18624270141530015026036766190979622193201973253702463
4010976646542647178366376613371237978468950*I*a*e^(867/2*I*c) + 60795232506
538055223176587897936433792261856200178968194469267330870452834002467117504
9604847740*I*a*e^(865/2*I*c) + 19575864696310598451382153078727701034973417
41976050146298563390984316998623891058083233354950720*I*a*e^(863/2*I*c) + 6
218376119870852927756620966644089546420244007571780243876369631090631048335
601455624785708312030*I*a*e^(861/2*I*c) + 194885761759582598684920866636691
35993891670326935036733424079659395654947395041671226077968039510*I*a*e^(85
9/2*I*c) + 6026593628490055787427805827112924362891485407231105426369995644
4497953786622754638704751947767300*I*a*e^(857/2*I*c) + 18390480722449191984
653021156617286527564614860938919747705723738915839721060770433508066830767
6320*I*a*e^(855/2*I*c) + 55383894059703720980142009333903146592834534246696
8134327170356436277614050695088234925854972461590*I*a*e^(853/2*I*c) + 16461
983593387429705294567317256050448984847501474521523234085808828867857964631
37774250844390402000*I*a*e^(851/2*I*c) + 4829780128388681541713419953174190
238612578362193415201128746557290101817249355474415799153209807700*I*a*e^(8
49/2*I*c) + 139880302353639240750491684435137322249209761735967189805879375
80972858259676577959002425056421304360*I*a*e^(847/2*I*c) + 3999507572369123
703819527876878059983261935298150260058394447451097584068838826531558201087
1954211560*I*a*e^(845/2*I*c) + 11290502937326806460867891716679720547384498
7041134184835386265805093053876406216627623784532569846520*I*a*e^(843/2*I*c
) + 31471121452937142499278871384574742584918973031441639870412126582700441
7977443472234197727267055736520*I*a*e^(841/2*I*c) + 86623957642181530750728
127464111311550190054206425835868194515351762023530664437686576191224355985
1440*I*a*e^(839/2*I*c) + 23546413672838444747557085543861571414604520196309
66364104906530187054447060414009835524070257156040960*I*a*e^(837/2*I*c) + 6
321293101377705534598501434563578869433768111389255715203589457357528663969
729197548539465525802001790*I*a*e^(835/2*I*c) + 167615577354420865588650781
906925881608190526290450076565027095315479927354840332703932411636818598565
40*I*a*e^(833/2*I*c) + 4390181120150456850569009393359897486332167308618300
3850263648062593209407109355666758585680043907834980*I*a*e^(831/2*I*c) + 11
359046846756325605798652651416795478177684709910176760999437070925021737758
6572896526050715752951812430*I*a*e^(829/2*I*c) + 29035145179318989889949274
155669704377722161796952473194288358946558861451206284482579042928487292316
1710*I*a*e^(827/2*I*c) + 73326261594425025070029099259126214159993758041023
2623396266193728726497598828508117264547235381193884620*I*a*e^(825/2*I*c) +
18296954886447219366136705626726114716446217590419862915929740000765974011
10348056876443996820050899846880*I*a*e^(823/2*I*c) + 4511403508274736689214
557264253227256924552084399115779078878703890873777350251660642170480830307
883930190*I*a*e^(821/2*I*c) + 109922736874110426596082787940287968274450210
55329801721849776762014980912735003773441507133726549972274140*I*a*e^(819/2

*I*c) + 2646887067379749675909233214252546196368008033957904180984157401017
4923923017960550488705783118611025231900*I*a*e^(817/2*I*c) + 62991788302738
889334378246177490929980502834395575555703321916458754517369753884106316753
560824690112805080*I*a*e^(815/2*I*c) + 148170372481752681412605856390708157
556929105970750484058252083383402807240476275164707994006284231733473540*I*
a*e^(813/2*I*c) + 344505081824778437645788999066343041845046916298803112223
744796287266870120059274579365813673652734652520860*I*a*e^(811/2*I*c) + 791
797833199627836948743904893083516778740662517693894919607742925640625105771
158237179198488030147594263020*I*a*e^(809/2*I*c) + 179905602323577179718681
091771937925464884402029134811419999527730672592178065188326666288499614699
9653568600*I*a*e^(807/2*I*c) + 40412318710901090466871534282586219688752552
62763820985144556783792101620081675023154167930533684581024874268*I*a*e^(80
5/2*I*c) + 8975285853701358274702317193638300392915140185413802501007132352
939624150862231577879738561005464285728950292*I*a*e^(803/2*I*c) + 197094604
905649653855618030405974776279198936239645498502512476341445027319617021654
80743287298391566662175368*I*a*e^(801/2*I*c) + 4279759582983167298530610807
902207660664832901864889720299363805718613562117178440851177655572421630342
7661062*I*a*e^(799/2*I*c) + 91898464215137332126596549205674856438317557197
351584304760723812330051881370624842695522772767931492290924348*I*a*e^(797/
2*I*c) + 195149410499491059972295971340541623205014478485259218183564112297
195495432447981697623978433168097927188489160*I*a*e^(795/2*I*c) + 409847262
333561884793176079911000774943237907896725050192873946316017656875993526712
458367191600832293291122032*I*a*e^(793/2*I*c) + 851330476367280675931975863
605407903147652289552972334476403837141273395148705506900158560019626231978
044203758*I*a*e^(791/2*I*c) + 174912593443959921315268323941947738188912530
5076480377326824972897585677418916594095435586707281927023930690932*I*a*e^(
789/2*I*c) + 35548069879901977764458692863155390035136489849940791238066624
96505394242907964145674445025619936991568879491783*I*a*e^(787/2*I*c) + 7146
743752116835696646457888685953635190050205364442589239754498947790079457620
288669958662869113747109252011972*I*a*e^(785/2*I*c) + 142142105345772785134
655638543704332319345315321319820871878648165896494954210260296992075052433
96560387206439630*I*a*e^(783/2*I*c) + 2796949758150298987804402115989629470
528185352041480180085623067166742199429392578144745102255719004249039363454
3*I*a*e^(781/2*I*c) + 54452642914272730607209059956928793329411850728280306
564009762144470353759860633440913049919720954752768197793167*I*a*e^(779/2*I
*c) + 104893986154804101296916258335583569488825031337940879705887720891017
009057737287233126607468682692414623104616138*I*a*e^(777/2*I*c) + 199942201
925298499861324281146334926533714538945870178594046781580871260287770985229
020625257582980635730225596574*I*a*e^(775/2*I*c) + 377142540527901948948638
950467507037490336862439884119528845344255236737968714764289755834224007993
516760655927631*I*a*e^(773/2*I*c) + 704009729937813921690706496710889899788
434054131439662107181131420712162600554109916450181425438322392204166968902
*I*a*e^(771/2*I*c) + 130061517427101052448113664695315832473694446405293294
6836830203015550268173675607727307084706325098884799728006340*I*a*e^(769/2*
I*c) + 23781592730321211106042463797053005815111645937589678722392818969981

73201558987639246325006007724766172710488108360*I*a*e^(767/2*I*c) + 4304076
708196868623182033776806377443199259967769349066155975950428294505280637590
876357992866456385230699336439762*I*a*e^(765/2*I*c) + 771065067832387504303
985863253313532066682438658369247383646577680905625502860759679978431131374
3491088689654262026*I*a*e^(763/2*I*c) + 13674110058458123094352164694691456
954602488602500355510203822923728942351461390874202410489991829018224022172
756144*I*a*e^(761/2*I*c) + 240065213305186194101177870502624116083503068842
59736598396370271394691940783119373097697849962988467299495366349818*I*a*e^(
759/2*I*c) + 4172599134954602125985964954763710995733952767863225515182161
583206688874425029726202125176968278133786685507865790*I*a*e^(757/2*I*c) +
71805309593088734534182328800742749493154891157519618233998054566080846925
267026007081349149718882656641899769699343*I*a*e^(755/2*I*c) + 122350052518
531164014106438492390774762115503639398988470214202703839744840033391227823
927738136823467053920486003100*I*a*e^(753/2*I*c) + 206430920167470779129078
232414834138915376857125870091984391255439368652648269437789361363756117101
262608074750774122*I*a*e^(751/2*I*c) + 344900353549768914526411942374597368
779545029739787329071887057800090344123223642583873423162351462269773310642
073463*I*a*e^(749/2*I*c) + 570671487070895844624376668699871118872005188740
963481132840070150675796254780502758777604671643556111867698875355315*I*a*e
^(747/2*I*c) + 935142309058644648996306800248685199060934241266676135556650
246097289205034092387373949739321723050371978110125781708*I*a*e^(745/2*I*c)
+ 151772675772323885245833852336431135734569046757961595531551453905021540
0801547691758586164717780698532261211210676134*I*a*e^(743/2*I*c) + 24398277
418473876406488597345779264755429792109177032067023126683130955186856176073
72855661697190253696743286780667183*I*a*e^(741/2*I*c) + 3885077898135324755
861645497801928788835406962443757329897190841925861451659303725355539278317
874676713417912602151785*I*a*e^(739/2*I*c) + 612830727974489757665370637383
400835846202652485602861930650162676637289161428815346565423045111826183623
4442144927690*I*a*e^(737/2*I*c) + 95764982457308878595876534947130519638685
080921098936198286314893286212748964728000968281166192564222030014273289380
60*I*a*e^(735/2*I*c) + 1482597913528423746226719438636827970156048051160166
4623844483834931515122932857057720991188592107170607681823985077157*I*a*e^(
733/2*I*c) + 22741305878635147307038153488672825947987978019457328121790385
481218602329797137379235963979112570181508105761299864241*I*a*e^(731/2*I*c)
+ 345626886583356492112358151163225379977564500741633872843777815593138933
78880959669959729851744028098328487235184728902*I*a*e^(729/2*I*c) + 5205042
453660871068507693566203030807490703686374070277936847572552205529179649231
0303721502418829383396474602416972540*I*a*e^(727/2*I*c) + 77676555142903920
918742869451767200086404378777218239528959225606553643216179633134107155344
214086969807897418659931017*I*a*e^(725/2*I*c) + 114875803385901547035330733
037349925264590791175334294610155096056593848065467484845287083746515024420
618289251515633198*I*a*e^(723/2*I*c) + 168369630997561227559333636030680505
097931214578041180941450461063477255836777741106503139414783875934618403506
189272070*I*a*e^(721/2*I*c) + 244578820412941222413395448190925169184058675
448514568342704435560427553614905283048112187005746964090261842716041958472

$*I*a*e^{(719/2*I*c)}$ + 352141077922736730546898110712009539372175536582481680
 338223417210222349307364684263727611860803222344128198352327695010*I*a*e^{(7
 17/2*I*c)} + 502550470992195878626411838079850006659779195814543518223999781
 294288056896824411098205470727732618356886370847708708442*I*a*e^{(715/2*I*c)}
 + 710934676236178199315293629653949575146187423228339879602954615038356642
 192952138658369772402012030511506524366087182336*I*a*e^{(713/2*I*c)} + 996983
 594413543237951663813404787394193696145753482819044943532904006038513638159
 184208380334024402798975173818169325844*I*a*e^{(711/2*I*c)} + 138603841629113
 966677521737046806457587286373607779128668239428274584755056608200145082656
 3620627885230711507370056591878*I*a*e^{(709/2*I*c)} + 19103432234307886023721
 991489053166839100219424150083892704298897051771392550811012732189464248660
 81924816129975411078465*I*a*e^{(707/2*I*c)} + 2610451243923981260104534330124
 548228672173877100030304998230362054126473659359168603476643211595958178462
 911317428591330*I*a*e^{(705/2*I*c)} + 353676461326065445325819276169145836984
 896738655835870701106999280831997418448425789700221226148531872501461065980
 6587718*I*a*e^{(703/2*I*c)} + 47511697379014573962071767119679331793636774785
 81181612846070387281280000757129577641826784358070259730216727235818690609*
 $I*a*e^{(701/2*I*c)}$ + 6328710189082709801343939529235301914241975683533849906
 310574474924327748271043712210167104784210984705855467961916306161*I*a*e^{(6
 99/2*I*c)} + 835921585525270059274636632763640027045000249563752770351589923
 0399148367849214592564997399354344495040435849157657695842*I*a*e^{(697/2*I*c
)} + 10948781640409755536653618878020227586096531153252918259760911769700775
 752607091211059932659598053974790570736751499078948*I*a*e^{(695/2*I*c)} + 142
 209625593650747633143084901254747868167601051690380654084614802880470310216
 63614568854283660670738808692795002194734697*I*a*e^{(693/2*I*c)} + 1831752639
 108958245055655762254832610159791452992357965388879209915336157123282457319
 5459835233402581137624623986434143640*I*a*e^{(691/2*I*c)} + 23398582340521584
 219692739918602629576737063867809667736348081290947722283666066820099655484
 755971820828136322171921666098*I*a*e^{(689/2*I*c)} + 296418870982327739739856
 543305793794838212447693239802111754473132194627045318804008018024008069296
 46379343015130723883672*I*a*e^{(687/2*I*c)} + 3724112127270207428677853672812
 764225881806807255976590945106476682385942094525927174030983042033484910583
 5645038354134788*I*a*e^{(685/2*I*c)} + 46402932547295212381676170516766288251
 068082667686289662170986814423296955583080165076111831556746240823052636895
 914088352*I*a*e^{(683/2*I*c)} + 573425601373144520885417340865419166341645485
 251082885818236246323558588723509662923863264405230221528792916946902782419
 78*I*a*e^{(681/2*I*c)} + 7027789085614364628701759332010686918276096400284793
 0991714351968190586465418479358035827449094009745978784506403119811120*I*a*
 $e^{(679/2*I*c)}$ + 85421852279646559159940111327081238178956474950230875118810
 643314639931537207781953980988720021516214669798986825724375032*I*a*e^{(677/
 2*I*c)} + 102973123976091249841486572077221888501123401210539358250850710575
 960244576128548550903800052937848163824888063710773611688*I*a*e^{(675/2*I*c)}
 + 123105242970350923938651678117286443754910359407420767699678173998603913
 981285412874706049223826034210909894133154305711632*I*a*e^{(673/2*I*c)} + 145
 954292246109699719912222123781738335390625966506127269098354886794694991875

083928883625876737543812581750368360348693528*I*a*e^(671/2*I*c) + 171605486
 957459662750676535846047739137214911399784738960265764988070463454234816351
 437243248376883372505719428491381980392*I*a*e^(669/2*I*c) + 200079105947663
 271955632509515980999406183444585533117797849122654482023351610633374676069
 055506407150129677270455507223800*I*a*e^(667/2*I*c) + 231316348116947702743
 937847858556087111914188372627836780990986990309150303501014036898617069788
 744248382411616930060590160*I*a*e^(665/2*I*c) + 265165814533423959664985597
 822349856950188516226578299292400272539208068973637723841893399770487082840
 158358266555486981880*I*a*e^(663/2*I*c) + 301371417263542762474942803033428
 113198447518244218355398012742691434651542769453942299897409407388624108299
 633805109409320*I*a*e^(661/2*I*c) + 339562583656041705384495512461799490148
 091494844161116673933307762871742676972723303982143114815055600538598674333
 818006140*I*a*e^(659/2*I*c) + 379247649666166689932538744439842684315020161
 725910584926467825646640883618440200155492765803014212451201418879737084175
 200*I*a*e^(657/2*I*c) + 419811308608661332767758302246251767306850733653256
 595228909948287744169176955320554102884264686683175846620329681501389720*I*
 a*e^(655/2*I*c) + 460516895758104311450607127871317475876181781180615971932
 371325200463116468437759451218270512614846167126411504887103639420*I*a*e<sup>(6
 53/2*I*c)</sup> + 500514141116875694467964701453317062463709842637219399054983606
 548122976960401801645044637308666501943921790160591726142140*I*a*e<sup>(651/2*I
 *c)</sup> + 538852813238202598104514114397598537283154717840682848350121553631835
 881042049299653608342888243843496016154319713358492360*I*a*e^(649/2*I*c) +
 574502411709033654260208328538715135728998551050899066786172225938409580464
 129402448537468149317355804595816458074515452280*I*a*e^(647/2*I*c) + 606377
 755149130644685483641133783894074517633951559003505177038260609373787904347
 095041198589577627025869566789105127854940*I*a*e^(645/2*I*c) + 633369970466
 23115163959349863305250081699258822289708061904408614478071012324018301393
 989621376114495936107929638109352240*I*a*e^(643/2*I*c) + 654382036818887115
 292417789242500921067425262240728690563625955515997936064156892229191882650
 976483089469657316496979497920*I*a*e^(641/2*I*c) + 668367696536351754028217
 381429124369355105991706596728061963354863236879295944678255966593383757885
 721698666786195936988720*I*a*e^(639/2*I*c) + 674372239032754395378205038535
 525134413991821067808642593147982202493843652677927082541781810369810638842
 234476194328917280*I*a*e^(637/2*I*c) + 671573416376600344255476890840084107
 068748497197932624701404034109653368013046774679359731432461763294766813316
 740570321920*I*a*e^(635/2*I*c) + 659320582553919912075191619269245382876605
 084236933842646778081417120118613397607349078853158222767507670564574204234
 442160*I*a*e^(633/2*I*c) + 637170080700014936844214998715409219682319023159
 649705213690092322129022799756117985062386207907676879268835925347571814760
 *I*a*e^(631/2*I*c) + 604914946234938282221086113012403617143413939275335944
 189009476407229605663075886937693800533384123491506489579681833036880*I*a*e
^(629/2*I*c) + 562607154534249611414703842333518335752632730873777259062662
 265415265197318757342578842160394845870139211686607258163271980*I*a*e<sup>(627/
 2*I*c)</sup> + 510570917219603298930927086412148133828632499451055506210033280538
 039998585805570450486684217149260136863823464698025514400*I*a*e^(625/2*I*c)

+ 449405910836006748962393603823185186535716033707509621536220013769001652
 160266321868773593038071461031867255150268333080200*I*a*e^(623/2*I*c) + 379
 979787117931179687380662746570800769009286292103738378140902975891504813248
 606746718458290904172947709059780815722352620*I*a*e^(621/2*I*c) + 303409839
 775241340348800685971884147465797827674549775052181699893605576672194645212
 093459076166299568495347142098625376060*I*a*e^(619/2*I*c) + 221034257892392
 428506776513729559457247579927221728582941832739190163959979535188907184118
 401352847011402341171183153330400*I*a*e^(617/2*I*c) + 134373946463544483939
 193158440319190954779730332932854832052915711911612344383418766608011863070
 070271134159756438724511480*I*a*e^(615/2*I*c) + 450864052506273528333921347
 261618114674346898319541826591071586007968674493473147224142350757139877682
 52905857943168960940*I*a*e^(613/2*I*c) - 4508640525062735283339213472616181
 146743468983195418265910715860079686744934731472241423507571398776825290585
 7943168960940*I*a*e^(611/2*I*c) - 13437394646354448393919315844031919095477
 973033293285483205291571191161234438341876660801186307007027113415975643872
 4511480*I*a*e^(609/2*I*c) - 22103425789239242850677651372955945724757992722
 172858294183273919016395997953518890718411840135284701140234117118315333040
 0*I*a*e^(607/2*I*c) - 30340983977524134034880068597188414746579782767454977
 5052181699893605576672194645212093459076166299568495347142098625376060*I*a*
 e^(605/2*I*c) - 37997978711793117968738066274657080076900928629210373837814
 0902975891504813248606746718458290904172947709059780815722352620*I*a*e^(603
 /2*I*c) - 44940591083600674896239360382318518653571603370750962153622001376
 9001652160266321868773593038071461031867255150268333080200*I*a*e^(601/2*I*c
) - 51057091721960329893092708641214813382863249945105550621003328053803999
 8585805570450486684217149260136863823464698025514400*I*a*e^(599/2*I*c) - 56
 260715453424961141470384233351833575263273087377725906266226541526519731875
 7342578842160394845870139211686607258163271980*I*a*e^(597/2*I*c) - 60491494
 623493828222108611301240361714341393927533594418900947640722960566307588693
 7693800533384123491506489579681833036880*I*a*e^(595/2*I*c) - 63717008070001
 493684421499871540921968231902315964970521369009232212902279975611798506238
 6207907676879268835925347571814760*I*a*e^(593/2*I*c) - 65932058255391991207
 519161926924538287660508423693384264677808141712011861339760734907885315822
 2767507670564574204234442160*I*a*e^(591/2*I*c) - 67157341637660034425547689
 084008410706874849719793262470140403410965336801304677467935973143246176329
 4766813316740570321920*I*a*e^(589/2*I*c) - 67437223903275439537820503853552
 513441399182106780864259314798220249384365267792708254178181036981063884223
 4476194328917280*I*a*e^(587/2*I*c) - 66836769653635175402821738142912436935
 510599170659672806196335486323687929594467825596659338375788572169866678619
 5936988720*I*a*e^(585/2*I*c) - 65438203681888711529241778924250092106742526
 224072869056362595551599793606415689222919188265097648308946965731649697949
 7920*I*a*e^(583/2*I*c) - 63336997046623115163959349863305250081699258822228
 9708061904408614478071012324018301393989621376114495936107929638109352240*I
 *a*e^(581/2*I*c) - 60637775514913064468548364113378389407451763395155900350
 5177038260609373787904347095041198589577627025869566789105127854940*I*a*e^(
 579/2*I*c) - 57450241170903365426020832853871513572899855105089906678617222

5938409580464129402448537468149317355804595816458074515452280*I*a*e^(577/2*I*c) - 53885281323820259810451411439759853728315471784068284835012155363183
5881042049299653608342888243843496016154319713358492360*I*a*e^(575/2*I*c) -
50051414111687569446796470145331706246370984263721939905498360654812297696
0401801645044637308666501943921790160591726142140*I*a*e^(573/2*I*c) - 46051
689575810431145060712787131747587618178118061597193237132520046311646843775
9451218270512614846167126411504887103639420*I*a*e^(571/2*I*c) - 41981130860
866133276775830224625176730685073365325659522890994828774416917695532055410
2884264686683175846620329681501389720*I*a*e^(569/2*I*c) - 37924764966616668
993253874443984268431502016172591058492646782564664088361844020015549276580
3014212451201418879737084175200*I*a*e^(567/2*I*c) - 33956258365604170538449
551246179949014809149484416111667393330776287174267697272330398214311481505
5600538598674333818006140*I*a*e^(565/2*I*c) - 30137141726354276247494280303
342811319844751824421835539801274269143465154276945394229989740940738862410
8299633805109409320*I*a*e^(563/2*I*c) - 26516581453342395966498559782234985
695018851622657829929240027253920806897363772384189339977048708284015835826
6555486981880*I*a*e^(561/2*I*c) - 23131634811694770274393784785855608711191
418837262783678099098699030915030350101403689861706978874424838241161693006
0590160*I*a*e^(559/2*I*c) - 20007910594766327195563250951598099940618344458
553311779784912265448202335161063337467606905550640715012967727045550722380
0*I*a*e^(557/2*I*c) - 17160548695745966275067653584604773913721491139978473
8960265764988070463454234816351437243248376883372505719428491381980392*I*a*
e^(555/2*I*c) - 14595429224610969971991222212378173833539062596650612726909
8354886794694991875083928883625876737543812581750368360348693528*I*a*e^(553
/2*I*c) - 12310524297035092393865167811728644375491035940742076769967817399
8603913981285412874706049223826034210909894133154305711632*I*a*e^(551/2*I*c
) - 10297312397609124984148657207722188850112340121053935825085071057596024
4576128548550903800052937848163824888063710773611688*I*a*e^(549/2*I*c) - 85
421852279646559159940111327081238178956474950230875118810643314639931537207
781953980988720021516214669798986825724375032*I*a*e^(547/2*I*c) - 702778908
561436462870175933201068691827609640028479309917143519681905864654184793580
35827449094009745978784506403119811120*I*a*e^(545/2*I*c) - 5734256013731445
208854173408654191663416454852510828858182362463235585887235096629238632644
0523022152879291694690278241978*I*a*e^(543/2*I*c) - 46402932547295212381676
170516766288251068082667686289662170986814423296955583080165076111831556746
240823052636895914088352*I*a*e^(541/2*I*c) - 372411212727020742867785367281
276422588180680725597659094510647668238594209452592717403098304203348491058
35645038354134788*I*a*e^(539/2*I*c) - 2964188709823277397398565433057937948
382124476932398021117544731321946270453188040080180240080692964637934301513
0723883672*I*a*e^(537/2*I*c) - 23398582340521584219692739918602629576737063
867809667736348081290947722283666066820099655484755971820828136322171921666
098*I*a*e^(535/2*I*c) - 183175263910895824505565576225483261015979145299235
79653888792099153361571232824573195459835233402581137624623986434143640*I*a
*e^(533/2*I*c) - 1422096255936507476331430849012547478681676010516903806540
8461480288047031021663614568854283660670738808692795002194734697*I*a*e^(531

/2*I*c) - 10948781640409755536653618878020227586096531153252918259760911769
700775752607091211059932659598053974790570736751499078948*I*a*e^(529/2*I*c)
- 835921585525270059274636632763640027045000249563752770351589923039914836
7849214592564997399354344495040435849157657695842*I*a*e^(527/2*I*c) - 63287
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/2*I*c) - 34450508182477843764578899906634304184504691629880311222374479628
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*c) - 733262615944250250700290992591262141599937580410232623396266193728726
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- 9747386308281902182329846941952263067390253415184368339278176002138446*I
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27934151729251738259408174007*I*a*e^(275/2*I*c) - 3548072343823171284468478
5105847570910860568249280080967679923825908*I*a*e^(273/2*I*c) - 52265900544
79657822483780963008017596290103718605086715148151861646*I*a*e^(271/2*I*c)
- 752979223311236135673287737101570895147168873416246633317325038975*I*a*e^
(269/2*I*c) - 1060529033630690131355782203401144980796096548044707937118871
13055*I*a*e^(267/2*I*c) - 1459710766342879044191244915105556978265555105498
8172538106007306*I*a*e^(265/2*I*c) - 19626290453792900877515775414919491704
74015064554905158361368638*I*a*e^(263/2*I*c) - 2576627521764579828166282791
34488280009873022281248928269109247*I*a*e^(261/2*I*c) - 3301518918344210573
1730952102686078554556811480087134425669934*I*a*e^(259/2*I*c) - 41268884219
83249887687507689195856719564200342340690922960020*I*a*e^(257/2*I*c) - 5029
99167177513108237013793834321163983016591115064062192504*I*a*e^(255/2*I*c)
- 59748408260336333152741949120162468054098096806527992694330*I*a*e^(253/2*
I*c) - 6913027812800192462704129875030929367293369791365329368306*I*a*e^(25
1/2*I*c) - 778664137844059492687126944618806483697276408481076541824*I*a*e^

$(249/2 * I * c) - 85332954326120674272311677629126536894715822244726480090 * I * a * e^{(247/2 * I * c)}$
 $- 9092846502857218261462182868864364624762389989516884454 * I * a * e^{(245/2 * I * c)}$
 $- 941492824021872538571407241795810657020217893835828847 * I * a * e^{(243/2 * I * c)}$
 $- 94660893371344177720163398371692357473361249542281324 * I * a * e^{(241/2 * I * c)}$
 $- 9235203345455625183635292584058316806336315160604010 * I * a * e^{(239/2 * I * c)}$
 $- 873599857808953513616291826808640309239785388357815 * I * a * e^{(237/2 * I * c)}$
 $- 80060329141110785684222094424093216574599312586835 * I * a * e^{(235/2 * I * c)}$
 $- 7102123372769126439065816004234917371096063247516 * I * a * e^{(233/2 * I * c)}$
 $- 609297262840711915743663065579736975124435730278 * I * a * e^{(231/2 * I * c)}$
 $- 50503236456021701342291384690765044913121286031 * I * a * e^{(229/2 * I * c)}$
 $- 4040258091712579222349027082245857300045844265 * I * a * e^{(227/2 * I * c)}$
 $- 311615599009998457055428626562611218440152906 * I * a * e^{(225/2 * I * c)}$
 $- 23143861022126290935795147446967794330788764 * I * a * e^{(223/2 * I * c)}$
 $- 1653132753283300154895933847581652791644965 * I * a * e^{(221/2 * I * c)}$
 $- 113407514349957436967142625811402104148561 * I * a * e^{(219/2 * I * c)}$
 $- 7461020183678979972532247003767832497030 * I * a * e^{(217/2 * I * c)}$
 $- 469985499033039677874623273935334240076 * I * a * e^{(215/2 * I * c)}$
 $- 28297555097893393021020676691580988233 * I * a * e^{(213/2 * I * c)}$
 $- 1625446980015332177755918014501608982 * I * a * e^{(211/2 * I * c)}$
 $- 88891629710203223556668933908311334 * I * a * e^{(209/2 * I * c)}$
 $- 4617746921046245570620730707402520 * I * a * e^{(207/2 * I * c)}$
 $- 227298420835979818145918471098570 * I * a * e^{(205/2 * I * c)}$
 $- 10572019483402811301975746281474 * I * a * e^{(203/2 * I * c)}$
 $- 463207036476152149238073238832 * I * a * e^{(201/2 * I * c)}$
 $- 19052217362358251291769232228 * I * a * e^{(199/2 * I * c)}$
 $- 732777588939374143249503406 * I * a * e^{(197/2 * I * c)}$
 $- 26237560685859258651673169 * I * a * e^{(195/2 * I * c)}$
 $- 870123185139944470654786 * I * a * e^{(193/2 * I * c)}$
 $- 26568646859265646058950 * I * a * e^{(191/2 * I * c)}$
 $- 741764252188078830689 * I * a * e^{(189/2 * I * c)}$
 $- 18778841824711012321 * I * a * e^{(187/2 * I * c)}$
 $- 426791859624382434 * I * a * e^{(185/2 * I * c)}$
 $- 8600339740332756 * I * a * e^{(183/2 * I * c)}$
 $- 151262256738489 * I * a * e^{(181/2 * I * c)}$
 $- 2274620402080 * I * a * e^{(179/2 * I * c)}$
 $- 28432755026 * I * a * e^{(177/2 * I * c)}$
 $- 283618504 * I * a * e^{(175/2 * I * c)}$
 $- 2116556 * I * a * e^{(173/2 * I * c)}$
 $- 10504 * I * a * e^{(171/2 * I * c)}$
 $- 26 * I * a * e^{(169/2 * I * c)}$
 $)/(e^{(531 * I * c)} + 432 * e^{(530 * I * c)} + 93096 * e^{(529 * I * c)} + 13343760 * e^{(528 * I * c)} + 1431118260 * e^{(527 * I * c)} + 122503723056 * e^{(526 * I * c)} + 8718181624155 * e^{(525 * I * c)} + 530563624556832 * e^{(524 * I * c)} + 28186192554792138 * e^{(523 * I * c)} + 1327882849274858880 * e^{(522 * I * c)} + 56169444526926562260 * e^{(521 * I * c)} + 2154864144781257856128 * e^{(520 * I * c)} + 75599817092670157806639 * e^{(519 * I * c)} + 2442455629894502983849104 * e^{(518 * I * c)} + 73099207817335597247098038 * e^{(517 * I * c)} + 2037031259470368160131922320 * e^{(516 * I * c)} + 53090127264630963470039804475 * e^{(515 * I * c)} + 1299146645993240318167826532288 * e^{(514 * I * c)} + 29952547749265499675257842032197 * e^{(513 * I * c)} + 652650253343206047453620559993840 * e^{(512 * I * c)} + 13477227799524701956579274210395326 * e^{(511 * I * c)} + 264410375780310742518099326419685040 * e^{(510 * I * c)} + 4939666610818025798809586352543471345 * e^{(509 * I * c)} + 88054927598941411145869950813388040256 * e^{(508 * I * c)} + 1500602747937397286405577818722691539392 * e^{(507 * I * c)} + 24489837337812338687718622491865013839488 * e^{(506 * I * c)} + 383360155801054824529764688213114368047154 * e^{(505 * I * c)} + 5764601046563151304213854710715346838447392 * e^{(504 * I * c)} + 83380839911837894453136303673785039051506805 * e^{(503 * I * c)} + 11615814137339717515336225119090469171887$

68400*e^(502*I*c) + 15603911277687607099721623771744933086920587272*e^(501*I*c) + 202347509724462171313966643580234078508179838320*e^(500*I*c) + 2535667460650279776834561566186591213109251642859*e^(499*I*c) + 30735366512830562160991166338490057308062762518496*e^(498*I*c) + 360688613036389349413809780004559963548775423325255*e^(497*I*c) + 4101545439937195793959956708442496709433800261224880*e^(496*I*c) + 45230940039830738332025694784646206844854827698075736*e^(495*I*c) + 484093410240488718655917025303662581091659126182344528*e^(494*I*c) + 5032024903401451824074213943766011922026507006311982753*e^(493*I*c) + 50836369508171099437019348610847391946736185108017183136*e^(492*I*c) + 499467506558531733671585862910572702811545035730398749530*e^(491*I*c) + 4775398607100853263534207733818266777478693412738731031680*e^(490*I*c) + 44456708175258821024400946210535004523775722190977468484496*e^(489*I*c) + 403212225957798188840846139960995624144491271694336796459584*e^(488*I*c) + 3564764890628724017088487996688178929195787613958545474804845*e^(487*I*c) + 30736217404321009965231037419663053962881035281709221697785072*e^(486*I*c) + 258585348715977270155829115684193411072034541491364393985491350*e^(485*I*c) + 2123702969188871318266718781223927067839949015727293884065388080*e^(484*I*c) + 17033886027390615741040977721655541665612162275485028584310890417*e^(483*I*c) + 133490210052026183779673313868332303530332906163247194627808410304*e^(482*I*c) + 1022536437468296737293065862705246449693687415559865844306888705423*e^(481*I*c) + 7659010520187549651777118357676871927081898989131125755798204236112*e^(480*I*c) + 56117081076341175384087570185188538660375932013674735519055227368366*e^(479*I*c) + 402349692266121158934003582839428785116904903936409545602519219664720*e^(478*I*c) + 2823905151936586678382525706564457280290098698638597987628380245881715*e^(477*I*c) + 19407979215594566593535008103303255257745408070082431338945184797463936*e^(476*I*c) + 130657660226560419335121434389938961884595434069984824307149332131747540*e^(475*I*c) + 861884851094991908764246805474672428603757315484453974713612812215428992*e^(474*I*c) + 5572551157328671121016216416307596161861955969011697222340926210112854418*e^(473*I*c) + 35324447206779018115378052820789411687581004582367431006205879633729015200*e^(472*I*c) + 219601281339515561500261478844190024870555261281946058839614044697037963695*e^(471*I*c) + 1339214374254245553564884406801945353385000254030655765953770237607180089968*e^(470*I*c) + 8013729580790752434361964945761543761469520791210746972675870481058674277844*e^(469*I*c) + 47065044611135158108487353367484243102698248838312635876283099427442745866704*e^(468*I*c) + 271361207503266570734486517077181014801775322183181055638619257836143271472358*e^(467*I*c) + 1536333238444927583532734556016494671674916578907116984548489078241693926940560*e^(466*I*c) + 8543013441126212334833540665069621472479085838041360564550722036723654297540205*e^(465*I*c) + 46668223548266017806854592468100570289355960869613650856575756758180182223308768*e^(464*I*c) + 250501028608928332469340456829902067712233644464602753159945727868485722395506952*e^(463*I*c) + 1321498055271300851429993866631619874424534425188183592049727687571032156435077280*e^(462*I*c) + 6852993223145736687328885311617795435592940841439866351079655652312894721972796266*e^(461*I*c) + 349410716132767046494779430433394502015040

75335160361865916029213860778606230624960*e^(460*I*c) + 1751931705006183002
 41515632381912285157790097816049220671217212220015297133400636060*e^(459*I*
 c) + 8639799336223303495562968200283955131987080649405057021260686529368007
 94826651264256*e^(458*I*c) + 4191542500656826148093339414544159143964478472
 492315931809171859902114109005939942952*e^(457*I*c) + 200080068030300471372
 93278250321597113540716201983333126349281186679153199068045257216*e^(456*I*
 c) + 9398691531306817914908360606568148278083606051053015461848694983946713
 1378859885998210*e^(455*I*c) + 43454667678028004534634449876389254079717510
 5756827515509297024187660299345484920192480*e^(454*I*c) + 19777929806658181
 35651300094326239158605448870806970860577325385028609983034534672318500*e^(
 453*I*c) + 8862752142756957285681340885764904597935349569355321815647721172
 537159186491471311666400*e^(452*I*c) + 391080312556018094765375353696118444
 40844903751605645023514572352045248104262933598850730*e^(451*I*c) + 1699563
 279699297677739020966526292532837045054771275445565344173766865409367060738
 47337600*e^(450*I*c) + 7275210107183942292917740738446942557987386670675353
 79759732795567942578751384250780476310*e^(449*I*c) + 3067974296431747364198
 159623962463671617006419626851426148418602934852907379021659761911840*e^(44
 8*I*c) + 127472196165033205413563430625628473686016221408567860254458145320
 37904111523242298235713300*e^(447*I*c) + 5219091220766182421581227185426974
 8071292843243227894769229690720010547141334131610989636000*e^(446*I*c) + 21
 059430138564847118432907888031750495336183995415942743400988466177725975254
 2647709150036990*e^(445*I*c) + 83757920692341193245878648676537353394654523
 9708990769488724813982189165104589895518909256320*e^(444*I*c) + 32838747605
 558186767263094803067344201550985839480744690141681718744421701096485216275
 38755920*e^(443*I*c) + 1269349693296492056507367363718128008854868250888025
 5337280065006566138696041797353216584528640*e^(442*I*c) + 48379489756434099
 843857791816589379406815042609340378747586437145781646245422045101230417309
 900*e^(441*I*c) + 181834661406177901315330129677145381166449188413194141169
 344354754920969034952610378945282257600*e^(440*I*c) + 674025530543133008894
 845775236625237450743114473544537818170447134607102575676676056675328961590
 *e^(439*I*c) + 246438219080743960907977422685567962936788570977643587663085
 1716253962696192341706239192878728160*e^(438*I*c) + 88882950287510246670442
 03837607976101480053134418614474620767522824868911959884352666444917404000*
 e^(437*I*c) + 3162664467472554773117679568752765357130596998592368839211216
 4915553242573269490908989570248533280*e^(436*I*c) + 11103414879700881944314
 389564446924229504986746431371096925761933889913379928561602006987261171085
 0*e^(435*I*c) + 38465584208066627445406307878483717499894905250097532216200
 3392549953413592461519365177908682078400*e^(434*I*c) + 13150521209306921221
 022971053276228423358707434285308910729835358622800944466077234738004774539
 14130*e^(433*I*c) + 4437210917843182347764349544443904699020056595069470847
 193617092114714077633077234972825351226979360*e^(432*I*c) + 147779550966171
 289987127451820714953621765069731830816502336052740516776249704643402427558
 40025673760*e^(431*I*c) + 4858425815314028044731483686877213139019541241904
 6732778458706015096881437076337910793584122475073760*e^(430*I*c) + 15768584
 552885091872146287786443509025758314941556132342738656289444759827793562980

0939237175625149830*e^(429*I*c) + 50529366312301525887848302573881281320339
7766845340065381261016353419722382620393032535960660921950400*e^(428*I*c) +
15987711010581926922705289996774447426856310062324561858449252201440023058
78120380828483988663574829100*e^(427*I*c) + 4995241956279138180205186744401
688024388272113921255663734956946927571305533146776898787878059685108480*e^
(426*I*c) + 154131112114860239372949708207973767160813447881633865435224219
39737507962125854981881879168348260330000*e^(425*I*c) + 4697022472711728182
645404501807067052255975662758034778453532001496348263235972944454188510227
4546002560*e^(424*I*c) + 14137993825355684328056550580740330413060613072543
4751745794079833141361748917639986145377066437210546190*e^(423*I*c) + 42035
802483514679858361121014594215468443794936564789908837252480215622288483958
0011688655664280691773600*e^(422*I*c) + 12346680418924099787800180817554402
16012582476396941937965899631953079203974222138794604328498972144766900*e^(
421*I*c) + 3582718002163296061414536703715109897107198252739284546149343102
348456124089657428594946438660859773886240*e^(420*I*c) + 102716025302028890
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75463677418470*e^(419*I*c) + 2909765106124745340664756978183691006216555985
2359052804259154165687125428752562385492373749486351714453120*e^(418*I*c) +
81452081413829111828875417564250054846037693312811480492909160758195989155
768107022568350953861815940704090*e^(417*I*c) + 225320532593220657767941109
289516248999794521015564134982827241710019675486694499689312466561907212627
820000*e^(416*I*c) + 616003116022979584937125701757887212998354300989991362
628038861093914561332071191909714949426587936910303300*e^(415*I*c) + 166447
503438721180939491774350293763897857493775476476398783587241044993069013157
2904279995484581013965001440*e^(414*I*c) + 44454122592954746250676595141983
129660154162999689303933456303459141097207405736188849805200100284514969962
10*e^(413*I*c) + 1173585692624511849311309100250160403234187698599908282352
0672241530200188223826392982302194084667538488665600*e^(412*I*c) + 30627581
054221957378390547289277609129572813931082733520247387226000020043538279468
776707958420892547870128680*e^(411*I*c) + 790191495587665692547839884872323
883529091449827471718567724632238089933670915034028764672701761243426996544
00*e^(410*I*c) + 2015579474247940980267724784620408833958675125629308629437
53568690084015585598010154781548625239409581907397500*e^(409*I*c) + 5083245
993010854601669786296830326614276544740829390480979396383915672987957883894
33842285751054665210868287680*e^(408*I*c) + 1267597017294812913400146276042
126929986480292870190399107554311079964227280196522475370108738477856311765
699610*e^(407*I*c) + 312568349317870174347970475030749017866629215072017936
3604335113528623329684606343185540756019935662148267863968*e^(406*I*c) + 76
217887919120470620388409177993746004288922581943676366829443560966814002463
12138001769285020661445991073249416*e^(405*I*c) + 1837980708400335976602764
921762114411609173557221662078886153580344970227380258835907670424184073351
3439114113248*e^(404*I*c) + 43834972142919377685378692233021063744554033100
928502737480438978976746989895784070951905237783490374305934542955*e^(403*I
*c) + 103399755467257436489847837640753754718204394473055795001467604326419
87655556873829531737211096115196005647730480*e^(402*I*c) + 241246021282440

061792908317783032876194801597133206052091286997043729145345755805710081489
 006741839439573984832678*e^(401*I*c) + 556756388711182340341026192734219546
 113651768317380539005893679049394714017063698565272728813669054779077208977
 840*e^(400*I*c) + 127103308293804895020136055483127034266234399127750461234
 2300366025046741742856580445289401786656311685859023084716*e^(399*I*c) + 28
 704961314123144518346747153535894395532944308085319334660862885437092462307
 69151392180699413405623017247753532944*e^(398*I*c) + 6413381895855925184758
 231451062556380328594938511006577015218119786536390213057284018202201631094
 434819584025113465*e^(397*I*c) + 141764836528757049572020133432411179043699
 77796653849959902524421980635189011634815653279605497783382888932766730080*
 e^(396*I*c) + 3100431920606941707706936314142348743182800909818474463567865
 2284177439464941651812564519144918003174108077634846014*e^(395*I*c) + 67091
 706130529669125019899210021576580237843462229535386295087076189297849995931
 360645605292130961496106707521506432*e^(394*I*c) + 143657687138044796942947
 119704259538458818199423516824674586293691056119209866358123637772245409530
 799230553767222252*e^(393*I*c) + 304384471106813336010284160123906370433888
 828490627422652551236966790916174520857759143930140187173492394981908258944
 *e^(392*I*c) + 638218892914533741505806399662488556066783600496091876408374
 974877448971778036074996245581124283460438065182071976085*e^(391*I*c) + 132
 431132498402742835522293814768237867286070881716174144874968959358802086084
 7508703702325320304649883120684987556400*e^(390*I*c) + 27195892834837439260
 408051010803419212445303112546072509291927739093315232266350358156728625692
 96693711643521070331394*e^(389*I*c) + 5527498849031178355861230009326668283
 926290082158467118000698502719379939045918344222192742145711257028040974074
 674736*e^(388*I*c) + 111194996453632010808810628248863384924253756584489779
 35535846349290425821570383090425411418521516670371372045206568345*e^(387*I*
 c) + 2214073500170860327091518076924139166203557875590397914890921360382255
 4792749183517160255571915875356439553717130797888*e^(386*I*c) + 43638300075
 815171025946211464465689618965773488660985857945657479854085108851857911222
 911989837615452608512356008400295*e^(385*I*c) + 851395332347864557795899594
 649006377607357297056212213800058372083691577946730496754287998178754314302
 46332625899630160*e^(384*I*c) + 1644375006769068927413232601543942785039545
 619360201335965818064493572402773494479273345171058099953000935492799312731
 78*e^(383*I*c) + 3144090358082258615655954369383549454454739910431297220467
 47228813030925204968503418566818838611866709807040793495364496*e^(382*I*c)
 + 5951576155004315149474792823365470538279260879164252634971877570294133854
 71835434198246807096214536895441388306027237899*e^(381*I*c) + 1115398285545
 601550533328045600184317993899359217996729818340704221801195667410485846996
 179056733558512452238583160792512*e^(380*I*c) + 206969828950086064346166576
 237380795751301942404117887190496055182941244934472243212567941795840300755
 1179298315947373776*e^(379*I*c) + 38026049967058911069646206338489648070370
 988545101822632430305972956307603535975319743247522663891931857608782741880
 13440*e^(378*I*c) + 6917838945214844278493330459361394923372333853619879637
 372673184942859712431066345726870422099893124890777678037369988150*e^(377*I
 *c) + 124621404405372580849285967098720668577570709431248685545009481547568

63454308032925408340311237850017814707896986969086816*e^(376*I*c) + 2223134
113180153534540639903772168684020839794195258013558464596674673665671627155
4826476282991066076564921432614339399735*e^(375*I*c) + 39274200414329861169
397944516225001081227433398585007206399231211907157795359719648241598754266
579840244551491476467899952*e^(374*I*c) + 687124660159856415124685861736597
477348795917100983546527861249360230739431410495736066485630053594117127648
95683903806088*e^(373*I*c) + 1190605918496605468347656932276764490675841482
488826784475048260772363334445134540951266687500572958111916433569089721914
40*e^(372*I*c) + 2043255572651860007674027102308478964597615839227636982354
33212833313077783041040074669379017394836761539649081690630811665*e^(371*I*
c) + 3473100538109352904194555605559573141295692107357459832343696599764133
74774078000173070075248654524917179128950507443058208*e^(370*I*c) + 5847495
736823045861793846288448833275814989698865403803788967679990756149640071746
00811092945356635118795824799369716742109*e^(369*I*c) + 9752103394440493187
572823117635177866732231755944579463832792646350850410049173002959042754331
44848532459919875479817581584*e^(368*I*c) + 1611092541400060525954859375264
194178347643471837078201446262435615142944587337833513586022729849523358436
493586042252995608*e^(367*I*c) + 263666241043079934044752228477824428374075
106865814072657644667120779832560683229593770506168629793029638233889257490
0819440*e^(366*I*c) + 42748269077205917525267113368208715008443456479223854
71534359333606189571832444641364132893108663576205133870672156264164115*e^(
365*I*c) + 6866425337518668262662693750908956965732924578142181630622157802
899880874681551031136314064199948604001529894566235238597088*e^(364*I*c) +
109272106034735448102797923478445360745888968062300411100895447316058631461
04181739039426674855453466097402330688331845602302*e^(363*I*c) + 1722950282
436764733440072199841759670394865739473880520939163659737038057239896471508
0095366818322029152193635869784095333760*e^(362*I*c) + 26917794701086615097
890120236890501105146799996021775195710866226228638984703456832694153230611
607263444183501026198563419616*e^(361*I*c) + 416704403753905436434182193422
717480400350714901190805852815224981888183759069003687012345313046331634463
19945130196476913600*e^(360*I*c) + 6392301943376198909061480128863509812319
944510230312261654464820899876780394445577704288655273849974718371313606910
4651812215*e^(359*I*c) + 97173055024742680058616722461368892661141295540263
493013032746083536157324268333390400308958318370219154887169702257444756176
*e^(358*I*c) + 146390448456351181218237382740374124191664819997746988076598
391862733629670142241546375533903130605297580105675355629160198162*e^(357*I
*c) + 218563166659649312247483640956272149212499115383828771029654283363972
585118090479413696638108156385244646591328454425745117584*e^(356*I*c) + 323
413178014841003714151138246079152576360976035058457890409937738723171537036
573043681997163745313602400139153046673668433091*e^(355*I*c) + 474323043563
100542377338629931966248129175976982332446018056009391154020438895903140822
967769494019446166779954024655344116288*e^(354*I*c) + 689518449328793559903
260418149974190253578340058895035589606468244680591556118170304005037563669
880057908765898949268614772285*e^(353*I*c) + 993555653649521127226443960820
233649386488510081892545866700444096661582790441241830855609577062039555625

090943332264901780720*e^(352*I*c) + 141916448142217657385823401389899962882
 232233330957373071631074383893583220145429361729317508645838962142530705175
 0612129761498*e^(351*I*c) + 20094961100926877381527820856837372227279688242
 990587392154460834993514676253344496707577640666906561509490149440439948228
 23920*e^(350*I*c) + 2820819298562215959107529807289628449621386798989436369
 393116069894018781201000275633104498398959346631795568022519974400130281*e^
 (349*I*c) + 392569765841577835276810394285601184021164276962171721799661439
 8473887186074391482638547212826538270453912634540299792270321024*e^(348*I*c
) + 54166628040524363495855959828183579538662584616443540182051589177425764
 25344364964596750653177677803492186817305171175032011500*e^(347*I*c) + 7410
 372612891152224364633296128043971657193280327754304382235672773781403023814
 127610355562505271969045177704726054907145784960*e^(346*I*c) + 100522095243
 695818275881549853455496780314457444999852082593854356097402724003014542468
 72041775159838468077381562338745636398374*e^(345*I*c) + 1352123041194543691
 555885470685179654156739981165687002156735297532681546781784653328912387169
 6056195231696146162720992221760992*e^(344*I*c) + 18035327338177455471177568
 594851682977978346449777193572087688510392426884519272991560851326393852241
 961470040819793627127923997*e^(343*I*c) + 238563985655628020306952781742126
 408332821541740064594582926440609479249073137359215616901539399060175186471
 82491616573724049744*e^(342*I*c) + 3129526368818983831377277587330726033411
 722725862950199235869563609266286606281984568906423581362297415012066892139
 1878398978380*e^(341*I*c) + 40715988963701918950020348336736423420513311359
 010485246919074652883393970805374470830156229705647312265477584256027212762
 941040*e^(340*I*c) + 525392233467407711425870923702570695360603196444395016
 67610482767955800276052892432152798814607975110366224945081428121888473324*
 e^(339*I*c) + 6724408796908070382370325719966304760689061048209049461949280
 2935130215979819469966383336788693900139115594646893784095418472336*e^(338*
 I*c) + 85368118430215312848231291739673735887746201851666299600392199418764
 750086828198719872744047767783667325326289221881974987582215*e^(337*I*c) +
 107504737406576916123480399169759633328321407419400510017498849830598621565
 428266546315933920821527544726380201659114903834605888*e^(336*I*c) + 134297
 742023479429904629616104559610096074758721068022704468938063017059688023363
 436458971534964665036319889119229809973806909680*e^(335*I*c) + 166432332922
 589195130558329266398753389823955737598527556096093062473559769545772321978
 969318904192572733997888230986469005970880*e^(334*I*c) + 204622295535729109
 519829916789867225319429705162560082648840394965423112809336591921290309392
 396263834674368977840527147037426908*e^(333*I*c) + 249593072282565866398389
 951509619202682634455487128714631461891770823201367527645793770203788784677
 343934971424317987895255031936*e^(332*I*c) + 302060638030868463461139442279
 360499718906917482524894483356220196138377050828911383056860425370161157201
 493696073712322595776808*e^(331*I*c) + 362706307563843231135699157418510732
 452420614013624168879312187645233450153927975793326834780741391203430153093
 712635355523960320*e^(330*I*c) + 432147856464086938023811561808678589594702
 047904674282297959658800170984456799067751878044806619012452636891350731618
 278545690160*e^(329*I*c) + 510907615111134507452623846147137141450275722444

316385531648429230851686635827717488464500331623385777400744950538410637735
936000*e^(328*I*c) + 599378484771733474809376142401554850207064972118137572
949257503651444541939309025276896049622515630263162184526394317285457368300
*e^(327*I*c) + 697789106925924614816713747684682785083659819027952244447043
355869741368500452561164024636073401929693105801105522738405349028160*e^(32
6*I*c) + 806169671327625532424575340089775733681994991576674446922354099714
615192085443245663852257001115644286660304979476023966071898200*e^(325*I*c)
+ 924320052867522584035777495761072351222534420784861960001821020509468146
356756433795246446491396141583854513687104334429566707520*e^(324*I*c) + 105
178210042883437194450817005121918711681634976695332263718214961087500422378
4784183284961906494422955462431208645690802526770780*e^(323*I*c) + 11878179
430793903161088023247981101290208227826600872485993676434812002060468221661
44285425922229375413676535071141005286431481600*e^(322*I*c) + 1331396114626
723035802462123531582050339749996014152452835305956367425370222758621753922
458727524856072950880960657564720475838500*e^(321*I*c) + 148118711708924666
246695569496567785552473031326055269082160265717621873742624552279532989146
4091005878304304075953693546767206080*e^(320*I*c) + 16355697446414219006578
863812890766532275801720566775874514020692343552836874896596139137619591407
73339736014790081814516625224440*e^(319*I*c) + 1792649078089632298936945728
481969334964391597506285088488350622937252533420980803144316431701452190522
716124797875257437516360640*e^(318*I*c) + 195028655078018191924499296120448
701005646036284521850167442376626632155879143691731787870223267921386828792
6294665202769722927380*e^(317*I*c) + 21061419034683443071125497612024845434
027942523524821998174104248696772627150982884376465186834879454627742236564
71345899082156800*e^(316*I*c) + 2257726219103856286812833012681573765496262
241420612932076143151171960854554124144699023009842080515157923529357189869
943515991200*e^(315*I*c) + 240246459556968608612000180303421105673944558862
194614138410616288624616181514976302503083487523406726777402343341826998243
1265280*e^(314*I*c) + 25377664154650303308154717466929885960699118946972250
529283204525421755871548480964833312098074301139430153983626696733379577557
20*e^(313*I*c) + 2661100647975783583828235139201441930178396643383423903583
862547255880772382049201015537214900832745601519737141849802506685264000*e^
(312*I*c) + 277007320715076864559750728138206549792496846605452741412233982
7333783770068305883487309979315983718403740872884345746380680204260*e^(311*
I*c) + 28625031263204617977706677807256441849912556231746261756790506721008
48988119391841466573417019247590580735265143427289340450811200*e^(310*I*c)
+ 2936494214351868498703239455426771104344827306267558916550877467232455153
286140521089582733932202553130712723836983468866230908800*e^(309*I*c) + 299
049894962254360853812938028386633519008711512485881814378761918695711190376
5723974899651518555144924290346242595167274383008960*e^(308*I*c) + 30233716
435082250271756031752129532190224850458507318453075190082773851547314612133
88035579159917590062343527464977286601165100620*e^(307*I*c) + 3034408355530
957075787731745322567981684616550162845473257679674280216947356785783843205
604202307836897073595410412575660465787520*e^(306*I*c) + 302337164350822502
717560317521295321902248504585073184530751900827738515473146121338803557915

9917590062343527464977286601165100620*e^(305*I*c) + 29904989496225436085381
 293802838663351900871151248588181437876191869571119037657239748996515185551
 44924290346242595167274383008960*e^(304*I*c) + 2936494214351868498703239455
 426771104344827306267558916550877467232455153286140521089582733932202553130
 712723836983468866230908800*e^(303*I*c) + 286250312632046179777066778072564
 418499125562317462617567905067210084898811939184146657341701924759058073526
 5143427289340450811200*e^(302*I*c) + 27700732071507686455975072813820654979
 249684660545274141223398273337837700683058834873099793159837184037408728843
 45746380680204260*e^(301*I*c) + 2661100647975783583828235139201441930178396
 643383423903583862547255880772382049201015537214900832745601519737141849802
 506685264000*e^(300*I*c) + 253776641546503033081547174669298859606991189469
 722505292832045254217558715484809648333120980743011394301539836266967333795
 7755720*e^(299*I*c) + 24024645955696860861200018030342110567394455886219461
 413841061628862461618151497630250308348752340672677740234334182699824312652
 80*e^(298*I*c) + 2257726219103856286812833012681573765496262241420612932076
 143151171960854554124144699023009842080515157923529357189869943515991200*e^
 (297*I*c) + 210614190346834430711254976120248454340279425235248219981741042
 4869677262715098288437646518683487945462774223656471345899082156800*e^(296*
 I*c) + 19502865507801819192449929612044870100564603628452185016744237662663
 21558791436917317878702232679213868287926294665202769722927380*e^(295*I*c)
 + 1792649078089632298936945728481969334964391597506285088488350622937252533
 420980803144316431701452190522716124797875257437516360640*e^(294*I*c) + 163
 556974464142190065788638128907665322758017205667758745140206923435528368748
 9659613913761959140773339736014790081814516625224440*e^(293*I*c) + 14811871
 170892466624669556949656778555247303132605526908216026571762187374262455227
 95329891464091005878304304075953693546767206080*e^(292*I*c) + 1331396114626
 723035802462123531582050339749996014152452835305956367425370222758621753922
 458727524856072950880960657564720475838500*e^(291*I*c) + 118781794307939031
 610880232479811012902082278266008724859936764348120020604682216614428542592
 2229375413676535071141005286431481600*e^(290*I*c) + 10517821004288343719445
 081700512191871168163497669533226371821496108750042237847841832849619064944
 22955462431208645690802526770780*e^(289*I*c) + 9243200528675225840357774957
 610723512225344207848619600018210205094681463567564337952464464913961415838
 54513687104334429566707520*e^(288*I*c) + 8061696713276255324245753400897757
 336819949915766744469223540997146151920854432456638522570011156442866603049
 79476023966071898200*e^(287*I*c) + 6977891069259246148167137476846827850836
 598190279522444470433558697413685004525611640246360734019296931058011055227
 38405349028160*e^(286*I*c) + 5993784847717334748093761424015548502070649721
 181375729492575036514445419393090252768960496225156302631621845263943172854
 57368300*e^(285*I*c) + 5109076151111345074526238461471371414502757224443163
 855316484292308516866358277174884645003316233857774007449505384106377359360
 00*e^(284*I*c) + 4321478564640869380238115618086785895947020479046742822979
 59658800170984456799067751878044806619012452636891350731618278545690160*e^(
 283*I*c) + 3627063075638432311356991574185107324524206140136241688793121876
 45233450153927975793326834780741391203430153093712635355523960320*e^(282*I*

c) + 3020606380308684634611394422793604997189069174825248944833562201961383
 77050828911383056860425370161157201493696073712322595776808*e^(281*I*c) + 2
 495930722825658663983899515096192026826344554871287146314618917708232013675
 27645793770203788784677343934971424317987895255031936*e^(280*I*c) + 2046222
 955357291095198299167898672253194297051625600826488403949654231128093365919
 21290309392396263834674368977840527147037426908*e^(279*I*c) + 1664323329225
 891951305583292663987533898239557375985275560960930624735597695457723219789
 69318904192572733997888230986469005970880*e^(278*I*c) + 1342977420234794299
 046296161045596100960747587210680227044689380630170596880233634364589715349
 64665036319889119229809973806909680*e^(277*I*c) + 1075047374065769161234803
 991697596333283214074194005100174988498305986215654282665463159339208215275
 44726380201659114903834605888*e^(276*I*c) + 8536811843021531284823129173967
 373588774620185166629960039219941876475008682819871987274404776778366732532
 6289221881974987582215*e^(275*I*c) + 67244087969080703823703257199663047606
 890610482090494619492802935130215979819469966383336788693900139115594646893
 784095418472336*e^(274*I*c) + 525392233467407711425870923702570695360603196
 444395016676104827679558002760528924321527988146079751103662249450814281218
 88473324*e^(273*I*c) + 4071598896370191895002034833673642342051331135901048
 524691907465288339397080537447083015622970564731226547758425602721276294104
 0*e^(272*I*c) + 31295263688189838313772775873307260334117227258629501992358
 695636092662866062819845689064235813622974150120668921391878398978380*e^(27
 1*I*c) + 238563985655628020306952781742126408332821541740064594582926440609
 47924907313735921561690153939906017518647182491616573724049744*e^(270*I*c)
 + 1803532733817745547117756859485168297797834644977719357208768851039242688
 4519272991560851326393852241961470040819793627127923997*e^(269*I*c) + 13521
 230411945436915558854706851796541567399811656870021567352975326815467817846
 533289123871696056195231696146162720992221760992*e^(268*I*c) + 100522095243
 695818275881549853455496780314457444999852082593854356097402724003014542468
 72041775159838468077381562338745636398374*e^(267*I*c) + 7410372612891152224
 364633296128043971657193280327754304382235672773781403023814127610355562505
 271969045177704726054907145784960*e^(266*I*c) + 541666280405243634958559598
 281835795386625846164435401820515891774257642534436496459675065317767780349
 2186817305171175032011500*e^(265*I*c) + 39256976584157783527681039428560118
 402116427696217172179966143984738871860743914826385472128265382704539126345
 40299792270321024*e^(264*I*c) + 2820819298562215959107529807289628449621386
 798989436369393116069894018781201000275633104498398959346631795568022519974
 400130281*e^(263*I*c) + 200949611009268773815278208568373722272796882429905
 873921544608349935146762533444967075776406669065615094901494404399482282392
 0*e^(262*I*c) + 1419164481422176573858234013898999628822322333095737307163
 10743838935832201454293617293175086458389621425307051750612129761498*e^(261
 *I*c) + 9935556536495211272264439608202336493864885100818925458667004440966
 61582790441241830855609577062039555625090943332264901780720*e^(260*I*c) + 6
 895184493287935599032604181499741902535783400588950355896064682446805915561
 18170304005037563669880057908765898949268614772285*e^(259*I*c) + 4743230435
 631005423773386299319662481291759769823324460180560093911540204388959031408

22967769494019446166779954024655344116288*e^(258*I*c) + 3234131780148410037
 141511382460791525763609760350584578904099377387231715370365730436819971637
 45313602400139153046673668433091*e^(257*I*c) + 2185631666596493122474836409
 562721492124991153838287710296542833639725851180904794136966381081563852446
 46591328454425745117584*e^(256*I*c) + 1463904484563511812182373827403741241
 916648199977469880765983918627336296701422415463755339031306052975801056753
 55629160198162*e^(255*I*c) + 9717305502474268005861672246136889266114129554
 026349301303274608353615732426833339040030895831837021915488716970225744475
 6176*e^(254*I*c) + 63923019433761989090614801288635098123199445102303122616
 544648208998767803944455777042886552738499747183713136069104651812215*e^(25
 3*I*c) + 416704403753905436434182193422717480400350714901190805852815224981
 88818375906900368701234531304633163446319945130196476913600*e^(252*I*c) + 2
 691779470108661509789012023689050110514679999602177519571086622622863898470
 3456832694153230611607263444183501026198563419616*e^(251*I*c) + 17229502824
 367647334400721998417596703948657394738805209391636597370380572398964715080
 095366818322029152193635869784095333760*e^(250*I*c) + 109272106034735448102
 797923478445360745888968062300411100895447316058631461041817390394266748554
 53466097402330688331845602302*e^(249*I*c) + 6866425337518668262662693750908
 956965732924578142181630622157802899880874681551031136314064199948604001529
 894566235238597088*e^(248*I*c) + 427482690772059175252671133682087150084434
 564792238547153435933360618957183244464136413289310866357620513387067215626
 4164115*e^(247*I*c) + 26366624104307993404475222847782442837407510686581407
 26576446671207798325606832295937705061686297930296382338892574900819440*e^(
 246*I*c) + 1611092541400060525954859375264194178347643471837078201446262435
 615142944587337833513586022729849523358436493586042252995608*e^(245*I*c) +
 975210339444049318757282311763517786673223175594457946383279264635085041004
 917300295904275433144848532459919875479817581584*e^(244*I*c) + 584749573682
 304586179384628844883327581498969886540380378896767999075614964007174600811
 092945356635118795824799369716742109*e^(243*I*c) + 347310053810935290419455
 560555957314129569210735745983234369659976413374774078000173070075248654524
 917179128950507443058208*e^(242*I*c) + 204325557265186000767402710230847896
 459761583922763698235433212833313077783041040074669379017394836761539649081
 690630811665*e^(241*I*c) + 119060591849660546834765693227676449067584148248
 882678447504826077236333444513454095126668750057295811191643356908972191440
 *e^(240*I*c) + 687124660159856415124685861736597477348795917100983546527861
 24936023073943141049573606648563005359411712764895683903806088*e^(239*I*c)
 + 3927420041432986116939794451622500108122743339858500720639923121190715779
 5359719648241598754266579840244551491476467899952*e^(238*I*c) + 22231341131
 801535345406399037721686840208397941952580135584645966746736656716271554826
 476282991066076564921432614339399735*e^(237*I*c) + 124621404405372580849285
 967098720668577570709431248685545009481547568634543080329254083403112378500
 17814707896986969086816*e^(236*I*c) + 6917838945214844278493330459361394923
 372333853619879637372673184942859712431066345726870422099893124890777678037
 369988150*e^(235*I*c) + 380260499670589110696462063384896480703709885451018
 2263243030597295630760353597531974324752266389193185760878274188013440*e^(2

34*I*c) + 20696982895008606434616657623738079575130194240411788719049605518
29412449344722432125679417958403007551179298315947373776*e^(233*I*c) + 1115
398285545601550533328045600184317993899359217996729818340704221801195667410
485846996179056733558512452238583160792512*e^(232*I*c) + 595157615500431514
947479282336547053827926087916425263497187757029413385471835434198246807096
214536895441388306027237899*e^(231*I*c) + 314409035808225861565595436938354
945445473991043129722046747228813030925204968503418566818838611866709807040
793495364496*e^(230*I*c) + 164437500676906892741323260154394278503954561936
020133596581806449357240277349447927334517105809995300093549279931273178*e^
(229*I*c) + 851395332347864557795899594649006377607357297056212213800058372
08369157794673049675428799817875431430246332625899630160*e^(228*I*c) + 4363
830007581517102594621146446568961896577348866098585794565747985408510885185
7911222911989837615452608512356008400295*e^(227*I*c) + 22140735001708603270
915180769241391662035578755903979148909213603822554792749183517160255571915
875356439553717130797888*e^(226*I*c) + 111194996453632010808810628248863384
924253756584489779355358463492904258215703830904254114185215166703713720452
06568345*e^(225*I*c) + 5527498849031178355861230009326668283926290082158467
118000698502719379939045918344222192742145711257028040974074674736*e^(224*I
*c) + 271958928348374392604080510108034192124453031125460725092919277390933
1523226635035815672862569296693711643521070331394*e^(223*I*c) + 13243113249
840274283552229381476823786728607088171617414487496895935880208608475087037
02325320304649883120684987556400*e^(222*I*c) + 6382188929145337415058063996
624885560667836004960918764083749748774489717780360749962455811242834604380
65182071976085*e^(221*I*c) + 3043844711068133360102841601239063704338888284
90627422652551236966790916174520857759143930140187173492394981908258944*e^(
220*I*c) + 1436576871380447969429471197042595384588181994235168246745862936
91056119209866358123637772245409530799230553767222252*e^(219*I*c) + 6709170
613052966912501989921002157658023784346222953538629508707618929784999593136
0645605292130961496106707521506432*e^(218*I*c) + 31004319206069417077069363
141423487431828009098184744635678652284177439464941651812564519144918003174
108077634846014*e^(217*I*c) + 141764836528757049572020133432411179043699777
96653849959902524421980635189011634815653279605497783382888932766730080*e^(
216*I*c) + 6413381895855925184758231451062556380328594938511006577015218119
786536390213057284018202201631094434819584025113465*e^(215*I*c) + 287049613
141231445183467471535358943955329443080853193346608628854370924623076915139
2180699413405623017247753532944*e^(214*I*c) + 12710330829380489502013605548
312703426623439912775046123423003660250467417428565804452894017866563116858
59023084716*e^(213*I*c) + 5567563887111823403410261927342195461136517683173
80539005893679049394714017063698565272728813669054779077208977840*e^(212*I*
c) + 2412460212824400617929083177830328761948015971332060520912869970437291
45345755805710081489006741839439573984832678*e^(211*I*c) + 1033997554672574
364898478376407537547182043944730557950014676043264198765555568738295317372
11096115196005647730480*e^(210*I*c) + 4383497214291937768537869223302106374
455403310092850273748043897897674698989578407095190523778349037430593454295
5*e^(209*I*c) + 18379807084003359766027649217621144116091735572216620788861

535803449702273802588359076704241840733513439114113248*e^(208*I*c) + 762178
 879191204706203884091779937460042889225819436763668294435609668140024631213
 8001769285020661445991073249416*e^(207*I*c) + 31256834931787017434797047503
 074901786662921507201793636043351135286233296846063431855407560199356621482
 67863968*e^(206*I*c) + 1267597017294812913400146276042126929986480292870190
 399107554311079964227280196522475370108738477856311765699610*e^(205*I*c) +
 508324599301085460166978629683032661427654474082939048097939638391567298795
 788389433842285751054665210868287680*e^(204*I*c) + 201557947424794098026772
 478462040883395867512562930862943753568690084015585598010154781548625239409
 581907397500*e^(203*I*c) + 790191495587665692547839884872323883529091449827
 47171856772463223808993367091503402876467270176124342699654400*e^(202*I*c)
 + 3062758105422195737839054728927760912957281393108273352024738722600002004
 3538279468776707958420892547870128680*e^(201*I*c) + 11735856926245118493113
 091002501604032341876985999082823520672241530200188223826392982302194084667
 538488665600*e^(200*I*c) + 444541225929547462506765951419831296601541629996
 8930393345630345914109720740573618884980520010028451496996210*e^(199*I*c) +
 16644750343872118093949177435029376389785749377547647639878358724104499306
 90131572904279995484581013965001440*e^(198*I*c) + 6160031160229795849371257
 017578872129983543009899913626280388610939145613320711919097149494265879369
 10303300*e^(197*I*c) + 2253205325932206577679411092895162489997945210155641
 34982827241710019675486694499689312466561907212627820000*e^(196*I*c) + 8145
 208141382911182887541756425005484603769331281148049290916075819598915576810
 7022568350953861815940704090*e^(195*I*c) + 29097651061247453406647569781836
 910062165559852359052804259154165687125428752562385492373749486351714453120
 *e^(194*I*c) + 102716025302028890024978135168494525909715128095290606651973
 01097052210064576088348023234671975463677418470*e^(193*I*c) + 3582718002163
 296061414536703715109897107198252739284546149343102348456124089657428594946
 438660859773886240*e^(192*I*c) + 123466804189240997878001808175544021601258
 2476396941937965899631953079203974222138794604328498972144766900*e^(191*I*c
) + 42035802483514679858361121014594215468443794936564789908837252480215622
 2884839580011688655664280691773600*e^(190*I*c) + 14137993825355684328056550
 580740330413060613072543475174579407983314136174891763998614537706643721054
 6190*e^(189*I*c) + 46970224727117281826454045018070670522559756627580347784
 535320014963482632359729444541885102274546002560*e^(188*I*c) + 154131112114
 860239372949708207973767160813447881633865435224219397375079621258549818818
 79168348260330000*e^(187*I*c) + 4995241956279138180205186744401688024388272
 113921255663734956946927571305533146776898787878059685108480*e^(186*I*c) +
 159877110105819269227052899967744474268563100623245618584492522014400230587
 8120380828483988663574829100*e^(185*I*c) + 50529366312301525887848302573881
 2813203397766845340065381261016353419722382620393032535960660921950400*e^(1
 84*I*c) + 15768584552885091872146287786443509025758314941556132342738656289
 4447598277935629800939237175625149830*e^(183*I*c) + 48584258153140280447314
 836868772131390195412419046732778458706015096881437076337910793584122475073
 760*e^(182*I*c) + 147779550966171289987127451820714953621765069731830816502
 33605274051677624970464340242755840025673760*e^(181*I*c) + 4437210917843182

347764349544443904699020056595069470847193617092114714077633077234972825351
226979360*e^(180*I*c) + 131505212093069212210229710532762284233587074342853
0891072983535862280094446607723473800477453914130*e^(179*I*c) + 38465584208
066627445406307878483717499894905250097532216200339254995341359246151936517
7908682078400*e^(178*I*c) + 11103414879700881944314389564446924229504986746
4313710969257619338899133799285616020069872611710850*e^(177*I*c) + 31626644
674725547731176795687527653571305969985923688392112164915553242573269490908
989570248533280*e^(176*I*c) + 888829502875102466704420383760797610148005313
4418614474620767522824868911959884352666444917404000*e^(175*I*c) + 24643821
908074396090797742268556796293678857097764358766308517162539626961923417062
39192878728160*e^(174*I*c) + 6740255305431330088948457752366252374507431144
73544537818170447134607102575676676056675328961590*e^(173*I*c) + 1818346614
061779013153301296771453811664491884131941411693443547549209690349526103789
45282257600*e^(172*I*c) + 4837948975643409984385779181658937940681504260934
0378747586437145781646245422045101230417309900*e^(171*I*c) + 12693496932964
920565073673637181280088548682508880255337280065006566138696041797353216584
528640*e^(170*I*c) + 328387476055581867672630948030673442015509858394807446
9014168171874442170109648521627538755920*e^(169*I*c) + 83757920692341193245
8786486765373533946545239708990769488724813982189165104589895518909256320*e
^(168*I*c) + 21059430138564847118432907888031750495336183995415942743400988
4661777259752542647709150036990*e^(167*I*c) + 52190912207661824215812271854
269748071292843243227894769229690720010547141334131610989636000*e^(166*I*c)
+ 127472196165033205413563430625628473686016221408567860254458145320379041
11523242298235713300*e^(165*I*c) + 3067974296431747364198159623962463671617
006419626851426148418602934852907379021659761911840*e^(164*I*c) + 727521010
718394229291774073844694255798738667067535379759732795567942578751384250780
476310*e^(163*I*c) + 169956327969929767773902096652629253283704505477127544
556534417376686540936706073847337600*e^(162*I*c) + 391080312556018094765375
35369611844440844903751605645023514572352045248104262933598850730*e^(161*I*
c) + 8862752142756957285681340885764904597935349569355321815647721172537159
186491471311666400*e^(160*I*c) + 197779298066581813565130009432623915860544
8870806970860577325385028609983034534672318500*e^(159*I*c) + 43454667678028
0045346344498763892540797175105756827515509297024187660299345484920192480*e
^(158*I*c) + 93986915313068179149083606065681482780836060510530154618486949
839467131378859885998210*e^(157*I*c) + 200080068030300471372932782503215971
13540716201983333126349281186679153199068045257216*e^(156*I*c) + 4191542500
656826148093339414544159143964478472492315931809171859902114109005939942952
*e^(155*I*c) + 863979933622330349556296820028395513198708064940505702126068
652936800794826651264256*e^(154*I*c) + 175193170500618300241515632381912285
157790097816049220671217212220015297133400636060*e^(153*I*c) + 349410716132
76704649477943043339450201504075335160361865916029213860778606230624960*e^(
152*I*c) + 6852993223145736687328885311617795435592940841439866351079655652
312894721972796266*e^(151*I*c) + 132149805527130085142999386663161987442453
4425188183592049727687571032156435077280*e^(150*I*c) + 25050102860892833246
9340456829902067712233644464602753159945727868485722395506952*e^(149*I*c) +

46668223548266017806854592468100570289355960869613650856575756758180182223
 308768*e^(148*I*c) + 854301344112621233483354066506962147247908583804136056
 4550722036723654297540205*e^(147*I*c) + 15363332384449275835327345560164946
 71674916578907116984548489078241693926940560*e^(146*I*c) + 2713612075032665
 70734486517077181014801775322183181055638619257836143271472358*e^(145*I*c)
 + 4706504461113515810848735336748424310269824883831263587628309942744274586
 6704*e^(144*I*c) + 80137295807907524343619649457615437614695207912107469726
 75870481058674277844*e^(143*I*c) + 1339214374254245553564884406801945353385
 000254030655765953770237607180089968*e^(142*I*c) + 219601281339515561500261
 478844190024870555261281946058839614044697037963695*e^(141*I*c) + 353244472
 06779018115378052820789411687581004582367431006205879633729015200*e^(140*I*
 c) + 5572551157328671121016216416307596161861955969011697222340926210112854
 418*e^(139*I*c) + 861884851094991908764246805474672428603757315484453974713
 612812215428992*e^(138*I*c) + 130657660226560419335121434389938961884595434
 069984824307149332131747540*e^(137*I*c) + 194079792155945665935350081033032
 55257745408070082431338945184797463936*e^(136*I*c) + 2823905151936586678382
 525706564457280290098698638597987628380245881715*e^(135*I*c) + 402349692266
 121158934003582839428785116904903936409545602519219664720*e^(134*I*c) + 561
 17081076341175384087570185188538660375932013674735519055227368366*e^(133*I*
 c) + 7659010520187549651777118357676871927081898989131125755798204236112*e^
 (132*I*c) + 102253643746829673729306586270524644969368741555986584430688870
 5423*e^(131*I*c) + 13349021005202618377967331386833230353033290616324719462
 7808410304*e^(130*I*c) + 17033886027390615741040977721655541665612162275485
 028584310890417*e^(129*I*c) + 212370296918887131826671878122392706783994901
 5727293884065388080*e^(128*I*c) + 25858534871597727015582911568419341107203
 4541491364393985491350*e^(127*I*c) + 30736217404321009965231037419663053962
 881035281709221697785072*e^(126*I*c) + 356476489062872401708848799668817892
 9195787613958545474804845*e^(125*I*c) + 40321222595779818884084613996099562
 4144491271694336796459584*e^(124*I*c) + 44456708175258821024400946210535004
 523775722190977468484496*e^(123*I*c) + 477539860710085326353420773381826677
 7478693412738731031680*e^(122*I*c) + 49946750655853173367158586291057270281
 1545035730398749530*e^(121*I*c) + 50836369508171099437019348610847391946736
 185108017183136*e^(120*I*c) + 503202490340145182407421394376601192202650700
 6311982753*e^(119*I*c) + 48409341024048871865591702530366258109165912618234
 4528*e^(118*I*c) + 45230940039830738332025694784646206844854827698075736*e^
 (117*I*c) + 4101545439937195793959956708442496709433800261224880*e^(116*I*c
) + 360688613036389349413809780004559963548775423325255*e^(115*I*c) + 30735
 366512830562160991166338490057308062762518496*e^(114*I*c) + 253566746065027
 9776834561566186591213109251642859*e^(113*I*c) + 20234750972446217131396664
 3580234078508179838320*e^(112*I*c) + 15603911277687607099721623771744933086
 920587272*e^(111*I*c) + 1161581413733971751533622511909046917188768400*e^(1
 10*I*c) + 83380839911837894453136303673785039051506805*e^(109*I*c) + 576460
 1046563151304213854710715346838447392*e^(108*I*c) + 38336015580105482452976
 4688213114368047154*e^(107*I*c) + 24489837337812338687718622491865013839488
 *e^(106*I*c) + 1500602747937397286405577818722691539392*e^(105*I*c) + 88054

927598941411145869950813388040256*e^(104*I*c) + 493966661081802579880958635
 2543471345*e^(103*I*c) + 264410375780310742518099326419685040*e^(102*I*c) +
 13477227799524701956579274210395326*e^(101*I*c) + 652650253343206047453620
 559993840*e^(100*I*c) + 29952547749265499675257842032197*e^(99*I*c) + 12991
 46645993240318167826532288*e^(98*I*c) + 53090127264630963470039804475*e^(97
 *I*c) + 2037031259470368160131922320*e^(96*I*c) + 7309920781733559724709803
 8*e^(95*I*c) + 2442455629894502983849104*e^(94*I*c) + 755998170926701578066
 39*e^(93*I*c) + 2154864144781257856128*e^(92*I*c) + 56169444526926562260*e^
 (91*I*c) + 1327882849274858880*e^(90*I*c) + 28186192554792138*e^(89*I*c) +
 530563624556832*e^(88*I*c) + 8718181624155*e^(87*I*c) + 122503723056*e^(86*
 I*c) + 1431118260*e^(85*I*c) + 13343760*e^(84*I*c) + 93096*e^(83*I*c) + 432
 *e^(82*I*c) + e^(81*I*c))*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))/((tan(1/4*d*x
 + c)^4*tan(1/2*c)^8 - 14*tan(1/4*d*x + c)^4*tan(1/2*c)^6 + 24*tan(1/4*d*x +
 c)^3*tan(1/2*c)^7 - 6*tan(1/4*d*x + c)^2*tan(1/2*c)^8 - 56*tan(1/4*d*x + c
)^3*tan(1/2*c)^5 + 84*tan(1/4*d*x + c)^2*tan(1/2*c)^6 - 24*tan(1/4*d*x + c)
 *tan(1/2*c)^7 + tan(1/2*c)^8 + 14*tan(1/4*d*x + c)^4*tan(1/2*c)^2 - 56*tan(
 1/4*d*x + c)^3*tan(1/2*c)^3 + 56*tan(1/4*d*x + c)*tan(1/2*c)^5 - 14*tan(1/2
 *c)^6 - tan(1/4*d*x + c)^4 + 24*tan(1/4*d*x + c)^3*tan(1/2*c) - 84*tan(1/4*
 d*x + c)^2*tan(1/2*c)^2 + 56*tan(1/4*d*x + c)*tan(1/2*c)^3 + 6*tan(1/4*d*x
 + c)^2 - 24*tan(1/4*d*x + c)*tan(1/2*c) + 14*tan(1/2*c)^2 - 1)^4*d)

Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \frac{91 a \sin\left(\frac{3c}{2} + \frac{3dx}{2}\right) \sqrt{a + a \cos(c + dx)} - 35 a \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a + a \cos(c + dx)}}{\frac{315 d \sqrt{\cos(c+dx)} \cos\left(\frac{c}{2} + \frac{dx}{2}\right)}{8} + \frac{315 d \sqrt{\cos(c+dx)} \cos\left(\frac{3c}{2} + \frac{3dx}{2}\right)}{8} + \frac{105 d \sqrt{\cos(c+dx)}}{8}}$$

[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2),x)

[Out] (91*a*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2) - 35*a*sin(c/2 + (d*x)/2)*(a + a*cos(c + d*x))^(1/2) + 26*a*sin((7*c)/2 + (7*d*x)/2)*(a + a*cos(c + d*x))^(1/2))/((315*d*cos(c + d*x)^(1/2)*cos(c/2 + (d*x)/2))/8 + (315*d*cos(c + d*x)^(1/2)*cos((3*c)/2 + (3*d*x)/2))/8 + (105*d*cos(c + d*x)^(1/2)*cos((5*c)/2 + (5*d*x)/2))/8 + (105*d*cos(c + d*x)^(1/2)*cos((7*c)/2 + (7*d*x)/2))/8)

3.213 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2} dx$

Optimal result	3116
Rubi [A] (verified)	3116
Mathematica [C] (warning: unable to verify)	3119
Maple [A] (verified)	3119
Fricas [A] (verification not implemented)	3120
Sympy [F(-1)]	3120
Maxima [B] (verification not implemented)	3120
Giac [F]	3125
Mupad [F(-1)]	3125

Optimal result

Integrand size = 25, antiderivative size = 200

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2} dx = \frac{163a^{5/2} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{163a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{163a^3 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{17a^3 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{4d}$$

[Out] $163/64*a^{(5/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+163/96*a^{(3)}*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+17/24*a^{(3)}*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+163/64*a^{(3)}*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}+1/4*a^{(2)}*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2842, 3060, 2849, 2853, 222}

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2} dx = \frac{163a^{5/2} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d} + \frac{17a^3 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}} + \frac{163a^3 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}} + \frac{163a^3 \sin(c+dx) \sqrt{\cos(c+dx)}}{64d\sqrt{a\cos(c+dx)+a}} + \frac{a^2 \sin(c+dx) \cos^{\frac{5}{2}}(c+dx) \sqrt{a\cos(c+dx)+a}}{4d}$$

[In] Int[Cos[c + d*x]^(3/2)*(a + a*cos[c + d*x])^(5/2),x]

[Out] (163*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*cos[c + d*x]])/(64*d) + (163*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqrt[a + a*cos[c + d*x]]) + (163*a^3*cos[c + d*x]^(3/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*cos[c + d*x]]) + (17*a^3*cos[c + d*x]^(5/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*cos[c + d*x]]) + (a^2*cos[c + d*x]^(5/2)*Sqrt[a + a*cos[c + d*x]]*Sin[c + d*x])/(4*d)

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2842

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2849

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[-2*b*cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 3060

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[-2*b*B*cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1))]/(b

```
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2 \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{4d} \\
&+ \frac{1}{4} \int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} \left(\frac{13a^2}{2} + \frac{17}{2} a^2 \cos(c + dx) \right) dx \\
&= \frac{17a^3 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{24d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{4d} \\
&+ \frac{1}{48} (163a^2) \int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{163a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{96d \sqrt{a + a \cos(c + dx)}} + \frac{17a^3 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{24d \sqrt{a + a \cos(c + dx)}} \\
&+ \frac{a^2 \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{4d} \\
&+ \frac{1}{64} (163a^2) \int \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)} dx \\
&= \frac{163a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{64d \sqrt{a + a \cos(c + dx)}} + \frac{163a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{96d \sqrt{a + a \cos(c + dx)}} \\
&+ \frac{17a^3 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{24d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{4d} \\
&+ \frac{1}{128} (163a^2) \int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{163a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{64d \sqrt{a + a \cos(c + dx)}} + \frac{163a^3 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{96d \sqrt{a + a \cos(c + dx)}} \\
&+ \frac{17a^3 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{24d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{4d} \\
&- \frac{(163a^2) \text{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{x^2}{a}}} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}} \right)}{64d}
\end{aligned}$$

$$\begin{aligned}
&= \frac{163a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{64d} + \frac{163a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{64d \sqrt{a+a \cos(c+dx)}} \\
&+ \frac{163a^3 \cos^3(c+dx) \sin(c+dx)}{96d \sqrt{a+a \cos(c+dx)}} + \frac{17a^3 \cos^5(c+dx) \sin(c+dx)}{24d \sqrt{a+a \cos(c+dx)}} \\
&+ \frac{a^2 \cos^5(c+dx) \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{4d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.99 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{\frac{5}{2}} dx = \frac{(a(1+\cos(c+dx)))^{\frac{5}{2}} \sec^4\left(\frac{1}{2}(c+dx)\right) (7(89+28 \cos(c+dx))+3 \cos(2(c+dx))) \text{Hypergeometric}}{dx}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2),x]

[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F1[-3/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] - 24*(3 + Cos[c + d*x])*Hypergeometric2F1[-1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 - 6*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{-1/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4*Tan[(c + d*x)/2])/(420*d)

Maple [A] (verified)

Time = 12.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.09

method	result
default	$ \frac{(48 \sin(dx+c) (\cos^3(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 184 \sin(dx+c) (\cos^2(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 326 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 48 \sin(dx+c) \cos^2(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 192d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}) a^{\frac{5}{2}}}{192d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}} $

[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/192/d*(48*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+184*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+326*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+489*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+489*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.68

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{5}{2}} dx = \frac{(48 a^2 \cos(dx + c)^3 + 184 a^2 \cos(dx + c)^2 + 326 a^2 \cos(dx + c) + 489 a^2) \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)}}{192 (d \cos(dx + c) + d)}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*((48*a^2*cos(d*x + c)^3 + 184*a^2*cos(d*x + c)^2 + 326*a^2*cos(d*x + c) + 489*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{5}{2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7450 vs. 2(168) = 336.

Time = 0.76 (sec) , antiderivative size = 7450, normalized size of antiderivative = 37.25

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{5}{2}} dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/768*(10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4*d*x + 4*c))^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c))^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c))^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c)
```

$$\begin{aligned}
&) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 3 * (2*a^2 * \cos(1/2 * \\
& \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + a^2 * \sin(4* \\
& d*x + 4*c) - 2 * (a^2 * \cos(4*d*x + 4*c) + a^2) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c) \\
&), \cos(4*d*x + 4*c))) * \cos(3/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) \\
& + 12 * (a^2 * \sin(4*d*x + 4*c)^3 + (a^2 * \cos(4*d*x + 4*c))^2 - a^2 * \cos(4*d*x + 4 \\
& *c)) * \sin(4*d*x + 4*c) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) \\
& + (8*a^2 * \cos(4*d*x + 4*c)^2 + 8*a^2 * \sin(4*d*x + 4*c)^2 - 3*a^2 * \cos(4*d*x + \\
& 4*c) + 32 * (a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4*c)^2 - 2*a^2 * \cos(4*d \\
& *x + 4*c) + a^2) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 3 \\
& 2 * (a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4*c)^2 + 2*a^2 * \cos(4*d*x + 4*c) \\
& + a^2) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 2 * (16*a^2 * \\
& \cos(4*d*x + 4*c)^2 + 16*a^2 * \sin(4*d*x + 4*c)^2 - 19*a^2 * \cos(4*d*x + 4*c) + \\
& 3*a^2) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 2 * (64*a^2 * \cos \\
& (1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + 19*a^2 \\
& * \sin(4*d*x + 4*c)) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin \\
& (3/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 12 * (4*a^2 * \cos(1/2 * \arct \\
& an2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x \\
& + 4*c)^2) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \cos(3/2 * \ar \\
& ctan2(\sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2 * \arctan2 \\
& (\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 1)) - (3*a^2 * \cos(4*d*x + 4*c)^3 - 8 \\
& *a^2 * \cos(4*d*x + 4*c)^2 + 4 * (3*a^2 * \cos(4*d*x + 4*c)^3 - 14*a^2 * \cos(4*d*x + \\
& 4*c)^2 + 19*a^2 * \cos(4*d*x + 4*c) + (3*a^2 * \cos(4*d*x + 4*c) - 8*a^2) * \sin(4*d \\
& *x + 4*c)^2 - 8*a^2) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 \\
& + (3*a^2 * \cos(4*d*x + 4*c) - 8*a^2) * \sin(4*d*x + 4*c)^2 + 4 * (3*a^2 * \cos(4*d*x \\
& + 4*c)^3 - 2*a^2 * \cos(4*d*x + 4*c)^2 - 13*a^2 * \cos(4*d*x + 4*c) + (3*a^2 * \cos \\
& (4*d*x + 4*c) - 8*a^2) * \sin(4*d*x + 4*c)^2 - 8*a^2) * \sin(1/2 * \arctan2(\sin(4*d* \\
& x + 4*c), \cos(4*d*x + 4*c)))^2 + (8*a^2 * \cos(4*d*x + 4*c)^2 + 8*a^2 * \sin(4*d* \\
& x + 4*c)^2 - 3*a^2 * \cos(4*d*x + 4*c) + 32 * (a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(\\
& 4*d*x + 4*c)^2 - 2*a^2 * \cos(4*d*x + 4*c) + a^2) * \cos(1/2 * \arctan2(\sin(4*d*x + \\
& 4*c), \cos(4*d*x + 4*c)))^2 + 32 * (a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4 \\
& *c)^2 + 2*a^2 * \cos(4*d*x + 4*c) + a^2) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos \\
& (4*d*x + 4*c)))^2 + 2 * (16*a^2 * \cos(4*d*x + 4*c)^2 + 16*a^2 * \sin(4*d*x + 4*c)^ \\
& 2 - 19*a^2 * \cos(4*d*x + 4*c) + 3*a^2) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(\\
& 4*d*x + 4*c))) - 2 * (64*a^2 * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4* \\
& c))) * \sin(4*d*x + 4*c) + 19*a^2 * \sin(4*d*x + 4*c)) * \sin(1/2 * \arctan2(\sin(4*d*x \\
& + 4*c), \cos(4*d*x + 4*c))) * \cos(3/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4 \\
& *c))) + 4 * (3*a^2 * \cos(4*d*x + 4*c)^3 - 11*a^2 * \cos(4*d*x + 4*c)^2 + 8*a^2 * \cos \\
& (4*d*x + 4*c) + (3*a^2 * \cos(4*d*x + 4*c) - 8*a^2) * \sin(4*d*x + 4*c)^2) * \cos(1/ \\
& 2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 3 * (2*a^2 * \cos(1/2 * \arctan2(s \\
& in(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + a^2 * \sin(4*d*x + 4*c) \\
& - 2 * (a^2 * \cos(4*d*x + 4*c) + a^2) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d \\
& *x + 4*c))) * \sin(3/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 4 * (4 * (3 \\
& *a^2 * \cos(4*d*x + 4*c) - 8*a^2) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x \\
& + 4*c))) * \sin(4*d*x + 4*c) + (3*a^2 * \cos(4*d*x + 4*c) - 8*a^2) * \sin(4*d*x + 4* \\
& c)) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(3/2 * \arctan2(s
\end{aligned}$$

$$\begin{aligned}
& 4*d*x + 4*c)^2 + 120*a^2 - 40*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c) \\
&)^2 + 2*a^2*\cos(4*d*x + 4*c) + a^2)*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4 \\
& *d*x + 4*c))) - 80*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 + 2*a^2 \\
& *\cos(4*d*x + 4*c) + a^2)*\cos(1/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c) \\
&))) * \sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 2*(6*a^2*\cos(4 \\
& *d*x + 4*c)^3 + 214*a^2*\cos(4*d*x + 4*c)^2 - 3*a^2*\sin(4*d*x + 4*c)*\sin(1/4 \\
& *\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 240*a^2*\cos(4*d*x + 4*c) + \\
& 2*(3*a^2*\cos(4*d*x + 4*c) + 110*a^2)*\sin(4*d*x + 4*c)^2 - (160*a^2*\cos(4*d* \\
& x + 4*c)^2 + 160*a^2*\sin(4*d*x + 4*c)^2 - 157*a^2*\cos(4*d*x + 4*c) - 3*a^2) \\
& *\cos(1/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) * \cos(1/2*\arctan2(\sin(\\
& 4*d*x + 4*c), \cos(4*d*x + 4*c))) - (80*a^2*\cos(4*d*x + 4*c)^2 + 80*a^2*\sin(\\
& 4*d*x + 4*c)^2 + 3*a^2*\cos(4*d*x + 4*c)) * \cos(1/4*\arctan2(\sin(4*d*x + 4*c), \\
& \cos(4*d*x + 4*c))) + 2*(320*a^2*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x \\
& + 4*c)))^2 * \sin(4*d*x + 4*c) + 157*a^2*\cos(1/4*\arctan2(\sin(4*d*x + 4*c), co \\
& s(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + 8*(80*a^2*\cos(1/4*\arctan2(\sin(4*d*x + 4 \\
& *c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) - (3*a^2*\cos(4*d*x + 4*c) + 110*a^ \\
& 2)*\sin(4*d*x + 4*c)) * \cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - \\
& 6*(a^2*\cos(4*d*x + 4*c) + 40*a^2)*\sin(4*d*x + 4*c) + 3*(a^2*\cos(4*d*x + 4* \\
& c) + a^2)*\sin(1/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) * \sin(1/2*arc \\
& tan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) * \sin(1/2*\arctan2(\sin(1/2*\arctan2(\\
& \sin(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos \\
& (4*d*x + 4*c)))) + 1))) * \sqrt{a} + 489*((a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d \\
& *x + 4*c)^2 + 4*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 - 2*a^2*co \\
& s(4*d*x + 4*c) + a^2)*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^ \\
& 2 + 4*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 + 2*a^2*\cos(4*d*x + \\
& 4*c) + a^2)*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 4*(a^2 \\
& *\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 - a^2*\cos(4*d*x + 4*c)) * \cos(1/ \\
& 2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 4*(4*a^2*\cos(1/2*\arctan2(s \\
& in(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + a^2*\sin(4*d*x + 4*c) \\
&) * \sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) * \arctan2(-(\cos(1/2*a \\
& rctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))))^2 + \sin(1/2*\arctan2(\sin(4*d*x + \\
& 4*c), \cos(4*d*x + 4*c))))^2 + 2*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x \\
& + 4*c))) + 1)^(1/4) * (\cos(1/2*\arctan2(\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos \\
& (4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) + 1)) \\
& * \sin(1/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - \cos(1/4*\arctan2(\sin \\
& (4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(1/2*\arctan2(\sin(1/2*\arctan2(\sin(4*d*x \\
& + 4*c), \cos(4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + \\
& 4*c))) + 1))), (\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + si \\
& n(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))))^2 + 2*\cos(1/2*\arctan2(si \\
& n(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 1)^(1/4) * (\cos(1/4*\arctan2(\sin(4*d*x + \\
& 4*c), \cos(4*d*x + 4*c))) * \cos(1/2*\arctan2(\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \\
& \cos(4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + \\
& 1)) + \sin(1/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(1/2*\arctan2(\\
& \sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4 \\
& *d*x + 4*c), \cos(4*d*x + 4*c)))) + 1)) + 1) - (a^2*\cos(4*d*x + 4*c)^2 + a^2
\end{aligned}$$

$$\begin{aligned}
& * \sin(4*d*x + 4*c)^2 + 4*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 - \\
& 2*a^2*\cos(4*d*x + 4*c) + a^2)*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + \\
& 4*c)))^2 + 4*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 + 2*a^2*\cos(\\
& 4*d*x + 4*c) + a^2)*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 \\
& + 4*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 - a^2*\cos(4*d*x + 4*c) \\
&)*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 4*(4*a^2*\cos(1/2*a \\
& rctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + a^2*\sin(4*d* \\
& x + 4*c)) * \sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \arctan2(-(c \\
& os(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + \sin(1/2*\arctan2(\sin \\
& (4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 2*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), c \\
& os(4*d*x + 4*c))) + 1)^{1/4} * (\cos(1/2*\arctan2(\sin(1/2*\arctan2(\sin(4*d*x + 4 \\
& *c), \cos(4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c) \\
&)) + 1)) * \sin(1/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - \cos(1/4*arc \\
& tan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(1/2*\arctan2(\sin(1/2*\arctan2(s \\
& in(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(\\
& 4*d*x + 4*c))) + 1))), (\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c) \\
&))^2 + \sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 2*\cos(1/2*ar \\
& ctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 1)^{1/4} * (\cos(1/4*\arctan2(\sin(\\
& 4*d*x + 4*c), \cos(4*d*x + 4*c))) * \cos(1/2*\arctan2(\sin(1/2*\arctan2(\sin(4*d*x \\
& + 4*c), \cos(4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4 \\
& *c))) + 1)) + \sin(1/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(1/2* \\
& arctan2(\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2*arcta \\
& n2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 1))) - 1) - (a^2*\cos(4*d*x + 4*c) \\
& ^2 + a^2*\sin(4*d*x + 4*c)^2 + 4*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4 \\
& *c)^2 - 2*a^2*\cos(4*d*x + 4*c) + a^2)*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos \\
& (4*d*x + 4*c)))^2 + 4*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 + 2* \\
& a^2*\cos(4*d*x + 4*c) + a^2)*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4 \\
& *c)))^2 + 4*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 - a^2*\cos(4*d* \\
& x + 4*c))*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 4*(4*a^2*c \\
& os(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + a^2* \\
& \sin(4*d*x + 4*c)) * \sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * arc \\
& tan2((\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + \sin(1/2*arct \\
& an2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 2*\cos(1/2*\arctan2(\sin(4*d*x + \\
& 4*c), \cos(4*d*x + 4*c))) + 1)^{1/4} * \sin(1/2*\arctan2(\sin(1/2*\arctan2(\sin(4*d \\
& *x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x \\
& + 4*c))) + 1)), (\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + s \\
& in(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 2*\cos(1/2*\arctan2(s \\
& in(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 1)^{1/4} * \cos(1/2*\arctan2(\sin(1/2*arct \\
& an2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4*d*x + 4*c), \\
& \cos(4*d*x + 4*c))) + 1)) + 1) + (a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + \\
& 4*c)^2 + 4*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 - 2*a^2*\cos(4*d \\
& *x + 4*c) + a^2)*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 4 \\
& *(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 + 2*a^2*\cos(4*d*x + 4*c) \\
& + a^2)*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 4*(a^2*\cos(\\
& 4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 - a^2*\cos(4*d*x + 4*c))*\cos(1/2*arc
\end{aligned}$$


```

tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))) + 1)), (cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))) +
1) - 1))*sqrt(a))/((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*
d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*
(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(co
s(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*
c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*d

```

Giac [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{5/2} dx = \int (a \cos(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{5/2} dx = \int \cos(c + dx)^{3/2} (a + a \cos(c + dx))^{5/2} dx$$

[In] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2), x)

3.214 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2} dx$

Optimal result	3126
Rubi [A] (verified)	3126
Mathematica [C] (warning: unable to verify)	3129
Maple [A] (verified)	3129
Fricas [A] (verification not implemented)	3129
Sympy [F(-1)]	3130
Maxima [B] (verification not implemented)	3130
Giac [F]	3132
Mupad [F(-1)]	3132

Optimal result

Integrand size = 25, antiderivative size = 160

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2} dx = \frac{25a^{5/2} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{25a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3 \cos^{3/2}(c+dx) \sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2 \cos^{3/2}(c+dx) \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{3d}$$

[Out] $25/8*a^{(5/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+13/12*a^3*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+25/8*a^3*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}+1/3*a^2*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2842, 3060, 2849, 2853, 222}

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2} dx = \frac{25a^{5/2} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d} + \frac{13a^3 \sin(c+dx) \cos^{3/2}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}} + \frac{25a^3 \sin(c+dx) \sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}} + \frac{a^2 \sin(c+dx) \cos^{3/2}(c+dx) \sqrt{a\cos(c+dx)+a}}{3d}$$

[In] Int[Sqrt[Cos[c + d*x]]*(a + a*cos[c + d*x])^(5/2),x]

[Out] (25*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*cos[c + d*x]])/(8*d) + (25*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*cos[c + d*x]]) + (13*a^3*cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*cos[c + d*x]]) + (a^2*cos[c + d*x]^(3/2)*Sqrt[a + a*cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2842

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2849

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[-2*b*cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 3060

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[-2*b*B*cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]

/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{3d} \\
 &+ \frac{1}{3} \int \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)} \left(\frac{9a^2}{2} + \frac{13}{2} a^2 \cos(c+dx) \right) dx \\
 &= \frac{13a^3 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)}} + \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{3d} \\
 &+ \frac{1}{8} (25a^2) \int \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)} dx \\
 &= \frac{25a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{8d \sqrt{a+a \cos(c+dx)}} + \frac{13a^3 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)}} \\
 &+ \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{3d} \\
 &+ \frac{1}{16} (25a^2) \int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
 &= \frac{25a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{8d \sqrt{a+a \cos(c+dx)}} + \frac{13a^3 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)}} \\
 &+ \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{3d} \\
 &- \frac{(25a^2) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}} \right)}{8d} \\
 &= \frac{25a^{5/2} \arcsin \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}} \right)}{8d} + \frac{25a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{8d \sqrt{a+a \cos(c+dx)}} \\
 &+ \frac{13a^3 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)}} + \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{3d}
 \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.87 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2} dx = \frac{(a(1+\cos(c+dx)))^{5/2} \sec^4\left(\frac{1}{2}(c+dx)\right) (7(89+28\cos(c+dx))+3\cos(2(c+dx))) \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2}, \frac{7}{2}, 2\sin\left(\frac{c+dx}{2}\right)\right]^2 - 8(3+\cos(c+dx)) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{2}, \frac{9}{2}, 2\sin\left(\frac{c+dx}{2}\right)\right]^2 * \sin(c+dx)^2 - 2\text{Csc}\left(\frac{c+dx}{2}\right)^2 \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{3}{2}, 2\right\}, \left\{1, \frac{9}{2}\right\}, 2\sin\left(\frac{c+dx}{2}\right)^2\right] * \sin(c+dx)^4 * \tan\left(\frac{c+dx}{2}\right)}{420*d}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2), x]

[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F1[-1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] - 8*(3 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 - 2*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{1/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4*Tan[(c + d*x)/2])/(420*d)

Maple [A] (verified)

Time = 12.50 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.14

method	result
default	$\frac{(8 \sin(dx+c) (\cos^2(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 34 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 75 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 75 \arctan(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{24d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(5/2), x, method=_RETURNVERBOSE)

[Out] 1/24/d*(8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+34*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+75*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+75*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2} dx = \frac{(8a^2 \cos(dx+c)^2 + 34a^2 \cos(dx+c) + 75a^2) \sqrt{a \cos(dx+c) + a} \sqrt{\cos(dx+c)} \sin(dx+c)}{24(d \cos(dx+c) + d)}$$

1)) - 1))*sqrt(a))/d

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2} dx = \int (a\cos(dx+c)+a)^{5/2} \sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2} dx = \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2} dx$$

[In] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2), x)

$$3.215 \quad \int \frac{(a+a \cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	3133
Rubi [A] (verified)	3133
Mathematica [C] (warning: unable to verify)	3135
Maple [A] (verified)	3135
Fricas [A] (verification not implemented)	3136
Sympy [F(-1)]	3136
Maxima [B] (verification not implemented)	3136
Giac [F(-1)]	3137
Mupad [F(-1)]	3137

Optimal result

Integrand size = 25, antiderivative size = 120

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}} dx = \frac{19a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4d} + \frac{9a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{4d \sqrt{a+a \cos(c+dx)}} + \frac{a^2 \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{2d}$$

[Out] $19/4*a^{(5/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+9/4*a^3*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}+1/2*a^2*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(a+a*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2842, 3060, 2853, 222}

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}} dx = \frac{19a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4d} + \frac{9a^3 \sin(c+dx) \sqrt{\cos(c+dx)}}{4d \sqrt{a \cos(c+dx)+a}} + \frac{a^2 \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}{2d}$$

[In] $\text{Int}[(a+a*\text{Cos}[c+d*x])^{(5/2)}/\text{Sqrt}[\text{Cos}[c+d*x]],x]$

[Out] $(19*a^{(5/2)}*\text{ArcSin}[(\text{Sqrt}[a]*\text{Sin}[c+d*x])/\text{Sqrt}[a+a*\text{Cos}[c+d*x]]])/(4*d) + (9*a^3*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(4*d*\text{Sqrt}[a+a*\text{Cos}[c+d*x]]) + (a^2*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(2*d)$

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_) + (f_.)
*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_) + (
f_.)*(x_)])*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]))], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2 \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{2d} \\
&+ \frac{1}{2} \int \frac{\sqrt{a + a \cos(c + dx)} \left(\frac{5a^2}{2} + \frac{9}{2} a^2 \cos(c + dx) \right)}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{9a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{4d \sqrt{a + a \cos(c + dx)}} + \frac{a^2 \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{2d} \\
&+ \frac{1}{8} (19a^2) \int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{9a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{4d \sqrt{a+a \cos(c+dx)}} + \frac{a^2 \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{2d} \\
&\quad - \frac{(19a^2) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4d} \\
&= \frac{19a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4d} + \frac{9a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{4d \sqrt{a+a \cos(c+dx)}} \\
&\quad + \frac{a^2 \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.80 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.52

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}} dx = \frac{(a(1+\cos(c+dx)))^{5/2} \sec^4\left(\frac{1}{2}(c+dx)\right) (7(89+28 \cos(c+dx)) + 3 \cos(2(c+dx)))}{4d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]

[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x]) + 3*Cos[2*(c + d*x)])*Hypergeometric2F1[1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 8*(3 + Cos[c + d*x])*Hypergeometric2F1[3/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 + 2*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{3/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4*Tan[(c + d*x)/2])/(420*d)

Maple [A] (verified)

Time = 13.84 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.43

method	result
default	$\frac{(2(\cos^2(dx+c)) \sin(dx+c) + 19 \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) + 11 \cos(dx+c) \sin(dx+c) + 19 \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})}{4d \sqrt{\cos(dx+c)} (1+\cos(dx+c))}$

[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4/d*(2*cos(d*x+c)^2*sin(d*x+c)+19*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+11*cos(d*x+c)*sin(d*x+c)+19*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/(1+cos(d*x+c))*a^2

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \frac{(2a^2 \cos(dx + c) + 11a^2) \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c) - 19}{4(d \cos(dx + c) + d)}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

```
[Out] 1/4*((2*a^2*cos(d*x + c) + 11*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1106 vs. 2(100) = 200.

Time = 0.47 (sec) , antiderivative size = 1106, normalized size of antiderivative = 9.22

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*d*x + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2*ar
```

```

ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))) + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d

```

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx$$

[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)

[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)

$$3.216 \quad \int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	3138
Rubi [A] (verified)	3138
Mathematica [C] (warning: unable to verify)	3140
Maple [A] (verified)	3140
Fricas [A] (verification not implemented)	3141
Sympy [F(-1)]	3141
Maxima [B] (verification not implemented)	3141
Giac [F(-1)]	3142
Mupad [F(-1)]	3142

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{5a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} - \frac{a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{d \sqrt{a+a \cos(c+dx)}} + \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

[Out] $5*a^{(5/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d-a^3*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}+2*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2841, 3060, 2853, 222}

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{5a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d} - \frac{a^3 \sin(c+dx) \sqrt{\cos(c+dx)}}{d \sqrt{a \cos(c+dx)+a}} + \frac{2a^2 \sin(c+dx) \sqrt{a \cos(c+dx)+a}}{d \sqrt{\cos(c+dx)}}$$

[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2), x]

[Out] $(5*a^{(5/2)}*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/d - (a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])$

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2841

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 3060

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
 &\quad - (2a) \int \frac{\left(-\frac{3a}{2} + \frac{1}{2}a \cos(c + dx)\right) \sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\
 &= -\frac{a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
 &\quad + \frac{1}{2} (5a^2) \int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{d \sqrt{a+a \cos(c+dx)}} + \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} \\
&\quad - \frac{(5a^2) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} \\
&= \frac{5a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} - \frac{a^3 \sqrt{\cos(c+dx)} \sin(c+dx)}{d \sqrt{a+a \cos(c+dx)}} \\
&\quad + \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.78 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.60

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{3/2}(c+dx)} dx = \frac{(a(1+\cos(c+dx)))^{5/2} \sec^4\left(\frac{1}{2}(c+dx)\right) (7(89+28 \cos(c+dx))+3 \cos(2(c+dx)))}{\dots}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2), x]

[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F1[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 24*(3 + Cos[c + d*x])*Hypergeometric2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 + 6*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{3/2, 2, 5/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)

Maple [A] (verified)

Time = 13.94 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.42

method	result
default	$ \frac{\left(5 \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) + \cos(dx+c) \sin(dx+c) + 5 \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)}{d(1+\cos(dx+c)) \sqrt{\cos(dx+c)}} $

[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/d*(5*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+cos(d*x+c)*sin(d*x+c)+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2*sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(1/2)*a^2

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.11

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{3/2}(c + dx)} dx = \frac{(a^2 \cos(dx + c) + 2a^2) \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c) - 5(a^2 \cos(dx + c) + 2a^2) \sqrt{\cos(dx + c)}}{d \cos(dx + c)^2 + d \cos(dx + c)}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")

```
[Out] ((a^2*cos(d*x + c) + 2*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{3/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 973 vs. 2(100) = 200.

Time = 0.50 (sec) , antiderivative size = 973, normalized size of antiderivative = 8.54

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{3/2}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")

```
[Out] 1/4*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))))
```

), $\cos(2dx + 2c) + 1$) + $\sin(dx + c) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 1 - a^2 \arctan2(-(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} (\cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \sin(dx + c) - \cos(dx + c) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))))$, $(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} (\cos(dx + c) \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + \sin(dx + c) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) - 1 - a^2 \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)))$, $(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 1 + a^2 \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)))$, $(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - 1) (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sqrt{a + 8(a^2 \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \sin(dx + c) - (a^2 \cos(dx + c) - a^2) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) \sqrt{a}} / ((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} d)$

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{3/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(a + a \cos(c + dx))^{5/2}}{\cos(c + dx)^{3/2}} dx$$

[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)

[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)

$$3.217 \quad \int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	3143
Rubi [A] (verified)	3143
Mathematica [C] (warning: unable to verify)	3145
Maple [A] (verified)	3145
Fricas [A] (verification not implemented)	3146
Sympy [F(-1)]	3146
Maxima [B] (verification not implemented)	3146
Giac [F(-1)]	3148
Mupad [F(-1)]	3148

Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{d} + \frac{14a^3 \sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} + \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] $2*a^{(5/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d+14/3*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}+2/3*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(3/2)}$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2841, 3059, 2853, 222}

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d} + \frac{14a^3 \sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}} + \frac{2a^2 \sin(c+dx)\sqrt{a \cos(c+dx)+a}}{3d \cos^{\frac{3}{2}}(c+dx)}$$

[In] $\text{Int}[(a+a*\text{Cos}[c+d*x])^{(5/2)}/\text{Cos}[c+d*x]^{(5/2)},x]$

[Out] $(2*a^{(5/2)}*\text{ArcSin}[(\text{Sqrt}[a]*\text{Sin}[c+d*x])/(\text{Sqrt}[a+a*\text{Cos}[c+d*x]])])/d + (14*a^3*\text{Sin}[c+d*x])/(3*d*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Cos}[c+d*x]]) + (2*a^2*\text{Sqrt}[a+a*\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(3*d*\text{Cos}[c+d*x]^{(3/2)})$

Rule 222

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2841

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2853

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} \\ &\quad - \frac{1}{3}(2a) \int \frac{\left(-\frac{7a}{2} - \frac{3}{2}a \cos(c + dx)\right) \sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{14a^3 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} \\ &\quad + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + a^2 \int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{14a^3 \sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{(2a^2) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&= \frac{2a^{5/2} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{14a^3 \sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.94 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.02

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{5/2}(c+dx)} dx = \frac{(a(1+\cos(c+dx)))^{5/2} \operatorname{csc}^3\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^5\left(\frac{c}{2} + \frac{dx}{2}\right) \left(256 \cos^4\left(\frac{1}{2}(c+dx)\right) {}_3F_2\left(\frac{5}{2}, \frac{3}{2}, \frac{7}{2}; \frac{9}{2}, \frac{11}{2}; z\right)\right)}{\cos^{5/2}(c+dx)}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]

[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Csc[c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^5*(256*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[c/2 + (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2, 2*Sin[c/2 + (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6*(2 - 3*Sin[c/2 + (d*x)/2]^2 + Sin[c/2 + (d*x)/2]^4) + (21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[c/2 + (d*x)/2]^2]]*(15 - 10*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4))/Sqrt[Sin[c/2 + (d*x)/2]^2] - 14*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(45 + 30*Sin[c/2 + (d*x)/2]^2)^2 - 31*Sin[c/2 + (d*x)/2]^4 + 12*Sin[c/2 + (d*x)/2]^6))/(672*d)

Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.44

method	result
default	$ \frac{2\left(3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c))\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+3\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)}{3d(1+\cos(dx+c))\cos(dx+c)^{\frac{3}{2}}} $

[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

```
[Out] 2/3/d*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+3*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+8*cos(d*x+c)*sin(d*x+c)+sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(3/2))*a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.11

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \frac{2 \left((8a^2 \cos(dx + c) + a^2) \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c) - 3 \left(d \cos(dx + c)^3 + a \right) \right)}{3 \left(d \cos(dx + c)^3 + a \right)}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*((8*a^2*cos(d*x + c) + a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. 2(100) = 200.

Time = 0.48 (sec) , antiderivative size = 1395, normalized size of antiderivative = 11.82

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
```

$$\begin{aligned}
& (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * ((\\
& 12a^2\cos(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) * \sin(2dx + 2c) \\
&) - 3a^2\sin(2dx + 2c) - 4*(3a^2\cos(2dx + 2c) + 4a^2) * \sin(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) * \cos(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + (12a^2\sin(2dx + 2c) * \sin(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) + 3a^2\cos(2dx + 2c) - a^2 + 4*(3a^2\cos(2dx + 2c) + 4a^2) * \cos(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) * \sin(3/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) * \sqrt{a} + 3*((a^2\cos(2dx + 2c)^2 + a^2\sin(2dx + 2c)^2 + 2a^2\cos(2dx + 2c) + a^2) * \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) * \sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - \cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) * \sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) * \cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) + \sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) * \sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) + 1) - (a^2\cos(2dx + 2c)^2 + a^2\sin(2dx + 2c)^2 + 2a^2\cos(2dx + 2c) + a^2) * \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) * \sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - \cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) * \sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c))))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) * \cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) + \sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) * \sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) - 1) - (a^2\cos(2dx + 2c)^2 + a^2\sin(2dx + 2c)^2 + 2a^2\cos(2dx + 2c) + a^2) * \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 1) + (a^2\cos(2dx + 2c)^2 + a^2\sin(2dx + 2c)^2 + 2a^2\cos(2dx + 2c) + a^2) * \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \sin(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \cos(1/2\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - 1)) * \sqrt{a} / ((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1) * d)
\end{aligned}$$

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \int \frac{(a + a \cos(c + dx))^{5/2}}{\cos(c + dx)^{5/2}} dx$$

```
[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)
```


$$3.218 \quad \int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	3149
Rubi [A] (verified)	3149
Mathematica [A] (verified)	3151
Maple [A] (verified)	3151
Fricas [A] (verification not implemented)	3151
Sympy [F(-1)]	3152
Maxima [A] (verification not implemented)	3152
Giac [F(-1)]	3152
Mupad [B] (verification not implemented)	3153

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{22a^3 \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{86a^3 \sin(c+dx)}{15d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}} + \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)}$$

[Out] $22/15*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(1/2)}+86/15*a^3*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}+2/5*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(5/2)}$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2841, 3059, 2850}

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{22a^3 \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} + \frac{86a^3 \sin(c+dx)}{15d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx) + a}} + \frac{2a^2 \sin(c+dx) \sqrt{a \cos(c+dx) + a}}{5d \cos^{\frac{5}{2}}(c+dx)}$$

[In] $\text{Int}[(a+a*\text{Cos}[c+d*x])^{(5/2)}/\text{Cos}[c+d*x]^{(7/2)},x]$

[Out] $(22*a^3*\text{Sin}[c+d*x])/(15*d*\text{Cos}[c+d*x]^{(3/2)}*\text{Sqrt}[a+a*\text{Cos}[c+d*x]]) + (86*a^3*\text{Sin}[c+d*x])/(15*d*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Cos}[c+d*x]]) + (2*a^2*\text{Sqrt}[a+a*\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(5*d*\text{Cos}[c+d*x]^{(5/2)})$

Rule 2841

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))

```

Rule 2850

```

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3059

```

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\
&\quad - \frac{1}{5}(2a) \int \frac{\left(-\frac{11a}{2} - \frac{7}{2}a \cos(c + dx)\right) \sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{22a^3 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\
&\quad + \frac{1}{15}(43a^2) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx
\end{aligned}$$

$$= \frac{22a^3 \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{86a^3 \sin(c+dx)}{15d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}} \\ + \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{a^2 \sqrt{a(1+\cos(c+dx))} (49+28 \cos(c+dx)+43 \cos(2(c+dx))) \tan\left(\frac{1}{2}(c+dx)\right)}{15d \cos^{\frac{5}{2}}(c+dx)}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]

[Out] (a^2*sqrt[a*(1 + Cos[c + d*x])]*(49 + 28*Cos[c + d*x] + 43*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 5.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2 \sin(dx+c) (43 (\cos^2(dx+c)) + 14 \cos(dx+c) + 3) \sqrt{a(1+\cos(dx+c))} a^2}{15d(1+\cos(dx+c)) \cos(dx+c)^{\frac{5}{2}}}$	65

[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/15/d*sin(d*x+c)*(43*cos(d*x+c)^2+14*cos(d*x+c)+3)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(5/2)*a^2

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.67

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{2(43a^2 \cos(dx+c)^2 + 14a^2 \cos(dx+c) + 3a^2) \sqrt{a \cos(dx+c) + a} \sqrt{\cos(dx+c)}}{15(d \cos(dx+c)^4 + d \cos(dx+c)^3)}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2), x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.25

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx = \frac{8 \left(\frac{15 \sqrt{2} a^{5/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sqrt{2} a^{5/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{28 \sqrt{2} a^{5/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{8 \sqrt{2} a^{5/2} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right)}{15 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{7/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{7/2}}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2), x, algorithm="maxima")

[Out] 8/15*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2))

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2), x, algorithm="giac")

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 16.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx = \frac{2 a^2 \sqrt{a (\cos(c + dx) + 1)} (98 \sin(c + dx) + 56 \sin(2c + 2dx) + 141 \sin(3c + 3dx) + 28 \sin(4c + 4dx) + 43 \sin(5c + 5dx))}{15 d \sqrt{\cos(c + dx)} (10 \cos(c + dx) + 8 \cos(2c + 2dx) + 5 \cos(3c + 3dx) + 2 \cos(4c + 4dx) + \cos(5c + 5dx) + 6)}$$

[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2),x)

[Out] (2*a^2*(a*(cos(c + d*x) + 1))^(1/2)*(98*sin(c + d*x) + 56*sin(2*c + 2*d*x) + 141*sin(3*c + 3*d*x) + 28*sin(4*c + 4*d*x) + 43*sin(5*c + 5*d*x)))/(15*d*cos(c + d*x)^(1/2)*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) + 2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))

$$3.219 \quad \int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	3154
Rubi [A] (verified)	3154
Mathematica [A] (verified)	3156
Maple [A] (verified)	3157
Fricas [A] (verification not implemented)	3157
Sympy [F(-1)]	3157
Maxima [A] (verification not implemented)	3158
Giac [C] (verification not implemented)	3158
Mupad [B] (verification not implemented)	3579

Optimal result

Integrand size = 25, antiderivative size = 161

$$\begin{aligned} \int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)} dx &= \frac{6a^3 \sin(c+dx)}{7d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} \\ &+ \frac{46a^3 \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{92a^3 \sin(c+dx)}{21d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}} \\ &+ \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} \end{aligned}$$

[Out] $\frac{6}{7} \frac{a^3 \sin(dx+c)}{d \cos(dx+c)^{5/2} (a+a \cos(dx+c))^{1/2}} + \frac{46}{21} \frac{a^3 \sin(dx+c)}{d \cos(dx+c)^{3/2} (a+a \cos(dx+c))^{1/2}} + \frac{92}{21} \frac{a^3 \sin(dx+c)}{d \cos(dx+c)^{1/2} (a+a \cos(dx+c))^{1/2}} + \frac{2}{7} \frac{a^2 \sin(dx+c) \sqrt{a+a \cos(dx+c)}}{d \cos(dx+c)^{7/2}}$

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2841, 3059, 2851, 2850}

$$\begin{aligned} \int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)} dx &= \frac{46a^3 \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} \\ &+ \frac{6a^3 \sin(c+dx)}{7d \cos^{\frac{5}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} + \frac{92a^3 \sin(c+dx)}{21d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx) + a}} \\ &+ \frac{2a^2 \sin(c+dx) \sqrt{a \cos(c+dx) + a}}{7d \cos^{\frac{7}{2}}(c+dx)} \end{aligned}$$

[In] Int[(a + a*cos[c + d*x])^(5/2)/cos[c + d*x]^(9/2),x]

[Out] (6*a^3*sin[c + d*x])/(7*d*cos[c + d*x]^(5/2)*sqrt[a + a*cos[c + d*x]]) + (4*6*a^3*sin[c + d*x])/(21*d*cos[c + d*x]^(3/2)*sqrt[a + a*cos[c + d*x]]) + (9*2*a^3*sin[c + d*x])/(21*d*sqrt[cos[c + d*x]]*sqrt[a + a*cos[c + d*x]]) + (2*a^2*sqrt[a + a*cos[c + d*x]]*sin[c + d*x])/(7*d*cos[c + d*x]^(7/2))

Rule 2841

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*cos[e + f*x]*(a + b*sin[e + f*x])^(m - 2)*((c + d*sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*sin[e + f*x])^(m - 2)*(c + d*sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2850

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(cos[e + f*x]/(f*(b*c + a*d)*sqrt[a + b*sin[e + f*x]]*sqrt[c + d*sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*cos[e + f*x]*((c + d*sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*sqrt[a + b*sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*sin[e + f*x]]*(c + d*sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rule 3059

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(B*c - A*d)*cos[e + f*x]*((c + d*sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d)*sqrt[a + b*sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*sin[e + f*x]]*(c + d*sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} \\
&\quad - \frac{1}{7}(2a) \int \frac{\left(-\frac{15a}{2} - \frac{11}{2}a \cos(c + dx)\right) \sqrt{a + a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{6a^3 \sin(c + dx)}{7d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} \\
&\quad + \frac{1}{7}(23a^2) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{6a^3 \sin(c + dx)}{7d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{46a^3 \sin(c + dx)}{21d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{1}{21}(46a^2) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{6a^3 \sin(c + dx)}{7d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{46a^3 \sin(c + dx)}{21d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{92a^3 \sin(c + dx)}{21d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \frac{a^2 \sqrt{a(1 + \cos(c + dx))} (29 + 93 \cos(c + dx) + 23 \cos(2(c + dx)) + 23 \cos(3(c + dx)))}{21d \cos^{7/2}(c + dx)}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2), x]

[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(29 + 93*Cos[c + d*x] + 23*Cos[2*(c + d*x)] + 23*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(21*d*Cos[c + d*x]^(7/2))

Maple [A] (verified)

Time = 5.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{2 \sin(dx+c) (46 \cos^3(dx+c) + 23 \cos^2(dx+c) + 12 \cos(dx+c) + 3) \sqrt{a(1+\cos(dx+c))} a^2}{21d(1+\cos(dx+c)) \cos(dx+c)^{\frac{7}{2}}}$	75

[In] `int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{2}{21} \frac{d \sin(dx+c) (46 \cos^3(dx+c) + 23 \cos^2(dx+c) + 12 \cos(dx+c) + 3) (a(1+\cos(dx+c)))^{1/2}}{(1+\cos(dx+c)) \cos(dx+c)^{7/2}} a^2$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \frac{2 (46 a^2 \cos(dx + c)^3 + 23 a^2 \cos(dx + c)^2 + 12 a^2 \cos(dx + c) + 3 a^2) \sqrt{a \cos(dx + c)}}{21 (d \cos(dx + c)^5 + d \cos(dx + c)^4)}$$

[In] `integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")`

[Out] $\frac{2}{21} \frac{(46 a^2 \cos^3(dx + c) + 23 a^2 \cos^2(dx + c) + 12 a^2 \cos(dx + c) + 3 a^2) \sqrt{a \cos(dx + c)}}{(d \cos^5(dx + c) + d \cos^4(dx + c)) \sin(dx + c)}$

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \text{Timed out}$$

[In] `integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2),x)`

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.51

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \frac{8 \left(\frac{21 \sqrt{2} a^{5/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{56 \sqrt{2} a^{5/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{63 \sqrt{2} a^{5/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{36 \sqrt{2} a^{5/2} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{8 \sqrt{2} a^{5/2} \sin(dx+c)^9}{(\cos(dx+c)+1)^9} \right)}{21 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{9/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{9/2} \left(\frac{2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 1 \right)}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")

[Out] 8/21*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 237.10 (sec) , antiderivative size = 98101, normalized size of antiderivative = 609.32

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="giac")

[Out] -67108864/21*sqrt(2)*sqrt(-tan(1/4*d*x + c)^4*tan(1/2*c)^8 + 14*tan(1/4*d*x + c)^4*tan(1/2*c)^6 - 24*tan(1/4*d*x + c)^3*tan(1/2*c)^7 + 6*tan(1/4*d*x + c)^2*tan(1/2*c)^8 + 56*tan(1/4*d*x + c)^3*tan(1/2*c)^5 - 84*tan(1/4*d*x + c)^2*tan(1/2*c)^6 + 24*tan(1/4*d*x + c)*tan(1/2*c)^7 - tan(1/2*c)^8 - 14*tan(1/4*d*x + c)^4*tan(1/2*c)^2 + 56*tan(1/4*d*x + c)^3*tan(1/2*c)^3 - 56*tan(1/4*d*x + c)*tan(1/2*c)^5 + 14*tan(1/2*c)^6 + tan(1/4*d*x + c)^4 - 24*tan(1/4*d*x + c)^3*tan(1/2*c) + 84*tan(1/4*d*x + c)^2*tan(1/2*c)^2 - 56*tan(1/4*d*x + c)*tan(1/2*c)^3 - 6*tan(1/4*d*x + c)^2 + 24*tan(1/4*d*x + c)*tan(1/2*c) - 14*tan(1/2*c)^2 + 1)*((((((((((((((((((23*I*a^2*e^(1027/2*I*c) + 8970*I*a^2*e^(1025/2*I*c) + 1744665*I*a^2*e^(1023/2*I*c) + 225643340*I*a^2*e^(1021/2*I*c) + 21830993145*I*a^2*e^(1019/2*I*c) + 1685352670794*I*a^2*e^(1017/2*I*c) + 108143463042754*I*a^2*e^(1015/2*I*c) + 5932441401249090*I*a^2*e^(1013/2*I*c) + 284015632092748725*I*a^2*e^(1011/2*I*c) + 12054885718630588825*I*a^2*e^(1009/2*I*c) + 459291145959804703779*I*a^2*e^(1007/2*I*c) + 15866421

411511793416437*I*a^2*e^(1005/2*I*c) + 501114476578787912641049*I*a^2*e^(10
 03/2*I*c) + 14570867105062952981500815*I*a^2*e^(1001/2*I*c) + 3923726363658
 91369041933300*I*a^2*e^(999/2*I*c) + 9835474114732862868439582838*I*a^2*e^(
 997/2*I*c) + 230518925632259863716881504052*I*a^2*e^(995/2*I*c) + 507141639
 8732103030201521505627*I*a^2*e^(993/2*I*c) + 105091018637393463590223550684
 728*I*a^2*e^(991/2*I*c) + 2057571551559319523757433019101275*I*a^2*e^(989/2
 *I*c) + 38167953019220461788377878471708140*I*a^2*e^(987/2*I*c) + 672482999
 427386036344407124264639911*I*a^2*e^(985/2*I*c) + 1127937434108902451695038
 9703822811442*I*a^2*e^(983/2*I*c) + 180469997815998782717102062578903704334
 *I*a^2*e^(981/2*I*c) + 2759687216371547310850322318637953260785*I*a^2*e^(97
 9/2*I*c) + 40401823984348761731551610542858793511300*I*a^2*e^(977/2*I*c) +
 567179508185660065903323118694229252909501*I*a^2*e^(975/2*I*c) + 7646420990
 625646107842349221267149768725400*I*a^2*e^(973/2*I*c) + 9913040189115061622
 7034640375461310347846749*I*a^2*e^(971/2*I*c) + 123742111851876855210771405
 3055438083342617820*I*a^2*e^(969/2*I*c) + 148903043462731256823069358460291
 08244114648650*I*a^2*e^(967/2*I*c) + 17291971385704661381306525573004218622
 9347958876*I*a^2*e^(965/2*I*c) + 193994372823371289858261550999658330105555
 4831933*I*a^2*e^(963/2*I*c) + 210454591666563299342031312342284106762563689
 47239*I*a^2*e^(961/2*I*c) + 22097743510955796389734819459141934959607508467
 1135*I*a^2*e^(959/2*I*c) + 224765758157689015832828145946305324913615514281
 8885*I*a^2*e^(957/2*I*c) + 221644173931247842567801070554974270573942880219
 10427*I*a^2*e^(955/2*I*c) + 21205974502313462941186749668142745993901806065
 9823083*I*a^2*e^(953/2*I*c) + 196992543655509626330354351334503037860435363
 5783621398*I*a^2*e^(951/2*I*c) + 177798612029937479756178056569168927717178
 74809673606262*I*a^2*e^(949/2*I*c) + 15601850139083301492402893352264558109
 1836687267573447170*I*a^2*e^(947/2*I*c) + 133186743835690094739408533396330
 8875669933795527159861115*I*a^2*e^(945/2*I*c) + 110672052589031643118494748
 72837148947402911102321523307256*I*a^2*e^(943/2*I*c) + 89567346975466416436
 560959012746672316379146573614093235883*I*a^2*e^(941/2*I*c) + 7063624656191
 02268226202879462703285306437535601820801093746*I*a^2*e^(939/2*I*c) + 54311
 58420289695690267628216417883201506781644825673663895125*I*a^2*e^(937/2*I*c
) + 40733825083879692084677912159273504330596927222466270616023040*I*a^2*e^
 (935/2*I*c) + 2981380798057955166367217467136663387761346390934173084580836
 40*I*a^2*e^(933/2*I*c) + 21304543896397841025392810964658996330528756254707
 54655905397856*I*a^2*e^(931/2*I*c) + 14869776488978491309444307452594685609
 833202276809994141174545956*I*a^2*e^(929/2*I*c) + 1014124524670115149879605
 08445679910794713086096737145187354675120*I*a^2*e^(927/2*I*c) + 67608737685
 7312474146771786049384206737469239071720613980492039300*I*a^2*e^(925/2*I*c)
 + 4407601795177120237389113481959743264257864428283490771370053462560*I*a^
 2*e^(923/2*I*c) + 281090883693732081557077857440001487224968658894703178673
 62512306740*I*a^2*e^(921/2*I*c) + 17542316219649612394009093118130642266705
 8440569170422851340745050152*I*a^2*e^(919/2*I*c) + 107168724187918360820483
 4854430988172125784220926155326017366664031320*I*a^2*e^(917/2*I*c) + 641106
 1389985712625839045934560628663538960416714378812599630238383260*I*a^2*e^(9
 15/2*I*c) + 375670642714870903983781671889666877915127767039192690108854011

42609920*I*a^2*e^(913/2*I*c) + 21568993975887401396718722083353373164324207
 4212236222605165544572554060*I*a^2*e^(911/2*I*c) + 121373283851051030642898
 2602384538617690483226714330573193357678961518000*I*a^2*e^(909/2*I*c) + 669
 5882317013495222345368298476838368511994443900460264534471085890483820*I*a^
 2*e^(907/2*I*c) + 362243658255481769079439605579958387100105565618941470113
 23135173581402240*I*a^2*e^(905/2*I*c) + 19222721348090494887203432115491092
 0985425803997803298273976235657509933640*I*a^2*e^(903/2*I*c) + 100082726787
 8708901757364273781353911409779752889226260010612599185707764320*I*a^2*e^(9
 01/2*I*c) + 511374503991778257094228867244440476224017707247445734826860865
 8998353643330*I*a^2*e^(899/2*I*c) + 256481940130631614458139173236114739489
 67661308908878336387655736333479132840*I*a^2*e^(897/2*I*c) + 12630226136697
 0813190120863068164729403993511960131743155255365483034776033710*I*a^2*e^(8
 95/2*I*c) + 610798290612478058634809110185506816479817707018034933796191469
 491921071289700*I*a^2*e^(893/2*I*c) + 2901413407805046277699928806426570164
 361980656563461341652112845236338748650110*I*a^2*e^(891/2*I*c) + 1354055460
 5087702310393396477276367400444112984077543019342063464929767322886040*I*a^
 2*e^(889/2*I*c) + 620962735163185935779611173057839829149089325652410656700
 71393675735767176645100*I*a^2*e^(887/2*I*c) + 27988619397464174037567262664
 2653063855265114960677901721413367956114946541063180*I*a^2*e^(885/2*I*c) +
 124012762440879058591428161804909999214977526014035569685290341969253022717
 7449210*I*a^2*e^(883/2*I*c) + 540256474462854504612952008124460438123572687
 3384595583804088637122848866439830910*I*a^2*e^(881/2*I*c) + 231451344052759
 63083954762748231025164257628027633977804093014188701364289864915790*I*a^2*
 e^(879/2*I*c) + 97526087003487022632582290289825035947901332365723366039562
 344307089798567346576790*I*a^2*e^(877/2*I*c) + 4042559161973144102059036313
 45838918131518622031814368289391512007823341054252319330*I*a^2*e^(875/2*I*c
) + 16486845255781261985040853913164070221877118197466968972726190217745665
 83748173498450*I*a^2*e^(873/2*I*c) + 66165787175526862693523980355038876126
 74609084260960128356927902046841481605353166520*I*a^2*e^(871/2*I*c) + 26134
 333584281766199506203706665359796711411159203716561602842513749476935891574
 081580*I*a^2*e^(869/2*I*c) + 1016101003988137282448021482325200966315174895
 01850198195876637177700803300250750696120*I*a^2*e^(867/2*I*c) + 38893191411
 963295484013014908614078945237385578780791021284415179604928900460540696715
 0*I*a^2*e^(865/2*I*c) + 146583001674253512451913146901065257757197591875644
 0541603851467254363529939075434269440*I*a^2*e^(863/2*I*c) + 544035755123037
 4902439985027394698228560509433313958273980496531279067808247723147105870*I
 *a^2*e^(861/2*I*c) + 198867587799113605322612027213532735821605971082743528
 59715885782505631373182736113224200*I*a^2*e^(859/2*I*c) + 71606217945850994
 71743068260276455922862272227808236824799962339103295121387601682173670*I*
 a^2*e^(857/2*I*c) + 2540058012870175403474010349780220858339706707380981477
 73886519381628953611819486811955620*I*a^2*e^(855/2*I*c) + 88776357742106247
 4940232214817331913856875978988239568134264776314007646535947673652724060*I
 *a^2*e^(853/2*I*c) + 305749026252213047211126953990398197866952487201346411
 2348409985533590524778523877236940090*I*a^2*e^(851/2*I*c) + 103776474298111
 268920821296539685875165453416594719110175099266389481283166180086952156183

60*I*a^2*e^(849/2*I*c) + 34717589857137672406923008566078790348191298872900
136928769377324783537177053188579391348690*I*a^2*e^(847/2*I*c) + 1144897513
265917816015872965826669331649064191233018567283685478610735438331877997532
83463360*I*a^2*e^(845/2*I*c) + 37221892415037889443572040902664005989095539
2590616282572164380128530253733901184626128906290*I*a^2*e^(843/2*I*c) + 119
314265750582670724834440624950185293640630911035594239694349009543441669669
8542207244495880*I*a^2*e^(841/2*I*c) + 377133056150951252461585059664034129
4357967299596271045598767418091013562271264018462771488980*I*a^2*e^(839/2*I
*c) + 117557492389849429622100629661757412192769384297540981500776815193118
75621804705720370211969800*I*a^2*e^(837/2*I*c) + 36141293667864514706613710
081812463630711904602288654112421060177161306607694059860383771279230*I*a^2
*e^(835/2*I*c) + 1095968214363242841726512152035509927963690094832507106773
45396805851498207559908266552991860510*I*a^2*e^(833/2*I*c) + 32785015633825
072564872192218982082848801710226763359158924377756236392724339435620243214
0538650*I*a^2*e^(831/2*I*c) + 967558857573420421200844825745019691984965901
119321858915203967518001948679909749229811961456530*I*a^2*e^(829/2*I*c) + 2
817373842739968143686897684606727718286414616842921650529046416318630866894
611374955142903521970*I*a^2*e^(827/2*I*c) + 8094990486752031102862847380062
157212388106523544569804503299552643657151717648240648270653869830*I*a^2*e^
(825/2*I*c) + 2295259936503095142465474029958186708878313715335541773803916
0015930697452167681087479690813393300*I*a^2*e^(823/2*I*c) + 642287093604098
124205861383565642110850082887627622096970058154814139059827243993231939647
45568820*I*a^2*e^(821/2*I*c) + 17739675542894941311253267670481610036345737
2470857284075358267979222962448762703682130780576952760*I*a^2*e^(819/2*I*c)
+ 483635269585920480609387707945457876743335983922410816057000799690442279
864698695936197638579602610*I*a^2*e^(817/2*I*c) + 1301613491181268974287090
549535283627357435997753232042703656582610084719317446750492963887848291340
*I*a^2*e^(815/2*I*c) + 3458383981160096213819834986181221606030887703758892
748920536238488574632981453327231642336638145090*I*a^2*e^(813/2*I*c) + 9072
482678531422390678482116667447595677867438372356824783887256763662854205131
381685197149710111720*I*a^2*e^(811/2*I*c) + 2350038180733536582371812906680
1316443196720042966976789038657507310743872020449940329840767115547150*I*a^
2*e^(809/2*I*c) + 601108690622843741594482098547268387114298912719254377848
21736413864552441235504659113309232113779360*I*a^2*e^(807/2*I*c) + 15184249
844321680132979221701134989863470387416943076510333347856618440977436539297
2733790654189070040*I*a^2*e^(805/2*I*c) + 378816002128920616369855929612160
874580768723711392251915842505525188562342470825520416422818082592000*I*a^2
*e^(803/2*I*c) + 9334473368592818432243553973420584946686982924440282529349
15487603996507357110331502736639692759358340*I*a^2*e^(801/2*I*c) + 22720048
811620935900466503561284482119532730047520542947942040259999751890733350464
66698628779651401360*I*a^2*e^(799/2*I*c) + 54628286678221586727361647210129
38562781809091770363563687018271454099131199171596676490634462439229220*I*a
^2*e^(797/2*I*c) + 12976133089122616927641373067671760628448690333305759931
883973052278026353267012367339299301873995466880*I*a^2*e^(795/2*I*c) + 3045
246785608650155466937068633484708402407505066968240527155699757540924494810

7709378067847213898960660*I*a^2*e^(793/2*I*c) + 706120625711697868994464587
 973763116513326309362048745112427355635163868880849095996197865807877112363
 60*I*a^2*e^(791/2*I*c) + 16178708438189461126289696260126574890420941225938
 9006374551841288165005493773030958210956760612646492280*I*a^2*e^(789/2*I*c)
 + 366307954798320205128798936155351391554335176467391622039978284149371380
 873241652456477113543820574273500*I*a^2*e^(787/2*I*c) + 8196247895171130191
 642817095414875203682334368894408831281121859152636556514559828805773471134
 10414307040*I*a^2*e^(785/2*I*c) + 18125017737117436736993992115331571295088
 23845114002513848872710211505465235145932640002862720411729752780*I*a^2*e^(
 783/2*I*c) + 39615443892690555191826925561670874009158112607811540466428348
 32166886887365224821817220970795789343876240*I*a^2*e^(781/2*I*c) + 85585699
 131774977720413736285672113976827764195153726973137506812103604497173350946
 87818153095679360516780*I*a^2*e^(779/2*I*c) + 18277470691534917701563233602
 298403971199737084099495749012388579821538776969597068131015951749218506666
 144*I*a^2*e^(777/2*I*c) + 3858660998287657496223400479535149465741817808337
 5590488901031957266204930635139096439961363048441151269800*I*a^2*e^(775/2*I
 *c) + 805359935544168628569903856943178293750286803058910337542718545982378
 50137678037212744640081812890524571520*I*a^2*e^(773/2*I*c) + 16618941002251
 115567274793379648778017515414556569019749473008667024648037226698449778484
 7689078576576553405*I*a^2*e^(771/2*I*c) + 339081049398958631695033084312249
 444034247861955469658173982280143780778138876895429955188412068983435311370
 *I*a^2*e^(769/2*I*c) + 6840964393238201555734771754472551627703517728713097
 06432815320177180262386714126080576227107839963492633507*I*a^2*e^(767/2*I*c
) + 13648077061320324179712693353613780904653490228983600018030481420618939
 27200900333187448316673412979345898752*I*a^2*e^(765/2*I*c) + 26927282448425
 364461722623574956155733196639443211460545174612226257404076090636468048967
 71692589250685909715*I*a^2*e^(763/2*I*c) + 52542053315628400861248402353101
 698809320987341776979631277027119955905761956900693691812471722966160542508
 10*I*a^2*e^(761/2*I*c) + 10140082596804607404376090989361337712629273671384
 446340003481136225061610661712978027889222103814938027534150*I*a^2*e^(759/2
 *I*c) + 1935629938092629464044933432048123474309129215431160763159842314939
 6817585242619148155405769562070657818017062*I*a^2*e^(757/2*I*c) + 365490412
 121312297832746648031039823017060530166350998938749984756716538698774385724
 22373857662116557803137691*I*a^2*e^(755/2*I*c) + 68269842357184501611988226
 541759877359893560816980579165757639885910403041554476683238744066404341119
 496497267*I*a^2*e^(753/2*I*c) + 1261558844450858809909057230508638945302599
 92110425146130569149222574918984083710347803558027793365144729446565*I*a^2*
 e^(751/2*I*c) + 23064195048734977698461337735625152416912708100691513109693
 2922360748741606056582939291771085884519360184891255*I*a^2*e^(749/2*I*c) +
 417203346852342786088688963220640917780075877592046434007891157690273764588
 009873939937985168264211297805134039*I*a^2*e^(747/2*I*c) + 7467281492243362
 913374727882275078265152540554796342982961634192168809916584030065162729746
 57126602569667620581*I*a^2*e^(745/2*I*c) + 13225410027687245271645902870752
 516183035559216180833489217915827253696179198591565450687109111482198342228
 62396*I*a^2*e^(743/2*I*c) + 23180048596133418845744953509005175801585656213

36318788947052214662319719223377004078504331647589087640360974954*I*a^2*e^(741/2*I*c) + 4020729692728752072547527964414583880720959971898567378088190721921140689114594980080070716969583769101530510140*I*a^2*e^(739/2*I*c) + 6902518665636037283762980242784909967883935371662357237821221528329174906800792394732946367748505381533617607645*I*a^2*e^(737/2*I*c) + 11728686977201016518903777818063543264965656103575795886869624447931763565472352750662022559534093518133411921208*I*a^2*e^(735/2*I*c) + 19726829299190651510301558556012003402191052348622782424435325329360695576352284999478792032516654385344097576701*I*a^2*e^(733/2*I*c) + 32844172588984923925745248170100614335810461284005739525160956863811444851393714424782905768489580238852810233892*I*a^2*e^(731/2*I*c) + 54135187069361775047519937605426565923507613794999731786828276683538558771749832815251100592417685414948864684145*I*a^2*e^(729/2*I*c) + 88338174640981237540185259392312475890335514451795608162633179634457749623643887095574804755049991966929176222830*I*a^2*e^(727/2*I*c) + 142722318968673080123248059644214027010647362985060273998965493470580757388030700770801323590848378479588489351378*I*a^2*e^(725/2*I*c) + 228316511183466357600740339397892651724763581113124858814232398699722508324249725657737970371965209398196125904775*I*a^2*e^(723/2*I*c) + 361669570834077260677316331292965514960294369836187403435553362802201139250475072743485753652885635408470996847372*I*a^2*e^(721/2*I*c) + 567339614895630128342983728404200845089288305505097140213560633981062751336174336795809192566239776955199921505915*I*a^2*e^(719/2*I*c) + 881369390315274220174241884381796152265055264249719308791502264814434849100759337832840344343735714440306099566040*I*a^2*e^(717/2*I*c) + 1356074964232792211022122776313085620131615916511397181422807416057396685245359582689147794647531935558283601984443*I*a^2*e^(715/2*I*c) + 2066556982160251267311485761192172322340476829807837784126886120148901191088967926521135564808688182641911560667924*I*a^2*e^(713/2*I*c) + 3119435182390404578526475400293555215484136600169446677359507312994761411131509471588414188813676704768199883865942*I*a^2*e^(711/2*I*c) + 4664404619442326161408337180614898813231496182593472259405080395954000367457095994601521782293014277832709054615380*I*a^2*e^(709/2*I*c) + 6909310380783860789281510926889703607841853587900716586154411151501397449721115415129140419091270756538802453583895*I*a^2*e^(707/2*I*c) + 10139527875816467343220793498549170075839065604546707385281954497494579711245464375474258366636437333914967133476201*I*a^2*e^(705/2*I*c) + 14742506081591720648724113911577950972875264495912180166443042116499971246321838780169739234187025835012002839150189*I*a^2*e^(703/2*I*c) + 2123836584523158555918531961644690235000015826170045512728950554645775595698170974068108854741604809260562872043619*I*a^2*e^(701/2*I*c) + 30317425197511637627728384831280057192585747834820863225726317028293946239909800510901728400285740011888506446492561*I*a^2*e^(699/2*I*c) + 42885426062905198376136115687779498313197624939417241023712107311560166080651136642711507677332263314929421901176965*I*a^2*e^(697/2*I*c) + 60117036809531954534774175328542432917785471258270317443583066712344018654100343282519010777886383221363137483769490*I*a^2*e^(695/2*I*c) + 83517876975248044258519777946946588944585187567768264892079917516487130082429368116942958987653732471395054399139538*I*a^2*e^(69

$3/2 * I * c) + 1149948298540085448006040505267361290686621708123406823856790208$
 $27942329161254533974586296625609880932042763111506754 * I * a^2 * e^{(691/2 * I * c)} +$
 $15693375545189563385847170380611690734635789954797344262825130014540666798$
 $973864045763035785690000058360558079161713 * I * a^2 * e^{(689/2 * I * c)} + 212282878$
 $705188354649502352775941442121310160088506940041461217611549577446696222647$
 $786382828325447861991528982057300 * I * a^2 * e^{(687/2 * I * c)} + 2846391060871606800$
 $731623460533476496613870616498866470992523092743038931742924481992332004821$
 $79969289319599583782705 * I * a^2 * e^{(685/2 * I * c)} + 37833333468850290590480261430$
 $311039994326548611578517874051472858642423749132402911463244178220938184533$
 $3938523265890 * I * a^2 * e^{(683/2 * I * c)} + 498509499153258403729144890980204589097$
 $433923264131841430599901592653873152072717512660804753287716928576078663461$
 $983 * I * a^2 * e^{(681/2 * I * c)} + 6511907152788140899870344218335142853224331708738$
 $43688369246449708975379749425293954979978533284470854614844971466368 * I * a^2 * e^{(679/2 * I * c)} +$
 $84332451270191474454340290972061610796241599215459523939992$
 $5409707630395189954029608070898352689818705209984508369520 * I * a^2 * e^{(677/2 * I * c)} +$
 $108279791731158921336755666005271059821490164985258768135079765765438$
 $7598767948898248611815354945578372200449025175360 * I * a^2 * e^{(675/2 * I * c)} + 137$
 $841218511280786821265588672751099560622640951984363764155174203766853911604$
 $1066140052440183078269143615845581155080 * I * a^2 * e^{(673/2 * I * c)} + 173980645966$
 $267455699286776317959942984209389154812532101128066698219989189285649266441$
 $5622347650053752088149727624160 * I * a^2 * e^{(671/2 * I * c)} + 217731969048282654771$
 $711229304304861992828592703831021692785468514017747785476203212307040869928$
 $3792898254962886739656 * I * a^2 * e^{(669/2 * I * c)} + 270178097219987688678575421108$
 $092666409622887102071045748248832472367036513555069245916876727204906676230$
 $0379877743040 * I * a^2 * e^{(667/2 * I * c)} + 332422018682049400516624062136424440385$
 $293496180132862103124536797010065849989260687633628462090240599145340816558$
 $8520 * I * a^2 * e^{(665/2 * I * c)} + 405549356101263502455389330044135493501084712983$
 $5154524985690016794545113963136786023289013915905378564692143643458640 * I * a^2 * e^{(663/2 * I * c)} +$
 $490582253753402966135019137348573887282442883805673193090$
 $6121670753804220745738659870244363280999523327382897200441520 * I * a^2 * e^{(661/2 * I * c)} +$
 $58842491824322513907976215991562151516085555252739287872752903351$
 $1932657541339616490316199475103405475041577880587000 * I * a^2 * e^{(659/2 * I * c)} +$
 $699801709577143537605745995805012474680123419462022952575050728512485982101$
 $0861715153517826996821338842938734860615040 * I * a^2 * e^{(657/2 * I * c)} + 825189325$
 $232861791713496500815678089607164024610954070331168646162697469777629068934$
 $8607898858162877878927869163809880 * I * a^2 * e^{(655/2 * I * c)} + 964745305573534195$
 $674410966098187224380314183001636114689823440121961412095824497833740990607$
 $0346233171019358264000480 * I * a^2 * e^{(653/2 * I * c)} + 111823577327573057886061252$
 $198327242557001856866976957507734569257457184696766716653323833754587819844$
 $03077867326956440 * I * a^2 * e^{(651/2 * I * c)} + 12849659528917295448887838389721821$
 $666429652280839717230872070976527285743131454112500472959858220404943909001$
 $977151360 * I * a^2 * e^{(649/2 * I * c)} + 1463717544059678626631832947276075907600136$
 $800889205562092713379905928249019089749450221782811925304884222407131239848$
 $0 * I * a^2 * e^{(647/2 * I * c)} + 165269738589748616391080233103711360543573746571987$
 $32212755546502892632722591536314813460778204894606881244239416717120 * I * a^2 *$

$e^{(645/2*I*c)} + 18495019957411734279807585585659028619914871376370221914181$
 $336130221442716020270657023135022033488496344464969637878700*I*a^2*e^{(643/2$
 $*I*c)} + 2051102444843872670037494084725650044873387925323161411761489184250$
 $6419291498065951049807036183243121806802376265432320*I*a^2*e^{(641/2*I*c)} +$
 $225385361156851285097390617380137664060069745810993642471804283519855464059$
 $04602601376439227176730279973640942509792180*I*a^2*e^{(639/2*I*c)} + 24535311$
 $108873854631583838561781463173989229567453522895600654095852502781216398579$
 $180366741352217131290346900784364200*I*a^2*e^{(637/2*I*c)} + 2645398140906918$
 $779129366718128208253140007832522532325943072293403735930148722069099152411$
 $4248730187335763372429125460*I*a^2*e^{(635/2*I*c)} + 282430314010148401244252$
 $742372382140499209187646122119213805129779128897295468555686691099785416535$
 $83678458630428337120*I*a^2*e^{(633/2*I*c)} + 29848075656762410777392478417988$
 $562936605721803803630324956331103418688840889616409755917532127762470680082$
 $194173437320*I*a^2*e^{(631/2*I*c)} + 3121341361032589865622950752731666120779$
 $125407165377491052692461158609085461581097250736613852938345287518553937166$
 $8680*I*a^2*e^{(629/2*I*c)} + 322838187691637519321622191504773951647461064909$
 $96239768011287301357561433297548231267227036903834777286313642263284780*I*a$
 $^2*e^{(627/2*I*c)} + 33006502605828069111234833523837274488458321604918549594$
 $915598999328301351391942676314725618514347956768945927684338260*I*a^2*e^{(62$
 $5/2*I*c)} + 3333317752892357232584661182806039255076186042496358040604028455$
 $3026862034226333729553143376429959067700689886330788900*I*a^2*e^{(623/2*I*c)}$
 $+ 332221305715899049824548255310152200708414431373762009302340011698341642$
 $97597217642962832109496476768915007520638843940*I*a^2*e^{(621/2*I*c)} + 32640$
 $210785137750395514344466824448907144608130646158918941737747797341437698189$
 $371184595982735305425435824684283776860*I*a^2*e^{(619/2*I*c)} + 3156462971757$
 $990011515613322118155603673009359681373455653706536893811895039416005506842$
 $6694882468474599065924339139340*I*a^2*e^{(617/2*I*c)} + 299844764403264089300$
 $993531197561297549816916660918748551593027415261824065408428430174430907554$
 $22103979691036978946000*I*a^2*e^{(615/2*I*c)} + 27901856653919484579343684902$
 $605106634990835693224277902772993509046816697554915822060373656496697112008$
 $237858612984680*I*a^2*e^{(613/2*I*c)} + 2533257934363435577954448716720210150$
 $053448362865394632903578214912387158727864289087840207127833296917349376745$
 $8002640*I*a^2*e^{(611/2*I*c)} + 223063337336447303794552102662515572156005721$
 $97223122288850891726285188766620799206359532978080306708567111158747156900*$
 $I*a^2*e^{(609/2*I*c)} + 18866322962402983332825118914013813247899665561112634$
 $287480400676793104541824749963087292981195024846572209729639189440*I*a^2*e^{($
 $607/2*I*c)} + 1506834767209077602337603303019631330294176309712540943549497$
 $8420582792711773103135974244949606016312505788823825840740*I*a^2*e^{(605/2*I$
 $*c)} + 109793609947359760449164031447559769661960407617052049037495768799372$
 $16168834924149129784251472577056488047960350809520*I*a^2*e^{(603/2*I*c)} + 66$
 $755444804312177590827259183014300018779913559371858760261598546140768866654$
 $20354664476764061095432856806716718924980*I*a^2*e^{(601/2*I*c)} + 22399805579$
 $503550425420743519255673800958480817601183862232025353384336700773356150077$
 $73599342750473469385817621308920*I*a^2*e^{(599/2*I*c)} - 22399805579503550425$
 $42074351925567380095848081760118386223202535338433670077335615007735993427$

50473469385817621308920*I*a^2*e^(597/2*I*c) - 66755444804312177590827259183
014300018779913559371858760261598546140768866654203546644767640610954328568
06716718924980*I*a^2*e^(595/2*I*c) - 10979360994735976044916403144755976966
196040761705204903749576879937216168834924149129784251472577056488047960350
809520*I*a^2*e^(593/2*I*c) - 1506834767209077602337603303019631330294176309
7125409435494978420582792711773103135974244949606016312505788823825840740*I
*a^2*e^(591/2*I*c) - 188663229624029833328251189140138132478996655611126342
87480400676793104541824749963087292981195024846572209729639189440*I*a^2*e^(
589/2*I*c) - 22306333733644730379455210266251557215600572197223122288850891
726285188766620799206359532978080306708567111158747156900*I*a^2*e^(587/2*I*
c) - 2533257934363435577954448716720210150053448362865394632903578214912387
1587278642890878402071278332969173493767458002640*I*a^2*e^(585/2*I*c) - 279
018566539194845793436849026051066349908356932242779027729935090468166975549
15822060373656496697112008237858612984680*I*a^2*e^(583/2*I*c) - 29984476440
326408930099353119756129754981691666091874855159302741526182406540842843017
443090755422103979691036978946000*I*a^2*e^(581/2*I*c) - 3156462971757990011
515613322118155603673009359681373455653706536893811895039416005506842669488
2468474599065924339139340*I*a^2*e^(579/2*I*c) - 326402107851377503955143444
668244489071446081306461589189417377477973414376981893711845959827353054254
35824684283776860*I*a^2*e^(577/2*I*c) - 33222130571589904982454825531015220
070841443137376200930234001169834164297597217642962832109496476768915007520
638843940*I*a^2*e^(575/2*I*c) - 3333317752892357232584661182806039255076186
042496358040604028455302686203422633372955314337642995906770068988633078890
0*I*a^2*e^(573/2*I*c) - 330065026058280691112348335238372744884583216049185
49594915598999328301351391942676314725618514347956768945927684338260*I*a^2*
e^(571/2*I*c) - 32283818769163751932162219150477395164746106490996239768011
287301357561433297548231267227036903834777286313642263284780*I*a^2*e^(569/2
*I*c) - 3121341361032589865622950752731666120779125407165377491052692461158
6090854615810972507366138529383452875185539371668680*I*a^2*e^(567/2*I*c) -
298480756567624107773924784179885629366057218038036303249563311034186888408
89616409755917532127762470680082194173437320*I*a^2*e^(565/2*I*c) - 28243031
401014840124425274237238214049920918764612211921380512977912889729546855568
669109978541653583678458630428337120*I*a^2*e^(563/2*I*c) - 2645398140906918
779129366718128208253140007832522532325943072293403735930148722069099152411
4248730187335763372429125460*I*a^2*e^(561/2*I*c) - 245353111088738546315838
385617814631739892295674535228956006540958525027812163985791803667413522171
31290346900784364200*I*a^2*e^(559/2*I*c) - 22538536115685128509739061738013
766406006974581099364247180428351985546405904602601376439227176730279973640
942509792180*I*a^2*e^(557/2*I*c) - 2051102444843872670037494084725650044873
387925323161411761489184250641929149806595104980703618324312180680237626543
2320*I*a^2*e^(555/2*I*c) - 184950199574117342798075855856590286199148713763
70221914181336130221442716020270657023135022033488496344464969637878700*I*a
^2*e^(553/2*I*c) - 16526973858974861639108023310371136054357374657198732212
755546502892632722591536314813460778204894606881244239416717120*I*a^2*e^(55
1/2*I*c) - 1463717544059678626631832947276075907600136800889205562092713379

9059282490190897494502217828119253048842224071312398480*I*a^2*e^(549/2*I*c)
- 128496595289172954488878383897218216664296522808397172308720709765272857
43131454112500472959858220404943909001977151360*I*a^2*e^(547/2*I*c) - 11182
357732757305788606125219832724255700185686697695750773456925745718469676671
665332383375458781984403077867326956440*I*a^2*e^(545/2*I*c) - 9647453055735
341956744109660981872243803141830016361146898234401219614120958244978337409
906070346233171019358264000480*I*a^2*e^(543/2*I*c) - 8251893252328617917134
965008156780896071640246109540703311686461626974697776290689348607898858162
877878927869163809880*I*a^2*e^(541/2*I*c) - 6998017095771435376057459958050
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*I*c) - 3324220186820494005166240621364244403852934961801328621031245367970
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175360*I*a^2*e^(521/2*I*c) - 8433245127019147445434029097206161079624159921
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^2*e^(519/2*I*c) - 65119071527881408998703442183351428532243317087384368836
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2*I*c) - 498509499153258403729144890980204589097433923264131841430599901592
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716068007316234605334764966138706164988664709925230927430389317429244819923
3200482179969289319599583782705*I*a^2*e^(511/2*I*c) - 212282878705188354649
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217081234068238567902082794232916125453397458629662560988093204276311150675
4*I*a^2*e^(505/2*I*c) - 835178769752480442585197779469465889445851875677682
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503/2*I*c) - 60117036809531954534774175328542432917785471258270317443583066

712344018654100343282519010777886383221363137483769490*I*a²*e^(501/2*I*c)
 - 4288542606290519837613611568777949831319762493941724102371210731156016608
 0651136642711507677332263314929421901176965*I*a²*e^(499/2*I*c) - 303174251
 975116376277283848312800571925857478348208632257263170282939462399098005109
 01728400285740011888506446492561*I*a²*e^(497/2*I*c) - 21238365845231585555
 918531961644690235000015826170045512728950554645775595698170974068108854741
 604809260562872043619*I*a²*e^(495/2*I*c) - 1474250608159172064872411391157
 795097287526449591218016644304211649997124632183878016973923418702583501200
 2839150189*I*a²*e^(493/2*I*c) - 101395278758164673432207934985491700758390
 65604546707385281954497494579711245464375474258366636437333914967133476201*
 I*a²*e^(491/2*I*c) - 69093103807838607892815109268897036078418535879007165
 86154411151501397449721115415129140419091270756538802453583895*I*a²*e<sup>(489
 /2*I*c)</sup> - 46644046194423261614083371806148988132314961825934722594050803959
 54000367457095994601521782293014277832709054615380*I*a²*e^(487/2*I*c) - 31
 194351823904045785264754002935552154841366001694466773595073129947614111315
 09471588414188813676704768199883865942*I*a²*e^(485/2*I*c) - 20665569821602
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 64808688182641911560667924*I*a²*e^(483/2*I*c) - 13560749642327922110221227
 763130856201316159165113971814228074160573966852453595826891477946475319355
 58283601984443*I*a²*e^(481/2*I*c) - 88136939031527422017424188438179615226
 505526424971930879150226481443484910075933783284034434373571444030609956604
 0*I*a²*e^(479/2*I*c) - 567339614895630128342983728404200845089288305505097
 140213560633981062751336174336795809192566239776955199921505915*I*a²*e<sup>(47
 7/2*I*c)</sup> - 3616695708340772606773163312929655149602943698361874034355533628
 02201139250475072743485753652885635408470996847372*I*a²*e^(475/2*I*c) - 22
 831651118346635760074033939789265172476358111312485881423239869972250832424
 9725657737970371965209398196125904775*I*a²*e^(473/2*I*c) - 142722318968673
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 9176222830*I*a²*e^(469/2*I*c) - 541351870693617750475199376054265659235076
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²*e^(467/2*I*c) - 32844172588984923925745248170100614335810461284005739525
 160956863811444851393714424782905768489580238852810233892*I*a²*e<sup>(465/2*I*
 c)</sup> - 1972682929919065151030155855601200340219105234862278242443532532936069
 5576352284999478792032516654385344097576701*I*a²*e^(463/2*I*c) - 117286869
 772010165189037778180635432649656561035757958868696244479317635654723527506
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 20959971898567378088190721921140689114594980080070716969583769101530510140*
 I*a²*e^(457/2*I*c) - 23180048596133418845744953509005175801585656213363187
 88947052214662319719223377004078504331647589087640360974954*I*a²*e<sup>(455/2*
 I*c)</sup> - 13225410027687245271645902870752516183035559216180833489217915827253
 69617919859156545068710911148219834222862396*I*a²*e^(453/2*I*c) - 74672814

922433629133747278822750782651525405547963429829616341921688099165840300651
6272974657126602569667620581*I*a^2*e^(451/2*I*c) - 417203346852342786088688
963220640917780075877592046434007891157690273764588009873939937985168264211
297805134039*I*a^2*e^(449/2*I*c) - 2306419504873497769846133773562515241691
27081006915131096932922360748741606056582939291771085884519360184891255*I*a
^2*e^(447/2*I*c) - 12615588444508588099090572305086389453025999211042514613
0569149222574918984083710347803558027793365144729446565*I*a^2*e^(445/2*I*c)
- 682698423571845016119882265417598773598935608169805791657576398859104030
41554476683238744066404341119496497267*I*a^2*e^(443/2*I*c) - 36549041212131
229783274664803103982301706053016635099893874998475671653869877438572422373
857662116557803137691*I*a^2*e^(441/2*I*c) - 1935629938092629464044933432048
123474309129215431160763159842314939681758524261914815540576956207065781801
7062*I*a^2*e^(439/2*I*c) - 101400825968046074043760909893613377126292736713
84446340003481136225061610661712978027889222103814938027534150*I*a^2*e^(437
/2*I*c) - 52542053315628400861248402353101698809320987341776979631277027119
95590576195690069369181247172296616054250810*I*a^2*e^(435/2*I*c) - 26927282
448425364461722623574956155733196639443211460545174612226257404076090636468
04896771692589250685909715*I*a^2*e^(433/2*I*c) - 13648077061320324179712693
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45898752*I*a^2*e^(431/2*I*c) - 68409643932382015557347717544725516277035177
2871309706432815320177180262386714126080576227107839963492633507*I*a^2*e^(4
29/2*I*c) - 339081049398958631695033084312249444034247861955469658173982280
143780778138876895429955188412068983435311370*I*a^2*e^(427/2*I*c) - 1661894
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97784847689078576576553405*I*a^2*e^(425/2*I*c) - 80535993554416862856990385
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571520*I*a^2*e^(423/2*I*c) - 3858660998287657496223400479535149465741817808
3375590488901031957266204930635139096439961363048441151269800*I*a^2*e^(421/
2*I*c) - 182774706915349177015632336022984039711997370840994957490123885798
21538776969597068131015951749218506666144*I*a^2*e^(419/2*I*c) - 85585699131
774977720413736285672113976827764195153726973137506812103604497173350946878
18153095679360516780*I*a^2*e^(417/2*I*c) - 39615443892690555191826925561670
87400915811260781154046642834832166886887365224821817220970795789343876240*
I*a^2*e^(415/2*I*c) - 18125017737117436736993992115331571295088238451140025
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1455982880577347113410414307040*I*a^2*e^(411/2*I*c) - 366307954798320205128
798936155351391554335176467391622039978284149371380873241652456477113543820
574273500*I*a^2*e^(409/2*I*c) - 1617870843818946112628969626012657489042094
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/2*I*c) - 70612062571169786899446458797376311651332630936204874511242735563
516386888084909599619786580787711236360*I*a^2*e^(405/2*I*c) - 3045246785608
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$e^{(401/2*I*c)}$ - 54628286678221586727361647210129385627818090917703635636870
 18271454099131199171596676490634462439229220*I*a^2*e^(399/2*I*c) - 22720048
 811620935900466503561284482119532730047520542947942040259999751890733350464
 66698628779651401360*I*a^2*e^(397/2*I*c) - 93344733685928184322435539734205
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 *e^(395/2*I*c) - 378816002128920616369855929612160874580768723711392251915
 842505525188562342470825520416422818082592000*I*a^2*e^(393/2*I*c) - 1518424
 984432168013297922170113498986347038741694307651033334785661844097743653929
 72733790654189070040*I*a^2*e^(391/2*I*c) - 60110869062284374159448209854726
 838711429891271925437784821736413864552441235504659113309232113779360*I*a^2
 *e^(389/2*I*c) - 2350038180733536582371812906680131644319672004296697678903
 8657507310743872020449940329840767115547150*I*a^2*e^(387/2*I*c) - 907248267
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 5197149710111720*I*a^2*e^(385/2*I*c) - 345838398116009621381983498618122160
 6030887703758892748920536238488574632981453327231642336638145090*I*a^2*e^(3
 83/2*I*c) - 130161349118126897428709054953528362735743599775323204270365658
 2610084719317446750492963887848291340*I*a^2*e^(381/2*I*c) - 483635269585920
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) - 64228709360409812420586138356564211085008288762762209697005815481413905
 982724399323193964745568820*I*a^2*e^(375/2*I*c) - 2295259936503095142465474
 0299581867088783137153355417738039160015930697452167681087479690813393300*I
 *a^2*e^(373/2*I*c) - 809499048675203110286284738006215721238810652354456980
 4503299552643657151717648240648270653869830*I*a^2*e^(371/2*I*c) - 281737384
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 965901119321858915203967518001948679909749229811961456530*I*a^2*e^(367/2*I*
 c) - 3278501563382507256487219221898208284880171022676335915892437775623639
 27243394356202432140538650*I*a^2*e^(365/2*I*c) - 10959682143632428417265121
 5203550992796369009483250710677345396805851498207559908266552991860510*I*a^2
 *e^(363/2*I*c) - 361412936678645147066137100818124636307119046022886541124
 21060177161306607694059860383771279230*I*a^2*e^(361/2*I*c) - 11755749238984
 942962210062966175741219276938429754098150077681519311875621804705720370211
 969800*I*a^2*e^(359/2*I*c) - 3771330561509512524615850596640341294357967299
 596271045598767418091013562271264018462771488980*I*a^2*e^(357/2*I*c) - 1193
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) - 11448975132659178160158729658266693316490641912330185672836854786107354
 3833187799753283463360*I*a^2*e^(351/2*I*c) - 347175898571376724069230085660
 78790348191298872900136928769377324783537177053188579391348690*I*a^2*e^(349
 /2*I*c) - 10377647429811126892082129653968587516545341659471911017509926638
 948128316618008695215618360*I*a^2*e^(347/2*I*c) - 3057490262522130472111269
 539903981978669524872013464112348409985533590524778523877236940090*I*a^2*e^

(345/2*I*c) - 8877635774210624749402322148173319138568759789882395681342647
76314007646535947673652724060*I*a^2*e^(343/2*I*c) - 25400580128701754034740
1034978022085833970670738098147773886519381628953611819486811955620*I*a^2*e
^(341/2*I*c) - 716062179458509947174306826027645592286227222278082368247999
62339103295121387601682173670*I*a^2*e^(339/2*I*c) - 19886758779911360532261
202721353273582160597108274352859715885782505631373182736113224200*I*a^2*e
(337/2*I*c) - 5440357551230374902439985027394698228560509433313958273980496
531279067808247723147105870*I*a^2*e^(335/2*I*c) - 1465830016742535124519131
469010652577571975918756440541603851467254363529939075434269440*I*a^2*e^(33
3/2*I*c) - 3889319141196329548401301490861407894523738557878079102128441517
96049289004605406967150*I*a^2*e^(331/2*I*c) - 10161010039881372824480214823
2520096631517489501850198195876637177700803300250750696120*I*a^2*e^(329/2*I
*c) - 261343335842817661995062037066653597967114111592037165616028425137494
76935891574081580*I*a^2*e^(327/2*I*c) - 66165787175526862693523980355038876
12674609084260960128356927902046841481605353166520*I*a^2*e^(325/2*I*c) - 16
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73498450*I*a^2*e^(323/2*I*c) - 40425591619731441020590363134583891813151862
2031814368289391512007823341054252319330*I*a^2*e^(321/2*I*c) - 975260870034
87022632582290289825035947901332365723366039562344307089798567346576790*I*a
^2*e^(319/2*I*c) - 23145134405275963083954762748231025164257628027633977804
093014188701364289864915790*I*a^2*e^(317/2*I*c) - 5402564744628545046129520
081244604381235726873384595583804088637122848866439830910*I*a^2*e^(315/2*I*
c) - 1240127624408790585914281618049099992149775260140355696852903419692530
227177449210*I*a^2*e^(313/2*I*c) - 2798861939746417403756726266426530638552
65114960677901721413367956114946541063180*I*a^2*e^(311/2*I*c) - 62096273516
318593577961117305783982914908932565241065670071393675735767176645100*I*a^2
*e^(309/2*I*c) - 1354055460508770231039339647727636740044411298407754301934
2063464929767322886040*I*a^2*e^(307/2*I*c) - 290141340780504627769992880642
6570164361980656563461341652112845236338748650110*I*a^2*e^(305/2*I*c) - 610
798290612478058634809110185506816479817707018034933796191469491921071289700
*I*a^2*e^(303/2*I*c) - 1263022613669708131901208630681647294039935119601317
43155255365483034776033710*I*a^2*e^(301/2*I*c) - 25648194013063161445813917
323611473948967661308908878336387655736333479132840*I*a^2*e^(299/2*I*c) - 5
113745039917782570942288672444404762240177072474457348268608658998353643330
*I*a^2*e^(297/2*I*c) - 1000827267878708901757364273781353911409779752889226
260010612599185707764320*I*a^2*e^(295/2*I*c) - 1922272134809049488720343211
54910920985425803997803298273976235657509933640*I*a^2*e^(293/2*I*c) - 36224
365825548176907943960557995838710010556561894147011323135173581402240*I*a^2
*e^(291/2*I*c) - 6695882317013495222345368298476838368511994443900460264534
471085890483820*I*a^2*e^(289/2*I*c) - 1213732838510510306428982602384538617
690483226714330573193357678961518000*I*a^2*e^(287/2*I*c) - 2156899397588740
13967187220833533731643242074212236222605165544572554060*I*a^2*e^(285/2*I*c
) - 37567064271487090398378167188966687791512776703919269010885401142609920
*I*a^2*e^(283/2*I*c) - 6411061389985712625839045934560628663538960416714378
812599630238383260*I*a^2*e^(281/2*I*c) - 1071687241879183608204834854430988

172125784220926155326017366664031320*I*a^2*e^(279/2*I*c) - 1754231621964961
 23940090931181306422667058440569170422851340745050152*I*a^2*e^(277/2*I*c) -
 28109088369373208155707785744000148722496865889470317867362512306740*I*a^2
 *e^(275/2*I*c) - 4407601795177120237389113481959743264257864428283490771370
 053462560*I*a^2*e^(273/2*I*c) - 6760873768573124741467717860493842067374692
 39071720613980492039300*I*a^2*e^(271/2*I*c) - 10141245246701151498796050844
 5679910794713086096737145187354675120*I*a^2*e^(269/2*I*c) - 148697764889784
 91309444307452594685609833202276809994141174545956*I*a^2*e^(267/2*I*c) - 21
 30454389639784102539281096465899633052875625470754655905397856*I*a^2*e^(265
 /2*I*c) - 298138079805795516636721746713666338776134639093417308458083640*I
 *a^2*e^(263/2*I*c) - 407338250838796920846779121592735043305969272224662706
 16023040*I*a^2*e^(261/2*I*c) - 54311584202896956902676282164178832015067816
 44825673663895125*I*a^2*e^(259/2*I*c) - 70636246561910226822620287946270328
 5306437535601820801093746*I*a^2*e^(257/2*I*c) - 895673469754664164365609590
 12746672316379146573614093235883*I*a^2*e^(255/2*I*c) - 11067205258903164311
 849474872837148947402911102321523307256*I*a^2*e^(253/2*I*c) - 1331867438356
 900947394085333963308875669933795527159861115*I*a^2*e^(251/2*I*c) - 1560185
 01390833014924028933522645581091836687267573447170*I*a^2*e^(249/2*I*c) - 17
 779861202993747975617805656916892771717874809673606262*I*a^2*e^(247/2*I*c)
 - 1969925436555096263303543513345030378604353635783621398*I*a^2*e^(245/2*I*
 c) - 212059745023134629411867496681427459939018060659823083*I*a^2*e^(243/2*
 I*c) - 22164417393124784256780107055497427057394288021910427*I*a^2*e^(241/2
 *I*c) - 2247657581576890158328281459463053249136155142818885*I*a^2*e^(239/2
 *I*c) - 220977435109557963897348194591419349596075084671135*I*a^2*e^(237/2*
 I*c) - 21045459166656329934203131234228410676256368947239*I*a^2*e^(235/2*I*
 c) - 1939943728233712898582615509996583301055554831933*I*a^2*e^(233/2*I*c)
 - 172919713857046613813065255730042186229347958876*I*a^2*e^(231/2*I*c) - 14
 890304346273125682306935846029108244114648650*I*a^2*e^(229/2*I*c) - 1237421
 118518768552107714053055438083342617820*I*a^2*e^(227/2*I*c) - 9913040189115
 0616227034640375461310347846749*I*a^2*e^(225/2*I*c) - 764642099062564610784
 2349221267149768725400*I*a^2*e^(223/2*I*c) - 567179508185660065903323118694
 229252909501*I*a^2*e^(221/2*I*c) - 4040182398434876173155161054285879351130
 0*I*a^2*e^(219/2*I*c) - 2759687216371547310850322318637953260785*I*a^2*e^(2
 17/2*I*c) - 180469997815998782717102062578903704334*I*a^2*e^(215/2*I*c) - 1
 1279374341089024516950389703822811442*I*a^2*e^(213/2*I*c) - 672482999427386
 036344407124264639911*I*a^2*e^(211/2*I*c) - 3816795301922046178837787847170
 8140*I*a^2*e^(209/2*I*c) - 2057571551559319523757433019101275*I*a^2*e^(207/
 2*I*c) - 105091018637393463590223550684728*I*a^2*e^(205/2*I*c) - 5071416398
 732103030201521505627*I*a^2*e^(203/2*I*c) - 230518925632259863716881504052*
 I*a^2*e^(201/2*I*c) - 9835474114732862868439582838*I*a^2*e^(199/2*I*c) - 39
 2372636365891369041933300*I*a^2*e^(197/2*I*c) - 14570867105062952981500815*
 I*a^2*e^(195/2*I*c) - 501114476578787912641049*I*a^2*e^(193/2*I*c) - 158664
 21411511793416437*I*a^2*e^(191/2*I*c) - 459291145959804703779*I*a^2*e^(189/
 2*I*c) - 12054885718630588825*I*a^2*e^(187/2*I*c) - 284015632092748725*I*a^
 2*e^(185/2*I*c) - 5932441401249090*I*a^2*e^(183/2*I*c) - 108143463042754*I*

$$\begin{aligned}
& a^{2e^{(181/2*I*c)}} - 1685352670794*I*a^{2e^{(179/2*I*c)}} - 21830993145*I*a^{2e^{(177/2*I*c)}} - 225643340*I*a^{2e^{(175/2*I*c)}} - 1744665*I*a^{2e^{(173/2*I*c)}} \\
& - 8970*I*a^{2e^{(171/2*I*c)}} - 23*I*a^{2e^{(169/2*I*c)}}) * \tan(1/4*d*x + c) / (e^{(517*I*c)} + 418*e^{(516*I*c)} + 87153*e^{(515*I*c)} + 12085216*e^{(514*I*c)} + 1253841160*e^{(513*I*c)} + 103818048048*e^{(512*I*c)} + 7146142307307*e^{(511*I*c)} + 420601518659718*e^{(510*I*c)} + 21608403021340047*e^{(509*I*c)} + 984382804329835768*e^{(508*I*c)} + 40261256699368950388*e^{(507*I*c)} + 1493326612293984160368*e^{(506*I*c)} + 50648660944512569972179*e^{(505*I*c)} + 1581796642397812408161814*e^{(504*I*c)} + 45759117183402579073139583*e^{(503*I*c)} + 1232445557346832245176696904*e^{(502*I*c)} + 3104222522074681615625020522*e^{(501*I*c)} + 734057263616388449968842366924*e^{(500*I*c)} + 16353164647151530240529137618111*e^{(499*I*c)} + 344277152012875134140739302960914*e^{(498*I*c)} + 6868329225263681349501997341320517*e^{(497*I*c)} + 130171193079172823835151430773360024*e^{(496*I*c)} + 2348998374244347079532766203075607598*e^{(495*I*c)} + 40443624781415311581857832389099634564*e^{(494*I*c)} + 665634670676210063754191847109971141414*e^{(493*I*c)} + 10490402669510897424624643766470754045064*e^{(492*I*c)} + 158566476113257562566117432227203884298856*e^{(491*I*c)} + 2302150411226234925855222345201500900533576*e^{(490*I*c)} + 32147887693375338817454482515377350383950278*e^{(489*I*c)} + 432333688644261557547944179250800440604964868*e^{(488*I*c)} + 5605927253067558551780452883689835514455118670*e^{(487*I*c)} + 70164515322544462906873548813748091084561870680*e^{(486*I*c)} + 84852202276512356496200136959676295361696315113*e^{(485*I*c)} + 9925490738534402272939987038714580495445431374618*e^{(484*I*c)} + 112391604542246650966429162063124338952554575234051*e^{(483*I*c)} + 1233096700139723365181997220750932590655287625342156*e^{(482*I*c)} + 13118781801172174729679339894318153694964675368481194*e^{(481*I*c)} + 135442594916636116191574650625331646238501101627937224*e^{(480*I*c)} + 1357990663161479842850642848032544982878359839580349899*e^{(479*I*c)} + 13231708870104896973800056733779919089340836756009580718*e^{(478*I*c)} + 125370496586921272662198050851269323171167338854081782959*e^{(477*I*c)} + 1155855412893594260345544966642687823630035899363232371472*e^{(476*I*c)} + 10375184499871175501909398956596684116802997082526660323524*e^{(475*I*c)} + 90722605722208814918642284639487187764607589706493970774776*e^{(474*I*c)} + 773204636991145775061462731028098506094432675788136295011259*e^{(473*I*c)} + 6426195485535248576425068136870465530087114003875716691383902*e^{(472*I*c)} + 52108117629177048660492400985175830987505700566877818954141639*e^{(471*I*c)} + 412430698299915190848067222327219435067747934091894670488982928*e^{(470*I*c)} + 3187749929744346497211536044751776582320958627923816470590659024*e^{(469*I*c)} + 24070801913529757101858022914372045864746991786182039740274325264*e^{(468*I*c)} + 177642829135119348577194437675802830239905460092687136494961404333*e^{(467*I*c)} + 1281817464914970810859604189828359000790789921169405304612211251818*e^{(466*I*c)} + 9046693523825682979044338963104263167672586826367911338826483549173*e^{(465*I*c)} + 62473550781053295317710774690247114124125187565731848441781904032672*e^{(464*I*c)} + 422276126632003687547754746555709988710527133086660161366353656787288*e^{(463*I*c)} + 2794709104475686611842790694973699164482254723977210209725661304403472*e^{(462*I*c)} + 181157684956157580767
\end{aligned}$$

10303055505625589254293659193314153418333944596408*e^(461*I*c) + 1150514818
 52080848873700388354521315567640365124003103691176697194292320*e^(460*I*c)
 + 716099497599058079895633338552940229192858196481597830078819711862600096*
 e^(459*I*c) + 4369442482910113914565353136069595862669338858053419381214131
 241925047008*e^(458*I*c) + 261439762799020214434719456650802545630568101835
 20401889800285493144867448*e^(457*I*c) + 1534360887450562541273272394615770
 71933130157764595997113973513183188399376*e^(456*I*c) + 8835009688217912026
 00774541927769200737689393513734789368397093333311961880*e^(455*I*c) + 4992
 519712457043983505377976607953988397368297591114957991804893688371867680*e^
 (454*I*c) + 276931165383432592259833826376479361226640338596151334898466646
 94361471028310*e^(453*I*c) + 1508223814314124137735664742100117468522974375
 97059186295243989481140398152780*e^(452*I*c) + 8066795436075891407593050107
 96189568269842021613388955218916278823182639488190*e^(451*I*c) + 4238125846
 763232586394188569858685826755328005548627437019301405851325887594480*e^(45
 0*I*c) + 218764828927139099280403456125787058051215087562266963170876518242
 52241418663320*e^(449*I*c) + 1109691996873209747499222595952504443412192185
 35349655762591192576535872151766080*e^(448*I*c) + 5532691288195286125029188
 69558947829098021956309349843584044631512291778800081490*e^(447*I*c) + 2711
 843239670717527605640490148833507130242448403978318523237721944200392830108
 580*e^(446*I*c) + 130698172034882898861932055083758183921249913823401603168
 86507181296548981014818410*e^(445*I*c) + 6194859665303550287956433881523431
 0660410902037882473161804774492916216575880077680*e^(444*I*c) + 28882075526
 4730654469968572021047109427318619508995802020689904590319476295408324280*e
 ^ (443*I*c) + 13247564123678374731574728211624836911209665019489539264922416
 43788264284546437221120*e^(442*I*c) + 5978992172944143218459161149299819706
 321732111578494525245228742976468409105395536290*e^(441*I*c) + 265568063890
 43407534496702369101545795994861757741414789944652712127566910185274123140*
 e^(440*I*c) + 1161045516835550437629115017121163993137330211326774811128240
 47246361794049635726479850*e^(439*I*c) + 4997075672538590843575963148137947
 68069337190915967491907488904933922677579665354338960*e^(438*I*c) + 2117589
 733466855707101501429210414722401838837940752841618541440888545729943138209
 036820*e^(437*I*c) + 883672064086047030569451402154796955129679409226698304
 4118375790025854584036796364768280*e^(436*I*c) + 36318369652302591732197444
 409798122022640824604130552506742586795183267354382847875885730*e^(435*I*c)
 + 147030816732276833163041582099592047512043725225353339238819165193000407
 629544745753221740*e^(434*I*c) + 586403466972683242741643328921560909375197
 453864243299571990964608857245771134145204174990*e^(433*I*c) + 230435107337
 384035737917859767306635201668278168913984209737666311848880384113193531364
 1840*e^(432*I*c) + 89232094473432967633318818816384717934996186706010260597
 30895962653291770229493028162575100*e^(431*I*c) + 3405405385129556915435234
 6722177172655187548910782008504718324168725029438589162349211628040*e^(430*
 I*c) + 12809891460168853967248054183040984770736750043860153680320449770111
 9911289087105659482783340*e^(429*I*c) + 47501057885760151927231661793842522
 2421786597241671026894318515408511467140969393115768793680*e^(428*I*c) + 17
 365742188181910718741974724501581238835642099506586391023371481227690806116

80719741726053840*e^(427*I*c) + 6259872156822252843650960708235034710201362
776057176647226323089751446565288850103898153859920*e^(426*I*c) + 222519591
767957777571673660360074802222113642321463998038643709633914912236872458234
57351580140*e^(425*I*c) + 7800980736802423987561373305885141712532711468107
0889640794249282633470580756557083923203377160*e^(424*I*c) + 26974580144021
129697268360186387895435796230852007659517712822762927324021520970821849736
3414140*e^(423*I*c) + 92008939302958903287460185002715932261252636844477148
9781974361078847528891468831038436064951920*e^(422*I*c) + 30961319716215201
623803015542414654517823620868102875377489029049859340201795657061771314216
14590*e^(421*I*c) + 1027936473066384084473957786246926260464886191429797258
9165243530651230690726244462479199894255180*e^(420*I*c) + 33675398872021568
375902384593982753362559801058104184627345411136262431943240778260721756991
027090*e^(419*I*c) + 108867995731829472826732905192034886797284621356445627
530909104429486741257822633476898356826454040*e^(418*I*c) + 347351473214713
780874352083129566601238765762775942366762733349952103889753982636403857556
867777300*e^(417*I*c) + 109385321448622035867403243450086667849977001130587
4172488975951612031456734608287095519501041975440*e^(416*I*c) + 34002325606
016516175216946808470898441980288316944174247948687793289505484181256054468
82081152636090*e^(415*I*c) + 1043411751657039596665369315558240210946034809
5473027807412321427346816928567197770376496170251803940*e^(414*I*c) + 31610
939331284692750694306443618414656095969520945215743004044560386895241801579
156543451940713351730*e^(413*I*c) + 945561802589319869193343034663656528268
58091314329189160736277175873841732196453379953705679466826880*e^(412*I*c)
+ 2792857558000352066798353688981654776448649877946653878274889338636337450
47373109049265172681702585720*e^(411*I*c) + 8146081877365305796702100252719
21415597183369881214299823291969785549876175969866367976653244974728560*e^(
410*I*c) + 2346518219239105142238141633073464768899155708935025778047637412
681781575765422219127409260159438712250*e^(409*I*c) + 667586629037114735850
376686566928901089354386983053870872494529158095117918829660615811125770696
8604740*e^(408*I*c) + 18759988218865563564163635735986073278255737257405706
279108891366378428467414559930481172863538598193890*e^(407*I*c) + 520751785
187932703864292633515443069511049935425005829381552416894081386752546080308
47907167748571734720*e^(406*I*c) + 1428017924502217624831808749188252741343
05133275417780084795034644763509333503150517345864659667189417080*e^(405*I*
c) + 3868762182342771656324517230499798892631152823746075416924431766739975
13742813591736171169652250611186480*e^(404*I*c) + 1035561982592002935226384
577908611548612111495080193573691339864706029186482466241805664949381049856
258510*e^(403*I*c) + 273889562479526560335522764656600088628077830508482570
2911938903656162004262736182657700406301914070062380*e^(402*I*c) + 71581246
868429414754738073636798397181727455815384090445033838526935969216224266967
40453944718143025248390*e^(401*I*c) + 1848740529900573269375272861187649089
0858357021974882371570623800186245137722660943641752976852924439870880*e^(4
00*I*c) + 47188220843466207695099506953573780357108897491422567898048199018
207708997005333860148836479527456156014520*e^(399*I*c) + 119041855403877964
948229577948370465600606623183045529526900430209270473212773847794935586074

714329479939280*e^(398*I*c) + 296825515282669589685318273280239050084555032
 203415941511962659596881615713799937680026497408305672297618840*e^(397*I*c)
 + 731584972206818362874729621403974444280010446301161527339760544815300951
 787985538419764656214582667219914080*e^(396*I*c) + 178244611493175185055635
 485663842190117441232229824949659165805393978719824656594597559557573419334
 8887952160*e^(395*I*c) + 42932064780080221260174889088518264947906207206601
 51451468181910917240027863968724539127659633517053002976480*e^(394*I*c) + 1
 022318202595486076721739030518645192356214547367429361991806349041148749612
 1804590274592702770571515456414680*e^(393*I*c) + 24068785139705277161193465
 644506143285241361037768216818922184400141048460210944696647752723371932874
 594597328*e^(392*I*c) + 560286834249035176584950138585345161671625910343679
 72498174660907450666778154353271630344650777885683547624184*e^(391*I*c) + 1
 289670800847547122460236808664883849832862590255331320446361090495451440295
 47003347761521666283977931640178464*e^(390*I*c) + 2935507435543427098081294
 535765623132997059826991874168629343739642556159671386762532763025915615235
 15603264403*e^(389*I*c) + 6607644731058690976914759738508379345110890331495
 86707982764263394766756649565279879146173318386505740391093990*e^(388*I*c)
 + 1470931146618934345515038362300100160482127749581443929904746910224777470
 198899052379114493999887003199419829579*e^(387*I*c) + 323849193136185147642
 332193353957909837773553920764146734623566582388704832694930560923158514374
 8690203615957136*e^(386*I*c) + 70521324141621979926023265245801430609853530
 54572933905524633121681021037340298366342203324325307072413739061024*e^(385
 *I*c) + 1518963421490880039641791172264375474804852010973481245910987881049
 3844381062650818971199637121458749456243274416*e^(384*I*c) + 32362731322419
 549410330088943640247460378328561316422931292427145902887913071643679502909
 055891236755143207382609*e^(383*I*c) + 682080330967936156837844096192442108
 186149916400415534244055278768932724966083242310981485024664539671577280789
 94*e^(382*I*c) + 1422131159648145176823866672767699094822716813187908898405
 01039441748635545362467679832449103520321953011780083069*e^(381*I*c) + 2933
 449200343007202870423834483428663138062854550400678230804455975459700234462
 31563554135133105493516316320059272*e^(380*I*c) + 5986501411122418589116765
 051805201503640032268413280814535970935877903386092124390855544668615826233
 50303061961052*e^(379*I*c) + 1208770358493658393089442222056935063283704108
 140593750226539846117737648216609559734831601248698274330296158612144*e^(37
 8*I*c) + 241496651681033850328907654920274051171005901179544713877346420569
 6455026442712426409599662771080264826008985061097*e^(377*I*c) + 47741411110
 660989702218453305949620164727142303742340606639568469509266426859469290641
 14194400360936223590725470146*e^(376*I*c) + 9339341958053494225251750965715
 057300707302083814774770306218224241022648247419956042957363055823830898547
 303219757*e^(375*I*c) + 180798200680288599703499386230072306765633142067088
 48499900139641237334763266479346963237936039328113185041591793848*e^(374*I*
 c) + 3463765717267169016765734453719708704888235485399327047206394307877360
 0446542963548348101269390443464480754513928502*e^(373*I*c) + 65674859268867
 300098827375812875225610654551686261103681664007007537115778097293533565243
 828873383722980353200611956*e^(372*I*c) + 123243941519332384741960072588103

506596406339253616391082062969960682419011745775738921817753391954462609323
881489157*e^(371*I*c) + 228911311738592780091492649162346834405867740776456
326108410928857257174707289268074347550225793244741923354395308214*e^(370*I
*c) + 420846342608949387277559021457924586578120966148561022647008499529468
452005980175119410628956210497609566002969884927*e^(369*I*c) + 765867795513
962781012558444628751418710940895281304790836743661582071650032154891482866
406314834433199455459798934952*e^(368*I*c) + 137967652979621207401710618806
658944835544650121089019510716486035022892858681553900306287502671193194194
7738690360722*e^(367*I*c) + 24604423758454226639270816309832607147349680919
05493027145639238827192254886349361126991457692409851120873307487457468*e^(
366*I*c) + 4343909696601932173357359687781579293701295681940827114215433175
336093967845908766740738240037114570667410936998017178*e^(365*I*c) + 759275
270014667896115309507358501547319702974653363333154979396147328576093580190
4155116764831560875947581048693527224*e^(364*I*c) + 13139771494104933881856
681151418293112242551521535686871181266579813877606348160261747201317735782
566021306798298336024*e^(363*I*c) + 225146757413080699615061655865028724304
219302106732643929972864856006401038672536048477155470605929676906537959511
42520*e^(362*I*c) + 3819901586758608797600299875662767499479544066790362502
9322346250133286489120875005013638128113893960349670280707161530*e^(361*I*c
) + 64175100693260066806238064886004597170740843300086839368616139164529108
049844675353111842725798658088840347241496099644*e^(360*I*c) + 106764832017
165594838085234189333528733587673329972530092661085186789939252915937090760
282232346919090426243399409323314*e^(359*I*c) + 175896258262755985757106812
613979301265801031595484353614904672865169442232075776580447184134141375995
770091499246759528*e^(358*I*c) + 286992943631231496557278010851576940896826
497466066327528801560677007112837431926735088120974861760511367008815728782
643*e^(357*I*c) + 463758288457367154544937678255005688733328145568049310423
995599886012800638619904022368378591108842602342094543682299102*e^(356*I*c)
+ 742228640908173124916937049462525617334148919679118270489831005497781951
221069955839623452499748653124658873553401442137*e^(355*I*c) + 117660072097
578696518987505089023109220461269697027743301453589578895677123079352038199
3106606880564628599822341722801012*e^(354*I*c) + 18475058564624515334452843
005713263237811625533045659718876707580910793067948218349281707731263646397
22071570131703785334*e^(353*I*c) + 2873610535922340187080835435582912277271
967977394720159791070274927714276869531467182688981041061381703885403497544
001592*e^(352*I*c) + 442767307910542531852431611298569365658485193610019245
7044455134483305045321452516347118488133224823670465103483954805161*e^(351*I
*c) + 67584804378885243725629359489638576266948555471955194861228775679817
18587262362871994967079401831957927901682582941234362*e^(350*I*c) + 1022042
377943463485133997529516339964170212224966366619305300830202609693215856833
8309418237395541351819026907953220681013*e^(349*I*c) + 15312837206662775379
347353212807682965712535652942631518286142403097738200270711195396582159028
513532779682154451996208592*e^(348*I*c) + 227316035661288411004195019470513
676668366524180772609139448107484730848918904101812854126048548766259195656
39521227223276*e^(347*I*c) + 3343589782793658130117117545961082945429816796

201741981007293673337850658442802420107245319345815533404669351674239071783
 2*e^(346*I*c) + 48733253505974923400852255563052101402196469313659554492725
 674754339375283010407167744366955828922837488705858532439654489*e^(345*I*c)
 + 703863497605948315670482240613950256985012022969663003767642203366977029
 61591099854055411376294871437468149528524796002762*e^(344*I*c) + 1007449618
 518537446117543009829801669624045538362229218684846942699661206076989070463
 43731011160948828100276729370132819357*e^(343*I*c) + 1429063191230555242465
 469284789542383713159258020223892364986521368398225020351556769709174190398
 34587967055588431566416784*e^(342*I*c) + 2009065871535788043803004695014416
 101745218512595419292098406889608594549085197748359058957576667708578886117
 38751858460424*e^(341*I*c) + 2799452444750398048229667304629608844921198748
 577911471240090794769204359417352933093054304386873331299124541967740701072
 64*e^(340*I*c) + 3866426730503800494573825628183169626519755509907792770487
 40238629858795018247356162888631015687664780101205287333082748791*e^(339*I*
 c) + 5293292527641139260039348369582435576725492389975607392144065991850478
 31955572583765358634395408771528009745467548382950094*e^(338*I*c) + 7183615
 963820582492091135444879010888683887440337132103324919713759067383415515404
 57264804304039664255915607349801911966551*e^(337*I*c) + 9664582753690377187
 477391307981516434835906841668322346880982911641606364181594521198157288093
 72125168836239364442397344064*e^(336*I*c) + 1289043515292933956480634330499
 677040181043935620106914267311067900030058398839787692376954090545278554544
 997710058754772400*e^(335*I*c) + 170458299670782280820467821816769300269866
 114771277235502145654381093006963718808588282475750060524696321081035170640
 5349408*e^(334*I*c) + 22348912763984394644786225783064348407246104844681778
 59822620658691921478645266653062563823553001228001009093606751066168944*e^(
 333*I*c) + 2905385722320057001953345274489482790856692529959823749532695963
 414164833366773128218607899328588608916176593772088622582464*e^(332*I*c) +
 374525759487665120465733498842622638814395450198683066422234922636107960954
 6822276067504899386703088982308185717143407211328*e^(331*I*c) + 47875274427
 809456851452048469715961653041694193282440732114595921296492550488768540598
 44720661078151288179612574986359194560*e^(330*I*c) + 6068949803156712248331
 871105329895471722806143008878014986559653687260694816550470195890004511965
 527567432722969707577202160*e^(329*I*c) + 762973181562782158046899242420700
 836643889673633302466186383810511044514894696232829763154703254341981182101
 5837863013682720*e^(328*I*c) + 95130322740195229542091131912682266422999120
 135256659402983810647978856909049931289480352274121440356338517795112193352
 77360*e^(327*I*c) + 1176421227487648408001090071467347449337127816055781198
 3724455826566055617658086479368641864908119643412413644803772131657280*e^(3
 26*I*c) + 14429816285208431204532978375375691965063154224649747551295851507
 389524083226976789688601369628399900747658579201929300744260*e^(325*I*c) +
 175562732712242923968872914031257162134914862611454785713767516901056560678
 38042151038271381300372757755676325408026834544840*e^(324*I*c) + 2118832140
 588288753961019837470686269589404922607709376413251251333619052397894969438
 7686059124526755048042957954264706637460*e^(323*I*c) + 25367176439119353621
 532260335983348154904982606125761711300683492963390816491583025705268737539

982149639300226512657426118880*e^(322*I*c) + 301284824145527032645590189530
881771560134374934382010784137698354483661481217545491975911299671707649697
00180348699207838960*e^(321*I*c) + 3550010310601964987627237679694948220958
137237103600501287780602748167280705994344524013631556850073237996658500567
8181937920*e^(320*I*c) + 41499832121963708043788523787401345541780088930538
206918853579026749273364671640037563488607716092887686471542838602788559660
*e^(319*I*c) + 481331176781840292165037485491103744789247190946356038928293
64863916553792278822957368285106328164715910598370871149079494360*e^(318*I*
c) + 5539091304497208621943268914633156608142795989696990021443429681773115
0863867056620768608187679709720152974148474907904177340*e^(317*I*c) + 63247
774101012179051794946075175569924076981338138483158042406747453874729387631
710544995247152912205118500597511052824347680*e^(316*I*c) + 716603298611733
955244419438892841091340911578446552456720842374024349446964649271318121906
59629511140639501743303863582092880*e^(315*I*c) + 8056624913068268418187620
188262351120636379033721801195411021064292776599764490382059542193687356531
4654415769070472655401600*e^(314*I*c) + 89883815801382382213973270477954602
744792877018051963347146307372464315121274929402347942874802899499538953561
056667668891020*e^(313*I*c) + 995122064720579659513403417380235485153364033
717178980408504709546575329772791134915068802907261111541019413860196895679
58040*e^(312*I*c) + 1093325373499662232039326785034263570798637070017282940
11042076530403923862654018978676516417314221089449922495612732870169660*e^(
311*I*c) + 1192097137020339270557553978236884444446474243245021853286263470
46599634721146573830681540495333543146776810911910410468628960*e^(310*I*c)
+ 1289950760115919034107638634270973299485861735745958627058491592809430464
58742663163454018491463855395649453952212899632198680*e^(309*I*c) + 1385297
945491510894513527695765434031263307472436800308324672058958190435681552392
64876762867172754338684027849855385453216080*e^(308*I*c) + 1476489208055453
334186231217678537773997829247483012287939243425749999379554217653701012351
22939557467548549202174550009604780*e^(307*I*c) + 1561859629535511961697382
188321736965098525515892107305783657274762594764744659554285023366737436864
99175698677875693611243400*e^(306*I*c) + 1639778160596077253752645598165058
478941877851014553603918974244829984153857876057653155092083377415901430785
72243505132706580*e^(305*I*c) + 1708698488689531011768603060531039943405303
903472600884326768425055551412938308389612759742689286664948457234625447091
02843680*e^(304*I*c) + 1767209299705546420045757700530957005953346598706827
32031975915532387577052414866323511140117680492929354517559479899220940360*
e^(303*I*c) + 1814081687709220598203685533166973216399848626282988285695602
73295630897626829345263592219034560853530733710529842148537901680*e^(302*I*
c) + 1848311519837489418176678501747082571381281721582694132877653585322407
73244336191900818557829905895684494889410451921524212840*e^(301*I*c) + 1869
154744365675149263514056231175032619875083519300838245664444356891392336834
11704641828762178799177848064220150818355261280*e^(300*I*c) + 1876153931685
100500714972805646035109124031329203120243708350626790376449902862853466735
07093452964351257962696133511725652320*e^(299*I*c) + 1869154744365675149263
514056231175032619875083519300838245664444356891392336834117046418287621787

99177848064220150818355261280*e^(298*I*c) + 1848311519837489418176678501747
 082571381281721582694132877653585322407732443361919008185578299058956844948
 89410451921524212840*e^(297*I*c) + 1814081687709220598203685533166973216399
 848626282988285695602732956308976268293452635922190345608535307337105298421
 48537901680*e^(296*I*c) + 1767209299705546420045757700530957005953346598706
 827320319759155323875770524148663235111401176804929293545175594798992209403
 60*e^(295*I*c) + 1708698488689531011768603060531039943405303903472600884326
 76842505555141293830838961275974268928666494845723462544709102843680*e^(294
 *I*c) + 1639778160596077253752645598165058478941877851014553603918974244829
 98415385787605765315509208337741590143078572243505132706580*e^(293*I*c) + 1
 561859629535511961697382188321736965098525515892107305783657274762594764744
 65955428502336673743686499175698677875693611243400*e^(292*I*c) + 1476489208
 055453334186231217678537773997829247483012287939243425749999379554217653701
 01235122939557467548549202174550009604780*e^(291*I*c) + 1385297945491510894
 513527695765434031263307472436800308324672058958190435681552392648767628671
 72754338684027849855385453216080*e^(290*I*c) + 1289950760115919034107638634
 270973299485861735745958627058491592809430464587426631634540184914638553956
 49453952212899632198680*e^(289*I*c) + 1192097137020339270557553978236884444
 446474243245021853286263470465996347211465738306815404953335431467768109119
 10410468628960*e^(288*I*c) + 1093325373499662232039326785034263570798637070
 017282940110420765304039238626540189786765164173142210894499224956127328701
 69660*e^(287*I*c) + 9951220647205796595134034173802354851533640337171789804
 0850470954657532977279113491506880290726111154101941386019689567958040*e^(2
 86*I*c) + 89883815801382382213973270477954602744792877018051963347146307372
 464315121274929402347942874802899499538953561056667668891020*e^(285*I*c) +
 805662491306826841818762018826235112063637903372180119541102106429277659976
 44903820595421936873565314654415769070472655401600*e^(284*I*c) + 7166032986
 117339552444194388928410913409115784465524567208423740243494469646492713181
 2190659629511140639501743303863582092880*e^(283*I*c) + 63247774101012179051
 794946075175569924076981338138483158042406747453874729387631710544995247152
 912205118500597511052824347680*e^(282*I*c) + 553909130449720862194326891463
 315660814279598969699002144342968177311508638670566207686081876797097201529
 74148474907904177340*e^(281*I*c) + 4813311767818402921650374854911037447892
 471909463560389282936486391655379227882295736828510632816471591059837087114
 9079494360*e^(280*I*c) + 41499832121963708043788523787401345541780088930538
 206918853579026749273364671640037563488607716092887686471542838602788559660
 *e^(279*I*c) + 355001031060196498762723767969494822095813723710360050128778
 06027481672807059943445240136315568500732379966585005678181937920*e^(278*I*
 c) + 3012848241455270326455901895308817715601343749343820107841376983544836
 6148121754549197591129967170764969700180348699207838960*e^(277*I*c) + 25367
 176439119353621532260335983348154904982606125761711300683492963390816491583
 025705268737539982149639300226512657426118880*e^(276*I*c) + 211883214058828
 875396101983747068626958940492260770937641325125133361905239789496943876860
 59124526755048042957954264706637460*e^(275*I*c) + 1755627327122429239688729
 140312571621349148626114547857137675169010565606783804215103827138130037275

7755676325408026834544840*e^(274*I*c) + 14429816285208431204532978375375691
965063154224649747551295851507389524083226976789688601369628399900747658579
201929300744260*e^(273*I*c) + 117642122748764840800109007146734744933712781
605578119837244558265660556176580864793686418649081196434124136448037721316
57280*e^(272*I*c) + 9513032274019522954209113191268226642299912013525665940
298381064797885690904993128948035227412144035633851779511219335277360*e^(27
1*I*c) + 762973181562782158046899242420700836643889673633302466186383810511
0445148946962328297631547032543419811821015837863013682720*e^(270*I*c) + 60
689498031567122483318711053298954717228061430088780149865596536872606948165
50470195890004511965527567432722969707577202160*e^(269*I*c) + 4787527442780
945685145204846971596165304169419328244073211459592129649255048876854059844
720661078151288179612574986359194560*e^(268*I*c) + 374525759487665120465733
498842622638814395450198683066422234922636107960954682227606750489938670308
8982308185717143407211328*e^(267*I*c) + 29053857223200570019533452744894827
908566925299598237495326959634141648333667731282186078993285886089161765937
72088622582464*e^(266*I*c) + 2234891276398439464478622578306434840724610484
468177859822620658691921478645266653062563823553001228001009093606751066168
944*e^(265*I*c) + 170458299670782280820467821816769300269866114771277235502
1456543810930069637188085882824757500605246963210810351706405349408*e^(264*
I*c) + 12890435152929339564806343304996770401810439356201069142673110679000
30058398839787692376954090545278554544997710058754772400*e^(263*I*c) + 9664
582753690377187477391307981516434835906841668322346880982911641606364181594
52119815728809372125168836239364442397344064*e^(262*I*c) + 7183615963820582
492091135444879010888683887440337132103324919713759067383415515404572648043
04039664255915607349801911966551*e^(261*I*c) + 5293292527641139260039348369
582435576725492389975607392144065991850478319555725837653586343954087715280
09745467548382950094*e^(260*I*c) + 3866426730503800494573825628183169626519
755509907792770487402386298587950182473561628886310156876647801012052873330
82748791*e^(259*I*c) + 2799452444750398048229667304629608844921198748577911
47124009079476920435941735293309305430438687333129912454196774070107264*e^(
258*I*c) + 2009065871535788043803004695014416101745218512595419292098406889
60859454908519774835905895757666770857888611738751858460424*e^(257*I*c) + 1
429063191230555242465469284789542383713159258020223892364986521368398225020
35155676970917419039834587967055588431566416784*e^(256*I*c) + 1007449618518
537446117543009829801669624045538362229218684846942699661206076989070463437
31011160948828100276729370132819357*e^(255*I*c) + 7038634976059483156704822
406139502569850120229696630037676422033669770296159109985405541137629487143
7468149528524796002762*e^(254*I*c) + 48733253505974923400852255563052101402
196469313659554492725674754339375283010407167744366955828922837488705858532
439654489*e^(253*I*c) + 334358978279365813011711754596108294542981679620174
19810072936733378506584428024201072453193458155334046693516742390717832*e^(
252*I*c) + 2273160356612884110041950194705136766683665241807726091394481074
8473084891890410181285412604854876625919565639521227223276*e^(251*I*c) + 15
312837206662775379347353212807682965712535652942631518286142403097738200270
711195396582159028513532779682154451996208592*e^(250*I*c) + 102204237794346

348513399752951633996417021222496636661930530083020260969321585683383094182
 37395541351819026907953220681013*e^(249*I*c) + 6758480437888524372562935948
 963857626694855547195519486122877567981718587262362871994967079401831957927
 901682582941234362*e^(248*I*c) + 442767307910542531852431611298569365658485
 193610019245704445513448330504532145251634711848813322482367046510348395480
 5161*e^(247*I*c) + 28736105359223401870808354355829122772719679773947201597
 91070274927714276869531467182688981041061381703885403497544001592*e^(246*I*
 c) + 1847505856462451533445284300571326323781162553304565971887670758091079
 306794821834928170773126364639722071570131703785334*e^(245*I*c) + 117660072
 097578696518987505089023109220461269697027743301453589578895677123079352038
 1993106606880564628599822341722801012*e^(244*I*c) + 74222864090817312491693
 704946252561733414891967911827048983100549778195122106995583962345249974865
 3124658873553401442137*e^(243*I*c) + 46375828845736715454493767825500568873
 332814556804931042399559988601280063861990402236837859110884260234209454368
 2299102*e^(242*I*c) + 28699294363123149655727801085157694089682649746606632
 7528801560677007112837431926735088120974861760511367008815728782643*e^(241*
 I*c) + 17589625826275598575710681261397930126580103159548435361490467286516
 9442232075776580447184134141375995770091499246759528*e^(240*I*c) + 10676483
 201716559483808523418933352873358767332997253009266108518678993925291593709
 0760282232346919090426243399409323314*e^(239*I*c) + 64175100693260066806238
 064886004597170740843300086839368616139164529108049844675353111842725798658
 088840347241496099644*e^(238*I*c) + 381990158675860879760029987566276749947
 954406679036250293223462501332864891208750050136381281138939603496702807071
 61530*e^(237*I*c) + 2251467574130806996150616558650287243042193021067326439
 2997286485600640103867253604847715547060592967690653795951142520*e^(236*I*c
) + 13139771494104933881856681151418293112242551521535686871181266579813877
 606348160261747201317735782566021306798298336024*e^(235*I*c) + 759275270014
 667896115309507358501547319702974653363333154979396147328576093580190415511
 6764831560875947581048693527224*e^(234*I*c) + 43439096966019321733573596877
 815792937012956819408271142154331753360939678459087667407382400371145706674
 10936998017178*e^(233*I*c) + 2460442375845422663927081630983260714734968091
 905493027145639238827192254886349361126991457692409851120873307487457468*e^
 (232*I*c) + 137967652979621207401710618806658944835544650121089019510716486
 0350228928586815539003062875026711931941947738690360722*e^(231*I*c) + 76586
 779551396278101255844462875141871094089528130479083674366158207165003215489
 1482866406314834433199455459798934952*e^(230*I*c) + 42084634260894938727755
 902145792458657812096614856102264700849952946845200598017511941062895621049
 7609566002969884927*e^(229*I*c) + 22891131173859278009149264916234683440586
 774077645632610841092885725717470728926807434755022579324474192335439530821
 4*e^(228*I*c) + 12324394151933238474196007258810350659640633925361639108206
 2969960682419011745775738921817753391954462609323881489157*e^(227*I*c) + 65
 674859268867300098827375812875225610654551686261103681664007007537115778097
 293533565243828873383722980353200611956*e^(226*I*c) + 346376571726716901676
 573445371970870488823548539932704720639430787736004465429635483481012693904
 43464480754513928502*e^(225*I*c) + 1807982006802885997034993862300723067656

331420670884849990013964123733476326647934696323793603932811318504159179384
 8*e^(224*I*c) + 93393419580534942252517509657150573007073020838147747703062
 18224241022648247419956042957363055823830898547303219757*e^(223*I*c) + 4774
 141111066098970221845330594962016472714230374234060663956846950926642685946
 929064114194400360936223590725470146*e^(222*I*c) + 241496651681033850328907
 654920274051171005901179544713877346420569645502644271242640959966277108026
 4826008985061097*e^(221*I*c) + 1208770358493658393089442220569350632837041
 08140593750226539846117737648216609559734831601248698274330296158612144*e^(220*I*c) + 5986501411122418589116765051805201503640032268413280814535970935
 87790338609212439085554466861582623350303061961052*e^(219*I*c) + 2933449200
 343007202870423834483428663138062854550400678230804455975459700234462315635
 54135133105493516316320059272*e^(218*I*c) + 1422131159648145176823866672767
 699094822716813187908898405010394417486355453624676798324491035203219530117
 80083069*e^(217*I*c) + 6820803309679361568378440961924421081861499164004155
 3424405527876893272496608324231098148502466453967157728078994*e^(216*I*c) +
 32362731322419549410330088943640247460378328561316422931292427145902887913
 071643679502909055891236755143207382609*e^(215*I*c) + 151896342149088003964
 179117226437547480485201097348124591098788104938443810626508189711996371214
 58749456243274416*e^(214*I*c) + 7052132414162197992602326524580143060985353
 054572933905524633121681021037340298366342203324325307072413739061024*e^(213*I*c) + 323849193136185147642332193353957909837773553920764146734623566582
 3887048326949305609231585143748690203615957136*e^(212*I*c) + 14709311466189
 343455150383623001001604821277495814439299047469102247774701988990523791144
 93999887003199419829579*e^(211*I*c) + 6607644731058690976914759738508379345
 11089033149586707982764263394766756649565279879146173318386505740391093990*
 e^(210*I*c) + 2935507435543427098081294535765623132997059826991874168629343
 73964255615967138676253276302591561523515603264403*e^(209*I*c) + 1289670800
 847547122460236808664883849832862590255331320446361090495451440295470033477
 61521666283977931640178464*e^(208*I*c) + 5602868342490351765849501385853451
 616716259103436797249817466090745066677815435327163034465077788568354762418
 4*e^(207*I*c) + 24068785139705277161193465644506143285241361037768216818922
 184400141048460210944696647752723371932874594597328*e^(206*I*c) + 102231820
 259548607672173903051864519235621454736742936199180634904114874961218045902
 74592702770571515456414680*e^(205*I*c) + 4293206478008022126017488908851826
 494790620720660151451468181910917240027863968724539127659633517053002976480
 *e^(204*I*c) + 178244611493175185055635485663842190117441232229824949659165
 8053939787198246565945975595575734193348887952160*e^(203*I*c) + 73158497220
 681836287472962140397444428001044630116152733976054481530095178798553841976
 4656214582667219914080*e^(202*I*c) + 29682551528266958968531827328023905008
 4555032203415941511962659596881615713799937680026497408305672297618840*e^(201*I*c) + 11904185540387796494822957794837046560060662318304552952690043020
 9270473212773847794935586074714329479939280*e^(200*I*c) + 47188220843466207
 695099506953573780357108897491422567898048199018207708997005333860148836479
 527456156014520*e^(199*I*c) + 184874052990057326937527286118764908908583570
 21974882371570623800186245137722660943641752976852924439870880*e^(198*I*c)

+ 7158124686842941475473807363679839718172745581538409044503383852693596921
 622426696740453944718143025248390*e^(197*I*c) + 273889562479526560335522764
 656600088628077830508482570291193890365616200426273618265770040630191407006
 2380*e^(196*I*c) + 10355619825920029352263845779086115486121114950801935736
 91339864706029186482466241805664949381049856258510*e^(195*I*c) + 3868762182
 342771656324517230499798892631152823746075416924431766739975137428135917361
 71169652250611186480*e^(194*I*c) + 1428017924502217624831808749188252741343
 05133275417780084795034644763509333503150517345864659667189417080*e<sup>(193*I*
 c)</sup> + 5207517851879327038642926335154430695110499354250058293815524168940813
 8675254608030847907167748571734720*e^(192*I*c) + 18759988218865563564163635
 735986073278255737257405706279108891366378428467414559930481172863538598193
 890*e^(191*I*c) + 667586629037114735850376686566928901089354386983053870872
 4945291580951179188296606158111257706968604740*e^(190*I*c) + 23465182192391
 051422381416330734647688991557089350257780476374126817815757654222191274092
 60159438712250*e^(189*I*c) + 8146081877365305796702100252719214155971833698
 81214299823291969785549876175969866367976653244974728560*e^(188*I*c) + 2792
 857558000352066798353688981654776448649877946653878274889338636337450473731
 09049265172681702585720*e^(187*I*c) + 9455618025893198691933430346636565282
 6858091314329189160736277175873841732196453379953705679466826880*e<sup>(186*I*c
)</sup> + 31610939331284692750694306443618414656095969520945215743004044560386895
 241801579156543451940713351730*e^(185*I*c) + 104341175165703959666536931555
 82402109460348095473027807412321427346816928567197770376496170251803940*e<sup>(
 184*I*c)</sup> + 3400232560601651617521694680847089844198028831694417424794868779
 328950548418125605446882081152636090*e^(183*I*c) + 109385321448622035867403
 24345008667849977001130587417248897595161203145673460828709551950104197544
 0*e^(182*I*c) + 34735147321471378087435208312956660123876576277594236676273
 3349952103889753982636403857556867777300*e^(181*I*c) + 10886799573182947282
 673290519203488679728462135644562753090910442948674125782263347689835682645
 4040*e^(180*I*c) + 33675398872021568375902384593982753362559801058104184627
 345411136262431943240778260721756991027090*e^(179*I*c) + 102793647306638408
 447395778624692626046488619142979725891652435306512306907262444624791998942
 55180*e^(178*I*c) + 3096131971621520162380301554241465451782362086810287537
 748902904985934020179565706177131421614590*e^(177*I*c) + 920089393029589032
 874601850027159322612526368444771489781974361078847528891468831038436064951
 920*e^(176*I*c) + 269745801440211296972683601863878954357962308520076595177
 128227629273240215209708218497363414140*e^(175*I*c) + 780098073680242398756
 13733058851417125327114681070889640794249282633470580756557083923203377160*
 e^(174*I*c) + 2225195917679577775716736603600748022221136423214639980386437
 0963391491223687245823457351580140*e^(173*I*c) + 62598721568222528436509607
 08235034710201362776057176647226323089751446565288850103898153859920*e<sup>(172
 *I*c)</sup> + 1736574218818191071874197472450158123883564209950658639102337148122
 769080611680719741726053840*e^(171*I*c) + 475010578857601519272316617938425
 222421786597241671026894318515408511467140969393115768793680*e^(170*I*c) +
 128098914601688539672480541830409847707367500438601536803204497701119911289
 087105659482783340*e^(169*I*c) + 340540538512955691543523467221771726551875

48910782008504718324168725029438589162349211628040*e^(168*I*c) + 8923209447
343296763331881881638471793499618670601026059730895962653291770229493028162
575100*e^(167*I*c) + 230435107337384035737917859767306635201668278168913984
2097376663118488803841131935313641840*e^(166*I*c) + 58640346697268324274164
3328921560909375197453864243299571990964608857245771134145204174990*e^(165*
I*c) + 1470308167322768331630415820995920475120437252253533923881916519300
0407629544745753221740*e^(164*I*c) + 36318369652302591732197444409798122022
640824604130552506742586795183267354382847875885730*e^(163*I*c) + 883672064
086047030569451402154796955129679409226698304411837579002585458403679636476
8280*e^(162*I*c) + 21175897334668557071015014292104147224018388379407528416
18541440888545729943138209036820*e^(161*I*c) + 4997075672538590843575963148
13794768069337190915967491907488904933922677579665354338960*e^(160*I*c) + 1
161045516835550437629115017121163993137330211326774811128240472463617940496
35726479850*e^(159*I*c) + 2655680638904340753449670236910154579599486175774
1414789944652712127566910185274123140*e^(158*I*c) + 59789921729441432184591
61149299819706321732111578494525245228742976468409105395536290*e^(157*I*c)
+ 1324756412367837473157472821162483691120966501948953926492241643788264284
546437221120*e^(156*I*c) + 288820755264730654469968572021047109427318619508
995802020689904590319476295408324280*e^(155*I*c) + 619485966530355028795643
38815234310660410902037882473161804774492916216575880077680*e^(154*I*c) + 1
306981720348828988619320550837581839212499138234016031688650718129654898101
4818410*e^(153*I*c) + 27118432396707175276056404901488335071302424484039783
18523237721944200392830108580*e^(152*I*c) + 5532691288195286125029188695589
47829098021956309349843584044631512291778800081490*e^(151*I*c) + 1109691996
87320974749922259595250444341219218535349655762591192576535872151766080*e^(
150*I*c) + 2187648289271390992804034561257870580512150875622669631708765182
4252241418663320*e^(149*I*c) + 42381258467632325863941885698586858267553280
05548627437019301405851325887594480*e^(148*I*c) + 8066795436075891407593050
10796189568269842021613388955218916278823182639488190*e^(147*I*c) + 1508223
81431412413773566474210011746852297437597059186295243989481140398152780*e^(
146*I*c) + 2769311653834325922598338263764793612266403385961513348984666469
4361471028310*e^(145*I*c) + 49925197124570439835053779766079539883973682975
91114957991804893688371867680*e^(144*I*c) + 8835009688217912026007745419277
69200737689393513734789368397093333311961880*e^(143*I*c) + 1534360887450562
54127327239461577071933130157764595997113973513183188399376*e^(142*I*c) + 2
6143976279902021443471945665080254563056810183520401889800285493144867448*e
^(141*I*c) + 43694424829101139145653531360695958626693388580534193812141312
41925047008*e^(140*I*c) + 7160994975990580798956333385529402291928581964815
97830078819711862600096*e^(139*I*c) + 1150514818520808488737003883545213155
67640365124003103691176697194292320*e^(138*I*c) + 1811576849561575807671030
3055505625589254293659193314153418333944596408*e^(137*I*c) + 27947091044756
86611842790694973699164482254723977210209725661304403472*e^(136*I*c) + 4222
76126632003687547754746555709988710527133086660161366353656787288*e^(135*I*
c) + 62473550781053295317710774690247114124125187565731848441781904032672*e
^(134*I*c) + 90466935238256829790443389631042631676725868263679113388264835

$49173e^{(133*I*c)} + 1281817464914970810859604189828359000790789921169405304$
 $612211251818e^{(132*I*c)} + 177642829135119348577194437675802830239905460092$
 $687136494961404333e^{(131*I*c)} + 240708019135297571018580229143720458647469$
 $91786182039740274325264e^{(130*I*c)} + 3187749929744346497211536044751776582$
 $320958627923816470590659024e^{(129*I*c)} + 412430698299915190848067222327219$
 $435067747934091894670488982928e^{(128*I*c)} + 521081176291770486604924009851$
 $75830987505700566877818954141639e^{(127*I*c)} + 6426195485535248576425068136$
 $870465530087114003875716691383902e^{(126*I*c)} + 773204636991145775061462731$
 $028098506094432675788136295011259e^{(125*I*c)} + 907226057222088149186422846$
 $39487187764607589706493970774776e^{(124*I*c)} + 1037518449987117550190939895$
 $6596684116802997082526660323524e^{(123*I*c)} + 11558554128935942603455449666$
 $42687823630035899363232371472e^{(122*I*c)} + 1253704965869212726621980508512$
 $69323171167338854081782959e^{(121*I*c)} + 1323170887010489697380005673377991$
 $9089340836756009580718e^{(120*I*c)} + 13579906631614798428506428480325449828$
 $78359839580349899e^{(119*I*c)} + 1354425949166361161915746506253316462385011$
 $01627937224e^{(118*I*c)} + 1311878180117217472967933989431815369496467536848$
 $1194e^{(117*I*c)} + 1233096700139723365181997220750932590655287625342156e^{($
 $116*I*c)} + 112391604542246650966429162063124338952554575234051e^{(115*I*c)}$
 $+ 9925490738534402272939987038714580495445431374618e^{(114*I*c)} + 848552202$
 $276512356496200136959676295361696315113e^{(113*I*c)} + 701645153225444629068$
 $73548813748091084561870680e^{(112*I*c)} + 5605927253067558551780452883689835$
 $514455118670e^{(111*I*c)} + 432333688644261557547944179250800440604964868e^{($
 $110*I*c)} + 32147887693375338817454482515377350383950278e^{(109*I*c)} + 2302$
 $150411226234925855222345201500900533576e^{(108*I*c)} + 158566476113257562566$
 $117432227203884298856e^{(107*I*c)} + 104904026695108974246246437664707540450$
 $64e^{(106*I*c)} + 665634670676210063754191847109971141414e^{(105*I*c)} + 4044$
 $3624781415311581857832389099634564e^{(104*I*c)} + 23489983742443470795327662$
 $03075607598e^{(103*I*c)} + 130171193079172823835151430773360024e^{(102*I*c)}$
 $+ 6868329225263681349501997341320517e^{(101*I*c)} + 344277152012875134140739$
 $302960914e^{(100*I*c)} + 16353164647151530240529137618111e^{(99*I*c)} + 73405$
 $7263616388449968842366924e^{(98*I*c)} + 3104222522074681615625020522e^{(97*$
 $I*c)} + 1232445557346832245176696904e^{(96*I*c)} + 45759117183402579073139583$
 $e^{(95*I*c)} + 1581796642397812408161814e^{(94*I*c)} + 5064866094451256997217$
 $9e^{(93*I*c)} + 1493326612293984160368e^{(92*I*c)} + 40261256699368950388e^{($
 $91*I*c)} + 984382804329835768e^{(90*I*c)} + 21608403021340047e^{(89*I*c)} + 42$
 $0601518659718e^{(88*I*c)} + 7146142307307e^{(87*I*c)} + 103818048048e^{(86*I*$
 $c)} + 1253841160e^{(85*I*c)} + 12085216e^{(84*I*c)} + 87153e^{(83*I*c)} + 418e$
 $^{(82*I*c)} + e^{(81*I*c)} - 14*(23*a^2e^{(1027/2*I*c)} + 8970*a^2e^{(1025/2*I*$
 $c)} + 1744665*a^2e^{(1023/2*I*c)} + 225643340*a^2e^{(1021/2*I*c)} + 2183099314$
 $5*a^2e^{(1019/2*I*c)} + 1685352670794*a^2e^{(1017/2*I*c)} + 108143463042714*a$
 $^2e^{(1015/2*I*c)} + 5932441401233490*a^2e^{(1013/2*I*c)} + 28401563208971452$
 $5*a^2e^{(1011/2*I*c)} + 12054885718238165655*a^2e^{(1009/2*I*c)} + 4592911459$
 $21837770879*a^2e^{(1007/2*I*c)} + 15866421408580747568967*a^2e^{(1005/2*I*c)}$
 $+ 501114476390712619058089*a^2e^{(1003/2*I*c)} + 14570867094745692063264465$
 $*a^2e^{(1001/2*I*c)} + 392372635871953337607706440*a^2e^{(999/2*I*c)} + 98354$

74093767985284835740178*a²*e^(997/2*I*c) + 230518924833500391396633471432*
a²*e^(995/2*I*c) + 5071416371138697123516748312797*a²*e^(993/2*I*c) + 105
091017765905750210205432017088*a²*e^(991/2*I*c) + 205757152621928017666056
9426269565*a²*e^(989/2*I*c) + 38167952336853964450194985322999760*a²*e<sup>(9
87/2*I*c)</sup> + 672482982322867333315410721290945657*a²*e^(985/2*I*c) + 112793
73940205552267581116020178794202*a²*e^(983/2*I*c) + 1804699889966560230083
85818021005185594*a²*e^(981/2*I*c) + 2759687033617389036404063195416999823
985*a²*e^(979/2*I*c) + 40401820406264183313387690283892201648160*a²*e<sup>(97
7/2*I*c)</sup> + 567179441813227622186529241088113209451509*a²*e^(975/2*I*c) + 7
646419821227105124391700712517502747187088*a²*e^(973/2*I*c) + 991303822775
32022515819177881629325075859509*a²*e^(971/2*I*c) + 1237420804707705093991
102638839493330163931640*a²*e^(969/2*I*c) + 148902995476946224064887889156
48999996474069170*a²*e^(967/2*I*c) + 1729196436077244411331733545659986054
01103776504*a²*e^(965/2*I*c) + 1939942742070099509831143493249507373979963
303677*a²*e^(963/2*I*c) + 210454458721348496801378614981824081260145633264
57*a²*e^(961/2*I*c) + 220977262761597134870041878241189142663942444889435*
a²*e^(959/2*I*c) + 2247655430280602939346801439987511004122770572057975*a²
*e^(957/2*I*c) + 22164391506926711131826332938481074378353565258441643*a²
*e^(955/2*I*c) + 212059444422641568370505689613891761234929288751535757*a²
*e^(953/2*I*c) + 1969922064355959547894504865149532487975181829580265482*a²
*e^(951/2*I*c) + 17779824621644501534677987130821297785034706339145224178*
a²*e^(949/2*I*c) + 1560181173081498739117537870655754044636737787248804115
10*a²*e^(947/2*I*c) + 1331863531921951594349497448051501144604555177632910
228645*a²*e^(945/2*I*c) + 110671667395938065379944325322023238925593588248
14410948544*a²*e^(943/2*I*c) + 8956697846475560153138642623244921735308706
0101994516952837*a²*e^(941/2*I*c) + 70635904260044009062362736864680387466
3354915637996034416454*a²*e^(939/2*I*c) + 5431127527768201077898230182311
22146548845466324910314438155*a²*e^(937/2*I*c) + 4073355402474013490085959
4350647954212567407648957747058481680*a²*e^(935/2*I*c) + 29813576609204982
1265508565759859757268901977404776167694840520*a²*e^(933/2*I*c) + 21304351
65596135408047552199368503071331826042021949871247131824*a²*e^(931/2*I*c)
+ 14869620923934928195860171826698271613545069265812546978930298844*a²*e<sup>(
929/2*I*c)</sup> + 10141122575476372516077609301456905198476636667938621489641703
2560*a²*e^(927/2*I*c) + 67607794586302974273678379566109514132606526811657
7325076514243900*a²*e^(925/2*I*c) + 44075310711231539469155754424078673682
73204436441682325952058880560*a²*e^(923/2*I*c) + 2810857079443600488986815
5095425291975524337124542327266338411863980*a²*e^(921/2*I*c) + 17541946418
1837861206705490491499611761897494030842873597094326137448*a²*e<sup>(919/2*I*c
)</sup> + 1071661434847365356402500490116585199491741089623963750947662209022120*
a²*e^(917/2*I*c) + 6410885411886208227235614369244284094773850732880341584
569571247989180*a²*e^(915/2*I*c) + 375658912654834216259223272376633939730
61791344075817295565545641550480*a²*e^(913/2*I*c) + 2156822939043670482294
52566525170165451086345847776055567286526724207180*a²*e^(911/2*I*c) + 1213
684086474480030032963524903872036826200642910143657468007690377301200*a²*e
^(909/2*I*c) + 669557812266523430522687007446585031832114379850814811985238

0345744581420*a²*e^(907/2*I*c) + 36222507823447452094316152383681805847086
 822685973455693062020799131961680*a²*e^(905/2*I*c) + 192216100832677211425
 583919374607410256140382963064998218563326540739057800*a²*e^(903/2*I*c) +
 100076216506457306558003970495070559462214153240314710070222810451820992248
 0*a²*e^(901/2*I*c) + 51133713404620801884705792163322470511998009337758134
 72986030153667768479970*a²*e^(899/2*I*c) + 2564609163526230666377598990704
 8193693026481072707785982065347970437174552640*a²*e^(897/2*I*c) + 12629066
 5976649423752795293300558365078090948368244878972263607020367969724510*a²*
 e^(895/2*I*c) + 61073557684864691322903531669001548120128321614764883763107
 2938937795953569020*a²*e^(893/2*I*c) + 29010807054572012234808075159505626
 90696054942137423459448133177752600175055070*a²*e^(891/2*I*c) + 1353882290
 2792275748563604516064479211324364455610559665374956958057411438898160*a²*
 e^(889/2*I*c) + 62087428059933702144851939557816743050943015221536726159024
 340696975430971481900*a²*e^(887/2*I*c) + 279841843449723329022256022542885
 182491449861432225949373308219431658133659304060*a²*e^(885/2*I*c) + 123990
 929743896845849802042104470487778663494423220810833085510785586972735255081
 0*a²*e^(883/2*I*c) + 54015092850750556638448628821387292768098169292999783
 33470094669272025087067952130*a²*e^(881/2*I*c) + 2314012260343947977239284
 4447344267578241288491282693326820133793119469871909054310*a²*e^{(879/2*I*c}
) + 97502706496344518739819416482817785708749119763377367851251579953471302
 421416590690*a²*e^(877/2*I*c) + 404148737461691129448272381486081487215931
 511785097115337433651585897523208487479570*a²*e^(875/2*I*c) + 164820164213
 9146705633894149783476374765855031698399268829056921671864832034920363790*a
²*e^(873/2*I*c) + 66144400820020510453479219244127878363376621124553998877
 45116058097858565172028583520*a²*e^(871/2*I*c) + 2612502097352610282419068
 2571375956088092948345240408465888776831062032840805359036420*a²*e<sup>(869/2*
 I*c)</sup> + 10157022331200088988995526991327474738371832329837740236016575815940
 8055448033759362720*a²*e^(867/2*I*c) + 38876396892010237458390588327513357
 0373138295169848377636334256622974927805402558286690*a²*e^(865/2*I*c) + 14
 651342290643008303482102337827023384371775676500548985537313589645208636961
 42827138160*a²*e^(863/2*I*c) + 5437521451042031455370282099308345101312396
 580420987248763253424089743393247174970193410*a²*e^(861/2*I*c) + 198753832
 701981230163980739547820481430867444791767062152949319392852315523776686667
 74800*a²*e^(859/2*I*c) + 7156131327668605954342040988322253447335243985577
 2274727763362387210317509376711352805370*a²*e^(857/2*I*c) + 25383132012926
 548867726804091248224918890371035127867297203271859140587613140010211574690
 0*a²*e^(855/2*I*c) + 88709614811178381979269807521163793735012825182063947
 1668845531985819368043596429487700180*a²*e^(853/2*I*c) + 30549765052290531
 86358196512899823679196539270073164577798894149381563584247883834020602010*
 a²*e^(851/2*I*c) + 1036832427833694706645534803655654449472691347656052168
 9641087749606367891595175422870841040*a²*e^(849/2*I*c) + 34683534777882172
 855590247839161459711508478268599595143978937459065098062910312662980093090
 *a²*e^(847/2*I*c) + 114367222654004146039785488306144607682565282217212089
 722715844480032864714063018623336102960*a²*e^(845/2*I*c) + 371784628277848
 407873740730843158415533102422282317148023178730401149797296961025637580441

$730*a^2*e^{(843/2*I*c)} + 119162602671200452022853596291671118196468103297935$
 $5080990640973734774508809765193594567308000*a^2*e^{(841/2*I*c)} + 37661117341$
 $863148447457366228609850041109045082888101894089219697224576875890481518758$
 $80889220*a^2*e^{(839/2*I*c)} + 1173805156223944291043665868661695293162135916$
 $3901525749755059400912331209404508874859876592800*a^2*e^{(837/2*I*c)} + 36082$
 $142955789372921795326276163720115936669443430427894923821823835712395825018$
 $234933157302110*a^2*e^{(835/2*I*c)} + 109401947851812253736939736757431402194$
 $995906470393287332344122103685909885397712947473326518610*a^2*e^{(833/2*I*c)}$
 $+ 327217244888055084262892186197327446953686436737681340634739093336235382$
 $375514336377928793244690*a^2*e^{(831/2*I*c)} + 965532218209251264607039716473$
 $737729829357676534904149014904268128459659120470520755736939192070*a^2*e^{(8$
 $29/2*I*c)} + 281097503606779152602078316141434903543061096416439129012877500$
 $5718676199562281038906617666401970*a^2*e^{(827/2*I*c)} + 80750674801709208568$
 $026954585195592726223421020352975266698203267805840061588744172988845366221$
 $70*a^2*e^{(825/2*I*c)} + 2289142246227600597089769949517582743299142275420964$
 $6711791861074723763534400400991756009136093340*a^2*e^{(823/2*I*c)} + 64043424$
 $833900148962660564937308584792588410556135883392172200976630674826538314163$
 $343570582742220*a^2*e^{(821/2*I*c)} + 176843212577304640940434087070526800206$
 $912147942357555568484158874194832893725593513293339771528640*a^2*e^{(819/2*I$
 $*c)} + 482003855527359842058748938929565762498717173867153381418816829773057$
 $750173701195879408932882129710*a^2*e^{(817/2*I*c)} + 129686978770094550352747$
 $818625343842928386699310567975195681220696431630642690809338359377106887518$
 $0*a^2*e^{(815/2*I*c)} + 34447742392644906310173946350452828676614441736472801$
 $34489449067279690770880154195712638143690670510*a^2*e^{(813/2*I*c)} + 9033952$
 $674591972024099323322407009219478763876009237692789450947388612631864264679$
 $797961473529847760*a^2*e^{(811/2*I*c)} + 233927347923584499864709149886416853$
 $48168613137145022740977224897697160854430483589336483946581829650*a^2*e^{(80$
 $9/2*I*c)} + 5981404789492439511671517626292625873582700958544166730832132595$
 $7072358824867958015638415996637992560*a^2*e^{(807/2*I*c)} + 15103468127596178$
 $167008188074186593145157080883927910480412370128563906374784352279321472766$
 $6247721320*a^2*e^{(805/2*I*c)} + 37664584457361554618416651099041615464594000$
 $2848766465441678202978044480377315238707708815985878943120*a^2*e^{(803/2*I*c$
 $) + 92769209802706548376575206531799625267482667453463111604771895974309785$
 $5192680766331861590769408198460*a^2*e^{(801/2*I*c)} + 22569366401976461724095$
 $887544833910482389125597195976929160987113401427554372850780614048807976783$
 $11440*a^2*e^{(799/2*I*c)} + 5423877415356140734240076530281376392307183648754$
 $518352919070721950593841907858843558278519237201933020*a^2*e^{(797/2*I*c)} +$
 $128767135259169892233668231930230540880904887524578931242343976286799924376$
 $62682878994494276660041769680*a^2*e^{(795/2*I*c)} + 3020188861789511923414776$
 $779708658681221749045460105123652572942343771647683389666175401865815402238$
 $9900*a^2*e^{(793/2*I*c)} + 69988368274664388580883736712738303159327351505142$
 $299304675566880142922981296353915575157471652388092680*a^2*e^{(791/2*I*c)} +$
 $160253943840621057078138437984495917828994512049873464602063929877936771758$
 $934087025740251939283794149320*a^2*e^{(789/2*I*c)} + 362585708102908727461692$
 $262308497693854332721503869703743775437736913882412243026230183024991966728$

272700*a²*e^(787/2*I*c) + 810698514587734599341419926912945883958489502890
 417749835477031467724072007792742636256985473924406832880*a²*e^(785/2*I*c)
 + 179135687128617945307626189529754871427562763762032201846805627904310478
 5643232974994540293493766790807180*a²*e^(783/2*I*c) + 39120632922983706718
 236562651806275103810426889069614510874217109674609809742431592402937733282
 59061298480*a²*e^(781/2*I*c) + 8444177149381193033478801248743170134902792
 413015392319772244917564145947358160863746469962582546517551020*a²*e<sup>(779/
 2*I*c)</sup> + 180161882031611515955708476802609030852814845475689392743272128585
 68842514711832067992981021945668432453744*a²*e^(777/2*I*c) + 3799695085003
 403823385083030620582717001776425358694149802829312074251648466449921908403
 6933062000158769320*a²*e^(775/2*I*c) + 79221081373658264272879848670833963
 158393802618546130507463440333508958081178065157747031476513568516431440*a²
 *e^(773/2*I*c) + 163291912624014896869889921756370257404344116207519220566
 929295476847426915494195734247976197944827166856605*a²*e^(771/2*I*c) + 332
 771401353095630876781482769166072007566368785944639929752492013340507467607
 203423204297878103391257399650*a²*e^(769/2*I*c) + 670517439437730975214058
 382921475629655071200139334454271221441426048415295599199783168621980211682
 925980307*a²*e^(767/2*I*c) + 133592516338887898994288754323787357347735407
 8078040384838340726239029587514889857191688753727386206824536152*a²*e<sup>(765/
 2*I*c)</sup> + 26320081955436011268865672586710800827931793309439329526480867362
 57575292194786895789417136105832466949897075*a²*e^(763/2*I*c) + 5128027932
 104560135559026593192544604592901120537915741822852980300115191592803605514
 077707054780313193678930*a²*e^(761/2*I*c) + 988089919880856328723361351985
 533201055999294177279451042419781976783115995510733928889251646631740223323
 8590*a²*e^(759/2*I*c) + 18830001036742739401007198462226884604435327942040
 227457903683414392625117454825138218461037881756069460095462*a²*e<sup>(757/2*I
 *c)</sup> + 354925185722847832617907210910079842132689358672913810999435476589968
 66612281989451531913916163009433157485571*a²*e^(755/2*I*c) + 6617297932881
 809399215054642382035162752852372860854002119931270260080224075665893897413
 8233183457153977060797*a²*e^(753/2*I*c) + 12204128126857543983447988302990
 699194420160067581866401325623015491396458064340669238189760339409304370344
 2025*a²*e^(751/2*I*c) + 22265879824279233418100220610928010409785016857978
 9527367599387695046282889021401012025548745828097666778231565*a²*e<sup>(749/2*
 I*c)</sup> + 40188785017504508015031388991007269452800944701646807261145319396663
 2487465676074617056051773919819622001323159*a²*e^(747/2*I*c) + 71767324920
 670553943767575873920097462584452902452927707470260940478945808166620375787
 5233986777843339431778971*a²*e^(745/2*I*c) + 12680329744048849958800906653
 423684542730040005610160238536791866755170095503645042488461800763709111728
 08894376*a²*e^(743/2*I*c) + 2216875962324492560656475252069459616280748019
 821115392295053079963518361922106809558395835470562396612703910894*a²*e<sup>(7
 41/2*I*c)</sup> + 383516964831217741295575895704558807255981323477748813823772353
 7853906921795690040312404465308498665963807731880*a²*e^(739/2*I*c) + 65657
 679593071530228829834619744537570730217235007718128652478013026176517587654
 98337855684754666047332109346955*a²*e^(737/2*I*c) + 1112422688241289810643
 238399018764325994794191948692614355578299900846301535590589525141555426051

1452162767106896*a²*e^(735/2*I*c) + 18653626462850532803035140588104717514
085431098848869419689186654357354383555540804832857748868154391812479528331
*a²*e^(733/2*I*c) + 309593486425765628601652998173661658629226486684257964
75504851207868785347857317439600867394648879054091070118592*a²*e<sup>(731/2*I*
c)</sup> + 5086061865420950083533559153160520120167735286246615994357148142901870
5622891681452604178883809245979957523278415*a²*e^(729/2*I*c) + 82710216196
192744659285946813603474744793948296346331753106207063703754012444364328700
859979459109222867113617510*a²*e^(727/2*I*c) + 133153018441395196034014770
358763691000098636960739548768552317876384874768681586531648288117265885968
911213464902*a²*e^(725/2*I*c) + 212218928188673385825585486875626217952905
358371974613119305643729514631087950307170160593893009749320293414137639*a²*
e^(723/2*I*c) + 334877229377355089426560502609284393492614010501825504426
666861674218991625217228236262755043572863766464617539152*a²*e^(721/2*I*c)
+ 523218326398909441228138386569661789123283174843980047262040319411958519
785242263748926347748867588273669418790115*a²*e^(719/2*I*c) + 809475688654
593052224501094178917252980035293905898453796392919564557037778617746976730
948724902348115224486645600*a²*e^(717/2*I*c) + 124015564057522683211246144
658763515180020658900739017222936903158267886075275533279331003594413534113
0684921794627*a²*e^(715/2*I*c) + 18816046186806990270245435156986520031264
26510004629919637670767851063752549093913764335909644924041423364457564056*
a²*e^(713/2*I*c) + 2827410762917657346065292926703346273000172053162448180
906376970875870523643742229301867770298371409250162486706446*a²*e<sup>(711/2*I*
c)</sup> + 420810587990096567104481160318687178813705026824971664290885292245016
6735904108996538151632302993343796013355257880*a²*e^(709/2*I*c) + 62036998
076732947111872108021144212850853086945744288084100595853011031037228772905
22605140847043745794303026775415*a²*e^(707/2*I*c) + 9059634894716266106041
250114654859819170770958994890643823267294971612713252112667925034180484391
205562621557500199*a²*e^(705/2*I*c) + 131067843765579140601235842829739716
168027656278860131867281252997375039972447886767293507155233679089682020168
14961*a²*e^(703/2*I*c) + 1878612345612180246044477763374134866227498929577
4122327946041136799723934764027545573154102942013646829238996755521*a²*e<sup>(
701/2*I*c)</sup> + 26678662604166841848095788924129301979448932840410184329153779
340794009872890361466821390436483345842294941747333409*a²*e^(699/2*I*c) +
375411345309312949907076283118878966417380012487360455442254727581286124794
35539648539936267245503594181132790075635*a²*e^(697/2*I*c) + 5234775479141
456200969042474636259615597005932613387746572682372750616929974976286787233
7382933080302575832354312830*a²*e^(695/2*I*c) + 72338117658183544125339619
278704570070398062503968469165923503150270252549867503793649405257860558789
674577875168502*a²*e^(693/2*I*c) + 990709333414297197602970045457468870551
580177755061961507795918979669303216746380846295952488464392179861861676915
66*a²*e^(691/2*I*c) + 1344828546045721626428840967446952957135004987386452
00651035548592354933239483430231951682574771378015247514707342287*a²*e<sup>(68
9/2*I*c)</sup> + 1809510824618191818695813304988792572428725718115718920483829817
86970569721390987257245905924108342179247496262397340*a²*e^(687/2*I*c) + 2
413577937113473282327609590718728464284134012731389363008530096007019884060

64584259031019225114956173896476656268335*a²*e^(685/2*I*c) + 3191537208183
 049874320021715335408800739785633976845781847779169877992931630884082047360
 76370214921003117423047033230*a²*e^(683/2*I*c) + 4184174731371814572412385
 090873293252824089396877216062965991125177653395872555871837976644605971129
 39981628640119137*a²*e^(681/2*I*c) + 5439064667128123519049147559829178351
 094822133364936621624653100349346061889628675519683252520558836292548902152
 35232*a²*e^(679/2*I*c) + 7010946890196828914166239682921267169075994030537
 12158434755550345947349948295539046888909742638931217537474025975440*a²*e^(677/2*I*c) + 8961920355975037209271807078077066028702648087726047622951692
 18466556192230960167844078573413391828541761332410051040*a²*e^(675/2*I*c)
 + 1136139693561537155978568944899860990716224618498598331698010751957196904
 760729746742387960564144749497838935456211960*a²*e^(673/2*I*c) + 142857608
 790754115407016752894453517980762876865018136822572838041989915471907902387
 6946406819487778508438319784170080*a²*e^(671/2*I*c) + 17817683182949669352
 805978484140547237298363548717084325768592352016467647943896189509505874913
 53081375055442537775544*a²*e^(669/2*I*c) + 2204504848819271258577686647437
 802673860228777835066862747405782838776390348098557012295819426306998948329
 682289436640*a²*e^(667/2*I*c) + 270594649944996307730421914875997577613545
 365348393733791756619433689219556830563155471311684966044174625838545572316
 0*a²*e^(665/2*I*c) + 32954345219222178101235110820882370320399432856468780
 34300290289132360253350516006721630787778671059075241422781406160*a²*e^(663/2*I*c) + 3982256683723412940987002608432103465539772199439132030360365812
 174562265353125356974005033580943527632143017811668560*a²*e^(661/2*I*c) +
 477537474879123150545968399666527814610204346354866288991622477572359598824
 4933194330547948361773890650352555746361400*a²*e^(659/2*I*c) + 56831194084
 038593110007516240036231488608208471207766480066528273851195374220703844734
 08108830451035508393726980174240*a²*e^(657/2*I*c) + 6712861418522615724988
 858473795623314248057954884136819874564233080140888033656717736078828058598
 240124888994298326360*a²*e^(655/2*I*c) + 787067024908872037443069512725257
 813818390760011548448756778960258982976295273259765824685217552123912881884
 8207047200*a²*e^(653/2*I*c) + 91609737380630587614977376948117073446973469
 19589075093557382262079852822162643836169596839715953389931229146306843160*
 a²*e^(651/2*I*c) + 1058623385911710506650733480513866864889318683528870371
 4430991946260066278599752585501637968876434411898083966268318240*a²*e^(649/2*I*c) + 12146654565504797783672415623355290385431521572754643028081311118
 774641507895483012327171848886038795599157607589336080*a²*e^(647/2*I*c) +
 138399376099471873989204752380174900209956108648629891563231150082467766771
 66646686425137922972069544356469424336874720*a²*e^(645/2*I*c) + 1566110116
 368601184782458239234044886066938476679972162717922203521376820514126055609
 5060384752632361819327032020892140*a²*e^(643/2*I*c) + 17602373941334170158
 602804303413344263296373051657482244301069046136457777485221759920683874338
 340816767315010883070640*a²*e^(641/2*I*c) + 196531744382527267525417924777
 777619934650225271457413465791247041112701698526103162375302138370767319391
 55254031315220*a²*e^(639/2*I*c) + 2180018094657529251098759558153408779545
 501567139438439807556810202379312678969165525747256028630827603152258911614

7800*a²*e^(637/2*I*c) + 24027493459410529878342393314596701203692315139519
374776263890070912654038803572108760194498231541018946484500110764820*a²*e^(635/2*I*c) + 263168836945024245212481243876288800464809215795717366607784
39611102049126719392867491978361605672355038563370490434320*a²*e^(633/2*I*c) + 2864812461220060601372002405432777931524031087082124165142024060554568
8144900537089198213263851453828234588917547711560*a²*e^(631/2*I*c) + 30999
386333592884159610215585305952854889407725459965085076104937976196911732638
644879528970536122548468911408679508520*a²*e^(629/2*I*c) + 333476816393137
132776489667910175466641478627744256121856468997745312213933869257141075472
25285273054290314596594977580*a²*e^(627/2*I*c) + 3566934156165823421939301
591416525880023144723499217512011162195375877388035613051502633901375966408
7801990290045546540*a²*e^(625/2*I*c) + 37940500212170537906421927439740074
329537276968507943576124882485983705492251107761060039819420913129160456600
166235380*a²*e^(623/2*I*c) + 401375680536466143754333426685279018978222583
06289397177277613987025021024009192926575679052108438711600692277041112940*
a²*e^(621/2*I*c) + 4223767435092749730816760791277049538441050990699929435
9254608598393831908627007418402687181252824823543923401086446140*a²*e^(619/2*I*c) + 44219062414309796885036555682850910798132641174189091355070247306
856269259520267158532714171765231548167835085083759540*a²*e^(617/2*I*c) +
460614252564034938067668169663307994388334771809988593995473930967657173021
18692459528278697654284187890865264198490880*a²*e^(615/2*I*c) + 4774617408
598129132026036222830018345928279440677646144908614295224316238252464662558
8139446166635147049927542868878520*a²*e^(613/2*I*c) + 49256637251142339677
939547027108976858345467610518344678311703270141917623803626587961704546454
017361035725620405883520*a²*e^(611/2*I*c) + 505781923678642695971460350025
649056223221849146426945287024152695150007033833953703993804355489314550466
04860814615100*a²*e^(609/2*I*c) + 5169833897799209701194910876351827860915
729120563316518750422956642026793826741359867266668160560685139381131276547
1840*a²*e^(607/2*I*c) + 52606722765605163245233385275200339644939530342158
370891740806422347556622042460358183525029339507310593911482225899900*a²*e^(605/2*I*c) + 532951247986600196698491744108009210277692661226623298727506
85463600092969548998627467943246663396973157934252963472160*a²*e^(603/2*I*c) + 5375743024611174648748124421967553620129596150938528123460343206055975
6788856444210733519160094322986825803224364734220*a²*e^(601/2*I*c) + 53989
590479579880111809597901378922286527227175566547926860482497477539014851169
842250286594884676270110583739674489880*a²*e^(599/2*I*c) + 539895904795798
801118095979013789222865272271755665479268604824974775390148511698422502865
94884676270110583739674489880*a²*e^(597/2*I*c) + 5375743024611174648748124
421967553620129596150938528123460343206055975678885644421073351916009432298
6825803224364734220*a²*e^(595/2*I*c) + 53295124798660019669849174410800921
027769266122662329872750685463600092969548998627467943246663396973157934252
963472160*a²*e^(593/2*I*c) + 526067227656051632452333852752003396449395303
42158370891740806422347556622042460358183525029339507310593911482225899900*
a²*e^(591/2*I*c) + 5169833897799209701194910876351827860915729120563316518
7504229566420267938267413598672666681605606851393811312765471840*a²*e⁽⁵⁸⁹

$/2*I*c) + 50578192367864269597146035002564905622322184914642694528702415269$
 $515000703383395370399380435548931455046604860814615100*a^2*e^(587/2*I*c) +$
 $492566372511423396779395470271089768583454676105183446783117032701419176238$
 $03626587961704546454017361035725620405883520*a^2*e^(585/2*I*c) + 4774617408$
 $598129132026036222830018345928279440677646144908614295224316238252464662558$
 $8139446166635147049927542868878520*a^2*e^(583/2*I*c) + 46061425256403493806$
 $766816966330799438833477180998859399547393096765717302118692459528278697654$
 $284187890865264198490880*a^2*e^(581/2*I*c) + 442190624143097968850365556828$
 $509107981326411741890913550702473068562692595202671585327141717652315481678$
 $35085083759540*a^2*e^(579/2*I*c) + 4223767435092749730816760791277049538441$
 $050990699929435925460859839383190862700741840268718125282482354392340108644$
 $6140*a^2*e^(577/2*I*c) + 40137568053646614375433342668527901897822258306289$
 $397177277613987025021024009192926575679052108438711600692277041112940*a^2*e$
 $^(575/2*I*c) + 379405002121705379064219274397400743295372769685079435761248$
 $82485983705492251107761060039819420913129160456600166235380*a^2*e^(573/2*I*$
 $c) + 3566934156165823421939301591416525880023144723499217512011162195375877$
 $3880356130515026339013759664087801990290045546540*a^2*e^(571/2*I*c) + 33347$
 $681639313713277648966791017546664147862774425612185646899774531221393386925$
 $714107547225285273054290314596594977580*a^2*e^(569/2*I*c) + 309993863335928$
 $841596102155853059528548894077254599650850761049379761969117326386448795289$
 $70536122548468911408679508520*a^2*e^(567/2*I*c) + 2864812461220060601372002$
 $405432777931524031087082124165142024060554568814490053708919821326385145382$
 $8234588917547711560*a^2*e^(565/2*I*c) + 26316883694502424521248124387628880$
 $046480921579571736660778439611102049126719392867491978361605672355038563370$
 $490434320*a^2*e^(563/2*I*c) + 240274934594105298783423933145967012036923151$
 $39519374776263890070912654038803572108760194498231541018946484500110764820*$
 $a^2*e^(561/2*I*c) + 2180018094657529251098759558153408779545501567139438439$
 $8075568102023793126789691655257472560286308276031522589116147800*a^2*e^(559$
 $/2*I*c) + 1965317443825272675254179247777761993465022527145741346579124704$
 $111270169852610316237530213837076731939155254031315220*a^2*e^(557/2*I*c) +$
 $176023739413341701586028043034133442632963730516574822443010690461364577774$
 $85221759920683874338340816767315010883070640*a^2*e^(555/2*I*c) + 1566110116$
 $368601184782458239234044886066938476679972162717922203521376820514126055609$
 $5060384752632361819327032020892140*a^2*e^(553/2*I*c) + 13839937609947187398$
 $920475238017490020995610864862989156323115008246776677166646686425137922972$
 $069544356469424336874720*a^2*e^(551/2*I*c) + 121466545655047977836724156233$
 $552903854315215727546430280813111187746415078954830123271718488860387955991$
 $57607589336080*a^2*e^(549/2*I*c) + 1058623385911710506650733480513866864889$
 $318683528870371443099194626006627859975258550163796887643441189808396626831$
 $8240*a^2*e^(547/2*I*c) + 91609737380630587614977376948117073446973469195890$
 $75093557382262079852822162643836169596839715953389931229146306843160*a^2*e$
 $^(545/2*I*c) + 7870670249088720374430695127252578138183907600115484487567789$
 $602589829762952732597658246852175521239128818848207047200*a^2*e^(543/2*I*c)$
 $+ 671286141852261572498885847379562331424805795488413681987456423308014088$
 $8033656717736078828058598240124888994298326360*a^2*e^(541/2*I*c) + 56831194$

084038593110007516240036231488608208471207766480066528273851195374220703844
73408108830451035508393726980174240*a^2*e^(539/2*I*c) + 4775374748791231505
459683996665278146102043463548662889916224775723595988244933194330547948361
773890650352555746361400*a^2*e^(537/2*I*c) + 398225668372341294098700260843
210346553977219943913203036036581217456226535312535697400503358094352763214
3017811668560*a^2*e^(535/2*I*c) + 32954345219222178101235110820882370320399
432856468780343002902891323602533505160067216307877786710590752414227814061
60*a^2*e^(533/2*I*c) + 2705946499449963077304219148759975776135453653483937
337917566194336892195568305631554713116849660441746258385455723160*a^2*e^(5
31/2*I*c) + 220450484881927125857768664743780267386022877783506686274740578
2838776390348098557012295819426306998948329682289436640*a^2*e^(529/2*I*c) +
17817683182949669352805978484140547237298363548717084325768592352016467647
94389618950950587491353081375055442537775544*a^2*e^(527/2*I*c) + 1428576087
907541154070167528944535179807628768650181368225728380419899154719079023876
94640681948778508438319784170080*a^2*e^(525/2*I*c) + 113613969356153715597
856894489986099071622461849859833169801075195719690476072974674238796056414
4749497838935456211960*a^2*e^(523/2*I*c) + 89619203559750372092718070780770
660287026480877260476229516921846655619223096016784407857341339182854176133
2410051040*a^2*e^(521/2*I*c) + 70109468901968289141662396829212671690759940
3053712158434755550345947349948295539046888909742638931217537474025975440*a
^2*e^(519/2*I*c) + 54390646671281235190491475598291783510948221333649366216
2465310034934606188962867551968325252055883629254890215235232*a^2*e^(517/2*
I*c) + 41841747313718145724123850908732932528240893968772160629659911251776
5339587255587183797664460597112939981628640119137*a^2*e^(515/2*I*c) + 31915
372081830498743200217153354088007397856339768457818477791698779929316308840
8204736076370214921003117423047033230*a^2*e^(513/2*I*c) + 24135779371134732
823276095907187284642841340127313893630085300960070198840606458425903101922
5114956173896476656268335*a^2*e^(511/2*I*c) + 18095108246181918186958133049
887925724287257181157189204838298178697056972139098725724590592410834217924
7496262397340*a^2*e^(509/2*I*c) + 13448285460457216264288409674469529571350
049873864520065103554859235493323948343023195168257477137801524751470734228
7*a^2*e^(507/2*I*c) + 99070933341429719760297004545746887055158017775506196
150779591897966930321674638084629595248846439217986186167691566*a^2*e^(505/
2*I*c) + 723381176581835441253396192787045700703980625039684691659235031502
70252549867503793649405257860558789674577875168502*a^2*e^(503/2*I*c) + 5234
775479141456200969042474636259615597005932613387746572682372750616929974976
2867872337382933080302575832354312830*a^2*e^(501/2*I*c) + 37541134530931294
990707628311887896641738001248736045544225472758128612479435539648539936267
245503594181132790075635*a^2*e^(499/2*I*c) + 266786626041668418480957889241
293019794489328404101843291537793407940098728903614668213904364833458422949
41747333409*a^2*e^(497/2*I*c) + 1878612345612180246044477763374134866227498
9295774122327946041136799723934764027545573154102942013646829238996755521*a
^2*e^(495/2*I*c) + 13106784376557914060123584282973971616802765627886013186
728125299737503997244788676729350715523367908968202016814961*a^2*e^(493/2*I
*c) + 905963489471626610604125011465485981917077095899489064382326729497161

2713252112667925034180484391205562621557500199*a²*e^(491/2*I*c) + 62036998
 076732947111872108021144212850853086945744288084100595853011031037228772905
 22605140847043745794303026775415*a²*e^(489/2*I*c) + 4208105879900965671044
 811603186871788137050268249716642908852922450166735904108996538151632302993
 343796013355257880*a²*e^(487/2*I*c) + 282741076291765734606529292670334627
 300017205316244818090637697087587052364374222930186777029837140925016248670
 6446*a²*e^(485/2*I*c) + 18816046186806990270245435156986520031264265100046
 29919637670767851063752549093913764335909644924041423364457564056*a²*e<sup>(48
 3/2*I*c)</sup> + 1240155640575226832112461446587635151800206589007390172229369031
 582678860752755332793310035944135341130684921794627*a²*e^(481/2*I*c) + 809
 475688654593052224501094178917252980035293905898453796392919564557037778617
 746976730948724902348115224486645600*a²*e^(479/2*I*c) + 523218326398909441
 228138386569661789123283174843980047262040319411958519785242263748926347748
 867588273669418790115*a²*e^(477/2*I*c) + 334877229377355089426560502609284
 393492614010501825504426666861674218991625217228236262755043572863766464617
 539152*a²*e^(475/2*I*c) + 212218928188673385825585486875626217952905358371
 974613119305643729514631087950307170160593893009749320293414137639*a²*e<sup>(4
 73/2*I*c)</sup> + 133153018441395196034014770358763691000098636960739548768552317
 876384874768681586531648288117265885968911213464902*a²*e^(471/2*I*c) + 827
 102161961927446592859468136034747447939482963463317531062070637037540124443
 64328700859979459109222867113617510*a²*e^(469/2*I*c) + 5086061865420950083
 533559153160520120167735286246615994357148142901870562289168145260417888380
 9245979957523278415*a²*e^(467/2*I*c) + 30959348642576562860165299817366165
 862922648668425796475504851207868785347857317439600867394648879054091070118
 592*a²*e^(465/2*I*c) + 186536264628505328030351405881047175140854310988488
 6941968918665435735438355540804832857748868154391812479528331*a²*e<sup>(463/2
 *I*c)</sup> + 1112422688241289810643238399018764325994794191948692614355578299900
 8463015355905895251415554260511452162767106896*a²*e^(461/2*I*c) + 65657679
 593071530228829834619744537570730217235007718128652478013026176517587654983
 37855684754666047332109346955*a²*e^(459/2*I*c) + 3835169648312177412955758
 957045588072559813234777488138237723537853906921795690040312404465308498665
 963807731880*a²*e^(457/2*I*c) + 221687596232449256065647525206945961628074
 8019821115392295053079963518361922106809558395835470562396612703910894*a²*
 e^(455/2*I*c) + 12680329744048849958800906653423684542730040005610160238536
 79186675517009550364504248846180076370911172808894376*a²*e^(453/2*I*c) + 7
 176732492067055394376757587392009746258445290245292770747026094047894580816
 66203757875233986777843339431778971*a²*e^(451/2*I*c) + 4018878501750450801
 503138899100726945280094470164680726114531939666324874656760746170560517739
 19819622001323159*a²*e^(449/2*I*c) + 2226587982427923341810022061092801040
 97850168579789527367599387695046282889021401012025548745828097666778231565*
 a²*e^(447/2*I*c) + 1220412812685754398344798830299069919442016006758186640
 13256230154913964580643406692381897603394093043703442025*a²*e^(445/2*I*c)
 + 6617297932881809399215054642382035162752852372860854002119931270260080224
 0756658938974138233183457153977060797*a²*e^(443/2*I*c) + 35492518572284783
 261790721091007984213268935867291381099943547658996866612281989451531913916

163009433157485571*a²*e^(441/2*I*c) + 188300010367427394010071984622268846
04435327942040227457903683414392625117454825138218461037881756069460095462*
a²*e^(439/2*I*c) + 9880899198808563287233613519855332010559992941772794510
424197819767831159955107339288892516466317402233238590*a²*e^(437/2*I*c) +
512802793210456013555902659319254460459290112053791574182285298030011519159
2803605514077707054780313193678930*a²*e^(435/2*I*c) + 26320081955436011268
865672586710800827931793309439329526480867362575752921947868957894171361058
32466949897075*a²*e^(433/2*I*c) + 1335925163388878989942887543237873573477
354078078040384838340726239029587514889857191688753727386206824536152*a²*e^(431/2*I*c) + 670517439437730975214058382921475629655071200139334454271221
441426048415295599199783168621980211682925980307*a²*e^(429/2*I*c) + 332771
401353095630876781482769166072007566368785944639929752492013340507467607203
423204297878103391257399650*a²*e^(427/2*I*c) + 163291912624014896869889921
756370257404344116207519220566929295476847426915494195734247976197944827166
856605*a²*e^(425/2*I*c) + 792210813736582642728798486708339631583938026185
4613050746344033508958081178065157747031476513568516431440*a²*e^(423/2*I*c) + 3799695085003403823385083030620582717001776425358694149802829312074251
6484664499219084036933062000158769320*a²*e^(421/2*I*c) + 18016188203161151
595570847680260903085281484547568939274327212858568842514711832067992981021
945668432453744*a²*e^(419/2*I*c) + 844417714938119303347880124874317013490
2792413015392319772244917564145947358160863746469962582546517551020*a²*e^(417/2*I*c) + 39120632922983706718236562651806275103810426889069614510874217
10967460980974243159240293773328259061298480*a²*e^(415/2*I*c) + 1791356871
286179453076261895297548714275627637620322018468056279043104785643232974994
540293493766790807180*a²*e^(413/2*I*c) + 810698514587734599341419926912945
883958489502890417749835477031467724072007792742636256985473924406832880*a²*e^(411/2*I*c) + 362585708102908727461692262308497693854332721503869703743
775437736913882412243026230183024991966728272700*a²*e^(409/2*I*c) + 160253
943840621057078138437984495917828994512049873464602063929877936771758934087
025740251939283794149320*a²*e^(407/2*I*c) + 699883682746643885808837367127
38303159327351505142299304675566880142922981296353915575157471652388092680*
a²*e^(405/2*I*c) + 3020188861789511923414776779708658681221749045460105123
6525729423437716476833896661754018658154022389900*a²*e^(403/2*I*c) + 12876
713525916989223366823193023054088090488752457893124234397628679992437662682
878994494276660041769680*a²*e^(401/2*I*c) + 542387741535614073424007653028
1376392307183648754518352919070721950593841907858843558278519237201933020*a²*e^(399/2*I*c) + 22569366401976461724095887544833910482389125597195976929
16098711340142755437285078061404880797678311440*a²*e^(397/2*I*c) + 9276920
980270654837657520653179962526748266745346311160477189597430978551926807663
31861590769408198460*a²*e^(395/2*I*c) + 3766458445736155461841665109904161
54645940002848766465441678202978044480377315238707708815985878943120*a²*e^(393/2*I*c) + 1510346812759617816700818807418659314515708088392791048041237
01285639063747843522793214727666247721320*a²*e^(391/2*I*c) + 5981404789492
439511671517626292625873582700958544166730832132595707235882486795801563841
5996637992560*a²*e^(389/2*I*c) + 23392734792358449986470914988641685348168

613137145022740977224897697160854430483589336483946581829650*a²*e^(387/2*I*c) + 903395267459197202409932332240700921947876387600923769278945094738861
 2631864264679797961473529847760*a²*e^(385/2*I*c) + 34447742392644906310173
 946350452828676614441736472801344894490672796907708801541957126381436906705
 10*a²*e^(383/2*I*c) + 1296869787700945503527478186253438429283866993105679
 751956812206964316306426908093383593771068875180*a²*e^(381/2*I*c) + 482003
 855527359842058748938929565762498717173867153381418816829773057750173701195
 879408932882129710*a²*e^(379/2*I*c) + 176843212577304640940434087070526800
 20691214794235755568484158874194832893725593513293339771528640*a²*e<sup>(377/
 2*I*c)</sup> + 640434248339001489626605649373085847925884105561358833921722009766
 30674826538314163343570582742220*a²*e^(375/2*I*c) + 2289142246227600597089
 769949517582743299142275420964671179186107472376353440040099175600913609334
 0*a²*e^(373/2*I*c) + 80750674801709208568026954585195592726223421020352975
 26669820326780584006158874417298884536622170*a²*e^(371/2*I*c) + 2810975036
 067791526020783161414349035430610964164391290128775005718676199562281038906
 617666401970*a²*e^(369/2*I*c) + 965532218209251264607039716473737729829357
 676534904149014904268128459659120470520755736939192070*a²*e^(367/2*I*c) +
 327217244888055084262892186197327446953686436737681340634739093336235382375
 514336377928793244690*a²*e^(365/2*I*c) + 109401947851812253736939736757431
 402194995906470393287332344122103685909885397712947473326518610*a²*e<sup>(363/
 2*I*c)</sup> + 360821429557893729217953262761637201159366694434304278949238218238
 35712395825018234933157302110*a²*e^(361/2*I*c) + 1173805156223944291043665
 8686616952931621359163901525749755059400912331209404508874859876592800*a²*
 e^(359/2*I*c) + 37661117341863148447457366228609850041109045082888101894089
 21969722457687589048151875880889220*a²*e^(357/2*I*c) + 1191626026712004520
 228535962916711181964681032979355080990640973734774508809765193594567308000
 *a²*e^(355/2*I*c) + 371784628277848407873740730843158415533102422282317148
 023178730401149797296961025637580441730*a²*e^(353/2*I*c) + 114367222654004
 146039785488306144607682565282217212089722715844480032864714063018623336102
 960*a²*e^(351/2*I*c) + 346835347778821728555902478391614597115084782685995
 95143978937459065098062910312662980093090*a²*e^(349/2*I*c) + 1036832427833
 694706645534803655654449472691347656052168964108774960636789159517542287084
 1040*a²*e^(347/2*I*c) + 30549765052290531863581965128998236791965392700731
 64577798894149381563584247883834020602010*a²*e^(345/2*I*c) + 8870961481117
 838197926980752116379373501282518206394716688455319858193680435964294877001
 80*a²*e^(343/2*I*c) + 2538313201292654886772680409124822491889037103512786
 72972032718591405876131400102115746900*a²*e^(341/2*I*c) + 7156131327668605
 9543420409883222534473352439855772274727763362387210317509376711352805370*a
²*e^(339/2*I*c) + 19875383270198123016398073954782048143086744479176706215
 294931939285231552377668666774800*a²*e^(337/2*I*c) + 543752145104203145537
 0282099308345101312396580420987248763253424089743393247174970193410*a²*e<sup>(
 335/2*I*c)</sup> + 14651342290643008303482102337827023384371775676500548985537313
 58964520863696142827138160*a²*e^(333/2*I*c) + 3887639689201023745839058832
 75133570373138295169848377636334256622974927805402558286690*a²*e<sup>(331/2*I*
 c)</sup> + 1015702233120008898899552699132747473837183232983774023601657581594080

55448033759362720*a²*e^(329/2*I*c) + 2612502097352610282419068257137595608
8092948345240408465888776831062032840805359036420*a²*e^(327/2*I*c) + 66144
400820020510453479219244127878363376621124553998877451160580978585651720285
83520*a²*e^(325/2*I*c) + 1648201642139146705633894149783476374765855031698
399268829056921671864832034920363790*a²*e^(323/2*I*c) + 404148737461691129
448272381486081487215931511785097115337433651585897523208487479570*a²*e<sup>(3
21/2*I*c)</sup> + 975027064963445187398194164828177857087491197633773678512515799
53471302421416590690*a²*e^(319/2*I*c) + 2314012260343947977239284444734426
7578241288491282693326820133793119469871909054310*a²*e^(317/2*I*c) + 54015
092850750556638448628821387292768098169292999783334700946692720250870679521
30*a²*e^(315/2*I*c) + 1239909297438968458498020421044704877786634944232208
108330855107855869727352550810*a²*e^(313/2*I*c) + 279841843449723329022256
022542885182491449861432225949373308219431658133659304060*a²*e^(311/2*I*c)
+ 620874280599337021448519395578167430509430152215367261590243406969754309
71481900*a²*e^(309/2*I*c) + 1353882290279227574856360451606447921132436445
5610559665374956958057411438898160*a²*e^(307/2*I*c) + 29010807054572012234
80807515950562690696054942137423459448133177752600175055070*a²*e<sup>(305/2*I*
c)</sup> + 6107355768486469132290353166900154812012832161476488376310729389377959
53569020*a²*e^(303/2*I*c) + 1262906659766494237527952933005583650780909483
68244878972263607020367969724510*a²*e^(301/2*I*c) + 2564609163526230666377
5989907048193693026481072707785982065347970437174552640*a²*e^(299/2*I*c) +
51133713404620801884705792163322470511998009337758134729860301536677684799
70*a²*e^(297/2*I*c) + 1000762165064573065580039704950705594622141532403147
100702228104518209922480*a²*e^(295/2*I*c) + 192216100832677211425583919374
607410256140382963064998218563326540739057800*a²*e^(293/2*I*c) + 362225078
23447452094316152383681805847086822685973455693062020799131961680*a²*e<sup>(29
1/2*I*c)</sup> + 6695578122665234305226870074465850318321143798508148119852380345
744581420*a²*e^(289/2*I*c) + 121368408647448003003296352490387203682620064
2910143657468007690377301200*a²*e^(287/2*I*c) + 21568229390436704822945256
6525170165451086345847776055567286526724207180*a²*e^(285/2*I*c) + 37565891
265483421625922327237663393973061791344075817295565545641550480*a²*e<sup>(283/
2*I*c)</sup> + 641088541188620822723561436924428409477385073288034158456957124798
9180*a²*e^(281/2*I*c) + 10716614348473653564025004901165851994917410896239
63750947662209022120*a²*e^(279/2*I*c) + 1754194641818378612067054904914996
11761897494030842873597094326137448*a²*e^(277/2*I*c) + 2810857079443600488
9868155095425291975524337124542327266338411863980*a²*e^(275/2*I*c) + 44075
31071123153946915575442407867368273204436441682325952058880560*a²*e<sup>(273/2
*I*c)</sup> + 676077945863029742736783795661095141326065268116577325076514243900*
a²*e^(271/2*I*c) + 1014112257547637251607760930145690519847663666793862148
96417032560*a²*e^(269/2*I*c) + 1486962092393492819586017182669827161354506
9265812546978930298844*a²*e^(267/2*I*c) + 21304351655961354080475521993685
03071331826042021949871247131824*a²*e^(265/2*I*c) + 2981357660920498212655
08565759859757268901977404776167694840520*a²*e^(263/2*I*c) + 4073355402474
0134900859594350647954212567407648957747058481680*a²*e^(261/2*I*c) + 54311
27527776820107789823018231122146548845466324910314438155*a²*e^(259/2*I*c)

+ 706359042600440090623627368646803874663354915637996034416454*a^2*e^(257/2
 *I*c) + 89566978464755601531386426232449217353087060101994516952837*a^2*e^(
 255/2*I*c) + 11067166739593806537994432532202323892559358824814410948544*a^
 2*e^(253/2*I*c) + 133186353192195159434949744805150114460455517763291022864
 5*a^2*e^(251/2*I*c) + 15601811730814987391175378706557540446367377872488041
 1510*a^2*e^(249/2*I*c) + 17779824621644501534677987130821297785034706339145
 224178*a^2*e^(247/2*I*c) + 196992206435595954789450486514953248797518182958
 0265482*a^2*e^(245/2*I*c) + 21205944442264156837050568961389176123492928875
 1535757*a^2*e^(243/2*I*c) + 22164391506926711131826332938481074378353565258
 441643*a^2*e^(241/2*I*c) + 224765543028060293934680143998751100412277057205
 7975*a^2*e^(239/2*I*c) + 22097726276159713487004187824118914266394244488943
 5*a^2*e^(237/2*I*c) + 21045445872134849680137861498182408126014563326457*a^
 2*e^(235/2*I*c) + 1939942742070099509831143493249507373979963303677*a^2*e^(
 233/2*I*c) + 172919643607724441133173354565998605401103776504*a^2*e^(231/2*
 I*c) + 14890299547694622406488788915648999996474069170*a^2*e^(229/2*I*c) +
 1237420804707705093991102638839493330163931640*a^2*e^(227/2*I*c) + 99130382
 277532022515819177881629325075859509*a^2*e^(225/2*I*c) + 764641982122710512
 4391700712517502747187088*a^2*e^(223/2*I*c) + 56717944181322762218652924108
 8113209451509*a^2*e^(221/2*I*c) + 40401820406264183313387690283892201648160
 *a^2*e^(219/2*I*c) + 2759687033617389036404063195416999823985*a^2*e^(217/2*
 I*c) + 180469988996656023008385818021005185594*a^2*e^(215/2*I*c) + 11279373
 940205552267581116020178794202*a^2*e^(213/2*I*c) + 672482982322867333315410
 721290945657*a^2*e^(211/2*I*c) + 38167952336853964450194985322999760*a^2*e^
 (209/2*I*c) + 2057571526219280176660569426269565*a^2*e^(207/2*I*c) + 105091
 017765905750210205432017088*a^2*e^(205/2*I*c) + 507141637113869712351674831
 2797*a^2*e^(203/2*I*c) + 230518924833500391396633471432*a^2*e^(201/2*I*c) +
 9835474093767985284835740178*a^2*e^(199/2*I*c) + 3923726358719533376077064
 40*a^2*e^(197/2*I*c) + 14570867094745692063264465*a^2*e^(195/2*I*c) + 50111
 4476390712619058089*a^2*e^(193/2*I*c) + 15866421408580747568967*a^2*e^(191/
 2*I*c) + 459291145921837770879*a^2*e^(189/2*I*c) + 12054885718238165655*a^2
 *e^(187/2*I*c) + 284015632089714525*a^2*e^(185/2*I*c) + 5932441401233490*a^
 2*e^(183/2*I*c) + 108143463042714*a^2*e^(181/2*I*c) + 1685352670794*a^2*e^(
 179/2*I*c) + 21830993145*a^2*e^(177/2*I*c) + 225643340*a^2*e^(175/2*I*c) +
 1744665*a^2*e^(173/2*I*c) + 8970*a^2*e^(171/2*I*c) + 23*a^2*e^(169/2*I*c))/
 (e^(517*I*c) + 418*e^(516*I*c) + 87153*e^(515*I*c) + 12085216*e^(514*I*c) +
 1253841160*e^(513*I*c) + 103818048048*e^(512*I*c) + 7146142307307*e^(511*I
 *c) + 420601518659718*e^(510*I*c) + 21608403021340047*e^(509*I*c) + 9843828
 04329835768*e^(508*I*c) + 40261256699368950388*e^(507*I*c) + 14933266122939
 84160368*e^(506*I*c) + 50648660944512569972179*e^(505*I*c) + 15817966423978
 12408161814*e^(504*I*c) + 45759117183402579073139583*e^(503*I*c) + 12324455
 57346832245176696904*e^(502*I*c) + 31042222522074681615625020522*e^(501*I*c
) + 734057263616388449968842366924*e^(500*I*c) + 16353164647151530240529137
 618111*e^(499*I*c) + 344277152012875134140739302960914*e^(498*I*c) + 686832
 9225263681349501997341320517*e^(497*I*c) + 13017119307917282383515143077336
 0024*e^(496*I*c) + 2348998374244347079532766203075607598*e^(495*I*c) + 4044

3624781415311581857832389099634564*e^(494*I*c) + 66563467067621006375419184
 7109971141414*e^(493*I*c) + 10490402669510897424624643766470754045064*e^(49
 2*I*c) + 158566476113257562566117432227203884298856*e^(491*I*c) + 230215041
 1226234925855222345201500900533576*e^(490*I*c) + 32147887693375338817454482
 515377350383950278*e^(489*I*c) + 432333688644261557547944179250800440604964
 868*e^(488*I*c) + 5605927253067558551780452883689835514455118670*e^(487*I*c
) + 70164515322544462906873548813748091084561870680*e^(486*I*c) + 848552202
 276512356496200136959676295361696315113*e^(485*I*c) + 992549073853440227293
 9987038714580495445431374618*e^(484*I*c) + 11239160454224665096642916206312
 4338952554575234051*e^(483*I*c) + 12330967001397233651819972207509325906552
 87625342156*e^(482*I*c) + 1311878180117217472967933989431815369496467536848
 1194*e^(481*I*c) + 135442594916636116191574650625331646238501101627937224*e
 ^ (480*I*c) + 1357990663161479842850642848032544982878359839580349899*e^(479
 *I*c) + 13231708870104896973800056733779919089340836756009580718*e^(478*I*c
) + 125370496586921272662198050851269323171167338854081782959*e^(477*I*c) +
 1155855412893594260345544966642687823630035899363232371472*e^(476*I*c) + 1
 0375184499871175501909398956596684116802997082526660323524*e^(475*I*c) + 90
 722605722208814918642284639487187764607589706493970774776*e^(474*I*c) + 773
 204636991145775061462731028098506094432675788136295011259*e^(473*I*c) + 642
 6195485535248576425068136870465530087114003875716691383902*e^(472*I*c) + 52
 108117629177048660492400985175830987505700566877818954141639*e^(471*I*c) +
 412430698299915190848067222327219435067747934091894670488982928*e^(470*I*c)
 + 3187749929744346497211536044751776582320958627923816470590659024*e^(469*
 I*c) + 24070801913529757101858022914372045864746991786182039740274325264*e^
 (468*I*c) + 177642829135119348577194437675802830239905460092687136494961404
 333*e^(467*I*c) + 128181746491497081085960418982835900079078992116940530461
 2211251818*e^(466*I*c) + 90466935238256829790443389631042631676725868263679
 11338826483549173*e^(465*I*c) + 6247355078105329531771077469024711412412518
 7565731848441781904032672*e^(464*I*c) + 42227612663200368754775474655570998
 8710527133086660161366353656787288*e^(463*I*c) + 27947091044756866118427906
 94973699164482254723977210209725661304403472*e^(462*I*c) + 1811576849561575
 8076710303055505625589254293659193314153418333944596408*e^(461*I*c) + 11505
 1481852080848873700388354521315567640365124003103691176697194292320*e^(460*
 I*c) + 71609949759905807989563333855294022919285819648159783007881971186260
 0096*e^(459*I*c) + 43694424829101139145653531360695958626693388580534193812
 14131241925047008*e^(458*I*c) + 2614397627990202144347194566508025456305681
 0183520401889800285493144867448*e^(457*I*c) + 15343608874505625412732723946
 1577071933130157764595997113973513183188399376*e^(456*I*c) + 88350096882179
 1202600774541927769200737689393513734789368397093333311961880*e^(455*I*c) +
 49925197124570439835053779766079539883973682975911149579918048936883718676
 80*e^(454*I*c) + 2769311653834325922598338263764793612266403385961513348984
 6664694361471028310*e^(453*I*c) + 15082238143141241377356647421001174685229
 7437597059186295243989481140398152780*e^(452*I*c) + 80667954360758914075930
 5010796189568269842021613388955218916278823182639488190*e^(451*I*c) + 42381
 25846763232586394188569858685826755328005548627437019301405851325887594480*

$e^{(450*I*c)} + 2187648289271390992804034561257870580512150875622669631708765$
 $1824252241418663320*e^{(449*I*c)} + 11096919968732097474992225959525044434121$
 $9218535349655762591192576535872151766080*e^{(448*I*c)} + 55326912881952861250$
 $2918869558947829098021956309349843584044631512291778800081490*e^{(447*I*c)} +$
 $27118432396707175276056404901488335071302424484039783185232377219442003928$
 $30108580*e^{(446*I*c)} + 1306981720348828988619320550837581839212499138234016$
 $0316886507181296548981014818410*e^{(445*I*c)} + 61948596653035502879564338815$
 $234310660410902037882473161804774492916216575880077680*e^{(444*I*c)} + 288820$
 $755264730654469968572021047109427318619508995802020689904590319476295408324$
 $280*e^{(443*I*c)} + 132475641236783747315747282116248369112096650194895392649$
 $2241643788264284546437221120*e^{(442*I*c)} + 59789921729441432184591611492998$
 $19706321732111578494525245228742976468409105395536290*e^{(441*I*c)} + 2655680$
 $638904340753449670236910154579599486175774141478994465271212756691018527412$
 $3140*e^{(440*I*c)} + 11610455168355504376291150171211639931373302113267748111$
 $2824047246361794049635726479850*e^{(439*I*c)} + 49970756725385908435759631481$
 $3794768069337190915967491907488904933922677579665354338960*e^{(438*I*c)} + 21$
 $175897334668557071015014292104147224018388379407528416185414408885457299431$
 $38209036820*e^{(437*I*c)} + 8836720640860470305694514021547969551296794092266$
 $983044118375790025854584036796364768280*e^{(436*I*c)} + 363183696523025917321$
 $97444409798122022640824604130552506742586795183267354382847875885730*e^{(435$
 $*I*c)} + 1470308167322768331630415820995920475120437252253533392388191651930$
 $00407629544745753221740*e^{(434*I*c)} + 5864034669726832427416433289215609093$
 $75197453864243299571990964608857245771134145204174990*e^{(433*I*c)} + 2304351$
 $073373840357379178597673066352016682781689139842097376663118488803841131935$
 $313641840*e^{(432*I*c)} + 892320944734329676333188188163847179349961867060102$
 $6059730895962653291770229493028162575100*e^{(431*I*c)} + 34054053851295569154$
 $352346722177172655187548910782008504718324168725029438589162349211628040*e^{($
 $430*I*c)} + 128098914601688539672480541830409847707367500438601536803204497$
 $701119911289087105659482783340*e^{(429*I*c)} + 475010578857601519272316617938$
 $425222421786597241671026894318515408511467140969393115768793680*e^{(428*I*c)}$
 $+ 173657421881819107187419747245015812388356420995065863910233714812276908$
 $0611680719741726053840*e^{(427*I*c)} + 62598721568222528436509607082350347102$
 $01362776057176647226323089751446565288850103898153859920*e^{(426*I*c)} + 2225$
 $19591767957775716736603600748022221136423214639980386437096339149122368724$
 $5823457351580140*e^{(425*I*c)} + 78009807368024239875613733058851417125327114$
 $681070889640794249282633470580756557083923203377160*e^{(424*I*c)} + 269745801$
 $440211296972683601863878954357962308520076595177128227629273240215209708218$
 $497363414140*e^{(423*I*c)} + 920089393029589032874601850027159322612526368444$
 $771489781974361078847528891468831038436064951920*e^{(422*I*c)} + 309613197162$
 $152016238030155424146545178236208681028753774890290498593402017956570617713$
 $1421614590*e^{(421*I*c)} + 10279364730663840844739577862469262604648861914297$
 $972589165243530651230690726244462479199894255180*e^{(420*I*c)} + 336753988720$
 $215683759023845939827533625598010581041846273454111362624319432407782607217$
 $56991027090*e^{(419*I*c)} + 1088679957318294728267329051920348867972846213564$
 $45627530909104429486741257822633476898356826454040*e^{(418*I*c)} + 3473514732$

147137808743520831295666012387657627759423667627333499521038897539826364038
57556867777300*e^(417*I*c) + 1093853214486220358674032434500866678499770011
305874172488975951612031456734608287095519501041975440*e^(416*I*c) + 340023
256060165161752169468084708984419802883169441742479486877932895054841812560
5446882081152636090*e^(415*I*c) + 10434117516570395966653693155582402109460
348095473027807412321427346816928567197770376496170251803940*e^(414*I*c) +
316109393312846927506943064436184146560959695209452157430040445603868952418
01579156543451940713351730*e^(413*I*c) + 9455618025893198691933430346636565
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I*c) + 27928575580003520667983536889816547764486498779466538782748893386363
3745047373109049265172681702585720*e^(411*I*c) + 81460818773653057967021002
527192141559718336988121429982329196978554987617596986636797665324497472856
0*e^(410*I*c) + 23465182192391051422381416330734647688991557089350257780476
37412681781575765422219127409260159438712250*e^(409*I*c) + 6675866290371147
358503766865669289010893543869830538708724945291580951179188296606158111257
706968604740*e^(408*I*c) + 187599882188655635641636357359860732782557372574
05706279108891366378428467414559930481172863538598193890*e^(407*I*c) + 5207
517851879327038642926335154430695110499354250058293815524168940813867525460
8030847907167748571734720*e^(406*I*c) + 14280179245022176248318087491882527
4134305133275417780084795034644763509333503150517345864659667189417080*e^(4
05*I*c) + 38687621823427716563245172304997988926311528237460754169244317667
3997513742813591736171169652250611186480*e^(404*I*c) + 10355619825920029352
263845779086115486121114950801935736913398647060291864824662418056649493810
49856258510*e^(403*I*c) + 2738895624795265603355227646566000886280778305084
825702911938903656162004262736182657700406301914070062380*e^(402*I*c) + 715
812468684294147547380736367983971817274558153840904450338385269359692162242
6696740453944718143025248390*e^(401*I*c) + 18487405299005732693752728611876
490890858357021974882371570623800186245137722660943641752976852924439870880
*e^(400*I*c) + 471882208434662076950995069535737803571088974914225678980481
99018207708997005333860148836479527456156014520*e^(399*I*c) + 1190418554038
779649482295779483704656006066231830455295269004302092704732127738477949355
86074714329479939280*e^(398*I*c) + 2968255152826695896853182732802390500845
55032203415941511962659596881615713799937680026497408305672297618840*e^(397
*I*c) + 7315849722068183628747296214039744442800104463011615273397605448153
00951787985538419764656214582667219914080*e^(396*I*c) + 1782446114931751850
556354856638421901174412322298249496591658053939787198246565945975595575734
193348887952160*e^(395*I*c) + 429320647800802212601748890885182649479062072
0660151451468181910917240027863968724539127659633517053002976480*e^(394*I*c
) + 10223182025954860767217390305186451923562145473674293619918063490411487
496121804590274592702770571515456414680*e^(393*I*c) + 240687851397052771611
934656445061432852413610377682168189221844001410484602109446966477527233719
32874594597328*e^(392*I*c) + 5602868342490351765849501385853451616716259103
4367972498174660907450666778154353271630344650777885683547624184*e^(391*I*c
) + 12896708008475471224602368086648838498328625902553313204463610904954514
4029547003347761521666283977931640178464*e^(390*I*c) + 29355074355434270980

812945357656231329970598269918741686293437396425561596713867625327630259156
1523515603264403*e^(389*I*c) + 66076447310586909769147597385083793451108903
3149586707982764263394766756649565279879146173318386505740391093990*e^(388*
I*c) + 14709311466189343455150383623001001604821277495814439299047469102247
77470198899052379114493999887003199419829579*e^(387*I*c) + 3238491931361851
476423321933539579098377735539207641467346235665823887048326949305609231585
143748690203615957136*e^(386*I*c) + 705213241416219799260232652458014306098
5353054572933905524633121681021037340298366342203324325307072413739061024*e
^(385*I*c) + 15189634214908800396417911722643754748048520109734812459109878
810493844381062650818971199637121458749456243274416*e^(384*I*c) + 323627313
224195494103300889436402474603783285613164229312924271459028879130716436795
02909055891236755143207382609*e^(383*I*c) + 6820803309679361568378440961924
421081861499164004155342440552787689327249660832423109814850246645396715772
8078994*e^(382*I*c) + 14221311596481451768238666727676990948227168131879088
9840501039441748635545362467679832449103520321953011780083069*e^(381*I*c) +
29334492003430072028704238344834286631380628545504006782308044559754597002
3446231563554135133105493516316320059272*e^(380*I*c) + 59865014111224185891
167650518052015036400322684132808145359709358779033860921243908555446686158
2623350303061961052*e^(379*I*c) + 12087703584936583930894422220569350632837
04108140593750226539846117737648216609559734831601248698274330296158612144*
e^(378*I*c) + 2414966516810338503289076549202740511710059011795447138773464
205696455026442712426409599662771080264826008985061097*e^(377*I*c) + 477414
111106609897022184533059496201647271423037423406066395684695092664268594692
9064114194400360936223590725470146*e^(376*I*c) + 93393419580534942252517509
657150573007073020838147747703062182242410226482474199560429573630558238308
98547303219757*e^(375*I*c) + 1807982006802885997034993862300723067656331420
6708848499900139641237334763266479346963237936039328113185041591793848*e^(3
74*I*c) + 34637657172671690167657344537197087048882354853993270472063943078
773600446542963548348101269390443464480754513928502*e^(373*I*c) + 656748592
688673000988273758128752256106545516862611036816640070075371157780972935335
65243828873383722980353200611956*e^(372*I*c) + 1232439415193323847419600725
881035065964063392536163910820629699606824190117457757389218177533919544626
09323881489157*e^(371*I*c) + 2289113117385927800914926491623468344058677407
76456326108410928857257174707289268074347550225793244741923354395308214*e^(
370*I*c) + 4208463426089493872775590214579245865781209661485610226470084995
29468452005980175119410628956210497609566002969884927*e^(369*I*c) + 7658677
955139627810125584446287514187109408952813047908367436615820716500321548914
82866406314834433199455459798934952*e^(368*I*c) + 1379676529796212074017106
188066589448355446501210890195107164860350228928586815539003062875026711931
941947738690360722*e^(367*I*c) + 246044237584542266392708163098326071473496
809190549302714563923882719225488634936112699145769240985112087330748745746
8*e^(366*I*c) + 43439096966019321733573596877815792937012956819408271142154
33175336093967845908766740738240037114570667410936998017178*e^(365*I*c) + 7
592752700146678961153095073585015473197029746533633331549793961473285760935
801904155116764831560875947581048693527224*e^(364*I*c) + 131397714941049338

818566811514182931122425515215356868711812665798138776063481602617472013177
35782566021306798298336024*e^(363*I*c) + 2251467574130806996150616558650287
243042193021067326439299728648560064010386725360484771554706059296769065379
5951142520*e^(362*I*c) + 38199015867586087976002998756627674994795440667903
625029322346250133286489120875005013638128113893960349670280707161530*e^(36
1*I*c) + 641751006932600668062380648860045971707408433000868393686161391645
29108049844675353111842725798658088840347241496099644*e^(360*I*c) + 1067648
320171655948380852341893335287335876733299725300926610851867899392529159370
90760282232346919090426243399409323314*e^(359*I*c) + 1758962582627559857571
068126139793012658010315954843536149046728651694422320757765804471841341413
75995770091499246759528*e^(358*I*c) + 2869929436312314965572780108515769408
968264974660663275288015606770071128374319267350881209748617605113670088157
28782643*e^(357*I*c) + 4637582884573671545449376782550056887333281455680493
10423995599886012800638619904022368378591108842602342094543682299102*e^(356
*I*c) + 7422286409081731249169370494625256173341489196791182704898310054977
81951221069955839623452499748653124658873553401442137*e^(355*I*c) + 1176600
720975786965189875050890231092204612696970277433014535895788956771230793520
381993106606880564628599822341722801012*e^(354*I*c) + 184750585646245153344
528430057132632378116255330456597188767075809107930679482183492817077312636
4639722071570131703785334*e^(353*I*c) + 28736105359223401870808354355829122
772719679773947201597910702749277142768695314671826889810410613817038854034
97544001592*e^(352*I*c) + 4427673079105425318524316112985693656584851936100
192457044455134483305045321452516347118488133224823670465103483954805161*e^
(351*I*c) + 675848043788852437256293594896385762669485554719551948612287756
7981718587262362871994967079401831957927901682582941234362*e^(350*I*c) + 10
220423779434634851339975295163399641702122249663666193053008302026096932158
568338309418237395541351819026907953220681013*e^(349*I*c) + 153128372066627
753793473532128076829657125356529426315182861424030977382002707111953965821
59028513532779682154451996208592*e^(348*I*c) + 2273160356612884110041950194
705136766683665241807726091394481074847308489189041018128541260485487662591
9565639521227223276*e^(347*I*c) + 33435897827936581301171175459610829454298
167962017419810072936733378506584428024201072453193458155334046693516742390
717832*e^(346*I*c) + 487332535059749234008522555630521014021964693136595544
92725674754339375283010407167744366955828922837488705858532439654489*e^(345
*I*c) + 7038634976059483156704822406139502569850120229696630037676422033669
7702961591099854055411376294871437468149528524796002762*e^(344*I*c) + 10074
496185185374461175430098298016696240455383622292186848469426996612060769890
7046343731011160948828100276729370132819357*e^(343*I*c) + 14290631912305552
424654692847895423837131592580202238923649865213683982250203515567697091741
9039834587967055588431566416784*e^(342*I*c) + 20090658715357880438030046950
144161017452185125954192920984068896085945490851977483590589575766677085788
8611738751858460424*e^(341*I*c) + 27994524447503980482296673046296088449211
987485779114712400907947692043594173529330930543043868733312991245419677407
0107264*e^(340*I*c) + 38664267305038004945738256281831696265197555099077927
7048740238629858795018247356162888631015687664780101205287333082748791*e^(3

39*I*c) + 52932925276411392600393483695824355767254923899756073921440659918
5047831955572583765358634395408771528009745467548382950094*e^(338*I*c) + 71
836159638205824920911354448790108886838874403371321033249197137590673834155
1540457264804304039664255915607349801911966551*e^(337*I*c) + 96645827536903
771874773913079815164348359068416683223468809829116416063641815945211981572
8809372125168836239364442397344064*e^(336*I*c) + 12890435152929339564806343
304996770401810439356201069142673110679000300583988397876923769540905452785
54544997710058754772400*e^(335*I*c) + 1704582996707822808204678218167693002
698661147712772355021456543810930069637188085882824757500605246963210810351
706405349408*e^(334*I*c) + 223489127639843946447862257830643484072461048446
817785982262065869192147864526665306256382355300122800100909360675106616894
4*e^(333*I*c) + 29053857223200570019533452744894827908566925299598237495326
95963414164833366773128218607899328588608916176593772088622582464*e^(332*I*
c) + 3745257594876651204657334988426226388143954501986830664222349226361079
609546822276067504899386703088982308185717143407211328*e^(331*I*c) + 478752
744278094568514520484697159616530416941932824407321145959212964925504887685
4059844720661078151288179612574986359194560*e^(330*I*c) + 60689498031567122
483318711053298954717228061430088780149865596536872606948165504701958900045
11965527567432722969707577202160*e^(329*I*c) + 7629731815627821580468992424
207008366438896736333024661863838105110445148946962328297631547032543419811
821015837863013682720*e^(328*I*c) + 951303227401952295420911319126822664229
991201352566594029838106479788569090499312894803522741214403563385177951121
9335277360*e^(327*I*c) + 11764212274876484080010900714673474493371278160557
811983724455826566055617658086479368641864908119643412413644803772131657280
*e^(326*I*c) + 144298162852084312045329783753756919650631542246497475512958
51507389524083226976789688601369628399900747658579201929300744260*e^(325*I*
c) + 1755627327122429239688729140312571621349148626114547857137675169010565
6067838042151038271381300372757755676325408026834544840*e^(324*I*c) + 21188
321405882887539610198374706862695894049226077093764132512513336190523978949
694387686059124526755048042957954264706637460*e^(323*I*c) + 253671764391193
536215322603359833481549049826061257617113006834929633908164915830257052687
37539982149639300226512657426118880*e^(322*I*c) + 3012848241455270326455901
895308817715601343749343820107841376983544836614812175454919759112996717076
4969700180348699207838960*e^(321*I*c) + 35500103106019649876272376796949482
209581372371036005012877806027481672807059943445240136315568500732379966585
005678181937920*e^(320*I*c) + 414998321219637080437885237874013455417800889
305382069188535790267492733646716400375634886077160928876864715428386027885
59660*e^(319*I*c) + 4813311767818402921650374854911037447892471909463560389
2829364863916553792278822957368285106328164715910598370871149079494360*e^(3
18*I*c) + 55390913044972086219432689146331566081427959896969900214434296817
731150863867056620768608187679709720152974148474907904177340*e^(317*I*c) +
632477741010121790517949460751755699240769813381384831580424067474538747293
87631710544995247152912205118500597511052824347680*e^(316*I*c) + 7166032986
117339552444194388928410913409115784465524567208423740243494469646492713181
2190659629511140639501743303863582092880*e^(315*I*c) + 80566249130682684181

876201882623511206363790337218011954110210642927765997644903820595421936873
565314654415769070472655401600*e^(314*I*c) + 898838158013823822139732704779
546027447928770180519633471463073724643151212749294023479428748028994995389
53561056667668891020*e^(313*I*c) + 9951220647205796595134034173802354851533
640337171789804085047095465753297727911349150688029072611115410194138601968
9567958040*e^(312*I*c) + 10933253734996622320393267850342635707986370700172
829401104207653040392386265401897867651641731422108944992249561273287016966
0*e^(311*I*c) + 11920971370203392705575539782368844444464742432450218532862
6347046599634721146573830681540495333543146776810911910410468628960*e^(310*
I*c) + 12899507601159190341076386342709732994858617357459586270584915928094
3046458742663163454018491463855395649453952212899632198680*e^(309*I*c) + 13
852979454915108945135276957654340312633074724368003083246720589581904356815
5239264876762867172754338684027849855385453216080*e^(308*I*c) + 14764892080
554533341862312176785377739978292474830122879392434257499993795542176537010
1235122939557467548549202174550009604780*e^(307*I*c) + 15618596295355119616
973821883217369650985255158921073057836572747625947647446595542850233667374
3686499175698677875693611243400*e^(306*I*c) + 16397781605960772537526455981
650584789418778510145536039189742448299841538578760576531550920833774159014
3078572243505132706580*e^(305*I*c) + 17086984886895310117686030605310399434
053039034726008843267684250555514129383083896127597426892866649484572346254
4709102843680*e^(304*I*c) + 17672092997055464200457577005309570059533465987
068273203197591553238757705241486632351114011768049292935451755947989922094
0360*e^(303*I*c) + 18140816877092205982036855331669732163998486262829882856
9560273295630897626829345263592219034560853530733710529842148537901680*e^(3
02*I*c) + 18483115198374894181766785017470825713812817215826941328776535853
2240773244336191900818557829905895684494889410451921524212840*e^(301*I*c) +
18691547443656751492635140562311750326198750835193008382456644443568913923
3683411704641828762178799177848064220150818355261280*e^(300*I*c) + 18761539
316851005007149728056460351091240313292031202437083506267903764499028628534
6673507093452964351257962696133511725652320*e^(299*I*c) + 18691547443656751
492635140562311750326198750835193008382456644443568913923368341170464182876
2178799177848064220150818355261280*e^(298*I*c) + 18483115198374894181766785
017470825713812817215826941328776535853224077324433619190081855782990589568
4494889410451921524212840*e^(297*I*c) + 18140816877092205982036855331669732
163998486262829882856956027329563089762682934526359221903456085353073371052
9842148537901680*e^(296*I*c) + 17672092997055464200457577005309570059533465
987068273203197591553238757705241486632351114011768049292935451755947989922
0940360*e^(295*I*c) + 17086984886895310117686030605310399434053039034726008
8432676842505555141293830838961275974268928666494845723462544709102843680*e
^(294*I*c) + 16397781605960772537526455981650584789418778510145536039189742
4482998415385787605765315509208337741590143078572243505132706580*e^(293*I*c
) + 15618596295355119616973821883217369650985255158921073057836572747625947
6474465955428502336673743686499175698677875693611243400*e^(292*I*c) + 14764
892080554533341862312176785377739978292474830122879392434257499993795542176
5370101235122939557467548549202174550009604780*e^(291*I*c) + 13852979454915

108945135276957654340312633074724368003083246720589581904356815523926487676
 2867172754338684027849855385453216080*e^(290*I*c) + 12899507601159190341076
 386342709732994858617357459586270584915928094304645874266316345401849146385
 5395649453952212899632198680*e^(289*I*c) + 11920971370203392705575539782368
 844444464742432450218532862634704659963472114657383068154049533354314677681
 0911910410468628960*e^(288*I*c) + 10933253734996622320393267850342635707986
 370700172829401104207653040392386265401897867651641731422108944992249561273
 2870169660*e^(287*I*c) + 99512206472057965951340341738023548515336403371717
 898040850470954657532977279113491506880290726111154101941386019689567958040
 *e^(286*I*c) + 898838158013823822139732704779546027447928770180519633471463
 07372464315121274929402347942874802899499538953561056667668891020*e^(285*I*
 c) + 8056624913068268418187620188262351120636379033721801195411021064292776
 5997644903820595421936873565314654415769070472655401600*e^(284*I*c) + 71660
 329861173395524441943889284109134091157844655245672084237402434944696464927
 131812190659629511140639501743303863582092880*e^(283*I*c) + 632477741010121
 790517949460751755699240769813381384831580424067474538747293876317105449952
 47152912205118500597511052824347680*e^(282*I*c) + 5539091304497208621943268
 914633156608142795989696990021443429681773115086386705662076860818767970972
 0152974148474907904177340*e^(281*I*c) + 48133117678184029216503748549110374
 478924719094635603892829364863916553792278822957368285106328164715910598370
 871149079494360*e^(280*I*c) + 414998321219637080437885237874013455417800889
 305382069188535790267492733646716400375634886077160928876864715428386027885
 59660*e^(279*I*c) + 3550010310601964987627237679694948220958137237103600501
 2877806027481672807059943445240136315568500732379966585005678181937920*e^(2
 78*I*c) + 30128482414552703264559018953088177156013437493438201078413769835
 448366148121754549197591129967170764969700180348699207838960*e^(277*I*c) +
 253671764391193536215322603359833481549049826061257617113006834929633908164
 91583025705268737539982149639300226512657426118880*e^(276*I*c) + 2118832140
 588288753961019837470686269589404922607709376413251251333619052397894969438
 7686059124526755048042957954264706637460*e^(275*I*c) + 17556273271224292396
 887291403125716213491486261145478571376751690105656067838042151038271381300
 372757755676325408026834544840*e^(274*I*c) + 144298162852084312045329783753
 756919650631542246497475512958515073895240832269767896886013696283999007476
 58579201929300744260*e^(273*I*c) + 1176421227487648408001090071467347449337
 127816055781198372445582656605561765808647936864186490811964341241364480377
 2131657280*e^(272*I*c) + 95130322740195229542091131912682266422999120135256
 65940298381064797885690904993128948035227412144035633851779511219335277360*
 e^(271*I*c) + 7629731815627821580468992424207008366438896736333024661863838
 105110445148946962328297631547032543419811821015837863013682720*e^(270*I*c)
 + 606894980315671224833187110532989547172280614300887801498655965368726069
 4816550470195890004511965527567432722969707577202160*e^(269*I*c) + 47875274
 427809456851452048469715961653041694193282440732114595921296492550488768540
 59844720661078151288179612574986359194560*e^(268*I*c) + 3745257594876651204
 657334988426226388143954501986830664222349226361079609546822276067504899386
 703088982308185717143407211328*e^(267*I*c) + 290538572232005700195334527448

948279085669252995982374953269596341416483336677312821860789932858860891617
6593772088622582464*e^(266*I*c) + 22348912763984394644786225783064348407246
104844681778598226206586919214786452666530625638235530012280010090936067510
66168944*e^(265*I*c) + 1704582996707822808204678218167693002698661147712772
355021456543810930069637188085882824757500605246963210810351706405349408*e^(
(264*I*c) + 128904351529293395648063433049967704018104393562010691426731106
7900030058398839787692376954090545278554544997710058754772400*e^(263*I*c) +
96645827536903771874773913079815164348359068416683223468809829116416063641
8159452119815728809372125168836239364442397344064*e^(262*I*c) + 71836159638
205824920911354448790108886838874403371321033249197137590673834155154045726
4804304039664255915607349801911966551*e^(261*I*c) + 52932925276411392600393
483695824355767254923899756073921440659918504783195557258376535863439540877
1528009745467548382950094*e^(260*I*c) + 38664267305038004945738256281831696
265197555099077927704874023862985879501824735616288863101568766478010120528
7333082748791*e^(259*I*c) + 27994524447503980482296673046296088449211987485
779114712400907947692043594173529330930543043868733312991245419677407010726
4*e^(258*I*c) + 20090658715357880438030046950144161017452185125954192920984
0688960859454908519774835905895757666770857888611738751858460424*e^(257*I*c
) + 14290631912305552424654692847895423837131592580202238923649865213683982
2502035155676970917419039834587967055588431566416784*e^(256*I*c) + 10074496
185185374461175430098298016696240455383622292186848469426996612060769890704
6343731011160948828100276729370132819357*e^(255*I*c) + 70386349760594831567
048224061395025698501202296966300376764220336697702961591099854055411376294
871437468149528524796002762*e^(254*I*c) + 487332535059749234008522555630521
014021964693136595544927256747543393752830104071677443669558289228374887058
58532439654489*e^(253*I*c) + 3343589782793658130117117545961082945429816796
201741981007293673337850658442802420107245319345815533404669351674239071783
2*e^(252*I*c) + 22731603566128841100419501947051367666836652418077260913944
810748473084891890410181285412604854876625919565639521227223276*e^(251*I*c)
+ 153128372066627753793473532128076829657125356529426315182861424030977382
00270711195396582159028513532779682154451996208592*e^(250*I*c) + 1022042377
943463485133997529516339964170212224966366619305300830202609693215856833830
9418237395541351819026907953220681013*e^(249*I*c) + 67584804378885243725629
359489638576266948555471955194861228775679817185872623628719949670794018319
57927901682582941234362*e^(248*I*c) + 4427673079105425318524316112985693656
584851936100192457044455134483305045321452516347118488133224823670465103483
954805161*e^(247*I*c) + 287361053592234018708083543558291227727196797739472
0159791070274927714276869531467182688981041061381703885403497544001592*e^(2
46*I*c) + 18475058564624515334452843005713263237811625533045659718876707580
91079306794821834928170773126364639722071570131703785334*e^(245*I*c) + 1176
600720975786965189875050890231092204612696970277433014535895788956771230793
520381993106606880564628599822341722801012*e^(244*I*c) + 742228640908173124
916937049462525617334148919679118270489831005497781951221069955839623452499
748653124658873553401442137*e^(243*I*c) + 463758288457367154544937678255005
688733328145568049310423995599886012800638619904022368378591108842602342094

543682299102*e^(242*I*c) + 286992943631231496557278010851576940896826497466
 066327528801560677007112837431926735088120974861760511367008815728782643*e^(
 (241*I*c) + 175896258262755985757106812613979301265801031595484353614904672
 865169442232075776580447184134141375995770091499246759528*e^(240*I*c) + 106
 764832017165594838085234189333528733587673329972530092661085186789939252915
 937090760282232346919090426243399409323314*e^(239*I*c) + 641751006932600668
 062380648860045971707408433000868393686161391645291080498446753531118427257
 98658088840347241496099644*e^(238*I*c) + 3819901586758608797600299875662767
 499479544066790362502932234625013328648912087500501363812811389396034967028
 0707161530*e^(237*I*c) + 22514675741308069961506165586502872430421930210673
 264392997286485600640103867253604847715547060592967690653795951142520*e^(23
 6*I*c) + 131397714941049338818566811514182931122425515215356868711812665798
 13877606348160261747201317735782566021306798298336024*e^(235*I*c) + 7592752
 700146678961153095073585015473197029746533633331549793961473285760935801904
 155116764831560875947581048693527224*e^(234*I*c) + 434390969660193217335735
 968778157929370129568194082711421543317533609396784590876674073824003711457
 0667410936998017178*e^(233*I*c) + 24604423758454226639270816309832607147349
 680919054930271456392388271922548863493611269914576924098511208733074874574
 68*e^(232*I*c) + 1379676529796212074017106188066589448355446501210890195107
 164860350228928586815539003062875026711931941947738690360722*e^(231*I*c) +
 765867795513962781012558444628751418710940895281304790836743661582071650032
 154891482866406314834433199455459798934952*e^(230*I*c) + 420846342608949387
 277559021457924586578120966148561022647008499529468452005980175119410628956
 210497609566002969884927*e^(229*I*c) + 228911311738592780091492649162346834
 405867740776456326108410928857257174707289268074347550225793244741923354395
 308214*e^(228*I*c) + 123243941519332384741960072588103506596406339253616391
 082062969960682419011745775738921817753391954462609323881489157*e^(227*I*c)
 + 656748592688673000988273758128752256106545516862611036816640070075371157
 78097293533565243828873383722980353200611956*e^(226*I*c) + 3463765717267169
 016765734453719708704888235485399327047206394307877360044654296354834810126
 9390443464480754513928502*e^(225*I*c) + 18079820068028859970349938623007230
 676563314206708848499900139641237334763266479346963237936039328113185041591
 793848*e^(224*I*c) + 933934195805349422525175096571505730070730208381477477
 0306218224241022648247419956042957363055823830898547303219757*e^(223*I*c) +
 47741411110660989702218453305949620164727142303742340606639568469509266426
 85946929064114194400360936223590725470146*e^(222*I*c) + 2414966516810338503
 289076549202740511710059011795447138773464205696455026442712426409599662771
 080264826008985061097*e^(221*I*c) + 120877035849365839308944222205693506328
 370410814059375022653984611773764821660955973483160124869827433029615861214
 4*e^(220*I*c) + 59865014111224185891167650518052015036400322684132808145359
 7093587790338609212439085554466861582623350303061961052*e^(219*I*c) + 29334
 492003430072028704238344834286631380628545504006782308044559754597002344623
 1563554135133105493516316320059272*e^(218*I*c) + 14221311596481451768238666
 727676990948227168131879088984050103944174863554536246767983244910352032195
 3011780083069*e^(217*I*c) + 68208033096793615683784409619244210818614991640

041553424405527876893272496608324231098148502466453967157728078994*e^(216*I*c) + 323627313224195494103300889436402474603783285613164229312924271459028
87913071643679502909055891236755143207382609*e^(215*I*c) + 1518963421490880
039641791172264375474804852010973481245910987881049384438106265081897119963
7121458749456243274416*e^(214*I*c) + 70521324141621979926023265245801430609
85353054572933905524633121681021037340298366342203324325307072413739061024*
e^(213*I*c) + 3238491931361851476423321933539579098377735539207641467346235
665823887048326949305609231585143748690203615957136*e^(212*I*c) + 147093114
661893434551503836230010016048212774958144392990474691022477747019889905237
9114493999887003199419829579*e^(211*I*c) + 66076447310586909769147597385083
793451108903314958670798276426339476675664956527987914617331838650574039109
3990*e^(210*I*c) + 29355074355434270980812945357656231329970598269918741686
2934373964255615967138676253276302591561523515603264403*e^(209*I*c) + 12896
708008475471224602368086648838498328625902553313204463610904954514402954700
3347761521666283977931640178464*e^(208*I*c) + 56028683424903517658495013858
534516167162591034367972498174660907450666778154353271630344650777885683547
624184*e^(207*I*c) + 240687851397052771611934656445061432852413610377682168
18922184400141048460210944696647752723371932874594597328*e^(206*I*c) + 1022
318202595486076721739030518645192356214547367429361991806349041148749612180
4590274592702770571515456414680*e^(205*I*c) + 42932064780080221260174889088
518264947906207206601514514681819109172400278639687245391276596335170530029
76480*e^(204*I*c) + 1782446114931751850556354856638421901174412322298249496
591658053939787198246565945975595575734193348887952160*e^(203*I*c) + 731584
972206818362874729621403974444280010446301161527339760544815300951787985538
419764656214582667219914080*e^(202*I*c) + 296825515282669589685318273280239
050084555032203415941511962659596881615713799937680026497408305672297618840
*e^(201*I*c) + 119041855403877964948229577948370465600606623183045529526900
430209270473212773847794935586074714329479939280*e^(200*I*c) + 471882208434
662076950995069535737803571088974914225678980481990182077089970053338601488
36479527456156014520*e^(199*I*c) + 1848740529900573269375272861187649089085
8357021974882371570623800186245137722660943641752976852924439870880*e^(198*
I*c) + 71581246868429414754738073636798397181727455815384090445033838526935
96921622426696740453944718143025248390*e^(197*I*c) + 2738895624795265603355
22764656600886280778305084825702911938903656162004262736182657700406301914
070062380*e^(196*I*c) + 103556198259200293522638457790861154861211149508019
3573691339864706029186482466241805664949381049856258510*e^(195*I*c) + 38687
621823427716563245172304997988926311528237460754169244317667399751374281359
1736171169652250611186480*e^(194*I*c) + 14280179245022176248318087491882527
4134305133275417780084795034644763509333503150517345864659667189417080*e^(1
93*I*c) + 52075178518793270386429263351544306951104993542500582938155241689
408138675254608030847907167748571734720*e^(192*I*c) + 187599882188655635641
636357359860732782557372574057062791088913663784284674145599304811728635385
98193890*e^(191*I*c) + 6675866290371147358503766865669289010893543869830538
708724945291580951179188296606158111257706968604740*e^(190*I*c) + 234651821
923910514223814163307346476889915570893502577804763741268178157576542221912

7409260159438712250*e^(189*I*c) + 81460818773653057967021002527192141559718
 3369881214299823291969785549876175969866367976653244974728560*e^(188*I*c) +
 27928575580003520667983536889816547764486498779466538782748893386363374504
 7373109049265172681702585720*e^(187*I*c) + 94556180258931986919334303466365
 652826858091314329189160736277175873841732196453379953705679466826880*e^(18
 6*I*c) + 316109393312846927506943064436184146560959695209452157430040445603
 86895241801579156543451940713351730*e^(185*I*c) + 1043411751657039596665369
 315558240210946034809547302780741232142734681692856719777037649617025180394
 0*e^(184*I*c) + 34002325606016516175216946808470898441980288316944174247948
 68779328950548418125605446882081152636090*e^(183*I*c) + 1093853214486220358
 674032434500866678499770011305874172488975951612031456734608287095519501041
 975440*e^(182*I*c) + 347351473214713780874352083129566601238765762775942366
 762733349952103889753982636403857556867777300*e^(181*I*c) + 108867995731829
 472826732905192034886797284621356445627530909104429486741257822633476898356
 826454040*e^(180*I*c) + 336753988720215683759023845939827533625598010581041
 84627345411136262431943240778260721756991027090*e^(179*I*c) + 1027936473066
 384084473957786246926260464886191429797258916524353065123069072624446247919
 9894255180*e^(178*I*c) + 30961319716215201623803015542414654517823620868102
 87537748902904985934020179565706177131421614590*e^(177*I*c) + 9200893930295
 890328746018500271593226125263684447714897819743610788475288914688310384360
 64951920*e^(176*I*c) + 2697458014402112969726836018638789543579623085200765
 95177128227629273240215209708218497363414140*e^(175*I*c) + 7800980736802423
 987561373305885141712532711468107088964079424928263347058075655708392320337
 7160*e^(174*I*c) + 2225195917679577757167366036007480222211364232146399803
 864370963391491223687245823457351580140*e^(173*I*c) + 625987215682225284365
 0960708235034710201362776057176647226323089751446565288850103898153859920*e
 ^ (172*I*c) + 17365742188181910718741974724501581238835642099506586391023371
 48122769080611680719741726053840*e^(171*I*c) + 4750105788576015192723166179
 38425222421786597241671026894318515408511467140969393115768793680*e^(170*I*
 c) + 1280989146016885396724805418304098477073675004386015368032044977011199
 11289087105659482783340*e^(169*I*c) + 3405405385129556915435234672217717265
 5187548910782008504718324168725029438589162349211628040*e^(168*I*c) + 89232
 094473432967633318818816384717934996186706010260597308959626532917702294930
 28162575100*e^(167*I*c) + 2304351073373840357379178597673066352016682781689
 139842097376663118488803841131935313641840*e^(166*I*c) + 586403466972683242
 741643328921560909375197453864243299571990964608857245771134145204174990*e^
 (165*I*c) + 147030816732276833163041582099592047512043725225353339238819165
 193000407629544745753221740*e^(164*I*c) + 363183696523025917321974444097981
 22022640824604130552506742586795183267354382847875885730*e^(163*I*c) + 8836
 720640860470305694514021547969551296794092266983044118375790025854584036796
 364768280*e^(162*I*c) + 211758973346685570710150142921041472240183883794075
 2841618541440888545729943138209036820*e^(161*I*c) + 49970756725385908435759
 6314813794768069337190915967491907488904933922677579665354338960*e^(160*I*c
) + 11610455168355504376291150171211639931373302113267748111282404724636179
 4049635726479850*e^(159*I*c) + 26556806389043407534496702369101545795994861

757741414789944652712127566910185274123140*e^(158*I*c) + 597899217294414321
8459161149299819706321732111578494525245228742976468409105395536290*e^(157*
I*c) + 13247564123678374731574728211624836911209665019489539264922416437882
64284546437221120*e^(156*I*c) + 2888207552647306544699685720210471094273186
19508995802020689904590319476295408324280*e^(155*I*c) + 6194859665303550287
9564338815234310660410902037882473161804774492916216575880077680*e^(154*I*c
) + 13069817203488289886193205508375818392124991382340160316886507181296548
981014818410*e^(153*I*c) + 271184323967071752760564049014883350713024244840
3978318523237721944200392830108580*e^(152*I*c) + 55326912881952861250291886
9558947829098021956309349843584044631512291778800081490*e^(151*I*c) + 11096
919968732097474992225959525044434121921853534965576259119257653587215176608
0*e^(150*I*c) + 21876482892713909928040345612578705805121508756226696317087
651824252241418663320*e^(149*I*c) + 423812584676323258639418856985868582675
5328005548627437019301405851325887594480*e^(148*I*c) + 80667954360758914075
9305010796189568269842021613388955218916278823182639488190*e^(147*I*c) + 15
082238143141241377356647421001174685229743759705918629524398948114039815278
0*e^(146*I*c) + 27693116538343259225983382637647936122664033859615133489846
664694361471028310*e^(145*I*c) + 499251971245704398350537797660795398839736
8297591114957991804893688371867680*e^(144*I*c) + 88350096882179120260077454
1927769200737689393513734789368397093333311961880*e^(143*I*c) + 15343608874
5056254127327239461577071933130157764595997113973513183188399376*e^(142*I*c
) + 26143976279902021443471945665080254563056810183520401889800285493144867
448*e^(141*I*c) + 436944248291011391456535313606959586266933885805341938121
4131241925047008*e^(140*I*c) + 71609949759905807989563333855294022919285819
6481597830078819711862600096*e^(139*I*c) + 11505148185208084887370038835452
1315567640365124003103691176697194292320*e^(138*I*c) + 18115768495615758076
710303055505625589254293659193314153418333944596408*e^(137*I*c) + 279470910
4475686611842790694973699164482254723977210209725661304403472*e^(136*I*c) +
422276126632003687547754746555709988710527133086660161366353656787288*e^(1
35*I*c) + 62473550781053295317710774690247114124125187565731848441781904032
672*e^(134*I*c) + 904669352382568297904433896310426316767258682636791133882
6483549173*e^(133*I*c) + 12818174649149708108596041898283590007907899211694
05304612211251818*e^(132*I*c) + 1776428291351193485771944376758028302399054
60092687136494961404333*e^(131*I*c) + 2407080191352975710185802291437204586
4746991786182039740274325264*e^(130*I*c) + 31877499297443464972115360447517
76582320958627923816470590659024*e^(129*I*c) + 4124306982999151908480672223
27219435067747934091894670488982928*e^(128*I*c) + 5210811762917704866049240
0985175830987505700566877818954141639*e^(127*I*c) + 64261954855352485764250
68136870465530087114003875716691383902*e^(126*I*c) + 7732046369911457750614
62731028098506094432675788136295011259*e^(125*I*c) + 9072260572220881491864
2284639487187764607589706493970774776*e^(124*I*c) + 10375184499871175501909
398956596684116802997082526660323524*e^(123*I*c) + 115585541289359426034554
4966642687823630035899363232371472*e^(122*I*c) + 12537049658692127266219805
0851269323171167338854081782959*e^(121*I*c) + 13231708870104896973800056733
779919089340836756009580718*e^(120*I*c) + 135799066316147984285064284803254

4982878359839580349899*e^(119*I*c) + 13544259491663611619157465062533164623
 8501101627937224*e^(118*I*c) + 13118781801172174729679339894318153694964675
 368481194*e^(117*I*c) + 123309670013972336518199722075093259065528762534215
 6*e^(116*I*c) + 112391604542246650966429162063124338952554575234051*e^(115*
 I*c) + 9925490738534402272939987038714580495445431374618*e^(114*I*c) + 8485
 52202276512356496200136959676295361696315113*e^(113*I*c) + 7016451532254446
 2906873548813748091084561870680*e^(112*I*c) + 56059272530675585517804528836
 89835514455118670*e^(111*I*c) + 4323336886442615575479441792508004406049648
 68*e^(110*I*c) + 32147887693375338817454482515377350383950278*e^(109*I*c) +
 2302150411226234925855222345201500900533576*e^(108*I*c) + 1585664761132575
 62566117432227203884298856*e^(107*I*c) + 1049040266951089742462464376647075
 4045064*e^(106*I*c) + 665634670676210063754191847109971141414*e^(105*I*c) +
 40443624781415311581857832389099634564*e^(104*I*c) + 234899837424434707953
 2766203075607598*e^(103*I*c) + 130171193079172823835151430773360024*e^(102*
 I*c) + 6868329225263681349501997341320517*e^(101*I*c) + 3442771520128751341
 40739302960914*e^(100*I*c) + 16353164647151530240529137618111*e^(99*I*c) +
 734057263616388449968842366924*e^(98*I*c) + 31042222522074681615625020522*e
 ^ (97*I*c) + 1232445557346832245176696904*e^(96*I*c) + 457591171834025790731
 39583*e^(95*I*c) + 1581796642397812408161814*e^(94*I*c) + 50648660944512569
 972179*e^(93*I*c) + 1493326612293984160368*e^(92*I*c) + 4026125669936895038
 8*e^(91*I*c) + 984382804329835768*e^(90*I*c) + 21608403021340047*e^(89*I*c)
 + 420601518659718*e^(88*I*c) + 7146142307307*e^(87*I*c) + 103818048048*e^(
 86*I*c) + 1253841160*e^(85*I*c) + 12085216*e^(84*I*c) + 87153*e^(83*I*c) +
 418*e^(82*I*c) + e^(81*I*c))) * tan(1/4*d*x + c) + 7*(-299*I*a^2*e^(1027/2*I*
 c) - 116610*I*a^2*e^(1025/2*I*c) - 22680645*I*a^2*e^(1023/2*I*c) - 29333634
 20*I*a^2*e^(1021/2*I*c) - 283802910885*I*a^2*e^(1019/2*I*c) - 2190958472032
 2*I*a^2*e^(1017/2*I*c) - 1405865019555002*I*a^2*e^(1015/2*I*c) - 7712173821
 5926170*I*a^2*e^(1013/2*I*c) - 3692203217145049425*I*a^2*e^(1011/2*I*c) - 1
 56713514334349191205*I*a^2*e^(1009/2*I*c) - 5970784896718122444327*I*a^2*e^
 (1007/2*I*c) - 206263478291032388361681*I*a^2*e^(1005/2*I*c) - 651448819176
 2735815405157*I*a^2*e^(1003/2*I*c) - 189421272159473056494074595*I*a^2*e^(1
 001/2*I*c) - 5100844262877818375740499940*I*a^2*e^(999/2*I*c) - 12786116307
 2229101394013407134*I*a^2*e^(997/2*I*c) - 2996746017244157830350053701476*I
 *a^2*e^(995/2*I*c) - 65928412631647739461683457820511*I*a^2*e^(993/2*I*c) -
 1366183224856297865521217009020824*I*a^2*e^(991/2*I*c) - 26748429663467970
 647815785552181215*I*a^2*e^(989/2*I*c) - 4961833756024532791076116853907677
 40*I*a^2*e^(987/2*I*c) - 8742278650463030069301003637232429883*I*a^2*e^(985
 /2*I*c) - 146631858416411858207561193721288320906*I*a^2*e^(983/2*I*c) - 234
 6109795219082040306865029327652810422*I*a^2*e^(981/2*I*c) - 358759301576956
 41006598626271371876133245*I*a^2*e^(979/2*I*c) - 52522364023363982941724850
 2076803689088820*I*a^2*e^(977/2*I*c) - 737333227893847787710951619084416318
 0831833*I*a^2*e^(975/2*I*c) - 99403449489614415589324261326783015782654200*
 I*a^2*e^(973/2*I*c) - 1288694832301854187496925699385880912905767417*I*a^2*
 e^(971/2*I*c) - 16086468264323655413475747304585608493929925740*I*a^2*e^(96
 9/2*I*c) - 193573860526473423179001809134869735140502053410*I*a^2*e^(967/2*

$I*c) - 2247954875096342235081391843579770658847258731308*I*a^2*e^(965/2*I*c)$
 $) - 25219248742824948157547271484589255811816930894969*I*a^2*e^(963/2*I*c)$
 $- 273590703261713668448365859396559199738283337615067*I*a^2*e^(961/2*I*c) -$
 $2872703209254401680718347285883520097429919909434035*I*a^2*e^(959/2*I*c) -$
 $29219505531617139745995198456026235957224788107224865*I*a^2*e^(957/2*I*c)$
 $- 288136908346804733688436834384090051687033252355064191*I*a^2*e^(955/2*I*c)$
 $) - 2756770672774248567550184031805577107703843954010093519*I*a^2*e^(953/2*$
 $I*c) - 25608963224785291572287952038770777479118266594544417374*I*a^2*e^(95$
 $1/2*I*c) - 231137463934351535240227752765265592838813655877791732766*I*a^2*$
 $e^(949/2*I*c) - 2028232835526354300983369892820970081423531403040583023770*$
 $I*a^2*e^(947/2*I*c) - 17314198559836475415651393365133642453745100461062396$
 $834495*I*a^2*e^(945/2*I*c) - 1438728978699584767118418083689734186092541395$
 $96808263965208*I*a^2*e^(943/2*I*c) - 11643681393163236067131954964955339034$
 $52490627065569725880879*I*a^2*e^(941/2*I*c) - 91826435809751223944820199495$
 $57981111651613557125350856577098*I*a^2*e^(939/2*I*c) - 70604441498160359313$
 $241260895971702149956086380303936468404945*I*a^2*e^(937/2*I*c) - 5295343038$
 $04314371317279287031587399342898319070257744555877120*I*a^2*e^(935/2*I*c) -$
 $3875748752953757251392321804098110389738719224033802668294149720*I*a^2*e^($
 $933/2*I*c) - 27695522491987336122297854656279067704143932579317971697894652$
 $128*I*a^2*e^(931/2*I*c) - 1933039822447819196915008963471641025408632532118$
 $17387366117379028*I*a^2*e^(929/2*I*c) - 13183373409046462891979618862923450$
 $09401456618543410019867356069360*I*a^2*e^(927/2*I*c) - 87889472217640335244$
 $46888793703125344784094679194612662969732267700*I*a^2*e^(925/2*I*c) - 57297$
 $408391078324425844220711696016060458004240084890338934689925280*I*a^2*e^(92$
 $3/2*I*c) - 3654077936361969733798789269469219931029712437325780546471234142$
 $14820*I*a^2*e^(921/2*I*c) - 22804271200781377193255138478742651480195418540$
 $31486848558248442727176*I*a^2*e^(919/2*I*c) - 13931417792509198161468797423$
 $752544838678599539275212332380442651357560*I*a^2*e^(917/2*I*c) - 8334027696$
 $2100363964566640495864516393854803017236184470442989394035180*I*a^2*e^(915/$
 $2*I*c) - 488348364369005961261801502808542403226214848682295131186689348564$
 $834560*I*a^2*e^(913/2*I*c) - 2803816222757253133472672279792586220272029806$
 $438905494766819989517225180*I*a^2*e^(911/2*I*c) - 1577755133492988474155898$
 $0231119513445377646781839885604883296740586671600*I*a^2*e^(909/2*I*c) - 870$
 $40382736241245048113135528917435772531399861517048611706176997728840060*I*a$
 $^2*e^(907/2*I*c) - 47087957289932945298631625353486851499258535581271904808$
 $5012389615234670720*I*a^2*e^(905/2*I*c) - 249873137703119593902009900676777$
 $5607036784147188475889323331541072591988520*I*a^2*e^(903/2*I*c) - 130094515$
 $20218054427987726909306388026728218430095197151626190204529199816160*I*a^2*$
 $e^(901/2*I*c) - 66471205999186848117890072662193021327677387770934414812726$
 $997704844663612890*I*a^2*e^(899/2*I*c) - 3333844415521753485758638516852800$
 $79397515408127201392843010509098144064832520*I*a^2*e^(897/2*I*c) - 16416972$
 $96378143299128733640940998805441384198489236819275463423360050416710230*I*a$
 $^2*e^(895/2*I*c) - 79391223920964898195760233657809090959109720434262147715$
 $45781246832910820414100*I*a^2*e^(893/2*I*c) - 37711714010161632851275194873$
 $586231922967566451955409746843374416439997598272230*I*a^2*e^(891/2*I*c) - 1$

759925414014648725917583014998348987406999100607813937748432025130626303367
53720*I*a^2*e^(889/2*I*c) - 80707446054953921000355859651553212989790297514
0321070941315505466030633289504700*I*a^2*e^(887/2*I*c) - 363763252490000796
1249646025024970707837559612619597442737493322095743154754032540*I*a^2*e^(8
85/2*I*c) - 161172874479982061427980838947631758408488571143435430472421389
53088137509988748530*I*a^2*e^(883/2*I*c) - 70212206304801678332513345599910
949246264802924017064833024979917894264890884744230*I*a^2*e^(881/2*I*c) - 3
007863800452871436289544996291365621932460978775098630558262520643265697436
30828470*I*a^2*e^(879/2*I*c) - 12673708756138164831723767151346119151550235
32375947527368816590522121089591841195870*I*a^2*e^(877/2*I*c) - 52531802189
01190370523307961292015004989360145697116713826069492059896349225432472890*
I*a^2*e^(875/2*I*c) - 21423226375734147493294111979827037040590614426574817
511241501723037567325263745727850*I*a^2*e^(873/2*I*c) - 8597268169139135497
9713197091334370055201000372937152633172947601816313260199706979160*I*a^2*e
^(871/2*I*c) - 339559768710652597013282888817312950668437949251721078395194
275059325692430095887234940*I*a^2*e^(869/2*I*c) - 1320132342599574776582487
491686376011020731128338564644972136377885389653577953327654360*I*a^2*e^(86
7/2*I*c) - 5052749693090074250020711633864893162784543894410325996308296849
332368152124619583542550*I*a^2*e^(865/2*I*c) - 1904184711831972629326580327
3951207945593799588235034178026656550799568684461605603777920*I*a^2*e^(863/
2*I*c) - 706678091924968389655322817331198722866713035743737606295549343337
98898169280995448949110*I*a^2*e^(861/2*I*c) - 25829986186907031921553951148
6716381312735629658913082894934911762706535490482149535095400*I*a^2*e^(859/
2*I*c) - 929980703633600952321625692946644952639044210651468835121797966088
502026440103109950142510*I*a^2*e^(857/2*I*c) - 3298577503461372465605345536
869123261456392605454980840206045785661659654642478147158578260*I*a^2*e^(85
5/2*I*c) - 1152754471021412729195932096628473834501981095596838526568360091
2288138678711188667077969580*I*a^2*e^(853/2*I*c) - 396969673391646744846986
05925414889836548997086240408750738793856030209166162894182626765170*I*a^2*
e^(851/2*I*c) - 13472244550279277538953701338689137282615560362808524038395
9392418361487884840921793997982680*I*a^2*e^(849/2*I*c) - 450645625803738882
960885871030566143953108988238530005053615795799976664205719692805608636970
*I*a^2*e^(847/2*I*c) - 1485908894655409129727579697390217111877691246727000
929093187863829367925384373668136922959680*I*a^2*e^(845/2*I*c) - 4830133064
761265676098823275890322270506766918770002889169425396499013319534724628237
025832970*I*a^2*e^(843/2*I*c) - 1548042334099588158474041602333287754558802
4374995660159497444149878426588554491651581857867240*I*a^2*e^(841/2*I*c) -
489225665813288279962433692883462704693078596446648027300850105971823469965
42663202606651415940*I*a^2*e^(839/2*I*c) - 15246953324765761349401422316907
9603947607129031097239090335734676330177814841868201155345866600*I*a^2*e^(8
37/2*I*c) - 468649433876940374716415334540852022756419671685290819126536222
202383329230675172080843145521190*I*a^2*e^(835/2*I*c) - 1420846196643578803
733575330926970974464675704020481618027446077982380731097941167611657029967
430*I*a^2*e^(833/2*I*c) - 4249342990738608701078109187168768367693341231356
030118163718909774173116310218460585993016680850*I*a^2*e^(831/2*I*c) - 1253

756283673259917976701035515246105680719195100500979645182339107900065576218
9067741192305394090*I*a^2*e^(829/2*I*c) - 364973263533159636054202981906874
65001055100758074304234792272161886339957870164613698417616283210*I*a^2*e^(
827/2*I*c) - 10483460908657262590728934622589025242928356504146032385140279
5840051940483234866884215706838405390*I*a^2*e^(825/2*I*c) - 297154478443352
847298744293536373570367631258373197290869967778261660884898898610815793932
187264900*I*a^2*e^(823/2*I*c) - 8312493331242709297495651233702193571656545
68742620039903429231671547436113371386430716266759961060*I*a^2*e^(821/2*I*c
) - 22950304190171440151107977321553753464773390795325750065806758694788583
21228765847919467823502616280*I*a^2*e^(819/2*I*c) - 62544569577108570849907
775078801967958260483428773913101155061676802330243540898930726814064752467
30*I*a^2*e^(817/2*I*c) - 16825577675009653410652812996723598120483470175375
332001250964015555125236873581387468224926998037020*I*a^2*e^(815/2*I*c) - 4
468523594698667379192522552599281379748585493930455912527936618154971335157
2630218142149059140721370*I*a^2*e^(813/2*I*c) - 117167084483123710863969807
938772155169956785317523090174862697344141293246236717353854714751253177160
*I*a^2*e^(811/2*I*c) - 3033387063507554872790571708825134747433602012003623
27954737042233540329160364416313115387680520970550*I*a^2*e^(809/2*I*c) - 77
546675678005380330048742944775088823732579762475791970674462847820393725583
3604141877857656968392480*I*a^2*e^(807/2*I*c) - 195768847717816482384418453
656564779457553292675546736005582093785170459597087760841904548288506591372
0*I*a^2*e^(805/2*I*c) - 488090481262949257818274970426572335837616686734800
3927103702683844870930164640446326414262466377824000*I*a^2*e^(803/2*I*c) -
120188852274055085277789182447130770488546641340585826414101184645320012379
95012608936579451133542506420*I*a^2*e^(801/2*I*c) - 29232457847784347194232
755870686399315171809346248346594946497316944533356199777722113539614776201
632080*I*a^2*e^(799/2*I*c) - 7023174283669148623240182143564669020476537017
0934364279337989127944150596825723542271716872404327490260*I*a^2*e^(797/2*I
*c) - 166685454178675867308847881643025971737125688220303037968682349925679
089331312174778117366039026285869440*I*a^2*e^(795/2*I*c) - 3908290134584787
137620058685317495505113483637554024253754752198632010424690276717209923242
63291992741380*I*a^2*e^(793/2*I*c) - 90537596448029304322156871761722280435
2981478608148290290910134804446811145030581376444150025327895167080*I*a^2*e
^(791/2*I*c) - 207229696847272763858236676321480935013103617973389480388160
1867867318900360235090208152423869380817193240*I*a^2*e^(789/2*I*c) - 468687
387315610187119076797456159860951344367020175746091423231250548462720987195
7976104328771649162041900*I*a^2*e^(787/2*I*c) - 104748971102058073678463750
667233366222445365231765611762172458395877791266303914821951105110432244350
73120*I*a^2*e^(785/2*I*c) - 23135457555667242015109562696724759269442147216
687397489819489071941696946645262869400576401992001928296540*I*a^2*e^(783/2
*I*c) - 5050036250415685947835384047283825798572043021687488308623030289699
6886794997397859935740848410975488426320*I*a^2*e^(781/2*I*c) - 108949410506
024382921420575473869228515942872496860505620768573056175175693027104049195
507535238812688471740*I*a^2*e^(779/2*I*c) - 2323244171972350620810225652359
599733291297040165125114926844072284475540053624755311811772573754281585766

$72 \cdot I \cdot a^2 \cdot e^{(777/2 \cdot I \cdot c)} - 48969955715529867639855570669017010328896220158706$
 $2038457047783309280898007016462695325519866776349668712200 \cdot I \cdot a^2 \cdot e^{(775/2 \cdot I \cdot c)}$
 $- 102036246834307522613193547374782135529005215241764645522016110223803$
 $2449230964279115841386608493376970117760 \cdot I \cdot a^2 \cdot e^{(773/2 \cdot I \cdot c)} - 210181226418$
 $206573041185958293800266062240235373058151016962305996082029052667361576247$
 $6646904104925062205465 \cdot I \cdot a^2 \cdot e^{(771/2 \cdot I \cdot c)} - 428028402472883733819557937783$
 $048952516777112736515505590589647226040774927042924678670664367895488463316$
 $3010 \cdot I \cdot a^2 \cdot e^{(769/2 \cdot I \cdot c)} - 861816491878892520390205950670276873300422354456$
 $4086461069581208848976475760787131715839621109117547446849991 \cdot I \cdot a^2 \cdot e^{(767/2 \cdot I \cdot c)}$
 $- 171571319853592779976394979501546747627791903510875678191956995177$
 $84666333383378336687113858692204044077794176 \cdot I \cdot a^2 \cdot e^{(765/2 \cdot I \cdot c)} - 33774289$
 $747978981424926739931160824797161112237362925619159566387573730808632464292$
 $717707940688773187398723895 \cdot I \cdot a^2 \cdot e^{(763/2 \cdot I \cdot c)} - 6574507261855051218987344$
 $731522085862393122331014252983980812185685066571852337666969183604277884663$
 $0737410930 \cdot I \cdot a^2 \cdot e^{(761/2 \cdot I \cdot c)} - 126560833582905534125457118283479588254886$
 $734066422681322743067453094955423272892820673108496357191409969596590 \cdot I \cdot a^2$
 $\cdot e^{(759/2 \cdot I \cdot c)} - 2409451272767662273203399400892982326810482391442487893249$
 $68088048819421447003749816077955279624100315116789806 \cdot I \cdot a^2 \cdot e^{(757/2 \cdot I \cdot c)} -$
 $45367323239426704394960592256932998094054962699938259984350566231168625730$
 $1966537920360838324667729133482235583 \cdot I \cdot a^2 \cdot e^{(755/2 \cdot I \cdot c)} - 844885322059161$
 $93236899351711720811183944355764632182261257568921304529568367117463318791$
 $743206771210437760631 \cdot I \cdot a^2 \cdot e^{(753/2 \cdot I \cdot c)} - 1556343096968573764756475235411$
 $933016733786313107239004864800578666621277030123112055440836769200504575169$
 $913345 \cdot I \cdot a^2 \cdot e^{(751/2 \cdot I \cdot c)} - 2835889022716793311093267108913411484117430817$
 $040975928639654911368211514698112927408762873660302882359781905835 \cdot I \cdot a^2 \cdot e^{(749/2 \cdot I \cdot c)}$
 $- 5111787155098231966381444413998413769738039840629789986636135$
 $861377096437110535352261196444661635160446805767627 \cdot I \cdot a^2 \cdot e^{(747/2 \cdot I \cdot c)} - 9$
 $115474553683826281436142305910668402582581988856745851085574498026708677442$
 $826803383401266387813162062594525953 \cdot I \cdot a^2 \cdot e^{(745/2 \cdot I \cdot c)} - 1608170803711723$
 $936488542495308324493651572492129938677860022595117949813117158952421904513$
 $8516435810850207340428 \cdot I \cdot a^2 \cdot e^{(743/2 \cdot I \cdot c)} - 280708048160262098107656410952$
 $750421279481280413894356297204856430240954064869242205425224481435633245184$
 $01964482 \cdot I \cdot a^2 \cdot e^{(741/2 \cdot I \cdot c)} - 48480920660514266867535157245728700672453465$
 $365901344447830053298267300849098214762864212092705639132905793681740 \cdot I \cdot a^2$
 $\cdot e^{(739/2 \cdot I \cdot c)} - 8285214296686085143161142011759637225646673944870189720446$
 $0441426498632548900216968337411640986907458522921325945 \cdot I \cdot a^2 \cdot e^{(737/2 \cdot I \cdot c)}$
 $- 140112609715644490114299623620223732634461647512701806908240383878654745$
 $399087836858804711629400762515425955916824 \cdot I \cdot a^2 \cdot e^{(735/2 \cdot I \cdot c)} - 2344850209$
 $804124673417431991453520294680763522663798546087735225937995659586211761414$
 $72761686768653754135750141593 \cdot I \cdot a^2 \cdot e^{(733/2 \cdot I \cdot c)} - 38836613077293651403504$
 $522610152872609997634306743502408489865605493216277496607446437043051044461$
 $6487043887402836 \cdot I \cdot a^2 \cdot e^{(731/2 \cdot I \cdot c)} - 636619675359008017622216008920407724$
 $708440107629987848134367181043106780412514399115423605660760788697651634379$
 $005 \cdot I \cdot a^2 \cdot e^{(729/2 \cdot I \cdot c)} - 1032893390108623451662629921902059825520150456341$
 $148954819241245986827172746045153810766991498955787562750756646550 \cdot I \cdot a^2 \cdot e^{(727/2 \cdot I \cdot c)}$

(727/2*I*c) - 1658792694072871479101869942544704973168924575943071103309570
774086262029542461228753343283112620767567415651784874*I*a^2*e^(725/2*I*c)
- 2637026704940057541717603416099047901577339805399244480403700320734657027
382051540741428399462562160665232399986075*I*a^2*e^(723/2*I*c) - 4149994810
365900522807468213278193091656141731116367453829137630709191482537488782374
639033936237089896768763566876*I*a^2*e^(721/2*I*c) - 6465709656300067812786
267514447106687902581433310919178038542426426780363215033938034518725151681
639996365849163135*I*a^2*e^(719/2*I*c) - 9973455781668332100482619120779044
796198884570518850362675620400800380525655955470978275300874201998244732697
737400*I*a^2*e^(717/2*I*c) - 1523215519724446998347723512048993425539959433
2170569196782170361245875795753404131530065929326205229838601927955519*I*a^
2*e^(715/2*I*c) - 230350339766078618487647778136258591386637475271317988770
08816594078301950324404686986975822232657903821541105390532*I*a^2*e^(713/2*
I*c) - 34494824856566822280833509701479402995499365014417407042892623384294
837884168792240702005587755169596414970227728126*I*a^2*e^(711/2*I*c) - 5115
434927366261590921623119062336482761125172264104514786908016584293008567702
5925218103510278265118135145553584580*I*a^2*e^(709/2*I*c) - 751278263211159
102200857958235109088145883877187557120298253868427887040377648408378649888
71197826414756226327584715*I*a^2*e^(707/2*I*c) - 10927856072355093095863814
815237730501887441255995315083939710515717152205210794377722809701572323901
6040062510748533*I*a^2*e^(705/2*I*c) - 157438648683134146474225508822869634
510407589789880501589161153804500657734549235665125080321402913947303819424
223977*I*a^2*e^(703/2*I*c) - 2246758701378169674485742909065720123536245722
47889623771866556695527535849562092439004233144825309318376200907596647*I*a
^2*e^(701/2*I*c) - 31761185459035984262486758829969439709205129575301223433
4896647966925083208128923978657573859261411691474063180579773*I*a^2*e^(699/
2*I*c) - 444793756576556805226598557121881255891420660178954005606190859767
907900375526699087281079775401935479298706556678625*I*a^2*e^(697/2*I*c) - 6
171189074154918820088936899923749860260586145951806369345963314131703797804
87585714587047788469214113451795516499370*I*a^2*e^(695/2*I*c) - 84830810258
463636248416282574357231621057004122495206444018770310837379429016187169353
1289023220368566861588408740234*I*a^2*e^(693/2*I*c) - 115541827126152536476
409983255601569565667174214839743077062191976959126206932489179734608971821
6295419176001175584602*I*a^2*e^(691/2*I*c) - 155937926574672215760111663377
054590568084528646619506804743775058210085021855893406344134616621928423629
2591756007869*I*a^2*e^(689/2*I*c) - 208553251562235328977529713768435604455
571784789590571202099167792635363269719613533615692265275394611066259818235
7060*I*a^2*e^(687/2*I*c) - 276414155191318382323224053831143486169805378254
0087737778740958755195255197468794265331998300489529793104153836938365*I*a^
2*e^(685/2*I*c) - 363083629960604990125971166822378351635954831561642609954
3877893492461453478630388967263756039510415511889625376389370*I*a^2*e^(683/
2*I*c) - 472694509125322570928470162025352958802736277391126716009458186085
5616546226572637107723406479754761335508976269914579*I*a^2*e^(681/2*I*c) -
609966124172952004987797301832329959414996250820384193576264039942887752609
9801926994421670829050753700797133864249984*I*a^2*e^(679/2*I*c) - 780198553

704215677064427541900442321556982842761256699474406159545507180383867270904
0326328594306655699615422670941360*I*a^2*e^(677/2*I*c) - 989238300147813523
003253928973088080293094948398241346146580458789389657623927099439810176311
7672285400912303258185280*I*a^2*e^(675/2*I*c) - 124340927124878766061808638
396410072037820869420527197275324884499542899154817680073003677169984302980
25515215263240040*I*a^2*e^(673/2*I*c) - 15494034073937889118907860466406475
286727040048377789279507477424950714627734498672369684614416130884899948506
496976480*I*a^2*e^(671/2*I*c) - 1914126253803615554481860995332970989493843
539604277817436110389256710689414700608853265805980848194465744446466781392
8*I*a^2*e^(669/2*I*c) - 234449427531097206161909944823962633447754673148656
64633925377711343676479143801423131782286928982641584906730652336320*I*a^2*
e^(667/2*I*c) - 28471827652223748549106708088925334267458310362833843507803
706954155729623190563034227809794821598285838730011875428360*I*a^2*e^(665/2
*I*c) - 3428325782509936146962902964699798701635741039313333787686078421859
4231344941134240798399418976200089374372937336277520*I*a^2*e^(663/2*I*c) -
409317259082598900650106764128980599350021675414437149684224475067236578143
62423616027467796037956594249470207911160560*I*a^2*e^(661/2*I*c) - 48457084
470569206202883594030058946835172276651843799267234795884834422497155701461
376425140347292769622287009253442200*I*a^2*e^(659/2*I*c) - 5688251126937328
787053919582243715200468010432185022939670892600501269384367580335094519646
3973920628412256283832232320*I*a^2*e^(657/2*I*c) - 662103806898148887211572
021781565247253004285579497310960300184699919224146809856652759478738246439
20689185621817195640*I*a^2*e^(655/2*I*c) - 76418222219392315087683756278405
775283650676201572604553792705921311480435170616914405792951857748812649327
303490604640*I*a^2*e^(653/2*I*c) - 8745497333836502049304089127544702923753
201710508893865734103007014108158654308529720698354449239039386927199860211
3720*I*a^2*e^(651/2*I*c) - 992377526151663889268259748362999901775488277659
52176761643470192559974239763217770479232350108190016210097408947946880*I*a
^2*e^(649/2*I*c) - 11164938668316917590534038681743634758101204034737897911
3869960210757779770685735228549707336610684327510689931586511440*I*a^2*e^(6
47/2*I*c) - 124536920158622880437723357410492491627484725712145537940168633
192047594624217812945579030363502701898045319958355946560*I*a^2*e^(645/2*I*
c) - 1377113190775873365939497477092207939280825288312178453390578303806872
21501394694136422854469574552691588871302472423100*I*a^2*e^(643/2*I*c) - 15
094854551394889612855844062282545059911318282687737316013226386285074538345
6730881605090985571977587756201763874677760*I*a^2*e^(641/2*I*c) - 163992134
042938389389225651587309049983478009502030995092591343379847364505252550567
737506346576435363268511946307432740*I*a^2*e^(639/2*I*c) - 1765573409723450
001100507041214753260252600880238729620605966668210110542353053983023539212
34166369742208455804477035400*I*a^2*e^(637/2*I*c) - 18833686709375576328739
945629259157824152792473316476470287071136509178913254197230435214674077890
0143326958742782477380*I*a^2*e^(635/2*I*c) - 199008077323035875418810143012
602047032565549819093659928410024126213956651006647088382356775180926199506
332461163896160*I*a^2*e^(633/2*I*c) - 2082415599585260055775536545795398042
955567297668130242148540641707936567254815862795262873257635689066072904521

25497960*I*a^2*e^(631/2*I*c) - 21571078886203198613131476087661950972269766
 910280469634443752416588230208522044131401041455911531050563271304180056324
 0*I*a^2*e^(629/2*I*c) - 221102578382862669574055349501933571612854998614867
 953951230095286508289213573934339976617529581891276161128355770449340*I*a^2
 *e^(627/2*I*c) - 2241279579615789317292097509967964922214639196992668295477
 01698682085463657608421983129180622041022833070349554614032580*I*a^2*e^(625
 /2*I*c) - 22453304535759074004336020741906348037449644508622430697289895272
 7492000443367189384084387014008320007336848252442351700*I*a^2*e^(623/2*I*c)
 - 222109467960437280366020062870258143910978561940461423361930719843483296
 905763027899463977958166886104616267999904808020*I*a^2*e^(621/2*I*c) - 2167
 038733501891618892539109704110311947248476488359496569667286836725639856090
 61164258525127094147128116084916863495980*I*a^2*e^(619/2*I*c) - 20822608470
 830635809364977157946585243436376677295277795951915747954625254498251389737
 0413187324342062073067119188597020*I*a^2*e^(617/2*I*c) - 196655494112094356
 859837485549688216718971873808032539175218270546486750286660449996087951876
 167053363772436503198669200*I*a^2*e^(615/2*I*c) - 1820453461131019930253756
 175671474723107663576873344456198356608227726178728431122956616730672791643
 98356825168900855240*I*a^2*e^(613/2*I*c) - 16452464294988338678087961173942
 563021667855512013685642442293393096887487465485642255833021631746677184972
 6211997355920*I*a^2*e^(611/2*I*c) - 144297497750869524271910782851992127993
 955010068529181184808921877382505548298335144664374339743032217233080067477
 001300*I*a^2*e^(609/2*I*c) - 1216398686319338567720172921903320083448166139
 25479968028867572670823450147938585266247649652554944275677884270413328320*
 I*a^2*e^(607/2*I*c) - 96893719441596910044889117012440950291020439770151527
 577177611508808188500572872477073629458560345895007381079831424020*I*a^2*e^
 (605/2*I*c) - 7045876637421058822315902686122344533334175343996063222175099
 8997297048156150655074881354085103291330810095125378469360*I*a^2*e^(603/2*I
 *c) - 427820779616692776756408623303751106018195326863658191098253482650889
 13439588364246424948792319286916984200949647432740*I*a^2*e^(601/2*I*c) - 14
 345893481329936950121653593657325353238574049422623814263012969352846730780
 039535900084812106747735276310173764055960*I*a^2*e^(599/2*I*c) + 1434589348
 132993695012165359365732535323857404942262381426301296935284673078003953590
 0084812106747735276310173764055960*I*a^2*e^(597/2*I*c) + 427820779616692776
 756408623303751106018195326863658191098253482650889134395883642464249487923
 19286916984200949647432740*I*a^2*e^(595/2*I*c) + 70458766374210588223159026
 861223445333341753439960632221750998997297048156150655074881354085103291330
 810095125378469360*I*a^2*e^(593/2*I*c) + 9689371944159691004488911701244095
 029102043977015152757717761150880818850057287247707362945856034589500738107
 9831424020*I*a^2*e^(591/2*I*c) + 121639868631933856772017292190332008344816
 613925479968028867572670823450147938585266247649652554944275677884270413328
 320*I*a^2*e^(589/2*I*c) + 1442974977508695242719107828519921279939550100685
 29181184808921877382505548298335144664374339743032217233080067477001300*I*a
 ^2*e^(587/2*I*c) + 16452464294988338678087961173942563021667855512013685642
 4422933930968874874654856422558330216317466771849726211997355920*I*a^2*e^(5
 85/2*I*c) + 182045346113101993025375617567147472310766357687334445619835660

$822772617872843112295661673067279164398356825168900855240 * I * a^2 * e^{(583/2 * I * c)}$
 $+ 1966554941120943568598374855496882167189718738080325391752182705464867$
 $50286660449996087951876167053363772436503198669200 * I * a^2 * e^{(581/2 * I * c)}$
 $+ 20$
 $822608470830635809364977157946585243436376677295277795951915747954625254498$
 $2513897370413187324342062073067119188597020 * I * a^2 * e^{(579/2 * I * c)}$
 $+ 216703873$
 $350189161889253910970411031194724847648835949656966728683672563985609061164$
 $258525127094147128116084916863495980 * I * a^2 * e^{(577/2 * I * c)}$
 $+ 2221094679604372$
 $803660200628702581439109785619404614233619307198434832969057630278994639779$
 $58166886104616267999904808020 * I * a^2 * e^{(575/2 * I * c)}$
 $+ 22453304535759074004336$
 $020741906348037449644508622430697289895272749200044336718938408438701400832$
 $0007336848252442351700 * I * a^2 * e^{(573/2 * I * c)}$
 $+ 224127957961578931729209750996$
 $796492221463919699266829547701698682085463657608421983129180622041022833070$
 $349554614032580 * I * a^2 * e^{(571/2 * I * c)}$
 $+ 2211025783828626695740553495019335716$
 $128549986148679539512300952865082892135739343399766175295818912761611283557$
 $70449340 * I * a^2 * e^{(569/2 * I * c)}$
 $+ 21571078886203198613131476087661950972269766$
 $910280469634443752416588230208522044131401041455911531050563271304180056324$
 $0 * I * a^2 * e^{(567/2 * I * c)}$
 $+ 208241559958526005577553654579539804295556729766813$
 $024214854064170793656725481586279526287325763568906607290452125497960 * I * a^2$
 $* e^{(565/2 * I * c)}$
 $+ 1990080773230358754188101430126020470325655498190936599284$
 $10024126213956651006647088382356775180926199506332461163896160 * I * a^2 * e^{(563$
 $/2 * I * c)}$
 $+ 18833686709375576328739945629259157824152792473316476470287071136$
 $5091789132541972304352146740778900143326958742782477380 * I * a^2 * e^{(561/2 * I * c)}$
 $+ 176557340972345000110050704121475326025260088023872962060596666821011054$
 $235305398302353921234166369742208455804477035400 * I * a^2 * e^{(559/2 * I * c)}$
 $+ 1639$
 $921340429383893892256515873090499834780095020309950925913433798473645052525$
 $50567737506346576435363268511946307432740 * I * a^2 * e^{(557/2 * I * c)}$
 $+ 15094854551$
 $394889612855844062282545059911318282687737316013226386285074538345673088160$
 $5090985571977587756201763874677760 * I * a^2 * e^{(555/2 * I * c)}$
 $+ 137711319077587336$
 $593949747709220793928082528831217845339057830380687221501394694136422854469$
 $574552691588871302472423100 * I * a^2 * e^{(553/2 * I * c)}$
 $+ 1245369201586228804377233$
 $574104924916274847257121455379401686331920475946242178129455790303635027018$
 $98045319958355946560 * I * a^2 * e^{(551/2 * I * c)}$
 $+ 11164938668316917590534038681743$
 $634758101204034737897911386996021075777977068573522854970733661068432751068$
 $9931586511440 * I * a^2 * e^{(549/2 * I * c)}$
 $+ 992377526151663889268259748362999901775$
 $488277659521767616434701925599742397632177704792323501081900162100974089479$
 $46880 * I * a^2 * e^{(547/2 * I * c)}$
 $+ 87454973338365020493040891275447029237532017105$
 $088938657341030070141081586543085297206983544492390393869271998602113720 * I *$
 $a^2 * e^{(545/2 * I * c)}$
 $+ 7641822221939231508768375627840577528365067620157260455$
 $3792705921311480435170616914405792951857748812649327303490604640 * I * a^2 * e^{(5$
 $43/2 * I * c)}$
 $+ 662103806898148887211572021781565247253004285579497310960300184$
 $69991922414680985665275947873824643920689185621817195640 * I * a^2 * e^{(541/2 * I * c)}$
 $) + 56882511269373287870539195822437152004680104321850229396708926005012693$
 $843675803350945196463973920628412256283832232320 * I * a^2 * e^{(539/2 * I * c)}$
 $+ 4845$
 $708447056920620288359403005894683517227665184379926723479588483442249715570$
 $1461376425140347292769622287009253442200 * I * a^2 * e^{(537/2 * I * c)}$
 $+ 409317259082$

598900650106764128980599350021675414437149684224475067236578143624236160274
67796037956594249470207911160560*I*a^2*e^(535/2*I*c) + 34283257825099361469
629029646997987016357410393133337876860784218594231344941134240798399418976
200089374372937336277520*I*a^2*e^(533/2*I*c) + 2847182765222374854910670808
892533426745831036283384350780370695415572962319056303422780979482159828583
8730011875428360*I*a^2*e^(531/2*I*c) + 234449427531097206161909944823962633
447754673148656646339253777113436764791438014231317822869289826415849067306
52336320*I*a^2*e^(529/2*I*c) + 19141262538036155544818609953329709894938435
396042778174361103892567106894147006088532658059808481944657444464667813928
*I*a^2*e^(527/2*I*c) + 1549403407393788911890786046640647528672704004837778
9279507477424950714627734498672369684614416130884899948506496976480*I*a^2*e
(525/2*I*c) + 124340927124878766061808638396410072037820869420527197275324
88449954289915481768007300367716998430298025515215263240040*I*a^2*e^(523/2*
I*c) + 98923830014781352300325392897308808029309494839824134614658045878938
96576239270994398101763117672285400912303258185280*I*a^2*e^(521/2*I*c) + 78
019855370421567706442754190044232155698284276125669947440615954550718038386
72709040326328594306655699615422670941360*I*a^2*e^(519/2*I*c) + 60996612417
295200498779730183232995941499625082038419357626403994288775260998019269944
21670829050753700797133864249984*I*a^2*e^(517/2*I*c) + 47269450912532257092
847016202535295880273627739112671600945818608556165462265726371077234064797
54761335508976269914579*I*a^2*e^(515/2*I*c) + 36308362996060499012597116682
237835163595483156164260995438778934924614534786303889672637560395104155118
89625376389370*I*a^2*e^(513/2*I*c) + 27641415519131838232322405383114348616
980537825400877377787409587551952551974687942653319983004895297931041538369
38365*I*a^2*e^(511/2*I*c) + 20855325156223532897752971376843560445557178478
95905712020991677926353632697196135336156922652753946110662598182357060*I*a
^2*e^(509/2*I*c) + 15593792657467221576011166337705459056808452864661950680
47437750582100850218558934063441346166219284236292591756007869*I*a^2*e^(507
/2*I*c) + 11554182712615253647640998325560156956566717421483974307706219197
69591262069324891797346089718216295419176001175584602*I*a^2*e^(505/2*I*c) +
84830810258463636248416282574357231621057004122495206444018770310837379429
0161871693531289023220368566861588408740234*I*a^2*e^(503/2*I*c) + 617118907
415491882008893689992374986026058614595180636934596331413170379780487585714
587047788469214113451795516499370*I*a^2*e^(501/2*I*c) + 4447937565765568052
265985571218812558914206601789540056061908597679079003755266990872810797754
01935479298706556678625*I*a^2*e^(499/2*I*c) + 31761185459035984262486758829
969439709205129575301223433489664796692508320812892397865757385926141169147
4063180579773*I*a^2*e^(497/2*I*c) + 224675870137816967448574290906572012353
624572247889623771866556695527535849562092439004233144825309318376200907596
647*I*a^2*e^(495/2*I*c) + 1574386486831341464742255088228696345104075897898
80501589161153804500657734549235665125080321402913947303819424223977*I*a^2*
e^(493/2*I*c) + 10927856072355093095863814815237730501887441255995315083939
7105157171522052107943777228097015723239016040062510748533*I*a^2*e^(491/2*I
*c) + 751278263211159102200857958235109088145883877187557120298253868427887
04037764840837864988871197826414756226327584715*I*a^2*e^(489/2*I*c) + 51154

349273662615909216231190623364827611251722641045147869080165842930085677025
 925218103510278265118135145553584580*I*a^2*e^(487/2*I*c) + 3449482485656682
 228083350970147940299549936501441740704289262338429483788416879224070200558
 7755169596414970227728126*I*a^2*e^(485/2*I*c) + 230350339766078618487647778
 13625859138663747527131798877008816594078301950324404686986975822326579038
 21541105390532*I*a^2*e^(483/2*I*c) + 15232155197244469983477235120489934255
 399594332170569196782170361245875795753404131530065929326205229838601927955
 519*I*a^2*e^(481/2*I*c) + 9973455781668332100482619120779044796198884570518
 850362675620400800380525655955470978275300874201998244732697737400*I*a^2*e^
 (479/2*I*c) + 6465709656300067812786267514447106687902581433310919178038542
 426426780363215033938034518725151681639996365849163135*I*a^2*e^(477/2*I*c)
 + 4149994810365900522807468213278193091656141731116367453829137630709191482
 537488782374639033936237089896768763566876*I*a^2*e^(475/2*I*c) + 2637026704
 940057541717603416099047901577339805399244480403700320734657027382051540741
 428399462562160665232399986075*I*a^2*e^(473/2*I*c) + 1658792694072871479101
 869942544704973168924575943071103309570774086262029542461228753343283112620
 767567415651784874*I*a^2*e^(471/2*I*c) + 1032893390108623451662629921902059
 825520150456341148954819241245986827172746045153810766991498955787562750756
 646550*I*a^2*e^(469/2*I*c) + 6366196753590080176222160089204077247084401076
 29987848134367181043106780412514399115423605660760788697651634379005*I*a^2*
 e^(467/2*I*c) + 38836613077293651403504522610152872609997634306743502408489
 8656054932162774966074464370430510444616487043887402836*I*a^2*e^(465/2*I*c)
 + 234485020980412467341743199145352029468076352266379854608773522593799565
 958621176141472761686768653754135750141593*I*a^2*e^(463/2*I*c) + 1401126097
 156444901142996236202237326344616475127018069082403838786547453990878368588
 04711629400762515425955916824*I*a^2*e^(461/2*I*c) + 82852142966860851431611
 420117596372256466739448701897204460441426498632548900216968337411640986907
 458522921325945*I*a^2*e^(459/2*I*c) + 4848092066051426686753515724572870067
 245346536590134444783005329826730084909821476286421209270563913290579368174
 0*I*a^2*e^(457/2*I*c) + 280708048160262098107656410952750421279481280413894
 35629720485643024095406486924220542522448143563324518401964482*I*a^2*e^(455
 /2*I*c) + 16081708037117239364885424953083244936515724921299386778600225951
 179498131171589524219045138516435810850207340428*I*a^2*e^(453/2*I*c) + 9115
 474553683826281436142305910668402582581988856745851085574498026708677442826
 803383401266387813162062594525953*I*a^2*e^(451/2*I*c) + 5111787155098231966
 381444413998413769738039840629789986636135861377096437110535352261196444661
 635160446805767627*I*a^2*e^(449/2*I*c) + 2835889022716793311093267108913411
 484117430817040975928639654911368211514698112927408762873660302882359781905
 835*I*a^2*e^(447/2*I*c) + 1556343096968573764756475235411933016733786313107
 239004864800578666621277030123112055440836769200504575169913345*I*a^2*e^(44
 5/2*I*c) + 8448853220591619323689935171172081111839443557646321822612575689
 21304529568367117463318791743206771210437760631*I*a^2*e^(443/2*I*c) + 45367
 323239426704394960592256932998094054962699938259984350566231168625730196653
 7920360838324667729133482235583*I*a^2*e^(441/2*I*c) + 240945127276766227320
 339940089298232681048239144248789324968088048819421447003749816077955279624

100315116789806*I*a²*e^(439/2*I*c) + 1265608335829055341254571182834795882
54886734066422681322743067453094955423272892820673108496357191409969596590*
I*a²*e^(437/2*I*c) + 65745072618550512189873447315220858623931223310142529
839808121856850665718523376669691836042778846630737410930*I*a²*e<sup>(435/2*I*
c)</sup> + 3377428974797898142492673993116082479716111223736292561915956638757373
0808632464292717707940688773187398723895*I*a²*e^(433/2*I*c) + 171571319853
592779976394979501546747627791903510875678191956995177846663333833783366871
13858692204044077794176*I*a²*e^(431/2*I*c) + 86181649187889252039020595067
027687330042235445640864610695812088489764757607871317158396211091175474468
49991*I*a²*e^(429/2*I*c) + 42802840247288373381955793778304895251677711273
65155055905896472260407749270429246786706643678954884633163010*I*a²*e<sup>(427
/2*I*c)</sup> + 21018122641820657304118595829380026606224023537305815101696230599
60820290526673615762476646904104925062205465*I*a²*e^(425/2*I*c) + 10203624
683430752261319354737478213552900521524176464552201611022380324492309642791
15841386608493376970117760*I*a²*e^(423/2*I*c) + 48969955715529867639855570
669017010328896220158706203845704778330928089800701646269532551986677634966
8712200*I*a²*e^(421/2*I*c) + 232324417197235062081022565235959973329129704
016512511492684407228447554005362475531181177257375428158576672*I*a²*e<sup>(41
9/2*I*c)</sup> + 1089494105060243829214205754738692285159428724968605056207685730
56175175693027104049195507535238812688471740*I*a²*e^(417/2*I*c) + 50500362
504156859478353840472838257985720430216874883086230302896996886794997397859
935740848410975488426320*I*a²*e^(415/2*I*c) + 2313545755566724201510956269
672475926944214721668739748981948907194169694664526286940057640199200192829
6540*I*a²*e^(413/2*I*c) + 104748971102058073678463750667233366222445365231
76561176217245839587779126630391482195110511043224435073120*I*a²*e<sup>(411/2*
I*c)</sup> + 46868738731561018711907679745615986095134436702017574609142323125054
84627209871957976104328771649162041900*I*a²*e^(409/2*I*c) + 20722969684727
276385823667632148093501310361797338948038816018678673189003602350902081524
23869380817193240*I*a²*e^(407/2*I*c) + 90537596448029304322156871761722280
4352981478608148290290910134804446811145030581376444150025327895167080*I*a<sup>^
2</sup>*e^(405/2*I*c) + 390829013458478713762005868531749550511348363755402425375
475219863201042469027671720992324263291992741380*I*a²*e^(403/2*I*c) + 1666
854541786758673088478816430259717371256882203030379686823499256790893313121
74778117366039026285869440*I*a²*e^(401/2*I*c) + 70231742836691486232401821
435646690204765370170934364279337989127944150596825723542271716872404327490
260*I*a²*e^(399/2*I*c) + 2923245784778434719423275587068639931517180934624
834659494649731694453335619977722113539614776201632080*I*a²*e^(397/2*I*c)
+ 120188852274055085277789182447130770488546641340585826414101184645320012
37995012608936579451133542506420*I*a²*e^(395/2*I*c) + 48809048126294925781
827497042657233583761668673480039271037026838448709301646404463264142624663
77824000*I*a²*e^(393/2*I*c) + 19576884771781648238441845365656477945755329
26755467360055820937851704595970877608419045482885065913720*I*a²*e<sup>(391/2*
I*c)</sup> + 77546675678005380330048742944775088823732579762475791970674462847820
3937255833604141877857656968392480*I*a²*e^(389/2*I*c) + 303338706350755487
279057170882513474743360201200362327954737042233540329160364416313115387680

$520970550 * I * a^2 * e^{(387/2 * I * c)} + 1171670844831237108639698079387721551699567$
 $85317523090174862697344141293246236717353854714751253177160 * I * a^2 * e^{(385/2 * I * c)}$
 $+ 44685235946986673791925225525992813797485854939304559125279366181549$
 $713351572630218142149059140721370 * I * a^2 * e^{(383/2 * I * c)} + 1682557767500965341$
 $065281299672359812048347017537533200125096401555512523687358138746822492699$
 $8037020 * I * a^2 * e^{(381/2 * I * c)} + 625445695771085708499077750788019679582604834$
 $2877391310115506167680233024354089893072681406475246730 * I * a^2 * e^{(379/2 * I * c)}$
 $+ 229503041901714401511079773215537534647733907953257500658067586947885832$
 $1228765847919467823502616280 * I * a^2 * e^{(377/2 * I * c)} + 831249333124270929749565$
 $123370219357165654568742620039903429231671547436113371386430716266759961060$
 $* I * a^2 * e^{(375/2 * I * c)} + 2971544784433528472987442935363735703676312583731972$
 $90869967778261660884898898610815793932187264900 * I * a^2 * e^{(373/2 * I * c)} + 10483$
 $460908657262590728934622589025242928356504146032385140279584005194048323486$
 $6884215706838405390 * I * a^2 * e^{(371/2 * I * c)} + 364973263533159636054202981906874$
 $65001055100758074304234792272161886339957870164613698417616283210 * I * a^2 * e^{(369/2 * I * c)}$
 $+ 12537562836732599179767010355152461056807191951005009796451823$
 $391079000655762189067741192305394090 * I * a^2 * e^{(367/2 * I * c)} + 4249342990738608$
 $701078109187168768367693341231356030118163718909774173116310218460585993016$
 $680850 * I * a^2 * e^{(365/2 * I * c)} + 1420846196643578803733575330926970974464675704$
 $020481618027446077982380731097941167611657029967430 * I * a^2 * e^{(363/2 * I * c)} + 4$
 $686494338769403747164153345408520227564196716852908191265362222023833292306$
 $75172080843145521190 * I * a^2 * e^{(361/2 * I * c)} + 15246953324765761349401422316907$
 $9603947607129031097239090335734676330177814841868201155345866600 * I * a^2 * e^{(359/2 * I * c)}$
 $+ 489225665813288279962433692883462704693078596446648027300850105$
 $97182346996542663202606651415940 * I * a^2 * e^{(357/2 * I * c)} + 15480423340995881584$
 $740416023332877545588024374995660159497444149878426588554491651581857867240$
 $* I * a^2 * e^{(355/2 * I * c)} + 4830133064761265676098823275890322270506766918770002$
 $889169425396499013319534724628237025832970 * I * a^2 * e^{(353/2 * I * c)} + 1485908894$
 $655409129727579697390217111877691246727000929093187863829367925384373668136$
 $922959680 * I * a^2 * e^{(351/2 * I * c)} + 4506456258037388829608858710305661439531089$
 $88238530005053615795799976664205719692805608636970 * I * a^2 * e^{(349/2 * I * c)} + 13$
 $472244550279277538953701338689137282615560362808524038395939241836148788484$
 $0921793997982680 * I * a^2 * e^{(347/2 * I * c)} + 396969673391646744846986059254148898$
 $36548997086240408750738793856030209166162894182626765170 * I * a^2 * e^{(345/2 * I * c)}$
 $+ 11527544710214127291959320966284738345019810955968385265683600912288138$
 $678711188667077969580 * I * a^2 * e^{(343/2 * I * c)} + 3298577503461372465605345536869$
 $123261456392605454980840206045785661659654642478147158578260 * I * a^2 * e^{(341/2 * I * c)}$
 $+ 9299807036336009523216256929466449526390442106514688351217979660885$
 $02026440103109950142510 * I * a^2 * e^{(339/2 * I * c)} + 25829986186907031921553951148$
 $6716381312735629658913082894934911762706535490482149535095400 * I * a^2 * e^{(337/2 * I * c)}$
 $+ 706678091924968389655322817331198722866713035743737606295549343337$
 $98898169280995448949110 * I * a^2 * e^{(335/2 * I * c)} + 19041847118319726293265803273$
 $951207945593799588235034178026656550799568684461605603777920 * I * a^2 * e^{(333/2 * I * c)}$
 $+ 5052749693090074250020711633864893162784543894410325996308296849332$
 $368152124619583542550 * I * a^2 * e^{(331/2 * I * c)} + 1320132342599574776582487491686$

376011020731128338564644972136377885389653577953327654360*I*a^2*e^(329/2*I*c) + 3395597687106525970132828888173129506684379492517210783951942750593256
 92430095887234940*I*a^2*e^(327/2*I*c) + 85972681691391354979713197091334370
 055201000372937152633172947601816313260199706979160*I*a^2*e^(325/2*I*c) + 2
 142322637573414749329411197982703704059061442657481751124150172303756732526
 3745727850*I*a^2*e^(323/2*I*c) + 525318021890119037052330796129201500498936
 0145697116713826069492059896349225432472890*I*a^2*e^(321/2*I*c) + 126737087
 561381648317237671513461191515502353237594752736881659052212108959184119587
 0*I*a^2*e^(319/2*I*c) + 300786380045287143628954499629136562193246097877509
 863055826252064326569743630828470*I*a^2*e^(317/2*I*c) + 7021220630480167833
 2513345599910949246264802924017064833024979917894264890884744230*I*a^2*e^(3
 15/2*I*c) + 161172874479982061427980838947631758408488571143435430472421389
 53088137509988748530*I*a^2*e^(313/2*I*c) + 36376325249000079612496460250249
 70707837559612619597442737493322095743154754032540*I*a^2*e^(311/2*I*c) + 80
 707446054953921000355859651553212989790297514032107094131550546603063328950
 4700*I*a^2*e^(309/2*I*c) + 175992541401464872591758301499834898740699910060
 781393774843202513062630336753720*I*a^2*e^(307/2*I*c) + 3771171401016163285
 1275194873586231922967566451955409746843374416439997598272230*I*a^2*e^(305/
 2*I*c) + 793912239209648981957602336578090909591097204342621477154578124683
 2910820414100*I*a^2*e^(303/2*I*c) + 164169729637814329912873364094099880544
 1384198489236819275463423360050416710230*I*a^2*e^(301/2*I*c) + 333384441552
 175348575863851685280079397515408127201392843010509098144064832520*I*a^2*e^
 (299/2*I*c) + 6647120599918684811789007266219302132767738777093441481272699
 7704844663612890*I*a^2*e^(297/2*I*c) + 130094515202180544279877269093063880
 26728218430095197151626190204529199816160*I*a^2*e^(295/2*I*c) + 24987313770
 3119593902009900676775607036784147188475889323331541072591988520*I*a^2*e^(
 293/2*I*c) + 47087957289932945298631625353486851499258535581271904808501238
 9615234670720*I*a^2*e^(291/2*I*c) + 870403827362412450481131355289174357725
 31399861517048611706176997728840060*I*a^2*e^(289/2*I*c) + 15777551334929884
 741558980231119513445377646781839885604883296740586671600*I*a^2*e^(287/2*I*
 c) + 2803816222757253133472672279792586220272029806438905494766819989517225
 180*I*a^2*e^(285/2*I*c) + 4883483643690059612618015028085424032262148486822
 95131186689348564834560*I*a^2*e^(283/2*I*c) + 83340276962100363964566640495
 864516393854803017236184470442989394035180*I*a^2*e^(281/2*I*c) + 1393141779
 2509198161468797423752544838678599539275212332380442651357560*I*a^2*e^(279/
 2*I*c) + 228042712007813771932551384787426514801954185403148684855824844272
 7176*I*a^2*e^(277/2*I*c) + 365407793636196973379878926946921993102971243732
 578054647123414214820*I*a^2*e^(275/2*I*c) + 5729740839107832442584422071169
 6016060458004240084890338934689925280*I*a^2*e^(273/2*I*c) + 878894722176403
 3524446888793703125344784094679194612662969732267700*I*a^2*e^(271/2*I*c) +
 1318337340904646289197961886292345009401456618543410019867356069360*I*a^2*e
 ^ (269/2*I*c) + 193303982244781919691500896347164102540863253211817387366117
 379028*I*a^2*e^(267/2*I*c) + 2769552249198733612229785465627906770414393257
 9317971697894652128*I*a^2*e^(265/2*I*c) + 387574875295375725139232180409811
 0389738719224033802668294149720*I*a^2*e^(263/2*I*c) + 529534303804314371317

279287031587399342898319070257744555877120*I*a^2*e^(261/2*I*c) + 7060444149
 8160359313241260895971702149956086380303936468404945*I*a^2*e^(259/2*I*c) +
 9182643580975122394482019949557981111651613557125350856577098*I*a^2*e^(257/
 2*I*c) + 1164368139316323606713195496495533903452490627065569725880879*I*a^
 2*e^(255/2*I*c) + 143872897869958476711841808368973418609254139596808263965
 208*I*a^2*e^(253/2*I*c) + 1731419855983647541565139336513364245374510046106
 2396834495*I*a^2*e^(251/2*I*c) + 202823283552635430098336989282097008142353
 1403040583023770*I*a^2*e^(249/2*I*c) + 231137463934351535240227752765265592
 838813655877791732766*I*a^2*e^(247/2*I*c) + 2560896322478529157228795203877
 0777479118266594544417374*I*a^2*e^(245/2*I*c) + 275677067277424856755018403
 1805577107703843954010093519*I*a^2*e^(243/2*I*c) + 288136908346804733688436
 834384090051687033252355064191*I*a^2*e^(241/2*I*c) + 2921950553161713974599
 5198456026235957224788107224865*I*a^2*e^(239/2*I*c) + 287270320925440168071
 8347285883520097429919909434035*I*a^2*e^(237/2*I*c) + 273590703261713668448
 365859396559199738283337615067*I*a^2*e^(235/2*I*c) + 2521924874282494815754
 7271484589255811816930894969*I*a^2*e^(233/2*I*c) + 224795487509634223508139
 1843579770658847258731308*I*a^2*e^(231/2*I*c) + 193573860526473423179001809
 134869735140502053410*I*a^2*e^(229/2*I*c) + 1608646826432365541347574730458
 5608493929925740*I*a^2*e^(227/2*I*c) + 128869483230185418749692569938588091
 2905767417*I*a^2*e^(225/2*I*c) + 994034494896144155893242613267830157826542
 00*I*a^2*e^(223/2*I*c) + 7373332278938477877109516190844163180831833*I*a^2*
 e^(221/2*I*c) + 525223640233639829417248502076803689088820*I*a^2*e^(219/2*I
 *c) + 35875930157695641006598626271371876133245*I*a^2*e^(217/2*I*c) + 23461
 09795219082040306865029327652810422*I*a^2*e^(215/2*I*c) + 14663185841641185
 8207561193721288320906*I*a^2*e^(213/2*I*c) + 874227865046303006930100363723
 2429883*I*a^2*e^(211/2*I*c) + 496183375602453279107611685390767740*I*a^2*e^
 (209/2*I*c) + 26748429663467970647815785552181215*I*a^2*e^(207/2*I*c) + 136
 6183224856297865521217009020824*I*a^2*e^(205/2*I*c) + 659284126316477394616
 83457820511*I*a^2*e^(203/2*I*c) + 2996746017244157830350053701476*I*a^2*e^(
 201/2*I*c) + 127861163072229101394013407134*I*a^2*e^(199/2*I*c) + 510084426
 2877818375740499940*I*a^2*e^(197/2*I*c) + 189421272159473056494074595*I*a^2
 *e^(195/2*I*c) + 6514488191762735815405157*I*a^2*e^(193/2*I*c) + 2062634782
 91032388361681*I*a^2*e^(191/2*I*c) + 5970784896718122444327*I*a^2*e^(189/2*
 I*c) + 156713514334349191205*I*a^2*e^(187/2*I*c) + 3692203217145049425*I*a^
 2*e^(185/2*I*c) + 77121738215926170*I*a^2*e^(183/2*I*c) + 1405865019555002*
 I*a^2*e^(181/2*I*c) + 21909584720322*I*a^2*e^(179/2*I*c) + 283802910885*I*a
 ^2*e^(177/2*I*c) + 2933363420*I*a^2*e^(175/2*I*c) + 22680645*I*a^2*e^(173/2
 *I*c) + 116610*I*a^2*e^(171/2*I*c) + 299*I*a^2*e^(169/2*I*c))/(e^(517*I*c)
 + 418*e^(516*I*c) + 87153*e^(515*I*c) + 12085216*e^(514*I*c) + 1253841160*e
 ^513*I*c) + 103818048048*e^(512*I*c) + 7146142307307*e^(511*I*c) + 4206015
 18659718*e^(510*I*c) + 21608403021340047*e^(509*I*c) + 984382804329835768*e
 ^508*I*c) + 40261256699368950388*e^(507*I*c) + 1493326612293984160368*e^(5
 06*I*c) + 50648660944512569972179*e^(505*I*c) + 1581796642397812408161814*e
 ^504*I*c) + 45759117183402579073139583*e^(503*I*c) + 123244555734683224517
 6696904*e^(502*I*c) + 31042222522074681615625020522*e^(501*I*c) + 734057263

616388449968842366924*e^(500*I*c) + 16353164647151530240529137618111*e^(499*I*c) + 344277152012875134140739302960914*e^(498*I*c) + 6868329225263681349501997341320517*e^(497*I*c) + 130171193079172823835151430773360024*e^(496*I*c) + 2348998374244347079532766203075607598*e^(495*I*c) + 40443624781415311581857832389099634564*e^(494*I*c) + 665634670676210063754191847109971141414*e^(493*I*c) + 10490402669510897424624643766470754045064*e^(492*I*c) + 158566476113257562566117432227203884298856*e^(491*I*c) + 2302150411226234925855222345201500900533576*e^(490*I*c) + 32147887693375338817454482515377350383950278*e^(489*I*c) + 432333688644261557547944179250800440604964868*e^(488*I*c) + 5605927253067558551780452883689835514455118670*e^(487*I*c) + 70164515322544462906873548813748091084561870680*e^(486*I*c) + 848552202276512356496200136959676295361696315113*e^(485*I*c) + 9925490738534402272939987038714580495445431374618*e^(484*I*c) + 112391604542246650966429162063124338952554575234051*e^(483*I*c) + 1233096700139723365181997220750932590655287625342156*e^(482*I*c) + 13118781801172174729679339894318153694964675368481194*e^(481*I*c) + 135442594916636116191574650625331646238501101627937224*e^(480*I*c) + 1357990663161479842850642848032544982878359839580349899*e^(479*I*c) + 13231708870104896973800056733779919089340836756009580718*e^(478*I*c) + 125370496586921272662198050851269323171167338854081782959*e^(477*I*c) + 1155855412893594260345544966642687823630035899363232371472*e^(476*I*c) + 10375184499871175501909398956596684116802997082526660323524*e^(475*I*c) + 90722605722208814918642284639487187764607589706493970774776*e^(474*I*c) + 773204636991145775061462731028098506094432675788136295011259*e^(473*I*c) + 6426195485535248576425068136870465530087114003875716691383902*e^(472*I*c) + 52108117629177048660492400985175830987505700566877818954141639*e^(471*I*c) + 412430698299915190848067222327219435067747934091894670488982928*e^(470*I*c) + 3187749929744346497211536044751776582320958627923816470590659024*e^(469*I*c) + 24070801913529757101858022914372045864746991786182039740274325264*e^(468*I*c) + 177642829135119348577194437675802830239905460092687136494961404333*e^(467*I*c) + 1281817464914970810859604189828359000790789921169405304612211251818*e^(466*I*c) + 9046693523825682979044338963104263167672586826367911338826483549173*e^(465*I*c) + 62473550781053295317710774690247114124125187565731848441781904032672*e^(464*I*c) + 422276126632003687547754746555709988710527133086660161366353656787288*e^(463*I*c) + 2794709104475686611842790694973699164482254723977210209725661304403472*e^(462*I*c) + 18115768495615758076710303055505625589254293659193314153418333944596408*e^(461*I*c) + 115051481852080848873700388354521315567640365124003103691176697194292320*e^(460*I*c) + 716099497599058079895633338552940229192858196481597830078819711862600096*e^(459*I*c) + 4369442482910113914565353136069595862669338858053419381214131241925047008*e^(458*I*c) + 26143976279902021443471945665080254563056810183520401889800285493144867448*e^(457*I*c) + 153436088745056254127327239461577071933130157764595997113973513183188399376*e^(456*I*c) + 883500968821791202600774541927769200737689393513734789368397093333311961880*e^(455*I*c) + 4992519712457043983505377976607953988397368297591114957991804893688371867680*e^(454*I*c) + 27693116538343259225983382637647936122664033859615133489846664694361471

028310*e^(453*I*c) + 150822381431412413773566474210011746852297437597059186
 295243989481140398152780*e^(452*I*c) + 806679543607589140759305010796189568
 269842021613388955218916278823182639488190*e^(451*I*c) + 423812584676323258
 6394188569858685826755328005548627437019301405851325887594480*e^(450*I*c) +
 21876482892713909928040345612578705805121508756226696317087651824252241418
 663320*e^(449*I*c) + 110969199687320974749922259595250444341219218535349655
 762591192576535872151766080*e^(448*I*c) + 553269128819528612502918869558947
 829098021956309349843584044631512291778800081490*e^(447*I*c) + 271184323967
 0717527605640490148833507130242448403978318523237721944200392830108580*e^(4
 46*I*c) + 13069817203488289886193205508375818392124991382340160316886507181
 296548981014818410*e^(445*I*c) + 619485966530355028795643388152343106604109
 02037882473161804774492916216575880077680*e^(444*I*c) + 2888207552647306544
 69968572021047109427318619508995802020689904590319476295408324280*e^(443*I*
 c) + 1324756412367837473157472821162483691120966501948953926492241643788264
 284546437221120*e^(442*I*c) + 597899217294414321845916114929981970632173211
 1578494525245228742976468409105395536290*e^(441*I*c) + 26556806389043407534
 496702369101545795994861757741414789944652712127566910185274123140*e^(440*I
 *c) + 116104551683555043762911501712116399313733021132677481112824047246361
 794049635726479850*e^(439*I*c) + 499707567253859084357596314813794768069337
 190915967491907488904933922677579665354338960*e^(438*I*c) + 211758973346685
 5707101501429210414722401838837940752841618541440888545729943138209036820*e
 ^ (437*I*c) + 88367206408604703056945140215479695512967940922669830441183757
 90025854584036796364768280*e^(436*I*c) + 3631836965230259173219744440979812
 2022640824604130552506742586795183267354382847875885730*e^(435*I*c) + 14703
 081673227683316304158209959204751204372522535333923881916519300040762954474
 5753221740*e^(434*I*c) + 58640346697268324274164332892156090937519745386424
 3299571990964608857245771134145204174990*e^(433*I*c) + 23043510733738403573
 79178597673066352016682781689139842097376663118488803841131935313641840*e^(
 432*I*c) + 8923209447343296763331881881638471793499618670601026059730895962
 653291770229493028162575100*e^(431*I*c) + 340540538512955691543523467221771
 72655187548910782008504718324168725029438589162349211628040*e^(430*I*c) + 1
 280989146016885396724805418304098477073675004386015368032044977011199112890
 87105659482783340*e^(429*I*c) + 4750105788576015192723166179384252224217865
 97241671026894318515408511467140969393115768793680*e^(428*I*c) + 1736574218
 818191071874197472450158123883564209950658639102337148122769080611680719741
 726053840*e^(427*I*c) + 625987215682225284365096070823503471020136277605717
 6647226323089751446565288850103898153859920*e^(426*I*c) + 22251959176795777
 757167366036007480222211364232146399803864370963391491223687245823457351580
 140*e^(425*I*c) + 780098073680242398756137330588514171253271146810708896407
 94249282633470580756557083923203377160*e^(424*I*c) + 2697458014402112969726
 83601863878954357962308520076595177128227629273240215209708218497363414140*
 e^(423*I*c) + 9200893930295890328746018500271593226125263684447714897819743
 61078847528891468831038436064951920*e^(422*I*c) + 3096131971621520162380301
 554241465451782362086810287537748902904985934020179565706177131421614590*e^
 (421*I*c) + 102793647306638408447395778624692626046488619142979725891652435

30651230690726244462479199894255180*e^(420*I*c) + 3367539887202156837590238
4593982753362559801058104184627345411136262431943240778260721756991027090*e
^(419*I*c) + 10886799573182947282673290519203488679728462135644562753090910
4429486741257822633476898356826454040*e^(418*I*c) + 34735147321471378087435
208312956660123876576277594236676273334995210388975398263640385755686777730
0*e^(417*I*c) + 10938532144862203586740324345008666784997700113058741724889
75951612031456734608287095519501041975440*e^(416*I*c) + 3400232560601651617
521694680847089844198028831694417424794868779328950548418125605446882081152
636090*e^(415*I*c) + 104341175165703959666536931555824021094603480954730278
07412321427346816928567197770376496170251803940*e^(414*I*c) + 3161093933128
469275069430644361841465609596952094521574300404456038689524180157915654345
1940713351730*e^(413*I*c) + 94556180258931986919334303466365652826858091314
329189160736277175873841732196453379953705679466826880*e^(412*I*c) + 279285
755800035206679835368898165477644864987794665387827488933863633745047373109
049265172681702585720*e^(411*I*c) + 814608187736530579670210025271921415597
183369881214299823291969785549876175969866367976653244974728560*e^(410*I*c)
+ 234651821923910514223814163307346476889915570893502577804763741268178157
5765422219127409260159438712250*e^(409*I*c) + 66758662903711473585037668656
69289010893543869830538708724945291580951179188296606158111257706968604740*
e^(408*I*c) + 1875998821886556356416363573598607327825573725740570627910889
1366378428467414559930481172863538598193890*e^(407*I*c) + 52075178518793270
386429263351544306951104993542500582938155241689408138675254608030847907167
748571734720*e^(406*I*c) + 142801792450221762483180874918825274134305133275
417780084795034644763509333503150517345864659667189417080*e^(405*I*c) + 386
876218234277165632451723049979889263115282374607541692443176673997513742813
591736171169652250611186480*e^(404*I*c) + 103556198259200293522638457790861
1548612111495080193573691339864706029186482466241805664949381049856258510*e
^(403*I*c) + 27388956247952656033552276465660008862807783050848257029119389
03656162004262736182657700406301914070062380*e^(402*I*c) + 7158124686842941
475473807363679839718172745581538409044503383852693596921622426696740453944
718143025248390*e^(401*I*c) + 184874052990057326937527286118764908908583570
21974882371570623800186245137722660943641752976852924439870880*e^(400*I*c)
+ 4718822084346620769509950695357378035710889749142256789804819901820770899
7005333860148836479527456156014520*e^(399*I*c) + 11904185540387796494822957
794837046560060662318304552952690043020927047321277384779493558607471432947
9939280*e^(398*I*c) + 29682551528266958968531827328023905008455503220341594
1511962659596881615713799937680026497408305672297618840*e^(397*I*c) + 73158
497220681836287472962140397444428001044630116152733976054481530095178798553
8419764656214582667219914080*e^(396*I*c) + 17824461149317518505563548566384
219011744123222982494965916580539397871982465659459755955757341933488879521
60*e^(395*I*c) + 4293206478008022126017488908851826494790620720660151451468
181910917240027863968724539127659633517053002976480*e^(394*I*c) + 102231820
259548607672173903051864519235621454736742936199180634904114874961218045902
74592702770571515456414680*e^(393*I*c) + 2406878513970527716119346564450614
328524136103776821681892218440014104846021094469664775272337193287459459732

$8 * e^{(392 * I * c)} + 56028683424903517658495013858534516167162591034367972498174$
 $660907450666778154353271630344650777885683547624184 * e^{(391 * I * c)} + 128967080$
 $084754712246023680866488384983286259025533132044636109049545144029547003347$
 $761521666283977931640178464 * e^{(390 * I * c)} + 293550743554342709808129453576562$
 $313299705982699187416862934373964255615967138676253276302591561523515603264$
 $403 * e^{(389 * I * c)} + 660764473105869097691475973850837934511089033149586707982$
 $764263394766756649565279879146173318386505740391093990 * e^{(388 * I * c)} + 147093$
 $114661893434551503836230010016048212774958144392990474691022477747019889905$
 $2379114493999887003199419829579 * e^{(387 * I * c)} + 32384919313618514764233219335$
 $395790983777355392076414673462356658238870483269493056092315851437486902036$
 $15957136 * e^{(386 * I * c)} + 7052132414162197992602326524580143060985353054572933$
 $905524633121681021037340298366342203324325307072413739061024 * e^{(385 * I * c)} +$
 $151896342149088003964179117226437547480485201097348124591098788104938443810$
 $62650818971199637121458749456243274416 * e^{(384 * I * c)} + 3236273132241954941033$
 $008894364024746037832856131642293129242714590288791307164367950290905589123$
 $6755143207382609 * e^{(383 * I * c)} + 68208033096793615683784409619244210818614991$
 $640041553424405527876893272496608324231098148502466453967157728078994 * e^{(38$
 $2 * I * c)} + 142213115964814517682386667276769909482271681318790889840501039441$
 $748635545362467679832449103520321953011780083069 * e^{(381 * I * c)} + 293344920034$
 $300720287042383448342866313806285455040067823080445597545970023446231563554$
 $135133105493516316320059272 * e^{(380 * I * c)} + 598650141112241858911676505180520$
 $150364003226841328081453597093587790338609212439085554466861582623350303061$
 $961052 * e^{(379 * I * c)} + 120877035849365839308944222205693506328370410814059375$
 $0226539846117737648216609559734831601248698274330296158612144 * e^{(378 * I * c)} +$
 $24149665168103385032890765492027405117100590117954471387734642056964550264$
 $42712426409599662771080264826008985061097 * e^{(377 * I * c)} + 4774141111066098970$
 $221845330594962016472714230374234060663956846950926642685946929064114194400$
 $360936223590725470146 * e^{(376 * I * c)} + 933934195805349422525175096571505730070$
 $730208381477477030621822424102264824741995604295736305582383089854730321975$
 $7 * e^{(375 * I * c)} + 18079820068028859970349938623007230676563314206708848499900$
 $139641237334763266479346963237936039328113185041591793848 * e^{(374 * I * c)} + 346$
 $376571726716901676573445371970870488823548539932704720639430787736004465429$
 $63548348101269390443464480754513928502 * e^{(373 * I * c)} + 6567485926886730009882$
 $737581287522561065455168626110368166400700753711577809729353356524382887338$
 $3722980353200611956 * e^{(372 * I * c)} + 12324394151933238474196007258810350659640$
 $633925361639108206296996068241901174577573892181775339195446260932388148915$
 $7 * e^{(371 * I * c)} + 22891131173859278009149264916234683440586774077645632610841$
 $0928857257174707289268074347550225793244741923354395308214 * e^{(370 * I * c)} + 42$
 $084634260894938727755902145792458657812096614856102264700849952946845200598$
 $0175119410628956210497609566002969884927 * e^{(369 * I * c)} + 76586779551396278101$
 $255844462875141871094089528130479083674366158207165003215489148286640631483$
 $4433199455459798934952 * e^{(368 * I * c)} + 13796765297962120740171061880665894483$
 $554465012108901951071648603502289285868155390030628750267119319419477386903$
 $60722 * e^{(367 * I * c)} + 2460442375845422663927081630983260714734968091905493027$
 $145639238827192254886349361126991457692409851120873307487457468 * e^{(366 * I * c)}$

+ 434390969660193217335735968778157929370129568194082711421543317533609396
7845908766740738240037114570667410936998017178*e^(365*I*c) + 75927527001466
789611530950735850154731970297465336333315497939614732857609358019041551167
64831560875947581048693527224*e^(364*I*c) + 1313977149410493388185668115141
829311224255152153568687118126657981387760634816026174720131773578256602130
6798298336024*e^(363*I*c) + 22514675741308069961506165586502872430421930210
673264392997286485600640103867253604847715547060592967690653795951142520*e^
(362*I*c) + 381990158675860879760029987566276749947954406679036250293223462
50133286489120875005013638128113893960349670280707161530*e^(361*I*c) + 6417
510069326006680623806488600459717074084330008683936861613916452910804984467
5353111842725798658088840347241496099644*e^(360*I*c) + 10676483201716559483
808523418933352873358767332997253009266108518678993925291593709076028223234
6919090426243399409323314*e^(359*I*c) + 17589625826275598575710681261397930
126580103159548435361490467286516944223207577658044718413414137599577009149
9246759528*e^(358*I*c) + 28699294363123149655727801085157694089682649746606
6327528801560677007112837431926735088120974861760511367008815728782643*e^(3
57*I*c) + 46375828845736715454493767825500568873332814556804931042399559988
6012800638619904022368378591108842602342094543682299102*e^(356*I*c) + 74222
864090817312491693704946252561733414891967911827048983100549778195122106995
5839623452499748653124658873553401442137*e^(355*I*c) + 11766007209757869651
898750508902310922046126969702774330145358957889567712307935203819931066068
80564628599822341722801012*e^(354*I*c) + 1847505856462451533445284300571326
323781162553304565971887670758091079306794821834928170773126364639722071570
131703785334*e^(353*I*c) + 287361053592234018708083543558291227727196797739
4720159791070274927714276869531467182688981041061381703885403497544001592*e
^(352*I*c) + 44276730791054253185243161129856936565848519361001924570444551
34483305045321452516347118488133224823670465103483954805161*e^(351*I*c) + 6
758480437888524372562935948963857626694855547195519486122877567981718587262
362871994967079401831957927901682582941234362*e^(350*I*c) + 102204237794346
348513399752951633996417021222496636661930530083020260969321585683383094182
37395541351819026907953220681013*e^(349*I*c) + 1531283720666277537934735321
280768296571253565294263151828614240309773820027071119539658215902851353277
9682154451996208592*e^(348*I*c) + 22731603566128841100419501947051367666836
652418077260913944810748473084891890410181285412604854876625919565639521227
223276*e^(347*I*c) + 334358978279365813011711754596108294542981679620174198
10072936733378506584428024201072453193458155334046693516742390717832*e^(346
*I*c) + 4873325350597492340085225556305210140219646931365955449272567475433
9375283010407167744366955828922837488705858532439654489*e^(345*I*c) + 70386
349760594831567048224061395025698501202296966300376764220336697702961591099
854055411376294871437468149528524796002762*e^(344*I*c) + 100744961851853744
611754300982980166962404553836222921868484694269966120607698907046343731011
160948828100276729370132819357*e^(343*I*c) + 142906319123055524246546928478
954238371315925802022389236498652136839822502035155676970917419039834587967
055588431566416784*e^(342*I*c) + 200906587153578804380300469501441610174521
851259541929209840688960859454908519774835905895757666770857888611738751858

460424*e^(341*I*c) + 279945244475039804822966730462960884492119874857791147
124009079476920435941735293309305430438687333129912454196774070107264*e^(34
0*I*c) + 386642673050380049457382562818316962651975550990779277048740238629
858795018247356162888631015687664780101205287333082748791*e^(339*I*c) + 529
329252764113926003934836958243557672549238997560739214406599185047831955572
583765358634395408771528009745467548382950094*e^(338*I*c) + 718361596382058
249209113544487901088868388744033713210332491971375906738341551540457264804
304039664255915607349801911966551*e^(337*I*c) + 966458275369037718747739130
798151643483590684166832234688098291164160636418159452119815728809372125168
836239364442397344064*e^(336*I*c) + 128904351529293395648063433049967704018
104393562010691426731106790003005839883978769237695409054527855454499771005
8754772400*e^(335*I*c) + 17045829967078228082046782181676930026986611477127
72355021456543810930069637188085882824757500605246963210810351706405349408*
e^(334*I*c) + 2234891276398439464478622578306434840724610484468177859822620
658691921478645266653062563823553001228001009093606751066168944*e^(333*I*c)
+ 290538572232005700195334527448948279085669252995982374953269596341416483
3366773128218607899328588608916176593772088622582464*e^(332*I*c) + 37452575
948766512046573349884262263881439545019868306642223492263610796095468222760
67504899386703088982308185717143407211328*e^(331*I*c) + 4787527442780945685
145204846971596165304169419328244073211459592129649255048876854059844720661
078151288179612574986359194560*e^(330*I*c) + 606894980315671224833187110532
989547172280614300887801498655965368726069481655047019589000451196552756743
2722969707577202160*e^(329*I*c) + 76297318156278215804689924242070083664388
967363330246618638381051104451489469623282976315470325434198118210158378630
13682720*e^(328*I*c) + 9513032274019522954209113191268226642299912013525665
940298381064797885690904993128948035227412144035633851779511219335277360*e^
(327*I*c) + 117642122748764840800109007146734744933712781605578119837244558
26566055617658086479368641864908119643412413644803772131657280*e^(326*I*c)
+ 1442981628520843120453297837537569196506315422464974755129585150738952408
3226976789688601369628399900747658579201929300744260*e^(325*I*c) + 17556273
271224292396887291403125716213491486261145478571376751690105656067838042151
038271381300372757755676325408026834544840*e^(324*I*c) + 211883214058828875
396101983747068626958940492260770937641325125133361905239789496943876860591
24526755048042957954264706637460*e^(323*I*c) + 2536717643911935362153226033
598334815490498260612576171130068349296339081649158302570526873753998214963
9300226512657426118880*e^(322*I*c) + 30128482414552703264559018953088177156
013437493438201078413769835448366148121754549197591129967170764969700180348
699207838960*e^(321*I*c) + 355001031060196498762723767969494822095813723710
360050128778060274816728070599434452401363155685007323799665850056781819379
20*e^(320*I*c) + 4149983212196370804378852378740134554178008893053820691885
3579026749273364671640037563488607716092887686471542838602788559660*e^(319*
I*c) + 48133117678184029216503748549110374478924719094635603892829364863916
553792278822957368285106328164715910598370871149079494360*e^(318*I*c) + 553
909130449720862194326891463315660814279598969699002144342968177311508638670
56620768608187679709720152974148474907904177340*e^(317*I*c) + 6324777410101

217905179494607517556992407698133813848315804240674745387472938763171054499
5247152912205118500597511052824347680*e^(316*I*c) + 71660329861173395524441
943889284109134091157844655245672084237402434944696464927131812190659629511
140639501743303863582092880*e^(315*I*c) + 805662491306826841818762018826235
112063637903372180119541102106429277659976449038205954219368735653146544157
69070472655401600*e^(314*I*c) + 8988381580138238221397327047795460274479287
701805196334714630737246431512127492940234794287480289949953895356105666766
8891020*e^(313*I*c) + 99512206472057965951340341738023548515336403371717898
040850470954657532977279113491506880290726111154101941386019689567958040*e^
(312*I*c) + 109332537349966223203932678503426357079863707001728294011042076
530403923862654018978676516417314221089449922495612732870169660*e^(311*I*c)
+ 11920971370203392705575539782368844444647424324502185328626347046599634
721146573830681540495333543146776810911910410468628960*e^(310*I*c) + 128995
076011591903410763863427097329948586173574595862705849159280943046458742663
163454018491463855395649453952212899632198680*e^(309*I*c) + 138529794549151
089451352769576543403126330747243680030832467205895819043568155239264876762
867172754338684027849855385453216080*e^(308*I*c) + 147648920805545333418623
121767853777399782924748301228793924342574999937955421765370101235122939557
467548549202174550009604780*e^(307*I*c) + 156185962953551196169738218832173
696509852551589210730578365727476259476474465955428502336673743686499175698
677875693611243400*e^(306*I*c) + 163977816059607725375264559816505847894187
785101455360391897424482998415385787605765315509208337741590143078572243505
132706580*e^(305*I*c) + 170869848868953101176860306053103994340530390347260
088432676842505555141293830838961275974268928666494845723462544709102843680
*e^(304*I*c) + 176720929970554642004575770053095700595334659870682732031975
915532387577052414866323511140117680492929354517559479899220940360*e^(303*I
*c) + 181408168770922059820368553316697321639984862628298828569560273295630
897626829345263592219034560853530733710529842148537901680*e^(302*I*c) + 184
831151983748941817667850174708257138128172158269413287765358532240773244336
191900818557829905895684494889410451921524212840*e^(301*I*c) + 186915474436
567514926351405623117503261987508351930083824566444435689139233683411704641
828762178799177848064220150818355261280*e^(300*I*c) + 187615393168510050071
497280564603510912403132920312024370835062679037644990286285346673507093452
964351257962696133511725652320*e^(299*I*c) + 186915474436567514926351405623
117503261987508351930083824566444435689139233683411704641828762178799177848
064220150818355261280*e^(298*I*c) + 184831151983748941817667850174708257138
128172158269413287765358532240773244336191900818557829905895684494889410451
921524212840*e^(297*I*c) + 181408168770922059820368553316697321639984862628
298828569560273295630897626829345263592219034560853530733710529842148537901
680*e^(296*I*c) + 176720929970554642004575770053095700595334659870682732031
975915532387577052414866323511140117680492929354517559479899220940360*e^(29
5*I*c) + 170869848868953101176860306053103994340530390347260088432676842505
555141293830838961275974268928666494845723462544709102843680*e^(294*I*c) +
163977816059607725375264559816505847894187785101455360391897424482998415385
787605765315509208337741590143078572243505132706580*e^(293*I*c) + 156185962

953551196169738218832173696509852551589210730578365727476259476474465955428
 502336673743686499175698677875693611243400*e^(292*I*c) + 147648920805545333
 418623121767853777399782924748301228793924342574999937955421765370101235122
 93955746754854920217455009604780*e^(291*I*c) + 138529794549151089451352769
 576543403126330747243680030832467205895819043568155239264876762867172754338
 684027849855385453216080*e^(290*I*c) + 128995076011591903410763863427097329
 948586173574595862705849159280943046458742663163454018491463855395649453952
 212899632198680*e^(289*I*c) + 119209713702033927055755397823688444444647424
 324502185328626347046599634721146573830681540495333543146776810911910410468
 628960*e^(288*I*c) + 109332537349966223203932678503426357079863707001728294
 011042076530403923862654018978676516417314221089449922495612732870169660*e^(287*I*c) + 995122064720579659513403417380235485153364033717178980408504709
 54657532977279113491506880290726111154101941386019689567958040*e^(286*I*c)
 + 8988381580138238221397327047795460274479287701805196334714630737246431512
 1274929402347942874802899499538953561056667668891020*e^(285*I*c) + 80566249
 130682684181876201882623511206363790337218011954110210642927765997644903820
 595421936873565314654415769070472655401600*e^(284*I*c) + 716603298611733955
 244419438892841091340911578446552456720842374024349446964649271318121906596
 29511140639501743303863582092880*e^(283*I*c) + 6324777410101217905179494607
 517556992407698133813848315804240674745387472938763171054499524715291220511
 8500597511052824347680*e^(282*I*c) + 55390913044972086219432689146331566081
 427959896969900214434296817731150863867056620768608187679709720152974148474
 907904177340*e^(281*I*c) + 481331176781840292165037485491103744789247190946
 356038928293648639165537922788229573682851063281647159105983708711490794943
 60*e^(280*I*c) + 4149983212196370804378852378740134554178008893053820691885
 3579026749273364671640037563488607716092887686471542838602788559660*e^(279*I*c) + 35500103106019649876272376796949482209581372371036005012877806027481
 672807059943445240136315568500732379966585005678181937920*e^(278*I*c) + 301
 284824145527032645590189530881771560134374934382010784137698354483661481217
 54549197591129967170764969700180348699207838960*e^(277*I*c) + 2536717643911
 935362153226033598334815490498260612576171130068349296339081649158302570526
 8737539982149639300226512657426118880*e^(276*I*c) + 21188321405882887539610
 198374706862695894049226077093764132512513336190523978949694387686059124526
 755048042957954264706637460*e^(275*I*c) + 175562732712242923968872914031257
 162134914862611454785713767516901056560678380421510382713813003727577556763
 25408026834544840*e^(274*I*c) + 1442981628520843120453297837537569196506315
 422464974755129585150738952408322697678968860136962839990074765857920192930
 0744260*e^(273*I*c) + 11764212274876484080010900714673474493371278160557811
 983724455826566055617658086479368641864908119643412413644803772131657280*e^(272*I*c) + 951303227401952295420911319126822664229991201352566594029838106
 4797885690904993128948035227412144035633851779511219335277360*e^(271*I*c) +
 76297318156278215804689924242070083664388967363330246618638381051104451489
 46962328297631547032543419811821015837863013682720*e^(270*I*c) + 6068949803
 156712248331871105329895471722806143008878014986559653687260694816550470195
 890004511965527567432722969707577202160*e^(269*I*c) + 478752744278094568514

520484697159616530416941932824407321145959212964925504887685405984472066107
8151288179612574986359194560*e^(268*I*c) + 37452575948766512046573349884262
263881439545019868306642223492263610796095468222760675048993867030889823081
85717143407211328*e^(267*I*c) + 2905385722320057001953345274489482790856692
529959823749532695963414164833366773128218607899328588608916176593772088622
582464*e^(266*I*c) + 223489127639843946447862257830643484072461048446817785
9822620658691921478645266653062563823553001228001009093606751066168944*e^(2
65*I*c) + 17045829967078228082046782181676930026986611477127723550214565438
10930069637188085882824757500605246963210810351706405349408*e^(264*I*c) + 1
289043515292933956480634330499677040181043935620106914267311067900030058398
839787692376954090545278554544997710058754772400*e^(263*I*c) + 966458275369
037718747739130798151643483590684166832234688098291164160636418159452119815
728809372125168836239364442397344064*e^(262*I*c) + 718361596382058249209113
544487901088868388744033713210332491971375906738341551540457264804304039664
255915607349801911966551*e^(261*I*c) + 529329252764113926003934836958243557
672549238997560739214406599185047831955572583765358634395408771528009745467
548382950094*e^(260*I*c) + 386642673050380049457382562818316962651975550990
779277048740238629858795018247356162888631015687664780101205287333082748791
*e^(259*I*c) + 279945244475039804822966730462960884492119874857791147124009
079476920435941735293309305430438687333129912454196774070107264*e^(258*I*c)
+ 200906587153578804380300469501441610174521851259541929209840688960859454
908519774835905895757666770857888611738751858460424*e^(257*I*c) + 142906319
123055524246546928478954238371315925802022389236498652136839822502035155676
970917419039834587967055588431566416784*e^(256*I*c) + 100744961851853744611
754300982980166962404553836222921868484694269966120607698907046343731011160
948828100276729370132819357*e^(255*I*c) + 703863497605948315670482240613950
256985012022969663003767642203366977029615910998540554113762948714374681495
28524796002762*e^(254*I*c) + 4873325350597492340085225556305210140219646931
365955449272567475433937528301040716774436695582892283748870585853243965448
9*e^(253*I*c) + 33435897827936581301171175459610829454298167962017419810072
936733378506584428024201072453193458155334046693516742390717832*e^(252*I*c)
+ 227316035661288411004195019470513676668366524180772609139448107484730848
91890410181285412604854876625919565639521227223276*e^(251*I*c) + 1531283720
666277537934735321280768296571253565294263151828614240309773820027071119539
6582159028513532779682154451996208592*e^(250*I*c) + 10220423779434634851339
975295163399641702122249663666193053008302026096932158568338309418237395541
351819026907953220681013*e^(249*I*c) + 675848043788852437256293594896385762
669485554719551948612287756798171858726236287199496707940183195792790168258
2941234362*e^(248*I*c) + 44276730791054253185243161129856936565848519361001
92457044455134483305045321452516347118488133224823670465103483954805161*e^(
247*I*c) + 2873610535922340187080835435582912277271967977394720159791070274
927714276869531467182688981041061381703885403497544001592*e^(246*I*c) + 184
750585646245153344528430057132632378116255330456597188767075809107930679482
1834928170773126364639722071570131703785334*e^(245*I*c) + 11766007209757869
651898750508902310922046126969702774330145358957889567712307935203819931066

06880564628599822341722801012*e^(244*I*c) + 7422286409081731249169370494625
 256173341489196791182704898310054977819512210699558396234524997486531246588
 73553401442137*e^(243*I*c) + 4637582884573671545449376782550056887333281455
 68049310423995599886012800638619904022368378591108842602342094543682299102*
 e^(242*I*c) + 2869929436312314965572780108515769408968264974660663275288015
 60677007112837431926735088120974861760511367008815728782643*e^(241*I*c) + 1
 758962582627559857571068126139793012658010315954843536149046728651694422320
 75776580447184134141375995770091499246759528*e^(240*I*c) + 1067648320171655
 948380852341893335287335876733299725300926610851867899392529159370907602822
 32346919090426243399409323314*e^(239*I*c) + 6417510069326006680623806488600
 459717074084330008683936861613916452910804984467535311184272579865808884034
 7241496099644*e^(238*I*c) + 38199015867586087976002998756627674994795440667
 903625029322346250133286489120875005013638128113893960349670280707161530*e^
 (237*I*c) + 225146757413080699615061655865028724304219302106732643929972864
 85600640103867253604847715547060592967690653795951142520*e^(236*I*c) + 1313
 977149410493388185668115141829311224255152153568687118126657981387760634816
 0261747201317735782566021306798298336024*e^(235*I*c) + 75927527001466789611
 530950735850154731970297465336333315497939614732857609358019041551167648315
 60875947581048693527224*e^(234*I*c) + 4343909696601932173357359687781579293
 701295681940827114215433175336093967845908766740738240037114570667410936998
 017178*e^(233*I*c) + 246044237584542266392708163098326071473496809190549302
 7145639238827192254886349361126991457692409851120873307487457468*e^(232*I*c
) + 13796765297962120740171061880665894483554465012108901951071648603502289
 28586815539003062875026711931941947738690360722*e^(231*I*c) + 7658677955139
 627810125584446287514187109408952813047908367436615820716500321548914828664
 06314834433199455459798934952*e^(230*I*c) + 4208463426089493872775590214579
 245865781209661485610226470084995294684520059801751194106289562104976095660
 02969884927*e^(229*I*c) + 2289113117385927800914926491623468344058677407764
 56326108410928857257174707289268074347550225793244741923354395308214*e^(228
 *I*c) + 1232439415193323847419600725881035065964063392536163910820629699606
 82419011745775738921817753391954462609323881489157*e^(227*I*c) + 6567485926
 886730009882737581287522561065455168626110368166400700753711577809729353356
 5243828873383722980353200611956*e^(226*I*c) + 34637657172671690167657344537
 197087048882354853993270472063943078773600446542963548348101269390443464480
 754513928502*e^(225*I*c) + 180798200680288599703499386230072306765633142067
 08848499900139641237334763266479346963237936039328113185041591793848*e^(224
 *I*c) + 9339341958053494225251750965715057300707302083814774770306218224241
 022648247419956042957363055823830898547303219757*e^(223*I*c) + 477414111106
 609897022184533059496201647271423037423406066395684695092664268594692906411
 4194400360936223590725470146*e^(222*I*c) + 24149665168103385032890765492027
 405117100590117954471387734642056964550264427124264095996627710802648260089
 85061097*e^(221*I*c) + 1208770358493658393089442222056935063283704108140593
 750226539846117737648216609559734831601248698274330296158612144*e^(220*I*c)
 + 598650141112241858911676505180520150364003226841328081453597093587790338
 609212439085554466861582623350303061961052*e^(219*I*c) + 293344920034300720

287042383448342866313806285455040067823080445597545970023446231563554135133
105493516316320059272*e^(218*I*c) + 142213115964814517682386667276769909482
271681318790889840501039441748635545362467679832449103520321953011780083069
*e^(217*I*c) + 682080330967936156837844096192442108186149916400415534244055
27876893272496608324231098148502466453967157728078994*e^(216*I*c) + 3236273
132241954941033008894364024746037832856131642293129242714590288791307164367
9502909055891236755143207382609*e^(215*I*c) + 15189634214908800396417911722
643754748048520109734812459109878810493844381062650818971199637121458749456
243274416*e^(214*I*c) + 705213241416219799260232652458014306098535305457293
3905524633121681021037340298366342203324325307072413739061024*e^(213*I*c) +
32384919313618514764233219335395790983777355392076414673462356658238870483
26949305609231585143748690203615957136*e^(212*I*c) + 1470931146618934345515
038362300100160482127749581443929904746910224777470198899052379114493999887
003199419829579*e^(211*I*c) + 660764473105869097691475973850837934511089033
149586707982764263394766756649565279879146173318386505740391093990*e^(210*I
*c) + 293550743554342709808129453576562313299705982699187416862934373964255
615967138676253276302591561523515603264403*e^(209*I*c) + 128967080084754712
246023680866488384983286259025533132044636109049545144029547003347761521666
283977931640178464*e^(208*I*c) + 560286834249035176584950138585345161671625
91034367972498174660907450666778154353271630344650777885683547624184*e^(207
*I*c) + 2406878513970527716119346564450614328524136103776821681892218440014
1048460210944696647752723371932874594597328*e^(206*I*c) + 10223182025954860
767217390305186451923562145473674293619918063490411487496121804590274592702
770571515456414680*e^(205*I*c) + 429320647800802212601748890885182649479062
0720660151451468181910917240027863968724539127659633517053002976480*e^(204*
I*c) + 17824461149317518505563548566384219011744123222982494965916580539397
87198246565945975595575734193348887952160*e^(203*I*c) + 7315849722068183628
747296214039744442800104463011615273397605448153009517879855384197646562145
82667219914080*e^(202*I*c) + 2968255152826695896853182732802390500845550322
03415941511962659596881615713799937680026497408305672297618840*e^(201*I*c)
+ 1190418554038779649482295779483704656006066231830455295269004302092704732
12773847794935586074714329479939280*e^(200*I*c) + 4718822084346620769509950
695357378035710889749142256789804819901820770899700533386014883647952745615
6014520*e^(199*I*c) + 18487405299005732693752728611876490890858357021974882
371570623800186245137722660943641752976852924439870880*e^(198*I*c) + 715812
468684294147547380736367983971817274558153840904450338385269359692162242669
6740453944718143025248390*e^(197*I*c) + 27388956247952656033552276465660008
86280778305084825702911938903656162004262736182657700406301914070062380*e^(
196*I*c) + 1035561982592002935226384577908611548612111495080193573691339864
706029186482466241805664949381049856258510*e^(195*I*c) + 386876218234277165
632451723049979889263115282374607541692443176673997513742813591736171169652
250611186480*e^(194*I*c) + 142801792450221762483180874918825274134305133275
417780084795034644763509333503150517345864659667189417080*e^(193*I*c) + 520
751785187932703864292633515443069511049935425005829381552416894081386752546
08030847907167748571734720*e^(192*I*c) + 1875998821886556356416363573598607

3278255737257405706279108891366378428467414559930481172863538598193890*e^(1
 91*I*c) + 66758662903711473585037668656692890108935438698305387087249452915
 80951179188296606158111257706968604740*e^(190*I*c) + 2346518219239105142238
 141633073464768899155708935025778047637412681781575765422219127409260159438
 712250*e^(189*I*c) + 814608187736530579670210025271921415597183369881214299
 823291969785549876175969866367976653244974728560*e^(188*I*c) + 279285755800
 035206679835368898165477644864987794665387827488933863633745047373109049265
 172681702585720*e^(187*I*c) + 945561802589319869193343034663656528268580913
 14329189160736277175873841732196453379953705679466826880*e^(186*I*c) + 3161
 093933128469275069430644361841465609596952094521574300404456038689524180157
 9156543451940713351730*e^(185*I*c) + 10434117516570395966653693155582402109
 460348095473027807412321427346816928567197770376496170251803940*e^(184*I*c)
 + 340023256060165161752169468084708984419802883169441742479486877932895054
 8418125605446882081152636090*e^(183*I*c) + 10938532144862203586740324345008
 66678499770011305874172488975951612031456734608287095519501041975440*e^(182
 *I*c) + 3473514732147137808743520831295666012387657627759423667627333499521
 03889753982636403857556867777300*e^(181*I*c) + 1088679957318294728267329051
 92034886797284621356445627530909104429486741257822633476898356826454040*e^(
 180*I*c) + 3367539887202156837590238459398275336255980105810418462734541113
 6262431943240778260721756991027090*e^(179*I*c) + 10279364730663840844739577
 862469262604648861914297972589165243530651230690726244462479199894255180*e^(
 178*I*c) + 309613197162152016238030155424146545178236208681028753774890290
 4985934020179565706177131421614590*e^(177*I*c) + 92008939302958903287460185
 0027159322612526368444771489781974361078847528891468831038436064951920*e^(1
 76*I*c) + 26974580144021129697268360186387895435796230852007659517712822762
 9273240215209708218497363414140*e^(175*I*c) + 78009807368024239875613733058
 851417125327114681070889640794249282633470580756557083923203377160*e^(174*I
 *c) + 22251959176795777571673660360074802222113642321463998038643709633914
 91223687245823457351580140*e^(173*I*c) + 6259872156822252843650960708235034
 710201362776057176647226323089751446565288850103898153859920*e^(172*I*c) +
 173657421881819107187419747245015812388356420995065863910233714812276908061
 1680719741726053840*e^(171*I*c) + 47501057885760151927231661793842522242178
 6597241671026894318515408511467140969393115768793680*e^(170*I*c) + 12809891
 460168853967248054183040984770736750043860153680320449770111991128908710565
 9482783340*e^(169*I*c) + 34054053851295569154352346722177172655187548910782
 008504718324168725029438589162349211628040*e^(168*I*c) + 892320944734329676
 3331881881638471793499618670601026059730895962653291770229493028162575100*e
 ^ (167*I*c) + 23043510733738403573791785976730663520166827816891398420973766
 63118488803841131935313641840*e^(166*I*c) + 5864034669726832427416433289215
 60909375197453864243299571990964608857245771134145204174990*e^(165*I*c) + 1
 470308167322768331630415820995920475120437252253533392388191651930004076295
 44745753221740*e^(164*I*c) + 3631836965230259173219744440979812202264082460
 4130552506742586795183267354382847875885730*e^(163*I*c) + 88367206408604703
 05694514021547969551296794092266983044118375790025854584036796364768280*e^(
 162*I*c) + 2117589733466855707101501429210414722401838837940752841618541440

888545729943138209036820*e^(161*I*c) + 499707567253859084357596314813794768
069337190915967491907488904933922677579665354338960*e^(160*I*c) + 116104551
683555043762911501712116399313733021132677481112824047246361794049635726479
850*e^(159*I*c) + 265568063890434075344967023691015457959948617577414147899
44652712127566910185274123140*e^(158*I*c) + 5978992172944143218459161149299
819706321732111578494525245228742976468409105395536290*e^(157*I*c) + 132475
641236783747315747282116248369112096650194895392649224164378826428454643722
1120*e^(156*I*c) + 28882075526473065446996857202104710942731861950899580202
0689904590319476295408324280*e^(155*I*c) + 61948596653035502879564338815234
310660410902037882473161804774492916216575880077680*e^(154*I*c) + 130698172
03488289886193205508375818392124991382340160316886507181296548981014818410*
e^(153*I*c) + 2711843239670717527605640490148833507130242448403978318523237
721944200392830108580*e^(152*I*c) + 553269128819528612502918869558947829098
021956309349843584044631512291778800081490*e^(151*I*c) + 110969199687320974
749922259595250444341219218535349655762591192576535872151766080*e^(150*I*c)
+ 218764828927139099280403456125787058051215087562266963170876518242522414
18663320*e^(149*I*c) + 4238125846763232586394188569858685826755328005548627
437019301405851325887594480*e^(148*I*c) + 806679543607589140759305010796189
568269842021613388955218916278823182639488190*e^(147*I*c) + 150822381431412
413773566474210011746852297437597059186295243989481140398152780*e^(146*I*c)
+ 276931165383432592259833826376479361226640338596151334898466646943614710
28310*e^(145*I*c) + 4992519712457043983505377976607953988397368297591114957
991804893688371867680*e^(144*I*c) + 883500968821791202600774541927769200737
689393513734789368397093333311961880*e^(143*I*c) + 153436088745056254127327
239461577071933130157764595997113973513183188399376*e^(142*I*c) + 261439762
79902021443471945665080254563056810183520401889800285493144867448*e^(141*I*
c) + 4369442482910113914565353136069595862669338858053419381214131241925047
008*e^(140*I*c) + 716099497599058079895633338552940229192858196481597830078
819711862600096*e^(139*I*c) + 115051481852080848873700388354521315567640365
124003103691176697194292320*e^(138*I*c) + 181157684956157580767103030555056
25589254293659193314153418333944596408*e^(137*I*c) + 2794709104475686611842
790694973699164482254723977210209725661304403472*e^(136*I*c) + 422276126632
003687547754746555709988710527133086660161366353656787288*e^(135*I*c) + 624
73550781053295317710774690247114124125187565731848441781904032672*e^(134*I*
c) + 9046693523825682979044338963104263167672586826367911338826483549173*e^
(133*I*c) + 128181746491497081085960418982835900079078992116940530461221125
1818*e^(132*I*c) + 17764282913511934857719443767580283023990546009268713649
4961404333*e^(131*I*c) + 24070801913529757101858022914372045864746991786182
039740274325264*e^(130*I*c) + 318774992974434649721153604475177658232095862
7923816470590659024*e^(129*I*c) + 41243069829991519084806722232721943506774
7934091894670488982928*e^(128*I*c) + 52108117629177048660492400985175830987
505700566877818954141639*e^(127*I*c) + 642619548553524857642506813687046553
0087114003875716691383902*e^(126*I*c) + 77320463699114577506146273102809850
6094432675788136295011259*e^(125*I*c) + 90722605722208814918642284639487187
764607589706493970774776*e^(124*I*c) + 103751844998711755019093989565966841

$16802997082526660323524 * e^{(123 * I * c)} + 1155855412893594260345544966642687823$
 $630035899363232371472 * e^{(122 * I * c)} + 125370496586921272662198050851269323171$
 $167338854081782959 * e^{(121 * I * c)} + 132317088701048969738000567337799190893408$
 $36756009580718 * e^{(120 * I * c)} + 1357990663161479842850642848032544982878359839$
 $580349899 * e^{(119 * I * c)} + 135442594916636116191574650625331646238501101627937$
 $224 * e^{(118 * I * c)} + 13118781801172174729679339894318153694964675368481194 * e^{($
 $117 * I * c)} + 1233096700139723365181997220750932590655287625342156 * e^{(116 * I * c)}$
 $+ 112391604542246650966429162063124338952554575234051 * e^{(115 * I * c)} + 992549$
 $0738534402272939987038714580495445431374618 * e^{(114 * I * c)} + 84855220227651235$
 $6496200136959676295361696315113 * e^{(113 * I * c)} + 70164515322544462906873548813$
 $748091084561870680 * e^{(112 * I * c)} + 560592725306755855178045288368983551445511$
 $8670 * e^{(111 * I * c)} + 432333688644261557547944179250800440604964868 * e^{(110 * I * c)}$
 $) + 32147887693375338817454482515377350383950278 * e^{(109 * I * c)} + 230215041122$
 $6234925855222345201500900533576 * e^{(108 * I * c)} + 15856647611325756256611743222$
 $7203884298856 * e^{(107 * I * c)} + 10490402669510897424624643766470754045064 * e^{(10$
 $6 * I * c)} + 665634670676210063754191847109971141414 * e^{(105 * I * c)} + 404436247814$
 $15311581857832389099634564 * e^{(104 * I * c)} + 2348998374244347079532766203075607$
 $598 * e^{(103 * I * c)} + 130171193079172823835151430773360024 * e^{(102 * I * c)} + 686832$
 $9225263681349501997341320517 * e^{(101 * I * c)} + 34427715201287513414073930296091$
 $4 * e^{(100 * I * c)} + 16353164647151530240529137618111 * e^{(99 * I * c)} + 7340572636163$
 $88449968842366924 * e^{(98 * I * c)} + 3104222522074681615625020522 * e^{(97 * I * c)} + 1$
 $232445557346832245176696904 * e^{(96 * I * c)} + 45759117183402579073139583 * e^{(95 * I$
 $* c)} + 1581796642397812408161814 * e^{(94 * I * c)} + 50648660944512569972179 * e^{(93 * I$
 $* c)} + 1493326612293984160368 * e^{(92 * I * c)} + 40261256699368950388 * e^{(91 * I * c)}$
 $+ 984382804329835768 * e^{(90 * I * c)} + 21608403021340047 * e^{(89 * I * c)} + 4206015186$
 $59718 * e^{(88 * I * c)} + 7146142307307 * e^{(87 * I * c)} + 103818048048 * e^{(86 * I * c)} + 125$
 $3841160 * e^{(85 * I * c)} + 12085216 * e^{(84 * I * c)} + 87153 * e^{(83 * I * c)} + 418 * e^{(82 * I * c}$
 $) + e^{(81 * I * c)})) * \tan(1/4 * d * x + c) + 28 * (299 * a^2 * e^{(1027/2 * I * c)} + 116610 * a^2$
 $* e^{(1025/2 * I * c)} + 22680645 * a^2 * e^{(1023/2 * I * c)} + 2933363420 * a^2 * e^{(1021/2 * I * c)}$
 $c) + 283802910885 * a^2 * e^{(1019/2 * I * c)} + 21909584720322 * a^2 * e^{(1017/2 * I * c)} +$
 $1405865019554882 * a^2 * e^{(1015/2 * I * c)} + 77121738215879370 * a^2 * e^{(1013/2 * I * c)}$
 $+ 3692203217135946825 * a^2 * e^{(1011/2 * I * c)} + 156713514333171921595 * a^2 * e^{(100$
 $9/2 * I * c)} + 5970784896604221606627 * a^2 * e^{(1007/2 * I * c)} + 20626347828223924323$
 $3771 * a^2 * e^{(1005/2 * I * c)} + 6514488191198508953598437 * a^2 * e^{(1003/2 * I * c)} + 18$
 $9421272128521178822066445 * a^2 * e^{(1001/2 * I * c)} + 5100844261395996953829648360$
 $* a^2 * e^{(999/2 * I * c)} + 127861163009333998455801820954 * a^2 * e^{(997/2 * I * c)} + 299$
 $6746014847853620317693731016 * a^2 * e^{(995/2 * I * c)} + 65928412548866286902779022$
 $351001 * a^2 * e^{(993/2 * I * c)} + 1366183222241782313586863286641184 * a^2 * e^{(991/2 * I$
 $* c)} + 26748429587445855729768539197182585 * a^2 * e^{(989/2 * I * c)} + 496183373555$
 $284804627911954521619600 * a^2 * e^{(987/2 * I * c)} + 874227859914729528548075880138$
 $4765381 * a^2 * e^{(985/2 * I * c)} + 146631857213698093058440158830415565266 * a^2 * e^{($
 $983/2 * I * c)} + 2346109768759347903572910732245570209842 * a^2 * e^{(981/2 * I * c)} + 3$
 $5875929609390406738394666219971221205725 * a^2 * e^{(979/2 * I * c)} + 52522362949838$
 $3939328992961505581135597600 * a^2 * e^{(977/2 * I * c)} + 73733320797991336390636586$
 $14527645894565137 * a^2 * e^{(975/2 * I * c)} + 9940344598096194358959846852073298354$

$8550704a^2e^{(973/2I*c)} + 1288694773452054046771640788950472807542901137a^2e^{(971/2I*c)} + 16086467322724552698501340975783163797842534840a^2e^{(969/2I*c)} + 193573846127814805336166214026334290497006915130a^2e^{(967/2I*c)} + 2247954664299349273623637155894975599108753667672a^2e^{(965/2I*c)} + 25219245783549721366067799114158978864399924058441a^2e^{(963/2I*c)} + 273590663366155266234104272169452688263007484513781a^2e^{(961/2I*c)} + 2872702692034937563897971496575074831246739185614575a^2e^{(959/2I*c)} + 29219499075263535843858862377648073071285310974657915a^2e^{(957/2I*c)} + 288136830654984406530837787337700439328183658059646399a^2e^{(955/2I*c)} + 2756769770542048744355911382380396555656358932078858681a^2e^{(953/2I*c)} + 25608953102811736020699671981400531591652082353456701666a^2e^{(951/2I*c)} + 231137354125616547233443142991124898825470770660317194554a^2e^{(949/2I*c)} + 2028231682527169820869698431068546224630661674135401655870a^2e^{(947/2I*c)} + 17314186832105700122787937106308399484490093525695416858785a^2e^{(945/2I*c)} + 143872782220632026353482783145282086606314049235674031853952a^2e^{(943/2I*c)} + 1164367032824623073366685138251309912040093060141928716087681a^2e^{(941/2I*c)} + 9182633302160212452918968603806828036823248995981353166570702a^2e^{(939/2I*c)} + 70604348724401170958829605205937063913034447082349890348162895a^2e^{(937/2I*c)} + 529533489706434824451250188592739498197462491764283044018575440a^2e^{(935/2I*c)} + 3875741803263271608578048125955066152785493897151672716099440360a^2e^{(933/2I*c)} + 27695464742706927996157891275342643431040906047696742940234352112a^2e^{(931/2I*c)} + 193303514872786115923379716628253419318944176435876056802589865772a^2e^{(929/2I*c)} + 1318333654990888899417746520374288649257223133573127905086467548080a^2e^{(927/2I*c)} + 8788918880786687559267555413796199314981595789787074901006933535500a^2e^{(925/2I*c)} + 57297195830579479880905283759887310015692255967368709515502818525680a^2e^{(923/2I*c)} + 365406237849518248372060960463581650995651842549690071352678851012540a^2e^{(921/2I*c)} + 2280416002497499972330270356628053500702221025770502602102780787687624a^2e^{(919/2I*c)} + 13931340194933935228013466294179987445068758219565436862349156508804360a^2e^{(917/2I*c)} + 83339747736469143532428073310762163525113789344043183602916613227872140a^2e^{(915/2I*c)} + 488344836125981471048169163657616516954158792795618978221198980949925840a^2e^{(913/2I*c)} + 2803793220826297874163429682131102503306549005265399219277240917413502940a^2e^{(911/2I*c)} + 15777404639975259272454933843539474544665236819117566997318672304666566800a^2e^{(909/2I*c)} + 87039467228531013705409191271452383389335356884921808641492423363762400060a^2e^{(907/2I*c)} + 470873979833165454444627659803101969430867153471663329279188629910819335440a^2e^{(905/2I*c)} + 2498697917579895977978723588576283933137296518743481104776932566391579455400a^2e^{(903/2I*c)} + 13009255453778977514917501132236907767356887691068368589411338908245552368240a^2e^{(901/2I*c)} + 66470080272037903682198219835488944949996749070187078822650198449544212324410a^2e^{(899/2I*c)} + 333378106740586138251610124958796714576935099619835073172307696551648259829120a^2e^{(897/2I*c)} + 1641662348095460363514356729639026504243315309578757871628322407537121464954630a^2e^{(895/2I*c)} + 79389333204974496359189696807583263073391162390718463$

30248311795038723075723660*a²*e^(893/2*I*c) + 3771071067074220374698026793
6076214521784550649912575518820552220031814445215110*a²*e^(891/2*I*c) + 17
598731744416001266512340274583125217315543205239553488481060367552361202279
1280*a²*e^(889/2*I*c) + 80704776818833353884668480346077979093105555510012
0589990758127536168861246233500*a²*e^(887/2*I*c) + 36374986460264255987452
79228803679517140090350241314216371570468174738174921307980*a²*e<sup>(885/2*I*
c)</sup> + 1611662816241146467989650283632477138962058485657487792246828268566428
3130175327730*a²*e^(883/2*I*c) + 70209017945411591200961651730058863557722
458877098381486938641100483455911795967290*a²*e^(881/2*I*c) + 300771234468
452520266562607329665922159115381500087728345930006292005623202669775630*a ²*e^(879/2*I*c) + 126730019194373732411932059934968365449334885021700732115
0739273112190995849189116570*a²*e^(877/2*I*c) + 52528560627818339382014908
69422444930464746423374588871765446690324009941198600912010*a²*e<sup>(875/2*I*
c)</sup> + 2142176528983705740386959616994147567864550649247186416870451808279751
3207707602578070*a²*e^(873/2*I*c) + 85966207795742771163110084083079326198
692050781198818319037362716785215369332723227360*a²*e^(871/2*I*c) + 339531
565165818157488111546236514739245172484736482729741196732548715077274621271
815060*a²*e^(869/2*I*c) + 132001151473743564403925218561453784232553131778
4101906639727300233020372137428050228960*a²*e^(867/2*I*c) + 50522405603727
72252508411884469329618226376927979062996145236110491773700805441539104570*
a²*e^(865/2*I*c) + 1903973670047703419191441787440869515565540598612972628
3796391182602876937826158174743280*a²*e^(863/2*I*c) + 70659202220402219699
327142472480242287876653764953302133360376448287226187487345644845530*a²*e
^(861/2*I*c) + 258265319998385920800570589348242938126445962121695949028377
963603738714515662920857912400*a²*e^(859/2*I*c) + 929844269839618243277627
786923243414623663776261220314871679199722455584698434216819569010*a²*e<sup>(8
57/2*I*c)</sup> + 329804705399395014662084250372152905831848535666007501658224801
7786240344190976379491032900*a²*e^(855/2*I*c) + 11525514338248122115806901
223631408865665060873992331731216717545839641395493543478555391940*a²*e<sup>(8
53/2*I*c)</sup> + 396893152747629625983440995776573121457424082278264312824228248
44089957174694332245240202130*a²*e^(851/2*I*c) + 1346940458317330848315741
70523497510104128836035403576353015982048577081438406747258639643920*a²*e ^(849/2*I*c) + 4505418159943504163295634826559092449793203344384059149684114
06156252958317001977289679320570*a²*e^(847/2*I*c) + 1485535118953138948865
978331717910599497456296177792374785590516233474655906439549342970114480*a ²*e^(845/2*I*c) + 482880723711149517085020202679552656003275504569154165385
0695089412640252061166703373641008090*a²*e^(843/2*I*c) + 15475789717397460
276234843111043950695013471030625427925175353797112876241958974587338558471
200*a²*e^(841/2*I*c) + 489066090725200576110713705079963402247766813797750
34925868200923794664214854540668184735223060*a²*e^(839/2*I*c) + 1524153741
060746229392242841536596015506271094363259266210144802293335348624678102171
19890170400*a²*e^(837/2*I*c) + 4684682618247749580917910719553579166169333
20342602739423768916749249443401418804336754943102630*a²*e^(835/2*I*c) + 1
420248786518187466718023048901731118377946854300811977671072304411278973969
440001583221282013130*a²*e^(833/2*I*c) + 424740092507737317611348631219014

5472181686422993933942447632846633095644440032003339271168690570*a²*e^(831/2*I*c) + 12531338227920856492074590326674099374453122868913516861854565249
779642744578491059412040960382510*a²*e^(829/2*I*c) + 364776537060422459446
888206046964942190813347671210579123319141378044153079733377939021772945992
10*a²*e^(827/2*I*c) + 1047732949197081814264563182467078417585755174410398
66971803838079319268269378919589734008354661810*a²*e^(825/2*I*c) + 2969660
051565635311059710749400502952109529633422921287734507155025354576844063789
25697100618058220*a²*e^(823/2*I*c) + 8306778908234055938810480280413755423
33955283073962626784718312376094416760253547591206118659958460*a²*e<sup>(821/2
*I*c)</sup> + 2293321308015586917932799317024982339168604649369171514848750864257
523843794511936002538498981041920*a²*e^(819/2*I*c) + 624941401292896228606
101040352900275360734236090608619203318510342840091307088905229294048471232
7830*a²*e^(817/2*I*c) + 16810896728225961874194498340222314518223276631636
193848187700774478509776074186756740832022063618940*a²*e^(815/2*I*c) + 446
430644715917174797040202765636425707178838266681451692476117610657225228549
88204793268950962007830*a²*e^(813/2*I*c) + 1170475435818151870626733214223
03561789600917165284254959414932144762194096889890432439125557135120080*a²
*e^(811/2*I*c) + 3030042922717630947457200399363345710168954253146935464958
93805386105536448400356863388712154386414250*a²*e^(809/2*I*c) + 7745434216
142034956009716154660779796523688812189335104242942400214093869560355261597
45861544320722480*a²*e^(807/2*I*c) + 1955172094293269164466209803317102552
595021647581151217877842297059127002495113810772654784644585060360*a²*e<sup>(8
05/2*I*c)</sup> + 487413507913275541434737569088780271179410565093886397040023000
2709422093992525472165364797296790138960*a²*e^(803/2*I*c) + 12000905703768
198174954282034631757209497411914073482804705515897689969997576557491508854
781156886995980*a²*e^(801/2*I*c) + 291853134486539174414648904873231895332
33580558386026734570615029457027933491512738753237134630607181520*a²*e<sup>(79
9/2*I*c)</sup> + 7010968653080637954896206585526731480367717649524179695153473166
9884594114550121278447472661295579702060*a²*e^(797/2*I*c) + 16637342101596
006574494683160512963808544116807135262064048346380029164027801434252046974
1317600337261840*a²*e^(795/2*I*c) + 39004127275497477700562769915484224698
8726024604162438024095285594852487855850372070546472734126022684700*a²*e<sup>(
793/2*I*c)</sup> + 90341197663448354263528999846283396366969515218525999665486805
1991473572372870248662847924612744264062440*a²*e^(791/2*I*c) + 20674608418
735299991184202651342837639644452126560315573995791853905721435236059086124
68666220745228526760*a²*e^(789/2*I*c) + 4675111642548746091915670754076032
371176136454036359982075221477297809925204476666662239031601034545996300*a²
*e^(787/2*I*c) + 104466389336129644073368725603779571652443519950184462203
21535598596368046659626683803611150049940561221040*a²*e^(785/2*I*c) + 2306
839351528818606448446560311796191128467375408418434749124204341353644953089
4861283359363464825007526940*a²*e^(783/2*I*c) + 50343125546713237985623313
646329330647858349495627492925205161566693001610650569967429902726126062669
565040*a²*e^(781/2*I*c) + 108585188157713942121077851789895722422594689018
720455070563476121301184727811574323466214291834201854672060*a²*e<sup>(779/2*I
*c)</sup> + 231490826585005787807753273508538765393237794891589480734288576865823

349858144380580905331957850509282551472*a^2*e^(777/2*I*c) + 487814432521111
 497585406719506802055636922087949888627025208454447300100038791012297859131
 526297355904933960*a^2*e^(775/2*I*c) + 101614982522492841528243011649242617
 1242192567909037312010368898174633451476277701838680759316125015139743120*a
 ^2*e^(773/2*I*c) + 20925092752473397254575539038059299937349385477218044541
 97071999505511719948252419379343549397905586122452665*a^2*e^(771/2*I*c) + 4
 259980611481175181529614616968079288266998347286916867765191606402809238614
 481985333675831806087956768872250*a^2*e^(769/2*I*c) + 857437043606127858832
 332973198205223244571782879175596317046623803379829879817966509635534643144
 4695120516791*a^2*e^(767/2*I*c) + 17063763943819201257154546007944190484800
 483541219625570334773139200258218933634528764590082974465232719019576*a^2*e
 ^ (765/2*I*c) + 335775328625086124049321435620567581695125431787234427844444
 13242416994607618240798840372204654901124952727575*a^2*e^(763/2*I*c) + 6533
 520985513974522546045816788264371546417470237990342441543633514171293293484
 7111707604519947532103535939690*a^2*e^(761/2*I*c) + 12571682379700564231734
 752649840179083753168561102364880492546544906525540804103230843956650070870
 0602697766710*a^2*e^(759/2*I*c) + 23922690108635111250191342017664138814183
 4680717243149148240541912365024035242396630141827067823768453577792206*a^2*
 e^(757/2*I*c) + 45021495315848243687744932353444363642498768967317657680863
 7277491307797447679834039022495671246539361123241623*a^2*e^(755/2*I*c) + 83
 800340563322626169800247516502885996183656047744566097833058888205480340879
 2968277041826209840110522910353801*a^2*e^(753/2*I*c) + 15428020946518486743
 869296775063361353014517577821624810293541913783856553098395903422957820702
 67248163797079525*a^2*e^(751/2*I*c) + 2809543518668204266823796801627118763
 645888741191307504647885398224587315004606447902339962521984923603337579865
 *a^2*e^(749/2*I*c) + 506109993897069624544009746293650406009823419486789109
 9800067879466606691927034473565491643439734586575243562587*a^2*e^(747/2*I*c
) + 90190365310541393641450361987298751815027889967211657109128538197992440
 35211696760046421655662466728741232371823*a^2*e^(745/2*I*c) + 1590024943779
 140593540429868995972129691455645302363562160215701542640088331196174250400
 3269238750112959349688008*a^2*e^(743/2*I*c) + 27733123170528060665047793029
 187981069336565834501591355485919460176359570456976926692677356655692104901
 055424742*a^2*e^(741/2*I*c) + 478593950514785774857771488256226664918441899
 89671287148895312750031063089597039280342277891313648282659506765160*a^2*e^
 (739/2*I*c) + 8172064679848532501937431778432275995784492227356990384764293
 3003662634138916360473885905820817778748179763625775*a^2*e^(737/2*I*c) + 13
 807505047291146001136685629516855528925718909732467031103395173509980199384
 6300774206061209409704454402575173168*a^2*e^(735/2*I*c) + 23085548667262670
 972353592787256885447107733250002236679808330690153794894813537159206238297
 2028832652892521530223*a^2*e^(733/2*I*c) + 38197032488897113782426041711935
 588050801140530650725271731911163896757370819281494759323796283751658736708
 3029056*a^2*e^(731/2*I*c) + 62546997108251205222144593922633931315752756168
 6453294385607721125006005737878528154829176814174073609304783133155*a^2*e^(
 729/2*I*c) + 10136635833813823155261199003857937805928973622595218808702566
 89578864566731937045334553766119426780595860617854190*a^2*e^(727/2*I*c) + 1

625979420531105682414675218243753902020818169524222534201660329925485571915
 419014974199094126837464169182082222286*a²*e^{-(725/2*I*c)} + 258162710700990
 073452206474459755120980230440691093473667883589900827910048538655151298617
 1247434443617529951824987*a²*e^{-(723/2*I*c)} + 40574472057365087204940150625
 708426775052741486448816168113267026281070947745142656809407130142563516596
 07202219536*a²*e^{-(721/2*I*c)} + 6312726327493419321171003213006926022587697
 863019991394721190170435857084298246006024922226846516328771260784350055*a²
 *e^{-(719/2*I*c)} + 972321278368853764330445117379093235276339211787819409971
 3212407827924571779656265406369365137858973995542668178560*a²*e^{-(717/2*I*c}
) + 14827078840958204423726546705512527357957361647396508632935988933241213
 590716537429203049741620280852461278755893511*a²*e^{-(715/2*I*c)} + 223861204
 043000044228760073517616478835216103343997859390380117904490678686278432262
 39824339519408830910225209114648*a²*e^{-(713/2*I*c)} + 3346602880864230042545
 312921309164181546635502663665294708326117788745948786647037069070866851406
 1085213215020765318*a²*e^{-(711/2*I*c)} + 49540060650503063037382280258699049
 436351964733180929934117445080596631561896216015780970984005090516521389882
 187000*a²*e^{-(709/2*I*c)} + 726207992189165652178758713813393428748038401335
 31376203309805775067797784602620988634808343998866415251767099657115*a²*e⁻⁽
 707/2*I*c) + 1054248537870287544696739608610518480770643791792014086875091
 67013295588338384945425067268491968039006968056581921707*a²*e^{-(705/2*I*c)}
 + 1515751703557392099463738763700576505493183205845090419520345899421443972
 07049663095445023255855799320406828152904813*a²*e^{-(703/2*I*c)} + 2158450037
 768053028611846925659270013831448502780905776912601373554991349743613134480
 26444380882959804366246287914973*a²*e^{-(701/2*I*c)} + 3044463932271468146139
 835397274447981735648583749804130804882711352566619133843133421838724023325
 78752912321480097437*a²*e^{-(699/2*I*c)} + 4253640612909728645993816052419769
 130262428923523341151858544049566443138054303154686859024852809072826580898
 81714375*a²*e^{-(697/2*I*c)} + 5887329114246524320364752186762673918338295870
 71733581552653599569115062655958634436180105642268527576118276535857990*a²
 *e^{-(695/2*I*c)} + 8072536581373423079001605955440784201898133383975139514595
 80649551593816718094436044450016924199349627155306092503566*a²*e^{-(693/2*I*}
 c) + 1096636578459444916376723286069829634240829040283071984682114965507192
 138829786266640857980171465345670412137298803158*a²*e^{-(691/2*I*c)} + 147605
 799556939063722335269724248321056986186278058992331714091490618616504496604
 1302091938311768459791677885305942531*a²*e^{-(689/2*I*c)} + 19686080741876624
 109603861249218077608394199124491180880794404793788306967638375921128173216
 48516678192250226251889260*a²*e^{-(687/2*I*c)} + 2601702122396645141345931033
 404570995027466204284274488653750753761498675668863970880304480234793529924
 712351195427555*a²*e^{-(685/2*I*c)} + 340742128545569404647432079815909007579
 872992486208020965032843806750401688159806770784439624253189019554430142698
 7190*a²*e^{-(683/2*I*c)} + 44227428395238267626454799570572435415401220549517
 03787862113937904048568048286850969056460218125357089349296008547181*a²*e⁻⁽
 681/2*I*c) + 5689615909970301520108319640363973134470190817749618201541040
 554671725803726189842972146004682392121289650040476771616*a²*e^{-(679/2*I*c)}
 + 725483608490846840464667844698960066507970803851351004856353722566938646

3363803374148622157625354861291347122905242320*a²*e^(677/2*I*c) + 91696728
 229777439771113208045411119046602732215064516519497072120559594660125051912
 20572773289655701792453230423105120*a²*e^(675/2*I*c) + 1148920746745549934
 448241874547815556301476112681612608053738346053162938630952263424011753683
 7763365043954166727248280*a²*e^(673/2*I*c) + 14271347645749226165714452377
 611116721623327963230348323460525493356318465738007926444717352691455157582
 924190074684640*a²*e^(671/2*I*c) + 175754962021387432374407024893184133486
 916710882090509569636135113669061260825212723946071155159361725255528913953
 95672*a²*e^(669/2*I*c) + 2146086962260379837386229537599197984426152702059
 4958201034758080773685660129127602861264204469990486648515185847661920*a²*
 e^(667/2*I*c) + 25984481821928790229019533220352810364053651428139405421011
 946723503931316803858639430225998735288638633019702866058680*a²*e<sup>(665/2*I
 *c)</sup> + 311988333994649835518980708243099585623900143132018775476726166507416
 08758923820402191204459236862231482346227933403280*a²*e^(663/2*I*c) + 3714
 937280284471372854337396169851190966768613631268053254500725448394335264764
 7881308788995351984175084901730678814480*a²*e^(661/2*I*c) + 43871822610861
 004795016564148131246645253380769718394461842295211859774351397481825053002
 145790551685375056214564880600*a²*e^(659/2*I*c) + 513894893405801540744982
 433433368737174296059038429613694759362607497318493136542768834775174834900
 16178128812433612320*a²*e^(657/2*I*c) + 5971069654214099733499822463934954
 037172606730532701585861325559900383461690575046424945947145442892976765968
 0543253880*a²*e^(655/2*I*c) + 68826496133929259333946349688756107181827094
 760687431541236793707609567157393356980015874728986445505375939321119773600
 *a²*e^(653/2*I*c) + 787088197407479900045877912467552903619923334159985208
 20275403809147240788340833226353369635194374316050644673527393080*a²*e<sup>(65
 1/2*I*c)</sup> + 8930922836251002488865020427401847962327893182922275165186927199
 7142940935939622499464941302023892783243036531939874720*a²*e^(649/2*I*c) +
 10055840372226804064251142934458723637594333986558344676841323547039010936
 7198695944619840576272225434857726514180226640*a²*e^(647/2*I*c) + 11236649
 759080445936801422982106157414808468612499049009642615813161923929281935665
 2354755191270559061866632206618123360*a²*e^(645/2*I*c) + 12462441665884060
 650876497209124758205675601440438158967894681087078534979153332713768691858
 2746592580804518495020202620*a²*e^(643/2*I*c) + 13720607154545038344270887
 404879438390091451812909804275801017418553270717230931330494545677429537124
 6074471543553060720*a²*e^(641/2*I*c) + 14997156173919430999584728565110821
 891121203463153131574047159901379426073295280466523230232892799172576152277
 0551792260*a²*e^(639/2*I*c) + 16277120705520291057493434540877352031721777
 289667694933591906272895139614138882801239915060213291933844861367859453900
 0*a²*e^(637/2*I*c) + 17545027470193203951906728672508040358898492677723748
 8421428048082175541862462852436250221253619940939635112553736957060*a²*e<sup>(
 635/2*I*c)</sup> + 18785419391079380803182410761746005914368278715083892101509503
 0626546459146628006540234377528430150128575756995169364560*a²*e<sup>(633/2*I*c
)</sup> + 19983400204498266916522184729468752220430459508834721990203664759739174
 0878128463392404040741222584961747206539785461480*a²*e^(631/2*I*c) + 21125
 173165589950328088911626057042795396260205184728323664763967057947249888119

5477410930866524432145488946958371101960*a²*e^(629/2*I*c) + 22198543089973
 567155340415906156873742212153996951840077939879427635377855440500118614320
 6492042172329004119602578495740*a²*e^(627/2*I*c) + 23193351279523436375472
 696826360991233102884214078718404407880905464893342408711586756776943726149
 9520146801998022772220*a²*e^(625/2*I*c) + 24101815333263099088298949586644
 564555630423082321126973108730787391177181463118017236056346056223133354877
 4668344293540*a²*e^(623/2*I*c) + 24918750414819434725909868038356555997935
 585867179616168647279674842644285963344362604891662152069336530849490595486
 3420*a²*e^(621/2*I*c) + 25641655035571186813578152461621262215024659683390
 8848844513195156955143995464929738486051818152589364681978419136195020*a²*
 e^(619/2*I*c) + 26270652450562973290005264246376104689736012808149252071960
 2062874621173097355113088395553861512842828529818716754053220*a²*e<sup>(617/2*
 I*c)</sup> + 26808287839028304188417404539860545935372851851755842090932439522551
 2576313595290722225856442317049534262750408159248640*a²*e^(615/2*I*c) + 27
 259190927757826416604429945939383077223969694779872067129066148813406385032
 4048969486969810581354040036172071321899160*a²*e^(613/2*I*c) + 27629622923
 023628717938300412394666859690541421012540515142680436801496772953432553495
 2787742280869431062101106717052160*a²*e^(611/2*I*c) + 27926934844149146717
 871461767007852072485044101499583790483891868945612576508612585857864549684
 9567201719177718577500300*a²*e^(609/2*I*c) + 28158970945497466772186667262
 401107988729716139977730660882276932284913638029650255132578719913962620739
 2754615890356320*a²*e^(607/2*I*c) + 28333455324151133802032867104842078622
 362525789288688458981537285116646961980068595574307376808759628667556270992
 1649100*a²*e^(605/2*I*c) + 28457401642077142070419359974824258973200896044
 1520037039680402737468153733442912103350926971108090433015340259175973280*a
²*e^(603/2*I*c) + 28536584939830721528882658066297886584159551927919722832
 5469882426298598034988143049654644002348988616277221990835896860*a²*e<sup>(601
 /2*I*c)</sup> + 28575110793326195218584481986699360496526973375720612020384053489
 8677511720614812627553159457956333256326887429379709240*a²*e^(599/2*I*c) +
 28575110793326195218584481986699360496526973375720612020384053489867751172
 0614812627553159457956333256326887429379709240*a²*e^(597/2*I*c) + 28536584
 939830721528882658066297886584159551927919722832546988242629859803498814304
 9654644002348988616277221990835896860*a²*e^(595/2*I*c) + 28457401642077142
 070419359974824258973200896044152003703968040273746815373344291210335092697
 1108090433015340259175973280*a²*e^(593/2*I*c) + 28333455324151133802032867
 104842078622362525789288688458981537285116646961980068595574307376808759628
 6675562709921649100*a²*e^(591/2*I*c) + 28158970945497466772186667262401107
 988729716139977730660882276932284913638029650255132578719913962620739275461
 5890356320*a²*e^(589/2*I*c) + 27926934844149146717871461767007852072485044
 101499583790483891868945612576508612585857864549684956720171917771857750030
 0*a²*e^(587/2*I*c) + 27629622923023628717938300412394666859690541421012540
 5151426804368014967729534325534952787742280869431062101106717052160*a²*e<sup>(
 585/2*I*c)</sup> + 27259190927757826416604429945939383077223969694779872067129066
 1488134063850324048969486969810581354040036172071321899160*a²*e<sup>(583/2*I*c
)</sup> + 26808287839028304188417404539860545935372851851755842090932439522551257

6313595290722225856442317049534262750408159248640*a²*e^(581/2*I*c) + 26270
 652450562973290005264246376104689736012808149252071960206287462117309735511
 3088395553861512842828529818716754053220*a²*e^(579/2*I*c) + 25641655035571
 186813578152461621262215024659683390884884451319515695514399546492973848605
 1818152589364681978419136195020*a²*e^(577/2*I*c) + 24918750414819434725909
 868038356555997935585867179616168647279674842644285963344362604891662152069
 3365308494905954863420*a²*e^(575/2*I*c) + 24101815333263099088298949586644
 564555630423082321126973108730787391177181463118017236056346056223133354877
 4668344293540*a²*e^(573/2*I*c) + 23193351279523436375472696826360991233102
 884214078718404407880905464893342408711586756776943726149952014680199802277
 2220*a²*e^(571/2*I*c) + 22198543089973567155340415906156873742212153996951
 8400779398794276353778554405001186143206492042172329004119602578495740*a²*
 e^(569/2*I*c) + 21125173165589950328088911626057042795396260205184728323664
 7639670579472498881195477410930866524432145488946958371101960*a²*e<sup>(567/2*
 I*c)</sup> + 19983400204498266916522184729468752220430459508834721990203664759739
 1740878128463392404040741222584961747206539785461480*a²*e^(565/2*I*c) + 18
 785419391079380803182410761746005914368278715083892101509503062654645914662
 8006540234377528430150128575756995169364560*a²*e^(563/2*I*c) + 17545027470
 193203951906728672508040358898492677723748842142804808217554186246285243625
 0221253619940939635112553736957060*a²*e^(561/2*I*c) + 16277120705520291057
 493434540877352031721777289667694933591906272895139614138882801239915060213
 2919338448613678594539000*a²*e^(559/2*I*c) + 14997156173919430999584728565
 110821891121203463153131574047159901379426073295280466523230232892799172576
 1522770551792260*a²*e^(557/2*I*c) + 13720607154545038344270887404879438390
 091451812909804275801017418553270717230931330494545677429537124607447154355
 3060720*a²*e^(555/2*I*c) + 12462441665884060650876497209124758205675601440
 4381589678946810870785349791533327137686918582746592580804518495020202620*a
²*e^(553/2*I*c) + 11236649759080445936801422982106157414808468612499049009
 6426158131619239292819356652354755191270559061866632206618123360*a²*e<sup>(551
 /2*I*c)</sup> + 10055840372226804064251142934458723637594333986558344676841323547
 0390109367198695944619840576272225434857726514180226640*a²*e^(549/2*I*c) +
 89309228362510024888650204274018479623278931829222751651869271997142940935
 939622499464941302023892783243036531939874720*a²*e^(547/2*I*c) + 787088197
 407479900045877912467552903619923334159985208202754038091472407883408332263
 53369635194374316050644673527393080*a²*e^(545/2*I*c) + 6882649613392925933
 394634968875610718182709476068743154123679370760956715739335698001587472898
 6445505375939321119773600*a²*e^(543/2*I*c) + 59710696542140997334998224639
 349540371726067305327015858613255599003834616905750464249459471454428929767
 659680543253880*a²*e^(541/2*I*c) + 513894893405801540744982433433368737174
 296059038429613694759362607497318493136542768834775174834900161781288124336
 12320*a²*e^(539/2*I*c) + 4387182261086100479501656414813124664525338076971
 8394461842295211859774351397481825053002145790551685375056214564880600*a²*
 e^(537/2*I*c) + 37149372802844713728543373961698511909667686136312680532545
 007254483943352647647881308788995351984175084901730678814480*a²*e<sup>(535/2*I
 *c)</sup> + 311988333994649835518980708243099585623900143132018775476726166507416

08758923820402191204459236862231482346227933403280*a²*e^(533/2*I*c) + 2598
448182192879022901953322035281036405365142813940542101194672350393131680385
8639430225998735288638633019702866058680*a²*e^(531/2*I*c) + 21460869622603
798373862295375991979844261527020594958201034758080773685660129127602861264
204469990486648515185847661920*a²*e^(529/2*I*c) + 175754962021387432374407
024893184133486916710882090509569636135113669061260825212723946071155159361
72525552891395395672*a²*e^(527/2*I*c) + 1427134764574922616571445237761111
672162332796323034832346052549335631846573800792644471735269145515758292419
0074684640*a²*e^(525/2*I*c) + 11489207467455499344482418745478155563014761
126816126080537383460531629386309522634240117536837763365043954166727248280
*a²*e^(523/2*I*c) + 91696728229777439771113208045411190466027322150645165
1949707212055959466012505191220572773289655701792453230423105120*a²*e<sup>(521
/2*I*c)</sup> + 72548360849084684046466784469896006650797080385135100485635372256
69386463363803374148622157625354861291347122905242320*a²*e^(519/2*I*c) + 5
689615909970301520108319640363973134470190817749618201541040554671725803726
189842972146004682392121289650040476771616*a²*e^(517/2*I*c) + 442274283952
382676264547995705724354154012205495170378786211393790404856804828685096905
6460218125357089349296008547181*a²*e^(515/2*I*c) + 34074212854556940464743
207981590900757987299248620802096503284380675040168815980677078443962425318
90195544301426987190*a²*e^(513/2*I*c) + 2601702122396645141345931033404570
995027466204284274488653750753761498675668863970880304480234793529924712351
195427555*a²*e^(511/2*I*c) + 196860807418766241096038612492180776083941991
2449118088079440479378830696763837592112817321648516678192250226251889260*a
^2*e^(509/2*I*c) + 14760579955693906372233526972424832105698618627805899233
17140914906186165044966041302091938311768459791677885305942531*a²*e<sup>(507/2
*I*c)</sup> + 1096636578459444916376723286069829634240829040283071984682114965507
192138829786266640857980171465345670412137298803158*a²*e^(505/2*I*c) + 807
253658137342307900160595544078420189813338397513951459580649551593816718094
436044450016924199349627155306092503566*a²*e^(503/2*I*c) + 588732911424652
432036475218676267391833829587071733581552653599569115062655958634436180105
642268527576118276535857990*a²*e^(501/2*I*c) + 425364061290972864599381605
241976913026242892352334115185854404956644313805430315468685902485280907282
658089881714375*a²*e^(499/2*I*c) + 304446393227146814613983539727444798173
564858374980413080488271135256661913384313342183872402332578752912321480097
437*a²*e^(497/2*I*c) + 215845003776805302861184692565927001383144850278090
577691260137355499134974361313448026444380882959804366246287914973*a²*e<sup>(4
95/2*I*c)</sup> + 151575170355739209946373876370057650549318320584509041952034589
942144397207049663095445023255855799320406828152904813*a²*e^(493/2*I*c) +
105424853787028754469673960861051848077064379179201408687509167013295588338
384945425067268491968039006968056581921707*a²*e^(491/2*I*c) + 726207992189
165652178758713813393428748038401335313762033098057750677977846026209886348
08343998866415251767099657115*a²*e^(489/2*I*c) + 4954006065050306303738228
025869904943635196473318092993411744508059663156189621601578097098400509051
6521389882187000*a²*e^(487/2*I*c) + 33466028808642300425453129213091641815
466355026636652947083261177887459487866470370690708668514061085213215020765

$318*a^2*e^{(485/2*I*c)} + 223861204043000044228760073517616478835216103343997$
 $85939038011790449067868627843226239824339519408830910225209114648*a^2*e^{(48$
 $3/2*I*c)} + 1482707884095820442372654670551252735795736164739650863293598893$
 $3241213590716537429203049741620280852461278755893511*a^2*e^{(481/2*I*c)} + 97$
 $232127836885376433044511737909323527633921178781940997132124078279245717796$
 $56265406369365137858973995542668178560*a^2*e^{(479/2*I*c)} + 6312726327493419$
 $321171003213006926022587697863019991394721190170435857084298246006024922226$
 $846516328771260784350055*a^2*e^{(477/2*I*c)} + 405744720573650872049401506257$
 $084267750527414864488161681132670262810709477451426568094071301425635165960$
 $7202219536*a^2*e^{(475/2*I*c)} + 25816271070099007345220647445975512098023044$
 $06910934736678835899008279100485386551512986171247434443617529951824987*a^2$
 $*e^{(473/2*I*c)} + 1625979420531105682414675218243753902020818169524222534201$
 $660329925485571915419014974199094126837464169182082222286*a^2*e^{(471/2*I*c)}$
 $+ 101366358338138231552611990038579378059289736225952188087025668957886456$
 $6731937045334553766119426780595860617854190*a^2*e^{(469/2*I*c)} + 62546997108$
 $251205222144593922633931315752756168645329438560772112500600573787852815482$
 $9176814174073609304783133155*a^2*e^{(467/2*I*c)} + 38197032488897113782426041$
 $711935588050801140530650725271731911163896757370819281494759323796283751658$
 $7367083029056*a^2*e^{(465/2*I*c)} + 23085548667262670972353592787256885447107$
 $7332500022366798083306901537948948135371592062382972028832652892521530223*a$
 $^2*e^{(463/2*I*c)} + 13807505047291146001136685629516855528925718909732467031$
 $1033951735099801993846300774206061209409704454402575173168*a^2*e^{(461/2*I*c}$
 $) + 81720646798485325019374317784322759957844922273569903847642933003662634$
 $138916360473885905820817778748179763625775*a^2*e^{(459/2*I*c)} + 478593950514$
 $785774857771488256226664918441899896712871488953127500310630895970392803422$
 $77891313648282659506765160*a^2*e^{(457/2*I*c)} + 2773312317052806066504779302$
 $918798106933656583450159135548591946017635957045697692669267735665569210490$
 $1055424742*a^2*e^{(455/2*I*c)} + 15900249437791405935404298689959721296914556$
 $453023635621602157015426400883311961742504003269238750112959349688008*a^2*e$
 $^{(453/2*I*c)} + 901903653105413936414503619872987518150278899672116571091285$
 $3819799244035211696760046421655662466728741232371823*a^2*e^{(451/2*I*c)} + 50$
 $610999389706962454400974629365040600982341948678910998000678794666066919270$
 $34473565491643439734586575243562587*a^2*e^{(449/2*I*c)} + 2809543518668204266$
 $823796801627118763645888741191307504647885398224587315004606447902339962521$
 $984923603337579865*a^2*e^{(447/2*I*c)} + 154280209465184867438692967750633613$
 $530145175778216248102935419137838565530983959034229578207026724816379707952$
 $5*a^2*e^{(445/2*I*c)} + 83800340563322626169800247516502885996183656047744566$
 $0978330588882054803408792968277041826209840110522910353801*a^2*e^{(443/2*I*c}$
 $) + 45021495315848243687744932353444363642498768967317657680863727749130779$
 $7447679834039022495671246539361123241623*a^2*e^{(441/2*I*c)} + 23922690108635$
 $111250191342017664138814183468071724314914824054191236502403524239663014182$
 $7067823768453577792206*a^2*e^{(439/2*I*c)} + 12571682379700564231734752649840$
 $179083753168561102364880492546544906525540804103230843956650070870060269776$
 $6710*a^2*e^{(437/2*I*c)} + 65335209855139745225460458167882643715464174702379$
 $903424415436335141712932934847111707604519947532103535939690*a^2*e^{(435/2*I$

$\ast c) + 335775328625086124049321435620567581695125431787234427844444132424169$
 $94607618240798840372204654901124952727575\ast a^2\ast e^{(433/2\ast I\ast c)} + 1706376394381$
 $920125715454600794419048480048354121962557033477313920025821893363452876459$
 $0082974465232719019576\ast a^2\ast e^{(431/2\ast I\ast c)} + 85743704360612785883233297319820$
 $522324457178287917559631704662380337982987981796650963553464314446951205167$
 $91\ast a^2\ast e^{(429/2\ast I\ast c)} + 4259980611481175181529614616968079288266998347286916$
 $867765191606402809238614481985333675831806087956768872250\ast a^2\ast e^{(427/2\ast I\ast c)}$
 $+ 209250927524733972545755390380592999373493854772180445419707199950551171$
 $9948252419379343549397905586122452665\ast a^2\ast e^{(425/2\ast I\ast c)} + 10161498252249284$
 $152824301164924261712421925679090373120103688981746334514762777018386807593$
 $16125015139743120\ast a^2\ast e^{(423/2\ast I\ast c)} + 4878144325211114975854067195068020556$
 $36922087949888627025208454447300100038791012297859131526297355904933960\ast a^2$
 $\ast e^{(421/2\ast I\ast c)} + 2314908265850057878077532735085387653932377948915894807342$
 $88576865823349858144380580905331957850509282551472\ast a^2\ast e^{(419/2\ast I\ast c)} + 1085$
 $851881577139421210778517898957224225946890187204550705634761213011847278115$
 $74323466214291834201854672060\ast a^2\ast e^{(417/2\ast I\ast c)} + 5034312554671323798562331$
 $364632933064785834949562749292520516156669300161065056996742990272612606266$
 $9565040\ast a^2\ast e^{(415/2\ast I\ast c)} + 23068393515288186064484465603117961911284673754$
 $084184347491242043413536449530894861283359363464825007526940\ast a^2\ast e^{(413/2\ast I$
 $\ast c)} + 104466389336129644073368725603779571652443519950184462203215355985963$
 $68046659626683803611150049940561221040\ast a^2\ast e^{(411/2\ast I\ast c)} + 4675111642548746$
 $091915670754076032371176136454036359982075221477297809925204476666662239031$
 $601034545996300\ast a^2\ast e^{(409/2\ast I\ast c)} + 206746084187352999911842026513428376396$
 $4445212656031557399579185390572143523605908612468666220745228526760\ast a^2\ast e^{($
 $407/2\ast I\ast c)} + 90341197663448354263528999846283396366969515218525999665486805$
 $1991473572372870248662847924612744264062440\ast a^2\ast e^{(405/2\ast I\ast c)} + 39004127275$
 $497477700562769915484224698872602460416243802409528559485248785585037207054$
 $6472734126022684700\ast a^2\ast e^{(403/2\ast I\ast c)} + 16637342101596006574494683160512963$
 $8085441168071352620640483463800291640278014342520469741317600337261840\ast a^2\ast$
 $e^{(401/2\ast I\ast c)} + 70109686530806379548962065855267314803677176495241796951534$
 $731669884594114550121278447472661295579702060\ast a^2\ast e^{(399/2\ast I\ast c)} + 291853134$
 $486539174414648904873231895332335805583860267345706150294570279334915127387$
 $53237134630607181520\ast a^2\ast e^{(397/2\ast I\ast c)} + 1200090570376819817495428203463175$
 $7209497411914073482804705515897689969997576557491508854781156886995980\ast a^2\ast$
 $e^{(395/2\ast I\ast c)} + 48741350791327554143473756908878027117941056509388639704002$
 $30002709422093992525472165364797296790138960\ast a^2\ast e^{(393/2\ast I\ast c)} + 1955172094$
 $293269164466209803317102552595021647581151217877842297059127002495113810772$
 $654784644585060360\ast a^2\ast e^{(391/2\ast I\ast c)} + 774543421614203495600971615466077979$
 $652368881218933510424294240021409386956035526159745861544320722480\ast a^2\ast e^{(3$
 $89/2\ast I\ast c)} + 303004292271763094745720039936334571016895425314693546495893805$
 $386105536448400356863388712154386414250\ast a^2\ast e^{(387/2\ast I\ast c)} + 117047543581815$
 $187062673321422303561789600917165284254959414932144762194096889890432439125$
 $557135120080\ast a^2\ast e^{(385/2\ast I\ast c)} + 446430644715917174797040202765636425707178$
 $83826668145169247611761065722522854988204793268950962007830\ast a^2\ast e^{(383/2\ast I\ast$
 $c)} + 1681089672822596187419449834022231451822327663163619384818770077447850$

9776074186756740832022063618940*a²*e^(381/2*I*c) + 62494140129289622860610
 104035290027536073423609060861920331851034284009130708890522929404847123278
 30*a²*e^(379/2*I*c) + 2293321308015586917932799317024982339168604649369171
 514848750864257523843794511936002538498981041920*a²*e^(377/2*I*c) + 830677
 89082340559388104802804137554233955283073962626784718312376094416760253547
 591206118659958460*a²*e^(375/2*I*c) + 296966005156563531105971074940050295
 210952963342292128773450715502535457684406378925697100618058220*a²*e<sup>(373/
 2*I*c)</sup> + 104773294919708181426456318246707841758575517441039866971803838079
 319268269378919589734008354661810*a²*e^(371/2*I*c) + 364776537060422459446
 888206046964942190813347671210579123319141378044153079733377939021772945992
 10*a²*e^(369/2*I*c) + 1253133822792085649207459032667409937445312286891351
 6861854565249779642744578491059412040960382510*a²*e^(367/2*I*c) + 42474009
 250773731761134863121901454721816864229939339424476328466330956444400320033
 39271168690570*a²*e^(365/2*I*c) + 1420248786518187466718023048901731118377
 946854300811977671072304411278973969440001583221282013130*a²*e^(363/2*I*c)
 + 468468261824774958091791071955357916616933320342602739423768916749249443
 401418804336754943102630*a²*e^(361/2*I*c) + 152415374106074622939224284153
 659601550627109436325926621014480229333534862467810217119890170400*a²*e<sup>(3
 59/2*I*c)</sup> + 489066090725200576110713705079963402247766813797750349258682009
 23794664214854540668184735223060*a²*e^(357/2*I*c) + 1547578971739746027623
 4843111043950695013471030625427925175353797112876241958974587338558471200*a
²*e^(355/2*I*c) + 48288072371114951708502020267955265600327550456915416538
 50695089412640252061166703373641008090*a²*e^(353/2*I*c) + 1485535118953138
 948865978331717910599497456296177792374785590516233474655906439549342970114
 480*a²*e^(351/2*I*c) + 450541815994350416329563482655909244979320334438405
 914968411406156252958317001977289679320570*a²*e^(349/2*I*c) + 134694045831
 733084831574170523497510104128836035403576353015982048577081438406747258639
 643920*a²*e^(347/2*I*c) + 396893152747629625983440995776573121457424082278
 26431282422824844089957174694332245240202130*a²*e^(345/2*I*c) + 1152551433
 824812211580690122363140886566506087399233173121671754583964139549354347855
 5391940*a²*e^(343/2*I*c) + 32980470539939501466208425037215290583184853566
 60075016582248017786240344190976379491032900*a²*e^(341/2*I*c) + 9298442698
 396182432776277869232434146236637762612203148716791997224555846984342168195
 69010*a²*e^(339/2*I*c) + 2582653199983859208005705893482429381264459621216
 95949028377963603738714515662920857912400*a²*e^(337/2*I*c) + 7065920222040
 221969932714247248024228787665376495330213336037644828722618748734564484553
 0*a²*e^(335/2*I*c) + 19039736700477034191914417874408695155655405986129726
 283796391182602876937826158174743280*a²*e^(333/2*I*c) + 505224056037277225
 2508411884469329618226376927979062996145236110491773700805441539104570*a²*
 e^(331/2*I*c) + 13200115147374356440392521856145378423255313177841019066397
 27300233020372137428050228960*a²*e^(329/2*I*c) + 3395315651658181574881115
 46236514739245172484736482729741196732548715077274621271815060*a²*e<sup>(327/2
 *I*c)</sup> + 8596620779574277116311008408307932619869205078119881831903736271678
 5215369332723227360*a²*e^(325/2*I*c) + 21421765289837057403869596169941475
 678645506492471864168704518082797513207707602578070*a²*e^(323/2*I*c) + 525

285606278183393820149086942244493046474642337458887176544669032400994119860
0912010*a²*e^(321/2*I*c) + 12673001919437373241193205993496836544933488502
17007321150739273112190995849189116570*a²*e^(319/2*I*c) + 3007712344684525
20266562607329665922159115381500087728345930006292005623202669775630*a²*e^(317/2*I*c) + 7020901794541159120096165173005886355772245887709838148693864
1100483455911795967290*a²*e^(315/2*I*c) + 16116628162411464679896502836324
771389620584856574877922468282685664283130175327730*a²*e^(313/2*I*c) + 363
749864602642559874527922880367951714009035024131421637157046817473817492130
7980*a²*e^(311/2*I*c) + 80704776818833353884668480346077979093105555510012
0589990758127536168861246233500*a²*e^(309/2*I*c) + 17598731744416001266512
3402745831252173155432052395534884810603675523612022791280*a²*e^(307/2*I*c)
) + 37710710670742203746980267936076214521784550649912575518820552220031814
445215110*a²*e^(305/2*I*c) + 793893332049744963591896968075832630733911623
9071846330248311795038723075723660*a²*e^(303/2*I*c) + 16416623480954603635
14356729639026504243315309578757871628322407537121464954630*a²*e^(301/2*I*c)
) + 3333781067405861382516101249587967145769350996198350731723076965516482
59829120*a²*e^(299/2*I*c) + 6647008027203790368219821983548894494999674907
0187078822650198449544212324410*a²*e^(297/2*I*c) + 13009255453778977514917
501132236907767356887691068368589411338908245552368240*a²*e^(295/2*I*c) +
249869791757989597797872358857628393313729651874348110477693256639157945540
0*a²*e^(293/2*I*c) + 47087397983316545444462765980310196943086715347166332
9279188629910819335440*a²*e^(291/2*I*c) + 87039467228531013705409191271452
383389335356884921808641492423363762400060*a²*e^(289/2*I*c) + 157774046399
75259272454933843539474544665236819117566997318672304666566800*a²*e^{(287/2}
*I*c) + 2803793220826297874163429682131102503306549005265399219277240917413
502940*a²*e^(285/2*I*c) + 488344836125981471048169163657616516954158792795
618978221198980949925840*a²*e^(283/2*I*c) + 833397477364691435324280733107
62163525113789344043183602916613227872140*a²*e^(281/2*I*c) + 1393134019493
3935228013466294179987445068758219565436862349156508804360*a²*e^{(279/2*I*c}
) + 2280416002497499972330270356628053500702221025770502602102780787687624*
a²*e^(277/2*I*c) + 3654062378495182483720609604635816509956518425496900713
52678851012540*a²*e^(275/2*I*c) + 5729719583057947988090528375988731001569
2255967368709515502818525680*a²*e^(273/2*I*c) + 87889188807866875592675554
13796199314981595789787074901006933535500*a²*e^(271/2*I*c) + 1318333654990
888899417746520374288649257223133573127905086467548080*a²*e^(269/2*I*c) +
193303514872786115923379716628253419318944176435876056802589865772*a²*e⁽²
67/2*I*c) + 276954647427069279961578912753426434310409060476967429402343521
12*a²*e^(265/2*I*c) + 3875741803263271608578048125955066152785493897151672
716099440360*a²*e^(263/2*I*c) + 529533489706434824451250188592739498197462
491764283044018575440*a²*e^(261/2*I*c) + 706043487244011709588296052059370
63913034447082349890348162895*a²*e^(259/2*I*c) + 9182633302160212452918968
603806828036823248995981353166570702*a²*e^(257/2*I*c) + 116436703282462307
336685138251309912040093060141928716087681*a²*e^(255/2*I*c) + 14387278222
0632026353482783145282086606314049235674031853952*a²*e^(253/2*I*c) + 17314
186832105700122787937106308399484490093525695416858785*a²*e^(251/2*I*c) +

2028231682527169820869698431068546224630661674135401655870*a²*e^(249/2*I*c)
) + 231137354125616547233443142991124898825470770660317194554*a²*e^(247/2*I*c)
 + 25608953102811736020699671981400531591652082353456701666*a²*e^(245/2*I*c)
 + 2756769770542048744355911382380396555656358932078858681*a²*e^(243/2*I*c)
 + 288136830654984406530837787337700439328183658059646399*a²*e^(241/2*I*c)
 + 29219499075263535843858862377648073071285310974657915*a²*e^(239/2*I*c)
 + 2872702692034937563897971496575074831246739185614575*a²*e^(237/2*I*c)
 + 273590663366155266234104272169452688263007484513781*a²*e^(235/2*I*c)
) + 25219245783549721366067799114158978864399924058441*a²*e^(233/2*I*c) +
 2247954664299349273623637155894975599108753667672*a²*e^(231/2*I*c) + 19357
 3846127814805336166214026334290497006915130*a²*e^(229/2*I*c) + 16086467322
 724552698501340975783163797842534840*a²*e^(227/2*I*c) + 128869477345205404
 6771640788950472807542901137*a²*e^(225/2*I*c) + 99403445980961943589598468
 520732983548550704*a²*e^(223/2*I*c) + 737333207979913363906365861452764589
 4565137*a²*e^(221/2*I*c) + 525223629498383939328992961505581135597600*a²*
 e^(219/2*I*c) + 35875929609390406738394666219971221205725*a²*e^(217/2*I*c)
 + 2346109768759347903572910732245570209842*a²*e^(215/2*I*c) + 14663185721
 3698093058440158830415565266*a²*e^(213/2*I*c) + 87422785991472952854807588
 01384765381*a²*e^(211/2*I*c) + 496183373555284804627911954521619600*a²*e^(209/2*I*c)
 + 26748429587445855729768539197182585*a²*e^(207/2*I*c) + 13661
 83222241782313586863286641184*a²*e^(205/2*I*c) + 6592841254886628690277902
 2351001*a²*e^(203/2*I*c) + 2996746014847853620317693731016*a²*e^(201/2*I*c)
 c) + 127861163009333998455801820954*a²*e^(199/2*I*c) + 5100844261395996953
 829648360*a²*e^(197/2*I*c) + 189421272128521178822066445*a²*e^(195/2*I*c)
 + 6514488191198508953598437*a²*e^(193/2*I*c) + 206263478282239243233771*a²*
 e^(191/2*I*c) + 5970784896604221606627*a²*e^(189/2*I*c) + 1567135143331
 71921595*a²*e^(187/2*I*c) + 3692203217135946825*a²*e^(185/2*I*c) + 771217
 38215879370*a²*e^(183/2*I*c) + 1405865019554882*a²*e^(181/2*I*c) + 219095
 84720322*a²*e^(179/2*I*c) + 283802910885*a²*e^(177/2*I*c) + 2933363420*a²*
 e^(175/2*I*c) + 22680645*a²*e^(173/2*I*c) + 116610*a²*e^(171/2*I*c) + 2
 99*a²*e^(169/2*I*c))/(e^(517*I*c) + 418*e^(516*I*c) + 87153*e^(515*I*c) +
 12085216*e^(514*I*c) + 1253841160*e^(513*I*c) + 103818048048*e^(512*I*c) +
 7146142307307*e^(511*I*c) + 420601518659718*e^(510*I*c) + 21608403021340047
 *e^(509*I*c) + 984382804329835768*e^(508*I*c) + 40261256699368950388*e<sup>(507
 *I*c)</sup> + 1493326612293984160368*e^(506*I*c) + 50648660944512569972179*e<sup>(505
 *I*c)</sup> + 1581796642397812408161814*e^(504*I*c) + 45759117183402579073139583*
 e^(503*I*c) + 1232445557346832245176696904*e^(502*I*c) + 310422225220746816
 15625020522*e^(501*I*c) + 734057263616388449968842366924*e^(500*I*c) + 1635
 3164647151530240529137618111*e^(499*I*c) + 34427715201287513414073930296091
 4*e^(498*I*c) + 6868329225263681349501997341320517*e^(497*I*c) + 1301711930
 79172823835151430773360024*e^(496*I*c) + 2348998374244347079532766203075607
 598*e^(495*I*c) + 40443624781415311581857832389099634564*e^(494*I*c) + 6656
 34670676210063754191847109971141414*e^(493*I*c) + 1049040266951089742462464
 3766470754045064*e^(492*I*c) + 158566476113257562566117432227203884298856*e<sup>(491
 *I*c)</sup> + 2302150411226234925855222345201500900533576*e^(490*I*c) + 3214

7887693375338817454482515377350383950278*e^(489*I*c) + 43233368864426155754
7944179250800440604964868*e^(488*I*c) + 56059272530675585517804528836898355
14455118670*e^(487*I*c) + 70164515322544462906873548813748091084561870680*e
^(486*I*c) + 848552202276512356496200136959676295361696315113*e^(485*I*c) +
9925490738534402272939987038714580495445431374618*e^(484*I*c) + 1123916045
42246650966429162063124338952554575234051*e^(483*I*c) + 1233096700139723365
181997220750932590655287625342156*e^(482*I*c) + 131187818011721747296793398
94318153694964675368481194*e^(481*I*c) + 1354425949166361161915746506253316
46238501101627937224*e^(480*I*c) + 1357990663161479842850642848032544982878
359839580349899*e^(479*I*c) + 132317088701048969738000567337799190893408367
56009580718*e^(478*I*c) + 1253704965869212726621980508512693231711673388540
81782959*e^(477*I*c) + 1155855412893594260345544966642687823630035899363232
371472*e^(476*I*c) + 103751844998711755019093989565966841168029970825266603
23524*e^(475*I*c) + 9072260572220881491864228463948718776460758970649397077
4776*e^(474*I*c) + 77320463699114577506146273102809850609443267578813629501
1259*e^(473*I*c) + 64261954855352485764250681368704655300871140038757166913
83902*e^(472*I*c) + 5210811762917704866049240098517583098750570056687781895
4141639*e^(471*I*c) + 41243069829991519084806722232721943506774793409189467
0488982928*e^(470*I*c) + 31877499297443464972115360447517765823209586279238
16470590659024*e^(469*I*c) + 2407080191352975710185802291437204586474699178
6182039740274325264*e^(468*I*c) + 17764282913511934857719443767580283023990
5460092687136494961404333*e^(467*I*c) + 12818174649149708108596041898283590
00790789921169405304612211251818*e^(466*I*c) + 9046693523825682979044338963
104263167672586826367911338826483549173*e^(465*I*c) + 624735507810532953177
10774690247114124125187565731848441781904032672*e^(464*I*c) + 4222761266320
03687547754746555709988710527133086660161366353656787288*e^(463*I*c) + 2794
709104475686611842790694973699164482254723977210209725661304403472*e^(462*I
*c) + 181157684956157580767103030555056255892542936591933141534183339445964
08*e^(461*I*c) + 1150514818520808488737003883545213155676403651240031036911
76697194292320*e^(460*I*c) + 7160994975990580798956333385529402291928581964
81597830078819711862600096*e^(459*I*c) + 4369442482910113914565353136069595
862669338858053419381214131241925047008*e^(458*I*c) + 261439762799020214434
71945665080254563056810183520401889800285493144867448*e^(457*I*c) + 1534360
88745056254127327239461577071933130157764595997113973513183188399376*e^(456
*I*c) + 8835009688217912026007745419277692007376893935137347893683970933333
11961880*e^(455*I*c) + 4992519712457043983505377976607953988397368297591114
957991804893688371867680*e^(454*I*c) + 276931165383432592259833826376479361
22664033859615133489846664694361471028310*e^(453*I*c) + 1508223814314124137
73566474210011746852297437597059186295243989481140398152780*e^(452*I*c) + 8
066795436075891407593050107961895682698420216133889552189162788231826394881
90*e^(451*I*c) + 4238125846763232586394188569858685826755328005548627437019
301405851325887594480*e^(450*I*c) + 218764828927139099280403456125787058051
21508756226696317087651824252241418663320*e^(449*I*c) + 1109691996873209747
49922259595250444341219218535349655762591192576535872151766080*e^(448*I*c)
+ 5532691288195286125029188695589478290980219563093498435840446315122917788

00081490*e^(447*I*c) + 2711843239670717527605640490148833507130242448403978
 318523237721944200392830108580*e^(446*I*c) + 130698172034882898861932055083
 75818392124991382340160316886507181296548981014818410*e^(445*I*c) + 6194859
 665303550287956433881523431066041090203788247316180477449291621657588007768
 0*e^(444*I*c) + 28882075526473065446996857202104710942731861950899580202068
 9904590319476295408324280*e^(443*I*c) + 13247564123678374731574728211624836
 91120966501948953926492241643788264284546437221120*e^(442*I*c) + 5978992172
 944143218459161149299819706321732111578494525245228742976468409105395536290
 *e^(441*I*c) + 265568063890434075344967023691015457959948617577414147899446
 52712127566910185274123140*e^(440*I*c) + 1161045516835550437629115017121163
 99313733021132677481112824047246361794049635726479850*e^(439*I*c) + 4997075
 672538590843575963148137947680693371909159674919074889049339226775796653543
 38960*e^(438*I*c) + 2117589733466855707101501429210414722401838837940752841
 618541440888545729943138209036820*e^(437*I*c) + 883672064086047030569451402
 1547969551296794092266983044118375790025854584036796364768280*e^(436*I*c) +
 36318369652302591732197444409798122022640824604130552506742586795183267354
 382847875885730*e^(435*I*c) + 147030816732276833163041582099592047512043725
 225353339238819165193000407629544745753221740*e^(434*I*c) + 586403466972683
 242741643328921560909375197453864243299571990964608857245771134145204174990
 *e^(433*I*c) + 230435107337384035737917859767306635201668278168913984209737
 6663118488803841131935313641840*e^(432*I*c) + 89232094473432967633318818816
 38471793499618670601026059730895962653291770229493028162575100*e^(431*I*c)
 + 3405405385129556915435234672217717265518754891078200850471832416872502943
 8589162349211628040*e^(430*I*c) + 12809891460168853967248054183040984770736
 7500438601536803204497701119911289087105659482783340*e^(429*I*c) + 47501057
 885760151927231661793842522242178659724167102689431851540851146714096939311
 5768793680*e^(428*I*c) + 17365742188181910718741974724501581238835642099506
 58639102337148122769080611680719741726053840*e^(427*I*c) + 6259872156822252
 843650960708235034710201362776057176647226323089751446565288850103898153859
 920*e^(426*I*c) + 222519591767957777571673660360074802222113642321463998038
 64370963391491223687245823457351580140*e^(425*I*c) + 7800980736802423987561
 3733058851417125327114681070889640794249282633470580756557083923203377160*e^(424*I*c)
 + 26974580144021129697268360186387895435796230852007659517712822
 7629273240215209708218497363414140*e^(423*I*c) + 92008939302958903287460185
 0027159322612526368444771489781974361078847528891468831038436064951920*e<sup>(4
 22*I*c)</sup> + 30961319716215201623803015542414654517823620868102875377489029049
 85934020179565706177131421614590*e^(421*I*c) + 1027936473066384084473957786
 2469262604648861914297972589165243530651230690726244462479199894255180*e<sup>(4
 20*I*c)</sup> + 33675398872021568375902384593982753362559801058104184627345411136
 262431943240778260721756991027090*e^(419*I*c) + 108867995731829472826732905
 192034886797284621356445627530909104429486741257822633476898356826454040*e<sup>(
 418*I*c)</sup> + 347351473214713780874352083129566601238765762775942366762733349
 952103889753982636403857556867777300*e^(417*I*c) + 109385321448622035867403
 243450086667849977001130587417248897595161203145673460828709551950104197544
 0*e^(416*I*c) + 34002325606016516175216946808470898441980288316944174247948

68779328950548418125605446882081152636090*e^(415*I*c) + 1043411751657039596
665369315558240210946034809547302780741232142734681692856719777037649617025
1803940*e^(414*I*c) + 31610939331284692750694306443618414656095969520945215
743004044560386895241801579156543451940713351730*e^(413*I*c) + 945561802589
319869193343034663656528268580913143291891607362771758738417321964533799537
05679466826880*e^(412*I*c) + 2792857558000352066798353688981654776448649877
94665387827488933863633745047373109049265172681702585720*e^(411*I*c) + 8146
081877365305796702100252719214155971833698812142998232919697855498761759698
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899155708935025778047637412681781575765422219127409260159438712250*e^(409*I
*c) + 667586629037114735850376686566928901089354386983053870872494529158095
1179188296606158111257706968604740*e^(408*I*c) + 18759988218865563564163635
735986073278255737257405706279108891366378428467414559930481172863538598193
890*e^(407*I*c) + 520751785187932703864292633515443069511049935425005829381
55241689408138675254608030847907167748571734720*e^(406*I*c) + 1428017924502
217624831808749188252741343051332754177800847950346447635093335031505173458
64659667189417080*e^(405*I*c) + 3868762182342771656324517230499798892631152
82374607541692443176673997513742813591736171169652250611186480*e^(404*I*c)
+ 1035561982592002935226384577908611548612111495080193573691339864706029186
482466241805664949381049856258510*e^(403*I*c) + 273889562479526560335522764
656600088628077830508482570291193890365616200426273618265770040630191407006
2380*e^(402*I*c) + 71581246868429414754738073636798397181727455815384090445
03383852693596921622426696740453944718143025248390*e^(401*I*c) + 1848740529
900573269375272861187649089085835702197488237157062380018624513772266094364
1752976852924439870880*e^(400*I*c) + 47188220843466207695099506953573780357
108897491422567898048199018207708997005333860148836479527456156014520*e^(39
9*I*c) + 119041855403877964948229577948370465600606623183045529526900430209
270473212773847794935586074714329479939280*e^(398*I*c) + 296825515282669589
685318273280239050084555032203415941511962659596881615713799937680026497408
305672297618840*e^(397*I*c) + 731584972206818362874729621403974444280010446
301161527339760544815300951787985538419764656214582667219914080*e^(396*I*c)
+ 178244611493175185055635485663842190117441232229824949659165805393978719
8246565945975595575734193348887952160*e^(395*I*c) + 42932064780080221260174
889088518264947906207206601514514681819109172400278639687245391276596335170
53002976480*e^(394*I*c) + 1022318202595486076721739030518645192356214547367
4293619918063490411487496121804590274592702770571515456414680*e^(393*I*c) +
24068785139705277161193465644506143285241361037768216818922184400141048460
210944696647752723371932874594597328*e^(392*I*c) + 560286834249035176584950
138585345161671625910343679724981746609074506667781543532716303446507778856
83547624184*e^(391*I*c) + 1289670800847547122460236808664883849832862590255
33132044636109049545144029547003347761521666283977931640178464*e^(390*I*c)
+ 2935507435543427098081294535765623132997059826991874168629343739642556159
67138676253276302591561523515603264403*e^(389*I*c) + 6607644731058690976914
759738508379345110890331495867079827642633947667566495652798791461733183865
05740391093990*e^(388*I*c) + 1470931146618934345515038362300100160482127749

581443929904746910224777470198899052379114493999887003199419829579*e^(387*I*c) + 323849193136185147642332193353957909837773553920764146734623566582388
 7048326949305609231585143748690203615957136*e^(386*I*c) + 70521324141621979
 926023265245801430609853530545729339055246331216810210373402983663422033243
 25307072413739061024*e^(385*I*c) + 1518963421490880039641791172264375474804
 8520109734812459109878810493844381062650818971199637121458749456243274416*e
 ^ (384*I*c) + 32362731322419549410330088943640247460378328561316422931292427
 145902887913071643679502909055891236755143207382609*e^(383*I*c) + 682080330
 967936156837844096192442108186149916400415534244055278768932724966083242310
 98148502466453967157728078994*e^(382*I*c) + 1422131159648145176823866672767
 699094822716813187908898405010394417486355453624676798324491035203219530117
 80083069*e^(381*I*c) + 2933449200343007202870423834483428663138062854550400
 67823080445597545970023446231563554135133105493516316320059272*e^(380*I*c)
 + 5986501411122418589116765051805201503640032268413280814535970935877903386
 09212439085554466861582623350303061961052*e^(379*I*c) + 1208770358493658393
 089442222056935063283704108140593750226539846117737648216609559734831601248
 698274330296158612144*e^(378*I*c) + 241496651681033850328907654920274051171
 005901179544713877346420569645502644271242640959966277108026482600898506109
 7*e^(377*I*c) + 47741411110660989702218453305949620164727142303742340606639
 56846950926642685946929064114194400360936223590725470146*e^(376*I*c) + 9339
 341958053494225251750965715057300707302083814774770306218224241022648247419
 956042957363055823830898547303219757*e^(375*I*c) + 180798200680288599703499
 386230072306765633142067088484999001396412373347632664793469632379360393281
 13185041591793848*e^(374*I*c) + 3463765717267169016765734453719708704888235
 4853993270472063943078773600446542963548348101269390443464480754513928502*e
 ^ (373*I*c) + 65674859268867300098827375812875225610654551686261103681664007
 007537115778097293533565243828873383722980353200611956*e^(372*I*c) + 123243
 941519332384741960072588103506596406339253616391082062969960682419011745775
 738921817753391954462609323881489157*e^(371*I*c) + 228911311738592780091492
 649162346834405867740776456326108410928857257174707289268074347550225793244
 741923354395308214*e^(370*I*c) + 420846342608949387277559021457924586578120
 966148561022647008499529468452005980175119410628956210497609566002969884927
 *e^(369*I*c) + 765867795513962781012558444628751418710940895281304790836743
 661582071650032154891482866406314834433199455459798934952*e^(368*I*c) + 137
 967652979621207401710618806658944835544650121089019510716486035022892858681
 5539003062875026711931941947738690360722*e^(367*I*c) + 24604423758454226639
 270816309832607147349680919054930271456392388271922548863493611269914576924
 09851120873307487457468*e^(366*I*c) + 4343909696601932173357359687781579293
 701295681940827114215433175336093967845908766740738240037114570667410936998
 017178*e^(365*I*c) + 759275270014667896115309507358501547319702974653363333
 1549793961473285760935801904155116764831560875947581048693527224*e^(364*I*c
) + 13139771494104933881856681151418293112242551521535686871181266579813877
 606348160261747201317735782566021306798298336024*e^(363*I*c) + 225146757413
 080699615061655865028724304219302106732643929972864856006401038672536048477
 15547060592967690653795951142520*e^(362*I*c) + 3819901586758608797600299875

662767499479544066790362502932234625013328648912087500501363812811389396034
9670280707161530*e^(361*I*c) + 64175100693260066806238064886004597170740843
300086839368616139164529108049844675353111842725798658088840347241496099644
*e^(360*I*c) + 106764832017165594838085234189333528733587673329972530092661
085186789939252915937090760282232346919090426243399409323314*e^(359*I*c) +
175896258262755985757106812613979301265801031595484353614904672865169442232
075776580447184134141375995770091499246759528*e^(358*I*c) + 286992943631231
496557278010851576940896826497466066327528801560677007112837431926735088120
974861760511367008815728782643*e^(357*I*c) + 463758288457367154544937678255
005688733328145568049310423995599886012800638619904022368378591108842602342
094543682299102*e^(356*I*c) + 742228640908173124916937049462525617334148919
679118270489831005497781951221069955839623452499748653124658873553401442137
*e^(355*I*c) + 117660072097578696518987505089023109220461269697027743301453
5895788956771230793520381993106606880564628599822341722801012*e^(354*I*c) +
18475058564624515334452843005713263237811625533045659718876707580910793067
94821834928170773126364639722071570131703785334*e^(353*I*c) + 2873610535922
340187080835435582912277271967977394720159791070274927714276869531467182688
981041061381703885403497544001592*e^(352*I*c) + 442767307910542531852431611
298569365658485193610019245704445513448330504532145251634711848813322482367
0465103483954805161*e^(351*I*c) + 67584804378885243725629359489638576266948
555471955194861228775679817185872623628719949670794018319579279016825829412
34362*e^(350*I*c) + 1022042377943463485133997529516339964170212224966366619
3053008302026096932158568338309418237395541351819026907953220681013*e^(349*
I*c) + 15312837206662775379347353212807682965712535652942631518286142403097
738200270711195396582159028513532779682154451996208592*e^(348*I*c) + 227316
035661288411004195019470513676668366524180772609139448107484730848918904101
81285412604854876625919565639521227223276*e^(347*I*c) + 3343589782793658130
117117545961082945429816796201741981007293673337850658442802420107245319345
8155334046693516742390717832*e^(346*I*c) + 48733253505974923400852255563052
101402196469313659554492725674754339375283010407167744366955828922837488705
858532439654489*e^(345*I*c) + 703863497605948315670482240613950256985012022
969663003767642203366977029615910998540554113762948714374681495285247960027
62*e^(344*I*c) + 1007449618518537446117543009829801669624045538362229218684
84694269966120607698907046343731011160948828100276729370132819357*e^(343*I*
c) + 1429063191230555242465469284789542383713159258020223892364986521368398
22502035155676970917419039834587967055588431566416784*e^(342*I*c) + 2009065
871535788043803004695014416101745218512595419292098406889608594549085197748
35905895757666770857888611738751858460424*e^(341*I*c) + 2799452444750398048
229667304629608844921198748577911471240090794769204359417352933093054304386
87333129912454196774070107264*e^(340*I*c) + 3866426730503800494573825628183
169626519755509907792770487402386298587950182473561628886310156876647801012
05287333082748791*e^(339*I*c) + 5293292527641139260039348369582435576725492
389975607392144065991850478319555725837653586343954087715280097454675483829
50094*e^(338*I*c) + 7183615963820582492091135444879010888683887440337132103
32491971375906738341551540457264804304039664255915607349801911966551*e^(337

$*I*c) + 9664582753690377187477391307981516434835906841668322346880982911641$
 $60636418159452119815728809372125168836239364442397344064*e^{(336*I*c)} + 1289$
 $043515292933956480634330499677040181043935620106914267311067900030058398839$
 $787692376954090545278554544997710058754772400*e^{(335*I*c)} + 170458299670782$
 $280820467821816769300269866114771277235502145654381093006963718808588282475$
 $7500605246963210810351706405349408*e^{(334*I*c)} + 22348912763984394644786225$
 $783064348407246104844681778598226206586919214786452666530625638235530012280$
 $01009093606751066168944*e^{(333*I*c)} + 2905385722320057001953345274489482790$
 $856692529959823749532695963414164833366773128218607899328588608916176593772$
 $088622582464*e^{(332*I*c)} + 374525759487665120465733498842622638814395450198$
 $683066422234922636107960954682227606750489938670308898230818571714340721132$
 $8*e^{(331*I*c)} + 47875274427809456851452048469715961653041694193282440732114$
 $59592129649255048876854059844720661078151288179612574986359194560*e^{(330*I*$
 $c)} + 6068949803156712248331871105329895471722806143008878014986559653687260$
 $694816550470195890004511965527567432722969707577202160*e^{(329*I*c)} + 762973$
 $181562782158046899242420700836643889673633302466186383810511044514894696232$
 $8297631547032543419811821015837863013682720*e^{(328*I*c)} + 95130322740195229$
 $542091131912682266422999120135256659402983810647978856909049931289480352274$
 $12144035633851779511219335277360*e^{(327*I*c)} + 1176421227487648408001090071$
 $467347449337127816055781198372445582656605561765808647936864186490811964341$
 $2413644803772131657280*e^{(326*I*c)} + 14429816285208431204532978375375691965$
 $063154224649747551295851507389524083226976789688601369628399900747658579201$
 $929300744260*e^{(325*I*c)} + 175562732712242923968872914031257162134914862611$
 $45478571376751690105656067838042151038271381300372757756763254080268345448$
 $40*e^{(324*I*c)} + 2118832140588288753961019837470686269589404922607709376413$
 $2512513336190523978949694387686059124526755048042957954264706637460*e^{(323*$
 $I*c)} + 25367176439119353621532260335983348154904982606125761711300683492963$
 $390816491583025705268737539982149639300226512657426118880*e^{(322*I*c)} + 301$
 $284824145527032645590189530881771560134374934382010784137698354483661481217$
 $54549197591129967170764969700180348699207838960*e^{(321*I*c)} + 3550010310601$
 $964987627237679694948220958137237103600501287780602748167280705994344524013$
 $6315568500732379966585005678181937920*e^{(320*I*c)} + 41499832121963708043788$
 $523787401345541780088930538206918853579026749273364671640037563488607716092$
 $887686471542838602788559660*e^{(319*I*c)} + 481331176781840292165037485491103$
 $744789247190946356038928293648639165537922788229573682851063281647159105983$
 $70871149079494360*e^{(318*I*c)} + 5539091304497208621943268914633156608142795$
 $989696990021443429681773115086386705662076860818767970972015297414847490790$
 $4177340*e^{(317*I*c)} + 63247774101012179051794946075175569924076981338138483$
 $158042406747453874729387631710544995247152912205118500597511052824347680*e^{(316*$
 $I*c)} + 716603298611733955244419438892841091340911578446552456720842374$
 $02434944696464927131812190659629511140639501743303863582092880*e^{(315*I*c)}$
 $+ 8056624913068268418187620188262351120636379033721801195411021064292776599$
 $7644903820595421936873565314654415769070472655401600*e^{(314*I*c)} + 89883815$
 $801382382213973270477954602744792877018051963347146307372464315121274929402$
 $347942874802899499538953561056667668891020*e^{(313*I*c)} + 995122064720579659$

513403417380235485153364033717178980408504709546575329772791134915068802907
26111154101941386019689567958040*e^(312*I*c) + 1093325373499662232039326785
034263570798637070017282940110420765304039238626540189786765164173142210894
49922495612732870169660*e^(311*I*c) + 1192097137020339270557553978236884444
446474243245021853286263470465996347211465738306815404953335431467768109119
10410468628960*e^(310*I*c) + 1289950760115919034107638634270973299485861735
745958627058491592809430464587426631634540184914638553956494539522128996321
98680*e^(309*I*c) + 1385297945491510894513527695765434031263307472436800308
32467205895819043568155239264876762867172754338684027849855385453216080*e^(
308*I*c) + 1476489208055453334186231217678537773997829247483012287939243425
74999937955421765370101235122939557467548549202174550009604780*e^(307*I*c)
+ 1561859629535511961697382188321736965098525515892107305783657274762594764
74465955428502336673743686499175698677875693611243400*e^(306*I*c) + 1639778
160596077253752645598165058478941877851014553603918974244829984153857876057
65315509208337741590143078572243505132706580*e^(305*I*c) + 1708698488689531
01176860306053103994340530390347260088432676842505551412938308389612759742
68928666494845723462544709102843680*e^(304*I*c) + 1767209299705546420045757
700530957005953346598706827320319759155323875770524148663235111401176804929
29354517559479899220940360*e^(303*I*c) + 1814081687709220598203685533166973
216399848626282988285695602732956308976268293452635922190345608535307337105
29842148537901680*e^(302*I*c) + 1848311519837489418176678501747082571381281
721582694132877653585322407732443361919008185578299058956844948894104519215
24212840*e^(301*I*c) + 1869154744365675149263514056231175032619875083519300
83824566444435689139233683411704641828762178799177848064220150818355261280*
e^(300*I*c) + 1876153931685100500714972805646035109124031329203120243708350
62679037644990286285346673507093452964351257962696133511725652320*e^(299*I*
c) + 1869154744365675149263514056231175032619875083519300838245664444356891
39233683411704641828762178799177848064220150818355261280*e^(298*I*c) + 1848
311519837489418176678501747082571381281721582694132877653585322407732443361
91900818557829905895684494889410451921524212840*e^(297*I*c) + 1814081687709
220598203685533166973216399848626282988285695602732956308976268293452635922
19034560853530733710529842148537901680*e^(296*I*c) + 1767209299705546420045
757700530957005953346598706827320319759155323875770524148663235111401176804
92929354517559479899220940360*e^(295*I*c) + 1708698488689531011768603060531
03994340530390347260088432676842505551412938308389612759742689286664948457
23462544709102843680*e^(294*I*c) + 1639778160596077253752645598165058478941
877851014553603918974244829984153857876057653155092083377415901430785722435
05132706580*e^(293*I*c) + 1561859629535511961697382188321736965098525515892
107305783657274762594764744659554285023366737436864991756986778756936112434
00*e^(292*I*c) + 1476489208055453334186231217678537773997829247483012287939
24342574999937955421765370101235122939557467548549202174550009604780*e^(291
*I*c) + 1385297945491510894513527695765434031263307472436800308324672058958
19043568155239264876762867172754338684027849855385453216080*e^(290*I*c) + 1
289950760115919034107638634270973299485861735745958627058491592809430464587
42663163454018491463855395649453952212899632198680*e^(289*I*c) + 1192097137

020339270557553978236884444446474243245021853286263470465996347211465738306
81540495333543146776810911910410468628960*e^(288*I*c) + 1093325373499662232
039326785034263570798637070017282940110420765304039238626540189786765164173
14221089449922495612732870169660*e^(287*I*c) + 9951220647205796595134034173
802354851533640337171789804085047095465753297727911349150688029072611115410
1941386019689567958040*e^(286*I*c) + 89883815801382382213973270477954602744
792877018051963347146307372464315121274929402347942874802899499538953561056
667668891020*e^(285*I*c) + 805662491306826841818762018826235112063637903372
180119541102106429277659976449038205954219368735653146544157690704726554016
00*e^(284*I*c) + 7166032986117339552444194388928410913409115784465524567208
4237402434944696464927131812190659629511140639501743303863582092880*e^(283*
I*c) + 63247774101012179051794946075175569924076981338138483158042406747453
874729387631710544995247152912205118500597511052824347680*e^(282*I*c) + 553
909130449720862194326891463315660814279598969699002144342968177311508638670
56620768608187679709720152974148474907904177340*e^(281*I*c) + 4813311767818
402921650374854911037447892471909463560389282936486391655379227882295736828
5106328164715910598370871149079494360*e^(280*I*c) + 41499832121963708043788
523787401345541780088930538206918853579026749273364671640037563488607716092
887686471542838602788559660*e^(279*I*c) + 355001031060196498762723767969494
822095813723710360050128778060274816728070599434452401363155685007323799665
85005678181937920*e^(278*I*c) + 3012848241455270326455901895308817715601343
749343820107841376983544836614812175454919759112996717076496970018034869920
7838960*e^(277*I*c) + 25367176439119353621532260335983348154904982606125761
711300683492963390816491583025705268737539982149639300226512657426118880*e^
(276*I*c) + 211883214058828875396101983747068626958940492260770937641325125
13336190523978949694387686059124526755048042957954264706637460*e^(275*I*c)
+ 1755627327122429239688729140312571621349148626114547857137675169010565606
7838042151038271381300372757755676325408026834544840*e^(274*I*c) + 14429816
285208431204532978375375691965063154224649747551295851507389524083226976789
688601369628399900747658579201929300744260*e^(273*I*c) + 117642122748764840
800109007146734744933712781605578119837244558265660556176580864793686418649
08119643412413644803772131657280*e^(272*I*c) + 9513032274019522954209113191
268226642299912013525665940298381064797885690904993128948035227412144035633
851779511219335277360*e^(271*I*c) + 762973181562782158046899242420700836643
889673633302466186383810511044514894696232829763154703254341981182101583786
3013682720*e^(270*I*c) + 60689498031567122483318711053298954717228061430088
78014986559653687260694816550470195890004511965527567432722969707577202160*
e^(269*I*c) + 4787527442780945685145204846971596165304169419328244073211459
592129649255048876854059844720661078151288179612574986359194560*e^(268*I*c)
+ 374525759487665120465733498842622638814395450198683066422234922636107960
9546822276067504899386703088982308185717143407211328*e^(267*I*c) + 29053857
223200570019533452744894827908566925299598237495326959634141648333667731282
18607899328588608916176593772088622582464*e^(266*I*c) + 2234891276398439464
478622578306434840724610484468177859822620658691921478645266653062563823553
001228001009093606751066168944*e^(265*I*c) + 170458299670782280820467821816

769300269866114771277235502145654381093006963718808588282475750060524696321
0810351706405349408*e^(264*I*c) + 12890435152929339564806343304996770401810
439356201069142673110679000300583988397876923769540905452785545449977100587
54772400*e^(263*I*c) + 9664582753690377187477391307981516434835906841668322
34688098291164160636418159452119815728809372125168836239364442397344064*e^(
262*I*c) + 7183615963820582492091135444879010888683887440337132103324919713
75906738341551540457264804304039664255915607349801911966551*e^(261*I*c) + 5
293292527641139260039348369582435576725492389975607392144065991850478319555
72583765358634395408771528009745467548382950094*e^(260*I*c) + 3866426730503
800494573825628183169626519755509907792770487402386298587950182473561628886
31015687664780101205287333082748791*e^(259*I*c) + 2799452444750398048229667
304629608844921198748577911471240090794769204359417352933093054304386873331
29912454196774070107264*e^(258*I*c) + 2009065871535788043803004695014416101
745218512595419292098406889608594549085197748359058957576667708578886117387
51858460424*e^(257*I*c) + 1429063191230555242465469284789542383713159258020
22389236498652136839822502035155676970917419039834587967055588431566416784*
e^(256*I*c) + 1007449618518537446117543009829801669624045538362229218684846
94269966120607698907046343731011160948828100276729370132819357*e^(255*I*c)
+ 7038634976059483156704822406139502569850120229696630037676422033669770296
1591099854055411376294871437468149528524796002762*e^(254*I*c) + 48733253505
974923400852255563052101402196469313659554492725674754339375283010407167744
366955828922837488705858532439654489*e^(253*I*c) + 334358978279365813011711
754596108294542981679620174198100729367333785065844280242010724531934581553
34046693516742390717832*e^(252*I*c) + 2273160356612884110041950194705136766
683665241807726091394481074847308489189041018128541260485487662591956563952
1227223276*e^(251*I*c) + 15312837206662775379347353212807682965712535652942
631518286142403097738200270711195396582159028513532779682154451996208592*e^(
250*I*c) + 102204237794346348513399752951633996417021222496636661930530083
02026096932158568338309418237395541351819026907953220681013*e^(249*I*c) + 6
758480437888524372562935948963857626694855547195519486122877567981718587262
362871994967079401831957927901682582941234362*e^(248*I*c) + 442767307910542
531852431611298569365658485193610019245704445513448330504532145251634711848
8133224823670465103483954805161*e^(247*I*c) + 28736105359223401870808354355
829122772719679773947201597910702749277142768695314671826889810410613817038
85403497544001592*e^(246*I*c) + 1847505856462451533445284300571326323781162
553304565971887670758091079306794821834928170773126364639722071570131703785
334*e^(245*I*c) + 117660072097578696518987505089023109220461269697027743301
4535895788956771230793520381993106606880564628599822341722801012*e^(244*I*c
) + 74222864090817312491693704946252561733414891967911827048983100549778195
1221069955839623452499748653124658873553401442137*e^(243*I*c) + 46375828845
736715454493767825500568873332814556804931042399559988601280063861990402236
8378591108842602342094543682299102*e^(242*I*c) + 28699294363123149655727801
085157694089682649746606632752880156067700711283743192673508812097486176051
1367008815728782643*e^(241*I*c) + 17589625826275598575710681261397930126580
103159548435361490467286516944223207577658044718413414137599577009149924675

9528*e^(240*I*c) + 10676483201716559483808523418933352873358767332997253009
2661085186789939252915937090760282232346919090426243399409323314*e^(239*I*c
) + 64175100693260066806238064886004597170740843300086839368616139164529108
049844675353111842725798658088840347241496099644*e^(238*I*c) + 381990158675
860879760029987566276749947954406679036250293223462501332864891208750050136
38128113893960349670280707161530*e^(237*I*c) + 2251467574130806996150616558
650287243042193021067326439299728648560064010386725360484771554706059296769
0653795951142520*e^(236*I*c) + 13139771494104933881856681151418293112242551
521535686871181266579813877606348160261747201317735782566021306798298336024
*e^(235*I*c) + 759275270014667896115309507358501547319702974653363333154979
3961473285760935801904155116764831560875947581048693527224*e^(234*I*c) + 43
439096966019321733573596877815792937012956819408271142154331753360939678459
08766740738240037114570667410936998017178*e^(233*I*c) + 2460442375845422663
927081630983260714734968091905493027145639238827192254886349361126991457692
409851120873307487457468*e^(232*I*c) + 137967652979621207401710618806658944
835544650121089019510716486035022892858681553900306287502671193194194773869
0360722*e^(231*I*c) + 76586779551396278101255844462875141871094089528130479
0836743661582071650032154891482866406314834433199455459798934952*e^(230*I*c
) + 42084634260894938727755902145792458657812096614856102264700849952946845
2005980175119410628956210497609566002969884927*e^(229*I*c) + 22891131173859
278009149264916234683440586774077645632610841092885725717470728926807434755
0225793244741923354395308214*e^(228*I*c) + 12324394151933238474196007258810
350659640633925361639108206296996068241901174577573892181775339195446260932
3881489157*e^(227*I*c) + 65674859268867300098827375812875225610654551686261
103681664007007537115778097293533565243828873383722980353200611956*e^(226*I
*c) + 346376571726716901676573445371970870488823548539932704720639430787736
00446542963548348101269390443464480754513928502*e^(225*I*c) + 1807982006802
885997034993862300723067656331420670884849990013964123733476326647934696323
7936039328113185041591793848*e^(224*I*c) + 93393419580534942252517509657150
573007073020838147747703062182242410226482474199560429573630558238308985473
03219757*e^(223*I*c) + 4774141111066098970221845330594962016472714230374234
060663956846950926642685946929064114194400360936223590725470146*e^(222*I*c)
+ 241496651681033850328907654920274051171005901179544713877346420569645502
6442712426409599662771080264826008985061097*e^(221*I*c) + 12087703584936583
930894422220569350632837041081405937502265398461177376482166095597348316012
48698274330296158612144*e^(220*I*c) + 5986501411122418589116765051805201503
640032268413280814535970935877903386092124390855544668615826233503030619610
52*e^(219*I*c) + 2933449200343007202870423834483428663138062854550400678230
80445597545970023446231563554135133105493516316320059272*e^(218*I*c) + 1422
131159648145176823866672767699094822716813187908898405010394417486355453624
67679832449103520321953011780083069*e^(217*I*c) + 6820803309679361568378440
961924421081861499164004155342440552787689327249660832423109814850246645396
7157728078994*e^(216*I*c) + 32362731322419549410330088943640247460378328561
316422931292427145902887913071643679502909055891236755143207382609*e^(215*I
*c) + 151896342149088003964179117226437547480485201097348124591098788104938

44381062650818971199637121458749456243274416*e^(214*I*c) + 7052132414162197
992602326524580143060985353054572933905524633121681021037340298366342203324
325307072413739061024*e^(213*I*c) + 323849193136185147642332193353957909837
7735539207641467346235665823887048326949305609231585143748690203615957136*e
^(212*I*c) + 14709311466189343455150383623001001604821277495814439299047469
10224777470198899052379114493999887003199419829579*e^(211*I*c) + 6607644731
058690976914759738508379345110890331495867079827642633947667566495652798791
46173318386505740391093990*e^(210*I*c) + 2935507435543427098081294535765623
132997059826991874168629343739642556159671386762532763025915615235156032644
03*e^(209*I*c) + 1289670800847547122460236808664883849832862590255331320446
36109049545144029547003347761521666283977931640178464*e^(208*I*c) + 5602868
342490351765849501385853451616716259103436797249817466090745066677815435327
1630344650777885683547624184*e^(207*I*c) + 24068785139705277161193465644506
143285241361037768216818922184400141048460210944696647752723371932874594597
328*e^(206*I*c) + 102231820259548607672173903051864519235621454736742936199
18063490411487496121804590274592702770571515456414680*e^(205*I*c) + 4293206
478008022126017488908851826494790620720660151451468181910917240027863968724
539127659633517053002976480*e^(204*I*c) + 178244611493175185055635485663842
190117441232229824949659165805393978719824656594597559557573419334888795216
0*e^(203*I*c) + 73158497220681836287472962140397444428001044630116152733976
0544815300951787985538419764656214582667219914080*e^(202*I*c) + 29682551528
266958968531827328023905008455503220341594151196265959688161571379993768002
6497408305672297618840*e^(201*I*c) + 11904185540387796494822957794837046560
0606623183045529526900430209270473212773847794935586074714329479939280*e^(2
00*I*c) + 47188220843466207695099506953573780357108897491422567898048199018
207708997005333860148836479527456156014520*e^(199*I*c) + 184874052990057326
937527286118764908908583570219748823715706238001862451377226609436417529768
52924439870880*e^(198*I*c) + 7158124686842941475473807363679839718172745581
538409044503383852693596921622426696740453944718143025248390*e^(197*I*c) +
27388956247952656033552276465660088628077830508482570291193890365616200426
2736182657700406301914070062380*e^(196*I*c) + 10355619825920029352263845779
086115486121114950801935736913398647060291864824662418056649493810498562585
10*e^(195*I*c) + 3868762182342771656324517230499798892631152823746075416924
43176673997513742813591736171169652250611186480*e^(194*I*c) + 1428017924502
217624831808749188252741343051332754177800847950346447635093335031505173458
64659667189417080*e^(193*I*c) + 5207517851879327038642926335154430695110499
3542500582938155241689408138675254608030847907167748571734720*e^(192*I*c) +
18759988218865563564163635735986073278255737257405706279108891366378428467
414559930481172863538598193890*e^(191*I*c) + 667586629037114735850376686566
9289010893543869830538708724945291580951179188296606158111257706968604740*e
^(190*I*c) + 23465182192391051422381416330734647688991557089350257780476374
12681781575765422219127409260159438712250*e^(189*I*c) + 8146081877365305796
702100252719214155971833698812142998232919697855498761759698663679766532449
74728560*e^(188*I*c) + 2792857558000352066798353688981654776448649877946653
87827488933863633745047373109049265172681702585720*e^(187*I*c) + 9455618025

893198691933430346636565282685809131432918916073627717587384173219645337995
 3705679466826880*e^(186*I*c) + 31610939331284692750694306443618414656095969
 520945215743004044560386895241801579156543451940713351730*e^(185*I*c) + 104
 341175165703959666536931555824021094603480954730278074123214273468169285671
 97770376496170251803940*e^(184*I*c) + 3400232560601651617521694680847089844
 198028831694417424794868779328950548418125605446882081152636090*e^(183*I*c)
 + 109385321448622035867403243450086667849977001130587417248897595161203145
 6734608287095519501041975440*e^(182*I*c) + 34735147321471378087435208312956
 6601238765762775942366762733349952103889753982636403857556867777300*e^(181*I*c)
 + 10886799573182947282673290519203488679728462135644562753090910442948
 6741257822633476898356826454040*e^(180*I*c) + 33675398872021568375902384593
 982753362559801058104184627345411136262431943240778260721756991027090*e^(179*I*c)
 + 102793647306638408447395778624692626046488619142979725891652435306
 51230690726244462479199894255180*e^(178*I*c) + 3096131971621520162380301554
 241465451782362086810287537748902904985934020179565706177131421614590*e^(177*I*c)
 + 920089393029589032874601850027159322612526368444771489781974361078
 847528891468831038436064951920*e^(176*I*c) + 269745801440211296972683601863
 878954357962308520076595177128227629273240215209708218497363414140*e^(175*I*c)
 + 780098073680242398756137330588514171253271146810708896407942492826334
 70580756557083923203377160*e^(174*I*c) + 2225195917679577775716736603600748
 0222211364232146399803864370963391491223687245823457351580140*e^(173*I*c) +
 62598721568222528436509607082350347102013627760571766472263230897514465652
 88850103898153859920*e^(172*I*c) + 1736574218818191071874197472450158123883
 564209950658639102337148122769080611680719741726053840*e^(171*I*c) + 475010
 578857601519272316617938425222421786597241671026894318515408511467140969393
 115768793680*e^(170*I*c) + 128098914601688539672480541830409847707367500438
 601536803204497701119911289087105659482783340*e^(169*I*c) + 340540538512955
 691543523467221771726551875489107820085047183241687250294385891623492116280
 40*e^(168*I*c) + 8923209447343296763331881881638471793499618670601026059730
 895962653291770229493028162575100*e^(167*I*c) + 230435107337384035737917859
 7673066352016682781689139842097376663118488803841131935313641840*e^(166*I*c)
) + 58640346697268324274164332892156090937519745386424329957199096460885724
 5771134145204174990*e^(165*I*c) + 14703081673227683316304158209959204751204
 3725225353339238819165193000407629544745753221740*e^(164*I*c) + 36318369652
 302591732197444409798122022640824604130552506742586795183267354382847875885
 730*e^(163*I*c) + 883672064086047030569451402154796955129679409226698304411
 8375790025854584036796364768280*e^(162*I*c) + 21175897334668557071015014292
 10414722401838837940752841618541440888545729943138209036820*e^(161*I*c) + 4
 997075672538590843575963148137947680693371909159674919074889049339226775796
 65354338960*e^(160*I*c) + 1161045516835550437629115017121163993137330211326
 77481112824047246361794049635726479850*e^(159*I*c) + 2655680638904340753449
 6702369101545795994861757741414789944652712127566910185274123140*e^(158*I*c)
) + 59789921729441432184591611492998197063217321115784945252452287429764684
 09105395536290*e^(157*I*c) + 1324756412367837473157472821162483691120966501
 948953926492241643788264284546437221120*e^(156*I*c) + 288820755264730654469

968572021047109427318619508995802020689904590319476295408324280*e^(155*I*c)
+ 619485966530355028795643388152343106604109020378824731618047744929162165
75880077680*e^(154*I*c) + 1306981720348828988619320550837581839212499138234
0160316886507181296548981014818410*e^(153*I*c) + 27118432396707175276056404
90148833507130242448403978318523237721944200392830108580*e^(152*I*c) + 5532
691288195286125029188695589478290980219563093498435840446315122917788000814
90*e^(151*I*c) + 1109691996873209747499222595952504443412192185353496557625
91192576535872151766080*e^(150*I*c) + 2187648289271390992804034561257870580
5121508756226696317087651824252241418663320*e^(149*I*c) + 42381258467632325
86394188569858685826755328005548627437019301405851325887594480*e^(148*I*c)
+ 8066795436075891407593050107961895682698420216133889552189162788231826394
88190*e^(147*I*c) + 1508223814314124137735664742100117468522974375970591862
95243989481140398152780*e^(146*I*c) + 2769311653834325922598338263764793612
2664033859615133489846664694361471028310*e^(145*I*c) + 49925197124570439835
05377976607953988397368297591114957991804893688371867680*e^(144*I*c) + 8835
00968821791202600774541927769200737689393513734789368397093333311961880*e^(
143*I*c) + 1534360887450562541273272394615770719331301577645959971139735131
83188399376*e^(142*I*c) + 2614397627990202144347194566508025456305681018352
0401889800285493144867448*e^(141*I*c) + 43694424829101139145653531360695958
62669338858053419381214131241925047008*e^(140*I*c) + 7160994975990580798956
33338552940229192858196481597830078819711862600096*e^(139*I*c) + 1150514818
52080848873700388354521315567640365124003103691176697194292320*e^(138*I*c)
+ 18115768495615758076710303055505625589254293659193314153418333944596408*e
^(137*I*c) + 27947091044756866118427906949736991644822547239772102097256613
04403472*e^(136*I*c) + 4222761266320036875477547465557099887105271330866601
61366353656787288*e^(135*I*c) + 6247355078105329531771077469024711412412518
7565731848441781904032672*e^(134*I*c) + 90466935238256829790443389631042631
67672586826367911338826483549173*e^(133*I*c) + 1281817464914970810859604189
828359000790789921169405304612211251818*e^(132*I*c) + 177642829135119348577
194437675802830239905460092687136494961404333*e^(131*I*c) + 240708019135297
57101858022914372045864746991786182039740274325264*e^(130*I*c) + 3187749929
744346497211536044751776582320958627923816470590659024*e^(129*I*c) + 412430
698299915190848067222327219435067747934091894670488982928*e^(128*I*c) + 521
08117629177048660492400985175830987505700566877818954141639*e^(127*I*c) + 6
426195485535248576425068136870465530087114003875716691383902*e^(126*I*c) +
773204636991145775061462731028098506094432675788136295011259*e^(125*I*c) +
90722605722208814918642284639487187764607589706493970774776*e^(124*I*c) + 1
0375184499871175501909398956596684116802997082526660323524*e^(123*I*c) + 11
55855412893594260345544966642687823630035899363232371472*e^(122*I*c) + 1253
70496586921272662198050851269323171167338854081782959*e^(121*I*c) + 1323170
8870104896973800056733779919089340836756009580718*e^(120*I*c) + 13579906631
61479842850642848032544982878359839580349899*e^(119*I*c) + 1354425949166361
16191574650625331646238501101627937224*e^(118*I*c) + 1311878180117217472967
9339894318153694964675368481194*e^(117*I*c) + 12330967001397233651819972207
50932590655287625342156*e^(116*I*c) + 1123916045422466509664291620631243389

$52554575234051e^{(115*I*c)} + 9925490738534402272939987038714580495445431374$
 $618e^{(114*I*c)} + 848552202276512356496200136959676295361696315113e^{(113*I$
 $*c)} + 70164515322544462906873548813748091084561870680e^{(112*I*c)} + 5605927$
 $253067558551780452883689835514455118670e^{(111*I*c)} + 432333688644261557547$
 $944179250800440604964868e^{(110*I*c)} + 321478876933753388174544825153773503$
 $83950278e^{(109*I*c)} + 2302150411226234925855222345201500900533576e^{(108*I$
 $*c)} + 158566476113257562566117432227203884298856e^{(107*I*c)} + 104904026695$
 $10897424624643766470754045064e^{(106*I*c)} + 6656346706762100637541918471099$
 $71141414e^{(105*I*c)} + 40443624781415311581857832389099634564e^{(104*I*c)} +$
 $2348998374244347079532766203075607598e^{(103*I*c)} + 1301711930791728238351$
 $51430773360024e^{(102*I*c)} + 6868329225263681349501997341320517e^{(101*I*c)}$
 $+ 344277152012875134140739302960914e^{(100*I*c)} + 163531646471515302405291$
 $37618111e^{(99*I*c)} + 734057263616388449968842366924e^{(98*I*c)} + 310422225$
 $22074681615625020522e^{(97*I*c)} + 1232445557346832245176696904e^{(96*I*c)} +$
 $45759117183402579073139583e^{(95*I*c)} + 1581796642397812408161814e^{(94*I*$
 $c)} + 50648660944512569972179e^{(93*I*c)} + 1493326612293984160368e^{(92*I*c)}$
 $+ 40261256699368950388e^{(91*I*c)} + 984382804329835768e^{(90*I*c)} + 216084$
 $03021340047e^{(89*I*c)} + 420601518659718e^{(88*I*c)} + 7146142307307e^{(87*I$
 $*c)} + 103818048048e^{(86*I*c)} + 1253841160e^{(85*I*c)} + 12085216e^{(84*I*c)}$
 $+ 87153e^{(83*I*c)} + 418e^{(82*I*c)} + e^{(81*I*c)}) * \tan(1/4*d*x + c) + 7*(3$
 $289*I*a^2e^{(1027/2*I*c)} + 1282710*I*a^2e^{(1025/2*I*c)} + 249487095*I*a^2e$
 $^{(1023/2*I*c)} + 32266997620*I*a^2e^{(1021/2*I*c)} + 3121832019735*I*a^2e^{(1$
 $019/2*I*c)} + 241005431923542*I*a^2e^{(1017/2*I*c)} + 15464515215103422*I*a^2$
 $*e^{(1015/2*I*c)} + 848339120374563870*I*a^2e^{(1013/2*I*c)} + 406142353884741$
 $75675*I*a^2e^{(1011/2*I*c)} + 1723848657662144174935*I*a^2e^{(1009/2*I*c)} +$
 $65678633862380668978797*I*a^2e^{(1007/2*I*c)} + 2268898261084114322780091*I*$
 $a^2e^{(1005/2*I*c)} + 71659370101867067314058647*I*a^2e^{(1003/2*I*c)} + 2083$
 $633993341511741962220545*I*a^2e^{(1001/2*I*c)} + 561092868718983694960933879$
 $80*I*a^2e^{(999/2*I*c)} + 1406472792955917865136489871114*I*a^2e^{(997/2*I*c$
 $) + 32964206157734965185866131506636*I*a^2e^{(995/2*I*c)} + 7252125378443701$
 $28223087979340181*I*a^2e^{(993/2*I*c)} + 15028015438558971323851950737244424$
 $*I*a^2e^{(991/2*I*c)} + 294232725284515750583337235833091605*I*a^2e^{(989/2*$
 $I*c)} + 5458017104331277636863728788108882260*I*a^2e^{(987/2*I*c)} + 96165064$
 $470879466516856468979981298953*I*a^2e^{(985/2*I*c)} + 1612950426544228644388$
 $946118660228217326*I*a^2e^{(983/2*I*c)} + 2580720739461026266619578662590249$
 $9776722*I*a^2e^{(981/2*I*c)} + 394635224423835765377875194556169798413535*I*$
 $a^2e^{(979/2*I*c)} + 5777459899431421909042575858969918573070140*I*a^2e^{(97$
 $7/2*I*c)} + 81106652413090837180607129215519047725478483*I*a^2e^{(975/2*I*c)}$
 $+ 1093437897602872607661796947121707741043900200*I*a^2e^{(973/2*I*c)} + 141$
 $75642370639692917078404160115322319156841267*I*a^2e^{(971/2*I*c)} + 17695113$
 $8352595690762855574906084921297520106020*I*a^2e^{(969/2*I*c)} + 212931227380$
 $3633942958758817457586335895472623670*I*a^2e^{(967/2*I*c)} + 247275008153416$
 $29941828329931180779105649314030308*I*a^2e^{(965/2*I*c)} + 27741169671260636$
 $7968860766543182757632328151469299*I*a^2e^{(963/2*I*c)} + 300949720391566796$
 $1566616687959439264027968787987497*I*a^2e^{(961/2*I*c)} + 315997284052108935$

23373814632423233128854032172756465*I*a^2*e^(959/2*I*c) + 32141447475846894
9963097519046442397490455658900239275*I*a^2*e^(957/2*I*c) + 316950495586183
0892959561795308811767842097316588802741*I*a^2*e^(955/2*I*c) + 303244653699
49159006596930561842561176581703526851670949*I*a^2*e^(953/2*I*c) + 28169846
0502942707632621647223155968135366263301340888714*I*a^2*e^(951/2*I*c) + 254
2510639040480710865133818142242368651712053811882453386*I*a^2*e^(949/2*I*c)
+ 22310545816063113198799373182025930998854858701529059606430*I*a^2*e^(947
/2*I*c) + 190456027772701631222167952225792270942344905110743699547445*I*a^
2*e^(945/2*I*c) + 158260033440758234375055969559220791274299437352600569720
5768*I*a^2*e^(943/2*I*c) + 128080347774618455483945008302899308595078582601
54054185630469*I*a^2*e^(941/2*I*c) + 10100894232160107305874381253939433955
0240476561660597100434478*I*a^2*e^(939/2*I*c) + 776647619316228317354588546
587427508115665747952666669113195515*I*a^2*e^(937/2*I*c) + 5824866485485956
160870802202075534098780512067011784177572657920*I*a^2*e^(935/2*I*c) + 4263
3143603942943140960289773464948350711612149585795367207109320*I*a^2*e^(933/
2*I*c) + 304649977276950113930284052476086398990639325603533022243244834208
*I*a^2*e^(931/2*I*c) + 2126337571797794379046038412612976575112869060832179
063823545402908*I*a^2*e^(929/2*I*c) + 1450166159360679490067964489724576105
2226002310796800725001239143760*I*a^2*e^(927/2*I*c) + 966780414697215163903
19544690830600478916298492500715925487578541500*I*a^2*e^(925/2*I*c) + 63026
8657433514150995959099261502648982435588218176946585350750330080*I*a^2*e^(9
23/2*I*c) + 401946498043070486464636812109863172714185982342999113956573183
1230220*I*a^2*e^(921/2*I*c) + 250845500422700380518750411245971355879039847
88805048794692954025826136*I*a^2*e^(919/2*I*c) + 15324456075184587400986237
4935321310982256761779852307795222356568491560*I*a^2*e^(917/2*I*c) + 916735
987815179627953686885573654147136590357095246361750239315192895780*I*a^2*e^
(915/2*I*c) + 5371784947478120709044419504203956773785248008646190849516027
804337557760*I*a^2*e^(913/2*I*c) + 3084167163686023594025629181950034538258
6813672641631674672477111150251380*I*a^2*e^(911/2*I*c) + 173551107925703126
215341839166039663518594417069855504146507661560225573200*I*a^2*e^(909/2*I*
c) + 9574319978225345417301011839913366667792723123325873202826499340865370
87060*I*a^2*e^(907/2*I*c) + 51796006917688205618395796412155403402403263740
97892582774073042443161947520*I*a^2*e^(905/2*I*c) + 27485598792264285077359
588674910772175227041673233676962838136429637468948920*I*a^2*e^(903/2*I*c)
+ 1431013510732931420092839332274408631508420725964231885979192395254186513
53760*I*a^2*e^(901/2*I*c) + 73116824755764274095454644780317230443812078292
2849282615551880122607710743390*I*a^2*e^(899/2*I*c) + 366714434063056662995
2111359806942570024766504496043462982986858382541115071320*I*a^2*e^(897/2*I
*c) + 180582039765624777920702824868131545497202755214229132567687359197264
39041572530*I*a^2*e^(895/2*I*c) + 87327823598100374994317077387410249071586
643589864753197028556930729086578763100*I*a^2*e^(893/2*I*c) + 4148154663438
23656661064248622633682876622904026735200531896157088593130927151330*I*a^2*
e^(891/2*I*c) + 19358482479665846727024539809700901486967605684655632297436
02617395081958731894120*I*a^2*e^(889/2*I*c) + 88774628719390268863029903134
37414184883499510782510846267398966612198923415358100*I*a^2*e^(887/2*I*c) +

40012171151122678514041860836408151256588288243462089966844126253521402025
 344585140*I*a^2*e^(885/2*I*c) + 1772813632700935277190161295627701522711616
 98446488654079612113477246381397761958630*I*a^2*e^(883/2*I*c) + 77229171606
 3274992440312252346975737223667501767420222475960558356343854661390320930*I
 *a^2*e^(881/2*I*c) + 330844802957560533379192984154489131546772530613357810
 8040786227714970623471206860370*I*a^2*e^(879/2*I*c) + 139401361487162679956
 07572857020772155406626631585810450884412302964119799649057708170*I*a^2*e^(
 877/2*I*c) + 57780655323004011661105950186801469816680778308881437669242770
 005565033316643354683390*I*a^2*e^(875/2*I*c) + 2356359852091749733863904016
 75836470550684688674441579086471016352980458180376995214350*I*a^2*e^(873/2*
 I*c) + 94561306896721567611120536596631011836125413574238589903670439632637
 8661255246170345160*I*a^2*e^(871/2*I*c) + 373478089910366692489689117273418
 3450725814252830409400966717277658895882383641210262740*I*a^2*e^(869/2*I*c)
 + 145198424329150757899071995292829628727523094108456259824807069742661564
 47145232907218760*I*a^2*e^(867/2*I*c) + 55573448001959612239936826324022972
 697759730966309101308647950811663281594543780392273650*I*a^2*e^(865/2*I*c)
 + 2094321349963134896477553415451998346478224385044127277546599944387708240
 07112166703016320*I*a^2*e^(863/2*I*c) + 77723095115454758659490558675908341
 8923490516228159852999058710437580204374586904243317010*I*a^2*e^(861/2*I*c)
 + 284083711975827938987199833034861027134073142157853888485619458708184061
 4185808628164270200*I*a^2*e^(859/2*I*c) + 102279652939493116554931333095316
 79349667296556076415563536820773718188117601532107888284410*I*a^2*e^(857/2*
 I*c) + 36277266293394668016579895100105489749256844935749788822739768560569
 739914792417941950684060*I*a^2*e^(855/2*I*c) + 1267758656269502633093199438
 38554954851498343676576355413893697564604056732997423489495758180*I*a^2*e^(
 853/2*I*c) + 43656439754789012384352203576673018502150289410752202696550338
 5942026891357953271442864432870*I*a^2*e^(851/2*I*c) + 148156739913925722074
 2984856810817337276885474019576266630035615865701122312638498716501009480*I
 *a^2*e^(849/2*I*c) + 495571453868242674634106948466054832300323257913018010
 6682728827347533547752576357707952254670*I*a^2*e^(847/2*I*c) + 163400020409
 121010059079712542157099577625582441862393568230137914540331843005750497672
 79180480*I*a^2*e^(845/2*I*c) + 53113740974357924077261924424954026967632012
 013437614126796673959257892104885022855537842837870*I*a^2*e^(843/2*I*c) + 1
 702227103368376103068077571938602281209875128249070937149286367440168099019
 48346647161290152440*I*a^2*e^(841/2*I*c) + 53793487181496539165714918185626
 6786362021448915902734170589600431759244861457731457099253162540*I*a^2*e^(8
 39/2*I*c) + 167644063558230403342502287676812257763552063871409056442363290
 4135623882124364564513385268391800*I*a^2*e^(837/2*I*c) + 515272076996608324
 557837671612610602268426810652231549431671612536298612235950428748170971264
 0290*I*a^2*e^(835/2*I*c) + 156213172695851629743500377016620739038320660530
 37757389465930231456447027071754543682438008358530*I*a^2*e^(833/2*I*c) + 46
 716792307241842863573677106083383910064862209310631692655943363496941035047
 443752352889335351750*I*a^2*e^(831/2*I*c) + 1378299071445552041326494556780
 33264359071427275235571619960841318324458723810685070899479318058190*I*a^2*
 e^(829/2*I*c) + 40120733298154037019072011843782949228393545866397197866634

6363722054305243088217239312307931028910*I*a^2*e^(827/2*I*c) + 115236006982
268734392696871710025839854804038946508854621186577486021454624759267672884
8874609796890*I*a^2*e^(825/2*I*c) + 326617631258352081771390516393508453174
2324869211867876237701962931500933286006887180358613164105900*I*a^2*e^(823/
2*I*c) + 913609187529074929225234328910909143817786137531872437362325135470
1581584015968521795430968115978060*I*a^2*e^(821/2*I*c) + 252224479792507085
550875551252828472101581796417818491918559131557380922733281522692045844070
17233480*I*a^2*e^(819/2*I*c) + 68731484148635694613385798142456583980432634
560150438395415555262011420422746898783679369530992622830*I*a^2*e^(817/2*I*
c) + 1848846881771967854498399170942797855546491583173725064987870958123783
85154076443311386842899167584820*I*a^2*e^(815/2*I*c) + 49097255278454453591
151296396450266754666635296270781711049215563134430195363915856962240437653
6434270*I*a^2*e^(813/2*I*c) + 128723590620437935187166578688363744192559416
7862898218962357652677116687693117677467659775845901537560*I*a^2*e^(811/2*I
*c) + 333224317057538727789737456584778173868120925773377480086621326290384
5566248518176236173736361200045650*I*a^2*e^(809/2*I*c) + 851775487746162175
921041758881837150696872910880460133173143067877735689119290858441136961745
7579050080*I*a^2*e^(807/2*I*c) + 215008275355181865049326112993018993479499
25586667928948128471751935533041051245303799007069546262478120*I*a^2*e^(805
/2*I*c) + 53599146402224802194955307591891208395245153076777159512871130579
368836314443143560221865548609573152000*I*a^2*e^(803/2*I*c) + 1319665076093
228473441012594458590529932180359928097996644967960497684759145055398738984
69402406413970620*I*a^2*e^(801/2*I*c) + 32092434143259237408969311139904211
2950775535658726081343425825933752868056210294766531099221236855879280*I*a^
2*e^(799/2*I*c) + 770910696497175215498318163165309645427022023760331201680
944186642421184373976839928023191662176768831260*I*a^2*e^(797/2*I*c) + 1829
350133963276644394553279026284426177848442275457686210158857944943819074798
783161324042498997317427840*I*a^2*e^(795/2*I*c) + 4288538652909887849130589
847351637643029848450778301964494714592052591197481731912471426589520243516
263980*I*a^2*e^(793/2*I*c) + 9932748629754582437382226365886429318241459110
941837582719271408317025926367392879382082082371215951644280*I*a^2*e^(791/2
*I*c) + 2273027144825386643612857177541483341370824822453166319864819982717
9865050434174612581384879203650868383240*I*a^2*e^(789/2*I*c) + 513974838431
951899636490507365837818251414043638341354472826870891225488030368470364153
43525663901500899300*I*a^2*e^(787/2*I*c) + 11484384707459065763145436387104
086390298961851919469966692757328156903646956181444034819603569946966845392
0*I*a^2*e^(785/2*I*c) + 253587837190809021767248009086668651415180026469779
516444785991183907648198192150423457499409024155250020340*I*a^2*e^(783/2*I*
c) + 5533879734013116424478230776166078762781271520718436706787860172327804
19838284554774403815733341601209348720*I*a^2*e^(781/2*I*c) + 11935402122039
701494213523414195660902374959249773818269715133234343103262930659468434933
63225350201134814740*I*a^2*e^(779/2*I*c) + 25443422316945175141616807168296
923287194380519393462329374227681331026938224878151333328318087823608786281
92*I*a^2*e^(777/2*I*c) + 53612973438697171084667990993019942224648313925343
17407061352455760155232727561073082302587812090643743399000*I*a^2*e^(775/2*

$I*c) + 11167208352144019218695116437660885213882258119098031252190023761494$
 $016791716779154090742792939091173967743360*I*a^2*e^(773/2*I*c) + 2299449489$
 $738542622097322849443718412050643625067644940051101254055986083704591615963$
 $2911464155526549381431315*I*a^2*e^(771/2*I*c) + 468092374417760880770454567$
 $779052158674705708504490008939122808175711829778508378763722302446100620444$
 $42700310*I*a^2*e^(769/2*I*c) + 94208774273445933221381421063663460535074325$
 $778510459824107482126170497962647591802858889673576512075885532301*I*a^2*e^$
 $(767/2*I*c) + 1874677922489089299561330574219460960879034070683175914767714$
 $47547750719179954808764060229352667984915509463936*I*a^2*e^(765/2*I*c) + 36$
 $885928504761333501929186360073885548626608316419076005935614969041716425351$
 $9144904038488540679789464913316445*I*a^2*e^(763/2*I*c) + 717656376155907065$
 $759936212018974356139973024595657687829172637355396336968070917450089165988$
 $836269386494950630*I*a^2*e^(761/2*I*c) + 1380756209106808396304845431595886$
 $388497415456016252009975967548003391565941821927276762064601455403004258892$
 $730*I*a^2*e^(759/2*I*c) + 2627149553317327769969373244307872444486689137143$
 $160099808964778201822442205684906256201229154604982146665759866*I*a^2*e^(75$
 $7/2*I*c) + 4943590548857470413264240596076664907046400834929991763856809374$
 $018881419830364727922611585201089620136026317813*I*a^2*e^(755/2*I*c) + 9200$
 $523816384367442545785438469850387610401307297165444408700381462596254604043$
 $289564337709717004801142921086301*I*a^2*e^(753/2*I*c) + 1693625331579961235$
 $698344925932719830217498344500133630880020014962896317376407304258660371213$
 $0511291927885438795*I*a^2*e^(751/2*I*c) + 308374984868313679573951518547565$
 $757139601431022052585735464853443500177526906924205744683016599956655578492$
 $52505*I*a^2*e^(749/2*I*c) + 55541830059974544592261992151514835449978278080$
 $976148376085630868939554780159003409328213277398065169847689092217*I*a^2*e^$
 $(747/2*I*c) + 9896067877381332477907331410532728990479916658567060282169308$
 $7358593921745023122200152775956139379846673397268683*I*a^2*e^(745/2*I*c) +$
 $174433041915826197511502890263763083656486064340032111281811122572047913024$
 $554638157278794872090651983244870819588*I*a^2*e^(743/2*I*c) + 3041869819363$
 $768437858778210112294716451155934227401963580661725454255121403686565694250$
 $63815737226926183384463382*I*a^2*e^(741/2*I*c) + 52483218772468292558321578$
 $816676278248909353367686948349840951656435278202419847295988245464078962606$
 $8104536529860*I*a^2*e^(739/2*I*c) + 895963859958152493547412864584913395734$
 $896005557389795660279941388150830218554870622219453991707279604334136069555$
 $*I*a^2*e^(737/2*I*c) + 1513467109188784722926918732075266628364282411928629$
 $648438218524262230687177044666639547409496828363953159086523784*I*a^2*e^(73$
 $5/2*I*c) + 2529823888499135589282113191325375524965955212119888415014985472$
 $650667601219344135752131106253007866458697018787603*I*a^2*e^(733/2*I*c) + 4$
 $184704755052826165974527300950420807062983014599647272123719797358506614705$
 $996884999995742988258778935888841569436*I*a^2*e^(731/2*I*c) + 6850450138887$
 $912000825100452191974506834308573363782773535329860915542003341587109658962$
 $844998052334052577634548575*I*a^2*e^(729/2*I*c) + 1109879385273004990046747$
 $762019025358821546539078513267660604535917754053262963983600772045533260403$
 $3960540691735410*I*a^2*e^(727/2*I*c) + 177974428455248389533708026871661195$
 $807307368722185909806084723583852985189274117637602325291805184924290277518$

50574*I*a^2*e^(725/2*I*c) + 28247984729479992301795480363153077415126408060
426867690538880688072317561634861489830544260490228406147427038550825*I*a^2
*e^(723/2*I*c) + 4438011397838597246564468236192505170269662779096049860977
2420753311935097096135857768437500727905258838167774746676*I*a^2*e^(721/2*I
*c) + 690213788319592291004146400087880699724418947841938490437682218370873
69885098447731719808741375563682226721172187125*I*a^2*e^(719/2*I*c) + 10626
652434334321766459036135723116162292100113522303870748347369043788287269069
7205296816108160492296802679988715880*I*a^2*e^(717/2*I*c) + 161975924681252
579129431400371746487242592407971142243019930752714928435249070757766463734
847432577359619977355420469*I*a^2*e^(715/2*I*c) + 2444383201892857707580722
095698191999119397721562170492707501294583055032330740932091911137683346822
84550770545377772*I*a^2*e^(713/2*I*c) + 36523870613735961660007069961908213
971375364026469046672070205356031160151611689552068306829366941704637447091
0362666*I*a^2*e^(711/2*I*c) + 540377176751496499428418826628855415406366794
419862095248202798863714210847135333597884562826909393930942090796205420*I*
a^2*e^(709/2*I*c) + 7916880141086010341969385719025570015048457121961260735
63496646347934496907150906085300660569889896692451445978865145*I*a^2*e^(707
/2*I*c) + 11486093101842118434753222468708644258031138830390813277433244721
66806173047836109990343345510402414879966726470291783*I*a^2*e^(705/2*I*c) +
16503506988059387126426648384162728269871092079416066005656104908217663632
03204649976483902793933869670965683695305667*I*a^2*e^(703/2*I*c) + 23484989
692457405718955800649188873297695524955302225343357743138268545799716265123
24706726915652887153906664457167917*I*a^2*e^(701/2*I*c) + 33100869759162217
418315389599083491362555095597323384379001005870122539675010204409604646176
64661519602047358728834783*I*a^2*e^(699/2*I*c) + 46211289433375953640884928
174432226421761105862506797685805582753867770819261106155854271748980070573
44787791811730155*I*a^2*e^(697/2*I*c) + 63905938123867227469017618560565444
985220804711816545861180508692134499872333614022221449832349755052032683384
04069070*I*a^2*e^(695/2*I*c) + 87547469413801408087671939659322353769637030
88465634077354465243009176340895808577057232750872517972049977460640893614*
I*a^2*e^(693/2*I*c) + 11881739200371933972343058576105853519924399145848229
504506452617232314491716995159718725808347159999045366193929126622*I*a^2*e^
(691/2*I*c) + 1597626214054121906639107390519685226609670955554168036011793
8187079996111298630046538666465022790399882273480095312159*I*a^2*e^(689/2*I
*c) + 212840204947745811406822158568352116473795328945261225223981686063833
10529659429739450732289641742111431990437516970220*I*a^2*e^(687/2*I*c) + 28
095702104111852259874464472424979134701203776699743241230943091889592515447
767332435351059967242467638490956039653215*I*a^2*e^(685/2*I*c) + 3675005520
070649333290795765914066612108818238970047299598953224138138848104104173705
3845102752164759225076976853447870*I*a^2*e^(683/2*I*c) + 476356198918911917
256834232268343221988791178397961063806977497695505656758649254888840194473
39957047967164208947031569*I*a^2*e^(681/2*I*c) + 61190611896503763789272072
878642053209624375112006324551704284531809128584929734442619573234048475266
772127108092721024*I*a^2*e^(679/2*I*c) + 7790043003847238849352329754319110
489671369996671445142523745610252403103028233213388140396177758373964809638

0071388560*I*a²*e^(677/2*I*c) + 982922628212793487848894044944258034604001
 166822608272604607919086727410065839247541721717494325250058525135537786236
 80*I*a²*e^(675/2*I*c) + 12292631113036298228293053180059041319389275547231
 3438669937885559695115359158329870216181319401680622539825771904344440*I*a²*
 e^(673/2*I*c) + 152383229842432070507148633325172656083380096888818957463
 018370238802480734804418338221079466921156239891439248449883680*I*a²*e<sup>(67
 1/2*I*c)</sup> + 1872475246197505524332546780238983050564730206045183344255446132
 85283731157047361688793462714993120367616167673996935608*I*a²*e<sup>(669/2*I*c
)</sup> + 22808682197921316199415479789836114095676782453865285024692419965722115
 0977419942700239602639983898674178986057439712320*I*a²*e^(667/2*I*c) + 275
 427157458731124798325279881980836380771483663374738836564636407094610672777
 421045705435059330328502535069932806121560*I*a²*e^(665/2*I*c) + 3297246905
 554797265674440656403433626477087249760509218203247780189699685460652368365
 77815026028465724065587612640991920*I*a²*e^(663/2*I*c) + 39133454374736488
 519864677465209818547306922352592846073370602598740993764739057934925217641
 6240917982911016910315770960*I*a²*e^(661/2*I*c) + 460477759764521721286996
 347373482165449790480861817007793686627820310832226867295442380560478494343
 204748239311356171400*I*a²*e^(659/2*I*c) + 5372076563188509565486762229023
 270847834089427969982383436982571112256868356666939740987247705299061639200
 11962226824320*I*a²*e^(657/2*I*c) + 62137710844722546741792097788275718792
 651909488663660530936664277826306939803561618339298244262228859462992430659
 9635240*I*a²*e^(655/2*I*c) + 712608482568001637742205756113658614513655700
 735605526081445312018223976994628396796883601028869346460296341948289617440
 *I*a²*e^(653/2*I*c) + 8102680616476679689723077165647936973126296046686156
 69311286553922242663375784497106242771219831211156675425686844562920*I*a²*
 e^(651/2*I*c) + 91344681916864046615113059314794982097348770693969631293685
 0746155847174085431933136538961202623251251021715718041418880*I*a²*e<sup>(649/
 2*I*c)</sup> + 102094930776104668548385169459917674415945192031111448291964132991
 5112719001444809615932039596196242112692638947129053040*I*a²*e^(647/2*I*c)
 + 113129222020148252022367827133514682471693009851984413955582514078689913
 0297554994321714798551723344393464571710164372160*I*a²*e^(645/2*I*c) + 124
 271385788920994479462688908227607597714184385846515949318348927192458876275
 3365507654571230494246610746617132263854100*I*a²*e^(643/2*I*c) + 135319531
 545142031343748956335691041467217034283009595675577063577546629106979211426
 6486247472277234973670415760683624960*I*a²*e^(641/2*I*c) + 146049367884779
 996348705967432073734364028858039127146662387688220750809943324883548267772
 0466676528226948603304177190540*I*a²*e^(639/2*I*c) + 156218696478710149062
 701349108127315919089774183855060924984234675569595242196603525208197237493
 9068238887193535889066200*I*a²*e^(637/2*I*c) + 165572993377065829377135776
 928411849394573449443880373012568963998141726044173358210452091309236216007
 4470891832981049580*I*a²*e^(635/2*I*c) + 173851932344383411300744333276661
 616571660527994618290071620087536756724352295900565383544565788873236442347
 7634536323360*I*a²*e^(633/2*I*c) + 180796650971197687193983930504271562272
 757863455378197045947177971268210876431987484817533881287532147836905817003
 0954360*I*a²*e^(631/2*I*c) + 186157514467476811042655717741694622592613091

247171965430695580653974971394224101753287950230192049439861293640446959404
 0*I*a^2*e^(629/2*I*c) + 189702097238899020730902311991586171144973137858939
 1881830453022468540443348039250865860183534862239996983661831471121940*I*a^
 2*e^(627/2*I*c) + 191223080811385847886687075371271455528953381639473530724
 3195218359518803129887560593449941686882293838559049940338313580*I*a^2*e^(6
 25/2*I*c) + 190545759745877670993082747286644726198717656358312900256802259
 2389115983255122396490881865521324931202362299671243884700*I*a^2*e^(623/2*I
 *c) + 187534855240355454931297264088788590265708968010259710129048270355555
 7920603504414035695853822453758049509430879646245020*I*a^2*e^(621/2*I*c) +
 182100358540327370257443285145091429165881478687762174019857551217909260269
 2929225363452883945313603152284886838427140580*I*a^2*e^(619/2*I*c) + 174202
 161575977048819805921731877834792604787042718224322086427889405315556512447
 0351463656875198719523915791426498304820*I*a^2*e^(617/2*I*c) + 163853278193
 465360522393967132407604027266053409198491793257536926005505495196321121321
 4142190996608510675175078391684400*I*a^2*e^(615/2*I*c) + 151121513222723615
 446480346518456156397747297546424335831184838243194252866215048146369599774
 8766198318465135571271910040*I*a^2*e^(613/2*I*c) + 136129495437596296420884
 500562907823940166903937012790115761231970738026554756831950765171087588772
 5773374076368465724720*I*a^2*e^(611/2*I*c) + 119053051215752961750868663864
 428963188407010559045915785840926999306532051405634624155942522334695954521
 3146168932415900*I*a^2*e^(609/2*I*c) + 100117955600417763499637658615844926
 543458106606233598267392054952982009468235335291879559480323088671161115356
 5466381120*I*a^2*e^(607/2*I*c) + 795951540844964362105765369799942907568549
 210445632322720475199221982324216278747224129251261321314554419313857978534
 620*I*a^2*e^(605/2*I*c) + 5779459984288740017111214347692428241353237506856
 55589997583403559962981849048418162235470912398160298542340000896820560*I*a
 ^2*e^(603/2*I*c) + 35057895912017819591634733445330903856400507887727722468
 2309521488800706637838812783226214194307553791389253059107552140*I*a^2*e^(6
 01/2*I*c) + 117499690284248794453528461426177116010601312991844343397745740
 049701798141893102911319654443453010624651381930666567560*I*a^2*e^(599/2*I*
 c) - 1174996902842487944535284614261771160106013129918443433977457400497017
 98141893102911319654443453010624651381930666567560*I*a^2*e^(597/2*I*c) - 35
 057895912017819591634733445330903856400507887727722468230952148880070663783
 8812783226214194307553791389253059107552140*I*a^2*e^(595/2*I*c) - 577945998
 42887400171112143476924282413532375068565589997583403559962981849048418162
 235470912398160298542340000896820560*I*a^2*e^(593/2*I*c) - 7959515408449643
 621057653697999429075685492104456323227204751992219823242162787472241292512
 61321314554419313857978534620*I*a^2*e^(591/2*I*c) - 10011795560041776349963
 765861584492654345810660623359826739205495298200946823533529187955948032308
 86711611153565466381120*I*a^2*e^(589/2*I*c) - 11905305121575296175086866386
 442896318840701055904591578584092699930653205140563462415594252233469595452
 13146168932415900*I*a^2*e^(587/2*I*c) - 13612949543759629642088450056290782
 394016690393701279011576123197073802655475683195076517108758877257733740763
 68465724720*I*a^2*e^(585/2*I*c) - 15112151322272361544648034651845615639774
 729754642433583118483824319425286621504814636959977487661983184651355712719

10040*I*a^2*e^(583/2*I*c) - 16385327819346536052239396713240760402726605340
91984917932575369260055054951963211213214142190996608510675175078391684400*
I*a^2*e^(581/2*I*c) - 17420216157597704881980592173187783479260478704271822
43220864278894053155565124470351463656875198719523915791426498304820*I*a^2*
e^(579/2*I*c) - 18210035854032737025744328514509142916588147868776217401985
75512179092602692929225363452883945313603152284886838427140580*I*a^2*e^(577
/2*I*c) - 18753485524035545493129726408878859026570896801025971012904827035
55557920603504414035695853822453758049509430879646245020*I*a^2*e^(575/2*I*c
) - 19054575974587767099308274728664472619871765635831290025680225923891159
83255122396490881865521324931202362299671243884700*I*a^2*e^(573/2*I*c) - 19
122308081138584788668707537127145552895338163947353072431952183595188031298
87560593449941686882293838559049940338313580*I*a^2*e^(571/2*I*c) - 18970209
723889902073090231199158617114497313785893918818304530224685404433480392508
65860183534862239996983661831471121940*I*a^2*e^(569/2*I*c) - 18615751446747
681104265571774169462259261309124717196543069558065397497139422410175328795
02301920494398612936404469594040*I*a^2*e^(567/2*I*c) - 18079665097119768719
398393050427156227275786345537819704594717797126821087643198748481753388128
75321478369058170030954360*I*a^2*e^(565/2*I*c) - 17385193234438341130074433
327666161657166052799461829007162008753675672435229590056538354456578887323
64423477634536323360*I*a^2*e^(563/2*I*c) - 16557299337706582937713577692841
184939457344944388037301256896399814172604417335821045209130923621600744708
91832981049580*I*a^2*e^(561/2*I*c) - 15621869647871014906270134910812731591
908977418385506092498423467556959524219660352520819723749390682388871935358
89066200*I*a^2*e^(559/2*I*c) - 14604936788477999634870596743207373436402885
803912714666238768822075080994332488354826777204666765282269486033041771905
40*I*a^2*e^(557/2*I*c) - 13531953154514203134374895633569104146721703428300
95956755770635775466291069792114266486247472277234973670415760683624960*I*a
^2*e^(555/2*I*c) - 12427138578892099447946268890822760759771418438584651594
93183489271924588762753365507654571230494246610746617132263854100*I*a^2*e^(
553/2*I*c) - 11312922202014825202236782713351468247169300985198441395558251
40786899130297554994321714798551723344393464571710164372160*I*a^2*e^(551/2*
I*c) - 10209493077610466854838516945991767441594519203111144829196413299151
12719001444809615932039596196242112692638947129053040*I*a^2*e^(549/2*I*c) -
91344681916864046615113059314794982097348770693969631293685074615584717408
5431933136538961202623251251021715718041418880*I*a^2*e^(547/2*I*c) - 810268
061647667968972307716564793697312629604668615669311286553922242663375784497
106242771219831211156675425686844562920*I*a^2*e^(545/2*I*c) - 7126084825680
016377422057561136586145136557007356055260814453120182239769946283967968836
01028869346460296341948289617440*I*a^2*e^(543/2*I*c) - 62137710844722546741
792097788275718792651909488663660530936664277826306939803561618339298244262
2288594629924306599635240*I*a^2*e^(541/2*I*c) - 537207656318850956548676222
902327084783408942796998238343698257111225686835666693974098724770529906163
920011962226824320*I*a^2*e^(539/2*I*c) - 4604777597645217212869963473734821
654497904808618170077936866278203108322268672954423805604784943432047482393
11356171400*I*a^2*e^(537/2*I*c) - 39133454374736488519864677465209818547306

922352592846073370602598740993764739057934925217641624091798291101691031577
0960*I*a^2*e^(535/2*I*c) - 329724690555479726567444065640343362647708724976
050921820324778018969968546065236836577815026028465724065587612640991920*I*
a^2*e^(533/2*I*c) - 2754271574587311247983252798819808363807714836633747388
36564636407094610672777421045705435059330328502535069932806121560*I*a^2*e^(
531/2*I*c) - 22808682197921316199415479789836114095676782453865285024692419
9657221150977419942700239602639983898674178986057439712320*I*a^2*e^(529/2*I
*c) - 187247524619750552433254678023898305056473020604518334425544613285283
731157047361688793462714993120367616167673996935608*I*a^2*e^(527/2*I*c) - 1
523832298424320705071486333251726560833800968888189574630183702388024807348
04418338221079466921156239891439248449883680*I*a^2*e^(525/2*I*c) - 12292631
113036298228293053180059041319389275547231343866993788555969511535915832987
0216181319401680622539825771904344440*I*a^2*e^(523/2*I*c) - 982922628212793
487848894044944258034604001166822608272604607919086727410065839247541721717
49432525005852513553778623680*I*a^2*e^(521/2*I*c) - 77900430038472388493523
297543191104896713699966714451425237456102524031030282332133881403961777583
739648096380071388560*I*a^2*e^(519/2*I*c) - 6119061189650376378927207287864
205320962437511200632455170428453180912858492973444261957323404847526677212
7108092721024*I*a^2*e^(517/2*I*c) - 476356198918911917256834232268343221988
791178397961063806977497695505656758649254888840194473399570479671642089470
31569*I*a^2*e^(515/2*I*c) - 36750055200706493332907957659140666121088182389
700472995989532241381388481041041737053845102752164759225076976853447870*I*
a^2*e^(513/2*I*c) - 2809570210411185225987446447242497913470120377669974324
1230943091889592515447767332435351059967242467638490956039653215*I*a^2*e^(5
11/2*I*c) - 212840204947745811406822158568352116473795328945261225223981686
06383310529659429739450732289641742111431990437516970220*I*a^2*e^(509/2*I*c
) - 15976262140541219066391073905196852266096709555541680360117938187079996
111298630046538666465022790399882273480095312159*I*a^2*e^(507/2*I*c) - 1188
173920037193397234305857610585351992439914584822950450645261723231449171699
5159718725808347159999045366193929126622*I*a^2*e^(505/2*I*c) - 875474694138
014080876719396593223537696370308846563407735446524300917634089580857705723
2750872517972049977460640893614*I*a^2*e^(503/2*I*c) - 639059381238672274690
176185605654449852208047118165458611805086921344998723336140222214498323497
5505203268338404069070*I*a^2*e^(501/2*I*c) - 462112894333759536408849281744
322264217611058625067976858055827538677708192611061558542717489800705734478
7791811730155*I*a^2*e^(499/2*I*c) - 331008697591622174183153895990834913625
550955973233843790010058701225396750102044096046461766466151960204735872883
4783*I*a^2*e^(497/2*I*c) - 234849896924574057189558006491888732976955249553
0222534335774313826854579971626512324706726915652887153906664457167917*I*a^
2*e^(495/2*I*c) - 165035069880593871264266483841627282698710920794160660056
5610490821766363203204649976483902793933869670965683695305667*I*a^2*e^(493/
2*I*c) - 114860931018421184347532224687086442580311388303908132774332447216
6806173047836109990343345510402414879966726470291783*I*a^2*e^(491/2*I*c) -
791688014108601034196938571902557001504845712196126073563496646347934496907
150906085300660569889896692451445978865145*I*a^2*e^(489/2*I*c) - 5403771767

514964994284188266288554154063667944198620952482027988637142108471353335978
 84562826909393930942090796205420*I*a^2*e^(487/2*I*c) - 36523870613735961660
 007069961908213971375364026469046672070205356031160151611689552068306829366
 9417046374470910362666*I*a^2*e^(485/2*I*c) - 244438320189285770758072209569
 819199911939772156217049270750129458305503233074093209191113768334682284550
 770545377772*I*a^2*e^(483/2*I*c) - 1619759246812525791294314003717464872425
 924079711422430199307527149284352490707577664637348474325773596199773554204
 69*I*a^2*e^(481/2*I*c) - 10626652434334321766459036135723116162292100113522
 3038707483473690437882872690697205296816108160492296802679988715880*I*a^2*e
 ^ (479/2*I*c) - 690213788319592291004146400087880699724418947841938490437682
 21837087369885098447731719808741375563682226721172187125*I*a^2*e^(477/2*I*c
) - 44380113978385972465644682361925051702696627790960498609772420753311935
 097096135857768437500727905258838167774746676*I*a^2*e^(475/2*I*c) - 2824798
 472947999230179548036315307741512640806042686769053888068807231756163486148
 9830544260490228406147427038550825*I*a^2*e^(473/2*I*c) - 177974428455248389
 533708026871661195807307368722185909806084723583852985189274117637602325291
 80518492429027751850574*I*a^2*e^(471/2*I*c) - 11098793852730049900467477620
 190253588215465390785132676606045359177540532629639836007720455332604033960
 540691735410*I*a^2*e^(469/2*I*c) - 6850450138887912000825100452191974506834
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 *I*a^2*e^(467/2*I*c) - 4184704755052826165974527300950420807062983014599647
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 5/2*I*c) - 2529823888499135589282113191325375524965955212119888415014985472
 650667601219344135752131106253007866458697018787603*I*a^2*e^(463/2*I*c) - 1
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 53991707279604334136069555*I*a^2*e^(459/2*I*c) - 52483218772468292558321578
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 8104536529860*I*a^2*e^(457/2*I*c) - 304186981936376843785877821011229471645
 115593422740196358066172545425512140368656569425063815737226926183384463382
 *I*a^2*e^(455/2*I*c) - 1744330419158261975115028902637630836564860643400321
 11281811122572047913024554638157278794872090651983244870819588*I*a^2*e^(453
 /2*I*c) - 98960678773813324779073314105327289904799166585670602821693087358
 593921745023122200152775956139379846673397268683*I*a^2*e^(451/2*I*c) - 5554
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 297165444408700381462596254604043289564337709717004801142921086301*I*a^2*e^
 (443/2*I*c) - 4943590548857470413264240596076664907046400834929991763856809
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33/2*I*c) - 187467792248908929956133057421946096087903407068317591476771447
547750719179954808764060229352667984915509463936*I*a^2*e^(431/2*I*c) - 9420
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a^2*e^(425/2*I*c) - 1116720835214401921869511643766088521388225811909803125
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- 536129734386971710846679909930199422246483139253431740706135245576015523
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0*I*a^2*e^(417/2*I*c) - 553387973401311642447823077616607876278127152071843
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c) - 2535878371908090217672480090866686514151800264697795164447859911839076
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I*a^2*e^(409/2*I*c) - 22730271448253866436128571775414833413708248224531663
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- 9932748629754582437382226365886429318241459110941837582719271408317025926
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848442275457686210158857944943819074798783161324042498997317427840*I*a^2*e^
(401/2*I*c) - 7709106964971752154983181631653096454270220237603312016809441
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*I*a^2*e^(395/2*I*c) - 5359914640222480219495530759189120839524515307677715
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6434270*I*a^2*e^(383/2*I*c) - 184884688177196785449839917094279785554649158
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c) - 6873148414863569461338579814245658398043263456015043839541555526201142
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793487181496539165714918185626678636202144891590273417058960043175924486145
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5/2*I*c) - 5311374097435792407726192442495402696763201201343761412679667395
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- 102279652939493116554931333095316793496672965560764155635368207737181881
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I*c) - 77723095115454758659490558675908341892349051622815985299905871043758
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199834647822438504412727754659994438770824007112166703016320*I*a^2*e^(333/2
*I*c) - 5557344800195961223993682632402297269775973096630910130864795081166

3281594543780392273650*I*a^2*e^(331/2*I*c) - 145198424329150757899071995292
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I*c) - 37347808991036669248968911727341834507258142528304094009667172776588
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- 235635985209174973386390401675836470550684688674441579086471016352980458
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3274992440312252346975737223667501767420222475960558356343854661390320930*I
*a^2*e^(315/2*I*c) - 177281363270093527719016129562770152271161698446488654
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1860836408151256588288243462089966844126253521402025344585140*I*a^2*e^(311/
2*I*c) - 887746287193902688630299031343741418488349951078251084626739896661
2198923415358100*I*a^2*e^(309/2*I*c) - 193584824796658467270245398097009014
8696760568465563229743602617395081958731894120*I*a^2*e^(307/2*I*c) - 414815
466343823656661064248622633682876622904026735200531896157088593130927151330
*I*a^2*e^(305/2*I*c) - 8732782359810037499431707738741024907158664358986475
3197028556930729086578763100*I*a^2*e^(303/2*I*c) - 180582039765624777920702
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) - 36671443406305666299521113598069425700247665044960434629829868583825411
15071320*I*a^2*e^(299/2*I*c) - 73116824755764274095454644780317230443812078
2922849282615551880122607710743390*I*a^2*e^(297/2*I*c) - 143101351073293142
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*I*c) - 2748559879226428507735958867491077217522704167323367696283813642963
7468948920*I*a^2*e^(293/2*I*c) - 517960069176882056183957964121554034024032
6374097892582774073042443161947520*I*a^2*e^(291/2*I*c) - 957431997822534541
730101183991336666779272312332587320282649934086537087060*I*a^2*e^(289/2*I*
c) - 1735511079257031262153418391660396635185944170698555041465076615602255
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504203956773785248008646190849516027804337557760*I*a^2*e^(283/2*I*c) - 9167
35987815179627953686885573654147136590357095246361750239315192895780*I*a^2*
e^(281/2*I*c) - 15324456075184587400986237493532131098225676177985230779522
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46368121098631727141859823429991139565731831230220*I*a^2*e^(275/2*I*c) - 63
0268657433514150995959099261502648982435588218176946585350750330080*I*a^2*e
^(273/2*I*c) - 966780414697215163903195446908306004789162984925007159254875
78541500*I*a^2*e^(271/2*I*c) - 14501661593606794900679644897245761052226002
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612976575112869060832179063823545402908*I*a^2*e^(267/2*I*c) - 3046499772769
50113930284052476086398990639325603533022243244834208*I*a^2*e^(265/2*I*c) -

42633143603942943140960289773464948350711612149585795367207109320*I*a^2*e^
 (263/2*I*c) - 5824866485485956160870802202075534098780512067011784177572657
 920*I*a^2*e^(261/2*I*c) - 7766476193162283173545885465874275081156657479526
 66669113195515*I*a^2*e^(259/2*I*c) - 10100894232160107305874381253939433955
 0240476561660597100434478*I*a^2*e^(257/2*I*c) - 128080347774618455483945008
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 43750559695592207912742994373526005697205768*I*a^2*e^(253/2*I*c) - 19045602
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 I*c) - 2542510639040480710865133818142242368651712053811882453386*I*a^2*e^(
 247/2*I*c) - 281698460502942707632621647223155968135366263301340888714*I*a^
 2*e^(245/2*I*c) - 30324465369949159006596930561842561176581703526851670949*
 I*a^2*e^(243/2*I*c) - 31695049558618308929595617953088117678420973165888027
 41*I*a^2*e^(241/2*I*c) - 32141447475846894996309751904644239749045565890023
 9275*I*a^2*e^(239/2*I*c) - 315997284052108935233738146324232331288540321727
 56465*I*a^2*e^(237/2*I*c) - 30094972039156679615666166879594392640279687879
 87497*I*a^2*e^(235/2*I*c) - 27741169671260636796886076654318275763232815146
 9299*I*a^2*e^(233/2*I*c) - 247275008153416299418283299311807791056493140303
 08*I*a^2*e^(231/2*I*c) - 2129312273803633942958758817457586335895472623670*
 I*a^2*e^(229/2*I*c) - 176951138352595690762855574906084921297520106020*I*a^
 2*e^(227/2*I*c) - 14175642370639692917078404160115322319156841267*I*a^2*e^(
 225/2*I*c) - 1093437897602872607661796947121707741043900200*I*a^2*e^(223/2*
 I*c) - 81106652413090837180607129215519047725478483*I*a^2*e^(221/2*I*c) - 5
 777459899431421909042575858969918573070140*I*a^2*e^(219/2*I*c) - 3946352244
 23835765377875194556169798413535*I*a^2*e^(217/2*I*c) - 25807207394610262666
 195786625902499776722*I*a^2*e^(215/2*I*c) - 1612950426544228644388946118660
 228217326*I*a^2*e^(213/2*I*c) - 96165064470879466516856468979981298953*I*a^
 2*e^(211/2*I*c) - 5458017104331277636863728788108882260*I*a^2*e^(209/2*I*c)
 - 294232725284515750583337235833091605*I*a^2*e^(207/2*I*c) - 1502801543855
 8971323851950737244424*I*a^2*e^(205/2*I*c) - 725212537844370128223087979340
 181*I*a^2*e^(203/2*I*c) - 32964206157734965185866131506636*I*a^2*e^(201/2*I
 *c) - 1406472792955917865136489871114*I*a^2*e^(199/2*I*c) - 561092868718983
 69496093387980*I*a^2*e^(197/2*I*c) - 2083633993341511741962220545*I*a^2*e^(
 195/2*I*c) - 71659370101867067314058647*I*a^2*e^(193/2*I*c) - 2268898261084
 114322780091*I*a^2*e^(191/2*I*c) - 65678633862380668978797*I*a^2*e^(189/2*I
 *c) - 1723848657662144174935*I*a^2*e^(187/2*I*c) - 40614235388474175675*I*a
 ^2*e^(185/2*I*c) - 848339120374563870*I*a^2*e^(183/2*I*c) - 154645152151034
 22*I*a^2*e^(181/2*I*c) - 241005431923542*I*a^2*e^(179/2*I*c) - 312183201973
 5*I*a^2*e^(177/2*I*c) - 32266997620*I*a^2*e^(175/2*I*c) - 249487095*I*a^2*e
 ^2*(173/2*I*c) - 1282710*I*a^2*e^(171/2*I*c) - 3289*I*a^2*e^(169/2*I*c))/(e^(
 517*I*c) + 418*e^(516*I*c) + 87153*e^(515*I*c) + 12085216*e^(514*I*c) + 125
 3841160*e^(513*I*c) + 103818048048*e^(512*I*c) + 7146142307307*e^(511*I*c)
 + 420601518659718*e^(510*I*c) + 21608403021340047*e^(509*I*c) + 98438280432
 9835768*e^(508*I*c) + 40261256699368950388*e^(507*I*c) + 149332661229398416
 0368*e^(506*I*c) + 50648660944512569972179*e^(505*I*c) + 158179664239781240

8161814*e^(504*I*c) + 45759117183402579073139583*e^(503*I*c) + 123244555734
6832245176696904*e^(502*I*c) + 31042222522074681615625020522*e^(501*I*c) +
734057263616388449968842366924*e^(500*I*c) + 163531646471515302405291376181
11*e^(499*I*c) + 344277152012875134140739302960914*e^(498*I*c) + 6868329225
263681349501997341320517*e^(497*I*c) + 130171193079172823835151430773360024
*e^(496*I*c) + 2348998374244347079532766203075607598*e^(495*I*c) + 40443624
781415311581857832389099634564*e^(494*I*c) + 665634670676210063754191847109
971141414*e^(493*I*c) + 10490402669510897424624643766470754045064*e^(492*I*
c) + 158566476113257562566117432227203884298856*e^(491*I*c) + 2302150411226
234925855222345201500900533576*e^(490*I*c) + 321478876933753388174544825153
77350383950278*e^(489*I*c) + 432333688644261557547944179250800440604964868*
e^(488*I*c) + 5605927253067558551780452883689835514455118670*e^(487*I*c) +
70164515322544462906873548813748091084561870680*e^(486*I*c) + 8485522022765
12356496200136959676295361696315113*e^(485*I*c) + 9925490738534402272939987
038714580495445431374618*e^(484*I*c) + 112391604542246650966429162063124338
952554575234051*e^(483*I*c) + 123309670013972336518199722075093259065528762
5342156*e^(482*I*c) + 13118781801172174729679339894318153694964675368481194
*e^(481*I*c) + 135442594916636116191574650625331646238501101627937224*e^(48
0*I*c) + 1357990663161479842850642848032544982878359839580349899*e^(479*I*c
) + 13231708870104896973800056733779919089340836756009580718*e^(478*I*c) +
125370496586921272662198050851269323171167338854081782959*e^(477*I*c) + 115
5855412893594260345544966642687823630035899363232371472*e^(476*I*c) + 10375
184499871175501909398956596684116802997082526660323524*e^(475*I*c) + 907226
05722208814918642284639487187764607589706493970774776*e^(474*I*c) + 7732046
36991145775061462731028098506094432675788136295011259*e^(473*I*c) + 6426195
485535248576425068136870465530087114003875716691383902*e^(472*I*c) + 521081
17629177048660492400985175830987505700566877818954141639*e^(471*I*c) + 4124
30698299915190848067222327219435067747934091894670488982928*e^(470*I*c) + 3
187749929744346497211536044751776582320958627923816470590659024*e^(469*I*c)
+ 24070801913529757101858022914372045864746991786182039740274325264*e^(468
*I*c) + 177642829135119348577194437675802830239905460092687136494961404333*
e^(467*I*c) + 1281817464914970810859604189828359000790789921169405304612211
251818*e^(466*I*c) + 904669352382568297904433896310426316767258682636791133
8826483549173*e^(465*I*c) + 62473550781053295317710774690247114124125187565
731848441781904032672*e^(464*I*c) + 422276126632003687547754746555709988710
527133086660161366353656787288*e^(463*I*c) + 279470910447568661184279069497
3699164482254723977210209725661304403472*e^(462*I*c) + 18115768495615758076
710303055505625589254293659193314153418333944596408*e^(461*I*c) + 115051481
852080848873700388354521315567640365124003103691176697194292320*e^(460*I*c)
+ 716099497599058079895633338552940229192858196481597830078819711862600096
*e^(459*I*c) + 436944248291011391456535313606959586266933885805341938121413
1241925047008*e^(458*I*c) + 26143976279902021443471945665080254563056810183
520401889800285493144867448*e^(457*I*c) + 153436088745056254127327239461577
071933130157764595997113973513183188399376*e^(456*I*c) + 883500968821791202
600774541927769200737689393513734789368397093333311961880*e^(455*I*c) + 499

2519712457043983505377976607953988397368297591114957991804893688371867680*e^(454*I*c) + 27693116538343259225983382637647936122664033859615133489846664
 694361471028310*e^(453*I*c) + 150822381431412413773566474210011746852297437
 597059186295243989481140398152780*e^(452*I*c) + 806679543607589140759305010
 796189568269842021613388955218916278823182639488190*e^(451*I*c) + 423812584
 6763232586394188569858685826755328005548627437019301405851325887594480*e<sup>(4
 50*I*c)</sup> + 21876482892713909928040345612578705805121508756226696317087651824
 252241418663320*e^(449*I*c) + 110969199687320974749922259595250444341219218
 535349655762591192576535872151766080*e^(448*I*c) + 553269128819528612502918
 869558947829098021956309349843584044631512291778800081490*e^(447*I*c) + 271
 184323967071752760564049014883350713024244840397831852323772194420039283010
 8580*e^(446*I*c) + 13069817203488289886193205508375818392124991382340160316
 886507181296548981014818410*e^(445*I*c) + 619485966530355028795643388152343
 10660410902037882473161804774492916216575880077680*e^(444*I*c) + 2888207552
 64730654469968572021047109427318619508995802020689904590319476295408324280*
 e^(443*I*c) + 1324756412367837473157472821162483691120966501948953926492241
 643788264284546437221120*e^(442*I*c) + 597899217294414321845916114929981970
 6321732111578494525245228742976468409105395536290*e^(441*I*c) + 26556806389
 043407534496702369101545795994861757741414789944652712127566910185274123140
 *e^(440*I*c) + 116104551683555043762911501712116399313733021132677481112824
 047246361794049635726479850*e^(439*I*c) + 499707567253859084357596314813794
 768069337190915967491907488904933922677579665354338960*e^(438*I*c) + 211758
 973346685570710150142921041472240183883794075284161854144088854572994313820
 9036820*e^(437*I*c) + 88367206408604703056945140215479695512967940922669830
 44118375790025854584036796364768280*e^(436*I*c) + 3631836965230259173219744
 4409798122022640824604130552506742586795183267354382847875885730*e<sup>(435*I*c
)</sup> + 14703081673227683316304158209959204751204372522535333923881916519300040
 7629544745753221740*e^(434*I*c) + 58640346697268324274164332892156090937519
 7453864243299571990964608857245771134145204174990*e^(433*I*c) + 23043510733
 738403573791785976730663520166827816891398420973766631184888038411319353136
 41840*e^(432*I*c) + 8923209447343296763331881881638471793499618670601026059
 730895962653291770229493028162575100*e^(431*I*c) + 340540538512955691543523
 46722177172655187548910782008504718324168725029438589162349211628040*e<sup>(430
 *I*c)</sup> + 1280989146016885396724805418304098477073675004386015368032044977011
 19911289087105659482783340*e^(429*I*c) + 4750105788576015192723166179384252
 22421786597241671026894318515408511467140969393115768793680*e^(428*I*c) + 1
 736574218818191071874197472450158123883564209950658639102337148122769080611
 680719741726053840*e^(427*I*c) + 625987215682225284365096070823503471020136
 2776057176647226323089751446565288850103898153859920*e^(426*I*c) + 22251959
 17679577757167366036007480222211364232146399803864370963391491223687245823
 457351580140*e^(425*I*c) + 780098073680242398756137330588514171253271146810
 70889640794249282633470580756557083923203377160*e^(424*I*c) + 2697458014402
 112969726836018638789543579623085200765951771282276292732402152097082184973
 63414140*e^(423*I*c) + 9200893930295890328746018500271593226125263684447714
 89781974361078847528891468831038436064951920*e^(422*I*c) + 3096131971621520

162380301554241465451782362086810287537748902904985934020179565706177131421
614590*e^(421*I*c) + 102793647306638408447395778624692626046488619142979725
89165243530651230690726244462479199894255180*e^(420*I*c) + 3367539887202156
837590238459398275336255980105810418462734541113626243194324077826072175699
1027090*e^(419*I*c) + 10886799573182947282673290519203488679728462135644562
7530909104429486741257822633476898356826454040*e^(418*I*c) + 34735147321471
378087435208312956660123876576277594236676273334995210388975398263640385755
6867777300*e^(417*I*c) + 10938532144862203586740324345008666784997700113058
74172488975951612031456734608287095519501041975440*e^(416*I*c) + 3400232560
601651617521694680847089844198028831694417424794868779328950548418125605446
882081152636090*e^(415*I*c) + 104341175165703959666536931555824021094603480
95473027807412321427346816928567197770376496170251803940*e^(414*I*c) + 3161
093933128469275069430644361841465609596952094521574300404456038689524180157
9156543451940713351730*e^(413*I*c) + 94556180258931986919334303466365652826
858091314329189160736277175873841732196453379953705679466826880*e^(412*I*c)
+ 279285755800035206679835368898165477644864987794665387827488933863633745
047373109049265172681702585720*e^(411*I*c) + 814608187736530579670210025271
921415597183369881214299823291969785549876175969866367976653244974728560*e^(
410*I*c) + 234651821923910514223814163307346476889915570893502577804763741
2681781575765422219127409260159438712250*e^(409*I*c) + 66758662903711473585
037668656692890108935438698305387087249452915809511791882966061581112577069
68604740*e^(408*I*c) + 1875998821886556356416363573598607327825573725740570
6279108891366378428467414559930481172863538598193890*e^(407*I*c) + 52075178
518793270386429263351544306951104993542500582938155241689408138675254608030
847907167748571734720*e^(406*I*c) + 142801792450221762483180874918825274134
305133275417780084795034644763509333503150517345864659667189417080*e^(405*I
*c) + 386876218234277165632451723049979889263115282374607541692443176673997
513742813591736171169652250611186480*e^(404*I*c) + 103556198259200293522638
457790861154861211149508019357369133986470602918648246624180566494938104985
6258510*e^(403*I*c) + 27388956247952656033552276465660008862807783050848257
02911938903656162004262736182657700406301914070062380*e^(402*I*c) + 7158124
686842941475473807363679839718172745581538409044503383852693596921622426696
740453944718143025248390*e^(401*I*c) + 184874052990057326937527286118764908
90858357021974882371570623800186245137722660943641752976852924439870880*e^(
400*I*c) + 4718822084346620769509950695357378035710889749142256789804819901
8207708997005333860148836479527456156014520*e^(399*I*c) + 11904185540387796
494822957794837046560060662318304552952690043020927047321277384779493558607
4714329479939280*e^(398*I*c) + 29682551528266958968531827328023905008455503
2203415941511962659596881615713799937680026497408305672297618840*e^(397*I*c
) + 73158497220681836287472962140397444428001044630116152733976054481530095
1787985538419764656214582667219914080*e^(396*I*c) + 17824461149317518505563
548566384219011744123222982494965916580539397871982465659459755955757341933
48887952160*e^(395*I*c) + 4293206478008022126017488908851826494790620720660
151451468181910917240027863968724539127659633517053002976480*e^(394*I*c) +
102231820259548607672173903051864519235621454736742936199180634904114874961

21804590274592702770571515456414680*e^(393*I*c) + 2406878513970527716119346
564450614328524136103776821681892218440014104846021094469664775272337193287
4594597328*e^(392*I*c) + 56028683424903517658495013858534516167162591034367
972498174660907450666778154353271630344650777885683547624184*e^(391*I*c) +
128967080084754712246023680866488384983286259025533132044636109049545144029
547003347761521666283977931640178464*e^(390*I*c) + 293550743554342709808129
453576562313299705982699187416862934373964255615967138676253276302591561523
515603264403*e^(389*I*c) + 660764473105869097691475973850837934511089033149
586707982764263394766756649565279879146173318386505740391093990*e^(388*I*c)
+ 147093114661893434551503836230010016048212774958144392990474691022477747
0198899052379114493999887003199419829579*e^(387*I*c) + 32384919313618514764
233219335395790983777355392076414673462356658238870483269493056092315851437
48690203615957136*e^(386*I*c) + 7052132414162197992602326524580143060985353
054572933905524633121681021037340298366342203324325307072413739061024*e^(38
5*I*c) + 151896342149088003964179117226437547480485201097348124591098788104
93844381062650818971199637121458749456243274416*e^(384*I*c) + 3236273132241
954941033008894364024746037832856131642293129242714590288791307164367950290
9055891236755143207382609*e^(383*I*c) + 68208033096793615683784409619244210
818614991640041553424405527876893272496608324231098148502466453967157728078
994*e^(382*I*c) + 142213115964814517682386667276769909482271681318790889840
501039441748635545362467679832449103520321953011780083069*e^(381*I*c) + 293
344920034300720287042383448342866313806285455040067823080445597545970023446
231563554135133105493516316320059272*e^(380*I*c) + 598650141112241858911676
50518052015036400322684132808145359709358779033860921243908554466861582623
350303061961052*e^(379*I*c) + 120877035849365839308944222205693506328370410
8140593750226539846117737648216609559734831601248698274330296158612144*e^(3
78*I*c) + 24149665168103385032890765492027405117100590117954471387734642056
96455026442712426409599662771080264826008985061097*e^(377*I*c) + 4774141111
066098970221845330594962016472714230374234060663956846950926642685946929064
114194400360936223590725470146*e^(376*I*c) + 933934195805349422525175096571
505730070730208381477477030621822424102264824741995604295736305582383089854
7303219757*e^(375*I*c) + 18079820068028859970349938623007230676563314206708
848499900139641237334763266479346963237936039328113185041591793848*e^(374*I
*c) + 346376571726716901676573445371970870488823548539932704720639430787736
00446542963548348101269390443464480754513928502*e^(373*I*c) + 6567485926886
730009882737581287522561065455168626110368166400700753711577809729353356524
3828873383722980353200611956*e^(372*I*c) + 12324394151933238474196007258810
350659640633925361639108206296996068241901174577573892181775339195446260932
3881489157*e^(371*I*c) + 22891131173859278009149264916234683440586774077645
6326108410928857257174707289268074347550225793244741923354395308214*e^(370*
I*c) + 42084634260894938727755902145792458657812096614856102264700849952946
8452005980175119410628956210497609566002969884927*e^(369*I*c) + 76586779551
396278101255844462875141871094089528130479083674366158207165003215489148286
6406314834433199455459798934952*e^(368*I*c) + 13796765297962120740171061880
665894483554465012108901951071648603502289285868155390030628750267119319419

47738690360722*e^(367*I*c) + 2460442375845422663927081630983260714734968091
905493027145639238827192254886349361126991457692409851120873307487457468*e^
(366*I*c) + 434390969660193217335735968778157929370129568194082711421543317
5336093967845908766740738240037114570667410936998017178*e^(365*I*c) + 75927
527001466789611530950735850154731970297465336333315497939614732857609358019
04155116764831560875947581048693527224*e^(364*I*c) + 1313977149410493388185
668115141829311224255152153568687118126657981387760634816026174720131773578
2566021306798298336024*e^(363*I*c) + 22514675741308069961506165586502872430
421930210673264392997286485600640103867253604847715547060592967690653795951
142520*e^(362*I*c) + 381990158675860879760029987566276749947954406679036250
29322346250133286489120875005013638128113893960349670280707161530*e^(361*I*
c) + 6417510069326006680623806488600459717074084330008683936861613916452910
8049844675353111842725798658088840347241496099644*e^(360*I*c) + 10676483201
716559483808523418933352873358767332997253009266108518678993925291593709076
0282232346919090426243399409323314*e^(359*I*c) + 17589625826275598575710681
261397930126580103159548435361490467286516944223207577658044718413414137599
5770091499246759528*e^(358*I*c) + 28699294363123149655727801085157694089682
649746606632752880156067700711283743192673508812097486176051136700881572878
2643*e^(357*I*c) + 46375828845736715454493767825500568873332814556804931042
3995599886012800638619904022368378591108842602342094543682299102*e^(356*I*c
) + 74222864090817312491693704946252561733414891967911827048983100549778195
1221069955839623452499748653124658873553401442137*e^(355*I*c) + 11766007209
757869651898750508902310922046126969702774330145358957889567712307935203819
93106606880564628599822341722801012*e^(354*I*c) + 1847505856462451533445284
300571326323781162553304565971887670758091079306794821834928170773126364639
722071570131703785334*e^(353*I*c) + 287361053592234018708083543558291227727
196797739472015979107027492771427686953146718268898104106138170388540349754
4001592*e^(352*I*c) + 44276730791054253185243161129856936565848519361001924
57044455134483305045321452516347118488133224823670465103483954805161*e^(351
*I*c) + 6758480437888524372562935948963857626694855547195519486122877567981
718587262362871994967079401831957927901682582941234362*e^(350*I*c) + 102204
237794346348513399752951633996417021222496636661930530083020260969321585683
38309418237395541351819026907953220681013*e^(349*I*c) + 1531283720666277537
934735321280768296571253565294263151828614240309773820027071119539658215902
8513532779682154451996208592*e^(348*I*c) + 22731603566128841100419501947051
367666836652418077260913944810748473084891890410181285412604854876625919565
639521227223276*e^(347*I*c) + 334358978279365813011711754596108294542981679
620174198100729367333785065844280242010724531934581553340466935167423907178
32*e^(346*I*c) + 4873325350597492340085225556305210140219646931365955449272
5674754339375283010407167744366955828922837488705858532439654489*e^(345*I*c
) + 70386349760594831567048224061395025698501202296966300376764220336697702
961591099854055411376294871437468149528524796002762*e^(344*I*c) + 100744961
851853744611754300982980166962404553836222921868484694269966120607698907046
343731011160948828100276729370132819357*e^(343*I*c) + 142906319123055524246
546928478954238371315925802022389236498652136839822502035155676970917419039

834587967055588431566416784*e^(342*I*c) + 200906587153578804380300469501441
610174521851259541929209840688960859454908519774835905895757666770857888611
738751858460424*e^(341*I*c) + 279945244475039804822966730462960884492119874
857791147124009079476920435941735293309305430438687333129912454196774070107
264*e^(340*I*c) + 386642673050380049457382562818316962651975550990779277048
740238629858795018247356162888631015687664780101205287333082748791*e^(339*I
*c) + 529329252764113926003934836958243557672549238997560739214406599185047
831955572583765358634395408771528009745467548382950094*e^(338*I*c) + 718361
596382058249209113544487901088868388744033713210332491971375906738341551540
457264804304039664255915607349801911966551*e^(337*I*c) + 966458275369037718
747739130798151643483590684166832234688098291164160636418159452119815728809
372125168836239364442397344064*e^(336*I*c) + 128904351529293395648063433049
967704018104393562010691426731106790003005839883978769237695409054527855454
4997710058754772400*e^(335*I*c) + 17045829967078228082046782181676930026986
611477127723550214565438109300696371880858828247575006052469632108103517064
05349408*e^(334*I*c) + 2234891276398439464478622578306434840724610484468177
859822620658691921478645266653062563823553001228001009093606751066168944*e^
(333*I*c) + 290538572232005700195334527448948279085669252995982374953269596
3414164833366773128218607899328588608916176593772088622582464*e^(332*I*c) +
37452575948766512046573349884262263881439545019868306642223492263610796095
46822276067504899386703088982308185717143407211328*e^(331*I*c) + 4787527442
780945685145204846971596165304169419328244073211459592129649255048876854059
844720661078151288179612574986359194560*e^(330*I*c) + 606894980315671224833
187110532989547172280614300887801498655965368726069481655047019589000451196
5527567432722969707577202160*e^(329*I*c) + 76297318156278215804689924242070
083664388967363330246618638381051104451489469623282976315470325434198118210
15837863013682720*e^(328*I*c) + 9513032274019522954209113191268226642299912
013525665940298381064797885690904993128948035227412144035633851779511219335
277360*e^(327*I*c) + 117642122748764840800109007146734744933712781605578119
83724455826566055617658086479368641864908119643412413644803772131657280*e^(
326*I*c) + 1442981628520843120453297837537569196506315422464974755129585150
7389524083226976789688601369628399900747658579201929300744260*e^(325*I*c) +
17556273271224292396887291403125716213491486261145478571376751690105656067
838042151038271381300372757755676325408026834544840*e^(324*I*c) + 211883214
058828875396101983747068626958940492260770937641325125133361905239789496943
87686059124526755048042957954264706637460*e^(323*I*c) + 2536717643911935362
153226033598334815490498260612576171130068349296339081649158302570526873753
9982149639300226512657426118880*e^(322*I*c) + 30128482414552703264559018953
088177156013437493438201078413769835448366148121754549197591129967170764969
700180348699207838960*e^(321*I*c) + 355001031060196498762723767969494822095
813723710360050128778060274816728070599434452401363155685007323799665850056
78181937920*e^(320*I*c) + 4149983212196370804378852378740134554178008893053
820691885357902674927336467164003756348860771609288768647154283860278855966
0*e^(319*I*c) + 48133117678184029216503748549110374478924719094635603892829
364863916553792278822957368285106328164715910598370871149079494360*e^(318*I

*c) + 553909130449720862194326891463315660814279598969699002144342968177311
50863867056620768608187679709720152974148474907904177340*e^(317*I*c) + 6324
777410101217905179494607517556992407698133813848315804240674745387472938763
1710544995247152912205118500597511052824347680*e^(316*I*c) + 71660329861173
395524441943889284109134091157844655245672084237402434944696464927131812190
659629511140639501743303863582092880*e^(315*I*c) + 805662491306826841818762
018826235112063637903372180119541102106429277659976449038205954219368735653
14654415769070472655401600*e^(314*I*c) + 8988381580138238221397327047795460
274479287701805196334714630737246431512127492940234794287480289949953895356
1056667668891020*e^(313*I*c) + 99512206472057965951340341738023548515336403
371717898040850470954657532977279113491506880290726111154101941386019689567
958040*e^(312*I*c) + 109332537349966223203932678503426357079863707001728294
011042076530403923862654018978676516417314221089449922495612732870169660*e^(
(311*I*c) + 119209713702033927055755397823688444444647424324502185328626347
046599634721146573830681540495333543146776810911910410468628960*e^(310*I*c)
+ 128995076011591903410763863427097329948586173574595862705849159280943046
458742663163454018491463855395649453952212899632198680*e^(309*I*c) + 138529
794549151089451352769576543403126330747243680030832467205895819043568155239
264876762867172754338684027849855385453216080*e^(308*I*c) + 147648920805545
333418623121767853777399782924748301228793924342574999937955421765370101235
122939557467548549202174550009604780*e^(307*I*c) + 156185962953551196169738
218832173696509852551589210730578365727476259476474465955428502336673743686
499175698677875693611243400*e^(306*I*c) + 163977816059607725375264559816505
847894187785101455360391897424482998415385787605765315509208337741590143078
572243505132706580*e^(305*I*c) + 170869848868953101176860306053103994340530
390347260088432676842505555141293830838961275974268928666494845723462544709
102843680*e^(304*I*c) + 176720929970554642004575770053095700595334659870682
732031975915532387577052414866323511140117680492929354517559479899220940360
*e^(303*I*c) + 181408168770922059820368553316697321639984862628298828569560
273295630897626829345263592219034560853530733710529842148537901680*e^(302*I
*c) + 184831151983748941817667850174708257138128172158269413287765358532240
773244336191900818557829905895684494889410451921524212840*e^(301*I*c) + 186
915474436567514926351405623117503261987508351930083824566444435689139233683
411704641828762178799177848064220150818355261280*e^(300*I*c) + 187615393168
510050071497280564603510912403132920312024370835062679037644990286285346673
507093452964351257962696133511725652320*e^(299*I*c) + 186915474436567514926
351405623117503261987508351930083824566444435689139233683411704641828762178
799177848064220150818355261280*e^(298*I*c) + 184831151983748941817667850174
708257138128172158269413287765358532240773244336191900818557829905895684494
889410451921524212840*e^(297*I*c) + 181408168770922059820368553316697321639
984862628298828569560273295630897626829345263592219034560853530733710529842
148537901680*e^(296*I*c) + 176720929970554642004575770053095700595334659870
682732031975915532387577052414866323511140117680492929354517559479899220940
360*e^(295*I*c) + 170869848868953101176860306053103994340530390347260088432
676842505555141293830838961275974268928666494845723462544709102843680*e^(29

$4*I*c) + 163977816059607725375264559816505847894187785101455360391897424482$
 $998415385787605765315509208337741590143078572243505132706580*e^(293*I*c) +$
 $156185962953551196169738218832173696509852551589210730578365727476259476474$
 $465955428502336673743686499175698677875693611243400*e^(292*I*c) + 147648920$
 $805545333418623121767853777399782924748301228793924342574999937955421765370$
 $101235122939557467548549202174550009604780*e^(291*I*c) + 138529794549151089$
 $451352769576543403126330747243680030832467205895819043568155239264876762867$
 $172754338684027849855385453216080*e^(290*I*c) + 128995076011591903410763863$
 $427097329948586173574595862705849159280943046458742663163454018491463855395$
 $649453952212899632198680*e^(289*I*c) + 119209713702033927055755397823688444$
 $444647424324502185328626347046599634721146573830681540495333543146776810911$
 $910410468628960*e^(288*I*c) + 109332537349966223203932678503426357079863707$
 $001728294011042076530403923862654018978676516417314221089449922495612732870$
 $169660*e^(287*I*c) + 995122064720579659513403417380235485153364033717178980$
 $40850470954657532977279113491506880290726111154101941386019689567958040*e^($
 $286*I*c) + 8988381580138238221397327047795460274479287701805196334714630737$
 $2464315121274929402347942874802899499538953561056667668891020*e^(285*I*c) +$
 $80566249130682684181876201882623511206363790337218011954110210642927765997$
 $644903820595421936873565314654415769070472655401600*e^(284*I*c) + 716603298$
 $611733955244419438892841091340911578446552456720842374024349446964649271318$
 $12190659629511140639501743303863582092880*e^(283*I*c) + 6324777410101217905$
 $179494607517556992407698133813848315804240674745387472938763171054499524715$
 $2912205118500597511052824347680*e^(282*I*c) + 55390913044972086219432689146$
 $331566081427959896969900214434296817731150863867056620768608187679709720152$
 $974148474907904177340*e^(281*I*c) + 481331176781840292165037485491103744789$
 $247190946356038928293648639165537922788229573682851063281647159105983708711$
 $49079494360*e^(280*I*c) + 4149983212196370804378852378740134554178008893053$
 $820691885357902674927336467164003756348860771609288768647154283860278855966$
 $0*e^(279*I*c) + 35500103106019649876272376796949482209581372371036005012877$
 $806027481672807059943445240136315568500732379966585005678181937920*e^(278*I$
 $*c) + 301284824145527032645590189530881771560134374934382010784137698354483$
 $66148121754549197591129967170764969700180348699207838960*e^(277*I*c) + 2536$
 $717643911935362153226033598334815490498260612576171130068349296339081649158$
 $3025705268737539982149639300226512657426118880*e^(276*I*c) + 21188321405882$
 $887539610198374706862695894049226077093764132512513336190523978949694387686$
 $059124526755048042957954264706637460*e^(275*I*c) + 175562732712242923968872$
 $914031257162134914862611454785713767516901056560678380421510382713813003727$
 $57755676325408026834544840*e^(274*I*c) + 1442981628520843120453297837537569$
 $196506315422464974755129585150738952408322697678968860136962839990074765857$
 $9201929300744260*e^(273*I*c) + 11764212274876484080010900714673474493371278$
 $160557811983724455826566055617658086479368641864908119643412413644803772131$
 $657280*e^(272*I*c) + 951303227401952295420911319126822664229991201352566594$
 $0298381064797885690904993128948035227412144035633851779511219335277360*e^(2$
 $71*I*c) + 76297318156278215804689924242070083664388967363330246618638381051$
 $10445148946962328297631547032543419811821015837863013682720*e^(270*I*c) + 6$

068949803156712248331871105329895471722806143008878014986559653687260694816
550470195890004511965527567432722969707577202160*e^(269*I*c) + 478752744278
094568514520484697159616530416941932824407321145959212964925504887685405984
4720661078151288179612574986359194560*e^(268*I*c) + 37452575948766512046573
349884262263881439545019868306642223492263610796095468222760675048993867030
88982308185717143407211328*e^(267*I*c) + 2905385722320057001953345274489482
790856692529959823749532695963414164833366773128218607899328588608916176593
772088622582464*e^(266*I*c) + 223489127639843946447862257830643484072461048
446817785982262065869192147864526665306256382355300122800100909360675106616
8944*e^(265*I*c) + 17045829967078228082046782181676930026986611477127723550
21456543810930069637188085882824757500605246963210810351706405349408*e^(264
*I*c) + 1289043515292933956480634330499677040181043935620106914267311067900
030058398839787692376954090545278554544997710058754772400*e^(263*I*c) + 966
458275369037718747739130798151643483590684166832234688098291164160636418159
452119815728809372125168836239364442397344064*e^(262*I*c) + 718361596382058
249209113544487901088868388744033713210332491971375906738341551540457264804
304039664255915607349801911966551*e^(261*I*c) + 529329252764113926003934836
958243557672549238997560739214406599185047831955572583765358634395408771528
009745467548382950094*e^(260*I*c) + 386642673050380049457382562818316962651
975550990779277048740238629858795018247356162888631015687664780101205287333
082748791*e^(259*I*c) + 279945244475039804822966730462960884492119874857791
147124009079476920435941735293309305430438687333129912454196774070107264*e^
(258*I*c) + 200906587153578804380300469501441610174521851259541929209840688
960859454908519774835905895757666770857888611738751858460424*e^(257*I*c) +
142906319123055524246546928478954238371315925802022389236498652136839822502
035155676970917419039834587967055588431566416784*e^(256*I*c) + 100744961851
853744611754300982980166962404553836222921868484694269966120607698907046343
731011160948828100276729370132819357*e^(255*I*c) + 703863497605948315670482
240613950256985012022969663003767642203366977029615910998540554113762948714
37468149528524796002762*e^(254*I*c) + 4873325350597492340085225556305210140
219646931365955449272567475433937528301040716774436695582892283748870585853
2439654489*e^(253*I*c) + 33435897827936581301171175459610829454298167962017
419810072936733378506584428024201072453193458155334046693516742390717832*e^
(252*I*c) + 227316035661288411004195019470513676668366524180772609139448107
48473084891890410181285412604854876625919565639521227223276*e^(251*I*c) + 1
531283720666277537934735321280768296571253565294263151828614240309773820027
0711195396582159028513532779682154451996208592*e^(250*I*c) + 10220423779434
634851339975295163399641702122249663666193053008302026096932158568338309418
237395541351819026907953220681013*e^(249*I*c) + 675848043788852437256293594
896385762669485554719551948612287756798171858726236287199496707940183195792
7901682582941234362*e^(248*I*c) + 44276730791054253185243161129856936565848
519361001924570444551344833050453214525163471184881332248236704651034839548
05161*e^(247*I*c) + 2873610535922340187080835435582912277271967977394720159
791070274927714276869531467182688981041061381703885403497544001592*e^(246*I
*c) + 184750585646245153344528430057132632378116255330456597188767075809107

9306794821834928170773126364639722071570131703785334*e^(245*I*c) + 11766007
209757869651898750508902310922046126969702774330145358957889567712307935203
81993106606880564628599822341722801012*e^(244*I*c) + 7422286409081731249169
370494625256173341489196791182704898310054977819512210699558396234524997486
53124658873553401442137*e^(243*I*c) + 4637582884573671545449376782550056887
333281455680493104239955998860128006386199040223683785911088426023420945436
82299102*e^(242*I*c) + 2869929436312314965572780108515769408968264974660663
27528801560677007112837431926735088120974861760511367008815728782643*e^(241
*I*c) + 1758962582627559857571068126139793012658010315954843536149046728651
69442232075776580447184134141375995770091499246759528*e^(240*I*c) + 1067648
320171655948380852341893335287335876733299725300926610851867899392529159370
90760282232346919090426243399409323314*e^(239*I*c) + 6417510069326006680623
806488600459717074084330008683936861613916452910804984467535311184272579865
8088840347241496099644*e^(238*I*c) + 38199015867586087976002998756627674994
795440667903625029322346250133286489120875005013638128113893960349670280707
161530*e^(237*I*c) + 225146757413080699615061655865028724304219302106732643
92997286485600640103867253604847715547060592967690653795951142520*e^(236*I*
c) + 1313977149410493388185668115141829311224255152153568687118126657981387
7606348160261747201317735782566021306798298336024*e^(235*I*c) + 75927527001
466789611530950735850154731970297465336333315497939614732857609358019041551
16764831560875947581048693527224*e^(234*I*c) + 4343909696601932173357359687
781579293701295681940827114215433175336093967845908766740738240037114570667
410936998017178*e^(233*I*c) + 246044237584542266392708163098326071473496809
1905493027145639238827192254886349361126991457692409851120873307487457468*e
^(232*I*c) + 13796765297962120740171061880665894483554465012108901951071648
60350228928586815539003062875026711931941947738690360722*e^(231*I*c) + 7658
677955139627810125584446287514187109408952813047908367436615820716500321548
91482866406314834433199455459798934952*e^(230*I*c) + 4208463426089493872775
590214579245865781209661485610226470084995294684520059801751194106289562104
97609566002969884927*e^(229*I*c) + 2289113117385927800914926491623468344058
677407764563261084109288572571747072892680743475502257932447419233543953082
14*e^(228*I*c) + 1232439415193323847419600725881035065964063392536163910820
62969960682419011745775738921817753391954462609323881489157*e^(227*I*c) + 6
567485926886730009882737581287522561065455168626110368166400700753711577809
7293533565243828873383722980353200611956*e^(226*I*c) + 34637657172671690167
657344537197087048882354853993270472063943078773600446542963548348101269390
443464480754513928502*e^(225*I*c) + 180798200680288599703499386230072306765
633142067088484999001396412373347632664793469632379360393281131850415917938
48*e^(224*I*c) + 9339341958053494225251750965715057300707302083814774770306
218224241022648247419956042957363055823830898547303219757*e^(223*I*c) + 477
414111106609897022184533059496201647271423037423406066395684695092664268594
6929064114194400360936223590725470146*e^(222*I*c) + 24149665168103385032890
765492027405117100590117954471387734642056964550264427124264095996627710802
64826008985061097*e^(221*I*c) + 1208770358493658393089442222056935063283704
108140593750226539846117737648216609559734831601248698274330296158612144*e^

(220*I*c) + 598650141112241858911676505180520150364003226841328081453597093
587790338609212439085554466861582623350303061961052*e^(219*I*c) + 293344920
034300720287042383448342866313806285455040067823080445597545970023446231563
554135133105493516316320059272*e^(218*I*c) + 142213115964814517682386667276
769909482271681318790889840501039441748635545362467679832449103520321953011
780083069*e^(217*I*c) + 682080330967936156837844096192442108186149916400415
53424405527876893272496608324231098148502466453967157728078994*e^(216*I*c)
+ 3236273132241954941033008894364024746037832856131642293129242714590288791
3071643679502909055891236755143207382609*e^(215*I*c) + 15189634214908800396
417911722643754748048520109734812459109878810493844381062650818971199637121
458749456243274416*e^(214*I*c) + 705213241416219799260232652458014306098535
3054572933905524633121681021037340298366342203324325307072413739061024*e^(2
13*I*c) + 32384919313618514764233219335395790983777355392076414673462356658
23887048326949305609231585143748690203615957136*e^(212*I*c) + 1470931146618
934345515038362300100160482127749581443929904746910224777470198899052379114
493999887003199419829579*e^(211*I*c) + 660764473105869097691475973850837934
511089033149586707982764263394766756649565279879146173318386505740391093990
*e^(210*I*c) + 293550743554342709808129453576562313299705982699187416862934
373964255615967138676253276302591561523515603264403*e^(209*I*c) + 128967080
084754712246023680866488384983286259025533132044636109049545144029547003347
761521666283977931640178464*e^(208*I*c) + 560286834249035176584950138585345
161671625910343679724981746609074506667781543532716303446507778856835476241
84*e^(207*I*c) + 2406878513970527716119346564450614328524136103776821681892
2184400141048460210944696647752723371932874594597328*e^(206*I*c) + 10223182
025954860767217390305186451923562145473674293619918063490411487496121804590
274592702770571515456414680*e^(205*I*c) + 429320647800802212601748890885182
649479062072066015145146818191091724002786396872453912765963351705300297648
0*e^(204*I*c) + 17824461149317518505563548566384219011744123222982494965916
58053939787198246565945975595575734193348887952160*e^(203*I*c) + 7315849722
068183628747296214039744442800104463011615273397605448153009517879855384197
64656214582667219914080*e^(202*I*c) + 2968255152826695896853182732802390500
84555032203415941511962659596881615713799937680026497408305672297618840*e^(
201*I*c) + 1190418554038779649482295779483704656006066231830455295269004302
09270473212773847794935586074714329479939280*e^(200*I*c) + 4718822084346620
769509950695357378035710889749142256789804819901820770899700533386014883647
9527456156014520*e^(199*I*c) + 18487405299005732693752728611876490890858357
021974882371570623800186245137722660943641752976852924439870880*e^(198*I*c)
+ 715812468684294147547380736367983971817274558153840904450338385269359692
1622426696740453944718143025248390*e^(197*I*c) + 27388956247952656033552276
465660008862807783050848257029119389036561620042627361826577004063019140700
62380*e^(196*I*c) + 1035561982592002935226384577908611548612111495080193573
691339864706029186482466241805664949381049856258510*e^(195*I*c) + 386876218
234277165632451723049979889263115282374607541692443176673997513742813591736
171169652250611186480*e^(194*I*c) + 142801792450221762483180874918825274134
305133275417780084795034644763509333503150517345864659667189417080*e^(193*I

$*c) + 520751785187932703864292633515443069511049935425005829381552416894081$
 $38675254608030847907167748571734720 * e^{(192 * I * c)} + 1875998821886556356416363$
 $573598607327825573725740570627910889136637842846741455993048117286353859819$
 $3890 * e^{(191 * I * c)} + 66758662903711473585037668656692890108935438698305387087$
 $24945291580951179188296606158111257706968604740 * e^{(190 * I * c)} + 2346518219239$
 $105142238141633073464768899155708935025778047637412681781575765422219127409$
 $260159438712250 * e^{(189 * I * c)} + 814608187736530579670210025271921415597183369$
 $881214299823291969785549876175969866367976653244974728560 * e^{(188 * I * c)} + 279$
 $285755800035206679835368898165477644864987794665387827488933863633745047373$
 $109049265172681702585720 * e^{(187 * I * c)} + 945561802589319869193343034663656528$
 $26858091314329189160736277175873841732196453379953705679466826880 * e^{(186 * I * c)}$
 $+ 3161093933128469275069430644361841465609596952094521574300404456038689$
 $5241801579156543451940713351730 * e^{(185 * I * c)} + 10434117516570395966653693155$
 $582402109460348095473027807412321427346816928567197770376496170251803940 * e^{(184 * I * c)}$
 $+ 340023256060165161752169468084708984419802883169441742479486877$
 $9328950548418125605446882081152636090 * e^{(183 * I * c)} + 10938532144862203586740$
 $324345008666784997700113058741724889759516120314567346082870955195010419754$
 $40 * e^{(182 * I * c)} + 3473514732147137808743520831295666012387657627759423667627$
 $33349952103889753982636403857556867777300 * e^{(181 * I * c)} + 1088679957318294728$
 $267329051920348867972846213564456275309091044294867412578226334768983568264$
 $54040 * e^{(180 * I * c)} + 3367539887202156837590238459398275336255980105810418462$
 $7345411136262431943240778260721756991027090 * e^{(179 * I * c)} + 10279364730663840$
 $844739577862469262604648861914297972589165243530651230690726244462479199894$
 $255180 * e^{(178 * I * c)} + 309613197162152016238030155424146545178236208681028753$
 $7748902904985934020179565706177131421614590 * e^{(177 * I * c)} + 92008939302958903$
 $287460185002715932261252636844477148978197436107884752889146883103843606495$
 $1920 * e^{(176 * I * c)} + 26974580144021129697268360186387895435796230852007659517$
 $7128227629273240215209708218497363414140 * e^{(175 * I * c)} + 78009807368024239875$
 $613733058851417125327114681070889640794249282633470580756557083923203377160$
 $* e^{(174 * I * c)} + 222519591767957777571673660360074802222113642321463998038643$
 $70963391491223687245823457351580140 * e^{(173 * I * c)} + 6259872156822252843650960$
 $708235034710201362776057176647226323089751446565288850103898153859920 * e^{(172 * I * c)}$
 $+ 173657421881819107187419747245015812388356420995065863910233714812$
 $2769080611680719741726053840 * e^{(171 * I * c)} + 47501057885760151927231661793842$
 $5222421786597241671026894318515408511467140969393115768793680 * e^{(170 * I * c)} +$
 $12809891460168853967248054183040984770736750043860153680320449770111991128$
 $9087105659482783340 * e^{(169 * I * c)} + 34054053851295569154352346722177172655187$
 $548910782008504718324168725029438589162349211628040 * e^{(168 * I * c)} + 892320944$
 $734329676333188188163847179349961867060102605973089596265329177022949302816$
 $2575100 * e^{(167 * I * c)} + 23043510733738403573791785976730663520166827816891398$
 $42097376663118488803841131935313641840 * e^{(166 * I * c)} + 5864034669726832427416$
 $43328921560909375197453864243299571990964608857245771134145204174990 * e^{(165 * I * c)}$
 $+ 1470308167322768331630415820995920475120437252253533392388191651930$
 $00407629544745753221740 * e^{(164 * I * c)} + 3631836965230259173219744440979812202$
 $2640824604130552506742586795183267354382847875885730 * e^{(163 * I * c)} + 88367206$

408604703056945140215479695512967940922669830441183757900258545840367963647
68280*e^(162*I*c) + 2117589733466855707101501429210414722401838837940752841
618541440888545729943138209036820*e^(161*I*c) + 499707567253859084357596314
813794768069337190915967491907488904933922677579665354338960*e^(160*I*c) +
116104551683555043762911501712116399313733021132677481112824047246361794049
635726479850*e^(159*I*c) + 265568063890434075344967023691015457959948617577
41414789944652712127566910185274123140*e^(158*I*c) + 5978992172944143218459
161149299819706321732111578494525245228742976468409105395536290*e^(157*I*c)
+ 132475641236783747315747282116248369112096650194895392649224164378826428
4546437221120*e^(156*I*c) + 28882075526473065446996857202104710942731861950
8995802020689904590319476295408324280*e^(155*I*c) + 61948596653035502879564
338815234310660410902037882473161804774492916216575880077680*e^(154*I*c) +
130698172034882898861932055083758183921249913823401603168865071812965489810
14818410*e^(153*I*c) + 2711843239670717527605640490148833507130242448403978
318523237721944200392830108580*e^(152*I*c) + 553269128819528612502918869558
947829098021956309349843584044631512291778800081490*e^(151*I*c) + 110969199
687320974749922259595250444341219218535349655762591192576535872151766080*e^
(150*I*c) + 218764828927139099280403456125787058051215087562266963170876518
24252241418663320*e^(149*I*c) + 4238125846763232586394188569858685826755328
005548627437019301405851325887594480*e^(148*I*c) + 806679543607589140759305
010796189568269842021613388955218916278823182639488190*e^(147*I*c) + 150822
381431412413773566474210011746852297437597059186295243989481140398152780*e^
(146*I*c) + 276931165383432592259833826376479361226640338596151334898466646
94361471028310*e^(145*I*c) + 4992519712457043983505377976607953988397368297
591114957991804893688371867680*e^(144*I*c) + 883500968821791202600774541927
769200737689393513734789368397093333311961880*e^(143*I*c) + 153436088745056
254127327239461577071933130157764595997113973513183188399376*e^(142*I*c) +
26143976279902021443471945665080254563056810183520401889800285493144867448*
e^(141*I*c) + 4369442482910113914565353136069595862669338858053419381214131
241925047008*e^(140*I*c) + 716099497599058079895633338552940229192858196481
597830078819711862600096*e^(139*I*c) + 115051481852080848873700388354521315
567640365124003103691176697194292320*e^(138*I*c) + 181157684956157580767103
03055505625589254293659193314153418333944596408*e^(137*I*c) + 2794709104475
686611842790694973699164482254723977210209725661304403472*e^(136*I*c) + 422
276126632003687547754746555709988710527133086660161366353656787288*e^(135*I
c) + 62473550781053295317710774690247114124125187565731848441781904032672
e^(134*I*c) + 9046693523825682979044338963104263167672586826367911338826483
549173*e^(133*I*c) + 128181746491497081085960418982835900079078992116940530
4612211251818*e^(132*I*c) + 17764282913511934857719443767580283023990546009
2687136494961404333*e^(131*I*c) + 24070801913529757101858022914372045864746
991786182039740274325264*e^(130*I*c) + 318774992974434649721153604475177658
2320958627923816470590659024*e^(129*I*c) + 41243069829991519084806722232721
9435067747934091894670488982928*e^(128*I*c) + 52108117629177048660492400985
175830987505700566877818954141639*e^(127*I*c) + 642619548553524857642506813
6870465530087114003875716691383902*e^(126*I*c) + 77320463699114577506146273

$1028098506094432675788136295011259 * e^{(125 * I * c)} + 90722605722208814918642284$
 $639487187764607589706493970774776 * e^{(124 * I * c)} + 103751844998711755019093989$
 $56596684116802997082526660323524 * e^{(123 * I * c)} + 1155855412893594260345544966$
 $642687823630035899363232371472 * e^{(122 * I * c)} + 125370496586921272662198050851$
 $269323171167338854081782959 * e^{(121 * I * c)} + 132317088701048969738000567337799$
 $19089340836756009580718 * e^{(120 * I * c)} + 1357990663161479842850642848032544982$
 $878359839580349899 * e^{(119 * I * c)} + 135442594916636116191574650625331646238501$
 $101627937224 * e^{(118 * I * c)} + 131187818011721747296793398943181536949646753684$
 $81194 * e^{(117 * I * c)} + 1233096700139723365181997220750932590655287625342156 * e^{(116 * I * c)}$
 $+ 112391604542246650966429162063124338952554575234051 * e^{(115 * I * c)}$
 $+ 9925490738534402272939987038714580495445431374618 * e^{(114 * I * c)} + 84855220$
 $2276512356496200136959676295361696315113 * e^{(113 * I * c)} + 70164515322544462906$
 $873548813748091084561870680 * e^{(112 * I * c)} + 560592725306755855178045288368983$
 $5514455118670 * e^{(111 * I * c)} + 432333688644261557547944179250800440604964868 * e^{(110 * I * c)}$
 $+ 32147887693375338817454482515377350383950278 * e^{(109 * I * c)} + 230$
 $2150411226234925855222345201500900533576 * e^{(108 * I * c)} + 15856647611325756256$
 $6117432227203884298856 * e^{(107 * I * c)} + 10490402669510897424624643766470754045$
 $064 * e^{(106 * I * c)} + 665634670676210063754191847109971141414 * e^{(105 * I * c)} + 404$
 $43624781415311581857832389099634564 * e^{(104 * I * c)} + 2348998374244347079532766$
 $203075607598 * e^{(103 * I * c)} + 130171193079172823835151430773360024 * e^{(102 * I * c)}$
 $+ 6868329225263681349501997341320517 * e^{(101 * I * c)} + 34427715201287513414073$
 $9302960914 * e^{(100 * I * c)} + 16353164647151530240529137618111 * e^{(99 * I * c)} + 7340$
 $57263616388449968842366924 * e^{(98 * I * c)} + 31042222522074681615625020522 * e^{(97$
 $* I * c)} + 1232445557346832245176696904 * e^{(96 * I * c)} + 4575911718340257907313958$
 $3 * e^{(95 * I * c)} + 1581796642397812408161814 * e^{(94 * I * c)} + 506486609445125699721$
 $79 * e^{(93 * I * c)} + 1493326612293984160368 * e^{(92 * I * c)} + 40261256699368950388 * e^{(91 * I * c)}$
 $+ 984382804329835768 * e^{(90 * I * c)} + 21608403021340047 * e^{(89 * I * c)} + 4$
 $20601518659718 * e^{(88 * I * c)} + 7146142307307 * e^{(87 * I * c)} + 103818048048 * e^{(86 * I$
 $* c)} + 1253841160 * e^{(85 * I * c)} + 12085216 * e^{(84 * I * c)} + 87153 * e^{(83 * I * c)} + 418 * e^{(82 * I * c)}$
 $+ e^{(81 * I * c)}) * \tan(1/4 * d * x + c) - 14 * (3289 * a^2 * e^{(1027/2 * I * c)} +$
 $1282710 * a^2 * e^{(1025/2 * I * c)} + 249487095 * a^2 * e^{(1023/2 * I * c)} + 32266997620 * a^2$
 $* e^{(1021/2 * I * c)} + 3121832019735 * a^2 * e^{(1019/2 * I * c)} + 241005431923542 * a^2 * e^{(1017/2 * I * c)}$
 $+ 15464515215103622 * a^2 * e^{(1015/2 * I * c)} + 848339120374641870 * a^2 * e^{(1013/2 * I * c)}$
 $+ 40614235388489346675 * a^2 * e^{(1011/2 * I * c)} + 17238486576641$
 $06290905 * a^2 * e^{(1009/2 * I * c)} + 65678633862570503690097 * a^2 * e^{(1007/2 * I * c)} +$
 $2268898261098769561120041 * a^2 * e^{(1005/2 * I * c)} + 71659370102807444959242727 * a^2 * e^{(1003/2 * I * c)}$
 $+ 2083633993393098160454111295 * a^2 * e^{(1001/2 * I * c)} + 56109$
 $286874368068446384618040 * a^2 * e^{(999/2 * I * c)} + 140647279306074281727813339969$
 $4 * a^2 * e^{(997/2 * I * c)} + 32964206161728793499031816369336 * a^2 * e^{(995/2 * I * c)} +$
 $725212537982338639553752606105971 * a^2 * e^{(993/2 * I * c)} + 150280154429164727843$
 $03355549568064 * a^2 * e^{(991/2 * I * c)} + 294232725411218343537913910694524115 * a^2$
 $* e^{(989/2 * I * c)} + 5458017107743192900932192332079754800 * a^2 * e^{(987/2 * I * c)} +$
 $96165064556404674374591793494770969751 * a^2 * e^{(985/2 * I * c)} + 1612950428548722$
 $021160870732573388748486 * a^2 * e^{(983/2 * I * c)} + 258072074387090234055646986543$
 $81610855782 * a^2 * e^{(981/2 * I * c)} + 394635225337657865294965990982357902779935 *$

$a^{2e^{(979/2I*c)}} + 5777459917323047305837478939343390617560800a^{2e^{(977/2I*c)}} + 81106652744979453503605904560553941804516827a^{2e^{(975/2I*c)}} + 1093437903450413476694775388085248143435843184a^{2e^{(973/2I*c)}} + 14175642468718517834053796420531668264840828827a^{2e^{(971/2I*c)}} + 176951139921850074633194549001040224465598018440a^{2e^{(969/2I*c)}} + 2129312297800033603965109459542894851379636241390a^{2e^{(967/2I*c)}} + 24727501166647061083289528750467452594464413496712a^{2e^{(965/2I*c)}} + 277411701644365486379380075778816997842958595635411a^{2e^{(963/2I*c)}} + 3009497270402664372603369055821186361937123336960151a^{2e^{(961/2I*c)}} + 31599729267161332823403075503125661623124842474211925a^{2e^{(959/2I*c)}} + 321414485517907071866518386845402404565806226718602745a^{2e^{(957/2I*c)}} + 3169505085332677220309823086926906379156042673077997829a^{2e^{(955/2I*c)}} + 30324466873468264078058992476301732181384084447966749251a^{2e^{(953/2I*c)}} + 281698477370386606708598633818891039165559192659067598486a^{2e^{(951/2I*c)}} + 2542510822024809049111292052295925694458486937681586168494a^{2e^{(949/2I*c)}} + 22310547737377338728195792669384697517889412712723300156650a^{2e^{(947/2I*c)}} + 190456047314980596262214455758830176412490479327548630395435a^{2e^{(945/2I*c)}} + 1582600527113724210945270692154487384537712932377861494537792a^{2e^{(943/2I*c)}} + 12808036621165917614961411495703359665564662395820821879432651a^{2e^{(941/2I*c)}} + 101008959448394278622065270281354488236767429874129774402372122a^{2e^{(939/2I*c)}} + 776647773894140039601026125724635184851400081828079008519495205a^{2e^{(937/2I*c)}} + 5824867841884974805556257455805728440177379367311530786366553840a^{2e^{(935/2I*c)}} + 42633155182758234213432657105359507039157758216710091608150751160a^{2e^{(933/2I*c)}} + 304650073489624069501319666840972753356711517554147295115707791952a^{2e^{(931/2I*c)}} + 2126338350434067101841066179986723736309721747170637151333282385892a^{2e^{(929/2I*c)}} + 14501667734089456372413097720073331427769744768582734443920820884880a^{2e^{(927/2I*c)}} + 96678088682187070566952755698159384671499301435475363946248645511300a^{2e^{(925/2I*c)}} + 630269011518917093786807783619832131991400363103105059485556854864080a^{2e^{(923/2I*c)}} + 4019467571972191918495205431223264437545950953166839320312436248766740a^{2e^{(921/2I*c)}} + 25084568560525486523793075765839786939668166684250998147267060999843864a^{2e^{(919/2I*c)}} + 153244689998280106303232133486628958245019231358231317009398623103014360a^{2e^{(917/2I*c)}} + 916736869251326841783602023183005234330809250515685392295995174302631940a^{2e^{(915/2I*c)}} + 5371790823547775986769311672178060666294358172540326849409957986665686640a^{2e^{(913/2I*c)}} + 30841709943145755974906466306871090919705337121093845564611475205831834740a^{2e^{(911/2I*c)}} + 173551352210830111542403729997736548659579690262161334684378354693653143600a^{2e^{(909/2I*c)}} + 957433522291942572603972061174768931380078714954598245160747546204280979860a^{2e^{(907/2I*c)}} + 5179610004567839676882734372075939558403884380512473251576883045496625080240a^{2e^{(905/2I*c)}} + 27485654500744845946086864693951416729958544421088007484546058264731062865400a^{2e^{(903/2I*c)}} + 143101677493167706049469001333839411556496168357433098121521463359375252809040a^{2e^{(901/2I*c)}} + 731170121584724033220964486773088649792495972559727457509380085597040672846910a^{2e^{(899/2I*c)}} + 36671548855755560353542103$

70922300362357210115667534077868226474380300702979520*a²*e^(897/2*I*c) + 1
 805826214706251079990263321103674819022743994744768011661354696943158714965
 7730*a²*e^(895/2*I*c) + 87328138277263357453687393632829255820603582731737
 822656013656737314066270769860*a²*e^(893/2*I*c) + 414817136098345744491539
 277362007380513271882803145751297848219367978399846839810*a²*e^(891/2*I*c)
 + 193585694088706675715864869610835837558634179385085506203995989160459324
 8887646480*a²*e^(889/2*I*c) + 88775072851962206182416554647787324999352216
 64373951650797658698877398128936202900*a²*e^(887/2*I*c) + 4001239388966249
 0616271399213614257399774009402473996666416480625157884768764896580*a²*e⁽
 885/2*I*c) + 17728246003277186589209994770321940050940172812087015693900726
 4944328352108674765830*a²*e^(883/2*I*c) + 77229701953334031851476725644580
 2696297496337802921019409004020665598966438156162590*a²*e^(881/2*I*c) + 33
 084732197038235375562437697635708153491799435857062717116045714440947701529
 19124730*a²*e^(879/2*I*c) + 1394025369616807940206425969265350761294521105
 2614130496704459947954824765033193710270*a²*e^(877/2*I*c) + 57781194331618
 050184075872423561791421849035243571600320288706532981609331680804189710*a²
 *e^(875/2*I*c) + 235638414403676636374964618535168235461531651215096600123
 550185747704196738532598229970*a²*e^(873/2*I*c) + 945623831015067465565761
 737751781712729579043386050190772281518980746360423316243590560*a²*e^{(871/}
 2*I*c) + 373482777754284296282094449715876412045906669724039458982566434651
 3551225394202631289660*a²*e^(869/2*I*c) + 14520043237771024867578892274833
 151999983805655774942432102559165991274651263356626027360*a²*e^(867/2*I*c)
 + 555742940074056571594107022251101282474230803333614786203427309125910893
 36970706714049470*a²*e^(865/2*I*c) + 2094356412464963422883748629580564387
 53891068972196618627904344080420177337078104676468880*a²*e^(863/2*I*c) + 7
 772452484638321644435794643177404249461896284949424535085804418007448285098
 67291417081630*a²*e^(861/2*I*c) + 2840894488629222010773847634980756855554
 160286708024741922802485707803328368153641110473200*a²*e^(859/2*I*c) + 102
 281918501206789312019757504315186539799825367591653596402987256155988989618
 78198815353510*a²*e^(857/2*I*c) + 3627814697373286004523746845042808477689
 0532748926686396879733480311191302936923550162289900*a²*e^(855/2*I*c) + 12
 677923591460860327846708769846048364283452347327321785486745897733577555808
 3027540308805740*a²*e^(853/2*I*c) + 43657709696171185576928771470335738988
 5645132886579731206499384230924672871218525038020148230*a²*e^(851/2*I*c) +
 14816145216650511407916681110461741561613773326596625530887987775226384369
 65972191593890153520*a²*e^(849/2*I*c) + 4955886749410854283637836408922370
 908861423693236555100736486744859805066952581996944150188670*a²*e^{(847/2*I}
 *c) + 163406219609422063383294877692249713526624644487836015598004157632318
 73879690981091170280400080*a²*e^(845/2*I*c) + 5311593939382952910273717641
 1367113005497399315270309341108454768136564764298294135564858847390*a²*e⁽
 843/2*I*c) + 17023039171380937558022108035506702691245677274619472930083614
 4395351294968707996644726411879200*a²*e^(841/2*I*c) + 53796131867127191193
 581204749764207246106941584902744735646183027124994216431926716790493201606
 0*a²*e^(839/2*I*c) + 16765303716452633937145844225799787825635321495142088
 00221991656698632081178644609104890307855200*a²*e^(837/2*I*c) + 5153020871

486711873416491815131975951900434246025142352516552818224273483297543776745
652059545730*a^2*e^(835/2*I*c) + 156223065664409210263436639434624267415625
14861595325201248149059018047548245480491740664747553230*a^2*e^(833/2*I*c)
+ 4672000737507884896003900021756385198292556710389242036797900363736884636
9023717951090246855155470*a^2*e^(831/2*I*c) + 13784020876187796043634965212
5396975133098136018690719338924949225993395730744805382442729064152410*a^2*
e^(829/2*I*c) + 40123988050979584686576221801497001662186042133402928513428
3286090912984697175449116164770568435310*a^2*e^(827/2*I*c) + 11524614778722
545432398884845151494358758349066560138411327899216487643850548107464116964
06101981510*a^2*e^(825/2*I*c) + 3266487923103038130999265140826940624050408
404456340195828586033626485345825342237364184398762915620*a^2*e^(823/2*I*c)
+ 913703632641901208311759228155356567277588504710020963574542295442139835
6217178283444278671884412660*a^2*e^(821/2*I*c) + 25225271662800212142969423
542567302484417429310194035755111310742077579689215072021281081661345709120
*a^2*e^(819/2*I*c) + 687398125609725309215114971069632517451775270564920953
01601973235793309670088282296954444560031434930*a^2*e^(817/2*I*c) + 1849089
239940393578550118977040296033236373497300831262371548539751029049877855639
33490666771915454740*a^2*e^(815/2*I*c) + 4910421418568783507156817560584451
38552698769297425826668396199882849855448274647622775780304699322930*a^2*e^
(813/2*I*c) + 1287433080457573572306673605052106758486917694203459437629697
821743582350653265565508291332729874085680*a^2*e^(811/2*I*c) + 333279451390
535968218626910840162366988772482405087981563250144139260512541568656551874
2717075933147150*a^2*e^(809/2*I*c) + 85192764511342537785361716307460782998
73324316380590414453537555043854967995509903575577929995103297680*a^2*e^(80
7/2*I*c) + 2150497228392055035861282930997456794673900631383363426467121398
9597677643264769478110594714212525857560*a^2*e^(805/2*I*c) + 53610291202997
129243387227510746438880417396133990475401200151836824511033359061830417818
144623930466160*a^2*e^(803/2*I*c) + 131996091090182003447057411825399119687
071252908752803875765221141863578132827930499563016110773453608580*a^2*e^(8
01/2*I*c) + 321001870082230626552157935960255206975703259672220019809311160
664723949929848535867562171221207552583920*a^2*e^(799/2*I*c) + 771111302353
098224996612507872558890266159399819221015025614890733856653497550312410883
732182309081849060*a^2*e^(797/2*I*c) + 182986267094803131328901419808732126
3556435400853890848564941806503546501752216732765684931337636766894640*a^2*
e^(795/2*I*c) + 42898317708769156046206216052503119586771288875695306799234
76007397181724403545145491954691454914942555700*a^2*e^(793/2*I*c) + 9935970
535949030878133346983171428555968257799381106714215413905033872603148050425
223025351154130410277240*a^2*e^(791/2*I*c) + 227381997341336530780277767348
923448083647508497915370273788561098000912781759166376916355824336395002615
60*a^2*e^(789/2*I*c) + 5141675317373213986665423945495911958975588547900983
5237894780038895835558829259413468345602675738630164100*a^2*e^(787/2*I*c) +
11489010685123829155133572826556236476512100511959149702463481561532806098
9866188627682839323290553007949840*a^2*e^(785/2*I*c) + 25369754009215492480
341260321336842031022521543381219390657970943834020256465972472182252505523
2970075634740*a^2*e^(783/2*I*c) + 55364497623850426876073693290820390895951

7626284011218937645206254241531004929708538056155465373941573477840*a²*e⁽
 781/2*I*c) + 11941350382182318927892141816101487084376646887192530245393316
 90089016746312190822555525802303755766475255060*a²*e^{(779/2*I*c) + 2545702}
 425248919092444745231903392706937806263515420686062286232533125040633932944
 013647476782825751916866192*a²*e^{(777/2*I*c) + 536437058177643067602336686}
 502662821685972963172417926385596859861894285657204297738586913679757410111
 2934360*a²*e^{(775/2*I*c) + 11174069580281343172329449999299750628302547624}
 765899734171290717678737916201127059517927335920301363381599920*a²*e^{(773/}
 2*I*c) + 230096320007156857245011890261110507422675235530575787327198436696
 36960476167526014756393531959162197925032115*a²*e^{(771/2*I*c) + 4684223968}
 783713426910236443923648845093519456603766005211936179651041190321991436923
 1181221570261150235170750*a²*e^{(769/2*I*c) + 94279883860223497927705813372}
 761408660272847636988935467257634598799635548470093870330610958925793769077
 443101*a²*e^{(767/2*I*c) + 187619225221186892362954324173464256888580245686}
 654501161811526819167428547859856880414713989939337716707545896*a²*e^{(765/}
 2*I*c) + 369178030253342447121644615776637010615775278423789324847337963965
 264696957387110053345923248330973169112504125*a²*e^{(763/2*I*c) + 718319539}
 770367264276023193923607996369471257717391142715655747378128411474061415600
 744631353742653947474979790*a²*e^{(761/2*I*c) + 138212008245370933387251058}
 741205952414997925496451106616160500947951412223145277797666935595477938732
 6179193890*a²*e^{(759/2*I*c) + 26299224000359092886055932289847481067460973}
 34648529702694924936483874523991116960016579495951972600623182611066*a²*e⁽
 757/2*I*c) + 4949163663099895843813162506453820232852919885253345976704821
 171775106226971407006064744321532373168942138457133*a²*e^{(755/2*I*c) + 921}
 159800087095861287271096006116330239579693186410364236421685386849465435222
 3629534260533174654764593528061971*a²*e^{(753/2*I*c) + 16958009786047484319}
 864637673956168568925607647419464421111187017418214448976233879833477597585
 335045518017550375*a²*e^{(751/2*I*c) + 308797603703517726930357308275844430}
 947259923870484962827851375803495400250683243429812243796212498170773512027
 55*a²*e^{(749/2*I*c) + 5562300342860253243302688841389823386423061808324476}
 3365901631495829738789432935549737175085734259798422054263257*a²*e^{(747/2*}
 I*c) + 99114849858421344389405236758955308432017258578325989313493313927809
 241858243075104249489142628498003793332896533*a²*e^{(745/2*I*c) + 174722600}
 888404804978530590100460003278180658978831905330154370982763984646513813160
 761146082864787793974499383768*a²*e^{(743/2*I*c) + 304724800478424739825811}
 157956168270645458485759770904702074659803850468461952919986071057980917692
 816270458765682*a²*e^{(741/2*I*c) + 525820101170441091857872327398452496414}
 525760936913287583911529580580538586559649846585541562686976417582636323800
 *a²*e^{(739/2*I*c) + 897758618407588265406502003227323745918017671431846078}
 493905129440539575848523829309505570503467433698687391599525*a²*e^{(737/2*I}
 *c) + 151669201186701211532104394665165894644929489976343801572574012074413
 1654354391550015561531757951384128294098404528*a²*e^{(735/2*I*c) + 25355554}
 133458355415663215796713320822307450379883912694516752439648444683306792410
 34930206578896711879155418196133*a²*e^{(733/2*I*c) + 4194780709047280483224}
 815833940022555816707903442864843083388983235625506900033313228628940959072

838355592388518976*a²*e^(731/2*I*c) + 686797215755123692175964784832304078
965366124147001472266808461611567475265507538689887990820919096579010590930
3585*a²*e^(729/2*I*c) + 11128936663080446107330089476544296777983319792144
897369857256870754469969651791477159202549359802639971419696605690*a²*e<sup>(7
27/2*I*c)</sup> + 178487413703785127991533047429031834258646683672567352293923626
69226980832673201354290319227799410874360671281422106*a²*e^(725/2*I*c) + 2
833435569095910075856346783902184507634209486006385880783671350359538756924
6628801974220943629593333922210878751177*a²*e^(723/2*I*c) + 44523991520646
586076906786564465923060604941985183775818546461478677519510288832394715366
256217063713589099772007856*a²*e^(721/2*I*c) + 692585172082884366515917933
309662610478617173192838529984034084594598199993193565540874370758585383154
37799030208205*a²*e^(719/2*I*c) + 1066532617297247229322755125788473457605
740958481386860472093607351888672979230548927882794820156294486734562372110
40*a²*e^(717/2*I*c) + 1626000312605035796776398697495577220992594081439694
83138470362784417124438163290111258560462574993542827637524092781*a²*e<sup>(71
5/2*I*c)</sup> + 2454349951719685029766850797579166546323520211218782365529260313
64907280287667538662828687293957579330538913455736808*a²*e^(713/2*I*c) + 3
668138702309215941431086549921542994120102019062944908064128698399563928602
71021521745294086086233837764981473476178*a²*e^(711/2*I*c) + 5428409594024
610830965653999053400760902549927897609443377744890862992358415570398824305
91018739442636384860204215400*a²*e^(709/2*I*c) + 7955022907770981714081348
695604512368434695645955161493365050851052422904247586076929456899240074360
94637703203085305*a²*e^(707/2*I*c) + 1154454335904857364247982557912578718
102746849691770314220186712706034099207927530450655444722398232085044436220
634697*a²*e^(705/2*I*c) + 165921734007899036629562342203845661808854868645
7548538185410690033650573056920057414962831294588203107486826119787423*a²*
e^(703/2*I*c) + 23618147825924090731401517179520551201564993752366198142604
03866479477744294234718960967782715041717115313047558623183*a²*e<sup>(701/2*I*
c)</sup> + 3329886438974832992528101094207114478285955872618100718692457613200121
080012023835679244861800931812439160323444845807*a²*e^(699/2*I*c) + 465028
053005713082784315362456098166784074181162617981958482480978359190894501250
6423661839456267739976826873672738365*a²*e^(697/2*I*c) + 64330993352691958
087170206396618494599861960856359687277765646258499326392809824960345969460
07808758161920095325761890*a²*e^(695/2*I*c) + 8816131688931211814057208115
533917355801787029335575600410977242399829559924610051615513646540527419139
301483310423786*a²*e^(693/2*I*c) + 119695549325675655931779914014876631911
499472637586508881324251962378813904578940095889045345302961783240309850725
73618*a²*e^(691/2*I*c) + 1610072907829930926448116373437511521483953182188
0188023924467778707809792051397971564268356677680703285577513151431841*a²*
e^(689/2*I*c) + 21458839249636384878533706734791925212847901427210525197864
886973863458734886342259576872792909047798523915929365691940*a²*e<sup>(687/2*I
*c)</sup> + 283390723752390156578013167975360696687889097326013890324653337128506
20264720283658988975362676026266822753938421168705*a²*e^(685/2*I*c) + 3708
594553993260963723097865204034276410360777816453731587009464079035003417707
1723193892067577540510415827321202078290*a²*e^(683/2*I*c) + 48095342669966

056020726545704006199266666160630361013755158878063661343210637596725104903
 439769999847624253161931778351*a²*e^(681/2*I*c) + 618147708540153199824482
 447394874356055527851229789104813784350420903665006327266287934972907658379
 77272563559609588576*a²*e^(679/2*I*c) + 7874132585546071069906792098596613
 272993668909384846081647513493184683001031856523986637858985635651761887673
 5439697520*a²*e^(677/2*I*c) + 99416866689402868205953100722816261101271808
 450248252257981611335676431923025899234859866205857872672547668167325989920
 *a²*e^(675/2*I*c) + 124419949314487403216172588535238853667583569695346393
 000362307045710566468085416110994554478758231547427229315129958280*a²*e<sup>(6
 73/2*I*c)</sup> + 154354165743697402995415216121916387446315508628912131928600309
 585745568868263338383595893423751250560255188896660980640*a²*e^(671/2*I*c)
 + 189832680846992441750979398054550765983447380706582698086505079016572222
 835305196482037082520548635494540196474773032392*a²*e^(669/2*I*c) + 231458
 954877865680481711467581492320475708179284410834307431449083296993015735283
 992533153783630057823894466466386789920*a²*e^(667/2*I*c) + 279803880980823
 081151507855346299192278245341814031460411506106372469276985146379946315124
 170832410518829080258705986280*a²*e^(665/2*I*c) + 335379844326285592736695
 46385701841843241318876648488111912395059240855249328246830729779350439108
 234296749061439256880*a²*e^(663/2*I*c) + 398612508602184099797618213464893
 583100423327557993330634195015291012852305358265371442241359113106146519054
 690230460080*a²*e^(661/2*I*c) + 469811437014079749298372016621303751653441
 794274933681143305269482262555422078791132625827649344909492073035868314640
 200*a²*e^(659/2*I*c) + 549140935306437649109502279093298286642071151104653
 437705824618506516690970915485474849970804454250926034882525081421920*a²*e
^(657/2*I*c) + 636592737563033962038907595159410207630226126748551544972393
 205357335273235966560353474541619072109609425303261030333480*a²*e<sup>(655/2*I
 *c)</sup> + 731962314882746930932643298184913142623544929508542538052238437217139
 684503440864983481224441799145110101150208078149600*a²*e^(653/2*I*c) + 834
 830647252737130370352256191399634154462308618312283624738448826183345582990
 700043901989251342801050174088405942776680*a²*e^(651/2*I*c) + 944553238998
 322012684234360243733605735140889390389676408061822904814729681532931792335
 624967472663525610160924422346720*a²*e^(649/2*I*c) + 106025796374953633514
 781711862919397290428499069890974784826574010293877175837317687855132853273
 3642490124903197641779440*a²*e^(647/2*I*c) + 11808529906611396788267529739
 649004013694888958024539109846840243868929535966239993555593680555049576275
 36059400058806560*a²*e^(645/2*I*c) + 1305045575264666719622445738569646486
 643942005634721936316646042608535115485389960589294914491574041407279419512
 731576020*a²*e^(643/2*I*c) + 143137191334499328625814579465878959112813573
 154334449563267522698427043075293835337740680975480964484566886833882197512
 0*a²*e^(641/2*I*c) + 15582375841235407475051527824265404338649299690182902
 39445758871815326594322852854852718664426477851831755925555671727660*a²*e ^(639/2*I*c) + 1683967390218535669432481192273624026413889498620534368790774
 587302981877714639969367635542513279277621696310502074133800*a²*e<sup>(637/2*I
 *c)</sup> + 180686268547401406135899278150063842182410552617971181977297359635900
 5838247462282270364980307784363004235593075525641260*a²*e^(635/2*I*c) + 19

252636223728394750051188080130802855312516229284947971945519780347026927574
76718025108186454783185469267626276190347760*a²*e^(633/2*I*c) + 2037613204
234302788388872974263559802922485435801951718645182637071550371893197987545
075984445655085553689119461288385080*a²*e^(631/2*I*c) + 214251964619046711
623814746757926022312992451440425137554782870765570734842808056050738631135
7900526669458781794087683160*a²*e^(629/2*I*c) + 22388133772086363214562711
695531110839152740444990636033971999650339788664043245120079613369281542808
08573579878469429140*a²*e^(627/2*I*c) + 2325595084506896663041199729630019
144950321875545686061358679706618715723878421925212744802710528140695848048
863263401620*a²*e^(625/2*I*c) + 240227152626534500276407910147939754731649
929031350785060489649742719688634225636482917330715633964895939957471168583
5340*a²*e^(623/2*I*c) + 24685764139519560439420221434147612346331971979438
06791819033461685747350263950586286250114393171484165699859371683668820*a²
*e^(621/2*I*c) + 2524574466857996282393547747052223049971409325335770864930
047451730298883199815748273420062646212064128261863603173113220*a²*e<sup>(619/
2*I*c)</sup> + 257064772383156388117355397683416885685254388955293074358989631221
3425786028231311572055034657555701798516652461881179020*a²*e^(617/2*I*c) +
26074642986772057436868337639434634440619010300129655886691393588320605098
97548972026199493982403179852911793212914653440*a²*e^(615/2*I*c) + 2635930
911076402644000121905925952148375148833390373058411708254060533702084460459
096459271430090085171142785221034332360*a²*e^(613/2*I*c) + 265713163140556
016230055392391322532989750689206902391066833032058176330388888172691591252
9432751510232297721132883711360*a²*e^(611/2*I*c) + 26722562616403225679443
657002462972657661599238234570150540782027602632669337097067668448077540895
75759625548721880960900*a²*e^(609/2*I*c) + 2682522557658080650589233969123
672190643044432210056127292023865687852378234863360381336201324629095652385
752662511996320*a²*e^(607/2*I*c) + 268909701530234476933529149937972078112
372732443614330857038266000225227119669039337313386144764268854277967640925
3164100*a²*e^(605/2*I*c) + 26930191466678412020042146362259063768904225037
79253729585442498782172066792698483998400631873111003925178781908847878880*
a²*e^(603/2*I*c) + 2695134040310534956999355762470654940028932561435586408
556892140495393036120495504431025989594669805132478153565562971060*a²*e<sup>(6
01/2*I*c)</sup> + 269603753121606409480357913437607430375456968923646477143498642
2320946498560245047737297861595509112192278615431723054440*a²*e<sup>(599/2*I*c
)</sup> + 26960375312160640948035791343760743037545696892364647714349864223209464
98560245047737297861595509112192278615431723054440*a²*e^(597/2*I*c) + 2695
134040310534956999355762470654940028932561435586408556892140495393036120495
504431025989594669805132478153565562971060*a²*e^(595/2*I*c) + 269301914666
784120200421463622590637689042250377925372958544249878217206679269848399840
0631873111003925178781908847878880*a²*e^(593/2*I*c) + 26890970153023447693
352914993797207811237273244361433085703826600022522711966903933731338614476
42688542779676409253164100*a²*e^(591/2*I*c) + 2682522557658080650589233969
123672190643044432210056127292023865687852378234863360381336201324629095652
385752662511996320*a²*e^(589/2*I*c) + 267225626164032256794436570024629726
576615992382345701505407820276026326693370970676684480775408957575962554872

$1880960900*a^2*e^{(587/2*I*c)} + 26571316314055601623005539239132253298975068$
 $920690239106683303205817633038888817269159125294327515102322977211328837113$
 $60*a^2*e^{(585/2*I*c)} + 2635930911076402644000121905925952148375148833390373$
 $058411708254060533702084460459096459271430090085171142785221034332360*a^2*e$
 $^{(583/2*I*c)} + 260746429867720574368683376394346344406190103001296558866913$
 $9358832060509897548972026199493982403179852911793212914653440*a^2*e^{(581/2*$
 $I*c)} + 25706477238315638811735539768341688568525438895529307435898963122134$
 $25786028231311572055034657555701798516652461881179020*a^2*e^{(579/2*I*c)} + 2$
 $524574466857996282393547747052223049971409325335770864930047451730298883199$
 $815748273420062646212064128261863603173113220*a^2*e^{(577/2*I*c)} + 246857641$
 $395195604394202214341476123463319719794380679181903346168574735026395058628$
 $6250114393171484165699859371683668820*a^2*e^{(575/2*I*c)} + 24022715262653450$
 $027640791014793975473164992903135078506048964974271968863422563648291733071$
 $56339648959399574711685835340*a^2*e^{(573/2*I*c)} + 2325595084506896663041199$
 $729630019144950321875545686061358679706618715723878421925212744802710528140$
 $695848048863263401620*a^2*e^{(571/2*I*c)} + 223881337720863632145627116955311$
 $108391527404449906360339719996503397886640432451200796133692815428080857357$
 $9878469429140*a^2*e^{(569/2*I*c)} + 21425196461904671162381474675792602231299$
 $245144042513755478287076557073484280805605073863113579005266694587817940876$
 $83160*a^2*e^{(567/2*I*c)} + 2037613204234302788388872974263559802922485435801$
 $951718645182637071550371893197987545075984445655085553689119461288385080*a^$
 $2*e^{(565/2*I*c)} + 192526362237283947500511880801308028553125162292849479719$
 $4551978034702692757476718025108186454783185469267626276190347760*a^2*e^{(563$
 $/2*I*c)} + 18068626854740140613589927815006384218241055261797118197729735963$
 $59005838247462282270364980307784363004235593075525641260*a^2*e^{(561/2*I*c)}$
 $+ 1683967390218535669432481192273624026413889498620534368790774587302981877$
 $714639969367635542513279277621696310502074133800*a^2*e^{(559/2*I*c)} + 155823$
 $758412354074750515278242654043386492996901829023944575887181532659432285285$
 $4852718664426477851831755925555671727660*a^2*e^{(557/2*I*c)} + 14313719133449$
 $932862581457946587895911281357315433444956326752269842704307529383533774068$
 $09754809644845668868338821975120*a^2*e^{(555/2*I*c)} + 1305045575264666719622$
 $445738569646486643942005634721936316646042608535115485389960589294914491574$
 $041407279419512731576020*a^2*e^{(553/2*I*c)} + 118085299066113967882675297396$
 $490040136948889580245391098468402438689295359662399935555936805550495762753$
 $6059400058806560*a^2*e^{(551/2*I*c)} + 10602579637495363351478171186291939729$
 $042849906989097478482657401029387717583731768785513285327336424901249031976$
 $41779440*a^2*e^{(549/2*I*c)} + 9445532389983220126842343602437336057351408893$
 $90389676408061822904814729681532931792335624967472663525610160924422346720*$
 $a^2*e^{(547/2*I*c)} + 8348306472527371303703522561913996341544623086183122836$
 $24738448826183345582990700043901989251342801050174088405942776680*a^2*e^{(54$
 $5/2*I*c)} + 7319623148827469309326432981849131426235449295085425380522384372$
 $17139684503440864983481224441799145110101150208078149600*a^2*e^{(543/2*I*c)}$
 $+ 6365927375630339620389075951594102076302261267485515449723932053573352732$
 $35966560353474541619072109609425303261030333480*a^2*e^{(541/2*I*c)} + 5491409$
 $353064376491095022790932982866420711511046534377058246185065166909709154854$

74849970804454250926034882525081421920*a²*e^(539/2*I*c) + 4698114370140797
492983720166213037516534417942749336811433052694822625554220787911326258276
49344909492073035868314640200*a²*e^(537/2*I*c) + 3986125086021840997976182
134648935831004233275579933306341950152910128523053582653714422413591131061
46519054690230460080*a²*e^(535/2*I*c) + 3353798443262855927366954638570184
184324131887664848811119123950592408552493282468307297793504391082342967490
61439256880*a²*e^(533/2*I*c) + 2798038809808230811515078553462991922782453
418140314604115061063724692769851463799463151241708324105188290802587059862
80*a²*e^(531/2*I*c) + 2314589548778656804817114675814923204757081792844108
34307431449083296993015735283992533153783630057823894466466386789920*a²*e^(529/2*I*c) + 1898326808469924417509793980545507659834473807065826980865050
79016572222835305196482037082520548635494540196474773032392*a²*e^(527/2*I*c) + 1543541657436974029954152161219163874463155086289121319286003095857455
68868263338383595893423751250560255188896660980640*a²*e^(525/2*I*c) + 1244
199493144874032161725885352388536675835696953463930003623070457105664680854
16110994554478758231547427229315129958280*a²*e^(523/2*I*c) + 9941686668940
286820595310072281626110127180845024825225798161133567643192302589923485986
6205857872672547668167325989920*a²*e^(521/2*I*c) + 78741325855460710699067
920985966132729936689093848460816475134931846830010318565239866378589856356
517618876735439697520*a²*e^(519/2*I*c) + 618147708540153199824482447394874
356055527851229789104813784350420903665006327266287934972907658379772725635
59609588576*a²*e^(517/2*I*c) + 4809534266996605602072654570400619926666616
063036101375515887806366134321063759672510490343976999984762425316193177835
1*a²*e^(515/2*I*c) + 37085945539932609637230978652040342764103607778164537
315870094640790350034177071723193892067577540510415827321202078290*a²*e^(513/2*I*c) + 283390723752390156578013167975360696687889097326013890324653337
12850620264720283658988975362676026266822753938421168705*a²*e^(511/2*I*c)
+ 2145883924963638487853370673479192521284790142721052519786488697386345873
4886342259576872792909047798523915929365691940*a²*e^(509/2*I*c) + 16100729
078299309264481163734375115214839531821880188023924467778707809792051397971
564268356677680703285577513151431841*a²*e^(507/2*I*c) + 119695549325675655
931779914014876631911499472637586508881324251962378813904578940095889045345
30296178324030985072573618*a²*e^(505/2*I*c) + 8816131688931211814057208115
533917355801787029335575600410977242399829559924610051615513646540527419139
301483310423786*a²*e^(503/2*I*c) + 643309933526919580871702063966184945998
619608563596872777656462584993263928098249603459694600780875816192009532576
1890*a²*e^(501/2*I*c) + 46502805300571308278431536245609816678407418116261
79819584824809783591908945012506423661839456267739976826873672738365*a²*e^(499/2*I*c) + 3329886438974832992528101094207114478285955872618100718692457
613200121080012023835679244861800931812439160323444845807*a²*e^(497/2*I*c)
+ 236181478259240907314015171795205512015649937523661981426040386647947774
4294234718960967782715041717115313047558623183*a²*e^(495/2*I*c) + 16592173
400789903662956234220384566180885486864575485381854106900336505730569200574
14962831294588203107486826119787423*a²*e^(493/2*I*c) + 1154454335904857364
247982557912578718102746849691770314220186712706034099207927530450655444722

398232085044436220634697*a²*e^(491/2*I*c) + 795502290777098171408134869560
451236843469564595516149336505085105242290424758607692945689924007436094637
703203085305*a²*e^(489/2*I*c) + 542840959402461083096565399905340076090254
992789760944337774489086299235841557039882430591018739442636384860204215400
*a²*e^(487/2*I*c) + 366813870230921594143108654992154299412010201906294490
806412869839956392860271021521745294086086233837764981473476178*a²*e<sup>(485/
2*I*c)</sup> + 245434995171968502976685079757916654632352021121878236552926031364
907280287667538662828687293957579330538913455736808*a²*e^(483/2*I*c) + 162
600031260503579677639869749557722099259408143969483138470362784417124438163
290111258560462574993542827637524092781*a²*e^(481/2*I*c) + 106653261729724
722932275512578847345760574095848138686047209360735188867297923054892788279
482015629448673456237211040*a²*e^(479/2*I*c) + 692585172082884366515917933
309662610478617173192838529984034084594598199993193565540874370758585383154
37799030208205*a²*e^(477/2*I*c) + 4452399152064658607690678656446592306060
494198518377581854646147867751951028883239471536625621706371358909977200785
6*a²*e^(475/2*I*c) + 28334355690959100758563467839021845076342094860063858
807836713503595387569246628801974220943629593333922210878751177*a²*e<sup>(473/
2*I*c)</sup> + 178487413703785127991533047429031834258646683672567352293923626692
26980832673201354290319227799410874360671281422106*a²*e^(471/2*I*c) + 1112
893666308044610733008947654429677798331979214489736985725687075446996965179
1477159202549359802639971419696605690*a²*e^(469/2*I*c) + 68679721575512369
217596478483230407896536612414700147226680846161156747526550753868988799082
09190965790105909303585*a²*e^(467/2*I*c) + 4194780709047280483224815833940
022555816707903442864843083388983235625506900033313228628940959072838355592
388518976*a²*e^(465/2*I*c) + 253555541334583554156632157967133208223074503
7988391269451675243964844468330679241034930206578896711879155418196133*a²*
e^(463/2*I*c) + 15166920118670121153210439466516589464492948997634380157257
40120744131654354391550015561531757951384128294098404528*a²*e^(461/2*I*c)
+ 8977586184075882654065020032273237459180176714318460784939051294405395758
48523829309505570503467433698687391599525*a²*e^(459/2*I*c) + 5258201011704
410918578723273984524964145257609369132875839115295805805385865596498465855
41562686976417582636323800*a²*e^(457/2*I*c) + 3047248004784247398258111579
561682706454584857597709047020746598038504684619529199860710579809176928162
70458765682*a²*e^(455/2*I*c) + 1747226008884048049785305901004600032781806
58978831905330154370982763984646513813160761146082864787793974499383768*a²
*e^(453/2*I*c) + 9911484985842134438940523675895530843201725857832598931349
3313927809241858243075104249489142628498003793332896533*a²*e^(451/2*I*c) +
55623003428602532433026888413898233864230618083244763365901631495829738789
432935549737175085734259798422054263257*a²*e^(449/2*I*c) + 308797603703517
726930357308275844430947259923870484962827851375803495400250683243429812243
79621249817077351202755*a²*e^(447/2*I*c) + 1695800978604748431986463767395
61685689256076474194644211118701741821444897623387983347759758533504551801
7550375*a²*e^(445/2*I*c) + 92115980008709586128727109600611633023957969318
64103642364216853868494654352223629534260533174654764593528061971*a²*e<sup>(44
3/2*I*c)</sup> + 4949163663099895843813162506453820232852919885253345976704821171

775106226971407006064744321532373168942138457133*a²*e^(441/2*I*c) + 262992
 240003590928860559322898474810674609733464852970269492493648387452399111696
 0016579495951972600623182611066*a²*e^(439/2*I*c) + 13821200824537093338725
 105874120595241499792549645110661616050094795141222314527779766693559547793
 87326179193890*a²*e^(437/2*I*c) + 7183195397703672642760231939236079963694
 71257717391142715655747378128411474061415600744631353742653947474979790*a²
 *e^(435/2*I*c) + 3691780302533424471216446157766370106157752784237893248473
 37963965264696957387110053345923248330973169112504125*a²*e^(433/2*I*c) + 1
 876192252211868923629543241734642568885802456866545011618115268191674285478
 59856880414713989939337716707545896*a²*e^(431/2*I*c) + 9427988386022349792
 770581337276140866027284763698893546725763459879963554847009387033061095892
 5793769077443101*a²*e^(429/2*I*c) + 46842239687837134269102364439236488450
 935194566037660052119361796510411903219914369231181221570261150235170750*a²
 *e^(427/2*I*c) + 230096320007156857245011890261110507422675235530575787327
 19843669636960476167526014756393531959162197925032115*a²*e^(425/2*I*c) + 1
 117406958028134317232944999929975062830254762476589973417129071767873791620
 1127059517927335920301363381599920*a²*e^(423/2*I*c) + 53643705817764306760
 233668650266282168597296317241792638559685986189428565720429773858691367975
 74101112934360*a²*e^(421/2*I*c) + 2545702425248919092444745231903392706937
 806263515420686062286232533125040633932944013647476782825751916866192*a²*e^(419/2*I*c) + 119413503821823189278921418161014870843766468871925302453933
 1690089016746312190822555525802303755766475255060*a²*e^(417/2*I*c) + 55364
 497623850426876073693290820390895951762628401121893764520625424153100492970
 8538056155465373941573477840*a²*e^(415/2*I*c) + 25369754009215492480341260
 321336842031022521543381219390657970943834020256465972472182252505523297007
 5634740*a²*e^(413/2*I*c) + 11489010685123829155133572826556236476512100511
 9591497024634815615328060989866188627682839323290553007949840*a²*e^(411/2*I*c) + 51416753173732139866654239454959119589755885479009835237894780038895
 835558829259413468345602675738630164100*a²*e^(409/2*I*c) + 227381997341336
 530780277767348923448083647508497915370273788561098000912781759166376916355
 82433639500261560*a²*e^(407/2*I*c) + 9935970535949030878133346983171428555
 968257799381106714215413905033872603148050425223025351154130410277240*a²*e^(405/2*I*c) + 428983177087691560462062160525031195867712888756953067992347
 6007397181724403545145491954691454914942555700*a²*e^(403/2*I*c) + 18298626
 709480313132890141980873212635564354008538908485649418065035465017522167327
 65684931337636766894640*a²*e^(401/2*I*c) + 7711113023530982249966125078725
 58890266159399819221015025614890733856653497550312410883732182309081849060*
 a²*e^(399/2*I*c) + 3210018700822306265521579359602552069757032596722200198
 09311160664723949929848535867562171221207552583920*a²*e^(397/2*I*c) + 1319
 960910901820034470574118253991196870712529087528038757652211418635781328279
 30499563016110773453608580*a²*e^(395/2*I*c) + 5361029120299712924338722751
 074643888041739613399047540120015183682451103335906183041781814462393046616
 0*a²*e^(393/2*I*c) + 21504972283920550358612829309974567946739006313833634
 264671213989597677643264769478110594714212525857560*a²*e^(391/2*I*c) + 851
 927645113425377853617163074607829987332431638059041445353755504385496799550

9903575577929995103297680*a²*e^(389/2*I*c) + 33327945139053596821862691084
 01623669887724824050879815632501441392605125415686565518742717075933147150*
 a²*e^(387/2*I*c) + 1287433080457573572306673605052106758486917694203459437
 629697821743582350653265565508291332729874085680*a²*e^(385/2*I*c) + 491042
 141856878350715681756058445138552698769297425826668396199882849855448274647
 622775780304699322930*a²*e^(383/2*I*c) + 184908923994039357855011897704029
 603323637349730083126237154853975102904987785563933490666771915454740*a²*e^(381/2*I*c) + 687398125609725309215114971069632517451775270564920953016019
 73235793309670088282296954444560031434930*a²*e^(379/2*I*c) + 2522527166280
 021214296942354256730248441742931019403575511131074207757968921507202128108
 1661345709120*a²*e^(377/2*I*c) + 91370363264190120831175922815535656727758
 85047100209635745422954421398356217178283444278671884412660*a²*e^(375/2*I*c) + 3266487923103038130999265140826940624050408404456340195828586033626485
 345825342237364184398762915620*a²*e^(373/2*I*c) + 115246147787225454323988
 848451514943587583490665601384113278992164876438505481074641169640610198151
 0*a²*e^(371/2*I*c) + 40123988050979584686576221801497001662186042133402928
 5134283286090912984697175449116164770568435310*a²*e^(369/2*I*c) + 13784020
 876187796043634965212539697513309813601869071933892494922599339573074480538
 2442729064152410*a²*e^(367/2*I*c) + 46720007375078848960039000217563851982
 925567103892420367979003637368846369023717951090246855155470*a²*e^(365/2*I*c) + 156223065664409210263436639434624267415625148615953252012481490590180
 47548245480491740664747553230*a²*e^(363/2*I*c) + 5153020871486711873416491
 815131975951900434246025142352516552818224273483297543776745652059545730*a²*e^(361/2*I*c) + 167653037164526339371458442257997878256353214951420880022
 1991656698632081178644609104890307855200*a²*e^(359/2*I*c) + 53796131867127
 191193581204749764207246106941584902744735646183027124994216431926716790493
 2016060*a²*e^(357/2*I*c) + 17023039171380937558022108035506702691245677274
 6194729300836144395351294968707996644726411879200*a²*e^(355/2*I*c) + 53115
 939393829529102737176411367113005497399315270309341108454768136564764298294
 135564858847390*a²*e^(353/2*I*c) + 163406219609422063383294877692249713526
 62464448783601559800415763231873879690981091170280400080*a²*e^(351/2*I*c)
 + 4955886749410854283637836408922370908861423693236555100736486744859805066
 952581996944150188670*a²*e^(349/2*I*c) + 148161452166505114079166811104617
 415616137733265966255308879877522638436965972191593890153520*a²*e^(347/2*I*c) + 43657709696171185576928771470335738988564513288657973120649938423092
 4672871218525038020148230*a²*e^(345/2*I*c) + 12677923591460860327846708769
 8460483642834523473273217854867458977335775558083027540308805740*a²*e^(343/2*I*c) + 36278146973732860045237468450428084776890532748926686396879733480
 311191302936923550162289900*a²*e^(341/2*I*c) + 102281918501206789312019757
 50431518653979982536759165359640298725615598898961878198815353510*a²*e^(339/2*I*c) + 2840894488629222010773847634980756855554160286708024741922802485
 707803328368153641110473200*a²*e^(337/2*I*c) + 777245248463832164443579464
 317740424946189628494942453508580441800744828509867291417081630*a²*e^(335/2*I*c) + 209435641246496342288374862958056438753891068972196618627904344080
 420177337078104676468880*a²*e^(333/2*I*c) + 555742940074056571594107022251

10128247423080333361478620342730912591089336970706714049470*a²*e^(331/2*I*c) + 1452004323777102486757889227483315199998380565577494243210255916599127
 4651263356626027360*a²*e^(329/2*I*c) + 37348277775428429628209444971587641
 20459066697240394589825664346513551225394202631289660*a²*e^(327/2*I*c) + 9
 456238310150674655657617377517817127295790433860501907722815189807463604233
 16243590560*a²*e^(325/2*I*c) + 2356384144036766363749646185351682354615316
 51215096600123550185747704196738532598229970*a²*e^(323/2*I*c) + 5778119433
 161805018407587242356179142184903524357160032028870653298160933168080418971
 0*a²*e^(321/2*I*c) + 13940253696168079402064259692653507612945211052614130
 496704459947954824765033193710270*a²*e^(319/2*I*c) + 330847321970382353755
 6243769763570815349179943585706271711604571444094770152919124730*a²*e<sup>(317
 /2*I*c)</sup> + 77229701953334031851476725644580269629749633780292101940900402066
 5598966438156162590*a²*e^(315/2*I*c) + 17728246003277186589209994770321940
 0509401728120870156939007264944328352108674765830*a²*e^(313/2*I*c) + 40012
 393889662490616271399213614257399774009402473996666416480625157884768764896
 580*a²*e^(311/2*I*c) + 887750728519622061824165546477873249993522166437395
 1650797658698877398128936202900*a²*e^(309/2*I*c) + 19358569408870667571586
 48696108358375586341793850855062039959891604593248887646480*a²*e<sup>(307/2*I*
 c)</sup> + 4148171360983457444915392773620073805132718828031457512978482193679783
 99846839810*a²*e^(305/2*I*c) + 8732813827726335745368739363282925582060358
 2731737822656013656737314066270769860*a²*e^(303/2*I*c) + 18058262147062510
 799902633211036748190227439947447680116613546969431587149657730*a²*e<sup>(301/
 2*I*c)</sup> + 366715488557555603535421037092230036235721011566753407786822647438
 0300702979520*a²*e^(299/2*I*c) + 73117012158472403322096448677308864979249
 5972559727457509380085597040672846910*a²*e^(297/2*I*c) + 14310167749316770
 6049469001333839411556496168357433098121521463359375252809040*a²*e<sup>(295/2*
 I*c)</sup> + 27485654500744845946086864693951416729958544421088007484546058264731
 062865400*a²*e^(293/2*I*c) + 517961000456783967688273437207593955840388438
 0512473251576883045496625080240*a²*e^(291/2*I*c) + 95743352229194257260397
 2061174768931380078714954598245160747546204280979860*a²*e^(289/2*I*c) + 17
 3551352210830111542403729997736548659579690262161334684378354693653143600*a
²*e^(287/2*I*c) + 30841709943145755974906466306871090919705337121093845564
 611475205831834740*a²*e^(285/2*I*c) + 537179082354777598676931167217806066
 6294358172540326849409957986665686640*a²*e^(283/2*I*c) + 91673686925132684
 1783602023183005234330809250515685392295995174302631940*a²*e^(281/2*I*c) +
 153244689998280106303232133486628958245019231358231317009398623103014360*a
²*e^(279/2*I*c) + 25084568560525486523793075765839786939668166684250998147
 267060999843864*a²*e^(277/2*I*c) + 401946757197219191849520543122326443754
 5950953166839320312436248766740*a²*e^(275/2*I*c) + 63026901151891709378680
 7783619832131991400363103105059485556854864080*a²*e^(273/2*I*c) + 96678088
 682187070566952755698159384671499301435475363946248645511300*a²*e<sup>(271/2*I
 c)</sup> + 14501667734089456372413097720073331427769744768582734443920820884880
 a²*e^(269/2*I*c) + 2126338350434067101841066179986723736309721747170637151
 333282385892*a²*e^(267/2*I*c) + 304650073489624069501319666840972753356711
 517554147295115707791952*a²*e^(265/2*I*c) + 426331551827582342134326571053

59507039157758216710091608150751160*a²*e^(263/2*I*c) + 5824867841884974805
 556257455805728440177379367311530786366553840*a²*e^(261/2*I*c) + 776647773
 894140039601026125724635184851400081828079008519495205*a²*e^(259/2*I*c) +
 101008959448394278622065270281354488236767429874129774402372122*a²*e<sup>(257/
 2*I*c)</sup> + 12808036621165917614961411495703359665564662395820821879432651*a²
 *e^(255/2*I*c) + 1582600527113724210945270692154487384537712932377861494537
 792*a²*e^(253/2*I*c) + 190456047314980596262214455758830176412490479327548
 630395435*a²*e^(251/2*I*c) + 223105477373773387281957926693846975178894127
 12723300156650*a²*e^(249/2*I*c) + 2542510822024809049111292052295925694458
 486937681586168494*a²*e^(247/2*I*c) + 281698477370386606708598633818891039
 165559192659067598486*a²*e^(245/2*I*c) + 303244668734682640780589924763017
 32181384084447966749251*a²*e^(243/2*I*c) + 3169505085332677220309823086926
 906379156042673077997829*a²*e^(241/2*I*c) + 321414485517907071866518386845
 402404565806226718602745*a²*e^(239/2*I*c) + 315997292671613328234030755031
 25661623124842474211925*a²*e^(237/2*I*c) + 3009497270402664372603369055821
 186361937123336960151*a²*e^(235/2*I*c) + 277411701644365486379380075778816
 997842958595635411*a²*e^(233/2*I*c) + 247275011666470610832895287504674525
 94464413496712*a²*e^(231/2*I*c) + 2129312297800033603965109459542894851379
 636241390*a²*e^(229/2*I*c) + 176951139921850074633194549001040224465598018
 440*a²*e^(227/2*I*c) + 14175642468718517834053796420531668264840828827*a²
 *e^(225/2*I*c) + 1093437903450413476694775388085248143435843184*a²*e<sup>(223/
 2*I*c)</sup> + 81106652744979453503605904560553941804516827*a²*e^(221/2*I*c) + 5
 777459917323047305837478939343390617560800*a²*e^(219/2*I*c) + 394635225337
 657865294965990982357902779935*a²*e^(217/2*I*c) + 258072074387090234055646
 98654381610855782*a²*e^(215/2*I*c) + 1612950428548722021160870732573388748
 486*a²*e^(213/2*I*c) + 96165064556404674374591793494770969751*a²*e<sup>(211/2
 *I*c)</sup> + 5458017107743192900932192332079754800*a²*e^(209/2*I*c) + 294232725
 411218343537913910694524115*a²*e^(207/2*I*c) + 150280154429164727843033555
 49568064*a²*e^(205/2*I*c) + 725212537982338639553752606105971*a²*e<sup>(203/2
 *I*c)</sup> + 32964206161728793499031816369336*a²*e^(201/2*I*c) + 14064727930607
 42817278133399694*a²*e^(199/2*I*c) + 56109286874368068446384618040*a²*e<sup>(
 197/2*I*c)</sup> + 2083633993393098160454111295*a²*e^(195/2*I*c) + 7165937010280
 7444959242727*a²*e^(193/2*I*c) + 2268898261098769561120041*a²*e<sup>(191/2*I*
 c)</sup> + 65678633862570503690097*a²*e^(189/2*I*c) + 1723848657664106290905*a²
 *e^(187/2*I*c) + 40614235388489346675*a²*e^(185/2*I*c) + 84833912037464187
 0*a²*e^(183/2*I*c) + 15464515215103622*a²*e^(181/2*I*c) + 241005431923542
 *a²*e^(179/2*I*c) + 3121832019735*a²*e^(177/2*I*c) + 32266997620*a²*e<sup>(1
 75/2*I*c)</sup> + 249487095*a²*e^(173/2*I*c) + 1282710*a²*e^(171/2*I*c) + 3289*
 a²*e^(169/2*I*c))/(e^(517*I*c) + 418*e^(516*I*c) + 87153*e^(515*I*c) + 120
 85216*e^(514*I*c) + 1253841160*e^(513*I*c) + 103818048048*e^(512*I*c) + 714
 6142307307*e^(511*I*c) + 420601518659718*e^(510*I*c) + 21608403021340047*e<sup>(
 509*I*c)</sup> + 984382804329835768*e^(508*I*c) + 40261256699368950388*e<sup>(507*I*
 c)</sup> + 1493326612293984160368*e^(506*I*c) + 50648660944512569972179*e<sup>(505*I*
 c)</sup> + 1581796642397812408161814*e^(504*I*c) + 45759117183402579073139583*e<sup>(
 503*I*c)</sup> + 1232445557346832245176696904*e^(502*I*c) + 310422225220746816156

25020522*e^(501*I*c) + 734057263616388449968842366924*e^(500*I*c) + 1635316
4647151530240529137618111*e^(499*I*c) + 344277152012875134140739302960914*e
^(498*I*c) + 6868329225263681349501997341320517*e^(497*I*c) + 1301711930791
72823835151430773360024*e^(496*I*c) + 2348998374244347079532766203075607598
*e^(495*I*c) + 40443624781415311581857832389099634564*e^(494*I*c) + 6656346
70676210063754191847109971141414*e^(493*I*c) + 1049040266951089742462464376
6470754045064*e^(492*I*c) + 158566476113257562566117432227203884298856*e^(4
91*I*c) + 2302150411226234925855222345201500900533576*e^(490*I*c) + 3214788
7693375338817454482515377350383950278*e^(489*I*c) + 43233368864426155754794
4179250800440604964868*e^(488*I*c) + 56059272530675585517804528836898355144
55118670*e^(487*I*c) + 70164515322544462906873548813748091084561870680*e^(4
86*I*c) + 84855220276512356496200136959676295361696315113*e^(485*I*c) + 99
25490738534402272939987038714580495445431374618*e^(484*I*c) + 1123916045422
46650966429162063124338952554575234051*e^(483*I*c) + 1233096700139723365181
997220750932590655287625342156*e^(482*I*c) + 131187818011721747296793398943
18153694964675368481194*e^(481*I*c) + 1354425949166361161915746506253316462
38501101627937224*e^(480*I*c) + 1357990663161479842850642848032544982878359
839580349899*e^(479*I*c) + 132317088701048969738000567337799190893408367560
09580718*e^(478*I*c) + 1253704965869212726621980508512693231711673388540817
82959*e^(477*I*c) + 1155855412893594260345544966642687823630035899363232371
472*e^(476*I*c) + 103751844998711755019093989565966841168029970825266603235
24*e^(475*I*c) + 9072260572220881491864228463948718776460758970649397077477
6*e^(474*I*c) + 77320463699114577506146273102809850609443267578813629501125
9*e^(473*I*c) + 64261954855352485764250681368704655300871140038757166913839
02*e^(472*I*c) + 5210811762917704866049240098517583098750570056687781895414
1639*e^(471*I*c) + 41243069829991519084806722232721943506774793409189467048
8982928*e^(470*I*c) + 31877499297443464972115360447517765823209586279238164
70590659024*e^(469*I*c) + 2407080191352975710185802291437204586474699178618
2039740274325264*e^(468*I*c) + 17764282913511934857719443767580283023990546
0092687136494961404333*e^(467*I*c) + 12818174649149708108596041898283590007
90789921169405304612211251818*e^(466*I*c) + 9046693523825682979044338963104
263167672586826367911338826483549173*e^(465*I*c) + 624735507810532953177107
74690247114124125187565731848441781904032672*e^(464*I*c) + 4222761266320036
87547754746555709988710527133086660161366353656787288*e^(463*I*c) + 2794709
104475686611842790694973699164482254723977210209725661304403472*e^(462*I*c)
+ 18115768495615758076710303055505625589254293659193314153418333944596408*
e^(461*I*c) + 1150514818520808488737003883545213155676403651240031036911766
97194292320*e^(460*I*c) + 7160994975990580798956333385529402291928581964815
97830078819711862600096*e^(459*I*c) + 4369442482910113914565353136069595862
669338858053419381214131241925047008*e^(458*I*c) + 261439762799020214434719
45665080254563056810183520401889800285493144867448*e^(457*I*c) + 1534360887
45056254127327239461577071933130157764595997113973513183188399376*e^(456*I*
c) + 8835009688217912026007745419277692007376893935137347893683970933333119
61880*e^(455*I*c) + 4992519712457043983505377976607953988397368297591114957
991804893688371867680*e^(454*I*c) + 276931165383432592259833826376479361226

64033859615133489846664694361471028310*e^(453*I*c) + 1508223814314124137735
66474210011746852297437597059186295243989481140398152780*e^(452*I*c) + 8066
79543607589140759305010796189568269842021613388955218916278823182639488190*
e^(451*I*c) + 4238125846763232586394188569858685826755328005548627437019301
405851325887594480*e^(450*I*c) + 218764828927139099280403456125787058051215
08756226696317087651824252241418663320*e^(449*I*c) + 1109691996873209747499
22259595250444341219218535349655762591192576535872151766080*e^(448*I*c) + 5
532691288195286125029188695589478290980219563093498435840446315122917788000
81490*e^(447*I*c) + 2711843239670717527605640490148833507130242448403978318
523237721944200392830108580*e^(446*I*c) + 130698172034882898861932055083758
18392124991382340160316886507181296548981014818410*e^(445*I*c) + 6194859665
3035502879564338815234310660410902037882473161804774492916216575880077680*e
^(444*I*c) + 28882075526473065446996857202104710942731861950899580202068990
4590319476295408324280*e^(443*I*c) + 13247564123678374731574728211624836911
20966501948953926492241643788264284546437221120*e^(442*I*c) + 5978992172944
143218459161149299819706321732111578494525245228742976468409105395536290*e^
(441*I*c) + 265568063890434075344967023691015457959948617577414147899446527
12127566910185274123140*e^(440*I*c) + 1161045516835550437629115017121163993
13733021132677481112824047246361794049635726479850*e^(439*I*c) + 4997075672
538590843575963148137947680693371909159674919074889049339226775796653543389
60*e^(438*I*c) + 2117589733466855707101501429210414722401838837940752841618
541440888545729943138209036820*e^(437*I*c) + 883672064086047030569451402154
7969551296794092266983044118375790025854584036796364768280*e^(436*I*c) + 36
318369652302591732197444409798122022640824604130552506742586795183267354382
847875885730*e^(435*I*c) + 147030816732276833163041582099592047512043725225
353339238819165193000407629544745753221740*e^(434*I*c) + 586403466972683242
741643328921560909375197453864243299571990964608857245771134145204174990*e^
(433*I*c) + 230435107337384035737917859767306635201668278168913984209737666
3118488803841131935313641840*e^(432*I*c) + 89232094473432967633318818816384
71793499618670601026059730895962653291770229493028162575100*e^(431*I*c) + 3
405405385129556915435234672217717265518754891078200850471832416872502943858
9162349211628040*e^(430*I*c) + 12809891460168853967248054183040984770736750
0438601536803204497701119911289087105659482783340*e^(429*I*c) + 47501057885
76015192723166179384252242178659724167102689431851540851146714096939311576
8793680*e^(428*I*c) + 17365742188181910718741974724501581238835642099506586
39102337148122769080611680719741726053840*e^(427*I*c) + 6259872156822252843
650960708235034710201362776057176647226323089751446565288850103898153859920
*e^(426*I*c) + 222519591767957777571673660360074802222113642321463998038643
70963391491223687245823457351580140*e^(425*I*c) + 7800980736802423987561373
3058851417125327114681070889640794249282633470580756557083923203377160*e^(4
24*I*c) + 26974580144021129697268360186387895435796230852007659517712822762
9273240215209708218497363414140*e^(423*I*c) + 92008939302958903287460185002
7159322612526368444771489781974361078847528891468831038436064951920*e^(422*
I*c) + 30961319716215201623803015542414654517823620868102875377489029049859
34020179565706177131421614590*e^(421*I*c) + 1027936473066384084473957786246

9262604648861914297972589165243530651230690726244462479199894255180*e^(420*I*c) + 33675398872021568375902384593982753362559801058104184627345411136262431943240778260721756991027090*e^(419*I*c) + 108867995731829472826732905192034886797284621356445627530909104429486741257822633476898356826454040*e^(418*I*c) + 347351473214713780874352083129566601238765762775942366762733349952103889753982636403857556867777300*e^(417*I*c) + 1093853214486220358674032434500866678499770011305874172488975951612031456734608287095519501041975440*e^(416*I*c) + 3400232560601651617521694680847089844198028831694417424794868779328950548418125605446882081152636090*e^(415*I*c) + 10434117516570395966653693155582402109460348095473027807412321427346816928567197770376496170251803940*e^(414*I*c) + 31610939331284692750694306443618414656095969520945215743004044560386895241801579156543451940713351730*e^(413*I*c) + 94556180258931986919334303466365652826858091314329189160736277175873841732196453379953705679466826880*e^(412*I*c) + 279285755800035206679835368898165477644864987794665387827488933863633745047373109049265172681702585720*e^(411*I*c) + 814608187736530579670210025271921415597183369881214299823291969785549876175969866367976653244974728560*e^(410*I*c) + 2346518219239105142238141633073464768899155708935025778047637412681781575765422219127409260159438712250*e^(409*I*c) + 6675866290371147358503766865669289010893543869830538708724945291580951179188296606158111257706968604740*e^(408*I*c) + 18759988218865563564163635735986073278255737257405706279108891366378428467414559930481172863538598193890*e^(407*I*c) + 52075178518793270386429263351544306951104993542500582938155241689408138675254608030847907167748571734720*e^(406*I*c) + 142801792450221762483180874918825274134305133275417780084795034644763509333503150517345864659667189417080*e^(405*I*c) + 386876218234277165632451723049979889263115282374607541692443176673997513742813591736171169652250611186480*e^(404*I*c) + 1035561982592002935226384577908611548612111495080193573691339864706029186482466241805664949381049856258510*e^(403*I*c) + 2738895624795265603355227646566000886280778305084825702911938903656162004262736182657700406301914070062380*e^(402*I*c) + 7158124686842941475473807363679839718172745581538409044503383852693596921622426696740453944718143025248390*e^(401*I*c) + 18487405299005732693752728611876490890858357021974882371570623800186245137722660943641752976852924439870880*e^(400*I*c) + 47188220843466207695099506953573780357108897491422567898048199018207708997005333860148836479527456156014520*e^(399*I*c) + 119041855403877964948229577948370465600606623183045529526900430209270473212773847794935586074714329479939280*e^(398*I*c) + 296825515282669589685318273280239050084555032203415941511962659596881615713799937680026497408305672297618840*e^(397*I*c) + 731584972206818362874729621403974444280010446301161527339760544815300951787985538419764656214582667219914080*e^(396*I*c) + 1782446114931751850556354856638421901174412322298249496591658053939787198246565945975595575734193348887952160*e^(395*I*c) + 4293206478008022126017488908851826494790620720660151451468181910917240027863968724539127659633517053002976480*e^(394*I*c) + 10223182025954860767217390305186451923562145473674293619918063490411487496121804590274592702770571515456414680*e^(393*I*c) + 24068785139705277161193465644506143285241361037768216818922184400141048460210

944696647752723371932874594597328*e^(392*I*c) + 560286834249035176584950138
585345161671625910343679724981746609074506667781543532716303446507778856835
47624184*e^(391*I*c) + 1289670800847547122460236808664883849832862590255331
32044636109049545144029547003347761521666283977931640178464*e^(390*I*c) + 2
935507435543427098081294535765623132997059826991874168629343739642556159671
38676253276302591561523515603264403*e^(389*I*c) + 6607644731058690976914759
738508379345110890331495867079827642633947667566495652798791461733183865057
40391093990*e^(388*I*c) + 1470931146618934345515038362300100160482127749581
443929904746910224777470198899052379114493999887003199419829579*e^(387*I*c)
+ 323849193136185147642332193353957909837773553920764146734623566582388704
8326949305609231585143748690203615957136*e^(386*I*c) + 70521324141621979926
023265245801430609853530545729339055246331216810210373402983663422033243253
07072413739061024*e^(385*I*c) + 1518963421490880039641791172264375474804852
0109734812459109878810493844381062650818971199637121458749456243274416*e^(3
84*I*c) + 32362731322419549410330088943640247460378328561316422931292427145
902887913071643679502909055891236755143207382609*e^(383*I*c) + 682080330967
936156837844096192442108186149916400415534244055278768932724966083242310981
48502466453967157728078994*e^(382*I*c) + 1422131159648145176823866672767699
094822716813187908898405010394417486355453624676798324491035203219530117800
83069*e^(381*I*c) + 2933449200343007202870423834483428663138062854550400678
23080445597545970023446231563554135133105493516316320059272*e^(380*I*c) + 5
986501411122418589116765051805201503640032268413280814535970935877903386092
12439085554466861582623350303061961052*e^(379*I*c) + 1208770358493658393089
442222056935063283704108140593750226539846117737648216609559734831601248698
274330296158612144*e^(378*I*c) + 241496651681033850328907654920274051171005
9011795447138773464205696455026442712426409599662771080264826008985061097*e
^(377*I*c) + 47741411110660989702218453305949620164727142303742340606639568
46950926642685946929064114194400360936223590725470146*e^(376*I*c) + 9339341
958053494225251750965715057300707302083814774770306218224241022648247419956
042957363055823830898547303219757*e^(375*I*c) + 180798200680288599703499386
230072306765633142067088484999001396412373347632664793469632379360393281131
85041591793848*e^(374*I*c) + 3463765717267169016765734453719708704888235485
3993270472063943078773600446542963548348101269390443464480754513928502*e^(3
73*I*c) + 65674859268867300098827375812875225610654551686261103681664007007
537115778097293533565243828873383722980353200611956*e^(372*I*c) + 123243941
519332384741960072588103506596406339253616391082062969960682419011745775738
921817753391954462609323881489157*e^(371*I*c) + 228911311738592780091492649
162346834405867740776456326108410928857257174707289268074347550225793244741
923354395308214*e^(370*I*c) + 420846342608949387277559021457924586578120966
148561022647008499529468452005980175119410628956210497609566002969884927*e^
(369*I*c) + 765867795513962781012558444628751418710940895281304790836743661
582071650032154891482866406314834433199455459798934952*e^(368*I*c) + 137967
652979621207401710618806658944835544650121089019510716486035022892858681553
9003062875026711931941947738690360722*e^(367*I*c) + 24604423758454226639270
816309832607147349680919054930271456392388271922548863493611269914576924098

51120873307487457468*e^(366*I*c) + 4343909696601932173357359687781579293701
295681940827114215433175336093967845908766740738240037114570667410936998017
178*e^(365*I*c) + 759275270014667896115309507358501547319702974653363333154
9793961473285760935801904155116764831560875947581048693527224*e^(364*I*c) +
13139771494104933881856681151418293112242551521535686871181266579813877606
348160261747201317735782566021306798298336024*e^(363*I*c) + 225146757413080
699615061655865028724304219302106732643929972864856006401038672536048477155
47060592967690653795951142520*e^(362*I*c) + 3819901586758608797600299875662
767499479544066790362502932234625013328648912087500501363812811389396034967
0280707161530*e^(361*I*c) + 64175100693260066806238064886004597170740843300
086839368616139164529108049844675353111842725798658088840347241496099644*e^(
360*I*c) + 106764832017165594838085234189333528733587673329972530092661085
186789939252915937090760282232346919090426243399409323314*e^(359*I*c) + 175
896258262755985757106812613979301265801031595484353614904672865169442232075
776580447184134141375995770091499246759528*e^(358*I*c) + 286992943631231496
557278010851576940896826497466066327528801560677007112837431926735088120974
861760511367008815728782643*e^(357*I*c) + 463758288457367154544937678255005
688733328145568049310423995599886012800638619904022368378591108842602342094
543682299102*e^(356*I*c) + 742228640908173124916937049462525617334148919679
118270489831005497781951221069955839623452499748653124658873553401442137*e^(
355*I*c) + 117660072097578696518987505089023109220461269697027743301453589
5788956771230793520381993106606880564628599822341722801012*e^(354*I*c) + 18
475058564624515334452843005713263237811625533045659718876707580910793067948
21834928170773126364639722071570131703785334*e^(353*I*c) + 2873610535922340
187080835435582912277271967977394720159791070274927714276869531467182688981
041061381703885403497544001592*e^(352*I*c) + 442767307910542531852431611298
569365658485193610019245704445513448330504532145251634711848813322482367046
5103483954805161*e^(351*I*c) + 67584804378885243725629359489638576266948555
471955194861228775679817185872623628719949670794018319579279016825829412343
62*e^(350*I*c) + 1022042377943463485133997529516339964170212224966366619305
3008302026096932158568338309418237395541351819026907953220681013*e^(349*I*c
) + 15312837206662775379347353212807682965712535652942631518286142403097738
200270711195396582159028513532779682154451996208592*e^(348*I*c) + 227316035
661288411004195019470513676668366524180772609139448107484730848918904101812
85412604854876625919565639521227223276*e^(347*I*c) + 3343589782793658130117
117545961082945429816796201741981007293673337850658442802420107245319345815
5334046693516742390717832*e^(346*I*c) + 48733253505974923400852255563052101
402196469313659554492725674754339375283010407167744366955828922837488705858
532439654489*e^(345*I*c) + 703863497605948315670482240613950256985012022969
66300376764220336697702961591099854055411376294871437468149528524796002762*
e^(344*I*c) + 1007449618518537446117543009829801669624045538362229218684846
94269966120607698907046343731011160948828100276729370132819357*e^(343*I*c)
+ 1429063191230555242465469284789542383713159258020223892364986521368398225
02035155676970917419039834587967055588431566416784*e^(342*I*c) + 2009065871
535788043803004695014416101745218512595419292098406889608594549085197748359

05895757666770857888611738751858460424*e^(341*I*c) + 2799452444750398048229
 667304629608844921198748577911471240090794769204359417352933093054304386873
 33129912454196774070107264*e^(340*I*c) + 3866426730503800494573825628183169
 626519755509907792770487402386298587950182473561628886310156876647801012052
 87333082748791*e^(339*I*c) + 5293292527641139260039348369582435576725492389
 975607392144065991850478319555725837653586343954087715280097454675483829500
 94*e^(338*I*c) + 7183615963820582492091135444879010888683887440337132103324
 91971375906738341551540457264804304039664255915607349801911966551*e^(337*I*
 c) + 9664582753690377187477391307981516434835906841668322346880982911641606
 36418159452119815728809372125168836239364442397344064*e^(336*I*c) + 1289043
 515292933956480634330499677040181043935620106914267311067900030058398839787
 692376954090545278554544997710058754772400*e^(335*I*c) + 170458299670782280
 820467821816769300269866114771277235502145654381093006963718808588282475750
 0605246963210810351706405349408*e^(334*I*c) + 22348912763984394644786225783
 064348407246104844681778598226206586919214786452666530625638235530012280010
 09093606751066168944*e^(333*I*c) + 2905385722320057001953345274489482790856
 692529959823749532695963414164833366773128218607899328588608916176593772088
 622582464*e^(332*I*c) + 374525759487665120465733498842622638814395450198683
 0664222349226361079609546822276067504899386703088982308185717143407211328*e
 ^ (331*I*c) + 47875274427809456851452048469715961653041694193282440732114595
 92129649255048876854059844720661078151288179612574986359194560*e^(330*I*c)
 + 6068949803156712248331871105329895471722806143008878014986559653687260694
 816550470195890004511965527567432722969707577202160*e^(329*I*c) + 762973181
 562782158046899242420700836643889673633302466186383810511044514894696232829
 7631547032543419811821015837863013682720*e^(328*I*c) + 95130322740195229542
 091131912682266422999120135256659402983810647978856909049931289480352274121
 44035633851779511219335277360*e^(327*I*c) + 1176421227487648408001090071467
 347449337127816055781198372445582656605561765808647936864186490811964341241
 3644803772131657280*e^(326*I*c) + 14429816285208431204532978375375691965063
 154224649747551295851507389524083226976789688601369628399900747658579201929
 300744260*e^(325*I*c) + 175562732712242923968872914031257162134914862611454
 78571376751690105656067838042151038271381300372757755676325408026834544840*
 e^(324*I*c) + 2118832140588288753961019837470686269589404922607709376413251
 2513336190523978949694387686059124526755048042957954264706637460*e^(323*I*c
) + 25367176439119353621532260335983348154904982606125761711300683492963390
 816491583025705268737539982149639300226512657426118880*e^(322*I*c) + 301284
 824145527032645590189530881771560134374934382010784137698354483661481217545
 49197591129967170764969700180348699207838960*e^(321*I*c) + 3550010310601964
 987627237679694948220958137237103600501287780602748167280705994344524013631
 5568500732379966585005678181937920*e^(320*I*c) + 41499832121963708043788523
 787401345541780088930538206918853579026749273364671640037563488607716092887
 686471542838602788559660*e^(319*I*c) + 481331176781840292165037485491103744
 789247190946356038928293648639165537922788229573682851063281647159105983708
 71149079494360*e^(318*I*c) + 5539091304497208621943268914633156608142795989
 696990021443429681773115086386705662076860818767970972015297414847490790417

7340*e^(317*I*c) + 63247774101012179051794946075175569924076981338138483158
042406747453874729387631710544995247152912205118500597511052824347680*e^(31
6*I*c) + 716603298611733955244419438892841091340911578446552456720842374024
34944696464927131812190659629511140639501743303863582092880*e^(315*I*c) + 8
056624913068268418187620188262351120636379033721801195411021064292776599764
4903820595421936873565314654415769070472655401600*e^(314*I*c) + 89883815801
382382213973270477954602744792877018051963347146307372464315121274929402347
942874802899499538953561056667668891020*e^(313*I*c) + 995122064720579659513
403417380235485153364033717178980408504709546575329772791134915068802907261
11154101941386019689567958040*e^(312*I*c) + 1093325373499662232039326785034
263570798637070017282940110420765304039238626540189786765164173142210894499
22495612732870169660*e^(311*I*c) + 1192097137020339270557553978236884444446
474243245021853286263470465996347211465738306815404953335431467768109119104
10468628960*e^(310*I*c) + 1289950760115919034107638634270973299485861735745
958627058491592809430464587426631634540184914638553956494539522128996321986
80*e^(309*I*c) + 1385297945491510894513527695765434031263307472436800308324
67205895819043568155239264876762867172754338684027849855385453216080*e^(308
*I*c) + 1476489208055453334186231217678537773997829247483012287939243425749
99937955421765370101235122939557467548549202174550009604780*e^(307*I*c) + 1
561859629535511961697382188321736965098525515892107305783657274762594764744
65955428502336673743686499175698677875693611243400*e^(306*I*c) + 1639778160
596077253752645598165058478941877851014553603918974244829984153857876057653
15509208337741590143078572243505132706580*e^(305*I*c) + 1708698488689531011
76860306053103994340530390347260088432676842505551412938308389612759742689
28666494845723462544709102843680*e^(304*I*c) + 1767209299705546420045757700
530957005953346598706827320319759155323875770524148663235111401176804929293
54517559479899220940360*e^(303*I*c) + 1814081687709220598203685533166973216
399848626282988285695602732956308976268293452635922190345608535307337105298
42148537901680*e^(302*I*c) + 1848311519837489418176678501747082571381281721
582694132877653585322407732443361919008185578299058956844948894104519215242
12840*e^(301*I*c) + 1869154744365675149263514056231175032619875083519300838
24566444435689139233683411704641828762178799177848064220150818355261280*e^(
300*I*c) + 1876153931685100500714972805646035109124031329203120243708350626
79037644990286285346673507093452964351257962696133511725652320*e^(299*I*c)
+ 1869154744365675149263514056231175032619875083519300838245664444356891392
33683411704641828762178799177848064220150818355261280*e^(298*I*c) + 1848311
519837489418176678501747082571381281721582694132877653585322407732443361919
00818557829905895684494889410451921524212840*e^(297*I*c) + 1814081687709220
598203685533166973216399848626282988285695602732956308976268293452635922190
34560853530733710529842148537901680*e^(296*I*c) + 1767209299705546420045757
700530957005953346598706827320319759155323875770524148663235111401176804929
29354517559479899220940360*e^(295*I*c) + 1708698488689531011768603060531039
94340530390347260088432676842505551412938308389612759742689286664948457234
62544709102843680*e^(294*I*c) + 1639778160596077253752645598165058478941877
851014553603918974244829984153857876057653155092083377415901430785722435051

$32706580 \cdot e^{(293 \cdot I \cdot c)} + 1561859629535511961697382188321736965098525515892107$
 $30578365727476259476474465955428502336673743686499175698677875693611243400 \cdot$
 $e^{(292 \cdot I \cdot c)} + 1476489208055453334186231217678537773997829247483012287939243$
 $42574999937955421765370101235122939557467548549202174550009604780 \cdot e^{(291 \cdot I \cdot$
 $c)} + 1385297945491510894513527695765434031263307472436800308324672058958190$
 $43568155239264876762867172754338684027849855385453216080 \cdot e^{(290 \cdot I \cdot c)} + 1289$
 $950760115919034107638634270973299485861735745958627058491592809430464587426$
 $63163454018491463855395649453952212899632198680 \cdot e^{(289 \cdot I \cdot c)} + 1192097137020$
 $33927055755397823688444446474243245021853286263470465996347211465738306815$
 $40495333543146776810911910410468628960 \cdot e^{(288 \cdot I \cdot c)} + 1093325373499662232039$
 $326785034263570798637070017282940110420765304039238626540189786765164173142$
 $21089449922495612732870169660 \cdot e^{(287 \cdot I \cdot c)} + 9951220647205796595134034173802$
 $354851533640337171789804085047095465753297727911349150688029072611115410194$
 $1386019689567958040 \cdot e^{(286 \cdot I \cdot c)} + 89883815801382382213973270477954602744792$
 $877018051963347146307372464315121274929402347942874802899499538953561056667$
 $668891020 \cdot e^{(285 \cdot I \cdot c)} + 805662491306826841818762018826235112063637903372180$
 $11954110210642927765997644903820595421936873565314654415769070472655401600 \cdot$
 $e^{(284 \cdot I \cdot c)} + 7166032986117339552444194388928410913409115784465524567208423$
 $7402434944696464927131812190659629511140639501743303863582092880 \cdot e^{(283 \cdot I \cdot c$
 $) + 63247774101012179051794946075175569924076981338138483158042406747453874$
 $729387631710544995247152912205118500597511052824347680 \cdot e^{(282 \cdot I \cdot c)} + 553909$
 $130449720862194326891463315660814279598969699002144342968177311508638670566$
 $20768608187679709720152974148474907904177340 \cdot e^{(281 \cdot I \cdot c)} + 4813311767818402$
 $921650374854911037447892471909463560389282936486391655379227882295736828510$
 $6328164715910598370871149079494360 \cdot e^{(280 \cdot I \cdot c)} + 41499832121963708043788523$
 $787401345541780088930538206918853579026749273364671640037563488607716092887$
 $686471542838602788559660 \cdot e^{(279 \cdot I \cdot c)} + 355001031060196498762723767969494822$
 $095813723710360050128778060274816728070599434452401363155685007323799665850$
 $05678181937920 \cdot e^{(278 \cdot I \cdot c)} + 3012848241455270326455901895308817715601343749$
 $343820107841376983544836614812175454919759112996717076496970018034869920783$
 $8960 \cdot e^{(277 \cdot I \cdot c)} + 25367176439119353621532260335983348154904982606125761711$
 $300683492963390816491583025705268737539982149639300226512657426118880 \cdot e^{(27$
 $6 \cdot I \cdot c)} + 211883214058828875396101983747068626958940492260770937641325125133$
 $36190523978949694387686059124526755048042957954264706637460 \cdot e^{(275 \cdot I \cdot c)} + 1$
 $755627327122429239688729140312571621349148626114547857137675169010565606783$
 $8042151038271381300372757755676325408026834544840 \cdot e^{(274 \cdot I \cdot c)} + 14429816285$
 $208431204532978375375691965063154224649747551295851507389524083226976789688$
 $601369628399900747658579201929300744260 \cdot e^{(273 \cdot I \cdot c)} + 117642122748764840800$
 $109007146734744933712781605578119837244558265660556176580864793686418649081$
 $19643412413644803772131657280 \cdot e^{(272 \cdot I \cdot c)} + 9513032274019522954209113191268$
 $226642299912013525665940298381064797885690904993128948035227412144035633851$
 $779511219335277360 \cdot e^{(271 \cdot I \cdot c)} + 762973181562782158046899242420700836643889$
 $673633302466186383810511044514894696232829763154703254341981182101583786301$
 $3682720 \cdot e^{(270 \cdot I \cdot c)} + 60689498031567122483318711053298954717228061430088780$
 $14986559653687260694816550470195890004511965527567432722969707577202160 \cdot e^{($

269*I*c) + 4787527442780945685145204846971596165304169419328244073211459592
129649255048876854059844720661078151288179612574986359194560*e^(268*I*c) +
374525759487665120465733498842622638814395450198683066422234922636107960954
6822276067504899386703088982308185717143407211328*e^(267*I*c) + 29053857223
200570019533452744894827908566925299598237495326959634141648333667731282186
07899328588608916176593772088622582464*e^(266*I*c) + 2234891276398439464478
622578306434840724610484468177859822620658691921478645266653062563823553001
228001009093606751066168944*e^(265*I*c) + 170458299670782280820467821816769
300269866114771277235502145654381093006963718808588282475750060524696321081
0351706405349408*e^(264*I*c) + 12890435152929339564806343304996770401810439
356201069142673110679000300583988397876923769540905452785545449977100587547
72400*e^(263*I*c) + 9664582753690377187477391307981516434835906841668322346
88098291164160636418159452119815728809372125168836239364442397344064*e^(262
*I*c) + 7183615963820582492091135444879010888683887440337132103324919713759
06738341551540457264804304039664255915607349801911966551*e^(261*I*c) + 5293
292527641139260039348369582435576725492389975607392144065991850478319555725
83765358634395408771528009745467548382950094*e^(260*I*c) + 3866426730503800
494573825628183169626519755509907792770487402386298587950182473561628886310
15687664780101205287333082748791*e^(259*I*c) + 2799452444750398048229667304
629608844921198748577911471240090794769204359417352933093054304386873331299
12454196774070107264*e^(258*I*c) + 2009065871535788043803004695014416101745
218512595419292098406889608594549085197748359058957576667708578886117387518
58460424*e^(257*I*c) + 1429063191230555242465469284789542383713159258020223
89236498652136839822502035155676970917419039834587967055588431566416784*e^(
256*I*c) + 1007449618518537446117543009829801669624045538362229218684846942
69966120607698907046343731011160948828100276729370132819357*e^(255*I*c) + 7
038634976059483156704822406139502569850120229696630037676422033669770296159
1099854055411376294871437468149528524796002762*e^(254*I*c) + 48733253505974
923400852255563052101402196469313659554492725674754339375283010407167744366
955828922837488705858532439654489*e^(253*I*c) + 334358978279365813011711754
596108294542981679620174198100729367333785065844280242010724531934581553340
46693516742390717832*e^(252*I*c) + 2273160356612884110041950194705136766683
665241807726091394481074847308489189041018128541260485487662591956563952122
7223276*e^(251*I*c) + 15312837206662775379347353212807682965712535652942631
518286142403097738200270711195396582159028513532779682154451996208592*e^(25
0*I*c) + 102204237794346348513399752951633996417021222496636661930530083020
26096932158568338309418237395541351819026907953220681013*e^(249*I*c) + 6758
480437888524372562935948963857626694855547195519486122877567981718587262362
871994967079401831957927901682582941234362*e^(248*I*c) + 442767307910542531
852431611298569365658485193610019245704445513448330504532145251634711848813
3224823670465103483954805161*e^(247*I*c) + 28736105359223401870808354355829
122772719679773947201597910702749277142768695314671826889810410613817038854
03497544001592*e^(246*I*c) + 1847505856462451533445284300571326323781162553
304565971887670758091079306794821834928170773126364639722071570131703785334
*e^(245*I*c) + 117660072097578696518987505089023109220461269697027743301453

5895788956771230793520381993106606880564628599822341722801012*e^(244*I*c) +
 74222864090817312491693704946252561733414891967911827048983100549778195122
 1069955839623452499748653124658873553401442137*e^(243*I*c) + 46375828845736
 715454493767825500568873332814556804931042399559988601280063861990402236837
 8591108842602342094543682299102*e^(242*I*c) + 28699294363123149655727801085
 157694089682649746606632752880156067700711283743192673508812097486176051136
 7008815728782643*e^(241*I*c) + 17589625826275598575710681261397930126580103
 159548435361490467286516944223207577658044718413414137599577009149924675952
 8*e^(240*I*c) + 10676483201716559483808523418933352873358767332997253009266
 1085186789939252915937090760282232346919090426243399409323314*e^(239*I*c) +
 64175100693260066806238064886004597170740843300086839368616139164529108049
 844675353111842725798658088840347241496099644*e^(238*I*c) + 381990158675860
 879760029987566276749947954406679036250293223462501332864891208750050136381
 28113893960349670280707161530*e^(237*I*c) + 2251467574130806996150616558650
 287243042193021067326439299728648560064010386725360484771554706059296769065
 3795951142520*e^(236*I*c) + 13139771494104933881856681151418293112242551521
 535686871181266579813877606348160261747201317735782566021306798298336024*e^(
 235*I*c) + 759275270014667896115309507358501547319702974653363333154979396
 1473285760935801904155116764831560875947581048693527224*e^(234*I*c) + 43439
 096966019321733573596877815792937012956819408271142154331753360939678459087
 66740738240037114570667410936998017178*e^(233*I*c) + 2460442375845422663927
 081630983260714734968091905493027145639238827192254886349361126991457692409
 851120873307487457468*e^(232*I*c) + 137967652979621207401710618806658944835
 544650121089019510716486035022892858681553900306287502671193194194773869036
 0722*e^(231*I*c) + 76586779551396278101255844462875141871094089528130479083
 6743661582071650032154891482866406314834433199455459798934952*e^(230*I*c) +
 42084634260894938727755902145792458657812096614856102264700849952946845200
 5980175119410628956210497609566002969884927*e^(229*I*c) + 22891131173859278
 009149264916234683440586774077645632610841092885725717470728926807434755022
 5793244741923354395308214*e^(228*I*c) + 12324394151933238474196007258810350
 659640633925361639108206296996068241901174577573892181775339195446260932388
 1489157*e^(227*I*c) + 65674859268867300098827375812875225610654551686261103
 681664007007537115778097293533565243828873383722980353200611956*e^(226*I*c)
 + 346376571726716901676573445371970870488823548539932704720639430787736004
 46542963548348101269390443464480754513928502*e^(225*I*c) + 1807982006802885
 997034993862300723067656331420670884849990013964123733476326647934696323793
 6039328113185041591793848*e^(224*I*c) + 93393419580534942252517509657150573
 007073020838147747703062182242410226482474199560429573630558238308985473032
 19757*e^(223*I*c) + 4774141111066098970221845330594962016472714230374234060
 663956846950926642685946929064114194400360936223590725470146*e^(222*I*c) +
 241496651681033850328907654920274051171005901179544713877346420569645502644
 2712426409599662771080264826008985061097*e^(221*I*c) + 12087703584936583930
 894422220569350632837041081405937502265398461177376482166095597348316012486
 98274330296158612144*e^(220*I*c) + 5986501411122418589116765051805201503640
 03226841328081453597093587790338609212439085554466861582623350303061961052*

$e^{(219*I*c)} + 2933449200343007202870423834483428663138062854550400678230804$
45597545970023446231563554135133105493516316320059272 $e^{(218*I*c)} + 1422131$
159648145176823866672767699094822716813187908898405010394417486355453624676
79832449103520321953011780083069 $e^{(217*I*c)} + 6820803309679361568378440961$
924421081861499164004155342440552787689327249660832423109814850246645396715
7728078994 $e^{(216*I*c)} + 32362731322419549410330088943640247460378328561316$
422931292427145902887913071643679502909055891236755143207382609 $e^{(215*I*c)}$
 $+ 151896342149088003964179117226437547480485201097348124591098788104938443$
81062650818971199637121458749456243274416 $e^{(214*I*c)} + 7052132414162197992$
602326524580143060985353054572933905524633121681021037340298366342203324325
307072413739061024 $e^{(213*I*c)} + 323849193136185147642332193353957909837773$
5539207641467346235665823887048326949305609231585143748690203615957136 $e^{(2$
12 $I*c)} + 14709311466189343455150383623001001604821277495814439299047469102$
24777470198899052379114493999887003199419829579 $e^{(211*I*c)} + 6607644731058$
690976914759738508379345110890331495867079827642633947667566495652798791461
73318386505740391093990 $e^{(210*I*c)} + 2935507435543427098081294535765623132$
99705982699187416862934373964255615967138676253276302591561523515603264403 $*$
 $e^{(209*I*c)} + 1289670800847547122460236808664883849832862590255331320446361$
09049545144029547003347761521666283977931640178464 $e^{(208*I*c)} + 5602868342$
490351765849501385853451616716259103436797249817466090745066677815435327163
0344650777885683547624184 $e^{(207*I*c)} + 24068785139705277161193465644506143$
285241361037768216818922184400141048460210944696647752723371932874594597328
 $*e^{(206*I*c)} + 102231820259548607672173903051864519235621454736742936199180$
63490411487496121804590274592702770571515456414680 $e^{(205*I*c)} + 4293206478$
008022126017488908851826494790620720660151451468181910917240027863968724539
127659633517053002976480 $e^{(204*I*c)} + 178244611493175185055635485663842190$
1174412322298249496591658053939787198246565945975595575734193348887952160 e
 $^{(203*I*c)} + 73158497220681836287472962140397444428001044630116152733976054$
4815300951787985538419764656214582667219914080 $e^{(202*I*c)} + 29682551528266$
958968531827328023905008455503220341594151196265959688161571379993768002649
7408305672297618840 $e^{(201*I*c)} + 11904185540387796494822957794837046560060$
6623183045529526900430209270473212773847794935586074714329479939280 $e^{(200*$
I*c) + 47188220843466207695099506953573780357108897491422567898048199018207
708997005333860148836479527456156014520 $e^{(199*I*c)} + 184874052990057326937$
527286118764908908583570219748823715706238001862451377226609436417529768529
24439870880 $e^{(198*I*c)} + 7158124686842941475473807363679839718172745581538$
409044503383852693596921622426696740453944718143025248390 $e^{(197*I*c)} + 273$
889562479526560335522764656600088628077830508482570291193890365616200426273
6182657700406301914070062380 $e^{(196*I*c)} + 10355619825920029352263845779086$
11548612111495080193573691339864706029186482466241805664949381049856258510 $*$
 $e^{(195*I*c)} + 3868762182342771656324517230499798892631152823746075416924431$
76673997513742813591736171169652250611186480 $e^{(194*I*c)} + 1428017924502217$
624831808749188252741343051332754177800847950346447635093335031505173458646
59667189417080 $e^{(193*I*c)} + 5207517851879327038642926335154430695110499354$
2500582938155241689408138675254608030847907167748571734720 $e^{(192*I*c)} + 18$

759988218865563564163635735986073278255737257405706279108891366378428467414
 559930481172863538598193890*e^(191*I*c) + 667586629037114735850376686566928
 9010893543869830538708724945291580951179188296606158111257706968604740*e<sup>(1
 90*I*c)</sup> + 23465182192391051422381416330734647688991557089350257780476374126
 81781575765422219127409260159438712250*e^(189*I*c) + 8146081877365305796702
 100252719214155971833698812142998232919697855498761759698663679766532449747
 28560*e^(188*I*c) + 2792857558000352066798353688981654776448649877946653878
 27488933863633745047373109049265172681702585720*e^(187*I*c) + 9455618025893
 198691933430346636565282685809131432918916073627717587384173219645337995370
 5679466826880*e^(186*I*c) + 31610939331284692750694306443618414656095969520
 945215743004044560386895241801579156543451940713351730*e^(185*I*c) + 104341
 175165703959666536931555824021094603480954730278074123214273468169285671977
 70376496170251803940*e^(184*I*c) + 3400232560601651617521694680847089844198
 028831694417424794868779328950548418125605446882081152636090*e^(183*I*c) +
 109385321448622035867403243450086667849977001130587417248897595161203145673
 4608287095519501041975440*e^(182*I*c) + 34735147321471378087435208312956660
 1238765762775942366762733349952103889753982636403857556867777300*e<sup>(181*I*c
)</sup> + 10886799573182947282673290519203488679728462135644562753090910442948674
 1257822633476898356826454040*e^(180*I*c) + 33675398872021568375902384593982
 753362559801058104184627345411136262431943240778260721756991027090*e<sup>(179*I
 *c)</sup> + 102793647306638408447395778624692626046488619142979725891652435306512
 30690726244462479199894255180*e^(178*I*c) + 3096131971621520162380301554241
 465451782362086810287537748902904985934020179565706177131421614590*e<sup>(177*I
 *c)</sup> + 920089393029589032874601850027159322612526368444771489781974361078847
 528891468831038436064951920*e^(176*I*c) + 269745801440211296972683601863878
 954357962308520076595177128227629273240215209708218497363414140*e^(175*I*c)
 + 780098073680242398756137330588514171253271146810708896407942492826334705
 80756557083923203377160*e^(174*I*c) + 2225195917679577775716736603600748022
 2211364232146399803864370963391491223687245823457351580140*e^(173*I*c) + 62
 598721568222528436509607082350347102013627760571766472263230897514465652888
 50103898153859920*e^(172*I*c) + 1736574218818191071874197472450158123883564
 209950658639102337148122769080611680719741726053840*e^(171*I*c) + 475010578
 857601519272316617938425222421786597241671026894318515408511467140969393115
 768793680*e^(170*I*c) + 128098914601688539672480541830409847707367500438601
 536803204497701119911289087105659482783340*e^(169*I*c) + 340540538512955691
 54352346722177172655187548910782008504718324168725029438589162349211628040*
 e^(168*I*c) + 8923209447343296763331881881638471793499618670601026059730895
 962653291770229493028162575100*e^(167*I*c) + 230435107337384035737917859767
 3066352016682781689139842097376663118488803841131935313641840*e^(166*I*c) +
 58640346697268324274164332892156090937519745386424329957199096460885724577
 1134145204174990*e^(165*I*c) + 14703081673227683316304158209959204751204372
 5225353339238819165193000407629544745753221740*e^(164*I*c) + 36318369652302
 591732197444409798122022640824604130552506742586795183267354382847875885730
 *e^(163*I*c) + 883672064086047030569451402154796955129679409226698304411837
 5790025854584036796364768280*e^(162*I*c) + 21175897334668557071015014292104

14722401838837940752841618541440888545729943138209036820*e^(161*I*c) + 4997
075672538590843575963148137947680693371909159674919074889049339226775796653
54338960*e^(160*I*c) + 1161045516835550437629115017121163993137330211326774
81112824047246361794049635726479850*e^(159*I*c) + 2655680638904340753449670
2369101545795994861757741414789944652712127566910185274123140*e^(158*I*c) +
59789921729441432184591611492998197063217321115784945252452287429764684091
05395536290*e^(157*I*c) + 1324756412367837473157472821162483691120966501948
953926492241643788264284546437221120*e^(156*I*c) + 288820755264730654469968
572021047109427318619508995802020689904590319476295408324280*e^(155*I*c) +
619485966530355028795643388152343106604109020378824731618047744929162165758
80077680*e^(154*I*c) + 1306981720348828988619320550837581839212499138234016
0316886507181296548981014818410*e^(153*I*c) + 27118432396707175276056404901
48833507130242448403978318523237721944200392830108580*e^(152*I*c) + 5532691
28819528612502918869558947829098021956309349843584044631512291778800081490*
e^(151*I*c) + 1109691996873209747499222595952504443412192185353496557625911
92576535872151766080*e^(150*I*c) + 2187648289271390992804034561257870580512
1508756226696317087651824252241418663320*e^(149*I*c) + 42381258467632325863
94188569858685826755328005548627437019301405851325887594480*e^(148*I*c) + 8
066795436075891407593050107961895682698420216133889552189162788231826394881
90*e^(147*I*c) + 1508223814314124137735664742100117468522974375970591862952
43989481140398152780*e^(146*I*c) + 2769311653834325922598338263764793612266
4033859615133489846664694361471028310*e^(145*I*c) + 49925197124570439835053
77976607953988397368297591114957991804893688371867680*e^(144*I*c) + 8835009
68821791202600774541927769200737689393513734789368397093333311961880*e^(143
*I*c) + 1534360887450562541273272394615770719331301577645959971139735131831
88399376*e^(142*I*c) + 2614397627990202144347194566508025456305681018352040
1889800285493144867448*e^(141*I*c) + 43694424829101139145653531360695958626
69338858053419381214131241925047008*e^(140*I*c) + 7160994975990580798956333
38552940229192858196481597830078819711862600096*e^(139*I*c) + 1150514818520
80848873700388354521315567640365124003103691176697194292320*e^(138*I*c) + 1
8115768495615758076710303055505625589254293659193314153418333944596408*e^(1
37*I*c) + 27947091044756866118427906949736991644822547239772102097256613044
03472*e^(136*I*c) + 4222761266320036875477547465557099887105271330866601613
66353656787288*e^(135*I*c) + 6247355078105329531771077469024711412412518756
5731848441781904032672*e^(134*I*c) + 90466935238256829790443389631042631676
72586826367911338826483549173*e^(133*I*c) + 1281817464914970810859604189828
359000790789921169405304612211251818*e^(132*I*c) + 177642829135119348577194
437675802830239905460092687136494961404333*e^(131*I*c) + 240708019135297571
01858022914372045864746991786182039740274325264*e^(130*I*c) + 3187749929744
346497211536044751776582320958627923816470590659024*e^(129*I*c) + 412430698
299915190848067222327219435067747934091894670488982928*e^(128*I*c) + 521081
17629177048660492400985175830987505700566877818954141639*e^(127*I*c) + 6426
195485535248576425068136870465530087114003875716691383902*e^(126*I*c) + 773
204636991145775061462731028098506094432675788136295011259*e^(125*I*c) + 907
22605722208814918642284639487187764607589706493970774776*e^(124*I*c) + 1037

5184499871175501909398956596684116802997082526660323524*e^(123*I*c) + 11558
 55412893594260345544966642687823630035899363232371472*e^(122*I*c) + 1253704
 96586921272662198050851269323171167338854081782959*e^(121*I*c) + 1323170887
 0104896973800056733779919089340836756009580718*e^(120*I*c) + 13579906631614
 79842850642848032544982878359839580349899*e^(119*I*c) + 1354425949166361161
 91574650625331646238501101627937224*e^(118*I*c) + 1311878180117217472967933
 9894318153694964675368481194*e^(117*I*c) + 12330967001397233651819972207509
 32590655287625342156*e^(116*I*c) + 1123916045422466509664291620631243389525
 54575234051*e^(115*I*c) + 9925490738534402272939987038714580495445431374618
 *e^(114*I*c) + 848552202276512356496200136959676295361696315113*e^(113*I*c)
 + 70164515322544462906873548813748091084561870680*e^(112*I*c) + 5605927253
 067558551780452883689835514455118670*e^(111*I*c) + 432333688644261557547944
 179250800440604964868*e^(110*I*c) + 321478876933753388174544825153773503839
 50278*e^(109*I*c) + 2302150411226234925855222345201500900533576*e^(108*I*c)
 + 158566476113257562566117432227203884298856*e^(107*I*c) + 104904026695108
 97424624643766470754045064*e^(106*I*c) + 6656346706762100637541918471099711
 41414*e^(105*I*c) + 40443624781415311581857832389099634564*e^(104*I*c) + 23
 48998374244347079532766203075607598*e^(103*I*c) + 1301711930791728238351514
 30773360024*e^(102*I*c) + 6868329225263681349501997341320517*e^(101*I*c) +
 344277152012875134140739302960914*e^(100*I*c) + 163531646471515302405291376
 18111*e^(99*I*c) + 734057263616388449968842366924*e^(98*I*c) + 310422225220
 74681615625020522*e^(97*I*c) + 1232445557346832245176696904*e^(96*I*c) + 45
 759117183402579073139583*e^(95*I*c) + 1581796642397812408161814*e^(94*I*c)
 + 50648660944512569972179*e^(93*I*c) + 1493326612293984160368*e^(92*I*c) +
 40261256699368950388*e^(91*I*c) + 984382804329835768*e^(90*I*c) + 216084030
 21340047*e^(89*I*c) + 420601518659718*e^(88*I*c) + 7146142307307*e^(87*I*c)
 + 103818048048*e^(86*I*c) + 1253841160*e^(85*I*c) + 12085216*e^(84*I*c) +
 87153*e^(83*I*c) + 418*e^(82*I*c) + e^(81*I*c))*tan(1/4*d*x + c) + 7*(-986
 7*I*a^2*e^(1027/2*I*c) - 3848130*I*a^2*e^(1025/2*I*c) - 748461285*I*a^2*e^(
 1023/2*I*c) - 96800992860*I*a^2*e^(1021/2*I*c) - 9365496059205*I*a^2*e^(101
 9/2*I*c) - 723016295770626*I*a^2*e^(1017/2*I*c) - 46393545645311706*I*a^2*e
 ^ (1015/2*I*c) - 2545017361124253210*I*a^2*e^(1013/2*I*c) - 1218427061655317
 58225*I*a^2*e^(1011/2*I*c) - 5171545973000559760165*I*a^2*e^(1009/2*I*c) -
 197035901588508817004391*I*a^2*e^(1007/2*I*c) - 6806694783357860712909393*I
 *a^2*e^(1005/2*I*c) - 214978110312371924676280101*I*a^2*e^(1003/2*I*c) - 62
 50901980395957795917828835*I*a^2*e^(1001/2*I*c) - 1683278606334769684822827
 79620*I*a^2*e^(999/2*I*c) - 4219418379622495018746735933182*I*a^2*e^(997/2*
 I*c) - 98892618501960556395579843057828*I*a^2*e^(995/2*I*c) - 2175637614526
 488309339795247719423*I*a^2*e^(993/2*I*c) - 4508404634705112156129133913129
 7112*I*a^2*e^(991/2*I*c) - 882698176765813429616327000431223295*I*a^2*e^(98
 9/2*I*c) - 16374051337559882202075446312226584700*I*a^2*e^(987/2*I*c) - 288
 495194028428088627568321559245818459*I*a^2*e^(985/2*I*c) - 4838851294065276
 461984778740285520947658*I*a^2*e^(983/2*I*c) - 7742162250134828024733423519
 1908463309686*I*a^2*e^(981/2*I*c) - 118390567985118721949628445198583036973
 3725*I*a^2*e^(979/2*I*c) - 17332379827117737513384038195005765450047540*I*a

$^2e^{(977/2*I*c)} - 243319959628953467711826163624006704408771129*I*a^2e^{(975/2*I*c)} - 3280313734912630390804532039053385555426092600*I*a^2e^{(973/2*I*c)} - 42526927818120262032236340231948484360039052121*I*a^2e^{(971/2*I*c)} - 530853426357042754469653748150767198919032400940*I*a^2e^{(969/2*I*c)} - 6387936994195976240622682990166245342613055517890*I*a^2e^{(967/2*I*c)} - 74182504975608450289636265368281851382711220649004*I*a^2e^{(965/2*I*c)} - 832235125649436217222715021488460783355491981492217*I*a^2e^{(963/2*I*c)} - 9028492090498513521931431245530169366369034596324891*I*a^2e^{(961/2*I*c)} - 94799191422336670911733284155867076030325977221341875*I*a^2e^{(959/2*I*c)} - 964243501752639006146026780418602842158889630673989345*I*a^2e^{(957/2*I*c)} - 9508515799900673113299221003726853644110325215605885183*I*a^2e^{(955/2*I*c)} - 90973406936806837531915685400953309110574252721276020367*I*a^2e^{(953/2*I*c)} - 845095502974670714598033224559068895202594191429069360222*I*a^2e^{(951/2*I*c)} - 7627533234852264511705859368402201230654030433440301675518*I*a^2e^{(949/2*I*c)} - 66931651284481661864618077305876269408561834596408636101210*I*a^2e^{(947/2*I*c)} - 571368224054265219726596099705196924252108483077143367419935*I*a^2e^{(945/2*I*c)} - 4747802391051472283183879516945405395337420791155799564552344*I*a^2e^{(943/2*I*c)} - 38424117610672655906936441589932580808631293346194215493375887*I*a^2e^{(941/2*I*c)} - 303026950314518079219169886639469213112529662619496065070362634*I*a^2e^{(939/2*I*c)} - 2329943971272634597068665479560383652668004620643027277203439985*I*a^2e^{(937/2*I*c)} - 17474609226004294346664529284651937762925604102208554097442050560*I*a^2e^{(935/2*I*c)} - 127899514211571855615342081558322168204136482117422337757176345560*I*a^2e^{(933/2*I*c)} - 913950624853444779964988596342201399080981813136131167642386677984*I*a^2e^{(931/2*I*c)} - 6379018324131616220652747415710905929685302237372129456527463165524*I*a^2e^{(929/2*I*c)} - 43505029014129136807566388318013609163815986027077804343349110468080*I*a^2e^{(927/2*I*c)} - 290034464520387036801727848589402597956276490946161717533961649595700*I*a^2e^{(925/2*I*c)} - 1890808523184465234870582011548986898368367578058256415408172123086240*I*a^2e^{(923/2*I*c)} - 12058413611979215014493808881094919327076136158958774603465013922951460*I*a^2e^{(921/2*I*c)} - 75253783547376958022568745986903775265291178425521737345338921401401608*I*a^2e^{(919/2*I*c)} - 459734613498525904391661453323499339021127870927849907055606144578837880*I*a^2e^{(917/2*I*c)} - 2750214314681421164420565775956331599212851388641230567209095240832869740*I*a^2e^{(915/2*I*c)} - 16115397185131183261784999068093482958330926537119548096569091610170990080*I*a^2e^{(913/2*I*c)} - 92525290960177968452743510383284189038302875808137991118047531970267330140*I*a^2e^{(911/2*I*c)} - 520655084297071213247874630919114875777150736266962384814615937366841583600*I*a^2e^{(909/2*I*c)} - 2872306980765485786798093527905670394316300311494879919370292601994164873980*I*a^2e^{(907/2*I*c)} - 15538869199972063791690049288524847158110979530065672277863507395517286098560*I*a^2e^{(905/2*I*c)} - 82457197940500065112831235778009337400577942437329270541936889611136248616360*I*a^2e^{(903/2*I*c)} - 429306406331604823645688251632236356131380665568461698695435155472754991584480*I*a^2e^{(901/2*I*c)} - 2193518253323607158598306760610194505563715717648737978249164699044796790178970*I*a^2e^{(899/2*I*c)} - 1$

100150905116991880340619030111007216914043473918839068068706937160009645646
1960*I*a^2*e^(897/2*I*c) - 541750313768436076293560450270587580326308933852
34188342863868414550489892855190*I*a^2*e^(895/2*I*c) - 26198574004344308038
8301874884500683905172501509414794896626641791046018314193300*I*a^2*e^(893/
2*I*c) - 124445844130409520749732783777934477929343261005269896448132005897
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1544047170905560703749866664890336325145752760*I*a^2*e^(889/2*I*c) - 266327
089901343246991998815245909258794444277345979118426257766971116140724628055
00*I*a^2*e^(887/2*I*c) - 12003812035094402972261579848458357881792093610304
2898479075195084456374447649531420*I*a^2*e^(885/2*I*c) - 531852003070453189
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(883/2*I*c) - 2316913417896533634148238479533816756801158698905671195529018
686134366565369411066790*I*a^2*e^(881/2*I*c) - 9925525882909518673569536731
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c) - 4182125688366367760446381399035690630569232443207281247902039184020973
8405992862342110*I*a^2*e^(877/2*I*c) - 173345856975272080307468888319573743
826748157949817083968790709575178522586695155600570*I*a^2*e^(875/2*I*c) - 7
069254940580017380084988687307754120182736498836563165198219602512939601118
42853827050*I*a^2*e^(873/2*I*c) - 28369169189394771245705015350036679506624
41685459586986859765033838975136651432973013080*I*a^2*e^(871/2*I*c) - 11204
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385410620*I*a^2*e^(869/2*I*c) - 4356097775980642550984171697832990426607531
3880276843623527207449861917293070562965518680*I*a^2*e^(867/2*I*c) - 166726
455941596831160623476351712587187594657642857127013830239484587313767680019
339582550*I*a^2*e^(865/2*I*c) - 6283217402529699146708974013748477212590031
78557296040399451247917033448679858171068226560*I*a^2*e^(863/2*I*c) - 23317
961812286764803587138748460406386913096709681823095244508892146725501813518
30378735030*I*a^2*e^(861/2*I*c) - 85229260483699889403559152526282219875527
53095020951377015329635773840197997200581857597800*I*a^2*e^(859/2*I*c) - 30
68553386459666418316194015078029015958251712763047637572948112622799853423
865147299102830*I*a^2*e^(857/2*I*c) - 1088381674585823184455849835282493949
20058359009481425820011589540335936121859362203566362580*I*a^2*e^(855/2*I*c
) - 38035197418842683642619864051339471341649460785516551807259419090622333
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9124812786168506645450981272566942196706295180922249225707977010*I*a^2*e^(8
51/2*I*c) - 444504319274789268390399853721138227585710412235922782253365493
8963716398520643447212245479640*I*a^2*e^(849/2*I*c) - 148683901006420511836
71971179387880003509146350007294634177899645088810313885195635392146744810*
I*a^2*e^(847/2*I*c) - 49024494337492279360818849127945363095454529548782519
402705804194570228377733166934824088655040*I*a^2*e^(845/2*I*c) - 1593571437
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4194243445208994753073600039579929266407037097651682920*I*a^2*e^(841/2*I*c)
- 161399625005227651497742238751482974339782549553805362337258500737267515
8127379199196038765006020*I*a^2*e^(839/2*I*c) - 502997233223228520128531881

1856011191917686754755796153099081679265022946497934343332346116036200*I*a²*e^{-(837/2*I*c)} - 154603381888035181010707962987414730510504499628085456326
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56750954584436945867670907710352859492970463590965208109494412332329295780*I
a²*e^{-(821/2*I*c)} - 75687878722014265825238066731591013702998100464324442
217272752916887964266256104313931049007060490840*I*a²*e^{-(819/2*I*c)} - 2062
550466642352923299538505218323676805884853392747978586557766976702788671365
67013953786924177858890*I*a²*e^{-(817/2*I*c)} - 55483047707198724665440844480
8030209487515325400605772819119408435792744646306336768559494858757362460*I
a²*e^{-(815/2*I*c)} - 147342444362109962377181703120951524343672463702574952
7267678685770355293167282547379994041612249536410*I*a²*e^{-(813/2*I*c)} - 386
314437516868018576356913292077827044669797836884278496544413991685735826342
8001264383566311006996680*I*a²*e^{-(811/2*I*c)} - 100007488425814211531157828
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50*I*a²*e^{-(809/2*I*c)} - 25564363098285969951461675934273364734821923616250
047177527687812898786857371780193438857338260097911840*I*a²*e^{-(807/2*I*c)}
- 6453273197782459147936919163715825933528964953360698297163136496405491168
1075637122490714561514235891960*I*a²*e^{-(805/2*I*c)} - 160878825079043394603
834949575837497625484370509752204556389520870042820153281247817248349563478
924960000*I*a²*e^{-(803/2*I*c)} - 3961156927720317702883047917185117312913684
06297439843435650750372236248577797783021338331228243750622260*I*a²*e<sup>-(801
/2*I*c)</sup> - 96333990133367005220184783649172964795326224863928850083854162444
7579232078571408017358275408623510613840*I*a²*e^{-(799/2*I*c)} - 231419987522
566986824406965158487639596366333938136712176597016461942695849656811645654
6662409068449840980*I*a²*e^{-(797/2*I*c)} - 549180314668753988053597656596379
5672164514086567208012015916026775639438732336258201804419618292545032320*I
a²*e^{-(795/2*I*c)} - 128750909685704951104826884973311195514817251151404663
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764548099437011716544543041268840*I*a²*e^{-(791/2*I*c)} - 6824899753441688984
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*e^{-(787/2*I*c)} - 3448716031646138840025055585349184630908375817774593116788
42067123286466226751144071133482776005430489425760*I*a²*e^{-(785/2*I*c)} - 76

157068141016866276950629713249017136732146629432296112698385566984533398678
3371033315463200057428630752220*I*a^2*e^(783/2*I*c) - 166205667443073414924
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3574957113980336688954791609736165974043249124955091096872997954793020*I*a^
2*e^(779/2*I*c) - 764306432142215085107655604542055474413839848522722800024
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617492222077204198418206928413023738918899830280644394498850906506327315474
5*I*a^2*e^(771/2*I*c) - 140672375713012687320389073579945533379367132007412
356558650081388469939218158614778502467163652527862550952130*I*a^2*e^(769/2
*I*c) - 2831541724454077422450878787538662877042027538054395604149601789061
36961891572928026855276213887121380071928103*I*a^2*e^(767/2*I*c) - 56352897
805763272096991770897155119738849844002918752049866812562736063687371997384
2729049383973395339489187328*I*a^2*e^(765/2*I*c) - 110895041237395917205685
994545909717581600207445777581219710119943087877137176487718794969468581971
8286221686935*I*a^2*e^(763/2*I*c) - 215791258592459333372869589796444940936
4172631286534684550661677201976202007165851929905826385985844876371677490*I
*a^2*e^(761/2*I*c) - 415245104359965804460096900332121399173917602775258904
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*c) - 790218349026871901495271289147853538925976831795670198761660396945228
8123018369851967117174545759607142995908398*I*a^2*e^(757/2*I*c) - 148725165
132286726845844543739529709855752443668839234988632043147587872364853959678
33146869064461137897493283199*I*a^2*e^(755/2*I*c) - 27684667755883080696165
241600850236290060527593753750250976925795644297762324894086602505304941640
081834882024183*I*a^2*e^(753/2*I*c) - 5097231322824610485284563027499470633
447420185121230070113720509369535675409863710427160861377451193096410578118
5*I*a^2*e^(751/2*I*c) - 928308113376605414018314259982460592969619502620219
41188955325593474604810499041616621349581953895002857873462955*I*a^2*e^(749
/2*I*c) - 16723812436760488425768739599809266279592755168930304104087967332
9504761663429237202212197395803292702881738952971*I*a^2*e^(747/2*I*c) - 298
048067652053494891789544111943306401708032136890389702630757761900213043371
027166375667728550358023100922727809*I*a^2*e^(745/2*I*c) - 5254939879412245
768339489479231903447058938212674523878231430709537350535886875076613143155
58209767708019478373004*I*a^2*e^(743/2*I*c) - 91664709431284914960140940755
641154562802047730145200725460409188806537847457937296000494471094027545632
6641248866*I*a^2*e^(741/2*I*c) - 158202062787581988673250065898762286534898
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*a^2*e^(739/2*I*c) - 270159546995359250505395026372937903793888340345141599
5229310843295475921159044641522890199450389533264819628536025*I*a^2*e^(737/
2*I*c) - 456509066991059164837134779602647589554670218072151783528080907829
7960907491881780744446079560657601476760082232152*I*a^2*e^(735/2*I*c) - 763

347349130754120948642458581148481719625483226104049697195217999448279842521
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323880522866518063344172714663818375801285677304841070409088315250448980118
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509311959418929183582885216898379036970362110971705665038962014129218543899
794341899845085*I*a^2*e^(729/2*I*c) - 3352990888500682268293154976719628483
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7590*I*a^2*e^(727/2*I*c) - 537910694584998904260122923727248823170880556157
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e^(725/2*I*c) - 85417618996417490452483371059823103161327605370266736083574
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c) - 1342665470057644847983235502917892846730721653892879442933623480246710
26925485137604645521152950533194618205062920988*I*a^2*e^(721/2*I*c) - 20892
721947321952554456586633742979365256651844601362539197681055721328802433993
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447898702348324001010813625753491002068478331883454364145448222725180431547
920764933095415393034588280*I*a^2*e^(717/2*I*c) - 4908693639020633756913108
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00227464127502047*I*a^2*e^(715/2*I*c) - 74123864476887296086214581322988427
643686096723451135697665601142973264698238420197726339483003974827374591876
3926276*I*a^2*e^(713/2*I*c) - 110829117874288088003949255863546681669181130
2866749288715555797951407508808250708041669875191682735463589899659311198*I
*a^2*e^(711/2*I*c) - 164088464461734084358144268349244158832586560261315262
569217345595900863555908606458185685041555583786399895054479300*I*a^2*e^(7
09/2*I*c) - 240577714439055714117813463067773244849650952054131834842019241
7807342381289784524672636122635696383857297613950049995*I*a^2*e^(707/2*I*c)
- 349309887710492687698237108032257440212635206127339745745586202481213457
7602605965839150066211359905342050250374117429*I*a^2*e^(705/2*I*c) - 502307
443186015730117888471656258174869062121735387850334388091202949158080461043
7496781598919605840393968558559789161*I*a^2*e^(703/2*I*c) - 715412996672345
730067340322771834762634518685264638245830985114944269590079262756662236828
7282262184380707453078413991*I*a^2*e^(701/2*I*c) - 100924786020063127364353
407717370680634544851192575840230472208882160787043159191976578217878926464
07702091249317303229*I*a^2*e^(699/2*I*c) - 14103212089664651841721385690411
495309067974792132984765752121078864973955276665921709606990805136055899318
515316302625*I*a^2*e^(697/2*I*c) - 1952283328672880331025077467311303310215
892334738969639245055633587494968652624921456871700212028594919841994338864
7210*I*a^2*e^(695/2*I*c) - 267730394333998274117237784491077008799869706368
35180213084845566895069119080487323194332830887134412610587215162894762*I*a
^2*e^(693/2*I*c) - 36375409931287795092607557958784126662059576747454408588
744309722082226791392845209172297521166997828928030420844023706*I*a^2*e^(69
1/2*I*c) - 4896646615878742114946199207175567373336856102462570532830088204
6960116093509913400520500730507564724324855396317174877*I*a^2*e^(689/2*I*c)
- 653123602074898065236571363863403753569295522760923767167642985299999533
07808042369566805167379162746143255282258954180*I*a^2*e^(687/2*I*c) - 86322

223651326073619711312568551997796065517198873894958063303051564972850936480
 127357633838565811305177606785202742045*I*a^2*e^(685/2*I*c) - 1130589802883
 564220385835139940220907469386070355433465119796591093989261756839556423349
 71876628296436495288367475025210*I*a^2*e^(683/2*I*c) - 14674621624473683474
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 2344517027436260129735987*I*a^2*e^(681/2*I*c) - 188769528710907069399315417
 787360030712701629061238948251068174174311765598652218110198035428223021692
 947533585962348672*I*a^2*e^(679/2*I*c) - 2406706654362267812491048707637320
 723073729459838418169127611392117063248071275964692668794775387149948760506
 30848911280*I*a^2*e^(677/2*I*c) - 30413274660453666225979217509555939007344
 516655283930371313173556486894852693903214644884373148639517109109679379882
 1440*I*a^2*e^(675/2*I*c) - 380954953495487600292955504287703099567666198405
 494548188510783430234085264993237349222897613346201432380448747964425320*I*
 a^2*e^(673/2*I*c) - 4730152954843393553865160495107424893668782016894758200
 57034050114095323134007835807393143129309159477656152691755564640*I*a^2*e^(
 671/2*I*c) - 58222044318581750206266482921312370263252473314620783459971606
 4885065789277590829360246460349955947394345054182441981224*I*a^2*e^(669/2*I
 *c) - 710442146558732233592026068873812784196608427957534833584445957207723
 692820312997979854292270461155374405948197659591360*I*a^2*e^(667/2*I*c) - 8
 594404367040659964920655309206627501878563640804789340222353178580696799516
 48889172789595706006814219368010815532681480*I*a^2*e^(665/2*I*c) - 10307746
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 54997979496639299345832099582375161360*I*a^2*e^(663/2*I*c) - 12257039776528
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 46506187045033600876523451824880*I*a^2*e^(661/2*I*c) - 14450810458031668137
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 45961071068493810560*I*a^2*e^(657/2*I*c) - 19578938349723584779059289990954
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 ^2*e^(649/2*I*c) - 32443298586951092651428287113773193999451013284950304274
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 647/2*I*c) - 36028305298407529565181939312745233961149440319768254370614299
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 I*c) - 39663748833127834823836467088030196693430004700156593977777724916107
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 09165508440803025209005356313475936021116327680*I*a^2*e^(641/2*I*c) - 46820
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39708012281597905204951327596457959200420*I*a^2*e^(639/2*I*c) - 50189962637
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02839567198319080*I*a^2*e^(631/2*I*c) - 60317329778848122601739294534272327
997929452223082034428048354862041610048278271424013436222327931550028652746
51684289320*I*a^2*e^(629/2*I*c) - 61588662421482231694403486393227657084883
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73020*I*a^2*e^(627/2*I*c) - 62202251230952204460675300839023987684115557416
54900638992044646455129937854804113318919111668831147496834175450404249540*
I*a^2*e^(625/2*I*c) - 62096571660597904142679194148630869282244387047293181
54672285436550356208263215568679904005246104294284315747952200822100*I*a^2*
e^(623/2*I*c) - 61222857792337143725906948970182085956642006810708599861702
74541517752605891024516685624452334324327807799672288064082260*I*a^2*e^(621
/2*I*c) - 59547257467401870889396732031512898770826022020504370086760285116
17860863938697514177299610327811386342958225508703594540*I*a^2*e^(619/2*I*c
) - 57052599204286374939405242600597907420683319525389105353847625378502538
23401148270433783507943120942720400689766599967260*I*a^2*e^(617/2*I*c) - 53
739690083650980637055279707516518897459636880832855309819571261998752476166
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00178554932777862617713413141739317320*I*a^2*e^(613/2*I*c) - 44756255352895
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49172606876659181697229192662160*I*a^2*e^(611/2*I*c) - 39181220751115193662
441268842368235396535288066011412234288694406185747436395037219120791405561
62185509947899164696134100*I*a^2*e^(609/2*I*c) - 32977503331460697218967225
115618795701660403555551597360783353756943479513821935596151070573301875361
37462087866642554560*I*a^2*e^(607/2*I*c) - 26235570810040910406795079057453
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765610370725372678514909790965099362497729563886351201929929584486288907771
39946480*I*a^2*e^(603/2*I*c) - 11565559961838689852000906392260895540213419
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20*I*a^2*e^(601/2*I*c) - 38769795201545572867941822935804912844942383041966
8428081882866237658365339074239012664180518005805128629710867652733080*I*a^
2*e^(599/2*I*c) + 387697952015455728679418229358049128449423830419668428081
882866237658365339074239012664180518005805128629710867652733080*I*a^2*e^(59
7/2*I*c) + 1156555996183868985200090639226089554021341932158192621923246957
977122810089035413564479870427981333952548182360473741220*I*a^2*e^(595/2*I*
c) + 1905972925948047400200417435675221383076561037072537267851490979096509
936249772956388635120192992958448628890777139946480*I*a^2*e^(593/2*I*c) + 2

623557081004091040679507905745376983741232629056220311681059841157626795431
 350015349450602987501509129301018716050208660*I*a^2*e^(591/2*I*c) + 3297750
 333146069721896722511561879570166040355555159736078335375694347951382193559
 615107057330187536137462087866642554560*I*a^2*e^(589/2*I*c) + 3918122075111
 519366244126884236823539653528806601141223428869440618574743639503721912079
 140556162185509947899164696134100*I*a^2*e^(587/2*I*c) + 4475625535289564972
 601839339977758010448497118081958608252277368750290974253606539667119249172
 606876659181697229192662160*I*a^2*e^(585/2*I*c) + 4962808094772787339747091
 515230276979874007394934794743163862907549639800176824606100178554932777862
 617713413141739317320*I*a^2*e^(583/2*I*c) + 5373969008365098063705527970751
 651889745963688083285530981957126199875247616605126027272850991193295122275
 535796115520400*I*a^2*e^(581/2*I*c) + 5705259920428637493940524260059790742
 068331952538910535384762537850253823401148270433783507943120942720400689766
 599967260*I*a^2*e^(579/2*I*c) + 5954725746740187088939673203151289877082602
 202050437008676028511617860863938697514177299610327811386342958225508703594
 540*I*a^2*e^(577/2*I*c) + 6122285779233714372590694897018208595664200681070
 859986170274541517752605891024516685624452334324327807799672288064082260*I*
 a^2*e^(575/2*I*c) + 6209657166059790414267919414863086928224438704729318154
 672285436550356208263215568679904005246104294284315747952200822100*I*a^2*e^
 (573/2*I*c) + 6220225123095220446067530083902398768411555741654900638992044
 646455129937854804113318919111668831147496834175450404249540*I*a^2*e^(571/2
 *I*c) + 6158866242148223169440348639322765708488301897861116767404404815878
 289922328995812629986104827051163389294175358746873020*I*a^2*e^(569/2*I*c)
 + 6031732977884812260173929453427232799792945222308203442804835486204161004
 827827142401343622232793155002865274651684289320*I*a^2*e^(567/2*I*c) + 5846
 008733717255034255828344364786950577635748702801782103469328105614678106958
 369335277201973424582273002839567198319080*I*a^2*e^(565/2*I*c) + 5609643858
 197182161541648275011752525586389733113799607955557467036661476245988801877
 065934658119475835050102401103094880*I*a^2*e^(563/2*I*c) + 5331083262965327
 698825985951212681674139396244116171023195725205327404493695541616243952433
 470941991883520791988803340740*I*a^2*e^(561/2*I*c) + 5018996263776862302367
 983716084120558289583785925497973599346198172442668983114339260693015366212
 494449899447639497797000*I*a^2*e^(559/2*I*c) + 4682018640135211402544193447
 675315778522163892489122131411483807638367574153671339708012281597905204951
 327596457959200420*I*a^2*e^(557/2*I*c) + 4328515848979025885689054474944368
 688597856212595758710745631586717058940809165508440803025209005356313475936
 021116327680*I*a^2*e^(555/2*I*c) + 3966374883312783482383646708803019669343
 00047001565939777772491610719951251704477398336756202715565781802459813301
 802300*I*a^2*e^(553/2*I*c) + 3602830529840752956518193931274523396114944031
 976825437061429976090308245501632173258267215963247319966703044426706223680
 *I*a^2*e^(551/2*I*c) + 3244329858695109265142828711377319399945101328495030
 427416903131503319022305572237606283736559593675791505521680740151120*I*a^2
 *e^(549/2*I*c) + 2896436789436624938470299068330946258001309969984703873411
 472869737850504382055706419008342552332959944508775701784058240*I*a^2*e^(54
 7/2*I*c) + 256377663643997703029604566982955668595380960160133353889974265

239086029092155773519578351942456055091409678154086303160*I*a^2*e^(545/2*I*c) + 2250018750681289401874467652562891487203326525709899095523732366806542
477845141215360711007405272102207680817427742759520*I*a^2*e^(543/2*I*c) + 1
957893834972358477905928999095419556405711958988853024935075157754339419656
153188117144093625692196108598042281972985720*I*a^2*e^(541/2*I*c) + 1689241
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020545030435533246445961071068493810560*I*a^2*e^(539/2*I*c) + 1445081045803
166813792264689037973866166202881538425312329021401301390889284254565543949
311378309541723432902867052706200*I*a^2*e^(537/2*I*c) + 1225703977652826279
11562521895660224455114447592406065362942222074085211001715038181853646506
187045033600876523451824880*I*a^2*e^(535/2*I*c) + 1030774619776542758151026
721954085608996661126865830135510680049965310137664141431154997979496639299
345832099582375161360*I*a^2*e^(533/2*I*c) + 8594404367040659964920655309206
627501878563640804789340222353178580696799516488891727895957060068142193680
10815532681480*I*a^2*e^(531/2*I*c) + 71044214655873223359202606887381278419
660842795753483358444595720772369282031299797985429227046115537440594819765
9591360*I*a^2*e^(529/2*I*c) + 582220443185817502062664829213123702632524733
146207834599716064885065789277590829360246460349955947394345054182441981224
*I*a^2*e^(527/2*I*c) + 4730152954843393553865160495107424893668782016894758
20057034050114095323134007835807393143129309159477656152691755564640*I*a^2*
e^(525/2*I*c) + 38095495349548760029295550428770309956766619840549454818851
0783430234085264993237349222897613346201432380448747964425320*I*a^2*e^(523/
2*I*c) + 304132746604536662259792175095559390073445166552839303713131735564
868948526939032146448843731486395171091096793798821440*I*a^2*e^(521/2*I*c)
+ 2406706654362267812491048707637320723073729459838418169127611392117063248
07127596469266879477538714994876050630848911280*I*a^2*e^(519/2*I*c) + 18876
952871090706939931541778736003071270162906123894825106817417431176559865221
8110198035428223021692947533585962348672*I*a^2*e^(517/2*I*c) + 146746216244
736834746047741749764395226222707303675074402068013240510446854646749820204
182172232344517027436260129735987*I*a^2*e^(515/2*I*c) + 1130589802883564220
385835139940220907469386070355433465119796591093989261756839556423349718766
28296436495288367475025210*I*a^2*e^(513/2*I*c) + 86322223651326073619711312
568551997796065517198873894958063303051564972850936480127357633838565811305
177606785202742045*I*a^2*e^(511/2*I*c) + 6531236020748980652365713638634037
535692955227609237671676429852999995330780804236956680516737916274614325528
2258954180*I*a^2*e^(509/2*I*c) + 489664661587874211494619920717556737333685
610246257053283008820469601160935099134005205007305075647243248553963171748
77*I*a^2*e^(507/2*I*c) + 36375409931287795092607557958784126662059576747454
40858874430972208226791392845209172297521166997828928030420844023706*I*a^2
*e^(505/2*I*c) + 2677303943339982741172377844910770087998697063683518021308
4845566895069119080487323194332830887134412610587215162894762*I*a^2*e^(503/
2*I*c) + 195228332867288033102507746731130331021589233473896963924505563358
74949686526249214568717002120285949198419943388647210*I*a^2*e^(501/2*I*c) +
14103212089664651841721385690411495309067974792132984765752121078864973955
276665921709606990805136055899318515316302625*I*a^2*e^(499/2*I*c) + 1009247

860200631273643534077173706806345448511925758402304722088821607870431591919
 7657821787892646407702091249317303229*I*a^2*e^(497/2*I*c) + 715412996672345
 730067340322771834762634518685264638245830985114944269590079262756662236828
 7282262184380707453078413991*I*a^2*e^(495/2*I*c) + 502307443186015730117888
 471656258174869062121735387850334388091202949158080461043749678159891960584
 0393968558559789161*I*a^2*e^(493/2*I*c) + 349309887710492687698237108032257
 440212635206127339745745586202481213457760260596583915006621135990534205025
 0374117429*I*a^2*e^(491/2*I*c) + 240577714439055714117813463067773244849650
 952054131834842019241780734238128978452467263612263569638385729761395004999
 5*I*a^2*e^(489/2*I*c) + 164088464461734084358144268349244158832586560261315
 2625692173455955900863555908606458185685041555583786399895054479300*I*a^2*e
 ^ (487/2*I*c) + 110829117874288088003949255863546681669181130286674928871555
 5797951407508808250708041669875191682735463589899659311198*I*a^2*e^(485/2*I
 *c) + 741238644768872960862145813229884276436860967234511356976656011429732
 646982384201977263394830039748273745918763926276*I*a^2*e^(483/2*I*c) + 4908
 693639020633756913108220724697291424820101761094624078736203613063207730728
 77532370567922640618200227464127502047*I*a^2*e^(481/2*I*c) + 32184963369252
 244789870234832400101081362575349100206847833188345436414544822272518043154
 7920764933095415393034588280*I*a^2*e^(479/2*I*c) + 208927219473219525544565
 866337429793652566518446013625391976810557213288024339930152549544168002925
 624951052677530015*I*a^2*e^(477/2*I*c) + 1342665470057644847983235502917892
 846730721653892879442933623480246710269254851376046455211529505331946182050
 62920988*I*a^2*e^(475/2*I*c) + 85417618996417490452483371059823103161327605
 370266736083574364508696427592725387814713600331423646619680648052044475*I*
 a^2*e^(473/2*I*c) + 5379106945849989042601229237272488231708805561575246835
 8198258128613020164914352269555347240301885068470565731734762*I*a^2*e^(471/
 2*I*c) + 335299088850068226829315497671962848308621901218358414812016207727
 26893006422670068438075620985878626866861887157590*I*a^2*e^(469/2*I*c) + 20
 686671000021215968489830509311959418929183582885216898379036970362110971705
 665038962014129218543899794341899845085*I*a^2*e^(467/2*I*c) + 1263169403449
 483238805228665180633441727146638183758012856773048410704090883152504489801
 1891962065924068550189725268*I*a^2*e^(465/2*I*c) + 763347349130754120948642
 458581148481719625483226104049697195217999448279842521515446999904354737102
 9896595546249849*I*a^2*e^(463/2*I*c) + 456509066991059164837134779602647589
 554670218072151783528080907829796090749188178074444607956065760147676008223
 2152*I*a^2*e^(461/2*I*c) + 270159546995359250505395026372937903793888340345
 1415995229310843295475921159044641522890199450389533264819628536025*I*a^2*e
 ^ (459/2*I*c) + 158202062787581988673250065898762286534898007216280070113086
 8690683669873547179234420581808071126591692483785355020*I*a^2*e^(457/2*I*c)
 + 916647094312849149601409407556411545628020477301452007254604091888065378
 474579372960004944710940275456326641248866*I*a^2*e^(455/2*I*c) + 5254939879
 412245768339489479231903447058938212674523878231430709537350535886875076613
 14315558209767708019478373004*I*a^2*e^(453/2*I*c) + 29804806765205349489178
 954411194330640170803213689038970263075776190021304337102716637566772855035
 8023100922727809*I*a^2*e^(451/2*I*c) + 167238124367604884257687395998092662

795927551689303041040879673329504761663429237202212197395803292702881738952
971*I*a^2*e^(449/2*I*c) + 9283081133766054140183142599824605929696195026202
1941188955325593474604810499041616621349581953895002857873462955*I*a^2*e^(4
47/2*I*c) + 509723132282461048528456302749947063344742018512123007011372050
93695356754098637104271608613774511930964105781185*I*a^2*e^(445/2*I*c) + 27
684667755883080696165241600850236290060527593753750250976925795644297762324
894086602505304941640081834882024183*I*a^2*e^(443/2*I*c) + 1487251651322867
268458445437395297098557524436688392349886320431475878723648539596783314686
9064461137897493283199*I*a^2*e^(441/2*I*c) + 790218349026871901495271289147
853538925976831795670198761660396945228812301836985196711717454575960714299
5908398*I*a^2*e^(439/2*I*c) + 415245104359965804460096900332121399173917602
7752589041806728036658263221899875572009537787169343651573979683470*I*a^2*e
^(437/2*I*c) + 215791258592459333372869589796444940936417263128653468455066
1677201976202007165851929905826385985844876371677490*I*a^2*e^(435/2*I*c) +
110895041237395917205685994545909717581600207445777581219710119943087877137
1764877187949694685819718286221686935*I*a^2*e^(433/2*I*c) + 563528978057632
720969917708971551197388498440029187520498668125627360636873719973842729049
383973395339489187328*I*a^2*e^(431/2*I*c) + 2831541724454077422450878787538
662877042027538054395604149601789061369618915729280268552762138871213800719
28103*I*a^2*e^(429/2*I*c) + 14067237571301268732038907357994553337936713200
7412356558650081388469939218158614778502467163652527862550952130*I*a^2*e^(4
27/2*I*c) + 690955667122722319724322397288994961749222207720419841820692841
30237389188998302806443944988509065063273154745*I*a^2*e^(425/2*I*c) + 33552
369185708773069292076239890437482410490941602744957472544576489334415521169
836149006133511757140035934080*I*a^2*e^(423/2*I*c) + 1610659553400442535055
634550846776406381699126024100836153672902222104936768062730161160906358480
1970379704200*I*a^2*e^(421/2*I*c) + 764306432142215085107655604542055474413
8398485227228000246848202365010891675711435238630995867297844214073376*I*a^
2*e^(419/2*I*c) + 358500567897480551605849967163259551339357495711398033668
8954791609736165974043249124955091096872997954793020*I*a^2*e^(417/2*I*c) +
166205667443073414924892879946898077373145654370121107055618121338992405598
8353226635860540433754964155111760*I*a^2*e^(415/2*I*c) + 761570681410168662
769506297132490171367321466294322961126983855669845333986783371033315463200
057428630752220*I*a^2*e^(413/2*I*c) + 3448716031646138840025055585349184630
90837581777459311678842067123286466226751144071133482776005430489425760*I*a
^2*e^(411/2*I*c) + 15433398044664752654560339343571798167516797662298543036
5188253022405140750956390615095093479110699588649900*I*a^2*e^(409/2*I*c) +
682489975344168898405747441723232848684360017115555657669816504708038746683
37015803960506067102299819184920*I*a^2*e^(407/2*I*c) + 29821871642675642979
884949769331439027860315600536250652213455370080842219764548099437011716544
543041268840*I*a^2*e^(405/2*I*c) + 1287509096857049511048268849733111955148
1725115140466354371165041532925663730313264282055462011603010654340*I*a^2*e
^(403/2*I*c) + 549180314668753988053597656596379567216451408656720801201591
6026775639438732336258201804419618292545032320*I*a^2*e^(401/2*I*c) + 231419
987522566986824406965158487639596366333938136712176597016461942695849656811

6456546662409068449840980*I*a²*e^(399/2*I*c) + 963339901333670052201847836
 491729647953262248639288500838541624447579232078571408017358275408623510613
 840*I*a²*e^(397/2*I*c) + 3961156927720317702883047917185117312913684062974
 39843435650750372236248577797783021338331228243750622260*I*a²*e^{(395/2*I*c}
) + 16087882507904339460383494957583749762548437050975220455638952087004282
 0153281247817248349563478924960000*I*a²*e^(393/2*I*c) + 645327319778245914
 793691916371582593352896495336069829716313649640549116810756371224907145615
 14235891960*I*a²*e^(391/2*I*c) + 25564363098285969951461675934273364734821
 923616250047177527687812898786857371780193438857338260097911840*I*a²*e⁽³⁸
 9/2*I*c) + 1000074884258142115311578285002841913064386956944913472196790825
 1115915010593127137857232140041620869750*I*a²*e^(387/2*I*c) + 386314437516
 868018576356913292077827044669797836884278496544413991685735826342800126438
 3566311006996680*I*a²*e^(385/2*I*c) + 147342444362109962377181703120951524
 3436724637025749527267678685770355293167282547379994041612249536410*I*a²*e
^(383/2*I*c) + 554830477071987246654408444808030209487515325400605772819119
 408435792744646306336768559494858757362460*I*a²*e^(381/2*I*c) + 2062550466
 642352923299538505218323676805884853392747978586557766976702788671365670139
 53786924177858890*I*a²*e^(379/2*I*c) + 75687878722014265825238066731591013
 702998100464324442217272752916887964266256104313931049007060490840*I*a²*e⁽
 377/2*I*c) + 2741514103484544395109493925675095458443694586767090771035285
 9492970463590965208109494412332329295780*I*a²*e^(375/2*I*c) + 980079316064
 566075944389460984891152942557969087433011532356902754100547673537917564006
 3961886333700*I*a²*e^(373/2*I*c) + 345781676701999202030946295835138625644
 9736593140969468279877327753792198494989629433408106651901070*I*a²*e^{(371/}
 2*I*c) + 120385831118903001859978475898311457410127655358396896577655106146
 3170435426403483179176840497456330*I*a²*e^(369/2*I*c) + 413564489143612695
 132947288092921397877659902127071578646579062379140019898506098486550619425
 904170*I*a²*e^(367/2*I*c) + 1401737032027635410158205056407283867183742759
 16041096078183091845413305288205080342342220046629650*I*a²*e^(365/2*I*c) +
 46871127019956579394527723949157745550097315795915660019622180811547871642
 339809799054651854945990*I*a²*e^(363/2*I*c) + 1546033818880351810107079629
 8741473051050449962808545632677281743199685497754720467992010224616870*I*a[^]
 2*e^(361/2*I*c) + 502997233223228520128531881185601119191768675475579615309
 9081679265022946497934343332346116036200*I*a²*e^(359/2*I*c) + 161399625005
 227651497742238751482974339782549553805362337258500737267515812737919919603
 8765006020*I*a²*e^(357/2*I*c) + 510723774383122473742590339083403953575864
 194243445208994753073600039579929266407037097651682920*I*a²*e^(355/2*I*c)
 + 1593571437033253029099589869710248950108651059517203782604973685085930558
 38453825165049611563210*I*a²*e^(353/2*I*c) + 49024494337492279360818849127
 945363095454529548782519402705804194570228377733166934824088655040*I*a²*e⁽
 351/2*I*c) + 1486839010064205118367197117938788000350914635000729463417789
 9645088810313885195635392146744810*I*a²*e^(349/2*I*c) + 444504319274789268
 390399853721138227585710412235922782253365493896371639852064344721224547964
 0*I*a²*e^(347/2*I*c) + 130978506840473201225164579044912481278616850664545
 0981272566942196706295180922249225707977010*I*a²*e^(345/2*I*c) + 380351974

188426836426198640513394713416494607855165518072594190906223337330504018165
258288940*I*a^2*e^(343/2*I*c) + 1088381674585823184455849835282493949200583
59009481425820011589540335936121859362203566362580*I*a^2*e^(341/2*I*c) + 30
685533864596666418316194015078029015958251712763047637572948112622799853423
865147299102830*I*a^2*e^(339/2*I*c) + 8522926048369988940355915252628221987
552753095020951377015329635773840197997200581857597800*I*a^2*e^(337/2*I*c)
+ 2331796181228676480358713874846040638691309670968182309524450889214672550
181351830378735030*I*a^2*e^(335/2*I*c) + 6283217402529699146708974013748477
21259003178557296040399451247917033448679858171068226560*I*a^2*e^(333/2*I*c
) + 16672645594159683116062347635171258718759465764285712701383023948458731
3767680019339582550*I*a^2*e^(331/2*I*c) + 435609777598064255098417169783299
04266075313880276843623527207449861917293070562965518680*I*a^2*e^(329/2*I*c
) + 11204681256258812161206109553481424902816807388805631619385616166895520
536176153385410620*I*a^2*e^(327/2*I*c) + 2836916918939477124570501535003667
950662441685459586986859765033838975136651432973013080*I*a^2*e^(325/2*I*c)
+ 7069254940580017380084988687307754120182736498836563165198219602512939601
11842853827050*I*a^2*e^(323/2*I*c) + 17334585697527208030746888831957374382
6748157949817083968790709575178522586695155600570*I*a^2*e^(321/2*I*c) + 418
212568836636776044638139903569063056923244320728124790203918402097384059928
62342110*I*a^2*e^(319/2*I*c) + 99255258829095186735695367313547359943179819
48112305002260477305616767418585528998710*I*a^2*e^(317/2*I*c) + 23169134178
96533634148238479533816756801158698905671195529018686134366565369411066790*
I*a^2*e^(315/2*I*c) + 53185200307045318933327613873998530720744532344199443
3993246593537700600251640914290*I*a^2*e^(313/2*I*c) + 120038120350944029722
615798484583578817920936103042898479075195084456374447649531420*I*a^2*e^(31
1/2*I*c) + 2663270899013432469919988152459092587944442773459791184262577669
7111614072462805500*I*a^2*e^(309/2*I*c) + 580760744355231221024154298187369
7011544047170905560703749866664890336325145752760*I*a^2*e^(307/2*I*c) + 124
44584413040952074973278377934477929343261005269896448132005897390334813136
4390*I*a^2*e^(305/2*I*c) + 261985740043443080388301874884500683905172501509
414794896626641791046018314193300*I*a^2*e^(303/2*I*c) + 5417503137684360762
9356045027058758032630893385234188342863868414550489892855190*I*a^2*e^(301/
2*I*c) + 110015090511699188034061903011100721691404347391883906806870693716
00096456461960*I*a^2*e^(299/2*I*c) + 21935182533236071585983067606101945055
63715717648737978249164699044796790178970*I*a^2*e^(297/2*I*c) + 42930640633
1604823645688251632236356131380665568461698695435155472754991584480*I*a^2*e
^(295/2*I*c) + 824571979405000651128312357780093374005779424373292705419368
89611136248616360*I*a^2*e^(293/2*I*c) + 15538869199972063791690049288524847
158110979530065672277863507395517286098560*I*a^2*e^(291/2*I*c) + 2872306980
765485786798093527905670394316300311494879919370292601994164873980*I*a^2*e
(289/2*I*c) + 5206550842970712132478746309191148757771507362669623848146159
37366841583600*I*a^2*e^(287/2*I*c) + 92525290960177968452743510383284189038
302875808137991118047531970267330140*I*a^2*e^(285/2*I*c) + 1611539718513118
3261784999068093482958330926537119548096569091610170990080*I*a^2*e^(283/2*I
*c) + 275021431468142116442056577595633159921285138864123056720909524083286

$9740 * I * a^{2 * e^{(281 / 2 * I * c)}} + 459734613498525904391661453323499339021127870927$
 $849907055606144578837880 * I * a^{2 * e^{(279 / 2 * I * c)}} + 7525378354737695802256874598$
 $6903775265291178425521737345338921401401608 * I * a^{2 * e^{(277 / 2 * I * c)}} + 120584136$
 $11979215014493808881094919327076136158958774603465013922951460 * I * a^{2 * e^{(275$
 $/ 2 * I * c)}} + 18908085231844652348705820115489868983683675780582564154081721230$
 $86240 * I * a^{2 * e^{(273 / 2 * I * c)}} + 29003446452038703680172784858940259795627649094$
 $6161717533961649595700 * I * a^{2 * e^{(271 / 2 * I * c)}} + 435050290141291368075663883180$
 $13609163815986027077804343349110468080 * I * a^{2 * e^{(269 / 2 * I * c)}} + 63790183241316$
 $16220652747415710905929685302237372129456527463165524 * I * a^{2 * e^{(267 / 2 * I * c)}} +$
 $913950624853444779964988596342201399080981813136131167642386677984 * I * a^{2 * e$
 $^{(265 / 2 * I * c)}} + 127899514211571855615342081558322168204136482117422337757176$
 $345560 * I * a^{2 * e^{(263 / 2 * I * c)}} + 1747460922600429434666452928465193776292560410$
 $2208554097442050560 * I * a^{2 * e^{(261 / 2 * I * c)}} + 232994397127263459706866547956038$
 $3652668004620643027277203439985 * I * a^{2 * e^{(259 / 2 * I * c)}} + 303026950314518079219$
 $169886639469213112529662619496065070362634 * I * a^{2 * e^{(257 / 2 * I * c)}} + 3842411761$
 $0672655906936441589932580808631293346194215493375887 * I * a^{2 * e^{(255 / 2 * I * c)}} +$
 $4747802391051472283183879516945405395337420791155799564552344 * I * a^{2 * e^{(253 /$
 $2 * I * c)}} + 571368224054265219726596099705196924252108483077143367419935 * I * a^{2$
 $* e^{(251 / 2 * I * c)}} + 6693165128448166186461807730587626940856183459640863610121$
 $0 * I * a^{2 * e^{(249 / 2 * I * c)}} + 762753323485226451170585936840220123065403043344030$
 $1675518 * I * a^{2 * e^{(247 / 2 * I * c)}} + 845095502974670714598033224559068895202594191$
 $429069360222 * I * a^{2 * e^{(245 / 2 * I * c)}} + 9097340693680683753191568540095330911057$
 $4252721276020367 * I * a^{2 * e^{(243 / 2 * I * c)}} + 950851579990067311329922100372685364$
 $4110325215605885183 * I * a^{2 * e^{(241 / 2 * I * c)}} + 964243501752639006146026780418602$
 $842158889630673989345 * I * a^{2 * e^{(239 / 2 * I * c)}} + 9479919142233667091173328415586$
 $7076030325977221341875 * I * a^{2 * e^{(237 / 2 * I * c)}} + 902849209049851352193143124553$
 $0169366369034596324891 * I * a^{2 * e^{(235 / 2 * I * c)}} + 832235125649436217222715021488$
 $460783355491981492217 * I * a^{2 * e^{(233 / 2 * I * c)}} + 7418250497560845028963626536828$
 $1851382711220649004 * I * a^{2 * e^{(231 / 2 * I * c)}} + 638793699419597624062268299016624$
 $5342613055517890 * I * a^{2 * e^{(229 / 2 * I * c)}} + 530853426357042754469653748150767198$
 $919032400940 * I * a^{2 * e^{(227 / 2 * I * c)}} + 4252692781812026203223634023194848436003$
 $9052121 * I * a^{2 * e^{(225 / 2 * I * c)}} + 328031373491263039080453203905338555542609260$
 $0 * I * a^{2 * e^{(223 / 2 * I * c)}} + 243319959628953467711826163624006704408771129 * I * a^{2$
 $* e^{(221 / 2 * I * c)}} + 17332379827117737513384038195005765450047540 * I * a^{2 * e^{(219 /$
 $2 * I * c)}} + 1183905679851187219496284451985830369733725 * I * a^{2 * e^{(217 / 2 * I * c)}} +$
 $77421622501348280247334235191908463309686 * I * a^{2 * e^{(215 / 2 * I * c)}} + 48388512940$
 $65276461984778740285520947658 * I * a^{2 * e^{(213 / 2 * I * c)}} + 28849519402842808862756$
 $8321559245818459 * I * a^{2 * e^{(211 / 2 * I * c)}} + 163740513375598822020754463122265847$
 $00 * I * a^{2 * e^{(209 / 2 * I * c)}} + 882698176765813429616327000431223295 * I * a^{2 * e^{(207 /$
 $2 * I * c)}} + 45084046347051121561291339131297112 * I * a^{2 * e^{(205 / 2 * I * c)}} + 21756376$
 $14526488309339795247719423 * I * a^{2 * e^{(203 / 2 * I * c)}} + 98892618501960556395579843$
 $057828 * I * a^{2 * e^{(201 / 2 * I * c)}} + 4219418379622495018746735933182 * I * a^{2 * e^{(199 / 2$
 $* I * c)}} + 168327860633476968482282779620 * I * a^{2 * e^{(197 / 2 * I * c)}} + 62509019803959$
 $57795917828835 * I * a^{2 * e^{(195 / 2 * I * c)}} + 214978110312371924676280101 * I * a^{2 * e^{(1$
 $93 / 2 * I * c)}} + 6806694783357860712909393 * I * a^{2 * e^{(191 / 2 * I * c)}} + 197035901588508$

$817004391 * I^2 * e^{(189/2 * I * c)} + 5171545973000559760165 * I^2 * e^{(187/2 * I * c)}$
 $+ 121842706165531758225 * I^2 * e^{(185/2 * I * c)} + 2545017361124253210 * I^2 * e^{(183/2 * I * c)}$
 $+ 46393545645311706 * I^2 * e^{(181/2 * I * c)} + 723016295770626 * I^2 * e^{(179/2 * I * c)}$
 $+ 9365496059205 * I^2 * e^{(177/2 * I * c)} + 96800992860 * I^2 * e^{(175/2 * I * c)}$
 $+ 748461285 * I^2 * e^{(173/2 * I * c)} + 3848130 * I^2 * e^{(171/2 * I * c)} + 9867 * I^2 * e^{(169/2 * I * c)}$
 $)/(e^{(517 * I * c)} + 418 * e^{(516 * I * c)} + 87153 * e^{(515 * I * c)} + 12085216 * e^{(514 * I * c)}$
 $+ 1253841160 * e^{(513 * I * c)} + 103818048048 * e^{(512 * I * c)} + 7146142307307 * e^{(511 * I * c)}$
 $+ 420601518659718 * e^{(510 * I * c)} + 21608403021340047 * e^{(509 * I * c)}$
 $+ 984382804329835768 * e^{(508 * I * c)} + 40261256699368950388 * e^{(507 * I * c)}$
 $+ 1493326612293984160368 * e^{(506 * I * c)} + 50648660944512569972179 * e^{(505 * I * c)}$
 $+ 1581796642397812408161814 * e^{(504 * I * c)} + 45759117183402579073139583 * e^{(503 * I * c)}$
 $+ 1232445557346832245176696904 * e^{(502 * I * c)} + 31042222522074681615625020522 * e^{(501 * I * c)}$
 $+ 734057263616388449968842366924 * e^{(500 * I * c)} + 16353164647151530240529137618111 * e^{(499 * I * c)}$
 $+ 344277152012875134140739302960914 * e^{(498 * I * c)}$
 $+ 6868329225263681349501997341320517 * e^{(497 * I * c)} + 130171193079172823835151430773360024 * e^{(496 * I * c)}$
 $+ 2348998374244347079532766203075607598 * e^{(495 * I * c)}$
 $+ 40443624781415311581857832389099634564 * e^{(494 * I * c)} + 665634670676210063754191847109971141414 * e^{(493 * I * c)}$
 $+ 10490402669510897424624643766470754045064 * e^{(492 * I * c)}$
 $+ 158566476113257562566117432227203884298856 * e^{(491 * I * c)}$
 $+ 2302150411226234925855222345201500900533576 * e^{(490 * I * c)} + 32147887693375338817454482515377350383950278 * e^{(489 * I * c)}$
 $+ 432333688644261557547944179250800440604964868 * e^{(488 * I * c)}$
 $+ 5605927253067558551780452883689835514455118670 * e^{(487 * I * c)}$
 $+ 70164515322544462906873548813748091084561870680 * e^{(486 * I * c)}$
 $+ 848552202276512356496200136959676295361696315113 * e^{(485 * I * c)}$
 $+ 9925490738534402272939987038714580495445431374618 * e^{(484 * I * c)}$
 $+ 112391604542246650966429162063124338952554575234051 * e^{(483 * I * c)}$
 $+ 1233096700139723365181997220750932590655287625342156 * e^{(482 * I * c)}$
 $+ 13118781801172174729679339894318153694964675368481194 * e^{(481 * I * c)}$
 $+ 135442594916636116191574650625331646238501101627937224 * e^{(480 * I * c)}$
 $+ 1357990663161479842850642848032544982878359839580349899 * e^{(479 * I * c)}$
 $+ 13231708870104896973800056733779919089340836756009580718 * e^{(478 * I * c)}$
 $+ 125370496586921272662198050851269323171167338854081782959 * e^{(477 * I * c)}$
 $+ 1155855412893594260345544966642687823630035899363232371472 * e^{(476 * I * c)}$
 $+ 10375184499871175501909398956596684116802997082526660323524 * e^{(475 * I * c)}$
 $+ 90722605722208814918642284639487187764607589706493970774776 * e^{(474 * I * c)}$
 $+ 773204636991145775061462731028098506094432675788136295011259 * e^{(473 * I * c)}$
 $+ 6426195485535248576425068136870465530087114003875716691383902 * e^{(472 * I * c)}$
 $+ 52108117629177048660492400985175830987505700566877818954141639 * e^{(471 * I * c)}$
 $+ 412430698299915190848067222327219435067747934091894670488982928 * e^{(470 * I * c)}$
 $+ 3187749929744346497211536044751776582320958627923816470590659024 * e^{(469 * I * c)}$
 $+ 24070801913529757101858022914372045864746991786182039740274325264 * e^{(468 * I * c)}$
 $+ 177642829135119348577194437675802830239905460092687136494961404333 * e^{(467 * I * c)}$
 $+ 1281817464914970810859604189828359000790789921169405304612211251818 * e^{(466 * I * c)}$
 $+ 9046693523825682979044338963104263167672586826367911338826483549173 * e^{(465 * I * c)}$
 $+ 62473550781053295317710774690247114124125187565731848441781904032672 * e^{(464 * I * c)} + 42227612663$

2003687547754746555709988710527133086660161366353656787288*e^(463*I*c) + 27
 94709104475686611842790694973699164482254723977210209725661304403472*e^(462
 *I*c) + 1811576849561575807671030305550562558925429365919331415341833394459
 6408*e^(461*I*c) + 11505148185208084887370038835452131556764036512400310369
 1176697194292320*e^(460*I*c) + 71609949759905807989563333855294022919285819
 6481597830078819711862600096*e^(459*I*c) + 43694424829101139145653531360695
 95862669338858053419381214131241925047008*e^(458*I*c) + 2614397627990202144
 3471945665080254563056810183520401889800285493144867448*e^(457*I*c) + 15343
 6088745056254127327239461577071933130157764595997113973513183188399376*e^(4
 56*I*c) + 88350096882179120260077454192776920073768939351373478936839709333
 3311961880*e^(455*I*c) + 49925197124570439835053779766079539883973682975911
 14957991804893688371867680*e^(454*I*c) + 2769311653834325922598338263764793
 6122664033859615133489846664694361471028310*e^(453*I*c) + 15082238143141241
 3773566474210011746852297437597059186295243989481140398152780*e^(452*I*c) +
 80667954360758914075930501079618956826984202161338895521891627882318263948
 8190*e^(451*I*c) + 42381258467632325863941885698586858267553280055486274370
 19301405851325887594480*e^(450*I*c) + 2187648289271390992804034561257870580
 5121508756226696317087651824252241418663320*e^(449*I*c) + 11096919968732097
 4749922259595250444341219218535349655762591192576535872151766080*e^(448*I*c
) + 55326912881952861250291886955894782909802195630934984358404463151229177
 8800081490*e^(447*I*c) + 27118432396707175276056404901488335071302424484039
 78318523237721944200392830108580*e^(446*I*c) + 1306981720348828988619320550
 8375818392124991382340160316886507181296548981014818410*e^(445*I*c) + 61948
 596653035502879564338815234310660410902037882473161804774492916216575880077
 680*e^(444*I*c) + 288820755264730654469968572021047109427318619508995802020
 689904590319476295408324280*e^(443*I*c) + 132475641236783747315747282116248
 3691120966501948953926492241643788264284546437221120*e^(442*I*c) + 59789921
 729441432184591611492998197063217321115784945252452287429764684091053955362
 90*e^(441*I*c) + 2655680638904340753449670236910154579599486175774141478994
 4652712127566910185274123140*e^(440*I*c) + 11610455168355504376291150171211
 6399313733021132677481112824047246361794049635726479850*e^(439*I*c) + 49970
 756725385908435759631481379476806933719091596749190748890493392267757966535
 4338960*e^(438*I*c) + 21175897334668557071015014292104147224018388379407528
 41618541440888545729943138209036820*e^(437*I*c) + 8836720640860470305694514
 021547969551296794092266983044118375790025854584036796364768280*e^(436*I*c)
 + 363183696523025917321974444097981220226408246041305525067425867951832673
 54382847875885730*e^(435*I*c) + 1470308167322768331630415820995920475120437
 25225353339238819165193000407629544745753221740*e^(434*I*c) + 5864034669726
 832427416433289215609093751974538642432995719909646088572457711341452041749
 90*e^(433*I*c) + 2304351073373840357379178597673066352016682781689139842097
 376663118488803841131935313641840*e^(432*I*c) + 892320944734329676333188188
 1638471793499618670601026059730895962653291770229493028162575100*e^(431*I*c
) + 34054053851295569154352346722177172655187548910782008504718324168725029
 438589162349211628040*e^(430*I*c) + 128098914601688539672480541830409847707
 367500438601536803204497701119911289087105659482783340*e^(429*I*c) + 475010

578857601519272316617938425222421786597241671026894318515408511467140969393
115768793680*e^(428*I*c) + 173657421881819107187419747245015812388356420995
0658639102337148122769080611680719741726053840*e^(427*I*c) + 62598721568222
528436509607082350347102013627760571766472263230897514465652888501038981538
59920*e^(426*I*c) + 2225195917679577775716736603600748022221136423214639980
3864370963391491223687245823457351580140*e^(425*I*c) + 78009807368024239875
613733058851417125327114681070889640794249282633470580756557083923203377160
*e^(424*I*c) + 269745801440211296972683601863878954357962308520076595177128
227629273240215209708218497363414140*e^(423*I*c) + 920089393029589032874601
850027159322612526368444771489781974361078847528891468831038436064951920*e^
(422*I*c) + 309613197162152016238030155424146545178236208681028753774890290
4985934020179565706177131421614590*e^(421*I*c) + 10279364730663840844739577
862469262604648861914297972589165243530651230690726244462479199894255180*e^
(420*I*c) + 336753988720215683759023845939827533625598010581041846273454111
36262431943240778260721756991027090*e^(419*I*c) + 1088679957318294728267329
05192034886797284621356445627530909104429486741257822633476898356826454040*
e^(418*I*c) + 3473514732147137808743520831295666012387657627759423667627333
49952103889753982636403857556867777300*e^(417*I*c) + 1093853214486220358674
032434500866678499770011305874172488975951612031456734608287095519501041975
440*e^(416*I*c) + 340023256060165161752169468084708984419802883169441742479
4868779328950548418125605446882081152636090*e^(415*I*c) + 10434117516570395
966653693155582402109460348095473027807412321427346816928567197770376496170
251803940*e^(414*I*c) + 316109393312846927506943064436184146560959695209452
15743004044560386895241801579156543451940713351730*e^(413*I*c) + 9455618025
893198691933430346636565282685809131432918916073627717587384173219645337995
3705679466826880*e^(412*I*c) + 27928575580003520667983536889816547764486498
7794665387827488933863633745047373109049265172681702585720*e^(411*I*c) + 81
460818773653057967021002527192141559718336988121429982329196978554987617596
9866367976653244974728560*e^(410*I*c) + 23465182192391051422381416330734647
68899155708935025778047637412681781575765422219127409260159438712250*e^(409
*I*c) + 6675866290371147358503766865669289010893543869830538708724945291580
951179188296606158111257706968604740*e^(408*I*c) + 187599882188655635641636
357359860732782557372574057062791088913663784284674145599304811728635385981
93890*e^(407*I*c) + 5207517851879327038642926335154430695110499354250058293
8155241689408138675254608030847907167748571734720*e^(406*I*c) + 14280179245
022176248318087491882527413430513327541778008479503464476350933350315051734
5864659667189417080*e^(405*I*c) + 38687621823427716563245172304997988926311
5282374607541692443176673997513742813591736171169652250611186480*e^(404*I*c
) + 10355619825920029352263845779086115486121114950801935736913398647060291
86482466241805664949381049856258510*e^(403*I*c) + 2738895624795265603355227
646566000886280778305084825702911938903656162004262736182657700406301914070
062380*e^(402*I*c) + 715812468684294147547380736367983971817274558153840904
4503383852693596921622426696740453944718143025248390*e^(401*I*c) + 18487405
299005732693752728611876490890858357021974882371570623800186245137722660943
641752976852924439870880*e^(400*I*c) + 471882208434662076950995069535737803

57108897491422567898048199018207708997005333860148836479527456156014520*e^(399*I*c) + 119041855403877964948229577948370465600606623183045529526900430209270473212773847794935586074714329479939280*e^(398*I*c) + 296825515282669589685318273280239050084555032203415941511962659596881615713799937680026497408305672297618840*e^(397*I*c) + 731584972206818362874729621403974444280010446301161527339760544815300951787985538419764656214582667219914080*e^(396*I*c) + 1782446114931751850556354856638421901174412322298249496591658053939787198246565945975595575734193348887952160*e^(395*I*c) + 4293206478008022126017488908851826494790620720660151451468181910917240027863968724539127659633517053002976480*e^(394*I*c) + 10223182025954860767217390305186451923562145473674293619918063490411487496121804590274592702770571515456414680*e^(393*I*c) + 24068785139705277161193465644506143285241361037768216818922184400141048460210944696647752723371932874594597328*e^(392*I*c) + 56028683424903517658495013858534516167162591034367972498174660907450666778154353271630344650777885683547624184*e^(391*I*c) + 128967080084754712246023680866488384983286259025533132044636109049545144029547003347761521666283977931640178464*e^(390*I*c) + 293550743554342709808129453576562313299705982699187416862934373964255615967138676253276302591561523515603264403*e^(389*I*c) + 660764473105869097691475973850837934511089033149586707982764263394766756649565279879146173318386505740391093990*e^(388*I*c) + 1470931146618934345515038362300100160482127749581443929904746910224777470198899052379114493999887003199419829579*e^(387*I*c) + 3238491931361851476423321933539579098377735539207641467346235665823887048326949305609231585143748690203615957136*e^(386*I*c) + 7052132414162197992602326524580143060985353054572933905524633121681021037340298366342203324325307072413739061024*e^(385*I*c) + 15189634214908800396417911722643754748048520109734812459109878810493844381062650818971199637121458749456243274416*e^(384*I*c) + 32362731322419549410330088943640247460378328561316422931292427145902887913071643679502909055891236755143207382609*e^(383*I*c) + 68208033096793615683784409619244210818614991640041553424405527876893272496608324231098148502466453967157728078994*e^(382*I*c) + 142213115964814517682386667276769909482271681318790889840501039441748635545362467679832449103520321953011780083069*e^(381*I*c) + 293344920034300720287042383448342866313806285455040067823080445597545970023446231563554135133105493516316320059272*e^(380*I*c) + 59865014111224185891167650518052015036400322684132808145359709358779033860921243908554466861582623350303061961052*e^(379*I*c) + 1208770358493658393089442222056935063283704108140593750226539846117737648216609559734831601248698274330296158612144*e^(378*I*c) + 2414966516810338503289076549202740511710059011795447138773464205696455026442712426409599662771080264826008985061097*e^(377*I*c) + 4774141111066098970221845330594962016472714230374234060663956846950926642685946929064114194400360936223590725470146*e^(376*I*c) + 9339341958053494225251750965715057300707302083814774770306218224241022648247419956042957363055823830898547303219757*e^(375*I*c) + 18079820068028859970349938623007230676563314206708848499900139641237334763266479346963237936039328113185041591793848*e^(374*I*c) + 34637657172671690167657344537197087048882354853993270472063943078773600446542963548348101269390443464480754513928502

$*e^{(373*I*c)} + 656748592688673000988273758128752256106545516862611036816640$
07007537115778097293533565243828873383722980353200611956 $*e^{(372*I*c)} + 1232$
439415193323847419600725881035065964063392536163910820629699606824190117457
75738921817753391954462609323881489157 $*e^{(371*I*c)} + 2289113117385927800914$
926491623468344058677407764563261084109288572571747072892680743475502257932
44741923354395308214 $*e^{(370*I*c)} + 4208463426089493872775590214579245865781$
209661485610226470084995294684520059801751194106289562104976095660029698849
27 $*e^{(369*I*c)} + 7658677955139627810125584446287514187109408952813047908367$
43661582071650032154891482866406314834433199455459798934952 $*e^{(368*I*c)} + 1$
379676529796212074017106188066589448355446501210890195107164860350228928586
815539003062875026711931941947738690360722 $*e^{(367*I*c)} + 246044237584542266$
392708163098326071473496809190549302714563923882719225488634936112699145769
2409851120873307487457468 $*e^{(366*I*c)} + 43439096966019321733573596877815792$
937012956819408271142154331753360939678459087667407382400371145706674109369
98017178 $*e^{(365*I*c)} + 7592752700146678961153095073585015473197029746533633$
331549793961473285760935801904155116764831560875947581048693527224 $*e^{(364*I$
 $*c)} + 131397714941049338818566811514182931122425515215356868711812665798138$
77606348160261747201317735782566021306798298336024 $*e^{(363*I*c)} + 2251467574$
130806996150616558650287243042193021067326439299728648560064010386725360484
7715547060592967690653795951142520 $*e^{(362*I*c)} + 38199015867586087976002998$
756627674994795440667903625029322346250133286489120875005013638128113893960
349670280707161530 $*e^{(361*I*c)} + 641751006932600668062380648860045971707408$
433000868393686161391645291080498446753531118427257986580888403472414960996
44 $*e^{(360*I*c)} + 1067648320171655948380852341893335287335876733299725300926$
61085186789939252915937090760282232346919090426243399409323314 $*e^{(359*I*c)}$
 $+ 1758962582627559857571068126139793012658010315954843536149046728651694422$
32075776580447184134141375995770091499246759528 $*e^{(358*I*c)} + 2869929436312$
314965572780108515769408968264974660663275288015606770071128374319267350881
20974861760511367008815728782643 $*e^{(357*I*c)} + 4637582884573671545449376782$
550056887333281455680493104239955998860128006386199040223683785911088426023
42094543682299102 $*e^{(356*I*c)} + 7422286409081731249169370494625256173341489$
196791182704898310054977819512210699558396234524997486531246588735534014421
37 $*e^{(355*I*c)} + 1176600720975786965189875050890231092204612696970277433014$
535895788956771230793520381993106606880564628599822341722801012 $*e^{(354*I*c)}$
 $+ 184750585646245153344528430057132632378116255330456597188767075809107930$
6794821834928170773126364639722071570131703785334 $*e^{(353*I*c)} + 28736105359$
223401870808354355829122772719679773947201597910702749277142768695314671826
88981041061381703885403497544001592 $*e^{(352*I*c)} + 4427673079105425318524316$
112985693656584851936100192457044455134483305045321452516347118488133224823
670465103483954805161 $*e^{(351*I*c)} + 675848043788852437256293594896385762669$
485554719551948612287756798171858726236287199496707940183195792790168258294
1234362 $*e^{(350*I*c)} + 10220423779434634851339975295163399641702122249663666$
193053008302026096932158568338309418237395541351819026907953220681013 $*e^{(34$
9 $I*c)} + 153128372066627753793473532128076829657125356529426315182861424030$
97738200270711195396582159028513532779682154451996208592 $*e^{(348*I*c)} + 2273$

160356612884110041950194705136766683665241807726091394481074847308489189041
0181285412604854876625919565639521227223276*e^(347*I*c) + 33435897827936581
301171175459610829454298167962017419810072936733378506584428024201072453193
458155334046693516742390717832*e^(346*I*c) + 487332535059749234008522555630
521014021964693136595544927256747543393752830104071677443669558289228374887
05858532439654489*e^(345*I*c) + 7038634976059483156704822406139502569850120
229696630037676422033669770296159109985405541137629487143746814952852479600
2762*e^(344*I*c) + 10074496185185374461175430098298016696240455383622292186
8484694269966120607698907046343731011160948828100276729370132819357*e^(343*
I*c) + 14290631912305552424654692847895423837131592580202238923649865213683
9822502035155676970917419039834587967055588431566416784*e^(342*I*c) + 20090
658715357880438030046950144161017452185125954192920984068896085945490851977
4835905895757666770857888611738751858460424*e^(341*I*c) + 27994524447503980
482296673046296088449211987485779114712400907947692043594173529330930543043
8687333129912454196774070107264*e^(340*I*c) + 38664267305038004945738256281
831696265197555099077927704874023862985879501824735616288863101568766478010
1205287333082748791*e^(339*I*c) + 52932925276411392600393483695824355767254
923899756073921440659918504783195557258376535863439540877152800974546754838
2950094*e^(338*I*c) + 71836159638205824920911354448790108886838874403371321
0332491971375906738341551540457264804304039664255915607349801911966551*e^(3
37*I*c) + 96645827536903771874773913079815164348359068416683223468809829116
4160636418159452119815728809372125168836239364442397344064*e^(336*I*c) + 12
890435152929339564806343304996770401810439356201069142673110679000300583988
39787692376954090545278554544997710058754772400*e^(335*I*c) + 1704582996707
822808204678218167693002698661147712772355021456543810930069637188085882824
757500605246963210810351706405349408*e^(334*I*c) + 223489127639843946447862
257830643484072461048446817785982262065869192147864526665306256382355300122
8001009093606751066168944*e^(333*I*c) + 29053857223200570019533452744894827
908566925299598237495326959634141648333667731282186078993285886089161765937
72088622582464*e^(332*I*c) + 3745257594876651204657334988426226388143954501
986830664222349226361079609546822276067504899386703088982308185717143407211
328*e^(331*I*c) + 478752744278094568514520484697159616530416941932824407321
1459592129649255048876854059844720661078151288179612574986359194560*e^(330*
I*c) + 60689498031567122483318711053298954717228061430088780149865596536872
60694816550470195890004511965527567432722969707577202160*e^(329*I*c) + 7629
731815627821580468992424207008366438896736333024661863838105110445148946962
328297631547032543419811821015837863013682720*e^(328*I*c) + 951303227401952
295420911319126822664229991201352566594029838106479788569090499312894803522
7412144035633851779511219335277360*e^(327*I*c) + 11764212274876484080010900
714673474493371278160557811983724455826566055617658086479368641864908119643
412413644803772131657280*e^(326*I*c) + 144298162852084312045329783753756919
650631542246497475512958515073895240832269767896886013696283999007476585792
01929300744260*e^(325*I*c) + 1755627327122429239688729140312571621349148626
114547857137675169010565606783804215103827138130037275775567632540802683454
4840*e^(324*I*c) + 21188321405882887539610198374706862695894049226077093764

132512513336190523978949694387686059124526755048042957954264706637460*e^(32
3*I*c) + 253671764391193536215322603359833481549049826061257617113006834929
63390816491583025705268737539982149639300226512657426118880*e^(322*I*c) + 3
012848241455270326455901895308817715601343749343820107841376983544836614812
1754549197591129967170764969700180348699207838960*e^(321*I*c) + 35500103106
019649876272376796949482209581372371036005012877806027481672807059943445240
136315568500732379966585005678181937920*e^(320*I*c) + 414998321219637080437
885237874013455417800889305382069188535790267492733646716400375634886077160
92887686471542838602788559660*e^(319*I*c) + 4813311767818402921650374854911
037447892471909463560389282936486391655379227882295736828510632816471591059
8370871149079494360*e^(318*I*c) + 55390913044972086219432689146331566081427
959896969900214434296817731150863867056620768608187679709720152974148474907
904177340*e^(317*I*c) + 632477741010121790517949460751755699240769813381384
83158042406747453874729387631710544995247152912205118500597511052824347680*
e^(316*I*c) + 7166032986117339552444194388928410913409115784465524567208423
7402434944696464927131812190659629511140639501743303863582092880*e^(315*I*c
) + 80566249130682684181876201882623511206363790337218011954110210642927765
997644903820595421936873565314654415769070472655401600*e^(314*I*c) + 898838
158013823822139732704779546027447928770180519633471463073724643151212749294
02347942874802899499538953561056667668891020*e^(313*I*c) + 9951220647205796
595134034173802354851533640337171789804085047095465753297727911349150688029
0726111154101941386019689567958040*e^(312*I*c) + 10933253734996622320393267
850342635707986370700172829401104207653040392386265401897867651641731422108
9449922495612732870169660*e^(311*I*c) + 11920971370203392705575539782368844
444464742432450218532862634704659963472114657383068154049533354314677681091
1910410468628960*e^(310*I*c) + 12899507601159190341076386342709732994858617
357459586270584915928094304645874266316345401849146385539564945395221289963
2198680*e^(309*I*c) + 13852979454915108945135276957654340312633074724368003
0832467205895819043568155239264876762867172754338684027849855385453216080*e
^(308*I*c) + 14764892080554533341862312176785377739978292474830122879392434
2574999937955421765370101235122939557467548549202174550009604780*e^(307*I*c
) + 15618596295355119616973821883217369650985255158921073057836572747625947
6474465955428502336673743686499175698677875693611243400*e^(306*I*c) + 16397
781605960772537526455981650584789418778510145536039189742448299841538578760
5765315509208337741590143078572243505132706580*e^(305*I*c) + 17086984886895
310117686030605310399434053039034726008843267684250555514129383083896127597
4268928666494845723462544709102843680*e^(304*I*c) + 17672092997055464200457
577005309570059533465987068273203197591553238757705241486632351114011768049
2929354517559479899220940360*e^(303*I*c) + 18140816877092205982036855331669
732163998486262829882856956027329563089762682934526359221903456085353073371
0529842148537901680*e^(302*I*c) + 18483115198374894181766785017470825713812
817215826941328776535853224077324433619190081855782990589568449488941045192
1524212840*e^(301*I*c) + 18691547443656751492635140562311750326198750835193
008382456644443568913923368341170464182876217879917784806422015081835526128
0*e^(300*I*c) + 18761539316851005007149728056460351091240313292031202437083

5062679037644990286285346673507093452964351257962696133511725652320*e^(299*I*c) + 18691547443656751492635140562311750326198750835193008382456644443568
 9139233683411704641828762178799177848064220150818355261280*e^(298*I*c) + 18
 483115198374894181766785017470825713812817215826941328776535853224077324433
 6191900818557829905895684494889410451921524212840*e^(297*I*c) + 18140816877
 092205982036855331669732163998486262829882856956027329563089762682934526359
 2219034560853530733710529842148537901680*e^(296*I*c) + 17672092997055464200
 457577005309570059533465987068273203197591553238757705241486632351114011768
 0492929354517559479899220940360*e^(295*I*c) + 17086984886895310117686030605
 31039943405303903472600884326768425055514129383083896127597426892866649484
 5723462544709102843680*e^(294*I*c) + 16397781605960772537526455981650584789
 418778510145536039189742448299841538578760576531550920833774159014307857224
 3505132706580*e^(293*I*c) + 15618596295355119616973821883217369650985255158
 921073057836572747625947647446595542850233667374368649917569867787569361124
 3400*e^(292*I*c) + 14764892080554533341862312176785377739978292474830122879
 3924342574999937955421765370101235122939557467548549202174550009604780*e^(2
 91*I*c) + 13852979454915108945135276957654340312633074724368003083246720589
 5819043568155239264876762867172754338684027849855385453216080*e^(290*I*c) +
 12899507601159190341076386342709732994858617357459586270584915928094304645
 8742663163454018491463855395649453952212899632198680*e^(289*I*c) + 11920971
 37020339270557553978236884444464742432450218532862634704659963472114657383
 0681540495333543146776810911910410468628960*e^(288*I*c) + 10933253734996622
 320393267850342635707986370700172829401104207653040392386265401897867651641
 7314221089449922495612732870169660*e^(287*I*c) + 99512206472057965951340341
 738023548515336403371717898040850470954657532977279113491506880290726111154
 101941386019689567958040*e^(286*I*c) + 898838158013823822139732704779546027
 447928770180519633471463073724643151212749294023479428748028994995389535610
 56667668891020*e^(285*I*c) + 8056624913068268418187620188262351120636379033
 721801195411021064292776599764490382059542193687356531465441576907047265540
 1600*e^(284*I*c) + 71660329861173395524441943889284109134091157844655245672
 084237402434944696464927131812190659629511140639501743303863582092880*e^(28
 3*I*c) + 632477741010121790517949460751755699240769813381384831580424067474
 53874729387631710544995247152912205118500597511052824347680*e^(282*I*c) + 5
 539091304497208621943268914633156608142795989696990021443429681773115086386
 7056620768608187679709720152974148474907904177340*e^(281*I*c) + 48133117678
 184029216503748549110374478924719094635603892829364863916553792278822957368
 285106328164715910598370871149079494360*e^(280*I*c) + 414998321219637080437
 885237874013455417800889305382069188535790267492733646716400375634886077160
 92887686471542838602788559660*e^(279*I*c) + 3550010310601964987627237679694
 948220958137237103600501287780602748167280705994344524013631556850073237996
 6585005678181937920*e^(278*I*c) + 30128482414552703264559018953088177156013
 437493438201078413769835448366148121754549197591129967170764969700180348699
 207838960*e^(277*I*c) + 253671764391193536215322603359833481549049826061257
 61711300683492963390816491583025705268737539982149639300226512657426118880*
 e^(276*I*c) + 2118832140588288753961019837470686269589404922607709376413251

2513336190523978949694387686059124526755048042957954264706637460*e^(275*I*c)
) + 17556273271224292396887291403125716213491486261145478571376751690105656
 067838042151038271381300372757755676325408026834544840*e^(274*I*c) + 144298
 162852084312045329783753756919650631542246497475512958515073895240832269767
 89688601369628399900747658579201929300744260*e^(273*I*c) + 1176421227487648
 408001090071467347449337127816055781198372445582656605561765808647936864186
 4908119643412413644803772131657280*e^(272*I*c) + 95130322740195229542091131
 912682266422999120135256659402983810647978856909049931289480352274121440356
 33851779511219335277360*e^(271*I*c) + 7629731815627821580468992424207008366
 438896736333024661863838105110445148946962328297631547032543419811821015837
 863013682720*e^(270*I*c) + 606894980315671224833187110532989547172280614300
 887801498655965368726069481655047019589000451196552756743272296970757720216
 0*e^(269*I*c) + 47875274427809456851452048469715961653041694193282440732114
 59592129649255048876854059844720661078151288179612574986359194560*e^(268*I*
 c) + 3745257594876651204657334988426226388143954501986830664222349226361079
 609546822276067504899386703088982308185717143407211328*e^(267*I*c) + 290538
 572232005700195334527448948279085669252995982374953269596341416483336677312
 8218607899328588608916176593772088622582464*e^(266*I*c) + 22348912763984394
 644786225783064348407246104844681778598226206586919214786452666530625638235
 53001228001009093606751066168944*e^(265*I*c) + 1704582996707822808204678218
 167693002698661147712772355021456543810930069637188085882824757500605246963
 210810351706405349408*e^(264*I*c) + 128904351529293395648063433049967704018
 104393562010691426731106790003005839883978769237695409054527855454499771005
 8754772400*e^(263*I*c) + 96645827536903771874773913079815164348359068416683
 2234688098291164160636418159452119815728809372125168836239364442397344064*e
 ^ (262*I*c) + 71836159638205824920911354448790108886838874403371321033249197
 1375906738341551540457264804304039664255915607349801911966551*e^(261*I*c) +
 52932925276411392600393483695824355767254923899756073921440659918504783195
 5572583765358634395408771528009745467548382950094*e^(260*I*c) + 38664267305
 038004945738256281831696265197555099077927704874023862985879501824735616288
 8631015687664780101205287333082748791*e^(259*I*c) + 27994524447503980482296
 673046296088449211987485779114712400907947692043594173529330930543043868733
 3129912454196774070107264*e^(258*I*c) + 20090658715357880438030046950144161
 017452185125954192920984068896085945490851977483590589575766677085788861173
 8751858460424*e^(257*I*c) + 14290631912305552424654692847895423837131592580
 202238923649865213683982250203515567697091741903983458796705558843156641678
 4*e^(256*I*c) + 10074496185185374461175430098298016696240455383622292186848
 4694269966120607698907046343731011160948828100276729370132819357*e^(255*I*c
) + 70386349760594831567048224061395025698501202296966300376764220336697702
 961591099854055411376294871437468149528524796002762*e^(254*I*c) + 487332535
 059749234008522555630521014021964693136595544927256747543393752830104071677
 44366955828922837488705858532439654489*e^(253*I*c) + 3343589782793658130117
 117545961082945429816796201741981007293673337850658442802420107245319345815
 5334046693516742390717832*e^(252*I*c) + 22731603566128841100419501947051367
 666836652418077260913944810748473084891890410181285412604854876625919565639

$521227223276 \cdot e^{(251 \cdot I \cdot c)} + 153128372066627753793473532128076829657125356529$
 $42631518286142403097738200270711195396582159028513532779682154451996208592 \cdot$
 $e^{(250 \cdot I \cdot c)} + 1022042377943463485133997529516339964170212224966366619305300$
 $8302026096932158568338309418237395541351819026907953220681013 \cdot e^{(249 \cdot I \cdot c)} +$
 $67584804378885243725629359489638576266948555471955194861228775679817185872$
 $62362871994967079401831957927901682582941234362 \cdot e^{(248 \cdot I \cdot c)} + 4427673079105$
 $425318524316112985693656584851936100192457044455134483305045321452516347118$
 $488133224823670465103483954805161 \cdot e^{(247 \cdot I \cdot c)} + 287361053592234018708083543$
 $558291227727196797739472015979107027492771427686953146718268898104106138170$
 $3885403497544001592 \cdot e^{(246 \cdot I \cdot c)} + 18475058564624515334452843005713263237811$
 $625533045659718876707580910793067948218349281707731263646397220715701317037$
 $85334 \cdot e^{(245 \cdot I \cdot c)} + 1176600720975786965189875050890231092204612696970277433$
 $014535895788956771230793520381993106606880564628599822341722801012 \cdot e^{(244 \cdot I$
 $\cdot c)} + 742228640908173124916937049462525617334148919679118270489831005497781$
 $951221069955839623452499748653124658873553401442137 \cdot e^{(243 \cdot I \cdot c)} + 463758288$
 $457367154544937678255005688733328145568049310423995599886012800638619904022$
 $368378591108842602342094543682299102 \cdot e^{(242 \cdot I \cdot c)} + 286992943631231496557278$
 $010851576940896826497466066327528801560677007112837431926735088120974861760$
 $511367008815728782643 \cdot e^{(241 \cdot I \cdot c)} + 175896258262755985757106812613979301265$
 $801031595484353614904672865169442232075776580447184134141375995770091499246$
 $759528 \cdot e^{(240 \cdot I \cdot c)} + 106764832017165594838085234189333528733587673329972530$
 $092661085186789939252915937090760282232346919090426243399409323314 \cdot e^{(239 \cdot I$
 $\cdot c)} + 641751006932600668062380648860045971707408433000868393686161391645291$
 $08049844675353111842725798658088840347241496099644 \cdot e^{(238 \cdot I \cdot c)} + 3819901586$
 $758608797600299875662767499479544066790362502932234625013328648912087500501$
 $3638128113893960349670280707161530 \cdot e^{(237 \cdot I \cdot c)} + 22514675741308069961506165$
 $586502872430421930210673264392997286485600640103867253604847715547060592967$
 $690653795951142520 \cdot e^{(236 \cdot I \cdot c)} + 131397714941049338818566811514182931122425$
 $515215356868711812665798138776063481602617472013177357825660213067982983360$
 $24 \cdot e^{(235 \cdot I \cdot c)} + 7592752700146678961153095073585015473197029746533633331549$
 $793961473285760935801904155116764831560875947581048693527224 \cdot e^{(234 \cdot I \cdot c)} +$
 $434390969660193217335735968778157929370129568194082711421543317533609396784$
 $5908766740738240037114570667410936998017178 \cdot e^{(233 \cdot I \cdot c)} + 24604423758454226$
 $639270816309832607147349680919054930271456392388271922548863493611269914576$
 $92409851120873307487457468 \cdot e^{(232 \cdot I \cdot c)} + 1379676529796212074017106188066589$
 $448355446501210890195107164860350228928586815539003062875026711931941947738$
 $690360722 \cdot e^{(231 \cdot I \cdot c)} + 765867795513962781012558444628751418710940895281304$
 $790836743661582071650032154891482866406314834433199455459798934952 \cdot e^{(230 \cdot I$
 $\cdot c)} + 420846342608949387277559021457924586578120966148561022647008499529468$
 $452005980175119410628956210497609566002969884927 \cdot e^{(229 \cdot I \cdot c)} + 228911311738$
 $592780091492649162346834405867740776456326108410928857257174707289268074347$
 $550225793244741923354395308214 \cdot e^{(228 \cdot I \cdot c)} + 123243941519332384741960072588$
 $103506596406339253616391082062969960682419011745775738921817753391954462609$
 $323881489157 \cdot e^{(227 \cdot I \cdot c)} + 656748592688673000988273758128752256106545516862$
 $61103681664007007537115778097293533565243828873383722980353200611956 \cdot e^{(226$

*I*c) + 3463765717267169016765734453719708704888235485399327047206394307877
3600446542963548348101269390443464480754513928502*e^(225*I*c) + 18079820068
028859970349938623007230676563314206708848499900139641237334763266479346963
237936039328113185041591793848*e^(224*I*c) + 933934195805349422525175096571
505730070730208381477477030621822424102264824741995604295736305582383089854
7303219757*e^(223*I*c) + 47741411110660989702218453305949620164727142303742
34060663956846950926642685946929064114194400360936223590725470146*e^(222*I*
c) + 2414966516810338503289076549202740511710059011795447138773464205696455
026442712426409599662771080264826008985061097*e^(221*I*c) + 120877035849365
83930894422205693506328370410814059375022653984611773764821660955973483160
1248698274330296158612144*e^(220*I*c) + 59865014111224185891167650518052015
036400322684132808145359709358779033860921243908555446686158262335030306196
1052*e^(219*I*c) + 29334492003430072028704238344834286631380628545504006782
3080445597545970023446231563554135133105493516316320059272*e^(218*I*c) + 14
221311596481451768238666727676990948227168131879088984050103944174863554536
2467679832449103520321953011780083069*e^(217*I*c) + 68208033096793615683784
409619244210818614991640041553424405527876893272496608324231098148502466453
967157728078994*e^(216*I*c) + 323627313224195494103300889436402474603783285
61316422931292427145902887913071643679502909055891236755143207382609*e^(215
*I*c) + 1518963421490880039641791172264375474804852010973481245910987881049
3844381062650818971199637121458749456243274416*e^(214*I*c) + 70521324141621
979926023265245801430609853530545729339055246331216810210373402983663422033
24325307072413739061024*e^(213*I*c) + 3238491931361851476423321933539579098
377735539207641467346235665823887048326949305609231585143748690203615957136
*e^(212*I*c) + 147093114661893434551503836230010016048212774958144392990474
6910224777470198899052379114493999887003199419829579*e^(211*I*c) + 66076447
310586909769147597385083793451108903314958670798276426339476675664956527987
9146173318386505740391093990*e^(210*I*c) + 29355074355434270980812945357656
231329970598269918741686293437396425561596713867625327630259156152351560326
4403*e^(209*I*c) + 12896708008475471224602368086648838498328625902553313204
4636109049545144029547003347761521666283977931640178464*e^(208*I*c) + 56028
683424903517658495013858534516167162591034367972498174660907450666778154353
271630344650777885683547624184*e^(207*I*c) + 240687851397052771611934656445
061432852413610377682168189221844001410484602109446966477527233719328745945
97328*e^(206*I*c) + 1022318202595486076721739030518645192356214547367429361
9918063490411487496121804590274592702770571515456414680*e^(205*I*c) + 42932
064780080221260174889088518264947906207206601514514681819109172400278639687
24539127659633517053002976480*e^(204*I*c) + 1782446114931751850556354856638
421901174412322298249496591658053939787198246565945975595575734193348887952
160*e^(203*I*c) + 731584972206818362874729621403974444280010446301161527339
760544815300951787985538419764656214582667219914080*e^(202*I*c) + 296825515
282669589685318273280239050084555032203415941511962659596881615713799937680
026497408305672297618840*e^(201*I*c) + 119041855403877964948229577948370465
600606623183045529526900430209270473212773847794935586074714329479939280*e^
(200*I*c) + 471882208434662076950995069535737803571088974914225678980481990

18207708997005333860148836479527456156014520*e^(199*I*c) + 1848740529900573
269375272861187649089085835702197488237157062380018624513772266094364175297
6852924439870880*e^(198*I*c) + 71581246868429414754738073636798397181727455
81538409044503383852693596921622426696740453944718143025248390*e^(197*I*c)
+ 2738895624795265603355227646566000886280778305084825702911938903656162004
262736182657700406301914070062380*e^(196*I*c) + 103556198259200293522638457
790861154861211149508019357369133986470602918648246624180566494938104985625
8510*e^(195*I*c) + 38687621823427716563245172304997988926311528237460754169
2443176673997513742813591736171169652250611186480*e^(194*I*c) + 14280179245
022176248318087491882527413430513327541778008479503464476350933350315051734
5864659667189417080*e^(193*I*c) + 52075178518793270386429263351544306951104
993542500582938155241689408138675254608030847907167748571734720*e^(192*I*c)
+ 187599882188655635641636357359860732782557372574057062791088913663784284
67414559930481172863538598193890*e^(191*I*c) + 6675866290371147358503766865
669289010893543869830538708724945291580951179188296606158111257706968604740
*e^(190*I*c) + 234651821923910514223814163307346476889915570893502577804763
7412681781575765422219127409260159438712250*e^(189*I*c) + 81460818773653057
967021002527192141559718336988121429982329196978554987617596986636797665324
4974728560*e^(188*I*c) + 27928575580003520667983536889816547764486498779466
5387827488933863633745047373109049265172681702585720*e^(187*I*c) + 94556180
258931986919334303466365652826858091314329189160736277175873841732196453379
953705679466826880*e^(186*I*c) + 316109393312846927506943064436184146560959
69520945215743004044560386895241801579156543451940713351730*e^(185*I*c) + 1
043411751657039596665369315558240210946034809547302780741232142734681692856
7197770376496170251803940*e^(184*I*c) + 34002325606016516175216946808470898
44198028831694417424794868779328950548418125605446882081152636090*e^(183*I*
c) + 1093853214486220358674032434500866678499770011305874172488975951612031
456734608287095519501041975440*e^(182*I*c) + 347351473214713780874352083129
566601238765762775942366762733349952103889753982636403857556867777300*e^(18
1*I*c) + 108867995731829472826732905192034886797284621356445627530909104429
486741257822633476898356826454040*e^(180*I*c) + 336753988720215683759023845
93982753362559801058104184627345411136262431943240778260721756991027090*e^(
179*I*c) + 1027936473066384084473957786246926260464886191429797258916524353
0651230690726244462479199894255180*e^(178*I*c) + 30961319716215201623803015
54241465451782362086810287537748902904985934020179565706177131421614590*e^(
177*I*c) + 9200893930295890328746018500271593226125263684447714897819743610
78847528891468831038436064951920*e^(176*I*c) + 2697458014402112969726836018
63878954357962308520076595177128227629273240215209708218497363414140*e^(175
*I*c) + 7800980736802423987561373305885141712532711468107088964079424928263
3470580756557083923203377160*e^(174*I*c) + 22251959176795777757167366036007
480222211364232146399803864370963391491223687245823457351580140*e^(173*I*c)
+ 625987215682225284365096070823503471020136277605717664722632308975144656
5288850103898153859920*e^(172*I*c) + 17365742188181910718741974724501581238
83564209950658639102337148122769080611680719741726053840*e^(171*I*c) + 4750
105788576015192723166179384252224217865972416710268943185154085114671409693

93115768793680*e^(170*I*c) + 1280989146016885396724805418304098477073675004
38601536803204497701119911289087105659482783340*e^(169*I*c) + 3405405385129
556915435234672217717265518754891078200850471832416872502943858916234921162
8040*e^(168*I*c) + 89232094473432967633318818816384717934996186706010260597
30895962653291770229493028162575100*e^(167*I*c) + 2304351073373840357379178
597673066352016682781689139842097376663118488803841131935313641840*e^(166*I
*c) + 586403466972683242741643328921560909375197453864243299571990964608857
245771134145204174990*e^(165*I*c) + 147030816732276833163041582099592047512
04372522535339238819165193000407629544745753221740*e^(164*I*c) + 363183696
523025917321974444097981220226408246041305525067425867951832673543828478758
85730*e^(163*I*c) + 8836720640860470305694514021547969551296794092266983044
118375790025854584036796364768280*e^(162*I*c) + 211758973346685570710150142
9210414722401838837940752841618541440888545729943138209036820*e^(161*I*c) +
49970756725385908435759631481379476806933719091596749190748890493392267757
9665354338960*e^(160*I*c) + 11610455168355504376291150171211639931373302113
2677481112824047246361794049635726479850*e^(159*I*c) + 26556806389043407534
496702369101545795994861757741414789944652712127566910185274123140*e^(158*I
*c) + 597899217294414321845916114929981970632173211157849452524522874297646
8409105395536290*e^(157*I*c) + 13247564123678374731574728211624836911209665
01948953926492241643788264284546437221120*e^(156*I*c) + 2888207552647306544
69968572021047109427318619508995802020689904590319476295408324280*e^(155*I*
c) + 6194859665303550287956433881523431066041090203788247316180477449291621
6575880077680*e^(154*I*c) + 13069817203488289886193205508375818392124991382
340160316886507181296548981014818410*e^(153*I*c) + 271184323967071752760564
0490148833507130242448403978318523237721944200392830108580*e^(152*I*c) + 55
326912881952861250291886955894782909802195630934984358404463151229177880008
1490*e^(151*I*c) + 11096919968732097474992225959525044434121921853534965576
2591192576535872151766080*e^(150*I*c) + 21876482892713909928040345612578705
805121508756226696317087651824252241418663320*e^(149*I*c) + 423812584676323
2586394188569858685826755328005548627437019301405851325887594480*e^(148*I*c
) + 80667954360758914075930501079618956826984202161338895521891627882318263
9488190*e^(147*I*c) + 15082238143141241377356647421001174685229743759705918
6295243989481140398152780*e^(146*I*c) + 27693116538343259225983382637647936
122664033859615133489846664694361471028310*e^(145*I*c) + 499251971245704398
3505377976607953988397368297591114957991804893688371867680*e^(144*I*c) + 88
3500968821791202600774541927769200737689393513734789368397093333311961880*e
^(143*I*c) + 15343608874505625412732723946157707193313015776459599711397351
3183188399376*e^(142*I*c) + 26143976279902021443471945665080254563056810183
520401889800285493144867448*e^(141*I*c) + 436944248291011391456535313606959
5862669338858053419381214131241925047008*e^(140*I*c) + 71609949759905807989
563333852940229192858196481597830078819711862600096*e^(139*I*c) + 11505148
1852080848873700388354521315567640365124003103691176697194292320*e^(138*I*c
) + 18115768495615758076710303055505625589254293659193314153418333944596408
*e^(137*I*c) + 279470910447568661184279069497369916448225472397721020972566
1304403472*e^(136*I*c) + 42227612663200368754775474655570998871052713308666

0161366353656787288*e^(135*I*c) + 62473550781053295317710774690247114124125
 187565731848441781904032672*e^(134*I*c) + 904669352382568297904433896310426
 3167672586826367911338826483549173*e^(133*I*c) + 12818174649149708108596041
 89828359000790789921169405304612211251818*e^(132*I*c) + 1776428291351193485
 77194437675802830239905460092687136494961404333*e^(131*I*c) + 2407080191352
 9757101858022914372045864746991786182039740274325264*e^(130*I*c) + 31877499
 29744346497211536044751776582320958627923816470590659024*e^(129*I*c) + 4124
 30698299915190848067222327219435067747934091894670488982928*e^(128*I*c) + 5
 2108117629177048660492400985175830987505700566877818954141639*e^(127*I*c) +
 6426195485535248576425068136870465530087114003875716691383902*e^(126*I*c)
 + 773204636991145775061462731028098506094432675788136295011259*e^(125*I*c)
 + 90722605722208814918642284639487187764607589706493970774776*e^(124*I*c) +
 10375184499871175501909398956596684116802997082526660323524*e^(123*I*c) +
 1155855412893594260345544966642687823630035899363232371472*e^(122*I*c) + 12
 5370496586921272662198050851269323171167338854081782959*e^(121*I*c) + 13231
 708870104896973800056733779919089340836756009580718*e^(120*I*c) + 135799066
 3161479842850642848032544982878359839580349899*e^(119*I*c) + 13544259491663
 6116191574650625331646238501101627937224*e^(118*I*c) + 13118781801172174729
 679339894318153694964675368481194*e^(117*I*c) + 123309670013972336518199722
 0750932590655287625342156*e^(116*I*c) + 11239160454224665096642916206312433
 8952554575234051*e^(115*I*c) + 99254907385344022729399870387145804954454313
 74618*e^(114*I*c) + 84855220276512356496200136959676295361696315113*e^(113
 *I*c) + 70164515322544462906873548813748091084561870680*e^(112*I*c) + 56059
 27253067558551780452883689835514455118670*e^(111*I*c) + 4323336886442615575
 47944179250800440604964868*e^(110*I*c) + 3214788769337533881745448251537735
 0383950278*e^(109*I*c) + 2302150411226234925855222345201500900533576*e^(108
 *I*c) + 158566476113257562566117432227203884298856*e^(107*I*c) + 1049040266
 9510897424624643766470754045064*e^(106*I*c) + 66563467067621006375419184710
 9971141414*e^(105*I*c) + 40443624781415311581857832389099634564*e^(104*I*c)
 + 2348998374244347079532766203075607598*e^(103*I*c) + 13017119307917282383
 5151430773360024*e^(102*I*c) + 6868329225263681349501997341320517*e^(101*I*
 c) + 344277152012875134140739302960914*e^(100*I*c) + 1635316464715153024052
 9137618111*e^(99*I*c) + 734057263616388449968842366924*e^(98*I*c) + 3104222
 2522074681615625020522*e^(97*I*c) + 1232445557346832245176696904*e^(96*I*c)
 + 45759117183402579073139583*e^(95*I*c) + 1581796642397812408161814*e^(94*
 I*c) + 50648660944512569972179*e^(93*I*c) + 1493326612293984160368*e^(92*I*
 c) + 40261256699368950388*e^(91*I*c) + 984382804329835768*e^(90*I*c) + 2160
 8403021340047*e^(89*I*c) + 420601518659718*e^(88*I*c) + 7146142307307*e^(87
 *I*c) + 103818048048*e^(86*I*c) + 1253841160*e^(85*I*c) + 12085216*e^(84*I*
 c) + 87153*e^(83*I*c) + 418*e^(82*I*c) + e^(81*I*c))*tan(1/4*d*x + c) + 8*
 (9867*a^2*e^(1027/2*I*c) + 3848130*a^2*e^(1025/2*I*c) + 748461285*a^2*e^(10
 23/2*I*c) + 96800992860*a^2*e^(1021/2*I*c) + 9365496059205*a^2*e^(1019/2*I*
 c) + 723016295770626*a^2*e^(1017/2*I*c) + 46393545645312066*a^2*e^(1015/2*I
 *c) + 2545017361124393610*a^2*e^(1013/2*I*c) + 121842706165559066025*a^2*e^
 (1011/2*I*c) + 5171545973004091569115*a^2*e^(1009/2*I*c) + 1970359015888505

19564291*a²*e^(1007/2*I*c) + 6806694783384240157395723*a²*e^(1005/2*I*c)
 + 214978110314064606438970181*a²*e^(1003/2*I*c) + 625090198048881354283481
 1885*a²*e^(1001/2*I*c) + 168327860637922441541183977320*a²*e^(999/2*I*c)
 + 4219418379811180891791273779482*a²*e^(997/2*I*c) + 988926185091494999778
 48839281608*a²*e^(995/2*I*c) + 2175637614774834148860646181993913*a²*e<sup>(9
 93/2*I*c)</sup> + 45084046354894731113654936481368992*a²*e^(991/2*I*c) + 8826981
 76993882170754638941630587545*a²*e^(989/2*I*c) + 1637405134370147041079523
 2052077859600*a²*e^(987/2*I*c) + 288495194182377907657065715223960353253*a
²*e^(985/2*I*c) + 4838851297673493785737825607702894317458*a²*e<sup>(983/2*I*
 c)</sup> + 77421622580729530039877656719012729205746*a²*e^(981/2*I*c) + 11839056
 81496154244790481704841511975812605*a²*e^(979/2*I*c) + 1733237985932470809
 3828663441537720252285600*a²*e^(977/2*I*c) + 24331996022639796544945661797
 8831797573560881*a²*e^(975/2*I*c) + 32803137454391362446300796802177674997
 15269552*a²*e^(973/2*I*c) + 4252692799468040081808204121518060738595837688
 1*a²*e^(971/2*I*c) + 530853429182039270333841634657382740409190158520*a²*
 e^(969/2*I*c) + 6387937037395462163597595720188397188000398246970*a²*e<sup>(96
 7/2*I*c)</sup> + 74182505608058306719063827296003070950486921176536*a²*e<sup>(965/2*
 I*c)</sup> + 832235134528204011410897179273991960185826206648233*a²*e<sup>(963/2*I*c
)</sup> + 9028492210199596456758771830837838651639222089434453*a²*e^(961/2*I*c)
 + 94799192974206012392427186208778348729309030336376975*a²*e^(959/2*I*c) +
 964243521124661527558501799663497136699025931317633435*a²*e^(957/2*I*c) +
 9508516033016066885210212047406599090079103658929512287*a²*e^(955/2*I*c)
 + 90973409644021201619968715710112262222236217410379753753*a²*e<sup>(953/2*I*c
)</sup> + 845095533347055389178362524561818615233275064627309969058*a²*e<sup>(951/2*
 I*c)</sup> + 7627533564356264505268759182677777495879640691370942733882*a²*e<sup>(94
 9/2*I*c)</sup> + 66931654744382785787972355191037205019780646018305304849150*a²*
 e^(947/2*I*c) + 57136825924759622801716432259879886251814562107922931909830
 5*a²*e^(945/2*I*c) + 47478027381094614767945239820058713839348868588872498
 99445376*a²*e^(943/2*I*c) + 3842412093130309269141789305668902719775998715
 4773450642875553*a²*e^(941/2*I*c) + 30302698116271675357927473971686907096
 5523974594833587483988366*a²*e^(939/2*I*c) + 23299442497098459472744224411
 03255660014398695628346348183060015*a²*e^(937/2*I*c) + 1747461166940741239
 8337320259117410186933692726353968273048317520*a²*e^(935/2*I*c) + 12789953
 5070952526512296560088751849450455540802802089156643701480*a²*e<sup>(933/2*I*c
)</sup> + 913950798194359635416152561783528812272662756335205591879244483056*a²*
 e^(931/2*I*c) + 63790197270645716153704815900673911785686800356211277725466
 22549676*a²*e^(929/2*I*c) + 4350504007884677173937543489236737613482035764
 0699616833255652702640*a²*e^(927/2*I*c) + 29003454960130919090411073655297
 0267303386666676203317563284374069900*a²*e^(925/2*I*c) + 18908091613354480
 47190792187685354505856885724920124943223790804192240*a²*e^(923/2*I*c) + 1
 2058418283043222327469841767802495482868812496873297080129187785276220*a²*
 e^(921/2*I*c) + 75253816928609973059371225239594968369651277542390457518108
 615791579592*a²*e^(919/2*I*c) + 459734846505027807935033990026603184232710
 156581030401969227246064835080*a²*e^(917/2*I*c) + 275021590392369377658766
 0311360995938373399507677533387719423983407767820*a²*e^(915/2*I*c) + 16115

407781051635972431360562753503561125657124843101942340833750431379920*a²*e^{-(913/2*I*c)} + 925253600441222539089961836187811384196766999322346250701735
 72656790224220*a²*e^{-(911/2*I*c)} + 5206555249152171385586465256467309791432
 72286503599511142866234900753030800*a²*e^{-(909/2*I*c)} + 2872309730845866743
 857089089793834185768860453051859152512886718432899387580*a²*e^{-(907/2*I*c)}
 + 155388860023750983926223336335851777050039427643419571677926729464905846
 80720*a²*e^{-(905/2*I*c)} + 8245729846693183179384299944105754752673624762543
 0643803799651404729037196200*a²*e^{-(903/2*I*c)} + 42930699545565691552902331
 9383740331787389876258528044568866617782668258091120*a²*e^{-(901/2*I*c)} + 21
 935216361583272147907315638045659778101602433822546371929909840313281201327
 30*a²*e^{-(899/2*I*c)} + 1100152808944845644054007798141013434062534318319264
 6912658929460020565020178560*a²*e^{-(897/2*I*c)} + 54175136420158870759776081
 878762183566234558535160745415613400901305789214741190*a²*e^{-(895/2*I*c)} +
 261986308398648944304761586627798288732821401867463656807224493760103478519
 469580*a²*e^{-(893/2*I*c)} + 124446145774508700568340258135130234074887149326
 8853526051502243461744066743087430*a²*e^{-(891/2*I*c)} + 58076231508882826570
 35726563472056066490098543042659092264931138827252796777795440*a²*e<sup>-(889/2
 *I*c)</sup> + 2663278925925429409999563585453790517239964770438993417533607361049
 4622102197100700*a²*e^{-(887/2*I*c)} + 12003852300760789346999100105954288505
 9361383953817197097130019148476286101342529740*a²*e^{-(885/2*I*c)} + 53185398
 624343902484464820844368189352328112116235379005196349061908702044767847749
 0*a²*e^{-(883/2*I*c)} + 23169230101668992483563829651438816385881286910311693
 42715746114596226696343685707770*a²*e^{-(881/2*I*c)} + 9925571456175007868044
 954512662122157945316385261733877625706791298570735980248218190*a²*e<sup>-(879/
 2*I*c)</sup> + 418214696078086696630592758657604232043406731740420127322863881502
 70004607442363966810*a²*e^{-(877/2*I*c)} + 1733468327028598352541440603456396
 67307060952339166178202747114201132722700081267981130*a²*e^{-(875/2*I*c)} + 7
 069298928172870212173237733840839918767937764495428805163323948981255356030
 91275389910*a²*e^{-(873/2*I*c)} + 2836936413066169670834440433496289616013064
 809877796049713791099298183967813067131443680*a²*e^{-(871/2*I*c)} + 112047661
 995564737603994580880619691142218662882525788135841357339841686118888072797
 04980*a²*e^{-(869/2*I*c)} + 4356134174496143245619429704996145134810103127968
 7284458749849229001908431128476748026080*a²*e^{-(867/2*I*c)} + 16672799000292
 546206639258943073014561649795795917738648698313155883657234229697497895641
 0*a²*e^{-(865/2*I*c)} + 62832810057976380292836471093718342426753895508796825
 0611858220771628940184317648588806640*a²*e^{-(863/2*I*c)} + 23318221269008375
 97004502462198397485462664047234985865673105368390469729222650473360964890*
 a²*e^{-(861/2*I*c)} + 8523030200555698023434125954726868445588548391969654048
 667935620974139552527924073673227600*a²*e^{-(859/2*I*c)} + 306859453525430924
 82702845869501577062965539970222689480264438633453524385996537638863076530*
 a²*e^{-(857/2*I*c)} + 1088397677409163120701510081479079419450467708773817754
 84587130539032295230513374581118741700*a²*e^{-(855/2*I*c)} + 3803581012271403
 286776687618843936964269229517706156850370891479241980521277969846923894572
 20*a²*e^{-(853/2*I*c)} + 1309808166769874221399887733462355768160270298416328
 496442284373072612082291023793255684692690*a²*e^{-(851/2*I*c)} + 444512894567

180347084498712979861252676253425520313764028313429183795321091999797850623
8508560*a²*e^(849/2*I*c) + 14868703654845358167864688325631664019537745751
307912266757843993755842178445925592260911134010*a²*e^(847/2*I*c) + 490256
236905547116371072305475099504134510113008960030655282357330538714407135273
84417816408240*a²*e^(845/2*I*c) + 1593611510437322170785202126279743690083
93404524099003327574913357421657095829547985344818982170*a²*e^(843/2*I*c)
+ 5107377846599813416034090367484253942454172957105621029119977487718777250
69645731797650605229600*a²*e^(841/2*I*c) + 1614044517509692628698926726691
711554483556138938275927980725902246471451300167215224536975484180*a²*e⁽⁸
39/2*I*c) + 503013621424727916492709072244653040032513798349205215250339726
4612305176456664098413388397453600*a²*e^(837/2*I*c) + 15460886627425884508
961370810425256795141491888489128568437523537118033479548555089427027219657
190*a²*e^(835/2*I*c) + 468729362513590454275729496388910119015672957486057
43880702066108160790194634165956243747535959690*a²*e^(833/2*I*c) + 1401795
872733355441214006539649487048219997533207692023158093435179394642227168749
91309588425294410*a²*e^(831/2*I*c) + 4135833571707610586197683750149474481
47331264219567666058810994571662705401249971101083839065381230*a²*e^{(829/2}
*I*c) + 1203917971578157296645777076870863707439458436180383807304624684176
423851533695303170101891807261930*a²*e^(827/2*I*c) + 345800280503438673512
743980082715643826253150767015076447826007342773287946204178151141215935775
6530*a²*e^(825/2*I*c) + 98013653204895775372030846775079420838712495252972
86835383796389305151429890843624056590011755946860*a²*e^(823/2*I*c) + 2741
687674089640072987146520227370040049594794347358669773626581835355652897742
3041810958346380289980*a²*e^(821/2*I*c) + 75693072940552179850368802526700
330728148475697452926304750162140888218008516416199886036847773583360*a²*e^{(819/2}
*I*c) + 206270381908349517479190349156917581993980630701580687347742
161171231054542461656321881524716493128790*a²*e^(817/2*I*c) + 554875148302
949454647011448410809413569126267931881001261866343075854476854621963920232
931497286644220*a²*e^(815/2*I*c) + 147355284549432805785850671058228532674
3293415958953146936295488654657940849117040291880353596346648790*a²*e⁽⁸¹³
/2*I*c) + 38635085916984055535033978460618702236165416022041363178158793702
80975938284468645658557553183035097040*a²*e^(811/2*I*c) + 1000176844449840
683429644028974003088418172797184999180929535566452409870168684292099752489
0661536393450*a²*e^(809/2*I*c) + 25567180322507257120585619430570092050160
262000190487155354567871662013819706851741289684023496304469040*a²*e^{(807/}
2*I*c) + 645404156407607621952393208620254905604210956337221684854156847130
60283455815144499151145622373940380680*a²*e^(805/2*I*c) + 1608995125284599
552313634176347568122610543894330517470874603675829342534787554129773435655
24272784358480*a²*e^(803/2*I*c) + 3961706813251755843197726989013283682929
02791228588983327071374204831719299590027443288379284314668153740*a²*e⁽⁸⁰
1/2*I*c) + 9634842120638867499160128646839175508864558019866476032886941641
87342312853923851439607672602293500951760*a²*e^(799/2*I*c) + 2314573830445
783175085160387552107047974992580875177277771045535577437479474533600582272
734498753096299180*a²*e^(797/2*I*c) + 549276005046531945002759964294515084
1777187786665642466832175421191164012050504122961907062600278172507920*a²*

$e^{(795/2*I*c)} + 12877509084394418602560805734901526225453783498663360455751$
 $206349831628844095133516016644770470613766067100*a^2*e^{(793/2*I*c)} + 298279$
 $066527631966338276448963205582224046466282391780008813641759084206077879129$
 $31438903408724374419711720*a^2*e^{(791/2*I*c)} + 6826387408872526087201389423$
 $029938383273582738291744504824598541069729430934346386249409263476217377315$
 $2680*a^2*e^{(789/2*I*c)} + 15437020316392925587372388710440654412454716753928$
 $9682542500628678342867087924444725771096903602651633852300*a^2*e^{(787/2*I*c)}$
 $) + 34495872849004463043768582576690085274232021127919548089000880129527426$
 $4001677025544610050440796333756569520*a^2*e^{(785/2*I*c)} + 76177770624508294$
 $623879645328180359115118734979167117124078273487340941805400730558031780615$
 $2784530927224220*a^2*e^{(783/2*I*c)} + 16625426837867801308552201362307056147$
 $63601437922169179759292847900888287650392006130085193519006957018065520*a^2$
 $*e^{(781/2*I*c)} + 3586132980406853273009614426136620444168040323250638139426$
 $350707897872734640524997436178532231665691424837180*a^2*e^{(779/2*I*c)} + 764$
 $564799385234166827225700327364483857149625184343788780399195581563364849871$
 $0528739228920550909832076966576*a^2*e^{(777/2*I*c)} + 16112447006542109020247$
 $003405135054882378494352850880906244115186916105715007393020759644056880735$
 $953339819080*a^2*e^{(775/2*I*c)} + 335654655654077755570940729937626333978538$
 $86595211631908837429624044246382645793193352761173849967036968399760*a^2*e^{(773/2*I*c)}$
 $+ 6912453504182182014696086456528580858014840924631827538971192$
 $3925453814611543634853224844727360427634946672345*a^2*e^{(771/2*I*c)} + 14073$
 $570567492355467628694583381812377238846295073495251790642704875900340966924$
 $0929211626086975689710204880250*a^2*e^{(769/2*I*c)} + 28329101961336997036639$
 $304743087996810506918354281673657670517009659134874129651255552716935460589$
 $3957646001303*a^2*e^{(767/2*I*c)} + 56382127901621204478348465744056778170231$
 $6218911354202346521165492358631341504712018051993565471351398126687288*a^2*$
 $e^{(765/2*I*c)} + 11095676030445535703079393328465475622364795469356585529958$
 $42131971428410665480149110838323663977569884246096375*a^2*e^{(763/2*I*c)} + 2$
 $159200924057075467180191415437627790720609282835675418577456503927360772787$
 $168002772269954154839989740930547370*a^2*e^{(761/2*I*c)} + 415510986011327403$
 $170503589556137282335648153836546257479272557010884295901529318522621284728$
 $0262524303114018870*a^2*e^{(759/2*I*c)} + 79076087608708714873833660689307825$
 $065630638433716509724353457270188231791939002127539354084027375839084892131$
 $98*a^2*e^{(757/2*I*c)} + 1488346251411126429985101277704576051398239260014305$
 $7294316507822087646214576864011182721754822647875076768550199*a^2*e^{(755/2*$
 $I*c)} + 27706505969452723903631310117930464578138489499047970896329350811820$
 $143485220527437209835414411792502873777351913*a^2*e^{(753/2*I*c)} + 510153989$
 $005486608142261569731084649051856497902279008167202757732034524561050174159$
 $76545022659656750774297629125*a^2*e^{(751/2*I*c)} + 9291487909477174387799535$
 $715302179402169034520215009539703668384587845708154251890193994158709966117$
 $2792407623865*a^2*e^{(749/2*I*c)} + 16740035531925763745740108220243908100673$
 $1470941354871537824741538495592535526802039060200311512888701307159198971*a$
 $^2*e^{(747/2*I*c)} + 29835771880478208633005298550674856542891830140048933893$
 $4495837264541379197068108175416231067446197128959008560399*a^2*e^{(745/2*I*c)}$
 $) + 52607860805714631324047570892285000882582687083323986398037568444399631$

7216442796531068155624883201334087280959304*a²*e^(743/2*I*c) + 91773894827
 795989476043585040004109958526769938661148648777454150514857025486935447100
 0327559644426920574492105446*a²*e^(741/2*I*c) + 15840379404423307489944270
 504437745228615615603405071870070645985213188214915657332420028631867109019
 19102738577000*a²*e^(739/2*I*c) + 2705282937843409731431305908735900619010
 005098568945719356146443354170715079330805197880740526704832092131340220175
 *a²*e^(737/2*I*c) + 457175964143471174640374586038614176140438528969816086
 5304281801217335925022420235958603301627684925619501359505584*a²*e^(735/2*I*c) + 76454077335312278299702010557632907221000823422073430608178188607874
 84975334278986476092554428384189995796332024399*a²*e^(733/2*I*c) + 1265282
 730763165698017429058901830868115362516610795212492857994684773761996019984
 7091173609876562903119933048840128*a²*e^(731/2*I*c) + 20723705462100228543
 564379129597172262819533630926489919140308482626095452518480544529805188182
 747817476292039529155*a²*e^(729/2*I*c) + 335941399424510451432448355193591
 058330198972721917514886609974044192211585287637126262233769979918846254064
 87226670*a²*e^(727/2*I*c) + 5390132929781224233104901286420525922095599412
 2957966304715639913733138144204227613778706330881783335399502133877518*a²*
 e^(725/2*I*c) + 85604970686858953781664880857907554866698081749894472747266
 687648213896807116363171841984054523474610308890194473531*a²*e^(723/2*I*c)
 + 134581685858500051457166986181305771256444280050023257223423071309453554
 352326832757059740492381058720694830011514768*a²*e^(721/2*I*c) + 209452012
 399444258194985474554170289235130008417571062903247768871514989000136201999
 506880510930373995894675234534215*a²*e^(719/2*I*c) + 322714910338350076746
 045321195011792421222997922965895264817760449140651671258010858297129703013
 615387970090639051520*a²*e^(717/2*I*c) + 492282044972901772475638599526690
 187432070459399637637990089398249675964961459629101488099939978916669693047
 617709543*a²*e^(715/2*I*c) + 743522657800607080438540387933205714505825130
 496084239140637610398839908817375402184750869198424041784284431826566424*a²*
 e^(713/2*I*c) + 111194850512634217145500457794122694896516956834684313495
 9745215920630475645218892480451066290591714849843147073830534*a²*e^(711/2*I*c) + 16466854370605885017917960835237196070683807484425195114163610915376
 95825885067462723229498159344267904597611202663800*a²*e^(709/2*I*c) + 2414
 891369610845294264548570382427882057103318209711416024324623451601940698438
 340504327720639504300166927877463171515*a²*e^(707/2*I*c) + 350728664670642
 619231218092920886850510103296049325432022207871877198417124428106255957304
 5526142805397508833277116491*a²*e^(705/2*I*c) + 50449584211711076387611751
 09467323650323322407477957464622341658119040966151572380873108952981985416
 05380682429000269*a²*e^(703/2*I*c) + 7187581498040295688064148446220796426
 180808096683233325069951704073208274385981090912272509225210297188762948371
 740349*a²*e^(701/2*I*c) + 101431595848015092216767521803238795078141897129
 9689167705933413557282408709556499065914430144489114916973805513194621*a²*
 e^(699/2*I*c) + 1417932866955815411610411486346627030043292825520536426962
 6100393447083705189919178506526410545635418865752499556686695*a²*e^(697/2*I*c) + 19636174483971881970585329590994393900907864647710674920340744073296
 793388281324402127652079005942678317631512574497670*a²*e^(695/2*I*c) + 269

403961217901287886061565729268177099341640059846118345875187123735434057853
59800212612917255715610912957174910777358*a^2*e^(693/2*I*c) + 3662049726136
637159821673050022634628088833521666007643601942347248266940142802940856636
3617139291005506869975456341654*a^2*e^(691/2*I*c) + 49322507203033015848718
132442452234982355078689460811116049412091260094352433061500872862319813121
631930280117212903523*a^2*e^(689/2*I*c) + 658255316595188761240378997316547
324878028071087093012924909023794790403024843180184693613802009619748937572
49507991020*a^2*e^(687/2*I*c) + 8705621754315442453011366540371817721900454
551478457216235415851248559164727884008992060781202408745718082124042499411
5*a^2*e^(685/2*I*c) + 11410100083295356807201357985483750187942435532105902
2690198540574186252466928860820946978421884052442697837445304298870*a^2*e^(
683/2*I*c) + 14821481020003709443684052332377239124351442222882405377877175
8636259259343111776220252287320307540021595261911983440653*a^2*e^(681/2*I*c
) + 19082474334443159943084184294828787580558933493866595429949535396397370
3044612402584139369333134538238828316723719229728*a^2*e^(679/2*I*c) + 24352
714244358175943674120063721720129145802610900762677145555349390136546257397
1992672632802197815055663084346195108560*a^2*e^(677/2*I*c) + 30807649507606
390446089294347868639276042634104789507220232648556940027885933178485287680
6071105563308692408147567281760*a^2*e^(675/2*I*c) + 38636470138438114278945
025361381879810743811311928079429380108947475491729650449068062782437865790
4258492234228073074840*a^2*e^(673/2*I*c) + 48038949687257444314784936219193
350510748888837634566851814946449042746860595193562891663875079535702122494
4593183133920*a^2*e^(671/2*I*c) + 59221121582155122720812669133369249523023
514909184814702609080243651228442553329266678961324164169720509822240474911
3176*a^2*e^(669/2*I*c) + 72389762559065411549118793423227424324484970043671
9045218875364503250870697409035170697796134919768543980827553791633760*a^2*
e^(667/2*I*c) + 87745724727075509631362938051666906796486620034872882921337
2622589781917768639588984441388351412385581993430228315462840*a^2*e^(665/2*
I*c) + 10547623791122394639948136127041258768172551089968129153625310481949
62354831651403688598001758947919324152376469450410640*a^2*e^(663/2*I*c) + 1
257463916593220813694471040548952951921946413668585469617434577903278283592
846903820096009154736822098426718690964820240*a^2*e^(661/2*I*c) + 148690096
311428001845203984190555458170637182894575824603998732024864250611393677000
8380775791193898326549381360036080600*a^2*e^(659/2*I*c) + 17440090819408481
040247316509101887385947524006798259941500269326620180266845163871548976868
76304034521493457700236681760*a^2*e^(657/2*I*c) + 2029231778437026925456547
313961801370821326629541185615597451330228868080901720885109396319507801220
852963035958792120440*a^2*e^(655/2*I*c) + 234244034388498836098397798294706
804034842683077521030340299222338297848848251816641603050314126515046113567
0698762448800*a^2*e^(653/2*I*c) + 26828674636321177250485473843459625648297
032645957641028598692431593679092203021053562971327962992435991181082961286
18040*a^2*e^(651/2*I*c) + 3049059928983217543219860668139335524538572312572
646500219769949634433042293748987411594579022817769730568722664428384160*a^
2*e^(649/2*I*c) + 343885534831037511160604780068832483337725044310356694154
7106368171397969065757184955404763761968717973478236699979978320*a^2*e^(647

$/2*I*c) + 38493869074239649140786476777445579800692189541340363554834972032$
 $51947701126711933295465546306108715552019091156430915680*a^2*e^(645/2*I*c)$
 $+ 4277118991923143907584646099317077382712626367847891663015318009218348769$
 $198119126809909884318912386982880800672549848060*a^2*e^(643/2*I*c) + 471791$
 $489840211675935138588841018808164406294317799148970637671212988777245247171$
 $6509762050150785180119633960611937301360*a^2*e^(641/2*I*c) + 51671360047848$
 $417431063667880187529545003790824296646069984909641361360960054563793348866$
 $22386143712847938577797648414980*a^2*e^(639/2*I*c) + 5619769752253037605696$
 $696087756007874489677081047508410730819825191049119012705663789664394519267$
 $641243087981362175345400*a^2*e^(637/2*I*c) + 607058174771615023970230634160$
 $172208975667617455517280065076442753855272194641854557614434268691578810875$
 $6356634786603780*a^2*e^(635/2*I*c) + 65142853790275937352387261370409802228$
 $683254415713018654499510420418294216628187907119028281903296974877810475582$
 $68403280*a^2*e^(633/2*I*c) + 6945720703138237818770883451264620784314167698$
 $411346710544665328150803572609078033981580941828564416238148863045581355240$
 $*a^2*e^(631/2*I*c) + 736003317033448987532863364682633427819793305957575663$
 $3955078756076566886317380923147551118884485049797192685638405417480*a^2*e^($
 $629/2*I*c) + 77528421143754056310868309820238442440954810616563242172170940$
 $82098098592347362371366056115586902822893825004393033879420*a^2*e^(627/2*I*$
 $c) + 8120388962888200037519047088615628291758404026429768323184937793172741$
 $348652963101033922655233990641205158758808681268860*a^2*e^(625/2*I*c) + 845$
 $965585334398975541548958919239232827581691721354383480368962635586316703506$
 $8688958052034223405284516133993569852906020*a^2*e^(623/2*I*c) + 87684467724$
 $243516706500617180236764284797184607212128463328591041318479299031101539080$
 $57541928127334342584801338549590460*a^2*e^(621/2*I*c) + 9045425408726938507$
 $106759246096468178735211155493083720253628384236827578516908858696948660167$
 $637572946357902815300931660*a^2*e^(619/2*I*c) + 929010650302159366185178110$
 $970376520179958598497492385882478276544507097138240216564822957444896894692$
 $7958434415014105060*a^2*e^(617/2*I*c) + 95028004299287924676676895683426191$
 $876362514863279062475529698356987304751075142590034152157228920643894777778$
 $96889496320*a^2*e^(615/2*I*c) + 9684513845272049894059630211867577805920257$
 $658610389543083420310181521224732382469117018412979994535865598339536803197$
 $080*a^2*e^(613/2*I*c) + 983681225712081330016788734343632112776761649862949$
 $3286164965625770868167956416865940531356033153055126362470897559374080*a^2*$
 $e^(611/2*I*c) + 99616530938130367253472944294584710837298119970125120273308$
 $40564875772300453968349385868405453624240211806875301852978700*a^2*e^(609/2$
 $*I*c) + 1006120003198815017122962983497645714730156321528253859746046862682$
 $3813306618535185930674447792987425681310609393392740960*a^2*e^(607/2*I*c) +$
 $10137630828571123640645947547344391084613029845660390823012151860006583466$
 $309742307221108338379545239032581029125009556300*a^2*e^(605/2*I*c) + 101929$
 $515398096567578770454801700478651479364119141535116491285953947928982427644$
 $99975681645730818164415605760655136116640*a^2*e^(603/2*I*c) + 1022882973811$
 $157962156348283494925404259647301175274555253980467287335356457278576530571$
 $8467559125115045008231345532769180*a^2*e^(601/2*I*c) + 10246458154525926955$
 $681721995657692250321163071754298198436472264795431060278155859430612231580$

414866282905464989775179320*a²*e^(599/2*I*c) + 102464581545259269556817219
 956576922503211630717542981984364722647954310602781558594306122315804148662
 82905464989775179320*a²*e^(597/2*I*c) + 1022882973811157962156348283494925
 404259647301175274555253980467287335356457278576530571846755912511504500823
 1345532769180*a²*e^(595/2*I*c) + 10192951539809656757877045480170047865147
 936411914153511649128595394792898242764499975681645730818164415605760655136
 116640*a²*e^(593/2*I*c) + 101376308285711236406459475473443910846130298456
 60390823012151860006583466309742307221108338379545239032581029125009556300*
 a²*e^(591/2*I*c) + 1006120003198815017122962983497645714730156321528253859
 7460468626823813306618535185930674447792987425681310609393392740960*a²*e⁽
 589/2*I*c) + 99616530938130367253472944294584710837298119970125120273308405
 64875772300453968349385868405453624240211806875301852978700*a²*e^{(587/2*I*}
 c) + 9836812257120813300167887343436321127767616498629493286164965625770868
 167956416865940531356033153055126362470897559374080*a²*e^(585/2*I*c) + 968
 451384527204989405963021186757780592025765861038954308342031018152122473238
 2469117018412979994535865598339536803197080*a²*e^(583/2*I*c) + 95028004299
 287924676676895683426191876362514863279062475529698356987304751075142590034
 1521572289206438947777896889496320*a²*e^(581/2*I*c) + 9290106503021593661
 851781109703765201799585984974923858824782765445070971382402165648229574448
 968946927958434415014105060*a²*e^(579/2*I*c) + 904542540872693850710675924
 609646817873521115549308372025362838423682757851690885869694866016763757294
 6357902815300931660*a²*e^(577/2*I*c) + 87684467724243516706500617180236764
 284797184607212128463328591041318479299031101539080575419281273343425848013
 38549590460*a²*e^(575/2*I*c) + 8459655853343989755415489589192392328275816
 917213543834803689626355863167035068688958052034223405284516133993569852906
 020*a²*e^(573/2*I*c) + 812038896288820003751904708861562829175840402642976
 8323184937793172741348652963101033922655233990641205158758808681268860*a²*
 e^(571/2*I*c) + 77528421143754056310868309820238442440954810616563242172170
 94082098098592347362371366056115586902822893825004393033879420*a²*e^{(569/2}
 *I*c) + 7360033170334489875328633646826334278197933059575756633955078756076
 566886317380923147551118884485049797192685638405417480*a²*e^(567/2*I*c) +
 694572070313823781877088345126462078431416769841134671054466532815080357260
 9078033981580941828564416238148863045581355240*a²*e^(565/2*I*c) + 65142853
 790275937352387261370409802228683254415713018654499510420418294216628187907
 11902828190329697487781047558268403280*a²*e^(563/2*I*c) + 6070581747716150
 239702306341601722089756676174555172800650764427538552721946418545576144342
 686915788108756356634786603780*a²*e^(561/2*I*c) + 561976975225303760569669
 608775600787448967708104750841073081982519104911901270566378966439451926764
 1243087981362175345400*a²*e^(559/2*I*c) + 51671360047848417431063667880187
 529545003790824296646069984909641361360960054563793348866223861437128479385
 77797648414980*a²*e^(557/2*I*c) + 4717914898402116759351385888410188081644
 062943177991489706376712129887772452471716509762050150785180119633960611937
 301360*a²*e^(555/2*I*c) + 427711899192314390758464609931707738271262636784
 7891663015318009218348769198119126809909884318912386982880800672549848060*a²*
 e^(553/2*I*c) + 38493869074239649140786476777445579800692189541340363554

83497203251947701126711933295465546306108715552019091156430915680*a²*e<sup>(55
1/2*I*c)</sup> + 3438855348310375111606047800688324833377250443103566941547106368
171397969065757184955404763761968717973478236699979978320*a²*e^(549/2*I*c)
+ 304905992898321754321986066813933552453857231257264650021976994963443304
2293748987411594579022817769730568722664428384160*a²*e^(547/2*I*c) + 26828
674636321177250485473843459625648297032645957641028598692431593679092203021
05356297132796299243599118108296128618040*a²*e^(545/2*I*c) + 2342440343884
988360983977982947068040348426830775210303402992223382978488482518166416030
503141265150461135670698762448800*a²*e^(543/2*I*c) + 202923177843702692545
654731396180137082132662954118561559745133022886808090172088510939631950780
1220852963035958792120440*a²*e^(541/2*I*c) + 17440090819408481040247316509
101887385947524006798259941500269326620180266845163871548976868763040345214
93457700236681760*a²*e^(539/2*I*c) + 1486900963114280018452039841905554581
706371828945758246039987320248642506113936770008380775791193898326549381360
036080600*a²*e^(537/2*I*c) + 125746391659322081369447104054895295192194641
366858546961743457790327828359284690382009600915473682209842671869096482024
0*a²*e^(535/2*I*c) + 10547623791122394639948136127041258768172551089968129
15362531048194962354831651403688598001758947919324152376469450410640*a²*e<sup>(
533/2*I*c)</sup> + 8774572472707550963136293805166690679648662003487288292133726
22589781917768639588984441388351412385581993430228315462840*a²*e<sup>(531/2*I*
c)</sup> + 7238976255906541154911879342322742432448497004367190452188753645032508
70697409035170697796134919768543980827553791633760*a²*e^(529/2*I*c) + 5922
112158215512272081266913336924952302351490918481470260908024365122844255332
92666789613241641697205098222404749113176*a²*e^(527/2*I*c) + 4803894968725
744431478493621919335051074888883763456685181494644904274686059519356289166
38750795357021224944593183133920*a²*e^(525/2*I*c) + 3863647013843811427894
502536138187981074381131192807942938010894747549172965044906806278243786579
04258492234228073074840*a²*e^(523/2*I*c) + 3080764950760639044608929434786
863927604263410478950722023264855694002788593317848528768060711055633086924
08147567281760*a²*e^(521/2*I*c) + 2435271424435817594367412006372172012914
580261090076267714555534939013654625739719926726328021978150556630843461951
08560*a²*e^(519/2*I*c) + 1908247433444315994308418429482878758055893349386
65954299495353963973703044612402584139369333134538238828316723719229728*a²
*e^(517/2*I*c) + 1482148102000370944368405233237723912435144222288240537787
71758636259259343111776220252287320307540021595261911983440653*a²*e<sup>(515/2
*I*c)</sup> + 1141010008329535680720135798548375018794243553210590226901985405741
86252466928860820946978421884052442697837445304298870*a²*e^(513/2*I*c) + 8
705621754315442453011366540371817721900454551478457216235415851248559164727
8840089920607812024087457180821240424994115*a²*e^(511/2*I*c) + 65825531659
518876124037899731654732487802807108709301292490902379479040302484318018469
361380200961974893757249507991020*a²*e^(509/2*I*c) + 493225072030330158487
181324424522349823550786894608111160494120912600943524330615008728623198131
21631930280117212903523*a²*e^(507/2*I*c) + 3662049726136637159821673050022
634628088833521666007643601942347248266940142802940856636361713929100550686
9975456341654*a²*e^(505/2*I*c) + 26940396121790128788606156572926817709934

164005984611834587518712373543405785359800212612917255715610912957174910777
 358*a²*e^(503/2*I*c) + 196361744839718819705853295909943939009078646477106
 74920340744073296793388281324402127652079005942678317631512574497670*a²*e^(501/2*I*c) + 1417932866955815411610411486346627030043292825520536426962610
 0393447083705189919178506526410545635418865752499556686695*a²*e^(499/2*I*c)
) + 10143159584801509221676752180323879507814189712996891677055933413557282
 408709556499065914430144489114916973805513194621*a²*e^(497/2*I*c) + 718758
 149804029568806414844622079642618080809668323332506995170407320827438598109
 0912272509225210297188762948371740349*a²*e^(495/2*I*c) + 50449584211711076
 387611751094673236503233222407477957464622341658119040966151572380873108952
 98198541605380682429000269*a²*e^(493/2*I*c) + 3507286646706426192312180929
 208868505101032960493254320222078718771984171244281062559573045526142805397
 508833277116491*a²*e^(491/2*I*c) + 241489136961084529426454857038242788205
 710331820971141602432462345160194069843834050432772063950430016692787746317
 1515*a²*e^(489/2*I*c) + 16466854370605885017917960835237196070683807484425
 19511416361091537695825885067462723229498159344267904597611202663800*a²*e^(487/2*I*c) + 1111948505126342171455004577941226948965169568346843134959745
 215920630475645218892480451066290591714849843147073830534*a²*e^(485/2*I*c)
 + 743522657800607080438540387933205714505825130496084239140637610398839908
 817375402184750869198424041784284431826566424*a²*e^(483/2*I*c) + 492282044
 972901772475638599526690187432070459399637637990089398249675964961459629101
 488099939978916669693047617709543*a²*e^(481/2*I*c) + 322714910338350076746
 045321195011792421222997922965895264817760449140651671258010858297129703013
 615387970090639051520*a²*e^(479/2*I*c) + 209452012399444258194985474554170
 289235130008417571062903247768871514989000136201999506880510930373995894675
 234534215*a²*e^(477/2*I*c) + 134581685858500051457166986181305771256444280
 050023257223423071309453554352326832757059740492381058720694830011514768*a²*e^(475/2*I*c) + 856049706868589537816648808579075548666980817498944727472
 66687648213896807116363171841984054523474610308890194473531*a²*e^(473/2*I*c)
 c) + 5390132929781224233104901286420525922095599412295796630471563991373313
 8144204227613778706330881783335399502133877518*a²*e^(471/2*I*c) + 33594139
 942451045143244835519359105833019897272191751488660997404419221158528763712
 626223376997991884625406487226670*a²*e^(469/2*I*c) + 207237054621002285435
 643791295971722628195336309264899191403084826260954525184805445298051881827
 47817476292039529155*a²*e^(467/2*I*c) + 1265282730763165698017429058901830
 868115362516610795212492857994684773761996019984709117360987656290311993304
 8840128*a²*e^(465/2*I*c) + 76454077335312278299702010557632907221000823422
 07343060817818860787484975334278986476092554428384189995796332024399*a²*e^(463/2*I*c) + 4571759641434711746403745860386141761404385289698160865304281
 801217335925022420235958603301627684925619501359505584*a²*e^(461/2*I*c) +
 270528293784340973143130590873590061901000509856894571935614644335417071507
 9330805197880740526704832092131340220175*a²*e^(459/2*I*c) + 15840379404423
 307489944270504437745228615615603405071870070645985213188214915657332420028
 63186710901919102738577000*a²*e^(457/2*I*c) + 9177389482779598947604358504
 00041099585267699386611486487745415051485702548693544710003275596444269205

74492105446*a²*e^(455/2*I*c) + 5260786080571463132404757089228500088258268
70833239863980375684443996317216442796531068155624883201334087280959304*a²
*e^(453/2*I*c) + 2983577188047820863300529855067485654289183014004893389344
95837264541379197068108175416231067446197128959008560399*a²*e^(451/2*I*c)
+ 1674003553192576374574010822024390810067314709413548715378247415384955925
35526802039060200311512888701307159198971*a²*e^(449/2*I*c) + 9291487909477
174387799535715302179402169034520215009539703668384587845708154251890193994
1587099661172792407623865*a²*e^(447/2*I*c) + 51015398900548660814226156973
108464905185649790227900816720275773203452456105017415976545022659656750774
297629125*a²*e^(445/2*I*c) + 277065059694527239036313101179304645781384894
99047970896329350811820143485220527437209835414411792502873777351913*a²*e^(443/2*I*c)
+ 1488346251411126429985101277704576051398239260014305729431650
7822087646214576864011182721754822647875076768550199*a²*e^(441/2*I*c) + 79
076087608708714873833660689307825065630638433716509724353457270188231791939
00212753935408402737583908489213198*a²*e^(439/2*I*c) + 4155109860113274031
705035895561372823356481538365462574792725570108842959015293185226212847280
262524303114018870*a²*e^(437/2*I*c) + 215920092405707546718019141543762779
072060928283567541857745650392736077278716800277226995415483998974093054737
0*a²*e^(435/2*I*c) + 11095676030445535703079393328465475622364795469356585
52995842131971428410665480149110838323663977569884246096375*a²*e^(433/2*I*c)
+ 5638212790162120447834846574405677817023162189113542023465211654923586
31341504712018051993565471351398126687288*a²*e^(431/2*I*c) + 2832910196133
699703663930474308799681050691835428167365767051700965913487412965125555271
69354605893957646001303*a²*e^(429/2*I*c) + 1407357056749235546762869458338
181237723884629507349525179064270487590034096692409292116260869756897102048
80250*a²*e^(427/2*I*c) + 6912453504182182014696086456528580858014840924631
8275389711923925453814611543634853224844727360427634946672345*a²*e^(425/2*I*c)
+ 33565465565407775557094072993762633397853886595211631908837429624044
246382645793193352761173849967036968399760*a²*e^(423/2*I*c) + 161124470065
421090202470034051350548823784943528508809062441151869161057150073930207596
44056880735953339819080*a²*e^(421/2*I*c) + 7645647993852341668272257003273
644838571496251843437887803991955815633648498710528739228920550909832076966
576*a²*e^(419/2*I*c) + 358613298040685327300961442613662044416804032325063
8139426350707897872734640524997436178532231665691424837180*a²*e^(417/2*I*c)
) + 16625426837867801308552201362307056147636014379221691797592928479008882
87650392006130085193519006957018065520*a²*e^(415/2*I*c) + 7617777062450829
462387964532818035911511873497916711712407827348734094180540073055803178061
52784530927224220*a²*e^(413/2*I*c) + 3449587284900446304376858257669008527
42320211279195480890008801295274264001677025544610050440796333756569520*a²
*e^(411/2*I*c) + 1543702031639292558737238871044065441245471675392896825425
00628678342867087924444725771096903602651633852300*a²*e^(409/2*I*c) + 6826
387408872526087201389423029938383273582738291744504824598541069729430934346
3862494092634762173773152680*a²*e^(407/2*I*c) + 29827906652763196633827644
896320558222404646628239178000881364175908420607787912931438903408724374419
711720*a²*e^(405/2*I*c) + 128775090843944186025608057349015262254537834986

63360455751206349831628844095133516016644770470613766067100*a²*e^(403/2*I*c) + 5492760050465319450027599642945150841777187786665642466832175421191164
 012050504122961907062600278172507920*a²*e^(401/2*I*c) + 231457383044578317
 508516038755210704797499258087517727777104553557743747947453360058227273449
 8753096299180*a²*e^(399/2*I*c) + 96348421206388674991601286468391755088645
 5801986647603288694164187342312853923851439607672602293500951760*a²*e^(397/2*I*c) + 39617068132517558431977269890132836829290279122858898332707137420
 4831719299590027443288379284314668153740*a²*e^(395/2*I*c) + 16089951252845
 995523136341763475681226105438943305174708746036758293425347875541297734356
 5524272784358480*a²*e^(393/2*I*c) + 64540415640760762195239320862025490560
 421095633722168485415684713060283455815144499151145622373940380680*a²*e^(391/2*I*c) + 255671803225072571205856194305700920501602620001904871553545678
 71662013819706851741289684023496304469040*a²*e^(389/2*I*c) + 1000176844449
 840683429644028974003088418172797184999180929535566452409870168684292099752
 4890661536393450*a²*e^(387/2*I*c) + 38635085916984055535033978460618702236
 16541602204136317815879370280975938284468645658557553183035097040*a²*e^(385/2*I*c) + 1473552845494328057858506710582285326743293415958953146936295488
 654657940849117040291880353596346648790*a²*e^(383/2*I*c) + 554875148302949
 454647011448410809413569126267931881001261866343075854476854621963920232931
 497286644220*a²*e^(381/2*I*c) + 206270381908349517479190349156917581993980
 630701580687347742161171231054542461656321881524716493128790*a²*e^(379/2*I*c) + 756930729405521798503688025267003307281484756974529263047501621408882
 18008516416199886036847773583360*a²*e^(377/2*I*c) + 2741687674089640072987
 146520227370040049594794347358669773626581835355652897742304181095834638028
 9980*a²*e^(375/2*I*c) + 98013653204895775372030846775079420838712495252972
 86835383796389305151429890843624056590011755946860*a²*e^(373/2*I*c) + 3458
 002805034386735127439800827156438262531507670150764478260073427732879462041
 781511412159357756530*a²*e^(371/2*I*c) + 120391797157815729664577707687086
 3707439458436180383807304624684176423851533695303170101891807261930*a²*e^(369/2*I*c) + 41358335717076105861976837501494744814733126421956766605881099
 4571662705401249971101083839065381230*a²*e^(367/2*I*c) + 14017958727333554
 412140065396494870482199975332076920231580934351793946422271687499130958842
 5294410*a²*e^(365/2*I*c) + 46872936251359045427572949638891011901567295748
 605743880702066108160790194634165956243747535959690*a²*e^(363/2*I*c) + 154
 608866274258845089613708104252567951414918884891285684375235371180334795485
 55089427027219657190*a²*e^(361/2*I*c) + 5030136214247279164927090722446530
 400325137983492052152503397264612305176456664098413388397453600*a²*e^(359/2*I*c) + 161404451750969262869892672669171155448355613893827592798072590224
 6471451300167215224536975484180*a²*e^(357/2*I*c) + 51073778465998134160340
 9036748425394245417295710562102911997748771877725069645731797650605229600*a²*e^(355/2*I*c) + 15936115104373221707852021262797436900839340452409900332
 7574913357421657095829547985344818982170*a²*e^(353/2*I*c) + 49025623690554
 711637107230547509950413451011300896003065528235733053871440713527384417816
 408240*a²*e^(351/2*I*c) + 148687036548453581678646883256316640195377457513
 07912266757843993755842178445925592260911134010*a²*e^(349/2*I*c) + 4445128

945671803470844987129798612526762534255203137640283134291837953210919997978
506238508560*a²*e^(347/2*I*c) + 130980816676987422139988773346235576816027
0298416328496442284373072612082291023793255684692690*a²*e^(345/2*I*c) + 38
035810122714032867766876188439369642692295177061568503708914792419805212779
6984692389457220*a²*e^(343/2*I*c) + 10883976774091631207015100814790794194
5046770877381775484587130539032295230513374581118741700*a²*e^(341/2*I*c) +
30685945352543092482702845869501577062965539970222689480264438633453524385
996537638863076530*a²*e^(339/2*I*c) + 852303020055569802343412595472686844
5588548391969654048667935620974139552527924073673227600*a²*e^(337/2*I*c) +
23318221269008375970045024621983974854626640472349858656731053683904697292
22650473360964890*a²*e^(335/2*I*c) + 6283281005797638029283647109371834242
67538955087968250611858220771628940184317648588806640*a²*e^(333/2*I*c) + 1
667279900029254620663925894307301456164979579591773864869831315588365723422
96974978956410*a²*e^(331/2*I*c) + 4356134174496143245619429704996145134810
1031279687284458749849229001908431128476748026080*a²*e^(329/2*I*c) + 11204
766199556473760399458088061969114221866288252578813584135733984168611888807
279704980*a²*e^(327/2*I*c) + 283693641306616967083444043349628961601306480
9877796049713791099298183967813067131443680*a²*e^(325/2*I*c) + 70692989281
728702121732377338408399187679377644954288051633239489812553560309127538991
0*a²*e^(323/2*I*c) + 17334683270285983525414406034563966730706095233916617
8202747114201132722700081267981130*a²*e^(321/2*I*c) + 41821469607808669663
059275865760423204340673174042012732286388150270004607442363966810*a²*e<sup>(3
19/2*I*c)</sup> + 992557145617500786804495451266212215794531638526173387762570679
1298570735980248218190*a²*e^(317/2*I*c) + 23169230101668992483563829651438
81638588128691031169342715746114596226696343685707770*a²*e^(315/2*I*c) + 5
318539862434390248446482084436818935232811211623537900519634906190870204476
78477490*a²*e^(313/2*I*c) + 1200385230076078934699910010595428850593613839
53817197097130019148476286101342529740*a²*e^(311/2*I*c) + 2663278925925429
4099995635854537905172399647704389934175336073610494622102197100700*a²*e<sup>(
309/2*I*c)</sup> + 58076231508882826570357265634720560664900985430426590922649311
3882725279677795440*a²*e^(307/2*I*c) + 1244461457745087005683402581351302
340748871493268853526051502243461744066743087430*a²*e^(305/2*I*c) + 261986
308398648944304761586627798288732821401867463656807224493760103478519469580
*a²*e^(303/2*I*c) + 541751364201588707597760818787621835662345585351607454
15613400901305789214741190*a²*e^(301/2*I*c) + 1100152808944845644054007798
1410134340625343183192646912658929460020565020178560*a²*e^(299/2*I*c) + 21
935216361583272147907315638045659778101602433822546371929909840313281201327
30*a²*e^(297/2*I*c) + 4293069954556569155290233193837403317873898762585280
44568866617782668258091120*a²*e^(295/2*I*c) + 8245729846693183179384299944
1057547526736247625430643803799651404729037196200*a²*e^(293/2*I*c) + 15538
886002375098392622333633585177705003942764341957167792672946490584680720*a ²*e^(291/2*I*c) + 287230973084586674385708908979383418576886045305185915251
2886718432899387580*a²*e^(289/2*I*c) + 52065552491521713855864652564673097
9143272286503599511142866234900753030800*a²*e^(287/2*I*c) + 92525360044122
253908996183618781138419676699932234625070173572656790224220*a²*e^{(285/2*I}

$*c) + 161154077810516359724313605627535035611256571248431019423408337504313$
 $79920*a^{2*e^{(283/2*I*c)}} + 2750215903923693776587660311360995938373399507677$
 $533387719423983407767820*a^{2*e^{(281/2*I*c)}} + 459734846505027807935033990026$
 $603184232710156581030401969227246064835080*a^{2*e^{(279/2*I*c)}} + 752538169286$
 $09973059371225239594968369651277542390457518108615791579592*a^{2*e^{(277/2*I*$
 $c)}} + 1205841828304322232746984176780249548286881249687329708012918778527622$
 $0*a^{2*e^{(275/2*I*c)}} + 18908091613354480471907921876853545058568857249201249$
 $43223790804192240*a^{2*e^{(273/2*I*c)}} + 2900345496013091909041107365529702673$
 $03386666676203317563284374069900*a^{2*e^{(271/2*I*c)}} + 4350504007884677173937$
 $5434892367376134820357640699616833255652702640*a^{2*e^{(269/2*I*c)}} + 63790197$
 $27064571615370481590067391178568680035621127772546622549676*a^{2*e^{(267/2*I*$
 $c)}} + 913950798194359635416152561783528812272662756335205591879244483056*a^{2$
 $*e^{(265/2*I*c)}} + 1278995350709525265122965600887518494504555408028020891566$
 $43701480*a^{2*e^{(263/2*I*c)}} + 1747461166940741239833732025911741018693369272$
 $6353968273048317520*a^{2*e^{(261/2*I*c)}} + 23299442497098459472744224411032556$
 $60014398695628346348183060015*a^{2*e^{(259/2*I*c)}} + 3030269811627167535792747$
 $39716869070965523974594833587483988366*a^{2*e^{(257/2*I*c)}} + 3842412093130309$
 $2691417893056689027197759987154773450642875553*a^{2*e^{(255/2*I*c)}} + 47478027$
 $38109461476794523982005871383934886858887249899445376*a^{2*e^{(253/2*I*c)}} + 5$
 $71368259247596228017164322598798862518145621079229319098305*a^{2*e^{(251/2*I*$
 $c)}} + 66931654744382785787972355191037205019780646018305304849150*a^{2*e^{(249$
 $/2*I*c)}} + 762753356435626450526875918267777495879640691370942733882*a^{2*e^{(247$
 $/2*I*c)}} + 845095533347055389178362524561818615233275064627309969058*a^{2$
 $*e^{(245/2*I*c)}} + 90973409644021201619968715710112262222236217410379753753*a$
 $^{2*e^{(243/2*I*c)}} + 9508516033016066885210212047406599090079103658929512287*$
 $a^{2*e^{(241/2*I*c)}} + 964243521124661527558501799663497136699025931317633435*$
 $a^{2*e^{(239/2*I*c)}} + 94799192974206012392427186208778348729309030336376975*a$
 $^{2*e^{(237/2*I*c)}} + 90284922101995964567587718308378386516392220894344453*a^{2$
 $*e^{(235/2*I*c)}} + 832235134528204011410897179273991960185826206648233*a^{2*e^{(233$
 $/2*I*c)}} + 74182505608058306719063827296003070950486921176536*a^{2*e^{(231$
 $/2*I*c)}} + 6387937037395462163597595720188397188000398246970*a^{2*e^{(229/2*I*$
 $c)}} + 530853429182039270333841634657382740409190158520*a^{2*e^{(227/2*I*c)}} + 4$
 $2526927994680400818082041215180607385958376881*a^{2*e^{(225/2*I*c)}} + 32803137$
 $45439136244630079680217767499715269552*a^{2*e^{(223/2*I*c)}} + 2433199602263979$
 $65449456617978831797573560881*a^{2*e^{(221/2*I*c)}} + 1733237985932470809382866$
 $3441537720252285600*a^{2*e^{(219/2*I*c)}} + 11839056814961542447904817048415119$
 $75812605*a^{2*e^{(217/2*I*c)}} + 77421622580729530039877656719012729205746*a^{2*$
 $e^{(215/2*I*c)}} + 4838851297673493785737825607702894317458*a^{2*e^{(213/2*I*c)}} +$
 $288495194182377907657065715223960353253*a^{2*e^{(211/2*I*c)}} + 1637405134370$
 $1470410795232052077859600*a^{2*e^{(209/2*I*c)}} + 88269817699388217075463894163$
 $0587545*a^{2*e^{(207/2*I*c)}} + 45084046354894731113654936481368992*a^{2*e^{(205/$
 $2*I*c)}} + 2175637614774834148860646181993913*a^{2*e^{(203/2*I*c)}} + 98892618509$
 $149499977848839281608*a^{2*e^{(201/2*I*c)}} + 4219418379811180891791273779482*a$
 $^{2*e^{(199/2*I*c)}} + 168327860637922441541183977320*a^{2*e^{(197/2*I*c)}} + 62509$
 $01980488813542834811885*a^{2*e^{(195/2*I*c)}} + 214978110314064606438970181*a^{2$

$e^{(193/2*I*c)} + 6806694783384240157395723*a^2*e^{(191/2*I*c)} + 197035901588$
 $850519564291*a^2*e^{(189/2*I*c)} + 5171545973004091569115*a^2*e^{(187/2*I*c)} +$
 $121842706165559066025*a^2*e^{(185/2*I*c)} + 2545017361124393610*a^2*e^{(183/2$
 $*I*c)} + 46393545645312066*a^2*e^{(181/2*I*c)} + 723016295770626*a^2*e^{(179/2*$
 $I*c)} + 9365496059205*a^2*e^{(177/2*I*c)} + 96800992860*a^2*e^{(175/2*I*c)} + 74$
 $8461285*a^2*e^{(173/2*I*c)} + 3848130*a^2*e^{(171/2*I*c)} + 9867*a^2*e^{(169/2*I$
 $*c)))/(e^{(517*I*c)} + 418*e^{(516*I*c)} + 87153*e^{(515*I*c)} + 12085216*e^{(514*I$
 $*c)} + 1253841160*e^{(513*I*c)} + 103818048048*e^{(512*I*c)} + 7146142307307*e^{($
 $511*I*c)} + 420601518659718*e^{(510*I*c)} + 21608403021340047*e^{(509*I*c)} + 98$
 $4382804329835768*e^{(508*I*c)} + 40261256699368950388*e^{(507*I*c)} + 149332661$
 $2293984160368*e^{(506*I*c)} + 50648660944512569972179*e^{(505*I*c)} + 158179664$
 $2397812408161814*e^{(504*I*c)} + 45759117183402579073139583*e^{(503*I*c)} + 123$
 $2445557346832245176696904*e^{(502*I*c)} + 31042222522074681615625020522*e^{(50$
 $1*I*c)} + 734057263616388449968842366924*e^{(500*I*c)} + 163531646471515302405$
 $29137618111*e^{(499*I*c)} + 344277152012875134140739302960914*e^{(498*I*c)} + 6$
 $868329225263681349501997341320517*e^{(497*I*c)} + 130171193079172823835151430$
 $773360024*e^{(496*I*c)} + 2348998374244347079532766203075607598*e^{(495*I*c)} +$
 $40443624781415311581857832389099634564*e^{(494*I*c)} + 665634670676210063754$
 $191847109971141414*e^{(493*I*c)} + 10490402669510897424624643766470754045064*$
 $e^{(492*I*c)} + 158566476113257562566117432227203884298856*e^{(491*I*c)} + 2302$
 $150411226234925855222345201500900533576*e^{(490*I*c)} + 321478876933753388174$
 $54482515377350383950278*e^{(489*I*c)} + 4323336886442615575479441792508004406$
 $04964868*e^{(488*I*c)} + 5605927253067558551780452883689835514455118670*e^{(48$
 $7*I*c)} + 70164515322544462906873548813748091084561870680*e^{(486*I*c)} + 8485$
 $52202276512356496200136959676295361696315113*e^{(485*I*c)} + 9925490738534402$
 $272939987038714580495445431374618*e^{(484*I*c)} + 112391604542246650966429162$
 $063124338952554575234051*e^{(483*I*c)} + 123309670013972336518199722075093259$
 $0655287625342156*e^{(482*I*c)} + 13118781801172174729679339894318153694964675$
 $368481194*e^{(481*I*c)} + 135442594916636116191574650625331646238501101627937$
 $224*e^{(480*I*c)} + 1357990663161479842850642848032544982878359839580349899*e$
 $^{(479*I*c)} + 13231708870104896973800056733779919089340836756009580718*e^{(47$
 $8*I*c)} + 125370496586921272662198050851269323171167338854081782959*e^{(477*I$
 $*c)} + 1155855412893594260345544966642687823630035899363232371472*e^{(476*I*c$
 $)} + 10375184499871175501909398956596684116802997082526660323524*e^{(475*I*c)}$
 $+ 90722605722208814918642284639487187764607589706493970774776*e^{(474*I*c)}$
 $+ 773204636991145775061462731028098506094432675788136295011259*e^{(473*I*c)}$
 $+ 6426195485535248576425068136870465530087114003875716691383902*e^{(472*I*c)}$
 $+ 52108117629177048660492400985175830987505700566877818954141639*e^{(471*I*$
 $c)} + 412430698299915190848067222327219435067747934091894670488982928*e^{(470$
 $*I*c)} + 3187749929744346497211536044751776582320958627923816470590659024*e^{($
 $469*I*c)} + 240708019135297571018580229143720458647469917861820397402743252$
 $64*e^{(468*I*c)} + 1776428291351193485771944376758028302399054600926871364949$
 $61404333*e^{(467*I*c)} + 1281817464914970810859604189828359000790789921169405$
 $304612211251818*e^{(466*I*c)} + 904669352382568297904433896310426316767258682$
 $6367911338826483549173*e^{(465*I*c)} + 62473550781053295317710774690247114124$

125187565731848441781904032672*e^(464*I*c) + 422276126632003687547754746555
 709988710527133086660161366353656787288*e^(463*I*c) + 279470910447568661184
 2790694973699164482254723977210209725661304403472*e^(462*I*c) + 18115768495
 615758076710303055505625589254293659193314153418333944596408*e^(461*I*c) +
 115051481852080848873700388354521315567640365124003103691176697194292320*e[^]
 (460*I*c) + 716099497599058079895633338552940229192858196481597830078819711
 862600096*e^(459*I*c) + 436944248291011391456535313606959586266933885805341
 9381214131241925047008*e^(458*I*c) + 26143976279902021443471945665080254563
 056810183520401889800285493144867448*e^(457*I*c) + 153436088745056254127327
 239461577071933130157764595997113973513183188399376*e^(456*I*c) + 883500968
 821791202600774541927769200737689393513734789368397093333311961880*e^(455*I
 *c) + 499251971245704398350537797660795398839736829759111495799180489368837
 1867680*e^(454*I*c) + 27693116538343259225983382637647936122664033859615133
 489846664694361471028310*e^(453*I*c) + 150822381431412413773566474210011746
 852297437597059186295243989481140398152780*e^(452*I*c) + 806679543607589140
 759305010796189568269842021613388955218916278823182639488190*e^(451*I*c) +
 423812584676323258639418856985868582675532800554862743701930140585132588759
 4480*e^(450*I*c) + 21876482892713909928040345612578705805121508756226696317
 087651824252241418663320*e^(449*I*c) + 110969199687320974749922259595250444
 341219218535349655762591192576535872151766080*e^(448*I*c) + 553269128819528
 612502918869558947829098021956309349843584044631512291778800081490*e^(447*I
 *c) + 271184323967071752760564049014883350713024244840397831852323772194420
 0392830108580*e^(446*I*c) + 13069817203488289886193205508375818392124991382
 340160316886507181296548981014818410*e^(445*I*c) + 619485966530355028795643
 38815234310660410902037882473161804774492916216575880077680*e^(444*I*c) + 2
 888207552647306544699685720210471094273186195089958020206899045903194762954
 08324280*e^(443*I*c) + 1324756412367837473157472821162483691120966501948953
 926492241643788264284546437221120*e^(442*I*c) + 597899217294414321845916114
 9299819706321732111578494525245228742976468409105395536290*e^(441*I*c) + 26
 556806389043407534496702369101545795994861757741414789944652712127566910185
 274123140*e^(440*I*c) + 116104551683555043762911501712116399313733021132677
 481112824047246361794049635726479850*e^(439*I*c) + 499707567253859084357596
 314813794768069337190915967491907488904933922677579665354338960*e^(438*I*c)
 + 211758973346685570710150142921041472240183883794075284161854144088854572
 9943138209036820*e^(437*I*c) + 88367206408604703056945140215479695512967940
 92266983044118375790025854584036796364768280*e^(436*I*c) + 3631836965230259
 1732197444409798122022640824604130552506742586795183267354382847875885730*e[^]
 (435*I*c) + 14703081673227683316304158209959204751204372522535333923881916
 5193000407629544745753221740*e^(434*I*c) + 58640346697268324274164332892156
 0909375197453864243299571990964608857245771134145204174990*e^(433*I*c) + 23
 043510733738403573791785976730663520166827816891398420973766631184888038411
 31935313641840*e^(432*I*c) + 8923209447343296763331881881638471793499618670
 601026059730895962653291770229493028162575100*e^(431*I*c) + 340540538512955
 691543523467221771726551875489107820085047183241687250294385891623492116280
 40*e^(430*I*c) + 1280989146016885396724805418304098477073675004386015368032

04497701119911289087105659482783340*e^(429*I*c) + 4750105788576015192723166
17938425222421786597241671026894318515408511467140969393115768793680*e^(428
*I*c) + 1736574218818191071874197472450158123883564209950658639102337148122
769080611680719741726053840*e^(427*I*c) + 625987215682225284365096070823503
4710201362776057176647226323089751446565288850103898153859920*e^(426*I*c) +
22251959176795777757167366036007480222211364232146399803864370963391491223
687245823457351580140*e^(425*I*c) + 780098073680242398756137330588514171253
27114681070889640794249282633470580756557083923203377160*e^(424*I*c) + 2697
458014402112969726836018638789543579623085200765951771282276292732402152097
08218497363414140*e^(423*I*c) + 9200893930295890328746018500271593226125263
68444771489781974361078847528891468831038436064951920*e^(422*I*c) + 3096131
971621520162380301554241465451782362086810287537748902904985934020179565706
177131421614590*e^(421*I*c) + 102793647306638408447395778624692626046488619
14297972589165243530651230690726244462479199894255180*e^(420*I*c) + 3367539
887202156837590238459398275336255980105810418462734541113626243194324077826
0721756991027090*e^(419*I*c) + 10886799573182947282673290519203488679728462
1356445627530909104429486741257822633476898356826454040*e^(418*I*c) + 34735
147321471378087435208312956660123876576277594236676273334995210388975398263
6403857556867777300*e^(417*I*c) + 10938532144862203586740324345008666784997
70011305874172488975951612031456734608287095519501041975440*e^(416*I*c) + 3
400232560601651617521694680847089844198028831694417424794868779328950548418
125605446882081152636090*e^(415*I*c) + 104341175165703959666536931555824021
09460348095473027807412321427346816928567197770376496170251803940*e^(414*I*
c) + 3161093933128469275069430644361841465609596952094521574300404456038689
5241801579156543451940713351730*e^(413*I*c) + 94556180258931986919334303466
365652826858091314329189160736277175873841732196453379953705679466826880*e^
(412*I*c) + 279285755800035206679835368898165477644864987794665387827488933
863633745047373109049265172681702585720*e^(411*I*c) + 814608187736530579670
210025271921415597183369881214299823291969785549876175969866367976653244974
728560*e^(410*I*c) + 234651821923910514223814163307346476889915570893502577
8047637412681781575765422219127409260159438712250*e^(409*I*c) + 66758662903
711473585037668656692890108935438698305387087249452915809511791882966061581
11257706968604740*e^(408*I*c) + 1875998821886556356416363573598607327825573
7257405706279108891366378428467414559930481172863538598193890*e^(407*I*c) +
52075178518793270386429263351544306951104993542500582938155241689408138675
254608030847907167748571734720*e^(406*I*c) + 142801792450221762483180874918
825274134305133275417780084795034644763509333503150517345864659667189417080
*e^(405*I*c) + 386876218234277165632451723049979889263115282374607541692443
176673997513742813591736171169652250611186480*e^(404*I*c) + 103556198259200
293522638457790861154861211149508019357369133986470602918648246624180566494
9381049856258510*e^(403*I*c) + 2738895624795265603355227646566008862807783
05084825702911938903656162004262736182657700406301914070062380*e^(402*I*c)
+ 7158124686842941475473807363679839718172745581538409044503383852693596921
622426696740453944718143025248390*e^(401*I*c) + 184874052990057326937527286
118764908908583570219748823715706238001862451377226609436417529768529244398

70880*e^(400*I*c) + 4718822084346620769509950695357378035710889749142256789
8048199018207708997005333860148836479527456156014520*e^(399*I*c) + 11904185
540387796494822957794837046560060662318304552952690043020927047321277384779
4935586074714329479939280*e^(398*I*c) + 29682551528266958968531827328023905
0084555032203415941511962659596881615713799937680026497408305672297618840*e
^(397*I*c) + 73158497220681836287472962140397444428001044630116152733976054
4815300951787985538419764656214582667219914080*e^(396*I*c) + 17824461149317
518505563548566384219011744123222982494965916580539397871982465659459755955
75734193348887952160*e^(395*I*c) + 4293206478008022126017488908851826494790
620720660151451468181910917240027863968724539127659633517053002976480*e^(39
4*I*c) + 102231820259548607672173903051864519235621454736742936199180634904
11487496121804590274592702770571515456414680*e^(393*I*c) + 2406878513970527
716119346564450614328524136103776821681892218440014104846021094469664775272
3371932874594597328*e^(392*I*c) + 56028683424903517658495013858534516167162
591034367972498174660907450666778154353271630344650777885683547624184*e^(39
1*I*c) + 128967080084754712246023680866488384983286259025533132044636109049
545144029547003347761521666283977931640178464*e^(390*I*c) + 293550743554342
709808129453576562313299705982699187416862934373964255615967138676253276302
591561523515603264403*e^(389*I*c) + 660764473105869097691475973850837934511
089033149586707982764263394766756649565279879146173318386505740391093990*e^
(388*I*c) + 147093114661893434551503836230010016048212774958144392990474691
0224777470198899052379114493999887003199419829579*e^(387*I*c) + 32384919313
618514764233219335395790983777355392076414673462356658238870483269493056092
31585143748690203615957136*e^(386*I*c) + 7052132414162197992602326524580143
060985353054572933905524633121681021037340298366342203324325307072413739061
024*e^(385*I*c) + 151896342149088003964179117226437547480485201097348124591
09878810493844381062650818971199637121458749456243274416*e^(384*I*c) + 3236
273132241954941033008894364024746037832856131642293129242714590288791307164
3679502909055891236755143207382609*e^(383*I*c) + 68208033096793615683784409
619244210818614991640041553424405527876893272496608324231098148502466453967
157728078994*e^(382*I*c) + 142213115964814517682386667276769909482271681318
790889840501039441748635545362467679832449103520321953011780083069*e^(381*I
*c) + 293344920034300720287042383448342866313806285455040067823080445597545
970023446231563554135133105493516316320059272*e^(380*I*c) + 598650141112241
858911676505180520150364003226841328081453597093587790338609212439085554466
861582623350303061961052*e^(379*I*c) + 12087703584936583930894422205693506
328370410814059375022653984611773764821660955973483160124869827433029615861
2144*e^(378*I*c) + 24149665168103385032890765492027405117100590117954471387
73464205696455026442712426409599662771080264826008985061097*e^(377*I*c) + 4
774141111066098970221845330594962016472714230374234060663956846950926642685
946929064114194400360936223590725470146*e^(376*I*c) + 933934195805349422525
175096571505730070730208381477477030621822424102264824741995604295736305582
3830898547303219757*e^(375*I*c) + 18079820068028859970349938623007230676563
314206708848499900139641237334763266479346963237936039328113185041591793848
*e^(374*I*c) + 346376571726716901676573445371970870488823548539932704720639

43078773600446542963548348101269390443464480754513928502*e^(373*I*c) + 6567
485926886730009882737581287522561065455168626110368166400700753711577809729
3533565243828873383722980353200611956*e^(372*I*c) + 12324394151933238474196
007258810350659640633925361639108206296996068241901174577573892181775339195
4462609323881489157*e^(371*I*c) + 22891131173859278009149264916234683440586
774077645632610841092885725717470728926807434755022579324474192335439530821
4*e^(370*I*c) + 42084634260894938727755902145792458657812096614856102264700
8499529468452005980175119410628956210497609566002969884927*e^(369*I*c) + 76
586779551396278101255844462875141871094089528130479083674366158207165003215
4891482866406314834433199455459798934952*e^(368*I*c) + 13796765297962120740
171061880665894483554465012108901951071648603502289285868155390030628750267
11931941947738690360722*e^(367*I*c) + 2460442375845422663927081630983260714
734968091905493027145639238827192254886349361126991457692409851120873307487
457468*e^(366*I*c) + 434390969660193217335735968778157929370129568194082711
4215433175336093967845908766740738240037114570667410936998017178*e^(365*I*c
) + 75927527001466789611530950735850154731970297465336333315497939614732857
60935801904155116764831560875947581048693527224*e^(364*I*c) + 1313977149410
493388185668115141829311224255152153568687118126657981387760634816026174720
1317735782566021306798298336024*e^(363*I*c) + 22514675741308069961506165586
502872430421930210673264392997286485600640103867253604847715547060592967690
653795951142520*e^(362*I*c) + 381990158675860879760029987566276749947954406
67903625029322346250133286489120875005013638128113893960349670280707161530*
e^(361*I*c) + 6417510069326006680623806488600459717074084330008683936861613
9164529108049844675353111842725798658088840347241496099644*e^(360*I*c) + 10
676483201716559483808523418933352873358767332997253009266108518678993925291
5937090760282232346919090426243399409323314*e^(359*I*c) + 17589625826275598
575710681261397930126580103159548435361490467286516944223207577658044718413
4141375995770091499246759528*e^(358*I*c) + 28699294363123149655727801085157
694089682649746606632752880156067700711283743192673508812097486176051136700
8815728782643*e^(357*I*c) + 46375828845736715454493767825500568873332814556
8049310423995599886012800638619904022368378591108842602342094543682299102*e
^(356*I*c) + 7422864090817312491693704946252561733414891967911827048983100
5497781951221069955839623452499748653124658873553401442137*e^(355*I*c) + 11
766007209757869651898750508902310922046126969702774330145358957889567712307
93520381993106606880564628599822341722801012*e^(354*I*c) + 1847505856462451
533445284300571326323781162553304565971887670758091079306794821834928170773
126364639722071570131703785334*e^(353*I*c) + 287361053592234018708083543558
291227727196797739472015979107027492771427686953146718268898104106138170388
5403497544001592*e^(352*I*c) + 44276730791054253185243161129856936565848519
361001924570444551344833050453214525163471184881332248236704651034839548051
61*e^(351*I*c) + 6758480437888524372562935948963857626694855547195519486122
877567981718587262362871994967079401831957927901682582941234362*e^(350*I*c)
+ 102204237794346348513399752951633996417021222496636661930530083020260969
32158568338309418237395541351819026907953220681013*e^(349*I*c) + 1531283720
666277537934735321280768296571253565294263151828614240309773820027071119539

6582159028513532779682154451996208592*e^(348*I*c) + 22731603566128841100419
 501947051367666836652418077260913944810748473084891890410181285412604854876
 625919565639521227223276*e^(347*I*c) + 334358978279365813011711754596108294
 542981679620174198100729367333785065844280242010724531934581553340466935167
 42390717832*e^(346*I*c) + 4873325350597492340085225556305210140219646931365
 9554492725674754339375283010407167744366955828922837488705858532439654489*e
 ^ (345*I*c) + 70386349760594831567048224061395025698501202296966300376764220
 336697702961591099854055411376294871437468149528524796002762*e^(344*I*c) +
 100744961851853744611754300982980166962404553836222921868484694269966120607
 698907046343731011160948828100276729370132819357*e^(343*I*c) + 142906319123
 055524246546928478954238371315925802022389236498652136839822502035155676970
 917419039834587967055588431566416784*e^(342*I*c) + 200906587153578804380300
 469501441610174521851259541929209840688960859454908519774835905895757666770
 857888611738751858460424*e^(341*I*c) + 279945244475039804822966730462960884
 492119874857791147124009079476920435941735293309305430438687333129912454196
 774070107264*e^(340*I*c) + 386642673050380049457382562818316962651975550990
 779277048740238629858795018247356162888631015687664780101205287333082748791
 *e^(339*I*c) + 529329252764113926003934836958243557672549238997560739214406
 599185047831955572583765358634395408771528009745467548382950094*e^(338*I*c)
 + 718361596382058249209113544487901088868388744033713210332491971375906738
 341551540457264804304039664255915607349801911966551*e^(337*I*c) + 966458275
 369037718747739130798151643483590684166832234688098291164160636418159452119
 815728809372125168836239364442397344064*e^(336*I*c) + 128904351529293395648
 063433049967704018104393562010691426731106790003005839883978769237695409054
 5278554544997710058754772400*e^(335*I*c) + 17045829967078228082046782181676
 930026986611477127723550214565438109300696371880858828247575006052469632108
 10351706405349408*e^(334*I*c) + 2234891276398439464478622578306434840724610
 484468177859822620658691921478645266653062563823553001228001009093606751066
 168944*e^(333*I*c) + 290538572232005700195334527448948279085669252995982374
 9532695963414164833366773128218607899328588608916176593772088622582464*e^(3
 32*I*c) + 37452575948766512046573349884262263881439545019868306642223492263
 61079609546822276067504899386703088982308185717143407211328*e^(331*I*c) + 4
 787527442780945685145204846971596165304169419328244073211459592129649255048
 876854059844720661078151288179612574986359194560*e^(330*I*c) + 606894980315
 671224833187110532989547172280614300887801498655965368726069481655047019589
 0004511965527567432722969707577202160*e^(329*I*c) + 76297318156278215804689
 924242070083664388967363330246618638381051104451489469623282976315470325434
 19811821015837863013682720*e^(328*I*c) + 9513032274019522954209113191268226
 642299912013525665940298381064797885690904993128948035227412144035633851779
 511219335277360*e^(327*I*c) + 117642122748764840800109007146734744933712781
 605578119837244558265660556176580864793686418649081196434124136448037721316
 57280*e^(326*I*c) + 1442981628520843120453297837537569196506315422464974755
 1295851507389524083226976789688601369628399900747658579201929300744260*e^(3
 25*I*c) + 17556273271224292396887291403125716213491486261145478571376751690
 105656067838042151038271381300372757755676325408026834544840*e^(324*I*c) +

211883214058828875396101983747068626958940492260770937641325125133361905239
78949694387686059124526755048042957954264706637460*e^(323*I*c) + 2536717643
911935362153226033598334815490498260612576171130068349296339081649158302570
5268737539982149639300226512657426118880*e^(322*I*c) + 30128482414552703264
559018953088177156013437493438201078413769835448366148121754549197591129967
170764969700180348699207838960*e^(321*I*c) + 355001031060196498762723767969
494822095813723710360050128778060274816728070599434452401363155685007323799
66585005678181937920*e^(320*I*c) + 4149983212196370804378852378740134554178
008893053820691885357902674927336467164003756348860771609288768647154283860
2788559660*e^(319*I*c) + 48133117678184029216503748549110374478924719094635
603892829364863916553792278822957368285106328164715910598370871149079494360
*e^(318*I*c) + 553909130449720862194326891463315660814279598969699002144342
96817731150863867056620768608187679709720152974148474907904177340*e^(317*I*
c) + 6324777410101217905179494607517556992407698133813848315804240674745387
4729387631710544995247152912205118500597511052824347680*e^(316*I*c) + 71660
329861173395524441943889284109134091157844655245672084237402434944696464927
131812190659629511140639501743303863582092880*e^(315*I*c) + 805662491306826
841818762018826235112063637903372180119541102106429277659976449038205954219
36873565314654415769070472655401600*e^(314*I*c) + 8988381580138238221397327
047795460274479287701805196334714630737246431512127492940234794287480289949
9538953561056667668891020*e^(313*I*c) + 99512206472057965951340341738023548
515336403371717898040850470954657532977279113491506880290726111154101941386
019689567958040*e^(312*I*c) + 109332537349966223203932678503426357079863707
001728294011042076530403923862654018978676516417314221089449922495612732870
169660*e^(311*I*c) + 119209713702033927055755397823688444444647424324502185
328626347046599634721146573830681540495333543146776810911910410468628960*e^
(310*I*c) + 128995076011591903410763863427097329948586173574595862705849159
280943046458742663163454018491463855395649453952212899632198680*e^(309*I*c)
+ 138529794549151089451352769576543403126330747243680030832467205895819043
568155239264876762867172754338684027849855385453216080*e^(308*I*c) + 147648
920805545333418623121767853777399782924748301228793924342574999937955421765
370101235122939557467548549202174550009604780*e^(307*I*c) + 156185962953551
196169738218832173696509852551589210730578365727476259476474465955428502336
673743686499175698677875693611243400*e^(306*I*c) + 163977816059607725375264
559816505847894187785101455360391897424482998415385787605765315509208337741
590143078572243505132706580*e^(305*I*c) + 170869848868953101176860306053103
994340530390347260088432676842505555141293830838961275974268928666494845723
462544709102843680*e^(304*I*c) + 176720929970554642004575770053095700595334
659870682732031975915532387577052414866323511140117680492929354517559479899
220940360*e^(303*I*c) + 181408168770922059820368553316697321639984862628298
828569560273295630897626829345263592219034560853530733710529842148537901680
*e^(302*I*c) + 184831151983748941817667850174708257138128172158269413287765
358532240773244336191900818557829905895684494889410451921524212840*e^(301*I
*c) + 186915474436567514926351405623117503261987508351930083824566444435689
139233683411704641828762178799177848064220150818355261280*e^(300*I*c) + 187

615393168510050071497280564603510912403132920312024370835062679037644990286
 285346673507093452964351257962696133511725652320*e^(299*I*c) + 186915474436
 567514926351405623117503261987508351930083824566444435689139233683411704641
 828762178799177848064220150818355261280*e^(298*I*c) + 184831151983748941817
 667850174708257138128172158269413287765358532240773244336191900818557829905
 895684494889410451921524212840*e^(297*I*c) + 181408168770922059820368553316
 697321639984862628298828569560273295630897626829345263592219034560853530733
 710529842148537901680*e^(296*I*c) + 176720929970554642004575770053095700595
 334659870682732031975915532387577052414866323511140117680492929354517559479
 899220940360*e^(295*I*c) + 170869848868953101176860306053103994340530390347
 260088432676842505555141293830838961275974268928666494845723462544709102843
 680*e^(294*I*c) + 163977816059607725375264559816505847894187785101455360391
 897424482998415385787605765315509208337741590143078572243505132706580*e^(29
 3*I*c) + 156185962953551196169738218832173696509852551589210730578365727476
 259476474465955428502336673743686499175698677875693611243400*e^(292*I*c) +
 147648920805545333418623121767853777399782924748301228793924342574999937955
 421765370101235122939557467548549202174550009604780*e^(291*I*c) + 138529794
 549151089451352769576543403126330747243680030832467205895819043568155239264
 876762867172754338684027849855385453216080*e^(290*I*c) + 128995076011591903
 410763863427097329948586173574595862705849159280943046458742663163454018491
 463855395649453952212899632198680*e^(289*I*c) + 119209713702033927055755397
 82368844444647424324502185328626347046599634721146573830681540495333543146
 776810911910410468628960*e^(288*I*c) + 109332537349966223203932678503426357
 079863707001728294011042076530403923862654018978676516417314221089449922495
 612732870169660*e^(287*I*c) + 995122064720579659513403417380235485153364033
 717178980408504709546575329772791134915068802907261111541019413860196895679
 58040*e^(286*I*c) + 8988381580138238221397327047795460274479287701805196334
 7146307372464315121274929402347942874802899499538953561056667668891020*e^(2
 85*I*c) + 80566249130682684181876201882623511206363790337218011954110210642
 927765997644903820595421936873565314654415769070472655401600*e^(284*I*c) +
 716603298611733955244419438892841091340911578446552456720842374024349446964
 64927131812190659629511140639501743303863582092880*e^(283*I*c) + 6324777410
 101217905179494607517556992407698133813848315804240674745387472938763171054
 4995247152912205118500597511052824347680*e^(282*I*c) + 55390913044972086219
 432689146331566081427959896969900214434296817731150863867056620768608187679
 709720152974148474907904177340*e^(281*I*c) + 481331176781840292165037485491
 103744789247190946356038928293648639165537922788229573682851063281647159105
 98370871149079494360*e^(280*I*c) + 4149983212196370804378852378740134554178
 008893053820691885357902674927336467164003756348860771609288768647154283860
 2788559660*e^(279*I*c) + 35500103106019649876272376796949482209581372371036
 005012877806027481672807059943445240136315568500732379966585005678181937920
 *e^(278*I*c) + 301284824145527032645590189530881771560134374934382010784137
 69835448366148121754549197591129967170764969700180348699207838960*e^(277*I*
 c) + 2536717643911935362153226033598334815490498260612576171130068349296339
 0816491583025705268737539982149639300226512657426118880*e^(276*I*c) + 21188

321405882887539610198374706862695894049226077093764132512513336190523978949
694387686059124526755048042957954264706637460*e^(275*I*c) + 175562732712242
923968872914031257162134914862611454785713767516901056560678380421510382713
81300372757755676325408026834544840*e^(274*I*c) + 1442981628520843120453297
837537569196506315422464974755129585150738952408322697678968860136962839990
0747658579201929300744260*e^(273*I*c) + 11764212274876484080010900714673474
493371278160557811983724455826566055617658086479368641864908119643412413644
803772131657280*e^(272*I*c) + 951303227401952295420911319126822664229991201
352566594029838106479788569090499312894803522741214403563385177951121933527
7360*e^(271*I*c) + 76297318156278215804689924242070083664388967363330246618
63838105110445148946962328297631547032543419811821015837863013682720*e^(270
*I*c) + 6068949803156712248331871105329895471722806143008878014986559653687
260694816550470195890004511965527567432722969707577202160*e^(269*I*c) + 478
752744278094568514520484697159616530416941932824407321145959212964925504887
6854059844720661078151288179612574986359194560*e^(268*I*c) + 37452575948766
512046573349884262263881439545019868306642223492263610796095468222760675048
99386703088982308185717143407211328*e^(267*I*c) + 2905385722320057001953345
274489482790856692529959823749532695963414164833366773128218607899328588608
916176593772088622582464*e^(266*I*c) + 223489127639843946447862257830643484
072461048446817785982262065869192147864526665306256382355300122800100909360
6751066168944*e^(265*I*c) + 17045829967078228082046782181676930026986611477
127723550214565438109300696371880858828247575006052469632108103517064053494
08*e^(264*I*c) + 1289043515292933956480634330499677040181043935620106914267
311067900030058398839787692376954090545278554544997710058754772400*e^(263*I
*c) + 966458275369037718747739130798151643483590684166832234688098291164160
636418159452119815728809372125168836239364442397344064*e^(262*I*c) + 718361
596382058249209113544487901088868388744033713210332491971375906738341551540
457264804304039664255915607349801911966551*e^(261*I*c) + 529329252764113926
003934836958243557672549238997560739214406599185047831955572583765358634395
408771528009745467548382950094*e^(260*I*c) + 386642673050380049457382562818
316962651975550990779277048740238629858795018247356162888631015687664780101
205287333082748791*e^(259*I*c) + 279945244475039804822966730462960884492119
874857791147124009079476920435941735293309305430438687333129912454196774070
107264*e^(258*I*c) + 200906587153578804380300469501441610174521851259541929
209840688960859454908519774835905895757666770857888611738751858460424*e^(25
7*I*c) + 142906319123055524246546928478954238371315925802022389236498652136
839822502035155676970917419039834587967055588431566416784*e^(256*I*c) + 100
744961851853744611754300982980166962404553836222921868484694269966120607698
907046343731011160948828100276729370132819357*e^(255*I*c) + 703863497605948
315670482240613950256985012022969663003767642203366977029615910998540554113
76294871437468149528524796002762*e^(254*I*c) + 4873325350597492340085225556
305210140219646931365955449272567475433937528301040716774436695582892283748
8705858532439654489*e^(253*I*c) + 33435897827936581301171175459610829454298
167962017419810072936733378506584428024201072453193458155334046693516742390
717832*e^(252*I*c) + 227316035661288411004195019470513676668366524180772609

13944810748473084891890410181285412604854876625919565639521227223276*e^(251
 *I*c) + 1531283720666277537934735321280768296571253565294263151828614240309
 7738200270711195396582159028513532779682154451996208592*e^(250*I*c) + 10220
 423779434634851339975295163399641702122249663666193053008302026096932158568
 338309418237395541351819026907953220681013*e^(249*I*c) + 675848043788852437
 256293594896385762669485554719551948612287756798171858726236287199496707940
 1831957927901682582941234362*e^(248*I*c) + 44276730791054253185243161129856
 936565848519361001924570444551344833050453214525163471184881332248236704651
 03483954805161*e^(247*I*c) + 2873610535922340187080835435582912277271967977
 394720159791070274927714276869531467182688981041061381703885403497544001592
 *e^(246*I*c) + 184750585646245153344528430057132632378116255330456597188767
 0758091079306794821834928170773126364639722071570131703785334*e^(245*I*c) +
 11766007209757869651898750508902310922046126969702774330145358957889567712
 30793520381993106606880564628599822341722801012*e^(244*I*c) + 7422286409081
 731249169370494625256173341489196791182704898310054977819512210699558396234
 52499748653124658873553401442137*e^(243*I*c) + 4637582884573671545449376782
 550056887333281455680493104239955998860128006386199040223683785911088426023
 42094543682299102*e^(242*I*c) + 2869929436312314965572780108515769408968264
 974660663275288015606770071128374319267350881209748617605113670088157287826
 43*e^(241*I*c) + 1758962582627559857571068126139793012658010315954843536149
 04672865169442232075776580447184134141375995770091499246759528*e^(240*I*c)
 + 1067648320171655948380852341893335287335876733299725300926610851867899392
 52915937090760282232346919090426243399409323314*e^(239*I*c) + 6417510069326
 006680623806488600459717074084330008683936861613916452910804984467535311184
 2725798658088840347241496099644*e^(238*I*c) + 38199015867586087976002998756
 627674994795440667903625029322346250133286489120875005013638128113893960349
 670280707161530*e^(237*I*c) + 225146757413080699615061655865028724304219302
 10673264392997286485600640103867253604847715547060592967690653795951142520*
 e^(236*I*c) + 1313977149410493388185668115141829311224255152153568687118126
 6579813877606348160261747201317735782566021306798298336024*e^(235*I*c) + 75
 927527001466789611530950735850154731970297465336333315497939614732857609358
 01904155116764831560875947581048693527224*e^(234*I*c) + 4343909696601932173
 357359687781579293701295681940827114215433175336093967845908766740738240037
 114570667410936998017178*e^(233*I*c) + 246044237584542266392708163098326071
 473496809190549302714563923882719225488634936112699145769240985112087330748
 7457468*e^(232*I*c) + 13796765297962120740171061880665894483554465012108901
 95107164860350228928586815539003062875026711931941947738690360722*e^(231*I*
 c) + 7658677955139627810125584446287514187109408952813047908367436615820716
 50032154891482866406314834433199455459798934952*e^(230*I*c) + 4208463426089
 493872775590214579245865781209661485610226470084995294684520059801751194106
 28956210497609566002969884927*e^(229*I*c) + 2289113117385927800914926491623
 468344058677407764563261084109288572571747072892680743475502257932447419233
 54395308214*e^(228*I*c) + 1232439415193323847419600725881035065964063392536
 16391082062969960682419011745775738921817753391954462609323881489157*e^(227
 *I*c) + 6567485926886730009882737581287522561065455168626110368166400700753

7115778097293533565243828873383722980353200611956*e^(226*I*c) + 34637657172
671690167657344537197087048882354853993270472063943078773600446542963548348
101269390443464480754513928502*e^(225*I*c) + 180798200680288599703499386230
072306765633142067088484999001396412373347632664793469632379360393281131850
41591793848*e^(224*I*c) + 9339341958053494225251750965715057300707302083814
774770306218224241022648247419956042957363055823830898547303219757*e^(223*I
*c) + 477414111106609897022184533059496201647271423037423406066395684695092
6642685946929064114194400360936223590725470146*e^(222*I*c) + 24149665168103
385032890765492027405117100590117954471387734642056964550264427124264095996
62771080264826008985061097*e^(221*I*c) + 1208770358493658393089442222056935
063283704108140593750226539846117737648216609559734831601248698274330296158
612144*e^(220*I*c) + 598650141112241858911676505180520150364003226841328081
453597093587790338609212439085554466861582623350303061961052*e^(219*I*c) +
293344920034300720287042383448342866313806285455040067823080445597545970023
446231563554135133105493516316320059272*e^(218*I*c) + 142213115964814517682
386667276769909482271681318790889840501039441748635545362467679832449103520
321953011780083069*e^(217*I*c) + 682080330967936156837844096192442108186149
91640041553424405527876893272496608324231098148502466453967157728078994*e^(
216*I*c) + 3236273132241954941033008894364024746037832856131642293129242714
5902887913071643679502909055891236755143207382609*e^(215*I*c) + 15189634214
908800396417911722643754748048520109734812459109878810493844381062650818971
199637121458749456243274416*e^(214*I*c) + 705213241416219799260232652458014
306098535305457293390552463312168102103734029836634220332432530707241373906
1024*e^(213*I*c) + 32384919313618514764233219335395790983777355392076414673
46235665823887048326949305609231585143748690203615957136*e^(212*I*c) + 1470
931146618934345515038362300100160482127749581443929904746910224777470198899
052379114493999887003199419829579*e^(211*I*c) + 660764473105869097691475973
850837934511089033149586707982764263394766756649565279879146173318386505740
391093990*e^(210*I*c) + 293550743554342709808129453576562313299705982699187
416862934373964255615967138676253276302591561523515603264403*e^(209*I*c) +
128967080084754712246023680866488384983286259025533132044636109049545144029
547003347761521666283977931640178464*e^(208*I*c) + 560286834249035176584950
138585345161671625910343679724981746609074506667781543532716303446507778856
83547624184*e^(207*I*c) + 2406878513970527716119346564450614328524136103776
8216818922184400141048460210944696647752723371932874594597328*e^(206*I*c) +
10223182025954860767217390305186451923562145473674293619918063490411487496
121804590274592702770571515456414680*e^(205*I*c) + 429320647800802212601748
890885182649479062072066015145146818191091724002786396872453912765963351705
3002976480*e^(204*I*c) + 17824461149317518505563548566384219011744123222982
49496591658053939787198246565945975595575734193348887952160*e^(203*I*c) + 7
315849722068183628747296214039744442800104463011615273397605448153009517879
85538419764656214582667219914080*e^(202*I*c) + 2968255152826695896853182732
802390500845550322034159415119626595968816157137999376800264974083056722976
18840*e^(201*I*c) + 1190418554038779649482295779483704656006066231830455295
26900430209270473212773847794935586074714329479939280*e^(200*I*c) + 4718822

084346620769509950695357378035710889749142256789804819901820770899700533386
0148836479527456156014520*e^(199*I*c) + 18487405299005732693752728611876490
890858357021974882371570623800186245137722660943641752976852924439870880*e^
(198*I*c) + 715812468684294147547380736367983971817274558153840904450338385
2693596921622426696740453944718143025248390*e^(197*I*c) + 27388956247952656
03355227646566008862807783050848257029119389036561620042627361826577004063
01914070062380*e^(196*I*c) + 1035561982592002935226384577908611548612111495
080193573691339864706029186482466241805664949381049856258510*e^(195*I*c) +
386876218234277165632451723049979889263115282374607541692443176673997513742
813591736171169652250611186480*e^(194*I*c) + 142801792450221762483180874918
825274134305133275417780084795034644763509333503150517345864659667189417080
*e^(193*I*c) + 520751785187932703864292633515443069511049935425005829381552
41689408138675254608030847907167748571734720*e^(192*I*c) + 1875998821886556
356416363573598607327825573725740570627910889136637842846741455993048117286
3538598193890*e^(191*I*c) + 66758662903711473585037668656692890108935438698
30538708724945291580951179188296606158111257706968604740*e^(190*I*c) + 2346
518219239105142238141633073464768899155708935025778047637412681781575765422
219127409260159438712250*e^(189*I*c) + 814608187736530579670210025271921415
597183369881214299823291969785549876175969866367976653244974728560*e^(188*I
*c) + 279285755800035206679835368898165477644864987794665387827488933863633
745047373109049265172681702585720*e^(187*I*c) + 945561802589319869193343034
66365652826858091314329189160736277175873841732196453379953705679466826880*
e^(186*I*c) + 3161093933128469275069430644361841465609596952094521574300404
4560386895241801579156543451940713351730*e^(185*I*c) + 10434117516570395966
653693155582402109460348095473027807412321427346816928567197770376496170251
803940*e^(184*I*c) + 340023256060165161752169468084708984419802883169441742
4794868779328950548418125605446882081152636090*e^(183*I*c) + 10938532144862
203586740324345008666784997700113058741724889759516120314567346082870955195
01041975440*e^(182*I*c) + 3473514732147137808743520831295666012387657627759
42366762733349952103889753982636403857556867777300*e^(181*I*c) + 1088679957
318294728267329051920348867972846213564456275309091044294867412578226334768
98356826454040*e^(180*I*c) + 3367539887202156837590238459398275336255980105
8104184627345411136262431943240778260721756991027090*e^(179*I*c) + 10279364
730663840844739577862469262604648861914297972589165243530651230690726244462
479199894255180*e^(178*I*c) + 309613197162152016238030155424146545178236208
6810287537748902904985934020179565706177131421614590*e^(177*I*c) + 92008939
302958903287460185002715932261252636844477148978197436107884752889146883103
8436064951920*e^(176*I*c) + 26974580144021129697268360186387895435796230852
0076595177128227629273240215209708218497363414140*e^(175*I*c) + 78009807368
024239875613733058851417125327114681070889640794249282633470580756557083923
203377160*e^(174*I*c) + 222519591767957777571673660360074802222113642321463
99803864370963391491223687245823457351580140*e^(173*I*c) + 6259872156822252
843650960708235034710201362776057176647226323089751446565288850103898153859
920*e^(172*I*c) + 173657421881819107187419747245015812388356420995065863910
2337148122769080611680719741726053840*e^(171*I*c) + 47501057885760151927231

6617938425222421786597241671026894318515408511467140969393115768793680*e^(170*I*c) + 12809891460168853967248054183040984770736750043860153680320449770
 1119911289087105659482783340*e^(169*I*c) + 34054053851295569154352346722177
 172655187548910782008504718324168725029438589162349211628040*e^(168*I*c) +
 892320944734329676333188188163847179349961867060102605973089596265329177022
 9493028162575100*e^(167*I*c) + 23043510733738403573791785976730663520166827
 81689139842097376663118488803841131935313641840*e^(166*I*c) + 5864034669726
 832427416433289215609093751974538642432995719909646088572457711341452041749
 90*e^(165*I*c) + 1470308167322768331630415820995920475120437252253533392388
 19165193000407629544745753221740*e^(164*I*c) + 3631836965230259173219744440
 9798122022640824604130552506742586795183267354382847875885730*e^(163*I*c) +
 88367206408604703056945140215479695512967940922669830441183757900258545840
 36796364768280*e^(162*I*c) + 2117589733466855707101501429210414722401838837
 940752841618541440888545729943138209036820*e^(161*I*c) + 499707567253859084
 357596314813794768069337190915967491907488904933922677579665354338960*e^(160*I*c) +
 116104551683555043762911501712116399313733021132677481112824047246
 361794049635726479850*e^(159*I*c) + 265568063890434075344967023691015457959
 94861757741414789944652712127566910185274123140*e^(158*I*c) + 5978992172944
 143218459161149299819706321732111578494525245228742976468409105395536290*e^(157*I*c) +
 132475641236783747315747282116248369112096650194895392649224164
 3788264284546437221120*e^(156*I*c) + 28882075526473065446996857202104710942
 7318619508995802020689904590319476295408324280*e^(155*I*c) + 61948596653035
 502879564338815234310660410902037882473161804774492916216575880077680*e^(154*I*c) +
 130698172034882898861932055083758183921249913823401603168865071812
 96548981014818410*e^(153*I*c) + 2711843239670717527605640490148833507130242
 448403978318523237721944200392830108580*e^(152*I*c) + 553269128819528612502
 918869558947829098021956309349843584044631512291778800081490*e^(151*I*c) +
 110969199687320974749922259595250444341219218535349655762591192576535872151
 766080*e^(150*I*c) + 218764828927139099280403456125787058051215087562266963
 17087651824252241418663320*e^(149*I*c) + 4238125846763232586394188569858685
 826755328005548627437019301405851325887594480*e^(148*I*c) + 806679543607589
 140759305010796189568269842021613388955218916278823182639488190*e^(147*I*c)
 + 150822381431412413773566474210011746852297437597059186295243989481140398
 152780*e^(146*I*c) + 276931165383432592259833826376479361226640338596151334
 89846664694361471028310*e^(145*I*c) + 4992519712457043983505377976607953988
 397368297591114957991804893688371867680*e^(144*I*c) + 883500968821791202600
 774541927769200737689393513734789368397093333311961880*e^(143*I*c) + 153436
 088745056254127327239461577071933130157764595997113973513183188399376*e^(142*I*c) +
 261439762799020214434719456650802545630568101835204018898002854931
 44867448*e^(141*I*c) + 4369442482910113914565353136069595862669338858053419
 381214131241925047008*e^(140*I*c) + 716099497599058079895633338552940229192
 858196481597830078819711862600096*e^(139*I*c) + 115051481852080848873700388
 354521315567640365124003103691176697194292320*e^(138*I*c) + 181157684956157
 58076710303055505625589254293659193314153418333944596408*e^(137*I*c) + 2794
 709104475686611842790694973699164482254723977210209725661304403472*e^(136*I

$*c) + 422276126632003687547754746555709988710527133086660161366353656787288$
 $*e^{(135*I*c)} + 624735507810532953177107746902471141241251875657318484417819$
 $04032672*e^{(134*I*c)} + 9046693523825682979044338963104263167672586826367911$
 $338826483549173*e^{(133*I*c)} + 128181746491497081085960418982835900079078992$
 $1169405304612211251818*e^{(132*I*c)} + 17764282913511934857719443767580283023$
 $9905460092687136494961404333*e^{(131*I*c)} + 24070801913529757101858022914372$
 $045864746991786182039740274325264*e^{(130*I*c)} + 318774992974434649721153604$
 $4751776582320958627923816470590659024*e^{(129*I*c)} + 41243069829991519084806$
 $7222327219435067747934091894670488982928*e^{(128*I*c)} + 52108117629177048660$
 $492400985175830987505700566877818954141639*e^{(127*I*c)} + 642619548553524857$
 $6425068136870465530087114003875716691383902*e^{(126*I*c)} + 77320463699114577$
 $5061462731028098506094432675788136295011259*e^{(125*I*c)} + 90722605722208814$
 $918642284639487187764607589706493970774776*e^{(124*I*c)} + 103751844998711755$
 $01909398956596684116802997082526660323524*e^{(123*I*c)} + 1155855412893594260$
 $345544966642687823630035899363232371472*e^{(122*I*c)} + 125370496586921272662$
 $198050851269323171167338854081782959*e^{(121*I*c)} + 132317088701048969738000$
 $56733779919089340836756009580718*e^{(120*I*c)} + 1357990663161479842850642848$
 $032544982878359839580349899*e^{(119*I*c)} + 135442594916636116191574650625331$
 $646238501101627937224*e^{(118*I*c)} + 131187818011721747296793398943181536949$
 $64675368481194*e^{(117*I*c)} + 1233096700139723365181997220750932590655287625$
 $342156*e^{(116*I*c)} + 112391604542246650966429162063124338952554575234051*e^{(115*I*c)}$
 $+ 9925490738534402272939987038714580495445431374618*e^{(114*I*c)} +$
 $848552202276512356496200136959676295361696315113*e^{(113*I*c)} + 70164515322$
 $544462906873548813748091084561870680*e^{(112*I*c)} + 560592725306755855178045$
 $2883689835514455118670*e^{(111*I*c)} + 43233368864426155754794417925080044060$
 $4964868*e^{(110*I*c)} + 32147887693375338817454482515377350383950278*e^{(109*I$
 $*c)} + 2302150411226234925855222345201500900533576*e^{(108*I*c)} + 15856647611$
 $3257562566117432227203884298856*e^{(107*I*c)} + 10490402669510897424624643766$
 $470754045064*e^{(106*I*c)} + 665634670676210063754191847109971141414*e^{(105*I$
 $*c)} + 40443624781415311581857832389099634564*e^{(104*I*c)} + 2348998374244347$
 $079532766203075607598*e^{(103*I*c)} + 130171193079172823835151430773360024*e^{(102*I*c)}$
 $+ 6868329225263681349501997341320517*e^{(101*I*c)} + 34427715201287$
 $5134140739302960914*e^{(100*I*c)} + 16353164647151530240529137618111*e^{(99*I*$
 $c)} + 734057263616388449968842366924*e^{(98*I*c)} + 31042222522074681615625020$
 $522*e^{(97*I*c)} + 1232445557346832245176696904*e^{(96*I*c)} + 4575911718340257$
 $9073139583*e^{(95*I*c)} + 1581796642397812408161814*e^{(94*I*c)} + 506486609445$
 $12569972179*e^{(93*I*c)} + 1493326612293984160368*e^{(92*I*c)} + 40261256699368$
 $950388*e^{(91*I*c)} + 984382804329835768*e^{(90*I*c)} + 21608403021340047*e^{(89$
 $*I*c)} + 420601518659718*e^{(88*I*c)} + 7146142307307*e^{(87*I*c)} + 10381804804$
 $8*e^{(86*I*c)} + 1253841160*e^{(85*I*c)} + 12085216*e^{(84*I*c)} + 87153*e^{(83*I*$
 $c)} + 418*e^{(82*I*c)} + e^{(81*I*c)}) * \tan(1/4*d*x + c) + 7*(9867*I*a^2*e^{(1027$
 $/2*I*c)} + 3848130*I*a^2*e^{(1025/2*I*c)} + 748461285*I*a^2*e^{(1023/2*I*c)} + 9$
 $6800992860*I*a^2*e^{(1021/2*I*c)} + 9365496059205*I*a^2*e^{(1019/2*I*c)} + 7230$
 $16295770626*I*a^2*e^{(1017/2*I*c)} + 46393545645311706*I*a^2*e^{(1015/2*I*c)} +$
 $2545017361124253210*I*a^2*e^{(1013/2*I*c)} + 121842706165531758225*I*a^2*e^{($

$1011/2 * I * c) + 5171545973000559760165 * I * a^2 * e^{(1009/2 * I * c)} + 197035901588508$
 $817004391 * I * a^2 * e^{(1007/2 * I * c)} + 6806694783357860712909393 * I * a^2 * e^{(1005/2 * I * c)}$
 $+ 214978110312371924676280101 * I * a^2 * e^{(1003/2 * I * c)} + 62509019803959577$
 $95917828835 * I * a^2 * e^{(1001/2 * I * c)} + 168327860633476968482282779620 * I * a^2 * e^{(999/2 * I * c)}$
 $+ 4219418379622495018746735933182 * I * a^2 * e^{(997/2 * I * c)} + 98892618$
 $501960556395579843057828 * I * a^2 * e^{(995/2 * I * c)} + 2175637614526488309339795247$
 $719423 * I * a^2 * e^{(993/2 * I * c)} + 45084046347051121561291339131297112 * I * a^2 * e^{(991/2 * I * c)}$
 $+ 882698176765813429616327000431223295 * I * a^2 * e^{(989/2 * I * c)} + 1637$
 $4051337559882202075446312226584700 * I * a^2 * e^{(987/2 * I * c)} + 288495194028428088$
 $627568321559245818459 * I * a^2 * e^{(985/2 * I * c)} + 4838851294065276461984778740285$
 $520947658 * I * a^2 * e^{(983/2 * I * c)} + 77421622501348280247334235191908463309686 * I$
 $* a^2 * e^{(981/2 * I * c)} + 1183905679851187219496284451985830369733725 * I * a^2 * e^{(979/2 * I * c)}$
 $+ 17332379827117737513384038195005765450047540 * I * a^2 * e^{(977/2 * I * c)}$
 $) + 243319959628953467711826163624006704408771129 * I * a^2 * e^{(975/2 * I * c)} + 328$
 $031373491263039080453203905338555426092600 * I * a^2 * e^{(973/2 * I * c)} + 425269278$
 $18120262032236340231948484360039052121 * I * a^2 * e^{(971/2 * I * c)} + 53085342635704$
 $2754469653748150767198919032400940 * I * a^2 * e^{(969/2 * I * c)} + 638793699419597624$
 $0622682990166245342613055517890 * I * a^2 * e^{(967/2 * I * c)} + 741825049756084502896$
 $36265368281851382711220649004 * I * a^2 * e^{(965/2 * I * c)} + 83223512564943621722271$
 $5021488460783355491981492217 * I * a^2 * e^{(963/2 * I * c)} + 902849209049851352193143$
 $1245530169366369034596324891 * I * a^2 * e^{(961/2 * I * c)} + 947991914223366709117332$
 $84155867076030325977221341875 * I * a^2 * e^{(959/2 * I * c)} + 96424350175263900614602$
 $6780418602842158889630673989345 * I * a^2 * e^{(957/2 * I * c)} + 950851579990067311329$
 $9221003726853644110325215605885183 * I * a^2 * e^{(955/2 * I * c)} + 909734069368068375$
 $31915685400953309110574252721276020367 * I * a^2 * e^{(953/2 * I * c)} + 84509550297467$
 $071459803322459068895202594191429069360222 * I * a^2 * e^{(951/2 * I * c)} + 762753323$
 $4852264511705859368402201230654030433440301675518 * I * a^2 * e^{(949/2 * I * c)} + 669$
 $31651284481661864618077305876269408561834596408636101210 * I * a^2 * e^{(947/2 * I * c)}$
 $) + 571368224054265219726596099705196924252108483077143367419935 * I * a^2 * e^{(945/2 * I * c)}$
 $+ 4747802391051472283183879516945405395337420791155799564552344 * I$
 $* a^2 * e^{(943/2 * I * c)} + 384241176106726559069364415899325808086312933461942154$
 $93375887 * I * a^2 * e^{(941/2 * I * c)} + 30302695031451807921916988663946921311252966$
 $2619496065070362634 * I * a^2 * e^{(939/2 * I * c)} + 232994397127263459706866547956038$
 $3652668004620643027277203439985 * I * a^2 * e^{(937/2 * I * c)} + 174746092260042943466$
 $64529284651937762925604102208554097442050560 * I * a^2 * e^{(935/2 * I * c)} + 12789951$
 $4211571855615342081558322168204136482117422337757176345560 * I * a^2 * e^{(933/2 * I$
 $* c)} + 913950624853444779964988596342201399080981813136131167642386677984 * I$
 $a^2 * e^{(931/2 * I * c)} + 6379018324131616220652747415710905929685302237372129456$
 $527463165524 * I * a^2 * e^{(929/2 * I * c)} + 4350502901412913680756638831801360916381$
 $5986027077804343349110468080 * I * a^2 * e^{(927/2 * I * c)} + 290034464520387036801727$
 $848589402597956276490946161717533961649595700 * I * a^2 * e^{(925/2 * I * c)} + 1890808$
 $523184465234870582011548986898368367578058256415408172123086240 * I * a^2 * e^{(92$
 $3/2 * I * c)} + 1205841361197921501449380888109491932707613615895877460346501392$
 $2951460 * I * a^2 * e^{(921/2 * I * c)} + 752537835473769580225687459869037752652911784$
 $25521737345338921401401608 * I * a^2 * e^{(919/2 * I * c)} + 45973461349852590439166145$

3323499339021127870927849907055606144578837880*I*a^2*e^(917/2*I*c) + 275021
4314681421164420565775956331599212851388641230567209095240832869740*I*a^2*e
^(915/2*I*c) + 161153971851311832617849990680934829583309265371195480965690
91610170990080*I*a^2*e^(913/2*I*c) + 92525290960177968452743510383284189038
302875808137991118047531970267330140*I*a^2*e^(911/2*I*c) + 5206550842970712
13247874630919114875777150736266962384814615937366841583600*I*a^2*e^(909/2*
I*c) + 28723069807654857867980935279056703943163003114948799193702926019941
64873980*I*a^2*e^(907/2*I*c) + 15538869199972063791690049288524847158110979
530065672277863507395517286098560*I*a^2*e^(905/2*I*c) + 8245719794050006511
2831235778009337400577942437329270541936889611136248616360*I*a^2*e^(903/2*I
*c) + 429306406331604823645688251632236356131380665568461698695435155472754
991584480*I*a^2*e^(901/2*I*c) + 2193518253323607158598306760610194505563715
717648737978249164699044796790178970*I*a^2*e^(899/2*I*c) + 1100150905116991
8803406190301110072169140434739188390680687069371600096456461960*I*a^2*e^(8
97/2*I*c) + 541750313768436076293560450270587580326308933852341883428638684
14550489892855190*I*a^2*e^(895/2*I*c) + 26198574004344308038830187488450068
3905172501509414794896626641791046018314193300*I*a^2*e^(893/2*I*c) + 124445
844130409520749732783777934477929343261005269896448132005897390334813136439
0*I*a^2*e^(891/2*I*c) + 580760744355231221024154298187369701154404717090556
070374986664890336325145752760*I*a^2*e^(889/2*I*c) + 266327089901343246991
99881524590925879444427734597911842625776697111614072462805500*I*a^2*e^(887
/2*I*c) + 12003812035094402972261579848458357881792093610304289847907519508
4456374447649531420*I*a^2*e^(885/2*I*c) + 531852003070453189333276138739985
307207445323441994433993246593537700600251640914290*I*a^2*e^(883/2*I*c) + 2
316913417896533634148238479533816756801158698905671195529018686134366565369
411066790*I*a^2*e^(881/2*I*c) + 9925525882909518673569536731354735994317981
948112305002260477305616767418585528998710*I*a^2*e^(879/2*I*c) + 4182125688
366367760446381399035690630569232443207281247902039184020973840599286234211
0*I*a^2*e^(877/2*I*c) + 173345856975272080307468888319573743826748157949817
083968790709575178522586695155600570*I*a^2*e^(875/2*I*c) + 7069254940580017
38008498868730775412018273649883656316519821960251293960111842853827050*I*a
^2*e^(873/2*I*c) + 28369169189394771245705015350036679506624416854595869868
59765033838975136651432973013080*I*a^2*e^(871/2*I*c) + 11204681256258812161
206109553481424902816807388805631619385616166895520536176153385410620*I*a^2
*e^(869/2*I*c) + 4356097775980642550984171697832990426607531388027684362352
7207449861917293070562965518680*I*a^2*e^(867/2*I*c) + 166726455941596831160
623476351712587187594657642857127013830239484587313767680019339582550*I*a^2
*e^(865/2*I*c) + 6283217402529699146708974013748477212590031785572960403994
51247917033448679858171068226560*I*a^2*e^(863/2*I*c) + 23317961812286764803
58713874846040638691309670968182309524450889214672550181351830378735030*I*a
^2*e^(861/2*I*c) + 85229260483699889403559152526282219875527530950209513770
15329635773840197997200581857597800*I*a^2*e^(859/2*I*c) + 30685533864596666
418316194015078029015958251712763047637572948112622799853423865147299102830
*I*a^2*e^(857/2*I*c) + 1088381674585823184455849835282493949200583590094814
25820011589540335936121859362203566362580*I*a^2*e^(855/2*I*c) + 38035197418

842683642619864051339471341649460785516551807259419090622333733050401816525
8288940*I*a^2*e^(853/2*I*c) + 130978506840473201225164579044912481278616850
6645450981272566942196706295180922249225707977010*I*a^2*e^(851/2*I*c) + 444
504319274789268390399853721138227585710412235922782253365493896371639852064
3447212245479640*I*a^2*e^(849/2*I*c) + 148683901006420511836719711793878800
03509146350007294634177899645088810313885195635392146744810*I*a^2*e^(847/2*
I*c) + 49024494337492279360818849127945363095454529548782519402705804194570
228377733166934824088655040*I*a^2*e^(845/2*I*c) + 1593571437033253029099589
86971024895010865105951720378260497368508593055838453825165049611563210*I*a
^2*e^(843/2*I*c) + 51072377438312247374259033908340395357586419424344520899
4753073600039579929266407037097651682920*I*a^2*e^(841/2*I*c) + 161399625005
227651497742238751482974339782549553805362337258500737267515812737919919603
8765006020*I*a^2*e^(839/2*I*c) + 502997233223228520128531881185601119191768
6754755796153099081679265022946497934343332346116036200*I*a^2*e^(837/2*I*c)
+ 154603381888035181010707962987414730510504499628085456326772817431996854
97754720467992010224616870*I*a^2*e^(835/2*I*c) + 46871127019956579394527723
949157745550097315795915660019622180811547871642339809799054651854945990*I*
a^2*e^(833/2*I*c) + 1401737032027635410158205056407283867183742759160410960
78183091845413305288205080342342220046629650*I*a^2*e^(831/2*I*c) + 41356448
914361269513294728809292139787765990212707157864657906237914001989850609848
6550619425904170*I*a^2*e^(829/2*I*c) + 120385831118903001859978475898311457
4101276553583968965776551061463170435426403483179176840497456330*I*a^2*e^(8
27/2*I*c) + 345781676701999202030946295835138625644973659314096946827987732
7753792198494989629433408106651901070*I*a^2*e^(825/2*I*c) + 980079316064566
075944389460984891152942557969087433011532356902754100547673537917564006396
1886333700*I*a^2*e^(823/2*I*c) + 274151410348454439510949392567509545844369
45867670907710352859492970463590965208109494412332329295780*I*a^2*e^(821/2*
I*c) + 75687878722014265825238066731591013702998100464324442217272752916887
964266256104313931049007060490840*I*a^2*e^(819/2*I*c) + 2062550466642352923
299538505218323676805884853392747978586557766976702788671365670139537869241
77858890*I*a^2*e^(817/2*I*c) + 55483047707198724665440844480803020948751532
5400605772819119408435792744646306336768559494858757362460*I*a^2*e^(815/2*I
*c) + 147342444362109962377181703120951524343672463702574952726767868577035
5293167282547379994041612249536410*I*a^2*e^(813/2*I*c) + 386314437516868018
576356913292077827044669797836884278496544413991685735826342800126438356631
1006996680*I*a^2*e^(811/2*I*c) + 100007488425814211531157828500284191306438
69569449134721967908251115915010593127137857232140041620869750*I*a^2*e^(809
/2*I*c) + 25564363098285969951461675934273364734821923616250047177527687812
898786857371780193438857338260097911840*I*a^2*e^(807/2*I*c) + 6453273197782
459147936919163715825933528964953360698297163136496405491168107563712249071
4561514235891960*I*a^2*e^(805/2*I*c) + 160878825079043394603834949575837497
625484370509752204556389520870042820153281247817248349563478924960000*I*a^2
*e^(803/2*I*c) + 3961156927720317702883047917185117312913684062974398434356
50750372236248577797783021338331228243750622260*I*a^2*e^(801/2*I*c) + 96333
990133367005220184783649172964795326224863928850083854162444757923207857140

8017358275408623510613840*I*a^2*e^(799/2*I*c) + 231419987522566986824406965
158487639596366333938136712176597016461942695849656811645654666240906844984
0980*I*a^2*e^(797/2*I*c) + 549180314668753988053597656596379567216451408656
7208012015916026775639438732336258201804419618292545032320*I*a^2*e^(795/2*I
*c) + 128750909685704951104826884973311195514817251151404663543711650415329
25663730313264282055462011603010654340*I*a^2*e^(793/2*I*c) + 29821871642675
642979884949769331439027860315600536250652213455370080842219764548099437011
716544543041268840*I*a^2*e^(791/2*I*c) + 6824899753441688984057474417232328
4868436001711555565766981650470803874668337015803960506067102299819184920*I
*a^2*e^(789/2*I*c) + 154333980446647526545603393435717981675167976622985430
365188253022405140750956390615095093479110699588649900*I*a^2*e^(787/2*I*c)
+ 3448716031646138840025055585349184630908375817774593116788420671232864662
26751144071133482776005430489425760*I*a^2*e^(785/2*I*c) + 76157068141016866
276950629713249017136732146629432296112698385566984533398678337103331546320
0057428630752220*I*a^2*e^(783/2*I*c) + 166205667443073414924892879946898077
3731456543701211070556181213389924055988353226635860540433754964155111760*I
*a^2*e^(781/2*I*c) + 358500567897480551605849967163259551339357495711398033
6688954791609736165974043249124955091096872997954793020*I*a^2*e^(779/2*I*c)
+ 764306432142215085107655604542055474413839848522722800024684820236501089
1675711435238630995867297844214073376*I*a^2*e^(777/2*I*c) + 161065955340044
253505563455084677640638169912602410083615367290222210493676806273016116090
63584801970379704200*I*a^2*e^(775/2*I*c) + 33552369185708773069292076239890
437482410490941602744957472544576489334415521169836149006133511757140035934
080*I*a^2*e^(773/2*I*c) + 6909556671227223197243223972889949617492222077204
1984182069284130237389188998302806443944988509065063273154745*I*a^2*e^(771/
2*I*c) + 140672375713012687320389073579945533379367132007412356558650081388
469939218158614778502467163652527862550952130*I*a^2*e^(769/2*I*c) + 2831541
724454077422450878787538662877042027538054395604149601789061369618915729280
26855276213887121380071928103*I*a^2*e^(767/2*I*c) + 56352897805763272096991
770897155119738849844002918752049866812562736063687371997384272904938397339
5339489187328*I*a^2*e^(765/2*I*c) + 110895041237395917205685994545909717581
6002074457775812197101199430878771371764877187949694685819718286221686935*I
*a^2*e^(763/2*I*c) + 215791258592459333372869589796444940936417263128653468
4550661677201976202007165851929905826385985844876371677490*I*a^2*e^(761/2*I
*c) + 415245104359965804460096900332121399173917602775258904180672803665826
3221899875572009537787169343651573979683470*I*a^2*e^(759/2*I*c) + 790218349
026871901495271289147853538925976831795670198761660396945228812301836985196
7117174545759607142995908398*I*a^2*e^(757/2*I*c) + 148725165132286726845844
543739529709855752443668839234988632043147587872364853959678331468690644611
37897493283199*I*a^2*e^(755/2*I*c) + 27684667755883080696165241600850236290
060527593753750250976925795644297762324894086602505304941640081834882024183
*I*a^2*e^(753/2*I*c) + 5097231322824610485284563027499470633447420185121230
0701137205093695356754098637104271608613774511930964105781185*I*a^2*e^(751/
2*I*c) + 928308113376605414018314259982460592969619502620219411889553255934
74604810499041616621349581953895002857873462955*I*a^2*e^(749/2*I*c) + 16723

812436760488425768739599809266279592755168930304104087967332950476166342923
7202212197395803292702881738952971*I*a^2*e^(747/2*I*c) + 298048067652053494
891789544111943306401708032136890389702630757761900213043371027166375667728
550358023100922727809*I*a^2*e^(745/2*I*c) + 5254939879412245768339489479231
903447058938212674523878231430709537350535886875076613143155582097677080194
78373004*I*a^2*e^(743/2*I*c) + 91664709431284914960140940755641154562802047
7301452007254604091888065378474579372960004944710940275456326641248866*I*a^
2*e^(741/2*I*c) + 158202062787581988673250065898762286534898007216280070113
0868690683669873547179234420581808071126591692483785355020*I*a^2*e^(739/2*I
*c) + 270159546995359250505395026372937903793888340345141599522931084329547
5921159044641522890199450389533264819628536025*I*a^2*e^(737/2*I*c) + 456509
066991059164837134779602647589554670218072151783528080907829796090749188178
0744446079560657601476760082232152*I*a^2*e^(735/2*I*c) + 763347349130754120
948642458581148481719625483226104049697195217999448279842521515446999904354
7371029896595546249849*I*a^2*e^(733/2*I*c) + 126316940344948323880522866518
063344172714663818375801285677304841070409088315250448980118919620659240685
50189725268*I*a^2*e^(731/2*I*c) + 20686671000021215968489830509311959418929
183582885216898379036970362110971705665038962014129218543899794341899845085
*I*a^2*e^(729/2*I*c) + 3352990888500682268293154976719628483086219012183584
1481201620772726893006422670068438075620985878626866861887157590*I*a^2*e^(7
27/2*I*c) + 537910694584998904260122923727248823170880556157524683581982581
28613020164914352269555347240301885068470565731734762*I*a^2*e^(725/2*I*c) +
85417618996417490452483371059823103161327605370266736083574364508696427592
725387814713600331423646619680648052044475*I*a^2*e^(723/2*I*c) + 1342665470
057644847983235502917892846730721653892879442933623480246710269254851376046
45521152950533194618205062920988*I*a^2*e^(721/2*I*c) + 20892721947321952554
456586633742979365256651844601362539197681055721328802433993015254954416800
2925624951052677530015*I*a^2*e^(719/2*I*c) + 321849633692522447898702348324
001010813625753491002068478331883454364145448222725180431547920764933095415
393034588280*I*a^2*e^(717/2*I*c) + 4908693639020633756913108220724697291424
820101761094624078736203613063207730728775323705679226406182002274641275020
47*I*a^2*e^(715/2*I*c) + 74123864476887296086214581322988427643686096723451
1356976656011429732646982384201977263394830039748273745918763926276*I*a^2*e
^(713/2*I*c) + 110829117874288088003949255863546681669181130286674928871555
5797951407508808250708041669875191682735463589899659311198*I*a^2*e^(711/2*I
*c) + 164088464461734084358144268349244158832586560261315262569217345595590
0863555908606458185685041555583786399895054479300*I*a^2*e^(709/2*I*c) + 240
577714439055714117813463067773244849650952054131834842019241780734238128978
4524672636122635696383857297613950049995*I*a^2*e^(707/2*I*c) + 349309887710
492687698237108032257440212635206127339745745586202481213457760260596583915
0066211359905342050250374117429*I*a^2*e^(705/2*I*c) + 502307443186015730117
888471656258174869062121735387850334388091202949158080461043749678159891960
5840393968558559789161*I*a^2*e^(703/2*I*c) + 715412996672345730067340322771
834762634518685264638245830985114944269590079262756662236828728226218438070
7453078413991*I*a^2*e^(701/2*I*c) + 100924786020063127364353407717370680634

544851192575840230472208882160787043159191976578217878926464077020912493173
03229*I*a^2*e^(699/2*I*c) + 14103212089664651841721385690411495309067974792
132984765752121078864973955276665921709606990805136055899318515316302625*I*
a^2*e^(697/2*I*c) + 1952283328672880331025077467311303310215892334738969639
2450556335874949686526249214568717002120285949198419943388647210*I*a^2*e^(6
95/2*I*c) + 267730394333998274117237784491077008799869706368351802130848455
66895069119080487323194332830887134412610587215162894762*I*a^2*e^(693/2*I*c
) + 36375409931287795092607557958784126662059576747454408588744309722082226
791392845209172297521166997828928030420844023706*I*a^2*e^(691/2*I*c) + 4896
646615878742114946199207175567373336856102462570532830088204696011609350991
3400520500730507564724324855396317174877*I*a^2*e^(689/2*I*c) + 653123602074
898065236571363863403753569295522760923767167642985299999533078080423695668
05167379162746143255282258954180*I*a^2*e^(687/2*I*c) + 86322223651326073619
711312568551997796065517198873894958063303051564972850936480127357633838565
811305177606785202742045*I*a^2*e^(685/2*I*c) + 1130589802883564220385835139
940220907469386070355433465119796591093989261756839556423349718766282964364
95288367475025210*I*a^2*e^(683/2*I*c) + 14674621624473683474604774174976439
522622270730367507440206801324051044685464674982020418217223234451702743626
0129735987*I*a^2*e^(681/2*I*c) + 188769528710907069399315417787360030712701
629061238948251068174174311765598652218110198035428223021692947533585962348
672*I*a^2*e^(679/2*I*c) + 2406706654362267812491048707637320723073729459838
41816912761139211706324807127596469266879477538714994876050630848911280*I*a
^2*e^(677/2*I*c) + 30413274660453666225979217509555939007344516655283930371
3131735564868948526939032146448843731486395171091096793798821440*I*a^2*e^(6
75/2*I*c) + 380954953495487600292955504287703099567666198405494548188510783
430234085264993237349222897613346201432380448747964425320*I*a^2*e^(673/2*I*
c) + 4730152954843393553865160495107424893668782016894758200570340501140953
23134007835807393143129309159477656152691755564640*I*a^2*e^(671/2*I*c) + 58
222044318581750206266482921312370263252473314620783459971606488506578927759
0829360246460349955947394345054182441981224*I*a^2*e^(669/2*I*c) + 710442146
558732233592026068873812784196608427957534833584445957207723692820312997979
854292270461155374405948197659591360*I*a^2*e^(667/2*I*c) + 8594404367040659
964920655309206627501878563640804789340222353178580696799516488891727895957
06006814219368010815532681480*I*a^2*e^(665/2*I*c) + 10307746197765427581510
267219540856089966611268658301355106800499653101376641414311549979794966392
99345832099582375161360*I*a^2*e^(663/2*I*c) + 12257039776528262791156252189
566022445511144475924060653629422220740852110017150381818536465061870450336
00876523451824880*I*a^2*e^(661/2*I*c) + 14450810458031668137922646890379738
661662028815384253123290214013013908892842545655439493113783095417234329028
67052706200*I*a^2*e^(659/2*I*c) + 16892412852670881120714741703490094181054
317802273206939347842482291303136287068760205450304355332464459610710684938
10560*I*a^2*e^(657/2*I*c) + 19578938349723584779059289990954195564057119589
88853024935075157754339419656153188117144093625692196108598042281972985720*
I*a^2*e^(655/2*I*c) + 22500187506812894018744676525628914872033265257098990
95523732366806542477845141215360711007405272102207680817427742759520*I*a^2*

$e^{(653/2*I*c)} + 25637766364399777030296045669829556685953809601601333538899$
 $74265239086029092155773519578351942456055091409678154086303160*I*a^2*e^{(651$
 $/2*I*c)} + 28964367894366249384702990683309462580013099699847038734114728697$
 $37850504382055706419008342552332959944508775701784058240*I*a^2*e^{(649/2*I*c$
 $)} + 32443298586951092651428287113773193999451013284950304274169031315033190$
 $22305572237606283736559593675791505521680740151120*I*a^2*e^{(647/2*I*c)} + 36$
 $028305298407529565181939312745233961149440319768254370614299760903082455016$
 $32173258267215963247319966703044426706223680*I*a^2*e^{(645/2*I*c)} + 39663748$
 $833127834823836467088030196693430004700156593977777724916107199512517044773$
 $98336756202715565781802459813301802300*I*a^2*e^{(643/2*I*c)} + 43285158489790$
 $258856890544749443686885978562125957587107456315867170589408091655084408030$
 $25209005356313475936021116327680*I*a^2*e^{(641/2*I*c)} + 46820186401352114025$
 $441934476753157785221638924891221314114838076383675741536713397080122815979$
 $05204951327596457959200420*I*a^2*e^{(639/2*I*c)} + 50189962637768623023679837$
 $160841205582895837859254979735993461981724426689831143392606930153662124944$
 $49899447639497797000*I*a^2*e^{(637/2*I*c)} + 53310832629653276988259859512126$
 $816741393962441161710231957252053274044936955416162439524334709419918835207$
 $91988803340740*I*a^2*e^{(635/2*I*c)} + 56096438581971821615416482750117525255$
 $863897331137996079555574670366614762459888018770659346581194758350501024011$
 $03094880*I*a^2*e^{(633/2*I*c)} + 58460087337172550342558283443647869505776357$
 $487028017821034693281056146781069583693352772019734245822730028395671983190$
 $80*I*a^2*e^{(631/2*I*c)} + 60317329778848122601739294534272327997929452223082$
 $03442804835486204161004827827142401343622232793155002865274651684289320*I*a$
 $^2*e^{(629/2*I*c)} + 61588662421482231694403486393227657084883018978611167674$
 $04404815878289922328995812629986104827051163389294175358746873020*I*a^2*e^{($
 $627/2*I*c)} + 62202251230952204460675300839023987684115557416549006389920446$
 $46455129937854804113318919111668831147496834175450404249540*I*a^2*e^{(625/2*$
 $I*c)} + 62096571660597904142679194148630869282244387047293181546722854365503$
 $56208263215568679904005246104294284315747952200822100*I*a^2*e^{(623/2*I*c)} +$
 $61222857792337143725906948970182085956642006810708599861702745415177526058$
 $91024516685624452334324327807799672288064082260*I*a^2*e^{(621/2*I*c)} + 59547$
 $257467401870889396732031512898770826022020504370086760285116178608639386975$
 $14177299610327811386342958225508703594540*I*a^2*e^{(619/2*I*c)} + 57052599204$
 $286374939405242600597907420683319525389105353847625378502538234011482704337$
 $83507943120942720400689766599967260*I*a^2*e^{(617/2*I*c)} + 53739690083650980$
 $637055279707516518897459636880832855309819571261998752476166051260272728509$
 $91193295122275535796115520400*I*a^2*e^{(615/2*I*c)} + 49628080947727873397470$
 $915152302769798740073949347947431638629075496398001768246061001785549327778$
 $62617713413141739317320*I*a^2*e^{(613/2*I*c)} + 44756255352895649726018393399$
 $777580104484971180819586082522773687502909742536065396671192491726068766591$
 $81697229192662160*I*a^2*e^{(611/2*I*c)} + 39181220751115193662441268842368235$
 $396535288066011412234288694406185747436395037219120791405561621855099478991$
 $64696134100*I*a^2*e^{(609/2*I*c)} + 32977503331460697218967225115618795701660$
 $403555551597360783353756943479513821935596151070573301875361374620878666425$
 $54560*I*a^2*e^{(607/2*I*c)} + 26235570810040910406795079057453769837412326290$

56220311681059841157626795431350015349450602987501509129301018716050208660*
 $I*a^2*e^{(605/2*I*c)}$ + 19059729259480474002004174356752213830765610370725372
 67851490979096509936249772956388635120192992958448628890777139946480*I*a^2*
 $e^{(603/2*I*c)}$ + 11565559961838689852000906392260895540213419321581926219232
 46957977122810089035413564479870427981333952548182360473741220*I*a^2*e^{(601
 /2*I*c)} + 38769795201545572867941822935804912844942383041966842808188286623
 7658365339074239012664180518005805128629710867652733080*I*a^2*e^{(599/2*I*c)}
 - 387697952015455728679418229358049128449423830419668428081882866237658365
 339074239012664180518005805128629710867652733080*I*a^2*e^{(597/2*I*c)} - 1156
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 413564479870427981333952548182360473741220*I*a^2*e^{(595/2*I*c)} - 1905972925
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 635120192992958448628890777139946480*I*a^2*e^{(593/2*I*c)} - 2623557081004091
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 72251156187957016604035555159736078335375694347951382193559615107057330187
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 229192662160*I*a^2*e^{(585/2*I*c)} - 4962808094772787339747091515230276979874
 007394934794743163862907549639800176824606100178554932777862617713413141739
 317320*I*a^2*e^{(583/2*I*c)} - 5373969008365098063705527970751651889745963688
 083285530981957126199875247616605126027272850991193295122275535796115520400
 *I*a^2*e^{(581/2*I*c)} - 5705259920428637493940524260059790742068331952538910
 535384762537850253823401148270433783507943120942720400689766599967260*I*a^2
 *e^{(579/2*I*c)} - 5954725746740187088939673203151289877082602202050437008676
 028511617860863938697514177299610327811386342958225508703594540*I*a^2*e^{(57
 7/2*I*c)} - 6122285779233714372590694897018208595664200681070859986170274541
 517752605891024516685624452334324327807799672288064082260*I*a^2*e^{(575/2*I*
 c)} - 6209657166059790414267919414863086928224438704729318154672285436550356
 208263215568679904005246104294284315747952200822100*I*a^2*e^{(573/2*I*c)} - 6
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 242148223169440348639322765708488301897861116767404404815878289922328995812
 629986104827051163389294175358746873020*I*a^2*e^{(569/2*I*c)} - 6031732977884
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 275011752525586389733113799607955557467036661476245988801877065934658119475
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497797000*I*a^2*e^(559/2*I*c) - 4682018640135211402544193447675315778522163
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420*I*a^2*e^(557/2*I*c) - 4328515848979025885689054474944368688597856212595
758710745631586717058940809165508440803025209005356313475936021116327680*I*
a^2*e^(555/2*I*c) - 3966374883312783482383646708803019669343000470015659397
777772491610719951251704477398336756202715565781802459813301802300*I*a^2*e^
(553/2*I*c) - 3602830529840752956518193931274523396114944031976825437061429
976090308245501632173258267215963247319966703044426706223680*I*a^2*e^(551/2
*I*c) - 3244329858695109265142828711377319399945101328495030427416903131503
319022305572237606283736559593675791505521680740151120*I*a^2*e^(549/2*I*c)
- 2896436789436624938470299068330946258001309969984703873411472869737850504
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776636439977703029604566982955668595380960160133353889974265239086029092155
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523451824880*I*a^2*e^(535/2*I*c) - 1030774619776542758151026721954085608996
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161360*I*a^2*e^(533/2*I*c) - 8594404367040659964920655309206627501878563640
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I*a^2*e^(531/2*I*c) - 71044214655873223359202606887381278419660842795753483
3584445957207723692820312997979854292270461155374405948197659591360*I*a^2*e
(529/2*I*c) - 582220443185817502062664829213123702632524733146207834599716
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*I*c) - 4730152954843393553865160495107424893668782016894758200570340501140
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4993237349222897613346201432380448747964425320*I*a^2*e^(523/2*I*c) - 304132
746604536662259792175095559390073445166552839303713131735564868948526939032
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267812491048707637320723073729459838418169127611392117063248071275964692668
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931541778736003071270162906123894825106817417431176559865221811019803542822
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74976439522622707303675074402068013240510446854646749820204182172232344517
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907469386070355433465119796591093989261756839556423349718766282964364952883
67475025210*I*a^2*e^(513/2*I*c) - 86322223651326073619711312568551997796065

517198873894958063303051564972850936480127357633838565811305177606785202742
045*I*a^2*e^(511/2*I*c) - 6531236020748980652365713638634037535692955227609
2376716764298529999953307808042369566805167379162746143255282258954180*I*a^
2*e^(509/2*I*c) - 489664661587874211494619920717556737333685610246257053283
00882046960116093509913400520500730507564724324855396317174877*I*a^2*e^(507
/2*I*c) - 36375409931287795092607557958784126662059576747454408588744309722
082226791392845209172297521166997828928030420844023706*I*a^2*e^(505/2*I*c)
- 2677303943339982741172377844910770087998697063683518021308484556689506911
9080487323194332830887134412610587215162894762*I*a^2*e^(503/2*I*c) - 195228
332867288033102507746731130331021589233473896963924505563358749496865262492
14568717002120285949198419943388647210*I*a^2*e^(501/2*I*c) - 14103212089664
651841721385690411495309067974792132984765752121078864973955276665921709606
990805136055899318515316302625*I*a^2*e^(499/2*I*c) - 1009247860200631273643
534077173706806345448511925758402304722088821607870431591919765782178789264
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834762634518685264638245830985114944269590079262756662236828728226218438070
7453078413991*I*a^2*e^(495/2*I*c) - 502307443186015730117888471656258174869
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9161*I*a^2*e^(493/2*I*c) - 349309887710492687698237108032257440212635206127
3397457455862024812134577602605965839150066211359905342050250374117429*I*a^
2*e^(491/2*I*c) - 240577714439055714117813463067773244849650952054131834842
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2*I*c) - 164088464461734084358144268349244158832586560261315262569217345595
5900863555908606458185685041555583786399895054479300*I*a^2*e^(487/2*I*c) -
110829117874288088003949255863546681669181130286674928871555579795140750880
8250708041669875191682735463589899659311198*I*a^2*e^(485/2*I*c) - 741238644
768872960862145813229884276436860967234511356976656011429732646982384201977
263394830039748273745918763926276*I*a^2*e^(483/2*I*c) - 4908693639020633756
913108220724697291424820101761094624078736203613063207730728775323705679226
40618200227464127502047*I*a^2*e^(481/2*I*c) - 32184963369252244789870234832
400101081362575349100206847833188345436414544822272518043154792076493309541
5393034588280*I*a^2*e^(479/2*I*c) - 208927219473219525544565866337429793652
566518446013625391976810557213288024339930152549544168002925624951052677530
015*I*a^2*e^(477/2*I*c) - 1342665470057644847983235502917892846730721653892
87944293362348024671026925485137604645521152950533194618205062920988*I*a^2*
e^(475/2*I*c) - 85417618996417490452483371059823103161327605370266736083574
364508696427592725387814713600331423646619680648052044475*I*a^2*e^(473/2*I*
c) - 5379106945849989042601229237272488231708805561575246835819825812861302
0164914352269555347240301885068470565731734762*I*a^2*e^(471/2*I*c) - 335299
088850068226829315497671962848308621901218358414812016207727268930064226700
68438075620985878626866861887157590*I*a^2*e^(469/2*I*c) - 20686671000021215
968489830509311959418929183582885216898379036970362110971705665038962014129
218543899794341899845085*I*a^2*e^(467/2*I*c) - 1263169403449483238805228665
180633441727146638183758012856773048410704090883152504489801189196206592406
8550189725268*I*a^2*e^(465/2*I*c) - 763347349130754120948642458581148481719

625483226104049697195217999448279842521515446999904354737102989659554624984
9*I*a^2*e^(463/2*I*c) - 456509066991059164837134779602647589554670218072151
7835280809078297960907491881780744446079560657601476760082232152*I*a^2*e^(4
61/2*I*c) - 270159546995359250505395026372937903793888340345141599522931084
3295475921159044641522890199450389533264819628536025*I*a^2*e^(459/2*I*c) -
158202062787581988673250065898762286534898007216280070113086869068366987354
7179234420581808071126591692483785355020*I*a^2*e^(457/2*I*c) - 916647094312
849149601409407556411545628020477301452007254604091888065378474579372960004
944710940275456326641248866*I*a^2*e^(455/2*I*c) - 5254939879412245768339489
479231903447058938212674523878231430709537350535886875076613143155582097677
08019478373004*I*a^2*e^(453/2*I*c) - 29804806765205349489178954411194330640
170803213689038970263075776190021304337102716637566772855035802310092272780
9*I*a^2*e^(451/2*I*c) - 167238124367604884257687395998092662795927551689303
041040879673329504761663429237202212197395803292702881738952971*I*a^2*e^(44
9/2*I*c) - 9283081133766054140183142599824605929696195026202194118895532559
3474604810499041616621349581953895002857873462955*I*a^2*e^(447/2*I*c) - 509
723132282461048528456302749947063344742018512123007011372050936953567540986
37104271608613774511930964105781185*I*a^2*e^(445/2*I*c) - 27684667755883080
696165241600850236290060527593753750250976925795644297762324894086602505304
941640081834882024183*I*a^2*e^(443/2*I*c) - 1487251651322867268458445437395
297098557524436688392349886320431475878723648539596783314686906446113789749
3283199*I*a^2*e^(441/2*I*c) - 790218349026871901495271289147853538925976831
7956701987616603969452288123018369851967117174545759607142995908398*I*a^2*e
(439/2*I*c) - 415245104359965804460096900332121399173917602775258904180672
8036658263221899875572009537787169343651573979683470*I*a^2*e^(437/2*I*c) -
21579125859245933372869589796444940936417263128653468455066167720197620200
7165851929905826385985844876371677490*I*a^2*e^(435/2*I*c) - 110895041237395
91720568599454590971758160020744577581219710119943087877137176487718794969
4685819718286221686935*I*a^2*e^(433/2*I*c) - 563528978057632720969917708971
551197388498440029187520498668125627360636873719973842729049383973395339489
187328*I*a^2*e^(431/2*I*c) - 2831541724454077422450878787538662877042027538
05439560414960178906136961891572928026855276213887121380071928103*I*a^2*e^(
429/2*I*c) - 14067237571301268732038907357994553337936713200741235655865008
1388469939218158614778502467163652527862550952130*I*a^2*e^(427/2*I*c) - 690
955667122722319724322397288994961749222207720419841820692841302373891889983
02806443944988509065063273154745*I*a^2*e^(425/2*I*c) - 33552369185708773069
292076239890437482410490941602744957472544576489334415521169836149006133511
757140035934080*I*a^2*e^(423/2*I*c) - 1610659553400442535055634550846776406
381699126024100836153672902221049367680627301611609063584801970379704200*I
*a^2*e^(421/2*I*c) - 764306432142215085107655604542055474413839848522722800
0246848202365010891675711435238630995867297844214073376*I*a^2*e^(419/2*I*c)
- 358500567897480551605849967163259551339357495711398033668895479160973616
5974043249124955091096872997954793020*I*a^2*e^(417/2*I*c) - 166205667443073
414924892879946898077373145654370121107055618121338992405598835322663586054
0433754964155111760*I*a^2*e^(415/2*I*c) - 761570681410168662769506297132490

171367321466294322961126983855669845333986783371033315463200057428630752220
 $*I*a^2*e^{(413/2*I*c)}$ - 3448716031646138840025055585349184630908375817774593
 11678842067123286466226751144071133482776005430489425760 $*I*a^2*e^{(411/2*I*c}$
 $)$ - 15433398044664752654560339343571798167516797662298543036518825302240514
 0750956390615095093479110699588649900 $*I*a^2*e^{(409/2*I*c)}$ - 682489975344168
 898405747441723232848684360017115555657669816504708038746683370158039605060
 67102299819184920 $*I*a^2*e^{(407/2*I*c)}$ - 29821871642675642979884949769331439
 027860315600536250652213455370080842219764548099437011716544543041268840 $*I*$
 $a^2*e^{(405/2*I*c)}$ - 1287509096857049511048268849733111955148172511514046635
 4371165041532925663730313264282055462011603010654340 $*I*a^2*e^{(403/2*I*c)}$ -
 549180314668753988053597656596379567216451408656720801201591602677563943873
 2336258201804419618292545032320 $*I*a^2*e^{(401/2*I*c)}$ - 231419987522566986824
 406965158487639596366333938136712176597016461942695849656811645654666240906
 8449840980 $*I*a^2*e^{(399/2*I*c)}$ - 963339901333670052201847836491729647953262
 248639288500838541624447579232078571408017358275408623510613840 $*I*a^2*e^{(39$
 $7/2*I*c)}$ - 3961156927720317702883047917185117312913684062974398434356507503
 72236248577797783021338331228243750622260 $*I*a^2*e^{(395/2*I*c)}$ - 16087882507
 904339460383494957583749762548437050975220455638952087004282015328124781724
 8349563478924960000 $*I*a^2*e^{(393/2*I*c)}$ - 645327319778245914793691916371582
 59335289649533606982971631364964054911681075637122490714561514235891960 $*I*a$
 $^2*e^{(391/2*I*c)}$ - 25564363098285969951461675934273364734821923616250047177
 527687812898786857371780193438857338260097911840 $*I*a^2*e^{(389/2*I*c)}$ - 1000
 074884258142115311578285002841913064386956944913472196790825111591501059312
 7137857232140041620869750 $*I*a^2*e^{(387/2*I*c)}$ - 386314437516868018576356913
 292077827044669797836884278496544413991685735826342800126438356631100699668
 $0*I*a^2*e^{(385/2*I*c)}$ - 147342444362109962377181703120951524343672463702574
 9527267678685770355293167282547379994041612249536410 $*I*a^2*e^{(383/2*I*c)}$ -
 554830477071987246654408444808030209487515325400605772819119408435792744646
 306336768559494858757362460 $*I*a^2*e^{(381/2*I*c)}$ - 2062550466642352923299538
 505218323676805884853392747978586557766976702788671365670139537869241778588
 $90*I*a^2*e^{(379/2*I*c)}$ - 75687878722014265825238066731591013702998100464324
 442217272752916887964266256104313931049007060490840 $*I*a^2*e^{(377/2*I*c)}$ - 2
 741514103484544395109493925675095458443694586767090771035285949297046359096
 5208109494412332329295780 $*I*a^2*e^{(375/2*I*c)}$ - 980079316064566075944389460
 9848911529425579690874330115323569027541005476735379175640063961886333700 $*I$
 $*a^2*e^{(373/2*I*c)}$ - 345781676701999202030946295835138625644973659314096946
 8279877327753792198494989629433408106651901070 $*I*a^2*e^{(371/2*I*c)}$ - 120385
 831118903001859978475898311457410127655358396896577655106146317043542640348
 3179176840497456330 $*I*a^2*e^{(369/2*I*c)}$ - 413564489143612695132947288092921
 397877659902127071578646579062379140019898506098486550619425904170 $*I*a^2*e^{$
 $(367/2*I*c)}$ - 1401737032027635410158205056407283867183742759160410960781830
 91845413305288205080342342220046629650 $*I*a^2*e^{(365/2*I*c)}$ - 46871127019956
 579394527723949157745550097315795915660019622180811547871642339809799054651
 854945990 $*I*a^2*e^{(363/2*I*c)}$ - 1546033818880351810107079629874147305105044
 9962808545632677281743199685497754720467992010224616870 $*I*a^2*e^{(361/2*I*c)}$

- 502997233223228520128531881185601119191768675475579615309908167926502294
 6497934343332346116036200*I*a^2*e^(359/2*I*c) - 161399625005227651497742238
 7514829743397825495538053623372585007372675158127379199196038765006020*I*a^2
 *e^(357/2*I*c) - 510723774383122473742590339083403953575864194243445208994
 753073600039579929266407037097651682920*I*a^2*e^(355/2*I*c) - 1593571437033
 253029099589869710248950108651059517203782604973685085930558384538251650496
 11563210*I*a^2*e^(353/2*I*c) - 49024494337492279360818849127945363095454529
 548782519402705804194570228377733166934824088655040*I*a^2*e^(351/2*I*c) - 1
 486839010064205118367197117938788000350914635000729463417789964508881031388
 5195635392146744810*I*a^2*e^(349/2*I*c) - 444504319274789268390399853721138
 2275857104122359227822533654938963716398520643447212245479640*I*a^2*e^(347/
 2*I*c) - 130978506840473201225164579044912481278616850664545098127256694219
 6706295180922249225707977010*I*a^2*e^(345/2*I*c) - 380351974188426836426198
 640513394713416494607855165518072594190906223337330504018165258288940*I*a^2
 *e^(343/2*I*c) - 1088381674585823184455849835282493949200583590094814258200
 11589540335936121859362203566362580*I*a^2*e^(341/2*I*c) - 30685533864596666
 418316194015078029015958251712763047637572948112622799853423865147299102830
 *I*a^2*e^(339/2*I*c) - 8522926048369988940355915252628221987552753095020951
 377015329635773840197997200581857597800*I*a^2*e^(337/2*I*c) - 2331796181228
 676480358713874846040638691309670968182309524450889214672550181351830378735
 030*I*a^2*e^(335/2*I*c) - 6283217402529699146708974013748477212590031785572
 96040399451247917033448679858171068226560*I*a^2*e^(333/2*I*c) - 16672645594
 159683116062347635171258718759465764285712701383023948458731376768001933958
 2550*I*a^2*e^(331/2*I*c) - 435609777598064255098417169783299042660753138802
 76843623527207449861917293070562965518680*I*a^2*e^(329/2*I*c) - 11204681256
 258812161206109553481424902816807388805631619385616166895520536176153385410
 620*I*a^2*e^(327/2*I*c) - 2836916918939477124570501535003667950662441685459
 586986859765033838975136651432973013080*I*a^2*e^(325/2*I*c) - 7069254940580
 01738008498868730775412018273649883656316519821960251293960111842853827050*
 I*a^2*e^(323/2*I*c) - 17334585697527208030746888831957374382674815794981708
 3968790709575178522586695155600570*I*a^2*e^(321/2*I*c) - 418212568836636776
 04463813990356906305692324432072812479020391840209738405992862342110*I*a^2*
 e^(319/2*I*c) - 99255258829095186735695367313547359943179819481123050022604
 77305616767418585528998710*I*a^2*e^(317/2*I*c) - 23169134178965336341482384
 79533816756801158698905671195529018686134366565369411066790*I*a^2*e^(315/2*
 I*c) - 53185200307045318933327613873998530720744532344199443399324659353770
 0600251640914290*I*a^2*e^(313/2*I*c) - 120038120350944029722615798484583578
 817920936103042898479075195084456374447649531420*I*a^2*e^(311/2*I*c) - 2663
 270899013432469919988152459092587944442773459791184262577669711161407246280
 5500*I*a^2*e^(309/2*I*c) - 580760744355231221024154298187369701154404717090
 5560703749866664890336325145752760*I*a^2*e^(307/2*I*c) - 124445844130409520
 7497327837779344779293432610052698964481320058973903348131364390*I*a^2*e^(3
 05/2*I*c) - 261985740043443080388301874884500683905172501509414794896626641
 791046018314193300*I*a^2*e^(303/2*I*c) - 5417503137684360762935604502705875
 8032630893385234188342863868414550489892855190*I*a^2*e^(301/2*I*c) - 110015

09051169918803406190301110072169140434739188390680687069371600096456461960*
 $I*a^2*e^{(299/2*I*c)}$ - 21935182533236071585983067606101945055637157176487379
 78249164699044796790178970*I*a^2*e^{(297/2*I*c)} - 42930640633160482364568825
 1632236356131380665568461698695435155472754991584480*I*a^2*e^{(295/2*I*c)} -
 824571979405000651128312357780093374005779424373292705419368896111362486163
 60*I*a^2*e^{(293/2*I*c)} - 15538869199972063791690049288524847158110979530065
 672277863507395517286098560*I*a^2*e^{(291/2*I*c)} - 2872306980765485786798093
 527905670394316300311494879919370292601994164873980*I*a^2*e^{(289/2*I*c)} - 5
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 $I*a^2*e^{(287/2*I*c)}$ - 92525290960177968452743510383284189038302875808137991
 118047531970267330140*I*a^2*e^{(285/2*I*c)} - 1611539718513118326178499906809
 3482958330926537119548096569091610170990080*I*a^2*e^{(283/2*I*c)} - 275021431
 4681421164420565775956331599212851388641230567209095240832869740*I*a^2*e^{(2
 81/2*I*c)} - 459734613498525904391661453323499339021127870927849907055606144
 578837880*I*a^2*e^{(279/2*I*c)} - 7525378354737695802256874598690377526529117
 8425521737345338921401401608*I*a^2*e^{(277/2*I*c)} - 120584136119792150144938
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 08523184465234870582011548986898368367578058256415408172123086240*I*a^2*e^{(
 273/2*I*c)} - 29003446452038703680172784858940259795627649094616171753396164
 9595700*I*a^2*e^{(271/2*I*c)} - 435050290141291368075663883180136091638159860
 27077804343349110468080*I*a^2*e^{(269/2*I*c)} - 63790183241316162206527474157
 10905929685302237372129456527463165524*I*a^2*e^{(267/2*I*c)} - 91395062485344
 4779964988596342201399080981813136131167642386677984*I*a^2*e^{(265/2*I*c)} -
 127899514211571855615342081558322168204136482117422337757176345560*I*a^2*e^{
 (263/2*I*c)} - 1747460922600429434666452928465193776292560410220855409744205
 0560*I*a^2*e^{(261/2*I*c)} - 232994397127263459706866547956038365266800462064
 3027277203439985*I*a^2*e^{(259/2*I*c)} - 303026950314518079219169886639469213
 112529662619496065070362634*I*a^2*e^{(257/2*I*c)} - 3842411761067265590693644
 1589932580808631293346194215493375887*I*a^2*e^{(255/2*I*c)} - 474780239105147
 2283183879516945405395337420791155799564552344*I*a^2*e^{(253/2*I*c)} - 571368
 224054265219726596099705196924252108483077143367419935*I*a^2*e^{(251/2*I*c)}
 - 66931651284481661864618077305876269408561834596408636101210*I*a^2*e^{(249/
 2*I*c)} - 7627533234852264511705859368402201230654030433440301675518*I*a^2*e
 ^{(247/2*I*c)} - 845095502974670714598033224559068895202594191429069360222*I*
 $a^2*e^{(245/2*I*c)}$ - 9097340693680683753191568540095330911057425272127602036
 7*I*a^2*e^{(243/2*I*c)} - 950851579990067311329922100372685364411032521560588
 5183*I*a^2*e^{(241/2*I*c)} - 964243501752639006146026780418602842158889630673
 989345*I*a^2*e^{(239/2*I*c)} - 9479919142233667091173328415586707603032597722
 1341875*I*a^2*e^{(237/2*I*c)} - 902849209049851352193143124553016936636903459
 6324891*I*a^2*e^{(235/2*I*c)} - 832235125649436217222715021488460783355491981
 492217*I*a^2*e^{(233/2*I*c)} - 7418250497560845028963626536828185138271122064
 9004*I*a^2*e^{(231/2*I*c)} - 638793699419597624062268299016624534261305551789
 0*I*a^2*e^{(229/2*I*c)} - 530853426357042754469653748150767198919032400940*I*
 $a^2*e^{(227/2*I*c)}$ - 42526927818120262032236340231948484360039052121*I*a^2*e
 ^{(225/2*I*c)} - 3280313734912630390804532039053385555426092600*I*a^2*e^{(223/

$2*I*c) - 243319959628953467711826163624006704408771129*I*a^2*e^{(221/2*I*c)}$
 $- 17332379827117737513384038195005765450047540*I*a^2*e^{(219/2*I*c)} - 118390$
 $5679851187219496284451985830369733725*I*a^2*e^{(217/2*I*c)} - 774216225013482$
 $80247334235191908463309686*I*a^2*e^{(215/2*I*c)} - 48388512940652764619847787$
 $40285520947658*I*a^2*e^{(213/2*I*c)} - 28849519402842808862756832155924581845$
 $9*I*a^2*e^{(211/2*I*c)} - 16374051337559882202075446312226584700*I*a^2*e^{(209$
 $/2*I*c)} - 882698176765813429616327000431223295*I*a^2*e^{(207/2*I*c)} - 450840$
 $46347051121561291339131297112*I*a^2*e^{(205/2*I*c)} - 21756376145264883093397$
 $95247719423*I*a^2*e^{(203/2*I*c)} - 98892618501960556395579843057828*I*a^2*e^{$
 $(201/2*I*c)} - 4219418379622495018746735933182*I*a^2*e^{(199/2*I*c)} - 1683278$
 $60633476968482282779620*I*a^2*e^{(197/2*I*c)} - 6250901980395957795917828835*$
 $I*a^2*e^{(195/2*I*c)} - 214978110312371924676280101*I*a^2*e^{(193/2*I*c)} - 680$
 $6694783357860712909393*I*a^2*e^{(191/2*I*c)} - 197035901588508817004391*I*a^2$
 $*e^{(189/2*I*c)} - 5171545973000559760165*I*a^2*e^{(187/2*I*c)} - 1218427061655$
 $31758225*I*a^2*e^{(185/2*I*c)} - 2545017361124253210*I*a^2*e^{(183/2*I*c)} - 46$
 $393545645311706*I*a^2*e^{(181/2*I*c)} - 723016295770626*I*a^2*e^{(179/2*I*c)} -$
 $9365496059205*I*a^2*e^{(177/2*I*c)} - 96800992860*I*a^2*e^{(175/2*I*c)} - 7484$
 $61285*I*a^2*e^{(173/2*I*c)} - 3848130*I*a^2*e^{(171/2*I*c)} - 9867*I*a^2*e^{(169$
 $/2*I*c)})/(e^{(517*I*c)} + 418*e^{(516*I*c)} + 87153*e^{(515*I*c)} + 12085216*e^{(5$
 $14*I*c)} + 1253841160*e^{(513*I*c)} + 103818048048*e^{(512*I*c)} + 7146142307307$
 $*e^{(511*I*c)} + 420601518659718*e^{(510*I*c)} + 21608403021340047*e^{(509*I*c)}$
 $+ 984382804329835768*e^{(508*I*c)} + 40261256699368950388*e^{(507*I*c)} + 14933$
 $26612293984160368*e^{(506*I*c)} + 50648660944512569972179*e^{(505*I*c)} + 15817$
 $96642397812408161814*e^{(504*I*c)} + 45759117183402579073139583*e^{(503*I*c)} +$
 $1232445557346832245176696904*e^{(502*I*c)} + 31042222522074681615625020522*e$
 $^{(501*I*c)} + 734057263616388449968842366924*e^{(500*I*c)} + 16353164647151530$
 $240529137618111*e^{(499*I*c)} + 344277152012875134140739302960914*e^{(498*I*c)}$
 $+ 6868329225263681349501997341320517*e^{(497*I*c)} + 13017119307917282383515$
 $1430773360024*e^{(496*I*c)} + 2348998374244347079532766203075607598*e^{(495*I$
 $c)} + 40443624781415311581857832389099634564*e^{(494*I*c)} + 66563467067621006$
 $3754191847109971141414*e^{(493*I*c)} + 10490402669510897424624643766470754045$
 $064*e^{(492*I*c)} + 158566476113257562566117432227203884298856*e^{(491*I*c)} +$
 $2302150411226234925855222345201500900533576*e^{(490*I*c)} + 32147887693375338$
 $817454482515377350383950278*e^{(489*I*c)} + 432333688644261557547944179250800$
 $440604964868*e^{(488*I*c)} + 5605927253067558551780452883689835514455118670*e$
 $^{(487*I*c)} + 70164515322544462906873548813748091084561870680*e^{(486*I*c)} +$
 $848552202276512356496200136959676295361696315113*e^{(485*I*c)} + 992549073853$
 $4402272939987038714580495445431374618*e^{(484*I*c)} + 11239160454224665096642$
 $9162063124338952554575234051*e^{(483*I*c)} + 12330967001397233651819972207509$
 $32590655287625342156*e^{(482*I*c)} + 1311878180117217472967933989431815369496$
 $4675368481194*e^{(481*I*c)} + 13544259491663611619157465062533164623850110162$
 $7937224*e^{(480*I*c)} + 13579906631614798428506428480325449828783598395803498$
 $99*e^{(479*I*c)} + 13231708870104896973800056733779919089340836756009580718*e$
 $^{(478*I*c)} + 125370496586921272662198050851269323171167338854081782959*e^{(4$
 $77*I*c)} + 1155855412893594260345544966642687823630035899363232371472*e^{(476$

$*I*c) + 10375184499871175501909398956596684116802997082526660323524*e^{(475*I*c)} + 90722605722208814918642284639487187764607589706493970774776*e^{(474*I*c)} + 773204636991145775061462731028098506094432675788136295011259*e^{(473*I*c)} + 6426195485535248576425068136870465530087114003875716691383902*e^{(472*I*c)} + 52108117629177048660492400985175830987505700566877818954141639*e^{(471*I*c)} + 412430698299915190848067222327219435067747934091894670488982928*e^{(470*I*c)} + 3187749929744346497211536044751776582320958627923816470590659024*e^{(469*I*c)} + 24070801913529757101858022914372045864746991786182039740274325264*e^{(468*I*c)} + 177642829135119348577194437675802830239905460092687136494961404333*e^{(467*I*c)} + 1281817464914970810859604189828359000790789921169405304612211251818*e^{(466*I*c)} + 9046693523825682979044338963104263167672586826367911338826483549173*e^{(465*I*c)} + 62473550781053295317710774690247114124125187565731848441781904032672*e^{(464*I*c)} + 422276126632003687547754746555709988710527133086660161366353656787288*e^{(463*I*c)} + 2794709104475686611842790694973699164482254723977210209725661304403472*e^{(462*I*c)} + 1811576849561575807671030305505625589254293659193314153418333944596408*e^{(461*I*c)} + 115051481852080848873700388354521315567640365124003103691176697194292320*e^{(460*I*c)} + 716099497599058079895633338552940229192858196481597830078819711862600096*e^{(459*I*c)} + 4369442482910113914565353136069595862669338858053419381214131241925047008*e^{(458*I*c)} + 26143976279902021443471945665080254563056810183520401889800285493144867448*e^{(457*I*c)} + 153436088745056254127327239461577071933130157764595997113973513183188399376*e^{(456*I*c)} + 88350968821791202600774541927769200737689393513734789368397093333311961880*e^{(455*I*c)} + 4992519712457043983505377976607953988397368297591114957991804893688371867680*e^{(454*I*c)} + 27693116538343259225983382637647936122664033859615133489846664694361471028310*e^{(453*I*c)} + 150822381431412413773566474210011746852297437597059186295243989481140398152780*e^{(452*I*c)} + 806679543607589140759305010796189568269842021613388955218916278823182639488190*e^{(451*I*c)} + 4238125846763232586394188569858685826755328005548627437019301405851325887594480*e^{(450*I*c)} + 21876482892713909928040345612578705805121508756226696317087651824252241418663320*e^{(449*I*c)} + 110969199687320974749922259595250444341219218535349655762591192576535872151766080*e^{(448*I*c)} + 553269128819528612502918869558947829098021956309349843584044631512291778800081490*e^{(447*I*c)} + 2711843239670717527605640490148833507130242448403978318523237721944200392830108580*e^{(446*I*c)} + 13069817203488289886193205508375818392124991382340160316886507181296548981014818410*e^{(445*I*c)} + 61948596653035502879564338815234310660410902037882473161804774492916216575880077680*e^{(444*I*c)} + 288820755264730654469968572021047109427318619508995802020689904590319476295408324280*e^{(443*I*c)} + 1324756412367837473157472821162483691120966501948953926492241643788264284546437221120*e^{(442*I*c)} + 5978992172944143218459161149299819706321732111578494525245228742976468409105395536290*e^{(441*I*c)} + 26556806389043407534496702369101545795994861757741414789944652712127566910185274123140*e^{(440*I*c)} + 116104551683555043762911501712116399313733021132677481112824047246361794049635726479850*e^{(439*I*c)} + 499707567253859084357596314813794768069337190915967491907488904933922677579665354338960*e^{(438*$

$I*c) + 21175897334668557071015014292104147224018388379407528416185414408885$
 $45729943138209036820*e^{(437*I*c)} + 8836720640860470305694514021547969551296$
 $794092266983044118375790025854584036796364768280*e^{(436*I*c)} + 363183696523$
 $025917321974444097981220226408246041305525067425867951832673543828478758857$
 $30*e^{(435*I*c)} + 1470308167322768331630415820995920475120437252253533392388$
 $19165193000407629544745753221740*e^{(434*I*c)} + 5864034669726832427416433289$
 $21560909375197453864243299571990964608857245771134145204174990*e^{(433*I*c)}$
 $+ 2304351073373840357379178597673066352016682781689139842097376663118488803$
 $841131935313641840*e^{(432*I*c)} + 892320944734329676333188188163847179349961$
 $8670601026059730895962653291770229493028162575100*e^{(431*I*c)} + 34054053851$
 $295569154352346722177172655187548910782008504718324168725029438589162349211$
 $628040*e^{(430*I*c)} + 128098914601688539672480541830409847707367500438601536$
 $803204497701119911289087105659482783340*e^{(429*I*c)} + 475010578857601519272$
 $316617938425222421786597241671026894318515408511467140969393115768793680*e^{(428*I*c)}$
 $+ 173657421881819107187419747245015812388356420995065863910233714$
 $8122769080611680719741726053840*e^{(427*I*c)} + 62598721568222528436509607082$
 $35034710201362776057176647226323089751446565288850103898153859920*e^{(426*I*c)}$
 $+ 222519591767957775716736603600748022221136423214639980386437096339149$
 $1223687245823457351580140*e^{(425*I*c)} + 78009807368024239875613733058851417$
 $125327114681070889640794249282633470580756557083923203377160*e^{(424*I*c)} +$
 $269745801440211296972683601863878954357962308520076595177128227629273240215$
 $209708218497363414140*e^{(423*I*c)} + 920089393029589032874601850027159322612$
 $526368444771489781974361078847528891468831038436064951920*e^{(422*I*c)} + 309$
 $613197162152016238030155424146545178236208681028753774890290498593402017956$
 $5706177131421614590*e^{(421*I*c)} + 10279364730663840844739577862469262604648$
 $861914297972589165243530651230690726244462479199894255180*e^{(420*I*c)} + 336$
 $753988720215683759023845939827533625598010581041846273454111362624319432407$
 $78260721756991027090*e^{(419*I*c)} + 1088679957318294728267329051920348867972$
 $84621356445627530909104429486741257822633476898356826454040*e^{(418*I*c)} + 3$
 $473514732147137808743520831295666012387657627759423667627333499521038897539$
 $82636403857556867777300*e^{(417*I*c)} + 1093853214486220358674032434500866678$
 $499770011305874172488975951612031456734608287095519501041975440*e^{(416*I*c)}$
 $+ 340023256060165161752169468084708984419802883169441742479486877932895054$
 $8418125605446882081152636090*e^{(415*I*c)} + 10434117516570395966653693155582$
 $402109460348095473027807412321427346816928567197770376496170251803940*e^{(414*I*c)}$
 $+ 316109393312846927506943064436184146560959695209452157430040445603$
 $86895241801579156543451940713351730*e^{(413*I*c)} + 9455618025893198691933430$
 $346636565282685809131432918916073627717587384173219645337995370567946682688$
 $0*e^{(412*I*c)} + 27928575580003520667983536889816547764486498779466538782748$
 $8933863633745047373109049265172681702585720*e^{(411*I*c)} + 81460818773653057$
 $967021002527192141559718336988121429982329196978554987617596986636797665324$
 $4974728560*e^{(410*I*c)} + 23465182192391051422381416330734647688991557089350$
 $25778047637412681781575765422219127409260159438712250*e^{(409*I*c)} + 6675866$
 $290371147358503766865669289010893543869830538708724945291580951179188296606$
 $158111257706968604740*e^{(408*I*c)} + 187599882188655635641636357359860732782$

55737257405706279108891366378428467414559930481172863538598193890*e^(407*I*c) + 5207517851879327038642926335154430695110499354250058293815524168940813
 8675254608030847907167748571734720*e^(406*I*c) + 14280179245022176248318087
 491882527413430513327541778008479503464476350933350315051734586465966718941
 7080*e^(405*I*c) + 38687621823427716563245172304997988926311528237460754169
 2443176673997513742813591736171169652250611186480*e^(404*I*c) + 10355619825
 920029352263845779086115486121114950801935736913398647060291864824662418056
 64949381049856258510*e^(403*I*c) + 2738895624795265603355227646566000886280
 778305084825702911938903656162004262736182657700406301914070062380*e^(402*I*c) + 715812468684294147547380736367983971817274558153840904450338385269359
 6921622426696740453944718143025248390*e^(401*I*c) + 18487405299005732693752
 728611876490890858357021974882371570623800186245137722660943641752976852924
 439870880*e^(400*I*c) + 471882208434662076950995069535737803571088974914225
 67898048199018207708997005333860148836479527456156014520*e^(399*I*c) + 1190
 418554038779649482295779483704656006066231830455295269004302092704732127738
 47794935586074714329479939280*e^(398*I*c) + 2968255152826695896853182732802
 390500845550322034159415119626595968816157137999376800264974083056722976188
 40*e^(397*I*c) + 7315849722068183628747296214039744442800104463011615273397
 60544815300951787985538419764656214582667219914080*e^(396*I*c) + 1782446114
 931751850556354856638421901174412322298249496591658053939787198246565945975
 595575734193348887952160*e^(395*I*c) + 429320647800802212601748890885182649
 4790620720660151451468181910917240027863968724539127659633517053002976480*e^(394*I*c) + 10223182025954860767217390305186451923562145473674293619918063
 490411487496121804590274592702770571515456414680*e^(393*I*c) + 240687851397
 052771611934656445061432852413610377682168189221844001410484602109446966477
 52723371932874594597328*e^(392*I*c) + 5602868342490351765849501385853451616
 7162591034367972498174660907450666778154353271630344650777885683547624184*e^(391*I*c) + 12896708008475471224602368086648838498328625902553313204463610
 9049545144029547003347761521666283977931640178464*e^(390*I*c) + 29355074355
 434270980812945357656231329970598269918741686293437396425561596713867625327
 6302591561523515603264403*e^(389*I*c) + 66076447310586909769147597385083793
 451108903314958670798276426339476675664956527987914617331838650574039109399
 0*e^(388*I*c) + 14709311466189343455150383623001001604821277495814439299047
 46910224777470198899052379114493999887003199419829579*e^(387*I*c) + 3238491
 931361851476423321933539579098377735539207641467346235665823887048326949305
 609231585143748690203615957136*e^(386*I*c) + 705213241416219799260232652458
 014306098535305457293390552463312168102103734029836634220332432530707241373
 9061024*e^(385*I*c) + 15189634214908800396417911722643754748048520109734812
 459109878810493844381062650818971199637121458749456243274416*e^(384*I*c) +
 323627313224195494103300889436402474603783285613164229312924271459028879130
 71643679502909055891236755143207382609*e^(383*I*c) + 6820803309679361568378
 440961924421081861499164004155342440552787689327249660832423109814850246645
 3967157728078994*e^(382*I*c) + 14221311596481451768238666727676990948227168
 1318790889840501039441748635545362467679832449103520321953011780083069*e^(381*I*c) + 29334492003430072028704238344834286631380628545504006782308044559

7545970023446231563554135133105493516316320059272*e^(380*I*c) + 59865014111
224185891167650518052015036400322684132808145359709358779033860921243908555
4466861582623350303061961052*e^(379*I*c) + 12087703584936583930894422220569
350632837041081405937502265398461177376482166095597348316012486982743302961
58612144*e^(378*I*c) + 2414966516810338503289076549202740511710059011795447
138773464205696455026442712426409599662771080264826008985061097*e^(377*I*c)
+ 477414111106609897022184533059496201647271423037423406066395684695092664
2685946929064114194400360936223590725470146*e^(376*I*c) + 93393419580534942
252517509657150573007073020838147747703062182242410226482474199560429573630
55823830898547303219757*e^(375*I*c) + 1807982006802885997034993862300723067
656331420670884849990013964123733476326647934696323793603932811318504159179
3848*e^(374*I*c) + 34637657172671690167657344537197087048882354853993270472
063943078773600446542963548348101269390443464480754513928502*e^(373*I*c) +
656748592688673000988273758128752256106545516862611036816640070075371157780
97293533565243828873383722980353200611956*e^(372*I*c) + 1232439415193323847
419600725881035065964063392536163910820629699606824190117457757389218177533
91954462609323881489157*e^(371*I*c) + 2289113117385927800914926491623468344
058677407764563261084109288572571747072892680743475502257932447419233543953
08214*e^(370*I*c) + 4208463426089493872775590214579245865781209661485610226
47008499529468452005980175119410628956210497609566002969884927*e^(369*I*c)
+ 7658677955139627810125584446287514187109408952813047908367436615820716500
32154891482866406314834433199455459798934952*e^(368*I*c) + 1379676529796212
074017106188066589448355446501210890195107164860350228928586815539003062875
026711931941947738690360722*e^(367*I*c) + 246044237584542266392708163098326
071473496809190549302714563923882719225488634936112699145769240985112087330
7487457468*e^(366*I*c) + 43439096966019321733573596877815792937012956819408
27114215433175336093967845908766740738240037114570667410936998017178*e^(365
*I*c) + 7592752700146678961153095073585015473197029746533633331549793961473
285760935801904155116764831560875947581048693527224*e^(364*I*c) + 131397714
941049338818566811514182931122425515215356868711812665798138776063481602617
47201317735782566021306798298336024*e^(363*I*c) + 2251467574130806996150616
558650287243042193021067326439299728648560064010386725360484771554706059296
7690653795951142520*e^(362*I*c) + 38199015867586087976002998756627674994795
440667903625029322346250133286489120875005013638128113893960349670280707161
530*e^(361*I*c) + 641751006932600668062380648860045971707408433000868393686
16139164529108049844675353111842725798658088840347241496099644*e^(360*I*c)
+ 1067648320171655948380852341893335287335876733299725300926610851867899392
52915937090760282232346919090426243399409323314*e^(359*I*c) + 1758962582627
559857571068126139793012658010315954843536149046728651694422320757765804471
84134141375995770091499246759528*e^(358*I*c) + 2869929436312314965572780108
515769408968264974660663275288015606770071128374319267350881209748617605113
67008815728782643*e^(357*I*c) + 4637582884573671545449376782550056887333281
455680493104239955998860128006386199040223683785911088426023420945436822991
02*e^(356*I*c) + 7422286409081731249169370494625256173341489196791182704898
31005497781951221069955839623452499748653124658873553401442137*e^(355*I*c)

+ 1176600720975786965189875050890231092204612696970277433014535895788956771
 230793520381993106606880564628599822341722801012*e^(354*I*c) + 184750585646
 245153344528430057132632378116255330456597188767075809107930679482183492817
 0773126364639722071570131703785334*e^(353*I*c) + 28736105359223401870808354
 355829122772719679773947201597910702749277142768695314671826889810410613817
 03885403497544001592*e^(352*I*c) + 4427673079105425318524316112985693656584
 851936100192457044455134483305045321452516347118488133224823670465103483954
 805161*e^(351*I*c) + 675848043788852437256293594896385762669485554719551948
 6122877567981718587262362871994967079401831957927901682582941234362*e^(350*
 I*c) + 10220423779434634851339975295163399641702122249663666193053008302026
 096932158568338309418237395541351819026907953220681013*e^(349*I*c) + 153128
 372066627753793473532128076829657125356529426315182861424030977382002707111
 95396582159028513532779682154451996208592*e^(348*I*c) + 2273160356612884110
 041950194705136766683665241807726091394481074847308489189041018128541260485
 4876625919565639521227223276*e^(347*I*c) + 33435897827936581301171175459610
 829454298167962017419810072936733378506584428024201072453193458155334046693
 516742390717832*e^(346*I*c) + 487332535059749234008522555630521014021964693
 136595544927256747543393752830104071677443669558289228374887058585324396544
 89*e^(345*I*c) + 7038634976059483156704822406139502569850120229696630037676
 4220336697702961591099854055411376294871437468149528524796002762*e^(344*I*c
) + 10074496185185374461175430098298016696240455383622292186848469426996612
 0607698907046343731011160948828100276729370132819357*e^(343*I*c) + 14290631
 912305552424654692847895423837131592580202238923649865213683982250203515567
 6970917419039834587967055588431566416784*e^(342*I*c) + 20090658715357880438
 030046950144161017452185125954192920984068896085945490851977483590589575766
 6770857888611738751858460424*e^(341*I*c) + 27994524447503980482296673046296
 088449211987485779114712400907947692043594173529330930543043868733312991245
 4196774070107264*e^(340*I*c) + 38664267305038004945738256281831696265197555
 099077927704874023862985879501824735616288863101568766478010120528733308274
 8791*e^(339*I*c) + 52932925276411392600393483695824355767254923899756073921
 4406599185047831955572583765358634395408771528009745467548382950094*e^(338*
 I*c) + 71836159638205824920911354448790108886838874403371321033249197137590
 6738341551540457264804304039664255915607349801911966551*e^(337*I*c) + 96645
 827536903771874773913079815164348359068416683223468809829116416063641815945
 2119815728809372125168836239364442397344064*e^(336*I*c) + 12890435152929339
 564806343304996770401810439356201069142673110679000300583988397876923769540
 90545278554544997710058754772400*e^(335*I*c) + 1704582996707822808204678218
 167693002698661147712772355021456543810930069637188085882824757500605246963
 210810351706405349408*e^(334*I*c) + 223489127639843946447862257830643484072
 461048446817785982262065869192147864526665306256382355300122800100909360675
 1066168944*e^(333*I*c) + 29053857223200570019533452744894827908566925299598
 23749532695963414164833366773128218607899328588608916176593772088622582464*
 e^(332*I*c) + 3745257594876651204657334988426226388143954501986830664222349
 226361079609546822276067504899386703088982308185717143407211328*e^(331*I*c)
 + 478752744278094568514520484697159616530416941932824407321145959212964925

5048876854059844720661078151288179612574986359194560*e^(330*I*c) + 60689498
031567122483318711053298954717228061430088780149865596536872606948165504701
95890004511965527567432722969707577202160*e^(329*I*c) + 7629731815627821580
468992424207008366438896736333024661863838105110445148946962328297631547032
543419811821015837863013682720*e^(328*I*c) + 951303227401952295420911319126
822664229991201352566594029838106479788569090499312894803522741214403563385
1779511219335277360*e^(327*I*c) + 11764212274876484080010900714673474493371
278160557811983724455826566055617658086479368641864908119643412413644803772
131657280*e^(326*I*c) + 144298162852084312045329783753756919650631542246497
47551295851507389524083226976789688601369628399900747658579201929300744260*
e^(325*I*c) + 1755627327122429239688729140312571621349148626114547857137675
1690105656067838042151038271381300372757755676325408026834544840*e^(324*I*c
) + 21188321405882887539610198374706862695894049226077093764132512513336190
523978949694387686059124526755048042957954264706637460*e^(323*I*c) + 253671
764391193536215322603359833481549049826061257617113006834929633908164915830
25705268737539982149639300226512657426118880*e^(322*I*c) + 3012848241455270
326455901895308817715601343749343820107841376983544836614812175454919759112
9967170764969700180348699207838960*e^(321*I*c) + 35500103106019649876272376
796949482209581372371036005012877806027481672807059943445240136315568500732
379966585005678181937920*e^(320*I*c) + 414998321219637080437885237874013455
417800889305382069188535790267492733646716400375634886077160928876864715428
38602788559660*e^(319*I*c) + 4813311767818402921650374854911037447892471909
463560389282936486391655379227882295736828510632816471591059837087114907949
4360*e^(318*I*c) + 55390913044972086219432689146331566081427959896969900214
434296817731150863867056620768608187679709720152974148474907904177340*e^(31
7*I*c) + 632477741010121790517949460751755699240769813381384831580424067474
53874729387631710544995247152912205118500597511052824347680*e^(316*I*c) + 7
166032986117339552444194388928410913409115784465524567208423740243494469646
4927131812190659629511140639501743303863582092880*e^(315*I*c) + 80566249130
682684181876201882623511206363790337218011954110210642927765997644903820595
421936873565314654415769070472655401600*e^(314*I*c) + 898838158013823822139
732704779546027447928770180519633471463073724643151212749294023479428748028
99499538953561056667668891020*e^(313*I*c) + 9951220647205796595134034173802
354851533640337171789804085047095465753297727911349150688029072611115410194
1386019689567958040*e^(312*I*c) + 10933253734996622320393267850342635707986
370700172829401104207653040392386265401897867651641731422108944992249561273
2870169660*e^(311*I*c) + 11920971370203392705575539782368844444464742432450
218532862634704659963472114657383068154049533354314677681091191041046862896
0*e^(310*I*c) + 12899507601159190341076386342709732994858617357459586270584
9159280943046458742663163454018491463855395649453952212899632198680*e^(309*
I*c) + 13852979454915108945135276957654340312633074724368003083246720589581
9043568155239264876762867172754338684027849855385453216080*e^(308*I*c) + 14
764892080554533341862312176785377739978292474830122879392434257499993795542
1765370101235122939557467548549202174550009604780*e^(307*I*c) + 15618596295
355119616973821883217369650985255158921073057836572747625947647446595542850

2336673743686499175698677875693611243400*e^(306*I*c) + 16397781605960772537
526455981650584789418778510145536039189742448299841538578760576531550920833
7741590143078572243505132706580*e^(305*I*c) + 17086984886895310117686030605
310399434053039034726008843267684250555514129383083896127597426892866649484
5723462544709102843680*e^(304*I*c) + 17672092997055464200457577005309570059
533465987068273203197591553238757705241486632351114011768049292935451755947
9899220940360*e^(303*I*c) + 18140816877092205982036855331669732163998486262
829882856956027329563089762682934526359221903456085353073371052984214853790
1680*e^(302*I*c) + 18483115198374894181766785017470825713812817215826941328
7765358532240773244336191900818557829905895684494889410451921524212840*e^(3
01*I*c) + 18691547443656751492635140562311750326198750835193008382456644443
5689139233683411704641828762178799177848064220150818355261280*e^(300*I*c) +
18761539316851005007149728056460351091240313292031202437083506267903764499
0286285346673507093452964351257962696133511725652320*e^(299*I*c) + 18691547
443656751492635140562311750326198750835193008382456644443568913923368341170
4641828762178799177848064220150818355261280*e^(298*I*c) + 18483115198374894
181766785017470825713812817215826941328776535853224077324433619190081855782
9905895684494889410451921524212840*e^(297*I*c) + 18140816877092205982036855
331669732163998486262829882856956027329563089762682934526359221903456085353
0733710529842148537901680*e^(296*I*c) + 17672092997055464200457577005309570
059533465987068273203197591553238757705241486632351114011768049292935451755
9479899220940360*e^(295*I*c) + 17086984886895310117686030605310399434053039
034726008843267684250555514129383083896127597426892866649484572346254470910
2843680*e^(294*I*c) + 16397781605960772537526455981650584789418778510145536
0391897424482998415385787605765315509208337741590143078572243505132706580*e
^(293*I*c) + 15618596295355119616973821883217369650985255158921073057836572
7476259476474465955428502336673743686499175698677875693611243400*e^(292*I*c
) + 14764892080554533341862312176785377739978292474830122879392434257499993
7955421765370101235122939557467548549202174550009604780*e^(291*I*c) + 13852
979454915108945135276957654340312633074724368003083246720589581904356815523
9264876762867172754338684027849855385453216080*e^(290*I*c) + 12899507601159
190341076386342709732994858617357459586270584915928094304645874266316345401
8491463855395649453952212899632198680*e^(289*I*c) + 11920971370203392705575
539782368844444464742432450218532862634704659963472114657383068154049533354
3146776810911910410468628960*e^(288*I*c) + 10933253734996622320393267850342
635707986370700172829401104207653040392386265401897867651641731422108944992
2495612732870169660*e^(287*I*c) + 99512206472057965951340341738023548515336
403371717898040850470954657532977279113491506880290726111154101941386019689
567958040*e^(286*I*c) + 898838158013823822139732704779546027447928770180519
63347146307372464315121274929402347942874802899499538953561056667668891020*
e^(285*I*c) + 8056624913068268418187620188262351120636379033721801195411021
0642927765997644903820595421936873565314654415769070472655401600*e^(284*I*c
) + 71660329861173395524441943889284109134091157844655245672084237402434944
696464927131812190659629511140639501743303863582092880*e^(283*I*c) + 632477
741010121790517949460751755699240769813381384831580424067474538747293876317

10544995247152912205118500597511052824347680*e^(282*I*c) + 5539091304497208
621943268914633156608142795989696990021443429681773115086386705662076860818
7679709720152974148474907904177340*e^(281*I*c) + 48133117678184029216503748
549110374478924719094635603892829364863916553792278822957368285106328164715
910598370871149079494360*e^(280*I*c) + 414998321219637080437885237874013455
417800889305382069188535790267492733646716400375634886077160928876864715428
38602788559660*e^(279*I*c) + 3550010310601964987627237679694948220958137237
103600501287780602748167280705994344524013631556850073237996658500567818193
7920*e^(278*I*c) + 30128482414552703264559018953088177156013437493438201078
413769835448366148121754549197591129967170764969700180348699207838960*e^(27
7*I*c) + 253671764391193536215322603359833481549049826061257617113006834929
63390816491583025705268737539982149639300226512657426118880*e^(276*I*c) + 2
118832140588288753961019837470686269589404922607709376413251251333619052397
8949694387686059124526755048042957954264706637460*e^(275*I*c) + 17556273271
224292396887291403125716213491486261145478571376751690105656067838042151038
271381300372757755676325408026834544840*e^(274*I*c) + 144298162852084312045
329783753756919650631542246497475512958515073895240832269767896886013696283
99900747658579201929300744260*e^(273*I*c) + 1176421227487648408001090071467
347449337127816055781198372445582656605561765808647936864186490811964341241
3644803772131657280*e^(272*I*c) + 95130322740195229542091131912682266422999
120135256659402983810647978856909049931289480352274121440356338517795112193
35277360*e^(271*I*c) + 7629731815627821580468992424207008366438896736333024
661863838105110445148946962328297631547032543419811821015837863013682720*e^
(270*I*c) + 606894980315671224833187110532989547172280614300887801498655965
3687260694816550470195890004511965527567432722969707577202160*e^(269*I*c) +
47875274427809456851452048469715961653041694193282440732114595921296492550
48876854059844720661078151288179612574986359194560*e^(268*I*c) + 3745257594
876651204657334988426226388143954501986830664222349226361079609546822276067
504899386703088982308185717143407211328*e^(267*I*c) + 290538572232005700195
334527448948279085669252995982374953269596341416483336677312821860789932858
8608916176593772088622582464*e^(266*I*c) + 22348912763984394644786225783064
348407246104844681778598226206586919214786452666530625638235530012280010090
93606751066168944*e^(265*I*c) + 1704582996707822808204678218167693002698661
147712772355021456543810930069637188085882824757500605246963210810351706405
349408*e^(264*I*c) + 128904351529293395648063433049967704018104393562010691
4267311067900030058398839787692376954090545278554544997710058754772400*e^(2
63*I*c) + 96645827536903771874773913079815164348359068416683223468809829116
4160636418159452119815728809372125168836239364442397344064*e^(262*I*c) + 71
836159638205824920911354448790108886838874403371321033249197137590673834155
1540457264804304039664255915607349801911966551*e^(261*I*c) + 52932925276411
392600393483695824355767254923899756073921440659918504783195557258376535863
4395408771528009745467548382950094*e^(260*I*c) + 38664267305038004945738256
281831696265197555099077927704874023862985879501824735616288863101568766478
0101205287333082748791*e^(259*I*c) + 27994524447503980482296673046296088449
211987485779114712400907947692043594173529330930543043868733312991245419677

4070107264*e^(258*I*c) + 20090658715357880438030046950144161017452185125954
1929209840688960859454908519774835905895757666770857888611738751858460424*e
^(257*I*c) + 14290631912305552424654692847895423837131592580202238923649865
2136839822502035155676970917419039834587967055588431566416784*e^(256*I*c) +
10074496185185374461175430098298016696240455383622292186848469426996612060
7698907046343731011160948828100276729370132819357*e^(255*I*c) + 70386349760
594831567048224061395025698501202296966300376764220336697702961591099854055
411376294871437468149528524796002762*e^(254*I*c) + 487332535059749234008522
555630521014021964693136595544927256747543393752830104071677443669558289228
37488705858532439654489*e^(253*I*c) + 3343589782793658130117117545961082945
429816796201741981007293673337850658442802420107245319345815533404669351674
2390717832*e^(252*I*c) + 22731603566128841100419501947051367666836652418077
260913944810748473084891890410181285412604854876625919565639521227223276*e^
(251*I*c) + 153128372066627753793473532128076829657125356529426315182861424
03097738200270711195396582159028513532779682154451996208592*e^(250*I*c) + 1
022042377943463485133997529516339964170212224966366619305300830202609693215
8568338309418237395541351819026907953220681013*e^(249*I*c) + 67584804378885
243725629359489638576266948555471955194861228775679817185872623628719949670
79401831957927901682582941234362*e^(248*I*c) + 4427673079105425318524316112
985693656584851936100192457044455134483305045321452516347118488133224823670
465103483954805161*e^(247*I*c) + 287361053592234018708083543558291227727196
797739472015979107027492771427686953146718268898104106138170388540349754400
1592*e^(246*I*c) + 18475058564624515334452843005713263237811625533045659718
87670758091079306794821834928170773126364639722071570131703785334*e^(245*I*
c) + 1176600720975786965189875050890231092204612696970277433014535895788956
771230793520381993106606880564628599822341722801012*e^(244*I*c) + 742228640
908173124916937049462525617334148919679118270489831005497781951221069955839
623452499748653124658873553401442137*e^(243*I*c) + 463758288457367154544937
678255005688733328145568049310423995599886012800638619904022368378591108842
602342094543682299102*e^(242*I*c) + 286992943631231496557278010851576940896
826497466066327528801560677007112837431926735088120974861760511367008815728
782643*e^(241*I*c) + 175896258262755985757106812613979301265801031595484353
614904672865169442232075776580447184134141375995770091499246759528*e^(240*I
*c) + 106764832017165594838085234189333528733587673329972530092661085186789
939252915937090760282232346919090426243399409323314*e^(239*I*c) + 641751006
932600668062380648860045971707408433000868393686161391645291080498446753531
11842725798658088840347241496099644*e^(238*I*c) + 3819901586758608797600299
875662767499479544066790362502932234625013328648912087500501363812811389396
0349670280707161530*e^(237*I*c) + 22514675741308069961506165586502872430421
930210673264392997286485600640103867253604847715547060592967690653795951142
520*e^(236*I*c) + 131397714941049338818566811514182931122425515215356868711
81266579813877606348160261747201317735782566021306798298336024*e^(235*I*c)
+ 7592752700146678961153095073585015473197029746533633331549793961473285760
935801904155116764831560875947581048693527224*e^(234*I*c) + 434390969660193
217335735968778157929370129568194082711421543317533609396784590876674073824

0037114570667410936998017178*e^(233*I*c) + 24604423758454226639270816309832
607147349680919054930271456392388271922548863493611269914576924098511208733
07487457468*e^(232*I*c) + 1379676529796212074017106188066589448355446501210
890195107164860350228928586815539003062875026711931941947738690360722*e^(23
1*I*c) + 765867795513962781012558444628751418710940895281304790836743661582
071650032154891482866406314834433199455459798934952*e^(230*I*c) + 420846342
608949387277559021457924586578120966148561022647008499529468452005980175119
410628956210497609566002969884927*e^(229*I*c) + 228911311738592780091492649
162346834405867740776456326108410928857257174707289268074347550225793244741
923354395308214*e^(228*I*c) + 123243941519332384741960072588103506596406339
253616391082062969960682419011745775738921817753391954462609323881489157*e^(
227*I*c) + 656748592688673000988273758128752256106545516862611036816640070
07537115778097293533565243828873383722980353200611956*e^(226*I*c) + 3463765
717267169016765734453719708704888235485399327047206394307877360044654296354
8348101269390443464480754513928502*e^(225*I*c) + 18079820068028859970349938
623007230676563314206708848499900139641237334763266479346963237936039328113
185041591793848*e^(224*I*c) + 933934195805349422525175096571505730070730208
3814774770306218224241022648247419956042957363055823830898547303219757*e^(2
23*I*c) + 47741411110660989702218453305949620164727142303742340606639568469
50926642685946929064114194400360936223590725470146*e^(222*I*c) + 2414966516
810338503289076549202740511710059011795447138773464205696455026442712426409
599662771080264826008985061097*e^(221*I*c) + 120877035849365839308944222205
693506328370410814059375022653984611773764821660955973483160124869827433029
6158612144*e^(220*I*c) + 59865014111224185891167650518052015036400322684132
8081453597093587790338609212439085554466861582623350303061961052*e^(219*I*c
) + 29334492003430072028704238344834286631380628545504006782308044559754597
0023446231563554135133105493516316320059272*e^(218*I*c) + 14221311596481451
768238666727676990948227168131879088984050103944174863554536246767983244910
3520321953011780083069*e^(217*I*c) + 68208033096793615683784409619244210818
614991640041553424405527876893272496608324231098148502466453967157728078994
*e^(216*I*c) + 323627313224195494103300889436402474603783285613164229312924
27145902887913071643679502909055891236755143207382609*e^(215*I*c) + 1518963
421490880039641791172264375474804852010973481245910987881049384438106265081
8971199637121458749456243274416*e^(214*I*c) + 70521324141621979926023265245
801430609853530545729339055246331216810210373402983663422033243253070724137
39061024*e^(213*I*c) + 3238491931361851476423321933539579098377735539207641
467346235665823887048326949305609231585143748690203615957136*e^(212*I*c) +
147093114661893434551503836230010016048212774958144392990474691022477747019
8899052379114493999887003199419829579*e^(211*I*c) + 66076447310586909769147
597385083793451108903314958670798276426339476675664956527987914617331838650
5740391093990*e^(210*I*c) + 29355074355434270980812945357656231329970598269
9187416862934373964255615967138676253276302591561523515603264403*e^(209*I*c
) + 12896708008475471224602368086648838498328625902553313204463610904954514
4029547003347761521666283977931640178464*e^(208*I*c) + 56028683424903517658
495013858534516167162591034367972498174660907450666778154353271630344650777

885683547624184*e^(207*I*c) + 240687851397052771611934656445061432852413610
 37768216818922184400141048460210944696647752723371932874594597328*e^(206*I*
 c) + 1022318202595486076721739030518645192356214547367429361991806349041148
 7496121804590274592702770571515456414680*e^(205*I*c) + 42932064780080221260
 174889088518264947906207206601514514681819109172400278639687245391276596335
 17053002976480*e^(204*I*c) + 1782446114931751850556354856638421901174412322
 298249496591658053939787198246565945975595575734193348887952160*e^(203*I*c)
 + 731584972206818362874729621403974444280010446301161527339760544815300951
 787985538419764656214582667219914080*e^(202*I*c) + 296825515282669589685318
 273280239050084555032203415941511962659596881615713799937680026497408305672
 297618840*e^(201*I*c) + 119041855403877964948229577948370465600606623183045
 529526900430209270473212773847794935586074714329479939280*e^(200*I*c) + 471
 882208434662076950995069535737803571088974914225678980481990182077089970053
 33860148836479527456156014520*e^(199*I*c) + 1848740529900573269375272861187
 649089085835702197488237157062380018624513772266094364175297685292443987088
 0*e^(198*I*c) + 71581246868429414754738073636798397181727455815384090445033
 83852693596921622426696740453944718143025248390*e^(197*I*c) + 2738895624795
 265603355227646566000886280778305084825702911938903656162004262736182657700
 406301914070062380*e^(196*I*c) + 103556198259200293522638457790861154861211
 1495080193573691339864706029186482466241805664949381049856258510*e^(195*I*c
) + 38687621823427716563245172304997988926311528237460754169244317667399751
 3742813591736171169652250611186480*e^(194*I*c) + 14280179245022176248318087
 491882527413430513327541778008479503464476350933350315051734586465966718941
 7080*e^(193*I*c) + 52075178518793270386429263351544306951104993542500582938
 155241689408138675254608030847907167748571734720*e^(192*I*c) + 187599882188
 655635641636357359860732782557372574057062791088913663784284674145599304811
 72863538598193890*e^(191*I*c) + 6675866290371147358503766865669289010893543
 869830538708724945291580951179188296606158111257706968604740*e^(190*I*c) +
 234651821923910514223814163307346476889915570893502577804763741268178157576
 5422219127409260159438712250*e^(189*I*c) + 81460818773653057967021002527192
 1415597183369881214299823291969785549876175969866367976653244974728560*e^(1
 88*I*c) + 27928575580003520667983536889816547764486498779466538782748893386
 3633745047373109049265172681702585720*e^(187*I*c) + 94556180258931986919334
 303466365652826858091314329189160736277175873841732196453379953705679466826
 880*e^(186*I*c) + 316109393312846927506943064436184146560959695209452157430
 04044560386895241801579156543451940713351730*e^(185*I*c) + 1043411751657039
 596665369315558240210946034809547302780741232142734681692856719777037649617
 0251803940*e^(184*I*c) + 34002325606016516175216946808470898441980288316944
 17424794868779328950548418125605446882081152636090*e^(183*I*c) + 1093853214
 486220358674032434500866678499770011305874172488975951612031456734608287095
 519501041975440*e^(182*I*c) + 347351473214713780874352083129566601238765762
 775942366762733349952103889753982636403857556867777300*e^(181*I*c) + 108867
 995731829472826732905192034886797284621356445627530909104429486741257822633
 476898356826454040*e^(180*I*c) + 336753988720215683759023845939827533625598
 01058104184627345411136262431943240778260721756991027090*e^(179*I*c) + 1027

936473066384084473957786246926260464886191429797258916524353065123069072624
4462479199894255180*e^(178*I*c) + 30961319716215201623803015542414654517823
62086810287537748902904985934020179565706177131421614590*e^(177*I*c) + 9200
893930295890328746018500271593226125263684447714897819743610788475288914688
31038436064951920*e^(176*I*c) + 2697458014402112969726836018638789543579623
08520076595177128227629273240215209708218497363414140*e^(175*I*c) + 7800980
736802423987561373305885141712532711468107088964079424928263347058075655708
3923203377160*e^(174*I*c) + 22251959176795777757167366036007480222211364232
146399803864370963391491223687245823457351580140*e^(173*I*c) + 625987215682
225284365096070823503471020136277605717664722632308975144656528885010389815
3859920*e^(172*I*c) + 17365742188181910718741974724501581238835642099506586
39102337148122769080611680719741726053840*e^(171*I*c) + 4750105788576015192
72316617938425222421786597241671026894318515408511467140969393115768793680*
e^(170*I*c) + 1280989146016885396724805418304098477073675004386015368032044
97701119911289087105659482783340*e^(169*I*c) + 3405405385129556915435234672
2177172655187548910782008504718324168725029438589162349211628040*e^(168*I*c
) + 89232094473432967633318818816384717934996186706010260597308959626532917
70229493028162575100*e^(167*I*c) + 2304351073373840357379178597673066352016
682781689139842097376663118488803841131935313641840*e^(166*I*c) + 586403466
972683242741643328921560909375197453864243299571990964608857245771134145204
174990*e^(165*I*c) + 147030816732276833163041582099592047512043725225353339
238819165193000407629544745753221740*e^(164*I*c) + 363183696523025917321974
44409798122022640824604130552506742586795183267354382847875885730*e^(163*I*
c) + 8836720640860470305694514021547969551296794092266983044118375790025854
584036796364768280*e^(162*I*c) + 211758973346685570710150142921041472240183
8837940752841618541440888545729943138209036820*e^(161*I*c) + 49970756725385
9084357596314813794768069337190915967491907488904933922677579665354338960*e
^(160*I*c) + 11610455168355504376291150171211639931373302113267748111282404
7246361794049635726479850*e^(159*I*c) + 26556806389043407534496702369101545
795994861757741414789944652712127566910185274123140*e^(158*I*c) + 597899217
294414321845916114929981970632173211157849452524522874297646840910539553629
0*e^(157*I*c) + 13247564123678374731574728211624836911209665019489539264922
41643788264284546437221120*e^(156*I*c) + 2888207552647306544699685720210471
09427318619508995802020689904590319476295408324280*e^(155*I*c) + 6194859665
3035502879564338815234310660410902037882473161804774492916216575880077680*e
^(154*I*c) + 13069817203488289886193205508375818392124991382340160316886507
181296548981014818410*e^(153*I*c) + 271184323967071752760564049014883350713
0242448403978318523237721944200392830108580*e^(152*I*c) + 55326912881952861
2502918869558947829098021956309349843584044631512291778800081490*e^(151*I*c
) + 11096919968732097474992225959525044434121921853534965576259119257653587
2151766080*e^(150*I*c) + 21876482892713909928040345612578705805121508756226
696317087651824252241418663320*e^(149*I*c) + 423812584676323258639418856985
8685826755328005548627437019301405851325887594480*e^(148*I*c) + 80667954360
7589140759305010796189568269842021613388955218916278823182639488190*e^(147*
I*c) + 15082238143141241377356647421001174685229743759705918629524398948114

0398152780*e^(146*I*c) + 27693116538343259225983382637647936122664033859615
 133489846664694361471028310*e^(145*I*c) + 499251971245704398350537797660795
 3988397368297591114957991804893688371867680*e^(144*I*c) + 88350096882179120
 2600774541927769200737689393513734789368397093333311961880*e^(143*I*c) + 15
 3436088745056254127327239461577071933130157764595997113973513183188399376*e
^(142*I*c) + 26143976279902021443471945665080254563056810183520401889800285
 493144867448*e^(141*I*c) + 436944248291011391456535313606959586266933885805
 3419381214131241925047008*e^(140*I*c) + 71609949759905807989563333855294022
 9192858196481597830078819711862600096*e^(139*I*c) + 11505148185208084887370
 0388354521315567640365124003103691176697194292320*e^(138*I*c) + 18115768495
 615758076710303055505625589254293659193314153418333944596408*e^(137*I*c) +
 2794709104475686611842790694973699164482254723977210209725661304403472*e<sup>(1
 36*I*c)</sup> + 42227612663200368754775474655570998871052713308666016136635365678
 7288*e^(135*I*c) + 62473550781053295317710774690247114124125187565731848441
 781904032672*e^(134*I*c) + 904669352382568297904433896310426316767258682636
 7911338826483549173*e^(133*I*c) + 12818174649149708108596041898283590007907
 89921169405304612211251818*e^(132*I*c) + 1776428291351193485771944376758028
 30239905460092687136494961404333*e^(131*I*c) + 2407080191352975710185802291
 4372045864746991786182039740274325264*e^(130*I*c) + 31877499297443464972115
 36044751776582320958627923816470590659024*e^(129*I*c) + 4124306982999151908
 48067222327219435067747934091894670488982928*e^(128*I*c) + 5210811762917704
 8660492400985175830987505700566877818954141639*e^(127*I*c) + 64261954855352
 48576425068136870465530087114003875716691383902*e^(126*I*c) + 7732046369911
 45775061462731028098506094432675788136295011259*e^(125*I*c) + 9072260572220
 8814918642284639487187764607589706493970774776*e^(124*I*c) + 10375184499871
 175501909398956596684116802997082526660323524*e^(123*I*c) + 115585541289359
 4260345544966642687823630035899363232371472*e^(122*I*c) + 12537049658692127
 2662198050851269323171167338854081782959*e^(121*I*c) + 13231708870104896973
 800056733779919089340836756009580718*e^(120*I*c) + 135799066316147984285064
 2848032544982878359839580349899*e^(119*I*c) + 13544259491663611619157465062
 5331646238501101627937224*e^(118*I*c) + 13118781801172174729679339894318153
 694964675368481194*e^(117*I*c) + 123309670013972336518199722075093259065528
 7625342156*e^(116*I*c) + 11239160454224665096642916206312433895255457523405
 1*e^(115*I*c) + 9925490738534402272939987038714580495445431374618*e<sup>(114*I*
 c)</sup> + 848552202276512356496200136959676295361696315113*e^(113*I*c) + 7016451
 5322544462906873548813748091084561870680*e^(112*I*c) + 56059272530675585517
 80452883689835514455118670*e^(111*I*c) + 4323336886442615575479441792508004
 40604964868*e^(110*I*c) + 32147887693375338817454482515377350383950278*e<sup>(1
 09*I*c)</sup> + 2302150411226234925855222345201500900533576*e^(108*I*c) + 1585664
 76113257562566117432227203884298856*e^(107*I*c) + 1049040266951089742462464
 3766470754045064*e^(106*I*c) + 665634670676210063754191847109971141414*e<sup>(1
 05*I*c)</sup> + 40443624781415311581857832389099634564*e^(104*I*c) + 234899837424
 4347079532766203075607598*e^(103*I*c) + 13017119307917282383515143077336002
 4*e^(102*I*c) + 6868329225263681349501997341320517*e^(101*I*c) + 3442771520
 12875134140739302960914*e^(100*I*c) + 16353164647151530240529137618111*e⁽⁹

$9*I*c) + 734057263616388449968842366924*e^{(98*I*c)} + 310422252207468161562$
 $5020522*e^{(97*I*c)} + 1232445557346832245176696904*e^{(96*I*c)} + 457591171834$
 $02579073139583*e^{(95*I*c)} + 1581796642397812408161814*e^{(94*I*c)} + 50648660$
 $944512569972179*e^{(93*I*c)} + 1493326612293984160368*e^{(92*I*c)} + 4026125669$
 $9368950388*e^{(91*I*c)} + 984382804329835768*e^{(90*I*c)} + 21608403021340047*e$
 $^{(89*I*c)} + 420601518659718*e^{(88*I*c)} + 7146142307307*e^{(87*I*c)} + 1038180$
 $48048*e^{(86*I*c)} + 1253841160*e^{(85*I*c)} + 12085216*e^{(84*I*c)} + 87153*e^{(8$
 $3*I*c)} + 418*e^{(82*I*c)} + e^{(81*I*c)})) * \tan(1/4*d*x + c) - 14*(3289*a^2*e^{(1$
 $027/2*I*c)} + 1282710*a^2*e^{(1025/2*I*c)} + 249487095*a^2*e^{(1023/2*I*c)} + 32$
 $266997620*a^2*e^{(1021/2*I*c)} + 3121832019735*a^2*e^{(1019/2*I*c)} + 241005431$
 $923542*a^2*e^{(1017/2*I*c)} + 15464515215103622*a^2*e^{(1015/2*I*c)} + 84833912$
 $0374641870*a^2*e^{(1013/2*I*c)} + 40614235388489346675*a^2*e^{(1011/2*I*c)} + 1$
 $723848657664106290905*a^2*e^{(1009/2*I*c)} + 65678633862570503690097*a^2*e^{(1$
 $007/2*I*c)} + 2268898261098769561120041*a^2*e^{(1005/2*I*c)} + 716593701028074$
 $44959242727*a^2*e^{(1003/2*I*c)} + 2083633993393098160454111295*a^2*e^{(1001/2$
 $*I*c)} + 56109286874368068446384618040*a^2*e^{(999/2*I*c)} + 14064727930607428$
 $17278133399694*a^2*e^{(997/2*I*c)} + 32964206161728793499031816369336*a^2*e^{($
 $995/2*I*c)} + 725212537982338639553752606105971*a^2*e^{(993/2*I*c)} + 15028015$
 $442916472784303355549568064*a^2*e^{(991/2*I*c)} + 294232725411218343537913910$
 $694524115*a^2*e^{(989/2*I*c)} + 5458017107743192900932192332079754800*a^2*e^{($
 $987/2*I*c)} + 96165064556404674374591793494770969751*a^2*e^{(985/2*I*c)} + 161$
 $2950428548722021160870732573388748486*a^2*e^{(983/2*I*c)} + 25807207438709023$
 $405564698654381610855782*a^2*e^{(981/2*I*c)} + 394635225337657865294965990982$
 $357902779935*a^2*e^{(979/2*I*c)} + 577745991732304730583747893934339061756080$
 $0*a^2*e^{(977/2*I*c)} + 81106652744979453503605904560553941804516827*a^2*e^{(9$
 $75/2*I*c)} + 1093437903450413476694775388085248143435843184*a^2*e^{(973/2*I*c$
 $) + 14175642468718517834053796420531668264840828827*a^2*e^{(971/2*I*c)} + 176$
 $951139921850074633194549001040224465598018440*a^2*e^{(969/2*I*c)} + 212931229$
 $7800033603965109459542894851379636241390*a^2*e^{(967/2*I*c)} + 24727501166647$
 $061083289528750467452594464413496712*a^2*e^{(965/2*I*c)} + 277411701644365486$
 $379380075778816997842958595635411*a^2*e^{(963/2*I*c)} + 300949727040266437260$
 $3369055821186361937123336960151*a^2*e^{(961/2*I*c)} + 31599729267161332823403$
 $075503125661623124842474211925*a^2*e^{(959/2*I*c)} + 321414485517907071866518$
 $386845402404565806226718602745*a^2*e^{(957/2*I*c)} + 316950508533267722030982$
 $3086926906379156042673077997829*a^2*e^{(955/2*I*c)} + 30324466873468264078058$
 $992476301732181384084447966749251*a^2*e^{(953/2*I*c)} + 281698477370386606708$
 $598633818891039165559192659067598486*a^2*e^{(951/2*I*c)} + 254251082202480904$
 $9111292052295925694458486937681586168494*a^2*e^{(949/2*I*c)} + 22310547737377$
 $338728195792669384697517889412712723300156650*a^2*e^{(947/2*I*c)} + 190456047$
 $314980596262214455758830176412490479327548630395435*a^2*e^{(945/2*I*c)} + 158$
 $2600527113724210945270692154487384537712932377861494537792*a^2*e^{(943/2*I*c$
 $) + 12808036621165917614961411495703359665564662395820821879432651*a^2*e^{(9$
 $41/2*I*c)} + 101008959448394278622065270281354488236767429874129774402372122$
 $*a^2*e^{(939/2*I*c)} + 776647773894140039601026125724635184851400081828079008$
 $519495205*a^2*e^{(937/2*I*c)} + 582486784188497480555625745580572844017737936$

7311530786366553840*a²*e^(935/2*I*c) + 42633155182758234213432657105359507
 039157758216710091608150751160*a²*e^(933/2*I*c) + 304650073489624069501319
 666840972753356711517554147295115707791952*a²*e^(931/2*I*c) + 212633835043
 4067101841066179986723736309721747170637151333282385892*a²*e^(929/2*I*c) +
 14501667734089456372413097720073331427769744768582734443920820884880*a²*e^(927/2*I*c) +
 966780886821870705669527556981593846714993014354753639462486
 45511300*a²*e^(925/2*I*c) + 6302690115189170937868077836198321319914003631
 03105059485556854864080*a²*e^(923/2*I*c) + 4019467571972191918495205431223
 264437545950953166839320312436248766740*a²*e^(921/2*I*c) + 250845685605254
 86523793075765839786939668166684250998147267060999843864*a²*e^(919/2*I*c)
 + 153244689998280106303232133486628958245019231358231317009398623103014360*
 a²*e^(917/2*I*c) + 9167368692513268417836020231830052343308092505156853922
 95995174302631940*a²*e^(915/2*I*c) + 5371790823547775986769311672178060666
 294358172540326849409957986665686640*a²*e^(913/2*I*c) + 308417099431457559
 74906466306871090919705337121093845564611475205831834740*a²*e^(911/2*I*c)
 + 1735513522108301115424037299977365486595796902621613346843783546936531436
 00*a²*e^(909/2*I*c) + 9574335222919425726039720611747689313800787149545982
 45160747546204280979860*a²*e^(907/2*I*c) + 5179610004567839676882734372075
 939558403884380512473251576883045496625080240*a²*e^(905/2*I*c) + 274856545
 00744845946086864693951416729958544421088007484546058264731062865400*a²*e^(903/2*I*c) +
 1431016774931677060494690013338394115564961683574330981215214
 63359375252809040*a²*e^(901/2*I*c) + 7311701215847240332209644867730886497
 92495972559727457509380085597040672846910*a²*e^(899/2*I*c) + 3667154885575
 556035354210370922300362357210115667534077868226474380300702979520*a²*e^(897/2*I*c) +
 180582621470625107999026332110367481902274399474476801166135469
 69431587149657730*a²*e^(895/2*I*c) + 8732813827726335745368739363282925582
 0603582731737822656013656737314066270769860*a²*e^(893/2*I*c) + 41481713609
 8345744491539277362007380513271882803145751297848219367978399846839810*a²*
 e^(891/2*I*c) + 19358569408870667571586486961083583755863417938508550620399
 59891604593248887646480*a²*e^(889/2*I*c) + 8877507285196220618241655464778
 732499935221664373951650797658698877398128936202900*a²*e^(887/2*I*c) + 400
 123938896624906162713992136142573997740094024739966664164806251578847687648
 96580*a²*e^(885/2*I*c) + 1772824600327718658920999477032194005094017281208
 70156939007264944328352108674765830*a²*e^(883/2*I*c) + 7722970195333403185
 14767256445802696297496337802921019409004020665598966438156162590*a²*e^(881/2*I*c) +
 3308473219703823537556243769763570815349179943585706271711604571
 444094770152919124730*a²*e^(879/2*I*c) + 139402536961680794020642596926535
 07612945211052614130496704459947954824765033193710270*a²*e^(877/2*I*c) + 5
 778119433161805018407587242356179142184903524357160032028870653298160933168
 0804189710*a²*e^(875/2*I*c) + 23563841440367663637496461853516823546153165
 1215096600123550185747704196738532598229970*a²*e^(873/2*I*c) + 94562383101
 506746556576173775178171272957904338605019077228151898074636042331624359056
 0*a²*e^(871/2*I*c) + 37348277775428429628209444971587641204590666972403945
 89825664346513551225394202631289660*a²*e^(869/2*I*c) + 1452004323777102486
 7578892274833151999983805655774942432102559165991274651263356626027360*a²*

$e^{(867/2*I*c)} + 55574294007405657159410702225110128247423080333361478620342$
730912591089336970706714049470*a²*e^(865/2*I*c) + 209435641246496342288374
862958056438753891068972196618627904344080420177337078104676468880*a²*e⁽⁸
63/2*I*c) + 777245248463832164443579464317740424946189628494942453508580441
800744828509867291417081630*a²*e^(861/2*I*c) + 284089448862922201077384763
4980756855554160286708024741922802485707803328368153641110473200*a²*e⁽⁸⁵⁹
/2*I*c) + 10228191850120678931201975750431518653979982536759165359640298725
615598898961878198815353510*a²*e^(857/2*I*c) + 362781469737328600452374684
50428084776890532748926686396879733480311191302936923550162289900*a²*e⁽⁸⁵
5/2*I*c) + 1267792359146086032784670876984604836428345234732732178548674589
77335775558083027540308805740*a²*e^(853/2*I*c) + 4365770969617118557692877
14703357389885645132886579731206499384230924672871218525038020148230*a²*e⁽⁸⁵¹
/2*I*c) + 1481614521665051140791668111046174156161377332659662553088798
777522638436965972191593890153520*a²*e^(849/2*I*c) + 495588674941085428363
7836408922370908861423693236555100736486744859805066952581996944150188670*a
²*e^(847/2*I*c) + 16340621960942206338329487769224971352662464448783601559
800415763231873879690981091170280400080*a²*e^(845/2*I*c) + 531159393938295
291027371764113671130054973993152703093411084547681365647642982941355648588
47390*a²*e^(843/2*I*c) + 1702303917138093755802210803550670269124567727461
94729300836144395351294968707996644726411879200*a²*e^(841/2*I*c) + 5379613
186712719119358120474976420724610694158490274473564618302712499421643192671
67904932016060*a²*e^(839/2*I*c) + 1676530371645263393714584422579978782563
532149514208800221991656698632081178644609104890307855200*a²*e^(837/2*I*c)
+ 515302087148671187341649181513197595190043424602514235251655281822427348
3297543776745652059545730*a²*e^(835/2*I*c) + 15622306566440921026343663943
462426741562514861595325201248149059018047548245480491740664747553230*a²*e
^(833/2*I*c) + 467200073750788489600390002175638519829255671038924203679790
03637368846369023717951090246855155470*a²*e^(831/2*I*c) + 1378402087618779
604363496521253969751330981360186907193389249492259933957307448053824427290
64152410*a²*e^(829/2*I*c) + 4012398805097958468657622180149700166218604213
34029285134283286090912984697175449116164770568435310*a²*e^(827/2*I*c) + 1
152461477872254543239888484515149435875834906656013841132789921648764385054
810746411696406101981510*a²*e^(825/2*I*c) + 326648792310303813099926514082
6940624050408404456340195828586033626485345825342237364184398762915620*a²*
e^(823/2*I*c) + 91370363264190120831175922815535656727758850471002096357454
22954421398356217178283444278671884412660*a²*e^(821/2*I*c) + 2522527166280
021214296942354256730248441742931019403575511131074207757968921507202128108
1661345709120*a²*e^(819/2*I*c) + 68739812560972530921511497106963251745177
527056492095301601973235793309670088282296954444560031434930*a²*e^{(817/2*I}
*c) + 184908923994039357855011897704029603323637349730083126237154853975102
904987785563933490666771915454740*a²*e^(815/2*I*c) + 491042141856878350715
681756058445138552698769297425826668396199882849855448274647622775780304699
322930*a²*e^(813/2*I*c) + 128743308045757357230667360505210675848691769420
3459437629697821743582350653265565508291332729874085680*a²*e^(811/2*I*c) +
33327945139053596821862691084016236698877248240508798156325014413926051254

15686565518742717075933147150*a²*e^(809/2*I*c) + 8519276451134253778536171
 630746078299873324316380590414453537555043854967995509903575577929995103297
 680*a²*e^(807/2*I*c) + 215049722839205503586128293099745679467390063138336
 34264671213989597677643264769478110594714212525857560*a²*e^(805/2*I*c) + 5
 361029120299712924338722751074643888041739613399047540120015183682451103335
 9061830417818144623930466160*a²*e^(803/2*I*c) + 13199609109018200344705741
 182539911968707125290875280387576522114186357813282793049956301611077345360
 8580*a²*e^(801/2*I*c) + 32100187008223062655215793596025520697570325967222
 0019809311160664723949929848535867562171221207552583920*a²*e^(799/2*I*c) +
 77111130235309822499661250787255889026615939981922101502561489073385665349
 7550312410883732182309081849060*a²*e^(797/2*I*c) + 18298626709480313132890
 141980873212635564354008538908485649418065035465017522167327656849313376367
 66894640*a²*e^(795/2*I*c) + 4289831770876915604620621605250311958677128887
 569530679923476007397181724403545145491954691454914942555700*a²*e<sup>(793/2*I
 *c)</sup> + 993597053594903087813334698317142855596825779938110671421541390503387
 2603148050425223025351154130410277240*a²*e^(791/2*I*c) + 22738199734133653
 078027776734892344808364750849791537027378856109800091278175916637691635582
 433639500261560*a²*e^(789/2*I*c) + 514167531737321398666542394549591195897
 55885479009835237894780038895835558829259413468345602675738630164100*a²*e ^(787/2*I*c) + 1148901068512382915513357282655623647651210051195914970246348
 15615328060989866188627682839323290553007949840*a²*e^(785/2*I*c) + 2536975
 400921549248034126032133684203102252154338121939065797094383402025646597247
 21822525055232970075634740*a²*e^(783/2*I*c) + 5536449762385042687607369329
 082039089595176262840112189376452062542415310049297085380561554653739415734
 77840*a²*e^(781/2*I*c) + 1194135038218231892789214181610148708437664688719
 253024539331690089016746312190822555525802303755766475255060*a²*e<sup>(779/2*I
 *c)</sup> + 254570242524891909244474523190339270693780626351542068606228623253312
 5040633932944013647476782825751916866192*a²*e^(777/2*I*c) + 53643705817764
 306760233668650266282168597296317241792638559685986189428565720429773858691
 36797574101112934360*a²*e^(775/2*I*c) + 1117406958028134317232944999929975
 062830254762476589973417129071767873791620112705951792733592030136338159992
 0*a²*e^(773/2*I*c) + 23009632000715685724501189026111050742267523553057578
 732719843669636960476167526014756393531959162197925032115*a²*e^(771/2*I*c)
 + 468422396878371342691023644392364884509351945660376600521193617965104119
 03219914369231181221570261150235170750*a²*e^(769/2*I*c) + 9427988386022349
 792770581337276140866027284763698893546725763459879963554847009387033061095
 8925793769077443101*a²*e^(767/2*I*c) + 18761922522118689236295432417346425
 688858024568665450116181152681916742854785985688041471398993933771670754589
 6*a²*e^(765/2*I*c) + 36917803025334244712164461577663701061577527842378932
 4847337963965264696957387110053345923248330973169112504125*a²*e<sup>(763/2*I*c
)</sup> + 71831953977036726427602319392360799636947125771739114271565574737812841
 1474061415600744631353742653947474979790*a²*e^(761/2*I*c) + 13821200824537
 093338725105874120595241499792549645110661616050094795141222314527779766693
 55954779387326179193890*a²*e^(759/2*I*c) + 2629922400035909288605593228984
 748106746097334648529702694924936483874523991116960016579495951972600623182

611066*a²*e^(757/2*I*c) + 494916366309989584381316250645382023285291988525
3345976704821171775106226971407006064744321532373168942138457133*a²*e<sup>(755
/2*I*c)</sup> + 92115980008709586128727109600611633023957969318641036423642168538
68494654352223629534260533174654764593528061971*a²*e^(753/2*I*c) + 1695800
97860474843198646376739561685689256076474194644211118701741821444897623387
9833477597585335045518017550375*a²*e^(751/2*I*c) + 30879760370351772693035
730827584443094725992387048496282785137580349540025068324342981224379621249
817077351202755*a²*e^(749/2*I*c) + 556230034286025324330268884138982338642
30618083244763365901631495829738789432935549737175085734259798422054263257*
a²*e^(747/2*I*c) + 9911484985842134438940523675895530843201725857832598931
3493313927809241858243075104249489142628498003793332896533*a²*e<sup>(745/2*I*c
)</sup> + 17472260088840480497853059010046000327818065897883190533015437098276398
4646513813160761146082864787793974499383768*a²*e^(743/2*I*c) + 30472480047
842473982581115795616827064545848575977090470207465980385046846195291998607
1057980917692816270458765682*a²*e^(741/2*I*c) + 52582010117044109185787232
739845249641452576093691328758391152958058053858655964984658554156268697641
7582636323800*a²*e^(739/2*I*c) + 89775861840758826540650200322732374591801
7671431846078493905129440539575848523829309505570503467433698687391599525*a
^2*e^(737/2*I*c) + 15166920118670121153210439466516589464492948997634380157
25740120744131654354391550015561531757951384128294098404528*a²*e<sup>(735/2*I*
c)</sup> + 253555413345835541566321579671332082230745037988391269451675243964844
468330679241034930206578896711879155418196133*a²*e^(733/2*I*c) + 419478070
904728048322481583394002255581670790344286484308338898323562550690003331322
8628940959072838355592388518976*a²*e^(731/2*I*c) + 68679721575512369217596
478483230407896536612414700147226680846161156747526550753868988799082091909
65790105909303585*a²*e^(729/2*I*c) + 1112893666308044610733008947654429677
798331979214489736985725687075446996965179147715920254935980263997141969660
5690*a²*e^(727/2*I*c) + 17848741370378512799153304742903183425864668367256
735229392362669226980832673201354290319227799410874360671281422106*a²*e<sup>(7
25/2*I*c)</sup> + 283343556909591007585634678390218450763420948600638588078367135
03595387569246628801974220943629593333922210878751177*a²*e^(723/2*I*c) + 4
452399152064658607690678656446592306060494198518377581854646147867751951028
8832394715366256217063713589099772007856*a²*e^(721/2*I*c) + 69258517208288
436651591793330966261047861717319283852998403408459459819999319356554087437
075858538315437799030208205*a²*e^(719/2*I*c) + 106653261729724722932275512
578847345760574095848138686047209360735188867297923054892788279482015629448
673456237211040*a²*e^(717/2*I*c) + 162600031260503579677639869749557722099
259408143969483138470362784417124438163290111258560462574993542827637524092
781*a²*e^(715/2*I*c) + 245434995171968502976685079757916654632352021121878
236552926031364907280287667538662828687293957579330538913455736808*a²*e<sup>(7
13/2*I*c)</sup> + 366813870230921594143108654992154299412010201906294490806412869
839956392860271021521745294086086233837764981473476178*a²*e^(711/2*I*c) +
542840959402461083096565399905340076090254992789760944337774489086299235841
557039882430591018739442636384860204215400*a²*e^(709/2*I*c) + 795502290777
098171408134869560451236843469564595516149336505085105242290424758607692945

689924007436094637703203085305*a²*e^(707/2*I*c) + 115445433590485736424798
 255791257871810274684969177031422018671270603409920792753045065544472239823
 2085044436220634697*a²*e^(705/2*I*c) + 16592173400789903662956234220384566
 180885486864575485381854106900336505730569200574149628312945882031074868261
 19787423*a²*e^(703/2*I*c) + 2361814782592409073140151717952055120156499375
 236619814260403866479477744294234718960967782715041717115313047558623183*a²
 *e^(701/2*I*c) + 332988643897483299252810109420711447828595587261810071869
 2457613200121080012023835679244861800931812439160323444845807*a²*e<sup>(699/2*
 I*c)</sup> + 46502805300571308278431536245609816678407418116261798195848248097835
 91908945012506423661839456267739976826873672738365*a²*e^(697/2*I*c) + 6433
 099335269195808717020639661849459986196085635968727776564625849932639280982
 496034596946007808758161920095325761890*a²*e^(695/2*I*c) + 881613168893121
 181405720811553391735580178702933557560041097724239982955992461005161551364
 6540527419139301483310423786*a²*e^(693/2*I*c) + 11969554932567565593177991
 401487663191149947263758650888132425196237881390457894009588904534530296178
 324030985072573618*a²*e^(691/2*I*c) + 161007290782993092644811637343751152
 148395318218801880239244677787078097920513979715642683566776807032855775131
 51431841*a²*e^(689/2*I*c) + 2145883924963638487853370673479192521284790142
 7210525197864886973863458734886342259576872792909047798523915929365691940*a
²*e^(687/2*I*c) + 28339072375239015657801316797536069668788909732601389032
 465333712850620264720283658988975362676026266822753938421168705*a²*e<sup>(685/
 2*I*c)</sup> + 370859455399326096372309786520403427641036077781645373158700946407
 90350034177071723193892067577540510415827321202078290*a²*e^(683/2*I*c) + 4
 809534266996605602072654570400619926666616063036101375515887806366134321063
 7596725104903439769999847624253161931778351*a²*e^(681/2*I*c) + 61814770854
 015319982448244739487435605552785122978910481378435042090366500632726628793
 497290765837977272563559609588576*a²*e^(679/2*I*c) + 787413258554607106990
 679209859661327299366890938484608164751349318468300103185652398663785898563
 56517618876735439697520*a²*e^(677/2*I*c) + 9941686668940286820595310072281
 626110127180845024825225798161133567643192302589923485986620585787267254766
 8167325989920*a²*e^(675/2*I*c) + 12441994931448740321617258853523885366758
 356969534639300036230704571056646808541611099455447875823154742722931512995
 8280*a²*e^(673/2*I*c) + 15435416574369740299541521612191638744631550862891
 2131928600309585745568868263338383595893423751250560255188896660980640*a²*
 e^(671/2*I*c) + 18983268084699244175097939805455076598344738070658269808650
 5079016572222835305196482037082520548635494540196474773032392*a²*e<sup>(669/2*
 I*c)</sup> + 23145895487786568048171146758149232047570817928441083430743144908329
 6993015735283992533153783630057823894466466386789920*a²*e^(667/2*I*c) + 27
 980388098082308115150785534629919227824534181403146041150610637246927698514
 6379946315124170832410518829080258705986280*a²*e^(665/2*I*c) + 33537984432
 628559273669546385701841843241318876648488111191239505924085524932824683072
 9779350439108234296749061439256880*a²*e^(663/2*I*c) + 39861250860218409979
 761821346489358310042332755799333063419501529101285230535826537144224135911
 3106146519054690230460080*a²*e^(661/2*I*c) + 46981143701407974929837201662
 13037516534417942749336811433052694822625542207879113262582764934490949207

3035868314640200*a²*e^(659/2*I*c) + 54914093530643764910950227909329828664
207115110465343770582461850651669097091548547484997080445425092603488252508
1421920*a²*e^(657/2*I*c) + 63659273756303396203890759515941020763022612674
8551544972393205357335273235966560353474541619072109609425303261030333480*a
²*e^(655/2*I*c) + 73196231488274693093264329818491314262354492950854253805
2238437217139684503440864983481224441799145110101150208078149600*a²*e<sup>(653
/2*I*c)</sup> + 83483064725273713037035225619139963415446230861831228362473844882
6183345582990700043901989251342801050174088405942776680*a²*e^(651/2*I*c) +
94455323899832201268423436024373360573514088939038967640806182290481472968
1532931792335624967472663525610160924422346720*a²*e^(649/2*I*c) + 10602579
637495363351478171186291939729042849906989097478482657401029387717583731768
78551328532733642490124903197641779440*a²*e^(647/2*I*c) + 1180852990661139
678826752973964900401369488895802453910984684024386892953596623999355559368
055504957627536059400058806560*a²*e^(645/2*I*c) + 130504557526466671962244
573856964648664394200563472193631664604260853511548538996058929491449157404
1407279419512731576020*a²*e^(643/2*I*c) + 14313719133449932862581457946587
895911281357315433444956326752269842704307529383533774068097548096448456688
68338821975120*a²*e^(641/2*I*c) + 1558237584123540747505152782426540433864
929969018290239445758871815326594322852854852718664426477851831755925555671
727660*a²*e^(639/2*I*c) + 168396739021853566943248119227362402641388949862
0534368790774587302981877714639969367635542513279277621696310502074133800*a
²*e^(637/2*I*c) + 18068626854740140613589927815006384218241055261797118197
72973596359005838247462282270364980307784363004235593075525641260*a²*e<sup>(63
5/2*I*c)</sup> + 1925263622372839475005118808013080285531251622928494797194551978
034702692757476718025108186454783185469267626276190347760*a²*e^(633/2*I*c)
+ 203761320423430278838887297426355980292248543580195171864518263707155037
1893197987545075984445655085553689119461288385080*a²*e^(631/2*I*c) + 21425
196461904671162381474675792602231299245144042513755478287076557073484280805
60507386311357900526669458781794087683160*a²*e^(629/2*I*c) + 2238813377208
636321456271169553111083915274044499063603397199965033978866404324512007961
336928154280808573579878469429140*a²*e^(627/2*I*c) + 232559508450689666304
119972963001914495032187554568606135867970661871572387842192521274480271052
8140695848048863263401620*a²*e^(625/2*I*c) + 24022715262653450027640791014
793975473164992903135078506048964974271968863422563648291733071563396489593
99574711685835340*a²*e^(623/2*I*c) + 2468576413951956043942022143414761234
633197197943806791819033461685747350263950586286250114393171484165699859371
683668820*a²*e^(621/2*I*c) + 252457446685799628239354774705222304997140932
533577086493004745173029888319981574827342006264621206412826186360317311322
0*a²*e^(619/2*I*c) + 25706477238315638811735539768341688568525438895529307
43589896312213425786028231311572055034657555701798516652461881179020*a²*e ^(617/2*I*c) + 2607464298677205743686833763943463444061901030012965588669139
358832060509897548972026199493982403179852911793212914653440*a²*e<sup>(615/2*I
*c)</sup> + 263593091107640264400012190592595214837514883339037305841170825406053
3702084460459096459271430090085171142785221034332360*a²*e^(613/2*I*c) + 26
571316314055601623005539239132253298975068920690239106683303205817633038888

81726915912529432751510232297721132883711360*a²*e^(611/2*I*c) + 2672256261
 640322567944365700246297265766159923823457015054078202760263266933709706766
 844807754089575759625548721880960900*a²*e^(609/2*I*c) + 268252255765808065
 058923396912367219064304443221005612729202386568785237823486336038133620132
 4629095652385752662511996320*a²*e^(607/2*I*c) + 26890970153023447693352914
 993797207811237273244361433085703826600022522711966903933731338614476426885
 42779676409253164100*a²*e^(605/2*I*c) + 2693019146667841202004214636225906
 376890422503779253729585442498782172066792698483998400631873111003925178781
 908847878880*a²*e^(603/2*I*c) + 269513404031053495699935576247065494002893
 256143558640855689214049539303612049550443102598959466980513247815356556297
 1060*a²*e^(601/2*I*c) + 26960375312160640948035791343760743037545696892364
 64771434986422320946498560245047737297861595509112192278615431723054440*a²
 *e^(599/2*I*c) + 2696037531216064094803579134376074303754569689236464771434
 986422320946498560245047737297861595509112192278615431723054440*a²*e<sup>(597/
 2*I*c)</sup> + 269513404031053495699935576247065494002893256143558640855689214049
 5393036120495504431025989594669805132478153565562971060*a²*e^(595/2*I*c) +
 26930191466678412020042146362259063768904225037792537295854424987821720667
 92698483998400631873111003925178781908847878880*a²*e^(593/2*I*c) + 2689097
 015302344769335291499379720781123727324436143308570382660002252271196690393
 373133861447642688542779676409253164100*a²*e^(591/2*I*c) + 268252255765808
 065058923396912367219064304443221005612729202386568785237823486336038133620
 1324629095652385752662511996320*a²*e^(589/2*I*c) + 26722562616403225679443
 657002462972657661599238234570150540782027602632669337097067668448077540895
 75759625548721880960900*a²*e^(587/2*I*c) + 2657131631405560162300553923913
 22532989750689206902391066833032058176330388881726915912529432751510232297
 721132883711360*a²*e^(585/2*I*c) + 263593091107640264400012190592595214837
 514883339037305841170825406053370208446045909645927143009008517114278522103
 4332360*a²*e^(583/2*I*c) + 26074642986772057436868337639434634440619010300
 12965588669139358832060509897548972026199493982403179852911793212914653440*
 a²*e^(581/2*I*c) + 2570647723831563881173553976834168856852543889552930743
 589896312213425786028231311572055034657555701798516652461881179020*a²*e<sup>(5
 79/2*I*c)</sup> + 252457446685799628239354774705222304997140932533577086493004745
 1730298883199815748273420062646212064128261863603173113220*a²*e<sup>(577/2*I*c
)</sup> + 24685764139519560439420221434147612346331971979438067918190334616857473
 50263950586286250114393171484165699859371683668820*a²*e^(575/2*I*c) + 2402
 271526265345002764079101479397547316499290313507850604896497427196886342256
 364829173307156339648959399574711685835340*a²*e^(573/2*I*c) + 232559508450
 689666304119972963001914495032187554568606135867970661871572387842192521274
 4802710528140695848048863263401620*a²*e^(571/2*I*c) + 22388133772086363214
 562711695531110839152740444990636033971999650339788664043245120079613369281
 54280808573579878469429140*a²*e^(569/2*I*c) + 2142519646190467116238147467
 579260223129924514404251375547828707655707348428080560507386311357900526669
 458781794087683160*a²*e^(567/2*I*c) + 203761320423430278838887297426355980
 292248543580195171864518263707155037189319798754507598444565508555368911946
 1288385080*a²*e^(565/2*I*c) + 19252636223728394750051188080130802855312516

229284947971945519780347026927574767180251081864547831854692676262761903477
60*a²*e^(563/2*I*c) + 1806862685474014061358992781500638421824105526179711
819772973596359005838247462282270364980307784363004235593075525641260*a²*e^(561/2*I*c) + 168396739021853566943248119227362402641388949862053436879077
4587302981877714639969367635542513279277621696310502074133800*a²*e^(559/2*I*c) + 15582375841235407475051527824265404338649299690182902394457588718153
26594322852854852718664426477851831755925555671727660*a²*e^(557/2*I*c) + 1
431371913344993286258145794658789591128135731543344495632675226984270430752
938353377406809754809644845668868338821975120*a²*e^(555/2*I*c) + 130504557
526466671962244573856964648664394200563472193631664604260853511548538996058
9294914491574041407279419512731576020*a²*e^(553/2*I*c) + 11808529906611396
788267529739649004013694888958024539109846840243868929535966239993555593680
55504957627536059400058806560*a²*e^(551/2*I*c) + 1060257963749536335147817
118629193972904284990698909747848265740102938771758373176878551328532733642
490124903197641779440*a²*e^(549/2*I*c) + 944553238998322012684234360243733
605735140889390389676408061822904814729681532931792335624967472663525610160
924422346720*a²*e^(547/2*I*c) + 834830647252737130370352256191399634154462
308618312283624738448826183345582990700043901989251342801050174088405942776
680*a²*e^(545/2*I*c) + 731962314882746930932643298184913142623544929508542
538052238437217139684503440864983481224441799145110101150208078149600*a²*e^(543/2*I*c) + 636592737563033962038907595159410207630226126748551544972393
205357335273235966560353474541619072109609425303261030333480*a²*e^(541/2*I*c) + 549140935306437649109502279093298286642071151104653437705824618506516
690970915485474849970804454250926034882525081421920*a²*e^(539/2*I*c) + 469
811437014079749298372016621303751653441794274933681143305269482262555422078
791132625827649344909492073035868314640200*a²*e^(537/2*I*c) + 398612508602
184099797618213464893583100423327557993330634195015291012852305358265371442
241359113106146519054690230460080*a²*e^(535/2*I*c) + 335379844326285592736
695463857018418432413188766484881111912395059240855249328246830729779350439
108234296749061439256880*a²*e^(533/2*I*c) + 279803880980823081151507855346
299192278245341814031460411506106372469276985146379946315124170832410518829
080258705986280*a²*e^(531/2*I*c) + 231458954877865680481711467581492320475
708179284410834307431449083296993015735283992533153783630057823894466466386
789920*a²*e^(529/2*I*c) + 189832680846992441750979398054550765983447380706
582698086505079016572222835305196482037082520548635494540196474773032392*a²*e^(527/2*I*c) + 154354165743697402995415216121916387446315508628912131928
600309585745568868263338383595893423751250560255188896660980640*a²*e^(525/2*I*c) + 124419949314487403216172588535238853667583569695346393000362307045
710566468085416110994554478758231547427229315129958280*a²*e^(523/2*I*c) +
994168666894028682059531007228162611012718084502482522579816113356764319230
25899234859866205857872672547668167325989920*a²*e^(521/2*I*c) + 7874132585
546071069906792098596613272993668909384846081647513493184683001031856523986
6378589856356517618876735439697520*a²*e^(519/2*I*c) + 61814770854015319982
448244739487435605552785122978910481378435042090366500632726628793497290765
837977272563559609588576*a²*e^(517/2*I*c) + 480953426699660560207265457040

061992666661606303610137551588780636613432106375967251049034397699998476242
53161931778351*a²*e^(515/2*I*c) + 3708594553993260963723097865204034276410
360777816453731587009464079035003417707172319389206757754051041582732120207
8290*a²*e^(513/2*I*c) + 28339072375239015657801316797536069668788909732601
389032465333712850620264720283658988975362676026266822753938421168705*a²*e^(511/2*I*c) + 214588392496363848785337067347919252128479014272105251978648
86973863458734886342259576872792909047798523915929365691940*a²*e^(509/2*I*c) + 1610072907829930926448116373437511521483953182188018802392446777870780
9792051397971564268356677680703285577513151431841*a²*e^(507/2*I*c) + 11969
554932567565593177991401487663191149947263758650888132425196237881390457894
009588904534530296178324030985072573618*a²*e^(505/2*I*c) + 881613168893121
181405720811553391735580178702933557560041097724239982955992461005161551364
6540527419139301483310423786*a²*e^(503/2*I*c) + 64330993352691958087170206
396618494599861960856359687277765646258499326392809824960345969460078087581
61920095325761890*a²*e^(501/2*I*c) + 4650280530057130827843153624560981667
840741811626179819584824809783591908945012506423661839456267739976826873672
738365*a²*e^(499/2*I*c) + 332988643897483299252810109420711447828595587261
8100718692457613200121080012023835679244861800931812439160323444845807*a²*
e^(497/2*I*c) + 23618147825924090731401517179520551201564993752366198142604
03866479477744294234718960967782715041717115313047558623183*a²*e^(495/2*I*c) + 1659217340078990366295623422038456618088548686457548538185410690033650
573056920057414962831294588203107486826119787423*a²*e^(493/2*I*c) + 115445
433590485736424798255791257871810274684969177031422018671270603409920792753
0450655444722398232085044436220634697*a²*e^(491/2*I*c) + 79550229077709817
140813486956045123684346956459551614933650508510524229042475860769294568992
4007436094637703203085305*a²*e^(489/2*I*c) + 54284095940246108309656539990
53400760902549927897609443377448908629923584155703988243059101873944263638
4860204215400*a²*e^(487/2*I*c) + 36681387023092159414310865499215429941201
020190629449080641286983995639286027102152174529408608623383776498147347617
8*a²*e^(485/2*I*c) + 24543499517196850297668507975791665463235202112187823
6552926031364907280287667538662828687293957579330538913455736808*a²*e^(483/2*I*c) + 16260003126050357967763986974955772209925940814396948313847036278
4417124438163290111258560462574993542827637524092781*a²*e^(481/2*I*c) + 10
665326172972472293227551257884734576057409584813868604720936073518886729792
3054892788279482015629448673456237211040*a²*e^(479/2*I*c) + 69258517208288
436651591793330966261047861717319283852998403408459459819999319356554087437
075858538315437799030208205*a²*e^(477/2*I*c) + 445239915206465860769067865
644659230606049419851837758185464614786775195102888323947153662562170637135
89099772007856*a²*e^(475/2*I*c) + 2833435569095910075856346783902184507634
209486006385880783671350359538756924662880197422094362959333392221087875117
7*a²*e^(473/2*I*c) + 17848741370378512799153304742903183425864668367256735
229392362669226980832673201354290319227799410874360671281422106*a²*e^(471/2*I*c) + 111289366630804461073300894765442967779833197921448973698572568707
54469969651791477159202549359802639971419696605690*a²*e^(469/2*I*c) + 6867
972157551236921759647848323040789653661241470014722668084616115674752655075

386898879908209190965790105909303585*a²*e^(467/2*I*c) + 419478070904728048
322481583394002255581670790344286484308338898323562550690003331322862894095
9072838355592388518976*a²*e^(465/2*I*c) + 25355554133458355415663215796713
320822307450379883912694516752439648444683306792410349302065788967118791554
18196133*a²*e^(463/2*I*c) + 1516692011867012115321043946651658946449294899
763438015725740120744131654354391550015561531757951384128294098404528*a²*e^(461/2*I*c) + 897758618407588265406502003227323745918017671431846078493905
129440539575848523829309505570503467433698687391599525*a²*e^(459/2*I*c) +
525820101170441091857872327398452496414525760936913287583911529580580538586
559649846585541562686976417582636323800*a²*e^(457/2*I*c) + 304724800478424
739825811157956168270645458485759770904702074659803850468461952919986071057
980917692816270458765682*a²*e^(455/2*I*c) + 174722600888404804978530590100
460003278180658978831905330154370982763984646513813160761146082864787793974
499383768*a²*e^(453/2*I*c) + 991148498584213443894052367589553084320172585
78325989313493313927809241858243075104249489142628498003793332896533*a²*e^(451/2*I*c) + 5562300342860253243302688841389823386423061808324476336590163
1495829738789432935549737175085734259798422054263257*a²*e^(449/2*I*c) + 30
879760370351772693035730827584443094725992387048496282785137580349540025068
324342981224379621249817077351202755*a²*e^(447/2*I*c) + 169580097860474843
198646376739561685689256076474194644211111870174182144489762338798334775975
85335045518017550375*a²*e^(445/2*I*c) + 9211598000870958612872710960061163
302395796931864103642364216853868494654352223629534260533174654764593528061
971*a²*e^(443/2*I*c) + 494916366309989584381316250645382023285291988525334
5976704821171775106226971407006064744321532373168942138457133*a²*e^(441/2*I*c) + 26299224000359092886055932289847481067460973346485297026949249364838
74523991116960016579495951972600623182611066*a²*e^(439/2*I*c) + 1382120082
453709333872510587412059524149979254964511066161605009479514122231452777976
669355954779387326179193890*a²*e^(437/2*I*c) + 718319539770367264276023193
923607996369471257717391142715655747378128411474061415600744631353742653947
474979790*a²*e^(435/2*I*c) + 369178030253342447121644615776637010615775278
423789324847337963965264696957387110053345923248330973169112504125*a²*e^(433/2*I*c) + 187619225221186892362954324173464256888580245686654501161811526
819167428547859856880414713989939337716707545896*a²*e^(431/2*I*c) + 942798
838602234979277058133727614086602728476369889354672576345987996355484700938
70330610958925793769077443101*a²*e^(429/2*I*c) + 4684223968783713426910236
443923648845093519456603766005211936179651041190321991436923118122157026115
0235170750*a²*e^(427/2*I*c) + 23009632000715685724501189026111050742267523
553057578732719843669636960476167526014756393531959162197925032115*a²*e^(425/2*I*c) + 111740695802813431723294499992997506283025476247658997341712907
17678737916201127059517927335920301363381599920*a²*e^(423/2*I*c) + 5364370
581776430676023366865026628216859729631724179263855968598618942856572042977
385869136797574101112934360*a²*e^(421/2*I*c) + 254570242524891909244474523
190339270693780626351542068606228623253312504063393294401364747678282575191
6866192*a²*e^(419/2*I*c) + 11941350382182318927892141816101487084376646887
1925302453933169008901674631219082255525802303755766475255060*a²*e^{(417/2}

$*I*c) + 5536449762385042687607369329082039089595176262840112189376452062542$
 $41531004929708538056155465373941573477840*a^2*e^{(415/2*I*c)} + 2536975400921$
 $549248034126032133684203102252154338121939065797094383402025646597247218225$
 $25055232970075634740*a^2*e^{(413/2*I*c)} + 1148901068512382915513357282655623$
 $64765121005119591497024634815615328060989866188627682839323290553007949840*$
 $a^2*e^{(411/2*I*c)} + 5141675317373213986665423945495911958975588547900983523$
 $7894780038895835558829259413468345602675738630164100*a^2*e^{(409/2*I*c)} + 22$
 $738199734133653078027776734892344808364750849791537027378856109800091278175$
 $916637691635582433639500261560*a^2*e^{(407/2*I*c)} + 993597053594903087813334$
 $698317142855596825779938110671421541390503387260314805042522302535115413041$
 $0277240*a^2*e^{(405/2*I*c)} + 42898317708769156046206216052503119586771288875$
 $69530679923476007397181724403545145491954691454914942555700*a^2*e^{(403/2*I*$
 $c)} + 1829862670948031313289014198087321263556435400853890848564941806503546$
 $501752216732765684931337636766894640*a^2*e^{(401/2*I*c)} + 771111302353098224$
 $996612507872558890266159399819221015025614890733856653497550312410883732182$
 $309081849060*a^2*e^{(399/2*I*c)} + 321001870082230626552157935960255206975703$
 $259672220019809311160664723949929848535867562171221207552583920*a^2*e^{(397/$
 $2*I*c)} + 131996091090182003447057411825399119687071252908752803875765221141$
 $863578132827930499563016110773453608580*a^2*e^{(395/2*I*c)} + 536102912029971$
 $292433872275107464388804173961339904754012001518368245110333590618304178181$
 $44623930466160*a^2*e^{(393/2*I*c)} + 2150497228392055035861282930997456794673$
 $9006313833634264671213989597677643264769478110594714212525857560*a^2*e^{(391$
 $/2*I*c)} + 85192764511342537785361716307460782998733243163805904144535375550$
 $43854967995509903575577929995103297680*a^2*e^{(389/2*I*c)} + 3332794513905359$
 $682186269108401623669887724824050879815632501441392605125415686565518742717$
 $075933147150*a^2*e^{(387/2*I*c)} + 128743308045757357230667360505210675848691$
 $7694203459437629697821743582350653265565508291332729874085680*a^2*e^{(385/2*$
 $I*c)} + 49104214185687835071568175605844513855269876929742582666839619988284$
 $9855448274647622775780304699322930*a^2*e^{(383/2*I*c)} + 18490892399403935785$
 $501189770402960332363734973008312623715485397510290498778556393349066677191$
 $5454740*a^2*e^{(381/2*I*c)} + 68739812560972530921511497106963251745177527056$
 $492095301601973235793309670088282296954444560031434930*a^2*e^{(379/2*I*c)} +$
 $252252716628002121429694235425673024844174293101940357551113107420775796892$
 $15072021281081661345709120*a^2*e^{(377/2*I*c)} + 9137036326419012083117592281$
 $553565672775885047100209635745422954421398356217178283444278671884412660*a^$
 $2*e^{(375/2*I*c)} + 326648792310303813099926514082694062405040840445634019582$
 $8586033626485345825342237364184398762915620*a^2*e^{(373/2*I*c)} + 11524614778$
 $722545432398884845151494358758349066560138411327899216487643850548107464116$
 $96406101981510*a^2*e^{(371/2*I*c)} + 4012398805097958468657622180149700166218$
 $60421334029285134283286090912984697175449116164770568435310*a^2*e^{(369/2*I*$
 $c)} + 1378402087618779604363496521253969751330981360186907193389249492259933$
 $95730744805382442729064152410*a^2*e^{(367/2*I*c)} + 4672000737507884896003900$
 $0217563851982925567103892420367979003637368846369023717951090246855155470*a$
 $^2*e^{(365/2*I*c)} + 15622306566440921026343663943462426741562514861595325201$
 $248149059018047548245480491740664747553230*a^2*e^{(363/2*I*c)} + 515302087148$

671187341649181513197595190043424602514235251655281822427348329754377674565
2059545730*a²*e^(361/2*I*c) + 16765303716452633937145844225799787825635321
49514208800221991656698632081178644609104890307855200*a²*e^(359/2*I*c) + 5
379613186712719119358120474976420724610694158490274473564618302712499421643
19267167904932016060*a²*e^(357/2*I*c) + 1702303917138093755802210803550670
26912456772746194729300836144395351294968707996644726411879200*a²*e<sup>(355/2
*I*c)</sup> + 5311593939382952910273717641136711300549739931527030934110845476813
6564764298294135564858847390*a²*e^(353/2*I*c) + 16340621960942206338329487
769224971352662464448783601559800415763231873879690981091170280400080*a²*e ^(351/2*I*c) + 495588674941085428363783640892237090886142369323655510073648
6744859805066952581996944150188670*a²*e^(349/2*I*c) + 14816145216650511407
91668111046174156161377332659662553088798777522638436965972191593890153520*
a²*e^(347/2*I*c) + 4365770969617118557692877147033573898856451328865797312
06499384230924672871218525038020148230*a²*e^(345/2*I*c) + 1267792359146086
032784670876984604836428345234732732178548674589773357755580830275403088057
40*a²*e^(343/2*I*c) + 3627814697373286004523746845042808477689053274892668
6396879733480311191302936923550162289900*a²*e^(341/2*I*c) + 10228191850120
678931201975750431518653979982536759165359640298725615598898961878198815353
510*a²*e^(339/2*I*c) + 284089448862922201077384763498075685555416028670802
4741922802485707803328368153641110473200*a²*e^(337/2*I*c) + 77724524846383
216444357946431774042494618962849494245350858044180074482850986729141708163
0*a²*e^(335/2*I*c) + 20943564124649634228837486295805643875389106897219661
8627904344080420177337078104676468880*a²*e^(333/2*I*c) + 55574294007405657
159410702225110128247423080333361478620342730912591089336970706714049470*a ²*e^(331/2*I*c) + 145200432377710248675788922748331519999838056557749424321
02559165991274651263356626027360*a²*e^(329/2*I*c) + 3734827777542842962820
944497158764120459066697240394589825664346513551225394202631289660*a²*e<sup>(3
27/2*I*c)</sup> + 945623831015067465565761737751781712729579043386050190772281518
980746360423316243590560*a²*e^(325/2*I*c) + 235638414403676636374964618535
168235461531651215096600123550185747704196738532598229970*a²*e<sup>(323/2*I*c)
+ 577811943316180501840758724235617914218490352435716003202887065329816093
31680804189710*a²*e^(321/2*I*c) + 1394025369616807940206425969265350761294
5211052614130496704459947954824765033193710270*a²*e^(319/2*I*c) + 33084732
197038235375562437697635708153491799435857062717116045714440947701529191247
30*a²*e^(317/2*I*c) + 7722970195333403185147672564458026962974963378029210
19409004020665598966438156162590*a²*e^(315/2*I*c) + 1772824600327718658920
99947703219400509401728120870156939007264944328352108674765830*a²*e<sup>(313/2
*I*c)</sup> + 4001239388966249061627139921361425739977400940247399666641648062515
7884768764896580*a²*e^(311/2*I*c) + 88775072851962206182416554647787324999
35221664373951650797658698877398128936202900*a²*e^(309/2*I*c) + 1935856940
887066757158648696108358375586341793850855062039959891604593248887646480*a ²*e^(307/2*I*c) + 414817136098345744491539277362007380513271882803145751297
848219367978399846839810*a²*e^(305/2*I*c) + 873281382772633574536873936328
29255820603582731737822656013656737314066270769860*a²*e^(303/2*I*c) + 1805
826214706251079990263321103674819022743994744768011661354696943158714965773</sup>

$0*a^2*e^{(301/2*I*c)} + 36671548855755560353542103709223003623572101156675340$
 $77868226474380300702979520*a^2*e^{(299/2*I*c)} + 7311701215847240332209644867$
 $73088649792495972559727457509380085597040672846910*a^2*e^{(297/2*I*c)} + 1431$
 $01677493167706049469001333839411556496168357433098121521463359375252809040*$
 $a^2*e^{(295/2*I*c)} + 2748565450074484594608686469395141672995854442108800748$
 $4546058264731062865400*a^2*e^{(293/2*I*c)} + 51796100045678396768827343720759$
 $39558403884380512473251576883045496625080240*a^2*e^{(291/2*I*c)} + 9574335222$
 $91942572603972061174768931380078714954598245160747546204280979860*a^2*e^{(28$
 $9/2*I*c)} + 1735513522108301115424037299977365486595796902621613346843783546$
 $93653143600*a^2*e^{(287/2*I*c)} + 3084170994314575597490646630687109091970533$
 $7121093845564611475205831834740*a^2*e^{(285/2*I*c)} + 53717908235477759867693$
 $11672178060666294358172540326849409957986665686640*a^2*e^{(283/2*I*c)} + 9167$
 $36869251326841783602023183005234330809250515685392295995174302631940*a^2*e^{($
 $281/2*I*c)} + 1532446899982801063032321334866289582450192313582313170093986$
 $23103014360*a^2*e^{(279/2*I*c)} + 2508456856052548652379307576583978693966816$
 $6684250998147267060999843864*a^2*e^{(277/2*I*c)} + 40194675719721919184952054$
 $31223264437545950953166839320312436248766740*a^2*e^{(275/2*I*c)} + 6302690115$
 $18917093786807783619832131991400363103105059485556854864080*a^2*e^{(273/2*I*$
 $c)} + 96678088682187070566952755698159384671499301435475363946248645511300*a$
 $^2*e^{(271/2*I*c)} + 14501667734089456372413097720073331427769744768582734443$
 $920820884880*a^2*e^{(269/2*I*c)} + 212633835043406710184106617998672373630972$
 $1747170637151333282385892*a^2*e^{(267/2*I*c)} + 30465007348962406950131966684$
 $0972753356711517554147295115707791952*a^2*e^{(265/2*I*c)} + 42633155182758234$
 $213432657105359507039157758216710091608150751160*a^2*e^{(263/2*I*c)} + 582486$
 $7841884974805556257455805728440177379367311530786366553840*a^2*e^{(261/2*I*c$
 $) + 776647773894140039601026125724635184851400081828079008519495205*a^2*e^{($
 $259/2*I*c)} + 10100895944839427862206527028135448823676742987412977440237212$
 $2*a^2*e^{(257/2*I*c)} + 12808036621165917614961411495703359665564662395820821$
 $879432651*a^2*e^{(255/2*I*c)} + 158260052711372421094527069215448738453771293$
 $2377861494537792*a^2*e^{(253/2*I*c)} + 19045604731498059626221445575883017641$
 $2490479327548630395435*a^2*e^{(251/2*I*c)} + 22310547737377338728195792669384$
 $697517889412712723300156650*a^2*e^{(249/2*I*c)} + 254251082202480904911129205$
 $2295925694458486937681586168494*a^2*e^{(247/2*I*c)} + 28169847737038660670859$
 $8633818891039165559192659067598486*a^2*e^{(245/2*I*c)} + 30324466873468264078$
 $058992476301732181384084447966749251*a^2*e^{(243/2*I*c)} + 316950508533267722$
 $0309823086926906379156042673077997829*a^2*e^{(241/2*I*c)} + 32141448551790707$
 $1866518386845402404565806226718602745*a^2*e^{(239/2*I*c)} + 31599729267161332$
 $823403075503125661623124842474211925*a^2*e^{(237/2*I*c)} + 300949727040266437$
 $2603369055821186361937123336960151*a^2*e^{(235/2*I*c)} + 27741170164436548637$
 $9380075778816997842958595635411*a^2*e^{(233/2*I*c)} + 24727501166647061083289$
 $528750467452594464413496712*a^2*e^{(231/2*I*c)} + 212931229780003360396510945$
 $9542894851379636241390*a^2*e^{(229/2*I*c)} + 17695113992185007463319454900104$
 $0224465598018440*a^2*e^{(227/2*I*c)} + 14175642468718517834053796420531668264$
 $840828827*a^2*e^{(225/2*I*c)} + 109343790345041347669477538808524814343584318$
 $4*a^2*e^{(223/2*I*c)} + 81106652744979453503605904560553941804516827*a^2*e^{(2$

$21/2 * I * c) + 5777459917323047305837478939343390617560800 * a^2 * e^{(219/2 * I * c)} +$
 $394635225337657865294965990982357902779935 * a^2 * e^{(217/2 * I * c)} + 25807207438$
 $709023405564698654381610855782 * a^2 * e^{(215/2 * I * c)} + 161295042854872202116087$
 $0732573388748486 * a^2 * e^{(213/2 * I * c)} + 96165064556404674374591793494770969751$
 $* a^2 * e^{(211/2 * I * c)} + 5458017107743192900932192332079754800 * a^2 * e^{(209/2 * I * c)}$
 $) + 294232725411218343537913910694524115 * a^2 * e^{(207/2 * I * c)} + 15028015442916$
 $47278430335549568064 * a^2 * e^{(205/2 * I * c)} + 725212537982338639553752606105971$
 $* a^2 * e^{(203/2 * I * c)} + 32964206161728793499031816369336 * a^2 * e^{(201/2 * I * c)} + 1$
 $406472793060742817278133399694 * a^2 * e^{(199/2 * I * c)} + 561092868743680684463846$
 $18040 * a^2 * e^{(197/2 * I * c)} + 2083633993393098160454111295 * a^2 * e^{(195/2 * I * c)} +$
 $71659370102807444959242727 * a^2 * e^{(193/2 * I * c)} + 2268898261098769561120041 * a^2$
 $* e^{(191/2 * I * c)} + 65678633862570503690097 * a^2 * e^{(189/2 * I * c)} + 1723848657664$
 $106290905 * a^2 * e^{(187/2 * I * c)} + 40614235388489346675 * a^2 * e^{(185/2 * I * c)} + 8483$
 $39120374641870 * a^2 * e^{(183/2 * I * c)} + 15464515215103622 * a^2 * e^{(181/2 * I * c)} + 24$
 $1005431923542 * a^2 * e^{(179/2 * I * c)} + 3121832019735 * a^2 * e^{(177/2 * I * c)} + 3226699$
 $7620 * a^2 * e^{(175/2 * I * c)} + 249487095 * a^2 * e^{(173/2 * I * c)} + 1282710 * a^2 * e^{(171/2$
 $* I * c)} + 3289 * a^2 * e^{(169/2 * I * c)}) / (e^{(517 * I * c)} + 418 * e^{(516 * I * c)} + 87153 * e^{(5$
 $15 * I * c)} + 12085216 * e^{(514 * I * c)} + 1253841160 * e^{(513 * I * c)} + 103818048048 * e^{(5$
 $12 * I * c)} + 7146142307307 * e^{(511 * I * c)} + 420601518659718 * e^{(510 * I * c)} + 2160840$
 $3021340047 * e^{(509 * I * c)} + 984382804329835768 * e^{(508 * I * c)} + 40261256699368950$
 $388 * e^{(507 * I * c)} + 1493326612293984160368 * e^{(506 * I * c)} + 50648660944512569972$
 $179 * e^{(505 * I * c)} + 1581796642397812408161814 * e^{(504 * I * c)} + 45759117183402579$
 $073139583 * e^{(503 * I * c)} + 1232445557346832245176696904 * e^{(502 * I * c)} + 31042222$
 $522074681615625020522 * e^{(501 * I * c)} + 734057263616388449968842366924 * e^{(500 * I$
 $* c)} + 16353164647151530240529137618111 * e^{(499 * I * c)} + 3442771520128751341407$
 $39302960914 * e^{(498 * I * c)} + 6868329225263681349501997341320517 * e^{(497 * I * c)} +$
 $130171193079172823835151430773360024 * e^{(496 * I * c)} + 234899837424434707953276$
 $6203075607598 * e^{(495 * I * c)} + 40443624781415311581857832389099634564 * e^{(494 * I$
 $* c)} + 665634670676210063754191847109971141414 * e^{(493 * I * c)} + 104904026695108$
 $97424624643766470754045064 * e^{(492 * I * c)} + 1585664761132575625661174322272038$
 $84298856 * e^{(491 * I * c)} + 2302150411226234925855222345201500900533576 * e^{(490 * I$
 $* c)} + 32147887693375338817454482515377350383950278 * e^{(489 * I * c)} + 4323336886$
 $44261557547944179250800440604964868 * e^{(488 * I * c)} + 5605927253067558551780452$
 $883689835514455118670 * e^{(487 * I * c)} + 701645153225444629068735488137480910845$
 $61870680 * e^{(486 * I * c)} + 848552202276512356496200136959676295361696315113 * e^{($
 $485 * I * c)} + 9925490738534402272939987038714580495445431374618 * e^{(484 * I * c)} +$
 $112391604542246650966429162063124338952554575234051 * e^{(483 * I * c)} + 123309670$
 $0139723365181997220750932590655287625342156 * e^{(482 * I * c)} + 13118781801172174$
 $729679339894318153694964675368481194 * e^{(481 * I * c)} + 135442594916636116191574$
 $650625331646238501101627937224 * e^{(480 * I * c)} + 135799066316147984285064284803$
 $2544982878359839580349899 * e^{(479 * I * c)} + 13231708870104896973800056733779919$
 $089340836756009580718 * e^{(478 * I * c)} + 125370496586921272662198050851269323171$
 $167338854081782959 * e^{(477 * I * c)} + 11558554128935942603454496664268782363003$
 $5899363232371472 * e^{(476 * I * c)} + 10375184499871175501909398956596684116802997$
 $082526660323524 * e^{(475 * I * c)} + 907226057222088149186422846394871877646075897$

06493970774776*e^(474*I*c) + 7732046369911457750614627310280985060944326757
 88136295011259*e^(473*I*c) + 6426195485535248576425068136870465530087114003
 875716691383902*e^(472*I*c) + 521081176291770486604924009851758309875057005
 66877818954141639*e^(471*I*c) + 4124306982999151908480672223272194350677479
 34091894670488982928*e^(470*I*c) + 3187749929744346497211536044751776582320
 958627923816470590659024*e^(469*I*c) + 240708019135297571018580229143720458
 64746991786182039740274325264*e^(468*I*c) + 1776428291351193485771944376758
 02830239905460092687136494961404333*e^(467*I*c) + 1281817464914970810859604
 189828359000790789921169405304612211251818*e^(466*I*c) + 904669352382568297
 9044338963104263167672586826367911338826483549173*e^(465*I*c) + 62473550781
 053295317710774690247114124125187565731848441781904032672*e^(464*I*c) + 422
 276126632003687547754746555709988710527133086660161366353656787288*e^(463*I
 *c) + 279470910447568661184279069497369916448225472397721020972566130440347
 2*e^(462*I*c) + 18115768495615758076710303055505625589254293659193314153418
 333944596408*e^(461*I*c) + 115051481852080848873700388354521315567640365124
 003103691176697194292320*e^(460*I*c) + 716099497599058079895633338552940229
 192858196481597830078819711862600096*e^(459*I*c) + 436944248291011391456535
 3136069595862669338858053419381214131241925047008*e^(458*I*c) + 26143976279
 902021443471945665080254563056810183520401889800285493144867448*e^(457*I*c)
 + 153436088745056254127327239461577071933130157764595997113973513183188399
 376*e^(456*I*c) + 883500968821791202600774541927769200737689393513734789368
 397093333311961880*e^(455*I*c) + 499251971245704398350537797660795398839736
 8297591114957991804893688371867680*e^(454*I*c) + 27693116538343259225983382
 637647936122664033859615133489846664694361471028310*e^(453*I*c) + 150822381
 431412413773566474210011746852297437597059186295243989481140398152780*e^(45
 2*I*c) + 806679543607589140759305010796189568269842021613388955218916278823
 182639488190*e^(451*I*c) + 423812584676323258639418856985868582675532800554
 8627437019301405851325887594480*e^(450*I*c) + 21876482892713909928040345612
 578705805121508756226696317087651824252241418663320*e^(449*I*c) + 110969199
 687320974749922259595250444341219218535349655762591192576535872151766080*e^
 (448*I*c) + 553269128819528612502918869558947829098021956309349843584044631
 512291778800081490*e^(447*I*c) + 271184323967071752760564049014883350713024
 2448403978318523237721944200392830108580*e^(446*I*c) + 13069817203488289886
 193205508375818392124991382340160316886507181296548981014818410*e^(445*I*c)
 + 619485966530355028795643388152343106604109020378824731618047744929162165
 75880077680*e^(444*I*c) + 2888207552647306544699685720210471094273186195089
 95802020689904590319476295408324280*e^(443*I*c) + 1324756412367837473157472
 821162483691120966501948953926492241643788264284546437221120*e^(442*I*c) +
 597899217294414321845916114929981970632173211157849452524522874297646840910
 5395536290*e^(441*I*c) + 26556806389043407534496702369101545795994861757741
 414789944652712127566910185274123140*e^(440*I*c) + 116104551683555043762911
 501712116399313733021132677481112824047246361794049635726479850*e^(439*I*c)
 + 499707567253859084357596314813794768069337190915967491907488904933922677
 579665354338960*e^(438*I*c) + 211758973346685570710150142921041472240183883
 7940752841618541440888545729943138209036820*e^(437*I*c) + 88367206408604703

05694514021547969551296794092266983044118375790025854584036796364768280*e^(436*I*c) + 36318369652302591732197444409798122022640824604130552506742586795183267354382847875885730*e^(435*I*c) + 14703081673227683316304158209959204751204372522535339238819165193000407629544745753221740*e^(434*I*c) + 586403466972683242741643328921560909375197453864243299571990964608857245771134145204174990*e^(433*I*c) + 2304351073373840357379178597673066352016682781689139842097376663118488803841131935313641840*e^(432*I*c) + 8923209447343296763331881881638471793499618670601026059730895962653291770229493028162575100*e^(431*I*c) + 34054053851295569154352346722177172655187548910782008504718324168725029438589162349211628040*e^(430*I*c) + 128098914601688539672480541830409847707367500438601536803204497701119911289087105659482783340*e^(429*I*c) + 475010578857601519272316617938425222421786597241671026894318515408511467140969393115768793680*e^(428*I*c) + 1736574218818191071874197472450158123883564209950658639102337148122769080611680719741726053840*e^(427*I*c) + 6259872156822252843650960708235034710201362776057176647226323089751446565288850103898153859920*e^(426*I*c) + 22251959176795777757167366036007480222211364232146399803864370963391491223687245823457351580140*e^(425*I*c) + 78009807368024239875613733058851417125327114681070889640794249282633470580756557083923203377160*e^(424*I*c) + 269745801440211296972683601863878954357962308520076595177128227629273240215209708218497363414140*e^(423*I*c) + 920089393029589032874601850027159322612526368444771489781974361078847528891468831038436064951920*e^(422*I*c) + 3096131971621520162380301554241465451782362086810287537748902904985934020179565706177131421614590*e^(421*I*c) + 10279364730663840844739577862469262604648861914297972589165243530651230690726244462479199894255180*e^(420*I*c) + 33675398872021568375902384593982753362559801058104184627345411136262431943240778260721756991027090*e^(419*I*c) + 108867995731829472826732905192034886797284621356445627530909104429486741257822633476898356826454040*e^(418*I*c) + 347351473214713780874352083129566601238765762775942366762733349952103889753982636403857556867777300*e^(417*I*c) + 1093853214486220358674032434500866678499770011305874172488975951612031456734608287095519501041975440*e^(416*I*c) + 3400232560601651617521694680847089844198028831694417424794868779328950548418125605446882081152636090*e^(415*I*c) + 10434117516570395966653693155582402109460348095473027807412321427346816928567197770376496170251803940*e^(414*I*c) + 31610939331284692750694306443618414656095969520945215743004044560386895241801579156543451940713351730*e^(413*I*c) + 94556180258931986919334303466365652826858091314329189160736277175873841732196453379953705679466826880*e^(412*I*c) + 279285755800035206679835368898165477644864987794665387827488933863633745047373109049265172681702585720*e^(411*I*c) + 814608187736530579670210025271921415597183369881214299823291969785549876175969866367976653244974728560*e^(410*I*c) + 2346518219239105142238141633073464768899155708935025778047637412681781575765422219127409260159438712250*e^(409*I*c) + 6675866290371147358503766865669289010893543869830538708724945291580951179188296606158111257706968604740*e^(408*I*c) + 18759988218865563564163635735986073278255737257405706279108891366378428467414559930481172863538598193890*e^(407*I*c) + 52075178518793270386429263351544306951104993542

500582938155241689408138675254608030847907167748571734720*e^(406*I*c) + 142
801792450221762483180874918825274134305133275417780084795034644763509333503
150517345864659667189417080*e^(405*I*c) + 386876218234277165632451723049979
889263115282374607541692443176673997513742813591736171169652250611186480*e^
(404*I*c) + 103556198259200293522638457790861154861211149508019357369133986
4706029186482466241805664949381049856258510*e^(403*I*c) + 27388956247952656
033552276465660008862807783050848257029119389036561620042627361826577004063
01914070062380*e^(402*I*c) + 7158124686842941475473807363679839718172745581
538409044503383852693596921622426696740453944718143025248390*e^(401*I*c) +
184874052990057326937527286118764908908583570219748823715706238001862451377
22660943641752976852924439870880*e^(400*I*c) + 4718822084346620769509950695
357378035710889749142256789804819901820770899700533386014883647952745615601
4520*e^(399*I*c) + 11904185540387796494822957794837046560060662318304552952
6900430209270473212773847794935586074714329479939280*e^(398*I*c) + 29682551
528266958968531827328023905008455503220341594151196265959688161571379993768
0026497408305672297618840*e^(397*I*c) + 73158497220681836287472962140397444
4280010446301161527339760544815300951787985538419764656214582667219914080*e
(396*I*c) + 17824461149317518505563548566384219011744123222982494965916580
53939787198246565945975595575734193348887952160*e^(395*I*c) + 4293206478008
022126017488908851826494790620720660151451468181910917240027863968724539127
659633517053002976480*e^(394*I*c) + 102231820259548607672173903051864519235
62145473674293619918063490411487496121804590274592702770571515456414680*e^(
393*I*c) + 2406878513970527716119346564450614328524136103776821681892218440
0141048460210944696647752723371932874594597328*e^(392*I*c) + 56028683424903
517658495013858534516167162591034367972498174660907450666778154353271630344
650777885683547624184*e^(391*I*c) + 128967080084754712246023680866488384983
286259025533132044636109049545144029547003347761521666283977931640178464*e^
(390*I*c) + 293550743554342709808129453576562313299705982699187416862934373
964255615967138676253276302591561523515603264403*e^(389*I*c) + 660764473105
869097691475973850837934511089033149586707982764263394766756649565279879146
173318386505740391093990*e^(388*I*c) + 147093114661893434551503836230010016
048212774958144392990474691022477747019889905237911449399988700319941982957
9*e^(387*I*c) + 32384919313618514764233219335395790983777355392076414673462
35665823887048326949305609231585143748690203615957136*e^(386*I*c) + 7052132
414162197992602326524580143060985353054572933905524633121681021037340298366
342203324325307072413739061024*e^(385*I*c) + 151896342149088003964179117226
437547480485201097348124591098788104938443810626508189711996371214587494562
43274416*e^(384*I*c) + 3236273132241954941033008894364024746037832856131642
2931292427145902887913071643679502909055891236755143207382609*e^(383*I*c) +
68208033096793615683784409619244210818614991640041553424405527876893272496
608324231098148502466453967157728078994*e^(382*I*c) + 142213115964814517682
386667276769909482271681318790889840501039441748635545362467679832449103520
321953011780083069*e^(381*I*c) + 293344920034300720287042383448342866313806
285455040067823080445597545970023446231563554135133105493516316320059272*e^
(380*I*c) + 598650141112241858911676505180520150364003226841328081453597093

587790338609212439085554466861582623350303061961052*e^(379*I*c) + 120877035
84936583930894422205693506328370410814059375022653984611773764821660955973
4831601248698274330296158612144*e^(378*I*c) + 24149665168103385032890765492
027405117100590117954471387734642056964550264427124264095996627710802648260
08985061097*e^(377*I*c) + 4774141111066098970221845330594962016472714230374
234060663956846950926642685946929064114194400360936223590725470146*e^(376*I
*c) + 933934195805349422525175096571505730070730208381477477030621822424102
2648247419956042957363055823830898547303219757*e^(375*I*c) + 18079820068028
859970349938623007230676563314206708848499900139641237334763266479346963237
936039328113185041591793848*e^(374*I*c) + 346376571726716901676573445371970
870488823548539932704720639430787736004465429635483481012693904434644807545
13928502*e^(373*I*c) + 6567485926886730009882737581287522561065455168626110
3681664007007537115778097293533565243828873383722980353200611956*e^(372*I*c
) + 12324394151933238474196007258810350659640633925361639108206296996068241
9011745775738921817753391954462609323881489157*e^(371*I*c) + 22891131173859
278009149264916234683440586774077645632610841092885725717470728926807434755
0225793244741923354395308214*e^(370*I*c) + 42084634260894938727755902145792
458657812096614856102264700849952946845200598017511941062895621049760956600
2969884927*e^(369*I*c) + 76586779551396278101255844462875141871094089528130
4790836743661582071650032154891482866406314834433199455459798934952*e^(368*
I*c) + 13796765297962120740171061880665894483554465012108901951071648603502
28928586815539003062875026711931941947738690360722*e^(367*I*c) + 2460442375
845422663927081630983260714734968091905493027145639238827192254886349361126
991457692409851120873307487457468*e^(366*I*c) + 434390969660193217335735968
778157929370129568194082711421543317533609396784590876674073824003711457066
7410936998017178*e^(365*I*c) + 75927527001466789611530950735850154731970297
46533633331549793961473285760935801904155116764831560875947581048693527224*
e^(364*I*c) + 1313977149410493388185668115141829311224255152153568687118126
6579813877606348160261747201317735782566021306798298336024*e^(363*I*c) + 22
514675741308069961506165586502872430421930210673264392997286485600640103867
253604847715547060592967690653795951142520*e^(362*I*c) + 381990158675860879
760029987566276749947954406679036250293223462501332864891208750050136381281
13893960349670280707161530*e^(361*I*c) + 6417510069326006680623806488600459
717074084330008683936861613916452910804984467535311184272579865808884034724
1496099644*e^(360*I*c) + 10676483201716559483808523418933352873358767332997
2530092661085186789939252915937090760282232346919090426243399409323314*e^(3
59*I*c) + 17589625826275598575710681261397930126580103159548435361490467286
5169442232075776580447184134141375995770091499246759528*e^(358*I*c) + 28699
294363123149655727801085157694089682649746606632752880156067700711283743192
6735088120974861760511367008815728782643*e^(357*I*c) + 46375828845736715454
493767825500568873332814556804931042399559988601280063861990402236837859110
8842602342094543682299102*e^(356*I*c) + 74222864090817312491693704946252561
733414891967911827048983100549778195122106995583962345249974865312465887355
3401442137*e^(355*I*c) + 11766007209757869651898750508902310922046126969702
77433014535895788956771230793520381993106606880564628599822341722801012*e^(

$354 \cdot I^c) + 1847505856462451533445284300571326323781162553304565971887670758$
 $091079306794821834928170773126364639722071570131703785334 \cdot e^{(353 \cdot I^c)} + 287$
 $361053592234018708083543558291227727196797739472015979107027492771427686953$
 $1467182688981041061381703885403497544001592 \cdot e^{(352 \cdot I^c)} + 44276730791054253$
 $185243161129856936565848519361001924570444551344833050453214525163471184881$
 $33224823670465103483954805161 \cdot e^{(351 \cdot I^c)} + 6758480437888524372562935948963$
 $857626694855547195519486122877567981718587262362871994967079401831957927901$
 $682582941234362 \cdot e^{(350 \cdot I^c)} + 102204237794346348513399752951633996417021222$
 $496636661930530083020260969321585683383094182373955413518190269079532206810$
 $13 \cdot e^{(349 \cdot I^c)} + 1531283720666277537934735321280768296571253565294263151828$
 $6142403097738200270711195396582159028513532779682154451996208592 \cdot e^{(348 \cdot I^c)}$
 $) + 22731603566128841100419501947051367666836652418077260913944810748473084$
 $891890410181285412604854876625919565639521227223276 \cdot e^{(347 \cdot I^c)} + 334358978$
 $279365813011711754596108294542981679620174198100729367333785065844280242010$
 $72453193458155334046693516742390717832 \cdot e^{(346 \cdot I^c)} + 4873325350597492340085$
 $225556305210140219646931365955449272567475433937528301040716774436695582892$
 $2837488705858532439654489 \cdot e^{(345 \cdot I^c)} + 70386349760594831567048224061395025$
 $698501202296966300376764220336697702961591099854055411376294871437468149528$
 $524796002762 \cdot e^{(344 \cdot I^c)} + 100744961851853744611754300982980166962404553836$
 $222921868484694269966120607698907046343731011160948828100276729370132819357$
 $\cdot e^{(343 \cdot I^c)} + 142906319123055524246546928478954238371315925802022389236498$
 $652136839822502035155676970917419039834587967055588431566416784 \cdot e^{(342 \cdot I^c)}$
 $+ 200906587153578804380300469501441610174521851259541929209840688960859454$
 $908519774835905895757666770857888611738751858460424 \cdot e^{(341 \cdot I^c)} + 279945244$
 $475039804822966730462960884492119874857791147124009079476920435941735293309$
 $305430438687333129912454196774070107264 \cdot e^{(340 \cdot I^c)} + 386642673050380049457$
 $382562818316962651975550990779277048740238629858795018247356162888631015687$
 $664780101205287333082748791 \cdot e^{(339 \cdot I^c)} + 529329252764113926003934836958243$
 $557672549238997560739214406599185047831955572583765358634395408771528009745$
 $467548382950094 \cdot e^{(338 \cdot I^c)} + 718361596382058249209113544487901088868388744$
 $033713210332491971375906738341551540457264804304039664255915607349801911966$
 $551 \cdot e^{(337 \cdot I^c)} + 966458275369037718747739130798151643483590684166832234688$
 $098291164160636418159452119815728809372125168836239364442397344064 \cdot e^{(336 \cdot I^c)}$
 $\cdot e^{(336 \cdot I^c)} + 128904351529293395648063433049967704018104393562010691426731106790003$
 $0058398839787692376954090545278554544997710058754772400 \cdot e^{(335 \cdot I^c)} + 17045$
 $829967078228082046782181676930026986611477127723550214565438109300696371880$
 $85882824757500605246963210810351706405349408 \cdot e^{(334 \cdot I^c)} + 2234891276398439$
 $464478622578306434840724610484468177859822620658691921478645266653062563823$
 $553001228001009093606751066168944 \cdot e^{(333 \cdot I^c)} + 290538572232005700195334527$
 $448948279085669252995982374953269596341416483336677312821860789932858860891$
 $6176593772088622582464 \cdot e^{(332 \cdot I^c)} + 37452575948766512046573349884262263881$
 $439545019868306642223492263610796095468222760675048993867030889823081857171$
 $43407211328 \cdot e^{(331 \cdot I^c)} + 4787527442780945685145204846971596165304169419328$
 $244073211459592129649255048876854059844720661078151288179612574986359194560$
 $\cdot e^{(330 \cdot I^c)} + 606894980315671224833187110532989547172280614300887801498655$

9653687260694816550470195890004511965527567432722969707577202160*e^(329*I*c)
) + 76297318156278215804689924242070083664388967363330246618638381051104451
48946962328297631547032543419811821015837863013682720*e^(328*I*c) + 9513032
274019522954209113191268226642299912013525665940298381064797885690904993128
948035227412144035633851779511219335277360*e^(327*I*c) + 117642122748764840
800109007146734744933712781605578119837244558265660556176580864793686418649
08119643412413644803772131657280*e^(326*I*c) + 1442981628520843120453297837
537569196506315422464974755129585150738952408322697678968860136962839990074
7658579201929300744260*e^(325*I*c) + 17556273271224292396887291403125716213
491486261145478571376751690105656067838042151038271381300372757755676325408
026834544840*e^(324*I*c) + 211883214058828875396101983747068626958940492260
770937641325125133361905239789496943876860591245267550480429579542647066374
60*e^(323*I*c) + 2536717643911935362153226033598334815490498260612576171130
0683492963390816491583025705268737539982149639300226512657426118880*e^(322*
I*c) + 30128482414552703264559018953088177156013437493438201078413769835448
366148121754549197591129967170764969700180348699207838960*e^(321*I*c) + 355
001031060196498762723767969494822095813723710360050128778060274816728070599
43445240136315568500732379966585005678181937920*e^(320*I*c) + 4149983212196
370804378852378740134554178008893053820691885357902674927336467164003756348
8607716092887686471542838602788559660*e^(319*I*c) + 48133117678184029216503
748549110374478924719094635603892829364863916553792278822957368285106328164
715910598370871149079494360*e^(318*I*c) + 553909130449720862194326891463315
660814279598969699002144342968177311508638670566207686081876797097201529741
48474907904177340*e^(317*I*c) + 6324777410101217905179494607517556992407698
133813848315804240674745387472938763171054499524715291220511850059751105282
4347680*e^(316*I*c) + 71660329861173395524441943889284109134091157844655245
672084237402434944696464927131812190659629511140639501743303863582092880*e^
(315*I*c) + 805662491306826841818762018826235112063637903372180119541102106
42927765997644903820595421936873565314654415769070472655401600*e^(314*I*c)
+ 8988381580138238221397327047795460274479287701805196334714630737246431512
1274929402347942874802899499538953561056667668891020*e^(313*I*c) + 99512206
472057965951340341738023548515336403371717898040850470954657532977279113491
50688029072611154101941386019689567958040*e^(312*I*c) + 109332537349966223
203932678503426357079863707001728294011042076530403923862654018978676516417
314221089449922495612732870169660*e^(311*I*c) + 119209713702033927055755397
823688444444647424324502185328626347046599634721146573830681540495333543146
776810911910410468628960*e^(310*I*c) + 128995076011591903410763863427097329
948586173574595862705849159280943046458742663163454018491463855395649453952
212899632198680*e^(309*I*c) + 138529794549151089451352769576543403126330747
243680030832467205895819043568155239264876762867172754338684027849855385453
216080*e^(308*I*c) + 147648920805545333418623121767853777399782924748301228
793924342574999937955421765370101235122939557467548549202174550009604780*e^
(307*I*c) + 156185962953551196169738218832173696509852551589210730578365727
476259476474465955428502336673743686499175698677875693611243400*e^(306*I*c)
+ 163977816059607725375264559816505847894187785101455360391897424482998415

385787605765315509208337741590143078572243505132706580*e^(305*I*c) + 170869
848868953101176860306053103994340530390347260088432676842505555141293830838
961275974268928666494845723462544709102843680*e^(304*I*c) + 176720929970554
642004575770053095700595334659870682732031975915532387577052414866323511140
117680492929354517559479899220940360*e^(303*I*c) + 181408168770922059820368
553316697321639984862628298828569560273295630897626829345263592219034560853
530733710529842148537901680*e^(302*I*c) + 184831151983748941817667850174708
257138128172158269413287765358532240773244336191900818557829905895684494889
410451921524212840*e^(301*I*c) + 186915474436567514926351405623117503261987
508351930083824566444435689139233683411704641828762178799177848064220150818
355261280*e^(300*I*c) + 187615393168510050071497280564603510912403132920312
024370835062679037644990286285346673507093452964351257962696133511725652320
*e^(299*I*c) + 186915474436567514926351405623117503261987508351930083824566
444435689139233683411704641828762178799177848064220150818355261280*e^(298*I
*c) + 184831151983748941817667850174708257138128172158269413287765358532240
773244336191900818557829905895684494889410451921524212840*e^(297*I*c) + 181
408168770922059820368553316697321639984862628298828569560273295630897626829
345263592219034560853530733710529842148537901680*e^(296*I*c) + 176720929970
554642004575770053095700595334659870682732031975915532387577052414866323511
140117680492929354517559479899220940360*e^(295*I*c) + 170869848868953101176
860306053103994340530390347260088432676842505555141293830838961275974268928
666494845723462544709102843680*e^(294*I*c) + 163977816059607725375264559816
505847894187785101455360391897424482998415385787605765315509208337741590143
078572243505132706580*e^(293*I*c) + 156185962953551196169738218832173696509
852551589210730578365727476259476474465955428502336673743686499175698677875
693611243400*e^(292*I*c) + 147648920805545333418623121767853777399782924748
301228793924342574999937955421765370101235122939557467548549202174550009604
780*e^(291*I*c) + 138529794549151089451352769576543403126330747243680030832
467205895819043568155239264876762867172754338684027849855385453216080*e^(29
0*I*c) + 128995076011591903410763863427097329948586173574595862705849159280
943046458742663163454018491463855395649453952212899632198680*e^(289*I*c) +
11920971370203392705575539782368844444647424324502185328626347046599634721
146573830681540495333543146776810911910410468628960*e^(288*I*c) + 109332537
349966223203932678503426357079863707001728294011042076530403923862654018978
676516417314221089449922495612732870169660*e^(287*I*c) + 995122064720579659
513403417380235485153364033717178980408504709546575329772791134915068802907
26111154101941386019689567958040*e^(286*I*c) + 8988381580138238221397327047
795460274479287701805196334714630737246431512127492940234794287480289949953
8953561056667668891020*e^(285*I*c) + 80566249130682684181876201882623511206
363790337218011954110210642927765997644903820595421936873565314654415769070
472655401600*e^(284*I*c) + 716603298611733955244419438892841091340911578446
552456720842374024349446964649271318121906596295111406395017433038635820928
80*e^(283*I*c) + 6324777410101217905179494607517556992407698133813848315804
2406747453874729387631710544995247152912205118500597511052824347680*e^(282*
I*c) + 55390913044972086219432689146331566081427959896969900214434296817731

150863867056620768608187679709720152974148474907904177340*e^(281*I*c) + 481
331176781840292165037485491103744789247190946356038928293648639165537922788
22957368285106328164715910598370871149079494360*e^(280*I*c) + 4149983212196
370804378852378740134554178008893053820691885357902674927336467164003756348
8607716092887686471542838602788559660*e^(279*I*c) + 35500103106019649876272
376796949482209581372371036005012877806027481672807059943445240136315568500
732379966585005678181937920*e^(278*I*c) + 301284824145527032645590189530881
771560134374934382010784137698354483661481217545491975911299671707649697001
80348699207838960*e^(277*I*c) + 2536717643911935362153226033598334815490498
260612576171130068349296339081649158302570526873753998214963930022651265742
6118880*e^(276*I*c) + 21188321405882887539610198374706862695894049226077093
764132512513336190523978949694387686059124526755048042957954264706637460*e^(
275*I*c) + 175562732712242923968872914031257162134914862611454785713767516
90105656067838042151038271381300372757755676325408026834544840*e^(274*I*c)
+ 1442981628520843120453297837537569196506315422464974755129585150738952408
3226976789688601369628399900747658579201929300744260*e^(273*I*c) + 11764212
274876484080010900714673474493371278160557811983724455826566055617658086479
368641864908119643412413644803772131657280*e^(272*I*c) + 951303227401952295
420911319126822664229991201352566594029838106479788569090499312894803522741
2144035633851779511219335277360*e^(271*I*c) + 76297318156278215804689924242
070083664388967363330246618638381051104451489469623282976315470325434198118
21015837863013682720*e^(270*I*c) + 6068949803156712248331871105329895471722
806143008878014986559653687260694816550470195890004511965527567432722969707
577202160*e^(269*I*c) + 478752744278094568514520484697159616530416941932824
4073211459592129649255048876854059844720661078151288179612574986359194560*e
^(268*I*c) + 37452575948766512046573349884262263881439545019868306642223492
26361079609546822276067504899386703088982308185717143407211328*e^(267*I*c)
+ 2905385722320057001953345274489482790856692529959823749532695963414164833
366773128218607899328588608916176593772088622582464*e^(266*I*c) + 223489127
639843946447862257830643484072461048446817785982262065869192147864526665306
2563823553001228001009093606751066168944*e^(265*I*c) + 17045829967078228082
046782181676930026986611477127723550214565438109300696371880858828247575006
05246963210810351706405349408*e^(264*I*c) + 1289043515292933956480634330499
677040181043935620106914267311067900030058398839787692376954090545278554544
997710058754772400*e^(263*I*c) + 966458275369037718747739130798151643483590
684166832234688098291164160636418159452119815728809372125168836239364442397
344064*e^(262*I*c) + 718361596382058249209113544487901088868388744033713210
332491971375906738341551540457264804304039664255915607349801911966551*e^(26
1*I*c) + 529329252764113926003934836958243557672549238997560739214406599185
047831955572583765358634395408771528009745467548382950094*e^(260*I*c) + 386
642673050380049457382562818316962651975550990779277048740238629858795018247
356162888631015687664780101205287333082748791*e^(259*I*c) + 279945244475039
804822966730462960884492119874857791147124009079476920435941735293309305430
438687333129912454196774070107264*e^(258*I*c) + 200906587153578804380300469
501441610174521851259541929209840688960859454908519774835905895757666770857

888611738751858460424*e^(257*I*c) + 142906319123055524246546928478954238371
 315925802022389236498652136839822502035155676970917419039834587967055588431
 566416784*e^(256*I*c) + 100744961851853744611754300982980166962404553836222
 921868484694269966120607698907046343731011160948828100276729370132819357*e^(255*I*c) + 703863497605948315670482240613950256985012022969663003767642203
 36697702961591099854055411376294871437468149528524796002762*e^(254*I*c) + 4
 873325350597492340085225556305210140219646931365955449272567475433937528301
 0407167744366955828922837488705858532439654489*e^(253*I*c) + 33435897827936
 581301171175459610829454298167962017419810072936733378506584428024201072453
 193458155334046693516742390717832*e^(252*I*c) + 227316035661288411004195019
 470513676668366524180772609139448107484730848918904101812854126048548766259
 19565639521227223276*e^(251*I*c) + 1531283720666277537934735321280768296571
 253565294263151828614240309773820027071119539658215902851353277968215445199
 6208592*e^(250*I*c) + 10220423779434634851339975295163399641702122249663666
 193053008302026096932158568338309418237395541351819026907953220681013*e^(249*I*c) + 675848043788852437256293594896385762669485554719551948612287756798
 1718587262362871994967079401831957927901682582941234362*e^(248*I*c) + 44276
 730791054253185243161129856936565848519361001924570444551344833050453214525
 16347118488133224823670465103483954805161*e^(247*I*c) + 2873610535922340187
 080835435582912277271967977394720159791070274927714276869531467182688981041
 061381703885403497544001592*e^(246*I*c) + 184750585646245153344528430057132
 632378116255330456597188767075809107930679482183492817077312636463972207157
 0131703785334*e^(245*I*c) + 11766007209757869651898750508902310922046126969
 70277433014535895788956771230793520381993106606880564628599822341722801012*
 e^(244*I*c) + 7422286409081731249169370494625256173341489196791182704898310
 05497781951221069955839623452499748653124658873553401442137*e^(243*I*c) + 4
 637582884573671545449376782550056887333281455680493104239955998860128006386
 19904022368378591108842602342094543682299102*e^(242*I*c) + 2869929436312314
 965572780108515769408968264974660663275288015606770071128374319267350881209
 74861760511367008815728782643*e^(241*I*c) + 1758962582627559857571068126139
 793012658010315954843536149046728651694422320757765804471841341413759957700
 91499246759528*e^(240*I*c) + 1067648320171655948380852341893335287335876733
 29972530092661085186789939252915937090760282232346919090426243399409323314*
 e^(239*I*c) + 6417510069326006680623806488600459717074084330008683936861613
 9164529108049844675353111842725798658088840347241496099644*e^(238*I*c) + 38
 199015867586087976002998756627674994795440667903625029322346250133286489120
 875005013638128113893960349670280707161530*e^(237*I*c) + 225146757413080699
 615061655865028724304219302106732643929972864856006401038672536048477155470
 60592967690653795951142520*e^(236*I*c) + 1313977149410493388185668115141829
 311224255152153568687118126657981387760634816026174720131773578256602130679
 8298336024*e^(235*I*c) + 75927527001466789611530950735850154731970297465336
 33331549793961473285760935801904155116764831560875947581048693527224*e^(234*I*c) + 4343909696601932173357359687781579293701295681940827114215433175336
 093967845908766740738240037114570667410936998017178*e^(233*I*c) + 246044237
 584542266392708163098326071473496809190549302714563923882719225488634936112

6991457692409851120873307487457468*e^(232*I*c) + 13796765297962120740171061
880665894483554465012108901951071648603502289285868155390030628750267119319
41947738690360722*e^(231*I*c) + 7658677955139627810125584446287514187109408
95281304790836743661582071650032154891482866406314834433199455459798934952*
e^(230*I*c) + 4208463426089493872775590214579245865781209661485610226470084
99529468452005980175119410628956210497609566002969884927*e^(229*I*c) + 2289
113117385927800914926491623468344058677407764563261084109288572571747072892
68074347550225793244741923354395308214*e^(228*I*c) + 1232439415193323847419
600725881035065964063392536163910820629699606824190117457757389218177533919
54462609323881489157*e^(227*I*c) + 6567485926886730009882737581287522561065
455168626110368166400700753711577809729353356524382887338372298035320061195
6*e^(226*I*c) + 34637657172671690167657344537197087048882354853993270472063
943078773600446542963548348101269390443464480754513928502*e^(225*I*c) + 180
798200680288599703499386230072306765633142067088484999001396412373347632664
79346963237936039328113185041591793848*e^(224*I*c) + 9339341958053494225251
750965715057300707302083814774770306218224241022648247419956042957363055823
830898547303219757*e^(223*I*c) + 477414111106609897022184533059496201647271
4230374234060663956846950926642685946929064114194400360936223590725470146*e
^(222*I*c) + 24149665168103385032890765492027405117100590117954471387734642
05696455026442712426409599662771080264826008985061097*e^(221*I*c) + 1208770
35849365839308944222056935063283704108140593750226539846117737648216609559
734831601248698274330296158612144*e^(220*I*c) + 598650141112241858911676505
180520150364003226841328081453597093587790338609212439085554466861582623350
303061961052*e^(219*I*c) + 293344920034300720287042383448342866313806285455
040067823080445597545970023446231563554135133105493516316320059272*e^(218*I
*c) + 142213115964814517682386667276769909482271681318790889840501039441748
635545362467679832449103520321953011780083069*e^(217*I*c) + 682080330967936
156837844096192442108186149916400415534244055278768932724966083242310981485
02466453967157728078994*e^(216*I*c) + 3236273132241954941033008894364024746
037832856131642293129242714590288791307164367950290905589123675514320738260
9*e^(215*I*c) + 15189634214908800396417911722643754748048520109734812459109
878810493844381062650818971199637121458749456243274416*e^(214*I*c) + 705213
241416219799260232652458014306098535305457293390552463312168102103734029836
6342203324325307072413739061024*e^(213*I*c) + 32384919313618514764233219335
395790983777355392076414673462356658238870483269493056092315851437486902036
15957136*e^(212*I*c) + 1470931146618934345515038362300100160482127749581443
92990474691022477470198899052379114493999887003199419829579*e^(211*I*c) +
660764473105869097691475973850837934511089033149586707982764263394766756649
565279879146173318386505740391093990*e^(210*I*c) + 293550743554342709808129
453576562313299705982699187416862934373964255615967138676253276302591561523
515603264403*e^(209*I*c) + 128967080084754712246023680866488384983286259025
533132044636109049545144029547003347761521666283977931640178464*e^(208*I*c)
+ 560286834249035176584950138585345161671625910343679724981746609074506667
78154353271630344650777885683547624184*e^(207*I*c) + 2406878513970527716119
346564450614328524136103776821681892218440014104846021094469664775272337193

2874594597328*e^(206*I*c) + 10223182025954860767217390305186451923562145473
 674293619918063490411487496121804590274592702770571515456414680*e^(205*I*c)
 + 429320647800802212601748890885182649479062072066015145146818191091724002
 7863968724539127659633517053002976480*e^(204*I*c) + 17824461149317518505563
 548566384219011744123222982494965916580539397871982465659459755955757341933
 48887952160*e^(203*I*c) + 7315849722068183628747296214039744442800104463011
 61527339760544815300951787985538419764656214582667219914080*e^(202*I*c) + 2
 968255152826695896853182732802390500845550322034159415119626595968816157137
 99937680026497408305672297618840*e^(201*I*c) + 1190418554038779649482295779
 483704656006066231830455295269004302092704732127738477949355860747143294799
 39280*e^(200*I*c) + 4718822084346620769509950695357378035710889749142256789
 8048199018207708997005333860148836479527456156014520*e^(199*I*c) + 18487405
 299005732693752728611876490890858357021974882371570623800186245137722660943
 641752976852924439870880*e^(198*I*c) + 715812468684294147547380736367983971
 8172745581538409044503383852693596921622426696740453944718143025248390*e^(1
 97*I*c) + 27388956247952656033552276465660008862807783050848257029119389036
 56162004262736182657700406301914070062380*e^(196*I*c) + 1035561982592002935
 226384577908611548612111495080193573691339864706029186482466241805664949381
 049856258510*e^(195*I*c) + 386876218234277165632451723049979889263115282374
 607541692443176673997513742813591736171169652250611186480*e^(194*I*c) + 142
 801792450221762483180874918825274134305133275417780084795034644763509333503
 150517345864659667189417080*e^(193*I*c) + 520751785187932703864292633515443
 06951104993542500582938155241689408138675254608030847907167748571734720*e^(
 192*I*c) + 1875998821886556356416363573598607327825573725740570627910889136
 6378428467414559930481172863538598193890*e^(191*I*c) + 66758662903711473585
 037668656692890108935438698305387087249452915809511791882966061581112577069
 68604740*e^(190*I*c) + 2346518219239105142238141633073464768899155708935025
 778047637412681781575765422219127409260159438712250*e^(189*I*c) + 814608187
 736530579670210025271921415597183369881214299823291969785549876175969866367
 976653244974728560*e^(188*I*c) + 279285755800035206679835368898165477644864
 987794665387827488933863633745047373109049265172681702585720*e^(187*I*c) +
 945561802589319869193343034663656528268580913143291891607362771758738417321
 96453379953705679466826880*e^(186*I*c) + 3161093933128469275069430644361841
 4656095969520945215743004044560386895241801579156543451940713351730*e^(185*
 I*c) + 10434117516570395966653693155582402109460348095473027807412321427346
 816928567197770376496170251803940*e^(184*I*c) + 340023256060165161752169468
 0847089844198028831694417424794868779328950548418125605446882081152636090*e
 ^ (183*I*c) + 10938532144862203586740324345008666784997700113058741724889759
 51612031456734608287095519501041975440*e^(182*I*c) + 3473514732147137808743
 520831295666012387657627759423667627333499521038897539826364038575568677773
 00*e^(181*I*c) + 1088679957318294728267329051920348867972846213564456275309
 09104429486741257822633476898356826454040*e^(180*I*c) + 3367539887202156837
 590238459398275336255980105810418462734541113626243194324077826072175699102
 7090*e^(179*I*c) + 10279364730663840844739577862469262604648861914297972589
 165243530651230690726244462479199894255180*e^(178*I*c) + 309613197162152016

238030155424146545178236208681028753774890290498593402017956570617713142161
4590*e^(177*I*c) + 92008939302958903287460185002715932261252636844477148978
1974361078847528891468831038436064951920*e^(176*I*c) + 26974580144021129697
268360186387895435796230852007659517712822762927324021520970821849736341414
0*e^(175*I*c) + 78009807368024239875613733058851417125327114681070889640794
249282633470580756557083923203377160*e^(174*I*c) + 22251959176795777571673
66036007480222211364232146399803864370963391491223687245823457351580140*e^(
173*I*c) + 6259872156822252843650960708235034710201362776057176647226323089
751446565288850103898153859920*e^(172*I*c) + 173657421881819107187419747245
0158123883564209950658639102337148122769080611680719741726053840*e^(171*I*c
) + 47501057885760151927231661793842522242178659724167102689431851540851146
7140969393115768793680*e^(170*I*c) + 12809891460168853967248054183040984770
7367500438601536803204497701119911289087105659482783340*e^(169*I*c) + 34054
053851295569154352346722177172655187548910782008504718324168725029438589162
349211628040*e^(168*I*c) + 892320944734329676333188188163847179349961867060
1026059730895962653291770229493028162575100*e^(167*I*c) + 23043510733738403
57379178597673066352016682781689139842097376663118488803841131935313641840*
e^(166*I*c) + 5864034669726832427416433289215609093751974538642432995719909
64608857245771134145204174990*e^(165*I*c) + 1470308167322768331630415820995
92047512043725225353339238819165193000407629544745753221740*e^(164*I*c) + 3
631836965230259173219744440979812202264082460413055250674258679518326735438
2847875885730*e^(163*I*c) + 88367206408604703056945140215479695512967940922
66983044118375790025854584036796364768280*e^(162*I*c) + 2117589733466855707
101501429210414722401838837940752841618541440888545729943138209036820*e^(16
1*I*c) + 499707567253859084357596314813794768069337190915967491907488904933
922677579665354338960*e^(160*I*c) + 116104551683555043762911501712116399313
733021132677481112824047246361794049635726479850*e^(159*I*c) + 265568063890
43407534496702369101545795994861757741414789944652712127566910185274123140*
e^(158*I*c) + 5978992172944143218459161149299819706321732111578494525245228
742976468409105395536290*e^(157*I*c) + 132475641236783747315747282116248369
1120966501948953926492241643788264284546437221120*e^(156*I*c) + 28882075526
4730654469968572021047109427318619508995802020689904590319476295408324280*e
^(155*I*c) + 61948596653035502879564338815234310660410902037882473161804774
492916216575880077680*e^(154*I*c) + 130698172034882898861932055083758183921
24991382340160316886507181296548981014818410*e^(153*I*c) + 2711843239670717
527605640490148833507130242448403978318523237721944200392830108580*e^(152*I
*c) + 553269128819528612502918869558947829098021956309349843584044631512291
778800081490*e^(151*I*c) + 110969199687320974749922259595250444341219218535
349655762591192576535872151766080*e^(150*I*c) + 218764828927139099280403456
12578705805121508756226696317087651824252241418663320*e^(149*I*c) + 4238125
846763232586394188569858685826755328005548627437019301405851325887594480*e^
(148*I*c) + 806679543607589140759305010796189568269842021613388955218916278
823182639488190*e^(147*I*c) + 150822381431412413773566474210011746852297437
597059186295243989481140398152780*e^(146*I*c) + 276931165383432592259833826
37647936122664033859615133489846664694361471028310*e^(145*I*c) + 4992519712

457043983505377976607953988397368297591114957991804893688371867680*e^(144*I*c) + 883500968821791202600774541927769200737689393513734789368397093333311961880*e^(143*I*c) + 153436088745056254127327239461577071933130157764595997113973513183188399376*e^(142*I*c) + 26143976279902021443471945665080254563056810183520401889800285493144867448*e^(141*I*c) + 4369442482910113914565353136069595862669338858053419381214131241925047008*e^(140*I*c) + 716099497599058079895633338552940229192858196481597830078819711862600096*e^(139*I*c) + 115051481852080848873700388354521315567640365124003103691176697194292320*e^(138*I*c) + 18115768495615758076710303055505625589254293659193314153418333944596408*e^(137*I*c) + 2794709104475686611842790694973699164482254723977210209725661304403472*e^(136*I*c) + 422276126632003687547754746555709988710527133086660161366353656787288*e^(135*I*c) + 62473550781053295317710774690247114124125187565731848441781904032672*e^(134*I*c) + 9046693523825682979044338963104263167672586826367911338826483549173*e^(133*I*c) + 1281817464914970810859604189828359000790789921169405304612211251818*e^(132*I*c) + 177642829135119348577194437675802830239905460092687136494961404333*e^(131*I*c) + 24070801913529757101858022914372045864746991786182039740274325264*e^(130*I*c) + 3187749929744346497211536044751776582320958627923816470590659024*e^(129*I*c) + 412430698299915190848067222327219435067747934091894670488982928*e^(128*I*c) + 52108117629177048660492400985175830987505700566877818954141639*e^(127*I*c) + 6426195485535248576425068136870465530087114003875716691383902*e^(126*I*c) + 773204636991145775061462731028098506094432675788136295011259*e^(125*I*c) + 90722605722208814918642284639487187764607589706493970774776*e^(124*I*c) + 10375184499871175501909398956596684116802997082526660323524*e^(123*I*c) + 1155855412893594260345544966642687823630035899363232371472*e^(122*I*c) + 125370496586921272662198050851269323171167338854081782959*e^(121*I*c) + 13231708870104896973800056733779919089340836756009580718*e^(120*I*c) + 1357990663161479842850642848032544982878359839580349899*e^(119*I*c) + 135442594916636116191574650625331646238501101627937224*e^(118*I*c) + 13118781801172174729679339894318153694964675368481194*e^(117*I*c) + 1233096700139723365181997220750932590655287625342156*e^(116*I*c) + 112391604542246650966429162063124338952554575234051*e^(115*I*c) + 9925490738534402272939987038714580495445431374618*e^(114*I*c) + 848552202276512356496200136959676295361696315113*e^(113*I*c) + 70164515322544462906873548813748091084561870680*e^(112*I*c) + 5605927253067558551780452883689835514455118670*e^(111*I*c) + 432333688644261557547944179250800440604964868*e^(110*I*c) + 32147887693375338817454482515377350383950278*e^(109*I*c) + 2302150411226234925855222345201500900533576*e^(108*I*c) + 158566476113257562566117432227203884298856*e^(107*I*c) + 10490402669510897424624643766470754045064*e^(106*I*c) + 665634670676210063754191847109971141414*e^(105*I*c) + 40443624781415311581857832389099634564*e^(104*I*c) + 2348998374244347079532766203075607598*e^(103*I*c) + 130171193079172823835151430773360024*e^(102*I*c) + 6868329225263681349501997341320517*e^(101*I*c) + 344277152012875134140739302960914*e^(100*I*c) + 16353164647151530240529137618111*e^(99*I*c) + 734057263616388449968842366924*e^(98*I*c) + 31042222522074681615625020522*e^(97*I*c) + 1232445557346832245176696904*e^

$(96*I*c) + 45759117183402579073139583*e^(95*I*c) + 158179664239781240816181$
 $4*e^(94*I*c) + 50648660944512569972179*e^(93*I*c) + 1493326612293984160368*$
 $e^(92*I*c) + 40261256699368950388*e^(91*I*c) + 984382804329835768*e^(90*I*c$
 $) + 21608403021340047*e^(89*I*c) + 420601518659718*e^(88*I*c) + 71461423073$
 $07*e^(87*I*c) + 103818048048*e^(86*I*c) + 1253841160*e^(85*I*c) + 12085216*$
 $e^(84*I*c) + 87153*e^(83*I*c) + 418*e^(82*I*c) + e^(81*I*c)))*tan(1/4*d*x +$
 $c) + 7*(-3289*I*a^2*e^(1027/2*I*c) - 1282710*I*a^2*e^(1025/2*I*c) - 249487$
 $095*I*a^2*e^(1023/2*I*c) - 32266997620*I*a^2*e^(1021/2*I*c) - 3121832019735$
 $*I*a^2*e^(1019/2*I*c) - 241005431923542*I*a^2*e^(1017/2*I*c) - 154645152151$
 $03422*I*a^2*e^(1015/2*I*c) - 848339120374563870*I*a^2*e^(1013/2*I*c) - 4061$
 $4235388474175675*I*a^2*e^(1011/2*I*c) - 1723848657662144174935*I*a^2*e^(100$
 $9/2*I*c) - 65678633862380668978797*I*a^2*e^(1007/2*I*c) - 22688982610841143$
 $22780091*I*a^2*e^(1005/2*I*c) - 71659370101867067314058647*I*a^2*e^(1003/2*$
 $I*c) - 2083633993341511741962220545*I*a^2*e^(1001/2*I*c) - 5610928687189836$
 $9496093387980*I*a^2*e^(999/2*I*c) - 1406472792955917865136489871114*I*a^2*e$
 $^(997/2*I*c) - 32964206157734965185866131506636*I*a^2*e^(995/2*I*c) - 72521$
 $2537844370128223087979340181*I*a^2*e^(993/2*I*c) - 150280154385589713238519$
 $50737244424*I*a^2*e^(991/2*I*c) - 294232725284515750583337235833091605*I*a^$
 $2*e^(989/2*I*c) - 5458017104331277636863728788108882260*I*a^2*e^(987/2*I*c)$
 $- 96165064470879466516856468979981298953*I*a^2*e^(985/2*I*c) - 16129504265$
 $44228644388946118660228217326*I*a^2*e^(983/2*I*c) - 25807207394610262666195$
 $786625902499776722*I*a^2*e^(981/2*I*c) - 3946352244238357653778751945561697$
 $98413535*I*a^2*e^(979/2*I*c) - 5777459899431421909042575858969918573070140*$
 $I*a^2*e^(977/2*I*c) - 81106652413090837180607129215519047725478483*I*a^2*e$
 $(975/2*I*c) - 1093437897602872607661796947121707741043900200*I*a^2*e^(973/2$
 $*I*c) - 14175642370639692917078404160115322319156841267*I*a^2*e^(971/2*I*c)$
 $- 176951138352595690762855574906084921297520106020*I*a^2*e^(969/2*I*c) - 2$
 $129312273803633942958758817457586335895472623670*I*a^2*e^(967/2*I*c) - 2472$
 $7500815341629941828329931180779105649314030308*I*a^2*e^(965/2*I*c) - 277411$
 $696712606367968860766543182757632328151469299*I*a^2*e^(963/2*I*c) - 3009497$
 $203915667961566616687959439264027968787987497*I*a^2*e^(961/2*I*c) - 3159972$
 $8405210893523373814632423233128854032172756465*I*a^2*e^(959/2*I*c) - 321414$
 $474758468949963097519046442397490455658900239275*I*a^2*e^(957/2*I*c) - 3169$
 $504955861830892959561795308811767842097316588802741*I*a^2*e^(955/2*I*c) - 3$
 $0324465369949159006596930561842561176581703526851670949*I*a^2*e^(953/2*I*c)$
 $- 281698460502942707632621647223155968135366263301340888714*I*a^2*e^(951/2$
 $*I*c) - 2542510639040480710865133818142242368651712053811882453386*I*a^2*e$
 $(949/2*I*c) - 22310545816063113198799373182025930998854858701529059606430*I$
 $*a^2*e^(947/2*I*c) - 190456027772701631222167952225792270942344905110743699$
 $547445*I*a^2*e^(945/2*I*c) - 1582600334407582343750559695592207912742994373$
 $526005697205768*I*a^2*e^(943/2*I*c) - 1280803477746184554839450083028993085$
 $9507858260154054185630469*I*a^2*e^(941/2*I*c) - 101008942321601073058743812$
 $539394339550240476561660597100434478*I*a^2*e^(939/2*I*c) - 7766476193162283$
 $1735458854658742750811566574795266669113195515*I*a^2*e^(937/2*I*c) - 58248$
 $66485485956160870802202075534098780512067011784177572657920*I*a^2*e^(935/2*$

$I*c) - 42633143603942943140960289773464948350711612149585795367207109320*I*$
 $a^2*e^(933/2*I*c) - 3046499772769501139302840524760863989906393256035330222$
 $43244834208*I*a^2*e^(931/2*I*c) - 21263375717977943790460384126129765751128$
 $69060832179063823545402908*I*a^2*e^(929/2*I*c) - 14501661593606794900679644$
 $897245761052226002310796800725001239143760*I*a^2*e^(927/2*I*c) - 9667804146$
 $9721516390319544690830600478916298492500715925487578541500*I*a^2*e^(925/2*I$
 $*c) - 630268657433514150995959099261502648982435588218176946585350750330080$
 $*I*a^2*e^(923/2*I*c) - 4019464980430704864646368121098631727141859823429991$
 $139565731831230220*I*a^2*e^(921/2*I*c) - 2508455004227003805187504112459713$
 $5587903984788805048794692954025826136*I*a^2*e^(919/2*I*c) - 153244560751845$
 $874009862374935321310982256761779852307795222356568491560*I*a^2*e^(917/2*I*$
 $c) - 9167359878151796279536868855736541471365903570952463617502393151928957$
 $80*I*a^2*e^(915/2*I*c) - 53717849474781207090444195042039567737852480086461$
 $90849516027804337557760*I*a^2*e^(913/2*I*c) - 30841671636860235940256291819$
 $500345382586813672641631674672477111150251380*I*a^2*e^(911/2*I*c) - 1735511$
 $07925703126215341839166039663518594417069855504146507661560225573200*I*a^2*$
 $e^(909/2*I*c) - 95743199782253454173010118399133666677927231233258732028264$
 $9934086537087060*I*a^2*e^(907/2*I*c) - 517960069176882056183957964121554034$
 $0240326374097892582774073042443161947520*I*a^2*e^(905/2*I*c) - 274855987922$
 $64285077359588674910772175227041673233676962838136429637468948920*I*a^2*e^($
 $903/2*I*c) - 14310135107329314200928393322744086315084207259642318859791923$
 $9525418651353760*I*a^2*e^(901/2*I*c) - 731168247557642740954546447803172304$
 $438120782922849282615551880122607710743390*I*a^2*e^(899/2*I*c) - 3667144340$
 $630566629952111359806942570024766504496043462982986858382541115071320*I*a^2$
 $*e^(897/2*I*c) - 1805820397656247779207028248681315454972027552142291325676$
 $8735919726439041572530*I*a^2*e^(895/2*I*c) - 873278235981003749943170773874$
 $10249071586643589864753197028556930729086578763100*I*a^2*e^(893/2*I*c) - 41$
 $481546634382365666106424862263368287662290402673520053189615708859313092715$
 $1330*I*a^2*e^(891/2*I*c) - 193584824796658467270245398097009014869676056846$
 $5563229743602617395081958731894120*I*a^2*e^(889/2*I*c) - 887746287193902688$
 $6302990313437414184883499510782510846267398966612198923415358100*I*a^2*e^(8$
 $87/2*I*c) - 400121711511226785140418608364081512565882882434620899668441262$
 $53521402025344585140*I*a^2*e^(885/2*I*c) - 17728136327009352771901612956277$
 $0152271161698446488654079612113477246381397761958630*I*a^2*e^(883/2*I*c) -$
 $772291716063274992440312252346975737223667501767420222475960558356343854661$
 $390320930*I*a^2*e^(881/2*I*c) - 3308448029575605333791929841544891315467725$
 $306133578108040786227714970623471206860370*I*a^2*e^(879/2*I*c) - 1394013614$
 $871626799560757285702077215540662663158581045088441230296411979964905770817$
 $0*I*a^2*e^(877/2*I*c) - 577806553230040116611059501868014698166807783088814$
 $37669242770005565033316643354683390*I*a^2*e^(875/2*I*c) - 23563598520917497$
 $3386390401675836470550684688674441579086471016352980458180376995214350*I*a^$
 $2*e^(873/2*I*c) - 945613068967215676111205365966310118361254135742385899036$
 $704396326378661255246170345160*I*a^2*e^(871/2*I*c) - 3734780899103666924896$
 $891172734183450725814252830409400966717277658895882383641210262740*I*a^2*e^$
 $(869/2*I*c) - 1451984243291507578990719952928296287275230941084562598248070$

6974266156447145232907218760*I*a^2*e^(867/2*I*c) - 555734480019596122399368
26324022972697759730966309101308647950811663281594543780392273650*I*a^2*e^(
865/2*I*c) - 20943213499631348964775534154519983464782243850441272775465999
4438770824007112166703016320*I*a^2*e^(863/2*I*c) - 777230951154547586594905
586759083418923490516228159852999058710437580204374586904243317010*I*a^2*e^(
861/2*I*c) - 2840837119758279389871998330348610271340731421578538884856194
587081840614185808628164270200*I*a^2*e^(859/2*I*c) - 1022796529394931165549
3133309531679349667296556076415563536820773718188117601532107888284410*I*a^
2*e^(857/2*I*c) - 362772662933946680165798951001054897492568449357497888227
39768560569739914792417941950684060*I*a^2*e^(855/2*I*c) - 12677586562695026
33093199438385495485149834367657635541389369756460405673299742348949575818
0*I*a^2*e^(853/2*I*c) - 436564397547890123843522035766730185021502894107522
026965503385942026891357953271442864432870*I*a^2*e^(851/2*I*c) - 1481567399
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501009480*I*a^2*e^(849/2*I*c) - 4955714538682426746341069484660548323003232
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/2*I*c) - 17022271033683761030680775719386022812098751282490709371492863674
4016809901948346647161290152440*I*a^2*e^(841/2*I*c) - 537934871814965391657
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*I*a^2*e^(839/2*I*c) - 1676440635582304033425022876768122577635520638714090
564423632904135623882124364564513385268391800*I*a^2*e^(837/2*I*c) - 5152720
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3832066053037757389465930231456447027071754543682438008358530*I*a^2*e^(833/
2*I*c) - 467167923072418428635736771060833839100648622093106316926559433634
96941035047443752352889335351750*I*a^2*e^(831/2*I*c) - 13782990714455520413
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8190*I*a^2*e^(829/2*I*c) - 401207332981540370190720118437829492283935458663
971978666346363722054305243088217239312307931028910*I*a^2*e^(827/2*I*c) - 1
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592676728848874609796890*I*a^2*e^(825/2*I*c) - 3266176312583520817713905163
935084531742324869211867876237701962931500933286006887180358613164105900*I*
a^2*e^(823/2*I*c) - 9136091875290749292252343289109091438177861375318724373
623251354701581584015968521795430968115978060*I*a^2*e^(821/2*I*c) - 2522244
797925070855508755512528284721015817964178184919185591315573809227332815226
9204584407017233480*I*a^2*e^(819/2*I*c) - 687314841486356946133857981424565
83980432634560150438395415555262011420422746898783679369530992622830*I*a^2*
e^(817/2*I*c) - 18488468817719678544983991709427978555464915831737250649878
7095812378385154076443311386842899167584820*I*a^2*e^(815/2*I*c) - 490972552
784544535911512963964502667546666352962707817110492155631344301953639158569
622404376536434270*I*a^2*e^(813/2*I*c) - 1287235906204379351871665786883637
441925594167862898218962357652677116687693117677467659775845901537560*I*a^2

$e^{(811/2 * I * c)}$ - 3332243170575387277897374565847781738681209257733774800866
 213262903845566248518176236173736361200045650 * $I * a^2 * e^{(809/2 * I * c)}$ - 8517754
 877461621759210417588818371506968729108804601331731430678777356891192908584
 411369617457579050080 * $I * a^2 * e^{(807/2 * I * c)}$ - 2150082753551818650493261129930
 1899347949925586667928948128471751935533041051245303799007069546262478120 * I
 $* a^2 * e^{(805/2 * I * c)}$ - 535991464022248021949553075918912083952451530767771595
 12871130579368836314443143560221865548609573152000 * $I * a^2 * e^{(803/2 * I * c)}$ - 13
 196650760932284734410125944585905299321803599280979966449679604976847591450
 5539873898469402406413970620 * $I * a^2 * e^{(801/2 * I * c)}$ - 320924341432592374089693
 111399042112950775535658726081343425825933752868056210294766531099221236855
 879280 * $I * a^2 * e^{(799/2 * I * c)}$ - 7709106964971752154983181631653096454270220237
 60331201680944186642421184373976839928023191662176768831260 * $I * a^2 * e^{(797/2 * I * c)}$
 $I * c)$ - 18293501339632766443945532790262844261778484422754576862101588579449
 43819074798783161324042498997317427840 * $I * a^2 * e^{(795/2 * I * c)}$ - 42885386529098
 878491305898473516376430298484507783019644947145920525911974817319124714265
 89520243516263980 * $I * a^2 * e^{(793/2 * I * c)}$ - 99327486297545824373822263658864293
 18241459110941837582719271408317025926367392879382082082371215951644280 * $I * a$
 $^2 * e^{(791/2 * I * c)}$ - 22730271448253866436128571775414833413708248224531663198
 648199827179865050434174612581384879203650868383240 * $I * a^2 * e^{(789/2 * I * c)}$ - 5
 139748384319518996364905073658378182514140436383413544728268708912254880303
 6847036415343525663901500899300 * $I * a^2 * e^{(787/2 * I * c)}$ - 114843847074590657631
 454363871040863902989618519194699666927573281569036469561814440348196035699
 469668453920 * $I * a^2 * e^{(785/2 * I * c)}$ - 2535878371908090217672480090866686514151
 80026469779516444785991183907648198192150423457499409024155250020340 * $I * a^2 * e$
 $^{(783/2 * I * c)}$ - 55338797340131164244782307761660787627812715207184367067878
 6017232780419838284554774403815733341601209348720 * $I * a^2 * e^{(781/2 * I * c)}$ - 119
 354021220397014942135234141956609023749592497738182697151332343431032629306
 5946843493363225350201134814740 * $I * a^2 * e^{(779/2 * I * c)}$ - 254434223169451751416
 168071682969232871943805193934623293742276813310269382248781513333283180878
 2360878628192 * $I * a^2 * e^{(777/2 * I * c)}$ - 536129734386971710846679909930199422246
 4831392534317407061352455760155232727561073082302587812090643743399000 * $I * a^$
 $2 * e^{(775/2 * I * c)}$ - 111672083521440192186951164376608852138822581190980312521
 90023761494016791716779154090742792939091173967743360 * $I * a^2 * e^{(773/2 * I * c)}$ -
 22994494897385426220973228494437184120506436250676449400511012540559860837
 045916159632911464155526549381431315 * $I * a^2 * e^{(771/2 * I * c)}$ - 4680923744177608
 807704545677790521586747057085044900089391228081757118297785083787637223024
 4610062044442700310 * $I * a^2 * e^{(769/2 * I * c)}$ - 942087742734459332213814210636634
 605350743257785104598241074821261704979626475918028588896735765120758855323
 01 * $I * a^2 * e^{(767/2 * I * c)}$ - 18746779224890892995613305742194609608790340706831
 7591476771447547750719179954808764060229352667984915509463936 * $I * a^2 * e^{(765/$
 $2 * I * c)}$ - 368859285047613335019291863600738855486266083164190760059356149690
 417164253519144904038488540679789464913316445 * $I * a^2 * e^{(763/2 * I * c)}$ - 7176563
 761559070657599362120189743561399730245956576878291726373553963369680709174
 50089165988836269386494950630 * $I * a^2 * e^{(761/2 * I * c)}$ - 13807562091068083963048
 454315958863884974154560162520099759675480033915659418219272767620646014554

03004258892730*I*a^2*e^(759/2*I*c) - 26271495533173277699693732443078724444
86689137143160099808964778201822442205684906256201229154604982146665759866*
I*a^2*e^(757/2*I*c) - 49435905488574704132642405960766649070464008349299917
63856809374018881419830364727922611585201089620136026317813*I*a^2*e^(755/2*
I*c) - 92005238163843674425457854384698503876104013072971654444087003814625
96254604043289564337709717004801142921086301*I*a^2*e^(753/2*I*c) - 16936253
315799612356983449259327198302174983445001336308800200149628963173764073042
586603712130511291927885438795*I*a^2*e^(751/2*I*c) - 3083749848683136795739
515185475657571396014310220525857354648534435001775269069242057446830165999
5665557849252505*I*a^2*e^(749/2*I*c) - 555418300599745445922619921515148354
499782780809761483760856308689395547801590034093282132773980651698476890922
17*I*a^2*e^(747/2*I*c) - 98960678773813324779073314105327289904799166585670
602821693087358593921745023122200152775956139379846673397268683*I*a^2*e^(74
5/2*I*c) - 1744330419158261975115028902637630836564860643400321112818111225
72047913024554638157278794872090651983244870819588*I*a^2*e^(743/2*I*c) - 30
418698193637684378587782101122947164511559342274019635806617254542551214036
8656569425063815737226926183384463382*I*a^2*e^(741/2*I*c) - 524832187724682
925583215788166762782489093533676869483498409516564352782024198472959882454
640789626068104536529860*I*a^2*e^(739/2*I*c) - 8959638599581524935474128645
849133957348960055573897956602799413881508302185548706222194539917072796043
34136069555*I*a^2*e^(737/2*I*c) - 15134671091887847229269187320752666283642
82411928629648438218524262230687177044666639547409496828363953159086523784*
I*a^2*e^(735/2*I*c) - 25298238884991355892821131913253755249659552121198884
15014985472650667601219344135752131106253007866458697018787603*I*a^2*e^(733
/2*I*c) - 41847047550528261659745273009504208070629830145996472721237197973
58506614705996884999995742988258778935888841569436*I*a^2*e^(731/2*I*c) - 68
504501388879120008251004521919745068343085733637827735353298609155420033415
87109658962844998052334052577634548575*I*a^2*e^(729/2*I*c) - 11098793852730
049900467477620190253588215465390785132676606045359177540532629639836007720
455332604033960540691735410*I*a^2*e^(727/2*I*c) - 1779744284552483895337080
268716611958073073687221859098060847235838529851892741176376023252918051849
2429027751850574*I*a^2*e^(725/2*I*c) - 282479847294799923017954803631530774
151264080604268676905388806880723175616348614898305442604902284061474270385
50825*I*a^2*e^(723/2*I*c) - 44380113978385972465644682361925051702696627790
960498609772420753311935097096135857768437500727905258838167774746676*I*a^2
*e^(721/2*I*c) - 6902137883195922910041464000878806997244189478419384904376
8221837087369885098447731719808741375563682226721172187125*I*a^2*e^(719/2*I
*c) - 106266524343343217664590361357231161622921001135223038707483473690437
882872690697205296816108160492296802679988715880*I*a^2*e^(717/2*I*c) - 1619
759246812525791294314003717464872425924079711422430199307527149284352490707
57766463734847432577359619977355420469*I*a^2*e^(715/2*I*c) - 24443832018928
577075807220956981919991193977215621704927075012945830550323307409320919111
3768334682284550770545377772*I*a^2*e^(713/2*I*c) - 365238706137359616600070
699619082139713753640264690466720702053560311601516116895520683068293669417
046374470910362666*I*a^2*e^(711/2*I*c) - 5403771767514964994284188266288554

154063667944198620952482027988637142108471353335978845628269093939309420907
 96205420*I*a²*e^(709/2*I*c) - 79168801410860103419693857190255700150484571
 2196126073563496646347934496907150906085300660569889896692451445978865145*I
 *a²*e^(707/2*I*c) - 114860931018421184347532224687086442580311388303908132
 7743324472166806173047836109990343345510402414879966726470291783*I*a²*e<sup>(7
 05/2*I*c)</sup> - 165035069880593871264266483841627282698710920794160660056561049
 0821766363203204649976483902793933869670965683695305667*I*a²*e^(703/2*I*c)
 - 234849896924574057189558006491888732976955249553022253433577431382685457
 9971626512324706726915652887153906664457167917*I*a²*e^(701/2*I*c) - 331008
 697591622174183153895990834913625550955973233843790010058701225396750102044
 0960464617664661519602047358728834783*I*a²*e^(699/2*I*c) - 462112894333759
 536408849281744322264217611058625067976858055827538677708192611061558542717
 4898007057344787791811730155*I*a²*e^(697/2*I*c) - 639059381238672274690176
 185605654449852208047118165458611805086921344998723336140222214498323497550
 5203268338404069070*I*a²*e^(695/2*I*c) - 875474694138014080876719396593223
 537696370308846563407735446524300917634089580857705723275087251797204997746
 0640893614*I*a²*e^(693/2*I*c) - 118817392003719339723430585761058535199243
 991458482295045064526172323144917169951597187258083471599990453661939291266
 22*I*a²*e^(691/2*I*c) - 15976262140541219066391073905196852266096709555541
 680360117938187079996111298630046538666465022790399882273480095312159*I*a²
 *e^(689/2*I*c) - 2128402049477458114068221585683521164737953289452612252239
 8168606383310529659429739450732289641742111431990437516970220*I*a²*e<sup>(687/
 2*I*c)</sup> - 280957021041118522598744644724249791347012037766997432412309430918
 89592515447767332435351059967242467638490956039653215*I*a²*e^(685/2*I*c) -
 36750055200706493332907957659140666121088182389700472995989532241381388481
 041041737053845102752164759225076976853447870*I*a²*e^(683/2*I*c) - 4763561
 989189119172568342322683432219887911783979610638069774976955056567586492548
 8884019447339957047967164208947031569*I*a²*e^(681/2*I*c) - 611906118965037
 637892720728786420532096243751120063245517042845318091285849297344426195732
 34048475266772127108092721024*I*a²*e^(679/2*I*c) - 77900430038472388493523
 297543191104896713699966714451425237456102524031030282332133881403961777583
 739648096380071388560*I*a²*e^(677/2*I*c) - 9829226282127934878488940449442
 580346040011668226082726046079190867274100658392475417217174943252500585251
 3553778623680*I*a²*e^(675/2*I*c) - 122926311130362982282930531800590413193
 892755472313438669937885559695115359158329870216181319401680622539825771904
 344440*I*a²*e^(673/2*I*c) - 1523832298424320705071486333251726560833800968
 88818957463018370238802480734804418338221079466921156239891439248449883680*
 I*a²*e^(671/2*I*c) - 18724752461975055243325467802389830505647302060451833
 4425544613285283731157047361688793462714993120367616167673996935608*I*a²*e
^(669/2*I*c) - 228086821979213161994154797898361140956767824538652850246924
 199657221150977419942700239602639983898674178986057439712320*I*a²*e<sup>(667/
 *I*c)</sup> - 2754271574587311247983252798819808363807714836633747388365646364070
 94610672777421045705435059330328502535069932806121560*I*a²*e^(665/2*I*c) -
 32972469055547972656744406564034336264770872497605092182032477801896996854
 6065236836577815026028465724065587612640991920*I*a²*e^(663/2*I*c) - 391334

543747364885198646774652098185473069223525928460733706025987409937647390579
349252176416240917982911016910315770960*I*a^2*e^(661/2*I*c) - 4604777597645
217212869963473734821654497904808618170077936866278203108322268672954423805
60478494343204748239311356171400*I*a^2*e^(659/2*I*c) - 53720765631885095654
867622290232708478340894279699823834369825711122568683566669397409872477052
9906163920011962226824320*I*a^2*e^(657/2*I*c) - 621377108447225467417920977
882757187926519094886636605309366642778263069398035616183392982442622288594
629924306599635240*I*a^2*e^(655/2*I*c) - 7126084825680016377422057561136586
145136557007356055260814453120182239769946283967968836010288693464602963419
48289617440*I*a^2*e^(653/2*I*c) - 81026806164766796897230771656479369731262
960466861566931128655392224266337578449710624277121983121115667542568684456
2920*I*a^2*e^(651/2*I*c) - 913446819168640466151130593147949820973487706939
696312936850746155847174085431933136538961202623251251021715718041418880*I*
a^2*e^(649/2*I*c) - 1020949307761046685483851694599176744159451920311114482
919641329915112719001444809615932039596196242112692638947129053040*I*a^2*e^
(647/2*I*c) - 1131292220201482520223678271335146824716930098519844139555825
140786899130297554994321714798551723344393464571710164372160*I*a^2*e^(645/2
*I*c) - 1242713857889209944794626889082276075977141843858465159493183489271
924588762753365507654571230494246610746617132263854100*I*a^2*e^(643/2*I*c)
- 1353195315451420313437489563356910414672170342830095956755770635775466291
069792114266486247472277234973670415760683624960*I*a^2*e^(641/2*I*c) - 1460
493678847799963487059674320737343640288580391271466623876882207508099433248
835482677720466676528226948603304177190540*I*a^2*e^(639/2*I*c) - 1562186964
787101490627013491081273159190897741838550609249842346755695952421966035252
081972374939068238887193535889066200*I*a^2*e^(637/2*I*c) - 1655729933770658
293771357769284118493945734494438803730125689639981417260441733582104520913
092362160074470891832981049580*I*a^2*e^(635/2*I*c) - 1738519323443834113007
443332766616165716605279946182900716200875367567243522959005653835445657888
732364423477634536323360*I*a^2*e^(633/2*I*c) - 1807966509711976871939839305
042715622727578634553781970459471779712682108764319874848175338812875321478
369058170030954360*I*a^2*e^(631/2*I*c) - 1861575144674768110426557177416946
225926130912471719654306955806539749713942241017532879502301920494398612936
404469594040*I*a^2*e^(629/2*I*c) - 1897020972388990207309023119915861711449
731378589391881830453022468540443348039250865860183534862239996983661831471
121940*I*a^2*e^(627/2*I*c) - 1912230808113858478866870753712714555289533816
394735307243195218359518803129887560593449941686882293838559049940338313580
*I*a^2*e^(625/2*I*c) - 1905457597458776709930827472866447261987176563583129
002568022592389115983255122396490881865521324931202362299671243884700*I*a^2
*e^(623/2*I*c) - 1875348552403554549312972640887885902657089680102597101290
48270355557920603504414035695853822453758049509430879646245020*I*a^2*e^(62
1/2*I*c) - 1821003585403273702574432851450914291658814786877621740198575512
179092602692929225363452883945313603152284886838427140580*I*a^2*e^(619/2*I*
c) - 1742021615759770488198059217318778347926047870427182243220864278894053
155565124470351463656875198719523915791426498304820*I*a^2*e^(617/2*I*c) - 1
638532781934653605223939671324076040272660534091984917932575369260055054951

963211213214142190996608510675175078391684400*I*a^2*e^(615/2*I*c) - 1511215
 132227236154464803465184561563977472975464243358311848382431942528662150481
 463695997748766198318465135571271910040*I*a^2*e^(613/2*I*c) - 1361294954375
 962964208845005629078239401669039370127901157612319707380265547568319507651
 710875887725773374076368465724720*I*a^2*e^(611/2*I*c) - 1190530512157529617
 508686638644289631884070105590459157858409269993065320514056346241559425223
 346959545213146168932415900*I*a^2*e^(609/2*I*c) - 1001179556004177634996376
 586158449265434581066062335982673920549529820094682353352918795594803230886
 711611153565466381120*I*a^2*e^(607/2*I*c) - 7959515408449643621057653697999
 429075685492104456323227204751992219823242162787472241292512613213145544193
 13857978534620*I*a^2*e^(605/2*I*c) - 57794599842887400171112143476924282413
 532375068565558999758340355996298184904841816223547091239816029854234000089
 6820560*I*a^2*e^(603/2*I*c) - 350578959120178195916347334453309038564005078
 877277224682309521488800706637838812783226214194307553791389253059107552140
 *I*a^2*e^(601/2*I*c) - 1174996902842487944535284614261771160106013129918443
 43397745740049701798141893102911319654443453010624651381930666567560*I*a^2*
 e^(599/2*I*c) + 11749969028424879445352846142617711601060131299184434339774
 5740049701798141893102911319654443453010624651381930666567560*I*a^2*e^(597/
 2*I*c) + 350578959120178195916347334453309038564005078877277224682309521488
 800706637838812783226214194307553791389253059107552140*I*a^2*e^(595/2*I*c)
 + 5779459984288740017111214347692428241353237506856555899975834035599629818
 49048418162235470912398160298542340000896820560*I*a^2*e^(593/2*I*c) + 79595
 154084496436210576536979994290756854921044563232272047519922198232421627874
 7224129251261321314554419313857978534620*I*a^2*e^(591/2*I*c) + 100117955600
 417763499637658615844926543458106606233598267392054952982009468235335291879
 5594803230886711611153565466381120*I*a^2*e^(589/2*I*c) + 119053051215752961
 750868663864428963188407010559045915785840926999306532051405634624155942522
 3346959545213146168932415900*I*a^2*e^(587/2*I*c) + 136129495437596296420884
 500562907823940166903937012790115761231970738026554756831950765171087588772
 5773374076368465724720*I*a^2*e^(585/2*I*c) + 151121513222723615446480346518
 456156397747297546424335831184838243194252866215048146369599774876619831846
 5135571271910040*I*a^2*e^(583/2*I*c) + 163853278193465360522393967132407604
 027266053409198491793257536926005505495196321121321414219099660851067517507
 8391684400*I*a^2*e^(581/2*I*c) + 174202161575977048819805921731877834792604
 787042718224322086427889405315556512447035146365687519871952391579142649830
 4820*I*a^2*e^(579/2*I*c) + 182100358540327370257443285145091429165881478687
 7621740198575512179092602692929225363452883945313603152284886838427140580*I
 *a^2*e^(577/2*I*c) + 187534855240355454931297264088788590265708968010259710
 129048270355557920603504414035695853822453758049509430879646245020*I*a^2*e
 ^ (575/2*I*c) + 190545759745877670993082747286644726198717656358312900256802
 2592389115983255122396490881865521324931202362299671243884700*I*a^2*e^(573/
 2*I*c) + 191223080811385847886687075371271455528953381639473530724319521835
 9518803129887560593449941686882293838559049940338313580*I*a^2*e^(571/2*I*c)
 + 189702097238899020730902311991586171144973137858939188183045302246854044
 3348039250865860183534862239996983661831471121940*I*a^2*e^(569/2*I*c) + 186

157514467476811042655717741694622592613091247171965430695580653974971394224
1017532879502301920494398612936404469594040*I*a^2*e^(567/2*I*c) + 180796650
971197687193983930504271562272757863455378197045947177971268210876431987484
8175338812875321478369058170030954360*I*a^2*e^(565/2*I*c) + 173851932344383
411300744333276661616571660527994618290071620087536756724352295900565383544
5657888732364423477634536323360*I*a^2*e^(563/2*I*c) + 165572993377065829377
135776928411849394573449443880373012568963998141726044173358210452091309236
2160074470891832981049580*I*a^2*e^(561/2*I*c) + 156218696478710149062701349
108127315919089774183855060924984234675569595242196603525208197237493906823
8887193535889066200*I*a^2*e^(559/2*I*c) + 146049367884779996348705967432073
734364028858039127146662387688220750809943324883548267772046667652822694860
3304177190540*I*a^2*e^(557/2*I*c) + 135319531545142031343748956335691041467
217034283009595675577063577546629106979211426648624747227723497367041576068
3624960*I*a^2*e^(555/2*I*c) + 124271385788920994479462688908227607597714184
385846515949318348927192458876275336550765457123049424661074661713226385410
0*I*a^2*e^(553/2*I*c) + 113129222020148252022367827133514682471693009851984
4139555825140786899130297554994321714798551723344393464571710164372160*I*a^
2*e^(551/2*I*c) + 102094930776104668548385169459917674415945192031111448291
9641329915112719001444809615932039596196242112692638947129053040*I*a^2*e^(5
49/2*I*c) + 913446819168640466151130593147949820973487706939696312936850746
155847174085431933136538961202623251251021715718041418880*I*a^2*e^(547/2*I*
c) + 8102680616476679689723077165647936973126296046686156693112865539222426
63375784497106242771219831211156675425686844562920*I*a^2*e^(545/2*I*c) + 71
260848256800163774220575611365861451365570073560552608144531201822397699462
8396796883601028869346460296341948289617440*I*a^2*e^(543/2*I*c) + 621377108
447225467417920977882757187926519094886636605309366642778263069398035616183
392982442622288594629924306599635240*I*a^2*e^(541/2*I*c) + 5372076563188509
565486762229023270847834089427969982383436982571112256868356666939740987247
70529906163920011962226824320*I*a^2*e^(539/2*I*c) + 46047775976452172128699
634737348216544979048086181700779368662782031083222686729544238056047849434
3204748239311356171400*I*a^2*e^(537/2*I*c) + 391334543747364885198646774652
098185473069223525928460733706025987409937647390579349252176416240917982911
016910315770960*I*a^2*e^(535/2*I*c) + 3297246905554797265674440656403433626
477087249760509218203247780189699685460652368365778150260284657240655876126
40991920*I*a^2*e^(533/2*I*c) + 27542715745873112479832527988198083638077148
366337473883656463640709461067277742104570543505933032850253506993280612156
0*I*a^2*e^(531/2*I*c) + 228086821979213161994154797898361140956767824538652
850246924199657221150977419942700239602639983898674178986057439712320*I*a^2
*e^(529/2*I*c) + 1872475246197505524332546780238983050564730206045183344255
44613285283731157047361688793462714993120367616167673996935608*I*a^2*e^(527
/2*I*c) + 15238322984243207050714863332517265608338009688881895746301837023
8802480734804418338221079466921156239891439248449883680*I*a^2*e^(525/2*I*c)
+ 122926311130362982282930531800590413193892755472313438669937885559695115
359158329870216181319401680622539825771904344440*I*a^2*e^(523/2*I*c) + 9829
226282127934878488940449442580346040011668226082726046079190867274100658392

4754172171749432525005852513553778623680*I*a²*e^(521/2*I*c) + 779004300384
 723884935232975431911048967136999667144514252374561025240310302823321338814
 03961777583739648096380071388560*I*a²*e^(519/2*I*c) + 61190611896503763789
 272072878642053209624375112006324551704284531809128584929734442619573234048
 475266772127108092721024*I*a²*e^(517/2*I*c) + 4763561989189119172568342322
 683432219887911783979610638069774976955056567586492548888401944733995704796
 7164208947031569*I*a²*e^(515/2*I*c) + 367500552007064933329079576591406661
 210881823897004729959895322413813884810410417370538451027521647592250769768
 53447870*I*a²*e^(513/2*I*c) + 28095702104111852259874464472424979134701203
 776699743241230943091889592515447767332435351059967242467638490956039653215
 *I*a²*e^(511/2*I*c) + 2128402049477458114068221585683521164737953289452612
 2522398168606383310529659429739450732289641742111431990437516970220*I*a²*e^(509/2*I*c) + 159762621405412190663910739051968522660967095555416803601179
 38187079996111298630046538666465022790399882273480095312159*I*a²*e^(507/2*I*c) + 11881739200371933972343058576105853519924399145848229504506452617232
 314491716995159718725808347159999045366193929126622*I*a²*e^(505/2*I*c) + 8
 754746941380140808767193965932235376963703088465634077354465243009176340895
 808577057232750872517972049977460640893614*I*a²*e^(503/2*I*c) + 6390593812
 386722746901761856056544498522080471181654586118050869213449987233361402222
 144983234975505203268338404069070*I*a²*e^(501/2*I*c) + 4621128943337595364
 088492817443222642176110586250679768580558275386777081926110615585427174898
 007057344787791811730155*I*a²*e^(499/2*I*c) + 3310086975916221741831538959
 908349136255509559732338437900100587012253967501020440960464617664661519602
 047358728834783*I*a²*e^(497/2*I*c) + 2348498969245740571895580064918887329
 769552495530222534335774313826854579971626512324706726915652887153906664457
 167917*I*a²*e^(495/2*I*c) + 1650350698805938712642664838416272826987109207
 941606600565610490821766363203204649976483902793933869670965683695305667*I*
 a²*e^(493/2*I*c) + 1148609310184211843475322246870864425803113883039081327
 743324472166806173047836109990343345510402414879966726470291783*I*a²*e^(491/2*I*c) + 7916880141086010341969385719025570015048457121961260735634966463
 47934496907150906085300660569889896692451445978865145*I*a²*e^(489/2*I*c) +
 54037717675149649942841882662885541540636679441986209524820279886371421084
 7135333597884562826909393930942090796205420*I*a²*e^(487/2*I*c) + 365238706
 137359616600070699619082139713753640264690466720702053560311601516116895520
 683068293669417046374470910362666*I*a²*e^(485/2*I*c) + 2444383201892857707
 580722095698191999119397721562170492707501294583055032330740932091911137683
 34682284550770545377772*I*a²*e^(483/2*I*c) + 16197592468125257912943140037
 174648724259240797114224301993075271492843524907075776646373484743257735961
 9977355420469*I*a²*e^(481/2*I*c) + 106266524343343217664590361357231161622
 921001135223038707483473690437882872690697205296816108160492296802679988715
 880*I*a²*e^(479/2*I*c) + 6902137883195922910041464000878806997244189478419
 3849043768221837087369885098447731719808741375563682226721172187125*I*a²*e^(477/2*I*c) + 443801139783859724656446823619250517026966277909604986097724
 20753311935097096135857768437500727905258838167774746676*I*a²*e^(475/2*I*c)
) + 28247984729479992301795480363153077415126408060426867690538880688072317

561634861489830544260490228406147427038550825*I*a^2*e^(473/2*I*c) + 1779744
284552483895337080268716611958073073687221859098060847235838529851892741176
3760232529180518492429027751850574*I*a^2*e^(471/2*I*c) + 110987938527300499
004674776201902535882154653907851326766060453591775405326296398360077204553
32604033960540691735410*I*a^2*e^(469/2*I*c) + 68504501388879120008251004521
919745068343085733637827735353298609155420033415871096589628449980523340525
77634548575*I*a^2*e^(467/2*I*c) + 41847047550528261659745273009504208070629
83014599647272123719797358506614705996884999995742988258778935888841569436*
I*a^2*e^(465/2*I*c) + 25298238884991355892821131913253755249659552121198884
15014985472650667601219344135752131106253007866458697018787603*I*a^2*e^(463
/2*I*c) + 15134671091887847229269187320752666283642824119286296484382185242
62230687177044666639547409496828363953159086523784*I*a^2*e^(461/2*I*c) + 89
596385995815249354741286458491339573489600555738979566027994138815083021855
4870622219453991707279604334136069555*I*a^2*e^(459/2*I*c) + 524832187724682
925583215788166762782489093533676869483498409516564352782024198472959882454
640789626068104536529860*I*a^2*e^(457/2*I*c) + 3041869819363768437858778210
112294716451155934227401963580661725454255121403686565694250638157372269261
83384463382*I*a^2*e^(455/2*I*c) + 17443304191582619751150289026376308365648
6064340032111281811122572047913024554638157278794872090651983244870819588*I
*a^2*e^(453/2*I*c) + 989606787738133247790733141053272899047991665856706028
21693087358593921745023122200152775956139379846673397268683*I*a^2*e^(451/2*
I*c) + 55541830059974544592261992151514835449978278080976148376085630868939
554780159003409328213277398065169847689092217*I*a^2*e^(449/2*I*c) + 3083749
848683136795739515185475657571396014310220525857354648534435001775269069242
0574468301659995665557849252505*I*a^2*e^(447/2*I*c) + 169362533157996123569
834492593271983021749834450013363088002001496289631737640730425866037121305
11291927885438795*I*a^2*e^(445/2*I*c) + 92005238163843674425457854384698503
876104013072971654444087003814625962546040432895643377097170048011429210863
01*I*a^2*e^(443/2*I*c) + 49435905488574704132642405960766649070464008349299
91763856809374018881419830364727922611585201089620136026317813*I*a^2*e^(441
/2*I*c) + 26271495533173277699693732443078724444866891371431600998089647782
01822442205684906256201229154604982146665759866*I*a^2*e^(439/2*I*c) + 13807
562091068083963048454315958863884974154560162520099759675480033915659418219
27276762064601455403004258892730*I*a^2*e^(437/2*I*c) + 71765637615590706575
993621201897435613997302459565768782917263735539633696807091745008916598883
6269386494950630*I*a^2*e^(435/2*I*c) + 368859285047613335019291863600738855
486266083164190760059356149690417164253519144904038488540679789464913316445
*I*a^2*e^(433/2*I*c) + 1874677922489089299561330574219460960879034070683175
91476771447547750719179954808764060229352667984915509463936*I*a^2*e^(431/2*
I*c) + 94208774273445933221381421063663460535074325778510459824107482126170
497962647591802858889673576512075885532301*I*a^2*e^(429/2*I*c) + 4680923744
177608807704545677790521586747057085044900089391228081757118297785083787637
2230244610062044442700310*I*a^2*e^(427/2*I*c) + 229944948973854262209732284
944371841205064362506764494005110125405598608370459161596329114641555265493
81431315*I*a^2*e^(425/2*I*c) + 11167208352144019218695116437660885213882258

119098031252190023761494016791716779154090742792939091173967743360*I*a^2*e^
 (423/2*I*c) + 5361297343869717108466799099301994222464831392534317407061352
 455760155232727561073082302587812090643743399000*I*a^2*e^(421/2*I*c) + 2544
 342231694517514161680716829692328719438051939346232937422768133102693822487
 815133332831808782360878628192*I*a^2*e^(419/2*I*c) + 1193540212203970149421
 352341419566090237495924977381826971513323434310326293065946843493363225350
 201134814740*I*a^2*e^(417/2*I*c) + 5533879734013116424478230776166078762781
 27152071843670678786017232780419838284554774403815733341601209348720*I*a^2*
 e^(415/2*I*c) + 25358783719080902176724800908666865141518002646977951644478
 5991183907648198192150423457499409024155250020340*I*a^2*e^(413/2*I*c) + 114
 843847074590657631454363871040863902989618519194699666927573281569036469561
 814440348196035699469668453920*I*a^2*e^(411/2*I*c) + 5139748384319518996364
 905073658378182514140436383413544728268708912254880303684703641534352566390
 1500899300*I*a^2*e^(409/2*I*c) + 227302714482538664361285717754148334137082
 48224531663198648199827179865050434174612581384879203650868383240*I*a^2*e^(
 407/2*I*c) + 99327486297545824373822263658864293182414591109418375827192714
 08317025926367392879382082082371215951644280*I*a^2*e^(405/2*I*c) + 42885386
 529098878491305898473516376430298484507783019644947145920525911974817319124
 71426589520243516263980*I*a^2*e^(403/2*I*c) + 18293501339632766443945532790
 262844261778484422754576862101588579449438190747987831613240424989973174278
 40*I*a^2*e^(401/2*I*c) + 77091069649717521549831816316530964542702202376033
 1201680944186642421184373976839928023191662176768831260*I*a^2*e^(399/2*I*c)
 + 320924341432592374089693111399042112950775535658726081343425825933752868
 056210294766531099221236855879280*I*a^2*e^(397/2*I*c) + 1319665076093228473
 441012594458590529932180359928097996644967960497684759145055398738984694024
 06413970620*I*a^2*e^(395/2*I*c) + 53599146402224802194955307591891208395245
 153076777159512871130579368836314443143560221865548609573152000*I*a^2*e^(39
 3/2*I*c) + 2150082753551818650493261129930189934794992558666792894812847175
 1935533041051245303799007069546262478120*I*a^2*e^(391/2*I*c) + 851775487746
 162175921041758881837150696872910880460133173143067877735689119290858441136
 9617457579050080*I*a^2*e^(389/2*I*c) + 333224317057538727789737456584778173
 8681209257733774800866213262903845566248518176236173736361200045650*I*a^2*e
 ^ (387/2*I*c) + 128723590620437935187166578688363744192559416786289821896235
 7652677116687693117677467659775845901537560*I*a^2*e^(385/2*I*c) + 490972552
 784544535911512963964502667546666352962707817110492155631344301953639158569
 622404376536434270*I*a^2*e^(383/2*I*c) + 1848846881771967854498399170942797
 85554649158317372506498787095812378385154076443311386842899167584820*I*a^2*
 e^(381/2*I*c) + 68731484148635694613385798142456583980432634560150438395415
 555262011420422746898783679369530992622830*I*a^2*e^(379/2*I*c) + 2522244797
 925070855508755512528284721015817964178184919185591315573809227332815226920
 4584407017233480*I*a^2*e^(377/2*I*c) + 913609187529074929225234328910909143
 8177861375318724373623251354701581584015968521795430968115978060*I*a^2*e^(3
 75/2*I*c) + 326617631258352081771390516393508453174232486921186787623770196
 2931500933286006887180358613164105900*I*a^2*e^(373/2*I*c) + 115236006982268
 734392696871710025839854804038946508854621186577486021454624759267672884887

4609796890*I*a^2*e^(371/2*I*c) + 401207332981540370190720118437829492283935
458663971978666346363722054305243088217239312307931028910*I*a^2*e^(369/2*I*
c) + 1378299071445552041326494556780332643590714272752355716199608413183244
58723810685070899479318058190*I*a^2*e^(367/2*I*c) + 46716792307241842863573
677106083383910064862209310631692655943363496941035047443752352889335351750
*I*a^2*e^(365/2*I*c) + 1562131726958516297435003770166207390383206605303775
7389465930231456447027071754543682438008358530*I*a^2*e^(363/2*I*c) + 515272
076996608324557837671612610602268426810652231549431671612536298612235950428
7481709712640290*I*a^2*e^(361/2*I*c) + 167644063558230403342502287676812257
7635520638714090564423632904135623882124364564513385268391800*I*a^2*e^(359/
2*I*c) + 537934871814965391657149181856266786362021448915902734170589600431
759244861457731457099253162540*I*a^2*e^(357/2*I*c) + 1702227103368376103068
07757193860228120987512824907093714928636744016809901948346647161290152440*
I*a^2*e^(355/2*I*c) + 53113740974357924077261924424954026967632012013437614
126796673959257892104885022855537842837870*I*a^2*e^(353/2*I*c) + 1634000204
091210100590797125421570995776255824418623935682301379145403318430057504976
7279180480*I*a^2*e^(351/2*I*c) + 495571453868242674634106948466054832300323
2579130180106682728827347533547752576357707952254670*I*a^2*e^(349/2*I*c) +
148156739913925722074298485681081733727688547401957626663003561586570112231
2638498716501009480*I*a^2*e^(347/2*I*c) + 436564397547890123843522035766730
185021502894107522026965503385942026891357953271442864432870*I*a^2*e^(345/2
*I*c) + 1267758656269502633093199438385549548514983436765763554138936975646
04056732997423489495758180*I*a^2*e^(343/2*I*c) + 36277266293394668016579895
100105489749256844935749788822739768560569739914792417941950684060*I*a^2*e^
(341/2*I*c) + 1022796529394931165549313330953167934966729655607641556353682
0773718188117601532107888284410*I*a^2*e^(339/2*I*c) + 284083711975827938987
1998330348610271340731421578538884856194587081840614185808628164270200*I*a^
2*e^(337/2*I*c) + 777230951154547586594905586759083418923490516228159852999
058710437580204374586904243317010*I*a^2*e^(335/2*I*c) + 2094321349963134896
47755341545199834647822438504412727754659994438770824007112166703016320*I*a
^2*e^(333/2*I*c) + 55573448001959612239936826324022972697759730966309101308
647950811663281594543780392273650*I*a^2*e^(331/2*I*c) + 1451984243291507578
9907199529282962872752309410845625982480706974266156447145232907218760*I*a^
2*e^(329/2*I*c) + 373478089910366692489689117273418345072581425283040940096
6717277658895882383641210262740*I*a^2*e^(327/2*I*c) + 945613068967215676111
205365966310118361254135742385899036704396326378661255246170345160*I*a^2*e^
(325/2*I*c) + 2356359852091749733863904016758364705506846886744415790864710
16352980458180376995214350*I*a^2*e^(323/2*I*c) + 57780655323004011661105950
186801469816680778308881437669242770005565033316643354683390*I*a^2*e^(321/2
*I*c) + 1394013614871626799560757285702077215540662663158581045088441230296
4119799649057708170*I*a^2*e^(319/2*I*c) + 330844802957560533379192984154489
1315467725306133578108040786227714970623471206860370*I*a^2*e^(317/2*I*c) +
772291716063274992440312252346975737223667501767420222475960558356343854661
390320930*I*a^2*e^(315/2*I*c) + 1772813632700935277190161295627701522711616
98446488654079612113477246381397761958630*I*a^2*e^(313/2*I*c) + 40012171151

122678514041860836408151256588288243462089966844126253521402025344585140*I*
 a^2*e^(311/2*I*c) + 8877462871939026886302990313437414184883499510782510846
 267398966612198923415358100*I*a^2*e^(309/2*I*c) + 1935848247966584672702453
 980970090148696760568465563229743602617395081958731894120*I*a^2*e^(307/2*I*
 c) + 4148154663438236566610642486226336828766229040267352005318961570885931
 30927151330*I*a^2*e^(305/2*I*c) + 87327823598100374994317077387410249071586
 643589864753197028556930729086578763100*I*a^2*e^(303/2*I*c) + 1805820397656
 2477792070282486813154549720275521422913256768735919726439041572530*I*a^2*e
 ^ (301/2*I*c) + 366714434063056662995211135980694257002476650449604346298298
 6858382541115071320*I*a^2*e^(299/2*I*c) + 731168247557642740954546447803172
 304438120782922849282615551880122607710743390*I*a^2*e^(297/2*I*c) + 1431013
 51073293142009283933227440863150842072596423188597919239525418651353760*I*a
 ^2*e^(295/2*I*c) + 27485598792264285077359588674910772175227041673233676962
 838136429637468948920*I*a^2*e^(293/2*I*c) + 5179600691768820561839579641215
 540340240326374097892582774073042443161947520*I*a^2*e^(291/2*I*c) + 9574319
 97822534541730101183991336666779272312332587320282649934086537087060*I*a^2*
 e^(289/2*I*c) + 17355110792570312621534183916603966351859441706985550414650
 7661560225573200*I*a^2*e^(287/2*I*c) + 308416716368602359402562918195003453
 82586813672641631674672477111150251380*I*a^2*e^(285/2*I*c) + 53717849474781
 20709044419504203956773785248008646190849516027804337557760*I*a^2*e^(283/2*
 I*c) + 91673598781517962795368688557365414713659035709524636175023931519289
 5780*I*a^2*e^(281/2*I*c) + 153244560751845874009862374935321310982256761779
 852307795222356568491560*I*a^2*e^(279/2*I*c) + 2508455004227003805187504112
 4597135587903984788805048794692954025826136*I*a^2*e^(277/2*I*c) + 401946498
 0430704864646368121098631727141859823429991139565731831230220*I*a^2*e^(275/
 2*I*c) + 630268657433514150995959099261502648982435588218176946585350750330
 080*I*a^2*e^(273/2*I*c) + 9667804146972151639031954469083060047891629849250
 0715925487578541500*I*a^2*e^(271/2*I*c) + 145016615936067949006796448972457
 61052226002310796800725001239143760*I*a^2*e^(269/2*I*c) + 21263375717977943
 79046038412612976575112869060832179063823545402908*I*a^2*e^(267/2*I*c) + 30
 4649977276950113930284052476086398990639325603533022243244834208*I*a^2*e^(2
 65/2*I*c) + 426331436039429431409602897734649483507116121495857953672071093
 20*I*a^2*e^(263/2*I*c) + 58248664854859561608708022020755340987805120670117
 84177572657920*I*a^2*e^(261/2*I*c) + 77664761931622831735458854658742750811
 5665747952666669113195515*I*a^2*e^(259/2*I*c) + 101008942321601073058743812
 539394339550240476561660597100434478*I*a^2*e^(257/2*I*c) + 1280803477746184
 5548394500830289930859507858260154054185630469*I*a^2*e^(255/2*I*c) + 158260
 0334407582343750559695592207912742994373526005697205768*I*a^2*e^(253/2*I*c)
 + 190456027772701631222167952225792270942344905110743699547445*I*a^2*e^(25
 1/2*I*c) + 22310545816063113198799373182025930998854858701529059606430*I*a^
 2*e^(249/2*I*c) + 254251063904048071086513381814224236865171205381188245338
 6*I*a^2*e^(247/2*I*c) + 281698460502942707632621647223155968135366263301340
 888714*I*a^2*e^(245/2*I*c) + 3032446536994915900659693056184256117658170352
 6851670949*I*a^2*e^(243/2*I*c) + 316950495586183089295956179530881176784209
 7316588802741*I*a^2*e^(241/2*I*c) + 321414474758468949963097519046442397490

455658900239275*I*a^2*e^(239/2*I*c) + 3159972840521089352337381463242323312
 8854032172756465*I*a^2*e^(237/2*I*c) + 300949720391566796156661668795943926
 4027968787987497*I*a^2*e^(235/2*I*c) + 277411696712606367968860766543182757
 632328151469299*I*a^2*e^(233/2*I*c) + 2472750081534162994182832993118077910
 5649314030308*I*a^2*e^(231/2*I*c) + 212931227380363394295875881745758633589
 5472623670*I*a^2*e^(229/2*I*c) + 176951138352595690762855574906084921297520
 106020*I*a^2*e^(227/2*I*c) + 1417564237063969291707840416011532231915684126
 7*I*a^2*e^(225/2*I*c) + 1093437897602872607661796947121707741043900200*I*a^
 2*e^(223/2*I*c) + 81106652413090837180607129215519047725478483*I*a^2*e^(221
 /2*I*c) + 5777459899431421909042575858969918573070140*I*a^2*e^(219/2*I*c) +
 394635224423835765377875194556169798413535*I*a^2*e^(217/2*I*c) + 258072073
 94610262666195786625902499776722*I*a^2*e^(215/2*I*c) + 16129504265442286443
 88946118660228217326*I*a^2*e^(213/2*I*c) + 96165064470879466516856468979981
 298953*I*a^2*e^(211/2*I*c) + 5458017104331277636863728788108882260*I*a^2*e^
 (209/2*I*c) + 294232725284515750583337235833091605*I*a^2*e^(207/2*I*c) + 15
 028015438558971323851950737244424*I*a^2*e^(205/2*I*c) + 7252125378443701282
 23087979340181*I*a^2*e^(203/2*I*c) + 32964206157734965185866131506636*I*a^2
 *e^(201/2*I*c) + 1406472792955917865136489871114*I*a^2*e^(199/2*I*c) + 5610
 9286871898369496093387980*I*a^2*e^(197/2*I*c) + 208363399334151174196222054
 5*I*a^2*e^(195/2*I*c) + 71659370101867067314058647*I*a^2*e^(193/2*I*c) + 22
 68898261084114322780091*I*a^2*e^(191/2*I*c) + 65678633862380668978797*I*a^2
 *e^(189/2*I*c) + 1723848657662144174935*I*a^2*e^(187/2*I*c) + 4061423538847
 4175675*I*a^2*e^(185/2*I*c) + 848339120374563870*I*a^2*e^(183/2*I*c) + 1546
 4515215103422*I*a^2*e^(181/2*I*c) + 241005431923542*I*a^2*e^(179/2*I*c) + 3
 121832019735*I*a^2*e^(177/2*I*c) + 32266997620*I*a^2*e^(175/2*I*c) + 249487
 095*I*a^2*e^(173/2*I*c) + 1282710*I*a^2*e^(171/2*I*c) + 3289*I*a^2*e^(169/2
 *I*c))/(e^(517*I*c) + 418*e^(516*I*c) + 87153*e^(515*I*c) + 12085216*e^(514
 *I*c) + 1253841160*e^(513*I*c) + 103818048048*e^(512*I*c) + 7146142307307*e
 ^((511*I*c) + 420601518659718*e^(510*I*c) + 21608403021340047*e^(509*I*c) +
 984382804329835768*e^(508*I*c) + 40261256699368950388*e^(507*I*c) + 1493326
 612293984160368*e^(506*I*c) + 50648660944512569972179*e^(505*I*c) + 1581796
 642397812408161814*e^(504*I*c) + 45759117183402579073139583*e^(503*I*c) + 1
 232445557346832245176696904*e^(502*I*c) + 31042222522074681615625020522*e^(
 501*I*c) + 734057263616388449968842366924*e^(500*I*c) + 1635316464715153024
 0529137618111*e^(499*I*c) + 344277152012875134140739302960914*e^(498*I*c) +
 6868329225263681349501997341320517*e^(497*I*c) + 1301711930791728238351514
 30773360024*e^(496*I*c) + 2348998374244347079532766203075607598*e^(495*I*c)
 + 40443624781415311581857832389099634564*e^(494*I*c) + 6656346706762100637
 54191847109971141414*e^(493*I*c) + 1049040266951089742462464376647075404506
 4*e^(492*I*c) + 158566476113257562566117432227203884298856*e^(491*I*c) + 23
 02150411226234925855222345201500900533576*e^(490*I*c) + 3214788769337533881
 7454482515377350383950278*e^(489*I*c) + 43233368864426155754794417925080044
 0604964868*e^(488*I*c) + 5605927253067558551780452883689835514455118670*e^(
 487*I*c) + 70164515322544462906873548813748091084561870680*e^(486*I*c) + 84
 8552202276512356496200136959676295361696315113*e^(485*I*c) + 99254907385344

02272939987038714580495445431374618*e^(484*I*c) + 1123916045422466509664291
62063124338952554575234051*e^(483*I*c) + 1233096700139723365181997220750932
590655287625342156*e^(482*I*c) + 131187818011721747296793398943181536949646
75368481194*e^(481*I*c) + 1354425949166361161915746506253316462385011016279
37224*e^(480*I*c) + 1357990663161479842850642848032544982878359839580349899
*e^(479*I*c) + 13231708870104896973800056733779919089340836756009580718*e^(
478*I*c) + 125370496586921272662198050851269323171167338854081782959*e^(477
*I*c) + 1155855412893594260345544966642687823630035899363232371472*e^(476*I
*c) + 10375184499871175501909398956596684116802997082526660323524*e^(475*I*
c) + 90722605722208814918642284639487187764607589706493970774776*e^(474*I*c
) + 773204636991145775061462731028098506094432675788136295011259*e^(473*I*c
) + 6426195485535248576425068136870465530087114003875716691383902*e^(472*I*
c) + 52108117629177048660492400985175830987505700566877818954141639*e^(471*I*
c) + 412430698299915190848067222327219435067747934091894670488982928*e^(4
70*I*c) + 3187749929744346497211536044751776582320958627923816470590659024*
e^(469*I*c) + 2407080191352975710185802291437204586474699178618203974027432
5264*e^(468*I*c) + 17764282913511934857719443767580283023990546009268713649
4961404333*e^(467*I*c) + 12818174649149708108596041898283590007907899211694
05304612211251818*e^(466*I*c) + 9046693523825682979044338963104263167672586
826367911338826483549173*e^(465*I*c) + 624735507810532953177107746902471141
24125187565731848441781904032672*e^(464*I*c) + 4222761266320036875477547465
55709988710527133086660161366353656787288*e^(463*I*c) + 2794709104475686611
842790694973699164482254723977210209725661304403472*e^(462*I*c) + 181157684
95615758076710303055505625589254293659193314153418333944596408*e^(461*I*c)
+ 115051481852080848873700388354521315567640365124003103691176697194292320*
e^(460*I*c) + 7160994975990580798956333385529402291928581964815978300788197
11862600096*e^(459*I*c) + 4369442482910113914565353136069595862669338858053
419381214131241925047008*e^(458*I*c) + 261439762799020214434719456650802545
63056810183520401889800285493144867448*e^(457*I*c) + 1534360887450562541273
27239461577071933130157764595997113973513183188399376*e^(456*I*c) + 8835009
68821791202600774541927769200737689393513734789368397093333311961880*e^(455
*I*c) + 4992519712457043983505377976607953988397368297591114957991804893688
371867680*e^(454*I*c) + 276931165383432592259833826376479361226640338596151
33489846664694361471028310*e^(453*I*c) + 1508223814314124137735664742100117
46852297437597059186295243989481140398152780*e^(452*I*c) + 8066795436075891
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+ 4238125846763232586394188569858685826755328005548627437019301405851325887
594480*e^(450*I*c) + 218764828927139099280403456125787058051215087562266963
17087651824252241418663320*e^(449*I*c) + 1109691996873209747499222595952504
44341219218535349655762591192576535872151766080*e^(448*I*c) + 5532691288195
28612502918869558947829098021956309349843584044631512291778800081490*e^(447
*I*c) + 2711843239670717527605640490148833507130242448403978318523237721944
200392830108580*e^(446*I*c) + 130698172034882898861932055083758183921249913
82340160316886507181296548981014818410*e^(445*I*c) + 6194859665303550287956
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28882075526473065446996857202104710942731861950899580202068990459031947629
 5408324280*e^(443*I*c) + 13247564123678374731574728211624836911209665019489
 53926492241643788264284546437221120*e^(442*I*c) + 5978992172944143218459161
 149299819706321732111578494525245228742976468409105395536290*e^(441*I*c) +
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 85274123140*e^(440*I*c) + 1161045516835550437629115017121163993137330211326
 77481112824047246361794049635726479850*e^(439*I*c) + 4997075672538590843575
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 c) + 2117589733466855707101501429210414722401838837940752841618541440888545
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 4092266983044118375790025854584036796364768280*e^(436*I*c) + 36318369652302
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 *e^(435*I*c) + 147030816732276833163041582099592047512043725225353339238819
 165193000407629544745753221740*e^(434*I*c) + 586403466972683242741643328921
 560909375197453864243299571990964608857245771134145204174990*e^(433*I*c) +
 230435107337384035737917859767306635201668278168913984209737666311848880384
 1131935313641840*e^(432*I*c) + 89232094473432967633318818816384717934996186
 70601026059730895962653291770229493028162575100*e^(431*I*c) + 3405405385129
 556915435234672217717265518754891078200850471832416872502943858916234921162
 8040*e^(430*I*c) + 12809891460168853967248054183040984770736750043860153680
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 6617938425222421786597241671026894318515408511467140969393115768793680*e^(4
 28*I*c) + 17365742188181910718741974724501581238835642099506586391023371481
 22769080611680719741726053840*e^(427*I*c) + 6259872156822252843650960708235
 034710201362776057176647226323089751446565288850103898153859920*e^(426*I*c)
 + 222519591767957777571673660360074802222113642321463998038643709633914912
 23687245823457351580140*e^(425*I*c) + 7800980736802423987561373305885141712
 5327114681070889640794249282633470580756557083923203377160*e^(424*I*c) + 26
 974580144021129697268360186387895435796230852007659517712822762927324021520
 9708218497363414140*e^(423*I*c) + 92008939302958903287460185002715932261252
 6368444771489781974361078847528891468831038436064951920*e^(422*I*c) + 30961
 319716215201623803015542414654517823620868102875377489029049859340201795657
 06177131421614590*e^(421*I*c) + 1027936473066384084473957786246926260464886
 1914297972589165243530651230690726244462479199894255180*e^(420*I*c) + 33675
 398872021568375902384593982753362559801058104184627345411136262431943240778
 260721756991027090*e^(419*I*c) + 108867995731829472826732905192034886797284
 621356445627530909104429486741257822633476898356826454040*e^(418*I*c) + 347
 351473214713780874352083129566601238765762775942366762733349952103889753982
 636403857556867777300*e^(417*I*c) + 109385321448622035867403243450086667849
 9770011305874172488975951612031456734608287095519501041975440*e^(416*I*c) +
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 18125605446882081152636090*e^(415*I*c) + 1043411751657039596665369315558240
 2109460348095473027807412321427346816928567197770376496170251803940*e^(414*
 I*c) + 31610939331284692750694306443618414656095969520945215743004044560386
 895241801579156543451940713351730*e^(413*I*c) + 945561802589319869193343034

66365652826858091314329189160736277175873841732196453379953705679466826880*
 $e^{(412*I*c)}$ + 2792857558000352066798353688981654776448649877946653878274889
 33863633745047373109049265172681702585720* $e^{(411*I*c)}$ + 8146081877365305796
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 74728560* $e^{(410*I*c)}$ + 2346518219239105142238141633073464768899155708935025
 778047637412681781575765422219127409260159438712250* $e^{(409*I*c)}$ + 667586629
 037114735850376686566928901089354386983053870872494529158095117918829660615
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 737257405706279108891366378428467414559930481172863538598193890* $e^{(407*I*c)}$
 + 520751785187932703864292633515443069511049935425005829381552416894081386
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 80* $e^{(405*I*c)}$ + 3868762182342771656324517230499798892631152823746075416924
 43176673997513742813591736171169652250611186480* $e^{(404*I*c)}$ + 1035561982592
 002935226384577908611548612111495080193573691339864706029186482466241805664
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 8305084825702911938903656162004262736182657700406301914070062380* $e^{(402*I*c)}$
) + 71581246868429414754738073636798397181727455815384090445033838526935969
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 861187649089085835702197488237157062380018624513772266094364175297685292443
 9870880* $e^{(400*I*c)}$ + 47188220843466207695099506953573780357108897491422567
 898048199018207708997005333860148836479527456156014520* $e^{(399*I*c)}$ + 119041
 855403877964948229577948370465600606623183045529526900430209270473212773847
 794935586074714329479939280* $e^{(398*I*c)}$ + 296825515282669589685318273280239
 050084555032203415941511962659596881615713799937680026497408305672297618840
 * $e^{(397*I*c)}$ + 731584972206818362874729621403974444280010446301161527339760
 544815300951787985538419764656214582667219914080* $e^{(396*I*c)}$ + 178244611493
 175185055635485663842190117441232229824949659165805393978719824656594597559
 5575734193348887952160* $e^{(395*I*c)}$ + 42932064780080221260174889088518264947
 90620720660151451468181910917240027863968724539127659633517053002976480* $e^{(}$
 $394*I*c)$ + 1022318202595486076721739030518645192356214547367429361991806349
 0411487496121804590274592702770571515456414680* $e^{(393*I*c)}$ + 24068785139705
 277161193465644506143285241361037768216818922184400141048460210944696647752
 723371932874594597328* $e^{(392*I*c)}$ + 560286834249035176584950138585345161671
 62591034367972498174660907450666778154353271630344650777885683547624184* $e^{(}$
 $391*I*c)$ + 1289670800847547122460236808664883849832862590255331320446361090
 49545144029547003347761521666283977931640178464* $e^{(390*I*c)}$ + 2935507435543
 427098081294535765623132997059826991874168629343739642556159671386762532763
 02591561523515603264403* $e^{(389*I*c)}$ + 6607644731058690976914759738508379345
 11089033149586707982764263394766756649565279879146173318386505740391093990*
 $e^{(388*I*c)}$ + 1470931146618934345515038362300100160482127749581443929904746
 910224777470198899052379114493999887003199419829579* $e^{(387*I*c)}$ + 323849193
 136185147642332193353957909837773553920764146734623566582388704832694930560
 9231585143748690203615957136* $e^{(386*I*c)}$ + 70521324141621979926023265245801
 430609853530545729339055246331216810210373402983663422033243253070724137390

61024*e^(385*I*c) + 1518963421490880039641791172264375474804852010973481245
9109878810493844381062650818971199637121458749456243274416*e^(384*I*c) + 32
362731322419549410330088943640247460378328561316422931292427145902887913071
643679502909055891236755143207382609*e^(383*I*c) + 682080330967936156837844
096192442108186149916400415534244055278768932724966083242310981485024664539
67157728078994*e^(382*I*c) + 1422131159648145176823866672767699094822716813
18790889840501039441748635545362467679832449103520321953011780083069*e^(381
*I*c) + 2933449200343007202870423834483428663138062854550400678230804455975
45970023446231563554135133105493516316320059272*e^(380*I*c) + 5986501411122
418589116765051805201503640032268413280814535970935877903386092124390855544
66861582623350303061961052*e^(379*I*c) + 120877035849365839308944222056935
063283704108140593750226539846117737648216609559734831601248698274330296158
612144*e^(378*I*c) + 241496651681033850328907654920274051171005901179544713
8773464205696455026442712426409599662771080264826008985061097*e^(377*I*c) +
47741411110660989702218453305949620164727142303742340606639568469509266426
85946929064114194400360936223590725470146*e^(376*I*c) + 9339341958053494225
251750965715057300707302083814774770306218224241022648247419956042957363055
823830898547303219757*e^(375*I*c) + 180798200680288599703499386230072306765
633142067088484999001396412373347632664793469632379360393281131850415917938
48*e^(374*I*c) + 3463765717267169016765734453719708704888235485399327047206
3943078773600446542963548348101269390443464480754513928502*e^(373*I*c) + 65
674859268867300098827375812875225610654551686261103681664007007537115778097
293533565243828873383722980353200611956*e^(372*I*c) + 123243941519332384741
960072588103506596406339253616391082062969960682419011745775738921817753391
954462609323881489157*e^(371*I*c) + 228911311738592780091492649162346834405
867740776456326108410928857257174707289268074347550225793244741923354395308
214*e^(370*I*c) + 420846342608949387277559021457924586578120966148561022647
008499529468452005980175119410628956210497609566002969884927*e^(369*I*c) +
765867795513962781012558444628751418710940895281304790836743661582071650032
154891482866406314834433199455459798934952*e^(368*I*c) + 137967652979621207
401710618806658944835544650121089019510716486035022892858681553900306287502
6711931941947738690360722*e^(367*I*c) + 24604423758454226639270816309832607
147349680919054930271456392388271922548863493611269914576924098511208733074
87457468*e^(366*I*c) + 4343909696601932173357359687781579293701295681940827
114215433175336093967845908766740738240037114570667410936998017178*e^(365*I
*c) + 759275270014667896115309507358501547319702974653363333154979396147328
5760935801904155116764831560875947581048693527224*e^(364*I*c) + 13139771494
104933881856681151418293112242551521535686871181266579813877606348160261747
201317735782566021306798298336024*e^(363*I*c) + 225146757413080699615061655
865028724304219302106732643929972864856006401038672536048477155470605929676
90653795951142520*e^(362*I*c) + 3819901586758608797600299875662767499479544
066790362502932234625013328648912087500501363812811389396034967028070716153
0*e^(361*I*c) + 64175100693260066806238064886004597170740843300086839368616
139164529108049844675353111842725798658088840347241496099644*e^(360*I*c) +
106764832017165594838085234189333528733587673329972530092661085186789939252

915937090760282232346919090426243399409323314*e^(359*I*c) + 175896258262755
 985757106812613979301265801031595484353614904672865169442232075776580447184
 134141375995770091499246759528*e^(358*I*c) + 286992943631231496557278010851
 576940896826497466066327528801560677007112837431926735088120974861760511367
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 568049310423995599886012800638619904022368378591108842602342094543682299102
 *e^(356*I*c) + 742228640908173124916937049462525617334148919679118270489831
 005497781951221069955839623452499748653124658873553401442137*e^(355*I*c) +
 117660072097578696518987505089023109220461269697027743301453589578895677123
 0793520381993106606880564628599822341722801012*e^(354*I*c) + 18475058564624
 515334452843005713263237811625533045659718876707580910793067948218349281707
 73126364639722071570131703785334*e^(353*I*c) + 2873610535922340187080835435
 582912277271967977394720159791070274927714276869531467182688981041061381703
 885403497544001592*e^(352*I*c) + 442767307910542531852431611298569365658485
 193610019245704445513448330504532145251634711848813322482367046510348395480
 5161*e^(351*I*c) + 67584804378885243725629359489638576266948555471955194861
 22877567981718587262362871994967079401831957927901682582941234362*e^(350*I*
 c) + 1022042377943463485133997529516339964170212224966366619305300830202609
 6932158568338309418237395541351819026907953220681013*e^(349*I*c) + 15312837
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 396582159028513532779682154451996208592*e^(348*I*c) + 227316035661288411004
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 76625919565639521227223276*e^(347*I*c) + 3343589782793658130117117545961082
 945429816796201741981007293673337850658442802420107245319345815533404669351
 6742390717832*e^(346*I*c) + 48733253505974923400852255563052101402196469313
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 *e^(345*I*c) + 703863497605948315670482240613950256985012022969663003767642
 20336697702961591099854055411376294871437468149528524796002762*e^(344*I*c)
 + 1007449618518537446117543009829801669624045538362229218684846942699661206
 07698907046343731011160948828100276729370132819357*e^(343*I*c) + 1429063191
 230555242465469284789542383713159258020223892364986521368398225020351556769
 70917419039834587967055588431566416784*e^(342*I*c) + 2009065871535788043803
 004695014416101745218512595419292098406889608594549085197748359058957576667
 70857888611738751858460424*e^(341*I*c) + 2799452444750398048229667304629608
 844921198748577911471240090794769204359417352933093054304386873331299124541
 96774070107264*e^(340*I*c) + 3866426730503800494573825628183169626519755509
 907792770487402386298587950182473561628886310156876647801012052873330827487
 91*e^(339*I*c) + 5293292527641139260039348369582435576725492389975607392144
 06599185047831955572583765358634395408771528009745467548382950094*e^(338*I*
 c) + 7183615963820582492091135444879010888683887440337132103324919713759067
 38341551540457264804304039664255915607349801911966551*e^(337*I*c) + 9664582
 753690377187477391307981516434835906841668322346880982911641606364181594521
 19815728809372125168836239364442397344064*e^(336*I*c) + 1289043515292933956
 480634330499677040181043935620106914267311067900030058398839787692376954090
 545278554544997710058754772400*e^(335*I*c) + 170458299670782280820467821816

769300269866114771277235502145654381093006963718808588282475750060524696321
0810351706405349408*e^(334*I*c) + 22348912763984394644786225783064348407246
104844681778598226206586919214786452666530625638235530012280010090936067510
66168944*e^(333*I*c) + 2905385722320057001953345274489482790856692529959823
749532695963414164833366773128218607899328588608916176593772088622582464*e^
(332*I*c) + 374525759487665120465733498842622638814395450198683066422234922
6361079609546822276067504899386703088982308185717143407211328*e^(331*I*c) +
47875274427809456851452048469715961653041694193282440732114595921296492550
48876854059844720661078151288179612574986359194560*e^(330*I*c) + 6068949803
156712248331871105329895471722806143008878014986559653687260694816550470195
890004511965527567432722969707577202160*e^(329*I*c) + 762973181562782158046
899242420700836643889673633302466186383810511044514894696232829763154703254
3419811821015837863013682720*e^(328*I*c) + 95130322740195229542091131912682
266422999120135256659402983810647978856909049931289480352274121440356338517
79511219335277360*e^(327*I*c) + 1176421227487648408001090071467347449337127
816055781198372445582656605561765808647936864186490811964341241364480377213
1657280*e^(326*I*c) + 14429816285208431204532978375375691965063154224649747
551295851507389524083226976789688601369628399900747658579201929300744260*e^
(325*I*c) + 175562732712242923968872914031257162134914862611454785713767516
90105656067838042151038271381300372757755676325408026834544840*e^(324*I*c)
+ 2118832140588288753961019837470686269589404922607709376413251251333619052
3978949694387686059124526755048042957954264706637460*e^(323*I*c) + 25367176
439119353621532260335983348154904982606125761711300683492963390816491583025
705268737539982149639300226512657426118880*e^(322*I*c) + 301284824145527032
645590189530881771560134374934382010784137698354483661481217545491975911299
67170764969700180348699207838960*e^(321*I*c) + 3550010310601964987627237679
694948220958137237103600501287780602748167280705994344524013631556850073237
9966585005678181937920*e^(320*I*c) + 41499832121963708043788523787401345541
780088930538206918853579026749273364671640037563488607716092887686471542838
602788559660*e^(319*I*c) + 481331176781840292165037485491103744789247190946
356038928293648639165537922788229573682851063281647159105983708711490794943
60*e^(318*I*c) + 5539091304497208621943268914633156608142795989696990021443
4296817731150863867056620768608187679709720152974148474907904177340*e^(317*
I*c) + 63247774101012179051794946075175569924076981338138483158042406747453
874729387631710544995247152912205118500597511052824347680*e^(316*I*c) + 716
603298611733955244419438892841091340911578446552456720842374024349446964649
27131812190659629511140639501743303863582092880*e^(315*I*c) + 8056624913068
268418187620188262351120636379033721801195411021064292776599764490382059542
1936873565314654415769070472655401600*e^(314*I*c) + 89883815801382382213973
270477954602744792877018051963347146307372464315121274929402347942874802899
499538953561056667668891020*e^(313*I*c) + 995122064720579659513403417380235
485153364033717178980408504709546575329772791134915068802907261111541019413
86019689567958040*e^(312*I*c) + 1093325373499662232039326785034263570798637
070017282940110420765304039238626540189786765164173142210894499224956127328
70169660*e^(311*I*c) + 1192097137020339270557553978236884444446474243245021

85328626347046599634721146573830681540495333543146776810911910410468628960*
 $e^{(310*I*c)}$ + 1289950760115919034107638634270973299485861735745958627058491
 59280943046458742663163454018491463855395649453952212899632198680* $e^{(309*I*c)}$
 + 1385297945491510894513527695765434031263307472436800308324672058958190
 43568155239264876762867172754338684027849855385453216080* $e^{(308*I*c)}$ + 1476
 489208055453334186231217678537773997829247483012287939243425749999379554217
 65370101235122939557467548549202174550009604780* $e^{(307*I*c)}$ + 1561859629535
 511961697382188321736965098525515892107305783657274762594764744659554285023
 36673743686499175698677875693611243400* $e^{(306*I*c)}$ + 1639778160596077253752
 645598165058478941877851014553603918974244829984153857876057653155092083377
 41590143078572243505132706580* $e^{(305*I*c)}$ + 1708698488689531011768603060531
 039943405303903472600884326768425055551412938308389612759742689286664948457
 23462544709102843680* $e^{(304*I*c)}$ + 1767209299705546420045757700530957005953
 346598706827320319759155323875770524148663235111401176804929293545175594798
 99220940360* $e^{(303*I*c)}$ + 1814081687709220598203685533166973216399848626282
 988285695602732956308976268293452635922190345608535307337105298421485379016
 80* $e^{(302*I*c)}$ + 1848311519837489418176678501747082571381281721582694132877
 65358532240773244336191900818557829905895684494889410451921524212840* $e^{(301}$
 $*I*c)}$ + 1869154744365675149263514056231175032619875083519300838245664444356
 89139233683411704641828762178799177848064220150818355261280* $e^{(300*I*c)}$ + 1
 876153931685100500714972805646035109124031329203120243708350626790376449902
 86285346673507093452964351257962696133511725652320* $e^{(299*I*c)}$ + 1869154744
 365675149263514056231175032619875083519300838245664444356891392336834117046
 41828762178799177848064220150818355261280* $e^{(298*I*c)}$ + 1848311519837489418
 176678501747082571381281721582694132877653585322407732443361919008185578299
 05895684494889410451921524212840* $e^{(297*I*c)}$ + 1814081687709220598203685533
 166973216399848626282988285695602732956308976268293452635922190345608535307
 33710529842148537901680* $e^{(296*I*c)}$ + 1767209299705546420045757700530957005
 953346598706827320319759155323875770524148663235111401176804929293545175594
 79899220940360* $e^{(295*I*c)}$ + 1708698488689531011768603060531039943405303903
 472600884326768425055551412938308389612759742689286664948457234625447091028
 43680* $e^{(294*I*c)}$ + 1639778160596077253752645598165058478941877851014553603
 91897424482998415385787605765315509208337741590143078572243505132706580* $e^{(}$
 $293*I*c)}$ + 1561859629535511961697382188321736965098525515892107305783657274
 76259476474465955428502336673743686499175698677875693611243400* $e^{(292*I*c)}$
 + 1476489208055453334186231217678537773997829247483012287939243425749999379
 55421765370101235122939557467548549202174550009604780* $e^{(291*I*c)}$ + 1385297
 945491510894513527695765434031263307472436800308324672058958190435681552392
 64876762867172754338684027849855385453216080* $e^{(290*I*c)}$ + 1289950760115919
 034107638634270973299485861735745958627058491592809430464587426631634540184
 91463855395649453952212899632198680* $e^{(289*I*c)}$ + 1192097137020339270557553
 978236884444446474243245021853286263470465996347211465738306815404953335431
 46776810911910410468628960* $e^{(288*I*c)}$ + 1093325373499662232039326785034263
 570798637070017282940110420765304039238626540189786765164173142210894499224
 95612732870169660* $e^{(287*I*c)}$ + 9951220647205796595134034173802354851533640

337171789804085047095465753297727911349150688029072611115410194138601968956
7958040*e^(286*I*c) + 89883815801382382213973270477954602744792877018051963
347146307372464315121274929402347942874802899499538953561056667668891020*e^
(285*I*c) + 805662491306826841818762018826235112063637903372180119541102106
42927765997644903820595421936873565314654415769070472655401600*e^(284*I*c)
+ 7166032986117339552444194388928410913409115784465524567208423740243494469
6464927131812190659629511140639501743303863582092880*e^(283*I*c) + 63247774
101012179051794946075175569924076981338138483158042406747453874729387631710
544995247152912205118500597511052824347680*e^(282*I*c) + 553909130449720862
194326891463315660814279598969699002144342968177311508638670566207686081876
79709720152974148474907904177340*e^(281*I*c) + 4813311767818402921650374854
911037447892471909463560389282936486391655379227882295736828510632816471591
0598370871149079494360*e^(280*I*c) + 41499832121963708043788523787401345541
780088930538206918853579026749273364671640037563488607716092887686471542838
602788559660*e^(279*I*c) + 355001031060196498762723767969494822095813723710
360050128778060274816728070599434452401363155685007323799665850056781819379
20*e^(278*I*c) + 3012848241455270326455901895308817715601343749343820107841
3769835448366148121754549197591129967170764969700180348699207838960*e^(277*
I*c) + 25367176439119353621532260335983348154904982606125761711300683492963
390816491583025705268737539982149639300226512657426118880*e^(276*I*c) + 211
883214058828875396101983747068626958940492260770937641325125133361905239789
49694387686059124526755048042957954264706637460*e^(275*I*c) + 1755627327122
429239688729140312571621349148626114547857137675169010565606783804215103827
1381300372757755676325408026834544840*e^(274*I*c) + 14429816285208431204532
978375375691965063154224649747551295851507389524083226976789688601369628399
900747658579201929300744260*e^(273*I*c) + 117642122748764840800109007146734
744933712781605578119837244558265660556176580864793686418649081196434124136
44803772131657280*e^(272*I*c) + 9513032274019522954209113191268226642299912
013525665940298381064797885690904993128948035227412144035633851779511219335
277360*e^(271*I*c) + 762973181562782158046899242420700836643889673633302466
1863838105110445148946962328297631547032543419811821015837863013682720*e^(2
70*I*c) + 60689498031567122483318711053298954717228061430088780149865596536
87260694816550470195890004511965527567432722969707577202160*e^(269*I*c) + 4
787527442780945685145204846971596165304169419328244073211459592129649255048
876854059844720661078151288179612574986359194560*e^(268*I*c) + 374525759487
665120465733498842622638814395450198683066422234922636107960954682227606750
4899386703088982308185717143407211328*e^(267*I*c) + 29053857223200570019533
452744894827908566925299598237495326959634141648333667731282186078993285886
08916176593772088622582464*e^(266*I*c) + 2234891276398439464478622578306434
840724610484468177859822620658691921478645266653062563823553001228001009093
606751066168944*e^(265*I*c) + 170458299670782280820467821816769300269866114
771277235502145654381093006963718808588282475750060524696321081035170640534
9408*e^(264*I*c) + 12890435152929339564806343304996770401810439356201069142
67311067900030058398839787692376954090545278554544997710058754772400*e^(263
*I*c) + 9664582753690377187477391307981516434835906841668322346880982911641

60636418159452119815728809372125168836239364442397344064*e^(262*I*c) + 7183
 615963820582492091135444879010888683887440337132103324919713759067383415515
 40457264804304039664255915607349801911966551*e^(261*I*c) + 5293292527641139
 260039348369582435576725492389975607392144065991850478319555725837653586343
 95408771528009745467548382950094*e^(260*I*c) + 3866426730503800494573825628
 183169626519755509907792770487402386298587950182473561628886310156876647801
 01205287333082748791*e^(259*I*c) + 2799452444750398048229667304629608844921
 198748577911471240090794769204359417352933093054304386873331299124541967740
 70107264*e^(258*I*c) + 2009065871535788043803004695014416101745218512595419
 29209840688960859454908519774835905895757666770857888611738751858460424*e^(
 257*I*c) + 1429063191230555242465469284789542383713159258020223892364986521
 36839822502035155676970917419039834587967055588431566416784*e^(256*I*c) + 1
 007449618518537446117543009829801669624045538362229218684846942699661206076
 98907046343731011160948828100276729370132819357*e^(255*I*c) + 7038634976059
 483156704822406139502569850120229696630037676422033669770296159109985405541
 1376294871437468149528524796002762*e^(254*I*c) + 48733253505974923400852255
 563052101402196469313659554492725674754339375283010407167744366955828922837
 488705858532439654489*e^(253*I*c) + 334358978279365813011711754596108294542
 981679620174198100729367333785065844280242010724531934581553340466935167423
 90717832*e^(252*I*c) + 2273160356612884110041950194705136766683665241807726
 0913944810748473084891890410181285412604854876625919565639521227223276*e^(2
 51*I*c) + 15312837206662775379347353212807682965712535652942631518286142403
 097738200270711195396582159028513532779682154451996208592*e^(250*I*c) + 102
 204237794346348513399752951633996417021222496636661930530083020260969321585
 68338309418237395541351819026907953220681013*e^(249*I*c) + 6758480437888524
 372562935948963857626694855547195519486122877567981718587262362871994967079
 401831957927901682582941234362*e^(248*I*c) + 442767307910542531852431611298
 569365658485193610019245704445513448330504532145251634711848813322482367046
 5103483954805161*e^(247*I*c) + 28736105359223401870808354355829122772719679
 773947201597910702749277142768695314671826889810410613817038854034975440015
 92*e^(246*I*c) + 1847505856462451533445284300571326323781162553304565971887
 670758091079306794821834928170773126364639722071570131703785334*e^(245*I*c)
 + 117660072097578696518987505089023109220461269697027743301453589578895677
 1230793520381993106606880564628599822341722801012*e^(244*I*c) + 74222864090
 817312491693704946252561733414891967911827048983100549778195122106995583962
 3452499748653124658873553401442137*e^(243*I*c) + 46375828845736715454493767
 825500568873332814556804931042399559988601280063861990402236837859110884260
 2342094543682299102*e^(242*I*c) + 28699294363123149655727801085157694089682
 649746606632752880156067700711283743192673508812097486176051136700881572878
 2643*e^(241*I*c) + 17589625826275598575710681261397930126580103159548435361
 4904672865169442232075776580447184134141375995770091499246759528*e^(240*I*c
) + 10676483201716559483808523418933352873358767332997253009266108518678993
 9252915937090760282232346919090426243399409323314*e^(239*I*c) + 64175100693
 260066806238064886004597170740843300086839368616139164529108049844675353111
 842725798658088840347241496099644*e^(238*I*c) + 381990158675860879760029987

566276749947954406679036250293223462501332864891208750050136381281138939603
49670280707161530*e^(237*I*c) + 2251467574130806996150616558650287243042193
021067326439299728648560064010386725360484771554706059296769065379595114252
0*e^(236*I*c) + 13139771494104933881856681151418293112242551521535686871181
266579813877606348160261747201317735782566021306798298336024*e^(235*I*c) +
759275270014667896115309507358501547319702974653363333154979396147328576093
5801904155116764831560875947581048693527224*e^(234*I*c) + 43439096966019321
733573596877815792937012956819408271142154331753360939678459087667407382400
37114570667410936998017178*e^(233*I*c) + 2460442375845422663927081630983260
714734968091905493027145639238827192254886349361126991457692409851120873307
487457468*e^(232*I*c) + 137967652979621207401710618806658944835544650121089
0195107164860350228928586815539003062875026711931941947738690360722*e^(231*
I*c) + 76586779551396278101255844462875141871094089528130479083674366158207
1650032154891482866406314834433199455459798934952*e^(230*I*c) + 42084634260
894938727755902145792458657812096614856102264700849952946845200598017511941
0628956210497609566002969884927*e^(229*I*c) + 22891131173859278009149264916
234683440586774077645632610841092885725717470728926807434755022579324474192
3354395308214*e^(228*I*c) + 12324394151933238474196007258810350659640633925
3616391082062969960682419011745775738921817753391954462609323881489157*e^(2
27*I*c) + 65674859268867300098827375812875225610654551686261103681664007007
537115778097293533565243828873383722980353200611956*e^(226*I*c) + 346376571
726716901676573445371970870488823548539932704720639430787736004465429635483
48101269390443464480754513928502*e^(225*I*c) + 1807982006802885997034993862
300723067656331420670884849990013964123733476326647934696323793603932811318
5041591793848*e^(224*I*c) + 93393419580534942252517509657150573007073020838
14774770306218224241022648247419956042957363055823830898547303219757*e^(223
*I*c) + 4774141111066098970221845330594962016472714230374234060663956846950
926642685946929064114194400360936223590725470146*e^(222*I*c) + 241496651681
033850328907654920274051171005901179544713877346420569645502644271242640959
9662771080264826008985061097*e^(221*I*c) + 12087703584936583930894422220569
350632837041081405937502265398461177376482166095597348316012486982743302961
58612144*e^(220*I*c) + 5986501411122418589116765051805201503640032268413280
81453597093587790338609212439085554466861582623350303061961052*e^(219*I*c)
+ 2933449200343007202870423834483428663138062854550400678230804455975459700
23446231563554135133105493516316320059272*e^(218*I*c) + 1422131159648145176
823866672767699094822716813187908898405010394417486355453624676798324491035
20321953011780083069*e^(217*I*c) + 6820803309679361568378440961924421081861
4991640041553424405527876893272496608324231098148502466453967157728078994*e
^(216*I*c) + 32362731322419549410330088943640247460378328561316422931292427
145902887913071643679502909055891236755143207382609*e^(215*I*c) + 151896342
149088003964179117226437547480485201097348124591098788104938443810626508189
71199637121458749456243274416*e^(214*I*c) + 7052132414162197992602326524580
143060985353054572933905524633121681021037340298366342203324325307072413739
061024*e^(213*I*c) + 323849193136185147642332193353957909837773553920764146
7346235665823887048326949305609231585143748690203615957136*e^(212*I*c) + 14

709311466189343455150383623001001604821277495814439299047469102247774701988
99052379114493999887003199419829579*e^(211*I*c) + 6607644731058690976914759
738508379345110890331495867079827642633947667566495652798791461733183865057
40391093990*e^(210*I*c) + 2935507435543427098081294535765623132997059826991
87416862934373964255615967138676253276302591561523515603264403*e^(209*I*c)
+ 1289670800847547122460236808664883849832862590255331320446361090495451440
29547003347761521666283977931640178464*e^(208*I*c) + 5602868342490351765849
501385853451616716259103436797249817466090745066677815435327163034465077788
5683547624184*e^(207*I*c) + 24068785139705277161193465644506143285241361037
768216818922184400141048460210944696647752723371932874594597328*e^(206*I*c)
+ 102231820259548607672173903051864519235621454736742936199180634904114874
96121804590274592702770571515456414680*e^(205*I*c) + 4293206478008022126017
488908851826494790620720660151451468181910917240027863968724539127659633517
053002976480*e^(204*I*c) + 178244611493175185055635485663842190117441232229
8249496591658053939787198246565945975595575734193348887952160*e^(203*I*c) +
73158497220681836287472962140397444428001044630116152733976054481530095178
7985538419764656214582667219914080*e^(202*I*c) + 29682551528266958968531827
328023905008455503220341594151196265959688161571379993768002649740830567229
7618840*e^(201*I*c) + 11904185540387796494822957794837046560060662318304552
9526900430209270473212773847794935586074714329479939280*e^(200*I*c) + 47188
220843466207695099506953573780357108897491422567898048199018207708997005333
860148836479527456156014520*e^(199*I*c) + 184874052990057326937527286118764
90890858357021974882371570623800186245137722660943641752976852924439870880*
e^(198*I*c) + 7158124686842941475473807363679839718172745581538409044503383
852693596921622426696740453944718143025248390*e^(197*I*c) + 273889562479526
560335522764656600088628077830508482570291193890365616200426273618265770040
6301914070062380*e^(196*I*c) + 10355619825920029352263845779086115486121114
95080193573691339864706029186482466241805664949381049856258510*e^(195*I*c)
+ 3868762182342771656324517230499798892631152823746075416924431766739975137
42813591736171169652250611186480*e^(194*I*c) + 1428017924502217624831808749
188252741343051332754177800847950346447635093335031505173458646596671894170
80*e^(193*I*c) + 5207517851879327038642926335154430695110499354250058293815
5241689408138675254608030847907167748571734720*e^(192*I*c) + 18759988218865
563564163635735986073278255737257405706279108891366378428467414559930481172
863538598193890*e^(191*I*c) + 667586629037114735850376686566928901089354386
9830538708724945291580951179188296606158111257706968604740*e^(190*I*c) + 23
465182192391051422381416330734647688991557089350257780476374126817815757654
22219127409260159438712250*e^(189*I*c) + 8146081877365305796702100252719214
15597183369881214299823291969785549876175969866367976653244974728560*e^(188
*I*c) + 2792857558000352066798353688981654776448649877946653878274889338636
33745047373109049265172681702585720*e^(187*I*c) + 9455618025893198691933430
346636565282685809131432918916073627717587384173219645337995370567946682688
0*e^(186*I*c) + 31610939331284692750694306443618414656095969520945215743004
044560386895241801579156543451940713351730*e^(185*I*c) + 104341175165703959
666536931555824021094603480954730278074123214273468169285671977703764961702

51803940*e^(184*I*c) + 3400232560601651617521694680847089844198028831694417
424794868779328950548418125605446882081152636090*e^(183*I*c) + 109385321448
622035867403243450086667849977001130587417248897595161203145673460828709551
9501041975440*e^(182*I*c) + 34735147321471378087435208312956660123876576277
5942366762733349952103889753982636403857556867777300*e^(181*I*c) + 10886799
573182947282673290519203488679728462135644562753090910442948674125782263347
6898356826454040*e^(180*I*c) + 33675398872021568375902384593982753362559801
058104184627345411136262431943240778260721756991027090*e^(179*I*c) + 102793
647306638408447395778624692626046488619142979725891652435306512306907262444
62479199894255180*e^(178*I*c) + 3096131971621520162380301554241465451782362
086810287537748902904985934020179565706177131421614590*e^(177*I*c) + 920089
393029589032874601850027159322612526368444771489781974361078847528891468831
038436064951920*e^(176*I*c) + 269745801440211296972683601863878954357962308
520076595177128227629273240215209708218497363414140*e^(175*I*c) + 780098073
680242398756137330588514171253271146810708896407942492826334705807565570839
23203377160*e^(174*I*c) + 2225195917679577775716736603600748022221136423214
6399803864370963391491223687245823457351580140*e^(173*I*c) + 62598721568222
528436509607082350347102013627760571766472263230897514465652888501038981538
59920*e^(172*I*c) + 1736574218818191071874197472450158123883564209950658639
102337148122769080611680719741726053840*e^(171*I*c) + 475010578857601519272
316617938425222421786597241671026894318515408511467140969393115768793680*e^(
170*I*c) + 128098914601688539672480541830409847707367500438601536803204497
701119911289087105659482783340*e^(169*I*c) + 340540538512955691543523467221
77172655187548910782008504718324168725029438589162349211628040*e^(168*I*c)
+ 8923209447343296763331881881638471793499618670601026059730895962653291770
229493028162575100*e^(167*I*c) + 230435107337384035737917859767306635201668
2781689139842097376663118488803841131935313641840*e^(166*I*c) + 58640346697
268324274164332892156090937519745386424329957199096460885724577113414520417
4990*e^(165*I*c) + 14703081673227683316304158209959204751204372522535333923
8819165193000407629544745753221740*e^(164*I*c) + 36318369652302591732197444
409798122022640824604130552506742586795183267354382847875885730*e^(163*I*c)
+ 883672064086047030569451402154796955129679409226698304411837579002585458
4036796364768280*e^(162*I*c) + 21175897334668557071015014292104147224018388
37940752841618541440888545729943138209036820*e^(161*I*c) + 4997075672538590
84357596314813794768069337190915967491907488904933922677579665354338960*e^(
160*I*c) + 1161045516835550437629115017121163993137330211326774811128240472
46361794049635726479850*e^(159*I*c) + 2655680638904340753449670236910154579
5994861757741414789944652712127566910185274123140*e^(158*I*c) + 59789921729
44143218459161149299819706321732111578494525245228742976468409105395536290*
e^(157*I*c) + 1324756412367837473157472821162483691120966501948953926492241
643788264284546437221120*e^(156*I*c) + 288820755264730654469968572021047109
427318619508995802020689904590319476295408324280*e^(155*I*c) + 619485966530
35502879564338815234310660410902037882473161804774492916216575880077680*e^(
154*I*c) + 1306981720348828988619320550837581839212499138234016031688650718
1296548981014818410*e^(153*I*c) + 27118432396707175276056404901488335071302

42448403978318523237721944200392830108580*e^(152*I*c) + 5532691288195286125
 02918869558947829098021956309349843584044631512291778800081490*e^(151*I*c)
 + 1109691996873209747499222595952504443412192185353496557625911925765358721
 51766080*e^(150*I*c) + 2187648289271390992804034561257870580512150875622669
 6317087651824252241418663320*e^(149*I*c) + 42381258467632325863941885698586
 85826755328005548627437019301405851325887594480*e^(148*I*c) + 8066795436075
 89140759305010796189568269842021613388955218916278823182639488190*e<sup>(147*I*
 c)</sup> + 1508223814314124137735664742100117468522974375970591862952439894811403
 98152780*e^(146*I*c) + 2769311653834325922598338263764793612266403385961513
 3489846664694361471028310*e^(145*I*c) + 49925197124570439835053779766079539
 88397368297591114957991804893688371867680*e^(144*I*c) + 8835009688217912026
 00774541927769200737689393513734789368397093333311961880*e^(143*I*c) + 1534
 36088745056254127327239461577071933130157764595997113973513183188399376*e<sup>(
 142*I*c)</sup> + 2614397627990202144347194566508025456305681018352040188980028549
 3144867448*e^(141*I*c) + 43694424829101139145653531360695958626693388580534
 19381214131241925047008*e^(140*I*c) + 7160994975990580798956333385529402291
 92858196481597830078819711862600096*e^(139*I*c) + 1150514818520808488737003
 88354521315567640365124003103691176697194292320*e^(138*I*c) + 1811576849561
 5758076710303055505625589254293659193314153418333944596408*e^(137*I*c) + 27
 94709104475686611842790694973699164482254723977210209725661304403472*e<sup>(136
 *I*c)</sup> + 4222761266320036875477547465557099887105271330866601613663536567872
 88*e^(135*I*c) + 6247355078105329531771077469024711412412518756573184844178
 1904032672*e^(134*I*c) + 90466935238256829790443389631042631676725868263679
 11338826483549173*e^(133*I*c) + 1281817464914970810859604189828359000790789
 921169405304612211251818*e^(132*I*c) + 177642829135119348577194437675802830
 239905460092687136494961404333*e^(131*I*c) + 240708019135297571018580229143
 72045864746991786182039740274325264*e^(130*I*c) + 3187749929744346497211536
 044751776582320958627923816470590659024*e^(129*I*c) + 412430698299915190848
 067222327219435067747934091894670488982928*e^(128*I*c) + 521081176291770486
 60492400985175830987505700566877818954141639*e^(127*I*c) + 6426195485535248
 576425068136870465530087114003875716691383902*e^(126*I*c) + 773204636991145
 775061462731028098506094432675788136295011259*e^(125*I*c) + 907226057222088
 14918642284639487187764607589706493970774776*e^(124*I*c) + 1037518449987117
 5501909398956596684116802997082526660323524*e^(123*I*c) + 11558554128935942
 60345544966642687823630035899363232371472*e^(122*I*c) + 1253704965869212726
 62198050851269323171167338854081782959*e^(121*I*c) + 1323170887010489697380
 0056733779919089340836756009580718*e^(120*I*c) + 13579906631614798428506428
 48032544982878359839580349899*e^(119*I*c) + 1354425949166361161915746506253
 31646238501101627937224*e^(118*I*c) + 1311878180117217472967933989431815369
 4964675368481194*e^(117*I*c) + 12330967001397233651819972207509325906552876
 25342156*e^(116*I*c) + 112391604542246650966429162063124338952554575234051*
 e^(115*I*c) + 9925490738534402272939987038714580495445431374618*e^(114*I*c)
 + 848552202276512356496200136959676295361696315113*e^(113*I*c) + 701645153
 22544462906873548813748091084561870680*e^(112*I*c) + 5605927253067558551780
 452883689835514455118670*e^(111*I*c) + 432333688644261557547944179250800440

$604964868e^{(110*I*c)} + 32147887693375338817454482515377350383950278e^{(109*I*c)} + 2302150411226234925855222345201500900533576e^{(108*I*c)} + 158566476113257562566117432227203884298856e^{(107*I*c)} + 10490402669510897424624643766470754045064e^{(106*I*c)} + 665634670676210063754191847109971141414e^{(105*I*c)} + 40443624781415311581857832389099634564e^{(104*I*c)} + 2348998374244347079532766203075607598e^{(103*I*c)} + 130171193079172823835151430773360024e^{(102*I*c)} + 6868329225263681349501997341320517e^{(101*I*c)} + 344277152012875134140739302960914e^{(100*I*c)} + 16353164647151530240529137618111e^{(99*I*c)} + 734057263616388449968842366924e^{(98*I*c)} + 3104222522074681615625020522e^{(97*I*c)} + 1232445557346832245176696904e^{(96*I*c)} + 45759117183402579073139583e^{(95*I*c)} + 1581796642397812408161814e^{(94*I*c)} + 50648660944512569972179e^{(93*I*c)} + 1493326612293984160368e^{(92*I*c)} + 40261256699368950388e^{(91*I*c)} + 984382804329835768e^{(90*I*c)} + 21608403021340047e^{(89*I*c)} + 420601518659718e^{(88*I*c)} + 7146142307307e^{(87*I*c)} + 103818048048e^{(86*I*c)} + 1253841160e^{(85*I*c)} + 12085216e^{(84*I*c)} + 87153e^{(83*I*c)} + 418e^{(82*I*c)} + e^{(81*I*c)}) * \tan(1/4*d*x + c) + 28*(299*a^2e^{(1027/2*I*c)} + 116610*a^2e^{(1025/2*I*c)} + 22680645*a^2e^{(1023/2*I*c)} + 2933363420*a^2e^{(1021/2*I*c)} + 283802910885*a^2e^{(1019/2*I*c)} + 21909584720322*a^2e^{(1017/2*I*c)} + 1405865019554882*a^2e^{(1015/2*I*c)} + 77121738215879370*a^2e^{(1013/2*I*c)} + 3692203217135946825*a^2e^{(1011/2*I*c)} + 156713514333171921595*a^2e^{(1009/2*I*c)} + 5970784896604221606627*a^2e^{(1007/2*I*c)} + 206263478282239243233771*a^2e^{(1005/2*I*c)} + 6514488191198508953598437*a^2e^{(1003/2*I*c)} + 189421272128521178822066445*a^2e^{(1001/2*I*c)} + 5100844261395996953829648360*a^2e^{(999/2*I*c)} + 127861163009333998455801820954*a^2e^{(997/2*I*c)} + 2996746014847853620317693731016*a^2e^{(995/2*I*c)} + 65928412548866286902779022351001*a^2e^{(993/2*I*c)} + 1366183222241782313586863286641184*a^2e^{(991/2*I*c)} + 26748429587445855729768539197182585*a^2e^{(989/2*I*c)} + 496183373555284804627911954521619600*a^2e^{(987/2*I*c)} + 8742278599147295285480758801384765381*a^2e^{(985/2*I*c)} + 146631857213698093058440158830415565266*a^2e^{(983/2*I*c)} + 2346109768759347903572910732245570209842*a^2e^{(981/2*I*c)} + 35875929609390406738394666219971221205725*a^2e^{(979/2*I*c)} + 525223629498383939328992961505581135597600*a^2e^{(977/2*I*c)} + 7373332079799133639063658614527645894565137*a^2e^{(975/2*I*c)} + 99403445980961943589598468520732983548550704*a^2e^{(973/2*I*c)} + 1288694773452054046771640788950472807542901137*a^2e^{(971/2*I*c)} + 16086467322724552698501340975783163797842534840*a^2e^{(969/2*I*c)} + 193573846127814805336166214026334290497006915130*a^2e^{(967/2*I*c)} + 2247954664299349273623637155894975599108753667672*a^2e^{(965/2*I*c)} + 25219245783549721366067799114158978864399924058441*a^2e^{(963/2*I*c)} + 273590663366155266234104272169452688263007484513781*a^2e^{(961/2*I*c)} + 2872702692034937563897971496575074831246739185614575*a^2e^{(959/2*I*c)} + 29219499075263535843858862377648073071285310974657915*a^2e^{(957/2*I*c)} + 288136830654984406530837787337700439328183658059646399*a^2e^{(955/2*I*c)} + 2756769770542048744355911382380396555656358932078858681*a^2e^{(953/2*I*c)} + 25608953102811736020699671981400531591652082353456701666*a^2e^{(951/2*I*c)} + 231137354125616547233443142991124898825470770660317194554*a^$

$2 * e^{(949/2 * I * c)} + 202823168252716982086969843106854622463066167413540165587$
 $0 * a^{2 * e^{(947/2 * I * c)}} + 17314186832105700122787937106308399484490093525695416$
 $858785 * a^{2 * e^{(945/2 * I * c)}} + 143872782220632026353482783145282086606314049235$
 $674031853952 * a^{2 * e^{(943/2 * I * c)}} + 116436703282462307336668513825130991204009$
 $3060141928716087681 * a^{2 * e^{(941/2 * I * c)}} + 91826333021602124529189686038068280$
 $36823248995981353166570702 * a^{2 * e^{(939/2 * I * c)}} + 7060434872440117095882960520$
 $5937063913034447082349890348162895 * a^{2 * e^{(937/2 * I * c)}} + 52953348970643482445$
 $1250188592739498197462491764283044018575440 * a^{2 * e^{(935/2 * I * c)}} + 38757418032$
 $63271608578048125955066152785493897151672716099440360 * a^{2 * e^{(933/2 * I * c)}} + 2$
 $7695464742706927996157891275342643431040906047696742940234352112 * a^{2 * e^{(931$
 $/2 * I * c)}} + 19330351487278611592337971662825341931894417643587605680258986577$
 $2 * a^{2 * e^{(929/2 * I * c)}} + 13183336549908888994177465203742886492572231335731279$
 $05086467548080 * a^{2 * e^{(927/2 * I * c)}} + 8788918880786687559267555413796199314981$
 $595789787074901006933535500 * a^{2 * e^{(925/2 * I * c)}} + 572971958305794798809052837$
 $59887310015692255967368709515502818525680 * a^{2 * e^{(923/2 * I * c)}} + 3654062378495$
 $18248372060960463581650995651842549690071352678851012540 * a^{2 * e^{(921/2 * I * c)}}$
 $+ 2280416002497499972330270356628053500702221025770502602102780787687624 * a^{2 * e^{(919/2 * I * c)}}$
 $+ 139313401949339352280134662941799874450687582195654368623$
 $49156508804360 * a^{2 * e^{(917/2 * I * c)}} + 8333974773646914353242807331076216352511$
 $3789344043183602916613227872140 * a^{2 * e^{(915/2 * I * c)}} + 48834483612598147104816$
 $9163657616516954158792795618978221198980949925840 * a^{2 * e^{(913/2 * I * c)}} + 28037$
 $93220826297874163429682131102503306549005265399219277240917413502940 * a^{2 * e^{(911/2 * I * c)}}$
 $+ 1577740463997525927245493384353947454466523681911756699731867$
 $2304666566800 * a^{2 * e^{(909/2 * I * c)}} + 87039467228531013705409191271452383389335$
 $356884921808641492423363762400060 * a^{2 * e^{(907/2 * I * c)}} + 470873979833165454444$
 $627659803101969430867153471663329279188629910819335440 * a^{2 * e^{(905/2 * I * c)}} +$
 $249869791757989597797872358857628393313729651874348110477693256639157945540$
 $0 * a^{2 * e^{(903/2 * I * c)}} + 13009255453778977514917501132236907767356887691068368$
 $589411338908245552368240 * a^{2 * e^{(901/2 * I * c)}} + 664700802720379036821982198354$
 $88944949996749070187078822650198449544212324410 * a^{2 * e^{(899/2 * I * c)}} + 3333781$
 $06740586138251610124958796714576935099619835073172307696551648259829120 * a^{2 * e^{(897/2 * I * c)}}$
 $+ 1641662348095460363514356729639026504243315309578757871628$
 $322407537121464954630 * a^{2 * e^{(895/2 * I * c)}} + 793893332049744963591896968075832$
 $6307339116239071846330248311795038723075723660 * a^{2 * e^{(893/2 * I * c)}} + 37710710$
 $670742203746980267936076214521784550649912575518820552220031814445215110 * a^{2 * e^{(891/2 * I * c)}}$
 $+ 175987317444160012665123402745831252173155432052395534884$
 $810603675523612022791280 * a^{2 * e^{(889/2 * I * c)}} + 807047768188333538846684803460$
 $779790931055555100120589990758127536168861246233500 * a^{2 * e^{(887/2 * I * c)}} + 363$
 $749864602642559874527922880367951714009035024131421637157046817473817492130$
 $7980 * a^{2 * e^{(885/2 * I * c)}} + 16116628162411464679896502836324771389620584856574$
 $877922468282685664283130175327730 * a^{2 * e^{(883/2 * I * c)}} + 702090179454115912009$
 $61651730058863557722458877098381486938641100483455911795967290 * a^{2 * e^{(881/2 * I * c)}}$
 $+ 3007712344684525202665626073296659221591153815000877283459300062920$
 $05623202669775630 * a^{2 * e^{(879/2 * I * c)}} + 1267300191943737324119320599349683654$
 $493348850217007321150739273112190995849189116570 * a^{2 * e^{(877/2 * I * c)}} + 525285$

606278183393820149086942244493046474642337458887176544669032400994119860091
2010*a²*e^(875/2*I*c) + 21421765289837057403869596169941475678645506492471
864168704518082797513207707602578070*a²*e^(873/2*I*c) + 859662077957427711
63110084083079326198692050781198818319037362716785215369332723227360*a²*e^(871/2*I*c) + 3395315651658181574881115462365147392451724847364827297411967
32548715077274621271815060*a²*e^(869/2*I*c) + 1320011514737435644039252185
614537842325531317784101906639727300233020372137428050228960*a²*e^(867/2*I*c) + 505224056037277225250841188446932961822637692797906299614523611049177
3700805441539104570*a²*e^(865/2*I*c) + 19039736700477034191914417874408695
155655405986129726283796391182602876937826158174743280*a²*e^(863/2*I*c) +
706592022204022196993271424724802422878766537649533021333603764482872261874
87345644845530*a²*e^(861/2*I*c) + 2582653199983859208005705893482429381264
45962121695949028377963603738714515662920857912400*a²*e^(859/2*I*c) + 9298
442698396182432776277869232434146236637762612203148716791997224555846984342
16819569010*a²*e^(857/2*I*c) + 3298047053993950146620842503721529058318485
356660075016582248017786240344190976379491032900*a²*e^(855/2*I*c) + 115255
143382481221158069012236314088656650608739923317312167175458396413954935434
78555391940*a²*e^(853/2*I*c) + 3968931527476296259834409957765731214574240
8227826431282422824844089957174694332245240202130*a²*e^(851/2*I*c) + 13469
404583173308483157417052349751010412883603540357635301598204857708143840674
7258639643920*a²*e^(849/2*I*c) + 45054181599435041632956348265590924497932
0334438405914968411406156252958317001977289679320570*a²*e^(847/2*I*c) + 14
855351189531389488659783317179105994974562961777923747855905162334746559064
39549342970114480*a²*e^(845/2*I*c) + 4828807237111495170850202026795526560
032755045691541653850695089412640252061166703373641008090*a²*e^(843/2*I*c)
+ 154757897173974602762348431110439506950134710306254279251753537971128762
41958974587338558471200*a²*e^(841/2*I*c) + 4890660907252005761107137050799
6340224776681379775034925868200923794664214854540668184735223060*a²*e^(839/2*I*c) + 15241537410607462293922428415365960155062710943632592662101448022
9333534862467810217119890170400*a²*e^(837/2*I*c) + 46846826182477495809179
1071955357916616933320342602739423768916749249443401418804336754943102630*a²*e^(835/2*I*c) + 14202487865181874667180230489017311183779468543008119776
71072304411278973969440001583221282013130*a²*e^(833/2*I*c) + 4247400925077
373176113486312190145472181686422993933942447632846633095644440032003339271
168690570*a²*e^(831/2*I*c) + 125313382279208564920745903266740993744531228
68913516861854565249779642744578491059412040960382510*a²*e^(829/2*I*c) + 3
647765370604224594468882060469649421908133476712105791233191413780441530797
3337793902177294599210*a²*e^(827/2*I*c) + 10477329491970818142645631824670
7841758575517441039866971803838079319268269378919589734008354661810*a²*e^(825/2*I*c) + 29696600515656353110597107494005029521095296334229212877345071
5502535457684406378925697100618058220*a²*e^(823/2*I*c) + 83067789082340559
388104802804137554233395528307396262678471831237609441676025354759120611865
9958460*a²*e^(821/2*I*c) + 22933213080155869179327993170249823391686046493
69171514848750864257523843794511936002538498981041920*a²*e^(819/2*I*c) + 6
249414012928962286061010403529002753607342360906086192033185103428400913070

889052292940484712327830*a²*e^(817/2*I*c) + 168108967282259618741944983402
22314518223276631636193848187700774478509776074186756740832022063618940*a²
*e^(815/2*I*c) + 4464306447159171747970402027656364257071788382666814516924
7611761065722522854988204793268950962007830*a²*e^(813/2*I*c) + 11704754358
181518706267332142230356178960091716528425495941493214476219409688989043243
9125557135120080*a²*e^(811/2*I*c) + 30300429227176309474572003993633457101
6895425314693546495893805386105536448400356863388712154386414250*a²*e<sup>(809
/2*I*c)</sup> + 77454342161420349560097161546607797965236888121893351042429424002
1409386956035526159745861544320722480*a²*e^(807/2*I*c) + 19551720942932691
644662098033171025525950216475811512178778422970591270024951138107726547846
44585060360*a²*e^(805/2*I*c) + 4874135079132755414347375690887802711794105
650938863970400230002709422093992525472165364797296790138960*a²*e<sup>(803/2*I
*c)</sup> + 120009057037681981749542820346317572094974119140734828047055158976899
69997576557491508854781156886995980*a²*e^(801/2*I*c) + 2918531344865391744
146489048732318953323358055838602673457061502945702793349151273875323713463
0607181520*a²*e^(799/2*I*c) + 70109686530806379548962065855267314803677176
495241796951534731669884594114550121278447472661295579702060*a²*e<sup>(797/2*I
*c)</sup> + 166373421015960065744946831605129638085441168071352620640483463800291
640278014342520469741317600337261840*a²*e^(795/2*I*c) + 390041272754974777
005627699154842246988726024604162438024095285594852487855850372070546472734
126022684700*a²*e^(793/2*I*c) + 903411976634483542635289998462833963669695
152185259996654868051991473572372870248662847924612744264062440*a²*e<sup>(791/
2*I*c)</sup> + 206746084187352999911842026513428376396444521265603155739957918539
0572143523605908612468666220745228526760*a²*e^(789/2*I*c) + 46751116425487
460919156707540760323711761364540363599820752214772978099252044766666622390
31601034545996300*a²*e^(787/2*I*c) + 1044663893361296440733687256037795716
5244351995018446220321535598596368046659626683803611150049940561221040*a²*
e^(785/2*I*c) + 23068393515288186064484465603117961911284673754084184347491
242043413536449530894861283359363464825007526940*a²*e^(783/2*I*c) + 503431
255467132379856233136463293306478583494956274929252051615666930016106505699
67429902726126062669565040*a²*e^(781/2*I*c) + 1085851881577139421210778517
898957224225946890187204550705634761213011847278115743234662142918342018546
72060*a²*e^(779/2*I*c) + 2314908265850057878077532735085387653932377948915
89480734288576865823349858144380580905331957850509282551472*a²*e<sup>(777/2*I*
c)</sup> + 4878144325211114975854067195068020556369220879498886270252084544473001
00038791012297859131526297355904933960*a²*e^(775/2*I*c) + 1016149825224928
415282430116492426171242192567909037312010368898174633451476277701838680759
316125015139743120*a²*e^(773/2*I*c) + 209250927524733972545755390380592999
3734938547721804454197071999505511719948252419379343549397905586122452665*a
²*e^(771/2*I*c) + 42599806114811751815296146169680792882669983472869168677
65191606402809238614481985333675831806087956768872250*a²*e^(769/2*I*c) + 8
574370436061278588323329731982052232445717828791755963170466238033798298798
179665096355346431444695120516791*a²*e^(767/2*I*c) + 170637639438192012571
545460079441904848004835412196255703347731392002582189336345287645900829744
65232719019576*a²*e^(765/2*I*c) + 3357753286250861240493214356205675816951

2543178723442784444413242416994607618240798840372204654901124952727575*a²*
e^{-(763/2*I*c)} + 65335209855139745225460458167882643715464174702379903424415
436335141712932934847111707604519947532103535939690*a²*e^{-(761/2*I*c)} + 125
716823797005642317347526498401790837531685611023648804925465449065255408041
032308439566500708700602697766710*a²*e^{-(759/2*I*c)} + 239226901086351112501
913420176641388141834680717243149148240541912365024035242396630141827067823
768453577792206*a²*e^{-(757/2*I*c)} + 450214953158482436877449323534443636424
987689673176576808637277491307797447679834039022495671246539361123241623*a²*
e^{-(755/2*I*c)} + 838003405633226261698002475165028859961836560477445660978
330588882054803408792968277041826209840110522910353801*a²*e^{-(753/2*I*c)} +
154280209465184867438692967750633613530145175778216248102935419137838565530
9839590342295782070267248163797079525*a²*e^{-(751/2*I*c)} + 28095435186682042
668237968016271187636458887411913075046478853982245873150046064479023399625
21984923603337579865*a²*e^{-(749/2*I*c)} + 5061099938970696245440097462936504
060098234194867891099800067879466606691927034473565491643439734586575243562
587*a²*e^{-(747/2*I*c)} + 901903653105413936414503619872987518150278899672116
5710912853819799244035211696760046421655662466728741232371823*a²*e<sup>-(745/2*
I*c)</sup> + 15900249437791405935404298689959721296914556453023635621602157015426
400883311961742504003269238750112959349688008*a²*e^{-(743/2*I*c)} + 277331231
705280606650477930291879810693365658345015913554859194601763595704569769266
92677356655692104901055424742*a²*e^{-(741/2*I*c)} + 4785939505147857748577714
882562266649184418998967128714889531275003106308959703928034227789131364828
2659506765160*a²*e^{-(739/2*I*c)} + 81720646798485325019374317784322759957844
922273569903847642933003662634138916360473885905820817778748179763625775*a²*
e^{-(737/2*I*c)} + 138075050472911460011366856295168555289257189097324670311
033951735099801993846300774206061209409704454402575173168*a²*e^{-(735/2*I*c)}
+ 230855486672626709723535927872568854471077332500022366798083306901537948
948135371592062382972028832652892521530223*a²*e^{-(733/2*I*c)} + 381970324888
971137824260417119355880508011405306507252717319111638967573708192814947593
237962837516587367083029056*a²*e^{-(731/2*I*c)} + 625469971082512052221445939
226339313157527561686453294385607721125006005737878528154829176814174073609
304783133155*a²*e^{-(729/2*I*c)} + 101366358338138231552611990038579378059289
7362259521880870256689578864566731937045334553766119426780595860617854190*a²*
e^{-(727/2*I*c)} + 16259794205311056824146752182437539020208181695242225342
01660329925485571915419014974199094126837464169182082222286*a²*e<sup>-(725/2*I*
c)</sup> + 2581627107009900734522064744597551209802304406910934736678835899008279
100485386551512986171247434443617529951824987*a²*e^{-(723/2*I*c)} + 405744720
573650872049401506257084267750527414864488161681132670262810709477451426568
0940713014256351659607202219536*a²*e^{-(721/2*I*c)} + 63127263274934193211710
032130069260225876978630199913947211901704358570842982460060249222268465163
28771260784350055*a²*e^{-(719/2*I*c)} + 9723212783688537643304451173790932352
763392117878194099713212407827924571779656265406369365137858973995542668178
560*a²*e^{-(717/2*I*c)} + 148270788409582044237265467055125273579573616473965
08632935988933241213590716537429203049741620280852461278755893511*a²*e<sup>-(71
5/2*I*c)</sup> + 2238612040430000442287600735176164788352161033439978593903801179

0449067868627843226239824339519408830910225209114648*a²*e^(713/2*I*c) + 33
 466028808642300425453129213091641815466355026636652947083261177887459487866
 470370690708668514061085213215020765318*a²*e^(711/2*I*c) + 495400606505030
 630373822802586990494363519647331809299341174450805966315618962160157809709
 84005090516521389882187000*a²*e^(709/2*I*c) + 7262079921891656521787587138
 133934287480384013353137620330980577506779778460262098863480834399886641525
 1767099657115*a²*e^(707/2*I*c) + 10542485378702875446967396086105184807706
 437917920140868750916701329558833838494542506726849196803900696805658192170
 7*a²*e^(705/2*I*c) + 15157517035573920994637387637005765054931832058450904
 1952034589942144397207049663095445023255855799320406828152904813*a²*e<sup>(703
 /2*I*c)</sup> + 21584500377680530286118469256592700138314485027809057769126013735
 5499134974361313448026444380882959804366246287914973*a²*e^(701/2*I*c) + 30
 444639322714681461398353972744479817356485837498041308048827113525666191338
 4313342183872402332578752912321480097437*a²*e^(699/2*I*c) + 42536406129097
 286459938160524197691302624289235233411518585440495664431380543031546868590
 2485280907282658089881714375*a²*e^(697/2*I*c) + 58873291142465243203647521
 867626739183382958707173358155265359956911506265595863443618010564226852757
 6118276535857990*a²*e^(695/2*I*c) + 80725365813734230790016059554407842018
 981333839751395145958064955159381671809443604445001692419934962715530609250
 3566*a²*e^(693/2*I*c) + 10966365784594449163767232860698296342408290402830
 71984682114965507192138829786266640857980171465345670412137298803158*a²*e<sup>(
 691/2*I*c)</sup> + 1476057995569390637223352697242483210569861862780589923317140
 914906186165044966041302091938311768459791677885305942531*a²*e^(689/2*I*c)
 + 196860807418766241096038612492180776083941991244911808807944047937883069
 6763837592112817321648516678192250226251889260*a²*e^(687/2*I*c) + 26017021
 223966451413459310334045709950274662042842744886537507537614986756688639708
 80304480234793529924712351195427555*a²*e^(685/2*I*c) + 3407421285455694046
 474320798159090075798729924862080209650328438067504016881598067707844396242
 531890195544301426987190*a²*e^(683/2*I*c) + 442274283952382676264547995705
 724354154012205495170378786211393790404856804828685096905646021812535708934
 9296008547181*a²*e^(681/2*I*c) + 56896159099703015201083196403639731344701
 908177496182015410405546717258037261898429721460046823921212896500404767716
 16*a²*e^(679/2*I*c) + 7254836084908468404646678446989600665079708038513510
 048563537225669386463363803374148622157625354861291347122905242320*a²*e<sup>(6
 77/2*I*c)</sup> + 916967282297774397711132080454111190466027322150645165194970721
 2055959466012505191220572773289655701792453230423105120*a²*e^(675/2*I*c) +
 11489207467455499344482418745478155563014761126816126080537383460531629386
 309522634240117536837763365043954166727248280*a²*e^(673/2*I*c) + 142713476
 457492261657144523776111167216233279632303483234605254933563184657380079264
 44717352691455157582924190074684640*a²*e^(671/2*I*c) + 1757549620213874323
 744070248931841334869167108820905095696361351136690612608252127239460711551
 5936172525552891395395672*a²*e^(669/2*I*c) + 21460869622603798373862295375
 991979844261527020594958201034758080773685660129127602861264204469990486648
 515185847661920*a²*e^(667/2*I*c) + 259844818219287902290195332203528103640
 536514281394054210119467235039313168038586394302259987352886386330197028660

58680*a²*e^(665/2*I*c) + 3119883339946498355189807082430995856239001431320
1877547672616650741608758923820402191204459236862231482346227933403280*a²*
e^(663/2*I*c) + 37149372802844713728543373961698511909667686136312680532545
007254483943352647647881308788995351984175084901730678814480*a²*e<sup>(661/2*I
*c)</sup> + 438718226108610047950165641481312466452533807697183944618422952118597
74351397481825053002145790551685375056214564880600*a²*e^(659/2*I*c) + 5138
948934058015407449824334333687371742960590384296136947593626074973184931365
4276883477517483490016178128812433612320*a²*e^(657/2*I*c) + 59710696542140
997334998224639349540371726067305327015858613255599003834616905750464249459
471454428929767659680543253880*a²*e^(655/2*I*c) + 688264961339292593339463
496887561071818270947606874315412367937076095671573933569800158747289864455
05375939321119773600*a²*e^(653/2*I*c) + 7870881974074799000458779124675529
036199233341599852082027540380914724078834083322635336963519437431605064467
3527393080*a²*e^(651/2*I*c) + 89309228362510024888650204274018479623278931
829222751651869271997142940935939622499464941302023892783243036531939874720
*a²*e^(649/2*I*c) + 100558403722268040642511429344587236375943339865583446
768413235470390109367198695944619840576272225434857726514180226640*a²*e<sup>(6
47/2*I*c)</sup> + 112366497590804459368014229821061574148084686124990490096426158
131619239292819356652354755191270559061866632206618123360*a²*e^(645/2*I*c)
+ 124624416658840606508764972091247582056756014404381589678946810870785349
791533327137686918582746592580804518495020202620*a²*e^(643/2*I*c) + 137206
071545450383442708874048794383900914518129098042758010174185532707172309313
304945456774295371246074471543553060720*a²*e^(641/2*I*c) + 149971561739194
309995847285651108218911212034631531315740471599013794260732952804665232302
328927991725761522770551792260*a²*e^(639/2*I*c) + 162771207055202910574934
345408773520317217772896676949335919062728951396141388828012399150602132919
338448613678594539000*a²*e^(637/2*I*c) + 175450274701932039519067286725080
403588984926777237488421428048082175541862462852436250221253619940939635112
553736957060*a²*e^(635/2*I*c) + 187854193910793808031824107617460059143682
787150838921015095030626546459146628006540234377528430150128575756995169364
560*a²*e^(633/2*I*c) + 199834002044982669165221847294687522204304595088347
219902036647597391740878128463392404040741222584961747206539785461480*a²*e
^(631/2*I*c) + 211251731655899503280889116260570427953962602051847283236647
639670579472498881195477410930866524432145488946958371101960*a²*e<sup>(629/2*I
*c)</sup> + 221985430899735671553404159061568737422121539969518400779398794276353
778554405001186143206492042172329004119602578495740*a²*e^(627/2*I*c) + 231
933512795234363754726968263609912331028842140787184044078809054648933424087
115867567769437261499520146801998022772220*a²*e^(625/2*I*c) + 241018153332
630990882989495866445645556304230823211269731087307873911771814631180172360
563460562231333548774668344293540*a²*e^(623/2*I*c) + 249187504148194347259
09868038356559979355858671796161686472796748426442859633443626048916621520
693365308494905954863420*a²*e^(621/2*I*c) + 256416550355711868135781524616
212622150246596833908848844513195156955143995464929738486051818152589364681
978419136195020*a²*e^(619/2*I*c) + 262706524505629732900052642463761046897
360128081492520719602062874621173097355113088395553861512842828529818716754

053220*a²*e^(617/2*I*c) + 268082878390283041884174045398605459353728518517
 558420909324395225512576313595290722225856442317049534262750408159248640*a²*e^(615/2*I*c) + 272591909277578264166044299459393830772239696947798720671
 290661488134063850324048969486969810581354040036172071321899160*a²*e^(613/2*I*c) + 276296229230236287179383004123946668596905414210125405151426804368
 014967729534325534952787742280869431062101106717052160*a²*e^(611/2*I*c) +
 279269348441491467178714617670078520724850441014995837904838918689456125765
 086125858578645496849567201719177718577500300*a²*e^(609/2*I*c) + 281589709
 454974667721866672624011079887297161399777306608822769322849136380296502551
 325787199139626207392754615890356320*a²*e^(607/2*I*c) + 283334553241511338
 020328671048420786223625257892886884589815372851166469619800685955743073768
 087596286675562709921649100*a²*e^(605/2*I*c) + 284574016420771420704193599
 748242589732008960441520037039680402737468153733442912103350926971108090433
 015340259175973280*a²*e^(603/2*I*c) + 285365849398307215288826580662978865
 841595519279197228325469882426298598034988143049654644002348988616277221990
 835896860*a²*e^(601/2*I*c) + 285751107933261952185844819866993604965269733
 757206120203840534898677511720614812627553159457956333256326887429379709240
 *a²*e^(599/2*I*c) + 285751107933261952185844819866993604965269733757206120
 203840534898677511720614812627553159457956333256326887429379709240*a²*e^(597/2*I*c) + 285365849398307215288826580662978865841595519279197228325469882
 426298598034988143049654644002348988616277221990835896860*a²*e^(595/2*I*c)
 + 284574016420771420704193599748242589732008960441520037039680402737468153
 733442912103350926971108090433015340259175973280*a²*e^(593/2*I*c) + 283334
 553241511338020328671048420786223625257892886884589815372851166469619800685
 955743073768087596286675562709921649100*a²*e^(591/2*I*c) + 281589709454974
 667721866672624011079887297161399777306608822769322849136380296502551325787
 199139626207392754615890356320*a²*e^(589/2*I*c) + 279269348441491467178714
 617670078520724850441014995837904838918689456125765086125858578645496849567
 201719177718577500300*a²*e^(587/2*I*c) + 276296229230236287179383004123946
 668596905414210125405151426804368014967729534325534952787742280869431062101
 106717052160*a²*e^(585/2*I*c) + 272591909277578264166044299459393830772239
 696947798720671290661488134063850324048969486969810581354040036172071321899
 160*a²*e^(583/2*I*c) + 268082878390283041884174045398605459353728518517558
 420909324395225512576313595290722225856442317049534262750408159248640*a²*e^(581/2*I*c) + 262706524505629732900052642463761046897360128081492520719602
 062874621173097355113088395553861512842828529818716754053220*a²*e^(579/2*I*c)
 *c) + 256416550355711868135781524616212622150246596833908848844513195156955
 143995464929738486051818152589364681978419136195020*a²*e^(577/2*I*c) + 249
 187504148194347259098680383565559979355858671796161686472796748426442859633
 443626048916621520693365308494905954863420*a²*e^(575/2*I*c) + 241018153332
 630990882989495866445645556304230823211269731087307873911771814631180172360
 563460562231333548774668344293540*a²*e^(573/2*I*c) + 231933512795234363754
 726968263609912331028842140787184044078809054648933424087115867567769437261
 499520146801998022772220*a²*e^(571/2*I*c) + 221985430899735671553404159061
 568737422121539969518400779398794276353778554405001186143206492042172329004

119602578495740*a²*e^(569/2*I*c) + 211251731655899503280889116260570427953
962602051847283236647639670579472498881195477410930866524432145488946958371
101960*a²*e^(567/2*I*c) + 199834002044982669165221847294687522204304595088
347219902036647597391740878128463392404040741222584961747206539785461480*a²*e^(565/2*I*c) + 187854193910793808031824107617460059143682787150838921015
095030626546459146628006540234377528430150128575756995169364560*a²*e^(563/2*I*c) + 175450274701932039519067286725080403588984926777237488421428048082
175541862462852436250221253619940939635112553736957060*a²*e^(561/2*I*c) +
162771207055202910574934345408773520317217772896676949335919062728951396141
388828012399150602132919338448613678594539000*a²*e^(559/2*I*c) + 149971561
739194309995847285651108218911212034631531315740471599013794260732952804665
232302328927991725761522770551792260*a²*e^(557/2*I*c) + 137206071545450383
442708874048794383900914518129098042758010174185532707172309313304945456774
295371246074471543553060720*a²*e^(555/2*I*c) + 124624416658840606508764972
091247582056756014404381589678946810870785349791533327137686918582746592580
804518495020202620*a²*e^(553/2*I*c) + 112366497590804459368014229821061574
148084686124990490096426158131619239292819356652354755191270559061866632206
618123360*a²*e^(551/2*I*c) + 100558403722268040642511429344587236375943339
865583446768413235470390109367198695944619840576272225434857726514180226640
*a²*e^(549/2*I*c) + 893092283625100248886502042740184796232789318292227516
51869271997142940935939622499464941302023892783243036531939874720*a²*e^(547/2*I*c) + 7870881974074799000458779124675529036199233341599852082027540380
9147240788340833226353369635194374316050644673527393080*a²*e^(545/2*I*c) +
68826496133929259333946349688756107181827094760687431541236793707609567157
393356980015874728986445505375939321119773600*a²*e^(543/2*I*c) + 597106965
421409973349982246393495403717260673053270158586132555990038346169057504642
49459471454428929767659680543253880*a²*e^(541/2*I*c) + 5138948934058015407
449824334333687371742960590384296136947593626074973184931365427688347751748
3490016178128812433612320*a²*e^(539/2*I*c) + 43871822610861004795016564148
131246645253380769718394461842295211859774351397481825053002145790551685375
056214564880600*a²*e^(537/2*I*c) + 371493728028447137285433739616985119096
676861363126805325450072544839433526476478813087889953519841750849017306788
14480*a²*e^(535/2*I*c) + 3119883339946498355189807082430995856239001431320
1877547672616650741608758923820402191204459236862231482346227933403280*a²*
e^(533/2*I*c) + 25984481821928790229019533220352810364053651428139405421011
946723503931316803858639430225998735288638633019702866058680*a²*e^(531/2*I*c) + 214608696226037983738622953759919798442615270205949582010347580807736
85660129127602861264204469990486648515185847661920*a²*e^(529/2*I*c) + 1757
549620213874323744070248931841334869167108820905095696361351136690612608252
1272394607115515936172525552891395395672*a²*e^(527/2*I*c) + 14271347645749
226165714452377611116721623327963230348323460525493356318465738007926444717
352691455157582924190074684640*a²*e^(525/2*I*c) + 114892074674554993444824
187454781555630147611268161260805373834605316293863095226342401175368377633
65043954166727248280*a²*e^(523/2*I*c) + 9169672822977743977111320804541111
904660273221506451651949707212055959466012505191220572773289655701792453230

423105120*a²*e^(521/2*I*c) + 725483608490846840464667844698960066507970803
8513510048563537225669386463363803374148622157625354861291347122905242320*a²*e^(519/2*I*c) + 56896159099703015201083196403639731344701908177496182015
41040554671725803726189842972146004682392121289650040476771616*a²*e^(517/2*I*c) + 4422742839523826762645479957057243541540122054951703787862113937904
048568048286850969056460218125357089349296008547181*a²*e^(515/2*I*c) + 340
742128545569404647432079815909007579872992486208020965032843806750401688159
8067707844396242531890195544301426987190*a²*e^(513/2*I*c) + 26017021223966
451413459310334045709950274662042842744886537507537614986756688639708803044
80234793529924712351195427555*a²*e^(511/2*I*c) + 1968608074187662410960386
124921807760839419912449118088079440479378830696763837592112817321648516678
192250226251889260*a²*e^(509/2*I*c) + 147605799556939063722335269724248321
056986186278058992331714091490618616504496604130209193831176845979167788530
5942531*a²*e^(507/2*I*c) + 10966365784594449163767232860698296342408290402
83071984682114965507192138829786266640857980171465345670412137298803158*a²
*e^(505/2*I*c) + 8072536581373423079001605955440784201898133383975139514595
80649551593816718094436044450016924199349627155306092503566*a²*e^(503/2*I*c)
+ 5887329114246524320364752186762673918338295870717335815526535995691150
62655958634436180105642268527576118276535857990*a²*e^(501/2*I*c) + 4253640
612909728645993816052419769130262428923523341151858544049566443138054303154
68685902485280907282658089881714375*a²*e^(499/2*I*c) + 3044463932271468146
139835397274447981735648583749804130804882711352566619133843133421838724023
32578752912321480097437*a²*e^(497/2*I*c) + 2158450037768053028611846925659
270013831448502780905776912601373554991349743613134480264443808829598043662
46287914973*a²*e^(495/2*I*c) + 1515751703557392099463738763700576505493183
20584509041952034589942144397207049663095445023255855799320406828152904813*
a²*e^(493/2*I*c) + 1054248537870287544696739608610518480770643791792014086
87509167013295588338384945425067268491968039006968056581921707*a²*e^(491/2*I*c)
+ 7262079921891656521787587138133934287480384013353137620330980577506
7797784602620988634808343998866415251767099657115*a²*e^(489/2*I*c) + 49540
060650503063037382280258699049436351964733180929934117445080596631561896216
015780970984005090516521389882187000*a²*e^(487/2*I*c) + 334660288086423004
254531292130916418154663550266366529470832611778874594878664703706907086685
14061085213215020765318*a²*e^(485/2*I*c) + 2238612040430000442287600735176
164788352161033439978593903801179044906786862784322623982433951940883091022
5209114648*a²*e^(483/2*I*c) + 14827078840958204423726546705512527357957361
647396508632935988933241213590716537429203049741620280852461278755893511*a²
*e^(481/2*I*c) + 972321278368853764330445117379093235276339211787819409971
3212407827924571779656265406369365137858973995542668178560*a²*e^(479/2*I*c)
) + 63127263274934193211710032130069260225876978630199913947211901704358570
84298246006024922226846516328771260784350055*a²*e^(477/2*I*c) + 4057447205
736508720494015062570842677505274148644881616811326702628107094774514265680
940713014256351659607202219536*a²*e^(475/2*I*c) + 258162710700990073452206
474459755120980230440691093473667883589900827910048538655151298617124743444
3617529951824987*a²*e^(473/2*I*c) + 16259794205311056824146752182437539020

208181695242225342016603299254855719154190149741990941268374641691820822222
86*a²*e^{-(471/2*I*c)} + 1013663583381382315526119900385793780592897362259521
880870256689578864566731937045334553766119426780595860617854190*a²*e<sup>-(469/
2*I*c)</sup> + 625469971082512052221445939226339313157527561686453294385607721125
006005737878528154829176814174073609304783133155*a²*e^{-(467/2*I*c)} + 381970
324888971137824260417119355880508011405306507252717319111638967573708192814
947593237962837516587367083029056*a²*e^{-(465/2*I*c)} + 230855486672626709723
535927872568854471077332500022366798083306901537948948135371592062382972028
832652892521530223*a²*e^{-(463/2*I*c)} + 138075050472911460011366856295168555
289257189097324670311033951735099801993846300774206061209409704454402575173
168*a²*e^{-(461/2*I*c)} + 817206467984853250193743177843227599578449222735699
03847642933003662634138916360473885905820817778748179763625775*a²*e<sup>-(459/2
*I*c)</sup> + 4785939505147857748577714882562266649184418998967128714889531275003
1063089597039280342277891313648282659506765160*a²*e^{-(457/2*I*c)} + 27733123
170528060665047793029187981069336565834501591355485919460176359570456976926
692677356655692104901055424742*a²*e^{-(455/2*I*c)} + 159002494377914059354042
986899597212969145564530236356216021570154264008833119617425040032692387501
12959349688008*a²*e^{-(453/2*I*c)} + 9019036531054139364145036198729875181502
788996721165710912853819799244035211696760046421655662466728741232371823*a ²*e^{-(451/2*I*c)} + 506109993897069624544009746293650406009823419486789109980
0067879466606691927034473565491643439734586575243562587*a²*e^{-(449/2*I*c)} +
28095435186682042668237968016271187636458887411913075046478853982245873150
04606447902339962521984923603337579865*a²*e^{-(447/2*I*c)} + 1542802094651848
674386929677506336135301451757782162481029354191378385655309839590342295782
070267248163797079525*a²*e^{-(445/2*I*c)} + 838003405633226261698002475165028
859961836560477445660978330588882054803408792968277041826209840110522910353
801*a²*e^{-(443/2*I*c)} + 450214953158482436877449323534443636424987689673176
576808637277491307797447679834039022495671246539361123241623*a²*e<sup>-(441/2*I
*c)</sup> + 239226901086351112501913420176641388141834680717243149148240541912365
024035242396630141827067823768453577792206*a²*e^{-(439/2*I*c)} + 125716823797
005642317347526498401790837531685611023648804925465449065255408041032308439
566500708700602697766710*a²*e^{-(437/2*I*c)} + 653352098551397452254604581678
826437154641747023799034244154363351417129329348471117076045199475321035359
39690*a²*e^{-(435/2*I*c)} + 3357753286250861240493214356205675816951254317872
3442784444413242416994607618240798840372204654901124952727575*a²*e<sup>-(433/2*
I*c)</sup> + 17063763943819201257154546007944190484800483541219625570334773139200
258218933634528764590082974465232719019576*a²*e^{-(431/2*I*c)} + 857437043606
127858832332973198205223244571782879175596317046623803379829879817966509635
5346431444695120516791*a²*e^{-(429/2*I*c)} + 42599806114811751815296146169680
792882669983472869168677651916064028092386144819853336758318060879567688722
50*a²*e^{-(427/2*I*c)} + 2092509275247339725457553903805929993734938547721804
454197071999505511719948252419379343549397905586122452665*a²*e^{-(425/2*I*c)}
+ 101614982522492841528243011649242617124219256790903731201036889817463345
1476277701838680759316125015139743120*a²*e^{-(423/2*I*c)} + 48781443252111149
758540671950680205563692208794988862702520845444730010003879101229785913152

$6297355904933960*a^2*e^{(421/2*I*c)} + 23149082658500578780775327350853876539$
 $3237794891589480734288576865823349858144380580905331957850509282551472*a^2*$
 $e^{(419/2*I*c)} + 10858518815771394212107785178989572242259468901872045507056$
 $3476121301184727811574323466214291834201854672060*a^2*e^{(417/2*I*c)} + 50343$
 $125546713237985623313646329330647858349495627492925205161566693001610650569$
 $967429902726126062669565040*a^2*e^{(415/2*I*c)} + 230683935152881860644844656$
 $031179619112846737540841843474912420434135364495308948612833593634648250075$
 $26940*a^2*e^{(413/2*I*c)} + 1044663893361296440733687256037795716524435199501$
 $8446220321535598596368046659626683803611150049940561221040*a^2*e^{(411/2*I*c}$
 $) + 46751116425487460919156707540760323711761364540363599820752214772978099$
 $2520447666662239031601034545996300*a^2*e^{(409/2*I*c)} + 2067460841873529999$
 $118420265134283763964445212656031557399579185390572143523605908612468666220$
 $745228526760*a^2*e^{(407/2*I*c)} + 903411976634483542635289998462833963669695$
 $152185259996654868051991473572372870248662847924612744264062440*a^2*e^{(405/}$
 $2*I*c)} + 390041272754974777005627699154842246988726024604162438024095285594$
 $852487855850372070546472734126022684700*a^2*e^{(403/2*I*c)} + 166373421015960$
 $065744946831605129638085441168071352620640483463800291640278014342520469741$
 $317600337261840*a^2*e^{(401/2*I*c)} + 701096865308063795489620658552673148036$
 $77176495241796951534731669884594114550121278447472661295579702060*a^2*e^{(39}$
 $9/2*I*c)} + 2918531344865391744146489048732318953323358055838602673457061502$
 $9457027933491512738753237134630607181520*a^2*e^{(397/2*I*c)} + 12000905703768$
 $198174954282034631757209497411914073482804705515897689969997576557491508854$
 $781156886995980*a^2*e^{(395/2*I*c)} + 487413507913275541434737569088780271179$
 $4105650938863970400230002709422093992525472165364797296790138960*a^2*e^{(393}$
 $/2*I*c)} + 19551720942932691644662098033171025525950216475811512178778422970$
 $59127002495113810772654784644585060360*a^2*e^{(391/2*I*c)} + 7745434216142034$
 $956009716154660779796523688812189335104242942400214093869560355261597458615$
 $44320722480*a^2*e^{(389/2*I*c)} + 3030042922717630947457200399363345710168954$
 $25314693546495893805386105536448400356863388712154386414250*a^2*e^{(387/2*I*$
 $c)} + 1170475435818151870626733214223035617896009171652842549594149321447621$
 $94096889890432439125557135120080*a^2*e^{(385/2*I*c)} + 4464306447159171747970$
 $402027656364257071788382666814516924761176106572252285498820479326895096200$
 $7830*a^2*e^{(383/2*I*c)} + 16810896728225961874194498340222314518223276631636$
 $193848187700774478509776074186756740832022063618940*a^2*e^{(381/2*I*c)} + 624$
 $941401292896228606101040352900275360734236090608619203318510342840091307088$
 $9052292940484712327830*a^2*e^{(379/2*I*c)} + 22933213080155869179327993170249$
 $82339168604649369171514848750864257523843794511936002538498981041920*a^2*e^{(}$
 $377/2*I*c)} + 8306778908234055938810480280413755423339552830739626267847183$
 $12376094416760253547591206118659958460*a^2*e^{(375/2*I*c)} + 2969660051565635$
 $311059710749400502952109529633422921287734507155025354576844063789256971006$
 $18058220*a^2*e^{(373/2*I*c)} + 1047732949197081814264563182467078417585755174$
 $41039866971803838079319268269378919589734008354661810*a^2*e^{(371/2*I*c)} + 3$
 $647765370604224594468882060469649421908133476712105791233191413780441530797$
 $3337793902177294599210*a^2*e^{(369/2*I*c)} + 12531338227920856492074590326674$
 $099374453122868913516861854565249779642744578491059412040960382510*a^2*e^{(3$

67/2*I*c) + 424740092507737317611348631219014547218168642299393394244763284
6633095644440032003339271168690570*a^2*e^(365/2*I*c) + 14202487865181874667
180230489017311183779468543008119776710723044112789739694400015832212820131
30*a^2*e^(363/2*I*c) + 4684682618247749580917910719553579166169333203426027
39423768916749249443401418804336754943102630*a^2*e^(361/2*I*c) + 1524153741
060746229392242841536596015506271094363259266210144802293335348624678102171
19890170400*a^2*e^(359/2*I*c) + 4890660907252005761107137050799634022477668
1379775034925868200923794664214854540668184735223060*a^2*e^(357/2*I*c) + 15
475789717397460276234843111043950695013471030625427925175353797112876241958
974587338558471200*a^2*e^(355/2*I*c) + 482880723711149517085020202679552656
0032755045691541653850695089412640252061166703373641008090*a^2*e^(353/2*I*c
) + 14855351189531389488659783317179105994974562961777923747855905162334746
55906439549342970114480*a^2*e^(351/2*I*c) + 4505418159943504163295634826559
09244979320334438405914968411406156252958317001977289679320570*a^2*e^(349/2
*I*c) + 1346940458317330848315741705234975101041288360354035763530159820485
77081438406747258639643920*a^2*e^(347/2*I*c) + 3968931527476296259834409957
7657312145742408227826431282422824844089957174694332245240202130*a^2*e^(345
/2*I*c) + 11525514338248122115806901223631408865665060873992331731216717545
839641395493543478555391940*a^2*e^(343/2*I*c) + 329804705399395014662084250
3721529058318485356660075016582248017786240344190976379491032900*a^2*e^(341
/2*I*c) + 92984426983961824327762778692324341462366377626122031487167919972
2455584698434216819569010*a^2*e^(339/2*I*c) + 25826531999838592080057058934
8242938126445962121695949028377963603738714515662920857912400*a^2*e^(337/2*
I*c) + 70659202220402219699327142472480242287876653764953302133360376448287
226187487345644845530*a^2*e^(335/2*I*c) + 190397367004770341919144178744086
95155655405986129726283796391182602876937826158174743280*a^2*e^(333/2*I*c)
+ 5052240560372772252508411884469329618226376927979062996145236110491773700
805441539104570*a^2*e^(331/2*I*c) + 132001151473743564403925218561453784232
5531317784101906639727300233020372137428050228960*a^2*e^(329/2*I*c) + 33953
156516581815748811154623651473924517248473648272974119673254871507727462127
1815060*a^2*e^(327/2*I*c) + 85966207795742771163110084083079326198692050781
198818319037362716785215369332723227360*a^2*e^(325/2*I*c) + 214217652898370
57403869596169941475678645506492471864168704518082797513207707602578070*a^2
*e^(323/2*I*c) + 5252856062781833938201490869422444930464746423374588871765
446690324009941198600912010*a^2*e^(321/2*I*c) + 126730019194373732411932059
9349683654493348850217007321150739273112190995849189116570*a^2*e^(319/2*I*c
) + 30077123446845252026656260732966592215911538150008772834593000629200562
3202669775630*a^2*e^(317/2*I*c) + 70209017945411591200961651730058863557722
458877098381486938641100483455911795967290*a^2*e^(315/2*I*c) + 161166281624
11464679896502836324771389620584856574877922468282685664283130175327730*a^2
*e^(313/2*I*c) + 3637498646026425598745279228803679517140090350241314216371
570468174738174921307980*a^2*e^(311/2*I*c) + 807047768188333538846684803460
77979093105555100120589990758127536168861246233500*a^2*e^(309/2*I*c) + 175
987317444160012665123402745831252173155432052395534884810603675523612022791
280*a^2*e^(307/2*I*c) + 377107106707422037469802679360762145217845506499125

75518820552220031814445215110*a²*e^(305/2*I*c) + 7938933320497449635918969
 680758326307339116239071846330248311795038723075723660*a²*e^(303/2*I*c) +
 164166234809546036351435672963902650424331530957875787162832240753712146495
 4630*a²*e^(301/2*I*c) + 33337810674058613825161012495879671457693509961983
 5073172307696551648259829120*a²*e^(299/2*I*c) + 66470080272037903682198219
 835488944949996749070187078822650198449544212324410*a²*e^(297/2*I*c) + 130
 09255453778977514917501132236907767356887691068368589411338908245552368240*
 a²*e^(295/2*I*c) + 2498697917579895977978723588576283933137296518743481104
 776932566391579455400*a²*e^(293/2*I*c) + 470873979833165454444627659803101
 969430867153471663329279188629910819335440*a²*e^(291/2*I*c) + 870394672285
 31013705409191271452383389335356884921808641492423363762400060*a²*e<sup>(289/2
 *I*c)</sup> + 1577740463997525927245493384353947454466523681911756699731867230466
 6566800*a²*e^(287/2*I*c) + 28037932208262978741634296821311025033065490052
 65399219277240917413502940*a²*e^(285/2*I*c) + 4883448361259814710481691636
 57616516954158792795618978221198980949925840*a²*e^(283/2*I*c) + 8333974773
 6469143532428073310762163525113789344043183602916613227872140*a²*e<sup>(281/2*
 I*c)</sup> + 13931340194933935228013466294179987445068758219565436862349156508804
 360*a²*e^(279/2*I*c) + 228041600249749997233027035662805350070222102577050
 2602102780787687624*a²*e^(277/2*I*c) + 36540623784951824837206096046358165
 0995651842549690071352678851012540*a²*e^(275/2*I*c) + 57297195830579479880
 905283759887310015692255967368709515502818525680*a²*e^(273/2*I*c) + 878891
 8880786687559267555413796199314981595789787074901006933535500*a²*e<sup>(271/2*
 I*c)</sup> + 1318333654990888899417746520374288649257223133573127905086467548080*
 a²*e^(269/2*I*c) + 1933035148727861159233797166282534193189441764358760568
 02589865772*a²*e^(267/2*I*c) + 2769546474270692799615789127534264343104090
 6047696742940234352112*a²*e^(265/2*I*c) + 38757418032632716085780481259550
 66152785493897151672716099440360*a²*e^(263/2*I*c) + 5295334897064348244512
 50188592739498197462491764283044018575440*a²*e^(261/2*I*c) + 7060434872440
 1170958829605205937063913034447082349890348162895*a²*e^(259/2*I*c) + 91826
 33302160212452918968603806828036823248995981353166570702*a²*e^(257/2*I*c)
 + 1164367032824623073366685138251309912040093060141928716087681*a²*e<sup>(255/
 2*I*c)</sup> + 143872782220632026353482783145282086606314049235674031853952*a²*e ^(253/2*I*c) + 17314186832105700122787937106308399484490093525695416858785*
 a²*e^(251/2*I*c) + 2028231682527169820869698431068546224630661674135401655
 870*a²*e^(249/2*I*c) + 231137354125616547233443142991124898825470770660317
 194554*a²*e^(247/2*I*c) + 256089531028117360206996719814005315916520823534
 56701666*a²*e^(245/2*I*c) + 2756769770542048744355911382380396555656358932
 078858681*a²*e^(243/2*I*c) + 288136830654984406530837787337700439328183658
 059646399*a²*e^(241/2*I*c) + 292194990752635358438588623776480730712853109
 74657915*a²*e^(239/2*I*c) + 2872702692034937563897971496575074831246739185
 614575*a²*e^(237/2*I*c) + 273590663366155266234104272169452688263007484513
 781*a²*e^(235/2*I*c) + 25219245783549721366067799114158978864399924058441*
 a²*e^(233/2*I*c) + 2247954664299349273623637155894975599108753667672*a²*e ^(231/2*I*c) + 193573846127814805336166214026334290497006915130*a²*e<sup>(229/
 2*I*c)</sup> + 16086467322724552698501340975783163797842534840*a²*e^(227/2*I*c)

+ 1288694773452054046771640788950472807542901137*a^2*e^(225/2*I*c) + 994034
 45980961943589598468520732983548550704*a^2*e^(223/2*I*c) + 7373332079799133
 639063658614527645894565137*a^2*e^(221/2*I*c) + 525223629498383939328992961
 505581135597600*a^2*e^(219/2*I*c) + 358759296093904067383946662199712212057
 25*a^2*e^(217/2*I*c) + 2346109768759347903572910732245570209842*a^2*e^(215/
 2*I*c) + 146631857213698093058440158830415565266*a^2*e^(213/2*I*c) + 874227
 8599147295285480758801384765381*a^2*e^(211/2*I*c) + 49618337355528480462791
 1954521619600*a^2*e^(209/2*I*c) + 26748429587445855729768539197182585*a^2*e
 ^((207/2*I*c) + 1366183222241782313586863286641184*a^2*e^(205/2*I*c) + 65928
 412548866286902779022351001*a^2*e^(203/2*I*c) + 299674601484785362031769373
 1016*a^2*e^(201/2*I*c) + 127861163009333998455801820954*a^2*e^(199/2*I*c) +
 5100844261395996953829648360*a^2*e^(197/2*I*c) + 1894212721285211788220664
 45*a^2*e^(195/2*I*c) + 6514488191198508953598437*a^2*e^(193/2*I*c) + 206263
 478282239243233771*a^2*e^(191/2*I*c) + 5970784896604221606627*a^2*e^(189/2*
 I*c) + 156713514333171921595*a^2*e^(187/2*I*c) + 3692203217135946825*a^2*e^
 (185/2*I*c) + 77121738215879370*a^2*e^(183/2*I*c) + 1405865019554882*a^2*e^
 (181/2*I*c) + 21909584720322*a^2*e^(179/2*I*c) + 283802910885*a^2*e^(177/2*
 I*c) + 2933363420*a^2*e^(175/2*I*c) + 22680645*a^2*e^(173/2*I*c) + 116610*a
 ^2*e^(171/2*I*c) + 299*a^2*e^(169/2*I*c))/(e^(517*I*c) + 418*e^(516*I*c) +
 87153*e^(515*I*c) + 12085216*e^(514*I*c) + 1253841160*e^(513*I*c) + 1038180
 48048*e^(512*I*c) + 7146142307307*e^(511*I*c) + 420601518659718*e^(510*I*c)
 + 21608403021340047*e^(509*I*c) + 984382804329835768*e^(508*I*c) + 4026125
 6699368950388*e^(507*I*c) + 1493326612293984160368*e^(506*I*c) + 5064866094
 4512569972179*e^(505*I*c) + 1581796642397812408161814*e^(504*I*c) + 4575911
 7183402579073139583*e^(503*I*c) + 1232445557346832245176696904*e^(502*I*c)
 + 31042222522074681615625020522*e^(501*I*c) + 73405726361638844996884236692
 4*e^(500*I*c) + 163531646471515302405291376181111*e^(499*I*c) + 344277152012
 875134140739302960914*e^(498*I*c) + 6868329225263681349501997341320517*e^(4
 97*I*c) + 130171193079172823835151430773360024*e^(496*I*c) + 23489983742443
 47079532766203075607598*e^(495*I*c) + 4044362478141531158185783238909963456
 4*e^(494*I*c) + 665634670676210063754191847109971141414*e^(493*I*c) + 10490
 402669510897424624643766470754045064*e^(492*I*c) + 158566476113257562566117
 432227203884298856*e^(491*I*c) + 230215041122623492585522234520150090053357
 6*e^(490*I*c) + 32147887693375338817454482515377350383950278*e^(489*I*c) +
 432333688644261557547944179250800440604964868*e^(488*I*c) + 560592725306755
 8551780452883689835514455118670*e^(487*I*c) + 70164515322544462906873548813
 748091084561870680*e^(486*I*c) + 848552202276512356496200136959676295361696
 315113*e^(485*I*c) + 9925490738534402272939987038714580495445431374618*e^(4
 84*I*c) + 112391604542246650966429162063124338952554575234051*e^(483*I*c) +
 1233096700139723365181997220750932590655287625342156*e^(482*I*c) + 1311878
 1801172174729679339894318153694964675368481194*e^(481*I*c) + 13544259491663
 6116191574650625331646238501101627937224*e^(480*I*c) + 13579906631614798428
 50642848032544982878359839580349899*e^(479*I*c) + 1323170887010489697380005
 6733779919089340836756009580718*e^(478*I*c) + 12537049658692127266219805085
 1269323171167338854081782959*e^(477*I*c) + 11558554128935942603455449666426

87823630035899363232371472*e^(476*I*c) + 1037518449987117550190939895659668
 4116802997082526660323524*e^(475*I*c) + 90722605722208814918642284639487187
 764607589706493970774776*e^(474*I*c) + 773204636991145775061462731028098506
 094432675788136295011259*e^(473*I*c) + 642619548553524857642506813687046553
 0087114003875716691383902*e^(472*I*c) + 52108117629177048660492400985175830
 987505700566877818954141639*e^(471*I*c) + 412430698299915190848067222327219
 435067747934091894670488982928*e^(470*I*c) + 318774992974434649721153604475
 1776582320958627923816470590659024*e^(469*I*c) + 24070801913529757101858022
 914372045864746991786182039740274325264*e^(468*I*c) + 177642829135119348577
 194437675802830239905460092687136494961404333*e^(467*I*c) + 128181746491497
 0810859604189828359000790789921169405304612211251818*e^(466*I*c) + 90466935
 23825682979044338963104263167672586826367911338826483549173*e^(465*I*c) + 6
 2473550781053295317710774690247114124125187565731848441781904032672*e^(464*
 I*c) + 42227612663200368754775474655570998871052713308666016136635365678728
 8*e^(463*I*c) + 27947091044756866118427906949736991644822547239772102097256
 61304403472*e^(462*I*c) + 1811576849561575807671030305550562558925429365919
 3314153418333944596408*e^(461*I*c) + 11505148185208084887370038835452131556
 7640365124003103691176697194292320*e^(460*I*c) + 71609949759905807989563333
 8552940229192858196481597830078819711862600096*e^(459*I*c) + 43694424829101
 13914565353136069595862669338858053419381214131241925047008*e^(458*I*c) + 2
 6143976279902021443471945665080254563056810183520401889800285493144867448*e
 ^ (457*I*c) + 15343608874505625412732723946157707193313015776459599711397351
 3183188399376*e^(456*I*c) + 88350096882179120260077454192776920073768939351
 3734789368397093333311961880*e^(455*I*c) + 49925197124570439835053779766079
 53988397368297591114957991804893688371867680*e^(454*I*c) + 2769311653834325
 9225983382637647936122664033859615133489846664694361471028310*e^(453*I*c) +
 15082238143141241377356647421001174685229743759705918629524398948114039815
 2780*e^(452*I*c) + 80667954360758914075930501079618956826984202161338895521
 8916278823182639488190*e^(451*I*c) + 42381258467632325863941885698586858267
 55328005548627437019301405851325887594480*e^(450*I*c) + 2187648289271390992
 8040345612578705805121508756226696317087651824252241418663320*e^(449*I*c) +
 11096919968732097474992225959525044434121921853534965576259119257653587215
 1766080*e^(448*I*c) + 55326912881952861250291886955894782909802195630934984
 3584044631512291778800081490*e^(447*I*c) + 27118432396707175276056404901488
 33507130242448403978318523237721944200392830108580*e^(446*I*c) + 1306981720
 3488289886193205508375818392124991382340160316886507181296548981014818410*e
 ^ (445*I*c) + 61948596653035502879564338815234310660410902037882473161804774
 492916216575880077680*e^(444*I*c) + 288820755264730654469968572021047109427
 318619508995802020689904590319476295408324280*e^(443*I*c) + 132475641236783
 7473157472821162483691120966501948953926492241643788264284546437221120*e^(4
 42*I*c) + 59789921729441432184591611492998197063217321115784945252452287429
 76468409105395536290*e^(441*I*c) + 2655680638904340753449670236910154579599
 4861757741414789944652712127566910185274123140*e^(440*I*c) + 11610455168355
 5043762911501712116399313733021132677481112824047246361794049635726479850*e
 ^ (439*I*c) + 49970756725385908435759631481379476806933719091596749190748890

4933922677579665354338960*e^(438*I*c) + 21175897334668557071015014292104147
22401838837940752841618541440888545729943138209036820*e^(437*I*c) + 8836720
640860470305694514021547969551296794092266983044118375790025854584036796364
768280*e^(436*I*c) + 363183696523025917321974444097981220226408246041305525
06742586795183267354382847875885730*e^(435*I*c) + 1470308167322768331630415
82099592047512043725225353339238819165193000407629544745753221740*e^(434*I*
c) + 5864034669726832427416433289215609093751974538642432995719909646088572
45771134145204174990*e^(433*I*c) + 2304351073373840357379178597673066352016
682781689139842097376663118488803841131935313641840*e^(432*I*c) + 892320944
734329676333188188163847179349961867060102605973089596265329177022949302816
2575100*e^(431*I*c) + 34054053851295569154352346722177172655187548910782008
504718324168725029438589162349211628040*e^(430*I*c) + 128098914601688539672
480541830409847707367500438601536803204497701119911289087105659482783340*e^
(429*I*c) + 475010578857601519272316617938425222421786597241671026894318515
408511467140969393115768793680*e^(428*I*c) + 173657421881819107187419747245
0158123883564209950658639102337148122769080611680719741726053840*e^(427*I*c
) + 62598721568222528436509607082350347102013627760571766472263230897514465
65288850103898153859920*e^(426*I*c) + 222519591767957775716736603600748022
2211364232146399803864370963391491223687245823457351580140*e^(425*I*c) + 78
009807368024239875613733058851417125327114681070889640794249282633470580756
557083923203377160*e^(424*I*c) + 269745801440211296972683601863878954357962
308520076595177128227629273240215209708218497363414140*e^(423*I*c) + 920089
393029589032874601850027159322612526368444771489781974361078847528891468831
038436064951920*e^(422*I*c) + 309613197162152016238030155424146545178236208
6810287537748902904985934020179565706177131421614590*e^(421*I*c) + 10279364
730663840844739577862469262604648861914297972589165243530651230690726244462
479199894255180*e^(420*I*c) + 336753988720215683759023845939827533625598010
58104184627345411136262431943240778260721756991027090*e^(419*I*c) + 1088679
957318294728267329051920348867972846213564456275309091044294867412578226334
76898356826454040*e^(418*I*c) + 3473514732147137808743520831295666012387657
62775942366762733349952103889753982636403857556867777300*e^(417*I*c) + 1093
853214486220358674032434500866678499770011305874172488975951612031456734608
287095519501041975440*e^(416*I*c) + 340023256060165161752169468084708984419
8028831694417424794868779328950548418125605446882081152636090*e^(415*I*c) +
10434117516570395966653693155582402109460348095473027807412321427346816928
567197770376496170251803940*e^(414*I*c) + 316109393312846927506943064436184
14656095969520945215743004044560386895241801579156543451940713351730*e^(413
*I*c) + 9455618025893198691933430346636565282685809131432918916073627717587
3841732196453379953705679466826880*e^(412*I*c) + 27928575580003520667983536
889816547764486498779466538782748893386363374504737310904926517268170258572
0*e^(411*I*c) + 81460818773653057967021002527192141559718336988121429982329
1969785549876175969866367976653244974728560*e^(410*I*c) + 23465182192391051
422381416330734647688991557089350257780476374126817815757654222191274092601
59438712250*e^(409*I*c) + 6675866290371147358503766865669289010893543869830
538708724945291580951179188296606158111257706968604740*e^(408*I*c) + 187599

882188655635641636357359860732782557372574057062791088913663784284674145599
 30481172863538598193890*e^(407*I*c) + 5207517851879327038642926335154430695
 1104993542500582938155241689408138675254608030847907167748571734720*e^(406*
 I*c) + 14280179245022176248318087491882527413430513327541778008479503464476
 3509333503150517345864659667189417080*e^(405*I*c) + 38687621823427716563245
 172304997988926311528237460754169244317667399751374281359173617116965225061
 1186480*e^(404*I*c) + 10355619825920029352263845779086115486121114950801935
 73691339864706029186482466241805664949381049856258510*e^(403*I*c) + 2738895
 62479526560335522764656600886280778305084825702911938903656162004262736182
 657700406301914070062380*e^(402*I*c) + 715812468684294147547380736367983971
 8172745581538409044503383852693596921622426696740453944718143025248390*e^(4
 01*I*c) + 18487405299005732693752728611876490890858357021974882371570623800
 186245137722660943641752976852924439870880*e^(400*I*c) + 471882208434662076
 950995069535737803571088974914225678980481990182077089970053338601488364795
 27456156014520*e^(399*I*c) + 1190418554038779649482295779483704656006066231
 83045529526900430209270473212773847794935586074714329479939280*e^(398*I*c)
 + 2968255152826695896853182732802390500845550322034159415119626595968816157
 13799937680026497408305672297618840*e^(397*I*c) + 7315849722068183628747296
 214039744442800104463011615273397605448153009517879855384197646562145826672
 19914080*e^(396*I*c) + 1782446114931751850556354856638421901174412322298249
 496591658053939787198246565945975595575734193348887952160*e^(395*I*c) + 429
 320647800802212601748890885182649479062072066015145146818191091724002786396
 8724539127659633517053002976480*e^(394*I*c) + 10223182025954860767217390305
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 414680*e^(393*I*c) + 240687851397052771611934656445061432852413610377682168
 18922184400141048460210944696647752723371932874594597328*e^(392*I*c) + 5602
 868342490351765849501385853451616716259103436797249817466090745066677815435
 3271630344650777885683547624184*e^(391*I*c) + 12896708008475471224602368086
 648838498328625902553313204463610904954514402954700334776152166628397793164
 0178464*e^(390*I*c) + 29355074355434270980812945357656231329970598269918741
 6862934373964255615967138676253276302591561523515603264403*e^(389*I*c) + 66
 076447310586909769147597385083793451108903314958670798276426339476675664956
 5279879146173318386505740391093990*e^(388*I*c) + 14709311466189343455150383
 623001001604821277495814439299047469102247774701988990523791144939998870031
 99419829579*e^(387*I*c) + 3238491931361851476423321933539579098377735539207
 641467346235665823887048326949305609231585143748690203615957136*e^(386*I*c)
 + 705213241416219799260232652458014306098535305457293390552463312168102103
 7340298366342203324325307072413739061024*e^(385*I*c) + 15189634214908800396
 417911722643754748048520109734812459109878810493844381062650818971199637121
 458749456243274416*e^(384*I*c) + 323627313224195494103300889436402474603783
 28561316422931292427145902887913071643679502909055891236755143207382609*e^(
 383*I*c) + 6820803309679361568378440961924421081861499164004155342440552787
 6893272496608324231098148502466453967157728078994*e^(382*I*c) + 14221311596
 481451768238666727676990948227168131879088984050103944174863554536246767983
 2449103520321953011780083069*e^(381*I*c) + 29334492003430072028704238344834

286631380628545504006782308044559754597002344623156355413513310549351631632
0059272*e^(380*I*c) + 59865014111224185891167650518052015036400322684132808
1453597093587790338609212439085554466861582623350303061961052*e^(379*I*c) +
12087703584936583930894422220569350632837041081405937502265398461177376482
16609559734831601248698274330296158612144*e^(378*I*c) + 2414966516810338503
289076549202740511710059011795447138773464205696455026442712426409599662771
080264826008985061097*e^(377*I*c) + 477414111106609897022184533059496201647
271423037423406066395684695092664268594692906411419440036093622359072547014
6*e^(376*I*c) + 93393419580534942252517509657150573007073020838147747703062
18224241022648247419956042957363055823830898547303219757*e^(375*I*c) + 1807
982006802885997034993862300723067656331420670884849990013964123733476326647
9346963237936039328113185041591793848*e^(374*I*c) + 34637657172671690167657
344537197087048882354853993270472063943078773600446542963548348101269390443
464480754513928502*e^(373*I*c) + 656748592688673000988273758128752256106545
51686261103681664007007537115778097293533565243828873383722980353200611956*
e^(372*I*c) + 1232439415193323847419600725881035065964063392536163910820629
69960682419011745775738921817753391954462609323881489157*e^(371*I*c) + 2289
113117385927800914926491623468344058677407764563261084109288572571747072892
68074347550225793244741923354395308214*e^(370*I*c) + 4208463426089493872775
590214579245865781209661485610226470084995294684520059801751194106289562104
97609566002969884927*e^(369*I*c) + 7658677955139627810125584446287514187109
408952813047908367436615820716500321548914828664063148344331994554597989349
52*e^(368*I*c) + 1379676529796212074017106188066589448355446501210890195107
164860350228928586815539003062875026711931941947738690360722*e^(367*I*c) +
246044237584542266392708163098326071473496809190549302714563923882719225488
6349361126991457692409851120873307487457468*e^(366*I*c) + 43439096966019321
733573596877815792937012956819408271142154331753360939678459087667407382400
37114570667410936998017178*e^(365*I*c) + 7592752700146678961153095073585015
473197029746533633331549793961473285760935801904155116764831560875947581048
693527224*e^(364*I*c) + 131397714941049338818566811514182931122425515215356
86871181266579813877606348160261747201317735782566021306798298336024*e^(363
*I*c) + 2251467574130806996150616558650287243042193021067326439299728648560
0640103867253604847715547060592967690653795951142520*e^(362*I*c) + 38199015
867586087976002998756627674994795440667903625029322346250133286489120875005
013638128113893960349670280707161530*e^(361*I*c) + 641751006932600668062380
648860045971707408433000868393686161391645291080498446753531118427257986580
88840347241496099644*e^(360*I*c) + 1067648320171655948380852341893335287335
876733299725300926610851867899392529159370907602822323469190904262433994093
23314*e^(359*I*c) + 1758962582627559857571068126139793012658010315954843536
14904672865169442232075776580447184134141375995770091499246759528*e^(358*I*
c) + 2869929436312314965572780108515769408968264974660663275288015606770071
12837431926735088120974861760511367008815728782643*e^(357*I*c) + 4637582884
573671545449376782550056887333281455680493104239955998860128006386199040223
68378591108842602342094543682299102*e^(356*I*c) + 7422286409081731249169370
494625256173341489196791182704898310054977819512210699558396234524997486531

24658873553401442137*e^(355*I*c) + 1176600720975786965189875050890231092204
 612696970277433014535895788956771230793520381993106606880564628599822341722
 801012*e^(354*I*c) + 184750585646245153344528430057132632378116255330456597
 1887670758091079306794821834928170773126364639722071570131703785334*e^(353*
 I*c) + 28736105359223401870808354355829122772719679773947201597910702749277
 14276869531467182688981041061381703885403497544001592*e^(352*I*c) + 4427673
 079105425318524316112985693656584851936100192457044455134483305045321452516
 347118488133224823670465103483954805161*e^(351*I*c) + 675848043788852437256
 293594896385762669485554719551948612287756798171858726236287199496707940183
 1957927901682582941234362*e^(350*I*c) + 10220423779434634851339975295163399
 641702122249663666193053008302026096932158568338309418237395541351819026907
 953220681013*e^(349*I*c) + 153128372066627753793473532128076829657125356529
 42631518286142403097738200270711195396582159028513532779682154451996208592*
 e^(348*I*c) + 2273160356612884110041950194705136766683665241807726091394481
 0748473084891890410181285412604854876625919565639521227223276*e^(347*I*c) +
 33435897827936581301171175459610829454298167962017419810072936733378506584
 428024201072453193458155334046693516742390717832*e^(346*I*c) + 487332535059
 749234008522555630521014021964693136595544927256747543393752830104071677443
 66955828922837488705858532439654489*e^(345*I*c) + 7038634976059483156704822
 406139502569850120229696630037676422033669770296159109985405541137629487143
 7468149528524796002762*e^(344*I*c) + 10074496185185374461175430098298016696
 240455383622292186848469426996612060769890704634373101116094882810027672937
 0132819357*e^(343*I*c) + 14290631912305552424654692847895423837131592580202
 2389236498652136839822502035155676970917419039834587967055588431566416784*e
 ^ (342*I*c) + 20090658715357880438030046950144161017452185125954192920984068
 8960859454908519774835905895757666770857888611738751858460424*e^(341*I*c) +
 27994524447503980482296673046296088449211987485779114712400907947692043594
 1735293309305430438687333129912454196774070107264*e^(340*I*c) + 38664267305
 038004945738256281831696265197555099077927704874023862985879501824735616288
 8631015687664780101205287333082748791*e^(339*I*c) + 52932925276411392600393
 483695824355767254923899756073921440659918504783195557258376535863439540877
 1528009745467548382950094*e^(338*I*c) + 71836159638205824920911354448790108
 886838874403371321033249197137590673834155154045726480430403966425591560734
 9801911966551*e^(337*I*c) + 96645827536903771874773913079815164348359068416
 683223468809829116416063641815945211981572880937212516883623936444239734406
 4*e^(336*I*c) + 12890435152929339564806343304996770401810439356201069142673
 11067900030058398839787692376954090545278554544997710058754772400*e^(335*I*
 c) + 1704582996707822808204678218167693002698661147712772355021456543810930
 069637188085882824757500605246963210810351706405349408*e^(334*I*c) + 223489
 127639843946447862257830643484072461048446817785982262065869192147864526665
 3062563823553001228001009093606751066168944*e^(333*I*c) + 29053857223200570
 019533452744894827908566925299598237495326959634141648333667731282186078993
 28588608916176593772088622582464*e^(332*I*c) + 3745257594876651204657334988
 426226388143954501986830664222349226361079609546822276067504899386703088982
 308185717143407211328*e^(331*I*c) + 478752744278094568514520484697159616530

416941932824407321145959212964925504887685405984472066107815128817961257498
6359194560*e^(330*I*c) + 60689498031567122483318711053298954717228061430088
78014986559653687260694816550470195890004511965527567432722969707577202160*
e^(329*I*c) + 7629731815627821580468992424207008366438896736333024661863838
105110445148946962328297631547032543419811821015837863013682720*e^(328*I*c)
+ 951303227401952295420911319126822664229991201352566594029838106479788569
0904993128948035227412144035633851779511219335277360*e^(327*I*c) + 11764212
274876484080010900714673474493371278160557811983724455826566055617658086479
368641864908119643412413644803772131657280*e^(326*I*c) + 144298162852084312
045329783753756919650631542246497475512958515073895240832269767896886013696
28399900747658579201929300744260*e^(325*I*c) + 1755627327122429239688729140
312571621349148626114547857137675169010565606783804215103827138130037275775
5676325408026834544840*e^(324*I*c) + 21188321405882887539610198374706862695
894049226077093764132512513336190523978949694387686059124526755048042957954
264706637460*e^(323*I*c) + 253671764391193536215322603359833481549049826061
257617113006834929633908164915830257052687375399821496393002265126574261188
80*e^(322*I*c) + 3012848241455270326455901895308817715601343749343820107841
3769835448366148121754549197591129967170764969700180348699207838960*e^(321*
I*c) + 35500103106019649876272376796949482209581372371036005012877806027481
672807059943445240136315568500732379966585005678181937920*e^(320*I*c) + 414
998321219637080437885237874013455417800889305382069188535790267492733646716
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5106328164715910598370871149079494360*e^(318*I*c) + 55390913044972086219432
689146331566081427959896969900214434296817731150863867056620768608187679709
720152974148474907904177340*e^(317*I*c) + 632477741010121790517949460751755
699240769813381384831580424067474538747293876317105449952471529122051185005
97511052824347680*e^(316*I*c) + 7166032986117339552444194388928410913409115
784465524567208423740243494469646492713181219065962951114063950174330386358
2092880*e^(315*I*c) + 80566249130682684181876201882623511206363790337218011
954110210642927765997644903820595421936873565314654415769070472655401600*e^
(314*I*c) + 898838158013823822139732704779546027447928770180519633471463073
72464315121274929402347942874802899499538953561056667668891020*e^(313*I*c)
+ 9951220647205796595134034173802354851533640337171789804085047095465753297
7279113491506880290726111154101941386019689567958040*e^(312*I*c) + 10933253
734996622320393267850342635707986370700172829401104207653040392386265401897
8676516417314221089449922495612732870169660*e^(311*I*c) + 11920971370203392
70557553978236884444464742432450218532862634704659963472114657383068154049
5333543146776810911910410468628960*e^(310*I*c) + 12899507601159190341076386
342709732994858617357459586270584915928094304645874266316345401849146385539
5649453952212899632198680*e^(309*I*c) + 13852979454915108945135276957654340
312633074724368003083246720589581904356815523926487676286717275433868402784
9855385453216080*e^(308*I*c) + 14764892080554533341862312176785377739978292
474830122879392434257499993795542176537010123512293955746754854920217455000
9604780*e^(307*I*c) + 15618596295355119616973821883217369650985255158921073

0578365727476259476474465955428502336673743686499175698677875693611243400*e^{^(306*I*c)} + 16397781605960772537526455981650584789418778510145536039189742
 4482998415385787605765315509208337741590143078572243505132706580*e^{^(305*I*c}
) + 17086984886895310117686030605310399434053039034726008843267684250555514
 1293830838961275974268928666494845723462544709102843680*e^{^(304*I*c)} + 17672
 092997055464200457577005309570059533465987068273203197591553238757705241486
 6323511140117680492929354517559479899220940360*e^{^(303*I*c)} + 18140816877092
 205982036855331669732163998486262829882856956027329563089762682934526359221
 9034560853530733710529842148537901680*e^{^(302*I*c)} + 18483115198374894181766
 785017470825713812817215826941328776535853224077324433619190081855782990589
 5684494889410451921524212840*e^{^(301*I*c)} + 18691547443656751492635140562311
 750326198750835193008382456644443568913923368341170464182876217879917784806
 4220150818355261280*e^{^(300*I*c)} + 18761539316851005007149728056460351091240
 313292031202437083506267903764499028628534667350709345296435125796269613351
 1725652320*e^{^(299*I*c)} + 18691547443656751492635140562311750326198750835193
 008382456644443568913923368341170464182876217879917784806422015081835526128
 0*e^{^(298*I*c)} + 18483115198374894181766785017470825713812817215826941328776
 5358532240773244336191900818557829905895684494889410451921524212840*e<sup>^(297*
 I*c)</sup> + 18140816877092205982036855331669732163998486262829882856956027329563
 0897626829345263592219034560853530733710529842148537901680*e^{^(296*I*c)} + 17
 672092997055464200457577005309570059533465987068273203197591553238757705241
 4866323511140117680492929354517559479899220940360*e^{^(295*I*c)} + 17086984886
 895310117686030605310399434053039034726008843267684250555514129383083896127
 5974268928666494845723462544709102843680*e^{^(294*I*c)} + 16397781605960772537
 526455981650584789418778510145536039189742448299841538578760576531550920833
 7741590143078572243505132706580*e^{^(293*I*c)} + 15618596295355119616973821883
 217369650985255158921073057836572747625947647446595542850233667374368649917
 5698677875693611243400*e^{^(292*I*c)} + 14764892080554533341862312176785377739
 978292474830122879392434257499993795542176537010123512293955746754854920217
 4550009604780*e^{^(291*I*c)} + 13852979454915108945135276957654340312633074724
 368003083246720589581904356815523926487676286717275433868402784985538545321
 6080*e^{^(290*I*c)} + 12899507601159190341076386342709732994858617357459586270
 5849159280943046458742663163454018491463855395649453952212899632198680*e<sup>^(2
 89*I*c)</sup> + 1192097137020339270557553978236884444464742432450218532862634704
 6599634721146573830681540495333543146776810911910410468628960*e^{^(288*I*c)} +
 10933253734996622320393267850342635707986370700172829401104207653040392386
 2654018978676516417314221089449922495612732870169660*e^{^(287*I*c)} + 99512206
 472057965951340341738023548515336403371717898040850470954657532977279113491
 506880290726111154101941386019689567958040*e^{^(286*I*c)} + 898838158013823822
 139732704779546027447928770180519633471463073724643151212749294023479428748
 02899499538953561056667668891020*e^{^(285*I*c)} + 8056624913068268418187620188
 262351120636379033721801195411021064292776599764490382059542193687356531465
 4415769070472655401600*e^{^(284*I*c)} + 71660329861173395524441943889284109134
 091157844655245672084237402434944696464927131812190659629511140639501743303
 863582092880*e^{^(283*I*c)} + 632477741010121790517949460751755699240769813381

384831580424067474538747293876317105449952471529122051185005975110528243476
80*e^(282*I*c) + 5539091304497208621943268914633156608142795989696990021443
4296817731150863867056620768608187679709720152974148474907904177340*e^(281*
I*c) + 48133117678184029216503748549110374478924719094635603892829364863916
553792278822957368285106328164715910598370871149079494360*e^(280*I*c) + 414
998321219637080437885237874013455417800889305382069188535790267492733646716
40037563488607716092887686471542838602788559660*e^(279*I*c) + 3550010310601
964987627237679694948220958137237103600501287780602748167280705994344524013
6315568500732379966585005678181937920*e^(278*I*c) + 30128482414552703264559
018953088177156013437493438201078413769835448366148121754549197591129967170
764969700180348699207838960*e^(277*I*c) + 253671764391193536215322603359833
481549049826061257617113006834929633908164915830257052687375399821496393002
26512657426118880*e^(276*I*c) + 2118832140588288753961019837470686269589404
922607709376413251251333619052397894969438768605912452675504804295795426470
6637460*e^(275*I*c) + 17556273271224292396887291403125716213491486261145478
571376751690105656067838042151038271381300372757755676325408026834544840*e^(
274*I*c) + 144298162852084312045329783753756919650631542246497475512958515
07389524083226976789688601369628399900747658579201929300744260*e^(273*I*c)
+ 1176421227487648408001090071467347449337127816055781198372445582656605561
7658086479368641864908119643412413644803772131657280*e^(272*I*c) + 95130322
740195229542091131912682266422999120135256659402983810647978856909049931289
48035227412144035633851779511219335277360*e^(271*I*c) + 7629731815627821580
468992424207008366438896736333024661863838105110445148946962328297631547032
543419811821015837863013682720*e^(270*I*c) + 606894980315671224833187110532
989547172280614300887801498655965368726069481655047019589000451196552756743
2722969707577202160*e^(269*I*c) + 47875274427809456851452048469715961653041
694193282440732114595921296492550488768540598447206610781512881796125749863
59194560*e^(268*I*c) + 3745257594876651204657334988426226388143954501986830
664222349226361079609546822276067504899386703088982308185717143407211328*e^(
267*I*c) + 290538572232005700195334527448948279085669252995982374953269596
3414164833366773128218607899328588608916176593772088622582464*e^(266*I*c) +
22348912763984394644786225783064348407246104844681778598226206586919214786
45266653062563823553001228001009093606751066168944*e^(265*I*c) + 1704582996
707822808204678218167693002698661147712772355021456543810930069637188085882
824757500605246963210810351706405349408*e^(264*I*c) + 128904351529293395648
063433049967704018104393562010691426731106790003005839883978769237695409054
5278554544997710058754772400*e^(263*I*c) + 96645827536903771874773913079815
164348359068416683223468809829116416063641815945211981572880937212516883623
9364442397344064*e^(262*I*c) + 71836159638205824920911354448790108886838874
403371321033249197137590673834155154045726480430403966425591560734980191196
6551*e^(261*I*c) + 52932925276411392600393483695824355767254923899756073921
4406599185047831955572583765358634395408771528009745467548382950094*e^(260*
I*c) + 38664267305038004945738256281831696265197555099077927704874023862985
8795018247356162888631015687664780101205287333082748791*e^(259*I*c) + 27994
524447503980482296673046296088449211987485779114712400907947692043594173529

3309305430438687333129912454196774070107264*e^(258*I*c) + 20090658715357880
438030046950144161017452185125954192920984068896085945490851977483590589575
7666770857888611738751858460424*e^(257*I*c) + 14290631912305552424654692847
895423837131592580202238923649865213683982250203515567697091741903983458796
7055588431566416784*e^(256*I*c) + 10074496185185374461175430098298016696240
455383622292186848469426996612060769890704634373101116094882810027672937013
2819357*e^(255*I*c) + 70386349760594831567048224061395025698501202296966300
376764220336697702961591099854055411376294871437468149528524796002762*e^(25
4*I*c) + 487332535059749234008522555630521014021964693136595544927256747543
39375283010407167744366955828922837488705858532439654489*e^(253*I*c) + 3343
589782793658130117117545961082945429816796201741981007293673337850658442802
4201072453193458155334046693516742390717832*e^(252*I*c) + 22731603566128841
100419501947051367666836652418077260913944810748473084891890410181285412604
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076829657125356529426315182861424030977382002707111953965821590285135327796
82154451996208592*e^(250*I*c) + 1022042377943463485133997529516339964170212
224966366619305300830202609693215856833830941823739554135181902690795322068
1013*e^(249*I*c) + 67584804378885243725629359489638576266948555471955194861
22877567981718587262362871994967079401831957927901682582941234362*e^(248*I*
c) + 4427673079105425318524316112985693656584851936100192457044455134483305
045321452516347118488133224823670465103483954805161*e^(247*I*c) + 287361053
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722801012*e^(244*I*c) + 742228640908173124916937049462525617334148919679118
270489831005497781951221069955839623452499748653124658873553401442137*e^(24
3*I*c) + 463758288457367154544937678255005688733328145568049310423995599886
012800638619904022368378591108842602342094543682299102*e^(242*I*c) + 286992
943631231496557278010851576940896826497466066327528801560677007112837431926
735088120974861760511367008815728782643*e^(241*I*c) + 175896258262755985757
106812613979301265801031595484353614904672865169442232075776580447184134141
375995770091499246759528*e^(240*I*c) + 106764832017165594838085234189333528
733587673329972530092661085186789939252915937090760282232346919090426243399
409323314*e^(239*I*c) + 641751006932600668062380648860045971707408433000868
39368616139164529108049844675353111842725798658088840347241496099644*e^(238
*I*c) + 3819901586758608797600299875662767499479544066790362502932234625013
3286489120875005013638128113893960349670280707161530*e^(237*I*c) + 22514675
741308069961506165586502872430421930210673264392997286485600640103867253604
847715547060592967690653795951142520*e^(236*I*c) + 131397714941049338818566
811514182931122425515215356868711812665798138776063481602617472013177357825
66021306798298336024*e^(235*I*c) + 7592752700146678961153095073585015473197
029746533633331549793961473285760935801904155116764831560875947581048693527
224*e^(234*I*c) + 434390969660193217335735968778157929370129568194082711421

5433175336093967845908766740738240037114570667410936998017178*e^(233*I*c) +
24604423758454226639270816309832607147349680919054930271456392388271922548
86349361126991457692409851120873307487457468*e^(232*I*c) + 1379676529796212
074017106188066589448355446501210890195107164860350228928586815539003062875
026711931941947738690360722*e^(231*I*c) + 765867795513962781012558444628751
418710940895281304790836743661582071650032154891482866406314834433199455459
798934952*e^(230*I*c) + 420846342608949387277559021457924586578120966148561
022647008499529468452005980175119410628956210497609566002969884927*e^(229*I
*c) + 228911311738592780091492649162346834405867740776456326108410928857257
174707289268074347550225793244741923354395308214*e^(228*I*c) + 123243941519
332384741960072588103506596406339253616391082062969960682419011745775738921
817753391954462609323881489157*e^(227*I*c) + 656748592688673000988273758128
752256106545516862611036816640070075371157780972935335652438288733837229803
53200611956*e^(226*I*c) + 3463765717267169016765734453719708704888235485399
3270472063943078773600446542963548348101269390443464480754513928502*e^(225*
I*c) + 18079820068028859970349938623007230676563314206708848499900139641237
334763266479346963237936039328113185041591793848*e^(224*I*c) + 933934195805
349422525175096571505730070730208381477477030621822424102264824741995604295
7363055823830898547303219757*e^(223*I*c) + 47741411110660989702218453305949
620164727142303742340606639568469509266426859469290641141944003609362235907
25470146*e^(222*I*c) + 2414966516810338503289076549202740511710059011795447
138773464205696455026442712426409599662771080264826008985061097*e^(221*I*c)
+ 12087703584936583930894422205693506328370410814059375022653984611773764
821660959734831601248698274330296158612144*e^(220*I*c) + 59865014111224185
891167650518052015036400322684132808145359709358779033860921243908555446686
1582623350303061961052*e^(219*I*c) + 29334492003430072028704238344834286631
380628545504006782308044559754597002344623156355413513310549351631632005927
2*e^(218*I*c) + 14221311596481451768238666727676990948227168131879088984050
1039441748635545362467679832449103520321953011780083069*e^(217*I*c) + 68208
033096793615683784409619244210818614991640041553424405527876893272496608324
231098148502466453967157728078994*e^(216*I*c) + 323627313224195494103300889
436402474603783285613164229312924271459028879130716436795029090558912367551
43207382609*e^(215*I*c) + 1518963421490880039641791172264375474804852010973
4812459109878810493844381062650818971199637121458749456243274416*e^(214*I*c
) + 70521324141621979926023265245801430609853530545729339055246331216810210
37340298366342203324325307072413739061024*e^(213*I*c) + 3238491931361851476
423321933539579098377735539207641467346235665823887048326949305609231585143
748690203615957136*e^(212*I*c) + 147093114661893434551503836230010016048212
7749581443929904746910224777470198899052379114493999887003199419829579*e^(2
11*I*c) + 66076447310586909769147597385083793451108903314958670798276426339
4766756649565279879146173318386505740391093990*e^(210*I*c) + 29355074355434
270980812945357656231329970598269918741686293437396425561596713867625327630
2591561523515603264403*e^(209*I*c) + 12896708008475471224602368086648838498
3286259025533132044636109049545144029547003347761521666283977931640178464*e
^(208*I*c) + 56028683424903517658495013858534516167162591034367972498174660

907450666778154353271630344650777885683547624184*e^(207*I*c) + 240687851397
052771611934656445061432852413610377682168189221844001410484602109446966477
52723371932874594597328*e^(206*I*c) + 1022318202595486076721739030518645192
3562145473674293619918063490411487496121804590274592702770571515456414680*e
^(205*I*c) + 42932064780080221260174889088518264947906207206601514514681819
10917240027863968724539127659633517053002976480*e^(204*I*c) + 1782446114931
751850556354856638421901174412322298249496591658053939787198246565945975595
575734193348887952160*e^(203*I*c) + 731584972206818362874729621403974444280
010446301161527339760544815300951787985538419764656214582667219914080*e^(20
2*I*c) + 296825515282669589685318273280239050084555032203415941511962659596
881615713799937680026497408305672297618840*e^(201*I*c) + 119041855403877964
948229577948370465600606623183045529526900430209270473212773847794935586074
714329479939280*e^(200*I*c) + 471882208434662076950995069535737803571088974
91422567898048199018207708997005333860148836479527456156014520*e^(199*I*c)
+ 1848740529900573269375272861187649089085835702197488237157062380018624513
7722660943641752976852924439870880*e^(198*I*c) + 71581246868429414754738073
636798397181727455815384090445033838526935969216224266967404539447181430252
48390*e^(197*I*c) + 2738895624795265603355227646566000886280778305084825702
911938903656162004262736182657700406301914070062380*e^(196*I*c) + 103556198
259200293522638457790861154861211149508019357369133986470602918648246624180
5664949381049856258510*e^(195*I*c) + 38687621823427716563245172304997988926
3115282374607541692443176673997513742813591736171169652250611186480*e^(194*
I*c) + 14280179245022176248318087491882527413430513327541778008479503464476
3509333503150517345864659667189417080*e^(193*I*c) + 52075178518793270386429
263351544306951104993542500582938155241689408138675254608030847907167748571
734720*e^(192*I*c) + 187599882188655635641636357359860732782557372574057062
79108891366378428467414559930481172863538598193890*e^(191*I*c) + 6675866290
371147358503766865669289010893543869830538708724945291580951179188296606158
111257706968604740*e^(190*I*c) + 234651821923910514223814163307346476889915
5708935025778047637412681781575765422219127409260159438712250*e^(189*I*c) +
81460818773653057967021002527192141559718336988121429982329196978554987617
5969866367976653244974728560*e^(188*I*c) + 27928575580003520667983536889816
5477644864987794665387827488933863633745047373109049265172681702585720*e^(1
87*I*c) + 94556180258931986919334303466365652826858091314329189160736277175
873841732196453379953705679466826880*e^(186*I*c) + 316109393312846927506943
064436184146560959695209452157430040445603868952418015791565434519407133517
30*e^(185*I*c) + 1043411751657039596665369315558240210946034809547302780741
2321427346816928567197770376496170251803940*e^(184*I*c) + 34002325606016516
175216946808470898441980288316944174247948687793289505484181256054468820811
52636090*e^(183*I*c) + 1093853214486220358674032434500866678499770011305874
172488975951612031456734608287095519501041975440*e^(182*I*c) + 347351473214
713780874352083129566601238765762775942366762733349952103889753982636403857
556867777300*e^(181*I*c) + 108867995731829472826732905192034886797284621356
445627530909104429486741257822633476898356826454040*e^(180*I*c) + 336753988
720215683759023845939827533625598010581041846273454111362624319432407782607

21756991027090*e^(179*I*c) + 1027936473066384084473957786246926260464886191
4297972589165243530651230690726244462479199894255180*e^(178*I*c) + 30961319
716215201623803015542414654517823620868102875377489029049859340201795657061
77131421614590*e^(177*I*c) + 9200893930295890328746018500271593226125263684
44771489781974361078847528891468831038436064951920*e^(176*I*c) + 2697458014
402112969726836018638789543579623085200765951771282276292732402152097082184
97363414140*e^(175*I*c) + 7800980736802423987561373305885141712532711468107
0889640794249282633470580756557083923203377160*e^(174*I*c) + 22251959176795
777757167366036007480222211364232146399803864370963391491223687245823457351
580140*e^(173*I*c) + 625987215682225284365096070823503471020136277605717664
7226323089751446565288850103898153859920*e^(172*I*c) + 17365742188181910718
74197472450158123883564209950658639102337148122769080611680719741726053840*
e^(171*I*c) + 4750105788576015192723166179384252224217865972416710268943185
15408511467140969393115768793680*e^(170*I*c) + 1280989146016885396724805418
30409847707367500438601536803204497701119911289087105659482783340*e^(169*I*
c) + 3405405385129556915435234672217717265518754891078200850471832416872502
9438589162349211628040*e^(168*I*c) + 89232094473432967633318818816384717934
99618670601026059730895962653291770229493028162575100*e^(167*I*c) + 2304351
073373840357379178597673066352016682781689139842097376663118488803841131935
313641840*e^(166*I*c) + 586403466972683242741643328921560909375197453864243
299571990964608857245771134145204174990*e^(165*I*c) + 147030816732276833163
041582099592047512043725225353339238819165193000407629544745753221740*e^(16
4*I*c) + 363183696523025917321974444097981220226408246041305525067425867951
83267354382847875885730*e^(163*I*c) + 8836720640860470305694514021547969551
296794092266983044118375790025854584036796364768280*e^(162*I*c) + 211758973
346685570710150142921041472240183883794075284161854144088854572994313820903
6820*e^(161*I*c) + 49970756725385908435759631481379476806933719091596749190
7488904933922677579665354338960*e^(160*I*c) + 11610455168355504376291150171
2116399313733021132677481112824047246361794049635726479850*e^(159*I*c) + 26
556806389043407534496702369101545795994861757741414789944652712127566910185
274123140*e^(158*I*c) + 597899217294414321845916114929981970632173211157849
4525245228742976468409105395536290*e^(157*I*c) + 13247564123678374731574728
21162483691120966501948953926492241643788264284546437221120*e^(156*I*c) + 2
888207552647306544699685720210471094273186195089958020206899045903194762954
08324280*e^(155*I*c) + 6194859665303550287956433881523431066041090203788247
3161804774492916216575880077680*e^(154*I*c) + 13069817203488289886193205508
375818392124991382340160316886507181296548981014818410*e^(153*I*c) + 271184
323967071752760564049014883350713024244840397831852323772194420039283010858
0*e^(152*I*c) + 55326912881952861250291886955894782909802195630934984358404
4631512291778800081490*e^(151*I*c) + 11096919968732097474992225959525044434
1219218535349655762591192576535872151766080*e^(150*I*c) + 21876482892713909
928040345612578705805121508756226696317087651824252241418663320*e^(149*I*c)
+ 423812584676323258639418856985868582675532800554862743701930140585132588
7594480*e^(148*I*c) + 80667954360758914075930501079618956826984202161338895
5218916278823182639488190*e^(147*I*c) + 15082238143141241377356647421001174

6852297437597059186295243989481140398152780*e^(146*I*c) + 27693116538343259
 225983382637647936122664033859615133489846664694361471028310*e^(145*I*c) +
 499251971245704398350537797660795398839736829759111495799180489368837186768
 0*e^(144*I*c) + 88350096882179120260077454192776920073768939351373478936839
 7093333311961880*e^(143*I*c) + 15343608874505625412732723946157707193313015
 7764595997113973513183188399376*e^(142*I*c) + 26143976279902021443471945665
 080254563056810183520401889800285493144867448*e^(141*I*c) + 436944248291011
 3914565353136069595862669338858053419381214131241925047008*e^(140*I*c) + 71
 609949759905807989563333852940229192858196481597830078819711862600096*e^(1
 39*I*c) + 11505148185208084887370038835452131556764036512400310369117669719
 4292320*e^(138*I*c) + 18115768495615758076710303055505625589254293659193314
 153418333944596408*e^(137*I*c) + 279470910447568661184279069497369916448225
 4723977210209725661304403472*e^(136*I*c) + 42227612663200368754775474655570
 9988710527133086660161366353656787288*e^(135*I*c) + 62473550781053295317710
 774690247114124125187565731848441781904032672*e^(134*I*c) + 904669352382568
 2979044338963104263167672586826367911338826483549173*e^(133*I*c) + 12818174
 64914970810859604189828359000790789921169405304612211251818*e^(132*I*c) + 1
 77642829135119348577194437675802830239905460092687136494961404333*e^(131*I*
 c) + 24070801913529757101858022914372045864746991786182039740274325264*e^(1
 30*I*c) + 3187749929744346497211536044751776582320958627923816470590659024*
 e^(129*I*c) + 4124306982999151908480672223272194350677479340918946704889829
 28*e^(128*I*c) + 5210811762917704866049240098517583098750570056687781895414
 1639*e^(127*I*c) + 64261954855352485764250681368704655300871140038757166913
 83902*e^(126*I*c) + 7732046369911457750614627310280985060944326757881362950
 11259*e^(125*I*c) + 9072260572220881491864228463948718776460758970649397077
 4776*e^(124*I*c) + 10375184499871175501909398956596684116802997082526660323
 524*e^(123*I*c) + 115585541289359426034554496664268782363003589936323237147
 2*e^(122*I*c) + 125370496586921272662198050851269323171167338854081782959*e
 ^ (121*I*c) + 13231708870104896973800056733779919089340836756009580718*e^(12
 0*I*c) + 1357990663161479842850642848032544982878359839580349899*e^(119*I*c
) + 135442594916636116191574650625331646238501101627937224*e^(118*I*c) + 13
 118781801172174729679339894318153694964675368481194*e^(117*I*c) + 123309670
 0139723365181997220750932590655287625342156*e^(116*I*c) + 11239160454224665
 0966429162063124338952554575234051*e^(115*I*c) + 99254907385344022729399870
 38714580495445431374618*e^(114*I*c) + 8485522022765123564962001369596762953
 61696315113*e^(113*I*c) + 70164515322544462906873548813748091084561870680*e
 ^ (112*I*c) + 5605927253067558551780452883689835514455118670*e^(111*I*c) + 4
 32333688644261557547944179250800440604964868*e^(110*I*c) + 3214788769337533
 8817454482515377350383950278*e^(109*I*c) + 23021504112262349258552223452015
 00900533576*e^(108*I*c) + 158566476113257562566117432227203884298856*e^(107
 *I*c) + 10490402669510897424624643766470754045064*e^(106*I*c) + 66563467067
 6210063754191847109971141414*e^(105*I*c) + 40443624781415311581857832389099
 634564*e^(104*I*c) + 2348998374244347079532766203075607598*e^(103*I*c) + 13
 0171193079172823835151430773360024*e^(102*I*c) + 68683292252636813495019973
 41320517*e^(101*I*c) + 344277152012875134140739302960914*e^(100*I*c) + 1635

$$\begin{aligned}
& 3164647151530240529137618111 * e^{(99 * I * c)} + 734057263616388449968842366924 * e^{(98 * I * c)} \\
& + 31042222522074681615625020522 * e^{(97 * I * c)} + 123244555734683224517 \\
& 6696904 * e^{(96 * I * c)} + 45759117183402579073139583 * e^{(95 * I * c)} + 15817966423978 \\
& 12408161814 * e^{(94 * I * c)} + 50648660944512569972179 * e^{(93 * I * c)} + 1493326612293 \\
& 984160368 * e^{(92 * I * c)} + 40261256699368950388 * e^{(91 * I * c)} + 984382804329835768 \\
& * e^{(90 * I * c)} + 21608403021340047 * e^{(89 * I * c)} + 420601518659718 * e^{(88 * I * c)} + 7 \\
& 146142307307 * e^{(87 * I * c)} + 103818048048 * e^{(86 * I * c)} + 1253841160 * e^{(85 * I * c)} + \\
& 12085216 * e^{(84 * I * c)} + 87153 * e^{(83 * I * c)} + 418 * e^{(82 * I * c)} + e^{(81 * I * c)})) * \tan \\
& (1/4 * d * x + c) + 7 * (299 * I * a^2 * e^{(1027/2 * I * c)} + 116610 * I * a^2 * e^{(1025/2 * I * c)} + \\
& 22680645 * I * a^2 * e^{(1023/2 * I * c)} + 2933363420 * I * a^2 * e^{(1021/2 * I * c)} + 28380291 \\
& 0885 * I * a^2 * e^{(1019/2 * I * c)} + 21909584720322 * I * a^2 * e^{(1017/2 * I * c)} + 140586501 \\
& 9555002 * I * a^2 * e^{(1015/2 * I * c)} + 77121738215926170 * I * a^2 * e^{(1013/2 * I * c)} + 369 \\
& 2203217145049425 * I * a^2 * e^{(1011/2 * I * c)} + 156713514334349191205 * I * a^2 * e^{(1009 \\
& /2 * I * c)} + 5970784896718122444327 * I * a^2 * e^{(1007/2 * I * c)} + 2062634782910323883 \\
& 61681 * I * a^2 * e^{(1005/2 * I * c)} + 6514488191762735815405157 * I * a^2 * e^{(1003/2 * I * c)} \\
& + 189421272159473056494074595 * I * a^2 * e^{(1001/2 * I * c)} + 510084426287781837574 \\
& 0499940 * I * a^2 * e^{(999/2 * I * c)} + 127861163072229101394013407134 * I * a^2 * e^{(997/2 \\
& * I * c)} + 2996746017244157830350053701476 * I * a^2 * e^{(995/2 * I * c)} + 6592841263164 \\
& 7739461683457820511 * I * a^2 * e^{(993/2 * I * c)} + 136618322485629786552121700902082 \\
& 4 * I * a^2 * e^{(991/2 * I * c)} + 26748429663467970647815785552181215 * I * a^2 * e^{(989/2 * \\
& I * c)} + 496183375602453279107611685390767740 * I * a^2 * e^{(987/2 * I * c)} + 874227865 \\
& 0463030069301003637232429883 * I * a^2 * e^{(985/2 * I * c)} + 146631858416411858207561 \\
& 193721288320906 * I * a^2 * e^{(983/2 * I * c)} + 2346109795219082040306865029327652810 \\
& 422 * I * a^2 * e^{(981/2 * I * c)} + 35875930157695641006598626271371876133245 * I * a^2 * e \\
& ^{(979/2 * I * c)} + 525223640233639829417248502076803689088820 * I * a^2 * e^{(977/2 * I * \\
& c)} + 7373332278938477877109516190844163180831833 * I * a^2 * e^{(975/2 * I * c)} + 9940 \\
& 3449489614415589324261326783015782654200 * I * a^2 * e^{(973/2 * I * c)} + 128869483230 \\
& 1854187496925699385880912905767417 * I * a^2 * e^{(971/2 * I * c)} + 160864682643236554 \\
& 13475747304585608493929925740 * I * a^2 * e^{(969/2 * I * c)} + 19357386052647342317900 \\
& 1809134869735140502053410 * I * a^2 * e^{(967/2 * I * c)} + 224795487509634223508139184 \\
& 3579770658847258731308 * I * a^2 * e^{(965/2 * I * c)} + 252192487428249481575472714845 \\
& 89255811816930894969 * I * a^2 * e^{(963/2 * I * c)} + 27359070326171366844836585939655 \\
& 9199738283337615067 * I * a^2 * e^{(961/2 * I * c)} + 287270320925440168071834728588352 \\
& 0097429919909434035 * I * a^2 * e^{(959/2 * I * c)} + 292195055316171397459951984560262 \\
& 35957224788107224865 * I * a^2 * e^{(957/2 * I * c)} + 28813690834680473368843683438409 \\
& 0051687033252355064191 * I * a^2 * e^{(955/2 * I * c)} + 275677067277424856755018403180 \\
& 5577107703843954010093519 * I * a^2 * e^{(953/2 * I * c)} + 256089632247852915722879520 \\
& 38770777479118266594544417374 * I * a^2 * e^{(951/2 * I * c)} + 23113746393435153524022 \\
& 7752765265592838813655877791732766 * I * a^2 * e^{(949/2 * I * c)} + 202823283552635430 \\
& 0983369892820970081423531403040583023770 * I * a^2 * e^{(947/2 * I * c)} + 173141985598 \\
& 36475415651393365133642453745100461062396834495 * I * a^2 * e^{(945/2 * I * c)} + 14387 \\
& 2897869958476711841808368973418609254139596808263965208 * I * a^2 * e^{(943/2 * I * c)} \\
& + 1164368139316323606713195496495533903452490627065569725880879 * I * a^2 * e^{(9 \\
& 41/2 * I * c)} + 9182643580975122394482019949557981111651613557125350856577098 * I \\
& * a^2 * e^{(939/2 * I * c)} + 706044414981603593132412608959717021499560863803039364
\end{aligned}$$

68404945*I*a^2*e^(937/2*I*c) + 52953430380431437131727928703158739934289831
 9070257744555877120*I*a^2*e^(935/2*I*c) + 387574875295375725139232180409811
 0389738719224033802668294149720*I*a^2*e^(933/2*I*c) + 276955224919873361222
 97854656279067704143932579317971697894652128*I*a^2*e^(931/2*I*c) + 19330398
 2244781919691500896347164102540863253211817387366117379028*I*a^2*e^(929/2*I
 *c) + 1318337340904646289197961886292345009401456618543410019867356069360*I
 *a^2*e^(927/2*I*c) + 878894722176403352444688879370312534478409467919461266
 2969732267700*I*a^2*e^(925/2*I*c) + 572974083910783244258442207116960160604
 58004240084890338934689925280*I*a^2*e^(923/2*I*c) + 36540779363619697337987
 8926946921993102971243732578054647123414214820*I*a^2*e^(921/2*I*c) + 228042
 7120078137719325513847874265148019541854031486848558248442727176*I*a^2*e^(9
 19/2*I*c) + 139314177925091981614687974237525448386785995392752123323804426
 51357560*I*a^2*e^(917/2*I*c) + 83340276962100363964566640495864516393854803
 017236184470442989394035180*I*a^2*e^(915/2*I*c) + 4883483643690059612618015
 02808542403226214848682295131186689348564834560*I*a^2*e^(913/2*I*c) + 28038
 16222757253133472672279792586220272029806438905494766819989517225180*I*a^2*
 e^(911/2*I*c) + 15777551334929884741558980231119513445377646781839885604883
 296740586671600*I*a^2*e^(909/2*I*c) + 8704038273624124504811313552891743577
 2531399861517048611706176997728840060*I*a^2*e^(907/2*I*c) + 470879572899329
 452986316253534868514992585355812719048085012389615234670720*I*a^2*e^(905/2
 *I*c) + 2498731377031195939020099006767775607036784147188475889323331541072
 591988520*I*a^2*e^(903/2*I*c) + 1300945152021805442798772690930638802672821
 8430095197151626190204529199816160*I*a^2*e^(901/2*I*c) + 664712059991868481
 17890072662193021327677387770934414812726997704844663612890*I*a^2*e^(899/2*
 I*c) + 33338444155217534857586385168528007939751540812720139284301050909814
 4064832520*I*a^2*e^(897/2*I*c) + 164169729637814329912873364094099880544138
 4198489236819275463423360050416710230*I*a^2*e^(895/2*I*c) + 793912239209648
 9819576023365780909095910972043426214771545781246832910820414100*I*a^2*e^(8
 93/2*I*c) + 377117140101616328512751948735862319229675664519554097468433744
 16439997598272230*I*a^2*e^(891/2*I*c) + 17599254140146487259175830149983489
 8740699910060781393774843202513062630336753720*I*a^2*e^(889/2*I*c) + 807074
 460549539210003558596515532129897902975140321070941315505466030633289504700
 *I*a^2*e^(887/2*I*c) + 3637632524900007961249646025024970707837559612619597
 442737493322095743154754032540*I*a^2*e^(885/2*I*c) + 1611728744799820614279
 8083894763175840848857114343543047242138953088137509988748530*I*a^2*e^(883/
 2*I*c) + 702122063048016783325133455999109492462648029240170648330249799178
 94264890884744230*I*a^2*e^(881/2*I*c) + 30078638004528714362895449962913656
 2193246097877509863055826252064326569743630828470*I*a^2*e^(879/2*I*c) + 126
 737087561381648317237671513461191515502353237594752736881659052212108959184
 1195870*I*a^2*e^(877/2*I*c) + 525318021890119037052330796129201500498936014
 5697116713826069492059896349225432472890*I*a^2*e^(875/2*I*c) + 214232263757
 34147493294111979827037040590614426574817511241501723037567325263745727850*
 I*a^2*e^(873/2*I*c) + 85972681691391354979713197091334370055201000372937152
 633172947601816313260199706979160*I*a^2*e^(871/2*I*c) + 3395597687106525970
 13282888817312950668437949251721078395194275059325692430095887234940*I*a^2*

$e^{(869/2*I*c)} + 13201323425995747765824874916863760110207311283385646449721$
 $36377885389653577953327654360*I*a^2*e^{(867/2*I*c)} + 50527496930900742500207$
 $11633864893162784543894410325996308296849332368152124619583542550*I*a^2*e^{($
 $865/2*I*c)} + 19041847118319726293265803273951207945593799588235034178026656$
 $550799568684461605603777920*I*a^2*e^{(863/2*I*c)} + 7066780919249683896553228$
 $1733119872286671303574373760629554934333798898169280995448949110*I*a^2*e^{(8$
 $61/2*I*c)} + 258299861869070319215539511486716381312735629658913082894934911$
 $762706535490482149535095400*I*a^2*e^{(859/2*I*c)} + 9299807036336009523216256$
 $92946644952639044210651468835121797966088502026440103109950142510*I*a^2*e^{($
 $857/2*I*c)} + 32985775034613724656053455368691232614563926054549808402060457$
 $85661659654642478147158578260*I*a^2*e^{(855/2*I*c)} + 11527544710214127291959$
 $320966284738345019810955968385265683600912288138678711188667077969580*I*a^2$
 $*e^{(853/2*I*c)} + 3969696733916467448469860592541488983654899708624040875073$
 $8793856030209166162894182626765170*I*a^2*e^{(851/2*I*c)} + 13472244502792775$
 $389537013386891372826155603628085240383959392418361487884840921793997982680$
 $*I*a^2*e^{(849/2*I*c)} + 4506456258037388829608858710305661439531089882385300$
 $05053615795799976664205719692805608636970*I*a^2*e^{(847/2*I*c)} + 14859088946$
 $554091297275796973902171118776912467270009290931878638293679253843736681369$
 $22959680*I*a^2*e^{(845/2*I*c)} + 48301330647612656760988232758903222705067669$
 $18770002889169425396499013319534724628237025832970*I*a^2*e^{(843/2*I*c)} + 15$
 $480423340995881584740416023332877545588024374995660159497444149878426588554$
 $491651581857867240*I*a^2*e^{(841/2*I*c)} + 4892256658132882799624336928834627$
 $0469307859644664802730085010597182346996542663202606651415940*I*a^2*e^{(839/$
 $2*I*c)} + 152469533247657613494014223169079603947607129031097239090335734676$
 $330177814841868201155345866600*I*a^2*e^{(837/2*I*c)} + 4686494338769403747164$
 $15334540852022756419671685290819126536222202383329230675172080843145521190*$
 $I*a^2*e^{(835/2*I*c)} + 14208461966435788037335753309269709744646757040204816$
 $18027446077982380731097941167611657029967430*I*a^2*e^{(833/2*I*c)} + 42493429$
 $907386087010781091871687683676933412313560301181637189097741731163102184605$
 $85993016680850*I*a^2*e^{(831/2*I*c)} + 12537562836732599179767010355152461056$
 $807191951005009796451823391079000655762189067741192305394090*I*a^2*e^{(829/2$
 $*I*c)} + 3649732635331596360542029819068746500105510075807430423479227216188$
 $6339957870164613698417616283210*I*a^2*e^{(827/2*I*c)} + 104834609086572625907$
 $289346225890252429283565041460323851402795840051940483234866884215706838405$
 $390*I*a^2*e^{(825/2*I*c)} + 2971544784433528472987442935363735703676312583731$
 $97290869967778261660884898898610815793932187264900*I*a^2*e^{(823/2*I*c)} + 83$
 $124933312427092974956512337021935716565456874262003990342923167154743611337$
 $1386430716266759961060*I*a^2*e^{(821/2*I*c)} + 229503041901714401511079773215$
 $5375346477339079532575006580675869478858321228765847919467823502616280*I*a^$
 $2*e^{(819/2*I*c)} + 625445695771085708499077750788019679582604834287739131011$
 $5506167680233024354089893072681406475246730*I*a^2*e^{(817/2*I*c)} + 168255776$
 $750096534106528129967235981204834701753753320012509640155551252368735813874$
 $68224926998037020*I*a^2*e^{(815/2*I*c)} + 44685235946986673791925225525992813$
 $797485854939304559125279366181549713351572630218142149059140721370*I*a^2*e^$
 $(813/2*I*c) + 1171670844831237108639698079387721551699567853175230901748626$

97344141293246236717353854714751253177160*I*a^2*e^(811/2*I*c) + 30333870635
 075548727905717088251347474336020120036232795473704223354032916036441631311
 5387680520970550*I*a^2*e^(809/2*I*c) + 775466756780053803300487429447750888
 237325797624757919706744628478203937255833604141877857656968392480*I*a^2*e^(
 807/2*I*c) + 1957688477178164823844184536565647794575532926755467360055820
 937851704595970877608419045482885065913720*I*a^2*e^(805/2*I*c) + 4880904812
 629492578182749704265723358376166867348003927103702683844870930164640446326
 414262466377824000*I*a^2*e^(803/2*I*c) + 1201888522740550852777891824471307
 7048854664134058582641410118464532001237995012608936579451133542506420*I*a^
 2*e^(801/2*I*c) + 292324578477843471942327558706863993151718093462483465949
 46497316944533356199777722113539614776201632080*I*a^2*e^(799/2*I*c) + 70231
 742836691486232401821435646690204765370170934364279337989127944150596825723
 542271716872404327490260*I*a^2*e^(797/2*I*c) + 1666854541786758673088478816
 430259717371256882203030379686823499256790893313121747781173660390262858694
 40*I*a^2*e^(795/2*I*c) + 39082901345847871376200586853174955051134836375540
 2425375475219863201042469027671720992324263291992741380*I*a^2*e^(793/2*I*c)
 + 905375964480293043221568717617222804352981478608148290290910134804446811
 145030581376444150025327895167080*I*a^2*e^(791/2*I*c) + 2072296968472727638
 582366763214809350131036179733894803881601867867318900360235090208152423869
 380817193240*I*a^2*e^(789/2*I*c) + 4686873873156101871190767974561598609513
 443670201757460914232312505484627209871957976104328771649162041900*I*a^2*e^(
 787/2*I*c) + 1047489711020580736784637506672333662224453652317656117621724
 5839587779126630391482195110511043224435073120*I*a^2*e^(785/2*I*c) + 231354
 575556672420151095626967247592694421472166873974898194890719416969466452628
 69400576401992001928296540*I*a^2*e^(783/2*I*c) + 50500362504156859478353840
 472838257985720430216874883086230302896996886794997397859935740848410975488
 426320*I*a^2*e^(781/2*I*c) + 1089494105060243829214205754738692285159428724
 96860505620768573056175175693027104049195507535238812688471740*I*a^2*e^(779
 /2*I*c) + 23232441719723506208102256523595997332912970401651251149268440722
 8447554005362475531181177257375428158576672*I*a^2*e^(777/2*I*c) + 489699557
 155298676398555706690170103288962201587062038457047783309280898007016462695
 325519866776349668712200*I*a^2*e^(775/2*I*c) + 1020362468343075226131935473
 747821355290052152417646455220161102238032449230964279115841386608493376970
 117760*I*a^2*e^(773/2*I*c) + 2101812264182065730411859582938002660622402353
 730581510169623059960820290526673615762476646904104925062205465*I*a^2*e^(77
 1/2*I*c) + 4280284024728837338195579377830489525167771127365155055905896472
 260407749270429246786706643678954884633163010*I*a^2*e^(769/2*I*c) + 8618164
 918788925203902059506702768733004223544564086461069581208848976475760787131
 715839621109117547446849991*I*a^2*e^(767/2*I*c) + 1715713198535927799763949
 795015467476277919035108756781919569951778466633338337833668711385869220404
 4077794176*I*a^2*e^(765/2*I*c) + 337742897479789814249267399311608247971611
 12237362925619159566387573730808632464292717707940688773187398723895*I*a^2*
 e^(763/2*I*c) + 65745072618550512189873447315220858623931223310142529839808
 121856850665718523376669691836042778846630737410930*I*a^2*e^(761/2*I*c) + 1
 265608335829055341254571182834795882548867340664226813227430674530949554232

72892820673108496357191409969596590*I*a^2*e^(759/2*I*c) + 24094512727676622
732033994008929823268104823914424878932496808804881942144700374981607795527
9624100315116789806*I*a^2*e^(757/2*I*c) + 453673232394267043949605922569329
980940549626999382599843505662311686257301966537920360838324667729133482235
583*I*a^2*e^(755/2*I*c) + 8448853220591619323689935171172081111839443557646
32182261257568921304529568367117463318791743206771210437760631*I*a^2*e^(753
/2*I*c) + 15563430969685737647564752354119330167337863131072390048648005786
66621277030123112055440836769200504575169913345*I*a^2*e^(751/2*I*c) + 28358
890227167933110932671089134114841174308170409759286396549113682115146981129
27408762873660302882359781905835*I*a^2*e^(749/2*I*c) + 51117871550982319663
814444139984137697380398406297899866361358613770964371105353522611964446616
35160446805767627*I*a^2*e^(747/2*I*c) + 91154745536838262814361423059106684
025825819888567458510855744980267086774428268033834012663878131620625945259
53*I*a^2*e^(745/2*I*c) + 16081708037117239364885424953083244936515724921299
386778600225951179498131171589524219045138516435810850207340428*I*a^2*e^(74
3/2*I*c) + 2807080481602620981076564109527504212794812804138943562972048564
3024095406486924220542522448143563324518401964482*I*a^2*e^(741/2*I*c) + 484
809206605142668675351572457287006724534653659013444478300532982673008490982
14762864212092705639132905793681740*I*a^2*e^(739/2*I*c) + 82852142966860851
431611420117596372256466739448701897204460441426498632548900216968337411640
986907458522921325945*I*a^2*e^(737/2*I*c) + 1401126097156444901142996236202
237326344616475127018069082403838786547453990878368588047116294007625154259
55916824*I*a^2*e^(735/2*I*c) + 23448502098041246734174319914535202946807635
2266379854608773522593799565958621176141472761686768653754135750141593*I*a^
2*e^(733/2*I*c) + 388366130772936514035045226101528726099976343067435024084
898656054932162774966074464370430510444616487043887402836*I*a^2*e^(731/2*I*
c) + 6366196753590080176222160089204077247084401076299878481343671810431067
80412514399115423605660760788697651634379005*I*a^2*e^(729/2*I*c) + 10328933
901086234516626299219020598255201504563411489548192412459868271727460451538
10766991498955787562750756646550*I*a^2*e^(727/2*I*c) + 16587926940728714791
018699425447049731689245759430711033095707740862620295424612287533432831126
20767567415651784874*I*a^2*e^(725/2*I*c) + 26370267049400575417176034160990
479015773398053992444804037003207346570273820515407414283994625621606652323
99986075*I*a^2*e^(723/2*I*c) + 41499948103659005228074682132781930916561417
31116367453829137630709191482537488782374639033936237089896768763566876*I*a
^2*e^(721/2*I*c) + 64657096563000678127862675144471066879025814333109191780
38542426426780363215033938034518725151681639996365849163135*I*a^2*e^(719/2*
I*c) + 99734557816683321004826191207790447961988845705188503626756204008003
80525655955470978275300874201998244732697737400*I*a^2*e^(717/2*I*c) + 15232
155197244469983477235120489934255399594332170569196782170361245875795753404
131530065929326205229838601927955519*I*a^2*e^(715/2*I*c) + 2303503397660786
184876477781362585913866374752713179887700881659407830195032440468698697582
2232657903821541105390532*I*a^2*e^(713/2*I*c) + 344948248565668222808335097
014794029954993650144174070428926233842948378841687922407020055877551695964
14970227728126*I*a^2*e^(711/2*I*c) + 51154349273662615909216231190623364827

611251722641045147869080165842930085677025925218103510278265118135145553584
580*I*a^2*e^(709/2*I*c) + 7512782632111591022008579582351090881458838771875
5712029825386842788704037764840837864988871197826414756226327584715*I*a^2*e
^(707/2*I*c) + 109278560723550930958638148152377305018874412559953150839397
105157171522052107943777228097015723239016040062510748533*I*a^2*e^(705/2*I*
c) + 1574386486831341464742255088228696345104075897898805015891611538045006
57734549235665125080321402913947303819424223977*I*a^2*e^(703/2*I*c) + 22467
587013781696744857429090657201235362457224788962377186655669552753584956209
2439004233144825309318376200907596647*I*a^2*e^(701/2*I*c) + 317611854590359
842624867588299694397092051295753012234334896647966925083208128923978657573
859261411691474063180579773*I*a^2*e^(699/2*I*c) + 4447937565765568052265985
571218812558914206601789540056061908597679079003755266990872810797754019354
79298706556678625*I*a^2*e^(697/2*I*c) + 61711890741549188200889368999237498
602605861459518063693459633141317037978048758571458704778846921411345179551
6499370*I*a^2*e^(695/2*I*c) + 848308102584636362484162825743572316210570041
224952064440187703108373794290161871693531289023220368566861588408740234*I*
a^2*e^(693/2*I*c) + 1155418271261525364764099832556015695656671742148397430
770621919769591262069324891797346089718216295419176001175584602*I*a^2*e^(69
1/2*I*c) + 1559379265746722157601116633770545905680845286466195068047437750
582100850218558934063441346166219284236292591756007869*I*a^2*e^(689/2*I*c)
+ 2085532515622353289775297137684356044555717847895905712020991677926353632
697196135336156922652753946110662598182357060*I*a^2*e^(687/2*I*c) + 2764141
551913183823232240538311434861698053782540087737778740958755195255197468794
265331998300489529793104153836938365*I*a^2*e^(685/2*I*c) + 3630836299606049
901259711668223783516359548315616426099543877893492461453478630388967263756
039510415511889625376389370*I*a^2*e^(683/2*I*c) + 4726945091253225709284701
620253529588027362773911267160094581860855616546226572637107723406479754761
335508976269914579*I*a^2*e^(681/2*I*c) + 6099661241729520049877973018323299
594149962508203841935762640399428877526099801926994421670829050753700797133
864249984*I*a^2*e^(679/2*I*c) + 7801985537042156770644275419004423215569828
427612566994744061595455071803838672709040326328594306655699615422670941360
*I*a^2*e^(677/2*I*c) + 9892383001478135230032539289730880802930949483982413
461465804587893896576239270994398101763117672285400912303258185280*I*a^2*e^
(675/2*I*c) + 1243409271248787660618086383964100720378208694205271972753248
8449954289915481768007300367716998430298025515215263240040*I*a^2*e^(673/2*I
*c) + 154940340739378891189078604664064752867270400483777892795074774249507
14627734498672369684614416130884899948506496976480*I*a^2*e^(671/2*I*c) + 19
141262538036155544818609953329709894938435396042778174361103892567106894147
006088532658059808481944657444464667813928*I*a^2*e^(669/2*I*c) + 2344494275
310972061619099448239626334477546731486566463392537771134367647914380142313
1782286928982641584906730652336320*I*a^2*e^(667/2*I*c) + 284718276522237485
491067080889253342674583103628338435078037069541557296231905630342278097948
21598285838730011875428360*I*a^2*e^(665/2*I*c) + 34283257825099361469629029
646997987016357410393133337876860784218594231344941134240798399418976200089
374372937336277520*I*a^2*e^(663/2*I*c) + 4093172590825989006501067641289805

993500216754144371496842244750672365781436242361602746779603795659424947020
7911160560*I*a^2*e^(661/2*I*c) + 484570844705692062028835940300589468351722
766518437992672347958848344224971557014613764251403472927696222870092534422
00*I*a^2*e^(659/2*I*c) + 56882511269373287870539195822437152004680104321850
229396708926005012693843675803350945196463973920628412256283832232320*I*a^2
*e^(657/2*I*c) + 6621038068981488872115720217815652472530042855794973109603
0018469991922414680985665275947873824643920689185621817195640*I*a^2*e^(655/
2*I*c) + 764182222193923150876837562784057752836506762015726045537927059213
11480435170616914405792951857748812649327303490604640*I*a^2*e^(653/2*I*c) +
87454973338365020493040891275447029237532017105088938657341030070141081586
543085297206983544492390393869271998602113720*I*a^2*e^(651/2*I*c) + 9923775
261516638892682597483629999017754882776595217676164347019255997423976321777
0479232350108190016210097408947946880*I*a^2*e^(649/2*I*c) + 111649386683169
175905340386817436347581012040347378979113869960210757779770685735228549707
336610684327510689931586511440*I*a^2*e^(647/2*I*c) + 1245369201586228804377
233574104924916274847257121455379401686331920475946242178129455790303635027
01898045319958355946560*I*a^2*e^(645/2*I*c) + 13771131907758733659394974770
922079392808252883121784533905783038068722150139469413642285446957455269158
8871302472423100*I*a^2*e^(643/2*I*c) + 150948545513948896128558440622825450
599113182826877373160132263862850745383456730881605090985571977587756201763
874677760*I*a^2*e^(641/2*I*c) + 1639921340429383893892256515873090499834780
095020309950925913433798473645052525505677375063465764353632685119463074327
40*I*a^2*e^(639/2*I*c) + 17655734097234500011005070412147532602526008802387
2962060596666821011054235305398302353921234166369742208455804477035400*I*a^
2*e^(637/2*I*c) + 188336867093755763287399456292591578241527924733164764702
870711365091789132541972304352146740778900143326958742782477380*I*a^2*e^(63
5/2*I*c) + 1990080773230358754188101430126020470325655498190936599284100241
26213956651006647088382356775180926199506332461163896160*I*a^2*e^(633/2*I*c
) + 20824155995852600557755365457953980429555672976681302421485406417079365
6725481586279526287325763568906607290452125497960*I*a^2*e^(631/2*I*c) + 215
710788862031986131314760876619509722697669102804696344437524165882302085220
441314010414559115310505632713041800563240*I*a^2*e^(629/2*I*c) + 2211025783
828626695740553495019335716128549986148679539512300952865082892135739343399
76617529581891276161128355770449340*I*a^2*e^(627/2*I*c) + 22412795796157893
172920975099679649222146391969926682954770169868208546365760842198312918062
2041022833070349554614032580*I*a^2*e^(625/2*I*c) + 224533045357590740043360
207419063480374496445086224306972898952727492000443367189384084387014008320
007336848252442351700*I*a^2*e^(623/2*I*c) + 2221094679604372803660200628702
581439109785619404614233619307198434832969057630278994639779581668861046162
67999904808020*I*a^2*e^(621/2*I*c) + 21670387335018916188925391097041103119
472484764883594965696672868367256398560906116425852512709414712811608491686
3495980*I*a^2*e^(619/2*I*c) + 208226084708306358093649771579465852434363766
772952777959519157479546252544982513897370413187324342062073067119188597020
*I*a^2*e^(617/2*I*c) + 1966554941120943568598374855496882167189718738080325
39175218270546486750286660449996087951876167053363772436503198669200*I*a^2*

$e^{(615/2*I*c)} + 18204534611310199302537561756714747231076635768733444561983$
 $5660822772617872843112295661673067279164398356825168900855240*I*a^2*e^{(613/$
 $2*I*c)} + 164524642949883386780879611739425630216678555120136856424422933930$
 $968874874654856422558330216317466771849726211997355920*I*a^2*e^{(611/2*I*c)}$
 $+ 1442974977508695242719107828519921279939550100685291811848089218773825055$
 $48298335144664374339743032217233080067477001300*I*a^2*e^{(609/2*I*c)} + 12163$
 $986863193385677201729219033200834481661392547996802886757267082345014793858$
 $5266247649652554944275677884270413328320*I*a^2*e^{(607/2*I*c)} + 968937194415$
 $969100448891170124409502910204397701515275771776115088081885005728724770736$
 $29458560345895007381079831424020*I*a^2*e^{(605/2*I*c)} + 70458766374210588223$
 $159026861223445333341753439960632221750998997297048156150655074881354085103$
 $291330810095125378469360*I*a^2*e^{(603/2*I*c)} + 4278207796166927767564086233$
 $037511060181953268636581910982534826508891343958836424642494879231928691698$
 $4200949647432740*I*a^2*e^{(601/2*I*c)} + 143458934813299369501216535936573253$
 $532385740494226238142630129693528467307800395359000848121067477352763101737$
 $64055960*I*a^2*e^{(599/2*I*c)} - 14345893481329936950121653593657325353238574$
 $049422623814263012969352846730780039535900084812106747735276310173764055960$
 $*I*a^2*e^{(597/2*I*c)} - 4278207796166927767564086233037511060181953268636581$
 $9109825348265088913439588364246424948792319286916984200949647432740*I*a^2*e$
 $^{(595/2*I*c)} - 704587663742105882231590268612234453333417534399606322217509$
 $98997297048156150655074881354085103291330810095125378469360*I*a^2*e^{(593/2*$
 $I*c)} - 96893719441596910044889117012440950291020439770151527577177611508808$
 $188500572872477073629458560345895007381079831424020*I*a^2*e^{(591/2*I*c)} - 1$
 $216398686319338567720172921903320083448166139254799680288675726708234501479$
 $38585266247649652554944275677884270413328320*I*a^2*e^{(589/2*I*c)} - 14429749$
 $775086952427191078285199212799395501006852918118480892187738250554829833514$
 $4664374339743032217233080067477001300*I*a^2*e^{(587/2*I*c)} - 164524642949883$
 $386780879611739425630216678555120136856424422933930968874874654856422558330$
 $216317466771849726211997355920*I*a^2*e^{(585/2*I*c)} - 1820453461131019930253$
 $756175671474723107663576873344456198356608227726178728431122956616730672791$
 $64398356825168900855240*I*a^2*e^{(583/2*I*c)} - 19665549411209435685983748554$
 $968821671897187380803253917521827054648675028666044999608795187616705336377$
 $2436503198669200*I*a^2*e^{(581/2*I*c)} - 208226084708306358093649771579465852$
 $434363766772952777959519157479546252544982513897370413187324342062073067119$
 $188597020*I*a^2*e^{(579/2*I*c)} - 2167038733501891618892539109704110311947248$
 $476488359496569667286836725639856090611642585251270941471281160849168634959$
 $80*I*a^2*e^{(577/2*I*c)} - 22210946796043728036602006287025814391097856194046$
 $1423361930719843483296905763027899463977958166886104616267999904808020*I*a^$
 $2*e^{(575/2*I*c)} - 224533045357590740043360207419063480374496445086224306972$
 $898952727492000443367189384084387014008320007336848252442351700*I*a^2*e^{(57$
 $3/2*I*c)} - 2241279579615789317292097509967964922214639196992668295477016986$
 $82085463657608421983129180622041022833070349554614032580*I*a^2*e^{(571/2*I*c$
 $)} - 22110257838286266957405534950193357161285499861486795395123009528650828$
 $9213573934339976617529581891276161128355770449340*I*a^2*e^{(569/2*I*c)} - 215$
 $710788862031986131314760876619509722697669102804696344437524165882302085220$

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26287325763568906607290452125497960*I*a^2*e^(565/2*I*c) - 19900807732303587
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143326958742782477380*I*a^2*e^(561/2*I*c) - 1765573409723450001100507041214
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7432740*I*a^2*e^(557/2*I*c) - 150948545513948896128558440622825450599113182
826877373160132263862850745383456730881605090985571977587756201763874677760
*I*a^2*e^(555/2*I*c) - 1377113190775873365939497477092207939280825288312178
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e^(553/2*I*c) - 12453692015862288043772335741049249162748472571214553794016
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2*I*c) - 111649386683169175905340386817436347581012040347378979113869960210
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- 9923775261516638892682597483629999017754882776595217676164347019255997423
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733383650204930408912754470292375320171050889386573410300701410815865430852
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*a^2*e^(535/2*I*c) - 342832578250993614696290296469979870163574103931333378
76860784218594231344941134240798399418976200089374372937336277520*I*a^2*e^(
533/2*I*c) - 28471827652223748549106708088925334267458310362833843507803706
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c) - 2344494275310972061619099448239626334477546731486566463392537771134367
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 *a^2*e^(517/2*I*c) - 472694509125322570928470162025352958802736277391126716
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 - 276414155191318382323224053831143486169805378254008773777874095875519525
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 99/2*I*c) - 317611854590359842624867588299694397092051295753012234334896647
 966925083208128923978657573859261411691474063180579773*I*a^2*e^(497/2*I*c)
 - 2246758701378169674485742909065720123536245722478896237718665566955275358
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 80*I*a^2*e^(487/2*I*c) - 34494824856566822280833509701479402995499365014417
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 - 152321551972444699834772351204899342553995943321705691967821703612458757
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 63566876*I*a^2*e^(475/2*I*c) - 26370267049400575417176034160990479015773398
 05399244480403700320734657027382051540741428399462562160665232399986075*I*a
 ^2*e^(473/2*I*c) - 16587926940728714791018699425447049731689245759430711033

09570774086262029542461228753343283112620767567415651784874*I*a^2*e^(471/2*I*c) - 10328933901086234516626299219020598255201504563411489548192412459868
27172746045153810766991498955787562750756646550*I*a^2*e^(469/2*I*c) - 63661
967535900801762221600892040772470844010762998784813436718104310678041251439
9115423605660760788697651634379005*I*a^2*e^(467/2*I*c) - 388366130772936514
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7512701806908240383878654745399087836858804711629400762515425955916824*I*a^
2*e^(461/2*I*c) - 828521429668608514316114201175963722564667394487018972044
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) - 48480920660514266867535157245728700672453465365901344447830053298267300
849098214762864212092705639132905793681740*I*a^2*e^(457/2*I*c) - 2807080481
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249530832449365157249212993867786002259511794981311715895242190451385164358
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82581988856745851085574498026708677442826803383401266387813162062594525953*
I*a^2*e^(451/2*I*c) - 51117871550982319663814444139984137697380398406297899
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I*c) - 28358890227167933110932671089134114841174308170409759286396549113682
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1210437760631*I*a^2*e^(443/2*I*c) - 453673232394267043949605922569329980940
549626999382599843505662311686257301966537920360838324667729133482235583*I*
a^2*e^(441/2*I*c) - 2409451272767662273203399400892982326810482391442487893
24968088048819421447003749816077955279624100315116789806*I*a^2*e^(439/2*I*c
) - 12656083358290553412545711828347958825488673406642268132274306745309495
5423272892820673108496357191409969596590*I*a^2*e^(437/2*I*c) - 657450726185
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31/2*I*c) - 861816491878892520390205950670276873300422354456408646106958120
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958293800266062240235373058151016962305996082029052667361576247664690410492
5062205465*I*a^2*e^(425/2*I*c) - 102036246834307522613193547374782135529005
2152417646455220161102238032449230964279115841386608493376970117760*I*a^2*e
^(423/2*I*c) - 489699557155298676398555706690170103288962201587062038457047

783309280898007016462695325519866776349668712200*I*a^2*e^(421/2*I*c) - 2323
244171972350620810225652359599733291297040165125114926844072284475540053624
75531181177257375428158576672*I*a^2*e^(419/2*I*c) - 10894941050602438292142
057547386922851594287249686050562076857305617517569302710404919550753523881
2688471740*I*a^2*e^(417/2*I*c) - 505003625041568594783538404728382579857204
30216874883086230302896996886794997397859935740848410975488426320*I*a^2*e^(
415/2*I*c) - 23135457555667242015109562696724759269442147216687397489819489
071941696946645262869400576401992001928296540*I*a^2*e^(413/2*I*c) - 1047489
711020580736784637506672333662224453652317656117621724583958777912663039148
2195110511043224435073120*I*a^2*e^(411/2*I*c) - 468687387315610187119076797
456159860951344367020175746091423231250548462720987195797610432877164916204
1900*I*a^2*e^(409/2*I*c) - 207229696847272763858236676321480935013103617973
3894803881601867867318900360235090208152423869380817193240*I*a^2*e^(407/2*I
*c) - 905375964480293043221568717617222804352981478608148290290910134804446
811145030581376444150025327895167080*I*a^2*e^(405/2*I*c) - 3908290134584787
137620058685317495505113483637554024253754752198632010424690276717209923242
63291992741380*I*a^2*e^(403/2*I*c) - 16668545417867586730884788164302597173
7125688220303037968682349925679089331312174778117366039026285869440*I*a^2*e
^(401/2*I*c) - 702317428366914862324018214356466902047653701709343642793379
89127944150596825723542271716872404327490260*I*a^2*e^(399/2*I*c) - 29232457
847784347194232755870686399315171809346248346594946497316944533356199777722
113539614776201632080*I*a^2*e^(397/2*I*c) - 1201888522740550852777891824471
3077048854664134058582641410118464532001237995012608936579451133542506420*I
*a^2*e^(395/2*I*c) - 488090481262949257818274970426572335837616686734800392
7103702683844870930164640446326414262466377824000*I*a^2*e^(393/2*I*c) - 195
768847717816482384418453656564779457553292675546736005582093785170459597087
7608419045482885065913720*I*a^2*e^(391/2*I*c) - 775466756780053803300487429
447750888237325797624757919706744628478203937255833604141877857656968392480
*I*a^2*e^(389/2*I*c) - 3033387063507554872790571708825134747433602012003623
27954737042233540329160364416313115387680520970550*I*a^2*e^(387/2*I*c) - 11
716708448312371086396980793877215516995678531752309017486269734414129324623
6717353854714751253177160*I*a^2*e^(385/2*I*c) - 446852359469866737919252255
25992813797485854939304559125279366181549713351572630218142149059140721370*
I*a^2*e^(383/2*I*c) - 16825577675009653410652812996723598120483470175375332
001250964015555125236873581387468224926998037020*I*a^2*e^(381/2*I*c) - 6254
456957710857084990777507880196795826048342877391310115506167680233024354089
893072681406475246730*I*a^2*e^(379/2*I*c) - 2295030419017144015110797732155
375346477339079532575006580675869478858321228765847919467823502616280*I*a^2
*e^(377/2*I*c) - 8312493331242709297495651233702193571656545687426200399034
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3932187264900*I*a^2*e^(373/2*I*c) - 104834609086572625907289346225890252429
283565041460323851402795840051940483234866884215706838405390*I*a^2*e^(371/2
*I*c) - 3649732635331596360542029819068746500105510075807430423479227216188
6339957870164613698417616283210*I*a^2*e^(369/2*I*c) - 125375628367325991797

670103551524610568071919510050097964518233910790006557621890677411923053940
90*I*a²*e^{-(367/2*I*c)} - 42493429907386087010781091871687683676933412313560
30118163718909774173116310218460585993016680850*I*a²*e^{-(365/2*I*c)} - 14208
461966435788037335753309269709744646757040204816180274460779823807310979411
67611657029967430*I*a²*e^{-(363/2*I*c)} - 46864943387694037471641533454085202
2756419671685290819126536222202383329230675172080843145521190*I*a²*e<sup>-(361/
2*I*c)</sup> - 152469533247657613494014223169079603947607129031097239090335734676
330177814841868201155345866600*I*a²*e^{-(359/2*I*c)} - 4892256658132882799624
3369288346270469307859644664802730085010597182346996542663202606651415940*I
*a²*e^{-(357/2*I*c)} - 154804233409958815847404160233328775455880243749956601
59497444149878426588554491651581857867240*I*a²*e^{-(355/2*I*c)} - 48301330647
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064562580373888296088587103056614395310898823853000505361579579997666420571
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c)</sup> - 3969696733916467448469860592541488983654899708624040875073879385603020
9166162894182626765170*I*a²*e^{-(345/2*I*c)} - 115275447102141272919593209662
84738345019810955968385265683600912288138678711188667077969580*I*a²*e<sup>-(343
/2*I*c)</sup> - 32985775034613724656053455368691232614563926054549808402060457856
61659654642478147158578260*I*a²*e^{-(341/2*I*c)} - 92998070363360095232162569
2946644952639044210651468835121797966088502026440103109950142510*I*a²*e<sup>-(3
39/2*I*c)</sup> - 258299861869070319215539511486716381312735629658913082894934911
762706535490482149535095400*I*a²*e^{-(337/2*I*c)} - 7066780919249683896553228
1733119872286671303574373760629554934333798898169280995448949110*I*a²*e<sup>-(3
35/2*I*c)</sup> - 190418471183197262932658032739512079455937995882350341780266565
50799568684461605603777920*I*a²*e^{-(333/2*I*c)} - 50527496930900742500207116
33864893162784543894410325996308296849332368152124619583542550*I*a²*e<sup>-(331
/2*I*c)</sup> - 13201323425995747765824874916863760110207311283385646449721363778
85389653577953327654360*I*a²*e^{-(329/2*I*c)} - 33955976871065259701328288881
7312950668437949251721078395194275059325692430095887234940*I*a²*e<sup>-(327/2*I
*c)</sup> - 859726816913913549797131970913343700552010003729371526331729476018163
13260199706979160*I*a²*e^{-(325/2*I*c)} - 21423226375734147493294111979827037
040590614426574817511241501723037567325263745727850*I*a²*e^{-(323/2*I*c)} - 5
253180218901190370523307961292015004989360145697116713826069492059896349225
432472890*I*a²*e^{-(321/2*I*c)} - 1267370875613816483172376715134611915155023
532375947527368816590522121089591841195870*I*a²*e^{-(319/2*I*c)} - 3007863800
45287143628954499629136562193246097877509863055826252064326569743630828470*
I*a²*e^{-(317/2*I*c)} - 70212206304801678332513345599910949246264802924017064
833024979917894264890884744230*I*a²*e^{-(315/2*I*c)} - 1611728744799820614279
8083894763175840848857114343543047242138953088137509988748530*I*a²*e<sup>-(313/
2*I*c)</sup> - 363763252490000796124964602502497070783755961261959744273749332209
5743154754032540*I*a²*e^{-(311/2*I*c)} - 807074460549539210003558596515532129
897902975140321070941315505466030633289504700*I*a²*e^{-(309/2*I*c)} - 1759925

41401464872591758301499834898740699910060781393774843202513062630336753720*
 $I*a^2*e^{(307/2*I*c)}$ - 37711714010161632851275194873586231922967566451955409
 746843374416439997598272230*I*a^2*e^{(305/2*I*c)} - 7939122392096489819576023
 365780909095910972043426214771545781246832910820414100*I*a^2*e^{(303/2*I*c)}
 - 1641697296378143299128733640940998805441384198489236819275463423360050416
 710230*I*a^2*e^{(301/2*I*c)} - 3333844415521753485758638516852800793975154081
 27201392843010509098144064832520*I*a^2*e^{(299/2*I*c)} - 66471205999186848117
 890072662193021327677387770934414812726997704844663612890*I*a^2*e^{(297/2*I*
 c)} - 1300945152021805442798772690930638802672821843009519715162619020452919
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 7188475889323331541072591988520*I*a^2*e^{(293/2*I*c)} - 470879572899329452986
 316253534868514992585355812719048085012389615234670720*I*a^2*e^{(291/2*I*c)}
 - 8704038273624124504811313552891743577253139986151704861170617699772884006
 0*I*a^2*e^{(289/2*I*c)} - 157775513349298847415589802311195134453776467818398
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 92586220272029806438905494766819989517225180*I*a^2*e^{(285/2*I*c)} - 48834836
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 83/2*I*c)} - 833402769621003639645666404958645163938548030172361844704429893
 94035180*I*a^2*e^{(281/2*I*c)} - 13931417792509198161468797423752544838678599
 539275212332380442651357560*I*a^2*e^{(279/2*I*c)} - 2280427120078137719325513
 847874265148019541854031486848558248442727176*I*a^2*e^{(277/2*I*c)} - 3654077
 93636196973379878926946921993102971243732578054647123414214820*I*a^2*e^{(275
 /2*I*c)} - 57297408391078324425844220711696016060458004240084890338934689925
 280*I*a^2*e^{(273/2*I*c)} - 8788947221764033524446888793703125344784094679194
 612662969732267700*I*a^2*e^{(271/2*I*c)} - 1318337340904646289197961886292345
 009401456618543410019867356069360*I*a^2*e^{(269/2*I*c)} - 1933039822447819196
 91500896347164102540863253211817387366117379028*I*a^2*e^{(267/2*I*c)} - 27695
 522491987336122297854656279067704143932579317971697894652128*I*a^2*e^{(265/2
 *I*c)} - 3875748752953757251392321804098110389738719224033802668294149720*I*
 $a^2*e^{(263/2*I*c)}$ - 5295343038043143713172792870315873993428983190702577445
 55877120*I*a^2*e^{(261/2*I*c)} - 70604441498160359313241260895971702149956086
 380303936468404945*I*a^2*e^{(259/2*I*c)} - 9182643580975122394482019949557981
 111651613557125350856577098*I*a^2*e^{(257/2*I*c)} - 1164368139316323606713195
 496495533903452490627065569725880879*I*a^2*e^{(255/2*I*c)} - 1438728978699584
 76711841808368973418609254139596808263965208*I*a^2*e^{(253/2*I*c)} - 17314198
 559836475415651393365133642453745100461062396834495*I*a^2*e^{(251/2*I*c)} - 2
 028232835526354300983369892820970081423531403040583023770*I*a^2*e^{(249/2*I*
 c)} - 231137463934351535240227752765265592838813655877791732766*I*a^2*e^{(247
 /2*I*c)} - 25608963224785291572287952038770777479118266594544417374*I*a^2*e^{
 (245/2*I*c)} - 2756770672774248567550184031805577107703843954010093519*I*a^2
 *e^{(243/2*I*c)} - 288136908346804733688436834384090051687033252355064191*I*a
 ^2*e^{(241/2*I*c)} - 29219505531617139745995198456026235957224788107224865*I*
 $a^2*e^{(239/2*I*c)}$ - 2872703209254401680718347285883520097429919909434035*I*
 $a^2*e^{(237/2*I*c)}$ - 273590703261713668448365859396559199738283337615067*I*a
 ^2*e^{(235/2*I*c)} - 25219248742824948157547271484589255811816930894969*I*a^2

$e^{(233/2*I*c)} - 2247954875096342235081391843579770658847258731308*I*a^2*e^{(231/2*I*c)} - 193573860526473423179001809134869735140502053410*I*a^2*e^{(229/2*I*c)} - 16086468264323655413475747304585608493929925740*I*a^2*e^{(227/2*I*c)} - 1288694832301854187496925699385880912905767417*I*a^2*e^{(225/2*I*c)} - 99403449489614415589324261326783015782654200*I*a^2*e^{(223/2*I*c)} - 7373332278938477877109516190844163180831833*I*a^2*e^{(221/2*I*c)} - 525223640233639829417248502076803689088820*I*a^2*e^{(219/2*I*c)} - 35875930157695641006598626271371876133245*I*a^2*e^{(217/2*I*c)} - 2346109795219082040306865029327652810422*I*a^2*e^{(215/2*I*c)} - 146631858416411858207561193721288320906*I*a^2*e^{(213/2*I*c)} - 8742278650463030069301003637232429883*I*a^2*e^{(211/2*I*c)} - 496183375602453279107611685390767740*I*a^2*e^{(209/2*I*c)} - 26748429663467970647815785552181215*I*a^2*e^{(207/2*I*c)} - 1366183224856297865521217009020824*I*a^2*e^{(205/2*I*c)} - 65928412631647739461683457820511*I*a^2*e^{(203/2*I*c)} - 2996746017244157830350053701476*I*a^2*e^{(201/2*I*c)} - 127861163072229101394013407134*I*a^2*e^{(199/2*I*c)} - 5100844262877818375740499940*I*a^2*e^{(197/2*I*c)} - 189421272159473056494074595*I*a^2*e^{(195/2*I*c)} - 6514488191762735815405157*I*a^2*e^{(193/2*I*c)} - 206263478291032388361681*I*a^2*e^{(191/2*I*c)} - 5970784896718122444327*I*a^2*e^{(189/2*I*c)} - 156713514334349191205*I*a^2*e^{(187/2*I*c)} - 3692203217145049425*I*a^2*e^{(185/2*I*c)} - 77121738215926170*I*a^2*e^{(183/2*I*c)} - 1405865019555002*I*a^2*e^{(181/2*I*c)} - 21909584720322*I*a^2*e^{(179/2*I*c)} - 283802910885*I*a^2*e^{(177/2*I*c)} - 2933363420*I*a^2*e^{(175/2*I*c)} - 22680645*I*a^2*e^{(173/2*I*c)} - 116610*I*a^2*e^{(171/2*I*c)} - 299*I*a^2*e^{(169/2*I*c)})/(e^{(517*I*c)} + 418*e^{(516*I*c)} + 87153*e^{(515*I*c)} + 12085216*e^{(514*I*c)} + 1253841160*e^{(513*I*c)} + 103818048048*e^{(512*I*c)} + 7146142307307*e^{(511*I*c)} + 420601518659718*e^{(510*I*c)} + 21608403021340047*e^{(509*I*c)} + 984382804329835768*e^{(508*I*c)} + 40261256699368950388*e^{(507*I*c)} + 1493326612293984160368*e^{(506*I*c)} + 50648660944512569972179*e^{(505*I*c)} + 1581796642397812408161814*e^{(504*I*c)} + 45759117183402579073139583*e^{(503*I*c)} + 1232445557346832245176696904*e^{(502*I*c)} + 31042222522074681615625020522*e^{(501*I*c)} + 734057263616388449968842366924*e^{(500*I*c)} + 16353164647151530240529137618111*e^{(499*I*c)} + 344277152012875134140739302960914*e^{(498*I*c)} + 6868329225263681349501997341320517*e^{(497*I*c)} + 130171193079172823835151430773360024*e^{(496*I*c)} + 2348998374244347079532766203075607598*e^{(495*I*c)} + 40443624781415311581857832389099634564*e^{(494*I*c)} + 665634670676210063754191847109971141414*e^{(493*I*c)} + 10490402669510897424624643766470754045064*e^{(492*I*c)} + 158566476113257562566117432227203884298856*e^{(491*I*c)} + 2302150411226234925855222345201500900533576*e^{(490*I*c)} + 32147887693375338817454482515377350383950278*e^{(489*I*c)} + 432333688644261557547944179250800440604964868*e^{(488*I*c)} + 5605927253067558551780452883689835514455118670*e^{(487*I*c)} + 70164515322544462906873548813748091084561870680*e^{(486*I*c)} + 848552202276512356496200136959676295361696315113*e^{(485*I*c)} + 9925490738534402272939987038714580495445431374618*e^{(484*I*c)} + 112391604542246650966429162063124338952554575234051*e^{(483*I*c)} + 1233096700139723365181997220750932590655287625342156*e^{(482*I*c)} + 13118781801172174729679339894318153694964675368481194*e^{(481*I*c)} + 1354425949166361161915746506$

25331646238501101627937224*e^(480*I*c) + 1357990663161479842850642848032544
 982878359839580349899*e^(479*I*c) + 132317088701048969738000567337799190893
 40836756009580718*e^(478*I*c) + 1253704965869212726621980508512693231711673
 38854081782959*e^(477*I*c) + 1155855412893594260345544966642687823630035899
 363232371472*e^(476*I*c) + 103751844998711755019093989565966841168029970825
 26660323524*e^(475*I*c) + 9072260572220881491864228463948718776460758970649
 3970774776*e^(474*I*c) + 77320463699114577506146273102809850609443267578813
 6295011259*e^(473*I*c) + 64261954855352485764250681368704655300871140038757
 16691383902*e^(472*I*c) + 5210811762917704866049240098517583098750570056687
 7818954141639*e^(471*I*c) + 41243069829991519084806722232721943506774793409
 1894670488982928*e^(470*I*c) + 31877499297443464972115360447517765823209586
 27923816470590659024*e^(469*I*c) + 2407080191352975710185802291437204586474
 6991786182039740274325264*e^(468*I*c) + 17764282913511934857719443767580283
 0239905460092687136494961404333*e^(467*I*c) + 12818174649149708108596041898
 28359000790789921169405304612211251818*e^(466*I*c) + 9046693523825682979044
 338963104263167672586826367911338826483549173*e^(465*I*c) + 624735507810532
 95317710774690247114124125187565731848441781904032672*e^(464*I*c) + 4222761
 26632003687547754746555709988710527133086660161366353656787288*e^(463*I*c)
 + 2794709104475686611842790694973699164482254723977210209725661304403472*e^
 (462*I*c) + 181157684956157580767103030555056255892542936591933141534183339
 44596408*e^(461*I*c) + 1150514818520808488737003883545213155676403651240031
 03691176697194292320*e^(460*I*c) + 7160994975990580798956333385529402291928
 58196481597830078819711862600096*e^(459*I*c) + 4369442482910113914565353136
 069595862669338858053419381214131241925047008*e^(458*I*c) + 261439762799020
 21443471945665080254563056810183520401889800285493144867448*e^(457*I*c) + 1
 53436088745056254127327239461577071933130157764595997113973513183188399376*
 e^(456*I*c) + 8835009688217912026007745419277692007376893935137347893683970
 93333311961880*e^(455*I*c) + 4992519712457043983505377976607953988397368297
 591114957991804893688371867680*e^(454*I*c) + 276931165383432592259833826376
 47936122664033859615133489846664694361471028310*e^(453*I*c) + 1508223814314
 12413773566474210011746852297437597059186295243989481140398152780*e^(452*I*
 c) + 8066795436075891407593050107961895682698420216133889552189162788231826
 39488190*e^(451*I*c) + 4238125846763232586394188569858685826755328005548627
 437019301405851325887594480*e^(450*I*c) + 218764828927139099280403456125787
 05805121508756226696317087651824252241418663320*e^(449*I*c) + 1109691996873
 20974749922259595250444341219218535349655762591192576535872151766080*e^(448
 *I*c) + 5532691288195286125029188695589478290980219563093498435840446315122
 91778800081490*e^(447*I*c) + 2711843239670717527605640490148833507130242448
 403978318523237721944200392830108580*e^(446*I*c) + 130698172034882898861932
 05508375818392124991382340160316886507181296548981014818410*e^(445*I*c) + 6
 194859665303550287956433881523431066041090203788247316180477449291621657588
 0077680*e^(444*I*c) + 28882075526473065446996857202104710942731861950899580
 2020689904590319476295408324280*e^(443*I*c) + 13247564123678374731574728211
 62483691120966501948953926492241643788264284546437221120*e^(442*I*c) + 5978
 992172944143218459161149299819706321732111578494525245228742976468409105395

536290*e^(441*I*c) + 265568063890434075344967023691015457959948617577414147
89944652712127566910185274123140*e^(440*I*c) + 1161045516835550437629115017
12116399313733021132677481112824047246361794049635726479850*e^(439*I*c) + 4
997075672538590843575963148137947680693371909159674919074889049339226775796
65354338960*e^(438*I*c) + 2117589733466855707101501429210414722401838837940
752841618541440888545729943138209036820*e^(437*I*c) + 883672064086047030569
4514021547969551296794092266983044118375790025854584036796364768280*e^(436*
I*c) + 36318369652302591732197444409798122022640824604130552506742586795183
267354382847875885730*e^(435*I*c) + 147030816732276833163041582099592047512
04372522535339238819165193000407629544745753221740*e^(434*I*c) + 586403466
972683242741643328921560909375197453864243299571990964608857245771134145204
174990*e^(433*I*c) + 230435107337384035737917859767306635201668278168913984
2097376663118488803841131935313641840*e^(432*I*c) + 89232094473432967633318
81881638471793499618670601026059730895962653291770229493028162575100*e^(431
*I*c) + 3405405385129556915435234672217717265518754891078200850471832416872
5029438589162349211628040*e^(430*I*c) + 12809891460168853967248054183040984
7707367500438601536803204497701119911289087105659482783340*e^(429*I*c) + 47
501057885760151927231661793842522242178659724167102689431851540851146714096
9393115768793680*e^(428*I*c) + 17365742188181910718741974724501581238835642
09950658639102337148122769080611680719741726053840*e^(427*I*c) + 6259872156
822252843650960708235034710201362776057176647226323089751446565288850103898
153859920*e^(426*I*c) + 222519591767957777571673660360074802222113642321463
99803864370963391491223687245823457351580140*e^(425*I*c) + 7800980736802423
987561373305885141712532711468107088964079424928263347058075655708392320337
7160*e^(424*I*c) + 26974580144021129697268360186387895435796230852007659517
7128227629273240215209708218497363414140*e^(423*I*c) + 92008939302958903287
460185002715932261252636844477148978197436107884752889146883103843606495192
0*e^(422*I*c) + 30961319716215201623803015542414654517823620868102875377489
02904985934020179565706177131421614590*e^(421*I*c) + 1027936473066384084473
957786246926260464886191429797258916524353065123069072624446247919989425518
0*e^(420*I*c) + 33675398872021568375902384593982753362559801058104184627345
411136262431943240778260721756991027090*e^(419*I*c) + 108867995731829472826
732905192034886797284621356445627530909104429486741257822633476898356826454
040*e^(418*I*c) + 347351473214713780874352083129566601238765762775942366762
733349952103889753982636403857556867777300*e^(417*I*c) + 109385321448622035
867403243450086667849977001130587417248897595161203145673460828709551950104
1975440*e^(416*I*c) + 34002325606016516175216946808470898441980288316944174
24794868779328950548418125605446882081152636090*e^(415*I*c) + 1043411751657
039596665369315558240210946034809547302780741232142734681692856719777037649
6170251803940*e^(414*I*c) + 31610939331284692750694306443618414656095969520
945215743004044560386895241801579156543451940713351730*e^(413*I*c) + 945561
802589319869193343034663656528268580913143291891607362771758738417321964533
79953705679466826880*e^(412*I*c) + 2792857558000352066798353688981654776448
64987794665387827488933863633745047373109049265172681702585720*e^(411*I*c)
+ 8146081877365305796702100252719214155971833698812142998232919697855498761

75969866367976653244974728560*e^(410*I*c) + 2346518219239105142238141633073
464768899155708935025778047637412681781575765422219127409260159438712250*e^
(409*I*c) + 667586629037114735850376686566928901089354386983053870872494529
1580951179188296606158111257706968604740*e^(408*I*c) + 18759988218865563564
163635735986073278255737257405706279108891366378428467414559930481172863538
598193890*e^(407*I*c) + 520751785187932703864292633515443069511049935425005
82938155241689408138675254608030847907167748571734720*e^(406*I*c) + 1428017
924502217624831808749188252741343051332754177800847950346447635093335031505
17345864659667189417080*e^(405*I*c) + 3868762182342771656324517230499798892
63115282374607541692443176673997513742813591736171169652250611186480*e^(404
*I*c) + 1035561982592002935226384577908611548612111495080193573691339864706
029186482466241805664949381049856258510*e^(403*I*c) + 273889562479526560335
522764656600088628077830508482570291193890365616200426273618265770040630191
4070062380*e^(402*I*c) + 71581246868429414754738073636798397181727455815384
09044503383852693596921622426696740453944718143025248390*e^(401*I*c) + 1848
740529900573269375272861187649089085835702197488237157062380018624513772266
0943641752976852924439870880*e^(400*I*c) + 47188220843466207695099506953573
780357108897491422567898048199018207708997005333860148836479527456156014520
*e^(399*I*c) + 119041855403877964948229577948370465600606623183045529526900
430209270473212773847794935586074714329479939280*e^(398*I*c) + 296825515282
669589685318273280239050084555032203415941511962659596881615713799937680026
497408305672297618840*e^(397*I*c) + 731584972206818362874729621403974444280
010446301161527339760544815300951787985538419764656214582667219914080*e^(39
6*I*c) + 178244611493175185055635485663842190117441232229824949659165805393
9787198246565945975595575734193348887952160*e^(395*I*c) + 42932064780080221
260174889088518264947906207206601514514681819109172400278639687245391276596
33517053002976480*e^(394*I*c) + 1022318202595486076721739030518645192356214
5473674293619918063490411487496121804590274592702770571515456414680*e^(393*
I*c) + 24068785139705277161193465644506143285241361037768216818922184400141
048460210944696647752723371932874594597328*e^(392*I*c) + 560286834249035176
584950138585345161671625910343679724981746609074506667781543532716303446507
77885683547624184*e^(391*I*c) + 1289670800847547122460236808664883849832862
59025533132044636109049545144029547003347761521666283977931640178464*e^(390
*I*c) + 2935507435543427098081294535765623132997059826991874168629343739642
55615967138676253276302591561523515603264403*e^(389*I*c) + 6607644731058690
976914759738508379345110890331495867079827642633947667566495652798791461733
18386505740391093990*e^(388*I*c) + 1470931146618934345515038362300100160482
127749581443929904746910224777470198899052379114493999887003199419829579*e^
(387*I*c) + 323849193136185147642332193353957909837773553920764146734623566
5823887048326949305609231585143748690203615957136*e^(386*I*c) + 70521324141
621979926023265245801430609853530545729339055246331216810210373402983663422
03324325307072413739061024*e^(385*I*c) + 1518963421490880039641791172264375
474804852010973481245910987881049384438106265081897119963712145874945624327
4416*e^(384*I*c) + 32362731322419549410330088943640247460378328561316422931
292427145902887913071643679502909055891236755143207382609*e^(383*I*c) + 682

080330967936156837844096192442108186149916400415534244055278768932724966083
24231098148502466453967157728078994*e^(382*I*c) + 1422131159648145176823866
672767699094822716813187908898405010394417486355453624676798324491035203219
53011780083069*e^(381*I*c) + 2933449200343007202870423834483428663138062854
55040067823080445597545970023446231563554135133105493516316320059272*e^(380
*I*c) + 5986501411122418589116765051805201503640032268413280814535970935877
90338609212439085554466861582623350303061961052*e^(379*I*c) + 1208770358493
65839308944222056935063283704108140593750226539846117737648216609559734831
601248698274330296158612144*e^(378*I*c) + 241496651681033850328907654920274
051171005901179544713877346420569645502644271242640959966277108026482600898
5061097*e^(377*I*c) + 47741411110660989702218453305949620164727142303742340
60663956846950926642685946929064114194400360936223590725470146*e^(376*I*c)
+ 9339341958053494225251750965715057300707302083814774770306218224241022648
247419956042957363055823830898547303219757*e^(375*I*c) + 180798200680288599
703499386230072306765633142067088484999001396412373347632664793469632379360
39328113185041591793848*e^(374*I*c) + 3463765717267169016765734453719708704
888235485399327047206394307877360044654296354834810126939044346448075451392
8502*e^(373*I*c) + 65674859268867300098827375812875225610654551686261103681
664007007537115778097293533565243828873383722980353200611956*e^(372*I*c) +
123243941519332384741960072588103506596406339253616391082062969960682419011
745775738921817753391954462609323881489157*e^(371*I*c) + 228911311738592780
091492649162346834405867740776456326108410928857257174707289268074347550225
793244741923354395308214*e^(370*I*c) + 420846342608949387277559021457924586
578120966148561022647008499529468452005980175119410628956210497609566002969
884927*e^(369*I*c) + 765867795513962781012558444628751418710940895281304790
836743661582071650032154891482866406314834433199455459798934952*e^(368*I*c)
+ 137967652979621207401710618806658944835544650121089019510716486035022892
8586815539003062875026711931941947738690360722*e^(367*I*c) + 24604423758454
226639270816309832607147349680919054930271456392388271922548863493611269914
57692409851120873307487457468*e^(366*I*c) + 4343909696601932173357359687781
579293701295681940827114215433175336093967845908766740738240037114570667410
936998017178*e^(365*I*c) + 759275270014667896115309507358501547319702974653
3633331549793961473285760935801904155116764831560875947581048693527224*e^(3
64*I*c) + 13139771494104933881856681151418293112242551521535686871181266579
813877606348160261747201317735782566021306798298336024*e^(363*I*c) + 225146
757413080699615061655865028724304219302106732643929972864856006401038672536
04847715547060592967690653795951142520*e^(362*I*c) + 3819901586758608797600
299875662767499479544066790362502932234625013328648912087500501363812811389
3960349670280707161530*e^(361*I*c) + 64175100693260066806238064886004597170
740843300086839368616139164529108049844675353111842725798658088840347241496
099644*e^(360*I*c) + 106764832017165594838085234189333528733587673329972530
092661085186789939252915937090760282232346919090426243399409323314*e^(359*I
*c) + 175896258262755985757106812613979301265801031595484353614904672865169
442232075776580447184134141375995770091499246759528*e^(358*I*c) + 286992943
631231496557278010851576940896826497466066327528801560677007112837431926735

088120974861760511367008815728782643*e^(357*I*c) + 463758288457367154544937
 678255005688733328145568049310423995599886012800638619904022368378591108842
 602342094543682299102*e^(356*I*c) + 742228640908173124916937049462525617334
 148919679118270489831005497781951221069955839623452499748653124658873553401
 442137*e^(355*I*c) + 117660072097578696518987505089023109220461269697027743
 3014535895788956771230793520381993106606880564628599822341722801012*e^(354*
 I*c) + 18475058564624515334452843005713263237811625533045659718876707580910
 79306794821834928170773126364639722071570131703785334*e^(353*I*c) + 2873610
 535922340187080835435582912277271967977394720159791070274927714276869531467
 182688981041061381703885403497544001592*e^(352*I*c) + 442767307910542531852
 431611298569365658485193610019245704445513448330504532145251634711848813322
 4823670465103483954805161*e^(351*I*c) + 67584804378885243725629359489638576
 266948555471955194861228775679817185872623628719949670794018319579279016825
 82941234362*e^(350*I*c) + 1022042377943463485133997529516339964170212224966
 3666193053008302026096932158568338309418237395541351819026907953220681013*e
 ^ (349*I*c) + 15312837206662775379347353212807682965712535652942631518286142
 403097738200270711195396582159028513532779682154451996208592*e^(348*I*c) +
 227316035661288411004195019470513676668366524180772609139448107484730848918
 90410181285412604854876625919565639521227223276*e^(347*I*c) + 3343589782793
 658130117117545961082945429816796201741981007293673337850658442802420107245
 3193458155334046693516742390717832*e^(346*I*c) + 48733253505974923400852255
 563052101402196469313659554492725674754339375283010407167744366955828922837
 488705858532439654489*e^(345*I*c) + 703863497605948315670482240613950256985
 012022969663003767642203366977029615910998540554113762948714374681495285247
 96002762*e^(344*I*c) + 1007449618518537446117543009829801669624045538362229
 21868484694269966120607698907046343731011160948828100276729370132819357*e^(
 343*I*c) + 1429063191230555242465469284789542383713159258020223892364986521
 36839822502035155676970917419039834587967055588431566416784*e^(342*I*c) + 2
 009065871535788043803004695014416101745218512595419292098406889608594549085
 19774835905895757666770857888611738751858460424*e^(341*I*c) + 2799452444750
 398048229667304629608844921198748577911471240090794769204359417352933093054
 30438687333129912454196774070107264*e^(340*I*c) + 3866426730503800494573825
 628183169626519755509907792770487402386298587950182473561628886310156876647
 80101205287333082748791*e^(339*I*c) + 5293292527641139260039348369582435576
 725492389975607392144065991850478319555725837653586343954087715280097454675
 48382950094*e^(338*I*c) + 7183615963820582492091135444879010888683887440337
 13210332491971375906738341551540457264804304039664255915607349801911966551*
 e^(337*I*c) + 9664582753690377187477391307981516434835906841668322346880982
 91164160636418159452119815728809372125168836239364442397344064*e^(336*I*c)
 + 1289043515292933956480634330499677040181043935620106914267311067900030058
 398839787692376954090545278554544997710058754772400*e^(335*I*c) + 170458299
 670782280820467821816769300269866114771277235502145654381093006963718808588
 2824757500605246963210810351706405349408*e^(334*I*c) + 22348912763984394644
 786225783064348407246104844681778598226206586919214786452666530625638235530
 01228001009093606751066168944*e^(333*I*c) + 2905385722320057001953345274489

482790856692529959823749532695963414164833366773128218607899328588608916176
593772088622582464*e^(332*I*c) + 374525759487665120465733498842622638814395
450198683066422234922636107960954682227606750489938670308898230818571714340
7211328*e^(331*I*c) + 47875274427809456851452048469715961653041694193282440
73211459592129649255048876854059844720661078151288179612574986359194560*e^(
330*I*c) + 6068949803156712248331871105329895471722806143008878014986559653
687260694816550470195890004511965527567432722969707577202160*e^(329*I*c) +
762973181562782158046899242420700836643889673633302466186383810511044514894
6962328297631547032543419811821015837863013682720*e^(328*I*c) + 95130322740
195229542091131912682266422999120135256659402983810647978856909049931289480
35227412144035633851779511219335277360*e^(327*I*c) + 1176421227487648408001
090071467347449337127816055781198372445582656605561765808647936864186490811
9643412413644803772131657280*e^(326*I*c) + 14429816285208431204532978375375
691965063154224649747551295851507389524083226976789688601369628399900747658
579201929300744260*e^(325*I*c) + 175562732712242923968872914031257162134914
862611454785713767516901056560678380421510382713813003727577556763254080268
34544840*e^(324*I*c) + 2118832140588288753961019837470686269589404922607709
3764132512513336190523978949694387686059124526755048042957954264706637460*e
^(323*I*c) + 25367176439119353621532260335983348154904982606125761711300683
492963390816491583025705268737539982149639300226512657426118880*e^(322*I*c)
+ 301284824145527032645590189530881771560134374934382010784137698354483661
48121754549197591129967170764969700180348699207838960*e^(321*I*c) + 3550010
310601964987627237679694948220958137237103600501287780602748167280705994344
5240136315568500732379966585005678181937920*e^(320*I*c) + 41499832121963708
043788523787401345541780088930538206918853579026749273364671640037563488607
716092887686471542838602788559660*e^(319*I*c) + 481331176781840292165037485
491103744789247190946356038928293648639165537922788229573682851063281647159
10598370871149079494360*e^(318*I*c) + 5539091304497208621943268914633156608
142795989696990021443429681773115086386705662076860818767970972015297414847
4907904177340*e^(317*I*c) + 63247774101012179051794946075175569924076981338
138483158042406747453874729387631710544995247152912205118500597511052824347
680*e^(316*I*c) + 716603298611733955244419438892841091340911578446552456720
84237402434944696464927131812190659629511140639501743303863582092880*e^(315
*I*c) + 8056624913068268418187620188262351120636379033721801195411021064292
7765997644903820595421936873565314654415769070472655401600*e^(314*I*c) + 89
883815801382382213973270477954602744792877018051963347146307372464315121274
929402347942874802899499538953561056667668891020*e^(313*I*c) + 995122064720
579659513403417380235485153364033717178980408504709546575329772791134915068
80290726111154101941386019689567958040*e^(312*I*c) + 1093325373499662232039
326785034263570798637070017282940110420765304039238626540189786765164173142
21089449922495612732870169660*e^(311*I*c) + 1192097137020339270557553978236
884444446474243245021853286263470465996347211465738306815404953335431467768
10911910410468628960*e^(310*I*c) + 1289950760115919034107638634270973299485
861735745958627058491592809430464587426631634540184914638553956494539522128
99632198680*e^(309*I*c) + 1385297945491510894513527695765434031263307472436

800308324672058958190435681552392648767628671727543386840278498553854532160
 80*e^(308*I*c) + 1476489208055453334186231217678537773997829247483012287939
 24342574999937955421765370101235122939557467548549202174550009604780*e^(307
 *I*c) + 1561859629535511961697382188321736965098525515892107305783657274762
 59476474465955428502336673743686499175698677875693611243400*e^(306*I*c) + 1
 639778160596077253752645598165058478941877851014553603918974244829984153857
 87605765315509208337741590143078572243505132706580*e^(305*I*c) + 1708698488
 689531011768603060531039943405303903472600884326768425055551412938308389612
 75974268928666494845723462544709102843680*e^(304*I*c) + 1767209299705546420
 045757700530957005953346598706827320319759155323875770524148663235111401176
 80492929354517559479899220940360*e^(303*I*c) + 1814081687709220598203685533
 166973216399848626282988285695602732956308976268293452635922190345608535307
 33710529842148537901680*e^(302*I*c) + 1848311519837489418176678501747082571
 381281721582694132877653585322407732443361919008185578299058956844948894104
 51921524212840*e^(301*I*c) + 1869154744365675149263514056231175032619875083
 519300838245664444356891392336834117046418287621787991778480642201508183552
 61280*e^(300*I*c) + 1876153931685100500714972805646035109124031329203120243
 70835062679037644990286285346673507093452964351257962696133511725652320*e^(
 299*I*c) + 1869154744365675149263514056231175032619875083519300838245664444
 35689139233683411704641828762178799177848064220150818355261280*e^(298*I*c)
 + 1848311519837489418176678501747082571381281721582694132877653585322407732
 44336191900818557829905895684494889410451921524212840*e^(297*I*c) + 1814081
 687709220598203685533166973216399848626282988285695602732956308976268293452
 63592219034560853530733710529842148537901680*e^(296*I*c) + 1767209299705546
 420045757700530957005953346598706827320319759155323875770524148663235111401
 17680492929354517559479899220940360*e^(295*I*c) + 1708698488689531011768603
 060531039943405303903472600884326768425055551412938308389612759742689286664
 94845723462544709102843680*e^(294*I*c) + 1639778160596077253752645598165058
 478941877851014553603918974244829984153857876057653155092083377415901430785
 72243505132706580*e^(293*I*c) + 1561859629535511961697382188321736965098525
 515892107305783657274762594764744659554285023366737436864991756986778756936
 11243400*e^(292*I*c) + 1476489208055453334186231217678537773997829247483012
 28793924342574999937955421765370101235122939557467548549202174550009604780*
 e^(291*I*c) + 1385297945491510894513527695765434031263307472436800308324672
 05895819043568155239264876762867172754338684027849855385453216080*e^(290*I*
 c) + 1289950760115919034107638634270973299485861735745958627058491592809430
 46458742663163454018491463855395649453952212899632198680*e^(289*I*c) + 1192
 09713702033927055755397823688444446474243245021853286263470465996347211465
 73830681540495333543146776810911910410468628960*e^(288*I*c) + 1093325373499
 662232039326785034263570798637070017282940110420765304039238626540189786765
 16417314221089449922495612732870169660*e^(287*I*c) + 9951220647205796595134
 034173802354851533640337171789804085047095465753297727911349150688029072611
 1154101941386019689567958040*e^(286*I*c) + 89883815801382382213973270477954
 602744792877018051963347146307372464315121274929402347942874802899499538953
 561056667668891020*e^(285*I*c) + 805662491306826841818762018826235112063637

903372180119541102106429277659976449038205954219368735653146544157690704726
55401600*e^(284*I*c) + 7166032986117339552444194388928410913409115784465524
5672084237402434944696464927131812190659629511140639501743303863582092880*e
^(283*I*c) + 63247774101012179051794946075175569924076981338138483158042406
747453874729387631710544995247152912205118500597511052824347680*e^(282*I*c)
+ 553909130449720862194326891463315660814279598969699002144342968177311508
63867056620768608187679709720152974148474907904177340*e^(281*I*c) + 4813311
767818402921650374854911037447892471909463560389282936486391655379227882295
7368285106328164715910598370871149079494360*e^(280*I*c) + 41499832121963708
043788523787401345541780088930538206918853579026749273364671640037563488607
716092887686471542838602788559660*e^(279*I*c) + 355001031060196498762723767
969494822095813723710360050128778060274816728070599434452401363155685007323
79966585005678181937920*e^(278*I*c) + 3012848241455270326455901895308817715
601343749343820107841376983544836614812175454919759112996717076496970018034
8699207838960*e^(277*I*c) + 25367176439119353621532260335983348154904982606
125761711300683492963390816491583025705268737539982149639300226512657426118
880*e^(276*I*c) + 211883214058828875396101983747068626958940492260770937641
32512513336190523978949694387686059124526755048042957954264706637460*e^(275
*I*c) + 1755627327122429239688729140312571621349148626114547857137675169010
5656067838042151038271381300372757755676325408026834544840*e^(274*I*c) + 14
429816285208431204532978375375691965063154224649747551295851507389524083226
976789688601369628399900747658579201929300744260*e^(273*I*c) + 117642122748
764840800109007146734744933712781605578119837244558265660556176580864793686
41864908119643412413644803772131657280*e^(272*I*c) + 9513032274019522954209
113191268226642299912013525665940298381064797885690904993128948035227412144
035633851779511219335277360*e^(271*I*c) + 762973181562782158046899242420700
836643889673633302466186383810511044514894696232829763154703254341981182101
5837863013682720*e^(270*I*c) + 60689498031567122483318711053298954717228061
430088780149865596536872606948165504701958900045119655275674327229697075772
02160*e^(269*I*c) + 4787527442780945685145204846971596165304169419328244073
211459592129649255048876854059844720661078151288179612574986359194560*e^(26
8*I*c) + 374525759487665120465733498842622638814395450198683066422234922636
1079609546822276067504899386703088982308185717143407211328*e^(267*I*c) + 29
053857223200570019533452744894827908566925299598237495326959634141648333667
73128218607899328588608916176593772088622582464*e^(266*I*c) + 2234891276398
439464478622578306434840724610484468177859822620658691921478645266653062563
823553001228001009093606751066168944*e^(265*I*c) + 170458299670782280820467
821816769300269866114771277235502145654381093006963718808588282475750060524
6963210810351706405349408*e^(264*I*c) + 12890435152929339564806343304996770
401810439356201069142673110679000300583988397876923769540905452785545449977
10058754772400*e^(263*I*c) + 9664582753690377187477391307981516434835906841
668322346880982911641606364181594521198157288093721251688362393644423973440
64*e^(262*I*c) + 7183615963820582492091135444879010888683887440337132103324
91971375906738341551540457264804304039664255915607349801911966551*e^(261*I*
c) + 5293292527641139260039348369582435576725492389975607392144065991850478

31955572583765358634395408771528009745467548382950094*e^(260*I*c) + 3866426
 730503800494573825628183169626519755509907792770487402386298587950182473561
 62888631015687664780101205287333082748791*e^(259*I*c) + 2799452444750398048
 229667304629608844921198748577911471240090794769204359417352933093054304386
 87333129912454196774070107264*e^(258*I*c) + 2009065871535788043803004695014
 416101745218512595419292098406889608594549085197748359058957576667708578886
 11738751858460424*e^(257*I*c) + 1429063191230555242465469284789542383713159
 258020223892364986521368398225020351556769709174190398345879670555884315664
 16784*e^(256*I*c) + 1007449618518537446117543009829801669624045538362229218
 68484694269966120607698907046343731011160948828100276729370132819357*e^(255
 *I*c) + 7038634976059483156704822406139502569850120229696630037676422033669
 7702961591099854055411376294871437468149528524796002762*e^(254*I*c) + 48733
 253505974923400852255563052101402196469313659554492725674754339375283010407
 167744366955828922837488705858532439654489*e^(253*I*c) + 334358978279365813
 011711754596108294542981679620174198100729367333785065844280242010724531934
 58155334046693516742390717832*e^(252*I*c) + 2273160356612884110041950194705
 136766683665241807726091394481074847308489189041018128541260485487662591956
 5639521227223276*e^(251*I*c) + 15312837206662775379347353212807682965712535
 652942631518286142403097738200270711195396582159028513532779682154451996208
 592*e^(250*I*c) + 102204237794346348513399752951633996417021222496636661930
 53008302026096932158568338309418237395541351819026907953220681013*e^(249*I*
 c) + 6758480437888524372562935948963857626694855547195519486122877567981718
 587262362871994967079401831957927901682582941234362*e^(248*I*c) + 442767307
 910542531852431611298569365658485193610019245704445513448330504532145251634
 7118488133224823670465103483954805161*e^(247*I*c) + 28736105359223401870808
 354355829122772719679773947201597910702749277142768695314671826889810410613
 81703885403497544001592*e^(246*I*c) + 1847505856462451533445284300571326323
 781162553304565971887670758091079306794821834928170773126364639722071570131
 703785334*e^(245*I*c) + 117660072097578696518987505089023109220461269697027
 7433014535895788956771230793520381993106606880564628599822341722801012*e^(2
 44*I*c) + 74222864090817312491693704946252561733414891967911827048983100549
 7781951221069955839623452499748653124658873553401442137*e^(243*I*c) + 46375
 828845736715454493767825500568873332814556804931042399559988601280063861990
 4022368378591108842602342094543682299102*e^(242*I*c) + 28699294363123149655
 727801085157694089682649746606632752880156067700711283743192673508812097486
 1760511367008815728782643*e^(241*I*c) + 17589625826275598575710681261397930
 126580103159548435361490467286516944223207577658044718413414137599577009149
 9246759528*e^(240*I*c) + 10676483201716559483808523418933352873358767332997
 2530092661085186789939252915937090760282232346919090426243399409323314*e^(2
 39*I*c) + 64175100693260066806238064886004597170740843300086839368616139164
 529108049844675353111842725798658088840347241496099644*e^(238*I*c) + 381990
 158675860879760029987566276749947954406679036250293223462501332864891208750
 05013638128113893960349670280707161530*e^(237*I*c) + 2251467574130806996150
 616558650287243042193021067326439299728648560064010386725360484771554706059
 2967690653795951142520*e^(236*I*c) + 13139771494104933881856681151418293112

242551521535686871181266579813877606348160261747201317735782566021306798298
336024*e^(235*I*c) + 759275270014667896115309507358501547319702974653363333
1549793961473285760935801904155116764831560875947581048693527224*e^(234*I*c
) + 43439096966019321733573596877815792937012956819408271142154331753360939
67845908766740738240037114570667410936998017178*e^(233*I*c) + 2460442375845
422663927081630983260714734968091905493027145639238827192254886349361126991
457692409851120873307487457468*e^(232*I*c) + 137967652979621207401710618806
658944835544650121089019510716486035022892858681553900306287502671193194194
7738690360722*e^(231*I*c) + 76586779551396278101255844462875141871094089528
1304790836743661582071650032154891482866406314834433199455459798934952*e^(2
30*I*c) + 42084634260894938727755902145792458657812096614856102264700849952
9468452005980175119410628956210497609566002969884927*e^(229*I*c) + 22891131
173859278009149264916234683440586774077645632610841092885725717470728926807
4347550225793244741923354395308214*e^(228*I*c) + 12324394151933238474196007
258810350659640633925361639108206296996068241901174577573892181775339195446
2609323881489157*e^(227*I*c) + 65674859268867300098827375812875225610654551
686261103681664007007537115778097293533565243828873383722980353200611956*e^
(226*I*c) + 346376571726716901676573445371970870488823548539932704720639430
78773600446542963548348101269390443464480754513928502*e^(225*I*c) + 1807982
006802885997034993862300723067656331420670884849990013964123733476326647934
6963237936039328113185041591793848*e^(224*I*c) + 93393419580534942252517509
657150573007073020838147747703062182242410226482474199560429573630558238308
98547303219757*e^(223*I*c) + 4774141111066098970221845330594962016472714230
374234060663956846950926642685946929064114194400360936223590725470146*e^(22
2*I*c) + 241496651681033850328907654920274051171005901179544713877346420569
6455026442712426409599662771080264826008985061097*e^(221*I*c) + 12087703584
93658393089442220569350632837041081405937502265398461177376482166095597348
31601248698274330296158612144*e^(220*I*c) + 5986501411122418589116765051805
201503640032268413280814535970935877903386092124390855544668615826233503030
61961052*e^(219*I*c) + 2933449200343007202870423834483428663138062854550400
67823080445597545970023446231563554135133105493516316320059272*e^(218*I*c)
+ 1422131159648145176823866672767699094822716813187908898405010394417486355
45362467679832449103520321953011780083069*e^(217*I*c) + 6820803309679361568
378440961924421081861499164004155342440552787689327249660832423109814850246
6453967157728078994*e^(216*I*c) + 32362731322419549410330088943640247460378
328561316422931292427145902887913071643679502909055891236755143207382609*e^
(215*I*c) + 151896342149088003964179117226437547480485201097348124591098788
10493844381062650818971199637121458749456243274416*e^(214*I*c) + 7052132414
162197992602326524580143060985353054572933905524633121681021037340298366342
203324325307072413739061024*e^(213*I*c) + 323849193136185147642332193353957
909837773553920764146734623566582388704832694930560923158514374869020361595
7136*e^(212*I*c) + 14709311466189343455150383623001001604821277495814439299
04746910224777470198899052379114493999887003199419829579*e^(211*I*c) + 6607
644731058690976914759738508379345110890331495867079827642633947667566495652
79879146173318386505740391093990*e^(210*I*c) + 2935507435543427098081294535

765623132997059826991874168629343739642556159671386762532763025915615235156
03264403*e^(209*I*c) + 1289670800847547122460236808664883849832862590255331
32044636109049545144029547003347761521666283977931640178464*e^(208*I*c) + 5
602868342490351765849501385853451616716259103436797249817466090745066677815
4353271630344650777885683547624184*e^(207*I*c) + 24068785139705277161193465
644506143285241361037768216818922184400141048460210944696647752723371932874
594597328*e^(206*I*c) + 102231820259548607672173903051864519235621454736742
93619918063490411487496121804590274592702770571515456414680*e^(205*I*c) + 4
293206478008022126017488908851826494790620720660151451468181910917240027863
968724539127659633517053002976480*e^(204*I*c) + 178244611493175185055635485
663842190117441232229824949659165805393978719824656594597559557573419334888
7952160*e^(203*I*c) + 73158497220681836287472962140397444428001044630116152
7339760544815300951787985538419764656214582667219914080*e^(202*I*c) + 29682
551528266958968531827328023905008455503220341594151196265959688161571379993
7680026497408305672297618840*e^(201*I*c) + 11904185540387796494822957794837
046560060662318304552952690043020927047321277384779493558607471432947993928
0*e^(200*I*c) + 47188220843466207695099506953573780357108897491422567898048
199018207708997005333860148836479527456156014520*e^(199*I*c) + 184874052990
057326937527286118764908908583570219748823715706238001862451377226609436417
52976852924439870880*e^(198*I*c) + 7158124686842941475473807363679839718172
745581538409044503383852693596921622426696740453944718143025248390*e^(197*I
*c) + 273889562479526560335522764656600088628077830508482570291193890365616
2004262736182657700406301914070062380*e^(196*I*c) + 10355619825920029352263
845779086115486121114950801935736913398647060291864824662418056649493810498
56258510*e^(195*I*c) + 3868762182342771656324517230499798892631152823746075
41692443176673997513742813591736171169652250611186480*e^(194*I*c) + 1428017
924502217624831808749188252741343051332754177800847950346447635093335031505
17345864659667189417080*e^(193*I*c) + 5207517851879327038642926335154430695
1104993542500582938155241689408138675254608030847907167748571734720*e^(192*I
*c) + 18759988218865563564163635735986073278255737257405706279108891366378
428467414559930481172863538598193890*e^(191*I*c) + 667586629037114735850376
686566928901089354386983053870872494529158095117918829660615811125770696860
4740*e^(190*I*c) + 23465182192391051422381416330734647688991557089350257780
47637412681781575765422219127409260159438712250*e^(189*I*c) + 8146081877365
305796702100252719214155971833698812142998232919697855498761759698663679766
53244974728560*e^(188*I*c) + 2792857558000352066798353688981654776448649877
94665387827488933863633745047373109049265172681702585720*e^(187*I*c) + 9455
618025893198691933430346636565282685809131432918916073627717587384173219645
3379953705679466826880*e^(186*I*c) + 31610939331284692750694306443618414656
095969520945215743004044560386895241801579156543451940713351730*e^(185*I*c)
+ 104341175165703959666536931555824021094603480954730278074123214273468169
28567197770376496170251803940*e^(184*I*c) + 3400232560601651617521694680847
089844198028831694417424794868779328950548418125605446882081152636090*e^(18
3*I*c) + 109385321448622035867403243450086667849977001130587417248897595161
2031456734608287095519501041975440*e^(182*I*c) + 34735147321471378087435208

3129566601238765762775942366762733349952103889753982636403857556867777300*e^{^(181*I*c)} + 10886799573182947282673290519203488679728462135644562753090910
4429486741257822633476898356826454040*e^{^(180*I*c)} + 33675398872021568375902
384593982753362559801058104184627345411136262431943240778260721756991027090
*e^{^(179*I*c)} + 102793647306638408447395778624692626046488619142979725891652
43530651230690726244462479199894255180*e^{^(178*I*c)} + 3096131971621520162380
301554241465451782362086810287537748902904985934020179565706177131421614590
*e^{^(177*I*c)} + 920089393029589032874601850027159322612526368444771489781974
361078847528891468831038436064951920*e^{^(176*I*c)} + 269745801440211296972683
601863878954357962308520076595177128227629273240215209708218497363414140*e^{^(175*I*c)} + 780098073680242398756137330588514171253271146810708896407942492
82633470580756557083923203377160*e^{^(174*I*c)} + 2225195917679577775716736603
6007480222211364232146399803864370963391491223687245823457351580140*e^{^(173*I*c)} + 62598721568222528436509607082350347102013627760571766472263230897514
46565288850103898153859920*e^{^(172*I*c)} + 1736574218818191071874197472450158
123883564209950658639102337148122769080611680719741726053840*e^{^(171*I*c)} +
475010578857601519272316617938425222421786597241671026894318515408511467140
969393115768793680*e^{^(170*I*c)} + 128098914601688539672480541830409847707367
500438601536803204497701119911289087105659482783340*e^{^(169*I*c)} + 340540538
512955691543523467221771726551875489107820085047183241687250294385891623492
11628040*e^{^(168*I*c)} + 8923209447343296763331881881638471793499618670601026
059730895962653291770229493028162575100*e^{^(167*I*c)} + 230435107337384035737
9178597673066352016682781689139842097376663118488803841131935313641840*e^{^(166*I*c)} + 58640346697268324274164332892156090937519745386424329957199096460
8857245771134145204174990*e^{^(165*I*c)} + 14703081673227683316304158209959204
751204372522535339238819165193000407629544745753221740*e^{^(164*I*c)} + 36318
369652302591732197444409798122022640824604130552506742586795183267354382847
875885730*e^{^(163*I*c)} + 883672064086047030569451402154796955129679409226698
3044118375790025854584036796364768280*e^{^(162*I*c)} + 21175897334668557071015
01429210414722401838837940752841618541440888545729943138209036820*e^{^(161*I*c)} + 4997075672538590843575963148137947680693371909159674919074889049339226
77579665354338960*e^{^(160*I*c)} + 1161045516835550437629115017121163993137330
21132677481112824047246361794049635726479850*e^{^(159*I*c)} + 2655680638904340
7534496702369101545795994861757741414789944652712127566910185274123140*e^{^(158*I*c)} + 59789921729441432184591611492998197063217321115784945252452287429
76468409105395536290*e^{^(157*I*c)} + 1324756412367837473157472821162483691120
966501948953926492241643788264284546437221120*e^{^(156*I*c)} + 288820755264730
654469968572021047109427318619508995802020689904590319476295408324280*e^{^(155*I*c)} + 619485966530355028795643388152343106604109020378824731618047744929
16216575880077680*e^{^(154*I*c)} + 1306981720348828988619320550837581839212499
1382340160316886507181296548981014818410*e^{^(153*I*c)} + 27118432396707175276
05640490148833507130242448403978318523237721944200392830108580*e^{^(152*I*c)}
+ 5532691288195286125029188695589478290980219563093498435840446315122917788
00081490*e^{^(151*I*c)} + 1109691996873209747499222595952504443412192185353496
55762591192576535872151766080*e^{^(150*I*c)} + 2187648289271390992804034561257

8705805121508756226696317087651824252241418663320*e^(149*I*c) + 42381258467
 63232586394188569858685826755328005548627437019301405851325887594480*e^(148
 *I*c) + 8066795436075891407593050107961895682698420216133889552189162788231
 82639488190*e^(147*I*c) + 1508223814314124137735664742100117468522974375970
 59186295243989481140398152780*e^(146*I*c) + 2769311653834325922598338263764
 7936122664033859615133489846664694361471028310*e^(145*I*c) + 49925197124570
 43983505377976607953988397368297591114957991804893688371867680*e^(144*I*c)
 + 8835009688217912026007745419277692007376893935137347893683970933333119618
 80*e^(143*I*c) + 1534360887450562541273272394615770719331301577645959971139
 73513183188399376*e^(142*I*c) + 2614397627990202144347194566508025456305681
 0183520401889800285493144867448*e^(141*I*c) + 43694424829101139145653531360
 69595862669338858053419381214131241925047008*e^(140*I*c) + 7160994975990580
 79895633338552940229192858196481597830078819711862600096*e^(139*I*c) + 1150
 51481852080848873700388354521315567640365124003103691176697194292320*e^(138
 *I*c) + 1811576849561575807671030305550562558925429365919331415341833394459
 6408*e^(137*I*c) + 27947091044756866118427906949736991644822547239772102097
 25661304403472*e^(136*I*c) + 4222761266320036875477547465557099887105271330
 86660161366353656787288*e^(135*I*c) + 6247355078105329531771077469024711412
 4125187565731848441781904032672*e^(134*I*c) + 90466935238256829790443389631
 04263167672586826367911338826483549173*e^(133*I*c) + 1281817464914970810859
 604189828359000790789921169405304612211251818*e^(132*I*c) + 177642829135119
 348577194437675802830239905460092687136494961404333*e^(131*I*c) + 240708019
 13529757101858022914372045864746991786182039740274325264*e^(130*I*c) + 3187
 749929744346497211536044751776582320958627923816470590659024*e^(129*I*c) +
 412430698299915190848067222327219435067747934091894670488982928*e^(128*I*c)
 + 52108117629177048660492400985175830987505700566877818954141639*e^(127*I*
 c) + 6426195485535248576425068136870465530087114003875716691383902*e^(126*I
 *c) + 773204636991145775061462731028098506094432675788136295011259*e^(125*I
 *c) + 90722605722208814918642284639487187764607589706493970774776*e^(124*I*
 c) + 10375184499871175501909398956596684116802997082526660323524*e^(123*I*c
) + 1155855412893594260345544966642687823630035899363232371472*e^(122*I*c)
 + 125370496586921272662198050851269323171167338854081782959*e^(121*I*c) + 1
 3231708870104896973800056733779919089340836756009580718*e^(120*I*c) + 13579
 90663161479842850642848032544982878359839580349899*e^(119*I*c) + 1354425949
 16636116191574650625331646238501101627937224*e^(118*I*c) + 1311878180117217
 4729679339894318153694964675368481194*e^(117*I*c) + 12330967001397233651819
 97220750932590655287625342156*e^(116*I*c) + 1123916045422466509664291620631
 24338952554575234051*e^(115*I*c) + 9925490738534402272939987038714580495445
 431374618*e^(114*I*c) + 848552202276512356496200136959676295361696315113*e^
 (113*I*c) + 70164515322544462906873548813748091084561870680*e^(112*I*c) + 5
 605927253067558551780452883689835514455118670*e^(111*I*c) + 432333688644261
 557547944179250800440604964868*e^(110*I*c) + 321478876933753388174544825153
 77350383950278*e^(109*I*c) + 2302150411226234925855222345201500900533576*e^
 (108*I*c) + 158566476113257562566117432227203884298856*e^(107*I*c) + 104904
 02669510897424624643766470754045064*e^(106*I*c) + 6656346706762100637541918

$47109971141414 * e^{(105 * I * c)} + 40443624781415311581857832389099634564 * e^{(104 * I * c)} + 2348998374244347079532766203075607598 * e^{(103 * I * c)} + 1301711930791728$
 $23835151430773360024 * e^{(102 * I * c)} + 6868329225263681349501997341320517 * e^{(101 * I * c)} + 344277152012875134140739302960914 * e^{(100 * I * c)} + 163531646471515302$
 $40529137618111 * e^{(99 * I * c)} + 734057263616388449968842366924 * e^{(98 * I * c)} + 310$
 $42222522074681615625020522 * e^{(97 * I * c)} + 1232445557346832245176696904 * e^{(96 * I * c)}$
 $+ 45759117183402579073139583 * e^{(95 * I * c)} + 1581796642397812408161814 * e^{(94 * I * c)}$
 $+ 50648660944512569972179 * e^{(93 * I * c)} + 1493326612293984160368 * e^{(92 * I * c)}$
 $+ 40261256699368950388 * e^{(91 * I * c)} + 984382804329835768 * e^{(90 * I * c)} + 21608403021340047 * e^{(89 * I * c)}$
 $+ 420601518659718 * e^{(88 * I * c)} + 7146142307307 * e^{(87 * I * c)} + 103818048048 * e^{(86 * I * c)} + 1253841160 * e^{(85 * I * c)} + 12085216 * e^{(84 * I * c)}$
 $+ 87153 * e^{(83 * I * c)} + 418 * e^{(82 * I * c)} + e^{(81 * I * c)}) * \tan(1/4 * d * x + c)$
 $- 14 * (23 * a^2 * e^{(1027/2 * I * c)} + 8970 * a^2 * e^{(1025/2 * I * c)} + 1744665 * a^2 * e^{(1023/2 * I * c)}$
 $+ 225643340 * a^2 * e^{(1021/2 * I * c)} + 21830993145 * a^2 * e^{(1019/2 * I * c)} + 1685352670794 * a^2 * e^{(1017/2 * I * c)}$
 $+ 108143463042714 * a^2 * e^{(1015/2 * I * c)} + 5932441401233490 * a^2 * e^{(1013/2 * I * c)} + 284015632089714525 * a^2 * e^{(1011/2 * I * c)} + 12054885718238165655 * a^2 * e^{(1009/2 * I * c)}$
 $+ 459291145921837770879 * a^2 * e^{(1007/2 * I * c)} + 15866421408580747568967 * a^2 * e^{(1005/2 * I * c)} + 501114476390712619058089 * a^2 * e^{(1003/2 * I * c)}$
 $+ 14570867094745692063264465 * a^2 * e^{(1001/2 * I * c)} + 392372635871953337607706440 * a^2 * e^{(999/2 * I * c)} + 9835474093767985284835740178 * a^2 * e^{(997/2 * I * c)}$
 $+ 230518924833500391396633471432 * a^2 * e^{(995/2 * I * c)} + 5071416371138697123516748312797 * a^2 * e^{(993/2 * I * c)} + 105091017765905750210205432017088 * a^2 * e^{(991/2 * I * c)}$
 $+ 2057571526219280176660569426269565 * a^2 * e^{(989/2 * I * c)} + 38167952336853964450194985322999760 * a^2 * e^{(987/2 * I * c)} + 672482982322867333315410721290945657 * a^2 * e^{(985/2 * I * c)} + 11279373940205552267581116020178794202 * a^2 * e^{(983/2 * I * c)}$
 $+ 180469988996656023008385818021005185594 * a^2 * e^{(981/2 * I * c)} + 2759687033617389036404063195416999823985 * a^2 * e^{(979/2 * I * c)} + 40401820406264183313387690283892201648160 * a^2 * e^{(977/2 * I * c)} + 567179441813227622186529241088113209451509 * a^2 * e^{(975/2 * I * c)} + 7646419821227105124391700712517502747187088 * a^2 * e^{(973/2 * I * c)} + 99130382277532022515819177881629325075859509 * a^2 * e^{(971/2 * I * c)} + 1237420804707705093991102638839493330163931640 * a^2 * e^{(969/2 * I * c)} + 14890299547694622406488788915648999996474069170 * a^2 * e^{(967/2 * I * c)} + 172919643607724441133173354565998605401103776504 * a^2 * e^{(965/2 * I * c)} + 1939942742070099509831143493249507373979963303677 * a^2 * e^{(963/2 * I * c)} + 21045445872134849680137861498182408126014563326457 * a^2 * e^{(961/2 * I * c)} + 20977262761597134870041878241189142663942444889435 * a^2 * e^{(959/2 * I * c)} + 2247655430280602939346801439987511004122770572057975 * a^2 * e^{(957/2 * I * c)} + 22164391506926711131826332938481074378353565258441643 * a^2 * e^{(955/2 * I * c)} + 212059444422641568370505689613891761234929288751535757 * a^2 * e^{(953/2 * I * c)} + 1969922064355959547894504865149532487975181829580265482 * a^2 * e^{(951/2 * I * c)} + 17779824621644501534677987130821297785034706339145224178 * a^2 * e^{(949/2 * I * c)} + 156018117308149873911753787065575404463673778724880411510 * a^2 * e^{(947/2 * I * c)} + 1331863531921951594349497448051501144604555177632910228645 * a^2 * e^{(945/2 * I * c)} + 11067166739593806537994432532202323892559358824814410948544 * a^2 * e^{(943/2 * I * c)} + 89566978464755601531386426232449217353087060101994516952837 * a^2 * e^{(941/2 * I * c)}$

$941/2 * I * c) + 706359042600440090623627368646803874663354915637996034416454 * a$
 $^2 * e^{(939/2 * I * c)} + 54311275277768201077898230182311221465488454663249103144$
 $38155 * a^2 * e^{(937/2 * I * c)} + 4073355402474013490085959435064795421256740764895$
 $7747058481680 * a^2 * e^{(935/2 * I * c)} + 29813576609204982126550856575985975726890$
 $1977404776167694840520 * a^2 * e^{(933/2 * I * c)} + 21304351655961354080475521993685$
 $03071331826042021949871247131824 * a^2 * e^{(931/2 * I * c)} + 1486962092393492819586$
 $0171826698271613545069265812546978930298844 * a^2 * e^{(929/2 * I * c)} + 10141122575$
 $4763725160776093014569051984766366679386214896417032560 * a^2 * e^{(927/2 * I * c)} +$
 $676077945863029742736783795661095141326065268116577325076514243900 * a^2 * e^{($
 $925/2 * I * c)} + 44075310711231539469155754424078673682732044364416823259520588$
 $80560 * a^2 * e^{(923/2 * I * c)} + 2810857079443600488986815509542529197552433712454$
 $2327266338411863980 * a^2 * e^{(921/2 * I * c)} + 17541946418183786120670549049149961$
 $1761897494030842873597094326137448 * a^2 * e^{(919/2 * I * c)} + 10716614348473653564$
 $02500490116585199491741089623963750947662209022120 * a^2 * e^{(917/2 * I * c)} + 6410$
 $885411886208227235614369244284094773850732880341584569571247989180 * a^2 * e^{(9$
 $15/2 * I * c)} + 375658912654834216259223272376633939730617913440758172955655456$
 $41550480 * a^2 * e^{(913/2 * I * c)} + 2156822939043670482294525665251701654510863458$
 $47776055567286526724207180 * a^2 * e^{(911/2 * I * c)} + 1213684086474480030032963524$
 $903872036826200642910143657468007690377301200 * a^2 * e^{(909/2 * I * c)} + 669557812$
 $2665234305226870074465850318321143798508148119852380345744581420 * a^2 * e^{(907$
 $/2 * I * c)} + 36222507823447452094316152383681805847086822685973455693062020799$
 $131961680 * a^2 * e^{(905/2 * I * c)} + 192216100832677211425583919374607410256140382$
 $963064998218563326540739057800 * a^2 * e^{(903/2 * I * c)} + 100076216506457306558003$
 $9704950705594622141532403147100702228104518209922480 * a^2 * e^{(901/2 * I * c)} + 51$
 $13371340462080188470579216332247051199800933775813472986030153667768479970 * a$
 $^2 * e^{(899/2 * I * c)} + 2564609163526230666377598990704819369302648107270778598$
 $2065347970437174552640 * a^2 * e^{(897/2 * I * c)} + 12629066597664942375279529330055$
 $8365078090948368244878972263607020367969724510 * a^2 * e^{(895/2 * I * c)} + 61073557$
 $6848646913229035316690015481201283216147648837631072938937795953569020 * a^2 * e$
 $^{(893/2 * I * c)} + 29010807054572012234808075159505626906960549421374234594481$
 $33177752600175055070 * a^2 * e^{(891/2 * I * c)} + 1353882290279227574856360451606447$
 $9211324364455610559665374956958057411438898160 * a^2 * e^{(889/2 * I * c)} + 62087428$
 $059933702144851939557816743050943015221536726159024340696975430971481900 * a^$
 $2 * e^{(887/2 * I * c)} + 279841843449723329022256022542885182491449861432225949373$
 $308219431658133659304060 * a^2 * e^{(885/2 * I * c)} + 123990929743896845849802042104$
 $4704877786634944232208108330855107855869727352550810 * a^2 * e^{(883/2 * I * c)} + 54$
 $015092850750556638448628821387292768098169292999783334700946692720250870679$
 $52130 * a^2 * e^{(881/2 * I * c)} + 2314012260343947977239284444734426757824128849128$
 $2693326820133793119469871909054310 * a^2 * e^{(879/2 * I * c)} + 97502706496344518739$
 $819416482817785708749119763377367851251579953471302421416590690 * a^2 * e^{(877/$
 $2 * I * c)} + 404148737461691129448272381486081487215931511785097115337433651585$
 $897523208487479570 * a^2 * e^{(875/2 * I * c)} + 164820164213914670563389414978347637$
 $4765855031698399268829056921671864832034920363790 * a^2 * e^{(873/2 * I * c)} + 66144$
 $400820020510453479219244127878363376621124553998877451160580978585651720285$
 $83520 * a^2 * e^{(871/2 * I * c)} + 2612502097352610282419068257137595608809294834524$

0408465888776831062032840805359036420*a²*e^(869/2*I*c) + 10157022331200088
9889955269913274747383718323298377402360165758159408055448033759362720*a²*
e^(867/2*I*c) + 38876396892010237458390588327513357037313829516984837763633
4256622974927805402558286690*a²*e^(865/2*I*c) + 14651342290643008303482102
33782702338437177567650054898553731358964520863696142827138160*a²*e<sup>(863/2
*I*c)</sup> + 5437521451042031455370282099308345101312396580420987248763253424089
743393247174970193410*a²*e^(861/2*I*c) + 198753832701981230163980739547820
48143086744479176706215294931939285231552377668666774800*a²*e^(859/2*I*c)
+ 7156131327668605954342040988322253447335243985577227472776336238721031750
9376711352805370*a²*e^(857/2*I*c) + 25383132012926548867726804091248224918
8903710351278672972032718591405876131400102115746900*a²*e^(855/2*I*c) + 88
709614811178381979269807521163793735012825182063947166884553198581936804359
6429487700180*a²*e^(853/2*I*c) + 30549765052290531863581965128998236791965
39270073164577798894149381563584247883834020602010*a²*e^(851/2*I*c) + 1036
832427833694706645534803655654449472691347656052168964108774960636789159517
5422870841040*a²*e^(849/2*I*c) + 34683534777882172855590247839161459711508
478268599595143978937459065098062910312662980093090*a²*e^(847/2*I*c) + 114
367222654004146039785488306144607682565282217212089722715844480032864714063
018623336102960*a²*e^(845/2*I*c) + 371784628277848407873740730843158415533
102422282317148023178730401149797296961025637580441730*a²*e^(843/2*I*c) +
119162602671200452022853596291671118196468103297935508099064097373477450880
9765193594567308000*a²*e^(841/2*I*c) + 37661117341863148447457366228609850
04110904508288810189408921969722457687589048151875880889220*a²*e<sup>(839/2*I*
c)</sup> + 1173805156223944291043665868661695293162135916390152574975505940091233
1209404508874859876592800*a²*e^(837/2*I*c) + 36082142955789372921795326276
163720115936669443430427894923821823835712395825018234933157302110*a²*e<sup>(8
35/2*I*c)</sup> + 109401947851812253736939736757431402194995906470393287332344122
103685909885397712947473326518610*a²*e^(833/2*I*c) + 327217244888055084262
892186197327446953686436737681340634739093336235382375514336377928793244690
*a²*e^(831/2*I*c) + 965532218209251264607039716473737729829357676534904149
014904268128459659120470520755736939192070*a²*e^(829/2*I*c) + 281097503606
779152602078316141434903543061096416439129012877500571867619956228103890661
7666401970*a²*e^(827/2*I*c) + 80750674801709208568026954585195592726223421
02035297526669820326780584006158874417298884536622170*a²*e^(825/2*I*c) + 2
289142246227600597089769949517582743299142275420964671179186107472376353440
0400991756009136093340*a²*e^(823/2*I*c) + 64043424833900148962660564937308
584792588410556135883392172200976630674826538314163343570582742220*a²*e<sup>(8
21/2*I*c)</sup> + 176843212577304640940434087070526800206912147942357555568484158
874194832893725593513293339771528640*a²*e^(819/2*I*c) + 482003855527359842
058748938929565762498717173867153381418816829773057750173701195879408932882
129710*a²*e^(817/2*I*c) + 129686978770094550352747818625343842928386699310
5679751956812206964316306426908093383593771068875180*a²*e^(815/2*I*c) + 34
447742392644906310173946350452828676614441736472801344894490672796907708801
54195712638143690670510*a²*e^(813/2*I*c) + 9033952674591972024099323322407
009219478763876009237692789450947388612631864264679797961473529847760*a²*e

$\wedge(811/2*I*c) + 233927347923584499864709149886416853481686131371450227409772$
 $24897697160854430483589336483946581829650*a^2*e^(809/2*I*c) + 5981404789492$
 $439511671517626292625873582700958544166730832132595707235882486795801563841$
 $5996637992560*a^2*e^(807/2*I*c) + 15103468127596178167008188074186593145157$
 $0808839279104804123701285639063747843522793214727666247721320*a^2*e^(805/2*$
 $I*c) + 37664584457361554618416651099041615464594000284876646544167820297804$
 $4480377315238707708815985878943120*a^2*e^(803/2*I*c) + 92769209802706548376$
 $575206531799625267482667453463111604771895974309785519268076633186159076940$
 $8198460*a^2*e^(801/2*I*c) + 22569366401976461724095887544833910482389125597$
 $19597692916098711340142755437285078061404880797678311440*a^2*e^(799/2*I*c)$
 $+ 5423877415356140734240076530281376392307183648754518352919070721950593841$
 $907858843558278519237201933020*a^2*e^(797/2*I*c) + 128767135259169892233668$
 $231930230540880904887524578931242343976286799924376626828789944942766600417$
 $69680*a^2*e^(795/2*I*c) + 3020188861789511923414776779708658681221749045460$
 $1051236525729423437716476833896661754018658154022389900*a^2*e^(793/2*I*c) +$
 $69988368274664388580883736712738303159327351505142299304675566880142922981$
 $296353915575157471652388092680*a^2*e^(791/2*I*c) + 160253943840621057078138$
 $437984495917828994512049873464602063929877936771758934087025740251939283794$
 $149320*a^2*e^(789/2*I*c) + 362585708102908727461692262308497693854332721503$
 $869703743775437736913882412243026230183024991966728272700*a^2*e^(787/2*I*c)$
 $+ 810698514587734599341419926912945883958489502890417749835477031467724072$
 $007792742636256985473924406832880*a^2*e^(785/2*I*c) + 179135687128617945307$
 $626189529754871427562763762032201846805627904310478564323297499454029349376$
 $6790807180*a^2*e^(783/2*I*c) + 39120632922983706718236562651806275103810426$
 $88906961451087421710967460980974243159240293773328259061298480*a^2*e^(781/2$
 $*I*c) + 8444177149381193033478801248743170134902792413015392319772244917564$
 $145947358160863746469962582546517551020*a^2*e^(779/2*I*c) + 180161882031611$
 $515955708476802609030852814845475689392743272128585688425147118320679929810$
 $21945668432453744*a^2*e^(777/2*I*c) + 3799695085003403823385083030620582717$
 $0017764253586941498028293120742516484664499219084036933062000158769320*a^2*$
 $e^(775/2*I*c) + 79221081373658264272879848670833963158393802618546130507463$
 $440333508958081178065157747031476513568516431440*a^2*e^(773/2*I*c) + 163291$
 $912624014896869889921756370257404344116207519220566929295476847426915494195$
 $734247976197944827166856605*a^2*e^(771/2*I*c) + 332771401353095630876781482$
 $769166072007566368785944639929752492013340507467607203423204297878103391257$
 $399650*a^2*e^(769/2*I*c) + 670517439437730975214058382921475629655071200139$
 $334454271221441426048415295599199783168621980211682925980307*a^2*e^(767/2*I$
 $*c) + 133592516338887898994288754323787357347735407807804038483834072623902$
 $9587514889857191688753727386206824536152*a^2*e^(765/2*I*c) + 26320081955436$
 $011268865672586710800827931793309439329526480867362575752921947868957894171$
 $36105832466949897075*a^2*e^(763/2*I*c) + 5128027932104560135559026593192544$
 $604592901120537915741822852980300115191592803605514077707054780313193678930$
 $*a^2*e^(761/2*I*c) + 988089919880856328723361351985533201055999294177279451$
 $0424197819767831159955107339288892516466317402233238590*a^2*e^(759/2*I*c) +$
 $18830001036742739401007198462226884604435327942040227457903683414392625117$

454825138218461037881756069460095462*a²*e^(757/2*I*c) + 354925185722847832
617907210910079842132689358672913810999435476589968666122819894515319139161
63009433157485571*a²*e^(755/2*I*c) + 6617297932881809399215054642382035162
7528523728608540021199312702600802240756658938974138233183457153977060797*a²*e^(753/2*I*c) + 12204128126857543983447988302990699194420160067581866401
3256230154913964580643406692381897603394093043703442025*a²*e^(751/2*I*c) +
22265879824279233418100220610928010409785016857978952736759938769504628288
9021401012025548745828097666778231565*a²*e^(749/2*I*c) + 40188785017504508
015031388991007269452800944701646807261145319396663248746567607461705605177
3919819622001323159*a²*e^(747/2*I*c) + 71767324920670553943767575873920097
462584452902452927707470260940478945808166620375787523398677784333943177897
1*a²*e^(745/2*I*c) + 12680329744048849958800906653423684542730040005610160
23853679186675517009550364504248846180076370911172808894376*a²*e^(743/2*I*c) + 2216875962324492560656475252069459616280748019821115392295053079963518
361922106809558395835470562396612703910894*a²*e^(741/2*I*c) + 383516964831
217741295575895704558807255981323477748813823772353785390692179569004031240
4465308498665963807731880*a²*e^(739/2*I*c) + 65657679593071530228829834619
744537570730217235007718128652478013026176517587654983378556847546660473321
09346955*a²*e^(737/2*I*c) + 1112422688241289810643238399018764325994794191
9486926143555782999008463015355905895251415554260511452162767106896*a²*e^(735/2*I*c) + 18653626462850532803035140588104717514085431098848869419689186
654357354383555540804832857748868154391812479528331*a²*e^(733/2*I*c) + 309
593486425765628601652998173661658629226486684257964755048512078687853478573
17439600867394648879054091070118592*a²*e^(731/2*I*c) + 5086061865420950083
533559153160520120167735286246615994357148142901870562289168145260417888380
9245979957523278415*a²*e^(729/2*I*c) + 82710216196192744659285946813603474
744793948296346331753106207063703754012444364328700859979459109222867113617
510*a²*e^(727/2*I*c) + 133153018441395196034014770358763691000098636960739
548768552317876384874768681586531648288117265885968911213464902*a²*e^(725/2*I*c) + 212218928188673385825585486875626217952905358371974613119305643729
514631087950307170160593893009749320293414137639*a²*e^(723/2*I*c) + 334877
229377355089426560502609284393492614010501825504426666861674218991625217228
236262755043572863766464617539152*a²*e^(721/2*I*c) + 523218326398909441228
138386569661789123283174843980047262040319411958519785242263748926347748867
588273669418790115*a²*e^(719/2*I*c) + 809475688654593052224501094178917252
980035293905898453796392919564557037778617746976730948724902348115224486645
600*a²*e^(717/2*I*c) + 124015564057522683211246144658763515180020658900739
0172229369031582678860752755332793310035944135341130684921794627*a²*e^(715/2*I*c) + 18816046186806990270245435156986520031264265100046299196376707678
51063752549093913764335909644924041423364457564056*a²*e^(713/2*I*c) + 2827
410762917657346065292926703346273000172053162448180906376970875870523643742
229301867770298371409250162486706446*a²*e^(711/2*I*c) + 420810587990096567
104481160318687178813705026824971664290885292245016673590410899653815163230
2993343796013355257880*a²*e^(709/2*I*c) + 62036998076732947111872108021144
212850853086945744288084100595853011031037228772905226051408470437457943030

26775415*a²*e^(707/2*I*c) + 9059634894716266106041250114654859819170770958
 994890643823267294971612713252112667925034180484391205562621557500199*a²*e^(705/2*I*c) + 131067843765579140601235842829739716168027656278860131867281
 25299737503997244788676729350715523367908968202016814961*a²*e^(703/2*I*c)
 + 1878612345612180246044477763374134866227498929577412232794604113679972393
 4764027545573154102942013646829238996755521*a²*e^(701/2*I*c) + 26678662604
 166841848095788924129301979448932840410184329153779340794009872890361466821
 390436483345842294941747333409*a²*e^(699/2*I*c) + 375411345309312949907076
 283118878966417380012487360455442254727581286124794355396485399362672455035
 94181132790075635*a²*e^(697/2*I*c) + 5234775479141456200969042474636259615
 597005932613387746572682372750616929974976286787233738293308030257583235431
 2830*a²*e^(695/2*I*c) + 72338117658183544125339619278704570070398062503968
 469165923503150270252549867503793649405257860558789674577875168502*a²*e<sup>(6
 93/2*I*c)</sup> + 990709333414297197602970045457468870551580177755061961507795918
 97966930321674638084629595248846439217986186167691566*a²*e^(691/2*I*c) + 1
 344828546045721626428840967446952957135004987386452006510355485923549332394
 83430231951682574771378015247514707342287*a²*e^(689/2*I*c) + 1809510824618
 191818695813304988792572428725718115718920483829817869705697213909872572459
 05924108342179247496262397340*a²*e^(687/2*I*c) + 2413577937113473282327609
 590718728464284134012731389363008530096007019884060645842590310192251149561
 73896476656268335*a²*e^(685/2*I*c) + 3191537208183049874320021715335408800
 739785633976845781847779169877992931630884082047360763702149210031174230470
 33230*a²*e^(683/2*I*c) + 4184174731371814572412385090873293252824089396877
 21606296599112517765339587255587183797664460597112939981628640119137*a²*e<sup>(
 681/2*I*c)</sup> + 5439064667128123519049147559829178351094822133364936621624653
 10034934606188962867551968325252055883629254890215235232*a²*e^(679/2*I*c)
 + 7010946890196828914166239682921267169075994030537121584347555503459473499
 48295539046888909742638931217537474025975440*a²*e^(677/2*I*c) + 8961920355
 975037209271807078077066028702648087726047622951692184665561922309601678440
 78573413391828541761332410051040*a²*e^(675/2*I*c) + 1136139693561537155978
 568944899860990716224618498598331698010751957196904760729746742387960564144
 749497838935456211960*a²*e^(673/2*I*c) + 142857608790754115407016752894453
 517980762876865018136822572838041989915471907902387694640681948777850843831
 9784170080*a²*e^(671/2*I*c) + 17817683182949669352805978484140547237298363
 54871708432576859235201646764794389618950950587491353081375055442537775544*
 a²*e^(669/2*I*c) + 2204504848819271258577686647437802673860228777835066862
 747405782838776390348098557012295819426306998948329682289436640*a²*e<sup>(667/
 2*I*c)</sup> + 270594649944996307730421914875997577613545365348393733791756619433
 6892195568305631554713116849660441746258385455723160*a²*e^(665/2*I*c) + 32
 954345219222178101235110820882370320399432856468780343002902891323602533505
 16006721630787778671059075241422781406160*a²*e^(663/2*I*c) + 3982256683723
 412940987002608432103465539772199439132030360365812174562265353125356974005
 033580943527632143017811668560*a²*e^(661/2*I*c) + 477537474879123150545968
 399666527814610204346354866288991622477572359598824493319433054794836177389
 0650352555746361400*a²*e^(659/2*I*c) + 56831194084038593110007516240036231

488608208471207766480066528273851195374220703844734081088304510355083937269
80174240*a²*e^(657/2*I*c) + 6712861418522615724988858473795623314248057954
884136819874564233080140888033656717736078828058598240124888994298326360*a²*e^(655/2*I*c) + 787067024908872037443069512725257813818390760011548448756
7789602589829762952732597658246852175521239128818848207047200*a²*e^(653/2*I*c) + 91609737380630587614977376948117073446973469195890750935573822620798
52822162643836169596839715953389931229146306843160*a²*e^(651/2*I*c) + 1058
623385911710506650733480513866864889318683528870371443099194626006627859975
2585501637968876434411898083966268318240*a²*e^(649/2*I*c) + 12146654565504
797783672415623355290385431521572754643028081311118774641507895483012327171
848886038795599157607589336080*a²*e^(647/2*I*c) + 138399376099471873989204
752380174900209956108648629891563231150082467766771666466864251379229720695
44356469424336874720*a²*e^(645/2*I*c) + 1566110116368601184782458239234044
886066938476679972162717922203521376820514126055609506038475263236181932703
2020892140*a²*e^(643/2*I*c) + 17602373941334170158602804303413344263296373
051657482244301069046136457777485221759920683874338340816767315010883070640
*a²*e^(641/2*I*c) + 19653174438252726752541792477777619934650225271457413
46579124704111270169852610316237530213837076731939155254031315220*a²*e^(639/2*I*c) + 2180018094657529251098759558153408779545501567139438439807556810
2023793126789691655257472560286308276031522589116147800*a²*e^(637/2*I*c) +
24027493459410529878342393314596701203692315139519374776263890070912654038
803572108760194498231541018946484500110764820*a²*e^(635/2*I*c) + 263168836
945024245212481243876288800464809215795717366607784396111020491267193928674
91978361605672355038563370490434320*a²*e^(633/2*I*c) + 2864812461220060601
372002405432777931524031087082124165142024060554568814490053708919821326385
1453828234588917547711560*a²*e^(631/2*I*c) + 30999386333592884159610215585
305952854889407725459965085076104937976196911732638644879528970536122548468
911408679508520*a²*e^(629/2*I*c) + 333476816393137132776489667910175466641
478627744256121856468997745312213933869257141075472252852730542903145965949
77580*a²*e^(627/2*I*c) + 3566934156165823421939301591416525880023144723499
2175120111621953758773880356130515026339013759664087801990290045546540*a²*
e^(625/2*I*c) + 37940500212170537906421927439740074329537276968507943576124
882485983705492251107761060039819420913129160456600166235380*a²*e^(623/2*I*c) + 401375680536466143754333426685279018978222583062893971772776139870250
21024009192926575679052108438711600692277041112940*a²*e^(621/2*I*c) + 4223
767435092749730816760791277049538441050990699929435925460859839383190862700
7418402687181252824823543923401086446140*a²*e^(619/2*I*c) + 44219062414309
796885036555682850910798132641174189091355070247306856269259520267158532714
171765231548167835085083759540*a²*e^(617/2*I*c) + 460614252564034938067668
169663307994388334771809988593995473930967657173021186924595282786976542841
87890865264198490880*a²*e^(615/2*I*c) + 4774617408598129132026036222830018
345928279440677646144908614295224316238252464662558813944616663514704992754
2868878520*a²*e^(613/2*I*c) + 49256637251142339677939547027108976858345467
610518344678311703270141917623803626587961704546454017361035725620405883520
*a²*e^(611/2*I*c) + 505781923678642695971460350025649056223221849146426945

28702415269515000703383395370399380435548931455046604860814615100*a²*e<sup>(60
 9/2*I*c)</sup> + 5169833897799209701194910876351827860915729120563316518750422956
 6420267938267413598672666681605606851393811312765471840*a²*e^(607/2*I*c) +
 52606722765605163245233385275200339644939530342158370891740806422347556622
 042460358183525029339507310593911482225899900*a²*e^(605/2*I*c) + 532951247
 986600196698491744108009210277692661226623298727506854636000929695489986274
 67943246663396973157934252963472160*a²*e^(603/2*I*c) + 5375743024611174648
 748124421967553620129596150938528123460343206055975678885644421073351916009
 4322986825803224364734220*a²*e^(601/2*I*c) + 53989590479579880111809597901
 378922286527227175566547926860482497477539014851169842250286594884676270110
 583739674489880*a²*e^(599/2*I*c) + 539895904795798801118095979013789222865
 272271755665479268604824974775390148511698422502865948846762701105837396744
 89880*a²*e^(597/2*I*c) + 5375743024611174648748124421967553620129596150938
 528123460343206059756788856444210733519160094322986825803224364734220*a²*
 e^(595/2*I*c) + 53295124798660019669849174410800921027769266122662329872750
 685463600092969548998627467943246663396973157934252963472160*a²*e<sup>(593/2*I
 *c)</sup> + 526067227656051632452333852752003396449395303421583708917408064223475
 56622042460358183525029339507310593911482225899900*a²*e^(591/2*I*c) + 5169
 833897799209701194910876351827860915729120563316518750422956642026793826741
 3598672666681605606851393811312765471840*a²*e^(589/2*I*c) + 50578192367864
 269597146035002564905622322184914642694528702415269515000703383395370399380
 435548931455046604860814615100*a²*e^(587/2*I*c) + 492566372511423396779395
 470271089768583454676105183446783117032701419176238036265879617045464540173
 61035725620405883520*a²*e^(585/2*I*c) + 4774617408598129132026036222830018
 345928279440677646144908614295224316238252464662558813944616663514704992754
 2868878520*a²*e^(583/2*I*c) + 46061425256403493806766816966330799438833477
 180998859399547393096765717302118692459528278697654284187890865264198490880
 *a²*e^(581/2*I*c) + 442190624143097968850365556828509107981326411741890913
 55070247306856269259520267158532714171765231548167835085083759540*a²*e<sup>(57
 9/2*I*c)</sup> + 4223767435092749730816760791277049538441050990699929435925460859
 8393831908627007418402687181252824823543923401086446140*a²*e^(577/2*I*c) +
 40137568053646614375433342668527901897822258306289397177277613987025021024
 009192926575679052108438711600692277041112940*a²*e^(575/2*I*c) + 379405002
 121705379064219274397400743295372769685079435761248824859837054922511077610
 60039819420913129160456600166235380*a²*e^(573/2*I*c) + 3566934156165823421
 939301591416525880023144723499217512011162195375877388035613051502633901375
 9664087801990290045546540*a²*e^(571/2*I*c) + 33347681639313713277648966791
 017546664147862774425612185646899774531221393386925714107547225285273054290
 314596594977580*a²*e^(569/2*I*c) + 309993863335928841596102155853059528548
 894077254599650850761049379761969117326386448795289705361225484689114086795
 08520*a²*e^(567/2*I*c) + 2864812461220060601372002405432777931524031087082
 1241651420240605545688144900537089198213263851453828234588917547711560*a²*
 e^(565/2*I*c) + 26316883694502424521248124387628880046480921579571736660778
 439611102049126719392867491978361605672355038563370490434320*a²*e<sup>(563/2*I
 *c)</sup> + 240274934594105298783423933145967012036923151395193747762638900709126

54038803572108760194498231541018946484500110764820*a²*e^(561/2*I*c) + 2180
018094657529251098759558153408779545501567139438439807556810202379312678969
1655257472560286308276031522589116147800*a²*e^(559/2*I*c) + 19653174438252
72675254179247777761993465022527145741346579124704111270169852610316237530
213837076731939155254031315220*a²*e^(557/2*I*c) + 176023739413341701586028
043034133442632963730516574822443010690461364577774852217599206838743383408
16767315010883070640*a²*e^(555/2*I*c) + 1566110116368601184782458239234044
886066938476679972162717922203521376820514126055609506038475263236181932703
2020892140*a²*e^(553/2*I*c) + 13839937609947187398920475238017490020995610
864862989156323115008246776677166646686425137922972069544356469424336874720
*a²*e^(551/2*I*c) + 121466545655047977836724156233552903854315215727546430
28081311118774641507895483012327171848886038795599157607589336080*a²*e<sup>(54
9/2*I*c)</sup> + 1058623385911710506650733480513866864889318683528870371443099194
6260066278599752585501637968876434411898083966268318240*a²*e^(547/2*I*c) +
91609737380630587614977376948117073446973469195890750935573822620798528221
62643836169596839715953389931229146306843160*a²*e^(545/2*I*c) + 7870670249
088720374430695127252578138183907600115484487567789602589829762952732597658
246852175521239128818848207047200*a²*e^(543/2*I*c) + 671286141852261572498
885847379562331424805795488413681987456423308014088803365671773607882805859
8240124888994298326360*a²*e^(541/2*I*c) + 56831194084038593110007516240036
231488608208471207766480066528273851195374220703844734081088304510355083937
26980174240*a²*e^(539/2*I*c) + 4775374748791231505459683996665278146102043
463548662889916224775723595988244933194330547948361773890650352555746361400
*a²*e^(537/2*I*c) + 398225668372341294098700260843210346553977219943913203
0360365812174562265353125356974005033580943527632143017811668560*a²*e<sup>(535
/2*I*c)</sup> + 32954345219222178101235110820882370320399432856468780343002902891
32360253350516006721630787778671059075241422781406160*a²*e^(533/2*I*c) + 2
705946499449963077304219148759975776135453653483937337917566194336892195568
305631554713116849660441746258385455723160*a²*e^(531/2*I*c) + 220450484881
927125857768664743780267386022877783506686274740578283877639034809855701229
5819426306998948329682289436640*a²*e^(529/2*I*c) + 17817683182949669352805
978484140547237298363548717084325768592352016467647943896189509505874913530
81375055442537775544*a²*e^(527/2*I*c) + 1428576087907541154070167528944535
179807628768650181368225728380419899154719079023876946406819487778508438319
784170080*a²*e^(525/2*I*c) + 113613969356153715597856894489986099071622461
8498598331698010751957196904760729746742387960564144749497838935456211960*a
²*e^(523/2*I*c) + 89619203559750372092718070780770660287026480877260476229
5169218466556192230960167844078573413391828541761332410051040*a²*e<sup>(521/2*
I*c)</sup> + 70109468901968289141662396829212671690759940305371215843475555034594
7349948295539046888909742638931217537474025975440*a²*e^(519/2*I*c) + 54390
646671281235190491475598291783510948221333649366216246531003493460618896286
7551968325252055883629254890215235232*a²*e^(517/2*I*c) + 41841747313718145
724123850908732932528240893968772160629659911251776533958725558718379766446
0597112939981628640119137*a²*e^(515/2*I*c) + 31915372081830498743200217153
354088007397856339768457818477791698779929316308840820473607637021492100311

7423047033230*a²*e^(513/2*I*c) + 24135779371134732823276095907187284642841
 340127313893630085300960070198840606458425903101922511495617389647665626833
 5*a²*e^(511/2*I*c) + 18095108246181918186958133049887925724287257181157189
 2048382981786970569721390987257245905924108342179247496262397340*a²*e<sup>(509
 /2*I*c)</sup> + 13448285460457216264288409674469529571350049873864520065103554859
 2354933239483430231951682574771378015247514707342287*a²*e^(507/2*I*c) + 99
 070933341429719760297004545746887055158017775506196150779591897966930321674
 638084629595248846439217986186167691566*a²*e^(505/2*I*c) + 723381176581835
 441253396192787045700703980625039684691659235031502702525498675037936494052
 57860558789674577875168502*a²*e^(503/2*I*c) + 5234775479141456200969042474
 636259615597005932613387746572682372750616929974976286787233738293308030257
 5832354312830*a²*e^(501/2*I*c) + 37541134530931294990707628311887896641738
 001248736045544225472758128612479435539648539936267245503594181132790075635
 *a²*e^(499/2*I*c) + 266786626041668418480957889241293019794489328404101843
 29153779340794009872890361466821390436483345842294941747333409*a²*e<sup>(497/2
 *I*c)</sup> + 1878612345612180246044477763374134866227498929577412232794604113679
 9723934764027545573154102942013646829238996755521*a²*e^(495/2*I*c) + 13106
 784376557914060123584282973971616802765627886013186728125299737503997244788
 676729350715523367908968202016814961*a²*e^(493/2*I*c) + 905963489471626610
 604125011465485981917077095899489064382326729497161271325211266792503418048
 4391205562621557500199*a²*e^(491/2*I*c) + 62036998076732947111872108021144
 212850853086945744288084100595853011031037228772905226051408470437457943030
 26775415*a²*e^(489/2*I*c) + 4208105879900965671044811603186871788137050268
 249716642908852922450166735904108996538151632302993343796013355257880*a²*e<sup>^
 (487/2*I*c)</sup> + 282741076291765734606529292670334627300017205316244818090637
 6970875870523643742229301867770298371409250162486706446*a²*e^(485/2*I*c) +
 18816046186806990270245435156986520031264265100046299196376707678510637525
 49093913764335909644924041423364457564056*a²*e^(483/2*I*c) + 1240155640575
 226832112461446587635151800206589007390172229369031582678860752755332793310
 035944135341130684921794627*a²*e^(481/2*I*c) + 809475688654593052224501094
 178917252980035293905898453796392919564557037778617746976730948724902348115
 224486645600*a²*e^(479/2*I*c) + 523218326398909441228138386569661789123283
 174843980047262040319411958519785242263748926347748867588273669418790115*a<sup>^
 2</sup>*e^(477/2*I*c) + 334877229377355089426560502609284393492614010501825504426
 666861674218991625217228236262755043572863766464617539152*a²*e^(475/2*I*c)
 + 212218928188673385825585486875626217952905358371974613119305643729514631
 087950307170160593893009749320293414137639*a²*e^(473/2*I*c) + 133153018441
 395196034014770358763691000098636960739548768552317876384874768681586531648
 288117265885968911213464902*a²*e^(471/2*I*c) + 827102161961927446592859468
 136034747447939482963463317531062070637037540124443643287008599794591092228
 67113617510*a²*e^(469/2*I*c) + 5086061865420950083533559153160520120167735
 2862466159943571481429018705622891681452604178883809245979957523278415*a²*
 e^(467/2*I*c) + 30959348642576562860165299817366165862922648668425796475504
 851207868785347857317439600867394648879054091070118592*a²*e^(465/2*I*c) +
 186536264628505328030351405881047175140854310988488694196891866543573543835

55540804832857748868154391812479528331*a²*e^(463/2*I*c) + 1112422688241289
810643238399018764325994794191948692614355578299900846301535590589525141555
4260511452162767106896*a²*e^(461/2*I*c) + 65657679593071530228829834619744
537570730217235007718128652478013026176517587654983378556847546660473321093
46955*a²*e^(459/2*I*c) + 3835169648312177412955758957045588072559813234777
488138237723537853906921795690040312404465308498665963807731880*a²*e<sup>(457/
2*I*c)</sup> + 221687596232449256065647525206945961628074801982111539229505307996
3518361922106809558395835470562396612703910894*a²*e^(455/2*I*c) + 12680329
744048849958800906653423684542730040005610160238536791866755170095503645042
48846180076370911172808894376*a²*e^(453/2*I*c) + 7176732492067055394376757
587392009746258445290245292770747026094047894580816662037578752339867778433
39431778971*a²*e^(451/2*I*c) + 4018878501750450801503138899100726945280094
47016468072611453193966632487465676074617056051773919819622001323159*a²*e ^(449/2*I*c) + 2226587982427923341810022061092801040978501685797895273675993
87695046282889021401012025548745828097666778231565*a²*e^(447/2*I*c) + 1220
412812685754398344798830299069919442016006758186640132562301549139645806434
06692381897603394093043703442025*a²*e^(445/2*I*c) + 6617297932881809399215
054642382035162752852372860854002119931270260080224075665893897413823318345
7153977060797*a²*e^(443/2*I*c) + 35492518572284783261790721091007984213268
935867291381099943547658996866612281989451531913916163009433157485571*a²*e ^(441/2*I*c) + 188300010367427394010071984622268846044353279420402274579036
83414392625117454825138218461037881756069460095462*a²*e^(439/2*I*c) + 9880
899198808563287233613519855332010559992941772794510424197819767831159955107
339288892516466317402233238590*a²*e^(437/2*I*c) + 512802793210456013555902
659319254460459290112053791574182285298030011519159280360551407770705478031
3193678930*a²*e^(435/2*I*c) + 26320081955436011268865672586710800827931793
30943932952648086736257575292194786895789417136105832466949897075*a²*e<sup>(43
3/2*I*c)</sup> + 1335925163388878989942887543237873573477354078078040384838340726
239029587514889857191688753727386206824536152*a²*e^(431/2*I*c) + 670517439
437730975214058382921475629655071200139334454271221441426048415295599199783
168621980211682925980307*a²*e^(429/2*I*c) + 332771401353095630876781482769
166072007566368785944639929752492013340507467607203423204297878103391257399
650*a²*e^(427/2*I*c) + 163291912624014896869889921756370257404344116207519
220566929295476847426915494195734247976197944827166856605*a²*e^(425/2*I*c)
+ 792210813736582642728798486708339631583938026185461305074634403335089580
81178065157747031476513568516431440*a²*e^(423/2*I*c) + 3799695085003403823
385083030620582717001776425358694149802829312074251648466449921908403693306
2000158769320*a²*e^(421/2*I*c) + 18016188203161151595570847680260903085281
484547568939274327212858568842514711832067992981021945668432453744*a²*e<sup>(4
19/2*I*c)</sup> + 844417714938119303347880124874317013490279241301539231977224491
7564145947358160863746469962582546517551020*a²*e^(417/2*I*c) + 39120632922
983706718236562651806275103810426889069614510874217109674609809742431592402
93773328259061298480*a²*e^(415/2*I*c) + 1791356871286179453076261895297548
714275627637620322018468056279043104785643232974994540293493766790807180*a²
*e^(413/2*I*c) + 810698514587734599341419926912945883958489502890417749835

477031467724072007792742636256985473924406832880*a²*e^(411/2*I*c) + 362585
708102908727461692262308497693854332721503869703743775437736913882412243026
230183024991966728272700*a²*e^(409/2*I*c) + 160253943840621057078138437984
495917828994512049873464602063929877936771758934087025740251939283794149320
*a²*e^(407/2*I*c) + 699883682746643885808837367127383031593273515051422993
04675566880142922981296353915575157471652388092680*a²*e^(405/2*I*c) + 3020
188861789511923414776779708658681221749045460105123652572942343771647683389
6661754018658154022389900*a²*e^(403/2*I*c) + 12876713525916989223366823193
023054088090488752457893124234397628679992437662682878994494276660041769680
*a²*e^(401/2*I*c) + 542387741535614073424007653028137639230718364875451835
2919070721950593841907858843558278519237201933020*a²*e^(399/2*I*c) + 22569
366401976461724095887544833910482389125597195976929160987113401427554372850
78061404880797678311440*a²*e^(397/2*I*c) + 9276920980270654837657520653179
96252674826674534631116047718959743097855192680766331861590769408198460*a²
*e^(395/2*I*c) + 3766458445736155461841665109904161546459400028487664654416
78202978044480377315238707708815985878943120*a²*e^(393/2*I*c) + 1510346812
759617816700818807418659314515708088392791048041237012856390637478435227932
14727666247721320*a²*e^(391/2*I*c) + 5981404789492439511671517626292625873
5827009585441667308321325957072358824867958015638415996637992560*a²*e<sup>(389
/2*I*c)</sup> + 23392734792358449986470914988641685348168613137145022740977224897
697160854430483589336483946581829650*a²*e^(387/2*I*c) + 903395267459197202
409932332240700921947876387600923769278945094738861263186426467979796147352
9847760*a²*e^(385/2*I*c) + 34447742392644906310173946350452828676614441736
47280134489449067279690770880154195712638143690670510*a²*e^(383/2*I*c) + 1
296869787700945503527478186253438429283866993105679751956812206964316306426
908093383593771068875180*a²*e^(381/2*I*c) + 482003855527359842058748938929
565762498717173867153381418816829773057750173701195879408932882129710*a²*e
^(379/2*I*c) + 176843212577304640940434087070526800206912147942357555568484
158874194832893725593513293339771528640*a²*e^(377/2*I*c) + 640434248339001
489626605649373085847925884105561358833921722009766306748265383141633435705
82742220*a²*e^(375/2*I*c) + 2289142246227600597089769949517582743299142275
4209646711791861074723763534400400991756009136093340*a²*e^(373/2*I*c) + 80
750674801709208568026954585195592726223421020352975266698203267805840061588
74417298884536622170*a²*e^(371/2*I*c) + 2810975036067791526020783161414349
035430610964164391290128775005718676199562281038906617666401970*a²*e<sup>(369/
2*I*c)</sup> + 965532218209251264607039716473737729829357676534904149014904268128
459659120470520755736939192070*a²*e^(367/2*I*c) + 327217244888055084262892
186197327446953686436737681340634739093336235382375514336377928793244690*a²
*e^(365/2*I*c) + 109401947851812253736939736757431402194995906470393287332
344122103685909885397712947473326518610*a²*e^(363/2*I*c) + 360821429557893
729217953262761637201159366694434304278949238218238357123958250182349331573
02110*a²*e^(361/2*I*c) + 1173805156223944291043665868661695293162135916390
1525749755059400912331209404508874859876592800*a²*e^(359/2*I*c) + 37661117
341863148447457366228609850041109045082888101894089219697224576875890481518
75880889220*a²*e^(357/2*I*c) + 1191626026712004520228535962916711181964681

032979355080990640973734774508809765193594567308000*a²*e^(355/2*I*c) + 371
784628277848407873740730843158415533102422282317148023178730401149797296961
025637580441730*a²*e^(353/2*I*c) + 114367222654004146039785488306144607682
565282217212089722715844480032864714063018623336102960*a²*e^(351/2*I*c) +
346835347778821728555902478391614597115084782685995951439789374590650980629
10312662980093090*a²*e^(349/2*I*c) + 1036832427833694706645534803655654449
4726913476560521689641087749606367891595175422870841040*a²*e^(347/2*I*c) +
30549765052290531863581965128998236791965392700731645777988941493815635842
47883834020602010*a²*e^(345/2*I*c) + 8870961481117838197926980752116379373
50128251820639471668845531985819368043596429487700180*a²*e^(343/2*I*c) + 2
538313201292654886772680409124822491889037103512786729720327185914058761314
00102115746900*a²*e^(341/2*I*c) + 7156131327668605954342040988322253447335
2439855772274727763362387210317509376711352805370*a²*e^(339/2*I*c) + 19875
383270198123016398073954782048143086744479176706215294931939285231552377668
666774800*a²*e^(337/2*I*c) + 543752145104203145537028209930834510131239658
0420987248763253424089743393247174970193410*a²*e^(335/2*I*c) + 14651342290
643008303482102337827023384371775676500548985537313589645208636961428271381
60*a²*e^(333/2*I*c) + 3887639689201023745839058832751335703731382951698483
77636334256622974927805402558286690*a²*e^(331/2*I*c) + 1015702233120008898
89955269913274747383718323298377402360165758159408055448033759362720*a²*e^(329/2*I*c) +
2612502097352610282419068257137595608809294834524040846588877
6831062032840805359036420*a²*e^(327/2*I*c) + 66144400820020510453479219244
12787836337662112455399887745116058097858565172028583520*a²*e^(325/2*I*c)
+ 1648201642139146705633894149783476374765855031698399268829056921671864832
034920363790*a²*e^(323/2*I*c) + 404148737461691129448272381486081487215931
511785097115337433651585897523208487479570*a²*e^(321/2*I*c) + 975027064963
44518739819416482817785708749119763377367851251579953471302421416590690*a²
*e^(319/2*I*c) + 2314012260343947977239284444734426757824128849128269332682
0133793119469871909054310*a²*e^(317/2*I*c) + 54015092850750556638448628821
38729276809816929299978333470094669272025087067952130*a²*e^(315/2*I*c) + 1
239909297438968458498020421044704877786634944232208108330855107855869727352
550810*a²*e^(313/2*I*c) + 279841843449723329022256022542885182491449861432
225949373308219431658133659304060*a²*e^(311/2*I*c) + 620874280599337021448
51939557816743050943015221536726159024340696975430971481900*a²*e<sup>(309/2*I*
c)</sup> + 1353882290279227574856360451606447921132436445561055966537495695805741
1438898160*a²*e^(307/2*I*c) + 29010807054572012234808075159505626906960549
42137423459448133177752600175055070*a²*e^(305/2*I*c) + 6107355768486469132
29035316690015481201283216147648837631072938937795953569020*a²*e<sup>(303/2*I*
c)</sup> + 1262906659766494237527952933005583650780909483682448789722636070203679
69724510*a²*e^(301/2*I*c) + 2564609163526230666377598990704819369302648107
2707785982065347970437174552640*a²*e^(299/2*I*c) + 51133713404620801884705
79216332247051199800933775813472986030153667768479970*a²*e^(297/2*I*c) + 1
000762165064573065580039704950705594622141532403147100702228104518209922480
*a²*e^(295/2*I*c) + 192216100832677211425583919374607410256140382963064998
218563326540739057800*a²*e^(293/2*I*c) + 362225078234474520943161523836818

05847086822685973455693062020799131961680*a²*e^(291/2*I*c) + 6695578122665
 234305226870074465850318321143798508148119852380345744581420*a²*e<sup>(289/2*I
 *c)</sup> + 121368408647448003003296352490387203682620064291014365746800769037730
 1200*a²*e^(287/2*I*c) + 21568229390436704822945256652517016545108634584777
 6055567286526724207180*a²*e^(285/2*I*c) + 37565891265483421625922327237663
 393973061791344075817295565545641550480*a²*e^(283/2*I*c) + 641088541188620
 8227235614369244284094773850732880341584569571247989180*a²*e^(281/2*I*c) +
 1071661434847365356402500490116585199491741089623963750947662209022120*a²
 *e^(279/2*I*c) + 1754194641818378612067054904914996117618974940308428735970
 94326137448*a²*e^(277/2*I*c) + 2810857079443600488986815509542529197552433
 7124542327266338411863980*a²*e^(275/2*I*c) + 44075310711231539469155754424
 07867368273204436441682325952058880560*a²*e^(273/2*I*c) + 6760779458630297
 42736783795661095141326065268116577325076514243900*a²*e^(271/2*I*c) + 1014
 11225754763725160776093014569051984766366679386214896417032560*a²*e<sup>(269/2
 *I*c)</sup> + 14869620923934928195860171826698271613545069265812546978930298844*a
²*e^(267/2*I*c) + 21304351655961354080475521993685030713318260420219498712
 47131824*a²*e^(265/2*I*c) + 2981357660920498212655085657598597572689019774
 04776167694840520*a²*e^(263/2*I*c) + 4073355402474013490085959435064795421
 2567407648957747058481680*a²*e^(261/2*I*c) + 54311275277768201077898230182
 31122146548845466324910314438155*a²*e^(259/2*I*c) + 7063590426004400906236
 27368646803874663354915637996034416454*a²*e^(257/2*I*c) + 8956697846475560
 1531386426232449217353087060101994516952837*a²*e^(255/2*I*c) + 11067166739
 593806537994432532202323892559358824814410948544*a²*e^(253/2*I*c) + 133186
 3531921951594349497448051501144604555177632910228645*a²*e^(251/2*I*c) + 15
 6018117308149873911753787065575404463673778724880411510*a²*e^(249/2*I*c) +
 17779824621644501534677987130821297785034706339145224178*a²*e^(247/2*I*c)
 + 1969922064355959547894504865149532487975181829580265482*a²*e<sup>(245/2*I*c
)</sup> + 212059444422641568370505689613891761234929288751535757*a²*e<sup>(243/2*I*c
)</sup> + 22164391506926711131826332938481074378353565258441643*a²*e^(241/2*I*c)
 + 2247655430280602939346801439987511004122770572057975*a²*e^(239/2*I*c) +
 220977262761597134870041878241189142663942444889435*a²*e^(237/2*I*c) + 21
 045445872134849680137861498182408126014563326457*a²*e^(235/2*I*c) + 193994
 2742070099509831143493249507373979963303677*a²*e^(233/2*I*c) + 17291964360
 7724441133173354565998605401103776504*a²*e^(231/2*I*c) + 14890299547694622
 406488788915648999996474069170*a²*e^(229/2*I*c) + 123742080470770509399110
 2638839493330163931640*a²*e^(227/2*I*c) + 99130382277532022515819177881629
 325075859509*a²*e^(225/2*I*c) + 764641982122710512439170071251750274718708
 8*a²*e^(223/2*I*c) + 567179441813227622186529241088113209451509*a²*e<sup>(221
 /2*I*c)</sup> + 40401820406264183313387690283892201648160*a²*e^(219/2*I*c) + 275
 9687033617389036404063195416999823985*a²*e^(217/2*I*c) + 18046998899665602
 3008385818021005185594*a²*e^(215/2*I*c) + 11279373940205552267581116020178
 794202*a²*e^(213/2*I*c) + 672482982322867333315410721290945657*a²*e<sup>(211/
 2*I*c)</sup> + 38167952336853964450194985322999760*a²*e^(209/2*I*c) + 2057571526
 219280176660569426269565*a²*e^(207/2*I*c) + 105091017765905750210205432017
 088*a²*e^(205/2*I*c) + 5071416371138697123516748312797*a²*e^(203/2*I*c) +

230518924833500391396633471432*a²*e^(201/2*I*c) + 98354740937679852848357
 40178*a²*e^(199/2*I*c) + 392372635871953337607706440*a²*e^(197/2*I*c) + 1
 4570867094745692063264465*a²*e^(195/2*I*c) + 501114476390712619058089*a²*
 e^(193/2*I*c) + 15866421408580747568967*a²*e^(191/2*I*c) + 459291145921837
 770879*a²*e^(189/2*I*c) + 12054885718238165655*a²*e^(187/2*I*c) + 2840156
 32089714525*a²*e^(185/2*I*c) + 5932441401233490*a²*e^(183/2*I*c) + 108143
 463042714*a²*e^(181/2*I*c) + 1685352670794*a²*e^(179/2*I*c) + 21830993145
 *a²*e^(177/2*I*c) + 225643340*a²*e^(175/2*I*c) + 1744665*a²*e^{(173/2*I*c}
) + 8970*a²*e^(171/2*I*c) + 23*a²*e^(169/2*I*c))/(e^(517*I*c) + 418*e<sup>(51
 6*I*c)</sup> + 87153*e^(515*I*c) + 12085216*e^(514*I*c) + 1253841160*e^(513*I*c)
 + 103818048048*e^(512*I*c) + 7146142307307*e^(511*I*c) + 420601518659718*e<sup>(
 510*I*c)</sup> + 21608403021340047*e^(509*I*c) + 984382804329835768*e^(508*I*c)
 + 40261256699368950388*e^(507*I*c) + 1493326612293984160368*e^(506*I*c) + 5
 0648660944512569972179*e^(505*I*c) + 1581796642397812408161814*e^(504*I*c)
 + 45759117183402579073139583*e^(503*I*c) + 1232445557346832245176696904*e<sup>(
 502*I*c)</sup> + 3104222522074681615625020522*e^(501*I*c) + 73405726361638844996
 8842366924*e^(500*I*c) + 16353164647151530240529137618111*e^(499*I*c) + 344
 277152012875134140739302960914*e^(498*I*c) + 686832922526368134950199734132
 0517*e^(497*I*c) + 130171193079172823835151430773360024*e^(496*I*c) + 23489
 98374244347079532766203075607598*e^(495*I*c) + 4044362478141531158185783238
 9099634564*e^(494*I*c) + 665634670676210063754191847109971141414*e^{(493*I*c}
) + 10490402669510897424624643766470754045064*e^(492*I*c) + 158566476113257
 562566117432227203884298856*e^(491*I*c) + 230215041122623492585522234520150
 0900533576*e^(490*I*c) + 32147887693375338817454482515377350383950278*e<sup>(48
 9*I*c)</sup> + 432333688644261557547944179250800440604964868*e^(488*I*c) + 560592
 7253067558551780452883689835514455118670*e^(487*I*c) + 70164515322544462906
 873548813748091084561870680*e^(486*I*c) + 848552202276512356496200136959676
 295361696315113*e^(485*I*c) + 992549073853440227293998703871458049544543137
 4618*e^(484*I*c) + 112391604542246650966429162063124338952554575234051*e<sup>(4
 83*I*c)</sup> + 1233096700139723365181997220750932590655287625342156*e^(482*I*c)
 + 13118781801172174729679339894318153694964675368481194*e^(481*I*c) + 13544
 2594916636116191574650625331646238501101627937224*e^(480*I*c) + 13579906631
 61479842850642848032544982878359839580349899*e^(479*I*c) + 1323170887010489
 6973800056733779919089340836756009580718*e^(478*I*c) + 12537049658692127266
 2198050851269323171167338854081782959*e^(477*I*c) + 11558554128935942603455
 44966642687823630035899363232371472*e^(476*I*c) + 1037518449987117550190939
 8956596684116802997082526660323524*e^(475*I*c) + 90722605722208814918642284
 639487187764607589706493970774776*e^(474*I*c) + 773204636991145775061462731
 028098506094432675788136295011259*e^(473*I*c) + 642619548553524857642506813
 6870465530087114003875716691383902*e^(472*I*c) + 52108117629177048660492400
 985175830987505700566877818954141639*e^(471*I*c) + 412430698299915190848067
 222327219435067747934091894670488982928*e^(470*I*c) + 318774992974434649721
 1536044751776582320958627923816470590659024*e^(469*I*c) + 24070801913529757
 101858022914372045864746991786182039740274325264*e^(468*I*c) + 177642829135
 119348577194437675802830239905460092687136494961404333*e^(467*I*c) + 128181

7464914970810859604189828359000790789921169405304612211251818*e^(466*I*c) +
 9046693523825682979044338963104263167672586826367911338826483549173*e^(465
 *I*c) + 6247355078105329531771077469024711412412518756573184844178190403267
 2*e^(464*I*c) + 42227612663200368754775474655570998871052713308666016136635
 3656787288*e^(463*I*c) + 27947091044756866118427906949736991644822547239772
 10209725661304403472*e^(462*I*c) + 1811576849561575807671030305550562558925
 4293659193314153418333944596408*e^(461*I*c) + 11505148185208084887370038835
 4521315567640365124003103691176697194292320*e^(460*I*c) + 71609949759905807
 9895633338552940229192858196481597830078819711862600096*e^(459*I*c) + 43694
 42482910113914565353136069595862669338858053419381214131241925047008*e^(458
 *I*c) + 2614397627990202144347194566508025456305681018352040188980028549314
 4867448*e^(457*I*c) + 15343608874505625412732723946157707193313015776459599
 7113973513183188399376*e^(456*I*c) + 88350096882179120260077454192776920073
 7689393513734789368397093333311961880*e^(455*I*c) + 49925197124570439835053
 77976607953988397368297591114957991804893688371867680*e^(454*I*c) + 2769311
 6538343259225983382637647936122664033859615133489846664694361471028310*e^(4
 53*I*c) + 15082238143141241377356647421001174685229743759705918629524398948
 1140398152780*e^(452*I*c) + 80667954360758914075930501079618956826984202161
 3388955218916278823182639488190*e^(451*I*c) + 42381258467632325863941885698
 58685826755328005548627437019301405851325887594480*e^(450*I*c) + 2187648289
 2713909928040345612578705805121508756226696317087651824252241418663320*e^(4
 49*I*c) + 11096919968732097474992225959525044434121921853534965576259119257
 6535872151766080*e^(448*I*c) + 55326912881952861250291886955894782909802195
 6309349843584044631512291778800081490*e^(447*I*c) + 27118432396707175276056
 40490148833507130242448403978318523237721944200392830108580*e^(446*I*c) + 1
 306981720348828988619320550837581839212499138234016031688650718129654898101
 4818410*e^(445*I*c) + 61948596653035502879564338815234310660410902037882473
 161804774492916216575880077680*e^(444*I*c) + 288820755264730654469968572021
 047109427318619508995802020689904590319476295408324280*e^(443*I*c) + 132475
 641236783747315747282116248369112096650194895392649224164378826428454643722
 1120*e^(442*I*c) + 59789921729441432184591611492998197063217321115784945252
 45228742976468409105395536290*e^(441*I*c) + 2655680638904340753449670236910
 1545795994861757741414789944652712127566910185274123140*e^(440*I*c) + 11610
 455168355504376291150171211639931373302113267748111282404724636179404963572
 6479850*e^(439*I*c) + 49970756725385908435759631481379476806933719091596749
 1907488904933922677579665354338960*e^(438*I*c) + 21175897334668557071015014
 29210414722401838837940752841618541440888545729943138209036820*e^(437*I*c)
 + 8836720640860470305694514021547969551296794092266983044118375790025854584
 036796364768280*e^(436*I*c) + 363183696523025917321974444097981220226408246
 04130552506742586795183267354382847875885730*e^(435*I*c) + 1470308167322768
 33163041582099592047512043725225353339238819165193000407629544745753221740*
 e^(434*I*c) + 5864034669726832427416433289215609093751974538642432995719909
 64608857245771134145204174990*e^(433*I*c) + 2304351073373840357379178597673
 066352016682781689139842097376663118488803841131935313641840*e^(432*I*c) +
 892320944734329676333188188163847179349961867060102605973089596265329177022

9493028162575100*e^(431*I*c) + 34054053851295569154352346722177172655187548
 910782008504718324168725029438589162349211628040*e^(430*I*c) + 128098914601
 688539672480541830409847707367500438601536803204497701119911289087105659482
 783340*e^(429*I*c) + 475010578857601519272316617938425222421786597241671026
 894318515408511467140969393115768793680*e^(428*I*c) + 173657421881819107187
 4197472450158123883564209950658639102337148122769080611680719741726053840*e
 ^ (427*I*c) + 62598721568222528436509607082350347102013627760571766472263230
 89751446565288850103898153859920*e^(426*I*c) + 222519591767957775716736603
 6007480222211364232146399803864370963391491223687245823457351580140*e^(425*
 I*c) + 78009807368024239875613733058851417125327114681070889640794249282633
 470580756557083923203377160*e^(424*I*c) + 269745801440211296972683601863878
 954357962308520076595177128227629273240215209708218497363414140*e^(423*I*c)
 + 920089393029589032874601850027159322612526368444771489781974361078847528
 891468831038436064951920*e^(422*I*c) + 309613197162152016238030155424146545
 1782362086810287537748902904985934020179565706177131421614590*e^(421*I*c) +
 10279364730663840844739577862469262604648861914297972589165243530651230690
 726244462479199894255180*e^(420*I*c) + 336753988720215683759023845939827533
 62559801058104184627345411136262431943240778260721756991027090*e^(419*I*c)
 + 1088679957318294728267329051920348867972846213564456275309091044294867412
 57822633476898356826454040*e^(418*I*c) + 3473514732147137808743520831295666
 01238765762775942366762733349952103889753982636403857556867777300*e^(417*I*
 c) + 1093853214486220358674032434500866678499770011305874172488975951612031
 456734608287095519501041975440*e^(416*I*c) + 340023256060165161752169468084
 7089844198028831694417424794868779328950548418125605446882081152636090*e^(4
 15*I*c) + 10434117516570395966653693155582402109460348095473027807412321427
 346816928567197770376496170251803940*e^(414*I*c) + 316109393312846927506943
 064436184146560959695209452157430040445603868952418015791565434519407133517
 30*e^(413*I*c) + 9455618025893198691933430346636565282685809131432918916073
 6277175873841732196453379953705679466826880*e^(412*I*c) + 27928575580003520
 667983536889816547764486498779466538782748893386363374504737310904926517268
 1702585720*e^(411*I*c) + 81460818773653057967021002527192141559718336988121
 4299823291969785549876175969866367976653244974728560*e^(410*I*c) + 23465182
 192391051422381416330734647688991557089350257780476374126817815757654222191
 27409260159438712250*e^(409*I*c) + 6675866290371147358503766865669289010893
 543869830538708724945291580951179188296606158111257706968604740*e^(408*I*c)
 + 187599882188655635641636357359860732782557372574057062791088913663784284
 67414559930481172863538598193890*e^(407*I*c) + 5207517851879327038642926335
 154430695110499354250058293815524168940813867525460803084790716774857173472
 0*e^(406*I*c) + 14280179245022176248318087491882527413430513327541778008479
 5034644763509333503150517345864659667189417080*e^(405*I*c) + 38687621823427
 716563245172304997988926311528237460754169244317667399751374281359173617116
 9652250611186480*e^(404*I*c) + 10355619825920029352263845779086115486121114
 95080193573691339864706029186482466241805664949381049856258510*e^(403*I*c)
 + 273889562479526560335522764656600886280778305084825702911938903656162004
 262736182657700406301914070062380*e^(402*I*c) + 715812468684294147547380736

367983971817274558153840904450338385269359692162242669674045394471814302524
8390*e^(401*I*c) + 18487405299005732693752728611876490890858357021974882371
570623800186245137722660943641752976852924439870880*e^(400*I*c) + 471882208
434662076950995069535737803571088974914225678980481990182077089970053338601
48836479527456156014520*e^(399*I*c) + 1190418554038779649482295779483704656
00606623183045529526900430209270473212773847794935586074714329479939280*e^(
398*I*c) + 2968255152826695896853182732802390500845550322034159415119626595
96881615713799937680026497408305672297618840*e^(397*I*c) + 7315849722068183
628747296214039744442800104463011615273397605448153009517879855384197646562
14582667219914080*e^(396*I*c) + 1782446114931751850556354856638421901174412
322298249496591658053939787198246565945975595575734193348887952160*e^(395*I
*c) + 429320647800802212601748890885182649479062072066015145146818191091724
0027863968724539127659633517053002976480*e^(394*I*c) + 10223182025954860767
217390305186451923562145473674293619918063490411487496121804590274592702770
571515456414680*e^(393*I*c) + 240687851397052771611934656445061432852413610
37768216818922184400141048460210944696647752723371932874594597328*e^(392*I*
c) + 5602868342490351765849501385853451616716259103436797249817466090745066
677815435327163034465077885683547624184*e^(391*I*c) + 12896708008475471224
602368086648838498328625902553313204463610904954514402954700334776152166628
3977931640178464*e^(390*I*c) + 29355074355434270980812945357656231329970598
2699187416862934373964255615967138676253276302591561523515603264403*e^(389*
I*c) + 66076447310586909769147597385083793451108903314958670798276426339476
6756649565279879146173318386505740391093990*e^(388*I*c) + 14709311466189343
455150383623001001604821277495814439299047469102247774701988990523791144939
99887003199419829579*e^(387*I*c) + 3238491931361851476423321933539579098377
735539207641467346235665823887048326949305609231585143748690203615957136*e^(
386*I*c) + 705213241416219799260232652458014306098535305457293390552463312
1681021037340298366342203324325307072413739061024*e^(385*I*c) + 15189634214
908800396417911722643754748048520109734812459109878810493844381062650818971
199637121458749456243274416*e^(384*I*c) + 323627313224195494103300889436402
474603783285613164229312924271459028879130716436795029090558912367551432073
82609*e^(383*I*c) + 6820803309679361568378440961924421081861499164004155342
4405527876893272496608324231098148502466453967157728078994*e^(382*I*c) + 14
221311596481451768238666727676990948227168131879088984050103944174863554536
2467679832449103520321953011780083069*e^(381*I*c) + 29334492003430072028704
238344834286631380628545504006782308044559754597002344623156355413513310549
3516316320059272*e^(380*I*c) + 59865014111224185891167650518052015036400322
6841328081453597093587790338609212439085554466861582623350303061961052*e^(3
79*I*c) + 1208770358493658393089442220569350632837041081405937502265398461
17737648216609559734831601248698274330296158612144*e^(378*I*c) + 2414966516
810338503289076549202740511710059011795447138773464205696455026442712426409
599662771080264826008985061097*e^(377*I*c) + 477414111106609897022184533059
496201647271423037423406066395684695092664268594692906411419440036093622359
0725470146*e^(376*I*c) + 93393419580534942252517509657150573007073020838147
74770306218224241022648247419956042957363055823830898547303219757*e^(375*I*

c) + 1807982006802885997034993862300723067656331420670884849990013964123733
4763266479346963237936039328113185041591793848*e^(374*I*c) + 34637657172671
690167657344537197087048882354853993270472063943078773600446542963548348101
269390443464480754513928502*e^(373*I*c) + 656748592688673000988273758128752
256106545516862611036816640070075371157780972935335652438288733837229803532
00611956*e^(372*I*c) + 1232439415193323847419600725881035065964063392536163
91082062969960682419011745775738921817753391954462609323881489157*e^(371*I*
c) + 2289113117385927800914926491623468344058677407764563261084109288572571
74707289268074347550225793244741923354395308214*e^(370*I*c) + 4208463426089
493872775590214579245865781209661485610226470084995294684520059801751194106
28956210497609566002969884927*e^(369*I*c) + 7658677955139627810125584446287
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59798934952*e^(368*I*c) + 1379676529796212074017106188066589448355446501210
890195107164860350228928586815539003062875026711931941947738690360722*e^(36
7*I*c) + 246044237584542266392708163098326071473496809190549302714563923882
7192254886349361126991457692409851120873307487457468*e^(366*I*c) + 43439096
966019321733573596877815792937012956819408271142154331753360939678459087667
40738240037114570667410936998017178*e^(365*I*c) + 7592752700146678961153095
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947581048693527224*e^(364*I*c) + 131397714941049338818566811514182931122425
515215356868711812665798138776063481602617472013177357825660213067982983360
24*e^(363*I*c) + 2251467574130806996150616558650287243042193021067326439299
7286485600640103867253604847715547060592967690653795951142520*e^(362*I*c) +
38199015867586087976002998756627674994795440667903625029322346250133286489
120875005013638128113893960349670280707161530*e^(361*I*c) + 641751006932600
668062380648860045971707408433000868393686161391645291080498446753531118427
25798658088840347241496099644*e^(360*I*c) + 1067648320171655948380852341893
335287335876733299725300926610851867899392529159370907602822323469190904262
43399409323314*e^(359*I*c) + 1758962582627559857571068126139793012658010315
95484353614904672865169442232075776580447184134141375995770091499246759528*
e^(358*I*c) + 2869929436312314965572780108515769408968264974660663275288015
60677007112837431926735088120974861760511367008815728782643*e^(357*I*c) + 4
637582884573671545449376782550056887333281455680493104239955998860128006386
19904022368378591108842602342094543682299102*e^(356*I*c) + 7422286409081731
249169370494625256173341489196791182704898310054977819512210699558396234524
99748653124658873553401442137*e^(355*I*c) + 1176600720975786965189875050890
231092204612696970277433014535895788956771230793520381993106606880564628599
822341722801012*e^(354*I*c) + 184750585646245153344528430057132632378116255
330456597188767075809107930679482183492817077312636463972207157013170378533
4*e^(353*I*c) + 28736105359223401870808354355829122772719679773947201597910
70274927714276869531467182688981041061381703885403497544001592*e^(352*I*c)
+ 4427673079105425318524316112985693656584851936100192457044455134483305045
321452516347118488133224823670465103483954805161*e^(351*I*c) + 675848043788
852437256293594896385762669485554719551948612287756798171858726236287199496
7079401831957927901682582941234362*e^(350*I*c) + 10220423779434634851339975

295163399641702122249663666193053008302026096932158568338309418237395541351
819026907953220681013*e^(349*I*c) + 153128372066627753793473532128076829657
125356529426315182861424030977382002707111953965821590285135327796821544519
96208592*e^(348*I*c) + 2273160356612884110041950194705136766683665241807726
0913944810748473084891890410181285412604854876625919565639521227223276*e^(3
47*I*c) + 33435897827936581301171175459610829454298167962017419810072936733
378506584428024201072453193458155334046693516742390717832*e^(346*I*c) + 487
332535059749234008522555630521014021964693136595544927256747543393752830104
07167744366955828922837488705858532439654489*e^(345*I*c) + 7038634976059483
156704822406139502569850120229696630037676422033669770296159109985405541137
6294871437468149528524796002762*e^(344*I*c) + 10074496185185374461175430098
298016696240455383622292186848469426996612060769890704634373101116094882810
0276729370132819357*e^(343*I*c) + 14290631912305552424654692847895423837131
592580202238923649865213683982250203515567697091741903983458796705558843156
6416784*e^(342*I*c) + 20090658715357880438030046950144161017452185125954192
9209840688960859454908519774835905895757666770857888611738751858460424*e^(3
41*I*c) + 27994524447503980482296673046296088449211987485779114712400907947
6920435941735293309305430438687333129912454196774070107264*e^(340*I*c) + 38
664267305038004945738256281831696265197555099077927704874023862985879501824
7356162888631015687664780101205287333082748791*e^(339*I*c) + 52932925276411
392600393483695824355767254923899756073921440659918504783195557258376535863
4395408771528009745467548382950094*e^(338*I*c) + 71836159638205824920911354
448790108886838874403371321033249197137590673834155154045726480430403966425
5915607349801911966551*e^(337*I*c) + 96645827536903771874773913079815164348
359068416683223468809829116416063641815945211981572880937212516883623936444
2397344064*e^(336*I*c) + 12890435152929339564806343304996770401810439356201
06914267311067900030058398839787692376954090545278554544997710058754772400*
e^(335*I*c) + 1704582996707822808204678218167693002698661147712772355021456
543810930069637188085882824757500605246963210810351706405349408*e^(334*I*c)
+ 223489127639843946447862257830643484072461048446817785982262065869192147
8645266653062563823553001228001009093606751066168944*e^(333*I*c) + 29053857
223200570019533452744894827908566925299598237495326959634141648333667731282
18607899328588608916176593772088622582464*e^(332*I*c) + 3745257594876651204
657334988426226388143954501986830664222349226361079609546822276067504899386
703088982308185717143407211328*e^(331*I*c) + 478752744278094568514520484697
159616530416941932824407321145959212964925504887685405984472066107815128817
9612574986359194560*e^(330*I*c) + 60689498031567122483318711053298954717228
061430088780149865596536872606948165504701958900045119655275674327229697075
77202160*e^(329*I*c) + 7629731815627821580468992424207008366438896736333024
661863838105110445148946962328297631547032543419811821015837863013682720*e^(
328*I*c) + 951303227401952295420911319126822664229991201352566594029838106
4797885690904993128948035227412144035633851779511219335277360*e^(327*I*c) +
11764212274876484080010900714673474493371278160557811983724455826566055617
658086479368641864908119643412413644803772131657280*e^(326*I*c) + 144298162
852084312045329783753756919650631542246497475512958515073895240832269767896

88601369628399900747658579201929300744260*e^(325*I*c) + 1755627327122429239
688729140312571621349148626114547857137675169010565606783804215103827138130
0372757755676325408026834544840*e^(324*I*c) + 21188321405882887539610198374
706862695894049226077093764132512513336190523978949694387686059124526755048
042957954264706637460*e^(323*I*c) + 253671764391193536215322603359833481549
049826061257617113006834929633908164915830257052687375399821496393002265126
57426118880*e^(322*I*c) + 3012848241455270326455901895308817715601343749343
820107841376983544836614812175454919759112996717076496970018034869920783896
0*e^(321*I*c) + 35500103106019649876272376796949482209581372371036005012877
806027481672807059943445240136315568500732379966585005678181937920*e^(320*I
*c) + 414998321219637080437885237874013455417800889305382069188535790267492
73364671640037563488607716092887686471542838602788559660*e^(319*I*c) + 4813
311767818402921650374854911037447892471909463560389282936486391655379227882
2957368285106328164715910598370871149079494360*e^(318*I*c) + 55390913044972
086219432689146331566081427959896969900214434296817731150863867056620768608
187679709720152974148474907904177340*e^(317*I*c) + 632477741010121790517949
460751755699240769813381384831580424067474538747293876317105449952471529122
05118500597511052824347680*e^(316*I*c) + 7166032986117339552444194388928410
913409115784465524567208423740243494469646492713181219065962951114063950174
3303863582092880*e^(315*I*c) + 80566249130682684181876201882623511206363790
337218011954110210642927765997644903820595421936873565314654415769070472655
401600*e^(314*I*c) + 898838158013823822139732704779546027447928770180519633
47146307372464315121274929402347942874802899499538953561056667668891020*e^(
313*I*c) + 9951220647205796595134034173802354851533640337171789804085047095
4657532977279113491506880290726111154101941386019689567958040*e^(312*I*c) +
10933253734996622320393267850342635707986370700172829401104207653040392386
2654018978676516417314221089449922495612732870169660*e^(311*I*c) + 11920971
370203392705575539782368844444464742432450218532862634704659963472114657383
0681540495333543146776810911910410468628960*e^(310*I*c) + 12899507601159190
341076386342709732994858617357459586270584915928094304645874266316345401849
1463855395649453952212899632198680*e^(309*I*c) + 13852979454915108945135276
957654340312633074724368003083246720589581904356815523926487676286717275433
8684027849855385453216080*e^(308*I*c) + 14764892080554533341862312176785377
739978292474830122879392434257499993795542176537010123512293955746754854920
2174550009604780*e^(307*I*c) + 15618596295355119616973821883217369650985255
158921073057836572747625947647446595542850233667374368649917569867787569361
1243400*e^(306*I*c) + 16397781605960772537526455981650584789418778510145536
0391897424482998415385787605765315509208337741590143078572243505132706580*e
^(305*I*c) + 17086984886895310117686030605310399434053039034726008843267684
2505555141293830838961275974268928666494845723462544709102843680*e^(304*I*c
) + 17672092997055464200457577005309570059533465987068273203197591553238757
7052414866323511140117680492929354517559479899220940360*e^(303*I*c) + 18140
816877092205982036855331669732163998486262829882856956027329563089762682934
5263592219034560853530733710529842148537901680*e^(302*I*c) + 18483115198374
894181766785017470825713812817215826941328776535853224077324433619190081855

7829905895684494889410451921524212840*e^(301*I*c) + 18691547443656751492635
140562311750326198750835193008382456644443568913923368341170464182876217879
9177848064220150818355261280*e^(300*I*c) + 18761539316851005007149728056460
351091240313292031202437083506267903764499028628534667350709345296435125796
2696133511725652320*e^(299*I*c) + 18691547443656751492635140562311750326198
750835193008382456644443568913923368341170464182876217879917784806422015081
8355261280*e^(298*I*c) + 18483115198374894181766785017470825713812817215826
941328776535853224077324433619190081855782990589568449488941045192152421284
0*e^(297*I*c) + 18140816877092205982036855331669732163998486262829882856956
0273295630897626829345263592219034560853530733710529842148537901680*e^(296*
I*c) + 17672092997055464200457577005309570059533465987068273203197591553238
7577052414866323511140117680492929354517559479899220940360*e^(295*I*c) + 17
086984886895310117686030605310399434053039034726008843267684250555514129383
0838961275974268928666494845723462544709102843680*e^(294*I*c) + 16397781605
960772537526455981650584789418778510145536039189742448299841538578760576531
5509208337741590143078572243505132706580*e^(293*I*c) + 15618596295355119616
973821883217369650985255158921073057836572747625947647446595542850233667374
3686499175698677875693611243400*e^(292*I*c) + 14764892080554533341862312176
785377739978292474830122879392434257499993795542176537010123512293955746754
8549202174550009604780*e^(291*I*c) + 13852979454915108945135276957654340312
633074724368003083246720589581904356815523926487676286717275433868402784985
5385453216080*e^(290*I*c) + 12899507601159190341076386342709732994858617357
459586270584915928094304645874266316345401849146385539564945395221289963219
8680*e^(289*I*c) + 11920971370203392705575539782368844444464742432450218532
8626347046599634721146573830681540495333543146776810911910410468628960*e^(2
88*I*c) + 10933253734996622320393267850342635707986370700172829401104207653
0403923862654018978676516417314221089449922495612732870169660*e^(287*I*c) +
99512206472057965951340341738023548515336403371717898040850470954657532977
279113491506880290726111154101941386019689567958040*e^(286*I*c) + 898838158
013823822139732704779546027447928770180519633471463073724643151212749294023
47942874802899499538953561056667668891020*e^(285*I*c) + 8056624913068268418
187620188262351120636379033721801195411021064292776599764490382059542193687
3565314654415769070472655401600*e^(284*I*c) + 71660329861173395524441943889
284109134091157844655245672084237402434944696464927131812190659629511140639
501743303863582092880*e^(283*I*c) + 632477741010121790517949460751755699240
769813381384831580424067474538747293876317105449952471529122051185005975110
52824347680*e^(282*I*c) + 5539091304497208621943268914633156608142795989696
990021443429681773115086386705662076860818767970972015297414847490790417734
0*e^(281*I*c) + 48133117678184029216503748549110374478924719094635603892829
364863916553792278822957368285106328164715910598370871149079494360*e^(280*I
*c) + 414998321219637080437885237874013455417800889305382069188535790267492
73364671640037563488607716092887686471542838602788559660*e^(279*I*c) + 3550
010310601964987627237679694948220958137237103600501287780602748167280705994
3445240136315568500732379966585005678181937920*e^(278*I*c) + 30128482414552
703264559018953088177156013437493438201078413769835448366148121754549197591

129967170764969700180348699207838960*e^(277*I*c) + 253671764391193536215322
603359833481549049826061257617113006834929633908164915830257052687375399821
49639300226512657426118880*e^(276*I*c) + 2118832140588288753961019837470686
269589404922607709376413251251333619052397894969438768605912452675504804295
7954264706637460*e^(275*I*c) + 17556273271224292396887291403125716213491486
261145478571376751690105656067838042151038271381300372757755676325408026834
544840*e^(274*I*c) + 144298162852084312045329783753756919650631542246497475
51295851507389524083226976789688601369628399900747658579201929300744260*e^(
273*I*c) + 1176421227487648408001090071467347449337127816055781198372445582
6566055617658086479368641864908119643412413644803772131657280*e^(272*I*c) +
95130322740195229542091131912682266422999120135256659402983810647978856909
04993128948035227412144035633851779511219335277360*e^(271*I*c) + 7629731815
627821580468992424207008366438896736333024661863838105110445148946962328297
631547032543419811821015837863013682720*e^(270*I*c) + 606894980315671224833
187110532989547172280614300887801498655965368726069481655047019589000451196
5527567432722969707577202160*e^(269*I*c) + 47875274427809456851452048469715
961653041694193282440732114595921296492550488768540598447206610781512881796
12574986359194560*e^(268*I*c) + 3745257594876651204657334988426226388143954
501986830664222349226361079609546822276067504899386703088982308185717143407
211328*e^(267*I*c) + 290538572232005700195334527448948279085669252995982374
9532695963414164833366773128218607899328588608916176593772088622582464*e^(2
66*I*c) + 22348912763984394644786225783064348407246104844681778598226206586
91921478645266653062563823553001228001009093606751066168944*e^(265*I*c) + 1
704582996707822808204678218167693002698661147712772355021456543810930069637
188085882824757500605246963210810351706405349408*e^(264*I*c) + 128904351529
293395648063433049967704018104393562010691426731106790003005839883978769237
6954090545278554544997710058754772400*e^(263*I*c) + 96645827536903771874773
913079815164348359068416683223468809829116416063641815945211981572880937212
5168836239364442397344064*e^(262*I*c) + 71836159638205824920911354448790108
886838874403371321033249197137590673834155154045726480430403966425591560734
9801911966551*e^(261*I*c) + 52932925276411392600393483695824355767254923899
756073921440659918504783195557258376535863439540877152800974546754838295009
4*e^(260*I*c) + 38664267305038004945738256281831696265197555099077927704874
0238629858795018247356162888631015687664780101205287333082748791*e^(259*I*c
) + 27994524447503980482296673046296088449211987485779114712400907947692043
5941735293309305430438687333129912454196774070107264*e^(258*I*c) + 20090658
715357880438030046950144161017452185125954192920984068896085945490851977483
5905895757666770857888611738751858460424*e^(257*I*c) + 14290631912305552424
654692847895423837131592580202238923649865213683982250203515567697091741903
9834587967055588431566416784*e^(256*I*c) + 10074496185185374461175430098298
016696240455383622292186848469426996612060769890704634373101116094882810027
6729370132819357*e^(255*I*c) + 70386349760594831567048224061395025698501202
296966300376764220336697702961591099854055411376294871437468149528524796002
762*e^(254*I*c) + 487332535059749234008522555630521014021964693136595544927
25674754339375283010407167744366955828922837488705858532439654489*e^(253*I*

c) + 3343589782793658130117117545961082945429816796201741981007293673337850
 6584428024201072453193458155334046693516742390717832*e^(252*I*c) + 22731603
 566128841100419501947051367666836652418077260913944810748473084891890410181
 285412604854876625919565639521227223276*e^(251*I*c) + 153128372066627753793
 473532128076829657125356529426315182861424030977382002707111953965821590285
 13532779682154451996208592*e^(250*I*c) + 1022042377943463485133997529516339
 964170212224966366619305300830202609693215856833830941823739554135181902690
 7953220681013*e^(249*I*c) + 67584804378885243725629359489638576266948555471
 95519486122877567981718587262362871994967079401831957927901682582941234362*
 e^(248*I*c) + 4427673079105425318524316112985693656584851936100192457044455
 134483305045321452516347118488133224823670465103483954805161*e^(247*I*c) +
 287361053592234018708083543558291227727196797739472015979107027492771427686
 9531467182688981041061381703885403497544001592*e^(246*I*c) + 18475058564624
 515334452843005713263237811625533045659718876707580910793067948218349281707
 73126364639722071570131703785334*e^(245*I*c) + 1176600720975786965189875050
 890231092204612696970277433014535895788956771230793520381993106606880564628
 599822341722801012*e^(244*I*c) + 742228640908173124916937049462525617334148
 919679118270489831005497781951221069955839623452499748653124658873553401442
 137*e^(243*I*c) + 463758288457367154544937678255005688733328145568049310423
 995599886012800638619904022368378591108842602342094543682299102*e^(242*I*c)
 + 286992943631231496557278010851576940896826497466066327528801560677007112
 837431926735088120974861760511367008815728782643*e^(241*I*c) + 175896258262
 755985757106812613979301265801031595484353614904672865169442232075776580447
 184134141375995770091499246759528*e^(240*I*c) + 106764832017165594838085234
 189333528733587673329972530092661085186789939252915937090760282232346919090
 426243399409323314*e^(239*I*c) + 641751006932600668062380648860045971707408
 433000868393686161391645291080498446753531118427257986580888403472414960996
 44*e^(238*I*c) + 3819901586758608797600299875662767499479544066790362502932
 2346250133286489120875005013638128113893960349670280707161530*e^(237*I*c) +
 22514675741308069961506165586502872430421930210673264392997286485600640103
 867253604847715547060592967690653795951142520*e^(236*I*c) + 131397714941049
 338818566811514182931122425515215356868711812665798138776063481602617472013
 17735782566021306798298336024*e^(235*I*c) + 7592752700146678961153095073585
 015473197029746533633331549793961473285760935801904155116764831560875947581
 048693527224*e^(234*I*c) + 434390969660193217335735968778157929370129568194
 0827114215433175336093967845908766740738240037114570667410936998017178*e^(2
 33*I*c) + 24604423758454226639270816309832607147349680919054930271456392388
 27192254886349361126991457692409851120873307487457468*e^(232*I*c) + 1379676
 529796212074017106188066589448355446501210890195107164860350228928586815539
 003062875026711931941947738690360722*e^(231*I*c) + 765867795513962781012558
 444628751418710940895281304790836743661582071650032154891482866406314834433
 199455459798934952*e^(230*I*c) + 420846342608949387277559021457924586578120
 966148561022647008499529468452005980175119410628956210497609566002969884927
 *e^(229*I*c) + 228911311738592780091492649162346834405867740776456326108410
 928857257174707289268074347550225793244741923354395308214*e^(228*I*c) + 123

243941519332384741960072588103506596406339253616391082062969960682419011745
775738921817753391954462609323881489157*e^(227*I*c) + 656748592688673000988
273758128752256106545516862611036816640070075371157780972935335652438288733
83722980353200611956*e^(226*I*c) + 3463765717267169016765734453719708704888
235485399327047206394307877360044654296354834810126939044346448075451392850
2*e^(225*I*c) + 18079820068028859970349938623007230676563314206708848499900
139641237334763266479346963237936039328113185041591793848*e^(224*I*c) + 933
934195805349422525175096571505730070730208381477477030621822424102264824741
9956042957363055823830898547303219757*e^(223*I*c) + 47741411110660989702218
453305949620164727142303742340606639568469509266426859469290641141944003609
36223590725470146*e^(222*I*c) + 2414966516810338503289076549202740511710059
011795447138773464205696455026442712426409599662771080264826008985061097*e^(
221*I*c) + 120877035849365839308944222205693506328370410814059375022653984
6117737648216609559734831601248698274330296158612144*e^(220*I*c) + 59865014
111224185891167650518052015036400322684132808145359709358779033860921243908
5554466861582623350303061961052*e^(219*I*c) + 29334492003430072028704238344
834286631380628545504006782308044559754597002344623156355413513310549351631
6320059272*e^(218*I*c) + 14221311596481451768238666727676990948227168131879
0889840501039441748635545362467679832449103520321953011780083069*e^(217*I*c
) + 68208033096793615683784409619244210818614991640041553424405527876893272
496608324231098148502466453967157728078994*e^(216*I*c) + 323627313224195494
103300889436402474603783285613164229312924271459028879130716436795029090558
91236755143207382609*e^(215*I*c) + 1518963421490880039641791172264375474804
8520109734812459109878810493844381062650818971199637121458749456243274416*e
^(214*I*c) + 70521324141621979926023265245801430609853530545729339055246331
21681021037340298366342203324325307072413739061024*e^(213*I*c) + 3238491931
361851476423321933539579098377735539207641467346235665823887048326949305609
231585143748690203615957136*e^(212*I*c) + 147093114661893434551503836230010
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9579*e^(211*I*c) + 66076447310586909769147597385083793451108903314958670798
2764263394766756649565279879146173318386505740391093990*e^(210*I*c) + 29355
074355434270980812945357656231329970598269918741686293437396425561596713867
6253276302591561523515603264403*e^(209*I*c) + 12896708008475471224602368086
648838498328625902553313204463610904954514402954700334776152166628397793164
0178464*e^(208*I*c) + 56028683424903517658495013858534516167162591034367972
498174660907450666778154353271630344650777885683547624184*e^(207*I*c) + 240
687851397052771611934656445061432852413610377682168189221844001410484602109
44696647752723371932874594597328*e^(206*I*c) + 1022318202595486076721739030
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6414680*e^(205*I*c) + 42932064780080221260174889088518264947906207206601514
51468181910917240027863968724539127659633517053002976480*e^(204*I*c) + 1782
446114931751850556354856638421901174412322298249496591658053939787198246565
945975595575734193348887952160*e^(203*I*c) + 731584972206818362874729621403
974444280010446301161527339760544815300951787985538419764656214582667219914
080*e^(202*I*c) + 296825515282669589685318273280239050084555032203415941511

962659596881615713799937680026497408305672297618840*e^(201*I*c) + 119041855
 403877964948229577948370465600606623183045529526900430209270473212773847794
 935586074714329479939280*e^(200*I*c) + 471882208434662076950995069535737803
 57108897491422567898048199018207708997005333860148836479527456156014520*e^(
 199*I*c) + 1848740529900573269375272861187649089085835702197488237157062380
 0186245137722660943641752976852924439870880*e^(198*I*c) + 71581246868429414
 754738073636798397181727455815384090445033838526935969216224266967404539447
 18143025248390*e^(197*I*c) + 2738895624795265603355227646566000886280778305
 084825702911938903656162004262736182657700406301914070062380*e^(196*I*c) +
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 2466241805664949381049856258510*e^(195*I*c) + 38687621823427716563245172304
 997988926311528237460754169244317667399751374281359173617116965225061118648
 0*e^(194*I*c) + 14280179245022176248318087491882527413430513327541778008479
 5034644763509333503150517345864659667189417080*e^(193*I*c) + 52075178518793
 270386429263351544306951104993542500582938155241689408138675254608030847907
 167748571734720*e^(192*I*c) + 187599882188655635641636357359860732782557372
 57405706279108891366378428467414559930481172863538598193890*e^(191*I*c) + 6
 675866290371147358503766865669289010893543869830538708724945291580951179188
 296606158111257706968604740*e^(190*I*c) + 234651821923910514223814163307346
 4768899155708935025778047637412681781575765422219127409260159438712250*e^(1
 89*I*c) + 81460818773653057967021002527192141559718336988121429982329196978
 5549876175969866367976653244974728560*e^(188*I*c) + 27928575580003520667983
 536889816547764486498779466538782748893386363374504737310904926517268170258
 5720*e^(187*I*c) + 94556180258931986919334303466365652826858091314329189160
 736277175873841732196453379953705679466826880*e^(186*I*c) + 316109393312846
 927506943064436184146560959695209452157430040445603868952418015791565434519
 40713351730*e^(185*I*c) + 1043411751657039596665369315558240210946034809547
 3027807412321427346816928567197770376496170251803940*e^(184*I*c) + 34002325
 606016516175216946808470898441980288316944174247948687793289505484181256054
 46882081152636090*e^(183*I*c) + 1093853214486220358674032434500866678499770
 011305874172488975951612031456734608287095519501041975440*e^(182*I*c) + 347
 351473214713780874352083129566601238765762775942366762733349952103889753982
 636403857556867777300*e^(181*I*c) + 108867995731829472826732905192034886797
 284621356445627530909104429486741257822633476898356826454040*e^(180*I*c) +
 336753988720215683759023845939827533625598010581041846273454111362624319432
 40778260721756991027090*e^(179*I*c) + 1027936473066384084473957786246926260
 4648861914297972589165243530651230690726244462479199894255180*e^(178*I*c) +
 30961319716215201623803015542414654517823620868102875377489029049859340201
 79565706177131421614590*e^(177*I*c) + 9200893930295890328746018500271593226
 12526368444771489781974361078847528891468831038436064951920*e^(176*I*c) + 2
 697458014402112969726836018638789543579623085200765951771282276292732402152
 09708218497363414140*e^(175*I*c) + 7800980736802423987561373305885141712532
 7114681070889640794249282633470580756557083923203377160*e^(174*I*c) + 22251
 95917679577775716736603600748022211364232146399803864370963391491223687245
 823457351580140*e^(173*I*c) + 625987215682225284365096070823503471020136277

6057176647226323089751446565288850103898153859920*e^(172*I*c) + 17365742188
181910718741974724501581238835642099506586391023371481227690806116807197417
26053840*e^(171*I*c) + 4750105788576015192723166179384252224217865972416710
26894318515408511467140969393115768793680*e^(170*I*c) + 1280989146016885396
72480541830409847707367500438601536803204497701119911289087105659482783340*
e^(169*I*c) + 3405405385129556915435234672217717265518754891078200850471832
4168725029438589162349211628040*e^(168*I*c) + 89232094473432967633318818816
38471793499618670601026059730895962653291770229493028162575100*e^(167*I*c)
+ 2304351073373840357379178597673066352016682781689139842097376663118488803
841131935313641840*e^(166*I*c) + 586403466972683242741643328921560909375197
453864243299571990964608857245771134145204174990*e^(165*I*c) + 147030816732
276833163041582099592047512043725225353339238819165193000407629544745753221
740*e^(164*I*c) + 363183696523025917321974444097981220226408246041305525067
42586795183267354382847875885730*e^(163*I*c) + 8836720640860470305694514021
547969551296794092266983044118375790025854584036796364768280*e^(162*I*c) +
211758973346685570710150142921041472240183883794075284161854144088854572994
3138209036820*e^(161*I*c) + 49970756725385908435759631481379476806933719091
5967491907488904933922677579665354338960*e^(160*I*c) + 11610455168355504376
2911501712116399313733021132677481112824047246361794049635726479850*e^(159*
I*c) + 26556806389043407534496702369101545795994861757741414789944652712127
566910185274123140*e^(158*I*c) + 597899217294414321845916114929981970632173
2111578494525245228742976468409105395536290*e^(157*I*c) + 13247564123678374
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*I*c) + 2888207552647306544699685720210471094273186195089958020206899045903
19476295408324280*e^(155*I*c) + 6194859665303550287956433881523431066041090
2037882473161804774492916216575880077680*e^(154*I*c) + 13069817203488289886
193205508375818392124991382340160316886507181296548981014818410*e^(153*I*c)
+ 271184323967071752760564049014883350713024244840397831852323772194420039
2830108580*e^(152*I*c) + 55326912881952861250291886955894782909802195630934
9843584044631512291778800081490*e^(151*I*c) + 11096919968732097474992225959
5250444341219218535349655762591192576535872151766080*e^(150*I*c) + 21876482
892713909928040345612578705805121508756226696317087651824252241418663320*e^(
149*I*c) + 423812584676323258639418856985868582675532800554862743701930140
5851325887594480*e^(148*I*c) + 80667954360758914075930501079618956826984202
1613388955218916278823182639488190*e^(147*I*c) + 15082238143141241377356647
4210011746852297437597059186295243989481140398152780*e^(146*I*c) + 27693116
538343259225983382637647936122664033859615133489846664694361471028310*e^(14
5*I*c) + 499251971245704398350537797660795398839736829759111495799180489368
8371867680*e^(144*I*c) + 88350096882179120260077454192776920073768939351373
4789368397093333311961880*e^(143*I*c) + 15343608874505625412732723946157707
1933130157764595997113973513183188399376*e^(142*I*c) + 26143976279902021443
471945665080254563056810183520401889800285493144867448*e^(141*I*c) + 436944
2482910113914565353136069595862669338858053419381214131241925047008*e^(140*
I*c) + 71609949759905807989563333855294022919285819648159783007881971186260
0096*e^(139*I*c) + 11505148185208084887370038835452131556764036512400310369

$1176697194292320 * e^{(138 * I * c)} + 18115768495615758076710303055505625589254293$
 $659193314153418333944596408 * e^{(137 * I * c)} + 279470910447568661184279069497369$
 $9164482254723977210209725661304403472 * e^{(136 * I * c)} + 42227612663200368754775$
 $4746555709988710527133086660161366353656787288 * e^{(135 * I * c)} + 62473550781053$
 $295317710774690247114124125187565731848441781904032672 * e^{(134 * I * c)} + 904669$
 $3523825682979044338963104263167672586826367911338826483549173 * e^{(133 * I * c)} +$
 $1281817464914970810859604189828359000790789921169405304612211251818 * e^{(132$
 $* I * c)} + 177642829135119348577194437675802830239905460092687136494961404333 *$
 $e^{(131 * I * c)} + 2407080191352975710185802291437204586474699178618203974027432$
 $5264 * e^{(130 * I * c)} + 31877499297443464972115360447517765823209586279238164705$
 $90659024 * e^{(129 * I * c)} + 4124306982999151908480672223272194350677479340918946$
 $70488982928 * e^{(128 * I * c)} + 5210811762917704866049240098517583098750570056687$
 $7818954141639 * e^{(127 * I * c)} + 64261954855352485764250681368704655300871140038$
 $75716691383902 * e^{(126 * I * c)} + 7732046369911457750614627310280985060944326757$
 $88136295011259 * e^{(125 * I * c)} + 9072260572220881491864228463948718776460758970$
 $6493970774776 * e^{(124 * I * c)} + 10375184499871175501909398956596684116802997082$
 $526660323524 * e^{(123 * I * c)} + 115585541289359426034554496664268782363003589936$
 $3232371472 * e^{(122 * I * c)} + 12537049658692127266219805085126932317116733885408$
 $1782959 * e^{(121 * I * c)} + 13231708870104896973800056733779919089340836756009580$
 $718 * e^{(120 * I * c)} + 1357990663161479842850642848032544982878359839580349899 * e$
 $^{(119 * I * c)} + 135442594916636116191574650625331646238501101627937224 * e^{(118 *}$
 $I * c)} + 13118781801172174729679339894318153694964675368481194 * e^{(117 * I * c)} +$
 $1233096700139723365181997220750932590655287625342156 * e^{(116 * I * c)} + 11239160$
 $4542246650966429162063124338952554575234051 * e^{(115 * I * c)} + 99254907385344022$
 $72939987038714580495445431374618 * e^{(114 * I * c)} + 8485522022765123564962001369$
 $59676295361696315113 * e^{(113 * I * c)} + 7016451532254446290687354881374809108456$
 $1870680 * e^{(112 * I * c)} + 5605927253067558551780452883689835514455118670 * e^{(111$
 $* I * c)} + 432333688644261557547944179250800440604964868 * e^{(110 * I * c)} + 3214788$
 $7693375338817454482515377350383950278 * e^{(109 * I * c)} + 23021504112262349258552$
 $22345201500900533576 * e^{(108 * I * c)} + 1585664761132575625661174322272038842988$
 $56 * e^{(107 * I * c)} + 10490402669510897424624643766470754045064 * e^{(106 * I * c)} + 66$
 $5634670676210063754191847109971141414 * e^{(105 * I * c)} + 40443624781415311581857$
 $832389099634564 * e^{(104 * I * c)} + 2348998374244347079532766203075607598 * e^{(103 *}$
 $I * c)} + 130171193079172823835151430773360024 * e^{(102 * I * c)} + 68683292252636813$
 $49501997341320517 * e^{(101 * I * c)} + 344277152012875134140739302960914 * e^{(100 * I *}$
 $c)} + 16353164647151530240529137618111 * e^{(99 * I * c)} + 734057263616388449968842$
 $366924 * e^{(98 * I * c)} + 3104222522074681615625020522 * e^{(97 * I * c)} + 123244555734$
 $6832245176696904 * e^{(96 * I * c)} + 45759117183402579073139583 * e^{(95 * I * c)} + 15817$
 $96642397812408161814 * e^{(94 * I * c)} + 50648660944512569972179 * e^{(93 * I * c)} + 1493$
 $326612293984160368 * e^{(92 * I * c)} + 40261256699368950388 * e^{(91 * I * c)} + 984382804$
 $329835768 * e^{(90 * I * c)} + 21608403021340047 * e^{(89 * I * c)} + 420601518659718 * e^{(88$
 $* I * c)} + 7146142307307 * e^{(87 * I * c)} + 103818048048 * e^{(86 * I * c)} + 1253841160 * e^{($
 $85 * I * c)} + 12085216 * e^{(84 * I * c)} + 87153 * e^{(83 * I * c)} + 418 * e^{(82 * I * c)} + e^{(81 * I$
 $* c)) * \tan(1/4 * d * x + c) + (-23 * I * a^2 * e^{(1027/2 * I * c)} - 8970 * I * a^2 * e^{(1025/2 * I$
 $* c)} - 1744665 * I * a^2 * e^{(1023/2 * I * c)} - 225643340 * I * a^2 * e^{(1021/2 * I * c)} - 21830$

993145*I*a^2*e^(1019/2*I*c) - 1685352670794*I*a^2*e^(1017/2*I*c) - 10814346
 3042754*I*a^2*e^(1015/2*I*c) - 5932441401249090*I*a^2*e^(1013/2*I*c) - 2840
 15632092748725*I*a^2*e^(1011/2*I*c) - 12054885718630588825*I*a^2*e^(1009/2*
 I*c) - 459291145959804703779*I*a^2*e^(1007/2*I*c) - 15866421411511793416437
 *I*a^2*e^(1005/2*I*c) - 501114476578787912641049*I*a^2*e^(1003/2*I*c) - 145
 70867105062952981500815*I*a^2*e^(1001/2*I*c) - 392372636365891369041933300*
 I*a^2*e^(999/2*I*c) - 9835474114732862868439582838*I*a^2*e^(997/2*I*c) - 23
 0518925632259863716881504052*I*a^2*e^(995/2*I*c) - 507141639873210303020152
 1505627*I*a^2*e^(993/2*I*c) - 105091018637393463590223550684728*I*a^2*e^(99
 1/2*I*c) - 2057571551559319523757433019101275*I*a^2*e^(989/2*I*c) - 3816795
 3019220461788377878471708140*I*a^2*e^(987/2*I*c) - 672482999427386036344407
 124264639911*I*a^2*e^(985/2*I*c) - 11279374341089024516950389703822811442*I
 *a^2*e^(983/2*I*c) - 180469997815998782717102062578903704334*I*a^2*e^(981/2
 *I*c) - 2759687216371547310850322318637953260785*I*a^2*e^(979/2*I*c) - 4040
 1823984348761731551610542858793511300*I*a^2*e^(977/2*I*c) - 567179508185660
 065903323118694229252909501*I*a^2*e^(975/2*I*c) - 7646420990625646107842349
 221267149768725400*I*a^2*e^(973/2*I*c) - 9913040189115061622703464037546131
 0347846749*I*a^2*e^(971/2*I*c) - 123742111851876855210771405305543808334261
 7820*I*a^2*e^(969/2*I*c) - 14890304346273125682306935846029108244114648650*
 I*a^2*e^(967/2*I*c) - 172919713857046613813065255730042186229347958876*I*a^
 2*e^(965/2*I*c) - 1939943728233712898582615509996583301055554831933*I*a^2*e
 ^ (963/2*I*c) - 21045459166656329934203131234228410676256368947239*I*a^2*e^(
 961/2*I*c) - 220977435109557963897348194591419349596075084671135*I*a^2*e^(9
 59/2*I*c) - 2247657581576890158328281459463053249136155142818885*I*a^2*e^(9
 57/2*I*c) - 22164417393124784256780107055497427057394288021910427*I*a^2*e^(
 955/2*I*c) - 212059745023134629411867496681427459939018060659823083*I*a^2*e
 ^ (953/2*I*c) - 1969925436555096263303543513345030378604353635783621398*I*a^
 2*e^(951/2*I*c) - 17779861202993747975617805656916892771717874809673606262*
 I*a^2*e^(949/2*I*c) - 15601850139083301492402893352264558109183668726757344
 7170*I*a^2*e^(947/2*I*c) - 133186743835690094739408533396330887566993379552
 7159861115*I*a^2*e^(945/2*I*c) - 110672052589031643118494748728371489474029
 11102321523307256*I*a^2*e^(943/2*I*c) - 89567346975466416436560959012746672
 316379146573614093235883*I*a^2*e^(941/2*I*c) - 7063624656191022682262028794
 62703285306437535601820801093746*I*a^2*e^(939/2*I*c) - 54311584202896956902
 67628216417883201506781644825673663895125*I*a^2*e^(937/2*I*c) - 40733825083
 879692084677912159273504330596927222466270616023040*I*a^2*e^(935/2*I*c) - 2
 98138079805795516636721746713666338776134639093417308458083640*I*a^2*e^(933
 /2*I*c) - 2130454389639784102539281096465899633052875625470754655905397856*
 I*a^2*e^(931/2*I*c) - 14869776488978491309444307452594685609833202276809994
 141174545956*I*a^2*e^(929/2*I*c) - 1014124524670115149879605084456799107947
 13086096737145187354675120*I*a^2*e^(927/2*I*c) - 67608737685731247414677178
 6049384206737469239071720613980492039300*I*a^2*e^(925/2*I*c) - 440760179517
 7120237389113481959743264257864428283490771370053462560*I*a^2*e^(923/2*I*c)
 - 28109088369373208155707785744000148722496865889470317867362512306740*I*a
 ^2*e^(921/2*I*c) - 17542316219649612394009093118130642266705844056917042285

1340745050152*I*a^2*e^(919/2*I*c) - 107168724187918360820483485443098817212
5784220926155326017366664031320*I*a^2*e^(917/2*I*c) - 641106138998571262583
9045934560628663538960416714378812599630238383260*I*a^2*e^(915/2*I*c) - 375
67064271487090398378167188966687791512776703919269010885401142609920*I*a^2*
e^(913/2*I*c) - 21568993975887401396718722083353373164324207421223622260516
5544572554060*I*a^2*e^(911/2*I*c) - 121373283851051030642898260238453861769
0483226714330573193357678961518000*I*a^2*e^(909/2*I*c) - 669588231701349522
2345368298476838368511994443900460264534471085890483820*I*a^2*e^(907/2*I*c)
- 362243658255481769079439605579958387100105565618941470113231351735814022
40*I*a^2*e^(905/2*I*c) - 19222721348090494887203432115491092098542580399780
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3781353911409779752889226260010612599185707764320*I*a^2*e^(901/2*I*c) - 511
3745039917782570942288672444404762240177072474457348268608658998353643330*I
*a^2*e^(899/2*I*c) - 256481940130631614458139173236114739489676613089088783
36387655736333479132840*I*a^2*e^(897/2*I*c) - 12630226136697081319012086306
8164729403993511960131743155255365483034776033710*I*a^2*e^(895/2*I*c) - 610
798290612478058634809110185506816479817707018034933796191469491921071289700
*I*a^2*e^(893/2*I*c) - 2901413407805046277699928806426570164361980656563461
341652112845236338748650110*I*a^2*e^(891/2*I*c) - 1354055460508770231039339
6477276367400444112984077543019342063464929767322886040*I*a^2*e^(889/2*I*c)
- 620962735163185935779611173057839829149089325652410656700713936757357671
76645100*I*a^2*e^(887/2*I*c) - 27988619397464174037567262664265306385526511
4960677901721413367956114946541063180*I*a^2*e^(885/2*I*c) - 124012762440879
0585914281618049099992149775260140355696852903419692530227177449210*I*a^2*e
^(883/2*I*c) - 540256474462854504612952008124460438123572687338459558380408
8637122848866439830910*I*a^2*e^(881/2*I*c) - 231451344052759630839547627482
31025164257628027633977804093014188701364289864915790*I*a^2*e^(879/2*I*c) -
97526087003487022632582290289825035947901332365723366039562344307089798567
346576790*I*a^2*e^(877/2*I*c) - 4042559161973144102059036313458389181315186
22031814368289391512007823341054252319330*I*a^2*e^(875/2*I*c) - 16486845255
78126198504085391316407022187711819746696897272619021774566583748173498450*
I*a^2*e^(873/2*I*c) - 66165787175526862693523980355038876126746090842609601
28356927902046841481605353166520*I*a^2*e^(871/2*I*c) - 26134333584281766199
506203706665359796711411159203716561602842513749476935891574081580*I*a^2*e^
(869/2*I*c) - 1016101003988137282448021482325200966315174895018501981958766
37177700803300250750696120*I*a^2*e^(867/2*I*c) - 38893191411963295484013014
9086140789452373855787807910212844151796049289004605406967150*I*a^2*e^(865/
2*I*c) - 146583001674253512451913146901065257757197591875644054160385146725
4363529939075434269440*I*a^2*e^(863/2*I*c) - 544035755123037490243998502739
4698228560509433313958273980496531279067808247723147105870*I*a^2*e^(861/2*I
*c) - 198867587799113605322612027213532735821605971082743528597158857825056
31373182736113224200*I*a^2*e^(859/2*I*c) - 71606217945850994717430682602764
559228622722227808236824799962339103295121387601682173670*I*a^2*e^(857/2*I*
c) - 2540058012870175403474010349780220858339706707380981477738865193816289
53611819486811955620*I*a^2*e^(855/2*I*c) - 88776357742106247494023221481733

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/2*I*c) - 34717589857137672406923008566078790348191298872900136928769377324
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e^(845/2*I*c) - 37221892415037889443572040902664005989095539259061628257216
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0*I*a^2*e^(841/2*I*c) - 377133056150951252461585059664034129435796729959627
1045598767418091013562271264018462771488980*I*a^2*e^(839/2*I*c) - 117557492
389849429622100629661757412192769384297540981500776815193118756218047057203
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- 1095968214363242841726512152035509927963690094832507106773453968058514982
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982082848801710226763359158924377562363927243394356202432140538650*I*a^2*e
^(831/2*I*c) - 967558857573420421200844825745019691984965901119321858915203
967518001948679909749229811961456530*I*a^2*e^(829/2*I*c) - 2817373842739968
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521970*I*a^2*e^(827/2*I*c) - 8094990486752031102862847380062157212388106523
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295259936503095142465474029958186708878313715335541773803916001593069745216
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e^(821/2*I*c) - 17739675542894941311253267670481610036345737247085728407535
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638579602610*I*a^2*e^(817/2*I*c) - 1301613491181268974287090549535283627357
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*I*c) - 3458383981160096213819834986181221606030887703758892748920536238488
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2966976789038657507310743872020449940329840767115547150*I*a^2*e^(809/2*I*c)
- 601108690622843741594482098547268387114298912719254377848217364138645524
41235504659113309232113779360*I*a^2*e^(807/2*I*c) - 15184249844321680132979
221701134989863470387416943076510333347856618440977436539297273379065418907
0040*I*a^2*e^(805/2*I*c) - 378816002128920616369855929612160874580768723711
392251915842505525188562342470825520416422818082592000*I*a^2*e^(803/2*I*c)
- 9334473368592818432243553973420584946686982924440282529349154876039965073
57110331502736639692759358340*I*a^2*e^(801/2*I*c) - 22720048811620935900466
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01360*I*a^2*e^(799/2*I*c) - 54628286678221586727361647210129385627818090917

70363563687018271454099131199171596676490634462439229220*I*a^2*e^(797/2*I*c)
) - 12976133089122616927641373067671760628448690333305759931883973052278026
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 466937068633484708402407505066968240527155699757540924494810770937806784721
 3898960660*I*a^2*e^(793/2*I*c) - 706120625711697868994464587973763116513326
 30936204874511242735563516386888084909599619786580787711236360*I*a^2*e^(791
 /2*I*c) - 16178708438189461126289696260126574890420941225938900637455184128
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 113543820574273500*I*a^2*e^(787/2*I*c) - 8196247895171130191642817095414875
 20368233436889440883128112185915263655651455982880577347113410414307040*I*a
 ^2*e^(785/2*I*c) - 18125017737117436736993992115331571295088238451140025138
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 24821817220970795789343876240*I*a^2*e^(781/2*I*c) - 85585699131774977720413
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 7/2*I*c) - 3858660998287657496223400479535149465741817808337559048890103195
 7266204930635139096439961363048441151269800*I*a^2*e^(775/2*I*c) - 805359935
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 648778017515414556569019749473008667024648037226698449778484768907857657655
 3405*I*a^2*e^(771/2*I*c) - 339081049398958631695033084312249444034247861955
 469658173982280143780778138876895429955188412068983435311370*I*a^2*e^(769/2
 *I*c) - 6840964393238201555734771754472551627703517728713097064328153201771
 80262386714126080576227107839963492633507*I*a^2*e^(767/2*I*c) - 13648077061
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 /2*I*c) - 10140082596804607404376090989361337712629273671384446340003481136
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 8155405769562070657818017062*I*a^2*e^(757/2*I*c) - 365490412121312297832746
 648031039823017060530166350998938749984756716538698774385724223738576621165
 57803137691*I*a^2*e^(755/2*I*c) - 68269842357184501611988226541759877359893
 560816980579165757639885910403041554476683238744066404341119496497267*I*a^2
 *e^(753/2*I*c) - 1261558844450858809909057230508638945302599921104251461305
 69149222574918984083710347803558027793365144729446565*I*a^2*e^(751/2*I*c) -
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 168264211297805134039*I*a^2*e^(747/2*I*c) - 7467281492243362913374727882275

078265152540554796342982961634192168809916584030065162729746571266025696676
20581*I*a^2*e^(745/2*I*c) - 13225410027687245271645902870752516183035559216
18083348921791582725369617919859156545068710911148219834222862396*I*a^2*e^(
743/2*I*c) - 23180048596133418845744953509005175801585656213363187889470522
14662319719223377004078504331647589087640360974954*I*a^2*e^(741/2*I*c) - 40
207296927287520725475279644145838807209599718985673780881907219211406891145
94980080070716969583769101530510140*I*a^2*e^(739/2*I*c) - 69025186656360372
837629802427849099678839353716623572378212215283291749068007923947329463677
48505381533617607645*I*a^2*e^(737/2*I*c) - 11728686977201016518903777818063
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921208*I*a^2*e^(735/2*I*c) - 1972682929919065151030155855601200340219105234
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^(733/2*I*c) - 328441725889849239257452481701006143358104612840057395251609
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I*a^2*e^(723/2*I*c) - 36166957083407726067731633129296551496029436983618740
3435553362802201139250475072743485753652885635408470996847372*I*a^2*e^(721/
2*I*c) - 567339614895630128342983728404200845089288305505097140213560633981
062751336174336795809192566239776955199921505915*I*a^2*e^(719/2*I*c) - 8813
693903152742201742418843817961522650552642497193087915022648144348491007593
37832840344343735714440306099566040*I*a^2*e^(717/2*I*c) - 13560749642327922
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I*a^2*e^(711/2*I*c) - 46644046194423261614083371806148988132314961825934722
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/2*I*c) - 69093103807838607892815109268897036078418535879007165861544111515
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139527875816467343220793498549170075839065604546707385281954497494579711245
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492561*I*a^2*e^(699/2*I*c) - 4288542606290519837613611568777949831319762493
9417241023712107311560166080651136642711507677332263314929421901176965*I*a^

$2 * e^{(697/2 * I * c)}$ - 601170368095319545347741753285424329177854712582703174435
 83066712344018654100343282519010777886383221363137483769490 * $I * a^2 * e^{(695/2 * I * c)}$ - 83517876975248044258519777946946588944585187567768264892079917516487
 130082429368116942958987653732471395054399139538 * $I * a^2 * e^{(693/2 * I * c)}$ - 1149
 948298540085448006040505267361290686621708123406823856790208279423291612545
 33974586296625609880932042763111506754 * $I * a^2 * e^{(691/2 * I * c)}$ - 15693375545189
 563385847170380611690734635789954797344262825130014540666798973864045763035
 7856900000058360558079161713 * $I * a^2 * e^{(689/2 * I * c)}$ - 212282878705188354649502
 352775941442121310160088506940041461217611549577446696222647786382828325447
 861991528982057300 * $I * a^2 * e^{(687/2 * I * c)}$ - 2846391060871606800731623460533476
 496613870616498866470992523092743038931742924481992332004821799692893195995
 83782705 * $I * a^2 * e^{(685/2 * I * c)}$ - 37833333468850290590480261430311039994326548
 6115785178740514728586424237491324029114632441782209381845333938523265890 * I
 $* a^2 * e^{(683/2 * I * c)}$ - 498509499153258403729144890980204589097433923264131841
 430599901592653873152072717512660804753287716928576078663461983 * $I * a^2 * e^{(68$
 $1/2 * I * c)}$ - 6511907152788140899870344218335142853224331708738436883692464497
 08975379749425293954979978533284470854614844971466368 * $I * a^2 * e^{(679/2 * I * c)}$ -
 84332451270191474454340290972061610796241599215459523939992540970763039518
 9954029608070898352689818705209984508369520 * $I * a^2 * e^{(677/2 * I * c)}$ - 108279791
 731158921336755666005271059821490164985258768135079765765438759876794889824
 8611815354945578372200449025175360 * $I * a^2 * e^{(675/2 * I * c)}$ - 137841218511280786
 821265588672751099560622640951984363764155174203766853911604106614005244018
 3078269143615845581155080 * $I * a^2 * e^{(673/2 * I * c)}$ - 173980645966267455699286776
 317959942984209389154812532101128066698219989189285649266441562234765005375
 2088149727624160 * $I * a^2 * e^{(671/2 * I * c)}$ - 217731969048282654771711229304304861
 992828592703831021692785468514017747785476203212307040869928379289825496288
 6739656 * $I * a^2 * e^{(669/2 * I * c)}$ - 270178097219987688678575421108092666409622887
 1020710457482488324723670365135550692459168767272049066762300379877743040 * I
 $* a^2 * e^{(667/2 * I * c)}$ - 332422018682049400516624062136424440385293496180132862
 1031245367970100658499892606876336284620902405991453408165588520 * $I * a^2 * e^{(6$
 $65/2 * I * c)}$ - 405549356101263502455389330044135493501084712983515452498569001
 6794545113963136786023289013915905378564692143643458640 * $I * a^2 * e^{(663/2 * I * c)}$
 - 490582253753402966135019137348573887282442883805673193090612167075380422
 0745738659870244363280999523327382897200441520 * $I * a^2 * e^{(661/2 * I * c)}$ - 588424
 91824322513907976215991562151516085555252739287872752903351193265754133961
 6490316199475103405475041577880587000 * $I * a^2 * e^{(659/2 * I * c)}$ - 699801709577143
 537605745995805012474680123419462022952575050728512485982101086171515351782
 6996821338842938734860615040 * $I * a^2 * e^{(657/2 * I * c)}$ - 825189325232861791713496
 500815678089607164024610954070331168646162697469777629068934860789885816287
 7878927869163809880 * $I * a^2 * e^{(655/2 * I * c)}$ - 964745305573534195674410966098187
 224380314183001636114689823440121961412095824497833740990607034623317101935
 8264000480 * $I * a^2 * e^{(653/2 * I * c)}$ - 111823577327573057886061252198327242557001
 856866976957507734569257457184696766716653323833754587819844030778673269564
 40 * $I * a^2 * e^{(651/2 * I * c)}$ - 12849659528917295448887838389721821666429652280839
 717230872070976527285743131454112500472959858220404943909001977151360 * $I * a^2$

$e^{(649/2*I*c)} - 1463717544059678626631832947276075907600136800889205562092$
 $7133799059282490190897494502217828119253048842224071312398480*I*a^2*e^{(647/}$
 $2*I*c)} - 165269738589748616391080233103711360543573746571987322127555465028$
 $92632722591536314813460778204894606881244239416717120*I*a^2*e^{(645/2*I*c)} -$
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 $27176730279973640942509792180*I*a^2*e^{(639/2*I*c)} - 24535311108873854631583$
 $838561781463173989229567453522895600654095852502781216398579180366741352217$
 $131290346900784364200*I*a^2*e^{(637/2*I*c)} - 2645398140906918779129366718128$
 $208253140007832522532325943072293403735930148722069099152411424873018733576$
 $3372429125460*I*a^2*e^{(635/2*I*c)} - 282430314010148401244252742372382140499$
 $209187646122119213805129779128897295468555686691099785416535836784586304283$
 $37120*I*a^2*e^{(633/2*I*c)} - 29848075656762410777392478417988562936605721803$
 $803630324956331103418688840889616409755917532127762470680082194173437320*I*$
 $a^2*e^{(631/2*I*c)} - 3121341361032589865622950752731666120779125407165377491$
 $0526924611586090854615810972507366138529383452875185539371668680*I*a^2*e^{(6$
 $29/2*I*c)} - 322838187691637519321622191504773951647461064909962397680112873$
 $01357561433297548231267227036903834777286313642263284780*I*a^2*e^{(627/2*I*c$
 $)} - 33006502605828069111234833523837274488458321604918549594915598999328301$
 $351391942676314725618514347956768945927684338260*I*a^2*e^{(625/2*I*c)} - 3333$
 $317752892357232584661182806039255076186042496358040604028455302686203422633$
 $3729553143376429959067700689886330788900*I*a^2*e^{(623/2*I*c)} - 332221305715$
 $899049824548255310152200708414431373762009302340011698341642975972176429628$
 $32109496476768915007520638843940*I*a^2*e^{(621/2*I*c)} - 32640210785137750395$
 $514344466824448907144608130646158918941737747797341437698189371184595982735$
 $305425435824684283776860*I*a^2*e^{(619/2*I*c)} - 3156462971757990011515613322$
 $118155603673009359681373455653706536893811895039416005506842669488246847459$
 $9065924339139340*I*a^2*e^{(617/2*I*c)} - 299844764403264089300993531197561297$
 $549816916660918748551593027415261824065408428430174430907554221039796910369$
 $78946000*I*a^2*e^{(615/2*I*c)} - 27901856653919484579343684902605106634990835$
 $693224277902772993509046816697554915822060373656496697112008237858612984680$
 $*I*a^2*e^{(613/2*I*c)} - 2533257934363435577954448716720210150053448362865394$
 $6329035782149123871587278642890878402071278332969173493767458002640*I*a^2*e$
 $^{(611/2*I*c)} - 223063337336447303794552102662515572156005721972231222888508$
 $91726285188766620799206359532978080306708567111158747156900*I*a^2*e^{(609/2*$
 $I*c)} - 18866322962402983332825118914013813247899665561112634287480400676793$
 $104541824749963087292981195024846572209729639189440*I*a^2*e^{(607/2*I*c)} - 1$
 $506834767209077602337603303019631330294176309712540943549497842058279271177$
 $3103135974244949606016312505788823825840740*I*a^2*e^{(605/2*I*c)} - 109793609$
 $947359760449164031447559769661960407617052049037495768799372161688349241491$
 $29784251472577056488047960350809520*I*a^2*e^{(603/2*I*c)} - 66755444804312177$
 $590827259183014300018779913559371858760261598546140768866654203546644767640$

61095432856806716718924980*I*a^2*e^(601/2*I*c) - 22399805579503550425420743
 519255673800958480817601183862232025353384336700773356150077735993427504734
 69385817621308920*I*a^2*e^(599/2*I*c) + 22399805579503550425420743519255673
 800958480817601183862232025353384336700773356150077735993427504734693858176
 21308920*I*a^2*e^(597/2*I*c) + 66755444804312177590827259183014300018779913
 55937185876026159854614076886665420354664476764061095432856806716718924980*
 I*a^2*e^(595/2*I*c) + 10979360994735976044916403144755976966196040761705204
 903749576879937216168834924149129784251472577056488047960350809520*I*a^2*e^
 (593/2*I*c) + 1506834767209077602337603303019631330294176309712540943549497
 8420582792711773103135974244949606016312505788823825840740*I*a^2*e^(591/2*I
 *c) + 188663229624029833328251189140138132478996655611126342874804006767931
 04541824749963087292981195024846572209729639189440*I*a^2*e^(589/2*I*c) + 22
 306333733644730379455210266251557215600572197223122288850891726285188766620
 799206359532978080306708567111158747156900*I*a^2*e^(587/2*I*c) + 2533257934
 363435577954448716720210150053448362865394632903578214912387158727864289087
 8402071278332969173493767458002640*I*a^2*e^(585/2*I*c) + 279018566539194845
 793436849026051066349908356932242779027729935090468166975549158220603736564
 96697112008237858612984680*I*a^2*e^(583/2*I*c) + 29984476440326408930099353
 119756129754981691666091874855159302741526182406540842843017443090755422103
 979691036978946000*I*a^2*e^(581/2*I*c) + 3156462971757990011515613322118155
 603673009359681373455653706536893811895039416005506842669488246847459906592
 4339139340*I*a^2*e^(579/2*I*c) + 326402107851377503955143444668244489071446
 081306461589189417377477973414376981893711845959827353054254358246842837768
 60*I*a^2*e^(577/2*I*c) + 33222130571589904982454825531015220070841443137376
 200930234001169834164297597217642962832109496476768915007520638843940*I*a^2
 *e^(575/2*I*c) + 3333317752892357232584661182806039255076186042496358040604
 0284553026862034226333729553143376429959067700689886330788900*I*a^2*e^(573/
 2*I*c) + 330065026058280691112348335238372744884583216049185495949155989993
 28301351391942676314725618514347956768945927684338260*I*a^2*e^(571/2*I*c) +
 32283818769163751932162219150477395164746106490996239768011287301357561433
 297548231267227036903834777286313642263284780*I*a^2*e^(569/2*I*c) + 3121341
 361032589865622950752731666120779125407165377491052692461158609085461581097
 2507366138529383452875185539371668680*I*a^2*e^(567/2*I*c) + 298480756567624
 107773924784179885629366057218038036303249563311034186888408896164097559175
 32127762470680082194173437320*I*a^2*e^(565/2*I*c) + 28243031401014840124425
 274237238214049920918764612211921380512977912889729546855568669109978541653
 583678458630428337120*I*a^2*e^(563/2*I*c) + 2645398140906918779129366718128
 208253140007832522532325943072293403735930148722069099152411424873018733576
 3372429125460*I*a^2*e^(561/2*I*c) + 245353111088738546315838385617814631739
 892295674535228956006540958525027812163985791803667413522171312903469007843
 64200*I*a^2*e^(559/2*I*c) + 22538536115685128509739061738013766406006974581
 099364247180428351985546405904602601376439227176730279973640942509792180*I*
 a^2*e^(557/2*I*c) + 2051102444843872670037494084725650044873387925323161411
 7614891842506419291498065951049807036183243121806802376265432320*I*a^2*e^(5
 55/2*I*c) + 184950199574117342798075855856590286199148713763702219141813361

30221442716020270657023135022033488496344464969637878700*I*a^2*e^(553/2*I*c
) + 16526973858974861639108023310371136054357374657198732212755546502892632
 722591536314813460778204894606881244239416717120*I*a^2*e^(551/2*I*c) + 1463
 717544059678626631832947276075907600136800889205562092713379905928249019089
 7494502217828119253048842224071312398480*I*a^2*e^(549/2*I*c) + 128496595289
 172954488878383897218216664296522808397172308720709765272857431314541125004
 72959858220404943909001977151360*I*a^2*e^(547/2*I*c) + 11182357732757305788
 606125219832724255700185686697695750773456925745718469676671665332383375458
 781984403077867326956440*I*a^2*e^(545/2*I*c) + 9647453055735341956744109660
 981872243803141830016361146898234401219614120958244978337409906070346233171
 019358264000480*I*a^2*e^(543/2*I*c) + 8251893252328617917134965008156780896
 071640246109540703311686461626974697776290689348607898858162877878927869163
 809880*I*a^2*e^(541/2*I*c) + 6998017095771435376057459958050124746801234194
 620229525750507285124859821010861715153517826996821338842938734860615040*I*
 a^2*e^(539/2*I*c) + 5884249182432251390797621599156215151608555552527392878
 727529033511932657541339616490316199475103405475041577880587000*I*a^2*e^(53
 7/2*I*c) + 4905822537534029661350191373485738872824428838056731930906121670
 753804220745738659870244363280999523327382897200441520*I*a^2*e^(535/2*I*c)
 + 4055493561012635024553893300441354935010847129835154524985690016794545113
 963136786023289013915905378564692143643458640*I*a^2*e^(533/2*I*c) + 3324220
 186820494005166240621364244403852934961801328621031245367970100658499892606
 876336284620902405991453408165588520*I*a^2*e^(531/2*I*c) + 2701780972199876
 886785754211080926664096228871020710457482488324723670365135550692459168767
 272049066762300379877743040*I*a^2*e^(529/2*I*c) + 2177319690482826547717112
 293043048619928285927038310216927854685140177477854762032123070408699283792
 898254962886739656*I*a^2*e^(527/2*I*c) + 1739806459662674556992867763179599
 429842093891548125321011280666982199891892856492664415622347650053752088149
 727624160*I*a^2*e^(525/2*I*c) + 1378412185112807868212655886727510995606226
 409519843637641551742037668539116041066140052440183078269143615845581155080
 *I*a^2*e^(523/2*I*c) + 1082797917311589213367556660052710598214901649852587
 681350797657654387598767948898248611815354945578372200449025175360*I*a^2*e^
 (521/2*I*c) + 8433245127019147445434029097206161079624159921545952393999254
 09707630395189954029608070898352689818705209984508369520*I*a^2*e^(519/2*I*c
) + 65119071527881408998703442183351428532243317087384368836924644970897537
 9749425293954979978533284470854614844971466368*I*a^2*e^(517/2*I*c) + 498509
 499153258403729144890980204589097433923264131841430599901592653873152072717
 512660804753287716928576078663461983*I*a^2*e^(515/2*I*c) + 3783333346885029
 059048026143031103999432654861157851787405147285864242374913240291146324417
 82209381845333938523265890*I*a^2*e^(513/2*I*c) + 28463910608716068007316234
 605334764966138706164988664709925230927430389317429244819923320048217996928
 9319599583782705*I*a^2*e^(511/2*I*c) + 212282878705188354649502352775941442
 121310160088506940041461217611549577446696222647786382828325447861991528982
 057300*I*a^2*e^(509/2*I*c) + 1569337554518956338584717038061169073463578995
 47973442628251300145406667989738640457630357856900000058360558079161713*I*a
 ^2*e^(507/2*I*c) + 11499482985400854480060405052673612906866217081234068238

5679020827942329161254533974586296625609880932042763111506754*I*a^2*e^(505/
2*I*c) + 835178769752480442585197779469465889445851875677682648920799175164
87130082429368116942958987653732471395054399139538*I*a^2*e^(503/2*I*c) + 60
117036809531954534774175328542432917785471258270317443583066712344018654100
343282519010777886383221363137483769490*I*a^2*e^(501/2*I*c) + 4288542606290
519837613611568777949831319762493941724102371210731156016608065113664271150
7677332263314929421901176965*I*a^2*e^(499/2*I*c) + 303174251975116376277283
848312800571925857478348208632257263170282939462399098005109017284002857400
11888506446492561*I*a^2*e^(497/2*I*c) + 21238365845231585555918531961644690
235000015826170045512728950554645775595698170974068108854741604809260562872
043619*I*a^2*e^(495/2*I*c) + 1474250608159172064872411391157795097287526449
5912180166443042116499971246321838780169739234187025835012002839150189*I*a^
2*e^(493/2*I*c) + 101395278758164673432207934985491700758390656045467073852
81954497494579711245464375474258366636437333914967133476201*I*a^2*e^(491/2*
I*c) + 69093103807838607892815109268897036078418535879007165861544111515013
97449721115415129140419091270756538802453583895*I*a^2*e^(489/2*I*c) + 46644
046194423261614083371806148988132314961825934722594050803959540003674570959
94601521782293014277832709054615380*I*a^2*e^(487/2*I*c) + 31194351823904045
785264754002935552154841366001694466773595073129947614111315094715884141888
13676704768199883865942*I*a^2*e^(485/2*I*c) + 20665569821602512673114857611
921723223404768298078377841268861201489011910889679265211355648086881826419
11560667924*I*a^2*e^(483/2*I*c) + 13560749642327922110221227763130856201316
15916511397181422807416057396685245359582689147794647531935558283601984443*
I*a^2*e^(481/2*I*c) + 88136939031527422017424188438179615226505526424971930
8791502264814434849100759337832840344343735714440306099566040*I*a^2*e^(479/
2*I*c) + 567339614895630128342983728404200845089288305505097140213560633981
062751336174336795809192566239776955199921505915*I*a^2*e^(477/2*I*c) + 3616
695708340772606773163312929655149602943698361874034355533628022011392504750
72743485753652885635408470996847372*I*a^2*e^(475/2*I*c) + 22831651118346635
760074033939789265172476358111312485881423239869972250832424972565773797037
1965209398196125904775*I*a^2*e^(473/2*I*c) + 142722318968673080123248059644
214027010647362985060273998965493470580757388030700770801323590848378479588
489351378*I*a^2*e^(471/2*I*c) + 8833817464098123754018525939231247589033551
4451795608162633179634457749623643887095574804755049991966929176222830*I*a^
2*e^(469/2*I*c) + 541351870693617750475199376054265659235076137949997317868
28276683538558771749832815251100592417685414948864684145*I*a^2*e^(467/2*I*c
) + 32844172588984923925745248170100614335810461284005739525160956863811444
851393714424782905768489580238852810233892*I*a^2*e^(465/2*I*c) + 1972682929
919065151030155855601200340219105234862278242443532532936069557635228499947
8792032516654385344097576701*I*a^2*e^(463/2*I*c) + 117286869772010165189037
778180635432649656561035757958868696244479317635654723527506620225595340935
18133411921208*I*a^2*e^(461/2*I*c) + 69025186656360372837629802427849099678
83935371662357237821221528329174906800792394732946367748505381533617607645*
I*a^2*e^(459/2*I*c) + 40207296927287520725475279644145838807209599718985673
78088190721921140689114594980080070716969583769101530510140*I*a^2*e^(457/2*

$I*c) + 23180048596133418845744953509005175801585656213363187889470522146623$
 $19719223377004078504331647589087640360974954*I*a^2*e^(455/2*I*c) + 13225410$
 $027687245271645902870752516183035559216180833489217915827253696179198591565$
 $45068710911148219834222862396*I*a^2*e^(453/2*I*c) + 74672814922433629133747$
 $278822750782651525405547963429829616341921688099165840300651627297465712660$
 $2569667620581*I*a^2*e^(451/2*I*c) + 417203346852342786088688963220640917780$
 $075877592046434007891157690273764588009873939937985168264211297805134039*I*$
 $a^2*e^(449/2*I*c) + 2306419504873497769846133773562515241691270810069151310$
 $96932922360748741606056582939291771085884519360184891255*I*a^2*e^(447/2*I*c$
 $) + 12615588444508588099090572305086389453025999211042514613056914922257491$
 $8984083710347803558027793365144729446565*I*a^2*e^(445/2*I*c) + 682698423571$
 $845016119882265417598773598935608169805791657576398859104030415544766832387$
 $44066404341119496497267*I*a^2*e^(443/2*I*c) + 36549041212131229783274664803$
 $103982301706053016635099893874998475671653869877438572422373857662116557803$
 $137691*I*a^2*e^(441/2*I*c) + 1935629938092629464044933432048123474309129215$
 $4311607631598423149396817585242619148155405769562070657818017062*I*a^2*e^(4$
 $39/2*I*c) + 101400825968046074043760909893613377126292736713844463400034811$
 $36225061610661712978027889222103814938027534150*I*a^2*e^(437/2*I*c) + 52542$
 $053315628400861248402353101698809320987341776979631277027119955905761956900$
 $69369181247172296616054250810*I*a^2*e^(435/2*I*c) + 26927282448425364461722$
 $623574956155733196639443211460545174612226257404076090636468048967716925892$
 $50685909715*I*a^2*e^(433/2*I*c) + 13648077061320324179712693353613780904653$
 $49022898360001803048142061893927200900333187448316673412979345898752*I*a^2*$
 $e^(431/2*I*c) + 68409643932382015557347717544725516277035177287130970643281$
 $5320177180262386714126080576227107839963492633507*I*a^2*e^(429/2*I*c) + 339$
 $081049398958631695033084312249444034247861955469658173982280143780778138876$
 $895429955188412068983435311370*I*a^2*e^(427/2*I*c) + 1661894100225111556727$
 $479337964877801751541455656901974947300866702464803722669844977848476890785$
 $76576553405*I*a^2*e^(425/2*I*c) + 80535993554416862856990385694317829375028$
 $680305891033754271854598237850137678037212744640081812890524571520*I*a^2*e^$
 $(423/2*I*c) + 3858660998287657496223400479535149465741817808337559048890103$
 $1957266204930635139096439961363048441151269800*I*a^2*e^(421/2*I*c) + 182774$
 $706915349177015632336022984039711997370840994957490123885798215387769695970$
 $68131015951749218506666144*I*a^2*e^(419/2*I*c) + 85585699131774977720413736$
 $285672113976827764195153726973137506812103604497173350946878181530956793605$
 $16780*I*a^2*e^(417/2*I*c) + 39615443892690555191826925561670874009158112607$
 $81154046642834832166886887365224821817220970795789343876240*I*a^2*e^(415/2*$
 $I*c) + 18125017737117436736993992115331571295088238451140025138488727102115$
 $05465235145932640002862720411729752780*I*a^2*e^(413/2*I*c) + 81962478951711$
 $301916428170954148752036823343688944088312811218591526365565145598288057734$
 $7113410414307040*I*a^2*e^(411/2*I*c) + 366307954798320205128798936155351391$
 $554335176467391622039978284149371380873241652456477113543820574273500*I*a^2$
 $*e^(409/2*I*c) + 1617870843818946112628969626012657489042094122593890063745$
 $51841288165005493773030958210956760612646492280*I*a^2*e^(407/2*I*c) + 70612$
 $062571169786899446458797376311651332630936204874511242735563516386888084909$

599619786580787711236360*I*a^2*e^(405/2*I*c) + 3045246785608650155466937068
 633484708402407505066968240527155699757540924494810770937806784721389896066
 0*I*a^2*e^(403/2*I*c) + 129761330891226169276413730676717606284486903333057
 59931883973052278026353267012367339299301873995466880*I*a^2*e^(401/2*I*c) +
 54628286678221586727361647210129385627818090917703635636870182714540991311
 99171596676490634462439229220*I*a^2*e^(399/2*I*c) + 22720048811620935900466
 503561284482119532730047520542947942040259999751890733350464666986287796514
 01360*I*a^2*e^(397/2*I*c) + 93344733685928184322435539734205849466869829244
 4028252934915487603996507357110331502736639692759358340*I*a^2*e^(395/2*I*c)
 + 378816002128920616369855929612160874580768723711392251915842505525188562
 342470825520416422818082592000*I*a^2*e^(393/2*I*c) + 1518424984432168013297
 922170113498986347038741694307651033334785661844097743653929727337906541890
 70040*I*a^2*e^(391/2*I*c) + 60110869062284374159448209854726838711429891271
 925437784821736413864552441235504659113309232113779360*I*a^2*e^(389/2*I*c)
 + 2350038180733536582371812906680131644319672004296697678903865750731074387
 2020449940329840767115547150*I*a^2*e^(387/2*I*c) + 907248267853142239067848
 211666744759567786743837235682478388725676366285420513138168519714971011172
 0*I*a^2*e^(385/2*I*c) + 345838398116009621381983498618122160603088770375889
 2748920536238488574632981453327231642336638145090*I*a^2*e^(383/2*I*c) + 130
 161349118126897428709054953528362735743599775323204270365658261008471931744
 6750492963887848291340*I*a^2*e^(381/2*I*c) + 483635269585920480609387707945
 457876743335983922410816057000799690442279864698695936197638579602610*I*a^2
 *e^(379/2*I*c) + 1773967554289494131125326767048161003634573724708572840753
 58267979222962448762703682130780576952760*I*a^2*e^(377/2*I*c) + 64228709360
 409812420586138356564211085008288762762209697005815481413905982724399323193
 964745568820*I*a^2*e^(375/2*I*c) + 2295259936503095142465474029958186708878
 3137153355417738039160015930697452167681087479690813393300*I*a^2*e^(373/2*I
 *c) + 809499048675203110286284738006215721238810652354456980450329955264365
 7151717648240648270653869830*I*a^2*e^(371/2*I*c) + 281737384273996814368689
 7684606727718286414616842921650529046416318630866894611374955142903521970*I
 *a^2*e^(369/2*I*c) + 967558857573420421200844825745019691984965901119321858
 915203967518001948679909749229811961456530*I*a^2*e^(367/2*I*c) + 3278501563
 382507256487219221898208284880171022676335915892437775623639272433943562024
 32140538650*I*a^2*e^(365/2*I*c) + 10959682143632428417265121520355099279636
 9009483250710677345396805851498207559908266552991860510*I*a^2*e^(363/2*I*c)
 + 361412936678645147066137100818124636307119046022886541124210601771613066
 07694059860383771279230*I*a^2*e^(361/2*I*c) + 11755749238984942962210062966
 175741219276938429754098150077681519311875621804705720370211969800*I*a^2*e^
 (359/2*I*c) + 3771330561509512524615850596640341294357967299596271045598767
 418091013562271264018462771488980*I*a^2*e^(357/2*I*c) + 1193142657505826707
 248344406249501852936406309110355942396943490095434416696698542207244495880
 *I*a^2*e^(355/2*I*c) + 3722189241503788944357204090266400598909553925906162
 82572164380128530253733901184626128906290*I*a^2*e^(353/2*I*c) + 11448975132
 659178160158729658266693316490641912330185672836854786107354383318779975328
 3463360*I*a^2*e^(351/2*I*c) + 347175898571376724069230085660787903481912988

72900136928769377324783537177053188579391348690*I*a^2*e^(349/2*I*c) + 10377
647429811126892082129653968587516545341659471911017509926638948128316618008
695215618360*I*a^2*e^(347/2*I*c) + 3057490262522130472111269539903981978669
524872013464112348409985533590524778523877236940090*I*a^2*e^(345/2*I*c) + 8
877635774210624749402322148173319138568759789882395681342647763140076465359
47673652724060*I*a^2*e^(343/2*I*c) + 25400580128701754034740103497802208583
3970670738098147773886519381628953611819486811955620*I*a^2*e^(341/2*I*c) +
716062179458509947174306826027645592286227222278082368247999623391032951213
87601682173670*I*a^2*e^(339/2*I*c) + 19886758779911360532261202721353273582
160597108274352859715885782505631373182736113224200*I*a^2*e^(337/2*I*c) + 5
440357551230374902439985027394698228560509433313958273980496531279067808247
723147105870*I*a^2*e^(335/2*I*c) + 1465830016742535124519131469010652577571
975918756440541603851467254363529939075434269440*I*a^2*e^(333/2*I*c) + 3889
319141196329548401301490861407894523738557878079102128441517960492890046054
06967150*I*a^2*e^(331/2*I*c) + 10161010039881372824480214823252009663151748
9501850198195876637177700803300250750696120*I*a^2*e^(329/2*I*c) + 261343335
842817661995062037066653597967114111592037165616028425137494769358915740815
80*I*a^2*e^(327/2*I*c) + 66165787175526862693523980355038876126746090842609
60128356927902046841481605353166520*I*a^2*e^(325/2*I*c) + 16486845255781261
98504085391316407022187711819746696897272619021774566583748173498450*I*a^2*
e^(323/2*I*c) + 40425591619731441020590363134583891813151862203181436828939
1512007823341054252319330*I*a^2*e^(321/2*I*c) + 975260870034870226325822902
89825035947901332365723366039562344307089798567346576790*I*a^2*e^(319/2*I*c
) + 23145134405275963083954762748231025164257628027633977804093014188701364
289864915790*I*a^2*e^(317/2*I*c) + 5402564744628545046129520081244604381235
726873384595583804088637122848866439830910*I*a^2*e^(315/2*I*c) + 1240127624
408790585914281618049099992149775260140355696852903419692530227177449210*I*
a^2*e^(313/2*I*c) + 2798861939746417403756726266426530638552651149606779017
21413367956114946541063180*I*a^2*e^(311/2*I*c) + 62096273516318593577961117
305783982914908932565241065670071393675735767176645100*I*a^2*e^(309/2*I*c)
+ 1354055460508770231039339647727636740044411298407754301934206346492976732
2886040*I*a^2*e^(307/2*I*c) + 290141340780504627769992880642657016436198065
6563461341652112845236338748650110*I*a^2*e^(305/2*I*c) + 610798290612478058
634809110185506816479817707018034933796191469491921071289700*I*a^2*e^(303/2
*I*c) + 1263022613669708131901208630681647294039935119601317431552553654830
34776033710*I*a^2*e^(301/2*I*c) + 25648194013063161445813917323611473948967
661308908878336387655736333479132840*I*a^2*e^(299/2*I*c) + 5113745039917782
570942288672444404762240177072474457348268608658998353643330*I*a^2*e^(297/2
*I*c) + 1000827267878708901757364273781353911409779752889226260010612599185
707764320*I*a^2*e^(295/2*I*c) + 1922272134809049488720343211549109209854258
03997803298273976235657509933640*I*a^2*e^(293/2*I*c) + 36224365825548176907
943960557995838710010556561894147011323135173581402240*I*a^2*e^(291/2*I*c)
+ 6695882317013495222345368298476838368511994443900460264534471085890483820
*I*a^2*e^(289/2*I*c) + 1213732838510510306428982602384538617690483226714330
573193357678961518000*I*a^2*e^(287/2*I*c) + 2156899397588740139671872208335

33731643242074212236222605165544572554060*I*a^2*e^(285/2*I*c) + 37567064271
 487090398378167188966687791512776703919269010885401142609920*I*a^2*e^(283/2
 *I*c) + 6411061389985712625839045934560628663538960416714378812599630238383
 260*I*a^2*e^(281/2*I*c) + 1071687241879183608204834854430988172125784220926
 155326017366664031320*I*a^2*e^(279/2*I*c) + 1754231621964961239400909311813
 06422667058440569170422851340745050152*I*a^2*e^(277/2*I*c) + 28109088369373
 208155707785744000148722496865889470317867362512306740*I*a^2*e^(275/2*I*c)
 + 4407601795177120237389113481959743264257864428283490771370053462560*I*a^2
 *e^(273/2*I*c) + 6760873768573124741467717860493842067374692390717206139804
 92039300*I*a^2*e^(271/2*I*c) + 10141245246701151498796050844567991079471308
 6096737145187354675120*I*a^2*e^(269/2*I*c) + 148697764889784913094443074525
 94685609833202276809994141174545956*I*a^2*e^(267/2*I*c) + 21304543896397841
 02539281096465899633052875625470754655905397856*I*a^2*e^(265/2*I*c) + 29813
 8079805795516636721746713666338776134639093417308458083640*I*a^2*e^(263/2*I
 *c) + 40733825083879692084677912159273504330596927222466270616023040*I*a^2*
 e^(261/2*I*c) + 54311584202896956902676282164178832015067816448256736638951
 25*I*a^2*e^(259/2*I*c) + 70636246561910226822620287946270328530643753560182
 0801093746*I*a^2*e^(257/2*I*c) + 895673469754664164365609590127466723163791
 46573614093235883*I*a^2*e^(255/2*I*c) + 11067205258903164311849474872837148
 947402911102321523307256*I*a^2*e^(253/2*I*c) + 1331867438356900947394085333
 963308875669933795527159861115*I*a^2*e^(251/2*I*c) + 1560185013908330149240
 28933522645581091836687267573447170*I*a^2*e^(249/2*I*c) + 17779861202993747
 975617805656916892771717874809673606262*I*a^2*e^(247/2*I*c) + 1969925436555
 096263303543513345030378604353635783621398*I*a^2*e^(245/2*I*c) + 2120597450
 23134629411867496681427459939018060659823083*I*a^2*e^(243/2*I*c) + 22164417
 393124784256780107055497427057394288021910427*I*a^2*e^(241/2*I*c) + 2247657
 581576890158328281459463053249136155142818885*I*a^2*e^(239/2*I*c) + 2209774
 35109557963897348194591419349596075084671135*I*a^2*e^(237/2*I*c) + 21045459
 166656329934203131234228410676256368947239*I*a^2*e^(235/2*I*c) + 1939943728
 233712898582615509996583301055554831933*I*a^2*e^(233/2*I*c) + 1729197138570
 46613813065255730042186229347958876*I*a^2*e^(231/2*I*c) + 14890304346273125
 682306935846029108244114648650*I*a^2*e^(229/2*I*c) + 1237421118518768552107
 714053055438083342617820*I*a^2*e^(227/2*I*c) + 9913040189115061622703464037
 5461310347846749*I*a^2*e^(225/2*I*c) + 764642099062564610784234922126714976
 8725400*I*a^2*e^(223/2*I*c) + 567179508185660065903323118694229252909501*I*
 a^2*e^(221/2*I*c) + 40401823984348761731551610542858793511300*I*a^2*e^(219/
 2*I*c) + 2759687216371547310850322318637953260785*I*a^2*e^(217/2*I*c) + 180
 469997815998782717102062578903704334*I*a^2*e^(215/2*I*c) + 1127937434108902
 4516950389703822811442*I*a^2*e^(213/2*I*c) + 672482999427386036344407124264
 639911*I*a^2*e^(211/2*I*c) + 38167953019220461788377878471708140*I*a^2*e^(2
 09/2*I*c) + 2057571551559319523757433019101275*I*a^2*e^(207/2*I*c) + 105091
 018637393463590223550684728*I*a^2*e^(205/2*I*c) + 5071416398732103030201521
 505627*I*a^2*e^(203/2*I*c) + 230518925632259863716881504052*I*a^2*e^(201/2*
 I*c) + 9835474114732862868439582838*I*a^2*e^(199/2*I*c) + 39237263636589136
 9041933300*I*a^2*e^(197/2*I*c) + 14570867105062952981500815*I*a^2*e^(195/2*

$I*c) + 501114476578787912641049*I*a^2*e^{(193/2*I*c)} + 158664214115117934164$
 $37*I*a^2*e^{(191/2*I*c)} + 459291145959804703779*I*a^2*e^{(189/2*I*c)} + 120548$
 $85718630588825*I*a^2*e^{(187/2*I*c)} + 284015632092748725*I*a^2*e^{(185/2*I*c)}$
 $+ 5932441401249090*I*a^2*e^{(183/2*I*c)} + 108143463042754*I*a^2*e^{(181/2*I*$
 $c)} + 1685352670794*I*a^2*e^{(179/2*I*c)} + 21830993145*I*a^2*e^{(177/2*I*c)} +$
 $225643340*I*a^2*e^{(175/2*I*c)} + 1744665*I*a^2*e^{(173/2*I*c)} + 8970*I*a^2*e^{$
 $(171/2*I*c)} + 23*I*a^2*e^{(169/2*I*c)})/(e^{(517*I*c)} + 418*e^{(516*I*c)} + 8715$
 $3*e^{(515*I*c)} + 12085216*e^{(514*I*c)} + 1253841160*e^{(513*I*c)} + 10381804804$
 $8*e^{(512*I*c)} + 7146142307307*e^{(511*I*c)} + 420601518659718*e^{(510*I*c)} + 2$
 $1608403021340047*e^{(509*I*c)} + 984382804329835768*e^{(508*I*c)} + 40261256699$
 $368950388*e^{(507*I*c)} + 1493326612293984160368*e^{(506*I*c)} + 50648660944512$
 $569972179*e^{(505*I*c)} + 1581796642397812408161814*e^{(504*I*c)} + 45759117183$
 $402579073139583*e^{(503*I*c)} + 1232445557346832245176696904*e^{(502*I*c)} + 31$
 $042222522074681615625020522*e^{(501*I*c)} + 734057263616388449968842366924*e^{$
 $(500*I*c)} + 16353164647151530240529137618111*e^{(499*I*c)} + 3442771520128751$
 $34140739302960914*e^{(498*I*c)} + 6868329225263681349501997341320517*e^{(497*I*$
 $*c)} + 130171193079172823835151430773360024*e^{(496*I*c)} + 234899837424434707$
 $9532766203075607598*e^{(495*I*c)} + 40443624781415311581857832389099634564*e^{$
 $(494*I*c)} + 665634670676210063754191847109971141414*e^{(493*I*c)} + 104904026$
 $69510897424624643766470754045064*e^{(492*I*c)} + 1585664761132575625661174322$
 $27203884298856*e^{(491*I*c)} + 2302150411226234925855222345201500900533576*e^{$
 $(490*I*c)} + 32147887693375338817454482515377350383950278*e^{(489*I*c)} + 4323$
 $33688644261557547944179250800440604964868*e^{(488*I*c)} + 5605927253067558551$
 $780452883689835514455118670*e^{(487*I*c)} + 701645153225444629068735488137480$
 $91084561870680*e^{(486*I*c)} + 8485522022765123564962001369596762953616963151$
 $13*e^{(485*I*c)} + 9925490738534402272939987038714580495445431374618*e^{(484*I*$
 $*c)} + 112391604542246650966429162063124338952554575234051*e^{(483*I*c)} + 123$
 $3096700139723365181997220750932590655287625342156*e^{(482*I*c)} + 13118781801$
 $172174729679339894318153694964675368481194*e^{(481*I*c)} + 135442594916636116$
 $191574650625331646238501101627937224*e^{(480*I*c)} + 135799066316147984285064$
 $2848032544982878359839580349899*e^{(479*I*c)} + 13231708870104896973800056733$
 $779919089340836756009580718*e^{(478*I*c)} + 125370496586921272662198050851269$
 $323171167338854081782959*e^{(477*I*c)} + 115585541289359426034554496664268782$
 $3630035899363232371472*e^{(476*I*c)} + 10375184499871175501909398956596684116$
 $802997082526660323524*e^{(475*I*c)} + 907226057222088149186422846394871877646$
 $07589706493970774776*e^{(474*I*c)} + 7732046369911457750614627310280985060944$
 $32675788136295011259*e^{(473*I*c)} + 6426195485535248576425068136870465530087$
 $114003875716691383902*e^{(472*I*c)} + 521081176291770486604924009851758309875$
 $05700566877818954141639*e^{(471*I*c)} + 4124306982999151908480672223272194350$
 $67747934091894670488982928*e^{(470*I*c)} + 3187749929744346497211536044751776$
 $582320958627923816470590659024*e^{(469*I*c)} + 240708019135297571018580229143$
 $72045864746991786182039740274325264*e^{(468*I*c)} + 1776428291351193485771944$
 $37675802830239905460092687136494961404333*e^{(467*I*c)} + 1281817464914970810$
 $859604189828359000790789921169405304612211251818*e^{(466*I*c)} + 904669352382$
 $5682979044338963104263167672586826367911338826483549173*e^{(465*I*c)} + 62473$

550781053295317710774690247114124125187565731848441781904032672*e^(464*I*c)
 + 422276126632003687547754746555709988710527133086660161366353656787288*e^(
 463*I*c) + 279470910447568661184279069497369916448225472397721020972566130
 4403472*e^(462*I*c) + 18115768495615758076710303055505625589254293659193314
 153418333944596408*e^(461*I*c) + 115051481852080848873700388354521315567640
 365124003103691176697194292320*e^(460*I*c) + 716099497599058079895633338552
 940229192858196481597830078819711862600096*e^(459*I*c) + 436944248291011391
 4565353136069595862669338858053419381214131241925047008*e^(458*I*c) + 26143
 976279902021443471945665080254563056810183520401889800285493144867448*e^(45
 7*I*c) + 153436088745056254127327239461577071933130157764595997113973513183
 188399376*e^(456*I*c) + 883500968821791202600774541927769200737689393513734
 789368397093333311961880*e^(455*I*c) + 499251971245704398350537797660795398
 8397368297591114957991804893688371867680*e^(454*I*c) + 27693116538343259225
 983382637647936122664033859615133489846664694361471028310*e^(453*I*c) + 150
 822381431412413773566474210011746852297437597059186295243989481140398152780
 *e^(452*I*c) + 806679543607589140759305010796189568269842021613388955218916
 278823182639488190*e^(451*I*c) + 423812584676323258639418856985868582675532
 8005548627437019301405851325887594480*e^(450*I*c) + 21876482892713909928040
 345612578705805121508756226696317087651824252241418663320*e^(449*I*c) + 110
 969199687320974749922259595250444341219218535349655762591192576535872151766
 080*e^(448*I*c) + 553269128819528612502918869558947829098021956309349843584
 044631512291778800081490*e^(447*I*c) + 271184323967071752760564049014883350
 7130242448403978318523237721944200392830108580*e^(446*I*c) + 13069817203488
 289886193205508375818392124991382340160316886507181296548981014818410*e^(44
 5*I*c) + 619485966530355028795643388152343106604109020378824731618047744929
 16216575880077680*e^(444*I*c) + 2888207552647306544699685720210471094273186
 19508995802020689904590319476295408324280*e^(443*I*c) + 1324756412367837473
 157472821162483691120966501948953926492241643788264284546437221120*e^(442*I
 *c) + 597899217294414321845916114929981970632173211157849452524522874297646
 8409105395536290*e^(441*I*c) + 26556806389043407534496702369101545795994861
 757741414789944652712127566910185274123140*e^(440*I*c) + 116104551683555043
 762911501712116399313733021132677481112824047246361794049635726479850*e^(43
 9*I*c) + 499707567253859084357596314813794768069337190915967491907488904933
 922677579665354338960*e^(438*I*c) + 211758973346685570710150142921041472240
 1838837940752841618541440888545729943138209036820*e^(437*I*c) + 88367206408
 604703056945140215479695512967940922669830441183757900258545840367963647682
 80*e^(436*I*c) + 3631836965230259173219744440979812202264082460413055250674
 2586795183267354382847875885730*e^(435*I*c) + 14703081673227683316304158209
 9592047512043725225353339238819165193000407629544745753221740*e^(434*I*c) +
 58640346697268324274164332892156090937519745386424329957199096460885724577
 1134145204174990*e^(433*I*c) + 23043510733738403573791785976730663520166827
 81689139842097376663118488803841131935313641840*e^(432*I*c) + 8923209447343
 296763331881881638471793499618670601026059730895962653291770229493028162575
 100*e^(431*I*c) + 340540538512955691543523467221771726551875489107820085047
 18324168725029438589162349211628040*e^(430*I*c) + 1280989146016885396724805

41830409847707367500438601536803204497701119911289087105659482783340*e^(429
*I*c) + 4750105788576015192723166179384252224217865972416710268943185154085
11467140969393115768793680*e^(428*I*c) + 1736574218818191071874197472450158
123883564209950658639102337148122769080611680719741726053840*e^(427*I*c) +
625987215682225284365096070823503471020136277605717664722632308975144656528
8850103898153859920*e^(426*I*c) + 22251959176795777757167366036007480222211
364232146399803864370963391491223687245823457351580140*e^(425*I*c) + 780098
073680242398756137330588514171253271146810708896407942492826334705807565570
83923203377160*e^(424*I*c) + 2697458014402112969726836018638789543579623085
20076595177128227629273240215209708218497363414140*e^(423*I*c) + 9200893930
295890328746018500271593226125263684447714897819743610788475288914688310384
36064951920*e^(422*I*c) + 3096131971621520162380301554241465451782362086810
287537748902904985934020179565706177131421614590*e^(421*I*c) + 102793647306
638408447395778624692626046488619142979725891652435306512306907262444624791
99894255180*e^(420*I*c) + 3367539887202156837590238459398275336255980105810
4184627345411136262431943240778260721756991027090*e^(419*I*c) + 10886799573
182947282673290519203488679728462135644562753090910442948674125782263347689
8356826454040*e^(418*I*c) + 34735147321471378087435208312956660123876576277
5942366762733349952103889753982636403857556867777300*e^(417*I*c) + 10938532
144862203586740324345008666784997700113058741724889759516120314567346082870
95519501041975440*e^(416*I*c) + 3400232560601651617521694680847089844198028
831694417424794868779328950548418125605446882081152636090*e^(415*I*c) + 104
341175165703959666536931555824021094603480954730278074123214273468169285671
97770376496170251803940*e^(414*I*c) + 3161093933128469275069430644361841465
6095969520945215743004044560386895241801579156543451940713351730*e^(413*I*c
) + 94556180258931986919334303466365652826858091314329189160736277175873841
732196453379953705679466826880*e^(412*I*c) + 279285755800035206679835368898
165477644864987794665387827488933863633745047373109049265172681702585720*e^
(411*I*c) + 814608187736530579670210025271921415597183369881214299823291969
785549876175969866367976653244974728560*e^(410*I*c) + 234651821923910514223
814163307346476889915570893502577804763741268178157576542221912740926015943
8712250*e^(409*I*c) + 66758662903711473585037668656692890108935438698305387
08724945291580951179188296606158111257706968604740*e^(408*I*c) + 1875998821
886556356416363573598607327825573725740570627910889136637842846741455993048
1172863538598193890*e^(407*I*c) + 52075178518793270386429263351544306951104
993542500582938155241689408138675254608030847907167748571734720*e^(406*I*c)
+ 142801792450221762483180874918825274134305133275417780084795034644763509
333503150517345864659667189417080*e^(405*I*c) + 386876218234277165632451723
049979889263115282374607541692443176673997513742813591736171169652250611186
480*e^(404*I*c) + 103556198259200293522638457790861154861211149508019357369
1339864706029186482466241805664949381049856258510*e^(403*I*c) + 27388956247
952656033552276465660008862807783050848257029119389036561620042627361826577
00406301914070062380*e^(402*I*c) + 7158124686842941475473807363679839718172
745581538409044503383852693596921622426696740453944718143025248390*e^(401*I
*c) + 184874052990057326937527286118764908908583570219748823715706238001862

45137722660943641752976852924439870880*e^(400*I*c) + 4718822084346620769509
950695357378035710889749142256789804819901820770899700533386014883647952745
6156014520*e^(399*I*c) + 11904185540387796494822957794837046560060662318304
5529526900430209270473212773847794935586074714329479939280*e^(398*I*c) + 29
682551528266958968531827328023905008455503220341594151196265959688161571379
9937680026497408305672297618840*e^(397*I*c) + 73158497220681836287472962140
397444428001044630116152733976054481530095178798553841976465621458266721991
4080*e^(396*I*c) + 17824461149317518505563548566384219011744123222982494965
91658053939787198246565945975595575734193348887952160*e^(395*I*c) + 4293206
478008022126017488908851826494790620720660151451468181910917240027863968724
539127659633517053002976480*e^(394*I*c) + 102231820259548607672173903051864
519235621454736742936199180634904114874961218045902745927027705715154564146
80*e^(393*I*c) + 2406878513970527716119346564450614328524136103776821681892
2184400141048460210944696647752723371932874594597328*e^(392*I*c) + 56028683
424903517658495013858534516167162591034367972498174660907450666778154353271
630344650777885683547624184*e^(391*I*c) + 128967080084754712246023680866488
384983286259025533132044636109049545144029547003347761521666283977931640178
464*e^(390*I*c) + 293550743554342709808129453576562313299705982699187416862
934373964255615967138676253276302591561523515603264403*e^(389*I*c) + 660764
473105869097691475973850837934511089033149586707982764263394766756649565279
879146173318386505740391093990*e^(388*I*c) + 147093114661893434551503836230
010016048212774958144392990474691022477747019889905237911449399988700319941
9829579*e^(387*I*c) + 32384919313618514764233219335395790983777355392076414
67346235665823887048326949305609231585143748690203615957136*e^(386*I*c) + 7
052132414162197992602326524580143060985353054572933905524633121681021037340
298366342203324325307072413739061024*e^(385*I*c) + 151896342149088003964179
117226437547480485201097348124591098788104938443810626508189711996371214587
49456243274416*e^(384*I*c) + 3236273132241954941033008894364024746037832856
1316422931292427145902887913071643679502909055891236755143207382609*e^(383*
I*c) + 68208033096793615683784409619244210818614991640041553424405527876893
272496608324231098148502466453967157728078994*e^(382*I*c) + 142213115964814
517682386667276769909482271681318790889840501039441748635545362467679832449
103520321953011780083069*e^(381*I*c) + 293344920034300720287042383448342866
313806285455040067823080445597545970023446231563554135133105493516316320059
272*e^(380*I*c) + 598650141112241858911676505180520150364003226841328081453
597093587790338609212439085554466861582623350303061961052*e^(379*I*c) + 120
87703584936583930894422205693506328370410814059375022653984611773764821660
9559734831601248698274330296158612144*e^(378*I*c) + 24149665168103385032890
765492027405117100590117954471387734642056964550264427124264095996627710802
64826008985061097*e^(377*I*c) + 4774141111066098970221845330594962016472714
230374234060663956846950926642685946929064114194400360936223590725470146*e^(
376*I*c) + 933934195805349422525175096571505730070730208381477477030621822
4241022648247419956042957363055823830898547303219757*e^(375*I*c) + 18079820
068028859970349938623007230676563314206708848499900139641237334763266479346
963237936039328113185041591793848*e^(374*I*c) + 346376571726716901676573445

371970870488823548539932704720639430787736004465429635483481012693904434644
80754513928502*e^(373*I*c) + 6567485926886730009882737581287522561065455168
6261103681664007007537115778097293533565243828873383722980353200611956*e^(3
72*I*c) + 12324394151933238474196007258810350659640633925361639108206296996
0682419011745775738921817753391954462609323881489157*e^(371*I*c) + 22891131
173859278009149264916234683440586774077645632610841092885725717470728926807
4347550225793244741923354395308214*e^(370*I*c) + 42084634260894938727755902
145792458657812096614856102264700849952946845200598017511941062895621049760
9566002969884927*e^(369*I*c) + 76586779551396278101255844462875141871094089
5281304790836743661582071650032154891482866406314834433199455459798934952*e
^(368*I*c) + 13796765297962120740171061880665894483554465012108901951071648
60350228928586815539003062875026711931941947738690360722*e^(367*I*c) + 2460
442375845422663927081630983260714734968091905493027145639238827192254886349
361126991457692409851120873307487457468*e^(366*I*c) + 434390969660193217335
735968778157929370129568194082711421543317533609396784590876674073824003711
4570667410936998017178*e^(365*I*c) + 75927527001466789611530950735850154731
970297465336333315497939614732857609358019041551167648315608759475810486935
27224*e^(364*I*c) + 1313977149410493388185668115141829311224255152153568687
1181266579813877606348160261747201317735782566021306798298336024*e^(363*I*c
) + 22514675741308069961506165586502872430421930210673264392997286485600640
103867253604847715547060592967690653795951142520*e^(362*I*c) + 381990158675
860879760029987566276749947954406679036250293223462501332864891208750050136
38128113893960349670280707161530*e^(361*I*c) + 6417510069326006680623806488
600459717074084330008683936861613916452910804984467535311184272579865808884
0347241496099644*e^(360*I*c) + 10676483201716559483808523418933352873358767
332997253009266108518678993925291593709076028223234691909042624339940932331
4*e^(359*I*c) + 17589625826275598575710681261397930126580103159548435361490
4672865169442232075776580447184134141375995770091499246759528*e^(358*I*c) +
28699294363123149655727801085157694089682649746606632752880156067700711283
7431926735088120974861760511367008815728782643*e^(357*I*c) + 46375828845736
715454493767825500568873332814556804931042399559988601280063861990402236837
8591108842602342094543682299102*e^(356*I*c) + 74222864090817312491693704946
252561733414891967911827048983100549778195122106995583962345249974865312465
8873553401442137*e^(355*I*c) + 11766007209757869651898750508902310922046126
969702774330145358957889567712307935203819931066068805646285998223417228010
12*e^(354*I*c) + 1847505856462451533445284300571326323781162553304565971887
670758091079306794821834928170773126364639722071570131703785334*e^(353*I*c)
+ 287361053592234018708083543558291227727196797739472015979107027492771427
6869531467182688981041061381703885403497544001592*e^(352*I*c) + 44276730791
054253185243161129856936565848519361001924570444551344833050453214525163471
18488133224823670465103483954805161*e^(351*I*c) + 6758480437888524372562935
948963857626694855547195519486122877567981718587262362871994967079401831957
927901682582941234362*e^(350*I*c) + 102204237794346348513399752951633996417
021222496636661930530083020260969321585683383094182373955413518190269079532
20681013*e^(349*I*c) + 1531283720666277537934735321280768296571253565294263

1518286142403097738200270711195396582159028513532779682154451996208592*e^(3
 48*I*c) + 22731603566128841100419501947051367666836652418077260913944810748
 473084891890410181285412604854876625919565639521227223276*e^(347*I*c) + 334
 358978279365813011711754596108294542981679620174198100729367333785065844280
 24201072453193458155334046693516742390717832*e^(346*I*c) + 4873325350597492
 340085225556305210140219646931365955449272567475433937528301040716774436695
 5828922837488705858532439654489*e^(345*I*c) + 70386349760594831567048224061
 395025698501202296966300376764220336697702961591099854055411376294871437468
 149528524796002762*e^(344*I*c) + 100744961851853744611754300982980166962404
 553836222921868484694269966120607698907046343731011160948828100276729370132
 819357*e^(343*I*c) + 142906319123055524246546928478954238371315925802022389
 236498652136839822502035155676970917419039834587967055588431566416784*e^(34
 2*I*c) + 200906587153578804380300469501441610174521851259541929209840688960
 859454908519774835905895757666770857888611738751858460424*e^(341*I*c) + 279
 945244475039804822966730462960884492119874857791147124009079476920435941735
 293309305430438687333129912454196774070107264*e^(340*I*c) + 386642673050380
 049457382562818316962651975550990779277048740238629858795018247356162888631
 015687664780101205287333082748791*e^(339*I*c) + 529329252764113926003934836
 958243557672549238997560739214406599185047831955572583765358634395408771528
 009745467548382950094*e^(338*I*c) + 718361596382058249209113544487901088868
 388744033713210332491971375906738341551540457264804304039664255915607349801
 911966551*e^(337*I*c) + 966458275369037718747739130798151643483590684166832
 234688098291164160636418159452119815728809372125168836239364442397344064*e^
 (336*I*c) + 128904351529293395648063433049967704018104393562010691426731106
 7900030058398839787692376954090545278554544997710058754772400*e^(335*I*c) +
 17045829967078228082046782181676930026986611477127723550214565438109300696
 37188085882824757500605246963210810351706405349408*e^(334*I*c) + 2234891276
 398439464478622578306434840724610484468177859822620658691921478645266653062
 563823553001228001009093606751066168944*e^(333*I*c) + 290538572232005700195
 334527448948279085669252995982374953269596341416483336677312821860789932858
 8608916176593772088622582464*e^(332*I*c) + 37452575948766512046573349884262
 26388143954501986830664223492263610796095468222760675048993867030889823081
 85717143407211328*e^(331*I*c) + 4787527442780945685145204846971596165304169
 419328244073211459592129649255048876854059844720661078151288179612574986359
 194560*e^(330*I*c) + 606894980315671224833187110532989547172280614300887801
 4986559653687260694816550470195890004511965527567432722969707577202160*e^(3
 29*I*c) + 76297318156278215804689924242070083664388967363330246618638381051
 10445148946962328297631547032543419811821015837863013682720*e^(328*I*c) + 9
 513032274019522954209113191268226642299912013525665940298381064797885690904
 993128948035227412144035633851779511219335277360*e^(327*I*c) + 117642122748
 764840800109007146734744933712781605578119837244558265660556176580864793686
 41864908119643412413644803772131657280*e^(326*I*c) + 1442981628520843120453
 297837537569196506315422464974755129585150738952408322697678968860136962839
 9900747658579201929300744260*e^(325*I*c) + 17556273271224292396887291403125
 716213491486261145478571376751690105656067838042151038271381300372757755676

325408026834544840*e^(324*I*c) + 211883214058828875396101983747068626958940
492260770937641325125133361905239789496943876860591245267550480429579542647
06637460*e^(323*I*c) + 2536717643911935362153226033598334815490498260612576
1711300683492963390816491583025705268737539982149639300226512657426118880*e
^(322*I*c) + 30128482414552703264559018953088177156013437493438201078413769
835448366148121754549197591129967170764969700180348699207838960*e^(321*I*c)
+ 355001031060196498762723767969494822095813723710360050128778060274816728
07059943445240136315568500732379966585005678181937920*e^(320*I*c) + 4149983
212196370804378852378740134554178008893053820691885357902674927336467164003
7563488607716092887686471542838602788559660*e^(319*I*c) + 48133117678184029
216503748549110374478924719094635603892829364863916553792278822957368285106
328164715910598370871149079494360*e^(318*I*c) + 553909130449720862194326891
463315660814279598969699002144342968177311508638670566207686081876797097201
52974148474907904177340*e^(317*I*c) + 6324777410101217905179494607517556992
407698133813848315804240674745387472938763171054499524715291220511850059751
1052824347680*e^(316*I*c) + 71660329861173395524441943889284109134091157844
655245672084237402434944696464927131812190659629511140639501743303863582092
880*e^(315*I*c) + 805662491306826841818762018826235112063637903372180119541
10210642927765997644903820595421936873565314654415769070472655401600*e^(314
*I*c) + 8988381580138238221397327047795460274479287701805196334714630737246
4315121274929402347942874802899499538953561056667668891020*e^(313*I*c) + 99
512206472057965951340341738023548515336403371717898040850470954657532977279
11349150688029072611154101941386019689567958040*e^(312*I*c) + 109332537349
966223203932678503426357079863707001728294011042076530403923862654018978676
516417314221089449922495612732870169660*e^(311*I*c) + 119209713702033927055
75539782368844444647424324502185328626347046599634721146573830681540495333
543146776810911910410468628960*e^(310*I*c) + 128995076011591903410763863427
097329948586173574595862705849159280943046458742663163454018491463855395649
453952212899632198680*e^(309*I*c) + 138529794549151089451352769576543403126
330747243680030832467205895819043568155239264876762867172754338684027849855
385453216080*e^(308*I*c) + 147648920805545333418623121767853777399782924748
301228793924342574999937955421765370101235122939557467548549202174550009604
780*e^(307*I*c) + 156185962953551196169738218832173696509852551589210730578
365727476259476474465955428502336673743686499175698677875693611243400*e^(30
6*I*c) + 163977816059607725375264559816505847894187785101455360391897424482
998415385787605765315509208337741590143078572243505132706580*e^(305*I*c) +
170869848868953101176860306053103994340530390347260088432676842505555141293
830838961275974268928666494845723462544709102843680*e^(304*I*c) + 176720929
970554642004575770053095700595334659870682732031975915532387577052414866323
511140117680492929354517559479899220940360*e^(303*I*c) + 181408168770922059
820368553316697321639984862628298828569560273295630897626829345263592219034
560853530733710529842148537901680*e^(302*I*c) + 184831151983748941817667850
174708257138128172158269413287765358532240773244336191900818557829905895684
494889410451921524212840*e^(301*I*c) + 186915474436567514926351405623117503
261987508351930083824566444435689139233683411704641828762178799177848064220

150818355261280*e^(300*I*c) + 187615393168510050071497280564603510912403132
 920312024370835062679037644990286285346673507093452964351257962696133511725
 652320*e^(299*I*c) + 186915474436567514926351405623117503261987508351930083
 824566444435689139233683411704641828762178799177848064220150818355261280*e^
 (298*I*c) + 184831151983748941817667850174708257138128172158269413287765358
 532240773244336191900818557829905895684494889410451921524212840*e^(297*I*c)
 + 181408168770922059820368553316697321639984862628298828569560273295630897
 626829345263592219034560853530733710529842148537901680*e^(296*I*c) + 176720
 929970554642004575770053095700595334659870682732031975915532387577052414866
 323511140117680492929354517559479899220940360*e^(295*I*c) + 170869848868953
 101176860306053103994340530390347260088432676842505555141293830838961275974
 268928666494845723462544709102843680*e^(294*I*c) + 163977816059607725375264
 559816505847894187785101455360391897424482998415385787605765315509208337741
 590143078572243505132706580*e^(293*I*c) + 156185962953551196169738218832173
 696509852551589210730578365727476259476474465955428502336673743686499175698
 677875693611243400*e^(292*I*c) + 147648920805545333418623121767853777399782
 924748301228793924342574999937955421765370101235122939557467548549202174550
 009604780*e^(291*I*c) + 138529794549151089451352769576543403126330747243680
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 *e^(290*I*c) + 128995076011591903410763863427097329948586173574595862705849
 159280943046458742663163454018491463855395649453952212899632198680*e^(289*I
 *c) + 119209713702033927055755397823688444444647424324502185328626347046599
 634721146573830681540495333543146776810911910410468628960*e^(288*I*c) + 109
 332537349966223203932678503426357079863707001728294011042076530403923862654
 018978676516417314221089449922495612732870169660*e^(287*I*c) + 995122064720
 579659513403417380235485153364033717178980408504709546575329772791134915068
 80290726111154101941386019689567958040*e^(286*I*c) + 8988381580138238221397
 327047795460274479287701805196334714630737246431512127492940234794287480289
 9499538953561056667668891020*e^(285*I*c) + 80566249130682684181876201882623
 511206363790337218011954110210642927765997644903820595421936873565314654415
 769070472655401600*e^(284*I*c) + 716603298611733955244419438892841091340911
 578446552456720842374024349446964649271318121906596295111406395017433038635
 82092880*e^(283*I*c) + 6324777410101217905179494607517556992407698133813848
 3158042406747453874729387631710544995247152912205118500597511052824347680*e
 ^ (282*I*c) + 55390913044972086219432689146331566081427959896969900214434296
 817731150863867056620768608187679709720152974148474907904177340*e^(281*I*c)
 + 481331176781840292165037485491103744789247190946356038928293648639165537
 92278822957368285106328164715910598370871149079494360*e^(280*I*c) + 4149983
 212196370804378852378740134554178008893053820691885357902674927336467164003
 7563488607716092887686471542838602788559660*e^(279*I*c) + 35500103106019649
 876272376796949482209581372371036005012877806027481672807059943445240136315
 568500732379966585005678181937920*e^(278*I*c) + 301284824145527032645590189
 530881771560134374934382010784137698354483661481217545491975911299671707649
 69700180348699207838960*e^(277*I*c) + 2536717643911935362153226033598334815
 490498260612576171130068349296339081649158302570526873753998214963930022651

2657426118880*e^(276*I*c) + 21188321405882887539610198374706862695894049226
077093764132512513336190523978949694387686059124526755048042957954264706637
460*e^(275*I*c) + 175562732712242923968872914031257162134914862611454785713
76751690105656067838042151038271381300372757755676325408026834544840*e^(274
*I*c) + 1442981628520843120453297837537569196506315422464974755129585150738
9524083226976789688601369628399900747658579201929300744260*e^(273*I*c) + 11
764212274876484080010900714673474493371278160557811983724455826566055617658
086479368641864908119643412413644803772131657280*e^(272*I*c) + 951303227401
952295420911319126822664229991201352566594029838106479788569090499312894803
5227412144035633851779511219335277360*e^(271*I*c) + 76297318156278215804689
924242070083664388967363330246618638381051104451489469623282976315470325434
19811821015837863013682720*e^(270*I*c) + 6068949803156712248331871105329895
471722806143008878014986559653687260694816550470195890004511965527567432722
969707577202160*e^(269*I*c) + 478752744278094568514520484697159616530416941
932824407321145959212964925504887685405984472066107815128817961257498635919
4560*e^(268*I*c) + 37452575948766512046573349884262263881439545019868306642
22349226361079609546822276067504899386703088982308185717143407211328*e^(267
*I*c) + 2905385722320057001953345274489482790856692529959823749532695963414
164833366773128218607899328588608916176593772088622582464*e^(266*I*c) + 223
489127639843946447862257830643484072461048446817785982262065869192147864526
6653062563823553001228001009093606751066168944*e^(265*I*c) + 17045829967078
228082046782181676930026986611477127723550214565438109300696371880858828247
57500605246963210810351706405349408*e^(264*I*c) + 1289043515292933956480634
330499677040181043935620106914267311067900030058398839787692376954090545278
554544997710058754772400*e^(263*I*c) + 966458275369037718747739130798151643
483590684166832234688098291164160636418159452119815728809372125168836239364
442397344064*e^(262*I*c) + 718361596382058249209113544487901088868388744033
713210332491971375906738341551540457264804304039664255915607349801911966551
*e^(261*I*c) + 529329252764113926003934836958243557672549238997560739214406
599185047831955572583765358634395408771528009745467548382950094*e^(260*I*c)
+ 386642673050380049457382562818316962651975550990779277048740238629858795
018247356162888631015687664780101205287333082748791*e^(259*I*c) + 279945244
475039804822966730462960884492119874857791147124009079476920435941735293309
305430438687333129912454196774070107264*e^(258*I*c) + 200906587153578804380
300469501441610174521851259541929209840688960859454908519774835905895757666
770857888611738751858460424*e^(257*I*c) + 142906319123055524246546928478954
238371315925802022389236498652136839822502035155676970917419039834587967055
588431566416784*e^(256*I*c) + 100744961851853744611754300982980166962404553
836222921868484694269966120607698907046343731011160948828100276729370132819
357*e^(255*I*c) + 703863497605948315670482240613950256985012022969663003767
64220336697702961591099854055411376294871437468149528524796002762*e^(254*I*
c) + 4873325350597492340085225556305210140219646931365955449272567475433937
5283010407167744366955828922837488705858532439654489*e^(253*I*c) + 33435897
827936581301171175459610829454298167962017419810072936733378506584428024201
072453193458155334046693516742390717832*e^(252*I*c) + 227316035661288411004

195019470513676668366524180772609139448107484730848918904101812854126048548
76625919565639521227223276*e^(251*I*c) + 1531283720666277537934735321280768
296571253565294263151828614240309773820027071119539658215902851353277968215
4451996208592*e^(250*I*c) + 10220423779434634851339975295163399641702122249
663666193053008302026096932158568338309418237395541351819026907953220681013
*e^(249*I*c) + 675848043788852437256293594896385762669485554719551948612287
7567981718587262362871994967079401831957927901682582941234362*e^(248*I*c) +
44276730791054253185243161129856936565848519361001924570444551344833050453
21452516347118488133224823670465103483954805161*e^(247*I*c) + 2873610535922
340187080835435582912277271967977394720159791070274927714276869531467182688
981041061381703885403497544001592*e^(246*I*c) + 184750585646245153344528430
057132632378116255330456597188767075809107930679482183492817077312636463972
2071570131703785334*e^(245*I*c) + 11766007209757869651898750508902310922046
126969702774330145358957889567712307935203819931066068805646285998223417228
01012*e^(244*I*c) + 7422286409081731249169370494625256173341489196791182704
89831005497781951221069955839623452499748653124658873553401442137*e^(243*I*
c) + 4637582884573671545449376782550056887333281455680493104239955998860128
00638619904022368378591108842602342094543682299102*e^(242*I*c) + 2869929436
312314965572780108515769408968264974660663275288015606770071128374319267350
88120974861760511367008815728782643*e^(241*I*c) + 1758962582627559857571068
126139793012658010315954843536149046728651694422320757765804471841341413759
95770091499246759528*e^(240*I*c) + 1067648320171655948380852341893335287335
876733299725300926610851867899392529159370907602822323469190904262433994093
23314*e^(239*I*c) + 6417510069326006680623806488600459717074084330008683936
8616139164529108049844675353111842725798658088840347241496099644*e^(238*I*c
) + 38199015867586087976002998756627674994795440667903625029322346250133286
489120875005013638128113893960349670280707161530*e^(237*I*c) + 225146757413
080699615061655865028724304219302106732643929972864856006401038672536048477
15547060592967690653795951142520*e^(236*I*c) + 1313977149410493388185668115
141829311224255152153568687118126657981387760634816026174720131773578256602
1306798298336024*e^(235*I*c) + 75927527001466789611530950735850154731970297
46533633331549793961473285760935801904155116764831560875947581048693527224*
e^(234*I*c) + 4343909696601932173357359687781579293701295681940827114215433
175336093967845908766740738240037114570667410936998017178*e^(233*I*c) + 246
044237584542266392708163098326071473496809190549302714563923882719225488634
9361126991457692409851120873307487457468*e^(232*I*c) + 13796765297962120740
171061880665894483554465012108901951071648603502289285868155390030628750267
11931941947738690360722*e^(231*I*c) + 7658677955139627810125584446287514187
109408952813047908367436615820716500321548914828664063148344331994554597989
34952*e^(230*I*c) + 4208463426089493872775590214579245865781209661485610226
47008499529468452005980175119410628956210497609566002969884927*e^(229*I*c)
+ 2289113117385927800914926491623468344058677407764563261084109288572571747
07289268074347550225793244741923354395308214*e^(228*I*c) + 1232439415193323
847419600725881035065964063392536163910820629699606824190117457757389218177
53391954462609323881489157*e^(227*I*c) + 6567485926886730009882737581287522

561065455168626110368166400700753711577809729353356524382887338372298035320
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472063943078773600446542963548348101269390443464480754513928502*e^(225*I*c)
+ 180798200680288599703499386230072306765633142067088484999001396412373347
63266479346963237936039328113185041591793848*e^(224*I*c) + 9339341958053494
225251750965715057300707302083814774770306218224241022648247419956042957363
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647271423037423406066395684695092664268594692906411419440036093622359072547
0146*e^(222*I*c) + 24149665168103385032890765492027405117100590117954471387
73464205696455026442712426409599662771080264826008985061097*e^(221*I*c) + 1
208770358493658393089442222056935063283704108140593750226539846117737648216
609559734831601248698274330296158612144*e^(220*I*c) + 598650141112241858911
676505180520150364003226841328081453597093587790338609212439085554466861582
623350303061961052*e^(219*I*c) + 293344920034300720287042383448342866313806
285455040067823080445597545970023446231563554135133105493516316320059272*e^(
218*I*c) + 142213115964814517682386667276769909482271681318790889840501039
441748635545362467679832449103520321953011780083069*e^(217*I*c) + 682080330
967936156837844096192442108186149916400415534244055278768932724966083242310
98148502466453967157728078994*e^(216*I*c) + 3236273132241954941033008894364
024746037832856131642293129242714590288791307164367950290905589123675514320
7382609*e^(215*I*c) + 15189634214908800396417911722643754748048520109734812
459109878810493844381062650818971199637121458749456243274416*e^(214*I*c) +
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0298366342203324325307072413739061024*e^(213*I*c) + 32384919313618514764233
219335395790983777355392076414673462356658238870483269493056092315851437486
90203615957136*e^(212*I*c) + 1470931146618934345515038362300100160482127749
58144392990474691022477470198899052379114493999887003199419829579*e^(211*I
*c) + 660764473105869097691475973850837934511089033149586707982764263394766
756649565279879146173318386505740391093990*e^(210*I*c) + 293550743554342709
808129453576562313299705982699187416862934373964255615967138676253276302591
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259025533132044636109049545144029547003347761521666283977931640178464*e^(20
8*I*c) + 560286834249035176584950138585345161671625910343679724981746609074
50666778154353271630344650777885683547624184*e^(207*I*c) + 2406878513970527
716119346564450614328524136103776821681892218440014104846021094469664775272
3371932874594597328*e^(206*I*c) + 10223182025954860767217390305186451923562
145473674293619918063490411487496121804590274592702770571515456414680*e^(20
5*I*c) + 429320647800802212601748890885182649479062072066015145146818191091
7240027863968724539127659633517053002976480*e^(204*I*c) + 17824461149317518
505563548566384219011744123222982494965916580539397871982465659459755955757
34193348887952160*e^(203*I*c) + 7315849722068183628747296214039744442800104
46301161527339760544815300951787985538419764656214582667219914080*e^(202*I*
c) + 2968255152826695896853182732802390500845550322034159415119626595968816
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29479939280*e^(200*I*c) + 4718822084346620769509950695357378035710889749142
 2567898048199018207708997005333860148836479527456156014520*e^(199*I*c) + 18
 487405299005732693752728611876490890858357021974882371570623800186245137722
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 983971817274558153840904450338385269359692162242669674045394471814302524839
 0*e^(197*I*c) + 27388956247952656033552276465660008862807783050848257029119
 38903656162004262736182657700406301914070062380*e^(196*I*c) + 1035561982592
 002935226384577908611548612111495080193573691339864706029186482466241805664
 949381049856258510*e^(195*I*c) + 386876218234277165632451723049979889263115
 282374607541692443176673997513742813591736171169652250611186480*e^(194*I*c)
 + 142801792450221762483180874918825274134305133275417780084795034644763509
 333503150517345864659667189417080*e^(193*I*c) + 520751785187932703864292633
 515443069511049935425005829381552416894081386752546080308479071677485717347
 20*e^(192*I*c) + 1875998821886556356416363573598607327825573725740570627910
 8891366378428467414559930481172863538598193890*e^(191*I*c) + 66758662903711
 473585037668656692890108935438698305387087249452915809511791882966061581112
 57706968604740*e^(190*I*c) + 2346518219239105142238141633073464768899155708
 935025778047637412681781575765422219127409260159438712250*e^(189*I*c) + 814
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 644864987794665387827488933863633745047373109049265172681702585720*e^(187*I
 *c) + 945561802589319869193343034663656528268580913143291891607362771758738
 41732196453379953705679466826880*e^(186*I*c) + 3161093933128469275069430644
 3618414656095969520945215743004044560386895241801579156543451940713351730*e
 ^^(185*I*c) + 10434117516570395966653693155582402109460348095473027807412321
 427346816928567197770376496170251803940*e^(184*I*c) + 340023256060165161752
 169468084708984419802883169441742479486877932895054841812560544688208115263
 6090*e^(183*I*c) + 10938532144862203586740324345008666784997700113058741724
 88975951612031456734608287095519501041975440*e^(182*I*c) + 3473514732147137
 808743520831295666012387657627759423667627333499521038897539826364038575568
 67777300*e^(181*I*c) + 1088679957318294728267329051920348867972846213564456
 27530909104429486741257822633476898356826454040*e^(180*I*c) + 3367539887202
 156837590238459398275336255980105810418462734541113626243194324077826072175
 6991027090*e^(179*I*c) + 10279364730663840844739577862469262604648861914297
 972589165243530651230690726244462479199894255180*e^(178*I*c) + 309613197162
 152016238030155424146545178236208681028753774890290498593402017956570617713
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 1489781974361078847528891468831038436064951920*e^(176*I*c) + 26974580144021
 129697268360186387895435796230852007659517712822762927324021520970821849736
 3414140*e^(175*I*c) + 78009807368024239875613733058851417125327114681070889
 640794249282633470580756557083923203377160*e^(174*I*c) + 222519591767957777
 571673660360074802222113642321463998038643709633914912236872458234573515801
 40*e^(173*I*c) + 6259872156822252843650960708235034710201362776057176647226
 323089751446565288850103898153859920*e^(172*I*c) + 173657421881819107187419
 7472450158123883564209950658639102337148122769080611680719741726053840*e^(1

71*I*c) + 47501057885760151927231661793842522242178659724167102689431851540
8511467140969393115768793680*e^(170*I*c) + 12809891460168853967248054183040
9847707367500438601536803204497701119911289087105659482783340*e^(169*I*c) +
34054053851295569154352346722177172655187548910782008504718324168725029438
589162349211628040*e^(168*I*c) + 892320944734329676333188188163847179349961
8670601026059730895962653291770229493028162575100*e^(167*I*c) + 23043510733
738403573791785976730663520166827816891398420973766631184888038411319353136
41840*e^(166*I*c) + 5864034669726832427416433289215609093751974538642432995
71990964608857245771134145204174990*e^(165*I*c) + 1470308167322768331630415
8209959204751204372522535339238819165193000407629544745753221740*e^(164*I*
c) + 3631836965230259173219744440979812202264082460413055250674258679518326
7354382847875885730*e^(163*I*c) + 88367206408604703056945140215479695512967
94092266983044118375790025854584036796364768280*e^(162*I*c) + 2117589733466
855707101501429210414722401838837940752841618541440888545729943138209036820
*e^(161*I*c) + 499707567253859084357596314813794768069337190915967491907488
904933922677579665354338960*e^(160*I*c) + 116104551683555043762911501712116
399313733021132677481112824047246361794049635726479850*e^(159*I*c) + 265568
063890434075344967023691015457959948617577414147899446527121275669101852741
23140*e^(158*I*c) + 5978992172944143218459161149299819706321732111578494525
245228742976468409105395536290*e^(157*I*c) + 132475641236783747315747282116
2483691120966501948953926492241643788264284546437221120*e^(156*I*c) + 28882
075526473065446996857202104710942731861950899580202068990459031947629540832
4280*e^(155*I*c) + 61948596653035502879564338815234310660410902037882473161
804774492916216575880077680*e^(154*I*c) + 130698172034882898861932055083758
18392124991382340160316886507181296548981014818410*e^(153*I*c) + 2711843239
670717527605640490148833507130242448403978318523237721944200392830108580*e^
(152*I*c) + 553269128819528612502918869558947829098021956309349843584044631
512291778800081490*e^(151*I*c) + 110969199687320974749922259595250444341219
218535349655762591192576535872151766080*e^(150*I*c) + 218764828927139099280
40345612578705805121508756226696317087651824252241418663320*e^(149*I*c) + 4
238125846763232586394188569858685826755328005548627437019301405851325887594
480*e^(148*I*c) + 806679543607589140759305010796189568269842021613388955218
916278823182639488190*e^(147*I*c) + 150822381431412413773566474210011746852
297437597059186295243989481140398152780*e^(146*I*c) + 276931165383432592259
83382637647936122664033859615133489846664694361471028310*e^(145*I*c) + 4992
519712457043983505377976607953988397368297591114957991804893688371867680*e^
(144*I*c) + 883500968821791202600774541927769200737689393513734789368397093
333311961880*e^(143*I*c) + 153436088745056254127327239461577071933130157764
595997113973513183188399376*e^(142*I*c) + 261439762799020214434719456650802
54563056810183520401889800285493144867448*e^(141*I*c) + 4369442482910113914
565353136069595862669338858053419381214131241925047008*e^(140*I*c) + 716099
497599058079895633338552940229192858196481597830078819711862600096*e^(139*I
*c) + 115051481852080848873700388354521315567640365124003103691176697194292
320*e^(138*I*c) + 181157684956157580767103030555056255892542936591933141534
18333944596408*e^(137*I*c) + 2794709104475686611842790694973699164482254723

$$\begin{aligned}
& 977210209725661304403472 * e^{(136 * I * c)} + 422276126632003687547754746555709988 \\
& 710527133086660161366353656787288 * e^{(135 * I * c)} + 624735507810532953177107746 \\
& 90247114124125187565731848441781904032672 * e^{(134 * I * c)} + 9046693523825682979 \\
& 044338963104263167672586826367911338826483549173 * e^{(133 * I * c)} + 128181746491 \\
& 4970810859604189828359000790789921169405304612211251818 * e^{(132 * I * c)} + 17764 \\
& 2829135119348577194437675802830239905460092687136494961404333 * e^{(131 * I * c)} + \\
& 24070801913529757101858022914372045864746991786182039740274325264 * e^{(130 * I \\
& * c)} + 3187749929744346497211536044751776582320958627923816470590659024 * e^{(1 \\
& 29 * I * c)} + 412430698299915190848067222327219435067747934091894670488982928 * e \\
& ^{(128 * I * c)} + 52108117629177048660492400985175830987505700566877818954141639 \\
& * e^{(127 * I * c)} + 642619548553524857642506813687046553008711400387571669138390 \\
& 2 * e^{(126 * I * c)} + 77320463699114577506146273102809850609443267578813629501125 \\
& 9 * e^{(125 * I * c)} + 90722605722208814918642284639487187764607589706493970774776 \\
& * e^{(124 * I * c)} + 10375184499871175501909398956596684116802997082526660323524 * \\
& e^{(123 * I * c)} + 1155855412893594260345544966642687823630035899363232371472 * e^{(\\
& 122 * I * c)} + 125370496586921272662198050851269323171167338854081782959 * e^{(12 \\
& 1 * I * c)} + 13231708870104896973800056733779919089340836756009580718 * e^{(120 * I * \\
& c)} + 1357990663161479842850642848032544982878359839580349899 * e^{(119 * I * c)} + \\
& 135442594916636116191574650625331646238501101627937224 * e^{(118 * I * c)} + 131187 \\
& 81801172174729679339894318153694964675368481194 * e^{(117 * I * c)} + 1233096700139 \\
& 723365181997220750932590655287625342156 * e^{(116 * I * c)} + 112391604542246650966 \\
& 429162063124338952554575234051 * e^{(115 * I * c)} + 992549073853440227293998703871 \\
& 4580495445431374618 * e^{(114 * I * c)} + 84855220227651235649620013695967629536169 \\
& 6315113 * e^{(113 * I * c)} + 70164515322544462906873548813748091084561870680 * e^{(11 \\
& 2 * I * c)} + 5605927253067558551780452883689835514455118670 * e^{(111 * I * c)} + 43233 \\
& 3688644261557547944179250800440604964868 * e^{(110 * I * c)} + 32147887693375338817 \\
& 454482515377350383950278 * e^{(109 * I * c)} + 230215041122623492585522234520150090 \\
& 0533576 * e^{(108 * I * c)} + 158566476113257562566117432227203884298856 * e^{(107 * I * c \\
&)} + 10490402669510897424624643766470754045064 * e^{(106 * I * c)} + 665634670676210 \\
& 063754191847109971141414 * e^{(105 * I * c)} + 404436247814153115818578323890996345 \\
& 64 * e^{(104 * I * c)} + 2348998374244347079532766203075607598 * e^{(103 * I * c)} + 130171 \\
& 193079172823835151430773360024 * e^{(102 * I * c)} + 686832922526368134950199734132 \\
& 0517 * e^{(101 * I * c)} + 344277152012875134140739302960914 * e^{(100 * I * c)} + 16353164 \\
& 647151530240529137618111 * e^{(99 * I * c)} + 734057263616388449968842366924 * e^{(98 * \\
& I * c)} + 31042222522074681615625020522 * e^{(97 * I * c)} + 1232445557346832245176696 \\
& 904 * e^{(96 * I * c)} + 45759117183402579073139583 * e^{(95 * I * c)} + 158179664239781240 \\
& 8161814 * e^{(94 * I * c)} + 50648660944512569972179 * e^{(93 * I * c)} + 14933266122939841 \\
& 60368 * e^{(92 * I * c)} + 40261256699368950388 * e^{(91 * I * c)} + 984382804329835768 * e^{(\\
& 90 * I * c)} + 21608403021340047 * e^{(89 * I * c)} + 420601518659718 * e^{(88 * I * c)} + 71461 \\
& 42307307 * e^{(87 * I * c)} + 103818048048 * e^{(86 * I * c)} + 1253841160 * e^{(85 * I * c)} + 120 \\
& 85216 * e^{(84 * I * c)} + 87153 * e^{(83 * I * c)} + 418 * e^{(82 * I * c)} + e^{(81 * I * c)}) * \text{sqrt}(a) \\
& * \text{sgn}(\cos(1/2 * d * x + 1/2 * c)) / ((\tan(1/4 * d * x + c) ^ 4 * \tan(1/2 * c) ^ 8 - 14 * \tan(1/4 * d \\
& * x + c) ^ 4 * \tan(1/2 * c) ^ 6 + 24 * \tan(1/4 * d * x + c) ^ 3 * \tan(1/2 * c) ^ 7 - 6 * \tan(1/4 * d * x \\
& + c) ^ 2 * \tan(1/2 * c) ^ 8 - 56 * \tan(1/4 * d * x + c) ^ 3 * \tan(1/2 * c) ^ 5 + 84 * \tan(1/4 * d * x \\
& + c) ^ 2 * \tan(1/2 * c) ^ 6 - 24 * \tan(1/4 * d * x + c) * \tan(1/2 * c) ^ 7 + \tan(1/2 * c) ^ 8 + 14 *
\end{aligned}$$

$$\tan(1/4*d*x + c)^4*\tan(1/2*c)^2 - 56*\tan(1/4*d*x + c)^3*\tan(1/2*c)^3 + 56*\tan(1/4*d*x + c)*\tan(1/2*c)^5 - 14*\tan(1/2*c)^6 - \tan(1/4*d*x + c)^4 + 24*\tan(1/4*d*x + c)^3*\tan(1/2*c) - 84*\tan(1/4*d*x + c)^2*\tan(1/2*c)^2 + 56*\tan(1/4*d*x + c)*\tan(1/2*c)^3 + 6*\tan(1/4*d*x + c)^2 - 24*\tan(1/4*d*x + c)*\tan(1/2*c) + 14*\tan(1/2*c)^2 - 1)^4*d$$

Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \frac{35 a^2 \sin\left(\frac{3c}{2} + \frac{3dx}{2}\right) \sqrt{a + a \cos(c + dx)}}{63 d \sqrt{\cos(c + dx)} \cos\left(\frac{c}{2} + \frac{dx}{2}\right)} - \frac{35 a^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a + a \cos(c + dx)}}{2} + \frac{23 a^2 \sin\left(\frac{7c}{2} + \frac{7dx}{2}\right) \sqrt{a + a \cos(c + dx)}}{63 d \sqrt{\cos(c + dx)} \cos\left(\frac{3c}{2} + \frac{3dx}{2}\right)} + \frac{21 d \sqrt{\cos(c + dx)} \cos\left(\frac{5c}{2} + \frac{5dx}{2}\right)}{8} + \frac{21 d \sqrt{\cos(c + dx)} \cos\left(\frac{7c}{2} + \frac{7dx}{2}\right)}{8}$$

[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2),x)

[Out] (35*a^2*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2) - (35*a^2*sin(c/2 + (d*x)/2)*(a + a*cos(c + d*x))^(1/2))/2 + (23*a^2*sin((7*c)/2 + (7*d*x)/2)*(a + a*cos(c + d*x))^(1/2))/2)/((63*d*cos(c + d*x)^(1/2)*cos(c/2 + (d*x)/2))/8 + (63*d*cos(c + d*x)^(1/2)*cos((3*c)/2 + (3*d*x)/2))/8 + (21*d*cos(c + d*x)^(1/2)*cos((5*c)/2 + (5*d*x)/2))/8 + (21*d*cos(c + d*x)^(1/2)*cos((7*c)/2 + (7*d*x)/2))/8)

$$3.220 \quad \int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)} dx$$

Optimal result	3580
Rubi [A] (verified)	3580
Mathematica [A] (verified)	3583
Maple [A] (verified)	3583
Fricas [A] (verification not implemented)	3583
Sympy [F(-1)]	3584
Maxima [A] (verification not implemented)	3584
Giac [F(-1)]	3584
Mupad [B] (verification not implemented)	3585

Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned} \int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)} dx &= \frac{38a^3 \sin(c+dx)}{63d \cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} \\ &+ \frac{146a^3 \sin(c+dx)}{105d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{584a^3 \sin(c+dx)}{315d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} \\ &+ \frac{1168a^3 \sin(c+dx)}{315d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}} + \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{9d \cos^{\frac{9}{2}}(c+dx)} \end{aligned}$$

[Out] 38/63*a^3*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2)+146/105*a^3*
sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2)+584/315*a^3*sin(d*x+c)
/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1168/315*a^3*sin(d*x+c)/d/cos(d*
x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)+2/9*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(9/2)

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00,
number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used
= {2841, 3059, 2851, 2850}

$$\begin{aligned} \int \frac{(a+a \cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)} dx &= \frac{584a^3 \sin(c+dx)}{315d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} \\ &+ \frac{146a^3 \sin(c+dx)}{105d \cos^{\frac{5}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} + \frac{38a^3 \sin(c+dx)}{63d \cos^{\frac{7}{2}}(c+dx) \sqrt{a \cos(c+dx) + a}} \\ &+ \frac{1168a^3 \sin(c+dx)}{315d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx) + a}} + \frac{2a^2 \sin(c+dx) \sqrt{a \cos(c+dx) + a}}{9d \cos^{\frac{9}{2}}(c+dx)} \end{aligned}$$

[In] Int[(a + a*cos[c + d*x])^(5/2)/cos[c + d*x]^(11/2),x]

[Out] (38*a^3*sin[c + d*x])/(63*d*cos[c + d*x]^(7/2)*sqrt[a + a*cos[c + d*x]]) + (146*a^3*sin[c + d*x])/(105*d*cos[c + d*x]^(5/2)*sqrt[a + a*cos[c + d*x]]) + (584*a^3*sin[c + d*x])/(315*d*cos[c + d*x]^(3/2)*sqrt[a + a*cos[c + d*x]]) + (1168*a^3*sin[c + d*x])/(315*d*sqrt[cos[c + d*x]]*sqrt[a + a*cos[c + d*x]]) + (2*a^2*sqrt[a + a*cos[c + d*x]]*sin[c + d*x])/(9*d*cos[c + d*x]^(9/2))

Rule 2841

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*cos[e + f*x]*(a + b*sin[e + f*x])^(m - 2)*((c + d*sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*sin[e + f*x])^(m - 2)*(c + d*sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2850

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(cos[e + f*x]/(f*(b*c + a*d)*sqrt[a + b*sin[e + f*x]]*sqrt[c + d*sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*cos[e + f*x]*((c + d*sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*sqrt[a + b*sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*sin[e + f*x]]*(c + d*sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rule 3059

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(B*c - A*d)*cos[e + f*x]*((c + d*sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d)*sqrt[a + b*sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*sin[e + f*x]]*(c + d*sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -

1]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{9d \cos^{\frac{9}{2}}(c + dx)} \\
&\quad - \frac{1}{9}(2a) \int \frac{\left(-\frac{19a}{2} - \frac{15}{2}a \cos(c + dx)\right) \sqrt{a + a \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx \\
&= \frac{38a^3 \sin(c + dx)}{63d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{9d \cos^{\frac{9}{2}}(c + dx)} \\
&\quad + \frac{1}{21}(73a^2) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{38a^3 \sin(c + dx)}{63d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{146a^3 \sin(c + dx)}{105d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{9d \cos^{\frac{9}{2}}(c + dx)} + \frac{1}{105}(292a^2) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{38a^3 \sin(c + dx)}{63d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{146a^3 \sin(c + dx)}{105d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{584a^3 \sin(c + dx)}{315d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{9d \cos^{\frac{9}{2}}(c + dx)} \\
&\quad + \frac{1}{315}(584a^2) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{38a^3 \sin(c + dx)}{63d \cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} + \frac{146a^3 \sin(c + dx)}{105d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{584a^3 \sin(c + dx)}{315d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
&\quad + \frac{1168a^3 \sin(c + dx)}{315d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sin(c + dx)}{9d \cos^{\frac{9}{2}}(c + dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.42

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{11/2}(c + dx)} dx = \frac{a^2 \sqrt{a(1 + \cos(c + dx))} (727 + 698 \cos(c + dx) + 803 \cos(2(c + dx)) + 146 \cos(3(c + dx)) + 146 \cos(4(c + dx))) \tan((c + dx)/2)}{315d \cos^{9/2}(c + dx)}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]

[Out] (a^2*sqrt[a*(1 + Cos[c + d*x])]*(727 + 698*Cos[c + d*x] + 803*Cos[2*(c + d*x)] + 146*Cos[3*(c + d*x)] + 146*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(315*d*Cos[c + d*x]^(9/2))

Maple [A] (verified)

Time = 5.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{2 \sin(dx+c) (584 (\cos^4(dx+c)) + 292 (\cos^3(dx+c)) + 219 (\cos^2(dx+c)) + 130 \cos(dx+c) + 35) \sqrt{a(1+\cos(dx+c))} a^2}{315d(1+\cos(dx+c)) \cos(dx+c)^{9/2}}$	85

[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)

[Out] 2/315/d*sin(d*x+c)*(584*cos(d*x+c)^4+292*cos(d*x+c)^3+219*cos(d*x+c)^2+130*cos(d*x+c)+35)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(9/2)*a^2

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{11/2}(c + dx)} dx = \frac{2 (584 a^2 \cos(dx + c)^4 + 292 a^2 \cos(dx + c)^3 + 219 a^2 \cos(dx + c)^2 + 130 a^2 \cos(dx + c) + 35 a^2) \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c)}{315 (d \cos(dx + c))^6 + d \cos(dx + c)^5}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")

[Out] 2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x + c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{\frac{11}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.44

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{\frac{11}{2}}(c + dx)} dx = \frac{8 \left(\frac{315 \sqrt{2} a^{\frac{5}{2}} \sin(dx+c)}{\cos(dx+c)+1} - \frac{945 \sqrt{2} a^{\frac{5}{2}} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1449 \sqrt{2} a^{\frac{5}{2}} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{1287 \sqrt{2} a^{\frac{5}{2}} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right)}{315 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{11}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{11}{2}} \left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} \right)}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")

[Out] 8/315*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{\frac{11}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 21.05 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.39

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{11/2}(c + dx)} dx = \frac{\sqrt{a + a \cos(c + dx)} \left(\frac{192 a^2 e^{\frac{c9i}{2} + \frac{dx9i}{2}}}{5} \right)}{12 \sqrt{\cos(c + dx)} e^{\frac{c9i}{2} + \frac{dx9i}{2}} \cos\left(\frac{c}{2} + \frac{dx}{2}\right) + 8 \sqrt{\cos(c + dx)} e^{\frac{c9i}{2} + \frac{dx9i}{2}} \cos\left(\frac{3c}{2}\right)}$$

[In] int((a + a*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2),x)

```
[Out] ((a + a*cos(c + d*x))^(1/2)*((192*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin(c/2 +
(d*x)/2))/(5*d) - (16*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((3*c)/2 + (3*d*x)/
2))/(3*d) + (1168*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2 + (5*d*x)/2))/
(35*d) + (2336*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((9*c)/2 + (9*d*x)/2))/(31
5*d)))/(12*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos(c/2 + (d*x)/2)
+ 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((3*c)/2 + (3*d*x)/2)
+ 8*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((5*c)/2 + (5*d*x)/2)
+ 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((7*c)/2 + (7*d*x)/2)
+ 2*cos(c + d*x)^(1/2)*exp((c*9i)/2 + (d*x*9i)/2)*cos((9*c)/2 + (9*d*x)/2)
)
```

$$3.221 \quad \int \frac{(a+a \cos(c+dx))^{3/2}}{\cos^{5/4}(c+dx)} dx$$

Optimal result	3586
Rubi [A] (verified)	3586
Mathematica [A] (verified)	3587
Maple [F]	3587
Fricas [A] (verification not implemented)	3587
Sympy [F]	3588
Maxima [B] (verification not implemented)	3588
Giac [F(-1)]	3588
Mupad [B] (verification not implemented)	3589

Optimal result

Integrand size = 25, antiderivative size = 38

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/4}(c + dx)} dx = \frac{4a^2 \sin(c + dx)}{d \sqrt[4]{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}$$

[Out] $4*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/4)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2841, 8}

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/4}(c + dx)} dx = \frac{4a^2 \sin(c + dx)}{d \sqrt[4]{\cos(c + dx)} \sqrt{a \cos(c + dx) + a}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}/\text{Cos}[c + d*x]^{(5/4)}, x]$

[Out] $(4*a^2*\text{Sin}[c + d*x])/(d*\text{Cos}[c + d*x]^{(1/4)}*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2841

$\text{Int}[(a + b*\sin[e + f*x])^{(m)}*((c + d)*\sin[e + f*x] + (f*(x))^{(n)}), x_Symbol] \rightarrow \text{Simp}[(-b^2)*(b*c - a*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-2)}*((c + d*\text{Sin}[e + f*x])^{(n+1)})/(d*f*(n+1)*(b*c + a*d)), x] + \text{Dist}[b^2/(d*(n+1)*(b*c + a*d)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-2)}*((c + d*\text{Sin}[e + f*x])^{(n+1)})/(d*f*(n+1)*(b*c + a*d)), x]$

2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{4a^2 \sin(c + dx)}{d^4 \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} - (4a) \int 0 dx \\ &= \frac{4a^2 \sin(c + dx)}{d^4 \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/4}(c + dx)} dx = \frac{2(a(1 + \cos(c + dx)))^{3/2} \sec^2\left(\frac{1}{2}(c + dx)\right) \tan\left(\frac{1}{2}(c + dx)\right)}{d^4 \sqrt{\cos(c + dx)}}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/4), x]

[Out] (2*(a*(1 + Cos[c + d*x]))^(3/2)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(d*Cos[c + d*x]^(1/4))

Maple [F]

$$\int \frac{(a + \cos(dx + c) a)^{3/2}}{\cos(dx + c)^{5/4}} dx$$

[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(5/4), x)

[Out] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(5/4), x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/4}(c + dx)} dx = \frac{4 \sqrt{a \cos(dx + c) + a} a \cos(dx + c)^{3/4} \sin(dx + c)}{d \cos(dx + c)^2 + d \cos(dx + c)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4), x, algorithm="fricas")

[Out] 4*sqrt(a*cos(d*x + c) + a)*a*cos(d*x + c)^(3/4)*sin(d*x + c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))

Sympy [F]

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/4}(c + dx)} dx = \int \frac{(a(\cos(c + dx) + 1))^{3/2}}{\cos^{5/4}(c + dx)} dx$$

[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/4), x)

[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/cos(c + d*x)**(5/4), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(34) = 68$.

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.18

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/4}(c + dx)} dx = \frac{4 \left(\frac{\sqrt{2}a^{3/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sqrt{2}a^{3/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{5/4} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{5/4} \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^{1/4}}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4), x, algorithm="maxima")

[Out] 4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^(1/4))

Giac [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/4}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4), x, algorithm="giac")

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/4}(c + dx)} dx = \frac{4 a \sin(c + dx) \sqrt{a (\cos(c + dx) + 1)}}{d \cos(c + dx)^{1/4} (\cos(c + dx) + 1)}$$

[In] int((a + a*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/4),x)

[Out] (4*a*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*cos(c + d*x)^(1/4)*(cos(c + d*x) + 1))

$$3.222 \quad \int \frac{\sqrt{a+a \cos(e+fx)}}{\sqrt{\cos(e+fx)}} dx$$

Optimal result	3590
Rubi [A] (verified)	3590
Mathematica [A] (verified)	3591
Maple [B] (verified)	3591
Fricas [A] (verification not implemented)	3592
Sympy [F]	3592
Maxima [B] (verification not implemented)	3592
Giac [F]	3593
Mupad [F(-1)]	3593

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{\sqrt{a+a \cos(e+fx)}}{\sqrt{\cos(e+fx)}} dx = \frac{2\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(e+fx)}{\sqrt{a+a \cos(e+fx)}}\right)}{f}$$

[Out] 2*arcsin(sin(f*x+e)*a^(1/2)/(a+a*cos(f*x+e))^(1/2))*a^(1/2)/f

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2853, 222}

$$\int \frac{\sqrt{a+a \cos(e+fx)}}{\sqrt{\cos(e+fx)}} dx = \frac{2\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(e+fx)}{\sqrt{a \cos(e+fx)+a}}\right)}{f}$$

[In] Int[Sqrt[a + a*Cos[e + f*x]]/Sqrt[Cos[e + f*x]],x]

[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + a*Cos[e + f*x]])/f

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos

$[e + f*x]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]]], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[d, a/b]$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(e+fx)}{\sqrt{a+a \cos(e+fx)}}\right)}{f} \\ &= \frac{2\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(e+fx)}{\sqrt{a+a \cos(e+fx)}}\right)}{f} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\begin{aligned} &\int \frac{\sqrt{a + a \cos(e + fx)}}{\sqrt{\cos(e + fx)}} dx \\ &= \frac{\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(e + fx)\right)\right) \sqrt{a(1 + \cos(e + fx))} \sec\left(\frac{1}{2}(e + fx)\right)}{f} \end{aligned}$$

[In] Integrate[Sqrt[a + a*Cos[e + f*x]]/Sqrt[Cos[e + f*x]],x]

[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sqrt[a*(1 + Cos[e + f*x])]*Sec[(e + f*x)/2])/f

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(31) = 62.

Time = 4.90 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{2\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}} \sqrt{a(1+\cos(fx+e))} \arctan\left(\tan(fx+e)\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\right)}{f\sqrt{\cos(fx+e)}}$	72

[In] int((a+cos(f*x+e)*a)^(1/2)/cos(f*x+e)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/f/cos(f*x+e)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(a*(1+cos(f*x+e)))^(1/2)*arctan(tan(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{a + a \cos(e + fx)}}{\sqrt{\cos(e + fx)}} dx$$

$$= \left[\frac{\sqrt{-a} \log \left(\frac{2a \cos(fx+e)^2 - 2\sqrt{a \cos(fx+e)} + a\sqrt{-a} \sqrt{\cos(fx+e)} \sin(fx+e) + a \cos(fx+e) - a}{\cos(fx+e)+1} \right)}{f}, \right. \\ \left. - \frac{2\sqrt{a} \arctan \left(\frac{\sqrt{a \cos(fx+e)} + a\sqrt{\cos(fx+e)}}{\sqrt{a} \sin(fx+e)} \right)}{f} \right]$$

[In] integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="fricas")

```
[Out] [sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(a*cos(f*x + e) + a)*sqrt(-a)*sqrt(cos(f*x + e))*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))/f, - 2*sqrt(a)*arctan(sqrt(a*cos(f*x + e) + a)*sqrt(cos(f*x + e))/(sqrt(a)*sin(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sqrt{a + a \cos(e + fx)}}{\sqrt{\cos(e + fx)}} dx = \int \frac{\sqrt{a(\cos(e + fx) + 1)}}{\sqrt{\cos(e + fx)}} dx$$

[In] integrate((a+a*cos(f*x+e))**(1/2)/cos(f*x+e)**(1/2),x)

[Out] Integral(sqrt(a*(cos(e + f*x) + 1))/sqrt(cos(e + f*x)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(31) = 62.

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.95

$$\int \frac{\sqrt{a + a \cos(e + fx)}}{\sqrt{\cos(e + fx)}} dx$$

$$= \frac{\sqrt{a} \arctan \left((\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2 \cos(2fx + 2e) + 1)^{\frac{1}{4}} \sin \left(\frac{1}{2} \arctan(\sin(2fx + 2e)) \right) \right)}{f}$$

[In] integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="maxima")

```
[Out] sqrt(a)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + si
n(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(f*
x + e))/f
```

Giac **[F]**

$$\int \frac{\sqrt{a + a \cos(e + fx)}}{\sqrt{\cos(e + fx)}} dx = \int \frac{\sqrt{a \cos(fx + e) + a}}{\sqrt{\cos(fx + e)}} dx$$

```
[In] integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(f*x + e) + a)/sqrt(cos(f*x + e)), x)
```

Mupad **[F(-1)]**

Timed out.

$$\int \frac{\sqrt{a + a \cos(e + fx)}}{\sqrt{\cos(e + fx)}} dx = \int \frac{\sqrt{a + a \cos(e + fx)}}{\sqrt{\cos(e + fx)}} dx$$

```
[In] int((a + a*cos(e + f*x))^(1/2)/cos(e + f*x)^(1/2),x)
```

```
[Out] int((a + a*cos(e + f*x))^(1/2)/cos(e + f*x)^(1/2), x)
```

$$3.223 \quad \int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx$$

Optimal result	3594
Rubi [A] (verified)	3594
Mathematica [A] (verified)	3595
Maple [B] (verified)	3595
Fricas [A] (verification not implemented)	3596
Sympy [F]	3596
Maxima [B] (verification not implemented)	3596
Giac [B] (verification not implemented)	3597
Mupad [F(-1)]	3597

Optimal result

Integrand size = 28, antiderivative size = 38

$$\int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx = -\frac{2\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(e + fx)}{\sqrt{a - a \cos(e + fx)}}\right)}{f}$$

[Out] $-2*\arcsin(\sin(f*x+e)*a^{(1/2)}/(a-a*\cos(f*x+e))^{(1/2)})*a^{(1/2)}/f$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2853, 222}

$$\int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx = -\frac{2\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(e + fx)}{\sqrt{a - a \cos(e + fx)}}\right)}{f}$$

[In] `Int[Sqrt[a - a*Cos[e + f*x]]/Sqrt[-Cos[e + f*x]],x]`

[Out] `(-2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a - a*Cos[e + f*x]])/f`

Rule 222

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

Rule 2853

`Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos`

$[e + f*x]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]]], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[d, a/b]$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, \frac{a \sin(e+fx)}{\sqrt{a-a \cos(e+fx)}}\right)}{f} \\ &= -\frac{2\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(e+fx)}{\sqrt{a-a \cos(e+fx)}}\right)}{f} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx = \frac{2 \arcsin\left(\sqrt{-\cos(e + fx)}\right) \sqrt{a - a \cos(e + fx)} \cot\left(\frac{1}{2}(e + fx)\right)}{f \sqrt{1 + \cos(e + fx)}}$$

[In] Integrate[Sqrt[a - a*Cos[e + f*x]]/Sqrt[-Cos[e + f*x]],x]

[Out] (2*ArcSin[Sqrt[-Cos[e + f*x]]]*Sqrt[a - a*Cos[e + f*x]]*Cot[(e + f*x)/2])/(f*Sqrt[1 + Cos[e + f*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(32) = 64.

Time = 3.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.18

method	result	size
default	$\frac{2\sqrt{-\frac{\cos(fx+e)}{1+\cos(fx+e)}} \sqrt{-a(\cos(fx+e)-1)} \arctan\left(\sqrt{-\frac{\cos(fx+e)}{1+\cos(fx+e)}}\right) (\cot(fx+e)+\csc(fx+e))}{f \sqrt{-\cos(fx+e)}}$	83

[In] int((a-cos(f*x+e)*a)^(1/2)/(-cos(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/f*(-cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-a*(cos(f*x+e)-1))^(1/2)*arctan((-cos(f*x+e)/(1+cos(f*x+e)))^(1/2)/(-cos(f*x+e))^(1/2)*(cot(f*x+e)+csc(f*x+e)))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 4.32

$$\int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx$$

$$= \left[\frac{\sqrt{-a} \log \left(\frac{4 \sqrt{-a \cos(fx+e)+a} (2 \cos(fx+e)^2 + 3 \cos(fx+e) + 1) \sqrt{-a} \sqrt{-\cos(fx+e)} - (8a \cos(fx+e)^2 + 8a \cos(fx+e) + a) \sin(fx+e)}{\sin(fx+e)} \right)}{2f} \right],$$

[In] integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="fricas")

```
[Out] [1/2*sqrt(-a)*log((4*sqrt(-a*cos(f*x + e) + a)*(2*cos(f*x + e)^2 + 3*cos(f*x + e) + 1)*sqrt(-a)*sqrt(-cos(f*x + e)) - (8*a*cos(f*x + e)^2 + 8*a*cos(f*x + e) + a)*sin(f*x + e))/sin(f*x + e))/f, sqrt(a)*arctan(1/2*sqrt(-a*cos(f*x + e) + a)*sqrt(-cos(f*x + e))*(2*cos(f*x + e) + 1)/(sqrt(a)*cos(f*x + e)*sin(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx = \int \frac{\sqrt{-a (\cos(e + fx) - 1)}}{\sqrt{-\cos(e + fx)}} dx$$

[In] integrate((a-a*cos(f*x+e))**(1/2)/(-cos(f*x+e))**(1/2),x)

[Out] Integral(sqrt(-a*(cos(e + f*x) - 1))/sqrt(-cos(e + f*x)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(32) = 64.

Time = 0.41 (sec) , antiderivative size = 420, normalized size of antiderivative = 11.05

$$\int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx$$

$$= \frac{\sqrt{-a} \left(\log \left(4 \sqrt{\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2 \cos(2fx + 2e) + 1} \cos \left(\frac{1}{2} \arctan(\sin(2fx + 2e)) \right) \right) \right)}{2f}$$

[In] integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{2}\sqrt{-a}(\log(4\sqrt{\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1})\cos(\frac{1}{2}\arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))^2 + 4\sqrt{\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1}\sin(\frac{1}{2}\arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))^2 + 8(\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1)^{1/4}\cos(\frac{1}{2}\arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) + 4) - \log(\cos(fx + e)^2 + \sin(fx + e)^2 + \sqrt{\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1})\cos(\frac{1}{2}\arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))^2 + \sin(\frac{1}{2}\arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))^2 + 2(\cos(2fx + 2e)^2 + \sin(2fx + 2e)^2 + 2\cos(2fx + 2e) + 1)^{1/4}(\cos(fx + e)\cos(\frac{1}{2}\arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1)) + \sin(fx + e)\sin(\frac{1}{2}\arctan2(\sin(2fx + 2e), \cos(2fx + 2e) + 1))))/f$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(32) = 64$.

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx = \frac{4\sqrt{a} \arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} + \frac{2\left(2\sqrt{2} - \sqrt{-\tan(\frac{1}{4}fx + \frac{1}{4}e)^4 + 6\tan(\frac{1}{4}fx + \frac{1}{4}e)^2 - 1}\right)}{\tan(\frac{1}{4}fx + \frac{1}{4}e)^2 - 3}\right)\right) \operatorname{sgn}\left(\sin\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{f}$$

[In] `integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="giac")`

[Out] $-4\sqrt{a}\arctan(-\frac{1}{2}\sqrt{2}(\sqrt{2} + \frac{2(2\sqrt{2} - \sqrt{-\tan(1/4fx + 1/4e)^4 + 6\tan(1/4fx + 1/4e)^2 - 1})}{\tan(1/4fx + 1/4e)^2 - 3}))\operatorname{sgn}(\sin(1/2fx + 1/2e))/f$

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx = \int \frac{\sqrt{a - a \cos(e + fx)}}{\sqrt{-\cos(e + fx)}} dx$$

[In] `int((a - a*cos(e + f*x))^(1/2)/(-cos(e + f*x))^(1/2),x)`

[Out] `int((a - a*cos(e + f*x))^(1/2)/(-cos(e + f*x))^(1/2), x)`

$$3.224 \quad \int \frac{\cos^5(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	3598
Rubi [A] (verified)	3598
Mathematica [A] (verified)	3601
Maple [A] (verified)	3601
Fricas [A] (verification not implemented)	3601
Sympy [F(-1)]	3602
Maxima [F]	3602
Giac [F(-1)]	3602
Mupad [F(-1)]	3603

Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{\cos^5(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{7 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4\sqrt{ad}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{a+a \cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{a+a \cos(c+dx)}}$$

[Out] 7/4*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)-1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2857, 3062, 3061, 2861, 211, 2853, 222}

$$\int \frac{\cos^5(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = \frac{7 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4\sqrt{ad}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} + \frac{\sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a \cos(c+dx)+a}} - \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{4d\sqrt{a \cos(c+dx)+a}}$$

[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]

[Out] $(7 \cdot \text{ArcSin}[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{a + a \cos[c + dx]}}]) / (4 \sqrt{a} d) - (\sqrt{2} \cdot \text{ArcTan}[\frac{\sqrt{a} \sin[c + dx]}{\sqrt{2} \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]}}]) / (\sqrt{a} d) - (\sqrt{\cos[c + dx]} \sin[c + dx]) / (4 d \sqrt{a + a \cos[c + dx]}) + (\cos[c + dx]^{3/2} \sin[c + dx]) / (2 d \sqrt{a + a \cos[c + dx]})$

Rule 211

$\text{Int}[(a_ + (b_ \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Rt}[a/b, 2]/a \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$

Rule 222

$\text{Int}[1/\sqrt{(a_ + (b_ \cdot x)^2)}, x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2] \cdot (x/\sqrt{a})]/\text{Rt}[-b, 2], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

Rule 2853

$\text{Int}[\sqrt{(a_ + (b_ \cdot \sin[e_ + (f_ \cdot x)])}/\sqrt{(d_ \cdot \sin[e_ + (f_ \cdot x)]) \cdot (x_)}], x_Symbol] \rightarrow \text{Dist}[-2/f, \text{Subst}[\text{Int}[1/\sqrt{1 - x^2/a}], x], x, b \cdot (\cos[e + fx]/\sqrt{a + b \sin[e + fx]})], x] /; \text{FreeQ}\{a, b, d, e, f\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[d, a/b]$

Rule 2857

$\text{Int}[(c_ + (d_ \cdot \sin[e_ + (f_ \cdot x)])^n)/\sqrt{(a_ + (b_ \cdot \sin[e_ + (f_ \cdot x)]) \cdot (x_)}], x_Symbol] \rightarrow \text{Simp}[-2d \cdot \cos[e + fx] \cdot (c + d \sin[e + fx])^{n-1} / (f \cdot (2n-1) \sqrt{a + b \sin[e + fx]})], x] - \text{Dist}[1/(b \cdot (2n-1)), \text{Int}[(c + d \sin[e + fx])^{n-2} / \sqrt{a + b \sin[e + fx]}] \cdot \text{Simp}[a \cdot c \cdot d - b \cdot (2d^2 \cdot (n-1) + c^2 \cdot (2n-1)) + d \cdot (a \cdot d - b \cdot c \cdot (4n-3)) \cdot \sin[e + fx], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2n]$

Rule 2861

$\text{Int}[1/(\sqrt{(a_ + (b_ \cdot \sin[e_ + (f_ \cdot x)])} \cdot \sqrt{(c_ + (d_ \cdot \sin[e_ + (f_ \cdot x)]) \cdot (x_)}), x_Symbol] \rightarrow \text{Dist}[-2 \cdot (a/f), \text{Subst}[\text{Int}[1/(2b^2 - (a \cdot c - b \cdot d) \cdot x^2)], x], x, b \cdot (\cos[e + fx]/(\sqrt{a + b \sin[e + fx]} \cdot \sqrt{c + d \sin[e + fx]})], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3061

$\text{Int}[(A_ + (B_ \cdot \sin[e_ + (f_ \cdot x)])]/\sqrt{(a_ + (b_ \cdot \sin[e_ + (f_ \cdot x)]) \cdot (x_)}), x_Symbol] \rightarrow \text{Dist}[(A \cdot b - a \cdot B)/b, \text{Int}[1/(\sqrt{a + b \sin[e + fx]} \cdot \sqrt{c + d \sin[e + fx]}), x], x] + \text{Dist}[B/b, \text{Int}[\sqrt{a + b \sin[e + fx]}/\sqrt{c + d \sin[e + fx]}], x], x]$

$x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3062

$\text{Int}[\{(a_) + (b_)*\sin[(e_) + (f_)*(x_)]\}^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)]\}^{(n_)}, x_Symbol] \ :> \ \text{Simp}[(-B)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*((c + d*\text{Sin}[e + f*x])^n/(f*(m + n + 1))), x] + \text{Dist}[1/(b*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(n - 1)}*\text{Simp}[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{EqQ}[m + 1/2, 0])$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{\sqrt{\cos(c+dx)}(-3a+a \cos(c+dx))}{\sqrt{a+a \cos(c+dx)}} dx}{4a} \\
 &= -\frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a + a \cos(c + dx)}} + \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{\frac{a^2}{2} - \frac{7}{2}a^2 \cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} dx}{4a^2} \\
 &= -\frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a + a \cos(c + dx)}} + \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{7 \int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{8a} - \int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{a + a \cos(c + dx)}} dx \\
 &= -\frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a + a \cos(c + dx)}} + \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \cos(c + dx)}} \\
 &\quad - \frac{7 \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4ad} \\
 &\quad + \frac{(2a) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{d} \\
 &= \frac{7 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{4\sqrt{ad}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} \\
 &\quad - \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a + a \cos(c + dx)}} + \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx =$$

$$\frac{\left(\arcsin\left(\sqrt{1-\cos(c+dx)}\right) + 8\arcsin\left(\sqrt{\cos(c+dx)}\right) - 4\sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right) - 2\sqrt{1-\cos(c+dx)}\right)}{4d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}$$

[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]

[Out] -1/4*((ArcSin[Sqrt[1 - Cos[c + d*x]]] + 8*ArcSin[Sqrt[Cos[c + d*x]]] - 4*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] - 2*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 12.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.04

method	result
default	$\frac{(2\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - \sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 7\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) + 8\arcsin(\cot(dx+c)))}{8d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}a}$

[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8/d*(2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+8*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{\sqrt{a\cos(dx+c)+a}(2\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c) - 7\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\right)}{4(ad\cos(dx+c)+ad)}$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{4} \sqrt{a \cos(dx + c) + a} (2 \cos(dx + c) - 1) \sqrt{\cos(dx + c)} \sin(dx + c) - 7 \sqrt{a} (\cos(dx + c) + 1) \arctan(\sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)}) / (\sqrt{a} \sin(dx + c)) + 4 \sqrt{2} (a \cos(dx + c) + a) \arctan(\sqrt{2} \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)}) / (\sqrt{a} \sin(dx + c)) / \sqrt{a} / (a d \cos(dx + c) + a d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{\sqrt{a \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{5/2}}{\sqrt{a + a \cos(c + dx)}} dx$$

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(1/2), x)
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```
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(1/2), x)
```

$$3.225 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	3604
Rubi [A] (verified)	3604
Mathematica [A] (verified)	3606
Maple [A] (verified)	3607
Fricas [A] (verification not implemented)	3607
Sympy [F]	3607
Maxima [F]	3608
Giac [F(-1)]	3608
Mupad [F(-1)]	3608

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = -\frac{\arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d \sqrt{a+a \cos(c+dx)}}$$

[Out] $-\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/d/a^{(1/2)}+\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2857, 3061, 2861, 211, 2853, 222}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx = -\frac{\arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{d \sqrt{a \cos(c+dx)+a}}$$

[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]

[Out] $-\frac{\text{ArcSin}[\sqrt{a}\sin[c + dx]]/\sqrt{a + a\cos[c + dx]}}{\sqrt{a}d} + \left(\sqrt{2}\text{ArcTan}\left[\frac{\sqrt{a}\sin[c + dx]}{\sqrt{2}\sqrt{\cos[c + dx]}\sqrt{a + a\cos[c + dx]}}\right]\right)/\sqrt{a}d + \frac{\sqrt{\cos[c + dx]}\sin[c + dx]}{d\sqrt{a + a\cos[c + dx]}}$

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2857

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]))], x] - Dist[1/(b*(2*n - 1)), Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3061

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]

$x]$, $x]$ /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2a} \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{2a} + \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} \\
 &\quad - \frac{(2a)\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
 &= -\frac{\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{ad}} \\
 &\quad + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\begin{aligned}
 &\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx \\
 &= \frac{\left(\arcsin\left(\sqrt{1-\cos(c+dx)}\right) + 2\arcsin\left(\sqrt{\cos(c+dx)}\right) - \sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right) + \sqrt{-((-1+\cos(c+dx))\cos(c+dx))}\right)}{d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]], x]

[Out] ((ArcSin[Sqrt[1 - Cos[c + d*x]]] + 2*ArcSin[Sqrt[Cos[c + d*x]]] - Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] + Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 12.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10

method	result
default	$\frac{(\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\sqrt{2}\arctan(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})-2\arcsin(\cot(dx+c)-\csc(dx+c)))(\sqrt{\cos(dx+c)}\sqrt{a(1+\cos(dx+c))})}{2d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}a}$

[In] `int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2}d*(\sin(dx+c)*2^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}-2^{(1/2)}*\arctan(\tan(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}-2*\arcsin(\cot(dx+c)-\csc(dx+c))))*\cos(dx+c)^{(1/2)}*(a*(1+\cos(dx+c)))^{(1/2)}/(1+\cos(dx+c))/(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*2^{(1/2)}/a$

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.12

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - \frac{\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{ad\cos(dx+c)+ad} + \dots$$

[In] `integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $(\sqrt{a}*(\cos(dx+c)+1)*\arctan(\sqrt{a*\cos(dx+c)+a}*\sqrt{\cos(dx+c)})/(\sqrt{a}*\sin(dx+c))) - \sqrt{2}*(a*\cos(dx+c)+a)*\arctan(\sqrt{2}*\sqrt{a*\cos(dx+c)+a}*\sqrt{\cos(dx+c)})/(\sqrt{a}*\sin(dx+c)))/\sqrt{a} + \sqrt{a}*\cos(dx+c)+a)*\sqrt{\cos(dx+c)}*\sin(dx+c)/(a*d*\cos(dx+c)+a*d)$

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a(\cos(c+dx)+1)}} dx$$

[In] `integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)`[Out] `Integral(cos(c+d*x)**(3/2)/sqrt(a*(cos(c+d*x)+1)),x)`

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{\sqrt{a \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{3/2}}{\sqrt{a + a \cos(c + dx)}} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(1/2), x)

$$3.226 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	3609
Rubi [A] (verified)	3609
Mathematica [A] (verified)	3611
Maple [A] (verified)	3611
Fricas [A] (verification not implemented)	3611
Sympy [F]	3612
Maxima [C] (verification not implemented)	3612
Giac [F]	3613
Mupad [F(-1)]	3613

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a \cos(c+dx)}} dx = \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}}$$

[Out] 2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2856, 2853, 222, 2861, 211}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a \cos(c+dx)}} dx = \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}}$$

[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2856

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{a} - \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}} dx \\
 &= -\frac{2 \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{ad} \\
 &\quad + \frac{(2a) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{d} \\
 &= \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{\left(-2 \arcsin\left(\sqrt{\cos(c+dx)}\right) + \sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\right)\right) \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}$$

[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]

[Out] ((-2*ArcSin[Sqrt[Cos[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Cos[c + d*x]])/Sqrt[1 - Cos[c + d*x]])*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)}\sqrt{a(1+\cos(dx+c))}(\sqrt{2}\arctan(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})+\arcsin(\cot(dx+c)-\csc(dx+c)))\sqrt{2}}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}a}}$	108

[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+arcsin(cot(d*x+c)-csc(d*x+c)))/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - 2\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{ad}$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] (sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/(a*d)

Giac [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(dx + c)}}{\sqrt{a \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(a*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{a + a \cos(c + dx)}} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(1/2), x)

$$3.227 \quad \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx$$

Optimal result	3614
Rubi [A] (verified)	3614
Mathematica [A] (verified)	3615
Maple [A] (verified)	3615
Fricas [A] (verification not implemented)	3616
Sympy [F]	3616
Maxima [C] (verification not implemented)	3616
Giac [F]	3617
Mupad [F(-1)]	3617

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx = \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{ad}}$$

[Out] arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2861, 211}

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx = \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{ad}}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]

[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{d} \\ &= \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} dx = \frac{2 \arctan\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right) \cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]

[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{a(1+\cos(dx+c))} \arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2}}{d\sqrt{\cos(dx+c)}a}$	67

[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)/a

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.84

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx$$

$$= \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}} \log\left(-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}}{2(\cos(dx+c)+1)}\right)}{\sqrt{a}} \right]$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(-1/a)
*sqrt(cos(d*x + c))*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*
x + c))*sqrt(a)))/(sqrt(a)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx = \int \frac{1}{\sqrt{a(\cos(c+dx)+1)}\sqrt{\cos(c+dx)}} dx$$

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x))), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 522, normalized size of antiderivative = 9.32

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{\sqrt{2} \arctan\left(\frac{\left(|e^{i dx+i c}+1\right|^4+\cos(dx+c)^4+\sin(dx+c)^4+2\left(\cos(dx+c)^2-\sin(dx+c)^2-2\cos(dx+c)+1\right)\left|e^{i dx+i c}+1\right|^2-4\cos(dx+c)^3+2\left(\cos(dx+c)+1\right)\right)}{\dots}}{\dots}$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)
)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x
+ I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)
*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arct
an2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I
*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/
abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((ab
s(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)
)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*c
os(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*
cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2(2*(cos(d
*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c)
+ 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x
+ I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a)*abs(e^(I*d*x +
I*c) + 1)))/(sqrt(a)*d)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx = \int \frac{1}{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}} dx$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(1/2)), x)
```

$$3.228 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	3618
Rubi [A] (verified)	3618
Mathematica [C] (warning: unable to verify)	3620
Maple [A] (verified)	3620
Fricas [A] (verification not implemented)	3620
Sympy [F]	3621
Maxima [C] (verification not implemented)	3621
Giac [F]	3622
Mupad [F(-1)]	3622

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} dx = -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx)}{d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}$$

[Out] $-\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2858, 12, 2861, 211}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} dx = \frac{2 \sin(c+dx)}{d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx) + a}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx) + a}}\right)}{\sqrt{ad}}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]), x]

[Out] $-\left(\left(\text{Sqrt}[2]*\text{ArcTan}\left[\frac{\text{Sqrt}[a]*\text{Sin}[c + d*x]}{\text{Sqrt}[2]*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]}\right]\right)/\left(\text{Sqrt}[a]*d\right)\right) + \left(2*\text{Sin}[c + d*x]\right)/\left(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]\right)$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 211

$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a) * \text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 2858

$\text{Int}[(c_*) + (d_*) * \sin[(e_*) + (f_*)(x_)])^{(n_*)} / \text{Sqrt}[(a_*) + (b_*) * \sin[(e_*) + (f_*)(x_)]], x_Symbol] \rightarrow \text{Simp}[(-d) * \text{Cos}[e + f*x] * ((c + d * \text{Sin}[e + f*x])^{(n + 1)} / (f * (n + 1) * (c^2 - d^2) * \text{Sqrt}[a + b * \text{Sin}[e + f*x]])), x] - \text{Dist}[1 / (2 * b * (n + 1) * (c^2 - d^2)), \text{Int}[(c + d * \text{Sin}[e + f*x])^{(n + 1)} * (\text{Simp}[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3) * \text{Sin}[e + f*x], x] / \text{Sqrt}[a + b * \text{Sin}[e + f*x]]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2861

$\text{Int}[1 / (\text{Sqrt}[(a_*) + (b_*) * \sin[(e_*) + (f_*)(x_)]) * \text{Sqrt}[(c_*) + (d_*) * \sin[(e_*) + (f_*)(x_)]]), x_Symbol] \rightarrow \text{Dist}[-2*(a/f), \text{Subst}[\text{Int}[1 / (2*b^2 - (a*c - b*d)*x^2), x], x, b * (\text{Cos}[e + f*x] / (\text{Sqrt}[a + b * \text{Sin}[e + f*x]] * \text{Sqrt}[c + d * \text{Sin}[e + f*x]])]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{a}{\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} dx}{a} \\
 &= \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} - \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} dx \\
 &= \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} + \frac{(2a) \text{Subst}\left(\int \frac{1}{2a^2 + ax^2} dx, x, -\frac{a \sin(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}\right)}{d} \\
 &= -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2} \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.55 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.94

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{2 \cos\left(\frac{1}{2}(c+dx)\right) \sin\left(\frac{1}{2}(c+dx)\right) \left(\frac{1}{2} \cos(c+dx)(2+\cos(c+dx)) \operatorname{csc}^4\left(\frac{1}{2}(c+dx)\right) \left(1-\cos(c+dx)\right) + \operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2(\cos(dx+c)^2+\cos(dx+c))\sqrt{a}}\right)\right)}{ad \cos(dx+c)^2 + ad \cos(dx+c)}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]

[Out] (2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 + Cos[c + d*x])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]])]/2 - (Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/10))/(d*Cos[c + d*x]^(3/2)*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 5.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

method	result
default	$\frac{\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\operatorname{csc}(dx+c))+\sqrt{2} \sin(dx+c)+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\operatorname{csc}(dx+c))\right)\sqrt{a(1+\cos(dx+c))}}{d(1+\cos(dx+c))\sqrt{\cos(dx+c)} a}$

[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+2^(1/2)*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(1/2)*2^(1/2)/a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.42

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx =$$

$$\frac{\sqrt{2}(a\cos(dx+c)^2+a\cos(dx+c)) \arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2(\cos(dx+c)^2+\cos(dx+c))\sqrt{a}}\right)}{\sqrt{a}} - 2\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)$$

$$ad \cos(dx+c)^2 + ad \cos(dx+c)$$


```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -(sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x +
c))*sqrt(a)))/sqrt(a) - 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(
d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a \cos(c + dx)}} dx = \int \frac{1}{\sqrt{a(\cos(c + dx) + 1)}\cos^{\frac{3}{2}}(c + dx)} dx$$

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3/2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 665, normalized size of antiderivative = 7.15

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a \cos(c + dx)}} dx$$

$$2 \cos\left(\frac{1}{2} \arctan(\sin(2 dx + 2c), \cos(2 dx + 2c) + 1)\right) \sin(dx + c) - 2(\cos(dx + c) - 1) \sin\left(\frac{1}{2} \arctan(\sin(2 dx + 2c), \cos(2 dx + 2c) + 1)\right)$$

=

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] (2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
2*(cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x
+ c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*
d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) +
1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*a
rctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e
^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) +
1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), (
(abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x
+ c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 -
```

$$4*\cos(d*x + c)^3 + 2*(\cos(d*x + c)^2 - 2*\cos(d*x + c) + 1)*\sin(d*x + c)^2 + 6*\cos(d*x + c)^2 - 4*\cos(d*x + c) + 1)^{(1/4)}*\sqrt{a}*\cos(1/2*\arctan2(2*(\cos(d*x + c) - 1)*\sin(d*x + c)/\text{abs}(e^{(I*d*x + I*c)} + 1)^2, (\text{abs}(e^{(I*d*x + I*c)} + 1)^2 + \cos(d*x + c)^2 - \sin(d*x + c)^2 - 2*\cos(d*x + c) + 1)/\text{abs}(e^{(I*d*x + I*c)} + 1)^2)) + \sqrt{a}*\cos(d*x + c) - \sqrt{a})/(\sqrt{a}*\text{abs}(e^{(I*d*x + I*c)} + 1))) / ((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{(1/4)}*\sqrt{a})*d$$

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}} dx = \int \frac{1}{\sqrt{a\cos(dx + c) + a}\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^{3/2}\sqrt{a + a\cos(c + dx)}} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(1/2)), x)

$$3.229 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	3623
Rubi [A] (verified)	3623
Mathematica [C] (warning: unable to verify)	3625
Maple [A] (verified)	3626
Fricas [A] (verification not implemented)	3626
Sympy [F]	3627
Maxima [C] (verification not implemented)	3627
Giac [F]	3628
Mupad [F(-1)]	3628

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} dx = \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} - \frac{2 \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}$$

[Out] arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+2/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)-2/3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2858, 3063, 12, 2861, 211}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} dx = \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx)+a}} - \frac{2 \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*cos[c + d*x]]),x]

[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*cos[c + d*x]])]/(Sqrt[a]*d) + (2*Sin[c + d*x])/(3*d*cos[c + d*x]^(3/2)*Sqrt[a + a*cos[c + d*x]]) - (2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2858

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{a - 2a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx}{3a} \\
 &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
 &\quad - \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} - \frac{2 \int -\frac{3a^2}{2 \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} dx}{3a^2} \\
 &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} \\
 &\quad + \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} dx \\
 &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} \\
 &\quad - \frac{(2a) \text{Subst}\left(\int \frac{1}{2a^2 + ax^2} dx, x, -\frac{a \sin(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}\right)}{d} \\
 &= \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2} \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
 &\quad - \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.16 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.61

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx =$$

$$\frac{2 \cot\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^4\left(\frac{c}{2} + \frac{dx}{2}\right) \left(12 \cos^4\left(\frac{1}{2}(c + dx)\right) {}_3F_2\left(2, 2, \frac{7}{2}; 1, \frac{9}{2}; \frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) \sin^8\left(\frac{c}{2} + \frac{dx}{2}\right) + 12 \right)}{d}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]

[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2])*Sin[c/2 + (d*x)/2]^8 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2

$$\begin{aligned} &+ (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)]*\text{Sin}[c/2 + (d*x)/2]^8*(4 - 7*\text{Sin} \\ &[c/2 + (d*x)/2]^2 + 3*\text{Sin}[c/2 + (d*x)/2]^4) + 7*(1 - 2*\text{Sin}[c/2 + (d*x)/2]^2) \\ &)^3*\text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)]*(15 - 20*\text{Sin}[c \\ &/2 + (d*x)/2]^2 + 8*\text{Sin}[c/2 + (d*x)/2]^4)*(ArcTanh[\text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^ \\ &2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)]*(3 - 6*\text{Sin}[c/2 + (d*x)/2]^2) + \text{Sqrt}[\text{Sin}[c \\ &/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)]*(-3 + 7*\text{Sin}[c/2 + (d*x)/2]^2 \\ &)))/((63*d*\text{Sqrt}[a*(1 + \text{Cos}[c + d*x])])*(1 - 2*\text{Sin}[c/2 + (d*x)/2]^2)^(7/2)) \end{aligned}$$

Maple [A] (verified)

Time = 5.85 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.21

method	result
default	$-\frac{\left(3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c))\arcsin(\cot(dx+c)-\csc(dx+c))+\sin(dx+c)\cos(dx+c)\sqrt{2+3\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))\right)}{3d(1+\cos(dx+c))\cos(dx+c)^{\frac{3}{2}}a}$

[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3/d*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))+sin(d*x+c)*cos(d*x+c)*2^(1/2)+3*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))-2^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(3/2)*2^(1/2)/a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx = \frac{2\sqrt{a\cos(dx+c)+a(\cos(dx+c)-1)}\sqrt{\cos(dx+c)}\sin(dx+c) - \frac{3\sqrt{2}(a\cos(dx+c)^3+a\cos(dx+c)^2)\arctan\left(\frac{\sqrt{2}\sin(dx+c)}{\sqrt{a\cos(dx+c)+a(\cos(dx+c)-1)}}\right)}{\sqrt{a}}}{3(ad\cos(dx+c))^3+ad\cos(dx+c)^2}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -1/3*(2*sqrt(a*cos(d*x+c)+a)*(cos(d*x+c)-1)*sqrt(cos(d*x+c))*sin(d*x+c)-3*sqrt(2)*(a*cos(d*x+c)^3+a*cos(d*x+c)^2)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x+c)+a)*sqrt(cos(d*x+c))*sin(d*x+c)/((cos(d*x+c)^2+cos(d*x+c))*sqrt(a)))/sqrt(a)/(a*d*cos(d*x+c)^3+a*d*cos(d*x+c)^2)

Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx = \int \frac{1}{\sqrt{a(\cos(c+dx)+1)}\cos^{\frac{5}{2}}(c+dx)} dx$$

[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(5/2)), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 818, normalized size of antiderivative = 6.24

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx = \text{Too large to display}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1))) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) - 3)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)*d)

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx = \int \frac{1}{\sqrt{a\cos(dx+c)+a}\cos^{\frac{5}{2}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx = \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx$$

[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(1/2)), x)

$$3.230 \quad \int \frac{1}{\cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	3629
Rubi [A] (verified)	3629
Mathematica [C] (warning: unable to verify)	3632
Maple [A] (verified)	3633
Fricas [A] (verification not implemented)	3633
Sympy [F(-1)]	3634
Maxima [C] (verification not implemented)	3634
Giac [F]	3635
Mupad [F(-1)]	3635

Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} dx = -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} - \frac{2 \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{26 \sin(c+dx)}{15d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}$$

[Out] $-\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+2/5*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)/(a+a*\cos(d*x+c))^{(1/2)}-2/15*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)/(a+a*\cos(d*x+c))^{(1/2)}+26/15*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)}}$

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

= {2858, 3063, 12, 2861, 211}

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx = -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{ad}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}} + \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}} + \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]), x]

[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2858

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&

EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{\int \frac{a - 4a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx}{5a} \\
 &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
 &\quad - \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{2 \int \frac{-\frac{13a^2}{2} + a^2 \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx}{15a^2} \\
 &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{26 \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} - \frac{4 \int \frac{15a^3}{4 \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} dx}{15a^3} \\
 &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{26 \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} - \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} dx \\
 &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} - \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{26 \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} \\
 &\quad + \frac{(2a) \text{Subst}\left(\int \frac{1}{2a^2 + ax^2} dx, x, -\frac{a \sin(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}}\right)}{d}
 \end{aligned}$$

$$= -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} - \frac{2 \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} + \frac{26 \sin(c+dx)}{15d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.59 (sec) , antiderivative size = 1540, normalized size of antiderivative = 9.11

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} dx = \frac{2 \cot\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^6\left(\frac{c}{2} + \frac{dx}{2}\right) \left(4725 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) - 48825 \sin^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 210105 \sin^6\left(\frac{c}{2} + \frac{dx}{2}\right) - 486630 \sin^8\left(\frac{c}{2} + \frac{dx}{2}\right) + 655812 \sin^{10}\left(\frac{c}{2} + \frac{dx}{2}\right) - 710 \operatorname{Hypergeometric2F1}\left[2, \frac{9}{2}, \frac{11}{2}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) - 40 \cos\left(\frac{c+dx}{2}\right) \operatorname{HypergeometricPFQ}\left[\{2, 2, 2, \frac{9}{2}\}, \{1, 1, \frac{11}{2}\}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) - 518760 \sin^{12}\left(\frac{c}{2} + \frac{dx}{2}\right) + 1770 \operatorname{Hypergeometric2F1}\left[2, \frac{9}{2}, \frac{11}{2}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 226656 \sin^{14}\left(\frac{c}{2} + \frac{dx}{2}\right) - 1500 \operatorname{Hypergeometric2F1}\left[2, \frac{9}{2}, \frac{11}{2}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) - 42048 \sin^{16}\left(\frac{c}{2} + \frac{dx}{2}\right) + 440 \operatorname{Hypergeometric2F1}\left[2, \frac{9}{2}, \frac{11}{2}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 4725 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] \sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}} - 56700 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}} + 291060 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}} + 833760 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}} - 833760 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}} + 1458000 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}} - 1598400 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}} + 1080000 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right)$$

[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]

[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]

$$\begin{aligned} & \text{in}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)] * \text{Sin}[c/2 + (d*x)/2]^2 * \text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)] - 414720 * \text{ArcTanh}[\text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)] * \text{Sin}[c/2 + (d*x)/2]^2] \\ & + 14 * \text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)] + 69120 * \text{ArcTanh}[\text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)] * \text{Sin}[c/2 + (d*x)/2]^2] \\ & + 16 * \text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)] + 60 * \text{Cos}[(c + d*x)/2]^4 * \text{HypergeometricPFQ}[\{2, 2, 9/2\}, \{1, 11/2\}, \text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)] * \text{Sin}[c/2 + (d*x)/2]^10 * (-5 + 4*\text{Sin}[c/2 + (d*x)/2]^2) \\ &) / (675 * d * \text{Sqrt}[a * (1 + \text{Cos}[c + d*x])] * (1 - 2*\text{Sin}[c/2 + (d*x)/2]^2)^{(7/2)} * (-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)) \end{aligned}$$

Maple [A] (verified)

Time = 5.85 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07

method	result
default	$\frac{(15(\cos^3(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))+13\sqrt{2}(\cos^2(dx+c))\sin(dx+c)+15\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c))\arcsin(\cot(dx+c)-\csc(dx+c))))^{(1/2)}\cos(dx+c)^2\arcsin(\cot(dx+c)-\csc(dx+c))-\sin(dx+c)\cos(dx+c)*2^{(1/2)}+3*2^{(1/2)}\sin(dx+c)*(a*(1+\cos(dx+c)))^{(1/2)}/(1+\cos(dx+c))/\cos(dx+c)^{(5/2)}*2^{(1/2)}/a}{15d(1+\cos(dx+c))\cos(dx+c)^{5/2}a}$

[In] int(1/cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/15/d*(15*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+13*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))-sin(d*x+c)*cos(d*x+c)*2^(1/2)+3*2^(1/2)*sin(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(5/2)*2^(1/2)/a

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.93

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{2\sqrt{a\cos(dx+c)+a}(13\cos(dx+c)^2-\cos(dx+c)+3)\sqrt{\cos(dx+c)}\sin(dx+c)-\frac{15\sqrt{2}(a\cos(dx+c)^4+\cos(dx+c))^{(1/2)}\arcsin(\cot(dx+c)-\csc(dx+c))}{15(ad\cos(dx+c)^4+ad\cos(dx+c)^3)}}{15(ad\cos(dx+c)^4+ad\cos(dx+c)^3)}$$

[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/15*(2*sqrt(a*cos(d*x+c)+a)*(13*cos(d*x+c)^2-cos(d*x+c)+3)*sqrt(cos(d*x+c))*sin(d*x+c)-15*sqrt(2)*(a*cos(d*x+c)^4+a*cos(d*x+c))^(1/2)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x+c)+a)*sqrt(cos(d*x+c))*sin(d*x+c)/((cos(d*x+c)^2+cos(d*x+c))*sqrt(a)))/sqrt(a)/(a*d*cos(d*x+c)^4+a*d*cos(d*x+c)^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 1006, normalized size of antiderivative = 5.95

$$\int \frac{1}{\cos^{\frac{7}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx = \text{Too large to display}$$

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1))) - 26*(cos(2*d*x + 2*c)^2*sin(d*x + c) + sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*cos(2*d*x + 2*c)*sin(d*x + c) + sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 24*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - 24*(cos(d*x + c) - 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*((13*cos(d*x + c) - 15)*cos(2*d*x + 2*c)^2 + (13*cos(d*x + c) - 15)*sin(2*d*x + 2*c)^2 + 2*(13*cos(d*x + c) - 15)*cos(2*d*x + 2*c) + 13*cos(d*x + c) - 15)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(7*cos(3/2*arct
```

$\text{an2}(\sin(2dx + 2c), \cos(2dx + 2c) + 1) \cdot \sin(dx + c) - (7\cos(dx + c) - 5) \cdot \sin\left(\frac{3}{2} \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) / ((\cos(2dx + 2c))^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{5/4} \cdot \sqrt{a} \cdot d$

Giac [F]

$$\int \frac{1}{\cos^{7/2}(c + dx) \sqrt{a + a \cos(c + dx)}} dx = \int \frac{1}{\sqrt{a \cos(dx + c) + a \cos(dx + c)}^{7/2}} dx$$

[In] integrate(1/cos(dx+c)^(7/2)/(a+a*cos(dx+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a*cos(dx + c) + a)*cos(dx + c)^(7/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{7/2}(c + dx) \sqrt{a + a \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^{7/2} \sqrt{a + a \cos(c + dx)}} dx$$

[In] int(1/(cos(c + dx)^(7/2)*(a + a*cos(c + dx))^(1/2)),x)

[Out] int(1/(cos(c + dx)^(7/2)*(a + a*cos(c + dx))^(1/2)), x)

$$3.231 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$$

Optimal result	3636
Rubi [A] (verified)	3636
Mathematica [A] (verified)	3638
Maple [A] (verified)	3639
Fricas [A] (verification not implemented)	3639
Sympy [F(-1)]	3640
Maxima [F]	3640
Giac [F(-1)]	3640
Mupad [F(-1)]	3640

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx = -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{7 \arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{4d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{1+\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{1+\cos(c+dx)}}$$

[Out] 7/4*arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))/d-arcsin(sin(d*x+c)/(1+cos(d*x+c)))^(1/2)/d+1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(1+cos(d*x+c))^(1/2)-1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(1+cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2857, 3062, 3061, 2860, 222, 2853}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx = -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d} + \frac{7 \arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{4d} + \frac{\sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}} - \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{4d\sqrt{\cos(c+dx)+1}}$$

[In] Int[Cos[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]

[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (7*ArcSin[Sin[c + d*x]/Sqrt[1 + Cos[c + d*x]]])/(4*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*

$d\sqrt{1 + \cos[c + dx]} + (\cos[c + dx]^{3/2}\sin[c + dx])/(2d\sqrt{1 + \cos[c + dx]})$

Rule 222

$\text{Int}[1/\sqrt{(a_+) + (b_+)(x_+)^2}, x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2](x/\sqrt{a})]/\text{Rt}[-b, 2], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

Rule 2853

$\text{Int}[\sqrt{(a_+) + (b_+)\sin[(e_+) + (f_+)(x_+)]}/\sqrt{(d_+)\sin[(e_+) + (f_+)(x_+)]}, x_Symbol] \rightarrow \text{Dist}[-2/f, \text{Subst}[\text{Int}[1/\sqrt{1 - x^2/a}], x], x, b(\cos[e + fx]/\sqrt{a + b\sin[e + fx]})], x] /; \text{FreeQ}\{a, b, d, e, f\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[d, a/b]$

Rule 2857

$\text{Int}[(c_+ + (d_+)\sin[(e_+) + (f_+)(x_+)])^{n_+}/\sqrt{(a_+) + (b_+)\sin[(e_+) + (f_+)(x_+)]}, x_Symbol] \rightarrow \text{Simp}[-2d\cos[e + fx]((c + d\sin[e + fx])^{n-1}/(f(2n-1)\sqrt{a + b\sin[e + fx]})), x] - \text{Dist}[1/(b(2n-1)), \text{Int}[(c + d\sin[e + fx])^{n-2}/\sqrt{a + b\sin[e + fx]}] * \text{Simp}[a*c*d - b(2d^2(n-1) + c^2(2n-1)) + d(ad - b*c(4n-3))\sin[e + fx], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2860

$\text{Int}[1/(\sqrt{(d_+)\sin[(e_+) + (f_+)(x_+)]} * \sqrt{(a_+) + (b_+)\sin[(e_+) + (f_+)(x_+)]}), x_Symbol] \rightarrow \text{Dist}[-\sqrt{2}/(\sqrt{a}*f), \text{Subst}[\text{Int}[1/\sqrt{1 - x^2}], x], x, b(\cos[e + fx]/(a + b\sin[e + fx]))], x] /; \text{FreeQ}\{a, b, d, e, f\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[d, a/b] \ \&\& \ \text{GtQ}[a, 0]$

Rule 3061

$\text{Int}[(A_+ + (B_+)\sin[(e_+) + (f_+)(x_+)]) / (\sqrt{(a_+) + (b_+)\sin[(e_+) + (f_+)(x_+)]} * \sqrt{(c_+) + (d_+)\sin[(e_+) + (f_+)(x_+)]}), x_Symbol] \rightarrow \text{Dist}[(A*b - a*B)/b, \text{Int}[1/(\sqrt{a + b\sin[e + fx]} * \sqrt{c + d\sin[e + fx]}), x], x] + \text{Dist}[B/b, \text{Int}[\sqrt{a + b\sin[e + fx]}/\sqrt{c + d\sin[e + fx]}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3062

$\text{Int}[(a_+ + (b_+)\sin[(e_+) + (f_+)(x_+)])^{m_+} * ((A_+ + (B_+)\sin[(e_+) + (f_+)(x_+)])^{n_+}), x_Symbol] \rightarrow \text{Simp}[(-B)\cos[e + fx] * (a + b\sin[e + fx])^m * ((c + d\sin[e + fx])^n / (f(m + n + 1))), x] + \text{Dist}[1/(b(m + n + 1)), \text{Int}[(a + b\sin[e + fx])^m * (c + d\sin[e + fx])^n, x], x]$

$n[e + f*x]^{(n - 1)} * \text{Simp}[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[n, 0] \&\& (\text{IntegerQ}[n] \mid\mid \text{EqQ}[m + 1/2, 0])$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 + \cos(c + dx)}} - \frac{1}{4} \int \frac{(-3 + \cos(c + dx))\sqrt{\cos(c + dx)}}{\sqrt{1 + \cos(c + dx)}} dx \\
 &= -\frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{1 + \cos(c + dx)}} + \frac{\cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 + \cos(c + dx)}} \\
 &\quad - \frac{1}{4} \int \frac{\frac{1}{2} - \frac{7}{2} \cos(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{1 + \cos(c + dx)}} dx \\
 &= -\frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{1 + \cos(c + dx)}} + \frac{\cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 + \cos(c + dx)}} \\
 &\quad + \frac{7}{8} \int \frac{\sqrt{1 + \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx - \int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{1 + \cos(c + dx)}} dx \\
 &= -\frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{1 + \cos(c + dx)}} + \frac{\cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 + \cos(c + dx)}} \\
 &\quad - \frac{7 \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{4d} + \frac{\sqrt{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \\
 &= -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{7 \arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{4d} \\
 &\quad - \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{1 + \cos(c + dx)}} + \frac{\cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 + \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \frac{\left(\arcsin\left(\sqrt{1 - \cos(c + dx)}\right) + 8 \arcsin\left(\sqrt{\cos(c + dx)}\right) - 4\sqrt{2} \arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right) - 2\sqrt{1 - \cos(c + dx)}\right)}{4d\sqrt{\sin^2(c + dx)}}$$

[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]], x]

```
[Out] -1/4*((ArcSin[Sqrt[1 - Cos[c + d*x]]] + 8*ArcSin[Sqrt[Cos[c + d*x]]] - 4*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] - 2*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Maple [A] (verified)

Time = 12.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.34

method	result
default	$\frac{(2 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 4 \arcsin(\cot(dx+c) - \csc(dx+c)) \sqrt{2} - \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 7 \arctan(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{8d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

```
[In] int(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+4*arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$$

$$= \frac{(2 \cos(dx+c) - 1) \sqrt{\cos(dx+c)+1} \sqrt{\cos(dx+c)} \sin(dx+c) + 4 (\sqrt{2} \cos(dx+c) + \sqrt{2}) \arctan\left(\frac{\sqrt{2} \cos(dx+c)}{\sin(dx+c)}\right)}{4(d \cos(dx+c) + d)}$$

```
[In] integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*cos(d*x + c) - 1)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c) + 4*(sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - 7*(cos(d*x + c) + 1)*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{\sqrt{\cos(dx + c) + 1}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/sqrt(cos(d*x + c) + 1), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{\frac{5}{2}}}{\sqrt{\cos(c + dx) + 1}} dx$$

[In] int(cos(c + d*x)^(5/2)/(cos(c + d*x) + 1)^(1/2),x)

[Out] int(cos(c + d*x)^(5/2)/(cos(c + d*x) + 1)^(1/2), x)

$$3.232 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$$

Optimal result	3641
Rubi [A] (verified)	3641
Mathematica [A] (warning: unable to verify)	3643
Maple [A] (verified)	3643
Fricas [A] (verification not implemented)	3644
Sympy [F]	3644
Maxima [F]	3644
Giac [F(-1)]	3645
Mupad [F(-1)]	3645

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} - \frac{\arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}$$

[Out] $-\arcsin(\sin(d*x+c)/(1+\cos(d*x+c))^{(1/2)})/d+\arcsin(\sin(d*x+c)/(1+\cos(d*x+c))) * 2^{(1/2)}/d+\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(1+\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2857, 3061, 2860, 222, 2853}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d} - \frac{\arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d} + \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^{(3/2)}/\text{Sqrt}[1 + \text{Cos}[c + d*x]], x]$

[Out] $(\text{Sqrt}[2]*\text{ArcSin}[\text{Sin}[c + d*x]/(1 + \text{Cos}[c + d*x])])/d - \text{ArcSin}[\text{Sin}[c + d*x]/\text{Sqrt}[1 + \text{Cos}[c + d*x]]]/d + (\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(d*\text{Sqrt}[1 + \text{Cos}[c + d*x]])$

Rule 222

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2853

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2857

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]))], x] - Dist[1/(b*(2*n - 1)), Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2860

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 3061

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{d\sqrt{1 + \cos(c + dx)}} - \frac{1}{2} \int \frac{-1 + \cos(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{1 + \cos(c + dx)}} dx \\ &= \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{d\sqrt{1 + \cos(c + dx)}} - \frac{1}{2} \int \frac{\sqrt{1 + \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\ &\quad + \int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{1 + \cos(c + dx)}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} \\
&\quad - \frac{\sqrt{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \\
&= \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} - \frac{\arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25

$$\begin{aligned}
&\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx \\
&= \frac{\left(\arcsin\left(\sqrt{1-\cos(c+dx)}\right) + 2\arcsin\left(\sqrt{\cos(c+dx)}\right) - \sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right) + \sqrt{-((-1+\cos(c+dx))\cos(c+dx))}\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]], x]

[Out] ((ArcSin[Sqrt[1 - Cos[c + d*x]]] + 2*ArcSin[Sqrt[Cos[c + d*x]]] - Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] + Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])

Maple [A] (verified)

Time = 12.86 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\left(\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2}-\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\cos(dx+c)}\sqrt{2+2\cos(dx+c)}\right)}{2d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2), x, method=_RETURNVERBOSE)

[Out] -1/2/d*(arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.47

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \frac{(\sqrt{2} \cos(dx + c) + \sqrt{2}) \arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right) - (\cos(dx + c) + 1) \arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d \cos(dx + c) + d}$$

```
[In] integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - (cos(d*x + c) + 1)*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{\cos(c + dx) + 1}} dx$$

```
[In] integrate(cos(d*x+c)**(3/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(cos(c + d*x) + 1), x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{\sqrt{\cos(dx + c) + 1}} dx$$

```
[In] integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(cos(d*x + c) + 1), x)
```


Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{3/2}}{\sqrt{\cos(c + dx) + 1}} dx$$

```
[In] int(cos(c + d*x)^(3/2)/(cos(c + d*x) + 1)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(cos(c + d*x) + 1)^(1/2), x)
```

3.233 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx$

Optimal result	3646
Rubi [A] (verified)	3646
Mathematica [A] (verified)	3647
Maple [B] (verified)	3648
Fricas [A] (verification not implemented)	3648
Sympy [F]	3648
Maxima [C] (verification not implemented)	3649
Giac [F]	3649
Mupad [F(-1)]	3650

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2 \arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d}$$

[Out] $2*\arcsin(\sin(d*x+c)/(1+\cos(d*x+c))^{(1/2)})/d-\arcsin(\sin(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2856, 2853, 222, 2860}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = \frac{2 \arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d} - \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

[In] `Int[Sqrt[Cos[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]`

[Out] $-\left(\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c + d*x]}{1 + \cos[c + d*x]}\right]}{d}\right) + \left(\frac{2 \operatorname{ArcSin}\left[\frac{\sin[c + d*x]}{\sqrt{1 + \cos[c + d*x]}}\right]}{d}\right)$

Rule 222

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

Rule 2853

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 2856

```
Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(a_) + (b_)*sin[(e_)
+ (f_)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
+ d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2860

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f
_)*(x_)]]), x_Symbol] :> Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= - \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx + \int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\ &= - \frac{2 \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} + \frac{\sqrt{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \\ &= - \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2 \arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.87

$$\begin{aligned} &\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx \\ &= - \frac{\left(2 \arcsin\left(\sqrt{\cos(c+dx)}\right) - \sqrt{2} \arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)\right) \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sin(c+dx)}{d \sqrt{-((-1+\cos(c+dx))\cos(c+dx))}} \end{aligned}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[1 + Cos[c + d*x]], x]
```

```
[Out] -(((2*ArcSin[Sqrt[Cos[c + d*x]]] - Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[S
in[(c + d*x)/2]^2]])*Sqrt[Cos[c + d*x]]/(1 + Cos[c + d*x]))*Sin[c + d*x])/(d
*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(50) = 100$.

Time = 2.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)}\sqrt{2+2\cos(dx+c)}\left(\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2+2\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}\right)\sqrt{2}}{2d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	108

[In] `int(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1/2/d*\cos(d*x+c)^{(1/2)}*(2+2*\cos(d*x+c))^{(1/2)}*(\arcsin(\cot(d*x+c)-\csc(d*x+c))*2^{(1/2)}+2*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}))/(1+\cos(d*x+c))/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)}}{d}$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right) - 2\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

[In] `integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $(\sqrt{2}*\arctan(\sqrt{2}*\sqrt{\cos(d*x+c)+1}*\sqrt{\cos(d*x+c)})/\sin(d*x+c) - 2*\arctan(\sqrt{\cos(d*x+c)+1}*\sqrt{\cos(d*x+c)})/\sin(d*x+c))/d$

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)+1}} dx$$

[In] `integrate(cos(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(cos(c+d*x))/sqrt(cos(c+d*x)+1), x)`

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 689, normalized size of antiderivative = 12.76

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = \sqrt{2} \arctan \left(\frac{(|2e^{i(dx+ic)}+2|^4 + 16 \cos(dx+c)^4 + 16 \sin(dx+c)^4 + 8(\cos(dx+c)^2 - \sin(dx+c)^2 - 2 \cos(dx+c)+1)|2e^{i(dx+ic)}+2|^2 - 64 \cos(dx+c))^{1/4} \sin(1/2 \arctan(8(\cos(dx+c)-1)\sin(dx+c)/\text{abs}(2e^{i(dx+ic)}+2)^2, (\text{abs}(2e^{i(dx+ic)}+2)^2 + 4\cos(dx+c)^2 - 4\sin(dx+c)^2 - 8\cos(dx+c)+4)/\text{abs}(2e^{i(dx+ic)}+2)^2)) + 2\sin(dx+c))/\text{abs}(2e^{i(dx+ic)}+2), ((\text{abs}(2e^{i(dx+ic)}+2)^4 + 16\cos(dx+c)^4 + 16\sin(dx+c)^4 + 8(\cos(dx+c)^2 - \sin(dx+c)^2 - 2\cos(dx+c)+1)\text{abs}(2e^{i(dx+ic)}+2)^2 - 64\cos(dx+c)^3 + 32(\cos(dx+c)^2 - 2\cos(dx+c)+1)\sin(dx+c)^2 + 96\cos(dx+c)^2 - 64\cos(dx+c)+16)^{1/4} \cos(1/2 \arctan(8(\cos(dx+c)-1)\sin(dx+c)/\text{abs}(2e^{i(dx+ic)}+2)^2, (\text{abs}(2e^{i(dx+ic)}+2)^2 + 4\cos(dx+c)^2 - 4\sin(dx+c)^2 - 8\cos(dx+c)+4)/\text{abs}(2e^{i(dx+ic)}+2)^2)) + 2\cos(dx+c)-2)/\text{abs}(2e^{i(dx+ic)}+2)) - \arctan(2(\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c)+1)^{1/4} \sin(1/2 \arctan(\sin(2dx+2c), \cos(2dx+2c)+1)) + \sin(dx+c), (\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c)+1)^{1/4} \cos(1/2 \arctan(\sin(2dx+2c), \cos(2dx+2c)+1)) + \cos(dx+c))}{d} \right)$$

[In] integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -(sqrt(2)*arctan2(((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x + c) + 16)^(1/4)*sin(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(d*x + c)/abs(2*e^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*cos(d*x + c)^2 - 4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) + 2)^2)) + 2*sin(d*x + c))/abs(2*e^(I*d*x + I*c) + 2), ((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x + c) + 16)^(1/4)*cos(1/2*arctan2(8*(cos(d*x + c) - 1)*sin(d*x + c)/abs(2*e^(I*d*x + I*c) + 2)^2, (abs(2*e^(I*d*x + I*c) + 2)^2 + 4*cos(d*x + c)^2 - 4*sin(d*x + c)^2 - 8*cos(d*x + c) + 4)/abs(2*e^(I*d*x + I*c) + 2)^2)) + 2*cos(d*x + c) - 2)/abs(2*e^(I*d*x + I*c) + 2)) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c)))/d

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{\cos(dx+c)+1}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(cos(d*x + c) + 1), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{\cos(c+dx)+1}} dx$$

```
[In] int(cos(c + d*x)^(1/2)/(cos(c + d*x) + 1)^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(cos(c + d*x) + 1)^(1/2), x)
```

$$3.234 \quad \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx$$

Optimal result	3651
Rubi [A] (verified)	3651
Mathematica [A] (verified)	3652
Maple [B] (verified)	3652
Fricas [B] (verification not implemented)	3653
Sympy [F]	3653
Maxima [C] (verification not implemented)	3653
Giac [F]	3654
Mupad [F(-1)]	3654

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}$$

[Out] arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2860, 222}

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]),x]

[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2860

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e

, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sqrt{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \\ &= \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx = \frac{2 \arctan\left(\frac{\sin(\frac{1}{2}(c+dx))}{\sqrt{\cos(c+dx)}}\right) \cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1+\cos(c+dx)}}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]), x]

[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2])/(d*Sqrt[1 + Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(25) = 50.

Time = 4.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

method	result	size
default	$-\frac{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{2+2\cos(dx+c)} \arcsin(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	61

[In] int(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2), x, method=_RETURNVERBOSE)

[Out] -1/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2+2*cos(d*x+c))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}\sin(dx+c)}{2(\cos(dx+c)^2+\cos(dx+c))}\right)}{d}$$

[In] integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c)^2 + cos(d*x + c)))/d

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)+1}\sqrt{\cos(c+dx)}} dx$$

[In] integrate(1/cos(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 505, normalized size of antiderivative = 18.70

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx$$

$$= \frac{\sqrt{2} \arctan\left(\frac{(|e^{i dx+i c}+1|^4+\cos(dx+c)^4+\sin(dx+c)^4+2(\cos(dx+c)^2-\sin(dx+c)^2-2\cos(dx+c)+1)|e^{i dx+i c}+1|^2-4\cos(dx+c)^3+2(\cos(dx+c)^2-\sin(dx+c)^2-2\cos(dx+c)+1))}{2(\cos(dx+c)^2+\cos(dx+c))}\right)}{d}$$

[In] integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] sqrt(2)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I

$*d*x + I*c) + 1)^2 + \cos(d*x + c)^2 - \sin(d*x + c)^2 - 2*\cos(d*x + c) + 1)/$
 $\text{abs}(e^{(I*d*x + I*c) + 1})^2)) + \sin(d*x + c))/\text{abs}(e^{(I*d*x + I*c) + 1}), ((\text{ab}$
 $\text{s}(e^{(I*d*x + I*c) + 1})^4 + \cos(d*x + c)^4 + \sin(d*x + c)^4 + 2*(\cos(d*x + c)$
 $)^2 - \sin(d*x + c)^2 - 2*\cos(d*x + c) + 1)*\text{abs}(e^{(I*d*x + I*c) + 1})^2 - 4*c$
 $\text{os}(d*x + c)^3 + 2*(\cos(d*x + c)^2 - 2*\cos(d*x + c) + 1)*\sin(d*x + c)^2 + 6*$
 $\cos(d*x + c)^2 - 4*\cos(d*x + c) + 1)^{(1/4)}*\cos(1/2*\arctan2(2*(\cos(d*x + c)$
 $- 1)*\sin(d*x + c)/\text{abs}(e^{(I*d*x + I*c) + 1})^2, (\text{abs}(e^{(I*d*x + I*c) + 1})^2 +$
 $\cos(d*x + c)^2 - \sin(d*x + c)^2 - 2*\cos(d*x + c) + 1)/\text{abs}(e^{(I*d*x + I*c)$
 $+ 1)^2)) + \cos(d*x + c) - 1)/\text{abs}(e^{(I*d*x + I*c) + 1}))/d$

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{1 + \cos(c + dx)}} dx = \int \frac{1}{\sqrt{\cos(dx + c) + 1}\sqrt{\cos(dx + c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{1 + \cos(c + dx)}} dx = \int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{\cos(c + dx) + 1}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(cos(c + d*x) + 1)^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(1/2)*(cos(c + d*x) + 1)^(1/2)), x)

$$3.235 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx$$

Optimal result	3655
Rubi [A] (verified)	3655
Mathematica [C] (warning: unable to verify)	3656
Maple [B] (verified)	3657
Fricas [B] (verification not implemented)	3657
Sympy [F]	3658
Maxima [C] (verification not implemented)	3658
Giac [F]	3659
Mupad [F(-1)]	3659

Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2 \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

[Out] $-\arcsin(\sin(d*x+c)/(1+\cos(d*x+c)))*2^{(1/2)}/d+2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(1+\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2858, 2860, 222}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \frac{2 \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}} - \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

[In] $\text{Int}[1/(\text{Cos}[c+d*x]^{(3/2)}*\text{Sqrt}[1+\text{Cos}[c+d*x]]),x]$

[Out] $-\left(\frac{\text{Sqrt}[2]*\text{ArcSin}[\text{Sin}[c+d*x]/(1+\text{Cos}[c+d*x])]}{d}\right) + \frac{2*\text{Sin}[c+d*x]}{d*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[1+\text{Cos}[c+d*x]]}$

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2860

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] := Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} - \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} dx \\ &= \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} + \frac{\sqrt{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \\ &= -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.87

$$\begin{aligned} &\int \frac{1}{\cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} dx \\ &= \frac{2 \cos\left(\frac{1}{2}(c + dx)\right) \sin\left(\frac{1}{2}(c + dx)\right) \left(\frac{1}{2} \cos(c + dx)(2 + \cos(c + dx)) \csc^4\left(\frac{1}{2}(c + dx)\right) \left(1 - \cos(c + dx) + \arctan\left(\frac{\sin(c + dx)}{1 + \cos(c + dx)}\right)\right)\right)}{\dots} \end{aligned}$$

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]), x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 + Cos[c + d*x])*Csc[
(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)
]/2)^2]))*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (Hypergeometric2F1[2,
5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/1
0))/(d*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(56) = 112.

Time = 4.94 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.13

method	result
default	$\frac{\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \cos(dx+c)\sqrt{2} \arcsin(\cot(dx+c)-\csc(dx+c))+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{2} \arcsin(\cot(dx+c)-\csc(dx+c))+2 \sin(dx+c)\right) \sqrt{2}}{2d(1+\cos(dx+c))\sqrt{\cos(dx+c)}}$

```
[In] int(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*2^(1/2)*arcsin(cot(d*x+
c)-csc(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arcsin(cot(d*x+c)-
csc(d*x+c))+2*sin(d*x+c))*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(
1/2)*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(56) = 112.

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)\sqrt{1 + \cos(c + dx)}} dx = \frac{(\sqrt{2} \cos(dx + c)^2 + \sqrt{2} \cos(dx + c)) \arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}\sin(dx+c)}{2(\cos(dx+c)^2+\cos(dx+c))}\right) - 2\sqrt{\cos(dx+c)+1}}{d \cos(dx+c)^2 + d \cos(dx+c)}$$

```
[In] integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(c
os(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c)^2 + cos(d*x
+ c))) - 2*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2 + d*cos(d*x + c))
```

SymPy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)+1}\cos^{\frac{3}{2}}(c+dx)} dx$$

[In] integrate(1/cos(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2), x)

[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**(3/2)), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 648, normalized size of antiderivative = 10.45

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx$$

$$2 \cos\left(\frac{1}{2} \arctan(\sin(2dx+2c), \cos(2dx+2c)+1)\right) \sin(dx+c) - 2(\cos(dx+c)-1) \sin\left(\frac{1}{2} \arctan(\sin$$

=

[In] integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] (2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - 2*(cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d*x + I*c) + 1)))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*d)

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(dx+c)+1}\cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \int \frac{1}{\cos(c+dx)^{3/2}\sqrt{\cos(c+dx)+1}} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(cos(c + d*x) + 1)^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(3/2)*(cos(c + d*x) + 1)^(1/2)), x)

$$3.236 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx$$

Optimal result	3660
Rubi [A] (verified)	3660
Mathematica [C] (warning: unable to verify)	3662
Maple [A] (verified)	3663
Fricas [A] (verification not implemented)	3663
Sympy [F]	3664
Maxima [C] (verification not implemented)	3664
Giac [F]	3665
Mupad [F(-1)]	3665

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2 \sin(c+dx)}{3d \sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

[Out] arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d+2/3*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2)-2/3*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2858, 3063, 12, 2860, 222}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d} + \frac{2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}} - \frac{2 \sin(c+dx)}{3d \sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]),x]

[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]/d + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]) - (2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2858

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2860

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]

Rule 3063

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} - \frac{1}{3} \int \frac{1 - 2 \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} dx \\
 &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} - \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} \\
 &\quad - \frac{2}{3} \int -\frac{3}{2 \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} dx \\
 &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} - \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} \\
 &\quad + \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} dx \\
 &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} - \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} \\
 &\quad - \frac{\sqrt{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \\
 &= \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} \\
 &\quad - \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.45 (sec) , antiderivative size = 471, normalized size of antiderivative = 4.81

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} dx =$$

$$2 \cot\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^4\left(\frac{c}{2} + \frac{dx}{2}\right) \left(12 \cos^4\left(\frac{1}{2}(c + dx)\right) {}_3F_2\left(2, 2, \frac{7}{2}; 1, \frac{9}{2}; \frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) \sin^8\left(\frac{c}{2} + \frac{dx}{2}\right) + 12 H\right)$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]),x]

[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin

$$\frac{[c/2 + (d*x)/2]^2 + 3*\text{Sin}[c/2 + (d*x)/2]^4 + 7*(1 - 2*\text{Sin}[c/2 + (d*x)/2]^2)^3*\text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)]*(15 - 20*\text{Sin}[c/2 + (d*x)/2]^2 + 8*\text{Sin}[c/2 + (d*x)/2]^4)*(ArcTanh[\text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)]*(3 - 6*\text{Sin}[c/2 + (d*x)/2]^2) + \text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)]*(-3 + 7*\text{Sin}[c/2 + (d*x)/2]^2)))]/(63*d*\text{Sqrt}[1 + \text{Cos}[c + d*x]]*(1 - 2*\text{Sin}[c/2 + (d*x)/2]^2)^{(7/2)})$$

Maple [A] (verified)

Time = 4.83 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.59

method	result
default	$-\frac{\left(3(\cos^2(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2}+3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \cos(dx+c)\sqrt{2} \arcsin(\cot(dx+c)-\csc(dx+c))\right)}{6d(1+\cos(dx+c)) \cos(dx+c)^{\frac{3}{2}}}$

[In] int(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out]
$$-1/6/d*(3*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\arcsin(\cot(d*x+c)-\csc(d*x+c))*2^{1/2}+3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\cos(d*x+c)*2^{1/2}*\arcsin(\cot(d*x+c)-\csc(d*x+c))+2*\cos(d*x+c)*\sin(d*x+c)-2*\sin(d*x+c))*(2+2*\cos(d*x+c))^{1/2}/(1+\cos(d*x+c))/\cos(d*x+c)^{(3/2)}*2^{1/2}$$

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \frac{2\sqrt{\cos(dx+c)+1}(\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c) - 3(\sqrt{2}\cos(dx+c)^3 + \sqrt{2}\cos(dx+c))}{3(d\cos(dx+c)^3 + d\cos(dx+c)^2)}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out]
$$-1/3*(2*\text{sqrt}(\cos(d*x+c)+1)*(\cos(d*x+c)-1)*\text{sqrt}(\cos(d*x+c))*\sin(d*x+c) - 3*(\text{sqrt}(2)*\cos(d*x+c)^3 + \text{sqrt}(2)*\cos(d*x+c)^2)*\arctan(1/2*\text{sqrt}(2)*\text{sqrt}(\cos(d*x+c)+1)*\text{sqrt}(\cos(d*x+c))*\sin(d*x+c)/(\cos(d*x+c)^2 + \cos(d*x+c))))/(d*\cos(d*x+c)^3 + d*\cos(d*x+c)^2)$$

Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)+1}\cos^{\frac{5}{2}}(c+dx)} dx$$

[In] integrate(1/cos(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2), x)

[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**(5/2)), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 801, normalized size of antiderivative = 8.17

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \text{Too large to display}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] 1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d*x + I*c) + 1)) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) - 3)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(dx+c)+1}\cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = \int \frac{1}{\cos(c+dx)^{5/2}\sqrt{\cos(c+dx)+1}} dx$$

[In] int(1/(cos(c + d*x)^(5/2)*(cos(c + d*x) + 1)^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(5/2)*(cos(c + d*x) + 1)^(1/2)), x)

$$3.237 \quad \int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx$$

Optimal result	3666
Rubi [A] (verified)	3666
Mathematica [C] (warning: unable to verify)	3669
Maple [A] (verified)	3670
Fricas [A] (verification not implemented)	3670
Sympy [F(-1)]	3671
Maxima [C] (verification not implemented)	3671
Giac [F]	3672
Mupad [F(-1)]	3672

Optimal result

Integrand size = 23, antiderivative size = 134

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2 \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} + \frac{26 \sin(c+dx)}{15d \sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

[Out] -arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)/d+2/5*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2)-2/15*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2)+26/15*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

= {2858, 3063, 12, 2860, 222}

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx = -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d} - \frac{2 \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}} + \frac{2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}} + \frac{26 \sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[1 + Cos[c + d*x]]),x]

[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*Sin[c + d*x])/((5*d*Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2858

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])], x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2860

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]

Rule 3063

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} - \frac{1}{5} \int \frac{1 - 4 \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} dx \\
&= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} - \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} \\
&\quad - \frac{2}{15} \int \frac{-\frac{13}{2} + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} dx \\
&= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} - \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} \\
&\quad + \frac{26 \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} - \frac{4}{15} \int \frac{15}{4 \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} dx \\
&= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} - \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} \\
&\quad + \frac{26 \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} - \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} dx \\
&= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} - \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} \\
&\quad + \frac{26 \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} + \frac{\sqrt{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \\
&= -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} \\
&\quad - \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} + \frac{26 \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}}
\end{aligned}$$

$$\frac{(-1 + 2*\sin[c/2 + (d*x)/2]^2)*\sin[c/2 + (d*x)/2]^{10}*(-5 + 4*\sin[c/2 + (d*x)/2]^2))/675*d*\sqrt{1 + \cos[c + d*x]}*(1 - 2*\sin[c/2 + (d*x)/2]^2)^{7/2}*(-1 + 2*\sin[c/2 + (d*x)/2]^2)}$$

Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.30

method	result
default	$\frac{(15(\cos^3(dx+c))\sqrt{2} \arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+15(\cos^2(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2}-30d(1+\cos(dx+c)) \cos(dx+c)^{\frac{5}{2}})}{30d(1+\cos(dx+c)) \cos(dx+c)^{\frac{5}{2}}}$

[In] int(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/30/d*(15*cos(d*x+c)^3*2^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)+26*cos(d*x+c)^2*sin(d*x+c)-2*cos(d*x+c)*sin(d*x+c)+6*sin(d*x+c))*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/cos(d*x+c)^(5/2)*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} dx$$

$$= \frac{2(13\cos(dx+c)^2 - \cos(dx+c) + 3)\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}\sin(dx+c) - 15(\sqrt{2}\cos(dx+c))^4}{15(d\cos(dx+c)^4 + d\cos(dx+c)^3)}$$

[In] integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/15*(2*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c) - 15*(sqrt(2)*cos(d*x + c)^4 + sqrt(2)*cos(d*x + c)^3)*arctan(1/2*sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c)^2 + cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(7/2)/(1+cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 989, normalized size of antiderivative = 7.38

$$\int \frac{1}{\cos^{\frac{7}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

```
[Out] -1/15*(15*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d*x + I*c) + 1) - 26*(cos(2*d*x + 2*c)^2*sin(d*x + c) + sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*cos(2*d*x + 2*c)*sin(d*x + c) + sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 24*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - 24*(cos(d*x + c) - 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*((13*cos(d*x + c) - 15)*cos(2*d*x + 2*c)^2 + (13*cos(d*x + c) - 15)*sin(2*d*x + 2*c)^2 + 2*(13*cos(d*x + c) - 15)*cos(2*d*x + 2*c) + 13*cos(d*x + c) - 15)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(7*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

+ 2*c) + 1))*sin(d*x + c) - (7*cos(d*x + c) - 5)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(5/4)*d)

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c + dx)\sqrt{1 + \cos(c + dx)}} dx = \int \frac{1}{\sqrt{\cos(dx + c) + 1} \cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(7/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c + dx)\sqrt{1 + \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^{7/2} \sqrt{\cos(c + dx) + 1}} dx$$

[In] int(1/(cos(c + d*x)^(7/2)*(cos(c + d*x) + 1)^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(7/2)*(cos(c + d*x) + 1)^(1/2)), x)

$$3.238 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{3}{2}}} dx$$

Optimal result	3673
Rubi [A] (verified)	3673
Mathematica [A] (verified)	3676
Maple [A] (verified)	3676
Fricas [A] (verification not implemented)	3677
Sympy [F(-1)]	3677
Maxima [F]	3677
Giac [F(-1)]	3678
Mupad [F(-1)]	3678

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{3}{2}}} dx = -\frac{3 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{\frac{3}{2}}d} + \frac{9 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{\frac{3}{2}}d} - \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d(a+a \cos(c+dx))^{\frac{3}{2}}} + \frac{3\sqrt{\cos(c+dx)} \sin(c+dx)}{2ad\sqrt{a+a \cos(c+dx)}}$$

[Out] $-3*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d-1/2*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}+9/4*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+3/2*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {2844, 3062, 3061, 2861, 211, 2853, 222}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = -\frac{3 \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d} + \frac{9 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}} + \frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}$$

[In] Int[Cos[c + d*x]^(5/2)/(a + a*cos[c + d*x])^(3/2), x]

[Out] (-3*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*cos[c + d*x]])/(a^(3/2)*d) + (9*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*cos[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*cos[c + d*x])^(3/2)) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*cos[c + d*x]])

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Ssin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3061

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3062

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}(\frac{3a}{2}-3a\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-\frac{3a^2}{2}+3a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2a^3} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} \\
 &\quad - \frac{3\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{2a^2} + \frac{9\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{9\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^2d} \\
&= -\frac{3\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{9\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} \\
&\quad - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.22

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{\left(-9\sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right) - 9\sqrt{2}\arctan\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)}}\right)\right)\cos(c+dx) + \dots}{\dots}$$

[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2), x]

[Out] ((-9*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]] - 9*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[c + d*x] + 4*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 6*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])] + 6*ArcSin[Sqrt[1 - Cos[c + d*x]]]*(1 + Cos[c + d*x]) + 18*ArcSin[Sqrt[Cos[c + d*x]]]*(1 + Cos[c + d*x]))*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [A] (verified)

Time = 11.47 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.38

method	result
default	$\frac{(2\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-6\sqrt{2}\arctan(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}))\cos(dx+c)-6\sqrt{2}a}{4d(1+\cos(dx+c))^{3/2}}$

[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/4/d*(2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-6*2^(1/2)*arctan(tan(d

$*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)-6*2^{(1/2)}*\arctan(\tan(d*x+c))*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}-9*\arcsin(\cot(d*x+c)-\csc(d*x+c))*\cos(d*x+c)-9*\arcsin(\cot(d*x+c)-\csc(d*x+c))*\cos(d*x+c)^{(1/2)}*(a*(1+\cos(d*x+c)))^{(1/2)}/(1+\cos(d*x+c))^{2}/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)}/a^{2}$

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \frac{9\sqrt{2}(\cos(dx+c)^2+2\cos(dx+c)+1)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos(dx+c)+a}}{4(a^2d\cos(dx+c)+a^2d)}$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] $-1/4*(9*\sqrt{2}*(\cos(d*x+c)^2+2*\cos(d*x+c)+1)*\sqrt{a}*\arctan(\sqrt{2}*\sqrt{a*\cos(d*x+c)+a}*\sqrt{\cos(d*x+c)})/(\sqrt{a}*\sin(d*x+c))) - 2*\sqrt{a*\cos(d*x+c)+a}*(2*\cos(d*x+c)+3)*\sqrt{\cos(d*x+c)}*\sin(d*x+c) - 12*(\cos(d*x+c)^2+2*\cos(d*x+c)+1)*\sqrt{a}*\arctan(\sqrt{a*\cos(d*x+c)+a}*\sqrt{\cos(d*x+c)})/(\sqrt{a}*\sin(d*x+c)))/(a^2*d*\cos(d*x+c)^2+2*a^2*d*\cos(d*x+c)+a^2*d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(cos(d*x+c)^(5/2)/(a*cos(d*x+c)+a)^(3/2),x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{3}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{\cos(c + dx)^{\frac{5}{2}}}{(a + a \cos(c + dx))^{\frac{3}{2}}} dx$$

[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(3/2), x)

$$3.239 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{3}{2}}} dx$$

Optimal result	3679
Rubi [A] (verified)	3679
Mathematica [A] (verified)	3681
Maple [A] (verified)	3682
Fricas [A] (verification not implemented)	3682
Sympy [F]	3682
Maxima [F]	3683
Giac [F(-1)]	3683
Mupad [F(-1)]	3683

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{3}{2}}} dx = \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{\frac{3}{2}}d} - \frac{5 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{\frac{3}{2}}d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a \cos(c+dx))^{\frac{3}{2}}}$$

[Out] $2*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d-5/4*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}-1/2*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(3/2)}$

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2844, 3061, 2861, 211, 2853, 222}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{3}{2}}} dx = \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{a^{\frac{3}{2}}d} - \frac{5 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{\frac{3}{2}}d} - \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a \cos(c+dx)+a)^{\frac{3}{2}}}$$

[In] $\text{Int}[\text{Cos}[c+d*x]^{(3/2)}/(a+a*\text{Cos}[c+d*x])^{(3/2)},x]$

[Out] $(2*\text{ArcSin}[(\text{Sqrt}[a]*\text{Sin}[c+d*x])/(\text{Sqrt}[a+a*\text{Cos}[c+d*x]])]/(a^{(3/2)}*d) - (5*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sin}[c+d*x])/(\text{Sqrt}[2]*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Cos}[c+d*x]])]/(2*\text{Sqrt}[2]*a^{(3/2)}*d) - \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a \cos(c+dx)+a)^{\frac{3}{2}}}$

$$\frac{c + d*x]]])/(2*\text{Sqrt}[2]*a^{(3/2)*d} - (\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(2*d*(a + a*\text{Cos}[c + d*x])^{(3/2)})$$

Rule 211

$$\text{Int}[\frac{(a_) + (b_)*(x_)^2}{(x_)^2}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Rt}[a/b, 2]}{a}*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$$

Rule 222

$$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$$

Rule 2844

$$\text{Int}[\frac{(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]}{(c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)]}, x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*((c + d*\text{Sin}[e + f*x])^{(n-1)/(a*f*(2*m+1))}), x] + \text{Dist}[1/(a*b*(2*m+1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m+1)}*(c + d*\text{Sin}[e + f*x])^{(n-2)}*\text{Simp}[b*(c^2*(m+1) + d^2*(n-1)) + a*c*d*(m-n+1) + d*(a*d*(m-n+1) + b*c*(m+n))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ (\text{IntegersQ}[2*m, 2*n] \ || \ (\text{IntegerQ}[m] \ \&\& \ \text{EqQ}[c, 0]))$$

Rule 2853

$$\text{Int}[\frac{\text{Sqrt}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]]}{\text{Sqrt}[(d_)*\text{sin}[(e_) + (f_)*(x_)]]}, x_Symbol] \rightarrow \text{Dist}[-2/f, \text{Subst}[\text{Int}[1/\text{Sqrt}[1 - x^2/a], x], x, b*(\text{Cos}[e + f*x]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]])], x] /; \text{FreeQ}\{a, b, d, e, f\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[d, a/b]$$

Rule 2861

$$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]]*\text{Sqrt}[(c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)]]), x_Symbol] \rightarrow \text{Dist}[-2*(a/f), \text{Subst}[\text{Int}[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(\text{Cos}[e + f*x]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$$

Rule 3061

$$\text{Int}[\frac{(A_) + (B_)*\text{sin}[(e_) + (f_)*(x_)]}{(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]}, x_Symbol] \rightarrow \text{Dist}[(A*b - a*B)/b, \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] + \text{Dist}[B/b, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[c + d*\text{Sin}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\frac{a}{2}-2a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
 &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{a^2} - \frac{5 \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
 &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5 \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d} \\
 &\quad - \frac{2 \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^2d} \\
 &= \frac{2 \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{5 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.18

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{\left(2 \arcsin\left(\sqrt{1-\cos(c+dx)}\right)(1+\cos(c+dx)) + 10 \arcsin\left(\sqrt{\cos(c+dx)}\right)(1+\cos(c+dx)) + \sqrt{2}\left(-\frac{5 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}\right)\right)}{4d\sqrt{1-\cos(c+dx)}(a(1+\cos(c+dx)))^{3/2}}$$

[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2), x]

[Out] -1/4*((2*ArcSin[Sqrt[1 - Cos[c + d*x]])*(1 + Cos[c + d*x]) + 10*ArcSin[Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x]) + Sqrt[2]*(-5*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*(1 + Cos[c + d*x]) + 2*Sqrt[Cos[c + d*x]*Sin[(c + d*x)/2]^2]])*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.52

method	result
default	$\frac{\left(4\sqrt{2} \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) \cos(dx+c) - \sin(dx+c)\sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 4\sqrt{2} \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) + 5 \arcsin\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)\right)}{4d(1+\cos(dx+c))^2 \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)

[Out] $\frac{1}{4}d*(4*2^{(1/2)}*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c))))^{(1/2)}*\cos(d*x+c) - \sin(d*x+c)*2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} + 4*2^{(1/2)}*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c))))^{(1/2)} + 5*\arcsin(\cot(d*x+c) - \csc(d*x+c))*\cos(d*x+c) + 5*\arcsin(\cot(d*x+c) - \csc(d*x+c))*\cos(d*x+c)^{(1/2)}*(a*(1+\cos(d*x+c)))^{(1/2)}/(1+\cos(d*x+c))^2/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)}/a^2$

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.36

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \frac{5\sqrt{2}(\cos(dx+c)^2 + 2\cos(dx+c) + 1)\sqrt{a} \arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{4d(1+\cos(dx+c))^2 \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] $\frac{1}{4}*(5*\sqrt{2}*(\cos(d*x+c)^2 + 2*\cos(d*x+c) + 1)*\sqrt{a}*\arctan(\sqrt{2}*\sqrt{a*\cos(d*x+c)+a}*\sqrt{\cos(d*x+c)})/(sqrt(a)*\sin(d*x+c))) - 8*(\cos(d*x+c)^2 + 2*\cos(d*x+c) + 1)*\sqrt{a}*\arctan(\sqrt{2}*\sqrt{a*\cos(d*x+c)+a}*\sqrt{\cos(d*x+c)})/(sqrt(a)*\sin(d*x+c)) - 2*\sqrt{a*\cos(d*x+c)+a}*\sqrt{\cos(d*x+c)}*\sin(d*x+c)/(a^2*d*\cos(d*x+c)^2 + 2*a^2*d*\cos(d*x+c) + a^2*d)$

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)

[Out] Integral(cos(c+d*x)**(3/2)/(a*(cos(c+d*x)+1))**(3/2),x)

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^{3/2}}{(a + a \cos(c + dx))^{3/2}} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(3/2), x)

3.240 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^{3/2}} dx$

Optimal result	3684
Rubi [A] (verified)	3684
Mathematica [A] (verified)	3686
Maple [A] (verified)	3686
Fricas [A] (verification not implemented)	3686
Sympy [F]	3687
Maxima [F]	3687
Giac [F(-1)]	3687
Mupad [F(-1)]	3687

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^{3/2}} dx = \frac{\arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}}$$

[Out] 1/4*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+1/2*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(3/2)

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2843, 12, 2861, 211}

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^{3/2}} dx = \frac{\arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{2d(a \cos(c+dx)+a)^{3/2}}$$

[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(3/2), x]

[Out] ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2843

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{a}{2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d} \\
 &= \frac{\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right) \sqrt{1+\cos(c+dx)} \left(\arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right) \sqrt{1+\cos(c+dx)}\right)}{2d(a(1+\cos(c+dx)))^{3/2}}$$

[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(3/2), x]

[Out] (Cos[(c + d*x)/2]*Sqrt[1 + Cos[c + d*x]]*(ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[1 + Cos[c + d*x]] + 2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2])/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\left(-\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)+\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(1+\cos(dx+c))}}{4d(1+\cos(dx+c))^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}a^2}$

[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2), x, method=_RETURNVERBOSE)

[Out] -1/4/d*(-sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^2

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \frac{\sqrt{2}(\cos(dx+c)^2 + 2\cos(dx+c) + 1)\sqrt{a} \arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}}{2(a\cos(dx+c)^2+a\cos(dx+c))}\right)}{4(a^2d\cos(dx+c))^2 + 2a^2d\cos(dx+c)}$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2), x, algorithm="fricas")

[Out] 1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a(\cos(c+dx)+1))^{3/2}} dx$$

[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2), x)

[Out] Integral(sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(3/2), x)

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{3/2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2), x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(3/2), x)

[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(3/2), x)

$$3.241 \quad \int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	3688
Rubi [A] (verified)	3688
Mathematica [A] (verified)	3690
Maple [B] (verified)	3690
Fricas [A] (verification not implemented)	3690
Sympy [F]	3691
Maxima [F]	3691
Giac [F(-1)]	3691
Mupad [F(-1)]	3691

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{3/2}} dx = \frac{3 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}}$$

[Out] $\frac{3}{4} \arctan\left(\frac{1}{2} \sin(dx+c) a^{1/2} 2^{1/2} / \cos(dx+c)^{1/2} / (a+a \cos(dx+c))^{1/2}\right) / a^{3/2} / d 2^{1/2} - \frac{1}{2} \sin(dx+c) \cos(dx+c)^{1/2} / d (a+a \cos(dx+c))^{3/2}$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2845, 12, 2861, 211}

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{3/2}} dx = \frac{3 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{2d(a \cos(c+dx) + a)^{3/2}}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]

[Out] $\frac{3 \text{ArcTan}\left[\frac{\sqrt{a} \sin[c + d*x]}{\sqrt{2} \sqrt{\cos[c + d*x]} \sqrt{a + a \cos[c + d*x]}}\right]}{2 \sqrt{2} a^{3/2} d} - \frac{\sqrt{\cos[c + d*x]} \sin[c + d*x]}{2 d (a + a \cos[c + d*x])^{3/2}}$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*sin[e + f*x]]*Sqrt[c + d*sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{3a}{2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
 &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3 \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
 &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{3 \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d} \\
 &= \frac{3 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right) \sqrt{\cos(c+dx)} \left(2 + 3\operatorname{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right) \cot^2\left(\frac{1}{2}(c+dx)\right) \sqrt{2-2\sec(c+dx)}\right)}{2d(a(1+\cos(c+dx)))^{3/2}}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]

[Out] -1/2*(Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*(2 + 3*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]])*Cot[(c + d*x)/2]^2*Sqrt[2 - 2*Sec[c + d*x]]*Sin[(c + d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(78) = 156.

Time = 4.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.87

method	result
default	$\frac{\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{a}{(\csc^2(dx+c)(1-\cos(dx+c))^2+1)}} \left(-\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} (\csc(dx+c)-\cot(dx+c))\right)}{4d \sqrt{-\frac{(\csc^2(dx+c)(1-\cos(dx+c))^2-1)}{(\csc^2(dx+c)(1-\cos(dx+c))^2+1)}} a^2}$

[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/4/d/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x+c)-cot(d*x+c))-3*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^2

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx = \frac{3\sqrt{2}(\cos(dx+c)^2 + 2\cos(dx+c) + 1)\sqrt{a} \arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)}}{2(a\cos(dx+c)+1)}\right)}{4(a^2d\cos(dx+c))^{3/2}}$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] $\frac{1}{4} \cdot (3 \sqrt{2} \cdot (\cos(dx + c))^2 + 2 \cos(dx + c) + 1) \sqrt{a} \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{a \cos(dx + c) + a} \sqrt{a} \sqrt{\cos(dx + c)} \sin(dx + c) / (a \cos(dx + c)^2 + a \cos(dx + c))\right) - 2 \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c) / (a^2 d \cos(dx + c)^2 + 2 a^2 d \cos(dx + c) + a^2 d)$

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^{3/2}} dx = \int \frac{1}{(a (\cos(c + dx) + 1))^{\frac{3}{2}} \sqrt{\cos(c + dx)}} dx$$

[In] `integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2), x)`

[Out] `Integral(1/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x))), x)`

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^{3/2}} dx = \int \frac{1}{(a \cos(dx + c) + a)^{\frac{3}{2}} \sqrt{\cos(dx + c)}} dx$$

[In] `integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2), x, algorithm="maxima")`

[Out] `integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)`

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] `integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2), x, algorithm="giac")`

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^{3/2}} dx = \int \frac{1}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^{3/2}} dx$$

[In] `int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)), x)`

[Out] `int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(3/2)), x)`

$$3.242 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	3692
Rubi [A] (verified)	3692
Mathematica [C] (warning: unable to verify)	3694
Maple [A] (verified)	3695
Fricas [A] (verification not implemented)	3695
Sympy [F]	3695
Maxima [F]	3696
Giac [F]	3696
Mupad [F(-1)]	3696

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} dx = -\frac{7 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{3/2}} + \frac{5 \sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}$$

[Out] $-7/4*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}-1/2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}/\cos(d*x+c)^{(1/2)}+5/2*\sin(d*x+c)/a/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2845, 3063, 12, 2861, 211}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} dx = -\frac{7 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{5 \sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}} - \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a \cos(c+dx)+a)^{3/2}}$$

[In] $\text{Int}[1/(\text{Cos}[c+d*x]^{(3/2)}*(a+a*\text{Cos}[c+d*x])^{(3/2)}),x]$

[Out] $(-7*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sin}[c+d*x])/(\text{Sqrt}[2]*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Cos}[c+d*x]])]/(2*\text{Sqrt}[2]*a^{(3/2)}*d) - \text{Sin}[c+d*x]/(2*d*\text{Sqrt}[\text{Cos}[c+d*x]]))$

$(a + a \cos[c + dx])^{3/2} + (5 \sin[c + dx]) / (2ad \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]})$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match}[\text{Q}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]]$

Rule 211

$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a) \text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 2845

$\text{Int}[(a_*) + (b_*) \sin[e_*) + (f_*)(x_)]^{(m_*)} ((c_*) + (d_*) \sin[e_*) + (f_*)(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[b^2 \cos[e + fx] (a + b \sin[e + fx])^m ((c + d \sin[e + fx])^{n+1} / (a f (2m+1) (b c - a d))), x] + \text{Dist}[1 / (a (2m+1) (b c - a d)), \text{Int}[(a + b \sin[e + fx])^{m+1} (c + d \sin[e + fx])^n \text{Simp}[b c (m+1) - a d (2m+n+2) + b d (m+n+2) \sin[e + fx], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \ \&\& \ \text{NeQ}[b c - a d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ (\text{IntegerS}[2m, 2n] \ || \ (\text{IntegerQ}[m] \ \&\& \ \text{EqQ}[c, 0]))$

Rule 2861

$\text{Int}[1 / (\sqrt{(a_*) + (b_*) \sin[e_*) + (f_*)(x_)}]) \sqrt{(c_*) + (d_*) \sin[e_*) + (f_*)(x_)}], x_Symbol] \rightarrow \text{Dist}[-2(a/f), \text{Subst}[\text{Int}[1 / (2b^2 - (ac - bd)x^2), x], x, b(\cos[e + fx] / (\sqrt{a + b \sin[e + fx]} \sqrt{c + d \sin[e + fx]})]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b c - a d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3063

$\text{Int}[(a_*) + (b_*) \sin[e_*) + (f_*)(x_)]^{(m_*)} ((A_*) + (B_*) \sin[e_*) + (f_*)(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(B c - A d) \cos[e + fx] (a + b \sin[e + fx])^m ((c + d \sin[e + fx])^{n+1} / (f(n+1)(c^2 - d^2))), x] + \text{Dist}[1 / (b(n+1)(c^2 - d^2)), \text{Int}[(a + b \sin[e + fx])^m (c + d \sin[e + fx])^{n+1} \text{Simp}[A(a d m + b c(n+1)) - B(a c m + b d(n+1)) + b(B c - A d)(m+n+2) \sin[e + fx], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b c - a d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{EqQ}[m + 1/2, 0])$

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{5a}{2}-a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
&= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{\int -\frac{7a^2}{4\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{a^3} \\
&= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{7\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
&= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{7\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d} \\
&= -\frac{7\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.60 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.80

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx = \frac{-\frac{35}{2}\cos^2(c+dx)\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)\left(78+108\cos(c+dx)\right)}{\dots}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x]

[Out] ((-35*Cos[c + d*x]^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4*(78 + 108*Cos[c + d*x] + 80*Cos[2*(c + d*x)] - 204*Cos[3*(c + d*x)] - 62*Cos[4*(c + d*x)] + 12*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Cos[(c + d*x)/2]^2*(64 + 55*Cos[c + d*x] + 64*Cos[2*(c + d*x)] + 17*Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])/2 - 768*Cos[(c + d*x)/2]^5*HypergeometricPFQ[{2, 2, 5/2}, {1, 9/2}, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[(c + d*x)/2]^3)/(3360*d*Cos[c + d*x]^(5/2)*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [A] (verified)

Time = 5.57 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

method	result
default	$\frac{\left(7\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (\cos^2(dx+c)) \arcsin(\cot(dx+c)-\csc(dx+c))+5\sin(dx+c) \cos(dx+c)\sqrt{2}+14\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))\right) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{4d\sqrt{\cos(dx+c)}(1+\cos(dx+c))}$

[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)

[Out] $\frac{1}{4}d\left(7\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{1/2}\cos(dx+c)^2\arcsin(\cot(dx+c)-\csc(dx+c))+5\sin(dx+c)\cos(dx+c)2^{1/2}+14\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{1/2}\arcsin(\cot(dx+c)-\csc(dx+c))+4\cdot 2^{1/2}\sin(dx+c)+7\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{1/2}\arcsin(\cot(dx+c)-\csc(dx+c))\right)\left(a(1+\cos(dx+c))\right)^{1/2}/\cos(dx+c)^{1/2}/(1+\cos(dx+c))^2\cdot 2^{1/2}/a^2$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.25

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \frac{7\sqrt{2}(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c))\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2(a\cos(dx+c)^2+a\cos(dx+c))}\right)-2}{4(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c))}$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] $-1/4\cdot(7\sqrt{2})\cdot(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c))\cdot\sqrt{a}\cdot\arctan(1/2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)/(a\cos(dx+c)^2+a\cos(dx+c)))-2\sqrt{a}\cdot(5\cos(dx+c)+4)\sqrt{\cos(dx+c)}\sin(dx+c)/(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c))$

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \int \frac{1}{(a(\cos(c+dx)+1))^{\frac{3}{2}}\cos^{\frac{3}{2}}(c+dx)} dx$$

[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)

[Out] Integral(1/((a*(cos(c+d*x)+1))**(3/2)*cos(c+d*x)**(3/2)), x)

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx = \int \frac{1}{\cos(c+dx)^{3/2}(a+a\cos(c+dx))^{3/2}} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)),x)

[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(3/2)), x)

$$3.243 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	3697
Rubi [A] (verified)	3697
Mathematica [C] (warning: unable to verify)	3700
Maple [A] (verified)	3700
Fricas [A] (verification not implemented)	3701
Sympy [F(-1)]	3701
Maxima [F]	3701
Giac [F]	3702
Mupad [F(-1)]	3702

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} dx = \frac{11 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}$$

$$- \frac{\sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}}$$

$$+ \frac{7 \sin(c+dx)}{6ad \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} - \frac{19 \sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}$$

[Out] $-1/2*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(3/2)}+11/4*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(3/2)}/d*2^{(1/2)}+7/6*\sin(d*x+c)/a/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(1/2)}-19/6*\sin(d*x+c)/a/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2845, 3063, 12, 2861, 211}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} dx = \frac{11 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}$$

$$+ \frac{7 \sin(c+dx)}{6ad \cos^{\frac{3}{2}}(c+dx)\sqrt{a \cos(c+dx)+a}} - \frac{\sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)(a \cos(c+dx)+a)^{3/2}}$$

$$- \frac{19 \sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*cos[c + d*x])^(3/2)),x]

[Out] (11*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + (7*Sin[c + d*x])/(6*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (19*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*sin[e + f*x]]*Sqrt[c + d*sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3063

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m

+ 1/2, 0])

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{7a}{2} - 2a \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} dx}{2a^2} \\
&= -\frac{\sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} \\
&\quad + \frac{7 \sin(c+dx)}{6ad \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} + \frac{\int \frac{-\frac{19a^2}{4} + \frac{7}{2}a^2 \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} dx}{3a^3} \\
&= -\frac{\sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} + \frac{7 \sin(c+dx)}{6ad \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{19 \sin(c+dx)}{6ad \sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} + \frac{2 \int \frac{33a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} dx}{3a^4} \\
&= -\frac{\sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} + \frac{7 \sin(c+dx)}{6ad \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{19 \sin(c+dx)}{6ad \sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} + \frac{11 \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} dx}{4a} \\
&= -\frac{\sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} + \frac{7 \sin(c+dx)}{6ad \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{19 \sin(c+dx)}{6ad \sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{11 \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{2d} \\
&= \frac{11 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} \\
&\quad + \frac{7 \sin(c+dx)}{6ad \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} - \frac{19 \sin(c+dx)}{6ad \sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.23 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.33

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \frac{\cot^3\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^4\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^2\left(\frac{1}{2}(c+dx)\right) \left(-80 \cos^6\left(\frac{1}{2}(c+dx)\right)\right)}{\dots}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x]

[Out] (Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^2*(-80*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 + 120*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2) + 21*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)])*(-392 + 2347*Sin[c/2 + (d*x)/2]^2 - 5391*Sin[c/2 + (d*x)/2]^4 + 5972*Sin[c/2 + (d*x)/2]^6 - 3232*Sin[c/2 + (d*x)/2]^8 + 696*Sin[c/2 + (d*x)/2]^10) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5880 + 37165*Sin[c/2 + (d*x)/2]^2 - 89856*Sin[c/2 + (d*x)/2]^4 + 103992*Sin[c/2 + (d*x)/2]^6 - 58336*Sin[c/2 + (d*x)/2]^8 + 12960*Sin[c/2 + (d*x)/2]^10))/ (945*d*(a*(1 + Cos[c + d*x])^(3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)))

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\left(33(\cos^3(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))+19\sqrt{2}(\cos^2(dx+c))\sin(dx+c)+66\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c))\arcsin(\cot(dx+c)-\csc(dx+c))\right)}{12d\cos^{\frac{5}{2}}(dx+c)(a+\cos(dx+c))^{\frac{3}{2}}}$

[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/12/d*(33*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+19*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+66*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))+12*sin(d*x+c)*cos(d*x+c)*2^(1/2)+33*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))-4*2^(1/2)*sin(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^2*2^(1/2)/a^2

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \frac{33\sqrt{2}(\cos(dx+c)^4 + 2\cos(dx+c)^3 + \cos(dx+c)^2)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\cos(dx+c)}\right) + 2\sqrt{a\cos(dx+c)+a}(19\cos(dx+c)^2 + 12\cos(dx+c) - 4)\sqrt{\cos(dx+c)}\sin(dx+c)}{(a^2d\cos(dx+c)^4 + 2a^2d\cos(dx+c)^3 + a^2d\cos(dx+c)^2)}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")

```
[Out] 1/12*(33*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(19*cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}} \cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx = \int \frac{1}{\cos(c+dx)^{5/2} (a+a\cos(c+dx))^{3/2}} dx$$

[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)),x)

[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(3/2)), x)

$$3.244 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	3703
Rubi [A] (verified)	3703
Mathematica [A] (verified)	3706
Maple [A] (verified)	3707
Fricas [A] (verification not implemented)	3707
Sympy [F(-1)]	3708
Maxima [F]	3708
Giac [F(-1)]	3708
Mupad [F(-1)]	3708

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = -\frac{5 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{5/2}d} + \frac{115 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} - \frac{15 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}} + \frac{35\sqrt{\cos(c+dx)} \sin(c+dx)}{16a^2d\sqrt{a+a \cos(c+dx)}}$$

```
[Out] -5*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d-1/4*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)-15/16*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)+115/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+35/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used

= {2844, 3056, 3062, 3061, 2861, 211, 2853, 222}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = -\frac{5 \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d} + \frac{115 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{35 \sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}} - \frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}} - \frac{15 \sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}$$

[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(5/2), x]

[Out] (-5*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/(a^(5/2)*d) + (115*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]/(16*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (15*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (35*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Ssin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\text{integral} = -\frac{\cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c + dx) \left(\frac{5a}{2} - 5a \cos(c + dx)\right)}{(a + a \cos(c + dx))^{3/2}} dx}{4a^2}$$

$$\begin{aligned}
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{45a^2}{4} - \frac{35}{2}a^2\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{35\sqrt{\cos(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-\frac{35a^3}{4} + 20a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{8a^5} \\
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{35\sqrt{\cos(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - \frac{5\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{2a^3} \\
&\quad + \frac{115\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{35\sqrt{\cos(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} + \frac{5\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^3d} \\
&\quad - \frac{115\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
&= -\frac{5\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{115\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\
&\quad - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{35\sqrt{\cos(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{\left(140\arcsin\left(\sqrt{1-\cos(c+dx)}\right)\cos^4\left(\frac{1}{2}(c+dx)\right) + 460\arcsin\left(\sqrt{\cos(c+dx)}\right)\right)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*cos[c + d*x])^(5/2), x]

```
[Out] ((140*ArcSin[Sqrt[1 - Cos[c + d*x]])*Cos[(c + d*x)/2]^4 + 460*ArcSin[Sqrt[Cos[c + d*x]])*Cos[(c + d*x)/2]^4 - 230*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^4 + 55*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 16*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2) + 35*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(16*d*Sqrt[1 - Cos[c + d*x]])*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] (verified)

Time = 12.74 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.61

method	result
default	$\frac{(16\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c)\sin(dx+c)-80\sqrt{2}\arctan(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})(\cos^2(dx+c))+55\sqrt{2}\cos(dx+c)\sin(dx+c))}{(16d\sqrt{1-\cos(dx+c)})^5(a(1+\cos(dx+c)))^5}$

```
[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d*(16*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-80*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2+55*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-115*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-160*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)+35*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-230*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-80*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-115*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \frac{115\sqrt{2}(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - 2\sqrt{a}\cos(dx+c)^{5/2}}{(16d\sqrt{1-\cos(dx+c)})^5(a(1+\cos(dx+c)))^5}$$

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(16*cos(d*x + c)^2 + 55*cos(d*x + c) + 35)*sqrt(cos(d*x + c))*sin(d*x + c) - 160*(cos(d*x + c)^3 + 3*cos
```

$(d*x + c)^2 + 3*\cos(d*x + c) + 1)*\sqrt{a}*\arctan(\sqrt{a*\cos(d*x + c) + a}*\sqrt{\cos(d*x + c)})/(\sqrt{a}*\sin(d*x + c)))/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(5/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(a \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(5/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2), x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\cos(c + dx)^{\frac{7}{2}}}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx$$

[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(5/2), x)

[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(5/2), x)

$$3.245 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{5}{2}}} dx$$

Optimal result	3709
Rubi [A] (verified)	3709
Mathematica [A] (verified)	3712
Maple [B] (verified)	3712
Fricas [A] (verification not implemented)	3713
Sympy [F(-1)]	3713
Maxima [F]	3713
Giac [F(-1)]	3714
Mupad [F(-1)]	3714

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{5}{2}}} dx = \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{\frac{5}{2}}d} - \frac{43 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{\frac{5}{2}}d} - \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4d(a+a \cos(c+dx))^{\frac{5}{2}}} - \frac{11\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a \cos(c+dx))^{\frac{3}{2}}}$$

[Out] $2*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(5/2)}/d-1/4*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(5/2)}-43/32*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}-11/16*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\cos(d*x+c))^{(3/2)}$

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2844, 3056, 3061, 2861, 211, 2853, 222}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{5}{2}}} dx = \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{a^{\frac{5}{2}}d} - \frac{43 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{\frac{5}{2}}d} - \frac{\sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{4d(a \cos(c+dx)+a)^{\frac{5}{2}}} - \frac{11 \sin(c+dx) \sqrt{\cos(c+dx)}}{16ad(a \cos(c+dx)+a)^{\frac{3}{2}}}$$

[In] Int[Cos[c + d*x]^(5/2)/(a + a*cos[c + d*x])^(5/2), x]

[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*cos[c + d*x]])/(a^(5/2)*d) - (43*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*cos[c + d*x]])]/(16*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*cos[c + d*x])^(5/2)) - (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*cos[c + d*x])^(3/2))

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim

$$\text{p}[(A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*((c + d*\text{Sin}[e + f*x])^n/(a*f*(2*m + 1))), x] - \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m+1}*(c + d*\text{Sin}[e + f*x])^{n-1}*\text{Simp}[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] || \text{EqQ}[c, 0])$$

Rule 3061

$$\text{Int}[(A_. + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)])/(\text{Sqrt}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])*\text{Sqrt}[(c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]), x_Symbol] := \text{Dist}[(A*b - a*B)/b, \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] + \text{Dist}[B/b, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[c + d*\text{Sin}[e + f*x]], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}(\frac{3a}{2}-4a\cos(c+dx))}{(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\frac{11a^2}{4}-8a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
 &\quad + \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{a^3} - \frac{43\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
 &\quad - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^3d} \\
 &\quad + \frac{43\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
 &= \frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{43\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\
 &\quad - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{5}{2}}} dx =$$

$$\frac{\left(44 \arcsin\left(\sqrt{1-\cos(c+dx)}\right) \cos^4\left(\frac{1}{2}(c+dx)\right) + 172 \arcsin\left(\sqrt{\cos(c+dx)}\right) \cos^4\left(\frac{1}{2}(c+dx)\right) - 86\sqrt{2} \arcsin\left(\sqrt{\frac{1-\cos(c+dx)}{2}}\right) \cos^4\left(\frac{1}{2}(c+dx)\right) - 16d\sqrt{1-\cos(c+dx)}\right)}{16d\sqrt{1-\cos(c+dx)}} + C$$

[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2), x]

[Out] -1/16*((44*ArcSin[Sqrt[1 - Cos[c + d*x]]]*Cos[(c + d*x)/2]^4 + 172*ArcSin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^4 - 86*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^4 + 15*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 11*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(143) = 286.

Time = 3.85 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\left(-\frac{\csc^2(dx+c)(1-\cos(dx+c))^2-1}{\csc^2(dx+c)(1-\cos(dx+c))^2+1}\right)^{\frac{5}{2}} \left(\csc^2(dx+c)(1-\cos(dx+c))^2+1\right)^3 \sqrt{\frac{a}{\csc^2(dx+c)(1-\cos(dx+c))^2+1}} \left(-2(\csc^3(dx+c))\sqrt{-\frac{1-\cos(dx+c)}{2}}\right)}{16d\sqrt{1-\cos(dx+c)}} + C$

[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2), x, method=_RETURNVERBOSE)

[Out] -1/32/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(5/2)/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(5/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^3*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-2*csc(d*x+c)^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^3+32*2^(1/2)*arctan(2^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(csc(d*x+c)-cot(d*x+c)))+13*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x+c)-cot(d*x+c))-43*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^3

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.30

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \frac{43\sqrt{2}(\cos(dx + c)^3 + 3\cos(dx + c)^2 + 3\cos(dx + c) + 1)\sqrt{a} \arctan\left(\frac{\sqrt{2}\sqrt{a}}{\dots}\right)}{\dots}$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

```
[Out] 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(15*cos(d*x + c) + 11)*sqrt(cos(d*x + c))*sin(d*x + c) - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\cos(c + dx)^{\frac{5}{2}}}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx$$

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(5/2), x)
```

$$3.246 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	3715
Rubi [A] (verified)	3715
Mathematica [A] (verified)	3717
Maple [A] (verified)	3718
Fricas [A] (verification not implemented)	3718
Sympy [F]	3718
Maxima [F]	3719
Giac [F(-1)]	3719
Mupad [F(-1)]	3719

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = \frac{3 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}}$$

[Out] 3/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(5/2)+7/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2844, 3057, 12, 2861, 211}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = \frac{3 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{7 \sin(c+dx) \sqrt{\cos(c+dx)}}{16ad(a \cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{4d(a \cos(c+dx)+a)^{5/2}}$$

[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]

[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4

$*d*(a + a*\cos[c + d*x])^{(5/2)} + (7*\sqrt{\cos[c + d*x]}*\sin[c + d*x])/(16*a*d*(a + a*\cos[c + d*x])^{(3/2)})$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 211

$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 2844

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)(x_)])^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)(x_)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*\cos[e + f*x]*(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^{(n-1)})/(a*f*(2*m+1)), x] + \text{Dist}[1/(a*b*(2*m+1)), \text{Int}[(a + b*\sin[e + f*x])^{(m+1)}*(c + d*\sin[e + f*x])^{(n-2)}*\text{Simp}[b*(c^2*(m+1) + d^2*(n-1)) + a*c*d*(m-n+1) + d*(a*d*(m-n+1) + b*c*(m+n))*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ (\text{IntegersQ}[2*m, 2*n] \ || \ (\text{IntegerQ}[m] \ \&\& \ \text{EqQ}[c, 0]))$

Rule 2861

$\text{Int}[1/(\sqrt{(a_*) + (b_*)*\sin[(e_*) + (f_*)(x_)]})*\sqrt{(c_*) + (d_*)*\sin[(e_*) + (f_*)(x_)]}), x_Symbol] \rightarrow \text{Dist}[-2*(a/f), \text{Subst}[\text{Int}[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(\cos[e + f*x]/(\sqrt{a + b*\sin[e + f*x]}*\sqrt{c + d*\sin[e + f*x]})]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3057

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)(x_)])^{(m_*)}*((A_*) + (B_*)*\sin[(e_*) + (f_*)(x_)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[b*(A*b - a*B)*\cos[e + f*x]*(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^{(n+1)})/(a*f*(2*m+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*(2*m+1)*(b*c - a*d)), \text{Int}[(a + b*\sin[e + f*x])^{(m+1)}*(c + d*\sin[e + f*x])^n*\text{Simp}[B*(a*c*m + b*d*(n+1)) + A*(b*c*(m+1) - a*d*(2*m+n+2)) + d*(A*b - a*B)*(m+n+2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ (\text{IntegerQ}[2*n] \ || \ \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\frac{a}{2}-3a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
 &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int -\frac{3a^2}{4\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
 &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
 &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
 &\quad - \frac{3\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
 &= \frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\
 &\quad - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{\cos^5\left(\frac{1}{2}(c+dx)\right) \left(3\arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right) \sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)} + \sqrt{2}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\right)}{4d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(a(1+\cos(c+dx)))}$$

[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2), x]

[Out] (Cos[(c + d*x)/2]^5*(3*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]*(5 - 2*Tan[(c + d*x)/2]^2))/(4*d*Sqrt[Cos[(c + d*x)/2]^2]*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\left(-7\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\arcsin(\cot(dx+c)-\csc(dx+c))(\cos^2(dx+c))-3\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+6\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)+3\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)^{1/2}(a(1+\cos(dx+c)))^{1/2}/(1+\cos(dx+c))^{3/2}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{32d(1+\cos(dx+c))^3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)

```
[Out] -1/32/d*(-7*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+3*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-3*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+6*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+3*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{3\sqrt{2}(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}}{2}\right)+2\sqrt{a\cos(dx+c)+a}(7\cos(dx+c)+3)\sqrt{\cos(dx+c)}\sin(dx+c)}{32(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d)}$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

```
[Out] 1/32*(3*sqrt(2)*(cos(d*x+c)^3+3*cos(d*x+c)^2+3*cos(d*x+c)+1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x+c)+a)*sqrt(a)*sqrt(cos(d*x+c))*sin(d*x+c)/(a*cos(d*x+c)^2+a*cos(d*x+c))) + 2*sqrt(a*cos(d*x+c)+a)*(7*cos(d*x+c)+3)*sqrt(cos(d*x+c))*sin(d*x+c)/(a^3*d*cos(d*x+c)^3+3*a^3*d*cos(d*x+c)^2+3*a^3*d*cos(d*x+c)+a^3*d)
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a(\cos(c+dx)+1))^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)

[Out] Integral(cos(c+d*x)**(3/2)/(a*(cos(c+d*x)+1))**(5/2), x)

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\cos(c + dx)^{\frac{3}{2}}}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(5/2), x)

$$3.247 \quad \int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	3720
Rubi [A] (verified)	3720
Mathematica [A] (verified)	3722
Maple [A] (verified)	3723
Fricas [A] (verification not implemented)	3723
Sympy [F]	3723
Maxima [F]	3724
Giac [F(-1)]	3724
Mupad [F(-1)]	3724

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^{5/2}} dx = \frac{5 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}}$$

[Out] 5/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(5/2)+1/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2843, 3057, 12, 2861, 211}

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^{5/2}} dx = \frac{5 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{16ad(a \cos(c+dx)+a)^{3/2}} + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{4d(a \cos(c+dx)+a)^{5/2}}$$

[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]

[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]/(16*Sqrt[2]*a^(5/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4

$*d*(a + a*\cos[c + d*x])^{5/2} + (\sqrt{\cos[c + d*x]}*\sin[c + d*x])/(16*a*d*(a + a*\cos[c + d*x])^{3/2})$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 211

$\text{Int}[(a_*) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 2843

$\text{Int}[(a_*) + (b_)*\sin[(e_*) + (f_)*(x_)])^{(m_)*((c_*) + (d_)*\sin[(e_*) + (f_)*(x_)])^{(n_)}}, x_Symbol] \rightarrow \text{Simp}[b*\cos[e + f*x]*(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^n/(a*f*(2*m + 1))), x] - \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^{(n - 1)}*\text{Simp}[a*d*n - b*c*(m + 1) - b*d*(m + n + 1)*\sin[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{LtQ}[0, n, 1] \ \&\& \ (\text{IntegersQ}[2*m, 2*n] \ || \ (\text{IntegerQ}[m] \ \&\& \ \text{EqQ}[c, 0]))$

Rule 2861

$\text{Int}[1/(\sqrt{(a_*) + (b_)*\sin[(e_*) + (f_)*(x_)])}*\sqrt{(c_*) + (d_)*\sin[(e_*) + (f_)*(x_)]}), x_Symbol] \rightarrow \text{Dist}[-2*(a/f), \text{Subst}[\text{Int}[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(\cos[e + f*x]/(\sqrt{a + b*\sin[e + f*x]}*\sqrt{c + d*\sin[e + f*x]})]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3057

$\text{Int}[(a_*) + (b_)*\sin[(e_*) + (f_)*(x_)])^{(m_)*((A_*) + (B_)*\sin[(e_*) + (f_)*(x_)])^{(n_)}}, x_Symbol] \rightarrow \text{Simp}[b*(A*b - a*B)*\cos[e + f*x]*(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^{(n + 1)})/(a*f*(2*m + 1)*(b*c - a*d)), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*(c + d*\sin[e + f*x])^n*\text{Simp}[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ (\text{IntegerQ}[2*n] \ || \ \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{a}{2} + a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{5a^2}{4\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{5 \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
 &\quad - \frac{5 \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
 &= \frac{5 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\
 &\quad + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right) \sqrt{\cos(c+dx)} \left(-2 + 6 \csc^2\left(\frac{1}{2}(c+dx)\right) - 5 \operatorname{arctanh}\left(\sqrt{-\sec(c+dx)}\right)\right)}{8d(a(1+\cos(c+dx)))^{5/2}}$$

[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(5/2), x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*(-2 + 6*Csc[(c + d*x)/2]^2 - 5*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Cot[(c + d*x)/2]^4*Sqrt[2 - 2*Sec[c + d*x]])*Sin[(c + d*x)/2]^3)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.42

method	result
default	$\frac{(\sqrt{2} \cos(dx+c) \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 5 \sin(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 5 \arcsin(\cot(dx+c) - \csc(dx+c)) (\cos^2(dx+c)) - 10 \arcsin(\cot(dx+c) - \csc(dx+c)) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})}{32d(1+\cos(dx+c))^3 \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] `int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)`

```
[Out] 1/32/d*(2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*
sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*arcsin(cot(d*x+c)-csc
(d*x+c))*cos(d*x+c)^2-10*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-5*arcsin(
cot(d*x+c)-csc(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*
x+c))^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx = \frac{5\sqrt{2}(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1)\sqrt{a} \arctan\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+5}\right) + 2\sqrt{a}\sin(dx+c)}{32(a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2 + 3a^3d\cos(dx+c) + a^3d)}$$

[In] `integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")`

```
[Out] 1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sq
rt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c)
+ a)*(cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a(\cos(c+dx)+1))^{5/2}} dx$$

[In] `integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)`[Out] `Integral(sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(5/2), x)`

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(5/2), x)

$$3.248 \quad \int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	3725
Rubi [A] (verified)	3725
Mathematica [A] (verified)	3727
Maple [B] (verified)	3728
Fricas [A] (verification not implemented)	3728
Sympy [F]	3729
Maxima [F]	3729
Giac [F(-1)]	3729
Mupad [F(-1)]	3729

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{5/2}} dx = \frac{19 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}}$$

[Out] $19/32*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}-1/4*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(5/2)}-9/16*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\cos(d*x+c))^{(3/2)}$

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2845, 3057, 12, 2861, 211}

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{5/2}} dx = \frac{19 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{9 \sin(c+dx) \sqrt{\cos(c+dx)}}{16ad(a \cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{4d(a \cos(c+dx)+a)^{5/2}}$$

[In] $\text{Int}[1/(\text{Sqrt}[\text{Cos}[c+d*x]]*(a+a*\text{Cos}[c+d*x])^{(5/2)}),x]$

[Out] $(19*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sin}[c+d*x])/(\text{Sqrt}[2]*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Cos}[c+d*x]])])/(16*\text{Sqrt}[2]*a^{(5/2)}*d) - (\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(($

$4*d*(a + a*\cos[c + d*x])^{5/2} - (9*\sqrt{\cos[c + d*x]}*\sin[c + d*x])/(16*a*d*(a + a*\cos[c + d*x])^{3/2})$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 211

$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 2845

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)(x_)])^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)(x_)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[b^2*\cos[e + f*x]*(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^{n+1}/(a*f*(2*m + 1)*(b*c - a*d))), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d)), \text{Int}[(a + b*\sin[e + f*x])^{m+1}*(c + d*\sin[e + f*x])^n*\text{Simp}[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ (\text{IntegerQ}[2*m, 2*n] \ || \ (\text{IntegerQ}[m] \ \&\& \ \text{EqQ}[c, 0]))$

Rule 2861

$\text{Int}[1/(\sqrt{(a_*) + (b_*)*\sin[(e_*) + (f_*)(x_)]})*\sqrt{(c_*) + (d_*)*\sin[(e_*) + (f_*)(x_)]}), x_Symbol] \rightarrow \text{Dist}[-2*(a/f), \text{Subst}[\text{Int}[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(\cos[e + f*x]/(\sqrt{a + b*\sin[e + f*x]}*\sqrt{c + d*\sin[e + f*x]})]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3057

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)(x_)])^{(m_*)}*((A_*) + (B_*)*\sin[(e_*) + (f_*)(x_)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[b*(A*b - a*B)*\cos[e + f*x]*(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^{n+1}/(a*f*(2*m + 1)*(b*c - a*d))), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d)), \text{Int}[(a + b*\sin[e + f*x])^{m+1}*(c + d*\sin[e + f*x])^n*\text{Simp}[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ (\text{IntegerQ}[2*n] \ || \ \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{7a}{2} - a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{19a^2}{4\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{19 \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad - \frac{19 \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
&= \frac{19 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} \\
&\quad - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx = \frac{\sec^2\left(\frac{1}{2}(c+dx)\right) \left(-76 \operatorname{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right) \cos^4\left(\frac{1}{2}(c+dx)\right) + \cos(c+dx)(13+9\cos(c+dx))\right)}{32\sqrt{2}a^2d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x]

[Out] -1/32*(Sec[(c + d*x)/2]^2*(-76*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^4 + Cos[c + d*x]*(13 + 9*Cos[c + d*x])*Sqrt[2 - 2*Sec[c + d*x]]*Tan[(c + d*x)/2])/(Sqrt[2]*a^2*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(112) = 224.

Time = 5.67 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.67

method	result
default	$\frac{\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{a}{(\csc^2(dx+c))(1-\cos(dx+c))^2+1}} \left(-2(\csc^3(dx+c)) \sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} (1-\cos(dx+c)) \right)}{32d \sqrt{-\frac{(\csc^2(dx+c))(1-\cos(dx+c))}{(\csc^2(dx+c))(1-\cos(dx+c))}}}$

[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/32/d/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-2*csc(d*x+c)^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^3-11*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x+c)-cot(d*x+c))-19*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^3

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{\cos(c+dx)(a+a\cos(c+dx))}^{5/2}} dx = \frac{19\sqrt{2}(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1)\sqrt{a}}{32(a^3d\cos(dx+c) + a^3d)}$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(9*cos(d*x + c) + 13)*sqrt(cos(d*x + c))*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx = \int \frac{1}{(a(\cos(c+dx)+1))^{5/2} \sqrt{\cos(c+dx)}} dx$$

[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2), x)

[Out] Integral(1/((a*(cos(c + d*x) + 1))**(5/2)*sqrt(cos(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx = \int \frac{1}{(a\cos(dx+c)+a)^{5/2} \sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2), x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)), x)

[Out] int(1/(cos(c + d*x)^(1/2)*(a + a*cos(c + d*x))^(5/2)), x)

$$3.249 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	3730
Rubi [A] (verified)	3730
Mathematica [C] (warning: unable to verify)	3733
Maple [A] (verified)	3733
Fricas [A] (verification not implemented)	3734
Sympy [F(-1)]	3734
Maxima [F]	3735
Giac [F]	3735
Mupad [F(-1)]	3735

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} dx =$$

$$\frac{75 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{5/2}}$$

$$- \frac{13 \sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{3/2}} + \frac{49 \sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}$$

[Out] -75/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2)-13/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+49/16*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2845, 3057, 3063, 12, 2861, 211}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} dx =$$

$$\frac{75 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{49 \sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}$$

$$- \frac{13 \sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a \cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a \cos(c+dx)+a)^{5/2}}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x]

[Out] (-75*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Sqrt[Cos[c + d*x]])*(a + a*Cos[c + d*x])^(5/2) - (13*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]])*(a + a*Cos[c + d*x])^(3/2) + (49*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]])*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*SIn[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3057

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]

&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sin(c + dx)}{4d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{5/2}} + \frac{\int \frac{\frac{9a}{2} - 2a\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^{3/2}} dx}{4a^2} \\
 &= -\frac{\sin(c + dx)}{4d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{5/2}} \\
 &\quad - \frac{13\sin(c + dx)}{16ad\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{3/2}} + \frac{\int \frac{\frac{49a^2}{4} - \frac{13}{2}a^2\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}} dx}{8a^4} \\
 &= -\frac{\sin(c + dx)}{4d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{5/2}} - \frac{13\sin(c + dx)}{16ad\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{3/2}} \\
 &\quad + \frac{49\sin(c + dx)}{16a^2d\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} + \frac{\int -\frac{75a^3}{8\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} dx}{4a^5} \\
 &= -\frac{\sin(c + dx)}{4d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{5/2}} - \frac{13\sin(c + dx)}{16ad\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{3/2}} \\
 &\quad + \frac{49\sin(c + dx)}{16a^2d\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} - \frac{75\int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} dx}{32a^2} \\
 &= -\frac{\sin(c + dx)}{4d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{5/2}} - \frac{13\sin(c + dx)}{16ad\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{3/2}} \\
 &\quad + \frac{49\sin(c + dx)}{16a^2d\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}} \\
 &\quad + \frac{75\text{Subst}\left(\int \frac{1}{2a^2 + ax^2} dx, x, -\frac{a\sin(c + dx)}{\sqrt{\cos(c + dx)}\sqrt{a + a\cos(c + dx)}}\right)}{16ad}
 \end{aligned}$$

+c)*2^(1/2)+225*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+32*2^(1/2)*sin(d*x+c)+75*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3/cos(d*x+c)^(1/2)*2^(1/2)/a^3

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} dx = \frac{75\sqrt{2}(\cos(dx+c)^4 + 3\cos(dx+c)^3 + 3\cos(dx+c)^2 + \cos(dx+c))\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a\cos(dx+c)}}{2(a\cos(dx+c)^2+a\cos(dx+c))}\right)}{32(a^3d\cos(dx+c)^4 + 3a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2 + a^3d\cos(dx+c))}$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -1/32*(75*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(49*cos(d*x + c)^2 + 85*cos(d*x + c) + 32)*sqrt(cos(d*x + c))*sin(d*x + c)/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}} \cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}} \cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{1}{\cos(c+dx)^{\frac{3}{2}}(a+a\cos(c+dx))^{\frac{5}{2}}} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)),x)

[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(5/2)), x)

$$3.250 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	3736
Rubi [A] (verified)	3736
Mathematica [C] (warning: unable to verify)	3739
Maple [A] (verified)	3740
Fricas [A] (verification not implemented)	3740
Sympy [F(-1)]	3741
Maxima [F]	3741
Giac [F]	3741
Mupad [F(-1)]	3742

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} dx = \frac{163 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{17 \sin(c+dx)} - \frac{4d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}}{16ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} + \frac{95 \sin(c+dx)}{48a^2d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)}} - \frac{299 \sin(c+dx)}{48a^2d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}$$

[Out] $-1/4*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(5/2)}-17/16*\sin(d*x+c)/a/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(3/2)}+163/32*\arctan(1/2*\sin(d*x+c)*a^{(1/2)*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(5/2)}/d*2^{(1/2)}+95/48*\sin(d*x+c)/a^2/d/\cos(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(1/2)}-299/48*\sin(d*x+c)/a^2/d/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2845, 3057, 3063, 12, 2861, 211}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} dx = \frac{163 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{95 \sin(c+dx)}{48a^2d \cos^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx)+a}} - \frac{299 \sin(c+dx)}{48a^2d \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}} - \frac{17 \sin(c+dx)}{16ad \cos^{\frac{3}{2}}(c+dx)(a \cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx)}{4d \cos^{\frac{3}{2}}(c+dx)(a \cos(c+dx)+a)^{5/2}}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*cos[c + d*x])^(5/2)),x]

[Out] (163*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*cos[c + d*x]])]/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*cos[c + d*x]^(3/2)*(a + a*cos[c + d*x])^(5/2)) - (17*Sin[c + d*x])/(16*a*d*cos[c + d*x]^(3/2)*(a + a*cos[c + d*x])^(3/2)) + (95*Sin[c + d*x])/(48*a^2*d*cos[c + d*x]^(3/2)*Sqrt[a + a*cos[c + d*x]]) - (299*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*sin[e + f*x]]*Sqrt[c + d*sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[

$b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}]$
 $\&\& \text{!GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] \mid\mid \text{EqQ}[c, 0])$

Rule 3063

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]])^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)])^{(n_.)}, x_Symbol] \text{:> Sim}$
 $\text{p}[(B*c - A*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*((c + d*\text{Sin}[e + f*x])^{(n + 1)})/(f*(n + 1)*(c^2 - d^2)), x] + \text{Dist}[1/(b*(n + 1)*(c^2 - d^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(n + 1)}*\text{Simp}[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*\text{Sin}[e + f*x], x], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[n, -1] \&\& (\text{IntegerQ}[n] \mid\mid \text{EqQ}[m + 1/2, 0])$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{5/2}} + \frac{\int \frac{\frac{11a}{2} - 3a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{3/2}} dx}{4a^2} \\ &= -\frac{\sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{5/2}} \\ &\quad - \frac{17 \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{3/2}} + \frac{\int \frac{\frac{95a^2}{4} - 17a^2 \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)\sqrt{a + a \cos(c + dx)}} dx}{8a^4} \\ &= -\frac{\sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{5/2}} - \frac{17 \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{3/2}} \\ &\quad + \frac{95 \sin(c + dx)}{48a^2d \cos^{\frac{3}{2}}(c + dx)\sqrt{a + a \cos(c + dx)}} + \frac{\int \frac{-\frac{299a^3}{8} + \frac{95}{4}a^3 \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a \cos(c + dx)}} dx}{12a^5} \\ &= -\frac{\sin(c + dx)}{4d \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{5/2}} - \frac{17 \sin(c + dx)}{16ad \cos^{\frac{3}{2}}(c + dx)(a + a \cos(c + dx))^{3/2}} \\ &\quad + \frac{95 \sin(c + dx)}{48a^2d \cos^{\frac{3}{2}}(c + dx)\sqrt{a + a \cos(c + dx)}} \\ &\quad - \frac{299 \sin(c + dx)}{48a^2d \sqrt{\cos(c + dx)}\sqrt{a + a \cos(c + dx)}} + \frac{\int \frac{489a^4}{16\sqrt{\cos(c + dx)}\sqrt{a + a \cos(c + dx)}} dx}{6a^6} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{4d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} - \frac{17 \sin(c+dx)}{16ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} \\
&\quad + \frac{95 \sin(c+dx)}{48a^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{299 \sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} + \frac{163 \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} dx}{32a^2} \\
&= -\frac{\sin(c+dx)}{4d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} - \frac{17 \sin(c+dx)}{16ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} \\
&\quad + \frac{95 \sin(c+dx)}{48a^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{299 \sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{163 \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{16ad} \\
&= \frac{163 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} \\
&\quad - \frac{17 \sin(c+dx)}{16ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} \\
&\quad + \frac{95 \sin(c+dx)}{48a^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{299 \sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 9.07 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.94

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} dx =$$

$$\cot^5\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^4\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^4\left(\frac{1}{2}(c+dx)\right) \left(640 \cos^8\left(\frac{1}{2}(c+dx)\right) {}_5F_4\left(2, 2, 2, 2, \frac{7}{2}; 1, 1, 1, \frac{13}{2}; \frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1+2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x]

[Out] -1/41580*(Cot[c/2 + (d*x)/2]^5*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^4*(640 *Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 13/2}, S

```

in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 -
1280*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 13/2}, Sin
[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6
+ 5*Sin[c/2 + (d*x)/2]^2) + 33*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*sqrt[Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-105*ArcTanh[Sqrt[Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Cos[(c + d*x)/2]^4*(-10935 + 72902
*Sin[c/2 + (d*x)/2]^2 - 188110*Sin[c/2 + (d*x)/2]^4 + 234156*Sin[c/2 + (d*x
)/2]^6 - 140732*Sin[c/2 + (d*x)/2]^8 + 33208*Sin[c/2 + (d*x)/2]^10) + Sqrt[
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-1148175 + 10333785*Si
n[c/2 + (d*x)/2]^2 - 38990350*Sin[c/2 + (d*x)/2]^4 + 79946462*Sin[c/2 + (d*
x)/2]^6 - 96281836*Sin[c/2 + (d*x)/2]^8 + 68243596*Sin[c/2 + (d*x)/2]^10 -
26448512*Sin[c/2 + (d*x)/2]^12 + 43444400*Sin[c/2 + (d*x)/2]^14)))/(d*(a*(1
+ Cos[c + d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))

```

Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\left(489\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))(\cos^4(dx+c))+299\sqrt{2}(\cos^3(dx+c))\sin(dx+c)+1467(\cos^3(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}{\dots}$

```
[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96/d*(489*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))
)*cos(d*x+c)^4+299*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+1467*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+503*2^(1/2)*cos(d
*x+c)^2*sin(d*x+c)+1467*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcs
in(cot(d*x+c)-csc(d*x+c))+160*sin(d*x+c)*cos(d*x+c)*2^(1/2)+489*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))-32*2^(1/2)*
sin(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^3*2^(1
/2)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.01

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{\frac{5}{2}}} dx = \frac{489\sqrt{2}(\cos(dx+c)^5 + 3\cos(dx+c)^4 + 3\cos(dx+c)^3 + \cos(dx+c)^2)}{\dots}$$

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```


[Out] $\frac{1}{96} \cdot (489 \sqrt{2}) \cdot (\cos(dx + c)^5 + 3 \cos(dx + c)^4 + 3 \cos(dx + c)^3 + \cos(dx + c)^2) \cdot \sqrt{a} \cdot \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{a \cos(dx + c) + a} \sqrt{a} \sqrt{\cos(dx + c)} \sin(dx + c) / (a \cos(dx + c)^2 + a \cos(dx + c))\right) - 2 \cdot (299 \cos(dx + c)^3 + 503 \cos(dx + c)^2 + 160 \cos(dx + c) - 32) \cdot \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c) / (a^3 d \cos(dx + c)^5 + 3 a^3 d \cos(dx + c)^4 + 3 a^3 d \cos(dx + c)^3 + a^3 d \cos(dx + c)^2)$

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] `integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{1}{(a \cos(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

[In] `integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)`

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{1}{(a \cos(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

[In] `integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")`

[Out] `integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{1}{\cos(c + dx)^{\frac{5}{2}}(a + a \cos(c + dx))^{\frac{5}{2}}} dx$$

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

$$3.251 \quad \int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	3743
Rubi [A] (verified)	3744
Mathematica [A] (verified)	3747
Maple [B] (verified)	3747
Fricas [A] (verification not implemented)	3748
Sympy [F(-1)]	3748
Maxima [F]	3749
Giac [F(-1)]	3749
Mupad [F(-1)]	3749

Optimal result

Integrand size = 25, antiderivative size = 254

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx = -\frac{7 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{7/2}d} + \frac{637 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2}} - \frac{7 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2}} - \frac{259 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2}} + \frac{189 \sqrt{\cos(c+dx)} \sin(c+dx)}{64a^3d \sqrt{a+a \cos(c+dx)}}$$

```
[Out] -7*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d-1/6*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-7/16*cos(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)-259/192*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)+637/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)+189/64*sin(d*x+c)*cos(d*x+c)^(1/2)/a^3/d/(a+a*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2844, 3056, 3062, 3061, 2861, 211, 2853, 222}

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = -\frac{7 \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d} + \frac{637 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{189 \sin(c+dx)\sqrt{\cos(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}} - \frac{259 \sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192a^2d(a\cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}} - \frac{7 \sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{16ad(a\cos(c+dx)+a)^{5/2}}$$

[In] Int[Cos[c + d*x]^(9/2)/(a + a*cos[c + d*x])^(7/2), x]

[Out] (-7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*cos[c + d*x]]]/(a^(7/2)*d) + (637*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*cos[c + d*x]])]/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(7/2)*Sin[c + d*x])/(6*d*(a + a*cos[c + d*x])^(7/2)) - (7*cos[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*cos[c + d*x])^(5/2)) - (259*cos[c + d*x]^(3/2)*Sin[c + d*x])/(192*a^2*d*(a + a*cos[c + d*x])^(3/2)) + (189*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*cos[c + d*x]])

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &

& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3061

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3062

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d

, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7a}{2}-7a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
&= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{105a^2}{4}-\frac{77}{2}a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
&= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{777a^3}{8}-\frac{567}{4}a^3\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
&= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{189\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-\frac{567a^4}{8}+168a^4\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{48a^7} \\
&= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{189\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{7\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{2a^4} + \frac{637\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
&= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{189\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{7\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^4d} \\
&\quad - \frac{637\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{7 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{7/2}d} + \frac{637 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} \\
&\quad - \frac{\cos^{7/2}(c+dx) \sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2}} - \frac{7 \cos^{5/2}(c+dx) \sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2}} \\
&\quad - \frac{259 \cos^{3/2}(c+dx) \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2}} + \frac{189\sqrt{\cos(c+dx)} \sin(c+dx)}{64a^3d\sqrt{a+a \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.01

$$\int \frac{\cos^{9/2}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx = \frac{\sqrt{a(1+\cos(c+dx))} \left(4536 \arcsin\left(\sqrt{1-\cos(c+dx)}\right) \cos^6\left(\frac{1}{2}(c+dx)\right) + 15288 \arcsin\left[\sqrt{\cos(c+dx)}\right] \cos\left[\frac{c+dx}{2}\right]^6 - 7644 \sqrt{2} \arctan\left[\frac{\sqrt{\cos(c+dx)}}{\sqrt{\sin\left[\frac{c+dx}{2}\right]^2}}\right] \cos\left[\frac{c+dx}{2}\right]^6 + 1442 \sqrt{1-\cos(c+dx)} \cos(c+dx)^{3/2} + 1099 \sqrt{1-\cos(c+dx)} \cos(c+dx)^{5/2} + 192 \sqrt{1-\cos(c+dx)} \cos(c+dx)^{7/2} + 567 \sqrt{-((-1+\cos(c+dx)) \cos(c+dx))} \sin(c+dx) \right)}{(192a^4d \sqrt{1-\cos(c+dx)} (1+\cos(c+dx))^4)}$$

[In] Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^(7/2), x]

[Out] (Sqrt[a*(1 + Cos[c + d*x])]*(4536*ArcSin[Sqrt[1 - Cos[c + d*x]])*Cos[(c + d*x)/2]^6 + 15288*ArcSin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^6 - 7644*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^6 + 1442*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 1099*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2) + 192*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(7/2) + 567*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])]*Sin[c + d*x])/(192*a^4*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^4)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(211) = 422.

Time = 12.47 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.76

method	result
default	$\frac{(192\sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (\cos^3(dx+c) \sin(dx+c) + 1099\sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (\cos^2(dx+c) \sin(dx+c) - 1344\sqrt{2} \arctan(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}))$

[In] int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^(7/2), x, method=_RETURNVERBOSE)

[Out] 1/384/d*(192*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*sin(d*x+c)+1099*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-1344*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^3+1442*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-1911*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3-4032*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2+567*sin(d*x+c)*2^(1/2)*(c

$$\cos(dx+c)/(1+\cos(dx+c))^{1/2} - 5733 \arcsin(\cot(dx+c) - \csc(dx+c)) \cos(dx+c)^2 - 4032 \cdot 2^{1/2} \arctan(\tan(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2}) \cos(dx+c) - 5733 \arcsin(\cot(dx+c) - \csc(dx+c)) \cos(dx+c) - 1344 \cdot 2^{1/2} \arctan(\tan(dx+c) \cdot (\cos(dx+c)/(1+\cos(dx+c)))^{1/2}) - 1911 \arcsin(\cot(dx+c) - \csc(dx+c)) \cos(dx+c)^{1/2} \cdot (a \cdot (1+\cos(dx+c)))^{1/2} / (1+\cos(dx+c))^4 / (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \cdot 2^{1/2} / a^4$$

Fricas [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.10

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx =$$

$$1911 \sqrt{2} (\cos(dx+c)^4 + 4 \cos(dx+c)^3 + 6 \cos(dx+c)^2 + 4 \cos(dx+c) + 1) \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a \cos(dx+c) + a}}{\sqrt{a} \sin(dx+c)}\right)$$

[In] integrate(cos(dx+c)^(9/2)/(a+a*cos(dx+c))^(7/2),x, algorithm="fricas")

[Out] -1/384*(1911*sqrt(2)*(cos(dx+c)^4 + 4*cos(dx+c)^3 + 6*cos(dx+c)^2 + 4*cos(dx+c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(dx+c) + a)*sqrt(cos(dx+c))/(sqrt(a)*sin(dx+c))) - 2*(192*cos(dx+c)^3 + 1099*cos(dx+c)^2 + 1442*cos(dx+c) + 567)*sqrt(a*cos(dx+c) + a)*sqrt(cos(dx+c))*sin(dx+c) - 2688*(cos(dx+c)^4 + 4*cos(dx+c)^3 + 6*cos(dx+c)^2 + 4*cos(dx+c) + 1)*sqrt(a)*arctan(sqrt(a*cos(dx+c) + a)*sqrt(cos(dx+c))/(sqrt(a)*sin(dx+c))))/(a^4*d*cos(dx+c)^4 + 4*a^4*d*cos(dx+c)^3 + 6*a^4*d*cos(dx+c)^2 + 4*a^4*d*cos(dx+c) + a^4*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(cos(dx+c)**(9/2)/(a+a*cos(dx+c))**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^{7/2}} dx = \int \frac{\cos(dx + c)^{\frac{9}{2}}}{(a \cos(dx + c) + a)^{\frac{7}{2}}} dx$$

[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(7/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^{7/2}} dx = \int \frac{\cos(c + dx)^{9/2}}{(a + a \cos(c + dx))^{7/2}} dx$$

[In] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^(7/2),x)

[Out] int(cos(c + d*x)^(9/2)/(a + a*cos(c + d*x))^(7/2), x)

$$3.252 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	3750
Rubi [A] (verified)	3750
Mathematica [A] (verified)	3753
Maple [B] (verified)	3754
Fricas [A] (verification not implemented)	3754
Sympy [F(-1)]	3755
Maxima [F]	3755
Giac [F(-1)]	3755
Mupad [F(-1)]	3755

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx = \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{a^{7/2}d} - \frac{177 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2}} - \frac{17 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{48ad(a+a \cos(c+dx))^{5/2}} - \frac{49 \sqrt{\cos(c+dx)} \sin(c+dx)}{64a^2d(a+a \cos(c+dx))^{3/2}}$$

[Out] 2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d-1/6*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)-17/48*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)-177/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)-49/64*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {2844, 3056, 3061, 2861, 211, 2853, 222}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \frac{2 \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d} - \frac{177 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{49 \sin(c+dx)\sqrt{\cos(c+dx)}}{64a^2d(a\cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}} - \frac{17 \sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{48ad(a\cos(c+dx)+a)^{5/2}}$$

[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(7/2), x]

[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])/(a^(7/2)*d) - (177*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - (17*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - (49*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2))

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Ssin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^n*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-6a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{51a^2}{4}-24a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{49\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\frac{147a^3}{8}-48a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{48a^6}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{49\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{a^4} \\
&\quad - \frac{177\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{49\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^4d} \\
&\quad + \frac{177\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d} \\
&= \frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} - \frac{177\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} \\
&\quad - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{49\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.07

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \sqrt{a(1+\cos(c+dx))} \left(1176 \arcsin\left(\sqrt{1-\cos(c+dx)}\right) \cos^6\left(\frac{1}{2}(c+dx)\right) + 4248 \arcsin\left(\sqrt{\cos(c+dx)}\right) \cos^5\left(\frac{1}{2}(c+dx)\right) \right)$$

[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(7/2),x]

[Out] -1/192*(Sqrt[a*(1 + Cos[c + d*x])]*(1176*ArcSin[Sqrt[1 - Cos[c + d*x]]]*Cos[(c + d*x)/2]^6 + 4248*ArcSin[Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]^6 - 2124*Sqrt[2]*ArcTan[Sqrt[Cos[c + d*x]]/Sqrt[Sin[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^6 + 362*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2) + 247*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2) + 147*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])*Sin[c + d*x])/(a^4*d*Sqrt[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^4)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(177) = 354.

Time = 3.92 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.77

method	result
default	$-\frac{\left(-\frac{\csc^2(dx+c)(1-\cos(dx+c))^2-1}{\csc^2(dx+c)(1-\cos(dx+c))^2+1}\right)^{\frac{7}{2}}\left(\csc^2(dx+c)(1-\cos(dx+c))^2+1\right)^4\sqrt{\frac{a}{\csc^2(dx+c)(1-\cos(dx+c))^2+1}}\left(8(\csc^5(dx+c))\sqrt{-(\csc^2(dx+c)(1-\cos(dx+c))^2+1)}\right)}{...}$

[In] `int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/384/d*(-(\csc(d*x+c)^2*(1-\cos(d*x+c))^2-1)/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1))^{\frac{7}{2}}/(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{7}{2}}*(\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^4*(a/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1))^{\frac{1}{2}}*(8*\csc(d*x+c)^5*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{1}{2}}*(1-\cos(d*x+c))^5-50*\csc(d*x+c)^3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{1}{2}}*(1-\cos(d*x+c))^3+384*2^{\frac{1}{2}}*\arctan(2^{\frac{1}{2}}*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{1}{2}}/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2-1)*(\csc(d*x+c)-\cot(d*x+c)))+189*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{1}{2}}*(\csc(d*x+c)-\cot(d*x+c))-531*\arcsin(\cot(d*x+c)-\csc(d*x+c)))*2^{\frac{1}{2}}/a^4$$

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.26

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{7}{2}}} dx = \frac{531\sqrt{2}(\cos(dx+c)^4 + 4\cos(dx+c)^3 + 6\cos(dx+c)^2 + 4\cos(dx+c) + 1)}{...}$$

[In] `integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")`

[Out]
$$1/384*(531*\sqrt{2}*(\cos(d*x+c)^4 + 4*\cos(d*x+c)^3 + 6*\cos(d*x+c)^2 + 4*\cos(d*x+c) + 1)*\sqrt{a}*\arctan(\sqrt{2}*\sqrt{a*\cos(d*x+c) + a}*\sqrt{\cos(d*x+c)})/(\sqrt{a}*\sin(d*x+c))) - 2*\sqrt{a*\cos(d*x+c) + a}*(247*\cos(d*x+c)^2 + 362*\cos(d*x+c) + 147)*\sqrt{\cos(d*x+c)}*\sin(d*x+c) - 768*(\cos(d*x+c)^4 + 4*\cos(d*x+c)^3 + 6*\cos(d*x+c)^2 + 4*\cos(d*x+c) + 1)*\sqrt{a}*\arctan(\sqrt{a*\cos(d*x+c) + a}*\sqrt{\cos(d*x+c)})/(\sqrt{a}*\sin(d*x+c)))/ (a^4*d*\cos(d*x+c)^4 + 4*a^4*d*\cos(d*x+c)^3 + 6*a^4*d*\cos(d*x+c)^2 + 4*a^4*d*\cos(d*x+c) + a^4*d)$$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(7/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(a \cos(dx + c) + a)^{\frac{7}{2}}} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2), x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2), x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \int \frac{\cos(c + dx)^{\frac{7}{2}}}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx$$

[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(7/2), x)

[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(7/2), x)

$$3.253 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	3756
Rubi [A] (verified)	3756
Mathematica [A] (warning: unable to verify)	3759
Maple [A] (verified)	3759
Fricas [A] (verification not implemented)	3760
Sympy [F(-1)]	3760
Maxima [F]	3760
Giac [F(-1)]	3761
Mupad [F(-1)]	3761

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx = \frac{5 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)} \sin(c+dx)}{48ad(a+a \cos(c+dx))^{5/2}} + \frac{67\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2}}$$

[Out] $-1/6*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(7/2)}+5/128*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(7/2)}/d*2^{(1/2)}-13/48*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\cos(d*x+c))^{(5/2)}+67/192*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d/(a+a*\cos(d*x+c))^{(3/2)}$

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2844, 3056, 3057, 12, 2861, 211}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx = \frac{5 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{67 \sin(c+dx) \sqrt{\cos(c+dx)}}{192a^2d(a \cos(c+dx) + a)^{3/2}} - \frac{\sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{6d(a \cos(c+dx) + a)^{7/2}} - \frac{13 \sin(c+dx) \sqrt{\cos(c+dx)}}{48ad(a \cos(c+dx) + a)^{5/2}}$$

[In] Int[Cos[c + d*x]^(5/2)/(a + a*cos[c + d*x])^(7/2), x]

[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*cos[c + d*x]])]/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a + a*cos[c + d*x])^(7/2)) - (13*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(48*a*d*(a + a*cos[c + d*x])^(5/2)) + (67*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*cos[c + d*x])^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2844

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n)/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int

egerQ[2*n] || EqQ[c, 0])

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}(\frac{3a}{2}-5a\cos(c+dx))}{(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\frac{13a^2}{4}-\frac{27}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
 &\quad + \frac{67\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} - \frac{\int -\frac{15a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
 &\quad + \frac{67\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
 &\quad + \frac{67\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} \\
 &\quad - \frac{5\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d} \\
 &= \frac{5\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} \\
 &\quad - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{67\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 1.79 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \frac{\cos^7\left(\frac{1}{2}(c+dx)\right) \sqrt{a(1+\cos(c+dx))} \left(15 \arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right) \sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\right)}{24a^4 d \sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}$$

[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2), x]

```
[Out] (Cos[(c + d*x)/2]^7*Sqrt[a*(1 + Cos[c + d*x])]*(15*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]*(33 - 26*Tan[(c + d*x)/2]^2 + 8*Tan[(c + d*x)/2]^4))/(24*a^4*d*Sqrt[Cos[(c + d*x)/2]^2]*(1 + Cos[c + d*x])^4)
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47

method	result
default	$-\frac{(-67\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c)\sin(dx+c)+15\arcsin(\cot(dx+c)-\csc(dx+c))(\cos^3(dx+c))-50\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{1+\cos(dx+c)}))}{24a^4d\sqrt{\cos^2(dx+c)}}$

[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2), x, method=_RETURNVERBOSE)

```
[Out] -1/384/d*(-67*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+15*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3-50*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+45*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-15*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+45*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+15*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \frac{15\sqrt{2}(\cos(dx + c)^4 + 4\cos(dx + c)^3 + 6\cos(dx + c)^2 + 4\cos(dx + c) + 1)\sqrt{a}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a\cos(dx + c) + a}\sqrt{\frac{\cos(dx + c)\sin(dx + c)}{a\cos(dx + c)^2 + a\cos(dx + c)}}\right) + 2\sqrt{a\cos(dx + c) + a}(67\cos(dx + c)^2 + 50\cos(dx + c) + 15)\sqrt{\cos(dx + c)\sin(dx + c)}}{384(a^4d\cos(dx + c)^4 + 4a^4d\cos(dx + c)^3 + 6a^4d\cos(dx + c)^2 + 4a^4d\cos(dx + c) + a^4d)}$$

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(15*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(67*cos(d*x + c)^2 + 50*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^{\frac{7}{2}}} dx$$

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \int \frac{\cos(c + dx)^{\frac{5}{2}}}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx$$

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(7/2), x)
```

$$3.254 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	3762
Rubi [A] (verified)	3762
Mathematica [A] (verified)	3764
Maple [A] (verified)	3765
Fricas [A] (verification not implemented)	3765
Sympy [F(-1)]	3766
Maxima [F]	3766
Giac [F(-1)]	3766
Mupad [F(-1)]	3766

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx = \frac{7 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2}} + \frac{17\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2}}$$

[Out] $7/128*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)})/a^{(7/2)}/d*2^{(1/2)}-1/6*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(7/2)}+3/16*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\cos(d*x+c))^{(5/2)}+17/192*2*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d/(a+a*\cos(d*x+c))^{(3/2)}$

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2844, 3057, 12, 2861, 211}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx = \frac{7 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{17 \sin(c+dx) \sqrt{\cos(c+dx)}}{192a^2d(a \cos(c+dx)+a)^{3/2}} + \frac{3 \sin(c+dx) \sqrt{\cos(c+dx)}}{16ad(a \cos(c+dx)+a)^{5/2}} - \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{6d(a \cos(c+dx)+a)^{7/2}}$$

[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2), x]

```
[Out] (7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6
*d*(a + a*Cos[c + d*x])^(7/2)) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a
*d*(a + a*Cos[c + d*x])^(5/2)) + (17*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a
^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 211

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :=> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2844

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] :=> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e
+ f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegerQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e
_) + (f_)*(x_)])], x_Symbol] :=> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :=> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\frac{a}{2}-4a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{-\frac{a^2}{4}-\frac{9}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad + \frac{17\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} - \frac{\int -\frac{21a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad + \frac{17\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{7\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad + \frac{17\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} \\
&\quad - \frac{7\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d} \\
&= \frac{7\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} \\
&\quad + \frac{3\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{17\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right) \left(672\text{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\cos^6\left(\frac{1}{2}(c+dx)\right) + 3072\sqrt{2}a^3\right)}{3072\sqrt{2}a^3}$$

[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2), x]

[Out] (Sec[(c + d*x)/2]^4*(672*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^6 + (140 + 135*Cos[c + d*x] + 140*Cos[2*(c + d*x)] + 17*Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a^3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47

method	result
default	$\frac{(17\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c)\sin(dx+c)+70\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-21\arcsin(\cot(dx+c)-\csc(dx+c)))(\cos^3(dx+c))}{(1+\cos(dx+c))^4}$

[In] `int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)`

```
[Out] 1/384/d*(17*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+70*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-21*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3+21*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-63*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-63*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-21*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \frac{21\sqrt{2}(\cos(dx+c)^4 + 4\cos(dx+c)^3 + 6\cos(dx+c)^2 + 4\cos(dx+c) + 1)}{384(a^4d\cos(dx+c) + \dots)}$$

[In] `integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")`

```
[Out] 1/384*(21*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(17*cos(d*x + c)^2 + 70*cos(d*x + c) + 21)*sqrt(cos(d*x + c))*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^{\frac{7}{2}}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx = \int \frac{\cos(c + dx)^{\frac{3}{2}}}{(a + a \cos(c + dx))^{\frac{7}{2}}} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(7/2),x)

[Out] int(cos(c + d*x)^(3/2)/(a + a*cos(c + d*x))^(7/2), x)

$$3.255 \quad \int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	3767
Rubi [A] (verified)	3767
Mathematica [A] (verified)	3769
Maple [A] (verified)	3770
Ericas [A] (verification not implemented)	3770
Sympy [F(-1)]	3771
Maxima [F]	3771
Giac [F(-1)]	3771
Mupad [F(-1)]	3771

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^{7/2}} dx = \frac{13 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2}} - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2}}$$

[Out] 13/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)+1/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(7/2)+1/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-5/192*a^2*d*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2843, 3057, 12, 2861, 211}

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a \cos(c+dx))^{7/2}} dx = \frac{13 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{5 \sin(c+dx) \sqrt{\cos(c+dx)}}{192a^2d(a \cos(c+dx)+a)^{3/2}} + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{16ad(a \cos(c+dx)+a)^{5/2}} + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{6d(a \cos(c+dx)+a)^{7/2}}$$

[In] Int[Sqrt[Cos[c + d*x]]/(a + a*cos[c + d*x])^(7/2),x]

```
[Out] (13*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]/(64*Sqrt[2]*a^(7/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2843

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{a}{2}+2a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
&= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{11a^2}{4}+\frac{3}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
&= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{39a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
&= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{13 \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
&= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} \\
&\quad - \frac{13 \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d} \\
&= \frac{13 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} \\
&\quad + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}} dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right) \sqrt{\cos(c+dx)} \sqrt{a(1+\cos(c+dx))} (73+4\cos(c+dx)-5\cos^2(c+dx))}{(a+a\cos(c+dx))^{7/2}}$$

[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*(73 + 4*Cos[c + d*x] - 5*Cos[2*(c + d*x)] - 156*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*Cot[(c + d*x)/2]^2*Sqrt[2 - 2*Sec[c + d*x]])*Sin[(c + d*x)/2])/(192*a^4*d*(1 + Cos[c + d*x])^4)

Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47

method	result
default	$-\frac{\left(5\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c)\sin(dx+c)-2\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+39\arcsin(\cot(dx+c)-\csc(dx+c))\right)(\cos^3(dx+c))}{\dots}$

[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)

```
[Out] -1/384/d*(5*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3-39*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+117*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+117*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+39*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(1/2)/(1+cos(d*x+c))^4/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}} dx = \frac{39\sqrt{2}(\cos(dx+c)^4 + 4\cos(dx+c)^3 + 6\cos(dx+c)^2 + 4\cos(dx+c) + 1)}{384(a^4d\cos(dx+c) + \dots)}$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")

```
[Out] 1/384*(39*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(5*cos(d*x + c)^2 - 2*cos(d*x + c) - 39)*sqrt(cos(d*x + c))*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{(a + a \cos(c + dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{(a + a \cos(c + dx))^{7/2}} dx = \int \frac{\sqrt{\cos(dx + c)}}{(a \cos(dx + c) + a)^{7/2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2), x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{(a + a \cos(c + dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2), x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{(a + a \cos(c + dx))^{7/2}} dx = \int \frac{\sqrt{\cos(c + dx)}}{(a + a \cos(c + dx))^{7/2}} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(7/2), x)

[Out] int(cos(c + d*x)^(1/2)/(a + a*cos(c + d*x))^(7/2), x)

$$3.256 \quad \int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	3772
Rubi [A] (verified)	3772
Mathematica [A] (verified)	3775
Maple [A] (verified)	3775
Fricas [A] (verification not implemented)	3776
Sympy [F(-1)]	3776
Maxima [F]	3776
Giac [F(-1)]	3777
Mupad [F(-1)]	3777

Optimal result

Integrand size = 25, antiderivative size = 177

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{7/2}} dx = \frac{63 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d}$$

$$- \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2}} - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2}}$$

$$- \frac{103\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2}}$$

[Out] 63/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(7/2)-5/16*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-103/192*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2845, 3057, 12, 2861, 211}

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{7/2}} dx = \frac{63 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

$$- \frac{103 \sin(c+dx) \sqrt{\cos(c+dx)}}{192a^2d(a \cos(c+dx)+a)^{3/2}} - \frac{5 \sin(c+dx) \sqrt{\cos(c+dx)}}{16ad(a \cos(c+dx)+a)^{5/2}}$$

$$- \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{6d(a \cos(c+dx)+a)^{7/2}}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]

[Out] (63*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)) - (103*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sina[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]

&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{11a}{2} - 2a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{73a^2}{4} - \frac{15}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{103\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{189a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{103\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{63 \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
&= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{103\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} \\
&\quad - \frac{63 \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d} \\
&= \frac{63 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} \\
&\quad - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{103\sqrt{\cos(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right) \left(-6048 \operatorname{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right) \cos^6\left(\frac{1}{2}(c+dx)\right) + (532 + 1089 \cos(c+dx))\sqrt{a}\right)}{3072\sqrt{2}a^3d\sqrt{-1+\cos(c+dx)}\sqrt{a}}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]

```
[Out] -1/3072*(Sec[(c + d*x)/2]^4*(-6048*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^6 + (532 + 1089*Cos[c + d*x] + 532*Cos[2*(c + d*x)]) + 103*Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]]*Tan[(c + d*x)/2])/(Sqrt[2]*a^3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] (verified)

Time = 5.66 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{a}{(\csc^2(dx+c))(1-\cos(dx+c))^2+1}} \left(8(\csc^5(dx+c))\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} (1-\cos(dx+c))\right)}{(1-\cos(dx+c))^2+1}$

[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)

```
[Out] -1/384/d/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(8*csc(d*x+c)^5*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^5+46*csc(d*x+c)^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^3+141*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x+c)-cot(d*x+c))+189*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} dx = \frac{189\sqrt{2}(\cos(dx+c)^4 + 4\cos(dx+c)^3 + 6\cos(dx+c)^2 + 4\cos(dx+c) + 1)\sqrt{a}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)\right) - 2\sqrt{a\cos(dx+c)+a}(103\cos(dx+c)^2 + 266\cos(dx+c) + 195)\sqrt{\cos(dx+c)}\sin(dx+c)}{(a^4d\cos(dx+c)^4 + 4a^4d\cos(dx+c)^3 + 6a^4d\cos(dx+c)^2 + 4a^4d\cos(dx+c) + a^4d)}$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(189*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(103*cos(d*x + c)^2 + 266*cos(d*x + c) + 195)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} dx = \int \frac{1}{(a\cos(dx+c)+a)^{7/2}\sqrt{\cos(dx+c)}} dx$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(cos(d*x + c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} dx$$

```
[In] int(1/(cos(c+d*x)^(1/2)*(a+a*cos(c+d*x))^(7/2)),x)
```

```
[Out] int(1/(cos(c+d*x)^(1/2)*(a+a*cos(c+d*x))^(7/2)), x)
```

$$3.257 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	3778
Rubi [A] (verified)	3778
Mathematica [C] (warning: unable to verify)	3782
Maple [A] (verified)	3782
Fricas [A] (verification not implemented)	3783
Sympy [F(-1)]	3783
Maxima [F]	3783
Giac [F]	3784
Mupad [F(-1)]	3784

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} dx = -\frac{363 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{7/2}} - \frac{19 \sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{5/2}} - \frac{199 \sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a+a \cos(c+dx))^{3/2}} + \frac{691 \sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}$$

[Out] -363/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)-1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/cos(d*x+c)^(1/2)-19/48*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2)-199/192*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+691/192*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

= {2845, 3057, 3063, 12, 2861, 211}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx =$$

$$\frac{363 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{691 \sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

$$- \frac{199 \sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

$$- \frac{19 \sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{5/2}} - \frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{7/2}}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)), x]

[Out] (-363*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*Sqrt[Cos[c + d*x]])*(a + a*Cos[c + d*x])^(7/2) - (19*Sin[c + d*x])/(48*a*d*Sqrt[Cos[c + d*x]])*(a + a*Cos[c + d*x])^(5/2) - (199*Sin[c + d*x])/(192*a^2*d*Sqrt[Cos[c + d*x]])*(a + a*Cos[c + d*x])^(3/2) + (691*Sin[c + d*x])/(192*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c

- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sin(c + dx)}{6d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{7/2}} + \frac{\int \frac{\frac{13a}{2} - 3a\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^{5/2}} dx}{6a^2} \\
 &= -\frac{\sin(c + dx)}{6d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{7/2}} \\
 &\quad - \frac{19\sin(c + dx)}{48ad\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{5/2}} + \frac{\int \frac{\frac{123a^2}{4} - 19a^2\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + a\cos(c + dx))^{3/2}} dx}{24a^4} \\
 &= -\frac{\sin(c + dx)}{6d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{7/2}} - \frac{19\sin(c + dx)}{48ad\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{5/2}} \\
 &\quad - \frac{199\sin(c + dx)}{192a^2d\sqrt{\cos(c + dx)}(a + a\cos(c + dx))^{3/2}} + \frac{\int \frac{\frac{691a^3}{8} - \frac{199}{4}a^3\cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)\sqrt{a + a\cos(c + dx)}} dx}{48a^6}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{199\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{691\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{\int -\frac{1089a^4}{16\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{24a^7} \\
&= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{199\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{691\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{363\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
&= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{199\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{691\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{363\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d} \\
&= -\frac{363\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} \\
&\quad - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{199\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{691\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.76 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.58

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx = \frac{2 \cos^7\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^6\left(\frac{1}{2}(c+dx)\right) \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\left(\frac{16 \cos^8\left(\frac{1}{2}(c+dx)\right) {}_5F_4\left(2, 2, 2, 2, 5/2, 1, 1, 1, 13/2, \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{3}\right)}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x]

[Out] (2*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*((16*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(3465*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) - (Csc[c/2 + (d*x)/2]^10*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(105*ArcTan h[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Cos[(c + d*x)/2]^6*(2187 - 12908*Sin[c/2 + (d*x)/2]^2 + 27986*Sin[c/2 + (d*x)/2]^4 - 26380*Sin[c/2 + (d*x)/2]^6 + 8752*Sin[c/2 + (d*x)/2]^8) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-229635 + 2120790*Sin[c/2 + (d*x)/2]^2 - 8267707*Sin[c/2 + (d*x)/2]^4 + 17646926*Sin[c/2 + (d*x)/2]^6 - 22251094*Sin[c/2 + (d*x)/2]^8 + 16548816*Sin[c/2 + (d*x)/2]^10 - 6712984*Sin[c/2 + (d*x)/2]^12 + 1144608*Sin[c/2 + (d*x)/2]^14))/1680))/(d*(a*(1 + Cos[c + d*x])^(7/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2)))

Maple [A] (verified)

Time = 5.44 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.49

method	result
default	$\frac{(1089\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))(\cos^4(dx+c)+691\sqrt{2}(\cos^3(dx+c)\sin(dx+c)+4356(\cos^3(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}))}{1680}}{d(a(1+\cos(dx+c))^{7/2}(1-2\sin^2(\frac{dx+c}{2}))^{3/2})}$

[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/384/d*(1089*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^4+691*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+4356*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+1874*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+6534*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))+1599*sin(d*x+c)*cos(d*x+c)*2^(1/2)+4356*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+384*2^(1/2)*sin(d*x+c)+1089*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-cs

$c(dx+c)) * (a*(1+\cos(dx+c)))^{1/2} / (1+\cos(dx+c))^4 / \cos(dx+c)^{1/2} * 2^{1/2} / a^4$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.10

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx =$$

$$\frac{1089\sqrt{2}(\cos(dx+c)^5 + 4\cos(dx+c)^4 + 6\cos(dx+c)^3 + 4\cos(dx+c)^2 + \cos(dx+c))\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}\cos(dx+c)}{a\cos(dx+c)+a}\right) - 384(a^4d\cos(dx+c)^5 + 4a^4d\cos(dx+c)^4 + 6a^4d\cos(dx+c)^3 + 4a^4d\cos(dx+c)^2 + a^4d\cos(dx+c))}{384(a^4d\cos(dx+c)^5 + 4a^4d\cos(dx+c)^4 + 6a^4d\cos(dx+c)^3 + 4a^4d\cos(dx+c)^2 + a^4d\cos(dx+c))}$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")

[Out] -1/384*(1089*sqrt(2)*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*(691*cos(d*x + c)^3 + 1874*cos(d*x + c)^2 + 1599*cos(d*x + c) + 384)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx = \int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}} \cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx = \int \frac{1}{\cos(c+dx)^{3/2} (a+a\cos(c+dx))^{7/2}} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)),x)

[Out] int(1/(cos(c + d*x)^(3/2)*(a + a*cos(c + d*x))^(7/2)), x)

$$3.258 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	3785
Rubi [A] (verified)	3786
Mathematica [C] (warning: unable to verify)	3789
Maple [A] (verified)	3790
Fricas [A] (verification not implemented)	3790
Sympy [F(-1)]	3791
Maxima [F(-1)]	3791
Giac [F]	3791
Mupad [F(-1)]	3791

Optimal result

Integrand size = 25, antiderivative size = 257

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} dx = \frac{1015 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sin(c+dx)}{23 \sin(c+dx)} - \frac{6d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}}{48ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} - \frac{109 \sin(c+dx)}{64a^2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} + \frac{193 \sin(c+dx)}{64a^3d \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} - \frac{629 \sin(c+dx)}{64a^3d \sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}$$

[Out] -1/6*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2)-23/48*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2)-109/64*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2)+1015/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))/a^(7/2)/d*2^(1/2)+193/64*sin(d*x+c)/a^3/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)-629/64*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2845, 3057, 3063, 12, 2861, 211}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx = \frac{1015 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{193\sin(c+dx)}{64a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}} - \frac{629\sin(c+dx)}{64a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}} - \frac{109\sin(c+dx)}{64a^2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{5/2}} - \frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)), x]

[Out] (1015*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)) - (23*Sin[c + d*x])/(48*a*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)) - (109*Sin[c + d*x])/(64*a^2*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + (193*Sin[c + d*x])/(64*a^3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (629*Sin[c + d*x])/(64*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sin(c+dx)}{6d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{15a}{2} - 4a \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} dx}{6a^2} \\ &= -\frac{\sin(c+dx)}{6d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} \\ &\quad - \frac{23 \sin(c+dx)}{48ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{189a^2}{4} - \frac{69}{2}a^2 \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} dx}{24a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{6d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} - \frac{23 \sin(c+dx)}{48ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} \\
&\quad - \frac{109 \sin(c+dx)}{64a^2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{1737a^3}{8} - \frac{327}{2}a^3 \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} dx}{48a^6} \\
&= -\frac{\sin(c+dx)}{6d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} - \frac{23 \sin(c+dx)}{48ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} \\
&\quad - \frac{109 \sin(c+dx)}{64a^2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} \\
&\quad + \frac{193 \sin(c+dx)}{64a^3d \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} + \frac{\int \frac{-\frac{5661a^4}{16} + \frac{1737}{8}a^4 \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} dx}{72a^7} \\
&= -\frac{\sin(c+dx)}{6d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} - \frac{23 \sin(c+dx)}{48ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} \\
&\quad - \frac{109 \sin(c+dx)}{64a^2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} + \frac{193 \sin(c+dx)}{64a^3d \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{629 \sin(c+dx)}{64a^3d \sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} + \frac{\int \frac{9135a^5}{32\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} dx}{36a^8} \\
&= -\frac{\sin(c+dx)}{6d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} - \frac{23 \sin(c+dx)}{48ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} \\
&\quad - \frac{109 \sin(c+dx)}{64a^2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} + \frac{193 \sin(c+dx)}{64a^3d \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{629 \sin(c+dx)}{64a^3d \sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} + \frac{1015 \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} dx}{128a^3} \\
&= -\frac{\sin(c+dx)}{6d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} - \frac{23 \sin(c+dx)}{48ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} \\
&\quad - \frac{109 \sin(c+dx)}{64a^2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} \\
&\quad + \frac{193 \sin(c+dx)}{64a^3d \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{629 \sin(c+dx)}{64a^3d \sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}} \\
&\quad - \frac{1015 \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{64a^2d}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1015 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sin(c+dx)}{6d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} \\
&\quad - \frac{23 \sin(c+dx)}{48ad \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{5/2}} - \frac{109 \sin(c+dx)}{64a^2d \cos^{\frac{3}{2}}(c+dx)(a+a \cos(c+dx))^{3/2}} \\
&\quad + \frac{193 \sin(c+dx)}{64a^3d \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a \cos(c+dx)}} - \frac{629 \sin(c+dx)}{64a^3d \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.28 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.70

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a \cos(c+dx))^{7/2}} dx = \frac{\cot^7\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^4\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^6\left(\frac{1}{2}(c+dx)\right) \left(-7680 \cos^{10}\left(\frac{1}{2}(c+dx)\right)\right)}{\dots}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x]

[Out] (Cot[c/2 + (d*x)/2]^7*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^6*(-7680*Cos[(c + d*x)/2]^10*HypergeometricPFQ[{2, 2, 2, 2, 2, 7/2}, {1, 1, 1, 1, 15/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 + 19200*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 15/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14*(-7 + 6*Sin[c/2 + (d*x)/2]^2) + 143*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(315*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Cos[(c + d*x)/2]^6*(351384 - 2928877*Sin[c/2 + (d*x)/2]^2 + 9953934*Sin[c/2 + (d*x)/2]^4 - 17629526*Sin[c/2 + (d*x)/2]^6 + 17139064*Sin[c/2 + (d*x)/2]^8 - 8670660*Sin[c/2 + (d*x)/2]^10 + 1793816*Sin[c/2 + (d*x)/2]^12) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-110685960 + 1291549455*Sin[c/2 + (d*x)/2]^2 - 6601900452*Sin[c/2 + (d*x)/2]^4 + 19406027859*Sin[c/2 + (d*x)/2]^6 - 36160322412*Sin[c/2 + (d*x)/2]^8 + 44313222590*Sin[c/2 + (d*x)/2]^10 - 35736693140*Sin[c/2 + (d*x)/2]^12 + 18305254212*Sin[c/2 + (d*x)/2]^14 - 5410719584*Sin[c/2 + (d*x)/2]^16 + 704274992*Sin[c/2 + (d*x)/2]^18)))/(3243240*d*(a*(1 + Cos[c + d*x]))^(7/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))

Maple [A] (verified)

Time = 5.86 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.37

method	result
default	$-\frac{\left(3045\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))(\cos^5(dx+c))+1887\sqrt{2}(\cos^4(dx+c))\sin(dx+c)+12180\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)^5+1887*2^{(1/2)}*\cos(dx+c)^4*\sin(dx+c)+12180*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\arcsin(\cot(dx+c)-\csc(dx+c))*\cos(dx+c)^4+5082*2^{(1/2)}*\cos(dx+c)^3*\sin(dx+c)+18270*\cos(dx+c)^3*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\arcsin(\cot(dx+c)-\csc(dx+c))+4251*2^{(1/2)}*\cos(dx+c)^2*\sin(dx+c)+12180*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\cos(dx+c)^2*\arcsin(\cot(dx+c)-\csc(dx+c))+896*\sin(dx+c)*\cos(dx+c)*2^{(1/2)}+3045*\cos(dx+c)*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*\arcsin(\cot(dx+c)-\csc(dx+c))-128*2^{(1/2)}*\sin(dx+c))*(a*(1+\cos(dx+c)))^{(1/2)}/(1+\cos(dx+c))^4/\cos(dx+c)^{(3/2)}*2^{(1/2)}/a^4$

[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)

[Out]
$$-1/384/d*(3045*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\arcsin(\cot(d*x+c)-\csc(d*x+c))*\cos(d*x+c)^5+1887*2^{(1/2)}*\cos(d*x+c)^4*\sin(d*x+c)+12180*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\arcsin(\cot(d*x+c)-\csc(d*x+c))*\cos(d*x+c)^4+5082*2^{(1/2)}*\cos(d*x+c)^3*\sin(d*x+c)+18270*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\arcsin(\cot(d*x+c)-\csc(d*x+c))+4251*2^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)+12180*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^2*\arcsin(\cot(d*x+c)-\csc(d*x+c))+896*\sin(d*x+c)*\cos(d*x+c)*2^{(1/2)}+3045*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\arcsin(\cot(d*x+c)-\csc(d*x+c))-128*2^{(1/2)}*\sin(d*x+c))*(a*(1+\cos(d*x+c)))^{(1/2)}/(1+\cos(d*x+c))^4/\cos(d*x+c)^{(3/2)}*2^{(1/2)}/a^4$$

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.98

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx = \frac{3045\sqrt{2}(\cos(dx+c)^6+4\cos(dx+c)^5+6\cos(dx+c)^4+4\cos(dx+c)^3+\cos(dx+c)^2)*\sqrt{a}\arctan(1/2\sqrt{2})\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)/(a\cos(dx+c)^2+a\cos(dx+c)) - 2*(1887*\cos(dx+c)^4+5082*\cos(dx+c)^3+4251*\cos(dx+c)^2+896*\cos(dx+c)-128)*\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}*\sin(dx+c)/(a^4*d*\cos(dx+c)^6+4*a^4*d*\cos(dx+c)^5+6*a^4*d*\cos(dx+c)^4+4*a^4*d*\cos(dx+c)^3+a^4*d*\cos(dx+c)^2)}{3045\sqrt{2}(\cos(dx+c)^6+4\cos(dx+c)^5+6\cos(dx+c)^4+4\cos(dx+c)^3+\cos(dx+c)^2)*\sqrt{a}\arctan(1/2\sqrt{2})\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)/(a\cos(dx+c)^2+a\cos(dx+c)) - 2*(1887*\cos(dx+c)^4+5082*\cos(dx+c)^3+4251*\cos(dx+c)^2+896*\cos(dx+c)-128)*\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}*\sin(dx+c)/(a^4*d*\cos(dx+c)^6+4*a^4*d*\cos(dx+c)^5+6*a^4*d*\cos(dx+c)^4+4*a^4*d*\cos(dx+c)^3+a^4*d*\cos(dx+c)^2)}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")

[Out]
$$1/384*(3045*\sqrt{2}*(\cos(d*x+c)^6+4*\cos(d*x+c)^5+6*\cos(d*x+c)^4+4*\cos(d*x+c)^3+\cos(d*x+c)^2)*\sqrt{a}\arctan(1/2*\sqrt{2})*\sqrt{a*\cos(d*x+c)+a}\sqrt{a}\sqrt{\cos(d*x+c)}*\sin(d*x+c)/(a*\cos(d*x+c)^2+a*\cos(d*x+c)) - 2*(1887*\cos(d*x+c)^4+5082*\cos(d*x+c)^3+4251*\cos(d*x+c)^2+896*\cos(d*x+c)-128)*\sqrt{a*\cos(d*x+c)+a}\sqrt{\cos(d*x+c)}*\sin(d*x+c)/(a^4*d*\cos(d*x+c)^6+4*a^4*d*\cos(d*x+c)^5+6*a^4*d*\cos(d*x+c)^4+4*a^4*d*\cos(d*x+c)^3+a^4*d*\cos(d*x+c)^2)$$

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)

[Out] Timed out

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] Timed out

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{7/2}} dx = \int \frac{1}{(a \cos(dx + c) + a)^{\frac{7}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{7/2}} dx = \int \frac{1}{\cos(c + dx)^{5/2} (a + a \cos(c + dx))^{7/2}} dx$$

[In] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)),x)

[Out] int(1/(cos(c + d*x)^(5/2)*(a + a*cos(c + d*x))^(7/2)), x)

$$3.259 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^{9/2}} dx$$

Optimal result	3792
Rubi [A] (verified)	3792
Mathematica [A] (verified)	3795
Maple [A] (verified)	3795
Fricas [A] (verification not implemented)	3796
Sympy [F(-1)]	3796
Maxima [F]	3797
Giac [F(-1)]	3797
Mupad [F(-1)]	3797

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^{9/2}} dx = \frac{35 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right)}{1024 \sqrt{2} a^{9/2} d} - \frac{\cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{8d(a+a \cos(c+dx))^{9/2}} - \frac{19 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{96ad(a+a \cos(c+dx))^{7/2}} - \frac{187 \sqrt{\cos(c+dx)} \sin(c+dx)}{768a^2 d(a+a \cos(c+dx))^{5/2}} + \frac{853 \sqrt{\cos(c+dx)} \sin(c+dx)}{3072a^3 d(a+a \cos(c+dx))^{3/2}}$$

[Out] $-1/8*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(9/2)}-19/96*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(7/2)}+35/2048*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(9/2)}/d*2^{(1/2)}-187/768*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d/(a+a*\cos(d*x+c))^{(5/2)}+853/3072*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^3/d/(a+a*\cos(d*x+c))^{(3/2)}$

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2844, 3056, 3057, 12, 2861, 211}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a \cos(c+dx))^{9/2}} dx = \frac{35 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{1024 \sqrt{2} a^{9/2} d} + \frac{853 \sin(c+dx) \sqrt{\cos(c+dx)}}{3072a^3 d(a \cos(c+dx)+a)^{3/2}} - \frac{187 \sin(c+dx) \sqrt{\cos(c+dx)}}{768a^2 d(a \cos(c+dx)+a)^{5/2}} - \frac{\sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{8d(a \cos(c+dx)+a)^{9/2}} - \frac{19 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{96ad(a \cos(c+dx)+a)^{7/2}}$$

[In] Int[Cos[c + d*x]^(7/2)/(a + a*cos[c + d*x])^(9/2),x]

[Out] (35*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*cos[c + d*x]])])/(1024*Sqrt[2]*a^(9/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(8*d*(a + a*cos[c + d*x])^(9/2)) - (19*cos[c + d*x]^(3/2)*Sin[c + d*x])/(96*a*d*(a + a*cos[c + d*x])^(7/2)) - (187*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(768*a^2*d*(a + a*cos[c + d*x])^(5/2)) + (853*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3072*a^3*d*(a + a*cos[c + d*x])^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n)/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&

NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-7a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{7/2}} dx}{8a^2} \\
 &= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{57a^2}{4}-\frac{65}{2}a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}} dx}{48a^4} \\
 &= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} \\
 &\quad - \frac{187\sqrt{\cos(c+dx)}\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\frac{187a^3}{8}-\frac{333}{4}a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{192a^6} \\
 &= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} \\
 &\quad - \frac{187\sqrt{\cos(c+dx)}\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}} + \frac{853\sqrt{\cos(c+dx)}\sin(c+dx)}{3072a^3d(a+a\cos(c+dx))^{3/2}} \\
 &\quad - \frac{\int -\frac{105a^4}{16\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{384a^8} \\
 &= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} \\
 &\quad - \frac{187\sqrt{\cos(c+dx)}\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}} + \frac{853\sqrt{\cos(c+dx)}\sin(c+dx)}{3072a^3d(a+a\cos(c+dx))^{3/2}} \\
 &\quad + \frac{35\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2048a^4}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{187\sqrt{\cos(c+dx)}\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}} \\
&\quad + \frac{853\sqrt{\cos(c+dx)}\sin(c+dx)}{3072a^3d(a+a\cos(c+dx))^{3/2}} - \frac{35\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024a^3d} \\
&= \frac{35\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024\sqrt{2}a^{9/2}d} \\
&\quad - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} \\
&\quad - \frac{187\sqrt{\cos(c+dx)}\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}} + \frac{853\sqrt{\cos(c+dx)}\sin(c+dx)}{3072a^3d(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.54 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}} dx = \frac{\cos^9\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\cos(c+dx))}\left(105\arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\right)}{(a+a\cos(c+dx))^{9/2}}$$

[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(9/2), x]

[Out] (Cos[(c + d*x)/2]^9*Sqrt[a*(1 + Cos[c + d*x])]*(105*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[(c + d*x)/2]^2 + Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]*(279 - 326*Tan[(c + d*x)/2]^2 + 200*Tan[(c + d*x)/2]^4 - 48*Tan[(c + d*x)/2]^6)))/(192*a^5*d*Sqrt[Cos[(c + d*x)/2]^2]*(1 + Cos[c + d*x])^5)

Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.61

method	result
default	$ -\frac{\left(-\frac{\csc^2(dx+c)(1-\cos(dx+c))^2-1}{\csc^2(dx+c)(1-\cos(dx+c))^2+1}\right)^{\frac{7}{2}}\left((\csc^2(dx+c)(1-\cos(dx+c))^2+1)\right)^4\sqrt{\frac{a}{\csc^2(dx+c)(1-\cos(dx+c))^2+1}}\left(48(\csc^7(dx+c))\sqrt{-\right)} $

[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(9/2), x, method=_RETURNVERBOSE)

[Out] -1/6144/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(7/2)/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(7/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^4*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(48*csc(d*x+c)^7*

```
(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^7-200*csc(d*x+c)^5*
(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^5+326*csc(d*x+c)^3*
(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^3-279*(-csc(d*x+c)^
2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x+c)-cot(d*x+c))+105*arcsin(cot(d*x+c)-c
sc(d*x+c)))*2^(1/2)/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.14

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}} dx = \frac{105\sqrt{2}(\cos(dx+c)^5 + 5\cos(dx+c)^4 + 10\cos(dx+c)^3 + 10\cos(dx+c)^2 + 5\cos(dx+c) + 1)\sqrt{a}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a\cos(dx+c)}\sin(dx+c)\right) + 2(853\cos(dx+c)^3 + 819\cos(dx+c)^2 + 455\cos(dx+c) + 105)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{(a^5d\cos(dx+c)^5 + 5a^5d\cos(dx+c)^4 + 10a^5d\cos(dx+c)^3 + 10a^5d\cos(dx+c)^2 + 5a^5d\cos(dx+c) + a^5d)}$$

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/6144*(105*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a
*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^
2 + a*cos(d*x + c))) + 2*(853*cos(d*x + c)^3 + 819*cos(d*x + c)^2 + 455*cos
(d*x + c) + 105)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/
(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 +
10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```


Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{9/2}} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(a \cos(dx + c) + a)^{\frac{9}{2}}} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(9/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{9/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{9/2}} dx = \int \frac{\cos(c + dx)^{7/2}}{(a + a \cos(c + dx))^{9/2}} dx$$

[In] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(9/2),x)

[Out] int(cos(c + d*x)^(7/2)/(a + a*cos(c + d*x))^(9/2), x)

$$3.260 \quad \int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^{9/2}} dx$$

Optimal result	3798
Rubi [A] (verified)	3798
Mathematica [A] (verified)	3801
Maple [A] (verified)	3801
Fricas [A] (verification not implemented)	3802
Sympy [F(-1)]	3802
Maxima [F]	3802
Giac [F(-1)]	3803
Mupad [F(-1)]	3803

Optimal result

Integrand size = 25, antiderivative size = 217

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^{9/2}} dx = \frac{45 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a \cos(c+dx)}}\right)}{1024\sqrt{2}a^{9/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{8d(a+a \cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)} \sin(c+dx)}{32ad(a+a \cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)} \sin(c+dx)}{256a^2d(a+a \cos(c+dx))^{5/2}} + \frac{73\sqrt{\cos(c+dx)} \sin(c+dx)}{1024a^3d(a+a \cos(c+dx))^{3/2}}$$

[Out] $-1/8*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(9/2)}+45/2048*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})/a^{(9/2)}/d*2^{(1/2)}-5/32*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a/d/(a+a*\cos(d*x+c))^{(7/2)}+33/256*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^2/d/(a+a*\cos(d*x+c))^{(5/2)}+73/1024*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/a^3/d/(a+a*\cos(d*x+c))^{(3/2)}$

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2844, 3056, 3057, 12, 2861, 211}

$$\int \frac{\cos^5(c+dx)}{(a+a \cos(c+dx))^{9/2}} dx = \frac{45 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{1024\sqrt{2}a^{9/2}d} + \frac{73 \sin(c+dx) \sqrt{\cos(c+dx)}}{1024a^3d(a \cos(c+dx)+a)^{3/2}} + \frac{33 \sin(c+dx) \sqrt{\cos(c+dx)}}{256a^2d(a \cos(c+dx)+a)^{5/2}} - \frac{\sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{8d(a \cos(c+dx)+a)^{9/2}} - \frac{5 \sin(c+dx) \sqrt{\cos(c+dx)}}{32ad(a \cos(c+dx)+a)^{7/2}}$$

[In] Int[Cos[c + d*x]^(5/2)/(a + a*cos[c + d*x])^(9/2),x]

[Out] (45*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])])/(1024*Sqrt[2]*a^(9/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*(a + a*cos[c + d*x])^(9/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(32*a*d*(a + a*cos[c + d*x])^(7/2)) + (33*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(256*a^2*d*(a + a*cos[c + d*x])^(5/2)) + (73*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(1024*a^3*d*(a + a*cos[c + d*x])^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n)/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&

NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}(\frac{3a}{2}-6a\cos(c+dx))}{(a+a\cos(c+dx))^{7/2}} dx}{8a^2} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\frac{15a^2}{4}-21a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx}{48a^4} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} \\
 &\quad + \frac{33\sqrt{\cos(c+dx)}\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\frac{21a^3}{8}-\frac{99}{4}a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{192a^6} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} \\
 &\quad + \frac{33\sqrt{\cos(c+dx)}\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}} + \frac{73\sqrt{\cos(c+dx)}\sin(c+dx)}{1024a^3d(a+a\cos(c+dx))^{3/2}} \\
 &\quad - \frac{\int -\frac{135a^4}{16\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{384a^8} \\
 &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} \\
 &\quad + \frac{33\sqrt{\cos(c+dx)}\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}} + \frac{73\sqrt{\cos(c+dx)}\sin(c+dx)}{1024a^3d(a+a\cos(c+dx))^{3/2}} \\
 &\quad + \frac{45\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2048a^4}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)}\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}} \\
&\quad + \frac{73\sqrt{\cos(c+dx)}\sin(c+dx)}{1024a^3d(a+a\cos(c+dx))^{3/2}} - \frac{45\text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024a^3d} \\
&= \frac{45\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024\sqrt{2}a^{9/2}d} \\
&\quad - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} \\
&\quad + \frac{33\sqrt{\cos(c+dx)}\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}} + \frac{73\sqrt{\cos(c+dx)}\sin(c+dx)}{1024a^3d(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.73

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}} dx = \frac{\sec^6\left(\frac{1}{2}(c+dx)\right) \left(5760\arctanh\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right)\right) \cos^8\left(\frac{1}{2}(c+dx)\right)}{(a+a\cos(c+dx))^{9/2}}$$

[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(9/2), x]

[Out] (Sec[(c + d*x)/2]^6*(5760*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]) *Cos[(c + d*x)/2]^8 + (999 + 2466*Cos[c + d*x] + 1072*Cos[2*(c + d*x)] + 702*Cos[3*(c + d*x)] + 73*Cos[4*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]]*Tan[(c + d*x)/2])/(65536*Sqrt[2]*a^4*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.61

method	result
default	$ \frac{\left(-\frac{\csc^2(dx+c)(1-\cos(dx+c))^2-1}{\csc^2(dx+c)(1-\cos(dx+c))^2+1}\right)^{\frac{5}{2}} \left((\csc^2(dx+c)(1-\cos(dx+c))^2+1)\right)^3 \sqrt{\frac{a}{\csc^2(dx+c)(1-\cos(dx+c))^2+1}} \left(16(\csc^7(dx+c))\sqrt{-(\csc^2(dx+c)(1-\cos(dx+c))^2+1)}\right)}{1024a^3d(a+a\cos(c+dx))^{3/2}} $

[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(9/2), x, method=_RETURNVERBOSE)

[Out] 1/2048/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(5/2)/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(5/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^3*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(16*csc(d*x+c)^7*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^7-24*csc(d*x+c)^5*(-

$$\text{csc}(d*x+c)^2*(1-\cos(d*x+c))^{2+1}^{(1/2)}*(1-\cos(d*x+c))^5-30*\text{csc}(d*x+c)^3*(-\text{csc}(d*x+c)^2*(1-\cos(d*x+c))^{2+1}^{(1/2)}*(1-\cos(d*x+c))^3+83*(-\text{csc}(d*x+c)^2*(1-\cos(d*x+c))^{2+1}^{(1/2)}*(\text{csc}(d*x+c)-\cot(d*x+c))-45*\arcsin(\cot(d*x+c)-\text{csc}(d*x+c)))*2^{(1/2)}/a^5$$

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.14

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}} dx = \frac{45\sqrt{2}(\cos(dx+c)^5 + 5\cos(dx+c)^4 + 10\cos(dx+c)^3 + 10\cos(dx+c)^2 + 10\cos(dx+c) + 1)\sqrt{a}\arctan(1/2\sqrt{2}\sqrt{a\cos(dx+c)+a})\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)/(a\cos(dx+c)^2 + a\cos(dx+c)) + 2(73\cos(dx+c)^3 + 351\cos(dx+c)^2 + 195\cos(dx+c) + 45)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)/(a^5d\cos(dx+c)^5 + 5a^5d\cos(dx+c)^4 + 10a^5d\cos(dx+c)^3 + 10a^5d\cos(dx+c)^2 + 5a^5d\cos(dx+c) + a^5d)}{2048}$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="fricas")

[Out] 1/2048*(45*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*(73*cos(d*x + c)^3 + 351*cos(d*x + c)^2 + 195*cos(d*x + c) + 45)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(9/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}} dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{9}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(9/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{9/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{9/2}} dx = \int \frac{\cos(c + dx)^{5/2}}{(a + a \cos(c + dx))^{9/2}} dx$$

```
[In] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(9/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + a*cos(c + d*x))^(9/2), x)
```

$$3.261 \quad \int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}} dx$$

Optimal result	3804
Rubi [A] (verified)	3804
Mathematica [A] (verified)	3805
Maple [B] (verified)	3805
Fricas [B] (verification not implemented)	3806
Sympy [F]	3806
Maxima [C] (verification not implemented)	3806
Giac [F]	3807
Mupad [F(-1)]	3807

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}} dx = \sqrt{2} \arcsin\left(\frac{\sin(x)}{1+\cos(x)}\right)$$

[Out] arcsin(sin(x)/(1+cos(x)))*2^(1/2)

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2860, 222}

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}} dx = \sqrt{2} \arcsin\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

[In] Int[1/(Sqrt[Cos[x]]*Sqrt[1 + Cos[x]]),x]

[Out] Sqrt[2]*ArcSin[Sin[x]/(1 + Cos[x])]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2860

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e

, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= -\left(\sqrt{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(x)}{1+\cos(x)}\right)\right) \\ &= \sqrt{2} \arcsin\left(\frac{\sin(x)}{1+\cos(x)}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}} dx = \frac{2 \arctan\left(\frac{\sin(\frac{x}{2})}{\sqrt{\cos(x)}}\right) \cos\left(\frac{x}{2}\right)}{\sqrt{1+\cos(x)}}$$

[In] Integrate[1/(Sqrt[Cos[x]]*Sqrt[1 + Cos[x]]), x]

[Out] (2*ArcTan[Sin[x/2]/Sqrt[Cos[x]]]*Cos[x/2])/Sqrt[1 + Cos[x]]

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 1.48 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

method	result	size
default	$-\frac{\sqrt{\frac{\cos(x)}{\cos(x)+1}} \sqrt{2\cos(x)+2} \arcsin(-\csc(x)+\cot(x))}{\sqrt{\cos(x)}}$	34

[In] int(1/cos(x)^(1/2)/(cos(x)+1)^(1/2), x, method=_RETURNVERBOSE)

[Out] -1/cos(x)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*(2*cos(x)+2)^(1/2)*arcsin(-csc(x)+cot(x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}} dx = \sqrt{2} \arctan \left(\frac{\sqrt{2}\sqrt{\cos(x)+1}\sqrt{\cos(x)}\sin(x)}{2(\cos(x)^2 + \cos(x))} \right)$$

[In] integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="fricas")

[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(cos(x) + 1)*sqrt(cos(x))*sin(x)/(cos(x)^2 + cos(x)))

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}} dx = \int \frac{1}{\sqrt{\cos(x)+1}\sqrt{\cos(x)}} dx$$

[In] integrate(1/cos(x)**(1/2)/(1+cos(x))**(1/2),x)

[Out] Integral(1/(sqrt(cos(x) + 1)*sqrt(cos(x))), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 306, normalized size of antiderivative = 19.12

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}} dx = \sqrt{2} \arctan \left(\frac{(|e^{ix} + 1|^4 + \cos(x)^4 + \sin(x)^4 + 2(\cos(x)^2 - \sin(x)^2 - 2\cos(x) + 1)|e^{ix} + 1|^2 - 4\cos(x))}{\dots} \right)$$

[In] integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="maxima")

[Out] sqrt(2)*arctan2(((abs(e^(I*x) + 1)^4 + cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 - sin(x)^2 - 2*cos(x) + 1)*abs(e^(I*x) + 1)^2 - 4*cos(x)^3 + 2*(cos(x)^2 - 2*cos(x) + 1)*sin(x)^2 + 6*cos(x)^2 - 4*cos(x) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(x) - 1)*sin(x)/abs(e^(I*x) + 1)^2, (abs(e^(I*x) + 1)^2 + cos(x)^2 - sin(x)^2 - 2*cos(x) + 1)/abs(e^(I*x) + 1)^2)) + sin(x))/abs(e^(I*x) + 1), ((abs(e^(I*x) + 1)^4 + cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 - sin(x)^2 - 2*cos(x) + 1)*abs(e^(I*x) + 1)^2 - 4*cos(x)^3 + 2*(cos(x)^2 - 2*cos(x) + 1)*sin(x)^2 + 6*cos(x)^2 - 4*cos(x) + 1)^(1/4)*cos(1/2*arctan2(2*(cos(x) - 1)*sin(x)/abs(e^(I*x) + 1)^2, (abs(e^(I*x) + 1)^2 + cos(x)^2 - sin(x)^2 - 2*cos(x) + 1)/abs(e^(I*x) + 1)^2)) + cos(x) - 1)/abs(e^(I*x) + 1))

Giac [F]

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}} dx = \int \frac{1}{\sqrt{\cos(x)+1}\sqrt{\cos(x)}} dx$$

[In] integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(cos(x) + 1)*sqrt(cos(x))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}} dx = \int \frac{1}{\sqrt{\cos(x)}\sqrt{\cos(x)+1}} dx$$

[In] int(1/(cos(x)^(1/2)*(cos(x) + 1)^(1/2)),x)

[Out] int(1/(cos(x)^(1/2)*(cos(x) + 1)^(1/2)), x)

$$3.262 \quad \int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}} dx$$

Optimal result	3808
Rubi [A] (verified)	3808
Mathematica [A] (verified)	3809
Maple [A] (verified)	3809
Fricas [A] (verification not implemented)	3810
Sympy [F]	3810
Maxima [C] (verification not implemented)	3810
Giac [F]	3811
Mupad [F(-1)]	3811

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}} dx = \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}$$

[Out] arctan(1/2*sin(x)*a^(1/2)*2^(1/2)/cos(x)^(1/2)/(a+a*cos(x))^(1/2))*2^(1/2)/a^(1/2)

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2861, 211}

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}} dx = \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{\cos(x)}\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

[In] Int[1/(Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]]),x]

[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]])])/Sqrt[a]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= - \left((2a) \text{Subst} \left(\int \frac{1}{2a^2 + ax^2} dx, x, -\frac{a \sin(x)}{\sqrt{\cos(x)} \sqrt{a + a \cos(x)}} \right) \right) \\ &= \frac{\sqrt{2} \arctan \left(\frac{\sqrt{a} \sin(x)}{\sqrt{2} \sqrt{\cos(x)} \sqrt{a + a \cos(x)}} \right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{\cos(x)} \sqrt{a + a \cos(x)}} dx = \frac{2 \arctan \left(\frac{\sin(\frac{x}{2})}{\sqrt{\cos(x)}} \right) \cos \left(\frac{x}{2} \right)}{\sqrt{a(1 + \cos(x))}}$$

[In] Integrate[1/(Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]]),x]

[Out] (2*ArcTan[Sin[x/2]/Sqrt[Cos[x]]]*Cos[x/2])/Sqrt[a*(1 + Cos[x])]

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\sqrt{\frac{\cos(x)}{\cos(x)+1}} \sqrt{a(\cos(x)+1)} \arcsin(-\csc(x)+\cot(x))\sqrt{2}}{\sqrt{\cos(x)} a}$	40

[In] int(1/cos(x)^(1/2)/(a+cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/cos(x)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*(a*(cos(x)+1))^(1/2)*arcsin(-csc(x)+cot(x))*2^(1/2)/a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}} dx$$

$$= \left[\frac{1}{2} \sqrt{2} \sqrt{-\frac{1}{a}} \log \left(-\frac{2\sqrt{2}\sqrt{a\cos(x)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(x)}\sin(x) - 3\cos(x)^2 - 2\cos(x) + 1}{\cos(x)^2 + 2\cos(x) + 1} \right), \frac{\sqrt{2}\arctan \left(\dots \right)}{\sqrt{a}} \right]$$

```
[In] integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(-1/a)*sqrt(cos(x))*sin(x) - 3*cos(x)^2 - 2*cos(x) + 1)/(cos(x)^2 + 2*cos(x) + 1)), sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(cos(x))*sin(x)/((cos(x)^2 + cos(x))*sqrt(a)))/sqrt(a)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}} dx = \int \frac{1}{\sqrt{a(\cos(x)+1)}\sqrt{\cos(x)}} dx$$

```
[In] integrate(1/cos(x)**(1/2)/(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(x) + 1))*sqrt(cos(x))), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 323, normalized size of antiderivative = 7.88

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}} dx$$

$$= \frac{\sqrt{2}\arctan \left(\frac{(|e^{ix}+1|^4 + \cos(x)^4 + \sin(x)^4 + 2(\cos(x)^2 - \sin(x)^2 - 2\cos(x)+1)|e^{ix}+1|^2 - 4\cos(x)^3 + 2(\cos(x)^2 - 2\cos(x)+1)\sin(x)^2 + 6\cos(x))}{|e^{ix}+1|} \right)}{\dots}$$

```
[In] integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan2(((abs(e^(I*x) + 1)^4 + cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 -
sin(x)^2 - 2*cos(x) + 1)*abs(e^(I*x) + 1)^2 - 4*cos(x)^3 + 2*(cos(x)^2 - 2*
cos(x) + 1)*sin(x)^2 + 6*cos(x)^2 - 4*cos(x) + 1)^(1/4)*sin(1/2*arctan2(2*(
cos(x) - 1)*sin(x)/abs(e^(I*x) + 1)^2, (abs(e^(I*x) + 1)^2 + cos(x)^2 - sin
(x)^2 - 2*cos(x) + 1)/abs(e^(I*x) + 1)^2)) + sin(x))/abs(e^(I*x) + 1), ((ab
s(e^(I*x) + 1)^4 + cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 - sin(x)^2 - 2*cos(x)
+ 1)*abs(e^(I*x) + 1)^2 - 4*cos(x)^3 + 2*(cos(x)^2 - 2*cos(x) + 1)*sin(x)^2
+ 6*cos(x)^2 - 4*cos(x) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2(2*(cos(x) - 1)*
sin(x)/abs(e^(I*x) + 1)^2, (abs(e^(I*x) + 1)^2 + cos(x)^2 - sin(x)^2 - 2*co
s(x) + 1)/abs(e^(I*x) + 1)^2)) + sqrt(a)*cos(x) - sqrt(a))/(sqrt(a)*abs(e^(
I*x) + 1)))/sqrt(a)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a + a\cos(x)}} dx = \int \frac{1}{\sqrt{a\cos(x) + a}\sqrt{\cos(x)}} dx$$

```
[In] integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(x) + a)*sqrt(cos(x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(x)}\sqrt{a + a\cos(x)}} dx = \int \frac{1}{\sqrt{\cos(x)}\sqrt{a + a\cos(x)}} dx$$

```
[In] int(1/(cos(x)^(1/2)*(a + a*cos(x))^(1/2)),x)
```

```
[Out] int(1/(cos(x)^(1/2)*(a + a*cos(x))^(1/2)), x)
```

3.263 $\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)} dx$

Optimal result	3812
Rubi [A] (verified)	3812
Mathematica [C] (verified)	3814
Maple [A] (verified)	3814
Fricas [A] (verification not implemented)	3814
Sympy [F]	3815
Maxima [B] (verification not implemented)	3815
Giac [F]	3816
Mupad [F(-1)]	3816

Optimal result

Integrand size = 26, antiderivative size = 129

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)} dx = -\frac{3\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}\right)}{4d} + \frac{3a \sqrt{\cos(c + dx)} \sin(c + dx)}{4d \sqrt{a - a \cos(c + dx)}} - \frac{a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d \sqrt{a - a \cos(c + dx)}}$$

[Out] $-3/4 * \operatorname{arctanh}(\sin(d*x+c) * a^{(1/2)} / \cos(d*x+c)^{(1/2)} / (a - a * \cos(d*x+c))^{(1/2)}) * a^{(1/2)} / d - 1/2 * a * \cos(d*x+c)^{(3/2)} * \sin(d*x+c) / d / (a - a * \cos(d*x+c))^{(1/2)} + 3/4 * a * \sin(d*x+c) * \cos(d*x+c)^{(1/2)} / d / (a - a * \cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2849, 2854, 213}

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)} dx = -\frac{3\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}\right)}{4d} - \frac{a \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{2d \sqrt{a - a \cos(c + dx)}} + \frac{3a \sin(c + dx) \sqrt{\cos(c + dx)}}{4d \sqrt{a - a \cos(c + dx)}}$$

[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]],x]

[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])]/(4*d) + (3*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a - a*Cos[c + d*x]]) - (a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a - a*Cos[c + d*x]])

Rule 213

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2849

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Ssin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Ssin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Ssin[e + f*x]]*(c + d*Ssin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2854

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a - a \cos(c + dx)}} - \frac{3}{4} \int \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)} dx \\
 &= \frac{3a\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a - a \cos(c + dx)}} - \frac{a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a - a \cos(c + dx)}} + \frac{3}{8} \int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{3a\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a - a \cos(c + dx)}} - \frac{a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a - a \cos(c + dx)}} \\
 &\quad + \frac{(3a)\text{Subst}\left(\int \frac{1}{-a+x^2} dx, x, \frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a \cos(c+dx)}}\right)}{4d} \\
 &= -\frac{3\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a \cos(c+dx)}}\right)}{4d} \\
 &\quad + \frac{3a\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a - a \cos(c + dx)}} - \frac{a \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a - a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.44

$$\int \cos^{\frac{3}{2}}(c+dx) \sqrt{a-a\cos(c+dx)} dx = \frac{e^{-\frac{3}{2}i(c+dx)} \left(\sqrt{1+e^{2i(c+dx)}} (1-2e^{i(c+dx)}-2e^{2i(c+dx)}+e^{3i(c+dx)}) + 3e^{2i(c+dx)} \operatorname{arcsinh}(e^{i(c+dx)}) + 3e^{2i(c+dx)} a \right)}{8d\sqrt{1+e^{2i(c+dx)}}$$

[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]],x]

[Out] -1/8*((Sqrt[1 + E^((2*I)*(c + d*x))])*(1 - 2*E^(I*(c + d*x)) - 2*E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x))) + 3*E^((2*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))]) + 3*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]*Csc[(c + d*x)/2]/(d*E^(((3*I)/2)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))])

Maple [A] (verified)

Time = 12.56 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.18

method	result
default	$-\frac{\csc(dx+c) \left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (\cos^2(dx+c) - \cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 3 \operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) - 3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \right) \sqrt{-a(\cos(dx+c) + \cos(dx+c))}}{4d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int(cos(d*x+c)^(3/2)*(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4/d*csc(d*x+c)*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arctanh((cos(d*x+c)/(1+cos(d*x+c))))^(1/2))-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-a*(cos(d*x+c)-1))^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.20

$$\int \cos^{\frac{3}{2}}(c+dx) \sqrt{a-a\cos(c+dx)} dx = \frac{3\sqrt{a} \log\left(\frac{4\sqrt{-a\cos(dx+c)+a} \left(2\cos^2(dx+c) + 3\cos(dx+c) + 1 \right) \sqrt{a}\sqrt{\cos(dx+c)} - (8a\cos(dx+c)^2 + 8a\cos(dx+c) + a) \sin(dx+c)}{\sin(dx+c)}\right)}{16d\sin(dx+c)}$$

$(2dx + 2c), \cos(2dx + 2c)) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))$, $(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} (\cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) - 1) + \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 1) - \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - 1))) / d$

Giac [F]

$$\int \cos^{3/2}(c + dx) \sqrt{a - a \cos(c + dx)} dx = \int \sqrt{-a \cos(dx + c) + a \cos(dx + c)}^{3/2} dx$$

[In] integrate(cos(dx+c)^(3/2)*(a-a*cos(dx+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-a*cos(dx + c) + a)*cos(dx + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \cos^{3/2}(c + dx) \sqrt{a - a \cos(c + dx)} dx = \int \cos(c + dx)^{3/2} \sqrt{a - a \cos(c + dx)} dx$$

[In] int(cos(c + dx)^(3/2)*(a - a*cos(c + dx))^(1/2),x)

[Out] int(cos(c + dx)^(3/2)*(a - a*cos(c + dx))^(1/2), x)

3.264 $\int \sqrt{\cos(c+dx)} \sqrt{a-a\cos(c+dx)} dx$

Optimal result	3817
Rubi [A] (verified)	3817
Mathematica [C] (verified)	3818
Maple [A] (verified)	3819
Fricas [A] (verification not implemented)	3819
Sympy [F]	3820
Maxima [B] (verification not implemented)	3820
Giac [F]	3821
Mupad [F(-1)]	3821

Optimal result

Integrand size = 26, antiderivative size = 85

$$\int \sqrt{\cos(c+dx)} \sqrt{a-a\cos(c+dx)} dx = \frac{\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a-a\cos(c+dx)}}\right)}{d} - \frac{a \sqrt{\cos(c+dx)} \sin(c+dx)}{d \sqrt{a-a\cos(c+dx)}}$$

[Out] $\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a-a*\cos(d*x+c))^{(1/2)})*a^{(1/2)}/d-a*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a-a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2849, 2854, 213}

$$\int \sqrt{\cos(c+dx)} \sqrt{a-a\cos(c+dx)} dx = \frac{\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a-a\cos(c+dx)}}\right)}{d} - \frac{a \sin(c+dx) \sqrt{\cos(c+dx)}}{d \sqrt{a-a\cos(c+dx)}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a-a*\operatorname{Cos}[c+d*x]],x]$

[Out] $(\operatorname{Sqrt}[a]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x])/(\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a-a*\operatorname{Cos}[c+d*x]])])/d - (a*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(d*\operatorname{Sqrt}[a-a*\operatorname{Cos}[c+d*x]])$

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]))], x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} - \frac{1}{2} \int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\ &= -\frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} - \frac{a\text{Subst}\left(\int \frac{1}{-a+x^2} dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} \\ &= \frac{\sqrt{a}\arctanh\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} - \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.79

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)} dx = \frac{i\left(\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}-e^{i(c+dx)}\operatorname{arcsinh}\left(e^{i(c+dx)}\right)-e^{i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{\cos(c+dx)}}{d(-1+e^{i(c+dx)})\sqrt{1+e^{2i(c+dx)}}}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]], x]
```

```
[Out] ((-I)*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))] - E^(I*(c + d*x))
)*ArcSinh[E^(I*(c + d*x))] - E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c +
d*x))]])*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]/(d*(-1 + E^(I*(c + d
*x)))*Sqrt[1 + E^((2*I)*(c + d*x))])
```

Maple [A] (verified)

Time = 12.95 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

method	result	size
default	$-\frac{\csc(dx+c)\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\left(\sqrt{\cos(dx+c)}\sqrt{-a(\cos(dx+c)-1)}\right)}{d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	120

```
[In] int(cos(d*x+c)^(1/2)*(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*csc(d*x+c)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)-arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)
^(1/2)*(-a*(cos(d*x+c)-1))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}dx$$

$$= \frac{\sqrt{a}\log\left(-\frac{4\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\sqrt{\cos(dx+c)}+\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)}{4d\sin(dx+c)}$$

```
[In] integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a)*log(-(4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) + (8*a*cos(d*x + c)^2 + 8*a*cos(d*x
+ c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*sqrt(-a*cos(d*x + c)
+ a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*sin(d*x + c))
```

SymPy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)} dx = \int \sqrt{-a(\cos(c+dx)-1)}\sqrt{\cos(c+dx)} dx$$

```
[In] integrate(cos(d*x+c)**(1/2)*(a-a*cos(d*x+c))**(1/2), x)
```

```
[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))*sqrt(cos(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(73) = 146$.

Time = 0.41 (sec) , antiderivative size = 795, normalized size of antiderivative = 9.35

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)} dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2), x, algorithm="maxima")
```

```
[Out] -1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(-a) + sqrt(-a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))) - 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```


Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)} dx = \int \sqrt{-a\cos(dx+c)+a}\sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)} dx = \int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)} dx$$

[In] int(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2), x)

$$3.265 \quad \int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

Optimal result	3822
Rubi [A] (verified)	3822
Mathematica [C] (verified)	3823
Maple [A] (verified)	3823
Fricas [A] (verification not implemented)	3824
Sympy [F]	3824
Maxima [B] (verification not implemented)	3824
Giac [B] (verification not implemented)	3825
Mupad [F(-1)]	3825

Optimal result

Integrand size = 26, antiderivative size = 48

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = -\frac{2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}\right)}{d}$$

[Out] $-2*\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a-a*\cos(d*x+c))^{(1/2)})*a^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.13 (sec), antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2854, 213}

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = -\frac{2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}\right)}{d}$$

[In] `Int[Sqrt[a - a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]`

[Out] `(-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/d`

Rule 213

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 2854

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]))], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(2a)\text{Subst}\left(\int \frac{1}{-a+x^2} dx, x, \frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a \cos(c+dx)}}\right)}{d} \\ &= -\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a \cos(c+dx)}}\right)}{d} \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \frac{e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)} (1 + e^{2i(c+dx)})} \left(\text{arcsinh}(e^{i(c+dx)}) + \text{arctanh}\left(\sqrt{1 + e^{2i(c+dx)}}\right) \right) \sqrt{a - a \cos(c + dx)} \text{cs}}{\sqrt{2}d\sqrt{1 + e^{2i(c+dx)}}$$

```
[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(Arc
Sinh[E^(I*(c + d*x))] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a - a*
Cos[c + d*x]]*Csc[(c + d*x)/2])/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))])
```

Maple [A] (verified)

Time = 4.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

method	result	size
default	$-\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-a(\cos(dx+c)-1)}\text{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cot(dx+c)+\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	79

```
[In] int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-a*(cos(d*x+c)-1))^(1/2)*arctanh((c
os(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)*(cot(d*x+c)+csc(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.23

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

$$= \left[\frac{\sqrt{a} \log \left(\frac{4 \sqrt{-a \cos(dx+c)+a} (2 \cos(dx+c)^2 + 3 \cos(dx+c)+1) \sqrt{a} \sqrt{\cos(dx+c)} - (8 a \cos(dx+c)^2 + 8 a \cos(dx+c)+a) \sin(dx+c)}{\sin(dx+c)} \right)}{2 d}, \frac{\sqrt{-a} \arctan \left(\frac{\sqrt{-a} \sqrt{\cos(dx+c)} - (8 a \cos(dx+c)^2 + 8 a \cos(dx+c)+a) \sin(dx+c)}{\sin(dx+c)} \right)}{2 d} \right]$$

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) - (8*a*cos(d*x + c)^2 + 8*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))/d, sqrt(-a)*arctan(1/2*sqrt(-a*cos(d*x + c) + a)*sqrt(-a)*(2*cos(d*x + c) + 1)/(a*sqrt(cos(d*x + c))*sin(d*x + c)))/d]
```

Sympy [F]

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \int \frac{\sqrt{-a (\cos(c + dx) - 1)}}{\sqrt{\cos(c + dx)}} dx$$

```
[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))/sqrt(cos(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(40) = 80.

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.08

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{\sqrt{-a} \arctan \left(\left(\cos(2 dx + 2 c)^2 + \sin(2 dx + 2 c)^2 + 2 \cos(2 dx + 2 c) + 1 \right)^{\frac{1}{4}} \sin \left(\frac{1}{2} \arctan(\sin(2 dx + 2 c)) \right) \right)}{2 d}$$

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

[Out] $\sqrt{-a} \arctan 2((\cos(2dx + 2c))^2 + \sin(2dx + 2c))^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + \sin(dx + c), (\cos(2dx + 2c))^2 + \sin(2dx + 2c))^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + \cos(dx + c))/d$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(40) = 80$.

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{2\sqrt{a} \log \left(\frac{2 \left(\tan(\frac{1}{4} dx + \frac{1}{4} c) \right)^2 + 2\sqrt{2} - \sqrt{\tan(\frac{1}{4} dx + \frac{1}{4} c)^4 - 6 \tan(\frac{1}{4} dx + \frac{1}{4} c)^2 + 1 + 1}}{-2 \tan(\frac{1}{4} dx + \frac{1}{4} c)^2 + 4\sqrt{2} + 2 \sqrt{\tan(\frac{1}{4} dx + \frac{1}{4} c)^4 - 6 \tan(\frac{1}{4} dx + \frac{1}{4} c)^2 + 1 - 2}} \right) \operatorname{sgn} \left(\sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{d}$$

[In] `integrate((a-a*cos(dx+c))^(1/2)/cos(dx+c)^(1/2),x, algorithm="giac")`

[Out] $2\sqrt{a} \log(2(\tan(1/4 dx + 1/4 c))^2 + 2\sqrt{2} - \sqrt{\tan(1/4 dx + 1/4 c)^4 - 6\tan(1/4 dx + 1/4 c)^2 + 1 + 1}) / \operatorname{abs}(-2\tan(1/4 dx + 1/4 c)^2 + 4\sqrt{2} + 2\sqrt{\tan(1/4 dx + 1/4 c)^4 - 6\tan(1/4 dx + 1/4 c)^2 + 1 - 2}) \operatorname{sgn}(\sin(1/2 dx + 1/2 c)) / d$

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \int \frac{\sqrt{a - a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

[In] `int((a - a*cos(c + dx))^(1/2)/cos(c + dx)^(1/2),x)`

[Out] `int((a - a*cos(c + dx))^(1/2)/cos(c + dx)^(1/2), x)`

$$3.266 \quad \int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx$$

Optimal result	3826
Rubi [A] (verified)	3826
Mathematica [A] (verified)	3827
Maple [A] (verified)	3827
Fricas [A] (verification not implemented)	3827
Sympy [F]	3828
Maxima [B] (verification not implemented)	3828
Giac [A] (verification not implemented)	3828
Mupad [B] (verification not implemented)	3829

Optimal result

Integrand size = 26, antiderivative size = 37

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2a \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}$$

[Out] 2*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2850}

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2a \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}$$

[In] Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(3/2), x]

[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])

Rule 2850

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\text{integral} = \frac{2a \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2\sqrt{a - a \cos(c + dx)} \cot\left(\frac{1}{2}(c + dx)\right)}{d\sqrt{\cos(c + dx)}}$$

[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]

[Out] (2*Sqrt[a - a*Cos[c + d*x]]*Cot[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 5.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\sqrt{-a(\cos(dx+c)-1)}(\cot(dx+c)+\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	40

[In] int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/d*(-a*(cos(d*x+c)-1))^(1/2)/cos(d*x+c)^(1/2)*(cot(d*x+c)+csc(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2\sqrt{-a \cos(dx + c) + a}(\cos(dx + c) + 1)}{d\sqrt{\cos(dx + c)} \sin(dx + c)}$$

[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] 2*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)/(d*sqrt(cos(d*x + c))*sin(d*x + c))

Sympy [F]

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{-a (\cos(c + dx) - 1)}}{\cos^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2), x)

[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))/cos(c + d*x)**(3/2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(33) = 66.

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2 \left(\sqrt{2} \sqrt{a} - \frac{\sqrt{2} \sqrt{a} \sin(dx+c)^2}{(\cos(dx+c)+1)^2} \right)}{d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{3}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{3}{2}}}$$

[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] 2*(sqrt(2)*sqrt(a) - sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = -\frac{2 \sqrt{2} \left(\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 - 1 \right) \sqrt{a} \operatorname{sgn} \left(\sin \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\sqrt{\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^4 - 6 \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 + 1} d}$$

[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="giac")

[Out] -2*sqrt(2)*(tan(1/4*d*x + 1/4*c)^2 - 1)*sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))/(sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)

Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = -\frac{2 \sin(c + dx) \sqrt{-a (\cos(c + dx) - 1)}}{d \sqrt{\cos(c + dx)} (\cos(c + dx) - 1)}$$

[In] `int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)`

[Out] `-(2*sin(c + d*x)*(-a*(cos(c + d*x) - 1))^(1/2))/(d*cos(c + d*x)^(1/2)*(cos(c + d*x) - 1))`

$$3.267 \quad \int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

Optimal result	3830
Rubi [A] (verified)	3830
Mathematica [A] (verified)	3831
Maple [A] (verified)	3831
Fricas [A] (verification not implemented)	3832
Sympy [F]	3832
Maxima [B] (verification not implemented)	3832
Giac [A] (verification not implemented)	3833
Mupad [B] (verification not implemented)	3833

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} - \frac{4a \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}$$

[Out] $2/3*a*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a-a*\cos(d*x+c))^{(1/2)}-4/3*a*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a-a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2851, 2850}

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} - \frac{4a \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}$$

[In] $\text{Int}[\text{Sqrt}[a - a*\text{Cos}[c + d*x]]/\text{Cos}[c + d*x]^{(5/2)}, x]$

[Out] $(2*a*\text{Sin}[c + d*x])/(3*d*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a - a*\text{Cos}[c + d*x]]) - (4*a*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a - a*\text{Cos}[c + d*x]])$

Rule 2850

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} - \frac{2}{3} \int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2a \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} - \frac{4a \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = -\frac{2(-1 + 2 \cos(c + dx)) \sqrt{a - a \cos(c + dx)} \cot\left(\frac{1}{2}(c + dx)\right)}{3d \cos^{\frac{3}{2}}(c + dx)}$$

[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]

[Out] (-2*(-1 + 2*Cos[c + d*x])*Sqrt[a - a*Cos[c + d*x]]*Cot[(c + d*x)/2])/(3*d*Cos[c + d*x]^(3/2))

Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2 \csc(dx+c) \sqrt{-a(\cos(dx+c)-1)} (-1+2(\cos^2(dx+c))+\cos(dx+c))}{3d \cos(dx+c)^{\frac{3}{2}}}$	51

[In] `int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$-2/3/d*\csc(d*x+c)*(-a*(\cos(d*x+c)-1))^(1/2)*(-1+2*\cos(d*x+c)^2+\cos(d*x+c))/\cos(d*x+c)^(3/2)$$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = -\frac{2 \sqrt{-a \cos(dx + c) + a} (2 \cos(dx + c)^2 + \cos(dx + c) - 1)}{3 d \cos(dx + c)^{\frac{3}{2}} \sin(dx + c)}$$

[In] `integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`

[Out]
$$-2/3*\sqrt{-a*\cos(d*x + c) + a}*(2*\cos(d*x + c)^2 + \cos(d*x + c) - 1)/(d*\cos(d*x + c)^(3/2)*\sin(d*x + c))$$

Sympy [F]

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{\sqrt{-a (\cos(c + dx) - 1)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

[In] `integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)`

[Out] `Integral(sqrt(-a*(cos(c + d*x) - 1))/cos(c + d*x)**(5/2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(67) = 134.

Time = 0.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = -\frac{2 \left(\sqrt{2}\sqrt{a} - \frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3\sqrt{2}\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^2}{3 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{5}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{5}{2}} \left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 1 \right)}$$

[In] `integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")`

[Out]
$$-2/3*(\sqrt{2}*\sqrt{a} - 4*\sqrt{2}*\sqrt{a}*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 3*\sqrt{2}*\sqrt{a}*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4)*(\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 1)^2/(d*(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)^(5/2)*(-\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)^(5/2)*(2*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + \sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + 1))$$

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{2\sqrt{2} \left(\left(\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 - 15 \right) \tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 + 15 \right) \tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 - 1}{3 \left(\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^4 - 6 \tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 + 1 \right)^{\frac{3}{2}}} \sqrt{a} \operatorname{sgn}\left(\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) dx$$

[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")

```
[Out] 2/3*sqrt(2)*(((tan(1/4*d*x + 1/4*c)^2 - 15)*tan(1/4*d*x + 1/4*c)^2 + 15)*tan(1/4*d*x + 1/4*c)^2 - 1)*sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))/((tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

Mupad [B] (verification not implemented)

Time = 14.73 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{4\sqrt{-a(\cos(c + dx) - 1)}(\sin(c + dx) - \sin(2c + 2dx) + \sin(3c + 3dx))}{3d\sqrt{\cos(c + dx)}(3\cos(c + dx) - 2\cos(2c + 2dx) + \cos(3c + 3dx) - 2)}$$

[In] int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)

```
[Out] (4*(-a*(cos(c + d*x) - 1))^(1/2)*(sin(c + d*x) - sin(2*c + 2*d*x) + sin(3*c + 3*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x) - 2))
```

$$3.268 \quad \int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

Optimal result	3834
Rubi [A] (verified)	3834
Mathematica [A] (verified)	3836
Maple [A] (verified)	3836
Fricas [A] (verification not implemented)	3836
Sympy [F(-1)]	3837
Maxima [B] (verification not implemented)	3837
Giac [A] (verification not implemented)	3837
Mupad [B] (verification not implemented)	3838

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx = \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} - \frac{8a \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{16a \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}$$

[Out] 2/5*a*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2)-8/15*a*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2)+16/15*a*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2851, 2850}

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx = -\frac{8a \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{16a \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}$$

[In] Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]

[Out] (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]) - (8*a*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) + (16*a*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])

Rule 2850

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} - \frac{4}{5} \int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} \\
 &\quad - \frac{8a \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{8}{15} \int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2a \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} - \frac{8a \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} \\
 &\quad + \frac{16a \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{2\sqrt{a - a \cos(c + dx)}(7 - 4 \cos(c + dx) + 4 \cos(2(c + dx))) \cot\left(\frac{1}{2}(c + dx)\right)}{15d \cos^{\frac{5}{2}}(c + dx)}$$

[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(7/2), x]

[Out] (2*Sqrt[a - a*Cos[c + d*x]]*(7 - 4*Cos[c + d*x] + 4*Cos[2*(c + d*x)])*Cot[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{2 \csc(dx+c) \sqrt{-a(\cos(dx+c)-1)} (3+8(\cos^3(dx+c))+4(\cos^2(dx+c))-\cos(dx+c))}{15d \cos(dx+c)^{\frac{5}{2}}}$	63

[In] int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] 2/15/d*csc(d*x+c)*(-a*(cos(d*x+c)-1))^(1/2)*(3+8*cos(d*x+c)^3+4*cos(d*x+c)^2-cos(d*x+c))/cos(d*x+c)^(5/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{2(8 \cos(dx + c)^3 + 4 \cos(dx + c)^2 - \cos(dx + c) + 3) \sqrt{-a \cos(dx + c) + a}}{15 d \cos(dx + c)^{\frac{5}{2}} \sin(dx + c)}$$

[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2), x, algorithm="fricas")

[Out] 2/15*(8*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(-a*cos(d*x + c) + a)/(d*cos(d*x + c)^(5/2)*sin(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(100) = 200.

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{2 \left(7\sqrt{2}\sqrt{a} - \frac{17\sqrt{2}\sqrt{a} \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{25\sqrt{2}\sqrt{a} \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{15\sqrt{2}\sqrt{a} \sin(dx+c)^6}{(\cos(dx+c)+1)^6} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^3}{15 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{7}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{7}{2}} \left(\frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 1 \right)}$$

[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 2/15*(7*sqrt(2)*sqrt(a) - 17*sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt(2)*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 15*sqrt(2)*sqrt(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx =$$

$$\frac{2\sqrt{2} \left(\left(\left(\left(7 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 75 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 430 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 430 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right) \right)}{15 \left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1 \right)}$$

[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] $-2/15\sqrt{2} * (((((7*\tan(1/4*d*x + 1/4*c)^2 - 75)*\tan(1/4*d*x + 1/4*c)^2 + 430)*\tan(1/4*d*x + 1/4*c)^2 - 430)*\tan(1/4*d*x + 1/4*c)^2 + 75)*\tan(1/4*d*x + 1/4*c)^2 - 7)*\sqrt{a}*\operatorname{sgn}(\sin(1/2*d*x + 1/2*c)) / ((\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1)^{(5/2)}*d)$

Mupad [B] (verification not implemented)

Time = 15.97 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{a - a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{8 \sqrt{2 a \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (7 \sin(c + dx) - 4 \sin(2c + 2dx) + 9 \sin(3c + 3dx) - 2 \sin(4c + 4dx) + 2 \sin(5c + 5dx))}}{15 d \sqrt{1 - 2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2} \left(-16 \sin(c + dx)^2 - 4 \sin(2c + 2dx)^2 + 20 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 10 \sin\left(\frac{3c}{2} + \frac{3dx}{2}\right)^2 - 16 \sin(c + dx)\right)}$$

[In] int((a - a*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)

[Out] $(8*(2*a*\sin(c/2 + (d*x)/2)^2)^{(1/2)}*(7*\sin(c + d*x) - 4*\sin(2*c + 2*d*x) + 9*\sin(3*c + 3*d*x) - 2*\sin(4*c + 4*d*x) + 2*\sin(5*c + 5*d*x)))/(15*d*(1 - 2*\sin(c/2 + (d*x)/2)^2)^{(1/2)}*(20*\sin(c/2 + (d*x)/2)^2 - 4*\sin(2*c + 2*d*x)^2 + 10*\sin((3*c)/2 + (3*d*x)/2)^2 + 2*\sin((5*c)/2 + (5*d*x)/2)^2 - 16*\sin(c + d*x)^2))$

3.269 $\int \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx) dx$

Optimal result	3839
Rubi [A] (verified)	3839
Mathematica [C] (verified)	3841
Maple [A] (verified)	3841
Fricas [A] (verification not implemented)	3841
Sympy [F]	3842
Maxima [B] (verification not implemented)	3842
Giac [F]	3843
Mupad [F(-1)]	3843

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx) dx = -\frac{3 \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

[Out] $-3/4*\operatorname{arctanh}(\sin(d*x+c)/(1-\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)})/d-1/2*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(1-\cos(d*x+c))^{(1/2)}+3/4*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(1-\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2849, 2854, 213}

$$\int \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx) dx = -\frac{3 \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} - \frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}} + \frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}}$$

[In] Int[Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2),x]

[Out] (-3*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/(4*d) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c + d*x]]) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 - Cos[c + d*x]])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2849

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2854

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 - \cos(c + dx)}} - \frac{3}{4} \int \sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)} dx \\
 &= \frac{3\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{1 - \cos(c + dx)}} - \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 - \cos(c + dx)}} + \frac{3}{8} \int \frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{3\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{1 - \cos(c + dx)}} - \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 - \cos(c + dx)}} \\
 &\quad + \frac{3 \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} \\
 &= -\frac{3 \arctanh\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} + \frac{3\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{1 - \cos(c + dx)}} - \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 - \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.59

$$\int \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx) dx = \frac{e^{-\frac{3}{2}i(c+dx)} \left(\sqrt{1 + e^{2i(c+dx)}} (1 - 2e^{i(c+dx)} - 2e^{2i(c+dx)} + e^{3i(c+dx)}) + 3e^{2i(c+dx)} \operatorname{arcsinh}(e^{i(c+dx)}) + 3e^{2i(c+dx)} \right)}{8d\sqrt{1 + e^{2i(c+dx)}}$$

[In] Integrate[Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2), x]

[Out] $-1/8*((\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}])*(1 - 2E^{(I*(c + d*x))} - 2E^{((2*I)*(c + d*x))} + E^{((3*I)*(c + d*x))}) + 3E^{((2*I)*(c + d*x))}*\operatorname{ArcSinh}[E^{(I*(c + d*x))}] + 3E^{((2*I)*(c + d*x))}*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}]])*\operatorname{Sqrt}[-((-1 + \operatorname{Cos}[c + d*x])*\operatorname{Cos}[c + d*x])]*\operatorname{Csc}[(c + d*x)/2])/(dE^{((3*I)/2)*(c + d*x)})*\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}]$

Maple [A] (verified)

Time = 13.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\operatorname{csc}(dx+c) \left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (\cos^2(dx+c) - \cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 3 \operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) - 3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \right) \sqrt{-2\cos(dx+c)}}{8d\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] $-1/8/d*\operatorname{csc}(d*x+c)*(2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^2-\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}+3*\operatorname{arctanh}((\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}))-3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*(-2*\cos(d*x+c)+2)^{(1/2)}*\cos(d*x+c)^{(1/2)}/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)}$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx) dx = \frac{2(2\cos(dx+c)^2 - \cos(dx+c) - 3)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)} - 3\log\left(-\frac{2(\cos(dx+c)+1)\sqrt{-\cos(dx+c)+1}}{\sin(dx+c)}\right)}{8d\sin(dx+c)}$$

[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out]
$$-1/8*(2*(2*\cos(dx + c)^2 - \cos(dx + c) - 3)*\sqrt{-\cos(dx + c) + 1}*\sqrt{\cos(dx + c)} - 3*\log(-(2*(\cos(dx + c) + 1)*\sqrt{-\cos(dx + c) + 1}*\sqrt{\cos(dx + c)} - (2*\cos(dx + c) + 1)*\sin(dx + c))/\sin(dx + c))*\sin(dx + c)))/(d*\sin(dx + c))$$

Sympy [F]

$$\int \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx) dx = \int \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx) dx$$

[In] integrate((1-cos(d*x+c))**(1/2)*cos(d*x+c)**(3/2),x)

[Out] Integral(sqrt(1 - cos(c + d*x))*cos(c + d*x)**(3/2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. 2(96) = 192.

Time = 0.40 (sec) , antiderivative size = 1305, normalized size of antiderivative = 11.45

$$\int \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx) dx = \text{Too large to display}$$

[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out]
$$\frac{1}{32}*(4*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{\frac{1}{4}}*(((\cos(2*d*x + 2*c) - 2)*\cos(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \sin(2*d*x + 2*c)*\sin(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + \cos(2*d*x + 2*c) - 2)*\cos(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - (\cos(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) * \sin(2*d*x + 2*c) - (\cos(2*d*x + 2*c) - 2)*\sin(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - \sin(2*d*x + 2*c))*\sin(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))) + 3*\log(\sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1}*\cos(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))^2 + \sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1}*\sin(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))^2 + 2*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{\frac{1}{4}}*\cos(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) + 1) - 3*\log(\sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1}*\cos(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))^2 + \sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1}*\sin(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))^2 - 2*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{\frac{1}{4}}*\cos(\frac{1}{2}*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))$$

1)) + 1) + 3*log(((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 3*log(((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1))/d

Giac [F]

$$\int \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx) dx = \int \sqrt{-\cos(dx + c) + 1} \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(-cos(d*x + c) + 1)*cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx) dx = \int \cos(c + dx)^{3/2} \sqrt{1 - \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2), x)

3.270 $\int \sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)} dx$

Optimal result	3844
Rubi [A] (verified)	3844
Mathematica [C] (verified)	3845
Maple [A] (verified)	3846
Fricas [A] (verification not implemented)	3846
Sympy [F]	3847
Maxima [B] (verification not implemented)	3847
Giac [F]	3848
Mupad [F(-1)]	3848

Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)} dx = \frac{\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}$$

[Out] $\operatorname{arctanh}(\sin(d*x+c)/(1-\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)})/d - \sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(1-\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2849, 2854, 213}

$$\int \sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)} dx = \frac{\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[1 - \operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]], x]$

[Out] $\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]/(\operatorname{Sqrt}[1 - \operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]])]/d - (\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/(d*\operatorname{Sqrt}[1 - \operatorname{Cos}[c + d*x]])$

Rule 213


```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2849

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} - \frac{1}{2} \int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\ &= -\frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \\ &= \frac{\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.15

$$\int \sqrt{1-\cos(c+dx)} \sqrt{\cos(c+dx)} dx = \frac{i\sqrt{2}\left(\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}-e^{i(c+dx)}\text{arcsinh}\left(e^{i(c+dx)}\right)-e^{i(c+dx)}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{\cos(c+dx)}}{d(-1+e^{i(c+dx)})\sqrt{1+e^{2i(c+dx)}}}$$

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]], x]
```

```
[Out] ((-I)*Sqrt[2]*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))] - E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))] - E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[Cos[c + d*x]*Sin[(c + d*x)/2]^2]/(d*(-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))])
```

Maple [A] (verified)

Time = 13.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.69

method	result	size
default	$-\frac{\csc(dx+c) \left(\cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - \operatorname{arctanh} \left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \right) \right) \sqrt{-2 \cos(dx+c)+2} (\sqrt{\cos(dx+c)}) \sqrt{2}}{2d \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	122

```
[In] int((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*csc(d*x+c)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(-2*cos(d*x+c)+2)^(1/2)*cos(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int \sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)} dx = \frac{2(\cos(dx + c) + 1) \sqrt{-\cos(dx + c) + 1} \sqrt{\cos(dx + c)} - \log \left(-\frac{2(\cos(dx + c) + 1) \sqrt{-\cos(dx + c) + 1} \sqrt{\cos(dx + c)} + (2 \cos(dx + c) + 1) \sqrt{\cos(dx + c)}}{\sin(dx + c)} \right)}{2d \sin(dx + c)}$$

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - log((-2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*cos(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c)/(d*sin(d*x + c))
```


$(1/4) * (\cos(dx + c) * \cos(1/2 * \arctan2(\sin(2*dx + 2*c), \cos(2*dx + 2*c) + 1)) + \sin(dx + c) * \sin(1/2 * \arctan2(\sin(2*dx + 2*c), \cos(2*dx + 2*c) + 1))) / d$

Giac [F]

$$\int \sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)} dx = \int \sqrt{-\cos(dx + c) + 1} \sqrt{\cos(dx + c)} dx$$

[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)} dx = \int \sqrt{\cos(c + dx)} \sqrt{1 - \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2), x)

$$3.271 \quad \int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	3849
Rubi [A] (verified)	3849
Mathematica [C] (verified)	3850
Maple [B] (verified)	3850
Fricas [A] (verification not implemented)	3851
Sympy [F]	3851
Maxima [B] (verification not implemented)	3851
Giac [B] (verification not implemented)	3852
Mupad [F(-1)]	3852

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx = -\frac{2\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

[Out] $-2*\operatorname{arctanh}(\sin(d*x+c)/(1-\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)})/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2854, 213}

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx = -\frac{2\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

[In] `Int[Sqrt[1 - Cos[c + d*x]]/Sqrt[Cos[c + d*x]], x]`

[Out] `(-2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d`

Rule 213

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)]*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rule 2854

`Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x`

```
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2 \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \\ &= -\frac{2 \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx = \frac{e^{\frac{1}{2}i(c+dx)} \sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})} \left(\operatorname{arcsinh}(e^{i(c+dx)}) + \operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right) \right) \sqrt{1-\cos(c+dx)} \operatorname{csc}(c+dx)}{\sqrt{2d}\sqrt{1+e^{2i(c+dx)}}$$

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Sqrt[Cos[c + d*x]], x]
```

```
[Out] -((E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(ArcSinh[E^(I*(c + d*x))] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[1 - Cos[c + d*x]]*Csc[(c + d*x)/2])/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(33) = 66.

Time = 4.75 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

method	result	size
default	$-\frac{\sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{-2 \cos(dx+c)+2} \operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) (\cot(dx+c)+\operatorname{csc}(dx+c))}{d \sqrt{\cos(dx+c)}}$	81

```
[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2), x, method=_RETURNVERBOSE)
```

```
[Out] -1/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*cos(d*x+c)+2)^(1/2)*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)*(cot(d*x+c)+csc(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \frac{\log\left(-\frac{2(\cos(dx+c)+1)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)} - (2\cos(dx+c)+1)\sin(dx+c)}{\sin(dx+c)}\right)}{d}$$

[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] log(-(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - (2*cos(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))/d

Sympy [F]

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \int \frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)

[Out] Integral(sqrt(1 - cos(c + d*x))/sqrt(cos(c + d*x)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(33) = 66.

Time = 0.36 (sec) , antiderivative size = 221, normalized size of antiderivative = 5.97

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{2 \operatorname{arsinh}(1) + \log\left(\cos(dx+c)^2 + \sin(dx+c)^2 + \sqrt{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c)}\right)}{d}$$

[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/2*(2*arsinh(1) + log(cos(d*x + c)^2 + sin(d*x + c)^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2) + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(33) = 66$.

Time = 0.60 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{2 \log \left(\frac{2 \left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 2\sqrt{2} - \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1} \right)}{\left| -2 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 4\sqrt{2} + 2 \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1} \right|} \right) \operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{d}$$

[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] 2*log(2*(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/abs(-2*tan(1/4*d*x + 1/4*c)^2 + 4*sqrt(2) + 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - 2))*sgn(sin(1/2*d*x + 1/2*c))/d

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \int \frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

[In] int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)

[Out] int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)

$$3.272 \quad \int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	3853
Rubi [A] (verified)	3853
Mathematica [A] (verified)	3854
Maple [A] (verified)	3854
Fricas [A] (verification not implemented)	3854
Sympy [F]	3855
Maxima [B] (verification not implemented)	3855
Giac [A] (verification not implemented)	3855
Mupad [B] (verification not implemented)	3856

Optimal result

Integrand size = 25, antiderivative size = 35

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{2 \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

[Out] $2*\sin(d*x+c)/d/(1-\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2850}

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{2 \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

[In] $\text{Int}[\text{Sqrt}[1 - \text{Cos}[c + d*x]]/\text{Cos}[c + d*x]^{(3/2)}, x]$

[Out] $(2*\text{Sin}[c + d*x])/(d*\text{Sqrt}[1 - \text{Cos}[c + d*x]]*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2850

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[-2*b^2*(\text{Cos}[e + f*x]/(f*(b*c + a*d)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])), x] /;$ $\text{FreeQ}\{a, b, c, d, e, f\}, x$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{EqQ}[a^2 - b^2, 0]$ && $\text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\text{integral} = \frac{2 \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2\sqrt{1 - \cos(c + dx)} \cot\left(\frac{1}{2}(c + dx)\right)}{d\sqrt{\cos(c + dx)}}$$

[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(3/2), x]

[Out] (2*Sqrt[1 - Cos[c + d*x]]*Cot[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\sqrt{-2 \cos(dx+c)+2} (\cot(dx+c)+\csc(dx+c))\sqrt{2}}{d\sqrt{\cos(dx+c)}}$	41

[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/d*(-2*cos(d*x+c)+2)^(1/2)/cos(d*x+c)^(1/2)*(cot(d*x+c)+csc(d*x+c))*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2(\cos(dx + c) + 1)\sqrt{-\cos(dx + c) + 1}}{d\sqrt{\cos(dx + c)}\sin(dx + c)}$$

[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="fricas")

[Out] 2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)/(d*sqrt(cos(d*x + c))*sin(d*x + c))

Sympy [F]

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2), x)

[Out] Integral(sqrt(1 - cos(c + d*x))/cos(c + d*x)**(3/2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(31) = 62.

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2 \left(\sqrt{2} - \frac{\sqrt{2} \sin(dx+c)^2}{(\cos(dx+c)+1)^2} \right)}{d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{3}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{3}{2}}}$$

[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] 2*(sqrt(2) - sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = -\frac{2\sqrt{2} \left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 1 \right) \operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{\sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1} d}$$

[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="giac")

[Out] -2*sqrt(2)*(tan(1/4*d*x + 1/4*c)^2 - 1)*sgn(sin(1/2*d*x + 1/2*c))/(sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)

Mupad [B] (verification not implemented)

Time = 14.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{1 - \cos(c + dx)}}$$

[In] `int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)`

[Out] `(2*sin(c + d*x))/(d*cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2))`

$$3.273 \quad \int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	3857
Rubi [A] (verified)	3857
Mathematica [A] (verified)	3858
Maple [A] (verified)	3858
Fricas [A] (verification not implemented)	3859
Sympy [F]	3859
Maxima [B] (verification not implemented)	3859
Giac [A] (verification not implemented)	3860
Mupad [B] (verification not implemented)	3860

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2 \sin(c+dx)}{3d\sqrt{1-\cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)} - \frac{4 \sin(c+dx)}{3d\sqrt{1-\cos(c+dx)} \sqrt{\cos(c+dx)}}$$

[Out] $2/3*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(1-\cos(d*x+c))^{(1/2)}-4/3*\sin(d*x+c)/d/(1-\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2851, 2850}

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2 \sin(c+dx)}{3d\sqrt{1-\cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)} - \frac{4 \sin(c+dx)}{3d\sqrt{1-\cos(c+dx)} \sqrt{\cos(c+dx)}}$$

[In] $\text{Int}[\text{Sqrt}[1 - \text{Cos}[c + d*x]]/\text{Cos}[c + d*x]^{(5/2)}, x]$

[Out] $(2*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[1 - \text{Cos}[c + d*x]]*\text{Cos}[c + d*x]^{(3/2)}) - (4*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[1 - \text{Cos}[c + d*x]]*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 2850

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2 \sin(c + dx)}{3d \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} - \frac{2}{3} \int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2 \sin(c + dx)}{3d \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} - \frac{4 \sin(c + dx)}{3d \sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = -\frac{2\sqrt{1 - \cos(c + dx)}(-1 + 2\cos(c + dx)) \cot\left(\frac{1}{2}(c + dx)\right)}{3d \cos^{\frac{3}{2}}(c + dx)}$$

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(5/2), x]
```

```
[Out] (-2*Sqrt[1 - Cos[c + d*x]]*(-1 + 2*Cos[c + d*x])*Cot[(c + d*x)/2])/(3*d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{\csc(dx+c)\sqrt{-2\cos(dx+c)+2}(-1+2(\cos^2(dx+c)+\cos(dx+c))\sqrt{2}}{3d\cos(dx+c)^{\frac{3}{2}}}$	53

[In] `int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $-1/3/d*\csc(d*x+c)*(-2*\cos(d*x+c)+2)^(1/2)*(-1+2*\cos(d*x+c)^2+\cos(d*x+c))/\cos(d*x+c)^(3/2)*2^(1/2)$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = -\frac{2(2 \cos(dx + c)^2 + \cos(dx + c) - 1) \sqrt{-\cos(dx + c) + 1}}{3 d \cos(dx + c)^{\frac{3}{2}} \sin(dx + c)}$$

[In] `integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] $-2/3*(2*\cos(d*x + c)^2 + \cos(d*x + c) - 1)*\sqrt{-\cos(d*x + c) + 1}/(d*\cos(d*x + c)^(3/2)*\sin(d*x + c))$

Sympy [F]

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

[In] `integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)`

[Out] `Integral(sqrt(1 - cos(c + d*x))/cos(c + d*x)**(5/2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(63) = 126.

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.19

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = -\frac{2 \left(\sqrt{2} - \frac{4\sqrt{2}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3\sqrt{2}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^2}{3d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{5}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{5}{2}} \left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 1 \right)}$$

[In] `integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] $-2/3*(\sqrt{2} - 4*\sqrt{2}*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 3*\sqrt{2}*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4)*(\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 1)^2/(d*(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)^(5/2)*(-\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)^(5/2)*(2*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + \sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + 1))$

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{2\sqrt{2} \left(\left(\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 - 15 \right) \tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 + 15 \right) \tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 - 1}{3 \left(\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^4 - 6 \tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 + 1 \right)^{\frac{3}{2}}} \operatorname{sgn}\left(\sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right) dx$$

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*(((tan(1/4*d*x + 1/4*c)^2 - 15)*tan(1/4*d*x + 1/4*c)^2 + 15)*tan(1/4*d*x + 1/4*c)^2 - 1)*sgn(sin(1/2*d*x + 1/2*c))/((tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(3/2)*d)
```

Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{4\sqrt{1 - \cos(c + dx)} (\sin(c + dx) - \sin(2c + 2dx) + \sin(3c + 3dx))}{3d\sqrt{\cos(c + dx)} (3\cos(c + dx) - 2\cos(2c + 2dx) + \cos(3c + 3dx) - 2)}$$

```
[In] int((1 - cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] (4*(1 - cos(c + d*x))^(1/2)*(sin(c + d*x) - sin(2*c + 2*d*x) + sin(3*c + 3*d*x)))/(3*d*cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x) - 2))
```


$$3.274 \quad \int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	3861
Rubi [A] (verified)	3861
Mathematica [A] (verified)	3863
Maple [A] (verified)	3863
Fricas [A] (verification not implemented)	3863
Sympy [F(-1)]	3864
Maxima [B] (verification not implemented)	3864
Giac [A] (verification not implemented)	3864
Mupad [B] (verification not implemented)	3865

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{2 \sin(c+dx)}{5d\sqrt{1-\cos(c+dx)} \cos^{\frac{5}{2}}(c+dx)} - \frac{8 \sin(c+dx)}{15d\sqrt{1-\cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)} + \frac{16 \sin(c+dx)}{15d\sqrt{1-\cos(c+dx)} \sqrt{\cos(c+dx)}}$$

[Out] 2/5*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2)-8/15*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2)+16/15*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2851, 2850}

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx = -\frac{8 \sin(c+dx)}{15d\sqrt{1-\cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)} + \frac{2 \sin(c+dx)}{5d\sqrt{1-\cos(c+dx)} \cos^{\frac{5}{2}}(c+dx)} + \frac{16 \sin(c+dx)}{15d\sqrt{1-\cos(c+dx)} \sqrt{\cos(c+dx)}}$$

[In] Int[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]

[Out] (2*Sin[c + d*x])/(5*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)) - (8*Sin[c + d*x])/(15*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) + (16*Sin[c + d*x])/(15*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])

Rule 2850

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \sin(c + dx)}{5d\sqrt{1 - \cos(c + dx)} \cos^{\frac{5}{2}}(c + dx)} - \frac{4}{5} \int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2 \sin(c + dx)}{5d\sqrt{1 - \cos(c + dx)} \cos^{\frac{5}{2}}(c + dx)} \\
 &\quad - \frac{8 \sin(c + dx)}{15d\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} + \frac{8}{15} \int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2 \sin(c + dx)}{5d\sqrt{1 - \cos(c + dx)} \cos^{\frac{5}{2}}(c + dx)} - \frac{8 \sin(c + dx)}{15d\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} \\
 &\quad + \frac{16 \sin(c + dx)}{15d\sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{2\sqrt{1 - \cos(c + dx)}(3 - 4\cos(c + dx) + 8\cos^2(c + dx)) \cot\left(\frac{1}{2}(c + dx)\right)}{15d \cos^{\frac{5}{2}}(c + dx)}$$

[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(7/2), x]

[Out] (2*Sqrt[1 - Cos[c + d*x]]*(3 - 4*Cos[c + d*x] + 8*Cos[c + d*x]^2)*Cot[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 5.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\csc(dx+c)\sqrt{-2\cos(dx+c)+2}(3+8(\cos^3(dx+c))+4(\cos^2(dx+c))-\cos(dx+c))\sqrt{2}}{15d \cos(dx+c)^{\frac{5}{2}}}$	65

[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] 1/15/d*csc(d*x+c)*(-2*cos(d*x+c)+2)^(1/2)*(3+8*cos(d*x+c)^3+4*cos(d*x+c)^2-cos(d*x+c))/cos(d*x+c)^(5/2)*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{2(8\cos(dx+c)^3 + 4\cos(dx+c)^2 - \cos(dx+c) + 3)\sqrt{-\cos(dx+c) + 1}}{15d \cos(dx+c)^{\frac{5}{2}} \sin(dx+c)}$$

[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2), x, algorithm="fricas")

[Out] 2/15*(8*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(-cos(d*x + c) + 1)/(d*cos(d*x + c)^(5/2)*sin(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(94) = 188.

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{2 \left(7\sqrt{2} - \frac{17\sqrt{2}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{25\sqrt{2}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{15\sqrt{2}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^3}{15 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{7}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{7}{2}} \left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 1 \right)}$$

[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 2/15*(7*sqrt(2) - 17*sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt(2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 15*sqrt(2)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))

Giac [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx =$$

$$\frac{2\sqrt{2} \left(\left(\left(\left(7 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 75 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 430 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 430 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right) \right)}{15 \left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1 \right)^{\frac{5}{2}} d}$$

[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] $-2/15\sqrt{2} * (((((7*\tan(1/4*d*x + 1/4*c)^2 - 75)*\tan(1/4*d*x + 1/4*c)^2 + 430)*\tan(1/4*d*x + 1/4*c)^2 - 430)*\tan(1/4*d*x + 1/4*c)^2 + 75)*\tan(1/4*d*x + 1/4*c)^2 - 7)*\text{sgn}(\sin(1/2*d*x + 1/2*c)) / ((\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1)^{(5/2)*d)}$

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{1 - \cos(c + dx)}}{\cos^{7/2}(c + dx)} dx$$

$$= \frac{8 \sqrt{2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2} (7 \sin(c + dx) - 4 \sin(2c + 2dx) + 9 \sin(3c + 3dx) - 2 \sin(4c + 4dx) + 2 \sin(5c + 5dx))}{15d \sqrt{1 - 2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2} \left(-16 \sin(c + dx)^2 - 4 \sin(2c + 2dx)^2 + 20 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 10 \sin\left(\frac{3c}{2} + \frac{3dx}{2}\right)\right)}$$

[In] $\text{int}((1 - \cos(c + d*x))^{(1/2)}/\cos(c + d*x)^{(7/2)}, x)$

[Out] $(8*(2*\sin(c/2 + (d*x)/2)^2)^{(1/2)}*(7*\sin(c + d*x) - 4*\sin(2*c + 2*d*x) + 9*\sin(3*c + 3*d*x) - 2*\sin(4*c + 4*d*x) + 2*\sin(5*c + 5*d*x)))/(15*d*(1 - 2*\sin(c/2 + (d*x)/2)^2)^{(1/2)}*(20*\sin(c/2 + (d*x)/2)^2 - 4*\sin(2*c + 2*d*x)^2 + 10*\sin((3*c)/2 + (3*d*x)/2)^2 + 2*\sin((5*c)/2 + (5*d*x)/2)^2 - 16*\sin(c + d*x)^2))$

$$3.275 \quad \int \frac{\cos^5(c+dx)}{\sqrt{a-a \cos(c+dx)}} dx$$

Optimal result	3866
Rubi [A] (verified)	3866
Mathematica [C] (verified)	3869
Maple [A] (verified)	3869
Fricas [A] (verification not implemented)	3870
Sympy [F(-1)]	3870
Maxima [F]	3871
Giac [B] (verification not implemented)	3871
Mupad [F(-1)]	3872

Optimal result

Integrand size = 26, antiderivative size = 185

$$\int \frac{\cos^5(c+dx)}{\sqrt{a-a \cos(c+dx)}} dx = \frac{7 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{4\sqrt{ad}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{a-a \cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{a-a \cos(c+dx)}}$$

[Out] 7/4*arctanh(sin(d*x+c)*a^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a-a*cos(d*x+c))^(1/2)+1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a-a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used

= {2857, 3062, 3061, 2861, 214, 2854, 213}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}} dx = \frac{7\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4\sqrt{ad}} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} + \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a-a\cos(c+dx)}}$$

[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a - a*Cos[c + d*x]], x]

[Out] (7*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])]/(4*Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a - a*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a - a*Cos[c + d*x]]))

Rule 213

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 214

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2854

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2857

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1)), Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -

$b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2861

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)]]), x_Symbol] \rightarrow \text{Dist}[-2*(a/f), \text{Subst}[\text{Int}[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(\text{Cos}[e + f*x]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rule 3061

$\text{Int}[(A_) + (B_)*\sin[(e_) + (f_)*(x_)]]/(\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)]]), x_Symbol] \rightarrow \text{Dist}[(A*b - a*B)/b, \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] + \text{Dist}[B/b, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[c + d*\text{Sin}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rule 3062

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-B)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{m_}*(c + d*\text{Sin}[e + f*x])^{n_}/(f*(m + n + 1)), x] + \text{Dist}[1/(b*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m_}*(c + d*\text{Sin}[e + f*x])^{n_ - 1}*\text{Simp}[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[n, 0] \&\& (\text{IntegerQ}[n] \mid\mid \text{EqQ}[m + 1/2, 0])$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a - a \cos(c + dx)}} + \frac{\int \frac{\sqrt{\cos(c+dx)}(3a+a \cos(c+dx))}{\sqrt{a-a \cos(c+dx)}} dx}{4a} \\ &= \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a - a \cos(c + dx)}} + \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a - a \cos(c + dx)}} - \frac{\int \frac{-\frac{a^2}{2} - \frac{7}{2}a^2 \cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a \cos(c+dx)}} dx}{4a^2} \\ &= \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{a - a \cos(c + dx)}} + \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{a - a \cos(c + dx)}} \\ &\quad - \frac{7 \int \frac{\sqrt{a-a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{8a} + \int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{a - a \cos(c + dx)}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} \\
&\quad - \frac{7\text{Subst}\left(\int \frac{1}{-a+x^2} dx, x, \frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4d} \\
&\quad - \frac{(2a)\text{Subst}\left(\int \frac{1}{2a^2-ax^2} dx, x, \frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} \\
&= \frac{7\text{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4\sqrt{ad}} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}} \\
&\quad + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.38

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}} dx = \frac{ie^{-2i(c+dx)}(-1+e^{i(c+dx)})\left(7\sqrt{2}e^{2i(c+dx)}\text{arcsinh}(e^{i(c+dx)})-16e^{2i(c+dx)}\text{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)\right)+\sqrt{2}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{8\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}}$$

[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a - a*Cos[c + d*x]],x]

[Out] ((-1/8*I)*(-1 + E^(I*(c + d*x))))*(7*Sqrt[2]*E^((2*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))] - 16*E^((2*I)*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) + Sqrt[2]*(Sqrt[1 + E^((2*I)*(c + d*x))])*(1 + 2*E^(I*(c + d*x)) + 2*E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x))) + 7*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[Cos[c + d*x]])/(Sqrt[2]*d*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Cos[c + d*x]])

Maple [A] (verified)

Time = 12.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.10

method	result
default	$\frac{\sin(dx+c)\left(2(\cos^2(dx+c))\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\cos(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7\sqrt{2}\arctanh\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}{8d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-a(\cos(dx+c)-1)}}$

[In] `int(cos(d*x+c)^(5/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{8}d\sin(dx+c)\cdot(2\cos(dx+c)^2)^{1/2}\cdot(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} + 3\cos(dx+c)\cdot 2^{1/2}\cdot(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} + 7\cdot 2^{1/2}\cdot\operatorname{arctanh}(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} + 2^{1/2}\cdot(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} - 8\cdot\operatorname{arctanh}(1/2\cdot 2^{1/2}/(\cos(dx+c)/(1+\cos(dx+c)))^{1/2})\cdot\cos(dx+c)^{1/2}/(1+\cos(dx+c))/(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}/(-a\cdot(\cos(dx+c)-1))^{1/2}\cdot 2^{1/2}$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}} dx$$

$$= \frac{4\sqrt{2}\sqrt{a}\log\left(-\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}-\sqrt{a}(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+7\sqrt{a}\log\left(-\frac{2\sqrt{-a\cos(dx+c)+a}}{\cos(dx+c)-1}\right)\sin(dx+c)}{\sin(dx+c)}$$

[In] `integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{8}\cdot(4\sqrt{2}\sqrt{a}\log(-2\sqrt{2}\sqrt{-a\cos(dx+c)+a}\cdot(\cos(dx+c)+1)\sqrt{\cos(dx+c)})/\sqrt{a}-(3\cos(dx+c)+1)\sin(dx+c))/((\cos(dx+c)-1)\sin(dx+c))\cdot\sin(dx+c)+7\sqrt{a}\log(-2\sqrt{-a\cos(dx+c)+a}\sqrt{a}\cdot(\cos(dx+c)+1)\sqrt{\cos(dx+c)})+(2a\cos(dx+c)+a)\sin(dx+c)/\sin(dx+c)\cdot\sin(dx+c)+2\sqrt{-a\cos(dx+c)+a}\cdot(2\cos(dx+c)^2+3\cos(dx+c)+1)\sqrt{\cos(dx+c)})/(a\cdot d\sin(dx+c))$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(5/2)/(a-a*cos(d*x+c))**(1/2),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{a - a \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{\sqrt{-a \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/sqrt(-a*cos(d*x + c) + a), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. 2(152) = 304.

Time = 6.91 (sec) , antiderivative size = 779, normalized size of antiderivative = 4.21

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{a - a \cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] $\frac{1}{4} * (2 * \sqrt{2} * \log(\tan(1/4 * d * x + 1/4 * c)^2 - \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1} + 1) / (\sqrt{a} * \text{sgn}(\sin(1/2 * d * x + 1/2 * c))) - 2 * \sqrt{2} * \log(\text{abs}(-\tan(1/4 * d * x + 1/4 * c)^2 + \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1} + 3)) / (\sqrt{a} * \text{sgn}(\sin(1/2 * d * x + 1/2 * c))) - 2 * \sqrt{2} * \log(\text{abs}(-\tan(1/4 * d * x + 1/4 * c)^2 + \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1} + 1)) / (\sqrt{a} * \text{sgn}(\sin(1/2 * d * x + 1/2 * c))) + 7 * \log(1/8 * \text{abs}(8 * \tan(1/4 * d * x + 1/4 * c)^2 - 16 * \sqrt{2} - 8 * \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1} + 8)) / (\tan(1/4 * d * x + 1/4 * c)^2 + 2 * \sqrt{2} - \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1} + 1)) / (\sqrt{a} * \text{sgn}(\sin(1/2 * d * x + 1/2 * c))) - 4 * \sqrt{2} * (17 * (\tan(1/4 * d * x + 1/4 * c)^2 - \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1})^7 * \sqrt{a} - 73 * (\tan(1/4 * d * x + 1/4 * c)^2 - \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1})^6 * \sqrt{a} + 157 * (\tan(1/4 * d * x + 1/4 * c)^2 - \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1})^5 * \sqrt{a} - 597 * (\tan(1/4 * d * x + 1/4 * c)^2 - \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1})^4 * \sqrt{a} + 1603 * (\tan(1/4 * d * x + 1/4 * c)^2 - \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1})^3 * \sqrt{a} - 875 * (\tan(1/4 * d * x + 1/4 * c)^2 - \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1})^2 * \sqrt{a} - 1585 * (\tan(1/4 * d * x + 1/4 * c)^2 - \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1}) * \sqrt{a} + 1737 * \sqrt{a}) / (((\tan(1/4 * d * x + 1/4 * c)^2 - \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1})^2 + 2 * \tan(1/4 * d * x + 1/4 * c)^2 - 2 * \sqrt{\tan(1/4 * d * x + 1/4 * c)^4 - 6 * \tan(1/4 * d * x + 1/4 * c)^2 + 1} - 7)^4 * a * \text{sgn}(\sin(1/2 * d * x + 1/2 * c)))) / d$

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{a - a \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{5/2}}{\sqrt{a - a \cos(c + dx)}} dx$$

```
[In] int(cos(c + d*x)^(5/2)/(a - a*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a - a*cos(c + d*x))^(1/2), x)
```

$$3.276 \quad \int \frac{\cos^3(c+dx)}{\sqrt{a-a \cos(c+dx)}} dx$$

Optimal result	3873
Rubi [A] (verified)	3873
Mathematica [C] (verified)	3876
Maple [A] (verified)	3876
Fricas [A] (verification not implemented)	3877
Sympy [F]	3877
Maxima [F]	3877
Giac [B] (verification not implemented)	3878
Mupad [F(-1)]	3878

Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{\cos^3(c+dx)}{\sqrt{a-a \cos(c+dx)}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d \sqrt{a-a \cos(c+dx)}}$$

[Out] $\operatorname{arctanh}(\sin(d*x+c)*a^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a-a*\cos(d*x+c))^{(1/2)})/d/a^{(1/2)} - \operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a-a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)} + \sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(a-a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

= {2857, 3061, 2861, 214, 2854, 213}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a - a*Cos[c + d*x]],x]

[Out] ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])]/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a - a*Cos[c + d*x]])

Rule 213

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 214

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2854

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2857

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n-1)/(f*(2*n-1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n-1)), Int[((c + d*Sin[e + f*x])^(n-2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d - b*(2*d^2*(n-1) + c^2*(2*n-1)) + d*(a*d - b*c*(4*n-3))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3061

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} + \frac{\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx}{2a} \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} - \frac{\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{2a} + \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{-a+x^2} dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} \\
 &\quad - \frac{(2a)\text{Subst}\left(\int \frac{1}{2a^2-ax^2} dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} \\
 &= \frac{\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}} \\
 &\quad - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.62

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}} dx = \frac{ie^{-i(c+dx)}(-1+e^{i(c+dx)})\left(\sqrt{2}e^{i(c+dx)}\operatorname{arcsinh}(e^{i(c+dx)})-4e^{i(c+dx)}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\left(1+e^{i(c+dx)}\right)\right)}{2\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+dx)}}$$

[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a - a*Cos[c + d*x]], x]

[Out] $((-1/2*I)*(-1 + E^{(I*(c + d*x))})*(\operatorname{Sqrt}[2]*E^{(I*(c + d*x))}*\operatorname{ArcSinh}[E^{(I*(c + d*x))}] - 4*E^{(I*(c + d*x))}*\operatorname{ArcTanh}[(1 + E^{(I*(c + d*x))}]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}]]]) + \operatorname{Sqrt}[2]*((1 + E^{(I*(c + d*x))})*\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}] + E^{(I*(c + d*x))}*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}]])))*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]/(\operatorname{Sqrt}[2]*d*E^{(I*(c + d*x))}*\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}])* \operatorname{Sqrt}[a - a*\operatorname{Cos}[c + d*x]]]$

Maple [A] (verified)

Time = 13.86 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sin(dx+c)\left(\cos(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{2}\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)-2\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\right)(\sqrt{\cos(dx+c)})}{2d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-a(\cos(dx+c)-1)}}$

[In] int(cos(d*x+c)^(3/2)/(a-cos(d*x+c)*a)^(1/2), x, method=_RETURNVERBOSE)

[Out] $1/2/d*\sin(d*x+c)*(\cos(d*x+c)*2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}+2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}+2^{(1/2)}*\operatorname{arctanh}((\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)})-2*\operatorname{arctanh}(1/2*2^{(1/2)}/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}))*\cos(d*x+c)^{(1/2)}/(1+\cos(d*x+c))/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}/(-a*(\cos(d*x+c)-1))^{(1/2)}*2^{(1/2)}$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.50

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a - a \cos(c + dx)}} dx$$

$$= \frac{\sqrt{2}\sqrt{a} \log\left(-\frac{2\sqrt{2}\sqrt{-a \cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)} - (3 \cos(dx+c)+1) \sin(dx+c)}{(\cos(dx+c)-1) \sin(dx+c)}\right) \sin(dx+c) + \sqrt{a} \log\left(-\frac{2\sqrt{-a \cos(dx+c)+a}}{\cos(dx+c)-1}\right)}{\sin(dx+c)}$$

[In] integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + sqrt(a)*log(-(2*sqrt(-a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(a*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a - a \cos(c + dx)}} dx = \int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{-a (\cos(c + dx) - 1)}} dx$$

[In] integrate(cos(d*x+c)**(3/2)/(a-a*cos(d*x+c))**(1/2),x)

[Out] Integral(cos(c + d*x)**(3/2)/sqrt(-a*(cos(c + d*x) - 1)), x)

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a - a \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{\sqrt{-a \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/sqrt(-a*cos(d*x + c) + a), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(118) = 236.

Time = 1.01 (sec) , antiderivative size = 581, normalized size of antiderivative = 4.12

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a - a \cos(c + dx)}} dx$$

$$\frac{\sqrt{2} \log\left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1}\right)}{\sqrt{a} \operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{\sqrt{2} \log\left(-\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1}\right)}{\sqrt{a} \operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}$$

[In] integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/2*(sqrt(2)*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) - sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3))/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) - sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1))/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) + 2*log(1/2*abs(2*tan(1/4*d*x + 1/4*c)^2 - 4*sqrt(2) - 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 2))/(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1))/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))) - 8*sqrt(2)*(3*(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^3*sqrt(a) - 7*(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^2*sqrt(a) + (tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))*sqrt(a) + 11*sqrt(a))/(((tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1))^2 + 2*tan(1/4*d*x + 1/4*c)^2 - 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - 7)^2*a*sgn(sin(1/2*d*x + 1/2*c))))/d

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a - a \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{3/2}}{\sqrt{a - a \cos(c + dx)}} dx$$

[In] int(cos(c + d*x)^(3/2)/(a - a*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(3/2)/(a - a*cos(c + d*x))^(1/2), x)

$$3.277 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a \cos(c+dx)}} dx$$

Optimal result	3879
Rubi [A] (verified)	3879
Mathematica [C] (verified)	3881
Maple [A] (verified)	3881
Fricas [A] (verification not implemented)	3882
Sympy [F]	3882
Maxima [F]	3882
Giac [B] (verification not implemented)	3883
Mupad [F(-1)]	3883

Optimal result

Integrand size = 26, antiderivative size = 107

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a \cos(c+dx)}} dx = \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}}$$

[Out] $2 * \operatorname{arctanh}(\sin(d*x+c) * a^{(1/2)} / \cos(d*x+c)^{(1/2)} / (a - a * \cos(d*x+c))^{(1/2)}) / d / a^{(1/2)} - \operatorname{arctanh}(1/2 * \sin(d*x+c) * a^{(1/2)} * 2^{(1/2)} / \cos(d*x+c)^{(1/2)} / (a - a * \cos(d*x+c))^{(1/2)}) * 2^{(1/2)} / d / a^{(1/2)}$

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2856, 2854, 213, 2861, 214}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a \cos(c+dx)}} dx = \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]] / \operatorname{Sqrt}[a - a * \operatorname{Cos}[c + d*x]], x]$

[Out] $(2 * \operatorname{ArcTanh}[(\operatorname{Sqrt}[a] * \operatorname{Sin}[c + d*x]) / (\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]] * \operatorname{Sqrt}[a - a * \operatorname{Cos}[c + d*x]])]) / (\operatorname{Sqrt}[a] * d) - (\operatorname{Sqrt}[2] * \operatorname{ArcTanh}[(\operatorname{Sqrt}[a] * \operatorname{Sin}[c + d*x]) / (\operatorname{Sqrt}[2] * \operatorname{Sqrt}[\operatorname{Cos}[c + d*x]] * \operatorname{Sqrt}[a - a * \operatorname{Cos}[c + d*x]])]) / (\operatorname{Sqrt}[a] * d)$

Rule 213

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 214

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2854

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2856

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{a} + \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx \\ &= -\frac{2\text{Subst}\left(\int \frac{1}{-a+x^2} dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} \\ &\quad -\frac{(2a)\text{Subst}\left(\int \frac{1}{2a^2-ax^2} dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} \end{aligned}$$

$$= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}} dx = \frac{i(-1+e^{i(c+dx)})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}\left(\operatorname{arcsinh}(e^{i(c+dx)})-\sqrt{2}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)\right)+\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+dx)}}$$

[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a - a*Cos[c + d*x]],x]

[Out] ((-I)*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]]) + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Cos[c + d*x]])

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\sin(dx+c)\left(-\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)+\sqrt{2}\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)(\sqrt{\cos(dx+c)})\sqrt{2}}{d(1+\cos(dx+c))\sqrt{-a(\cos(dx+c)-1)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	116

[In] int(cos(d*x+c)^(1/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d*sin(d*x+c)*(-arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2^(1/2)*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(-a*(cos(d*x+c)-1))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}} dx$$

$$= \frac{\sqrt{2}\sqrt{a} \log\left(-\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a(\cos(dx+c)+1)}\sqrt{\cos(dx+c)} - (3\cos(dx+c)+1)\sin(dx+c)}{\sqrt{a}(\cos(dx+c)-1)\sin(dx+c)}\right) + 2\sqrt{a} \log\left(-\frac{2\sqrt{-a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{2ad}$$

```
[In] integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))) + 2*sqrt(a)*log(-(2*sqrt(-a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)))/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-a(\cos(c+dx)-1)}} dx$$

```
[In] integrate(cos(d*x+c)**(1/2)/(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(-a*(cos(c + d*x) - 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-a\cos(dx+c)+a}} dx$$

```
[In] integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-a*cos(d*x + c) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(88) = 176.

Time = 0.67 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.51

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}} dx = \sqrt{2} \left(2\sqrt{2} \log \left(\frac{2 \left(\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 + 2\sqrt{2} - \sqrt{\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^4 - 6\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 + 1} \right)}{\left| -2\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 + 4\sqrt{2} + 2\sqrt{\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^4 - 6\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 + 1} \right|} \right) - \log \left(\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 - \sqrt{\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^4 - 6\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 + 1} \right) \right)$$

[In] integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*(2*sqrt(2)*log(2*(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/abs(-2*tan(1/4*d*x + 1/4*c)^2 + 4*sqrt(2) + 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - 2)) - log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1) + log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) + log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)))/(sqrt(a)*d*sgn(sin(1/2*d*x + 1/2*c)))

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}} dx$$

[In] int(cos(c + d*x)^(1/2)/(a - a*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)/(a - a*cos(c + d*x))^(1/2), x)

$$3.278 \quad \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx$$

Optimal result	3884
Rubi [A] (verified)	3884
Mathematica [C] (verified)	3885
Maple [A] (verified)	3885
Fricas [A] (verification not implemented)	3886
Sympy [F]	3886
Maxima [C] (verification not implemented)	3886
Giac [B] (verification not implemented)	3887
Mupad [F(-1)]	3887

Optimal result

Integrand size = 26, antiderivative size = 58

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}}$$

[Out] $-\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)/(a-a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)})$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2861, 214}

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a-a*\operatorname{Cos}[c+d*x]]),x]$

[Out] $-\left(\left(\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[\left(\operatorname{Sqrt}[a]*\operatorname{Sin}[c+d*x]\right)/\left(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a-a*\operatorname{Cos}[c+d*x]]\right)]\right)\right)/\left(\operatorname{Sqrt}[a]*d\right)$

Rule 214

$\operatorname{Int}[\left((a_.) + (b_.)*(x_)^2\right)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[\left(\operatorname{Rt}[-a/b, 2]/a\right)*\operatorname{ArcTanh}[x/\operatorname{Rt}[-a/b, 2]], x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 2861


```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{2a^2-ax^2} dx, x, \frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a \cos(c+dx)}}\right)}{d} \\ &= -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}} \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.03

$$\begin{aligned} &\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a \cos(c+dx)}} dx \\ &= \frac{i(-1 + e^{i(c+dx)}) \sqrt{e^{-i(c+dx)}(1 + e^{2i(c+dx)})} \text{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{d\sqrt{1 + e^{2i(c+dx)}}\sqrt{a - a \cos(c + dx)}} \end{aligned}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]),x]

[Out] (I*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.99 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}}{d\sqrt{\cos(dx+c)}\sqrt{-a(\cos(dx+c)-1)}}$	80

[In] int(1/cos(d*x+c)^(1/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] $-1/d*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\operatorname{arctanh}(1/2*2^{(1/2)}/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)})/\cos(d*x+c)^{(1/2)}/(-a*(\cos(d*x+c)-1))^{(1/2)}*2^{(1/2)}$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.48

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx$$

$$= \left[\frac{\sqrt{2} \log \left(-\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a(\cos(dx+c)+1)}\sqrt{\cos(dx+c)} - (3\cos(dx+c)+1)\sin(dx+c)}{\sqrt{a}(\cos(dx+c)-1)\sin(dx+c)} \right)}{2\sqrt{ad}}, \frac{\sqrt{2}\sqrt{-\frac{1}{a}} \operatorname{arctan} \left(\frac{\sqrt{2}\sqrt{-a\cos(dx+c)+a}\sqrt{\sin(dx+c)}}{\sin(dx+c)} \right)}{d} \right]$$

[In] `integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $[1/2*\sqrt{2}*\log(-2*\sqrt{2}*\sqrt{-a*\cos(d*x+c)+a}*(\cos(d*x+c)+1)*\sqrt{\cos(d*x+c)}/\sqrt{a} - (3*\cos(d*x+c)+1)*\sin(d*x+c))/((\cos(d*x+c)-1)*\sin(d*x+c)))/(\sqrt{a}*d), \sqrt{2}*\sqrt{-1/a}*\operatorname{arctan}(\sqrt{2}*\sqrt{-a*\cos(d*x+c)+a}*\sqrt{-1/a}*\sqrt{\cos(d*x+c)}/\sin(d*x+c))/d]$

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-a(\cos(c+dx)-1)}\sqrt{\cos(c+dx)}} dx$$

[In] `integrate(1/cos(d*x+c)**(1/2)/(a-a*cos(d*x+c))**(1/2),x)`

[Out] `Integral(1/(sqrt(-a*(cos(c+d*x)-1))*sqrt(cos(c+d*x))), x)`

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.60

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx =$$

$$\frac{\sqrt{2} \operatorname{arctan} \left(\frac{2\sqrt{2}(\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c)+1)^{\frac{1}{4}} \sin(\frac{1}{2} \operatorname{arctan}(\sin(2dx+2c), \cos(2dx+2c)+1))}{\sqrt{a}|e^{i(dx+ic)}-1|} \right)}{\sqrt{-ad}}$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -sqrt(2)*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt(-a)*abs(e^(I*d*x + I*c) - 1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1)))/(sqrt(-a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(47) = 94.

Time = 0.57 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx$$

$$= \frac{\sqrt{2} \left(\log \left(\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 - \sqrt{\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^4 - 6 \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 + 1 + 1} \right) - \log \left(\left| -\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right) \right| \right) \right)}{\sqrt{a}}$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(2)*(log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1) - log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) - log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)))/(sqrt(a)*d*sgn(sin(1/2*d*x + 1/2*c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

$$3.279 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx$$

Optimal result	3888
Rubi [A] (verified)	3888
Mathematica [C] (verified)	3890
Maple [A] (verified)	3890
Fricas [A] (verification not implemented)	3891
Sympy [F]	3891
Maxima [C] (verification not implemented)	3891
Giac [B] (verification not implemented)	3892
Mupad [F(-1)]	3892

Optimal result

Integrand size = 26, antiderivative size = 95

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

[Out] $-\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)/(a-a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)/(a-a*\cos(d*x+c))^{(1/2)})/2)$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2858, 12, 2861, 214}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx = \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}}$$

[In] $\text{Int}[1/(\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[a - a*\text{Cos}[c + d*x]]), x]$

[Out] $-\left(\left(\text{Sqrt}[2]*\text{ArcTanh}[\left(\text{Sqrt}[a]*\text{Sin}[c + d*x]\right)/\left(\text{Sqrt}[2]*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a - a*\text{Cos}[c + d*x]]\right)]\right)/\left(\text{Sqrt}[a]*d\right)\right) + \left(2*\text{Sin}[c + d*x]\right)/\left(d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a - a*\text{Cos}[c + d*x]]\right)$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 214

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2858

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} + \frac{\int \frac{a}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} dx}{a} \\
 &= \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} + \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} dx \\
 &= \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} - \frac{(2a) \text{Subst}\left(\int \frac{1}{2a^2 - ax^2} dx, x, \frac{a \sin(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}\right)}{d} \\
 &= -\frac{\sqrt{2} \arctanh\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2} \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.65

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx$$

$$= \frac{2 \left(-\frac{e^{-\frac{1}{2}i(c+dx)}(1+e^{2i(c+dx)})\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}} + 2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right) \right) \sin\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]),x]

[Out] (2*(-(((1 + E^((2*I)*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x)))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*E^((I/2)*(c + d*x)))) + 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}+\sqrt{2}}\right)\sin(dx+c)\sqrt{2}}{d\sqrt{-a(\cos(dx+c)-1)}\sqrt{\cos(dx+c)}}$	85

[In] int(1/cos(d*x+c)^(3/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(-arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2^(1/2))*sin(d*x+c)/(-a*(cos(d*x+c)-1))^(1/2)/cos(d*x+c)^(1/2)*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.60

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx$$

$$= \frac{\sqrt{2}\sqrt{a}\cos(dx+c)\log\left(-\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a(\cos(dx+c)+1)}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{\sqrt{a}(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4\sqrt{-a}}{2ad\cos(dx+c)\sin(dx+c)}$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] 1/2*(sqrt(2)*sqrt(a)*cos(d*x + c)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)
*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d
*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))*sin(d*x + c) + 4*sqrt(-a*cos(d*
x + c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)*sin(d*
x + c))
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-a(\cos(c+dx)-1)}\cos^{\frac{3}{2}}(c+dx)} dx$$

[In] integrate(1/cos(d*x+c)**(3/2)/(a-a*cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(-a*(cos(c + d*x) - 1))*cos(c + d*x)**(3/2)), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.69

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx$$

$$= \frac{2\cos\left(\frac{1}{2}\arctan\left(\frac{\sin(2dx+2c)}{\cos(2dx+2c)+1}\right)\right)\sin(dx+c)-2(\cos(dx+c)+1)\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(2dx+2c)}{\cos(2dx+2c)+1}\right)\right)}{2ad\cos(dx+c)\sin(dx+c)}$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")

```
[Out] (2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) -
2*(cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt(2)*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt(-a)*abs(e^(I*d*x + I*c)
- 1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1)))/((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(-a)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(80) = 160.

Time = 0.70 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.94

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} dx = \frac{4 \left(\frac{\sqrt{2} \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2}{\sqrt{a} \operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{\sqrt{2}}{\sqrt{a} \operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} \right)}{\sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1}} - \frac{\sqrt{2} \log\left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1}\right)}{\sqrt{a} \operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} + \frac{\sqrt{2} \log\left(\dots\right)}{\dots}$$

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(4*(sqrt(2)*tan(1/4*d*x + 1/4*c)^2/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c)))
- sqrt(2)/(sqrt(a)*sgn(sin(1/2*d*x + 1/2*c))))/sqrt(tan(1/4*d*x + 1/4*c)^4
- 6*tan(1/4*d*x + 1/4*c)^2 + 1) - sqrt(2)*log(tan(1/4*d*x + 1/4*c)^2 - sqr
t(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/(sqrt(a)*sgn(
sin(1/2*d*x + 1/2*c))) + sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan
(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3))/(sqrt(a)*sgn(sin(
1/2*d*x + 1/2*c))) + sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4
*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1))/(sqrt(a)*sgn(sin(1/2*
d*x + 1/2*c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^{3/2} \sqrt{a - a \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(3/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a - a*cos(c + d*x))^(1/2)), x)
```


$$3.280 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a-a \cos(c+dx)}} dx$$

Optimal result	3893
Rubi [A] (verified)	3893
Mathematica [C] (verified)	3895
Maple [A] (verified)	3896
Fricas [A] (verification not implemented)	3896
Sympy [F]	3896
Maxima [C] (verification not implemented)	3897
Giac [B] (verification not implemented)	3897
Mupad [F(-1)]	3898

Optimal result

Integrand size = 26, antiderivative size = 135

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a-a \cos(c+dx)}} dx = -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx) \sqrt{a-a \cos(c+dx)}} + \frac{2 \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}$$

[Out] $-\operatorname{arctanh}(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a-a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}/d/a^{(1/2)}+2/3*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(a-a*\cos(d*x+c))^{(1/2)}+2/3*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(a-a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2858, 3063, 12, 2861, 214}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a-a \cos(c+dx)}} dx = -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx) \sqrt{a-a \cos(c+dx)}} + \frac{2 \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]),x]

[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])]/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) + (2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 214

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2858

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3063

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{\int \frac{a + 2a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} dx}{3a} \\
 &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} \\
 &\quad + \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} - \frac{2 \int -\frac{3a^2}{2 \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} dx}{3a^2} \\
 &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} \\
 &\quad + \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} dx \\
 &= \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} \\
 &\quad - \frac{(2a) \text{Subst}\left(\int \frac{1}{2a^2 - ax^2} dx, x, \frac{a \sin(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}\right)}{d} \\
 &= -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2} \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}\right)}{\sqrt{ad}} \\
 &\quad + \frac{2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{2 \sin(c + dx)}{3d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.27

$$\begin{aligned}
 &\int \frac{1}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} dx \\
 &= \frac{2 \left(-\frac{3e^{-\frac{3}{2}i(c+dx)}(1+e^{2i(c+dx)})^2 \operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{2\sqrt{2}} + 2\sqrt{1+e^{2i(c+dx)}} \cos\left(\frac{1}{2}(c+dx)\right) (1+\cos(c+dx)) \right)}{3d\sqrt{1+e^{2i(c+dx)}} \cos^{\frac{3}{2}}(c+dx) \sqrt{a-a\cos(c+dx)}}
 \end{aligned}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]),x]

[Out] (2*((-3*(1 + E^((2*I)*(c + d*x)))^2*ArcTanh[(1 + E^(I*(c + d*x))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))) + 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x]))*Sin[(c + d*x)/2])/(3*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]])

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\left(-3 \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right) + \sqrt{2} \cos(dx+c) + \sqrt{2}\right) \sin(dx+c) \sqrt{2}}{3d\sqrt{-a(\cos(dx+c)-1)} \cos(dx+c)^{\frac{3}{2}}}$	102

[In] `int(1/cos(d*x+c)^(5/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{3}d*(-3*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*\operatorname{arctanh}(1/2*2^(1/2)/(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2))+2^(1/2)*\cos(d*x+c)+2^(1/2))*\sin(d*x+c)/(-a*(\cos(d*x+c)-1))^(1/2)/\cos(d*x+c)^(3/2)*2^(1/2)}$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.22

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx$$

$$= \frac{3\sqrt{2}\sqrt{a}\cos(dx+c)^2 \log\left(-\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}-\sqrt{a}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right) \sin(dx+c) + 4\sqrt{-a\cos(dx+c)+a(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}}{6ad\cos(dx+c)^2\sin(dx+c)}$$

[In] `integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{6}*(3*\sqrt{2}*\sqrt{a}*\cos(d*x+c)^2*\log(-(2*\sqrt{2}*\sqrt{-a*\cos(d*x+c)+a}*(\cos(d*x+c)+1)*\sqrt{\cos(d*x+c)})/\sqrt{a}-(3*\cos(d*x+c)+1)*\sin(d*x+c))/((\cos(d*x+c)-1)*\sin(d*x+c))*\sin(d*x+c)+4*\sqrt{-a*\cos(d*x+c)+a}*(\cos(d*x+c)^2+2*\cos(d*x+c)+1)*\sqrt{\cos(d*x+c)})/(a*d*\cos(d*x+c)^2*\sin(d*x+c))$

Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-a(\cos(c+dx)-1)} \cos^{\frac{5}{2}}(c+dx)} dx$$

[In] `integrate(1/cos(d*x+c)**(5/2)/(a-a*cos(d*x+c))**(1/2),x)`

[Out] `Integral(1/(sqrt(-a*(cos(c+d*x)-1))*cos(c+d*x)**(5/2)), x)`

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.73

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx =$$

$$3(\sqrt{2}\cos(2dx+2c)^2 + \sqrt{2}\sin(2dx+2c)^2 + 2\sqrt{2}\cos(2dx+2c) + \sqrt{2}) \arctan\left(\frac{2\sqrt{2}(\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2dx+2c), \cos(2dx+2c) + 1))}{\dots}\right)$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - sqrt(-a)*abs(e^(I*d*x + I*c) - 1) + 2*sqrt(a))/(a*abs(e^(I*d*x + I*c) - 1))) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) + 3)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))) - 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(-a)*d)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(112) = 224.

Time = 0.69 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.74

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx =$$

$$\sqrt{2}\left(\frac{8\left(\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-3\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+3\right)\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2-1}{\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^4-6\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2+1\right)^{\frac{3}{2}}}\right) - 3 \log\left(\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2 - \sqrt{\tan\left(\frac{1}{4}dx+\frac{1}{4}c\right)^2}\right)$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")

```
[Out] -1/6*sqrt(2)*(8*(((tan(1/4*d*x + 1/4*c)^2 - 3)*tan(1/4*d*x + 1/4*c)^2 + 3)*
tan(1/4*d*x + 1/4*c)^2 - 1)/(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c
)^2 + 1)^(3/2) - 3*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4
- 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1) + 3*log(abs(-tan(1/4*d*x + 1/4*c)^2 +
sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) + 3*log(
abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x +
1/4*c)^2 + 1) + 1)))/(sqrt(a)*d*sgn(sin(1/2*d*x + 1/2*c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^{\frac{5}{2}} \sqrt{a - a \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(5/2)*(a - a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

$$3.281 \quad \int \frac{1}{\cos^{\frac{7}{2}}(c+dx) \sqrt{a-a \cos(c+dx)}} dx$$

Optimal result	3899
Rubi [A] (verified)	3899
Mathematica [C] (verified)	3902
Maple [A] (verified)	3902
Fricas [A] (verification not implemented)	3903
Sympy [F(-1)]	3903
Maxima [C] (verification not implemented)	3903
Giac [A] (verification not implemented)	3904
Mupad [F(-1)]	3905

Optimal result

Integrand size = 26, antiderivative size = 173

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx) \sqrt{a-a \cos(c+dx)}} dx = -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx) \sqrt{a-a \cos(c+dx)}} + \frac{2 \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx) \sqrt{a-a \cos(c+dx)}} + \frac{26 \sin(c+dx)}{15d \sqrt{\cos(c+dx)} \sqrt{a-a \cos(c+dx)}}$$

```
[Out] -arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+2/5*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2)+2/15*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2)+26/15*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used

= {2858, 3063, 12, 2861, 214}

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a - a*Cos[c + d*x]]), x]

[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]) + (2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 214

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2858

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&

EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{\int \frac{a + 4a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} dx}{5a} \\
 &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} \\
 &\quad + \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} - \frac{2 \int \frac{-\frac{13a^2}{2} - a^2 \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} dx}{15a^2} \\
 &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} \\
 &\quad + \frac{26 \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} + \frac{4 \int \frac{15a^3}{4 \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} dx}{15a^3} \\
 &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} \\
 &\quad + \frac{26 \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} + \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} dx \\
 &= \frac{2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} + \frac{2 \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} \\
 &\quad + \frac{26 \sin(c + dx)}{15d \sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}} \\
 &\quad - \frac{(2a) \text{Subst}\left(\int \frac{1}{2a^2 - ax^2} dx, x, \frac{a \sin(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a - a \cos(c + dx)}}\right)}{d}
 \end{aligned}$$

$$= -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{ad}} + \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.26

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx$$

$$= \frac{e^{-\frac{5}{2}i(c+dx)}\left(2\sqrt{1+e^{2i(c+dx)}}(13+15e^{i(c+dx)}+40e^{2i(c+dx)}+40e^{3i(c+dx)}+15e^{4i(c+dx)}+13e^{5i(c+dx)})-15\sqrt{2}(1+e^{i(c+dx)})\right)}{60d\sqrt{1+e^{2i(c+dx)}}\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a - a*Cos[c + d*x]]),x]

[Out] ((2*Sqrt[1 + E^((2*I)*(c + d*x))])*(13 + 15*E^(I*(c + d*x)) + 40*E^((2*I)*(c + d*x)) + 40*E^((3*I)*(c + d*x)) + 15*E^((4*I)*(c + d*x)) + 13*E^((5*I)*(c + d*x))) - 15*Sqrt[2]*(1 + E^((2*I)*(c + d*x))))^3*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])*Sin[(c + d*x)/2])/(60*d*E^((5*I)/2)*(c + d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]])

Maple [A] (verified)

Time = 6.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{\left(15\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)(\cos^2(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-13\sqrt{2}(\cos^2(dx+c))-\sqrt{2}\cos(dx+c)-3\sqrt{2}\right)\sin(dx+c)\sqrt{2}}{15d\sqrt{-a(\cos(dx+c)-1)}\cos(dx+c)^{\frac{5}{2}}}$	120

[In] int(1/cos(d*x+c)^(7/2)/(a-cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/15/d*(15*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-13*2^(1/2)*cos(d*x+c)^2-2^(1/2)*cos(d*x+c)-3*2^(1/2))*sin(d*x+c)/(-a*(cos(d*x+c)-1))^(1/2)/cos(d*x+c)^(5/2)*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx$$

$$= \frac{15\sqrt{2}\sqrt{a}\cos(dx+c)^3 \log\left(-\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a(\cos(dx+c)+1)}\sqrt{\cos(dx+c)} - (3\cos(dx+c)+1)\sin(dx+c)}{\sqrt{a}(\cos(dx+c)-1)\sin(dx+c)}\right) \sin(dx+c) + 4(1 - \cos(dx+c))}{30ad\cos(dx+c)^3 \sin(dx+c)}$$

[In] integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/30*(15*sqrt(2)*sqrt(a)*cos(d*x + c)^3*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))*sin(d*x + c) + 4*(13*cos(d*x + c)^3 + 14*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt(-a*cos(d*x + c) + a)*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3*sin(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(7/2)/(a-a*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 692, normalized size of antiderivative = 4.00

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} dx = \text{Too large to display}$$

[In] integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/15*(15*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(2*sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(sqrt(a)*abs(e^(I*d*x + I*c) - 1)), 2*(sqrt(2)*

$(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sqrt{a} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - \sqrt{-a} \operatorname{abs}(e^{(I dx + I c)} - 1) + 2 \sqrt{a} / (a \operatorname{abs}(e^{(I dx + I c)} - 1)) - 26 (\cos(2dx + 2c)^2 \sin(dx + c) + \sin(2dx + 2c)^2 \sin(dx + c) + 2 \cos(2dx + 2c) \sin(dx + c) + \sin(dx + c)) \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 24 \cos(5/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \sin(dx + c) - 24 (\cos(dx + c) + 1) \sin(5/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 2 ((13 \cos(dx + c) + 15) \cos(2dx + 2c)^2 + (13 \cos(dx + c) + 15) \sin(2dx + 2c)^2 + 2 (13 \cos(dx + c) + 15) \cos(2dx + 2c) + 13 \cos(dx + c) + 15) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - 4 \sqrt{\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2 \cos(2dx + 2c) + 1} (7 \cos(3/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \sin(dx + c) - (7 \cos(dx + c) + 5) \sin(3/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) / ((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2 \cos(2dx + 2c) + 1)^{5/4} \sqrt{-a} d)$

Giac [A] (verification not implemented)

none

Time = 0.79 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.53

$$\int \frac{1}{\cos^{7/2}(c + dx) \sqrt{a - a \cos(c + dx)}} dx = \frac{\sqrt{2} \left(\frac{4 \left(\left(\left(\left(\left(17 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 165 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 650 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 650 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 165 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 17 \right)}{\left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1 \right)^{5/2}} \right)}{\sqrt{2}} - 15$$

[In] integrate(1/cos(dx+c)^(7/2)/(a-a*cos(dx+c))^(1/2),x, algorithm="giac")

[Out] $-1/30 \sqrt{2} * (4 * (((((17 * \tan(1/4 * dx + 1/4 * c)^2 - 165) * \tan(1/4 * dx + 1/4 * c)^2 + 650) * \tan(1/4 * dx + 1/4 * c)^2 - 650) * \tan(1/4 * dx + 1/4 * c)^2 + 165) * \tan(1/4 * dx + 1/4 * c)^2 - 17) / (\tan(1/4 * dx + 1/4 * c)^4 - 6 * \tan(1/4 * dx + 1/4 * c)^2 + 1)^{5/2} - 15 * \log(\tan(1/4 * dx + 1/4 * c)^2 - \sqrt{\tan(1/4 * dx + 1/4 * c)^4 - 6 * \tan(1/4 * dx + 1/4 * c)^2 + 1}) + 15 * \log(\operatorname{abs}(-\tan(1/4 * dx + 1/4 * c)^2 + \sqrt{\tan(1/4 * dx + 1/4 * c)^4 - 6 * \tan(1/4 * dx + 1/4 * c)^2 + 1}) + 3)) + 15 * \log(\operatorname{abs}(-\tan(1/4 * dx + 1/4 * c)^2 + \sqrt{\tan(1/4 * dx + 1/4 * c)^4 - 6 * \tan(1/4 * dx + 1/4 * c)^2 + 1}) + 1))) / (\sqrt{a} * d * \operatorname{sgn}(\sin(1/2 * dx + 1/2 * c)))$

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c + dx) \sqrt{a - a \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^{7/2} \sqrt{a - a \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(7/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(7/2)*(a - a*cos(c + d*x))^(1/2)), x)
```

$$3.282 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx$$

Optimal result	3906
Rubi [A] (verified)	3906
Mathematica [C] (verified)	3909
Maple [A] (verified)	3909
Fricas [A] (verification not implemented)	3910
Sympy [F(-1)]	3910
Maxima [F]	3910
Giac [B] (verification not implemented)	3911
Mupad [F(-1)]	3911

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx = \frac{7 \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

[Out] 7/4*arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-arctanh(1/2*sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1/2)/d+1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(1-cos(d*x+c))^(1/2)+1/4*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(1-cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2857, 3062, 3061, 2861, 212, 2854, 213}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx = \frac{7 \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}} + \frac{\sin(c+dx) \sqrt{\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}}$$

[In] Int[Cos[c + d*x]^(5/2)/Sqrt[1 - Cos[c + d*x]],x]

[Out] (7*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/(4*d) - (Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x]/(4*d*Sqrt[1 - Cos[c + d*x]])) + (Cos[c + d*x]^(3/2)*Sin[c + d*x]/(2*d*Sqrt[1 - Cos[c + d*x]]))

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 213

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1)*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2854

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2857

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1)), Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3062

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 - \cos(c + dx)}} + \frac{1}{4} \int \frac{\sqrt{\cos(c + dx)}(3 + \cos(c + dx))}{\sqrt{1 - \cos(c + dx)}} dx \\
&= \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{1 - \cos(c + dx)}} + \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 - \cos(c + dx)}} \\
&\quad - \frac{1}{4} \int \frac{-\frac{1}{2} - \frac{7}{2} \cos(c + dx)}{\sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}} dx \\
&= \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{1 - \cos(c + dx)}} + \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 - \cos(c + dx)}} \\
&\quad - \frac{7}{8} \int \frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx + \int \frac{1}{\sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}} dx \\
&= \frac{\sqrt{\cos(c + dx)} \sin(c + dx)}{4d\sqrt{1 - \cos(c + dx)}} + \frac{\cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2d\sqrt{1 - \cos(c + dx)}} \\
&\quad - \frac{7 \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} \\
&\quad - \frac{2 \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}
\end{aligned}$$

$$= \frac{7 \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \\ + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.58

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx = \frac{ie^{-2i(c+dx)}(-1+e^{i(c+dx)})\left(7\sqrt{2}e^{2i(c+dx)}\operatorname{arcsinh}(e^{i(c+dx)})-16e^{2i(c+dx)}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)\right)+\sqrt{2}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{8\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}$$

[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[1 - Cos[c + d*x]], x]

[Out] $((-1/8I)*(-1 + E^{(I*(c + d*x))})*(7*\operatorname{Sqrt}[2]*E^{((2*I)*(c + d*x))}*\operatorname{ArcSinh}[E^{(I*(c + d*x))}] - 16*E^{((2*I)*(c + d*x))}*\operatorname{ArcTanh}[(1 + E^{(I*(c + d*x))})/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}]]]) + \operatorname{Sqrt}[2]*(\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}])*(1 + 2*E^{(I*(c + d*x))} + 2*E^{((2*I)*(c + d*x))} + E^{((3*I)*(c + d*x))}) + 7*E^{((2*I)*(c + d*x))}*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}]]])*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]])/(\operatorname{Sqrt}[2]*d*E^{((2*I)*(c + d*x))}*\operatorname{Sqrt}[1 + E^{((2*I)*(c + d*x))}]*\operatorname{Sqrt}[1 - \operatorname{Cos}[c + d*x]])$

Maple [A] (verified)

Time = 13.83 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sin(dx+c) \left(-2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (\cos^2(dx+c)) + 4 \operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right) \sqrt{2-3\cos(dx+c)} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 7 \operatorname{arctanh}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right) \right)}{4d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-2\cos(dx+c)+2}}$

[In] int(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2), x, method=_RETURNVERBOSE)

[Out] $-1/4/d*\sin(d*x+c)*(-2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^{2+4*\operatorname{arctanh}(1/2*2^{(1/2)}/(\cos(d*x+c)/(1+\cos(d*x+c))))^{(1/2)}}*2^{(1/2)}-3*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}-(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}-7*\operatorname{arctanh}(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}))*\cos(d*x+c)^{(1/2)}/(1+\cos(d*x+c))/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}/(-2*\cos(d*x+c)+2)^{(1/2)}*2^{(1/2)}$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.48

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx$$

$$= \frac{4\sqrt{2} \log\left(-\frac{2(\sqrt{2}\cos(dx+c)+\sqrt{2})\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right) \sin(dx+c) + 2(2\cos(dx+c) + 1)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)} + 7\log(2(\sqrt{-\cos(dx+c)+1})\sqrt{\cos(dx+c)} + \sin(dx+c))/\sin(dx+c) - 7\log(2(\sqrt{-\cos(dx+c)+1})\sqrt{\cos(dx+c)} - \sin(dx+c))/\sin(dx+c)) \sin(dx+c)}{d\sin(dx+c)}$$

[In] integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] 1/8*(4*sqrt(2)*log(-2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))*sin(d*x + c) + 2*(2*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + 7*log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 7*log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - sin(d*x + c))/sin(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(1-cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{-\cos(dx+c)+1}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/sqrt(-cos(d*x + c) + 1), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. 2(136) = 272.

Time = 6.63 (sec) , antiderivative size = 747, normalized size of antiderivative = 4.64

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{1 - \cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] $\frac{1}{4} \cdot (2 \cdot \sqrt{2} \cdot \log(\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 - \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1} + 1) / \text{sgn}(\sin(\frac{1}{2}d*x + \frac{1}{2}c)) - 2 \cdot \sqrt{2} \cdot \log(\text{abs}(-\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1} + 3)) / \text{sgn}(\sin(\frac{1}{2}d*x + \frac{1}{2}c)) - 2 \cdot \sqrt{2} \cdot \log(\text{abs}(-\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1} + 1)) / \text{sgn}(\sin(\frac{1}{2}d*x + \frac{1}{2}c)) - 7 \cdot \log(\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 2 \cdot \sqrt{2} - \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1} + 1) / \text{sgn}(\sin(\frac{1}{2}d*x + \frac{1}{2}c)) + 7 \cdot \log(\text{abs}(-\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 2 \cdot \sqrt{2} + \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1} - 1)) / \text{sgn}(\sin(\frac{1}{2}d*x + \frac{1}{2}c)) - 4 \cdot \sqrt{2} \cdot (17 \cdot (\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 - \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1}))^7 - 73 \cdot (\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 - \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1}))^6 + 157 \cdot (\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 - \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1}))^5 - 597 \cdot (\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 - \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1}))^4 + 1603 \cdot (\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 - \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1}))^3 - 875 \cdot (\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 - \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1}))^2 - 1585 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1585 \cdot \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1} + 1737) / (((\tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 - \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1}))^2 + 2 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 - 2 \cdot \sqrt{\tan(\frac{1}{4}d*x + \frac{1}{4}c)^4 - 6 \cdot \tan(\frac{1}{4}d*x + \frac{1}{4}c)^2 + 1} - 7))^4 \cdot \text{sgn}(\sin(\frac{1}{2}d*x + \frac{1}{2}c)))) / d$

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{\sqrt{1 - \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{5/2}}{\sqrt{1 - \cos(c + dx)}} dx$$

[In] int(cos(c + d*x)^(5/2)/(1 - cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(5/2)/(1 - cos(c + d*x))^(1/2), x)

$$3.283 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx$$

Optimal result	3912
Rubi [A] (verified)	3912
Mathematica [C] (verified)	3914
Maple [A] (verified)	3915
Fricas [B] (verification not implemented)	3915
Sympy [F]	3916
Maxima [F]	3916
Giac [B] (verification not implemented)	3916
Mupad [F(-1)]	3917

Optimal result

Integrand size = 25, antiderivative size = 118

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx = \frac{\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}$$

[Out] $\operatorname{arctanh}(\sin(d*x+c)/(1-\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)})/d - \operatorname{arctanh}(1/2*\sin(d*x+c)*2^{(1/2)}/(1-\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)})*2^{(1/2)}/d + \sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(1-\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2857, 3061, 2861, 212, 2854, 213}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx = \frac{\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

[In] Int[Cos[c + d*x]^(3/2)/Sqrt[1 - Cos[c + d*x]],x]

[Out] ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d - (Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]])

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 213

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2854

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2857

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(b*(2*n - 1)), Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3061

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dis

t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} + \frac{1}{2} \int \frac{1+\cos(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} - \frac{1}{2} \int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
 &\quad + \int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx \\
 &= \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \\
 &\quad - \frac{2\text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \\
 &= \frac{\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \\
 &\quad - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.92

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx = \frac{ie^{-i(c+dx)}(-1+e^{i(c+dx)})\left(\sqrt{2}e^{i(c+dx)}\text{arcsinh}(e^{i(c+dx)})-4e^{i(c+dx)}\text{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\left((1+e^{i(c+dx)})\sqrt{1-\cos(c+dx)}\right)\right)}{2\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+dx)}}$$

[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[1 - Cos[c + d*x]], x]

[Out] ((-1/2*I)*(-1 + E^(I*(c + d*x)))*(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))] - 4*E^(I*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) + Sqrt[2]*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])) * Sqrt[Cos[c + d*x]] / (Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 - Cos[c + d*x]])

Maple [A] (verified)

Time = 13.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

method	result
default	$\frac{\sin(dx+c) \left(\cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - \operatorname{arctanh} \left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}} \right) \sqrt{2} + \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + \operatorname{arctanh} \left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \right) \right) (\sqrt{\cos(dx+c)}) \sqrt{d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{-2\cos(dx+c)+2}}}{d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{-2\cos(dx+c)+2}}$

[In] `int(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

```
[Out] 1/d*sin(d*x+c)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2*cos(d*x+c)+2)^(1/2)*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(103) = 206.

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.91

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx$$

$$\sqrt{2} \log \left(-\frac{2(\sqrt{2}\cos(dx+c)+\sqrt{2})\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)} \right) \sin(dx+c) + 2(\cos(dx+c) +$$

[In] `integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")`

```
[Out] 1/2*(sqrt(2)*log(-(2*(sqrt(2)*cos(d*x+c)+sqrt(2))*sqrt(-cos(d*x+c)+1)*sqrt(cos(d*x+c))-(3*cos(d*x+c)+1)*sin(d*x+c))/((cos(d*x+c)-1)*sin(d*x+c)))*sin(d*x+c)+2*(cos(d*x+c)+1)*sqrt(-cos(d*x+c)+1)*sqrt(cos(d*x+c))+log(2*(sqrt(-cos(d*x+c)+1)*sqrt(cos(d*x+c))+sin(d*x+c))/sin(d*x+c))*sin(d*x+c)-log(2*(sqrt(-cos(d*x+c)+1)*sqrt(cos(d*x+c))-sin(d*x+c))/sin(d*x+c))*sin(d*x+c))/(d*sin(d*x+c))
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx$$

[In] integrate(cos(d*x+c)**(3/2)/(1-cos(d*x+c))**(1/2),x)

[Out] Integral(cos(c + d*x)**(3/2)/sqrt(1 - cos(c + d*x)), x)

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{-\cos(dx+c)+1}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/sqrt(-cos(d*x + c) + 1), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(103) = 206.

Time = 0.94 (sec) , antiderivative size = 560, normalized size of antiderivative = 4.75

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}} dx$$

$$\frac{\sqrt{2} \log\left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1} + 1\right)}{\operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{\sqrt{2} \log\left(\left| -\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1} \right| + 1\right)}{\operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}$$

=

[In] integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/2*(sqrt(2)*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/sgn(sin(1/2*d*x + 1/2*c)) - sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3))/sgn(sin(1/2*d*x + 1/2*c)) - sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1))/sgn(sin(1/2*d*x + 1/2*c)) - 2*log(tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/sgn(sin(1/2*d*x + 1/2*c)) + 2*log(abs(-tan(1/4*d*x + 1/4*c)^2 + 2*sqrt(2) + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - 1))/sgn(sin(1/2*d*x +

$$\frac{1}{2}c)) - 8\sqrt{2}*(3*(\tan(1/4*d*x + 1/4*c)^2 - \sqrt{\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1})^3 - 7*(\tan(1/4*d*x + 1/4*c)^2 - \sqrt{\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1})^2 + \tan(1/4*d*x + 1/4*c)^2 - \sqrt{\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1} + 11)/((\tan(1/4*d*x + 1/4*c)^2 - \sqrt{\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1})^2 + 2*\tan(1/4*d*x + 1/4*c)^2 - 2*\sqrt{\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1} - 7)^2*\text{sgn}(\sin(1/2*d*x + 1/2*c))))/d$$

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{1 - \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{3/2}}{\sqrt{1 - \cos(c + dx)}} dx$$

[In] int(cos(c + d*x)^(3/2)/(1 - cos(c + d*x))^(1/2), x)

[Out] int(cos(c + d*x)^(3/2)/(1 - cos(c + d*x))^(1/2), x)

$$3.284 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}} dx$$

Optimal result	3918
Rubi [A] (verified)	3918
Mathematica [C] (verified)	3920
Maple [A] (verified)	3920
Fricas [B] (verification not implemented)	3920
Sympy [F]	3921
Maxima [F]	3921
Giac [B] (verification not implemented)	3921
Mupad [F(-1)]	3922

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}} dx = \frac{2\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

[Out] 2*arctanh(sin(d*x+c)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))/d-arctanh(1/2*sin(d*x+c)*2^(1/2)/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2))*2^(1/2)/d

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2856, 2854, 213, 2861, 212}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}} dx = \frac{2\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[1 - Cos[c + d*x]],x]

[Out] (2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d - (Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 213

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2854

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2856

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{1}{\sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}} dx - \int \frac{\sqrt{1 - \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\
 &= -\frac{2 \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)} \sqrt{\cos(c+dx)}}\right)}{d} - \frac{2 \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)} \sqrt{\cos(c+dx)}}\right)}{d} \\
 &= \frac{2 \arctanh\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)} \sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2} \arctanh\left(\frac{\sin(c+dx)}{\sqrt{2} \sqrt{1-\cos(c+dx)} \sqrt{\cos(c+dx)}}\right)}{d}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}} dx = \frac{i(-1+e^{i(c+dx)})\sqrt{e^{-i(c+dx)}(1+e^{2i(c+dx)})}\left(\operatorname{arcsinh}(e^{i(c+dx)})-\sqrt{2}\operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)\right)+\operatorname{arctanh}\left(\sqrt{\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}}\right)}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+dx)}}$$

[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[1 - Cos[c + d*x]], x]

[Out] ((-1)*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 - Cos[c + d*x]])

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.36

method	result	size
default	$\frac{\sin(dx+c)\left(\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}-2\operatorname{arctanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)(\sqrt{\cos(dx+c)})\sqrt{2}}{d(1+\cos(dx+c))\sqrt{-2\cos(dx+c)+2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	116

[In] int(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2), x, method=_RETURNVERBOSE)

[Out] -1/d*sin(d*x+c)*(arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*2^(1/2)-2*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)/(1+cos(d*x+c))/(-2*cos(d*x+c)+2)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(74) = 148.

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}} dx = \frac{\sqrt{2}\log\left(\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)+2\log\left(\frac{2\left(\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}\right)}{\sin(dx+c)}\right)}{2d}$$

[In] integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{2} \sqrt{2} \log(-2 \sqrt{2} \cos(dx + c) + \sqrt{2}) \sqrt{-\cos(dx + c) + 1} \sqrt{\cos(dx + c)} - (3 \cos(dx + c) + 1) \sin(dx + c) / ((\cos(dx + c) - 1) \sin(dx + c)) + 2 \log(2 \sqrt{-\cos(dx + c) + 1} \sqrt{\cos(dx + c) + \sin(dx + c)}) / \sin(dx + c) - 2 \log(2 \sqrt{-\cos(dx + c) + 1} \sqrt{\cos(dx + c) - \sin(dx + c)}) / \sin(dx + c)) / d$

Sympy [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{1 - \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{1 - \cos(c + dx)}} dx$$

[In] integrate(cos(d*x+c)**(1/2)/(1-cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(cos(c + d*x))/sqrt(1 - cos(c + d*x)), x)

Maxima [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{1 - \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(dx + c)}}{\sqrt{-\cos(dx + c) + 1}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(-cos(d*x + c) + 1), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(74) = 148$.

Time = 0.58 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.13

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{1 - \cos(c + dx)}} dx = \frac{\sqrt{2} \left(2 \sqrt{2} \log \left(\frac{2 \left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 2 \sqrt{2} - \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1 + 1} \right)}{-2 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 4 \sqrt{2} + 2 \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1 - 2} \right)}{\dots} - \log \left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - \dots \right) \right)}{\dots}$$

[In] integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] $-1/2 \sqrt{2} (2 \sqrt{2} \log(2 (\tan(1/4 dx + 1/4 c))^2 + 2 \sqrt{2} - \sqrt{\tan(1/4 dx + 1/4 c)^4 - 6 \tan(1/4 dx + 1/4 c)^2 + 1 + 1}) - \log(\tan(1/4 dx + 1/4 c)^2 - \dots) - \dots) / \text{abs}(-2 \tan(1/4 dx + 1/4 c)^2 + 1) + 1$

```
x + 1/4*c)^2 + 4*sqrt(2) + 2*sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x +
1/4*c)^2 + 1) - 2)) - log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)
)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1) + log(abs(-tan(1/4*d*x + 1/4*c)^2
+ sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) + log(a
bs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x +
1/4*c)^2 + 1) + 1)))/(d*sgn(sin(1/2*d*x + 1/2*c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{1 - \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{1 - \cos(c + dx)}} dx$$

```
[In] int(cos(c + d*x)^(1/2)/(1 - cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(1 - cos(c + d*x))^(1/2), x)
```

$$3.285 \quad \int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

Optimal result	3923
Rubi [A] (verified)	3923
Mathematica [C] (verified)	3924
Maple [B] (verified)	3924
Fricas [B] (verification not implemented)	3925
Sympy [F]	3925
Maxima [C] (verification not implemented)	3925
Giac [B] (verification not implemented)	3926
Mupad [F(-1)]	3926

Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

[Out] $-\operatorname{arctanh}(1/2*\sin(d*x+c)*2^{(1/2)}/(1-\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)})*2^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2861, 212}

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[1-\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]),x]$

[Out] $-\left(\left(\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[1-\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])]\right)\right)/d$

Rule 212

$\operatorname{Int}[\left((a_.) + (b_.)*(x_.)^2\right)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[\left(1/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2])\right)*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*(x/\operatorname{Rt}[a, 2])], x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2\text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \\ &= -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.34

$$\begin{aligned} &\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx \\ &= \frac{ie^{-i(c+dx)}(-1+e^{i(c+dx)})\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}d\sqrt{-((-1+\cos(c+dx))\cos(c+dx))}} \end{aligned}$$

[In] Integrate[1/(Sqrt[1 - Cos[c + d*x])*Sqrt[Cos[c + d*x]]), x]

[Out] (I*(-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(40) = 80.

Time = 5.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

method	result	size
default	$\frac{4\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c)-1)\sin(dx+c)\text{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)}{d\sqrt{\cos(dx+c)}(-2\cos(dx+c)+2)^{\frac{3}{2}}}$	84

[In] `int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $4/d * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * (\cos(dx+c)-1) * \sin(dx+c) * \operatorname{arctanh}(1/2 * 2^{1/2} / ((\cos(dx+c)/(1+\cos(dx+c)))^{1/2}) / \cos(dx+c)^{1/2} / (-2 * \cos(dx+c) + 2)^{3/2})$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \frac{\sqrt{2} \log\left(-\frac{2(\sqrt{2}\cos(dx+c)+\sqrt{2})\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2d}$$

[In] `integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] $1/2 * \sqrt{2} * \log(-2 * (\sqrt{2} * \cos(dx+c) + \sqrt{2}) * \sqrt{-\cos(dx+c) + 1} * \sqrt{\cos(dx+c)} - (3 * \cos(dx+c) + 1) * \sin(dx+c)) / ((\cos(dx+c) - 1) * \sin(dx+c))) / d$

Sympy [F]

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

[In] `integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)`

[Out] `Integral(1/(sqrt(1-cos(c+d*x))*sqrt(cos(c+d*x))), x)`

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 5.28

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \frac{\sqrt{2} \log\left(4\left(\left|i e^{i dx+i c}-i\right|^2+2 \sqrt{\cos(2 dx+2 c)^2+\sin(2 dx+2 c)^2+2 \cos(2 dx+2 c)+1}\left(\cos\left(\frac{1}{2} \arctan(\sin(2 dx+2 c), \cos(2 dx+2 c)+1)\right)^2+\sin\left(\frac{1}{2} \arctan(\sin(2 dx+2 c), \cos(2 dx+2 c)+1)\right)\right)\right)}{\dots}$$

[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{2}\sqrt{2}\log(4*(\operatorname{abs}(I*e^{(I*d*x + I*c)} - I)^2 + 2*\sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2} + 2*\cos(2*d*x + 2*c) + 1)*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)))^2 - 2*(\sqrt{2}*\operatorname{abs}(I*e^{(I*d*x + I*c)} - I)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))) - 2*\sqrt{2}*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)))*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{(1/4)} + 4)/\operatorname{abs}(I*e^{(I*d*x + I*c)} - I)^2/d$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(40) = 80$.

Time = 0.48 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.38

$$\int \frac{1}{\sqrt{1 - \cos(c + dx)}\sqrt{\cos(c + dx)}} dx = \frac{\sqrt{2} \left(\log \left(\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 - \sqrt{\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^4 - 6 \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 + 1 + 1} \right) - \log \left(\left| -\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right) \right| \right) \right)}{\dots}$$

[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{2}\sqrt{2}*(\log(\tan(1/4*d*x + 1/4*c)^2 - \sqrt{\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1 + 1}) + 1) - \log(\operatorname{abs}(-\tan(1/4*d*x + 1/4*c)^2 + \sqrt{\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1 + 1})) - \log(\operatorname{abs}(-\tan(1/4*d*x + 1/4*c)^2 + \sqrt{\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1 + 1})))/(d*\operatorname{sgn}(\sin(1/2*d*x + 1/2*c)))$

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1 - \cos(c + dx)}\sqrt{\cos(c + dx)}} dx = \int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{1 - \cos(c + dx)}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(1/2)*(1 - cos(c + d*x))^(1/2)), x)

$$3.286 \quad \int \frac{1}{\sqrt{1-\cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	3927
Rubi [A] (verified)	3927
Mathematica [C] (verified)	3928
Maple [A] (verified)	3929
Fricas [A] (verification not implemented)	3929
Sympy [F]	3930
Maxima [C] (verification not implemented)	3930
Giac [B] (verification not implemented)	3931
Mupad [F(-1)]	3931

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{1}{\sqrt{1-\cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)} dx = -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{2 \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

[Out] $-\operatorname{arctanh}(1/2*\sin(d*x+c)*2^{(1/2)}/(1-\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)})*2^{(1/2)}/d+2*\sin(d*x+c)/d/(1-\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2858, 2861, 212}

$$\int \frac{1}{\sqrt{1-\cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)} dx = \frac{2 \sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[1-\operatorname{Cos}[c+d*x]])*\operatorname{Cos}[c+d*x]^{(3/2)}],x]$

[Out] $-\left(\left(\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[1-\operatorname{Cos}[c+d*x]])*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]]\right)\right)/d + (2*\operatorname{Sin}[c+d*x])/(d*\operatorname{Sqrt}[1-\operatorname{Cos}[c+d*x]])*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]$

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2858

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2 \sin(c + dx)}{d\sqrt{1 - \cos(c + dx)}\sqrt{\cos(c + dx)}} + \int \frac{1}{\sqrt{1 - \cos(c + dx)}\sqrt{\cos(c + dx)}} dx \\ &= \frac{2 \sin(c + dx)}{d\sqrt{1 - \cos(c + dx)}\sqrt{\cos(c + dx)}} - \frac{2 \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \\ &= -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{2 \sin(c + dx)}{d\sqrt{1 - \cos(c + dx)}\sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.83

$$\begin{aligned} &\int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{2 \left(-\frac{e^{-\frac{1}{2}i(c+dx)}(1+e^{2i(c+dx)}) \operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}} + 2\sqrt{1+e^{2i(c+dx)}} \cos\left(\frac{1}{2}(c+dx)\right) \right) \sin\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{-((-1+\cos(c+dx))\cos(c+dx))}} \end{aligned}$$

[In] Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]

[Out] (2*(-(((1 + E^((2*I)*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*E^((I/2)*(c + d*x)))) + 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2])*Sin[(c + d*x)/2])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])]

Maple [A] (verified)

Time = 5.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{\left(\operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}-2}\right)\sin(dx+c)\sqrt{2}}{d\sqrt{\cos(dx+c)}\sqrt{-2\cos(dx+c)+2}}$	85

[In] int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/d*(arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2)*sin(d*x+c)/cos(d*x+c)^(1/2)/(-2*cos(d*x+c)+2)^(1/2)*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{\sqrt{2} \cos(dx + c) \log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right) \sin(dx + c) + 4(\cos(dx + c) + 1)\sqrt{\cos(dx + c)}}{2d \cos(dx + c) \sin(dx + c)}$$

[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] 1/2*(sqrt(2)*cos(d*x + c)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c)))/(cos(d*x + c) - 1)*sin(d*x + c))*sin(d*x + c) + 4*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*cos(d*x + c)*sin(d*x + c))

Sympy [F]

$$\int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2), x)

[Out] Integral(1/(sqrt(1 - cos(c + d*x))*cos(c + d*x)**(3/2)), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 400, normalized size of antiderivative = 4.82

$$\int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} dx =$$

$$\sqrt{2} \left(2 \sqrt{2} \sin(dx + c) \sin\left(\frac{1}{2} \arctan(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) + 2(\sqrt{2} \cos(dx + c) + \sqrt{2}) \cos\left(\frac{1}{2} \arctan(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) \right) \frac{1}{\sqrt{2} \cos(dx + c) \sqrt{\cos(2dx + 2c) + 1}}$$

[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] -1/2*sqrt(2)*(2*sqrt(2)*sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*(sqrt(2)*cos(d*x + c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*log(4*(abs(I*e^(I*d*x + I*c) - I)^2 + 2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2) - 2*(sqrt(2)*abs(I*e^(I*d*x + I*c) - I)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4) + 4)/abs(I*e^(I*d*x + I*c) - I)^2)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*d)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(72) = 144.

Time = 0.56 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.18

$$\int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} dx =$$

$$\frac{4 \left(\frac{\sqrt{2} \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2}{\operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} - \frac{\sqrt{2}}{\operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} \right)}{\sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1}} - \frac{\sqrt{2} \log\left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1}\right)}{\operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)} + \frac{\sqrt{2} \log\left(\left| -\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1}\right| \right)}{\operatorname{sgn}\left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}$$

[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] -1/2*(4*(sqrt(2)*tan(1/4*d*x + 1/4*c)^2/sgn(sin(1/2*d*x + 1/2*c)) - sqrt(2)/sgn(sin(1/2*d*x + 1/2*c)))/sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) - sqrt(2)*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1)/sgn(sin(1/2*d*x + 1/2*c)) + sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3))/sgn(sin(1/2*d*x + 1/2*c)) + sqrt(2)*log(abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1))/sgn(sin(1/2*d*x + 1/2*c)))/d

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\cos(c + dx)^{\frac{3}{2}} \sqrt{1 - \cos(c + dx)}} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(3/2)*(1 - cos(c + d*x))^(1/2)), x)

$$3.287 \quad \int \frac{1}{\sqrt{1-\cos(c+dx)} \cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	3932
Rubi [A] (verified)	3932
Mathematica [C] (verified)	3934
Maple [A] (verified)	3935
Fricas [A] (verification not implemented)	3935
Sympy [F]	3935
Maxima [C] (verification not implemented)	3936
Giac [B] (verification not implemented)	3936
Mupad [F(-1)]	3937

Optimal result

Integrand size = 25, antiderivative size = 122

$$\int \frac{1}{\sqrt{1-\cos(c+dx)} \cos^{\frac{5}{2}}(c+dx)} dx = -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{2 \sin(c+dx)}{3d\sqrt{1-\cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)} + \frac{2 \sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

[Out] $-\operatorname{arctanh}(1/2*\sin(d*x+c)*2^{(1/2)/(1-\cos(d*x+c))^{(1/2)/\cos(d*x+c)^{(1/2)}}*2^{(1/2)/d+2/3*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)/(1-\cos(d*x+c))^{(1/2)+2/3*\sin(d*x+c)/d/(1-\cos(d*x+c))^{(1/2)/\cos(d*x+c)^{(1/2)}}}$

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2858, 3063, 12, 2861, 212}

$$\int \frac{1}{\sqrt{1-\cos(c+dx)} \cos^{\frac{5}{2}}(c+dx)} dx = -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{2 \sin(c+dx)}{3d\sqrt{1-\cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)} + \frac{2 \sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

[In] Int[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)),x]

[Out] -((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]))/d) + (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) + (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 212

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2858

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2 \sin(c + dx)}{3d\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} + \frac{1}{3} \int \frac{1 + 2 \cos(c + dx)}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2 \sin(c + dx)}{3d\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} + \frac{2 \sin(c + dx)}{3d\sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}} \\
&\quad - \frac{2}{3} \int -\frac{1}{2\sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}} dx \\
&= \frac{2 \sin(c + dx)}{3d\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} + \frac{2 \sin(c + dx)}{3d\sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}} \\
&\quad + \int \frac{1}{\sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}} dx \\
&= \frac{2 \sin(c + dx)}{3d\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} + \frac{2 \sin(c + dx)}{3d\sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}} \\
&\quad - \frac{2 \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \\
&= -\frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} \\
&\quad + \frac{2 \sin(c + dx)}{3d\sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} + \frac{2 \sin(c + dx)}{3d\sqrt{1 - \cos(c + dx)} \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.39

$$\begin{aligned}
&\int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{5}{2}}(c + dx)} dx \\
&2 \left(-\frac{3e^{-\frac{3}{2}i(c+dx)}(1+e^{2i(c+dx)})^2 \operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{2\sqrt{2}} + 2\sqrt{1 + e^{2i(c+dx)}} \cos\left(\frac{1}{2}(c + dx)\right) (1 + \cos(c + dx)) \right) \sin \\
&= \frac{\hspace{15em}}{3d\sqrt{1 + e^{2i(c+dx)}} \sqrt{1 - \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

[In] Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)), x]

[Out] (2*((-3*(1 + E^((2*I)*(c + d*x)))^2*ArcTanh[(1 + E^(I*(c + d*x))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])])/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))) + 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x]))*Sin[(c + d*x)/2])/(3*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2))

Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\left(3 \cos(dx+c) \operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 2 \cos(dx+c) - 2\right) \sin(dx+c) \sqrt{2}}{3d \cos(dx+c)^{\frac{3}{2}} \sqrt{-2 \cos(dx+c)+2}}$	100

[In] `int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out] `-1/3/d*(3*cos(d*x+c)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2*cos(d*x+c)-2)*sin(d*x+c)/cos(d*x+c)^(3/2)/(-2*cos(d*x+c)+2)^(1/2)*2^(1/2)`

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{1-\cos(c+dx)} \cos^{\frac{5}{2}}(c+dx)} dx$$

$$= \frac{3\sqrt{2} \cos(dx+c)^2 \log\left(-\frac{2(\sqrt{2}\cos(dx+c)+\sqrt{2})\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right) \sin(dx+c) + 4}{6d \cos(dx+c)^2 \sin(dx+c)}$$

[In] `integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] `1/6*(3*sqrt(2)*cos(d*x + c)^2*log(-2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))*sin(d*x + c) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2*sin(d*x + c))`

Sympy [F]

$$\int \frac{1}{\sqrt{1-\cos(c+dx)} \cos^{\frac{5}{2}}(c+dx)} dx = \int \frac{1}{\sqrt{1-\cos(c+dx)} \cos^{\frac{5}{2}}(c+dx)} dx$$

[In] `integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)`[Out] `Integral(1/(sqrt(1 - cos(c + d*x))*cos(c + d*x)**(5/2)), x)`

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 563, normalized size of antiderivative = 4.61

$$\int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{3 (\cos(2 dx + 2 c)^2 + \sin(2 dx + 2 c)^2 + 2 \cos(2 dx + 2 c) + 1) \log \left(\frac{4 \left(|i e^{(i dx + i c)} - i|^2 + 2 \sqrt{\cos(2 dx + 2 c)^2 + \sin(2 dx + 2 c)^2} \right)}{\dots} \right)}{\dots}$$

[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] 1/3*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(4*(abs(I*e^(I*d*x + I*c) - I)^2 + 2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2) - 2*(sqrt(2)*abs(I*e^(I*d*x + I*c) - I)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4) + 4)/abs(I*e^(I*d*x + I*c) - I)^2 - 2*(sqrt(2)*sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(d*x + c) + 3*sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4) - 4*(sqrt(2)*sin(d*x + c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(d*x + c) - sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4))/((sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*d)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(103) = 206.

Time = 0.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{5}{2}}(c + dx)} dx =$$

$$\frac{\sqrt{2} \left(\frac{8 \left(\left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 3 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 3 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 1 \right)}{\left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1 \right)^{\frac{3}{2}}} - 3 \log \left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - \sqrt{\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1} \right) \right)}{\dots}$$

[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")

```
[Out] -1/6*sqrt(2)*(8*((tan(1/4*d*x + 1/4*c)^2 - 3)*tan(1/4*d*x + 1/4*c)^2 + 3)*
tan(1/4*d*x + 1/4*c)^2 - 1)/(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c
)^2 + 1)^(3/2) - 3*log(tan(1/4*d*x + 1/4*c)^2 - sqrt(tan(1/4*d*x + 1/4*c)^4
- 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 1) + 3*log(abs(-tan(1/4*d*x + 1/4*c)^2 +
sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1) + 3)) + 3*log(
abs(-tan(1/4*d*x + 1/4*c)^2 + sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x +
1/4*c)^2 + 1) + 1)))/(d*sgn(sin(1/2*d*x + 1/2*c)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1 - \cos(c + dx)} \cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{\cos(c + dx)^{\frac{5}{2}} \sqrt{1 - \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(5/2)*(1 - cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(1 - cos(c + d*x))^(1/2)), x)
```

3.288 $\int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{a + a \cos(c + dx)} dx$

Optimal result	3938
Rubi [A] (verified)	3938
Mathematica [F]	3939
Maple [F]	3940
Fricas [F]	3940
Sympy [F(-1)]	3940
Maxima [F]	3940
Giac [F]	3941
Mupad [F(-1)]	3941

Optimal result

Integrand size = 25, antiderivative size = 78

$$\int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{a + a \cos(c + dx)} dx$$

$$= \frac{2^{5/6} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{4}{3}, \frac{1}{6}, \frac{3}{2}, 1 - \cos(c + dx), \frac{1}{2}(1 - \cos(c + dx))\right) \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{d(1 + \cos(c + dx))^{5/6}}$$

[Out] $2^{(5/6)} * \operatorname{AppellF1}(1/2, -4/3, 1/6, 3/2, 1 - \cos(d*x+c), 1/2 - 1/2 * \cos(d*x+c)) * (a + a * \cos(d*x+c))^{(1/3)} * \sin(d*x+c) / d / (1 + \cos(d*x+c))^{(5/6)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2866, 2864, 138}

$$\int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{a + a \cos(c + dx)} dx$$

$$= \frac{2^{5/6} \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{4}{3}, \frac{1}{6}, \frac{3}{2}, 1 - \cos(c + dx), \frac{1}{2}(1 - \cos(c + dx))\right)}{d(\cos(c + dx) + 1)^{5/6}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]^{(4/3)} * (a + a * \operatorname{Cos}[c + d*x])^{(1/3)}, x]$

[Out] $(2^{(5/6)} * \operatorname{AppellF1}[1/2, -4/3, 1/6, 3/2, 1 - \operatorname{Cos}[c + d*x], (1 - \operatorname{Cos}[c + d*x])/2] * (a + a * \operatorname{Cos}[c + d*x])^{(1/3)} * \operatorname{Sin}[c + d*x]) / (d * (1 + \operatorname{Cos}[c + d*x])^{(5/6)})$

Rule 138

$\operatorname{Int}[(b_*) * (x_*)^{(m_*)} * ((c_*) + (d_*) * (x_*)^{(n_*)} * ((e_*) + (f_*) * (x_*)^{(p_*)}), x_*$
 Symbol] $\rightarrow \operatorname{Simp}[c^n * e^p * (b*x)^{(m+1)} / (b*(m+1))] * \operatorname{AppellF1}[m+1, -n, -p,$

$m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; \text{FreeQ}\{b, c, d, e, f, m, n, p\}, x\} \&$
 $\& \text{!IntegerQ}[m] \&\& \text{!IntegerQ}[n] \&\& \text{GtQ}[c, 0] \&\& (\text{IntegerQ}[p] \mid\mid \text{GtQ}[e, 0])$

Rule 2864

$\text{Int}[(d_* \sin[e_*] + f_*(x_*))^n * (a_* + (b_* \sin[e_*] + f_*(x_*))^m), x_Symbol] \text{:> Dist}[(-b)*(d/b)^n * (\text{Cos}[e + f*x] / (f*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])*\text{Sqrt}[a - b*\text{Sin}[e + f*x]]), \text{Subst}[\text{Int}[(a - x)^n * ((2*a - x)^{m - 1/2}) / \text{Sqrt}[x]], x], x, a - b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, d, e, f, m, n\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{!IntegerQ}[m] \&\& \text{GtQ}[a, 0] \&\& \text{GtQ}[d/b, 0]$

Rule 2866

$\text{Int}[(d_* \sin[e_*] + f_*(x_*))^n * (a_* + (b_* \sin[e_*] + f_*(x_*))^m), x_Symbol] \text{:> Dist}[a^{\text{IntPart}[m]} * ((a + b*\text{Sin}[e + f*x])^{\text{FracPart}[m]} / (1 + (b/a)*\text{Sin}[e + f*x])^{\text{FracPart}[m]}), \text{Int}[(1 + (b/a)*\text{Sin}[e + f*x])^m * (d*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, d, e, f, m, n\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{!IntegerQ}[m] \&\& \text{!GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt[3]{a + a \cos(c + dx)} \int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{1 + \cos(c + dx)} dx}{\sqrt[3]{1 + \cos(c + dx)}} \\ &= \frac{\left(\sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)\right) \text{Subst}\left(\int \frac{(1-x)^{4/3}}{\sqrt[5]{2-x}\sqrt{x}} dx, x, 1 - \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}(1 + \cos(c + dx))^{5/6}} \\ &= \frac{2^{5/6} \text{AppellF1}\left(\frac{1}{2}, -\frac{4}{3}, \frac{1}{6}, \frac{3}{2}, 1 - \cos(c + dx), \frac{1}{2}(1 - \cos(c + dx))\right) \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{d(1 + \cos(c + dx))^{5/6}} \end{aligned}$$

Mathematica [F]

$$\int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{a + a \cos(c + dx)} dx = \int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{a + a \cos(c + dx)} dx$$

[In] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3),x]

[Out] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3), x]

Maple [F]

$$\int \left(\cos^{\frac{4}{3}}(dx + c) \right) (a + \cos(dx + c) a)^{\frac{1}{3}} dx$$

[In] `int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(1/3),x)`

[Out] `int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(1/3),x)`

Fricas [F]

$$\int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{a + a \cos(c + dx)} dx = \int (a \cos(dx + c) + a)^{\frac{1}{3}} \cos(dx + c)^{\frac{4}{3}} dx$$

[In] `integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")`

[Out] `integral((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)`

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{a + a \cos(c + dx)} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(4/3)*(a+a*cos(d*x+c))**(1/3),x)`

[Out] Timed out

Maxima [F]

$$\int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{a + a \cos(c + dx)} dx = \int (a \cos(dx + c) + a)^{\frac{1}{3}} \cos(dx + c)^{\frac{4}{3}} dx$$

[In] `integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")`

[Out] `integrate((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)`

Giac [F]

$$\int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{a + a \cos(c + dx)} dx = \int (a \cos(dx + c) + a)^{\frac{1}{3}} \cos(dx + c)^{\frac{4}{3}} dx$$

[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c + dx) \sqrt[3]{a + a \cos(c + dx)} dx = \int \cos(c + dx)^{\frac{4}{3}} (a + a \cos(c + dx))^{\frac{1}{3}} dx$$

[In] int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(1/3),x)

[Out] int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(1/3), x)

3.289 $\int \cos^{\frac{4}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx$

Optimal result	3942
Rubi [A] (verified)	3942
Mathematica [F]	3943
Maple [F]	3944
Fricas [F(-1)]	3944
Sympy [F(-1)]	3944
Maxima [F]	3944
Giac [F(-1)]	3945
Mupad [F(-1)]	3945

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \cos^{\frac{4}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \frac{2\sqrt{2} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{4}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \cos(c + dx), \frac{1}{2}(1 - \cos(c + dx))\right) (a + a \cos(c + dx))^{2/3} \sin(c + dx)}{d(1 + \cos(c + dx))^{7/6}}$$

[Out] $2*2^{(1/6)}*\operatorname{AppellF1}(1/2, -4/3, -1/6, 3/2, 1 - \cos(d*x+c), 1/2 - 1/2*\cos(d*x+c))*(a+a*\cos(d*x+c))^{(2/3)}*\sin(d*x+c)/d/(1+\cos(d*x+c))^{(7/6)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2866, 2864, 138}

$$\int \cos^{\frac{4}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \frac{2\sqrt{2} \sin(c + dx)(a \cos(c + dx) + a)^{2/3} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{4}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \cos(c + dx), \frac{1}{2}(1 - \cos(c + dx))\right)}{d(\cos(c + dx) + 1)^{7/6}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]^{(4/3)}*(a + a*\operatorname{Cos}[c + d*x])^{(2/3)}, x]$

[Out] $(2*2^{(1/6)}*\operatorname{AppellF1}[1/2, -4/3, -1/6, 3/2, 1 - \operatorname{Cos}[c + d*x], (1 - \operatorname{Cos}[c + d*x])/2]*(a + a*\operatorname{Cos}[c + d*x])^{(2/3)}*\operatorname{Sin}[c + d*x])/(d*(1 + \operatorname{Cos}[c + d*x])^{(7/6)})$

Rule 138

$\operatorname{Int}[(b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*))^{(n_*)}*((e_*) + (f_*)*(x_*))^{(p_*)}, x_*$
 Symbol] $\rightarrow \operatorname{Simp}[c^n * e^p * (b*x)^{(m+1)} / (b*(m+1))] * \operatorname{AppellF1}[m+1, -n, -p,$

$m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; \text{FreeQ}\{b, c, d, e, f, m, n, p\}, x\} \&$
 $\& \text{!IntegerQ}[m] \&\& \text{!IntegerQ}[n] \&\& \text{GtQ}[c, 0] \&\& (\text{IntegerQ}[p] \mid\mid \text{GtQ}[e, 0])$

Rule 2864

$\text{Int}[\{(d_)*\sin[(e_)] + (f_)*(x_)]\}^{(n_)}*((a_)] + (b_)*\sin[(e_)] + (f_)*(x_)]\}^{(m_)}, x_Symbol] :> \text{Dist}[(-b)*(d/b)^n*(\text{Cos}[e + f*x]/(f*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])*\text{Sqrt}[a - b*\text{Sin}[e + f*x]])), \text{Subst}[\text{Int}[(a - x)^n*((2*a - x)^{(m - 1/2})/\text{Sqrt}[x]), x], x, a - b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, d, e, f, m, n\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{!IntegerQ}[m] \&\& \text{GtQ}[a, 0] \&\& \text{GtQ}[d/b, 0]$

Rule 2866

$\text{Int}[\{(d_)*\sin[(e_)] + (f_)*(x_)]\}^{(n_)}*((a_)] + (b_)*\sin[(e_)] + (f_)*(x_)]\}^{(m_)}, x_Symbol] :> \text{Dist}[a^{\text{IntPart}[m]}*((a + b*\text{Sin}[e + f*x])^{\text{FracPart}[m]}/(1 + (b/a)*\text{Sin}[e + f*x])^{\text{FracPart}[m]}), \text{Int}[(1 + (b/a)*\text{Sin}[e + f*x])^m*(d*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, d, e, f, m, n\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{!IntegerQ}[m] \&\& \text{!GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(a + a \cos(c + dx))^{2/3} \int \cos^{4/3}(c + dx)(1 + \cos(c + dx))^{2/3} dx}{(1 + \cos(c + dx))^{2/3}} \\ &= \frac{((a + a \cos(c + dx))^{2/3} \sin(c + dx)) \text{Subst}\left(\int \frac{(1-x)^{4/3} \sqrt[6]{2-x}}{\sqrt{x}} dx, x, 1 - \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}(1 + \cos(c + dx))^{7/6}} \\ &= \frac{2\sqrt[6]{2} \text{AppellF1}\left(\frac{1}{2}, -\frac{4}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \cos(c + dx), \frac{1}{2}(1 - \cos(c + dx))\right) (a + a \cos(c + dx))^{2/3} \sin(c + dx)}{d(1 + \cos(c + dx))^{7/6}} \end{aligned}$$

Mathematica [F]

$$\int \cos^{4/3}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \int \cos^{4/3}(c + dx)(a + a \cos(c + dx))^{2/3} dx$$

[In] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3), x]

[Out] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3), x]

Maple [F]

$$\int \left(\cos^{\frac{4}{3}}(dx + c) \right) (a + \cos(dx + c) a)^{\frac{2}{3}} dx$$

[In] `int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(2/3),x)`

[Out] `int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(2/3),x)`

Fricas [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(4/3)*(a+a*cos(d*x+c))**(2/3),x)`

[Out] Timed out

Maxima [F]

$$\int \cos^{\frac{4}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \int (a \cos(dx + c) + a)^{\frac{2}{3}} \cos(dx + c)^{\frac{4}{3}} dx$$

[In] `integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")`

[Out] `integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(4/3), x)`

Giac [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \int \cos(c + dx)^{4/3} (a + a \cos(c + dx))^{2/3} dx$$

```
[In] int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)^(4/3)*(a + a*cos(c + d*x))^(2/3), x)
```

3.290 $\int \cos^{\frac{5}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx$

Optimal result	3946
Rubi [A] (verified)	3946
Mathematica [F]	3947
Maple [F]	3948
Fricas [F]	3948
Sympy [F(-1)]	3948
Maxima [F]	3948
Giac [F]	3949
Mupad [F(-1)]	3949

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \cos^{\frac{5}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \frac{2\sqrt[6]{2} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{5}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \cos(c + dx), \frac{1}{2}(1 - \cos(c + dx))\right) (a + a \cos(c + dx))^{2/3} \sin(c + dx)}{d(1 + \cos(c + dx))^{7/6}}$$

[Out] $2*2^{(1/6)}*\operatorname{AppellF1}(1/2, -5/3, -1/6, 3/2, 1 - \cos(d*x+c), 1/2 - 1/2*\cos(d*x+c))*(a+a*\cos(d*x+c))^{(2/3)}*\sin(d*x+c)/d/(1+\cos(d*x+c))^{(7/6)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2866, 2864, 138}

$$\int \cos^{\frac{5}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \frac{2\sqrt[6]{2} \sin(c + dx)(a \cos(c + dx) + a)^{2/3} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{5}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \cos(c + dx), \frac{1}{2}(1 - \cos(c + dx))\right)}{d(\cos(c + dx) + 1)^{7/6}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]^{(5/3)}*(a + a*\operatorname{Cos}[c + d*x])^{(2/3)}, x]$

[Out] $(2*2^{(1/6)}*\operatorname{AppellF1}[1/2, -5/3, -1/6, 3/2, 1 - \operatorname{Cos}[c + d*x], (1 - \operatorname{Cos}[c + d*x])/2]*(a + a*\operatorname{Cos}[c + d*x])^{(2/3)}*\operatorname{Sin}[c + d*x])/d*(1 + \operatorname{Cos}[c + d*x])^{(7/6)}$

Rule 138

$\operatorname{Int}[(b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*))^{(n_*)}*((e_*) + (f_*)*(x_*))^{(p_*)}, x_*$
 Symbol] $\rightarrow \operatorname{Simp}[c^n * e^p * (b*x)^{(m+1)} / (b*(m+1))] * \operatorname{AppellF1}[m+1, -n, -p,$

$m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; \text{FreeQ}\{b, c, d, e, f, m, n, p\}, x\} \&$
 $\& \text{!IntegerQ}[m] \&\& \text{!IntegerQ}[n] \&\& \text{GtQ}[c, 0] \&\& (\text{IntegerQ}[p] \mid \mid \text{GtQ}[e, 0])$

Rule 2864

$\text{Int}[\{(d_)*\sin[(e_)] + (f_)*(x_)]\}^{(n_)}*((a_)] + (b_)*\sin[(e_)] + (f_)*(x_)]\}^{(m_)}, x_Symbol] \text{:> Dist}[(-b)*(d/b)^n*(\text{Cos}[e + f*x]/(f*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])*\text{Sqrt}[a - b*\text{Sin}[e + f*x]])), \text{Subst}[\text{Int}[(a - x)^n*((2*a - x)^{(m - 1/2})/\text{Sqrt}[x]), x], x, a - b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, d, e, f, m, n\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{!IntegerQ}[m] \&\& \text{GtQ}[a, 0] \&\& \text{GtQ}[d/b, 0]$

Rule 2866

$\text{Int}[\{(d_)*\sin[(e_)] + (f_)*(x_)]\}^{(n_)}*((a_)] + (b_)*\sin[(e_)] + (f_)*(x_)]\}^{(m_)}, x_Symbol] \text{:> Dist}[a^{\text{IntPart}[m]}*((a + b*\text{Sin}[e + f*x])^{\text{FracPart}[m]}/(1 + (b/a)*\text{Sin}[e + f*x])^{\text{FracPart}[m]}), \text{Int}[(1 + (b/a)*\text{Sin}[e + f*x])^m*(d*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, d, e, f, m, n\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{!IntegerQ}[m] \&\& \text{!GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(a + a \cos(c + dx))^{2/3} \int \cos^{5/3}(c + dx)(1 + \cos(c + dx))^{2/3} dx}{(1 + \cos(c + dx))^{2/3}} \\ &= \frac{((a + a \cos(c + dx))^{2/3} \sin(c + dx)) \text{Subst}\left(\int \frac{(1-x)^{5/3} \sqrt[6]{2-x}}{\sqrt{x}} dx, x, 1 - \cos(c + dx)\right)}{d\sqrt{1 - \cos(c + dx)}(1 + \cos(c + dx))^{7/6}} \\ &= \frac{2\sqrt[6]{2} \text{AppellF1}\left(\frac{1}{2}, -\frac{5}{3}, -\frac{1}{6}, \frac{3}{2}, 1 - \cos(c + dx), \frac{1}{2}(1 - \cos(c + dx))\right) (a + a \cos(c + dx))^{2/3} \sin(c + dx)}{d(1 + \cos(c + dx))^{7/6}} \end{aligned}$$

Mathematica [F]

$$\int \cos^{5/3}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \int \cos^{5/3}(c + dx)(a + a \cos(c + dx))^{2/3} dx$$

[In] Integrate[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3), x]

[Out] Integrate[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3), x]

Maple [F]

$$\int \left(\cos^{\frac{5}{3}}(dx + c) \right) (a + \cos(dx + c) a)^{\frac{2}{3}} dx$$

[In] `int(cos(d*x+c)^(5/3)*(a+cos(d*x+c)*a)^(2/3),x)`

[Out] `int(cos(d*x+c)^(5/3)*(a+cos(d*x+c)*a)^(2/3),x)`

Fricas [F]

$$\int \cos^{\frac{5}{3}}(c + dx)(a + a \cos(c + dx))^{\frac{2}{3}} dx = \int (a \cos(dx + c) + a)^{\frac{2}{3}} \cos(dx + c)^{\frac{5}{3}} dx$$

[In] `integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")`

[Out] `integral((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)`

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{3}}(c + dx)(a + a \cos(c + dx))^{\frac{2}{3}} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(5/3)*(a+a*cos(d*x+c))**(2/3),x)`

[Out] Timed out

Maxima [F]

$$\int \cos^{\frac{5}{3}}(c + dx)(a + a \cos(c + dx))^{\frac{2}{3}} dx = \int (a \cos(dx + c) + a)^{\frac{2}{3}} \cos(dx + c)^{\frac{5}{3}} dx$$

[In] `integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")`

[Out] `integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)`

Giac [F]

$$\int \cos^{\frac{5}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \int (a \cos(dx + c) + a)^{\frac{2}{3}} \cos(dx + c)^{\frac{5}{3}} dx$$

[In] integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{3}}(c + dx)(a + a \cos(c + dx))^{2/3} dx = \int \cos(c + dx)^{5/3} (a + a \cos(c + dx))^{2/3} dx$$

[In] int(cos(c + d*x)^(5/3)*(a + a*cos(c + d*x))^(2/3),x)

[Out] int(cos(c + d*x)^(5/3)*(a + a*cos(c + d*x))^(2/3), x)

3.291 $\int (a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$

Optimal result	3950
Rubi [A] (verified)	3950
Mathematica [C] (verified)	3953
Maple [B] (verified)	3953
Fricas [C] (verification not implemented)	3954
Sympy [F(-1)]	3954
Maxima [F]	3955
Giac [F]	3955
Mupad [F(-1)]	3955

Optimal result

Integrand size = 21, antiderivative size = 151

$$\begin{aligned} & \int (a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx \\ &= -\frac{6a \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\ & \quad + \frac{2a \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} + \frac{6a \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\ & \quad + \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} \end{aligned}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*sec(d*x+c)^(5/2)*sin(d*x+c)/d+6/5
*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {3317, 3872, 3853, 3856, 2720, 2719}

$$\int (a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{2a \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} + \frac{2a \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d}$$

$$+ \frac{6a \sin(c + dx) \sqrt{\sec(c + dx)}}{5d} + \frac{2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

$$- \frac{6a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2), x]

[Out] (-6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (6*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sec^{\frac{5}{2}}(c + dx)(a + a \sec(c + dx)) dx \\
&= a \int \sec^{\frac{5}{2}}(c + dx) dx + a \int \sec^{\frac{7}{2}}(c + dx) dx \\
&= \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad + \frac{1}{3}a \int \sqrt{\sec(c + dx)} dx + \frac{1}{5}(3a) \int \sec^{\frac{3}{2}}(c + dx) dx \\
&= \frac{6a \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad - \frac{1}{5}(3a) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{3} \left(a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2a \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{6a \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\
&\quad + \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad - \frac{1}{5} \left(3a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{6a \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
&\quad + \frac{2a \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{6a \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\
&\quad + \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.77

$$\int (a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{a(1 + \cos(c + dx)) \sec^2\left(\frac{1}{2}(c + dx)\right) \left(i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9(1 + e^{2i(c+dx)}) + 9(-1 + e^{2ic})\sqrt{1 + e^{2i(c+dx)}}\right)\right)}{15(d - dE^{((2*I)*c)})}$$

[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2), x]

[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((I*Sqrt[2]*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*(9*(1 + E^((2*I)*(c + d*x)))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/E^(I*(c + d*x)) + (1 - E^((2*I)*c))*Sqrt[Sec[c + d*x]]*(9*Cos[d*x]*Csc[c] + (5 + 3*Sec[c + d*x])*Tan[c + d*x]))/(15*(d - d*E^((2*I)*c)))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(179) = 358.

Time = 9.73 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.54

method	result
default	$4\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} a \left(-\frac{\cos(\frac{dx}{2} + \frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}}{12(\cos^2(\frac{dx}{2} + \frac{c}{2}) - \frac{1}{2})^2} + \frac{7\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2(\cos^2(\frac{dx}{2} + \frac{c}{2}))}}{15\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2}))}} \right)$
parts	$2a\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \left(24\cos(\frac{dx}{2} + \frac{c}{2})(\sin^6(\frac{dx}{2} + \frac{c}{2})) - 12\sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2}))} - 1\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} E(\cos(\frac{dx}{2} + \frac{c}{2})) \right)$

[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/12*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic F(cos(1/2*d*x+1/2*c), 2^(1/2))-1/40*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)

$$\frac{c^2)^{1/2} - 3/10 * (\sin(1/2 * dx + 1/2 * c)^2)^{1/2} * (-2 * \cos(1/2 * dx + 1/2 * c)^2 + 1)^{1/2}}{(-2 * \sin(1/2 * dx + 1/2 * c)^4 + \sin(1/2 * dx + 1/2 * c)^2)^{1/2} * (\text{EllipticF}(\cos(1/2 * dx + 1/2 * c), 2^{1/2}) - \text{EllipticE}(\cos(1/2 * dx + 1/2 * c), 2^{1/2}))} / \sin(1/2 * dx + 1/2 * c) / (2 * \cos(1/2 * dx + 1/2 * c)^2 - 1)^{1/2} / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int (a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{-5i \sqrt{2} a \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} a \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{\dots}$$

[In] integrate((a+a*cos(dx+c))*sec(dx+c)^(7/2),x, algorithm="fricas")

[Out] 1/15*(-5*I*sqrt(2)*a*cos(dx + c)^2*weierstrassPInverse(-4, 0, cos(dx + c) + I*sin(dx + c)) + 5*I*sqrt(2)*a*cos(dx + c)^2*weierstrassPInverse(-4, 0, cos(dx + c) - I*sin(dx + c)) - 9*I*sqrt(2)*a*cos(dx + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(dx + c) + I*sin(dx + c))) + 9*I*sqrt(2)*a*cos(dx + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(dx + c) - I*sin(dx + c))) + 2*(9*a*cos(dx + c)^2 + 5*a*cos(dx + c) + 3*a)*sin(dx + c)/sqrt(cos(dx + c)))/(d*cos(dx + c)^2)

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(dx+c))*sec(dx+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx = \int (a \cos(dx + c) + a) \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)

Giac [F]

$$\int (a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx = \int (a \cos(dx + c) + a) \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{7/2} (a + a \cos(c + dx)) dx$$

[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)),x)

[Out] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x)), x)

3.292 $\int (a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$

Optimal result	3956
Rubi [A] (verified)	3956
Mathematica [C] (verified)	3958
Maple [B] (verified)	3959
Fricas [C] (verification not implemented)	3960
Sympy [F(-1)]	3960
Maxima [F]	3960
Giac [F]	3961
Mupad [F(-1)]	3961

Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned} & \int (a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx \\ &= -\frac{2a\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)\sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{2a\sqrt{\cos(c + dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)\sqrt{\sec(c + dx)}}{3d} \\ & \quad + \frac{2a\sqrt{\sec(c + dx)}\sin(c + dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{3d} \end{aligned}$$

[Out] $2/3*a*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2*a*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {3317, 3872, 3853, 3856, 2719, 2720}

$$\int (a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{2a \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d} + \frac{2a \sin(c + dx) \sqrt{\sec(c + dx)}}{d}$$

$$+ \frac{2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

$$- \frac{2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2), x]

[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx)) dx \\
&= a \int \sec^{\frac{3}{2}}(c + dx) dx + a \int \sec^{\frac{5}{2}}(c + dx) dx \\
&= \frac{2a \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad + \frac{1}{3} a \int \sqrt{\sec(c + dx)} dx - a \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2a \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad + \frac{1}{3} \left(a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad - \left(a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{2a \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2a \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{2a \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.86 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.07

$$\begin{aligned}
&\int (a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx \\
&= \frac{a(1 + \cos(c + dx)) \sec^2\left(\frac{1}{2}(c + dx)\right) \left(i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \left(3(1 + e^{2i(c+dx)}) + 3(-1 + e^{2ic}) \sqrt{1 + e^{2i(c+dx)}} \right) \right)}{1}
\end{aligned}$$

[In] Integrate[(a + a*cos[c + d*x])*Sec[c + d*x]^(5/2),x]

[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((I*Sqrt[2]*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/E^(I*(c + d*x)) - (-1 + E^((2*I)*c))*Sqrt[Sec[c + d*x]]*(3*Cos[d*x]*Csc[c] + Tan[c + d*x]))/(3*(d - d*E^((2*I)*c)))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(159) = 318$.

Time = 8.44 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.99

method	result
default	$2\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} a \left(12(\sin^4(\frac{dx}{2} + \frac{c}{2})) \cos(\frac{dx}{2} + \frac{c}{2}) - 2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} F(\cos(\frac{dx}{2} + \frac{c}{2}), 2^{(1/2)}) \right)$
parts	$\frac{2a \left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} F(\cos(\frac{dx}{2} + \frac{c}{2}), \sqrt{2}) (\sin^2(\frac{dx}{2} + \frac{c}{2})) - 2(\sin^2(\frac{dx}{2} + \frac{c}{2})) \cos(\frac{dx}{2} + \frac{c}{2}) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right)}{3\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})} (2(\cos^2(\frac{dx}{2} + \frac{c}{2})))}$

[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out]
$$\frac{-2/3 * (-(-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * a / (4 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 4 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 1) / \sin(1/2 * d * x + 1/2 * c) ^ 3 * (12 * \sin(1/2 * d * x + 1/2 * c) ^ 4 * \cos(1/2 * d * x + 1/2 * c) - 2 * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 6 * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 8 * \sin(1/2 * d * x + 1/2 * c) ^ 2 * \cos(1/2 * d * x + 1/2 * c) + (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) + 3 * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2))) * (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.36

$$\int (a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{-i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3 i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 3 i \sqrt{2} a \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 2 * (3 a \cos(dx + c) + a) \sin(dx + c) / \sqrt{\cos(dx + c)}}{d \cos(dx + c)}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + I*sqrt(2)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a*cos(d*x + c)*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a
*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x +
c)))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx = \int (a \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int (a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx = \int (a \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{5/2} (a + a \cos(c + dx)) dx$$

[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)),x)

[Out] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x)), x)

3.293 $\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	3962
Rubi [A] (verified)	3962
Mathematica [C] (verified)	3964
Maple [A] (verified)	3965
Fricas [C] (verification not implemented)	3965
Sympy [F]	3966
Maxima [F]	3966
Giac [F]	3966
Mupad [F(-1)]	3966

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

$$= -\frac{2a \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d}$$

$$+ \frac{2a \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a \sqrt{\sec(c + dx)} \sin(c + dx)}{d}$$

[Out] $2*a*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.11 (sec), antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3317, 3872, 3856, 2720, 3853, 2719}

$$\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{2a \sin(c + dx) \sqrt{\sec(c + dx)}}{d} + \frac{2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

$$- \frac{2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])* \operatorname{Sec}[c + d*x]^{(3/2)}, x]$

[Out] $(-2*a*\sqrt{\cos[c + d*x]}*EllipticE[(c + d*x)/2, 2]*\sqrt{\sec[c + d*x]})/d + (2*a*\sqrt{\cos[c + d*x]}*EllipticF[(c + d*x)/2, 2]*\sqrt{\sec[c + d*x]})/d + (2*a*\sqrt{\sec[c + d*x]}*\sin[c + d*x])/d$

Rule 2719

$\text{Int}[\sqrt{\sin[(c_.) + (d_.)*(x_)]}], x_Symbol] \rightarrow \text{Simp}[(2/d)*EllipticE[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\sqrt{\sin[(c_.) + (d_.)*(x_)]}], x_Symbol] \rightarrow \text{Simp}[(2/d)*EllipticF[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3317

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_.)})^{(p_.)}], x_Symbol] \rightarrow \text{Dist}[d^{(n*p)}, \text{Int}[(d*\text{Csc}[e + f*x])^{(m - n*p)}*(b + a*\text{Csc}[e + f*x]^n)^p, x], x] /; \text{FreeQ}\{a, b, d, e, f, m, n, p\}, x] \&\amp; !\text{IntegerQ}[m] \&\amp; \text{IntegersQ}[n, p]$

Rule 3853

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.)^{(n_.)}], x_Symbol] \rightarrow \text{Simp}[(-b)*\cos[c + d*x]*(b*\text{Csc}[c + d*x])^{(n - 1)}/(d*(n - 1)), x] + \text{Dist}[b^2*(n - 2)/(n - 1), \text{Int}[(b*\text{Csc}[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\amp; \text{GtQ}[n, 1] \& \& \text{IntegerQ}[2*n]$

Rule 3856

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.)^{(n_.)}], x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^n*\sin[c + d*x]^n, \text{Int}[1/\sin[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\amp; \text{EqQ}[n^2, 1/4]$

Rule 3872

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \sqrt{\sec(c + dx)}(a + a \sec(c + dx)) dx \\ &= a \int \sqrt{\sec(c + dx)} dx + a \int \sec^{\frac{3}{2}}(c + dx) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - a \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\
&\quad + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} \\
&\quad - \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.55 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx = \frac{2iae^{-i(c+dx)}\left(-1 + \sqrt{1 + e^{2i(c+dx)}}\right)\operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) + e^{i(c+dx)}\sqrt{1 + e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right)}{d}$$

[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2), x]

[Out] ((-2*I)*a*(-1 + Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c + d*x]]/(d*E^(I*(c + d*x)))

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

method	result
default	$\frac{2a \left(2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \cos \left(\frac{dx}{2} + \frac{c}{2} \right) - \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} F \left(\cos \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) - \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \right)}{\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1}} d$
parts	$\frac{2a \left(-2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \right)}{\sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right)} \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1}} d$

```
[In] int((a+cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{-i \sqrt{2} a \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{\sin(dx + c) \sqrt{\cos(dx + c)}}$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I
*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sq
rt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) + 2*a*sin(d*x + c)/sqrt(cos(d*x + c)))/
d
```

Sympy [F]

$$\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx = a \left(\int \cos(c + dx) \sec^{\frac{3}{2}}(c + dx) dx + \int \sec^{\frac{3}{2}}(c + dx) dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(3/2), x))
```

Maxima [F]

$$\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx = \int (a \cos(dx + c) + a) \sec(dx + c)^{\frac{3}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx = \int (a \cos(dx + c) + a) \sec(dx + c)^{\frac{3}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{3/2} (a + a \cos(c + dx)) dx$$

```
[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)),x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x)), x)
```

3.294 $\int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx$

Optimal result	3967
Rubi [A] (verified)	3967
Mathematica [C] (verified)	3969
Maple [A] (verified)	3969
Fricas [C] (verification not implemented)	3970
Sympy [F]	3970
Maxima [F]	3970
Giac [F]	3971
Mupad [F(-1)]	3971

Optimal result

Integrand size = 21, antiderivative size = 75

$$\begin{aligned} & \int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx \\ &= \frac{2a \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{2a \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \end{aligned}$$

[Out] $2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.10 (sec), antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3317, 3872, 3856, 2719, 2720}

$$\begin{aligned} & \int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx \\ &= \frac{2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} \\ & \quad + \frac{2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} \end{aligned}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])*Sqrt[\operatorname{Sec}[c + d*x]], x]$

[Out] $(2*a*\sqrt{\cos[c + d*x]}*EllipticE[(c + d*x)/2, 2]*\sqrt{\sec[c + d*x]})/d + (2*a*\sqrt{\cos[c + d*x]}*EllipticF[(c + d*x)/2, 2]*\sqrt{\sec[c + d*x]})/d$

Rule 2719

$\text{Int}[\sqrt{\sin[(c_.) + (d_.)*(x_.)]}, x_Symbol] \rightarrow \text{Simp}[(2/d)*EllipticE[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\sqrt{\sin[(c_.) + (d_.)*(x_.)]}, x_Symbol] \rightarrow \text{Simp}[(2/d)*EllipticF[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3317

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[d^{(n*p)}, \text{Int}[(d*\text{Csc}[e + f*x])^{(m - n*p)}*(b + a*\text{Csc}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, d, e, f, m, n, p\}, x] \&\& !\text{IntegerQ}[m] \&\& \text{IntegersQ}[n, p]$

Rule 3856

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^n*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{EqQ}[n^2, 1/4]$

Rule 3872

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{a + a \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx \\
 &= a \int \frac{1}{\sqrt{\sec(c + dx)}} dx + a \int \sqrt{\sec(c + dx)} dx \\
 &= \left(a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad + \left(a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{2a \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
 &\quad + \frac{2a \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.59 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.88

$$\int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx = \frac{2ia \left(1 + e^{2i(c+dx)} - 2\sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) + 2e^{i(c+dx)} \sqrt{1 + e^{2i(c+dx)}}\right)}{d(1 + e^{2i(c+dx)}) \sqrt{\sec(c + dx)}}$$

[In] Integrate[(a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]

[Out] ((-2*I)*a*(1 + E^((2*I)*(c + d*x)) - 2*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])/(d*(1 + E^((2*I)*(c + d*x)))*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.00

method	result
default	$\frac{2\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \left(F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}$
parts	$\frac{2a\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d} + \frac{2a\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}$
risch	$\frac{i(e^{2i(dx+c)} + 1)a\sqrt{2} \sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)} + 1}} e^{-i(dx+c)}}{d} - \frac{i \left(\sqrt{-i(e^{i(dx+c)} + i)} \sqrt{2} \sqrt{i(e^{i(dx+c)} - i)} \sqrt{i e^{i(dx+c)}} F\left(\sqrt{-i(e^{i(dx+c)} + i)}, \frac{\sqrt{2}}{2}\right) \right)}{\sqrt{e^{3i(dx+c)} + e^{i(dx+c)}}}$

[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

$$= \frac{-i \sqrt{2} a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} a \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{d}$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] (-I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d

Sympy [F]

$$\int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx = a \left(\int \cos(c + dx) \sqrt{\sec(c + dx)} dx + \int \sqrt{\sec(c + dx)} dx \right)$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(1/2),x)

[Out] a*(Integral(cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))

Maxima [F]

$$\int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx = \int (a \cos(dx + c) + a) \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)

Giac [F]

$$\int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx = \int (a \cos(dx + c) + a) \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx)) \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} (a + a \cos(c + dx)) dx$$

[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)),x)

[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x)), x)

3.295 $\int \frac{a+a \cos(c+dx)}{\sqrt{\sec(c+dx)}} dx$

Optimal result	3972
Rubi [A] (verified)	3972
Mathematica [C] (verified)	3974
Maple [A] (verified)	3974
Fricas [C] (verification not implemented)	3975
Sympy [F]	3976
Maxima [F]	3976
Giac [F]	3976
Mupad [F(-1)]	3976

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = \frac{2a\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} + \frac{2a \sin(c + dx)}{3d\sqrt{\sec(c + dx)}}$$

[Out] $2/3*a*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3317, 3872, 3854, 3856, 2720, 2719}

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = \frac{2a \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} + \frac{2a\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2a\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])/Sqrt[\operatorname{Sec}[c + d*x]], x]$

[Out] $(2*a*\sqrt{\cos[c + d*x]}*EllipticE[(c + d*x)/2, 2]*\sqrt{\sec[c + d*x]})/d + (2*a*\sqrt{\cos[c + d*x]}*EllipticF[(c + d*x)/2, 2]*\sqrt{\sec[c + d*x]})/(3*d) + (2*a*\sin[c + d*x])/(3*d*\sqrt{\sec[c + d*x]})$

Rule 2719

$\text{Int}[\sqrt{\sin[(c_.) + (d_.)*(x_)]}, x_Symbol] \rightarrow \text{Simp}[(2/d)*EllipticE[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\sqrt{\sin[(c_.) + (d_.)*(x_)]}, x_Symbol] \rightarrow \text{Simp}[(2/d)*EllipticF[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3317

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)^{(m_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[d^{(n*p)}, \text{Int}[(d*\text{Csc}[e + f*x])^{(m - n*p)}*(b + a*\text{Csc}[e + f*x])^n]^p, x] /; \text{FreeQ}\{a, b, d, e, f, m, n, p\}, x] \&\& !\text{IntegerQ}[m] \&\& \text{IntegersQ}[n, p]$

Rule 3854

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[\cos[c + d*x]*((b*\text{Csc}[c + d*x])^{(n + 1)})/(b*d^n), x] + \text{Dist}[(n + 1)/(b^2*n), \text{Int}[(b*\text{Csc}[c + d*x])^{(n + 2)}], x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 3856

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.)^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^n*\sin[c + d*x]^n, \text{Int}[1/\sin[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{EqQ}[n^2, 1/4]$

Rule 3872

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}], x], x] /; \text{FreeQ}\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{a + a \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx \\ &= a \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx + a \int \frac{1}{\sqrt{\sec(c + dx)}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2a \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{1}{3} a \int \sqrt{\sec(c + dx)} dx \\
&\quad + \left(a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{2a \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{1}{3} \left(a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2a \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2a \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} + \frac{2a \sin(c + dx)}{3d \sqrt{\sec(c + dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.93 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.39

$$\begin{aligned}
&\int \frac{a + a \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{ae^{-2ic}(-i \cos(2c) + \sin(2c)) \left(6 - \frac{12 \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1} \right)}{3d \sqrt{\sec(c + dx)}}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]

[Out] (a*((-I)*Cos[2*c] + Sin[2*c])*(6 - (12*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + (2*I)*Sin[c + d*x]))/(3*d*E^((2*I)*c)*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 5.66 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.23

method	result
default	$\frac{2\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right) a\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$\frac{2a\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}d$

[In] `int((a+cos(d*x+c)*a)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-2/3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)-2*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+(s$$

$$\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticF}(\cos(1$$

$$/2*d*x+1/2*c),2^(1/2))-3*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)$$

$$^2-1)^(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*\sin(1/2*d*x+1/2*c)^4$$

$$+\sin(1/2*d*x+1/2*c)^2)^(1/2)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^($$

$$1/2)/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{2a\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}a\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + i\sqrt{2}a\text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\sin(dx+c))}{d}$$

[In] `integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]
$$1/3*(2*a*\sqrt{\cos(dx+c)}*\sin(dx+c) - I*\sqrt{2}*a*\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + I*\sin(dx+c)) + I*\sqrt{2}*a*\text{weierstrassPInverse}(-4, 0, \cos(dx+c) - I*\sin(dx+c)) + 3*I*\sqrt{2}*a*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + I*\sin(dx+c))) - 3*I*\sqrt{2}*a*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - I*\sin(dx+c))))/d$$

Sympy [F]

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = a \left(\int \frac{\cos(c + dx)}{\sqrt{\sec(c + dx)}} dx + \int \frac{1}{\sqrt{\sec(c + dx)}} dx \right)$$

[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)

[Out] a*(Integral(cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(sec(c + d*x)), x))

Maxima [F]

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = \int \frac{a \cos(dx + c) + a}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)

Giac [F]

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = \int \frac{a \cos(dx + c) + a}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = \int \frac{a + a \cos(c + dx)}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)

$$3.296 \quad \int \frac{a+a \cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	3977
Rubi [A] (verified)	3977
Mathematica [C] (verified)	3979
Maple [A] (verified)	3980
Fricas [C] (verification not implemented)	3980
Sympy [F]	3981
Maxima [F]	3981
Giac [F]	3981
Mupad [F(-1)]	3981

Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{a+a \cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{3d \sqrt{\sec(c+dx)}}$$

[Out] 2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3317, 3872, 3854, 3856, 2719, 2720}

$$\int \frac{a+a \cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{3d \sqrt{\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} + \frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])/Sec[c + d*x]^(3/2), x]

[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\text{integral} = \int \frac{a + a \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx$$

$$\begin{aligned}
&= a \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)} dx + a \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{3d \sqrt{\sec(c+dx)}} + \frac{1}{3} a \int \sqrt{\sec(c+dx)} dx + \frac{1}{5} (3a) \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\
&= \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{3d \sqrt{\sec(c+dx)}} \\
&\quad + \frac{1}{3} \left(a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&\quad + \frac{1}{5} \left(3a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&= \frac{6a \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{2a \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{3d \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.74 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.76

$$\int \frac{a + a \cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{iae^{-3i(c+dx)}(1 + \cos(c+dx)) \left(-3 - 10e^{i(c+dx)} + 33e^{2i(c+dx)} + 39e^{4i(c+dx)} + 10e^{5i(c+dx)} + 3e^{6i(c+dx)} - 72 \right)}{\dots}$$

[In] Integrate[(a + a*Cos[c + d*x])/Sec[c + d*x]^(3/2), x]

[Out] ((-1/120*I)*a*(1 + Cos[c + d*x])*(-3 - 10*E^(I*(c + d*x)) + 33*E^((2*I)*(c + d*x)) + 39*E^((4*I)*(c + d*x)) + 10*E^((5*I)*(c + d*x)) + 3*E^((6*I)*(c + d*x)) - 72*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) + 40*E^((3*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])]*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]])/(d*E^((3*I)*(c + d*x)))

Maple [A] (verified)

Time = 6.65 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.72

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} a \left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-28\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\right)}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$\frac{2a\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d$

```
[In] int((a+cos(d*x+c)*a)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{-5i\sqrt{2}\text{awierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i\sqrt{2}\text{awierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 9\sqrt{2}\text{awierstrassZeta}(-4, 0, \text{awierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 9\sqrt{2}\text{awierstrassZeta}(-4, 0, \text{awierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2*(3*a*\cos(dx + c)^2 + 5*a*\cos(dx + c))*\sin(dx + c)/\sqrt{\cos(dx + c)}}{d}$$

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*a*cos(d*x + c)^2 + 5*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```


Sympy [F]

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx = a \left(\int \frac{\cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \right)$$

[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)**(3/2), x)

[Out] a*(Integral(cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(-3/2), x))

Maxima [F]

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{a \cos(dx + c) + a}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2), x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{a \cos(dx + c) + a}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2), x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{a + a \cos(c + dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{3}{2}}} dx$$

[In] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)

[Out] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)

$$3.297 \quad \int \frac{a+a \cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	3982
Rubi [A] (verified)	3982
Mathematica [C] (verified)	3984
Maple [A] (verified)	3985
Fricas [C] (verification not implemented)	3985
Sympy [F]	3986
Maxima [F]	3986
Giac [F]	3986
Mupad [F(-1)]	3986

Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{a+a \cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx = \frac{6a \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} + \frac{10a \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} + \frac{2a \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{10a \sin(c+dx)}{21d \sqrt{\sec(c+dx)}}$$

[Out] 2/7*a*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10/21*a*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+10/21*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3317, 3872, 3854, 3856, 2720, 2719}

$$\int \frac{a+a \cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx = \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{10a \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{10a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} + \frac{6a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])/Sec[c + d*x]^(5/2), x]

[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\text{integral} = \int \frac{a + a \sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)} dx$$

$$\begin{aligned}
&= a \int \frac{1}{\sec^{\frac{7}{2}}(c+dx)} dx + a \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2a \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3a) \int \frac{1}{\sqrt{\sec(c+dx)}} dx + \frac{1}{7}(5a) \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2a \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{10a \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{1}{21}(5a) \int \sqrt{\sec(c+dx)} dx \\
&\quad + \frac{1}{5} \left(3a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&= \frac{6a \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} + \frac{2a \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{10a \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{1}{21} \left(5a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{6a \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{10a \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{2a \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{2a \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{10a \sin(c+dx)}{21d \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.60 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.31

$$\int \frac{a + a \cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx$$

$$ae^{-4i(c+dx)} \sqrt{\sec(c+dx)} (\cos(4(c+dx)) + i \sin(4(c+dx))) \left(-504i \cos(c+dx) + 504ie^{-i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \right)$$

[In] Integrate[(a + a*Cos[c + d*x])/Sec[c + d*x]^(5/2), x]

[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)])*((-504*I)*Cos[c + d*x] + ((504*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) - (200*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]) + 42*Sin[c + d*x] + 130*Sin[2*(c + d*x)] + 42*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(420*d*E^((4*I)*(c + d*x)))

Maple [A] (verified)

Time = 9.77 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.79

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}a\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-528\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+448\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{105\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}}$
parts	$-\frac{2a\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

[In] int((a+cos(d*x+c)*a)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-122*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{-25i\sqrt{2}a\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i\sqrt{2}a\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 63i\sqrt{2}a\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 63i\sqrt{2}a\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2*(15*a*cos(dx + c)^3 + 21*a*cos(dx + c)^2 + 25*a*cos(dx + c))*sin(dx + c)/sqrt(cos(dx + c))}{105}$$

[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")

```
[Out] 1/105*(-25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*I*sqrt(2)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*a*cos(d*x + c)^3 + 21*a*cos(d*x + c)^2 + 25*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx = a \left(\int \frac{\cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx + \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx \right)$$

[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)

[Out] a*(Integral(cos(c + d*x)/sec(c + d*x)**(5/2), x) + Integral(sec(c + d*x)**(-5/2), x))

Maxima [F]

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{a \cos(dx + c) + a}{\sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{a \cos(dx + c) + a}{\sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{a + a \cos(c + dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{5}{2}}} dx$$

[In] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)

[Out] int((a + a*cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)

3.298 $\int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx$

Optimal result	3987
Rubi [A] (verified)	3987
Mathematica [C] (verified)	3990
Maple [B] (verified)	3990
Fricas [C] (verification not implemented)	3991
Sympy [F(-1)]	3992
Maxima [F]	3992
Giac [F]	3992
Mupad [F(-1)]	3992

Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}
 & \int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx \\
 &= -\frac{16a^2 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &+ \frac{4a^2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
 &+ \frac{16a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\
 &+ \frac{4a^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d}
 \end{aligned}$$

```
[Out] 4/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+16/5*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d-16/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
)*sec(d*x+c)^(1/2)/d+4/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3873, 3853, 3856, 2720, 4131, 2719}

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{2a^2 \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} + \frac{4a^2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d}$$

$$+ \frac{16a^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

$$+ \frac{4a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

$$- \frac{16a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]

[Out] (-16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (16*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] & IntegerQ[2*n]

Rule 3856


```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^(n)*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_.)), x_Symbol] := Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2 dx \\
&= (2a^2) \int \sec^{\frac{5}{2}}(c + dx) dx + \int \sec^{\frac{3}{2}}(c + dx) (a^2 + a^2 \sec^2(c + dx)) dx \\
&= \frac{4a^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad + \frac{1}{3}(2a^2) \int \sqrt{\sec(c + dx)} dx + \frac{1}{5}(8a^2) \int \sec^{\frac{3}{2}}(c + dx) dx \\
&= \frac{16a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{4a^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad + \frac{2a^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} - \frac{1}{5}(8a^2) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + \frac{1}{3} \left(2a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{4a^2 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{16a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\
&\quad + \frac{4a^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad - \frac{1}{5} \left(8a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx
\end{aligned}$$

$$= -\frac{16a^2 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{\sec(c+dx)}}{5d} + \frac{4a^2 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} + \frac{16a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{4a^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d} + \frac{2a^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.47 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.62

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{a^2(1 + \cos(c + dx))^2 \sec^4\left(\frac{1}{2}(c + dx)\right) \left(-\frac{2i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (12(1+e^{2i(c+dx)})+12(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}) \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -E^{\left(\frac{2i}{1+e^{2i(c+dx)}}\right)}\right]}{\dots} \right)}{\dots}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2), x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c)) + Sqrt[Sec[c + d*x]]*(24*Cos[d*x]*Csc[c] + (10 + 3*Sec[c + d*x])*Tan[c + d*x]))/(30*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(189) = 378.

Time = 20.62 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.40

method	result
default	$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a^2} \left(-\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}{12\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2}\right)^2} + \frac{17\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{30\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} \right)$
parts	Expression too large to display

[In] `int((a+cos(d*x+c))*a)^2*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)`

[Out] $-8*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^2*(-1/12*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^2+17/30*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-4/5*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)/(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-2/5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-1/80*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^3/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx =$$

$$2 \left(5i \sqrt{2} a^2 \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^2 \cos(dx + c) \right)$$

[In] `integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="fricas")`

[Out] $-2/15*(5*I*\sqrt{2})*a^2*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) - 5*I*\sqrt{2}*a^2*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + 12*I*\sqrt{2}*a^2*\cos(d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) - 12*I*\sqrt{2}*a^2*\cos(d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) - (24*a^2*\cos(d*x + c)^2 + 10*a^2*\cos(d*x + c) + 3*a^2)*\sin(d*x + c)/\sqrt{\cos(d*x + c)}/(d*\cos(d*x + c)^2)$

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)

Giac [F]

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{7/2} (a + a \cos(c + dx))^2 dx$$

[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2, x)

3.299 $\int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx$

Optimal result	3993
Rubi [A] (verified)	3993
Mathematica [C] (verified)	3996
Maple [B] (verified)	3996
Fricas [C] (verification not implemented)	3997
Sympy [F(-1)]	3997
Maxima [F]	3998
Giac [F]	3998
Mupad [F(-1)]	3998

Optimal result

Integrand size = 23, antiderivative size = 131

$$\begin{aligned} & \int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx \\ &= -\frac{4a^2 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{8a^2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\ & \quad + \frac{4a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \end{aligned}$$

[Out] $2/3*a^2*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+4*a^2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-4*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+8/3*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3873, 3853, 3856, 2719, 4131, 2720}

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{2a^2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d} + \frac{4a^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{d}$$

$$+ \frac{8a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

$$- \frac{4a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]

[Out] (-4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_.)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2 dx \\
&= (2a^2) \int \sec^{\frac{3}{2}}(c+dx) dx + \int \sqrt{\sec(c+dx)}(a^2+a^2\sec^2(c+dx)) dx \\
&= \frac{4a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} \\
&\quad + \frac{1}{3}(4a^2) \int \sqrt{\sec(c+dx)} dx - (2a^2) \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\
&= \frac{4a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} \\
&\quad + \frac{1}{3}\left(4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&\quad - \left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{8a^2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad + \frac{4a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.06 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.91

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{a^2(1 + \cos(c + dx))^2 \sec^4\left(\frac{1}{2}(c + dx)\right) \left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right)}{\text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -E^{\left(\frac{1}{2}(c+dx)\right)}\right]}\right)}{\dots}$$

[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2), x]

[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) + 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c] + Tan[c + d*x]))/(6*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(167) = 334.

Time = 19.14 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.83

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2\left(12\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\dots}$
parts	$-\frac{2a^2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\dots\right)}$

[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^(5/2), x, method=_RETURNVERBOSE)

[Out] -4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*sin(1/2*d*x+1/2*c)^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*sin(1/2*d*x+1/2*c)^2-7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-

$1)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$
 $2) * (2 * \sin(1/2*d*x+1/2*c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * (-2 * \sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} / (2 * \cos(1/2*d*x+1/2*c)^2 - 1)^{(1/2)} / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx = \frac{2 \left(2i \sqrt{2} a^2 \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 2i \sqrt{2} a^2 \cos(dx + c) \right)}{-}$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] -2/3*(2*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 2*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*a^2*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (6*a^2*cos(d*x + c) + a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)

Giac [F]

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{5/2} (a + a \cos(c + dx))^2 dx$$

[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^2, x)

3.300 $\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	3999
Rubi [A] (verified)	3999
Mathematica [A] (verified)	4001
Maple [B] (verified)	4001
Fricas [C] (verification not implemented)	4002
Sympy [F(-1)]	4002
Maxima [F]	4002
Giac [F]	4003
Mupad [F(-1)]	4003

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{4a^2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{d}$$

[Out] $2*a^2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+4*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.13 (sec), antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3317, 3873, 3856, 2720, 4128}

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{2a^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{d} + \frac{4a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^2*\operatorname{Sec}[c + d*x]^{(3/2)}, x]$

[Out] $(4*a^2*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{EllipticF}[(c + d*x)/2, 2]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]])/d + (2*a^2*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]*\operatorname{Sin}[c + d*x])/d$

Rule 2720

$\operatorname{Int}[1/\operatorname{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \operatorname{Simp}[(2/d)*\operatorname{EllipticF}[(1/2)*(c - \operatorname{Pi}/2 + d*x), 2], x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4128

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(a + a \sec(c + dx))^2}{\sqrt{\sec(c + dx)}} dx \\
&= (2a^2) \int \sqrt{\sec(c + dx)} dx + \int \frac{a^2 + a^2 \sec^2(c + dx)}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \left(2a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{4a^2 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{2a^2 \sqrt{\sec(c + dx)} \left(2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sin(c + dx) \right)}{d}$$

[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]

[Out] (2*a^2*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]))/d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(84) = 168.

Time = 5.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.89

method	result
default	$\frac{4a^2 \left(-\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}$
parts	$\frac{2a^2 \left(-2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}$

[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -4*a^2*(-cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.20

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx = \frac{2 \left(i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{d}$$

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
- I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
- a^2*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{3/2} (a + a \cos(c + dx))^2 dx$$

[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2, x)

3.301 $\int (a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$

Optimal result	4004
Rubi [A] (verified)	4004
Mathematica [C] (verified)	4006
Maple [A] (verified)	4006
Fricas [C] (verification not implemented)	4007
Sympy [F]	4008
Maxima [F]	4008
Giac [F]	4008
Mupad [F(-1)]	4008

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int (a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$$

$$= \frac{4a^2 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d}$$

$$+ \frac{8a^2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} + \frac{2a^2 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}}$$

[Out] $2/3*a^2*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+4*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+8/3*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3873, 3856, 2719, 4130, 2720}

$$\int (a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$$

$$= \frac{2a^2 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{8a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

$$+ \frac{4a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^2*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]], x]$

[Out] $(4a^2\sqrt{\cos[c + dx]} \text{EllipticE}[(c + dx)/2, 2] \sqrt{\sec[c + dx]})/d + (8a^2\sqrt{\cos[c + dx]} \text{EllipticF}[(c + dx)/2, 2] \sqrt{\sec[c + dx]})/(3d) + (2a^2\sin[c + dx])/(3d\sqrt{\sec[c + dx]})$

Rule 2719

$\text{Int}[\sqrt{\sin[(c_.) + (d_.)x]}, x_Symbol] \rightarrow \text{Simp}[(2/d)\text{EllipticE}[(1/2)(c - \pi/2 + dx), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\sqrt{\sin[(c_.) + (d_.)x]}, x_Symbol] \rightarrow \text{Simp}[(2/d)\text{EllipticF}[(1/2)(c - \pi/2 + dx), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3317

$\text{Int}[(\csc[(e_.) + (f_.)x])(d_.)^{(m_.)}((a_.) + (b_.)\sin[(e_.) + (f_.)x])^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[d^{(n*p)}, \text{Int}[(d*\csc[e + f*x])^{(m - n*p)}(b + a*\csc[e + f*x]^n)^p, x], x] /; \text{FreeQ}\{a, b, d, e, f, m, n, p\}, x \ \&\& \text{IntegerQ}[m] \ \&\& \text{IntegersQ}[n, p]$

Rule 3856

$\text{Int}[(\csc[(c_.) + (d_.)x])(b_.)^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\csc[c + dx])^n \sin[c + dx]^n, \text{Int}[1/\sin[c + dx]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \text{EqQ}[n^2, 1/4]$

Rule 3873

$\text{Int}[(\csc[(e_.) + (f_.)x])(d_.)^{(n_.)}(\csc[(e_.) + (f_.)x])(b_.) + (a_.)^2, x_Symbol] \rightarrow \text{Dist}[2*a*(b/d), \text{Int}[(d*\csc[e + f*x])^{(n + 1)}, x], x] + \text{Int}[(d*\csc[e + f*x])^n(a^2 + b^2*\csc[e + f*x]^2), x] /; \text{FreeQ}\{a, b, d, e, f, n\}, x]$

Rule 4130

$\text{Int}[(\csc[(e_.) + (f_.)x])(b_.)^{(m_.)}(\csc[(e_.) + (f_.)x])^2(C_.) + (A_)), x_Symbol] \rightarrow \text{Simp}[A*\cot[e + f*x]*((b*\csc[e + f*x])^m/(f*m)), x] + \text{Dist}[(C*m + A*(m + 1))/(b^2*m), \text{Int}[(b*\csc[e + f*x])^{(m + 2)}, x], x] /; \text{FreeQ}\{b, e, f, A, C\}, x \ \&\& \text{NeQ}[C*m + A*(m + 1), 0] \ \&\& \text{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{(a + a \sec(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx \\ &= (2a^2) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \int \frac{a^2 + a^2 \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2a^2 \sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(4a^2) \int \sqrt{\sec(c+dx)} dx \\
&\quad + \left(2a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx \\
&= \frac{4a^2 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} + \frac{2a^2 \sin(c+dx)}{3d\sqrt{\sec(c+dx)}} \\
&\quad + \frac{1}{3} \left(4a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{4a^2 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{8a^2 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} + \frac{2a^2 \sin(c+dx)}{3d\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.19

$$\begin{aligned}
&\int (a + a \cos(c+dx))^2 \sqrt{\sec(c+dx)} dx \\
&= \frac{a^2 \left(\frac{{}_2F_1\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2 \left(-6i - 4i\sqrt{1+e^{2i(c+dx)}} \right) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right) \right)}{3d\sqrt{\sec(c+dx)}}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]], x]

[Out] (a^2*(((24*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*(-6*I - (4*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c + d*x])))/(3*d*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.13

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}} a^2\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{1}{2}+\frac{\cos(dx+c)}{2}}\right)$
parts	$-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}d$

[In] `int((a+cos(d*x+c))*a)^2*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-4/3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)-\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+2*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^(1/2))-3*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^(1/2)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.25

$$\int (a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$$

$$2 \left(a^2 \sqrt{\cos(dx + c)} \sin(dx + c) - 2i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right) / d$$

[In] `integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]
$$2/3*(a^2*\sqrt{\cos(dx + c)}*\sin(dx + c) - 2*I*\sqrt{2}*a^2*\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c)) + 2*I*\sqrt{2}*a^2*\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c)) + 3*I*\sqrt{2}*a^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c))) - 3*I*\sqrt{2}*a^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c))))/d$$

Sympy [F]

$$\int (a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx = a^2 \left(\int 2 \cos(c + dx) \sqrt{\sec(c + dx)} dx \right. \\ \left. + \int \cos^2(c + dx) \sqrt{\sec(c + dx)} dx \right. \\ \left. + \int \sqrt{\sec(c + dx)} dx \right)$$

[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(1/2),x)

[Out] a**2*(Integral(2*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x)**2*sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))

Maxima [F]

$$\int (a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx = \int (a \cos(dx + c) + a)^2 \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)

Giac [F]

$$\int (a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx = \int (a \cos(dx + c) + a)^2 \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} (a + a \cos(c + dx))^2 dx$$

[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2, x)

$$3.302 \quad \int \frac{(a+a \cos(c+dx))^2}{\sqrt{\sec(c+dx)}} dx$$

Optimal result	4009
Rubi [A] (verified)	4009
Mathematica [C] (verified)	4011
Maple [A] (verified)	4012
Fricas [C] (verification not implemented)	4012
Sympy [F]	4013
Maxima [F]	4013
Giac [F]	4013
Mupad [F(-1)]	4014

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(a+a \cos(c+dx))^2}{\sqrt{\sec(c+dx)}} dx = \frac{16a^2 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\ + \frac{4a^2 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} \\ + \frac{2a^2 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2 \sin(c+dx)}{3d \sqrt{\sec(c+dx)}}$$

[Out] $2/5*a^2*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+4/3*a^2*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}$
 $+16/5*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2$
 $*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+4/3*a^2*(\cos(1/2*d$
 $*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})$
 $*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00,
 number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used
 = {3317, 3873, 3854, 3856, 2720, 4130, 2719}

$$\int \frac{(a+a \cos(c+dx))^2}{\sqrt{\sec(c+dx)}} dx = \frac{2a^2 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2 \sin(c+dx)}{3d \sqrt{\sec(c+dx)}} \\ + \frac{4a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{16a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]

[Out] (16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3873

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4130

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +

Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(a + a \sec(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &= (2a^2) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{a^2 + a^2 \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2a^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{4a^2 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
 &\quad + \frac{1}{3} (2a^2) \int \sqrt{\sec(c + dx)} dx + \frac{1}{5} (8a^2) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{2a^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{4a^2 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
 &\quad + \frac{1}{3} \left(2a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad + \frac{1}{5} \left(8a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{16a^2 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &\quad + \frac{4a^2 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
 &\quad + \frac{2a^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{4a^2 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\begin{aligned}
 &\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{a^2 \left(-96i + \frac{192i \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 40i \sqrt{1+e^{2i(c+dx)}} \text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right) \right)}{30d \sqrt{\sec(c + dx)}}
 \end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]], x]

```
[Out] (a^2*(-96*I + ((192*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (40*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 40*Sin[c + d*x] + 6*Sin[2*(c + d*x)]))/(30*d*Sqrt[Sec[c + d*x]])
```

Maple [A] (verified)

Time = 8.36 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.85

method	result
default	$\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}a^2\left(-12\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-13\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$
parts	$\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$

```
[In] int((a+cos(d*x+c)*a)^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-13*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.16

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx =$$

$$2 \left(5i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))
```



```
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (3*a^2*cos(d*x + c)^2 + 10*a^2*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx = a^2 \left(\int \frac{2 \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx + \int \frac{\cos^2(c + dx)}{\sqrt{\sec(c + dx)}} dx + \int \frac{1}{\sqrt{\sec(c + dx)}} dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^2}{\sqrt{\sec(dx + c)}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^2}{\sqrt{\sec(dx + c)}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + a \cos(c + dx))^2}{\sqrt{\frac{1}{\cos(c + dx)}}} dx$$

```
[In] int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

```
[Out] int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)
```

$$3.303 \quad \int \frac{(a+a \cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4015
Rubi [A] (verified)	4015
Mathematica [C] (verified)	4017
Maple [A] (verified)	4018
Fricas [C] (verification not implemented)	4018
Sympy [F]	4019
Maxima [F]	4019
Giac [F]	4019
Mupad [F(-1)]	4020

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(a + a \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{12a^2 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{8a^2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{7d} + \frac{2a^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{4a^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{8a^2 \sin(c + dx)}{7d \sqrt{\sec(c + dx)}}$$

[Out] 2/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/7*a^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+8/7*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3317, 3873, 3854, 3856, 2719, 4130, 2720}

$$\int \frac{(a + a \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{4a^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{8a^2 \sin(c + dx)}{7d \sqrt{\sec(c + dx)}} + \frac{8a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{7d} + \frac{12a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]

[Out] (12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3873

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4130

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +

Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(a + a \sec(c + dx))^2}{\sec^{\frac{7}{2}}(c + dx)} dx \\
 &= (2a^2) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx + \int \frac{a^2 + a^2 \sec^2(c + dx)}{\sec^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2a^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{4a^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
 &\quad + \frac{1}{5}(6a^2) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{7}(12a^2) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2a^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{4a^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{8a^2 \sin(c + dx)}{7d \sqrt{\sec(c + dx)}} + \frac{1}{7}(4a^2) \int \sqrt{\sec(c + dx)} dx \\
 &\quad + \frac{1}{5} \left(6a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{12a^2 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{2a^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
 &\quad + \frac{4a^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{8a^2 \sin(c + dx)}{7d \sqrt{\sec(c + dx)}} + \frac{1}{7} \left(4a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{12a^2 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &\quad + \frac{8a^2 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{7d} \\
 &\quad + \frac{2a^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{4a^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{8a^2 \sin(c + dx)}{7d \sqrt{\sec(c + dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.69 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93

$$\begin{aligned}
 &\int \frac{(a + a \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{a^2 \left(\frac{672i \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2 \left(-168i - 80i \sqrt{1 + e^{2i(c+dx)}} \text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right) \right) \right)}{140d \sqrt{\sec(c + dx)}}
 \end{aligned}$$

[In] Integrate[(a + a*cos[c + d*x])^2/Sec[c + d*x]^(3/2), x]

[Out] (a^2*((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*(-168*I - (80*I)*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 85*Sin[c + d*x] + 28*Sin[2*(c + d*x)] + 5*Sin[3*(c + d*x)]))/(140*d*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 10.48 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$\frac{4\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2\left(40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-116\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+126\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-35\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{2a^2\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$
parts	

[In] int((a+cos(d*x+c)*a)^2/sec(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-39*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{(a + a \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx =$$

$$2 \left(10i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} a^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2), x, algorithm="fricas")

```
[Out] -2/35*(10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) - 10*I*sqrt(2)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*a^2*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (5*a^2*cos(d
*x + c)^3 + 14*a^2*cos(d*x + c)^2 + 20*a^2*cos(d*x + c))*sin(d*x + c)/sqrt(
cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx = a^2 \left(\int \frac{2 \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{\cos^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2), x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(cos(c + d*
x)**2/sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^2}{\sec(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2), x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^2}{\sec(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2), x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a + a \cos(c + dx))^2}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

```
[In] int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```

```
[Out] int((a + a*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)
```


3.304 $\int (a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx$

Optimal result	4021
Rubi [A] (verified)	4022
Mathematica [C] (verified)	4024
Maple [B] (verified)	4025
Fricas [C] (verification not implemented)	4025
Sympy [F(-1)]	4026
Maxima [F]	4026
Giac [F]	4026
Mupad [F(-1)]	4027

Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}
 & \int (a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx \\
 &= -\frac{28a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &+ \frac{52a^3 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{21d} \\
 &+ \frac{28a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{52a^3 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{21d} \\
 &+ \frac{6a^3 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2a^3 \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{7d}
 \end{aligned}$$

```
[Out] 52/21*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+6/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)
/d+2/7*a^3*sec(d*x+c)^(7/2)*sin(d*x+c)/d+28/5*a^3*sin(d*x+c)*sec(d*x+c)^(1/
2)/d-28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+52/21*a^3*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3876, 3853, 3856, 2719, 2720}

$$\begin{aligned} & \int (a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx \\ &= \frac{2a^3 \sin(c + dx) \sec^{\frac{7}{2}}(c + dx)}{7d} + \frac{6a^3 \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} \\ &+ \frac{52a^3 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{21d} + \frac{28a^3 \sin(c + dx) \sqrt{\sec(c + dx)}}{5d} \\ &+ \frac{52a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} \\ &- \frac{28a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} \end{aligned}$$

[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]

[Out] (-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &

& IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3876

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I GtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3 dx \\
 &= \int \left(a^3 \sec^{\frac{3}{2}}(c + dx) + 3a^3 \sec^{\frac{5}{2}}(c + dx) + 3a^3 \sec^{\frac{7}{2}}(c + dx) + a^3 \sec^{\frac{9}{2}}(c + dx) \right) dx \\
 &= a^3 \int \sec^{\frac{3}{2}}(c + dx) dx + a^3 \int \sec^{\frac{9}{2}}(c + dx) dx \\
 &\quad + (3a^3) \int \sec^{\frac{5}{2}}(c + dx) dx + (3a^3) \int \sec^{\frac{7}{2}}(c + dx) dx \\
 &= \frac{2a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^3 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d} \\
 &\quad + \frac{6a^3 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2a^3 \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{7d} \\
 &\quad + \frac{1}{7}(5a^3) \int \sec^{\frac{5}{2}}(c + dx) dx - a^3 \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &\quad + a^3 \int \sqrt{\sec(c + dx)} dx + \frac{1}{5}(9a^3) \int \sec^{\frac{3}{2}}(c + dx) dx \\
 &= \frac{28a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{52a^3 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{21d} \\
 &\quad + \frac{6a^3 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2a^3 \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{7d} \\
 &\quad + \frac{1}{21}(5a^3) \int \sqrt{\sec(c + dx)} dx - \frac{1}{5}(9a^3) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &\quad + \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad - \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{2a^3 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} \\
&+ \frac{2a^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&+ \frac{28a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{52a^3 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{21d} \\
&+ \frac{6a^3 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{2a^3 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{7d} \\
&+ \frac{1}{21} \left(5a^3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&- \frac{1}{5} \left(9a^3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{28a^3 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&+ \frac{52a^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} \\
&+ \frac{28a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{52a^3 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{21d} \\
&+ \frac{6a^3 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{2a^3 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{7d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.13 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.49

$$\int (a + a \cos(c+dx))^3 \sec^{\frac{9}{2}}(c+dx) dx$$

$$= \frac{a^3 (1 + \cos(c+dx))^3 \sec^6\left(\frac{1}{2}(c+dx)\right) \left(-\frac{2i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (147(1+e^{2i(c+dx)})+147(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}) \operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -E^{\left(\frac{2i}{1+e^{2i(c+dx)}}\right)}\right]}{d} \right)}{420}$$

[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2), x]

[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(294*Cos[d*x]*Csc[c] + (80 + 63*Cos[c + d*x] + 65*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c + d*x]))/(420*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(211) = 422$.

Time = 64.02 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.35

method	result
default	$16\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} a^3 \left(-\frac{13\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{168\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2} + \frac{53\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{105\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}} \right)$
parts	Expression too large to display

[In] `int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)`

[Out]
$$-16*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^3*(-13/168*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^2+53/105*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1/448*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^4-7/10*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)/(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-7/20*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-3/160*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^3/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx =$$

$$2 \left(65i \sqrt{2} a^3 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 65i \sqrt{2} a^3 \cos(dx + c) \right)$$

[In] `integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="fricas")`

[Out]
$$-2/105*(65*I*\sqrt{2})*a^3*\cos(d*x + c)^3*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) - 65*I*\sqrt{2})*a^3*\cos(d*x + c)^3*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + 147*I*\sqrt{2})*a^3*\cos(d*x + c)^3*$$

```
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c))) - 147*I*sqrt(2)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (294*a^3*cos(d*x + c)^3
+ 130*a^3*cos(d*x + c)^2 + 63*a^3*cos(d*x + c) + 15*a^3)*sin(d*x + c)/sqrt(
cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{9}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

Giac [F]

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{9}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{9/2} (a + a \cos(c + dx))^3 dx$$

```
[In] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3, x)
```

3.305 $\int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx$

Optimal result	4028
Rubi [A] (verified)	4028
Mathematica [C] (verified)	4031
Maple [B] (verified)	4032
Fricas [C] (verification not implemented)	4032
Sympy [F(-1)]	4033
Maxima [F]	4033
Giac [F]	4033
Mupad [F(-1)]	4033

Optimal result

Integrand size = 23, antiderivative size = 157

$$\begin{aligned}
 & \int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx \\
 &= -\frac{36a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &+ \frac{4a^3 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \\
 &+ \frac{36a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\
 &+ \frac{2a^3 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d} + \frac{2a^3 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d}
 \end{aligned}$$

```
[Out] 2*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d+3
6/5*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d-36/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {3317, 3876, 3856, 2720, 3853, 2719}

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{2a^3 \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} + \frac{2a^3 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{d}$$

$$+ \frac{36a^3 \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

$$+ \frac{4a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

$$- \frac{36a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2), x]

[Out] (-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n], x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.))^m], x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{\sec(c + dx)}(a + a \sec(c + dx))^3 dx \\
&= \int \left(a^3 \sqrt{\sec(c + dx)} + 3a^3 \sec^{\frac{3}{2}}(c + dx) + 3a^3 \sec^{\frac{5}{2}}(c + dx) + a^3 \sec^{\frac{7}{2}}(c + dx) \right) dx \\
&= a^3 \int \sqrt{\sec(c + dx)} dx + a^3 \int \sec^{\frac{7}{2}}(c + dx) dx \\
&\quad + (3a^3) \int \sec^{\frac{3}{2}}(c + dx) dx + (3a^3) \int \sec^{\frac{5}{2}}(c + dx) dx \\
&= \frac{6a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^3 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d} \\
&\quad + \frac{2a^3 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{1}{5} (3a^3) \int \sec^{\frac{3}{2}}(c + dx) dx \\
&\quad + a^3 \int \sqrt{\sec(c + dx)} dx - (3a^3) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2a^3 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{36a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{2a^3 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d} \\
&\quad + \frac{2a^3 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} - \frac{1}{5} (3a^3) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad - \left(3a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx
\end{aligned}$$

$$\begin{aligned}
&= -\frac{6a^3 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{4a^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{36a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} \\
&\quad + \frac{2a^3 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{d} + \frac{2a^3 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad - \frac{1}{5} \left(3a^3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{36a^3 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{4a^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{36a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} \\
&\quad + \frac{2a^3 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{d} + \frac{2a^3 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.49 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.65

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx$$

$$= a^3 (1 + \cos(c + dx))^3 \sec^6\left(\frac{1}{2}(c + dx)\right) \left(-\frac{2i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (9(1+e^{2i(c+dx)})+9(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}})}{\operatorname{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -E^{\left(\frac{1}{2}(c+dx)\right)}\right]} \right)$$

[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2), x]

[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x)))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c] + (5 + Sec[c + d*x])*Tan[c + d*x]))/(20*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(189) = 378.

Time = 64.27 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.46

method	result
default	$16\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} a^3 \left(\frac{7\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1} F(\cos(\frac{dx}{2} + \frac{c}{2}), \sqrt{2})}{10\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}} - \frac{\cos(\frac{dx}{2} + \frac{c}{2}) \sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}}{16(\cos^2(\frac{dx}{2} + \frac{c}{2}))} \right)$
parts	Expression too large to display

[In] `int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)`

[Out]
$$-16*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a^3*(7/10*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1/16*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^2-9/10*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)/(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}-9/20*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))-1/160*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^3/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx =$$

$$2 \left(5i \sqrt{2} a^3 \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c) \right)$$

[In] `integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="fricas")`

[Out]
$$-2/5*(5*I*\text{sqrt}(2)*a^3*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) - 5*I*\text{sqrt}(2)*a^3*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + 9*I*\text{sqrt}(2)*a^3*\cos(d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) - 9*I*\text{sqrt}(2)*a^3*\cos(d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) - (18*a^3*\cos(d*x + c)^2 + 5*a^3*\cos(d*x + c) + a^3)*\sin(d*x + c)/\text{sqrt}(\cos(d*x + c)))/(d*\cos(d*x + c)^2)$$

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{7}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

Giac [F]

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{7}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{7/2} (a + a \cos(c + dx))^3 dx$$

```
[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3, x)
```

3.306 $\int (a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx$

Optimal result	4034
Rubi [A] (verified)	4034
Mathematica [C] (verified)	4037
Maple [B] (verified)	4037
Fricas [C] (verification not implemented)	4038
Sympy [F(-1)]	4038
Maxima [F]	4039
Giac [F]	4039
Mupad [F(-1)]	4039

Optimal result

Integrand size = 23, antiderivative size = 131

$$\begin{aligned} & \int (a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx \\ &= -\frac{4a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{20a^3 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\ & \quad + \frac{6a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^3 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \end{aligned}$$

[Out] $2/3*a^3*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+6*a^3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-4$
 $*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+$
 $1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+20/3*a^3*(\cos(1/2*d*x+1$
 $/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos$
 $(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00,
 number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {3317, 3876, 3856, 2719, 2720, 3853}

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{2a^3 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d} + \frac{6a^3 \sin(c + dx) \sqrt{\sec(c + dx)}}{d}$$

$$+ \frac{20a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

$$- \frac{4a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2), x]

[Out] (-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3876

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n_*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m_], x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I GtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(a + a \sec(c + dx))^3}{\sqrt{\sec(c + dx)}} dx \\
&= \int \left(\frac{a^3}{\sqrt{\sec(c + dx)}} + 3a^3 \sqrt{\sec(c + dx)} + 3a^3 \sec^{\frac{3}{2}}(c + dx) + a^3 \sec^{\frac{5}{2}}(c + dx) \right) dx \\
&= a^3 \int \frac{1}{\sqrt{\sec(c + dx)}} dx + a^3 \int \sec^{\frac{5}{2}}(c + dx) dx \\
&\quad + (3a^3) \int \sqrt{\sec(c + dx)} dx + (3a^3) \int \sec^{\frac{3}{2}}(c + dx) dx \\
&= \frac{6a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^3 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad + \frac{1}{3} a^3 \int \sqrt{\sec(c + dx)} dx - (3a^3) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&\quad + \left(3a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{6a^3 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{6a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^3 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad + \frac{1}{3} \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad - \left(3a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx
\end{aligned}$$

$$= -\frac{4a^3 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} + \frac{20a^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} + \frac{6a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{d} + \frac{2a^3 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.20

$$\int (a + a \cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx) dx = \frac{ia^3 \sec^{\frac{3}{2}}(c+dx) \left(-6 - 6 \cos(2(c+dx)) + 6e^{-2i(c+dx)} (1 + e^{2i(c+dx)})^{3/2} \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -\right)\right)}{d}$$

[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2), x]

[Out] ((-1/3*I)*a^3*Sec[c + d*x]^(3/2)*(-6 - 6*Cos[2*(c + d*x)] + (6*(1 + E^((2*I)*(c + d*x)))^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) / E^((2*I)*(c + d*x)) + 20*sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + (2*I)*Sin[c + d*x] + (9*I)*Sin[2*(c + d*x)])) / d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(167) = 334.

Time = 62.79 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.83

method	result
default	$-\frac{4\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} a^3 \left(18(\sin^4(\frac{dx}{2} + \frac{c}{2})) \cos(\frac{dx}{2} + \frac{c}{2}) - 10\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2}))} - 1 F\left(\dots\right)\right)}{d}$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(5/2), x, method=_RETURNVERBOSE)

[Out]
$$-\frac{4}{3} * (-(-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * a ^ 3 / (4 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 4 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 1) / \sin(1/2 * d * x + 1/2 * c) ^ 3 * (18 * \sin(1/2 * d * x + 1/2 * c) ^ 4 * \cos(1/2 * d * x + 1/2 * c) - 10 * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \operatorname{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 6 * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \operatorname{Ellip}$$

```
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-10*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.37

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx =$$

$$2 \left(5i \sqrt{2} a^3 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \cos(dx + c) \right) \sqrt{\cos(dx + c)}$$

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*a^3*cos(d*x + c)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I
*sqrt(2)*a^3*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c))) - (9*a^3*cos(d*x + c) + a^3)*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)

Giac [F]

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{\frac{5}{2}} (a + a \cos(c + dx))^3 dx$$

[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3, x)

3.307 $\int (a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	4040
Rubi [A] (verified)	4040
Mathematica [C] (verified)	4043
Maple [A] (verified)	4043
Fricas [C] (verification not implemented)	4044
Sympy [F(-1)]	4044
Maxima [F]	4044
Giac [F]	4045
Mupad [F(-1)]	4045

Optimal result

Integrand size = 23, antiderivative size = 131

$$\begin{aligned} & \int (a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx \\ &= \frac{4a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{20a^3 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\ & \quad + \frac{2a^3 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{2a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} \end{aligned}$$

[Out] $2/3*a^3*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2*a^3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+4*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+20/3*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3876, 3854, 3856, 2720, 2719, 3853}

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{2a^3 \sin(c + dx) \sqrt{\sec(c + dx)}}{d} + \frac{2a^3 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{20a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{4a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2), x]

[Out] (4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1)/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

]

Rule 3856

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^n*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{EqQ}[n^2, 1/4]$

Rule 3876

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^m, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[(a + b*\text{csc}[e + f*x])^m*(d*\text{csc}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, d, e, f, m, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{I GtQ}[m, 0] \&\& \text{RationalQ}[n]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(a + a \sec(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &= \int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c + dx)} + \frac{3a^3}{\sqrt{\sec(c + dx)}} + 3a^3 \sqrt{\sec(c + dx)} + a^3 \sec^{\frac{3}{2}}(c + dx) \right) dx \\
 &= a^3 \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx + a^3 \int \sec^{\frac{3}{2}}(c + dx) dx \\
 &\quad + (3a^3) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + (3a^3) \int \sqrt{\sec(c + dx)} dx \\
 &= \frac{2a^3 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{2a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{1}{3} a^3 \int \sqrt{\sec(c + dx)} dx \\
 &\quad - a^3 \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \left(3a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad + \left(3a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{6a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
 &\quad + \frac{6a^3 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \\
 &\quad + \frac{2a^3 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{2a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} \\
 &\quad + \frac{1}{3} \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad - \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{4a^3 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} \\
&+ \frac{20a^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} \\
&+ \frac{2a^3 \sin(c+dx)}{3d \sqrt{\sec(c+dx)}} + \frac{2a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03

$$\begin{aligned}
&\int (a + a \cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx) dx \\
&= \frac{a^3 \left(\frac{24i \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2 \left(-6i - 10i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right) \right) \right)}{3d \sqrt{\sec(c+dx)}}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2), x]

[Out] (a^3*((24*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*(-6*I - (10*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c + d*x] + 3*Tan[c + d*x]))/(3*d*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 7.91 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.31

method	result
default	$ \frac{4a^3 \left(2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \cos \left(\frac{dx}{2} + \frac{c}{2} \right) - 4 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \cos \left(\frac{dx}{2} + \frac{c}{2} \right) + 5 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} F \left(\cos \left(\frac{dx}{2} + \frac{c}{2} \right), \sqrt{2} \right) \right)}{3 \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} d} $
parts	$ \frac{2a^3 \left(-2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} d \right)}{\sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} d} $

[In] int((a+cos(d*x+c))*a)^3*sec(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c))^4*cos(1/2*d*x+1/2*c)-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx =$$

$$2 \left(5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3I \sqrt{2} a^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I \sin(dx + c))) + 3I \sqrt{2} a^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I \sin(dx + c))) - (a^3 \cos(dx + c) + 3a^3 \sin(dx + c) / \sqrt{\cos(dx + c)}) \right) / d$$

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c) + 3*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```


Giac [F]

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{3/2} (a + a \cos(c + dx))^3 dx$$

[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3, x)

3.308 $\int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx$

Optimal result	4046
Rubi [A] (verified)	4046
Mathematica [C] (verified)	4049
Maple [A] (verified)	4049
Fricas [C] (verification not implemented)	4050
Sympy [F]	4050
Maxima [F]	4051
Giac [F]	4051
Mupad [F(-1)]	4051

Optimal result

Integrand size = 23, antiderivative size = 131

$$\begin{aligned} & \int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx \\ &= \frac{36a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\ & \quad + \frac{4a^3 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{2a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{d \sqrt{\sec(c + dx)}} \end{aligned}$$

```
[Out] 2/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+3
6/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4*a^3*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {3317, 3876, 3854, 3856, 2719, 2720}

$$\int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx$$

$$= \frac{2a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{d \sqrt{\sec(c + dx)}}$$

$$+ \frac{4a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

$$+ \frac{36a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]

[Out] (36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3876

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I GtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(a + a \sec(c + dx))^3}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \int \left(\frac{a^3}{\sec^{\frac{5}{2}}(c + dx)} + \frac{3a^3}{\sec^{\frac{3}{2}}(c + dx)} + \frac{3a^3}{\sqrt{\sec(c + dx)}} + a^3 \sqrt{\sec(c + dx)} \right) dx \\
&= a^3 \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx + a^3 \int \sqrt{\sec(c + dx)} dx \\
&\quad + (3a^3) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx + (3a^3) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{d \sqrt{\sec(c + dx)}} + \frac{1}{5} (3a^3) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + a^3 \int \sqrt{\sec(c + dx)} dx + \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad + \left(3a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{6a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2a^3 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
&\quad + \frac{2a^3 \sin(c + dx)}{d \sqrt{\sec(c + dx)}} + \frac{1}{5} \left(3a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&\quad + \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{36a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
&\quad + \frac{4a^3 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{d \sqrt{\sec(c + dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx$$

$$= \frac{a^3 \left(\frac{144i \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2 \left(-36i - 20i\sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right) \right) \right)}{10d\sqrt{\sec(c+dx)}}$$

[In] Integrate[(a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]

[Out] (a^3*(((144*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*(-36*I - (20*I)*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 10*Sin[c + d*x] + Sin[2*(c + d*x)])))/(10*d*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 6.86 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.91

method	result
default	$-\frac{4\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3\left(-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$
parts	$-\frac{2a^3\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-2a^3\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$

[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.19

$$\int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx =$$

$$2 \left(5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 5i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right) / d$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -2/5*(5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (a^3*cos(d*x + c)^2 + 5*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d

Sympy [F]

$$\int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx = a^3 \left(\int 3 \cos(c + dx) \sqrt{\sec(c + dx)} dx \right.$$

$$+ \int 3 \cos^2(c + dx) \sqrt{\sec(c + dx)} dx$$

$$+ \int \cos^3(c + dx) \sqrt{\sec(c + dx)} dx$$

$$\left. + \int \sqrt{\sec(c + dx)} dx \right)$$

[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(1/2),x)

[Out] a**3*(Integral(3*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(3*cos(c + d*x)**2*sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x)**3*sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))

Maxima [F]

$$\int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx = \int (a \cos(dx + c) + a)^3 \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)

Giac [F]

$$\int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx = \int (a \cos(dx + c) + a)^3 \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} (a + a \cos(c + dx))^3 dx$$

[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3, x)

3.309 $\int \frac{(a+a \cos(c+dx))^3}{\sqrt{\sec(c+dx)}} dx$

Optimal result	4052
Rubi [A] (verified)	4052
Mathematica [C] (verified)	4055
Maple [A] (verified)	4055
Fricas [C] (verification not implemented)	4056
Sympy [F]	4056
Maxima [F]	4057
Giac [F]	4057
Mupad [F(-1)]	4057

Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx = \frac{28a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{52a^3 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{21d} + \frac{2a^3 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{52a^3 \sin(c + dx)}{21d \sqrt{\sec(c + dx)}}$$

[Out] 2/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)+6/5*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+52/21*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)+28/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+52/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3876, 3854, 3856, 2720, 2719}

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx = \frac{6a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{52a^3 \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{52a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{28a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]

[Out] (28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3876

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(a + a \sec(c + dx))^3}{\sec^{\frac{7}{2}}(c + dx)} dx \\
&= \int \left(\frac{a^3}{\sec^{\frac{7}{2}}(c + dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c + dx)} + \frac{3a^3}{\sec^{\frac{3}{2}}(c + dx)} + \frac{a^3}{\sqrt{\sec(c + dx)}} \right) dx \\
&= a^3 \int \frac{1}{\sec^{\frac{7}{2}}(c + dx)} dx + a^3 \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + (3a^3) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx + (3a^3) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a^3 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{d \sqrt{\sec(c + dx)}} + \frac{1}{7} (5a^3) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&\quad + a^3 \int \sqrt{\sec(c + dx)} dx + \frac{1}{5} (9a^3) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{2a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a^3 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&\quad + \frac{6a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{52a^3 \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{1}{21} (5a^3) \int \sqrt{\sec(c + dx)} dx \\
&\quad + \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad + \frac{1}{5} \left(9a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{28a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
&\quad + \frac{2a^3 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2a^3 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{52a^3 \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{1}{21} \left(5a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{28a^3 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
&\quad + \frac{52a^3 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{21d} \\
&\quad + \frac{2a^3 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{52a^3 \sin(c + dx)}{21d \sqrt{\sec(c + dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{a^3 \left(-2352i + \frac{4704i \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 1040i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right) \right)}{420d \sqrt{\sec(c + dx)}}$$

[In] Integrate[(a + a*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]

[Out] (a^3*(-2352*I + ((4704*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (1040*I)*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1070*Sin[c + d*x] + 252*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 11.25 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{4\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3\left(120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+602\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-208*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+65*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx =$$

$$2 \left(65i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 65i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)$$

```
[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 65*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 147*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*a^3*cos(d*x + c)^3 + 63*a^3*cos(d*x + c)^2 + 130*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx = a^3 \left(\int \frac{3 \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx + \int \frac{3 \cos^2(c + dx)}{\sqrt{\sec(c + dx)}} dx + \int \frac{\cos^3(c + dx)}{\sqrt{\sec(c + dx)}} dx + \int \frac{1}{\sqrt{\sec(c + dx)}} dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(3*cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x)**3/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^3}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^3}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + a \cos(c + dx))^3}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)

$$3.310 \quad \int \frac{(a+a \cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4058
Rubi [A] (verified)	4059
Mathematica [C] (verified)	4061
Maple [A] (verified)	4062
Fricas [C] (verification not implemented)	4062
Sympy [F]	4063
Maxima [F]	4063
Giac [F]	4063
Mupad [F(-1)]	4063

Optimal result

Integrand size = 23, antiderivative size = 187

$$\int \frac{(a+a \cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{68a^3 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{15d}$$

$$+ \frac{44a^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d}$$

$$+ \frac{2a^3 \sin(c+dx)}{9d \sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3 \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{68a^3 \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)} + \frac{44a^3 \sin(c+dx)}{21d \sqrt{\sec(c+dx)}}$$

```
[Out] 2/9*a^3*sin(d*x+c)/d/sec(d*x+c)^(7/2)+6/7*a^3*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+68/45*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)+44/21*a^3*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+68/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
+44/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3876, 3854, 3856, 2719, 2720}

$$\int \frac{(a + a \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{68a^3 \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{44a^3 \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{44a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{68a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d}$$

[In] Int[(a + a*Cos[c + d*x])^3/Sec[c + d*x]^(3/2), x]

[Out] (68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^n*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{EqQ}[n^2, 1/4]$

Rule 3876

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^m), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[(a + b*\text{csc}[e + f*x])^m*(d*\text{csc}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, d, e, f, m, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{I GtQ}[m, 0] \&\& \text{RationalQ}[n]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(a + a \sec(c + dx))^3}{\sec^{\frac{9}{2}}(c + dx)} dx \\
 &= \int \left(\frac{a^3}{\sec^{\frac{9}{2}}(c + dx)} + \frac{3a^3}{\sec^{\frac{7}{2}}(c + dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c + dx)} + \frac{a^3}{\sec^{\frac{3}{2}}(c + dx)} \right) dx \\
 &= a^3 \int \frac{1}{\sec^{\frac{9}{2}}(c + dx)} dx + a^3 \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &\quad + (3a^3) \int \frac{1}{\sec^{\frac{7}{2}}(c + dx)} dx + (3a^3) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2a^3 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a^3 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
 &\quad + \frac{1}{3} a^3 \int \sqrt{\sec(c + dx)} dx + \frac{1}{9} (7a^3) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &\quad + \frac{1}{5} (9a^3) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{7} (15a^3) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2a^3 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{6a^3 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{68a^3 \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{44a^3 \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
 &\quad + \frac{1}{15} (7a^3) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{7} (5a^3) \int \sqrt{\sec(c + dx)} dx \\
 &\quad + \frac{1}{3} \left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad + \frac{1}{5} \left(9a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{18a^3 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{2a^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} + \frac{2a^3 \sin(c+dx)}{9d \sec^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{6a^3 \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{68a^3 \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)} + \frac{44a^3 \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} \\
&\quad + \frac{1}{15} \left(7a^3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx \\
&\quad + \frac{1}{7} \left(5a^3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{68a^3 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{15d} \\
&\quad + \frac{44a^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{2a^3 \sin(c+dx)}{9d \sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3 \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{68a^3 \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)} + \frac{44a^3 \sin(c+dx)}{21d \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \frac{(a + a \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{a^3 \left(-11424i + \frac{22848i \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 5280i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right) \right)}{2520d \sqrt{\sec(c+dx)}}$$

[In] Integrate[(a + a*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]

[Out] (a^3*(-11424*I + ((22848*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (5280*I)*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 5820*Sin[c + d*x] + 2044*Sin[2*(c + d*x)] + 540*Sin[3*(c + d*x)] + 70*Sin[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 12.57 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.39

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}a^3\left(560\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-600\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+212\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+66\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{315\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-357*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{(a + a \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx =$$

$$2 \left(165i \sqrt{2} a^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 165i \sqrt{2} a^3 \text{weierstrassPInverse}(\dots) \right)$$

[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")

```
[Out] -2/315*(165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 165*I*sqrt(2)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*I*sqrt(2)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*a^3*cos(d*x + c)^4 + 135*a^3*cos(d*x + c)^3 + 238*a^3*cos(d*x + c)^2 + 330*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx = a^3 \left(\int \frac{3 \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{3 \cos^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx \right. \\ \left. + \int \frac{\cos^3(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \right)$$

[In] integrate((a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)

[Out] a**3*(Integral(3*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(3*cos(c + d*x)**2/sec(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**3/sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(-3/2), x))

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^3}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^3}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a + a \cos(c + dx))^3}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

[In] int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)

[Out] int((a + a*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)

3.311 $\int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx$

Optimal result	4064
Rubi [A] (verified)	4065
Mathematica [C] (verified)	4068
Maple [B] (verified)	4068
Fricas [C] (verification not implemented)	4069
Sympy [F(-1)]	4069
Maxima [F]	4070
Giac [F]	4070
Mupad [F(-1)]	4070

Optimal result

Integrand size = 23, antiderivative size = 187

$$\begin{aligned}
 & \int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx \\
 &= -\frac{64a^4 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &+ \frac{136a^4 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{21d} \\
 &+ \frac{64a^4 \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{94a^4 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{21d} \\
 &+ \frac{8a^4 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2a^4 \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{7d}
 \end{aligned}$$

```
[Out] 94/21*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+8/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)
/d+2/7*a^4*sec(d*x+c)^(7/2)*sin(d*x+c)/d+64/5*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)
/d-64/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+136/21*a^4*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3876, 3856, 2720, 3853, 2719}

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{2a^4 \sin(c + dx) \sec^{\frac{7}{2}}(c + dx)}{7d} + \frac{8a^4 \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d}$$

$$+ \frac{94a^4 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{21d} + \frac{64a^4 \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

$$+ \frac{136a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d}$$

$$- \frac{64a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(9/2), x]

[Out] (-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &

& IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3876

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \sqrt{\sec(c + dx)}(a + a \sec(c + dx))^4 dx \\
 &= \int \left(a^4 \sqrt{\sec(c + dx)} + 4a^4 \sec^{\frac{3}{2}}(c + dx) + 6a^4 \sec^{\frac{5}{2}}(c + dx) + 4a^4 \sec^{\frac{7}{2}}(c + dx) \right. \\
 &\quad \left. + a^4 \sec^{\frac{9}{2}}(c + dx) \right) dx \\
 &= a^4 \int \sqrt{\sec(c + dx)} dx + a^4 \int \sec^{\frac{9}{2}}(c + dx) dx + (4a^4) \int \sec^{\frac{3}{2}}(c + dx) dx \\
 &\quad + (4a^4) \int \sec^{\frac{7}{2}}(c + dx) dx + (6a^4) \int \sec^{\frac{5}{2}}(c + dx) dx \\
 &= \frac{8a^4 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{4a^4 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d} \\
 &\quad + \frac{8a^4 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2a^4 \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{7d} \\
 &\quad + \frac{1}{7}(5a^4) \int \sec^{\frac{5}{2}}(c + dx) dx + (2a^4) \int \sqrt{\sec(c + dx)} dx \\
 &\quad + \frac{1}{5}(12a^4) \int \sec^{\frac{3}{2}}(c + dx) dx - (4a^4) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &\quad + \left(a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{64a^4 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{94a^4 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{21d} \\
&\quad + \frac{8a^4 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{2a^4 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{1}{21} (5a^4) \int \sqrt{\sec(c+dx)} dx - \frac{1}{5} (12a^4) \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\
&\quad + \left(2a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&\quad - \left(4a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx \\
&= - \frac{8a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{6a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{64a^4 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{94a^4 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{21d} \\
&\quad + \frac{8a^4 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{2a^4 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{1}{21} \left(5a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&\quad - \frac{1}{5} \left(12a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx \\
&= - \frac{64a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{136a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{64a^4 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{94a^4 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{21d} \\
&\quad + \frac{8a^4 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} + \frac{2a^4 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{7d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.67 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.45

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{a^4(1 + \cos(c + dx))^4 \sec^8\left(\frac{1}{2}(c + dx)\right) \left(-\frac{4i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(168(1+e^{2i(c+dx)})+168(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right)}{168(1+e^{2i(c+dx)})+168(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}}\right)}{168(1+e^{2i(c+dx)})+168(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}}$$

[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(9/2),x]

[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(((−4*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(168*(1 + E^((2*I)*(c + d*x)))) + 168*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) + 85*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(672*Cos[d*x]*Csc[c] + (235 + 84*Sec[c + d*x] + 15*Sec[c + d*x]^2)*Tan[c + d*x]))/(840*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(211) = 422.

Time = 202.27 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.35

method	result
default	$- \frac{32\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} a^4 \left(\frac{253\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1} F(\cos(\frac{dx}{2} + \frac{c}{2}), \sqrt{2})}{420\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}} - \frac{47\cos(\frac{dx}{2} + \frac{c}{2})\sqrt{-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1}}{672(\cos(\frac{dx}{2} + \frac{c}{2}) + \sin(\frac{dx}{2} + \frac{c}{2}))} \right)}{672(\cos(\frac{dx}{2} + \frac{c}{2}) + \sin(\frac{dx}{2} + \frac{c}{2}))}$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(253/420*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-47/672*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^2)^(1/2))

$*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{1/2} * (\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{1/2}) - \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{1/2})) - 1/80 * \cos(1/2*d*x+1/2*c) * (-2 * \sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{1/2} / (\cos(1/2*d*x+1/2*c)^2 - 1/2)^3 - 1/896 * \cos(1/2*d*x+1/2*c) * (-2 * \sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{1/2} / (\cos(1/2*d*x+1/2*c)^2 - 1/2)^4 / \sin(1/2*d*x+1/2*c) / (2 * \cos(1/2*d*x+1/2*c)^2 - 1)^{1/2} / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx =$$

$$2 \left(170i \sqrt{2} a^4 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 170i \sqrt{2} a^4 \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right) / (d * \cos(dx + c)^3)$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] $-2/105 * (170 * I * \sqrt{2} * a^4 * \cos(dx + c)^3 * \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I * \sin(dx + c)) - 170 * I * \sqrt{2} * a^4 * \cos(dx + c)^3 * \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I * \sin(dx + c)) + 336 * I * \sqrt{2} * a^4 * \cos(dx + c)^3 * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I * \sin(dx + c))) - 336 * I * \sqrt{2} * a^4 * \cos(dx + c)^3 * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I * \sin(dx + c))) - (672 * a^4 * \cos(dx + c)^3 + 235 * a^4 * \cos(dx + c)^2 + 84 * a^4 * \cos(dx + c) + 15 * a^4) * \sin(dx + c) / \sqrt{\cos(dx + c)}) / (d * \cos(dx + c)^3)$

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^4 \sec(dx + c)^{\frac{9}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(9/2), x)

Giac [F]

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^4 \sec(dx + c)^{\frac{9}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{9}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{9/2} (a + a \cos(c + dx))^4 dx$$

[In] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^4,x)

[Out] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^4, x)

3.312 $\int (a + a \cos(c + dx))^4 \sec^{\frac{7}{2}}(c + dx) dx$

Optimal result	4071
Rubi [A] (verified)	4072
Mathematica [C] (verified)	4074
Maple [B] (verified)	4075
Fricas [C] (verification not implemented)	4076
Sympy [F(-1)]	4076
Maxima [F]	4076
Giac [F]	4077
Mupad [F(-1)]	4077

Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}
 & \int (a + a \cos(c + dx))^4 \sec^{\frac{7}{2}}(c + dx) dx \\
 &= -\frac{56a^4 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &+ \frac{32a^4 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
 &+ \frac{66a^4 \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\
 &+ \frac{8a^4 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a^4 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d}
 \end{aligned}$$

```
[Out] 8/3*a^4*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^4*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+66/5*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d-56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
)*sec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3876, 3856, 2719, 2720, 3853}

$$\begin{aligned} & \int (a + a \cos(c + dx))^4 \sec^{\frac{7}{2}}(c + dx) dx \\ &= \frac{2a^4 \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} + \frac{8a^4 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d} \\ &+ \frac{66a^4 \sin(c + dx) \sqrt{\sec(c + dx)}}{5d} \\ &+ \frac{32a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\ &- \frac{56a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} \end{aligned}$$

[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2),x]

[Out] (-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &

& IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3876

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I GtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(a + a \sec(c + dx))^4}{\sqrt{\sec(c + dx)}} dx \\
 &= \int \left(\frac{a^4}{\sqrt{\sec(c + dx)}} + 4a^4 \sqrt{\sec(c + dx)} + 6a^4 \sec^{\frac{3}{2}}(c + dx) + 4a^4 \sec^{\frac{5}{2}}(c + dx) \right. \\
 &\quad \left. + a^4 \sec^{\frac{7}{2}}(c + dx) \right) dx \\
 &= a^4 \int \frac{1}{\sqrt{\sec(c + dx)}} dx + a^4 \int \sec^{\frac{7}{2}}(c + dx) dx + (4a^4) \int \sqrt{\sec(c + dx)} dx \\
 &\quad + (4a^4) \int \sec^{\frac{5}{2}}(c + dx) dx + (6a^4) \int \sec^{\frac{3}{2}}(c + dx) dx \\
 &= \frac{12a^4 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{8a^4 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
 &\quad + \frac{2a^4 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{1}{5} (3a^4) \int \sec^{\frac{3}{2}}(c + dx) dx \\
 &\quad + \frac{1}{3} (4a^4) \int \sqrt{\sec(c + dx)} dx - (6a^4) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &\quad + \left(a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
 &\quad + \left(4a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} \\
&+ \frac{8a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&+ \frac{66a^4 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{8a^4 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d} \\
&+ \frac{2a^4 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} - \frac{1}{5} (3a^4) \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\
&+ \frac{1}{3} \left(4a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&- \left(6a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{10a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} \\
&+ \frac{32a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} \\
&+ \frac{66a^4 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} \\
&+ \frac{8a^4 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d} + \frac{2a^4 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} \\
&- \frac{1}{5} \left(3a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{56a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&+ \frac{32a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} \\
&+ \frac{66a^4 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} \\
&+ \frac{8a^4 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d} + \frac{2a^4 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.66 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.73

$$\int (a + a \cos(c+dx))^4 \sec^{\frac{7}{2}}(c+dx) dx$$

$$= \frac{a^4 (1 + \cos(c+dx))^4 \sec^8\left(\frac{1}{2}(c+dx)\right) \left(-\frac{8i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (21(1+e^{2i(c+dx)})+21(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}})}{\operatorname{Hypergeometric} \right)}{1}$$

[In] Integrate[(a + a*cos[c + d*x])^4*Sec[c + d*x]^(7/2),x]

[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(((-8*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) + 20*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(-3*(-61 + 5*Cos[2*c])*Cos[d*x]*Csc[c] + 30*Cos[c]*Sin[d*x] + 2*(20 + 3*Sec[c + d*x])*Tan[c + d*x]))/(240*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(189) = 378.

Time = 202.29 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.40

method	result
default	$- \frac{32 \sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} a^4 \left(\frac{41 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1} F(\cos(\frac{dx}{2} + \frac{c}{2}), \sqrt{2})}{60 \sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}} - 7 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right)}{1}$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(41/60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/24*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-33/40*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-1/320*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{7}{2}}(c + dx) dx =$$

$$2 \left(40i \sqrt{2} a^4 \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 40i \sqrt{2} a^4 \cos(dx + c) \right)$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] -2/15*(40*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 40*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 42*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 42*I*sqrt(2)*a^4*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (99*a^4*cos(d*x + c)^2 + 20*a^4*cos(d*x + c) + 3*a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{7}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{7}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^4 \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(7/2), x)

Giac [F]

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{7}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^4 \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{7}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{7/2} (a + a \cos(c + dx))^4 dx$$

[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^4,x)

[Out] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^4, x)

3.313 $\int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx$

Optimal result	4078
Rubi [A] (verified)	4078
Mathematica [A] (verified)	4081
Maple [B] (verified)	4081
Fricas [C] (verification not implemented)	4082
Sympy [F(-1)]	4082
Maxima [F]	4082
Giac [F]	4083
Mupad [F(-1)]	4083

Optimal result

Integrand size = 23, antiderivative size = 118

$$\begin{aligned} & \int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx \\ &= \frac{40a^4 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} + \frac{2a^4 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\ & \quad + \frac{8a^4 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^4 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \end{aligned}$$

[Out] $2/3*a^4*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/3*a^4*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+8*a^4*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+40/3*a^4*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3317, 3876, 3854, 3856, 2720, 2719, 3853}

$$\begin{aligned} & \int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx \\ &= \frac{2a^4 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d} + \frac{8a^4 \sin(c + dx) \sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{2a^4 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{40a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \end{aligned}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^4*\operatorname{Sec}[c + d*x]^{(5/2)}, x]$

[Out] $(40a^4\sqrt{\cos[c + dx]} \operatorname{EllipticF}[(c + dx)/2, 2] \sqrt{\sec[c + dx]}) / (3d) + (2a^4 \sin[c + dx]) / (3d \sqrt{\sec[c + dx]}) + (8a^4 \sqrt{\sec[c + dx]} \sin[c + dx]) / d + (2a^4 \sec[c + dx]^{3/2} \sin[c + dx]) / (3d)$

Rule 2719

$\operatorname{Int}[\sqrt{\sin[(c_.) + (d_.)x]}], x_Symbol] \rightarrow \operatorname{Simp}[(2/d) \operatorname{EllipticE}[(1/2)(c - \pi/2 + dx), 2], x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2720

$\operatorname{Int}[1/\sqrt{\sin[(c_.) + (d_.)x]}], x_Symbol] \rightarrow \operatorname{Simp}[(2/d) \operatorname{EllipticF}[(1/2)(c - \pi/2 + dx), 2], x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3317

$\operatorname{Int}[(\operatorname{csc}[e_.] + (f_.)x)(d_.)^{(m_.)}((a_.) + (b_.)\sin[e_.] + (f_.)x)^{(n_.)}]{p_., x_Symbol] \rightarrow \operatorname{Dist}[d^{(n*p)}, \operatorname{Int}[(d \operatorname{Csc}[e + fx])^{(m - n*p)}(b + a \operatorname{Csc}[e + fx]^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, d, e, f, m, n, p\}, x] \&\& !\operatorname{IntegerQ}[m] \&\& \operatorname{IntegersQ}[n, p]$

Rule 3853

$\operatorname{Int}[(\operatorname{csc}[c_.] + (d_.)x)(b_.)^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(-b) \cos[c + dx] * ((b \operatorname{Csc}[c + dx])^{(n - 1)} / (d(n - 1))), x] + \operatorname{Dist}[b^2 * ((n - 2) / (n - 1)), \operatorname{Int}[(b \operatorname{Csc}[c + dx])^{(n - 2)}, x], x] /; \operatorname{FreeQ}\{b, c, d\}, x] \&\& \operatorname{GtQ}[n, 1] \&\& \operatorname{IntegerQ}[2*n]$

Rule 3854

$\operatorname{Int}[(\operatorname{csc}[c_.] + (d_.)x)(b_.)^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[\cos[c + dx] * ((b \operatorname{Csc}[c + dx])^{(n + 1)} / (b*d*n)), x] + \operatorname{Dist}[(n + 1) / (b^2*n), \operatorname{Int}[(b \operatorname{Csc}[c + dx])^{(n + 2)}, x], x] /; \operatorname{FreeQ}\{b, c, d\}, x] \&\& \operatorname{LtQ}[n, -1] \&\& \operatorname{IntegerQ}[2*n]$

Rule 3856

$\operatorname{Int}[(\operatorname{csc}[c_.] + (d_.)x)(b_.)^{(n_.)}, x_Symbol] \rightarrow \operatorname{Dist}[(b \operatorname{Csc}[c + dx])^n \sin[c + dx]^n, \operatorname{Int}[1/\sin[c + dx]^n, x], x] /; \operatorname{FreeQ}\{b, c, d\}, x] \&\& \operatorname{EqQ}[n^2, 1/4]$

Rule 3876

$\operatorname{Int}[(\operatorname{csc}[e_.] + (f_.)x)(d_.)^{(n_.)}(\operatorname{csc}[e_.] + (f_.)x)(b_.) + (a_.)^{(m_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[(a + b \operatorname{csc}[e + fx])^m (d \operatorname{csc}[e + fx])^n], x] /; \operatorname{FreeQ}\{a, b, d, e, f, m, n\}, x] \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{I} \operatorname{GtQ}[m, 0] \&\& \operatorname{RationalQ}[n]$

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(a + a \sec(c + dx))^4}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \int \left(\frac{a^4}{\sec^{\frac{3}{2}}(c + dx)} + \frac{4a^4}{\sqrt{\sec(c + dx)}} + 6a^4 \sqrt{\sec(c + dx)} + 4a^4 \sec^{\frac{3}{2}}(c + dx) \right. \\
&\quad \left. + a^4 \sec^{\frac{5}{2}}(c + dx) \right) dx \\
&= a^4 \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx + a^4 \int \sec^{\frac{5}{2}}(c + dx) dx + (4a^4) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + (4a^4) \int \sec^{\frac{3}{2}}(c + dx) dx + (6a^4) \int \sqrt{\sec(c + dx)} dx \\
&= \frac{2a^4 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{8a^4 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^4 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad + 2 \left(\frac{1}{3} a^4 \int \sqrt{\sec(c + dx)} dx \right) - (4a^4) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + \left(4a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&\quad + \left(6a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{8a^4 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{12a^4 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a^4 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{8a^4 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^4 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad + 2 \left(\frac{1}{3} \left(a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \right) \\
&\quad - \left(4a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{40a^4 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} + \frac{2a^4 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{8a^4 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^4 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{a^4 \sec^{\frac{3}{2}}(c + dx) \left(80 \cos^{\frac{3}{2}}(c + dx) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 5 \sin(c + dx) + 24 \sin(2(c + dx)) + \sin(3(c + dx)) \right)}{6d}$$

[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(5/2),x]

[Out] (a^4*Sec[c + d*x]^(3/2)*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[c + d*x] + 24*Sin[2*(c + d*x)] + Sin[3*(c + d*x)])/(6*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(128) = 256.

Time = 202.99 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.47

method	result
default	$8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} a^4 \left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-14\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right) + 3\left(4\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out]
$$\frac{8}{3} \left(-(-2\cos(1/2*d*x+1/2*c)^2+1)\sin(1/2*d*x+1/2*c)^2 \right)^{1/2} a^4 / (4\sin(1/2*d*x+1/2*c)^4 - 4\sin(1/2*d*x+1/2*c)^2 + 1) / \sin(1/2*d*x+1/2*c)^3 \left(2\cos(1/2*d*x+1/2*c)\sin(1/2*d*x+1/2*c)^6 - 14\sin(1/2*d*x+1/2*c)^4\cos(1/2*d*x+1/2*c) + 10\sin(1/2*d*x+1/2*c)^2 \right)^{1/2} \left(2\sin(1/2*d*x+1/2*c)^2 - 1 \right)^{1/2} \operatorname{EllipticF}\left(\cos(1/2*d*x+1/2*c), 2^{1/2}\right) \sin(1/2*d*x+1/2*c)^2 + 7\sin(1/2*d*x+1/2*c)^2 \cos(1/2*d*x+1/2*c) - 5\left(\sin(1/2*d*x+1/2*c)^2\right)^{1/2} \left(2\sin(1/2*d*x+1/2*c)^2 - 1 \right)^{1/2} \operatorname{EllipticF}\left(\cos(1/2*d*x+1/2*c), 2^{1/2}\right) \left(-2\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2 \right)^{1/2} / (2\cos(1/2*d*x+1/2*c)^2 - 1)^{1/2} / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx =$$

$$\frac{2 \left(10i \sqrt{2} a^4 \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 10i \sqrt{2} a^4 \cos(dx + c) \right)}{3 d \cos(dx + c)}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(10*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 10*I*sqrt(2)*a^4*cos(d*x + c)*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - (a^4*cos(d*x + c)^2 + 12*a^4*cos(d*x +
c) + a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^4 \sec(dx + c)^{\frac{5}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^4 \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{5/2} (a + a \cos(c + dx))^4 dx$$

[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^4,x)

[Out] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^4, x)

3.314 $\int (a + a \cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	4084
Rubi [A] (verified)	4084
Mathematica [C] (verified)	4087
Maple [A] (verified)	4088
Fricas [C] (verification not implemented)	4088
Sympy [F(-1)]	4089
Maxima [F]	4089
Giac [F]	4089
Mupad [F(-1)]	4089

Optimal result

Integrand size = 23, antiderivative size = 159

$$\begin{aligned} & \int (a + a \cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx) dx \\ &= \frac{56a^4 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\ & \quad + \frac{32a^4 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\ & \quad + \frac{2a^4 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{8a^4 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{2a^4 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} \end{aligned}$$

```
[Out] 2/5*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/3*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+2*a^4*sin(d*x+c)*sec(d*x+c)^(1/2)/d+56/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+32/3*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3876, 3854, 3856, 2719, 2720, 3853}

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{2a^4 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a^4 \sin(c + dx) \sqrt{\sec(c + dx)}}{d} + \frac{8a^4 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}}$$

$$+ \frac{32a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

$$+ \frac{56a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(3/2), x]

[Out] (56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

]

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))^(m_.), x_Symbol] := Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(a + a \sec(c + dx))^4}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \int \left(\frac{a^4}{\sec^{\frac{5}{2}}(c + dx)} + \frac{4a^4}{\sec^{\frac{3}{2}}(c + dx)} + \frac{6a^4}{\sqrt{\sec(c + dx)}} + 4a^4 \sqrt{\sec(c + dx)} + a^4 \sec^{\frac{3}{2}}(c \right. \\
&\quad \left. + dx) \right) dx \\
&= a^4 \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx + a^4 \int \sec^{\frac{3}{2}}(c + dx) dx + (4a^4) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&\quad + (4a^4) \int \sqrt{\sec(c + dx)} dx + (6a^4) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2a^4 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{8a^4 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2a^4 \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{1}{5} (3a^4) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad - a^4 \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{3} (4a^4) \int \sqrt{\sec(c + dx)} dx \\
&\quad + \left(4a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad + \left(6a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{12a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{8a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2a^4 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4 \sin(c+dx)}{3d \sqrt{\sec(c+dx)}} + \frac{2a^4 \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
&\quad + \frac{1}{5} \left(3a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&\quad - \left(a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&\quad + \frac{1}{3} \left(4a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{56a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{32a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} \\
&\quad + \frac{2a^4 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4 \sin(c+dx)}{3d \sqrt{\sec(c+dx)}} + \frac{2a^4 \sqrt{\sec(c+dx)} \sin(c+dx)}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int (a + a \cos(c+dx))^4 \sec^{\frac{3}{2}}(c+dx) dx \\
&= \frac{a^4 \left(-336i + \frac{672i \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 320i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right) \right)}{30d \sqrt{\sec(c+dx)}}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(3/2),x]

[Out] (a^4*(-336*I + ((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (320*I)*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 80*Sin[c + d*x] + 3*Sec[c + d*x]*Sin[3*(c + d*x)] + 63*Tan[c + d*x]))/(30*d*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 10.71 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.22

method	result
default	$\frac{8a^4 \left(6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 26 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 19 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 20 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 1}}{15 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 1}}$
parts	Expression too large to display

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8/15*a^4*(6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-26*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.02

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx) dx =$$

$$\frac{2 \left(40i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 40i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{\dots}$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/15*(40*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 40*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 42*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 42*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*a^4*cos(d*x + c)^2 + 20*a^4*cos(d*x + c) + 15*a^4)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^4 \sec(dx + c)^{\frac{3}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx) dx = \int (a \cos(dx + c) + a)^4 \sec(dx + c)^{\frac{3}{2}} dx$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{\frac{3}{2}} (a + a \cos(c + dx))^4 dx$$

```
[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^4,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^4, x)
```

3.315 $\int (a + a \cos(c + dx))^4 \sqrt{\sec(c + dx)} dx$

Optimal result	4090
Rubi [A] (verified)	4090
Mathematica [C] (verified)	4093
Maple [A] (verified)	4094
Fricas [C] (verification not implemented)	4094
Sympy [F]	4095
Maxima [F]	4095
Giac [F]	4095
Mupad [F(-1)]	4096

Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned} & \int (a + a \cos(c + dx))^4 \sqrt{\sec(c + dx)} dx \\ &= \frac{64a^4 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\ & \quad + \frac{136a^4 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{21d} \\ & \quad + \frac{2a^4 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{8a^4 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{94a^4 \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \end{aligned}$$

[Out] $2/7*a^4*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}+8/5*a^4*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}+94/21*a^4*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+64/5*a^4*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+136/21*a^4*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {3317, 3876, 3854, 3856, 2720, 2719}

$$\int (a + a \cos(c + dx))^4 \sqrt{\sec(c + dx)} dx$$

$$= \frac{8a^4 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a^4 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{94a^4 \sin(c + dx)}{21d \sqrt{\sec(c + dx)}}$$

$$+ \frac{136a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d}$$

$$+ \frac{64a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(a + a*Cos[c + d*x])^4*Sqrt[Sec[c + d*x]], x]

[Out] (64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*((b + a*Csc[e + f*x]^n)^p, x), x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3876

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n_*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m_], x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I GtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(a + a \sec(c + dx))^4}{\sec^{\frac{7}{2}}(c + dx)} dx \\
&= \int \left(\frac{a^4}{\sec^{\frac{7}{2}}(c + dx)} + \frac{4a^4}{\sec^{\frac{5}{2}}(c + dx)} + \frac{6a^4}{\sec^{\frac{3}{2}}(c + dx)} + \frac{4a^4}{\sqrt{\sec(c + dx)}} + a^4 \sqrt{\sec(c + dx)} \right) dx \\
&= a^4 \int \frac{1}{\sec^{\frac{7}{2}}(c + dx)} dx + a^4 \int \sqrt{\sec(c + dx)} dx + (4a^4) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&\quad + (4a^4) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + (6a^4) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a^4 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{8a^4 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{4a^4 \sin(c + dx)}{d \sqrt{\sec(c + dx)}} + \frac{1}{7} (5a^4) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&\quad + (2a^4) \int \sqrt{\sec(c + dx)} dx + \frac{1}{5} (12a^4) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + \left(a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad + \left(4a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{8a^4 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2a^4 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a^4 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
&\quad + \frac{8a^4 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{94a^4 \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{1}{21} (5a^4) \int \sqrt{\sec(c + dx)} dx \\
&\quad + \left(2a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad + \frac{1}{5} \left(12a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{64a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{6a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2a^4 \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{94a^4 \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} \\
&\quad + \frac{1}{21} \left(5a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{64a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{136a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{2a^4 \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{94a^4 \sin(c+dx)}{21d \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\begin{aligned}
&\int (a + a \cos(c+dx))^4 \sqrt{\sec(c+dx)} dx \\
&= \frac{a^4 \left(-5376i + \frac{10752i \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2720i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right) \right)}{420d \sqrt{\sec(c+dx)}}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^4*Sqrt[Sec[c + d*x]],x]

[Out] (a^4*(-5376*I + ((10752*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (2720*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1910*Sin[c + d*x] + 336*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 11.15 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.69

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}} a^4 \left(60\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-258\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+448\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] -8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+85*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int (a + a \cos(c + dx))^4 \sqrt{\sec(c + dx)} dx =$$

$$-\frac{2 \left(170i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 170i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) \right)}{d}$$

[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="fricas")

```
[Out] -2/105*(170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 170*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 336*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 336*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*a^4*cos(d*x + c)^3 + 84*a^4*cos(d*x + c)^2 + 235*a^4*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int (a + a \cos(c + dx))^4 \sqrt{\sec(c + dx)} dx = a^4 \left(\int 4 \cos(c + dx) \sqrt{\sec(c + dx)} dx \right. \\ \left. + \int 6 \cos^2(c + dx) \sqrt{\sec(c + dx)} dx \right. \\ \left. + \int 4 \cos^3(c + dx) \sqrt{\sec(c + dx)} dx \right. \\ \left. + \int \cos^4(c + dx) \sqrt{\sec(c + dx)} dx \right. \\ \left. + \int \sqrt{\sec(c + dx)} dx \right)$$

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(1/2),x)
```

```
[Out] a**4*(Integral(4*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(6*cos(c + d
*x)**2*sqrt(sec(c + d*x)), x) + Integral(4*cos(c + d*x)**3*sqrt(sec(c + d*x
)), x) + Integral(cos(c + d*x)**4*sqrt(sec(c + d*x)), x) + Integral(sqrt(se
c(c + d*x)), x))
```

Maxima [F]

$$\int (a + a \cos(c + dx))^4 \sqrt{\sec(c + dx)} dx = \int (a \cos(dx + c) + a)^4 \sqrt{\sec(dx + c)} dx$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int (a + a \cos(c + dx))^4 \sqrt{\sec(c + dx)} dx = \int (a \cos(dx + c) + a)^4 \sqrt{\sec(dx + c)} dx$$

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^4 \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} (a + a \cos(c + dx))^4 dx$$

```
[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^4, x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^4, x)
```

$$3.316 \quad \int \frac{(a+a \cos(c+dx))^4}{\sqrt{\sec(c+dx)}} dx$$

Optimal result	4097
Rubi [A] (verified)	4097
Mathematica [C] (verified)	4100
Maple [A] (verified)	4101
Fricas [C] (verification not implemented)	4101
Sympy [F]	4102
Maxima [F]	4102
Giac [F]	4102
Mupad [F(-1)]	4103

Optimal result

Integrand size = 23, antiderivative size = 187

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\sec(c + dx)}} dx = \frac{152a^4 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{15d} + \frac{32a^4 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{7d} + \frac{2a^4 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{8a^4 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{122a^4 \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{32a^4 \sin(c + dx)}{7d \sqrt{\sec(c + dx)}}$$

```
[Out] 2/9*a^4*sin(d*x+c)/d/sec(d*x+c)^(7/2)+8/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+122/45*a^4*sin(d*x+c)/d/sec(d*x+c)^(3/2)+32/7*a^4*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+152/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
+32/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {3317, 3876, 3854, 3856, 2719, 2720}

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\sec(c + dx)}} dx = \frac{122a^4 \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{8a^4 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)}$$

$$+ \frac{2a^4 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{32a^4 \sin(c + dx)}{7d \sqrt{\sec(c + dx)}}$$

$$+ \frac{32a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{7d}$$

$$+ \frac{152a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d}$$

[In] Int[(a + a*Cos[c + d*x])^4/Sqrt[Sec[c + d*x]],x]

[Out] (152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3876

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n_.*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m_., x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !GtQ[m, 0] && RationalQ[n]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(a + a \sec(c + dx))^4}{\sec^{\frac{9}{2}}(c + dx)} dx \\
&= \int \left(\frac{a^4}{\sec^{\frac{9}{2}}(c + dx)} + \frac{4a^4}{\sec^{\frac{7}{2}}(c + dx)} + \frac{6a^4}{\sec^{\frac{5}{2}}(c + dx)} + \frac{4a^4}{\sec^{\frac{3}{2}}(c + dx)} + \frac{a^4}{\sqrt{\sec(c + dx)}} \right) dx \\
&= a^4 \int \frac{1}{\sec^{\frac{9}{2}}(c + dx)} dx + a^4 \int \frac{1}{\sqrt{\sec(c + dx)}} dx + (4a^4) \int \frac{1}{\sec^{\frac{7}{2}}(c + dx)} dx \\
&\quad + (4a^4) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx + (6a^4) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2a^4 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{8a^4 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{12a^4 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{8a^4 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{1}{9}(7a^4) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx + \frac{1}{3}(4a^4) \int \sqrt{\sec(c + dx)} dx \\
&\quad + \frac{1}{7}(20a^4) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx + \frac{1}{5}(18a^4) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + \left(a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{2a^4 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2a^4 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
&\quad + \frac{8a^4 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{122a^4 \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{32a^4 \sin(c + dx)}{7d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{1}{15}(7a^4) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{21}(20a^4) \int \sqrt{\sec(c + dx)} dx \\
&\quad + \frac{1}{3} \left(4a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad + \frac{1}{5} \left(18a^4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{46a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{8a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} + \frac{2a^4 \sin(c+dx)}{9d \sec^{\frac{7}{2}}(c+dx)} \\
&\quad + \frac{8a^4 \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{122a^4 \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)} + \frac{32a^4 \sin(c+dx)}{7d \sqrt{\sec(c+dx)}} \\
&\quad + \frac{1}{15} \left(7a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx \\
&\quad + \frac{1}{21} \left(20a^4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{152a^4 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{15d} \\
&\quad + \frac{32a^4 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{7d} \\
&\quad + \frac{2a^4 \sin(c+dx)}{9d \sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4 \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{122a^4 \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)} + \frac{32a^4 \sin(c+dx)}{7d \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\begin{aligned}
&\int \frac{(a + a \cos(c+dx))^4}{\sqrt{\sec(c+dx)}} dx \\
&= \frac{a^4 \left(-25536i + \frac{51072i \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 11520i \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -e^{2i(c+dx)}\right) \right)}{2520d \sqrt{\sec(c+dx)}}
\end{aligned}$$

[In] Integrate[(a + a*Cos[c + d*x])^4/Sqrt[Sec[c + d*x]],x]

[Out] (a^4*(-25536*I + ((51072*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (11520*I)*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 12240*Sin[c + d*x] + 3556*Sin[2*(c + d*x)] + 720*Sin[3*(c + d*x)] + 70*Sin[4*(c + d*x)])/(2520*d*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 16.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.39

method	result
default	$-\frac{8\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{315\sqrt{-2\left(\sin^4\right)}} a^4 \left(280\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+34\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+72\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)$
parts	Expression too large to display

[In] int((a+cos(d*x+c))*a^4/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] -8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*cos(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*cos(1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-399*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\sec(c + dx)}} dx =$$

$$2 \left(360i \sqrt{2} a^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - 360i \sqrt{2} a^4 \text{weierstrassPInverse} \right)$$

[In] integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")

```
[Out] -2/315*(360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 360*I*sqrt(2)*a^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 798*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 798*I*sqrt(2)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*a^4*cos(d*x + c)^4 + 180*a^4*cos(d*x + c)^3 + 427*a^4*cos(d*x + c)^2 + 720*a^4*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\sec(c + dx)}} dx = a^4 \left(\int \frac{4 \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx + \int \frac{6 \cos^2(c + dx)}{\sqrt{\sec(c + dx)}} dx \right. \\ \left. + \int \frac{4 \cos^3(c + dx)}{\sqrt{\sec(c + dx)}} dx + \int \frac{\cos^4(c + dx)}{\sqrt{\sec(c + dx)}} dx \right. \\ \left. + \int \frac{1}{\sqrt{\sec(c + dx)}} dx \right)$$

[In] integrate((a+a*cos(d*x+c))**4/sec(d*x+c)**(1/2),x)

[Out] a**4*(Integral(4*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(6*cos(c + d*x)**2/sqrt(sec(c + d*x)), x) + Integral(4*cos(c + d*x)**3/sqrt(sec(c + d*x))), x) + Integral(cos(c + d*x)**4/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(sec(c + d*x)), x))

Maxima [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^4}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^4}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^4}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + a \cos(c + dx))^4}{\sqrt{\frac{1}{\cos(c + dx)}}} dx$$

```
[In] int((a + a*cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

```
[Out] int((a + a*cos(c + d*x))^4/(1/cos(c + d*x))^(1/2), x)
```

3.317 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	4104
Rubi [A] (verified)	4104
Mathematica [C] (verified)	4107
Maple [B] (verified)	4107
Fricas [C] (verification not implemented)	4108
Sympy [F(-1)]	4109
Maxima [F]	4109
Giac [F]	4109
Mupad [F(-1)]	4109

Optimal result

Integrand size = 23, antiderivative size = 164

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a \cos(c+dx)} dx = \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3ad} - \frac{3\sqrt{\sec(c+dx)} \sin(c+dx)}{ad} + \frac{5 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{d(a+a \sec(c+dx))}$$

[Out] $5/3*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d-\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))-3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d+3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d+5/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3903, 3872, 3853, 3856, 2719, 2720}

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)} dx = -\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)} + \frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad} - \frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3ad} + \frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{ad}$$

[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x]), x]

[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) - (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (5*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(m_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(n_)]^(p_), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] & IntegerQ[2*n]

Rule 3856

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n * Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :=> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :=> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sec^{\frac{7}{2}}(c + dx)}{a + a \sec(c + dx)} dx \\
&= -\frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} - \frac{\int \sec^{\frac{3}{2}}(c + dx) \left(\frac{3a}{2} - \frac{5}{2}a \sec(c + dx)\right) dx}{a^2} \\
&= -\frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} - \frac{3 \int \sec^{\frac{3}{2}}(c + dx) dx}{2a} + \frac{5 \int \sec^{\frac{5}{2}}(c + dx) dx}{2a} \\
&= -\frac{3\sqrt{\sec(c + dx)} \sin(c + dx)}{ad} + \frac{5 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3ad} \\
&\quad - \frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} + \frac{5 \int \sqrt{\sec(c + dx)} dx}{6a} + \frac{3 \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{2a} \\
&= -\frac{3\sqrt{\sec(c + dx)} \sin(c + dx)}{ad} + \frac{5 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3ad} \\
&\quad - \frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} + \frac{\left(5\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{6a} \\
&\quad + \frac{\left(3\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{2a}
\end{aligned}$$

$$\begin{aligned}
&= \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} \\
&+ \frac{5\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} \\
&- \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} \\
&+ \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.51 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.74

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)} dx$$

$$= \cos^2\left(\frac{1}{2}(c+dx)\right) \left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9(1+e^{2i(c+dx)})+9(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right)\operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{-1+e^{2ic}} \right)$$

[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]

[Out] (Cos[(c + d*x)/2]^2*((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x)))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c] + Sec[c + d*x]*(-5*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + Tan[(c + d*x)/2]))) / (3*a*d*(1 + Cos[c + d*x]))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(198) = 396.

Time = 5.17 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.52

method	result
default	$\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(10\cos\left(\frac{dx}{2}+\frac{c}{2}\right)F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}\right)\right)$

[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] $\frac{1}{3} * (-(-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / a / \cos(1/2 * d * x + 1/2 * c) / \sin(1/2 * d * x + 1/2 * c)^3 / (4 * \sin(1/2 * d * x + 1/2 * c)^4 - 4 * \sin(1/2 * d * x + 1/2 * c)^2 + 1) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (10 * \cos(1/2 * d * x + 1/2 * c) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * \sin(1/2 * d * x + 1/2 * c)^2 - 18 * \cos(1/2 * d * x + 1/2 * c) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * \sin(1/2 * d * x + 1/2 * c)^2 - 36 * \sin(1/2 * d * x + 1/2 * c)^6 - 5 * \cos(1/2 * d * x + 1/2 * c) * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) + 9 * \cos(1/2 * d * x + 1/2 * c) * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) + 44 * \sin(1/2 * d * x + 1/2 * c)^4 - 11 * \sin(1/2 * d * x + 1/2 * c)^2) / (2 * \cos(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.51

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx =$$

$$\frac{5(i\sqrt{2}\cos(dx+c)^2 + i\sqrt{2}\cos(dx+c))\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 5(-$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] $-1/6 * (5 * (I * \sqrt{2} * \cos(dx + c)^2 + I * \sqrt{2} * \cos(dx + c)) * \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I * \sin(dx + c)) + 5 * (-I * \sqrt{2} * \cos(dx + c)^2 - I * \sqrt{2} * \cos(dx + c)) * \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I * \sin(dx + c)) + 9 * (-I * \sqrt{2} * \cos(dx + c)^2 - I * \sqrt{2} * \cos(dx + c)) * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I * \sin(dx + c))) + 9 * (I * \sqrt{2} * \cos(dx + c)^2 + I * \sqrt{2} * \cos(dx + c)) * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I * \sin(dx + c))) + 2 * (9 * \cos(dx + c)^2 + 4 * \cos(dx + c) - 2) * \sin(dx + c) / \sqrt{\cos(dx + c)}) / (a * d * \cos(dx + c)^2 + a * d * \cos(dx + c))$

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{a \cos(dx + c) + a} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{a \cos(dx + c) + a} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{a + a \cos(c + dx)} dx$$

[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x)),x)

[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x)), x)

3.318 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	4110
Rubi [A] (verified)	4110
Mathematica [C] (verified)	4112
Maple [A] (verified)	4113
Fricas [C] (verification not implemented)	4113
Sympy [F]	4114
Maxima [F]	4114
Giac [F]	4114
Mupad [F(-1)]	4115

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a \cos(c+dx)} dx = -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a \sec(c+dx))}$$

[Out] $-\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))+3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d-3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d-(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d$

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3317, 3903, 3872, 3856, 2720, 3853, 2719}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a \cos(c+dx)} dx = -\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a \sec(c+dx)+a)} + \frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} - \frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{ad}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]

[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^p, x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a + b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n

- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sec^{\frac{5}{2}}(c + dx)}{a + a \sec(c + dx)} dx \\
&= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} - \frac{\int \sqrt{\sec(c + dx)} \left(\frac{a}{2} - \frac{3}{2}a \sec(c + dx)\right) dx}{a^2} \\
&= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} - \frac{\int \sqrt{\sec(c + dx)} dx}{2a} + \frac{3 \int \sec^{\frac{3}{2}}(c + dx) dx}{2a} \\
&= \frac{3\sqrt{\sec(c + dx)} \sin(c + dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} \\
&\quad - \frac{3 \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{2a} - \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{2a} \\
&= -\frac{\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{ad} + \frac{3\sqrt{\sec(c + dx)} \sin(c + dx)}{ad} \\
&\quad - \frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))} - \frac{\left(3\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{2a} \\
&= -\frac{3\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{ad} \\
&\quad - \frac{\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{ad} \\
&\quad + \frac{3\sqrt{\sec(c + dx)} \sin(c + dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{d(a + a \sec(c + dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.88

$$\begin{aligned}
&\int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx \\
&\cos^2\left(\frac{1}{2}(c + dx)\right) \left(-\frac{2i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right) \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{d(-1+e^{2ic})} \right) \\
&= \frac{\hspace{15em}}{a(1 + \cos(c + dx))}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]

[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))]/(1 + E^((2*I)*(c + d*x))))*(3*(1 + E^((2*I)*(c + d*x)))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c] - 2*Tan[(c + d*x)/2]))/d)/(a*(1 + Cos[c + d*x]))

Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.86

method	result
default	$-\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\left(F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - 3E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) - 3E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}$

[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

[Out] -(-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.44

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx$$

$$= \frac{(i\sqrt{2}\cos(dx + c) + i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i\sin(dx + c)) + (-i\sqrt{2}\cos(dx + c) - i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i\sin(dx + c))}{2(a + a \cos(c + dx))}$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/2*((I*sqr(2)*cos(d*x + c) + I*sqr(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (-I*sqr(2)*cos(d*x + c) - I*sqr(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(I*sqr(2)*cos(d*x + c) + I*sqr(2))*cos(d*x + c)/2

+ I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c))/(a*d*cos(d*x + c) + a*d)

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\cos(c+dx)+1} dx}{a}$$

[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)

[Out] Integral(sec(c + d*x)**(3/2)/(cos(c + d*x) + 1), x)/a

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{a \cos(dx + c) + a} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{a \cos(dx + c) + a} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{3}{2}}}{a+a\cos(c+dx)} dx$$

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x)), x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x)), x)
```

3.319 $\int \frac{\sqrt{\sec(c+dx)}}{a+a \cos(c+dx)} dx$

Optimal result	4116
Rubi [A] (verified)	4116
Mathematica [C] (verified)	4118
Maple [A] (verified)	4119
Fricas [C] (verification not implemented)	4119
Sympy [F]	4120
Maxima [F]	4120
Giac [F]	4120
Mupad [F(-1)]	4120

Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a \cos(c+dx)} dx = \frac{\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))}$$

[Out] $-\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))+(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d+(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3903, 3872, 3856, 2719, 2720}

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a \cos(c+dx)} dx = -\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a \sec(c+dx) + a)} + \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} + \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]/(a + a*\operatorname{Cos}[c + d*x]), x]$

[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3903

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a + b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1]

Rubi steps

$$\text{integral} = \int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + a \sec(c + dx)} dx$$

$$\begin{aligned}
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))} - \frac{\int \frac{-\frac{a}{2} - \frac{1}{2} a \sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{a^2} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a} + \frac{\int \sqrt{\sec(c+dx)} dx}{2a} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{2a} \\
&= \frac{\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{ad} \\
&\quad + \frac{\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.92 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a \cos(c+dx)} dx = \frac{4i \cos^2\left(\frac{1}{2}(c+dx)\right) \left(1 + e^{2i(c+dx)} - (1 + e^{i(c+dx)}) \sqrt{1 + e^{2i(c+dx)}}\right) \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{ad(1 + e^{i(c+dx)})}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x]),x]

[Out] ((-4*I)*Cos[(c + d*x)/2]^2*(1 + E^((2*I)*(c + d*x)) - (1 + E^(I*(c + d*x))))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sqrt[Sec[c + d*x]])/(a*d*(1 + E^(I*(c + d*x)))^3)

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.82

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x,method=_RETURNVERBOSE)

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c)
)* (2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)} dx$$

$$= \frac{(-i\sqrt{2}\cos(dx+c) - i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + (i\sqrt{2}\cos(dx+c) + i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\sin(dx+c))}{2(a\cos(dx+c) + a)}$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")

```
[Out] 1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) +
I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*sqrt(
cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)} dx = \frac{\int \frac{\sqrt{\sec(c+dx)}}{\cos(c+dx)+1} dx}{a}$$

[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)

[Out] Integral(sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x)/a

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)} dx = \int \frac{\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)} dx = \int \frac{\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{a+a\cos(c+dx)} dx$$

[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x)),x)

[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x)), x)

$$3.320 \quad \int \frac{1}{(a+a \cos(c+dx)) \sqrt{\sec(c+dx)}} dx$$

Optimal result	4121
Rubi [A] (verified)	4121
Mathematica [C] (verified)	4123
Maple [A] (verified)	4124
Fricas [C] (verification not implemented)	4124
Sympy [F]	4125
Maxima [F]	4125
Giac [F]	4125
Mupad [F(-1)]	4125

Optimal result

Integrand size = 23, antiderivative size = 110

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx)) \sqrt{\sec(c+dx)}} dx \\ &= -\frac{\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{ad} \\ & \quad + \frac{\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))} \end{aligned}$$

[Out] $\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))-(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d+(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3905, 3872, 3856, 2719, 2720}

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx)) \sqrt{\sec(c+dx)}} dx \\ &= \frac{\sin(c+dx) \sqrt{\sec(c+dx)}}{d(a \sec(c+dx) + a)} + \frac{\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad} \\ & \quad - \frac{\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} \end{aligned}$$

[In] $\operatorname{Int}[1/((a + a*\operatorname{Cos}[c + d*x])*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]]), x]$

[Out] $-\left(\sqrt{\cos[c + dx]} \operatorname{EllipticE}\left[\frac{c + dx}{2}, 2\right] \sqrt{\sec[c + dx]}\right) / (a \cdot d) + \left(\sqrt{\cos[c + dx]} \operatorname{EllipticF}\left[\frac{c + dx}{2}, 2\right] \sqrt{\sec[c + dx]}\right) / (a \cdot d) + \left(\sqrt{\sec[c + dx]} \sin[c + dx]\right) / (d \cdot (a + a \cdot \sec[c + dx]))$

Rule 2719

$\operatorname{Int}\left[\sqrt{\sin[c] + (d \cdot x)}\right], x_{\text{Symbol}}] \rightarrow \operatorname{Simp}\left[\frac{2}{d} \operatorname{EllipticE}\left[\frac{1}{2}(c - \pi/2 + dx), 2\right], x\right] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2720

$\operatorname{Int}\left[1/\sqrt{\sin[c] + (d \cdot x)}\right], x_{\text{Symbol}}] \rightarrow \operatorname{Simp}\left[\frac{2}{d} \operatorname{EllipticF}\left[\frac{1}{2}(c - \pi/2 + dx), 2\right], x\right] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3317

$\operatorname{Int}\left[(\csc[e] + (f \cdot x) \cdot (d \cdot x))^m \cdot ((a) + (b \cdot \sin[e] + (f \cdot x) \cdot x)^n)^p, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}\left[d^{n \cdot p}, \operatorname{Int}\left[(d \cdot \csc[e + f \cdot x])^{m - n \cdot p} \cdot (b + a \cdot \csc[e + f \cdot x])^n\right]^p, x\right] /; \operatorname{FreeQ}\{a, b, d, e, f, m, n, p\}, x] \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{IntegersQ}[n, p]$

Rule 3856

$\operatorname{Int}\left[(\csc[c] + (d \cdot x) \cdot (b \cdot x))^n, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}\left[b \cdot \csc[c + dx]^n, \operatorname{Int}\left[1/\sin[c + dx]^n, x\right], x\right] /; \operatorname{FreeQ}\{b, c, d\}, x] \&\& \operatorname{EqQ}[n^2, 1/4]$

Rule 3872

$\operatorname{Int}\left[(\csc[e] + (f \cdot x) \cdot (d \cdot x))^n \cdot (\csc[e] + (f \cdot x) \cdot (b \cdot x) + (a \cdot x)), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}\left[a, \operatorname{Int}\left[(d \cdot \csc[e + f \cdot x])^n, x\right], x\right] + \operatorname{Dist}\left[b/d, \operatorname{Int}\left[(d \cdot \csc[e + f \cdot x])^{n+1}, x\right], x\right] /; \operatorname{FreeQ}\{a, b, d, e, f, n\}, x]$

Rule 3905

$\operatorname{Int}\left[(\csc[e] + (f \cdot x) \cdot (d \cdot x))^n / (\csc[e] + (f \cdot x) \cdot (b \cdot x) + (a \cdot x)), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}\left[(-b) \cdot d \cdot \cot[e + f \cdot x] \cdot ((d \cdot \csc[e + f \cdot x])^{n-1} / (a \cdot f \cdot (a + b \cdot \csc[e + f \cdot x])))\right], x\right] + \operatorname{Dist}\left[d \cdot ((n - 1) / (a \cdot b)), \operatorname{Int}\left[(d \cdot \csc[e + f \cdot x])^{n-1} \cdot (a - b \cdot \csc[e + f \cdot x])\right], x\right] /; \operatorname{FreeQ}\{a, b, d, e, f, n\}, x] \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\text{integral} = \int \frac{\sqrt{\sec(c + dx)}}{a + a \sec(c + dx)} dx$$

$$\begin{aligned}
&= \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))} - \frac{\int \frac{a-a \sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{2a^2} \\
&= \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a} + \frac{\int \sqrt{\sec(c+dx)} dx}{2a} \\
&= \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{2a} \\
&= -\frac{\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{ad} \\
&\quad + \frac{\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.91 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.65

$$\int \frac{1}{(a+a \cos(c+dx))\sqrt{\sec(c+dx)}} dx = \frac{4i \cos^2\left(\frac{1}{2}(c+dx)\right) \left(-1 - e^{2i(c+dx)} + (1 + e^{i(c+dx)}) \sqrt{1 + e^{2i(c+dx)}}\right) \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{ad(1 + e^{i(c+dx)})}$$

[In] Integrate[1/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]

[Out] ((-4*I)*Cos[(c + d*x)/2]^2*(-1 - E^((2*I)*(c + d*x)) + (1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c + d*x]]/(a*d*(1 + E^(I*(c + d*x)))^3)

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

[In] int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{1}{(a + a \cos(c + dx)) \sqrt{\sec(c + dx)}} dx$$

$$= \frac{(-i\sqrt{2}\cos(dx+c) - i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + (i\sqrt{2}\cos(dx+c) + i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\sin(dx+c))}{(a*d*\cos(d*x+c) + a*d)}$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

```
[Out] 1/2*((-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c)
- I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + (I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(
cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```


Sympy [F]

$$\int \frac{1}{(a + a \cos(c + dx))\sqrt{\sec(c + dx)}} dx = \int \frac{1}{\cos(c+dx)\sqrt{\sec(c+dx)+\sqrt{\sec(c+dx)}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(1/2), x)

[Out] Integral(1/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x)/a

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))\sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a \cos(dx + c) + a)\sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2), x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))\sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a \cos(dx + c) + a)\sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2), x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))\sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + a \cos(c + dx))} dx$$

[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))), x)

[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))), x)

$$3.321 \quad \int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4126
Rubi [A] (verified)	4126
Mathematica [C] (verified)	4128
Maple [A] (verified)	4129
Fricas [C] (verification not implemented)	4129
Sympy [F]	4130
Maxima [F]	4130
Giac [F]	4130
Mupad [F(-1)]	4130

Optimal result

Integrand size = 23, antiderivative size = 112

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{3}{2}}(c+dx)} dx \\ &= \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{ad} \\ & \quad - \frac{\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a \sec(c+dx))} \end{aligned}$$

[Out] $-\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))+3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d-(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a/d$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3904, 3872, 3856, 2719, 2720}

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{3}{2}}(c+dx)} dx \\ &= -\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a \sec(c+dx)+a)} - \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad} \\ & \quad + \frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{ad} \end{aligned}$$

[In] Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]

[Out] (3* $\sqrt{\cos[c + dx]}$)* $\text{EllipticE}[(c + dx)/2, 2]$ * $\sqrt{\sec[c + dx]}$)/(a*d) -
 ($\sqrt{\cos[c + dx]}$)* $\text{EllipticF}[(c + dx)/2, 2]$ * $\sqrt{\sec[c + dx]}$)/(a*d) -
 ($\sqrt{\sec[c + dx]}$)* $\sin[c + dx]$)/(d*(a + a* $\sec[c + dx]$))

Rule 2719

$\text{Int}[\sqrt{\sin[(c_.) + (d_.)(x_)]}, x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticE}[(1/2)*(c - \pi/2 + dx), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\sqrt{\sin[(c_.) + (d_.)(x_)]}, x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticF}[(1/2)*(c - \pi/2 + dx), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3317

$\text{Int}[(\csc[(e_.) + (f_.)(x_)]*(d_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)(x_)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[d^{(n*p)}, \text{Int}[(d*\csc[e + f*x])^{(m - n*p)}*(b + a*\csc[e + f*x]^n)^p, x], x] /; \text{FreeQ}\{a, b, d, e, f, m, n, p\}, x] \&\& !\text{IntegerQ}[m] \&\& \text{IntegersQ}[n, p]$

Rule 3856

$\text{Int}[(\csc[(c_.) + (d_.)(x_)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\csc[c + dx])^n*\sin[c + dx]^n, \text{Int}[1/\sin[c + dx]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{EqQ}[n^2, 1/4]$

Rule 3872

$\text{Int}[(\csc[(e_.) + (f_.)(x_)]*(d_.))^{(n_.)}*(\csc[(e_.) + (f_.)(x_)]*(b_.) + (a_.)), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\csc[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\csc[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, d, e, f, n\}, x]$

Rule 3904

$\text{Int}[(\csc[(e_.) + (f_.)(x_)]*(d_.))^{(n_.)}/(\csc[(e_.) + (f_.)(x_)]*(b_.) + (a_.)), x_Symbol] \rightarrow \text{Simp}[\text{Cot}[e + f*x]*((d*\csc[e + f*x])^n/(f*(a + b*\csc[e + f*x]))), x] - \text{Dist}[1/a^2, \text{Int}[(d*\csc[e + f*x])^n*(a*(n - 1) - b*n*\csc[e + f*x]), x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{\sqrt{\sec(c + dx)}(a + a \sec(c + dx))} dx \\ &= -\frac{\sqrt{\sec(c + dx)} \sin(c + dx)}{d(a + a \sec(c + dx))} - \frac{\int \frac{-\frac{3a}{2} + \frac{1}{2}a \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx}{a^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)} dx}{2a} + \frac{3 \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a} \\
&\quad + \frac{\left(3\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{2a} \\
&= \frac{3\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{ad} \\
&\quad - \frac{\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{d(a+a \sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.47 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.78

$$\int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{3}{2}}(c+dx)} dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c+dx)\right) \left(\frac{2i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}}\right) \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) + d(-1+e^{2ic})}{d(-1+e^{2ic})} \right)}{d(-1+e^{2ic})}$$

[In] Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)), x]

[Out] (Cos[(c + d*x)/2]^2*((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x)))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])/((d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))) - ((Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 2*Cos[(c + 3*d*x)/2] + Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/(2*d))/(a*(1 + Cos[c + d*x]))

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)+3E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

```
[In] int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.66

$$\int \frac{1}{(a + a \cos(dx + c)) \sec^{\frac{3}{2}}(dx + c)} dx$$

$$= \frac{(i\sqrt{2} \cos(dx + c) + i\sqrt{2}) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + (-i\sqrt{2} \cos(dx + c) - i\sqrt{2}) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3(-i\sqrt{2} \cos(dx + c) - i\sqrt{2}) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 3(i\sqrt{2} \cos(dx + c) + i\sqrt{2}) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) - 2\sqrt{\cos(dx + c)} \sin(dx + c)}{(a \cos(dx + c) + a)d}$$

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + (-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*cos(d*x + c
) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*sq
rt(cos(d*x + c))*sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx = \frac{\int \frac{1}{\cos(c+dx) \sec^{\frac{3}{2}}(c+dx) + \sec^{\frac{3}{2}}(c+dx)} dx}{a}$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)

[Out] Integral(1/(cos(c + d*x)*sec(c + d*x)**(3/2) + sec(c + d*x)**(3/2)), x)/a

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + a \cos(c + dx))} dx$$

[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))),x)

[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))), x)

$$3.322 \quad \int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	4131
Rubi [A] (verified)	4131
Mathematica [C] (verified)	4134
Maple [A] (verified)	4134
Fricas [C] (verification not implemented)	4135
Sympy [F(-1)]	4135
Maxima [F]	4135
Giac [F]	4136
Mupad [F(-1)]	4136

Optimal result

Integrand size = 23, antiderivative size = 140

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{5}{2}}(c+dx)} dx \\ &= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} \\ & \quad + \frac{5\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} \\ & \quad + \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a \sec(c+dx))} \end{aligned}$$

[Out] 5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-sin(d*x+c)/d/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2)-3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3904, 3872, 3854, 3856, 2720, 2719}

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{5 \sin(c + dx)}{3ad \sqrt{\sec(c + dx)}} - \frac{\sin(c + dx)}{d \sqrt{\sec(c + dx)} (a \sec(c + dx) + a)}$$

$$+ \frac{5 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3ad}$$

$$- \frac{3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{ad}$$

[In] Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]

[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + (5*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e + f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} dx \\
&= -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{5a}{2} + \frac{3}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx}{a^2} \\
&= -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{3 \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a} + \frac{5 \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)} dx}{2a} \\
&= \frac{5 \sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} \\
&\quad + \frac{5 \int \sqrt{\sec(c+dx)} dx}{6a} - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{2a} \\
&= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{ad} \\
&\quad + \frac{5 \sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} \\
&\quad + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{6a} \\
&= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{ad} \\
&\quad + \frac{5\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{3ad} \\
&\quad + \frac{5 \sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.07 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.23

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx$$

$$\cos^2\left(\frac{1}{2}(c + dx)\right) \left(-\frac{2i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (9(1+e^{2i(c+dx)})+9(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}) \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{-1+e^{2ic}} \right)$$

[In] Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]

[Out] (Cos[(c + d*x)/2]^2*((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + 2*Sqrt[Sec[c + d*x]]*(3*(2 + Cos[2*c])*Cos[d*x]*Csc[c] + Cos[2*d*x]*Sin[2*c] - 3*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 6*Cos[c]*Sin[d*x] + Cos[2*c]*Sin[2*d*x] - 3*Tan[c/2]))/(3*a*d*(1 + Cos[c + d*x]))

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.54

method	result
default	$-\frac{\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(5F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) + 9E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) - 3a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}}{3a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}}$

[In] int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx =$$

$$\frac{5 (i \sqrt{2} \cos(dx + c) + i \sqrt{2}) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 (-i \sqrt{2} \cos(dx + c) + i \sqrt{2}) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{(a + a \cos(dx + c)) \sec^{\frac{5}{2}}(dx + c)}$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] -1/6*(5*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*cos(d*x + c)^2 + 5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + a \cos(c + dx))} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))),x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))), x)

$$3.323 \quad \int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	4137
Rubi [A] (verified)	4137
Mathematica [C] (verified)	4140
Maple [A] (verified)	4140
Fricas [C] (verification not implemented)	4141
Sympy [F(-1)]	4141
Maxima [F]	4141
Giac [F]	4142
Mupad [F(-1)]	4142

Optimal result

Integrand size = 23, antiderivative size = 168

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx)) \sec^{\frac{7}{2}}(c+dx)} dx \\ &= \frac{21 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5ad} \\ & \quad - \frac{5 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3ad} + \frac{7 \sin(c+dx)}{5ad \sec^{\frac{3}{2}}(c+dx)} \\ & \quad - \frac{5 \sin(c+dx)}{3ad \sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d \sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))} \end{aligned}$$

```
[Out] 7/5*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)-sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(
d*x+c))-5/3*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+21/5*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3904, 3872, 3854, 3856, 2719, 2720}

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)} dx$$

$$= -\frac{\sin(c + dx)}{d \sec^{\frac{3}{2}}(c + dx)(a \sec(c + dx) + a)} + \frac{7 \sin(c + dx)}{5ad \sec^{\frac{3}{2}}(c + dx)}$$

$$- \frac{5 \sin(c + dx)}{3ad \sqrt{\sec(c + dx)}} - \frac{5 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3ad}$$

$$+ \frac{21 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5ad}$$

[In] Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2)),x]

[Out] (21*sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(5*a*d) - (5*sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(3*a*d) + (7*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])/(3*a*d*sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3904

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e + f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} dx \\
&= -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{7a}{2} + \frac{5}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx}{a^2} \\
&= -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{5 \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)} dx}{2a} + \frac{7 \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)} dx}{2a} \\
&= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} \\
&\quad - \frac{5 \int \sqrt{\sec(c+dx)} dx}{6a} + \frac{21 \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{10a} \\
&= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} \\
&\quad - \frac{(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{6a} \\
&\quad + \frac{(21\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}) \int \sqrt{\cos(c+dx)} dx}{10a} \\
&= \frac{21\sqrt{\cos(c+dx)}E(\frac{1}{2}(c+dx)|2)\sqrt{\sec(c+dx)}}{5ad} \\
&\quad - \frac{5\sqrt{\cos(c+dx)}\text{EllipticF}(\frac{1}{2}(c+dx),2)\sqrt{\sec(c+dx)}}{3ad} + \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.22 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.03

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{\cos^2\left(\frac{1}{2}(c + dx)\right) \left(\frac{8i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (63(1+e^{2i(c+dx)})+63(-1+e^{2ic}) \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right) - 1 + e^{2ic}}{1+e^{2ic}} \right)}{\dots}$$

[In] Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2)),x]

[Out] (Cos[(c + d*x)/2]^2*((8*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))])*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*(17 + 11*Cos[2*c])*Cos[d*x]*Csc[c] + 4*(10*Cos[2*d*x]*Sin[2*c] - 3*Cos[3*d*x]*Sin[3*c] - 30*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 99*Cos[c]*Sin[d*x] + 10*Cos[2*c]*Sin[2*d*x] - 3*Cos[3*c]*Sin[3*d*x] - 30*Tan[c/2])))/(60*a*d*(1 + Cos[c + d*x]))

Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(-\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(25F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) + 63E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) + 15a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2}\right)}{15a \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2}\right)}$

[In] int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)^4+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)} dx =$$

$$25 (-i \sqrt{2} \cos(dx + c) - i \sqrt{2}) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25 (i \sqrt{2} \cos$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] -1/30*(25*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 63*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(6*cos(d*x + c)^3 - 4*cos(d*x + c)^2 - 25*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))/(a*d*cos(d*x + c) + a*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2)), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a) \sec(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2} (a + a \cos(c + dx))} dx$$

[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))),x)

[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))), x)

$$3.324 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx$$

Optimal result	4143
Rubi [A] (verified)	4143
Mathematica [C] (verified)	4146
Maple [A] (verified)	4147
Fricas [C] (verification not implemented)	4147
Sympy [F(-1)]	4148
Maxima [F]	4148
Giac [F]	4148
Mupad [F(-1)]	4149

Optimal result

Integrand size = 23, antiderivative size = 202

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^2} dx = \frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{3a^2d} - \frac{7\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a \sec(c+dx))^2}$$

```
[Out] 10/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d-7/3*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-7*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d+7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used

= {3317, 3901, 4104, 3872, 3853, 3856, 2719, 2720}

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx = -\frac{7\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)} + \frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}$$

$$- \frac{7\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}$$

$$+ \frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d}$$

$$+ \frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2d}$$

$$- \frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]

[Out] (7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - (7*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (10*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d) - (7*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3901

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])

Rule 4104

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{\sec^{\frac{9}{2}}(c + dx)}{(a + a \sec(c + dx))^2} dx \\
 &= -\frac{\sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} - \frac{\int \frac{\sec^{\frac{5}{2}}(c + dx) \left(\frac{5a}{2} - \frac{9}{2} a \sec(c + dx)\right)}{a + a \sec(c + dx)} dx}{3a^2} \\
 &= -\frac{7 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{3a^2 d(1 + \sec(c + dx))} - \frac{\sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} \\
 &\quad - \frac{\int \sec^{\frac{3}{2}}(c + dx) \left(\frac{21a^2}{2} - 15a^2 \sec(c + dx)\right) dx}{3a^4}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{7 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3a^2 d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&\quad - \frac{7 \int \sec^{\frac{3}{2}}(c+dx) dx}{2a^2} + \frac{5 \int \sec^{\frac{5}{2}}(c+dx) dx}{a^2} \\
&= -\frac{7\sqrt{\sec(c+dx)} \sin(c+dx)}{a^2 d} + \frac{10 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2 d} \\
&\quad - \frac{7 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3a^2 d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&\quad + \frac{5 \int \sqrt{\sec(c+dx)} dx}{3a^2} + \frac{7 \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a^2} \\
&= -\frac{7\sqrt{\sec(c+dx)} \sin(c+dx)}{a^2 d} + \frac{10 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2 d} \\
&\quad - \frac{7 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3a^2 d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&\quad + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3a^2} \\
&\quad + \frac{\left(7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{2a^2} \\
&= \frac{7\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{a^2 d} \\
&\quad + \frac{10\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3a^2 d} \\
&\quad - \frac{7\sqrt{\sec(c+dx)} \sin(c+dx)}{a^2 d} + \frac{10 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2 d} \\
&\quad - \frac{7 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3a^2 d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{3d(a+a\sec(c+dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.01 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.42

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx = \frac{e^{-\frac{1}{2}i(4c+3dx)}(-1+e^{ic})\cos\left(\frac{1}{2}(c+dx)\right)\csc\left(\frac{c}{2}\right)\left(-10-37e^{i(c+dx)}-65e^{2i(c+dx)}-82e^{3i(c+dx)}-68e^{4i(c+dx)}\right)}{\dots}$$

[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]

```
[Out] -1/12*((-1 + E^(I*c))*Cos[(c + d*x)/2]*Csc[c/2]*(-10 - 37*E^(I*(c + d*x)) -
65*E^((2*I)*(c + d*x)) - 82*E^((3*I)*(c + d*x)) - 68*E^((4*I)*(c + d*x)) -
53*E^((5*I)*(c + d*x)) - 21*E^((6*I)*(c + d*x)) + (10*I)*(1 + E^(I*(c + d*
x))))^3*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2] + 7*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^3*(1 + E^((2*I)*(c + d*x)))^(3
/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sqrt[Sec[c + d*
x]])/(a^2*d*E^((I/2)*(4*c + 3*d*x))*(1 + E^((2*I)*(c + d*x)))*(1 + Cos[c +
d*x]))^2)
```

Maple [A] (verified)

Time = 9.83 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))}}{\left(\frac{\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}}{3 \cos(\frac{dx}{2} + \frac{c}{2})^3} + \frac{6\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}}{\cos(\frac{dx}{2} + \frac{c}{2})} - 22\sqrt{\frac{1}{2} - \cos(\frac{dx}{2} + \frac{c}{2})} \right)}$

```
[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1
/2)^2+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2
+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.62

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx =$$

$$\frac{10(i\sqrt{2}\cos(dx + c)^3 + 2i\sqrt{2}\cos(dx + c)^2 + i\sqrt{2}\cos(dx + c))\text{weierstrassPInverse}(-4, 0, \cos(dx + c))}{\dots}$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out]
$$-1/6*(10*(I*\sqrt{2}*\cos(dx + c)^3 + 2*I*\sqrt{2}*\cos(dx + c)^2 + I*\sqrt{2}*\cos(dx + c))*\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c)) + 10*(-I*\sqrt{2}*\cos(dx + c)^3 - 2*I*\sqrt{2}*\cos(dx + c)^2 - I*\sqrt{2}*\cos(dx + c))*\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c)) + 21*(-I*\sqrt{2}*\cos(dx + c)^3 - 2*I*\sqrt{2}*\cos(dx + c)^2 - I*\sqrt{2}*\cos(dx + c))*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c))) + 21*(I*\sqrt{2}*\cos(dx + c)^3 + 2*I*\sqrt{2}*\cos(dx + c)^2 + I*\sqrt{2}*\cos(dx + c))*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c))) + 2*(21*\cos(dx + c)^3 + 32*\cos(dx + c)^2 + 8*\cos(dx + c) - 2)*\sin(dx + c)/\sqrt{\cos(dx + c)})/(a^2*d*\cos(dx + c)^3 + 2*a^2*d*\cos(dx + c)^2 + a^2*d*\cos(dx + c))$$

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{(a + a \cos(c + dx))^2} dx$$

```
[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^2, x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^2, x)
```

$$3.325 \quad \int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^2} dx$$

Optimal result	4150
Rubi [A] (verified)	4150
Mathematica [C] (verified)	4153
Maple [A] (verified)	4154
Fricas [C] (verification not implemented)	4154
Sympy [F]	4155
Maxima [F]	4155
Giac [F]	4155
Mupad [F(-1)]	4155

Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^2} dx = -\frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a \sec(c+dx))^2}$$

```
[Out] -5/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*sec(d*x+c)^(5/2)*
sin(d*x+c)/d/(a+a*sec(d*x+c))^2+4*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d-4*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-5/3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used

= {3317, 3901, 4104, 3872, 3856, 2720, 3853, 2719}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx = -\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)} + \frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}$$

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d}$$

$$-\frac{4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{a^2d}$$

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2, x]

[Out] (-4*sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(a^2*d) - (5*sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(3*a^2*d) + (4*sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) - (5*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*((b + a*Csc[e + f*x]^n)^p), x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] & IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_))^(m_), x_Symbol] := Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] := Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sec^{\frac{7}{2}}(c + dx)}{(a + a \sec(c + dx))^2} dx \\
&= -\frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c + dx) \left(\frac{3a}{2} - \frac{7}{2}a \sec(c + dx)\right)}{a + a \sec(c + dx)} dx}{3a^2} \\
&= -\frac{5 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3a^2 d(1 + \sec(c + dx))} - \frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} \\
&\quad - \frac{\int \sqrt{\sec(c + dx)} \left(\frac{5a^2}{2} - 6a^2 \sec(c + dx)\right) dx}{3a^4} \\
&= -\frac{5 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3a^2 d(1 + \sec(c + dx))} - \frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} \\
&\quad - \frac{5 \int \sqrt{\sec(c + dx)} dx}{6a^2} + \frac{2 \int \sec^{\frac{3}{2}}(c + dx) dx}{a^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{4\sqrt{\sec(c+dx)} \sin(c+dx)}{a^2 d} - \frac{5 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2 d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&\quad - \frac{2 \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{a^2} - \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{6a^2} \\
&= -\frac{5\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3a^2 d} \\
&\quad + \frac{4\sqrt{\sec(c+dx)} \sin(c+dx)}{a^2 d} - \frac{5 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2 d(1+\sec(c+dx))} \\
&\quad - \frac{\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{a^2} \\
&= -\frac{4\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{a^2 d} \\
&\quad - \frac{5\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3a^2 d} \\
&\quad + \frac{4\sqrt{\sec(c+dx)} \sin(c+dx)}{a^2 d} \\
&\quad - \frac{5 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2 d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{3d(a+a\sec(c+dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.18 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.43

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx = \frac{e^{-idx} \cos\left(\frac{1}{2}(c+dx)\right) \sqrt{\sec(c+dx)} \left(-4ie^{-i(c+dx)}(1+e^{i(c+dx)})^3 \sqrt{1+e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, -E^{\left(\frac{1}{2}(c+dx)\right)}\right)\right)}{(a+a\cos(c+dx))^2}$$

[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]

[Out] -1/6*(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-4*I)*(1 + E^(I*(c + d*x))))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 40*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(29 + 50*Cos[c + d*x] + 17*Cos[2*(c + d*x)] + (12*I)*Sin[c + d*x] + (7*I)*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.30

method	result
default	$-\frac{2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(5F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-12E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$

```
[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.58

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2} dx =$$

$$\frac{5(-i\sqrt{2}\cos(dx+c)^2-2i\sqrt{2}\cos(dx+c)-i\sqrt{2})\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{(a+a\cos(c+dx))^2}$$

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(5*(-I*sqrt(2)*cos(d*x+c)^2-2*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c))+5*(I*sqrt(2)*cos(d*x+c)^2+2*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))+12*(I*sqrt(2)*cos(d*x+c)^2+2*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c)))+12*(-I*sqrt(2)*cos(d*x+c)^2-2*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c)))-2*(12*cos(d*x+c)^2+19*cos(d*x+c)+5)
```

$d*x + c) + 6)*\sin(d*x + c)/\sqrt{\cos(d*x + c))}/(a^2*d*\cos(d*x + c)^2 + 2*a^2*d*\cos(d*x + c) + a^2*d)$

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1} \frac{dx}{a^2}$$

[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)

[Out] Integral(sec(c + d*x)**(3/2)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{3}{2}}}{(a + a \cos(c + dx))^2} dx$$

[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^2, x)

3.326 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^2} dx$

Optimal result	4156
Rubi [A] (verified)	4157
Mathematica [C] (verified)	4159
Maple [A] (verified)	4159
Fricas [C] (verification not implemented)	4160
Sympy [F]	4160
Maxima [F]	4161
Giac [F]	4161
Mupad [F(-1)]	4161

Optimal result

Integrand size = 23, antiderivative size = 149

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^2} dx = \frac{\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{a^2 d} + \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3a^2 d} - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{a^2 d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d(a+a \sec(c+dx))^2}$$

```
[Out] -1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))+cos(1/2*d*x+1/2*c)^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```


Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3317, 3901, 4104, 3872, 3856, 2719, 2720}

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2} dx = -\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)} + \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d} + \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^2, x]

[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^ (n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3901

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^ (n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^ (m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(
a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
+ 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0
] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^ (n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^ (m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \sec(c + dx))^2} dx \\
&= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}(\frac{a}{2} - \frac{5}{2}a \sec(c+dx))}{a+a \sec(c+dx)} dx}{3a^2} \\
&= -\frac{\sqrt{\sec(c + dx)} \sin(c + dx)}{a^2 d(1 + \sec(c + dx))} - \frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} - \frac{\int \frac{-\frac{3a^2}{2} - a^2 \sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{3a^4} \\
&= -\frac{\sqrt{\sec(c + dx)} \sin(c + dx)}{a^2 d(1 + \sec(c + dx))} - \frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} \\
&\quad + \frac{\int \sqrt{\sec(c + dx)} dx}{3a^2} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{a^2 d (1 + \sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d(a + a \sec(c+dx))^2} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3a^2} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{2a^2} \\
&= \frac{\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{a^2 d} \\
&\quad + \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3a^2 d} \\
&\quad - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{a^2 d (1 + \sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d(a + a \sec(c+dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{\sec(c+dx)}}{(a + a \cos(c+dx))^2} dx$$

$$e^{-idx} \cos\left(\frac{1}{2}(c+dx)\right) \sqrt{\sec(c+dx)} \left(-ie^{-i(c+dx)}(1 + e^{i(c+dx)})^3 \sqrt{1 + e^{2i(c+dx)}} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \dots\right)\right)$$

[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^2,x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*(1 + E^(I*(c + d*x))))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(5 + 14*Cos[c + d*x] + 5*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2])/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

method	result
default	$ \frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(12\left(\cos^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 4\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\right)\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6a^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)^3 \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} $

```
[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2} dx =$$

$$\frac{2(i\sqrt{2}\cos(dx+c)^2 + 2i\sqrt{2}\cos(dx+c) + i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))}{-}$$

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^2 + 4*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2} dx = \frac{\int \frac{\sqrt{\sec(c+dx)}}{\cos^2(c+dx)+2\cos(c+dx)+1} dx}{a^2}$$

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2} dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2} dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{(a+a\cos(c+dx))^2} dx$$

[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^2, x)

$$3.327 \quad \int \frac{1}{(a+a \cos(c+dx))^2 \sqrt{\sec(c+dx)}} dx$$

Optimal result	4162
Rubi [A] (verified)	4162
Mathematica [A] (verified)	4164
Maple [B] (verified)	4164
Fricas [C] (verification not implemented)	4165
Sympy [F]	4165
Maxima [F]	4165
Giac [F]	4166
Mupad [F(-1)]	4166

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{1}{(a+a \cos(c+dx))^2 \sqrt{\sec(c+dx)}} dx$$

$$= \frac{\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3a^2 d} + \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d(a+a \sec(c+dx))^2}$$

[Out] 1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3317, 3900, 21, 3856, 2720}

$$\int \frac{1}{(a+a \cos(c+dx))^2 \sqrt{\sec(c+dx)}} dx$$

$$= \frac{\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2 d} + \frac{\sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3d(a \sec(c+dx) + a)^2}$$

[In] Int[1/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]

[Out] (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 21

```
Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplifierQ[c + d*x,
  a + b*x])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3317

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(m_)*((a_) + (b_)*sin[(e_) + (f_)*(x
_)])^(n_)]^(p_), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
!IntegerQ[m] && IntegersQ[n, p]
```

Rule 3856

```
Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3900

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)*(csc[(e_) + (f_)*(x_)]*(b_) + (
a_))^(m_), x_Symbol] := Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \sec(c + dx))^2} dx \\
&= \frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{\int \frac{\sqrt{\sec(c + dx)} \left(\frac{a}{2} + \frac{1}{2}a \sec(c + dx)\right)}{a + a \sec(c + dx)} dx}{3a^2} \\
&= \frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{\int \sqrt{\sec(c + dx)} dx}{6a^2} \\
&= \frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d(a + a \sec(c + dx))^2} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{6a^2}
\end{aligned}$$

$$= \frac{\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3a^2d} + \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+a\cos(c+dx))^2 \sqrt{\sec(c+dx)}} dx$$

$$= \frac{\cos\left(\frac{1}{2}(c+dx)\right) \sqrt{\sec(c+dx)} \left(4\cos^3\left(\frac{1}{2}(c+dx)\right) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)}{3a^2d(1+\cos(c+dx))^2}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]), x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(4*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(93) = 186.

Time = 3.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.44

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{6a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d$

[In] int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(1/2), x, method=_RETURNVERBOSE)

[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.95

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx$$

$$= \frac{(-i \sqrt{2} \cos(dx + c))^2 - 2i \sqrt{2} \cos(dx + c) - i \sqrt{2}}{6(a^2 d)} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/6*((-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx$$

$$= \frac{\int \frac{1}{\cos^2(c+dx)\sqrt{\sec(c+dx)} + 2\cos(c+dx)\sqrt{\sec(c+dx)} + \sqrt{\sec(c+dx)}} dx}{a^2}$$

[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)

[Out] Integral(1/(cos(c + d*x)**2*sqrt(sec(c + d*x)) + 2*cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x)/a**2

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a \cos(dx + c) + a)^2 \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a \cos(dx + c) + a)^2 \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + a \cos(c + dx))^2} dx$$

[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2),x)

[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^2), x)

$$3.328 \quad \int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4167
Rubi [A] (verified)	4167
Mathematica [C] (verified)	4170
Maple [A] (verified)	4170
Fricas [C] (verification not implemented)	4171
Sympy [F]	4171
Maxima [F]	4172
Giac [F]	4172
Mupad [F(-1)]	4172

Optimal result

Integrand size = 23, antiderivative size = 149

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)} dx \\ &= -\frac{\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{a^2 d} \\ & \quad + \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3a^2 d} \\ & \quad + \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{a^2 d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d(a+a \sec(c+dx))^2} \end{aligned}$$

```
[Out] -1/3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2+sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3902, 4104, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{\sin(c + dx) \sqrt{\sec(c + dx)}}{a^2 d (\sec(c + dx) + 1)} + \frac{2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3a^2 d}$$

$$- \frac{\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{a^2 d} - \frac{\sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d(a \sec(c + dx) + a)^2}$$

[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]

[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^m*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^p, x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3902

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])

```

Rule 4104

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2} dx \\
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}(-\frac{5a}{2}+\frac{1}{2}a\sec(c+dx))}{a+a\sec(c+dx)} dx}{3a^2} \\
&= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\frac{3a^2}{2}-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{3a^4} \\
&= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)} dx}{3a^2} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a^2} \\
&= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3a^2} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{2a^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{a^2d} \\
&\quad + \frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{3a^2d} \\
&\quad + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+a\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)} dx$$

$$\frac{e^{-idx} \cos\left(\frac{1}{2}(c+dx)\right) \sqrt{\sec(c+dx)} \left(16 \cos^3\left(\frac{1}{2}(c+dx)\right) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \left(\cos\left(\frac{1}{2}(c+dx)\right) + \dots\right)\right)}{\dots}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(-7 - 10*Cos[c + d*x] - 7*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*(c + d*x)) + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

method	result
default	$ -\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\dots\right)\right)}{6a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)} $

[In] int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x+1/2*c)^3)

$*c)^2-1)/a^2/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)^3/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx = \frac{2(i\sqrt{2}\cos(dx+c)^2 + 2i\sqrt{2}\cos(dx+c) + i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))}{\dots}$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] $-1/6*(2*(I*\sqrt{2}*\cos(d*x + c)^2 + 2*I*\sqrt{2}*\cos(d*x + c) + I*\sqrt{2}))*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + 2*(-I*\sqrt{2}*\cos(d*x + c)^2 - 2*I*\sqrt{2}*\cos(d*x + c) - I*\sqrt{2})*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + 3*(I*\sqrt{2}*\cos(d*x + c)^2 + 2*I*\sqrt{2}*\cos(d*x + c) + I*\sqrt{2})*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) + 3*(-I*\sqrt{2}*\cos(d*x + c)^2 - 2*I*\sqrt{2}*\cos(d*x + c) - I*\sqrt{2})*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) - 2*(3*\cos(d*x + c)^2 + 2*\cos(d*x + c))*\sin(d*x + c)/\sqrt{\cos(d*x + c)}/(a^2*d*\cos(d*x + c)^2 + 2*a^2*d*\cos(d*x + c) + a^2*d)$

Sympy [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx = \frac{\int \frac{1}{\cos^2(c+dx) \sec^{\frac{3}{2}}(c+dx) + 2 \cos(c+dx) \sec^{\frac{3}{2}}(c+dx) + \sec^{\frac{3}{2}}(c+dx)} dx}{a^2}$$

[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)

[Out] $\text{Integral}(1/(\cos(c + d*x)**2*\sec(c + d*x)**(3/2) + 2*\cos(c + d*x)*\sec(c + d*x)**(3/2) + \sec(c + d*x)**(3/2)), x)/a**2$

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + a \cos(c + dx))^2} dx$$

[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2),x)

[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^2), x)

$$3.329 \quad \int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	4173
Rubi [A] (verified)	4173
Mathematica [C] (verified)	4176
Maple [A] (verified)	4176
Fricas [C] (verification not implemented)	4177
Sympy [F(-1)]	4177
Maxima [F]	4178
Giac [F]	4178
Mupad [F(-1)]	4178

Optimal result

Integrand size = 23, antiderivative size = 152

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)} dx \\ &= \frac{4\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{a^2 d} \\ & \quad - \frac{5\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3a^2 d} \\ & \quad - \frac{5\sqrt{\sec(c+dx)} \sin(c+dx)}{3a^2 d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{3d(a+a \sec(c+dx))^2} \end{aligned}$$

[Out] $-5/3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^2/d/(1+\sec(d*x+c))-1/3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\sec(d*x+c))^2+4*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/d-5/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^2/d$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3902, 4105, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx$$

$$= -\frac{5 \sin(c + dx) \sqrt{\sec(c + dx)}}{3a^2 d (\sec(c + dx) + 1)} - \frac{5 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3a^2 d}$$

$$+ \frac{4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{a^2 d} - \frac{\sin(c + dx) \sqrt{\sec(c + dx)}}{3d(a \sec(c + dx) + a)^2}$$

[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]

[Out] (4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) - (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3902

```

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])

```

Rule 4105

```

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} dx \\
&= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{7a}{2} + \frac{3}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} dx}{3a^2} \\
&= -\frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-6a^2 + \frac{5}{2}a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{3a^4} \\
&= -\frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&\quad - \frac{5\int \sqrt{\sec(c+dx)} dx}{6a^2} + \frac{2\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{a^2} \\
&= -\frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} \\
&\quad - \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{6a^2} \\
&\quad + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)} dx}{a^2}
\end{aligned}$$

$$= \frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} - \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.66 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.70

$$\int \frac{1}{(a+a\cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)} dx = \frac{e^{-idx} \cos\left(\frac{1}{2}(c+dx)\right) \csc\left(\frac{c}{2}\right) \sec\left(\frac{c}{2}\right) \sqrt{\sec(c+dx)} \sin(c)(\cos(dx)+i\sin(dx)) \left(-24i \cos\left(\frac{1}{2}(c+dx)\right) - 18\right)}{\dots}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]

[Out] -1/6*(Cos[(c + d*x)/2]*Csc[c/2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c]*(Cos[d*x] + I*Sin[d*x])*((-24*I)*Cos[(c + d*x)/2] - (18*I)*Cos[(3*(c + d*x))/2] - (6*I)*Cos[(5*(c + d*x))/2] + 20*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((2*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) + Sin[(c + d*x)/2] + 2*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.69

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\dots}$

[In] int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d*x+1/2*c)^5)

$$\frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx =$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.82

$$\frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx =$$

$$\frac{5(-i\sqrt{2}\cos(dx+c)^2 - 2i\sqrt{2}\cos(dx+c) - i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))}{(a^2 d \cos(dx+c)^2 + 2a^2 d \cos(dx+c) + a^2 d)}$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] -1/6*(5*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 12*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 12*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(6*cos(d*x + c)^2 + 5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + a \cos(c + dx))^2} dx$$

[In] int(1/((1/cos(c + d*x))^5/2*(a + a*cos(c + d*x))^2),x)

[Out] int(1/((1/cos(c + d*x))^5/2*(a + a*cos(c + d*x))^2), x)

$$3.330 \quad \int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	4179
Rubi [A] (verified)	4179
Mathematica [C] (verified)	4182
Maple [A] (verified)	4183
Fricas [C] (verification not implemented)	4183
Sympy [F(-1)]	4184
Maxima [F]	4184
Giac [F]	4184
Mupad [F(-1)]	4184

Optimal result

Integrand size = 23, antiderivative size = 178

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{7}{2}}(c+dx)} dx \\ &= -\frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\ & \quad + \frac{10\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} \\ & \quad - \frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} \end{aligned}$$

```
[Out] 10/3*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)-7/3*sin(d*x+c)/a^2/d/(1+sec(d*x+c))/
sec(d*x+c)^(1/2)-1/3*sin(d*x+c)/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-7*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+10/3*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used

= {3317, 3902, 4105, 3872, 3854, 3856, 2720, 2719}

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{10 \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} - \frac{7 \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)} (\sec(c + dx) + 1)}$$

$$+ \frac{10 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3a^2 d}$$

$$- \frac{7 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{a^2 d} - \frac{\sin(c + dx)}{3d \sqrt{\sec(c + dx)} (a \sec(c + dx) + a)^2}$$

[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)),x]

[Out] (-7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (7*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3902

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_.), x_Symbol] := Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e
+ f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e +
f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[
m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(-A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
+ f*x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} dx \\
&= -\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{9a}{2} + \frac{5}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} dx}{3a^2} \\
&= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} \\
&\quad - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \frac{\int \frac{-15a^2 + \frac{21}{2}a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx}{3a^4} \\
&= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} \\
&\quad - \frac{7\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a^2} + \frac{5\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)} dx}{a^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{10 \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} - \frac{7 \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}(1 + \sec(c + dx))} \\
&\quad - \frac{\sin(c + dx)}{3d \sqrt{\sec(c + dx)}(a + a \sec(c + dx))^2} + \frac{5 \int \sqrt{\sec(c + dx)} dx}{3a^2} \\
&\quad - \frac{\left(7 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{2a^2} \\
&= -\frac{7 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{a^2 d} + \frac{10 \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} \\
&\quad - \frac{7 \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}(1 + \sec(c + dx))} - \frac{\sin(c + dx)}{3d \sqrt{\sec(c + dx)}(a + a \sec(c + dx))^2} \\
&\quad + \frac{\left(5 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3a^2} \\
&= -\frac{7 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{a^2 d} \\
&\quad + \frac{10 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3a^2 d} + \frac{10 \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}} \\
&\quad - \frac{7 \sin(c + dx)}{3a^2 d \sqrt{\sec(c + dx)}(1 + \sec(c + dx))} - \frac{\sin(c + dx)}{3d \sqrt{\sec(c + dx)}(a + a \sec(c + dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.63 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{e^{-idx} \cos\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)} (\cos(dx) + i \sin(dx)) \left(-84i \cos\left(\frac{1}{2}(c + dx)\right) - 63i \cos\left(\frac{3}{2}(c + dx)\right) - 21i\right)}{1}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)),x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x]))*((-84*I)*Cos[(c + d*x)/2] - (63*I)*Cos[(3*(c + d*x))/2] - (21*I)*Cos[(5*(c + d*x))/2] + 80*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((7*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) + 3*Sin[(c + d*x)/2] + 10*Sin[(3*(c + d*x))/2] + 12*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2])/((6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)

Maple [A] (verified)

Time = 5.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.52

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(16\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\right)}{6a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

[In] int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+a\cos(c+dx))^2 \sec^{\frac{7}{2}}(c+dx)} dx =$$

$$\frac{10(i\sqrt{2}\cos(dx+c)^2+2i\sqrt{2}\cos(dx+c)+i\sqrt{2})\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d)}$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")

```
[Out] -1/6*(10*(I*sqrt(2)*cos(d*x+c)^2+2*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c))+10*(-I*sqrt(2)*cos(d*x+c)^2-2*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))+21*(I*sqrt(2)*cos(d*x+c)^2+2*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c)))+21*(-I*sqrt(2)*cos(d*x+c)^2-2*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c)))-2*(2*cos(d*x+c)^3+13*cos(d*x+c)^2+10*cos(d*x+c))*sin(d*x+c)/sqrt(cos(d*x+c)))/(a^2*d*cos(d*x+c)^2+2*a^2*d*cos(d*x+c)+a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2)), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2} (a + a \cos(c + dx))^2} dx$$

[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2),x)

[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^2), x)

$$3.331 \quad \int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	4185
Rubi [A] (verified)	4186
Mathematica [C] (verified)	4188
Maple [A] (verified)	4189
Fricas [C] (verification not implemented)	4189
Sympy [F(-1)]	4190
Maxima [F]	4190
Giac [F]	4190
Mupad [F(-1)]	4191

Optimal result

Integrand size = 23, antiderivative size = 200

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx))^2 \sec^{\frac{9}{2}}(c+dx)} dx \\ &= \frac{56 \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5a^2 d} \\ & \quad - \frac{5 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{a^2 d} \\ & \quad + \frac{56 \sin(c+dx)}{15a^2 d \sec^{\frac{3}{2}}(c+dx)} - \frac{5 \sin(c+dx)}{a^2 d \sqrt{\sec(c+dx)}} \\ & \quad - \frac{3 \sin(c+dx)}{a^2 d \sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d \sec^{\frac{3}{2}}(c+dx)(a+a \sec(c+dx))^2} \end{aligned}$$

```
[Out] 56/15*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)-3*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)
/(1+sec(d*x+c))-1/3*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2-5*sin(
d*x+c)/a^2/d/sec(d*x+c)^(1/2)+56/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^2/d-5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3317, 3902, 4105, 3872, 3854, 3856, 2719, 2720}

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx)} dx$$

$$= -\frac{3 \sin(c + dx)}{a^2 d \sec^{\frac{3}{2}}(c + dx) (\sec(c + dx) + 1)} + \frac{56 \sin(c + dx)}{15 a^2 d \sec^{\frac{3}{2}}(c + dx)}$$

$$- \frac{5 \sin(c + dx)}{a^2 d \sqrt{\sec(c + dx)}} - \frac{5 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{a^2 d}$$

$$+ \frac{56 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5 a^2 d} - \frac{\sin(c + dx)}{3 d \sec^{\frac{3}{2}}(c + dx) (a \sec(c + dx) + a)^2}$$

[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)),x]

[Out] (56*sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(5*a^2*d) - (5*sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(a^2*d) + (56*Sin[c + d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])/(a^2*d*sqrt[Sec[c + d*x]]) - (3*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3902

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.), x_Symbol] := Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])

Rule 4105

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] := Simp[(-A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m + 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} dx \\
 &= -\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{11a}{2} + \frac{7}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} dx}{3a^2} \\
 &= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} \\
 &\quad - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\int \frac{-28a^2 + \frac{45}{2}a^2\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx}{3a^4}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{3 \sin(c + dx)}{a^2 d \sec^{\frac{3}{2}}(c + dx)(1 + \sec(c + dx))} - \frac{\sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2} \\
&\quad - \frac{15 \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)} dx}{2a^2} + \frac{28 \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)} dx}{3a^2} \\
&= \frac{56 \sin(c + dx)}{15a^2 d \sec^{\frac{3}{2}}(c + dx)} - \frac{5 \sin(c + dx)}{a^2 d \sqrt{\sec(c + dx)}} - \frac{3 \sin(c + dx)}{a^2 d \sec^{\frac{3}{2}}(c + dx)(1 + \sec(c + dx))} \\
&\quad - \frac{\sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2} - \frac{5 \int \sqrt{\sec(c + dx)} dx}{2a^2} + \frac{28 \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{5a^2} \\
&= \frac{56 \sin(c + dx)}{15a^2 d \sec^{\frac{3}{2}}(c + dx)} - \frac{5 \sin(c + dx)}{a^2 d \sqrt{\sec(c + dx)}} \\
&\quad - \frac{3 \sin(c + dx)}{a^2 d \sec^{\frac{3}{2}}(c + dx)(1 + \sec(c + dx))} - \frac{\sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2} \\
&\quad - \frac{\left(5 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a^2} \\
&\quad + \frac{\left(28 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{5a^2} \\
&= \frac{56 \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5a^2 d} \\
&\quad - \frac{5 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{a^2 d} \\
&\quad + \frac{56 \sin(c + dx)}{15a^2 d \sec^{\frac{3}{2}}(c + dx)} - \frac{5 \sin(c + dx)}{a^2 d \sqrt{\sec(c + dx)}} \\
&\quad - \frac{3 \sin(c + dx)}{a^2 d \sec^{\frac{3}{2}}(c + dx)(1 + \sec(c + dx))} - \frac{\sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.88 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx)} dx$$

$$e^{-idx} \cos\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)} (\cos(dx) + i \sin(dx)) \left(1344i \cos\left(\frac{1}{2}(c + dx)\right) + 1008i \cos\left(\frac{3}{2}(c + dx)\right) + \dots\right)$$

[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)),x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x]))*((1344*I)*Cos[(c + d*x)/2] + (1008*I)*Cos[(3*(c + d*x))/2] + (336*I)*Cos[(5*(c + d*x))/2])

- 1200*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (112*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]/E^((I/2)*(c + d*x)) - 34*Sin[(c + d*x)/2] - 148*Sin[(3*(c + d*x))/2] - 168*Sin[(5*(c + d*x))/2] - 11*Sin[(7*(c + d*x))/2] + 3*Sin[(9*(c + d*x))/2]))/(60*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)

Maple [A] (verified)

Time = 5.86 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.42

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(96\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+120\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-150\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{30a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}$

[In] int(1/(a+cos(d*x+c))*a^2/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.48

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx)} dx = \frac{75(-i\sqrt{2}\cos(dx+c)^2 - 2i\sqrt{2}\cos(dx+c) - i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))}{(a + a \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx)}$$

[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] -1/30*(75*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 75*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 168*(-I*sqrt(2)*cos(d*x + c)^2 - 2

```
*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 168*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(6*cos(d*x + c)^4 - 8*cos(d*x + c)^3 - 94*cos(d*x + c)^2 - 75*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{9}{2}}} dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{9}{2}}} dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{9/2} (a + a \cos(c + dx))^2} dx$$

```
[In] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^2), x)
```

```
[Out] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^2), x)
```

$$3.332 \quad \int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^3} dx$$

Optimal result	4192
Rubi [A] (verified)	4193
Mathematica [C] (verified)	4196
Maple [B] (verified)	4196
Fricas [C] (verification not implemented)	4197
Sympy [F]	4197
Maxima [F]	4198
Giac [F]	4198
Mupad [F(-1)]	4198

Optimal result

Integrand size = 23, antiderivative size = 221

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{6a^3d} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a \sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a \sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a^3+a^3 \sec(c+dx))}$$

```
[Out] -1/5*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-8/15*sec(d*x+c)^(5/2)
*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-13/6*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a^3+
a^3*sec(d*x+c))+49/10*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d-49/10*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3317, 3901, 4104, 3872, 3856, 2720, 3853, 2719}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx = -\frac{13\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d(a^3\sec(c+dx)+a^3)} + \frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{6a^3d} - \frac{49\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{10a^3d} - \frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3} - \frac{8\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3, x]

[Out] (-49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) - (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (8*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (13*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x])^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &

& IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n], x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3901

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m], x_Symbol] := Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])

Rule 4104

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m*(csc[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := Simp[d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{\sec^{\frac{9}{2}}(c + dx)}{(a + a \sec(c + dx))^3} dx \\ &= -\frac{\sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{\int \frac{\sec^{\frac{5}{2}}(c + dx) \left(\frac{5a}{2} - \frac{11}{2} a \sec(c + dx)\right)}{(a + a \sec(c + dx))^2} dx}{5a^2} \\ &= -\frac{\sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{8 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c + dx) \left(12a^2 - \frac{41}{2} a^2 \sec(c + dx)\right)}{a + a \sec(c + dx)} dx}{15a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&\quad - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a^3+a^3\sec(c+dx))} - \frac{\int\sqrt{\sec(c+dx)}\left(\frac{65a^3}{4}-\frac{147}{4}a^3\sec(c+dx)\right)dx}{15a^6} \\
&= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&\quad - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a^3+a^3\sec(c+dx))} - \frac{13\int\sqrt{\sec(c+dx)}dx}{12a^3} + \frac{49\int\sec^{\frac{3}{2}}(c+dx)dx}{20a^3} \\
&= \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\
&\quad - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a^3+a^3\sec(c+dx))} \\
&\quad - \frac{49\int\frac{1}{\sqrt{\sec(c+dx)}}dx}{20a^3} - \frac{\left(13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{12a^3} \\
&= -\frac{13\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} \\
&\quad + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\
&\quad - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a^3+a^3\sec(c+dx))} \\
&\quad - \frac{\left(49\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\sqrt{\cos(c+dx)}dx}{20a^3} \\
&= -\frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad - \frac{13\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} \\
&\quad + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\
&\quad - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a^3+a^3\sec(c+dx))}
\end{aligned}$$

$$2)^{(1/2)} * (65 * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) - 147 * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)})) * \cos(1/2 * d * x + 1/2 * c) + 588 * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \sin(1/2 * d * x + 1/2 * c)^8 - 1634 * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \sin(1/2 * d * x + 1/2 * c)^6 + 1488 * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \sin(1/2 * d * x + 1/2 * c)^4 - 439 * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \sin(1/2 * d * x + 1/2 * c)^2 / a^3 / \cos(1/2 * d * x + 1/2 * c)^5 / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / \sin(1/2 * d * x + 1/2 * c) / (2 * \cos(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.60

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx =$$

$$\frac{65(-i\sqrt{2}\cos(dx+c)^3 - 3i\sqrt{2}\cos(dx+c)^2 - 3i\sqrt{2}\cos(dx+c) - i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c)) + \dots}{(a + a \cos(c + dx))^3}$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] -1/60*(65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(147*cos(d*x + c)^3 + 376*cos(d*x + c)^2 + 295*cos(d*x + c) + 60)*sin(d*x + c)/sqrt(cos(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1} dx}{a^3}$$

[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)

[Out] Integral(sec(c + d*x)**(3/2)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x)/a**3

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^3} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^3} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{(a + a \cos(c + dx))^3} dx$$

[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^3, x)

$$3.333 \quad \int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^3} dx$$

Optimal result	4199
Rubi [A] (verified)	4199
Mathematica [C] (verified)	4202
Maple [A] (verified)	4203
Fricas [C] (verification not implemented)	4203
Sympy [F]	4204
Maxima [F]	4204
Giac [F]	4204
Mupad [F(-1)]	4204

Optimal result

Integrand size = 23, antiderivative size = 195

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^3} dx = \frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a \sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a \sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d(a^3+a^3 \sec(c+dx))}$$

```
[Out] -1/5*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-2/5*sec(d*x+c)^(3/2)*
sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-9/10*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a
^3*sec(d*x+c))+9/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/2*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3901, 4104, 3872, 3856, 2719, 2720}

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3} dx = -\frac{9\sin(c+dx)\sqrt{\sec(c+dx)}}{10d(a^3\sec(c+dx)+a^3)} + \frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{2a^3d} + \frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{10a^3d} - \frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3} - \frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5ad(a\sec(c+dx)+a)^2}$$

[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^3,x]

[Out] (9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*a^3*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (9*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x, x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3901

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.)^{(n_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_))^{(m_)}, x_Symbol] :> \text{Simp}[(-d^2) \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot ((d \cdot \text{Csc}[e + f \cdot x])^{(n - 2)} / (f \cdot (2 \cdot m + 1))), x] + \text{Dist}[d^2 / (a \cdot b \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{(m + 1)} \cdot (d \cdot \text{Csc}[e + f \cdot x])^{(n - 2)} \cdot (b \cdot (n - 2) + a \cdot (m - n + 2) \cdot \text{Csc}[e + f \cdot x]), x], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{GtQ}[n, 2] \&\& (\text{IntegersQ}[2 \cdot m, 2 \cdot n] \mid \mid \text{IntegerQ}[m])$

Rule 4104

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.)^{(n_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_))^{(m_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (B_.) + (A_))], x_Symbol] :> \text{Simp}[d \cdot (A \cdot b - a \cdot B) \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot ((d \cdot \text{Csc}[e + f \cdot x])^{(n - 1)} / (a \cdot f \cdot (2 \cdot m + 1))), x] - \text{Dist}[1 / (a \cdot b \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{(m + 1)} \cdot (d \cdot \text{Csc}[e + f \cdot x])^{(n - 1)} \cdot \text{Simp}[A \cdot (a \cdot d \cdot (n - 1)) - B \cdot (b \cdot d \cdot (n - 1)) - d \cdot (a \cdot B \cdot (m - n + 1) + A \cdot b \cdot (m + n)) \cdot \text{Csc}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A \cdot b - a \cdot B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{\sec^{\frac{7}{2}}(c + dx)}{(a + a \sec(c + dx))^3} dx \\
 &= -\frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{\int \frac{\sec^{\frac{3}{2}}(c + dx) (\frac{3a}{2} - \frac{9}{2} a \sec(c + dx))}{(a + a \sec(c + dx))^2} dx}{5a^2} \\
 &= -\frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5ad(a + a \sec(c + dx))^2} - \frac{\int \frac{\sqrt{\sec(c + dx)} (3a^2 - \frac{21}{2} a^2 \sec(c + dx))}{a + a \sec(c + dx)} dx}{15a^4} \\
 &= -\frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5ad(a + a \sec(c + dx))^2} \\
 &\quad - \frac{9 \sqrt{\sec(c + dx)} \sin(c + dx)}{10d(a^3 + a^3 \sec(c + dx))} - \frac{\int \frac{-\frac{27a^3}{4} - \frac{15}{4} a^3 \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx}{15a^6} \\
 &= -\frac{\sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5ad(a + a \sec(c + dx))^2} \\
 &\quad - \frac{9 \sqrt{\sec(c + dx)} \sin(c + dx)}{10d(a^3 + a^3 \sec(c + dx))} + \frac{\int \sqrt{\sec(c + dx)} dx}{4a^3} + \frac{9 \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{20a^3}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} \\
&\quad - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d(a^3+a^3\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{4a^3} \\
&\quad + \frac{\left(9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\sqrt{\cos(c+dx)}dx}{20a^3} \\
&= \frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad + \frac{\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{2a^3d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\
&\quad - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d(a^3+a^3\sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.78 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3} dx$$

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-3ie^{-2i(c+dx)}(1+e^{i(c+dx)})^5\sqrt{1+e^{2i(c+dx)}}\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}\right)\right)}{1}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^3, x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((−3I)*(1 + E^(I*(c + d*x))))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, −E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] − I*Sin[(c + d*x)/2]) + (2*I)*(34 + 69*Cos[c + d*x] + 34*Cos[2*(c + d*x)] + 7*Cos[3*(c + d*x)] + (2*I)*Sin[c + d*x] + (6*I)*Sin[2*(c + d*x)] + (2*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.37

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{20a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

```
[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2))*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1/2*c)^4+cos(1/2*d*x+1/2*c)^2+1/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3} dx =$$

$$5(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2})\text{weierstrassPInverse}(-4,0,\cos(dx+c))$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

```
[Out] -1/20*(5*(I*sqrt(2)*cos(d*x+c)^3+3*I*sqrt(2)*cos(d*x+c)^2+3*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c))+5*(-I*sqrt(2)*cos(d*x+c)^3-3*I*sqrt(2)*cos(d*x+c)^2-3*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c))+9*(-I*sqrt(2)*cos(d*x+c)^3-3*I*sqrt(2)*cos(d*x+c)^2-3*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)+I*sin(d*x+c)))+9*(I*sqrt(2)*cos(d*x+c)^3+3*I*sqrt(2)*cos(d*x+c)^2+3*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassZeta(-4,0,weierstrassPInverse(-4,0,cos(d*x+c)-I*sin(d*x+c)))+2*(9*cos(d*x+c)^3+22*cos(d*x+c)^2+15*cos(d*x+c))*sin(d*x+c)/sqrt(cos(d*x+c))/(a^3*d*cos(d*x+c)^3+3*a^3*d*cos(d*x+c)^2+3*a^3*d*cos(d*x+c)+a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3} dx = \frac{\int \frac{\sqrt{\sec(c+dx)}}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1} dx}{a^3}$$

[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)

[Out] Integral(sqrt(sec(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x) + 1), x)/a**3

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3} dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^3} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3} dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^3} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{(a+a\cos(c+dx))^3} dx$$

[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^3, x)

3.334 $\int \frac{1}{(a+a \cos(c+dx))^3 \sqrt{\sec(c+dx)}} dx$

Optimal result	4205
Rubi [A] (verified)	4205
Mathematica [C] (verified)	4208
Maple [A] (verified)	4209
Fricas [C] (verification not implemented)	4209
Sympy [F]	4210
Maxima [F]	4210
Giac [F]	4210
Mupad [F(-1)]	4211

Optimal result

Integrand size = 23, antiderivative size = 195

$$\int \frac{1}{(a+a \cos(c+dx))^3 \sqrt{\sec(c+dx)}} dx$$

$$= \frac{\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{6a^3d} - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d(a+a \sec(c+dx))^3}$$

$$- \frac{4\sqrt{\sec(c+dx)} \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a^3+a^3 \sec(c+dx))}$$

```
[Out] -1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-4/15*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))+1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used

= {3317, 3901, 4104, 4105, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx$$

$$= \frac{\sin(c + dx) \sqrt{\sec(c + dx)}}{6d(a^3 \sec(c + dx) + a^3)} + \frac{\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{6a^3 d}$$

$$+ \frac{\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{10a^3 d}$$

$$- \frac{\sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{5d(a \sec(c + dx) + a)^3} - \frac{4 \sin(c + dx) \sqrt{\sec(c + dx)}}{15ad(a \sec(c + dx) + a)^2}$$

[In] Int[1/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]

[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(10*a^3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(6*a^3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x])))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3901

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.)^{(n_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_))^{(m_)}, x_Symbol] :> \text{Simp}[(-d^2) \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot ((d \cdot \text{Csc}[e + f \cdot x])^{(n - 2)} / (f \cdot (2 \cdot m + 1))), x] + \text{Dist}[d^2 / (a \cdot b \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{(m + 1)} \cdot (d \cdot \text{Csc}[e + f \cdot x])^{(n - 2)} \cdot (b \cdot (n - 2) + a \cdot (m - n + 2) \cdot \text{Csc}[e + f \cdot x]), x], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{GtQ}[n, 2] \&\& (\text{IntegersQ}[2 \cdot m, 2 \cdot n] \mid \mid \text{IntegerQ}[m])$

Rule 4104

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.)^{(n_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_))^{(m_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (B_.) + (A_)), x_Symbol] :> \text{Simp}[d \cdot (A \cdot b - a \cdot B) \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot ((d \cdot \text{Csc}[e + f \cdot x])^{(n - 1)} / (a \cdot f \cdot (2 \cdot m + 1))), x] - \text{Dist}[1 / (a \cdot b \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{(m + 1)} \cdot (d \cdot \text{Csc}[e + f \cdot x])^{(n - 1)} \cdot \text{Simp}[A \cdot (a \cdot d \cdot (n - 1)) - B \cdot (b \cdot d \cdot (n - 1)) - d \cdot (a \cdot B \cdot (m - n + 1) + A \cdot b \cdot (m + n)) \cdot \text{Csc}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A \cdot b - a \cdot B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{GtQ}[n, 0]$

Rule 4105

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.)^{(n_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_))^{(m_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (B_.) + (A_)), x_Symbol] :> \text{Simp}[(-A \cdot b - a \cdot B) \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot ((d \cdot \text{Csc}[e + f \cdot x])^n / (b \cdot f \cdot (2 \cdot m + 1))), x] - \text{Dist}[1 / (a^2 \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{(m + 1)} \cdot (d \cdot \text{Csc}[e + f \cdot x])^n \cdot \text{Simp}[b \cdot B \cdot n - a \cdot A \cdot (2 \cdot m + n + 1) + (A \cdot b - a \cdot B) \cdot (m + n + 1) \cdot \text{Csc}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[A \cdot b - a \cdot B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \sec(c + dx))^3} dx \\ &= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{\int \frac{\sqrt{\sec(c + dx)} \left(\frac{a}{2} - \frac{7}{2} a \sec(c + dx)\right)}{(a + a \sec(c + dx))^2} dx}{5a^2} \\ &= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{4\sqrt{\sec(c + dx)} \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} - \frac{\int \frac{-2a^2 - \frac{9}{2}a^2 \sec(c + dx)}{\sqrt{\sec(c + dx)}(a + a \sec(c + dx))} dx}{15a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&\quad + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a^3+a^3\sec(c+dx))} - \frac{\int \frac{-\frac{3a^3}{4}-\frac{5}{4}a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{15a^6} \\
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&\quad + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a^3+a^3\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{20a^3} + \frac{\int \sqrt{\sec(c+dx)} dx}{12a^3} \\
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&\quad + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a^3+a^3\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)} dx}{20a^3} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{12a^3} \\
&= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad + \frac{\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{6a^3d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\
&\quad - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a^3+a^3\sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.62 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}} dx$$

$$= \frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right)\operatorname{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{-1+e^{2ic}}\right)}{\dots}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]

[Out] (2*Cos[(c + d*x)/2]^6*((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x)))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))])*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]) -

$$5E^{(I(c+d*x))*(-1+E^{((2*I)*c)})}*\text{Sqrt}[1+E^{((2*I)*(c+d*x))}]*\text{Hypergeometric2F1}[1/4, 1/2, 5/4, -E^{((2*I)*(c+d*x))}]/(E^{(I(c+d*x))*(-1+E^{((2*I)*c}))} - ((36*\text{Cos}[(c-d*x)/2] + 9*\text{Cos}[(3*c+d*x)/2] + 7*\text{Cos}[(c+3*d*x)/2] + 26*\text{Cos}[(5*c+3*d*x)/2] + 10*\text{Cos}[(3*c+5*d*x)/2] + 5*\text{Cos}[(7*c+5*d*x)/2] + 3*\text{Cos}[(5*c+7*d*x)/2]))*\text{Csc}[c/2]*\text{Sec}[c/2]*\text{Sec}[(c+d*x)/2]^5*\text{Sqrt}[\text{Sec}[c+d*x]]/32)/(15*a^3*d*(1+\text{Cos}[c+d*x])^3)$$

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+1F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{60a^3\cos\left(\frac{dx}{2}+\frac{c}{2}\right)^5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

[In] int(1/(a+cos(d*x+c))*a^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2))*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}} dx =$$

$$\frac{5(i\sqrt{2}\cos(dx+c)^3+3i\sqrt{2}\cos(dx+c)^2+3i\sqrt{2}\cos(dx+c)+i\sqrt{2})\text{weierstrassPInverse}(-4,0,\cos(dx+c))}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -1/60*(5*(I*sqrt(2)*cos(d*x+c)^3+3*I*sqrt(2)*cos(d*x+c)^2+3*I*sqrt(2)*cos(d*x+c)+I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c))+I*sin(d*x+c)+5*(-I*sqrt(2)*cos(d*x+c)^3-3*I*sqrt(2)*cos(d*x+c)^2-3*I*sqrt(2)*cos(d*x+c)-I*sqrt(2))*weierstrassPInverse(-4,0,cos(d*x+c))

) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - 5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx$$

$$= \frac{\int \frac{1}{\cos^3(c+dx)\sqrt{\sec(c+dx)}+3\cos^2(c+dx)\sqrt{\sec(c+dx)}+3\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}} dx}{a^3}$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)

[Out] Integral(1/(cos(c + d*x)**3*sqrt(sec(c + d*x)) + 3*cos(c + d*x)**2*sqrt(sec(c + d*x)) + 3*cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x)/a**3

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + a \cos(c + dx))^3} dx$$

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3), x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^3), x)
```

$$3.335 \quad \int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4212
Rubi [A] (verified)	4212
Mathematica [C] (verified)	4215
Maple [A] (verified)	4216
Fricas [C] (verification not implemented)	4216
Sympy [F(-1)]	4217
Maxima [F]	4217
Giac [F]	4217
Mupad [F(-1)]	4218

Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)} dx \\ &= -\frac{\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{10a^3d} \\ & \quad + \frac{\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{6a^3d} + \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d(a+a \sec(c+dx))^3} \\ & \quad - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a^3+a^3 \sec(c+dx))} \end{aligned}$$

[Out] 1/5*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))-1/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used

= {3317, 3900, 4104, 4105, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{\sin(c + dx) \sqrt{\sec(c + dx)}}{6d(a^3 \sec(c + dx) + a^3)} + \frac{\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{6a^3 d}$$

$$- \frac{\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{10a^3 d}$$

$$+ \frac{\sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{5d(a \sec(c + dx) + a)^3} - \frac{\sin(c + dx) \sqrt{\sec(c + dx)}}{15ad(a \sec(c + dx) + a)^2}$$

[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]

[Out] -1/10*(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^p, x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[

$(d * \text{Csc}[e + f * x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3900

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)](d_.))^{(n_.)}(\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.))^{(m_.)}, x_Symbol] :> \text{Simp}[b * d * \text{Cot}[e + f * x] * (a + b * \text{Csc}[e + f * x])^m * ((d * \text{Csc}[e + f * x])^{(n - 1)} / (a * f * (2 * m + 1))), x] - \text{Dist}[d / (a * b * (2 * m + 1)), \text{Int}[(a + b * \text{Csc}[e + f * x])^{(m + 1)} * (d * \text{Csc}[e + f * x])^{(n - 1)} * (a * (n - 1) - b * (m + n) * \text{Csc}[e + f * x]), x], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{LtQ}[1, n, 2] \&\& (\text{IntegersQ}[2 * m, 2 * n] \parallel \text{IntegerQ}[m])$

Rule 4104

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)](d_.))^{(n_.)}(\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.))^{(m_.)}(\text{csc}[(e_.) + (f_.)(x_.)](B_.) + (A_.)), x_Symbol] :> \text{Simp}[d * (A * b - a * B) * \text{Cot}[e + f * x] * (a + b * \text{Csc}[e + f * x])^m * ((d * \text{Csc}[e + f * x])^{(n - 1)} / (a * f * (2 * m + 1))), x] - \text{Dist}[1 / (a * b * (2 * m + 1)), \text{Int}[(a + b * \text{Csc}[e + f * x])^{(m + 1)} * (d * \text{Csc}[e + f * x])^{(n - 1)} * \text{Simp}[A * (a * d * (n - 1)) - B * (b * d * (n - 1)) - d * (a * B * (m - n + 1) + A * b * (m + n)) * \text{Csc}[e + f * x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A * b - a * B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{GtQ}[n, 0]$

Rule 4105

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)](d_.))^{(n_.)}(\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.))^{(m_.)}(\text{csc}[(e_.) + (f_.)(x_.)](B_.) + (A_.)), x_Symbol] :> \text{Simp}[(- (A * b - a * B)) * \text{Cot}[e + f * x] * (a + b * \text{Csc}[e + f * x])^m * ((d * \text{Csc}[e + f * x])^n / (b * f * (2 * m + 1))), x] - \text{Dist}[1 / (a^2 * (2 * m + 1)), \text{Int}[(a + b * \text{Csc}[e + f * x])^{(m + 1)} * (d * \text{Csc}[e + f * x])^n * \text{Simp}[b * B * n - a * A * (2 * m + n + 1) + (A * b - a * B) * (m + n + 1) * \text{Csc}[e + f * x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[A * b - a * B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \sec(c + dx))^3} dx \\ &= \frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} + \frac{\int \frac{\sqrt{\sec(c + dx)} \left(\frac{a}{2} + \frac{3}{2} a \sec(c + dx)\right)}{(a + a \sec(c + dx))^2} dx}{5a^2} \\ &= \frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{\sqrt{\sec(c + dx)} \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} + \frac{\int \frac{\frac{a^2}{2} + 3a^2 \sec(c + dx)}{\sqrt{\sec(c + dx)}(a + a \sec(c + dx))} dx}{15a^4} \end{aligned}$$

$$\begin{aligned}
&= \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d(a+a \sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} \\
&\quad + \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a^3+a^3 \sec(c+dx))} + \frac{\int \frac{-\frac{3a^3}{4} + \frac{5}{4}a^3 \sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{15a^6} \\
&= \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d(a+a \sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} \\
&\quad + \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a^3+a^3 \sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{20a^3} + \frac{\int \sqrt{\sec(c+dx)} dx}{12a^3} \\
&= \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d(a+a \sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} \\
&\quad + \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a^3+a^3 \sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{20a^3} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{12a^3} \\
&= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad + \frac{\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} + \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d(a+a \sec(c+dx))^3} \\
&\quad - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a^3+a^3 \sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.74 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.86

$$\int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)} dx$$

$$= 2 \cos^6\left(\frac{1}{2}(c+dx)\right) \left(-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3(1+e^{2i(c+dx)})+3(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}}\right)\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-E^{((2*I)*(c+dx))}\right)}{-1+e^{2ic}} \right)$$

[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]

[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x)))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])

$$+ 5E^{(I*(c + d*x))*(-1 + E^{((2*I)*c)})}*\text{Sqrt}[1 + E^{((2*I)*(c + d*x))}]*\text{Hypergeometric2F1}[1/4, 1/2, 5/4, -E^{((2*I)*(c + d*x))}]/(E^{(I*(c + d*x))*(-1 + E^{((2*I)*c)})} + ((36*\text{Cos}[(c - d*x)/2] + 9*\text{Cos}[(3*c + d*x)/2] + 17*\text{Cos}[(c + 3*d*x)/2] + 16*\text{Cos}[(5*c + 3*d*x)/2] + 20*\text{Cos}[(3*c + 5*d*x)/2] - 5*\text{Cos}[(7*c + 5*d*x)/2] + 3*\text{Cos}[(5*c + 7*d*x)/2])*\text{Csc}[c/2]*\text{Sec}[c/2]*\text{Sec}[(c + d*x)/2]^5*\text{Sqrt}[\text{Sec}[c + d*x]]/32)/(15*a^3*d*(1 + \text{Cos}[c + d*x])^3)$$

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \left(12\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 10\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\right)}{60a^3 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)^5 \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}$

[In] int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out]
$$-1/60*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(12*\cos(1/2*d*x+1/2*c)^8+10*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*2*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(1/2*d*x+1/2*c)^5+6*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*\cos(1/2*d*x+1/2*c)^5*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-2*\cos(1/2*d*x+1/2*c)^6-24*\cos(1/2*d*x+1/2*c)^4+17*\cos(1/2*d*x+1/2*c)^2-3)/a^3/\cos(1/2*d*x+1/2*c)^5/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx =$$

$$5(i\sqrt{2} \cos(dx + c)^3 + 3i\sqrt{2} \cos(dx + c)^2 + 3i\sqrt{2} \cos(dx + c) + i\sqrt{2}) \text{weierstrassPInverse}(-4, 0, \cos(dx + c))$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out]
$$-1/60*(5*(I*\text{sqrt}(2)*\cos(d*x + c)^3 + 3*I*\text{sqrt}(2)*\cos(d*x + c)^2 + 3*I*\text{sqrt}(2)*\cos(d*x + c) + I*\text{sqrt}(2))*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + 5*(-I*\text{sqrt}(2)*\cos(d*x + c)^3 - 3*I*\text{sqrt}(2)*\cos(d*x + c)^2 - 3*I*\text{sqrt}(2)*\cos(d*x + c) - I*\text{sqrt}(2))*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c))$$

) - I*sin(d*x + c)) + 3*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*cos(d*x + c)^3 + 14*cos(d*x + c)^2 + 5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{3}{2}} (a + a \cos(c + dx))^3} dx$$

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3), x)
```

```
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^3), x)
```

$$3.336 \quad \int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	4219
Rubi [A] (verified)	4219
Mathematica [C] (verified)	4222
Maple [A] (verified)	4223
Fricas [C] (verification not implemented)	4223
Sympy [F(-1)]	4224
Maxima [F]	4224
Giac [F]	4224
Mupad [F(-1)]	4224

Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)} dx \\ &= -\frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ & \quad + \frac{\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{2a^3d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a \sec(c+dx))^3} \\ & \quad + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a \sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a^3+a^3 \sec(c+dx))} \end{aligned}$$

[Out] $-1/5*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\sec(d*x+c))^{3+2/5}*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d/(a+a*\sec(d*x+c))^{2+1/2}*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a^3+a^3*\sec(d*x+c))-9/10*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d+1/2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^3/d$

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used

= {3317, 3902, 4104, 4105, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{\sin(c + dx) \sqrt{\sec(c + dx)}}{2d(a^3 \sec(c + dx) + a^3)} + \frac{\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{2a^3 d}$$

$$- \frac{9 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{10a^3 d}$$

$$- \frac{\sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{5d(a \sec(c + dx) + a)^3} + \frac{2 \sin(c + dx) \sqrt{\sec(c + dx)}}{5ad(a \sec(c + dx) + a)^2}$$

[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]

[Out] (-9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*a^3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a^3 + a^3*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[

$(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3902

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] :> \text{Simp}[(-\text{Cot}[e + f*x])*(a + b*\text{Csc}[e + f*x])^m*((d*\text{Csc}[e + f*x])^n/(f*(2*m + 1))), x] + \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*\text{Csc}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& (\text{IntegersQ}[2*m, 2*n] || \text{IntegerQ}[m])$

Rule 4104

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[d*(A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*((d*\text{Csc}[e + f*x])^{(n - 1)})/(a*f*(2*m + 1))), x] - \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^{(n - 1)}*\text{Simp}[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& \text{GtQ}[n, 0]$

Rule 4105

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] :> \text{Simp}[(-A*b - a*B)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*((d*\text{Csc}[e + f*x])^n/(b*f*(2*m + 1))), x] - \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m + 1)}*(d*\text{Csc}[e + f*x])^n*\text{Simp}[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[A*b - a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{\sqrt{\sec(c + dx)}}{(a + a \sec(c + dx))^3} dx \\ &= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{\int \frac{\sqrt{\sec(c + dx)}(-\frac{9a}{2} + \frac{3}{2}a \sec(c + dx))}{(a + a \sec(c + dx))^2} dx}{5a^2} \\ &= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d(a + a \sec(c + dx))^3} + \frac{2\sqrt{\sec(c + dx)} \sin(c + dx)}{5ad(a + a \sec(c + dx))^2} - \frac{\int \frac{3a^2 - \frac{9}{2}a^2 \sec(c + dx)}{\sqrt{\sec(c + dx)}(a + a \sec(c + dx))} dx}{15a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} \\
&\quad + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a^3+a^3\sec(c+dx))} - \frac{\int \frac{\frac{27a^3}{4} - \frac{15}{4}a^3\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{15a^6} \\
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} \\
&\quad + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a^3+a^3\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)} dx}{4a^3} - \frac{9 \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{20a^3} \\
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} \\
&\quad + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a^3+a^3\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{4a^3} \\
&\quad - \frac{\left(9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{20a^3} \\
&= -\frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad + \frac{\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{2a^3d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} \\
&\quad + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a^3+a^3\sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.30 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.39

$$\begin{aligned}
&\int \frac{1}{(a+a\cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{e^{-idx} \cos\left(\frac{1}{2}(c+dx)\right) \sqrt{\sec(c+dx)} \left(160 \cos^5\left(\frac{1}{2}(c+dx)\right) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \left(\cos\left(\frac{1}{2}(c+dx)\right) + \sqrt{\cos(c+dx)}\right) + \dots\right)}{\dots}
\end{aligned}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(-68 - 128*Cos[c + d*x] - 68*Cos[2*(c + d*x)] - 24*Cos[3*(c + d*x)] + (3*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + (6*I)*Sin[c + d*x] + (8*I)*Sin[2*(c + d*x)] + (6*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 5.17 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(36\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{20a^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}$

[In] int(1/(a+cos(d*x+c))*a^3/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx =$$

$$5(i\sqrt{2}\cos(dx+c)^3 + 3i\sqrt{2}\cos(dx+c)^2 + 3i\sqrt{2}\cos(dx+c) + i\sqrt{2})\text{weierstrassPInverse}(-4, 0, \cos(dx+c))$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")

```
[Out] -1/20*(5*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(9*cos(d*x + c)^3 + 12*cos(d*x + c)^2 + 5*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + a \cos(c + dx))^3} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3),x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^3), x)

$$3.337 \quad \int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	4225
Rubi [A] (verified)	4225
Mathematica [C] (verified)	4228
Maple [A] (verified)	4229
Fricas [C] (verification not implemented)	4229
Sympy [F(-1)]	4230
Maxima [F]	4230
Giac [F]	4230
Mupad [F(-1)]	4230

Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{7}{2}}(c+dx)} dx \\ &= \frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{10a^3d} \\ & \quad - \frac{13\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{6a^3d} - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{5d(a+a \sec(c+dx))^3} \\ & \quad - \frac{8\sqrt{\sec(c+dx)} \sin(c+dx)}{15ad(a+a \sec(c+dx))^2} - \frac{13\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a^3+a^3 \sec(c+dx))} \end{aligned}$$

[Out] -1/5*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3-8/15*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2-13/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))+49/10*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-13/6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3902, 4105, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)} dx$$

$$= -\frac{13 \sin(c + dx) \sqrt{\sec(c + dx)}}{6d (a^3 \sec(c + dx) + a^3)} - \frac{13 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{6a^3 d}$$

$$+ \frac{49 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{10a^3 d}$$

$$- \frac{8 \sin(c + dx) \sqrt{\sec(c + dx)}}{15ad (a \sec(c + dx) + a)^2} - \frac{\sin(c + dx) \sqrt{\sec(c + dx)}}{5d (a \sec(c + dx) + a)^3}$$

[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]

[Out] (49*sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(10*a^3*d) - (13*sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(6*a^3*d) - (sqrt[Sec[c + d*x]]*sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (8*sqrt[Sec[c + d*x]]*sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (13*sqrt[Sec[c + d*x]]*sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))

Rule 2719

Int[sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[

$(d \cdot \text{Csc}[e + f \cdot x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3902

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.))^{(n_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_.))^{(m_)}, x_ \text{Symbol}] :> \text{Simp}[(-\text{Cot}[e + f \cdot x]) \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot ((d \cdot \text{Csc}[e + f \cdot x])^n / (f \cdot (2 \cdot m + 1))), x] + \text{Dist}[1 / (a^2 \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{(m + 1)} \cdot (d \cdot \text{Csc}[e + f \cdot x])^n \cdot (a \cdot (2 \cdot m + n + 1) - b \cdot (m + n + 1) \cdot \text{Csc}[e + f \cdot x]), x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& (\text{IntegersQ}[2 \cdot m, 2 \cdot n] \mid \mid \text{IntegerQ}[m])$

Rule 4105

$\text{Int}[(\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (d_.))^{(n_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (b_.) + (a_.))^{(m_)} \cdot (\text{csc}[(e_.) + (f_.) \cdot (x_)] \cdot (B_.) + (A_.)), x_ \text{Symbol}] :> \text{Simp}[(- (A \cdot b - a \cdot B)) \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot ((d \cdot \text{Csc}[e + f \cdot x])^n / (b \cdot f \cdot (2 \cdot m + 1))), x] - \text{Dist}[1 / (a^2 \cdot (2 \cdot m + 1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{(m + 1)} \cdot (d \cdot \text{Csc}[e + f \cdot x])^n \cdot \text{Simp}[b \cdot B \cdot n - a \cdot A \cdot (2 \cdot m + n + 1) + (A \cdot b - a \cdot B) \cdot (m + n + 1) \cdot \text{Csc}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[A \cdot b - a \cdot B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{1}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^3} dx \\
 &= -\frac{\sqrt{\sec(c + dx)} \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{\int \frac{-\frac{11a}{2} + \frac{5}{2}a \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))^2} dx}{5a^2} \\
 &= -\frac{\sqrt{\sec(c + dx)} \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{8\sqrt{\sec(c + dx)} \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} - \frac{\int \frac{-\frac{41a^2}{2} + 12a^2 \sec(c + dx)}{\sqrt{\sec(c + dx)} (a + a \sec(c + dx))} dx}{15a^4} \\
 &= -\frac{\sqrt{\sec(c + dx)} \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{8\sqrt{\sec(c + dx)} \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
 &\quad - \frac{13\sqrt{\sec(c + dx)} \sin(c + dx)}{6d(a^3 + a^3 \sec(c + dx))} - \frac{\int \frac{-\frac{147a^3}{4} + \frac{65}{4}a^3 \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx}{15a^6} \\
 &= -\frac{\sqrt{\sec(c + dx)} \sin(c + dx)}{5d(a + a \sec(c + dx))^3} - \frac{8\sqrt{\sec(c + dx)} \sin(c + dx)}{15ad(a + a \sec(c + dx))^2} \\
 &\quad - \frac{13\sqrt{\sec(c + dx)} \sin(c + dx)}{6d(a^3 + a^3 \sec(c + dx))} - \frac{13 \int \sqrt{\sec(c + dx)} dx}{12a^3} + \frac{49 \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{20a^3}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)} \sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&\quad - \frac{13\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a^3+a^3\sec(c+dx))} - \frac{\left(13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{12a^3} \\
&\quad + \frac{\left(49\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{20a^3} \\
&= \frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad - \frac{13\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{6a^3d} \\
&\quad - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)} \sin(c+dx)}{15ad(a+a\sec(c+dx))^2} \\
&\quad - \frac{13\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a^3+a^3\sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.06 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+a\cos(c+dx))^3 \sec^{\frac{7}{2}}(c+dx)} dx$$

$$= \frac{2 \cos^6\left(\frac{1}{2}(c+dx)\right) \left(\frac{2i\sqrt{2}e^{-i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} (147(1+e^{2i(c+dx)})+147(-1+e^{2ic})\sqrt{1+e^{2i(c+dx)}})}{1+e^{2ic}} \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, -e^{2i(c+dx)}\right)}{-1+e^{2ic}} \right)}{1}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]

[Out] (2*Cos[(c + d*x)/2]^6*((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) - ((1134*Cos[(c - d*x)/2] + 1071*Cos[(3*c + d*x)/2] + 923*Cos[(c + 3*d*x)/2] + 694*Cos[(5*c + 3*d*x)/2] + 470*Cos[(3*c + 5*d*x)/2] + 265*Cos[(7*c + 5*d*x)/2] + 117*Cos[(5*c + 7*d*x)/2] + 30*Cos[(9*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]]/32)/(15*a^3*d*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 5.62 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.38

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(348\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{60a^3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+c}}$

```
[In] int(1/(a+cos(d*x+c))*a^3/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.81

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)} dx =$$

$$65 \left(-i \sqrt{2} \cos(dx + c)^3 - 3i \sqrt{2} \cos(dx + c)^2 - 3i \sqrt{2} \cos(dx + c) - i \sqrt{2} \right) \text{weierstrassPInverse}(-4, 0, c$$

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/60*(65*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 65*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 147*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 147*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(87*cos(d*x + c)^3 + 146*cos(d*x + c)^2 + 65*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2} (a + a \cos(c + dx))^3} dx$$

[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3),x)

[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^3), x)

$$3.338 \quad \int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	4231
Rubi [A] (verified)	4232
Mathematica [C] (verified)	4235
Maple [A] (verified)	4235
Fricas [C] (verification not implemented)	4236
Sympy [F(-1)]	4236
Maxima [F]	4237
Giac [F]	4237
Mupad [F(-1)]	4237

Optimal result

Integrand size = 23, antiderivative size = 221

$$\begin{aligned} & \int \frac{1}{(a+a \cos(c+dx))^3 \sec^{\frac{9}{2}}(c+dx)} dx \\ &= -\frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\ & \quad + \frac{11\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{2a^3d} \\ & \quad + \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a \sec(c+dx))^3} \\ & \quad - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a \sec(c+dx))^2} - \frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}(a^3+a^3 \sec(c+dx))} \end{aligned}$$

```
[Out] 11/2*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)-1/5*sin(d*x+c)/d/(a+a*sec(d*x+c))^3/
sec(d*x+c)^(1/2)-2/3*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2)-119
/30*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/sec(d*x+c)^(1/2)-119/10*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d+11/2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3317, 3902, 4105, 3872, 3854, 3856, 2720, 2719}

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx)} dx$$

$$= \frac{11 \sin(c + dx)}{2a^3 d \sqrt{\sec(c + dx)}} - \frac{119 \sin(c + dx)}{30d \sqrt{\sec(c + dx)} (a^3 \sec(c + dx) + a^3)}$$

$$+ \frac{11 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{2a^3 d}$$

$$- \frac{119 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{10a^3 d}$$

$$- \frac{2 \sin(c + dx)}{3ad \sqrt{\sec(c + dx)} (a \sec(c + dx) + a)^2} - \frac{\sin(c + dx)}{5d \sqrt{\sec(c + dx)} (a \sec(c + dx) + a)^3}$$

[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)),x]

[Out] (-119*sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(10*a^3*d) + (11*sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(5*d*sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d*sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +

$d*x])^{(n + 2), x], x] /; FreeQ[\{b, c, d\}, x] \&\& LtQ[n, -1] \&\& IntegerQ[2*n]$

Rule 3856

$Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^{(n_.), x_Symbol] :> Dist[(b*Csc[c + d*x])^{n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[\{b, c, d\}, x] \&\& EqQ[n^2, 1/4]$

Rule 3872

$Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^{(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^{(n + 1)}, x], x] /; FreeQ[\{a, b, d, e, f, n\}, x]$

Rule 3902

$Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^{(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^{(m_.), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^{(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f*x]), x], x] /; FreeQ[\{a, b, d, e, f, n\}, x] \&\& EqQ[a^2 - b^2, 0] \&\& LtQ[m, -1] \&\& (IntegersQ[2*m, 2*n] || IntegerQ[m])$

Rule 4105

$Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^{(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^{(m_.)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_.)), x_Symbol] :> Simp[(-A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m + 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^{(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[\{a, b, d, e, f, A, B, n\}, x] \&\& NeQ[A*b - a*B, 0] \&\& EqQ[a^2 - b^2, 0] \&\& LtQ[m, -2^(-1)] \&\& !GtQ[n, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^3} dx \\ &= -\frac{\sin(c + dx)}{5d\sqrt{\sec(c + dx)}(a + a \sec(c + dx))^3} - \frac{\int \frac{-\frac{13a}{2} + \frac{7}{2}a \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)(a + a \sec(c + dx))^2} dx}{5a^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} \\
&\quad -\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{69a^2}{2}+25a^2\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} dx}{15a^4} \\
&= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} \\
&\quad -\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}(a^3+a^3\sec(c+dx))} - \frac{\int \frac{-\frac{495a^3}{4}+\frac{357}{4}a^3\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx}{15a^6} \\
&= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} \\
&\quad -\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}(a^3+a^3\sec(c+dx))} - \frac{119\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{20a^3} + \frac{33\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)} dx}{4a^3} \\
&= \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} \\
&\quad -\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}(a^3+a^3\sec(c+dx))} \\
&\quad + \frac{11\int \sqrt{\sec(c+dx)} dx}{4a^3} - \frac{\left(119\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)} dx}{20a^3} \\
&= -\frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad + \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} \\
&\quad -\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}(a^3+a^3\sec(c+dx))} \\
&\quad + \frac{\left(11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{4a^3} \\
&= -\frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{10a^3d} \\
&\quad + \frac{11\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{2a^3d} \\
&\quad + \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} \\
&\quad -\frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}(a^3+a^3\sec(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.35 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx)} dx$$

$$= \frac{e^{-idx} \cos\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)} (\cos(dx) + i \sin(dx)) \left(-5355i \cos\left(\frac{1}{2}(c + dx)\right) - 3927i \cos\left(\frac{3}{2}(c + dx)\right)\right)}{\dots}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)),x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-5355*I)*Cos[(c + d*x)/2] - (3927*I)*Cos[(3*(c + d*x))/2] - (1785*I)*Cos[(5*(c + d*x))/2] - (357*I)*Cos[(7*(c + d*x))/2] + 5280*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]])*EllipticF[(c + d*x)/2, 2] + ((119*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(((3*I)/2)*(c + d*x)) + 193*Sin[(c + d*x)/2] + 579*Sin[(3*(c + d*x))/2] + 555*Sin[(5*(c + d*x))/2] + 227*Sin[(7*(c + d*x))/2] + 10*Sin[(9*(c + d*x))/2]))/(120*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 5.57 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\dots} \left(160\left(\cos^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 468\left(\cos^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 330\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right) / 60a^3 \sqrt{-2\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$

[In] int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.64

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx)} dx =$$

$$\frac{165 (i \sqrt{2} \cos(dx + c)^3 + 3i \sqrt{2} \cos(dx + c)^2 + 3i \sqrt{2} \cos(dx + c) + i \sqrt{2}) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{(a^3 d \cos(dx + c)^3 + 3a^3 d \cos(dx + c)^2 + 3a^3 d \cos(dx + c) + a^3 d)}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/60*(165*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 165*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 357*(I*sqrt(2)*cos(d*x + c)^3 + 3*I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 357*(-I*sqrt(2)*cos(d*x + c)^3 - 3*I*sqrt(2)*cos(d*x + c)^2 - 3*I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(20*cos(d*x + c)^4 + 237*cos(d*x + c)^3 + 376*cos(d*x + c)^2 + 165*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```


Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{9}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2)), x)

Giac [F]

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{9}{2}}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{9/2} (a + a \cos(c + dx))^3} dx$$

[In] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3),x)

[Out] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^3), x)

3.339 $\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) dx$

Optimal result	4238
Rubi [A] (verified)	4238
Mathematica [A] (verified)	4240
Maple [A] (verified)	4241
Fricas [A] (verification not implemented)	4241
Sympy [F(-1)]	4241
Maxima [B] (verification not implemented)	4242
Giac [A] (verification not implemented)	4242
Mupad [B] (verification not implemented)	4243

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) dx = \frac{32a \sqrt{\sec(c + dx)} \sin(c + dx)}{35d \sqrt{a + a \cos(c + dx)}} + \frac{16a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{35d \sqrt{a + a \cos(c + dx)}} + \frac{12a \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{35d \sqrt{a + a \cos(c + dx)}} + \frac{2a \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{7d \sqrt{a + a \cos(c + dx)}}$$

[Out] 16/35*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+12/35*a*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+32/35*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used

= {4307, 2851, 2850}

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) dx = \frac{2a \sin(c + dx) \sec^{\frac{7}{2}}(c + dx)}{7d\sqrt{a \cos(c + dx) + a}} + \frac{12a \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{35d\sqrt{a \cos(c + dx) + a}} + \frac{16a \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{35d\sqrt{a \cos(c + dx) + a}} + \frac{32a \sin(c + dx) \sqrt{\sec(c + dx)}}{35d\sqrt{a \cos(c + dx) + a}}$$

[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2), x]

[Out] (32*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (12*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])

Rule 2850

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rule 4307

Int[(csc[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)} dx \\
&= \frac{2a \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7} \left(6\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx \\
&= \frac{12a \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{1}{35} \left(24\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{16a \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{12a \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{2a \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{1}{35} \left(16\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{32a \sqrt{\sec(c+dx)} \sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{16a \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{12a \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\begin{aligned}
&\int \sqrt{a+a\cos(c+dx)} \sec^{\frac{9}{2}}(c+dx) dx \\
&= \frac{2\sqrt{a(1+\cos(c+dx))}(9+18\cos(c+dx)+4\cos(2(c+dx))+4\cos(3(c+dx))) \sec^{\frac{7}{2}}(c+dx) \tan\left(\frac{1}{2}(c+dx)\right)}{35d}
\end{aligned}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2), x]

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(9 + 18*Cos[c + d*x] + 4*Cos[2*(c + d*x)] + 4*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(35*d)

Maple [A] (verified)

Time = 6.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2 \cot(dx+c) \left(\sec^{\frac{9}{2}}(dx+c) \sqrt{a(1+\cos(dx+c))} (16(\cos^4(dx+c)) - 8(\cos^3(dx+c)) - 2(\cos^2(dx+c)) - \cos(dx+c) - 5) \right)}{35d}$	72

[In] `int(sec(d*x+c)^(9/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`[Out] `-2/35/d*cot(d*x+c)*sec(d*x+c)^(9/2)*(a*(1+cos(d*x+c)))^(1/2)*(16*cos(d*x+c)^4-8*cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c)-5)`**Fricas [A] (verification not implemented)**

none

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{2 (16 \cos(dx + c)^3 + 8 \cos(dx + c)^2 + 6 \cos(dx + c) + 5) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{35 (d \cos(dx + c)^4 + d \cos(dx + c)^3) \sqrt{\cos(dx + c)}}$$

[In] `integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")`[Out] `2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))`**Sympy [F(-1)]**

Timed out.

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) dx = \text{Timed out}$$

[In] `integrate(sec(d*x+c)**(9/2)*(a+a*cos(d*x+c))**(1/2),x)`

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(129) = 258.

Time = 0.33 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.85

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{2 \left(\frac{35 \sqrt{2} \sqrt{a} \sin(dx+c)}{\cos(dx+c)+1} - \frac{70 \sqrt{2} \sqrt{a} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{84 \sqrt{2} \sqrt{a} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{58 \sqrt{2} \sqrt{a} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{9 \sqrt{2} \sqrt{a} \sin(dx+c)^9}{(\cos(dx+c)+1)^9} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} \right)}{35 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{9}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{9}{2}} \left(\frac{4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{4 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8} \right)}$$

[In] integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 2/35*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^4/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{4 \sqrt{2} \left(\left(\left(\left(7 \left(5 \left(\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 - 10 \right) \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 + 267 \right) \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 - 3684 \right) \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 + 1869 \right) \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 - 350 \right) \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 + 35 \right) \sqrt{a} \operatorname{sgn}(\cos(1/2 * d * x + 1/2 * c)) \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right) / \left(\left(\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^4 - 6 \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 + 1 \right)^{7/2} * d \right)}{35 \left(\tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^4 - 6 \tan \left(\frac{1}{4} dx + \frac{1}{4} c \right)^2 + 1 \right)^{7/2} * d}$$

[In] integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 4/35*sqrt(2)*((((7*(5*(tan(1/4*d*x + 1/4*c)^2 - 10)*tan(1/4*d*x + 1/4*c)^2 + 267)*tan(1/4*d*x + 1/4*c)^2 - 3684)*tan(1/4*d*x + 1/4*c)^2 + 1869)*tan(1/4*d*x + 1/4*c)^2 - 350)*tan(1/4*d*x + 1/4*c)^2 + 35)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/((tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)^(7/2)*d)

Mupad [B] (verification not implemented)

Time = 18.85 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{14 \sin\left(\frac{3c}{2} + \frac{3dx}{2}\right) \sqrt{a + a \cos(c + dx)} \sqrt{\frac{2e^{c1i+dx1i}}{e^{c2i+dx2i}+1}} + 4 \sin\left(\frac{7c}{2} + \frac{7dx}{2}\right) \sqrt{a + a \cos(c + dx)} \sqrt{\frac{2e^{c1i+dx1i}}{e^{c2i+dx2i}+1}}}{\frac{105d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)}{8} + \frac{105d \cos\left(\frac{3c}{2} + \frac{3dx}{2}\right)}{8} + \frac{35d \cos\left(\frac{5c}{2} + \frac{5dx}{2}\right)}{8} + \frac{35d \cos\left(\frac{7c}{2} + \frac{7dx}{2}\right)}{8}}$$

[In] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(1/2),x)

```
[Out] (14*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) + 4*sin((7*c)/2 + (7*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2))/((105*d*cos(c/2 + (d*x)/2))/8 + (105*d*cos((3*c)/2 + (3*d*x)/2))/8 + (35*d*cos((5*c)/2 + (5*d*x)/2))/8 + (35*d*cos((7*c)/2 + (7*d*x)/2))/8)
```

3.340 $\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx$

Optimal result	4244
Rubi [A] (verified)	4244
Mathematica [A] (verified)	4246
Maple [A] (verified)	4246
Fricas [A] (verification not implemented)	4246
Sympy [F(-1)]	4247
Maxima [B] (verification not implemented)	4247
Giac [A] (verification not implemented)	4247
Mupad [B] (verification not implemented)	4248

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx = \frac{16a \sqrt{\sec(c + dx)} \sin(c + dx)}{15d \sqrt{a + a \cos(c + dx)}} + \frac{8a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{15d \sqrt{a + a \cos(c + dx)}} + \frac{2a \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d \sqrt{a + a \cos(c + dx)}}$$

[Out] 8/15*a*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/5*a*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+16/15*a*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4307, 2851, 2850}

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx = \frac{2a \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d \sqrt{a \cos(c + dx) + a}} + \frac{8a \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{15d \sqrt{a \cos(c + dx) + a}} + \frac{16a \sin(c + dx) \sqrt{\sec(c + dx)}}{15d \sqrt{a \cos(c + dx) + a}}$$

[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]

[Out] $(16*a*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(15*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (8*a*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(15*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (2*a*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 2850

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]]/((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[-2*b^2*(\text{Cos}[e + f*x]/(f*(b*c + a*d)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rule 2851

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]]*((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*\text{Cos}[e + f*x]*((c + d*\text{Sin}[e + f*x])^{(n + 1)})/(f*(n + 1)*(c^2 - d^2)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] + \text{Dist}[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[n, -1] \&\& \text{NeQ}[2*n + 3, 0] \&\& \text{IntegerQ}[2*n]$

Rule 4307

$\text{Int}[(\text{csc}[(a_) + (b_)*(x_)]*(c_))^{(m_)}*(u_), x_Symbol] \rightarrow \text{Dist}[(c*\text{Csc}[a + b*x])^m*(c*\text{Sin}[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u]/(c*\text{Sin}[a + b*x])^m, x], x] /; \text{FreeQ}\{a, b, c, m\}, x] \&\& !\text{IntegerQ}[m] \&\& \text{KnownSineIntegrandQ}[u, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx \\ &= \frac{2a \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d \sqrt{a + a \cos(c + dx)}} + \frac{1}{5} \left(4 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \\ &= \frac{8a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{15d \sqrt{a + a \cos(c + dx)}} + \frac{2a \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d \sqrt{a + a \cos(c + dx)}} \\ &\quad + \frac{1}{15} \left(8 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{16a \sqrt{\sec(c + dx)} \sin(c + dx)}{15d \sqrt{a + a \cos(c + dx)}} + \frac{8a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{15d \sqrt{a + a \cos(c + dx)}} + \frac{2a \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d \sqrt{a + a \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{2\sqrt{a(1 + \cos(c + dx))(7 + 4 \cos(c + dx) + 4 \cos(2(c + dx)))} \sec^{\frac{5}{2}}(c + dx) \tan\left(\frac{1}{2}(c + dx)\right)}{15d}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2), x]

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(7 + 4*Cos[c + d*x] + 4*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)

Maple [A] (verified)

Time = 6.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.54

method	result	size
default	$-\frac{2 \cot(dx+c) \left(\sec^{\frac{7}{2}}(dx+c) \sqrt{a(1+\cos(dx+c))} (8(\cos^3(dx+c))-4(\cos^2(dx+c))-\cos(dx+c)-3) \right)}{15d}$	62

[In] int(sec(d*x+c)^(7/2)*(a+cos(d*x+c)*a)^(1/2), x, method=_RETURNVERBOSE)

[Out] -2/15/d*cot(d*x+c)*sec(d*x+c)^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*(8*cos(d*x+c)^3-4*cos(d*x+c)^2-cos(d*x+c)-3)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{2\sqrt{a \cos(dx + c) + a}(8 \cos(dx + c)^2 + 4 \cos(dx + c) + 3) \sin(dx + c)}{15(d \cos(dx + c)^3 + d \cos(dx + c)^2) \sqrt{\cos(dx + c)}}$$

[In] integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] 2/15*sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x + c)))

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(7/2)*(a+a*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(97) = 194.

Time = 0.31 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.06

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{2 \left(\frac{15\sqrt{2}\sqrt{a} \sin(dx+c)}{\cos(dx+c)+1} - \frac{25\sqrt{2}\sqrt{a} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{17\sqrt{2}\sqrt{a} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{7\sqrt{2}\sqrt{a} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^3}{15 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{7}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{7}{2}} \left(\frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 1 \right)}$$

[In] integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 2/15*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{4\sqrt{2} \left(\left(\left(5 \left(3 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 20 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 282 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 100 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right) \right)}{15 \left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1 \right)^{\frac{5}{2}} d}$$

[In] integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] $\frac{4}{15}\sqrt{2} * (((5*(3*\tan(1/4*d*x + 1/4*c))^2 - 20)*\tan(1/4*d*x + 1/4*c))^2 + 282)*\tan(1/4*d*x + 1/4*c)^2 - 100)*\tan(1/4*d*x + 1/4*c)^2 + 15)*\sqrt{a}*\operatorname{sgn}(\cos(1/2*d*x + 1/2*c))*\tan(1/4*d*x + 1/4*c)/((\tan(1/4*d*x + 1/4*c))^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1)^{(5/2)*d}$

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{8 \sqrt{a (\cos(c + dx) + 1)} \sqrt{\frac{1}{\cos(c + dx)}} (7 \sin(c + dx) + 4 \sin(2c + 2dx) + 9 \sin(3c + 3dx) + 2 \sin(4c + 4dx) + \cos(5c + 5dx))}{15d (10 \cos(c + dx) + 8 \cos(2c + 2dx) + 5 \cos(3c + 3dx) + 2 \cos(4c + 4dx) + \cos(5c + 5dx))}$$

[In] `int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(1/2),x)`

[Out] $(8*(a*(\cos(c + d*x) + 1))^{(1/2)}*(1/\cos(c + d*x))^{(1/2)}*(7*\sin(c + d*x) + 4*\sin(2*c + 2*d*x) + 9*\sin(3*c + 3*d*x) + 2*\sin(4*c + 4*d*x) + 2*\sin(5*c + 5*d*x)))/(15*d*(10*\cos(c + d*x) + 8*\cos(2*c + 2*d*x) + 5*\cos(3*c + 3*d*x) + 2*\cos(4*c + 4*d*x) + \cos(5*c + 5*d*x) + 6))$

3.341 $\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx$

Optimal result	4249
Rubi [A] (verified)	4249
Mathematica [A] (verified)	4250
Maple [A] (verified)	4251
Fricas [A] (verification not implemented)	4251
Sympy [F(-1)]	4251
Maxima [B] (verification not implemented)	4252
Giac [A] (verification not implemented)	4252
Mupad [B] (verification not implemented)	4253

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx = \frac{4a \sqrt{\sec(c + dx)} \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} + \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}}$$

[Out] $2/3*a*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+4/3*a*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4307, 2851, 2850}

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx = \frac{2a \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d \sqrt{a \cos(c + dx) + a}} + \frac{4a \sin(c + dx) \sqrt{\sec(c + dx)}}{3d \sqrt{a \cos(c + dx) + a}}$$

[In] $\text{Int}[\text{Sqrt}[a + a*\text{Cos}[c + d*x]]*\text{Sec}[c + d*x]^{(5/2)}, x]$

[Out] $(4*a*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (2*a*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 2850

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \\ &= \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} + \frac{1}{3} \left(2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\ &= \frac{4a \sqrt{\sec(c + dx)} \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} + \frac{2a \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\begin{aligned} &\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx \\ &= \frac{2\sqrt{a(1 + \cos(c + dx))(1 + 2\cos(c + dx))} \sec^{\frac{3}{2}}(c + dx) \tan\left(\frac{1}{2}(c + dx)\right)}{3d} \end{aligned}$$

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2), x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 2*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Tan[(c + d*x)/2])/(3*d)
```

Maple [A] (verified)

Time = 6.51 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{2 \cot(dx+c) \left(\sec^{\frac{5}{2}}(dx+c)\right) \sqrt{a(1+\cos(dx+c))} (2(\cos^2(dx+c))-\cos(dx+c)-1)}{3d}$	52

[In] `int(sec(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `-2/3/d*cot(d*x+c)*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*(2*cos(d*x+c)^2-cos(d*x+c)-1)`

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{2 \sqrt{a \cos(dx + c) + a} (2 \cos(dx + c) + 1) \sin(dx + c)}{3 (d \cos(dx + c))^2 + d \cos(dx + c)} \sqrt{\cos(dx + c)}$$

[In] `integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `2/3*sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 1)*sin(d*x + c)/((d*cos(d*x + c))^2 + d*cos(d*x + c))*sqrt(cos(d*x + c))`

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx = \text{Timed out}$$

[In] `integrate(sec(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)`

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.47

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{2 \left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} - \frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^2}{3d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{5}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{5}{2}} \left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 1 \right)}$$

[In] integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] $\frac{2/3*(3*\sqrt{2}*\sqrt{a}*\sin(d*x + c)/(\cos(d*x + c) + 1) - 4*\sqrt{2}*\sqrt{a}*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + \sqrt{2}*\sqrt{a}*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5)*(\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 1)^2/(d*(\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)^{(5/2)}*(-\sin(d*x + c)/(\cos(d*x + c) + 1) + 1)^{(5/2)}*(2*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + \sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + 1))}$

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{4\sqrt{2} \left(\left(3 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 - 10 \right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 3 \right) \sqrt{a} \operatorname{sgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)}{3 \left(\tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^4 - 6 \tan\left(\frac{1}{4} dx + \frac{1}{4} c\right)^2 + 1 \right)^{\frac{3}{2}} d}$$

[In] integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] $\frac{4/3*\sqrt{2}*((3*\tan(1/4*d*x + 1/4*c)^2 - 10)*\tan(1/4*d*x + 1/4*c)^2 + 3)*\sqrt{a}*\operatorname{sgn}(\cos(1/2*d*x + 1/2*c))*\tan(1/4*d*x + 1/4*c)/((\tan(1/4*d*x + 1/4*c)^4 - 6*\tan(1/4*d*x + 1/4*c)^2 + 1)^{(3/2)}*d)}$

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{4 \sqrt{a (\cos(c + dx) + 1)} \sqrt{\frac{1}{\cos(c + dx)}} (\sin(c + dx) + \sin(2c + 2dx) + \sin(3c + 3dx))}{3d (3 \cos(c + dx) + 2 \cos(2c + 2dx) + \cos(3c + 3dx) + 2)}$$

[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(1/2),x)

[Out] (4*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(2*c + 2*d*x) + sin(3*c + 3*d*x)))/(3*d*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x) + 2))

3.342 $\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	4254
Rubi [A] (verified)	4254
Mathematica [A] (verified)	4255
Maple [A] (verified)	4255
Fricas [A] (verification not implemented)	4256
Sympy [F(-1)]	4256
Maxima [B] (verification not implemented)	4256
Giac [A] (verification not implemented)	4257
Mupad [B] (verification not implemented)	4257

Optimal result

Integrand size = 25, antiderivative size = 36

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \frac{2a \sqrt{\sec(c + dx)} \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}}$$

[Out] $2*a*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4307, 2850}

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \frac{2a \sin(c + dx) \sqrt{\sec(c + dx)}}{d \sqrt{a \cos(c + dx) + a}}$$

[In] `Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]`

[Out] `(2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])`

Rule 2850

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
```

] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx \\ &= \frac{2a\sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \sqrt{a+a\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) dx = \frac{2\sqrt{a(1+\cos(c+dx))} \sqrt{\sec(c+dx)} \tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2), x]

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/d

Maple [A] (verified)

Time = 5.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{2 \cot(dx+c) \left(\sec^{\frac{3}{2}}(dx+c) \right) (\cos(dx+c)-1) \sqrt{a(1+\cos(dx+c))}}{d}$	40

[In] int(sec(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(1/2), x, method=_RETURNVERBOSE)

[Out] -2/d*cot(d*x+c)*sec(d*x+c)^(3/2)*(cos(d*x+c)-1)*(a*(1+cos(d*x+c)))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \frac{2 \sqrt{a \cos(dx + c) + a \sin(dx + c)}}{(d \cos(dx + c) + d) \sqrt{\cos(dx + c)}}$$

[In] integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(32) = 64.

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.72

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \frac{2 \left(\frac{\sqrt{2}\sqrt{a} \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sqrt{2}\sqrt{a} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{3}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{3}{2}}}$$

[In] integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 2*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \frac{4\sqrt{2}\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)}{\sqrt{\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^4 - 6\tan\left(\frac{1}{4}dx + \frac{1}{4}c\right)^2 + 1}d}$$

[In] integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 4*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x + 1/4*c)/(sqrt(tan(1/4*d*x + 1/4*c)^4 - 6*tan(1/4*d*x + 1/4*c)^2 + 1)*d)

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \frac{2 \sin(c + dx) \sqrt{a (\cos(c + dx) + 1)} \sqrt{\frac{1}{\cos(c + dx)}}}{d (\cos(c + dx) + 1)}$$

[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2),x)

[Out] (2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2))/(d*(cos(c + d*x) + 1))

3.343 $\int \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} dx$

Optimal result	4258
Rubi [A] (verified)	4258
Mathematica [A] (verified)	4259
Maple [A] (verified)	4260
Fricas [A] (verification not implemented)	4260
Sympy [F]	4260
Maxima [B] (verification not implemented)	4261
Giac [F]	4261
Mupad [F(-1)]	4261

Optimal result

Integrand size = 25, antiderivative size = 57

$$\int \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} dx$$

$$= \frac{2\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{d}$$

[Out] $2*\arcsin(\sin(d*x+c)*a^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)}}*a^{(1/2)*\cos(d*x+c)^{(1/2)}}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4307, 2853, 222}

$$\int \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} dx$$

$$= \frac{2\sqrt{a} \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d}$$

[In] `Int[Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]`

[Out] $(2*\text{Sqrt}[a]*\text{ArcSin}[(\text{Sqrt}[a]*\text{Sin}[c + d*x])/\text{Sqrt}[a + a*\text{Cos}[c + d*x]])*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[c + d*x]])/d$

Rule 222

`Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

Rule 2853

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*
*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\ &= \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right)}{d} \\ &= \frac{2\sqrt{a} \arcsin \left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\begin{aligned} &\int \sqrt{a+a\cos(c+dx)} \sqrt{\sec(c+dx)} dx \\ &= \frac{\sqrt{2} \arcsin \left(\sqrt{2} \sin \left(\frac{1}{2}(c+dx) \right) \right) \sqrt{\cos(c+dx)} \sqrt{a(1+\cos(c+dx))} \sec \left(\frac{1}{2}(c+dx) \right) \sqrt{\sec(c+dx)}}{d} \end{aligned}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]

[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/d

Maple [A] (verified)

Time = 6.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.54

method	result	size
default	$\frac{2(\sqrt{\sec(dx+c)}\sqrt{a(1+\cos(dx+c))})\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos(dx+c)}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	88

[In] `int(sec(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $2/d*\sec(d*x+c)^{(1/2)}*(a*(1+\cos(d*x+c)))^{(1/2)}*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)/(1+\cos(d*x+c))/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)})$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.09

$$\int \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} dx$$

$$= \left[\frac{\sqrt{-a} \log \left(\frac{2a \cos(dx+c)^2 - 2\sqrt{a} \cos(dx+c) + a\sqrt{-a} \sqrt{\cos(dx+c)} \sin(dx+c) + a \cos(dx+c) - a}{\cos(dx+c)+1} \right)}{d}, \right. \\ \left. - \frac{2\sqrt{a} \arctan \left(\frac{\sqrt{a} \cos(dx+c) + a\sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)} \right)}{d} \right]$$

[In] `integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `[sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sqrt(cos(d*x + c))*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))/d, - 2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/d]`

Sympy [F]

$$\int \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} dx = \int \sqrt{a (\cos(c + dx) + 1)} \sqrt{\sec(c + dx)} dx$$

[In] `integrate(sec(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x)), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(47) = 94.

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.56

$$\int \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} dx$$

$$= \frac{\sqrt{a} \arctan\left(\left(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1\right)^{\frac{1}{4}} \sin\left(\frac{1}{2} \arctan(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) + \sin(dx + c), \left(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1\right)^{\frac{1}{4}} \cos\left(\frac{1}{2} \arctan(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) + \cos(dx + c)\right)}{d}$$

[In] integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*x + c))/d

Giac [F]

$$\int \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} dx = \int \sqrt{a \cos(dx + c) + a} \sqrt{\sec(dx + c)} dx$$

[In] integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} \sqrt{a + a \cos(c + dx)} dx$$

[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2),x)

[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2), x)

3.344 $\int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\sec(c+dx)}} dx$

Optimal result	4262
Rubi [A] (verified)	4262
Mathematica [A] (verified)	4264
Maple [A] (verified)	4264
Fricas [A] (verification not implemented)	4264
Sympy [F]	4265
Maxima [B] (verification not implemented)	4265
Giac [F]	4266
Mupad [F(-1)]	4266

Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\sec(c+dx)}} dx = \frac{\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} + \frac{a \sin(c+dx)}{d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}$$

[Out] $a*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}/\sec(d*x+c)^{(1/2)}+\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*a^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4307, 2849, 2853, 222}

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\sec(c+dx)}} dx = \frac{\sqrt{a} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d} + \frac{a \sin(c+dx)}{d \sqrt{\sec(c+dx)} \sqrt{a \cos(c+dx)+a}}$$

[In] $\text{Int}[\text{Sqrt}[a + a*\text{Cos}[c + d*x]]/\text{Sqrt}[\text{Sec}[c + d*x]],x]$

[Out] $(\text{Sqrt}[a]*\text{ArcSin}[(\text{Sqrt}[a]*\text{Sin}[c + d*x])/\text{Sqrt}[a + a*\text{Cos}[c + d*x]])*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[c + d*x]]/d + (a*\text{Sin}[c + d*x])/(d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2849

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*SIN[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*SIN[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*SIN[e + f*x]]*(c + d*SIN[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*SIN[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*SIN[a + b*x])^m, Int[ActivateTrig[u]/(c*SIN[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)} dx \\
 &= \frac{a \sin(c+dx)}{d \sqrt{a+a\cos(c+dx)} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{1}{2} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
 &= \frac{a \sin(c+dx)}{d \sqrt{a+a\cos(c+dx)} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right)}{d} \\
 &= \frac{\sqrt{a} \arcsin \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} + \frac{a \sin(c+dx)}{d \sqrt{a+a\cos(c+dx)} \sqrt{\sec(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \frac{\sqrt{\cos(c + dx)} \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)} \left(\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right)\right) + 2\sqrt{\cos(c + dx)}}{2d}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]

[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)

Maple [A] (verified)

Time = 14.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + \arctan(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}))\sqrt{a(1+\cos(dx+c))}}{d(1+\cos(dx+c))\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	108

[In] int((a+cos(d*x+c)*a)^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \frac{\sqrt{a}(\cos(dx + c) + 1) \arctan\left(\frac{\sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)}}{\sqrt{a} \sin(dx + c)}\right) - \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c)}{d \cos(dx + c) + d}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)

SymPy [F]

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \int \frac{\sqrt{a (\cos(c + dx) + 1)}}{\sqrt{\sec(c + dx)}} dx$$

[In] integrate((a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)

[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/sqrt(sec(c + d*x)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(78) = 156$.

Time = 0.41 (sec) , antiderivative size = 791, normalized size of antiderivative = 8.60

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{4} * (2 * (\cos(2 * d * x + 2 * c) ^ 2 + \sin(2 * d * x + 2 * c) ^ 2 + 2 * \cos(2 * d * x + 2 * c) + 1) ^ (1/4) * (\cos(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))), \sin(d * x + c) - (\cos(d * x + c) - 1) * \sin(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1)))) * \sqrt{a} + \sqrt{a} * (\arctan(2 * (-\cos(2 * d * x + 2 * c) ^ 2 + \sin(2 * d * x + 2 * c) ^ 2 + 2 * \cos(2 * d * x + 2 * c) + 1) ^ (1/4) * (\cos(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))), \sin(d * x + c) - \cos(d * x + c) * \sin(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))))), (\cos(2 * d * x + 2 * c) ^ 2 + \sin(2 * d * x + 2 * c) ^ 2 + 2 * \cos(2 * d * x + 2 * c) + 1) ^ (1/4) * (\cos(d * x + c) * \cos(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))) + \sin(d * x + c) * \sin(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1)))) + 1) - \arctan(2 * (-\cos(2 * d * x + 2 * c) ^ 2 + \sin(2 * d * x + 2 * c) ^ 2 + 2 * \cos(2 * d * x + 2 * c) + 1) ^ (1/4) * (\cos(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))), \sin(d * x + c) - \cos(d * x + c) * \sin(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))))), (\cos(2 * d * x + 2 * c) ^ 2 + \sin(2 * d * x + 2 * c) ^ 2 + 2 * \cos(2 * d * x + 2 * c) + 1) ^ (1/4) * (\cos(d * x + c) * \cos(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))) + \sin(d * x + c) * \sin(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1)))) - 1) - \arctan(2 * ((\cos(2 * d * x + 2 * c) ^ 2 + \sin(2 * d * x + 2 * c) ^ 2 + 2 * \cos(2 * d * x + 2 * c) + 1) ^ (1/4) * \sin(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))), (\cos(2 * d * x + 2 * c) ^ 2 + \sin(2 * d * x + 2 * c) ^ 2 + 2 * \cos(2 * d * x + 2 * c) + 1) ^ (1/4) * \cos(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))) + 1) + \arctan(2 * ((\cos(2 * d * x + 2 * c) ^ 2 + \sin(2 * d * x + 2 * c) ^ 2 + 2 * \cos(2 * d * x + 2 * c) + 1) ^ (1/4) * \sin(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))), (\cos(2 * d * x + 2 * c) ^ 2 + \sin(2 * d * x + 2 * c) ^ 2 + 2 * \cos(2 * d * x + 2 * c) + 1) ^ (1/4) * \cos(1/2 * \arctan(2 * \sin(2 * d * x + 2 * c) / (\cos(2 * d * x + 2 * c) + 1))) - 1))) / d$

Giac [F]

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \int \frac{\sqrt{a \cos(dx + c) + a}}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2), x)

$$3.345 \quad \int \frac{\sqrt{a+a \cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4267
Rubi [A] (verified)	4267
Mathematica [A] (verified)	4269
Maple [A] (verified)	4269
Fricas [A] (verification not implemented)	4270
Sympy [F]	4270
Maxima [B] (verification not implemented)	4270
Giac [F]	4271
Mupad [F(-1)]	4272

Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{3\sqrt{a} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{4d} + \frac{a \sin(c+dx)}{2d \sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} + \frac{3a \sin(c+dx)}{4d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}$$

[Out] 1/2*a*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+3/4*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+3/4*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4307, 2849, 2853, 222}

$$\int \frac{\sqrt{a+a \cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{3\sqrt{a} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4d} + \frac{a \sin(c+dx)}{2d \sec^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx)+a}} + \frac{3a \sin(c+dx)}{4d \sqrt{\sec(c+dx)} \sqrt{a \cos(c+dx)+a}}$$

[In] Int[Sqrt[a + a*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]

[Out] (3*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(4*d) + (a*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (3*a*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2849

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 4307

Int[(csc[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)} dx \\
 &= \frac{a \sin(c + dx)}{2d \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} \\
 &\quad + \frac{1}{4} \left(3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)} dx \\
 &= \frac{a \sin(c + dx)}{2d \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} + \frac{3a \sin(c + dx)}{4d \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}} \\
 &\quad + \frac{1}{8} \left(3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{a \sin(c + dx)}{2d \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} + \frac{3a \sin(c + dx)}{4d \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}} \\
&\quad \left(3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{x^2}{a}}} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}} \right) \\
&\quad \frac{3\sqrt{a} \arcsin \left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{4d} \\
&+ \frac{a \sin(c + dx)}{2d \sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} + \frac{3a \sin(c + dx)}{4d \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82

$$\begin{aligned}
&\int \frac{\sqrt{a + a \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{\sqrt{\cos(c + dx)} \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)} \left(3\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c + dx)\right)\right) + 2\sqrt{2} \right)}{8d}
\end{aligned}$$

[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Sec[c + d*x]^(3/2), x]

[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))) / (8*d)

Maple [A] (verified)

Time = 14.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07

method	result	si
default	$\frac{\sqrt{a(1+\cos(dx+c))} \left(2 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 3 \tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 3 \sec(dx+c) \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) \right)}{4d(1+\cos(dx+c)) \sec(dx+c)^{\frac{3}{2}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	14

[In] int((a+cos(d*x+c)*a)^(1/2)/sec(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/4/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*sec(d*x+c)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))/((1+cos(d*x+c)))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{3\sqrt{a}(\cos(dx + c) + 1) \arctan\left(\frac{\sqrt{a \cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) - \frac{\sqrt{a \cos(dx+c)+a}(2 \cos(dx+c)^2 + 3 \cos(dx+c)) \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4(d \cos(dx + c) + d)}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*(2*cos(d*x +
c)^2 + 3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d
)
```

Sympy [F]

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{a (\cos(c + dx) + 1)}}{\sec^{\frac{3}{2}}(c + dx)} dx$$

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/sec(c + d*x)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(112) = 224.

Time = 0.43 (sec) , antiderivative size = 1059, normalized size of antiderivative = 7.79

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
*c)))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
```

```

*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 1)))/d

```

Giac [F]

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{a \cos(dx + c) + a}}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{a + a \cos(c + dx)}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

```
[In] int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)
```

```
[Out] int((a + a*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)
```

3.346 $\int (a + a \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx$

Optimal result	4273
Rubi [A] (verified)	4273
Mathematica [A] (verified)	4275
Maple [A] (verified)	4276
Fricas [A] (verification not implemented)	4276
Sympy [F(-1)]	4276
Maxima [A] (verification not implemented)	4277
Giac [F(-1)]	4277
Mupad [B] (verification not implemented)	4277

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int (a + a \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \frac{208a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{105d \sqrt{a + a \cos(c + dx)}} + \frac{104a^2 \sec^{3/2}(c + dx) \sin(c + dx)}{105d \sqrt{a + a \cos(c + dx)}} + \frac{26a^2 \sec^{5/2}(c + dx) \sin(c + dx)}{35d \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sec^{7/2}(c + dx) \sin(c + dx)}{7d \sqrt{a + a \cos(c + dx)}}$$

[Out] $104/105*a^2*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+26/35*a^2*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/7*a^2*\sec(d*x+c)^{(7/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+208/105*a^2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2841, 21, 2851, 2850}

$$\int (a + a \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \frac{2a^2 \sin(c + dx) \sec^{7/2}(c + dx)}{7d \sqrt{a \cos(c + dx) + a}} + \frac{26a^2 \sin(c + dx) \sec^{5/2}(c + dx)}{35d \sqrt{a \cos(c + dx) + a}} + \frac{104a^2 \sin(c + dx) \sec^{3/2}(c + dx)}{105d \sqrt{a \cos(c + dx) + a}} + \frac{208a^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{105d \sqrt{a \cos(c + dx) + a}}$$

[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2), x]

[Out] (208*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (104*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (26*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])

Rule 21

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :> Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 2841

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Ssin[e + f*x])^(m - 2)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2850

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Ssin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Ssin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Ssin[e + f*x]]*(c + d*Ssin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Ssin[a + b*x])^m, Int[ActivateTrig[u]/(c*Ssin[a + b*x])^m, x], x

] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{9/2}(c+dx)} dx \\
 &= \frac{2a^2 \sec^{7/2}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} \\
 &\quad - \frac{1}{7} \left(2a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{13a}{2} - \frac{13}{2}a \cos(c+dx)}{\cos^{7/2}(c+dx) \sqrt{a+a\cos(c+dx)}} dx \\
 &= \frac{2a^2 \sec^{7/2}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7} \left(13a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{7/2}(c+dx)} dx \\
 &= \frac{26a^2 \sec^{5/2}(c+dx) \sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sec^{7/2}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{1}{35} \left(52a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{5/2}(c+dx)} dx \\
 &= \frac{104a^2 \sec^{3/2}(c+dx) \sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{26a^2 \sec^{5/2}(c+dx) \sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{2a^2 \sec^{7/2}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{1}{105} \left(104a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{3/2}(c+dx)} dx \\
 &= \frac{208a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{104a^2 \sec^{3/2}(c+dx) \sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{26a^2 \sec^{5/2}(c+dx) \sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sec^{7/2}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.45

$$\int (a+a\cos(c+dx))^{3/2} \sec^{9/2}(c+dx) dx = \frac{2a\sqrt{a(1+\cos(c+dx))}(41+117\cos(c+dx)+26\cos(2(c+dx))+26\cos(3(c+dx)))\sec^{7/2}(c+dx)}{105d}$$

[In] Integrate[(a+a*Cos[c+d*x])^(3/2)*Sec[c+d*x]^(9/2),x]

[Out] (2*a*Sqrt[a*(1+Cos[c+d*x])]*(41+117*Cos[c+d*x]+26*Cos[2*(c+d*x)]+26*Cos[3*(c+d*x)])*Sec[c+d*x]^(7/2)*Tan[(c+d*x)/2])/(105*d)

Maple [A] (verified)

Time = 5.97 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

method	result	size
default	$-\frac{2 \cot(dx+c) \sqrt{a(1+\cos(dx+c))} \left(\sec^{\frac{9}{2}}(dx+c)\right) (104(\cos^4(dx+c)) - 52(\cos^3(dx+c)) - 13(\cos^2(dx+c)) - 24 \cos(dx+c) - 15)a}{105d}$	73

[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] -2/105/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(9/2)*(104*cos(d*x+c)^4-52*cos(d*x+c)^3-13*cos(d*x+c)^2-24*cos(d*x+c)-15)*a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.53

$$\int (a + a \cos(c + dx))^{3/2} \sec^{\frac{9}{2}}(c + dx) dx = \frac{2(104 a \cos(dx + c)^3 + 52 a \cos(dx + c)^2 + 39 a \cos(dx + c) + 15 a) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{105 (d \cos(dx + c)^4 + d \cos(dx + c)^3) \sqrt{\cos(dx + c)}}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] 2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^{\frac{9}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.63

$$\int (a + a \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \frac{4 \left(\frac{105 \sqrt{2} a^{3/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{245 \sqrt{2} a^{3/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{273 \sqrt{2} a^{3/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{171 \sqrt{2} a^{3/2} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{38 \sqrt{2} a^{3/2} \sin(dx+c)^9}{(\cos(dx+c)+1)^9} \right)}{105 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{9/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{9/2} \left(\frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 1 \right)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")

```
[Out] 4/105*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

Giac [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 18.01 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.37

$$\int (a + a \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \frac{-35 a \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a + a \cos(c + dx)} \sqrt{\frac{2 e^{c \cdot 1 + dx \cdot 1}}{e^{c \cdot 2i + dx \cdot 2i} + 1}} + 91 a \sin\left(\frac{3c}{2} + \frac{3dx}{2}\right) \sqrt{a + a \cos(c + dx)}}{\frac{315 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)}{8} + \frac{315 d \cos\left(\frac{3c}{2} + \frac{3dx}{2}\right)}{8} + \frac{105 d \cos\left(\frac{5c}{2} + \frac{5dx}{2}\right)}{8}}$$

[In] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(3/2),x)

```
[Out] (91*a*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*
x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) - 35*a*sin(c/2 + (d*x)/2)*(a + a*cos
(c + d*x))^(1/2)*((2*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) +
26*a*sin((7*c)/2 + (7*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*x
*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2))/((315*d*cos(c/2 + (d*x)/2))/8 + (315
*d*cos((3*c)/2 + (3*d*x)/2))/8 + (105*d*cos((5*c)/2 + (5*d*x)/2))/8 + (105*
d*cos((7*c)/2 + (7*d*x)/2))/8)
```

3.347 $\int (a + a \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx$

Optimal result	4279
Rubi [A] (verified)	4279
Mathematica [A] (verified)	4281
Maple [A] (verified)	4281
Fricas [A] (verification not implemented)	4282
Sympy [F(-1)]	4282
Maxima [B] (verification not implemented)	4282
Giac [F(-1)]	4283
Mupad [B] (verification not implemented)	4283

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int (a + a \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \frac{12a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{5d \sqrt{a + a \cos(c + dx)}} + \frac{6a^2 \sec^{3/2}(c + dx) \sin(c + dx)}{5d \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sec^{5/2}(c + dx) \sin(c + dx)}{5d \sqrt{a + a \cos(c + dx)}}$$

[Out] $6/5*a^2*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/5*a^2*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+12/5*a^2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2841, 21, 2851, 2850}

$$\int (a + a \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \frac{2a^2 \sin(c + dx) \sec^{5/2}(c + dx)}{5d \sqrt{a \cos(c + dx) + a}} + \frac{6a^2 \sin(c + dx) \sec^{3/2}(c + dx)}{5d \sqrt{a \cos(c + dx) + a}} + \frac{12a^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{5d \sqrt{a \cos(c + dx) + a}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}*\text{Sec}[c + d*x]^{(7/2)}, x]$

[Out] $(12*a^2*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (6*a^2*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (2*a^2*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx$$

$$\begin{aligned}
&= \frac{2a^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{1}{5} \left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{-\frac{9a}{2} - \frac{9}{2}a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx \\
&= \frac{2a^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{5} \left(9a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{6a^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{1}{5} \left(6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{12a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{6a^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

$$\int (a+a\cos(c+dx))^{3/2} \sec^{\frac{7}{2}}(c+dx) dx = \frac{2a\sqrt{a(1+\cos(c+dx))}(4+3\cos(c+dx)+3\cos(2(c+dx)))\sec^{\frac{5}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{5d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2), x]

[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(5*d)

Maple [A] (verified)

Time = 6.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{2 \cot(dx+c)\sqrt{a(1+\cos(dx+c))}\left(\sec^{\frac{7}{2}}(dx+c)\right)\left(6\cos^3(dx+c)-3\cos^2(dx+c)-2\cos(dx+c)-1\right)a}{5d}$	63

[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] -2/5/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(7/2)*(6*cos(d*x+c)^3-3*cos(d*x+c)^2-2*cos(d*x+c)-1)*a

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.60

$$\int (a + a \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \frac{2(6a \cos(dx + c)^2 + 3a \cos(dx + c) + a) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{5(d \cos(dx + c)^3 + d \cos(dx + c)^2) \sqrt{\cos(dx + c)}}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x + c)))

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(103) = 206.

Time = 0.32 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int (a + a \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \frac{4 \left(\frac{5\sqrt{2}a^{3/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{10\sqrt{2}a^{3/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{7\sqrt{2}a^{3/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{2\sqrt{2}a^{3/2} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right) \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^2}{5d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{7/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{7/2} \left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 1 \right)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 4/5*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))

Giac [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int (a + a \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \frac{4a \sqrt{a(\cos(c + dx) + 1)} \sqrt{\frac{1}{\cos(c + dx)}} (8 \sin(c + dx) + 6 \sin(2c + 2dx) + 11 \sin(3c + 3dx) + 3 \sin(4c + 4dx) + 3 \sin(5c + 5dx))}{5d(10 \cos(c + dx) + 8 \cos(2c + 2dx) + 5 \cos(3c + 3dx) + 2 \cos(4c + 4dx) + \cos(5c + 5dx) + 6)}$$

```
[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] (4*a*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(8*sin(c + d*x) +
6*sin(2*c + 2*d*x) + 11*sin(3*c + 3*d*x) + 3*sin(4*c + 4*d*x) + 3*sin(5*c +
5*d*x)))/(5*d*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c + 3*d*x) +
2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

3.348 $\int (a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx$

Optimal result	4284
Rubi [A] (verified)	4284
Mathematica [A] (verified)	4286
Maple [A] (verified)	4286
Fricas [A] (verification not implemented)	4286
Sympy [F(-1)]	4287
Maxima [A] (verification not implemented)	4287
Giac [F(-1)]	4287
Mupad [B] (verification not implemented)	4288

Optimal result

Integrand size = 25, antiderivative size = 81

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \frac{10a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sec^{3/2}(c + dx) \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}}$$

[Out] $2/3*a^2*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+10/3*a^2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4307, 2841, 21, 2850}

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \frac{2a^2 \sin(c + dx) \sec^{3/2}(c + dx)}{3d \sqrt{a \cos(c + dx) + a}} + \frac{10a^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{3d \sqrt{a \cos(c + dx) + a}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}*\text{Sec}[c + d*x]^{(5/2)}, x]$

[Out] $(10*a^2*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (2*a^2*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(3*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 21


```
Int[(u_.)*((a_.) + (b_.)*(v_.))^(m_.)*((c_.) + (d_.)*(v_.))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 2841

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_.)])^(n_.), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_.)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx \\
&= \frac{2a^2 \sec^{3/2}(c + dx) \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} \\
&\quad - \frac{1}{3} \left(2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{-\frac{5a}{2} - \frac{5}{2}a \cos(c + dx)}{\cos^{3/2}(c + dx) \sqrt{a + a \cos(c + dx)}} dx \\
&= \frac{2a^2 \sec^{3/2}(c + dx) \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} + \frac{1}{3} \left(5a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \cos(c + dx)}}{\cos^{3/2}(c + dx)} dx \\
&= \frac{10a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sec^{3/2}(c + dx) \sin(c + dx)}{3d \sqrt{a + a \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \frac{2a\sqrt{a(1 + \cos(c + dx))}(1 + 5 \cos(c + dx)) \sec^{3/2}(c + dx) \tan\left(\frac{1}{2}(c + dx)\right)}{3d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2), x]

[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 5*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Tan[(c + d*x)/2])/(3*d)

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{2 \cot(dx+c) \sqrt{a(1+\cos(dx+c))} (\sec^{5/2}(dx+c)) (5(\cos^2(dx+c)) - 4 \cos(dx+c) - 1) a}{3d}$	53

[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(5/2), x, method=_RETURNVERBOSE)

[Out] -2/3/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(5/2)*(5*cos(d*x+c)^2 - 4*cos(d*x+c) - 1)*a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \frac{2(5a \cos(dx + c) + a) \sqrt{a \cos(dx + c) + a \sin(dx + c)}}{3(d \cos(dx + c)^2 + d \cos(dx + c)) \sqrt{\cos(dx + c)}}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2), x, algorithm="fricas")

[Out] 2/3*(5*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c))^2 + d*cos(d*x + c))*sqrt(cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \frac{4 \left(\frac{3 \sqrt{2} a^{3/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{5 \sqrt{2} a^{3/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{2 \sqrt{2} a^{3/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{3 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{5/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{5/2}}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] 4/3*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2))

Giac [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \frac{2a \sqrt{a(\cos(c + dx) + 1)} \sqrt{\frac{1}{\cos(c + dx)}} (5 \sin(c + dx) + 2 \sin(2c + 2dx) + 5 \sin(3c + 3dx))}{3d(3 \cos(c + dx) + 2 \cos(2c + 2dx) + \cos(3c + 3dx) + 2)}$$

[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2),x)

[Out] (2*a*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(5*sin(c + d*x) + 2*sin(2*c + 2*d*x) + 5*sin(3*c + 3*d*x)))/(3*d*(3*cos(c + d*x) + 2*cos(2*c + 2*d*x) + cos(3*c + 3*d*x) + 2))

3.349 $\int (a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx$

Optimal result	4289
Rubi [A] (verified)	4289
Mathematica [A] (verified)	4291
Maple [A] (verified)	4291
Fricas [A] (verification not implemented)	4292
Sympy [F(-1)]	4292
Maxima [B] (verification not implemented)	4292
Giac [F]	4293
Mupad [F(-1)]	4294

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int (a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \frac{2a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} + \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{d \sqrt{a+a \cos(c+dx)}}$$

[Out] $2*a^{(3/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2*a^2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2841, 21, 2853, 222}

$$\int (a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \frac{2a^{3/2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d} + \frac{2a^2 \sin(c+dx) \sqrt{\sec(c+dx)}}{d \sqrt{a \cos(c+dx) + a}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}*\text{Sec}[c + d*x]^{(3/2)}, x]$

```
[Out] (2*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)])*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{(a + a \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx$$

$$\begin{aligned}
&= \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{-\frac{a}{2} - \frac{1}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx \\
&= \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&= \frac{2a^{3/2} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int (a+a\cos(c+dx))^{3/2} \sec^{\frac{3}{2}}(c+dx) dx = \frac{a\sqrt{a(1+\cos(c+dx))} \sec\left(\frac{1}{2}(c+dx)\right) \sqrt{\sec(c+dx)} \left(\sqrt{2} \arcsin\left(\sqrt{2} \sin\left(\frac{1}{2}(c+dx)\right)\right)\right) \sqrt{\cos(c+dx)}}{d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]

[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/d

Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

method	result
default	$\frac{2\left(\sec^{\frac{3}{2}}(dx+c)\right)\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) + \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)}{d(1+\cos(dx+c))}$

[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] $2/d*\sec(d*x+c)^{(3/2)}*(\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}))+(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}))+\sin(d*x+c))*\cos(d*x+c)*(a*(1+\cos(d*x+c)))^{(1/2)}/(1+\cos(d*x+c))*a$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \frac{2 \left((a \cos(dx + c) + a) \sqrt{a} \arctan \left(\frac{\sqrt{a \cos(dx+c) + a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)} \right) - \frac{\sqrt{a \cos(dx+c) + a} \sin(dx+c)}{\sqrt{\cos(dx+c)}} \right)}{d \cos(dx + c) + d}$$

[In] `integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")`

[Out] $-2*((a*\cos(d*x + c) + a)*\sqrt{a}*\arctan(\sqrt{a*\cos(d*x + c) + a}*\sqrt{\cos(d*x + c)})/(\sqrt{a}*\sin(d*x + c))) - \sqrt{a*\cos(d*x + c) + a}*a*\sin(d*x + c)/\sqrt{\cos(d*x + c)})/(d*\cos(d*x + c) + d)$

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \text{Timed out}$$

[In] `integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(3/2),x)`

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. $2(82) = 164$.

Time = 0.43 (sec) , antiderivative size = 997, normalized size of antiderivative = 10.39

$$\int (a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \text{Too large to display}$$

[In] `integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")`

[Out] $1/2*((a*\arctan2((\cos(2*d*x + 2*c))^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{(1/4)}*(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))), \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))) - \cos(1/2*\arctan2(\sin(2*d$


```

*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*d)

```

Giac [F]

$$\int (a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \int (a \cos(dx + c) + a)^{3/2} \sec(dx + c)^{3/2} dx$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{3/2} (a + a \cos(c + dx))^{3/2} dx$$

```
[In] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2), x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2), x)
```

3.350 $\int (a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx$

Optimal result	4295
Rubi [A] (verified)	4295
Mathematica [A] (verified)	4297
Maple [A] (verified)	4297
Fricas [A] (verification not implemented)	4298
Sympy [F(-1)]	4298
Maxima [B] (verification not implemented)	4298
Giac [F(-1)]	4299
Mupad [F(-1)]	4299

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int (a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \frac{3a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} + \frac{a^2 \sin(c+dx)}{d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}$$

[Out] $a^2 \sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}/\sec(d*x+c)^{(1/2)}+3*a^{(3/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2842, 21, 2853, 222}

$$\int (a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \frac{3a^{3/2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d} + \frac{a^2 \sin(c+dx)}{d \sqrt{\sec(c+dx)} \sqrt{a \cos(c+dx)+a}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}*\text{Sqrt}[\text{Sec}[c + d*x]], x]$

```
[Out] (3*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2853

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)])*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx$$

$$\begin{aligned}
&= \frac{a^2 \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{3a^2}{2} + \frac{3}{2}a^2 \cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx \\
&= \frac{a^2 \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{1}{2} \left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{a^2 \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&= \frac{3a^{3/2} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a^2 \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int (a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \frac{a \sqrt{\cos(c + dx)} \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)} \left(3\sqrt{2} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\right)}{2d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]

[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)

Maple [A] (verified)

Time = 15.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{\left(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 3 \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right) \left(\sqrt{\sec(dx+c)}\sqrt{a(1+\cos(dx+c))} \cos(dx+c)a\right)}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	117

[In] `int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $1/d*(\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}+3*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}))*\sec(d*x+c)^{1/2}*(a*(1+\cos(d*x+c)))^{1/2}*\cos(d*x+c)/(1+\cos(d*x+c))/(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int (a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \frac{\sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)} \sin(dx + c) - 3(a \cos(dx + c) + a) \sqrt{a} \arctan(\frac{\sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)}}{\sqrt{a} \sin(dx + c)})}{d \cos(dx + c) + d}$$

[In] `integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] $(\sqrt{a*\cos(d*x + c) + a}*a*\sqrt{\cos(d*x + c)}*\sin(d*x + c) - 3*(a*\cos(d*x + c) + a)*\sqrt{a}*\arctan(\sqrt{a*\cos(d*x + c) + a}*\sqrt{\cos(d*x + c)}/(\sqrt{a}*\sin(d*x + c))))/(d*\cos(d*x + c) + d)$

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \text{Timed out}$$

[In] `integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(1/2),x)`

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(81) = 162$.

Time = 0.43 (sec) , antiderivative size = 803, normalized size of antiderivative = 8.45

$$\int (a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \text{Too large to display}$$

[In] `integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] $1/4*(2*(a*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(d*x + c) - (a*\cos(d*x + c) - a)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2$

$$\begin{aligned}
 & *c) + 1))) * (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \sqrt{a} + 3 * (a * \arctan2(-(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) * \sin(dx + c) - \cos(dx + c) * \sin(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(dx + c) * \cos(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + \sin(dx + c) * \sin(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) + 1) - a * \arctan2(-(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) * \sin(dx + c) - \cos(dx + c) * \sin(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * (\cos(dx + c) * \cos(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + \sin(dx + c) * \sin(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)))) - 1) - a * \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \sin(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \cos(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 1) + a * \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \sin(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} * \cos(1/2 * \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - 1)) * \sqrt{a}) / d
 \end{aligned}$$

Giac [**F(-1)**]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(dx+c))^(3/2)*sec(dx+c)^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [**F(-1)**]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} (a + a \cos(c + dx))^{3/2} dx$$

[In] int((1/cos(c + dx))^(1/2)*(a + a*cos(c + dx))^(3/2),x)

[Out] int((1/cos(c + dx))^(1/2)*(a + a*cos(c + dx))^(3/2), x)

3.351 $\int \frac{(a+a \cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}} dx$

Optimal result	4300
Rubi [A] (verified)	4300
Mathematica [A] (verified)	4302
Maple [A] (verified)	4303
Fricas [A] (verification not implemented)	4303
Sympy [F]	4303
Maxima [B] (verification not implemented)	4304
Giac [F]	4305
Mupad [F(-1)]	4305

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \frac{7a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{4d} + \frac{a^2 \sin(c + dx)}{2d \sqrt{a + a \cos(c + dx)} \sec^{3/2}(c + dx)} + \frac{7a^2 \sin(c + dx)}{4d \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}}$$

[Out] $1/2*a^2*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(1/2)}+7/4*a^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}/\sec(d*x+c)^{(1/2)}+7/4*a^{(3/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2842, 21, 2849, 2853, 222}

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \frac{7a^{3/2} \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4d} + \frac{a^2 \sin(c + dx)}{2d \sec^{3/2}(c + dx) \sqrt{a \cos(c + dx) + a}} + \frac{7a^2 \sin(c + dx)}{4d \sqrt{\sec(c + dx)} \sqrt{a \cos(c + dx) + a}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}/\text{Sqrt}[\text{Sec}[c + d*x]],x]$

[Out] $(7*a^{(3/2)}*\text{ArcSin}[(\text{Sqrt}[a]*\text{Sin}[c + d*x])/(\text{Sqrt}[a + a*\text{Cos}[c + d*x]])]*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[c + d*x]])/(4*d) + (a^2*\text{Sin}[c + d*x])/(2*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

$\cos[c + d*x]]*\text{Sec}[c + d*x]^{(3/2)} + (7*a^2*\text{Sin}[c + d*x])/(4*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 21

$\text{Int}[(u_.)*((a_.) + (b_.)*(v_))^{(m_.)}*((c_.) + (d_.)*(v_))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b/d)^m, \text{Int}[u*(c + d*v)^{(m + n)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& (!\text{IntegerQ}[n] \parallel \text{SimplerQ}[c + d*x, a + b*x])$

Rule 222

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*(x/\text{Sqrt}[a])]/\text{Rt}[-b, 2], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{GtQ}[a, 0] \&\& \text{NegQ}[b]$

Rule 2842

$\text{Int}[(a_.) + (b_.)*\text{sin}[e_.) + (f_.)*(x_)]^{(m_.)}*((c_.) + (d_.)*\text{sin}[e_.) + (f_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(-b^2)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m - 2)}*((c + d*\text{Sin}[e + f*x])^{(n + 1)}/(d*f*(m + n))), x] + \text{Dist}[1/(d*(m + n)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m - 2)}*(c + d*\text{Sin}[e + f*x])^n*\text{Simp}[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 1] \&\& !\text{LtQ}[n, -1] \&\& (\text{IntegersQ}[2*m, 2*n] \parallel \text{IntegerQ}[m + 1/2] \parallel (\text{IntegerQ}[m] \&\& \text{EqQ}[c, 0]))$

Rule 2849

$\text{Int}[\text{Sqrt}[(a_.) + (b_.)*\text{sin}[e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*\text{sin}[e_.) + (f_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[-2*b*\text{Cos}[e + f*x]*((c + d*\text{Sin}[e + f*x])^n/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])), x] + \text{Dist}[2*n*((b*c + a*d)/(b*(2*n + 1))), \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^{(n - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[n, 0] \&\& \text{IntegerQ}[2*n]$

Rule 2853

$\text{Int}[\text{Sqrt}[(a_.) + (b_.)*\text{sin}[e_.) + (f_.)*(x_)]/\text{Sqrt}[(d_.)*\text{sin}[e_.) + (f_.)*(x_)]], x_Symbol] \rightarrow \text{Dist}[-2/f, \text{Subst}[\text{Int}[1/\text{Sqrt}[1 - x^2/a], x], x, b*(\text{Cos}[e + f*x]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]])], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[d, a/b]$

Rule 4307

$\text{Int}[(\text{csc}[(a_.) + (b_.)*(x_)]*(c_.)^{(m_.)}*(u_.), x_Symbol] \rightarrow \text{Dist}[(c*\text{Csc}[a + b*x])^m*(c*\text{Sin}[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u]/(c*\text{Sin}[a + b*x])^m, x], x]$

] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} (a + a \cos(c+dx))^{3/2} dx \\
 &= \frac{a^2 \sin(c+dx)}{2d \sqrt{a+a \cos(c+dx)} \sec^{3/2}(c+dx)} \\
 &\quad + \frac{1}{2} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)} \left(\frac{7a^2}{2} + \frac{7}{2} a^2 \cos(c+dx) \right)}{\sqrt{a+a \cos(c+dx)}} dx \\
 &= \frac{a^2 \sin(c+dx)}{2d \sqrt{a+a \cos(c+dx)} \sec^{3/2}(c+dx)} \\
 &\quad + \frac{1}{4} \left(7a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)} dx \\
 &= \frac{a^2 \sin(c+dx)}{2d \sqrt{a+a \cos(c+dx)} \sec^{3/2}(c+dx)} + \frac{7a^2 \sin(c+dx)}{4d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{1}{8} \left(7a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
 &= \frac{a^2 \sin(c+dx)}{2d \sqrt{a+a \cos(c+dx)} \sec^{3/2}(c+dx)} + \frac{7a^2 \sin(c+dx)}{4d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(7a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}} \right)}{4d} \\
 &= \frac{7a^{3/2} \arcsin \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{4d} \\
 &\quad + \frac{a^2 \sin(c+dx)}{2d \sqrt{a+a \cos(c+dx)} \sec^{3/2}(c+dx)} + \frac{7a^2 \sin(c+dx)}{4d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \frac{(a + a \cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}} dx = \frac{a \sqrt{\cos(c+dx)} \sqrt{a(1 + \cos(c+dx))} \sec \left(\frac{1}{2}(c+dx) \right) \sqrt{\sec(c+dx)} \left(7\sqrt{2} \arcsin \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}} \right) + 2\sqrt{2} \sin \left(\frac{3}{2}(c+dx) \right) \right)}{8d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]], x]

[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(7*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(6*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(8*d)

Maple [A] (verified)

Time = 14.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(2 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 7 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 7 \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right) \sqrt{a(1+\cos(dx+c))} a}{4d(1+\cos(dx+c)) \sqrt{\sec(dx+c)} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int((a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4/d*(2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \frac{7(a \cos(dx + c) + a) \sqrt{a} \arctan\left(\frac{\sqrt{a \cos(dx+c) + a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) - \frac{(2a \cos(dx+c)^2 + 7a \cos(dx+c)) \sqrt{a \cos(dx+c) + a} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4(d \cos(dx + c) + d)}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -1/4*(7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*a*cos(d*x + c)^2 + 7*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)

Sympy [F]

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a(\cos(c + dx) + 1))^{3/2}}{\sqrt{\sec(c + dx)}} dx$$

[In] integrate((a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)

[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/sqrt(sec(c + d*x)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. 2(116) = 232.

Time = 0.43 (sec) , antiderivative size = 1080, normalized size of antiderivative = 7.71

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 6*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^{3/2}}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + a \cos(c + dx))^{3/2}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)

$$3.352 \quad \int \frac{(a+a \cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4306
Rubi [A] (verified)	4306
Mathematica [A] (verified)	4309
Maple [A] (verified)	4309
Fricas [A] (verification not implemented)	4310
Sympy [F]	4310
Maxima [B] (verification not implemented)	4310
Giac [F]	4312
Mupad [F(-1)]	4312

Optimal result

Integrand size = 25, antiderivative size = 180

$$\int \frac{(a+a \cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{11a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{8d}$$

$$+ \frac{a^2 \sin(c+dx)}{3d \sqrt{a+a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2 \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)}$$

$$+ \frac{11a^2 \sin(c+dx)}{8d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}$$

[Out] $1/3*a^2*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}/(a+a*\cos(d*x+c))^{(1/2)}+11/12*a^2*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(1/2)}+11/8*a^2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}/\sec(d*x+c)^{(1/2)}+11/8*a^{(3/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2842, 21, 2849, 2853, 222}

$$\int \frac{(a+a \cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{11a^{3/2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{8d}$$

$$+ \frac{11a^2 \sin(c+dx)}{12d \sec^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx)+a}}$$

$$+ \frac{a^2 \sin(c+dx)}{3d \sec^{\frac{5}{2}}(c+dx) \sqrt{a \cos(c+dx)+a}} + \frac{11a^2 \sin(c+dx)}{8d \sqrt{\sec(c+dx)} \sqrt{a \cos(c+dx)+a}}$$

[In] Int[(a + a*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]

[Out] (11*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(8*d) + (a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (11*a^2*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (11*a^2*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplifierQ[c + d*x,
a + b*x])

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2842

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2849

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \cos^{\frac{3}{2}}(c+dx) (a + a \cos(c+dx))^{3/2} dx \\
&= \frac{a^2 \sin(c+dx)}{3d \sqrt{a+a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{1}{3} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx) \left(\frac{11a^2}{2} + \frac{11}{2} a^2 \cos(c+dx) \right)}{\sqrt{a+a \cos(c+dx)}} dx \\
&= \frac{a^2 \sin(c+dx)}{3d \sqrt{a+a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{1}{6} \left(11a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \cos^{\frac{3}{2}}(c+dx) \sqrt{a+a \cos(c+dx)} dx \\
&= \frac{a^2 \sin(c+dx)}{3d \sqrt{a+a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2 \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{1}{8} \left(11a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)} dx \\
&= \frac{a^2 \sin(c+dx)}{3d \sqrt{a+a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2 \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{11a^2 \sin(c+dx)}{8d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{1}{16} \left(11a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{a^2 \sin(c+dx)}{3d \sqrt{a+a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2 \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{11a^2 \sin(c+dx)}{8d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(11a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}} \right)}{8d}
\end{aligned}$$

$$\begin{aligned}
&= \frac{11a^{3/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{8d} \\
&\quad + \frac{a^2 \sin(c+dx)}{3d \sqrt{a+a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2 \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{11a^2 \sin(c+dx)}{8d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.70

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{a \sqrt{\cos(c + dx)} \sqrt{a(1 + \cos(c + dx))} \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)} \left(33\sqrt{2} \arcsin\left(\frac{\sqrt{2} \sin\left(\frac{c + dx}{2}\right)}{\sqrt{1 + \cos(c + dx)}}\right) + 2\sqrt{\cos(c + dx)} \left(26\sin\left(\frac{c + dx}{2}\right) + 9\sin\left(\frac{3(c + dx)}{2}\right) + 2\sin\left(\frac{5(c + dx)}{2}\right)\right)\right)}{48d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]

[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(33*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(26*Sin[(c + d*x)/2] + 9*Sin[(3*(c + d*x))/2] + 2*Sin[(5*(c + d*x))/2])))/(48*d)

Maple [A] (verified)

Time = 14.72 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.99

method	result
default	$ \frac{\sqrt{a(1+\cos(dx+c))} \left(8 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 22 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 33 \tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 33 \sec(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \right)}{24d(1+\cos(dx+c)) \sec(dx+c)^{\frac{3}{2}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}} $

[In] int((a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/24/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+22*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*sec(d*x+c)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.68

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{33(a \cos(dx + c) + a)\sqrt{a} \arctan\left(\frac{\sqrt{a \cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) - \frac{(8a \cos(dx+c)^3 + 22a \cos(dx+c)^2 + 33a \cos(dx+c))\sqrt{a \cos(dx+c)}}{\sqrt{\cos(dx+c)}}}{24(d \cos(dx + c) + d)}$$

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(33*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a*cos(d*x + c)^3 + 22*a*cos(d*x + c)^2 + 33*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a(\cos(c + dx) + 1))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(c + dx)} dx$$

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/sec(c + d*x)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1942 vs. 2(150) = 300.

Time = 0.53 (sec) , antiderivative size = 1942, normalized size of antiderivative = 10.79

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sec^{\frac{3}{2}}(c + dx)} dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1))*sin(3*d*x + 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1))*((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
```


c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))/d

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^{3/2}}{\sec(dx + c)^{3/2}} dx$$

[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{3/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(a + a \cos(c + dx))^{3/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

[In] int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2),x)

[Out] int((a + a*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)

3.353 $\int (a + a \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx$

Optimal result	4313
Rubi [A] (verified)	4313
Mathematica [A] (verified)	4316
Maple [A] (verified)	4316
Fricas [A] (verification not implemented)	4316
Sympy [F(-1)]	4317
Maxima [A] (verification not implemented)	4317
Giac [F(-1)]	4317
Mupad [B] (verification not implemented)	4318

Optimal result

Integrand size = 25, antiderivative size = 201

$$\int (a + a \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \frac{1168a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{315d \sqrt{a + a \cos(c + dx)}} + \frac{584a^3 \sec^{3/2}(c + dx) \sin(c + dx)}{315d \sqrt{a + a \cos(c + dx)}} + \frac{146a^3 \sec^{5/2}(c + dx) \sin(c + dx)}{105d \sqrt{a + a \cos(c + dx)}} + \frac{38a^3 \sec^{7/2}(c + dx) \sin(c + dx)}{63d \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sec^{9/2}(c + dx) \sin(c + dx)}{9d}$$

[Out] $584/315*a^3*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+146/105*a^3*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+38/63*a^3*\sec(d*x+c)^{(7/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/9*a^2*\sec(d*x+c)^{(9/2)}*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d+1168/315*a^3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2841, 3059, 2851, 2850}

$$\int (a + a \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \frac{38a^3 \sin(c + dx) \sec^{7/2}(c + dx)}{63d \sqrt{a \cos(c + dx) + a}} + \frac{146a^3 \sin(c + dx) \sec^{5/2}(c + dx)}{105d \sqrt{a \cos(c + dx) + a}} + \frac{584a^3 \sin(c + dx) \sec^{3/2}(c + dx)}{315d \sqrt{a \cos(c + dx) + a}} + \frac{1168a^3 \sin(c + dx) \sqrt{\sec(c + dx)}}{315d \sqrt{a \cos(c + dx) + a}} + \frac{2a^2 \sin(c + dx) \sec^{9/2}(c + dx) \sqrt{a \cos(c + dx) + a}}{9d}$$

[In] Int[(a + a*cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2), x]

[Out] (1168*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*cos[c + d*x]]) + (584*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqrt[a + a*cos[c + d*x]]) + (146*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*cos[c + d*x]]) + (38*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*cos[c + d*x]]) + (2*a^2*Sqrt[a + a*cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)

Rule 2841

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2850

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rule 3059

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -

1]

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{11/2}(c+dx)} dx \\
&= \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{9/2}(c+dx) \sin(c+dx)}{9d} \\
&\quad - \frac{1}{9} \left(2a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\left(-\frac{19a}{2} - \frac{15}{2}a\cos(c+dx)\right) \sqrt{a+a\cos(c+dx)}}{\cos^{9/2}(c+dx)} dx \\
&= \frac{38a^3 \sec^{7/2}(c+dx) \sin(c+dx)}{63d \sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{9/2}(c+dx) \sin(c+dx)}{9d} \\
&\quad + \frac{1}{21} \left(73a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{7/2}(c+dx)} dx \\
&= \frac{146a^3 \sec^{5/2}(c+dx) \sin(c+dx)}{105d \sqrt{a+a\cos(c+dx)}} + \frac{38a^3 \sec^{7/2}(c+dx) \sin(c+dx)}{63d \sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{9/2}(c+dx) \sin(c+dx)}{9d} \\
&\quad + \frac{1}{105} \left(292a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{5/2}(c+dx)} dx \\
&= \frac{584a^3 \sec^{3/2}(c+dx) \sin(c+dx)}{315d \sqrt{a+a\cos(c+dx)}} + \frac{146a^3 \sec^{5/2}(c+dx) \sin(c+dx)}{105d \sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{38a^3 \sec^{7/2}(c+dx) \sin(c+dx)}{63d \sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{9/2}(c+dx) \sin(c+dx)}{9d} \\
&\quad + \frac{1}{315} \left(584a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{3/2}(c+dx)} dx \\
&= \frac{1168a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{315d \sqrt{a+a\cos(c+dx)}} + \frac{584a^3 \sec^{3/2}(c+dx) \sin(c+dx)}{315d \sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{146a^3 \sec^{5/2}(c+dx) \sin(c+dx)}{105d \sqrt{a+a\cos(c+dx)}} + \frac{38a^3 \sec^{7/2}(c+dx) \sin(c+dx)}{63d \sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{9/2}(c+dx) \sin(c+dx)}{9d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.42

$$\int (a + a \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \frac{a^2 \sqrt{a(1 + \cos(c + dx))} (727 + 698 \cos(c + dx) + 803 \cos(2(c + dx)) + 146 \cos(3(c + dx)) + 146 \cos(4(c + dx)))}{315d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2), x]

[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(727 + 698*Cos[c + d*x] + 803*Cos[2*(c + d*x)] + 146*Cos[3*(c + d*x)] + 146*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/2)*Tan[(c + d*x)/2])/(315*d)

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

$$\frac{2 \cot(dx + c) \sqrt{a(1 + \cos(dx + c))} \left(\sec^{11/2}(dx + c) \right) (584(\cos^5(dx + c)) - 292(\cos^4(dx + c)) - 73(\cos^3(dx + c)))}{315d}$$

[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(11/2), x)

[Out] -2/315/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(11/2)*(584*cos(d*x+c)^5-292*cos(d*x+c)^4-73*cos(d*x+c)^3-89*cos(d*x+c)^2-95*cos(d*x+c)-35)*a^2

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53

$$\int (a + a \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \frac{2(584a^2 \cos(dx + c)^4 + 292a^2 \cos(dx + c)^3 + 219a^2 \cos(dx + c)^2 + 130a^2 \cos(dx + c) + 35a^2) \sqrt{\cos(dx + c)}}{315(d \cos(dx + c)^5 + d \cos(dx + c)^4) \sqrt{\cos(dx + c)}}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2), x, algorithm="fricas")

[Out] 2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x + c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(d*x + c)))

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(11/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.44

$$\int (a + a \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \frac{8 \left(\frac{315 \sqrt{2} a^{5/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{945 \sqrt{2} a^{5/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{1449 \sqrt{2} a^{5/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{1287 \sqrt{2} a^{5/2} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{572 \sqrt{2} a^{5/2} \sin(dx+c)^9}{(\cos(dx+c)+1)^9} \right)}{315 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{11/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{11/2} \left(\frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} \right)}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="maxima")

[Out] 8/315*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*sqrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))

Giac [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="giac")

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 18.61 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.52

$$\int (a + a \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \frac{\sqrt{\frac{1}{\frac{e^{-c \cdot 1i - dx \cdot 1i}}{2} + \frac{e^{c \cdot 1i + dx \cdot 1i}}{2}}}}{\left(\frac{192 a^2 e^{\frac{c \cdot 9i}{2} + \frac{dx \cdot 9i}{2}} \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a + a \cos(c + dx)}}{5d} - \frac{16 a^2 e^{\frac{c \cdot 9i}{2} + \frac{dx \cdot 9i}{2}} \sin\left(\frac{3c}{2} + \frac{3dx}{2}\right) \sqrt{a + a \cos(c + dx)}}{3d} \right)}{12 e^{\frac{c \cdot 9i}{2} + \frac{dx \cdot 9i}{2}} \cos\left(\frac{c}{2} + \frac{dx}{2}\right) + 8 e^{\frac{c \cdot 9i}{2} + \frac{dx \cdot 9i}{2}} \cos\left(\frac{3c}{2} + \frac{3dx}{2}\right) + 8 e^{\frac{c \cdot 9i}{2} + \frac{dx \cdot 9i}{2}} \cos\left(\frac{5c}{2} + \frac{5dx}{2}\right) + 2 e^{\frac{c \cdot 9i}{2} + \frac{dx \cdot 9i}{2}} \cos\left(\frac{7c}{2} + \frac{7dx}{2}\right) + 2 e^{\frac{c \cdot 9i}{2} + \frac{dx \cdot 9i}{2}} \cos\left(\frac{9c}{2} + \frac{9dx}{2}\right)}$$

[In] int((1/cos(c + d*x))^(11/2)*(a + a*cos(c + d*x))^(5/2),x)

```
[Out] ((1/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((192*a^2*exp((c
*9i)/2 + (d*x*9i)/2)*sin(c/2 + (d*x)/2)*(a + a*cos(c + d*x))^(1/2))/(5*d) -
(16*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c +
d*x))^(1/2))/(3*d) + (1168*a^2*exp((c*9i)/2 + (d*x*9i)/2)*sin((5*c)/2 + (5
*d*x)/2)*(a + a*cos(c + d*x))^(1/2))/(35*d) + (2336*a^2*exp((c*9i)/2 + (d*x
*9i)/2)*sin((9*c)/2 + (9*d*x)/2)*(a + a*cos(c + d*x))^(1/2))/(315*d)))/(12*
exp((c*9i)/2 + (d*x*9i)/2)*cos(c/2 + (d*x)/2) + 8*exp((c*9i)/2 + (d*x*9i)/2
)*cos((3*c)/2 + (3*d*x)/2) + 8*exp((c*9i)/2 + (d*x*9i)/2)*cos((5*c)/2 + (5*
d*x)/2) + 2*exp((c*9i)/2 + (d*x*9i)/2)*cos((7*c)/2 + (7*d*x)/2) + 2*exp((c*
9i)/2 + (d*x*9i)/2)*cos((9*c)/2 + (9*d*x)/2))
```

3.354 $\int (a + a \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx$

Optimal result	4319
Rubi [A] (verified)	4319
Mathematica [A] (verified)	4321
Maple [A] (verified)	4322
Fricas [A] (verification not implemented)	4322
Sympy [F(-1)]	4322
Maxima [A] (verification not implemented)	4323
Giac [F(-1)]	4323
Mupad [B] (verification not implemented)	4323

Optimal result

Integrand size = 25, antiderivative size = 161

$$\int (a + a \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \frac{92a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{21d \sqrt{a + a \cos(c + dx)}} + \frac{46a^3 \sec^{3/2}(c + dx) \sin(c + dx)}{21d \sqrt{a + a \cos(c + dx)}} + \frac{6a^3 \sec^{5/2}(c + dx) \sin(c + dx)}{7d \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sec^{7/2}(c + dx) \sin(c + dx)}{7d}$$

[Out] 46/21*a^3*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+6/7*a^3*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)+2/7*a^2*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d+92/21*a^3*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2841, 3059, 2851, 2850}

$$\int (a + a \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \frac{6a^3 \sin(c + dx) \sec^{5/2}(c + dx)}{7d \sqrt{a \cos(c + dx) + a}} + \frac{46a^3 \sin(c + dx) \sec^{3/2}(c + dx)}{21d \sqrt{a \cos(c + dx) + a}} + \frac{92a^3 \sin(c + dx) \sqrt{\sec(c + dx)}}{21d \sqrt{a \cos(c + dx) + a}} + \frac{2a^2 \sin(c + dx) \sec^{7/2}(c + dx) \sqrt{a \cos(c + dx) + a}}{7d}$$

[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]

[Out] (92*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) + (46*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)

Rule 2841

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2850

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2851

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rule 3059

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{9/2}(c+dx)} dx \\
 &= \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d} \\
 &\quad - \frac{1}{7} \left(2a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\left(-\frac{15a}{2} - \frac{11}{2}a\cos(c+dx)\right) \sqrt{a+a\cos(c+dx)}}{\cos^{7/2}(c+dx)} dx \\
 &= \frac{6a^3 \sec^{5/2}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d} \\
 &\quad + \frac{1}{7} \left(23a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{5/2}(c+dx)} dx \\
 &= \frac{46a^3 \sec^{3/2}(c+dx) \sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{6a^3 \sec^{5/2}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d} \\
 &\quad + \frac{1}{21} \left(46a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{3/2}(c+dx)} dx \\
 &= \frac{92a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{46a^3 \sec^{3/2}(c+dx) \sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{6a^3 \sec^{5/2}(c+dx) \sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 5.52 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46

$$\int (a+a\cos(c+dx))^{5/2} \sec^{9/2}(c+dx) dx = \frac{a^2 \sqrt{a(1+\cos(c+dx))} (29 + 93\cos(c+dx) + 23\cos(2(c+dx)) + 23\cos(3(c+dx))) \sec^{7/2}(c+dx)}{21d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]

[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(29 + 93*Cos[c + d*x] + 23*Cos[2*(c + d*x)] + 23*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(21*d)

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.47

$$\frac{2 \cot(dx + c) \sqrt{a(1 + \cos(dx + c))} \left(\sec^{\frac{9}{2}}(dx + c) \right) (46 \cos^4(dx + c) - 23 \cos^3(dx + c) - 11 \cos^2(dx + c) + dx)}{21d}$$

[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(9/2),x)

[Out] -2/21/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(9/2)*(46*cos(d*x+c)^4-23*cos(d*x+c)^3-11*cos(d*x+c)^2-9*cos(d*x+c)-3)*a^2

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int (a + a \cos(c + dx))^{5/2} \sec^{\frac{9}{2}}(c + dx) dx = \frac{2 (46 a^2 \cos(dx + c)^3 + 23 a^2 \cos(dx + c)^2 + 12 a^2 \cos(dx + c) + 3 a^2) \sqrt{a \cos(dx + c) + a \sin(dx + c)}}{21 (d \cos(dx + c)^4 + d \cos(dx + c)^3) \sqrt{\cos(dx + c)}}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] 2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c) + 3*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos(d*x + c)))

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{\frac{9}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.51

$$\int (a + a \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \frac{8 \left(\frac{21 \sqrt{2} a^{5/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{56 \sqrt{2} a^{5/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{63 \sqrt{2} a^{5/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{36 \sqrt{2} a^{5/2} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{8 \sqrt{2} a^{5/2} \sin(dx+c)^9}{(\cos(dx+c)+1)^9} \right) \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{9/2} \left(\frac{2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 1 \right)}{21 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{9/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{9/2} \left(\frac{2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 1 \right)}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")

```
[Out] 8/21*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2))*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

Giac [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 18.31 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.41

$$\int (a + a \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \frac{-\frac{35 a^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a + a \cos(c + dx)} \sqrt{\frac{2 e^{c 1 i + dx 1 i}}{e^{c 2 i + dx 2 i} + 1}}}{2} + 35 a^2 \sin\left(\frac{3c}{2} + \frac{3dx}{2}\right) \sqrt{a + a \cos(c + dx)} \sqrt{\frac{2 e^{c 1 i + dx 1 i}}{e^{c 2 i + dx 2 i} + 1}}}{\frac{63 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)}{8} + \frac{63 d \cos\left(\frac{3c}{2} + \frac{3dx}{2}\right)}{8} + \frac{21 d \cos\left(\frac{5c}{2} + \frac{5dx}{2}\right)}{8} + \frac{21 d \cos\left(\frac{7c}{2} + \frac{7dx}{2}\right)}{8} + \frac{8 d \cos\left(\frac{9c}{2} + \frac{9dx}{2}\right)}{8}}$$

[In] int((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(5/2),x)

```
[Out] (35*a^2*sin((3*c)/2 + (3*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp(c*1i +
d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2) - (35*a^2*sin(c/2 + (d*x)/2)*(a +
a*cos(c + d*x))^(1/2)*((2*exp(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/
2))/2 + (23*a^2*sin((7*c)/2 + (7*d*x)/2)*(a + a*cos(c + d*x))^(1/2)*((2*exp
(c*1i + d*x*1i))/(exp(c*2i + d*x*2i) + 1))^(1/2))/2)/((63*d*cos(c/2 + (d*x)
/2))/8 + (63*d*cos((3*c)/2 + (3*d*x)/2))/8 + (21*d*cos((5*c)/2 + (5*d*x)/2)
)/8 + (21*d*cos((7*c)/2 + (7*d*x)/2))/8)
```


3.355 $\int (a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx$

Optimal result	4325
Rubi [A] (verified)	4325
Mathematica [A] (verified)	4327
Maple [A] (verified)	4327
Fricas [A] (verification not implemented)	4328
Sympy [F(-1)]	4328
Maxima [A] (verification not implemented)	4328
Giac [F(-1)]	4329
Mupad [B] (verification not implemented)	4329

Optimal result

Integrand size = 25, antiderivative size = 121

$$\int (a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \frac{86a^3 \sqrt{\sec(c + dx)} \sin(c + dx)}{15d \sqrt{a + a \cos(c + dx)}} + \frac{22a^3 \sec^{3/2}(c + dx) \sin(c + dx)}{15d \sqrt{a + a \cos(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sec^{5/2}(c + dx) \sin(c + dx)}{5d}$$

[Out] $22/15*a^3*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/5*a^2*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d+86/15*a^3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4307, 2841, 3059, 2850}

$$\int (a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \frac{22a^3 \sin(c + dx) \sec^{3/2}(c + dx)}{15d \sqrt{a \cos(c + dx) + a}} + \frac{86a^3 \sin(c + dx) \sqrt{\sec(c + dx)}}{15d \sqrt{a \cos(c + dx) + a}} + \frac{2a^2 \sin(c + dx) \sec^{5/2}(c + dx) \sqrt{a \cos(c + dx) + a}}{5d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(5/2)}*\text{Sec}[c + d*x]^{(7/2)}, x]$

[Out] $(86*a^3*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/((15*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (22*a^3*\text{Sec}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/((15*d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]) + (2*a^2*\text{Sqrt}[a + a*\text{Cos}[c + d*x]]*\text{Sec}[c + d*x]^{(5/2)}*\text{Sin}[c + d*x]))/(5*d)$

Rule 2841

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegerQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2850

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(3/2), x_Symbol] := Simp[-2*b^2*(Cos[e + f*x]/(f*(b*c + a*d)*Sq
rt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx \\ &= \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sec^{5/2}(c + dx) \sin(c + dx)}{5d} \\ &\quad - \frac{1}{5} \left(2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\left(-\frac{11a}{2} - \frac{7}{2} a \cos(c + dx) \right) \sqrt{a + a \cos(c + dx)}}{\cos^{5/2}(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{22a^3 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{1}{15} \left(43a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{86a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{22a^3 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

$$\int (a+a\cos(c+dx))^{5/2} \sec^{7/2}(c+dx) dx = \frac{a^2 \sqrt{a(1+\cos(c+dx))} (49+28\cos(c+dx)+43\cos(2(c+dx))) \sec^{5/2}(c+dx) \tan\left(\frac{1}{2}(c+dx)\right)}{15d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2), x]

[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(49 + 28*Cos[c + d*x] + 43*Cos[2*(c + d*x)])*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.54

$$\frac{2 \cot(dx+c) \sqrt{a(1+\cos(dx+c))} \left(\sec^{7/2}(dx+c) \right) (43(\cos^3(dx+c)) - 29(\cos^2(dx+c)) - 11 \cos(dx+c) - 3)}{15d}$$

[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(7/2), x)

[Out] -2/15/d*cot(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)*sec(d*x+c)^(7/2)*(43*cos(d*x+c)^3-29*cos(d*x+c)^2-11*cos(d*x+c)-3)*a^2

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.67

$$\int (a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \frac{2(43a^2 \cos(dx + c)^2 + 14a^2 \cos(dx + c) + 3a^2) \sqrt{a \cos(dx + c) + a \sin(dx + c)}}{15(d \cos(dx + c)^3 + d \cos(dx + c)^2) \sqrt{\cos(dx + c)}}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x + c)))

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.25

$$\int (a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \frac{8 \left(\frac{15\sqrt{2}a^{5/2} \sin(dx+c)}{\cos(dx+c)+1} - \frac{35\sqrt{2}a^{5/2} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{28\sqrt{2}a^{5/2} \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{8\sqrt{2}a^{5/2} \sin(dx+c)^7}{(\cos(dx+c)+1)^7} \right)}{15 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{7/2} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{7/2}}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 8/15*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2))

Giac [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13

$$\int (a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \frac{2 a^2 \sqrt{a (\cos(c + dx) + 1)} \sqrt{\frac{1}{\cos(c + dx)}} (98 \sin(c + dx) + 56 \sin(2c + 2dx) + 141 \sin(3c + 3dx) + 28 \sin(4c + 4dx) + 43 \sin(5c + 5dx))}{15 d (10 \cos(c + dx) + 8 \cos(2c + 2dx) + 5 \cos(3c + 3dx) + 2 \cos(4c + 4dx) + \cos(5c + 5dx) + 6)}$$

```
[In] int((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] (2*a^2*(a*(cos(c + d*x) + 1))^(1/2)*(1/cos(c + d*x))^(1/2)*(98*sin(c + d*x)
+ 56*sin(2*c + 2*d*x) + 141*sin(3*c + 3*d*x) + 28*sin(4*c + 4*d*x) + 43*si
n(5*c + 5*d*x)))/(15*d*(10*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 5*cos(3*c +
3*d*x) + 2*cos(4*c + 4*d*x) + cos(5*c + 5*d*x) + 6))
```

3.356 $\int (a + a \cos(c + dx))^{5/2} \sec^{\frac{5}{2}}(c + dx) dx$

Optimal result	4330
Rubi [A] (verified)	4330
Mathematica [C] (warning: unable to verify)	4332
Maple [A] (verified)	4333
Fricas [A] (verification not implemented)	4333
Sympy [F(-1)]	4334
Maxima [B] (verification not implemented)	4334
Giac [F(-1)]	4335
Mupad [F(-1)]	4335

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int (a + a \cos(c + dx))^{5/2} \sec^{\frac{5}{2}}(c + dx) dx = \frac{2a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} + \frac{14a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{3d \sqrt{a+a \cos(c+dx)}} + \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d}$$

[Out] $2/3*a^2*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d+2*a^{(5/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+14/3*a^3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2841, 3059, 2853, 222}

$$\int (a + a \cos(c + dx))^{5/2} \sec^{\frac{5}{2}}(c + dx) dx = \frac{2a^{5/2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{d} + \frac{14a^3 \sin(c+dx) \sqrt{\sec(c+dx)}}{3d \sqrt{a \cos(c+dx)+a}} + \frac{2a^2 \sin(c+dx) \sec^{\frac{3}{2}}(c+dx) \sqrt{a \cos(c+dx)+a}}{3d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(5/2)}*\text{Sec}[c + d*x]^{(5/2)}, x]$

```
[Out] (2*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (14*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2841

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2853

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)
*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])], x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{5/2}(c+dx)} dx \\
&= \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{3d} \\
&\quad - \frac{1}{3} \left(2a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\left(-\frac{7a}{2} - \frac{3}{2}a\cos(c+dx)\right) \sqrt{a+a\cos(c+dx)}}{\cos^{3/2}(c+dx)} dx \\
&= \frac{14a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{3d \sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{3d} \\
&\quad + \left(a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{14a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{3d \sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{3d} \\
&\quad - \frac{\left(2a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right)}{d} \\
&= \frac{2a^{5/2} \arcsin \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{14a^3 \sqrt{\sec(c+dx)} \sin(c+dx)}{3d \sqrt{a+a\cos(c+dx)}} + \frac{2a^2 \sqrt{a+a\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.19 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.93

$$\int (a+a\cos(c+dx))^{5/2} \sec^{5/2}(c+dx) dx = \frac{(a(1+\cos(c+dx)))^{5/2} \csc^3\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^5\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}} \sqrt{1-2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)} \left(256 \cos^4\left(\frac{c}{2} + \frac{dx}{2}\right) \right)}{d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]

[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Csc[c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^5*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(256*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[c/2 + (

$d*x)/2]^2]*\text{Sin}[c/2 + (d*x)/2]^6 + 512*\text{Hypergeometric2F1}[3/2, 7/2, 9/2, 2*\text{Sin}[c/2 + (d*x)/2]^2]*\text{Sin}[c/2 + (d*x)/2]^6*(2 - 3*\text{Sin}[c/2 + (d*x)/2]^2 + \text{Sin}[c/2 + (d*x)/2]^4) + (21*\text{Sqrt}[2]*\text{ArcSin}[\text{Sqrt}[2]*\text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2]]*(15 - 10*\text{Sin}[c/2 + (d*x)/2]^2 + 3*\text{Sin}[c/2 + (d*x)/2]^4))/\text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2] - 14*\text{Sqrt}[1 - 2*\text{Sin}[c/2 + (d*x)/2]^2]*(45 + 30*\text{Sin}[c/2 + (d*x)/2]^2 - 31*\text{Sin}[c/2 + (d*x)/2]^4 + 12*\text{Sin}[c/2 + (d*x)/2]^6)))/(672*d)$

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$2\left(\sec^{\frac{5}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\left(3\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cos^3(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)$$

[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(5/2),x)

[Out] 2/3/d*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))*(3*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+8*cos(d*x+c)^2*sin(d*x+c)+cos(d*x+c)*sin(d*x+c))*a^2

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int (a + a \cos(c + dx))^{5/2} \sec^{\frac{5}{2}}(c + dx) dx = \frac{2\left(3\left(a^2 \cos(dx+c)^2 + a^2 \cos(dx+c)\right)\sqrt{a} \arctan\left(\frac{\sqrt{a \cos(dx+c)+a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) - \frac{(8a^2 \cos(dx+c)+a^2)\sqrt{a \cos(dx+c)}}{\sqrt{\cos(dx+c)}}\right)}{3(d \cos(dx+c))^2 + d \cos(dx+c)}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] -2/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a^2*cos(d*x + c) + a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. 2(116) = 232.

Time = 0.44 (sec) , antiderivative size = 1395, normalized size of antiderivative = 10.11

$$\int (a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx) dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(2*d*x + 2*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
```

$$\begin{aligned}
& (2dx + 2c) + 1)^{1/4} (\cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))) - 1) - (a^2 \cos(2dx + 2c)^2 + a^2 \sin(2dx + 2c)^2 + 2a^2 \cos(2dx + 2c) + a^2) \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 1) + (a^2 \cos(2dx + 2c)^2 + a^2 \sin(2dx + 2c)^2 + 2a^2 \cos(2dx + 2c) + a^2) \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - 1)) \sqrt{a} / ((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1) d)
\end{aligned}$$

Giac [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{5/2} (a + a \cos(c + dx))^{5/2} dx$$

[In] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2),x)

[Out] int((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2), x)

3.357 $\int (a + a \cos(c + dx))^{5/2} \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	4336
Rubi [A] (verified)	4337
Mathematica [C] (warning: unable to verify)	4339
Maple [A] (verified)	4339
Fricas [A] (verification not implemented)	4340
Sympy [F(-1)]	4340
Maxima [B] (verification not implemented)	4340
Giac [F(-1)]	4341
Mupad [F(-1)]	4341

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int (a + a \cos(c + dx))^{5/2} \sec^{\frac{3}{2}}(c + dx) dx = \frac{5a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} - \frac{a^3 \sin(c+dx)}{d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}} + \frac{2a^2 \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d}$$

```
[Out] -a^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+5*a^(5/2)*arcsin(
sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d+2*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00,
 number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used
 = {4307, 2841, 3060, 2853, 222}

$$\int (a + a \cos(c + dx))^{5/2} \sec^{\frac{3}{2}}(c + dx) dx = \frac{5a^{5/2} \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right) - \frac{a^3 \sin(c + dx)}{d \sqrt{\sec(c + dx)} \sqrt{a \cos(c + dx) + a}} + \frac{2a^2 \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a \cos(c + dx) + a}}{d}}$$

[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]

[Out] (5*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/d - (a^3*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2841

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 3060

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x]
/; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{(a + a \cos(c + dx))^{5/2}}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{d} \\
&\quad - \left(2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\left(-\frac{3a}{2} + \frac{1}{2}a \cos(c + dx) \right) \sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\
&= -\frac{a^3 \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{d} \\
&\quad + \frac{1}{2} \left(5a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + a \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\
&= -\frac{a^3 \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}} + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{d} \\
&\quad - \frac{\left(5a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{x^2}{a}}} dx, x, -\frac{a \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}} \right)}{d} \\
&= \frac{5a^{5/2} \arcsin \left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a + a \cos(c + dx)}} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{d} \\
&\quad - \frac{a^3 \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2a^2 \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.97 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.51

$$\int (a + a \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \frac{\sqrt{\cos(c + dx)}(a(1 + \cos(c + dx)))^{5/2} \sec^4\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)}(7(89 + 28 \cos(c + dx)) + 3 \cos(c + dx))}{d(1 + \cos(c + dx))}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]

[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(7*(89 + 28*Cos[c + d*x]) + 3*Cos[2*(c + d*x)])*Hypergeometric2F1[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 24*(3 + Cos[c + d*x])*Hypergeometric2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 + 6*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{3/2, 2, 5/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4*Tan[(c + d*x)/2])/(420*d)

Maple [A] (verified)

Time = 16.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.25

method	result
default	$\frac{(\sec^{\frac{3}{2}}(dx+c)) \left(5 \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) + \cos(dx+c) \sin(dx+c) + 5 \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) \right)}{d(1+\cos(dx+c))}$

[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/d*sec(d*x+c)^(3/2)*(5*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+cos(d*x+c)*sin(d*x+c)+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2*sin(d*x+c)*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))*a^2

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int (a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx =$$

$$\frac{5(a^2 \cos(dx + c) + a^2) \sqrt{a} \arctan\left(\frac{\sqrt{a \cos(dx+c) + a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) - \frac{(a^2 \cos(dx+c) + 2a^2) \sqrt{a \cos(dx+c) + a} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d \cos(dx + c) + d}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -(5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) - (a^2*cos(d*x + c) + 2*a^2)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 973 vs. 2(116) = 232.

Time = 0.43 (sec) , antiderivative size = 973, normalized size of antiderivative = 7.26

$$\int (a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*
x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c
```


), $\cos(2dx + 2c) + 1$) + $\sin(dx + c) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 1 - a^2 \arctan2(-(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} (\cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \sin(dx + c) - \cos(dx + c) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} (\cos(dx + c) \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + \sin(dx + c) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) - 1 - a^2 \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) + 1 + a^2 \arctan2((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))), (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) - 1)) (\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} \sqrt{a + 8(a^2 \cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)) \sin(dx + c) - (a^2 \cos(dx + c) - a^2) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1))) \sqrt{a}} / ((\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{1/4} d)$

Giac **[F(-1)]**

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx = \text{Timed out}$$

[In] `integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="giac")`

[Out] Timed out

Mupad **[F(-1)]**

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{3/2} (a + a \cos(c + dx))^{5/2} dx$$

[In] `int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2),x)`

[Out] `int((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2), x)`

3.358 $\int (a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx$

Optimal result	4342
Rubi [A] (verified)	4342
Mathematica [C] (warning: unable to verify)	4344
Maple [A] (verified)	4345
Fricas [A] (verification not implemented)	4345
Sympy [F(-1)]	4345
Maxima [B] (verification not implemented)	4346
Giac [F(-1)]	4347
Mupad [F(-1)]	4347

Optimal result

Integrand size = 25, antiderivative size = 140

$$\int (a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \frac{19a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{4d} + \frac{9a^3 \sin(c+dx)}{4d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}} + \frac{a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{2d \sqrt{\sec(c+dx)}}$$

[Out] $9/4*a^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^(1/2)/\sec(d*x+c)^(1/2)+1/2*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^(1/2)/d/\sec(d*x+c)^(1/2)+19/4*a^(5/2)*\arcsin(\sin(d*x+c)*a^(1/2)/(a+a*\cos(d*x+c))^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d$

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2842, 3060, 2853, 222}

$$\int (a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \frac{19a^{5/2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{4d} + \frac{9a^3 \sin(c+dx)}{4d \sqrt{\sec(c+dx)} \sqrt{a \cos(c+dx)+a}} + \frac{a^2 \sin(c+dx) \sqrt{a \cos(c+dx)+a}}{2d \sqrt{\sec(c+dx)}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^(5/2)*\text{Sqrt}[\text{Sec}[c + d*x]], x]$

```
[Out] (19*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (9*a^3*SIN[c + d*x])/(4*d*Sqrt[a +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
d*x])/(2*d*Sqrt[Sec[c + d*x]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2842

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*SIN[e + f*x
])^(m - 2)*((c + d*SIN[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*SIN[e + f*x])^(m - 2)*(c + d*SIN[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*SIN[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2853

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)
*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*SIN[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[-2*b*B*Cos[e + f*x]*((c + d*SIN[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*SIN[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*SIN[e + f*x]]*(c + d*SIN[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*SIN[a + b*x])^m, Int[ActivateTrig[u]/(c*SIN[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{a^2 \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{2d\sqrt{\sec(c+dx)}} \\
&\quad + \frac{1}{2} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)} \left(\frac{5a^2}{2} + \frac{9}{2} a^2 \cos(c+dx) \right)}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{9a^3 \sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2 \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{2d\sqrt{\sec(c+dx)}} \\
&\quad + \frac{1}{8} \left(19a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{9a^3 \sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2 \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{2d\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(19a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right)}{4d} \\
&= \frac{19a^{5/2} \arcsin \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{4d} \\
&\quad + \frac{9a^3 \sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2 \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{2d\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.95 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.44

$$\int (a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)} dx = \frac{\sqrt{\cos(c+dx)} (a(1+\cos(c+dx)))^{5/2} \sec^4\left(\frac{1}{2}(c+dx)\right) \sqrt{\sec(c+dx)} (7(89+28\cos(c+dx))^{3/2} + 14(89+28\cos(c+dx))^{1/2} \sin(c+dx) + 7\sin^2(c+dx))}{420d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]

[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(7*(89 + 28*Cos[c + d*x]) + 3*Cos[2*(c + d*x)])*Hypergeometric2F1[1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 8*(3 + Cos[c + d*x])*Hypergeometric2F1[3/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 + 2*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{3/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)

Maple [A] (verified)

Time = 16.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.10

method	result
default	$\frac{\left(2 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 11 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 19 \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right) (\sqrt{\sec(dx+c)}) \sqrt{a(1+\cos(dx+c))}}{4d(1+\cos(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] $\frac{1}{4}d*(2*\sin(d*x+c)*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}+11*\sin(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}+19*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}))*\sec(d*x+c)^{1/2}*(a*(1+\cos(d*x+c)))^{1/2}*\cos(d*x+c)/(1+\cos(d*x+c))/(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int (a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \frac{19(a^2 \cos(dx + c) + a^2) \sqrt{a} \arctan\left(\frac{\sqrt{a \cos(dx+c) + a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) - \frac{(2a^2 \cos(dx+c)^2 + 11a^2 \cos(dx+c)) \sqrt{a \cos(dx+c) + a} \sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4(d \cos(dx + c) + d)}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] $-1/4*(19*(a^2*\cos(d*x + c) + a^2)*\sqrt{a}*\arctan(\sqrt{a*\cos(d*x + c) + a}*\sqrt{\cos(d*x + c)})/(\sqrt{a}*\sin(d*x + c))) - (2*a^2*\cos(d*x + c)^2 + 11*a^2*\cos(d*x + c))*\sqrt{a*\cos(d*x + c) + a}*\sin(d*x + c)/\sqrt{\cos(d*x + c)}}/(d*\cos(d*x + c) + d)$

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(1/2),x)

[Out] Timed out

Giac [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} (a + a \cos(c + dx))^{5/2} dx$$

```
[In] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(5/2), x)
```

$$3.359 \quad \int \frac{(a+a \cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}} dx$$

Optimal result	4348
Rubi [A] (verified)	4348
Mathematica [C] (warning: unable to verify)	4351
Maple [A] (verified)	4351
Fricas [A] (verification not implemented)	4352
Sympy [F(-1)]	4352
Maxima [B] (verification not implemented)	4352
Giac [F]	4354
Mupad [F(-1)]	4354

Optimal result

Integrand size = 25, antiderivative size = 180

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}} dx = \frac{25a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{8d}$$

$$+ \frac{13a^3 \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)} \sec^{3/2}(c+dx)}$$

$$+ \frac{a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{3d \sec^{3/2}(c+dx)} + \frac{25a^3 \sin(c+dx)}{8d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}$$

[Out] 13/12*a^3*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2)+1/3*a^2*sin(d*x+c)*(a+a*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+25/8*a^3*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+25/8*a^(5/2)*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2842, 3060, 2849, 2853, 222}

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}} dx = \frac{25a^{5/2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{8d}$$

$$+ \frac{13a^3 \sin(c+dx)}{12d \sec^{3/2}(c+dx) \sqrt{a \cos(c+dx)+a}}$$

$$+ \frac{25a^3 \sin(c+dx)}{8d \sqrt{\sec(c+dx)} \sqrt{a \cos(c+dx)+a}} + \frac{a^2 \sin(c+dx) \sqrt{a \cos(c+dx)+a}}{3d \sec^{3/2}(c+dx)}$$

[In] Int[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]

[Out] (25*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(8*d) + (13*a^3*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (25*a^3*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2842

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2849

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 3060

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1))]/(b

```
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^m*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} (a + a \cos(c+dx))^{5/2} dx \\
&= \frac{a^2 \sqrt{a + a \cos(c+dx)} \sin(c+dx)}{3d \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{1}{3} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} \sqrt{a + a \cos(c+dx)} \left(\frac{9a^2}{2} \right. \\
&\quad \quad \quad \left. + \frac{13}{2} a^2 \cos(c+dx) \right) dx \\
&= \frac{13a^3 \sin(c+dx)}{12d \sqrt{a + a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} + \frac{a^2 \sqrt{a + a \cos(c+dx)} \sin(c+dx)}{3d \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{1}{8} \left(25a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} \sqrt{a + a \cos(c+dx)} dx \\
&= \frac{13a^3 \sin(c+dx)}{12d \sqrt{a + a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} + \frac{a^2 \sqrt{a + a \cos(c+dx)} \sin(c+dx)}{3d \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{25a^3 \sin(c+dx)}{8d \sqrt{a + a \cos(c+dx)} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{1}{16} \left(25a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a + a \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{13a^3 \sin(c+dx)}{12d \sqrt{a + a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} + \frac{a^2 \sqrt{a + a \cos(c+dx)} \sin(c+dx)}{3d \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{25a^3 \sin(c+dx)}{8d \sqrt{a + a \cos(c+dx)} \sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(25a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}} \right)}{8d}
\end{aligned}$$

$$= \frac{25a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{8d} + \frac{13a^3 \sin(c+dx)}{12d \sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} + \frac{a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{3d \sec^{\frac{3}{2}}(c+dx)} + \frac{25a^3 \sin(c+dx)}{8d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.00 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.12

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \frac{\sqrt{\cos(c + dx)}(a(1 + \cos(c + dx)))^{5/2} \sec^4\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)}(7(89 + \dots))}{\dots}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]

[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F1[-1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] - 8*(3 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 - 2*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{1/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)

Maple [A] (verified)

Time = 14.78 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02

method	result
default	$\frac{(8 \sin(dx+c) (\cos^2(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 34 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 75 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 75 \arctan(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})) \sqrt{\sec(dx+c)} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{24d(1+\cos(dx+c)) \sqrt{\sec(dx+c)} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int((a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/24/d*(8*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+34*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+75*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+75*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.74

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx =$$

$$\frac{75 (a^2 \cos(dx + c) + a^2) \sqrt{a} \arctan\left(\frac{\sqrt{a \cos(dx+c)} + a \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) - \frac{(8 a^2 \cos(dx+c)^3 + 34 a^2 \cos(dx+c)^2 + 75 a^2 \cos(dx+c)) \sqrt{a \cos(dx+c)}}{\sqrt{\cos(dx+c)}}}{24 (d \cos(dx + c) + d)}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(75*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a^2*cos(d*x + c)^3 + 34*a^2*cos(d*x + c)^2 + 75*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. 2(150) = 300.

Time = 0.54 (sec) , antiderivative size = 1964, normalized size of antiderivative = 10.91

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1))*sin(3*d*x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 1))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))
```


$$\begin{aligned}
& + 3*c))^{1/2} + \sin(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^{1/2} + 2*\cos \\
& (2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)^{1/4}*\cos(1/2*\arctan \\
& 2(\sin(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))), \cos(2/3*\arctan2(\sin \\
& (3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)) + 1) + a^2*\arctan2((\cos(2/3*\arctan2 \\
& (\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^{1/2} + \sin(2/3*\arctan2(\sin(3*d*x + 3*c), \\
& \cos(3*d*x + 3*c)))^{1/2} + 2*\cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c \\
&))) + 1)^{1/4}*\sin(1/2*\arctan2(\sin(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x \\
& + 3*c))), \cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + 1)), (\cos(\\
& 2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c)))^{1/2} + \sin(2/3*\arctan2(\sin(3* \\
& d*x + 3*c), \cos(3*d*x + 3*c)))^{1/2} + 2*\cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(\\
& 3*d*x + 3*c))) + 1)^{1/4}*\cos(1/2*\arctan2(\sin(2/3*\arctan2(\sin(3*d*x + 3*c), \\
& \cos(3*d*x + 3*c))), \cos(2/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))) + \\
& 1)) - 1))*\sqrt{a})/d
\end{aligned}$$

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a \cos(dx + c) + a)^{5/2}}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + a \cos(c + dx))^{5/2}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)

$$3.360 \quad \int \frac{(a+a \cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4355
Rubi [A] (verified)	4355
Mathematica [C] (warning: unable to verify)	4358
Maple [A] (verified)	4359
Fricas [A] (verification not implemented)	4359
Sympy [F(-1)]	4359
Maxima [B] (verification not implemented)	4360
Giac [F]	4365
Mupad [F(-1)]	4365

Optimal result

Integrand size = 25, antiderivative size = 220

$$\int \frac{(a+a \cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{163a^{5/2} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64d}$$

$$+ \frac{17a^3 \sin(c+dx)}{24d \sqrt{a+a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{a^2 \sqrt{a+a \cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{163a^3 \sin(c+dx)}{96d \sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} + \frac{163a^3 \sin(c+dx)}{64d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}$$

[Out] $17/24*a^3*\sin(d*x+c)/d/\sec(d*x+c)^{(5/2)}/(a+a*\cos(d*x+c))^{(1/2)}+163/96*a^3*\sin(d*x+c)/d/\sec(d*x+c)^{(3/2)}/(a+a*\cos(d*x+c))^{(1/2)}+1/4*a^2*\sin(d*x+c)*(a+a*\cos(d*x+c))^{(1/2)}/d/\sec(d*x+c)^{(5/2)}+163/64*a^3*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}/\sec(d*x+c)^{(1/2)}+163/64*a^{(5/2)}*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

= {4307, 2842, 3060, 2849, 2853, 222}

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sec^3(c + dx)} dx = \frac{163a^{5/2} \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{64d}$$

$$+ \frac{163a^3 \sin(c + dx)}{96d \sec^3(c + dx) \sqrt{a \cos(c + dx) + a}} + \frac{17a^3 \sin(c + dx)}{24d \sec^5(c + dx) \sqrt{a \cos(c + dx) + a}}$$

$$+ \frac{163a^3 \sin(c + dx)}{64d \sqrt{\sec(c + dx)} \sqrt{a \cos(c + dx) + a}} + \frac{a^2 \sin(c + dx) \sqrt{a \cos(c + dx) + a}}{4d \sec^5(c + dx)}$$

[In] Int[(a + a*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2), x]

[Out] (163*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(64*d) + (17*a^3*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (163*a^3*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (163*a^3*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2842

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2849

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[2*n*((b*c + a*d)/(b*(2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]

Rule 2853


```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*
(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos
[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]))], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \cos^{\frac{3}{2}}(c+dx) (a + a \cos(c+dx))^{5/2} dx \\
&= \frac{a^2 \sqrt{a + a \cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} + \frac{1}{4} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \cos^{\frac{3}{2}}(c \\
&\quad + dx) \sqrt{a + a \cos(c+dx)} \left(\frac{13a^2}{2} + \frac{17}{2} a^2 \cos(c+dx) \right) dx \\
&= \frac{17a^3 \sin(c+dx)}{24d \sqrt{a + a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{a^2 \sqrt{a + a \cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{1}{48} \left(163a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \cos^{\frac{3}{2}}(c+dx) \sqrt{a + a \cos(c+dx)} dx \\
&= \frac{17a^3 \sin(c+dx)}{24d \sqrt{a + a \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{a^2 \sqrt{a + a \cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{163a^3 \sin(c+dx)}{96d \sqrt{a + a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{1}{64} \left(163a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} \sqrt{a + a \cos(c+dx)} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{17a^3 \sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{a^2 \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} \\
&+ \frac{163a^3 \sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} + \frac{163a^3 \sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)} \sqrt{\sec(c+dx)}} \\
&+ \frac{1}{128} \left(163a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{17a^3 \sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{a^2 \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} \\
&+ \frac{163a^3 \sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} + \frac{163a^3 \sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)} \sqrt{\sec(c+dx)}} \\
&\quad \left(163a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right) \\
&\quad - \frac{163a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64d} \\
&= \frac{163a^{5/2} \arcsin \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64d} \\
&+ \frac{17a^3 \sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} + \frac{a^2 \sqrt{a+a\cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} \\
&+ \frac{163a^3 \sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} + \frac{163a^3 \sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)} \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.03 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{\sqrt{\cos(c+dx)}(a(1+\cos(c+dx)))^{5/2} \sec^4\left(\frac{1}{2}(c+dx)\right) \sqrt{\sec(c+dx)}(7(89+28\cos(c+dx))+3\cos(2(c+dx))) \text{Hypergeometric2F1}\left[-\frac{3}{2}, \frac{1}{2}, \frac{7}{2}, 2\sin^2\left(\frac{c+dx}{2}\right)\right] - 24(3+\cos(c+dx)) \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{3}{2}, \frac{9}{2}, 2\sin^2\left(\frac{c+dx}{2}\right)\right] \sin^2(c+dx) - 6\text{Csc}\left(\frac{c+dx}{2}\right) \text{HypergeometricPFQ}\left\{-\frac{1}{2}, \frac{3}{2}, 2\right\}, \{1, \frac{9}{2}\}, 2\sin^2\left(\frac{c+dx}{2}\right) \sin^4(c+dx) \tan\left(\frac{c+dx}{2}\right)}{420d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2), x]

[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F1[-3/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] - 24*(3 + Cos[c + d*x])*Hypergeometric2F1[-1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 - 6*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{-1/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4*Tan[(c + d*x)/2])/(420*d)

Maple [A] (verified)

Time = 13.83 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))} \left(48 \sin(dx+c) (\cos^2(dx+c)) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 184 \sin(dx+c) \cos(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 326 \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \right)}{192d(1+\cos(dx+c)) \sec(dx+c)^{\frac{3}{2}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] `int((a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)`

```
[Out] 1/192/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))/sec(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(48*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+184*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+326*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+489*tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+489*sec(d*x+c)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.66

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{489(a^2 \cos(dx + c) + a^2) \sqrt{a} \arctan\left(\frac{\sqrt{a \cos(dx+c)+a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) - \frac{(48a^2 \cos(dx+c)^4 + 184a^2 \cos(dx+c)^3 + 326a^2 \cos(dx+c)^2 + 489a^2 \cos(dx+c)) \sqrt{\cos(dx+c)}}{\sqrt{\cos(dx+c)}}}{192(d \cos(dx + c) + d)}$$

[In] `integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")`

```
[Out] -1/192*(489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (48*a^2*cos(d*x + c)^4 + 184*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sec^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

[In] `integrate((a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)`

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7450 vs. $2(184) = 368$.

Time = 0.70 (sec) , antiderivative size = 7450, normalized size of antiderivative = 33.86

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sec^{3/2}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] 1/768*(10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(2*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*2*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*a^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 19*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 12*(4*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))) + 1) - (3*a^2*cos(4*d*x + 4*c)^3 - 8*a^2*cos(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 14*a^2*cos(4*d*x + 4*c)^2 + 19*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 2*a^2*cos(4*d*x + 4*c)^2 - 13*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(

$$\begin{aligned}
& 4*d*x + 4*c)^2 - 2*a^2*\cos(4*d*x + 4*c) + a^2)*\cos(1/2*\arctan2(\sin(4*d*x + \\
& 4*c), \cos(4*d*x + 4*c)))^2 + 32*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4 \\
& *c)^2 + 2*a^2*\cos(4*d*x + 4*c) + a^2)*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos \\
& (4*d*x + 4*c)))^2 + 2*(16*a^2*\cos(4*d*x + 4*c)^2 + 16*a^2*\sin(4*d*x + 4*c)^ \\
& 2 - 19*a^2*\cos(4*d*x + 4*c) + 3*a^2)*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(\\
& 4*d*x + 4*c))) - 2*(64*a^2*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4* \\
& c)))*\sin(4*d*x + 4*c) + 19*a^2*\sin(4*d*x + 4*c))*\sin(1/2*\arctan2(\sin(4*d*x \\
& + 4*c), \cos(4*d*x + 4*c))))*\cos(3/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4 \\
& *c))) + 4*(3*a^2*\cos(4*d*x + 4*c)^3 - 11*a^2*\cos(4*d*x + 4*c)^2 + 8*a^2*\cos \\
& (4*d*x + 4*c) + (3*a^2*\cos(4*d*x + 4*c) - 8*a^2)*\sin(4*d*x + 4*c)^2)*\cos(1/ \\
& 2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 3*(2*a^2*\cos(1/2*\arctan2(s \\
& in(4*d*x + 4*c), \cos(4*d*x + 4*c)))*\sin(4*d*x + 4*c) + a^2*\sin(4*d*x + 4*c) \\
& - 2*(a^2*\cos(4*d*x + 4*c) + a^2)*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d \\
& *x + 4*c))))*\sin(3/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 4*(4*(3 \\
& *a^2*\cos(4*d*x + 4*c) - 8*a^2)*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x \\
& + 4*c)))*\sin(4*d*x + 4*c) + (3*a^2*\cos(4*d*x + 4*c) - 8*a^2)*\sin(4*d*x + 4* \\
& c))*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))))*\sin(3/2*\arctan2(s \\
& in(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2*\arctan2(\sin(4* \\
& d*x + 4*c), \cos(4*d*x + 4*c))) + 1))*\sqrt{a} - 6*(\cos(1/2*\arctan2(\sin(4*d* \\
& x + 4*c), \cos(4*d*x + 4*c)))^2 + \sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d* \\
& x + 4*c)))^2 + 2*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 1)^ \\
& (1/4)*((3*a^2*\cos(4*d*x + 4*c)^2*\sin(4*d*x + 4*c) + 3*a^2*\sin(4*d*x + 4*c)^ \\
& 3 + 3*a^2*\cos(1/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))*\sin(4*d*x + \\
& 4*c) - 160*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 + 2*a^2*\cos(4*d \\
& *x + 4*c) + a^2)*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^3 + 4 \\
& *(3*a^2*\sin(4*d*x + 4*c)^3 + 3*(a^2*\cos(4*d*x + 4*c)^2 - 2*a^2*\cos(4*d*x + \\
& 4*c) + a^2)*\sin(4*d*x + 4*c) - 160*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x \\
& + 4*c)^2 - 2*a^2*\cos(4*d*x + 4*c) + a^2)*\sin(1/4*\arctan2(\sin(4*d*x + 4*c), \\
& \cos(4*d*x + 4*c))))*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 \\
& + 4*(3*a^2*\sin(4*d*x + 4*c)^3 + 160*a^2*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), c \\
& os(4*d*x + 4*c)))*\sin(4*d*x + 4*c) + (3*a^2*\cos(4*d*x + 4*c)^2 + 6*a^2*\cos(\\
& 4*d*x + 4*c) + 43*a^2)*\sin(4*d*x + 4*c) - 160*(a^2*\cos(4*d*x + 4*c)^2 + a^2 \\
& *\sin(4*d*x + 4*c)^2 + 2*a^2*\cos(4*d*x + 4*c) + a^2)*\sin(1/4*\arctan2(\sin(4*d \\
& *x + 4*c), \cos(4*d*x + 4*c))))*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x \\
& + 4*c)))^2 + 2*(6*a^2*\sin(4*d*x + 4*c)^3 + 3*a^2*\cos(1/4*\arctan2(\sin(4*d*x \\
& + 4*c), \cos(4*d*x + 4*c)))*\sin(4*d*x + 4*c) + 6*(a^2*\cos(4*d*x + 4*c)^2 - a \\
& ^2*\cos(4*d*x + 4*c))*\sin(4*d*x + 4*c) - (320*a^2*\cos(4*d*x + 4*c)^2 + 320*a \\
& ^2*\sin(4*d*x + 4*c)^2 - 317*a^2*\cos(4*d*x + 4*c) - 3*a^2)*\sin(1/4*\arctan2(s \\
& in(4*d*x + 4*c), \cos(4*d*x + 4*c))))*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(\\
& 4*d*x + 4*c))) - 2*(20*a^2*\cos(4*d*x + 4*c)^2 + 26*a^2*\sin(4*d*x + 4*c)^2 - \\
& 317*a^2*\sin(4*d*x + 4*c)*\sin(1/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c \\
&))) + 80*(a^2*\cos(4*d*x + 4*c)^2 + a^2*\sin(4*d*x + 4*c)^2 - 2*a^2*\cos(4*d*x \\
& + 4*c) + a^2)*\cos(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 8*(\\
& 10*a^2*\cos(4*d*x + 4*c)^2 + 13*a^2*\sin(4*d*x + 4*c)^2 - 160*a^2*\sin(4*d*x + \\
& 4*c)*\sin(1/4*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 10*a^2*\cos(4*d
\end{aligned}$$

$$\begin{aligned}
& *x + 4*c)) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 3*(a^2 * \cos(4*d*x + 4*c) + a^2) * \cos(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) \\
& * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - (160*a^2 * \cos(4*d*x + 4*c)^2 + 160*a^2 * \sin(4*d*x + 4*c)^2 + 3*a^2 * \cos(4*d*x + 4*c)) * \sin(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) \\
& * \cos(1/2 * \arctan2(\sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) + 1)) - (3*a^2 * \cos(4*d*x + 4*c)^3 + 120*a^2 * \cos(4*d*x + 4*c)^2 - 160*(a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4*c)^2 - 2*a^2 * \cos(4*d*x + 4*c) + a^2) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^3 - 3*a^2 * \sin(4*d*x + 4*c) * \sin(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 4*(3*a^2 * \cos(4*d*x + 4*c)^3 + 74*a^2 * \cos(4*d*x + 4*c)^2 - 197*a^2 * \cos(4*d*x + 4*c) + (3*a^2 * \cos(4*d*x + 4*c) + 80*a^2) * \sin(4*d*x + 4*c)^2 + 120*a^2 - 80*(a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4*c)^2 - 2*a^2 * \cos(4*d*x + 4*c) + a^2) * \cos(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 3*(a^2 * \cos(4*d*x + 4*c) + 40*a^2) * \sin(4*d*x + 4*c)^2 + 4*(3*a^2 * \cos(4*d*x + 4*c)^3 + 126*a^2 * \cos(4*d*x + 4*c)^2 + 243*a^2 * \cos(4*d*x + 4*c) + 3*(a^2 * \cos(4*d*x + 4*c) + 40*a^2) * \sin(4*d*x + 4*c)^2 + 120*a^2 - 40*(a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4*c)^2 + 2*a^2 * \cos(4*d*x + 4*c) + a^2) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 80*(a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4*c)^2 + 2*a^2 * \cos(4*d*x + 4*c) + a^2) * \cos(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 2*(6*a^2 * \cos(4*d*x + 4*c)^3 + 214*a^2 * \cos(4*d*x + 4*c)^2 - 3*a^2 * \sin(4*d*x + 4*c) * \sin(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 240*a^2 * \cos(4*d*x + 4*c) + 2*(3*a^2 * \cos(4*d*x + 4*c) + 110*a^2) * \sin(4*d*x + 4*c)^2 - (160*a^2 * \cos(4*d*x + 4*c)^2 + 160*a^2 * \sin(4*d*x + 4*c)^2 - 157*a^2 * \cos(4*d*x + 4*c) - 3*a^2) * \cos(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - (80*a^2 * \cos(4*d*x + 4*c)^2 + 80*a^2 * \sin(4*d*x + 4*c)^2 + 3*a^2 * \cos(4*d*x + 4*c)) * \cos(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) + 2*(320*a^2 * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 * \sin(4*d*x + 4*c) + 157*a^2 * \cos(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) + 8*(80*a^2 * \cos(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(4*d*x + 4*c) - (3*a^2 * \cos(4*d*x + 4*c) + 110*a^2) * \sin(4*d*x + 4*c)) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 6*(a^2 * \cos(4*d*x + 4*c) + 40*a^2) * \sin(4*d*x + 4*c) + 3*(a^2 * \cos(4*d*x + 4*c) + a^2) * \sin(1/4 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) * \sin(1/2 * \arctan2(\sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))), \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))) + 1))) * \sqrt{a} + 489*((a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4*c)^2 + 4*(a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4*c)^2 - 2*a^2 * \cos(4*d*x + 4*c) + a^2) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 4*(a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4*c)^2 + 2*a^2 * \cos(4*d*x + 4*c) + a^2) * \sin(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c)))^2 + 4*(a^2 * \cos(4*d*x + 4*c)^2 + a^2 * \sin(4*d*x + 4*c)^2 - a^2 * \cos(4*d*x + 4*c)) * \cos(1/2 * \arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))) - 4*(4*a^2 * \cos(1/2 * \arctan2(s
\end{aligned}$$

Giac [F]

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(a \cos(dx + c) + a)^{5/2}}{\sec(dx + c)^{3/2}} dx$$

[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a \cos(c + dx))^{5/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(a + a \cos(c + dx))^{5/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

[In] int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2),x)

[Out] int((a + a*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)

3.361 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$

Optimal result	4366
Rubi [A] (verified)	4366
Mathematica [C] (warning: unable to verify)	4369
Maple [A] (verified)	4370
Fricas [A] (verification not implemented)	4370
Sympy [F(-1)]	4371
Maxima [C] (verification not implemented)	4371
Giac [F]	4372
Mupad [F(-1)]	4372

Optimal result

Integrand size = 23, antiderivative size = 154

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx = -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} + \frac{26\sqrt{\sec(c+dx)} \sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d\sqrt{1+\cos(c+dx)}}$$

[Out] $-2/15*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(1+\cos(d*x+c))^{(1/2)}+2/5*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(1+\cos(d*x+c))^{(1/2)}-\arcsin(\sin(d*x+c)/(1+\cos(d*x+c)))*2^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+26/15*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/(1+\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4307, 2858, 3063, 12, 2860, 222}

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx = -\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d} + \frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{\cos(c+dx)+1}} - \frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{\cos(c+dx)+1}} + \frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{\cos(c+dx)+1}}$$

[In] Int[Sec[c + d*x]^(7/2)/Sqrt[1 + Cos[c + d*x]],x]

[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/d) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d*x]/(15*d*Sqrt[1 + Cos[c + d*x]])) - (2*Sec[c + d*x]^(3/2)*Sin[c + d*x]/(15*d*Sqrt[1 + Cos[c + d*x]])) + (2*Sec[c + d*x]^(5/2)*Sin[c + d*x]/(5*d*Sqrt[1 + Cos[c + d*x]]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2858

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2860

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]

Rule 3063

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{7}{2}}(c+dx) \sqrt{1+\cos(c+dx)}} dx \\
&= \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{1+\cos(c+dx)}} \\
&\quad - \frac{1}{5} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1-4\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{1+\cos(c+dx)}} dx \\
&= -\frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d \sqrt{1+\cos(c+dx)}} + \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{1+\cos(c+dx)}} \\
&\quad - \frac{1}{15} \left(2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{13}{2} + \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{1+\cos(c+dx)}} dx \\
&= \frac{26 \sqrt{\sec(c+dx)} \sin(c+dx)}{15d \sqrt{1+\cos(c+dx)}} - \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d \sqrt{1+\cos(c+dx)}} + \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{1+\cos(c+dx)}} \\
&\quad - \frac{1}{15} \left(4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{15}{4 \sqrt{\cos(c+dx)} \sqrt{1+\cos(c+dx)}} dx \\
&= \frac{26 \sqrt{\sec(c+dx)} \sin(c+dx)}{15d \sqrt{1+\cos(c+dx)}} - \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d \sqrt{1+\cos(c+dx)}} \\
&\quad + \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{1+\cos(c+dx)}} - \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{1+\cos(c+dx)}} dx \\
&= \frac{26 \sqrt{\sec(c+dx)} \sin(c+dx)}{15d \sqrt{1+\cos(c+dx)}} - \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d \sqrt{1+\cos(c+dx)}} \\
&\quad + \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{1+\cos(c+dx)}} + \frac{\left(\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)} \right)}{d} \\
&= -\frac{\sqrt{2} \arcsin \left(\frac{\sin(c+dx)}{1+\cos(c+dx)} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{26 \sqrt{\sec(c+dx)} \sin(c+dx)}{15d \sqrt{1+\cos(c+dx)}} \\
&\quad - \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d \sqrt{1+\cos(c+dx)}} + \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{1+\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.27 (sec) , antiderivative size = 1540, normalized size of antiderivative = 10.00

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx =$$

$$2 \cot\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^6\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{1}{1-2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)^{7/2} \left(4725 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) - 48825 \sin^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 210105 \sin^6\left(\frac{c}{2} + \frac{dx}{2}\right) - 486630 \sin^8\left(\frac{c}{2} + \frac{dx}{2}\right) + 655812 \sin^{10}\left(\frac{c}{2} + \frac{dx}{2}\right) - 710 \operatorname{Hypergeometric2F1}\left[2, \frac{9}{2}, \frac{11}{2}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right] - 40 \cos\left(\frac{c+dx}{2}\right) \operatorname{HypergeometricPFQ}\left[\{2, 2, 2, \frac{9}{2}\}, \{1, 1, \frac{11}{2}\}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right] - 518760 \sin^{12}\left(\frac{c}{2} + \frac{dx}{2}\right) + 770 \operatorname{Hypergeometric2F1}\left[2, \frac{9}{2}, \frac{11}{2}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right] + 226656 \sin^{14}\left(\frac{c}{2} + \frac{dx}{2}\right) - 1500 \operatorname{Hypergeometric2F1}\left[2, \frac{9}{2}, \frac{11}{2}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right] - 42048 \sin^{16}\left(\frac{c}{2} + \frac{dx}{2}\right) + 440 \operatorname{Hypergeometric2F1}\left[2, \frac{9}{2}, \frac{11}{2}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\right] + 4725 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] - 56700 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] + 291060 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] - 833760 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] + 1458000 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] - 1598400 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] + 1080000 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] - 414720 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] + 69120 \operatorname{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right] + 60 \cos\left(\frac{c+dx}{2}\right) \operatorname{HypergeometricPFQ}\left[\{2,$$

[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[1 + Cos[c + d*x]],x]

[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^(7/2)*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2,

2, 9/2}, {1, 11/2}, $\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)]*\text{Sin}[c/2 + (d*x)/2]^10*(-5 + 4*\text{Sin}[c/2 + (d*x)/2]^2)))/(675*d*\text{Sqrt}[1 + \text{Cos}[c + d*x]]*(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2))$

Maple [A] (verified)

Time = 5.94 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.18

method	result
default	$\frac{(\sec^{\frac{7}{2}}(dx+c))\sqrt{2+2\cos(dx+c)}(15(\cos^4(dx+c))\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))+15(\cos^3(dx+c))\sqrt{2}\arcsin(\cot(dx+c)-\csc(dx+c))))}{30d(1+\cos(dx+c))}$

[In] `int(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{30}d*\sec(d*x+c)^{(7/2)}*(2+2*\cos(d*x+c))^{(1/2)}/(1+\cos(d*x+c))*(15*\cos(d*x+c)^4*2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\arcsin(\cot(d*x+c)-\csc(d*x+c))+15*\cos(d*x+c)^3*2^{(1/2)}*\arcsin(\cot(d*x+c)-\csc(d*x+c))*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}+26*\sin(d*x+c)*\cos(d*x+c)^3-2*\cos(d*x+c)^2*\sin(d*x+c)+6*\cos(d*x+c)*\sin(d*x+c))*2^{(1/2)}$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$$

$$= \frac{15(\sqrt{2}\cos(dx+c)^3 + \sqrt{2}\cos(dx+c)^2) \arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right) + \frac{2(13\cos(dx+c)^2 - \cos(dx+c) + 3)\sqrt{\cos(dx+c)}}{\sqrt{\cos(dx+c)}}}{15(d\cos(dx+c)^3 + d\cos(dx+c)^2)}$$

[In] `integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{15}*(15*(\text{sqrt}(2)*\cos(d*x + c)^3 + \text{sqrt}(2)*\cos(d*x + c)^2)*\arctan(\text{sqrt}(2)*\text{sqrt}(\cos(d*x + c) + 1)*\text{sqrt}(\cos(d*x + c))/\sin(d*x + c)) + 2*(13*\cos(d*x + c)^2 - \cos(d*x + c) + 3)*\text{sqrt}(\cos(d*x + c) + 1)*\sin(d*x + c)/\text{sqrt}(\cos(d*x + c)))/(d*\cos(d*x + c)^3 + d*\cos(d*x + c)^2)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(7/2)/(1+cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 989, normalized size of antiderivative = 6.42

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/15*(15*(\sqrt{2}*\cos(2*d*x + 2*c)^2 + \sqrt{2}*\sin(2*d*x + 2*c)^2 + 2*\sqrt{2} \\ & * \cos(2*d*x + 2*c) + \sqrt{2})*(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + \\ & 2*\cos(2*d*x + 2*c) + 1)^{1/4}*\arctan2(((\text{abs}(e^{(I*d*x + I*c)} + 1)^4 + \cos(d*x + c)^4 + \sin(d*x + c)^4 + \\ & 2*(\cos(d*x + c)^2 - \sin(d*x + c)^2 - 2*\cos(d*x + c) + 1)*\text{abs}(e^{(I*d*x + I*c)} + 1)^2 - 4*\cos(d*x + c)^3 + \\ & 2*(\cos(d*x + c)^2 - 2*\cos(d*x + c) + 1)*\sin(d*x + c)^2 + 6*\cos(d*x + c)^2 - 4*\cos(d*x + c) + \\ & 1)^{1/4}*\sin(1/2*\arctan2(2*(\cos(d*x + c) - 1)*\sin(d*x + c)/\text{abs}(e^{(I*d*x + I*c)} + 1)^2, \\ & (\text{abs}(e^{(I*d*x + I*c)} + 1)^2 + \cos(d*x + c)^2 - \sin(d*x + c)^2 - 2*\cos(d*x + c) + 1)/\text{abs}(e^{(I*d*x + I*c)} + 1)^2)) \\ & + \sin(d*x + c))/\text{abs}(e^{(I*d*x + I*c)} + 1), ((\text{abs}(e^{(I*d*x + I*c)} + 1)^4 + \cos(d*x + c)^4 + \sin(d*x + c)^4 + \\ & 2*(\cos(d*x + c)^2 - \sin(d*x + c)^2 - 2*\cos(d*x + c) + 1)*\text{abs}(e^{(I*d*x + I*c)} + 1)^2 - 4*\cos(d*x + c)^3 + \\ & 2*(\cos(d*x + c)^2 - 2*\cos(d*x + c) + 1)*\sin(d*x + c)^2 + 6*\cos(d*x + c)^2 - 4*\cos(d*x + c) + 1)^{1/4}*\cos(1/2*\ar \\ & \text{ctan2}(2*(\cos(d*x + c) - 1)*\sin(d*x + c)/\text{abs}(e^{(I*d*x + I*c)} + 1)^2, (\text{abs}(e^{(I*d*x + I*c)} + 1)^2 + \cos(d*x + c)^2 - \sin(d*x + c)^2 - \\ & 2*\cos(d*x + c) + 1)/\text{abs}(e^{(I*d*x + I*c)} + 1)^2)) + \cos(d*x + c) - 1)/\text{abs}(e^{(I*d*x + I*c)} + 1) \\ &) - 26*(\cos(2*d*x + 2*c)^2*\sin(d*x + c) + \sin(2*d*x + 2*c)^2*\sin(d*x + c) + \\ & 2*\cos(2*d*x + 2*c)*\sin(d*x + c) + \sin(d*x + c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) \\ & + 24*\cos(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(d*x + c) - 24*(\cos(d*x + c) - 1)*\sin(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) \\ & + 2*((13*\cos(d*x + c) - 15)*\cos(2*d*x + 2*c)^2 + (13*\cos(d*x + c) - 15)*\sin(2*d*x + 2*c)^2 + 2*(13*\cos(d*x + c) - 15)*\cos(2*d*x + 2*c) + 13*\cos(d*x + c) - 15)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) - 4*\sqrt{\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2} \end{aligned}$$

$2 + 2*\cos(2*d*x + 2*c) + 1)*(7*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))*\sin(d*x + c) - (7*\cos(d*x + c) - 5)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1))))/((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^(5/4)*d)$

Giac [F]

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{\frac{7}{2}}}{\sqrt{\cos(dx + c) + 1}} dx$$

[In] integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(7/2)/sqrt(cos(d*x + c) + 1), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\sqrt{\cos(c + dx) + 1}} dx$$

[In] int((1/cos(c + d*x))^(7/2)/(cos(c + d*x) + 1)^(1/2),x)

[Out] int((1/cos(c + d*x))^(7/2)/(cos(c + d*x) + 1)^(1/2), x)

$$3.362 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$$

Optimal result	4373
Rubi [A] (verified)	4373
Mathematica [C] (warning: unable to verify)	4375
Maple [A] (verified)	4376
Fricas [A] (verification not implemented)	4376
Sympy [F(-1)]	4377
Maxima [C] (verification not implemented)	4377
Giac [F]	4378
Mupad [F(-1)]	4378

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} - \frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}}$$

[Out] $2/3*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(1+\cos(d*x+c))^{(1/2)}+\arcsin(\sin(d*x+c)/(1+\cos(d*x+c)))*2^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d-2/3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(1+\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4307, 2858, 3063, 12, 2860, 222}

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d} + \frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{\cos(c+dx)+1}} - \frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{\cos(c+dx)+1}}$$

[In] $\text{Int}[\text{Sec}[c+d*x]^{(5/2)}/\text{Sqrt}[1+\text{Cos}[c+d*x]],x]$

[Out] $(\text{Sqrt}[2]*\text{ArcSin}[\text{Sin}[c+d*x]/(1+\text{Cos}[c+d*x])]*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[\text{Sec}[c+d*x]])/d - (2*\text{Sqrt}[\text{Sec}[c+d*x]]*\text{Sin}[c+d*x])/(3*d*\text{Sqrt}[1+\text{Cos}[c+d*x]]) + (2*\text{Sec}[c+d*x]^{(3/2)}*\text{Sin}[c+d*x])/(3*d*\text{Sqrt}[1+\text{Cos}[c+d*x]])$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2858

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2860

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)])*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 4307

Int[(csc[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{5}{2}}(c+dx) \sqrt{1+\cos(c+dx)}} dx \\
 &= \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d \sqrt{1+\cos(c+dx)}} \\
 &\quad - \frac{1}{3} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1-2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{1+\cos(c+dx)}} dx \\
 &= -\frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{3d \sqrt{1+\cos(c+dx)}} + \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d \sqrt{1+\cos(c+dx)}} \\
 &\quad - \frac{1}{3} \left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int -\frac{3}{2\sqrt{\cos(c+dx)} \sqrt{1+\cos(c+dx)}} dx \\
 &= -\frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{3d \sqrt{1+\cos(c+dx)}} + \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d \sqrt{1+\cos(c+dx)}} \\
 &\quad + \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{1+\cos(c+dx)}} dx \\
 &= -\frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{3d \sqrt{1+\cos(c+dx)}} + \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d \sqrt{1+\cos(c+dx)}} \\
 &\quad - \frac{\left(\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)} \right)}{d} \\
 &= \frac{\sqrt{2} \arcsin \left(\frac{\sin(c+dx)}{1+\cos(c+dx)} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} \\
 &\quad - \frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{3d \sqrt{1+\cos(c+dx)}} + \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d \sqrt{1+\cos(c+dx)}}
 \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.45 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.01

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx =$$

$$2 \cot \left(\frac{c}{2} + \frac{dx}{2} \right) \csc^4 \left(\frac{c}{2} + \frac{dx}{2} \right) \left(\frac{1}{1-2 \sin^2 \left(\frac{c}{2} + \frac{dx}{2} \right)} \right)^{7/2} \left(12 \cos^4 \left(\frac{1}{2}(c+dx) \right) {}_3F_2 \left(2, 2, \frac{7}{2}; 1, \frac{9}{2}; \frac{\sin^2 \left(\frac{c}{2} + \frac{dx}{2} \right)}{-1+2 \sin^2 \left(\frac{c}{2} + \frac{dx}{2} \right)} \right) \right)$$

[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]

[Out] $(-2*\cot[c/2 + (d*x)/2]*\csc[c/2 + (d*x)/2]^4*((1 - 2*\sin[c/2 + (d*x)/2]^2)^{-1})^{7/2}*(12*\cos[(c + d*x)/2]^4*\text{HypergeometricPFQ}[\{2, 2, 7/2\}, \{1, 9/2\}, \sin[c/2 + (d*x)/2]^2/(-1 + 2*\sin[c/2 + (d*x)/2]^2)]*\sin[c/2 + (d*x)/2]^8 + 12*\text{Hypergeometric2F1}[2, 7/2, 9/2, \sin[c/2 + (d*x)/2]^2/(-1 + 2*\sin[c/2 + (d*x)/2]^2)]*\sin[c/2 + (d*x)/2]^8*(4 - 7*\sin[c/2 + (d*x)/2]^2 + 3*\sin[c/2 + (d*x)/2]^4) + 7*(1 - 2*\sin[c/2 + (d*x)/2]^2)^3*\sqrt{\sin[c/2 + (d*x)/2]^2/(-1 + 2*\sin[c/2 + (d*x)/2]^2)}*(15 - 20*\sin[c/2 + (d*x)/2]^2 + 8*\sin[c/2 + (d*x)/2]^4)*(ArcTanh[\sqrt{\sin[c/2 + (d*x)/2]^2/(-1 + 2*\sin[c/2 + (d*x)/2]^2)}])*(3 - 6*\sin[c/2 + (d*x)/2]^2) + \sqrt{\sin[c/2 + (d*x)/2]^2/(-1 + 2*\sin[c/2 + (d*x)/2]^2)}*(-3 + 7*\sin[c/2 + (d*x)/2]^2)))/(63*d*\sqrt{1 + \cos[c + d*x]})$

Maple [A] (verified)

Time = 6.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.41

method	result
default	$-\frac{(\sec^{\frac{5}{2}}(dx+c))\sqrt{2+2\cos(dx+c)}(3(\cos^3(dx+c))\sqrt{2}\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3(\cos^2(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})}{6d(1+\cos(dx+c))}$

[In] int(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] $-1/6/d*\sec(d*x+c)^{5/2}*(2+2*\cos(d*x+c))^{1/2}/(1+\cos(d*x+c))*(3*\cos(d*x+c)^3*2^{1/2}*\arcsin(\cot(d*x+c)-\csc(d*x+c))*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}+3*\cos(d*x+c)^2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\arcsin(\cot(d*x+c)-\csc(d*x+c)))*2^{1/2}+2*\cos(d*x+c)^2*\sin(d*x+c)-2*\cos(d*x+c)*\sin(d*x+c))*2^{1/2}$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \frac{3(\sqrt{2}\cos(dx+c)^2 + \sqrt{2}\cos(dx+c))\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right) + \frac{2\sqrt{\cos(dx+c)+1}(\cos(dx+c)-1)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3(d\cos(dx+c)^2 + d\cos(dx+c))}$$

[In] integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $-1/3*(3*(\sqrt{2}*\cos(d*x + c)^2 + \sqrt{2}*\cos(d*x + c))*\arctan(\sqrt{2}*\sqrt{\cos(d*x + c) + 1}*\sqrt{\cos(d*x + c)}/\sin(d*x + c)) + 2*\sqrt{\cos(d*x + c) + 1}*(\cos(d*x + c) - 1)*\sin(d*x + c)/\sqrt{\cos(d*x + c)})/(d*\cos(d*x + c)^2 + d*\cos(d*x + c))$

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 801, normalized size of antiderivative = 6.79

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d*x + I*c) + 1)) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) - 3)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - (cos(d*x + c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{\sqrt{\cos(dx + c) + 1}} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/sqrt(cos(d*x + c) + 1), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\sqrt{\cos(c + dx) + 1}} dx$$

[In] int((1/cos(c + d*x))^(5/2)/(cos(c + d*x) + 1)^(1/2),x)

[Out] int((1/cos(c + d*x))^(5/2)/(cos(c + d*x) + 1)^(1/2), x)

$$3.363 \quad \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$$

Optimal result	4379
Rubi [A] (verified)	4379
Mathematica [C] (warning: unable to verify)	4381
Maple [A] (verified)	4381
Fricas [A] (verification not implemented)	4381
Sympy [F]	4382
Maxima [C] (verification not implemented)	4382
Giac [F]	4383
Mupad [F(-1)]	4383

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx = -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} + \frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}$$

[Out] $-\arcsin(\sin(d*x+c)/(1+\cos(d*x+c)))*2^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(1+\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4307, 2858, 2860, 222}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx = \frac{2 \sin(c+dx) \sqrt{\sec(c+dx)}}{d \sqrt{\cos(c+dx)+1}} - \frac{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

[In] $\text{Int}[\text{Sec}[c+d*x]^{(3/2)}/\text{Sqrt}[1+\text{Cos}[c+d*x]],x]$

[Out] $-\left(\left(\text{Sqrt}[2]*\text{ArcSin}[\text{Sin}[c+d*x]/(1+\text{Cos}[c+d*x])]*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[\text{Sec}[c+d*x]]\right)/d\right) + \left(2*\text{Sqrt}[\text{Sec}[c+d*x]]*\text{Sin}[c+d*x]\right)/\left(d*\text{Sqrt}[1+\text{Cos}[c+d*x]]\right)$

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2858

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2860

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)}} dx \\
 &= \frac{2\sqrt{\sec(c + dx)} \sin(c + dx)}{d\sqrt{1 + \cos(c + dx)}} \\
 &\quad - \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)}} dx \\
 &= \frac{2\sqrt{\sec(c + dx)} \sin(c + dx)}{d\sqrt{1 + \cos(c + dx)}} \\
 &\quad + \frac{\left(\sqrt{2} \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)} \right)}{d} \\
 &= -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)} \right) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}}{d} + \frac{2\sqrt{\sec(c + dx)} \sin(c + dx)}{d\sqrt{1 + \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.17

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$$

$$= \frac{2 \cos\left(\frac{1}{2}(c+dx)\right) \sec^{\frac{3}{2}}(c+dx) \sin\left(\frac{1}{2}(c+dx)\right) \left(\frac{1}{2} \cos(c+dx)(2+\cos(c+dx)) \csc^4\left(\frac{1}{2}(c+dx)\right) \left(1-\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{\dots}$$

[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]

[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 + Cos[c + d*x])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]]))/2 - (Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/10)/(d*Sqrt[1 + Cos[c + d*x]])

Maple [A] (verified)

Time = 5.88 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

method	result
default	$\frac{\left(\sec^{\frac{3}{2}}(dx+c)\right)\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \cos(dx+c)\sqrt{2} \arcsin(\cot(dx+c)-\csc(dx+c))+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{2} \arcsin(\cot(dx+c)-\csc(dx+c))+2\sin(dx+c)\right)}{2d(1+\cos(dx+c))}$

[In] int(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/d*sec(d*x+c)^(3/2)*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*2^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+2*sin(d*x+c))*cos(d*x+c)*(2+2*cos(d*x+c))^(1/2))/(1+cos(d*x+c))*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx$$

$$= \frac{(\sqrt{2} \cos(dx+c) + \sqrt{2}) \arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right) + \frac{2\sqrt{\cos(dx+c)+1}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d \cos(dx+c) + d}$$

[In] integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] ((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*sqrt(cos(d*x + c) + 1)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{\cos(c + dx) + 1}} dx$$

[In] integrate(sec(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)**(3/2)/sqrt(cos(c + d*x) + 1), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 648, normalized size of antiderivative = 7.90

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx$$

$$2 \cos\left(\frac{1}{2} \arctan\left(\frac{\sin(2 dx + 2c)}{\cos(2 dx + 2c) + 1}\right)\right) \sin(dx + c) - 2(\cos(dx + c) - 1) \sin\left(\frac{1}{2} \arctan\left(\frac{\sin(2 dx + 2c)}{\cos(2 dx + 2c) + 1}\right)\right)$$

[In] integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] (2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - 2*(cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 +

$6*\cos(d*x + c)^2 - 4*\cos(d*x + c) + 1)^{(1/4)}*\cos(1/2*\arctan2(2*(\cos(d*x + c) - 1)*\sin(d*x + c)/\text{abs}(e^{(I*d*x + I*c)} + 1)^2, (\text{abs}(e^{(I*d*x + I*c)} + 1)^2 + \cos(d*x + c)^2 - \sin(d*x + c)^2 - 2*\cos(d*x + c) + 1)/\text{abs}(e^{(I*d*x + I*c)} + 1)^2)) + \cos(d*x + c) - 1)/\text{abs}(e^{(I*d*x + I*c)} + 1)))/((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1)^{(1/4)}*d)$

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{\sqrt{\cos(dx + c) + 1}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/sqrt(cos(d*x + c) + 1), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\sqrt{\cos(c + dx) + 1}} dx$$

[In] int((1/cos(c + d*x))^(3/2)/(cos(c + d*x) + 1)^(1/2),x)

[Out] int((1/cos(c + d*x))^(3/2)/(cos(c + d*x) + 1)^(1/2), x)

$$3.364 \quad \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx$$

Optimal result	4384
Rubi [A] (verified)	4384
Mathematica [A] (verified)	4385
Maple [A] (verified)	4385
Fricas [A] (verification not implemented)	4386
Sympy [F]	4386
Maxima [C] (verification not implemented)	4386
Giac [F]	4387
Mupad [F(-1)]	4387

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d}$$

[Out] arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4307, 2860, 222}

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = \frac{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]

[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2860

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{1+\cos(c+dx)}} dx \\ &= - \frac{\left(\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)} \right)}{d} \\ &= \frac{\sqrt{2} \arcsin \left(\frac{\sin(c+dx)}{1+\cos(c+dx)} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = \frac{2 \arctan \left(\frac{\sin(\frac{1}{2}(c+dx))}{\sqrt{\cos(c+dx)}} \right) \cos \left(\frac{1}{2}(c+dx) \right) \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{\sec(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]])*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]]/d
```

Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

method	result	size
default	$-\frac{\sqrt{2+2\cos(dx+c)} \arcsin(\cot(dx+c)-\csc(dx+c))(\sqrt{\sec(dx+c)} \cos(dx+c))}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	77

```
[In] int(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

[Out] $-1/d*(2+2*\cos(d*x+c))^{(1/2)}*\arcsin(\cot(d*x+c)-\csc(d*x+c))*\sec(d*x+c)^{(1/2)}*\cos(d*x+c)/(1+\cos(d*x+c))/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

[In] `integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $-\sqrt{2}*\arctan(\sqrt{2}*\sqrt{\cos(d*x+c)+1}*\sqrt{\cos(d*x+c)})/\sin(d*x+c))/d$

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\cos(c+dx)+1}} dx$$

[In] `integrate(sec(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(sec(c+d*x))/sqrt(cos(c+d*x)+1), x)`

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 505, normalized size of antiderivative = 10.74

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx$$

$$= \sqrt{2} \arctan \left(\frac{(|e^{(i dx+i c)+1}|^4 + \cos(dx+c)^4 + \sin(dx+c)^4 + 2(\cos(dx+c)^2 - \sin(dx+c)^2 - 2\cos(dx+c)+1)|e^{(i dx+i c)+1}|^2 - 4\cos(dx+c)^3 + 2(\cos(dx+c)+1))}{\dots} \right)$$

[In] `integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] $\sqrt{2}*\arctan2(((\text{abs}(e^{(I*d*x + I*c)} + 1))^4 + \cos(d*x + c)^4 + \sin(d*x + c)^4 + 2*(\cos(d*x + c)^2 - \sin(d*x + c)^2 - 2*\cos(d*x + c) + 1)*\text{abs}(e^{(I*d*x$

+ I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1) *sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + cos(d*x + c) - 1)/abs(e^(I*d*x + I*c) + 1))/d

Giac [F]

$$\int \frac{\sqrt{\sec(c + dx)}}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\sqrt{\sec(dx + c)}}{\sqrt{\cos(dx + c) + 1}} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/sqrt(cos(d*x + c) + 1), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c + dx)}}{\sqrt{1 + \cos(c + dx)}} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{\sqrt{\cos(c + dx) + 1}} dx$$

[In] int((1/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1)^(1/2),x)

[Out] int((1/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1)^(1/2), x)

$$3.365 \quad \int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} dx$$

Optimal result	4388
Rubi [A] (verified)	4388
Mathematica [C] (verified)	4390
Maple [A] (verified)	4390
Fricas [A] (verification not implemented)	4391
Sympy [F]	4391
Maxima [C] (verification not implemented)	4391
Giac [F]	4392
Mupad [F(-1)]	4392

Optimal result

Integrand size = 23, antiderivative size = 94

$$\begin{aligned} & \int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} dx \\ &= -\frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} \\ & \quad + \frac{2 \arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} \end{aligned}$$

[Out] 2*arcsin(sin(d*x+c)/(1+cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d-arcsin(sin(d*x+c)/(1+cos(d*x+c)))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4307, 2856, 2853, 222, 2860}

$$\begin{aligned} & \int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} dx \\ &= \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d} \\ & \quad - \frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d} \end{aligned}$$

[In] Int[1/(Sqrt[1 + Cos[c + d*x]]*Sqrt[Sec[c + d*x]]), x]

[Out] $-\left(\frac{\sqrt{2} \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{1+\cos[c+dx]}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d}\right) + \frac{2 \operatorname{ArcSin}\left[\frac{\sin[c+dx]}{\sqrt{1+\cos[c+dx]}}\right] \sqrt{\cos[c+dx]} \sqrt{\sec[c+dx]}}{d}$

Rule 222

$\operatorname{Int}\left[\frac{1}{\sqrt{a+(b \cdot x)^2}}, x_{\text{Symbol}}\right] \rightarrow \operatorname{Simp}\left[\operatorname{ArcSin}\left[\frac{\operatorname{Rt}[-b, 2] \cdot (x/\sqrt{a})}{\operatorname{Rt}[-b, 2], x}\right]\right] / \operatorname{Rt}[-b, 2], x \text{ ; FreeQ}\{a, b, x\} \ \&\& \ \operatorname{GtQ}[a, 0] \ \&\& \ \operatorname{NegQ}[b]$

Rule 2853

$\operatorname{Int}\left[\frac{\sqrt{a+(b \cdot \sin[e+fx])+(f \cdot x)}}{\sqrt{d \cdot \sin[e+fx]+(f \cdot x)^2}}, x_{\text{Symbol}}\right] \rightarrow \operatorname{Dist}\left[-\frac{2}{f}, \operatorname{Subst}\left[\operatorname{Int}\left[\frac{1}{\sqrt{1-x^2/a}}, x\right], x, b \cdot \left(\frac{\cos[e+fx]}{\sqrt{a+b \cdot \sin[e+fx]}}\right)\right], x\right] \text{ ; FreeQ}\{a, b, d, e, f, x\} \ \&\& \ \operatorname{EqQ}[a^2-b^2, 0] \ \&\& \ \operatorname{EqQ}[d, a/b]$

Rule 2856

$\operatorname{Int}\left[\frac{\sqrt{c+(d \cdot \sin[e+fx])+(f \cdot x)}}{\sqrt{a+(b \cdot \sin[e+fx])+(f \cdot x)^2}}, x_{\text{Symbol}}\right] \rightarrow \operatorname{Dist}\left[\frac{d}{b}, \operatorname{Int}\left[\frac{\sqrt{a+b \cdot \sin[e+fx]}}{\sqrt{c+d \cdot \sin[e+fx]}}\right], x\right] + \operatorname{Dist}\left[\frac{b \cdot c - a \cdot d}{b}, \operatorname{Int}\left[\frac{1}{\sqrt{a+b \cdot \sin[e+fx]} \sqrt{c+d \cdot \sin[e+fx]}}\right], x\right] \text{ ; FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \operatorname{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \operatorname{EqQ}[a^2-b^2, 0] \ \&\& \ \operatorname{NeQ}[c^2-d^2, 0]$

Rule 2860

$\operatorname{Int}\left[\frac{1}{\sqrt{d \cdot \sin[e+fx]+(f \cdot x)} \sqrt{a+(b \cdot \sin[e+fx])+(f \cdot x)^2}}, x_{\text{Symbol}}\right] \rightarrow \operatorname{Dist}\left[-\frac{\sqrt{2}}{\sqrt{a} \cdot f}, \operatorname{Subst}\left[\operatorname{Int}\left[\frac{1}{\sqrt{1-x^2}}, x\right], x, b \cdot \left(\frac{\cos[e+fx]}{a+b \cdot \sin[e+fx]}\right)\right], x\right] \text{ ; FreeQ}\{a, b, d, e, f, x\} \ \&\& \ \operatorname{EqQ}[a^2-b^2, 0] \ \&\& \ \operatorname{EqQ}[d, a/b] \ \&\& \ \operatorname{GtQ}[a, 0]$

Rule 4307

$\operatorname{Int}\left[(\csc[a+(b \cdot x)] \cdot (c \cdot x))^m \cdot (u), x_{\text{Symbol}}\right] \rightarrow \operatorname{Dist}\left[(c \cdot \csc[a+b \cdot x])^m \cdot (c \cdot \sin[a+b \cdot x])^m, \operatorname{Int}\left[\operatorname{ActivateTrig}[u] / (c \cdot \sin[a+b \cdot x])^m, x\right], x\right] \text{ ; FreeQ}\{a, b, c, m, x\} \ \&\& \ \text{!IntegerQ}[m] \ \&\& \ \operatorname{KnownSineIntegrandQ}[u, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}} dx \\ &= -\left(\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{1+\cos(c+dx)}} dx\right) \\ &\quad + \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \end{aligned}$$

$$\begin{aligned}
&= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}dx,x,-\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} \\
&\quad +\frac{\left(\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}}dx,x,-\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \\
&= -\frac{\sqrt{2}\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} \\
&\quad +\frac{2\arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.82

$$\begin{aligned}
&\int\frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}dx \\
&= \frac{i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-\operatorname{arcsinh}\left(e^{i(c+dx)}\right)+\sqrt{2}\operatorname{arctanh}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)\right)+\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{d\sqrt{1+\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[1/(Sqrt[1 + Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]

[Out] (I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[(c + d*x)/2])/(d*E^((I/2)*(c + d*x))*Sqrt[1 + Cos[c + d*x]])

Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\sqrt{2+2\cos(dx+c)}\left(\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{2+2\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}\right)\sqrt{2}}{2d(1+\cos(dx+c))\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	108

[In] int(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/d*(2+2*cos(d*x+c))^(1/2)*(arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)+2*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))/(1+cos(d*x+c))/sec(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{1 + \cos(c + dx)} \sqrt{\sec(c + dx)}} dx$$

$$= \frac{\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right) - 2 \arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

[In] integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] (sqrt(2)*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/d

Sympy [F]

$$\int \frac{1}{\sqrt{1 + \cos(c + dx)} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\cos(c + dx) + 1} \sqrt{\sec(c + dx)}} dx$$

[In] integrate(1/sec(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*sqrt(sec(c + d*x))), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 689, normalized size of antiderivative = 7.33

$$\int \frac{1}{\sqrt{1 + \cos(c + dx)} \sqrt{\sec(c + dx)}} dx =$$

$$\sqrt{2} \arctan\left(\frac{(|2e^{i dx + i c} + 2|^4 + 16 \cos(dx+c)^4 + 16 \sin(dx+c)^4 + 8 (\cos(dx+c)^2 - \sin(dx+c)^2 - 2 \cos(dx+c) + 1) |2e^{i dx + i c} + 2|^2 - 64 \cos(dx+c))}{\dots}\right)$$

[In] integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -(sqrt(2)*arctan2(((abs(2*e^(I*d*x + I*c) + 2)^4 + 16*cos(d*x + c)^4 + 16*sin(d*x + c)^4 + 8*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(2*e^(I*d*x + I*c) + 2)^2 - 64*cos(d*x + c)^3 + 32*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 96*cos(d*x + c)^2 - 64*cos(d*x + c) + 16)^(

$$\frac{1}{4} \sin\left(\frac{1}{2} \arctan\left(\frac{2(\cos(dx+c)-1)\sin(dx+c)}{\sqrt{2e^{I dx+I c}+2}}\right)\right) + 2)^2, \left(\sqrt{2e^{I dx+I c}+2}\right)^2 + 4\cos(dx+c)^2 - 4\sin(dx+c)^2 - 8\cos(dx+c) + 4\right) / \sqrt{2e^{I dx+I c}+2} + 2\sin(dx+c) / \sqrt{2e^{I dx+I c}+2}, \left(\left(\sqrt{2e^{I dx+I c}+2}\right)^4 + 16\cos(dx+c)^4 + 16\sin(dx+c)^4 + 8(\cos(dx+c)^2 - \sin(dx+c)^2 - 2\cos(dx+c) + 1)\sqrt{2e^{I dx+I c}+2} - 64\cos(dx+c)^3 + 32(\cos(dx+c)^2 - 2\cos(dx+c) + 1)\sin(dx+c)^2 + 96\cos(dx+c)^2 - 64\cos(dx+c) + 16\right)^{1/4} \cos\left(\frac{1}{2} \arctan\left(\frac{2(\cos(dx+c)-1)\sin(dx+c)}{\sqrt{2e^{I dx+I c}+2}}\right)\right) + 2)^2, \left(\sqrt{2e^{I dx+I c}+2}\right)^2 + 4\cos(dx+c)^2 - 4\sin(dx+c)^2 - 8\cos(dx+c) + 4\right) / \sqrt{2e^{I dx+I c}+2} + 2\cos(dx+c) - 2) / \sqrt{2e^{I dx+I c}+2} - \arctan\left(\frac{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c) + 1}{\cos(2dx+2c) + 1}\right)^{1/4} \sin\left(\frac{1}{2} \arctan\left(\frac{\sin(2dx+2c)}{\cos(2dx+2c) + 1}\right)\right) + \sin(dx+c), \left(\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c) + 1\right)^{1/4} \cos\left(\frac{1}{2} \arctan\left(\frac{\sin(2dx+2c)}{\cos(2dx+2c) + 1}\right)\right) + \cos(dx+c)\right) / d$$

Giac [F]

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(dx+c)+1}\sqrt{\sec(dx+c)}} dx$$

[In] integrate(1/sec(dx+c)^(1/2)/(1+cos(dx+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(cos(dx+c)+1)*sqrt(sec(dx+c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)+1}\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int(1/((cos(c+dx)+1)^(1/2)*(1/cos(c+dx))^(1/2)),x)

[Out] int(1/((cos(c+dx)+1)^(1/2)*(1/cos(c+dx))^(1/2)), x)

$$3.366 \quad \int \frac{1}{\sqrt{1+\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4393
Rubi [A] (verified)	4393
Mathematica [C] (verified)	4396
Maple [A] (verified)	4396
Fricas [A] (verification not implemented)	4397
Sympy [F]	4397
Maxima [F]	4397
Giac [F(-1)]	4398
Mupad [F(-1)]	4398

Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{1}{\sqrt{1+\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} dx = \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} - \frac{\arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} + \frac{\sin(c+dx)}{d \sqrt{1+\cos(c+dx)} \sqrt{\sec(c+dx)}}$$

[Out] $\sin(d*x+c)/d/(1+\cos(d*x+c))^{(1/2)}/\sec(d*x+c)^{(1/2)}-\arcsin(\sin(d*x+c)/(1+\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+\arcsin(\sin(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4307, 2857, 3061, 2860, 222, 2853}

$$\int \frac{1}{\sqrt{1+\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} dx = \frac{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d} - \frac{\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d} + \frac{\sin(c+dx)}{d \sqrt{\cos(c+dx)+1} \sqrt{\sec(c+dx)}}$$

[In] Int[1/(Sqrt[1 + Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]

[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (ArcSin[Sin[c + d*x]/Sqrt[1 + Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + Sin[c + d*x]/(d*Sqrt[1 + Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2857

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(b*(2*n - 1)), Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2860

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x^2], x], x, b*(Cos[e + f*x]/(a + b*Sin[e + f*x]))], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]

Rule 3061

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}} dx \\
&= \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{1}{2} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-1+\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx \\
&= \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{1}{2} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx \\
&\quad + \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} dx \\
&= \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} \\
&\quad - \frac{\left(\sqrt{2}\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \\
&= \frac{\sqrt{2} \arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} \\
&\quad - \frac{\arcsin\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{1 + \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \frac{i e^{-2i(c+dx)} (1 + e^{i(c+dx)}) \left(1 - e^{i(c+dx)} + e^{2i(c+dx)} - e^{3i(c+dx)} + e^{i(c+dx)} \sqrt{1 + e^{2i(c+dx)}} \operatorname{arcsinh}(e^{i(c+dx)}) + 2\sqrt{2} e^{i(c+dx)}\right)}{4d\sqrt{1 + \cos(c + dx)}}$$

[In] Integrate[1/(Sqrt[1 + Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]

[Out] ((I/4)*(1 + E^(I*(c + d*x)))*(1 - E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) - E^((3*I)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) - E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[1 + Cos[c + d*x]])

Maple [A] (verified)

Time = 13.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{\left(\arcsin(\cot(dx+c)) - \csc(dx+c)\right)\sqrt{2-\sin(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{2+2\cos(dx+c)}\sqrt{2}}{2d\sqrt{\sec(dx+c)}(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	133

[In] int(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d/sec(d*x+c)^(1/2)*(arcsin(cot(d*x+c))-csc(d*x+c))*2^(1/2)-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1 + \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \frac{(\sqrt{2} \cos(dx + c) + \sqrt{2}) \arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right) - (\cos(dx + c) + 1) \arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d \cos(dx + c) + d}$$

[In] integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] -((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - (cos(d*x + c) + 1)*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1 + \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{\cos(c + dx) + 1} \sec^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate(1/sec(d*x+c)**(3/2)/(1+cos(d*x+c))^(1/2),x)

[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*sec(c + d*x)**(3/2)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{1 + \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{\cos(dx + c) + 1} \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*sec(d*x + c)^(3/2)), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1 + \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1 + \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{\cos(c + dx) + 1} \left(\frac{1}{\cos(c + dx)}\right)^{\frac{3}{2}}} dx$$

```
[In] int(1/((cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((cos(c + d*x) + 1)^(1/2)*(1/cos(c + d*x))^(3/2)), x)
```

$$3.367 \quad \int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	4399
Rubi [A] (verified)	4399
Mathematica [C] (warning: unable to verify)	4402
Maple [A] (verified)	4403
Fricas [A] (verification not implemented)	4403
Sympy [F(-1)]	4404
Maxima [C] (verification not implemented)	4404
Giac [F]	4405
Mupad [F(-1)]	4405

Optimal result

Integrand size = 25, antiderivative size = 189

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

$$= -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{\sqrt{ad}}$$

$$+ \frac{26 \sqrt{\sec(c+dx)} \sin(c+dx)}{15d \sqrt{a+a \cos(c+dx)}} - \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d \sqrt{a+a \cos(c+dx)}} + \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{a+a \cos(c+dx)}}$$

[Out] $-2/15*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+2/5*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}-\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/a^{(1/2)}+26/15*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used

= {4307, 2858, 3063, 12, 2861, 211}

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= -\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{ad}}$$

$$+ \frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}} - \frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}} + \frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}$$

[In] Int[Sec[c + d*x]^(7/2)/Sqrt[a + a*Cos[c + d*x]],x]

[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2858

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 4307

Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{7}{2}}(c+dx) \sqrt{a+a\cos(c+dx)}} dx \\
 &= \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{a+a\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{a-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+a\cos(c+dx)}} dx}{5a} \\
 &= -\frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d \sqrt{a+a\cos(c+dx)}} + \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{a+a\cos(c+dx)}} \\
 &\quad - \frac{\left(2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{13a^2}{2} + a^2 \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+a\cos(c+dx)}} dx}{15a^2} \\
 &= \frac{26 \sqrt{\sec(c+dx)} \sin(c+dx)}{15d \sqrt{a+a\cos(c+dx)}} - \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d \sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{a+a\cos(c+dx)}} \\
 &\quad - \frac{\left(4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{15a^3}{4 \sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx}{15a^3} \\
 &= \frac{26 \sqrt{\sec(c+dx)} \sin(c+dx)}{15d \sqrt{a+a\cos(c+dx)}} - \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15d \sqrt{a+a\cos(c+dx)}} \\
 &\quad + \frac{2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d \sqrt{a+a\cos(c+dx)}} - \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} \\
&+ \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int\frac{1}{2a^2+ax^2}dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&= -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{ad}} \\
&+ \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} \\
&- \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.21 (sec) , antiderivative size = 1542, normalized size of antiderivative = 8.16

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx = \frac{2\cot\left(\frac{c}{2} + \frac{dx}{2}\right)\csc^6\left(\frac{c}{2} + \frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)^{7/2}\left(4725\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right) - 48825\sin^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 210105\sin^6\left(\frac{c}{2} + \frac{dx}{2}\right) - 486630\sin^8\left(\frac{c}{2} + \frac{dx}{2}\right) + 655812\sin^{10}\left(\frac{c}{2} + \frac{dx}{2}\right) - 710\text{Hypergeometric2F1}\left[2, 9/2, 11/2, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)/(-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right))\right]\sin^{10}\left(\frac{c}{2} + \frac{dx}{2}\right) - 40\cos\left[\frac{c+dx}{2}\right]^6\text{HypergeometricPFQ}\left[\{2, 2, 2, 9/2\}, \{1, 1, 11/2\}, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)/(-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right))\right]\sin^{10}\left(\frac{c}{2} + \frac{dx}{2}\right) - 518760\sin^{12}\left(\frac{c}{2} + \frac{dx}{2}\right) + 1770\text{Hypergeometric2F1}\left[2, 9/2, 11/2, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)/(-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right))\right]\sin^{12}\left(\frac{c}{2} + \frac{dx}{2}\right) + 226656\sin^{14}\left(\frac{c}{2} + \frac{dx}{2}\right) - 1500\text{Hypergeometric2F1}\left[2, 9/2, 11/2, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)/(-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right))\right]\sin^{14}\left(\frac{c}{2} + \frac{dx}{2}\right) - 42048\sin^{16}\left(\frac{c}{2} + \frac{dx}{2}\right) + 440\text{Hypergeometric2F1}\left[2, 9/2, 11/2, \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)/(-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right))\right]\sin^{16}\left(\frac{c}{2} + \frac{dx}{2}\right) + 4725\text{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right]\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}} - 56700\text{ArcTanh}\left[\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right]\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{\frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1 + 2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}}\right)}{\sqrt{a+a\cos(c+dx)}}$$

[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[a + a*Cos[c + d*x]], x]

[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^7/2*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]

$$\begin{aligned} &] + 291060 \operatorname{ArcTanh}[\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &]]*\operatorname{Sin}[c/2 + (d*x)/2]^4*\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2)] \\ &]^2] - 833760 \operatorname{ArcTanh}[\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &]]*\operatorname{Sin}[c/2 + (d*x)/2]^6*\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2)] \\ &] + 1458000 \operatorname{ArcTanh}[\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &]]*\operatorname{Sin}[c/2 + (d*x)/2]^8*\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &] - 1598400 \operatorname{ArcTanh}[\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &]]*\operatorname{Sin}[c/2 + (d*x)/2]^10*\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &] + 1080000 \operatorname{ArcTanh}[\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &]]*\operatorname{Sin}[c/2 + (d*x)/2]^12*\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &] - 414720 \operatorname{ArcTanh}[\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &]]*\operatorname{Sin}[c/2 + (d*x)/2]^14*\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &] + 69120 \operatorname{ArcTanh}[\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &]]*\operatorname{Sin}[c/2 + (d*x)/2]^16*\operatorname{Sqrt}[\operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \\ &] + 60 \operatorname{Cos}[(c + d*x)/2]^4 \operatorname{HypergeometricPFQ}[\{2, 2, 9/2\}, \{1, 11/2\}, \operatorname{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2)] * \operatorname{Sin}[c/2 + (d*x)/2]^10 * (-5 + 4*\operatorname{Sin}[c/2 + (d*x)/2]^2) / (675*d*\operatorname{Sqrt}[a*(1 + \operatorname{Cos}[c + d*x])]) * (-1 + 2*\operatorname{Sin}[c/2 + (d*x)/2]^2) \end{aligned}$$

Maple [A] (verified)

Time = 6.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.99

method	result
default	$\frac{(\sec^{\frac{7}{2}}(dx+c))\sqrt{a(1+\cos(dx+c))}\left(15\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))(\cos^4(dx+c)+15(\cos^3(dx+c)))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}{15d(1+\cos(dx+c))}$

[In] int(sec(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out]
$$\frac{1}{15d} \frac{\sec^{\frac{7}{2}}(dx+c) \cdot (a + \cos(dx+c))^{1/2}}{(1 + \cos(dx+c)) \cdot (15 \cdot (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \arcsin(\cot(dx+c) - \csc(dx+c)) \cdot \cos(dx+c)^4 + 15 \cdot \cos(dx+c)^3 \cdot (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} \cdot \arcsin(\cot(dx+c) - \csc(dx+c)) + 13 \cdot 2^{1/2} \cdot \cos(dx+c)^3 \cdot \sin(dx+c) - 2^{1/2} \cdot \cos(dx+c)^2 \cdot \sin(dx+c) + 3 \cdot \sin(dx+c) \cdot \cos(dx+c) \cdot 2^{1/2}) \cdot 2^{1/2}}{a}$$

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.75

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$= \frac{15\sqrt{2}\left(a\cos(dx+c)^3 + a\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) + \frac{2\sqrt{a\cos(dx+c)+a}\left(13\cos(dx+c)^2 - \cos(dx+c) + 3\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{45\left(ad\cos(dx+c)^3 + ad\cos(dx+c)^2\right)}$$

[In] integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/15*(15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 1006, normalized size of antiderivative = 5.32

$$\int \frac{\sec^{\frac{7}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/15*(15*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1))) - 26*(cos(2*d*x + 2*c)^2*sin(d*x + c) + s

$$\begin{aligned} & \sin(2dx + 2c)^2 \sin(dx + c) + 2\cos(2dx + 2c) \sin(dx + c) + \sin(dx + c) \\ & \cos\left(\frac{1}{2}\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) + 24\cos\left(\frac{5}{2}\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) \sin(dx + c) \\ & - 24(\cos(dx + c) - 1) \sin\left(\frac{5}{2}\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) + 2\left(\left(13\cos(dx + c) - 15\right)\cos(2dx + 2c)^2 + \left(13\cos(dx + c) - 15\right)\sin(2dx + 2c)^2 + 2\left(13\cos(dx + c) - 15\right)\cos(2dx + 2c) + 13\cos(dx + c) - 15\right) \sin\left(\frac{1}{2}\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) - 4\sqrt{\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1} \left(7\cos\left(\frac{3}{2}\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) \sin(dx + c) - \left(7\cos(dx + c) - 5\right) \sin\left(\frac{3}{2}\arctan2(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right)\right) / \left(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1\right)^{5/4} \sqrt{a} dx \end{aligned}$$

Giac [F]

$$\int \frac{\sec^{7/2}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{7/2}}{\sqrt{a \cos(dx + c) + a}} dx$$

[In] integrate(sec(dx+c)^(7/2)/(a+a*cos(dx+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(dx + c)^(7/2)/sqrt(a*cos(dx + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{7/2}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{7/2}}{\sqrt{a + a \cos(c + dx)}} dx$$

[In] int((1/cos(c + d*x))^(7/2)/(a + a*cos(c + d*x))^(1/2),x)

[Out] int((1/cos(c + d*x))^(7/2)/(a + a*cos(c + d*x))^(1/2), x)

$$3.368 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	4406
Rubi [A] (verified)	4406
Mathematica [C] (warning: unable to verify)	4409
Maple [A] (verified)	4409
Fricas [A] (verification not implemented)	4410
Sympy [F(-1)]	4410
Maxima [C] (verification not implemented)	4410
Giac [F]	4411
Mupad [F(-1)]	4411

Optimal result

Integrand size = 25, antiderivative size = 151

$$\begin{aligned} & \int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx \\ &= \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{\sqrt{ad}} \\ & \quad - \frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{3d\sqrt{a+a \cos(c+dx)}} + \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a \cos(c+dx)}} \end{aligned}$$

[Out] $2/3*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}+\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/a^{(1/2)}-2/3*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2858, 3063, 12, 2861, 211}

$$\begin{aligned} & \int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx \\ &= \frac{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} \\ & \quad + \frac{2 \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a \cos(c+dx)+a}} - \frac{2 \sin(c+dx) \sqrt{\sec(c+dx)}}{3d\sqrt{a \cos(c+dx)+a}} \end{aligned}$$

[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2858

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+a\cos(c+dx)}} dx \\
&= \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{a-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+a\cos(c+dx)}} dx}{3a} \\
&= -\frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int -\frac{3a^2}{2\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx}{3a^2} \\
&= -\frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx \\
&= -\frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(2a\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} \right)}{d} \\
&= \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{\sqrt{ad}} \\
&\quad - \frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.46 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.15

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx =$$

$$2 \cot\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^4\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{1}{1-2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)^{7/2} \left(12 \cos^4\left(\frac{1}{2}(c+dx)\right) {}_3F_2\left(2, 2, \frac{7}{2}; 1, \frac{9}{2}; \frac{\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{-1+2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)\right)$$

[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^(7/2)*(12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)])*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2)))/(63*d*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 6.80 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.11

method	result
default	$-\frac{\left(\sec^{\frac{5}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\left(3\cos^3(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))+\sqrt{2}(\cos^2(dx+c))\sin(dx+c)+3\right)}{3d(1+\cos(dx+c))a}$

[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3/d*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))*(3*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))-sin(d*x+c)*cos(d*x+c)*2^(1/2))*2^(1/2)/a

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{3\sqrt{2}(a \cos(dx+c)^2 + a \cos(dx+c)) \arctan\left(\frac{\sqrt{2}\sqrt{a \cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) + 2\sqrt{a \cos(dx+c)+a}(\cos(dx+c)-1) \sin(dx+c)}{3(ad \cos(dx+c)^2 + ad \cos(dx+c))\sqrt{a} \sqrt{\cos(dx+c)}}$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] -1/3*(3*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 818, normalized size of antiderivative = 5.42

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

```
[Out] 1/3*(3*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) +
```

$1)^{1/4} \sin(1/2 \arctan(2(\cos(dx+c)-1)\sin(dx+c)/\sqrt{e^{i(dx+c)}+1})) / \sqrt{e^{i(dx+c)}+1}$, $(\sqrt{e^{i(dx+c)}+1} + \cos(dx+c)^2 - \sin(dx+c)^2 - 2\cos(dx+c) + 1) / \sqrt{e^{i(dx+c)}+1}$, $(\sqrt{e^{i(dx+c)}+1} + \cos(dx+c)^4 + \sin(dx+c)^4 + 2(\cos(dx+c)^2 - \sin(dx+c)^2 - 2\cos(dx+c) + 1)\sqrt{a}\cos(1/2 \arctan(2(\cos(dx+c)-1)\sin(dx+c)/\sqrt{e^{i(dx+c)}+1}))^2, (\sqrt{e^{i(dx+c)}+1} + \cos(dx+c)^2 - \sin(dx+c)^2 - 2\cos(dx+c) + 1) / \sqrt{e^{i(dx+c)}+1}$, $\sqrt{a}\cos(dx+c) - \sqrt{a} / (\sqrt{a}\sqrt{e^{i(dx+c)}+1}) - 2(\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c) + 1)^{3/4} (\cos(1/2 \arctan(\sin(2dx+2c), \cos(2dx+2c) + 1))\sin(dx+c) - (\cos(dx+c) - 3)\sin(1/2 \arctan(\sin(2dx+2c), \cos(2dx+2c) + 1))) - 4(\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c) + 1)^{1/4} (\cos(3/2 \arctan(\sin(2dx+2c), \cos(2dx+2c) + 1))\sin(dx+c) - (\cos(dx+c) + 1)\sin(3/2 \arctan(\sin(2dx+2c), \cos(2dx+2c) + 1)))) / ((\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c) + 1)\sqrt{a}d)$

Giac [F]

$$\int \frac{\sec^{5/2}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx = \int \frac{\sec(dx+c)^{5/2}}{\sqrt{a\cos(dx+c)+a}} dx$$

[In] integrate(sec(dx+c)^(5/2)/(a+a*cos(dx+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(dx+c)^(5/2)/sqrt(a*cos(dx+c)+a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{5/2}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\sqrt{a+a\cos(c+dx)}} dx$$

[In] int((1/cos(c+dx))^(5/2)/(a+a*cos(c+dx))^(1/2),x)

[Out] int((1/cos(c+dx))^(5/2)/(a+a*cos(c+dx))^(1/2), x)

$$3.369 \quad \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	4412
Rubi [A] (verified)	4412
Mathematica [C] (warning: unable to verify)	4414
Maple [A] (verified)	4415
Fricas [A] (verification not implemented)	4415
Sympy [F]	4415
Maxima [C] (verification not implemented)	4416
Giac [F]	4416
Mupad [F(-1)]	4417

Optimal result

Integrand size = 25, antiderivative size = 113

$$\begin{aligned} & \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx \\ &= -\frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{\sqrt{ad}} \\ & \quad + \frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a \cos(c+dx)}} \end{aligned}$$

[Out] $-\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/a^{(1/2)}+2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2858, 12, 2861, 211}

$$\begin{aligned} & \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx \\ &= \frac{2 \sin(c+dx) \sqrt{\sec(c+dx)}}{d\sqrt{a \cos(c+dx)+a}} \\ & \quad - \frac{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} \end{aligned}$$

[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]

[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2858

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx \\ &= \frac{2 \sqrt{\sec(c + dx)} \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}} - \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{a}{\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} dx}{a} \end{aligned}$$

$$\begin{aligned}
&= \frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx \\
&= \frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(2a\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} \right)}{d} \\
&= -\frac{\sqrt{2} \arctan \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{\sqrt{ad}} \\
&\quad + \frac{2\sqrt{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.54 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.59

$$\begin{aligned}
&\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx \\
&= \frac{2 \cos\left(\frac{1}{2}(c+dx)\right) \sec^{\frac{3}{2}}(c+dx) \sin\left(\frac{1}{2}(c+dx)\right) \left(\frac{1}{2} \cos(c+dx)(2+\cos(c+dx)) \csc^4\left(\frac{1}{2}(c+dx)\right) (1-\cos(c+dx))\right)}{\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 + Cos[c + d*x])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)])*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (Hypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/10)/(d*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 6.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.20

method	result
default	$\frac{\left(\sec^{\frac{3}{2}}(dx+c)\right)\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))+\sqrt{2}\sin(dx+c)+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))\right)}{d(1+\cos(dx+c))a}$

[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] $1/d*\sec(d*x+c)^{(3/2)}*(\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\arcsin(\cot(d*x+c)-\csc(d*x+c))+2^{(1/2)}*\sin(d*x+c)+(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\arcsin(\cot(d*x+c)-\csc(d*x+c)))*\cos(d*x+c)*(a*(1+\cos(d*x+c)))^{(1/2)}/(1+\cos(d*x+c))*2^{(1/2)}/a$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}} + \frac{2\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{ad\cos(dx+c)+ad}$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $(\sqrt{2}*(a*\cos(d*x+c)+a)*\arctan(\sqrt{2}*\sqrt{a*\cos(d*x+c)+a}*\sqrt{\cos(d*x+c)})/(\sqrt{a}*\sin(d*x+c)))/\sqrt{a}+2*\sqrt{a*\cos(d*x+c)+a}*\sin(d*x+c)/\sqrt{\cos(d*x+c)})/(a*d*\cos(d*x+c)+a*d)$

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a(\cos(c+dx)+1)}} dx$$

[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(sec(c+d*x)**(3/2)/sqrt(a*(cos(c+d*x)+1)),x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 665, normalized size of antiderivative = 5.88

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx$$

$$2 \cos\left(\frac{1}{2} \arctan(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right) \sin(dx + c) - 2(\cos(dx + c) - 1) \sin\left(\frac{1}{2} \arctan(\sin(2dx + 2c), \cos(2dx + 2c) + 1)\right)$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] (2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - 2*(cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))) - sqrt(2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1))))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)*d)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{\sqrt{a \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\sqrt{a + a \cos(c + dx)}} dx$$

```
[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(1/2), x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(1/2), x)
```

$$3.370 \quad \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	4418
Rubi [A] (verified)	4418
Mathematica [A] (verified)	4419
Maple [A] (verified)	4420
Fricas [A] (verification not implemented)	4420
Sympy [F]	4421
Maxima [C] (verification not implemented)	4421
Giac [F]	4422
Mupad [F(-1)]	4422

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a \cos(c+dx)}} dx = \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{2}\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}}$$

[Out] $\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)}*2^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)}}*2^{(1/2)/d/a^{(1/2)}}$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4307, 2861, 211}

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a \cos(c+dx)}} dx = \frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}}$$

[In] $\text{Int}[\text{Sqrt}[\text{Sec}[c + d*x]]/\text{Sqrt}[a + a*\text{Cos}[c + d*x]], x]$

[Out] $(\text{Sqrt}[2]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sin}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])]*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[c + d*x]])/(\text{Sqrt}[a]*d)$

Rule 211

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx \\ &= - \frac{\left(2a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} \right)}{d} \\ &= \frac{\sqrt{2} \arctan \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{\sqrt{ad}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\begin{aligned} &\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}} dx \\ &= \frac{2 \arctan \left(\frac{\sin(\frac{1}{2}(c+dx))}{\sqrt{\cos(c+dx)}} \right) \cos \left(\frac{1}{2}(c+dx) \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{d \sqrt{a(1+\cos(c+dx))}} \end{aligned}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]])*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(d*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 6.87 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.48

method	result	size
default	$-\frac{\arcsin(\cot(dx+c)-\csc(dx+c))(\sqrt{\sec(dx+c)})\sqrt{a(1+\cos(dx+c))}\cos(dx+c)\sqrt{2}}{d(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}a}$	83

[In] `int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `-1/d*arcsin(cot(d*x+c)-csc(d*x+c))*sec(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(1+cos(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a`

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}} \log\left(-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, \right.$$

$$\left. -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{ad}} \right]$$

[In] `integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x,algorithm="fricas")`

[Out] `[1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(d*x+c)+a)*sqrt(-1/a)*sqrt(cos(d*x+c))*sin(d*x+c)-3*cos(d*x+c)^2-2*cos(d*x+c)+1)/(cos(d*x+c)^2+2*cos(d*x+c)+1))/d,-sqrt(2)*arctan(sqrt(2)*sqrt(a*cos(d*x+c)+a)*sqrt(cos(d*x+c))/(sqrt(a)*sin(d*x+c)))/(sqrt(a)*d)]`

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}} dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a(\cos(c+dx)+1)}} dx$$

[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2), x)

[Out] Integral(sqrt(sec(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 522, normalized size of antiderivative = 9.32

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}} dx$$

$$= \frac{\sqrt{2} \arctan \left(\frac{|e^{(i dx + i c)} + 1|^4 + \cos(dx+c)^4 + \sin(dx+c)^4 + 2(\cos(dx+c)^2 - \sin(dx+c)^2 - 2\cos(dx+c) + 1)|e^{(i dx + i c)} + 1|^2 - 4\cos(dx+c)^3 + 2(\cos(dx+c)^2 - \sin(dx+c)^2 - 2\cos(dx+c) + 1)}{\dots} \right)}{\dots}$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] sqrt(2)*arctan2(((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sin(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sin(d*x + c))/abs(e^(I*d*x + I*c) + 1), ((abs(e^(I*d*x + I*c) + 1)^4 + cos(d*x + c)^4 + sin(d*x + c)^4 + 2*(cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)*abs(e^(I*d*x + I*c) + 1)^2 - 4*cos(d*x + c)^3 + 2*(cos(d*x + c)^2 - 2*cos(d*x + c) + 1)*sin(d*x + c)^2 + 6*cos(d*x + c)^2 - 4*cos(d*x + c) + 1)^(1/4)*sqrt(a)*cos(1/2*arctan2(2*(cos(d*x + c) - 1)*sin(d*x + c)/abs(e^(I*d*x + I*c) + 1)^2, (abs(e^(I*d*x + I*c) + 1)^2 + cos(d*x + c)^2 - sin(d*x + c)^2 - 2*cos(d*x + c) + 1)/abs(e^(I*d*x + I*c) + 1)^2)) + sqrt(a)*cos(d*x + c) - sqrt(a))/(sqrt(a)*abs(e^(I*d*x + I*c) + 1)))/(sqrt(a)*d)

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}} dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{a\cos(dx+c)+a}} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/sqrt(a*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{\sqrt{a+a\cos(c+dx)}} dx$$

[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(1/2),x)

[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(1/2), x)

$$3.371 \quad \int \frac{1}{\sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}} dx$$

Optimal result	4423
Rubi [A] (verified)	4423
Mathematica [C] (verified)	4425
Maple [A] (verified)	4426
Fricas [A] (verification not implemented)	4426
Sympy [F]	4426
Maxima [C] (verification not implemented)	4427
Giac [F]	4427
Mupad [F(-1)]	4428

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \frac{1}{\sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}} dx = \frac{2 \arctan\left(\frac{\sqrt{a} \sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sqrt{\sec(c+dx)} \sin(c+dx)}{\sqrt{2} \sqrt{a+a \cos(c+dx)}}\right)}{\sqrt{ad}}$$

[Out] $2*\arctan(\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)})/d/a^{(1/2)}-\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*\sec(d*x+c)^{(1/2)*2^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)})}*2^{(1/2)/d/a^{(1/2)}}$

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2856, 2853, 222, 2861, 211}

$$\int \frac{1}{\sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}} dx = \frac{2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}} - \frac{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{\sqrt{ad}}$$

[In] Int[1/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]

[Out] $(2 \operatorname{ArcSin}[\sqrt{a} \sin[c + dx]] / \sqrt{a + a \cos[c + dx]}) \sqrt{\cos[c + dx]} \sqrt{\sec[c + dx]} / (\sqrt{a} d) - (\sqrt{2} \operatorname{ArcTan}[\sqrt{a} \sin[c + dx]] / (\sqrt{2} \sqrt{\cos[c + dx]} \sqrt{a + a \cos[c + dx]})) \sqrt{\cos[c + dx]} \sqrt{\sec[c + dx]} / (\sqrt{a} d)$

Rule 211

$\operatorname{Int}[(a_) + (b_)(x_)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Rt}[a/b, 2]/a \operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{PosQ}[a/b]$

Rule 222

$\operatorname{Int}[1/\sqrt{(a_) + (b_)(x_)^2}, x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSin}[\operatorname{Rt}[-b, 2] \cdot (x/\sqrt{a})] / \operatorname{Rt}[-b, 2], x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{GtQ}[a, 0] \ \&\& \ \operatorname{NegQ}[b]$

Rule 2853

$\operatorname{Int}[\sqrt{(a_) + (b_)\sin[(e_) + (f_)(x_)]} / \sqrt{(d_)\sin[(e_) + (f_)(x_)]}, x_Symbol] \rightarrow \operatorname{Dist}[-2/f, \operatorname{Subst}[\operatorname{Int}[1/\sqrt{1 - x^2/a}], x], x, b \cdot (\cos[e + fx] / \sqrt{a + b \sin[e + fx]])], x] /; \operatorname{FreeQ}\{a, b, d, e, f, x\} \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{EqQ}[d, a/b]$

Rule 2856

$\operatorname{Int}[\sqrt{(c_) + (d_)\sin[(e_) + (f_)(x_)]} / \sqrt{(a_) + (b_)\sin[(e_) + (f_)(x_)]}, x_Symbol] \rightarrow \operatorname{Dist}[d/b, \operatorname{Int}[\sqrt{a + b \sin[e + fx]} / \sqrt{c + d \sin[e + fx]], x], x] + \operatorname{Dist}[(b \cdot c - a \cdot d)/b, \operatorname{Int}[1/(\sqrt{a + b \sin[e + fx]} \sqrt{c + d \sin[e + fx]])], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \operatorname{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 2861

$\operatorname{Int}[1/(\sqrt{(a_) + (b_)\sin[(e_) + (f_)(x_)]} \sqrt{(c_) + (d_)\sin[(e_) + (f_)(x_)]}), x_Symbol] \rightarrow \operatorname{Dist}[-2 \cdot (a/f), \operatorname{Subst}[\operatorname{Int}[1/(2 \cdot b^2 - (a \cdot c - b \cdot d) \cdot x^2)], x], x, b \cdot (\cos[e + fx] / (\sqrt{a + b \sin[e + fx]} \sqrt{c + d \sin[e + fx]}))], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \operatorname{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 4307

$\operatorname{Int}[(\csc[(a_) + (b_)(x_)] \cdot (c_))^{(m_)} \cdot (u_), x_Symbol] \rightarrow \operatorname{Dist}[(c \cdot \csc[a + bx])^m \cdot (c \cdot \sin[a + bx])^m, \operatorname{Int}[\operatorname{ActivateTrig}[u] / (c \cdot \sin[a + bx])^m, x], x] /; \operatorname{FreeQ}\{a, b, c, m, x\} \ \&\& \ !\operatorname{IntegerQ}[m] \ \&\& \ \operatorname{KnownSineIntegrandQ}[u, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}} dx \\
 &= - \left(\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx \right) \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{a} \\
 &= - \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right)}{ad} \\
 &\quad + \frac{\left(2a\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} \right)}{d} \\
 &= \frac{2 \arcsin \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{\sqrt{ad}} \\
 &\quad - \frac{\sqrt{2} \arctan \left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{\sqrt{ad}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.65

$$\begin{aligned}
 &\int \frac{1}{\sqrt{a+a\cos(c+dx)} \sqrt{\sec(c+dx)}} dx \\
 &= \frac{i\sqrt{2}e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left(-\text{arcsinh}(e^{i(c+dx)}) + \sqrt{2} \text{arctanh} \left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}} \right) \right) + \text{arctanh}(\sqrt{1+e^{2i(c+dx)}})}{d\sqrt{a(1+\cos(c+dx))}}
 \end{aligned}$$

[In] Integrate[1/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]

[Out] (I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[(c + d*x)/2])/(d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{\sqrt{a(1+\cos(dx+c))} \left(\sqrt{2} \arctan \left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \right) + \arcsin(\cot(dx+c) - \csc(dx+c)) \right) \sqrt{2}}{d(1+\cos(dx+c)) \sqrt{\sec(dx+c)} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} a}$	108

[In] `int(1/sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{d} \frac{\sqrt{a(1+\cos(dx+c))} \left(\sqrt{2} \arctan \left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \right) + \arcsin(\cot(dx+c) - \csc(dx+c)) \right) \sqrt{2}}{\sqrt{\sec(dx+c)} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} a}$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} dx = \frac{\sqrt{2}\sqrt{a} \arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - 2\sqrt{a} \arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{ad}$$

[In] `integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] $\frac{(\sqrt{2}\sqrt{a}\arctan(\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)})/(\sqrt{a}\sin(dx+c))) - 2\sqrt{a}\arctan(\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)})/(\sqrt{a}\sin(dx+c)))}{(a*d)}$

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} dx = \int \frac{1}{\sqrt{a(\cos(c+dx)+1)}\sqrt{\sec(c+dx)}} dx$$

[In] `integrate(1/sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)`

[Out] `Integral(1/(sqrt(a*(cos(c+d*x)+1))*sqrt(sec(c+d*x))), x)`

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 698, normalized size of antiderivative = 6.65

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}} dx = \sqrt{2} \sqrt{a} \arctan \left(\frac{(|2e^{(i dx + i c)} + 2|^4 + 16 \cos(dx + c)^4 + 16 \sin(dx + c)^4 + 8 (\cos(dx + c)^2 - \sin(dx + c)^2 - 2 \cos(dx + c) + 1) |2e^{(i dx + i c)} + 2|^2 - 6}{\dots} \right)$$

[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] $-(\sqrt{2} \sqrt{a} \arctan2(((\text{abs}(2e^{(I*d*x + I*c)} + 2)^4 + 16 \cos(d*x + c)^4 + 16 \sin(d*x + c)^4 + 8 * (\cos(d*x + c)^2 - \sin(d*x + c)^2 - 2 \cos(d*x + c) + 1) * \text{abs}(2e^{(I*d*x + I*c)} + 2)^2 - 64 \cos(d*x + c)^3 + 32 * (\cos(d*x + c)^2 - 2 \cos(d*x + c) + 1) * \sin(d*x + c)^2 + 96 \cos(d*x + c)^2 - 64 \cos(d*x + c) + 16)^{1/4} * \sin(1/2 * \arctan2(8 * (\cos(d*x + c) - 1) * \sin(d*x + c) / \text{abs}(2e^{(I*d*x + I*c)} + 2)^2, (\text{abs}(2e^{(I*d*x + I*c)} + 2)^2 + 4 \cos(d*x + c)^2 - 4 \sin(d*x + c)^2 - 8 \cos(d*x + c) + 4) / \text{abs}(2e^{(I*d*x + I*c)} + 2)^2))) + 2 * \sin(d*x + c) / \text{abs}(2e^{(I*d*x + I*c)} + 2), ((\text{abs}(2e^{(I*d*x + I*c)} + 2)^4 + 16 \cos(d*x + c)^4 + 16 \sin(d*x + c)^4 + 8 * (\cos(d*x + c)^2 - \sin(d*x + c)^2 - 2 \cos(d*x + c) + 1) * \text{abs}(2e^{(I*d*x + I*c)} + 2)^2 - 64 \cos(d*x + c)^3 + 32 * (\cos(d*x + c)^2 - 2 \cos(d*x + c) + 1) * \sin(d*x + c)^2 + 96 \cos(d*x + c)^2 - 64 \cos(d*x + c) + 16)^{1/4} * \cos(1/2 * \arctan2(8 * (\cos(d*x + c) - 1) * \sin(d*x + c) / \text{abs}(2e^{(I*d*x + I*c)} + 2)^2, (\text{abs}(2e^{(I*d*x + I*c)} + 2)^2 + 4 \cos(d*x + c)^2 - 4 \sin(d*x + c)^2 - 8 \cos(d*x + c) + 4) / \text{abs}(2e^{(I*d*x + I*c)} + 2)^2))) + 2 * \cos(d*x + c) - 2) / \text{abs}(2e^{(I*d*x + I*c)} + 2)) - \sqrt{a} * \arctan2((\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2 * \cos(2*d*x + 2*c) + 1)^{1/4} * \sin(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) + \sin(d*x + c), (\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2 * \cos(2*d*x + 2*c) + 1)^{1/4} * \cos(1/2 * \arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c) + 1)) + \cos(d*x + c))) / (a*d)$

Giac [F]

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{a \cos(dx + c) + a} \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} \sqrt{a + a \cos(c + dx)}} dx$$

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(1/2)), x)
```


$$3.372 \quad \int \frac{1}{\sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4429
Rubi [A] (verified)	4429
Mathematica [C] (verified)	4432
Maple [A] (verified)	4433
Fricas [A] (verification not implemented)	4433
Sympy [F]	4433
Maxima [F]	4434
Giac [F(-1)]	4434
Mupad [F(-1)]	4434

Optimal result

Integrand size = 25, antiderivative size = 168

$$\begin{aligned} & \int \frac{1}{\sqrt{a+a \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} dx \\ &= -\frac{\arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{\sqrt{ad}} \\ & \quad + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{\sqrt{ad}} \\ & \quad + \frac{\sin(c+dx)}{d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}} \end{aligned}$$

[Out] $\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}/\sec(d*x+c)^{(1/2)}-\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/a^{(1/2)}+\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*2^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/a^{(1/2)}$

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {4307, 2857, 3061, 2861, 211, 2853, 222}

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx$$

$$= -\frac{\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{\sqrt{ad}}$$

$$+ \frac{\sqrt{2} \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \arctan\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2} \sqrt{\cos(c + dx)} \sqrt{a \cos(c + dx) + a}}\right)}{\sqrt{ad}}$$

$$+ \frac{\sin(c + dx)}{d \sqrt{\sec(c + dx)} \sqrt{a \cos(c + dx) + a}}$$

[In] Int[1/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]

[Out] -((ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]))

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2857

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1)), Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}} dx \\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2a} \\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad + \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{2a}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} \\
&\quad - \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&= -\frac{\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{ad}} \\
&\quad + \frac{\sqrt{2} \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{ad}} \\
&\quad + \frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} dx$$

$$= \frac{ie^{-2i(c+dx)}(1+e^{i(c+dx)})\left(1-e^{i(c+dx)}+e^{2i(c+dx)}-e^{3i(c+dx)}+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arcsinh}(e^{i(c+dx)})+2\sqrt{2}e^{i(c+dx)}\right)}{4d\sqrt{a(1+\cos(c+dx))}}$$

[In] Integrate[1/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]

[Out] ((I/4)*(1 + E^(I*(c + d*x)))*(1 - E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) - E^((3*I)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))])*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) - E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[Sec[c + d*x]]/(d*E^((2*I)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 13.82 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\left(-\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+2\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(1+\cos(dx+c))}\sqrt{2}}{2d\sqrt{\sec(dx+c)}(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}a}$

[In] int(1/sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out]
$$-1/2/d/\sec(d*x+c)^{(1/2)}*(-\sin(d*x+c)*2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}+2^{(1/2)}*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}+2*\arcsin(\cot(d*x+c)-\csc(d*x+c)))*(a*(1+\cos(d*x+c)))^{(1/2)/(1+\cos(d*x+c))}/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)}/a$$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} dx$$

$$= \frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - \frac{\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{ad\cos(dx+c)+ad} + v$$

[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out]
$$\left(\frac{\sqrt{a}(\cos(dx+c)+1)\arctan(\sqrt{a\cos(dx+c)+a})\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)} - \sqrt{2}(a\cos(dx+c)+a)\arctan(\sqrt{2}\sqrt{a\cos(dx+c)+a})\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)/\sqrt{a} + \sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}/(a*d\cos(dx+c)+a*d)$$

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} dx = \int \frac{1}{\sqrt{a(\cos(c+dx)+1)}\sec^{\frac{3}{2}}(c+dx)} dx$$

[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(a*(cos(c+d*x)+1))*sec(c+d*x)**(3/2)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{a \cos(dx + c) + a} \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + a \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} \sqrt{a + a \cos(c + dx)}} dx$$

[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)),x)

[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(1/2)), x)

$$3.373 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	4435
Rubi [A] (verified)	4435
Mathematica [C] (warning: unable to verify)	4438
Maple [A] (verified)	4438
Fricas [A] (verification not implemented)	4439
Sympy [F(-1)]	4439
Maxima [F]	4440
Giac [F]	4440
Mupad [F(-1)]	4440

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{11 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{6ad\sqrt{a+a \cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}} + \frac{7 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{6ad\sqrt{a+a \cos(c+dx)}}$$

[Out] $-1/2*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}+7/6*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(1/2)}+11/4*\arctan(1/2*\sin(d*x+c)*a^{(1/2)})*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d*2^{(1/2)}-19/6*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2845, 3063, 12, 2861, 211}

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{11 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{7 \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a \cos(c+dx)+a}} - \frac{\sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{2d(a \cos(c+dx)+a)^{3/2}} - \frac{19 \sin(c+dx) \sqrt{\sec(c+dx)}}{6ad\sqrt{a \cos(c+dx)+a}}$$

[In] $\text{Int}[\text{Sec}[c+d*x]^{(5/2)}/(a+a*\text{Cos}[c+d*x])^{(3/2)},x]$

```
[Out] (11*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - (19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]]) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (7*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^m*(c + d*Ssin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```


Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx \\
&= -\frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{7a}{2} - 2a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
&= -\frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{19a^2}{4} + \frac{7}{2}a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx}{3a^3} \\
&= -\frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{33a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{3a^4} \\
&= -\frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(11\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
&= -\frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(11\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d}
\end{aligned}$$

$$= \frac{11 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{6ad\sqrt{a+a \cos(c+dx)}} - \frac{\sec^{3/2}(c+dx) \sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2}} + \frac{7 \sec^{3/2}(c+dx) \sin(c+dx)}{6ad\sqrt{a+a \cos(c+dx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.00 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.00

$$\int \frac{\sec^{5/2}(c+dx)}{(a+a \cos(c+dx))^{3/2}} dx = \frac{\cot^3\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^4\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^2\left(\frac{1}{2}(c+dx)\right) \left(\frac{1}{1-2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)^{7/2} \left(-80 \cos^6\left(\frac{1}{2}(c+dx)\right) + \dots\right)}{(a+a \cos(c+dx))^{3/2}}$$

[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2), x]

[Out] (Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^2*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^(7/2)*(-80*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 + 120*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2) + 21*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-15*ArcTan h[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*(-392 + 2347*Sin[c/2 + (d*x)/2]^2 - 5391*Sin[c/2 + (d*x)/2]^4 + 5972*Sin[c/2 + (d*x)/2]^6 - 3232*Sin[c/2 + (d*x)/2]^8 + 696*Sin[c/2 + (d*x)/2]^10) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5880 + 37165*Sin[c/2 + (d*x)/2]^2 - 89856*Sin[c/2 + (d*x)/2]^4 + 103992*Sin[c/2 + (d*x)/2]^6 - 58336*Sin[c/2 + (d*x)/2]^8 + 12960*Sin[c/2 + (d*x)/2]^10)))/(945*d*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [A] (verified)

Time = 6.64 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.18

method	result
default	$-\frac{\left(\sec^{5/2}(dx+c)\right) \sqrt{a(1+\cos(dx+c))} \left(33\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))(\cos^4(dx+c))+19\sqrt{2}(\cos^3(dx+c)) \sin(dx+c)+\dots\right)}{(a+\cos(dx+c)*a)^{3/2}}$

[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2), x, method=_RETURNVERBOSE)

```
[Out] -1/12/d*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^2*(33*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^4+19
*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+66*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+12*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+33
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c
))-4*sin(d*x+c)*cos(d*x+c)*2^(1/2))*2^(1/2)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{3}{2}}} dx =$$

$$\frac{33\sqrt{2}(\cos(dx+c))^3 + 2\cos(dx+c)^2 + \cos(dx+c))\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) + \frac{2\sqrt{a\cos(dx+c)}}{12(a^2d\cos(dx+c))^3 + 2a^2d\cos(dx+c)^2 + a^2d\cos(dx+c)}}{12(a^2d\cos(dx+c))^3 + 2a^2d\cos(dx+c)^2 + a^2d\cos(dx+c)}$$

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(33*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a
)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(19*cos(d*x + c)^2 + 12*cos(d*x + c)
- 4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(
d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{5}{2}}}{(a + a \cos(c + dx))^{\frac{3}{2}}} dx$$

[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(3/2),x)

[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(3/2), x)

$$3.374 \quad \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{3}{2}}} dx$$

Optimal result	4441
Rubi [A] (verified)	4441
Mathematica [C] (warning: unable to verify)	4444
Maple [A] (verified)	4444
Fricas [A] (verification not implemented)	4445
Sympy [F]	4445
Maxima [F]	4445
Giac [F]	4446
Mupad [F(-1)]	4446

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{3}{2}}} dx =$$

$$\frac{7 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{2\sqrt{2}a^{\frac{3}{2}}d}$$

$$- \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{2d(a+a \cos(c+dx))^{\frac{3}{2}}} + \frac{5\sqrt{\sec(c+dx)} \sin(c+dx)}{2ad\sqrt{a+a \cos(c+dx)}}$$

[Out] $-1/2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d/(a+a*\cos(d*x+c))^{(3/2)}-7/4*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d*2^{(1/2)}+5/2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2845, 3063, 12, 2861, 211}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{3}{2}}} dx =$$

$$\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{\frac{3}{2}}d}$$

$$+ \frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}} - \frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\cos(c+dx)+a)^{\frac{3}{2}}}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + a*cos[c + d*x])^(3/2), x]

[Out] (-7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*cos[c + d*x])^(3/2)) + (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*sin[e + f*x]]*Sqrt[c + d*sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3063

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2), Int[(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{5a}{2} - a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)} \sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int -\frac{7a^2}{4\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{a^3} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)} \sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(7\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)} \sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(7\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d} \\
&= -\frac{7 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} \\
&\quad - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)} \sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.59

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \frac{-\frac{35}{2} \cot\left(\frac{1}{2}(c+dx)\right) \csc^4\left(\frac{1}{2}(c+dx)\right) \left(78 + 108 \cos(c+dx) + 80 \cos(2(c+dx))\right)}{(a+a\cos(c+dx))^{\frac{3}{2}}}$$

[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2), x]

[Out] ((-35*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4*(78 + 108*Cos[c + d*x] + 80*Cos[2*(c + d*x)] - 204*Cos[3*(c + d*x)] - 62*Cos[4*(c + d*x)] + 12*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^2*(64 + 55*Cos[c + d*x] + 64*Cos[2*(c + d*x)] + 17*Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Sqrt[Sec[c + d*x]])/2 - 768*Cos[(c + d*x)/2]^5*HypergeometricPFQ[{2, 2, 5/2}, {1, 9/2}, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sec[c + d*x]^(5/2)*Sin[(c + d*x)/2]^3)/(3360*d*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [A] (verified)

Time = 6.53 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

method	result
default	$\frac{\left(7\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (\cos^2(dx+c) \arcsin(\cot(dx+c)-\csc(dx+c))+5\sin(dx+c) \cos(dx+c)\sqrt{2}+14\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c)))\right)}{4d(1+\cos(dx+c))^{3/2}}$

[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/4/d*(7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))+5*sin(d*x+c)*cos(d*x+c)*2^(1/2)+14*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+4*2^(1/2)*sin(d*x+c)+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c)))*sec(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(1+cos(d*x+c))^2*2^(1/2)/a^2

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \frac{7\sqrt{2}(\cos(dx+c)^2+2\cos(dx+c)+1)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{4(a^2d\cos(dx+c))^2+2a^2d\cos(dx+c)+a^2d}$$

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*sq
rt(a*cos(d*x + c) + a)*(5*cos(d*x + c) + 4)*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}} dx$$

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{3}{2}}} dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}} dx$$

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{(a + a \cos(c + dx))^{3/2}} dx$$

[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(3/2),x)

[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(3/2), x)

$$3.375 \quad \int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^{3/2}} dx$$

Optimal result	4447
Rubi [A] (verified)	4447
Mathematica [A] (verified)	4449
Maple [A] (verified)	4449
Fricas [A] (verification not implemented)	4450
Sympy [F]	4450
Maxima [F]	4450
Giac [F(-1)]	4451
Mupad [F(-1)]	4451

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^{3/2}} dx = \frac{3 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] $-1/2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}/\sec(d*x+c)^{(1/2)}+3/4*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2845, 12, 2861, 211}

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^{3/2}} dx = \frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a \cos(c+dx)+a)^{3/2}}$$

[In] $\text{Int}[\text{Sqrt}[\text{Sec}[c+d*x]]/(a+a*\text{Cos}[c+d*x])^{(3/2)},x]$

[Out] $(3*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sin}[c+d*x])/(\text{Sqrt}[2]*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+a*\text{Cos}[c+d*x]])]*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[\text{Sec}[c+d*x]])/(2*\text{Sqrt}[2]*a^{(3/2)}*d) - \text{Sin}[c+d*x]/(2*d*(a+a*\text{Cos}[c+d*x])^{(3/2)}*\text{Sqrt}[\text{Sec}[c+d*x]])$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} (a + a \cos(c + dx))^{3/2}} dx \\ &= -\frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} \\ &\quad + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{3a}{2\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} dx}{2a^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
&= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d} \\
&= \frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} \\
&\quad - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \frac{\left(2+3\arctanh\left(\sqrt{-\sec(c+dx)}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cot^2\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\right)\tan\left(\frac{1}{2}(c+dx)\right)}{4ad\sqrt{a(1+\cos(c+dx))}\sqrt{\sec(c+dx)}}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(3/2), x]

[Out] -1/4*((2 + 3*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)])*Cot[(c + d*x)/2]^2*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(a*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 6.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\left(\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)+3\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(1+\cos(dx+c))}}{4d(1+\cos(dx+c))^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}a^2}$

[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2), x, method=_RETURNVERBOSE)

[Out] -1/4/d*(sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+3*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x

$+c)))^{1/2} * \sec(dx+c)^{1/2} * \cos(dx+c) / (1+\cos(dx+c))^{3/2} / (\cos(dx+c) / (1+\cos(dx+c)))^{1/2} * 2^{1/2} / a^2$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \frac{3\sqrt{2}(\cos(dx+c)^2 + 2\cos(dx+c) + 1)\sqrt{a} \arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) + 2\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{4(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c) + a^2d)}$$

[In] integrate(sec(dx+c)^(1/2)/(a+a*cos(dx+c))^(3/2),x, algorithm="fricas")

[Out] -1/4*(3*sqrt(2)*(cos(dx+c)^2 + 2*cos(dx+c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(dx+c)+a)*sqrt(cos(dx+c))/(sqrt(a)*sin(dx+c))) + 2*sqrt(a*cos(dx+c)+a)*sqrt(cos(dx+c))*sin(dx+c)/(a^2*d*cos(dx+c)^2 + 2*a^2*d*cos(dx+c) + a^2*d)

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\sec(c+dx)}}{(a(\cos(c+dx)+1))^{3/2}} dx$$

[In] integrate(sec(dx+c)**(1/2)/(a+a*cos(dx+c))**(3/2),x)

[Out] Integral(sqrt(sec(c+dx))/(a*(cos(c+dx)+1))**(3/2),x)

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{3/2}} dx$$

[In] integrate(sec(dx+c)^(1/2)/(a+a*cos(dx+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(sec(dx+c))/(a*cos(dx+c)+a)^(3/2),x)

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{(a+a\cos(c+dx))^{3/2}} dx$$

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(3/2), x)
```

$$3.376 \quad \int \frac{1}{(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} dx$$

Optimal result	4452
Rubi [A] (verified)	4452
Mathematica [A] (verified)	4454
Maple [A] (verified)	4454
Fricas [A] (verification not implemented)	4455
Sympy [F]	4455
Maxima [F]	4455
Giac [F(-1)]	4456
Mupad [F(-1)]	4456

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{1}{(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} dx = \frac{\arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] 1/2*sin(d*x+c)/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+1/4*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2843, 12, 2861, 211}

$$\int \frac{1}{(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} dx = \frac{\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a \cos(c+dx)+a)^{3/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]

[Out] (ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 211

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

Rule 2843

`Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))`

Rule 2861

`Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]`

Rule 4307

`Int[(csc[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]`

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}} dx \\ &= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\ &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{a}{2\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx}{2a^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
&= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d} \\
&= \frac{\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} \\
&\quad + \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} dx = \frac{\cos\left(\frac{1}{2}(c+dx)\right) \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{\sec(c+dx)} \left(\arcsin\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)\right)}{2ad\sqrt{a(1+\cos(c+dx))}}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]

[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]]* (ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[1 + Cos[c + d*x]] + 2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]))/(2*a*d*Sqrt[a*(1 + Cos[c + d*x])])

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12

method	result
default	$-\frac{\left(-\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)+\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(1+\cos(dx+c))}\sqrt{2}}{4d(1+\cos(dx+c))^2\sqrt{\sec(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}a^2}$

[In] int(1/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/4/d*(-sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))

$$\left. \right)^{(1/2)} / (1 + \cos(dx+c))^{3/2} / \sec(dx+c)^{(1/2)} / (\cos(dx+c) / (1 + \cos(dx+c)))^{(1/2)} \\ \left. \right) * 2^{(1/2)} / a^2$$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \frac{\sqrt{2}(\cos(dx + c)^2 + 2 \cos(dx + c) + 1) \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a \cos(dx+c) + a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) - 2 \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)}}{4(a^2 d \cos(dx + c)^2 + 2 a^2 d \cos(dx + c) + a^2 d)}$$

[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a (\cos(c + dx) + 1))^{3/2} \sqrt{\sec(c + dx)}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)

[Out] Integral(1/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a \cos(dx + c) + a)^{3/2} \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + a \cos(c + dx))^{3/2}} dx$$

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

$$3.377 \quad \int \frac{1}{(a+a \cos(c+dx))^{3/2} \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4457
Rubi [A] (verified)	4457
Mathematica [C] (verified)	4460
Maple [A] (verified)	4460
Fricas [A] (verification not implemented)	4461
Sympy [F(-1)]	4461
Maxima [F]	4461
Giac [F(-1)]	4462
Mupad [F(-1)]	4462

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{1}{(a+a \cos(c+dx))^{3/2} \sec^{\frac{3}{2}}(c+dx)} dx = \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{a^{3/2} d} - \frac{5 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{2\sqrt{2} a^{3/2} d} - \frac{\sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] $-1/2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}/\sec(d*x+c)^{(1/2)}+2*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d-5/4*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4307, 2844, 3061, 2861, 211, 2853, 222}

$$\int \frac{1}{(a+a \cos(c+dx))^{3/2} \sec^{\frac{3}{2}}(c+dx)} dx = \frac{2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a \cos(c+dx)+a}}\right)}{a^{3/2} d} - \frac{5 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{2\sqrt{2} a^{3/2} d} - \frac{\sin(c+dx)}{2d \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{3/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]

[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(a^(3/2)*d) - (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Ssin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3061

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dis

$t[(A*b - a*B)/b, \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] + \text{Dist}[B/b, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[c + d*\text{Sin}[e + f*x]], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 4307

$\text{Int}[(\text{csc}[a_.] + (b_.)*(x_.)]*(c_.)^{(m_.)}*(u_.), x_Symbol] \text{:>} \text{Dist}[(c*\text{Csc}[a + b*x])^m*(c*\text{Sin}[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u]/(c*\text{Sin}[a + b*x])^m, x], x] /;$
 $\text{FreeQ}\{a, b, c, m\}, x \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ \text{KnownSineIntegrandQ}[u, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}} dx \\
 &= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{a}{2} - 2a\cos(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx}{2a^2} \\
 &= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{a^2} \\
 &\quad - \frac{\left(5\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx}{4a} \\
 &= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(5\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}}\right)}{2d} \\
 &\quad - \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^2d} \\
 &= \frac{2 \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{a^{3/2}d} \\
 &\quad - \frac{5 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} \\
 &\quad - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.67 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx)} dx = \frac{ie^{-\frac{1}{2}i(c+dx)} \cos^3\left(\frac{1}{2}(c + dx)\right) \left(4e^{i(c+dx)} \operatorname{arcsinh}(e^{i(c+dx)}) + 5\sqrt{2}e^{i(c+dx)} \operatorname{arctanh}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right) - 4e^{i(c+dx)} \operatorname{arctanh}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}(a(1+\cos(c+dx)))}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]

[Out] ((-I)*Cos[(c + d*x)/2]^3*(4*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))] + 5*sqrt[2]*E^(I*(c + d*x))*ArcTanh[(1 - E^(I*(c + d*x)))/(sqrt[2]*sqrt[1 + E^((2*I)*(c + d*x))]]] - 4*E^(I*(c + d*x))*ArcTanh[sqrt[1 + E^((2*I)*(c + d*x))]] - I*sqrt[1 + E^((2*I)*(c + d*x))]*Tan[(c + d*x)/2] + sqrt[1 + E^((2*I)*(c + d*x))]*Tan[(c + d*x)/2]^2)/(d*E^((I/2)*(c + d*x))*sqrt[1 + E^((2*I)*(c + d*x))]*(a*(1 + Cos[c + d*x]))^(3/2)*sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\sqrt{a(1+\cos(dx+c))} \left(\tan(dx+c)\sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 4\sqrt{2} \arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) - 5 \arcsin(\cot(dx+c) - \csc(dx+c)) - 4 \sec(dx+c) \right)}{4d(1+\cos(dx+c))^2 \sec(dx+c)^{\frac{3}{2}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int(1/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^2/sec(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-4*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-5*arcsin(cot(d*x+c)-csc(d*x+c))-4*sec(d*x+c)*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-5*sec(d*x+c)*arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)/a^2

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx)} dx = \frac{5\sqrt{2}(\cos(dx + c)^2 + 2\cos(dx + c) + 1)\sqrt{a} \arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx + c) + a}}{\sqrt{a}\sin(dx + c)}\right) - 8(\cos(dx + c)^2 + 2\cos(dx + c) + 1)\sqrt{a} \arctan\left(\frac{\sqrt{a\cos(dx + c) + a}}{\sin(dx + c)}\right) - 2\sqrt{a\cos(dx + c) + a}\sqrt{\cos(dx + c)}\sin(dx + c)/(a^2 d \cos(dx + c)^2 + 2a^2 d \cos(dx + c) + a^2 d)}{a^2 d}$$

[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 8*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{3/2} \sec(dx + c)^{3/2}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + a \cos(c + dx))^{3/2}} dx$$

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(3/2)), x)
```

$$3.378 \quad \int \frac{1}{(a+a \cos(c+dx))^{3/2} \sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	4463
Rubi [A] (verified)	4463
Mathematica [C] (verified)	4466
Maple [A] (verified)	4467
Fricas [A] (verification not implemented)	4467
Sympy [F(-1)]	4468
Maxima [F]	4468
Giac [F(-1)]	4468
Mupad [F(-1)]	4468

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{1}{(a+a \cos(c+dx))^{3/2} \sec^{\frac{5}{2}}(c+dx)} dx =$$

$$\frac{3 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{a^{3/2} d}$$

$$+ \frac{9 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{2\sqrt{2} a^{3/2} d}$$

$$- \frac{\sin(c+dx)}{2d(a+a \cos(c+dx))^{3/2} \sec^{\frac{3}{2}}(c+dx)} + \frac{3 \sin(c+dx)}{2ad \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}$$

[Out] $-1/2*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(3/2)}/\sec(d*x+c)^{(3/2)}+3/2*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(1/2)}/\sec(d*x+c)^{(1/2)}-3*\arcsin(\sin(d*x+c)*a^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d+9/4*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(3/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used

= {4307, 2844, 3062, 3061, 2861, 211, 2853, 222}

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx)} dx =$$

$$\frac{3\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{a^{3/2}d}$$

$$+ \frac{9\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \arctan\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2}\sqrt{\cos(c + dx)}\sqrt{a \cos(c + dx) + a}}\right)}{2\sqrt{2}a^{3/2}d}$$

$$- \frac{\sin(c + dx)}{2d \sec^{3/2}(c + dx)(a \cos(c + dx) + a)^{3/2}} + \frac{3 \sin(c + dx)}{2ad \sqrt{\sec(c + dx)}\sqrt{a \cos(c + dx) + a}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]

[Out] (-3*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(a^(3/2)*d) + (9*ArcTan[(Sqrt[a]*Sin[c + d*x])/Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (3*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos

$[e + f*x]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]]], x] /; \text{FreeQ}\{a, b, d, e, f\}, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[d, a/b]$

Rule 2861

$\text{Int}[1/(\text{Sqrt}[a_] + (b_)*\text{sin}[(e_) + (f_)*(x_)])*\text{Sqrt}[(c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])], x_Symbol] \rightarrow \text{Dist}[-2*(a/f), \text{Subst}[\text{Int}[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(\text{Cos}[e + f*x]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]))], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rule 3061

$\text{Int}[(A_) + (B_)*\text{sin}[(e_) + (f_)*(x_)]) / (\text{Sqrt}[a_] + (b_)*\text{sin}[(e_) + (f_)*(x_)])*\text{Sqrt}[(c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)]), x_Symbol] \rightarrow \text{Dist}[(A*b - a*B)/b, \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] + \text{Dist}[B/b, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[c + d*\text{Sin}[e + f*x]], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rule 3062

$\text{Int}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\text{sin}[(e_) + (f_)*(x_)])*((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-B)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m_)}*((c + d*\text{Sin}[e + f*x])^{(n_)} / (f*(m + n + 1))), x] + \text{Dist}[1/(b*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m_)}*(c + d*\text{Sin}[e + f*x])^{(n_ - 1)}*\text{Simp}[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[n, 0] \&\& (\text{IntegerQ}[n] \parallel \text{EqQ}[m + 1/2, 0])$

Rule 4307

$\text{Int}[(\text{csc}[a_] + (b_)*(x_)]*(c_)^{(m_)}*(u_), x_Symbol] \rightarrow \text{Dist}[(c*\text{Csc}[a + b*x])^{(m_)}*(c*\text{Sin}[a + b*x])^{(m_)}, \text{Int}[\text{ActivateTrig}[u]/(c*\text{Sin}[a + b*x])^{(m_)}, x], x] /; \text{FreeQ}\{a, b, c, m\}, x\} \&\& !\text{IntegerQ}[m] \&\& \text{KnownSineIntegrandQ}[u, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{3/2}} dx \\ &= \frac{\sin(c + dx)}{2d(a + a \cos(c + dx))^{3/2} \sec^{\frac{3}{2}}(c + dx)} \\ &\quad - \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\cos(c + dx)} \left(\frac{3a}{2} - 3a \cos(c + dx) \right)}{\sqrt{a + a \cos(c + dx)}} dx}{2a^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{3/2}(c+dx)} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{-\frac{3a^2}{2}+3a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2a^3} \\
&= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{3/2}(c+dx)} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{2a^2} \\
&\quad + \frac{\left(9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a} \\
&= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{3/2}(c+dx)} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d} \\
&\quad + \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^2d} \\
&= -\frac{3\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} \\
&\quad + \frac{9\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} \\
&\quad - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{3/2}(c+dx)} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.71 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{5/2}(c+dx)} dx = \frac{\cos^3\left(\frac{1}{2}(c+dx)\right) \left(3i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\right) \left(2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right) + 9\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\right)}{2a^{3/2}d}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]

[Out] (Cos[(c + d*x)/2]^3*(((3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(2*ArcSinh[E^(I*(c + d*x))]] + 3*Sqrt[2]*ArcTanh[(1 - E^(I*(c + d*x))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])))/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2))

- 2*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/E^((I/2)*(c + d*x)) + Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(-Sin[(c + d*x)/2] + 2*Sin[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))

Maple [A] (verified)

Time = 15.05 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.12

method	result
default	$\frac{(2\sqrt{2} \cos(dx+c) \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 3 \sin(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 6\sqrt{2} \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) \cos(dx+c) - 6\sqrt{2}}{4d\sqrt{\sec(dx+c)}(1+\cos(dx+c))^{3/2}}$

[In] int(1/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/4/d/sec(d*x+c)^(1/2)*(2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-6*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)-6*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-9*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-9*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^2

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{\frac{5}{2}}(c + dx)} dx = \frac{9\sqrt{2}(\cos(dx+c)^2 + 2\cos(dx+c) + 1)\sqrt{a} \arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - 12(\cos(dx+c)^2 + 2\cos(dx+c) + 1)\sqrt{a}}{4(a^2d\cos(dx+c)^2 + 2a^2d)}$$

[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] -1/4*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{3/2} \sec(dx + c)^{5/2}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{3/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + a \cos(c + dx))^{3/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2)),x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(3/2)), x)

$$3.379 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{5}{2}}} dx$$

Optimal result	4469
Rubi [A] (verified)	4469
Mathematica [C] (warning: unable to verify)	4473
Maple [A] (verified)	4473
Fricas [A] (verification not implemented)	4474
Sympy [F(-1)]	4474
Maxima [F]	4474
Giac [F]	4475
Mupad [F(-1)]	4475

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{\frac{5}{2}}} dx = \frac{163 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{16\sqrt{2}a^{\frac{5}{2}}d} - \frac{299\sqrt{\sec(c+dx)} \sin(c+dx)}{48a^2d\sqrt{a+a \cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4d(a+a \cos(c+dx))^{\frac{5}{2}}} - \frac{17 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{16ad(a+a \cos(c+dx))^{\frac{3}{2}}} + \frac{95 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{48a^2d\sqrt{a+a \cos(c+dx)}}$$

[Out] $-1/4*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(5/2)}-17/16*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(3/2)}+95/48*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^{(1/2)}+163/32*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)})/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(5/2)}/d*2^{(1/2)}-299/48*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^2/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {4307, 2845, 3057, 3063, 12, 2861, 211}

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{5}{2}}} dx = \frac{163\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

$$+ \frac{95\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}} - \frac{299\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}$$

$$- \frac{17\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2), x]

[Out] (163*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - (299*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]]) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (17*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (95*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&

EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3063

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 4307

Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{5}{2}}} dx \\ &= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{4d(a + a \cos(c + dx))^{\frac{5}{2}}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\frac{11a}{2} - 3a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + a \cos(c + dx))^{\frac{3}{2}}} dx}{4a^2} \\ &= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{4d(a + a \cos(c + dx))^{\frac{5}{2}}} - \frac{17 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{16ad(a + a \cos(c + dx))^{\frac{3}{2}}} \\ &\quad + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\frac{95a^2}{4} - 17a^2 \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx}{8a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{95\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{-\frac{299a^3}{8} + \frac{95}{4}a^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx}{12a^5} \\
&= -\frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{95\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{489a^4}{16\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{6a^6} \\
&= -\frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{95\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(163\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
&= -\frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{95\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(163\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
&= \frac{163 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} \\
&\quad - \frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{95\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.37 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.70

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{5}{2}}} dx =$$

$$\cot^5\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^4\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^4\left(\frac{1}{2}(c+dx)\right) \left(\frac{1}{1-2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)^{7/2} \left(640 \cos^8\left(\frac{1}{2}(c+dx)\right) {}_5F_4\left(2, 2, 2, 2, \frac{7}{2}; 1, \dots\right)\right)$$

[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]

[Out] -1/41580*(Cot[c/2 + (d*x)/2]^5*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^4*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^7/2*(640*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 - 1280*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[c/2 + (d*x)/2]^2) + 33*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-10935 + 72902*Sin[c/2 + (d*x)/2]^2 - 188110*Sin[c/2 + (d*x)/2]^4 + 234156*Sin[c/2 + (d*x)/2]^6 - 140732*Sin[c/2 + (d*x)/2]^8 + 33208*Sin[c/2 + (d*x)/2]^10) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-1148175 + 10333785*Sin[c/2 + (d*x)/2]^2 - 38990350*Sin[c/2 + (d*x)/2]^4 + 79946462*Sin[c/2 + (d*x)/2]^6 - 96281836*Sin[c/2 + (d*x)/2]^8 + 68243596*Sin[c/2 + (d*x)/2]^10 - 26448512*Sin[c/2 + (d*x)/2]^12 + 43444400*Sin[c/2 + (d*x)/2]^14)))/(d*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.25

method	result
default	$-\frac{\left(\sec^{\frac{5}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\left(489\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))\cos^5(dx+c)+1467\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))\cos^4(dx+c)+299\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos^3(dx+c)+503\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos^2(dx+c)+1467\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)+489\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}{d(a(1+\cos(dx+c)))^{\frac{5}{2}}}$

[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/96/d*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3*(489*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^5+1467*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^4+299*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+503*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+1467*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+489*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c)))/d

$\text{in}(d*x+c)+489*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\cos(d*x+c)^2*\arcsin(\cot(d*x+c)-\csc(d*x+c))+160*2^{(1/2)}*\cos(d*x+c)^2*\sin(d*x+c)-32*\sin(d*x+c)*\cos(d*x+c)*2^{(1/2)})*2^{(1/2)}/a^3$

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.82

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{5}{2}}} dx =$$

$$\frac{489\sqrt{2}(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c))\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{96(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c))}$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -1/96*(489*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(299*cos(d*x + c)^3 + 503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{5}{2}}}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx$$

[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(5/2),x)

[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(5/2), x)

$$3.380 \quad \int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	4476
Rubi [A] (verified)	4476
Mathematica [C] (warning: unable to verify)	4479
Maple [A] (verified)	4480
Fricas [A] (verification not implemented)	4480
Sympy [F(-1)]	4481
Maxima [F]	4481
Giac [F]	4481
Mupad [F(-1)]	4481

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{\sec^3(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx =$$

$$\frac{75 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}$$

$$- \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)} \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2}}$$

$$+ \frac{49\sqrt{\sec(c+dx)} \sin(c+dx)}{16a^2d\sqrt{a+a \cos(c+dx)}}$$

```
[Out] -1/4*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(5/2)-13/16*sin(d*x+c)*
sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(3/2)-75/32*arctan(1/2*sin(d*x+c)*a^(
1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+49/16*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(a+a
*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {4307, 2845, 3057, 3063, 12, 2861, 211}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx =$$

$$\frac{75\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

$$+ \frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}$$

$$- \frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2), x]

[Out] (-75*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (13*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si

$n[e + f*x]]))], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3057

$\text{Int}[(a + (b \cdot \sin[e + f \cdot x]))^m \cdot ((A + (B \cdot \sin[e + f \cdot x]) + (f \cdot x)) \cdot ((c + d \cdot \sin[e + f \cdot x]))^n), x_Symbol] \rightarrow \text{Simp}[b \cdot (A \cdot b - a \cdot B) \cdot \text{Cos}[e + f \cdot x] \cdot (a + b \cdot \sin[e + f \cdot x])^m \cdot ((c + d \cdot \sin[e + f \cdot x]))^{n+1} / (a \cdot f \cdot (2 \cdot m + 1) \cdot (b \cdot c - a \cdot d)), x] + \text{Dist}[1 / (a \cdot (2 \cdot m + 1) \cdot (b \cdot c - a \cdot d)), \text{Int}[(a + b \cdot \sin[e + f \cdot x])^{m+1} \cdot (c + d \cdot \sin[e + f \cdot x])^n \cdot \text{Simp}[B \cdot (a \cdot c \cdot m + b \cdot d \cdot (n + 1)) + A \cdot (b \cdot c \cdot (m + 1) - a \cdot d \cdot (2 \cdot m + n + 2)) + d \cdot (A \cdot b - a \cdot B) \cdot (m + n + 2) \cdot \sin[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2 \cdot m] \ \&\& \ (\text{IntegerQ}[2 \cdot n] \ || \ \text{EqQ}[c, 0])$

Rule 3063

$\text{Int}[(a + (b \cdot \sin[e + f \cdot x]))^m \cdot ((A + (B \cdot \sin[e + f \cdot x]) + (f \cdot x)) \cdot ((c + d \cdot \sin[e + f \cdot x]))^n), x_Symbol] \rightarrow \text{Simp}[(B \cdot c - A \cdot d) \cdot \text{Cos}[e + f \cdot x] \cdot (a + b \cdot \sin[e + f \cdot x])^m \cdot ((c + d \cdot \sin[e + f \cdot x]))^{n+1} / (f \cdot (n + 1) \cdot (c^2 - d^2)), x] + \text{Dist}[1 / (b \cdot (n + 1) \cdot (c^2 - d^2)), \text{Int}[(a + b \cdot \sin[e + f \cdot x])^m \cdot (c + d \cdot \sin[e + f \cdot x])^{n+1} \cdot \text{Simp}[A \cdot (a \cdot d \cdot m + b \cdot c \cdot (n + 1)) - B \cdot (a \cdot c \cdot m + b \cdot d \cdot (n + 1)) + b \cdot (B \cdot c - A \cdot d) \cdot (m + n + 2) \cdot \sin[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{EqQ}[m + 1/2, 0])$

Rule 4307

$\text{Int}[(\text{csc}[a + b \cdot x] \cdot (c + d \cdot \sin[a + b \cdot x]))^m \cdot (u), x_Symbol] \rightarrow \text{Dist}[(c \cdot \text{Csc}[a + b \cdot x])^m \cdot (c \cdot \sin[a + b \cdot x])^m, \text{Int}[\text{ActivateTrig}[u] / (c \cdot \sin[a + b \cdot x])^m, x], x] /; \text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ \text{KnownSineIntegrandQ}[u, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx) (a + a \cos(c + dx))^{5/2}} dx \\ &= -\frac{\sqrt{\sec(c + dx)} \sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\frac{9a}{2} - 2a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) (a + a \cos(c + dx))^{3/2}} dx}{4a^2} \\ &= -\frac{\sqrt{\sec(c + dx)} \sin(c + dx)}{4d(a + a \cos(c + dx))^{5/2}} - \frac{13\sqrt{\sec(c + dx)} \sin(c + dx)}{16ad(a + a \cos(c + dx))^{3/2}} \\ &\quad + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\frac{49a^2}{4} - \frac{13}{2}a^2 \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + a \cos(c + dx)}} dx}{8a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{49\sqrt{\sec(c+dx)} \sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int -\frac{75a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{4a^5} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{49\sqrt{\sec(c+dx)} \sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(75\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)} \sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(75\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
&= -\frac{75 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} \\
&\quad - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)} \sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{49\sqrt{\sec(c+dx)} \sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.53 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.58

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{2\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{3/2}}{8\cos^6\left(\frac{1}{2}(c+dx)\right)}$$

[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2), x]

[Out] (2*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^(3/2)*((8*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2

, 2, 5/2}, {1, 1, 11/2}, $\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2) * \text{Sin}[c/2 + (d*x)/2]^2/(315*(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)) + (\text{Csc}[c/2 + (d*x)/2]^8*(1 - 2*\text{Sin}[c/2 + (d*x)/2]^2)^2*\text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)]*(-15*\text{ArcTanh}[\text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)])*\text{Cos}[(c + d*x)/2]^4*(-343 + 1465*\text{Sin}[c/2 + (d*x)/2]^2 - 2021*\text{Sin}[c/2 + (d*x)/2]^4 + 824*\text{Sin}[c/2 + (d*x)/2]^6) + \text{Sqrt}[\text{Sin}[c/2 + (d*x)/2]^2/(-1 + 2*\text{Sin}[c/2 + (d*x)/2]^2)]*(-5145 + 33980*\text{Sin}[c/2 + (d*x)/2]^2 - 87764*\text{Sin}[c/2 + (d*x)/2]^4 + 109737*\text{Sin}[c/2 + (d*x)/2]^6 - 66122*\text{Sin}[c/2 + (d*x)/2]^8 + 15344*\text{Sin}[c/2 + (d*x)/2]^10))/120)/(d*(a*(1 + \text{Cos}[c + d*x]))^(5/2))$

Maple [A] (verified)

Time = 6.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.35

method	result
default	$\frac{(75(\cos^3(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))+49\sqrt{2}(\cos^2(dx+c))\sin(dx+c)+225\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c))\arcsin(\cot(dx+c)-\csc(dx+c))))^{1/2} \cos(dx+c)^2 \arcsin(\cot(dx+c)-\csc(dx+c))+85\sin(dx+c)\cos(dx+c)^2)^{1/2}+225\cos(dx+c)(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \arcsin(\cot(dx+c)-\csc(dx+c))+32)^{1/2} \sin(dx+c)+75(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \arcsin(\cot(dx+c)-\csc(dx+c)))*\sec(dx+c)^{3/2}*(a*(1+\cos(dx+c)))^{1/2} \cos(dx+c)/(1+\cos(dx+c))^3)^{1/2}/a^3$

[In] `int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{32}d*(75*\cos(d*x+c)^3*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\arcsin(\cot(d*x+c)-\csc(d*x+c))+49*2^{1/2}*\cos(d*x+c)^2*\sin(d*x+c)+225*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\cos(d*x+c)^2*\arcsin(\cot(d*x+c)-\csc(d*x+c))+85*\sin(d*x+c)*\cos(d*x+c)^2)^{1/2}+225*\cos(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\arcsin(\cot(d*x+c)-\csc(d*x+c))+32*2^{1/2}*\sin(d*x+c)+75*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\arcsin(\cot(d*x+c)-\csc(d*x+c)))*\sec(d*x+c)^{3/2}*(a*(1+\cos(d*x+c)))^{1/2}*\cos(d*x+c)/(1+\cos(d*x+c))^3)^{1/2}/a^3$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx = \frac{75\sqrt{2}(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}}{\sin(dx+c)}\right)+2\sqrt{a\cos(dx+c)+a}(49\cos(dx+c)^2+85\cos(dx+c)+32)\sin(dx+c)/\sqrt{\cos(dx+c)}}{32(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d)}$$

[In] `integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] $\frac{1}{32}*(75*\sqrt{2}*(\cos(d*x+c)^3+3*\cos(d*x+c)^2+3*\cos(d*x+c)+1)*\sqrt{a}*\arctan(\sqrt{2}*\sqrt{a*\cos(d*x+c)+a}*\sqrt{\cos(d*x+c)})/(\sqrt{a}*\sin(d*x+c))+2*\sqrt{a*\cos(d*x+c)+a}*(49*\cos(d*x+c)^2+85*\cos(d*x+c)+32)*\sin(d*x+c)/\sqrt{\cos(d*x+c)})/(a^3*d*\cos(d*x+c)^3+3*a^3*d*\cos(d*x+c)^2+3*a^3*d*\cos(d*x+c)+a^3*d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(a \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2), x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{3}{2}}}{(a + a \cos(c + dx))^{\frac{5}{2}}} dx$$

[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(5/2), x)

[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(5/2), x)

$$3.381 \quad \int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	4482
Rubi [A] (verified)	4482
Mathematica [A] (verified)	4485
Maple [A] (verified)	4485
Fricas [A] (verification not implemented)	4485
Sympy [F(-1)]	4486
Maxima [F]	4486
Giac [F(-1)]	4486
Mupad [F(-1)]	4487

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^{5/2}} dx = \frac{19 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{9 \sin(c+dx)} - \frac{4d(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}}{16ad(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] $-1/4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(5/2)}/\sec(d*x+c)^{(1/2)}-9/16*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(3/2)}/\sec(d*x+c)^{(1/2)}+19/32*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(5/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2845, 3057, 12, 2861, 211}

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^{5/2}} dx = \frac{19 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{9 \sin(c+dx)} - \frac{16ad \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{3/2}}{4d \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{5/2}}$$

[In] $\text{Int}[\text{Sqrt}[\text{Sec}[c + d*x]]/(a + a*\text{Cos}[c + d*x])^{(5/2)}, x]$

[Out] $(19*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sin}[c + d*x])/(\text{Sqrt}[2]*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])]*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[c + d*x]])/(16*\text{Sqrt}[2]*a^{(5/2)}*d)$

$$-\frac{\sin[c + dx]}{(4d(a + a\cos[c + dx])^{5/2}\sqrt{\sec[c + dx]})} - \frac{(9\sin[c + dx])}{(16ad(a + a\cos[c + dx])^{3/2}\sqrt{\sec[c + dx]})}$$

Rule 12

$$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$$

Rule 211

$$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$$

Rule 2845

$$\text{Int}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_)]])^{(m_*)}((c_*) + (d_*)\sin[(e_*) + (f_*)(x_)]])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[b^2\cos[e + fx]*(a + b\sin[e + fx])^m((c + d\sin[e + fx])^{n+1}/(af(2m+1)(bc-ad))), x] + \text{Dist}[1/(a(2m+1)(bc-ad)), \text{Int}[(a + b\sin[e + fx])^{m+1}(c + d\sin[e + fx])^n\text{Simp}[b*c*(m+1) - a*d*(2m+n+2) + b*d*(m+n+2)\sin[e + fx], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ (\text{IntegerSqrt}[2m, 2n] \ || \ (\text{IntegerQ}[m] \ \&\& \ \text{EqQ}[c, 0]))$$

Rule 2861

$$\text{Int}[1/(\sqrt{(a_*) + (b_*)\sin[(e_*) + (f_*)(x_)]})*\sqrt{(c_*) + (d_*)\sin[(e_*) + (f_*)(x_)]})], x_Symbol] \rightarrow \text{Dist}[-2*(a/f), \text{Subst}[\text{Int}[1/(2b^2 - (ac - bd)*x^2), x], x, b*(\cos[e + fx]/(\sqrt{a + b\sin[e + fx]})*\sqrt{c + d\sin[e + fx]})]], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$$

Rule 3057

$$\text{Int}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_)]])^{(m_*)}((A_*) + (B_*)\sin[(e_*) + (f_*)(x_)]])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[b*(A*b - a*B)*\cos[e + fx]*(a + b\sin[e + fx])^m((c + d\sin[e + fx])^{n+1}/(af(2m+1)(bc-ad))), x] + \text{Dist}[1/(a(2m+1)(bc-ad)), \text{Int}[(a + b\sin[e + fx])^{m+1}(c + d\sin[e + fx])^n\text{Simp}[B*(a*c*m + b*d*(n+1)) + A*(b*c*(m+1) - a*d*(2m+n+2)) + d*(A*b - a*B)*(m+n+2)*\sin[e + fx], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2m] \ \&\& \ (\text{IntegerQ}[2n] \ || \ \text{EqQ}[c, 0])$$

Rule 4307

$$\text{Int}[(\csc[(a_*) + (b_*)(x_)]*(c_*)^{(m_*)}*(u_)), x_Symbol] \rightarrow \text{Dist}[(c*\csc[a + b*x])^m*(c*\sin[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u]/(c*\sin[a + b*x])^m, x], x]$$

] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} (a+a\cos(c+dx))^{5/2}} dx \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{7a}{2} - a\cos(c+dx)}{\sqrt{\cos(c+dx)} (a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{19a^2}{4\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(19\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(19\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
 &= \frac{19 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} \\
 &\quad - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx = \frac{\left(76\operatorname{arctanh}\left(\sqrt{-\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)}\right) - \cos(c+dx)(13+9\cos(c+dx))\right)}{64\sqrt{2}a^2d\sqrt{a(1+\cos(c+dx))}}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]

[Out] ((76*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]] - Cos[c + d*x]*(13 + 9*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[2 - 2*Sec[c + d*x]])*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(64*Sqrt[2]*a^2*d*Sqrt[a*(1 + Cos[c + d*x])])*Sqrt[1 - Sec[c + d*x]])

Maple [A] (verified)

Time = 6.55 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\left(9\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+19\arcsin(\cot(dx+c)-\csc(dx+c))(\cos^2(dx+c))+13\sin(dx+c)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+38a\arcsin(\cot(dx+c)-\csc(dx+c))\right)\sqrt{a(1+\cos(dx+c))}}{32d(1+\cos(dx+c))^3\sqrt{a}}$

[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/32/d*(9*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+19*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+13*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+38*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+19*a*arcsin(cot(d*x+c)-csc(d*x+c))*sec(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(1+cos(d*x+c))^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx = \frac{19\sqrt{2}(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\sqrt{a}\cos(dx+c)}{\sqrt{a}}}{32(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3)}$$

[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] $-1/32*(19*\sqrt{2}*(\cos(dx + c)^3 + 3*\cos(dx + c)^2 + 3*\cos(dx + c) + 1)*\sqrt{a}*\arctan(\sqrt{2}*\sqrt{a*\cos(dx + c) + a}*\sqrt{\cos(dx + c)})/(\sqrt{a}*\sin(dx + c))) + 2*\sqrt{a*\cos(dx + c) + a}*(9*\cos(dx + c)^2 + 13*\cos(dx + c))*\sin(dx + c)/\sqrt{\cos(dx + c)})/(a^3*d*\cos(dx + c)^3 + 3*a^3*d*\cos(dx + c)^2 + 3*a^3*d*\cos(dx + c) + a^3*d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c + dx)}}{(a + a \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] `integrate(sec(dx+c)**(1/2)/(a+a*cos(dx+c))**(5/2), x)`

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt{\sec(c + dx)}}{(a + a \cos(c + dx))^{5/2}} dx = \int \frac{\sqrt{\sec(dx + c)}}{(a \cos(dx + c) + a)^{5/2}} dx$$

[In] `integrate(sec(dx+c)^(1/2)/(a+a*cos(dx+c))^(5/2), x, algorithm="maxima")`

[Out] `integrate(sqrt(sec(dx + c))/(a*cos(dx + c) + a)^(5/2), x)`

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c + dx)}}{(a + a \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] `integrate(sec(dx+c)^(1/2)/(a+a*cos(dx+c))^(5/2), x, algorithm="giac")`

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{(a+a\cos(c+dx))^{5/2}} dx$$

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(5/2), x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(5/2), x)
```

$$3.382 \quad \int \frac{1}{(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} dx$$

Optimal result	4488
Rubi [A] (verified)	4488
Mathematica [A] (verified)	4491
Maple [A] (verified)	4491
Fricas [A] (verification not implemented)	4491
Sympy [F(-1)]	4492
Maxima [F]	4492
Giac [F(-1)]	4492
Mupad [F(-1)]	4492

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} dx = \frac{5 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] 1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+1/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+5/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2843, 3057, 12, 2861, 211}

$$\int \frac{1}{(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} dx = \frac{5\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a \cos(c+dx)+a)^{3/2}} + \frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a \cos(c+dx)+a)^{5/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]), x]

[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)

+ Sin[c + d*x]/(4*d*(a + a*cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + Sin[c + d*x]/(16*a*d*(a + a*cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2843

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*(m + 1) - b*d*(m + n + 1)*sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*sin[e + f*x]])*Sqrt[c + d*sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 4307

Int[(csc[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*sin[a + b*x])^m, Int[ActivateTrig[u]/(c*sin[a + b*x])^m, x], x]

] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}} dx \\
 &= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{a}{2} + a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
 &= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{5a^2}{4\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
 &= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(5\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
 &= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(5\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} \right)}{16ad} \\
 &= \frac{5 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} \right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} \\
 &\quad + \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \frac{-5 \operatorname{arctanh}\left(\sqrt{-\sec(c + dx) \sin^2\left(\frac{1}{2}(c + dx)\right)}\right) \cot\left(\frac{1}{2}(c + dx)\right)}{32a^2 d \sqrt{a(1 + \cos(c + dx))}}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]

[Out] (-5*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]*Sqrt[2 - 2*Sec[c + d*x]] + 48*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 2*Tan[(c + d*x)/2]^3)/(32*a^2*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.24

method	result
default	$\frac{\left(\sqrt{2} \cos(dx+c) \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 5 \sin(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 5 \arcsin(\cot(dx+c) - \csc(dx+c)) (\cos^2(dx+c)) - 10 \arcsin\left(\frac{\sqrt{2} \cos(dx+c) \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 5 \sin(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{32d(1+\cos(dx+c))^3 \sqrt{\sec(dx+c)} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)}{32d(1+\cos(dx+c))^3 \sqrt{\sec(dx+c)} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$

[In] int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/32/d*(2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-10*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-5*arcsin(cot(d*x+c)-csc(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3/sec(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \frac{5 \sqrt{2} (\cos(dx + c)^3 + 3 \cos(dx + c)^2 + 3 \cos(dx + c) + 1) \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a \cos(dx+c) + a} \sqrt{\cos(dx+c)}}{\sqrt{a} \sin(dx+c)}\right) - \frac{2 \sqrt{a \cos(dx+c) + a}}{\sqrt{a}}}{32 (a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 + 3 a^3 d \cos(dx + c) + a^3 d)}$$

[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] -1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*

$\sin(dx + c))) - 2\sqrt{a\cos(dx + c) + a}(\cos(dx + c)^2 + 5\cos(dx + c))\sin(dx + c)/\sqrt{\cos(dx + c)})/(a^3d\cos(dx + c)^3 + 3a^3d\cos(dx + c)^2 + 3a^3d\cos(dx + c) + a^3d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(dx+c))**(5/2)/sec(dx+c)**(1/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a \cos(dx + c) + a)^{5/2} \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+a*cos(dx+c))^(5/2)/sec(dx+c)^(1/2), x, algorithm="maxima")

[Out] integrate(1/((a*cos(dx + c) + a)^(5/2)*sqrt(sec(dx + c))), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(dx+c))^(5/2)/sec(dx+c)^(1/2), x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + a \cos(c + dx))^{5/2}} dx$$

[In] int(1/((1/cos(c + dx))^(1/2)*(a + a*cos(c + dx))^(5/2)), x)

[Out] int(1/((1/cos(c + dx))^(1/2)*(a + a*cos(c + dx))^(5/2)), x)

$$3.383 \quad \int \frac{1}{(a+a \cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4493
Rubi [A] (verified)	4493
Mathematica [A] (verified)	4496
Maple [A] (verified)	4496
Fricas [A] (verification not implemented)	4496
Sympy [F(-1)]	4497
Maxima [F]	4497
Giac [F(-1)]	4497
Mupad [F(-1)]	4498

Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{(a+a \cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} dx = \frac{3 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{7 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] $-1/4*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(5/2)}/\sec(d*x+c)^{(1/2)}+7/16*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(3/2)}/\sec(d*x+c)^{(1/2)}+3/32*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(5/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2844, 3057, 12, 2861, 211}

$$\int \frac{1}{(a+a \cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} dx = \frac{3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{7 \sin(c+dx)}{16ad \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{3/2}} - \frac{\sin(c+dx)}{4d \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{5/2}}$$

[In] $\text{Int}[1/((a+a*\text{Cos}[c+d*x])^{(5/2)}*\text{Sec}[c+d*x]^{(3/2)}),x]$

[Out] $(3*\text{ArcTan}[(\text{Sqrt}[a]*\text{Sin}[c+d*x])/(\text{Sqrt}[2]*\text{Sqrt}[\text{Cos}[c+d*x]])*\text{Sqrt}[a+a*\text{Cos}[c+d*x]])]*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[\text{Sec}[c+d*x]])/(16*\text{Sqrt}[2]*a^{(5/2)}*d)$

$$- \frac{\sin[c + dx]}{(4d(a + a\cos[c + dx])^{5/2}\sqrt{\sec[c + dx]}} + \frac{(7\sin[c + dx])}{(16ad(a + a\cos[c + dx])^{3/2}\sqrt{\sec[c + dx]}}$$

Rule 12

$$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$$

Rule 211

$$\text{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$$

Rule 2844

$$\text{Int}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_)])^{(m_*)}((c_*) + (d_*)\sin[(e_*) + (f_*)(x_)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*((c + d*\text{Sin}[e + f*x])^{n-1}/(a*f*(2*m + 1))), x] + \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m+1}*(c + d*\text{Sin}[e + f*x])^{n-2}*\text{imp}[b*(c^2*(m+1) + d^2*(n-1)) + a*c*d*(m-n+1) + d*(a*d*(m-n+1) + b*c*(m+n))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ (\text{IntegersQ}[2*m, 2*n] \ || \ (\text{IntegerQ}[m] \ \&\& \ \text{EqQ}[c, 0]))$$

Rule 2861

$$\text{Int}[1/(\sqrt{(a_*) + (b_*)\sin[(e_*) + (f_*)(x_)]})*\sqrt{(c_*) + (d_*)\sin[(e_*) + (f_*)(x_)]}), x_Symbol] \rightarrow \text{Dist}[-2*(a/f), \text{Subst}[\text{Int}[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(\text{Cos}[e + f*x]/(\sqrt{a + b*\text{Sin}[e + f*x]})*\sqrt{c + d*\text{Sin}[e + f*x]})], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$$

Rule 3057

$$\text{Int}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_)])^{(m_*)}((A_*) + (B_*)\sin[(e_*) + (f_*)(x_)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[b*(A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*((c + d*\text{Sin}[e + f*x])^{n+1}/(a*f*(2*m + 1)*(b*c - a*d))), x] + \text{Dist}[1/(a*(2*m + 1)*(b*c - a*d)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m+1}*(c + d*\text{Sin}[e + f*x])^n*\text{Simp}[B*(a*c*m + b*d*(n+1)) + A*(b*c*(m+1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m+n+2)*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ (\text{IntegerQ}[2*n] \ || \ \text{EqQ}[c, 0])$$

Rule 4307

$$\text{Int}[(\text{csc}[(a_*) + (b_*)(x_)]*(c_*))^{(m_*)}(u_), x_Symbol] \rightarrow \text{Dist}[(c*\text{Csc}[a + b*x])^m*(c*\text{Sin}[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u]/(c*\text{Sin}[a + b*x])^m, x], x$$

] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{a}{2} - 3a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int -\frac{3a^2}{4\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(3\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(3\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
 &= \frac{3\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} \\
 &\quad - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = \frac{\sqrt{\cos(c + dx)}(1 + \cos(c + dx))^{3/2} \sec\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)}}{(a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]

[Out] (Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(6*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]^2*Sqrt[1 + Cos[c + d*x]] - Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(Sin[(c + d*x)/2] - 7*Sin[(3*(c + d*x))/2]))/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))} \left(7 \sin(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 3 \tan(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 3 \arcsin(\cot(dx+c) - \csc(dx+c)) \cos(dx+c) - 6 \arcsin(\cot(dx+c) - \csc(dx+c)) \right)}{32d(1+\cos(dx+c))^3 \sec(dx+c)^{3/2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} a^3}$

[In] int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/32/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3/sec(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(7*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-6*arcsin(cot(d*x+c)-csc(d*x+c))-3*sec(d*x+c)*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^3

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = \frac{3\sqrt{2}(\cos(dx+c))^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1}{32(a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2 + 3a^3d\cos(dx+c) + a^3d)} \sqrt{a} \arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - \frac{2\sqrt{a}\cos(dx+c)}{32(a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c)^2 + 3a^3d\cos(dx+c) + a^3d)}$$

[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x,algorithm="fricas")

[Out] $-1/32*(3*\sqrt{2}*(\cos(dx + c)^3 + 3*\cos(dx + c)^2 + 3*\cos(dx + c) + 1)*\sqrt{a}*\arctan(\sqrt{2}*\sqrt{a*\cos(dx + c) + a}*\sqrt{\cos(dx + c)})/(\sqrt{a}*\sin(dx + c))) - 2*\sqrt{a*\cos(dx + c) + a}*(7*\cos(dx + c)^2 + 3*\cos(dx + c))*\sin(dx + c)/\sqrt{\cos(dx + c)})/(a^3*d*\cos(dx + c)^3 + 3*a^3*d*\cos(dx + c)^2 + 3*a^3*d*\cos(dx + c) + a^3*d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = \text{Timed out}$$

[In] `integrate(1/(a+a*cos(dx+c))**(5/2)/sec(dx+c)**(3/2),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{5/2} \sec(dx + c)^{3/2}} dx$$

[In] `integrate(1/(a+a*cos(dx+c))^(5/2)/sec(dx+c)^(3/2),x, algorithm="maxima")`

[Out] `integrate(1/((a*cos(dx + c) + a)^(5/2)*sec(dx + c)^(3/2)), x)`

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = \text{Timed out}$$

[In] `integrate(1/(a+a*cos(dx+c))^(5/2)/sec(dx+c)^(3/2),x, algorithm="giac")`

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + a \cos(c + dx))^{5/2}} dx$$

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(5/2)), x)
```

$$3.384 \quad \int \frac{1}{(a+a \cos(c+dx))^{5/2} \sec^2(c+dx)} dx$$

Optimal result	4499
Rubi [A] (verified)	4499
Mathematica [C] (verified)	4502
Maple [A] (verified)	4503
Fricas [A] (verification not implemented)	4503
Sympy [F(-1)]	4504
Maxima [F]	4504
Giac [F(-1)]	4504
Mupad [F(-1)]	4504

Optimal result

Integrand size = 25, antiderivative size = 214

$$\int \frac{1}{(a+a \cos(c+dx))^{5/2} \sec^2(c+dx)} dx = \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{a^{5/2} d} - \frac{43 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{16\sqrt{2} a^{5/2} d} - \frac{\sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2} \sec^3(c+dx)} - \frac{11 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] -1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)-11/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d-43/32*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used

= {4307, 2844, 3056, 3061, 2861, 211, 2853, 222}

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx = \frac{2\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{a^{5/2}d}$$

$$- \frac{43\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \arctan\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2}\sqrt{\cos(c + dx)}\sqrt{a \cos(c + dx) + a}}\right)}{16\sqrt{2}a^{5/2}d}$$

$$- \frac{\sin(c + dx)}{4d \sec^{3/2}(c + dx)(a \cos(c + dx) + a)^{5/2}} - \frac{11 \sin(c + dx)}{16ad\sqrt{\sec(c + dx)}(a \cos(c + dx) + a)^{3/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)), x]

[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(a^(5/2)*d) - (43*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) - (11*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3061

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}} dx \\ &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} \\ &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)} \left(\frac{3a}{2} - 4a\cos(c+dx) \right)}{(a+a\cos(c+dx))^{3/2}} dx}{4a^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{3/2}(c+dx)} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{11a^2-8a^2\cos(c+dx)}{4}}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
&= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{3/2}(c+dx)} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{a^3} \\
&\quad - \frac{\left(43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2} \\
&= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{3/2}(c+dx)} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^3d} \\
&\quad + \frac{\left(43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
&= \frac{2\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} \\
&\quad - \frac{43\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} \\
&\quad - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{3/2}(c+dx)} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.74

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{5/2}(c+dx)} dx = \frac{e^{-\frac{1}{2}i(c+dx)}\left(\frac{1}{16}ie^{-2i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\right)\left(-43(1+e^{i(c+dx)})^4\sqrt{1+e^{2i(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]

[Out] (((I/16)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-43*(1 + E^(I*(c + d*x)))^4*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) + Sqrt[2]*(-15 - 7*E^(I*(c + d*x)) - 8

$$*E^{\left((2*I)*(c + d*x)\right)} + 8*E^{\left((3*I)*(c + d*x)\right)} + 7*E^{\left((4*I)*(c + d*x)\right)} + 15*E^{\left((5*I)*(c + d*x)\right)} + 16*(1 + E^{(I*(c + d*x))})^4*\text{Sqrt}[1 + E^{\left((2*I)*(c + d*x)\right)}] * \text{ArcTanh}[\text{Sqrt}[1 + E^{\left((2*I)*(c + d*x)\right)}]] * \text{Cos}[(c + d*x)/2] / E^{\left((2*I)*(c + d*x)\right)} - (16*I)*\text{Sqrt}[2]*\text{Sqrt}[E^{(I*(c + d*x))} / (1 + E^{\left((2*I)*(c + d*x)\right)})] * \text{Sqrt}[1 + E^{\left((2*I)*(c + d*x)\right)}] * \text{ArcSinh}[E^{(I*(c + d*x))}] * \text{Cos}[(c + d*x)/2]^5 / (4*d*E^{\left((I/2)*(c + d*x)\right)} * (a*(1 + \text{Cos}[c + d*x]))^{(5/2)})$$

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{a(1+\cos(dx+c))} \left(15 \tan(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 32\sqrt{2} \arctan\left(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right) + 11 \tan(dx+c) \sec(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \right)}{a^3}$

[In] int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out]
$$-1/32/d*(a*(1+\cos(d*x+c)))^{(1/2)}/(1+\cos(d*x+c))^3/\sec(d*x+c)^{(5/2)}/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*(15*\tan(d*x+c)*2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}-32*2^{(1/2)}*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)})+11*\tan(d*x+c)*\sec(d*x+c)*2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}-43*\arcsin(\cot(d*x+c)-\csc(d*x+c))-64*\sec(d*x+c)*2^{(1/2)}*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)})-86*\sec(d*x+c)*\arcsin(\cot(d*x+c)-\csc(d*x+c))-32*\sec(d*x+c)^2*2^{(1/2)}*\arctan(\tan(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)})-43*\sec(d*x+c)^2*\arcsin(\cot(d*x+c)-\csc(d*x+c)))*2^{(1/2)}/a^3$$

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx = \frac{43 \sqrt{2} (\cos(dx + c)^3 + 3 \cos(dx + c)^2 + 3 \cos(dx + c) + 1) \sqrt{a}}{(a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)}$$

[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out]
$$1/32*(43*\text{sqrt}(2)*(\cos(d*x + c)^3 + 3*\cos(d*x + c)^2 + 3*\cos(d*x + c) + 1)*\text{sqrt}(a)*\arctan(\text{sqrt}(2)*\text{sqrt}(a*\cos(d*x + c) + a)*\text{sqrt}(\cos(d*x + c)))/(\text{sqrt}(a)*\sin(d*x + c))) - 64*(\cos(d*x + c)^3 + 3*\cos(d*x + c)^2 + 3*\cos(d*x + c) + 1)*\text{sqrt}(a)*\arctan(\text{sqrt}(a*\cos(d*x + c) + a)*\text{sqrt}(\cos(d*x + c)))/(\text{sqrt}(a)*\sin(d*x + c))) - 2*\text{sqrt}(a*\cos(d*x + c) + a)*(15*\cos(d*x + c)^2 + 11*\cos(d*x + c))*\sin(d*x + c)/\text{sqrt}(\cos(d*x + c)))/(a^3*d*\cos(d*x + c)^3 + 3*a^3*d*\cos(d*x + c)^2 + 3*a^3*d*\cos(d*x + c) + a^3*d)$$

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{5/2} \sec(dx + c)^{5/2}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + a \cos(c + dx))^{5/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)),x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(5/2)), x)

$$3.385 \quad \int \frac{1}{(a+a \cos(c+dx))^{5/2} \sec^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	4505
Rubi [A] (verified)	4506
Mathematica [C] (verified)	4509
Maple [A] (verified)	4510
Fricas [A] (verification not implemented)	4510
Sympy [F(-1)]	4511
Maxima [F]	4511
Giac [F(-1)]	4511
Mupad [F(-1)]	4511

Optimal result

Integrand size = 25, antiderivative size = 254

$$\int \frac{1}{(a+a \cos(c+dx))^{5/2} \sec^{\frac{7}{2}}(c+dx)} dx =$$

$$\frac{5 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{a^{5/2} d}$$

$$+ \frac{115 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{16\sqrt{2} a^{5/2} d}$$

$$- \frac{\sin(c+dx)}{4d(a+a \cos(c+dx))^{5/2} \sec^{\frac{5}{2}}(c+dx)} - \frac{15 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{3/2} \sec^{\frac{3}{2}}(c+dx)}$$

$$+ \frac{35 \sin(c+dx)}{16a^2 d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}$$

```
[Out] -1/4*sin(d*x+c)/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)-15/16*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)+35/16*sin(d*x+c)/a^2/d/(a+a*cos
(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-5*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c)
)^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d+115/32*arctan(1/2*sin(
d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4307, 2844, 3056, 3062, 3061, 2861, 211, 2853, 222}

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx)} dx =$$

$$-\frac{5\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}$$

$$+ \frac{115\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}$$

$$+ \frac{35\sin(c + dx)}{16a^2d\sqrt{\sec(c + dx)}\sqrt{a\cos(c + dx) + a}}$$

$$- \frac{15\sin(c + dx)}{16ad\sec^{3/2}(c + dx)(a\cos(c + dx) + a)^{3/2}} - \frac{\sin(c + dx)}{4d\sec^{5/2}(c + dx)(a\cos(c + dx) + a)^{5/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)), x]

[Out] (-5*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(a^(5/2)*d) + (115*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) - (15*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (35*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&

NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3061

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3062

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m

+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^m*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sine[a + b*x])^m, Int[ActivateTrig[u]/(c*Sine[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{\frac{5}{2}}} dx \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{\frac{5}{2}} \sec^{\frac{5}{2}}(c+dx)} \\
 &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx) \left(\frac{5a}{2} - 5a\cos(c+dx) \right)}{(a+a\cos(c+dx))^{\frac{3}{2}}} dx}{4a^2} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{\frac{5}{2}} \sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{\frac{3}{2}} \sec^{\frac{3}{2}}(c+dx)} \\
 &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)} \left(\frac{45a^2}{4} - \frac{35}{2}a^2\cos(c+dx) \right)}{\sqrt{a+a\cos(c+dx)}} dx}{8a^4} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{\frac{5}{2}} \sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{\frac{3}{2}} \sec^{\frac{3}{2}}(c+dx)} \\
 &\quad + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{35a^3}{4} + 20a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{8a^5} \\
 &= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{\frac{5}{2}} \sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{\frac{3}{2}} \sec^{\frac{3}{2}}(c+dx)} \\
 &\quad + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(5\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{2a^3} \\
 &\quad + \frac{\left(115\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{32a^2}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{5/2}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{3/2}(c+dx)} \\
&\quad + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a}}}dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^3d} \\
&\quad - \frac{\left(115\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int\frac{1}{2a^2+ax^2}dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16ad} \\
&= -\frac{5\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} \\
&\quad + \frac{115\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} \\
&\quad - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{5/2}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{3/2}(c+dx)} \\
&\quad + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.50 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.62

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{7/2}(c+dx)} dx = \frac{e^{-\frac{1}{2}i(c+dx)}\left(40i\sqrt{2}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\operatorname{arcsinh}(e^{i(c+dx)})\cos\right)}{1}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)),x]

[Out] ((40*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))*Cos[(c + d*x)/2]^5 + (115*I)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[(c + d*x)/2]^5 - ((I/16)*(-8 - 47*E^(I*(c + d*x)) - 39*E^((2*I)*(c + d*x)) - 16*E^((3*I)*(c + d*x)) + 16*E^((4*I)*(c + d*x)) + 39*E^((5*I)*(c + d*x)) + 47*E^((6*I)*(c + d*x)) + 8*E^((7*I)*(c + d*x)) + 40*E^(I*(c + d*x)))*(1 + E^(I*(c + d*x)))^4*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]/E^((3*I)*(c + d*x)))/(4*d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 14.71 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(16\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c))\sin(dx+c)-80\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cos^2(dx+c))+55\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}{\dots}$

```
[In] int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/sec(d*x+c)^(1/2)*(16*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-80*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)^2+55*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-115*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-160*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)+35*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-230*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-80*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))-115*arcsin(cot(d*x+c)-csc(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx)} dx =$$

$$\frac{115\sqrt{2}(\cos(dx+c))^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right) - 160}{32(a^3d\cos(d$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^2 + 35*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(7/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{5/2} \sec(dx + c)^{7/2}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2), x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2)), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2), x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{5/2} \sec^{7/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2} (a + a \cos(c + dx))^{5/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2)), x)

[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(5/2)), x)

$$3.386 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	4512
Rubi [A] (verified)	4512
Mathematica [C] (warning: unable to verify)	4516
Maple [A] (verified)	4517
Fricas [A] (verification not implemented)	4517
Sympy [F(-1)]	4518
Maxima [F(-1)]	4518
Giac [F]	4518
Mupad [F(-1)]	4518

Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx = \frac{1015 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{629\sqrt{\sec(c+dx)} \sin(c+dx)}{64a^3d\sqrt{a+a \cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2}} - \frac{23 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{48ad(a+a \cos(c+dx))^{5/2}} - \frac{109 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{64a^2d(a+a \cos(c+dx))^{3/2}} + \frac{193 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{64a^3d\sqrt{a+a \cos(c+dx)}}$$

[Out] $-1/6*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(7/2)}-23/48*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(5/2)}-109/64*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^{(3/2)}+193/64*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/a^3/d/(a+a*\cos(d*x+c))^{(1/2)}+1015/128*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(7/2)}/d*2^{(1/2)}-629/64*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/a^3/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {4307, 2845, 3057, 3063, 12, 2861, 211}

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \frac{1015\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

$$+ \frac{193\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^3d\sqrt{a\cos(c+dx)+a}} - \frac{629\sin(c+dx)\sqrt{\sec(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}$$

$$- \frac{109\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^2d(a\cos(c+dx)+a)^{3/2}} - \frac{23\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48ad(a\cos(c+dx)+a)^{5/2}} - \frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2), x]

[Out] (1015*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) - (629*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*x]]) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - (23*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - (109*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + (193*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si

$n[e + f*x]]))], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rule 3057

$\text{Int}[(a + (b \cdot \sin(e + f \cdot x)))^m \cdot ((A + (B \cdot \sin(e + f \cdot x) + (f \cdot x))) \cdot ((c + d \cdot \sin(e + f \cdot x)))^n), x_Symbol] \rightarrow \text{Simp}[b \cdot (A \cdot b - a \cdot B) \cdot \text{Cos}[e + f \cdot x] \cdot (a + b \cdot \sin[e + f \cdot x])^m \cdot ((c + d \cdot \sin[e + f \cdot x])^{n+1}) / (a \cdot f \cdot (2 \cdot m + 1) \cdot (b \cdot c - a \cdot d)), x] + \text{Dist}[1 / (a \cdot (2 \cdot m + 1) \cdot (b \cdot c - a \cdot d)), \text{Int}[(a + b \cdot \sin[e + f \cdot x])^{m+1} \cdot (c + d \cdot \sin[e + f \cdot x])^n \cdot \text{Simp}[B \cdot (a \cdot c \cdot m + b \cdot d \cdot (n + 1)) + A \cdot (b \cdot c \cdot (m + 1) - a \cdot d \cdot (2 \cdot m + n + 2)) + d \cdot (A \cdot b - a \cdot B) \cdot (m + n + 2) \cdot \sin[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ !\text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2 \cdot m] \ \&\& \ (\text{IntegerQ}[2 \cdot n] \ || \ \text{EqQ}[c, 0])$

Rule 3063

$\text{Int}[(a + (b \cdot \sin(e + f \cdot x)))^m \cdot ((A + (B \cdot \sin(e + f \cdot x) + (f \cdot x))) \cdot ((c + d \cdot \sin(e + f \cdot x)))^n), x_Symbol] \rightarrow \text{Simp}[(B \cdot c - A \cdot d) \cdot \text{Cos}[e + f \cdot x] \cdot (a + b \cdot \sin[e + f \cdot x])^m \cdot ((c + d \cdot \sin[e + f \cdot x])^{n+1}) / (f \cdot (n + 1) \cdot (c^2 - d^2)), x] + \text{Dist}[1 / (b \cdot (n + 1) \cdot (c^2 - d^2)), \text{Int}[(a + b \cdot \sin[e + f \cdot x])^m \cdot (c + d \cdot \sin[e + f \cdot x])^{n+1} \cdot \text{Simp}[A \cdot (a \cdot d \cdot m + b \cdot c \cdot (n + 1)) - B \cdot (a \cdot c \cdot m + b \cdot d \cdot (n + 1)) + b \cdot (B \cdot c - A \cdot d) \cdot (m + n + 2) \cdot \sin[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{EqQ}[m + 1/2, 0])$

Rule 4307

$\text{Int}[(\text{csc}[a + b \cdot x] \cdot (c + d \cdot \sin[a + b \cdot x]))^m \cdot (u), x_Symbol] \rightarrow \text{Dist}[(c \cdot \text{Csc}[a + b \cdot x])^m \cdot (c \cdot \sin[a + b \cdot x])^m, \text{Int}[\text{ActivateTrig}[u] / (c \cdot \sin[a + b \cdot x])^m, x], x] /; \text{FreeQ}[\{a, b, c, m\}, x] \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ \text{KnownSineIntegrandQ}[u, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx) (a + a \cos(c + dx))^{7/2}} dx \\ &= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{6d(a + a \cos(c + dx))^{7/2}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\frac{15a}{2} - 4a \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) (a + a \cos(c + dx))^{5/2}} dx}{6a^2} \\ &= -\frac{\sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{6d(a + a \cos(c + dx))^{7/2}} - \frac{23 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{48ad(a + a \cos(c + dx))^{5/2}} \\ &\quad + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\frac{189a^2}{4} - \frac{69}{2}a^2 \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) (a + a \cos(c + dx))^{3/2}} dx}{24a^4} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{1737a^3}{8} - \frac{327}{2}a^3\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} + \frac{193\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{-\frac{5661a^4}{16} + \frac{1737}{8}a^4\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx}{72a^7} \\
&= -\frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} \\
&\quad - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{193\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{9135a^5}{32\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{36a^8} \\
&= -\frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} \\
&\quad - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{193\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(1015\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} \\
&\quad - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{193\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(1015\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d} \\
&= \frac{1015 \arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} \\
&\quad - \frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} + \frac{193\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.86 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.51

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \frac{\cot^7\left(\frac{c}{2} + \frac{dx}{2}\right) \csc^4\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^6\left(\frac{1}{2}(c+dx)\right) \left(\frac{1}{1-2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)^{7/2} \left(-7680 \cos^{10}\right)}{1}$$

[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2), x]

[Out] (Cot[c/2 + (d*x)/2]^7*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^6*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^(7/2)*(-7680*Cos[(c + d*x)/2]^10*HypergeometricPFQ[{2, 2, 2, 2, 2, 7/2}, {1, 1, 1, 1, 15/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 + 19200*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 15/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14*(-7 + 6*Sin[c/2 + (d*x)/2]^2) + 143*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(315*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*(351384 - 2928877*Sin[c/2 + (d*x)/2]^2 + 9953934*Sin[c/2 + (d*x)/2]^4 - 17629526*Sin[c/2 + (d*x)/2]^6 + 17139064*Sin[c/2 + (d*x)/2]^8 - 8670660*Sin[c/2 + (d*x)/2]^10 + 1793816*Sin[c/2 + (d*x)/2]^12) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-110685

960 + 1291549455*Sin[c/2 + (d*x)/2]^2 - 6601900452*Sin[c/2 + (d*x)/2]^4 + 19406027859*Sin[c/2 + (d*x)/2]^6 - 36160322412*Sin[c/2 + (d*x)/2]^8 + 44313222590*Sin[c/2 + (d*x)/2]^10 - 35736693140*Sin[c/2 + (d*x)/2]^12 + 18305254212*Sin[c/2 + (d*x)/2]^14 - 5410719584*Sin[c/2 + (d*x)/2]^16 + 704274992*Sin[c/2 + (d*x)/2]^18)))/(3243240*d*(a*(1 + Cos[c + d*x]))^(7/2))

Maple [A] (verified)

Time = 6.68 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.30

method	result
default	$-\frac{\left(\sec^{\frac{5}{2}}(dx+c)\right)\sqrt{a(1+\cos(dx+c))}\left(3045(\cos^6(dx+c))\arcsin(\cot(dx+c)-\csc(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+12180\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)^5+1887\cos(dx+c)^5\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+18270\cos(dx+c)^4\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))\cos(dx+c)^4+5082\cos(dx+c)^4\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+12180\cos(dx+c)^3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))+4251\cos(dx+c)^3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+3045\cos(dx+c)^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin(\cot(dx+c)-\csc(dx+c))+896\cos(dx+c)^2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-128\sin(dx+c)\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)}{384(a^4d\cos(dx+c)^5+4a^4d\cos(dx+c)^4+6a^4d\cos(dx+c)^3+4a^4d\cos(dx+c)^2+a^4d\cos(dx+c))\sqrt{a(1+\cos(dx+c))}}$

[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/384/d*sec(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4*(3045*cos(d*x+c)^6*arcsin(cot(d*x+c)-csc(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+12180*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^5+1887*cos(d*x+c)^5*2^(1/2)*sin(d*x+c)+18270*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^4+5082*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)+12180*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+4251*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+3045*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))+896*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-128*sin(d*x+c)*cos(d*x+c)*2^(1/2))*2^(1/2)/a^4

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.83

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \frac{3045\sqrt{2}(\cos(dx+c)^5+4\cos(dx+c)^4+6\cos(dx+c)^3+4\cos(dx+c)^2+\cos(dx+c))\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\right)+2(1887\cos(dx+c)^4+5082\cos(dx+c)^3+4251\cos(dx+c)^2+896\cos(dx+c)-128)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{384(a^4d\cos(dx+c)^5+4a^4d\cos(dx+c)^4+6a^4d\cos(dx+c)^3+4a^4d\cos(dx+c)^2+a^4d\cos(dx+c))}$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")

[Out] -1/384*(3045*sqrt(2)*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(1887*cos(d*x + c)^4 + 5082*cos(d*x + c)^3 + 4251*cos(d*x + c)^2 + 896*cos(d*x + c) - 128)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2), x)

[Out] Timed out

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2), x, algorithm="maxima")

[Out] Timed out

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{7/2}} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{(a \cos(dx + c) + a)^{\frac{7}{2}}} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2), x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{7/2}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{(a + a \cos(c + dx))^{7/2}} dx$$

[In] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(7/2), x)

[Out] int((1/cos(c + d*x))^(5/2)/(a + a*cos(c + d*x))^(7/2), x)

$$3.387 \quad \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	4519
Rubi [A] (verified)	4520
Mathematica [C] (warning: unable to verify)	4523
Maple [A] (verified)	4523
Fricas [A] (verification not implemented)	4524
Sympy [F(-1)]	4524
Maxima [F]	4524
Giac [F]	4525
Mupad [F(-1)]	4525

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a \cos(c+dx))^{7/2}} dx =$$

$$\frac{363 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64 \sqrt{2} a^{7/2} d}$$

$$- \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2}} - \frac{19 \sqrt{\sec(c+dx)} \sin(c+dx)}{48ad(a+a \cos(c+dx))^{5/2}}$$

$$- \frac{199 \sqrt{\sec(c+dx)} \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2}} + \frac{691 \sqrt{\sec(c+dx)} \sin(c+dx)}{192a^3d \sqrt{a+a \cos(c+dx)}}$$

```
[Out] -1/6*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^(7/2)-19/48*sin(d*x+c)*
sec(d*x+c)^(1/2)/a/d/(a+a*cos(d*x+c))^(5/2)-199/192*sin(d*x+c)*sec(d*x+c)^(
1/2)/a^2/d/(a+a*cos(d*x+c))^(3/2)-363/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(
1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^(7/2)/d*2^(1/2)+691/192*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d/(a+a*cos(d
*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4307, 2845, 3057, 3063, 12, 2861, 211}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx =$$

$$\frac{363\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

$$+ \frac{691\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^3d\sqrt{a\cos(c+dx)+a}} - \frac{199\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}$$

$$- \frac{19\sin(c+dx)\sqrt{\sec(c+dx)}}{48ad(a\cos(c+dx)+a)^{5/2}} - \frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d(a\cos(c+dx)+a)^{7/2}}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2), x]

[Out] (-363*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - (19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - (199*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + (691*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]])]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSqrt[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3057

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3063

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} dx \\ &= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{13a}{2} - 3a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{123a^2}{4} - 19a^2 \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{199\sqrt{\sec(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{691a^3}{8} - \frac{199}{4}a^3 \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{199\sqrt{\sec(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{691\sqrt{\sec(c+dx)} \sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int -\frac{1089a^4}{16\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{24a^7} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{199\sqrt{\sec(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{691\sqrt{\sec(c+dx)} \sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} \\
&\quad - \frac{\left(363\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
&= -\frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{199\sqrt{\sec(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{691\sqrt{\sec(c+dx)} \sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}} \\
&\quad + \frac{\left(363\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a \sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d} \\
&= -\frac{363 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} \\
&\quad - \frac{\sqrt{\sec(c+dx)} \sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)} \sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} \\
&\quad - \frac{199\sqrt{\sec(c+dx)} \sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \frac{691\sqrt{\sec(c+dx)} \sin(c+dx)}{192a^3d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.64 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.37

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \frac{2 \cos^7\left(\frac{c}{2} + \frac{dx}{2}\right) \sec^6\left(\frac{1}{2}(c+dx)\right) \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\frac{1}{1-2\sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)^{3/2} \left(\frac{16 \cos^8\left(\frac{1}{2}(c+dx)\right)}{\dots}\right)}{\dots}$$

[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]

[Out] (2*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1))^3/2*((16*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(3465*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) - (Csc[c/2 + (d*x)/2]^10*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*(2187 - 12908*Sin[c/2 + (d*x)/2]^2 + 27986*Sin[c/2 + (d*x)/2]^4 - 26380*Sin[c/2 + (d*x)/2]^6 + 8752*Sin[c/2 + (d*x)/2]^8) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-229635 + 2120790*Sin[c/2 + (d*x)/2]^2 - 8267707*Sin[c/2 + (d*x)/2]^4 + 17646926*Sin[c/2 + (d*x)/2]^6 - 22251094*Sin[c/2 + (d*x)/2]^8 + 16548816*Sin[c/2 + (d*x)/2]^10 - 6712984*Sin[c/2 + (d*x)/2]^12 + 1144608*Sin[c/2 + (d*x)/2]^14))/1680)/(d*(a*(1 + Cos[c + d*x]))^(7/2))

Maple [A] (verified)

Time = 6.65 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.39

method	result
default	$\frac{(1089\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \arcsin(\cot(dx+c)-\csc(dx+c))(\cos^4(dx+c))+691\sqrt{2}(\cos^3(dx+c))\sin(dx+c)+4356(\cos^3(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})}{\dots}$

[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/384/d*(1089*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^4+691*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+4356*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+1874*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+6534*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))+1599*sin(d*x+c)*cos(d*x+c)*2^(1/2)+4356*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-csc(d*x+c))+384*2^(1/2)*sin(d*x+c)+1089*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin(cot(d*x+c)-cs

$c(dx+c)) \cdot \sec(dx+c)^{3/2} \cdot (a \cdot (1+\cos(dx+c)))^{1/2} \cdot \cos(dx+c) / (1+\cos(dx+c))^{4 \cdot 2^{1/2}} / a^4$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \frac{1089\sqrt{2}(\cos(dx+c)^4 + 4\cos(dx+c)^3 + 6\cos(dx+c)^2 + 4\cos(dx+c) + 1) \cdot \sqrt{a} \cdot \arctan(\sqrt{2}\sqrt{a\cos(dx+c)+a}) \cdot \sqrt{\cos(dx+c)}}{384(a^4d\cos(dx+c)^4 + 4a^4d\cos(dx+c)^3 + 6a^4d\cos(dx+c)^2 + 4a^4d\cos(dx+c) + a^4d)}$$

[In] integrate(sec(dx+c)^(3/2)/(a+a*cos(dx+c))^(7/2),x, algorithm="fricas")

[Out] 1/384*(1089*sqrt(2)*(cos(dx+c)^4 + 4*cos(dx+c)^3 + 6*cos(dx+c)^2 + 4*cos(dx+c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(dx+c)+a)*sqrt(cos(dx+c))/(sqrt(a)*sin(dx+c))) + 2*(691*cos(dx+c)^3 + 1874*cos(dx+c)^2 + 1599*cos(dx+c) + 384)*sqrt(a*cos(dx+c)+a)*sin(dx+c)/sqrt(cos(dx+c)))/(a^4*d*cos(dx+c)^4 + 4*a^4*d*cos(dx+c)^3 + 6*a^4*d*cos(dx+c)^2 + 4*a^4*d*cos(dx+c) + a^4*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(sec(dx+c)**(3/2)/(a+a*cos(dx+c))**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}} dx$$

[In] integrate(sec(dx+c)^(3/2)/(a+a*cos(dx+c))^(7/2),x, algorithm="maxima")

[Out] integrate(sec(dx+c)^(3/2)/(a*cos(dx+c)+a)^(7/2),x)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{(a+a\cos(c+dx))^{7/2}} dx$$

[In] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(7/2),x)

[Out] int((1/cos(c + d*x))^(3/2)/(a + a*cos(c + d*x))^(7/2), x)

$$3.388 \quad \int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^{7/2}} dx$$

Optimal result	4526
Rubi [A] (verified)	4526
Mathematica [A] (verified)	4529
Maple [A] (verified)	4529
Fricas [A] (verification not implemented)	4530
Sympy [F(-1)]	4530
Maxima [F]	4530
Giac [F(-1)]	4531
Mupad [F(-1)]	4531

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^{7/2}} dx = \frac{63 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64 \sqrt{2} a^{7/2} d} - \frac{\sin(c+dx)}{5 \sin(c+dx)} - \frac{6d(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}}{103 \sin(c+dx)} - \frac{16ad(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}}{192a^2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] -1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)-5/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-103/192*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+63/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2845, 3057, 12, 2861, 211}

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^{7/2}} dx = \frac{63 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{64 \sqrt{2} a^{7/2} d} - \frac{103 \sin(c+dx)}{192a^2d \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{3/2}} - \frac{5 \sin(c+dx)}{16ad \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{5/2}} - \frac{\sin(c+dx)}{6d \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{7/2}}$$

[In] Int[Sqrt[Sec[c + d*x]]/(a + a*cos[c + d*x])^(7/2), x]

[Out] (63*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]])*Sqrt[a + a*Cos[c + d*x]])*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d - Sin[c + d*x]/(6*d*(a + a*cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) - (5*Sin[c + d*x])/(16*a*d*(a + a*cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (103*Sin[c + d*x])/(192*a^2*d*(a + a*cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2845

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b^2*cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*sin[e + f*x]])*Sqrt[c + d*sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*((c + d*sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]

&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} dx \\
 &= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{11a}{2} - 2a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
 &= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{73a^2}{4} - \frac{15}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
 &= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
 &\quad - \frac{103\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{189a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
 &= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
 &\quad - \frac{103\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(63\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
 &= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
 &\quad - \frac{103\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(63\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{63 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} \\
&\quad - \frac{\sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} \\
&\quad - \frac{5 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
&\quad - \frac{103 \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a \cos(c+dx))^{7/2}} dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right) \left(-2(493+532 \cos(c+dx)+103 \cos(2(c+dx)))\sqrt{2-2 \sec(c+dx)}\right)}{3072\sqrt{2}a^3d\sqrt{a(1+\cos(c+dx))}}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(7/2), x]

[Out] (Sec[(c + d*x)/2]^4*(-2*(493 + 532*Cos[c + d*x] + 103*Cos[2*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]] + 6048*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*Sec[c + d*x])*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a^3*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])

Maple [A] (verified)

Time = 6.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.35

method	result
default	$-\frac{(103\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c)\sin(dx+c)+266\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+189\arcsin(\cot(dx+c)-\csc(dx+c))))}{3072\sqrt{2}a^3d\sqrt{a(1+\cos(dx+c))}}$

[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2), x, method=_RETURNVERBOSE)

[Out] -1/384/d*(103*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+266*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+189*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3+195*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+567*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+567*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+189*arcsin(cot(d*x+c)-csc(d*x+c)))*sec(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)/(1+cos(d*x+c))^4/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}} dx =$$

$$\frac{189\sqrt{2}(\cos(dx+c)^4 + 4\cos(dx+c)^3 + 6\cos(dx+c)^2 + 4\cos(dx+c) + 1)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{384(a^4d\cos(dx+c)^4 + 4a^4d\cos(dx+c)^3 + 6a^4d\cos(dx+c)^2 + 4a^4d\cos(dx+c) + a^4d)}$$

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(189*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(103*cos(d*x + c)^3 + 266*cos(d*x
+ c)^2 + 195*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x
+ c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}} dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}} dx = \int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{7/2}} dx$$

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}} dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{(a+a\cos(c+dx))^{7/2}} dx$$

```
[In] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(7/2),x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + a*cos(c + d*x))^(7/2), x)
```

$$3.389 \quad \int \frac{1}{(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} dx$$

Optimal result	4532
Rubi [A] (verified)	4532
Mathematica [A] (verified)	4535
Maple [A] (verified)	4535
Fricas [A] (verification not implemented)	4536
Sympy [F(-1)]	4536
Maxima [F]	4536
Giac [F(-1)]	4537
Mupad [F(-1)]	4537

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} dx = \frac{13 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d}$$

$$+ \frac{\sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}}$$

$$- \frac{5 \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] 1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+1/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)-5/192*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+13/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2843, 3057, 12, 2861, 211}

$$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} dx = \frac{13 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

$$- \frac{5 \sin(c+dx)}{192a^2d \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{3/2}}$$

$$+ \frac{\sin(c+dx)}{16ad \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{5/2}} + \frac{\sin(c+dx)}{6d \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{7/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]

[Out] (13*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d + Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + Sin[c + d*x]/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (5*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2843

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]

&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}} dx \\
 &= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{a}{2} + 2a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
 &= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{11a^2}{4} + \frac{3}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
 &= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{5\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{39a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
 &= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{5\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(13\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
 &= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{5\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(13\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{13 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64 \sqrt{2} a^{7/2} d} \\
&+ \frac{\sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} \\
&+ \frac{\sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
&- \frac{5 \sin(c+dx)}{192a^2 d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} dx = \frac{-312 \operatorname{arctanh}\left(\sqrt{-\sec(c+dx)} \sin^2\left(\frac{1}{2}(c+dx)\right)\right) \cot\left(\frac{1}{2}(c+dx)\right)}{3072a^3}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]

[Out] (-312*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]*Sqrt[2 - 2*Sec[c + d*x]] + (73 + 4*Cos[c + d*x] - 5*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^6*Sin[c + d*x])/(3072*a^3*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\left(5\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos^2(dx+c))\sin(dx+c)-2\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+39\arcsin(\cot(dx+c)-\csc(dx+c))\right)(\cos^3(dx+c))}{3072a^3d\sqrt{a(1+\cos(dx+c))}\sqrt{\sec(dx+c)}}$

[In] int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/384/d*(5*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3-39*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+117*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+117*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+39*arcsin(cot(d*x+c)-csc(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/sec(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)/a^4

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sqrt{\sec(c + dx)}} dx =$$

$$\frac{39 \sqrt{2} (\cos(dx + c)^4 + 4 \cos(dx + c)^3 + 6 \cos(dx + c)^2 + 4 \cos(dx + c) + 1) \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a \cos(dx + c) + a} \sqrt{c}}{\sqrt{a} \sin(dx + c)}\right)}{384 (a^4 d \cos(dx + c)^4 + 4 a^4 d \cos(dx + c)^3 + 6 a^4 d \cos(dx + c)^2 + 4 a^4 d \cos(dx + c) + a^4 d)}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/384*(39*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(5*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 39*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a \cos(dx + c) + a)^{7/2} \sqrt{\sec(dx + c)}} dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + a \cos(c + dx))^{7/2}} dx$$

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

$$3.390 \quad \int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	4538
Rubi [A] (verified)	4538
Mathematica [A] (verified)	4541
Maple [A] (verified)	4541
Fricas [A] (verification not implemented)	4542
Sympy [F(-1)]	4542
Maxima [F]	4542
Giac [F(-1)]	4543
Mupad [F(-1)]	4543

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{3}{2}}(c+dx)} dx = \frac{7 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d}$$

$$- \frac{\sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{3 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}}$$

$$+ \frac{17 \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] $-1/6*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(7/2)}/\sec(d*x+c)^{(1/2)}+3/16*\sin(d*x+c)/a$
 $/d/(a+a*\cos(d*x+c))^{(5/2)}/\sec(d*x+c)^{(1/2)}+17/192*\sin(d*x+c)/a^2/d/(a+a*\cos$
 $(d*x+c))^{(3/2)}/\sec(d*x+c)^{(1/2)}+7/128*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}$
 $/\cos(d*x+c)^{(1/2)/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}$
 $/a^{(7/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00,
 number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used
 = {4307, 2844, 3057, 12, 2861, 211}

$$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{3}{2}}(c+dx)} dx = \frac{7\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a \cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

$$+ \frac{17 \sin(c+dx)}{192a^2d \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{3/2}}$$

$$+ \frac{3 \sin(c+dx)}{16ad \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{5/2}} - \frac{\sin(c+dx)}{6d \sqrt{\sec(c+dx)} (a \cos(c+dx) + a)^{7/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]

[Out] (7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d - Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + (3*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (17*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3057

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]

&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx \\
 &= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{a}{2} - 4a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
 &= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{a^2}{4} - \frac{9}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
 &= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{17\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int -\frac{21a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
 &= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{17\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(7\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
 &= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{17\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(7\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{7 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} \\
&\quad - \frac{\sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{3 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{17 \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{3/2}(c+dx)} dx = \frac{\sec^4\left(\frac{1}{2}(c+dx)\right) \left(2(59+140 \cos(c+dx)+17 \cos(2(c+dx)))\sqrt{\sec(c+dx)}\right)}{3072\sqrt{2}a^3}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]

[Out] (Sec[(c + d*x)/2]^4*(2*(59 + 140*Cos[c + d*x] + 17*Cos[2*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]] + 672*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)])*Cos[(c + d*x)/2]^6*Sec[c + d*x]*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a^3*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.27

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))} \left(17\sqrt{2} \cos(dx+c) \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 70 \sin(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 21 \arcsin(\cot(dx+c) - \csc(dx+c))\right)}{384 a^{7/2} (1+\cos(dx+c))^{1/2} \sec(dx+c)^{3/2}}$

[In] int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/384/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/sec(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(17*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+70*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-21*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+21*tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-63*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-63*a*arcsin(cot(d*x+c)-csc(d*x+c))-21*sec(d*x+c)*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^4

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{3/2}(c + dx)} dx =$$

$$\frac{21 \sqrt{2} (\cos(dx + c)^4 + 4 \cos(dx + c)^3 + 6 \cos(dx + c)^2 + 4 \cos(dx + c) + 1) \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a \cos(dx + c) + a} \sqrt{c}}{\sqrt{a} \sin(dx + c)}\right)}{384 (a^4 d \cos(dx + c)^4 + 4 a^4 d \cos(dx + c)^3 + 6 a^4 d \cos(dx + c)^2$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/384*(21*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(17*cos(d*x + c)^3 + 70*cos(d*x + c
)^2 + 21*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x +
c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{3/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{7/2} \sec(dx + c)^{3/2}} dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{3/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + a \cos(c + dx))^{7/2}} dx$$

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + a*cos(c + d*x))^(7/2)), x)
```

$$3.391 \quad \int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	4544
Rubi [A] (verified)	4544
Mathematica [A] (verified)	4547
Maple [A] (verified)	4548
Fricas [A] (verification not implemented)	4548
Sympy [F(-1)]	4549
Maxima [F]	4549
Giac [F(-1)]	4549
Mupad [F(-1)]	4549

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{5}{2}}(c+dx)} dx = \frac{5 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{\sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2} \sec^{\frac{3}{2}}(c+dx)} - \frac{13 \sin(c+dx)}{48ad(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{67 \sin(c+dx)}{192a^2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] $-1/6*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(7/2)}/\sec(d*x+c)^{(3/2)}-13/48*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(5/2)}/\sec(d*x+c)^{(1/2)}+67/192*\sin(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^{(3/2)}/\sec(d*x+c)^{(1/2)}+5/128*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(7/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4307, 2844, 3056, 3057, 12, 2861, 211}

$$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{5}{2}}(c+dx)} dx = \frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{67\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}} - \frac{\sin(c+dx)}{6d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{7/2}} - \frac{13\sin(c+dx)}{48ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{5/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x]

[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d - Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)) - (13*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (67*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n)/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int

egerQ[2*n] || EqQ[c, 0])

Rule 3057

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx \\
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sec^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)} \left(\frac{3a}{2} - 5a\cos(c+dx) \right)}{(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sec^{\frac{3}{2}}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{13a^2}{4} - \frac{27}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sec^{\frac{3}{2}}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{67\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int -\frac{15a^3}{8\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{48a^6}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{3/2}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{67\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{3/2}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{67\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d} \\
&= \frac{5\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} \\
&\quad - \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{3/2}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{67\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{5/2}(c+dx)} dx = \frac{\cos^7\left(\frac{1}{2}(c+dx)\right) \sqrt{\cos(c+dx)}\sqrt{a(1+\cos(c+dx))}\sqrt{\sec(c+dx)}}{\dots}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)), x]

[Out] (Cos[(c + d*x)/2]^7*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]])*(15*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]*(33 - 26*Tan[(c + d*x)/2]^2 + 8*Tan[(c + d*x)/2]^4))/(24*a^4*d*Sqrt[Cos[(c + d*x)/2]^2]*(1 + Cos[c + d*x])^4)

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.27

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))} \left(67 \sin(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 50 \tan(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 15 \arcsin(\cot(dx+c) - \csc(dx+c)) \cos(dx+c) + 1 \right)}{384d}$

[In] int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

```
[Out] 1/384/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^4/sec(d*x+c)^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(67*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+50*tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-15*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+15*tan(d*x+c)*sec(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-45*arcsin(cot(d*x+c)-csc(d*x+c))-45*sec(d*x+c)*arcsin(cot(d*x+c)-csc(d*x+c))-15*sec(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{5/2}(c + dx)} dx = \frac{15 \sqrt{2} (\cos(dx + c)^4 + 4 \cos(dx + c)^3 + 6 \cos(dx + c)^2 + 4 \cos(dx + c) + 1) \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a \cos(dx+c) + a} \sqrt{c}}{\sqrt{a} \sin(dx+c)}\right)}{384 (a^4 d \cos(dx + c)^4 + 4 a^4 d \cos(dx + c)^3 + 6 a^4 d \cos(dx + c)^2 + 4 a^4 d \cos(dx + c) + a^4 d)}$$

[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")

```
[Out] -1/384*(15*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(67*cos(d*x + c)^3 + 50*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```


Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(5/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{7/2} \sec(dx + c)^{5/2}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2), x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5/2)), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2), x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + a \cos(c + dx))^{7/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2)), x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(7/2)), x)

$$3.392 \quad \int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^2(c+dx)} dx$$

Optimal result	4550
Rubi [A] (verified)	4551
Mathematica [C] (verified)	4554
Maple [A] (warning: unable to verify)	4555
Fricas [A] (verification not implemented)	4555
Sympy [F(-1)]	4556
Maxima [F]	4556
Giac [F(-1)]	4556
Mupad [F(-1)]	4556

Optimal result

Integrand size = 25, antiderivative size = 254

$$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^2(c+dx)} dx = \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{a^{7/2} d}$$

$$- \frac{177 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64 \sqrt{2} a^{7/2} d}$$

$$- \frac{\sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2} \sec^{5/2}(c+dx)} - \frac{17 \sin(c+dx)}{48ad(a+a \cos(c+dx))^{5/2} \sec^{3/2}(c+dx)}$$

$$- \frac{49 \sin(c+dx)}{64a^2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

```
[Out] -1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2)-17/48*sin(d*x+c)/
a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2)-49/64*sin(d*x+c)/a^2/d/(a+a*cos
(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c)
)^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d-177/128*arctan(1/2*sin
(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4307, 2844, 3056, 3061, 2861, 211, 2853, 222}

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^2(c + dx)} dx = \frac{2\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{a^{7/2}d} - \frac{177\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \arctan\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2}\sqrt{\cos(c + dx)}\sqrt{a \cos(c + dx) + a}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{49 \sin(c + dx)}{64a^2d\sqrt{\sec(c + dx)}(a \cos(c + dx) + a)^{3/2}} - \frac{17 \sin(c + dx)}{48ad \sec^3(c + dx)(a \cos(c + dx) + a)^{5/2}} - \frac{\sin(c + dx)}{6d \sec^5(c + dx)(a \cos(c + dx) + a)^{7/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]

[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) - (177*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)) - (17*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)) - (49*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &

& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x])]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^n, x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3061

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 4307

Int[(csc[(a_) + (b_)*(x_)])*(c_)^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}} dx \\
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sec^{\frac{5}{2}}(c+dx)} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx) \left(\frac{5a}{2} - 6a\cos(c+dx) \right)}{(a+a\cos(c+dx))^{5/2}} dx}{6a^2} \\
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)} \left(\frac{51a^2}{4} - 24a^2\cos(c+dx) \right)}{(a+a\cos(c+dx))^{3/2}} dx}{24a^4} \\
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{49\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{147a^3}{8} - 48a^3\cos(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx}{48a^6} \\
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{49\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{a^4} \\
&\quad - \frac{\left(177\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2} \sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{49\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}} \right)}{a^4d} \\
&\quad + \frac{\left(177\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \text{Subst} \left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+a\cos(c+dx)}} \right)}{64a^2d}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{a^{7/2} d} \\
&\quad - \frac{177 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64 \sqrt{2} a^{7/2} d} \\
&\quad - \frac{\sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2} \sec^{5/2}(c+dx)} - \frac{17 \sin(c+dx)}{48ad(a+a \cos(c+dx))^{5/2} \sec^{3/2}(c+dx)} \\
&\quad - \frac{49 \sin(c+dx)}{64a^2d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.94 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.79

$$\begin{aligned}
&\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{7/2}(c+dx)} dx = \\
&\quad i e^{-\frac{1}{2}i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}} \sqrt{1+e^{2i(c+dx)}} \left(64 \operatorname{arcsinh}(e^{i(c+dx)}) + \frac{177 \operatorname{arctanh}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}} - 64 \operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right) \right) \\
&\quad + \frac{4\sqrt{2}d(a(1+\cos(c+dx)))^{7/2}}{\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right) \sqrt{\sec(c+dx)}} \left(-\frac{247 \cos\left(\frac{dx}{2}\right) \sin\left(\frac{c}{2}\right)}{12d} - \frac{247 \cos\left(\frac{c}{2}\right) \sin\left(\frac{dx}{2}\right)}{12d} + \frac{379 \sec\left(\frac{c}{2}\right) \sec^2\left(\frac{c}{2}+\frac{dx}{2}\right) \sin\left(\frac{dx}{2}\right)}{24d} - \frac{41 \sec\left(\frac{c}{2}\right) \sec^4\left(\frac{c}{2}+\frac{dx}{2}\right) \sin\left(\frac{dx}{2}\right)}{12d} \right) \\
&\quad (a(1+\cos(c+dx)))^{7/2}
\end{aligned}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]

[Out] ((-1/4*I)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(64*ArcSinh[E^(I*(c + d*x))]) + (177*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])/Sqrt[2] - 64*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^7)/(Sqrt[2]*d*E^((I/2)*(c + d*x)))*(a*(1 + Cos[c + d*x]))^(7/2) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d*x]]*((-247*Cos[(d*x)/2]*Sin[c/2])/(12*d) - (247*Cos[c/2]*Sin[(d*x)/2])/(12*d) + (379*Sec[c/2]*Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(24*d) - (41*Sec[c/2]*Sec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(12*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(3*d) + (379*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - (41*Sec[c/2 + (d*x)/2]^3*Tan[c/2])/(12*d) + (Sec[c/2 + (d*x)/2]^5*Tan[c/2])/(3*d)))/(a*(1 + Cos[c + d*x]))^(7/2)

Maple [A] (warning: unable to verify)

Time = 4.38 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.58

method	result
default	$\frac{\left(\left(\csc^2(dx+c)\right)\left(1-\cos(dx+c)\right)^2+1\right)^4\left(\left(\csc^2(dx+c)\right)\left(1-\cos(dx+c)\right)^2-1\right)\sqrt{\frac{a}{\left(\csc^2(dx+c)\right)\left(1-\cos(dx+c)\right)^2+1}}\left(8\left(\csc^5(dx+c)\right)\sqrt{-\left(\csc^2(dx+c)\right)\left(1-\cos(dx+c)\right)^2-1}\right)}{\dots}$

[In] int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

```
[Out] 1/384/d/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^7/2/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(9/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^4*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(a/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(8*csc(d*x+c)^5*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^5-50*csc(d*x+c)^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(1-cos(d*x+c))^3+384*2^(1/2)*arctan(2^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(csc(d*x+c)-cot(d*x+c))))+189*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc(d*x+c)-cot(d*x+c))-531*arcsin(cot(d*x+c)-csc(d*x+c)))*2^(1/2)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{7/2}(c + dx)} dx = \frac{531 \sqrt{2} (\cos(dx + c)^4 + 4 \cos(dx + c)^3 + 6 \cos(dx + c)^2 + 4 \cos(dx + c) + 1) \sqrt{a} \arctan(\sqrt{2} \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)})}{(a^4 d \cos(dx + c)^4 + 4 a^4 d \cos(dx + c)^3 + 6 a^4 d \cos(dx + c)^2 + 4 a^4 d \cos(dx + c) + a^4 d)}$$

[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")

```
[Out] 1/384*(531*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 768*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(247*cos(d*x + c)^3 + 362*cos(d*x + c)^2 + 147*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{7/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{7/2} \sec(dx + c)^{7/2}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(7/2)), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{7/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2} (a + a \cos(c + dx))^{7/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2)),x)

[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(7/2)), x)

$$3.393 \quad \int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	4557
Rubi [A] (verified)	4558
Mathematica [C] (verified)	4562
Maple [A] (verified)	4562
Fricas [A] (verification not implemented)	4563
Sympy [F(-1)]	4563
Maxima [F]	4564
Giac [F(-1)]	4564
Mupad [F(-1)]	4564

Optimal result

Integrand size = 25, antiderivative size = 294

$$\int \frac{1}{(a+a \cos(c+dx))^{7/2} \sec^{\frac{9}{2}}(c+dx)} dx =$$

$$\frac{7 \arcsin\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{a^{7/2} d}$$

$$+ \frac{637 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{64 \sqrt{2} a^{7/2} d}$$

$$- \frac{\sin(c+dx)}{6d(a+a \cos(c+dx))^{7/2} \sec^{\frac{7}{2}}(c+dx)} - \frac{7 \sin(c+dx)}{16ad(a+a \cos(c+dx))^{5/2} \sec^{\frac{5}{2}}(c+dx)}$$

$$- \frac{192a^2 d (a+a \cos(c+dx))^{3/2} \sec^{\frac{3}{2}}(c+dx)}{259 \sin(c+dx)}$$

$$+ \frac{189 \sin(c+dx)}{64a^3 d \sqrt{a+a \cos(c+dx)} \sqrt{\sec(c+dx)}}$$

[Out] -1/6*sin(d*x+c)/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2)-7/16*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2)-259/192*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2)+189/64*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)-7*arcsin(sin(d*x+c)*a^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d+637/128*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(7/2)/d*2^(1/2)

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4307, 2844, 3056, 3062, 3061, 2861, 211, 2853, 222}

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{9/2}(c + dx)} dx =$$

$$-\frac{7\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \arcsin\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{a \cos(c + dx) + a}}\right)}{a^{7/2}d}$$

$$+ \frac{637\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)} \arctan\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2}\sqrt{\cos(c + dx)}\sqrt{a \cos(c + dx) + a}}\right)}{64\sqrt{2}a^{7/2}d}$$

$$+ \frac{189 \sin(c + dx)}{64a^3d\sqrt{\sec(c + dx)}\sqrt{a \cos(c + dx) + a}} - \frac{259 \sin(c + dx)}{192a^2d \sec^{3/2}(c + dx)(a \cos(c + dx) + a)^{3/2}}$$

$$- \frac{7 \sin(c + dx)}{16ad \sec^{5/2}(c + dx)(a \cos(c + dx) + a)^{5/2}} - \frac{\sin(c + dx)}{6d \sec^{7/2}(c + dx)(a \cos(c + dx) + a)^{7/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(9/2)),x]

[Out] (-7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(a^(7/2)*d) + (637*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)) - (7*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) - (259*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (189*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2844

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1))

+ b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&
 & GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2853

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(d_)*sin[(e_) + (f_)*
 (x_)]], x_Symbol] := Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, b(Cos
 [e + f*x]/Sqrt[a + b*Ssin[e + f*x]])], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
 Q[a^2 - b^2, 0] && EqQ[d, a/b]

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e
 .) + (f)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
 - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Si
 n[e + f*x]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
 (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
 p[(A*b - a*B)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^n/(
 a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m +
 1)*(c + d*Ssin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; Free
 Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
 egerQ[2*n] || EqQ[c, 0])

Rule 3061

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) +
 (f_)*(x_)]]*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dis
 t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Ssin[e + f*x]]/Sqrt[c + d*Ssin[e + f*x]],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
 2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3062

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
 (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
 p[(-B)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^n/(f*(m +
 n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Ssin[e + f*x])^m*(c + d*Si

$n[e + f*x]^{(n - 1)} * \text{Simp}[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n)) * \text{Sin}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[n, 0] \&\& (\text{IntegerQ}[n] \mid\mid \text{EqQ}[m + 1/2, 0])$

Rule 4307

$\text{Int}[(\text{csc}[a_.] + (b_.) * (x_)] * (c_.)^{(m_.)} * (u_), x_Symbol] :> \text{Dist}[(c * \text{Csc}[a + b*x])^m * (c * \text{Sin}[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u] / (c * \text{Sin}[a + b*x])^m, x], x] /; \text{FreeQ}\{a, b, c, m\}, x\} \&\& !\text{IntegerQ}[m] \&\& \text{KnownSineIntegrandQ}[u, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + a \cos(c + dx))^{7/2}} dx \\
 &= -\frac{\sin(c + dx)}{6d(a + a \cos(c + dx))^{7/2} \sec^{\frac{7}{2}}(c + dx)} \\
 &\quad - \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\cos^{\frac{5}{2}}(c + dx) \left(\frac{7a}{2} - 7a \cos(c + dx) \right)}{(a + a \cos(c + dx))^{5/2}} dx}{6a^2} \\
 &= -\frac{\sin(c + dx)}{6d(a + a \cos(c + dx))^{7/2} \sec^{\frac{7}{2}}(c + dx)} - \frac{7 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{5/2} \sec^{\frac{5}{2}}(c + dx)} \\
 &\quad - \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c + dx) \left(\frac{105a^2}{4} - \frac{77}{2} a^2 \cos(c + dx) \right)}{(a + a \cos(c + dx))^{3/2}} dx}{24a^4} \\
 &= -\frac{\sin(c + dx)}{6d(a + a \cos(c + dx))^{7/2} \sec^{\frac{7}{2}}(c + dx)} - \frac{7 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{5/2} \sec^{\frac{5}{2}}(c + dx)} \\
 &\quad - \frac{259 \sin(c + dx)}{192a^2 d (a + a \cos(c + dx))^{3/2} \sec^{\frac{3}{2}}(c + dx)} \\
 &\quad - \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\cos(c + dx)} \left(\frac{777a^3}{8} - \frac{567}{4} a^3 \cos(c + dx) \right)}{\sqrt{a + a \cos(c + dx)}} dx}{48a^6} \\
 &= -\frac{\sin(c + dx)}{6d(a + a \cos(c + dx))^{7/2} \sec^{\frac{7}{2}}(c + dx)} - \frac{7 \sin(c + dx)}{16ad(a + a \cos(c + dx))^{5/2} \sec^{\frac{5}{2}}(c + dx)} \\
 &\quad - \frac{259 \sin(c + dx)}{192a^2 d (a + a \cos(c + dx))^{3/2} \sec^{\frac{3}{2}}(c + dx)} \\
 &\quad + \frac{189 \sin(c + dx)}{64a^3 d \sqrt{a + a \cos(c + dx)} \sqrt{\sec(c + dx)}} \\
 &\quad - \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{-\frac{567a^4}{8} + 168a^4 \cos(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a + a \cos(c + dx)}} dx}{48a^7}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{7/2}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{5/2}(c+dx)} \\
&\quad - \frac{259\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sec^{3/2}(c+dx)} \\
&+ \frac{189\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx}{2a^4} \\
&\quad + \frac{\left(637\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{128a^3} \\
&= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{7/2}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{5/2}(c+dx)} \\
&\quad - \frac{259\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sec^{3/2}(c+dx)} + \frac{189\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^4d} \\
&\quad - \frac{\left(637\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64a^2d} \\
&= -\frac{7\arcsin\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} \\
&\quad + \frac{637\arctan\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} \\
&\quad - \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{7/2}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{5/2}(c+dx)} \\
&\quad - \frac{259\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}\sec^{3/2}(c+dx)} \\
&\quad + \frac{189\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{9/2}(c + dx)} dx = \frac{e^{-\frac{1}{2}i(c+dx)} \left(-\frac{1}{64} i e^{-4i(c+dx)} \sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\right) \left(-1911 e^{i(c+dx)} (1 + e^{i(c+dx)})\right)}{\dots}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(9/2)),x]

[Out] ((((-1/64*I)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-1911*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^6*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x))]/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]) + Sqrt[2]*(-96 - 1003*E^(I*(c + d*x)) - 2169*E^((2*I)*(c + d*x)) - 2297*E^((3*I)*(c + d*x)) - 779*E^((4*I)*(c + d*x)) + 779*E^((5*I)*(c + d*x)) + 2297*E^((6*I)*(c + d*x)) + 2169*E^((7*I)*(c + d*x)) + 1003*E^((8*I)*(c + d*x)) + 96*E^((9*I)*(c + d*x)) + 672*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^6*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Cos[(c + d*x)/2])/E^((4*I)*(c + d*x)) + (672*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Cos[(c + d*x)/2]^7)*Sqrt[a*(1 + Cos[c + d*x])])/(24*a^4*d*E^((I/2)*(c + d*x))*(1 + Cos[c + d*x])^4)

Maple [A] (verified)

Time = 15.02 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.52

method	result
default	$\frac{(192\sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (\cos^3(dx+c)) \sin(dx+c) + 1099\sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (\cos^2(dx+c)) \sin(dx+c) - 1344\sqrt{2} \arctan(\tan(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}})) \dots}{\dots}$

[In] int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] 1/384/d/sec(d*x+c)^(1/2)*(192*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*sin(d*x+c)+1099*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-1344*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)^3+1442*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-1911*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^3-4032*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)^2+567*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5733*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2-4032*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))*cos(d*x+c)-5733*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)-1344*2^(1/2)*arctan(tan(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))))^(1/2))-1911*arcsin(co

$t(d*x+c)-\csc(d*x+c))*(a*(1+\cos(d*x+c)))^{(1/2)}/(1+\cos(d*x+c))^4/(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)*2^{(1/2)}/a^4$

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{9/2}(c + dx)} dx =$$

$$1911 \sqrt{2} (\cos(dx + c)^4 + 4 \cos(dx + c)^3 + 6 \cos(dx + c)^2 + 4 \cos(dx + c) + 1) \sqrt{a} \arctan \left(\frac{\sqrt{2} \sqrt{a \cos(dx + c)}}{\sqrt{a} \sin(dx + c)} \right)$$

[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] -1/384*(1911*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2688*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(192*cos(d*x + c)^4 + 1099*cos(d*x + c)^3 + 1442*cos(d*x + c)^2 + 567*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{9/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{9/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{7/2} \sec(dx + c)^{9/2}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(9/2)), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{9/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{7/2} \sec^{9/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{9/2} (a + a \cos(c + dx))^{7/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(7/2)),x)

[Out] int(1/((1/cos(c + d*x))^(9/2)*(a + a*cos(c + d*x))^(7/2)), x)

$$3.394 \quad \int \frac{1}{(a+a \cos(c+dx))^{9/2} \sec^2(c+dx)} dx$$

Optimal result	4565
Rubi [A] (verified)	4565
Mathematica [A] (verified)	4569
Maple [A] (verified)	4569
Fricas [A] (verification not implemented)	4570
Sympy [F(-1)]	4570
Maxima [F]	4570
Giac [F(-1)]	4571
Mupad [F(-1)]	4571

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{1}{(a+a \cos(c+dx))^{9/2} \sec^2(c+dx)} dx = \frac{45 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{1024 \sqrt{2} a^{9/2} d} - \frac{\sin(c+dx)}{8d(a+a \cos(c+dx))^{9/2} \sec^3(c+dx)} - \frac{5 \sin(c+dx)}{32ad(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} + \frac{33 \sin(c+dx)}{256a^2 d(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{73 \sin(c+dx)}{1024a^3 d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

```
[Out] -1/8*sin(d*x+c)/d/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(3/2)-5/32*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2)+33/256*sin(d*x+c)/a^2/d/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2)+73/1024*sin(d*x+c)/a^3/d/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+45/2048*arctan(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(9/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {4307, 2844, 3056, 3057, 12, 2861, 211}

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{5/2}(c + dx)} dx = \frac{45 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \arctan\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2} \sqrt{\cos(c + dx)} \sqrt{a \cos(c + dx) + a}}\right)}{1024 \sqrt{2} a^{9/2} d}$$

$$+ \frac{73 \sin(c + dx)}{1024 a^3 d \sqrt{\sec(c + dx)} (a \cos(c + dx) + a)^{3/2}} + \frac{33 \sin(c + dx)}{256 a^2 d \sqrt{\sec(c + dx)} (a \cos(c + dx) + a)^{5/2}}$$

$$- \frac{\sin(c + dx)}{8 d \sec^{3/2}(c + dx) (a \cos(c + dx) + a)^{9/2}} - \frac{5 \sin(c + dx)}{32 a d \sqrt{\sec(c + dx)} (a \cos(c + dx) + a)^{7/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)),x]

[Out] (45*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(1024*Sqrt[2]*a^(9/2)*d) - Sin[c + d*x]/(8*d*(a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])/(32*a*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + (33*Sin[c + d*x])/(256*a^2*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (73*Sin[c + d*x])/(1024*a^3*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2844

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&

EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + a \cos(c + dx))^{9/2}} dx \\ &= -\frac{\sin(c + dx)}{8d(a + a \cos(c + dx))^{9/2} \sec^{\frac{3}{2}}(c + dx)} \\ &\quad - \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\cos(c + dx)} \left(\frac{3a}{2} - 6a \cos(c + dx) \right)}{(a + a \cos(c + dx))^{7/2}} dx}{8a^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{3/2}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{15a^2}{4}-21a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} dx}{48a^4} \\
&= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{3/2}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{33\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{21a^3}{8}-\frac{99}{4}a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{192a^6} \\
&= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{3/2}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{33\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{73\sin(c+dx)}{1024a^3d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int -\frac{135a^4}{16\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{384a^8} \\
&= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{3/2}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{33\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{73\sin(c+dx)}{1024a^3d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(45\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2048a^4} \\
&= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{3/2}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{33\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{73\sin(c+dx)}{1024a^3d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(45\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024a^3d}
\end{aligned}$$

$$\begin{aligned}
&= \frac{45 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{1024 \sqrt{2} a^{9/2} d} \\
&\quad - \frac{\sin(c+dx)}{8d(a+a \cos(c+dx))^{9/2} \sec^{3/2}(c+dx)} - \frac{5 \sin(c+dx)}{32ad(a+a \cos(c+dx))^{7/2} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{33 \sin(c+dx)}{256a^2 d (a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{73 \sin(c+dx)}{1024a^3 d (a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+a \cos(c+dx))^{9/2} \sec^{5/2}(c+dx)} dx = \frac{\sec^6\left(\frac{1}{2}(c+dx)\right) \left(2(882+999 \cos(c+dx)) + 702 \cos(2(c+dx))\right)}{(a+a \cos(c+dx))^{9/2} \sec^{5/2}(c+dx)}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)), x]

[Out] (Sec[(c + d*x)/2]^6*(2*(882 + 999*Cos[c + d*x] + 702*Cos[2*(c + d*x)] + 73*Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]] + 5760*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Cos[(c + d*x)/2]^8*Sec[c + d*x]*Tan[(c + d*x)/2])/(65536*Sqrt[2]*a^4*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.32

method	result
default	$\sqrt{a(1+\cos(dx+c))} \left(73\sqrt{2} \cos(dx+c) \sin(dx+c) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 351 \sin(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 45 \arcsin(\cot(dx+c) - \csc(dx+c)) \right)$

[In] int(1/(a+cos(d*x+c)*a)^(9/2)/sec(d*x+c)^(5/2), x, method=_RETURNVERBOSE)

[Out] 1/2048/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^5/sec(d*x+c)^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(73*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+351*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-45*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)^2+195*tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-180*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+45*tan(d*x+c)*sec(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-270*arcsin(cot(d*x+c)-csc(d*x+c))-180*sec(d*x+c)*arcsin(cot(d*x+c)-csc(d*x+c))-45*sec(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)/a^5

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{5/2}(c + dx)} dx =$$

$$\frac{45 \sqrt{2} (\cos(dx + c)^5 + 5 \cos(dx + c)^4 + 10 \cos(dx + c)^3 + 10 \cos(dx + c)^2 + 5 \cos(dx + c) + 1) \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{a \cos(dx + c) + a} \sqrt{\cos(dx + c)}}{\sqrt{a} \sin(dx + c)}\right) - 2(73 \cos(dx + c)^4 + 351 \cos(dx + c)^3 + 195 \cos(dx + c)^2 + 45 \cos(dx + c)) \sqrt{a \cos(dx + c) + a} \sin(dx + c) / \sqrt{\cos(dx + c)}}{2048 (a^5 d \cos(dx + c)^5 + 5 a^5 d \cos(dx + c)^4 + 10 a^5 d \cos(dx + c)^3 + 10 a^5 d \cos(dx + c)^2 + 5 a^5 d \cos(dx + c) + a^5 d)}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2048*(45*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(73*cos(d*x + c)^4 + 351*cos(d*x + c)^3 + 195*cos(d*x + c)^2 + 45*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{5/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(9/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{9/2} \sec(dx + c)^{5/2}} dx$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{5/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + a \cos(c + dx))^{9/2}} dx$$

```
[In] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(9/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + a*cos(c + d*x))^(9/2)), x)
```

$$3.395 \quad \int \frac{1}{(a+a \cos(c+dx))^{9/2} \sec^2(c+dx)} dx$$

Optimal result	4572
Rubi [A] (verified)	4572
Mathematica [A] (verified)	4576
Maple [A] (verified)	4576
Fricas [A] (verification not implemented)	4577
Sympy [F(-1)]	4577
Maxima [F]	4578
Giac [F(-1)]	4578
Mupad [F(-1)]	4578

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{1}{(a+a \cos(c+dx))^{9/2} \sec^2(c+dx)} dx = \frac{35 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{1024 \sqrt{2} a^{9/2} d} - \frac{\sin(c+dx)}{8d(a+a \cos(c+dx))^{9/2} \sec^{5/2}(c+dx)} - \frac{19 \sin(c+dx)}{96ad(a+a \cos(c+dx))^{7/2} \sec^{3/2}(c+dx)} - \frac{187 \sin(c+dx)}{768a^2d(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{853 \sin(c+dx)}{3072a^3d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

[Out] $-1/8*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(9/2)}/\sec(d*x+c)^{(5/2)}-19/96*\sin(d*x+c)/a/d/(a+a*\cos(d*x+c))^{(7/2)}/\sec(d*x+c)^{(3/2)}-187/768*\sin(d*x+c)/a^2/d/(a+a*\cos(d*x+c))^{(5/2)}/\sec(d*x+c)^{(1/2)}+853/3072*\sin(d*x+c)/a^3/d/(a+a*\cos(d*x+c))^{(3/2)}/\sec(d*x+c)^{(1/2)}+35/2048*\arctan(1/2*\sin(d*x+c)*a^{(1/2)}*2^{(1/2)}/\cos(d*x+c)^{(1/2)}/(a+a*\cos(d*x+c))^{(1/2)}*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/a^{(9/2)}/d*2^{(1/2)}$

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used

= {4307, 2844, 3056, 3057, 12, 2861, 211}

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^7(c + dx)} dx = \frac{35 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \arctan\left(\frac{\sqrt{a} \sin(c + dx)}{\sqrt{2} \sqrt{\cos(c + dx)} \sqrt{a \cos(c + dx) + a}}\right)}{1024 \sqrt{2} a^{9/2} d}$$

$$+ \frac{853 \sin(c + dx)}{3072 a^3 d \sqrt{\sec(c + dx)} (a \cos(c + dx) + a)^{3/2}} - \frac{187 \sin(c + dx)}{768 a^2 d \sqrt{\sec(c + dx)} (a \cos(c + dx) + a)^{5/2}}$$

$$- \frac{19 \sin(c + dx)}{96 a d \sec^3(c + dx) (a \cos(c + dx) + a)^{7/2}} - \frac{\sin(c + dx)}{8 d \sec^5(c + dx) (a \cos(c + dx) + a)^{9/2}}$$

[In] Int[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(7/2)),x]

[Out] (35*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(1024*Sqrt[2]*a^(9/2)*d) - Sin[c + d*x]/(8*d*(a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)) - (19*Sin[c + d*x])/(96*a*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)) - (187*Sin[c + d*x])/(768*a^2*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (853*Sin[c + d*x])/(3072*a^3*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2844

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*(2*m + 1)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 2)*Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1) + b*c*(m + n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 2861

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c - b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&

EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3056

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + a \cos(c + dx))^{9/2}} dx \\ &= -\frac{\sin(c + dx)}{8d(a + a \cos(c + dx))^{9/2} \sec^{\frac{5}{2}}(c + dx)} \\ &\quad - \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c + dx) \left(\frac{5a}{2} - 7a \cos(c + dx) \right)}{(a + a \cos(c + dx))^{7/2}} dx}{8a^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{5/2}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{3/2}(c+dx)} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\sqrt{\cos(c+dx)}\left(\frac{57a^2}{4} - \frac{65}{2}a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}} dx}{48a^4} \\
&= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{5/2}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{3/2}(c+dx)} \\
&\quad - \frac{187\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{187a^3}{8} - \frac{333}{4}a^3\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} dx}{192a^6} \\
&= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{5/2}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{3/2}(c+dx)} \\
&\quad - \frac{187\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{853\sin(c+dx)}{3072a^3d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int -\frac{105a^4}{16\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{384a^8} \\
&= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{5/2}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{3/2}(c+dx)} \\
&\quad - \frac{187\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{853\sin(c+dx)}{3072a^3d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(35\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} dx}{2048a^4} \\
&= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{5/2}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{3/2}(c+dx)} \\
&\quad - \frac{187\sin(c+dx)}{768a^2d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{853\sin(c+dx)}{3072a^3d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(35\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \text{Subst}\left(\int \frac{1}{2a^2+ax^2} dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024a^3d}
\end{aligned}$$

$$= \frac{35 \arctan\left(\frac{\sqrt{a} \sin(c+dx)}{\sqrt{2} \sqrt{\cos(c+dx)} \sqrt{a+a \cos(c+dx)}}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}}{1024 \sqrt{2} a^{9/2} d} - \frac{\sin(c+dx)}{8d(a+a \cos(c+dx))^{9/2} \sec^{5/2}(c+dx)} - \frac{19 \sin(c+dx)}{96ad(a+a \cos(c+dx))^{7/2} \sec^{3/2}(c+dx)} - \frac{187 \sin(c+dx)}{768a^2d(a+a \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} + \frac{853 \sin(c+dx)}{3072a^3d(a+a \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 6.06 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.67

$$\int \frac{1}{(a+a \cos(c+dx))^{9/2} \sec^{7/2}(c+dx)} dx = \frac{2 \cos^9\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{\frac{1}{1-2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)}} \sqrt{1-2 \sin^2\left(\frac{c}{2} + \frac{dx}{2}\right)} \left(\frac{35}{128} \arcsin\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)\right)}{(a+a \cos(c+dx))^{9/2} \sec^{7/2}(c+dx)}$$

[In] Integrate[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(7/2)),x]

[Out] (2*Cos[c/2 + (d*x)/2]^9*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*((35*ArcSin[Sin[c/2 + (d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]])/128 + (93*Sin[c/2 + (d*x)/2]*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/((128*Sqrt[Cos[(c + d*x)/2]^2]) - (163*Sin[c/2 + (d*x)/2]^3*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/(192*(Cos[(c + d*x)/2]^2)^(3/2)) + (25*Sin[c/2 + (d*x)/2]^5*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/(48*(Cos[(c + d*x)/2]^2)^(5/2)) - (Sin[c/2 + (d*x)/2]^7*Sqrt[1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2])/(8*(Cos[(c + d*x)/2]^2)^(7/2)))/(d*(a*(1 + Cos[c + d*x]))^(9/2))

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.32

method	result
default	$\frac{\sqrt{a(1+\cos(dx+c))} \left(853 \sin(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} + 819 \tan(dx+c) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - 105 \arcsin(\cot(dx+c) - \csc(dx+c)) \cos(dx+c) \right)}{6144 d (a(1+\cos(dx+c)))^{1/2} (1+\cos(dx+c))^5 \sec(dx+c)^{7/2} (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} (853 \sin(dx+c) 2^{1/2} (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} + 819 \tan(dx+c) 2^{1/2} (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} - 105 \arcsin(\cot(dx+c) - \csc(dx+c)) \cos(dx+c))^{1/2}}$

[In] int(1/(a+cos(d*x+c)*a)^(9/2)/sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/6144/d*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^5/sec(d*x+c)^(7/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(853*sin(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+819*tan(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-105*arcsin(cot(d*x+c)-csc(d*x+c))*cos(d*x+c))^(1/2)

```
in(cot(d*x+c)-csc(d*x+c))*cos(d*x+c)+455*tan(d*x+c)*sec(d*x+c)*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)-420*arcsin(cot(d*x+c)-csc(d*x+c))+105*tan(d*x
+c)*sec(d*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-630*sec(d*x+c)*a
rcsin(cot(d*x+c)-csc(d*x+c))-420*sec(d*x+c)^2*arcsin(cot(d*x+c)-csc(d*x+c))
-105*sec(d*x+c)^3*arcsin(cot(d*x+c)-csc(d*x+c))*2^(1/2)/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{7/2}(c + dx)} dx =$$

$$\frac{105 \sqrt{2} (\cos(dx + c)^5 + 5 \cos(dx + c)^4 + 10 \cos(dx + c)^3 + 10 \cos(dx + c)^2 + 5 \cos(dx + c) + 1) \sqrt{a} \arcsin\left(\frac{\cos(dx + c) - a}{\sqrt{a \cos(dx + c) + a}}\right) - 2(853 \cos(dx + c)^4 + 819 \cos(dx + c)^3 + 455 \cos(dx + c)^2 + 105 \cos(dx + c)) \sqrt{a \cos(dx + c) + a} \sin(dx + c) / \sqrt{\cos(dx + c)}}{6144 (a^5 d \cos(dx + c)^5 + 5 a^5 d \cos(dx + c)^4 + 10 a^5 d \cos(dx + c)^3 + 10 a^5 d \cos(dx + c)^2 + 5 a^5 d \cos(dx + c) + a^5 d)}$$

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/6144*(105*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos
s(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(853*cos(d*x
+ c)^4 + 819*cos(d*x + c)^3 + 455*cos(d*x + c)^2 + 105*cos(d*x + c))*sqrt(
a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^5*d*cos(d*x + c)^5
+ 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^
2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{7/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+a*cos(d*x+c))**(9/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{7/2}(c + dx)} dx = \int \frac{1}{(a \cos(dx + c) + a)^{9/2} \sec(dx + c)^{7/2}} dx$$

[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(7/2)), x)

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + a \cos(c + dx))^{9/2} \sec^{7/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{7/2} (a + a \cos(c + dx))^{9/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(9/2)),x)

[Out] int(1/((1/cos(c + d*x))^(7/2)*(a + a*cos(c + d*x))^(9/2)), x)

3.396 $\int (a + a \cos(c + dx))^{3/2} \sec^{5/4}(c + dx) dx$

Optimal result	4579
Rubi [A] (verified)	4579
Mathematica [A] (verified)	4580
Maple [F]	4581
Fricas [A] (verification not implemented)	4581
Sympy [F(-1)]	4581
Maxima [B] (verification not implemented)	4581
Giac [F(-1)]	4582
Mupad [B] (verification not implemented)	4582

Optimal result

Integrand size = 25, antiderivative size = 38

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/4}(c + dx) dx = \frac{4a^2 \sqrt[4]{\sec(c + dx)} \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}}$$

[Out] $4*a^2*\sec(d*x+c)^{(1/4)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4307, 2841, 8}

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/4}(c + dx) dx = \frac{4a^2 \sin(c + dx) \sqrt[4]{\sec(c + dx)}}{d \sqrt{a \cos(c + dx) + a}}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(3/2)}*\text{Sec}[c + d*x]^{(5/4)}, x]$

[Out] $(4*a^2*\text{Sec}[c + d*x]^{(1/4)}*\text{Sin}[c + d*x])/(d*\text{Sqrt}[a + a*\text{Cos}[c + d*x]])$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2841

$\text{Int}[(a + (b_*)*\sin[(e_*) + (f_*)*(x_*)])^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(-b^2)*(b*c - a*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m - 2)}*((c + d*\text{Sin}[e + f*x])^{(n + 1)})/(d*f*(n + 1)*(b*c + a*d)), x] + \text{Dist}[b^2/(d*(n + 1)*(b*c + a*d)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m -$

```

2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))

```

Rule 4307

```

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt[4]{\cos(c+dx)} \sqrt[4]{\sec(c+dx)} \right) \int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{5/4}(c+dx)} dx \\
&= \frac{4a^2 \sqrt[4]{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \left(4a \sqrt[4]{\cos(c+dx)} \sqrt[4]{\sec(c+dx)} \right) \int 0 dx \\
&= \frac{4a^2 \sqrt[4]{\sec(c+dx)} \sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\begin{aligned}
&\int (a+a\cos(c+dx))^{3/2} \sec^{5/4}(c \\
&+ dx) dx = \frac{2(a(1+\cos(c+dx)))^{3/2} \sec^2\left(\frac{1}{2}(c+dx)\right) \sqrt[4]{\sec(c+dx)} \tan\left(\frac{1}{2}(c+dx)\right)}{d}
\end{aligned}$$

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/4), x]
```

```
[Out] (2*(a*(1 + Cos[c + d*x]))^(3/2)*Sec[(c + d*x)/2]^2*Sec[c + d*x]^(1/4)*Tan[(
c + d*x)/2])/d
```


Maple [F]

$$\int (a + \cos(dx + c) a)^{\frac{3}{2}} \left(\sec^{\frac{5}{4}}(dx + c) \right) dx$$

[In] `int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(5/4),x)`

[Out] `int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(5/4),x)`

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int (a + a \cos(c + dx))^{\frac{3}{2}} \sec^{\frac{5}{4}}(c + dx) dx = \frac{4 \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{(d \cos(dx + c) + d) \cos(dx + c)^{\frac{1}{4}}}$$

[In] `integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="fricas")`

[Out] `4*sqrt(a*cos(d*x + c) + a)*a*sin(d*x + c)/((d*cos(d*x + c) + d)*cos(d*x + c)^(1/4))`

Sympy [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{\frac{3}{2}} \sec^{\frac{5}{4}}(c + dx) dx = \text{Timed out}$$

[In] `integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/4),x)`

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(34) = 68.

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.18

$$\int (a + a \cos(c + dx))^{\frac{3}{2}} \sec^{\frac{5}{4}}(c + dx) dx = \frac{4 \left(\frac{\sqrt{2} a^{\frac{3}{2}} \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sqrt{2} a^{\frac{3}{2}} \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{5}{4}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1} + 1 \right)^{\frac{5}{4}} \left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)^{\frac{1}{4}}}$$

[In] `integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="maxima")`

[Out] $4*(\sqrt{2})*a^{(3/2)}*\sin(dx + c)/(\cos(dx + c) + 1) - \sqrt{2}*a^{(3/2)}*\sin(dx + c)^3/(\cos(dx + c) + 1)^3/(d*(\sin(dx + c)/(\cos(dx + c) + 1) + 1)^{(5/4)}*(-\sin(dx + c)/(\cos(dx + c) + 1) + 1)^{(5/4)}*(\sin(dx + c)^2/(\cos(dx + c) + 1)^2 + 1)^{(1/4)})$

Giac [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/4}(c + dx) dx = \text{Timed out}$$

[In] `integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="giac")`

[Out] Timed out

Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a + a \cos(c + dx))^{3/2} \sec^{5/4}(c + dx) dx = \frac{4 a \sin(c + dx) \sqrt{a (\cos(c + dx) + 1)} \left(\frac{1}{\cos(c + dx)}\right)^{1/4}}{d (\cos(c + dx) + 1)}$$

[In] `int((1/cos(c + d*x))^(5/4)*(a + a*cos(c + d*x))^(3/2),x)`

[Out] $(4*a*\sin(c + d*x)*(a*(\cos(c + d*x) + 1))^{(1/2)}*(1/\cos(c + d*x))^{(1/4)})/(d*(\cos(c + d*x) + 1))$

3.397 $\int \cos^m(c + dx)(a + a \cos(c + dx))^4 dx$

Optimal result	4583
Rubi [A] (verified)	4584
Mathematica [A] (verified)	4587
Maple [F]	4587
Fricas [F]	4587
Sympy [F(-1)]	4588
Maxima [F]	4588
Giac [F]	4588
Mupad [F(-1)]	4588

Optimal result

Integrand size = 21, antiderivative size = 302

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^4 dx = \frac{a^4(55 + 29m + 4m^2) \cos^{1+m}(c + dx) \sin(c + dx)}{d(2 + m)(3 + m)(4 + m)} + \frac{\cos^{1+m}(c + dx) (a^2 + a^2 \cos(c + dx))^2 \sin(c + dx)}{d(4 + m)} + \frac{2(5 + m) \cos^{1+m}(c + dx) (a^4 + a^4 \cos(c + dx)) \sin(c + dx)}{d(3 + m)(4 + m)} - \frac{a^4(35 + 40m + 8m^2) \cos^{1+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1 + m)(2 + m)(4 + m) \sqrt{\sin^2(c + dx)}} - \frac{4a^4(5 + 2m) \cos^{2+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 + m)(3 + m) \sqrt{\sin^2(c + dx)}}$$

```
[Out] a^4*(4*m^2+29*m+55)*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(4+m)/(m^2+5*m+6)+cos(d*x+c)^(1+m)*(a^2+a^2*cos(d*x+c))^2*sin(d*x+c)/d/(4+m)+2*(5+m)*cos(d*x+c)^(1+m)*(a^4+a^4*cos(d*x+c))*sin(d*x+c)/d/(3+m)/(4+m)-a^4*(8*m^2+40*m+35)*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m], [3/2+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/d/(m^3+7*m^2+14*m+8)/(sin(d*x+c)^2)^(1/2)-4*a^4*(5+2*m)*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m], [2+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2842, 3055, 3047, 3102, 2827, 2722}

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^4 dx =$$

$$\frac{a^4(8m^2 + 40m + 35) \sin(c + dx) \cos^{m+1}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \cos^2(c + dx)\right)}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c + dx)}} -$$

$$\frac{4a^4(2m+5) \sin(c + dx) \cos^{m+2}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \cos^2(c + dx)\right)}{d(m+2)(m+3)\sqrt{\sin^2(c + dx)}} +$$

$$\frac{a^4(4m^2 + 29m + 55) \sin(c + dx) \cos^{m+1}(c + dx)}{d(m+2)(m+3)(m+4)} +$$

$$\frac{2(m+5) \sin(c + dx) (a^4 \cos(c + dx) + a^4) \cos^{m+1}(c + dx)}{d(m+3)(m+4)} +$$

$$\frac{\sin(c + dx) (a^2 \cos(c + dx) + a^2)^2 \cos^{m+1}(c + dx)}{d(m+4)}$$

[In] Int[Cos[c + d*x]^m*(a + a*cos[c + d*x])^4,x]

[Out] (a^4*(55 + 29*m + 4*m^2)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)*(4 + m)) + (Cos[c + d*x]^(1 + m)*(a^2 + a^2*cos[c + d*x])^2*sin[c + d*x])/ (d*(4 + m)) + (2*(5 + m)*Cos[c + d*x]^(1 + m)*(a^4 + a^4*cos[c + d*x])*Sin[c + d*x])/ (d*(3 + m)*(4 + m)) - (a^4*(35 + 40*m + 8*m^2)*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/ (d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c + d*x]^2]) - (4*a^4*(5 + 2*m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/ (d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)]^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2842

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))

```

Rule 3047

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

```

Rule 3055

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
+ 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
+ b*d*(n + 1) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
&& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

```

Rule 3102

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cos^{1+m}(c + dx) (a^2 + a^2 \cos(c + dx))^2 \sin(c + dx)}{d(4 + m)} \\
&+ \frac{\int \cos^m(c + dx) (a + a \cos(c + dx))^2 (a^2(5 + 2m) + 2a^2(5 + m) \cos(c + dx)) dx}{4 + m}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\cos^{1+m}(c+dx) (a^2 + a^2 \cos(c+dx))^2 \sin(c+dx)}{d(4+m)} \\
&+ \frac{2(5+m) \cos^{1+m}(c+dx) (a^4 + a^4 \cos(c+dx)) \sin(c+dx)}{d(3+m)(4+m)} \\
&+ \frac{\int \cos^m(c+dx) (a + a \cos(c+dx)) (a^3(25 + 23m + 4m^2) + a^3(55 + 29m + 4m^2) \cos(c+dx)) dx}{12 + 7m + m^2} \\
&= \frac{\cos^{1+m}(c+dx) (a^2 + a^2 \cos(c+dx))^2 \sin(c+dx)}{d(4+m)} \\
&+ \frac{2(5+m) \cos^{1+m}(c+dx) (a^4 + a^4 \cos(c+dx)) \sin(c+dx)}{d(3+m)(4+m)} \\
&+ \frac{\int \cos^m(c+dx) (a^4(25 + 23m + 4m^2) + (a^4(25 + 23m + 4m^2) + a^4(55 + 29m + 4m^2)) \cos(c+dx)) dx}{12 + 7m + m^2} \\
&= \frac{a^4(55 + 29m + 4m^2) \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)(12 + 7m + m^2)} \\
&+ \frac{\cos^{1+m}(c+dx) (a^2 + a^2 \cos(c+dx))^2 \sin(c+dx)}{d(4+m)} \\
&+ \frac{2(5+m) \cos^{1+m}(c+dx) (a^4 + a^4 \cos(c+dx)) \sin(c+dx)}{d(3+m)(4+m)} \\
&+ \frac{\int \cos^m(c+dx) (a^4(3+m)(35 + 40m + 8m^2) + 4a^4(2+m)(4+m)(5 + 2m) \cos(c+dx)) dx}{24 + 26m + 9m^2 + m^3} \\
&= \frac{a^4(55 + 29m + 4m^2) \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)(12 + 7m + m^2)} \\
&+ \frac{\cos^{1+m}(c+dx) (a^2 + a^2 \cos(c+dx))^2 \sin(c+dx)}{d(4+m)} \\
&+ \frac{2(5+m) \cos^{1+m}(c+dx) (a^4 + a^4 \cos(c+dx)) \sin(c+dx)}{d(3+m)(4+m)} \\
&+ \frac{(4a^4(5 + 2m)) \int \cos^{1+m}(c+dx) dx}{3+m} + \frac{(a^4(35 + 40m + 8m^2)) \int \cos^m(c+dx) dx}{8 + 6m + m^2} \\
&= \frac{a^4(55 + 29m + 4m^2) \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)(12 + 7m + m^2)} \\
&+ \frac{\cos^{1+m}(c+dx) (a^2 + a^2 \cos(c+dx))^2 \sin(c+dx)}{d(4+m)} \\
&+ \frac{2(5+m) \cos^{1+m}(c+dx) (a^4 + a^4 \cos(c+dx)) \sin(c+dx)}{d(3+m)(4+m)} \\
&- \frac{a^4(35 + 40m + 8m^2) \cos^{1+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{d(1+m)(8 + 6m + m^2) \sqrt{\sin^2(c+dx)}} \\
&- \frac{4a^4(5 + 2m) \cos^{2+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{d(2+m)(3+m) \sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.93

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^4 dx$$

$$= \frac{a^2 \cos^{1+m}(c + dx) \csc(c + dx) \left((a + a \cos(c + dx))^2 \sin^2(c + dx) - \frac{a^2(5+2m) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right)}{1+m} \right)}{d}$$

[In] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^4,x]

[Out] (a^2*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*((a + a*Cos[c + d*x])^2*Sin[c + d*x]^2 - (a^2*(5 + 2*m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(1 + m) - (2*a^2*(10 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(2 + m) - (a^2*(25 + 6*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(3 + m) - (2*a^2*(5 + m)*Cos[c + d*x]^3*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4 + m))/(d*(4 + m))

Maple [F]

$$\int (\cos^m(dx + c))(a + \cos(dx + c)a)^4 dx$$

[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^4,x)

[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^4,x)

Fricas [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^4 dx = \int (a \cos(dx + c) + a)^4 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="fricas")

[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2 + 4*a^4*cos(d*x + c) + a^4)*cos(d*x + c)^m, x)

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^4 dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^4 dx = \int (a \cos(dx + c) + a)^4 \cos(dx + c)^m dx$$

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^4 dx = \int (a \cos(dx + c) + a)^4 \cos(dx + c)^m dx$$

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^4 dx = \int \cos(c + dx)^m (a + a \cos(c + dx))^4 dx$$

```
[In] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^4,x)
```

```
[Out] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^4, x)
```


3.398 $\int \cos^m(c + dx)(a + a \cos(c + dx))^3 dx$

Optimal result	4589
Rubi [A] (verified)	4589
Mathematica [A] (verified)	4592
Maple [F]	4592
Fricas [F]	4592
Sympy [F(-1)]	4593
Maxima [F]	4593
Giac [F]	4593
Mupad [F(-1)]	4593

Optimal result

Integrand size = 21, antiderivative size = 232

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^3 dx$$

$$= \frac{a^3(7 + 2m) \cos^{1+m}(c + dx) \sin(c + dx)}{d(2 + m)(3 + m)} + \frac{\cos^{1+m}(c + dx) (a^3 + a^3 \cos(c + dx)) \sin(c + dx)}{d(3 + m)}$$

$$- \frac{a^3(5 + 4m) \cos^{1+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1 + m)(2 + m) \sqrt{\sin^2(c + dx)}}$$

$$- \frac{a^3(11 + 4m) \cos^{2+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 + m)(3 + m) \sqrt{\sin^2(c + dx)}}$$

```
[Out] a^3*(7+2*m)*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)/(3+m)+cos(d*x+c)^(1+m)*(a^3
+a^3*cos(d*x+c))*sin(d*x+c)/d/(3+m)-a^3*(5+4*m)*cos(d*x+c)^(1+m)*hypergeom(
[1/2, 1/2+1/2*m], [3/2+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/d/(1+m)/(2+m)/(sin(d*
x+c)^2)^(1/2)-a^3*(11+4*m)*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m], [2+1/2
*m], cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

= {2842, 3047, 3102, 2827, 2722}

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^3 dx$$

$$= -\frac{a^3(4m + 5) \sin(c + dx) \cos^{m+1}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \cos^2(c + dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c + dx)}} - \frac{a^3(4m + 11) \sin(c + dx) \cos^{m+2}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \cos^2(c + dx)\right)}{d(m+2)(m+3)\sqrt{\sin^2(c + dx)}} + \frac{a^3(2m + 7) \sin(c + dx) \cos^{m+1}(c + dx)}{d(m+2)(m+3)} + \frac{\sin(c + dx)(a^3 \cos(c + dx) + a^3) \cos^{m+1}(c + dx)}{d(m+3)}$$

[In] Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^3,x]

[Out] (a^3*(7 + 2*m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (Cos[c + d*x]^(1 + m)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(d*(3 + m)) - (a^3*(5 + 4*m)*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (a^3*(11 + 4*m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2842

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c

, 0]))

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\cos^{1+m}(c+dx)(a^3+a^3\cos(c+dx))\sin(c+dx)}{d(3+m)} \\
 &+ \frac{\int \cos^m(c+dx)(a+a\cos(c+dx))(2a^2(2+m)+a^2(7+2m)\cos(c+dx)) dx}{3+m} \\
 &= \frac{\cos^{1+m}(c+dx)(a^3+a^3\cos(c+dx))\sin(c+dx)}{d(3+m)} \\
 &+ \frac{\int \cos^m(c+dx)(2a^3(2+m)+(2a^3(2+m)+a^3(7+2m))\cos(c+dx)+a^3(7+2m)\cos^2(c+dx)) dx}{3+m} \\
 &= \frac{a^3(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{\cos^{1+m}(c+dx)(a^3+a^3\cos(c+dx))\sin(c+dx)}{d(3+m)} \\
 &+ \frac{\int \cos^m(c+dx)(a^3(3+m)(5+4m)+a^3(2+m)(11+4m)\cos(c+dx)) dx}{6+5m+m^2} \\
 &= \frac{a^3(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{\cos^{1+m}(c+dx)(a^3+a^3\cos(c+dx))\sin(c+dx)}{d(3+m)} \\
 &+ \frac{(a^3(5+4m))\int \cos^m(c+dx) dx}{2+m} + \frac{(a^3(11+4m))\int \cos^{1+m}(c+dx) dx}{3+m} \\
 &= \frac{a^3(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{\cos^{1+m}(c+dx)(a^3+a^3\cos(c+dx))\sin(c+dx)}{d(3+m)} \\
 &- \frac{a^3(5+4m)\cos^{1+m}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)(2+m)\sqrt{\sin^2(c+dx)}} \\
 &- \frac{a^3(11+4m)\cos^{2+m}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c+dx)\right)\sin(c+dx)}{d(2+m)(3+m)\sqrt{\sin^2(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.72

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^3 dx = \frac{a^3 \cos^{1+m}(c + dx) \sin(c + dx) \left(\frac{(15+17m+4m^2) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right)}{\sqrt{\sin^2(c+dx)}} - (1+m) \left(3(3+m) + \dots \right) \right)}{d(1 + \dots)}$$

[In] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^3,x]

[Out] -((a^3*Cos[c + d*x]^(1 + m)*Sin[c + d*x]*(((15 + 17*m + 4*m^2)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2])/Sqrt[Sin[c + d*x]^2] - (1 + m)*(3*(3 + m) + (2 + m)*Cos[c + d*x] - (11 + 4*m)*Cot[c + d*x]*Csc[c + d*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))) / (d*(1 + m)*(2 + m)*(3 + m))

Maple [F]

$$\int (\cos^m(dx + c))(a + \cos(dx + c)a)^3 dx$$

[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^3,x)

[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^3,x)

Fricas [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^3 dx = \int (a \cos(dx + c) + a)^3 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3)*cos(d*x + c)^m, x)

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^3 dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^3 dx = \int (a \cos(dx + c) + a)^3 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)

Giac [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^3 dx = \int (a \cos(dx + c) + a)^3 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^3 dx = \int \cos(c + dx)^m (a + a \cos(c + dx))^3 dx$$

[In] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^3,x)

[Out] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^3, x)

3.399 $\int \cos^m(c + dx)(a + a \cos(c + dx))^2 dx$

Optimal result	4594
Rubi [A] (verified)	4594
Mathematica [A] (verified)	4596
Maple [F]	4596
Fricas [F]	4597
Sympy [F]	4597
Maxima [F]	4597
Giac [F]	4597
Mupad [F(-1)]	4598

Optimal result

Integrand size = 21, antiderivative size = 173

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^2 dx$$

$$= \frac{a^2 \cos^{1+m}(c + dx) \sin(c + dx)}{d(2 + m)}$$

$$- \frac{a^2(3 + 2m) \cos^{1+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1 + m)(2 + m) \sqrt{\sin^2(c + dx)}}$$

$$- \frac{2a^2 \cos^{2+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 + m) \sqrt{\sin^2(c + dx)}}$$

[Out] a^2*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)-a^2*(3+2*m)*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m], [3/2+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/d/(1+m)/(2+m)/(sin(d*x+c)^2)^(1/2)-2*a^2*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m], [2+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

= {2842, 2827, 2722}

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2 dx =$$

$$\frac{a^2(2m+3)\sin(c+dx)\cos^{m+1}(c+dx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}$$

$$\frac{2a^2\sin(c+dx)\cos^{m+2}(c+dx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \cos^2(c+dx)\right)}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

$$+ \frac{a^2\sin(c+dx)\cos^{m+1}(c+dx)}{d(m+2)}$$

[In] Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^2,x]

[Out] (a^2*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) - (a^2*(3 + 2*m)*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*a^2*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2842

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n, -1] && (IntegerQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c, 0]))

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a^2 \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)} \\
 &+ \frac{\int \cos^m(c+dx) (a^2(3+2m) + 2a^2(2+m) \cos(c+dx)) dx}{2+m} \\
 &= \frac{a^2 \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)} + (2a^2) \int \cos^{1+m}(c+dx) dx + \frac{(a^2(3+2m)) \int \cos^m(c+dx) dx}{2+m} \\
 &= \frac{a^2 \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)} \\
 &- \frac{a^2(3+2m) \cos^{1+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{d(1+m)(2+m) \sqrt{\sin^2(c+dx)}} \\
 &- \frac{2a^2 \cos^{2+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{d(2+m) \sqrt{\sin^2(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.80

$$\begin{aligned}
 &\int \cos^m(c+dx) (a + a \cos(c+dx))^2 dx \\
 &= \frac{a^2 \cos^{1+m}(c+dx) \csc(c+dx) \left(- \left((3+2m) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right) \sqrt{\sin^2(c+dx)} \right. \right.}{d(1+m)(2+m)}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^m*(a + a*cos[c + d*x])^2,x]

[Out] (a^2*cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-(3 + 2*m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]) + (1 + m)*(Sin[c + d*x]^2 - 2*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(d*(1 + m)*(2 + m))

Maple [F]

$$\int (\cos^m(dx+c)) (a + \cos(dx+c) a)^2 dx$$

[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^2,x)

[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^2,x)

Fricas [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^2 dx = \int (a \cos(dx + c) + a)^2 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*cos(d*x + c)^m, x)

Sympy [F]

$$\begin{aligned} \int \cos^m(c + dx)(a + a \cos(c + dx))^2 dx = a^2 & \left(\int 2 \cos(c + dx) \cos^m(c + dx) dx \right. \\ & + \int \cos^2(c + dx) \cos^m(c + dx) dx \\ & \left. + \int \cos^m(c + dx) dx \right) \end{aligned}$$

[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**2,x)

[Out] a**2*(Integral(2*cos(c + d*x)*cos(c + d*x)**m, x) + Integral(cos(c + d*x)**2*cos(c + d*x)**m, x) + Integral(cos(c + d*x)**m, x))

Maxima [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^2 dx = \int (a \cos(dx + c) + a)^2 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)

Giac [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^2 dx = \int (a \cos(dx + c) + a)^2 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + a \cos(c + dx))^2 dx = \int \cos(c + dx)^m (a + a \cos(c + dx))^2 dx$$

```
[In] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^m*(a + a*cos(c + d*x))^2, x)
```

3.400 $\int \cos^m(c + dx)(a + a \cos(c + dx)) dx$

Optimal result	4599
Rubi [A] (verified)	4599
Mathematica [A] (verified)	4600
Maple [F]	4601
Fricas [F]	4601
Sympy [F]	4601
Maxima [F]	4601
Giac [F]	4602
Mupad [F(-1)]	4602

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \cos^m(c + dx)(a + a \cos(c + dx)) dx$$

$$= -\frac{a \cos^{1+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1+m)\sqrt{\sin^2(c + dx)}} - \frac{a \cos^{2+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2+m)\sqrt{\sin^2(c + dx)}}$$

[Out] $-a \cos(d*x+c)^{(1+m)} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2} + \frac{1}{2} * m\right], \left[\frac{3}{2} + \frac{1}{2} * m\right], \cos(d*x+c)^2\right) * \sin(d*x+c) / d / (1+m) / (\sin(d*x+c)^2)^{(1/2)} - a \cos(d*x+c)^{(2+m)} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2} + \frac{1}{2} * m\right], \left[2 + \frac{1}{2} * m\right], \cos(d*x+c)^2\right) * \sin(d*x+c) / d / (2+m) / (\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2827, 2722}

$$\int \cos^m(c + dx)(a + a \cos(c + dx)) dx$$

$$= -\frac{a \sin(c + dx) \cos^{m+1}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \cos^2(c + dx)\right)}{d(m+1)\sqrt{\sin^2(c + dx)}} - \frac{a \sin(c + dx) \cos^{m+2}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \cos^2(c + dx)\right)}{d(m+2)\sqrt{\sin^2(c + dx)}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]^m * (a + a * \operatorname{Cos}[c + d*x]), x]$

```
[Out] -((a*cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (a*cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_.)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \cos^m(c + dx) dx + a \int \cos^{1+m}(c + dx) dx \\ &= -\frac{a \cos^{1+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1+m)\sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{a \cos^{2+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2+m)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \cos^m(c + dx)(a + a \cos(c + dx)) dx = -\frac{a \cos^{1+m}(c + dx) \csc(c + dx) ((2 + m) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) + (1 + m) \cos(c + dx))}{d(1 + m)(2 + m)}$$

```
[In] Integrate[Cos[c + d*x]^m*(a + a*cos[c + d*x]),x]
```

```
[Out] -((a*cos[c + d*x]^(1 + m)*Csc[c + d*x]*((2 + m)*Hypergeometric2F1[1/2, (1 +
m)/2, (3 + m)/2, Cos[c + d*x]^2] + (1 + m)*Cos[c + d*x]*Hypergeometric2F1[
1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m
)*(2 + m))
```

Maple [F]

$$\int (\cos^m(dx + c)) (a + \cos(dx + c) a) dx$$

[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*a),x)

[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*a),x)

Fricas [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx)) dx = \int (a \cos(dx + c) + a) \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)

Sympy [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx)) dx = a \left(\int \cos(c + dx) \cos^m(c + dx) dx + \int \cos^m(c + dx) dx \right)$$

[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c)),x)

[Out] a*(Integral(cos(c + d*x)*cos(c + d*x)**m, x) + Integral(cos(c + d*x)**m, x))

Maxima [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx)) dx = \int (a \cos(dx + c) + a) \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)

Giac [F]

$$\int \cos^m(c + dx)(a + a \cos(c + dx)) dx = \int (a \cos(dx + c) + a) \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + a \cos(c + dx)) dx = \int \cos(c + dx)^m (a + a \cos(c + dx)) dx$$

[In] int(cos(c + d*x)^m*(a + a*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^m*(a + a*cos(c + d*x)), x)

3.401 $\int \frac{\cos^m(c+dx)}{a+a \cos(c+dx)} dx$

Optimal result	4603
Rubi [A] (verified)	4603
Mathematica [A] (verified)	4605
Maple [F]	4605
Fricas [F]	4605
Sympy [F]	4606
Maxima [F]	4606
Giac [F(-2)]	4606
Mupad [F(-1)]	4606

Optimal result

Integrand size = 21, antiderivative size = 156

$$\begin{aligned} & \int \frac{\cos^m(c+dx)}{a+a \cos(c+dx)} dx \\ &= \frac{\cos^m(c+dx) \sin(c+dx)}{d(a+a \cos(c+dx))} \\ & \quad - \frac{\cos^m(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{ad\sqrt{\sin^2(c+dx)}} \\ & \quad + \frac{m \cos^{1+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{ad(1+m)\sqrt{\sin^2(c+dx)}} \end{aligned}$$

[Out] $\cos(d*x+c)^m*\sin(d*x+c)/d/(a+a*\cos(d*x+c))- \cos(d*x+c)^m*\operatorname{hypergeom}([1/2, 1/2*m], [1+1/2*m], \cos(d*x+c)^2)*\sin(d*x+c)/a/d/(\sin(d*x+c)^2)^{(1/2)+m*\cos(d*x+c)^{(1+m)*\operatorname{hypergeom}([1/2, 1/2+1/2*m], [3/2+1/2*m], \cos(d*x+c)^2)*\sin(d*x+c)/a/d/(1+m)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

= {2848, 2827, 2722}

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)} dx$$

$$= \frac{m \sin(c+dx) \cos^{m+1}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \cos^2(c+dx)\right)}{ad(m+1)\sqrt{\sin^2(c+dx)}} - \frac{\sin(c+dx) \cos^m(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m}{2}, \frac{m+2}{2}, \cos^2(c+dx)\right)}{ad\sqrt{\sin^2(c+dx)}} + \frac{\sin(c+dx) \cos^m(c+dx)}{d(a\cos(c+dx)+a)}$$

[In] Int[Cos[c + d*x]^m/(a + a*Cos[c + d*x]),x]

[Out] (Cos[c + d*x]^m*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) - (Cos[c + d*x]^m*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*sqrt[Sin[c + d*x]^2]) + (m*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*(1 + m)*sqrt[Sin[c + d*x]^2])

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2848

Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b)*Cos[e + f*x]*((c + d*Sin[e + f*x])^n/(a*f*(a + b*Sin[e + f*x]))), x] + Dist[d*(n/(a*b)), Int[(c + d*Sin[e + f*x])^(n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\text{integral} = \frac{\cos^m(c+dx) \sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{m \int \cos^{-1+m}(c+dx)(a-a\cos(c+dx)) dx}{a^2}$$

$$\begin{aligned}
&= \frac{\cos^m(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{m \int \cos^{-1+m}(c+dx) dx}{a} - \frac{m \int \cos^m(c+dx) dx}{a} \\
&= \frac{\cos^m(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} \\
&\quad - \frac{\cos^m(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{ad\sqrt{\sin^2(c+dx)}} \\
&\quad + \frac{m \cos^{1+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{ad(1+m)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)} dx = \frac{\cos^m(c+dx) \cot\left(\frac{1}{2}(c+dx)\right) \left(-((1+m)(-1+\cos(c+dx))) - (1+m) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m}{2}, \frac{2+m}{2}, \cos^2(c+dx)\right)\right)}{ad(1+m)}$$

[In] Integrate[Cos[c + d*x]^m/(a + a*cos[c + d*x]), x]

[Out] (Cos[c + d*x]^m*Cot[(c + d*x)/2]*(-((1 + m)*(-1 + Cos[c + d*x]))) - (1 + m)*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2] + m*cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(a*d*(1 + m)*(1 + Cos[c + d*x]))

Maple [F]

$$\int \frac{\cos^m(dx+c)}{a+\cos(dx+c)a} dx$$

[In] int(cos(d*x+c)^m/(a+cos(d*x+c)*a), x)

[Out] int(cos(d*x+c)^m/(a+cos(d*x+c)*a), x)

Fricas [F]

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)} dx = \int \frac{\cos(dx+c)^m}{a\cos(dx+c)+a} dx$$

[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)), x, algorithm="fricas")

[Out] integral(cos(d*x + c)^m/(a*cos(d*x + c) + a), x)

Sympy [F]

$$\int \frac{\cos^m(c + dx)}{a + a \cos(c + dx)} dx = \frac{\int \frac{\cos^m(c+dx)}{\cos(c+dx)+1} dx}{a}$$

[In] integrate(cos(d*x+c)**m/(a+a*cos(d*x+c)),x)

[Out] Integral(cos(c + d*x)**m/(cos(c + d*x) + 1), x)/a

Maxima [F]

$$\int \frac{\cos^m(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\cos(dx + c)^m}{a \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^m/(a*cos(d*x + c) + a), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^m(c + dx)}{a + a \cos(c + dx)} dx = \text{Exception raised: TypeError}$$

[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx);;OUTPUT:Unable to divide, perhaps due to rounding error%%{1,[0,1,0]%%} / %%{2,[0,0,1]%%} Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c + dx)}{a + a \cos(c + dx)} dx = \int \frac{\cos(c + dx)^m}{a + a \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^m/(a + a*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^m/(a + a*cos(c + d*x)), x)

$$3.402 \quad \int \frac{\cos^m(c+dx)}{(a+a \cos(c+dx))^2} dx$$

Optimal result	4607
Rubi [A] (verified)	4607
Mathematica [A] (verified)	4609
Maple [F]	4610
Fricas [F]	4610
Sympy [F]	4610
Maxima [F]	4611
Giac [F(-2)]	4611
Mupad [F(-1)]	4611

Optimal result

Integrand size = 21, antiderivative size = 229

$$\int \frac{\cos^m(c+dx)}{(a+a \cos(c+dx))^2} dx$$

$$= -\frac{2(1-m) \cos^{1+m}(c+dx) \sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{1+m}(c+dx) \sin(c+dx)}{3d(a+a \cos(c+dx))^2}$$

$$+ \frac{(1-2m)m \cos^{1+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{3a^2d(1+m)\sqrt{\sin^2(c+dx)}}$$

$$- \frac{2(1-m)(1+m) \cos^{2+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{3a^2d(2+m)\sqrt{\sin^2(c+dx)}}$$

[Out] $-2/3*(1-m)*\cos(d*x+c)^{(1+m)}*\sin(d*x+c)/a^2/d/(1+\cos(d*x+c))-1/3*\cos(d*x+c)^{(1+m)}*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^2+1/3*(1-2*m)*m*\cos(d*x+c)^{(1+m)}*\operatorname{hypergeom}([1/2, 1/2+1/2*m], [3/2+1/2*m], \cos(d*x+c)^2)*\sin(d*x+c)/a^2/d/(1+m)/(\sin(d*x+c)^2)^{(1/2)}-2/3*(1-m)*(1+m)*\cos(d*x+c)^{(2+m)}*\operatorname{hypergeom}([1/2, 1+1/2*m], [2+1/2*m], \cos(d*x+c)^2)*\sin(d*x+c)/a^2/d/(2+m)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

= {2845, 3057, 2827, 2722}

$$\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2} dx$$

$$= \frac{(1-2m)m \sin(c+dx) \cos^{m+1}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \cos^2(c+dx)\right)}{3a^2 d(m+1) \sqrt{\sin^2(c+dx)}}$$

$$- \frac{2(1-m)(m+1) \sin(c+dx) \cos^{m+2}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \cos^2(c+dx)\right)}{3a^2 d(m+2) \sqrt{\sin^2(c+dx)}}$$

$$- \frac{2(1-m) \sin(c+dx) \cos^{m+1}(c+dx)}{3a^2 d(\cos(c+dx)+1)} - \frac{\sin(c+dx) \cos^{m+1}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

[In] Int[Cos[c + d*x]^m/(a + a*Cos[c + d*x])^2,x]

[Out] (-2*(1 - m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((1 - 2*m)*m*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(3*a^2*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) - (2*(1 - m)*(1 + m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(3*a^2*d*(2 + m)*Sqrt[Sin[c + d*x]^2])

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2845

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && !GtQ[n, 0] && (IntegerSQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))

Rule 3057

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^m(c+dx)(a(2-m)+am\cos(c+dx))}{a+a\cos(c+dx)} dx}{3a^2} \\
&= -\frac{2(1-m)\cos^{1+m}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad + \frac{\int \cos^m(c+dx)(-a^2(1-2m)m+2a^2(1-m)(1+m)\cos(c+dx)) dx}{3a^4} \\
&= -\frac{2(1-m)\cos^{1+m}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad - \frac{((1-2m)m)\int \cos^m(c+dx) dx}{3a^2} + \frac{(2(1-m)(1+m))\int \cos^{1+m}(c+dx) dx}{3a^2} \\
&= -\frac{2(1-m)\cos^{1+m}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} \\
&\quad + \frac{(1-2m)m\cos^{1+m}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right)\sin(c+dx)}{3a^2d(1+m)\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{2(1-m)(1+m)\cos^{2+m}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c+dx)\right)\sin(c+dx)}{3a^2d(2+m)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.82

$$\begin{aligned}
&\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2} dx \\
&= \frac{\cos^{1+m}(c+dx)\csc(c+dx)\left(-\sin^2(c+dx) - \frac{(1+\cos(c+dx))(-2(-1+m)(1+m)(2+m)\sin^2(c+dx)-(1+\cos(c+dx))((1-2m) \dots)}{3a^2d(1- \dots)}\right)}{3a^2d(1- \dots)}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x])^2,x]

```
[Out] (Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-Sin[c + d*x]^2 - ((1 + Cos[c + d*x])*(
-2*(-1 + m)*(1 + m)*(2 + m)*Sin[c + d*x]^2 - (1 + Cos[c + d*x])*((1 - 2*m)*
m*(2 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2] + 2*
(-1 + m)*(1 + m)^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2
, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]))/((1 + m)*(2 + m)))/(3*a^2*d*(1 +
Cos[c + d*x])^2)
```

Maple [F]

$$\int \frac{\cos^m(dx + c)}{(a + \cos(dx + c)a)^2} dx$$

```
[In] int(cos(d*x+c)^m/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] int(cos(d*x+c)^m/(a+cos(d*x+c)*a)^2,x)
```

Fricas [F]

$$\int \frac{\cos^m(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^m}{(a \cos(dx + c) + a)^2} dx$$

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^m/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\cos^m(c + dx)}{(a + a \cos(c + dx))^2} dx = \frac{\int \frac{\cos^m(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1} dx}{a^2}$$

```
[In] integrate(cos(d*x+c)**m/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**m/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

Maxima [F]

$$\int \frac{\cos^m(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^m}{(a \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^m/(a*cos(d*x + c) + a)^2, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{\cos^m(c + dx)}{(a + a \cos(c + dx))^2} dx = \text{Exception raised: TypeError}$$

[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to rounding error%%{1,[0,1,2,0]%%}+%%{1,[0,1,0,0]%%} / %%{4,[0,0,0,2]%%} Error: Ba

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c + dx)}{(a + a \cos(c + dx))^2} dx = \int \frac{\cos(c + dx)^m}{(a + a \cos(c + dx))^2} dx$$

[In] int(cos(c + d*x)^m/(a + a*cos(c + d*x))^2,x)

[Out] int(cos(c + d*x)^m/(a + a*cos(c + d*x))^2, x)

3.403 $\int \cos^7(c + dx)(a + b \cos(c + dx)) dx$

Optimal result	4612
Rubi [A] (verified)	4612
Mathematica [A] (verified)	4614
Maple [A] (verified)	4615
Fricas [A] (verification not implemented)	4616
Sympy [B] (verification not implemented)	4616
Maxima [A] (verification not implemented)	4617
Giac [A] (verification not implemented)	4617
Mupad [B] (verification not implemented)	4618

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int \cos^7(c + dx)(a + b \cos(c + dx)) dx = \frac{35bx}{128} + \frac{a \sin(c + dx)}{d} + \frac{35b \cos(c + dx) \sin(c + dx)}{128d} + \frac{35b \cos^3(c + dx) \sin(c + dx)}{192d} + \frac{7b \cos^5(c + dx) \sin(c + dx)}{48d} + \frac{b \cos^7(c + dx) \sin(c + dx)}{8d} - \frac{a \sin^3(c + dx)}{d} + \frac{3a \sin^5(c + dx)}{5d} - \frac{a \sin^7(c + dx)}{7d}$$

[Out] 35/128*b*x+a*sin(d*x+c)/d+35/128*b*cos(d*x+c)*sin(d*x+c)/d+35/192*b*cos(d*x+c)^3*sin(d*x+c)/d+7/48*b*cos(d*x+c)^5*sin(d*x+c)/d+1/8*b*cos(d*x+c)^7*sin(d*x+c)/d-a*sin(d*x+c)^3/d+3/5*a*sin(d*x+c)^5/d-1/7*a*sin(d*x+c)^7/d

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used

= {2827, 2713, 2715, 8}

$$\int \cos^7(c + dx)(a + b \cos(c + dx)) dx = -\frac{a \sin^7(c + dx)}{7d} + \frac{3a \sin^5(c + dx)}{5d} - \frac{a \sin^3(c + dx)}{d}$$

$$+ \frac{a \sin(c + dx)}{d} + \frac{b \sin(c + dx) \cos^7(c + dx)}{8d}$$

$$+ \frac{7b \sin(c + dx) \cos^5(c + dx)}{48d}$$

$$+ \frac{35b \sin(c + dx) \cos^3(c + dx)}{192d}$$

$$+ \frac{35b \sin(c + dx) \cos(c + dx)}{128d} + \frac{35bx}{128}$$

[In] Int[Cos[c + d*x]^7*(a + b*Cos[c + d*x]),x]

[Out] (35*b*x)/128 + (a*Sin[c + d*x])/d + (35*b*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35*b*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (7*b*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (b*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= a \int \cos^7(c+dx) dx + b \int \cos^8(c+dx) dx \\
&= \frac{b \cos^7(c+dx) \sin(c+dx)}{8d} + \frac{1}{8}(7b) \int \cos^6(c+dx) dx \\
&\quad - \frac{a \text{Subst}(\int (1-3x^2+3x^4-x^6) dx, x, -\sin(c+dx))}{d} \\
&= \frac{a \sin(c+dx)}{d} + \frac{7b \cos^5(c+dx) \sin(c+dx)}{48d} + \frac{b \cos^7(c+dx) \sin(c+dx)}{8d} \\
&\quad - \frac{a \sin^3(c+dx)}{d} + \frac{3a \sin^5(c+dx)}{5d} - \frac{a \sin^7(c+dx)}{7d} + \frac{1}{48}(35b) \int \cos^4(c+dx) dx \\
&= \frac{a \sin(c+dx)}{d} + \frac{35b \cos^3(c+dx) \sin(c+dx)}{192d} + \frac{7b \cos^5(c+dx) \sin(c+dx)}{48d} \\
&\quad + \frac{b \cos^7(c+dx) \sin(c+dx)}{8d} - \frac{a \sin^3(c+dx)}{d} + \frac{3a \sin^5(c+dx)}{5d} \\
&\quad - \frac{a \sin^7(c+dx)}{7d} + \frac{1}{64}(35b) \int \cos^2(c+dx) dx \\
&= \frac{a \sin(c+dx)}{d} + \frac{35b \cos(c+dx) \sin(c+dx)}{128d} + \frac{35b \cos^3(c+dx) \sin(c+dx)}{192d} \\
&\quad + \frac{7b \cos^5(c+dx) \sin(c+dx)}{48d} + \frac{b \cos^7(c+dx) \sin(c+dx)}{8d} \\
&\quad - \frac{a \sin^3(c+dx)}{d} + \frac{3a \sin^5(c+dx)}{5d} - \frac{a \sin^7(c+dx)}{7d} + \frac{1}{128}(35b) \int 1 dx \\
&= \frac{35bx}{128} + \frac{a \sin(c+dx)}{d} + \frac{35b \cos(c+dx) \sin(c+dx)}{128d} + \frac{35b \cos^3(c+dx) \sin(c+dx)}{192d} \\
&\quad + \frac{7b \cos^5(c+dx) \sin(c+dx)}{48d} + \frac{b \cos^7(c+dx) \sin(c+dx)}{8d} \\
&\quad - \frac{a \sin^3(c+dx)}{d} + \frac{3a \sin^5(c+dx)}{5d} - \frac{a \sin^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\begin{aligned}
\int \cos^7(c+dx)(a+b\cos(c+dx)) dx &= \frac{35b(c+dx)}{128d} + \frac{a \sin(c+dx)}{d} - \frac{a \sin^3(c+dx)}{d} \\
&\quad + \frac{3a \sin^5(c+dx)}{5d} - \frac{a \sin^7(c+dx)}{7d} \\
&\quad + \frac{7b \sin(2(c+dx))}{32d} + \frac{7b \sin(4(c+dx))}{128d} \\
&\quad + \frac{b \sin(6(c+dx))}{96d} + \frac{b \sin(8(c+dx))}{1024d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^7*(a + b*Cos[c + d*x]),x]

[Out] (35*b*(c + d*x))/(128*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d) + (7*b*Sin[2*(c + d*x)])/(32*d) + (7*b*Sin[4*(c + d*x)])/(128*d) + (b*Sin[6*(c + d*x)])/(96*d) + (b*Sin[8*(c + d*x)])/(1024*d)

Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.67

method	result
derivativedivides	$b \left(\frac{\left(\cos^7(dx+c) + \frac{7(\cos^5(dx+c))}{6} + \frac{35(\cos^3(dx+c))}{24} + \frac{35 \cos(dx+c)}{16} \right) \sin(dx+c)}{8} + \frac{35dx}{128} + \frac{35c}{128} \right) + \frac{a \left(\frac{16}{5} + \cos^6(dx+c) + \frac{6(\cos^4(dx+c))}{5} \right)}{7d}$
default	$b \left(\frac{\left(\cos^7(dx+c) + \frac{7(\cos^5(dx+c))}{6} + \frac{35(\cos^3(dx+c))}{24} + \frac{35 \cos(dx+c)}{16} \right) \sin(dx+c)}{8} + \frac{35dx}{128} + \frac{35c}{128} \right) + \frac{a \left(\frac{16}{5} + \cos^6(dx+c) + \frac{6(\cos^4(dx+c))}{5} \right)}{7d}$
parts	$a \left(\frac{16}{5} + \cos^6(dx+c) + \frac{6(\cos^4(dx+c))}{5} + \frac{8(\cos^2(dx+c))}{5} \right) \sin(dx+c) + \frac{b \left(\frac{\left(\cos^7(dx+c) + \frac{7(\cos^5(dx+c))}{6} + \frac{35(\cos^3(dx+c))}{24} + \frac{35 \cos(dx+c)}{16} \right) \sin(dx+c)}{8} + \frac{35dx}{128} + \frac{35c}{128} \right)}{d}$
parallelrisch	$\frac{29400bxd + 58800a \sin(dx+c) + 105b \sin(8dx+8c) + 240a \sin(7dx+7c) + 1120b \sin(6dx+6c) + 2352a \sin(5dx+5c) + 5880 \sin(4dx+4c)}{107520d}$
risch	$\frac{35bx}{128} + \frac{35a \sin(dx+c)}{64d} + \frac{b \sin(8dx+8c)}{1024d} + \frac{a \sin(7dx+7c)}{448d} + \frac{b \sin(6dx+6c)}{96d} + \frac{7a \sin(5dx+5c)}{320d} + \frac{7b \sin(4dx+4c)}{128d}$
norman	$\frac{35bx}{128} + \frac{35bx \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{16} + \frac{245bx \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{32} + \frac{245bx \left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{16} + \frac{1225bx \left(\tan^8\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{64} + \frac{245bx \left(\tan^{10}\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{16}$

[In] int(cos(d*x+c)^7*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] 1/d*(b*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c)+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.65

$$\int \cos^7(c + dx)(a + b \cos(c + dx)) dx$$

$$= \frac{3675 b dx + (1680 b \cos(dx + c))^7 + 1920 a \cos(dx + c)^6 + 1960 b \cos(dx + c)^5 + 2304 a \cos(dx + c)^4 + 2450 b \cos(dx + c)^3 + 3072 a \cos(dx + c)^2 + 3675 b \cos(dx + c) + 6144 a}{13440 d} \sin(dx + c)$$

[In] integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="fricas")

```
[Out] 1/13440*(3675*b*d*x + (1680*b*cos(d*x + c)^7 + 1920*a*cos(d*x + c)^6 + 1960
*b*cos(d*x + c)^5 + 2304*a*cos(d*x + c)^4 + 2450*b*cos(d*x + c)^3 + 3072*a*
cos(d*x + c)^2 + 3675*b*cos(d*x + c) + 6144*a)*sin(d*x + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(141) = 282.

Time = 0.67 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.91

$$\int \cos^7(c + dx)(a + b \cos(c + dx)) dx$$

$$= \begin{cases} \frac{16a \sin^7(c+dx)}{35d} + \frac{8a \sin^5(c+dx) \cos^2(c+dx)}{5d} + \frac{2a \sin^3(c+dx) \cos^4(c+dx)}{d} + \frac{a \sin(c+dx) \cos^6(c+dx)}{d} + \frac{35bx \sin^8(c+dx)}{128} + \frac{35bx \sin^6(c+dx)}{128} \\ x(a + b \cos(c)) \cos^7(c) \end{cases}$$

[In] integrate(cos(d*x+c)**7*(a+b*cos(d*x+c)),x)

```
[Out] Piecewise((16*a*sin(c + d*x)**7/(35*d) + 8*a*sin(c + d*x)**5*cos(c + d*x)**
2/(5*d) + 2*a*sin(c + d*x)**3*cos(c + d*x)**4/d + a*sin(c + d*x)*cos(c + d*
x)**6/d + 35*b*x*sin(c + d*x)**8/128 + 35*b*x*sin(c + d*x)**6*cos(c + d*x)*
**2/32 + 105*b*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*b*x*sin(c + d*x)**2
*cos(c + d*x)**6/32 + 35*b*x*cos(c + d*x)**8/128 + 35*b*sin(c + d*x)**7*cos
(c + d*x)/(128*d) + 385*b*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 511*b*s
in(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*b*sin(c + d*x)*cos(c + d*x)**7/
(128*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**7, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.70

$$\int \cos^7(c + dx)(a + b \cos(c + dx)) dx = \frac{3072 (5 \sin(dx + c)^7 - 21 \sin(dx + c)^5 + 35 \sin(dx + c)^3 - 35 \sin(dx + c))a + 35 (128 \sin(2dx + 2c)^3 - 840 \sin(2dx + 2c) - 168 \sin(4dx + 4c) - 768 \sin(2dx + 2c))b}{107520 d}$$

[In] integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="maxima")

```
[Out] -1/107520*(3072*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a + 35*(128*sin(2*d*x + 2*c)^3 - 840*d*x - 840*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*b)/d
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81

$$\int \cos^7(c + dx)(a + b \cos(c + dx)) dx = \frac{35}{128} bx + \frac{b \sin(8dx + 8c)}{1024 d} + \frac{a \sin(7dx + 7c)}{448 d} + \frac{b \sin(6dx + 6c)}{96 d} + \frac{7a \sin(5dx + 5c)}{320 d} + \frac{7b \sin(4dx + 4c)}{128 d} + \frac{7a \sin(3dx + 3c)}{64 d} + \frac{7b \sin(2dx + 2c)}{32 d} + \frac{35a \sin(dx + c)}{64 d}$$

[In] integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="giac")

```
[Out] 35/128*b*x + 1/1024*b*sin(8*d*x + 8*c)/d + 1/448*a*sin(7*d*x + 7*c)/d + 1/96*b*sin(6*d*x + 6*c)/d + 7/320*a*sin(5*d*x + 5*c)/d + 7/128*b*sin(4*d*x + 4*c)/d + 7/64*a*sin(3*d*x + 3*c)/d + 7/32*b*sin(2*d*x + 2*c)/d + 35/64*a*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 17.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17

$$\int \cos^7(c + dx)(a + b \cos(c + dx)) dx = \frac{35 b x}{128} + \frac{(2a - \frac{93b}{64}) \tan(\frac{c}{2} + \frac{dx}{2})^{15} + (6a - \frac{91b}{192}) \tan(\frac{c}{2} + \frac{dx}{2})^{13} + (\frac{106a}{5} - \frac{1799b}{192}) \tan(\frac{c}{2} + \frac{dx}{2})^{11} + (\frac{1026a}{35} + \frac{1085b}{192}) \tan(\frac{c}{2} + \frac{dx}{2})^9 + (2a - \frac{93b}{64}) \tan(\frac{c}{2} + \frac{dx}{2})^7 + (6a - \frac{91b}{192}) \tan(\frac{c}{2} + \frac{dx}{2})^5 + (\frac{106a}{5} - \frac{1799b}{192}) \tan(\frac{c}{2} + \frac{dx}{2})^3 + (\frac{1026a}{35} + \frac{1085b}{192}) \tan(\frac{c}{2} + \frac{dx}{2})}{(d(\tan(\frac{c}{2} + \frac{dx}{2})^2 + 1))^8}$$

[In] int(cos(c + d*x)^7*(a + b*cos(c + d*x)),x)

```
[Out] (35*b*x)/128 + (tan(c/2 + (d*x)/2)*(2*a + (93*b)/64) + tan(c/2 + (d*x)/2)^15*(2*a - (93*b)/64) + tan(c/2 + (d*x)/2)^3*(6*a + (91*b)/192) + tan(c/2 + (d*x)/2)^13*(6*a - (91*b)/192) + tan(c/2 + (d*x)/2)^5*((106*a)/5 + (1799*b)/192) + tan(c/2 + (d*x)/2)^11*((106*a)/5 - (1799*b)/192) + tan(c/2 + (d*x)/2)^9*((1026*a)/35 - (1085*b)/192) + tan(c/2 + (d*x)/2)^7*((1026*a)/35 + (1085*b)/192))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

3.404 $\int \cos^6(c + dx)(a + b \cos(c + dx)) dx$

Optimal result	4619
Rubi [A] (verified)	4619
Mathematica [A] (verified)	4621
Maple [A] (verified)	4622
Fricas [A] (verification not implemented)	4622
Sympy [A] (verification not implemented)	4623
Maxima [A] (verification not implemented)	4623
Giac [A] (verification not implemented)	4623
Mupad [B] (verification not implemented)	4624

Optimal result

Integrand size = 19, antiderivative size = 128

$$\int \cos^6(c + dx)(a + b \cos(c + dx)) dx = \frac{5ax}{16} + \frac{b \sin(c + dx)}{d} + \frac{5a \cos(c + dx) \sin(c + dx)}{16d} + \frac{5a \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{a \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{b \sin^3(c + dx)}{d} + \frac{3b \sin^5(c + dx)}{5d} - \frac{b \sin^7(c + dx)}{7d}$$

[Out] 5/16*a*x+b*sin(d*x+c)/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d-b*sin(d*x+c)^3/d+3/5*b*sin(d*x+c)^5/d-1/7*b*sin(d*x+c)^7/d

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 2715, 8, 2713}

$$\int \cos^6(c + dx)(a + b \cos(c + dx)) dx = \frac{a \sin(c + dx) \cos^5(c + dx)}{6d} + \frac{5a \sin(c + dx) \cos^3(c + dx)}{24d} + \frac{5a \sin(c + dx) \cos(c + dx)}{16d} + \frac{5ax}{16} - \frac{b \sin^7(c + dx)}{7d} + \frac{3b \sin^5(c + dx)}{5d} - \frac{b \sin^3(c + dx)}{d} + \frac{b \sin(c + dx)}{d}$$

[In] Int[Cos[c + d*x]^6*(a + b*Cos[c + d*x]),x]

[Out] (5*a*x)/16 + (b*Sin[c + d*x])/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (b*Sin[c + d*x]^3)/d + (3*b*Sin[c + d*x]^5)/(5*d) - (b*Sin[c + d*x]^7)/(7*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \cos^6(c + dx) dx + b \int \cos^7(c + dx) dx \\
 &= \frac{a \cos^5(c + dx) \sin(c + dx)}{6d} + \frac{1}{6}(5a) \int \cos^4(c + dx) dx \\
 &\quad - \frac{b \text{Subst}\left(\int (1 - 3x^2 + 3x^4 - x^6) dx, x, -\sin(c + dx)\right)}{d} \\
 &= \frac{b \sin(c + dx)}{d} + \frac{5a \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{a \cos^5(c + dx) \sin(c + dx)}{6d} \\
 &\quad - \frac{b \sin^3(c + dx)}{d} + \frac{3b \sin^5(c + dx)}{5d} - \frac{b \sin^7(c + dx)}{7d} + \frac{1}{8}(5a) \int \cos^2(c + dx) dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{b \sin(c+dx)}{d} + \frac{5a \cos(c+dx) \sin(c+dx)}{16d} + \frac{5a \cos^3(c+dx) \sin(c+dx)}{24d} \\
&\quad + \frac{a \cos^5(c+dx) \sin(c+dx)}{6d} - \frac{b \sin^3(c+dx)}{d} \\
&\quad + \frac{3b \sin^5(c+dx)}{5d} - \frac{b \sin^7(c+dx)}{7d} + \frac{1}{16}(5a) \int 1 dx \\
&= \frac{5ax}{16} + \frac{b \sin(c+dx)}{d} + \frac{5a \cos(c+dx) \sin(c+dx)}{16d} + \frac{5a \cos^3(c+dx) \sin(c+dx)}{24d} \\
&\quad + \frac{a \cos^5(c+dx) \sin(c+dx)}{6d} - \frac{b \sin^3(c+dx)}{d} + \frac{3b \sin^5(c+dx)}{5d} - \frac{b \sin^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\begin{aligned}
&\int \cos^6(c+dx)(a+b \cos(c+dx)) dx \\
&= \frac{6720b \sin(c+dx) - 6720b \sin^3(c+dx) + 4032b \sin^5(c+dx) - 960b \sin^7(c+dx) + 35a(60c + 60dx + 45 \sin[2(c+dx)] + 9 \sin[4(c+dx)] + \sin[6(c+dx)])}{6720d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^6*(a + b*Cos[c + d*x]),x]

[Out] (6720*b*Sin[c + d*x] - 6720*b*Sin[c + d*x]^3 + 4032*b*Sin[c + d*x]^5 - 960*b*Sin[c + d*x]^7 + 35*a*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)]))/(6720*d)

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{b \left(\frac{16}{5} + \cos^6(dx+c) + \frac{6(\cos^4(dx+c))}{5} + \frac{8(\cos^2(dx+c))}{5} \right) \sin(dx+c)}{7} + a \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + 5 \right)$
default	$\frac{b \left(\frac{16}{5} + \cos^6(dx+c) + \frac{6(\cos^4(dx+c))}{5} + \frac{8(\cos^2(dx+c))}{5} \right) \sin(dx+c)}{7} + a \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + 5 \right)$
parts	$a \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{b \left(\frac{16}{5} + \cos^6(dx+c) + \frac{6(\cos^4(dx+c))}{5} + \frac{8(\cos^2(dx+c))}{5} \right) \sin(dx+c)}{7d}$
parallelrisch	$\frac{2100axd + 3675b \sin(dx+c) + 15b \sin(7dx+7c) + 35a \sin(6dx+6c) + 147b \sin(5dx+5c) + 315 \sin(4dx+4c)a + 735b \sin(3dx+3c)}{6720d}$
risch	$\frac{5ax}{16} + \frac{35b \sin(dx+c)}{64d} + \frac{b \sin(7dx+7c)}{448d} + \frac{a \sin(6dx+6c)}{192d} + \frac{7b \sin(5dx+5c)}{320d} + \frac{3a \sin(4dx+4c)}{64d} + \frac{7b \sin(3dx+3c)}{64d}$
norman	$\frac{\frac{5ax}{16} + \frac{35ax \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{16} + \frac{105ax \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{16} + \frac{175ax \left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{16} + \frac{175ax \left(\tan^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{16} + \frac{105ax \left(\tan^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{16} + 35}{16}$

```
[In] int(cos(d*x+c)^6*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*b*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x
+5/16*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.67

$$\int \cos^6(c+dx)(a+b \cos(c+dx)) dx$$

$$= \frac{525 adx + (240 b \cos(dx+c)^6 + 280 a \cos(dx+c)^5 + 288 b \cos(dx+c)^4 + 350 a \cos(dx+c)^3 + 384 b \cos(dx+c)^2 + 525 a \cos(dx+c) + 768 b) \sin(dx+c)}{1680 d}$$

```
[In] integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(525*a*d*x + (240*b*cos(d*x + c)^6 + 280*a*cos(d*x + c)^5 + 288*b*cos(d*x + c)^4 + 350*a*cos(d*x + c)^3 + 384*b*cos(d*x + c)^2 + 525*a*cos(d*x + c) + 768*b)*sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.86

$$\int \cos^6(c + dx)(a + b \cos(c + dx)) dx$$

$$= \left\{ \begin{array}{l} \frac{5ax \sin^6(c+dx)}{16} + \frac{15ax \sin^4(c+dx) \cos^2(c+dx)}{16} + \frac{15ax \sin^2(c+dx) \cos^4(c+dx)}{16} + \frac{5ax \cos^6(c+dx)}{16} + \frac{5a \sin^5(c+dx) \cos(c+dx)}{16d} + \frac{5a \sin^4(c+dx) \cos^2(c+dx)}{16d} + \frac{5a \sin^3(c+dx) \cos^3(c+dx)}{16d} + \frac{5a \sin^2(c+dx) \cos^4(c+dx)}{16d} + \frac{5a \sin(c+dx) \cos^5(c+dx)}{16d} + \frac{5a \cos^6(c+dx)}{16d} \\ x(a + b \cos(c)) \cos^6(c) \end{array} \right.$$

[In] integrate(cos(d*x+c)**6*(a+b*cos(d*x+c)),x)

[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 16*b*sin(c + d*x)**7/(35*d) + 8*b*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*b*sin(c + d*x)**3*cos(c + d*x)**4/d + b*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**6, True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int \cos^6(c + dx)(a + b \cos(c + dx)) dx =$$

$$\frac{35(4 \sin(2dx + 2c)^3 - 60dx - 60c - 9 \sin(4dx + 4c) - 48 \sin(2dx + 2c))a + 192(5 \sin(dx + c)^7 - 21 \sin(dx + c)^5 + 35 \sin(dx + c)^3 - 35 \sin(dx + c))b}{6720d}$$

[In] integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] -1/6720*(35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a + 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*b)/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \cos^6(c + dx)(a + b \cos(c + dx)) dx = \frac{5}{16} ax + \frac{b \sin(7 dx + 7 c)}{448 d} + \frac{a \sin(6 dx + 6 c)}{192 d} + \frac{7 b \sin(5 dx + 5 c)}{320 d} + \frac{3 a \sin(4 dx + 4 c)}{64 d} + \frac{7 b \sin(3 dx + 3 c)}{64 d} + \frac{15 a \sin(2 dx + 2 c)}{64 d} + \frac{35 b \sin(dx + c)}{64 d}$$

[In] integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] 5/16*a*x + 1/448*b*sin(7*d*x + 7*c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 7/320*b*sin(5*d*x + 5*c)/d + 3/64*a*sin(4*d*x + 4*c)/d + 7/64*b*sin(3*d*x + 3*c)/d + 15/64*a*sin(2*d*x + 2*c)/d + 35/64*b*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 17.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.20

$$\int \cos^6(c + dx)(a + b \cos(c + dx)) dx = \frac{5 a x}{16} + \frac{(2 b - \frac{11 a}{8}) \tan(\frac{c}{2} + \frac{d x}{2})^{13} + (4 b - \frac{7 a}{6}) \tan(\frac{c}{2} + \frac{d x}{2})^{11} + (\frac{86 b}{5} - \frac{85 a}{24}) \tan(\frac{c}{2} + \frac{d x}{2})^9 + \frac{424 b \tan(\frac{c}{2} + \frac{d x}{2})^7}{35}}{d \left(\tan(\frac{c}{2} + \frac{d x}{2})^2 + 1 \right)^7}$$

[In] int(cos(c + d*x)^6*(a + b*cos(c + d*x)),x)

[Out] (5*a*x)/16 + (tan(c/2 + (d*x)/2)*((11*a)/8 + 2*b) + tan(c/2 + (d*x)/2)^3*((7*a)/6 + 4*b) - tan(c/2 + (d*x)/2)^11*((7*a)/6 - 4*b) - tan(c/2 + (d*x)/2)^13*((11*a)/8 - 2*b) + tan(c/2 + (d*x)/2)^5*((85*a)/24 + (86*b)/5) - tan(c/2 + (d*x)/2)^9*((85*a)/24 - (86*b)/5) + (424*b*tan(c/2 + (d*x)/2)^7)/35)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)

3.405 $\int \cos^5(c + dx)(a + b \cos(c + dx)) dx$

Optimal result	4625
Rubi [A] (verified)	4625
Mathematica [A] (verified)	4627
Maple [A] (verified)	4627
Fricas [A] (verification not implemented)	4628
Sympy [B] (verification not implemented)	4628
Maxima [A] (verification not implemented)	4629
Giac [A] (verification not implemented)	4629
Mupad [B] (verification not implemented)	4630

Optimal result

Integrand size = 19, antiderivative size = 114

$$\int \cos^5(c + dx)(a + b \cos(c + dx)) dx = \frac{5bx}{16} + \frac{a \sin(c + dx)}{d} + \frac{5b \cos(c + dx) \sin(c + dx)}{16d} + \frac{5b \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{b \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin^5(c + dx)}{5d}$$

[Out] 5/16*b*x+a*sin(d*x+c)/d+5/16*b*cos(d*x+c)*sin(d*x+c)/d+5/24*b*cos(d*x+c)^3*sin(d*x+c)/d+1/6*b*cos(d*x+c)^5*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^5/d

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 2713, 2715, 8}

$$\int \cos^5(c + dx)(a + b \cos(c + dx)) dx = \frac{a \sin^5(c + dx)}{5d} - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin(c + dx)}{d} + \frac{b \sin(c + dx) \cos^5(c + dx)}{6d} + \frac{5b \sin(c + dx) \cos^3(c + dx)}{24d} + \frac{5b \sin(c + dx) \cos(c + dx)}{16d} + \frac{5bx}{16}$$

[In] Int[Cos[c + d*x]^5*(a + b*Cos[c + d*x]),x]

[Out] (5*b*x)/16 + (a*Sin[c + d*x])/d + (5*b*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*b*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^5)/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \cos^5(c + dx) dx + b \int \cos^6(c + dx) dx \\
 &= \frac{b \cos^5(c + dx) \sin(c + dx)}{6d} + \frac{1}{6}(5b) \int \cos^4(c + dx) dx \\
 &\quad - \frac{a \text{Subst}\left(\int (1 - 2x^2 + x^4) dx, x, -\sin(c + dx)\right)}{d} \\
 &= \frac{a \sin(c + dx)}{d} + \frac{5b \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{b \cos^5(c + dx) \sin(c + dx)}{6d} \\
 &\quad - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin^5(c + dx)}{5d} + \frac{1}{8}(5b) \int \cos^2(c + dx) dx \\
 &= \frac{a \sin(c + dx)}{d} + \frac{5b \cos(c + dx) \sin(c + dx)}{16d} + \frac{5b \cos^3(c + dx) \sin(c + dx)}{24d} \\
 &\quad + \frac{b \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{2a \sin^3(c + dx)}{3d} + \frac{a \sin^5(c + dx)}{5d} + \frac{1}{16}(5b) \int 1 dx
 \end{aligned}$$

$$= \frac{5bx}{16} + \frac{a \sin(c+dx)}{d} + \frac{5b \cos(c+dx) \sin(c+dx)}{16d} + \frac{5b \cos^3(c+dx) \sin(c+dx)}{24d}$$

$$+ \frac{b \cos^5(c+dx) \sin(c+dx)}{6d} - \frac{2a \sin^3(c+dx)}{3d} + \frac{a \sin^5(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int \cos^5(c+dx)(a+b \cos(c+dx)) dx$$

$$= \frac{960a \sin(c+dx) - 640a \sin^3(c+dx) + 192a \sin^5(c+dx) + 5b(60c + 60dx + 45 \sin(2(c+dx))) + 9 \sin(4(c+dx))}{960d}$$

[In] Integrate[Cos[c + d*x]^5*(a + b*Cos[c + d*x]), x]

[Out] (960*a*Sin[c + d*x] - 640*a*Sin[c + d*x]^3 + 192*a*Sin[c + d*x]^5 + 5*b*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])/(960*d)

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

method	result
derivativedivides	$b \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{a \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5}$
default	$b \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right) + \frac{a \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5}$
parallelrisc	$\frac{300bxd+600a \sin(dx+c)+5b \sin(6dx+6c)+12a \sin(5dx+5c)+45 \sin(4dx+4c)b+100a \sin(3dx+3c)+225 \sin(2dx+2c)b}{960d}$
parts	$a \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c) + b \left(\frac{\left(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15 \cos(dx+c)}{8} \right) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right)$
risc	$\frac{5bx}{16} + \frac{5a \sin(dx+c)}{8d} + \frac{b \sin(6dx+6c)}{192d} + \frac{a \sin(5dx+5c)}{80d} + \frac{3b \sin(4dx+4c)}{64d} + \frac{5a \sin(3dx+3c)}{48d} + \frac{15b \sin(2dx+2c)}{64d}$
norman	$\frac{5bx}{16} + \frac{15bx \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{8} + \frac{75bx \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{16} + \frac{25bx \left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{4} + \frac{75bx \left(\tan^8\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{16} + \frac{15bx \left(\tan^{10}\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{8} + \frac{5bx \left(\tan^{12}\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{16}$

[In] int(cos(d*x+c)^5*(a+cos(d*x+c)*b), x, method=_RETURNVERBOSE)

[Out] $1/d*(b*(1/6*(\cos(dx+c)^5+5/4*\cos(dx+c)^3+15/8*\cos(dx+c))*\sin(dx+c)+5/16*d*x+5/16*c)+1/5*a*(8/3+\cos(dx+c)^4+4/3*\cos(dx+c)^2)*\sin(dx+c))$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \cos^5(c+dx)(a+b\cos(c+dx)) dx$$

$$= \frac{75 b dx + (40 b \cos(dx+c)^5 + 48 a \cos(dx+c)^4 + 50 b \cos(dx+c)^3 + 64 a \cos(dx+c)^2 + 75 b \cos(dx+c))}{240 d}$$

[In] `integrate(cos(dx+c)^5*(a+b*cos(dx+c)),x, algorithm="fricas")`

[Out] $1/240*(75*b*d*x + (40*b*\cos(dx + c)^5 + 48*a*\cos(dx + c)^4 + 50*b*\cos(dx + c)^3 + 64*a*\cos(dx + c)^2 + 75*b*\cos(dx + c) + 128*a)*\sin(dx + c))/d$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(107) = 214.

Time = 0.34 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.89

$$\int \cos^5(c+dx)(a+b\cos(c+dx)) dx$$

$$= \begin{cases} \frac{8a \sin^5(c+dx)}{15d} + \frac{4a \sin^3(c+dx) \cos^2(c+dx)}{3d} + \frac{a \sin(c+dx) \cos^4(c+dx)}{d} + \frac{5bx \sin^6(c+dx)}{16} + \frac{15bx \sin^4(c+dx) \cos^2(c+dx)}{16} + \frac{15bx \sin^2(c+dx) \cos^4(c+dx)}{16} \\ x(a + b \cos(c)) \cos^5(c) \end{cases}$$

[In] `integrate(cos(dx+c)**5*(a+b*cos(dx+c)),x)`

[Out] `Piecewise((8*a*sin(c + dx)**5/(15*d) + 4*a*sin(c + dx)**3*cos(c + dx)**2/(3*d) + a*sin(c + dx)*cos(c + dx)**4/d + 5*b*x*sin(c + dx)**6/16 + 15*b*x*sin(c + dx)**4*cos(c + dx)**2/16 + 15*b*x*sin(c + dx)**2*cos(c + dx)**4/16 + 5*b*x*cos(c + dx)**6/16 + 5*b*sin(c + dx)**5*cos(c + dx)/(16*d) + 5*b*sin(c + dx)**3*cos(c + dx)**3/(6*d) + 11*b*sin(c + dx)*cos(c + dx)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**5, True))`

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \cos^5(c + dx)(a + b \cos(c + dx)) dx$$

$$= \frac{64 (3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))a - 5 (4 \sin(2dx + 2c)^3 - 60 dx - 60 c - 9 \sin(4dx + 4c) - 48 \sin(2dx + 2c))b}{960 d}$$

[In] integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="maxima")

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a - 5*(4
*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2
*c))*b)/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int \cos^5(c + dx)(a + b \cos(c + dx)) dx = \frac{5}{16} bx + \frac{b \sin(6 dx + 6 c)}{192 d} + \frac{a \sin(5 dx + 5 c)}{80 d}$$

$$+ \frac{3 b \sin(4 dx + 4 c)}{64 d} + \frac{5 a \sin(3 dx + 3 c)}{48 d}$$

$$+ \frac{15 b \sin(2 dx + 2 c)}{64 d} + \frac{5 a \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="giac")

```
[Out] 5/16*b*x + 1/192*b*sin(6*d*x + 6*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 3/64*b*
sin(4*d*x + 4*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 15/64*b*sin(2*d*x + 2*c)/d
+ 5/8*a*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\begin{aligned}
\int \cos^5(c + dx)(a + b \cos(c + dx)) dx = & \frac{5bx}{16} + \frac{8a \sin(c + dx)}{15d} \\
& + \frac{5b \cos(c + dx) \sin(c + dx)}{16d} \\
& + \frac{4a \cos(c + dx)^2 \sin(c + dx)}{15d} \\
& + \frac{a \cos(c + dx)^4 \sin(c + dx)}{5d} \\
& + \frac{5b \cos(c + dx)^3 \sin(c + dx)}{24d} \\
& + \frac{b \cos(c + dx)^5 \sin(c + dx)}{6d}
\end{aligned}$$

[In] int(cos(c + d*x)^5*(a + b*cos(c + d*x)),x)

```
[Out] (5*b*x)/16 + (8*a*sin(c + d*x))/(15*d) + (5*b*cos(c + d*x)*sin(c + d*x))/(16*d) + (4*a*cos(c + d*x)^2*sin(c + d*x))/(15*d) + (a*cos(c + d*x)^4*sin(c + d*x))/(5*d) + (5*b*cos(c + d*x)^3*sin(c + d*x))/(24*d) + (b*cos(c + d*x)^5*sin(c + d*x))/(6*d)
```

3.406 $\int \cos^4(c + dx)(a + b \cos(c + dx)) dx$

Optimal result	4631
Rubi [A] (verified)	4631
Mathematica [A] (verified)	4633
Maple [A] (verified)	4633
Fricas [A] (verification not implemented)	4634
Sympy [A] (verification not implemented)	4634
Maxima [A] (verification not implemented)	4634
Giac [A] (verification not implemented)	4635
Mupad [B] (verification not implemented)	4635

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \cos^4(c + dx)(a + b \cos(c + dx)) dx = \frac{3ax}{8} + \frac{b \sin(c + dx)}{d} + \frac{3a \cos(c + dx) \sin(c + dx)}{8d} + \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{2b \sin^3(c + dx)}{3d} + \frac{b \sin^5(c + dx)}{5d}$$

[Out] $3/8*a*x+b*\sin(d*x+c)/d+3/8*a*\cos(d*x+c)*\sin(d*x+c)/d+1/4*a*\cos(d*x+c)^3*\sin(d*x+c)/d-2/3*b*\sin(d*x+c)^3/d+1/5*b*\sin(d*x+c)^5/d$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 2715, 8, 2713}

$$\int \cos^4(c + dx)(a + b \cos(c + dx)) dx = \frac{a \sin(c + dx) \cos^3(c + dx)}{4d} + \frac{3a \sin(c + dx) \cos(c + dx)}{8d} + \frac{3ax}{8} + \frac{b \sin^5(c + dx)}{5d} - \frac{2b \sin^3(c + dx)}{3d} + \frac{b \sin(c + dx)}{d}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^4*(a + b*\text{Cos}[c + d*x]), x]$

[Out] $(3*a*x)/8 + (b*\text{Sin}[c + d*x])/d + (3*a*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (a*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(4*d) - (2*b*\text{Sin}[c + d*x]^3)/(3*d) + (b*\text{Sin}[c + d*x]^5)/(5*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2715

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \cos^4(c + dx) dx + b \int \cos^5(c + dx) dx \\
 &= \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{1}{4}(3a) \int \cos^2(c + dx) dx \\
 &\quad - \frac{b \text{Subst}(\int (1 - 2x^2 + x^4) dx, x, -\sin(c + dx))}{d} \\
 &= \frac{b \sin(c + dx)}{d} + \frac{3a \cos(c + dx) \sin(c + dx)}{8d} + \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} \\
 &\quad - \frac{2b \sin^3(c + dx)}{3d} + \frac{b \sin^5(c + dx)}{5d} + \frac{1}{8}(3a) \int 1 dx \\
 &= \frac{3ax}{8} + \frac{b \sin(c + dx)}{d} + \frac{3a \cos(c + dx) \sin(c + dx)}{8d} \\
 &\quad + \frac{a \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{2b \sin^3(c + dx)}{3d} + \frac{b \sin^5(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \cos^4(c + dx)(a + b \cos(c + dx)) dx = \frac{3a(c + dx)}{8d} + \frac{b \sin(c + dx)}{d} - \frac{2b \sin^3(c + dx)}{3d} + \frac{b \sin^5(c + dx)}{5d} + \frac{a \sin(2(c + dx))}{4d} + \frac{a \sin(4(c + dx))}{32d}$$

`[In] Integrate[Cos[c + d*x]^4*(a + b*Cos[c + d*x]), x]`

```
[Out] (3*a*(c + d*x))/(8*d) + (b*Sin[c + d*x])/d - (2*b*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^5)/(5*d) + (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

method	result
parallelrisch	$\frac{180axd+300b \sin(dx+c)+6b \sin(5dx+5c)+15 \sin(4dx+4c)a+50b \sin(3dx+3c)+120 \sin(2dx+2c)a}{480d}$
derivativedivides	$\frac{b \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + a \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right)}{d}$
default	$\frac{b \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + a \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right)}{d}$
parts	$a \left(\frac{(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{b \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5d}$
risch	$\frac{3ax}{8} + \frac{5b \sin(dx+c)}{8d} + \frac{b \sin(5dx+5c)}{80d} + \frac{a \sin(4dx+4c)}{32d} + \frac{5b \sin(3dx+3c)}{48d} + \frac{a \sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8} + \frac{15ax(\tan^2(\frac{dx}{2} + \frac{c}{2}))}{8} + \frac{15ax(\tan^4(\frac{dx}{2} + \frac{c}{2}))}{4} + \frac{15ax(\tan^6(\frac{dx}{2} + \frac{c}{2}))}{4} + \frac{15ax(\tan^8(\frac{dx}{2} + \frac{c}{2}))}{8} + \frac{3ax(\tan^{10}(\frac{dx}{2} + \frac{c}{2}))}{8} + \frac{116b}{8}}{(1 + \tan^2(\frac{dx}{2}))^5}$

`[In] int(cos(d*x+c)^4*(a+cos(d*x+c)*b), x, method=_RETURNVERBOSE)`

```
[Out] 1/480*(180*a*x*d+300*b*sin(d*x+c)+6*b*sin(5*d*x+5*c)+15*sin(4*d*x+4*c)*a+50*b*sin(3*d*x+3*c)+120*sin(2*d*x+2*c)*a)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \cos^4(c + dx)(a + b \cos(c + dx)) dx$$

$$= \frac{45 adx + (24b \cos(dx + c)^4 + 30a \cos(dx + c)^3 + 32b \cos(dx + c)^2 + 45a \cos(dx + c) + 64b) \sin(dx + c)}{120 d}$$

`[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="fricas")``[Out] 1/120*(45*a*d*x + (24*b*cos(d*x + c)^4 + 30*a*cos(d*x + c)^3 + 32*b*cos(d*x + c)^2 + 45*a*cos(d*x + c) + 64*b)*sin(d*x + c))/d`**Sympy [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

$$\int \cos^4(c + dx)(a + b \cos(c + dx)) dx$$

$$= \begin{cases} \frac{3ax \sin^4(c+dx)}{8} + \frac{3ax \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{3ax \cos^4(c+dx)}{8} + \frac{3a \sin^3(c+dx) \cos(c+dx)}{8d} + \frac{5a \sin(c+dx) \cos^3(c+dx)}{8d} + \frac{8b \sin^5(c+dx)}{15d} \\ x(a + b \cos(c)) \cos^4(c) \end{cases}$$

`[In] integrate(cos(d*x+c)**4*(a+b*cos(d*x+c)),x)``[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*b*sin(c + d*x)**5/(15*d) + 4*b*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + b*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**4, True))`**Maxima [A] (verification not implemented)**

none

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \cos^4(c + dx)(a + b \cos(c + dx)) dx$$

$$= \frac{15(12 dx + 12c + \sin(4 dx + 4c) + 8 \sin(2 dx + 2c))a + 32(3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))b}{480 d}$$

`[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="maxima")``[Out] 1/480*(15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*b)/d`

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int \cos^4(c + dx)(a + b \cos(c + dx)) dx = \frac{3}{8} ax + \frac{b \sin(5 dx + 5 c)}{80 d} + \frac{a \sin(4 dx + 4 c)}{32 d} + \frac{5 b \sin(3 dx + 3 c)}{48 d} + \frac{a \sin(2 dx + 2 c)}{4 d} + \frac{5 b \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] 3/8*a*x + 1/80*b*sin(5*d*x + 5*c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 5/48*b*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 5/8*b*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25

$$\int \cos^4(c + dx)(a + b \cos(c + dx)) dx = \frac{3 a x}{8} + \frac{(2 b - \frac{5 a}{4}) \tan(\frac{c}{2} + \frac{d x}{2})^9 + (\frac{8 b}{3} - \frac{a}{2}) \tan(\frac{c}{2} + \frac{d x}{2})^7 + \frac{116 b \tan(\frac{c}{2} + \frac{d x}{2})^5}{15} + (\frac{a}{2} + \frac{8 b}{3}) \tan(\frac{c}{2} + \frac{d x}{2})^3 + (\frac{5 a}{4} + \frac{b}{2}) \tan(\frac{c}{2} + \frac{d x}{2})}{d \left(\tan(\frac{c}{2} + \frac{d x}{2})^2 + 1 \right)^5}$$

[In] int(cos(c + d*x)^4*(a + b*cos(c + d*x)),x)

[Out] (3*a*x)/8 + (tan(c/2 + (d*x)/2)*((5*a)/4 + 2*b) + tan(c/2 + (d*x)/2)^3*(a/2 + (8*b)/3) - tan(c/2 + (d*x)/2)^9*((5*a)/4 - 2*b) - tan(c/2 + (d*x)/2)^7*(a/2 - (8*b)/3) + (116*b*tan(c/2 + (d*x)/2)^5)/15)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)

3.407 $\int \cos^3(c + dx)(a + b \cos(c + dx)) dx$

Optimal result	4636
Rubi [A] (verified)	4636
Mathematica [A] (verified)	4638
Maple [A] (verified)	4638
Fricas [A] (verification not implemented)	4639
Sympy [B] (verification not implemented)	4639
Maxima [A] (verification not implemented)	4639
Giac [A] (verification not implemented)	4640
Mupad [B] (verification not implemented)	4640

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \cos^3(c + dx)(a + b \cos(c + dx)) dx = \frac{3bx}{8} + \frac{a \sin(c + dx)}{d} + \frac{3b \cos(c + dx) \sin(c + dx)}{8d} + \frac{b \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{a \sin^3(c + dx)}{3d}$$

[Out] $3/8*b*x+a*\sin(d*x+c)/d+3/8*b*\cos(d*x+c)*\sin(d*x+c)/d+1/4*b*\cos(d*x+c)^3*\sin(d*x+c)/d-1/3*a*\sin(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 2713, 2715, 8}

$$\int \cos^3(c + dx)(a + b \cos(c + dx)) dx = -\frac{a \sin^3(c + dx)}{3d} + \frac{a \sin(c + dx)}{d} + \frac{b \sin(c + dx) \cos^3(c + dx)}{4d} + \frac{3b \sin(c + dx) \cos(c + dx)}{8d} + \frac{3bx}{8}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^3*(a + b*\text{Cos}[c + d*x]), x]$

[Out] $(3*b*x)/8 + (a*\text{Sin}[c + d*x])/d + (3*b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (b*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(4*d) - (a*\text{Sin}[c + d*x]^3)/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2715

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \cos^3(c + dx) dx + b \int \cos^4(c + dx) dx \\
 &= \frac{b \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{1}{4}(3b) \int \cos^2(c + dx) dx \\
 &\quad - \frac{a \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{d} \\
 &= \frac{a \sin(c + dx)}{d} + \frac{3b \cos(c + dx) \sin(c + dx)}{8d} \\
 &\quad + \frac{b \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{a \sin^3(c + dx)}{3d} + \frac{1}{8}(3b) \int 1 dx \\
 &= \frac{3bx}{8} + \frac{a \sin(c + dx)}{d} + \frac{3b \cos(c + dx) \sin(c + dx)}{8d} \\
 &\quad + \frac{b \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{a \sin^3(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \cos^3(c + dx)(a + b \cos(c + dx)) dx = \frac{3b(c + dx)}{8d} + \frac{a \sin(c + dx)}{d} - \frac{a \sin^3(c + dx)}{3d} + \frac{b \sin(2(c + dx))}{4d} + \frac{b \sin(4(c + dx))}{32d}$$

[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x]),x]

[Out] (3*b*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result
parallelrisc	$\frac{36bxd+72a \sin(dx+c)+3 \sin(4dx+4c)b+8a \sin(3dx+3c)+24 \sin(2dx+2c)b}{96d}$
derivativedivides	$\frac{b \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{a(2+\cos^2(dx+c)) \sin(dx+c)}{3}}{d}$
default	$\frac{b \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{a(2+\cos^2(dx+c)) \sin(dx+c)}{3}}{d}$
parts	$\frac{a(2+\cos^2(dx+c)) \sin(dx+c)}{3d} + \frac{b \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right)}{d}$
risc	$\frac{3bx}{8} + \frac{3a \sin(dx+c)}{4d} + \frac{b \sin(4dx+4c)}{32d} + \frac{a \sin(3dx+3c)}{12d} + \frac{b \sin(2dx+2c)}{4d}$
norman	$\frac{3bx}{8} + \frac{3bx \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{2} + \frac{9bx \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{4} + \frac{3bx \left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{2} + \frac{3bx \left(\tan^8\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{8} + \frac{(8a-5b) \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{4d} + \frac{(8a+5b) \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{4d} + \frac{1}{(1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^4}$

[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] 1/96*(36*b*x*d+72*a*sin(d*x+c)+3*sin(4*d*x+4*c)*b+8*a*sin(3*d*x+3*c)+24*sin(2*d*x+2*c)*b)/d

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \cos^3(c + dx)(a + b \cos(c + dx)) dx$$

$$= \frac{9 b dx + (6 b \cos(dx + c))^3 + 8 a \cos(dx + c)^2 + 9 b \cos(dx + c) + 16 a) \sin(dx + c)}{24 d}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/24*(9*b*d*x + (6*b*cos(d*x + c))^3 + 8*a*cos(d*x + c)^2 + 9*b*cos(d*x + c) + 16*a)*sin(d*x + c)/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(70) = 140.

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.89

$$\int \cos^3(c + dx)(a + b \cos(c + dx)) dx$$

$$= \begin{cases} \frac{2a \sin^3(c+dx)}{3d} + \frac{a \sin(c+dx) \cos^2(c+dx)}{d} + \frac{3bx \sin^4(c+dx)}{8} + \frac{3bx \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{3bx \cos^4(c+dx)}{8} + \frac{3b \sin^3(c+dx) \cos(c+dx)}{8d} \\ x(a + b \cos(c)) \cos^3(c) \end{cases}$$

[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c)),x)

[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d + 3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b*x*cos(c + d*x)**4/8 + 3*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \cos^3(c + dx)(a + b \cos(c + dx)) dx =$$

$$\frac{32 (\sin(dx + c)^3 - 3 \sin(dx + c))a - 3(12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c))b}{96 d}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b)/d

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \cos^3(c + dx)(a + b \cos(c + dx)) dx = \frac{3}{8}bx + \frac{b \sin(4dx + 4c)}{32d} + \frac{a \sin(3dx + 3c)}{12d} + \frac{b \sin(2dx + 2c)}{4d} + \frac{3a \sin(dx + c)}{4d}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] 3/8*b*x + 1/32*b*sin(4*d*x + 4*c)/d + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*b*sin(2*d*x + 2*c)/d + 3/4*a*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 14.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \cos^3(c + dx)(a + b \cos(c + dx)) dx = \frac{3bx}{8} + \frac{2a \sin(c + dx)}{3d} + \frac{3b \cos(c + dx) \sin(c + dx)}{8d} + \frac{a \cos(c + dx)^2 \sin(c + dx)}{3d} + \frac{b \cos(c + dx)^3 \sin(c + dx)}{4d}$$

[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x)),x)

[Out] (3*b*x)/8 + (2*a*sin(c + d*x))/(3*d) + (3*b*cos(c + d*x)*sin(c + d*x))/(8*d) + (a*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (b*cos(c + d*x)^3*sin(c + d*x))/(4*d)

3.408 $\int \cos^2(c + dx)(a + b \cos(c + dx)) dx$

Optimal result	4641
Rubi [A] (verified)	4641
Mathematica [A] (verified)	4642
Maple [A] (verified)	4643
Fricas [A] (verification not implemented)	4643
Sympy [A] (verification not implemented)	4644
Maxima [A] (verification not implemented)	4644
Giac [A] (verification not implemented)	4644
Mupad [B] (verification not implemented)	4645

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cos^2(c + dx)(a + b \cos(c + dx)) dx = \frac{ax}{2} + \frac{b \sin(c + dx)}{d} + \frac{a \cos(c + dx) \sin(c + dx)}{2d} - \frac{b \sin^3(c + dx)}{3d}$$

[Out] $1/2*a*x+b*\sin(d*x+c)/d+1/2*a*\cos(d*x+c)*\sin(d*x+c)/d-1/3*b*\sin(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 2715, 8, 2713}

$$\int \cos^2(c + dx)(a + b \cos(c + dx)) dx = \frac{a \sin(c + dx) \cos(c + dx)}{2d} + \frac{ax}{2} - \frac{b \sin^3(c + dx)}{3d} + \frac{b \sin(c + dx)}{d}$$

[In] `Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x]),x]`

[Out] $(a*x)/2 + (b*\sin[c + d*x])/d + (a*\cos[c + d*x]*\sin[c + d*x])/(2*d) - (b*\sin[c + d*x]^3)/(3*d)$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \cos^2(c + dx) dx + b \int \cos^3(c + dx) dx \\ &= \frac{a \cos(c + dx) \sin(c + dx)}{2d} + \frac{1}{2}a \int 1 dx - \frac{b \text{Subst}\left(\int (1 - x^2) dx, x, -\sin(c + dx)\right)}{d} \\ &= \frac{ax}{2} + \frac{b \sin(c + dx)}{d} + \frac{a \cos(c + dx) \sin(c + dx)}{2d} - \frac{b \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \cos^2(c + dx)(a + b \cos(c + dx)) dx = \frac{a(c + dx)}{2d} + \frac{b \sin(c + dx)}{d} - \frac{b \sin^3(c + dx)}{3d} + \frac{a \sin(2(c + dx))}{4d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

method	result
parallelrisc	$\frac{6axd+b\sin(3dx+3c)+3\sin(2dx+2c)a+9b\sin(dx+c)}{12d}$
risc	$\frac{ax}{2} + \frac{3b\sin(dx+c)}{4d} + \frac{b\sin(3dx+3c)}{12d} + \frac{a\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{b(2+\cos^2(dx+c))\sin(dx+c)}{3} + a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)$ d
default	$\frac{b(2+\cos^2(dx+c))\sin(dx+c)}{3} + a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)$ d
parts	$a\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + \frac{b(2+\cos^2(dx+c))\sin(dx+c)}{3d}$
norman	$\frac{(a+2b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{ax}{2} + \frac{3ax(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))}{2} + \frac{3ax(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right))}{2} + \frac{ax(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right))}{2} + \frac{4b(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right))}{3d} - \frac{(a-2b)(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right))}{d}}{(1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^3}$

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(6*a*x*d+b*sin(3*d*x+3*c)+3*sin(2*d*x+2*c)*a+9*b*sin(d*x+c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \cos^2(c+dx)(a+b\cos(c+dx))dx$$

$$= \frac{3adx + (2b\cos(dx+c))^2 + 3a\cos(dx+c) + 4b\sin(dx+c)}{6d}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*d*x + (2*b*cos(d*x + c))^2 + 3*a*cos(d*x + c) + 4*b)*sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \cos^2(c + dx)(a + b \cos(c + dx)) dx$$

$$= \begin{cases} \frac{ax \sin^2(c+dx)}{2} + \frac{ax \cos^2(c+dx)}{2} + \frac{a \sin(c+dx) \cos(c+dx)}{2d} + \frac{2b \sin^3(c+dx)}{3d} + \frac{b \sin(c+dx) \cos^2(c+dx)}{d} & \text{for } d \neq 0 \\ x(a + b \cos(c)) \cos^2(c) & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c)),x)

[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*b*sin(c + d*x)**3/(3*d) + b*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \cos^2(c + dx)(a + b \cos(c + dx)) dx$$

$$= \frac{3(2dx + 2c + \sin(2dx + 2c))a - 4(\sin(dx + c)^3 - 3\sin(dx + c))b}{12d}$$

[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a - 4*(sin(d*x + c)^3 - 3*sin(d*x + c))*b)/d

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \cos^2(c + dx)(a + b \cos(c + dx)) dx = \frac{1}{2}ax + \frac{b \sin(3dx + 3c)}{12d}$$

$$+ \frac{a \sin(2dx + 2c)}{4d} + \frac{3b \sin(dx + c)}{4d}$$

[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] 1/2*a*x + 1/12*b*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 3/4*b*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 14.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \cos^2(c + dx)(a + b \cos(c + dx)) dx = \frac{ax}{2} + \frac{2b \sin(c + dx)}{3d} + \frac{a \cos(c + dx) \sin(c + dx)}{2d} + \frac{b \cos(c + dx)^2 \sin(c + dx)}{3d}$$

[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x)),x)

[Out] (a*x)/2 + (2*b*sin(c + d*x))/(3*d) + (a*cos(c + d*x)*sin(c + d*x))/(2*d) + (b*cos(c + d*x)^2*sin(c + d*x))/(3*d)

3.409 $\int \cos(c + dx)(a + b \cos(c + dx)) dx$

Optimal result	4646
Rubi [A] (verified)	4646
Mathematica [A] (verified)	4647
Maple [A] (verified)	4647
Fricas [A] (verification not implemented)	4647
Sympy [B] (verification not implemented)	4648
Maxima [A] (verification not implemented)	4648
Giac [A] (verification not implemented)	4648
Mupad [B] (verification not implemented)	4649

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \cos(c + dx)(a + b \cos(c + dx)) dx = \frac{bx}{2} + \frac{a \sin(c + dx)}{d} + \frac{b \cos(c + dx) \sin(c + dx)}{2d}$$

[Out] $1/2*b*x+a*\sin(d*x+c)/d+1/2*b*\cos(d*x+c)*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.02 (sec), antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2813}

$$\int \cos(c + dx)(a + b \cos(c + dx)) dx = \frac{a \sin(c + dx)}{d} + \frac{b \sin(c + dx) \cos(c + dx)}{2d} + \frac{bx}{2}$$

[In] $\text{Int}[\text{Cos}[c + d*x]*(a + b*\text{Cos}[c + d*x]), x]$

[Out] $(b*x)/2 + (a*\text{Sin}[c + d*x])/d + (b*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rule 2813

$\text{Int}[(a + (b_*)\sin[(e_*) + (f_*)(x)])*((c_*) + (d_*)\sin[(e_*) + (f_*)(x)]), x_Symbol] \rightarrow \text{Simp}[(2*a*c + b*d)*(x/2), x] + (-\text{Simp}[(b*c + a*d)*(\text{Cos}[e + f*x]/f), x] - \text{Simp}[b*d*\text{Cos}[e + f*x]*(\text{Sin}[e + f*x]/(2*f)), x]) /;$ Free Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\text{integral} = \frac{bx}{2} + \frac{a \sin(c + dx)}{d} + \frac{b \cos(c + dx) \sin(c + dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos(c + dx)(a + b \cos(c + dx)) dx = \frac{4a \sin(c + dx) + b(2(c + dx) + \sin(2(c + dx)))}{4d}$$

`[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x]),x]``[Out] (4*a*Sin[c + d*x] + b*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)`**Maple [A] (verified)**

Time = 0.80 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{bx}{2} + \frac{a \sin(dx+c)}{d} + \frac{b \sin(2dx+2c)}{4d}$	32
parallelrisch	$\frac{2bxd + \sin(2dx+2c)b + 4a \sin(dx+c)}{4d}$	32
derivativedivides	$b \left(\frac{\cos(dx+c) \sin(dx+c) + \frac{dx}{2} + \frac{c}{2}}{d} \right) + a \sin(dx+c)$	38
default	$b \left(\frac{\cos(dx+c) \sin(dx+c) + \frac{dx}{2} + \frac{c}{2}}{d} \right) + a \sin(dx+c)$	38
parts	$\frac{b \left(\frac{\cos(dx+c) \sin(dx+c) + \frac{dx}{2} + \frac{c}{2}}{d} \right)}{d} + \frac{a \sin(dx+c)}{d}$	40
norman	$\frac{bx \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \frac{(2a-b) \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} + \frac{(2a+b) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d} + \frac{bx}{2} + \frac{bx \left(\tan^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{2}}{\left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2}$	91

`[In] int(cos(d*x+c)*(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)``[Out] 1/2*b*x+a*sin(d*x+c)/d+1/4*b/d*sin(2*d*x+2*c)`**Fricas [A] (verification not implemented)**

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \cos(c + dx)(a + b \cos(c + dx)) dx = \frac{bdx + (b \cos(dx + c) + 2a) \sin(dx + c)}{2d}$$

`[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="fricas")``[Out] 1/2*(b*d*x + (b*cos(d*x + c) + 2*a)*sin(d*x + c))/d`

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \cos(c + dx)(a + b \cos(c + dx)) dx$$

$$= \begin{cases} \frac{a \sin(c+dx)}{d} + \frac{bx \sin^2(c+dx)}{2} + \frac{bx \cos^2(c+dx)}{2} + \frac{b \sin(c+dx) \cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a + b \cos(c)) \cos(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)/d + b*x*sin(c + d*x)**2/2 + b*x*cos(c + d*x)**2/2 + b*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \cos(c + dx)(a + b \cos(c + dx)) dx = \frac{(2 dx + 2 c + \sin(2 dx + 2 c))b + 4 a \sin(dx + c)}{4 d}$$

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*b + 4*a*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos(c + dx)(a + b \cos(c + dx)) dx = \frac{1}{2} bx + \frac{b \sin(2 dx + 2 c)}{4 d} + \frac{a \sin(dx + c)}{d}$$

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*b*x + 1/4*b*sin(2*d*x + 2*c)/d + a*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 14.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \cos(c + dx)(a + b \cos(c + dx)) dx = \frac{bx}{2} + \frac{b \sin(2c + 2dx)}{4d} + \frac{a \sin(c + dx)}{d}$$

[In] int(cos(c + d*x)*(a + b*cos(c + d*x)),x)

[Out] (b*x)/2 + (b*sin(2*c + 2*d*x))/(4*d) + (a*sin(c + d*x))/d

3.410 $\int (a + b \cos(c + dx)) dx$

Optimal result	4650
Rubi [A] (verified)	4650
Mathematica [A] (verified)	4651
Maple [A] (verified)	4651
Fricas [A] (verification not implemented)	4651
Sympy [A] (verification not implemented)	4652
Maxima [A] (verification not implemented)	4652
Giac [A] (verification not implemented)	4652
Mupad [B] (verification not implemented)	4652

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int (a + b \cos(c + dx)) dx = ax + \frac{b \sin(c + dx)}{d}$$

[Out] a*x+b*sin(d*x+c)/d

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2717}

$$\int (a + b \cos(c + dx)) dx = ax + \frac{b \sin(c + dx)}{d}$$

[In] Int[a + b*Cos[c + d*x],x]

[Out] a*x + (b*Sin[c + d*x])/d

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= ax + b \int \cos(c + dx) dx \\ &= ax + \frac{b \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int (a + b \cos(c + dx)) dx = ax + \frac{b \cos(dx) \sin(c)}{d} + \frac{b \cos(c) \sin(dx)}{d}$$

`[In] Integrate[a + b*Cos[c + d*x],x]``[Out] a*x + (b*Cos[d*x]*Sin[c])/d + (b*Cos[c]*Sin[d*x])/d`**Maple [A] (verified)**

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax + \frac{b \sin(dx+c)}{d}$	16
risch	$ax + \frac{b \sin(dx+c)}{d}$	16
parallelrisch	$ax + \frac{b \sin(dx+c)}{d}$	16
parts	$ax + \frac{b \sin(dx+c)}{d}$	16
derivativedivides	$\frac{a(dx+c)+b \sin(dx+c)}{d}$	21
norman	$\frac{ax+ax \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \frac{2b \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d}}{1+\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right)}$	50

`[In] int(a+cos(d*x+c)*b,x,method=_RETURNVERBOSE)``[Out] a*x+b*sin(d*x+c)/d`**Fricas [A] (verification not implemented)**

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + b \cos(c + dx)) dx = \frac{adx + b \sin(dx + c)}{d}$$

`[In] integrate(a+b*cos(d*x+c),x, algorithm="fricas")``[Out] (a*d*x + b*sin(d*x + c))/d`

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + b \cos(c + dx)) dx = ax + b \left(\begin{cases} \frac{\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x \cos(c) & \text{otherwise} \end{cases} \right)$$

[In] integrate(a+b*cos(d*x+c),x)

[Out] a*x + b*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + b \cos(c + dx)) dx = ax + \frac{b \sin(dx + c)}{d}$$

[In] integrate(a+b*cos(d*x+c),x, algorithm="maxima")

[Out] a*x + b*sin(d*x + c)/d

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + b \cos(c + dx)) dx = ax + \frac{b \sin(dx + c)}{d}$$

[In] integrate(a+b*cos(d*x+c),x, algorithm="giac")

[Out] a*x + b*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + b \cos(c + dx)) dx = \frac{b \sin(c + dx) + a dx}{d}$$

[In] int(a + b*cos(c + d*x),x)

[Out] (b*sin(c + d*x) + a*d*x)/d

3.411 $\int (a + b \cos(c + dx)) \sec(c + dx) dx$

Optimal result	4653
Rubi [A] (verified)	4653
Mathematica [A] (verified)	4654
Maple [A] (verified)	4654
Fricas [B] (verification not implemented)	4655
Sympy [B] (verification not implemented)	4655
Maxima [A] (verification not implemented)	4655
Giac [B] (verification not implemented)	4656
Mupad [B] (verification not implemented)	4656

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int (a + b \cos(c + dx)) \sec(c + dx) dx = bx + \frac{a \operatorname{arctanh}(\sin(c + dx))}{d}$$

[Out] `b*x+a*arctanh(sin(d*x+c))/d`

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2814, 3855}

$$\int (a + b \cos(c + dx)) \sec(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{d} + bx$$

[In] `Int[(a + b*Cos[c + d*x])*Sec[c + d*x],x]`

[Out] `b*x + (a*ArcTanh[Sin[c + d*x]])/d`

Rule 2814

`Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \text{integral} &= bx + a \int \sec(c + dx) dx \\ &= bx + \frac{a \operatorname{arctanh}(\sin(c + dx))}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a + b \cos(c + dx)) \sec(c + dx) dx = bx + \frac{a \operatorname{arctanh}(\sin(c + dx))}{d}$$

[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x],x]

[Out] b*x + (a*ArcTanh[Sin[c + d*x]])/d

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result	size
derivativedivides	$\frac{a \ln(\sec(dx+c)+\tan(dx+c))+b(dx+c)}{d}$	29
default	$\frac{a \ln(\sec(dx+c)+\tan(dx+c))+b(dx+c)}{d}$	29
parts	$\frac{a \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{b(dx+c)}{d}$	31
parallelrisch	$\frac{bx d - a \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + a \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d}$	39
risch	$bx + \frac{a \ln(e^{i(dx+c)} + i)}{d} - \frac{a \ln(e^{i(dx+c)} - i)}{d}$	42
norman	$\frac{bx + bx \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{a \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d} - \frac{a \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d}$	71

[In] int((a+cos(d*x+c)*b)*sec(d*x+c),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*ln(sec(d*x+c)+tan(d*x+c))+b*(d*x+c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int (a + b \cos(c + dx)) \sec(c + dx) dx = \frac{2 b dx + a \log(\sin(dx + c) + 1) - a \log(-\sin(dx + c) + 1)}{2 d}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")

[Out] 1/2*(2*b*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.

Time = 2.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int (a + b \cos(c + dx)) \sec(c + dx) dx = a \left(\begin{array}{ll} \frac{x \tan(c) \sec(c)}{\tan(c) + \sec(c)} + \frac{x \sec^2(c)}{\tan(c) + \sec(c)} & \text{for } d = 0 \\ \frac{\log(\tan(c + dx) + \sec(c + dx))}{d} & \text{otherwise} \end{array} \right) + bx$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x)

[Out] a*Piecewise((x*tan(c)*sec(c)/(tan(c) + sec(c)) + x*sec(c)**2/(tan(c) + sec(c)), Eq(d, 0)), (log(tan(c + d*x) + sec(c + d*x))/d, True)) + b*x

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int (a + b \cos(c + dx)) \sec(c + dx) dx = \frac{(dx + c)b + a \log(\sec(dx + c) + \tan(dx + c))}{d}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")

[Out] ((d*x + c)*b + a*log(sec(d*x + c) + tan(d*x + c)))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(16) = 32.

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.69

$$\int (a + b \cos(c + dx)) \sec(c + dx) dx$$

$$= \frac{(dx + c)b + a \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|) - a \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{d}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")

[Out] ((d*x + c)*b + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)))/d

Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int (a + b \cos(c + dx)) \sec(c + dx) dx = \frac{2 a \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d} + \frac{2 b \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{d x}{2}\right)}{\cos\left(\frac{c}{2} + \frac{d x}{2}\right)}\right)}{d}$$

[In] int((a + b*cos(c + d*x))/cos(c + d*x),x)

[Out] (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d

3.412 $\int (a + b \cos(c + dx)) \sec^2(c + dx) dx$

Optimal result	4657
Rubi [A] (verified)	4657
Mathematica [A] (verified)	4658
Maple [A] (verified)	4658
Fricas [B] (verification not implemented)	4659
Sympy [F]	4659
Maxima [A] (verification not implemented)	4660
Giac [B] (verification not implemented)	4660
Mupad [B] (verification not implemented)	4660

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int (a + b \cos(c + dx)) \sec^2(c + dx) dx = \frac{\operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a \tan(c + dx)}{d}$$

[Out] `b*arctanh(sin(d*x+c))/d+a*tan(d*x+c)/d`

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 3852, 8, 3855}

$$\int (a + b \cos(c + dx)) \sec^2(c + dx) dx = \frac{a \tan(c + dx)}{d} + \frac{\operatorname{arctanh}(\sin(c + dx))}{d}$$

[In] `Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^2,x]`

[Out] `(b*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d`

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2827

`Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \sec^2(c + dx) dx + b \int \sec(c + dx) dx \\ &= \frac{\text{barctanh}(\sin(c + dx))}{d} - \frac{a \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\ &= \frac{\text{barctanh}(\sin(c + dx))}{d} + \frac{a \tan(c + dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (a + b \cos(c + dx)) \sec^2(c + dx) dx = \frac{\text{barctanh}(\sin(c + dx))}{d} + \frac{a \tan(c + dx)}{d}$$

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{\tan(dx+c)a+b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	30
default	$\frac{\tan(dx+c)a+b\ln(\sec(dx+c)+\tan(dx+c))}{d}$	30
parts	$\frac{a \tan(dx+c)}{d} + \frac{b \ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
risch	$\frac{2ia}{d(e^{2i(dx+c)}+1)} + \frac{\ln(e^{i(dx+c)}+i)b}{d} - \frac{\ln(e^{i(dx+c)}-i)b}{d}$	59
parallelrisc	$\frac{-b \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) \cos(dx+c)+b \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) \cos(dx+c)+a \sin(dx+c)}{d \cos(dx+c)}$	63
norman	$\frac{-\frac{2a \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} - \frac{2a \left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right) \left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{b \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d} - \frac{b \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	101

[In] `int((a+cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOSE)`

[Out] `1/d*(tan(d*x+c)*a+b*ln(sec(d*x+c)+tan(d*x+c)))`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int (a + b \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{b \cos(dx + c) \log(\sin(dx + c) + 1) - b \cos(dx + c) \log(-\sin(dx + c) + 1) + 2a \sin(dx + c)}{2d \cos(dx + c)}$$

[In] `integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")`

[Out] `1/2*(b*cos(d*x + c)*log(sin(d*x + c) + 1) - b*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))`

Sympy [F]

$$\int (a + b \cos(c + dx)) \sec^2(c + dx) dx = \int (a + b \cos(c + dx)) \sec^2(c + dx) dx$$

[In] `integrate((a+b*cos(d*x+c))*sec(d*x+c)**2,x)`

[Out] `Integral((a + b*cos(c + d*x))*sec(c + d*x)**2, x)`

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int (a + b \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{b(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 2a \tan(dx + c)}{2d}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")

[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*a*tan(d*x + c))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int (a + b \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{b \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - b \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - \frac{2a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1}}{d}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")

[Out] (b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d

Mupad [B] (verification not implemented)

Time = 13.81 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int (a + b \cos(c + dx)) \sec^2(c + dx) dx = \frac{2b \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{2a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

[In] int((a + b*cos(c + d*x))/cos(c + d*x)^2,x)

[Out] (2*b*atanh(tan(c/2 + (d*x)/2)))/d - (2*a*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))

3.413 $\int (a + b \cos(c + dx)) \sec^3(c + dx) dx$

Optimal result	4661
Rubi [A] (verified)	4661
Mathematica [A] (verified)	4662
Maple [A] (verified)	4663
Fricas [A] (verification not implemented)	4663
Sympy [F]	4664
Maxima [A] (verification not implemented)	4664
Giac [B] (verification not implemented)	4664
Mupad [B] (verification not implemented)	4665

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int (a + b \cos(c + dx)) \sec^3(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{b \tan(c + dx)}{d} + \frac{a \sec(c + dx) \tan(c + dx)}{2d}$$

[Out] $1/2*a*\operatorname{arctanh}(\sin(d*x+c))/d+b*\tan(d*x+c)/d+1/2*a*\sec(d*x+c)*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.05 (sec), antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2827, 3853, 3855, 3852, 8}

$$\int (a + b \cos(c + dx)) \sec^3(c + dx) dx = \frac{a \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a \tan(c + dx) \sec(c + dx)}{2d} + \frac{b \tan(c + dx)}{d}$$

[In] $\operatorname{Int}[(a + b*\operatorname{Cos}[c + d*x])* \operatorname{Sec}[c + d*x]^3, x]$

[Out] $(a*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (b*\operatorname{Tan}[c + d*x])/d + (a*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \sec^3(c + dx) dx + b \int \sec^2(c + dx) dx \\ &= \frac{a \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2}a \int \sec(c + dx) dx - \frac{b \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\ &= \frac{a \arctanh(\sin(c + dx))}{2d} + \frac{b \tan(c + dx)}{d} + \frac{a \sec(c + dx) \tan(c + dx)}{2d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int (a + b \cos(c + dx)) \sec^3(c + dx) dx = \frac{a \arctanh(\sin(c + dx))}{2d} + \frac{b \tan(c + dx)}{d} + \frac{a \sec(c + dx) \tan(c + dx)}{2d}$$

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right) + \tan(dx+c)b}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right) + \tan(dx+c)b}{d}$
parts	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d} + \frac{b\tan(dx+c)}{d}$
parallelrisch	$\frac{-a(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+a(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2a\sin(dx+c)+2\sin(2dx+2c)b}{2d(1+\cos(2dx+2c))}$
risch	$-\frac{i(ae^{3i(dx+c)}-2be^{2i(dx+c)}-ae^{i(dx+c)}-2b)}{d(e^{2i(dx+c)}+1)^2} + \frac{a\ln(e^{i(dx+c)}+i)}{2d} - \frac{a\ln(e^{i(dx+c)}-i)}{2d}$
norman	$\frac{\frac{(a-2b)\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{2a\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} - \frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d} + \frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$

```
[In] int((a+cos(d*x+c)*b)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+tan(d*x+c)*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int (a + b \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \frac{a \cos(dx + c)^2 \log(\sin(dx + c) + 1) - a \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2(2b \cos(dx + c) + a) \sin(dx + c)}{4d \cos(dx + c)^2}$$

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*b*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int (a + b \cos(c + dx)) \sec^3(c + dx) dx = \int (a + b \cos(c + dx)) \sec^3(c + dx) dx$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**3,x)

[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int (a + b \cos(c + dx)) \sec^3(c + dx) dx$$

$$= -\frac{a \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1) \right) - 4b \tan(dx+c)}{4d}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")

[Out] -1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 4*b*tan(d*x + c))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\int (a + b \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \frac{a \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - a \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) + \frac{2 \left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 - 2b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)^2}}{2d}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")

[Out] 1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d

Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.72

$$\int (a + b \cos(c + dx)) \sec^3(c + dx) dx = \frac{a \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} + \frac{(a - 2b) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (a + 2b) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)}$$

`[In] int((a + b*cos(c + d*x))/cos(c + d*x)^3,x)`

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^3*(a - 2*b) + tan(c/2
+ (d*x)/2)*(a + 2*b))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 +
1))
```

3.414 $\int (a + b \cos(c + dx)) \sec^4(c + dx) dx$

Optimal result	4666
Rubi [A] (verified)	4666
Mathematica [A] (verified)	4667
Maple [A] (verified)	4668
Fricas [A] (verification not implemented)	4668
Sympy [F]	4669
Maxima [A] (verification not implemented)	4669
Giac [B] (verification not implemented)	4669
Mupad [B] (verification not implemented)	4670

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (a + b \cos(c + dx)) \sec^4(c + dx) dx = \frac{\operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a \tan(c + dx)}{d} + \frac{b \sec(c + dx) \tan(c + dx)}{2d} + \frac{a \tan^3(c + dx)}{3d}$$

[Out] $1/2*b*\operatorname{arctanh}(\sin(d*x+c))/d+a*\tan(d*x+c)/d+1/2*b*\sec(d*x+c)*\tan(d*x+c)/d+1/3*a*\tan(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 3852, 3853, 3855}

$$\int (a + b \cos(c + dx)) \sec^4(c + dx) dx = \frac{a \tan^3(c + dx)}{3d} + \frac{a \tan(c + dx)}{d} + \frac{\operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{b \tan(c + dx) \sec(c + dx)}{2d}$$

[In] $\operatorname{Int}[(a + b*\operatorname{Cos}[c + d*x])*Sec[c + d*x]^4, x]$

[Out] $(b*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(2*d) + (a*\operatorname{Tan}[c + d*x])/d + (b*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(2*d) + (a*\operatorname{Tan}[c + d*x]^3)/(3*d)$

Rule 2827

$\operatorname{Int}[(b*\sin[e_.] + (f_.)*(x_.)]^m*((c_.) + (d_.)*\sin[e_.] + (f_.)*(x_.)], x_Symbol] \rightarrow \operatorname{Dist}[c, \operatorname{Int}[(b*\sin[e + f*x])^m, x], x] + \operatorname{Dist}[d/b, \operatorname{Int}[($

$b \sin[e + f x]^{m+1}, x, x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 3852

$\text{Int}[\text{csc}[(c_.) + (d_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rule 3853

$\text{Int}[(\text{csc}[(c_.) + (d_.)(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d x]*(b*\text{Csc}[c + d x])^{(n - 1)}/(d*(n - 1)), x] + \text{Dist}[b^2*((n - 2)/(n - 1)), \text{Int}[(b*\text{Csc}[c + d x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \& \& \text{IntegerQ}[2*n]$

Rule 3855

$\text{Int}[\text{csc}[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{ArcTanh}[\text{Cos}[c + d x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \sec^4(c + dx) dx + b \int \sec^3(c + dx) dx \\ &= \frac{b \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2} b \int \sec(c + dx) dx - \frac{a \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d} \\ &= \frac{\text{barctanh}(\sin(c + dx))}{2d} + \frac{a \tan(c + dx)}{d} + \frac{b \sec(c + dx) \tan(c + dx)}{2d} + \frac{a \tan^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int (a + b \cos(c + dx)) \sec^4(c + dx) dx = \frac{\text{barctanh}(\sin(c + dx))}{2d} + \frac{b \sec(c + dx) \tan(c + dx)}{2d} + \frac{a(\tan(c + dx) + \frac{1}{3} \tan^3(c + dx))}{d}$$

[In] Integrate[(a + b Cos[c + d x])*Sec[c + d x]^4, x]

[Out] (b*ArcTanh[Sin[c + d x]])/(2*d) + (b*Sec[c + d x]*Tan[c + d x])/(2*d) + (a*(Tan[c + d x] + Tan[c + d x]^3/3))/d

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-a \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + b \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right)}{d}$
default	$\frac{-a \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + b \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right)}{d}$
parts	$-\frac{a \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d} + \frac{b \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right)}{d}$
risch	$-\frac{i(3b e^{5i(dx+c)} - 12a e^{2i(dx+c)} - 3b e^{i(dx+c)} - 4a)}{3d(e^{2i(dx+c)} + 1)^3} + \frac{\ln(e^{i(dx+c)} + i)b}{2d} - \frac{\ln(e^{i(dx+c)} - i)b}{2d}$
parallelrisc	$\frac{-9b \left(\frac{\cos(3dx+3c)}{3} + \cos(dx+c) \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 9b \left(\frac{\cos(3dx+3c)}{3} + \cos(dx+c) \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) + 6 \sin(2dx+2c)b}{6d(\cos(3dx+3c) + 3 \cos(dx+c))}$
norman	$\frac{\frac{(2a-3b) \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} - \frac{(2a-b) \left(\tan^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{(2a+b) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d} - \frac{(2a+3b) \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d}}{\left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)^3} - \frac{b \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)}{2d}$

[In] int((a+cos(d*x+c)*b)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int (a + b \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{3b \cos(dx+c)^3 \log(\sin(dx+c)+1) - 3b \cos(dx+c)^3 \log(-\sin(dx+c)+1) + 2(4a \cos(dx+c)^2 + 3b \cos(dx+c)) \sin(dx+c)}{12d \cos(dx+c)^3}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")

[Out] 1/12*(3*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*b*cos(d*x + c) + 2*a)*sin(d*x + c))/(d*cos(d*x + c)^3)

Sympy [F]

$$\int (a + b \cos(c + dx)) \sec^4(c + dx) dx = \int (a + b \cos(c + dx)) \sec^4(c + dx) dx$$

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int (a + b \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{4 (\tan(dx + c)^3 + 3 \tan(dx + c))a - 3b \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right)}{12d}$$

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(57) = 114.

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.94

$$\int (a + b \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{3b \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 3b \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(6a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 3b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 4a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 6a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 3b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{\left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 - 1 \right)^3}}{6d}$$

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 - 4*a*tan(1/2*d*x + 1/2*c)^3 + 6*a*tan(1/2*d*x + 1/2*c) + 3*b*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.76

$$\int (a + b \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{b \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{(2a - b) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 - \frac{4a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{3} + (2a + b) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1 \right)}$$

[In] int((a + b*cos(c + d*x))/cos(c + d*x)^4,x)

```
[Out] (b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^5*(2*a - b) + tan(c/2
+ (d*x)/2)*(2*a + b) - (4*a*tan(c/2 + (d*x)/2)^3)/3)/(d*(3*tan(c/2 + (d*x)
/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.415 $\int (a + b \cos(c + dx)) \sec^5(c + dx) dx$

Optimal result	4671
Rubi [A] (verified)	4671
Mathematica [A] (verified)	4673
Maple [A] (verified)	4673
Fricas [A] (verification not implemented)	4674
Sympy [F]	4674
Maxima [A] (verification not implemented)	4674
Giac [B] (verification not implemented)	4675
Mupad [B] (verification not implemented)	4675

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int (a + b \cos(c + dx)) \sec^5(c + dx) dx = \frac{3a \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{b \tan(c + dx)}{d} + \frac{3a \sec(c + dx) \tan(c + dx)}{8d} + \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{b \tan^3(c + dx)}{3d}$$

[Out] $3/8*a*\operatorname{arctanh}(\sin(d*x+c))/d+b*\tan(d*x+c)/d+3/8*a*\sec(d*x+c)*\tan(d*x+c)/d+1/4*a*\sec(d*x+c)^3*\tan(d*x+c)/d+1/3*b*\tan(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 3853, 3855, 3852}

$$\int (a + b \cos(c + dx)) \sec^5(c + dx) dx = \frac{3a \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a \tan(c + dx) \sec^3(c + dx)}{4d} + \frac{3a \tan(c + dx) \sec(c + dx)}{8d} + \frac{b \tan^3(c + dx)}{3d} + \frac{b \tan(c + dx)}{d}$$

[In] $\operatorname{Int}[(a + b*\operatorname{Cos}[c + d*x])* \operatorname{Sec}[c + d*x]^5, x]$

[Out] $(3*a*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/(8*d) + (b*\operatorname{Tan}[c + d*x])/d + (3*a*\operatorname{Sec}[c + d*x]*\operatorname{Tan}[c + d*x])/(8*d) + (a*\operatorname{Sec}[c + d*x]^3*\operatorname{Tan}[c + d*x])/(4*d) + (b*\operatorname{Tan}[c + d*x]^3)/(3*d)$

Rule 2827

$\text{Int}[(b \cdot \sin[e + f \cdot x])^m \cdot (c + d \cdot \sin[e + f \cdot x])], x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b \cdot \sin[e + f \cdot x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b \cdot \sin[e + f \cdot x])^{m+1}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 3852

$\text{Int}[\csc[c + d \cdot x]^n, x_Symbol] \rightarrow \text{Dist}[-d^{-1}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}, x], x], x, \text{Cot}[c + d \cdot x]], x] /; \text{FreeQ}\{c, d\}, x \&\& \text{IGtQ}[n/2, 0]$

Rule 3853

$\text{Int}[(\csc[c + d \cdot x] \cdot (b \cdot \csc[c + d \cdot x])^n), x_Symbol] \rightarrow \text{Simp}[(-b) \cdot \cos[c + d \cdot x] \cdot ((b \cdot \csc[c + d \cdot x])^{n-1} / (d \cdot (n-1))), x] + \text{Dist}[b^2 \cdot ((n-2)/(n-1)), \text{Int}[(b \cdot \csc[c + d \cdot x])^{n-2}, x], x] /; \text{FreeQ}\{b, c, d\}, x \&\& \text{GtQ}[n, 1] \& \& \text{IntegerQ}[2 \cdot n]$

Rule 3855

$\text{Int}[\csc[c + d \cdot x], x_Symbol] \rightarrow \text{Simp}[-\text{ArcTanh}[\cos[c + d \cdot x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \sec^5(c + dx) dx + b \int \sec^4(c + dx) dx \\
 &= \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{4}(3a) \int \sec^3(c + dx) dx \\
 &\quad - \frac{b \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d} \\
 &= \frac{b \tan(c + dx)}{d} + \frac{3a \sec(c + dx) \tan(c + dx)}{8d} + \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} \\
 &\quad + \frac{b \tan^3(c + dx)}{3d} + \frac{1}{8}(3a) \int \sec(c + dx) dx \\
 &= \frac{3a \arctanh(\sin(c + dx))}{8d} + \frac{b \tan(c + dx)}{d} + \frac{3a \sec(c + dx) \tan(c + dx)}{8d} \\
 &\quad + \frac{a \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{b \tan^3(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int (a + b \cos(c + dx)) \sec^5(c + dx) dx$$

$$= \frac{9a \operatorname{arctanh}(\sin(c + dx)) + \tan(c + dx) (9a \sec(c + dx) + 6a \sec^3(c + dx) + 8b(3 + \tan^2(c + dx)))}{24d}$$

[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^5,x]

[Out] (9*a*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(9*a*Sec[c + d*x] + 6*a*Sec[c + d*x]^3 + 8*b*(3 + Tan[c + d*x]^2)))/(24*d)

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right) - b \left(- \frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d}$
default	$\frac{a \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right) - b \left(- \frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d}$
parts	$\frac{a \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right)}{d} - \frac{b \left(- \frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d}$
risch	$\frac{i(9a e^{7i(dx+c)} + 33a e^{5i(dx+c)} - 48b e^{4i(dx+c)} - 33a e^{3i(dx+c)} - 64b e^{2i(dx+c)} - 9a e^{i(dx+c)} - 16b)}{12d(e^{2i(dx+c)} + 1)^4} - \frac{3a \ln(e^{i(dx+c)} - i)}{8d}$
parallelrisc	$\frac{-18 \left(\frac{3}{4} + \frac{\cos(4dx+4c)}{4} + \cos(2dx+2c) \right) a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 18 \left(\frac{3}{4} + \frac{\cos(4dx+4c)}{4} + \cos(2dx+2c) \right) a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)}{12d(\cos(4dx+4c) + 4 \cos(2dx+2c) + 3)}$
norman	$\frac{\frac{3a \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{2d} + \frac{2(3a-2b) \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} + \frac{2(3a+2b) \left(\tan^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} + \frac{(5a-8b) \left(\tan^9 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{4d} + \frac{(8b+5a) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d}}{\left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)^4}$

[In] int((a+cos(d*x+c)*b)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-b*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int (a + b \cos(c + dx)) \sec^5(c + dx) dx$$

$$= \frac{9 a \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 9 a \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2(16 b \cos(dx + c)^3 + 9 a \cos(dx + c)^2 + 8 b \cos(dx + c) + 6 a) \sin(dx + c)}{48 d \cos(dx + c)^4}$$

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*b*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*b*cos(d*x + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int (a + b \cos(c + dx)) \sec^5(c + dx) dx = \int (a + b \cos(c + dx)) \sec^5(c + dx) dx$$

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int (a + b \cos(c + dx)) \sec^5(c + dx) dx$$

$$= \frac{16(\tan(dx + c)^3 + 3 \tan(dx + c))b - 3a \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right)}{48 d}$$

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*b - 3*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(77) = 154.

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.93

$$\int (a + b \cos(c + dx)) \sec^5(c + dx) dx$$

$$= \frac{9a \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 9a \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2\left(15a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 24b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 9a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 40b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 9a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 40b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 15a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 24b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1^4}{d}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")

[Out] 1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2*d*x + 1/2*c)^7 + 9*a*tan(1/2*d*x + 1/2*c)^5 + 40*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*tan(1/2*d*x + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*d*x + 1/2*c) + 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4/d

Mupad [B] (verification not implemented)

Time = 16.75 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

$$\int (a + b \cos(c + dx)) \sec^5(c + dx) dx$$

$$= \frac{\left(\frac{5a}{4} - 2b\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{3a}{4} + \frac{10b}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{3a}{4} - \frac{10b}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{5a}{4} + 2b\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + \frac{3a \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{4d}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)}$$

[In] int((a + b*cos(c + d*x))/cos(c + d*x)^5,x)

[Out] (tan(c/2 + (d*x)/2)*((5*a)/4 + 2*b) + tan(c/2 + (d*x)/2)^7*((5*a)/4 - 2*b) + tan(c/2 + (d*x)/2)^3*((3*a)/4 - (10*b)/3) + tan(c/2 + (d*x)/2)^5*((3*a)/4 + (10*b)/3))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d)

3.416 $\int (a + b \cos(c + dx)) \sec^6(c + dx) dx$

Optimal result	4676
Rubi [A] (verified)	4676
Mathematica [A] (verified)	4678
Maple [A] (verified)	4678
Fricas [A] (verification not implemented)	4679
Sympy [F]	4679
Maxima [A] (verification not implemented)	4679
Giac [A] (verification not implemented)	4680
Mupad [B] (verification not implemented)	4680

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int (a + b \cos(c + dx)) \sec^6(c + dx) dx = \frac{3b \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a \tan(c + dx)}{d} + \frac{3b \sec(c + dx) \tan(c + dx)}{8d} + \frac{b \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{2a \tan^3(c + dx)}{3d} + \frac{a \tan^5(c + dx)}{5d}$$

[Out] $\frac{3}{8}b \operatorname{arctanh}(\sin(dx+c))/d + a \tan(dx+c)/d + \frac{3}{8}b \sec(dx+c) \tan(dx+c)/d + \frac{1}{4}b \sec(dx+c)^3 \tan(dx+c)/d + \frac{2}{3}a \tan(dx+c)^3/d + \frac{1}{5}a \tan(dx+c)^5/d$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2827, 3852, 3853, 3855}

$$\int (a + b \cos(c + dx)) \sec^6(c + dx) dx = \frac{a \tan^5(c + dx)}{5d} + \frac{2a \tan^3(c + dx)}{3d} + \frac{a \tan(c + dx)}{d} + \frac{3b \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{b \tan(c + dx) \sec^3(c + dx)}{4d} + \frac{3b \tan(c + dx) \sec(c + dx)}{8d}$$

[In] `Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^6,x]`

[Out] $(3*b*ArcTanh[\sin[c + d*x]])/(8*d) + (a*\tan[c + d*x])/d + (3*b*\sec[c + d*x]*\tan[c + d*x])/(8*d) + (b*\sec[c + d*x]^3*\tan[c + d*x])/(4*d) + (2*a*\tan[c + d*x]^3)/(3*d) + (a*\tan[c + d*x]^5)/(5*d)$

Rule 2827

$\text{Int}[(b_* \sin[e_*] + (f_*)*(x_*))^{(m_*)}*((c_*) + (d_*)*\sin[e_*] + (f_*)*(x_*))], x_Symbol] := \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m + 1)}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 3852

$\text{Int}[\csc[(c_*) + (d_*)*(x_)]^{(n_*)}, x_Symbol] := \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

$\text{Int}[(\csc[(c_*) + (d_*)*(x_)]*(b_*))^{(n_*)}, x_Symbol] := \text{Simp}[(-b)*\cos[c + d*x]*((b*\csc[c + d*x])^{(n - 1)}/(d*(n - 1))), x] + \text{Dist}[b^2*((n - 2)/(n - 1)), \text{Int}[(b*\csc[c + d*x])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] & IntegerQ[2*n]

Rule 3855

$\text{Int}[\csc[(c_*) + (d_*)*(x_)], x_Symbol] := \text{Simp}[-\text{ArcTanh}[\cos[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \sec^6(c + dx) dx + b \int \sec^5(c + dx) dx \\
 &= \frac{b \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{4}(3b) \int \sec^3(c + dx) dx \\
 &\quad - \frac{a \text{Subst}(\int (1 + 2x^2 + x^4) dx, x, -\tan(c + dx))}{d} \\
 &= \frac{a \tan(c + dx)}{d} + \frac{3b \sec(c + dx) \tan(c + dx)}{8d} + \frac{b \sec^3(c + dx) \tan(c + dx)}{4d} \\
 &\quad + \frac{2a \tan^3(c + dx)}{3d} + \frac{a \tan^5(c + dx)}{5d} + \frac{1}{8}(3b) \int \sec(c + dx) dx \\
 &= \frac{3b \text{arctanh}(\sin(c + dx))}{8d} + \frac{a \tan(c + dx)}{d} + \frac{3b \sec(c + dx) \tan(c + dx)}{8d} \\
 &\quad + \frac{b \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{2a \tan^3(c + dx)}{3d} + \frac{a \tan^5(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int (a + b \cos(c + dx)) \sec^6(c + dx) dx = \frac{3b \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{3b \sec(c + dx) \tan(c + dx)}{8d} + \frac{b \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{a(\tan(c + dx) + \frac{2}{3} \tan^3(c + dx) + \frac{1}{5} \tan^5(c + dx))}{d}$$

`[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^6,x]`

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (3*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) +
(b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

method	result
derivativedivides	$-a \left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c) + b \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right)$
default	$-a \left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c) + b \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right)$
parts	$-\frac{a \left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c)}{d} + \frac{b \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c) + \tan(dx+c))}{8} \right)}{d}$
risch	$-\frac{i(45b e^{9i(dx+c)} + 210b e^{7i(dx+c)} - 640a e^{4i(dx+c)} - 210b e^{3i(dx+c)} - 320a e^{2i(dx+c)} - 45b e^{i(dx+c)} - 64a)}{60d(e^{2i(dx+c)} + 1)^5} - \frac{3 \ln(e^{i(dx+c)} + \tan(dx+c))}{8d}$
parallelrisc	$-225b \left(\frac{\cos(5dx+5c)}{10} + \frac{\cos(3dx+3c)}{2} + \cos(dx+c) \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 225b \left(\frac{\cos(5dx+5c)}{10} + \frac{\cos(3dx+3c)}{2} + \cos(dx+c) \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)$
norman	$\frac{(8a-9b) \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{12d} - \frac{(8a-5b) \left(\tan^{11} \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{4d} - \frac{(8a+5b) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d} + \frac{(8a+9b) \left(\tan^9 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{12d} - \frac{(152a-15b) \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{30d} - \frac{1}{(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right)) \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)^5}$

`[In] int((a+cos(d*x+c)*b)*sec(d*x+c)^6,x,method=_RETURNVERBOSE)`

```
[Out] 1/d*(-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+b*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int (a + b \cos(c + dx)) \sec^6(c + dx) dx$$

$$= \frac{45 b \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 45 b \cos(dx + c)^5 \log(-\sin(dx + c) + 1) + 2(64 a \cos(dx + c)^4 + 45 b \cos(dx + c)^3 + 32 a \cos(dx + c)^2 + 30 b \cos(dx + c) + 24 a) \sin(dx + c)}{240 d \cos(dx + c)^5}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fricas")

```
[Out] 1/240*(45*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*b*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(64*a*cos(d*x + c)^4 + 45*b*cos(d*x + c)^3 + 32*a*cos(d*x + c)^2 + 30*b*cos(d*x + c) + 24*a)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int (a + b \cos(c + dx)) \sec^6(c + dx) dx = \int (a + b \cos(c + dx)) \sec^6(c + dx) dx$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**6,x)

[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**6, x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int (a + b \cos(c + dx)) \sec^6(c + dx) dx$$

$$= \frac{16(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c))a - 15b \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log(\sin(dx+c) + 1) + 3 \log(\sin(dx+c) - 1) \right)}{240 d}$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="maxima")

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 15*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.76

$$\int (a + b \cos(c + dx)) \sec^6(c + dx) dx$$

$$45 b \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 45 b \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(120 a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^9 - 75 b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^9 - 16 \right)}{\dots}$$

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/120*(45*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*a*tan(1/2*d*x + 1/2*c)^9 - 75*b*tan(1/2*d*x + 1/2*c)^9 - 160*a*tan(1/2*d*x + 1/2*c)^7 + 30*b*tan(1/2*d*x + 1/2*c)^7 + 464*a*tan(1/2*d*x + 1/2*c)^5 - 160*a*tan(1/2*d*x + 1/2*c)^3 - 30*b*tan(1/2*d*x + 1/2*c)^3 + 120*a*tan(1/2*d*x + 1/2*c) + 75*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 17.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.78

$$\int (a + b \cos(c + dx)) \sec^6(c + dx) dx = \frac{3 b \operatorname{atanh} \left(\tan \left(\frac{c}{2} + \frac{dx}{2} \right) \right)}{4 d} - \frac{\left(2 a - \frac{5 b}{4} \right) \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^9 + \left(\frac{b}{2} - \frac{8 a}{3} \right) \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^7 + \frac{116 a \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^5}{15} + \left(-\frac{8 a}{3} - \frac{b}{2} \right) \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^3 + \left(2 a - \frac{5 b}{4} \right) \tan \left(\frac{c}{2} + \frac{dx}{2} \right)}{d \left(\tan \left(\frac{c}{2} + \frac{dx}{2} \right)^{10} - 5 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^8 + 10 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^6 - 10 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^4 + 5 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)^2 - 1 \right)}$$

```
[In] int((a + b*cos(c + d*x))/cos(c + d*x)^6,x)
```

```
[Out] (3*b*atanh(tan(c/2 + (d*x)/2)))/(4*d) - (tan(c/2 + (d*x)/2)*(2*a + (5*b)/4) - tan(c/2 + (d*x)/2)^3*((8*a)/3 + b/2) + tan(c/2 + (d*x)/2)^9*(2*a - (5*b)/4) - tan(c/2 + (d*x)/2)^7*((8*a)/3 - b/2) + (116*a*tan(c/2 + (d*x)/2)^5)/15)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.417 $\int \cos^4(c + dx)(a + b \cos(c + dx))^2 dx$

Optimal result	4681
Rubi [A] (verified)	4681
Mathematica [A] (verified)	4683
Maple [A] (verified)	4684
Fricas [A] (verification not implemented)	4684
Sympy [B] (verification not implemented)	4685
Maxima [A] (verification not implemented)	4685
Giac [A] (verification not implemented)	4686
Mupad [B] (verification not implemented)	4686

Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \cos^4(c + dx)(a + b \cos(c + dx))^2 dx = \frac{1}{16}(6a^2 + 5b^2)x + \frac{2ab \sin(c + dx)}{d} + \frac{(6a^2 + 5b^2) \cos(c + dx) \sin(c + dx)}{16d} + \frac{(6a^2 + 5b^2) \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{b^2 \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{4ab \sin^3(c + dx)}{3d} + \frac{2ab \sin^5(c + dx)}{5d}$$

[Out] 1/16*(6*a^2+5*b^2)*x+2*a*b*sin(d*x+c)/d+1/16*(6*a^2+5*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*(6*a^2+5*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*b^2*cos(d*x+c)^5*sin(d*x+c)/d-4/3*a*b*sin(d*x+c)^3/d+2/5*a*b*sin(d*x+c)^5/d

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

= {2868, 2713, 3093, 2715, 8}

$$\int \cos^4(c+dx)(a+b\cos(c+dx))^2 dx = \frac{(6a^2+5b^2)\sin(c+dx)\cos^3(c+dx)}{24d} + \frac{(6a^2+5b^2)\sin(c+dx)\cos(c+dx)}{16d} + \frac{1}{16}x(6a^2+5b^2) + \frac{2ab\sin^5(c+dx)}{5d} - \frac{4ab\sin^3(c+dx)}{3d} + \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin(c+dx)\cos^5(c+dx)}{6d}$$

[In] Int[Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2,x]

[Out] ((6*a^2 + 5*b^2)*x)/16 + (2*a*b*Sin[c + d*x])/d + ((6*a^2 + 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((6*a^2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (4*a*b*Sin[c + d*x]^3)/(3*d) + (2*a*b*Sin[c + d*x]^5)/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2868

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3093

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f

$(m + 2))$, $x]$ + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (2ab) \int \cos^5(c + dx) dx + \int \cos^4(c + dx) (a^2 + b^2 \cos^2(c + dx)) dx \\
 &= \frac{b^2 \cos^5(c + dx) \sin(c + dx)}{6d} + \frac{1}{6} (6a^2 + 5b^2) \int \cos^4(c + dx) dx \\
 &\quad - \frac{(2ab) \text{Subst}(\int (1 - 2x^2 + x^4) dx, x, -\sin(c + dx))}{d} \\
 &= \frac{2ab \sin(c + dx)}{d} + \frac{(6a^2 + 5b^2) \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{b^2 \cos^5(c + dx) \sin(c + dx)}{6d} \\
 &\quad - \frac{4ab \sin^3(c + dx)}{3d} + \frac{2ab \sin^5(c + dx)}{5d} + \frac{1}{8} (6a^2 + 5b^2) \int \cos^2(c + dx) dx \\
 &= \frac{2ab \sin(c + dx)}{d} + \frac{(6a^2 + 5b^2) \cos(c + dx) \sin(c + dx)}{16d} + \frac{(6a^2 + 5b^2) \cos^3(c + dx) \sin(c + dx)}{24d} \\
 &\quad + \frac{b^2 \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{4ab \sin^3(c + dx)}{3d} + \frac{2ab \sin^5(c + dx)}{5d} + \frac{1}{16} (6a^2 + 5b^2) \int 1 dx \\
 &= \frac{1}{16} (6a^2 + 5b^2) x + \frac{2ab \sin(c + dx)}{d} + \frac{(6a^2 + 5b^2) \cos(c + dx) \sin(c + dx)}{16d} \\
 &\quad + \frac{(6a^2 + 5b^2) \cos^3(c + dx) \sin(c + dx)}{24d} \\
 &\quad + \frac{b^2 \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{4ab \sin^3(c + dx)}{3d} + \frac{2ab \sin^5(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.82

$$\begin{aligned}
 &\int \cos^4(c + dx)(a + b \cos(c + dx))^2 dx \\
 &= \frac{1920ab \sin(c + dx) - 1280ab \sin^3(c + dx) + 384ab \sin^5(c + dx) + 5(72a^2c + 60b^2c + 72a^2dx + 60b^2dx + (48a^2 + 45b^2) \sin[2(c + dx)] + (6a^2 + 9b^2) \sin[4(c + dx)] + b^2 \sin[6(c + dx)])}{960d}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2,x]

[Out] (1920*a*b*Sin[c + d*x] - 1280*a*b*Sin[c + d*x]^3 + 384*a*b*Sin[c + d*x]^5 + 5*(72*a^2*c + 60*b^2*c + 72*a^2*d*x + 60*b^2*d*x + (48*a^2 + 45*b^2)*Sin[2*(c + d*x)] + (6*a^2 + 9*b^2)*Sin[4*(c + d*x)] + b^2*Sin[6*(c + d*x)]))/(960*d)

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result
parallelrisc	$\frac{(240a^2+225b^2) \sin(2dx+2c)+(30a^2+45b^2) \sin(4dx+4c)+200ab \sin(3dx+3c)+24ab \sin(5dx+5c)+5b^2 \sin(6dx+6c)+1200ab \sin(dx+c)}{960d}$
derivativedivides	$a^2 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx + \frac{3c}{8}}{8} \right) + \frac{2ab \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + b^2 \left(\frac{\cos^5(dx+c) + \frac{5 \cos^3(dx+c)}{4} + \frac{15 \cos(dx+c)}{8}}{6} \right) \sin(dx+c)$
default	$a^2 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx + \frac{3c}{8}}{8} \right) + \frac{2ab \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5} + b^2 \left(\frac{\cos^5(dx+c) + \frac{5 \cos^3(dx+c)}{4} + \frac{15 \cos(dx+c)}{8}}{6} \right) \sin(dx+c)$
parts	$a^2 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx + \frac{3c}{8}}{8} \right) + \frac{b^2 \left(\frac{\cos^5(dx+c) + \frac{5 \cos^3(dx+c)}{4} + \frac{15 \cos(dx+c)}{8}}{6} \right) \sin(dx+c)}{6} + \frac{5dx}{16}$
risc	$\frac{3a^2x}{8} + \frac{5b^2x}{16} + \frac{5ab \sin(dx+c)}{4d} + \frac{b^2 \sin(6dx+6c)}{192d} + \frac{ab \sin(5dx+5c)}{40d} + \frac{\sin(4dx+4c)a^2}{32d} + \frac{3 \sin(4dx+4c)b^2}{64d} + \frac{5ab \sin(dx+c)}{16d}$
norman	$\frac{\left(\frac{3a^2}{8} + \frac{5b^2}{16} \right) x + \left(\frac{3a^2}{8} + \frac{5b^2}{16} \right) x \left(\tan^{12} \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \left(\frac{9a^2}{4} + \frac{15b^2}{8} \right) x \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \left(\frac{9a^2}{4} + \frac{15b^2}{8} \right) x \left(\tan^{10} \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \left(\frac{15a^2}{8} + \frac{15b^2}{16} \right) x \left(\tan^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \frac{5ab \sin(dx+c)}{16d}$

```
[In] int(cos(d*x+c)^4*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/960*((240*a^2+225*b^2)*sin(2*d*x+2*c)+(30*a^2+45*b^2)*sin(4*d*x+4*c)+200*
a*b*sin(3*d*x+3*c)+24*a*b*sin(5*d*x+5*c)+5*b^2*sin(6*d*x+6*c)+1200*a*b*sin(
d*x+c)+360*d*x*(a^2+5/6*b^2))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int \cos^4(c+dx)(a+b \cos(c+dx))^2 dx$$

$$= \frac{15(6a^2+5b^2)dx + (40b^2 \cos(dx+c))^5 + 96ab \cos(dx+c)^4 + 128ab \cos(dx+c)^2 + 10(6a^2+5b^2) \cos(dx+c)}{240d}$$

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(6*a^2+5*b^2)*d*x+(40*b^2*cos(d*x+c))^5+96*a*b*cos(d*x+c)
)^4+128*a*b*cos(d*x+c)^2+10*(6*a^2+5*b^2)*cos(d*x+c)^3+256*a*b
+15*(6*a^2+5*b^2)*cos(d*x+c)*sin(d*x+c))/d
```


Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(141) = 282$.

Time = 0.37 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.29

$$\int \cos^4(c + dx)(a + b \cos(c + dx))^2 dx$$

$$= \left\{ \begin{array}{l} \frac{3a^2 x \sin^4(c+dx)}{8} + \frac{3a^2 x \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{3a^2 x \cos^4(c+dx)}{8} + \frac{3a^2 \sin^3(c+dx) \cos(c+dx)}{8d} + \frac{5a^2 \sin(c+dx) \cos^3(c+dx)}{8d} + 1 \\ x(a + b \cos(c))^2 \cos^4(c) \end{array} \right.$$

[In] integrate(cos(d*x+c)**4*(a+b*cos(d*x+c))**2,x)

[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 16*a*b*sin(c + d*x)**5/(15*d) + 8*a*b*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 2*a*b*sin(c + d*x)*cos(c + d*x)**4/d + 5*b**2*x*sin(c + d*x)**6/16 + 15*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*b**2*x*cos(c + d*x)**6/16 + 5*b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c)**4, True))

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.80

$$\int \cos^4(c + dx)(a + b \cos(c + dx))^2 dx$$

$$= \frac{30(12dx + 12c + \sin(4dx + 4c)) + 8\sin(2dx + 2c)a^2 + 128(3\sin(dx + c)^5 - 10\sin(dx + c)^3 + 15\sin(dx + c))a^2 + 128(3\sin(dx + c)^5 - 10\sin(dx + c)^3 + 15\sin(dx + c))ab - 5(4\sin(2dx + 2c)^3 - 60dx - 60c - 9\sin(4dx + 4c) - 48\sin(2dx + 2c))b^2}{960d}$$

[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] 1/960*(30*(12*d*x + 12*c + sin(4*d*x + 4*c)) + 8*sin(2*d*x + 2*c))*a^2 + 128*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a*b - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^2/d

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$\int \cos^4(c + dx)(a + b \cos(c + dx))^2 dx = \frac{1}{16} (6a^2 + 5b^2)x + \frac{b^2 \sin(6dx + 6c)}{192d} + \frac{ab \sin(5dx + 5c)}{40d} + \frac{5ab \sin(3dx + 3c)}{24d} + \frac{5ab \sin(dx + c)}{4d} + \frac{(2a^2 + 3b^2) \sin(4dx + 4c)}{64d} + \frac{(16a^2 + 15b^2) \sin(2dx + 2c)}{64d}$$

[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] 1/16*(6*a^2 + 5*b^2)*x + 1/192*b^2*sin(6*d*x + 6*c)/d + 1/40*a*b*sin(5*d*x + 5*c)/d + 5/24*a*b*sin(3*d*x + 3*c)/d + 5/4*a*b*sin(d*x + c)/d + 1/64*(2*a^2 + 3*b^2)*sin(4*d*x + 4*c)/d + 1/64*(16*a^2 + 15*b^2)*sin(2*d*x + 2*c)/d

Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95

$$\int \cos^4(c + dx)(a + b \cos(c + dx))^2 dx = \frac{3a^2x}{8} + \frac{5b^2x}{16} + \frac{a^2 \sin(2c + 2dx)}{4d} + \frac{a^2 \sin(4c + 4dx)}{32d} + \frac{15b^2 \sin(2c + 2dx)}{64d} + \frac{3b^2 \sin(4c + 4dx)}{64d} + \frac{b^2 \sin(6c + 6dx)}{192d} + \frac{5ab \sin(c + dx)}{4d} + \frac{5ab \sin(3c + 3dx)}{24d} + \frac{ab \sin(5c + 5dx)}{40d}$$

[In] int(cos(c + d*x)^4*(a + b*cos(c + d*x))^2,x)

[Out] (3*a^2*x)/8 + (5*b^2*x)/16 + (a^2*sin(2*c + 2*d*x))/(4*d) + (a^2*sin(4*c + 4*d*x))/(32*d) + (15*b^2*sin(2*c + 2*d*x))/(64*d) + (3*b^2*sin(4*c + 4*d*x))/(64*d) + (b^2*sin(6*c + 6*d*x))/(192*d) + (5*a*b*sin(c + d*x))/(4*d) + (5*a*b*sin(3*c + 3*d*x))/(24*d) + (a*b*sin(5*c + 5*d*x))/(40*d)

3.418 $\int \cos^3(c + dx)(a + b \cos(c + dx))^2 dx$

Optimal result	4687
Rubi [A] (verified)	4687
Mathematica [A] (verified)	4689
Maple [A] (verified)	4689
Fricas [A] (verification not implemented)	4690
Sympy [B] (verification not implemented)	4691
Maxima [A] (verification not implemented)	4691
Giac [A] (verification not implemented)	4692
Mupad [B] (verification not implemented)	4692

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^2 dx = \frac{3abx}{4} + \frac{(a^2 + b^2) \sin(c + dx)}{d} + \frac{3ab \cos(c + dx) \sin(c + dx)}{4d} + \frac{ab \cos^3(c + dx) \sin(c + dx)}{2d} - \frac{(a^2 + 2b^2) \sin^3(c + dx)}{3d} + \frac{b^2 \sin^5(c + dx)}{5d}$$

[Out] $\frac{3}{4}abx + \frac{(a^2 + b^2) \sin(dx + c)}{d} + \frac{3}{4}ab \cos(dx + c) \sin(dx + c) / d + \frac{1}{2}ab \cos(dx + c)^3 \sin(dx + c) / d - \frac{1}{3}(a^2 + 2b^2) \sin(dx + c)^3 / d + \frac{1}{5}b^2 \sin(dx + c)^5 / d$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2868, 2715, 8, 3092, 380}

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^2 dx = -\frac{(a^2 + 2b^2) \sin^3(c + dx)}{3d} + \frac{(a^2 + b^2) \sin(c + dx)}{d} + \frac{ab \sin(c + dx) \cos^3(c + dx)}{2d} + \frac{3ab \sin(c + dx) \cos(c + dx)}{4d} + \frac{3abx}{4} + \frac{b^2 \sin^5(c + dx)}{5d}$$

[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^2,x]

[Out] (3*a*b*x)/4 + ((a^2 + b^2)*Sin[c + d*x])/d + (3*a*b*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a*b*Cos[c + d*x]^3*Ssin[c + d*x])/(2*d) - ((a^2 + 2*b^2)*Sin[c + d*x]^3)/(3*d) + (b^2*Ssin[c + d*x]^5)/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 380

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n-1)/(d*n)), x] + Dist[b^2*((n-1)/n), Int[(b*Ssin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2868

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Ssin[e + f*x])^(m+1), x], x] + Int[(b*Ssin[e + f*x])^m*(c^2 + d^2*Ssin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3092

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Dist[-f^(-1), Subst[Int[(1 - x^2)^((m-1)/2)*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m+1)/2, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= (2ab) \int \cos^4(c + dx) dx + \int \cos^3(c + dx) (a^2 + b^2 \cos^2(c + dx)) dx \\ &= \frac{ab \cos^3(c + dx) \sin(c + dx)}{2d} + \frac{1}{2}(3ab) \int \cos^2(c + dx) dx \\ &\quad - \frac{\text{Subst}(\int (1 - x^2) (a^2 + b^2 - b^2 x^2) dx, x, -\sin(c + dx))}{d} \end{aligned}$$

$$\begin{aligned}
&= \frac{3ab \cos(c+dx) \sin(c+dx)}{4d} + \frac{ab \cos^3(c+dx) \sin(c+dx)}{2d} + \frac{1}{4}(3ab) \int 1 dx \\
&\quad \text{Subst}\left(\int \left(a^2 \left(1 + \frac{b^2}{a^2}\right) - (a^2 + 2b^2)x^2 + b^2x^4\right) dx, x, -\sin(c+dx)\right) \\
&\quad \underline{\hspace{10em} d \hspace{10em}} \\
&= \frac{3abx}{4} + \frac{(a^2 + b^2) \sin(c+dx)}{d} + \frac{3ab \cos(c+dx) \sin(c+dx)}{4d} \\
&\quad + \frac{ab \cos^3(c+dx) \sin(c+dx)}{2d} - \frac{(a^2 + 2b^2) \sin^3(c+dx)}{3d} + \frac{b^2 \sin^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.77

$$\begin{aligned}
&\int \cos^3(c+dx)(a+b\cos(c+dx))^2 dx \\
&= \frac{240(a^2 + b^2) \sin(c+dx) - 80(a^2 + 2b^2) \sin^3(c+dx) + 48b^2 \sin^5(c+dx) + 15ab(12(c+dx) + 8 \sin(2(c+dx)))}{240d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^2,x]

[Out] (240*(a^2 + b^2)*Sin[c + d*x] - 80*(a^2 + 2*b^2)*Sin[c + d*x]^3 + 48*b^2*Sin[c + d*x]^5 + 15*a*b*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(240*d)

Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a^2(2+\cos^2(dx+c))\sin(dx+c)}{3} + 2ab \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2})\sin(dx+c)}{4} + \frac{3dx+3c}{8} \right) + \frac{b^2 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5d}$
default	$\frac{a^2(2+\cos^2(dx+c))\sin(dx+c)}{3} + 2ab \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2})\sin(dx+c)}{4} + \frac{3dx+3c}{8} \right) + \frac{b^2 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5d}$
parts	$\frac{a^2(2+\cos^2(dx+c))\sin(dx+c)}{3d} + \frac{b^2 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right) \sin(dx+c)}{5d} + \frac{2ab \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2})\sin(dx+c)}{4} \right)}{d}$
parallelrisch	$\frac{180abxd + 180a^2 \sin(dx+c) + 150 \sin(dx+c)b^2 + 3b^2 \sin(5dx+5c) + 15ab \sin(4dx+4c) + 20a^2 \sin(3dx+3c) + 25b^2 \sin(3dx+3c)}{240d}$
risch	$\frac{3abx}{4} + \frac{3a^2 \sin(dx+c)}{4d} + \frac{5b^2 \sin(dx+c)}{8d} + \frac{b^2 \sin(5dx+5c)}{80d} + \frac{ab \sin(4dx+4c)}{16d} + \frac{a^2 \sin(3dx+3c)}{12d} + \frac{5 \sin(3dx+3c)b^2}{48d}$
norman	$\frac{3abx}{4} + \frac{4(25a^2+29b^2)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d} + \frac{(4a^2-5ab+4b^2)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d} + \frac{(4a^2+5ab+4b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d} + \frac{(16a^2-3ab+8b^2)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}$

[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/5*b^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^2 dx$$

$$= \frac{45abdx + (12b^2 \cos(dx+c))^4 + 30ab \cos(dx+c)^3 + 45ab \cos(dx+c) + 4(5a^2 + 4b^2) \cos(dx+c)^2 + 40}{60d}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/60*(45*a*b*d*x + (12*b^2*cos(d*x + c))^4 + 30*a*b*cos(d*x + c)^3 + 45*a*b*cos(d*x + c) + 4*(5*a^2 + 4*b^2)*cos(d*x + c)^2 + 40*a^2 + 32*b^2)*sin(d*x + c))/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(104) = 208$.

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.99

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^2 dx$$

$$= \begin{cases} \frac{2a^2 \sin^3(c+dx)}{3d} + \frac{a^2 \sin(c+dx) \cos^2(c+dx)}{d} + \frac{3abx \sin^4(c+dx)}{4} + \frac{3abx \sin^2(c+dx) \cos^2(c+dx)}{2} + \frac{3abx \cos^4(c+dx)}{4} + \frac{3ab \sin^3(c+dx)}{4d} \\ x(a + b \cos(c))^2 \cos^3(c) \end{cases}$$

[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**2,x)

[Out] Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*a*b*x*sin(c + d*x)**4/4 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 3*a*b*x*cos(c + d*x)**4/4 + 3*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 5*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 8*b**2*sin(c + d*x)**5/(15*d) + 4*b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + b**2*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^2 dx =$$

$$\frac{80 (\sin(dx + c)^3 - 3 \sin(dx + c))a^2 - 15(12dx + 12c + \sin(4dx + 4c) + 8 \sin(2dx + 2c))ab - 16 (\sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))b^2}{240d}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] -1/240*(80*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b - 16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*b^2)/d

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^2 dx = \frac{3}{4} abx + \frac{b^2 \sin(5 dx + 5 c)}{80 d} + \frac{ab \sin(4 dx + 4 c)}{16 d} + \frac{ab \sin(2 dx + 2 c)}{2 d} + \frac{(4 a^2 + 5 b^2) \sin(3 dx + 3 c)}{48 d} + \frac{(6 a^2 + 5 b^2) \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="giac")

```
[Out] 3/4*a*b*x + 1/80*b^2*sin(5*d*x + 5*c)/d + 1/16*a*b*sin(4*d*x + 4*c)/d + 1/2
*a*b*sin(2*d*x + 2*c)/d + 1/48*(4*a^2 + 5*b^2)*sin(3*d*x + 3*c)/d + 1/8*(6*
a^2 + 5*b^2)*sin(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.05

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^2 dx = \frac{3 a^2 \sin(c + dx)}{4 d} + \frac{5 b^2 \sin(c + dx)}{8 d} + \frac{3 a b x}{4} + \frac{a^2 \sin(3 c + 3 d x)}{12 d} + \frac{5 b^2 \sin(3 c + 3 d x)}{48 d} + \frac{b^2 \sin(5 c + 5 d x)}{80 d} + \frac{a b \sin(2 c + 2 d x)}{2 d} + \frac{a b \sin(4 c + 4 d x)}{16 d}$$

[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^2,x)

```
[Out] (3*a^2*sin(c + d*x))/(4*d) + (5*b^2*sin(c + d*x))/(8*d) + (3*a*b*x)/4 + (a^
2*sin(3*c + 3*d*x))/(12*d) + (5*b^2*sin(3*c + 3*d*x))/(48*d) + (b^2*sin(5*c
+ 5*d*x))/(80*d) + (a*b*sin(2*c + 2*d*x))/(2*d) + (a*b*sin(4*c + 4*d*x))/(
16*d)
```


3.419 $\int \cos^2(c + dx)(a + b \cos(c + dx))^2 dx$

Optimal result	4693
Rubi [A] (verified)	4693
Mathematica [A] (verified)	4695
Maple [A] (verified)	4695
Fricas [A] (verification not implemented)	4696
Sympy [B] (verification not implemented)	4696
Maxima [A] (verification not implemented)	4696
Giac [A] (verification not implemented)	4697
Mupad [B] (verification not implemented)	4697

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^2 dx = \frac{1}{8}(4a^2 + 3b^2)x + \frac{2ab \sin(c + dx)}{d} + \frac{(4a^2 + 3b^2) \cos(c + dx) \sin(c + dx)}{8d} + \frac{b^2 \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{2ab \sin^3(c + dx)}{3d}$$

[Out] 1/8*(4*a^2+3*b^2)*x+2*a*b*sin(d*x+c)/d+1/8*(4*a^2+3*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/4*b^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2868, 2713, 3093, 2715, 8}

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^2 dx = \frac{(4a^2 + 3b^2) \sin(c + dx) \cos(c + dx)}{8d} + \frac{1}{8}x(4a^2 + 3b^2) - \frac{2ab \sin^3(c + dx)}{3d} + \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \sin(c + dx) \cos^3(c + dx)}{4d}$$

[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2,x]

[Out] ((4*a^2 + 3*b^2)*x)/8 + (2*a*b*Sin[c + d*x])/d + ((4*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*b*Sin[c + d*x]^3)/(3*d)

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2713

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2715

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 2868

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3093

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= (2ab) \int \cos^3(c + dx) dx + \int \cos^2(c + dx) (a^2 + b^2 \cos^2(c + dx)) dx \\
 &= \frac{b^2 \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{1}{4}(4a^2 + 3b^2) \int \cos^2(c + dx) dx \\
 &\quad - \frac{(2ab) \text{Subst}\left(\int (1 - x^2) dx, x, -\sin(c + dx)\right)}{d} \\
 &= \frac{2ab \sin(c + dx)}{d} + \frac{(4a^2 + 3b^2) \cos(c + dx) \sin(c + dx)}{8d} \\
 &\quad + \frac{b^2 \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{2ab \sin^3(c + dx)}{3d} + \frac{1}{8}(4a^2 + 3b^2) \int 1 dx
 \end{aligned}$$

$$= \frac{1}{8}(4a^2 + 3b^2)x + \frac{2ab \sin(c + dx)}{d} + \frac{(4a^2 + 3b^2) \cos(c + dx) \sin(c + dx)}{8d} \\ + \frac{b^2 \cos^3(c + dx) \sin(c + dx)}{4d} - \frac{2ab \sin^3(c + dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^2 dx \\ = \frac{48a^2c + 36b^2c + 48a^2dx + 36b^2dx + 192ab \sin(c + dx) - 64ab \sin^3(c + dx) + 24(a^2 + b^2) \sin(2(c + dx))}{96d}$$

[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2,x]

[Out] (48*a^2*c + 36*b^2*c + 48*a^2*d*x + 36*b^2*d*x + 192*a*b*Sin[c + d*x] - 64*a*b*Sin[c + d*x]^3 + 24*(a^2 + b^2)*Sin[2*(c + d*x)] + 3*b^2*Sin[4*(c + d*x)])/(96*d)

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{24(a^2+b^2) \sin(2dx+2c)+16ab \sin(3dx+3c)+3 \sin(4dx+4c)b^2+144ab \sin(dx+c)+48d\left(a^2+\frac{3b^2}{4}\right)x}{96d}$
derivativedivides	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2ab(2+\cos^2(dx+c))\sin(dx+c)}{3}+b^2\left(\frac{\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}}{4}\sin(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\frac{2ab(2+\cos^2(dx+c))\sin(dx+c)}{3}+b^2\left(\frac{\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}}{4}\sin(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{a^2x}{2} + \frac{3b^2x}{8} + \frac{3ab \sin(dx+c)}{2d} + \frac{\sin(4dx+4c)b^2}{32d} + \frac{ab \sin(3dx+3c)}{6d} + \frac{\sin(2dx+2c)a^2}{4d} + \frac{\sin(2dx+2c)b^2}{4d}$
parts	$a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right) + \frac{b^2\left(\frac{\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}}{4}\sin(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d} + \frac{2ab(2+\cos^2(dx+c))\sin(dx+c)}{3d}$
norman	$\left(\frac{a^2}{2}+\frac{3b^2}{8}\right)x + \left(2a^2+\frac{3b^2}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \left(2a^2+\frac{3b^2}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \left(3a^2+\frac{9b^2}{4}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \left(\frac{a^2}{2}+\frac{3b^2}{8}\right)x$

[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/96*(24*(a^2+b^2)*sin(2*d*x+2*c)+16*a*b*sin(3*d*x+3*c)+3*sin(4*d*x+4*c)*b^2+144*a*b*sin(d*x+c)+48*d*(a^2+3/4*b^2)*x)/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^2 dx$$

$$= \frac{3(4a^2 + 3b^2)dx + (6b^2 \cos(dx + c))^3 + 16ab \cos(dx + c)^2 + 32ab + 3(4a^2 + 3b^2) \cos(dx + c) \sin(dx + c)}{24d}$$

[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/24*(3*(4*a^2 + 3*b^2)*d*x + (6*b^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^2 + 32*a*b + 3*(4*a^2 + 3*b^2)*cos(d*x + c))*sin(d*x + c))/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(92) = 184.

Time = 0.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.09

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^2 dx$$

$$= \begin{cases} \frac{a^2 x \sin^2(c+dx)}{2} + \frac{a^2 x \cos^2(c+dx)}{2} + \frac{a^2 \sin(c+dx) \cos(c+dx)}{2d} + \frac{4ab \sin^3(c+dx)}{3d} + \frac{2ab \sin(c+dx) \cos^2(c+dx)}{d} + \frac{3b^2 x \sin^4(c+dx)}{8} + \dots \\ x(a + b \cos(c))^2 \cos^2(c) \end{cases}$$

[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**2,x)

[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 4*a*b*sin(c + d*x)**3/(3*d) + 2*a*b*sin(c + d*x)*cos(c + d*x)**2/d + 3*b**2*x*sin(c + d*x)**4/8 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b**2*x*cos(c + d*x)**4/8 + 3*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c)**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^2 dx$$

$$= \frac{24(2dx + 2c + \sin(2dx + 2c))a^2 - 64(\sin(dx + c)^3 - 3\sin(dx + c))ab + 3(12dx + 12c + \sin(4dx + 4c))b^2}{96d}$$

[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 64*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b^2)/d

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^2 dx = \frac{1}{8} (4a^2 + 3b^2)x + \frac{b^2 \sin(4dx + 4c)}{32d} + \frac{ab \sin(3dx + 3c)}{6d} + \frac{3ab \sin(dx + c)}{2d} + \frac{(a^2 + b^2) \sin(2dx + 2c)}{4d}$$

[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] 1/8*(4*a^2 + 3*b^2)*x + 1/32*b^2*sin(4*d*x + 4*c)/d + 1/6*a*b*sin(3*d*x + 3*c)/d + 3/2*a*b*sin(d*x + c)/d + 1/4*(a^2 + b^2)*sin(2*d*x + 2*c)/d

Mupad [B] (verification not implemented)

Time = 14.97 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^2 dx = \frac{a^2 x}{2} + \frac{3b^2 x}{8} + \frac{a^2 \sin(2c + 2dx)}{4d} + \frac{b^2 \sin(2c + 2dx)}{4d} + \frac{b^2 \sin(4c + 4dx)}{32d} + \frac{3ab \sin(c + dx)}{2d} + \frac{ab \sin(3c + 3dx)}{6d}$$

[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^2,x)

[Out] (a^2*x)/2 + (3*b^2*x)/8 + (a^2*sin(2*c + 2*d*x))/(4*d) + (b^2*sin(2*c + 2*d*x))/(4*d) + (b^2*sin(4*c + 4*d*x))/(32*d) + (3*a*b*sin(c + d*x))/(2*d) + (a*b*sin(3*c + 3*d*x))/(6*d)

3.420 $\int \cos(c + dx)(a + b \cos(c + dx))^2 dx$

Optimal result	4698
Rubi [A] (verified)	4698
Mathematica [A] (verified)	4699
Maple [A] (verified)	4700
Fricas [A] (verification not implemented)	4700
Sympy [A] (verification not implemented)	4701
Maxima [A] (verification not implemented)	4701
Giac [A] (verification not implemented)	4701
Mupad [B] (verification not implemented)	4702

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \cos(c + dx)(a + b \cos(c + dx))^2 dx = abx + \frac{2(a^2 + b^2) \sin(c + dx)}{3d} + \frac{ab \cos(c + dx) \sin(c + dx)}{3d} + \frac{(a + b \cos(c + dx))^2 \sin(c + dx)}{3d}$$

[Out] a*b*x+2/3*(a^2+b^2)*sin(d*x+c)/d+1/3*a*b*cos(d*x+c)*sin(d*x+c)/d+1/3*(a+b*cos(d*x+c))^2*sin(d*x+c)/d

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2832, 2813}

$$\int \cos(c + dx)(a + b \cos(c + dx))^2 dx = \frac{2(a^2 + b^2) \sin(c + dx)}{3d} + \frac{\sin(c + dx)(a + b \cos(c + dx))^2}{3d} + \frac{ab \sin(c + dx) \cos(c + dx)}{3d} + abx$$

[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2,x]

[Out] a*b*x + (2*(a^2 + b^2)*Sin[c + d*x])/(3*d) + (a*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + ((a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)

Rule 2813

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*
*(x_)]), x_Symbol] := Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(a + b \cos(c + dx))^2 \sin(c + dx)}{3d} + \frac{1}{3} \int (2b + 2a \cos(c + dx))(a + b \cos(c + dx)) dx \\ &= abx + \frac{2(a^2 + b^2) \sin(c + dx)}{3d} + \frac{ab \cos(c + dx) \sin(c + dx)}{3d} + \frac{(a + b \cos(c + dx))^2 \sin(c + dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\begin{aligned} &\int \cos(c + dx)(a + b \cos(c + dx))^2 dx \\ &= \frac{3(4a^2 + 3b^2) \sin(c + dx) + b(12a(c + dx) + 6a \sin(2(c + dx)) + b \sin(3(c + dx)))}{12d} \end{aligned}$$

```
[In] Integrate[Cos[c + d*x]*(a + b*cos[c + d*x])^2,x]
```

```
[Out] (3*(4*a^2 + 3*b^2)*Sin[c + d*x] + b*(12*a*(c + d*x) + 6*a*Sin[2*(c + d*x)]
+ b*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

method	result
parallelrisc	$\frac{12abd+b^2 \sin(3dx+3c)+6ab \sin(2dx+2c)+12a^2 \sin(dx+c)+9 \sin(dx+c)b^2}{12d}$
derivativedivides	$\frac{a^2 \sin(dx+c)+2ab \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + \frac{b^2 (2+\cos^2(dx+c)) \sin(dx+c)}{3}}{d}$
default	$\frac{a^2 \sin(dx+c)+2ab \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + \frac{b^2 (2+\cos^2(dx+c)) \sin(dx+c)}{3}}{d}$
risc	$abx + \frac{a^2 \sin(dx+c)}{d} + \frac{3b^2 \sin(dx+c)}{4d} + \frac{\sin(3dx+3c)b^2}{12d} + \frac{ab \sin(2dx+2c)}{2d}$
parts	$\frac{b^2 (2+\cos^2(dx+c)) \sin(dx+c)}{3d} + \frac{a^2 \sin(dx+c)}{d} + \frac{2ab \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d}$
norman	$\frac{abx+abx \left(\tan^6 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \frac{4(3a^2+b^2) \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} + \frac{2(a^2-ab+b^2) \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} + \frac{2(a^2+ab+b^2) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d} + 3abx \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{\left(1+\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^3}$

[In] int(cos(d*x+c)*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/12*(12*a*b*x*d+b^2*sin(3*d*x+3*c)+6*a*b*sin(2*d*x+2*c)+12*a^2*sin(d*x+c)+9*sin(d*x+c)*b^2)/d

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \cos(c+dx)(a+b \cos(c+dx))^2 dx$$

$$= \frac{3abd x + (b^2 \cos(dx+c)^2 + 3ab \cos(dx+c) + 3a^2 + 2b^2) \sin(dx+c)}{3d}$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/3*(3*a*b*d*x + (b^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) + 3*a^2 + 2*b^2)*sin(d*x + c))/d

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.51

$$\int \cos(c + dx)(a + b \cos(c + dx))^2 dx$$

$$= \begin{cases} \frac{a^2 \sin(c+dx)}{d} + abx \sin^2(c + dx) + abx \cos^2(c + dx) + \frac{ab \sin(c+dx) \cos(c+dx)}{d} + \frac{2b^2 \sin^3(c+dx)}{3d} + \frac{b^2 \sin(c+dx) \cos^2(c+dx)}{d} \\ x(a + b \cos(c))^2 \cos(c) \end{cases}$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2,x)

[Out] Piecewise((a**2*sin(c + d*x)/d + a*b*x*sin(c + d*x)**2 + a*b*x*cos(c + d*x)**2 + a*b*sin(c + d*x)*cos(c + d*x)/d + 2*b**2*sin(c + d*x)**3/(3*d) + b**2*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c), True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \cos(c + dx)(a + b \cos(c + dx))^2 dx$$

$$= \frac{3(2dx + 2c + \sin(2dx + 2c))ab - 2(\sin(dx + c)^3 - 3\sin(dx + c))b^2 + 6a^2 \sin(dx + c)}{6d}$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] 1/6*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b - 2*(sin(d*x + c)^3 - 3*sin(d*x + c))*b^2 + 6*a^2*sin(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \cos(c + dx)(a + b \cos(c + dx))^2 dx = abx + \frac{b^2 \sin(3dx + 3c)}{12d} + \frac{ab \sin(2dx + 2c)}{2d}$$

$$+ \frac{(4a^2 + 3b^2) \sin(dx + c)}{4d}$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] a*b*x + 1/12*b^2*sin(3*d*x + 3*c)/d + 1/2*a*b*sin(2*d*x + 2*c)/d + 1/4*(4*a^2 + 3*b^2)*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \cos(c + dx)(a + b \cos(c + dx))^2 dx = \frac{a^2 \sin(c + dx)}{d} + \frac{2b^2 \sin(c + dx)}{3d} + abx + \frac{b^2 \cos(c + dx)^2 \sin(c + dx)}{3d} + \frac{ab \cos(c + dx) \sin(c + dx)}{d}$$

```
[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x))/d + (2*b^2*sin(c + d*x))/(3*d) + a*b*x + (b^2*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (a*b*cos(c + d*x)*sin(c + d*x))/d
```

3.421 $\int (a + b \cos(c + dx))^2 dx$

Optimal result	4703
Rubi [A] (verified)	4703
Mathematica [A] (verified)	4704
Maple [A] (verified)	4704
Fricas [A] (verification not implemented)	4704
Sympy [A] (verification not implemented)	4705
Maxima [A] (verification not implemented)	4705
Giac [A] (verification not implemented)	4705
Mupad [B] (verification not implemented)	4706

Optimal result

Integrand size = 12, antiderivative size = 50

$$\int (a + b \cos(c + dx))^2 dx = \frac{1}{2}(2a^2 + b^2)x + \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \cos(c + dx) \sin(c + dx)}{2d}$$

[Out] $1/2*(2*a^2+b^2)*x+2*a*b*\sin(d*x+c)/d+1/2*b^2*\cos(d*x+c)*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2723}

$$\int (a + b \cos(c + dx))^2 dx = \frac{1}{2}x(2a^2 + b^2) + \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \sin(c + dx) \cos(c + dx)}{2d}$$

[In] $\text{Int}[(a + b*\text{Cos}[c + d*x])^2, x]$

[Out] $((2*a^2 + b^2)*x)/2 + (2*a*b*\text{Sin}[c + d*x])/d + (b^2*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rule 2723

$\text{Int}[(a + b*\sin[(c + d*x)])^2, x_Symbol] \rightarrow \text{Simp}[(2*a^2 + b^2)*(x/2), x] + (-\text{Simp}[2*a*b*(\text{Cos}[c + d*x]/d), x] - \text{Simp}[b^2*\text{Cos}[c + d*x]*(\text{Sin}[c + d*x]/(2*d)), x]) /;$ FreeQ[{a, b, c, d}, x]

Rubi steps

$$\text{integral} = \frac{1}{2}(2a^2 + b^2)x + \frac{2ab \sin(c + dx)}{d} + \frac{b^2 \cos(c + dx) \sin(c + dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (a + b \cos(c + dx))^2 dx = \frac{2(2a^2 + b^2)(c + dx) + 8ab \sin(c + dx) + b^2 \sin(2(c + dx))}{4d}$$

`[In] Integrate[(a + b*Cos[c + d*x])^2,x]``[Out] (2*(2*a^2 + b^2)*(c + d*x) + 8*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*x)])/(4*d)`**Maple [A] (verified)**

Time = 0.95 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result	s
risch	$a^2x + \frac{b^2x}{2} + \frac{2ab \sin(dx+c)}{d} + \frac{\sin(2dx+2c)b^2}{4d}$	4
parallelrisc	$\frac{\sin(2dx+2c)b^2 + 8ab \sin(dx+c) + 4d(a^2 + \frac{b^2}{2})x}{4d}$	4
parts	$a^2x + \frac{b^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d} + \frac{2ab \sin(dx+c)}{d}$	4
derivativedivides	$\frac{b^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 2ab \sin(dx+c) + a^2(dx+c)}{d}$	5
default	$\frac{b^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 2ab \sin(dx+c) + a^2(dx+c)}{d}$	5
norman	$\frac{\left(a^2 + \frac{b^2}{2}\right)x + \left(a^2 + \frac{b^2}{2}\right)x \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (2a^2 + b^2)x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{b(4a-b) \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} + \frac{b(4a+b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}$	1

`[In] int((a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)``[Out] a^2*x+1/2*b^2*x+2*a*b*sin(d*x+c)/d+1/4/d*sin(2*d*x+2*c)*b^2`**Fricas [A] (verification not implemented)**

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (a + b \cos(c + dx))^2 dx = \frac{(2a^2 + b^2)dx + (b^2 \cos(dx + c) + 4ab) \sin(dx + c)}{2d}$$

`[In] integrate((a+b*cos(d*x+c))^2,x, algorithm="fricas")``[Out] 1/2*((2*a^2 + b^2)*d*x + (b^2*cos(d*x + c) + 4*a*b)*sin(d*x + c))/d`

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int (a + b \cos(c + dx))^2 dx = \begin{cases} a^2 x + \frac{2ab \sin(c+dx)}{d} + \frac{b^2 x \sin^2(c+dx)}{2} + \frac{b^2 x \cos^2(c+dx)}{2} + \frac{b^2 \sin(c+dx) \cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a + b \cos(c))^2 & \text{otherwise} \end{cases}$$

[In] integrate((a+b*cos(d*x+c))**2,x)

[Out] Piecewise((a**2*x + 2*a*b*sin(c + d*x)/d + b**2*x*sin(c + d*x)**2/2 + b**2*x*cos(c + d*x)**2/2 + b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a + b*cos(c))**2, True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int (a + b \cos(c + dx))^2 dx = a^2 x + \frac{(2 dx + 2 c + \sin(2 dx + 2 c))b^2}{4 d} + \frac{2 ab \sin(dx + c)}{d}$$

[In] integrate((a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] a^2*x + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*b^2/d + 2*a*b*sin(d*x + c)/d

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int (a + b \cos(c + dx))^2 dx = \frac{1}{2} (2 a^2 + b^2) x + \frac{b^2 \sin(2 dx + 2 c)}{4 d} + \frac{2 ab \sin(dx + c)}{d}$$

[In] integrate((a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] 1/2*(2*a^2 + b^2)*x + 1/4*b^2*sin(2*d*x + 2*c)/d + 2*a*b*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int (a + b \cos(c + dx))^2 dx = a^2 x + \frac{b^2 x}{2} + \frac{b^2 \sin(2c + 2dx)}{4d} + \frac{2ab \sin(c + dx)}{d}$$

[In] int((a + b*cos(c + d*x))^2,x)

[Out] a^2*x + (b^2*x)/2 + (b^2*sin(2*c + 2*d*x))/(4*d) + (2*a*b*sin(c + d*x))/d

3.422 $\int (a + b \cos(c + dx))^2 \sec(c + dx) dx$

Optimal result	4707
Rubi [A] (verified)	4707
Mathematica [A] (verified)	4708
Maple [A] (verified)	4708
Fricas [A] (verification not implemented)	4709
Sympy [F]	4709
Maxima [A] (verification not implemented)	4710
Giac [B] (verification not implemented)	4710
Mupad [B] (verification not implemented)	4710

Optimal result

Integrand size = 19, antiderivative size = 33

$$\int (a + b \cos(c + dx))^2 \sec(c + dx) dx = 2abx + \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{b^2 \sin(c + dx)}{d}$$

[Out] $2*a*b*x + a^2*\operatorname{arctanh}(\sin(d*x+c))/d + b^2*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2825, 2814, 3855}

$$\int (a + b \cos(c + dx))^2 \sec(c + dx) dx = \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + 2abx + \frac{b^2 \sin(c + dx)}{d}$$

[In] $\operatorname{Int}[(a + b*\operatorname{Cos}[c + d*x])^2*\operatorname{Sec}[c + d*x], x]$

[Out] $2*a*b*x + (a^2*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (b^2*\operatorname{Sin}[c + d*x])/d$

Rule 2814

$\operatorname{Int}[(a + b*\sin[(e + f*x)])/(c + d*\sin[(e + f*x)]), x_Symbol] := \operatorname{Simp}[b*(x/d), x] - \operatorname{Dist}[(b*c - a*d)/d, \operatorname{Int}[1/(c + d*\sin[e + f*x]), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0]$

Rule 2825

$\operatorname{Int}[(a + b*\sin[(e + f*x)]^2)/(c + d*\sin[(e + f*x)]), x_Symbol] := \operatorname{Simp}[(-b^2)*(Cos[e + f*x]/(d*f)), x] + \operatorname{Dist}[1/d, \operatorname{Int}[\operatorname{Simp}[a^2*d - b*(b*c - 2*a*d)*\sin[e + f*x], x]/(c + d*\sin[e + f*x]), x],$

`x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{b^2 \sin(c + dx)}{d} + \int (a^2 + 2ab \cos(c + dx)) \sec(c + dx) dx \\ &= 2abx + \frac{b^2 \sin(c + dx)}{d} + a^2 \int \sec(c + dx) dx \\ &= 2abx + \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{b^2 \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\begin{aligned} \int (a + b \cos(c + dx))^2 \sec(c + dx) dx &= 2abx + \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{d} \\ &\quad + \frac{b^2 \cos(dx) \sin(c)}{d} + \frac{b^2 \cos(c) \sin(dx)}{d} \end{aligned}$$

`[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x],x]`

`[Out] 2*a*b*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (b^2*Cos[d*x]*Sin[c])/d + (b^2*Cos[c]*Sin[d*x])/d`

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2ab(dx+c)+\sin(dx+c)b^2}{d}$
default	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))+2ab(dx+c)+\sin(dx+c)b^2}{d}$
parts	$\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{b^2 \sin(dx+c)}{d} + \frac{2ab(dx+c)}{d}$
parallelrisch	$\frac{2abxd - a^2 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + a^2 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + \sin(dx+c)b^2}{d}$
risch	$2abx - \frac{ib^2 e^{i(dx+c)}}{2d} + \frac{ib^2 e^{-i(dx+c)}}{2d} + \frac{a^2 \ln(e^{i(dx+c)+i})}{d} - \frac{a^2 \ln(e^{i(dx+c)-i})}{d}$
norman	$\frac{2abx + \frac{2b^2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{2b^2 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} + 4abx \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2abx \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} + \frac{a^2 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d}$

[In] `int((a+cos(d*x+c)*b)^2*sec(d*x+c),x,method=_RETURNVERBOSE)`

[Out] $1/d*(a^2*\ln(\sec(d*x+c)+\tan(d*x+c))+2*a*b*(d*x+c)+\sin(d*x+c)*b^2)$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int (a + b \cos(c + dx))^2 \sec(c + dx) dx$$

$$= \frac{4 abdx + a^2 \log(\sin(dx + c) + 1) - a^2 \log(-\sin(dx + c) + 1) + 2 b^2 \sin(dx + c)}{2d}$$

[In] `integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="fricas")`

[Out] $1/2*(4*a*b*d*x + a^2*\log(\sin(d*x + c) + 1) - a^2*\log(-\sin(d*x + c) + 1) + 2*b^2*\sin(d*x + c))/d$

Sympy [F]

$$\int (a + b \cos(c + dx))^2 \sec(c + dx) dx = \int (a + b \cos(c + dx))^2 \sec(c + dx) dx$$

[In] `integrate((a+b*cos(d*x+c))**2*sec(d*x+c),x)`

[Out] `Integral((a + b*cos(c + d*x))**2*sec(c + d*x), x)`

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int (a + b \cos(c + dx))^2 \sec(c + dx) dx$$

$$= \frac{2(dx + c)ab + a^2 \log(\sec(dx + c) + \tan(dx + c)) + b^2 \sin(dx + c)}{d}$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="maxima")

[Out] (2*(d*x + c)*a*b + a^2*log(sec(d*x + c) + tan(d*x + c)) + b^2*sin(d*x + c))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(33) = 66.

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int (a + b \cos(c + dx))^2 \sec(c + dx) dx$$

$$= \frac{2(dx + c)ab + a^2 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|) - a^2 \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|) + \frac{2b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1}}{d}$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="giac")

[Out] (2*(d*x + c)*a*b + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d

Mupad [B] (verification not implemented)

Time = 14.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.21

$$\int (a + b \cos(c + dx))^2 \sec(c + dx) dx = \frac{b^2 \sin(c + dx)}{d} + \frac{2a^2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

$$+ \frac{4ab \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

[In] int((a + b*cos(c + d*x))^2/cos(c + d*x),x)

[Out] (b^2*sin(c + d*x))/d + (2*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d

3.423 $\int (a + b \cos(c + dx))^2 \sec^2(c + dx) dx$

Optimal result	4711
Rubi [A] (verified)	4711
Mathematica [A] (verified)	4712
Maple [A] (verified)	4712
Fricas [B] (verification not implemented)	4713
Sympy [F]	4713
Maxima [A] (verification not implemented)	4714
Giac [B] (verification not implemented)	4714
Mupad [B] (verification not implemented)	4714

Optimal result

Integrand size = 21, antiderivative size = 33

$$\int (a + b \cos(c + dx))^2 \sec^2(c + dx) dx = b^2 x + \frac{2ab \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \tan(c + dx)}{d}$$

[Out] $b^2*x + 2*a*b*\operatorname{arctanh}(\sin(d*x+c))/d + a^2*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.08 (sec), antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2868, 3855, 3091, 8}

$$\int (a + b \cos(c + dx))^2 \sec^2(c + dx) dx = \frac{a^2 \tan(c + dx)}{d} + \frac{2ab \operatorname{arctanh}(\sin(c + dx))}{d} + b^2 x$$

[In] $\operatorname{Int}[(a + b*\operatorname{Cos}[c + d*x])^2*\operatorname{Sec}[c + d*x]^2, x]$

[Out] $b^2*x + (2*a*b*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (a^2*\operatorname{Tan}[c + d*x])/d$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2868

$\operatorname{Int}[(b*\sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])^2, x_Symbol] \rightarrow \operatorname{Dist}[2*c*(d/b), \operatorname{Int}[(b*\sin[e + f*x])^{m+1}, x], x] + \operatorname{Int}[(b*\sin[e + f*x])^m*(c^2 + d^2*\sin[e + f*x]^2), x] /; \operatorname{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= (2ab) \int \sec(c + dx) dx + \int (a^2 + b^2 \cos^2(c + dx)) \sec^2(c + dx) dx \\ &= \frac{2ab \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \tan(c + dx)}{d} + b^2 \int 1 dx \\ &= b^2 x + \frac{2ab \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2 \tan(c + dx)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int (a + b \cos(c + dx))^2 \sec^2(c + dx) dx = \frac{b^2 dx + 2ab \operatorname{arctanh}(\sin(c + dx)) + a^2 \tan(c + dx)}{d}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

```
[Out] (b^2*d*x + 2*a*b*ArcTanh[Sin[c + d*x]] + a^2*Tan[c + d*x])/d
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

method	result
derivativdivides	$\frac{a^2 \tan(dx+c) + 2ab \ln(\sec(dx+c) + \tan(dx+c)) + b^2(dx+c)}{d}$
default	$\frac{a^2 \tan(dx+c) + 2ab \ln(\sec(dx+c) + \tan(dx+c)) + b^2(dx+c)}{d}$
parts	$\frac{a^2 \tan(dx+c)}{d} + \frac{b^2(dx+c)}{d} + \frac{2ab \ln(\sec(dx+c) + \tan(dx+c))}{d}$
risch	$b^2x + \frac{2ia^2}{d(e^{2i(dx+c)}+1)} + \frac{2 \ln(e^{i(dx+c)}+i)ab}{d} - \frac{2 \ln(e^{i(dx+c)}-i)ab}{d}$
parallelrisc	$\frac{b^2 dx \cos(dx+c) - 2ab \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \cos(dx+c) + 2ab \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) \cos(dx+c) + a^2 \sin(dx+c)}{d \cos(dx+c)}$
norman	$\frac{b^2x \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + b^2x \left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - b^2x - \frac{2a^2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} - \frac{4a^2 \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} - \frac{2a^2 \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} - b^2x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}$

[In] `int((a+cos(d*x+c)*b)^2*sec(d*x+c)^2,x,method=_RETURNVERBOSE)`

[Out] `1/d*(a^2*tan(d*x+c)+2*a*b*ln(sec(d*x+c)+tan(d*x+c))+b^2*(d*x+c))`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(33) = 66$.

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int (a + b \cos(c + dx))^2 \sec^2(c + dx) dx$$

$$= \frac{b^2 dx \cos(dx + c) + ab \cos(dx + c) \log(\sin(dx + c) + 1) - ab \cos(dx + c) \log(-\sin(dx + c) + 1) + a^2 \sin(dx + c)}{d \cos(dx + c)}$$

[In] `integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="fricas")`

[Out] `(b^2*d*x*cos(d*x + c) + a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))`

Sympy [F]

$$\int (a + b \cos(c + dx))^2 \sec^2(c + dx) dx = \int (a + b \cos(c + dx))^2 \sec^2(c + dx) dx$$

[In] `integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**2,x)`

[Out] `Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**2, x)`

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int (a + b \cos(c + dx))^2 \sec^2(c + dx) dx$$

$$= \frac{(dx + c)b^2 + ab(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + a^2 \tan(dx + c)}{d}$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="maxima")

[Out] ((d*x + c)*b^2 + a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + a^2*tan(d*x + c))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(33) = 66.

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.33

$$\int (a + b \cos(c + dx))^2 \sec^2(c + dx) dx$$

$$= \frac{(dx + c)b^2 + 2ab \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 2ab \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1}}{d}$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="giac")

[Out] ((d*x + c)*b^2 + 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d

Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 181, normalized size of antiderivative = 5.48

$$\int (a + b \cos(c + dx))^2 \sec^2(c + dx) dx = \frac{2b^2 \operatorname{atan}\left(\frac{64b^6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{256a^2 b^4 + 64b^6} + \frac{256a^2 b^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{256a^2 b^4 + 64b^6}\right)}{d}$$

$$- \frac{2a^2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

$$+ \frac{4ab \operatorname{atanh}\left(\frac{128ab^5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{512a^3 b^3 + 128ab^5} + \frac{512a^3 b^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{512a^3 b^3 + 128ab^5}\right)}{d}$$

[In] $\text{int}((a + b\cos(c + dx))^2/\cos(c + dx)^2, x)$

[Out] $(2b^2\text{atan}((64b^6\tan(c/2 + (dx)/2))/(64b^6 + 256a^2b^4) + (256a^2b^4\tan(c/2 + (dx)/2))/(64b^6 + 256a^2b^4)))/d - (2a^2\tan(c/2 + (dx)/2))/(d(\tan(c/2 + (dx)/2)^2 - 1)) + (4ab\text{atanh}((128ab^5\tan(c/2 + (dx)/2))/(128ab^5 + 512a^3b^3) + (512a^3b^3\tan(c/2 + (dx)/2))/(128ab^5 + 512a^3b^3)))/d$

3.424 $\int (a + b \cos(c + dx))^2 \sec^3(c + dx) dx$

Optimal result	4716
Rubi [A] (verified)	4716
Mathematica [A] (verified)	4717
Maple [A] (verified)	4718
Fricas [A] (verification not implemented)	4718
Sympy [F]	4719
Maxima [A] (verification not implemented)	4719
Giac [B] (verification not implemented)	4719
Mupad [B] (verification not implemented)	4720

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int (a + b \cos(c + dx))^2 \sec^3(c + dx) dx = \frac{(a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{2ab \tan(c + dx)}{d} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d}$$

[Out] 1/2*(a^2+2*b^2)*arctanh(sin(d*x+c))/d+2*a*b*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*tan(d*x+c)/d

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2868, 3852, 8, 3091, 3855}

$$\int (a + b \cos(c + dx))^2 \sec^3(c + dx) dx = \frac{(a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a^2 \tan(c + dx) \sec(c + dx)}{2d} + \frac{2ab \tan(c + dx)}{d}$$

[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3,x]

[Out] ((a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2868


```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= (2ab) \int \sec^2(c + dx) dx + \int (a^2 + b^2 \cos^2(c + dx)) \sec^3(c + dx) dx \\
 &= \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2}(a^2 + 2b^2) \int \sec(c + dx) dx \\
 &\quad - \frac{(2ab) \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d} \\
 &= \frac{(a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{2ab \tan(c + dx)}{d} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\begin{aligned}
 \int (a + b \cos(c + dx))^2 \sec^3(c + dx) dx &= \frac{a^2 \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{b^2 \operatorname{arctanh}(\sin(c + dx))}{d} \\
 &\quad + \frac{2ab \tan(c + dx)}{d} + \frac{a^2 \sec(c + dx) \tan(c + dx)}{2d}
 \end{aligned}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (b^2*ArcTanh[Sin[c + d*x]])/d + (2*a*b*Tan[c + d*x])/d + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right) + 2ab \tan(dx+c) + b^2 \ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right) + 2ab \tan(dx+c) + b^2 \ln(\sec(dx+c)+\tan(dx+c))}{d}$
parts	$\frac{a^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d} + \frac{b^2 \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{2ab \tan(dx+c)}{d}$
parallelrisch	$\frac{-(a^2+2b^2)(1+\cos(2dx+2c)) \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + (a^2+2b^2)(1+\cos(2dx+2c)) \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) + 2a^2 \sin(dx+c) + 4ab \cos(dx+c)}{2d(1+\cos(2dx+2c))}$
risch	$-\frac{ia(ae^{3i(dx+c)}-4be^{2i(dx+c)}-ae^{i(dx+c)}-4b)}{d(e^{2i(dx+c)}+1)^2} + \frac{a^2 \ln(e^{i(dx+c)}+i)}{2d} + \frac{\ln(e^{i(dx+c)}+i)b^2}{d} - \frac{a^2 \ln(e^{i(dx+c)}-i)}{2d} - \frac{\ln(e^{i(dx+c)}-i)b^2}{d}$
norman	$\frac{\frac{a(a-4b)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{a(a+4b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{a(3a-4b)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{a(3a+4b)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 \left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} - \frac{(a^2+2b^2) \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$

[In] int((a+cos(d*x+c)*b)^2*sec(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2*a*b*tan(d*x+c)+b^2*ln(sec(d*x+c)+tan(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.58

$$\int (a + b \cos(c + dx))^2 \sec^3(c + dx) dx$$

$$= \frac{(a^2 + 2b^2) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (a^2 + 2b^2) \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2(4ab \cos(dx + c) + a^2) \sin(dx + c)}{4d \cos(dx + c)^2}$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="fricas")

[Out] 1/4*((a^2 + 2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^2 + 2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(4*a*b*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)

Sympy [F]

$$\int (a + b \cos(c + dx))^2 \sec^3(c + dx) dx = \int (a + b \cos(c + dx))^2 \sec^3(c + dx) dx$$

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**3,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int (a + b \cos(c + dx))^2 \sec^3(c + dx) dx = \frac{a^2 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1) \right) - 2b^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1))}{4d}$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 8*a*b*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(55) = 110.

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.15

$$\int (a + b \cos(c + dx))^2 \sec^3(c + dx) dx = \frac{(a^2 + 2b^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - (a^2 + 2b^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2(a^2 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 - 4ab \tan(\frac{1}{2}dx + \frac{1}{2}c))}{2d}}{2d}$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^3 + a^2*tan(1/2*d*x + 1/2*c) + 4*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int (a + b \cos(c + dx))^2 \sec^3(c + dx) dx$$

$$= \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (a^2 + 2b^2)}{d} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (4ab - a^2) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (a^2 + 4ba)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^3,x)

[Out] (atanh(tan(c/2 + (d*x)/2))*(a^2 + 2*b^2))/d - (tan(c/2 + (d*x)/2)^3*(4*a*b - a^2) - tan(c/2 + (d*x)/2)*(4*a*b + a^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))

3.425 $\int (a + b \cos(c + dx))^2 \sec^4(c + dx) dx$

Optimal result	4721
Rubi [A] (verified)	4721
Mathematica [A] (verified)	4723
Maple [A] (verified)	4724
Fricas [A] (verification not implemented)	4724
Sympy [F]	4725
Maxima [A] (verification not implemented)	4725
Giac [B] (verification not implemented)	4725
Mupad [B] (verification not implemented)	4726

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int (a + b \cos(c + dx))^2 \sec^4(c + dx) dx = \frac{ab \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{(2a^2 + 3b^2) \tan(c + dx)}{3d} + \frac{ab \sec(c + dx) \tan(c + dx)}{d} + \frac{a^2 \sec^2(c + dx) \tan(c + dx)}{3d}$$

[Out] $a*b*\operatorname{arctanh}(\sin(d*x+c))/d+1/3*(2*a^2+3*b^2)*\tan(d*x+c)/d+a*b*\sec(d*x+c)*\tan(d*x+c)/d+1/3*a^2*\sec(d*x+c)^2*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.11 (sec), antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2868, 3853, 3855, 3091, 3852, 8}

$$\int (a + b \cos(c + dx))^2 \sec^4(c + dx) dx = \frac{(2a^2 + 3b^2) \tan(c + dx)}{3d} + \frac{a^2 \tan(c + dx) \sec^2(c + dx)}{3d} + \frac{ab \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{ab \tan(c + dx) \sec(c + dx)}{d}$$

[In] $\operatorname{Int}[(a + b*\operatorname{Cos}[c + d*x])^2*\operatorname{Sec}[c + d*x]^4,x]$

[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + ((2*a^2 + 3*b^2)*Tan[c + d*x])/(3*d) + (a*b*Sec[c + d*x]*Tan[c + d*x])/d + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2868

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3091

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3852

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= (2ab) \int \sec^3(c + dx) dx + \int (a^2 + b^2 \cos^2(c + dx)) \sec^4(c + dx) dx \\ &= \frac{ab \sec(c + dx) \tan(c + dx)}{d} + \frac{a^2 \sec^2(c + dx) \tan(c + dx)}{3d} \\ &\quad + (ab) \int \sec(c + dx) dx + \frac{1}{3} (2a^2 + 3b^2) \int \sec^2(c + dx) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{a b \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a b \sec(c + dx) \tan(c + dx)}{d} \\
&\quad + \frac{a^2 \sec^2(c + dx) \tan(c + dx)}{3d} - \frac{(2a^2 + 3b^2) \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{3d} \\
&= \frac{a b \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{(2a^2 + 3b^2) \tan(c + dx)}{3d} \\
&\quad + \frac{a b \sec(c + dx) \tan(c + dx)}{d} + \frac{a^2 \sec^2(c + dx) \tan(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\begin{aligned}
\int (a + b \cos(c + dx))^2 \sec^4(c + dx) dx &= \frac{a b \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{b^2 \tan(c + dx)}{d} \\
&\quad + \frac{a b \sec(c + dx) \tan(c + dx)}{d} \\
&\quad + \frac{a^2 (\tan(c + dx) + \frac{1}{3} \tan^3(c + dx))}{d}
\end{aligned}$$

[In] Integrate[(a + b*cos[c + d*x])^2*Sec[c + d*x]^4,x]

[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*Tan[c + d*x])/d + (a*b*Sec[c + d*x]*Tan[c + d*x])/d + (a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-a^2 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + 2ab \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) + b^2 \tan(dx+c)}{d}$
default	$\frac{-a^2 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + 2ab \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) + b^2 \tan(dx+c)}{d}$
parts	$-\frac{a^2 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d} + \frac{b^2 \tan(dx+c)}{d} + \frac{ab \sec(dx+c) \tan(dx+c)}{d} + \frac{ab \ln(\sec(dx+c) + \tan(dx+c))}{d}$
risch	$-\frac{2i(3ab e^{5i(dx+c)} - 3b^2 e^{4i(dx+c)} - 6a^2 e^{2i(dx+c)} - 6b^2 e^{2i(dx+c)} - 3ab e^{i(dx+c)} - 2a^2 - 3b^2)}{3d(e^{2i(dx+c)} + 1)^3} + \frac{\ln(e^{i(dx+c)} + i) ab}{d} - \frac{\ln(e^{i(dx+c)} - i) ab}{d}$
parallelrisc	$\frac{-9b \left(\frac{\cos(3dx+3c)}{3} + \cos(dx+c) \right) a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 9b \left(\frac{\cos(3dx+3c)}{3} + \cos(dx+c) \right) a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) + (2a^2 + 3b^2) \sin(dx+c)}{3d(\cos(3dx+3c) + 3 \cos(dx+c))}$
norman	$\frac{\frac{4(a^2 - 3b^2) \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} - \frac{2(a^2 - ab + b^2) \left(\tan^9 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{2(a^2 + ab + b^2) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d} - \frac{4a(2a - 3b) \left(\tan^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d} - \frac{4a(2a + 3b) \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3d}}{\left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2 \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right)^3}$

[In] int((a+cos(d*x+c)*b)^2*sec(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*a*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+b^2*tan(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int (a + b \cos(c + dx))^2 \sec^4(c + dx) dx$$

$$= \frac{3ab \cos(dx+c)^3 \log(\sin(dx+c)+1) - 3ab \cos(dx+c)^3 \log(-\sin(dx+c)+1) + 2(3ab \cos(dx+c) + a^2 \sin(dx+c))}{6d \cos(dx+c)^3}$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="fricas")

[Out] 1/6*(3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(3*a*b*cos(d*x + c) + (2*a^2 + 3*b^2)*cos(d*x + c)^2 + a^2*sin(d*x + c)))/(d*cos(d*x + c)^3)

Sympy [F]

$$\int (a + b \cos(c + dx))^2 \sec^4(c + dx) dx = \int (a + b \cos(c + dx))^2 \sec^4(c + dx) dx$$

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**4,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int (a + b \cos(c + dx))^2 \sec^4(c + dx) dx$$

$$= \frac{2 (\tan(dx + c)^3 + 3 \tan(dx + c)) a^2 - 3 ab \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right)}{6 d}$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*b^2*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(76) = 152.

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.22

$$\int (a + b \cos(c + dx))^2 \sec^4(c + dx) dx$$

$$= \frac{3 ab \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) - 3 ab \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - 1 \right| \right) - \frac{2 \left(3 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 3 ab \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 + 3 b^2 \right)}{3 d}}{3 d}$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*a^2*tan(1/2*d*x + 1/2*c)^3 - 6*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 17.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.76

$$\int (a + b \cos(c + dx))^2 \sec^4(c + dx) dx = \frac{2ab \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{(2a^2 - 2ab + 2b^2) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(-\frac{4a^2}{3} - 4b^2\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (2a^2 + 2ab + 2b^2) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^4,x)

```
[Out] (2*a*b*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^5*(2*a^2 - 2*a*b
+ 2*b^2) - tan(c/2 + (d*x)/2)^3*((4*a^2)/3 + 4*b^2) + tan(c/2 + (d*x)/2)*(2
*a*b + 2*a^2 + 2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4
+ tan(c/2 + (d*x)/2)^6 - 1))
```

3.426 $\int (a + b \cos(c + dx))^2 \sec^5(c + dx) dx$

Optimal result	4727
Rubi [A] (verified)	4727
Mathematica [A] (verified)	4729
Maple [A] (verified)	4730
Fricas [A] (verification not implemented)	4730
Sympy [F]	4731
Maxima [A] (verification not implemented)	4731
Giac [B] (verification not implemented)	4731
Mupad [B] (verification not implemented)	4732

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int (a + b \cos(c + dx))^2 \sec^5(c + dx) dx = \frac{(3a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{2ab \tan(c + dx)}{d} + \frac{(3a^2 + 4b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{a^2 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{2ab \tan^3(c + dx)}{3d}$$

[Out] $1/8*(3*a^2+4*b^2)*\operatorname{arctanh}(\sin(d*x+c))/d+2*a*b*\tan(d*x+c)/d+1/8*(3*a^2+4*b^2)*\sec(d*x+c)*\tan(d*x+c)/d+1/4*a^2*\sec(d*x+c)^3*\tan(d*x+c)/d+2/3*a*b*\tan(d*x+c)^3/d$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2868, 3852, 3091, 3853, 3855}

$$\int (a + b \cos(c + dx))^2 \sec^5(c + dx) dx = \frac{(3a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{(3a^2 + 4b^2) \tan(c + dx) \sec(c + dx)}{8d} + \frac{a^2 \tan(c + dx) \sec^3(c + dx)}{4d} + \frac{2ab \tan^3(c + dx)}{3d} + \frac{2ab \tan(c + dx)}{d}$$

[In] $\operatorname{Int}[(a + b*\operatorname{Cos}[c + d*x])^2*\operatorname{Sec}[c + d*x]^5,x]$

[Out] $((3a^2 + 4b^2) \operatorname{ArcTanh}[\sin[c + dx]])/(8d) + (2ab \tan[c + dx])/d + ((3a^2 + 4b^2) \sec[c + dx] \tan[c + dx])/(8d) + (a^2 \sec[c + dx]^3 \tan[c + dx])/(4d) + (2ab \tan[c + dx]^3)/(3d)$

Rule 2868

$\operatorname{Int}[(b \sin[e] + f x)^m ((c) + (d \sin[e] + f x) x)^2, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[2c(d/b), \operatorname{Int}[(b \sin[e + fx])^{m+1}, x], x] + \operatorname{Int}[(b \sin[e + fx])^m (c^2 + d^2 \sin[e + fx]^2), x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 3091

$\operatorname{Int}[(b \sin[e] + f x)^m ((A) + (C \sin[e] + f x) x)^2, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[A \cos[e + fx] ((b \sin[e + fx])^{m+1} / (b f (m + 1))), x] + \operatorname{Dist}[(A(m + 2) + C(m + 1)) / (b^2 (m + 1)), \operatorname{Int}[(b \sin[e + fx])^{m+2}, x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3852

$\operatorname{Int}[\csc[(c) + (d) x]^n, x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[-d^{-1}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}, x], x], x, \cot[c + dx]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

$\operatorname{Int}[(\csc[(c) + (d) x] (b))^n, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(-b) \cos[c + dx] ((b \csc[c + dx])^{n-1} / (d(n-1))), x] + \operatorname{Dist}[b^2 ((n-2)/(n-1)), \operatorname{Int}[(b \csc[c + dx])^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

$\operatorname{Int}[\csc[(c) + (d) x], x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[-\operatorname{ArcTanh}[\cos[c + dx]]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= (2ab) \int \sec^4(c + dx) dx + \int (a^2 + b^2 \cos^2(c + dx)) \sec^5(c + dx) dx \\ &= \frac{a^2 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{4} (3a^2 + 4b^2) \int \sec^3(c + dx) dx \\ &\quad - \frac{(2ab) \operatorname{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{d} \end{aligned}$$

$$\begin{aligned}
&= \frac{2ab \tan(c+dx)}{d} + \frac{(3a^2+4b^2) \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{a^2 \sec^3(c+dx) \tan(c+dx)}{4d} + \frac{2ab \tan^3(c+dx)}{3d} + \frac{1}{8}(3a^2+4b^2) \int \sec(c+dx) dx \\
&= \frac{(3a^2+4b^2) \operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{2ab \tan(c+dx)}{d} \\
&\quad + \frac{(3a^2+4b^2) \sec(c+dx) \tan(c+dx)}{8d} \\
&\quad + \frac{a^2 \sec^3(c+dx) \tan(c+dx)}{4d} + \frac{2ab \tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\begin{aligned}
&\int (a+b \cos(c+dx))^2 \sec^5(c+dx) dx \\
&= \frac{3(3a^2+4b^2) \operatorname{arctanh}(\sin(c+dx)) + \tan(c+dx) (3(3a^2+4b^2) \sec(c+dx) + 6a^2 \sec^3(c+dx) + 16ab(3 + \tan^2(c+dx)))}{24d}
\end{aligned}$$

[In] Integrate[(a + b*cos[c + d*x])^2*Sec[c + d*x]^5,x]

[Out] (3*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(3*a^2 + 4*b^2)*Sec[c + d*x] + 6*a^2*Sec[c + d*x]^3 + 16*a*b*(3 + Tan[c + d*x]^2)))/(24*d)

Maple [A] (verified)

Time = 3.76 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^2 \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right) - 2ab \left(- \frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + b^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
default	$\frac{a^2 \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right) - 2ab \left(- \frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + b^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
parts	$\frac{a^2 \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right)}{d} + \frac{b^2 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d}$
parallelrisch	$\frac{-36 \left(a^2 + \frac{4b^2}{3} \right) \left(\frac{3}{4} + \frac{\cos(4dx+4c)}{4} + \cos(2dx+2c) \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 36 \left(a^2 + \frac{4b^2}{3} \right) \left(\frac{3}{4} + \frac{\cos(4dx+4c)}{4} + \cos(2dx+2c) \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)}{24d(\cos(4dx+4c)+4 \cos(2dx+2c))}$
risch	$\frac{i(9a^2 e^{7i(dx+c)} + 12b^2 e^{7i(dx+c)} + 33a^2 e^{5i(dx+c)} + 12b^2 e^{5i(dx+c)} - 96ab e^{4i(dx+c)} - 33a^2 e^{3i(dx+c)} - 12b^2 e^{3i(dx+c)} - 128ab e^{i(dx+c)})}{12d(e^{2i(dx+c)}+1)^4}$
norman	$\frac{\frac{(5a^2-16ab+4b^2)(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right))}{4d} + \frac{(5a^2+16ab+4b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d} + \frac{(21a^2-16ab-12b^2)(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right))}{6d} + \frac{(21a^2+16ab-12b^2)(\tan\left(\frac{dx}{2}+\frac{c}{2}\right))}{6d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 \left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

[In] int((a+cos(d*x+c)*b)^2*sec(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-2*a*b*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int (a + b \cos(c + dx))^2 \sec^5(c + dx) dx$$

$$= \frac{3(3a^2 + 4b^2) \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 3(3a^2 + 4b^2) \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2 \cos(dx + c)^4}{48d \cos(dx + c)^4}$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="fricas")

[Out] 1/48*(3*(3*a^2 + 4*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*a^2 + 4*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(32*a*b*cos(d*x + c)^3 + 16*a*b*cos(d*x + c) + 3*(3*a^2 + 4*b^2)*cos(d*x + c)^2 + 6*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)

Sympy [F]

$$\int (a + b \cos(c + dx))^2 \sec^5(c + dx) dx = \int (a + b \cos(c + dx))^2 \sec^5(c + dx) dx$$

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**5,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.31

$$\int (a + b \cos(c + dx))^2 \sec^5(c + dx) dx$$

$$= \frac{32 (\tan(dx + c)^3 + 3 \tan(dx + c)) ab - 3 a^2 \left(\frac{2 (3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right)}{48 d}$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b - 3*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(102) = 204.

Time = 0.32 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.35

$$\int (a + b \cos(c + dx))^2 \sec^5(c + dx) dx$$

$$= \frac{3(3a^2 + 4b^2) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 3(3a^2 + 4b^2) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) + \frac{2(15a^2 \tan(\frac{1}{2} dx + \frac{1}{2} c) + 15a^2 \tan(\frac{1}{2} dx + \frac{1}{2} c) - 1)}{48d}}{48 d}$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^2 + 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*a^2 + 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*a^2*tan(1/2*d*x + 1/2*c)^7 - 15*a^2*tan(1/2*d*x + 1/2*c) - 15*a^2*tan(1/2*d*x + 1/2*c) + 1)/d
```

$$\frac{1/2*d*x + 1/2*c)^5 + 80*a*b*\tan(1/2*d*x + 1/2*c)^5 - 12*b^2*\tan(1/2*d*x + 1/2*c)^5 + 9*a^2*\tan(1/2*d*x + 1/2*c)^3 - 80*a*b*\tan(1/2*d*x + 1/2*c)^3 - 12*b^2*\tan(1/2*d*x + 1/2*c)^3 + 15*a^2*\tan(1/2*d*x + 1/2*c) + 48*a*b*\tan(1/2*d*x + 1/2*c) + 12*b^2*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^4}{d}$$

Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int (a + b \cos(c + dx))^2 \sec^5(c + dx) dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) \left(\frac{3a^2}{4} + b^2\right)}{d} + \frac{\left(\frac{5a^2}{4} - 4ab + b^2\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{3a^2}{4} + \frac{20ab}{3} - b^2\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{3a^2}{4} - \frac{20ab}{3} - b^2\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)}$$

[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^5,x)

[Out] (atanh(tan(c/2 + (d*x)/2))*((3*a^2)/4 + b^2))/d + (tan(c/2 + (d*x)/2)^5*((20*a*b)/3 + (3*a^2)/4 - b^2) + tan(c/2 + (d*x)/2)*(4*a*b + (5*a^2)/4 + b^2) + tan(c/2 + (d*x)/2)^7*((5*a^2)/4 - 4*a*b + b^2) - tan(c/2 + (d*x)/2)^3*((20*a*b)/3 - (3*a^2)/4 + b^2))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))

3.427 $\int (a + b \cos(c + dx))^2 \sec^6(c + dx) dx$

Optimal result	4733
Rubi [A] (verified)	4733
Mathematica [A] (verified)	4735
Maple [A] (verified)	4736
Fricas [A] (verification not implemented)	4736
Sympy [F(-1)]	4737
Maxima [A] (verification not implemented)	4737
Giac [B] (verification not implemented)	4737
Mupad [B] (verification not implemented)	4738

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (a + b \cos(c + dx))^2 \sec^6(c + dx) dx = \frac{3ab \operatorname{arctanh}(\sin(c + dx))}{4d} + \frac{(4a^2 + 5b^2) \tan(c + dx)}{5d} + \frac{3ab \sec(c + dx) \tan(c + dx)}{4d} + \frac{ab \sec^3(c + dx) \tan(c + dx)}{2d} + \frac{a^2 \sec^4(c + dx) \tan(c + dx)}{5d} + \frac{(4a^2 + 5b^2) \tan^3(c + dx)}{15d}$$

[Out] $\frac{3}{4}ab \operatorname{arctanh}(\sin(dx+c))/d + \frac{1}{5}(4a^2+5b^2) \tan(dx+c)/d + \frac{3}{4}ab \sec(dx+c) \tan(dx+c)/d + \frac{1}{2}ab \sec(dx+c)^3 \tan(dx+c)/d + \frac{1}{5}a^2 \sec(dx+c)^4 \tan(dx+c)/d + \frac{1}{15}(4a^2+5b^2) \tan(dx+c)^3/d$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

= {2868, 3853, 3855, 3091, 3852}

$$\int (a + b \cos(c + dx))^2 \sec^6(c + dx) dx = \frac{(4a^2 + 5b^2) \tan^3(c + dx)}{15d} + \frac{(4a^2 + 5b^2) \tan(c + dx)}{5d} + \frac{a^2 \tan(c + dx) \sec^4(c + dx)}{5d} + \frac{3ab \operatorname{arctanh}(\sin(c + dx))}{4d} + \frac{ab \tan(c + dx) \sec^3(c + dx)}{2d} + \frac{3ab \tan(c + dx) \sec(c + dx)}{4d}$$

[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^6,x]

[Out] (3*a*b*ArcTanh[Sin[c + d*x]]/(4*d) + ((4*a^2 + 5*b^2)*Tan[c + d*x])/(5*d) + (3*a*b*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a*b*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) + (a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((4*a^2 + 5*b^2)*Tan[c + d*x]^3)/(15*d)

Rule 2868

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*SIN[e + f*x])^(m + 1), x], x] + Int[(b*SIN[e + f*x])^m*(c^2 + d^2*SIN[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3091

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[A*Cos[e + f*x]*((b*SIN[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*SIN[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3852

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= (2ab) \int \sec^5(c + dx) dx + \int (a^2 + b^2 \cos^2(c + dx)) \sec^6(c + dx) dx \\
 &= \frac{ab \sec^3(c + dx) \tan(c + dx)}{2d} + \frac{a^2 \sec^4(c + dx) \tan(c + dx)}{5d} \\
 &\quad + \frac{1}{2}(3ab) \int \sec^3(c + dx) dx + \frac{1}{5}(4a^2 + 5b^2) \int \sec^4(c + dx) dx \\
 &= \frac{3ab \sec(c + dx) \tan(c + dx)}{4d} + \frac{ab \sec^3(c + dx) \tan(c + dx)}{2d} + \frac{a^2 \sec^4(c + dx) \tan(c + dx)}{5d} \\
 &\quad + \frac{1}{4}(3ab) \int \sec(c + dx) dx - \frac{(4a^2 + 5b^2) \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{5d} \\
 &= \frac{3ab \arctanh(\sin(c + dx))}{4d} + \frac{(4a^2 + 5b^2) \tan(c + dx)}{5d} + \frac{3ab \sec(c + dx) \tan(c + dx)}{4d} \\
 &\quad + \frac{ab \sec^3(c + dx) \tan(c + dx)}{2d} + \frac{a^2 \sec^4(c + dx) \tan(c + dx)}{5d} + \frac{(4a^2 + 5b^2) \tan^3(c + dx)}{15d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\begin{aligned}
 &\int (a + b \cos(c + dx))^2 \sec^6(c + dx) dx \\
 &= \frac{45ab \arctanh(\sin(c + dx)) + \tan(c + dx) (60(a^2 + b^2) + 45ab \sec(c + dx) + 30ab \sec^3(c + dx) + 20(2a^2 + b^2) \tan^2(c + dx) + 12a^2 \tan^4(c + dx))}{60d}
 \end{aligned}$$

`[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^6,x]`

`[Out] (45*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(60*(a^2 + b^2) + 45*a*b*Sec[c + d*x] + 30*a*b*Sec[c + d*x]^3 + 20*(2*a^2 + b^2)*Tan[c + d*x]^2 + 12*a^2*Tan[c + d*x]^4))/(60*d)`

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sec^6(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int (a + b \cos(c + dx))^2 \sec^6(c + dx) dx$$

$$= \frac{8(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c))a^2 + 40(\tan(dx + c)^3 + 3 \tan(dx + c))b^2 - 15 a b \log\left(\frac{\tan(dx + c) + 1}{\tan(dx + c) - 1}\right)}{120 d}$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + 40*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^2 - 15*a*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(123) = 246.

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.01

$$\int (a + b \cos(c + dx))^2 \sec^6(c + dx) dx$$

$$= \frac{45 ab \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 45 ab \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - 2\left(60 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 - 75 ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 60 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 80 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 80 ab \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 80 b^2\right)}{120 d}$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/60*(45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*d*x + 1/2*c)^7 + 60*b^2*tan(1/2*d*x + 1/2*c)^5 - 80*a^2*tan(1/2*d*x + 1/2*c)^3 + 80*a*b*tan(1/2*d*x + 1/2*c) - 80*b^2))
```

$$30ab \tan(1/2 dx + 1/2 c)^7 - 160b^2 \tan(1/2 dx + 1/2 c)^7 + 232a^2 \tan(1/2 dx + 1/2 c)^5 + 200b^2 \tan(1/2 dx + 1/2 c)^5 - 80a^2 \tan(1/2 dx + 1/2 c)^3 - 30ab \tan(1/2 dx + 1/2 c)^3 - 160b^2 \tan(1/2 dx + 1/2 c)^3 + 60a^2 \tan(1/2 dx + 1/2 c) + 75ab \tan(1/2 dx + 1/2 c) + 60b^2 \tan(1/2 dx + 1/2 c) / (\tan(1/2 dx + 1/2 c)^2 - 1)^5 / d$$

Mupad [B] (verification not implemented)

Time = 18.54 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.64

$$\int (a + b \cos(c + dx))^2 \sec^6(c + dx) dx = \frac{3ab \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{2d} - \frac{\left(2a^2 - \frac{5ab}{2} + 2b^2\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(-\frac{8a^2}{3} + ab - \frac{16b^2}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{116a^2}{15} + \frac{20b^2}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^6,x)

[Out] (3*a*b*atanh(tan(c/2 + (d*x)/2)))/(2*d) - (tan(c/2 + (d*x)/2)^5*((116*a^2)/15 + (20*b^2)/3) + tan(c/2 + (d*x)/2)^9*(2*a^2 - (5*a*b)/2 + 2*b^2) - tan(c/2 + (d*x)/2)^3*(a*b + (8*a^2)/3 + (16*b^2)/3) - tan(c/2 + (d*x)/2)^7*((8*a^2)/3 - a*b + (16*b^2)/3) + tan(c/2 + (d*x)/2)*((5*a*b)/2 + 2*a^2 + 2*b^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))

3.428 $\int \cos^3(c + dx)(a + b \cos(c + dx))^3 dx$

Optimal result	4739
Rubi [A] (verified)	4739
Mathematica [A] (verified)	4742
Maple [A] (verified)	4742
Fricas [A] (verification not implemented)	4743
Sympy [B] (verification not implemented)	4744
Maxima [A] (verification not implemented)	4744
Giac [A] (verification not implemented)	4745
Mupad [B] (verification not implemented)	4745

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^3 dx = \frac{9}{8}a^2bx + \frac{5b^3x}{16} + \frac{a(a^2 + 3b^2) \sin(c + dx)}{d} + \frac{b(18a^2 + 5b^2) \cos(c + dx) \sin(c + dx)}{16d} + \frac{b(18a^2 + 5b^2) \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{b^3 \cos^5(c + dx) \sin(c + dx)}{6d} - \frac{a(a^2 + 6b^2) \sin^3(c + dx)}{3d} + \frac{3ab^2 \sin^5(c + dx)}{5d}$$

```
[Out] 9/8*a^2*b*x+5/16*b^3*x+a*(a^2+3*b^2)*sin(d*x+c)/d+1/16*b*(18*a^2+5*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*b*(18*a^2+5*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*b^3*cos(d*x+c)^5*sin(d*x+c)/d-1/3*a*(a^2+6*b^2)*sin(d*x+c)^3/d+3/5*a*b^2*sin(d*x+c)^5/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {2872, 3102, 2827, 2713, 2715, 8}

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^3 dx = -\frac{a(5a^2 + 12b^2) \sin^3(c + dx)}{15d} + \frac{a(5a^2 + 12b^2) \sin(c + dx)}{5d} + \frac{b(18a^2 + 5b^2) \sin(c + dx) \cos^3(c + dx)}{24d} + \frac{b(18a^2 + 5b^2) \sin(c + dx) \cos(c + dx)}{16d} + \frac{1}{16}bx(18a^2 + 5b^2) + \frac{b^2 \sin(c + dx) \cos^4(c + dx)(a + b \cos(c + dx))}{6d} + \frac{13ab^2 \sin(c + dx) \cos^4(c + dx)}{30d}$$

[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^3,x]

[Out] (b*(18*a^2 + 5*b^2)*x)/16 + (a*(5*a^2 + 12*b^2)*Sin[c + d*x])/(5*d) + (b*(18*a^2 + 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (b*(18*a^2 + 5*b^2)*Cos[c + d*x]^3*Ssin[c + d*x])/(24*d) + (13*a*b^2*Cos[c + d*x]^4*Ssin[c + d*x])/(30*d) + (b^2*Cos[c + d*x]^4*(a + b*Cos[c + d*x])*Sin[c + d*x])/(6*d) - (a*(5*a^2 + 12*b^2)*Sin[c + d*x]^3)/(15*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*
x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Ssin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \cos^4(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{6d} + \frac{1}{6} \int \cos^3(c + dx) (2a(3a^2 + 2b^2) \\
&\quad + b(18a^2 + 5b^2) \cos(c + dx) + 13ab^2 \cos^2(c + dx)) dx \\
&= \frac{13ab^2 \cos^4(c + dx) \sin(c + dx)}{30d} + \frac{b^2 \cos^4(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{6d} \\
&\quad + \frac{1}{30} \int \cos^3(c + dx) (6a(5a^2 + 12b^2) + 5b(18a^2 + 5b^2) \cos(c + dx)) dx \\
&= \frac{13ab^2 \cos^4(c + dx) \sin(c + dx)}{30d} + \frac{b^2 \cos^4(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{6d} \\
&\quad + \frac{1}{6} (b(18a^2 + 5b^2)) \int \cos^4(c + dx) dx + \frac{1}{5} (a(5a^2 + 12b^2)) \int \cos^3(c + dx) dx \\
&= \frac{b(18a^2 + 5b^2) \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{13ab^2 \cos^4(c + dx) \sin(c + dx)}{30d} \\
&\quad + \frac{b^2 \cos^4(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{6d} \\
&\quad + \frac{1}{8} (b(18a^2 + 5b^2)) \int \cos^2(c + dx) dx \\
&\quad - \frac{(a(5a^2 + 12b^2)) \text{Subst}(\int (1 - x^2) dx, x, -\sin(c + dx))}{5d}
\end{aligned}$$

$$\begin{aligned}
&= \frac{a(5a^2 + 12b^2) \sin(c + dx)}{5d} + \frac{b(18a^2 + 5b^2) \cos(c + dx) \sin(c + dx)}{16d} \\
&\quad + \frac{b(18a^2 + 5b^2) \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{13ab^2 \cos^4(c + dx) \sin(c + dx)}{30d} \\
&\quad + \frac{b^2 \cos^4(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{6d} \\
&\quad - \frac{a(5a^2 + 12b^2) \sin^3(c + dx)}{15d} + \frac{1}{16}(b(18a^2 + 5b^2)) \int 1 dx \\
&= \frac{1}{16}b(18a^2 + 5b^2) x + \frac{a(5a^2 + 12b^2) \sin(c + dx)}{5d} + \frac{b(18a^2 + 5b^2) \cos(c + dx) \sin(c + dx)}{16d} \\
&\quad + \frac{b(18a^2 + 5b^2) \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{13ab^2 \cos^4(c + dx) \sin(c + dx)}{30d} \\
&\quad + \frac{b^2 \cos^4(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{6d} - \frac{a(5a^2 + 12b^2) \sin^3(c + dx)}{15d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int \cos^3(c + dx)(a + b \cos(c + dx))^3 dx \\
&= \frac{1080a^2bc + 300b^3c + 1080a^2bdx + 300b^3dx + 360a(2a^2 + 5b^2) \sin(c + dx) + 45(16a^2b + 5b^3) \sin(2(c + dx))}{60d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^3,x]

[Out] (1080*a^2*b*c + 300*b^3*c + 1080*a^2*b*d*x + 300*b^3*d*x + 360*a*(2*a^2 + 5*b^2)*Sin[c + d*x] + 45*(16*a^2*b + 5*b^3)*Sin[2*(c + d*x)] + 80*a^3*Ssin[3*(c + d*x)] + 300*a*b^2*Ssin[3*(c + d*x)] + 90*a^2*b*Ssin[4*(c + d*x)] + 45*b^3*Ssin[4*(c + d*x)] + 36*a*b^2*Ssin[5*(c + d*x)] + 5*b^3*Ssin[6*(c + d*x)])/(90*d)

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{(720a^2b+225b^3) \sin(2dx+2c)+(80a^3+300ab^2) \sin(3dx+3c)+(90a^2b+45b^3) \sin(4dx+4c)+36ab^2 \sin(5dx+5c)+5b^3 \sin(6dx+6c)+(720a^3+1800ab^2) \sin(dx+c)+1080b^3d(a^2+5/18b^2)x}{960d}$
derivativdivides	$\frac{a^3(2+\cos^2(dx+c)) \sin(dx+c)}{3} + 3a^2b \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{3ab^2 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right)}{5}$
default	$\frac{a^3(2+\cos^2(dx+c)) \sin(dx+c)}{3} + 3a^2b \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{3ab^2 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right)}{5}$
parts	$\frac{a^3(2+\cos^2(dx+c)) \sin(dx+c)}{3d} + \frac{b^3 \left(\frac{(\cos^5(dx+c) + \frac{5(\cos^3(dx+c))}{4} + \frac{15\cos(dx+c)}{8}) \sin(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16} \right)}{d} + \frac{3ab^2 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right)}{5}$
risch	$\frac{9a^2bx}{8} + \frac{5b^3x}{16} + \frac{3a^3 \sin(dx+c)}{4d} + \frac{15ab^2 \sin(dx+c)}{8d} + \frac{b^3 \sin(6dx+6c)}{192d} + \frac{3ab^2 \sin(5dx+5c)}{80d} + \frac{3 \sin(4dx+4c)a^2}{32d}$
norman	$\frac{(\frac{9}{8}a^2b + \frac{5}{16}b^3)x + (\frac{9}{8}a^2b + \frac{5}{16}b^3)x \left(\tan^{12} \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + (\frac{27}{4}a^2b + \frac{15}{8}b^3)x \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + (\frac{27}{4}a^2b + \frac{15}{8}b^3)x \left(\tan^{10} \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{240d}$

[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] 1/960*((720*a^2*b+225*b^3)*sin(2*d*x+2*c)+(80*a^3+300*a*b^2)*sin(3*d*x+3*c)+(90*a^2*b+45*b^3)*sin(4*d*x+4*c)+36*a*b^2*sin(5*d*x+5*c)+5*b^3*sin(6*d*x+6*c)+(720*a^3+1800*a*b^2)*sin(d*x+c)+1080*b^3*d*(a^2+5/18*b^2)*x)/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^3 dx = \frac{15(18a^2b+5b^3)dx + (40b^3\cos(dx+c))^5 + 144ab^2\cos(dx+c)^4 + 10(18a^2b+5b^3)\cos(dx+c)^3 + 160a^3 + 384ab^2 + 16(5a^3+12ab^2)\cos(dx+c)^2 + 15(18a^2b+5b^3)\cos(dx+c)}{240d} \sin(dx+c)$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/240*(15*(18*a^2*b + 5*b^3)*d*x + (40*b^3*cos(d*x + c))^5 + 144*a*b^2*cos(d*x + c)^4 + 10*(18*a^2*b + 5*b^3)*cos(d*x + c)^3 + 160*a^3 + 384*a*b^2 + 16*(5*a^3 + 12*a*b^2)*cos(d*x + c)^2 + 15*(18*a^2*b + 5*b^3)*cos(d*x + c))*sin(d*x + c)/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(158) = 316.

Time = 0.37 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.31

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^3 dx$$

$$= \begin{cases} \frac{2a^3 \sin^3(c+dx)}{3d} + \frac{a^3 \sin(c+dx) \cos^2(c+dx)}{d} + \frac{9a^2 b x \sin^4(c+dx)}{8} + \frac{9a^2 b x \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{9a^2 b x \cos^4(c+dx)}{8} + \frac{9a^2 b \sin^3(c+dx)}{8} \\ x(a + b \cos(c))^3 \cos^3(c) \end{cases}$$

[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**3,x)

[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d + 9*a**2*b*x*sin(c + d*x)**4/8 + 9*a**2*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 9*a**2*b*x*cos(c + d*x)**4/8 + 9*a**2*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 15*a**2*b*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*a*b**2*sin(c + d*x)**5/(5*d) + 4*a*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*a*b**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*b**3*x*sin(c + d*x)**6/16 + 15*b**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*b**3*x*cos(c + d*x)**6/16 + 5*b**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b**3*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))**3*cos(c)**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^3 dx =$$

$$\frac{320 (\sin(dx + c)^3 - 3 \sin(dx + c)) a^3 - 90 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(2 dx + 2 c)) a^2 b - 192 b^3 \sin(dx + c)}{d}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] -1/960*(320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a*b^2 + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^3)/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.88

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^3 dx = \frac{b^3 \sin(6 dx + 6 c)}{192 d} + \frac{3 ab^2 \sin(5 dx + 5 c)}{80 d} + \frac{1}{16} (18 a^2 b + 5 b^3) x + \frac{3(2 a^2 b + b^3) \sin(4 dx + 4 c)}{64 d} + \frac{(4 a^3 + 15 ab^2) \sin(3 dx + 3 c)}{48 d} + \frac{3(16 a^2 b + 5 b^3) \sin(2 dx + 2 c)}{64 d} + \frac{3(2 a^3 + 5 ab^2) \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] 1/192*b^3*sin(6*d*x + 6*c)/d + 3/80*a*b^2*sin(5*d*x + 5*c)/d + 1/16*(18*a^2*b + 5*b^3)*x + 3/64*(2*a^2*b + b^3)*sin(4*d*x + 4*c)/d + 1/48*(4*a^3 + 15*a*b^2)*sin(3*d*x + 3*c)/d + 3/64*(16*a^2*b + 5*b^3)*sin(2*d*x + 2*c)/d + 3/8*(2*a^3 + 5*a*b^2)*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 15.87 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.24

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^3 dx = \frac{\left(2 a^3 - \frac{15 a^2 b}{4} + 6 a b^2 - \frac{11 b^3}{8}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} + \left(\frac{22 a^3}{3} - \frac{21 a^2 b}{4} + 14 a b^2 + \frac{5 b^3}{24}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(12 a^3 - \frac{15 a^2 b}{4} + 6 a b^2 - \frac{11 b^3}{8}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{22 a^3}{3} - \frac{21 a^2 b}{4} + 14 a b^2 + \frac{5 b^3}{24}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(12 a^3 - \frac{15 a^2 b}{4} + 6 a b^2 - \frac{11 b^3}{8}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(\frac{22 a^3}{3} - \frac{21 a^2 b}{4} + 14 a b^2 + \frac{5 b^3}{24}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + \frac{b \operatorname{atan}\left(\frac{b \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (18 a^2 + 5 b^2)}{8 \left(\frac{9 a^2 b}{4} + \frac{5 b^3}{8}\right)}\right) (18 a^2 + 5 b^2)}{8 d} + \frac{b (18 a^2 + 5 b^2) \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{8 d}$$

[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^3,x)

[Out] (tan(c/2 + (d*x)/2)^11*(6*a*b^2 - (15*a^2*b)/4 + 2*a^3 - (11*b^3)/8) + tan(c/2 + (d*x)/2)^3*(14*a*b^2 + (21*a^2*b)/4 + (22*a^3)/3 - (5*b^3)/24) + tan(c/2 + (d*x)/2)^7*(12*a^3 - (15*a^2*b)/4 + 6*a*b^2 - (11*b^3)/8) + tan(c/2 + (d*x)/2)^5*(12*a^3 - (15*a^2*b)/4 + 6*a*b^2 - (11*b^3)/8) + tan(c/2 + (d*x)/2)^9*(12*a^3 - (15*a^2*b)/4 + 6*a*b^2 - (11*b^3)/8) + tan(c/2 + (d*x)/2)^11*(12*a^3 - (15*a^2*b)/4 + 6*a*b^2 - (11*b^3)/8)

$$\begin{aligned}
& c/2 + (d*x)/2)^9*(14*a*b^2 - (21*a^2*b)/4 + (22*a^3)/3 + (5*b^3)/24) + \tan(\\
& c/2 + (d*x)/2)^5*((156*a*b^2)/5 + (3*a^2*b)/2 + 12*a^3 + (15*b^3)/4) + \tan(\\
& c/2 + (d*x)/2)^7*((156*a*b^2)/5 - (3*a^2*b)/2 + 12*a^3 - (15*b^3)/4) + \tan(\\
& c/2 + (d*x)/2)*(6*a*b^2 + (15*a^2*b)/4 + 2*a^3 + (11*b^3)/8))/(d*(6*\tan(c/2 \\
& + (d*x)/2)^2 + 15*\tan(c/2 + (d*x)/2)^4 + 20*\tan(c/2 + (d*x)/2)^6 + 15*\tan(\\
& c/2 + (d*x)/2)^8 + 6*\tan(c/2 + (d*x)/2)^10 + \tan(c/2 + (d*x)/2)^12 + 1)) + \\
& (b*\operatorname{atan}((b*\tan(c/2 + (d*x)/2)*(18*a^2 + 5*b^2))/(8*((9*a^2*b)/4 + (5*b^3)/8 \\
&))*(18*a^2 + 5*b^2)))/(8*d) - (b*(18*a^2 + 5*b^2)*(atan(\tan(c/2 + (d*x)/2)) \\
& - (d*x)/2)))/(8*d)
\end{aligned}$$

3.429 $\int \cos^2(c + dx)(a + b \cos(c + dx))^3 dx$

Optimal result	4747
Rubi [A] (verified)	4747
Mathematica [A] (verified)	4749
Maple [A] (verified)	4750
Fricas [A] (verification not implemented)	4750
Sympy [A] (verification not implemented)	4751
Maxima [A] (verification not implemented)	4751
Giac [A] (verification not implemented)	4752
Mupad [B] (verification not implemented)	4752

Optimal result

Integrand size = 21, antiderivative size = 180

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^3 dx = \frac{1}{8}a(4a^2 + 9b^2)x - \frac{(3a^4 - 52a^2b^2 - 16b^4) \sin(c + dx)}{30bd} - \frac{a(6a^2 - 71b^2) \cos(c + dx) \sin(c + dx)}{120d} - \frac{(3a^2 - 16b^2)(a + b \cos(c + dx))^2 \sin(c + dx)}{60bd} - \frac{a(a + b \cos(c + dx))^3 \sin(c + dx)}{20bd} + \frac{(a + b \cos(c + dx))^4 \sin(c + dx)}{5bd}$$

```
[Out] 1/8*a*(4*a^2+9*b^2)*x-1/30*(3*a^4-52*a^2*b^2-16*b^4)*sin(d*x+c)/b/d-1/120*a
*(6*a^2-71*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/60*(3*a^2-16*b^2)*(a+b*cos(d*x+c)
)^2*sin(d*x+c)/b/d-1/20*a*(a+b*cos(d*x+c))^3*sin(d*x+c)/b/d+1/5*(a+b*cos(d*
x+c))^4*sin(d*x+c)/b/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

= {2870, 2832, 2813}

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^3 dx = -\frac{(3a^2 - 16b^2) \sin(c + dx)(a + b \cos(c + dx))^2}{60bd} - \frac{a(6a^2 - 71b^2) \sin(c + dx) \cos(c + dx)}{120d} + \frac{1}{8}ax(4a^2 + 9b^2) - \frac{(3a^4 - 52a^2b^2 - 16b^4) \sin(c + dx)}{30bd} + \frac{\sin(c + dx)(a + b \cos(c + dx))^4}{5bd} - \frac{a \sin(c + dx)(a + b \cos(c + dx))^3}{20bd}$$

[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3,x]

[Out] (a*(4*a^2 + 9*b^2)*x)/8 - ((3*a^4 - 52*a^2*b^2 - 16*b^4)*Sin[c + d*x])/(30*b*d) - (a*(6*a^2 - 71*b^2)*Cos[c + d*x]*Sin[c + d*x])/(120*d) - ((3*a^2 - 16*b^2)*(a + b*Cos[c + d*x])^2*SIN[c + d*x])/(60*b*d) - (a*(a + b*Cos[c + d*x])^3*SIN[c + d*x])/(20*b*d) + ((a + b*Cos[c + d*x])^4*SIN[c + d*x])/(5*b*d)

Rule 2813

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Cos[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*SIN[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*SIN[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2870

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*SIN[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*SIN[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{(a + b \cos(c + dx))^4 \sin(c + dx)}{5bd} + \frac{\int (4b - a \cos(c + dx))(a + b \cos(c + dx))^3 dx}{5b} \\
 &= -\frac{a(a + b \cos(c + dx))^3 \sin(c + dx)}{20bd} + \frac{(a + b \cos(c + dx))^4 \sin(c + dx)}{5bd} \\
 &\quad + \frac{\int (a + b \cos(c + dx))^2 (13ab - (3a^2 - 16b^2) \cos(c + dx)) dx}{20b} \\
 &= -\frac{(3a^2 - 16b^2)(a + b \cos(c + dx))^2 \sin(c + dx)}{60bd} \\
 &\quad - \frac{a(a + b \cos(c + dx))^3 \sin(c + dx)}{20bd} + \frac{(a + b \cos(c + dx))^4 \sin(c + dx)}{5bd} \\
 &\quad + \frac{\int (a + b \cos(c + dx))(b(33a^2 + 32b^2) - a(6a^2 - 71b^2) \cos(c + dx)) dx}{60b} \\
 &= \frac{1}{8}a(4a^2 + 9b^2)x - \frac{(3a^4 - 52a^2b^2 - 16b^4) \sin(c + dx)}{30bd} \\
 &\quad - \frac{a(6a^2 - 71b^2) \cos(c + dx) \sin(c + dx)}{120d} \\
 &\quad - \frac{(3a^2 - 16b^2)(a + b \cos(c + dx))^2 \sin(c + dx)}{60bd} \\
 &\quad - \frac{a(a + b \cos(c + dx))^3 \sin(c + dx)}{20bd} + \frac{(a + b \cos(c + dx))^4 \sin(c + dx)}{5bd}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.72

$$\begin{aligned}
 &\int \cos^2(c + dx)(a + b \cos(c + dx))^3 dx \\
 &= \frac{240a^3c + 540ab^2c + 240a^3dx + 540ab^2dx + 60b(18a^2 + 5b^2) \sin(c + dx) + 120(a^3 + 3ab^2) \sin(2(c + dx))}{480d}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3,x]

[Out] (240*a^3*c + 540*a*b^2*c + 240*a^3*d*x + 540*a*b^2*d*x + 60*b*(18*a^2 + 5*b^2)*Sin[c + d*x] + 120*(a^3 + 3*a*b^2)*Sin[2*(c + d*x)] + 120*a^2*b*Ssin[3*(c + d*x)] + 50*b^3*Ssin[3*(c + d*x)] + 45*a*b^2*Ssin[4*(c + d*x)] + 6*b^3*Ssin[5*(c + d*x)]/(480*d)

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{(120a^3+360ab^2)\sin(2dx+2c)+(120a^2b+50b^3)\sin(3dx+3c)+45ab^2\sin(4dx+4c)+6b^3\sin(5dx+5c)+(1080a^2b+300b^3)\sin(dx+c)}{480d}$
derivativedivides	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2b(2+\cos^2(dx+c))\sin(dx+c)+3ab^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2b(2+\cos^2(dx+c))\sin(dx+c)+3ab^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
parts	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{b^3\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\cos^2(dx+c)}{3}\right)\sin(dx+c)}{5d} + \frac{3ab^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}\right)}{d}$
risch	$\frac{a^3x}{2} + \frac{9ab^2x}{8} + \frac{9\sin(dx+c)a^2b}{4d} + \frac{5\sin(dx+c)b^3}{8d} + \frac{b^3\sin(5dx+5c)}{80d} + \frac{3ab^2\sin(4dx+4c)}{32d} + \frac{\sin(3dx+3c)a^2b}{4d} + \frac{\sin(dx+c)a^3}{4d}$
norman	$\frac{\left(\frac{1}{2}a^3+\frac{9}{8}ab^2\right)x+(5a^3+\frac{45}{4}ab^2)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(5a^3+\frac{45}{4}ab^2\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^3+\frac{9}{8}ab^2\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^3+\frac{9}{8}ab^2\right)x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{120d}$

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/480*((120*a^3+360*a*b^2)*sin(2*d*x+2*c)+(120*a^2*b+50*b^3)*sin(3*d*x+3*c)
+45*a*b^2*sin(4*d*x+4*c)+6*b^3*sin(5*d*x+5*c)+(1080*a^2*b+300*b^3)*sin(d*x+
c)+240*(a^2+9/4*b^2)*d*x*a)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.61

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3 dx$$

$$= \frac{15(4a^3+9ab^2)dx + (24b^3\cos(dx+c)^4 + 90ab^2\cos(dx+c)^3 + 240a^2b + 64b^3 + 8(15a^2b + 4b^3)\cos(dx+c))}{120d}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(4*a^3 + 9*a*b^2)*d*x + (24*b^3*cos(d*x + c)^4 + 90*a*b^2*cos(d*x
+ c)^3 + 240*a^2*b + 64*b^3 + 8*(15*a^2*b + 4*b^3)*cos(d*x + c)^2 + 15*(4*
a^3 + 9*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```


Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.69

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^3 dx = \frac{b^3 \sin(5 dx + 5 c)}{80 d} + \frac{3 ab^2 \sin(4 dx + 4 c)}{32 d} + \frac{1}{8} (4 a^3 + 9 ab^2)x + \frac{(12 a^2 b + 5 b^3) \sin(3 dx + 3 c)}{48 d} + \frac{(a^3 + 3 ab^2) \sin(2 dx + 2 c)}{4 d} + \frac{(18 a^2 b + 5 b^3) \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] 1/80*b^3*sin(5*d*x + 5*c)/d + 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/8*(4*a^3 + 9*a*b^2)*x + 1/48*(12*a^2*b + 5*b^3)*sin(3*d*x + 3*c)/d + 1/4*(a^3 + 3*a*b^2)*sin(2*d*x + 2*c)/d + 1/8*(18*a^2*b + 5*b^3)*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 15.87 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.77

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^3 dx = \frac{\left(-a^3 + 6 a^2 b - \frac{15 a b^2}{4} + 2 b^3\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(-2 a^3 + 16 a^2 b - \frac{3 a b^2}{2} + \frac{8 b^3}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(20 a^2 b + \dots\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + \dots\right)} + \frac{a \operatorname{atan}\left(\frac{a \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (4 a^2 + 9 b^2)}{4 (a^3 + \frac{9 a b^2}{4})}\right) (4 a^2 + 9 b^2)}{4 d} - \frac{a (4 a^2 + 9 b^2) \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{4 d}$$

[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^3,x)

[Out] (tan(c/2 + (d*x)/2)^3*((3*a*b^2)/2 + 16*a^2*b + 2*a^3 + (8*b^3)/3) - tan(c/2 + (d*x)/2)^7*((3*a*b^2)/2 - 16*a^2*b + 2*a^3 - (8*b^3)/3) + tan(c/2 + (d*x)/2)*((15*a*b^2)/4 + 6*a^2*b + a^3 + 2*b^3) + tan(c/2 + (d*x)/2)^5*(20*a^2*b + (116*b^3)/15) - tan(c/2 + (d*x)/2)^9*((15*a*b^2)/4 - 6*a^2*b + a^3 - 2*b^3))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (a*atan((a*tan(c/2 + (d*x)/2)*(4*a^2 + 9*b^2))/(4*((9*a*b^2)/4 + a^3)))*(4*a^2 + 9*b^2))/(4*d) - (a*(4*a^2 + 9*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d)

3.430 $\int \cos(c + dx)(a + b \cos(c + dx))^3 dx$

Optimal result	4753
Rubi [A] (verified)	4753
Mathematica [A] (verified)	4754
Maple [A] (verified)	4755
Fricas [A] (verification not implemented)	4755
Sympy [B] (verification not implemented)	4756
Maxima [A] (verification not implemented)	4756
Giac [A] (verification not implemented)	4757
Mupad [B] (verification not implemented)	4757

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \cos(c + dx)(a + b \cos(c + dx))^3 dx = \frac{3}{8}b(4a^2 + b^2)x + \frac{a(a^2 + 4b^2)\sin(c + dx)}{2d} + \frac{b(2a^2 + 3b^2)\cos(c + dx)\sin(c + dx)}{8d} + \frac{a(a + b \cos(c + dx))^2 \sin(c + dx)}{4d} + \frac{(a + b \cos(c + dx))^3 \sin(c + dx)}{4d}$$

[Out] $\frac{3}{8}b(4a^2 + b^2)x + \frac{1}{2}a(a^2 + 4b^2)\frac{\sin(dx + c)}{d} + \frac{1}{8}b(2a^2 + 3b^2)\cos(dx + c)\frac{\sin(dx + c)}{d} + \frac{1}{4}a(a + b\cos(dx + c))^2\frac{\sin(dx + c)}{d} + \frac{1}{4}(a + b\cos(dx + c))^3\frac{\sin(dx + c)}{d}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2832, 2813}

$$\int \cos(c + dx)(a + b \cos(c + dx))^3 dx = \frac{a(a^2 + 4b^2)\sin(c + dx)}{2d} + \frac{b(2a^2 + 3b^2)\sin(c + dx)\cos(c + dx)}{8d} + \frac{3}{8}bx(4a^2 + b^2) + \frac{\sin(c + dx)(a + b \cos(c + dx))^3}{4d} + \frac{a \sin(c + dx)(a + b \cos(c + dx))^2}{4d}$$

[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^3,x]

[Out] (3*b*(4*a^2 + b^2)*x)/8 + (a*(a^2 + 4*b^2)*Sin[c + d*x])/(2*d) + (b*(2*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(a + b*Cos[c + d*x])^2*SIN[c + d*x])/(4*d) + ((a + b*Cos[c + d*x])^3*SIN[c + d*x])/(4*d)

Rule 2813

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Cos[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*SIN[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*SIN[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(a + b \cos(c + dx))^3 \sin(c + dx)}{4d} + \frac{1}{4} \int (3b + 3a \cos(c + dx))(a + b \cos(c + dx))^2 dx \\ &= \frac{a(a + b \cos(c + dx))^2 \sin(c + dx)}{4d} + \frac{(a + b \cos(c + dx))^3 \sin(c + dx)}{4d} \\ &\quad + \frac{1}{12} \int (a + b \cos(c + dx)) (15ab + 3(2a^2 + 3b^2) \cos(c + dx)) dx \\ &= \frac{3}{8} b(4a^2 + b^2) x + \frac{a(a^2 + 4b^2) \sin(c + dx)}{2d} + \frac{b(2a^2 + 3b^2) \cos(c + dx) \sin(c + dx)}{8d} \\ &\quad + \frac{a(a + b \cos(c + dx))^2 \sin(c + dx)}{4d} + \frac{(a + b \cos(c + dx))^3 \sin(c + dx)}{4d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\begin{aligned} &\int \cos(c + dx)(a + b \cos(c + dx))^3 dx \\ &= \frac{8a(4a^2 + 9b^2) \sin(c + dx) + b(48a^2c + 12b^2c + 48a^2dx + 12b^2dx + 8(3a^2 + b^2) \sin(2(c + dx))) + 8ab \sin(3(c + dx))}{32d} \end{aligned}$$

[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^3,x]

[Out] (8*a*(4*a^2 + 9*b^2)*Sin[c + d*x] + b*(48*a^2*c + 12*b^2*c + 48*a^2*d*x + 12*b^2*d*x + 8*(3*a^2 + b^2)*Sin[2*(c + d*x)] + 8*a*b*SIN[3*(c + d*x)] + b^2*SIN[4*(c + d*x)])/(32*d)

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{a^3 \sin(dx+c) + 3a^2 b \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + a b^2 (2 + \cos^2(dx+c)) \sin(dx+c) + b^3 \left(\frac{\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}}{4} \right) \sin(dx+c)}{d}$
default	$\frac{a^3 \sin(dx+c) + 3a^2 b \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + a b^2 (2 + \cos^2(dx+c)) \sin(dx+c) + b^3 \left(\frac{\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}}{4} \right) \sin(dx+c)}{d}$
parallelrisch	$\frac{48a^2 b dx + 12b^3 dx + 32a^3 \sin(dx+c) + 72 \sin(dx+c) a b^2 + \sin(4dx+4c) b^3 + 8 \sin(3dx+3c) a b^2 + 24 \sin(2dx+2c) a^2 b + 8 \sin(dx+c) a^3}{32d}$
parts	$\frac{a^3 \sin(dx+c)}{d} + \frac{b^3 \left(\frac{\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2}}{4} \right) \sin(dx+c) + \frac{3dx}{8} + \frac{3c}{8}}{d} + \frac{a b^2 (2 + \cos^2(dx+c)) \sin(dx+c)}{d} + \frac{3a^2 b \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d}$
risch	$\frac{3a^2 b x}{2} + \frac{3b^3 x}{8} + \frac{a^3 \sin(dx+c)}{d} + \frac{9a b^2 \sin(dx+c)}{4d} + \frac{\sin(4dx+4c) b^3}{32d} + \frac{\sin(3dx+3c) a b^2}{4d} + \frac{3 \sin(2dx+2c) a^2 b}{4d} + \frac{3 \sin(dx+c) a^3}{4d}$
norman	$\left(\frac{3}{2} a^2 b + \frac{3}{8} b^3 \right) x + (6a^2 b + \frac{3}{2} b^3) x \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + (6a^2 b + \frac{3}{2} b^3) x \left(\tan^6 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + (9a^2 b + \frac{9}{4} b^3) x \left(\tan^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \left(\frac{3}{2} a^2 b + \frac{3}{8} b^3 \right) x$

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*sin(d*x+c)+3*a^2*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

$$\int \cos(c+dx)(a+b \cos(c+dx))^3 dx$$

$$= \frac{3(4a^2b+b^3)dx + (2b^3 \cos(dx+c))^3 + 8ab^2 \cos(dx+c)^2 + 8a^3 + 16ab^2 + 3(4a^2b+b^3) \cos(dx+c) \sin(dx+c)}{8d}$$

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(4*a^2*b + b^3)*d*x + (2*b^3*cos(d*x + c))^3 + 8*a*b^2*cos(d*x + c)^2 + 8*a^3 + 16*a*b^2 + 3*(4*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(109) = 218.

Time = 0.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.93

$$\int \cos(c + dx)(a + b \cos(c + dx))^3 dx$$

$$= \begin{cases} \frac{a^3 \sin(c+dx)}{d} + \frac{3a^2bx \sin^2(c+dx)}{2} + \frac{3a^2bx \cos^2(c+dx)}{2} + \frac{3a^2b \sin(c+dx) \cos(c+dx)}{2d} + \frac{2ab^2 \sin^3(c+dx)}{d} + \frac{3ab^2 \sin(c+dx) \cos^2(c+dx)}{d} \\ x(a + b \cos(c))^3 \cos(c) \end{cases}$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**3,x)

[Out] Piecewise((a**3*sin(c + d*x)/d + 3*a**2*b*x*sin(c + d*x)**2/2 + 3*a**2*b*x*cos(c + d*x)**2/2 + 3*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*a*b**2*sin(c + d*x)**3/d + 3*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*b**3*x*sin(c + d*x)**4/8 + 3*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b**3*x*cos(c + d*x)**4/8 + 3*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*b**3*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**3*cos(c), True))

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \cos(c + dx)(a + b \cos(c + dx))^3 dx$$

$$= \frac{24(2dx + 2c + \sin(2dx + 2c))a^2b - 32(\sin(dx + c)^3 - 3\sin(dx + c))ab^2 + (12dx + 12c + \sin(4dx + 4c))b^3 + 32a^3\sin(dx + c)}{32d}$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] 1/32*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b^2 + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b^3 + 32*a^3*sin(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int \cos(c + dx)(a + b \cos(c + dx))^3 dx = \frac{b^3 \sin(4 dx + 4 c)}{32 d} + \frac{ab^2 \sin(3 dx + 3 c)}{4 d} + \frac{3}{8} (4 a^2 b + b^3) x + \frac{(3 a^2 b + b^3) \sin(2 dx + 2 c)}{4 d} + \frac{(4 a^3 + 9 a b^2) \sin(dx + c)}{4 d}$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] 1/32*b^3*sin(4*d*x + 4*c)/d + 1/4*a*b^2*sin(3*d*x + 3*c)/d + 3/8*(4*a^2*b + b^3)*x + 1/4*(3*a^2*b + b^3)*sin(2*d*x + 2*c)/d + 1/4*(4*a^3 + 9*a*b^2)*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 16.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.31

$$\int \cos(c + dx)(a + b \cos(c + dx))^3 dx = \frac{\left(2 a^3 - 3 a^2 b + 6 a b^2 - \frac{5 b^3}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(6 a^3 - 3 a^2 b + 10 a b^2 + \frac{3 b^3}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(6 a^3 + 3 a^2 b - 3 a b^2 - \frac{b^3}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(2 a^3 - 3 a^2 b + 6 a b^2 - \frac{5 b^3}{4}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)} + \frac{3 b \operatorname{atan}\left(\frac{3 b \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (4 a^2 + b^2)}{4 \left(3 a^2 b + \frac{3 b^3}{4}\right)}\right) (4 a^2 + b^2)}{4 d} - \frac{3 b (4 a^2 + b^2) \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{4 d}$$

[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^3,x)

[Out] (tan(c/2 + (d*x)/2)^7*(6*a*b^2 - 3*a^2*b + 2*a^3 - (5*b^3)/4) + tan(c/2 + (d*x)/2)^3*(10*a*b^2 + 3*a^2*b + 6*a^3 - (3*b^3)/4) + tan(c/2 + (d*x)/2)^5*(10*a*b^2 - 3*a^2*b + 6*a^3 + (3*b^3)/4) + tan(c/2 + (d*x)/2)*(6*a*b^2 + 3*a^2*b + 2*a^3 + (5*b^3)/4))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*b*atan((3*b*tan(c/2 + (d*x)/2)*(4*a^2 + b^2))/(4*(3*a^2*b + (3*b^3)/4)))*(4*a^2 + b^2))/(4*d) - (3*b*(4*a^2 + b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d)

3.431 $\int (a + b \cos(c + dx))^3 dx$

Optimal result	4758
Rubi [A] (verified)	4758
Mathematica [A] (verified)	4759
Maple [A] (verified)	4760
Fricas [A] (verification not implemented)	4760
Sympy [A] (verification not implemented)	4761
Maxima [A] (verification not implemented)	4761
Giac [A] (verification not implemented)	4761
Mupad [B] (verification not implemented)	4762

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int (a + b \cos(c + dx))^3 dx = a^3 x + \frac{3}{2} ab^2 x + \frac{b(3a^2 + b^2) \sin(c + dx)}{d} + \frac{3ab^2 \cos(c + dx) \sin(c + dx)}{2d} - \frac{b^3 \sin^3(c + dx)}{3d}$$

[Out] $a^3 x + \frac{3}{2} a b^2 x + b(3 a^2 + b^2) \sin(d x + c) / d + \frac{3}{2} a b^2 \cos(d x + c) \sin(d x + c) / d - \frac{1}{3} b^3 \sin^3(d x + c) / d$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2735, 2813}

$$\int (a + b \cos(c + dx))^3 dx = \frac{2b(4a^2 + b^2) \sin(c + dx)}{3d} + \frac{1}{2} ax(2a^2 + 3b^2) + \frac{5ab^2 \sin(c + dx) \cos(c + dx)}{6d} + \frac{b \sin(c + dx)(a + b \cos(c + dx))^2}{3d}$$

[In] Int[(a + b*cos[c + d*x])^3, x]

[Out] $(a*(2*a^2 + 3*b^2)*x)/2 + (2*b*(4*a^2 + b^2)*Sin[c + d*x])/(3*d) + (5*a*b^2*\cos[c + d*x]*\sin[c + d*x])/(6*d) + (b*(a + b*\cos[c + d*x])^2*\sin[c + d*x])/(3*d)$

Rule 2735

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*
(x_)]), x_Symbol] := Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{b(a + b \cos(c + dx))^2 \sin(c + dx)}{3d} \\ &+ \frac{1}{3} \int (a + b \cos(c + dx)) (3a^2 + 2b^2 + 5ab \cos(c + dx)) dx \\ &= \frac{1}{2} a(2a^2 + 3b^2) x + \frac{2b(4a^2 + b^2) \sin(c + dx)}{3d} \\ &+ \frac{5ab^2 \cos(c + dx) \sin(c + dx)}{6d} + \frac{b(a + b \cos(c + dx))^2 \sin(c + dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int (a + b \cos(c + dx))^3 dx = \frac{12a^3c + 18ab^2c + 12a^3dx + 18ab^2dx + 9b(4a^2 + b^2) \sin(c + dx) + 9ab^2 \sin(2(c + dx)) + b^3 \sin(3(c + dx))}{12d}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (12*a^3*c + 18*a*b^2*c + 12*a^3*d*x + 18*a*b^2*d*x + 9*b*(4*a^2 + b^2)*Sin[
c + d*x] + 9*a*b^2*Ssin[2*(c + d*x)] + b^3*Ssin[3*(c + d*x)])/(12*d)
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

method	result
parallelrisc	$\frac{9 \sin(2dx+2c) a b^2 + b^3 \sin(3dx+3c) + 9(4a^2b+b^3) \sin(dx+c) + 12\left(a^2 + \frac{3b^2}{2}\right) dx a}{12d}$
derivativdivides	$\frac{\frac{b^3(2+\cos^2(dx+c)) \sin(dx+c)}{3} + 3a b^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + 3 \sin(dx+c) a^2 b + a^3(dx+c)}{d}$
default	$\frac{\frac{b^3(2+\cos^2(dx+c)) \sin(dx+c)}{3} + 3a b^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + 3 \sin(dx+c) a^2 b + a^3(dx+c)}{d}$
parts	$a^3 x + \frac{b^3(2+\cos^2(dx+c)) \sin(dx+c)}{3d} + \frac{3a b^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{3 \sin(dx+c) a^2 b}{d}$
risc	$a^3 x + \frac{3a b^2 x}{2} + \frac{3 \sin(dx+c) a^2 b}{d} + \frac{3 \sin(dx+c) b^3}{4d} + \frac{\sin(3dx+3c) b^3}{12d} + \frac{3 \sin(2dx+2c) a b^2}{4d}$
norman	$\frac{(a^3 + \frac{3}{2} a b^2) x + (a^3 + \frac{3}{2} a b^2) x \left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (3a^3 + \frac{9}{2} a b^2) x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (3a^3 + \frac{9}{2} a b^2) x \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{b(6a^2 - 3b^2)}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^3}}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^3}$

```
[In] int((a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(9*sin(2*d*x+2*c)*a*b^2+b^3*sin(3*d*x+3*c)+9*(4*a^2*b+b^3)*sin(d*x+c)+12*(a^2+3/2*b^2)*d*x*a)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int (a + b \cos(c + dx))^3 dx$$

$$= \frac{3(2a^3 + 3ab^2)dx + (2b^3 \cos(dx + c)^2 + 9ab^2 \cos(dx + c) + 18a^2b + 4b^3) \sin(dx + c)}{6d}$$

```
[In] integrate((a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*a^3 + 3*a*b^2)*d*x + (2*b^3*cos(d*x + c)^2 + 9*a*b^2*cos(d*x + c) + 18*a^2*b + 4*b^3)*sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.68

$$\int (a + b \cos(c + dx))^3 dx$$

$$= \begin{cases} a^3 x + \frac{3a^2 b \sin(c+dx)}{d} + \frac{3ab^2 x \sin^2(c+dx)}{2} + \frac{3ab^2 x \cos^2(c+dx)}{2} + \frac{3ab^2 \sin(c+dx) \cos(c+dx)}{2d} + \frac{2b^3 \sin^3(c+dx)}{3d} + \frac{b^3 \sin(c+dx) \cos(c+dx)}{d} \\ x(a + b \cos(c))^3 \end{cases}$$

[In] integrate((a+b*cos(d*x+c))**3,x)

[Out] Piecewise((a**3*x + 3*a**2*b*sin(c + d*x)/d + 3*a*b**2*x*sin(c + d*x)**2/2 + 3*a*b**2*x*cos(c + d*x)**2/2 + 3*a*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*b**3*sin(c + d*x)**3/(3*d) + b**3*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a + b*cos(c))**3, True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int (a + b \cos(c + dx))^3 dx = a^3 x + \frac{3(2dx + 2c + \sin(2dx + 2c))ab^2}{4d} - \frac{(\sin(dx + c))^3 - 3\sin(dx + c)b^3}{3d} + \frac{3a^2 b \sin(dx + c)}{d}$$

[In] integrate((a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] a^3*x + 3/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b^2/d - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*b^3/d + 3*a^2*b*sin(d*x + c)/d

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int (a + b \cos(c + dx))^3 dx = \frac{b^3 \sin(3dx + 3c)}{12d} + \frac{3ab^2 \sin(2dx + 2c)}{4d} + \frac{1}{2}(2a^3 + 3ab^2)x + \frac{3(4a^2b + b^3) \sin(dx + c)}{4d}$$

[In] integrate((a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] 1/12*b^3*sin(3*d*x + 3*c)/d + 3/4*a*b^2*sin(2*d*x + 2*c)/d + 1/2*(2*a^3 + 3*a*b^2)*x + 3/4*(4*a^2*b + b^3)*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int (a + b \cos(c + dx))^3 dx = a^3 x + \frac{3b^3 \sin(c + dx)}{4d} + \frac{b^3 \sin(3c + 3dx)}{12d} + \frac{3ab^2 x}{2} + \frac{3ab^2 \sin(2c + 2dx)}{4d} + \frac{3a^2 b \sin(c + dx)}{d}$$

[In] int((a + b*cos(c + d*x))^3,x)

[Out] a^3*x + (3*b^3*sin(c + d*x))/(4*d) + (b^3*sin(3*c + 3*d*x))/(12*d) + (3*a*b^2*x)/2 + (3*a*b^2*sin(2*c + 2*d*x))/(4*d) + (3*a^2*b*sin(c + d*x))/d

3.432 $\int (a + b \cos(c + dx))^3 \sec(c + dx) dx$

Optimal result	4763
Rubi [A] (verified)	4763
Mathematica [A] (verified)	4765
Maple [A] (verified)	4765
Fricas [A] (verification not implemented)	4766
Sympy [F]	4766
Maxima [A] (verification not implemented)	4766
Giac [B] (verification not implemented)	4767
Mupad [B] (verification not implemented)	4767

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (a + b \cos(c + dx))^3 \sec(c + dx) dx = \frac{1}{2}b(6a^2 + b^2)x + \frac{a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{5ab^2 \sin(c + dx)}{2d} + \frac{b^2(a + b \cos(c + dx)) \sin(c + dx)}{2d}$$

[Out] $1/2*b*(6*a^2+b^2)*x+a^3*\operatorname{arctanh}(\sin(d*x+c))/d+5/2*a*b^2*\sin(d*x+c)/d+1/2*b^2*(a+b*\cos(d*x+c))*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.13 (sec), antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2872, 3102, 2814, 3855}

$$\int (a + b \cos(c + dx))^3 \sec(c + dx) dx = \frac{a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{1}{2}bx(6a^2 + b^2) + \frac{5ab^2 \sin(c + dx)}{2d} + \frac{b^2 \sin(c + dx)(a + b \cos(c + dx))}{2d}$$

[In] $\operatorname{Int}[(a + b*\operatorname{Cos}[c + d*x])^3*\operatorname{Sec}[c + d*x], x]$

[Out] $(b*(6*a^2 + b^2)*x)/2 + (a^3*\operatorname{ArcTanh}[\operatorname{Sin}[c + d*x]])/d + (5*a*b^2*\operatorname{Sin}[c + d*x])/(2*d) + (b^2*(a + b*\operatorname{Cos}[c + d*x])*\operatorname{Sin}[c + d*x])/(2*d)$

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{b^2(a + b \cos(c + dx)) \sin(c + dx)}{2d} \\
 &+ \frac{1}{2} \int (2a^3 + b(6a^2 + b^2) \cos(c + dx) + 5ab^2 \cos^2(c + dx)) \sec(c + dx) dx \\
 &= \frac{5ab^2 \sin(c + dx)}{2d} + \frac{b^2(a + b \cos(c + dx)) \sin(c + dx)}{2d} \\
 &+ \frac{1}{2} \int (2a^3 + b(6a^2 + b^2) \cos(c + dx)) \sec(c + dx) dx \\
 &= \frac{1}{2} b(6a^2 + b^2) x + \frac{5ab^2 \sin(c + dx)}{2d} + \frac{b^2(a + b \cos(c + dx)) \sin(c + dx)}{2d} + a^3 \int \sec(c \\
 &\quad + dx) dx
 \end{aligned}$$

$$= \frac{1}{2}b(6a^2 + b^2)x + \frac{a^3 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{5ab^2 \sin(c + dx)}{2d} + \frac{b^2(a + b \cos(c + dx)) \sin(c + dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int (a + b \cos(c + dx))^3 \sec(c + dx) dx = \frac{2b(6a^2 + b^2)(c + dx) - 4a^3 \log(\cos(\frac{1}{2}(c + dx)) - \sin(\frac{1}{2}(c + dx))) + 4a^3 \log(\cos(\frac{1}{2}(c + dx)) + \sin(\frac{1}{2}(c + dx)))}{4d}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x],x]

[Out] (2*b*(6*a^2 + b^2)*(c + d*x) - 4*a^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*a*b^2*Sin[c + d*x] + b^3*Sin[2*(c + d*x)])/(4*d)

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+3a^2b(dx+c)+3 \sin(dx+c)ab^2+b^3 \left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$
default	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+3a^2b(dx+c)+3 \sin(dx+c)ab^2+b^3 \left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$
parallelrisc	$\frac{12a^2bdx+2b^3dx+12 \sin(dx+c)ab^2+4 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a^3-4 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^3+\sin(2dx+2c)b^3}{4d}$
parts	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{b^3 \left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{3ab^2 \sin(dx+c)}{d} + \frac{3a^2b(dx+c)}{d}$
risc	$3a^2bx + \frac{b^3x}{2} - \frac{3iab^2e^{i(dx+c)}}{2d} + \frac{3ia^2b^2e^{-i(dx+c)}}{2d} + \frac{a^3 \ln(e^{i(dx+c)}+i)}{d} - \frac{a^3 \ln(e^{i(dx+c)}-i)}{d} + \frac{\sin(2dx+2c)b^3}{4d}$
norman	$\frac{(3a^2b+\frac{1}{2}b^3)x+(3a^2b+\frac{1}{2}b^3)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(9a^2b+\frac{3}{2}b^3)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(9a^2b+\frac{3}{2}b^3)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b^2(6a-1/2\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right))^3}$

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c),x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^2*b*(d*x+c)+3*sin(d*x+c)*a*b^2+b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int (a + b \cos(c + dx))^3 \sec(c + dx) dx$$

$$= \frac{a^3 \log(\sin(dx + c) + 1) - a^3 \log(-\sin(dx + c) + 1) + (6a^2b + b^3)dx + (b^3 \cos(dx + c) + 6ab^2) \sin(dx + c)}{2d}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="fricas")

[Out] 1/2*(a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) + (6*a^2*b + b^3)*d*x + (b^3*cos(d*x + c) + 6*a*b^2)*sin(d*x + c))/d

Sympy [F]

$$\int (a + b \cos(c + dx))^3 \sec(c + dx) dx = \int (a + b \cos(c + dx))^3 \sec(c + dx) dx$$

[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c),x)

[Out] Integral((a + b*cos(c + d*x))**3*sec(c + d*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (a + b \cos(c + dx))^3 \sec(c + dx) dx$$

$$= \frac{12(dx + c)a^2b + (2dx + 2c + \sin(2dx + 2c))b^3 + 4a^3 \log(\sec(dx + c) + \tan(dx + c)) + 12ab^2 \sin(dx + c)}{4d}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="maxima")

[Out] 1/4*(12*(d*x + c)*a^2*b + (2*d*x + 2*c + sin(2*d*x + 2*c))*b^3 + 4*a^3*log(sec(d*x + c) + tan(d*x + c)) + 12*a*b^2*sin(d*x + c))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(67) = 134.

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.88

$$\int (a + b \cos(c + dx))^3 \sec(c + dx) dx$$

$$= \frac{2a^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 2a^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + (6a^2b + b^3)(dx + c) + \frac{2(6ab^2 \tan(\frac{1}{2}dx - 1/2c))}{\tan^2(\frac{1}{2}dx + \frac{1}{2}c) + 1}}{2d}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="giac")

[Out] 1/2*(2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + (6*a^2*b + b^3)*(d*x + c) + 2*(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d

Mupad [B] (verification not implemented)

Time = 14.79 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.68

$$\int (a + b \cos(c + dx))^3 \sec(c + dx) dx = \frac{2a^3 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{b^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

$$+ \frac{b^3 \sin(2c + 2dx)}{4d} + \frac{3ab^2 \sin(c + dx)}{d}$$

$$+ \frac{6a^2b \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

[In] int((a + b*cos(c + d*x))^3/cos(c + d*x),x)

[Out] (2*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(2*c + 2*d*x))/(4*d) + (3*a*b^2*sin(c + d*x))/d + (6*a^2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d

3.433 $\int (a + b \cos(c + dx))^3 \sec^2(c + dx) dx$

Optimal result	4768
Rubi [A] (verified)	4768
Mathematica [A] (verified)	4770
Maple [A] (verified)	4770
Fricas [A] (verification not implemented)	4771
Sympy [F]	4771
Maxima [A] (verification not implemented)	4771
Giac [A] (verification not implemented)	4772
Mupad [B] (verification not implemented)	4772

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int (a + b \cos(c + dx))^3 \sec^2(c + dx) dx = 3ab^2x + \frac{3a^2b \operatorname{arctanh}(\sin(c + dx))}{d} - \frac{b(a^2 - b^2) \sin(c + dx)}{d} + \frac{a^2(a + b \cos(c + dx)) \tan(c + dx)}{d}$$

[Out] 3*a*b^2*x+3*a^2*b*arctanh(sin(d*x+c))/d-b*(a^2-b^2)*sin(d*x+c)/d+a^2*(a+b*cos(d*x+c))*tan(d*x+c)/d

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2871, 3102, 2814, 3855}

$$\int (a + b \cos(c + dx))^3 \sec^2(c + dx) dx = \frac{3a^2b \operatorname{arctanh}(\sin(c + dx))}{d} - \frac{b(a^2 - b^2) \sin(c + dx)}{d} + \frac{a^2 \tan(c + dx)(a + b \cos(c + dx))}{d} + 3ab^2x$$

[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2,x]

[Out] 3*a*b^2*x + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b*(a^2 - b^2)*Sin[c + d*x])/d + (a^2*(a + b*Cos[c + d*x])*Tan[c + d*x])/d

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a^2(a + b \cos(c + dx)) \tan(c + dx)}{d} \\
 &+ \int (3a^2b + 3ab^2 \cos(c + dx) - b(a^2 - b^2) \cos^2(c + dx)) \sec(c + dx) dx \\
 &= -\frac{b(a^2 - b^2) \sin(c + dx)}{d} + \frac{a^2(a + b \cos(c + dx)) \tan(c + dx)}{d} \\
 &+ \int (3a^2b + 3ab^2 \cos(c + dx)) \sec(c + dx) dx \\
 &= 3ab^2x - \frac{b(a^2 - b^2) \sin(c + dx)}{d} + \frac{a^2(a + b \cos(c + dx)) \tan(c + dx)}{d} + (3a^2b) \int \sec(c + dx) dx
 \end{aligned}$$

$$= 3ab^2x + \frac{3a^2b \operatorname{arctanh}(\sin(c+dx))}{d} - \frac{b(a^2-b^2)\sin(c+dx)}{d} + \frac{a^2(a+b\cos(c+dx))\tan(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.29

$$\int (a+b\cos(c+dx))^3 \sec^2(c+dx) dx = \frac{3ab(bc+bdx - a \log(\cos(\frac{1}{2}(c+dx)) - \sin(\frac{1}{2}(c+dx))) + a \log(\cos(\frac{1}{2}(c+dx)) + \sin(\frac{1}{2}(c+dx))))}{d} + b^3$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2,x]

[Out] (3*a*b*(b*c + b*d*x - a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + b^3*Sin[c + d*x] + a^3*Tan[c + d*x])/d

Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result
derivativdivides	$\frac{a^3 \tan(dx+c) + 3a^2b \ln(\sec(dx+c) + \tan(dx+c)) + 3ab^2(dx+c) + b^3 \sin(dx+c)}{d}$
default	$\frac{a^3 \tan(dx+c) + 3a^2b \ln(\sec(dx+c) + \tan(dx+c)) + 3ab^2(dx+c) + b^3 \sin(dx+c)}{d}$
parts	$\frac{a^3 \tan(dx+c)}{d} + \frac{\sin(dx+c)b^3}{d} + \frac{3a^2b \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{3ab^2(dx+c)}{d}$
parallelrisch	$\frac{6ab^2dx \cos(dx+c) - 6 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) a^2b \cos(dx+c) + 6 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) a^2b \cos(dx+c) + 2a^3 \sin(dx+c) + \sin(2dx+c)}{2d \cos(dx+c)}$
risch	$3ab^2x - \frac{ib^3e^{i(dx+c)}}{2d} + \frac{ib^3e^{-i(dx+c)}}{2d} + \frac{2ia^3}{d(e^{2i(dx+c)}+1)} - \frac{3a^2b \ln(e^{i(dx+c)}-i)}{d} + \frac{3a^2b \ln(e^{i(dx+c)}+i)}{d}$
norman	$\frac{-3ab^2x - \frac{2(a^3-b^3)(\tan^7(\frac{dx}{2} + \frac{c}{2}))}{d} - \frac{2(a^3+b^3)\tan(\frac{dx}{2} + \frac{c}{2})}{d} - \frac{2(3a^3-b^3)(\tan^5(\frac{dx}{2} + \frac{c}{2}))}{d} - \frac{2(3a^3+b^3)(\tan^3(\frac{dx}{2} + \frac{c}{2}))}{d} - 6ab^2x}{(1+\tan^2(\frac{dx}{2} + \frac{c}{2}))^3(\tan^2(\frac{dx}{2} + \frac{c}{2})-1)}$

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*tan(d*x+c)+3*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3*a*b^2*(d*x+c)+b^3*sin(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int (a + b \cos(c + dx))^3 \sec^2(c + dx) dx$$

$$= \frac{6 ab^2 dx \cos(dx + c) + 3 a^2 b \cos(dx + c) \log(\sin(dx + c) + 1) - 3 a^2 b \cos(dx + c) \log(-\sin(dx + c) + 1)}{2 d \cos(dx + c)}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="fricas")

```
[Out] 1/2*(6*a*b^2*d*x*cos(d*x + c) + 3*a^2*b*cos(d*x + c)*log(sin(d*x + c) + 1)
- 3*a^2*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(b^3*cos(d*x + c) + a^3)*
sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int (a + b \cos(c + dx))^3 \sec^2(c + dx) dx = \int (a + b \cos(c + dx))^3 \sec^2(c + dx) dx$$

[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**2,x)

[Out] Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int (a + b \cos(c + dx))^3 \sec^2(c + dx) dx$$

$$= \frac{6(dx + c)ab^2 + 3a^2b(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 2b^3 \sin(dx + c) + 2a^3 \tan(dx + c)}{2d}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="maxima")

```
[Out] 1/2*(6*(d*x + c)*a*b^2 + 3*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1)) + 2*b^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90

$$\int (a + b \cos(c + dx))^3 \sec^2(c + dx) dx$$

$$= \frac{3(dx + c)ab^2 + 3a^2b \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3a^2b \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(a^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - b^3\right)}{d}}{d}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="giac")

```
[Out] (3*(d*x + c)*a*b^2 + 3*a^2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 14.83 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.43

$$\int (a + b \cos(c + dx))^3 \sec^2(c + dx) dx = \frac{b^3 \sin(c + dx)}{d} + \frac{a^3 \sin(c + dx)}{d \cos(c + dx)}$$

$$+ \frac{6ab^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

$$+ \frac{6a^2b \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^2,x)

```
[Out] (b^3*sin(c + d*x))/d + (a^3*sin(c + d*x))/(d*cos(c + d*x)) + (6*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```


3.434 $\int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx$

Optimal result	4773
Rubi [A] (verified)	4773
Mathematica [A] (verified)	4775
Maple [A] (verified)	4775
Fricas [A] (verification not implemented)	4776
Sympy [F]	4776
Maxima [A] (verification not implemented)	4776
Giac [A] (verification not implemented)	4777
Mupad [B] (verification not implemented)	4777

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx = b^3 x + \frac{a(a^2 + 6b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{5a^2 b \tan(c + dx)}{2d} + \frac{a^2 (a + b \cos(c + dx)) \sec(c + dx) \tan(c + dx)}{2d}$$

[Out] $b^3 x + \frac{1}{2} a (a^2 + 6 b^2) \operatorname{arctanh}(\sin(d x + c)) / d + \frac{5}{2} a^2 b \tan(d x + c) / d + \frac{1}{2} a^2 (a + b \cos(d x + c)) \sec(d x + c) \tan(d x + c) / d$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2871, 3100, 2814, 3855}

$$\int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx = \frac{a(a^2 + 6b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{5a^2 b \tan(c + dx)}{2d} + \frac{a^2 \tan(c + dx) \sec(c + dx) (a + b \cos(c + dx))}{2d} + b^3 x$$

[In] $\text{Int}[(a + b \cos[c + d x])^3 \sec[c + d x]^3, x]$

[Out] $b^3 x + (a(a^2 + 6b^2) \operatorname{ArcTanh}[\sin[c + d x]]) / (2d) + (5a^2 b \tan[c + d x]) / (2d) + (a^2 (a + b \cos[c + d x]) \sec[c + d x] \tan[c + d x]) / (2d)$

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{a^2(a + b \cos(c + dx)) \sec(c + dx) \tan(c + dx)}{2d} \\ &+ \frac{1}{2} \int (5a^2b + a(a^2 + 6b^2) \cos(c + dx) + 2b^3 \cos^2(c + dx)) \sec^2(c + dx) dx \\ &= \frac{5a^2b \tan(c + dx)}{2d} + \frac{a^2(a + b \cos(c + dx)) \sec(c + dx) \tan(c + dx)}{2d} \\ &+ \frac{1}{2} \int (a(a^2 + 6b^2) + 2b^3 \cos(c + dx)) \sec(c + dx) dx \end{aligned}$$

$$\begin{aligned}
&= b^3 x + \frac{5a^2 b \tan(c + dx)}{2d} + \frac{a^2(a + b \cos(c + dx)) \sec(c + dx) \tan(c + dx)}{2d} \\
&\quad + \frac{1}{2}(a(a^2 + 6b^2)) \int \sec(c + dx) dx \\
&= b^3 x + \frac{a(a^2 + 6b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{5a^2 b \tan(c + dx)}{2d} \\
&\quad + \frac{a^2(a + b \cos(c + dx)) \sec(c + dx) \tan(c + dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\begin{aligned}
&\int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx \\
&= \frac{2b^3 dx + a(a^2 + 6b^2) \operatorname{arctanh}(\sin(c + dx)) + a^2(6b + a \sec(c + dx)) \tan(c + dx)}{2d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3,x]

[Out] (2*b^3*d*x + a*(a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]] + a^2*(6*b + a*Sec[c + d*x])*Tan[c + d*x])/(2*d)

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) + 3a^2 b \tan(dx+c) + 3a b^2 \ln(\sec(dx+c) + \tan(dx+c)) + b^3(dx+c)}{d}$
default	$\frac{a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) + 3a^2 b \tan(dx+c) + 3a b^2 \ln(\sec(dx+c) + \tan(dx+c)) + b^3(dx+c)}{d}$
parts	$\frac{a^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right)}{d} + \frac{b^3(dx+c)}{d} + \frac{3a^2 b \tan(dx+c)}{d} + \frac{3a b^2 \ln(\sec(dx+c) + \tan(dx+c))}{d}$
parallelrisch	$\frac{-a(a^2 + 6b^2)(1 + \cos(2dx + 2c)) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + a(a^2 + 6b^2)(1 + \cos(2dx + 2c)) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + 2b^3 dx \cos(2dx + 2c)}{2d(1 + \cos(2dx + 2c))}$
risch	$b^3 x - \frac{ia^2(a e^{3i(dx+c)} - 6b e^{2i(dx+c)} - a e^{i(dx+c)} - 6b)}{d(e^{2i(dx+c)} + 1)^2} + \frac{a^3 \ln(e^{i(dx+c)} + i)}{2d} + \frac{3a \ln(e^{i(dx+c)} + i) b^2}{d} - \frac{a^3 \ln(e^{i(dx+c)} + i)}{2d}$
norman	$\frac{b^3 x + b^3 x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + b^3 x \left(\tan^8\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + b^3 x \left(\tan^{10}\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + \frac{a^2(a-6b) \left(\tan^9\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{d} + \frac{a^2(a+6b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3}$

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] $1/d*(a^3*(1/2*\sec(d*x+c)*\tan(d*x+c)+1/2*\ln(\sec(d*x+c)+\tan(d*x+c)))+3*a^2*b*\tan(d*x+c)+3*a*b^2*\ln(\sec(d*x+c)+\tan(d*x+c))+b^3*(d*x+c))$

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.42

$$\int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx$$

$$= \frac{4 b^3 dx \cos(dx + c)^2 + (a^3 + 6 ab^2) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (a^3 + 6 ab^2) \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2*(6*a^2*b*\cos(dx + c) + a^3)*\sin(dx + c)}{4 d \cos(dx + c)^2}$$

[In] `integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="fricas")`

[Out] $1/4*(4*b^3*d*x*\cos(d*x + c)^2 + (a^3 + 6*a*b^2)*\cos(d*x + c)^2*\log(\sin(d*x + c) + 1) - (a^3 + 6*a*b^2)*\cos(d*x + c)^2*\log(-\sin(d*x + c) + 1) + 2*(6*a^2*b*\cos(d*x + c) + a^3)*\sin(d*x + c))/(d*\cos(d*x + c)^2)$

Sympy [F]

$$\int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx = \int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx$$

[In] `integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**3,x)`

[Out] `Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**3, x)`

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx$$

$$= \frac{4(dx + c)b^3 - a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1)\right) + 6ab^2(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1))}{4d}$$

[In] `integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="maxima")`

[Out] $1/4*(4*(d*x + c)*b^3 - a^3*(2*\sin(d*x + c)/(\sin(d*x + c)^2 - 1) - \log(\sin(d*x + c) + 1) + \log(\sin(d*x + c) - 1)) + 6*a*b^2*(\log(\sin(d*x + c) + 1) - \log(\sin(d*x + c) - 1)) + 12*a^2*b*\tan(d*x + c))/d$

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.81

$$\int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx$$

$$= \frac{2(dx + c)b^3 + (a^3 + 6ab^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - (a^3 + 6ab^2) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2(a^3 \tan^3(\frac{1}{2}dx + \frac{1}{2}c) - 6a^2b \tan(\frac{1}{2}dx + \frac{1}{2}c) + a^3)}{2d}}{2d}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="giac")

```
[Out] 1/2*(2*(d*x + c)*b^3 + (a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
(a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x +
1/2*c)^3 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + 6*a^
2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int (a + b \cos(c + dx))^3 \sec^3(c + dx) dx = \frac{a^3 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

$$+ \frac{2b^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{a^3 \sin(c + dx)}{2d \cos(c + dx)^2}$$

$$+ \frac{6ab^2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{3a^2 b \sin(c + dx)}{d \cos(c + dx)}$$

[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^3,x)

```
[Out] (a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b^3*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*sin(c + d*x))/(2*d*cos(c + d*x)^2)
+ (6*a*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a^2*b*sin(
c + d*x))/(d*cos(c + d*x))
```

3.435 $\int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx$

Optimal result	4778
Rubi [A] (verified)	4778
Mathematica [A] (verified)	4780
Maple [A] (verified)	4781
Fricas [A] (verification not implemented)	4781
Sympy [F]	4782
Maxima [A] (verification not implemented)	4782
Giac [B] (verification not implemented)	4782
Mupad [B] (verification not implemented)	4783

Optimal result

Integrand size = 21, antiderivative size = 109

$$\int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx = \frac{b(3a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a(2a^2 + 9b^2) \tan(c + dx)}{3d} + \frac{7a^2b \sec(c + dx) \tan(c + dx)}{6d} + \frac{a^2(a + b \cos(c + dx)) \sec^2(c + dx) \tan(c + dx)}{3d}$$

[Out] 1/2*b*(3*a^2+2*b^2)*arctanh(sin(d*x+c))/d+1/3*a*(2*a^2+9*b^2)*tan(d*x+c)/d+7/6*a^2*b*sec(d*x+c)*tan(d*x+c)/d+1/3*a^2*(a+b*cos(d*x+c))*sec(d*x+c)^2*tan(d*x+c)/d

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2871, 3100, 2827, 3852, 8, 3855}

$$\int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx = \frac{b(3a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a(2a^2 + 9b^2) \tan(c + dx)}{3d} + \frac{7a^2b \tan(c + dx) \sec(c + dx)}{6d} + \frac{a^2 \tan(c + dx) \sec^2(c + dx)(a + b \cos(c + dx))}{3d}$$

[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^4,x]

[Out] (b*(3*a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]]/(2*d) + (a*(2*a^2 + 9*b^2)*Tan[c + d*x])/(3*d) + (7*a^2*b*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m - 2)*((c + d*SIN[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*SIN[e + f*x])^(m - 3)*(c + d*SIN[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3100

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*SIN[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2(a + b \cos(c + dx)) \sec^2(c + dx) \tan(c + dx)}{3d} \\
&+ \frac{1}{3} \int (7a^2b + a(2a^2 + 9b^2) \cos(c + dx) + b(a^2 + 3b^2) \cos^2(c + dx)) \sec^3(c + dx) dx \\
&= \frac{7a^2b \sec(c + dx) \tan(c + dx)}{6d} + \frac{a^2(a + b \cos(c + dx)) \sec^2(c + dx) \tan(c + dx)}{3d} \\
&+ \frac{1}{6} \int (2a(2a^2 + 9b^2) + 3b(3a^2 + 2b^2) \cos(c + dx)) \sec^2(c + dx) dx \\
&= \frac{7a^2b \sec(c + dx) \tan(c + dx)}{6d} + \frac{a^2(a + b \cos(c + dx)) \sec^2(c + dx) \tan(c + dx)}{3d} \\
&+ \frac{1}{2} (b(3a^2 + 2b^2)) \int \sec(c + dx) dx + \frac{1}{3} (a(2a^2 + 9b^2)) \int \sec^2(c + dx) dx \\
&= \frac{b(3a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{7a^2b \sec(c + dx) \tan(c + dx)}{6d} \\
&+ \frac{a^2(a + b \cos(c + dx)) \sec^2(c + dx) \tan(c + dx)}{3d} \\
&- \frac{(a(2a^2 + 9b^2)) \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{3d} \\
&= \frac{b(3a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a(2a^2 + 9b^2) \tan(c + dx)}{3d} \\
&+ \frac{7a^2b \sec(c + dx) \tan(c + dx)}{6d} + \frac{a^2(a + b \cos(c + dx)) \sec^2(c + dx) \tan(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\begin{aligned}
&\int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx \\
&= \frac{(9a^2b + 6b^3) \operatorname{arctanh}(\sin(c + dx)) + a \tan(c + dx) (6a^2 + 18b^2 + 9ab \sec(c + dx) + 2a^2 \tan^2(c + dx))}{6d}
\end{aligned}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

```
[Out] ((9*a^2*b + 6*b^3)*ArcTanh[Sin[c + d*x]] + a*Tan[c + d*x]*(6*a^2 + 18*b^2 + 9*a*b*Sec[c + d*x] + 2*a^2*Tan[c + d*x]^2))/(6*d)
```


Maple [A] (verified)

Time = 4.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-a^3 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + 3a^2 b \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) + 3a b^2 \tan(dx+c) + b^3 \ln(\sec(dx+c) + \tan(dx+c))}{d}$
default	$\frac{-a^3 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + 3a^2 b \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) + 3a b^2 \tan(dx+c) + b^3 \ln(\sec(dx+c) + \tan(dx+c))}{d}$
parts	$-\frac{a^3 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d} + \frac{b^3 \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{3a^2 b \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right)}{d}$
parallelrisc	$\frac{-27b \left(a^2 + \frac{2b^2}{3} \right) \left(\frac{\cos(3dx+3c)}{3} + \cos(dx+c) \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 27b \left(a^2 + \frac{2b^2}{3} \right) \left(\frac{\cos(3dx+3c)}{3} + \cos(dx+c) \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)}{6d(\cos(3dx+3c) + 3 \cos(dx+c))}$
risc	$-\frac{ia(9ab e^{5i(dx+c)} - 18b^2 e^{4i(dx+c)} - 12a^2 e^{2i(dx+c)} - 36b^2 e^{2i(dx+c)} - 9ab e^{i(dx+c)} - 4a^2 - 18b^2)}{3d(e^{2i(dx+c)} + 1)^3} - \frac{3a^2 b \ln(e^{i(dx+c)} - i)}{2d}$
norman	$\frac{-\frac{2a(2a^2 - 3ab - 6b^2) \left(\tan^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{a(2a^2 - 3ab + 6b^2) \left(\tan^{11} \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{2a(2a^2 + 3ab - 6b^2) \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} - \frac{a(2a^2 + 3ab + 6b^2) \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d}}{\left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^3 \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}$

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^2*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*tan(d*x+c)+b^3*ln(sec(d*x+c)+tan(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

$$\int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx$$

$$= \frac{3(3a^2b + 2b^3) \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 3(3a^2b + 2b^3) \cos(dx + c)^3 \log(-\sin(dx + c) + 1)}{12d \cos(dx + c)^3}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="fricas")

[Out] 1/12*(3*(3*a^2*b + 2*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(3*a^2*b + 2*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(9*a^2*b*cos(d*x + c) + 2*a^3 + 2*(2*a^3 + 9*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)

Sympy [F]

$$\int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx = \int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx$$

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**4,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04

$$\int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx$$

$$= \frac{4 (\tan(dx + c)^3 + 3 \tan(dx + c)) a^3 - 9 a^2 b \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c) + 1) + \log(\sin(dx+c) - 1) \right) + 6 b^3 (\log(\sin(dx+c) + 1) - \log(\sin(dx+c) - 1)) + 36 a b^2 \tan(dx+c)}{12 d}$$

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*a*b^2*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(101) = 202.

Time = 0.31 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.88

$$\int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx$$

$$= \frac{3(3a^2b + 2b^3) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 3(3a^2b + 2b^3) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) - \frac{2(6a^3 \tan(\frac{1}{2} dx + \frac{1}{2} c) + 3a^2b \tan^2(\frac{1}{2} dx + \frac{1}{2} c) + 2b^3 \tan^3(\frac{1}{2} dx + \frac{1}{2} c))}{(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - 1)^3}}{d}$$

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(3*a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*a^3*tan(1/2*d*x + 1/2*c)^3 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2*d*x + 1/2*c) + 9*a^2*b*tan(1/2*d*x + 1/2*c) + 18*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3/d
```

Mupad [B] (verification not implemented)

Time = 16.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.44

$$\int (a + b \cos(c + dx))^3 \sec^4(c + dx) dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (3a^2b + 2b^3)}{d} - \frac{(2a^3 - 3a^2b + 6ab^2) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(-\frac{4a^3}{3} - 12ab^2\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + (2a^3 + 3a^2b + 6ab^2) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^4,x)

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(3*a^2*b + 2*b^3))/d - (tan(c/2 + (d*x)/2)^5*(6*a*b^2 - 3*a^2*b + 2*a^3) - tan(c/2 + (d*x)/2)^3*(12*a*b^2 + (4*a^3)/3) + tan(c/2 + (d*x)/2)*(6*a*b^2 + 3*a^2*b + 2*a^3))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.436 $\int (a + b \cos(c + dx))^3 \sec^5(c + dx) dx$

Optimal result	4784
Rubi [A] (verified)	4784
Mathematica [A] (verified)	4787
Maple [A] (verified)	4787
Fricas [A] (verification not implemented)	4788
Sympy [F(-1)]	4788
Maxima [A] (verification not implemented)	4788
Giac [B] (verification not implemented)	4789
Mupad [B] (verification not implemented)	4789

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int (a + b \cos(c + dx))^3 \sec^5(c + dx) dx = \frac{3a(a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{b(2a^2 + b^2) \tan(c + dx)}{d} + \frac{3a(a^2 + 4b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{3a^2 b \sec^2(c + dx) \tan(c + dx)}{4d} + \frac{a^2(a + b \cos(c + dx)) \sec^3(c + dx) \tan(c + dx)}{4d}$$

[Out] $\frac{3}{8}a*(a^2+4*b^2)*\operatorname{arctanh}(\sin(d*x+c))/d+b*(2*a^2+b^2)*\tan(d*x+c)/d+\frac{3}{8}a*(a^2+4*b^2)*\sec(d*x+c)*\tan(d*x+c)/d+\frac{3}{4}a^2*b*\sec(d*x+c)^2*\tan(d*x+c)/d+\frac{1}{4}a^2*(a+b*\cos(d*x+c))*\sec(d*x+c)^3*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {2871, 3100, 2827, 3853, 3855, 3852, 8}

$$\int (a + b \cos(c + dx))^3 \sec^5(c + dx) dx = \frac{3a(a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{b(2a^2 + b^2) \tan(c + dx)}{d} + \frac{3a(a^2 + 4b^2) \tan(c + dx) \sec(c + dx)}{8d} + \frac{3a^2 b \tan(c + dx) \sec^2(c + dx)}{4d} + \frac{a^2 \tan(c + dx) \sec^3(c + dx)(a + b \cos(c + dx))}{4d}$$

[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5,x]

[Out] (3*a*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]]/(8*d) + (b*(2*a^2 + b^2)*Tan[c + d*x])/d + (3*a*(a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^2*b*Sec[c + d*x]^2*Tan[c + d*x])/(4*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m - 2)*((c + d*SIN[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*SIN[e + f*x])^(m - 3)*(c + d*SIN[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*SIN[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*SIN[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3100

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[(-(A*b^2

- a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :=> Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :=> Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :=> Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a^2(a + b \cos(c + dx)) \sec^3(c + dx) \tan(c + dx)}{4d} \\
 &+ \frac{1}{4} \int (9a^2b + 3a(a^2 + 4b^2) \cos(c + dx) + 2b(a^2 + 2b^2) \cos^2(c + dx)) \sec^4(c + dx) dx \\
 &= \frac{3a^2b \sec^2(c + dx) \tan(c + dx)}{4d} + \frac{a^2(a + b \cos(c + dx)) \sec^3(c + dx) \tan(c + dx)}{4d} \\
 &+ \frac{1}{12} \int (9a(a^2 + 4b^2) + 12b(2a^2 + b^2) \cos(c + dx)) \sec^3(c + dx) dx \\
 &= \frac{3a^2b \sec^2(c + dx) \tan(c + dx)}{4d} + \frac{a^2(a + b \cos(c + dx)) \sec^3(c + dx) \tan(c + dx)}{4d} \\
 &+ (b(2a^2 + b^2)) \int \sec^2(c + dx) dx + \frac{1}{4}(3a(a^2 + 4b^2)) \int \sec^3(c + dx) dx \\
 &= \frac{3a(a^2 + 4b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{3a^2b \sec^2(c + dx) \tan(c + dx)}{4d} \\
 &+ \frac{a^2(a + b \cos(c + dx)) \sec^3(c + dx) \tan(c + dx)}{4d} \\
 &+ \frac{1}{8}(3a(a^2 + 4b^2)) \int \sec(c + dx) dx - \frac{(b(2a^2 + b^2)) \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d}
 \end{aligned}$$

$$= \frac{3a(a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{b(2a^2 + b^2) \tan(c + dx)}{d} + \frac{3a(a^2 + 4b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{3a^2 b \sec^2(c + dx) \tan(c + dx)}{4d} + \frac{a^2(a + b \cos(c + dx)) \sec^3(c + dx) \tan(c + dx)}{4d}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

$$\int (a + b \cos(c + dx))^3 \sec^5(c + dx) dx$$

$$= \frac{3a(a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx)) + \tan(c + dx) (3a(a^2 + 4b^2) \sec(c + dx) + 2a^3 \sec^3(c + dx) + 8b(3a^2 + 4b^2) \sec^2(c + dx) \tan(c + dx))}{8d}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5,x]

[Out] (3*a*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*a*(a^2 + 4*b^2)*Sec[c + d*x] + 2*a^3*Sec[c + d*x]^3 + 8*b*(3*a^2 + b^2 + a^2*Tan[c + d*x]^2)))/(8*d)

Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.94

method	result
derivativedivides	$a^3 \left(- \left(- \frac{(\sec^3(dx+c))}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right) - 3a^2 b \left(- \frac{2}{3} - \frac{(\sec^2(dx+c))}{3} \right) \tan(dx+c) + \frac{d}{d}$
default	$a^3 \left(- \left(- \frac{(\sec^3(dx+c))}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right) - 3a^2 b \left(- \frac{2}{3} - \frac{(\sec^2(dx+c))}{3} \right) \tan(dx+c) + \frac{d}{d}$
parts	$a^3 \left(- \left(- \frac{(\sec^3(dx+c))}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right) + \frac{b^3 \tan(dx+c)}{d} - \frac{3a^2 b \left(- \frac{2}{3} - \frac{(\sec^2(dx+c))}{3} \right) \tan(dx+c)}{d}$
parallelrisc	$-6 \left(\frac{3}{4} + \frac{\cos(4dx+4c)}{4} + \cos(2dx+2c) \right) (a^2+4b^2) a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 6 \left(\frac{3}{4} + \frac{\cos(4dx+4c)}{4} + \cos(2dx+2c) \right) (a^2+4b^2) a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) + \frac{4d(\cos(4dx+4c))}{4d(\cos(4dx+4c))}$
risc	$- \frac{i(3a^3 e^{7i(dx+c)} + 12a^2 b^2 e^{7i(dx+c)} - 8b^3 e^{6i(dx+c)} + 11a^3 e^{5i(dx+c)} + 12a^2 b^2 e^{5i(dx+c)} - 48a^2 b e^{4i(dx+c)} - 24b^3 e^{4i(dx+c)} - 11a^3 e^{3i(dx+c)} + 12a^2 b^2 e^{3i(dx+c)} - 8b^3 e^{3i(dx+c)} + 11a^3 e^{i(dx+c)} + 12a^2 b^2 e^{i(dx+c)} - 48a^2 b e^{i(dx+c)} - 24b^3 e^{i(dx+c)} - 11a^3)}{4d(e^{2i(dx+c)} + 1)}$
norman	$\frac{a(7a^2 - 12b^2) \left(\tan^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d} + \frac{(5a^3 - 24a^2 b + 12a b^2 - 8b^3) \left(\tan^{13} \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{4d} + \frac{(5a^3 + 24a^2 b + 12a b^2 + 8b^3) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d} + \frac{(27a^3 - 81a^2 b + 108a b^2 - 72b^3)}{4d} + \frac{d}{d} + \frac{d}{d}$

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] $1/d*(a^3*(-(-1/4*\sec(d*x+c)^3-3/8*\sec(d*x+c))*\tan(d*x+c)+3/8*\ln(\sec(d*x+c)+\tan(d*x+c)))-3*a^2*b*(-2/3-1/3*\sec(d*x+c)^2)*\tan(d*x+c)+3*a*b^2*(1/2*\sec(d*x+c)*\tan(d*x+c)+1/2*\ln(\sec(d*x+c)+\tan(d*x+c)))+\tan(d*x+c)*b^3)$

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.05

$$\int (a + b \cos(c + dx))^3 \sec^5(c + dx) dx$$

$$= \frac{3(a^3 + 4ab^2) \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 3(a^3 + 4ab^2) \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2}{16 d \cos(dx + c)}$$

[In] `integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="fricas")`

[Out] $1/16*(3*(a^3 + 4*a*b^2)*\cos(d*x + c)^4*\log(\sin(d*x + c) + 1) - 3*(a^3 + 4*a*b^2)*\cos(d*x + c)^4*\log(-\sin(d*x + c) + 1) + 2*(8*a^2*b*\cos(d*x + c) + 8*(2*a^2*b + b^3)*\cos(d*x + c)^3 + 2*a^3 + 3*(a^3 + 4*a*b^2)*\cos(d*x + c)^2)*\sin(d*x + c))/(d*\cos(d*x + c)^4)$

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^5(c + dx) dx = \text{Timed out}$$

[In] `integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**5,x)`

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19

$$\int (a + b \cos(c + dx))^3 \sec^5(c + dx) dx$$

$$= \frac{16(\tan(dx + c)^3 + 3 \tan(dx + c))a^2b - a^3 \left(\frac{2(3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right)}{16 d \cos(dx + c)}$$

[In] `integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="maxima")`

[Out] $1/16*(16*(\tan(dx + c))^3 + 3*\tan(dx + c))*a^2*b - a^3*(2*(3*\sin(dx + c))^3 - 5*\sin(dx + c))/(\sin(dx + c)^4 - 2*\sin(dx + c)^2 + 1) - 3*\log(\sin(dx + c) + 1) + 3*\log(\sin(dx + c) - 1) - 12*a*b^2*(2*\sin(dx + c))/(\sin(dx + c)^2 - 1) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) + 16*b^3*\tan(dx + c))/d$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(125) = 250$.

Time = 0.31 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.48

$$\int (a + b \cos(c + dx))^3 \sec^5(c + dx) dx$$

$$= \frac{3(a^3 + 4ab^2) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 3(a^3 + 4ab^2) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) + \frac{2(5a^3 \tan(\frac{1}{2} dx + \frac{1}{2} c))^7}{\dots}}{\dots}$$

[In] `integrate((a+b*cos(dx+c))^3*sec(dx+c)^5,x, algorithm="giac")`

[Out] $1/8*(3*(a^3 + 4*a*b^2)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) - 3*(a^3 + 4*a*b^2)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*\tan(1/2*d*x + 1/2*c))^7 - 2*4*a^2*b*\tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*\tan(1/2*d*x + 1/2*c)^7 - 8*b^3*\tan(1/2*d*x + 1/2*c)^7 + 3*a^3*\tan(1/2*d*x + 1/2*c)^5 + 40*a^2*b*\tan(1/2*d*x + 1/2*c)^5 - 12*a*b^2*\tan(1/2*d*x + 1/2*c)^5 + 24*b^3*\tan(1/2*d*x + 1/2*c)^5 + 3*a^3*\tan(1/2*d*x + 1/2*c)^3 - 40*a^2*b*\tan(1/2*d*x + 1/2*c)^3 - 12*a*b^2*\tan(1/2*d*x + 1/2*c)^3 - 24*b^3*\tan(1/2*d*x + 1/2*c)^3 + 5*a^3*\tan(1/2*d*x + 1/2*c) + 24*a^2*b*\tan(1/2*d*x + 1/2*c) + 12*a*b^2*\tan(1/2*d*x + 1/2*c) + 8*b^3*\tan(1/2*d*x + 1/2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^4/d$

Mupad [B] (verification not implemented)

Time = 17.99 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.68

$$\int (a + b \cos(c + dx))^3 \sec^5(c + dx) dx$$

$$= \frac{\left(\frac{5a^3}{4} - 6a^2b + 3ab^2 - 2b^3\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{3a^3}{4} + 10a^2b - 3ab^2 + 6b^3\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{3a^3}{4} - 10a^2b + 3ab^2 - 2b^3\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \frac{3a \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (a^2 + 4b^2)}{4d}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)}$$

[In] `int((a + b*cos(c + dx))^3/cos(c + dx)^5,x)`

[Out] $(\tan(c/2 + (dx)/2))^7*(3*a*b^2 - 6*a^2*b + (5*a^3)/4 - 2*b^3) - \tan(c/2 + (dx)/2)^5*(3*a*b^2 + 10*a^2*b - (3*a^3)/4 + 6*b^3) + \tan(c/2 + (dx)/2)^3*(3*a*b^2 + 10*a^2*b - (3*a^3)/4 + 6*b^3) + \tan(c/2 + (dx)/2)^1*(3*a*b^2 + 10*a^2*b - (3*a^3)/4 + 6*b^3)$

$$\frac{10a^2b - 3ab^2 + \frac{3a^3}{4} + 6b^3 + \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)(3a^2b^2 + 6a^2b + \frac{5a^3}{4} + 2b^3)}{d(6\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^4 - 4\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^2 - 4\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^6 + \tan\left(\frac{c}{2} + \frac{d*x}{2}\right)^8 + 1)} + \frac{3a \operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{d*x}{2}\right)\right)(a^2 + 4b^2)}{4d}$$

3.437 $\int (a + b \cos(c + dx))^3 \sec^6(c + dx) dx$

Optimal result	4791
Rubi [A] (verified)	4791
Mathematica [A] (verified)	4794
Maple [A] (verified)	4794
Fricas [A] (verification not implemented)	4795
Sympy [F(-1)]	4795
Maxima [A] (verification not implemented)	4795
Giac [B] (verification not implemented)	4796
Mupad [B] (verification not implemented)	4796

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int (a + b \cos(c + dx))^3 \sec^6(c + dx) dx = \frac{b(9a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a(4a^2 + 15b^2) \tan(c + dx)}{5d} + \frac{b(9a^2 + 4b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{11a^2 b \sec^3(c + dx) \tan(c + dx)}{20d} + \frac{a^2(a + b \cos(c + dx)) \sec^4(c + dx) \tan(c + dx)}{5d} + \frac{a(4a^2 + 15b^2) \tan^3(c + dx)}{15d}$$

```
[Out] 1/8*b*(9*a^2+4*b^2)*arctanh(sin(d*x+c))/d+1/5*a*(4*a^2+15*b^2)*tan(d*x+c)/d
+1/8*b*(9*a^2+4*b^2)*sec(d*x+c)*tan(d*x+c)/d+11/20*a^2*b*sec(d*x+c)^3*tan(d
*x+c)/d+1/5*a^2*(a+b*cos(d*x+c))*sec(d*x+c)^4*tan(d*x+c)/d+1/15*a*(4*a^2+15
*b^2)*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {2871, 3100, 2827, 3852, 3853, 3855}

$$\int (a + b \cos(c + dx))^3 \sec^6(c + dx) dx = \frac{b(9a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a(4a^2 + 15b^2) \tan^3(c + dx)}{15d} + \frac{a(4a^2 + 15b^2) \tan(c + dx)}{5d} + \frac{b(9a^2 + 4b^2) \tan(c + dx) \sec(c + dx)}{8d} + \frac{11a^2b \tan(c + dx) \sec^3(c + dx)}{20d} + \frac{a^2 \tan(c + dx) \sec^4(c + dx)(a + b \cos(c + dx))}{5d}$$

[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^6,x]

[Out] (b*(9*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]]/(8*d) + (a*(4*a^2 + 15*b^2)*Tan[c + d*x])/(5*d) + (b*(9*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (11*a^2*b*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (a*(4*a^2 + 15*b^2)*Tan[c + d*x]^3)/(15*d)

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(- (b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3100

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[(- (A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*

$(a^2 - b^2))$, $x]$ + Dist[$1/(b*(m + 1)*(a^2 - b^2))$, Int[($a + b*\text{Sin}[e + f*x]$) $^{(m + 1)*\text{Simp}[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*\text{Sin}[e + f*x]$, $x]$, $x]$ /; FreeQ[{ a, b, e, f, A, B, C }, $x]$ && LtQ[$m, -1]$ && NeQ[$a^2 - b^2, 0]$

Rule 3852

Int[csc[($c_.$) + ($d_.$)*($x_.$)] $^{(n_.)}$, $x_Symbol]$:= Dist[$-d^{-1}$, Subst[Int[ExpandIntegrand[($1 + x^2$) $^{(n/2 - 1)}$, $x]$, $x]$, Cot[$c + d*x]$], $x]$ /; FreeQ[{ c, d }, $x]$ && IGtQ[$n/2, 0]$

Rule 3853

Int[(csc[($c_.$) + ($d_.$)*($x_.$)]*($b_.$)) $^{(n_.)}$, $x_Symbol]$:= Simp[$(-b)*\text{Cos}[c + d*x]*((b*\text{Csc}[c + d*x]) $^{(n - 1)}$ /($d*(n - 1)$))$, $x]$ + Dist[$b^2*((n - 2)/(n - 1))$, Int[($b*\text{Csc}[c + d*x]$) $^{(n - 2)}$, $x]$, $x]$ /; FreeQ[{ b, c, d }, $x]$ && GtQ[$n, 1]$ & IntegerQ[$2*n$]

Rule 3855

Int[csc[($c_.$) + ($d_.$)*($x_.$)], $x_Symbol]$:= Simp[$-\text{ArcTanh}[\text{Cos}[c + d*x]]/d$, $x]$ /; FreeQ[{ c, d }, $x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a^2(a + b \cos(c + dx)) \sec^4(c + dx) \tan(c + dx)}{5d} \\
 &+ \frac{1}{5} \int (11a^2b + a(4a^2 + 15b^2) \cos(c + dx) + b(3a^2 + 5b^2) \cos^2(c + dx)) \sec^5(c + dx) dx \\
 &= \frac{11a^2b \sec^3(c + dx) \tan(c + dx)}{20d} + \frac{a^2(a + b \cos(c + dx)) \sec^4(c + dx) \tan(c + dx)}{5d} \\
 &+ \frac{1}{20} \int (4a(4a^2 + 15b^2) + 5b(9a^2 + 4b^2) \cos(c + dx)) \sec^4(c + dx) dx \\
 &= \frac{11a^2b \sec^3(c + dx) \tan(c + dx)}{20d} + \frac{a^2(a + b \cos(c + dx)) \sec^4(c + dx) \tan(c + dx)}{5d} \\
 &+ \frac{1}{4}(b(9a^2 + 4b^2)) \int \sec^3(c + dx) dx + \frac{1}{5}(a(4a^2 + 15b^2)) \int \sec^4(c + dx) dx \\
 &= \frac{b(9a^2 + 4b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{11a^2b \sec^3(c + dx) \tan(c + dx)}{20d} \\
 &+ \frac{a^2(a + b \cos(c + dx)) \sec^4(c + dx) \tan(c + dx)}{5d} \\
 &+ \frac{1}{8}(b(9a^2 + 4b^2)) \int \sec(c + dx) dx \\
 &- \frac{(a(4a^2 + 15b^2)) \text{Subst}(\int (1 + x^2) dx, x, -\tan(c + dx))}{5d}
 \end{aligned}$$

$$= \frac{b(9a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a(4a^2 + 15b^2) \tan(c + dx)}{5d} + \frac{b(9a^2 + 4b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{11a^2b \sec^3(c + dx) \tan(c + dx)}{20d} + \frac{a^2(a + b \cos(c + dx)) \sec^4(c + dx) \tan(c + dx)}{5d} + \frac{a(4a^2 + 15b^2) \tan^3(c + dx)}{15d}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int (a + b \cos(c + dx))^3 \sec^6(c + dx) dx$$

$$= \frac{15b(9a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx)) + \tan(c + dx) (15b(9a^2 + 4b^2) \sec(c + dx) + 90a^2b \sec^3(c + dx) + 8a(15b(9a^2 + 4b^2) \sec(c + dx) \tan(c + dx) + 11a^2b \sec^3(c + dx) \tan(c + dx) + a^2(a + b \cos(c + dx)) \sec^4(c + dx) \tan(c + dx)))}{120d}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^6,x]

[Out] (15*b*(9*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*b*(9*a^2 + 4*b^2)*Sec[c + d*x] + 90*a^2*b*Sec[c + d*x]^3 + 8*a*(15*(a^2 + 3*b^2) + 5*(2*a^2 + 3*b^2)*Tan[c + d*x]^2 + 3*a^2*Tan[c + d*x]^4)))/(120*d)

Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-a^3 \left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c) + 3a^2b \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3\sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3\ln(\sec(dx+c))}{d} \right)$
default	$-a^3 \left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c) + 3a^2b \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3\sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3\ln(\sec(dx+c))}{d} \right)$
parts	$\frac{a^3 \left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c)}{d} + \frac{b^3 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right)}{d} + \frac{3a^2b \left(\frac{\sec(dx+c)}{8} + \frac{\tan(dx+c)}{4} \right)}{d}$
parallelrisc	$-1350b \left(a^2 + \frac{4b^2}{9} \right) \left(\frac{\cos(5dx+5c)}{10} + \frac{\cos(3dx+3c)}{2} + \cos(dx+c) \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 1350b \left(a^2 + \frac{4b^2}{9} \right) \left(\frac{\cos(5dx+5c)}{10} + \frac{\cos(3dx+3c)}{2} + \cos(dx+c) \right)$
risc	$-\frac{i(135a^2b e^{9i(dx+c)} + 60b^3 e^{9i(dx+c)} + 630a^2b e^{7i(dx+c)} + 120b^3 e^{7i(dx+c)} - 720a b^2 e^{6i(dx+c)} - 640a^3 e^{4i(dx+c)} - 1680a b^2 e^{4i(dx+c)})}{60d(e^{2i(dx+c)} + 1)}$
norman	$-\frac{(8a^3 - 15a^2b + 24ab^2 - 4b^3) \left(\tan^{15} \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{4d} - \frac{(8a^3 + 15a^2b + 24ab^2 + 4b^3) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d} - \frac{(40a^3 - 117a^2b + 24ab^2 - 12b^3) \left(\tan^{13} \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{12d}$

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^6,x,method=_RETURNVERBOSE)

[Out] $1/d*(-a^3*(-8/15-1/5*\sec(dx+c)^4-4/15*\sec(dx+c)^2)*\tan(dx+c)+3*a^2*b*(-(-1/4*\sec(dx+c)^3-3/8*\sec(dx+c))*\tan(dx+c)+3/8*\ln(\sec(dx+c)+\tan(dx+c)))-3*a*b^2*(-2/3-1/3*\sec(dx+c)^2)*\tan(dx+c)+b^3*(1/2*\sec(dx+c)*\tan(dx+c)+1/2*\ln(\sec(dx+c)+\tan(dx+c))))$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.01

$$\int (a + b \cos(c + dx))^3 \sec^6(c + dx) dx$$

$$= \frac{15(9a^2b + 4b^3) \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 15(9a^2b + 4b^3) \cos(dx + c)^5 \log(-\sin(dx + c) + 1)}{1}$$

[In] `integrate((a+b*cos(dx+c))^3*sec(dx+c)^6,x, algorithm="fricas")`

[Out] $1/240*(15*(9*a^2*b + 4*b^3)*\cos(dx + c)^5*\log(\sin(dx + c) + 1) - 15*(9*a^2*b + 4*b^3)*\cos(dx + c)^5*\log(-\sin(dx + c) + 1) + 2*(16*(4*a^3 + 15*a*b^2)*\cos(dx + c)^4 + 90*a^2*b*\cos(dx + c) + 15*(9*a^2*b + 4*b^3)*\cos(dx + c)^3 + 24*a^3 + 8*(4*a^3 + 15*a*b^2)*\cos(dx + c)^2*\sin(dx + c))/(d*\cos(dx + c)^5)$

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^6(c + dx) dx = \text{Timed out}$$

[In] `integrate((a+b*cos(dx+c))**3*sec(dx+c)**6,x)`

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.07

$$\int (a + b \cos(c + dx))^3 \sec^6(c + dx) dx$$

$$= \frac{16(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c))a^3 + 240(\tan(dx + c)^3 + 3 \tan(dx + c))ab^2 - \dots}{1}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="maxima")

[Out] $\frac{1}{240} * (16 * (3 * \tan(d * x + c))^5 + 10 * \tan(d * x + c)^3 + 15 * \tan(d * x + c)) * a^3 + 240 * (\tan(d * x + c)^3 + 3 * \tan(d * x + c)) * a * b^2 - 45 * a^2 * b * (2 * (3 * \sin(d * x + c))^3 - 5 * \sin(d * x + c)) / (\sin(d * x + c)^4 - 2 * \sin(d * x + c)^2 + 1) - 3 * \log(\sin(d * x + c) + 1) + 3 * \log(\sin(d * x + c) - 1) - 60 * b^3 * (2 * \sin(d * x + c)) / (\sin(d * x + c)^2 - 1) - \log(\sin(d * x + c) + 1) + \log(\sin(d * x + c) - 1)) / d$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(157) = 314.

Time = 0.35 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.17

$$\int (a + b \cos(c + dx))^3 \sec^6(c + dx) dx$$

$$= \frac{15(9a^2b + 4b^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 15(9a^2b + 4b^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2(120a^3 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 1)}{\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1}}{d}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="giac")

[Out] $\frac{1}{120} * (15 * (9 * a^2 * b + 4 * b^3) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) + 1)) - 15 * (9 * a^2 * b + 4 * b^3) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) - 1)) - 2 * (120 * a^3 * \tan(1/2 * d * x + 1/2 * c)^9 - 225 * a^2 * b * \tan(1/2 * d * x + 1/2 * c)^9 + 360 * a * b^2 * \tan(1/2 * d * x + 1/2 * c)^9 - 60 * b^3 * \tan(1/2 * d * x + 1/2 * c)^9 - 160 * a^3 * \tan(1/2 * d * x + 1/2 * c)^7 + 90 * a^2 * b * \tan(1/2 * d * x + 1/2 * c)^7 - 960 * a * b^2 * \tan(1/2 * d * x + 1/2 * c)^7 + 120 * b^3 * \tan(1/2 * d * x + 1/2 * c)^7 + 464 * a^3 * \tan(1/2 * d * x + 1/2 * c)^5 + 1200 * a * b^2 * \tan(1/2 * d * x + 1/2 * c)^5 - 160 * a^3 * \tan(1/2 * d * x + 1/2 * c)^3 - 90 * a^2 * b * \tan(1/2 * d * x + 1/2 * c)^3 - 960 * a * b^2 * \tan(1/2 * d * x + 1/2 * c)^3 - 120 * b^3 * \tan(1/2 * d * x + 1/2 * c)^3 + 120 * a^3 * \tan(1/2 * d * x + 1/2 * c) + 225 * a^2 * b * \tan(1/2 * d * x + 1/2 * c) + 360 * a * b^2 * \tan(1/2 * d * x + 1/2 * c) + 60 * b^3 * \tan(1/2 * d * x + 1/2 * c)) / (\tan(1/2 * d * x + 1/2 * c)^2 - 1)^5) / d$

Mupad [B] (verification not implemented)

Time = 17.67 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.54

$$\int (a + b \cos(c + dx))^3 \sec^6(c + dx) dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) \left(\frac{9a^2b}{4} + b^3\right)}{d} - \frac{\left(2a^3 - \frac{15a^2b}{4} + 6ab^2 - b^3\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(-\frac{8a^3}{3} + \frac{3a^2b}{2} - 16ab^2 + 2b^3\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{116a^3}{15} + 2b^3\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 - 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^6,x)


```
[Out] (atanh(tan(c/2 + (d*x)/2))*((9*a^2*b)/4 + b^3))/d - (tan(c/2 + (d*x)/2)^9*(
6*a*b^2 - (15*a^2*b)/4 + 2*a^3 - b^3) - tan(c/2 + (d*x)/2)^3*(16*a*b^2 + (3
*a^2*b)/2 + (8*a^3)/3 + 2*b^3) - tan(c/2 + (d*x)/2)^7*(16*a*b^2 - (3*a^2*b)
/2 + (8*a^3)/3 - 2*b^3) + tan(c/2 + (d*x)/2)*(6*a*b^2 + (15*a^2*b)/4 + 2*a^
3 + b^3) + tan(c/2 + (d*x)/2)^5*(20*a*b^2 + (116*a^3)/15))/(d*(5*tan(c/2 +
(d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2
+ (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.438 $\int \cos^3(c + dx)(a + b \cos(c + dx))^4 dx$

Optimal result	4798
Rubi [A] (verified)	4799
Mathematica [A] (verified)	4802
Maple [A] (verified)	4803
Fricas [A] (verification not implemented)	4803
Sympy [B] (verification not implemented)	4804
Maxima [A] (verification not implemented)	4804
Giac [A] (verification not implemented)	4805
Mupad [B] (verification not implemented)	4805

Optimal result

Integrand size = 21, antiderivative size = 247

$$\begin{aligned}
 \int \cos^3(c + dx)(a + b \cos(c + dx))^4 dx = & \frac{1}{4}ab(6a^2 + 5b^2)x \\
 & + \frac{(35a^4 + 168a^2b^2 + 24b^4) \sin(c + dx)}{35d} \\
 & + \frac{ab(6a^2 + 5b^2) \cos(c + dx) \sin(c + dx)}{4d} \\
 & + \frac{ab(6a^2 + 5b^2) \cos^3(c + dx) \sin(c + dx)}{6d} \\
 & + \frac{b^2(37a^2 + 6b^2) \cos^4(c + dx) \sin(c + dx)}{35d} \\
 & + \frac{8ab^3 \cos^5(c + dx) \sin(c + dx)}{21d} \\
 & + \frac{b^2 \cos^4(c + dx)(a + b \cos(c + dx))^2 \sin(c + dx)}{7d} \\
 & - \frac{(35a^4 + 168a^2b^2 + 24b^4) \sin^3(c + dx)}{105d}
 \end{aligned}$$

```
[Out] 1/4*a*b*(6*a^2+5*b^2)*x+1/35*(35*a^4+168*a^2*b^2+24*b^4)*sin(d*x+c)/d+1/4*a
*b*(6*a^2+5*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/6*a*b*(6*a^2+5*b^2)*cos(d*x+c)^3
*sin(d*x+c)/d+1/35*b^2*(37*a^2+6*b^2)*cos(d*x+c)^4*sin(d*x+c)/d+8/21*a*b^3*
cos(d*x+c)^5*sin(d*x+c)/d+1/7*b^2*cos(d*x+c)^4*(a+b*cos(d*x+c))^2*sin(d*x+c
)/d-1/105*(35*a^4+168*a^2*b^2+24*b^4)*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2872, 3112, 3102, 2827, 2713, 2715, 8}

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^4 dx = \frac{b^2(37a^2 + 6b^2) \sin(c + dx) \cos^4(c + dx)}{35d} + \frac{ab(6a^2 + 5b^2) \sin(c + dx) \cos^3(c + dx)}{6d} + \frac{ab(6a^2 + 5b^2) \sin(c + dx) \cos(c + dx)}{4d} + \frac{1}{4}abx(6a^2 + 5b^2) - \frac{(35a^4 + 168a^2b^2 + 24b^4) \sin^3(c + dx)}{105d} + \frac{(35a^4 + 168a^2b^2 + 24b^4) \sin(c + dx)}{35d} + \frac{8ab^3 \sin(c + dx) \cos^5(c + dx)}{21d} + \frac{b^2 \sin(c + dx) \cos^4(c + dx)(a + b \cos(c + dx))^2}{7d}$$

[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^4,x]

[Out] (a*b*(6*a^2 + 5*b^2)*x)/4 + ((35*a^4 + 168*a^2*b^2 + 24*b^4)*Sin[c + d*x])/(35*d) + (a*b*(6*a^2 + 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a*b*(6*a^2 + 5*b^2)*Cos[c + d*x]^3*SIN[c + d*x])/(6*d) + (b^2*(37*a^2 + 6*b^2)*Cos[c + d*x]^4*SIN[c + d*x])/(35*d) + (8*a*b^3*Cos[c + d*x]^5*SIN[c + d*x])/(21*d) + (b^2*Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2*SIN[c + d*x])/(7*d) - ((35*a^4 + 168*a^2*b^2 + 24*b^4)*Sin[c + d*x]^3)/(105*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2

*n]

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m - 2)*((c + d*SIN[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*SIN[e + f*x])^(m - 3)*(c + d*SIN[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*SIN[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*SIN[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])*(A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*d*COS[e + f*x]*SIN[e + f*x]*((a + b*SIN[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*SIN[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{b^2 \cos^4(c + dx)(a + b \cos(c + dx))^2 \sin(c + dx)}{7d} + \frac{1}{7} \int \cos^3(c + dx)(a + b \cos(c + dx)) (a(7a^2 + 4b^2) + 3b(7a^2 + 2b^2) \cos(c + dx) + 16ab^2 \cos^2(c + dx)) dx$$

$$\begin{aligned}
&= \frac{8ab^3 \cos^5(c+dx) \sin(c+dx)}{21d} + \frac{b^2 \cos^4(c+dx)(a+b \cos(c+dx))^2 \sin(c+dx)}{7d} \\
&\quad + \frac{1}{42} \int \cos^3(c+dx) (6a^2(7a^2+4b^2) + 28ab(6a^2+5b^2) \cos(c+dx) \\
&\qquad\qquad\qquad + 6b^2(37a^2+6b^2) \cos^2(c+dx)) dx \\
&= \frac{b^2(37a^2+6b^2) \cos^4(c+dx) \sin(c+dx)}{35d} + \frac{8ab^3 \cos^5(c+dx) \sin(c+dx)}{21d} \\
&\quad + \frac{b^2 \cos^4(c+dx)(a+b \cos(c+dx))^2 \sin(c+dx)}{7d} \\
&\quad + \frac{1}{210} \int \cos^3(c+dx) (6(35a^4+168a^2b^2+24b^4) + 140ab(6a^2+5b^2) \cos(c+dx)) dx \\
&= \frac{b^2(37a^2+6b^2) \cos^4(c+dx) \sin(c+dx)}{35d} + \frac{8ab^3 \cos^5(c+dx) \sin(c+dx)}{21d} \\
&\quad + \frac{b^2 \cos^4(c+dx)(a+b \cos(c+dx))^2 \sin(c+dx)}{7d} \\
&\quad + \frac{1}{3} (2ab(6a^2+5b^2)) \int \cos^4(c+dx) dx \\
&\quad + \frac{1}{35} (35a^4+168a^2b^2+24b^4) \int \cos^3(c+dx) dx \\
&= \frac{ab(6a^2+5b^2) \cos^3(c+dx) \sin(c+dx)}{6d} + \frac{b^2(37a^2+6b^2) \cos^4(c+dx) \sin(c+dx)}{35d} \\
&\quad + \frac{8ab^3 \cos^5(c+dx) \sin(c+dx)}{21d} + \frac{b^2 \cos^4(c+dx)(a+b \cos(c+dx))^2 \sin(c+dx)}{7d} \\
&\quad + \frac{1}{2} (ab(6a^2+5b^2)) \int \cos^2(c+dx) dx \\
&\quad - \frac{(35a^4+168a^2b^2+24b^4) \text{Subst}(\int (1-x^2) dx, x, -\sin(c+dx))}{35d} \\
&= \frac{(35a^4+168a^2b^2+24b^4) \sin(c+dx)}{35d} + \frac{ab(6a^2+5b^2) \cos(c+dx) \sin(c+dx)}{4d} \\
&\quad + \frac{ab(6a^2+5b^2) \cos^3(c+dx) \sin(c+dx)}{6d} + \frac{b^2(37a^2+6b^2) \cos^4(c+dx) \sin(c+dx)}{35d} \\
&\quad + \frac{8ab^3 \cos^5(c+dx) \sin(c+dx)}{21d} + \frac{b^2 \cos^4(c+dx)(a+b \cos(c+dx))^2 \sin(c+dx)}{7d} \\
&\quad - \frac{(35a^4+168a^2b^2+24b^4) \sin^3(c+dx)}{105d} + \frac{1}{4} (ab(6a^2+5b^2)) \int 1 dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4}ab(6a^2 + 5b^2)x + \frac{(35a^4 + 168a^2b^2 + 24b^4)\sin(c + dx)}{35d} \\
&\quad + \frac{ab(6a^2 + 5b^2)\cos(c + dx)\sin(c + dx)}{4d} + \frac{ab(6a^2 + 5b^2)\cos^3(c + dx)\sin(c + dx)}{6d} \\
&\quad + \frac{b^2(37a^2 + 6b^2)\cos^4(c + dx)\sin(c + dx)}{35d} + \frac{8ab^3\cos^5(c + dx)\sin(c + dx)}{21d} \\
&\quad + \frac{b^2\cos^4(c + dx)(a + b\cos(c + dx))^2\sin(c + dx)}{7d} \\
&\quad - \frac{(35a^4 + 168a^2b^2 + 24b^4)\sin^3(c + dx)}{105d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.73

$$\begin{aligned}
&\int \cos^3(c + dx)(a + b\cos(c + dx))^4 dx \\
&= \frac{1680ab(6a^2 + 5b^2)(c + dx) + 105(48a^4 + 240a^2b^2 + 35b^4)\sin(c + dx) + 420ab(16a^2 + 15b^2)\sin(2(c + dx))}{6720d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^4,x]

[Out] (1680*a*b*(6*a^2 + 5*b^2)*(c + d*x) + 105*(48*a^4 + 240*a^2*b^2 + 35*b^4)*Sin[c + d*x] + 420*a*b*(16*a^2 + 15*b^2)*Sin[2*(c + d*x)] + 35*(16*a^4 + 120*a^2*b^2 + 21*b^4)*Sin[3*(c + d*x)] + 420*a*b*(2*a^2 + 3*b^2)*Sin[4*(c + d*x)] + 21*b^2*(24*a^2 + 7*b^2)*Sin[5*(c + d*x)] + 140*a*b^3*Ssin[6*(c + d*x)] + 15*b^4*Ssin[7*(c + d*x)])/(6720*d)

Maple [A] (verified)

Time = 5.95 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.71

method	result
parallelrisch	$(560a^4+4200a^2b^2+735b^4) \sin(3dx+3c)+(6720a^3b+6300ab^3) \sin(2dx+2c)+(840a^3b+1260ab^3) \sin(4dx+4c)+(504a^2b^2+147ab^4) \sin(5dx+5c)+140ab^3 \sin(6dx+6c)+15b^4 \sin(7dx+7c)+(5040a^4+25200a^2b^2+3675b^4) \sin(dx+c)+10080b^4(a^2+5/6b^2)dx/a/d$
derivativedivides	$\frac{a^4(2+\cos^2(dx+c)) \sin(dx+c)}{3} + 4a^3b \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx + \frac{3c}{8}}{8} \right) + \frac{6a^2b^2 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right)}{5}$
default	$\frac{a^4(2+\cos^2(dx+c)) \sin(dx+c)}{3} + 4a^3b \left(\frac{(\cos^3(dx+c) + \frac{3\cos(dx+c)}{2}) \sin(dx+c)}{4} + \frac{3dx + \frac{3c}{8}}{8} \right) + \frac{6a^2b^2 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3} \right)}{5}$
parts	$\frac{a^4(2+\cos^2(dx+c)) \sin(dx+c)}{3d} + \frac{b^4 \left(\frac{16}{5} + \cos^6(dx+c) + \frac{6(\cos^4(dx+c))}{5} + \frac{8(\cos^2(dx+c))}{5} \right) \sin(dx+c)}{7d} + \frac{4ab^3 \left(\frac{\cos^5(dx+c)}{5} \right)}{5}$
risch	$\frac{3a^3bx}{2} + \frac{5ab^3x}{4} + \frac{3a^4 \sin(dx+c)}{4d} + \frac{15 \sin(dx+c)a^2b^2}{4d} + \frac{35 \sin(dx+c)b^4}{64d} + \frac{b^4 \sin(7dx+7c)}{448d} + \frac{ab^3 \sin(6dx+6c)}{48d}$
norman	$\frac{(\frac{3}{2}a^3b + \frac{5}{4}ab^3)x + (\frac{3}{2}a^3b + \frac{5}{4}ab^3)x \left(\tan^{14} \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + (\frac{21}{2}a^3b + \frac{35}{4}ab^3)x \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + (\frac{21}{2}a^3b + \frac{35}{4}ab^3)x \left(\tan^{12} \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d}$

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6720*((560*a^4+4200*a^2*b^2+735*b^4)*sin(3*d*x+3*c)+(6720*a^3*b+6300*a*b^3)*sin(2*d*x+2*c)+(840*a^3*b+1260*a*b^3)*sin(4*d*x+4*c)+(504*a^2*b^2+147*b^4)*sin(5*d*x+5*c)+140*a*b^3*sin(6*d*x+6*c)+15*b^4*sin(7*d*x+7*c)+(5040*a^4+25200*a^2*b^2+3675*b^4)*sin(d*x+c)+10080*b*(a^2+5/6*b^2)*d*x/a/d
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.69

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^4 dx$$

$$= \frac{105(6a^3b+5ab^3)dx + (60b^4\cos(dx+c))^6 + 280ab^3\cos(dx+c)^5 + 72(7a^2b^2+b^4)\cos(dx+c)^4 + 280a^4 + 1344a^2b^2}{d}$$

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/420*(105*(6*a^3*b + 5*a*b^3)*d*x + (60*b^4*cos(d*x + c))^6 + 280*a*b^3*cos(d*x + c)^5 + 72*(7*a^2*b^2 + b^4)*cos(d*x + c)^4 + 280*a^4 + 1344*a^2*b^2)
```


$n(d*x + c)^3 + 15*\sin(d*x + c)*a^2*b^2 + 35*(4*\sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*\sin(4*d*x + 4*c) - 48*\sin(2*d*x + 2*c))*a*b^3 + 48*(5*\sin(d*x + c)^7 - 21*\sin(d*x + c)^5 + 35*\sin(d*x + c)^3 - 35*\sin(d*x + c))*b^4)/d$

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.80

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^4 dx = \frac{b^4 \sin(7 dx + 7 c)}{448 d} + \frac{ab^3 \sin(6 dx + 6 c)}{48 d} + \frac{1}{4} (6 a^3 b + 5 ab^3) x + \frac{(24 a^2 b^2 + 7 b^4) \sin(5 dx + 5 c)}{320 d} + \frac{(2 a^3 b + 3 ab^3) \sin(4 dx + 4 c)}{16 d} + \frac{(16 a^4 + 120 a^2 b^2 + 21 b^4) \sin(3 dx + 3 c)}{192 d} + \frac{(16 a^3 b + 15 ab^3) \sin(2 dx + 2 c)}{16 d} + \frac{(48 a^4 + 240 a^2 b^2 + 35 b^4) \sin(dx + c)}{64 d}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out] $1/448*b^4*\sin(7*d*x + 7*c)/d + 1/48*a*b^3*\sin(6*d*x + 6*c)/d + 1/4*(6*a^3*b + 5*a*b^3)*x + 1/320*(24*a^2*b^2 + 7*b^4)*\sin(5*d*x + 5*c)/d + 1/16*(2*a^3*b + 3*a*b^3)*\sin(4*d*x + 4*c)/d + 1/192*(16*a^4 + 120*a^2*b^2 + 21*b^4)*\sin(3*d*x + 3*c)/d + 1/16*(16*a^3*b + 15*a*b^3)*\sin(2*d*x + 2*c)/d + 1/64*(48*a^4 + 240*a^2*b^2 + 35*b^4)*\sin(d*x + c)/d$

Mupad [B] (verification not implemented)

Time = 15.71 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.93

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^4 dx = \frac{\left(2 a^4 - 5 a^3 b + 12 a^2 b^2 - \frac{11 a b^3}{2} + 2 b^4\right) \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^{13} + \left(\frac{28 a^4}{3} - 12 a^3 b + 40 a^2 b^2 - \frac{14 a b^3}{3} + 4 b^4\right) \tan\left(\frac{c}{2} + \frac{d x}{2}\right)}{2 d} + \frac{a b \operatorname{atan}\left(\frac{a b \tan\left(\frac{c}{2} + \frac{d x}{2}\right) (6 a^2 + 5 b^2)}{2 (3 a^3 b + \frac{5 a b^3}{2})}\right) (6 a^2 + 5 b^2)}{2 d} - \frac{a b (6 a^2 + 5 b^2) \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right)\right) - \frac{d x}{2}\right)}{2 d}$$

[In] $\text{int}(\cos(c + d*x)^3*(a + b*\cos(c + d*x))^4,x)$

[Out] $(\tan(c/2 + (d*x)/2)^7*(24*a^4 + (424*b^4)/35 + (624*a^2*b^2)/5) + \tan(c/2 + (d*x)/2)^{13}*(2*a^4 - 5*a^3*b - (11*a*b^3)/2 + 2*b^4 + 12*a^2*b^2) + \tan(c/2 + (d*x)/2)^3*((14*a*b^3)/3 + 12*a^3*b + (28*a^4)/3 + 4*b^4 + 40*a^2*b^2) + \tan(c/2 + (d*x)/2)^{11}*((28*a^4)/3 - 12*a^3*b - (14*a*b^3)/3 + 4*b^4 + 40*a^2*b^2) + \tan(c/2 + (d*x)/2)^5*((85*a*b^3)/6 + 9*a^3*b + (58*a^4)/3 + (86*b^4)/5 + (452*a^2*b^2)/5) + \tan(c/2 + (d*x)/2)^9*((58*a^4)/3 - 9*a^3*b - (85*a*b^3)/6 + (86*b^4)/5 + (452*a^2*b^2)/5) + \tan(c/2 + (d*x)/2)*((11*a*b^3)/2 + 5*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(7*\tan(c/2 + (d*x)/2)^2 + 21*\tan(c/2 + (d*x)/2)^4 + 35*\tan(c/2 + (d*x)/2)^6 + 35*\tan(c/2 + (d*x)/2)^8 + 21*\tan(c/2 + (d*x)/2)^{10} + 7*\tan(c/2 + (d*x)/2)^{12} + \tan(c/2 + (d*x)/2)^{14} + 1)) + (a*b*\text{atan}((a*b*\tan(c/2 + (d*x)/2)*(6*a^2 + 5*b^2))/(2*((5*a*b^3)/2 + 3*a^3*b)))*(6*a^2 + 5*b^2))/(2*d) - (a*b*(6*a^2 + 5*b^2)*(\text{atan}(\tan(c/2 + (d*x)/2)) - (d*x)/2))/(2*d)$

3.439 $\int \cos^2(c + dx)(a + b \cos(c + dx))^4 dx$

Optimal result	4807
Rubi [A] (verified)	4808
Mathematica [A] (verified)	4810
Maple [A] (verified)	4810
Fricas [A] (verification not implemented)	4811
Sympy [B] (verification not implemented)	4811
Maxima [A] (verification not implemented)	4812
Giac [A] (verification not implemented)	4812
Mupad [B] (verification not implemented)	4813

Optimal result

Integrand size = 21, antiderivative size = 235

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^4 dx = \frac{1}{16}(8a^4 + 36a^2b^2 + 5b^4)x - \frac{a(4a^4 - 121a^2b^2 - 128b^4) \sin(c + dx)}{60bd} - \frac{(8a^4 - 178a^2b^2 - 75b^4) \cos(c + dx) \sin(c + dx)}{240d} - \frac{a(4a^2 - 53b^2)(a + b \cos(c + dx))^2 \sin(c + dx)}{120bd} - \frac{(4a^2 - 25b^2)(a + b \cos(c + dx))^3 \sin(c + dx)}{120bd} - \frac{a(a + b \cos(c + dx))^4 \sin(c + dx)}{30bd} + \frac{(a + b \cos(c + dx))^5 \sin(c + dx)}{6bd}$$

[Out] 1/16*(8*a^4+36*a^2*b^2+5*b^4)*x-1/60*a*(4*a^4-121*a^2*b^2-128*b^4)*sin(d*x+c)/b/d-1/240*(8*a^4-178*a^2*b^2-75*b^4)*cos(d*x+c)*sin(d*x+c)/d-1/120*a*(4*a^2-53*b^2)*(a+b*cos(d*x+c))^2*sin(d*x+c)/b/d-1/120*(4*a^2-25*b^2)*(a+b*cos(d*x+c))^3*sin(d*x+c)/b/d-1/30*a*(a+b*cos(d*x+c))^4*sin(d*x+c)/b/d+1/6*(a+b*cos(d*x+c))^5*sin(d*x+c)/b/d

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2870, 2832, 2813}

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^4 dx = -\frac{(4a^2 - 25b^2) \sin(c + dx)(a + b \cos(c + dx))^3}{120bd} - \frac{a(4a^2 - 53b^2) \sin(c + dx)(a + b \cos(c + dx))^2}{120bd} - \frac{a(4a^4 - 121a^2b^2 - 128b^4) \sin(c + dx)}{60bd} - \frac{(8a^4 - 178a^2b^2 - 75b^4) \sin(c + dx) \cos(c + dx)}{240d} + \frac{1}{16}x(8a^4 + 36a^2b^2 + 5b^4) + \frac{\sin(c + dx)(a + b \cos(c + dx))^5}{6bd} - \frac{a \sin(c + dx)(a + b \cos(c + dx))^4}{30bd}$$

[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4,x]

[Out] ((8*a^4 + 36*a^2*b^2 + 5*b^4)*x)/16 - (a*(4*a^4 - 121*a^2*b^2 - 128*b^4)*Sin[c + d*x])/(60*b*d) - ((8*a^4 - 178*a^2*b^2 - 75*b^4)*Cos[c + d*x]*Sin[c + d*x])/(240*d) - (a*(4*a^2 - 53*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(120*b*d) - ((4*a^2 - 25*b^2)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(120*b*d) - (a*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(30*b*d) + ((a + b*Cos[c + d*x])^5*Sin[c + d*x])/(6*b*d)

Rule 2813

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Cos[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2870

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^2, x_Symbol] := Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && !LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(a + b \cos(c + dx))^5 \sin(c + dx)}{6bd} + \frac{\int (5b - a \cos(c + dx))(a + b \cos(c + dx))^4 dx}{6b} \\
&= -\frac{a(a + b \cos(c + dx))^4 \sin(c + dx)}{30bd} + \frac{(a + b \cos(c + dx))^5 \sin(c + dx)}{6bd} \\
&\quad + \frac{\int (a + b \cos(c + dx))^3 (21ab - (4a^2 - 25b^2) \cos(c + dx)) dx}{30b} \\
&= -\frac{(4a^2 - 25b^2)(a + b \cos(c + dx))^3 \sin(c + dx)}{120bd} \\
&\quad - \frac{a(a + b \cos(c + dx))^4 \sin(c + dx)}{30bd} + \frac{(a + b \cos(c + dx))^5 \sin(c + dx)}{6bd} \\
&\quad + \frac{\int (a + b \cos(c + dx))^2 (3b(24a^2 + 25b^2) - 3a(4a^2 - 53b^2) \cos(c + dx)) dx}{120b} \\
&= -\frac{a(4a^2 - 53b^2)(a + b \cos(c + dx))^2 \sin(c + dx)}{120bd} \\
&\quad - \frac{(4a^2 - 25b^2)(a + b \cos(c + dx))^3 \sin(c + dx)}{120bd} \\
&\quad - \frac{a(a + b \cos(c + dx))^4 \sin(c + dx)}{30bd} + \frac{(a + b \cos(c + dx))^5 \sin(c + dx)}{6bd} \\
&\quad + \frac{\int (a + b \cos(c + dx)) (3ab(64a^2 + 181b^2) - 3(8a^4 - 178a^2b^2 - 75b^4) \cos(c + dx)) dx}{360b} \\
&= \frac{1}{16} (8a^4 + 36a^2b^2 + 5b^4) x - \frac{a(4a^4 - 121a^2b^2 - 128b^4) \sin(c + dx)}{60bd} \\
&\quad - \frac{(8a^4 - 178a^2b^2 - 75b^4) \cos(c + dx) \sin(c + dx)}{240d} \\
&\quad - \frac{a(4a^2 - 53b^2)(a + b \cos(c + dx))^2 \sin(c + dx)}{120bd} \\
&\quad - \frac{(4a^2 - 25b^2)(a + b \cos(c + dx))^3 \sin(c + dx)}{120bd} \\
&\quad - \frac{a(a + b \cos(c + dx))^4 \sin(c + dx)}{30bd} + \frac{(a + b \cos(c + dx))^5 \sin(c + dx)}{6bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^4 dx$$

$$= \frac{60(8a^4 + 36a^2b^2 + 5b^4)(c + dx) + 480ab(6a^2 + 5b^2) \sin(c + dx) + 15(16a^4 + 96a^2b^2 + 15b^4) \sin(2(c + dx))}{9}$$

`[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4,x]`

```
[Out] (60*(8*a^4 + 36*a^2*b^2 + 5*b^4)*(c + d*x) + 480*a*b*(6*a^2 + 5*b^2)*Sin[c + d*x] + 15*(16*a^4 + 96*a^2*b^2 + 15*b^4)*Sin[2*(c + d*x)] + 80*a*b*(4*a^2 + 5*b^2)*Sin[3*(c + d*x)] + 45*b^2*(4*a^2 + b^2)*Sin[4*(c + d*x)] + 48*a*b^3*Ssin[5*(c + d*x)] + 5*b^4*Ssin[6*(c + d*x)])/(960*d)
```

Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

method	result
parallelrisch	$\frac{(240a^4 + 1440a^2b^2 + 225b^4) \sin(2dx + 2c) + (320a^3b + 400ab^3) \sin(3dx + 3c) + (180a^2b^2 + 45b^4) \sin(4dx + 4c) + 48ab^3 \sin(5dx + 5c) + 5b^4 \sin(6dx + 6c)}{960d}$
derivativdivides	$a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + \frac{4a^3b(2 + \cos^2(dx+c)) \sin(dx+c)}{3} + 6a^2b^2 \left(\frac{\cos^3(dx+c) + \frac{3}{2} \cos(dx+c)}{4} \sin(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right)$
default	$a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + \frac{4a^3b(2 + \cos^2(dx+c)) \sin(dx+c)}{3} + 6a^2b^2 \left(\frac{\cos^3(dx+c) + \frac{3}{2} \cos(dx+c)}{4} \sin(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right)$
parts	$\frac{a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d} + \frac{b^4 \left(\frac{\cos^5(dx+c) + \frac{5}{4} \cos^3(dx+c) + \frac{15}{8} \cos(dx+c)}{6} \sin(dx+c) + \frac{5dx}{16} + \frac{5c}{16} \right)}{d} + \frac{4ab^3 \left(\frac{8}{3} + \dots \right)}{d}$
risch	$\frac{a^4x}{2} + \frac{9x a^2b^2}{4} + \frac{5b^4x}{16} + \frac{3 \sin(dx+c) a^3b}{d} + \frac{5 \sin(dx+c) a b^3}{2d} + \frac{b^4 \sin(6dx+6c)}{192d} + \frac{a b^3 \sin(5dx+5c)}{20d} + \frac{3 \sin(4dx+4c)}{16}$
norman	$\frac{(\frac{1}{2}a^4 + \frac{9}{4}a^2b^2 + \frac{5}{16}b^4)x + (3a^4 + \frac{27}{2}a^2b^2 + \frac{15}{8}b^4)x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + (3a^4 + \frac{27}{2}a^2b^2 + \frac{15}{8}b^4)x \left(\tan^{10}\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + (10a^4 + 45a^2b^2 + 5b^4)x}{d}$

`[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)`

```
[Out] 1/960*((240*a^4+1440*a^2*b^2+225*b^4)*sin(2*d*x+2*c)+(320*a^3*b+400*a*b^3)*sin(3*d*x+3*c)+(180*a^2*b^2+45*b^4)*sin(4*d*x+4*c)+48*a*b^3*sin(5*d*x+5*c)+5*b^4*sin(6*d*x+6*c)+(2880*a^3*b+2400*a*b^3)*sin(d*x+c)+480*d*(a^4+9/2*a^2*b^2+5/8*b^4)*x)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.64

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^4 dx$$

$$= \frac{15(8a^4 + 36a^2b^2 + 5b^4)dx + (40b^4 \cos(dx + c))^5 + 192ab^3 \cos(dx + c)^4 + 640a^3b + 512ab^3 + 10(36a^2b^2 + 5b^4) \cos(dx + c)^3 + 64(5a^3b + 4a^2b^3) \cos(dx + c)^2 + 15(8a^4 + 36a^2b^2 + 5b^4) \cos(dx + c) \sin(dx + c)}{2d}$$

[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x, algorithm="fricas")

```
[Out] 1/240*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*d*x + (40*b^4*cos(d*x + c)^5 + 192*a
*b^3*cos(d*x + c)^4 + 640*a^3*b + 512*a*b^3 + 10*(36*a^2*b^2 + 5*b^4)*cos(d
*x + c)^3 + 64*(5*a^3*b + 4*a*b^3)*cos(d*x + c)^2 + 15*(8*a^4 + 36*a^2*b^2
+ 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. 2(211) = 422.

Time = 0.39 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.95

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^4 dx$$

$$= \begin{cases} \frac{a^4 x \sin^2(c+dx)}{2} + \frac{a^4 x \cos^2(c+dx)}{2} + \frac{a^4 \sin(c+dx) \cos(c+dx)}{2d} + \frac{8a^3 b \sin^3(c+dx)}{3d} + \frac{4a^3 b \sin(c+dx) \cos^2(c+dx)}{d} + \frac{9a^2 b^2 x \sin^4(c+dx)}{4} \\ x(a + b \cos(c))^4 \cos^2(c) \end{cases}$$

[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**4,x)

```
[Out] Piecewise((a**4*x*sin(c + d*x)**2/2 + a**4*x*cos(c + d*x)**2/2 + a**4*sin(c
+ d*x)*cos(c + d*x)/(2*d) + 8*a**3*b*sin(c + d*x)**3/(3*d) + 4*a**3*b*sin(
c + d*x)*cos(c + d*x)**2/d + 9*a**2*b**2*x*sin(c + d*x)**4/4 + 9*a**2*b**2*
x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*a**2*b**2*x*cos(c + d*x)**4/4 + 9*a
**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 15*a**2*b**2*sin(c + d*x)*cos
(c + d*x)**3/(4*d) + 32*a*b**3*sin(c + d*x)**5/(15*d) + 16*a*b**3*sin(c + d
*x)**3*cos(c + d*x)**2/(3*d) + 4*a*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 5*
b**4*x*sin(c + d*x)**6/16 + 15*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 +
15*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*b**4*x*cos(c + d*x)**6/16
+ 5*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*b**4*sin(c + d*x)**3*cos(c
+ d*x)**3/(6*d) + 11*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)),
(x*(a + b*cos(c))**4*cos(c)**2, True))
```


Mupad [B] (verification not implemented)

Time = 14.46 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.91

$$\begin{aligned}
\int \cos^2(c + dx)(a + b \cos(c + dx))^4 dx = & \frac{a^4 x}{2} + \frac{5 b^4 x}{16} + \frac{9 a^2 b^2 x}{4} + \frac{a^4 \sin(2c + 2dx)}{4d} \\
& + \frac{15 b^4 \sin(2c + 2dx)}{64d} + \frac{3 b^4 \sin(4c + 4dx)}{64d} \\
& + \frac{b^4 \sin(6c + 6dx)}{192d} + \frac{5 a b^3 \sin(3c + 3dx)}{12d} \\
& + \frac{a^3 b \sin(3c + 3dx)}{3d} + \frac{a b^3 \sin(5c + 5dx)}{20d} \\
& + \frac{3 a^2 b^2 \sin(2c + 2dx)}{2d} + \frac{3 a^2 b^2 \sin(4c + 4dx)}{16d} \\
& + \frac{5 a b^3 \sin(c + dx)}{2d} + \frac{3 a^3 b \sin(c + dx)}{d}
\end{aligned}$$

[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^4,x)

```
[Out] (a^4*x)/2 + (5*b^4*x)/16 + (9*a^2*b^2*x)/4 + (a^4*sin(2*c + 2*d*x))/(4*d) +
(15*b^4*sin(2*c + 2*d*x))/(64*d) + (3*b^4*sin(4*c + 4*d*x))/(64*d) + (b^4*
sin(6*c + 6*d*x))/(192*d) + (5*a*b^3*sin(3*c + 3*d*x))/(12*d) + (a^3*b*sin(
3*c + 3*d*x))/(3*d) + (a*b^3*sin(5*c + 5*d*x))/(20*d) + (3*a^2*b^2*sin(2*c
+ 2*d*x))/(2*d) + (3*a^2*b^2*sin(4*c + 4*d*x))/(16*d) + (5*a*b^3*sin(c + d*
x))/(2*d) + (3*a^3*b*sin(c + d*x))/d
```

3.440 $\int \cos(c + dx)(a + b \cos(c + dx))^4 dx$

Optimal result	4814
Rubi [A] (verified)	4814
Mathematica [A] (verified)	4816
Maple [A] (verified)	4816
Fricas [A] (verification not implemented)	4817
Sympy [A] (verification not implemented)	4818
Maxima [A] (verification not implemented)	4818
Giac [A] (verification not implemented)	4819
Mupad [B] (verification not implemented)	4819

Optimal result

Integrand size = 19, antiderivative size = 170

$$\int \cos(c + dx)(a + b \cos(c + dx))^4 dx = \frac{1}{2}ab(4a^2 + 3b^2)x + \frac{2(3a^4 + 28a^2b^2 + 4b^4)\sin(c + dx)}{15d}$$

$$+ \frac{ab(6a^2 + 29b^2)\cos(c + dx)\sin(c + dx)}{30d}$$

$$+ \frac{(3a^2 + 4b^2)(a + b\cos(c + dx))^2\sin(c + dx)}{15d}$$

$$+ \frac{a(a + b\cos(c + dx))^3\sin(c + dx)}{5d}$$

$$+ \frac{(a + b\cos(c + dx))^4\sin(c + dx)}{5d}$$

```
[Out] 1/2*a*b*(4*a^2+3*b^2)*x+2/15*(3*a^4+28*a^2*b^2+4*b^4)*sin(d*x+c)/d+1/30*a*b
*(6*a^2+29*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/15*(3*a^2+4*b^2)*(a+b*cos(d*x+c))
^2*sin(d*x+c)/d+1/5*a*(a+b*cos(d*x+c))^3*sin(d*x+c)/d+1/5*(a+b*cos(d*x+c))^
4*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00,
 number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used

= {2832, 2813}

$$\int \cos(c + dx)(a + b \cos(c + dx))^4 dx = \frac{(3a^2 + 4b^2) \sin(c + dx)(a + b \cos(c + dx))^2}{15d} + \frac{ab(6a^2 + 29b^2) \sin(c + dx) \cos(c + dx)}{30d} + \frac{1}{2} abx(4a^2 + 3b^2) + \frac{2(3a^4 + 28a^2b^2 + 4b^4) \sin(c + dx)}{15d} + \frac{\sin(c + dx)(a + b \cos(c + dx))^4}{5d} + \frac{a \sin(c + dx)(a + b \cos(c + dx))^3}{5d}$$

[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^4,x]

[Out] (a*b*(4*a^2 + 3*b^2)*x)/2 + (2*(3*a^4 + 28*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(15*d) + (a*b*(6*a^2 + 29*b^2)*Cos[c + d*x]*Sin[c + d*x])/(30*d) + ((3*a^2 + 4*b^2)*(a + b*Cos[c + d*x])^2*SIN[c + d*x])/(15*d) + (a*(a + b*Cos[c + d*x])^3*SIN[c + d*x])/(5*d) + ((a + b*Cos[c + d*x])^4*SIN[c + d*x])/(5*d)

Rule 2813

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Cos[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*SIN[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*SIN[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(a + b \cos(c + dx))^4 \sin(c + dx)}{5d} + \frac{1}{5} \int (4b + 4a \cos(c + dx))(a + b \cos(c + dx))^3 dx \\ &= \frac{a(a + b \cos(c + dx))^3 \sin(c + dx)}{5d} + \frac{(a + b \cos(c + dx))^4 \sin(c + dx)}{5d} \\ &\quad + \frac{1}{20} \int (a + b \cos(c + dx))^2 (28ab + 4(3a^2 + 4b^2) \cos(c + dx)) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{(3a^2 + 4b^2)(a + b \cos(c + dx))^2 \sin(c + dx)}{15d} \\
&\quad + \frac{a(a + b \cos(c + dx))^3 \sin(c + dx)}{5d} + \frac{(a + b \cos(c + dx))^4 \sin(c + dx)}{5d} \\
&\quad + \frac{1}{60} \int (a + b \cos(c + dx)) (4b(27a^2 + 8b^2) + 4a(6a^2 + 29b^2) \cos(c + dx)) dx \\
&= \frac{1}{2} ab(4a^2 + 3b^2) x + \frac{2(3a^4 + 28a^2b^2 + 4b^4) \sin(c + dx)}{15d} \\
&\quad + \frac{ab(6a^2 + 29b^2) \cos(c + dx) \sin(c + dx)}{30d} \\
&\quad + \frac{(3a^2 + 4b^2)(a + b \cos(c + dx))^2 \sin(c + dx)}{15d} \\
&\quad + \frac{a(a + b \cos(c + dx))^3 \sin(c + dx)}{5d} + \frac{(a + b \cos(c + dx))^4 \sin(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\begin{aligned}
&\int \cos(c + dx)(a + b \cos(c + dx))^4 dx \\
&= \frac{30(8a^4 + 36a^2b^2 + 5b^4) \sin(c + dx) + b(480a^3c + 360ab^2c + 480a^3dx + 360ab^2dx + 240a(a^2 + b^2) \sin(2(c + dx)))}{240d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^4,x]

[Out] (30*(8*a^4 + 36*a^2*b^2 + 5*b^4)*Sin[c + d*x] + b*(480*a^3*c + 360*a*b^2*c + 480*a^3*d*x + 360*a*b^2*d*x + 240*a*(a^2 + b^2)*Sin[2*(c + d*x)] + 5*(24*a^2*b + 5*b^3)*Sin[3*(c + d*x)] + 30*a*b^2*Ssin[4*(c + d*x)] + 3*b^3*Ssin[5*(c + d*x)]))/(240*d)

Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{240(a^3b+ab^3)\sin(2dx+2c)+5(24a^2b^2+5b^4)\sin(3dx+3c)+30\sin(4dx+4c)ab^3+3\sin(5dx+5c)b^4+30(8a^4+36a^2b^2+5b^4)\sin(dx+c)}{240d}$
derivativedivides	$\frac{a^4\sin(dx+c)+4a^3b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2b^2(2+\cos^2(dx+c))\sin(dx+c)+4ab^3\left(\frac{\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}}{4}\right)}{d}$
default	$\frac{a^4\sin(dx+c)+4a^3b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2b^2(2+\cos^2(dx+c))\sin(dx+c)+4ab^3\left(\frac{\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}}{4}\right)}{d}$
parts	$\frac{b^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5d} + \frac{a^4\sin(dx+c)}{d} + \frac{4ab^3\left(\frac{\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}}{4}\right)\sin(dx+c)+\frac{3dx}{8}}{d}$
risch	$2a^3bx + \frac{3ab^3x}{2} + \frac{a^4\sin(dx+c)}{d} + \frac{9\sin(dx+c)a^2b^2}{2d} + \frac{5\sin(dx+c)b^4}{8d} + \frac{\sin(5dx+5c)b^4}{80d} + \frac{\sin(4dx+4c)ab^3}{8d} +$
norman	$\frac{(2a^3b+\frac{3}{2}ab^3)x+(2a^3b+\frac{3}{2}ab^3)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10a^3b+\frac{15}{2}ab^3)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(10a^3b+\frac{15}{2}ab^3)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30d}$

[In] `int(cos(d*x+c)*(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{240}*(240*(a^3*b+a*b^3)*\sin(2*d*x+2*c)+5*(24*a^2*b^2+5*b^4)*\sin(3*d*x+3*c)+30*\sin(4*d*x+4*c)*a*b^3+3*\sin(5*d*x+5*c)*b^4+30*(8*a^4+36*a^2*b^2+5*b^4)*\sin(dx+c)+480*b*(a^2+3/4*b^2)*d*x*a)/d$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.71

$$\int \cos(c+dx)(a+b\cos(c+dx))^4 dx$$

$$= \frac{15(4a^3b+3ab^3)dx + (6b^4\cos(dx+c))^4 + 30ab^3\cos(dx+c)^3 + 30a^4 + 120a^2b^2 + 16b^4 + 4(15a^2b^2 + 2b^4)\cos(dx+c)^2 + 15(4a^3b+3ab^3)\cos(dx+c)\sin(dx+c)}{30d}$$

[In] `integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="fricas")`

[Out] $\frac{1}{30}*(15*(4*a^3*b+3*a*b^3)*d*x+(6*b^4*\cos(d*x+c))^4+30*a*b^3*\cos(d*x+c)^3+30*a^4+120*a^2*b^2+16*b^4+4*(15*a^2*b^2+2*b^4)*\cos(d*x+c)^2+15*(4*a^3*b+3*a*b^3)*\cos(d*x+c)*\sin(d*x+c))/d$

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.77

$$\int \cos(c + dx)(a + b \cos(c + dx))^4 dx$$

$$= \begin{cases} \frac{a^4 \sin(c+dx)}{d} + 2a^3bx \sin^2(c + dx) + 2a^3bx \cos^2(c + dx) + \frac{2a^3b \sin(c+dx) \cos(c+dx)}{d} + \frac{4a^2b^2 \sin^3(c+dx)}{d} + \frac{6a^2b^2 \sin(c+dx) \cos^2(c+dx)}{d} \\ x(a + b \cos(c))^4 \cos(c) \end{cases}$$

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)/d + 2*a**3*b*x*sin(c + d*x)**2 + 2*a**3*b*x*cos(c + d*x)**2 + 2*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*a**2*b**2*sin(c + d*x)**3/d + 6*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*a*b**3*x*sin(c + d*x)**4/2 + 3*a*b**3*x*cos(c + d*x)**4/2 + 3*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*a*b**3*x*cos(c + d*x)**3*cos(c + d*x)/(2*d) + 5*a*b**3*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 8*b**4*sin(c + d*x)**5/(15*d) + 4*b**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + b**4*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))**4*cos(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

$$\int \cos(c + dx)(a + b \cos(c + dx))^4 dx$$

$$= \frac{120(2dx + 2c + \sin(2dx + 2c))a^3b - 240(\sin(dx + c)^3 - 3\sin(dx + c))a^2b^2 + 15(12dx + 12c + \sin(4dx + 4c))a^2b^2 + 8\sin(2dx + 2c)a^2b^2 + 8(3\sin(dx + c)^5 - 10\sin(dx + c)^3 + 15\sin(dx + c))b^4 + 120a^4\sin(dx + c)}{d}$$

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3*b - 240*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2*b^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c)) + 8*sin(2*d*x + 2*c))*a^2*b^2 + 8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*b^4 + 120*a^4*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.79

$$\int \cos(c + dx)(a + b \cos(c + dx))^4 dx = \frac{b^4 \sin(5 dx + 5 c)}{80 d} + \frac{ab^3 \sin(4 dx + 4 c)}{8 d} + \frac{1}{2} (4 a^3 b + 3 ab^3) x + \frac{(24 a^2 b^2 + 5 b^4) \sin(3 dx + 3 c)}{48 d} + \frac{(a^3 b + ab^3) \sin(2 dx + 2 c)}{d} + \frac{(8 a^4 + 36 a^2 b^2 + 5 b^4) \sin(dx + c)}{8 d}$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/80*b^4*sin(5*d*x + 5*c)/d + 1/8*a*b^3*sin(4*d*x + 4*c)/d + 1/2*(4*a^3*b + 3*a*b^3)*x + 1/48*(24*a^2*b^2 + 5*b^4)*sin(3*d*x + 3*c)/d + (a^3*b + a*b^3)*sin(2*d*x + 2*c)/d + 1/8*(8*a^4 + 36*a^2*b^2 + 5*b^4)*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 15.58 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.14

$$\int \cos(c + dx)(a + b \cos(c + dx))^4 dx = \frac{(2 a^4 - 4 a^3 b + 12 a^2 b^2 - 5 a b^3 + 2 b^4) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(8 a^4 - 8 a^3 b + 32 a^2 b^2 - 2 a b^3 + \frac{8 b^4}{3}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 5 a b \operatorname{atan}\left(\frac{a b \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (4 a^2 + 3 b^2)}{4 a^3 b + 3 a b^3}\right) (4 a^2 + 3 b^2) + a b (4 a^2 + 3 b^2) \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{d}$$

[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^4,x)

[Out] (tan(c/2 + (d*x)/2)^5*(12*a^4 + (116*b^4)/15 + 40*a^2*b^2) + tan(c/2 + (d*x)/2)^9*(2*a^4 - 4*a^3*b - 5*a*b^3 + 2*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(2*a*b^3 + 8*a^3*b + 8*a^4 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(8*a^4 - 8*a^3*b - 2*a*b^3 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)*(5*a*b^3 + 4*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)/2)^5*(12*a^4 + (116*b^4)/15 + 40*a^2*b^2) + tan(c/2 + (d*x)/2)^9*(2*a^4 - 4*a^3*b - 5*a*b^3 + 2*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(2*a*b^3 + 8*a^3*b + 8*a^4 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(8*a^4 - 8*a^3*b - 2*a*b^3 + (8*b^4)/3 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)*(5*a*b^3 + 4*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))

$$\begin{aligned} & /2)^2 + 10*\tan(c/2 + (d*x)/2)^4 + 10*\tan(c/2 + (d*x)/2)^6 + 5*\tan(c/2 + (d* \\ & x)/2)^8 + \tan(c/2 + (d*x)/2)^{10} + 1)) + (a*b*atan((a*b*\tan(c/2 + (d*x)/2)*(\\ & 4*a^2 + 3*b^2))/(3*a*b^3 + 4*a^3*b))*(4*a^2 + 3*b^2))/d - (a*b*(4*a^2 + 3*b \\ & ^2)*(atan(\tan(c/2 + (d*x)/2)) - (d*x)/2))/d \end{aligned}$$

3.441 $\int (a + b \cos(c + dx))^4 dx$

Optimal result	4821
Rubi [A] (verified)	4821
Mathematica [A] (verified)	4823
Maple [A] (verified)	4823
Fricas [A] (verification not implemented)	4824
Sympy [A] (verification not implemented)	4824
Maxima [A] (verification not implemented)	4824
Giac [A] (verification not implemented)	4825
Mupad [B] (verification not implemented)	4825

Optimal result

Integrand size = 12, antiderivative size = 137

$$\int (a + b \cos(c + dx))^4 dx = \frac{1}{8}(8a^4 + 24a^2b^2 + 3b^4)x + \frac{ab(19a^2 + 16b^2) \sin(c + dx)}{6d} + \frac{b^2(26a^2 + 9b^2) \cos(c + dx) \sin(c + dx)}{24d} + \frac{7ab(a + b \cos(c + dx))^2 \sin(c + dx)}{12d} + \frac{b(a + b \cos(c + dx))^3 \sin(c + dx)}{4d}$$

[Out] $\frac{1}{8}(8a^4 + 24a^2b^2 + 3b^4)x + \frac{ab(19a^2 + 16b^2) \sin(dx + c)}{6d} + \frac{b^2(26a^2 + 9b^2) \cos(dx + c) \sin(dx + c)}{24d} + \frac{7ab(a + b \cos(dx + c))^2 \sin(dx + c)}{12d} + \frac{b(a + b \cos(dx + c))^3 \sin(dx + c)}{4d}$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2735, 2832, 2813}

$$\int (a + b \cos(c + dx))^4 dx = \frac{ab(19a^2 + 16b^2) \sin(c + dx)}{6d} + \frac{b^2(26a^2 + 9b^2) \sin(c + dx) \cos(c + dx)}{24d} + \frac{1}{8}x(8a^4 + 24a^2b^2 + 3b^4) + \frac{b \sin(c + dx)(a + b \cos(c + dx))^3}{4d} + \frac{7ab \sin(c + dx)(a + b \cos(c + dx))^2}{12d}$$

[In] Int[(a + b*cos[c + d*x])^4, x]

[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 + 16*b^2)*Sin[c + d*x])/(6*d) + (b^2*(26*a^2 + 9*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (7*a*b*(a + b*cos[c + d*x])^2*sin[c + d*x])/(12*d) + (b*(a + b*cos[c + d*x])^3*sin[c + d*x])/(4*d)

Rule 2735

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2813

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Cos[e + f*x]/f), x] - Simp[b*d*cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*sin[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{b(a + b \cos(c + dx))^3 \sin(c + dx)}{4d} \\
 &+ \frac{1}{4} \int (a + b \cos(c + dx))^2 (4a^2 + 3b^2 + 7ab \cos(c + dx)) dx \\
 &= \frac{7ab(a + b \cos(c + dx))^2 \sin(c + dx)}{12d} + \frac{b(a + b \cos(c + dx))^3 \sin(c + dx)}{4d} \\
 &+ \frac{1}{12} \int (a + b \cos(c + dx)) (a(12a^2 + 23b^2) + b(26a^2 + 9b^2) \cos(c + dx)) dx \\
 &= \frac{1}{8} (8a^4 + 24a^2b^2 + 3b^4) x + \frac{ab(19a^2 + 16b^2) \sin(c + dx)}{6d} \\
 &+ \frac{b^2(26a^2 + 9b^2) \cos(c + dx) \sin(c + dx)}{24d} \\
 &+ \frac{7ab(a + b \cos(c + dx))^2 \sin(c + dx)}{12d} + \frac{b(a + b \cos(c + dx))^3 \sin(c + dx)}{4d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int (a + b \cos(c + dx))^4 dx$$

$$= \frac{12(8a^4 + 24a^2b^2 + 3b^4)(c + dx) + 96ab(4a^2 + 3b^2) \sin(c + dx) + 24b^2(6a^2 + b^2) \sin(2(c + dx)) + 32ab^3 \sin(3(c + dx)) + 3b^4 \sin(4(c + dx))}{96d}$$

[In] Integrate[(a + b*Cos[c + d*x])^4,x]

[Out] (12*(8*a^4 + 24*a^2*b^2 + 3*b^4)*(c + d*x) + 96*a*b*(4*a^2 + 3*b^2)*Sin[c + d*x] + 24*b^2*(6*a^2 + b^2)*Sin[2*(c + d*x)] + 32*a*b^3*Ssin[3*(c + d*x)] + 3*b^4*Ssin[4*(c + d*x)])/(96*d)

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{24(6a^2b^2 + b^4) \sin(2dx + 2c) + 32 \sin(3dx + 3c) a b^3 + 3 \sin(4dx + 4c) b^4 + 96(4a^3b + 3a b^3) \sin(dx + c) + 96d(a^4 + 3a^2b^2 + \frac{3}{8}b^4)x}{96d}$
derivativedivides	$b^4 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx + 3c}{8} \right) + \frac{4a b^3 (2 + \cos^2(dx+c)) \sin(dx+c)}{3} + 6a^2 b^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)$
default	$b^4 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx + 3c}{8} \right) + \frac{4a b^3 (2 + \cos^2(dx+c)) \sin(dx+c)}{3} + 6a^2 b^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)$
parts	$a^4 x + \frac{b^4 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx + 3c}{8} \right)}{d} + \frac{6a^2 b^2 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right)}{d} + \frac{4 \sin(dx+c) a b^3}{d}$
risch	$a^4 x + 3x a^2 b^2 + \frac{3b^4 x}{8} + \frac{4 \sin(dx+c) a^3 b}{d} + \frac{3 \sin(dx+c) a b^3}{d} + \frac{\sin(4dx+4c) b^4}{32d} + \frac{\sin(3dx+3c) a b^3}{3d} + \frac{3 \sin(2dx+2c) a^2 b^2}{3d}$
norman	$\frac{(a^4 + 3a^2b^2 + \frac{3}{8}b^4)x + (a^4 + 3a^2b^2 + \frac{3}{8}b^4)x \left(\tan^8\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + (4a^4 + 12a^2b^2 + \frac{3}{2}b^4)x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + (4a^4 + 12a^2b^2 + \frac{3}{2}b^4)}{96d}$

[In] int((a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)

[Out] 1/96*(24*(6*a^2*b^2+b^4)*sin(2*d*x+2*c)+32*sin(3*d*x+3*c)*a*b^3+3*sin(4*d*x+4*c)*b^4+96*(4*a^3*b+3*a*b^3)*sin(d*x+c)+96*d*(a^4+3*a^2*b^2+3/8*b^4)*x)/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int (a + b \cos(c + dx))^4 dx$$

$$= \frac{3(8a^4 + 24a^2b^2 + 3b^4)dx + (6b^4 \cos(dx + c)^3 + 32ab^3 \cos(dx + c)^2 + 96a^3b + 64ab^3 + 9(8a^2b^2 + b^4) \cos(dx + c))}{24d}$$

```
[In] integrate((a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*d*x + (6*b^4*cos(d*x + c)^3 + 32*a*b^3*cos(d*x + c)^2 + 96*a^3*b + 64*a*b^3 + 9*(8*a^2*b^2 + b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.75

$$\int (a + b \cos(c + dx))^4 dx$$

$$= \begin{cases} a^4x + \frac{4a^3b \sin(c+dx)}{d} + 3a^2b^2x \sin^2(c + dx) + 3a^2b^2x \cos^2(c + dx) + \frac{3a^2b^2 \sin(c+dx) \cos(c+dx)}{d} + \frac{8ab^3 \sin^3(c+dx)}{3d} + \\ x(a + b \cos(c))^4 \end{cases}$$

```
[In] integrate((a+b*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*sin(c + d*x)/d + 3*a**2*b**2*x*sin(c + d*x)**2 + 3*a**2*b**2*x*cos(c + d*x)**2 + 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)/d + 8*a*b**3*sin(c + d*x)**3/(3*d) + 4*a*b**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b**4*x*cos(c + d*x)**4/8 + 3*b**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.81

$$\int (a + b \cos(c + dx))^4 dx = a^4 x + \frac{3(2dx + 2c + \sin(2dx + 2c))a^2 b^2}{2d} - \frac{4(\sin(dx + c)^3 - 3\sin(dx + c))ab^3}{3d} + \frac{(12dx + 12c + \sin(4dx + 4c) + 8\sin(2dx + 2c))b^4}{32d} + \frac{4a^3 b \sin(dx + c)}{d}$$

[In] integrate((a+b*cos(d*x+c))^4,x, algorithm="maxima")

[Out] a^4*x + 3/2*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b^2/d - 4/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b^3/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b^4/d + 4*a^3*b*sin(d*x + c)/d

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int (a + b \cos(c + dx))^4 dx = \frac{b^4 \sin(4dx + 4c)}{32d} + \frac{ab^3 \sin(3dx + 3c)}{3d} + \frac{1}{8}(8a^4 + 24a^2b^2 + 3b^4)x + \frac{(6a^2b^2 + b^4)\sin(2dx + 2c)}{4d} + \frac{(4a^3b + 3ab^3)\sin(dx + c)}{d}$$

[In] integrate((a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/32*b^4*sin(4*d*x + 4*c)/d + 1/3*a*b^3*sin(3*d*x + 3*c)/d + 1/8*(8*a^4 + 24*a^2*b^2 + 3*b^4)*x + 1/4*(6*a^2*b^2 + b^4)*sin(2*d*x + 2*c)/d + (4*a^3*b + 3*a*b^3)*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int (a + b \cos(c + dx))^4 dx = a^4 x + \frac{3b^4 x}{8} + 3a^2 b^2 x + \frac{b^4 \sin(2c + 2dx)}{4d} + \frac{b^4 \sin(4c + 4dx)}{32d} + \frac{ab^3 \sin(3c + 3dx)}{3d} + \frac{3a^2 b^2 \sin(2c + 2dx)}{2d} + \frac{3ab^3 \sin(c + dx)}{d} + \frac{4a^3 b \sin(c + dx)}{d}$$

[In] int((a + b*cos(c + d*x))^4,x)

[Out] $a^4x + \frac{3b^4x}{8} + 3a^2b^2x + \frac{b^4\sin(2c + 2dx)}{4d} + \frac{b^4\sin(4c + 4dx)}{32d} + \frac{ab^3\sin(3c + 3dx)}{3d} + \frac{3a^2b^2\sin(2c + 2dx)}{2d} + \frac{3ab^3\sin(c + dx)}{d} + \frac{4a^3b\sin(c + dx)}{d}$

3.442 $\int (a + b \cos(c + dx))^4 \sec(c + dx) dx$

Optimal result	4827
Rubi [A] (verified)	4827
Mathematica [A] (verified)	4829
Maple [A] (verified)	4830
Fricas [A] (verification not implemented)	4830
Sympy [F]	4831
Maxima [A] (verification not implemented)	4831
Giac [B] (verification not implemented)	4831
Mupad [B] (verification not implemented)	4832

Optimal result

Integrand size = 19, antiderivative size = 107

$$\int (a + b \cos(c + dx))^4 \sec(c + dx) dx = 2ab(2a^2 + b^2)x + \frac{a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{b^2(17a^2 + 2b^2) \sin(c + dx)}{3d} + \frac{4ab^3 \cos(c + dx) \sin(c + dx)}{3d} + \frac{b^2(a + b \cos(c + dx))^2 \sin(c + dx)}{3d}$$

[Out] $2*a*b*(2*a^2+b^2)*x+a^4*\operatorname{arctanh}(\sin(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*\sin(d*x+c)/d+4/3*a*b^3*\cos(d*x+c)*\sin(d*x+c)/d+1/3*b^2*(a+b*\cos(d*x+c))^2*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2872, 3112, 3102, 2814, 3855}

$$\int (a + b \cos(c + dx))^4 \sec(c + dx) dx = \frac{a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{b^2(17a^2 + 2b^2) \sin(c + dx)}{3d} + 2abx(2a^2 + b^2) + \frac{4ab^3 \sin(c + dx) \cos(c + dx)}{3d} + \frac{b^2 \sin(c + dx)(a + b \cos(c + dx))^2}{3d}$$

[In] $\operatorname{Int}[(a + b*\operatorname{Cos}[c + d*x])^4*\operatorname{Sec}[c + d*x], x]$

[Out] $2*a*b*(2*a^2 + b^2)*x + (a^4*ArcTanh[\sin[c + d*x]])/d + (b^2*(17*a^2 + 2*b^2)*\sin[c + d*x])/(3*d) + (4*a*b^3*\cos[c + d*x]*\sin[c + d*x])/(3*d) + (b^2*(a + b*\cos[c + d*x])^2*\sin[c + d*x])/(3*d)$

Rule 2814

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2872

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 3102

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 3112

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d))*(m + 3))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2(a + b \cos(c + dx))^2 \sin(c + dx)}{3d} + \frac{1}{3} \int (a + b \cos(c + dx)) (3a^3 \\
&\quad + b(9a^2 + 2b^2) \cos(c + dx) + 8ab^2 \cos^2(c + dx)) \sec(c + dx) dx \\
&= \frac{4ab^3 \cos(c + dx) \sin(c + dx)}{3d} + \frac{b^2(a + b \cos(c + dx))^2 \sin(c + dx)}{3d} + \frac{1}{6} \int (6a^4 \\
&\quad + 12ab(2a^2 + b^2) \cos(c + dx) + 2b^2(17a^2 + 2b^2) \cos^2(c + dx)) \sec(c + dx) dx \\
&= \frac{b^2(17a^2 + 2b^2) \sin(c + dx)}{3d} + \frac{4ab^3 \cos(c + dx) \sin(c + dx)}{3d} \\
&\quad + \frac{b^2(a + b \cos(c + dx))^2 \sin(c + dx)}{3d} \\
&\quad + \frac{1}{6} \int (6a^4 + 12ab(2a^2 + b^2) \cos(c + dx)) \sec(c + dx) dx \\
&= 2ab(2a^2 + b^2) x + \frac{b^2(17a^2 + 2b^2) \sin(c + dx)}{3d} + \frac{4ab^3 \cos(c + dx) \sin(c + dx)}{3d} \\
&\quad + \frac{b^2(a + b \cos(c + dx))^2 \sin(c + dx)}{3d} + a^4 \int \sec(c + dx) dx \\
&= 2ab(2a^2 + b^2) x + \frac{a^4 \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{b^2(17a^2 + 2b^2) \sin(c + dx)}{3d} \\
&\quad + \frac{4ab^3 \cos(c + dx) \sin(c + dx)}{3d} + \frac{b^2(a + b \cos(c + dx))^2 \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.20

$$\begin{aligned}
&\int (a + b \cos(c + dx))^4 \sec(c + dx) dx \\
&= \frac{24ab(2a^2 + b^2)(c + dx) - 12a^4 \log(\cos(\frac{1}{2}(c + dx)) - \sin(\frac{1}{2}(c + dx))) + 12a^4 \log(\cos(\frac{1}{2}(c + dx)) + \sin(\frac{1}{2}(c + dx)))}{12d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x], x]

[Out] (24*a*b*(2*a^2 + b^2)*(c + d*x) - 12*a^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*b^2*(8*a^2 + b^2)*Sin[c + d*x] + 12*a*b^3*Sin[2*(c + d*x)] + b^4*Sin[3*(c + d*x)])/(12*d)

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))+4a^3b(dx+c)+6 \sin(dx+c)a^2b^2+4a b^3 \left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + \frac{b^4(2+\cos^2(dx+c)) \sin(dx+c)}{3}}{d}$
default	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))+4a^3b(dx+c)+6 \sin(dx+c)a^2b^2+4a b^3 \left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + \frac{b^4(2+\cos^2(dx+c)) \sin(dx+c)}{3}}{d}$
parallelrisc	$\frac{-12a^4 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + 12a^4 \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + 12 \sin(2dx+2c)a b^3 + \sin(3dx+3c)b^4 + 9(8a^2b^2+b^4) \sin(dx+c)+4a^2b^2}{12d}$
parts	$\frac{a^4 \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{b^4(2+\cos^2(dx+c)) \sin(dx+c)}{3d} + \frac{4a b^3 \left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} + \frac{6 \sin(dx+c)a^2b^2}{d}$
risc	$4a^3bx + 2a b^3x - \frac{3ie^{i(dx+c)}a^2b^2}{d} - \frac{3ie^{i(dx+c)}b^4}{8d} + \frac{3ie^{-i(dx+c)}a^2b^2}{d} + \frac{3ie^{-i(dx+c)}b^4}{8d} + \frac{a^4 \ln(e^{i(dx+c)}+i)}{d} - \frac{a^4 \ln(e^{-i(dx+c)}-i)}{d}$
norman	$\frac{(4a^3b+2a b^3)x + (4a^3b+2a b^3)x \left(\tan^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (16a^3b+8a b^3)x \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (16a^3b+8a b^3)x \left(\tan^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (2a^2b^2+2a b^3)x \left(\tan^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{6d}$

[In] int((a+cos(d*x+c)*b)^4*sec(d*x+c),x,method=_RETURNVERBOSE)

[Out] 1/d*(a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*b*(d*x+c)+6*sin(d*x+c)*a^2*b^2+4*a*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+1/3*b^4*(2+cos(d*x+c)^2)*sin(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int (a + b \cos(c + dx))^4 \sec(c + dx) dx$$

$$= \frac{3a^4 \log(\sin(dx+c)+1) - 3a^4 \log(-\sin(dx+c)+1) + 12(2a^3b + ab^3)dx + 2(b^4 \cos(dx+c)^2 + 6ab^3 \cos(dx+c))}{6d}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c),x, algorithm="fricas")

[Out] 1/6*(3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) + 1) + 12*(2*a^3*b + a*b^3)*d*x + 2*(b^4*cos(d*x + c)^2 + 6*a*b^3*cos(d*x + c) + 18*a^2*b^2 + 2*b^4)*sin(d*x + c))/d

Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.48

$$\begin{aligned}
\int (a + b \cos(c + dx))^4 \sec(c + dx) dx = & \frac{3b^4 \sin(c + dx)}{4d} + \frac{2a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} \\
& + \frac{b^4 \sin(3c + 3dx)}{12d} + \frac{ab^3 \sin(2c + 2dx)}{d} \\
& + \frac{6a^2 b^2 \sin(c + dx)}{d} + \frac{4ab^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} \\
& + \frac{8a^3 b \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}
\end{aligned}$$

[In] int((a + b*cos(c + d*x))^4/cos(c + d*x),x)

```
[Out] (3*b^4*sin(c + d*x))/(4*d) + (2*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^4*sin(3*c + 3*d*x))/(12*d) + (a*b^3*sin(2*c + 2*d*x))/d + (6*a^2*b^2*sin(c + d*x))/d + (4*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.443 $\int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx$

Optimal result	4833
Rubi [A] (verified)	4833
Mathematica [A] (verified)	4835
Maple [A] (verified)	4836
Fricas [A] (verification not implemented)	4836
Sympy [F]	4837
Maxima [A] (verification not implemented)	4837
Giac [A] (verification not implemented)	4837
Mupad [B] (verification not implemented)	4838

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx = \frac{1}{2}b^2(12a^2 + b^2)x + \frac{4a^3b \operatorname{arctanh}(\sin(c + dx))}{d} - \frac{2ab(a^2 - 2b^2) \sin(c + dx)}{d} - \frac{b^2(2a^2 - b^2) \cos(c + dx) \sin(c + dx)}{2d} + \frac{a^2(a + b \cos(c + dx))^2 \tan(c + dx)}{d}$$

[Out] 1/2*b^2*(12*a^2+b^2)*x+4*a^3*b*arctanh(sin(d*x+c))/d-2*a*b*(a^2-2*b^2)*sin(d*x+c)/d-1/2*b^2*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/d+a^2*(a+b*cos(d*x+c))^2*tan(d*x+c)/d

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2871, 3112, 3102, 2814, 3855}

$$\int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx = \frac{4a^3b \operatorname{arctanh}(\sin(c + dx))}{d} - \frac{2ab(a^2 - 2b^2) \sin(c + dx)}{d} - \frac{b^2(2a^2 - b^2) \sin(c + dx) \cos(c + dx)}{2d} + \frac{1}{2}b^2x(12a^2 + b^2) + \frac{a^2 \tan(c + dx)(a + b \cos(c + dx))^2}{d}$$

[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2,x]

[Out] (b^2*(12*a^2 + b^2)*x)/2 + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*(a^2 - 2*b^2)*Sin[c + d*x])/d - (b^2*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/d

Rule 2814

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

Rule 3102

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Ssin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 3112

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])*(A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Ssin[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Ssin[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a^2(a + b \cos(c + dx))^2 \tan(c + dx)}{d} + \int (a + b \cos(c + dx)) (4a^2b + 3ab^2 \cos(c + dx) \\
 &\quad - b(2a^2 - b^2) \cos^2(c + dx)) \sec(c + dx) dx \\
 &= -\frac{b^2(2a^2 - b^2) \cos(c + dx) \sin(c + dx)}{2d} + \frac{a^2(a + b \cos(c + dx))^2 \tan(c + dx)}{d} \\
 &\quad + \frac{1}{2} \int (8a^3b + b^2(12a^2 + b^2) \cos(c + dx) - 4ab(a^2 - 2b^2) \cos^2(c + dx)) \sec(c + dx) dx \\
 &= -\frac{2ab(a^2 - 2b^2) \sin(c + dx)}{d} - \frac{b^2(2a^2 - b^2) \cos(c + dx) \sin(c + dx)}{2d} \\
 &\quad + \frac{a^2(a + b \cos(c + dx))^2 \tan(c + dx)}{d} \\
 &\quad + \frac{1}{2} \int (8a^3b + b^2(12a^2 + b^2) \cos(c + dx)) \sec(c + dx) dx \\
 &= \frac{1}{2} b^2(12a^2 + b^2) x - \frac{2ab(a^2 - 2b^2) \sin(c + dx)}{d} - \frac{b^2(2a^2 - b^2) \cos(c + dx) \sin(c + dx)}{2d} \\
 &\quad + \frac{a^2(a + b \cos(c + dx))^2 \tan(c + dx)}{d} + (4a^3b) \int \sec(c + dx) dx \\
 &= \frac{1}{2} b^2(12a^2 + b^2) x + \frac{4a^3b \operatorname{arctanh}(\sin(c + dx))}{d} - \frac{2ab(a^2 - 2b^2) \sin(c + dx)}{d} \\
 &\quad - \frac{b^2(2a^2 - b^2) \cos(c + dx) \sin(c + dx)}{2d} + \frac{a^2(a + b \cos(c + dx))^2 \tan(c + dx)}{d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx = \frac{2b(b(12a^2 + b^2)(c + dx) - 8a^3 \log(\cos(\frac{1}{2}(c + dx)) - \sin(\frac{1}{2}(c + dx))) + 8a^3 \log(\cos(\frac{1}{2}(c + dx)) + \sin(\frac{1}{2}(c + dx))))}{4d}$$

[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2,x]

[Out] (2*b*(b*(12*a^2 + b^2)*(c + d*x) - 8*a^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]) + 8*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 16*a*b^3*Sin[c + d*x] + b^4*Sin[2*(c + d*x)] + 4*a^4*Tan[c + d*x])/(4*d)

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a^4 \tan(dx+c) + 4a^3 b \ln(\sec(dx+c) + \tan(dx+c)) + 6a^2 b^2 (dx+c) + 4 \sin(dx+c) a b^3 + b^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx+c}{2} \right)}{d}$
default	$\frac{a^4 \tan(dx+c) + 4a^3 b \ln(\sec(dx+c) + \tan(dx+c)) + 6a^2 b^2 (dx+c) + 4 \sin(dx+c) a b^3 + b^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx+c}{2} \right)}{d}$
parts	$\frac{a^4 \tan(dx+c)}{d} + \frac{b^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx+c}{2} \right)}{d} + \frac{4a^3 b \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{6a^2 b^2 (dx+c)}{d} + \frac{4 \sin(dx+c) a b^3}{d}$
parallelrisc	$\frac{-32a^3 b \ln\left(\tan\left(\frac{dx+c}{2}\right) - 1\right) \cos(dx+c) + 32a^3 b \ln\left(\tan\left(\frac{dx+c}{2}\right) + 1\right) \cos(dx+c) + 16 \sin(2dx+2c) a b^3 + \sin(3dx+3c) b^4 + 48 \cos(dx+c)}{8d \cos(dx+c)}$
risc	$6x a^2 b^2 + \frac{b^4 x}{2} - \frac{ib^4 e^{2i(dx+c)}}{8d} - \frac{2ia b^3 e^{i(dx+c)}}{d} + \frac{2ia b^3 e^{-i(dx+c)}}{d} + \frac{ib^4 e^{-2i(dx+c)}}{8d} + \frac{2ia^4}{d(e^{2i(dx+c)}+1)} + \frac{4a^3 b^2}{d}$
norman	$\frac{(-6a^2 b^2 - \frac{1}{2} b^4) x + (-18a^2 b^2 - \frac{3}{2} b^4) x \left(\tan^2\left(\frac{dx+c}{2}\right) \right) + (6a^2 b^2 + \frac{1}{2} b^4) x \left(\tan^{10}\left(\frac{dx+c}{2}\right) \right) + (18a^2 b^2 + \frac{3}{2} b^4) x \left(\tan^8\left(\frac{dx+c}{2}\right) \right)}{2d \cos(dx+c)}$

[In] int((a+cos(d*x+c)*b)^4*sec(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^4*tan(d*x+c)+4*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6*a^2*b^2*(d*x+c)+4*sin(d*x+c)*a*b^3+b^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx$$

$$= \frac{4 a^3 b \cos(dx + c) \log(\sin(dx + c) + 1) - 4 a^3 b \cos(dx + c) \log(-\sin(dx + c) + 1) + (12 a^2 b^2 + b^4) dx \cos(dx + c)}{2 d \cos(dx + c)}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="fricas")

[Out] 1/2*(4*a^3*b*cos(d*x + c)*log(sin(d*x + c) + 1) - 4*a^3*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + (12*a^2*b^2 + b^4)*d*x*cos(d*x + c) + (b^4*cos(d*x + c))^2 + 8*a*b^3*cos(d*x + c) + 2*a^4)*sin(d*x + c)/(d*cos(d*x + c))

Sympy [F]

$$\int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx = \int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx$$

[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**2,x)

[Out] Integral((a + b*cos(c + d*x))**4*sec(c + d*x)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.79

$$\int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx$$

$$= \frac{24(dx + c)a^2b^2 + (2dx + 2c + \sin(2dx + 2c))b^4 + 8a^3b(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1))}{4d}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="maxima")

[Out] 1/4*(24*(d*x + c)*a^2*b^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*b^4 + 8*a^3*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a*b^3*sin(d*x + c) + 4*a^4*tan(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

$$\int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx$$

$$= \frac{8a^3b \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 8a^3b \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{4a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1} + (12a^2b^2 + b^4)(dx + c)}{2d}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="giac")

[Out] 1/2*(8*a^3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^3*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + (12*a^2*b^2 + b^4)*(d*x + c) + 2*(8*a*b^3*tan(1/2*d*x + 1/2*c)^3 - b^4*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^3*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2/d

Mupad [B] (verification not implemented)

Time = 14.55 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32

$$\begin{aligned}
\int (a + b \cos(c + dx))^4 \sec^2(c + dx) dx = & \frac{b^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{a^4 \sin(c + dx)}{d \cos(c + dx)} \\
& + \frac{12 a^2 b^2 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{4 a b^3 \sin(c + dx)}{d} \\
& + \frac{b^4 \cos(c + dx) \sin(c + dx)}{2 d} \\
& + \frac{8 a^3 b \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}
\end{aligned}$$

[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^2,x)

```
[Out] (b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^4*sin(c + d*x))/(d
*cos(c + d*x)) + (12*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
+ (4*a*b^3*sin(c + d*x))/d + (b^4*cos(c + d*x)*sin(c + d*x))/(2*d) + (8*a^
3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.444 $\int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx$

Optimal result	4839
Rubi [A] (verified)	4839
Mathematica [A] (verified)	4841
Maple [A] (verified)	4842
Fricas [A] (verification not implemented)	4842
Sympy [F]	4843
Maxima [A] (verification not implemented)	4843
Giac [A] (verification not implemented)	4843
Mupad [B] (verification not implemented)	4844

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx = 4ab^3x + \frac{a^2(a^2 + 12b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} - \frac{b^2(a^2 - 2b^2) \sin(c + dx)}{2d} + \frac{3a^3b \tan(c + dx)}{d} + \frac{a^2(a + b \cos(c + dx))^2 \sec(c + dx) \tan(c + dx)}{2d}$$

[Out] $4*a*b^3*x + 1/2*a^2*(a^2+12*b^2)*\operatorname{arctanh}(\sin(d*x+c))/d - 1/2*b^2*(a^2-2*b^2)*\sin(d*x+c)/d + 3*a^3*b*\tan(d*x+c)/d + 1/2*a^2*(a+b*\cos(d*x+c))^2*\sec(d*x+c)*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2871, 3110, 3102, 2814, 3855}

$$\int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx = \frac{3a^3b \tan(c + dx)}{d} + \frac{a^2(a^2 + 12b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} - \frac{b^2(a^2 - 2b^2) \sin(c + dx)}{2d} + \frac{a^2 \tan(c + dx) \sec(c + dx) (a + b \cos(c + dx))^2}{2d} + 4ab^3x$$

[In] Int[(a + b*cos[c + d*x])^4*Sec[c + d*x]^3,x]

[Out] 4*a*b^3*x + (a^2*(a^2 + 12*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*(a^2 - 2*b^2)*Sin[c + d*x])/(2*d) + (3*a^3*b*Tan[c + d*x])/d + (a^2*(a + b*cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)

Rule 2814

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Ssin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(- (b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3102

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Ssin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 3110

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])*(A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(- (b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*((a + b*Ssin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*Simp[b*(m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a^2(a + b \cos(c + dx))^2 \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2} \int (a + b \cos(c + dx)) (6a^2b \\
 &\quad + a(a^2 + 6b^2) \cos(c + dx) - b(a^2 - 2b^2) \cos^2(c + dx)) \sec^2(c + dx) dx \\
 &= \frac{3a^3b \tan(c + dx)}{d} + \frac{a^2(a + b \cos(c + dx))^2 \sec(c + dx) \tan(c + dx)}{2d} \\
 &\quad - \frac{1}{2} \int (-a^2(a^2 + 12b^2) - 8ab^3 \cos(c + dx) + b^2(a^2 - 2b^2) \cos^2(c + dx)) \sec(c + dx) dx \\
 &= -\frac{b^2(a^2 - 2b^2) \sin(c + dx)}{2d} + \frac{3a^3b \tan(c + dx)}{d} \\
 &\quad + \frac{a^2(a + b \cos(c + dx))^2 \sec(c + dx) \tan(c + dx)}{2d} \\
 &\quad - \frac{1}{2} \int (-a^2(a^2 + 12b^2) - 8ab^3 \cos(c + dx)) \sec(c + dx) dx \\
 &= 4ab^3x - \frac{b^2(a^2 - 2b^2) \sin(c + dx)}{2d} + \frac{3a^3b \tan(c + dx)}{d} \\
 &\quad + \frac{a^2(a + b \cos(c + dx))^2 \sec(c + dx) \tan(c + dx)}{2d} \\
 &\quad + \frac{1}{2} (a^2(a^2 + 12b^2)) \int \sec(c + dx) dx \\
 &= 4ab^3x + \frac{a^2(a^2 + 12b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} - \frac{b^2(a^2 - 2b^2) \sin(c + dx)}{2d} \\
 &\quad + \frac{3a^3b \tan(c + dx)}{d} + \frac{a^2(a + b \cos(c + dx))^2 \sec(c + dx) \tan(c + dx)}{2d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.61

$$\begin{aligned}
 &\int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx \\
 &= \frac{a \left(16b^3c + 16b^3dx - 2a(a^2 + 12b^2) \log \left(\cos \left(\frac{1}{2}(c + dx) \right) - \sin \left(\frac{1}{2}(c + dx) \right) \right) + 2a(a^2 + 12b^2) \log \left(\cos \left(\frac{1}{2}(c + dx) \right) \right) \right)}{d}
 \end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3,x]

[Out] $(a*(16*b^3*c + 16*b^3*d*x - 2*a*(a^2 + 12*b^2)*\text{Log}[\text{Cos}[(c + d*x)/2] - \text{Sin}[(c + d*x)/2]] + 2*a*(a^2 + 12*b^2)*\text{Log}[\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2]] + a^3/(\text{Cos}[(c + d*x)/2] - \text{Sin}[(c + d*x)/2])^2 - a^3/(\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2])^2 + 4*b^4*\text{Sin}[c + d*x] + 16*a^3*b*\text{Tan}[c + d*x])/(4*d)$

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right) + 4a^3 b \tan(dx+c) + 6a^2 b^2 \ln(\sec(dx+c)+\tan(dx+c)) + 4a b^3 (dx+c) + \sin(dx+c)}{d}$
default	$\frac{a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right) + 4a^3 b \tan(dx+c) + 6a^2 b^2 \ln(\sec(dx+c)+\tan(dx+c)) + 4a b^3 (dx+c) + \sin(dx+c)}{d}$
parts	$\frac{a^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2} \right)}{d} + \frac{\sin(dx+c)b^4}{d} + \frac{4a^3 b \tan(dx+c)}{d} + \frac{6a^2 b^2 \ln(\sec(dx+c)+\tan(dx+c))}{d}$
parallelrisch	$\frac{-a^2(a^2+12b^2)(1+\cos(2dx+2c)) \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + a^2(a^2+12b^2)(1+\cos(2dx+2c)) \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) + 8a b^3 dx \cos(dx+c)}{2d(1+\cos(2dx+2c))}$
risch	$4a b^3 x - \frac{ie^{i(dx+c)}b^4}{2d} + \frac{ie^{-i(dx+c)}b^4}{2d} - \frac{ia^3(ae^{3i(dx+c)}-8be^{2i(dx+c)}-ae^{i(dx+c)}-8b)}{d(e^{2i(dx+c)}+1)^2} + \frac{a^4 \ln(e^{i(dx+c)}+i)}{2d} + \frac{6a \ln(e^{i(dx+c)}-i)}{2d}$
norman	$\frac{(a^4-8a^3b+2b^4)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{(a^4+8a^3b+2b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{(5a^4-24a^3b+2b^4)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{(5a^4+24a^3b+2b^4)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$

[In] `int((a+cos(d*x+c)*b)^4*sec(d*x+c)^3,x,method=_RETURNVERBOSE)`

[Out] $1/d*(a^4*(1/2*\sec(d*x+c)*\tan(d*x+c)+1/2*\ln(\sec(d*x+c)+\tan(d*x+c)))+4*a^3*b*\tan(d*x+c)+6*a^2*b^2*\ln(\sec(d*x+c)+\tan(d*x+c))+4*a*b^3*(d*x+c)+\sin(d*x+c)*b^4)$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx$$

$$= \frac{16 ab^3 dx \cos(dx + c)^2 + (a^4 + 12 a^2 b^2) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (a^4 + 12 a^2 b^2) \cos(dx + c)^2 \log(\sin(dx + c) - 1)}{4 d \cos(dx + c)^2}$$

[In] `integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="fricas")`

[Out] $1/4*(16*a*b^3*d*x*\cos(d*x + c)^2 + (a^4 + 12*a^2*b^2)*\cos(d*x + c)^2*\log(\sin(d*x + c) + 1) - (a^4 + 12*a^2*b^2)*\cos(d*x + c)^2*\log(-\sin(d*x + c) + 1) + 2*(2*b^4*\cos(d*x + c)^2 + 8*a^3*b*\cos(d*x + c) + a^4)*\sin(d*x + c))/(d*\cos(d*x + c)^2)$

Sympy [F]

$$\int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx = \int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx$$

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**3,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**4*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx$$

$$= \frac{16(dx + c)ab^3 - a^4 \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right) + 12a^2b^2(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 4b^4 \sin(dx + c) + 16a^3b \tan(dx + c)}{4d}$$

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a*b^3 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*b^4*sin(d*x + c) + 16*a^3*b*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

$$\int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx$$

$$= \frac{8(dx + c)ab^3 + \frac{4b^4 \tan(\frac{1}{2}dx + \frac{1}{2}c)}{\tan(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1} + (a^4 + 12a^2b^2) \log(|\tan(\frac{1}{2}dx + \frac{1}{2}c) + 1|) - (a^4 + 12a^2b^2) \log(|\tan(\frac{1}{2}dx + \frac{1}{2}c) - 1|) + 2(a^4 \tan(\frac{1}{2}dx + \frac{1}{2}c)^3 - 8a^3b \tan(\frac{1}{2}dx + \frac{1}{2}c) + a^4 \tan(\frac{1}{2}dx + \frac{1}{2}c) + 8a^3b \tan(\frac{1}{2}dx + \frac{1}{2}c))}{2d}$$

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a*b^3 + 4*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + (a^4 + 12*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (a^4 + 12*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*b*tan(1/2*d*x + 1/2*c) + a^4*tan(1/2*d*x + 1/2*c) + 8*a^3*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2/d
```

Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.41

$$\begin{aligned}
\int (a + b \cos(c + dx))^4 \sec^3(c + dx) dx &= \frac{b^4 \sin(c + dx)}{d} + \frac{a^4 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} \\
&+ \frac{a^4 \sin(c + dx)}{2d \cos(c + dx)^2} + \frac{12 a^2 b^2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} \\
&+ \frac{8 a b^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{4 a^3 b \sin(c + dx)}{d \cos(c + dx)}
\end{aligned}$$

[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^3,x)

```
[Out] (b^4*sin(c + d*x))/d + (a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
+ (a^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (12*a^2*b^2*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d + (4*a^3*b*sin(c + d*x))/(d*cos(c + d*x))
```


3.445 $\int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx$

Optimal result	4845
Rubi [A] (verified)	4845
Mathematica [A] (verified)	4848
Maple [A] (verified)	4848
Fricas [A] (verification not implemented)	4849
Sympy [F]	4849
Maxima [A] (verification not implemented)	4849
Giac [B] (verification not implemented)	4850
Mupad [B] (verification not implemented)	4850

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx = b^4 x + \frac{2ab(a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2(2a^2 + 17b^2) \tan(c + dx)}{3d} + \frac{4a^3 b \sec(c + dx) \tan(c + dx)}{3d} + \frac{a^2(a + b \cos(c + dx))^2 \sec^2(c + dx) \tan(c + dx)}{3d}$$

[Out] $b^4 x + 2 a b (a^2 + 2 b^2) \operatorname{arctanh}(\sin(d x + c)) / d + 1 / 3 a^2 (2 a^2 + 17 b^2) \tan(d x + c) / d + 4 / 3 a^3 b \sec(d x + c) \tan(d x + c) / d + 1 / 3 a^2 (a + b \cos(d x + c))^2 \sec(d x + c)^2 \tan(d x + c) / d$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2871, 3110, 3100, 2814, 3855}

$$\int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx = \frac{4a^3 b \tan(c + dx) \sec(c + dx)}{3d} + \frac{2ab(a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{d} + \frac{a^2(2a^2 + 17b^2) \tan(c + dx)}{3d} + \frac{a^2 \tan(c + dx) \sec^2(c + dx) (a + b \cos(c + dx))^2}{3d} + b^4 x$$

[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^4,x]

[Out] b^4*x + (2*a*b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 + 17*b^2)*Tan[c + d*x])/(3*d) + (4*a^3*b*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (a^2*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)

Rule 2814

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sine + f*x)], x, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(- (b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Sine + f*x)^(m - 2)*((c + d*Sine + f*x)^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sine + f*x)^(m - 3)*(c + d*Sine + f*x)^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sine + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sine + f*x]^2, x], x, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

Rule 3100

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[(- (A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sine + f*x)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sine + f*x)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sine + f*x], x], x, x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 3110

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])*(A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[(- (b*c - a*d))* (A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sine + f*x)^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sine + f*x)^(m + 1)*Simp[b*(m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))*Sine + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sine + f*x]^2, x], x, x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.67

$$\int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx$$

$$= \frac{3b^4 dx + 6ab(a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx)) + 3a^2(a^2 + 6b^2 + 2ab \sec(c + dx)) \tan(c + dx) + a^4 \tan^3(c + dx)}{3d}$$

[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^4,x]

[Out] (3*b^4*d*x + 6*a*b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]] + 3*a^2*(a^2 + 6*b^2 + 2*a*b*Sec[c + d*x])*Tan[c + d*x] + a^4*Tan[c + d*x]^3)/(3*d)

Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-a^4 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + 4a^3 b \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) + 6a^2 b^2 \tan(dx+c) + 4a b^3 \ln(\sec(dx+c) + \tan(dx+c))}{d}$
default	$\frac{-a^4 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + 4a^3 b \left(\frac{\sec(dx+c) \tan(dx+c)}{2} + \frac{\ln(\sec(dx+c) + \tan(dx+c))}{2} \right) + 6a^2 b^2 \tan(dx+c) + 4a b^3 \ln(\sec(dx+c) + \tan(dx+c))}{d}$
parts	$-\frac{a^4 \left(-\frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c)}{d} + \frac{b^4(dx+c)}{d} + \frac{2a^3 b \sec(dx+c) \tan(dx+c)}{d} + \frac{2a^3 b \ln(\sec(dx+c) + \tan(dx+c))}{d}$
risch	$b^4 x - \frac{4ia^2(3ab e^{5i(dx+c)} - 9b^2 e^{4i(dx+c)} - 3a^2 e^{2i(dx+c)} - 18b^2 e^{2i(dx+c)} - 3ab e^{i(dx+c)} - a^2 - 9b^2)}{3d(e^{2i(dx+c)} + 1)^3} + \frac{2a^3 b \ln(e^{i(dx+c)} + i)}{d}$
parallelrisc	$\frac{-18b \left(\frac{\cos(3dx+3c)}{3} + \cos(dx+c) \right) (a^2 + 2b^2) a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 18b \left(\frac{\cos(3dx+3c)}{3} + \cos(dx+c) \right) (a^2 + 2b^2) a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)}{3d(\cos(3dx+3c) + 1)}$
norman	$b^4 x \left(\tan^{12} \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + b^4 x \left(\tan^{14} \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - b^4 x - b^4 x \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 3b^4 x \left(\tan^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + 3b^4 x \left(\tan^6 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 3b^4 x$

[In] int((a+cos(d*x+c)*b)^4*sec(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(-a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a^3*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+6*a^2*b^2*tan(d*x+c)+4*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+b^4*(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.20

$$\int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx$$

$$= \frac{3b^4 dx \cos(dx + c)^3 + 3(a^3b + 2ab^3) \cos(dx + c)^3 \log(\sin(dx + c) + 1) - 3(a^3b + 2ab^3) \cos(dx + c)^3 \log(\sin(dx + c) - 1) + 6a^3b \cos(dx + c) + a^4 + 2(a^4 + 9a^2b^2) \cos(dx + c)^2 \sin(dx + c)}{3d \cos(dx + c)^3}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="fricas")

```
[Out] 1/3*(3*b^4*d*x*cos(d*x + c)^3 + 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (6*a^3*b*cos(d*x + c) + a^4 + 2*(a^4 + 9*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx = \int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx$$

[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**4,x)

[Out] Integral((a + b*cos(c + d*x))**4*sec(c + d*x)**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx$$

$$= \frac{(\tan(dx + c)^3 + 3 \tan(dx + c))a^4 + 3(dx + c)b^4 - 3a^3b \left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2 - 1} - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1) \right) + 6a^3b^3(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1)) + 18a^2b^2 \tan(dx + c)}{3d}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="maxima")

```
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 3*(d*x + c)*b^4 - 3*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*a^2*b^2*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(109) = 218.

Time = 0.34 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.92

$$\int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx$$

$$= \frac{3(dx + c)b^4 + 6(a^3b + 2ab^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 6(a^3b + 2ab^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2}{3} \dots}{\dots}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="giac")

[Out] 1/3*(3*(d*x + c)*b^4 + 6*(a^3*b + 2*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*(a^3*b + 2*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*a^4*tan(1/2*d*x + 1/2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*a^4*tan(1/2*d*x + 1/2*c)^3 - 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^4*tan(1/2*d*x + 1/2*c) + 6*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3/d

Mupad [B] (verification not implemented)

Time = 14.88 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

$$\int (a + b \cos(c + dx))^4 \sec^4(c + dx) dx = \frac{2b^4 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2a^4 \sin(c + dx)}{3d \cos(c + dx)} + \frac{a^4 \sin(c + dx)}{3d \cos(c + dx)^3} + \frac{8ab^3 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{4a^3b \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} + \frac{2a^3b \sin(c + dx)}{d \cos(c + dx)^2} + \frac{6a^2b^2 \sin(c + dx)}{d \cos(c + dx)}$$

[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^4,x)

[Out] (2*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a^4*sin(c + d*x))/(3*d*cos(c + d*x)) + (a^4*sin(c + d*x))/(3*d*cos(c + d*x)^3) + (8*a*b^3*a*tanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*a^3*b*sin(c + d*x))/(d*cos(c + d*x)^2) + (6*a^2*b^2*sin(c + d*x))/(d*cos(c + d*x))

3.446 $\int (a + b \cos(c + dx))^4 \sec^5(c + dx) dx$

Optimal result	4851
Rubi [A] (verified)	4851
Mathematica [A] (verified)	4854
Maple [A] (verified)	4855
Fricas [A] (verification not implemented)	4855
Sympy [F(-1)]	4856
Maxima [A] (verification not implemented)	4856
Giac [B] (verification not implemented)	4856
Mupad [B] (verification not implemented)	4857

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int (a + b \cos(c + dx))^4 \sec^5(c + dx) dx = \frac{(3a^4 + 24a^2b^2 + 8b^4) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{4ab(2a^2 + 3b^2) \tan(c + dx)}{3d} + \frac{a^2(3a^2 + 22b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{5a^3b \sec^2(c + dx) \tan(c + dx)}{6d} + \frac{a^2(a + b \cos(c + dx))^2 \sec^3(c + dx) \tan(c + dx)}{4d}$$

[Out] $\frac{1}{8}*(3*a^4+24*a^2*b^2+8*b^4)*\operatorname{arctanh}(\sin(d*x+c))/d+4/3*a*b*(2*a^2+3*b^2)*\tan(d*x+c)/d+1/8*a^2*(3*a^2+22*b^2)*\sec(d*x+c)*\tan(d*x+c)/d+5/6*a^3*b*\sec(d*x+c)^2*\tan(d*x+c)/d+1/4*a^2*(a+b*\cos(d*x+c))^2*\sec(d*x+c)^3*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {2871, 3110, 3100, 2827, 3852, 8, 3855}

$$\int (a + b \cos(c + dx))^4 \sec^5(c + dx) dx = \frac{5a^3 b \tan(c + dx) \sec^2(c + dx)}{6d} + \frac{4ab(2a^2 + 3b^2) \tan(c + dx)}{3d} + \frac{a^2(3a^2 + 22b^2) \tan(c + dx) \sec(c + dx)}{8d} + \frac{a^2 \tan(c + dx) \sec^3(c + dx) (a + b \cos(c + dx))^2}{4d} + \frac{(3a^4 + 24a^2 b^2 + 8b^4) \operatorname{arctanh}(\sin(c + dx))}{8d}$$

[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^5,x]

[Out] ((3*a^4 + 24*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]]/(8*d) + (4*a*b*(2*a^2 + 3*b^2)*Tan[c + d*x])/(3*d) + (a^2*(3*a^2 + 22*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (5*a^3*b*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a^2*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2871

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(- (b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m - 2)*((c + d*SIN[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*SIN[e + f*x])^(m - 3)*(c + d*SIN[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*SIN[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*SIN[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3100

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(- (A*b^2

$$- a*b*B + a^2*C)) * \text{Cos}[e + f*x] * ((a + b*\text{Sin}[e + f*x])^{m+1} / (b*f*(m+1)*(a^2 - b^2))), x] + \text{Dist}[1/(b*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m+1} * \text{Simp}[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m+1))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$$

Rule 3110

$$\text{Int}[(a_.) + (b_.) * \text{sin}[(e_.) + (f_.) * (x_)]])^{m_}) * ((c_.) + (d_.) * \text{sin}[(e_.) + (f_.) * (x_)]]) * ((A_.) + (B_.) * \text{sin}[(e_.) + (f_.) * (x_)] + (C_.) * \text{sin}[(e_.) + (f_.) * (x_)]^2), x_Symbol] := \text{Simp}[(-b*c - a*d) * (A*b^2 - a*b*B + a^2*C) * \text{Cos}[e + f*x] * ((a + b*\text{Sin}[e + f*x])^{m+1} / (b^2*f*(m+1)*(a^2 - b^2))), x] - \text{Dist}[1/(b^2*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m+1} * \text{Simp}[b*(m+1) * ((b*B - a*C) * (b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m+1) - a*b*c*(m+2)) + (b*c - a*d) * (A*b^2*(m+2) + C*(a^2 + b^2*(m+1))) * \text{Sin}[e + f*x] - b*C*d*(m+1) * (a^2 - b^2) * \text{Sin}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1]$$

Rule 3852

$$\text{Int}[\text{csc}[(c_.) + (d_.) * (x_)]^{n_}), x_Symbol] := \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$$

Rule 3855

$$\text{Int}[\text{csc}[(c_.) + (d_.) * (x_)], x_Symbol] := \text{Simp}[-\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{a^2(a + b \cos(c + dx))^2 \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{4} \int (a + b \cos(c + dx)) (10a^2b \\ &\quad + 3a(a^2 + 4b^2) \cos(c + dx) + b(a^2 + 4b^2) \cos^2(c + dx)) \sec^4(c + dx) dx \\ &= \frac{5a^3b \sec^2(c + dx) \tan(c + dx)}{6d} + \frac{a^2(a + b \cos(c + dx))^2 \sec^3(c + dx) \tan(c + dx)}{4d} \\ &\quad - \frac{1}{12} \int (-3a^2(3a^2 + 22b^2) - 16ab(2a^2 + 3b^2) \cos(c + dx) \\ &\quad \quad - 3b^2(a^2 + 4b^2) \cos^2(c + dx)) \sec^3(c + dx) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{a^2(3a^2 + 22b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{5a^3b \sec^2(c + dx) \tan(c + dx)}{6d} \\
&\quad + \frac{a^2(a + b \cos(c + dx))^2 \sec^3(c + dx) \tan(c + dx)}{4d} \\
&\quad - \frac{1}{24} \int (-32ab(2a^2 + 3b^2) - 3(3a^4 + 24a^2b^2 + 8b^4) \cos(c + dx)) \sec^2(c + dx) dx \\
&= \frac{a^2(3a^2 + 22b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{5a^3b \sec^2(c + dx) \tan(c + dx)}{6d} \\
&\quad + \frac{a^2(a + b \cos(c + dx))^2 \sec^3(c + dx) \tan(c + dx)}{4d} \\
&\quad + \frac{1}{3}(4ab(2a^2 + 3b^2)) \int \sec^2(c + dx) dx - \frac{1}{8}(-3a^4 - 24a^2b^2 - 8b^4) \int \sec(c + dx) dx \\
&= \frac{(3a^4 + 24a^2b^2 + 8b^4) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{a^2(3a^2 + 22b^2) \sec(c + dx) \tan(c + dx)}{8d} \\
&\quad + \frac{5a^3b \sec^2(c + dx) \tan(c + dx)}{6d} + \frac{a^2(a + b \cos(c + dx))^2 \sec^3(c + dx) \tan(c + dx)}{4d} \\
&\quad - \frac{(4ab(2a^2 + 3b^2)) \operatorname{Subst}(\int 1 dx, x, -\tan(c + dx))}{3d} \\
&= \frac{(3a^4 + 24a^2b^2 + 8b^4) \operatorname{arctanh}(\sin(c + dx))}{8d} + \frac{4ab(2a^2 + 3b^2) \tan(c + dx)}{3d} \\
&\quad + \frac{a^2(3a^2 + 22b^2) \sec(c + dx) \tan(c + dx)}{8d} + \frac{5a^3b \sec^2(c + dx) \tan(c + dx)}{6d} \\
&\quad + \frac{a^2(a + b \cos(c + dx))^2 \sec^3(c + dx) \tan(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\begin{aligned}
&\int (a + b \cos(c + dx))^4 \sec^5(c + dx) dx \\
&= \frac{3(3a^4 + 24a^2b^2 + 8b^4) \operatorname{arctanh}(\sin(c + dx)) + a \tan(c + dx) (9a(a^2 + 8b^2) \sec(c + dx) + 6a^3 \sec^3(c + dx) + 6a^2 \tan(c + dx)^2)}{24d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^5,x]

[Out] (3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]] + a*Tan[c + d*x]*(9*a*(a^2 + 8*b^2)*Sec[c + d*x] + 6*a^3*Sec[c + d*x]^3 + 32*b*(3*(a^2 + b^2) + a^2*Tan[c + d*x]^2)))/(24*d)

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a^4 \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right) - 4a^3 b \left(- \frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + \dots}{d}$
default	$\frac{a^4 \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right) - 4a^3 b \left(- \frac{2}{3} - \frac{\sec^2(dx+c)}{3} \right) \tan(dx+c) + \dots}{d}$
parts	$\frac{a^4 \left(- \left(- \frac{\sec^3(dx+c)}{4} - \frac{3 \sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3 \ln(\sec(dx+c)+\tan(dx+c))}{8} \right)}{d} + \frac{b^4 \ln(\sec(dx+c)+\tan(dx+c))}{d} - \dots$
parallelrisc	$-36 \left(\frac{3}{4} + \frac{\cos(4dx+4c)}{4} + \cos(2dx+2c) \right) (a^4 + 8a^2b^2 + \frac{8}{3}b^4) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 36 \left(\frac{3}{4} + \frac{\cos(4dx+4c)}{4} + \cos(2dx+2c) \right) (a^4 + 8a^2b^2 + \frac{8}{3}b^4) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)$
risc	$- \frac{ia(9a^3e^{7i(dx+c)} + 72ab^2e^{7i(dx+c)} - 96b^3e^{6i(dx+c)} + 33a^3e^{5i(dx+c)} + 72ab^2e^{5i(dx+c)} - 192a^2be^{4i(dx+c)} - 288b^3e^{4i(dx+c)} - 12d(e^{2i(dx+c)} - 1))}{12d(e^{2i(dx+c)} - 1)}$
norman	$\frac{a(5a^3 - 32a^2b + 24ab^2 - 32b^3) \left(\tan^{15} \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{4d} + \frac{a(5a^3 + 32a^2b + 24ab^2 + 32b^3) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{4d} + \frac{a(45a^3 - 32a^2b + 24ab^2 + 96b^3) \left(\tan^{11} \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{4d}$

```
[In] int((a+cos(d*x+c)*b)^4*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-4*a^3*b*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*a^2*b^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*tan(d*x+c)*a*b^3+b^4*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.06

$$\int (a + b \cos(c + dx))^4 \sec^5(c + dx) dx$$

$$= \frac{3(3a^4 + 24a^2b^2 + 8b^4) \cos(dx + c)^4 \log(\sin(dx + c) + 1) - 3(3a^4 + 24a^2b^2 + 8b^4) \cos(dx + c)^4 \log(-\sin(dx + c) + 1) + 2(32a^3b \cos(dx + c) + 6a^4 + 32(2a^3b + 3a^2b^2) \cos(dx + c)^3 + 9(a^4 + 8a^2b^2) \cos(dx + c)^2) \sin(dx + c)}{(d \cos(dx + c))^4}$$

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(32*a^3*b*cos(d*x + c) + 6*a^4 + 32*(2*a^3*b + 3*a^2*b^2)*cos(d*x + c)^3 + 9*(a^4 + 8*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^4 \sec^5(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**5,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21

$$\int (a + b \cos(c + dx))^4 \sec^5(c + dx) dx$$

$$= \frac{64 (\tan(dx + c)^3 + 3 \tan(dx + c)) a^3 b - 3 a^4 \left(\frac{2 (3 \sin(dx+c)^3 - 5 \sin(dx+c))}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log(\sin(dx + c) + 1) + 3 \log(\sin(dx + c) - 1) \right)}{d}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="maxima")

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3*b - 3*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*a*b^3*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(144) = 288.

Time = 0.34 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.34

$$\int (a + b \cos(c + dx))^4 \sec^5(c + dx) dx$$

$$= \frac{3(3a^4 + 24a^2b^2 + 8b^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3(3a^4 + 24a^2b^2 + 8b^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{d}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="giac")

```
[Out] 1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^4
```

$$\begin{aligned} & 4*\tan(1/2*d*x + 1/2*c)^7 - 96*a^3*b*\tan(1/2*d*x + 1/2*c)^7 + 72*a^2*b^2*\tan \\ & (1/2*d*x + 1/2*c)^7 - 96*a*b^3*\tan(1/2*d*x + 1/2*c)^7 + 9*a^4*\tan(1/2*d*x + \\ & 1/2*c)^5 + 160*a^3*b*\tan(1/2*d*x + 1/2*c)^5 - 72*a^2*b^2*\tan(1/2*d*x + 1/2 \\ & *c)^5 + 288*a*b^3*\tan(1/2*d*x + 1/2*c)^5 + 9*a^4*\tan(1/2*d*x + 1/2*c)^3 - 1 \\ & 60*a^3*b*\tan(1/2*d*x + 1/2*c)^3 - 72*a^2*b^2*\tan(1/2*d*x + 1/2*c)^3 - 288*a \\ & *b^3*\tan(1/2*d*x + 1/2*c)^3 + 15*a^4*\tan(1/2*d*x + 1/2*c) + 96*a^3*b*\tan(1/ \\ & 2*d*x + 1/2*c) + 72*a^2*b^2*\tan(1/2*d*x + 1/2*c) + 96*a*b^3*\tan(1/2*d*x + 1 \\ & /2*c))/(\tan(1/2*d*x + 1/2*c)^2 - 1)^4/d \end{aligned}$$

Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.59

$$\begin{aligned} & \int (a + b \cos(c + dx))^4 \sec^5(c + dx) dx \\ & = \frac{\left(\frac{5a^4}{4} - 8a^3b + 6a^2b^2 - 8ab^3\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(\frac{3a^4}{4} + \frac{40a^3b}{3} - 6a^2b^2 + 24ab^3\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(\frac{3a^4}{4} - \right. \\ & \left. d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right) \right. \\ & \left. + \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) \left(\frac{3a^4}{4} + 6a^2b^2 + 2b^4\right)}{d} \right. \end{aligned}$$

[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^5,x)

[Out] (tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3*b + (5*a^4)/4 + 6*a^2*b^2) - tan(c/2 + (d*x)/2)^7*(8*a*b^3 + 8*a^3*b - (5*a^4)/4 - 6*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(24*a*b^3 + (40*a^3*b)/3 - (3*a^4)/4 + 6*a^2*b^2) + tan(c/2 + (d*x)/2)^5*(24*a*b^3 + (40*a^3*b)/3 + (3*a^4)/4 - 6*a^2*b^2))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x)/2))*((3*a^4)/4 + 2*b^4 + 6*a^2*b^2))/d

3.447 $\int (a + b \cos(c + dx))^4 \sec^6(c + dx) dx$

Optimal result	4858
Rubi [A] (verified)	4858
Mathematica [A] (verified)	4861
Maple [A] (verified)	4862
Fricas [A] (verification not implemented)	4862
Sympy [F(-1)]	4863
Maxima [A] (verification not implemented)	4863
Giac [B] (verification not implemented)	4863
Mupad [B] (verification not implemented)	4864

Optimal result

Integrand size = 21, antiderivative size = 188

$$\int (a + b \cos(c + dx))^4 \sec^6(c + dx) dx = \frac{ab(3a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{(8a^4 + 60a^2b^2 + 15b^4) \tan(c + dx)}{15d} + \frac{ab(3a^2 + 4b^2) \sec(c + dx) \tan(c + dx)}{2d} + \frac{a^2(4a^2 + 27b^2) \sec^2(c + dx) \tan(c + dx)}{15d} + \frac{3a^3b \sec^3(c + dx) \tan(c + dx)}{5d} + \frac{a^2(a + b \cos(c + dx))^2 \sec^4(c + dx) \tan(c + dx)}{5d}$$

[Out] 1/2*a*b*(3*a^2+4*b^2)*arctanh(sin(d*x+c))/d+1/15*(8*a^4+60*a^2*b^2+15*b^4)*tan(d*x+c)/d+1/2*a*b*(3*a^2+4*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/15*a^2*(4*a^2+27*b^2)*sec(d*x+c)^2*tan(d*x+c)/d+3/5*a^3*b*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a^2*(a+b*cos(d*x+c))^2*sec(d*x+c)^4*tan(d*x+c)/d

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used

= {2871, 3110, 3100, 2827, 3853, 3855, 3852, 8}

$$\int (a + b \cos(c + dx))^4 \sec^6(c + dx) dx = \frac{3a^3b \tan(c + dx) \sec^3(c + dx)}{5d} + \frac{ab(3a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{2d} + \frac{a^2(4a^2 + 27b^2) \tan(c + dx) \sec^2(c + dx)}{15d} + \frac{ab(3a^2 + 4b^2) \tan(c + dx) \sec(c + dx)}{2d} + \frac{a^2 \tan(c + dx) \sec^4(c + dx) (a + b \cos(c + dx))^2}{5d} + \frac{(8a^4 + 60a^2b^2 + 15b^4) \tan(c + dx)}{15d}$$

[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^6,x]

[Out] (a*b*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]]/(2*d) + ((8*a^4 + 60*a^2*b^2 + 15*b^4)*Tan[c + d*x])/(15*d) + (a*b*(3*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a^2*(4*a^2 + 27*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) + (3*a^3*b*Sec[c + d*x]^3*Tan[c + d*x])/(5*d) + (a^2*(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2827

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)])], x_Symbol] := Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2871

Int[((a_.) + (b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(- (A*b^2
- a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] := Simp[(- (b*c - a*d))* (A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
+ 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{a^2(a + b \cos(c + dx))^2 \sec^4(c + dx) \tan(c + dx)}{5d} + \frac{1}{5} \int (a + b \cos(c + dx)) (12a^2b + a(4a^2 + 15b^2) \cos(c + dx) + b(2a^2 + 5b^2) \cos^2(c + dx)) \sec^5(c + dx) dx$$

$$\begin{aligned}
&= \frac{3a^3b \sec^3(c+dx) \tan(c+dx)}{5d} + \frac{a^2(a+b \cos(c+dx))^2 \sec^4(c+dx) \tan(c+dx)}{5d} \\
&\quad - \frac{1}{20} \int (-4a^2(4a^2+27b^2) - 20ab(3a^2+4b^2) \cos(c+dx) \\
&\quad\quad\quad - 4b^2(2a^2+5b^2) \cos^2(c+dx)) \sec^4(c+dx) dx \\
&= \frac{a^2(4a^2+27b^2) \sec^2(c+dx) \tan(c+dx)}{15d} + \frac{3a^3b \sec^3(c+dx) \tan(c+dx)}{5d} \\
&\quad + \frac{a^2(a+b \cos(c+dx))^2 \sec^4(c+dx) \tan(c+dx)}{5d} \\
&\quad - \frac{1}{60} \int (-60ab(3a^2+4b^2) - 4(8a^4+60a^2b^2+15b^4) \cos(c+dx)) \sec^3(c+dx) dx \\
&= \frac{a^2(4a^2+27b^2) \sec^2(c+dx) \tan(c+dx)}{15d} + \frac{3a^3b \sec^3(c+dx) \tan(c+dx)}{5d} \\
&\quad + \frac{a^2(a+b \cos(c+dx))^2 \sec^4(c+dx) \tan(c+dx)}{5d} \\
&\quad + (ab(3a^2+4b^2)) \int \sec^3(c+dx) dx - \frac{1}{15} (-8a^4 - 60a^2b^2 - 15b^4) \int \sec^2(c+dx) dx \\
&= \frac{ab(3a^2+4b^2) \sec(c+dx) \tan(c+dx)}{2d} + \frac{a^2(4a^2+27b^2) \sec^2(c+dx) \tan(c+dx)}{15d} \\
&\quad + \frac{3a^3b \sec^3(c+dx) \tan(c+dx)}{5d} + \frac{a^2(a+b \cos(c+dx))^2 \sec^4(c+dx) \tan(c+dx)}{5d} \\
&\quad + \frac{1}{2} (ab(3a^2+4b^2)) \int \sec(c+dx) dx \\
&\quad - \frac{(8a^4+60a^2b^2+15b^4) \text{Subst}(\int 1 dx, x, -\tan(c+dx))}{15d} \\
&= \frac{ab(3a^2+4b^2) \operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{(8a^4+60a^2b^2+15b^4) \tan(c+dx)}{15d} \\
&\quad + \frac{ab(3a^2+4b^2) \sec(c+dx) \tan(c+dx)}{2d} + \frac{a^2(4a^2+27b^2) \sec^2(c+dx) \tan(c+dx)}{15d} \\
&\quad + \frac{3a^3b \sec^3(c+dx) \tan(c+dx)}{5d} + \frac{a^2(a+b \cos(c+dx))^2 \sec^4(c+dx) \tan(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.66

$$\begin{aligned}
&\int (a+b \cos(c+dx))^4 \sec^6(c+dx) dx \\
&= \frac{15ab(3a^2+4b^2) \operatorname{arctanh}(\sin(c+dx)) + \tan(c+dx) (30(a^4+6a^2b^2+b^4) + 15ab(3a^2+4b^2) \sec(c+dx) + \dots}{30d}
\end{aligned}$$

[In] Integrate[(a + b*cos[c + d*x])^4*Sec[c + d*x]^6,x]

[Out] $(15*a*b*(3*a^2 + 4*b^2)*ArcTanh[\sin[c + d*x]] + \tan[c + d*x]*(30*(a^4 + 6*a^2*b^2 + b^4) + 15*a*b*(3*a^2 + 4*b^2)*Sec[c + d*x] + 30*a^3*b*Sec[c + d*x]^3 + 20*a^2*(a^2 + 3*b^2)*\tan[c + d*x]^2 + 6*a^4*\tan[c + d*x]^4))/(30*d)$

Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-a^4 \left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c) + 4a^3b \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3\sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3\ln(\sec(dx+c))}{8} \right)$
default	$-a^4 \left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c) + 4a^3b \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3\sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3\ln(\sec(dx+c))}{8} \right)$
parts	$-\frac{a^4 \left(-\frac{8}{15} - \frac{\sec^4(dx+c)}{5} - \frac{4(\sec^2(dx+c))}{15} \right) \tan(dx+c)}{d} + \frac{b^4 \tan(dx+c)}{d} + \frac{4a^3b \left(-\left(-\frac{\sec^3(dx+c)}{4} - \frac{3\sec(dx+c)}{8} \right) \tan(dx+c) + \frac{3\ln(\sec(dx+c))}{8} \right)}{d}$
parallelrisc	$-450b \left(\frac{\cos(5dx+5c)}{10} + \frac{\cos(3dx+3c)}{2} + \cos(dx+c) \right) \left(a^2 + \frac{4b^2}{3} \right) a \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 450b \left(\frac{\cos(5dx+5c)}{10} + \frac{\cos(3dx+3c)}{2} + \cos(dx+c) \right)$
risc	$i(-45a^3b e^{9i(dx+c)} - 60a b^3 e^{9i(dx+c)} + 30b^4 e^{8i(dx+c)} - 210a^3 b e^{7i(dx+c)} - 120b^3 a e^{7i(dx+c)} + 360a^2 b^2 e^{6i(dx+c)} + 120b^4 e^{6i(dx+c)} - 120a^3 b e^{5i(dx+c)} - 60a b^3 e^{5i(dx+c)} + 30b^4 e^{4i(dx+c)} - 210a^3 b e^{3i(dx+c)} - 120b^3 a e^{3i(dx+c)} + 360a^2 b^2 e^{2i(dx+c)} + 120b^4 e^{2i(dx+c)} - 120a^3 b e^{i(dx+c)} - 60a b^3 e^{i(dx+c)} + 30b^4 e^{i(dx+c)})$

[In] `int((a+cos(d*x+c)*b)^4*sec(d*x+c)^6,x,method=_RETURNVERBOSE)`

[Out] $1/d*(-a^4*(-8/15-1/5*\sec(d*x+c)^4-4/15*\sec(d*x+c)^2)*\tan(d*x+c)+4*a^3*b*(-(-1/4*\sec(d*x+c)^3-3/8*\sec(d*x+c))*\tan(d*x+c)+3/8*\ln(\sec(d*x+c)+\tan(d*x+c))))-6*a^2*b^2*(-2/3-1/3*\sec(d*x+c)^2)*\tan(d*x+c)+4*a*b^3*(1/2*\sec(d*x+c)*\tan(d*x+c)+1/2*\ln(\sec(d*x+c)+\tan(d*x+c)))+\tan(d*x+c)*b^4)$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.97

$$\int (a + b \cos(c + dx))^4 \sec^6(c + dx) dx$$

$$= \frac{15(3a^3b + 4ab^3) \cos(dx + c)^5 \log(\sin(dx + c) + 1) - 15(3a^3b + 4ab^3) \cos(dx + c)^5 \log(-\sin(dx + c) + 1) + 2(30a^3b \cos(dx + c) + 2(8a^4 + 60a^2b^2 + 15b^4) \cos(dx + c)^4 + 6a^4 + 15(3a^3b + 4a^2b^3) \cos(dx + c)^3 + 4(2a^4 + 15a^2b^2) \cos(dx + c)^2) \sin(dx + c)}{(d \cos(dx + c))^5}$$

[In] `integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="fricas")`

[Out] $1/60*(15*(3*a^3*b + 4*a*b^3)*\cos(d*x + c)^5*\log(\sin(d*x + c) + 1) - 15*(3*a^3*b + 4*a^2*b^3)*\cos(d*x + c)^5*\log(-\sin(d*x + c) + 1) + 2*(30*a^3*b*\cos(d*x + c) + 2*(8*a^4 + 60*a^2*b^2 + 15*b^4)*\cos(d*x + c)^4 + 6*a^4 + 15*(3*a^3*b + 4*a^2*b^3)*\cos(d*x + c)^3 + 4*(2*a^4 + 15*a^2*b^2)*\cos(d*x + c)^2)*\sin(d*x + c))/(d*\cos(d*x + c)^5)$

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^4 \sec^6(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**6,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04

$$\int (a + b \cos(c + dx))^4 \sec^6(c + dx) dx$$

$$4(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c))a^4 + 120(\tan(dx + c)^3 + 3 \tan(dx + c))a^2b^2 -$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="maxima")

```
[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2 - 15*a^3*b*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*b^4*tan(d*x +
c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(176) = 352.

Time = 0.33 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.45

$$\int (a + b \cos(c + dx))^4 \sec^6(c + dx) dx$$

$$15(3a^3b + 4ab^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 15(3a^3b + 4ab^3) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2(30a^4 \tan^2(\frac{1}{2}dx + \frac{1}{2}c) + 15a^3b \tan(\frac{1}{2}dx + \frac{1}{2}c) + 15a^2b^2)}{30a^4 \tan^2(\frac{1}{2}dx + \frac{1}{2}c) + 15a^3b \tan(\frac{1}{2}dx + \frac{1}{2}c) + 15a^2b^2}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="giac")

```
[Out] 1/30*(15*(3*a^3*b + 4*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*a^3
*b + 4*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*a^4*tan(1/2*d*x +
```

$$\begin{aligned} & \frac{1}{2}c)^9 - 75a^3b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 180a^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 - 60ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 + 30b^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 - 40a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 30a^3b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 480a^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 120ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 - 120b^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^7 + 116a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 600a^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 180b^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 - 40a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 30a^3b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 480a^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 120ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 120b^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 30a^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 75a^3b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 180a^2b^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 60ab^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 30b^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) \Big/ (\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1)^5 \Big/ d \end{aligned}$$

Mupad [B] (verification not implemented)

Time = 18.75 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.62

$$\int (a + b \cos(c + dx))^4 \sec^6(c + dx) dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) (3a^3b + 4ab^3)}{d} - \frac{(2a^4 - 5a^3b + 12a^2b^2 - 4ab^3 + 2b^4) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + \left(-\frac{8a^4}{3} + 2a^3b - 32a^2b^2 + 8ab^3 - 8b^4\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 1\right)}$$

[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^6,x)

[Out] (atanh(tan(c/2 + (d*x)/2))*(4*a*b^3 + 3*a^3*b))/d - (tan(c/2 + (d*x)/2)^5*((116*a^4)/15 + 12*b^4 + 40*a^2*b^2) + tan(c/2 + (d*x)/2)^9*(2*a^4 - 5*a^3*b - 4*a*b^3 + 2*b^4 + 12*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(8*a*b^3 + 2*a^3*b + (8*a^4)/3 + 8*b^4 + 32*a^2*b^2) - tan(c/2 + (d*x)/2)^7*((8*a^4)/3 - 2*a^3*b - 8*a*b^3 + 8*b^4 + 32*a^2*b^2) + tan(c/2 + (d*x)/2)*(4*a*b^3 + 5*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))

3.448 $\int (a + b \cos(c + dx))^4 \sec^7(c + dx) dx$

Optimal result	4865
Rubi [A] (verified)	4866
Mathematica [A] (verified)	4869
Maple [A] (verified)	4869
Fricas [A] (verification not implemented)	4870
Sympy [F(-1)]	4870
Maxima [A] (verification not implemented)	4870
Giac [B] (verification not implemented)	4871
Mupad [B] (verification not implemented)	4872

Optimal result

Integrand size = 21, antiderivative size = 222

$$\int (a + b \cos(c + dx))^4 \sec^7(c + dx) dx = \frac{(5a^4 + 36a^2b^2 + 8b^4) \operatorname{arctanh}(\sin(c + dx))}{16d} + \frac{4ab(4a^2 + 5b^2) \tan(c + dx)}{5d} + \frac{(5a^4 + 36a^2b^2 + 8b^4) \sec(c + dx) \tan(c + dx)}{16d} + \frac{a^2(5a^2 + 32b^2) \sec^3(c + dx) \tan(c + dx)}{24d} + \frac{7a^3b \sec^4(c + dx) \tan(c + dx)}{15d} + \frac{a^2(a + b \cos(c + dx))^2 \sec^5(c + dx) \tan(c + dx)}{6d} + \frac{4ab(4a^2 + 5b^2) \tan^3(c + dx)}{15d}$$

```
[Out] 1/16*(5*a^4+36*a^2*b^2+8*b^4)*arctanh(sin(d*x+c))/d+4/5*a*b*(4*a^2+5*b^2)*tan(d*x+c)/d+1/16*(5*a^4+36*a^2*b^2+8*b^4)*sec(d*x+c)*tan(d*x+c)/d+1/24*a^2*(5*a^2+32*b^2)*sec(d*x+c)^3*tan(d*x+c)/d+7/15*a^3*b*sec(d*x+c)^4*tan(d*x+c)/d+1/6*a^2*(a+b*cos(d*x+c))^2*sec(d*x+c)^5*tan(d*x+c)/d+4/15*a*b*(4*a^2+5*b^2)*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2871, 3110, 3100, 2827, 3852, 3853, 3855}

$$\int (a + b \cos(c + dx))^4 \sec^7(c + dx) dx = \frac{7a^3 b \tan(c + dx) \sec^4(c + dx)}{15d} + \frac{4ab(4a^2 + 5b^2) \tan^3(c + dx)}{15d} + \frac{4ab(4a^2 + 5b^2) \tan(c + dx)}{5d} + \frac{a^2(5a^2 + 32b^2) \tan(c + dx) \sec^3(c + dx)}{24d} + \frac{a^2 \tan(c + dx) \sec^5(c + dx)(a + b \cos(c + dx))^2}{6d} + \frac{(5a^4 + 36a^2b^2 + 8b^4) \operatorname{arctanh}(\sin(c + dx))}{16d} + \frac{(5a^4 + 36a^2b^2 + 8b^4) \tan(c + dx) \sec(c + dx)}{16d}$$

[In] Int[(a + b*cos[c + d*x])^4*Sec[c + d*x]^7,x]

[Out] ((5*a^4 + 36*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]]/(16*d) + (4*a*b*(4*a^2 + 5*b^2)*Tan[c + d*x])/(5*d) + ((5*a^4 + 36*a^2*b^2 + 8*b^4)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^2*(5*a^2 + 32*b^2)*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (7*a^3*b*Sec[c + d*x]^4*Tan[c + d*x])/(15*d) + (a^2*(a + b*cos[c + d*x])^2*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a*b*(4*a^2 + 5*b^2)*Tan[c + d*x]^3)/(15*d)

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*sin[e + f*x])^(m - 2)*((c + d*sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*sin[e + f*x])^(m - 3)*(c + d*sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b

$^2, 0]$ && $\text{NeQ}[c^2 - d^2, 0]$ && $\text{GtQ}[m, 2]$ && $\text{LtQ}[n, -1]$ && $(\text{IntegerQ}[m] \mid \mid \text{IntegersQ}[2*m, 2*n])$

Rule 3100

$\text{Int}[(a_. + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)] + (C_.)*\sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] \rightarrow \text{Simp}[(-(A*b^2 - a*b*B + a^2*C))*\text{Cos}[e + f*x]*((a + b*\sin[e + f*x])^{(m + 1)})/(b*f*(m + 1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*\text{Simp}[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3110

$\text{Int}[(a_. + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)] + (C_.)*\sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] \rightarrow \text{Simp}[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*((a + b*\sin[e + f*x])^{(m + 1)})/(b^2*f*(m + 1)*(a^2 - b^2)), x] - \text{Dist}[1/(b^2*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\sin[e + f*x])^{(m + 1)}*\text{Simp}[b*(m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))*\sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*\sin[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1]$

Rule 3852

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rule 3853

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[c + d*x]*((b*Csc[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \text{Dist}[b^2*((n - 2)/(n - 1)), \text{Int}[(b*Csc[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 3855

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2(a+b\cos(c+dx))^2 \sec^5(c+dx) \tan(c+dx)}{6d} + \frac{1}{6} \int (a+b\cos(c+dx)) (14a^2b \\
&\quad + a(5a^2+18b^2)\cos(c+dx) + 3b(a^2+2b^2)\cos^2(c+dx)) \sec^6(c+dx) dx \\
&= \frac{7a^3b \sec^4(c+dx) \tan(c+dx)}{15d} + \frac{a^2(a+b\cos(c+dx))^2 \sec^5(c+dx) \tan(c+dx)}{6d} \\
&\quad - \frac{1}{30} \int (-5a^2(5a^2+32b^2) - 24ab(4a^2+5b^2)\cos(c+dx) \\
&\quad\quad - 15b^2(a^2+2b^2)\cos^2(c+dx)) \sec^5(c+dx) dx \\
&= \frac{a^2(5a^2+32b^2) \sec^3(c+dx) \tan(c+dx)}{24d} + \frac{7a^3b \sec^4(c+dx) \tan(c+dx)}{15d} \\
&\quad + \frac{a^2(a+b\cos(c+dx))^2 \sec^5(c+dx) \tan(c+dx)}{6d} \\
&\quad - \frac{1}{120} \int (-96ab(4a^2+5b^2) - 15(5a^4+36a^2b^2+8b^4)\cos(c+dx)) \sec^4(c+dx) dx \\
&= \frac{a^2(5a^2+32b^2) \sec^3(c+dx) \tan(c+dx)}{24d} + \frac{7a^3b \sec^4(c+dx) \tan(c+dx)}{15d} \\
&\quad + \frac{a^2(a+b\cos(c+dx))^2 \sec^5(c+dx) \tan(c+dx)}{6d} \\
&\quad + \frac{1}{5}(4ab(4a^2+5b^2)) \int \sec^4(c+dx) dx - \frac{1}{8}(-5a^4-36a^2b^2-8b^4) \int \sec^3(c+dx) dx \\
&= \frac{(5a^4+36a^2b^2+8b^4) \sec(c+dx) \tan(c+dx)}{16d} \\
&\quad + \frac{a^2(5a^2+32b^2) \sec^3(c+dx) \tan(c+dx)}{24d} + \frac{7a^3b \sec^4(c+dx) \tan(c+dx)}{15d} \\
&\quad + \frac{a^2(a+b\cos(c+dx))^2 \sec^5(c+dx) \tan(c+dx)}{6d} \\
&\quad - \frac{1}{16}(-5a^4-36a^2b^2-8b^4) \int \sec(c+dx) dx \\
&\quad - \frac{(4ab(4a^2+5b^2)) \text{Subst}(\int (1+x^2) dx, x, -\tan(c+dx))}{5d} \\
&= \frac{(5a^4+36a^2b^2+8b^4) \operatorname{arctanh}(\sin(c+dx))}{16d} + \frac{4ab(4a^2+5b^2) \tan(c+dx)}{5d} \\
&\quad + \frac{(5a^4+36a^2b^2+8b^4) \sec(c+dx) \tan(c+dx)}{16d} \\
&\quad + \frac{a^2(5a^2+32b^2) \sec^3(c+dx) \tan(c+dx)}{24d} + \frac{7a^3b \sec^4(c+dx) \tan(c+dx)}{15d} \\
&\quad + \frac{a^2(a+b\cos(c+dx))^2 \sec^5(c+dx) \tan(c+dx)}{6d} + \frac{4ab(4a^2+5b^2) \tan^3(c+dx)}{15d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.69

$$\int (a + b \cos(c + dx))^4 \sec^7(c + dx) dx$$

$$= \frac{15(5a^4 + 36a^2b^2 + 8b^4) \operatorname{arctanh}(\sin(c + dx)) + \tan(c + dx) (15(5a^4 + 36a^2b^2 + 8b^4) \sec(c + dx) + 10a^2(5$$

[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^7,x]

[Out] (15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*Sec[c + d*x] + 10*a^2*(5*a^2 + 36*b^2)*Sec[c + d*x]^3 + 40*a^4*Sec[c + d*x]^5 + 64*a*b*(15*(a^2 + b^2) + 5*(2*a^2 + b^2)*Tan[c + d*x]^2 + 3*a^2*Tan[c + d*x]^4)))/(240*d)

Maple [A] (verified)

Time = 6.02 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.94

method	result
derivativedivides	$a^4 \left(- \left(- \frac{(\sec^5(dx+c))}{6} - \frac{5(\sec^3(dx+c))}{24} - \frac{5 \sec(dx+c)}{16} \right) \tan(dx+c) + \frac{5 \ln(\sec(dx+c) + \tan(dx+c))}{16} \right) - 4a^3b \left(- \frac{8}{15} - \frac{(\sec^4(dx+c))}{5} \right)$
default	$a^4 \left(- \left(- \frac{(\sec^5(dx+c))}{6} - \frac{5(\sec^3(dx+c))}{24} - \frac{5 \sec(dx+c)}{16} \right) \tan(dx+c) + \frac{5 \ln(\sec(dx+c) + \tan(dx+c))}{16} \right) - 4a^3b \left(- \frac{8}{15} - \frac{(\sec^4(dx+c))}{5} \right)$
parts	$a^4 \left(- \left(- \frac{(\sec^5(dx+c))}{6} - \frac{5(\sec^3(dx+c))}{24} - \frac{5 \sec(dx+c)}{16} \right) \tan(dx+c) + \frac{5 \ln(\sec(dx+c) + \tan(dx+c))}{16} \right) + \frac{b^4 \left(\frac{\sec(dx+c) \tan(dx+c)}{2} \right)}{d}$
parallelrisch	$\frac{-1125(a^4 + \frac{36}{5}a^2b^2 + \frac{8}{5}b^4) \left(\frac{\cos(6dx+6c)}{15} + \frac{2 \cos(4dx+4c)}{5} + \cos(2dx+2c) + \frac{2}{3} \right) \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + 1125(a^4 + \frac{36}{5}a^2b^2 + \frac{8}{5}b^4)}{d}$
risch	$\frac{i(-640ab^3 - 512a^3b - 3072a^3be^{2i(dx+c)} - 5120a^3be^{6i(dx+c)} - 3840ab^3e^{2i(dx+c)} - 540a^2b^2e^{i(dx+c)} - 3060a^2b^2e^{3i(dx+c)})}{d}$

[In] int((a+cos(d*x+c)*b)^4*sec(d*x+c)^7,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^4*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c)))-4*a^3*b*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+6*a^2*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-4*a*b^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.98

$$\int (a + b \cos(c + dx))^4 \sec^7(c + dx) dx$$

$$= \frac{15(5a^4 + 36a^2b^2 + 8b^4) \cos(dx + c)^6 \log(\sin(dx + c) + 1) - 15(5a^4 + 36a^2b^2 + 8b^4) \cos(dx + c)^6 \log(-\sin(dx + c) + 1) + 2(128(4a^3b + 5ab^3) \cos(dx + c)^5 + 192a^3b \cos(dx + c) + 15(5a^4 + 36a^2b^2 + 8b^4) \cos(dx + c)^4 + 40a^4 + 64(4a^3b + 5ab^3) \cos(dx + c)^3 + 10(5a^4 + 36a^2b^2) \cos(dx + c)^2 \sin(dx + c))}{(d \cos(dx + c))^6}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="fricas")

```
[Out] 1/480*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1)
- 15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) +
2*(128*(4*a^3*b + 5*a*b^3)*cos(d*x + c)^5 + 192*a^3*b*cos(d*x + c) + 15*(5*
a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 40*a^4 + 64*(4*a^3*b + 5*a*b^3)*
cos(d*x + c)^3 + 10*(5*a^4 + 36*a^2*b^2)*cos(d*x + c)^2*sin(d*x + c))/(d*c
os(d*x + c)^6)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^4 \sec^7(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**7,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.24

$$\int (a + b \cos(c + dx))^4 \sec^7(c + dx) dx$$

$$= \frac{128(3 \tan(dx + c)^5 + 10 \tan(dx + c)^3 + 15 \tan(dx + c))a^3b + 640(\tan(dx + c)^3 + 3 \tan(dx + c))ab^3 - 5a^4(2(15 \sin(dx + c)^5 + 10 \sin(dx + c)^3 + 15 \sin(dx + c)) \cos(dx + c)^6 \log(\sin(dx + c) + 1) - 15 \cos(dx + c)^6 \log(-\sin(dx + c) + 1))}{(d \cos(dx + c))^6}$$

[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="maxima")

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3*b +
640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^3 - 5*a^4*(2*(15*sin(d*x + c)^5 +
10*sin(d*x + c)^3 + 15*sin(d*x + c))*cos(d*x + c)^6*log(sin(d*x + c) + 1) -
15*cos(d*x + c)^6*log(-sin(d*x + c) + 1)))/d^6
```

$$- 40\sin(dx + c)^3 + 33\sin(dx + c))/(\sin(dx + c)^6 - 3\sin(dx + c)^4 + 3\sin(dx + c)^2 - 1) - 15\log(\sin(dx + c) + 1) + 15\log(\sin(dx + c) - 1) - 180a^2b^2(2(3\sin(dx + c)^3 - 5\sin(dx + c)))/(\sin(dx + c)^4 - 2\sin(dx + c)^2 + 1) - 3\log(\sin(dx + c) + 1) + 3\log(\sin(dx + c) - 1) - 120b^4(2\sin(dx + c))/(\sin(dx + c)^2 - 1) - \log(\sin(dx + c) + 1) + \log(\sin(dx + c) - 1))/d$$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(208) = 416.

Time = 0.36 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.67

$$\int (a + b \cos(c + dx))^4 \sec^7(c + dx) dx$$

$$15(5a^4 + 36a^2b^2 + 8b^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 15(5a^4 + 36a^2b^2 + 8b^4) \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) -$$

[In] integrate((a+b*cos(dx+c))^4*sec(dx+c)^7,x, algorithm="giac")

[Out] 1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(16*5*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*b^4*tan(1/2*d*x + 1/2*c)^11 + 25*a^4*tan(1/2*d*x + 1/2*c)^9 + 2240*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 3520*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 360*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*b^4*tan(1/2*d*x + 1/2*c)^7 + 450*a^4*tan(1/2*d*x + 1/2*c)^5 + 4992*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 5760*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*b^4*tan(1/2*d*x + 1/2*c)^3 + 165*a^4*tan(1/2*d*x + 1/2*c) + 960*a^3*b*tan(1/2*d*x + 1/2*c) + 900*a^2*b^2*tan(1/2*d*x + 1/2*c) + 960*a*b^3*tan(1/2*d*x + 1/2*c) + 120*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d

Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.67

$$\int (a + b \cos(c + dx))^4 \sec^7(c + dx) dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) \left(\frac{5a^4}{8} + \frac{9a^2b^2}{2} + b^4\right)}{d} + \frac{\left(\frac{11a^4}{8} - 8a^3b + \frac{15a^2b^2}{2} - 8ab^3 + b^4\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} + \left(\frac{5a^4}{24} + \frac{56a^3b}{3} - \frac{21a^2b^2}{2} + \frac{88ab^3}{3} - 3b^4\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^9}{d}$$

[In] int((a + b*cos(c + d*x))^4/cos(c + d*x)^7,x)

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((5*a^4)/8 + b^4 + (9*a^2*b^2)/2))/d + (tan(c/2 + (d*x)/2)^9*((88*a*b^3)/3 + (56*a^3*b)/3 + (5*a^4)/24 - 3*b^4 - (21*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^3*((88*a*b^3)/3 + (56*a^3*b)/3 - (5*a^4)/24 + 3*b^4 + (21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*(48*a*b^3 + (208*a^3*b)/5 + (15*a^4)/4 + 2*b^4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)^7*((15*a^4)/4 - (208*a^3*b)/5 - 48*a*b^3 + 2*b^4 + 3*a^2*b^2) + tan(c/2 + (d*x)/2)*(8*a*b^3 + 8*a^3*b + (11*a^4)/8 + b^4 + (15*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^11*((11*a^4)/8 - 8*a^3*b - 8*a*b^3 + b^4 + (15*a^2*b^2)/2))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.449 $\int \frac{\cos^5(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	4873
Rubi [A] (verified)	4873
Mathematica [A] (verified)	4876
Maple [A] (verified)	4876
Fricas [A] (verification not implemented)	4877
Sympy [F(-1)]	4877
Maxima [F(-2)]	4878
Giac [B] (verification not implemented)	4878
Mupad [B] (verification not implemented)	4879

Optimal result

Integrand size = 21, antiderivative size = 193

$$\int \frac{\cos^5(c+dx)}{a+b \cos(c+dx)} dx = \frac{(8a^4 + 4a^2b^2 + 3b^4)x}{8b^5} - \frac{2a^5 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^5\sqrt{a+bd}} - \frac{a(3a^2 + 2b^2) \sin(c+dx)}{3b^4d} + \frac{(4a^2 + 3b^2) \cos(c+dx) \sin(c+dx)}{8b^3d} - \frac{a \cos^2(c+dx) \sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx) \sin(c+dx)}{4bd}$$

[Out] $1/8*(8*a^4+4*a^2*b^2+3*b^4)*x/b^5-1/3*a*(3*a^2+2*b^2)*\sin(d*x+c)/b^4/d+1/8*(4*a^2+3*b^2)*\cos(d*x+c)*\sin(d*x+c)/b^3/d-1/3*a*\cos(d*x+c)^2*\sin(d*x+c)/b^2/d+1/4*\cos(d*x+c)^3*\sin(d*x+c)/b/d-2*a^5*\arctan((a-b)^(1/2)*\tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^5/d/(a-b)^(1/2)/(a+b)^(1/2)$

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2872, 3128, 3102, 2814, 2738, 211}

$$\int \frac{\cos^5(c+dx)}{a+b \cos(c+dx)} dx = -\frac{2a^5 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\sqrt{a+bd}} - \frac{a(3a^2 + 2b^2) \sin(c+dx)}{3b^4d} + \frac{(4a^2 + 3b^2) \sin(c+dx) \cos(c+dx)}{8b^3d} + \frac{x(8a^4 + 4a^2b^2 + 3b^4)}{8b^5} - \frac{a \sin(c+dx) \cos^2(c+dx)}{3b^2d} + \frac{\sin(c+dx) \cos^3(c+dx)}{4bd}$$

[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x]),x]

```
[Out] ((8*a^4 + 4*a^2*b^2 + 3*b^4)*x)/(8*b^5) - (2*a^5*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^5*Sqrt[a + b]*d) - (a*(3*a^2 + 2*b^2)*Sin[c + d*x])/(3*b^4*d) + ((4*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*b^3*d) - (a*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*b*d)
```

Rule 211

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & NeQ[c, 0])))
```

Rule 3102

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 3128

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_) + (C_.)*sin[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\int \frac{\cos^2(c+dx)(3a+3b\cos(c+dx)-4a\cos^2(c+dx))}{a+b\cos(c+dx)} dx}{4b} \\
&= -\frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} \\
&\quad + \frac{\int \frac{\cos(c+dx)(-8a^2+ab\cos(c+dx)+3(4a^2+3b^2)\cos^2(c+dx))}{a+b\cos(c+dx)} dx}{12b^2} \\
&= \frac{(4a^2+3b^2)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d} \\
&\quad + \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\int \frac{3a(4a^2+3b^2)-b(4a^2-9b^2)\cos(c+dx)-8a(3a^2+2b^2)\cos^2(c+dx)}{a+b\cos(c+dx)} dx}{24b^3} \\
&= -\frac{a(3a^2+2b^2)\sin(c+dx)}{3b^4d} + \frac{(4a^2+3b^2)\cos(c+dx)\sin(c+dx)}{8b^3d} \\
&\quad - \frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} \\
&\quad + \frac{\int \frac{3ab(4a^2+3b^2)+3(8a^4+4a^2b^2+3b^4)\cos(c+dx)}{a+b\cos(c+dx)} dx}{24b^4} \\
&= \frac{(8a^4+4a^2b^2+3b^4)x}{8b^5} - \frac{a(3a^2+2b^2)\sin(c+dx)}{3b^4d} + \frac{(4a^2+3b^2)\cos(c+dx)\sin(c+dx)}{8b^3d} \\
&\quad - \frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} - \frac{a^5 \int \frac{1}{a+b\cos(c+dx)} dx}{b^5} \\
&= \frac{(8a^4+4a^2b^2+3b^4)x}{8b^5} - \frac{a(3a^2+2b^2)\sin(c+dx)}{3b^4d} \\
&\quad + \frac{(4a^2+3b^2)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d} \\
&\quad + \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} - \frac{(2a^5) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^5d}
\end{aligned}$$

$$= \frac{(8a^4 + 4a^2b^2 + 3b^4)x}{8b^5} - \frac{2a^5 \arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^5\sqrt{a+bd}}$$

$$- \frac{a(3a^2 + 2b^2)\sin(c+dx)}{3b^4d} + \frac{(4a^2 + 3b^2)\cos(c+dx)\sin(c+dx)}{8b^3d}$$

$$- \frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4bd}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.79

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)} dx$$

$$= \frac{12(8a^4 + 4a^2b^2 + 3b^4)(c+dx) + \frac{192a^5 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 24ab(4a^2 + 3b^2)\sin(c+dx) + 24b^2(a^2 + 3b^2)\sin^2(c+dx)}{96b^5d}$$

[In] Integrate[Cos[c + d*x]^5/(a + b*cos[c + d*x]), x]

[Out] (12*(8*a^4 + 4*a^2*b^2 + 3*b^4)*(c + d*x) + (192*a^5*ArcTanh[((a - b)*Tan[(c + d*x)/2]])/Sqrt[-a^2 + b^2])/Sqrt[-a^2 + b^2] - 24*a*b*(4*a^2 + 3*b^2)*Sin[c + d*x] + 24*b^2*(a^2 + b^2)*Sin[2*(c + d*x)] - 8*a*b^3*Ssin[3*(c + d*x)] + 3*b^4*Ssin[4*(c + d*x)]/(96*b^5*d)

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{2a^5 \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^5\sqrt{(a-b)(a+b)}} + \frac{2\left(-a^3b - \frac{1}{2}a^2b^2 - ab^3 - \frac{5}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \left(-3a^3b - \frac{5}{3}ab^3 + \frac{3}{8}b^4 - \frac{1}{2}a^2b^2\right)\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))\sqrt{(a-b)(a+b)}}$
default	$-\frac{2a^5 \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^5\sqrt{(a-b)(a+b)}} + \frac{2\left(-a^3b - \frac{1}{2}a^2b^2 - ab^3 - \frac{5}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \left(-3a^3b - \frac{5}{3}ab^3 + \frac{3}{8}b^4 - \frac{1}{2}a^2b^2\right)\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))\sqrt{(a-b)(a+b)}}$
risch	$\frac{xa^4}{b^5} + \frac{xa^2}{2b^3} + \frac{3x}{8b} + \frac{ia^3e^{i(dx+c)}}{2b^4d} + \frac{3ia^2e^{i(dx+c)}}{8b^2d} - \frac{ia^3e^{-i(dx+c)}}{2b^4d} - \frac{3ia^2e^{-i(dx+c)}}{8b^2d} - \frac{a^5 \ln\left(\frac{e^{i(dx+c)} + \sqrt{-a^2+ib^2}}{\sqrt{-a^2+ib^2}}\right)}{\sqrt{-a^2+ib^2}db^5}$

[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*b), x, method=_RETURNVERBOSE)

[Out] 1/d*(-2*a^5/b^5/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))+2/b^5*(((a^3*b-1/2*a^2*b^2-a*b^3-5/8*b^4)*tan(1/2*d*x+1/2*c))^7+(-3*a^3*b-5/3*a*b^3+3/8*b^4-1/2*a^2*b^2)*tan(1/2*d*x+1/2*c))^5+(1/2*a^2*b

$$\frac{\cos^5(c+dx)}{a+b\cos(c+dx)} dx = \frac{12\sqrt{-a^2+b^2}a^5 \log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right) - 3(8a^6 - 4a^4b^2 - a^2b^4 - 3b^6)dx + (24a^5b - 8a^3b^3 - 16ab^5 - 6(a^2b^4 - b^6)\cos(dx+c)^3 + 8(a^3b^3 - ab^5)\cos(dx+c)^2 - 3(4a^4b^2 - a^2b^4 - 3b^6)\cos(dx+c))\sin(dx+c)}{24\sqrt{a^2-b^2}a^5 \arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right) - 3(8a^6 - 4a^4b^2 - a^2b^4 - 3b^6)dx + (24a^5b - 8a^3b^3 - 16ab^5 - 6(a^2b^4 - b^6)\cos(dx+c)^3 + 8(a^3b^3 - ab^5)\cos(dx+c)^2 - 3(4a^4b^2 - a^2b^4 - 3b^6)\cos(dx+c))\sin(dx+c)}$$

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.48

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)} dx = \frac{12\sqrt{-a^2+b^2}a^5 \log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right) - 3(8a^6 - 4a^4b^2 - a^2b^4 - 3b^6)dx + (24a^5b - 8a^3b^3 - 16ab^5 - 6(a^2b^4 - b^6)\cos(dx+c)^3 + 8(a^3b^3 - ab^5)\cos(dx+c)^2 - 3(4a^4b^2 - a^2b^4 - 3b^6)\cos(dx+c))\sin(dx+c)}{24\sqrt{a^2-b^2}a^5 \arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right) - 3(8a^6 - 4a^4b^2 - a^2b^4 - 3b^6)dx + (24a^5b - 8a^3b^3 - 16ab^5 - 6(a^2b^4 - b^6)\cos(dx+c)^3 + 8(a^3b^3 - ab^5)\cos(dx+c)^2 - 3(4a^4b^2 - a^2b^4 - 3b^6)\cos(dx+c))\sin(dx+c)}$$

[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] [-1/24*(12*sqrt(-a^2 + b^2)*a^5*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 3*(8*a^6 - 4*a^4*b^2 - a^2*b^4 - 3*b^6)*d*x + (24*a^5*b - 8*a^3*b^3 - 16*a*b^5 - 6*(a^2*b^4 - b^6)*cos(d*x + c)^3 + 8*(a^3*b^3 - a*b^5)*cos(d*x + c)^2 - 3*(4*a^4*b^2 - a^2*b^4 - 3*b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 - b^7)*d), -1/24*(2*sqrt(a^2 - b^2)*a^5*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*(8*a^6 - 4*a^4*b^2 - a^2*b^4 - 3*b^6)*d*x + (24*a^5*b - 8*a^3*b^3 - 16*a*b^5 - 6*(a^2*b^4 - b^6)*cos(d*x + c)^3 + 8*(a^3*b^3 - a*b^5)*cos(d*x + c)^2 - 3*(4*a^4*b^2 - a^2*b^4 - 3*b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 - b^7)*d)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c)),x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c + dx)}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(174) = 348.

Time = 0.29 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.04

$$\int \frac{\cos^5(c + dx)}{a + b \cos(c + dx)} dx$$

$$= \frac{48 \left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}} \right) \right) a^5}{\sqrt{a^2 - b^2} b^5} + \frac{3(8a^4 + 4a^2b^2 + 3b^4)(dx+c)}{b^5} - \frac{2(24a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7}{b^5} - \dots}{b^5}$$

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^5/(sqrt(a^2
- b^2)*b^5) + 3*(8*a^4 + 4*a^2*b^2 + 3*b^4)*(d*x + c)/b^5 - 2*(24*a^3*tan(
1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*a*b^2*tan(1/2*d*x
+ 1/2*c)^7 + 15*b^3*tan(1/2*d*x + 1/2*c)^7 + 72*a^3*tan(1/2*d*x + 1/2*c)^5
+ 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*b^
3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*b*tan(1/2
*d*x + 1/2*c)^3 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*b^3*tan(1/2*d*x + 1/2
*c)^3 + 24*a^3*tan(1/2*d*x + 1/2*c) - 12*a^2*b*tan(1/2*d*x + 1/2*c) + 24*a*
b^2*tan(1/2*d*x + 1/2*c) - 15*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2
*c)^2 + 1)^4*b^4))/d
```

Mupad [B] (verification not implemented)

Time = 15.66 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.46

$$\int \frac{\cos^5(c + dx)}{a + b \cos(c + dx)} dx = \frac{\sin(2c + 2dx)}{4bd} + \frac{\sin(4c + 4dx)}{32bd} + \frac{3 \operatorname{atan}\left(\frac{40 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a^4 b^6 + 15 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a^2 b^8 + 9 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) b^{10}}{b \cos\left(\frac{c}{2} + \frac{dx}{2}\right) (40 a^4 b^5 + 15 a^2 b^7 + 9 b^9)}\right)}{4bd} - \frac{a \sin(3c + 3dx)}{12b^2d} - \frac{a^3 \sin(c + dx)}{b^4d} + \frac{a^2 \sin(2c + 2dx)}{4b^3d} + \frac{a^2 \operatorname{atan}\left(\frac{40 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a^4 b^6 + 15 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a^2 b^8 + 9 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) b^{10}}{b \cos\left(\frac{c}{2} + \frac{dx}{2}\right) (40 a^4 b^5 + 15 a^2 b^7 + 9 b^9)}\right)}{b^3d} + \frac{2a^4 \operatorname{atan}\left(\frac{40 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a^4 b^6 + 15 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a^2 b^8 + 9 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) b^{10}}{b \cos\left(\frac{c}{2} + \frac{dx}{2}\right) (40 a^4 b^5 + 15 a^2 b^7 + 9 b^9)}\right)}{b^5d} - \frac{3a \sin(c + dx)}{4b^2d} - \frac{a^5 \operatorname{atan}\left(\frac{(a \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - b \sin\left(\frac{c}{2} + \frac{dx}{2}\right)) i}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2}}\right) 2i}{b^5d \sqrt{b^2 - a^2}}$$

[In] int(cos(c + d*x)^5/(a + b*cos(c + d*x)),x)

[Out] sin(2*c + 2*d*x)/(4*b*d) + sin(4*c + 4*d*x)/(32*b*d) + (3*atan((9*b^10*sin(c/2 + (d*x)/2) + 15*a^2*b^8*sin(c/2 + (d*x)/2) + 40*a^4*b^6*sin(c/2 + (d*x)/2))/(b*cos(c/2 + (d*x)/2)*(9*b^9 + 15*a^2*b^7 + 40*a^4*b^5)))/(4*b*d) - (a*sin(3*c + 3*d*x))/(12*b^2*d) - (a^3*sin(c + d*x))/(b^4*d) + (a^2*sin(2*c + 2*d*x))/(4*b^3*d) + (a^2*atan((9*b^10*sin(c/2 + (d*x)/2) + 15*a^2*b^8*sin(c/2 + (d*x)/2) + 40*a^4*b^6*sin(c/2 + (d*x)/2))/(b*cos(c/2 + (d*x)/2)*(9*b^9 + 15*a^2*b^7 + 40*a^4*b^5)))/(b^3*d) + (2*a^4*atan((9*b^10*sin(c/2 + (d*x)/2) + 15*a^2*b^8*sin(c/2 + (d*x)/2) + 40*a^4*b^6*sin(c/2 + (d*x)/2))/(b*cos(c/2 + (d*x)/2)*(9*b^9 + 15*a^2*b^7 + 40*a^4*b^5)))/(b^5*d) - (3*a*sin(c + d*x))/(4*b^2*d) - (a^5*atan(((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))*i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^5*d*(b^2 - a^2)^(1/2))

3.450 $\int \frac{\cos^4(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	4880
Rubi [A] (verified)	4880
Mathematica [A] (verified)	4883
Maple [A] (verified)	4883
Fricas [A] (verification not implemented)	4884
Sympy [F(-1)]	4884
Maxima [F(-2)]	4884
Giac [A] (verification not implemented)	4885
Mupad [B] (verification not implemented)	4885

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{\cos^4(c+dx)}{a+b \cos(c+dx)} dx = -\frac{a(2a^2+b^2)x}{2b^4} + \frac{2a^4 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}} + \frac{(3a^2+2b^2)\sin(c+dx)}{3b^3d} - \frac{a \cos(c+dx) \sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx) \sin(c+dx)}{3bd}$$

[Out] $-1/2*a*(2*a^2+b^2)*x/b^4+1/3*(3*a^2+2*b^2)*\sin(d*x+c)/b^3/d-1/2*a*\cos(d*x+c)*\sin(d*x+c)/b^2/d+1/3*\cos(d*x+c)^2*\sin(d*x+c)/b/d+2*a^4*\arctan((a-b)^(1/2)*\tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^4/d/(a-b)^(1/2)/(a+b)^(1/2)$

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2872, 3128, 3102, 2814, 2738, 211}

$$\int \frac{\cos^4(c+dx)}{a+b \cos(c+dx)} dx = \frac{2a^4 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4 d \sqrt{a-b} \sqrt{a+b}} - \frac{ax(2a^2+b^2)}{2b^4} + \frac{(3a^2+2b^2)\sin(c+dx)}{3b^3d} - \frac{a \sin(c+dx) \cos(c+dx)}{2b^2d} + \frac{\sin(c+dx) \cos^2(c+dx)}{3bd}$$

[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x]),x]

```
[Out] -1/2*(a*(2*a^2 + b^2)*x)/b^4 + (2*a^4*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + ((3*a^2 + 2*b^2)*Sin[c + d
*x])/(3*b^3*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (Cos[c + d*x]^2*
Sin[c + d*x])/(3*b*d)
```

Rule 211

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rule 3128

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)])^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cos^2(c + dx) \sin(c + dx)}{3bd} + \frac{\int \frac{\cos(c+dx)(2a+2b \cos(c+dx)-3a \cos^2(c+dx))}{a+b \cos(c+dx)} dx}{3b} \\
&= -\frac{a \cos(c + dx) \sin(c + dx)}{2b^2d} + \frac{\cos^2(c + dx) \sin(c + dx)}{3bd} \\
&\quad + \frac{\int \frac{-3a^2+ab \cos(c+dx)+2(3a^2+2b^2) \cos^2(c+dx)}{a+b \cos(c+dx)} dx}{6b^2} \\
&= \frac{(3a^2 + 2b^2) \sin(c + dx)}{3b^3d} - \frac{a \cos(c + dx) \sin(c + dx)}{2b^2d} \\
&\quad + \frac{\cos^2(c + dx) \sin(c + dx)}{3bd} + \frac{\int \frac{-3a^2b-3a(2a^2+b^2) \cos(c+dx)}{a+b \cos(c+dx)} dx}{6b^3} \\
&= -\frac{a(2a^2 + b^2)x}{2b^4} + \frac{(3a^2 + 2b^2) \sin(c + dx)}{3b^3d} - \frac{a \cos(c + dx) \sin(c + dx)}{2b^2d} \\
&\quad + \frac{\cos^2(c + dx) \sin(c + dx)}{3bd} + \frac{a^4 \int \frac{1}{a+b \cos(c+dx)} dx}{b^4} \\
&= -\frac{a(2a^2 + b^2)x}{2b^4} + \frac{(3a^2 + 2b^2) \sin(c + dx)}{3b^3d} - \frac{a \cos(c + dx) \sin(c + dx)}{2b^2d} \\
&\quad + \frac{\cos^2(c + dx) \sin(c + dx)}{3bd} + \frac{(2a^4) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{b^4d} \\
&= -\frac{a(2a^2 + b^2)x}{2b^4} + \frac{2a^4 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}d} + \frac{(3a^2 + 2b^2) \sin(c + dx)}{3b^3d} \\
&\quad - \frac{a \cos(c + dx) \sin(c + dx)}{2b^2d} + \frac{\cos^2(c + dx) \sin(c + dx)}{3bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{\cos^4(c + dx)}{a + b \cos(c + dx)} dx$$

$$= \frac{-6a(2a^2 + b^2)(c + dx) - \frac{24a^4 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + 3b(4a^2 + 3b^2)\sin(c + dx) - 3ab^2 \sin(2(c + dx))}{12b^4 d}$$

[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x]),x]

[Out] $(-6*a*(2*a^2 + b^2)*(c + d*x) - (24*a^4*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 3*b*(4*a^2 + 3*b^2)*Sin[c + d*x] - 3*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)]/(12*b^4*d)$

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{2a^4 \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^4 \sqrt{(a-b)(a+b)}} - \frac{2\left(\frac{(-a^2b - \frac{1}{2}ab^2 - b^3)\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-2a^2b - \frac{2}{3}b^3)\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-a^2b - b^3 + \frac{1}{2}ab^2)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^3}{b^4}$
default	$\frac{2a^4 \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^4 \sqrt{(a-b)(a+b)}} - \frac{2\left(\frac{(-a^2b - \frac{1}{2}ab^2 - b^3)\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-2a^2b - \frac{2}{3}b^3)\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-a^2b - b^3 + \frac{1}{2}ab^2)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^3}{b^4}$
risch	$-\frac{a^3x}{b^4} - \frac{ax}{2b^2} - \frac{ie^{i(dx+c)}a^2}{2b^3d} - \frac{3ie^{i(dx+c)}}{8bd} + \frac{ie^{-i(dx+c)}a^2}{2b^3d} + \frac{3ie^{-i(dx+c)}}{8bd} - \frac{a^4 \ln\left(e^{i(dx+c)} + \frac{ia^2 - ib^2 + a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d b^4}$

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] $1/d*(2*a^4/b^4/((a-b)*(a+b))^{(1/2)}*\arctan((a-b)*\tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^{(1/2)})-2/b^4*(((a^2*b-1/2*a*b^2-b^3)*\tan(1/2*d*x+1/2*c)^5+(-2*a^2*b-2/3*b^3)*\tan(1/2*d*x+1/2*c)^3+(-a^2*b-b^3+1/2*a*b^2)*\tan(1/2*d*x+1/2*c))/(1+\tan(1/2*d*x+1/2*c)^2)^3+1/2*a*(2*a^2+b^2)*\arctan(\tan(1/2*d*x+1/2*c)))$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.70

$$\int \frac{\cos^4(c + dx)}{a + b \cos(c + dx)} dx$$

$$= \left[\frac{3\sqrt{-a^2 + b^2}a^4 \log\left(\frac{2ab \cos(dx+c) + (2a^2 - b^2) \cos(dx+c)^2 + 2\sqrt{-a^2 + b^2}(a \cos(dx+c) + b) \sin(dx+c) - a^2 + 2b^2}{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}\right) + 3(2a^5 - a^3b^2)}{6(a^2b^4)} \right]$$

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*(2*a^5 - a^3*b^2 - a*b^4)*d*x - (6*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^2*b^3 - b^5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c)/((a^2*b^4 - b^6)*d), 1/6*(6*sqrt(a^2 - b^2)*a^4*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*(2*a^5 - a^3*b^2 - a*b^4)*d*x + (6*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^2*b^3 - b^5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c)/((a^2*b^4 - b^6)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{a + b \cos(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c + dx)}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)
```


Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.68

$$\int \frac{\cos^4(c + dx)}{a + b \cos(c + dx)} dx = \frac{12 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}} \right) \right) a^4}{\sqrt{a^2 - b^2} b^4} + \frac{3(2a^3 + ab^2)(dx+c)}{b^4} - \frac{2(6a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 3a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 3a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 3a^2)}{b^4}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^4/(sqrt(a^2 - b^2)*b^4) + 3*(2*a^3 + a*b^2)*(d*x + c)/b^4 - 2*(6*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 15.34 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37

$$\int \frac{\cos^4(c + dx)}{a + b \cos(c + dx)} dx = \frac{3 \sin(c + dx)}{4bd} + \frac{\sin(3c + 3dx)}{12bd} - \frac{a \sin(2c + 2dx)}{4b^2d} + \frac{a^2 \sin(c + dx)}{b^3d} - \frac{2a^3 \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{b^4d} - \frac{a \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{b^2d} + \frac{a^4 \operatorname{atan}\left(\frac{(a \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - b \sin\left(\frac{c}{2} + \frac{dx}{2}\right)) \operatorname{li}}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2}}\right) \operatorname{li}}{b^4d \sqrt{b^2 - a^2}}$$

```
[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x)),x)
```

```
[Out] (3*sin(c + d*x))/(4*b*d) + sin(3*c + 3*d*x)/(12*b*d) - (a*sin(2*c + 2*d*x))/(4*b^2*d) + (a^2*sin(c + d*x))/(b^3*d) - (2*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^4*d) - (a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^2*d) + (a^4*atan(((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))*li)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))))*li)/(b^4*d*(b^2 - a^2)^(1/2))
```

3.451 $\int \frac{\cos^3(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	4886
Rubi [A] (verified)	4886
Mathematica [A] (verified)	4888
Maple [A] (verified)	4888
Fricas [A] (verification not implemented)	4889
Sympy [F(-1)]	4889
Maxima [F(-2)]	4890
Giac [A] (verification not implemented)	4890
Mupad [B] (verification not implemented)	4890

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\cos^3(c+dx)}{a+b \cos(c+dx)} dx = \frac{(2a^2 + b^2)x}{2b^3} - \frac{2a^3 \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{\sqrt{a-b} b^3 \sqrt{a+b} d} - \frac{a \sin(c+dx)}{b^2 d} + \frac{\cos(c+dx) \sin(c+dx)}{2bd}$$

[Out] $\frac{1}{2}*(2*a^2+b^2)*x/b^3 - a*\sin(d*x+c)/b^2/d + 1/2*\cos(d*x+c)*\sin(d*x+c)/b/d - 2*a^3*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/b^3/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2872, 3102, 2814, 2738, 211}

$$\int \frac{\cos^3(c+dx)}{a+b \cos(c+dx)} dx = -\frac{2a^3 \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{b^3 d \sqrt{a-b} \sqrt{a+b}} + \frac{x(2a^2 + b^2)}{2b^3} - \frac{a \sin(c+dx)}{b^2 d} + \frac{\sin(c+dx) \cos(c+dx)}{2bd}$$

[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x]),x]

[Out] $((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*Sin[c + d*x])/(b^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)$

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2872

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & NeQ[c, 0])))

Rule 3102

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\cos(c + dx) \sin(c + dx)}{2bd} + \frac{\int \frac{a+b \cos(c+dx)-2a \cos^2(c+dx)}{a+b \cos(c+dx)} dx}{2b} \\ &= -\frac{a \sin(c + dx)}{b^2d} + \frac{\cos(c + dx) \sin(c + dx)}{2bd} + \frac{\int \frac{ab+(2a^2+b^2) \cos(c+dx)}{a+b \cos(c+dx)} dx}{2b^2} \end{aligned}$$

$$\begin{aligned}
 &= \frac{(2a^2 + b^2)x}{2b^3} - \frac{a \sin(c + dx)}{b^2 d} + \frac{\cos(c + dx) \sin(c + dx)}{2bd} - \frac{a^3 \int \frac{1}{a+b \cos(c+dx)} dx}{b^3} \\
 &= \frac{(2a^2 + b^2)x}{2b^3} - \frac{a \sin(c + dx)}{b^2 d} + \frac{\cos(c + dx) \sin(c + dx)}{2bd} \\
 &\quad - \frac{(2a^3) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{b^3 d} \\
 &= \frac{(2a^2 + b^2)x}{2b^3} - \frac{2a^3 \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{\sqrt{a-b} b^3 \sqrt{a+b} d} - \frac{a \sin(c + dx)}{b^2 d} + \frac{\cos(c + dx) \sin(c + dx)}{2bd}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\begin{aligned}
 &\int \frac{\cos^3(c + dx)}{a + b \cos(c + dx)} dx \\
 &= \frac{2(2a^2 + b^2)(c + dx) + \frac{8a^3 \operatorname{arctanh}\left(\frac{(a-b) \tan(\frac{1}{2}(c+dx))}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 4ab \sin(c + dx) + b^2 \sin(2(c + dx))}{4b^3 d}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^3/(a + b*cos[c + d*x]),x]

[Out] (2*(2*a^2 + b^2)*(c + d*x) + (8*a^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 4*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*x)]/(4*b^3*d)

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

method	result
derivativedivides	$ \frac{2a^3 \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3 \sqrt{(a-b)(a+b)}} + \frac{2\left((-ab - \frac{1}{2}b^2\right)\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \left(-ab + \frac{1}{2}b^2\right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^2} + (2a^2 + b^2) \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d b^3} $
default	$ \frac{2a^3 \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^3 \sqrt{(a-b)(a+b)}} + \frac{2\left((-ab - \frac{1}{2}b^2\right)\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \left(-ab + \frac{1}{2}b^2\right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^2} + (2a^2 + b^2) \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d b^3} $
risch	$ \frac{x a^2}{b^3} + \frac{x}{2b} + \frac{ia e^{i(dx+c)}}{2b^2 d} - \frac{ia e^{-i(dx+c)}}{2b^2 d} - \frac{a^3 \ln\left(\frac{e^{i(dx+c)} - ia^2 - ib^2 - a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2} d b^3} + \frac{a^3 \ln\left(\frac{e^{i(dx+c)} + ia^2 - ib^2 + a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2} d b^3} $

[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] $1/d*(-2*a^3/b^3/((a-b)*(a+b))^{1/2}*\arctan((a-b)*\tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^{1/2}))+2/b^3*(((a-b)-1/2*b^2)*\tan(1/2*d*x+1/2*c)^3+(-a*b+1/2*b^2)*\tan(1/2*d*x+1/2*c))/(1+\tan(1/2*d*x+1/2*c)^2)+1/2*(2*a^2+b^2)*\arctan(\tan(1/2*d*x+1/2*c))$

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.04

$$\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)} dx$$

$$= \left[\frac{\sqrt{-a^2+b^2}a^3 \log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right) - (2a^4 - a^2b^2 - b^4)dx + (2a^3b - 2ab^3 - (a^2b^2 - b^4)\cos(dx+c))}{2(a^2b^3 - b^5)d} \right]$$

[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] $[-1/2*(\sqrt{-a^2+b^2})*a^3*\log((2*a*b*\cos(d*x+c)+(2*a^2-b^2)*\cos(d*x+c)^2-2*\sqrt{-a^2+b^2}*(a*\cos(d*x+c)+b)*\sin(d*x+c)-a^2+2*b^2)/(b^2*\cos(d*x+c)^2+2*a*b*\cos(d*x+c)+a^2))-(2*a^4-a^2*b^2-b^4)*d*x+(2*a^3*b-2*a*b^3-(a^2*b^2-b^4)*\cos(d*x+c))*\sin(d*x+c)]/((a^2*b^3-b^5)*d), -1/2*(2*\sqrt{a^2-b^2})*a^3*\arctan(-(a*\cos(d*x+c)+b)/(\sqrt{a^2-b^2}*\sin(d*x+c)))-(2*a^4-a^2*b^2-b^4)*d*x+(2*a^3*b-2*a*b^3-(a^2*b^2-b^4)*\cos(d*x+c))*\sin(d*x+c)]/((a^2*b^3-b^5)*d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c)),x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c + dx)}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.61

$$\int \frac{\cos^3(c + dx)}{a + b \cos(c + dx)} dx = \frac{4 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right) a^3}{\sqrt{a^2 - b^2} b^3} + \frac{(2a^2 + b^2)(dx+c)}{b^3} - \frac{2 \left(2a \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + b \tan(\frac{1}{2} dx + \frac{1}{2} c) \right)}{2d \left(\tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 1 \right)}$$

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^3/(sqrt(a^2 -
b^2)*b^3) + (2*a^2 + b^2)*(d*x + c)/b^3 - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 +
b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2)/d
```

Mupad [B] (verification not implemented)

Time = 15.56 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.53

$$\int \frac{\cos^3(c + dx)}{a + b \cos(c + dx)} dx = \frac{\operatorname{atan} \left(\frac{\sin(\frac{c}{2} + \frac{dx}{2})}{\cos(\frac{c}{2} + \frac{dx}{2})} \right)}{bd} + \frac{\sin(2c + 2dx)}{4bd} + \frac{2a^2 \operatorname{atan} \left(\frac{\sin(\frac{c}{2} + \frac{dx}{2})}{\cos(\frac{c}{2} + \frac{dx}{2})} \right)}{b^3 d} - \frac{a \sin(c + dx)}{b^2 d} - \frac{a^3 \operatorname{atan} \left(\frac{(a \sin(\frac{c}{2} + \frac{dx}{2}) - b \sin(\frac{c}{2} + \frac{dx}{2})) \operatorname{li} 1}{\cos(\frac{c}{2} + \frac{dx}{2}) \sqrt{b^2 - a^2}} \right)}{b^3 d \sqrt{b^2 - a^2}} 2i$$

```
[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x)),x)
```

```
[Out] atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(b*d) + sin(2*c + 2*d*x)/(4*b*d)
+ (2*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d) - (a*sin(c
+ d*x))/(b^2*d) - (a^3*atan((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))*
1i)/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)))*2i)/(b^3*d*(b^2 - a^2)^(1/2))
```

3.452 $\int \frac{\cos^2(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	4892
Rubi [A] (verified)	4892
Mathematica [A] (verified)	4894
Maple [A] (verified)	4894
Fricas [A] (verification not implemented)	4894
Sympy [B] (verification not implemented)	4895
Maxima [F(-2)]	4896
Giac [A] (verification not implemented)	4896
Mupad [B] (verification not implemented)	4897

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{\cos^2(c+dx)}{a+b \cos(c+dx)} dx = -\frac{ax}{b^2} + \frac{2a^2 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b} b^2 \sqrt{a+b}} + \frac{\sin(c+dx)}{bd}$$

[Out] $-\frac{ax}{b^2} + \frac{\sin(dx+c)}{bd} + \frac{2a^2 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b} b^2 \sqrt{a+b}}$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2825, 12, 2814, 2738, 211}

$$\int \frac{\cos^2(c+dx)}{a+b \cos(c+dx)} dx = \frac{2a^2 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2 d \sqrt{a-b} \sqrt{a+b}} - \frac{ax}{b^2} + \frac{\sin(c+dx)}{bd}$$

[In] $\text{Int}[\text{Cos}[c + d*x]^2/(a + b*\text{Cos}[c + d*x]), x]$

[Out] $-\frac{ax}{b^2} + \frac{2a^2 \text{ArcTan}\left[\frac{\sqrt{a-b} \tan\left[\frac{c+d*x}{2}\right]}{\sqrt{a+b}}\right]}{b^2 d \sqrt{a-b} \sqrt{a+b}} + \frac{\sin[c + d*x]}{bd}$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2825

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sin(c + dx)}{bd} - \frac{\int \frac{a \cos(c+dx)}{a+b \cos(c+dx)} dx}{b} \\
 &= \frac{\sin(c + dx)}{bd} - \frac{a \int \frac{\cos(c+dx)}{a+b \cos(c+dx)} dx}{b} \\
 &= -\frac{ax}{b^2} + \frac{\sin(c + dx)}{bd} + \frac{a^2 \int \frac{1}{a+b \cos(c+dx)} dx}{b^2} \\
 &= -\frac{ax}{b^2} + \frac{\sin(c + dx)}{bd} + \frac{(2a^2) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{b^2d} \\
 &= -\frac{ax}{b^2} + \frac{2a^2 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+bd}} + \frac{\sin(c + dx)}{bd}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)} dx = \frac{-a(c+dx) - \frac{2a^2 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + b\sin(c+dx)}{b^2 d}$$

[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x]),x]

[Out] $(-a*(c + d*x)) - (2*a^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + b*Sin[c + d*x]/(b^2*d)$ **Maple [A] (verified)**

Time = 0.98 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{2\left(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2} + \frac{2a^2\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}$
default	$\frac{2\left(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2} + \frac{2a^2\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}$
risch	$-\frac{ax}{b^2} - \frac{ie^{i(dx+c)}}{2bd} + \frac{ie^{-i(dx+c)}}{2bd} - \frac{a^2\ln\left(e^{i(dx+c)} + \frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}db^2} + \frac{a^2\ln\left(e^{i(dx+c)} - \frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}db^2}$

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] $1/d*(-2/b^2*(-b*\tan(1/2*d*x+1/2*c)/(1+\tan(1/2*d*x+1/2*c)^2)+a*\arctan(\tan(1/2*d*x+1/2*c)))+2*a^2/b^2/((a-b)*(a+b))^(1/2)*\arctan((a-b)*\tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))$ **Fricas [A] (verification not implemented)**

none

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.54

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)} dx = \left[\frac{\sqrt{-a^2+b^2}a^2 \log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right) + 2(a^3-ab^2)dx}{2(a^2b^2-b^4)d} \right]$$

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^2*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(a^3 - a*b^2)*d*x -
2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d), (sqrt(a^2 - b^2)*a^2*ar
ctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^3 - a*b^2)*
d*x + (a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1744 vs. 2(65) = 130.

Time = 65.92 (sec) , antiderivative size = 1744, normalized size of antiderivative = 22.95

$$\int \frac{\cos^2(c + dx)}{a + b \cos(c + dx)} dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c)),x)
[Out] Piecewise((zoo*x*cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-d*x*tan(c/2 + d
*x/2)**2/(b*d*tan(c/2 + d*x/2)**2 + b*d) - d*x/(b*d*tan(c/2 + d*x/2)**2 + b
*d) + tan(c/2 + d*x/2)**3/(b*d*tan(c/2 + d*x/2)**2 + b*d) + 3*tan(c/2 + d*x
/2)/(b*d*tan(c/2 + d*x/2)**2 + b*d), Eq(a, b)), (d*x*tan(c/2 + d*x/2)**3/(b
*d*tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + d*x*tan(c/2 + d*x/2)/(b*d*
tan(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + 3*tan(c/2 + d*x/2)**2/(b*d*ta
n(c/2 + d*x/2)**3 + b*d*tan(c/2 + d*x/2)) + 1/(b*d*tan(c/2 + d*x/2)**3 + b*
d*tan(c/2 + d*x/2)), Eq(a, -b)), ((x*sin(c + d*x)**2/2 + x*cos(c + d*x)**2/
2 + sin(c + d*x)*cos(c + d*x)/(2*d))/a, Eq(b, 0)), (x*cos(c)**2/(a + b*cos(
c)), Eq(d, 0)), (-a**2*d*x*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2
/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt
(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*
x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - a**2*d*x*sqrt(-a/(a - b) -
b/(a - b))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*
b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*t
an(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) + a**2*log(-sqrt(
-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(a*b**2*d*s
qrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b)
- b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b*
**3*d*sqrt(-a/(a - b) - b/(a - b))) + a**2*log(-sqrt(-a/(a - b) - b/(a - b))
+ tan(c/2 + d*x/2))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2
)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(
a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - a**2*l
og(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(a*
b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/
(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)
```

```

**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - a**2*log(sqrt(-a/(a - b) - b/(
a - b)) + tan(c/2 + d*x/2))/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2
+ d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b
) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) +
a*b*d*x*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2/(a*b**2*d*sqrt(-a
/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a
- b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*s
qrt(-a/(a - b) - b/(a - b))) + a*b*d*x*sqrt(-a/(a - b) - b/(a - b))/(a*b**2
*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a -
b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2
- b**3*d*sqrt(-a/(a - b) - b/(a - b))) + 2*a*b*sqrt(-a/(a - b) - b/(a - b))
*tan(c/2 + d*x/2)/(a*b**2*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**
2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*d*sqrt(-a/(a - b) - b/(a -
b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b) - b/(a - b))) - 2*b**2*sq
rt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)/(a*b**2*d*sqrt(-a/(a - b) - b/(
a - b))*tan(c/2 + d*x/2)**2 + a*b**2*d*sqrt(-a/(a - b) - b/(a - b)) - b**3*
d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**3*d*sqrt(-a/(a - b)
- b/(a - b))), True))

```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c + dx)}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\int \frac{\cos^2(c + dx)}{a + b \cos(c + dx)} dx = \frac{2 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}} \right) \right) a^2}{\sqrt{a^2 - b^2} b^2} + \frac{(dx+c)a}{b^2} - \frac{2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1\right) b}$$

d

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] $-(2*(\pi*\text{floor}(1/2*(d*x + c)/\pi + 1/2)*\text{sgn}(-2*a + 2*b) + \arctan(-(a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{a^2 - b^2}))*a^2/(\sqrt{a^2 - b^2}) + (d*x + c)*a/b^2 - 2*\tan(1/2*d*x + 1/2*c)/((\tan(1/2*d*x + 1/2*c)^2 + 1)*b))/d$

Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.50

$$\int \frac{\cos^2(c + dx)}{a + b \cos(c + dx)} dx$$

$$= \frac{\sin(c + dx)}{bd} - \frac{2a \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{b^2 d}$$

$$- \frac{a^2 \operatorname{atan}\left(\frac{1i \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a^2 b - 2i \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a b^2 + 1i \sin\left(\frac{c}{2} + \frac{dx}{2}\right) b^3}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right) (b^2 - a^2)^{3/2} + a^2 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2} - a b \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2}}\right)}{b^2 d \sqrt{b^2 - a^2}} 2i$$

[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x)),x)

[Out] $\sin(c + d*x)/(b*d) - (2*a*\operatorname{atan}(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(b^2*d) - (a^2*\operatorname{atan}((b^3*\sin(c/2 + (d*x)/2)*1i - a*b^2*\sin(c/2 + (d*x)/2)*2i + a^2*b*\sin(c/2 + (d*x)/2)*1i)/(\cos(c/2 + (d*x)/2)*(b^2 - a^2)^{(3/2)} + a^2*\cos(c/2 + (d*x)/2)*(b^2 - a^2)^{(1/2)} - a*b*\cos(c/2 + (d*x)/2)*(b^2 - a^2)^{(1/2)}))*2i)/(b^2*d*(b^2 - a^2)^{(1/2)})$

3.453 $\int \frac{\cos(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	4898
Rubi [A] (verified)	4898
Mathematica [A] (verified)	4899
Maple [A] (verified)	4899
Fricas [A] (verification not implemented)	4900
Sympy [B] (verification not implemented)	4900
Maxima [F(-2)]	4901
Giac [B] (verification not implemented)	4902
Mupad [B] (verification not implemented)	4902

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{\cos(c+dx)}{a+b \cos(c+dx)} dx = \frac{x}{b} - \frac{2a \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b} b \sqrt{a+b}}$$

[Out] x/b-2*a*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b/d/(a-b)^(1/2)/(a+b)^(1/2)

Rubi [A] (verified)

Time = 0.09 (sec), antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2814, 2738, 211}

$$\int \frac{\cos(c+dx)}{a+b \cos(c+dx)} dx = \frac{x}{b} - \frac{2a \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x]),x]

[Out] x/b - (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (

$a - b)e^{2x^2}$, $x]$, x , $\text{Tan}[(c + dx)/2]/e]$, $x]$ /; $\text{FreeQ}\{a, b, c, d\}, x]$
 $\&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2814

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)x]/((c_.) + (d_.)\sin[(e_.) + (f_.)x])]$, $x_Symbol]$ \rightarrow $\text{Simp}[b(x/d), x]$ - $\text{Dist}[(b*c - a*d)/d, \text{Int}[1/(c + d*\sin[e + f*x]), x], x]$ /; $\text{FreeQ}\{a, b, c, d, e, f\}, x]$ $\&\& \text{NeQ}[b*c - a*d, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{x}{b} - \frac{a \int \frac{1}{a+b \cos(c+dx)} dx}{b} \\ &= \frac{x}{b} - \frac{(2a) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd} \\ &= \frac{x}{b} - \frac{2a \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b} \sqrt{a+bd}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{\cos(c+dx)}{a+b \cos(c+dx)} dx = \frac{c+dx + \frac{2a \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{bd}$$

[In] $\text{Integrate}[\text{Cos}[c + d*x]/(a + b*\text{Cos}[c + d*x]), x]$

[Out] $(c + d*x + (2*a*\text{ArcTanh}[(a - b)*\text{Tan}[(c + d*x)/2]]/\text{Sqrt}[-a^2 + b^2]))/\text{Sqrt}[-a^2 + b^2]/(b*d)$

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{b} - \frac{2a \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}}{d}$	65
default	$\frac{\frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{b} - \frac{2a \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}}{d}$	65
risch	$\frac{x}{b} - \frac{a \ln\left(e^{i(dx+c)} - \frac{ia^2 - ib^2 - a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2} db} + \frac{a \ln\left(e^{i(dx+c)} + \frac{ia^2 - ib^2 + a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2} db}$	152

[In] `int(cos(d*x+c)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)`

[Out] $1/d*(2/b*\arctan(\tan(1/2*d*x+1/2*c))-2/b*a/((a-b)*(a+b))^{(1/2)*\arctan((a-b)*\tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^{(1/2)})}$

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.78

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)} dx = \left[\frac{2(a^2-b^2)dx - \sqrt{-a^2+b^2}a \log\left(\frac{2ab\cos(dx+c) + (2a^2-b^2)\cos(dx+c)^2 - 2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c) - a^2 + 2b^2}{b^2\cos(dx+c)^2 + 2ab\cos(dx+c) + a^2}\right)}{2(a^2b-b^3)d}, (a^2$$

[In] `integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas")`

[Out] $[1/2*(2*(a^2 - b^2)*d*x - \sqrt{-a^2 + b^2}*a*\log((2*a*b*\cos(d*x + c) + (2*a^2 - b^2)*\cos(d*x + c)^2 - 2*\sqrt{-a^2 + b^2}*(a*\cos(d*x + c) + b)*\sin(d*x + c) - a^2 + 2*b^2)/(b^2*\cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + a^2)))/((a^2 * b - b^3)*d), ((a^2 - b^2)*d*x - \sqrt{a^2 - b^2}*a*\arctan(-(a*\cos(d*x + c) + b)/(\sqrt{a^2 - b^2}*\sin(d*x + c))))/((a^2*b - b^3)*d)]$

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(48) = 96$.

Time = 12.67 (sec) , antiderivative size = 320, normalized size of antiderivative = 5.42

$$\int \frac{\cos(c + dx)}{a + b \cos(c + dx)} dx$$

$$= \begin{cases} \tilde{\infty}x \\ \frac{x}{b} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{bd} \\ \frac{x}{b} + \frac{1}{bd \tan\left(\frac{c}{2} + \frac{dx}{2}\right)} \\ \frac{\sin(c+dx)}{ad} \\ \frac{x \cos(c)}{a+b \cos(c)} \\ \frac{adx \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}}{abd \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b^2 d \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} - \frac{a \log\left(-\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{abd \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b^2 d \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} + \frac{a \log\left(\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{abd \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - b^2 d \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} - \frac{bdx}{abd \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} \end{cases}$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x)

[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b - tan(c/2 + d*x/2)/(b*d), Eq(a, b)), (x/b + 1/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), (sin(c + d*x)/(a*d), Eq(b, 0)), (x*cos(c)/(a + b*cos(c)), Eq(d, 0)), (a*d*x*sqrt(-a/(a - b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))) - b*d*x*sqrt(-a/(a - b) - b/(a - b))/(a*b*d*sqrt(-a/(a - b) - b/(a - b)) - b**2*d*sqrt(-a/(a - b) - b/(a - b))), True))

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c + dx)}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(50) = 100.

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.07

$$\int \frac{\cos(c + dx)}{a + b \cos(c + dx)} dx = \frac{(\sqrt{a^2 - b^2}(2a - b)|a - b| + \sqrt{a^2 - b^2}|a - b||b|) \left(\pi \left\lfloor \frac{dx + c}{2\pi} + \frac{1}{2} \right\rfloor + \arctan \left(\frac{2\sqrt{\frac{1}{2}} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{\frac{2a + \sqrt{-4(a+b)(a-b) + 4a^2}}{a-b}}} \right) \right) + \left(\pi \left\lfloor \frac{dx + c}{2\pi} + \frac{1}{2} \right\rfloor + \arctan \left(\frac{2\sqrt{\frac{1}{2}} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{\frac{2a - \sqrt{-4(a+b)(a-b) + 4a^2}}{a-b}}} \right) \right)}{(a^2 - 2ab + b^2)b^2 + (a^3 - 2a^2b + ab^2)|b|} + \frac{\left(\pi \left\lfloor \frac{dx + c}{2\pi} + \frac{1}{2} \right\rfloor + \arctan \left(\frac{2\sqrt{\frac{1}{2}} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{\frac{2a - \sqrt{-4(a+b)(a-b) + 4a^2}}{a-b}}} \right) \right)}{b^2 - a|b|}$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] -((sqrt(a^2 - b^2)*(2*a - b)*abs(a - b) + sqrt(a^2 - b^2)*abs(a - b)*abs(b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a + sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))/(a^2 - 2*a*b + b^2)*b^2 + (a^3 - 2*a^2*b + a*b^2)*abs(b)) + (pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a - sqrt(-4*(a + b)*(a - b) + 4*a^2))/(a - b))))*(2*a - b - abs(b))/(b^2 - a*abs(b))/d

Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.68

$$\int \frac{\cos(c + dx)}{a + b \cos(c + dx)} dx = \frac{2 \operatorname{atan} \left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)} \right)}{bd} + \frac{2a \operatorname{atanh} \left(\frac{a \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - b \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2}} \right)}{bd \sqrt{b^2 - a^2}}$$

[In] int(cos(c + d*x)/(a + b*cos(c + d*x)),x)

[Out] (2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*a*atanh((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))))/(b*d*(b^2 - a^2)^(1/2))

3.454 $\int \frac{1}{a+b \cos(c+dx)} dx$

Optimal result	4903
Rubi [A] (verified)	4903
Mathematica [A] (verified)	4904
Maple [A] (verified)	4904
Fricas [A] (verification not implemented)	4905
Sympy [B] (verification not implemented)	4905
Maxima [F(-2)]	4906
Giac [A] (verification not implemented)	4906
Mupad [B] (verification not implemented)	4906

Optimal result

Integrand size = 12, antiderivative size = 49

$$\int \frac{1}{a+b \cos(c+dx)} dx = \frac{2 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+bd}}$$

[Out] 2*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/d/(a-b)^(1/2)/(a+b)^(1/2)

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2738, 211}

$$\int \frac{1}{a+b \cos(c+dx)} dx = \frac{2 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

[In] Int[(a + b*Cos[c + d*x])^(-1),x]

[Out] (2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*d)

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (

$a - b) * e^{2*x^2}, x], x, \text{Tan}[(c + d*x)/2]/e], x]] /; \text{FreeQ}[\{a, b, c, d\}, x]$
 $\&\& \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2 \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d} \\ &= \frac{2 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b} \sqrt{a+bd}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{1}{a + b \cos(c + dx)} dx = -\frac{2 \arctanh\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2} d}$$

[In] Integrate[(a + b*Cos[c + d*x])^(-1),x]

[Out] (-2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^2]*d)

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{2 \arctan\left(\frac{(a-b) \tan\left(\frac{dx+c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{d \sqrt{(a-b)(a+b)}}$	44
default	$\frac{2 \arctan\left(\frac{(a-b) \tan\left(\frac{dx+c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{d \sqrt{(a-b)(a+b)}}$	44
risch	$-\frac{\ln\left(e^{i(dx+c)} + \frac{ia^2 - ib^2 + a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2} d} + \frac{\ln\left(e^{i(dx+c)} - \frac{ia^2 - ib^2 - a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2} d}$	139

[In] int(1/(a*cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] 2/d/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.57

$$\int \frac{1}{a + b \cos(c + dx)} dx$$

$$= \left[-\frac{\sqrt{-a^2 + b^2} \log\left(\frac{2ab \cos(dx+c) + (2a^2 - b^2) \cos(dx+c)^2 + 2\sqrt{-a^2 + b^2}(a \cos(dx+c) + b) \sin(dx+c) - a^2 + 2b^2}{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}\right)}{2(a^2 - b^2)d}, \frac{\arctan\left(-\frac{a \cos(dx+c) + b}{\sqrt{a^2 - b^2} \sin(dx+c)}\right)}{\sqrt{a^2 - b^2}d} \right]$$

[In] integrate(1/(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))/((a^2 - b^2)*d), arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))/(sqrt(a^2 - b^2)*d)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(41) = 82.

Time = 1.98 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.51

$$\int \frac{1}{a + b \cos(c + dx)} dx$$

$$= \begin{cases} \frac{\infty x}{\cos(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{bd} & \text{for } a = b \\ \frac{1}{bd \tan\left(\frac{c}{2} + \frac{dx}{2}\right)} & \text{for } a = -b \\ \frac{x}{a + b \cos(c)} & \text{for } d = 0 \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - bd\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} - \frac{\log\left(\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} - bd\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

[In] integrate(1/(a+b*cos(d*x+c)),x)

[Out] Piecewise((zoo*x/cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (tan(c/2 + d*x/2)/(b*d), Eq(a, b)), (1/(b*d*tan(c/2 + d*x/2)), Eq(a, -b)), (x/(a + b*cos(c)), Eq(d, 0)), (log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a - b)) - b*d*sqrt(-a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a - b)) - b*d*sqrt(-a/(a - b) - b/(a - b))), True))

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{1}{a + b \cos(c + dx)} dx = -\frac{2 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a + 2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{\sqrt{a^2 - b^2} d}$$

```
[In] integrate(1/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{1}{a + b \cos(c + dx)} dx = \frac{2 \operatorname{atan} \left(\frac{\tan \left(\frac{c}{2} + \frac{dx}{2} \right) (a-b)}{\sqrt{a^2 - b^2}} \right)}{d \sqrt{a^2 - b^2}}$$

```
[In] int(1/(a + b*cos(c + d*x)),x)
```

```
[Out] (2*atan((tan(c/2 + (d*x)/2)*(a - b))/(a^2 - b^2)^(1/2)))/(d*(a^2 - b^2)^(1/2))
```

3.455 $\int \frac{\sec(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	4907
Rubi [A] (verified)	4907
Mathematica [A] (verified)	4908
Maple [A] (verified)	4909
Fricas [A] (verification not implemented)	4909
Sympy [F]	4910
Maxima [F(-2)]	4910
Giac [B] (verification not implemented)	4910
Mupad [B] (verification not implemented)	4911

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{\sec(c+dx)}{a+b \cos(c+dx)} dx = -\frac{2b \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d} + \frac{\operatorname{arctanh}(\sin(c+dx))}{ad}$$

[Out] $\operatorname{arctanh}(\sin(d*x+c))/a/d-2*b*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2826, 3855, 2738, 211}

$$\int \frac{\sec(c+dx)}{a+b \cos(c+dx)} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{ad} - \frac{2b \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

[In] $\text{Int}[\text{Sec}[c + d*x]/(a + b*\text{Cos}[c + d*x]), x]$

[Out] $(-2*b*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[(c + d*x)/2])/(\text{Sqrt}[a + b])]/(a*\text{Sqrt}[a - b]*\text{Sqrt}[a + b]*d) + \text{ArcTanh}[\text{Sin}[c + d*x]]/(a*d)$

Rule 211

$\text{Int}[(a_+ + (b_-)*(x_-)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$

Rule 2738

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}-\frac{2b\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}}{d}$
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}-\frac{2b\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a\sqrt{(a-b)(a+b)}}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a}}{d}$
risch	$-\frac{b\ln\left(\frac{e^{i(dx+c)}-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}b}\right)}{\sqrt{-a^2+b^2}da}+\frac{b\ln\left(\frac{e^{i(dx+c)}+ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}da}-\frac{\ln(e^{i(dx+c)}-i)}{da}+\frac{\ln(e^{i(dx+c)}+i)}{da}$

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a*ln(tan(1/2*d*x+1/2*c)-1)-2/a*b/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))+1/a*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 4.09

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)} dx$$

$$= \left[\frac{\sqrt{-a^2+b^2}b \log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right) - (a^2-b^2)\log(\sin(dx+c)+1)}{2(a^3-ab^2)d} \right. \\ \left. - \frac{2\sqrt{a^2-b^2}b \arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right) - (a^2-b^2)\log(\sin(dx+c)+1) + (a^2-b^2)\log(-\sin(dx+c))}{2(a^3-ab^2)d} \right]$$

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (a^2 - b^2)*log(sin(d*x + c) + 1) + (a^2 - b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d), -1/2*(2*sqrt(a^2 - b^2)*b*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^2 - b^2)*log(sin(d*x + c) + 1) + (a^2 - b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{\sec(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\sec(c + dx)}{a + b \cos(c + dx)} dx$$

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c + dx)}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(59) = 118.

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.75

$$\int \frac{\sec(c + dx)}{a + b \cos(c + dx)} dx = \frac{2 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(2a-2b) + \arctan \left(\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right) b}{\sqrt{a^2 - b^2} a} - \frac{\log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a} + \frac{\log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a}$$

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*a
) - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2*d*x + 1/2*c) - 1
))/a)/d
```

Mupad [B] (verification not implemented)

Time = 15.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.46

$$\int \frac{\sec(c + dx)}{a + b \cos(c + dx)} dx = \frac{2 \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{a d} + \frac{2 b \operatorname{atanh}\left(\frac{a \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - b \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2}}\right)}{a d \sqrt{b^2 - a^2}}$$

[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))),x)

[Out] (2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d) + (2*b*atanh((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))))/(a*d*(b^2 - a^2)^(1/2))

3.456 $\int \frac{\sec^2(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	4912
Rubi [A] (verified)	4912
Mathematica [A] (verified)	4914
Maple [A] (verified)	4914
Fricas [B] (verification not implemented)	4915
Sympy [F]	4915
Maxima [F(-2)]	4916
Giac [B] (verification not implemented)	4916
Mupad [B] (verification not implemented)	4916

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sec^2(c+dx)}{a+b \cos(c+dx)} dx$$

$$= \frac{2b^2 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2 \sqrt{a-b} \sqrt{a+b}} - \frac{b \operatorname{arctanh}(\sin(c+dx))}{a^2 d} + \frac{\tan(c+dx)}{ad}$$

[Out] $-b \operatorname{arctanh}(\sin(d*x+c))/a^2/d + 2*b^2 \operatorname{arctan}((a-b)^{(1/2)} * \tan(1/2*d*x + 1/2*c)/(a+b)^{(1/2)})/a^2/d / (a-b)^{(1/2)} / (a+b)^{(1/2)} + \tan(d*x+c)/a/d$

Rubi [A] (verified)

Time = 0.22 (sec), antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2881, 12, 2826, 3855, 2738, 211}

$$\int \frac{\sec^2(c+dx)}{a+b \cos(c+dx)} dx$$

$$= \frac{2b^2 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2 d \sqrt{a-b} \sqrt{a+b}} - \frac{b \operatorname{arctanh}(\sin(c+dx))}{a^2 d} + \frac{\tan(c+dx)}{ad}$$

[In] $\text{Int}[\text{Sec}[c + d*x]^2/(a + b*\text{Cos}[c + d*x]), x]$

[Out] $(2*b^2*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[(c + d*x)/2])/(\text{Sqrt}[a + b])]/(a^2*\text{Sqrt}[a - b]*\text{Sqrt}[a + b]*d) - (b*\text{ArcTanh}[\text{Sin}[c + d*x]])/(a^2*d) + \text{Tan}[c + d*x]/(a*d)$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 211

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

Rule 2738

`Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

Rule 2826

`Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]`

Rule 2881

`Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))`

Rule 3855

`Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\text{integral} = \frac{\tan(c + dx)}{ad} - \frac{\int \frac{b \sec(c+dx)}{a+b \cos(c+dx)} dx}{a}$$

$$\begin{aligned}
&= \frac{\tan(c+dx)}{ad} - \frac{b \int \frac{\sec(c+dx)}{a+b \cos(c+dx)} dx}{a} \\
&= \frac{\tan(c+dx)}{ad} - \frac{b \int \sec(c+dx) dx}{a^2} + \frac{b^2 \int \frac{1}{a+b \cos(c+dx)} dx}{a^2} \\
&= -\frac{\operatorname{barctanh}(\sin(c+dx))}{a^2 d} + \frac{\tan(c+dx)}{ad} + \frac{(2b^2) \operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2 d} \\
&= \frac{2b^2 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2 \sqrt{a-b} \sqrt{a+b} d} - \frac{\operatorname{barctanh}(\sin(c+dx))}{a^2 d} + \frac{\tan(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

$$\begin{aligned}
&\int \frac{\sec^2(c+dx)}{a+b \cos(c+dx)} dx \\
&= \frac{2b^2 \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{b \left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{a^2 d}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x]),x]

[Out] ((-2*b^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a*Tan[c + d*x])/(a^2*d)

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.45

method	result
derivativedivides	$ \frac{-\frac{1}{a \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{a^2} + \frac{2b^2 \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^2 \sqrt{(a-b)(a+b)}}}{d} - \frac{1}{a \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^2} $
default	$ \frac{-\frac{1}{a \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{a^2} + \frac{2b^2 \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^2 \sqrt{(a-b)(a+b)}}}{d} - \frac{1}{a \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^2} $
risch	$ \frac{2i}{da(e^{2i(dx+c)}+1)} + \frac{b \ln(e^{i(dx+c)}-i)}{a^2 d} - \frac{b^2 \ln\left(e^{i(dx+c)} + \frac{ia^2 - ib^2 + a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2} da^2} + \frac{b^2 \ln\left(e^{i(dx+c)} - \frac{ia^2 - ib^2 - a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2} da^2} $

[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] $1/d*(-1/a/(\tan(1/2*d*x+1/2*c)+1)-b/a^2*\ln(\tan(1/2*d*x+1/2*c)+1)+2*b^2/a^2/((a-b)*(a+b))^{(1/2)*\arctan((a-b)*\tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^{(1/2)})-1/a/(\tan(1/2*d*x+1/2*c)-1)+b/a^2*\ln(\tan(1/2*d*x+1/2*c)-1))$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(76) = 152$.

Time = 0.31 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.49

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)} dx = \left[-\frac{\sqrt{-a^2+b^2}b^2 \cos(dx+c) \log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right) + (c)}{2(a^4} \right.$$

[In] `integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fricas")`

[Out] $[-1/2*(\sqrt{-a^2+b^2})*b^2*\cos(d*x+c)*\log((2*a*b*\cos(d*x+c)+(2*a^2-b^2)*\cos(d*x+c)^2+2*\sqrt{-a^2+b^2}*(a*\cos(d*x+c)+b)*\sin(d*x+c)-a^2+2*b^2)/(b^2*\cos(d*x+c)^2+2*a*b*\cos(d*x+c)+a^2))+ (a^2*b-b^3)*\cos(d*x+c)*\log(\sin(d*x+c)+1)-(a^2*b-b^3)*\cos(d*x+c)*\log(-\sin(d*x+c)+1)-2*(a^3-a*b^2)*\sin(d*x+c)/((a^4-a^2*b^2)*d*\cos(d*x+c)), 1/2*(2*\sqrt{a^2-b^2})*b^2*\arctan(-(a*\cos(d*x+c)+b)/(\sqrt{a^2-b^2}*\sin(d*x+c)))*\cos(d*x+c)-(a^2*b-b^3)*\cos(d*x+c)*\log(\sin(d*x+c)+1)+(a^2*b-b^3)*\cos(d*x+c)*\log(-\sin(d*x+c)+1)+2*(a^3-a*b^2)*\sin(d*x+c)/((a^4-a^2*b^2)*d*\cos(d*x+c))]$

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)} dx = \int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)} dx$$

[In] `integrate(sec(d*x+c)**2/(a+b*cos(d*x+c)),x)`

[Out] `Integral(sec(c+d*x)**2/(a+b*cos(c+d*x)), x)`

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c + dx)}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(76) = 152.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

$$\int \frac{\sec^2(c + dx)}{a + b \cos(c + dx)} dx = \frac{2 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}} \right) \right) b^2}{\sqrt{a^2 - b^2} a^2} + \frac{b \log(|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1|)}{a^2} - \frac{b \log(|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1|)}{a^2}$$

d

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] $-(2*(\pi*\text{floor}(1/2*(d*x + c)/\pi + 1/2)*\operatorname{sgn}(-2*a + 2*b) + \arctan(-(a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{a^2 - b^2}))*b^2/(\sqrt{a^2 - b^2}*a^2) + b*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^2 - b*\log(\operatorname{abs}(\tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*\tan(1/2*d*x + 1/2*c)/((\tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d$

Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.81

$$\int \frac{\sec^2(c + dx)}{a + b \cos(c + dx)} dx = \frac{a^3 \sin(c + dx) - a b^2 \sin(c + dx)}{a^2 d \cos(c + dx) (a^2 - b^2)}$$

$$2 a^2 b \operatorname{atanh} \left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)} \right) - 2 b^3 \operatorname{atanh} \left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)} \right) + 2 b^2 \operatorname{atanh} \left(\frac{a^5 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2} + 2 b^3 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) (b^2 - a^2)^3}{a^2 d (a^2 - b^2)} \right)$$

[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))),x)


```
[Out] (a^3*sin(c + d*x) - a*b^2*sin(c + d*x))/(a^2*d*cos(c + d*x)*(a^2 - b^2)) -
(2*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 2*b^3*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)) + 2*b^2*atanh((a^5*sin(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2) + 2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 2*b^5*sin(c/2 +
(d*x)/2)*(b^2 - a^2)^(1/2) + 3*a^2*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
) - a^3*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^4*b*sin(c/2 + (d*x)/2
*(b^2 - a^2)^(1/2)))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)^2))*(b^2 - a^2)^(1/2
)/(a^2*d*(a^2 - b^2))
```

3.457 $\int \frac{\sec^3(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	4918
Rubi [A] (verified)	4918
Mathematica [A] (verified)	4920
Maple [A] (verified)	4921
Fricas [A] (verification not implemented)	4921
Sympy [F]	4922
Maxima [F(-2)]	4922
Giac [A] (verification not implemented)	4922
Mupad [B] (verification not implemented)	4923

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{\sec^3(c+dx)}{a+b \cos(c+dx)} dx = -\frac{2b^3 \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^3 \sqrt{a-b} \sqrt{a+b}} + \frac{(a^2 + 2b^2) \operatorname{arctanh}(\sin(c+dx))}{2a^3 d} - \frac{b \tan(c+dx)}{a^2 d} + \frac{\sec(c+dx) \tan(c+dx)}{2ad}$$

[Out] $1/2*(a^2+2*b^2)*\operatorname{arctanh}(\sin(d*x+c))/a^3/d-2*b^3*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a^3/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}-b*\tan(d*x+c)/a^2/d+1/2*\sec(d*x+c)*\tan(d*x+c)/a/d$

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2881, 3134, 3080, 3855, 2738, 211}

$$\int \frac{\sec^3(c+dx)}{a+b \cos(c+dx)} dx = -\frac{2b^3 \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^3 d \sqrt{a-b} \sqrt{a+b}} - \frac{b \tan(c+dx)}{a^2 d} + \frac{(a^2 + 2b^2) \operatorname{arctanh}(\sin(c+dx))}{2a^3 d} + \frac{\tan(c+dx) \sec(c+dx)}{2ad}$$

[In] $\text{Int}[\text{Sec}[c + d*x]^3/(a + b*\text{Cos}[c + d*x]), x]$

[Out] $(-2*b^3*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[(c + d*x)/2])/(\text{Sqrt}[a + b])]/(a^3*\text{Sqrt}[a - b]*\text{Sqrt}[a + b]*d) + ((a^2 + 2*b^2)*\text{ArcTanh}[\text{Sin}[c + d*x]])/(2*a^3*d) - (b*\text{Tan}[c + d*x])/(a^2*d) + (\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/(2*a*d)$

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3080

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[

n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{(-2b+a\cos(c+dx)+b\cos^2(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)} dx}{2a} \\
 &= -\frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{(a^2+2b^2+ab\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)} dx}{2a^2} \\
 &= -\frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} - \frac{b^3 \int \frac{1}{a+b\cos(c+dx)} dx}{a^3} + \frac{(a^2+2b^2) \int \sec(c+dx) dx}{2a^3} \\
 &= \frac{(a^2+2b^2)\operatorname{arctanh}(\sin(c+dx))}{2a^3d} - \frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} \\
 &\quad - \frac{(2b^3)\operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d} \\
 &= -\frac{2b^3\operatorname{arctan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}} + \frac{(a^2+2b^2)\operatorname{arctanh}(\sin(c+dx))}{2a^3d} \\
 &\quad - \frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.00

$$\begin{aligned}
 &\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)} dx \\
 &= \frac{sb^3\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - 2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - 4b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \sin
 \end{aligned}$$

[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x]),x]

[Out] ((8*b^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*a*b*Tan[c + d*x])/(4*a^3*d)

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{2b^3 \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3 \sqrt{(a-b)(a+b)}} - \frac{1}{2a\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{-a-2b}{2a^2\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{(a^2+2b^2)\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2a^3} + \frac{1}{2a\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{-a+2b}{2a^2\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{(a^2+2b^2)\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2a^3}$
default	$\frac{2b^3 \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{a^3 \sqrt{(a-b)(a+b)}} - \frac{1}{2a\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} - \frac{-a-2b}{2a^2\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{(a^2+2b^2)\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{2a^3} + \frac{1}{2a\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{-a+2b}{2a^2\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{(a^2+2b^2)\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2a^3}$
risch	$-\frac{i(ae^{3i(dx+c)} + 2be^{2i(dx+c)} - ae^{i(dx+c)} + 2b)}{da^2(e^{2i(dx+c)} + 1)^2} - \frac{\ln(e^{i(dx+c)} - i)}{2da} - \frac{\ln(e^{i(dx+c)} - i)b^2}{a^3d} + \frac{\ln(e^{i(dx+c)} + i)}{2ad} + \frac{\ln(e^{i(dx+c)} + i)b^2}{a^3d}$

[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] 1/d*(-2*b^3/a^3/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/2/a/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-a-2*b)/a^2/(tan(1/2*d*x+1/2*c)+1)+1/2*(a^2+2*b^2)/a^3*ln(tan(1/2*d*x+1/2*c)+1)+1/2/a/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-a-2*b)/a^2/(tan(1/2*d*x+1/2*c)-1)+1/2/a^3*(-a^2-2*b^2)*ln(tan(1/2*d*x+1/2*c)-1))

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.86

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)} dx$$

$$= \left[\frac{2\sqrt{-a^2+b^2}b^3 \cos(dx+c)^2 \log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)}{4\sqrt{a^2-b^2}b^3 \arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right) \cos(dx+c)^2 - (a^4+a^2b^2-2b^4)\cos(dx+c)^2 \log(\sin(dx+c))} \right] - 4(a^5 \dots)$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] [-1/4*(2*sqrt(-a^2+b^2)*b^3*cos(d*x+c)^2*log((2*a*b*cos(d*x+c)+(2*a^2-b^2)*cos(d*x+c)^2-2*sqrt(-a^2+b^2)*(a*cos(d*x+c)+b)*sin(d*x+c)-a^2+2*b^2)/(b^2*cos(d*x+c)^2+2*a*b*cos(d*x+c)+a^2))- (a^4+a^2*b^2-2*b^4)*cos(d*x+c)^2*log(sin(d*x+c)+1)+(a^4+a^2*b^2-2*b^4)*cos(d*x+c)^2*log(-sin(d*x+c)+1)-2*(a^4-a^2*b^2-2*(a^3*b

- a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2), -
 1/4*(4*sqrt(a^2 - b^2)*b^3*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (a^4 + a^2*b^2 - 2*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^4 + a^2*b^2 - 2*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(a^4 - a^2*b^2 - 2*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2)]

Sympy [F]

$$\int \frac{\sec^3(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\sec^3(c + dx)}{a + b \cos(c + dx)} dx$$

[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c)),x)

[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c + dx)}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.77

$$\int \frac{\sec^3(c + dx)}{a + b \cos(c + dx)} dx$$

$$= \frac{4 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}} \right) \right) b^3}{\sqrt{a^2 - b^2} a^3} + \frac{(a^2 + 2b^2) \log\left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1 \right|\right)}{a^3} - \frac{(a^2 + 2b^2) \log\left(\left| \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1 \right|\right)}{a^3}$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac")

3.458 $\int \frac{\sec^4(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	4924
Rubi [A] (verified)	4924
Mathematica [A] (verified)	4927
Maple [A] (verified)	4927
Fricas [A] (verification not implemented)	4928
Sympy [F]	4928
Maxima [F(-2)]	4929
Giac [B] (verification not implemented)	4929
Mupad [B] (verification not implemented)	4930

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{\sec^4(c+dx)}{a+b \cos(c+dx)} dx = \frac{2b^4 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4 \sqrt{a-b} \sqrt{a+b}} - \frac{b(a^2+2b^2) \operatorname{arctanh}(\sin(c+dx))}{2a^4 d} + \frac{(2a^2+3b^2) \tan(c+dx)}{3a^3 d} - \frac{b \sec(c+dx) \tan(c+dx)}{2a^2 d} + \frac{\sec^2(c+dx) \tan(c+dx)}{3ad}$$

[Out] $-1/2*b*(a^2+2*b^2)*\operatorname{arctanh}(\sin(dx+c))/a^4/d+2*b^4*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2}))/a^4/d/(a-b)^{(1/2)}/(a+b)^{(1/2)}+1/3*(2*a^2+3*b^2)*\tan(dx+c)/a^3/d-1/2*b*\sec(dx+c)*\tan(dx+c)/a^2/d+1/3*\sec(dx+c)^2*\tan(dx+c)/a/d$

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2881, 3134, 3080, 3855, 2738, 211}

$$\int \frac{\sec^4(c+dx)}{a+b \cos(c+dx)} dx = \frac{2b^4 \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4 d \sqrt{a-b} \sqrt{a+b}} - \frac{b \tan(c+dx) \sec(c+dx)}{2a^2 d} - \frac{b(a^2+2b^2) \operatorname{arctanh}(\sin(c+dx))}{2a^4 d} + \frac{(2a^2+3b^2) \tan(c+dx)}{3a^3 d} + \frac{\tan(c+dx) \sec^2(c+dx)}{3ad}$$

[In] $\text{Int}[\text{Sec}[c+d*x]^4/(a+b*\text{Cos}[c+d*x]),x]$

[Out] $(2b^4 \operatorname{ArcTan}[\sqrt{a-b} \tan[(c+dx)/2]] / \sqrt{a+b}) / (a^4 \sqrt{a-b} \sqrt{a+b} d - (b(a^2 + 2b^2) \operatorname{ArcTanh}[\sin[c+dx]]) / (2a^4 d) + ((2a^2 + 3b^2) \tan[c+dx]) / (3a^3 d) - (b \sec[c+dx] \tan[c+dx]) / (2a^2 d) + (\sec[c+dx]^2 \tan[c+dx]) / (3a d))$

Rule 211

$\operatorname{Int}[(a_ + (b_)(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2]/a) \operatorname{ArcTan}[x/\operatorname{Rt}[a/b, 2]], x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{PosQ}[a/b]$

Rule 2738

$\operatorname{Int}[(a_ + (b_)\sin[\pi/2 + (c_ + (d_)(x_)])^{-1}, x_Symbol] \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\tan[(c+dx)/2], x]\}, \operatorname{Dist}[2(e/d), \operatorname{Subst}[\operatorname{Int}[1/(a+b+(a-b)e^2x^2)], x], x, \tan[(c+dx)/2]/e], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

Rule 2881

$\operatorname{Int}[(a_ + (b_)\sin[(e_ + (f_)(x_)])^{(m_)}((c_ + (d_)\sin[(e_ + (f_)(x_)])^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[(-b^2)\cos[e+fx](a+b\sin[e+fx])^{(m+1)}((c+d\sin[e+fx])^{(n+1)})/(f(m+1)(b*c-a*d)(a^2-b^2))], x] + \operatorname{Dist}[1/((m+1)(b*c-a*d)(a^2-b^2)), \operatorname{Int}[(a+b\sin[e+fx])^{(m+1)}(c+d\sin[e+fx])^n \operatorname{Simp}[a(b*c-a*d)(m+1)+b^2*d(m+n+2)-(b^2*c+b(b*c-a*d)(m+1))\sin[e+fx]-b^2*d(m+n+3)\sin[e+fx]^2, x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n\}, x] \ \&\& \operatorname{NeQ}[b*c-a*d, 0] \ \&\& \operatorname{NeQ}[a^2-b^2, 0] \ \&\& \operatorname{NeQ}[c^2-d^2, 0] \ \&\& \operatorname{LtQ}[m, -1] \ \&\& \operatorname{IntegersQ}[2*m, 2*n] \ \&\& ((\operatorname{EqQ}[a, 0] \ \&\& \operatorname{IntegerQ}[m] \ \&\& \operatorname{!IntegerQ}[n]) \ || \ \operatorname{!(IntegerQ}[2*n] \ \&\& \operatorname{LtQ}[n, -1] \ \&\& ((\operatorname{IntegerQ}[n] \ \&\& \operatorname{!IntegerQ}[m]) \ || \ \operatorname{EqQ}[a, 0]))]$

Rule 3080

$\operatorname{Int}[(A_ + (B_)\sin[(e_ + (f_)(x_)])/((A_ + (b_)\sin[(e_ + (f_)(x_)])) * ((c_ + (d_)\sin[(e_ + (f_)(x_)]))), x_Symbol] \rightarrow \operatorname{Dist}[(A*b - a*B)/(b*c - a*d), \operatorname{Int}[1/(a+b\sin[e+fx]), x], x] + \operatorname{Dist}[(B*c - A*d)/(b*c - a*d), \operatorname{Int}[1/(c+d\sin[e+fx]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{NeQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 3134

$\operatorname{Int}[(a_ + (b_)\sin[(e_ + (f_)(x_)])^{(m_)}((c_ + (d_)\sin[(e_ + (f_)(x_)] + (A_ + (B_)\sin[(e_ + (f_)(x_)] + (C_)\sin[(e_ + (f_)(x_)]^2), x_Symbol] \rightarrow \operatorname{Simp}[(-A*b^2 - a*b*B + a^2*C)\cos[e+fx](a+b\sin[e+fx])^{(m+1)}((c+d\sin[e+fx])^{(n+1)})/(f(m+1)(b*c-a*d)(a^2-b^2))], x] + \operatorname{Dist}[1/((m+1)(b*c-a*d)(a^2-b^2)), \operatorname{Int}[(a+b\sin[e+fx])^{(m+1)}(c+d\sin[e+fx])^n \operatorname{Simp}[(m+1)(b*c-a*d)(a*A - b*B + a*C) + d(A*b^2 - a*b*B + a^2*C)(m+n+2) - (c(A*b^2 - a$

*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sec^2(c + dx) \tan(c + dx)}{3ad} + \frac{\int \frac{(-3b+2a \cos(c+dx)+2b \cos^2(c+dx)) \sec^3(c+dx)}{a+b \cos(c+dx)} dx}{3a} \\
 &= -\frac{b \sec(c + dx) \tan(c + dx)}{2a^2d} + \frac{\sec^2(c + dx) \tan(c + dx)}{3ad} \\
 &\quad + \frac{\int \frac{(2(2a^2+3b^2)+ab \cos(c+dx)-3b^2 \cos^2(c+dx)) \sec^2(c+dx)}{a+b \cos(c+dx)} dx}{6a^2} \\
 &= \frac{(2a^2 + 3b^2) \tan(c + dx)}{3a^3d} - \frac{b \sec(c + dx) \tan(c + dx)}{2a^2d} \\
 &\quad + \frac{\sec^2(c + dx) \tan(c + dx)}{3ad} + \frac{\int \frac{(-3b(a^2+2b^2)-3ab^2 \cos(c+dx)) \sec(c+dx)}{a+b \cos(c+dx)} dx}{6a^3} \\
 &= \frac{(2a^2 + 3b^2) \tan(c + dx)}{3a^3d} - \frac{b \sec(c + dx) \tan(c + dx)}{2a^2d} + \frac{\sec^2(c + dx) \tan(c + dx)}{3ad} \\
 &\quad + \frac{b^4 \int \frac{1}{a+b \cos(c+dx)} dx}{a^4} - \frac{(b(a^2 + 2b^2)) \int \sec(c + dx) dx}{2a^4} \\
 &= -\frac{b(a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{2a^4d} + \frac{(2a^2 + 3b^2) \tan(c + dx)}{3a^3d} - \frac{b \sec(c + dx) \tan(c + dx)}{2a^2d} \\
 &\quad + \frac{\sec^2(c + dx) \tan(c + dx)}{3ad} + \frac{(2b^4) \operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{a^4d} \\
 &= \frac{2b^4 \operatorname{arctan}\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}} - \frac{b(a^2 + 2b^2) \operatorname{arctanh}(\sin(c + dx))}{2a^4d} \\
 &\quad + \frac{(2a^2 + 3b^2) \tan(c + dx)}{3a^3d} - \frac{b \sec(c + dx) \tan(c + dx)}{2a^2d} + \frac{\sec^2(c + dx) \tan(c + dx)}{3ad}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.64

$$\int \frac{\sec^4(c + dx)}{a + b \cos(c + dx)} dx$$

$$= -\frac{24b^4 \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{1}{2} \sec^3(c + dx) (9b(a^2 + 2b^2) \cos(c + dx) (\log(\cos(\frac{1}{2}(c + dx))) - \sin(\frac{1}{2}(c + dx))))$$

[In] Integrate[Sec[c + d*x]^4/(a + b*Cos[c + d*x]),x]

[Out] $((-24*b^4*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (Sec[c + d*x]^3*(9*b*(a^2 + 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*b*(a^2 + 2*b^2)*Cos[3*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*a*(4*a^2 + 3*b^2 - 3*a*b*Cos[c + d*x] + (2*a^2 + 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/2)/(12*a^4*d)$

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.61

method	result
derivativedivides	$-\frac{1}{3a(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)^3} - \frac{-a-b}{2a^2(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)^2} - \frac{2a^2+ab+2b^2}{2a^3(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)} - \frac{b(a^2+2b^2) \ln(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)}{2a^4} + \frac{2b^4 \operatorname{arctan}\left(\frac{(a-b) \tan(\frac{dx}{2} + \frac{c}{2})}{\sqrt{(a-b)(a+b)}}\right)}{a^4 \sqrt{(a-b)(a+b)}}$
default	$-\frac{1}{3a(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)^3} - \frac{-a-b}{2a^2(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)^2} - \frac{2a^2+ab+2b^2}{2a^3(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)} - \frac{b(a^2+2b^2) \ln(\tan(\frac{dx}{2} + \frac{c}{2}) + 1)}{2a^4} + \frac{2b^4 \operatorname{arctan}\left(\frac{(a-b) \tan(\frac{dx}{2} + \frac{c}{2})}{\sqrt{(a-b)(a+b)}}\right)}{a^4 \sqrt{(a-b)(a+b)}}$
risch	$\frac{i(3abe^{5i(dx+c)} + 6b^2e^{4i(dx+c)} + 12a^2e^{2i(dx+c)} + 12b^2e^{2i(dx+c)} - 3abe^{i(dx+c)} + 4a^2 + 6b^2)}{3a^3d(e^{2i(dx+c)} + 1)^3} + \frac{b \ln(e^{i(dx+c)} - i)}{2a^2d} + \frac{b^3 \ln(e^{i(dx+c)} + i)}{2a^2d}$

[In] int(sec(d*x+c)^4/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] $1/d*(-1/3/a/(\tan(1/2*d*x+1/2*c)+1)^3-1/2*(-a-b)/a^2/(\tan(1/2*d*x+1/2*c)+1)^2-1/2*(2*a^2+a*b+2*b^2)/a^3/(\tan(1/2*d*x+1/2*c)+1)-1/2*b*(a^2+2*b^2)/a^4*\ln(\tan(1/2*d*x+1/2*c)+1)+2*b^4/a^4/((a-b)*(a+b))^(1/2)*\operatorname{arctan}((a-b)*\tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/3/a/(\tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)/a^2/(\tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*a^2+a*b+2*b^2)/a^3/(\tan(1/2*d*x+1/2*c)-1)+1/2*b*(a^2+2*b^2)/a^4*\ln(\tan(1/2*d*x+1/2*c)-1))$

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.41

$$\int \frac{\sec^4(c + dx)}{a + b \cos(c + dx)} dx$$

$$= \left[\frac{6 \sqrt{-a^2 + b^2} b^4 \cos(dx + c)^3 \log\left(\frac{2ab \cos(dx+c) + (2a^2 - b^2) \cos(dx+c)^2 + 2\sqrt{-a^2 + b^2}(a \cos(dx+c) + b) \sin(dx+c) - a^2 + 2b^2}{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}\right) + 3\left(a^4 b + a^2 b^3 - 2b^5\right) \cos(dx+c)^3 \log(\sin(dx+c) + 1) - 3\left(a^4 b + a^2 b^3 - 2b^5\right) \cos(dx+c)^3 \log(-\sin(dx+c) + 1) - 2\left(2a^5 - 2a^3 b^2 + 2\left(2a^5 + a^3 b^2 - 3a b^4\right) \cos(dx+c)^2 - 3\left(a^4 b - a^2 b^3\right) \cos(dx+c)\right) \sin(dx+c)}{\left(a^6 - a^4 b^2\right) d \cos(dx+c)^3}, \frac{1}{12} \left(12 \sqrt{a^2 - b^2} b^4 \arctan\left(-\frac{a \cos(dx+c) + b}{\sqrt{a^2 - b^2} \sin(dx+c)}\right) \cos(dx+c)^3 - 3\left(a^4 b + a^2 b^3 - 2b^5\right) \cos(dx+c)^3 \log(\sin(dx+c) + 1) + 3\left(a^4 b + a^2 b^3 - 2b^5\right) \cos(dx+c)^3 \log(-\sin(dx+c) + 1) + 2\left(2a^5 - 2a^3 b^2 + 2\left(2a^5 + a^3 b^2 - 3a b^4\right) \cos(dx+c)^2 - 3\left(a^4 b - a^2 b^3\right) \cos(dx+c)\right) \sin(dx+c)}{\left(a^6 - a^4 b^2\right) d \cos(dx+c)^3} \right]$$

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*sqrt(-a^2 + b^2)*b^4*cos(d*x + c)^3*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*a^5 - 2*a^3*b^2 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c)/((a^6 - a^4*b^2)*d*cos(d*x + c)^3), 1/12*(12*sqrt(a^2 - b^2)*b^4*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) *cos(d*x + c)^3 - 3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*a^5 - 2*a^3*b^2 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c)/((a^6 - a^4*b^2)*d*cos(d*x + c)^3)]
```

Sympy [F]

$$\int \frac{\sec^4(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\sec^4(c + dx)}{a + b \cos(c + dx)} dx$$

```
[In] integrate(sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*cos(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c + dx)}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(140) = 280.

Time = 0.34 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.82

$$\int \frac{\sec^4(c + dx)}{a + b \cos(c + dx)} dx = \frac{12 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}} \right) \right) b^4}{\sqrt{a^2 - b^2} a^4} + \frac{3(a^2 b + 2b^3) \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) + 1|)}{a^4} - \frac{3(a^2 b + 2b^3) \log(|\tan(\frac{1}{2} dx + \frac{1}{2} c) - 1|)}{a^4}$$

[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out]
$$-1/6*(12*(\pi*\text{floor}(1/2*(d*x + c)/\pi + 1/2)*\text{sgn}(-2*a + 2*b) + \arctan(-(a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{a^2 - b^2}))) * b^4 / (\sqrt{a^2 - b^2} * a^4) + 3*(a^2*b + 2*b^3) * \log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) / a^4 - 3*(a^2*b + 2*b^3) * \log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) / a^4 + 2*(6*a^2*\tan(1/2*d*x + 1/2*c)^5 + 3*a*b*\tan(1/2*d*x + 1/2*c)^5 + 6*b^2*\tan(1/2*d*x + 1/2*c)^5 - 4*a^2*\tan(1/2*d*x + 1/2*c)^3 - 12*b^2*\tan(1/2*d*x + 1/2*c)^3 + 6*a^2*\tan(1/2*d*x + 1/2*c) - 3*a*b*\tan(1/2*d*x + 1/2*c) + 6*b^2*\tan(1/2*d*x + 1/2*c)) / ((\tan(1/2*d*x + 1/2*c)^2 - 1)^3 * a^3) / d$$

Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 991, normalized size of antiderivative = 6.31

$$\int \frac{\sec^4(c + dx)}{a + b \cos(c + dx)} dx = \text{Too large to display}$$

```
[In] int(1/(cos(c + d*x)^4*(a + b*cos(c + d*x))),x)
```

```
[Out] (a^5*(sin(c + d*x)/2 + sin(3*c + 3*d*x)/6) - a^4*((b*sin(2*c + 2*d*x))/4 +
(b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4 + (3*b*
cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 - a^2*((3*b^3
*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 - (b^3*sin(2*
c + 2*d*x))/4 + (b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c +
3*d*x))/4) - a^3*((b^2*sin(c + d*x))/4 - (b^2*sin(3*c + 3*d*x))/12) - a*((
b^4*sin(c + d*x))/4 + (b^4*sin(3*c + 3*d*x))/4) + (3*b^5*cos(c + d*x)*atanh
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (b^5*atanh(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (3*b^4*atanh((a^9*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*b^9*
sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^7*sin(c/2 + (d*x)/2)*(b^2 -
a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3*a^5*b^4*sin
(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6*b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2
)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^8*b*sin(c/2 +
(d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(a^7 - 3*a^3*
b^4 + 2*a^5*b^2)))*cos(c + d*x)*(b^2 - a^2)^(1/2))/2 + (b^4*atanh((a^9*sin(
c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*b^7*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/
2) - 8*b^9*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^7*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) + 3*a^4*b^5*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 3*
a^5*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 2*a^6*b^3*sin(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2) + 2*a^7*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^8*b
*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a*b^2 - a^3)*(a
^7 - 3*a^3*b^4 + 2*a^5*b^2)))*cos(3*c + 3*d*x)*(b^2 - a^2)^(1/2))/2)/(a^4*d
*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^2 - b^2))
```

$$3.459 \quad \int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	4931
Rubi [A] (verified)	4932
Mathematica [C] (verified)	4935
Maple [A] (verified)	4935
Fricas [A] (verification not implemented)	4936
Sympy [F(-1)]	4936
Maxima [F(-2)]	4937
Giac [A] (verification not implemented)	4937
Mupad [B] (verification not implemented)	4938

Optimal result

Integrand size = 21, antiderivative size = 266

$$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^2} dx = -\frac{a(4a^2+b^2)x}{b^5} + \frac{2a^4(4a^2-5b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2} b^5 (a+b)^{3/2} d}$$

$$+ \frac{(12a^4 - 7a^2b^2 - 2b^4) \sin(c+dx)}{3b^4(a^2 - b^2)d}$$

$$- \frac{a(2a^2 - b^2) \cos(c+dx) \sin(c+dx)}{b^3(a^2 - b^2)d}$$

$$+ \frac{(4a^2 - b^2) \cos^2(c+dx) \sin(c+dx)}{3b^2(a^2 - b^2)d}$$

$$- \frac{a^2 \cos^3(c+dx) \sin(c+dx)}{b(a^2 - b^2)d(a+b \cos(c+dx))}$$

```
[Out] -a*(4*a^2+b^2)*x/b^5+2*a^4*(4*a^2-5*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2
*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^5/(a+b)^(3/2)/d+1/3*(12*a^4-7*a^2*b^2-2*b^4)
*sin(d*x+c)/b^4/(a^2-b^2)/d-a*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/b^3/(a^2-b^
2)/d+1/3*(4*a^2-b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(a^2-b^2)/d-a^2*cos(d*x+c)
^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2871, 3128, 3102, 2814, 2738, 211}

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2} dx = -\frac{a^2 \sin(c+dx) \cos^3(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))} + \frac{(4a^2-b^2) \sin(c+dx) \cos^2(c+dx)}{3b^2d(a^2-b^2)} - \frac{ax(4a^2+b^2)}{b^5} - \frac{a(2a^2-b^2) \sin(c+dx) \cos(c+dx)}{b^3d(a^2-b^2)} + \frac{2a^4(4a^2-5b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{b^5d(a-b)^{3/2}(a+b)^{3/2}} + \frac{(12a^4-7a^2b^2-2b^4) \sin(c+dx)}{3b^4d(a^2-b^2)}$$

[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^2,x]

[Out] -((a*(4*a^2 + b^2)*x)/b^5) + (2*a^4*(4*a^2 - 5*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^5*(a + b)^(3/2)*d) + ((12*a^4 - 7*a^2*b^2 - 2*b^4)*Sin[c + d*x])/(3*b^4*(a^2 - b^2)*d) - (a*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((4*a^2 - b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2814

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*SIN[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2871


```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
+ a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])

```

Rule 3102

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]

```

Rule 3128

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2 \cos^3(c + dx) \sin(c + dx)}{b(a^2 - b^2) d(a + b \cos(c + dx))} - \frac{\int \frac{\cos^2(c + dx)(3a^2 - ab \cos(c + dx) - (4a^2 - b^2) \cos^2(c + dx))}{a + b \cos(c + dx)} dx}{b(a^2 - b^2)} \\
&= \frac{(4a^2 - b^2) \cos^2(c + dx) \sin(c + dx)}{3b^2(a^2 - b^2) d} - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{b(a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad - \frac{\int \frac{\cos(c + dx)(-2a(4a^2 - b^2) + b(a^2 + 2b^2) \cos(c + dx) + 6a(2a^2 - b^2) \cos^2(c + dx))}{a + b \cos(c + dx)} dx}{3b^2(a^2 - b^2)}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{a(2a^2 - b^2) \cos(c + dx) \sin(c + dx)}{b^3 (a^2 - b^2) d} \\
&\quad + \frac{(4a^2 - b^2) \cos^2(c + dx) \sin(c + dx)}{3b^2 (a^2 - b^2) d} - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{b (a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad - \frac{\int \frac{6a^2(2a^2 - b^2) - 2ab(2a^2 + b^2) \cos(c + dx) - 2(12a^4 - 7a^2b^2 - 2b^4) \cos^2(c + dx)}{a + b \cos(c + dx)} dx}{6b^3 (a^2 - b^2)} \\
&= \frac{(12a^4 - 7a^2b^2 - 2b^4) \sin(c + dx)}{3b^4 (a^2 - b^2) d} - \frac{a(2a^2 - b^2) \cos(c + dx) \sin(c + dx)}{b^3 (a^2 - b^2) d} \\
&\quad + \frac{(4a^2 - b^2) \cos^2(c + dx) \sin(c + dx)}{3b^2 (a^2 - b^2) d} - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{b (a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad - \frac{\int \frac{6a^2b(2a^2 - b^2) + 6a(a^2 - b^2)(4a^2 + b^2) \cos(c + dx)}{a + b \cos(c + dx)} dx}{6b^4 (a^2 - b^2)} \\
&= -\frac{a(4a^2 + b^2) x}{b^5} + \frac{(12a^4 - 7a^2b^2 - 2b^4) \sin(c + dx)}{3b^4 (a^2 - b^2) d} \\
&\quad - \frac{a(2a^2 - b^2) \cos(c + dx) \sin(c + dx)}{b^3 (a^2 - b^2) d} + \frac{(4a^2 - b^2) \cos^2(c + dx) \sin(c + dx)}{3b^2 (a^2 - b^2) d} \\
&\quad - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{b (a^2 - b^2) d(a + b \cos(c + dx))} + \frac{(a^4(4a^2 - 5b^2)) \int \frac{1}{a + b \cos(c + dx)} dx}{b^5 (a^2 - b^2)} \\
&= -\frac{a(4a^2 + b^2) x}{b^5} + \frac{(12a^4 - 7a^2b^2 - 2b^4) \sin(c + dx)}{3b^4 (a^2 - b^2) d} \\
&\quad - \frac{a(2a^2 - b^2) \cos(c + dx) \sin(c + dx)}{b^3 (a^2 - b^2) d} \\
&\quad + \frac{(4a^2 - b^2) \cos^2(c + dx) \sin(c + dx)}{3b^2 (a^2 - b^2) d} - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{b (a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad + \frac{(2a^4(4a^2 - 5b^2)) \text{Subst}\left(\int \frac{1}{a + b + (a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{b^5 (a^2 - b^2) d} \\
&= -\frac{a(4a^2 + b^2) x}{b^5} + \frac{2a^4(4a^2 - 5b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2} b^5 (a+b)^{3/2} d} \\
&\quad + \frac{(12a^4 - 7a^2b^2 - 2b^4) \sin(c + dx)}{3b^4 (a^2 - b^2) d} - \frac{a(2a^2 - b^2) \cos(c + dx) \sin(c + dx)}{b^3 (a^2 - b^2) d} \\
&\quad + \frac{(4a^2 - b^2) \cos^2(c + dx) \sin(c + dx)}{3b^2 (a^2 - b^2) d} - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{b (a^2 - b^2) d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.66

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2} dx = \frac{-12a(2a-ib)(2a+ib)(c+dx) + \frac{24a^4(4a^2-5b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} + 9b(4a^2+b^2)\sin(c+dx) + \frac{12b^5d}{(a-b)^2}}{12b^5d}$$

[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^2,x]

[Out] (-12*a*(2*a - I*b)*(2*a + I*b)*(c + d*x) + (24*a^4*(4*a^2 - 5*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 9*b*(4*a^2 + b^2)*Sin[c + d*x] + (12*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) - 6*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)]/(12*b^5*d)

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{2a^4 \left(\frac{ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2-b^2)\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+a+b} + \frac{(4a^2-5b^2) \operatorname{arctan}\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}} \right)}{b^5} - \frac{2 \left(\frac{-3a^2b-ab^2-b^3}{d} \right)}{d}$
default	$\frac{2a^4 \left(\frac{ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2-b^2)\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+a+b} + \frac{(4a^2-5b^2) \operatorname{arctan}\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}} \right)}{b^5} - \frac{2 \left(\frac{-3a^2b-ab^2-b^3}{d} \right)}{d}$
risch	$-\frac{4a^3x}{b^5} - \frac{ax}{b^3} + \frac{iae^{2i(dx+c)}}{4b^3d} - \frac{3ie^{i(dx+c)}a^2}{2b^4d} - \frac{3ie^{i(dx+c)}}{8b^2d} + \frac{3ie^{-i(dx+c)}a^2}{2b^4d} + \frac{3ie^{-i(dx+c)}}{8b^2d} - \frac{iae^{-2i(dx+c)}}{4b^3d}$

[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(2/b^5*a^4*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(4*a^2-5*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/b^5*(((3*a^2*b-a*b^2-b^3)*tan(1/2*d*x+1/2*c)^5+(-6*a^2*b-2/3*b^3)*tan(1/2*d*x+1/2*c)^3+(-3*a^2*b+a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^3+a*(4*a^2+b^2)*arctan(tan(1/2*d*x+1/2*c))))

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 747, normalized size of antiderivative = 2.81

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^2} dx$$

$$= \frac{6(4a^7b - 7a^5b^3 + 2a^3b^5 + ab^7)dx \cos(dx + c) + 6(4a^8 - 7a^6b^2 + 2a^4b^4 + a^2b^6)dx + 3(4a^7 - 5a^5b^2 + \dots)}{3(4a^7b - 7a^5b^3 + 2a^3b^5 + ab^7)dx \cos(dx + c) + 3(4a^8 - 7a^6b^2 + 2a^4b^4 + a^2b^6)dx - 3(4a^7 - 5a^5b^2 + \dots)}$$

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(4*a^7*b - 7*a^5*b^3 + 2*a^3*b^5 + a*b^7)*d*x*cos(d*x + c) + 6*(4*a^8 - 7*a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*d*x + 3*(4*a^7 - 5*a^5*b^2 + (4*a^6*b - 5*a^4*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(12*a^7*b - 19*a^5*b^3 + 5*a^3*b^5 + 2*a*b^7 + (a^4*b^4 - 2*a^2*b^6 + b^8)*cos(d*x + c)^3 - 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^2 + 2*(3*a^6*b^2 - 5*a^4*b^4 + a^2*b^6 + b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c) + (a^5*b^5 - 2*a^3*b^7 + a*b^9)*d), -1/3*(3*(4*a^7*b - 7*a^5*b^3 + 2*a^3*b^5 + a*b^7)*d*x*cos(d*x + c) + 3*(4*a^8 - 7*a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*d*x - 3*(4*a^7 - 5*a^5*b^2 + (4*a^6*b - 5*a^4*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (12*a^7*b - 19*a^5*b^3 + 5*a^3*b^5 + 2*a*b^7 + (a^4*b^4 - 2*a^2*b^6 + b^8)*cos(d*x + c)^3 - 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^2 + 2*(3*a^6*b^2 - 5*a^4*b^4 + a^2*b^6 + b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c) + (a^5*b^5 - 2*a^3*b^7 + a*b^9)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.25

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^2} dx$$

$$= \frac{6 a^5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{(a^2 b^4 - b^6) \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a + b\right)} - \frac{6 (4 a^6 - 5 a^4 b^2) \left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2 a + 2 b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^2 b^5 - b^7) \sqrt{a^2 - b^2}}$$

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^5*tan(1/2*d*x + 1/2*c)/((a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - 6*(4*a^6 - 5*a^4*b^2)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^5 - b^7)*sqrt(a^2 - b^2)) -
3*(4*a^3 + a*b^2)*(d*x + c)/b^5 + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a*b*t
an(1/2*d*x + 1/2*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*tan(1/2*d*x +
1/2*c)^3 + 2*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c) - 3*a
*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c
)^2 + 1)^3*b^4))/d
```

Mupad [B] (verification not implemented)

Time = 22.00 (sec) , antiderivative size = 3852, normalized size of antiderivative = 14.48

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Too large to display}$$

[In] int(cos(c + d*x)^5/(a + b*cos(c + d*x))^2,x)

[Out] - ((2*tan(c/2 + (d*x)/2)^3*(8*a*b^4 - 6*a^4*b - 36*a^5 - b^5 + 7*a^2*b^3 + 19*a^3*b^2))/(3*b^4*(a + b)*(a - b)) - (2*tan(c/2 + (d*x)/2)^7*(4*a^5 - 2*a^4*b + b^5 + a^2*b^3 - 3*a^3*b^2))/(b^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)^5*(8*a*b^4 + 6*a^4*b - 36*a^5 + b^5 - 7*a^2*b^3 + 19*a^3*b^2))/(3*b^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)*(b^5 - 4*a^5 - 2*a^4*b + a^2*b^3 + 3*a^3*b^2))/(b^4*(a + b)*(a - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^8*(a - b) + tan(c/2 + (d*x)/2)^2*(4*a + 2*b) + tan(c/2 + (d*x)/2)^6*(4*a - 2*b) + 6*a*tan(c/2 + (d*x)/2)^4) - (2*a*atan(((a*(4*a^2 + b^2))*((32*tan(c/2 + (d*x)/2)*(32*a^12 - 32*a^11*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^5 + 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2)))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (a*(4*a^2 + b^2))*((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 + 2*a^7*b^11 - 4*a^8*b^10)))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (a*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a*b^15 - 2*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10)*32i)/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8))) * i) / b^5) / b^5 + (a*(4*a^2 + b^2))*((32*tan(c/2 + (d*x)/2)*(32*a^12 - 32*a^11*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^5 + 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2)))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (a*(4*a^2 + b^2))*((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 + 2*a^7*b^11 - 4*a^8*b^10)))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (a*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a*b^15 - 2*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10)*32i)/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8))) * i) / b^5) / b^5) / ((64*(64*a^14 - 32*a^13*b + 5*a^6*b^8 - 5*a^7*b^7 + 31*a^8*b^6 - 6*a^9*b^5 + 12*a^10*b^4 + 48*a^11*b^3 - 112*a^12*b^2)))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (a*(4*a^2 + b^2))*((32*tan(c/2 + (d*x)/2)*(32*a^12 - 32*a^11*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^5 + 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2)))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (a*(4*a^2 + b^2))*((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 + 2*a^7*b^11 - 4*a^8*b^10)))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (a*tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a*b^15 - 2*a^2*b^14 - 4*a^3*b^13 + 4*a^4*b^12 + 2*a^5*b^11 - 2*a^6*b^10)*32i)/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8))) * i) / b^5) / b^5 + (a*(4*a^2 + b^2))*((32*tan(c/2 + (d*x)/2)*(32*a^12 - 32*a^11*b + a^2*b^10 - 2*a^3*b^9 + 7*a^4*b^8 - 12*a^5*b^7 + 7*a^6*b^6 - 2*a^7*b^5 + 2*a^8*b^4 + 48*a^9*b^3 - 48*a^10*b^2)))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (a*(4*a^2 + b^2))*((32*(a*b^17 + a^3*b^15 - 5*a^4*b^14 - 4*a^5*b^13 + 9*a^6*b^12 + 2*a^7*b^11 - 4*a^8*b^10)))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (

$$)^3(a - b)^3)^{(1/2)*2i)/(d*(b^{11} - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))$$

$$3.460 \quad \int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	4941
Rubi [A] (verified)	4941
Mathematica [A] (verified)	4944
Maple [A] (verified)	4944
Fricas [A] (verification not implemented)	4945
Sympy [F(-1)]	4945
Maxima [F(-2)]	4946
Giac [A] (verification not implemented)	4946
Mupad [B] (verification not implemented)	4946

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^2} dx = \frac{(6a^2+b^2)x}{2b^4} - \frac{2a^3(3a^2-4b^2) \operatorname{arctanh}\left(\frac{(a-b)\sin(c+dx)}{\sqrt{-a^2+b^2}(1+\cos(c+dx))}\right)}{b^4(-a^2+b^2)^{3/2}d}$$

$$- \frac{2a \sin(c+dx)}{b^3d} + \frac{\cos(c+dx) \sin(c+dx)}{2b^2d}$$

$$- \frac{a^4 \sin(c+dx)}{b^3(a^2-b^2)d(a+b \cos(c+dx))}$$

[Out] 1/2*(6*a^2+b^2)*x/b^4-2*a^3*(3*a^2-4*b^2)*arctanh((a-b)*sin(d*x+c)/(1+cos(d*x+c)))/(-a^2+b^2)^(1/2)/b^4/(-a^2+b^2)^(3/2)/d-2*a*sin(d*x+c)/b^3/d+1/2*cos(d*x+c)*sin(d*x+c)/b^2/d-a^4*sin(d*x+c)/b^3/(a^2-b^2)/d/(a+b*cos(d*x+c))

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2871, 3128, 3102, 2814, 2738, 211}

$$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^2} dx = -\frac{a^2 \sin(c+dx) \cos^2(c+dx)}{bd(a^2-b^2)(a+b \cos(c+dx))}$$

$$+ \frac{(3a^2-b^2) \sin(c+dx) \cos(c+dx)}{2b^2d(a^2-b^2)}$$

$$+ \frac{x(6a^2+b^2)}{2b^4} - \frac{a(3a^2-2b^2) \sin(c+dx)}{b^3d(a^2-b^2)}$$

$$- \frac{2a^3(3a^2-4b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}$$

[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]

[Out] ((6*a^2 + b^2)*x)/(2*b^4) - (2*a^3*(3*a^2 - 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) - (a*(3*a^2 - 2*b^2)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((3*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(-1), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2871

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3102

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)^2], x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]

&& !LtQ[m, -1]

Rule 3128

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{a^2 \cos^2(c + dx) \sin(c + dx)}{b(a^2 - b^2)d(a + b \cos(c + dx))} - \frac{\int \frac{\cos(c+dx)(2a^2 - ab \cos(c+dx) - (3a^2 - b^2) \cos^2(c+dx))}{a + b \cos(c+dx)} dx}{b(a^2 - b^2)} \\
 &= \frac{(3a^2 - b^2) \cos(c + dx) \sin(c + dx)}{2b^2(a^2 - b^2)d} - \frac{a^2 \cos^2(c + dx) \sin(c + dx)}{b(a^2 - b^2)d(a + b \cos(c + dx))} \\
 &\quad - \frac{\int \frac{-a(3a^2 - b^2) + b(a^2 + b^2) \cos(c+dx) + 2a(3a^2 - 2b^2) \cos^2(c+dx)}{a + b \cos(c+dx)} dx}{2b^2(a^2 - b^2)} \\
 &= -\frac{a(3a^2 - 2b^2) \sin(c + dx)}{b^3(a^2 - b^2)d} + \frac{(3a^2 - b^2) \cos(c + dx) \sin(c + dx)}{2b^2(a^2 - b^2)d} \\
 &\quad - \frac{a^2 \cos^2(c + dx) \sin(c + dx)}{b(a^2 - b^2)d(a + b \cos(c + dx))} - \frac{\int \frac{-ab(3a^2 - b^2) - (a^2 - b^2)(6a^2 + b^2) \cos(c+dx)}{a + b \cos(c+dx)} dx}{2b^3(a^2 - b^2)} \\
 &= \frac{(6a^2 + b^2)x}{2b^4} - \frac{a(3a^2 - 2b^2) \sin(c + dx)}{b^3(a^2 - b^2)d} + \frac{(3a^2 - b^2) \cos(c + dx) \sin(c + dx)}{2b^2(a^2 - b^2)d} \\
 &\quad - \frac{a^2 \cos^2(c + dx) \sin(c + dx)}{b(a^2 - b^2)d(a + b \cos(c + dx))} - \frac{(a^3(3a^2 - 4b^2)) \int \frac{1}{a + b \cos(c+dx)} dx}{b^4(a^2 - b^2)} \\
 &= \frac{(6a^2 + b^2)x}{2b^4} - \frac{a(3a^2 - 2b^2) \sin(c + dx)}{b^3(a^2 - b^2)d} \\
 &\quad + \frac{(3a^2 - b^2) \cos(c + dx) \sin(c + dx)}{2b^2(a^2 - b^2)d} - \frac{a^2 \cos^2(c + dx) \sin(c + dx)}{b(a^2 - b^2)d(a + b \cos(c + dx))} \\
 &\quad - \frac{(2a^3(3a^2 - 4b^2)) \text{Subst}\left(\int \frac{1}{a + b + (a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{b^4(a^2 - b^2)d}
 \end{aligned}$$

$$= \frac{(6a^2 + b^2)x}{2b^4} - \frac{2a^3(3a^2 - 4b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2} b^4 (a+b)^{3/2} d} - \frac{a(3a^2 - 2b^2) \sin(c+dx)}{b^3 (a^2 - b^2) d}$$

$$+ \frac{(3a^2 - b^2) \cos(c+dx) \sin(c+dx)}{2b^2 (a^2 - b^2) d} - \frac{a^2 \cos^2(c+dx) \sin(c+dx)}{b (a^2 - b^2) d (a+b \cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^2} dx$$

$$= \frac{2(6a^2 + b^2)(c+dx) - \frac{8a^3(3a^2-4b^2) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} - 8ab \sin(c+dx) - \frac{4a^4 b \sin(c+dx)}{(a-b)(a+b)(a+b \cos(c+dx))} + b^2 \sin^2(c+dx)}{4b^4 d}$$

[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]

[Out] (2*(6*a^2 + b^2)*(c + d*x) - (8*a^3*(3*a^2 - 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - 8*a*b*Sin[c + d*x] - (4*a^4*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + b^2*Sin[2*(c + d*x)]/(4*b^4*d)

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.31

method	result
derivativedivides	$2a^3 \left(\frac{ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2-b^2) \left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{a-b} \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{a+b} \right)} + \frac{(3a^2-4b^2) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b) \sqrt{(a-b)(a+b)}} \right) \frac{2 \left((-2ab - \frac{1}{2}b^2) \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \right)}{d}$
default	$2a^3 \left(\frac{ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2-b^2) \left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{a-b} \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{a+b} \right)} + \frac{(3a^2-4b^2) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b) \sqrt{(a-b)(a+b)}} \right) \frac{2 \left((-2ab - \frac{1}{2}b^2) \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right) \right) \right)}{d}$
risch	$\frac{3x a^2}{b^4} + \frac{x}{2b^2} - \frac{ie^{2i(dx+c)}}{8b^2 d} + \frac{ia e^{i(dx+c)}}{b^3 d} - \frac{ia e^{-i(dx+c)}}{b^3 d} + \frac{ie^{-2i(dx+c)}}{8b^2 d} - \frac{2ia^4 (a e^{i(dx+c)} + b)}{b^4 (a^2 - b^2) d (b e^{2i(dx+c)} + 2a e^{i(dx+c)} + b)}$

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-2*a^3/b^4*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2-a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(3*a^2-4*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*ar

ctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))+2/b^4*(((-2*a*b-1/2*b^2)*tan(1/2*d*x+1/2*c)^3+(-2*a*b+1/2*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(6*a^2+b^2)*arctan(tan(1/2*d*x+1/2*c)))

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.92

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2} dx$$

$$= \left[\frac{(6a^6b - 11a^4b^3 + 4a^2b^5 + b^7)dx \cos(dx+c) + (6a^7 - 11a^5b^2 + 4a^3b^4 + ab^6)dx - (3a^6 - 4a^4b^2 + (3a^5b - 4a^3b^3)\cos(dx+c))\sqrt{-a^2+b^2}\log((2ab\cos(dx+c) + (2a^2 - b^2)\cos(dx+c))^2 - 2\sqrt{-a^2+b^2}(a\cos(dx+c) + b)\sin(dx+c) - a^2 + 2b^2)/(b^2\cos(dx+c)^2 + 2ab\cos(dx+c) + a^2)) - (6a^6b - 10a^4b^3 + 4a^2b^5 - (a^4b^3 - 2a^2b^5 + b^7)\cos(dx+c)^2 + 3(a^5b^2 - 2a^3b^4 + ab^6)\cos(dx+c))\sin(dx+c)}{(a^4b^5 - 2a^2b^7 + b^9)d\cos(dx+c) + (a^5b^4 - 2a^3b^6 + ab^8)d} \right]$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*((6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d*x*cos(d*x + c) + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d*x - (3*a^6 - 4*a^4*b^2 + (3*a^5*b - 4*a^3*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c))^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*a^6*b - 10*a^4*b^3 + 4*a^2*b^5 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2 + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d), 1/2*((6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d*x*cos(d*x + c) + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d*x - 2*(3*a^6 - 4*a^4*b^2 + (3*a^5*b - 4*a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*a^6*b - 10*a^4*b^3 + 4*a^2*b^5 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2 + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.58

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^2} dx = \frac{4 a^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{(a^2 b^3 - b^5) \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a + b\right)} - \frac{4 (3 a^5 - 4 a^3 b^2) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(-2 a + 2 b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^2 b^4 - b^6) \sqrt{a^2 - b^2}}$$

$2 d$

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a^4*tan(1/2*d*x + 1/2*c)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - 4*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/2
*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*
tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) -
(6*a^2 + b^2)*(d*x + c)/b^4 + 2*(4*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(1/2*d*
x + 1/2*c)^3 + 4*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/((tan(1/2
*d*x + 1/2*c)^2 + 1)^2*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 21.50 (sec) , antiderivative size = 3751, normalized size of antiderivative = 22.60

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Too large to display}$$

```
[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^2,x)
```


$$\begin{aligned}
& \left(-(a+b)^3(a-b)^3 \right)^{1/2} / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4) * 1i \\
& / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4) + (a^3(3a^2 - 4b^2) * (-(a+b)^3(a-b)^3)^{1/2} * ((8 \tan(c/2 + (d*x)/2) * (72a^{10} - 72a^9b - 2a^8b^2 + b^{10} + 11a^2b^8 - 20a^3b^7 + 23a^4b^6 - 26a^5b^5 + 17a^6b^4 + 120a^7b^3 - 120a^8b^2)) / (a^3b^6 - a^2b^7 - a^3b^6) - (a^3((8(2b^{15} + 6a^2b^{13} - 16a^3b^{12} - 14a^4b^{11} + 28a^5b^{10} + 6a^6b^9 - 12a^7b^8)) / (a^3b^{11} + b^{12} - a^2b^{10} - a^3b^9) + (8a^3 \tan(c/2 + (d*x)/2) * (3a^2 - 4b^2) * (-(a+b)^3(a-b)^3)^{1/2} * (8a^3b^{13} - 8a^2b^{12} - 16a^3b^{11} + 16a^4b^{10} + 8a^5b^9 - 8a^6b^8)) / ((a^3b^8 + b^9 - a^2b^7 - a^3b^6) * (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4))) * (3a^2 - 4b^2) * (-(a+b)^3(a-b)^3)^{1/2}) / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4)) * 1i) / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4) / ((16(108a^{11} - 54a^{10}b + 4a^3b^8 - 4a^4b^7 + 41a^5b^6 - 9a^6b^5 + 63a^7b^4 + 81a^8b^3 - 216a^9b^2)) / (a^3b^{11} + b^{12} - a^2b^{10} - a^3b^9) - (a^3(3a^2 - 4b^2) * (-(a+b)^3(a-b)^3)^{1/2} * ((8 \tan(c/2 + (d*x)/2) * (72a^{10} - 72a^9b - 2a^8b^2 + b^{10} + 11a^2b^8 - 20a^3b^7 + 23a^4b^6 - 26a^5b^5 + 17a^6b^4 + 120a^7b^3 - 120a^8b^2)) / (a^3b^6 - a^2b^7 - a^3b^6) + (a^3((8(2b^{15} + 6a^2b^{13} - 16a^3b^{12} - 14a^4b^{11} + 28a^5b^{10} + 6a^6b^9 - 12a^7b^8)) / (a^3b^{11} + b^{12} - a^2b^{10} - a^3b^9) - (8a^3 \tan(c/2 + (d*x)/2) * (3a^2 - 4b^2) * (-(a+b)^3(a-b)^3)^{1/2} * (8a^3b^{13} - 8a^2b^{12} - 16a^3b^{11} + 16a^4b^{10} + 8a^5b^9 - 8a^6b^8)) / ((a^3b^8 + b^9 - a^2b^7 - a^3b^6) * (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4))) * (3a^2 - 4b^2) * (-(a+b)^3(a-b)^3)^{1/2}) / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4)) / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4) + (a^3(3a^2 - 4b^2) * (-(a+b)^3(a-b)^3)^{1/2} * ((8 \tan(c/2 + (d*x)/2) * (72a^{10} - 72a^9b - 2a^8b^2 + b^{10} + 11a^2b^8 - 20a^3b^7 + 23a^4b^6 - 26a^5b^5 + 17a^6b^4 + 120a^7b^3 - 120a^8b^2)) / (a^3b^6 - a^2b^7 - a^3b^6) - (a^3((8(2b^{15} + 6a^2b^{13} - 16a^3b^{12} - 14a^4b^{11} + 28a^5b^{10} + 6a^6b^9 - 12a^7b^8)) / (a^3b^{11} + b^{12} - a^2b^{10} - a^3b^9) + (8a^3 \tan(c/2 + (d*x)/2) * (3a^2 - 4b^2) * (-(a+b)^3(a-b)^3)^{1/2} * (8a^3b^{13} - 8a^2b^{12} - 16a^3b^{11} + 16a^4b^{10} + 8a^5b^9 - 8a^6b^8)) / ((a^3b^8 + b^9 - a^2b^7 - a^3b^6) * (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4))) * (3a^2 - 4b^2) * (-(a+b)^3(a-b)^3)^{1/2}) / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4)) / (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4)) * (3a^2 - 4b^2) * (-(a+b)^3(a-b)^3)^{1/2} * 2i) / (d * (b^{10} - 3a^2b^8 + 3a^4b^6 - a^6b^4))
\end{aligned}$$

3.461 $\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^2} dx$

Optimal result	4949
Rubi [A] (verified)	4949
Mathematica [A] (verified)	4951
Maple [A] (verified)	4952
Fricas [A] (verification not implemented)	4952
Sympy [F(-1)]	4953
Maxima [F(-2)]	4953
Giac [B] (verification not implemented)	4953
Mupad [B] (verification not implemented)	4954

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^2} dx = -\frac{2ax}{b^3} + \frac{2a^2(2a^2-3b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2} b^3 (a+b)^{3/2} d} + \frac{(2a^2-b^2) \sin(c+dx)}{b^2 (a^2-b^2) d} - \frac{a^2 \cos(c+dx) \sin(c+dx)}{b (a^2-b^2) d (a+b \cos(c+dx))}$$

[Out] $-2*a*x/b^3+2*a^2*(2*a^2-3*b^2)*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/(a-b)^{(3/2)}/b^3/(a+b)^{(3/2)}/d+(2*a^2-b^2)*\sin(d*x+c)/b^2/(a^2-b^2)/d-a^2*\cos(d*x+c)*\sin(d*x+c)/b/(a^2-b^2)/d/(a+b*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2871, 3102, 2814, 2738, 211}

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^2} dx = \frac{2a^2(2a^2-3b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3 d (a-b)^{3/2} (a+b)^{3/2}} + \frac{(2a^2-b^2) \sin(c+dx)}{b^2 d (a^2-b^2)} - \frac{a^2 \sin(c+dx) \cos(c+dx)}{b d (a^2-b^2) (a+b \cos(c+dx))} - \frac{2ax}{b^3}$$

[In] $\text{Int}[\text{Cos}[c+d*x]^3/(a+b*\text{Cos}[c+d*x])^2,x]$

[Out] $(-2*a*x)/b^3 + (2*a^2*(2*a^2-3*b^2)*\text{ArcTan}[(\text{Sqrt}[a-b]*\text{Tan}[(c+d*x)/2])/\text{Sqrt}[a+b]])/((a-b)^{(3/2)}*b^3*(a+b)^{(3/2)*d} + ((2*a^2-b^2)*\text{Sin}[c +$

$d*x]/(b^2*(a^2 - b^2)*d) - (a^2*\cos[c + d*x]*\sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*\cos[c + d*x]))$

Rule 211

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a) * \text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$

Rule 2738

$\text{Int}[(a + (b \cdot \sin[\pi/2 + (c + d \cdot x)])^{-1}, x_Symbol] \rightarrow \text{With}[e = \text{FreeFactors}[\text{Tan}[(c + d \cdot x)/2], x], \text{Dist}[2*(e/d), \text{Subst}[\text{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \text{Tan}[(c + d \cdot x)/2]/e], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 2814

$\text{Int}[(a + (b \cdot \sin[(e + f \cdot x)])/(c + (d \cdot \sin[(e + f \cdot x)]) \cdot x)], x_Symbol] \rightarrow \text{Simp}[b*(x/d), x] - \text{Dist}[(b*c - a*d)/d, \text{Int}[1/(c + d*\sin[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 2871

$\text{Int}[(a + (b \cdot \sin[(e + f \cdot x)])^m * (c + (d \cdot \sin[(e + f \cdot x)]) \cdot x)^n), x_Symbol] \rightarrow \text{Simp}[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\cos[e + f*x]*(a + b*\sin[e + f*x])^{m-2} * (c + d*\sin[e + f*x])^{n+1} / (d*f*(n+1)*(c^2 - d^2)), x] + \text{Dist}[1/(d*(n+1)*(c^2 - d^2)), \text{Int}[(a + b*\sin[e + f*x])^{m-3} * (c + d*\sin[e + f*x])^{n+1} * \text{Simp}[b*(m-2)*(b*c - a*d)^2 + a*d*(n+1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n+1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n+2)*(b*c - a*d)^2)*\sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*\sin[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[m, 2] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegersQ}[2*m, 2*n])$

Rule 3102

$\text{Int}[(a + (b \cdot \sin[(e + f \cdot x)])^m * ((A + (B \cdot \sin[(e + f \cdot x)] + (C \cdot \sin[(e + f \cdot x)]^2)), x_Symbol] \rightarrow \text{Simp}[(-C)*\cos[e + f*x]*(a + b*\sin[e + f*x])^{m+1} / (b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a + b*\sin[e + f*x])^m * \text{Simp}[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C, m\}, x \ \&\& \ !\text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{a^2 \cos(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} - \frac{\int \frac{a^2-ab\cos(c+dx)-(2a^2-b^2)\cos^2(c+dx)}{a+b\cos(c+dx)} dx}{b(a^2-b^2)} \\
 &= \frac{(2a^2-b^2)\sin(c+dx)}{b^2(a^2-b^2)d} - \frac{a^2 \cos(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} - \frac{\int \frac{a^2b+2a(a^2-b^2)\cos(c+dx)}{a+b\cos(c+dx)} dx}{b^2(a^2-b^2)} \\
 &= -\frac{2ax}{b^3} + \frac{(2a^2-b^2)\sin(c+dx)}{b^2(a^2-b^2)d} - \frac{a^2 \cos(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} \\
 &\quad + \frac{(a^2(2a^2-3b^2)) \int \frac{1}{a+b\cos(c+dx)} dx}{b^3(a^2-b^2)} \\
 &= -\frac{2ax}{b^3} + \frac{(2a^2-b^2)\sin(c+dx)}{b^2(a^2-b^2)d} - \frac{a^2 \cos(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} \\
 &\quad + \frac{(2a^2(2a^2-3b^2)) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^3(a^2-b^2)d} \\
 &= -\frac{2ax}{b^3} + \frac{2a^2(2a^2-3b^2) \arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} \\
 &\quad + \frac{(2a^2-b^2)\sin(c+dx)}{b^2(a^2-b^2)d} - \frac{a^2 \cos(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.73

$$\begin{aligned}
 &\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2} dx \\
 &= \frac{-2a(c+dx) + \frac{2a^2(2a^2-3b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} + \left(b + \frac{a^3b}{(a-b)(a+b)(a+b\cos(c+dx))}\right) \sin(c+dx)}{b^3d}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]

[Out] (-2*a*(c + d*x) + (2*a^2*(2*a^2 - 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + (b + (a^3*b)/((a - b)*(a + b)*(a + b*Cos[c + d*x]))) * Sin[c + d*x]/(b^3*d)

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{2a^2 \left(\frac{ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2-b^2)\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+a+b} + \frac{(2a^2-3b^2) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}} \right)}{b^3} - \frac{2 \left(-\frac{b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{2a^2}{b} \right)}{d}$
default	$\frac{2a^2 \left(\frac{ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2-b^2)\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+a+b} + \frac{(2a^2-3b^2) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}} \right)}{b^3} - \frac{2 \left(-\frac{b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{2a^2}{b} \right)}{d}$
risch	$-\frac{2ax}{b^3} - \frac{ie^{i(dx+c)}}{2b^2d} + \frac{ie^{-i(dx+c)}}{2b^2d} + \frac{2ia^3(ae^{i(dx+c)}+b)}{b^3(a^2-b^2)d(b e^{2i(dx+c)}+2a e^{i(dx+c)}+b)} - \frac{2a^4 \ln\left(e^{i(dx+c)} + \frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}(a+b)(a-b)db^3}$

[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(2/b^3*a^2*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(2*a^2-3*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/b^3*(-b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+2*a*arctan(tan(1/2*d*x+1/2*c))))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.57

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2} dx$$

$$= \frac{4(a^5b - 2a^3b^3 + ab^5)dx \cos(dx+c) + 4(a^6 - 2a^4b^2 + a^2b^4)dx + (2a^5 - 3a^3b^2 + (2a^4b - 3a^2b^3) \cos(dx+c))}{2(a^5b - 2a^3b^3 + ab^5)dx \cos(dx+c) + 2(a^6 - 2a^4b^2 + a^2b^4)dx - (2a^5 - 3a^3b^2 + (2a^4b - 3a^2b^3) \cos(dx+c))} + \frac{(a^4b^4 - 2a^2b^6 + b^8)d \cos(dx+c)}{2}$$

[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] [-1/2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 4*(a^6 - 2*a^4*b^2 + a^2*b^4)*d*x + (2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c) + a) + (2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x + c)))/d]

$$d*x + c)^2 + 2*a*b*\cos(d*x + c) + a^2)) - 2*(2*a^5*b - 3*a^3*b^3 + a*b^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*\cos(d*x + c))*\sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*\cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d), -(2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*\cos(d*x + c) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d*x - (2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*\cos(d*x + c))*\sqrt{a^2 - b^2}*\arctan(-(a*\cos(d*x + c) + b)/(\sqrt{a^2 - b^2}*\sin(d*x + c)))) - (2*a^5*b - 3*a^3*b^3 + a*b^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*\cos(d*x + c))*\sin(d*x + c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*d*\cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d)]$$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(146) = 292.

Time = 0.37 (sec) , antiderivative size = 847, normalized size of antiderivative = 5.46

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^2} dx$$

$$= \frac{(4a^6b^2 - 2a^5b^3 - 9a^4b^4 + 4a^3b^5 + 5a^2b^6 - 2ab^7 + 2a^3| -a^2b^3 + b^5 | -a^2b| -a^2b^3 + b^5 | -2ab^2| -a^2b^3 + b^5 |)}{a^3b^2| -a^2b^3 + b^5 | -ab^4| -a^2b^3 + b^5 | + (a^2b^3 - b^5)^2} \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] + \arctan \left(\frac{\sqrt{2a^3b^2 - 2ab^4 + \sqrt{\dots}}}{\dots} \right) \right)$$

[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] ((4*a^6*b^2 - 2*a^5*b^3 - 9*a^4*b^4 + 4*a^3*b^5 + 5*a^2*b^6 - 2*a*b^7 + 2*a^3*abs(-a^2*b^3 + b^5) - a^2*b*abs(-a^2*b^3 + b^5) - 2*a*b^2*abs(-a^2*b^3 + b^5))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a^3*b^2 - 2*a*b^4 + sqrt(-4*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*(a^3*b^2 - a^2*b^3 - a*b^4 + b^5) + 4*(a^3*b^2 - a*b^4)^2))/(a^3*b^2 - a^2*b^3 - a*b^4 + b^5))))/(a^3*b^2*abs(-a^2*b^3 + b^5) - a*b^4*abs(-a^2*b^3 + b^5) + (a^2*b^3 - b^5)^2) - ((2*a^3 - a^2*b - 2*a*b^2)*sqrt(a^2 - b^2)*abs(-a^2*b^3 + b^5)*abs(-a + b) - (4*a^6*b^2 - 2*a^5*b^3 - 9*a^4*b^4 + 4*a^3*b^5 + 5*a^2*b^6 - 2*a*b^7)*sqrt(a^2 - b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(2*sqrt(1/2)*tan(1/2*d*x + 1/2*c)/sqrt((2*a^3*b^2 - 2*a*b^4 - sqrt(-4*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*(a^3*b^2 - a^2*b^3 - a*b^4 + b^5) + 4*(a^3*b^2 - a*b^4)^2))/(a^3*b^2 - a^2*b^3 - a*b^4 + b^5))))/(a^2*b^3 - b^5)^2*(a^2 - 2*a*b + b^2) - (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*abs(-a^2*b^3 + b^5)) + 2*(2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4))/d

Mupad [B] (verification not implemented)

Time = 20.65 (sec) , antiderivative size = 3180, normalized size of antiderivative = 20.52

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Too large to display}$$

[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^2,x)

[Out] - ((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + a^2*b - 2*a^3 - b^3))/(b^2*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)*(a*b^2 - a^2*b - 2*a^3 + b^3))/(b^2*(a + b)*(a - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*(a - b) + 2*a*tan(c/2 + (d*x)/2)^2)) - (4*a*atan(((2*a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)*64i)/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*2i)/b^3))/b^3 + (2*a*((32*tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a*tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)*64i)/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*2i)/b^3))/b^3)/((64*(8*a^8 - 4*a^7*b + 12*a^4*b^4 + 6*a^5*b^3 - 20

$$\begin{aligned}
& *a^6*b^2)/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a*((32*\tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a*\tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)*64i)/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4))*2i)/b^3 + (a*((32*\tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (a*\tan(c/2 + (d*x)/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)*64i)/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4))*2i)/b^3))/b^3*d) - (a^2*a*\tan(((a^2*((32*\tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a^2*(2*a^2 - 3*b^2))*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a^2*\tan(c/2 + (d*x)/2)*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a^2*((32*\tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a^2*(2*a^2 - 3*b^2))*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a^2*\tan(c/2 + (d*x)/2)*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(8*a^8 - 4*a^7*b + 12*a^4*b^4 + 6*a^5*b^3 - 20*a^6*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (a^2*((32*\tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a^2*(2*a^2 - 3*b^2))*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*a^2*\tan(c/2 + (d*x)/2)*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6)))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(-(a + b)^3*(a - b)^3)^(1/2))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/b^3 + (a^2*((32*\tan(c/2 + (d*x)/2)*(8*a^8 - 8*a^7*b + 4*a^2*b^6 - 8*a^3*b^5 + 5*a^4*b^4 + 16*a^5*b^3 - 16*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (a^2*(2*a^2 - 3*b^2))*((32*(2*a*b^11 - 3*a^2*b^10 - 3*a^3*b^9 + 5*a^4*b^8 + a^5*b^7 - 2*a^6*b^6)))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a^2*\tan(c/2 + (d*x)/2)*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 +
\end{aligned}$$

$$\frac{4a^4b^8 + 2a^5b^7 - 2a^6b^6}{(ab^6 + b^7 - a^2b^5 - a^3b^4)(b^9 - 3a^2b^7 + 3a^4b^5 - a^6b^3)} \cdot \frac{-(a+b)^3(a-b)^3^{(1/2)}}{(b^9 - 3a^2b^7 + 3a^4b^5 - a^6b^3)} \cdot \frac{(2a^2 - 3b^2) \cdot -(a+b)^3(a-b)^3^{(1/2)}}{(b^9 - 3a^2b^7 + 3a^4b^5 - a^6b^3)} \cdot \frac{(2a^2 - 3b^2) \cdot -(a+b)^3(a-b)^3^{(1/2)} \cdot 2i}{d(b^9 - 3a^2b^7 + 3a^4b^5 - a^6b^3)}$$

$$3.462 \quad \int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	4957
Rubi [A] (verified)	4957
Mathematica [A] (verified)	4959
Maple [A] (verified)	4959
Fricas [B] (verification not implemented)	4960
Sympy [F(-1)]	4960
Maxima [F(-2)]	4960
Giac [A] (verification not implemented)	4961
Mupad [B] (verification not implemented)	4961

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^2} dx = \frac{x}{b^2} - \frac{2a(a^2 - 2b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{3/2} b^2 (a+b)^{3/2} d} - \frac{a^2 \sin(c+dx)}{b(a^2 - b^2) d (a+b \cos(c+dx))}$$

[Out] x/b^2-2*a*(a^2-2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^2/(a+b)^(3/2)/d-a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2869, 2814, 2738, 211}

$$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^2} dx = -\frac{2a(a^2 - 2b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{b^2 d (a-b)^{3/2} (a+b)^{3/2}} - \frac{a^2 \sin(c+dx)}{bd(a^2 - b^2)(a+b \cos(c+dx))} + \frac{x}{b^2}$$

[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]

[Out] x/b^2 - (2*a*(a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - (a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2869

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) + c^2*(m + 2)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{a^2 \sin(c + dx)}{b(a^2 - b^2)d(a + b \cos(c + dx))} + \frac{\int \frac{ab + (a^2 - b^2) \cos(c + dx)}{a + b \cos(c + dx)} dx}{b(a^2 - b^2)} \\
 &= \frac{x}{b^2} - \frac{a^2 \sin(c + dx)}{b(a^2 - b^2)d(a + b \cos(c + dx))} - \frac{(a(a^2 - 2b^2)) \int \frac{1}{a + b \cos(c + dx)} dx}{b^2(a^2 - b^2)} \\
 &= \frac{x}{b^2} - \frac{a^2 \sin(c + dx)}{b(a^2 - b^2)d(a + b \cos(c + dx))} \\
 &\quad - \frac{(2a(a^2 - 2b^2)) \text{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{b^2(a^2 - b^2)d} \\
 &= \frac{x}{b^2} - \frac{2a(a^2 - 2b^2) \arctan\left(\frac{\sqrt{a - b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b}}\right)}{(a - b)^{3/2} b^2 (a + b)^{3/2} d} - \frac{a^2 \sin(c + dx)}{b(a^2 - b^2)d(a + b \cos(c + dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2} dx$$

$$= \frac{c+dx - \frac{2a(a^2-2b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} - \frac{a^2b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{b^2d}$$

[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]

[Out] (c + d*x - (2*a*(a^2 - 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]))/(-a^2 + b^2)^(3/2) - (a^2*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/(b^2*d)

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{b^2} - \frac{2a \left(\frac{ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2 - b^2) \left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a - b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a + b} \right) + \frac{(a^2 - 2b^2) \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d b^2}$
default	$\frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{b^2} - \frac{2a \left(\frac{ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2 - b^2) \left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a - b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a + b} \right) + \frac{(a^2 - 2b^2) \arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{d b^2}$
risch	$\frac{x}{b^2} - \frac{2ia^2(ae^{i(dx+c)}+b)}{b^2(a^2-b^2)d(be^{2i(dx+c)}+2ae^{i(dx+c)}+b)} - \frac{a^3 \ln\left(\frac{e^{i(dx+c)} - ia^2 - ib^2 - a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}(a+b)(a-b)db^2} + \frac{2a \ln\left(\frac{e^{i(dx+c)} - ia^2 - ib^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}(a+b)}$

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(2/b^2*arctan(tan(1/2*d*x+1/2*c))-2*a/b^2*(a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(a^2-2*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(99) = 198$.

Time = 0.29 (sec) , antiderivative size = 470, normalized size of antiderivative = 4.35

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^2} dx$$

$$= \frac{2(a^4b - 2a^2b^3 + b^5)dx \cos(dx + c) + 2(a^5 - 2a^3b^2 + ab^4)dx - (a^4 - 2a^2b^2 + (a^3b - 2ab^3) \cos(dx + c))}{2((a^4b^3 - 2a^2b^5 + b^7)d \cos(dx + c))}$$

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*(2*(a^4*b - 2*a^2*b^3 + b^5)*d*x*cos(d*x + c) + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x - (a^4 - 2*a^2*b^2 + (a^3*b - 2*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d), ((a^4*b - 2*a^2*b^3 + b^5)*d*x*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d*x - (a^4 - 2*a^2*b^2 + (a^3*b - 2*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^4*b - a^2*b^3)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.62

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^2} dx =$$

$$\frac{2a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{(a^2 b - b^3) \left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a + b\right)} - \frac{2(a^3 - 2ab^2) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^2 b^2 - b^4) \sqrt{a^2 - b^2}}$$

$$d$$

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="giac")

```
[Out] -(2*a^2*tan(1/2*d*x + 1/2*c)/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - 2*(a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) - (d*x + c)/b^2)/d
```

Mupad [B] (verification not implemented)

Time = 20.89 (sec) , antiderivative size = 2872, normalized size of antiderivative = 26.59

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Too large to display}$$

[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^2,x)

```
[Out] (2*atan((((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))*1i)/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))/b^2 - (((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))*1i)/b^2 - (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))/b^2)/((64*(2*a*b^4 - a^4*b + a^5 + 2*a^2*b^3 - 3*a^3*b^2))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (((32*(2*a*b^8 - b^9 + a^2*b^7 - 3*a^3*b^6 + a^5*b^4))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4)*32i)/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))*1i)/b^2 + (32*tan(c/2 + (d*x)/2)*(2*a^6 - 2*a^5*b - 2*a*b^5 + b^6 + 3*a^2*b^4 + 4*a^3*b^3 - 5*a^4*b^2))/(a*b^4 + b^5 - a^2*b^3
```


$$3.463 \quad \int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	4963
Rubi [A] (verified)	4963
Mathematica [A] (verified)	4964
Maple [A] (verified)	4965
Fricas [A] (verification not implemented)	4965
Sympy [F(-1)]	4966
Maxima [F(-2)]	4966
Giac [A] (verification not implemented)	4966
Mupad [B] (verification not implemented)	4967

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^2} dx = -\frac{2b \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{a \sin(c+dx)}{(a^2-b^2)d(a+b \cos(c+dx))}$$

[Out] $-2*b*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/(a-b)^{(3/2)/(a+b)^{(3/2)/d+a*\sin(d*x+c)/(a^2-b^2)/d/(a+b*\cos(d*x+c))}$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2833, 12, 2738, 211}

$$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^2} dx = \frac{a \sin(c+dx)}{d(a^2-b^2)(a+b \cos(c+dx))} - \frac{2b \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

[In] $\text{Int}[\text{Cos}[c + d*x]/(a + b*\text{Cos}[c + d*x])^2, x]$

[Out] $(-2*b*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[(c + d*x)/2])/(\text{Sqrt}[a + b])]/((a - b)^{(3/2)}*(a + b)^{(3/2)*d} + (a*\text{Sin}[c + d*x])/((a^2 - b^2)*d*(a + b*\text{Cos}[c + d*x])))$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!Match}[Q[u, (b_)*(v_)] /; \text{FreeQ}[b, x]]$

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2833

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a \sin(c + dx)}{(a^2 - b^2) d(a + b \cos(c + dx))} + \frac{\int \frac{b}{a + b \cos(c + dx)} dx}{-a^2 + b^2} \\
 &= \frac{a \sin(c + dx)}{(a^2 - b^2) d(a + b \cos(c + dx))} - \frac{b \int \frac{1}{a + b \cos(c + dx)} dx}{a^2 - b^2} \\
 &= \frac{a \sin(c + dx)}{(a^2 - b^2) d(a + b \cos(c + dx))} - \frac{(2b) \text{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{(a^2 - b^2) d} \\
 &= -\frac{2b \arctan\left(\frac{\sqrt{a - b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b}}\right)}{(a - b)^{3/2}(a + b)^{3/2}d} + \frac{a \sin(c + dx)}{(a^2 - b^2) d(a + b \cos(c + dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^2} dx = \frac{2b \operatorname{arctanh}\left(\frac{(a - b) \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{-a^2 + b^2}}\right)}{(-a^2 + b^2)^{3/2}} + \frac{a \sin(c + dx)}{(a - b)(a + b)(a + b \cos(c + dx))}$$

[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^2,x]

[Out] ((-2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]/(-a^2 + b^2)^(3/2) + (a*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x]))) / d

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{2a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2 - b^2) \left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a - b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + a + b \right)} - \frac{2b \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b) \sqrt{(a-b)(a+b)}}$
default	$\frac{2a \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2 - b^2) \left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a - b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + a + b \right)} - \frac{2b \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b) \sqrt{(a-b)(a+b)}}$
risch	$-\frac{2ia(ae^{i(dx+c)}+b)}{b(-a^2+b^2)d(b e^{2i(dx+c)}+2a e^{i(dx+c)}+b)} - \frac{b \ln\left(e^{i(dx+c)} + \frac{-ia^2+ib^2+a\sqrt{-a^2+b^2}}{\sqrt{-a^2+b^2}b}\right)}{\sqrt{-a^2+b^2}(a+b)(a-b)d} + \frac{b \ln\left(e^{i(dx+c)} + \frac{ia^2-ib^2+a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}(a+b)(a-b)d}$

[In] int(cos(d*x+c)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(2*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)-2*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.78

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2} dx$$

$$= \left[\frac{(b^2 \cos(dx+c) + ab)\sqrt{-a^2+b^2} \log\left(\frac{2ab\cos(dx+c) + (2a^2-b^2)\cos(dx+c)^2 + 2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c) - a^2 + 2b^2}{b^2 \cos(dx+c)^2 + 2ab\cos(dx+c) + a^2}\right)}{2((a^4b - 2a^2b^3 + b^5)d\cos(dx+c) + (a^5 - 2a^3b^2 + ab^4)d)} - \frac{(b^2 \cos(dx+c) + ab)\sqrt{a^2-b^2} \arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right) - (a^3 - ab^2)\sin(dx+c)}{(a^4b - 2a^2b^3 + b^5)d\cos(dx+c) + (a^5 - 2a^3b^2 + ab^4)d} \right]$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*((b^2*cos(d*x + c) + a*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), -(b^2*cos(d*x + c) + a*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^3 - a*b^2)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.59

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^2} dx$$

$$= \frac{2 \left(\frac{\left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}}\right)\right) b}{(a^2 - b^2)^{\frac{3}{2}}} + \frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a + b\right)(a^2 - b^2)} \right)}{d}$$

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b/(a^2 - b^2)^(3/2)
+ a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
c)^2 + a + b)*(a^2 - b^2)))/d
```

Mupad [B] (verification not implemented)

Time = 15.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^2} dx = \frac{2 a \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d (a + b) (a - b) \left((a - b) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a + b \right)} - \frac{2 b \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2 a - 2 b)}{2 \sqrt{a+b} \sqrt{a-b}}\right)}{d (a + b)^{3/2} (a - b)^{3/2}}$$

[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^2,x)

```
[Out] (2*a*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b)) - (2*b*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b))/(2*(a + b)^(1/2)*(a - b)^(1/2))))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.464 $\int \frac{1}{(a+b \cos(c+dx))^2} dx$

Optimal result	4968
Rubi [A] (verified)	4968
Mathematica [A] (verified)	4969
Maple [A] (verified)	4970
Fricas [A] (verification not implemented)	4970
Sympy [B] (verification not implemented)	4971
Maxima [F(-2)]	4972
Giac [A] (verification not implemented)	4973
Mupad [B] (verification not implemented)	4973

Optimal result

Integrand size = 12, antiderivative size = 86

$$\int \frac{1}{(a+b \cos(c+dx))^2} dx = \frac{2a \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b \sin(c+dx)}{(a^2-b^2)d(a+b \cos(c+dx))}$$

[Out] $2*a*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/(a-b)^{(3/2)/(a+b)^{(3/2)/d}-b*\sin(d*x+c)/(a^2-b^2)/d/(a+b*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2743, 12, 2738, 211}

$$\int \frac{1}{(a+b \cos(c+dx))^2} dx = \frac{2a \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}} - \frac{b \sin(c+dx)}{d(a^2-b^2)(a+b \cos(c+dx))}$$

[In] $\text{Int}[(a + b*\text{Cos}[c + d*x])^{-2}, x]$

[Out] $(2*a*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[(c + d*x)/2])/\text{Sqrt}[a + b]])/((a - b)^{(3/2)}*(a + b)^{(3/2)*d} - (b*\text{Sin}[c + d*x])/((a^2 - b^2)*d*(a + b*\text{Cos}[c + d*x]))$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{b \sin(c + dx)}{(a^2 - b^2) d(a + b \cos(c + dx))} - \frac{\int \frac{a}{a + b \cos(c + dx)} dx}{-a^2 + b^2} \\
 &= -\frac{b \sin(c + dx)}{(a^2 - b^2) d(a + b \cos(c + dx))} + \frac{a \int \frac{1}{a + b \cos(c + dx)} dx}{a^2 - b^2} \\
 &= -\frac{b \sin(c + dx)}{(a^2 - b^2) d(a + b \cos(c + dx))} + \frac{(2a) \text{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{(a^2 - b^2) d} \\
 &= \frac{2a \arctan\left(\frac{\sqrt{a - b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b}}\right)}{(a - b)^{3/2}(a + b)^{3/2} d} - \frac{b \sin(c + dx)}{(a^2 - b^2) d(a + b \cos(c + dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a + b \cos(c + dx))^2} dx = \frac{2a \operatorname{arctanh}\left(\frac{(a - b) \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{-a^2 + b^2}}\right)}{(-a^2 + b^2)^{3/2}} - \frac{b \sin(c + dx)}{(a - b)(a + b)(a + b \cos(c + dx))}$$

[In] Integrate[(a + b*Cos[c + d*x])^(-2), x]

[Out] ((2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - (b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x]))/d

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{2b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2 - b^2) \left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a - b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + a + b \right)} + \frac{2a \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}$
default	$\frac{2b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2 - b^2) \left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a - b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + a + b \right)} + \frac{2a \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}$
risch	$\frac{2i(a e^{i(dx+c)} + b)}{d(-a^2 + b^2)(b e^{2i(dx+c)} + 2a e^{i(dx+c)} + b)} - \frac{a \ln\left(e^{i(dx+c)} + \frac{ia^2 - ib^2 + a\sqrt{-a^2 + b^2}}{b\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}(a+b)(a-b)d} + \frac{a \ln\left(e^{i(dx+c)} - \frac{ia^2 - ib^2 - a\sqrt{-a^2 + b^2}}{b\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}(a+b)(a-b)d}$

```
[In] int(1/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/(a^2-b^2)*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+2*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.72

$$\int \frac{1}{(a + b \cos(c + dx))^2} dx$$

$$= \left[\frac{(ab \cos(dx + c) + a^2)\sqrt{-a^2 + b^2} \log\left(\frac{2ab \cos(dx+c) + (2a^2 - b^2) \cos(dx+c)^2 - 2\sqrt{-a^2 + b^2}(a \cos(dx+c) + b) \sin(dx+c) - a^2 + 2b^2}{b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2}\right)}{2((a^4b - 2a^2b^3 + b^5)d \cos(dx + c) + (a^5 - 2a^3b^2 + ab^4)d)} \right]$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*cos(d*x + c) + a^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(a^2*b - b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d), ((a*b*cos(d*x + c) + a^2)*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^2*b - b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)]
```



```

- 2*a**2*b**2*d*sqrt(-a/(a - b) - b/(a - b)) + 2*a*b**3*d*sqrt(-a/(a - b) -
  b/(a - b))*tan(c/2 + d*x/2)**2 - b**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c
/2 + d*x/2)**2 + b**4*d*sqrt(-a/(a - b) - b/(a - b))) + a*b*log(-sqrt(-a/(a
- b) - b/(a - b)) + tan(c/2 + d*x/2))/(a**4*d*sqrt(-a/(a - b) - b/(a - b))
*tan(c/2 + d*x/2)**2 + a**4*d*sqrt(-a/(a - b) - b/(a - b)) - 2*a**3*b*d*sq
rt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - 2*a**2*b**2*d*sqrt(-a/(a -
b) - b/(a - b)) + 2*a*b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)*
**2 - b**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + b**4*d*sqrt(
-a/(a - b) - b/(a - b))) + a*b*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 +
d*x/2))*tan(c/2 + d*x/2)**2/(a**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 +
d*x/2)**2 + a**4*d*sqrt(-a/(a - b) - b/(a - b)) - 2*a**3*b*d*sqrt(-a/(a -
b) - b/(a - b))*tan(c/2 + d*x/2)**2 - 2*a**2*b**2*d*sqrt(-a/(a - b) - b/(a
- b)) + 2*a*b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**4*
d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + b**4*d*sqrt(-a/(a - b)
- b/(a - b))) - a*b*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(
a**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + a**4*d*sqrt(-a/(a
- b) - b/(a - b)) - 2*a**3*b*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/
2)**2 - 2*a**2*b**2*d*sqrt(-a/(a - b) - b/(a - b)) + 2*a*b**3*d*sqrt(-a/(a
- b) - b/(a - b))*tan(c/2 + d*x/2)**2 - b**4*d*sqrt(-a/(a - b) - b/(a - b))
*tan(c/2 + d*x/2)**2 + b**4*d*sqrt(-a/(a - b) - b/(a - b))) + 2*b**2*sqrt(-
a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)/(a**4*d*sqrt(-a/(a - b) - b/(a - b)
))*tan(c/2 + d*x/2)**2 + a**4*d*sqrt(-a/(a - b) - b/(a - b)) - 2*a**3*b*d*sq
rt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 - 2*a**2*b**2*d*sqrt(-a/(a -
b) - b/(a - b)) + 2*a*b**3*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)
**2 - b**4*d*sqrt(-a/(a - b) - b/(a - b))*tan(c/2 + d*x/2)**2 + b**4*d*sqrt
(-a/(a - b) - b/(a - b))), True)

```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a + b \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```


Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a + b \cos(c + dx))^2} dx = \frac{2 \left(\frac{\left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}}\right)\right) a}{(a^2 - b^2)^{\frac{3}{2}}} \right) + \frac{b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + a + b\right) (a^2 - b^2)^{\frac{3}{2}}}}{d}$$

[In] integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="giac")

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a/(a^2 - b^2)^(3/2) + b*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2 - b^2)))/d
```

Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a + b \cos(c + dx))^2} dx = \frac{2 a \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2 a - 2 b)}{2 \sqrt{a+b} \sqrt{a-b}}\right)}{d (a+b)^{3/2} (a-b)^{3/2}} - \frac{2 b \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d (a+b) (a-b) \left(\left(a-b\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a + b\right)}$$

[In] int(1/(a + b*cos(c + d*x))^2,x)

```
[Out] (2*a*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b))/(2*(a + b)^(1/2)*(a - b)^(1/2)))/((d*(a + b)^(3/2)*(a - b)^(3/2)) - (2*b*tan(c/2 + (d*x)/2))/(d*(a + b)*(a - b)*(a + b + tan(c/2 + (d*x)/2)^2*(a - b))))
```

3.465 $\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^2} dx$

Optimal result	4974
Rubi [A] (verified)	4974
Mathematica [A] (verified)	4976
Maple [A] (verified)	4977
Fricas [B] (verification not implemented)	4977
Sympy [F]	4978
Maxima [F(-2)]	4978
Giac [A] (verification not implemented)	4978
Mupad [B] (verification not implemented)	4979

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^2} dx = -\frac{2b(2a^2 - b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{b^2 \sin(c+dx)}{a(a^2 - b^2)d(a+b \cos(c+dx))}$$

[Out] $-2*b*(2*a^2-b^2)*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a^2/(a-b)^{(3/2)/(a+b)^{(3/2)/d}+\operatorname{arctanh}(\sin(d*x+c))/a^2/d+b^2*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2881, 3080, 3855, 2738, 211}

$$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^2} dx = -\frac{2b(2a^2 - b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}} + \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{b^2 \sin(c+dx)}{ad(a^2 - b^2)(a+b \cos(c+dx))}$$

[In] $\text{Int}[\text{Sec}[c + d*x]/(a + b*\text{Cos}[c + d*x])^2, x]$

[Out] $(-2*b*(2*a^2 - b^2)*\text{ArcTan}[(\text{Sqrt}[a - b]*\text{Tan}[(c + d*x)/2])/(\text{Sqrt}[a + b])]/(a^2*(a - b)^{(3/2)*(a + b)^{(3/2)*d}) + \text{ArcTanh}[\text{Sin}[c + d*x]]/(a^2*d) + (b^2*\text{Sin}[c + d*x])/(a*(a^2 - b^2)*d*(a + b*\text{Cos}[c + d*x]))$

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3080

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3855

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{b^2 \sin(c + dx)}{a(a^2 - b^2)d(a + b \cos(c + dx))} + \frac{\int \frac{(a^2 - b^2 - ab \cos(c + dx)) \sec(c + dx)}{a + b \cos(c + dx)} dx}{a(a^2 - b^2)} \\ &= \frac{b^2 \sin(c + dx)}{a(a^2 - b^2)d(a + b \cos(c + dx))} + \frac{\int \sec(c + dx) dx}{a^2} - \frac{(b(2a^2 - b^2)) \int \frac{1}{a + b \cos(c + dx)} dx}{a^2(a^2 - b^2)} \end{aligned}$$

$$\begin{aligned}
&= \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2 d} + \frac{b^2 \sin(c+dx)}{a(a^2-b^2)d(a+b\cos(c+dx))} \\
&\quad - \frac{(2b(2a^2-b^2)) \operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2(a^2-b^2)d} \\
&= -\frac{2b(2a^2-b^2) \operatorname{arctan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} \\
&\quad + \frac{\operatorname{arctanh}(\sin(c+dx))}{a^2 d} + \frac{b^2 \sin(c+dx)}{a(a^2-b^2)d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

$$\begin{aligned}
&\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2} dx \\
&= \frac{2b(-2a^2+b^2) \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right) \\
&\quad \frac{b^2 \sin(c+dx)}{a^2 d}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^2,x]

[Out] ((2*b*(-2*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x]))/(a^2*d)

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+ \frac{(2a^2-b^2)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{d}}$
default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2}-\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+ \frac{(2a^2-b^2)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{d}}$
risch	$\frac{2ib(ae^{i(dx+c)}+b)}{(-a^2+b^2)da(b e^{2i(dx+c)}+2a e^{i(dx+c)}+b)}-\frac{2b\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}(a+b)(a-b)d}+\frac{b^3\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-a\sqrt{-a^2+b^2}}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}(a+b)(a-b)d}}$

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2*ln(tan(1/2*d*x+1/2*c)-1)-2*b/a^2*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(2*a^2-b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))+1/a^2*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(109) = 218.

Time = 0.47 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.02

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2} dx$$

$$= \left[\frac{(2a^3b-ab^3+(2a^2b^2-b^4)\cos(dx+c))\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)}{2(2a^3b-ab^3+(2a^2b^2-b^4)\cos(dx+c))\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)} - (a^5-2a^3b^2+ab^4) + \dots \right]$$

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b
```

$$\begin{aligned} &^5) \cos(dx + c) \log(\sin(dx + c) + 1) + (a^5 - 2a^3b^2 + ab^4 + (a^4b \\ &- 2a^2b^3 + b^5) \cos(dx + c) \log(-\sin(dx + c) + 1) - 2(a^3b^2 - ab^4) \\ &^4) \sin(dx + c) / ((a^6b - 2a^4b^3 + a^2b^5) d \cos(dx + c) + (a^7 - 2a^5b^2 \\ &+ a^3b^4) d), -1/2(2(2a^3b - ab^3 + (2a^2b^2 - b^4) \cos(dx \\ &+ c)) \sqrt{a^2 - b^2} \arctan(-(a \cos(dx + c) + b) / (\sqrt{a^2 - b^2} \sin(dx \\ &x + c))) - (a^5 - 2a^3b^2 + ab^4 + (a^4b - 2a^2b^3 + b^5) \cos(dx + c \\ &)) \log(\sin(dx + c) + 1) + (a^5 - 2a^3b^2 + ab^4 + (a^4b - 2a^2b^3 + \\ &b^5) \cos(dx + c)) \log(-\sin(dx + c) + 1) - 2(a^3b^2 - ab^4) \sin(dx + c \\ &)) / ((a^6b - 2a^4b^3 + a^2b^5) d \cos(dx + c) + (a^7 - 2a^5b^2 + a^3b^4) \\ &d) \end{aligned}$$

Sympy [F]

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^2} dx$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**2,x)

[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**2, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.68

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^2} dx = \frac{2b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(a^3 - ab^2) (a \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - b \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + a + b)} - \frac{2(2a^2b - b^3) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(2a-2b) + \arctan \left(\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^4 - a^2b^2) \sqrt{a^2 - b^2}}$$

d

$$\begin{aligned}
& a^8 - a^2 b^6 + 3a^4 b^4 - 3a^6 b^2) + (b((32 \tan(c/2 + (d*x)/2) * (a^6 - \\
& 2a^5 b - 2a^4 b^2 + 2b^6 - 5a^2 b^4 + 4a^3 b^3 + 3a^4 b^2)) / (a^4 b + a^5 \\
& - a^2 b^3 - a^3 b^2) - (b(2a^2 - b^2) * ((32(2a^8 b - a^9 + a^4 b^5 - 3 \\
& a^6 b^3 + a^7 b^2)) / (a^5 b + a^6 - a^3 b^3 - a^4 b^2) - (32b \tan(c/2 + (d \\
& *x)/2) * (2a^2 - b^2) * (-a + b)^3 * (a - b)^3)^{(1/2)} * (2a^9 b - 2a^4 b^6 + 2 \\
& a^5 b^5 + 4a^6 b^4 - 4a^7 b^3 - 2a^8 b^2)) / ((a^4 b + a^5 - a^2 b^3 - a^3 \\
& b^2) * (a^8 - a^2 b^6 + 3a^4 b^4 - 3a^6 b^2))) * (-a + b)^3 * (a - b)^3)^{(1/2)} \\
&)) / (a^8 - a^2 b^6 + 3a^4 b^4 - 3a^6 b^2)) * (2a^2 - b^2) * (-a + b)^3 * (a - \\
& b)^3)^{(1/2)} * i) / (a^8 - a^2 b^6 + 3a^4 b^4 - 3a^6 b^2)) / ((64(2a^4 b - a \\
& b^4 + b^5 - 3a^2 b^3 + 2a^3 b^2)) / (a^5 b + a^6 - a^3 b^3 - a^4 b^2) - (b \\
& ((32 \tan(c/2 + (d*x)/2) * (a^6 - 2a^5 b - 2a^4 b^2 + 2b^6 - 5a^2 b^4 + 4a^3 \\
& b^3 + 3a^4 b^2)) / (a^4 b + a^5 - a^2 b^3 - a^3 b^2) + (b(2a^2 - b^2) * ((\\
& 32(2a^8 b - a^9 + a^4 b^5 - 3a^6 b^3 + a^7 b^2)) / (a^5 b + a^6 - a^3 b^3 \\
& - a^4 b^2) + (32b \tan(c/2 + (d*x)/2) * (2a^2 - b^2) * (-a + b)^3 * (a - b)^3)^{(1/2)} * (2a^9 b - 2a^4 b^6 + 2a^5 b^5 + 4a^6 b^4 - 4a^7 b^3 - 2a^8 b^2) \\
&)) / ((a^4 b + a^5 - a^2 b^3 - a^3 b^2) * (a^8 - a^2 b^6 + 3a^4 b^4 - 3a^6 b^2 \\
&))) * (-a + b)^3 * (a - b)^3)^{(1/2)})) / (a^8 - a^2 b^6 + 3a^4 b^4 - 3a^6 b^2)) * \\
& (2a^2 - b^2) * (-a + b)^3 * (a - b)^3)^{(1/2)})) / (a^8 - a^2 b^6 + 3a^4 b^4 - 3 \\
& a^6 b^2) + (b((32 \tan(c/2 + (d*x)/2) * (a^6 - 2a^5 b - 2a^4 b^2 + 2b^6 - 5a^2 b^4 + 4a^3 \\
& a^2 b^4 + 4a^3 b^3 + 3a^4 b^2)) / (a^4 b + a^5 - a^2 b^3 - a^3 b^2) - (b(2 \\
& a^2 - b^2) * ((32(2a^8 b - a^9 + a^4 b^5 - 3a^6 b^3 + a^7 b^2)) / (a^5 b + \\
& a^6 - a^3 b^3 - a^4 b^2) - (32b \tan(c/2 + (d*x)/2) * (2a^2 - b^2) * (-a + b) \\
& ^3 * (a - b)^3)^{(1/2)} * (2a^9 b - 2a^4 b^6 + 2a^5 b^5 + 4a^6 b^4 - 4a^7 b^3 \\
& - 2a^8 b^2)) / ((a^4 b + a^5 - a^2 b^3 - a^3 b^2) * (a^8 - a^2 b^6 + 3a^4 b^4 \\
& - 3a^6 b^2))) * (-a + b)^3 * (a - b)^3)^{(1/2)})) / (a^8 - a^2 b^6 + 3a^4 b^4 \\
& - 3a^6 b^2)) * (2a^2 - b^2) * (-a + b)^3 * (a - b)^3)^{(1/2)})) / (a^8 - a^2 b^6 + \\
& 3a^4 b^4 - 3a^6 b^2))) * (2a^2 - b^2) * (-a + b)^3 * (a - b)^3)^{(1/2)} * 2i) / (d \\
& (a^8 - a^2 b^6 + 3a^4 b^4 - 3a^6 b^2)) - (2b^2 \tan(c/2 + (d*x)/2)) / (d * (a \\
& + b) * (a * b - a^2) * (a + b + \tan(c/2 + (d*x)/2)^2 * (a - b)))
\end{aligned}$$

$$3.466 \quad \int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	4981
Rubi [A] (verified)	4981
Mathematica [A] (verified)	4984
Maple [A] (verified)	4984
Fricas [B] (verification not implemented)	4985
Sympy [F]	4985
Maxima [F(-2)]	4986
Giac [B] (verification not implemented)	4986
Mupad [B] (verification not implemented)	4987

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^2} dx = \frac{2b^2(3a^2 - 2b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} - \frac{2b \operatorname{arctanh}(\sin(c+dx))}{a^3 d} + \frac{(a^2 - 2b^2) \tan(c+dx)}{a^2(a^2 - b^2)d} + \frac{b^2 \tan(c+dx)}{a(a^2 - b^2)d(a+b \cos(c+dx))}$$

[Out] $2*b^2*(3*a^2-2*b^2)*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a^3/(a-b)^{(3/2)/(a+b)^{(3/2)/d}-2*b*\operatorname{arctanh}(\sin(d*x+c))/a^3/d+(a^2-2*b^2)*\tan(d*x+c)/a^2/(a^2-b^2)/d+b^2*\tan(d*x+c)/a/(a^2-b^2)/d/(a+b*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2881, 3134, 3080, 3855, 2738, 211}

$$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^2} dx = -\frac{2b \operatorname{arctanh}(\sin(c+dx))}{a^3 d} + \frac{(a^2 - 2b^2) \tan(c+dx)}{a^2 d(a^2 - b^2)} + \frac{b^2 \tan(c+dx)}{ad(a^2 - b^2)(a+b \cos(c+dx))} + \frac{2b^2(3a^2 - 2b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^3 d(a-b)^{3/2}(a+b)^{3/2}}$$

[In] $\text{Int}[\text{Sec}[c + d*x]^2/(a + b*\text{Cos}[c + d*x])^2, x]$

```
[Out] (2*b^2*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - (2*b*ArcTanh[Sin[c + d*x]]/(a^3*d) +
((a^2 - 2*b^2)*Tan[c + d*x])/(a^2*(a^2 - b^2)*d) + (b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3855

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{b^2 \tan(c + dx)}{a(a^2 - b^2)d(a + b \cos(c + dx))} + \frac{\int \frac{(a^2 - 2b^2 - ab \cos(c + dx) + b^2 \cos^2(c + dx)) \sec^2(c + dx)}{a + b \cos(c + dx)} dx}{a(a^2 - b^2)} \\
 &= \frac{(a^2 - 2b^2) \tan(c + dx)}{a^2(a^2 - b^2)d} + \frac{b^2 \tan(c + dx)}{a(a^2 - b^2)d(a + b \cos(c + dx))} \\
 &\quad + \frac{\int \frac{(-2b(a^2 - b^2) + ab^2 \cos(c + dx)) \sec(c + dx)}{a + b \cos(c + dx)} dx}{a^2(a^2 - b^2)} \\
 &= \frac{(a^2 - 2b^2) \tan(c + dx)}{a^2(a^2 - b^2)d} + \frac{b^2 \tan(c + dx)}{a(a^2 - b^2)d(a + b \cos(c + dx))} \\
 &\quad - \frac{(2b) \int \sec(c + dx) dx}{a^3} + \frac{(b^2(3a^2 - 2b^2)) \int \frac{1}{a + b \cos(c + dx)} dx}{a^3(a^2 - b^2)} \\
 &= -\frac{2b \operatorname{arctanh}(\sin(c + dx))}{a^3 d} + \frac{(a^2 - 2b^2) \tan(c + dx)}{a^2(a^2 - b^2)d} \\
 &\quad + \frac{b^2 \tan(c + dx)}{a(a^2 - b^2)d(a + b \cos(c + dx))} \\
 &\quad + \frac{(2b^2(3a^2 - 2b^2)) \operatorname{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{a^3(a^2 - b^2)d} \\
 &= \frac{2b^2(3a^2 - 2b^2) \arctan\left(\frac{\sqrt{a - b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b}}\right)}{a^3(a - b)^{3/2}(a + b)^{3/2}d} - \frac{2b \operatorname{arctanh}(\sin(c + dx))}{a^3 d} \\
 &\quad + \frac{(a^2 - 2b^2) \tan(c + dx)}{a^2(a^2 - b^2)d} + \frac{b^2 \tan(c + dx)}{a(a^2 - b^2)d(a + b \cos(c + dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^2} dx$$

$$= \frac{2b^2(-3a^2+2b^2) \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right) + 2b \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - 2b \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3 d}$$

[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]

[Out] ((-2*b^2*(-3*a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a*b^3*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + a*Tan[c + d*x]/(a^3*d)

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{2b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^3} - \frac{1}{a^2\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{1}{a^2\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{2b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{a^3} + \frac{2b^2 \left(-\frac{ab \tan\left(\frac{dx}{2}\right)}{(a^2 - b^2)\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a} \right)}{d}$
default	$\frac{\frac{2b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^3} - \frac{1}{a^2\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{1}{a^2\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{2b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{a^3} + \frac{2b^2 \left(-\frac{ab \tan\left(\frac{dx}{2}\right)}{(a^2 - b^2)\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a} \right)}{d}$
risch	$\frac{2i(-ab^2e^{3i(dx+c)} + a^2be^{2i(dx+c)} - 2b^3e^{i(dx+c)} + 2a^3e^{i(dx+c)} - 3ab^2e^{i(dx+c)} + a^2b - 2b^3)}{(a^2 - b^2)d a^2 (be^{2i(dx+c)} + 2ae^{i(dx+c)} + b)(e^{2i(dx+c)} + 1)} - \frac{3b^2 \ln\left(\frac{e^{i(dx+c)} + ia^2 - ib^2 + a}{b\sqrt{-a^2}}\right)}{\sqrt{-a^2 + b^2}(a+b)(a-b)}$

[In] int(sec(d*x+c)^2/(a*cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(2*b/a^3*ln(tan(1/2*d*x+1/2*c)-1)-1/a^2/(tan(1/2*d*x+1/2*c)-1)-1/a^2/(tan(1/2*d*x+1/2*c)+1)-2*b/a^3*ln(tan(1/2*d*x+1/2*c)+1)+2*b^2/a^3*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(3*a^2-2*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(146) = 292$.

Time = 0.47 (sec) , antiderivative size = 750, normalized size of antiderivative = 4.84

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^2} dx$$

$$= \left[-\frac{((3a^2b^3 - 2b^5) \cos(dx + c))^2 + (3a^3b^2 - 2ab^4) \cos(dx + c) \sqrt{-a^2 + b^2} \log\left(\frac{2ab \cos(dx+c) + (2a^2 - b^2) \cos(dx+c)}{b^2 \cos(c)}\right)}{\dots} \right]$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out]
$$[-1/2 * (((3*a^2*b^3 - 2*b^5) * \cos(d*x + c))^2 + (3*a^3*b^2 - 2*a*b^4) * \cos(d*x + c)) * \sqrt{-a^2 + b^2} * \log((2*a*b*\cos(d*x + c) + (2*a^2 - b^2) * \cos(d*x + c))^2 + 2*\sqrt{-a^2 + b^2} * (a*\cos(d*x + c) + b) * \sin(d*x + c) - a^2 + 2*b^2) / (b^2 * \cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + a^2)) + 2 * ((a^4*b^2 - 2*a^2*b^4 + b^6) * \cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5) * \cos(d*x + c)) * \log(\sin(d*x + c) + 1) - 2 * ((a^4*b^2 - 2*a^2*b^4 + b^6) * \cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5) * \cos(d*x + c)) * \log(-\sin(d*x + c) + 1) - 2 * (a^6 - 2*a^4*b^2 + a^2*b^4 + (a^5*b - 3*a^3*b^3 + 2*a*b^5) * \cos(d*x + c)) * \sin(d*x + c)) / ((a^7*b - 2*a^5*b^3 + a^3*b^5) * d * \cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4) * d * \cos(d*x + c)), (((3*a^2*b^3 - 2*b^5) * \cos(d*x + c))^2 + (3*a^3*b^2 - 2*a*b^4) * \cos(d*x + c)) * \sqrt{a^2 - b^2} * \arctan(-(a*\cos(d*x + c) + b) / (\sqrt{a^2 - b^2} * \sin(d*x + c))) - ((a^4*b^2 - 2*a^2*b^4 + b^6) * \cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5) * \cos(d*x + c)) * \log(\sin(d*x + c) + 1) + ((a^4*b^2 - 2*a^2*b^4 + b^6) * \cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5) * \cos(d*x + c)) * \log(-\sin(d*x + c) + 1) + (a^6 - 2*a^4*b^2 + a^2*b^4 + (a^5*b - 3*a^3*b^3 + 2*a*b^5) * \cos(d*x + c)) * \sin(d*x + c)) / ((a^7*b - 2*a^5*b^3 + a^3*b^5) * d * \cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4) * d * \cos(d*x + c))]$$

Sympy [F]

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^2} dx$$

[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)

[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**2, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(146) = 292.

Time = 0.33 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.14

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^2} dx = \frac{2 \left(\frac{(3a^2b^2 - 2b^4) \left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^5 - a^3b^2)\sqrt{a^2 - b^2}} \right) + a^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - a^2 b \tan(\frac{1}{2} dx + \frac{1}{2} c)^3}{(a^5 - a^3b^2)\sqrt{a^2 - b^2}}$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] -2*((3*a^2*b^2 - 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*b^3*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*tan(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^4 - a^2*b^2)) + b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - b*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)/d

$$\begin{aligned}
& 2 + (d*x)/2) * (8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5 \\
& *b^3 + 4*a^6*b^2) / (a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b^2*(3*a^2 - 2*b^2) \\
& * ((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2) \\
&) / (a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b^2*tan(c/2 + (d*x)/2)*(3*a^2 - 2 \\
& *b^2)*(-(a + b)^3*(a - b)^3)^{(1/2)}*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^ \\
& 8*b^4 - 4*a^9*b^3 - 2*a^10*b^2)) / ((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - \\
& a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))) * (-(a + b)^3*(a - b)^3)^{(1/2)} / (a^9 - a^3 \\
& *b^6 + 3*a^5*b^4 - 3*a^7*b^2)) * (3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^{(1/2)} \\
& * 1i) / (a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)) / ((64*(8*b^8 - 4*a*b^7 - 20*a^ \\
& 2*b^6 + 6*a^3*b^5 + 12*a^4*b^4)) / (a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^2*(\\
& (32*tan(c/2 + (d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b \\
& ^4 - 8*a^5*b^3 + 4*a^6*b^2)) / (a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b^2*(3*a^ \\
& 2 - 2*b^2)*((32*(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3 \\
& *a^10*b^2)) / (a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b^2*tan(c/2 + (d*x)/2)* \\
& (3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^{(1/2)}*(2*a^11*b - 2*a^6*b^6 + 2*a^7* \\
& b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2)) / ((a^6*b + a^7 - a^4*b^3 - a^5*b^ \\
& 2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))) * (-(a + b)^3*(a - b)^3)^{(1/2)} / \\
& (a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)) * (3*a^2 - 2*b^2)*(-(a + b)^3*(a - b \\
&)^3)^{(1/2)} / (a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) - (b^2*((32*tan(c/2 + (\\
& d*x)/2)*(8*b^8 - 8*a*b^7 - 16*a^2*b^6 + 16*a^3*b^5 + 5*a^4*b^4 - 8*a^5*b^3 \\
& + 4*a^6*b^2)) / (a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b^2*(3*a^2 - 2*b^2)*((32 \\
& *(2*a^11*b - 2*a^6*b^6 + a^7*b^5 + 5*a^8*b^4 - 3*a^9*b^3 - 3*a^10*b^2)) / (a^ \\
& 8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b^2*tan(c/2 + (d*x)/2)*(3*a^2 - 2*b^2) \\
& *(-(a + b)^3*(a - b)^3)^{(1/2)}*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 \\
& - 4*a^9*b^3 - 2*a^10*b^2)) / ((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b \\
& ^6 + 3*a^5*b^4 - 3*a^7*b^2))) * (-(a + b)^3*(a - b)^3)^{(1/2)} / (a^9 - a^3*b^6 \\
& + 3*a^5*b^4 - 3*a^7*b^2)) * (3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^{(1/2)} / (a^ \\
& 9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)) * (3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^ \\
& 3)^{(1/2)} * 2i) / (d*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))
\end{aligned}$$

$$3.467 \quad \int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	4989
Rubi [A] (verified)	4989
Mathematica [A] (verified)	4992
Maple [A] (verified)	4993
Fricas [B] (verification not implemented)	4993
Sympy [F]	4994
Maxima [F(-2)]	4994
Giac [A] (verification not implemented)	4995
Mupad [B] (verification not implemented)	4995

Optimal result

Integrand size = 21, antiderivative size = 217

$$\int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^2} dx = -\frac{2b^3(4a^2-3b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} + \frac{(a^2+6b^2) \operatorname{arctanh}(\sin(c+dx))}{2a^4d} - \frac{b(2a^2-3b^2) \tan(c+dx)}{a^3(a^2-b^2)d} + \frac{(a^2-3b^2) \sec(c+dx) \tan(c+dx)}{2a^2(a^2-b^2)d} + \frac{b^2 \sec(c+dx) \tan(c+dx)}{a(a^2-b^2)d(a+b \cos(c+dx))}$$

```
[Out] -2*b^3*(4*a^2-3*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4
/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*(a^2+6*b^2)*arctanh(sin(d*x+c))/a^4/d-b*(2*a
^2-3*b^2)*tan(d*x+c)/a^3/(a^2-b^2)/d+1/2*(a^2-3*b^2)*sec(d*x+c)*tan(d*x+c)/
a^2/(a^2-b^2)/d+b^2*sec(d*x+c)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {2881, 3134, 3080, 3855, 2738, 211}

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2} dx = \frac{(a^2-3b^2)\tan(c+dx)\sec(c+dx)}{2a^2d(a^2-b^2)} + \frac{b^2\tan(c+dx)\sec(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))} - \frac{2b^3(4a^2-3b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}} + \frac{(a^2+6b^2)\operatorname{arctanh}(\sin(c+dx))}{2a^4d} - \frac{b(2a^2-3b^2)\tan(c+dx)}{a^3d(a^2-b^2)}$$

[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]

[Out] (-2*b^3*(4*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (b*(2*a^2 - 3*b^2)*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2 - 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*SIN[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*SIN[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m + 1)*((c + d*SIN[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{b^2 \sec(c + dx) \tan(c + dx)}{a(a^2 - b^2) d(a + b \cos(c + dx))} + \frac{\int \frac{(a^2 - 3b^2 - ab \cos(c + dx) + 2b^2 \cos^2(c + dx)) \sec^3(c + dx)}{a + b \cos(c + dx)} dx}{a(a^2 - b^2)} \\
 &= \frac{(a^2 - 3b^2) \sec(c + dx) \tan(c + dx)}{2a^2(a^2 - b^2) d} + \frac{b^2 \sec(c + dx) \tan(c + dx)}{a(a^2 - b^2) d(a + b \cos(c + dx))} \\
 &\quad + \frac{\int \frac{(-2b(2a^2 - 3b^2) + a(a^2 + b^2) \cos(c + dx) + b(a^2 - 3b^2) \cos^2(c + dx)) \sec^2(c + dx)}{a + b \cos(c + dx)} dx}{2a^2(a^2 - b^2)} \\
 &= -\frac{b(2a^2 - 3b^2) \tan(c + dx)}{a^3(a^2 - b^2) d} + \frac{(a^2 - 3b^2) \sec(c + dx) \tan(c + dx)}{2a^2(a^2 - b^2) d} \\
 &\quad + \frac{b^2 \sec(c + dx) \tan(c + dx)}{a(a^2 - b^2) d(a + b \cos(c + dx))} + \frac{\int \frac{(a^4 + 5a^2b^2 - 6b^4 + ab(a^2 - 3b^2) \cos(c + dx)) \sec(c + dx)}{a + b \cos(c + dx)} dx}{2a^3(a^2 - b^2)}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{b(2a^2 - 3b^2) \tan(c + dx)}{a^3 (a^2 - b^2) d} + \frac{(a^2 - 3b^2) \sec(c + dx) \tan(c + dx)}{2a^2 (a^2 - b^2) d} \\
&\quad + \frac{b^2 \sec(c + dx) \tan(c + dx)}{a (a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad - \frac{(b^3(4a^2 - 3b^2)) \int \frac{1}{a+b \cos(c+dx)} dx}{a^4 (a^2 - b^2)} + \frac{(a^2 + 6b^2) \int \sec(c + dx) dx}{2a^4} \\
&= \frac{(a^2 + 6b^2) \operatorname{arctanh}(\sin(c + dx))}{2a^4 d} - \frac{b(2a^2 - 3b^2) \tan(c + dx)}{a^3 (a^2 - b^2) d} \\
&\quad + \frac{(a^2 - 3b^2) \sec(c + dx) \tan(c + dx)}{2a^2 (a^2 - b^2) d} + \frac{b^2 \sec(c + dx) \tan(c + dx)}{a (a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad - \frac{(2b^3(4a^2 - 3b^2)) \operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{a^4 (a^2 - b^2) d} \\
&= -\frac{2b^3(4a^2 - 3b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4 (a-b)^{3/2} (a+b)^{3/2} d} \\
&\quad + \frac{(a^2 + 6b^2) \operatorname{arctanh}(\sin(c + dx))}{2a^4 d} - \frac{b(2a^2 - 3b^2) \tan(c + dx)}{a^3 (a^2 - b^2) d} \\
&\quad + \frac{(a^2 - 3b^2) \sec(c + dx) \tan(c + dx)}{2a^2 (a^2 - b^2) d} + \frac{b^2 \sec(c + dx) \tan(c + dx)}{a (a^2 - b^2) d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.31

$$\begin{aligned}
&\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^2} dx \\
&= \frac{8b^3(-4a^2 + 3b^2) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} - 2a^2 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) - 12b^2 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]

[Out] ((8*b^3*(-4*a^2 + 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 12*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + a^2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*a*b^4*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) - 8*a*b*Tan[c + d*x]/(4*a^4*d)

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{1}{2a^2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{-a-4b}{2a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{(-a^2-6b^2) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2a^4} - 2b^3 \left(-\frac{ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2-b^2) \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a - b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \right)$
default	$\frac{1}{2a^2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{-a-4b}{2a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{(-a^2-6b^2) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2a^4} - 2b^3 \left(-\frac{ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a^2-b^2) \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a - b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \right)$
risch	$-\frac{i(a^3 e^{5i(dx+c)} b - 3b^3 a e^{5i(dx+c)} + 2a^4 e^{4i(dx+c)} + 2a^2 b^2 e^{4i(dx+c)} - 6b^4 e^{4i(dx+c)} + 8b a^3 e^{3i(dx+c)} - 12b^3 a e^{3i(dx+c)} - 2a^4 e^{2i(dx+c)} - 2a^2 b^2 e^{2i(dx+c)} - 2b^4 e^{2i(dx+c)} - 2a^3 b e^{2i(dx+c)} - 2a b^3 e^{2i(dx+c)} - 2a^2 b^2 e^{2i(dx+c)} - 2a b^3 e^{2i(dx+c)} - 2a^2 b^2 e^{2i(dx+c)} - 2b^4 e^{2i(dx+c)})}{d a^3 (e^{2i(dx+c)} + 1)^2 (a^2 - b^2) (b e^{2i(dx+c)} + 2a)}$

[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2/a^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-a-4*b)/a^3/(tan(1/2*d*x+1/2*c)-1)+1/2/a^4*(-a^2-6*b^2)*ln(tan(1/2*d*x+1/2*c)-1)-2*b^3/a^4*(-a*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)+(4*a^2-3*b^2)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-1/2/a^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-a-4*b)/a^3/(tan(1/2*d*x+1/2*c)+1)+1/2*(a^2+6*b^2)/a^4*ln(tan(1/2*d*x+1/2*c)+1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(204) = 408.

Time = 0.71 (sec) , antiderivative size = 899, normalized size of antiderivative = 4.14

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2} dx$$

$$= \left[-\frac{2((4a^2b^4-3b^6)\cos(dx+c)^3+(4a^3b^3-3ab^5)\cos(dx+c)^2)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)}{b^2\cos(dx+c)}\right)}{4((4a^2b^4-3b^6)\cos(dx+c)^3+(4a^3b^3-3ab^5)\cos(dx+c)^2)\sqrt{a^2-b^2}} \arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right) - \right]$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] [-1/4*(2*((4*a^2*b^4-3*b^6)*cos(d*x+c)^3+(4*a^3*b^3-3*a*b^5)*cos(d*x+c)^2)*sqrt(-a^2+b^2)*log((2*a*b*cos(d*x+c)+(2*a^2-b^2)*cos(d*x+c))

```

+ c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2
)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((a^6*b + 4*a^4*b^3 -
11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^
6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((a^6*b + 4*a^4*b^3 - 11*a^2*b^5
+ 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) - 2*(a^7 - 2*a^5*b^2 + a^3*b^4 - 2*(2*a^5*b
^2 - 5*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2 - 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*
cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3
+ (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2), -1/4*(4*((4*a^2*b^4 - 3*b
^6)*cos(d*x + c)^3 + (4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*
arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((a^6*b + 4*
a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^
4 + 6*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((a^6*b + 4*a^4*b^3 -
11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^
6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^7 - 2*a^5*b^2 + a^3*b^4 -
2*(2*a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2 - 3*(a^6*b - 2*a^4*b^3 +
a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(
d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2)]

```

Sympy [F]

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^2} dx$$

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^2} dx$$

$$= \frac{4b^4 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{(a^5 - a^3b^2)\left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a + b\right)} + \frac{4(4a^2b^3 - 3b^5)\left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^6 - a^4b^2)\sqrt{a^2 - b^2}}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^4*tan(1/2*d*x + 1/2*c)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 4*(4*a^2*b^3 - 3*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) + (a^2 + 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - (a^2 + 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) - 4*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 21.33 (sec) , antiderivative size = 3699, normalized size of antiderivative = 17.05

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Too large to display}$$

```
[In] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^2),x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)*(3*a*b^3 - 3*a^3*b + a^4 + 6*b^4 - 5*a^2*b^2))/((a^3*b - a^4)*(a + b)) + (tan(c/2 + (d*x)/2)^5*(3*a^3*b - 3*a*b^3 + a^4 + 6*b^4 - 5*a^2*b^2))/((a^3*b - a^4)*(a + b)) + (2*tan(c/2 + (d*x)/2)^3*(a^4 - 6*b^4 + 3*a^2*b^2))/(a*(a^2*b - a^3)*(a + b)))/(d*(a + b - tan(c/2 + (d*x)/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2)^4*(a - 3*b) + tan(c/2 + (d*x)/2)^6*(a - b)) - (atan((((a^2 + 6*b^2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2)))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - ((a^2 + 6*b^2)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2)))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2 + 6*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))))/(2*a^4)*1i)/(2*a^4) + ((a^2 + 6*b^2)*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b -
```

$$\begin{aligned}
& 72*a*b^9 + 72*b^{10} - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26*a^5*b^5 + \\
& 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2)/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + \\
& ((a^2 + 6*b^2)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11* \\
& *b^4 - 16*a^12*b^3 + 6*a^13*b^2)))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (4 \\
& *tan(c/2 + (d*x)/2)*(a^2 + 6*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^ \\
& 10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)) \\
&))/(2*a^4))*1i)/(2*a^4))/((16*(108*b^11 - 54*a*b^10 - 216*a^2*b^9 + 81*a^3* \\
& b^8 + 63*a^4*b^7 - 9*a^5*b^6 + 41*a^6*b^5 - 4*a^7*b^4 + 4*a^8*b^3))/(a^11*b \\
& + a^12 - a^9*b^3 - a^10*b^2) - ((a^2 + 6*b^2)*((8*tan(c/2 + (d*x)/2)*(a^10 \\
& - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - \\
& 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 \\
& - a^7*b^2) - ((a^2 + 6*b^2)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10* \\
& b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2)))/(a^11*b + a^12 - a^9*b^3 - a \\
& ^10*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2 + 6*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^ \\
& 9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^6*b^ \\
& 3 - a^7*b^2))))/(2*a^4))/((2*a^4)) + ((a^2 + 6*b^2)*((8*tan(c/2 + (d*x)/2)*(\\
& a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^ \\
& 6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6* \\
& b^3 - a^7*b^2) + ((a^2 + 6*b^2)*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a \\
& ^10*b^5 - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2)))/(a^11*b + a^12 - a^9*b^3 \\
& - a^10*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2 + 6*b^2)*(8*a^13*b - 8*a^8*b^6 + \\
& 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/(a^4*(a^8*b + a^9 - a^ \\
& 6*b^3 - a^7*b^2))))/(2*a^4))/((2*a^4))*(a^2 + 6*b^2)*1i)/(a^4*d) - (b^3*at \\
& an(((b^3*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))*((8*tan(c/2 + (d*x)/2 \\
&)*(a^10 - 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4 \\
& *b^6 - 26*a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a \\
& ^6*b^3 - a^7*b^2) + (b^3*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 \\
& - 14*a^11*b^4 - 16*a^12*b^3 + 6*a^13*b^2)))/(a^11*b + a^12 - a^9*b^3 - a^10 \\
& *b^2) + (8*b^3*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1 \\
& /2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12* \\
& b^2)))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^ \\
& 8*b^2)))*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^10 - a^4*b^6 + 3* \\
& a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + (b^3*(\\
& 4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))*((8*tan(c/2 + (d*x)/2)*(a^10 - \\
& 2*a^9*b - 72*a*b^9 + 72*b^10 - 120*a^2*b^8 + 120*a^3*b^7 + 17*a^4*b^6 - 26* \\
& a^5*b^5 + 23*a^6*b^4 - 20*a^7*b^3 + 11*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a \\
& ^7*b^2) - (b^3*((8*(2*a^15 - 12*a^8*b^7 + 6*a^9*b^6 + 28*a^10*b^5 - 14*a^11 \\
& *b^4 - 16*a^12*b^3 + 6*a^13*b^2)))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (8 \\
& *b^3*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*(8*a^1 \\
& 3*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)))/((a^ \\
& 8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(\\
& 4*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - \\
& 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))/((16*(108*b^11 - \\
& 54*a*b^10 - 216*a^2*b^9 + 81*a^3*b^8 + 63*a^4*b^7 - 9*a^5*b^6 + 41*a^6*b^5 \\
& - 4*a^7*b^4 + 4*a^8*b^3))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (b^3*(4*a^
\end{aligned}$$

$$\begin{aligned}
& (2 - 3b^2) * (-(a + b)^3 * (a - b)^3)^{(1/2)} * ((8 * \tan(c/2 + (d*x)/2) * (a^{10} - 2a^9b - 72a^8b^2 + 72b^{10} - 120a^2b^8 + 120a^3b^7 + 17a^4b^6 - 26a^5b^5 + 23a^6b^4 - 20a^7b^3 + 11a^8b^2)) / (a^8b + a^9 - a^6b^3 - a^7b^2) + (b^3 * ((8 * (2a^{15} - 12a^8b^7 + 6a^9b^6 + 28a^{10}b^5 - 14a^{11}b^4 - 16a^{12}b^3 + 6a^{13}b^2)) / (a^{11}b + a^{12} - a^9b^3 - a^{10}b^2) + (8b^3 * \tan(c/2 + (d*x)/2) * (4a^2 - 3b^2) * (-(a + b)^3 * (a - b)^3)^{(1/2)} * (8a^{13}b - 8a^8b^6 + 8a^9b^5 + 16a^{10}b^4 - 16a^{11}b^3 - 8a^{12}b^2)) / ((a^8b + a^9 - a^6b^3 - a^7b^2) * (a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2))) * (4a^2 - 3b^2) * (-(a + b)^3 * (a - b)^3)^{(1/2)}) / (a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)) / (a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2) - (b^3 * (4a^2 - 3b^2) * (-(a + b)^3 * (a - b)^3)^{(1/2)} * ((8 * \tan(c/2 + (d*x)/2) * (a^{10} - 2a^9b - 72a^8b^2 + 72b^{10} - 120a^2b^8 + 120a^3b^7 + 17a^4b^6 - 26a^5b^5 + 23a^6b^4 - 20a^7b^3 + 11a^8b^2)) / (a^8b + a^9 - a^6b^3 - a^7b^2) - (b^3 * ((8 * (2a^{15} - 12a^8b^7 + 6a^9b^6 + 28a^{10}b^5 - 14a^{11}b^4 - 16a^{12}b^3 + 6a^{13}b^2)) / (a^{11}b + a^{12} - a^9b^3 - a^{10}b^2) - (8b^3 * \tan(c/2 + (d*x)/2) * (4a^2 - 3b^2) * (-(a + b)^3 * (a - b)^3)^{(1/2)} * (8a^{13}b - 8a^8b^6 + 8a^9b^5 + 16a^{10}b^4 - 16a^{11}b^3 - 8a^{12}b^2)) / ((a^8b + a^9 - a^6b^3 - a^7b^2) * (a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2))) * (4a^2 - 3b^2) * (-(a + b)^3 * (a - b)^3)^{(1/2)}) / (a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)) / (a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2)) * (4a^2 - 3b^2) * (-(a + b)^3 * (a - b)^3)^{(1/2)} * 2i) / (d * (a^{10} - a^4b^6 + 3a^6b^4 - 3a^8b^2))
\end{aligned}$$

3.468 $\int \frac{\sec^4(c+dx)}{(a+b \cos(c+dx))^2} dx$

Optimal result	4998
Rubi [A] (verified)	4999
Mathematica [A] (verified)	5002
Maple [A] (verified)	5003
Fricas [A] (verification not implemented)	5003
Sympy [F]	5004
Maxima [F(-2)]	5004
Giac [A] (verification not implemented)	5005
Mupad [B] (verification not implemented)	5005

Optimal result

Integrand size = 21, antiderivative size = 270

$$\int \frac{\sec^4(c+dx)}{(a+b \cos(c+dx))^2} dx = \frac{2b^4(5a^2 - 4b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b(a^2 + 4b^2) \operatorname{arctanh}(\sin(c+dx))}{a^5d} + \frac{(2a^4 + 7a^2b^2 - 12b^4) \tan(c+dx)}{3a^4(a^2 - b^2)d} - \frac{b(a^2 - 2b^2) \sec(c+dx) \tan(c+dx)}{a^3(a^2 - b^2)d} + \frac{(a^2 - 4b^2) \sec^2(c+dx) \tan(c+dx)}{3a^2(a^2 - b^2)d} + \frac{b^2 \sec^2(c+dx) \tan(c+dx)}{a(a^2 - b^2)d(a+b \cos(c+dx))}$$

```
[Out] 2*b^4*(5*a^2-4*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/
(a-b)^(3/2)/(a+b)^(3/2)/d-b*(a^2+4*b^2)*arctanh(sin(d*x+c))/a^5/d+1/3*(2*a^
4+7*a^2*b^2-12*b^4)*tan(d*x+c)/a^4/(a^2-b^2)/d-b*(a^2-2*b^2)*sec(d*x+c)*tan
(d*x+c)/a^3/(a^2-b^2)/d+1/3*(a^2-4*b^2)*sec(d*x+c)^2*tan(d*x+c)/a^2/(a^2-b^
2)/d+b^2*sec(d*x+c)^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2881, 3134, 3080, 3855, 2738, 211}

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2} dx = \frac{(a^2-4b^2)\tan(c+dx)\sec^2(c+dx)}{3a^2d(a^2-b^2)} + \frac{b^2\tan(c+dx)\sec^2(c+dx)}{ad(a^2-b^2)(a+b\cos(c+dx))} + \frac{2b^4(5a^2-4b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}} - \frac{b(a^2+4b^2)\operatorname{arctanh}(\sin(c+dx))}{a^5d} + \frac{(2a^4+7a^2b^2-12b^4)\tan(c+dx)}{3a^4d(a^2-b^2)} - \frac{b(a^2-2b^2)\tan(c+dx)\sec(c+dx)}{a^3d(a^2-b^2)}$$

[In] Int[Sec[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]

[Out] (2*b^4*(5*a^2 - 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(a^5*d) + ((2*a^4 + 7*a^2*b^2 - 12*b^4)*Tan[c + d*x])/(3*a^4*(a^2 - b^2)*d) - (b*(a^2 - 2*b^2)*Sec[c + d*x]*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2 - 4*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2

```

)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

```

Rule 3080

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3134

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))

```

Rule 3855

```

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \sec^2(c + dx) \tan(c + dx)}{a(a^2 - b^2) d(a + b \cos(c + dx))} + \frac{\int \frac{(a^2 - 4b^2 - ab \cos(c + dx) + 3b^2 \cos^2(c + dx)) \sec^4(c + dx)}{a + b \cos(c + dx)} dx}{a(a^2 - b^2)} \\
&= \frac{(a^2 - 4b^2) \sec^2(c + dx) \tan(c + dx)}{3a^2(a^2 - b^2) d} + \frac{b^2 \sec^2(c + dx) \tan(c + dx)}{a(a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{(-6b(a^2 - 2b^2) + a(2a^2 + b^2) \cos(c + dx) + 2b(a^2 - 4b^2) \cos^2(c + dx)) \sec^3(c + dx)}{a + b \cos(c + dx)} dx}{3a^2(a^2 - b^2)}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{b(a^2 - 2b^2) \sec(c + dx) \tan(c + dx)}{a^3 (a^2 - b^2) d} \\
&\quad + \frac{(a^2 - 4b^2) \sec^2(c + dx) \tan(c + dx)}{3a^2 (a^2 - b^2) d} + \frac{b^2 \sec^2(c + dx) \tan(c + dx)}{a (a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{(2(2a^4 + 7a^2b^2 - 12b^4) - 2ab(a^2 + 2b^2) \cos(c + dx) - 6b^2(a^2 - 2b^2) \cos^2(c + dx)) \sec^2(c + dx)}{a + b \cos(c + dx)} dx}{6a^3 (a^2 - b^2)} \\
&= \frac{(2a^4 + 7a^2b^2 - 12b^4) \tan(c + dx)}{3a^4 (a^2 - b^2) d} - \frac{b(a^2 - 2b^2) \sec(c + dx) \tan(c + dx)}{a^3 (a^2 - b^2) d} \\
&\quad + \frac{(a^2 - 4b^2) \sec^2(c + dx) \tan(c + dx)}{3a^2 (a^2 - b^2) d} + \frac{b^2 \sec^2(c + dx) \tan(c + dx)}{a (a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{(-6b(a^4 + 3a^2b^2 - 4b^4) - 6ab^2(a^2 - 2b^2) \cos(c + dx)) \sec(c + dx)}{a + b \cos(c + dx)} dx}{6a^4 (a^2 - b^2)} \\
&= \frac{(2a^4 + 7a^2b^2 - 12b^4) \tan(c + dx)}{3a^4 (a^2 - b^2) d} - \frac{b(a^2 - 2b^2) \sec(c + dx) \tan(c + dx)}{a^3 (a^2 - b^2) d} \\
&\quad + \frac{(a^2 - 4b^2) \sec^2(c + dx) \tan(c + dx)}{3a^2 (a^2 - b^2) d} + \frac{b^2 \sec^2(c + dx) \tan(c + dx)}{a (a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad + \frac{(b^4(5a^2 - 4b^2)) \int \frac{1}{a + b \cos(c + dx)} dx}{a^5 (a^2 - b^2)} - \frac{(b(a^2 + 4b^2)) \int \sec(c + dx) dx}{a^5} \\
&= -\frac{b(a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{a^5 d} + \frac{(2a^4 + 7a^2b^2 - 12b^4) \tan(c + dx)}{3a^4 (a^2 - b^2) d} \\
&\quad - \frac{b(a^2 - 2b^2) \sec(c + dx) \tan(c + dx)}{a^3 (a^2 - b^2) d} \\
&\quad + \frac{(a^2 - 4b^2) \sec^2(c + dx) \tan(c + dx)}{3a^2 (a^2 - b^2) d} + \frac{b^2 \sec^2(c + dx) \tan(c + dx)}{a (a^2 - b^2) d(a + b \cos(c + dx))} \\
&\quad + \frac{(2b^4(5a^2 - 4b^2)) \operatorname{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{a^5 (a^2 - b^2) d} \\
&= \frac{2b^4(5a^2 - 4b^2) \arctan\left(\frac{\sqrt{a - b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b}}\right)}{a^5 (a - b)^{3/2} (a + b)^{3/2} d} - \frac{b(a^2 + 4b^2) \operatorname{arctanh}(\sin(c + dx))}{a^5 d} \\
&\quad + \frac{(2a^4 + 7a^2b^2 - 12b^4) \tan(c + dx)}{3a^4 (a^2 - b^2) d} - \frac{b(a^2 - 2b^2) \sec(c + dx) \tan(c + dx)}{a^3 (a^2 - b^2) d} \\
&\quad + \frac{(a^2 - 4b^2) \sec^2(c + dx) \tan(c + dx)}{3a^2 (a^2 - b^2) d} + \frac{b^2 \sec^2(c + dx) \tan(c + dx)}{a (a^2 - b^2) d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.48 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.85

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2} dx = & -\frac{2b^4(5a^2-4b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{a^5(a^2-b^2)\sqrt{-a^2+b^2}d} \\
& + \frac{(a^2b+4b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} \\
& + \frac{(-a^2b-4b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} \\
& + \frac{a-6b}{12a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\
& + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{6a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} \\
& + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{6a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} \\
& + \frac{-a+6b}{12a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} \\
& + \frac{2a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)}{3a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\
& + \frac{2a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)}{3a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} \\
& - \frac{b^5\sin(c+dx)}{a^4(a-b)(a+b)d(a+b\cos(c+dx))}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]

```

[Out] (-2*b^4*(5*a^2 - 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]/(a^5*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d) + ((a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]/(a^5*d) + ((-a^2*b) - 4*b^3)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]/(a^5*d) + (a - 6*b)/(12*a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + Sin[(c + d*x)/2]/(6*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + Sin[(c + d*x)/2]/(6*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + (-a + 6*b)/(12*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2])/(3*a^4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2])/(3*a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) - (b^5*Sin[c + d*x])/(a^4*(a - b)*(a + b)*d*(a + b*Cos[c + d*x]))

```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{1}{3a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+2b}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a^2+ab+3b^2}{a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\left(a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^5}-\frac{1}{3a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{\dots}$
default	$\frac{-\frac{1}{3a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+2b}{2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a^2+ab+3b^2}{a^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{b\left(a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^5}-\frac{1}{3a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{\dots}$
risch	$\frac{2i\left(3a^3b^2e^{7i(dx+c)}-6ab^4e^{7i(dx+c)}+6ba^4e^{6i(dx+c)}+3a^2b^3e^{6i(dx+c)}-12b^5e^{6i(dx+c)}+21a^3b^2e^{5i(dx+c)}-30ab^4e^{5i(dx+c)}\right)}{\dots}$

[In] `int(sec(d*x+c)^4/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{d}\left(-\frac{1}{3}\frac{1}{a^2}\left(\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)-1\right)^{-3}-\frac{1}{2}\frac{a+2b}{a^3}\left(\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)-1\right)^{-2}-\frac{a^2+ab+3b^2}{a^4}\left(\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)-1\right)^{-1}+b\frac{a^2+4b^2}{a^5}\ln\left(\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)-1\right)-\frac{1}{3}\frac{1}{a^2}\left(\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)+1\right)^{-3}-\frac{1}{2}\frac{-a-2b}{a^3}\left(\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)+1\right)^{-2}-\frac{a^2+ab+3b^2}{a^4}\left(\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)+1\right)^{-1}-b\frac{a^2+4b^2}{a^5}\ln\left(\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)+1\right)+2b^4\frac{1}{a^5}\left(-\frac{ab}{a^2-b^2}\right)\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\left(\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\frac{a-b\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+a+b}{(a-b)(a+b)}+\frac{5a^2-4b^2}{(a-b)(a+b)}\arctan\left(\frac{(a-b)\tan\left(\frac{1}{2}d*x+\frac{1}{2}c\right)}{(a-b)(a+b)}\right)\right)$

Fricas [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.71

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2} dx = \text{Too large to display}$$

[In] `integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="fricas")`

[Out] $[-\frac{1}{6}\left(3\left(5a^2b^5-4b^7\right)\cos\left(d*x+c\right)^4+\left(5a^3b^4-4ab^6\right)\cos\left(d*x+c\right)^3\right)\sqrt{-a^2+b^2}\log\left(\left(2ab\cos\left(d*x+c\right)+\left(2a^2-b^2\right)\cos\left(d*x+c\right)^2+2\sqrt{-a^2+b^2}\left(a\cos\left(d*x+c\right)+b\right)\sin\left(d*x+c\right)-a^2+2b^2\right)\left(b^2\cos\left(d*x+c\right)^2+2ab\cos\left(d*x+c\right)+a^2\right)\right)+3\left(\left(a^6b^2+2a^4b^4-7a^2b^6+4b^8\right)\cos\left(d*x+c\right)^4+\left(a^7b+2a^5b^3-7a^3b^5+4ab^7\right)\cos\left(d*x+c\right)^3\right)\log\left(\sin\left(d*x+c\right)+1\right)-3\left(\left(a^6b^2+2a^4b^4-7a^2b^6+4b^8\right)\cos\left(d*x+c\right)^4+\left(a^7b+2a^5b^3-7a^3b^5+4ab^7\right)\cos\left(d*x+c\right)^3\right)$

$$\begin{aligned} & * \cos(dx + c)^3 * \log(-\sin(dx + c) + 1) - 2*(a^8 - 2*a^6*b^2 + a^4*b^4 + (2 \\ & * a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*\cos(dx + c)^3 + 2*(a^8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*\cos(dx + c)^2 - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5) \\ &)*\cos(dx + c))*\sin(dx + c))/((a^9*b - 2*a^7*b^3 + a^5*b^5)*d*\cos(dx + c) \\ & ^4 + (a^{10} - 2*a^8*b^2 + a^6*b^4)*d*\cos(dx + c)^3), 1/6*(6*((5*a^2*b^5 - 4 \\ & * b^7)*\cos(dx + c)^4 + (5*a^3*b^4 - 4*a*b^6)*\cos(dx + c)^3)*\sqrt{a^2 - b^2} \\ &)*\arctan(-(a*\cos(dx + c) + b)/(\sqrt{a^2 - b^2}*\sin(dx + c))) - 3*((a^6*b^2 + 2* \\ & a^4*b^4 - 7*a^2*b^6 + 4*b^8)*\cos(dx + c)^4 + (a^7*b + 2*a^5*b^3 - 7* \\ & a^3*b^5 + 4*a*b^7)*\cos(dx + c)^3)*\log(\sin(dx + c) + 1) + 3*((a^6*b^2 + 2* \\ & a^4*b^4 - 7*a^2*b^6 + 4*b^8)*\cos(dx + c)^4 + (a^7*b + 2*a^5*b^3 - 7*a^3*b^5 \\ & + 4*a*b^7)*\cos(dx + c)^3)*\log(-\sin(dx + c) + 1) + 2*(a^8 - 2*a^6*b^2 + \\ & a^4*b^4 + (2*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*\cos(dx + c)^3 + 2* \\ & (a^8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*\cos(dx + c)^2 - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*\cos(dx + c))*\sin(dx + c))/((a^9*b - 2*a^7*b^3 + a^5*b^5)*d* \\ & \cos(dx + c)^4 + (a^{10} - 2*a^8*b^2 + a^6*b^4)*d*\cos(dx + c)^3)] \end{aligned}$$

Sympy [F]

$$\int \frac{\sec^4(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\sec^4(c + dx)}{(a + b \cos(c + dx))^2} dx$$

[In] integrate(sec(dx+c)**4/(a+b*cos(dx+c))**2,x)

[Out] Integral(sec(c + dx)**4/(a + b*cos(c + dx))**2, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^4(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

[In] integrate(sec(dx+c)^4/(a+b*cos(dx+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.36

$$\int \frac{\sec^4(c + dx)}{(a + b \cos(c + dx))^2} dx =$$

$$\frac{6b^5 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{(a^6 - a^4b^2)\left(a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a + b\right)} + \frac{6(5a^2b^4 - 4b^6)\left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{a^2 - b^2}}\right)\right)}{(a^7 - a^5b^2)\sqrt{a^2 - b^2}}$$

[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="giac")

```
[Out] -1/3*(6*b^5*tan(1/2*d*x + 1/2*c)/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 6*(5*a^2*b^4 - 4*b^6)*(pi*floor(1/2
*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*
tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - a^5*b^2)*sqrt(a^2 - b^2)) +
3*(a^2*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 3*(a^2*b + 4*b^
3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5
+ 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*a^2*tan(
1/2*d*x + 1/2*c)^3 - 18*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/
2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 - 1)^3*a^4))/d
```

Mupad [B] (verification not implemented)

Time = 21.28 (sec) , antiderivative size = 3843, normalized size of antiderivative = 14.23

$$\int \frac{\sec^4(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Too large to display}$$

[In] int(1/(cos(c + d*x)^4*(a + b*cos(c + d*x))^2),x)

```
[Out] ((2*tan(c/2 + (d*x)/2)^7*(a^5 - 2*a*b^4 + 4*b^5 - 3*a^2*b^3 + a^3*b^2))/(a^
4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)^3*(6*a*b^4 - 8*a^4*b + a^5 + 36*
b^5 - 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)
/2)^5*(6*a*b^4 + 8*a^4*b + a^5 - 36*b^5 + 19*a^2*b^3 - 7*a^3*b^2))/(3*a^4*(
a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)*(a^5 - 2*a*b^4 - 4*b^5 + 3*a^2*b^3
+ a^3*b^2))/(a^4*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^8*(a - b
- tan(c/2 + (d*x)/2)^2*(2*a + 4*b) + tan(c/2 + (d*x)/2)^6*(2*a - 4*b) + 6*
b*tan(c/2 + (d*x)/2)^4)) + (b*atan(((b*(a^2 + 4*b^2))*((32*tan(c/2 + (d*x)/2
)*(32*b^12 - 32*a*b^11 - 48*a^2*b^10 + 48*a^3*b^9 + 2*a^4*b^8 - 2*a^5*b^7 +
7*a^6*b^6 - 12*a^7*b^5 + 7*a^8*b^4 - 2*a^9*b^3 + a^10*b^2)))/(a^10*b + a^11
- a^8*b^3 - a^9*b^2) + (b*(a^2 + 4*b^2))*((32*(a^17*b - 4*a^10*b^8 + 2*a^11
```

$$\begin{aligned}
& *b^7 + 9a^{12}b^6 - 4a^{13}b^5 - 5a^{14}b^4 + a^{15}b^3) / (a^{14}b + a^{15} - a^{12}b^3 - a^{13}b^2) + (32b \tan(c/2 + (dx)/2) * (a^2 + 4b^2) * (2a^{15}b - 2a^{10}b^6 + 2a^{11}b^5 + 4a^{12}b^4 - 4a^{13}b^3 - 2a^{14}b^2)) / (a^5 * (a^{10}b + a^{11} - a^8b^3 - a^9b^2))) / a^5 + (b * (a^2 + 4b^2) * ((32 \tan(c/2 + (dx)/2) * (32b^{12} - 32a * b^{11} - 48a^2 * b^{10} + 48a^3 * b^9 + 2a^4 * b^8 - 2a^5 * b^7 + 7a^6 * b^6 - 12a^7 * b^5 + 7a^8 * b^4 - 2a^9 * b^3 + a^{10} * b^2)) / (a^{10}b + a^{11} - a^8b^3 - a^9b^2) - (b * (a^2 + 4b^2) * ((32 * (a^{17} * b - 4a^{10} * b^8 + 2a^{11} * b^7 + 9a^{12} * b^6 - 4a^{13} * b^5 - 5a^{14} * b^4 + a^{15} * b^3)) / (a^{14}b + a^{15} - a^{12}b^3 - a^{13}b^2) - (32b \tan(c/2 + (dx)/2) * (a^2 + 4b^2) * (2a^{15}b - 2a^{10}b^6 + 2a^{11}b^5 + 4a^{12}b^4 - 4a^{13}b^3 - 2a^{14}b^2)) / (a^5 * (a^{10}b + a^{11} - a^8b^3 - a^9b^2)))))) / a^5) / ((64 * (64b^{14} - 32a * b^{13} - 112a^2 * b^{12} + 48a^3 * b^{11} + 12a^4 * b^{10} - 6a^5 * b^9 + 31a^6 * b^8 - 5a^7 * b^7 + 5a^8 * b^6)) / (a^{14}b + a^{15} - a^{12}b^3 - a^{13}b^2) + (b * (a^2 + 4b^2) * ((32 \tan(c/2 + (dx)/2) * (32b^{12} - 32a * b^{11} - 48a^2 * b^{10} + 48a^3 * b^9 + 2a^4 * b^8 - 2a^5 * b^7 + 7a^6 * b^6 - 12a^7 * b^5 + 7a^8 * b^4 - 2a^9 * b^3 + a^{10} * b^2)) / (a^{10}b + a^{11} - a^8b^3 - a^9b^2) + (b * (a^2 + 4b^2) * ((32 * (a^{17} * b - 4a^{10} * b^8 + 2a^{11} * b^7 + 9a^{12} * b^6 - 4a^{13} * b^5 - 5a^{14} * b^4 + a^{15} * b^3)) / (a^{14}b + a^{15} - a^{12}b^3 - a^{13}b^2) + (32b \tan(c/2 + (dx)/2) * (a^2 + 4b^2) * (2a^{15}b - 2a^{10}b^6 + 2a^{11}b^5 + 4a^{12}b^4 - 4a^{13}b^3 - 2a^{14}b^2)) / (a^5 * (a^{10}b + a^{11} - a^8b^3 - a^9b^2)))))) / a^5) / a^5 - (b * (a^2 + 4b^2) * ((32 \tan(c/2 + (dx)/2) * (32b^{12} - 32a * b^{11} - 48a^2 * b^{10} + 48a^3 * b^9 + 2a^4 * b^8 - 2a^5 * b^7 + 7a^6 * b^6 - 12a^7 * b^5 + 7a^8 * b^4 - 2a^9 * b^3 + a^{10} * b^2)) / (a^{10}b + a^{11} - a^8b^3 - a^9b^2) - (b * (a^2 + 4b^2) * ((32 * (a^{17} * b - 4a^{10} * b^8 + 2a^{11} * b^7 + 9a^{12} * b^6 - 4a^{13} * b^5 - 5a^{14} * b^4 + a^{15} * b^3)) / (a^{14}b + a^{15} - a^{12}b^3 - a^{13}b^2) - (32b \tan(c/2 + (dx)/2) * (a^2 + 4b^2) * (2a^{15}b - 2a^{10}b^6 + 2a^{11}b^5 + 4a^{12}b^4 - 4a^{13}b^3 - 2a^{14}b^2)) / (a^5 * (a^{10}b + a^{11} - a^8b^3 - a^9b^2)))))) / a^5) / a^5) * (a^2 + 4b^2) * 2i) / (a^5 * d) + (b^4 * atan(((b^4 * (5a^2 - 4b^2) * (-a + b)^3 * (a - b)^3)^(1/2) * ((32 \tan(c/2 + (dx)/2) * (32b^{12} - 32a * b^{11} - 48a^2 * b^{10} + 48a^3 * b^9 + 2a^4 * b^8 - 2a^5 * b^7 + 7a^6 * b^6 - 12a^7 * b^5 + 7a^8 * b^4 - 2a^9 * b^3 + a^{10} * b^2)) / (a^{10}b + a^{11} - a^8b^3 - a^9b^2) + (b^4 * ((32 * (a^{17} * b - 4a^{10} * b^8 + 2a^{11} * b^7 + 9a^{12} * b^6 - 4a^{13} * b^5 - 5a^{14} * b^4 + a^{15} * b^3)) / (a^{14}b + a^{15} - a^{12}b^3 - a^{13}b^2) + (32b^4 \tan(c/2 + (dx)/2) * (5a^2 - 4b^2) * (-a + b)^3 * (a - b)^3)^(1/2) * (2a^{15}b - 2a^{10}b^6 + 2a^{11}b^5 + 4a^{12}b^4 - 4a^{13}b^3 - 2a^{14}b^2)) / ((a^{10}b + a^{11} - a^8b^3 - a^9b^2) * (a^{11} - a^5b^6 + 3a^7b^4 - 3a^9b^2)))) * (5a^2 - 4b^2) * (-a + b)^3 * (a - b)^3)^(1/2)) / (a^{11} - a^5b^6 + 3a^7b^4 - 3a^9b^2)) * i) / (a^{11} - a^5b^6 + 3a^7b^4 - 3a^9b^2) + (b^4 * (5a^2 - 4b^2) * (-a + b)^3 * (a - b)^3)^(1/2) * ((32 \tan(c/2 + (dx)/2) * (32b^{12} - 32a * b^{11} - 48a^2 * b^{10} + 48a^3 * b^9 + 2a^4 * b^8 - 2a^5 * b^7 + 7a^6 * b^6 - 12a^7 * b^5 + 7a^8 * b^4 - 2a^9 * b^3 + a^{10} * b^2)) / (a^{10}b + a^{11} - a^8b^3 - a^9b^2) - (b^4 * ((32 * (a^{17} * b - 4a^{10} * b^8 + 2a^{11} * b^7 + 9a^{12} * b^6 - 4a^{13} * b^5 - 5a^{14} * b^4 + a^{15} * b^3)) / (a^{14}b + a^{15} - a^{12}b^3 - a^{13}b^2) - (32b^4 \tan(c/2 + (dx)/2) * (5a^2 - 4b^2) * (-a + b)^3 * (a - b)^3)^(1/2) * (2a^{15}b - 2a^{10}b^6 + 2a^{11}b^5 + 4a^{12}b^4 - 4a^{13}b^3 - 2a^{14}b^2)) / ((a^{10}b + a^{11} - a^8b^3
\end{aligned}$$

$$\begin{aligned}
& - a^9 b^2) (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2)) (5 a^2 - 4 b^2) (- (a \\
& + b)^3 (a - b)^3)^{(1/2)} / (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2) * i) / (a^{11} \\
& - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2) / ((64 (64 b^{14} - 32 a b^{13} - 112 a^2 b^{12} \\
& + 48 a^3 b^{11} + 12 a^4 b^{10} - 6 a^5 b^9 + 31 a^6 b^8 - 5 a^7 b^7 + 5 a^8 b^6)) / (a^{14} b + a^{15} - a^{12} b^3 - a^{13} b^2) + (b^4 (5 a^2 - 4 b^2) (- (a + \\
& b)^3 (a - b)^3)^{(1/2)} ((32 \tan(c/2 + (d*x)/2) (32 b^{12} - 32 a b^{11} - 48 a^2 b^{10} \\
& + 48 a^3 b^9 + 2 a^4 b^8 - 2 a^5 b^7 + 7 a^6 b^6 - 12 a^7 b^5 + 7 a^8 b^4 - 2 a^9 b^3 + a^{10} b^2)) / (a^{10} b + a^{11} - a^8 b^3 - a^9 b^2) + (b^4 (\\
& (32 (a^{17} b - 4 a^{10} b^8 + 2 a^{11} b^7 + 9 a^{12} b^6 - 4 a^{13} b^5 - 5 a^{14} b^4 + a^{15} b^3)) / (a^{14} b + a^{15} - a^{12} b^3 - a^{13} b^2) + (32 b^4 \tan(c/2 + (d \\
& *x)/2) (5 a^2 - 4 b^2) (- (a + b)^3 (a - b)^3)^{(1/2)} (2 a^{15} b - 2 a^{10} b^6 \\
& + 2 a^{11} b^5 + 4 a^{12} b^4 - 4 a^{13} b^3 - 2 a^{14} b^2)) / ((a^{10} b + a^{11} - a^8 b^3 - a^9 b^2) (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2)) (5 a^2 - 4 b^2) * \\
& (- (a + b)^3 (a - b)^3)^{(1/2)} / (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2)) / (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2) - (b^4 (5 a^2 - 4 b^2) (- (a + b)^3 (\\
& a - b)^3)^{(1/2)} ((32 \tan(c/2 + (d*x)/2) (32 b^{12} - 32 a b^{11} - 48 a^2 b^{10} \\
& + 48 a^3 b^9 + 2 a^4 b^8 - 2 a^5 b^7 + 7 a^6 b^6 - 12 a^7 b^5 + 7 a^8 b^4 - 2 a^9 b^3 + a^{10} b^2)) / (a^{10} b + a^{11} - a^8 b^3 - a^9 b^2) - (b^4 ((32 (a^{17} b - 4 a^{10} b^8 + 2 a^{11} b^7 + 9 a^{12} b^6 - 4 a^{13} b^5 - 5 a^{14} b^4 + a^{15} b^3)) / (a^{14} b + a^{15} - a^{12} b^3 - a^{13} b^2) - (32 b^4 \tan(c/2 + (d*x)/2) * \\
& (5 a^2 - 4 b^2) (- (a + b)^3 (a - b)^3)^{(1/2)} (2 a^{15} b - 2 a^{10} b^6 + 2 a^{11} b^5 + 4 a^{12} b^4 - 4 a^{13} b^3 - 2 a^{14} b^2)) / ((a^{10} b + a^{11} - a^8 b^3 - a^9 b^2) (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2)) (5 a^2 - 4 b^2) (- (a + \\
& b)^3 (a - b)^3)^{(1/2)} / (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2)) / (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2)) (5 a^2 - 4 b^2) (- (a + b)^3 (a - b)^3)^{(1/2)} * i) / (d (a^{11} - a^5 b^6 + 3 a^7 b^4 - 3 a^9 b^2))
\end{aligned}$$

$$3.469 \quad \int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^3} dx$$

Optimal result	5008
Rubi [A] (verified)	5009
Mathematica [A] (verified)	5012
Maple [A] (verified)	5013
Fricas [A] (verification not implemented)	5013
Sympy [F(-1)]	5014
Maxima [F(-2)]	5014
Giac [B] (verification not implemented)	5015
Mupad [B] (verification not implemented)	5016

Optimal result

Integrand size = 21, antiderivative size = 300

$$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^3} dx = \frac{(12a^2 + b^2)x}{2b^5} - \frac{a^3(12a^4 - 29a^2b^2 + 20b^4) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^5(a+b)^{5/2}d} - \frac{3a(4a^4 - 7a^2b^2 + 2b^4) \sin(c+dx)}{2b^4(a^2 - b^2)^2 d} + \frac{(6a^4 - 10a^2b^2 + b^4) \cos(c+dx) \sin(c+dx)}{2b^3(a^2 - b^2)^2 d} - \frac{a^2 \cos^3(c+dx) \sin(c+dx)}{2b(a^2 - b^2)d(a+b \cos(c+dx))^2} - \frac{a^2(4a^2 - 7b^2) \cos^2(c+dx) \sin(c+dx)}{2b^2(a^2 - b^2)^2 d(a+b \cos(c+dx))}$$

```
[Out] 1/2*(12*a^2+b^2)*x/b^5-a^3*(12*a^4-29*a^2*b^2+20*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^5/(a+b)^(5/2)/d-3/2*a*(4*a^4-7*a^2*b^2+2*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d+1/2*(6*a^4-10*a^2*b^2+b^4)*cos(d*x+c)*sin(d*x+c)/b^3/(a^2-b^2)^2/d-1/2*a^2*cos(d*x+c)^3*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/2*a^2*(4*a^2-7*b^2)*cos(d*x+c)^2*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2871, 3126, 3128, 3102, 2814, 2738, 211}

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^3} dx = -\frac{a^2 \sin(c + dx) \cos^3(c + dx)}{2bd(a^2 - b^2)(a + b \cos(c + dx))^2} - \frac{a^2(4a^2 - 7b^2) \sin(c + dx) \cos^2(c + dx)}{2b^2d(a^2 - b^2)^2(a + b \cos(c + dx))} + \frac{x(12a^2 + b^2)}{2b^5} - \frac{3a(4a^4 - 7a^2b^2 + 2b^4) \sin(c + dx)}{2b^4d(a^2 - b^2)^2} + \frac{(6a^4 - 10a^2b^2 + b^4) \sin(c + dx) \cos(c + dx)}{2b^3d(a^2 - b^2)^2} - \frac{a^3(12a^4 - 29a^2b^2 + 20b^4) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{5/2}(a+b)^{5/2}}$$

[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^3,x]

[Out] ((12*a^2 + b^2)*x)/(2*b^5) - (a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*d) - (3*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^2*d) + ((6*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)^2*d) - (a^2*Cos[c + d*x]^3*Ssin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(4*a^2 - 7*b^2)*Cos[c + d*x]^2*Ssin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2814

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Ssin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Co
s[e + f*x]*(a + b*sin[e + f*x])^(m - 2)*((c + d*sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*sin[
e + f*x])^(m - 3)*(c + d*sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
+ a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-c^2*C - B*c*d + A*d^2)*Cos[e + f*x
]*(a + b*sin[e + f*x])^(m)*((c + d*sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*sin[e + f*x])^(m -
1)*(c + d*sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))*Sin[e + f*x]
+ b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*sin[e + f*x
])^(m)*((c + d*sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*sin[e + f*x])^(m - 1)*(c + d*sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B))*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
```

$n + 2)) * \text{Sin}[e + f * x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C, n\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 0] \&\& !(\text{IGtQ}[n, 0] \&\& (! \text{IntegerQ}[m] || (\text{EqQ}[a, 0] \&\& \text{NeQ}[c, 0])))$

Rubi steps

integral

$$\begin{aligned}
&= -\frac{a^2 \cos^3(c + dx) \sin(c + dx)}{2b(a^2 - b^2)d(a + b \cos(c + dx))^2} - \frac{\int \frac{\cos^2(c + dx)(3a^2 - 2ab \cos(c + dx) - 2(2a^2 - b^2) \cos^2(c + dx))}{(a + b \cos(c + dx))^2} dx}{2b(a^2 - b^2)} \\
&= -\frac{a^2 \cos^3(c + dx) \sin(c + dx)}{2b(a^2 - b^2)d(a + b \cos(c + dx))^2} - \frac{a^2(4a^2 - 7b^2) \cos^2(c + dx) \sin(c + dx)}{2b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{\cos(c + dx)(-2a^2(4a^2 - 7b^2) + ab(a^2 - 4b^2) \cos(c + dx) + 2(6a^4 - 10a^2b^2 + b^4) \cos^2(c + dx))}{a + b \cos(c + dx)} dx}{2b^2(a^2 - b^2)^2} \\
&= \frac{(6a^4 - 10a^2b^2 + b^4) \cos(c + dx) \sin(c + dx)}{2b^3(a^2 - b^2)^2 d} \\
&\quad - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{2b(a^2 - b^2)d(a + b \cos(c + dx))^2} - \frac{a^2(4a^2 - 7b^2) \cos^2(c + dx) \sin(c + dx)}{2b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{2a(6a^4 - 10a^2b^2 + b^4) - 2b(2a^4 - 4a^2b^2 - b^4) \cos(c + dx) - 6a(4a^4 - 7a^2b^2 + 2b^4) \cos^2(c + dx)}{a + b \cos(c + dx)} dx}{4b^3(a^2 - b^2)^2} \\
&= -\frac{3a(4a^4 - 7a^2b^2 + 2b^4) \sin(c + dx)}{2b^4(a^2 - b^2)^2 d} + \frac{(6a^4 - 10a^2b^2 + b^4) \cos(c + dx) \sin(c + dx)}{2b^3(a^2 - b^2)^2 d} \\
&\quad - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{2b(a^2 - b^2)d(a + b \cos(c + dx))^2} - \frac{a^2(4a^2 - 7b^2) \cos^2(c + dx) \sin(c + dx)}{2b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{2ab(6a^4 - 10a^2b^2 + b^4) + 2(a^2 - b^2)^2(12a^2 + b^2) \cos(c + dx)}{a + b \cos(c + dx)} dx}{4b^4(a^2 - b^2)^2} \\
&= \frac{(12a^2 + b^2)x}{2b^5} - \frac{3a(4a^4 - 7a^2b^2 + 2b^4) \sin(c + dx)}{2b^4(a^2 - b^2)^2 d} \\
&\quad + \frac{(6a^4 - 10a^2b^2 + b^4) \cos(c + dx) \sin(c + dx)}{2b^3(a^2 - b^2)^2 d} - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{2b(a^2 - b^2)d(a + b \cos(c + dx))^2} \\
&\quad - \frac{a^2(4a^2 - 7b^2) \cos^2(c + dx) \sin(c + dx)}{2b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))} - \frac{(a^3(12a^4 - 29a^2b^2 + 20b^4)) \int \frac{1}{a + b \cos(c + dx)} dx}{2b^5(a^2 - b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(12a^2 + b^2)x}{2b^5} - \frac{3a(4a^4 - 7a^2b^2 + 2b^4)\sin(c + dx)}{2b^4(a^2 - b^2)^2 d} \\
&+ \frac{(6a^4 - 10a^2b^2 + b^4)\cos(c + dx)\sin(c + dx)}{2b^3(a^2 - b^2)^2 d} \\
&- \frac{a^2\cos^3(c + dx)\sin(c + dx)}{2b(a^2 - b^2)d(a + b\cos(c + dx))^2} - \frac{a^2(4a^2 - 7b^2)\cos^2(c + dx)\sin(c + dx)}{2b^2(a^2 - b^2)^2 d(a + b\cos(c + dx))} \\
&- \frac{(a^3(12a^4 - 29a^2b^2 + 20b^4))\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{b^5(a^2 - b^2)^2 d} \\
&= \frac{(12a^2 + b^2)x}{2b^5} - \frac{a^3(12a^4 - 29a^2b^2 + 20b^4)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^5(a+b)^{5/2}d} \\
&- \frac{3a(4a^4 - 7a^2b^2 + 2b^4)\sin(c + dx)}{2b^4(a^2 - b^2)^2 d} + \frac{(6a^4 - 10a^2b^2 + b^4)\cos(c + dx)\sin(c + dx)}{2b^3(a^2 - b^2)^2 d} \\
&- \frac{a^2\cos^3(c + dx)\sin(c + dx)}{2b(a^2 - b^2)d(a + b\cos(c + dx))^2} - \frac{a^2(4a^2 - 7b^2)\cos^2(c + dx)\sin(c + dx)}{2b^2(a^2 - b^2)^2 d(a + b\cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.66

$$\int \frac{\cos^5(c + dx)}{(a + b\cos(c + dx))^3} dx$$

$$= \frac{2(12a^2 + b^2)(c + dx) + \frac{4a^3(12a^4 - 29a^2b^2 + 20b^4)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - 12ab\sin(c + dx) + \frac{2a^5b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{4b^5d}$$

[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^3,x]

[Out] (2*(12*a^2 + b^2)*(c + d*x) + (4*a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 12*a*b*Sin[c + d*x] + (2*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (2*a^4*b*(-7*a^2 + 10*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])) + b^2*Sin[2*(c + d*x)]/(4*b^5*d)


```

^5*b^5 - 6*a^3*b^7 - (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(d*x + c)^
3 + 4*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*cos(d*x + c)^2 + (18*a^8*b^
2 - 50*a^6*b^4 + 43*a^4*b^6 - 11*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6
*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + 2*(a^7*b^6 - 3*a^5
*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5 - 3*a^6*b^7 + 3*a^4*b
^9 - a^2*b^11)*d), 1/2*((12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 -
b^10)*d*x*cos(d*x + c)^2 + 2*(12*a^9*b - 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b
^7 - a*b^9)*d*x*cos(d*x + c) + (12*a^10 - 35*a^8*b^2 + 33*a^6*b^4 - 9*a^4*b
^6 - a^2*b^8)*d*x - (12*a^9 - 29*a^7*b^2 + 20*a^5*b^4 + (12*a^7*b^2 - 29*a^
5*b^4 + 20*a^3*b^6)*cos(d*x + c)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4*b^5)
*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2
)*sin(d*x + c))) - (12*a^9*b - 33*a^7*b^3 + 27*a^5*b^5 - 6*a^3*b^7 - (a^6*b
^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(d*x + c)^3 + 4*(a^7*b^3 - 3*a^5*b^5
+ 3*a^3*b^7 - a*b^9)*cos(d*x + c)^2 + (18*a^8*b^2 - 50*a^6*b^4 + 43*a^4*b^6
- 11*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^
11 - b^13)*d*cos(d*x + c)^2 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)
*d*cos(d*x + c) + (a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d)]

```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. 2(281) = 562.

Time = 0.50 (sec) , antiderivative size = 1735, normalized size of antiderivative = 5.78

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out]
$$-1/2 * ((12*a^6 - 6*a^5*b - 23*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 - a*b^5 + b^6) * \sqrt{a^2 - b^2} * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9) * \text{abs}(-a + b) + (24*a^{11}*b^4 - 12*a^{10}*b^5 - 100*a^9*b^6 + 47*a^8*b^7 + 158*a^7*b^8 - 68*a^6*b^9 - 111*a^5*b^{10} + 42*a^4*b^{11} + 28*a^3*b^{12} - 8*a^2*b^{13} + a*b^{14} - b^{15}) * \sqrt{a^2 - b^2} * \text{abs}(-a + b)) * (\pi * \text{floor}(1/2*(d*x + c)/\pi + 1/2) + \arctan(2*\tan(1/2*d*x + 1/2*c)/\sqrt{(4*a^5*b^4 - 8*a^3*b^6 + 4*a*b^8 + \sqrt{-16*(a^5*b^4 + a^4*b^5 - 2*a^3*b^6 - 2*a^2*b^7 + a*b^8 + b^9)*(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9) + 16*(a^5*b^4 - 2*a^3*b^6 + a*b^8)^2})}) / (a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9))) / ((a^4*b^5 - 2*a^2*b^7 + b^9)^2 * (a^2 - 2*a*b + b^2) + (a^7*b^4 - 2*a^6*b^5 - a^5*b^6 + 4*a^4*b^7 - a^3*b^8 - 2*a^2*b^9 + a*b^{10}) * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9)) - (24*a^{11}*b^4 - 12*a^{10}*b^5 - 100*a^9*b^6 + 47*a^8*b^7 + 158*a^7*b^8 - 68*a^6*b^9 - 111*a^5*b^{10} + 42*a^4*b^{11} + 28*a^3*b^{12} - 8*a^2*b^{13} + a*b^{14} - b^{15} - 12*a^6 * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9) + 6*a^5*b * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9) + 23*a^4*b^2 * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9) - 10*a^3*b^3 * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9) - 10*a^2*b^4 * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9) + a*b^5 * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9) - b^6 * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9)) * (\pi * \text{floor}(1/2*(d*x + c)/\pi + 1/2) + \arctan(2*\tan(1/2*d*x + 1/2*c)/\sqrt{(4*a^5*b^4 - 8*a^3*b^6 + 4*a*b^8 - \sqrt{-16*(a^5*b^4 + a^4*b^5 - 2*a^3*b^6 - 2*a^2*b^7 + a*b^8 + b^9)*(a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9) + 16*(a^5*b^4 - 2*a^3*b^6 + a*b^8)^2})}) / (a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + 2*a^2*b^7 + a*b^8 - b^9))) / (a^5*b^4 * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9) - 2*a^3*b^6 * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9) + a*b^8 * \text{abs}(a^4*b^5 - 2*a^2*b^7 + b^9) - (a^4*b^5 - 2*a^2*b^7 + b^9)^2) + 2*(12*a^7*\tan(1/2*d*x + 1/2*c)^7 - 18*a^6*b*\tan(1/2*d*x + 1/2*c)^7 - 17*a^5*b^2*\tan(1/2*d*x + 1/2*c)^7 + 33*a^4*b^3*\tan(1/2*d*x + 1/2*c)^7 - 2*a^3*b^4*\tan(1/2*d*x + 1/2*c)^7 - 13*a^2*b^5*\tan(1/2*d*x + 1/2*c)^7 + 4*a*b^6*\tan(1/2*d*x + 1/2*c)^7 + b^7*\tan(1/2*d*x + 1/2*c)^7 + 36*a^7*\tan(1/2*d*x + 1/2*c)^5 - 18*a^6*b*\tan(1/2*d*x + 1/2*c)^5 - 67*a^5*b^2*\tan(1/2*d*x + 1/2*c)^5 + 29*a^4*b^3*\tan(1/2*d*x + 1/2*c)^5 + 26*a^3*b^4*\tan(1/2*d*x + 1/2*c)^5 - 5*a^2*b^5*\tan(1/2*d*x + 1/2*c)^5 - 4*a*b^6*\tan(1/2*d*x + 1/2*c)^5 - 3*b^7*\tan(1/2*d*x + 1/2*c)^5 + 36*a^7*\tan(1/2*d*x + 1/2*c)^3 + 18*a^6*b*\tan(1/2*d*x + 1/2*c)^3 - 67*a^5*b^2*\tan(1/2*d*x + 1/2*c)^3 - 29*a^4*b^3*\tan(1/2*d*x + 1/2*c)^3 + 26*a^3*b^4*\tan(1/2*d*x + 1/2*c)^3 + 5*a^2*b^5*\tan(1/2*d*x + 1/2*c)^3 - 4*a*b^6*\tan(1/2*d*x + 1/2*c)^3 + 3*b^7*\tan(1/2*d*x + 1/2*c)^3 + 12*a^7*\tan(1/2*d*x + 1/2*c) + 18*a^6*b*\tan(1/2*d*x + 1/2*c$$

$$\begin{aligned}
& 2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10} - a^6*b^9 - a^7*b^8)) * (12*a^4 + 20*b^4 - 29*a^2*b^2) / (2*(b^{15} - 5*a^2*b^{13} + 10*a^4*b^{11} - 10*a^6*b^9 + 5*a^8*b^7 - a^{10}*b^5)) * (- (a + b)^5 * (a - b)^5)^{(1/2)} * (12*a^4 + 20*b^4 - 29*a^2*b^2) * i) / (2*(b^{15} - 5*a^2*b^{13} + 10*a^4*b^{11} - 10*a^6*b^9 + 5*a^8*b^7 - a^{10}*b^5)) + (a^3 * ((8*\tan(c/2 + (d*x)/2) * (288*a^{14} - 288*a^{13}*b - 2*a*b^{13} + b^{14} + 21*a^2*b^{12} - 40*a^3*b^{11} + 74*a^4*b^{10} - 108*a^5*b^9 + 18*a^6*b^8 + 872*a^7*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^{10}*b^4 + 1104*a^{11}*b^3 - 1104*a^{12}*b^2)) / (a*b^{14} + b^{15} - 3*a^2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10} - a^6*b^9 - a^7*b^8) - (a^3 * (- (a + b)^5 * (a - b)^5)^{(1/2)} * ((4*(4*b^{21} + 28*a^2*b^{19} - 80*a^3*b^{18} - 120*a^4*b^{17} + 276*a^5*b^{16} + 164*a^6*b^{15} - 360*a^7*b^{14} - 100*a^8*b^{13} + 212*a^9*b^{12} + 24*a^{10}*b^{11} - 48*a^{11}*b^{10})) / (a*b^{18} + b^{19} - 3*a^2*b^{17} - 3*a^3*b^{16} + 3*a^4*b^{15} + 3*a^5*b^{14} - a^6*b^{13} - a^7*b^{12}) + (4*a^3*\tan(c/2 + (d*x)/2) * (- (a + b)^5 * (a - b)^5)^{(1/2)} * (12*a^4 + 20*b^4 - 29*a^2*b^2) * (8*a*b^{19} - 8*a^2*b^{18} - 32*a^3*b^{17} + 32*a^4*b^{16} + 48*a^5*b^{15} - 48*a^6*b^{14} - 32*a^7*b^{13} + 32*a^8*b^{12} + 8*a^9*b^{11} - 8*a^{10}*b^{10})) / ((b^{15} - 5*a^2*b^{13} + 10*a^4*b^{11} - 10*a^6*b^9 + 5*a^8*b^7 - a^{10}*b^5)) * (a*b^{14} + b^{15} - 3*a^2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10} - a^6*b^9 - a^7*b^8)) * (12*a^4 + 20*b^4 - 29*a^2*b^2) / (2*(b^{15} - 5*a^2*b^{13} + 10*a^4*b^{11} - 10*a^6*b^9 + 5*a^8*b^7 - a^{10}*b^5)) / ((8*(1728*a^{15} - 864*a^{14}*b + 20*a^3*b^{12} - 20*a^4*b^{11} + 411*a^5*b^{10} - 11*a^6*b^9 + 1314*a^7*b^8 + 2326*a^8*b^7 - 7829*a^9*b^6 - 4770*a^{10}*b^5 + 11700*a^{11}*b^4 + 3456*a^{12}*b^3 - 7344*a^{13}*b^2)) / (a*b^{18} + b^{19} - 3*a^2*b^{17} - 3*a^3*b^{16} + 3*a^4*b^{15} + 3*a^5*b^{14} - a^6*b^{13} - a^7*b^{12}) - (a^3 * ((8*\tan(c/2 + (d*x)/2) * (288*a^{14} - 288*a^{13}*b - 2*a*b^{13} + b^{14} + 21*a^2*b^{12} - 40*a^3*b^{11} + 74*a^4*b^{10} - 108*a^5*b^9 + 18*a^6*b^8 + 872*a^7*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^{10}*b^4 + 1104*a^{11}*b^3 - 1104*a^{12}*b^2)) / (a*b^{14} + b^{15} - 3*a^2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10} - a^6*b^9 - a^7*b^8) + (a^3 * (- (a + b)^5 * (a - b)^5)^{(1/2)} * ((4*(4*b^{21} + 28*a^2*b^{19} - 80*a^3*b^{18} - 120*a^4*b^{17} + 276*a^5*b^{16} + 164*a^6*b^{15} - 360*a^7*b^{14} - 100*a^8*b^{13} + 212*a^9*b^{12} + 24*a^{10}*b^{11} - 48*a^{11}*b^{10})) / (a*b^{18} + b^{19} - 3*a^2*b^{17} - 3*a^3*b^{16} + 3*a^4*b^{15} + 3*a^5*b^{14} - a^6*b^{13} - a^7*b^{12}) - (4*a^3*\tan(c/2 + (d*x)/2) * (- (a + b)^5 * (a - b)^5)^{(1/2)} * (12*a^4 + 20*b^4 - 29*a^2*b^2) * (8*a*b^{19} - 8*a^2*b^{18} - 32*a^3*b^{17} + 32*a^4*b^{16} + 48*a^5*b^{15} - 48*a^6*b^{14} - 32*a^7*b^{13} + 32*a^8*b^{12} + 8*a^9*b^{11} - 8*a^{10}*b^{10})) / ((b^{15} - 5*a^2*b^{13} + 10*a^4*b^{11} - 10*a^6*b^9 + 5*a^8*b^7 - a^{10}*b^5)) * (a*b^{14} + b^{15} - 3*a^2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10} - a^6*b^9 - a^7*b^8)) * (12*a^4 + 20*b^4 - 29*a^2*b^2) / (2*(b^{15} - 5*a^2*b^{13} + 10*a^4*b^{11} - 10*a^6*b^9 + 5*a^8*b^7 - a^{10}*b^5)) * (- (a + b)^5 * (a - b)^5)^{(1/2)} * (12*a^4 + 20*b^4 - 29*a^2*b^2) * i) / (2*(b^{15} - 5*a^2*b^{13} + 10*a^4*b^{11} - 10*a^6*b^9 + 5*a^8*b^7 - a^{10}*b^5)) + (a^3 * ((8*\tan(c/2 + (d*x)/2) * (288*a^{14} - 288*a^{13}*b - 2*a*b^{13} + b^{14} + 21*a^2*b^{12} - 40*a^3*b^{11} + 74*a^4*b^{10} - 108*a^5*b^9 + 18*a^6*b^8 + 872*a^7*b^7 - 827*a^8*b^6 - 1538*a^9*b^5 + 1538*a^{10}*b^4 + 1104*a^{11}*b^3 - 1104*a^{12}*b^2)) / (a*b^{14} + b^{15} - 3*a^2*b^{13} - 3*a^3*b^{12} + 3*a^4*b^{11} + 3*a^5*b^{10}
\end{aligned}$$

$$\begin{aligned}
& - a^6 b^9 - a^7 b^8) - (a^3 (-a + b)^5 (a - b)^5)^{1/2} \left((4(4b^{21} + 28a^2 b^{19} - 80a^3 b^{18} - 120a^4 b^{17} + 276a^5 b^{16} + 164a^6 b^{15} - 360a^7 b^{14} - 100a^8 b^{13} + 212a^9 b^{12} + 24a^{10} b^{11} - 48a^{11} b^{10})) / (a^{18} + b^{19} - 3a^2 b^{17} - 3a^3 b^{16} + 3a^4 b^{15} + 3a^5 b^{14} - a^6 b^{13} - a^7 b^{12}) + (4a^3 \tan(c/2 + (d*x)/2) (-a + b)^5 (a - b)^5)^{1/2} (12a^4 + 20b^4 - 29a^2 b^2) (8a^{19} b - 8a^{18} b^2 - 32a^{17} b^3 + 32a^{16} b^4 + 48a^{15} b^5 - 48a^{14} b^6 - 32a^{13} b^7 + 32a^{12} b^8 + 8a^{11} b^9 - 8a^{10} b^{10}) / ((b^{15} - 5a^2 b^{13} + 10a^4 b^{11} - 10a^6 b^9 + 5a^8 b^7 - a^{10} b^5) (a^{14} b + b^{15} - 3a^2 b^{13} - 3a^3 b^{12} + 3a^4 b^{11} + 3a^5 b^{10} - a^6 b^9 - a^7 b^8)) (12a^4 + 20b^4 - 29a^2 b^2) / (2(b^{15} - 5a^2 b^{13} + 10a^4 b^{11} - 10a^6 b^9 + 5a^8 b^7 - a^{10} b^5)) (-a + b)^5 (a - b)^5)^{1/2} (12a^4 + 20b^4 - 29a^2 b^2) / (2(b^{15} - 5a^2 b^{13} + 10a^4 b^{11} - 10a^6 b^9 + 5a^8 b^7 - a^{10} b^5)) (-a + b)^5 (a - b)^5)^{1/2} (12a^4 + 20b^4 - 29a^2 b^2) * i) / (d(b^{15} - 5a^2 b^{13} + 10a^4 b^{11} - 10a^6 b^9 + 5a^8 b^7 - a^{10} b^5))
\end{aligned}$$

3.470 $\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^3} dx$

Optimal result	5020
Rubi [A] (verified)	5020
Mathematica [A] (verified)	5023
Maple [A] (verified)	5023
Fricas [B] (verification not implemented)	5024
Sympy [F(-1)]	5025
Maxima [F(-2)]	5025
Giac [A] (verification not implemented)	5025
Mupad [B] (verification not implemented)	5026

Optimal result

Integrand size = 21, antiderivative size = 221

$$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^3} dx = -\frac{3ax}{b^4} + \frac{3a^2(2a^4 - 5a^2b^2 + 4b^4) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d}$$

$$+ \frac{(3a^2 - 2b^2) \sin(c+dx)}{2b^3(a^2 - b^2)d} - \frac{a^2 \cos^2(c+dx) \sin(c+dx)}{2b(a^2 - b^2)d(a+b \cos(c+dx))^2}$$

$$+ \frac{3a^3(a^2 - 2b^2) \sin(c+dx)}{2b^3(a^2 - b^2)^2 d(a+b \cos(c+dx))}$$

[Out] $-3ax/b^4 + 3a^2(2a^4 - 5a^2b^2 + 4b^4) \arctan((a-b)^{1/2} \tan(1/2 dx + 1/2 * c) / (a+b)^{1/2}) / (a-b)^{5/2} / b^4 / (a+b)^{5/2} / d + 1/2 * (3a^2 - 2b^2) * \sin(dx + c) / b^3 / (a^2 - b^2) / d - 1/2 * a^2 * \cos(dx + c)^2 * \sin(dx + c) / b / (a^2 - b^2) / d / (a+b * \cos(dx + c))^2 + 3/2 * a^3 * (a^2 - 2b^2) * \sin(dx + c) / b^3 / (a^2 - b^2)^2 / d / (a+b * \cos(dx + c))$

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2871, 3110, 3102, 2814, 2738, 211}

$$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^3} dx = -\frac{a^2 \sin(c+dx) \cos^2(c+dx)}{2bd(a^2 - b^2)(a+b \cos(c+dx))^2} + \frac{(3a^2 - 2b^2) \sin(c+dx)}{2b^3d(a^2 - b^2)}$$

$$+ \frac{3a^2(2a^4 - 5a^2b^2 + 4b^4) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}$$

$$+ \frac{3a^3(a^2 - 2b^2) \sin(c+dx)}{2b^3d(a^2 - b^2)^2(a+b \cos(c+dx))} - \frac{3ax}{b^4}$$

[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^3,x]

[Out] $(-3*a*x)/b^4 + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*\text{ArcTan}[\text{Sqrt}[a - b]*\text{Tan}[(c + d*x)/2]]/\text{Sqrt}[a + b])/((a - b)^{(5/2)}*b^4*(a + b)^{(5/2)}*d) + ((3*a^2 - 2*b^2)*\text{Sin}[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (a^2*\text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*\text{Cos}[c + d*x])^2) + (3*a^3*(a^2 - 2*b^2)*\text{Sin}[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*\text{Cos}[c + d*x]))$

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2871

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

Rule 3102

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]

&& !LtQ[m, -1]

Rule 3110

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rubi steps

integral

$$\begin{aligned}
 &= -\frac{a^2 \cos^2(c + dx) \sin(c + dx)}{2b(a^2 - b^2) d(a + b \cos(c + dx))^2} - \frac{\int \frac{\cos(c+dx)(2a^2 - 2ab \cos(c+dx) - (3a^2 - 2b^2) \cos^2(c+dx))}{(a+b \cos(c+dx))^2} dx}{2b(a^2 - b^2)} \\
 &= -\frac{a^2 \cos^2(c + dx) \sin(c + dx)}{2b(a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{3a^3(a^2 - 2b^2) \sin(c + dx)}{2b^3(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
 &\quad - \frac{\int \frac{3a^2b(a^2 - 2b^2) + a(3a^2 - 4b^2)(a^2 - b^2) \cos(c+dx) - b(3a^2 - 2b^2)(a^2 - b^2) \cos^2(c+dx)}{a+b \cos(c+dx)} dx}{2b^3(a^2 - b^2)^2} \\
 &= \frac{(3a^2 - 2b^2) \sin(c + dx)}{2b^3(a^2 - b^2) d} - \frac{a^2 \cos^2(c + dx) \sin(c + dx)}{2b(a^2 - b^2) d(a + b \cos(c + dx))^2} \\
 &\quad + \frac{3a^3(a^2 - 2b^2) \sin(c + dx)}{2b^3(a^2 - b^2)^2 d(a + b \cos(c + dx))} - \frac{\int \frac{3a^2b^2(a^2 - 2b^2) + 6ab(a^2 - b^2)^2 \cos(c+dx)}{a+b \cos(c+dx)} dx}{2b^4(a^2 - b^2)^2} \\
 &= -\frac{3ax}{b^4} + \frac{(3a^2 - 2b^2) \sin(c + dx)}{2b^3(a^2 - b^2) d} - \frac{a^2 \cos^2(c + dx) \sin(c + dx)}{2b(a^2 - b^2) d(a + b \cos(c + dx))^2} \\
 &\quad + \frac{3a^3(a^2 - 2b^2) \sin(c + dx)}{2b^3(a^2 - b^2)^2 d(a + b \cos(c + dx))} + \frac{(3a^2(2a^4 - 5a^2b^2 + 4b^4)) \int \frac{1}{a+b \cos(c+dx)} dx}{2b^4(a^2 - b^2)^2} \\
 &= -\frac{3ax}{b^4} + \frac{(3a^2 - 2b^2) \sin(c + dx)}{2b^3(a^2 - b^2) d} - \frac{a^2 \cos^2(c + dx) \sin(c + dx)}{2b(a^2 - b^2) d(a + b \cos(c + dx))^2} \\
 &\quad + \frac{3a^3(a^2 - 2b^2) \sin(c + dx)}{2b^3(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
 &\quad + \frac{(3a^2(2a^4 - 5a^2b^2 + 4b^4)) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{b^4(a^2 - b^2)^2 d}
 \end{aligned}$$

$$= -\frac{3ax}{b^4} + \frac{3a^2(2a^4 - 5a^2b^2 + 4b^4) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d} + \frac{(3a^2 - 2b^2) \sin(c+dx)}{2b^3(a^2 - b^2)d} - \frac{a^2 \cos^2(c+dx) \sin(c+dx)}{2b(a^2 - b^2)d(a+b \cos(c+dx))^2} + \frac{3a^3(a^2 - 2b^2) \sin(c+dx)}{2b^3(a^2 - b^2)^2 d(a+b \cos(c+dx))}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.80

$$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^3} dx$$

$$= \frac{-6a(c+dx) - \frac{6a^2(2a^4-5a^2b^2+4b^4) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + 2b \sin(c+dx) - \frac{a^4 b \sin(c+dx)}{(a-b)(a+b)(a+b \cos(c+dx))^2} + \frac{a^5}{(a-b)^2}}{2b^4 d}$$

[In] Integrate[Cos[c + d*x]^4/(a + b*cos[c + d*x])^3,x]

[Out] (-6*a*(c + d*x) - (6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + 2*b*Sin[c + d*x] - (a^4*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*cos[c + d*x])^2) + (a^3*b*(5*a^2 - 8*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*cos[c + d*x]))/(2*b^4*d)

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.20

method	result
derivativedivides	$2a^2 \left(\frac{\frac{(4a^2-ab-8b^2)ab \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2(a-b)(a^2+2ab+b^2)} + \frac{(4a^2+ab-8b^2)ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)}}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a-b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+a+b} \right)^2 + \frac{3(2a^4-5a^2b^2+4b^4) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2(a^4-2a^2b^2+b^4)\sqrt{(a-b)(a+b)}} \right) \frac{d}{b^4}$
default	$2a^2 \left(\frac{\frac{(4a^2-ab-8b^2)ab \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2(a-b)(a^2+2ab+b^2)} + \frac{(4a^2+ab-8b^2)ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)}}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a-b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+a+b} \right)^2 + \frac{3(2a^4-5a^2b^2+4b^4) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{2(a^4-2a^2b^2+b^4)\sqrt{(a-b)(a+b)}} \right) \frac{d}{b^4}$
risch	$-\frac{3ax}{b^4} - \frac{ie^{i(dx+c)}}{2b^3d} + \frac{ie^{-i(dx+c)}}{2b^3d} + \frac{ia^3(6ba^3e^{3i(dx+c)} - 9b^3ae^{3i(dx+c)} + 10a^4e^{2i(dx+c)} - 11b^2a^2e^{2i(dx+c)} - 8b^4e^{2i(dx+c)} + 2a^5e^{i(dx+c)})}{b^4(a^2-b^2)^2d(b^2e^{2i(dx+c)} + 2ae^{i(dx+c)})}$

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

```
[Out] 1/d*(2*a^2/b^4*((1/2*(4*a^2-a*b-8*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(4*a^2+a*b-8*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+3/2*(2*a^4-5*a^2*b^2+4*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/b^4*(-b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+3*a*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(206) = 412.

Time = 0.34 (sec) , antiderivative size = 1029, normalized size of antiderivative = 4.66

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^3} dx$$

$$= \frac{\begin{aligned} &12(a^7b^2 - 3a^5b^4 + 3a^3b^6 - ab^8)dx \cos(dx + c)^2 + 24(a^8b - 3a^6b^3 + 3a^4b^5 - a^2b^7)dx \cos(dx + c) + 12(a^9 - 3a^7b^2 + 3a^5b^4 - a^3b^6)dx \\ &+ 3*(2a^8 - 5a^6b^2 + 4a^4b^4 + (2a^6b^2 - 5a^4b^4 + 4a^2b^6)*\cos(dx + c)^2 + 2*(2a^7b - 5a^5b^3 + 4a^3b^5)*\cos(dx + c)) * \sqrt{-a^2 + b^2} * \log((2a*b*\cos(dx + c) + (2a^2 - b^2)*\cos(dx + c)^2 + 2*\sqrt{-a^2 + b^2}*(a*\cos(dx + c) + b)*\sin(dx + c) - a^2 + 2*b^2)/(b^2*\cos(dx + c)^2 + 2*a*b*\cos(dx + c) + a^2)) - 2*(6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*\cos(dx + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*\cos(dx + c))*\sin(dx + c)) / ((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*\cos(dx + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*\cos(dx + c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*\cos(dx + c)^2 + 12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*\cos(dx + c) + 6*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*x - 3*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*\cos(dx + c)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*\cos(dx + c))*\sqrt{a^2 - b^2} * \arctan(-(a*\cos(dx + c) + b)/(\sqrt{a^2 - b^2}*\sin(dx + c))) - (6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*\cos(dx + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*\cos(dx + c))*\sin(dx + c)) / ((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*\cos(dx + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*\cos(dx + c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d)}{6(a^7b^2 - 3a^5b^4 + 3a^3b^6 - ab^8)dx \cos(dx + c)^2 + 12(a^8b - 3a^6b^3 + 3a^4b^5 - a^2b^7)dx \cos(dx + c) + 6(a^9 - 3a^7b^2 + 3a^5b^4 - a^3b^6)dx + 3*(2a^8 - 5a^6b^2 + 4a^4b^4 + (2a^6b^2 - 5a^4b^4 + 4a^2b^6)*\cos(dx + c)^2 + 2*(2a^7b - 5a^5b^3 + 4a^3b^5)*\cos(dx + c))*\sqrt{-a^2 + b^2} * \log((2a*b*\cos(dx + c) + (2a^2 - b^2)*\cos(dx + c)^2 + 2*\sqrt{-a^2 + b^2}*(a*\cos(dx + c) + b)*\sin(dx + c) - a^2 + 2*b^2)/(b^2*\cos(dx + c)^2 + 2*a*b*\cos(dx + c) + a^2)) - 2*(6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*\cos(dx + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*\cos(dx + c))*\sin(dx + c)) / ((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*\cos(dx + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*\cos(dx + c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d)}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(12*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c)^2 + 24*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c) + 12*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*x + 3*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos(d*x + c)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(6*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c)^2 + 12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c) + 6*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*x - 3*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos(d*x + c)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Exception raised: ValueError}$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.60

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^3} dx = \frac{3(2a^6 - 5a^4b^2 + 4a^2b^4) \left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^4b^4 - 2a^2b^6 + b^8)\sqrt{a^2 - b^2}} - \frac{4a^6 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 5a^5b \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 4a^4b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c) - 5a^3b^3}{(a^4b^4 - 2a^2b^6 + b^8)\sqrt{a^2 - b^2}}$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] $-(3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*a^6*tan(1/2*d*x + 1/2*c)^3 - 5*a^5*b*tan(1/2*d*x + 1/2*c)^2 + 4*a^4*b^2*tan(1/2*d*x + 1/2*c) - 5*a^3*b^3*tan(1/2*d*x + 1/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*a^6*tan(1/2*d*x + 1/2*c)^3 - 5*a^5*b*tan(1/2*d*x + 1/2*c)^2 + 4*a^4*b^2*tan(1/2*d*x + 1/2*c) - 5*a^3*b^3*tan(1/2*d*x + 1/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2))$

$$\begin{aligned}
& 4*b^{12} + 3*a^5*b^{11} - a^6*b^{10} - a^7*b^9) - (a*\tan(c/2 + (d*x)/2)*(8*a*b^{17} \\
& - 8*a^2*b^{16} - 32*a^3*b^{15} + 32*a^4*b^{14} + 48*a^5*b^{13} - 48*a^6*b^{12} - 32* \\
& a^7*b^{11} + 32*a^8*b^{10} + 8*a^9*b^9 - 8*a^{10}*b^8)*24i)/(b^4*(a*b^{12} + b^{13} - \\
& 3*a^2*b^{11} - 3*a^3*b^{10} + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6))) * 3i) \\
& /b^4) * 3i)/b^4 + (a*((8*\tan(c/2 + (d*x)/2)*(72*a^{12} - 72*a^{11}*b + 36*a^2*b^{10} \\
& 0 - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441 \\
& *a^8*b^4 + 288*a^9*b^3 - 288*a^{10}*b^2)))/(a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3 \\
& *b^{10} + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (a*((24*(4*a*b^{17} - 8* \\
& a^2*b^{16} - 12*a^3*b^{15} + 26*a^4*b^{14} + 14*a^5*b^{13} - 32*a^6*b^{12} - 8*a^7*b^{11} \\
& + 18*a^8*b^{10} + 2*a^9*b^9 - 4*a^{10}*b^8)))/(a*b^{15} + b^{16} - 3*a^2*b^{14} - 3 \\
& *a^3*b^{13} + 3*a^4*b^{12} + 3*a^5*b^{11} - a^6*b^{10} - a^7*b^9) + (a*\tan(c/2 + (d \\
& *x)/2)*(8*a*b^{17} - 8*a^2*b^{16} - 32*a^3*b^{15} + 32*a^4*b^{14} + 48*a^5*b^{13} - 4 \\
& 8*a^6*b^{12} - 32*a^7*b^{11} + 32*a^8*b^{10} + 8*a^9*b^9 - 8*a^{10}*b^8)*24i)/(b^4* \\
& (a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 \\
& - a^7*b^6))) * 3i)/b^4) * 3i)/b^4)))/(b^4*d) - (a^2*atan(((a^2*(-(a + b)^5*(a - \\
& b)^5)^{(1/2)}*((8*\tan(c/2 + (d*x)/2)*(72*a^{12} - 72*a^{11}*b + 36*a^2*b^{10} - 72 \\
& *a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 432*a^7*b^5 + 441*a^8*b \\
& ^4 + 288*a^9*b^3 - 288*a^{10}*b^2)))/(a*b^{12} + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} \\
& + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (3*a^2*((24*(4*a*b^{17} - 8*a^ \\
& 2*b^{16} - 12*a^3*b^{15} + 26*a^4*b^{14} + 14*a^5*b^{13} - 32*a^6*b^{12} - 8*a^7*b^{11} \\
& + 18*a^8*b^{10} + 2*a^9*b^9 - 4*a^{10}*b^8)))/(a*b^{15} + b^{16} - 3*a^2*b^{14} - 3*a \\
& ^3*b^{13} + 3*a^4*b^{12} + 3*a^5*b^{11} - a^6*b^{10} - a^7*b^9) - (12*a^2*\tan(c/2 + \\
& (d*x)/2)*(-(a + b)^5*(a - b)^5)^{(1/2)}*(2*a^4 + 4*b^4 - 5*a^2*b^2)*(8*a*b^{17} \\
& 7 - 8*a^2*b^{16} - 32*a^3*b^{15} + 32*a^4*b^{14} + 48*a^5*b^{13} - 48*a^6*b^{12} - 32 \\
& *a^7*b^{11} + 32*a^8*b^{10} + 8*a^9*b^9 - 8*a^{10}*b^8)))/((b^{14} - 5*a^2*b^{12} + 10 \\
& *a^4*b^{10} - 10*a^6*b^8 + 5*a^8*b^6 - a^{10}*b^4)*(a*b^{12} + b^{13} - 3*a^2*b^{11} \\
& - 3*a^3*b^{10} + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(-(a + b)^5*(a \\
& - b)^5)^{(1/2)}*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^{14} - 5*a^2*b^{12} + 10*a^4*b \\
& ^{10} - 10*a^6*b^8 + 5*a^8*b^6 - a^{10}*b^4)))*(2*a^4 + 4*b^4 - 5*a^2*b^2)*3i)/ \\
& (2*(b^{14} - 5*a^2*b^{12} + 10*a^4*b^{10} - 10*a^6*b^8 + 5*a^8*b^6 - a^{10}*b^4)) + \\
& (a^2*(-(a + b)^5*(a - b)^5)^{(1/2)}*((8*\tan(c/2 + (d*x)/2)*(72*a^{12} - 72*a^{11} \\
& 1*b + 36*a^2*b^{10} - 72*a^3*b^9 + 36*a^4*b^8 + 288*a^5*b^7 - 288*a^6*b^6 - 4 \\
& 32*a^7*b^5 + 441*a^8*b^4 + 288*a^9*b^3 - 288*a^{10}*b^2)))/(a*b^{12} + b^{13} - 3* \\
& a^2*b^{11} - 3*a^3*b^{10} + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (3*a^2 \\
& *((24*(4*a*b^{17} - 8*a^2*b^{16} - 12*a^3*b^{15} + 26*a^4*b^{14} + 14*a^5*b^{13} - 32 \\
& *a^6*b^{12} - 8*a^7*b^{11} + 18*a^8*b^{10} + 2*a^9*b^9 - 4*a^{10}*b^8)))/(a*b^{15} + b \\
& ^{16} - 3*a^2*b^{14} - 3*a^3*b^{13} + 3*a^4*b^{12} + 3*a^5*b^{11} - a^6*b^{10} - a^7*b^ \\
& 9) + (12*a^2*\tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^{(1/2)}*(2*a^4 + 4*b^4 \\
& - 5*a^2*b^2)*(8*a*b^{17} - 8*a^2*b^{16} - 32*a^3*b^{15} + 32*a^4*b^{14} + 48*a^5*b \\
& ^{13} - 48*a^6*b^{12} - 32*a^7*b^{11} + 32*a^8*b^{10} + 8*a^9*b^9 - 8*a^{10}*b^8)))/((\\
& b^{14} - 5*a^2*b^{12} + 10*a^4*b^{10} - 10*a^6*b^8 + 5*a^8*b^6 - a^{10}*b^4)*(a*b^{12} \\
& + b^{13} - 3*a^2*b^{11} - 3*a^3*b^{10} + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7* \\
& b^6)))*(-(a + b)^5*(a - b)^5)^{(1/2)}*(2*a^4 + 4*b^4 - 5*a^2*b^2))/(2*(b^{14} - \\
& 5*a^2*b^{12} + 10*a^4*b^{10} - 10*a^6*b^8 + 5*a^8*b^6 - a^{10}*b^4)))*(2*a^4 + 4 \\
& *b^4 - 5*a^2*b^2)*3i)/(2*(b^{14} - 5*a^2*b^{12} + 10*a^4*b^{10} - 10*a^6*b^8 + 5*
\end{aligned}$$

$$\begin{aligned}
& a^8 b^6 - a^{10} b^4) / ((48(36 a^{12} - 18 a^{11} b + 72 a^4 b^8 + 72 a^5 b^7 - \\
& 234 a^6 b^6 - 126 a^7 b^5 + 288 a^8 b^4 + 81 a^9 b^3 - 162 a^{10} b^2)) / (a b \\
& ^{15} + b^{16} - 3 a^2 b^{14} - 3 a^3 b^{13} + 3 a^4 b^{12} + 3 a^5 b^{11} - a^6 b^{10} - \\
& a^7 b^9) - (3 a^2 (- (a + b)^5 (a - b)^5)^{1/2} ((8 \tan(c/2 + (d x)/2) (72 a \\
& ^{12} - 72 a^{11} b + 36 a^2 b^{10} - 72 a^3 b^9 + 36 a^4 b^8 + 288 a^5 b^7 - 28 \\
& 8 a^6 b^6 - 432 a^7 b^5 + 441 a^8 b^4 + 288 a^9 b^3 - 288 a^{10} b^2)) / (a b^{1 \\
& 2} + b^{13} - 3 a^2 b^{11} - 3 a^3 b^{10} + 3 a^4 b^9 + 3 a^5 b^8 - a^6 b^7 - a^7 a \\
& b^6) + (3 a^2 ((24(4 a^* b^{17} - 8 a^2 b^{16} - 12 a^3 b^{15} + 26 a^4 b^{14} + 14 a \\
& ^5 b^{13} - 32 a^6 b^{12} - 8 a^7 b^{11} + 18 a^8 b^{10} + 2 a^9 b^9 - 4 a^{10} b^8) \\
&)) / (a b^{15} + b^{16} - 3 a^2 b^{14} - 3 a^3 b^{13} + 3 a^4 b^{12} + 3 a^5 b^{11} - a^6 a \\
& b^{10} - a^7 b^9) - (12 a^2 \tan(c/2 + (d x)/2) (- (a + b)^5 (a - b)^5)^{1/2} (\\
& 2 a^4 + 4 b^4 - 5 a^2 b^2) (8 a^* b^{17} - 8 a^2 b^{16} - 32 a^3 b^{15} + 32 a^4 b^ \\
& ^{14} + 48 a^5 b^{13} - 48 a^6 b^{12} - 32 a^7 b^{11} + 32 a^8 b^{10} + 8 a^9 b^9 - 8 a \\
& ^{10} b^8)) / ((b^{14} - 5 a^2 b^{12} + 10 a^4 b^{10} - 10 a^6 b^8 + 5 a^8 b^6 - a^{1 \\
& 0} b^4) (a b^{12} + b^{13} - 3 a^2 b^{11} - 3 a^3 b^{10} + 3 a^4 b^9 + 3 a^5 b^8 - a \\
& ^6 b^7 - a^7 b^6)) (- (a + b)^5 (a - b)^5)^{1/2} (2 a^4 + 4 b^4 - 5 a^2 b^2 \\
&)) / (2 (b^{14} - 5 a^2 b^{12} + 10 a^4 b^{10} - 10 a^6 b^8 + 5 a^8 b^6 - a^{10} b^4) \\
&)) (2 a^4 + 4 b^4 - 5 a^2 b^2)) / (2 (b^{14} - 5 a^2 b^{12} + 10 a^4 b^{10} - 10 a^ \\
& 6 b^8 + 5 a^8 b^6 - a^{10} b^4)) + (3 a^2 (- (a + b)^5 (a - b)^5)^{1/2} ((8 \tan \\
& (c/2 + (d x)/2) (72 a^{12} - 72 a^{11} b + 36 a^2 b^{10} - 72 a^3 b^9 + 36 a^4 b \\
& ^8 + 288 a^5 b^7 - 288 a^6 b^6 - 432 a^7 b^5 + 441 a^8 b^4 + 288 a^9 b^3 - \\
& 288 a^{10} b^2)) / (a b^{12} + b^{13} - 3 a^2 b^{11} - 3 a^3 b^{10} + 3 a^4 b^9 + 3 a^5 \\
& * b^8 - a^6 b^7 - a^7 b^6) - (3 a^2 ((24(4 a^* b^{17} - 8 a^2 b^{16} - 12 a^3 b^{1 \\
& 5} + 26 a^4 b^{14} + 14 a^5 b^{13} - 32 a^6 b^{12} - 8 a^7 b^{11} + 18 a^8 b^{10} + 2 a \\
& ^9 b^9 - 4 a^{10} b^8)) / (a b^{15} + b^{16} - 3 a^2 b^{14} - 3 a^3 b^{13} + 3 a^4 b^{1 \\
& 2} + 3 a^5 b^{11} - a^6 b^{10} - a^7 b^9) + (12 a^2 \tan(c/2 + (d x)/2) (- (a + b) \\
& ^5 (a - b)^5)^{1/2} (2 a^4 + 4 b^4 - 5 a^2 b^2) (8 a^* b^{17} - 8 a^2 b^{16} - 32 \\
& a^3 b^{15} + 32 a^4 b^{14} + 48 a^5 b^{13} - 48 a^6 b^{12} - 32 a^7 b^{11} + 32 a^8 a \\
& b^{10} + 8 a^9 b^9 - 8 a^{10} b^8)) / ((b^{14} - 5 a^2 b^{12} + 10 a^4 b^{10} - 10 a^6 a \\
& b^8 + 5 a^8 b^6 - a^{10} b^4) (a b^{12} + b^{13} - 3 a^2 b^{11} - 3 a^3 b^{10} + 3 a^ \\
& 4 b^9 + 3 a^5 b^8 - a^6 b^7 - a^7 b^6)) (- (a + b)^5 (a - b)^5)^{1/2} (2 a^ \\
& 4 + 4 b^4 - 5 a^2 b^2)) / (2 (b^{14} - 5 a^2 b^{12} + 10 a^4 b^{10} - 10 a^6 b^8 + \\
& 5 a^8 b^6 - a^{10} b^4)) (2 a^4 + 4 b^4 - 5 a^2 b^2)) / (2 (b^{14} - 5 a^2 b^{12} \\
& + 10 a^4 b^{10} - 10 a^6 b^8 + 5 a^8 b^6 - a^{10} b^4)) (- (a + b)^5 (a - b)^5 \\
&)^{1/2} (2 a^4 + 4 b^4 - 5 a^2 b^2) * 3i) / (d (b^{14} - 5 a^2 b^{12} + 10 a^4 b^{10} \\
& - 10 a^6 b^8 + 5 a^8 b^6 - a^{10} b^4))
\end{aligned}$$

$$3.471 \quad \int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^3} dx$$

Optimal result	5029
Rubi [A] (verified)	5029
Mathematica [A] (verified)	5031
Maple [A] (verified)	5032
Fricas [B] (verification not implemented)	5032
Sympy [F(-1)]	5033
Maxima [F(-2)]	5033
Giac [A] (verification not implemented)	5034
Mupad [B] (verification not implemented)	5034

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^3} dx = \frac{x}{b^3} - \frac{a(2a^4 - 5a^2b^2 + 6b^4) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{5/2} b^3 (a+b)^{5/2} d}$$

$$- \frac{a^2 \cos(c+dx) \sin(c+dx)}{2b(a^2 - b^2) d (a+b \cos(c+dx))^2}$$

$$- \frac{a^2(2a^2 - 5b^2) \sin(c+dx)}{2b^2(a^2 - b^2)^2 d (a+b \cos(c+dx))}$$

[Out] x/b^3-a*(2*a^4-5*a^2*b^2+6*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^3/(a+b)^(5/2)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/2*a^2*(2*a^2-5*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2871, 3100, 2814, 2738, 211}

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^3} dx = -\frac{a^2(2a^2 - 5b^2) \sin(c+dx)}{2b^2 d (a^2 - b^2)^2 (a+b \cos(c+dx))}$$

$$- \frac{a^2 \sin(c+dx) \cos(c+dx)}{2bd(a^2 - b^2) (a+b \cos(c+dx))^2}$$

$$- \frac{a(2a^4 - 5a^2b^2 + 6b^4) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{b^3 d (a-b)^{5/2} (a+b)^{5/2}} + \frac{x}{b^3}$$

[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]

[Out] x/b^3 - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(2*a^2 - 5*b^2)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2814

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3100

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)^2]), x_Symbol] := Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B

, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{a^2 \cos(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{\int \frac{a^2-2ab\cos(c+dx)-2(a^2-b^2)\cos^2(c+dx)}{(a+b\cos(c+dx))^2} dx}{2b(a^2-b^2)} \\
 &= -\frac{a^2 \cos(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} \\
 &\quad - \frac{a^2(2a^2-5b^2)\sin(c+dx)}{2b^2(a^2-b^2)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{ab(a^2-4b^2)+2(a^2-b^2)\cos(c+dx)}{a+b\cos(c+dx)} dx}{2b^2(a^2-b^2)^2} \\
 &= \frac{x}{b^3} - \frac{a^2 \cos(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{a^2(2a^2-5b^2)\sin(c+dx)}{2b^2(a^2-b^2)^2 d(a+b\cos(c+dx))} \\
 &\quad - \frac{(a(2a^4-5a^2b^2+6b^4)) \int \frac{1}{a+b\cos(c+dx)} dx}{2b^3(a^2-b^2)^2} \\
 &= \frac{x}{b^3} - \frac{a^2 \cos(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{a^2(2a^2-5b^2)\sin(c+dx)}{2b^2(a^2-b^2)^2 d(a+b\cos(c+dx))} \\
 &\quad - \frac{(a(2a^4-5a^2b^2+6b^4)) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^3(a^2-b^2)^2 d} \\
 &= \frac{x}{b^3} - \frac{a(2a^4-5a^2b^2+6b^4) \arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} \\
 &\quad - \frac{a^2 \cos(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{a^2(2a^2-5b^2)\sin(c+dx)}{2b^2(a^2-b^2)^2 d(a+b\cos(c+dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.83

$$\begin{aligned}
 &\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3} dx \\
 &= \frac{2(c+dx) + \frac{2a(2a^4-5a^2b^2+6b^4) \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - \frac{a^2b(2a^3-5ab^2+3b(a^2-2b^2)\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}}{2b^3d}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]

[Out] (2*(c + d*x) + (2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - (a^2*b*(2*a^3 - 5*a*b^2 + 3*b*(a^2 - 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2)/(2*b^3*d)

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.30

method	result
derivativedivides	$2a \left(\frac{\left(\frac{2a^2-ab-6b^2}{2(a-b)} \right) ab \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \left(\frac{2a^2+ab-6b^2}{2(a+b)} \right) ab \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{\left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - b \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + a + b} + \frac{(2a^4 - 5a^2b^2 + 6b^4) \arctan \left(\frac{(a-b) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{\sqrt{(a-b)(a+b)}} \right)}{2(a^4 - 2a^2b^2 + b^4) \sqrt{(a-b)(a+b)}} \right) \frac{d}{b^3} + 2a$
default	$2a \left(\frac{\left(\frac{2a^2-ab-6b^2}{2(a-b)} \right) ab \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \left(\frac{2a^2+ab-6b^2}{2(a+b)} \right) ab \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{\left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - b \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + a + b} + \frac{(2a^4 - 5a^2b^2 + 6b^4) \arctan \left(\frac{(a-b) \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{\sqrt{(a-b)(a+b)}} \right)}{2(a^4 - 2a^2b^2 + b^4) \sqrt{(a-b)(a+b)}} \right) \frac{d}{b^3} + 2a$
risch	$\frac{x}{b^3} - \frac{ia^2(4b a^3 e^{3i(dx+c)} - 7b^3 a e^{3i(dx+c)} + 6a^4 e^{2i(dx+c)} - 9b^2 a^2 e^{2i(dx+c)} - 6b^4 e^{2i(dx+c)} + 8b a^3 e^{i(dx+c)} - 17 e^{i(dx+c)} b^3 a + \dots)}{b^3(a^2 - b^2)^2 d (b e^{2i(dx+c)} + 2a e^{i(dx+c)} + b)^2}$

[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-2*a/b^3*((1/2*(2*a^2-a*b-6*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(2*a^2+a*b-6*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(2*a^4-5*a^2*b^2+6*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))+2/b^3*arctan(tan(1/2*d*x+1/2*c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(166) = 332.

Time = 0.31 (sec) , antiderivative size = 913, normalized size of antiderivative = 5.10

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^3} dx = \left[\frac{4(a^6 b^2 - 3a^4 b^4 + 3a^2 b^6 - b^8) dx \cos(dx + c)^2 + 8(a^7 b - 3a^5 b^3 + 3a^3 b^5 - ab^7) dx \cos(dx + c) + 4(a^8 - 3a^6 b^2 + 3a^4 b^4 - a^2 b^6) dx - (2a^7 - 5a^5 b^2 + 6a^3 b^4 + (2a^5 b^2 - 5a^3 b^4 + 6a b^6) \cos(dx + c)^2 + 2*(2a^6 b - 5a^4 b^3 + 6a^2 b^5) \cos(dx + c)) \sqrt{-a^2 + b^2} \log((2a b \cos(dx + c) + (2a^2 - b^2) \cos(dx + c))^2 - 2 \sqrt{-a^2 + b^2} (a \cos(dx + c) + b) \sin(dx + c) - a^2 + 2b^2)}{b^3(a^2 - b^2)^2 d (b e^{2i(dx+c)} + 2a e^{i(dx+c)} + b)^2} \right]$$

[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] [1/4*(4*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 + 8*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x - (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4 + (2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c))^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)

$$\frac{1}{(b^2 \cos(dx + c)^2 + 2ab \cos(dx + c) + a^2)} - \frac{2(2a^7b - 7a^5b^3 + 5a^3b^5 + 3(a^6b^2 - 3a^4b^4 + 2a^2b^6) \cos(dx + c)) \sin(dx + c)}{((a^6b^5 - 3a^4b^7 + 3a^2b^9 - b^{11})d \cos(dx + c)^2 + 2(a^7b^4 - 3a^5b^6 + 3a^3b^8 - ab^{10})d \cos(dx + c) + (a^8b^3 - 3a^6b^5 + 3a^4b^7 - a^2b^9)d)}$$

$$+ \frac{1}{2} \frac{(2(a^6b^2 - 3a^4b^4 + 3a^2b^6 - b^8)d \cos(dx + c)^2 + 4(a^7b - 3a^5b^3 + 3a^3b^5 - ab^7)d \cos(dx + c) + 2(a^8 - 3a^6b^2 + 3a^4b^4 - a^2b^6)d \cos(dx + c) - (2a^7 - 5a^5b^2 + 6a^3b^4 + (2a^5b^2 - 5a^3b^4 + 6ab^6) \cos(dx + c)^2 + 2(2a^6b - 5a^4b^3 + 6a^2b^5) \cos(dx + c)) \sqrt{a^2 - b^2} \arctan(-a \cos(dx + c) + b) / (\sqrt{a^2 - b^2} \sin(dx + c)))}{(a^6b^5 - 3a^4b^7 + 3a^2b^9 - b^{11})d \cos(dx + c)^2 + 2(a^7b^4 - 3a^5b^6 + 3a^3b^8 - ab^{10})d \cos(dx + c) + (a^8b^3 - 3a^6b^5 + 3a^4b^7 - a^2b^9)d}$$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(dx+c)**3/(a+b*cos(dx+c))**3,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Exception raised: ValueError}$$

[In] integrate(cos(dx+c)^3/(a+b*cos(dx+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.78

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^3} dx$$

$$\frac{(2a^5 - 5a^3b^2 + 6ab^4) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^4b^3 - 2a^2b^5 + b^7)\sqrt{a^2 - b^2}} - \frac{2a^5 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 3a^4b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(a^4b^3 - 2a^2b^5 + b^7)\sqrt{a^2 - b^2}}$$

[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] ((2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 - b^2)) - (2*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^5*tan(1/2*d*x + 1/2*c) + 3*a^4*b*tan(1/2*d*x + 1/2*c) - 5*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*a^2*b^3*tan(1/2*d*x + 1/2*c))/(a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + (d*x + c)/b^3)/d

Mupad [B] (verification not implemented)

Time = 23.09 (sec) , antiderivative size = 5102, normalized size of antiderivative = 28.50

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Too large to display}$$

[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^3,x)

[Out] (2*atan(((((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6)))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6)*8i)/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))*1i)/b^3 + (8*tan(c/2 + (d*x)/2)*(8*a^10 - 8*a^9*b - 8*a*b^9 + 4*b^10 + 24*a^2*b^8 + 32*a^3*b^7 - 52*a^4*b^6 - 48*a^5*b^5 + 57*a^6*b^4 + 32*a^7*b^3 - 32*a^8*b^2))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))/b^3 - (((((8*(12*a*b^14 - 4*b^15 + 8*a^2*b^13 - 34*a^3*b^12 - 6*a^4*b^11 + 36*a^5*b^10 + 4*a^6*b^9 - 18*a^7*b^8 - 2*a^8*b^7 + 4*a^9*b^6)))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (tan(c/2 + (d*x)/2)*(8*a*b^15

$$\begin{aligned}
& - 8a^2b^{14} - 32a^3b^{13} + 32a^4b^{12} + 48a^5b^{11} - 48a^6b^{10} - 32a^7b^9 + 32a^8b^8 + 8a^9b^7 - 8a^{10}b^6) * 8i) / (b^3(a^2b^9 + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4))) * 1i) / b^3 \\
& - (8 * \tan(c/2 + (d*x)/2) * (8a^{10} - 8a^9b - 8a^8b^9 + 4b^{10} + 24a^2b^8 + 32a^3b^7 - 52a^4b^6 - 48a^5b^5 + 57a^6b^4 + 32a^7b^3 - 32a^8b^2)) / (a^2b^9 + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4)) / b^3) / ((((((8 * (12a^2b^{14} - 4b^{15} + 8a^2b^{13} - 34a^3b^{12} - 6a^4b^{11} + 36a^5b^{10} + 4a^6b^9 - 18a^7b^8 - 2a^8b^7 + 4a^9b^6) / (a^2b^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6) - (\tan(c/2 + (d*x)/2) * (8a^{15} - 8a^2b^{14} - 32a^3b^{13} + 32a^4b^{12} + 48a^5b^{11} - 48a^6b^{10} - 32a^7b^9 + 32a^8b^8 + 8a^9b^7 - 8a^{10}b^6) * 8i) / (b^3(a^2b^9 + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4))) * 1i) / b^3 + (8 * \tan(c/2 + (d*x)/2) * (8a^{10} - 8a^9b - 8a^8b^9 + 4b^{10} + 24a^2b^8 + 32a^3b^7 - 52a^4b^6 - 48a^5b^5 + 57a^6b^4 + 32a^7b^3 - 32a^8b^2)) / (a^2b^9 + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4))) * 1i) / b^3 + ((((((8 * (12a^2b^{14} - 4b^{15} + 8a^2b^{13} - 34a^3b^{12} - 6a^4b^{11} + 36a^5b^{10} + 4a^6b^9 - 18a^7b^8 - 2a^8b^7 + 4a^9b^6) / (a^2b^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6) + (\tan(c/2 + (d*x)/2) * (8a^{15} - 8a^2b^{14} - 32a^3b^{13} + 32a^4b^{12} + 48a^5b^{11} - 48a^6b^{10} - 32a^7b^9 + 32a^8b^8 + 8a^9b^7 - 8a^{10}b^6) * 8i) / (b^3(a^2b^9 + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4))) * 1i) / b^3 - (8 * \tan(c/2 + (d*x)/2) * (8a^{10} - 8a^9b - 8a^8b^9 + 4b^{10} + 24a^2b^8 + 32a^3b^7 - 52a^4b^6 - 48a^5b^5 + 57a^6b^4 + 32a^7b^3 - 32a^8b^2)) / (a^2b^9 + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4))) * 1i) / b^3 + (16 * (12a^2b^8 - 2a^8b + 4a^9 + 24a^2b^7 - 34a^3b^6 - 26a^4b^5 + 36a^5b^4 + 13a^6b^3 - 18a^7b^2)) / (a^2b^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6)))) / (b^3*d) + (((\tan(c/2 + (d*x)/2)^3 * (a^3b - 2a^4 + 6a^2b^2)) / ((a^2b^2 - b^3) * (a + b)^2) - (\tan(c/2 + (d*x)/2) * (a^3b + 2a^4 - 6a^2b^2)) / ((a + b) * (b^4 - 2a^2b^3 + a^2b^2))) / (d * (2a^2b + \tan(c/2 + (d*x)/2)^2 * (2a^2 - 2b^2) + \tan(c/2 + (d*x)/2)^4 * (a^2 - 2a^2b + b^2) + a^2 + b^2)) + (a * \operatorname{atan}(((a * ((8 * \tan(c/2 + (d*x)/2) * (8a^{10} - 8a^9b - 8a^8b^9 + 4b^{10} + 24a^2b^8 + 32a^3b^7 - 52a^4b^6 - 48a^5b^5 + 57a^6b^4 + 32a^7b^3 - 32a^8b^2)) / (a^2b^9 + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4) + (a * ((8 * (12a^2b^{14} - 4b^{15} + 8a^2b^{13} - 34a^3b^{12} - 6a^4b^{11} + 36a^5b^{10} + 4a^6b^9 - 18a^7b^8 - 2a^8b^7 + 4a^9b^6) / (a^2b^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6) - (4a * \tan(c/2 + (d*x)/2) * (-(a + b)^5 * (a - b)^5)^{(1/2) * (2a^4 + 6b^4 - 5a^2b^2)) * (8a^{15} - 8a^2b^{14} - 32a^3b^{13} + 32a^4b^{12} + 48a^5b^{11} - 48a^6b^{10} - 32a^7b^9 + 32a^8b^8 + 8a^9b^7 - 8a^{10}b^6)) / ((b^{13} - 5a^2b^{11} + 10a^4b^9 - 10a^6b^7 + 5a^8b^5 - a^{10}b^3) * (a^2b^9 + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4))) * (-(a + b)^5 * (a - b)^5)^{(1/2) * (2a^4 + 6b^4 - 5a^2b^2)) / (2 * (b^{13} - 5a^2b^{11} + 10a^4b^9 - 10a^6b^7 + 5a^8b^5 - a^{10}b^3))) * (-
\end{aligned}$$

$$\begin{aligned}
& (a + b)^5(a - b)^5)^{(1/2)} * (2a^4 + 6b^4 - 5a^2b^2) * i) / (2(b^{13} - 5a^2 \\
& * b^{11} + 10a^4b^9 - 10a^6b^7 + 5a^8b^5 - a^{10}b^3)) + (a * ((8 * \tan(c/2 + \\
& (d*x)/2) * (8a^{10} - 8a^9b - 8a^8b^9 + 4b^{10} + 24a^2b^8 + 32a^3b^7 - \\
& 52a^4b^6 - 48a^5b^5 + 57a^6b^4 + 32a^7b^3 - 32a^8b^2)) / (a^2b^{10} + \\
& b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4) - \\
& (a * ((8 * (12a^2b^{14} - 4b^{15} + 8a^2b^{13} - 34a^3b^{12} - 6a^4b^{11} + 36a^5 \\
& b^{10} + 4a^6b^9 - 18a^7b^8 - 2a^8b^7 + 4a^9b^6)) / (a^2b^{12} + b^{13} - \\
& 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6) + (4a \\
& * \tan(c/2 + (d*x)/2) * (-a + b)^5(a - b)^5)^{(1/2)} * (2a^4 + 6b^4 - 5a^2b^2 \\
&) * (8a^2b^{15} - 8a^2b^{14} - 32a^3b^{13} + 32a^4b^{12} + 48a^5b^{11} - 48a^6 \\
& * b^{10} - 32a^7b^9 + 32a^8b^8 + 8a^9b^7 - 8a^{10}b^6)) / ((b^{13} - 5a^2b \\
& ^{11} + 10a^4b^9 - 10a^6b^7 + 5a^8b^5 - a^{10}b^3) * (a^2b^{10} + b^{11} - 3a^2 \\
& b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4))) * (-a + b)^ \\
& 5(a - b)^5)^{(1/2)} * (2a^4 + 6b^4 - 5a^2b^2) / (2(b^{13} - 5a^2b^{11} + 10a^4 \\
& b^9 - 10a^6b^7 + 5a^8b^5 - a^{10}b^3))) * (-a + b)^5(a - b)^5)^{(1/2)} \\
& * (2a^4 + 6b^4 - 5a^2b^2) * i) / (2(b^{13} - 5a^2b^{11} + 10a^4b^9 - 10a^6 \\
& b^7 + 5a^8b^5 - a^{10}b^3))) / ((16 * (12a^2b^8 - 2a^8b + 4a^9 + 24a^2b \\
& ^7 - 34a^3b^6 - 26a^4b^5 + 36a^5b^4 + 13a^6b^3 - 18a^7b^2)) / (a^2b^{12} + \\
& b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7 \\
& * b^6) + (a * ((8 * \tan(c/2 + (d*x)/2) * (8a^{10} - 8a^9b - 8a^8b^9 + 4b^{10} + 24 \\
& a^2b^8 + 32a^3b^7 - 52a^4b^6 - 48a^5b^5 + 57a^6b^4 + 32a^7b^3 - \\
& 32a^8b^2)) / (a^2b^{10} + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 \\
& - a^6b^5 - a^7b^4) + (a * ((8 * (12a^2b^{14} - 4b^{15} + 8a^2b^{13} - 34a^3b^{12} - 6a^4b^{11} + 36a^5b^{10} + 4a^6b^9 - 18a^7b^8 - 2a^8b^7 + 4a^9 \\
& * b^6)) / (a^2b^{12} + b^{13} - 3a^2b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a \\
& ^6b^7 - a^7b^6) - (4a * \tan(c/2 + (d*x)/2) * (-a + b)^5(a - b)^5)^{(1/2)} * (2 \\
& a^4 + 6b^4 - 5a^2b^2) * (8a^2b^{15} - 8a^2b^{14} - 32a^3b^{13} + 32a^4b^{12} + 48a^5b^{11} - 48a^6b^{10} - 32a^7b^9 + 32a^8b^8 + 8a^9b^7 - 8a^{10} \\
& b^6)) / ((b^{13} - 5a^2b^{11} + 10a^4b^9 - 10a^6b^7 + 5a^8b^5 - a^{10}b^3) * (a^2b^{10} + b^{11} - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 \\
& - a^7b^4))) * (-a + b)^5(a - b)^5)^{(1/2)} * (2a^4 + 6b^4 - 5a^2b^2) / (2 * \\
& (b^{13} - 5a^2b^{11} + 10a^4b^9 - 10a^6b^7 + 5a^8b^5 - a^{10}b^3))) * (-a \\
& + b)^5(a - b)^5)^{(1/2)} * (2a^4 + 6b^4 - 5a^2b^2) / (2(b^{13} - 5a^2b^{11} \\
& + 10a^4b^9 - 10a^6b^7 + 5a^8b^5 - a^{10}b^3)) - (a * ((8 * \tan(c/2 + (d*x) \\
&)/2) * (8a^{10} - 8a^9b - 8a^8b^9 + 4b^{10} + 24a^2b^8 + 32a^3b^7 - 52a^4 \\
& b^6 - 48a^5b^5 + 57a^6b^4 + 32a^7b^3 - 32a^8b^2)) / (a^2b^{10} + b^{11} \\
& - 3a^2b^9 - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4) - (a * (\\
& (8 * (12a^2b^{14} - 4b^{15} + 8a^2b^{13} - 34a^3b^{12} - 6a^4b^{11} + 36a^5b^{10} \\
& + 4a^6b^9 - 18a^7b^8 - 2a^8b^7 + 4a^9b^6)) / (a^2b^{12} + b^{13} - 3a^2 \\
& * b^{11} - 3a^3b^{10} + 3a^4b^9 + 3a^5b^8 - a^6b^7 - a^7b^6) + (4a * \tan(\\
& c/2 + (d*x)/2) * (-a + b)^5(a - b)^5)^{(1/2)} * (2a^4 + 6b^4 - 5a^2b^2) * (8 \\
& a^2b^{15} - 8a^2b^{14} - 32a^3b^{13} + 32a^4b^{12} + 48a^5b^{11} - 48a^6b^{10} \\
& - 32a^7b^9 + 32a^8b^8 + 8a^9b^7 - 8a^{10}b^6)) / ((b^{13} - 5a^2b^{11} + \\
& 10a^4b^9 - 10a^6b^7 + 5a^8b^5 - a^{10}b^3) * (a^2b^{10} + b^{11} - 3a^2b^9 \\
& - 3a^3b^8 + 3a^4b^7 + 3a^5b^6 - a^6b^5 - a^7b^4))) * (-a + b)^5(a
\end{aligned}$$

$$\begin{aligned}
& - b)^5)^{(1/2)} * (2*a^4 + 6*b^4 - 5*a^2*b^2)) / (2*(b^{13} - 5*a^2*b^{11} + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^{10}*b^3))) * (- (a + b)^5 * (a - b)^5)^{(1/2)} * (2*a^4 + 6*b^4 - 5*a^2*b^2)) / (2*(b^{13} - 5*a^2*b^{11} + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^{10}*b^3))) * (- (a + b)^5 * (a - b)^5)^{(1/2)} * (2*a^4 + 6*b^4 - 5*a^2*b^2) * i) / (d*(b^{13} - 5*a^2*b^{11} + 10*a^4*b^9 - 10*a^6*b^7 + 5*a^8*b^5 - a^{10}*b^3))
\end{aligned}$$

3.472 $\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^3} dx$

Optimal result	5038
Rubi [A] (verified)	5038
Mathematica [A] (verified)	5040
Maple [A] (verified)	5041
Fricas [A] (verification not implemented)	5041
Sympy [F(-1)]	5042
Maxima [F(-2)]	5042
Giac [A] (verification not implemented)	5042
Mupad [B] (verification not implemented)	5043

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^3} dx = \frac{(a^2 + 2b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{a^2 \sin(c+dx)}{2b(a^2 - b^2)d(a+b \cos(c+dx))^2} + \frac{a(a^2 - 4b^2) \sin(c+dx)}{2b(a^2 - b^2)^2 d(a+b \cos(c+dx))}$$

[Out] (a^2+2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*a*(a^2-4*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2869, 2833, 12, 2738, 211}

$$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^3} dx = \frac{(a^2 + 2b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}} - \frac{a^2 \sin(c+dx)}{2bd(a^2 - b^2)(a+b \cos(c+dx))^2} + \frac{a(a^2 - 4b^2) \sin(c+dx)}{2bd(a^2 - b^2)^2 (a+b \cos(c+dx))}$$

[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]

[Out] $((a^2 + 2*b^2)*\text{ArcTan}[\text{Sqrt}[a - b]*\text{Tan}[(c + d*x)/2]]/\text{Sqrt}[a + b])/((a - b)^{(5/2)}*(a + b)^{(5/2)*d} - (a^2*\text{Sin}[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*\text{Cos}[c + d*x])^2) + (a*(a^2 - 4*b^2)*\text{Sin}[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*\text{Cos}[c + d*x]))$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 211

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]/a)*\text{ArcTan}[x/\text{Rt}[a/b, 2]], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 2738

$\text{Int}[(a_ + (b_)*\text{sin}[\text{Pi}/2 + (c_.) + (d_)*(x_)])^{-1}, x_Symbol] \rightarrow \text{With}[\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Dist}[2*(e/d), \text{Subst}[\text{Int}[1/(a + b + (a - b)*e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 2833

$\text{Int}[(a_ + (b_)*\text{sin}[(e_.) + (f_)*(x_)])^{(m_)*((c_.) + (d_)*\text{sin}[(e_.) + (f_)*(x_)])}, x_Symbol] \rightarrow \text{Simp}[(-b*c - a*d)*\text{Cos}[e + f*x]*((a + b*\text{Sin}[e + f*x])^{(m + 1)/(f*(m + 1)*(a^2 - b^2)}), x] + \text{Dist}[1/((m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)*\text{Simp}[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[2*m]$

Rule 2869

$\text{Int}[(a_ + (b_)*\text{sin}[(e_.) + (f_)*(x_)])^{(m_)*((c_.) + (d_)*\text{sin}[(e_.) + (f_)*(x_)])^2}, x_Symbol] \rightarrow \text{Simp}[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\text{Cos}[e + f*x]*((a + b*\text{Sin}[e + f*x])^{(m + 1)/(b*f*(m + 1)*(a^2 - b^2)}), x] - \text{Dist}[1/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)*\text{Simp}[b*(m + 1)*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) + c^2*(m + 2)))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -1]$

Rubi steps

$$\text{integral} = -\frac{a^2 \sin(c + dx)}{2b(a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{\int \frac{2ab + (a^2 - 2b^2) \cos(c + dx)}{(a + b \cos(c + dx))^2} dx}{2b(a^2 - b^2)}$$

$$\begin{aligned}
&= -\frac{a^2 \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{a(a^2-4b^2)\sin(c+dx)}{2b(a^2-b^2)^2d(a+b\cos(c+dx))} + \frac{\int \frac{b(a^2+2b^2)}{a+b\cos(c+dx)} dx}{2b(a^2-b^2)^2} \\
&= -\frac{a^2 \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} \\
&\quad + \frac{a(a^2-4b^2)\sin(c+dx)}{2b(a^2-b^2)^2d(a+b\cos(c+dx))} + \frac{(a^2+2b^2)\int \frac{1}{a+b\cos(c+dx)} dx}{2(a^2-b^2)^2} \\
&= -\frac{a^2 \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{a(a^2-4b^2)\sin(c+dx)}{2b(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad + \frac{(a^2+2b^2)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)^2d} \\
&= \frac{(a^2+2b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{a^2 \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} \\
&\quad + \frac{a(a^2-4b^2)\sin(c+dx)}{2b(a^2-b^2)^2d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\begin{aligned}
&\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3} dx \\
&\quad - \frac{2(a^2+2b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{a(-3ab+(a^2-4b^2)\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} \\
&= \frac{\hspace{10em}}{2d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]

[Out] ((-2*(a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (a*(-3*a*b + (a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2)/(2*d)

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{(a+4b)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^2+2ab+b^2)}+\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^2-2ab+b^2)}\left(a^2+2b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}{d}$
default	$\frac{-\frac{(a+4b)a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^2+2ab+b^2)}+\frac{(a-4b)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^2-2ab+b^2)}\left(a^2+2b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^2}+\frac{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a-b)(a+b)}}{d}$
risch	$\frac{ia(2ba^3e^{3i(dx+c)}-5b^3ae^{3i(dx+c)}+2a^4e^{2i(dx+c)}-7b^2a^2e^{2i(dx+c)}-4b^4e^{2i(dx+c)}+2ba^3e^{i(dx+c)}-11e^{i(dx+c)}b^3a+a^2b^2)}{b^2(a^2-b^2)^2d(b e^{2i(dx+c)}+2a e^{i(dx+c)}+b)^2}$

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(2*(-1/2*(a+4*b)*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c))^3+1/2*(a-4*b)*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+(a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2))*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 587, normalized size of antiderivative = 3.94

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3} dx$$

$$= \left[-\frac{(a^4+2a^2b^2+(a^2b^2+2b^4)\cos(dx+c)^2+2(a^3b+2ab^3)\cos(dx+c))\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)+2a^2-b^2}{4((a^6b^2-3a^4b^4+3a^2b^6-b^8)d\cos(dx+c)^2+2(a^7b^2-3a^5b^4+3a^3b^6-b^8)d\cos(dx+c)+2(a^8-3a^6b^2+3a^4b^4-a^2b^6)*d)}\right)}{4((a^6b^2-3a^4b^4+3a^2b^6-b^8)d\cos(dx+c)^2+2(a^7b^2-3a^5b^4+3a^3b^6-b^8)d\cos(dx+c)+2(a^8-3a^6b^2+3a^4b^4-a^2b^6)*d)} \right]$$

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] [-1/4*((a^4+2*a^2*b^2+(a^2*b^2+2*b^4)*cos(d*x+c)^2+2*(a^3*b+2*a*b^3)*cos(d*x+c))*sqrt(-a^2+b^2)*log((2*a*b*cos(d*x+c)+(2*a^2-b^2)*cos(d*x+c)^2+2*sqrt(-a^2+b^2)*(a*cos(d*x+c)+b)*sin(d*x+c)-a^2+2*b^2)/(b^2*cos(d*x+c)^2+2*a*b*cos(d*x+c)+a^2))+2*(3*a^4*b-3*a^2*b^3-(a^5-5*a^3*b^2+4*a*b^4)*cos(d*x+c))*sin(d*x+c))/((a^6*b^2-3*a^4*b^4+3*a^2*b^6-b^8)*d*cos(d*x+c)^2+2*(a^7*b-3*a^5*b^3+3*a^3*b^5-a*b^7)*d*cos(d*x+c)+(a^8-3*a^6*b^2+3*a^4*b^4-a^2*b^6)*d), 1/2*((a^4+2*a^2*b^2+(a^2*b^2+2*b^4)*cos(d*x+c)^2+2*(a^3*b+2*a*b^3)*cos(d*x+c))*sqrt(a^2-b^2)*arctan(-(a*cos(d*x+c)+b)/(sqrt(a^2-b^2)*sin(d*x+c)))-(3*a^4*b-3*a^2*b^3-(a^5-5*a^3*b^2+4*a*b^4)*cos(d*x+c))*sin(d*x+c))/((a^6*b^2-3*a^4*b^4+3*a^2*b^6-b^8)*d*cos(d*x+c)^2+2*(a^7*b-3*a^5*b^3+3*a^3*b^5-a*b^7)*d*cos(d*x+c)+(a^8-3*a^6*b^2+3*a^4*b^4-a^2*b^6)*d)]

$b^4 \cos(dx + c) \sin(dx + c) / ((a^6 b^2 - 3a^4 b^4 + 3a^2 b^6 - b^8) d \cos(dx + c)^2 + 2(a^7 b - 3a^5 b^3 + 3a^3 b^5 - a b^7) d \cos(dx + c) + (a^8 - 3a^6 b^2 + 3a^4 b^4 - a^2 b^6) d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Exception raised: ValueError}$$

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^3} dx = \frac{\left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(\frac{-a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right) (a^2 + 2b^2)}{(a^4 - 2a^2 b^2 + b^4) \sqrt{a^2 - b^2}} + \frac{a^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 3a^2 b \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 4ab^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(a^4 - 2a^2 b^2 + b^4)}$$

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] $-(\pi \lfloor \frac{1}{2}(dx + c) / \pi + \frac{1}{2} \rfloor \operatorname{sgn}(-2a + 2b) + \arctan(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}})) * (a^2 + 2b^2) / ((a^4 - 2a^2 b^2 + b^4) \sqrt{a^2 - b^2}) + \frac{a^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 3a^2 b \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 4ab^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(a^4 - 2a^2 b^2 + b^4)}$

$$- 2a^2b^2 + b^4) \sqrt{a^2 - b^2}) + (a^3 \tan(1/2 dx + 1/2 c)^3 + 3a^2b \tan(1/2 dx + 1/2 c)^3 - 4ab^2 \tan(1/2 dx + 1/2 c)^3 - a^3 \tan(1/2 dx + 1/2 c) + 3a^2b \tan(1/2 dx + 1/2 c) + 4ab^2 \tan(1/2 dx + 1/2 c)) / ((a^4 - 2a^2b^2 + b^4) (a \tan(1/2 dx + 1/2 c)^2 - b \tan(1/2 dx + 1/2 c)^2 + a + b)^2) / d$$

Mupad [B] (verification not implemented)

Time = 17.06 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.36

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^3} dx$$

$$= \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2a - 2b) (a^2 - 2ab + b^2)}{2\sqrt{a+b} (a-b)^{5/2}}\right) (a^2 + 2b^2)}{d (a+b)^{5/2} (a-b)^{5/2}} - \frac{\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (a^2 + 4ba)}{(a+b)^2 (a-b)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (4ab - a^2)}{(a+b) (a^2 - 2ab + b^2)}}{d \left(2ab + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (2a^2 - 2b^2) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (a^2 - 2ab + b^2) + a^2 + b^2\right)}$$

[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^3,x)

[Out] (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(a - b)^(5/2)))*(a^2 + 2*b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2)) - ((tan(c/2 + (d*x)/2)^3*(4*a*b + a^2))/((a + b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(4*a*b - a^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2))

3.473 $\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^3} dx$

Optimal result	5044
Rubi [A] (verified)	5044
Mathematica [A] (verified)	5046
Maple [A] (verified)	5046
Fricas [B] (verification not implemented)	5047
Sympy [F(-1)]	5047
Maxima [F(-2)]	5048
Giac [B] (verification not implemented)	5048
Mupad [B] (verification not implemented)	5049

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^3} dx = -\frac{3ab \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a \sin(c+dx)}{2(a^2-b^2)d(a+b \cos(c+dx))^2} + \frac{(a^2+2b^2) \sin(c+dx)}{2(a^2-b^2)^2 d(a+b \cos(c+dx))}$$

[Out] $-3*a*b*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/(a-b)^{(5/2)/(a+b)^{(5/2)/d+1/2*a*\sin(d*x+c)/(a^2-b^2)/d/(a+b*\cos(d*x+c))^2+1/2*(a^2+2*b^2)*\sin(d*x+c)/(a^2-b^2)^2/d/(a+b*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2833, 12, 2738, 211}

$$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^3} dx = \frac{(a^2+2b^2) \sin(c+dx)}{2d(a^2-b^2)^2(a+b \cos(c+dx))} + \frac{a \sin(c+dx)}{2d(a^2-b^2)(a+b \cos(c+dx))^2} - \frac{3ab \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

[In] $\text{Int}[\text{Cos}[c+d*x]/(a+b*\text{Cos}[c+d*x])^3,x]$

[Out] $(-3*a*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^{(5/2)}*(a + b)^{(5/2)*d} + (a*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((a^2 + 2*b^2)*Sin[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2833

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{a \sin(c + dx)}{2(a^2 - b^2) d(a + b \cos(c + dx))^2} - \frac{\int \frac{2b - a \cos(c + dx)}{(a + b \cos(c + dx))^2} dx}{2(a^2 - b^2)} \\
 &= \frac{a \sin(c + dx)}{2(a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{(a^2 + 2b^2) \sin(c + dx)}{2(a^2 - b^2)^2 d(a + b \cos(c + dx))} + \frac{\int -\frac{3ab}{a + b \cos(c + dx)} dx}{2(a^2 - b^2)^2} \\
 &= \frac{a \sin(c + dx)}{2(a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{(a^2 + 2b^2) \sin(c + dx)}{2(a^2 - b^2)^2 d(a + b \cos(c + dx))} - \frac{(3ab) \int \frac{1}{a + b \cos(c + dx)} dx}{2(a^2 - b^2)^2} \\
 &= \frac{a \sin(c + dx)}{2(a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{(a^2 + 2b^2) \sin(c + dx)}{2(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
 &\quad - \frac{(3ab) \text{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{(a^2 - b^2)^2 d}
 \end{aligned}$$

$$= -\frac{3ab \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a \sin(c+dx)}{2(a^2-b^2)d(a+b \cos(c+dx))^2}$$

$$+ \frac{(a^2+2b^2) \sin(c+dx)}{2(a^2-b^2)^2 d(a+b \cos(c+dx))}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^3} dx$$

$$= \frac{6ab \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{(a(2a^2+b^2)+b(a^2+2b^2) \cos(c+dx)) \sin(c+dx)}{(a+b \cos(c+dx))^2}$$

$$2(a-b)^2(a+b)^2d$$

[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^3,x]

[Out] ((6*a*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + ((a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Cos[c + d*x])*Sin[c + d*x]/(a + b*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{2 \left(-\frac{(2a^2+ab+2b^2) \tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{2(a-b)(a^2+2ab+b^2)} - \frac{(2a^2-ab+2b^2) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)} \right)}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) a - b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + a + b \right)^2 \right) d} - \frac{3ab \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a^4-2a^2b^2+b^4) \sqrt{(a-b)(a+b)}}$
default	$\frac{2 \left(-\frac{(2a^2+ab+2b^2) \tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{2(a-b)(a^2+2ab+b^2)} - \frac{(2a^2-ab+2b^2) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)} \right)}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) a - b \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + a + b \right)^2 \right) d} - \frac{3ab \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a^4-2a^2b^2+b^4) \sqrt{(a-b)(a+b)}}$
risch	$\frac{i(3b^3 a e^{3i(dx+c)} + 2a^4 e^{2i(dx+c)} + 5b^2 a^2 e^{2i(dx+c)} + 2b^4 e^{2i(dx+c)} + 4b a^3 e^{i(dx+c)} + 5e^{i(dx+c)} b^3 a + a^2 b^2 + 2b^4)}{b(a^2-b^2)^2 d (b e^{2i(dx+c)} + 2a e^{i(dx+c)} + b)^2} - \frac{3ab \ln\left(e^{i(dx+c)}\right)}{2\sqrt{-a^2+b^2}}$

[In] int(cos(d*x+c)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-2*(-1/2*(2*a^2+a*b+2*b^2)/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(2*a^2-a*b+2*b^2)/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2-3*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(121) = 242$.

Time = 0.29 (sec) , antiderivative size = 555, normalized size of antiderivative = 4.14

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3} dx$$

$$= \left[\frac{3(ab^3\cos(dx+c)^2 + 2a^2b^2\cos(dx+c) + a^3b)\sqrt{-a^2+b^2} \log\left(\frac{2ab\cos(dx+c) + (2a^2-b^2)\cos(dx+c)^2 - 2\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2 + 2ab\cos(dx+c) + a^2}\right) - (2a^5 - a^3b^2 - ab^5)\sqrt{a^2-b^2} \arctan\left(\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right) - (2a^5 - a^3b^2 - ab^5)\sqrt{a^2-b^2} \arctan\left(\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)}{4((a^6b^2 - 3a^4b^4 + 3a^2b^6 - b^8)d\cos(dx+c)^2 + 2(a^7b - 3a^5b^3 + 3a^3b^5 - ab^7)d\cos(dx+c) + (a^8 - 3a^6b^2 + 3a^4b^4 - a^2b^6)d)} \right]$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] $[-1/4*(3*(a*b^3*\cos(d*x + c)^2 + 2*a^2*b^2*\cos(d*x + c) + a^3*b)*\sqrt{-a^2 + b^2}*\log((2*a*b*\cos(d*x + c) + (2*a^2 - b^2)*\cos(d*x + c)^2 - 2*\sqrt{-a^2 + b^2}*(a*\cos(d*x + c) + b)*\sin(d*x + c) - a^2 + 2*b^2)/(b^2*\cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + a^2)) - 2*(2*a^5 - a^3*b^2 - a*b^4 + (a^4*b + a^2*b^3 - 2*b^5)*\cos(d*x + c))*\sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*\cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*\cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), -1/2*(3*(a*b^3*\cos(d*x + c)^2 + 2*a^2*b^2*\cos(d*x + c) + a^3*b)*\sqrt{a^2 - b^2}*\arctan(-(a*\cos(d*x + c) + b)/(\sqrt{a^2 - b^2}*\sin(d*x + c)))) - (2*a^5 - a^3*b^2 - a*b^4 + (a^4*b + a^2*b^3 - 2*b^5)*\cos(d*x + c))*\sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*\cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*\cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(121) = 242.

Time = 0.32 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.02

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^3} dx$$

$$= \frac{3 \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right) ab}{(a^4 - 2a^2b^2 + b^4)\sqrt{a^2 - b^2}} + \frac{2a^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - a^2b \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + ab^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(a^4 - 2a^2b^2 + b^4)}$$

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a*b/((a^4 - 2*a^2*b
^2 + b^4)*sqrt(a^2 - b^2)) + (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*
d*x + 1/2*c)^3 + a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2*c)^
3 + 2*a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) + a*b^2*tan(1/2
*d*x + 1/2*c) + 2*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.54

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^3} dx$$

$$= \frac{\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (2a^2 + ab + 2b^2)}{(a+b)^2 (a-b)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2a^2 - ab + 2b^2)}{(a+b)(a^2 - 2ab + b^2)}}{d \left(2ab + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (2a^2 - 2b^2) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (a^2 - 2ab + b^2) + a^2 + b^2 \right)}$$

$$- \frac{3ab \operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2a - 2b) (a^2 - 2ab + b^2)}{2\sqrt{a+b}(a-b)^{5/2}}\right)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

```
[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(a*b + 2*a^2 + 2*b^2))/((a + b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(2*a^2 - a*b + 2*b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (3*a*b*atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)*(a - b)^(5/2))))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.474 $\int \frac{1}{(a+b \cos(c+dx))^3} dx$

Optimal result	5050
Rubi [A] (verified)	5050
Mathematica [A] (verified)	5052
Maple [A] (verified)	5052
Fricas [B] (verification not implemented)	5053
Sympy [F(-1)]	5054
Maxima [F(-2)]	5054
Giac [B] (verification not implemented)	5054
Mupad [B] (verification not implemented)	5055

Optimal result

Integrand size = 12, antiderivative size = 133

$$\int \frac{1}{(a+b \cos(c+dx))^3} dx = \frac{(2a^2 + b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b \sin(c+dx)}{2(a^2 - b^2) d(a+b \cos(c+dx))^2} - \frac{3ab \sin(c+dx)}{2(a^2 - b^2)^2 d(a+b \cos(c+dx))}$$

[Out] (2*a^2+b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-3/2*a*b*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2743, 2833, 12, 2738, 211}

$$\int \frac{1}{(a+b \cos(c+dx))^3} dx = \frac{(2a^2 + b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}} - \frac{3ab \sin(c+dx)}{2d(a^2 - b^2)^2 (a+b \cos(c+dx))} - \frac{b \sin(c+dx)}{2d(a^2 - b^2) (a+b \cos(c+dx))^2}$$

[In] Int[(a + b*cos[c + d*x])^(-3), x]

[Out] $((2*a^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^{(5/2)}*(a + b)^{(5/2)*d} - (b*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (3*a*b*Sin[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2833

Int[((a_) + (b_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{b \sin(c + dx)}{2(a^2 - b^2) d(a + b \cos(c + dx))^2} - \frac{\int \frac{-2a + b \cos(c + dx)}{(a + b \cos(c + dx))^2} dx}{2(a^2 - b^2)} \\ &= -\frac{b \sin(c + dx)}{2(a^2 - b^2) d(a + b \cos(c + dx))^2} - \frac{3ab \sin(c + dx)}{2(a^2 - b^2)^2 d(a + b \cos(c + dx))} + \frac{\int \frac{2a^2 + b^2}{a + b \cos(c + dx)} dx}{2(a^2 - b^2)^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{b \sin(c + dx)}{2(a^2 - b^2) d(a + b \cos(c + dx))^2} \\
&\quad - \frac{3ab \sin(c + dx)}{2(a^2 - b^2)^2 d(a + b \cos(c + dx))} + \frac{(2a^2 + b^2) \int \frac{1}{a + b \cos(c + dx)} dx}{2(a^2 - b^2)^2} \\
&= -\frac{b \sin(c + dx)}{2(a^2 - b^2) d(a + b \cos(c + dx))^2} - \frac{3ab \sin(c + dx)}{2(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad + \frac{(2a^2 + b^2) \text{Subst}\left(\int \frac{1}{a + b + (a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{(a^2 - b^2)^2 d} \\
&= \frac{(2a^2 + b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a+b}}\right)}{(a - b)^{5/2}(a + b)^{5/2}d} \\
&\quad - \frac{b \sin(c + dx)}{2(a^2 - b^2) d(a + b \cos(c + dx))^2} - \frac{3ab \sin(c + dx)}{2(a^2 - b^2)^2 d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \frac{1}{(a + b \cos(c + dx))^3} dx \\
&= \frac{2(2a^2 + b^2) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{-a^2 + b^2}}\right)}{(-a^2 + b^2)^{5/2}} + \frac{b(-4a^2 + b^2 - 3ab \cos(c + dx)) \sin(c + dx)}{(a - b)^2 (a + b)^2 (a + b \cos(c + dx))^2} \\
&= \frac{\hspace{15em}}{2d}
\end{aligned}$$

[In] Integrate[(a + b*cos[c + d*x])^(-3),x]

[Out] ((-2*(2*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + (b*(-4*a^2 + b^2 - 3*a*b*cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*cos[c + d*x])^2)/(2*d)

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{-\frac{(4a+b)b \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - (4a-b)b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a-b)(a^2+2ab+b^2)} - \frac{(4a-b)b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a+b)(a^2-2ab+b^2)} + \frac{(2a^2+b^2) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a^4-2a^2b^2+b^4) \sqrt{(a-b)(a+b)}}}{\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+a+b\right)^2} + \frac{(2a^2+b^2) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a^4-2a^2b^2+b^4) \sqrt{(a-b)(a+b)}}$
default	$\frac{-\frac{(4a+b)b \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - (4a-b)b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a-b)(a^2+2ab+b^2)} - \frac{(4a-b)b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a+b)(a^2-2ab+b^2)} + \frac{(2a^2+b^2) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a^4-2a^2b^2+b^4) \sqrt{(a-b)(a+b)}}}{d}$
risch	$-\frac{i(2a^2be^{3i(dx+c)}+b^3e^{3i(dx+c)}+6a^3e^{2i(dx+c)}+3ab^2e^{2i(dx+c)}+10a^2be^{i(dx+c)}-b^3e^{i(dx+c)}+3ab^2)}{(a^2-b^2)^2d(be^{2i(dx+c)}+2ae^{i(dx+c)}+b)^2} - \frac{a^2 \ln\left(e^{i(dx+c)}\right)}{\sqrt{-a^2+b^2}}$

[In] int(1/(a+cos(d*x+c))*b^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(4*a-b)*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(120) = 240.

Time = 0.29 (sec) , antiderivative size = 585, normalized size of antiderivative = 4.40

$$\int \frac{1}{(a+b \cos(c+dx))^3} dx$$

$$= \left[-\frac{(2a^4+a^2b^2+(2a^2b^2+b^4)\cos(dx+c)^2+2(2a^3b+ab^3)\cos(dx+c))\sqrt{-a^2+b^2} \log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{(b^2\cos(dx+c)^2+2a*b*\cos(dx+c)+a^2)}\right)+2*(4*a^4*b-5*a^2*b^3+b^5+3*(a^3*b^2-a*b^4)*\cos(dx+c))*\sin(dx+c)}{4*((a^6b^2-3a^4b^4+3a^2b^6-b^8)d\cos(dx+c)^2+2(a^7b^2-3a^4*b^4+3a^2*b^6-b^8)*d*\cos(dx+c)^2+2*(a^7*b-3a^5*b^3+3a^3*b^5-a*b^7)*d*\cos(dx+c)+(a^8-3a^6*b^2+3a^4*b^4-a^2*b^6)*d)} \right]$$

[In] integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] [-1/4*((2*a^4+a^2*b^2+(2*a^2*b^2+b^4)*cos(d*x+c)^2+2*(2*a^3*b+a*b^3)*cos(d*x+c))*sqrt(-a^2+b^2)*log((2*a*b*cos(d*x+c)+(2*a^2-b^2)*cos(d*x+c)^2+2*sqrt(-a^2+b^2)*(a*cos(d*x+c)+b)*sin(d*x+c)-a^2+2*b^2)/(b^2*cos(d*x+c)^2+2*a*b*cos(d*x+c)+a^2))+2*(4*a^4*b-5*a^2*b^3+b^5+3*(a^3*b^2-a*b^4)*cos(d*x+c))*sin(d*x+c)/((a^6*b^2-3*a^4*b^4+3*a^2*b^6-b^8)*d*cos(d*x+c)^2+2*(a^7*b-3*a^5*b^3+3*a^3*b^5-a*b^7)*d*cos(d*x+c)+(a^8-3*a^6*b^2+3*a^4*b^4-a^2*b^6)*d), 1/2*((2*a^4+a^2*b^2+(2*a^2*b^2+b^4)*cos(d*x+c)^2+2*(2*a^3*b+a*b^3)*cos(d*x+c))*sqrt(a^2-b^2)*arctan(-(a*cos(d*x+c)+b)/(sqrt(a^2-b^2)*sin(d*x+c)))-(4*a^4*b-5*a^2*b^3+b^5+3*(a^3*b^2-a*b^4)*cos(d*x+c))*sin(d*x+c)/((a^6*b^2-3*a^4*b^4+3*a^2*b^6-b^8)*d*cos(d*x+c)^2+2*(a^7*b-3*a^5*b^3+3*a^3*b^5-a*b^7)*d*cos(d*x+c)+(a^8-3*a^6*b^2+3*a^4*b^4-a^2*b^6)*d)]

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a + b \cos(c + dx))^3} dx = \text{Exception raised: ValueError}$$

[In] integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(120) = 240.

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.89

$$\int \frac{1}{(a + b \cos(c + dx))^3} dx = \frac{\left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}} \right) \right) (2a^2 + b^2)}{(a^4 - 2a^2b^2 + b^4)\sqrt{a^2 - b^2}} + \frac{4a^2b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 3ab^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{(a^4 - 2a^2b^2 + b^4)} \frac{1}{d}$$

[In] integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(2*a^2 + b^2)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (4*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^2*b*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c)))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2)/d

Mupad [B] (verification not implemented)

Time = 16.63 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a + b \cos(c + dx))^3} dx$$

$$= \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)(2a - 2b)(a^2 - 2ab + b^2)}{2\sqrt{a+b}(a-b)^{5/2}}\right)(2a^2 + b^2)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

$$- \frac{\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3(b^2 + 4ab)}{(a+b)^2(a-b)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)(4ab - b^2)}{(a+b)(a^2 - 2ab + b^2)}}{d\left(2ab + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2(2a^2 - 2b^2) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4(a^2 - 2ab + b^2) + a^2 + b^2\right)}$$

`[In] int(1/(a + b*cos(c + d*x))^3,x)`

```
[Out] (atan((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2)
*(a - b)^(5/2))))*(2*a^2 + b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2)) - ((tan(c/2
+ (d*x)/2)^3*(4*a*b + b^2))/((a + b)^2*(a - b)) + (tan(c/2 + (d*x)/2)*(4*a
*b - b^2))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*
(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2))
```

3.475 $\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^3} dx$

Optimal result	5056
Rubi [A] (verified)	5056
Mathematica [A] (verified)	5059
Maple [A] (verified)	5059
Fricas [B] (verification not implemented)	5060
Sympy [F]	5060
Maxima [F(-2)]	5061
Giac [B] (verification not implemented)	5061
Mupad [B] (verification not implemented)	5062

Optimal result

Integrand size = 19, antiderivative size = 182

$$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^3} dx = -\frac{b(6a^4 - 5a^2b^2 + 2b^4) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\operatorname{arctanh}(\sin(c+dx))}{a^3d} + \frac{b^2 \sin(c+dx)}{2a(a^2-b^2)d(a+b \cos(c+dx))^2} + \frac{b^2(5a^2-2b^2) \sin(c+dx)}{2a^2(a^2-b^2)^2d(a+b \cos(c+dx))}$$

[Out] $-b*(6*a^4-5*a^2*b^2+2*b^4)*\arctan((a-b)^{(1/2)}*\tan(1/2*d*x+1/2*c)/(a+b)^{(1/2)})/a^3/(a-b)^{(5/2)/(a+b)^{(5/2)/d}+\operatorname{arctanh}(\sin(d*x+c))/a^3/d+1/2*b^2*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\cos(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*\sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2881, 3134, 3080, 3855, 2738, 211}

$$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^3} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^3d} + \frac{b^2(5a^2-2b^2) \sin(c+dx)}{2a^2d(a^2-b^2)^2(a+b \cos(c+dx))} + \frac{b^2 \sin(c+dx)}{2ad(a^2-b^2)(a+b \cos(c+dx))^2} - \frac{b(6a^4-5a^2b^2+2b^4) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}$$

[In] Int[Sec[c + d*x]/(a + b*cos[c + d*x])^3,x]

[Out] -((b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + ArcTanh[Sin[c + d*x]]/(a^3*d) + (b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*cos[c + d*x])^2) + (b^2*(5*a^2 - 2*b^2)*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*cos[c + d*x]))

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3080

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

Int[((A_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[

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(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)
*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
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Rule 3855

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Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
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Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{\int \frac{(2(a^2 - b^2) - 2ab \cos(c + dx) + b^2 \cos^2(c + dx)) \sec(c + dx)}{(a + b \cos(c + dx))^2} dx}{2a(a^2 - b^2)} \\
&= \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{b^2(5a^2 - 2b^2) \sin(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{(2(a^2 - b^2)^2 - ab(4a^2 - b^2) \cos(c + dx)) \sec(c + dx)}{a + b \cos(c + dx)} dx}{2a^2(a^2 - b^2)^2} \\
&= \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{b^2(5a^2 - 2b^2) \sin(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \sec(c + dx) dx}{a^3} - \frac{(b(6a^4 - 5a^2b^2 + 2b^4)) \int \frac{1}{a + b \cos(c + dx)} dx}{2a^3(a^2 - b^2)^2} \\
&= \frac{\arctanh(\sin(c + dx))}{a^3 d} + \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d(a + b \cos(c + dx))^2} \\
&\quad + \frac{b^2(5a^2 - 2b^2) \sin(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad - \frac{(b(6a^4 - 5a^2b^2 + 2b^4)) \text{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{a^3(a^2 - b^2)^2 d} \\
&= -\frac{b(6a^4 - 5a^2b^2 + 2b^4) \arctan\left(\frac{\sqrt{a - b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b}}\right)}{a^3(a - b)^{5/2}(a + b)^{5/2} d} + \frac{\arctanh(\sin(c + dx))}{a^3 d} \\
&\quad + \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{b^2(5a^2 - 2b^2) \sin(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^3} dx$$

$$= \frac{2b(6a^4 - 5a^2b^2 + 2b^4) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - 2 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 2 \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^3d}$$

[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^3,x]

[Out] ((2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*(6*a^3 - 3*a*b^2 + b*(5*a^2 - 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x]^2))/(2*a^3*d)

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.38

method	result
derivativedivides	$-\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^3} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{a^3} - \frac{2b \left(\frac{-\frac{(6a^2+ab-2b^2)ab \tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{2(a-b)(a^2+2ab+b^2)} - \frac{(6a^2-ab-2b^2)ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)} \right)}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - a - b\right)\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + a + b\right)^2} + \frac{(6a^4 - 5a^2b^2 + 2b^4)}{a^3}$
default	$-\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^3} + \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{a^3} - \frac{2b \left(\frac{-\frac{(6a^2+ab-2b^2)ab \tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{2(a-b)(a^2+2ab+b^2)} - \frac{(6a^2-ab-2b^2)ab \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)} \right)}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) - a - b\right)\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right) + a + b\right)^2} + \frac{(6a^4 - 5a^2b^2 + 2b^4)}{a^3}$
risch	$\frac{ib(4ba^3e^{3i(dx+c)} - b^3ae^{3i(dx+c)} + 10a^4e^{2i(dx+c)} + b^2a^2e^{2i(dx+c)} - 2b^4e^{2i(dx+c)} + 16ba^3e^{i(dx+c)} - 7e^{i(dx+c)}b^3a + 5a^2b^2 - 7a^2b^2)}{a^2d(a^2-b^2)^2(b e^{2i(dx+c)} + 2a e^{i(dx+c)} + b)^2}$

[In] int(sec(d*x+c)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/a^3*ln(tan(1/2*d*x+1/2*c)-1)+1/a^3*ln(tan(1/2*d*x+1/2*c)+1)-2*b/a^3*((-1/2*(6*a^2+a*b-2*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(6*a^2-a*b-2*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(6*a^4-5*a^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(169) = 338.

Time = 0.80 (sec) , antiderivative size = 1142, normalized size of antiderivative = 6.27

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Too large to display}$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] [-1/4*((6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b^6 + (5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d), -1/2*((6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b^6 + (5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d)]

Sympy [F]

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^3} dx$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**3,x)

[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**3, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Exception raised: ValueError}$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(169) = 338.

Time = 0.32 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.89

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^3} dx$$

$$\frac{(6a^4b - 5a^2b^3 + 2b^5) \left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^7 - 2a^5b^2 + a^3b^4)\sqrt{a^2 - b^2}} + \frac{6a^3b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 5a^2b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\dots}$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] ((6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 - b^2)) + (6*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) + 5*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*a*b^4*tan(1/2*d*x + 1/2*c) - 2*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)/d

Mupad [B] (verification not implemented)

Time = 23.98 (sec) , antiderivative size = 5090, normalized size of antiderivative = 27.97

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Too large to display}$$

[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^3),x)

[Out] - (atan(((((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2)))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (8*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))/a^3 - (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2)))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))*1i)/a^3 - (((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2)))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))/a^3 + (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2)))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2))*1i)/a^3)/((((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2)))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (8*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))/a^3 - (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2)))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))/a^3 + (((8*(12*a^14*b - 4*a^15 + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^10*b^5 - 6*a^11*b^4 - 34*a^12*b^3 + 8*a^13*b^2)))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)))/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))/a^3 + (8*tan(c/2 + (d*x)/2)*(4*a^10 - 8*a^9*b - 8*a*b^9 + 8*b^10 - 32*a^2*b^8 + 32

$$\begin{aligned}
& *a^3*b^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2) \\
& / (a^{10}*b + a^{11} - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3 \\
& *a^9*b^2) / a^3 - (16*(12*a^8*b - 2*a*b^8 + 4*b^9 - 18*a^2*b^7 + 13*a^3*b^6 \\
& + 36*a^4*b^5 - 26*a^5*b^4 - 34*a^6*b^3 + 24*a^7*b^2)) / (a^{12}*b + a^{13} - a^6* \\
& b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2)) * 2i) / (a^3 \\
& *d) - ((\tan(c/2 + (d*x)/2)^3*(a*b^3 - 2*b^4 + 6*a^2*b^2)) / ((a^2*b - a^3)*(a \\
& + b)^2) + (\tan(c/2 + (d*x)/2)*(a*b^3 + 2*b^4 - 6*a^2*b^2)) / ((a + b)*(a^4 - \\
& 2*a^3*b + a^2*b^2))) / (d*(2*a*b + \tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + \tan \\
& (c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (b*\operatorname{atan}(((b*((8*\tan(\\
& c/2 + (d*x)/2)*(4*a^{10} - 8*a^9*b - 8*a*b^9 + 8*b^{10} - 32*a^2*b^8 + 32*a^3*b \\
& ^7 + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2)) / (a^{10} \\
& *b + a^{11} - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b \\
& ^2) - (b*((8*(12*a^{14}*b - 4*a^{15} + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a \\
& ^9*b^6 + 36*a^{10}*b^5 - 6*a^{11}*b^4 - 34*a^{12}*b^3 + 8*a^{13}*b^2)) / (a^{12}*b + a^{13} \\
& - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) - \\
& (4*b*\tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^{(1/2)}*(6*a^4 + 2*b^4 - 5*a^ \\
& 2*b^2)*(8*a^{15}*b - 8*a^6*b^{10} + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^ \\
& 10*b^6 + 48*a^{11}*b^5 + 32*a^{12}*b^4 - 32*a^{13}*b^3 - 8*a^{14}*b^2)) / ((a^{13} - a^ \\
& 3*b^{10} + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2)*(a^{10}*b + a^{11} - \\
& a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*(-(a \\
& + b)^5*(a - b)^5)^{(1/2)}*(6*a^4 + 2*b^4 - 5*a^2*b^2)) / (2*(a^{13} - a^3*b^{10} + \\
& 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2)))*(-(a + b)^5*(a - b)^5)^ \\
& (1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)*i) / (2*(a^{13} - a^3*b^{10} + 5*a^5*b^8 - 10* \\
& a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2)) + (b*((8*\tan(c/2 + (d*x)/2)*(4*a^{10} - 8 \\
& *a^9*b - 8*a*b^9 + 8*b^{10} - 32*a^2*b^8 + 32*a^3*b^7 + 57*a^4*b^6 - 48*a^5*b \\
& ^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2)) / (a^{10}*b + a^{11} - a^4*b^7 - a^5* \\
& b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + (b*((8*(12*a^{14}*b - \\
& 4*a^{15} + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9*b^6 + 36*a^{10}*b^5 - 6*a \\
& ^{11}*b^4 - 34*a^{12}*b^3 + 8*a^{13}*b^2)) / (a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 + 3 \\
& *a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) + (4*b*\tan(c/2 + (d*x)/2)*(\\
& -(a + b)^5*(a - b)^5)^{(1/2)}*(6*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^{15}*b - 8*a^6*b \\
& ^{10} + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^{10}*b^6 + 48*a^{11}*b^5 + 32* \\
& a^{12}*b^4 - 32*a^{13}*b^3 - 8*a^{14}*b^2)) / ((a^{13} - a^3*b^{10} + 5*a^5*b^8 - 10*a^ \\
& 7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2)*(a^{10}*b + a^{11} - a^4*b^7 - a^5*b^6 + 3*a^6 \\
& *b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*(-(a + b)^5*(a - b)^5)^{(1/2)}*(6 \\
& *a^4 + 2*b^4 - 5*a^2*b^2)) / (2*(a^{13} - a^3*b^{10} + 5*a^5*b^8 - 10*a^7*b^6 + 1 \\
& 0*a^9*b^4 - 5*a^{11}*b^2)))*(-(a + b)^5*(a - b)^5)^{(1/2)}*(6*a^4 + 2*b^4 - 5*a \\
& ^2*b^2)*i) / (2*(a^{13} - a^3*b^{10} + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a \\
& ^{11}*b^2)) / ((16*(12*a^8*b - 2*a*b^8 + 4*b^9 - 18*a^2*b^7 + 13*a^3*b^6 + 36* \\
& a^4*b^5 - 26*a^5*b^4 - 34*a^6*b^3 + 24*a^7*b^2)) / (a^{12}*b + a^{13} - a^6*b^7 - \\
& a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) + (b*((8*\tan(c/ \\
& 2 + (d*x)/2)*(4*a^{10} - 8*a^9*b - 8*a*b^9 + 8*b^{10} - 32*a^2*b^8 + 32*a^3*b^7 \\
& + 57*a^4*b^6 - 48*a^5*b^5 - 52*a^6*b^4 + 32*a^7*b^3 + 24*a^8*b^2)) / (a^{10}*b \\
& + a^{11} - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) \\
&) - (b*((8*(12*a^{14}*b - 4*a^{15} + 4*a^6*b^9 - 2*a^7*b^8 - 18*a^8*b^7 + 4*a^9
\end{aligned}$$

$$\begin{aligned}
& *b^6 + 36a^{10}b^5 - 6a^{11}b^4 - 34a^{12}b^3 + 8a^{13}b^2)) / (a^{12}b + a^{13} \\
& - a^6b^7 - a^7b^6 + 3a^8b^5 + 3a^9b^4 - 3a^{10}b^3 - 3a^{11}b^2) - (\\
& 4b \tan(c/2 + (d*x)/2) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (6a^4 + 2b^4 - 5a^2 * \\
& b^2) * (8a^{15}b - 8a^6b^{10} + 8a^7b^9 + 32a^8b^8 - 32a^9b^7 - 48a^{10} \\
& *b^6 + 48a^{11}b^5 + 32a^{12}b^4 - 32a^{13}b^3 - 8a^{14}b^2)) / ((a^{13} - a^3 * \\
& b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2) * (a^{10}b + a^{11} - a \\
& ^4b^7 - a^5b^6 + 3a^6b^5 + 3a^7b^4 - 3a^8b^3 - 3a^9b^2))) * (-a + \\
& b)^5 * (a - b)^5)^{(1/2)} * (6a^4 + 2b^4 - 5a^2 * b^2)) / (2 * (a^{13} - a^3 * b^{10} + 5a \\
& ^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2))) * (-a + b)^5 * (a - b)^5)^{(1 \\
& /2)} * (6a^4 + 2b^4 - 5a^2 * b^2)) / (2 * (a^{13} - a^3 * b^{10} + 5a^5b^8 - 10a^7b^6 \\
& + 10a^9b^4 - 5a^{11}b^2)) - (b * ((8 * \tan(c/2 + (d*x)/2) * (4a^{10} - 8a^9 * \\
& b - 8a * b^9 + 8b^{10} - 32a^2b^8 + 32a^3b^7 + 57a^4b^6 - 48a^5b^5 - \\
& 52a^6b^4 + 32a^7b^3 + 24a^8b^2)) / (a^{10}b + a^{11} - a^4b^7 - a^5b^6 + \\
& 3a^6b^5 + 3a^7b^4 - 3a^8b^3 - 3a^9b^2) + (b * ((8 * (12a^{14}b - 4a^{15} \\
& + 4a^6b^9 - 2a^7b^8 - 18a^8b^7 + 4a^9b^6 + 36a^{10}b^5 - 6a^{11}b^4 - \\
& 34a^{12}b^3 + 8a^{13}b^2)) / (a^{12}b + a^{13} - a^6b^7 - a^7b^6 + 3a^8 * \\
& b^5 + 3a^9b^4 - 3a^{10}b^3 - 3a^{11}b^2) + (4b * \tan(c/2 + (d*x)/2) * (-a + \\
& b)^5 * (a - b)^5)^{(1/2)} * (6a^4 + 2b^4 - 5a^2 * b^2) * (8a^{15}b - 8a^6b^{10} + \\
& 8a^7b^9 + 32a^8b^8 - 32a^9b^7 - 48a^{10}b^6 + 48a^{11}b^5 + 32a^{12} * \\
& b^4 - 32a^{13}b^3 - 8a^{14}b^2)) / ((a^{13} - a^3 * b^{10} + 5a^5b^8 - 10a^7b^6 \\
& + 10a^9b^4 - 5a^{11}b^2) * (a^{10}b + a^{11} - a^4b^7 - a^5b^6 + 3a^6b^5 \\
& + 3a^7b^4 - 3a^8b^3 - 3a^9b^2))) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (6a^4 \\
& + 2b^4 - 5a^2 * b^2)) / (2 * (a^{13} - a^3 * b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9 \\
& *b^4 - 5a^{11}b^2))) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (6a^4 + 2b^4 - 5a^2 * b^2 \\
&)) / (2 * (a^{13} - a^3 * b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2) \\
&)) * (-a + b)^5 * (a - b)^5)^{(1/2)} * (6a^4 + 2b^4 - 5a^2 * b^2) * 1i) / (d * (a^{13} - \\
& a^3 * b^{10} + 5a^5b^8 - 10a^7b^6 + 10a^9b^4 - 5a^{11}b^2))
\end{aligned}$$

3.476 $\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^3} dx$

Optimal result	5065
Rubi [A] (verified)	5065
Mathematica [A] (verified)	5068
Maple [A] (verified)	5069
Fricas [B] (verification not implemented)	5069
Sympy [F]	5070
Maxima [F(-2)]	5070
Giac [A] (verification not implemented)	5071
Mupad [B] (verification not implemented)	5071

Optimal result

Integrand size = 21, antiderivative size = 232

$$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^3} dx = \frac{3b^2(4a^4 - 5a^2b^2 + 2b^4) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d} - \frac{3b \operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{(2a^4 - 11a^2b^2 + 6b^4) \tan(c+dx)}{2a^3(a^2 - b^2)^2d} + \frac{b^2 \tan(c+dx)}{2a(a^2 - b^2)d(a+b \cos(c+dx))^2} + \frac{3b^2(2a^2 - b^2) \tan(c+dx)}{2a^2(a^2 - b^2)^2d(a+b \cos(c+dx))}$$

```
[Out] 3*b^2*(4*a^4-5*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(5/2)/(a+b)^(5/2)/d-3*b*arctanh(sin(d*x+c))/a^4/d+1/2*(2*a^4-11*a^2*b^2+6*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+3/2*b^2*(2*a^2-b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {2881, 3134, 3080, 3855, 2738, 211}

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3} dx = -\frac{3b\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{3b^2(2a^2-b^2)\tan(c+dx)}{2a^2d(a^2-b^2)^2(a+b\cos(c+dx))} \\ + \frac{b^2\tan(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2} \\ + \frac{3b^2(4a^4-5a^2b^2+2b^4)\operatorname{arctan}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}} \\ + \frac{(2a^4-11a^2b^2+6b^4)\tan(c+dx)}{2a^3d(a^2-b^2)^2}$$

[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]

[Out] (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) - (3*b*ArcTanh[Sin[c + d*x]]/(a^4*d) + ((2*a^4 - 11*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b^2*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*b^2*(2*a^2 - b^2)*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \tan(c + dx)}{2a(a^2 - b^2)d(a + b \cos(c + dx))^2} + \frac{\int \frac{(2a^2 - 3b^2 - 2ab \cos(c + dx) + 2b^2 \cos^2(c + dx)) \sec^2(c + dx)}{(a + b \cos(c + dx))^2} dx}{2a(a^2 - b^2)} \\
&= \frac{b^2 \tan(c + dx)}{2a(a^2 - b^2)d(a + b \cos(c + dx))^2} + \frac{3b^2(2a^2 - b^2) \tan(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{(2a^4 - 11a^2b^2 + 6b^4 - ab(4a^2 - b^2) \cos(c + dx) + 3b^2(2a^2 - b^2) \cos^2(c + dx)) \sec^2(c + dx)}{a + b \cos(c + dx)} dx}{2a^2(a^2 - b^2)^2} \\
&= \frac{(2a^4 - 11a^2b^2 + 6b^4) \tan(c + dx)}{2a^3(a^2 - b^2)^2 d} + \frac{b^2 \tan(c + dx)}{2a(a^2 - b^2)d(a + b \cos(c + dx))^2} \\
&\quad + \frac{3b^2(2a^2 - b^2) \tan(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{(-6b(a^2 - b^2)^2 + 3ab^2(2a^2 - b^2) \cos(c + dx)) \sec(c + dx)}{a + b \cos(c + dx)} dx}{2a^3(a^2 - b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(2a^4 - 11a^2b^2 + 6b^4) \tan(c + dx)}{2a^3 (a^2 - b^2)^2 d} + \frac{b^2 \tan(c + dx)}{2a (a^2 - b^2) d(a + b \cos(c + dx))^2} \\
&+ \frac{3b^2(2a^2 - b^2) \tan(c + dx)}{2a^2 (a^2 - b^2)^2 d(a + b \cos(c + dx))} - \frac{(3b) \int \sec(c + dx) dx}{a^4} \\
&+ \frac{(3b^2(4a^4 - 5a^2b^2 + 2b^4)) \int \frac{1}{a+b \cos(c+dx)} dx}{2a^4 (a^2 - b^2)^2} \\
&= -\frac{3b \operatorname{arctanh}(\sin(c + dx))}{a^4 d} + \frac{(2a^4 - 11a^2b^2 + 6b^4) \tan(c + dx)}{2a^3 (a^2 - b^2)^2 d} \\
&+ \frac{b^2 \tan(c + dx)}{2a (a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{3b^2(2a^2 - b^2) \tan(c + dx)}{2a^2 (a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&+ \frac{(3b^2(4a^4 - 5a^2b^2 + 2b^4)) \operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{a^4 (a^2 - b^2)^2 d} \\
&= \frac{3b^2(4a^4 - 5a^2b^2 + 2b^4) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4 (a-b)^{5/2} (a+b)^{5/2} d} \\
&- \frac{3b \operatorname{arctanh}(\sin(c + dx))}{a^4 d} + \frac{(2a^4 - 11a^2b^2 + 6b^4) \tan(c + dx)}{2a^3 (a^2 - b^2)^2 d} \\
&+ \frac{b^2 \tan(c + dx)}{2a (a^2 - b^2) d(a + b \cos(c + dx))^2} + \frac{3b^2(2a^2 - b^2) \tan(c + dx)}{2a^2 (a^2 - b^2)^2 d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.88

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^3} dx = \frac{6b^2(4a^4 - 5a^2b^2 + 2b^4) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - 6b \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) + 6b \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right) + 2a^4 d$$

[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]

[Out] -1/2*((6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 6*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^3*(8*a^3 - 5*a*b^2 + b*(7*a^2 - 4*b^2))*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) - 2*a*Tan[c + d*x]/(a^4*d)

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{1}{a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{3b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^4} - \frac{1}{a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{3b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{a^4} + \frac{2b^2 \left(\frac{-\frac{(8a^2+ab-4b^2)ab(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right))}{2(a-b)(a^2+2ab+b^2)} \right)}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$
default	$\frac{-\frac{1}{a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{3b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^4} - \frac{1}{a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{3b \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{a^4} + \frac{2b^2 \left(\frac{-\frac{(8a^2+ab-4b^2)ab(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right))}{2(a-b)(a^2+2ab+b^2)} \right)}{d}$
risch	$\frac{i(-6b^3a^3e^{5i(dx+c)} + 3b^5e^{5i(dx+c)}a - 12a^4b^2e^{4i(dx+c)} - 3a^2b^4e^{4i(dx+c)} + 6b^6e^{4i(dx+c)} + 8ba^5e^{3i(dx+c)} - 44b^3a^3e^{3i(dx+c)} + \dots)}{(a^2-b^2)^2da}$

[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/a^3/(tan(1/2*d*x+1/2*c)-1)+3*b/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/a^3/(tan(1/2*d*x+1/2*c)+1)-3*b/a^4*ln(tan(1/2*d*x+1/2*c)+1)+2*b^2/a^4*((-1/2*(8*a^2+a*b-4*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(8*a^2-a*b-4*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^2+3/2*(4*a^4-5*a^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(217) = 434.

Time = 0.80 (sec) , antiderivative size = 1346, normalized size of antiderivative = 5.80

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Too large to display}$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] [-1/4*(3*((4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(d*x + c)^3 + 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(d*x + c)^2 + (4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 6*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x + c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c)^2 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x

```

+ c))*log(sin(d*x + c) + 1) - 6*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c
os(d*x + c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c)^2
+ (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c)
+ 1) - 2*(2*a^9 - 6*a^7*b^2 + 6*a^5*b^4 - 2*a^3*b^6 + (2*a^7*b^2 - 13*a^5*
b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4
*b^5 - 9*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^10*b^2 - 3*a^8*b^4 + 3*a^
6*b^6 - a^4*b^8)*d*cos(d*x + c)^3 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5
*b^7)*d*cos(d*x + c)^2 + (a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*
x + c)), 1/2*(3*((4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(d*x + c)^3 + 2*(4*a^5*
b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(d*x + c)^2 + (4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*
b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 -
b^2)*sin(d*x + c))) - 3*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x +
c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c)^2 + (a^8*b
- 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3
*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x + c)^3 + 2*(a^7*b^2 - 3*a
^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c)^2 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5
- a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*a^9 - 6*a^7*b^2 + 6*a
^5*b^4 - 2*a^3*b^6 + (2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(d*
x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(d*x + c))*si
n(d*x + c))/((a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d*cos(d*x + c)^3
+ 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c)^2 + (a^12 - 3
*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c))]

```

Sympy [F]

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^3} dx$$

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.64

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^3} dx = \frac{3(4a^4b^2 - 5a^2b^4 + 2b^6) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan\left(\frac{-a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^8 - 2a^6b^2 + a^4b^4)\sqrt{a^2 - b^2}} + \frac{8a^3b^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 7a^2b^4 \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + 6ab^5 \tan(\frac{1}{2} dx + \frac{1}{2} c) - 5b^6}{(a^8 - 2a^6b^2 + a^4b^4)\sqrt{a^2 - b^2}}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(a^2 - b^2)) + (8*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 5*a*b^5*tan(1/2*d*x + 1/2*c) + 4*b^6*tan(1/2*d*x + 1/2*c)^2 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c) + 7*a^2*b^4*tan(1/2*d*x + 1/2*c) - 5*a*b^5*tan(1/2*d*x + 1/2*c) - 4*b^6*tan(1/2*d*x + 1/2*c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + 3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3*b*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 5347, normalized size of antiderivative = 23.05

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Too large to display}$$

```
[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^3),x)
```

```
[Out] (b*atan(((b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288*a^2*b^10 + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^10*b^2)))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (3*b*((24*(4*a^17*b - 4*a^8*b^10 + 2*a^9*b^9 + 18*a^10*b^8 - 8*a^11*b^7 - 32*a^12*b^6 + 14*a^13*b^5 + 26*a^14*b^4 - 12*a^15*b^3 - 8*a^16*b^2)))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (24*b*tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2)))/(a^4*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2))))/a^4)*3i)/a^4 + (b*((8*tan(c/2 + (d*x)/2)*(72*b^12 - 72*a*b^11 - 288
```

$$\begin{aligned}
& *a^2*b^{10} + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7 \\
& *b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^{10}*b^2)/(a^{12}*b + a^{13} - a^6*b^7 - a \\
& ^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) + (3*b*((24*(4*a^ \\
& 17*b - 4*a^8*b^{10} + 2*a^9*b^9 + 18*a^{10}*b^8 - 8*a^{11}*b^7 - 32*a^{12}*b^6 + 14 \\
& *a^{13}*b^5 + 26*a^{14}*b^4 - 12*a^{15}*b^3 - 8*a^{16}*b^2))/(a^{15}*b + a^{16} - a^9*b \\
& ^7 - a^{10}*b^6 + 3*a^{11}*b^5 + 3*a^{12}*b^4 - 3*a^{13}*b^3 - 3*a^{14}*b^2) + (24*b* \\
& \tan(c/2 + (d*x)/2)*(8*a^{17}*b - 8*a^8*b^{10} + 8*a^9*b^9 + 32*a^{10}*b^8 - 32*a^ \\
& 11*b^7 - 48*a^{12}*b^6 + 48*a^{13}*b^5 + 32*a^{14}*b^4 - 32*a^{15}*b^3 - 8*a^{16}*b^2 \\
&))/(a^4*(a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10} \\
& *b^3 - 3*a^{11}*b^2))))/a^4)*3i)/a^4)/((48*(36*b^{12} - 18*a*b^{11} - 162*a^2*b^{10} \\
& + 81*a^3*b^9 + 288*a^4*b^8 - 126*a^5*b^7 - 234*a^6*b^6 + 72*a^7*b^5 + 72* \\
& a^8*b^4))/(a^{15}*b + a^{16} - a^9*b^7 - a^{10}*b^6 + 3*a^{11}*b^5 + 3*a^{12}*b^4 - 3 \\
& *a^{13}*b^3 - 3*a^{14}*b^2) - (3*b*((8*\tan(c/2 + (d*x)/2)*(72*b^{12} - 72*a*b^{11} \\
& - 288*a^2*b^{10} + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 28 \\
& 8*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^{10}*b^2))/(a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) - (3*b*((24*(4*a^ \\
& 17*b - 4*a^8*b^{10} + 2*a^9*b^9 + 18*a^{10}*b^8 - 8*a^{11}*b^7 - 32*a^{12}*b^6 \\
& + 14*a^{13}*b^5 + 26*a^{14}*b^4 - 12*a^{15}*b^3 - 8*a^{16}*b^2))/(a^{15}*b + a^{16} - \\
& a^9*b^7 - a^{10}*b^6 + 3*a^{11}*b^5 + 3*a^{12}*b^4 - 3*a^{13}*b^3 - 3*a^{14}*b^2) - (\\
& 24*b*\tan(c/2 + (d*x)/2)*(8*a^{17}*b - 8*a^8*b^{10} + 8*a^9*b^9 + 32*a^{10}*b^8 - \\
& 32*a^{11}*b^7 - 48*a^{12}*b^6 + 48*a^{13}*b^5 + 32*a^{14}*b^4 - 32*a^{15}*b^3 - 8*a^{16} \\
& *b^2))/(a^4*(a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3 \\
& *a^{10}*b^3 - 3*a^{11}*b^2))))/a^4)/a^4 + (3*b*((8*\tan(c/2 + (d*x)/2)*(72*b^{12} \\
& - 72*a*b^{11} - 288*a^2*b^{10} + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288 \\
& *a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^{10}*b^2))/(a^{12}*b + \\
& a^{13} - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) \\
& + (3*b*((24*(4*a^{17}*b - 4*a^8*b^{10} + 2*a^9*b^9 + 18*a^{10}*b^8 - 8*a^{11}*b^7 \\
& - 32*a^{12}*b^6 + 14*a^{13}*b^5 + 26*a^{14}*b^4 - 12*a^{15}*b^3 - 8*a^{16}*b^2))/(a^{15} \\
& *b + a^{16} - a^9*b^7 - a^{10}*b^6 + 3*a^{11}*b^5 + 3*a^{12}*b^4 - 3*a^{13}*b^3 - 3* \\
& a^{14}*b^2) + (24*b*\tan(c/2 + (d*x)/2)*(8*a^{17}*b - 8*a^8*b^{10} + 8*a^9*b^9 + 3 \\
& 2*a^{10}*b^8 - 32*a^{11}*b^7 - 48*a^{12}*b^6 + 48*a^{13}*b^5 + 32*a^{14}*b^4 - 32*a^{15} \\
& *b^3 - 8*a^{16}*b^2))/(a^4*(a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + \\
& 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2))))/a^4)/a^4)*6i)/(a^4*d) - ((\tan(c/2 \\
& + (d*x)/2)^5*(3*a*b^4 - 2*a^4*b + 2*a^5 - 6*b^5 + 12*a^2*b^3 - 4*a^3*b^2)) \\
& /((a^3*b - a^4)*(a + b)^2) - (\tan(c/2 + (d*x)/2)*(3*a*b^4 + 2*a^4*b + 2*a^5 \\
& + 6*b^5 - 12*a^2*b^3 - 4*a^3*b^2))/((a + b)*(a^5 - 2*a^4*b + a^3*b^2)) + (\\
& 2*\tan(c/2 + (d*x)/2)^3*(2*a^6 - 6*b^6 + 13*a^2*b^4 - 6*a^4*b^2))/(a*(a^2*b \\
& - a^3)*(a + b)^2*(a - b))/(d*(2*a*b - \tan(c/2 + (d*x)/2)^2*(2*a*b - a^2 + \\
& 3*b^2) - \tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - \tan(c/2 + (\\
& d*x)/2)^4*(2*a*b + a^2 - 3*b^2))) + (b^2*atan(((b^2*(-(a + b)^5*(a - b)^5)^ \\
& (1/2))*((8*\tan(c/2 + (d*x)/2)*(72*b^{12} - 72*a*b^{11} - 288*a^2*b^{10} + 288*a^3* \\
& b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - \\
& 72*a^9*b^3 + 36*a^{10}*b^2))/(a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + \\
& 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) - (3*b^2*((24*(4*a^{17}*b - 4*a^8*b^{10} \\
& + 2*a^9*b^9 + 18*a^{10}*b^8 - 8*a^{11}*b^7 - 32*a^{12}*b^6 + 14*a^{13}*b^5 + 26*a^{14}
\end{aligned}$$

$$\begin{aligned}
& 4*b^4 - 12*a^{15}*b^3 - 8*a^{16}*b^2)) / (a^{15}*b + a^{16} - a^9*b^7 - a^{10}*b^6 + 3* \\
& a^{11}*b^5 + 3*a^{12}*b^4 - 3*a^{13}*b^3 - 3*a^{14}*b^2) - (12*b^2*\tan(c/2 + (d*x)/ \\
& 2)*(-(a + b)^5*(a - b)^5)^{(1/2)}*(4*a^4 + 2*b^4 - 5*a^2*b^2)*(8*a^{17}*b - 8*a \\
& ^8*b^{10} + 8*a^9*b^9 + 32*a^{10}*b^8 - 32*a^{11}*b^7 - 48*a^{12}*b^6 + 48*a^{13}*b^5 \\
& + 32*a^{14}*b^4 - 32*a^{15}*b^3 - 8*a^{16}*b^2)) / ((a^{14} - a^4*b^{10} + 5*a^6*b^8 - \\
& 10*a^8*b^6 + 10*a^{10}*b^4 - 5*a^{12}*b^2)*(a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 \\
& + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2)))*(-(a + b)^5*(a - b)^5) \\
& ^{(1/2)}*(4*a^4 + 2*b^4 - 5*a^2*b^2)) / (2*(a^{14} - a^4*b^{10} + 5*a^6*b^8 - 10*a^8* \\
& b^6 + 10*a^{10}*b^4 - 5*a^{12}*b^2)))*(4*a^4 + 2*b^4 - 5*a^2*b^2)*3i) / (2*(a^{14} \\
& - a^4*b^{10} + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^{10}*b^4 - 5*a^{12}*b^2)) + (b^2*(\\
& -(a + b)^5*(a - b)^5)^{(1/2)}*((8*\tan(c/2 + (d*x)/2)*(72*b^{12} - 72*a*b^{11} - 2 \\
& 88*a^2*b^{10} + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^6*b^6 + 288*a \\
& ^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^{10}*b^2)) / (a^{12}*b + a^{13} - a^6*b^7 - \\
& a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) + (3*b^2*((24*(\\
& 4*a^{17}*b - 4*a^8*b^{10} + 2*a^9*b^9 + 18*a^{10}*b^8 - 8*a^{11}*b^7 - 32*a^{12}*b^6 \\
& + 14*a^{13}*b^5 + 26*a^{14}*b^4 - 12*a^{15}*b^3 - 8*a^{16}*b^2)) / (a^{15}*b + a^{16} - a \\
& ^9*b^7 - a^{10}*b^6 + 3*a^{11}*b^5 + 3*a^{12}*b^4 - 3*a^{13}*b^3 - 3*a^{14}*b^2) + (1 \\
& 2*b^2*\tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^{(1/2)}*(4*a^4 + 2*b^4 - 5*a^2* \\
& b^2)*(8*a^{17}*b - 8*a^8*b^{10} + 8*a^9*b^9 + 32*a^{10}*b^8 - 32*a^{11}*b^7 - 48* \\
& a^{12}*b^6 + 48*a^{13}*b^5 + 32*a^{14}*b^4 - 32*a^{15}*b^3 - 8*a^{16}*b^2)) / ((a^{14} - \\
& a^4*b^{10} + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^{10}*b^4 - 5*a^{12}*b^2)*(a^{12}*b + a^{13} \\
& - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2)))* \\
& (-(a + b)^5*(a - b)^5)^{(1/2)}*(4*a^4 + 2*b^4 - 5*a^2*b^2)) / (2*(a^{14} - a^4*b^ \\
& 10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^{10}*b^4 - 5*a^{12}*b^2)))*(4*a^4 + 2*b^4 - \\
& 5*a^2*b^2)*3i) / (2*(a^{14} - a^4*b^{10} + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^{10}*b^4 - \\
& 5*a^{12}*b^2)) / ((48*(36*b^{12} - 18*a*b^{11} - 162*a^2*b^{10} + 81*a^3*b^9 + 288* \\
& a^4*b^8 - 126*a^5*b^7 - 234*a^6*b^6 + 72*a^7*b^5 + 72*a^8*b^4)) / (a^{15}*b + a \\
& ^{16} - a^9*b^7 - a^{10}*b^6 + 3*a^{11}*b^5 + 3*a^{12}*b^4 - 3*a^{13}*b^3 - 3*a^{14}*b^ \\
& 2) - (3*b^2*(-(a + b)^5*(a - b)^5)^{(1/2)}*((8*\tan(c/2 + (d*x)/2)*(72*b^{12} - \\
& 72*a*b^{11} - 288*a^2*b^{10} + 288*a^3*b^9 + 441*a^4*b^8 - 432*a^5*b^7 - 288*a^ \\
& 6*b^6 + 288*a^7*b^5 + 36*a^8*b^4 - 72*a^9*b^3 + 36*a^{10}*b^2)) / (a^{12}*b + a^{13} \\
& - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3*a^{11}*b^2) - \\
& (3*b^2*((24*(4*a^{17}*b - 4*a^8*b^{10} + 2*a^9*b^9 + 18*a^{10}*b^8 - 8*a^{11}*b^7 - \\
& 32*a^{12}*b^6 + 14*a^{13}*b^5 + 26*a^{14}*b^4 - 12*a^{15}*b^3 - 8*a^{16}*b^2)) / (a^{15} \\
& *b + a^{16} - a^9*b^7 - a^{10}*b^6 + 3*a^{11}*b^5 + 3*a^{12}*b^4 - 3*a^{13}*b^3 - 3*a \\
& ^{14}*b^2) - (12*b^2*\tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^{(1/2)}*(4*a^4 + \\
& 2*b^4 - 5*a^2*b^2)*(8*a^{17}*b - 8*a^8*b^{10} + 8*a^9*b^9 + 32*a^{10}*b^8 - 32*a \\
& ^{11}*b^7 - 48*a^{12}*b^6 + 48*a^{13}*b^5 + 32*a^{14}*b^4 - 32*a^{15}*b^3 - 8*a^{16}*b^ \\
& 2)) / ((a^{14} - a^4*b^{10} + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^{10}*b^4 - 5*a^{12}*b^2)* \\
& (a^{12}*b + a^{13} - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^{10}*b^3 - 3 \\
& *a^{11}*b^2)))*(-(a + b)^5*(a - b)^5)^{(1/2)}*(4*a^4 + 2*b^4 - 5*a^2*b^2)) / (2*(\\
& a^{14} - a^4*b^{10} + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^{10}*b^4 - 5*a^{12}*b^2)))*(4*a \\
& ^4 + 2*b^4 - 5*a^2*b^2)) / (2*(a^{14} - a^4*b^{10} + 5*a^6*b^8 - 10*a^8*b^6 + 10* \\
& a^{10}*b^4 - 5*a^{12}*b^2)) + (3*b^2*(-(a + b)^5*(a - b)^5)^{(1/2)}*((8*\tan(c/2 + \\
& (d*x)/2)*(72*b^{12} - 72*a*b^{11} - 288*a^2*b^{10} + 288*a^3*b^9 + 441*a^4*b^8 -
\end{aligned}$$

$$\begin{aligned}
& (432a^5b^7 - 288a^6b^6 + 288a^7b^5 + 36a^8b^4 - 72a^9b^3 + 36a^{10}b^2) / (a^{12}b + a^{13} - a^6b^7 - a^7b^6 + 3a^8b^5 + 3a^9b^4 - 3a^{10}b^3 - 3a^{11}b^2) + (3b^2 * ((24(4a^{17}b - 4a^8b^{10} + 2a^9b^9 + 18a^{10}b^8 - 8a^{11}b^7 - 32a^{12}b^6 + 14a^{13}b^5 + 26a^{14}b^4 - 12a^{15}b^3 - 8a^{16}b^2))) / (a^{15}b + a^{16} - a^9b^7 - a^{10}b^6 + 3a^{11}b^5 + 3a^{12}b^4 - 3a^{13}b^3 - 3a^{14}b^2) + (12b^2 * \tan(c/2 + (d*x)/2) * (-(a+b)^5 * (a-b)^5)^{(1/2)} * (4a^4 + 2b^4 - 5a^2b^2) * (8a^{17}b - 8a^8b^{10} + 8a^9b^9 + 32a^{10}b^8 - 32a^{11}b^7 - 48a^{12}b^6 + 48a^{13}b^5 + 32a^{14}b^4 - 32a^{15}b^3 - 8a^{16}b^2))) / ((a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2) * (a^{12}b + a^{13} - a^6b^7 - a^7b^6 + 3a^8b^5 + 3a^9b^4 - 3a^{10}b^3 - 3a^{11}b^2))) * (-(a+b)^5 * (a-b)^5)^{(1/2)} * (4a^4 + 2b^4 - 5a^2b^2)) / (2 * (a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2))) * (4a^4 + 2b^4 - 5a^2b^2)) / (2 * (a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2))) * (-(a+b)^5 * (a-b)^5)^{(1/2)} * (4a^4 + 2b^4 - 5a^2b^2) * 3i) / (d * (a^{14} - a^4b^{10} + 5a^6b^8 - 10a^8b^6 + 10a^{10}b^4 - 5a^{12}b^2))
\end{aligned}$$

$$3.477 \quad \int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^3} dx$$

Optimal result	5075
Rubi [A] (verified)	5076
Mathematica [A] (verified)	5079
Maple [A] (verified)	5080
Fricas [B] (verification not implemented)	5080
Sympy [F]	5081
Maxima [F(-2)]	5082
Giac [B] (verification not implemented)	5082
Mupad [B] (verification not implemented)	5083

Optimal result

Integrand size = 21, antiderivative size = 305

$$\int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^3} dx = -\frac{b^3(20a^4 - 29a^2b^2 + 12b^4) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d}$$

$$+ \frac{(a^2 + 12b^2) \operatorname{arctanh}(\sin(c+dx))}{2a^5d}$$

$$- \frac{3b(2a^4 - 7a^2b^2 + 4b^4) \tan(c+dx)}{2a^4(a^2 - b^2)^2d}$$

$$+ \frac{(a^4 - 10a^2b^2 + 6b^4) \sec(c+dx) \tan(c+dx)}{2a^3(a^2 - b^2)^2d}$$

$$+ \frac{b^2 \sec(c+dx) \tan(c+dx)}{2a(a^2 - b^2)d(a+b \cos(c+dx))^2}$$

$$+ \frac{b^2(7a^2 - 4b^2) \sec(c+dx) \tan(c+dx)}{2a^2(a^2 - b^2)^2d(a+b \cos(c+dx))}$$

```
[Out] -b^3*(20*a^4-29*a^2*b^2+12*b^4)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*(a^2+12*b^2)*arctanh(sin(d*x+c))/a^5/d-3/2*b*(2*a^4-7*a^2*b^2+4*b^4)*tan(d*x+c)/a^4/(a^2-b^2)^2/d+1/2*(a^4-10*a^2*b^2+6*b^4)*sec(d*x+c)*tan(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*sec(d*x+c)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2+1/2*b^2*(7*a^2-4*b^2)*sec(d*x+c)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2881, 3134, 3080, 3855, 2738, 211}

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3} dx = \frac{b^2(7a^2-4b^2)\tan(c+dx)\sec(c+dx)}{2a^2d(a^2-b^2)^2(a+b\cos(c+dx))} + \frac{b^2\tan(c+dx)\sec(c+dx)}{2ad(a^2-b^2)(a+b\cos(c+dx))^2} + \frac{(a^2+12b^2)\operatorname{arctanh}(\sin(c+dx))}{2a^5d} - \frac{3b(2a^4-7a^2b^2+4b^4)\tan(c+dx)}{2a^4d(a^2-b^2)^2} - \frac{b^3(20a^4-29a^2b^2+12b^4)\operatorname{arctan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}} + \frac{(a^4-10a^2b^2+6b^4)\tan(c+dx)\sec(c+dx)}{2a^3d(a^2-b^2)^2}$$

[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]

[Out] -((b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((a^2 + 12*b^2)*ArcTan[Sin[c + d*x]]/(2*a^5*d) - (3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Tan[c + d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^2 + 6*b^4)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b^2*(7*a^2 - 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f


```
x]^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

integral

$$= \frac{b^2 \sec(c + dx) \tan(c + dx)}{2a(a^2 - b^2)d(a + b \cos(c + dx))^2} + \frac{\int \frac{(2(a^2 - 2b^2) - 2ab \cos(c + dx) + 3b^2 \cos^2(c + dx)) \sec^3(c + dx)}{(a + b \cos(c + dx))^2} dx}{2a(a^2 - b^2)}$$

$$\begin{aligned}
&= \frac{b^2 \sec(c+dx) \tan(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{b^2(7a^2-4b^2)\sec(c+dx)\tan(c+dx)}{2a^2(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad + \frac{\int \frac{(2(a^4-10a^2b^2+6b^4)-ab(4a^2-b^2)\cos(c+dx)+2b^2(7a^2-4b^2)\cos^2(c+dx))\sec^3(c+dx)}{a+b\cos(c+dx)} dx}{2a^2(a^2-b^2)^2} \\
&= \frac{(a^4-10a^2b^2+6b^4)\sec(c+dx)\tan(c+dx)}{2a^3(a^2-b^2)^2d} \\
&\quad + \frac{b^2\sec(c+dx)\tan(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{b^2(7a^2-4b^2)\sec(c+dx)\tan(c+dx)}{2a^2(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad + \frac{\int \frac{(-6b(2a^4-7a^2b^2+4b^4)+2a(a^4+4a^2b^2-2b^4)\cos(c+dx)+2b(a^4-10a^2b^2+6b^4)\cos^2(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)} dx}{4a^3(a^2-b^2)^2} \\
&= -\frac{3b(2a^4-7a^2b^2+4b^4)\tan(c+dx)}{2a^4(a^2-b^2)^2d} + \frac{(a^4-10a^2b^2+6b^4)\sec(c+dx)\tan(c+dx)}{2a^3(a^2-b^2)^2d} \\
&\quad + \frac{b^2\sec(c+dx)\tan(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{b^2(7a^2-4b^2)\sec(c+dx)\tan(c+dx)}{2a^2(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad + \frac{\int \frac{(2(a^2-b^2)^2(a^2+12b^2)+2ab(a^4-10a^2b^2+6b^4)\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)} dx}{4a^4(a^2-b^2)^2} \\
&= -\frac{3b(2a^4-7a^2b^2+4b^4)\tan(c+dx)}{2a^4(a^2-b^2)^2d} + \frac{(a^4-10a^2b^2+6b^4)\sec(c+dx)\tan(c+dx)}{2a^3(a^2-b^2)^2d} \\
&\quad + \frac{b^2\sec(c+dx)\tan(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{b^2(7a^2-4b^2)\sec(c+dx)\tan(c+dx)}{2a^2(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad + \frac{(a^2+12b^2)\int \sec(c+dx) dx}{2a^5} - \frac{(b^3(20a^4-29a^2b^2+12b^4))\int \frac{1}{a+b\cos(c+dx)} dx}{2a^5(a^2-b^2)^2} \\
&= \frac{(a^2+12b^2)\operatorname{arctanh}(\sin(c+dx))}{2a^5d} - \frac{3b(2a^4-7a^2b^2+4b^4)\tan(c+dx)}{2a^4(a^2-b^2)^2d} \\
&\quad + \frac{(a^4-10a^2b^2+6b^4)\sec(c+dx)\tan(c+dx)}{2a^3(a^2-b^2)^2d} \\
&\quad + \frac{b^2\sec(c+dx)\tan(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{b^2(7a^2-4b^2)\sec(c+dx)\tan(c+dx)}{2a^2(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad - \frac{(b^3(20a^4-29a^2b^2+12b^4))\operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^5(a^2-b^2)^2d}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{b^3(20a^4 - 29a^2b^2 + 12b^4) \arctan\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d} + \frac{(a^2 + 12b^2) \operatorname{arctanh}(\sin(c+dx))}{2a^5d} \\
&\quad - \frac{3b(2a^4 - 7a^2b^2 + 4b^4) \tan(c+dx)}{2a^4(a^2 - b^2)^2d} + \frac{(a^4 - 10a^2b^2 + 6b^4) \sec(c+dx) \tan(c+dx)}{2a^3(a^2 - b^2)^2d} \\
&\quad + \frac{b^2 \sec(c+dx) \tan(c+dx)}{2a(a^2 - b^2)d(a+b\cos(c+dx))^2} + \frac{b^2(7a^2 - 4b^2) \sec(c+dx) \tan(c+dx)}{2a^2(a^2 - b^2)^2d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.52 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.40

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3} dx &= \frac{b^3(20a^4 - 29a^2b^2 + 12b^4) \operatorname{arctanh}\left(\frac{(a-b)\tan(\frac{1}{2}(c+dx))}{\sqrt{-a^2+b^2}}\right)}{a^5(a^2 - b^2)^2\sqrt{-a^2+b^2}d} \\
&\quad + \frac{(-a^2 - 12b^2) \log(\cos(\frac{1}{2}(c+dx)) - \sin(\frac{1}{2}(c+dx)))}{2a^5d} \\
&\quad + \frac{(a^2 + 12b^2) \log(\cos(\frac{1}{2}(c+dx)) + \sin(\frac{1}{2}(c+dx)))}{2a^5d} \\
&\quad + \frac{1}{4a^3d(\cos(\frac{1}{2}(c+dx)) - \sin(\frac{1}{2}(c+dx)))^2} \\
&\quad - \frac{3b\sin(\frac{1}{2}(c+dx))}{a^4d(\cos(\frac{1}{2}(c+dx)) - \sin(\frac{1}{2}(c+dx)))} \\
&\quad - \frac{1}{4a^3d(\cos(\frac{1}{2}(c+dx)) + \sin(\frac{1}{2}(c+dx)))^2} \\
&\quad - \frac{3b\sin(\frac{1}{2}(c+dx))}{a^4d(\cos(\frac{1}{2}(c+dx)) + \sin(\frac{1}{2}(c+dx)))} \\
&\quad + \frac{b^4\sin(c+dx)}{2a^3(a-b)(a+b)d(a+b\cos(c+dx))^2} \\
&\quad + \frac{3(3a^2b^4\sin(c+dx) - 2b^6\sin(c+dx))}{2a^4(a-b)^2(a+b)^2d(a+b\cos(c+dx))}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]

[Out] (b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(a^5*(a^2 - b^2)^2*Sqrt[-a^2 + b^2]*d) + ((-a^2 - 12*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*a^5*d) + ((a^2 + 12*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(2*a^5*d) + 1/(4*a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - (3*b*Sin[(c + d*x)/2])/(a^4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - 1/(4*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (3*b*Sin[(c + d*x)/2])/(a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b^4*Sin[c + d*x])/(2*a^3*(a - b)*(a + b)*d*(a + b*Cos[c + d*x])^2) + (3*(3*a^2*b^4*Sin[c + d*x] - 2*b^6*Sin[c + d*x]))/(2*a^4*(a - b)^2*(a + b)^2*d*(a + b*Cos[c + d*x]))

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{1}{2a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{-a-6b}{2a^4 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{(-a^2-12b^2) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2a^5} - \frac{2b^3 \left(-\frac{(10a^2+ab-6b^2)ab \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2(a-b)(a^2+2ab+b^2)} - \frac{((\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^{a-b} (\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)))}{2a^3} \right)}{((\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^{a-b} (\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)))}$
default	$\frac{1}{2a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} - \frac{-a-6b}{2a^4 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{(-a^2-12b^2) \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{2a^5} - \frac{2b^3 \left(-\frac{(10a^2+ab-6b^2)ab \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2(a-b)(a^2+2ab+b^2)} - \frac{((\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^{a-b} (\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)))}{2a^3} \right)}{((\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right))^{a-b} (\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)))}$
risch	Expression too large to display

[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] $1/d*(1/2/a^3/(\tan(1/2*d*x+1/2*c)-1)^2-1/2*(-a-6*b)/a^4/(\tan(1/2*d*x+1/2*c)-1)+1/2/a^5*(-a^2-12*b^2)*\ln(\tan(1/2*d*x+1/2*c)-1)-2*b^3/a^5*((-1/2*(10*a^2+a*b-6*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*\tan(1/2*d*x+1/2*c))^3-1/2*(10*a^2-a*b-6*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*\tan(1/2*d*x+1/2*c))/(\tan(1/2*d*x+1/2*c)^2*a-b*\tan(1/2*d*x+1/2*c)^2+a+b)^2+1/2*(20*a^4-29*a^2*b^2+12*b^4)/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^{1/2}*\arctan((a-b)*\tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^{1/2})))-1/2/a^3/(\tan(1/2*d*x+1/2*c)+1)^2-1/2*(-a-6*b)/a^4/(\tan(1/2*d*x+1/2*c)+1)+1/2*(a^2+12*b^2)/a^5*\ln(\tan(1/2*d*x+1/2*c)+1))$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(286) = 572.

Time = 1.47 (sec) , antiderivative size = 1524, normalized size of antiderivative = 5.00

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3} dx = \text{Too large to display}$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] $[-1/4*((20*a^4*b^5 - 29*a^2*b^7 + 12*b^9)*\cos(d*x + c)^4 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*\cos(d*x + c)^3 + (20*a^6*b^3 - 29*a^4*b^5 + 12*a^2*b^7)*\cos(d*x + c)^2)*\sqrt{-a^2 + b^2}*\log((2*a*b*\cos(d*x + c) + (2*a^2 - b^2)*\cos(d*x + c)^2 - 2*\sqrt{-a^2 + b^2}*(a*\cos(d*x + c) + b)*\sin(d*x + c) - a^2 + 2*b^2)/(b^2*\cos(d*x + c)^2 + 2*a*b*\cos(d*x + c) + a^2)) - ((a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*\cos(d*x + c)^4 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*\cos(d*x + c)^3 + (a^10 + 9$

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*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*
x + c) + 1) + ((a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*co
s(d*x + c)^4 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*c
os(d*x + c)^3 + (a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*c
os(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a
^4*b^6 - 3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3*b^7 - 4*a*b^9)*cos(d*x + c)^3 -
(11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c)^2 - 4*(a^9
*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11*b^
2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 3*a^10*
b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^11*b^2 + 3*a^9*b^
4 - a^7*b^6)*d*cos(d*x + c)^2), -1/4*(2*((20*a^4*b^5 - 29*a^2*b^7 + 12*b^9)
*cos(d*x + c)^4 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*cos(d*x + c)^3 + (
20*a^6*b^3 - 29*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arcta
n(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((a^8*b^2 + 9*a^6
*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(d*x + c)^4 + 2*(a^9*b + 9*a^7
*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(d*x + c)^3 + (a^10 + 9*a^8*b
^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1) + ((a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(d*x
+ c)^4 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(d*x
+ c)^3 + (a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) - 2*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6
- 3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3*b^7 - 4*a*b^9)*cos(d*x + c)^3 - (11*a^
8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c)^2 - 4*(a^9*b - 3
*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11*b^2 - 3*
a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 3*a^10*b^3 +
3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^
7*b^6)*d*cos(d*x + c)^2)]

```

Sympy [F]

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^3} dx$$

[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**3,x)

[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**3, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Exception raised: ValueError}$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(286) = 572.

Time = 0.35 (sec) , antiderivative size = 801, normalized size of antiderivative = 2.63

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Too large to display}$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{2} * (2 * (20 * a^4 * b^3 - 29 * a^2 * b^5 + 12 * b^7) * (\pi * \text{floor}(1/2 * (d * x + c)) / \pi + 1/2) * \text{sgn}(-2 * a + 2 * b) + \arctan(-(a * \tan(1/2 * d * x + 1/2 * c) - b * \tan(1/2 * d * x + 1/2 * c)) / \sqrt{a^2 - b^2})) / ((a^9 - 2 * a^7 * b^2 + a^5 * b^4) * \sqrt{a^2 - b^2}) + 2 * (a^7 * \tan(1/2 * d * x + 1/2 * c)^7 + 4 * a^6 * b * \tan(1/2 * d * x + 1/2 * c)^7 - 13 * a^5 * b^2 * \tan(1/2 * d * x + 1/2 * c)^7 - 2 * a^4 * b^3 * \tan(1/2 * d * x + 1/2 * c)^7 + 33 * a^3 * b^4 * \tan(1/2 * d * x + 1/2 * c)^7 - 17 * a^2 * b^5 * \tan(1/2 * d * x + 1/2 * c)^7 - 18 * a * b^6 * \tan(1/2 * d * x + 1/2 * c)^7 + 12 * b^7 * \tan(1/2 * d * x + 1/2 * c)^7 + 3 * a^7 * \tan(1/2 * d * x + 1/2 * c)^5 + 4 * a^6 * b * \tan(1/2 * d * x + 1/2 * c)^5 + 5 * a^5 * b^2 * \tan(1/2 * d * x + 1/2 * c)^5 - 26 * a^4 * b^3 * \tan(1/2 * d * x + 1/2 * c)^5 - 29 * a^3 * b^4 * \tan(1/2 * d * x + 1/2 * c)^5 + 67 * a^2 * b^5 * \tan(1/2 * d * x + 1/2 * c)^5 + 18 * a * b^6 * \tan(1/2 * d * x + 1/2 * c)^5 - 36 * b^7 * \tan(1/2 * d * x + 1/2 * c)^5 + 3 * a^7 * \tan(1/2 * d * x + 1/2 * c)^3 - 4 * a^6 * b * \tan(1/2 * d * x + 1/2 * c)^3 + 5 * a^5 * b^2 * \tan(1/2 * d * x + 1/2 * c)^3 + 26 * a^4 * b^3 * \tan(1/2 * d * x + 1/2 * c)^3 - 29 * a^3 * b^4 * \tan(1/2 * d * x + 1/2 * c)^3 - 67 * a^2 * b^5 * \tan(1/2 * d * x + 1/2 * c)^3 + 18 * a * b^6 * \tan(1/2 * d * x + 1/2 * c)^3 + 36 * b^7 * \tan(1/2 * d * x + 1/2 * c)^3 + a^7 * \tan(1/2 * d * x + 1/2 * c) - 4 * a^6 * b * \tan(1/2 * d * x + 1/2 * c) - 13 * a^5 * b^2 * \tan(1/2 * d * x + 1/2 * c) + 2 * a^4 * b^3 * \tan(1/2 * d * x + 1/2 * c) + 33 * a^3 * b^4 * \tan(1/2 * d * x + 1/2 * c) + 17 * a^2 * b^5 * \tan(1/2 * d * x + 1/2 * c) - 18 * a * b^6 * \tan(1/2 * d * x + 1/2 * c) - 12 * b^7 * \tan(1/2 * d * x + 1/2 * c)) / ((a^8 - 2 * a^6 * b^2 + a^4 * b^4) * (a * \tan(1/2 * d * x + 1/2 * c)^4 - b * \tan(1/2 * d * x + 1/2 * c)^4 + 2 * b * \tan(1/2 * d * x + 1/2 * c)^2 - a - b)^2) + (a^2 + 12 * b^2) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) + 1)) / a^5 - (a^2 + 12 * b^2) * \log(\text{abs}(\tan(1/2 * d * x + 1/2 * c) - 1)) / a^5) / d$

Mupad [B] (verification not implemented)

Time = 23.25 (sec) , antiderivative size = 5910, normalized size of antiderivative = 19.38

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Too large to display}$$

[In] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^3),x)

[Out] ((tan(c/2 + (d*x)/2)^3*(18*a*b^6 - 4*a^6*b + 3*a^7 + 36*b^7 - 67*a^2*b^5 - 29*a^3*b^4 + 26*a^4*b^3 + 5*a^5*b^2))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) + (tan(c/2 + (d*x)/2)^5*(18*a*b^6 + 4*a^6*b + 3*a^7 - 36*b^7 + 67*a^2*b^5 - 29*a^3*b^4 - 26*a^4*b^3 + 5*a^5*b^2))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) - (tan(c/2 + (d*x)/2)^7*(6*a*b^5 + 5*a^5*b + a^6 - 12*b^6 + 23*a^2*b^4 - 10*a^3*b^3 - 8*a^4*b^2))/((a^4*b - a^5)*(a + b)^2) - (tan(c/2 + (d*x)/2)*(6*a*b^5 + 5*a^5*b - a^6 + 12*b^6 - 23*a^2*b^4 - 10*a^3*b^3 + 8*a^4*b^2))/((a + b)*(a^6 - 2*a^5*b + a^4*b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) - tan(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2) + tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (atan(((a^2 + 12*b^2)*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2)))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - ((a^2 + 12*b^2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2)))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2 + 12*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2)))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*1i)/(2*a^5) + ((a^2 + 12*b^2)*((8*tan(c/2 + (d*x)/2)*(a^14 - 2*a^13*b - 288*a*b^13 + 288*b^14 - 1104*a^2*b^12 + 1104*a^3*b^11 + 1538*a^4*b^10 - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^10*b^4 - 40*a^11*b^3 + 21*a^12*b^2)))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + ((a^2 + 12*b^2)*((4*(4*a^21 - 48*a^10*b^11 + 24*a^11*b^10 + 212*a^12*b^9 - 100*a^13*b^8 - 360*a^14*b^7 + 164*a^15*b^6 + 276*a^16*b^5 - 120*a^17*b^4 - 80*a^18*b^3 + 28*a^19*b^2)))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*tan(c/2 + (d*x)/2)*(a^2 + 12*b^2)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2)))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2))))/(2*a^5))*1i)/(2*a^5))/((8*(1728*b^15 - 864*a*b^14 - 7344*a^2*b^13 + 3456*a^3*b^12 + 11700*a^4*b^11 - 4770*a^5*b^10 - 7829*a^6*b^9 + 2326*a^7*b^8 + 1314*a^8*b^7 - 11*a^9*b^6 + 411*a^1

$$\begin{aligned}
& 0*b^5 - 20*a^{11}*b^4 + 20*a^{12}*b^3)/(a^{18}*b + a^{19} - a^{12}*b^7 - a^{13}*b^6 + \\
& 3*a^{14}*b^5 + 3*a^{15}*b^4 - 3*a^{16}*b^3 - 3*a^{17}*b^2) - ((a^2 + 12*b^2)*((8*\tan \\
& n(c/2 + (d*x)/2)*(a^{14} - 2*a^{13}*b - 288*a*b^{13} + 288*b^{14} - 1104*a^2*b^{12} + \\
& 1104*a^3*b^{11} + 1538*a^4*b^{10} - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + \\
& 18*a^8*b^6 - 108*a^9*b^5 + 74*a^{10}*b^4 - 40*a^{11}*b^3 + 21*a^{12}*b^2))/(a^{14} \\
& *b + a^{15} - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 - 3*a^{13} \\
& *b^2) - ((a^2 + 12*b^2)*((4*(4*a^{21} - 48*a^{10}*b^{11} + 24*a^{11}*b^{10} + 212*a^{12} \\
& *b^9 - 100*a^{13}*b^8 - 360*a^{14}*b^7 + 164*a^{15}*b^6 + 276*a^{16}*b^5 - 120*a^{17} \\
& *b^4 - 80*a^{18}*b^3 + 28*a^{19}*b^2)))/(a^{18}*b + a^{19} - a^{12}*b^7 - a^{13}*b^6 \\
& + 3*a^{14}*b^5 + 3*a^{15}*b^4 - 3*a^{16}*b^3 - 3*a^{17}*b^2) - (4*\tan(c/2 + (d*x)/2) \\
& *(a^2 + 12*b^2)*(8*a^{19}*b - 8*a^{10}*b^{10} + 8*a^{11}*b^9 + 32*a^{12}*b^8 - 32*a^{13} \\
& *b^7 - 48*a^{14}*b^6 + 48*a^{15}*b^5 + 32*a^{16}*b^4 - 32*a^{17}*b^3 - 8*a^{18}*b^2) \\
&))/(a^5*(a^{14}*b + a^{15} - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12} \\
& *b^3 - 3*a^{13}*b^2)))/(2*a^5)))/(2*a^5) + ((a^2 + 12*b^2)*((8*\tan(c/2 + (d \\
& *x)/2)*(a^{14} - 2*a^{13}*b - 288*a*b^{13} + 288*b^{14} - 1104*a^2*b^{12} + 1104*a^3 \\
& *b^{11} + 1538*a^4*b^{10} - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - \\
& 108*a^9*b^5 + 74*a^{10}*b^4 - 40*a^{11}*b^3 + 21*a^{12}*b^2))/(a^{14}*b + a^{15} \\
& - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 - 3*a^{13}*b^2) + \\
& ((a^2 + 12*b^2)*((4*(4*a^{21} - 48*a^{10}*b^{11} + 24*a^{11}*b^{10} + 212*a^{12}*b^9 - \\
& 100*a^{13}*b^8 - 360*a^{14}*b^7 + 164*a^{15}*b^6 + 276*a^{16}*b^5 - 120*a^{17}*b^4 - \\
& 80*a^{18}*b^3 + 28*a^{19}*b^2)))/(a^{18}*b + a^{19} - a^{12}*b^7 - a^{13}*b^6 + 3*a^{14} \\
& *b^5 + 3*a^{15}*b^4 - 3*a^{16}*b^3 - 3*a^{17}*b^2) + (4*\tan(c/2 + (d*x)/2)*(a^2 + \\
& 12*b^2)*(8*a^{19}*b - 8*a^{10}*b^{10} + 8*a^{11}*b^9 + 32*a^{12}*b^8 - 32*a^{13}*b^7 - \\
& 48*a^{14}*b^6 + 48*a^{15}*b^5 + 32*a^{16}*b^4 - 32*a^{17}*b^3 - 8*a^{18}*b^2))/(a^5*(\\
& a^{14}*b + a^{15} - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 - \\
& 3*a^{13}*b^2)))/(2*a^5)))/(2*a^5))*((a^2 + 12*b^2)*i)/(a^5*d) - (b^3*atan((\\
& b^3*((8*\tan(c/2 + (d*x)/2)*(a^{14} - 2*a^{13}*b - 288*a*b^{13} + 288*b^{14} - 1104 \\
& *a^2*b^{12} + 1104*a^3*b^{11} + 1538*a^4*b^{10} - 1538*a^5*b^9 - 827*a^6*b^8 + 87 \\
& 2*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5 + 74*a^{10}*b^4 - 40*a^{11}*b^3 + 21*a^{12} \\
& *b^2))/(a^{14}*b + a^{15} - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12} \\
& *b^3 - 3*a^{13}*b^2) - (b^3*(-(a + b)^5*(a - b)^5)^{(1/2)*((4*(4*a^{21} - 48*a^{10} \\
& *b^{11} + 24*a^{11}*b^{10} + 212*a^{12}*b^9 - 100*a^{13}*b^8 - 360*a^{14}*b^7 + 164*a^{15} \\
& *b^6 + 276*a^{16}*b^5 - 120*a^{17}*b^4 - 80*a^{18}*b^3 + 28*a^{19}*b^2))/(a^{18}*b \\
& + a^{19} - a^{12}*b^7 - a^{13}*b^6 + 3*a^{14}*b^5 + 3*a^{15}*b^4 - 3*a^{16}*b^3 - 3*a^{17} \\
& *b^2) - (4*b^3*\tan(c/2 + (d*x)/2)*(-(a + b)^5*(a - b)^5)^{(1/2)*(20*a^4 + 1 \\
& 2*b^4 - 29*a^2*b^2)*(8*a^{19}*b - 8*a^{10}*b^{10} + 8*a^{11}*b^9 + 32*a^{12}*b^8 - 32 \\
& *a^{13}*b^7 - 48*a^{14}*b^6 + 48*a^{15}*b^5 + 32*a^{16}*b^4 - 32*a^{17}*b^3 - 8*a^{18} \\
& *b^2))/((a^{15} - a^5*b^{10} + 5*a^7*b^8 - 10*a^9*b^6 + 10*a^{11}*b^4 - 5*a^{13}*b^2) \\
& *(a^{14}*b + a^{15} - a^8*b^7 - a^9*b^6 + 3*a^{10}*b^5 + 3*a^{11}*b^4 - 3*a^{12}*b^3 \\
& - 3*a^{13}*b^2)))*(20*a^4 + 12*b^4 - 29*a^2*b^2))/(2*(a^{15} - a^5*b^{10} + 5*a^7 \\
& *b^8 - 10*a^9*b^6 + 10*a^{11}*b^4 - 5*a^{13}*b^2)))*(-(a + b)^5*(a - b)^5)^{(1/ \\
& 2)*(20*a^4 + 12*b^4 - 29*a^2*b^2)*i)/(2*(a^{15} - a^5*b^{10} + 5*a^7*b^8 - 10 \\
& a^9*b^6 + 10*a^{11}*b^4 - 5*a^{13}*b^2)) + (b^3*((8*\tan(c/2 + (d*x)/2)*(a^{14} - \\
& 2*a^{13}*b - 288*a*b^{13} + 288*b^{14} - 1104*a^2*b^{12} + 1104*a^3*b^{11} + 1538*a^4 \\
& *b^{10} - 1538*a^5*b^9 - 827*a^6*b^8 + 872*a^7*b^7 + 18*a^8*b^6 - 108*a^9*b^5
\end{aligned}$$

$$\begin{aligned}
&^4 - 32a^{17}b^3 - 8a^{18}b^2)) / ((a^{15} - a^5b^{10} + 5a^7b^8 - 10a^9b^6 \\
&+ 10a^{11}b^4 - 5a^{13}b^2) * (a^{14}b + a^{15} - a^8b^7 - a^9b^6 + 3a^{10}b^5 \\
&+ 3a^{11}b^4 - 3a^{12}b^3 - 3a^{13}b^2)) * (20a^4 + 12b^4 - 29a^2b^2)) / \\
&(2 * (a^{15} - a^5b^{10} + 5a^7b^8 - 10a^9b^6 + 10a^{11}b^4 - 5a^{13}b^2)) * \\
&(-(a + b)^5 * (a - b)^5)^{(1/2)} * (20a^4 + 12b^4 - 29a^2b^2)) / (2 * (a^{15} - a^5 \\
&*b^{10} + 5a^7b^8 - 10a^9b^6 + 10a^{11}b^4 - 5a^{13}b^2))) * (-(a + b)^5 * (\\
&a - b)^5)^{(1/2)} * (20a^4 + 12b^4 - 29a^2b^2) * 1i) / (d * (a^{15} - a^5 * b^{10} + 5 * \\
&a^7 * b^8 - 10 * a^9 * b^6 + 10 * a^{11} * b^4 - 5 * a^{13} * b^2))
\end{aligned}$$

$$3.478 \quad \int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^4} dx$$

Optimal result	5087
Rubi [A] (verified)	5088
Mathematica [A] (verified)	5091
Maple [A] (verified)	5091
Fricas [B] (verification not implemented)	5092
Sympy [F(-1)]	5093
Maxima [F(-2)]	5093
Giac [A] (verification not implemented)	5093
Mupad [B] (verification not implemented)	5094

Optimal result

Integrand size = 21, antiderivative size = 307

$$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^4} dx = -\frac{4ax}{b^5} + \frac{a^2(8a^6 - 28a^4b^2 + 35a^2b^4 - 20b^6) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^5(a+b)^{7/2}d} + \frac{(12a^4 - 23a^2b^2 + 6b^4) \sin(c+dx)}{6b^4(a^2 - b^2)^2 d} - \frac{a^2 \cos^3(c+dx) \sin(c+dx)}{3b(a^2 - b^2)d(a+b \cos(c+dx))^3} - \frac{a^2(4a^2 - 9b^2) \cos^2(c+dx) \sin(c+dx)}{6b^2(a^2 - b^2)^2 d(a+b \cos(c+dx))^2} + \frac{a^3(4a^4 - 11a^2b^2 + 12b^4) \sin(c+dx)}{2b^4(a^2 - b^2)^3 d(a+b \cos(c+dx))}$$

```
[Out] -4*a*x/b^5+a^2*(8*a^6-28*a^4*b^2+35*a^2*b^4-20*b^6)*arctan((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7/2)/d+1/6*(12*a^4-23*a^
2*b^2+6*b^4)*sin(d*x+c)/b^4/(a^2-b^2)^2/d-1/3*a^2*cos(d*x+c)^3*sin(d*x+c)/b
/(a^2-b^2)/d/(a+b*cos(d*x+c))^3-1/6*a^2*(4*a^2-9*b^2)*cos(d*x+c)^2*sin(d*x+
c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/2*a^3*(4*a^4-11*a^2*b^2+12*b^4)*s
in(d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2871, 3126, 3110, 3102, 2814, 2738, 211}

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4} dx = -\frac{a^2 \sin(c+dx) \cos^3(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^3} - \frac{a^2(4a^2-9b^2) \sin(c+dx) \cos^2(c+dx)}{6b^2d(a^2-b^2)^2(a+b\cos(c+dx))^2} + \frac{(12a^4-23a^2b^2+6b^4) \sin(c+dx)}{6b^4d(a^2-b^2)^2} + \frac{a^2(8a^6-28a^4b^2+35a^2b^4-20b^6) \arctan\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{b^5d(a-b)^{7/2}(a+b)^{7/2}} + \frac{a^3(4a^4-11a^2b^2+12b^4) \sin(c+dx)}{2b^4d(a^2-b^2)^3(a+b\cos(c+dx))} - \frac{4ax}{b^5}$$

[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^4,x]

[Out] (-4*a*x)/b^5 + (a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d) + ((12*a^4 - 23*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - (a^2*(4*a^2 - 9*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + (a^3*(4*a^4 - 11*a^2*b^2 + 12*b^4)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2814

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Co
s[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[
e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
+ a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Ssin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])*(A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] := Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Ssin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*Simp[b*(m
+ 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Ssin[e + f*x])^m*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m -
1)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))]*Sin[e + f*x]
```

+ b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

Rubi steps

integral

$$\begin{aligned}
&= -\frac{a^2 \cos^3(c + dx) \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} - \frac{\int \frac{\cos^2(c + dx)(3a^2 - 3ab \cos(c + dx) - (4a^2 - 3b^2) \cos^2(c + dx))}{(a + b \cos(c + dx))^3} dx}{3b(a^2 - b^2)} \\
&= -\frac{a^2 \cos^3(c + dx) \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} - \frac{a^2(4a^2 - 9b^2) \cos^2(c + dx) \sin(c + dx)}{6b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{\int \frac{\cos(c + dx)(-2a^2(4a^2 - 9b^2) + 2ab(a^2 - 6b^2) \cos(c + dx) + (12a^4 - 23a^2b^2 + 6b^4) \cos^2(c + dx))}{(a + b \cos(c + dx))^2} dx}{6b^2(a^2 - b^2)^2} \\
&= -\frac{a^2 \cos^3(c + dx) \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} \\
&\quad - \frac{a^2(4a^2 - 9b^2) \cos^2(c + dx) \sin(c + dx)}{6b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} + \frac{a^3(4a^4 - 11a^2b^2 + 12b^4) \sin(c + dx)}{2b^4(a^2 - b^2)^3 d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{-3a^2b(4a^4 - 11a^2b^2 + 12b^4) - a(12a^6 - 37a^4b^2 + 43a^2b^4 - 18b^6) \cos(c + dx) + b(a^2 - b^2)(12a^4 - 23a^2b^2 + 6b^4) \cos^2(c + dx)}{a + b \cos(c + dx)} dx}{6b^4(a^2 - b^2)^3} \\
&= \frac{(12a^4 - 23a^2b^2 + 6b^4) \sin(c + dx)}{6b^4(a^2 - b^2)^2 d} - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} \\
&\quad - \frac{a^2(4a^2 - 9b^2) \cos^2(c + dx) \sin(c + dx)}{6b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} + \frac{a^3(4a^4 - 11a^2b^2 + 12b^4) \sin(c + dx)}{2b^4(a^2 - b^2)^3 d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{-3a^2b^2(4a^4 - 11a^2b^2 + 12b^4) - 24ab(a^2 - b^2)^3 \cos(c + dx)}{a + b \cos(c + dx)} dx}{6b^5(a^2 - b^2)^3} \\
&= -\frac{4ax}{b^5} + \frac{(12a^4 - 23a^2b^2 + 6b^4) \sin(c + dx)}{6b^4(a^2 - b^2)^2 d} - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} \\
&\quad - \frac{a^2(4a^2 - 9b^2) \cos^2(c + dx) \sin(c + dx)}{6b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} + \frac{a^3(4a^4 - 11a^2b^2 + 12b^4) \sin(c + dx)}{2b^4(a^2 - b^2)^3 d(a + b \cos(c + dx))} \\
&\quad + \frac{(a^2(8a^6 - 28a^4b^2 + 35a^2b^4 - 20b^6)) \int \frac{1}{a + b \cos(c + dx)} dx}{2b^5(a^2 - b^2)^3} \\
&= -\frac{4ax}{b^5} + \frac{(12a^4 - 23a^2b^2 + 6b^4) \sin(c + dx)}{6b^4(a^2 - b^2)^2 d} - \frac{a^2 \cos^3(c + dx) \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} \\
&\quad - \frac{a^2(4a^2 - 9b^2) \cos^2(c + dx) \sin(c + dx)}{6b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} + \frac{a^3(4a^4 - 11a^2b^2 + 12b^4) \sin(c + dx)}{2b^4(a^2 - b^2)^3 d(a + b \cos(c + dx))} \\
&\quad + \frac{(a^2(8a^6 - 28a^4b^2 + 35a^2b^4 - 20b^6)) \text{Subst}\left(\int \frac{1}{a + b + \frac{1}{(a-b)x^2}} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{b^5(a^2 - b^2)^3 d}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{4ax}{b^5} + \frac{a^2(8a^6 - 28a^4b^2 + 35a^2b^4 - 20b^6) \arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^5(a+b)^{7/2}d} \\
&\quad + \frac{(12a^4 - 23a^2b^2 + 6b^4) \sin(c+dx)}{6b^4(a^2 - b^2)^2 d} - \frac{a^2 \cos^3(c+dx) \sin(c+dx)}{3b(a^2 - b^2) d(a+b \cos(c+dx))^3} \\
&\quad - \frac{a^2(4a^2 - 9b^2) \cos^2(c+dx) \sin(c+dx)}{6b^2(a^2 - b^2)^2 d(a+b \cos(c+dx))^2} + \frac{a^3(4a^4 - 11a^2b^2 + 12b^4) \sin(c+dx)}{2b^4(a^2 - b^2)^3 d(a+b \cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.08 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.78

$$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^4} dx$$

$$= \frac{-24a(c+dx) + \frac{6a^2(8a^6 - 28a^4b^2 + 35a^2b^4 - 20b^6) \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + 6b \sin(c+dx) + \frac{2a^5b \sin(c+dx)}{(a-b)(a+b)(a+b \cos(c+dx))}}{6b^5d}$$

[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^4,x]

[Out] $(-24*a*(c + d*x) + (6*a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*b^6)*\operatorname{ArcTan}(\frac{(a-b)*\operatorname{Tan}[(c + d*x)/2]}{\operatorname{Sqrt}[-a^2 + b^2]}))/(-a^2 + b^2)^{(7/2)} + 6*b*\operatorname{Sin}[c + d*x] + (2*a^5*b*\operatorname{Sin}[c + d*x])/((a-b)*(a+b)*(a+b*\operatorname{Cos}[c + d*x]))^3) + (5*a^4*b*(-2*a^2 + 3*b^2)*\operatorname{Sin}[c + d*x])/((a-b)^2*(a+b)^2*(a+b*\operatorname{Cos}[c + d*x])^2) + (a^3*b*(26*a^4 - 71*a^2*b^2 + 60*b^4)*\operatorname{Sin}[c + d*x])/((a-b)^3*(a+b)^3*(a+b*\operatorname{Cos}[c + d*x])))/(6*b^5*d)$

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.29

method	result
derivativedivides	$ 2a^2 \left(\frac{(6a^4 - 2a^3b - 18a^2b^2 + 5ab^3 + 20b^4)ab \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2(a-b)(a^3 + 3a^2b + 3ab^2 + b^3)} + \frac{2(9a^4 - 29a^2b^2 + 30b^4)ab \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3(a^2 - 2ab + b^2)(a^2 + 2ab + b^2)} + \frac{(6a^4 + 2a^3b - 18a^2b^2 - 5ab^3)}{2(a+b)(a^3 - 3a^2b + 3ab^2 - b^3)} \right) \frac{1}{\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a + b\right)^3} $
default	$ 2a^2 \left(\frac{(6a^4 - 2a^3b - 18a^2b^2 + 5ab^3 + 20b^4)ab \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2(a-b)(a^3 + 3a^2b + 3ab^2 + b^3)} + \frac{2(9a^4 - 29a^2b^2 + 30b^4)ab \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3(a^2 - 2ab + b^2)(a^2 + 2ab + b^2)} + \frac{(6a^4 + 2a^3b - 18a^2b^2 - 5ab^3)}{2(a+b)(a^3 - 3a^2b + 3ab^2 - b^3)} \right) \frac{1}{\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a + b\right)^3} $
risch	Expression too large to display

[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)

```
[Out] 1/d*(2*a^2/b^5*((1/2*(6*a^4-2*a^3*b-18*a^2*b^2+5*a*b^3+20*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(9*a^4-29*a^2*b^2+30*b^4)*a*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(6*a^4+2*a^3*b-18*a^2*b^2-5*a*b^3+20*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^3+1/2*(8*a^6-2*8*a^4*b^2+35*a^2*b^4-20*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))-2/b^5*(-b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+4*a*arctan(tan(1/2*d*x+1/2*c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(290) = 580.

Time = 0.37 (sec) , antiderivative size = 1593, normalized size of antiderivative = 5.19

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(48*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*d*x*cos(d*x + c)^3 + 144*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*d*x*cos(d*x + c)^2 + 144*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(d*x + c) + 48*(a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*d*x + 3*(8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6 + (8*a^8*b^3 - 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c)^3 + 3*(8*a^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + 3*(8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(24*a^11*b - 92*a^9*b^3 + 133*a^7*b^5 - 71*a^5*b^7 + 6*a^3*b^9 + 6*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cos(d*x + c)^3 + (44*a^9*b^3 - 169*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x + c)^2 + 3*(20*a^10*b^2 - 77*a^8*b^4 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*cos(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d), -1/6*(24*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*d*x*cos(d*x + c)^3 + 72*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*d*x*cos(d*x + c)^2 + 72*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(d*x + c) + 24*(a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*d*x - 3*(8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6 + (8*a^8*b^3 - 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c)^3 + 3*(8*a^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + 3*(8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c))*sqrt(a^2 - b
```


$$\begin{aligned} &^2) \arctan(- (a \cos(dx + c) + b) / (\sqrt{a^2 - b^2} \sin(dx + c))) - (24a^{11} \\ &*b - 92a^9b^3 + 133a^7b^5 - 71a^5b^7 + 6a^3b^9 + 6(a^8b^4 - 4a^6 \\ &*b^6 + 6a^4b^8 - 4a^2b^{10} + b^{12}) \cos(dx + c)^3 + (44a^9b^3 - 169a^7 \\ &*b^5 + 239a^5b^7 - 132a^3b^9 + 18a^1b^{11}) \cos(dx + c)^2 + 3(20a^{10} \\ &b^2 - 77a^8b^4 + 110a^6b^6 - 59a^4b^8 + 6a^2b^{10}) \cos(dx + c) \sin \\ &(dx + c) / ((a^8b^8 - 4a^6b^{10} + 6a^4b^{12} - 4a^2b^{14} + b^{16}) d \cos(d \\ &*x + c)^3 + 3(a^9b^7 - 4a^7b^9 + 6a^5b^{11} - 4a^3b^{13} + a^1b^{15}) d \cos \\ &(dx + c)^2 + 3(a^{10}b^6 - 4a^8b^8 + 6a^6b^{10} - 4a^4b^{12} + a^2b^{14} \\ &) d \cos(dx + c) + (a^{11}b^5 - 4a^9b^7 + 6a^7b^9 - 4a^5b^{11} + a^3b^{13} \\ &+ a^1b^{15}) d] \end{aligned}$$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Timed out}$$

[In] integrate(cos(dx+c)**5/(a+b*cos(dx+c))**4,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Exception raised: ValueError}$$

[In] integrate(cos(dx+c)^5/(a+b*cos(dx+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.83

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^4} dx = \frac{3(8a^8 - 28a^6b^2 + 35a^4b^4 - 20a^2b^6) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^6b^5 - 3a^4b^7 + 3a^2b^9 - b^{11}) \sqrt{a^2 - b^2}} - \frac{18a^9 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5}{\dots}$$

[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out]
$$-1/3*(3*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 - 20*a^2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*sqrt(a^2 - b^2)) - (18*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*a^8*b*tan(1/2*d*x + 1/2*c)^5 - 24*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 24*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 105*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 60*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*a^9*tan(1/2*d*x + 1/2*c)^3 - 152*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 236*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 120*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a^9*tan(1/2*d*x + 1/2*c) + 42*a^8*b*tan(1/2*d*x + 1/2*c) - 24*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*a^6*b^3*tan(1/2*d*x + 1/2*c) - 24*a^5*b^4*tan(1/2*d*x + 1/2*c) + 105*a^4*b^5*tan(1/2*d*x + 1/2*c) + 60*a^3*b^6*tan(1/2*d*x + 1/2*c))/(a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3 + 12*(d*x + c)*a/b^5 - 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^4))/d$$

Mupad [B] (verification not implemented)

Time = 24.95 (sec) , antiderivative size = 7494, normalized size of antiderivative = 24.41

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Too large to display}$$

[In] int(cos(c + d*x)^5/(a + b*cos(c + d*x))^4,x)

[Out]
$$-((\tan(c/2 + (d*x)/2))^3*(12*a^7*b - 72*a^8 - 18*b^8 + 72*a^2*b^6 + 60*a^3*b^5 - 273*a^4*b^4 - 47*a^5*b^3 + 236*a^6*b^2))/(3*b^4*(a + b)^2*(a - b)^3) - (\tan(c/2 + (d*x)/2))^5*(12*a^7*b + 72*a^8 + 18*b^8 - 72*a^2*b^6 + 60*a^3*b^5 + 273*a^4*b^4 - 47*a^5*b^3 - 236*a^6*b^2))/(3*b^4*(a + b)^3*(a - b)^2) + (\tan(c/2 + (d*x)/2)*(2*a*b^6 - 4*a^6*b - 8*a^7 + 2*b^7 - 6*a^2*b^5 - 26*a^3*b^4 + 11*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)*(a - b)^3) + (\tan(c/2 + (d*x)/2))^7*(2*a*b^6 + 4*a^6*b - 8*a^7 - 2*b^7 + 6*a^2*b^5 - 26*a^3*b^4 - 11*a^4*b^3 + 24*a^5*b^2))/(b^4*(a + b)^3*(a - b)))/(d*(3*a*b^2 + 3*a^2*b - \tan(c/2 + (d*x)/2))^4*(6*a*b^2 - 6*a^3) + \tan(c/2 + (d*x)/2)^2*(6*a^2*b + 4*a^3 - 2*b^3) + \tan(c/2 + (d*x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b^3 + \tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (8*a*atan(((4*a*((8*\tan(c/2 + (d*x)/2)*(128*a^16 - 128*a^15*b + 64*a^2*b^14 - 128*a^3*b^13 + 80*a^4*b^12 + 768*a^5*b^11 - 824*a^6*b^10 - 1920*a^7*b^9 + 2025*a^8*b^8 + 2560*a^9*b^7 - 2600*a^10*b^6 - 1920*a^11*b^5 + 1920*a^12*b^4 + 768*a^13*b^3 - 768*a^14*b^2)))/(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8) + (a*((16*(8*a*b^23 - 20*a^2*b^22 - 36*a^3*b^21 + 95*a^4*b^20 + 73*a^5*b^19 - 193*a^6*b^18 - 87*a^7*b^17 + 217*a^8*b^16 + 63*a^9*b^15 - 143*a^10*b^14 - 25*a^11*b^13 + 52*a^12*b^12 + 4*a^13*b^11 - 8*a^14*b^10)))/(a*b^22$$

$$\begin{aligned}
& + b^{23} - 5a^2b^{21} - 5a^3b^{20} + 10a^4b^{19} + 10a^5b^{18} - 10a^6b^{17} \\
& - 10a^7b^{16} + 5a^8b^{15} + 5a^9b^{14} - a^{10}b^{13} - a^{11}b^{12}) - (a \tan(\\
& c/2 + (dx)/2) * (8a^2b^{23} - 8a^2b^{22} - 48a^3b^{21} + 48a^4b^{20} + 120a^5 \\
& * b^{19} - 120a^6b^{18} - 160a^7b^{17} + 160a^8b^{16} + 120a^9b^{15} - 120a^{10} \\
& * b^{14} - 48a^{11}b^{13} + 48a^{12}b^{12} + 8a^{13}b^{11} - 8a^{14}b^{10}) * 32i) / (b^5 \\
& * (a^2b^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10 \\
& a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8))) * 4 \\
& i) / b^5) / b^5 + (4a * ((8 \tan(c/2 + (dx)/2) * (128a^{16} - 128a^{15}b + 64a^{12} \\
& b^{14} - 128a^3b^{13} + 80a^4b^{12} + 768a^5b^{11} - 824a^6b^{10} - 1920a^7 \\
& b^9 + 2025a^8b^8 + 2560a^9b^7 - 2600a^{10}b^6 - 1920a^{11}b^5 + 1920a^{12} \\
& b^4 + 768a^{13}b^3 - 768a^{14}b^2)) / (a^2b^{18} + b^{19} - 5a^2b^{17} - 5a^3 \\
& b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + \\
& 5a^9b^{10} - a^{10}b^9 - a^{11}b^8) - (a * ((16 * (8a^2b^{23} - 20a^2b^{22} - 36a^3 \\
& b^{21} + 95a^4b^{20} + 73a^5b^{19} - 193a^6b^{18} - 87a^7b^{17} + 217a^8 \\
& b^{16} + 63a^9b^{15} - 143a^{10}b^{14} - 25a^{11}b^{13} + 52a^{12}b^{12} + 4a^{13}b^{11} \\
& - 8a^{14}b^{10})) / (a^2b^{22} + b^{23} - 5a^2b^{21} - 5a^3b^{20} + 10a^4b^{19} \\
& + 10a^5b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15} + 5a^9b^{14} - a^{10} \\
& b^{13} - a^{11}b^{12}) + (a \tan(c/2 + (dx)/2) * (8a^2b^{23} - 8a^2b^{22} - 48a^3b^{21} \\
& + 48a^4b^{20} + 120a^5b^{19} - 120a^6b^{18} - 160a^7b^{17} + 160a^8b^{16} \\
& + 120a^9b^{15} - 120a^{10}b^{14} - 48a^{11}b^{13} + 48a^{12}b^{12} + 8a^{13}b^{11} \\
& - 8a^{14}b^{10}) * 32i) / (b^5 * (a^2b^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} + 10a^4 \\
& b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} \\
& - a^{10}b^9 - a^{11}b^8))) * 4i) / b^5) / ((32 * (128a^{16} - 64a^{15}b + 320a^{12} \\
& b^{12} + 480a^5b^{11} - 1520a^6b^{10} - 1280a^7b^9 + 3088a^8b^8 + 1602 \\
& * a^9b^7 - 3472a^{10}b^6 - 1088a^{11}b^5 + 2288a^{12}b^4 + 400a^{13}b^3 - 8 \\
& 32a^{14}b^2)) / (a^2b^{22} + b^{23} - 5a^2b^{21} - 5a^3b^{20} + 10a^4b^{19} + 10a^5 \\
& b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15} + 5a^9b^{14} - a^{10}b^{13} - \\
& a^{11}b^{12}) - (a * ((8 \tan(c/2 + (dx)/2) * (128a^{16} - 128a^{15}b + 64a^{12}b^{14} \\
& - 128a^3b^{13} + 80a^4b^{12} + 768a^5b^{11} - 824a^6b^{10} - 1920a^7b^9 \\
& + 2025a^8b^8 + 2560a^9b^7 - 2600a^{10}b^6 - 1920a^{11}b^5 + 1920a^{12} \\
& b^4 + 768a^{13}b^3 - 768a^{14}b^2)) / (a^2b^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} \\
& + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9 \\
& b^{10} - a^{10}b^9 - a^{11}b^8) + (a * ((16 * (8a^2b^{23} - 20a^2b^{22} - 36a^3 \\
& b^{21} + 95a^4b^{20} + 73a^5b^{19} - 193a^6b^{18} - 87a^7b^{17} + 217a^8 \\
& b^{16} + 63a^9b^{15} - 143a^{10}b^{14} - 25a^{11}b^{13} + 52a^{12}b^{12} + 4a^{13}b^{11} \\
& - 8a^{14}b^{10})) / (a^2b^{22} + b^{23} - 5a^2b^{21} - 5a^3b^{20} + 10a^4b^{19} + 1 \\
& 0a^5b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15} + 5a^9b^{14} - a^{10}b^{13} \\
& - a^{11}b^{12}) - (a \tan(c/2 + (dx)/2) * (8a^2b^{23} - 8a^2b^{22} - 48a^3b^{21} \\
& + 48a^4b^{20} + 120a^5b^{19} - 120a^6b^{18} - 160a^7b^{17} + 160a^8b^{16} \\
& + 120a^9b^{15} - 120a^{10}b^{14} - 48a^{11}b^{13} + 48a^{12}b^{12} + 8a^{13}b^{11} \\
& - 8a^{14}b^{10}) * 32i) / (b^5 * (a^2b^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} + 10a^4 \\
& b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} - \\
& a^{10}b^9 - a^{11}b^8))) * 4i) / b^5) * 4i) / b^5 + (a * ((8 \tan(c/2 + (dx)/2) * (128a^{16} \\
& - 128a^{15}b + 64a^{12}b^{14} - 128a^3b^{13} + 80a^4b^{12} + 768a^5b^{11} - \\
& 824a^6b^{10} - 1920a^7b^9 + 2025a^8b^8 + 2560a^9b^7 - 2600a^{10}b^6
\end{aligned}$$

$$\begin{aligned}
& - 1920a^{11}b^5 + 1920a^{12}b^4 + 768a^{13}b^3 - 768a^{14}b^2) / (a^2b^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8) - (a^2((16(8a^8b^{23} - 20a^2b^{22} - 36a^3b^{21} + 95a^4b^{20} + 73a^5b^{19} - 193a^6b^{18} - 87a^7b^{17} + 217a^8b^{16} + 63a^9b^{15} - 143a^{10}b^{14} - 25a^{11}b^{13} + 52a^{12}b^{12} + 4a^{13}b^{11} - 8a^{14}b^{10}))) / (a^2b^{22} + b^{23} - 5a^2b^{21} - 5a^3b^{20} + 10a^4b^{19} + 10a^5b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15} + 5a^9b^{14} - a^{10}b^{13} - a^{11}b^{12}) + (a \tan(c/2 + (dx)/2) * (8a^8b^3 - 8a^2b^{22} - 48a^3b^{21} + 48a^4b^{20} + 120a^5b^{19} - 120a^6b^{18} - 160a^7b^{17} + 160a^8b^{16} + 120a^9b^{15} - 120a^{10}b^{14} - 48a^{11}b^{13} + 48a^{12}b^{12} + 8a^{13}b^{11} - 8a^{14}b^{10}) * 32i) / (b^5(a^2b^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8))) * 4i) / b^5) / b^5) / (b^5 * d) - (a^2 * \operatorname{atan}(((a^2 * ((8 \tan(c/2 + (dx)/2) * (128a^{16} - 128a^{15}b + 64a^2b^{14} - 128a^3b^{13} + 80a^4b^{12} + 768a^5b^{11} - 824a^6b^{10} - 1920a^7b^9 + 2025a^8b^8 + 2560a^9b^7 - 2600a^{10}b^6 - 1920a^{11}b^5 + 1920a^{12}b^4 + 768a^{13}b^3 - 768a^{14}b^2)) / (a^2b^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8) + (a^2 * ((16(8a^8b^{23} - 20a^2b^{22} - 36a^3b^{21} + 95a^4b^{20} + 73a^5b^{19} - 193a^6b^{18} - 87a^7b^{17} + 217a^8b^{16} + 63a^9b^{15} - 143a^{10}b^{14} - 25a^{11}b^{13} + 52a^{12}b^{12} + 4a^{13}b^{11} - 8a^{14}b^{10}))) / (a^2b^{22} + b^{23} - 5a^2b^{21} - 5a^3b^{20} + 10a^4b^{19} + 10a^5b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15} + 5a^9b^{14} - a^{10}b^{13} - a^{11}b^{12}) - (4a^2 \tan(c/2 + (dx)/2) * (-(a + b)^7 * (a - b)^7)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2)) * (8a^8b^{23} - 8a^2b^{22} - 48a^3b^{21} + 48a^4b^{20} + 120a^5b^{19} - 120a^6b^{18} - 160a^7b^{17} + 160a^8b^{16} + 120a^9b^{15} - 120a^{10}b^{14} - 48a^{11}b^{13} + 48a^{12}b^{12} + 8a^{13}b^{11} - 8a^{14}b^{10}))) / ((b^{19} - 7a^2b^{17} + 21a^4b^{15} - 35a^6b^{13} + 35a^8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 - a^{14}b^5)) * (a^2b^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8))) * (-(a + b)^7 * (a - b)^7)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2)) / (2 * (b^{19} - 7a^2b^{17} + 21a^4b^{15} - 35a^6b^{13} + 35a^8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 - a^{14}b^5))) * (-(a + b)^7 * (a - b)^7)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2) * 1i) / (2 * (b^{19} - 7a^2b^{17} + 21a^4b^{15} - 35a^6b^{13} + 35a^8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 - a^{14}b^5)) + (a^2 * ((8 \tan(c/2 + (dx)/2) * (128a^{16} - 128a^{15}b + 64a^2b^{14} - 128a^3b^{13} + 80a^4b^{12} + 768a^5b^{11} - 824a^6b^{10} - 1920a^7b^9 + 2025a^8b^8 + 2560a^9b^7 - 2600a^{10}b^6 - 1920a^{11}b^5 + 1920a^{12}b^4 + 768a^{13}b^3 - 768a^{14}b^2)) / (a^2b^{18} + b^{19} - 5a^2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8) - (a^2 * ((16(8a^8b^{23} - 20a^2b^{22} - 36a^3b^{21} + 95a^4b^{20} + 73a^5b^{19} - 193a^6b^{18} - 87a^7b^{17} + 217a^8b^{16} + 63a^9b^{15} - 143a^{10}b^{14} - 25a^{11}b^{13} + 52a^{12}b^{12} + 4a^{13}b^{11} - 8a^{14}b^{10}))) / (a^2b^{22} + b^{23} - 5a^2b^{21} - 5a^3b^{20} + 10a^4b^{19} + 10a^5b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15}
\end{aligned}$$

$$\begin{aligned}
& + 5a^9b^{14} - a^{10}b^{13} - a^{11}b^{12}) + (4a^2 \tan(c/2 + (d*x)/2) * (-(a + b) \\
& ^7 * (a - b)^7)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2) * (8ab^{23} - \\
& 8a^2b^{22} - 48a^3b^{21} + 48a^4b^{20} + 120a^5b^{19} - 120a^6b^{18} - 160a \\
& ^7b^{17} + 160a^8b^{16} + 120a^9b^{15} - 120a^{10}b^{14} - 48a^{11}b^{13} + 48a \\
& ^{12}b^{12} + 8a^{13}b^{11} - 8a^{14}b^{10})) / ((b^{19} - 7a^2b^{17} + 21a^4b^{15} - \\
& 35a^6b^{13} + 35a^8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 - a^{14}b^5) * (ab^{18} + \\
& b^{19} - 5a^2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - \\
& 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8))) * (-(a + b)^7 \\
& * (a - b)^7)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2)) / (2 * (b^{19} - 7a \\
& ^2b^{17} + 21a^4b^{15} - 35a^6b^{13} + 35a^8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 \\
& - a^{14}b^5))) * (-(a + b)^7 * (a - b)^7)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 \\
& - 28a^4b^2) * i) / (2 * (b^{19} - 7a^2b^{17} + 21a^4b^{15} - 35a^6b^{13} + 35a^ \\
& 8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 - a^{14}b^5))) / ((32 * (128a^{16} - 64a^{15}b \\
& + 320a^4b^{12} + 480a^5b^{11} - 1520a^6b^{10} - 1280a^7b^9 + 3088a^8b^8 \\
& + 1602a^9b^7 - 3472a^{10}b^6 - 1088a^{11}b^5 + 2288a^{12}b^4 + 400a^{13}b^3 \\
& - 832a^{14}b^2)) / (ab^{22} + b^{23} - 5a^2b^{21} - 5a^3b^{20} + 10a^4b^{19} \\
& + 10a^5b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15} + 5a^9b^{14} - a^{10} \\
& * b^{13} - a^{11}b^{12}) - (a^2 * ((8 * \tan(c/2 + (d*x)/2) * (128a^{16} - 128a^{15}b + 6 \\
& 4a^2b^{14} - 128a^3b^{13} + 80a^4b^{12} + 768a^5b^{11} - 824a^6b^{10} - 192 \\
& 0a^7b^9 + 2025a^8b^8 + 2560a^9b^7 - 2600a^{10}b^6 - 1920a^{11}b^5 + 1 \\
& 920a^{12}b^4 + 768a^{13}b^3 - 768a^{14}b^2)) / (ab^{18} + b^{19} - 5a^2b^{17} - \\
& 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} + 5a^8b^{11} \\
& + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8) + (a^2 * ((16 * (8ab^{23} - 20a^2b^2 \\
& 2 - 36a^3b^{21} + 95a^4b^{20} + 73a^5b^{19} - 193a^6b^{18} - 87a^7b^{17} + \\
& 217a^8b^{16} + 63a^9b^{15} - 143a^{10}b^{14} - 25a^{11}b^{13} + 52a^{12}b^{12} + \\
& 4a^{13}b^{11} - 8a^{14}b^{10})) / (ab^{22} + b^{23} - 5a^2b^{21} - 5a^3b^{20} + 10a^ \\
& ^4b^{19} + 10a^5b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15} + 5a^9b^{14} \\
& - a^{10}b^{13} - a^{11}b^{12}) - (4a^2 * \tan(c/2 + (d*x)/2) * (-(a + b)^7 * (a - b)^7 \\
&)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2) * (8ab^{23} - 8a^2b^{22} - \\
& 48a^3b^{21} + 48a^4b^{20} + 120a^5b^{19} - 120a^6b^{18} - 160a^7b^{17} + 1 \\
& 60a^8b^{16} + 120a^9b^{15} - 120a^{10}b^{14} - 48a^{11}b^{13} + 48a^{12}b^{12} + \\
& 8a^{13}b^{11} - 8a^{14}b^{10})) / ((b^{19} - 7a^2b^{17} + 21a^4b^{15} - 35a^6b^{13} \\
& + 35a^8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 - a^{14}b^5) * (ab^{18} + b^{19} - 5a^ \\
& 2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^{12} \\
& + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8))) * (-(a + b)^7 * (a - b)^7)^ \\
& (1/2) * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2)) / (2 * (b^{19} - 7a^2b^{17} + 2 \\
& 1a^4b^{15} - 35a^6b^{13} + 35a^8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 - a^{14}b^ \\
& 5))) * (-(a + b)^7 * (a - b)^7)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2 \\
&)) / (2 * (b^{19} - 7a^2b^{17} + 21a^4b^{15} - 35a^6b^{13} + 35a^8b^{11} - 21a^1 \\
& 0b^9 + 7a^{12}b^7 - a^{14}b^5)) + (a^2 * ((8 * \tan(c/2 + (d*x)/2) * (128a^{16} - 1 \\
& 28a^{15}b + 64a^2b^{14} - 128a^3b^{13} + 80a^4b^{12} + 768a^5b^{11} - 824a^ \\
& ^6b^{10} - 1920a^7b^9 + 2025a^8b^8 + 2560a^9b^7 - 2600a^{10}b^6 - 1920 \\
& * a^{11}b^5 + 1920a^{12}b^4 + 768a^{13}b^3 - 768a^{14}b^2)) / (ab^{18} + b^{19} - \\
& 5a^2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} - 10a^7b^ \\
& b^{12} + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8) - (a^2 * ((16 * (8ab^{23}
\end{aligned}$$

$$\begin{aligned}
& - 20a^2b^{22} - 36a^3b^{21} + 95a^4b^{20} + 73a^5b^{19} - 193a^6b^{18} - 8 \\
& 7a^7b^{17} + 217a^8b^{16} + 63a^9b^{15} - 143a^{10}b^{14} - 25a^{11}b^{13} + 52 \\
& a^{12}b^{12} + 4a^{13}b^{11} - 8a^{14}b^{10}) / (a^2b^{22} + b^{23} - 5a^2b^{21} - 5a^3 \\
& b^{20} + 10a^4b^{19} + 10a^5b^{18} - 10a^6b^{17} - 10a^7b^{16} + 5a^8b^{15} \\
& + 5a^9b^{14} - a^{10}b^{13} - a^{11}b^{12}) + (4a^2 \tan(c/2 + (dx)/2) * (-(a + b) \\
&)^7 * (a - b)^7)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2) * (8a^2b^{23} - \\
& 8a^2b^{22} - 48a^3b^{21} + 48a^4b^{20} + 120a^5b^{19} - 120a^6b^{18} - 160 \\
& a^7b^{17} + 160a^8b^{16} + 120a^9b^{15} - 120a^{10}b^{14} - 48a^{11}b^{13} + 48 \\
& a^{12}b^{12} + 8a^{13}b^{11} - 8a^{14}b^{10}) / ((b^{19} - 7a^2b^{17} + 21a^4b^{15} \\
& - 35a^6b^{13} + 35a^8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 - a^{14}b^5) * (a^2b^{18} \\
& + b^{19} - 5a^2b^{17} - 5a^3b^{16} + 10a^4b^{15} + 10a^5b^{14} - 10a^6b^{13} \\
& - 10a^7b^{12} + 5a^8b^{11} + 5a^9b^{10} - a^{10}b^9 - a^{11}b^8)) * (-(a + b)^7 * \\
& (a - b)^7)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2) / (2 * (b^{19} - 7 \\
& a^2b^{17} + 21a^4b^{15} - 35a^6b^{13} + 35a^8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 - \\
& a^{14}b^5)) * (-(a + b)^7 * (a - b)^7)^{(1/2)} * (8a^6 - 20b^6 + 35a^2b^4 \\
& - 28a^4b^2) / (2 * (b^{19} - 7a^2b^{17} + 21a^4b^{15} - 35a^6b^{13} + 35a^8b^{11} \\
& - 21a^{10}b^9 + 7a^{12}b^7 - a^{14}b^5))) * (-(a + b)^7 * (a - b)^7)^{(1/2)} \\
& * (8a^6 - 20b^6 + 35a^2b^4 - 28a^4b^2) * i) / (d * (b^{19} - 7a^2b^{17} + 21a^4b^{15} \\
& - 35a^6b^{13} + 35a^8b^{11} - 21a^{10}b^9 + 7a^{12}b^7 - a^{14}b^5) \\
&)
\end{aligned}$$

$$3.479 \quad \int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^4} dx$$

Optimal result	5099
Rubi [A] (verified)	5099
Mathematica [A] (verified)	5102
Maple [A] (verified)	5103
Fricas [B] (verification not implemented)	5103
Sympy [F(-1)]	5104
Maxima [F(-2)]	5104
Giac [B] (verification not implemented)	5105
Mupad [B] (verification not implemented)	5105

Optimal result

Integrand size = 21, antiderivative size = 250

$$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^4} dx = \frac{x}{b^4} - \frac{a(2a^6 - 7a^4b^2 + 8a^2b^4 - 8b^6) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d}$$

$$- \frac{a^2 \cos^2(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b \cos(c+dx))^3}$$

$$+ \frac{a^3(3a^2-8b^2) \sin(c+dx)}{6b^3(a^2-b^2)^2 d(a+b \cos(c+dx))^2}$$

$$- \frac{a^2(9a^4-28a^2b^2+34b^4) \sin(c+dx)}{6b^3(a^2-b^2)^3 d(a+b \cos(c+dx))}$$

```
[Out] x/b^4-a*(2*a^6-7*a^4*b^2+8*a^2*b^4-8*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^4/(a+b)^(7/2)/d-1/3*a^2*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a^3*(3*a^2-8*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2-1/6*a^2*(9*a^4-28*a^2*b^2+34*b^4)*sin(d*x+c)/b^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {2871, 3110, 3100, 2814, 2738, 211}

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4} dx = -\frac{a^2 \sin(c+dx) \cos^2(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^3} - \frac{a^2(9a^4-28a^2b^2+34b^4)\sin(c+dx)}{6b^3d(a^2-b^2)^3(a+b\cos(c+dx))} + \frac{a^3(3a^2-8b^2)\sin(c+dx)}{6b^3d(a^2-b^2)^2(a+b\cos(c+dx))^2} - \frac{a(2a^6-7a^4b^2+8a^2b^4-8b^6)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}} + \frac{x}{b^4}$$

[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^4,x]

[Out] x/b^4 - (a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) - (a^2*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a^3*(3*a^2 - 8*b^2)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - (a^2*(9*a^4 - 28*a^2*b^2 + 34*b^4)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2814

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2871

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +

$b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*\text{Sin}[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*\text{Sin}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 2] \&\& \text{LtQ}[n, -1] \&\& (\text{IntegerQ}[m] \mid\mid \text{IntegersQ}[2*m, 2*n])$

Rule 3100

$\text{Int}[(a_. + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> \text{Simp}[(-A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*((a + b*\text{Sin}[e + f*x])^{(m + 1)})/(b*f*(m + 1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*\text{Simp}[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x\} \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3110

$\text{Int}[(a_. + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)])*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> \text{Simp}[(-b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*((a + b*\text{Sin}[e + f*x])^{(m + 1)})/(b^2*f*(m + 1)*(a^2 - b^2)), x] - \text{Dist}[1/(b^2*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*\text{Simp}[b*(m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))*\text{Sin}[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2))*\text{Sin}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, C\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1]$

Rubi steps

integral

$$\begin{aligned}
 &= -\frac{a^2 \cos^2(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^3} - \frac{\int \frac{\cos(c+dx)(2a^2 - 3ab \cos(c+dx) - 3(a^2 - b^2) \cos^2(c+dx))}{(a + b \cos(c+dx))^3} dx}{3b(a^2 - b^2)} \\
 &= -\frac{a^2 \cos^2(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^3} + \frac{a^3(3a^2 - 8b^2) \sin(c + dx)}{6b^3(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
 &\quad - \frac{\int \frac{2a^2b(3a^2 - 8b^2) + a(3a^4 - 10a^2b^2 + 12b^4) \cos(c+dx) - 6b(a^2 - b^2)^2 \cos^2(c+dx)}{(a + b \cos(c+dx))^2} dx}{6b^3(a^2 - b^2)^2} \\
 &= -\frac{a^2 \cos^2(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^3} + \frac{a^3(3a^2 - 8b^2) \sin(c + dx)}{6b^3(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
 &\quad - \frac{a^2(9a^4 - 28a^2b^2 + 34b^4) \sin(c + dx)}{6b^3(a^2 - b^2)^3 d(a + b \cos(c + dx))} + \frac{\int \frac{3ab^2(a^4 - 2a^2b^2 + 6b^4) + 6b(a^2 - b^2)^3 \cos(c+dx)}{a + b \cos(c+dx)} dx}{6b^4(a^2 - b^2)^3}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{x}{b^4} - \frac{a^2 \cos^2(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} + \frac{a^3(3a^2-8b^2)\sin(c+dx)}{6b^3(a^2-b^2)^2 d(a+b\cos(c+dx))^2} \\
&\quad - \frac{a^2(9a^4-28a^2b^2+34b^4)\sin(c+dx)}{6b^3(a^2-b^2)^3 d(a+b\cos(c+dx))} - \frac{(a(2a^6-7a^4b^2+8a^2b^4-8b^6)) \int \frac{1}{a+b\cos(c+dx)} dx}{2b^4(a^2-b^2)^3} \\
&= \frac{x}{b^4} - \frac{a^2 \cos^2(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} + \frac{a^3(3a^2-8b^2)\sin(c+dx)}{6b^3(a^2-b^2)^2 d(a+b\cos(c+dx))^2} \\
&\quad - \frac{a^2(9a^4-28a^2b^2+34b^4)\sin(c+dx)}{6b^3(a^2-b^2)^3 d(a+b\cos(c+dx))} \\
&\quad - \frac{(a(2a^6-7a^4b^2+8a^2b^4-8b^6)) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^4(a^2-b^2)^3 d} \\
&= \frac{x}{b^4} - \frac{a(2a^6-7a^4b^2+8a^2b^4-8b^6) \arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d} \\
&\quad - \frac{a^2 \cos^2(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} + \frac{a^3(3a^2-8b^2)\sin(c+dx)}{6b^3(a^2-b^2)^2 d(a+b\cos(c+dx))^2} \\
&\quad - \frac{a^2(9a^4-28a^2b^2+34b^4)\sin(c+dx)}{6b^3(a^2-b^2)^3 d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.91

$$\begin{aligned}
&\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4} dx \\
&= \frac{6(c+dx) - \frac{6a(2a^6-7a^4b^2+8a^2b^4-8b^6)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} - \frac{2a^4b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^3} + \frac{a^3b(7a^2-12b^2)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}}{6b^4d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^4,x]

[Out] (6*(c + d*x) - (6*a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]/(-a^2 + b^2)^(7/2) - (2*a^4*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + (a^3*b*(7*a^2 - 12*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + (a^2*b*(-11*a^4 + 32*a^2*b^2 - 36*b^4)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Cos[c + d*x])))/(6*b^4*d)

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{b^4} - \frac{2a \left(\frac{(2a^4 - a^3b - 6a^2b^2 + 4ab^3 + 12b^4)ab \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2(a-b)(a^3 + 3a^2b + 3ab^2 + b^3)} + \frac{2(3a^4 - 11a^2b^2 + 18b^4)ab \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3(a^2 - 2ab + b^2)(a^2 + 2ab + b^2)} + \frac{(2a^4 - a^3b - 6a^2b^2 + 4ab^3 + 12b^4)ab \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{(a^2 - 2ab + b^2)(a^2 + 2ab + b^2)} \right)}{\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a + b\right)^3}$
default	$\frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{b^4} - \frac{2a \left(\frac{(2a^4 - a^3b - 6a^2b^2 + 4ab^3 + 12b^4)ab \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2(a-b)(a^3 + 3a^2b + 3ab^2 + b^3)} + \frac{2(3a^4 - 11a^2b^2 + 18b^4)ab \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3(a^2 - 2ab + b^2)(a^2 + 2ab + b^2)} + \frac{(2a^4 - a^3b - 6a^2b^2 + 4ab^3 + 12b^4)ab \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{(a^2 - 2ab + b^2)(a^2 + 2ab + b^2)} \right)}{\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a + b\right)^3}$
risch	Expression too large to display

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)

```
[Out] 1/d*(2/b^4*arctan(tan(1/2*d*x+1/2*c))-2*a/b^4*((1/2*(2*a^4-a^3*b-6*a^2*b^2+
4*a*b^3+12*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/
3*(3*a^4-11*a^2*b^2+18*b^4)*a*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3+1/2*(2*a^4+a^3*b-6*a^2*b^2-4*a*b^3+12*b^4)*a*b/(a+b)/(a^3-3*a^2*b
+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2
*c)^2+a+b)^3+1/2*(2*a^6-7*a^4*b^2+8*a^2*b^4-8*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4
-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/
2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(235) = 470.

Time = 0.36 (sec) , antiderivative size = 1445, normalized size of antiderivative = 5.78

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="fricas")

```
[Out] [1/12*(12*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*x*cos(d*x
+ c)^3 + 36*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*x*cos(
d*x + c)^2 + 36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*
cos(d*x + c) + 12*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x
- 3*(2*a^10 - 7*a^8*b^2 + 8*a^6*b^4 - 8*a^4*b^6 + (2*a^7*b^3 - 7*a^5*b^5 +
8*a^3*b^7 - 8*a*b^9)*cos(d*x + c)^3 + 3*(2*a^8*b^2 - 7*a^6*b^4 + 8*a^4*b^6
- 8*a^2*b^8)*cos(d*x + c)^2 + 3*(2*a^9*b - 7*a^7*b^3 + 8*a^5*b^5 - 8*a^3*b^
7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*c
```

```

os(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
+ 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6*a^10*b - 2
3*a^8*b^3 + 43*a^6*b^5 - 26*a^4*b^7 + (11*a^8*b^3 - 43*a^6*b^5 + 68*a^4*b^7
- 36*a^2*b^9)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3
*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^
2*b^13 + b^15)*d*cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a
^3*b^12 + a*b^14)*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 -
4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 -
4*a^5*b^10 + a^3*b^12)*d), 1/6*(6*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2
*b^9 + b^11)*d*x*cos(d*x + c)^3 + 18*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10)*d*x*cos(d*x + c)^2 + 18*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
4*a^4*b^7 + a^2*b^9)*d*x*cos(d*x + c) + 6*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4
*a^5*b^6 + a^3*b^8)*d*x - 3*(2*a^10 - 7*a^8*b^2 + 8*a^6*b^4 - 8*a^4*b^6 + (
2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9)*cos(d*x + c)^3 + 3*(2*a^8*b^2
- 7*a^6*b^4 + 8*a^4*b^6 - 8*a^2*b^8)*cos(d*x + c)^2 + 3*(2*a^9*b - 7*a^7*b^
3 + 8*a^5*b^5 - 8*a^3*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x
+ c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*a^10*b - 23*a^8*b^3 + 43*a^
6*b^5 - 26*a^4*b^7 + (11*a^8*b^3 - 43*a^6*b^5 + 68*a^4*b^7 - 36*a^2*b^9)*co
s(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3*b^8)*cos(d*x + c
))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*
cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)
*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*
b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3
*b^12)*d)]

```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(235) = 470.

Time = 0.35 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.12

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^4} dx$$

$$\frac{3(2a^7 - 7a^5b^2 + 8a^3b^4 - 8ab^6) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^6b^4 - 3a^4b^6 + 3a^2b^8 - b^{10})\sqrt{a^2 - b^2}} - \frac{6a^8 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 15a^7b \tan(\frac{1}{2} dx + \frac{1}{2} c)^4 + \dots}{\dots}$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/3*(3*(2*a^7 - 7*a^5*b^2 + 8*a^3*b^4 - 8*a*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(a^2 - b^2)) - (6*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 6*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 45*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^1 - 60*a^3*b^5*tan(1/2*d*x + 1/2*c)^0 + 36*a^2*b^6*tan(1/2*d*x + 1/2*c)^-1 + 12*a^8*tan(1/2*d*x + 1/2*c)^3 - 56*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 72*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 6*a^8*tan(1/2*d*x + 1/2*c) + 15*a^7*b*tan(1/2*d*x + 1/2*c) - 6*a^6*b^2*tan(1/2*d*x + 1/2*c) - 45*a^5*b^3*tan(1/2*d*x + 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b^5*tan(1/2*d*x + 1/2*c) + 36*a^2*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 3*(d*x + c)/b^4)/d

Mupad [B] (verification not implemented)

Time = 27.41 (sec) , antiderivative size = 7247, normalized size of antiderivative = 28.99

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Too large to display}$$

[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^4,x)

[Out] (2*atan((((8*(16*a*b^20 - 4*b^21 + 12*a^2*b^19 - 64*a^3*b^18 - 20*a^4*b^17 + 110*a^5*b^16 + 30*a^6*b^15 - 110*a^7*b^14 - 30*a^8*b^13 + 70*a^9*b^12 + 14*a^10*b^11 - 26*a^11*b^10 - 2*a^12*b^9 + 4*a^13*b^8)))/(a*b^19 + b^20 - 5

$$\begin{aligned}
& a^2b^{18} - 5a^3b^{17} + 10a^4b^{16} + 10a^5b^{15} - 10a^6b^{14} - 10a^7b^{13} + 5a^8b^{12} + 5a^9b^{11} - a^{10}b^{10} - a^{11}b^9) - (\tan(c/2 + (d*x)/2) \\
& * (8a^2b^{21} - 8a^2b^{20} - 48a^3b^{19} + 48a^4b^{18} + 120a^5b^{17} - 120a^6b^{16} - 160a^7b^{15} + 160a^8b^{14} + 120a^9b^{13} - 120a^{10}b^{12} - 48a^{11}b^{11} + 48a^{12}b^{10} + 8a^{13}b^9 - 8a^{14}b^8)*8i)/(b^4*(a^2b^{16} + b^{17} - \\
& 5a^2b^{15} - 5a^3b^{14} + 10a^4b^{13} + 10a^5b^{12} - 10a^6b^{11} - 10a^7b^{10} + 5a^8b^9 + 5a^9b^8 - a^{10}b^7 - a^{11}b^6))) * i)/b^4 + (8*\tan(c/2 \\
& + (d*x)/2)*(8a^{14} - 8a^{13}b - 8a^2b^{13} + 4b^{14} + 44a^2b^{12} + 48a^3b^{11} - 92a^4b^{10} - 120a^5b^9 + 156a^6b^8 + 160a^7b^7 - 164a^8b^6 - \\
& 120a^9b^5 + 117a^{10}b^4 + 48a^{11}b^3 - 48a^{12}b^2))/(a^2b^{16} + b^{17} - 5a^2b^{15} - 5a^3b^{14} + 10a^4b^{13} + 10a^5b^{12} - 10a^6b^{11} - 10a^7b^{10} + 5a^8b^9 + 5a^9b^8 - a^{10}b^7 - a^{11}b^6))/b^4 - (((((8*(16a^2b^{20} \\
& - 4b^{21} + 12a^2b^{19} - 64a^3b^{18} - 20a^4b^{17} + 110a^5b^{16} + 30a^6b^{15} - 110a^7b^{14} - 30a^8b^{13} + 70a^9b^{12} + 14a^{10}b^{11} - 26a^{11}b^{10} - 2a^{12}b^9 + 4a^{13}b^8)))/(a^2b^{19} + b^{20} - 5a^2b^{18} - 5a^3b^{17} + \\
& 10a^4b^{16} + 10a^5b^{15} - 10a^6b^{14} - 10a^7b^{13} + 5a^8b^{12} + 5a^9b^{11} - a^{10}b^{10} - a^{11}b^9) + (\tan(c/2 + (d*x)/2)*(8a^2b^{21} - 8a^2b^{20} - \\
& 48a^3b^{19} + 48a^4b^{18} + 120a^5b^{17} - 120a^6b^{16} - 160a^7b^{15} + 160a^8b^{14} + 120a^9b^{13} - 120a^{10}b^{12} - 48a^{11}b^{11} + 48a^{12}b^{10} + \\
& 8a^{13}b^9 - 8a^{14}b^8)*8i)/(b^4*(a^2b^{16} + b^{17} - 5a^2b^{15} - 5a^3b^{14} + 10a^4b^{13} + 10a^5b^{12} - 10a^6b^{11} - 10a^7b^{10} + 5a^8b^9 + 5a^9b^8 - a^{10}b^7 - a^{11}b^6))) * i)/b^4 - (8*\tan(c/2 + (d*x)/2)*(8a^{14} - 8a^{13}b - 8a^2b^{13} + 4b^{14} + 44a^2b^{12} + 48a^3b^{11} - 92a^4b^{10} - 120a^5b^9 + 156a^6b^8 + 160a^7b^7 - 164a^8b^6 - 120a^9b^5 + 117a^{10}b^4 + 48a^{11}b^3 - 48a^{12}b^2))/(a^2b^{16} + b^{17} - 5a^2b^{15} - 5a^3b^{14} + 10a^4b^{13} + 10a^5b^{12} - 10a^6b^{11} - 10a^7b^{10} + 5a^8b^9 + 5a^9b^8 - a^{10}b^7 - a^{11}b^6))/b^4)/((((((8*(16a^2b^{20} - 4b^{21} + 12a^2b^{19} - 64a^3b^{18} - 20a^4b^{17} + 110a^5b^{16} + 30a^6b^{15} - 110a^7b^{14} - 30a^8b^{13} + 70a^9b^{12} + 14a^{10}b^{11} - 26a^{11}b^{10} - 2a^{12}b^9 + 4a^{13}b^8)))/(a^2b^{19} + b^{20} - 5a^2b^{18} - 5a^3b^{17} + 10a^4b^{16} + 10a^5b^{15} - 10a^6b^{14} - 10a^7b^{13} + 5a^8b^{12} + 5a^9b^{11} - a^{10}b^{10} - a^{11}b^9) - (\tan(c/2 + (d*x)/2)*(8a^2b^{21} - 8a^2b^{20} - 48a^3b^{19} + 48a^4b^{18} + 120a^5b^{17} - 120a^6b^{16} - 160a^7b^{15} + 160a^8b^{14} + 120a^9b^{13} - 120a^{10}b^{12} - 48a^{11}b^{11} + 48a^{12}b^{10} + 8a^{13}b^9 - 8a^{14}b^8)*8i)/(b^4*(a^2b^{16} + b^{17} - 5a^2b^{15} - 5a^3b^{14} + 10a^4b^{13} + 10a^5b^{12} - 10a^6b^{11} - 10a^7b^{10} + 5a^8b^9 + 5a^9b^8 - a^{10}b^7 - a^{11}b^6))) * i)/b^4 + (8*\tan(c/2 + (d*x)/2)*(8a^{14} - 8a^{13}b - 8a^2b^{13} + 4b^{14} + 44a^2b^{12} + 48a^3b^{11} - 92a^4b^{10} - 120a^5b^9 + 156a^6b^8 + 160a^7b^7 - 164a^8b^6 - 120a^9b^5 + 117a^{10}b^4 + 48a^{11}b^3 - 48a^{12}b^2))/(a^2b^{16} + b^{17} - 5a^2b^{15} - 5a^3b^{14} + 10a^4b^{13} + 10a^5b^{12} - 10a^6b^{11} - 10a^7b^{10} + 5a^8b^9 + 5a^9b^8 - a^{10}b^7 - a^{11}b^6))) * i)/b^4 + ((((((8*(16a^2b^{20} - 4b^{21} + 12a^2b^{19} - 64a^3b^{18} - 20a^4b^{17} + 110a^5b^{16} + 30a^6b^{15} - 110a^7b^{14} - 30a^8b^{13} + 70a^9b^{12} + 14a^{10}b^{11} - 26a^{11}b^{10} - 2a^{12}b^9 + 4a^{13}b^8)))/(a^2b^{19} + b^{20} - 5a^2b^{18} - 5a^3b^{17} + 10a^4b^{16} + 10a^5b^{15} - 10a^6b^{14} - 10
\end{aligned}$$

$$\begin{aligned}
& *a^7*b^{13} + 5*a^8*b^{12} + 5*a^9*b^{11} - a^{10}*b^{10} - a^{11}*b^9) + (\tan(c/2 + (d \\
& *x)/2)*(8*a*b^{21} - 8*a^2*b^{20} - 48*a^3*b^{19} + 48*a^4*b^{18} + 120*a^5*b^{17} - \\
& 120*a^6*b^{16} - 160*a^7*b^{15} + 160*a^8*b^{14} + 120*a^9*b^{13} - 120*a^{10}*b^{12} - \\
& 48*a^{11}*b^{11} + 48*a^{12}*b^{10} + 8*a^{13}*b^9 - 8*a^{14}*b^8)*8i)/(b^4*(a*b^{16} + \\
& b^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + 10*a^4*b^{13} + 10*a^5*b^{12} - 10*a^6*b^{11} - \\
& 10*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - a^{10}*b^7 - a^{11}*b^6))*1i)/b^4 - (8*t \\
& an(c/2 + (d*x)/2)*(8*a^{14} - 8*a^{13}*b - 8*a*b^{13} + 4*b^{14} + 44*a^2*b^{12} + 48 \\
& *a^3*b^{11} - 92*a^4*b^{10} - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8 \\
& *b^6 - 120*a^9*b^5 + 117*a^{10}*b^4 + 48*a^{11}*b^3 - 48*a^{12}*b^2))/(a*b^{16} + b \\
& ^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + 10*a^4*b^{13} + 10*a^5*b^{12} - 10*a^6*b^{11} - 1 \\
& 0*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - a^{10}*b^7 - a^{11}*b^6))*1i)/b^4 + (16*(1 \\
& 6*a*b^{12} - 2*a^{12}*b + 4*a^{13} + 48*a^2*b^{11} - 64*a^3*b^{10} - 64*a^4*b^9 + 110 \\
& *a^5*b^8 + 66*a^6*b^7 - 110*a^7*b^6 - 34*a^8*b^5 + 70*a^9*b^4 + 11*a^{10}*b^3 \\
& - 26*a^{11}*b^2))/(a*b^{19} + b^{20} - 5*a^2*b^{18} - 5*a^3*b^{17} + 10*a^4*b^{16} + 1 \\
& 0*a^5*b^{15} - 10*a^6*b^{14} - 10*a^7*b^{13} + 5*a^8*b^{12} + 5*a^9*b^{11} - a^{10}*b^{1 \\
& 0} - a^{11}*b^9)))/(b^4*d) - ((\tan(c/2 + (d*x)/2)^5*(2*a^6 - a^5*b + 12*a^2*b \\
& ^4 + 4*a^3*b^3 - 6*a^4*b^2))/((a*b^3 - b^4)*(a + b)^3) + (4*\tan(c/2 + (d*x) \\
& /2)^3*(3*a^6 + 18*a^2*b^4 - 11*a^4*b^2))/((3*(a + b)^2*(b^5 - 2*a*b^4 + a^2* \\
& b^3)) + (\tan(c/2 + (d*x)/2)*(a^5*b + 2*a^6 + 12*a^2*b^4 - 4*a^3*b^3 - 6*a^4 \\
& *b^2))/((a + b)*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)))/(d*(3*a*b^2 - \tan(c \\
& /2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - \tan(c/2 + (d*x)/2)^2* \\
& (3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + \tan(c/2 + (d*x) \\
& /2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (a*atan(((a*((8*\tan(c/2 + (d*x)/2 \\
&)*(8*a^{14} - 8*a^{13}*b - 8*a*b^{13} + 4*b^{14} + 44*a^2*b^{12} + 48*a^3*b^{11} - 92*a \\
& ^4*b^{10} - 120*a^5*b^9 + 156*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b \\
& ^5 + 117*a^{10}*b^4 + 48*a^{11}*b^3 - 48*a^{12}*b^2))/(a*b^{16} + b^{17} - 5*a^2*b^{15} \\
& - 5*a^3*b^{14} + 10*a^4*b^{13} + 10*a^5*b^{12} - 10*a^6*b^{11} - 10*a^7*b^{10} + 5*a \\
& ^8*b^9 + 5*a^9*b^8 - a^{10}*b^7 - a^{11}*b^6) + (a*((8*(16*a*b^{20} - 4*b^{21} + 12 \\
& *a^2*b^{19} - 64*a^3*b^{18} - 20*a^4*b^{17} + 110*a^5*b^{16} + 30*a^6*b^{15} - 110*a^ \\
& 7*b^{14} - 30*a^8*b^{13} + 70*a^9*b^{12} + 14*a^{10}*b^{11} - 26*a^{11}*b^{10} - 2*a^{12}*b \\
& ^9 + 4*a^{13}*b^8))/(a*b^{19} + b^{20} - 5*a^2*b^{18} - 5*a^3*b^{17} + 10*a^4*b^{16} + \\
& 10*a^5*b^{15} - 10*a^6*b^{14} - 10*a^7*b^{13} + 5*a^8*b^{12} + 5*a^9*b^{11} - a^{10}*b^{ \\
& 10} - a^{11}*b^9) - (4*a*\tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^{(1/2)}*(2*a^ \\
& 6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*(8*a*b^{21} - 8*a^2*b^{20} - 48*a^3*b^{19} + 4 \\
& 8*a^4*b^{18} + 120*a^5*b^{17} - 120*a^6*b^{16} - 160*a^7*b^{15} + 160*a^8*b^{14} + 12 \\
& 0*a^9*b^{13} - 120*a^{10}*b^{12} - 48*a^{11}*b^{11} + 48*a^{12}*b^{10} + 8*a^{13}*b^9 - 8*a \\
& ^{14}*b^8))/((b^{18} - 7*a^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 2 \\
& 1*a^{10}*b^8 + 7*a^{12}*b^6 - a^{14}*b^4)*(a*b^{16} + b^{17} - 5*a^2*b^{15} - 5*a^3*b^{1 \\
& 4} + 10*a^4*b^{13} + 10*a^5*b^{12} - 10*a^6*b^{11} - 10*a^7*b^{10} + 5*a^8*b^9 + 5*a \\
& ^9*b^8 - a^{10}*b^7 - a^{11}*b^6))*(-(a + b)^7*(a - b)^7)^{(1/2)}*(2*a^6 - 8*b^6 \\
& + 8*a^2*b^4 - 7*a^4*b^2))/(2*(b^{18} - 7*a^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{1 \\
& 2} + 35*a^8*b^{10} - 21*a^{10}*b^8 + 7*a^{12}*b^6 - a^{14}*b^4)))*(-(a + b)^7*(a - b \\
&)^7)^{(1/2)}*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*1i)/(2*(b^{18} - 7*a^2*b^{1 \\
& 6} + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10}*b^8 + 7*a^{12}*b^6 - a^ \\
& ^{14}*b^4)) + (a*((8*\tan(c/2 + (d*x)/2)*(8*a^{14} - 8*a^{13}*b - 8*a*b^{13} + 4*b^{14}
\end{aligned}$$

$$\begin{aligned}
& 18 - 7a^2b^{16} + 21a^4b^{14} - 35a^6b^{12} + 35a^8b^{10} - 21a^{10}b^8 + 7 \\
& *a^{12}b^6 - a^{14}b^4) - (a*((8*\tan(c/2 + (d*x)/2)*(8*a^{14} - 8*a^{13}b - 8*a \\
& *b^{13} + 4*b^{14} + 44*a^2*b^{12} + 48*a^3*b^{11} - 92*a^4*b^{10} - 120*a^5*b^9 + 15 \\
& 6*a^6*b^8 + 160*a^7*b^7 - 164*a^8*b^6 - 120*a^9*b^5 + 117*a^{10}b^4 + 48*a^{11} \\
& b^3 - 48*a^{12}b^2)))/(a*b^{16} + b^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + 10*a^4*b^{13} \\
& + 10*a^5*b^{12} - 10*a^6*b^{11} - 10*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - a^{10} \\
& b^7 - a^{11}b^6) - (a*((8*(16*a*b^{20} - 4*b^{21} + 12*a^2*b^{19} - 64*a^3*b^{18} - \\
& 20*a^4*b^{17} + 110*a^5*b^{16} + 30*a^6*b^{15} - 110*a^7*b^{14} - 30*a^8*b^{13} + 70* \\
& a^9*b^{12} + 14*a^{10}b^{11} - 26*a^{11}b^{10} - 2*a^{12}b^9 + 4*a^{13}b^8)))/(a*b^{19} \\
& + b^{20} - 5*a^2*b^{18} - 5*a^3*b^{17} + 10*a^4*b^{16} + 10*a^5*b^{15} - 10*a^6*b^{14} \\
& - 10*a^7*b^{13} + 5*a^8*b^{12} + 5*a^9*b^{11} - a^{10}b^{10} - a^{11}b^9) + (4*a*\tan(\\
& c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^{(1/2)}*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7* \\
& a^4*b^2)*(8*a*b^{21} - 8*a^2*b^{20} - 48*a^3*b^{19} + 48*a^4*b^{18} + 120*a^5*b^{17} \\
& - 120*a^6*b^{16} - 160*a^7*b^{15} + 160*a^8*b^{14} + 120*a^9*b^{13} - 120*a^{10}b^{12} \\
& - 48*a^{11}b^{11} + 48*a^{12}b^{10} + 8*a^{13}b^9 - 8*a^{14}b^8))/((b^{18} - 7*a^2*b \\
& ^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10}b^8 + 7*a^{12}b^6 - \\
& a^{14}b^4)*(a*b^{16} + b^{17} - 5*a^2*b^{15} - 5*a^3*b^{14} + 10*a^4*b^{13} + 10*a^5*b \\
& ^{12} - 10*a^6*b^{11} - 10*a^7*b^{10} + 5*a^8*b^9 + 5*a^9*b^8 - a^{10}b^7 - a^{11}b \\
& ^6)))*(-(a + b)^7*(a - b)^7)^{(1/2)}*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)) \\
& /((2*(b^{18} - 7*a^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10} \\
& b^8 + 7*a^{12}b^6 - a^{14}b^4)))*(-(a + b)^7*(a - b)^7)^{(1/2)}*(2*a^6 - 8*b^6 \\
& + 8*a^2*b^4 - 7*a^4*b^2))/((2*(b^{18} - 7*a^2*b^{16} + 21*a^4*b^{14} - 35*a^6*b^{12} \\
& + 35*a^8*b^{10} - 21*a^{10}b^8 + 7*a^{12}b^6 - a^{14}b^4)))))*(-(a + b)^7*(a - b \\
&)^7)^{(1/2)}*(2*a^6 - 8*b^6 + 8*a^2*b^4 - 7*a^4*b^2)*i)/(d*(b^{18} - 7*a^2*b^{16} \\
& + 21*a^4*b^{14} - 35*a^6*b^{12} + 35*a^8*b^{10} - 21*a^{10}b^8 + 7*a^{12}b^6 - a^{14} \\
& b^4))
\end{aligned}$$

3.480 $\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^4} dx$

Optimal result	5110
Rubi [A] (verified)	5110
Mathematica [A] (verified)	5113
Maple [A] (verified)	5113
Fricas [A] (verification not implemented)	5114
Sympy [F(-1)]	5115
Maxima [F(-2)]	5115
Giac [A] (verification not implemented)	5116
Mupad [B] (verification not implemented)	5116

Optimal result

Integrand size = 21, antiderivative size = 222

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^4} dx = -\frac{b(3a^2+2b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2 \cos(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b \cos(c+dx))^3} - \frac{a^2(2a^2-7b^2) \sin(c+dx)}{6b^2(a^2-b^2)^2 d(a+b \cos(c+dx))^2} + \frac{a(2a^4-5a^2b^2+18b^4) \sin(c+dx)}{6b^2(a^2-b^2)^3 d(a+b \cos(c+dx))}$$

```
[Out] -b*(3*a^2+2*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3-1/6*a^2*(2*a^2-7*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*a*(2*a^4-5*a^2*b^2+18*b^4)*sin(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {2871, 3100, 2833, 12, 2738, 211}

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^4} dx = -\frac{b(3a^2 + 2b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} - \frac{a^2(2a^2 - 7b^2) \sin(c + dx)}{6b^2d(a^2 - b^2)^2(a + b \cos(c + dx))^2} - \frac{a^2 \sin(c + dx) \cos(c + dx)}{3bd(a^2 - b^2)(a + b \cos(c + dx))^3} + \frac{a(2a^4 - 5a^2b^2 + 18b^4) \sin(c + dx)}{6b^2d(a^2 - b^2)^3(a + b \cos(c + dx))}$$

[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^4,x]

[Out] -((b*(3*a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (a^2*Cos[c + d*x]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - (a^2*(2*a^2 - 7*b^2)*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4)*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2833

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Ssin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rule 2871

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(- (b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
+ a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])

```

Rule 3100

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[(- (A*b^2
- a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2 \cos(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^3} - \frac{\int \frac{a^2 - 3ab \cos(c + dx) - (2a^2 - 3b^2) \cos^2(c + dx)}{(a + b \cos(c + dx))^3} dx}{3b(a^2 - b^2)} \\
&= -\frac{a^2 \cos(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^3} - \frac{a^2(2a^2 - 7b^2) \sin(c + dx)}{6b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{\int \frac{2ab(a^2 - 6b^2) + (2a^4 - 3a^2b^2 + 6b^4) \cos(c + dx)}{(a + b \cos(c + dx))^2} dx}{6b^2(a^2 - b^2)^2} \\
&= -\frac{a^2 \cos(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^3} - \frac{a^2(2a^2 - 7b^2) \sin(c + dx)}{6b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{a(2a^4 - 5a^2b^2 + 18b^4) \sin(c + dx)}{6b^2(a^2 - b^2)^3 d(a + b \cos(c + dx))} - \frac{\int \frac{3b^3(3a^2 + 2b^2)}{a + b \cos(c + dx)} dx}{6b^2(a^2 - b^2)^3} \\
&= -\frac{a^2 \cos(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^3} - \frac{a^2(2a^2 - 7b^2) \sin(c + dx)}{6b^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{a(2a^4 - 5a^2b^2 + 18b^4) \sin(c + dx)}{6b^2(a^2 - b^2)^3 d(a + b \cos(c + dx))} - \frac{(b(3a^2 + 2b^2)) \int \frac{1}{a + b \cos(c + dx)} dx}{2(a^2 - b^2)^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{a^2 \cos(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} - \frac{a^2(2a^2-7b^2)\sin(c+dx)}{6b^2(a^2-b^2)^2d(a+b\cos(c+dx))^2} \\
&\quad + \frac{a(2a^4-5a^2b^2+18b^4)\sin(c+dx)}{6b^2(a^2-b^2)^3d(a+b\cos(c+dx))} \\
&\quad - \frac{(b(3a^2+2b^2)) \operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)^3d} \\
&= -\frac{b(3a^2+2b^2) \arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2 \cos(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^3} \\
&\quad - \frac{a^2(2a^2-7b^2)\sin(c+dx)}{6b^2(a^2-b^2)^2d(a+b\cos(c+dx))^2} + \frac{a(2a^4-5a^2b^2+18b^4)\sin(c+dx)}{6b^2(a^2-b^2)^3d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\begin{aligned}
&\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4} dx \\
&= \frac{6b(3a^2+2b^2) \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + \frac{a(4a^4+11a^2b^2+3ab(a^2+9b^2)\cos(c+dx)+(2a^4-5a^2b^2+18b^4)\cos^2(c+dx))\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3} \\
&= \frac{\hspace{15em}}{6d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^3/(a + b*cos[c + d*x])^4, x]

[Out] ((-6*b*(3*a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (a*(4*a^4 + 11*a^2*b^2 + 3*a*b*(a^2 + 9*b^2)*Cos[c + d*x] + (2*a^4 - 5*a^2*b^2 + 18*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a + b*cos[c + d*x])^3)/(6*d)

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{2 \left(-\frac{(2a^2+3ab+6b^2)a \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{2(a-b)(a^3+3a^2b+3ab^2+b^3)} - \frac{2(a^2+9b^2)a \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3(a^2-2ab+b^2)(a^2+2ab+b^2)} - \frac{(2a^2-3ab+6b^2)a \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{2(a+b)(a^3-3a^2b+3ab^2-b^3)} \right)}{\left(\left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - b \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + a + b \right)^3} - \frac{b(3a^2+2b^2) \arctan \left(\frac{a-b \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{a+b} \right)}{(a^6-3a^4b^2+3a^2b^4-d)}$
default	$\frac{2 \left(-\frac{(2a^2+3ab+6b^2)a \left(\tan^5 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{2(a-b)(a^3+3a^2b+3ab^2+b^3)} - \frac{2(a^2+9b^2)a \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{3(a^2-2ab+b^2)(a^2+2ab+b^2)} - \frac{(2a^2-3ab+6b^2)a \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{2(a+b)(a^3-3a^2b+3ab^2-b^3)} \right)}{\left(\left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a - b \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + a + b \right)^3} - \frac{b(3a^2+2b^2) \arctan \left(\frac{a-b \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{a+b} \right)}{(a^6-3a^4b^2+3a^2b^4-d)}$
risch	$ia(6a^5e^{5i(dx+c)}b^2 - 18a^3b^4e^{5i(dx+c)} + 27a^6e^{5i(dx+c)} + 12a^6be^{4i(dx+c)} - 36a^4b^3e^{4i(dx+c)} + 81a^2b^5e^{4i(dx+c)} + 18b^7e^{4i(dx+c)})$

[In] `int(cos(d*x+c)^3/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{d} \left(-2 \left(-\frac{1}{2} (2a^2+3ab+6b^2) \frac{a}{(a-b)} \left(\frac{a^3+3a^2b+3ab^2+b^3}{a^2+2ab+b^2} \right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)^5 - \frac{2}{3} (a^2+9b^2) \frac{a}{(a^2-2ab+b^2)} \left(\frac{a^2+2ab+b^2}{a^2+2ab+b^2} \right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)^3 - \frac{1}{2} (2a^2-3ab+6b^2) \frac{a}{(a+b)} \left(\frac{a^3-3a^2b+3ab^2-b^3}{a^2+2ab+b^2} \right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right) / \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)^2 a - b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)^2 + a + b \right)^3 - b (3a^2+2b^2) / (a^6-3a^4b^2+3a^2b^4-b^6) / \left((a-b)(a+b) \right)^{1/2} \arctan \left(\frac{(a-b) \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{(a-b)(a+b)} \right) \right)$

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 893, normalized size of antiderivative = 4.02

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^4} dx$$

$$= \frac{\left[3(3a^5b+2a^3b^3+(3a^2b^4+2b^6) \cos(dx+c)^3 + 3(3a^3b^3+2ab^5) \cos(dx+c)^2 + 3(3a^4b^2+2a^2b^4) \cos(dx+c) \right)}{12((a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11})d \cos(dx+c)^3 + (a^9b^2-4a^7b^4+3a^5b^6-3a^3b^8+ab^{10})d \cos(dx+c)^2 + (a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11})d \cos(dx+c) + (a^9b^2-4a^7b^4+3a^5b^6-3a^3b^8+ab^{10})d)} + \frac{3(3a^5b+2a^3b^3+(3a^2b^4+2b^6) \cos(dx+c)^3 + 3(3a^3b^3+2ab^5) \cos(dx+c)^2 + 3(3a^4b^2+2a^2b^4) \cos(dx+c))}{6((a^8b^3-4a^6b^5+6a^4b^7-4a^2b^9+b^{11})d \cos(dx+c)^3 + (a^9b^2-4a^7b^4+3a^5b^6-3a^3b^8+ab^{10})d)}$$

[In] `integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="fricas")`

[Out] $\frac{1}{12} \left(3 \left(3a^5b + 2a^3b^3 + (3a^2b^4 + 2b^6) \cos(dx+c) \right)^3 + 3 \left(3a^3b^3 + 2ab^5 \right) \cos(dx+c)^2 + 3 \left(3a^4b^2 + 2a^2b^4 \right) \cos(dx+c) \right) \sqrt{-a^2+b^2} \log \left(\frac{2ab \cos(dx+c) + (2a^2-b^2) \cos(dx+c)^2 + 2 \sqrt{-a^2+b^2} (a \cos(dx+c) + b) \sin(dx+c) - a^2 + 2b^2}{(b^2 \cos(dx+c)^2 + 2ab \cos(dx+c) + a^2)} \right) + 2 \left(4a^7 + 7a^5b^2 - 11a^3b^4 + (2a^7 - 7a^5b^2 + 23a^3b^4 - 18a^2b^6) \cos(dx+c)^2 + 3(a^6b + 8a^4b^3 - 9a^2b^5) \cos(dx+c) \right) \sin(dx+c) \right) / \left((a^8b^3 - 4a^6b^5 + 6a^4b^7 - 4a^2b^9 + b^{11})d \cos(dx+c)^3 + (a^9b^2 - 4a^7b^4 + 3a^5b^6 - 3a^3b^8 + ab^{10})d \right)$

$$\begin{aligned}
& 6a^4b^7 - 4a^2b^9 + b^{11})d\cos(dx + c)^3 + 3(a^9b^2 - 4a^7b^4 + \\
& 6a^5b^6 - 4a^3b^8 + ab^{10})d\cos(dx + c)^2 + 3(a^{10}b - 4a^8b^3 + \\
& 6a^6b^5 - 4a^4b^7 + a^2b^9)d\cos(dx + c) + (a^{11} - 4a^9b^2 + 6a^7 \\
& b^4 - 4a^5b^6 + a^3b^8)d, -1/6(3(3a^5b + 2a^3b^3 + (3a^2b^4 + \\
& 2b^6)\cos(dx + c)^3 + 3(3a^3b^3 + 2ab^5)\cos(dx + c)^2 + 3(3a^4 \\
& b^2 + 2a^2b^4)\cos(dx + c))\sqrt{a^2 - b^2}\arctan(-(a\cos(dx + c) + b) \\
& /(\sqrt{a^2 - b^2}\sin(dx + c))) - (4a^7 + 7a^5b^2 - 11a^3b^4 + (2a^7 \\
& - 7a^5b^2 + 23a^3b^4 - 18ab^6)\cos(dx + c)^2 + 3(a^6b + 8a^4b^3 \\
& - 9a^2b^5)\cos(dx + c))\sin(dx + c))/((a^8b^3 - 4a^6b^5 + 6a^4b^7 \\
& - 4a^2b^9 + b^{11})d\cos(dx + c)^3 + 3(a^9b^2 - 4a^7b^4 + 6a^5b^6 \\
& - 4a^3b^8 + ab^{10})d\cos(dx + c)^2 + 3(a^{10}b - 4a^8b^3 + 6a^6b^5 \\
& - 4a^4b^7 + a^2b^9)d\cos(dx + c) + (a^{11} - 4a^9b^2 + 6a^7b^4 - 4a^ \\
& ^5b^6 + a^3b^8)d)]
\end{aligned}$$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Timed out}$$

[In] integrate(cos(dx+c)**3/(a+b*cos(dx+c))**4,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Exception raised: ValueError}$$

[In] integrate(cos(dx+c)^3/(a+b*cos(dx+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.80

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^4} dx$$

$$= \frac{3(3a^2b + 2b^3) \left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \arctan\left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}}\right) \right)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\sqrt{a^2 - b^2}} + \frac{6a^5 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 3a^4b \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 6a^3b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 27a^2b^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 18a^2b^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 4a^5 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 32a^3b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 36a^3b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 + 6a^5 \tan(\frac{1}{2} dx + \frac{1}{2} c) + 3a^4b \tan(\frac{1}{2} dx + \frac{1}{2} c) + 6a^3b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c) + 27a^2b^3 \tan(\frac{1}{2} dx + \frac{1}{2} c) + 18a^2b^4 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6) \left(a \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 - b \tan(\frac{1}{2} dx + \frac{1}{2} c)^2 + a + b \right)^3} / d$$

[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out] $\frac{1}{3} * (3 * (3 * a^2 * b + 2 * b^3) * (\pi * \text{floor}(1/2 * (d * x + c) / \pi + 1/2) * \text{sgn}(-2 * a + 2 * b) + \arctan(-(a * \tan(1/2 * d * x + 1/2 * c) - b * \tan(1/2 * d * x + 1/2 * c)) / \sqrt{a^2 - b^2}))) / ((a^6 - 3 * a^4 * b^2 + 3 * a^2 * b^4 - b^6) * \sqrt{a^2 - b^2}) + (6 * a^5 * \tan(1/2 * d * x + 1/2 * c)^5 - 3 * a^4 * b * \tan(1/2 * d * x + 1/2 * c)^5 + 6 * a^3 * b^2 * \tan(1/2 * d * x + 1/2 * c)^5 - 27 * a^2 * b^3 * \tan(1/2 * d * x + 1/2 * c)^5 + 18 * a^2 * b^3 * \tan(1/2 * d * x + 1/2 * c)^5 + 4 * a^5 * \tan(1/2 * d * x + 1/2 * c)^3 + 32 * a^3 * b^2 * \tan(1/2 * d * x + 1/2 * c)^3 - 36 * a^3 * b^2 * \tan(1/2 * d * x + 1/2 * c)^3 + 6 * a^5 * \tan(1/2 * d * x + 1/2 * c) + 3 * a^4 * b * \tan(1/2 * d * x + 1/2 * c) + 6 * a^3 * b^2 * \tan(1/2 * d * x + 1/2 * c) + 27 * a^2 * b^3 * \tan(1/2 * d * x + 1/2 * c) + 18 * a^2 * b^4 * \tan(1/2 * d * x + 1/2 * c)) / ((a^6 - 3 * a^4 * b^2 + 3 * a^2 * b^4 - b^6) * (a * \tan(1/2 * d * x + 1/2 * c)^2 - b * \tan(1/2 * d * x + 1/2 * c)^2 + a + b)^3) / d$

Mupad [B] (verification not implemented)

Time = 18.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.70

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^4} dx$$

$$= \frac{\frac{4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (a^3 + 9ab^2)}{3(a+b)^2(a^2 - 2ab + b^2)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (2a^3 + 3a^2b + 6ab^2)}{(a+b)^3(a-b)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{(a+b)(a^3 - b^3)}}{d \left(3ab^2 - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (-3a^3 + 3a^2b + 3ab^2 - 3b^3) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (-3a^3 - 3a^2b + 3ab^2 + 3b^3) + b \operatorname{atan}\left(\frac{b \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (3a^2 + 2b^2) (2a - 2b) (a^3 - 3a^2b + 3ab^2 - b^3)}{2(3a^2b + 2b^3) \sqrt{a+b} (a-b)^{7/2}}\right) (3a^2 + 2b^2) \right)}{d(a+b)^{7/2} (a-b)^{7/2}}$$

[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^4,x)

[Out] $((4 * \tan(c/2 + (d * x) / 2)^3 * (9 * a * b^2 + a^3)) / (3 * (a + b)^2 * (a^2 - 2 * a * b + b^2)) + (\tan(c/2 + (d * x) / 2)^5 * (6 * a * b^2 + 3 * a^2 * b + 2 * a^3)) / ((a + b)^3 * (a - b)) + (\tan(c/2 + (d * x) / 2) * (6 * a * b^2 - 3 * a^2 * b + 2 * a^3)) / ((a + b) * (3 * a * b^2 - 3 * a^2 * b + a^3 - b^3))) / (d * (3 * a * b^2 - \tan(c/2 + (d * x) / 2)^4 * (3 * a * b^2 + 3 * a^2 * b - 3$

$$\begin{aligned}
& *a^3 - 3*b^3) - \tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + \\
& 3*a^2*b + a^3 + b^3 + \tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) \\
&) - (b*\operatorname{atan}((b*\tan(c/2 + (d*x)/2)*(3*a^2 + 2*b^2)*(2*a - 2*b)*(3*a*b^2 - 3* \\
& a^2*b + a^3 - b^3)))/(2*(3*a^2*b + 2*b^3)*(a + b)^{(1/2)}*(a - b)^{(7/2)})))*(3*a \\
& ^2 + 2*b^2))/(d*(a + b)^{(7/2)}*(a - b)^{(7/2)})
\end{aligned}$$

$$3.481 \quad \int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^4} dx$$

Optimal result	5118
Rubi [A] (verified)	5118
Mathematica [A] (verified)	5121
Maple [A] (verified)	5121
Fricas [B] (verification not implemented)	5122
Sympy [F(-1)]	5122
Maxima [F(-2)]	5123
Giac [B] (verification not implemented)	5123
Mupad [B] (verification not implemented)	5124

Optimal result

Integrand size = 21, antiderivative size = 206

$$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^4} dx = \frac{a(a^2+4b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2 \sin(c+dx)}{3b(a^2-b^2)d(a+b \cos(c+dx))^3} + \frac{a(a^2-6b^2) \sin(c+dx)}{6b(a^2-b^2)^2 d(a+b \cos(c+dx))^2} + \frac{(a^4-10a^2b^2-6b^4) \sin(c+dx)}{6b(a^2-b^2)^3 d(a+b \cos(c+dx))}$$

[Out] a*(a^2+4*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*a*(a^2-6*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*(a^4-10*a^2*b^2-6*b^4)*sin(d*x+c)/b/(a^2-b^2)^3/d/(a+b*cos(d*x+c))

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

= {2869, 2833, 12, 2738, 211}

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^4} dx = \frac{a(a^2 + 4b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} - \frac{a^2 \sin(c + dx)}{3bd(a^2 - b^2)(a + b \cos(c + dx))^3} + \frac{a(a^2 - 6b^2) \sin(c + dx)}{6bd(a^2 - b^2)^2(a + b \cos(c + dx))^2} + \frac{(a^4 - 10a^2b^2 - 6b^4) \sin(c + dx)}{6bd(a^2 - b^2)^3(a + b \cos(c + dx))}$$

[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]

[Out] (a*(a^2 + 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]/((a - b)^(7/2)*(a + b)^(7/2)*d) - (a^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a*(a^2 - 6*b^2)*Sin[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Sin[c + d*x])/(6*b*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2833

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rule 2869

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^2, x_Symbol] :> Simp[(- (b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
+ c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2 \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{\int \frac{3ab + (a^2 - 3b^2) \cos(c + dx)}{(a + b \cos(c + dx))^3} dx}{3b(a^2 - b^2)} \\
&= -\frac{a^2 \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{a(a^2 - 6b^2) \sin(c + dx)}{6b(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad - \frac{\int \frac{-2b(2a^2 + 3b^2) - a(a^2 - 6b^2) \cos(c + dx)}{(a + b \cos(c + dx))^2} dx}{6b(a^2 - b^2)^2} \\
&= -\frac{a^2 \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{a(a^2 - 6b^2) \sin(c + dx)}{6b(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{(a^4 - 10a^2b^2 - 6b^4) \sin(c + dx)}{6b(a^2 - b^2)^3 d(a + b \cos(c + dx))} + \frac{\int \frac{3ab(a^2 + 4b^2)}{a + b \cos(c + dx)} dx}{6b(a^2 - b^2)^3} \\
&= -\frac{a^2 \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{a(a^2 - 6b^2) \sin(c + dx)}{6b(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{(a^4 - 10a^2b^2 - 6b^4) \sin(c + dx)}{6b(a^2 - b^2)^3 d(a + b \cos(c + dx))} + \frac{(a^2 + 4b^2) \int \frac{1}{a + b \cos(c + dx)} dx}{2(a^2 - b^2)^3} \\
&= -\frac{a^2 \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{a(a^2 - 6b^2) \sin(c + dx)}{6b(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{(a^4 - 10a^2b^2 - 6b^4) \sin(c + dx)}{6b(a^2 - b^2)^3 d(a + b \cos(c + dx))} \\
&\quad + \frac{(a^2 + 4b^2) \text{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{(a^2 - b^2)^3 d} \\
&= \frac{a(a^2 + 4b^2) \arctan\left(\frac{\sqrt{a - b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b}}\right)}{(a - b)^{7/2}(a + b)^{7/2}d} - \frac{a^2 \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^3} \\
&\quad + \frac{a(a^2 - 6b^2) \sin(c + dx)}{6b(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} + \frac{(a^4 - 10a^2b^2 - 6b^4) \sin(c + dx)}{6b(a^2 - b^2)^3 d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4} dx$$

$$= \frac{6a(a^2+4b^2)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + \frac{(-13a^4b-2a^2b^3+3a(a^4-9a^2b^2-2b^4))\cos(c+dx)+b(a^4-10a^2b^2-6b^4)\cos^2(c+dx)\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}$$

$6d$

`[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]`

```
[Out] (((6*a*(a^2 + 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + ((-13*a^4*b - 2*a^2*b^3 + 3*a*(a^4 - 9*a^2*b^2 - 2*b^4)*Cos[c + d*x] + b*(a^4 - 10*a^2*b^2 - 6*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{(a^3+6a^2b+2ab^2+2b^3)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^3+3a^2b+3ab^2+b^3)} - \frac{4(7a^2+3b^2)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2-2ab+b^2)(a^2+2ab+b^2)} + \frac{(a^3-6a^2b+2ab^2-2b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^3-3a^2b+3ab^2-b^3)} a(a^2+4b^2)\operatorname{arctan}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^3} + \frac{d}{(a^6-3a^4b^2+3a^2b^4-b^6)}$
default	$\frac{\frac{(a^3+6a^2b+2ab^2+2b^3)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)(a^3+3a^2b+3ab^2+b^3)} - \frac{4(7a^2+3b^2)b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3(a^2-2ab+b^2)(a^2+2ab+b^2)} + \frac{(a^3-6a^2b+2ab^2-2b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)(a^3-3a^2b+3ab^2-b^3)} a(a^2+4b^2)\operatorname{arctan}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b\right)^3} + \frac{d}{(a^6-3a^4b^2+3a^2b^4-b^6)}$
risch	$\frac{i(-3a^3b^4e^{5i(dx+c)}-12ab^6e^{5i(dx+c)}+6a^6be^{4i(dx+c)}-33a^4b^3e^{4i(dx+c)}-42a^2b^5e^{4i(dx+c)}-6b^7e^{4i(dx+c)}+4a^7e^{3i(dx+c)})}{d}$

`[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)`

```
[Out] 1/d*(2*(-1/2*(a^3+6*a^2*b+2*a*b^2+2*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(7*a^2+3*b^2)*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(a^3-6*a^2*b+2*a*b^2-2*b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^3+a*(a^2+4*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(191) = 382.

Time = 0.31 (sec) , antiderivative size = 893, normalized size of antiderivative = 4.33

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^4} dx$$

$$= \frac{3(a^6 + 4a^4b^2 + (a^3b^3 + 4ab^5) \cos(dx + c)^3 + 3(a^4b^2 + 4a^2b^4) \cos(dx + c)^2 + 3(a^5b + 4a^3b^3) \cos(dx + c) + 3a^6b^0 \cos^2(dx + c))}{12((a^8b^3 - 4a^6b^5 + 6a^4b^7 - 4a^2b^9 + b^{11})d \cos(dx + c) + \dots)}$$

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fricas")

[Out] [1/12*(3*(a^6 + 4*a^4*b^2 + (a^3*b^3 + 4*a*b^5)*cos(d*x + c)^3 + 3*(a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^2 + 3*(a^5*b + 4*a^3*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5 - (a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6*b^7)*cos(d*x + c)^2 - 3*(a^7 - 10*a^5*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), 1/6*(3*(a^6 + 4*a^4*b^2 + (a^3*b^3 + 4*a*b^5)*cos(d*x + c)^3 + 3*(a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^2 + 3*(a^5*b + 4*a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5 - (a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6*b^7)*cos(d*x + c)^2 - 3*(a^7 - 10*a^5*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**4,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(191) = 382.

Time = 0.33 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.07

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^4} dx = \frac{3(a^3 + 4ab^2) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\sqrt{a^2 - b^2}} + \frac{3a^5 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 12a^4b \tan(\frac{1}{2} dx + \frac{1}{2} c)^5}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\sqrt{a^2 - b^2}}$$

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 4*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2))
)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (3*a^5*tan(1/2*d*
x + 1/2*c)^5 + 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1
/2*c)^5 + 12*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^
5 + 6*b^5*tan(1/2*d*x + 1/2*c)^5 + 28*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 16*a^2
*b^3*tan(1/2*d*x + 1/2*c)^3 - 12*b^5*tan(1/2*d*x + 1/2*c)^3 - 3*a^5*tan(1/2
*d*x + 1/2*c) + 12*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 12*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*a*b^4*tan(1/2*d*x + 1/2*c) + 6*b
^5*tan(1/2*d*x + 1/2*c))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*
x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3)/d
```

Mupad [B] (verification not implemented)

Time = 18.37 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.85

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^4} dx = \frac{a \operatorname{atan}\left(\frac{a \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (a^2 + 4b^2) (2a - 2b) (a^3 - 3a^2b + 3ab^2 - b^3)}{2\sqrt{a+b}(a-b)^{7/2}(a^3 + 4ab^2)}\right) (a^2 + 4b^2)}{d(a+b)^{7/2}(a-b)^{7/2}} - \frac{\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (a^3 + 6a^2b + 2ab^2 + 2b^3)}{(a+b)^3(a-b)} + \frac{4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (7a^2b + 3b^3)}{3(a+b)^2(a^2 - 2ab + b^2)} - \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{(a+b)}}{d \left(3ab^2 - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (-3a^3 + 3a^2b + 3ab^2 - 3b^3) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (-3a^3 - 3a^2b + 3ab^2 + 3b^3)\right)}$$

[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^4,x)

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7/2)*(4*a*b^2 + a^3)))*(a^2 + 4*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((tan(c/2 + (d*x)/2)^5*(2*a*b^2 + 6*a^2*b + a^3 + 2*b^3))/((a + b)^3*(a - b)) + (4*tan(c/2 + (d*x)/2)^3*(7*a^2*b + 3*b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) - (tan(c/2 + (d*x)/2)*(2*a*b^2 - 6*a^2*b + a^3 - 2*b^3))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
```


$$3.482 \quad \int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^4} dx$$

Optimal result	5125
Rubi [A] (verified)	5125
Mathematica [A] (verified)	5127
Maple [A] (verified)	5128
Fricas [B] (verification not implemented)	5128
Sympy [F(-1)]	5129
Maxima [F(-2)]	5129
Giac [B] (verification not implemented)	5130
Mupad [B] (verification not implemented)	5130

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^4} dx = -\frac{b(4a^2+b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a \sin(c+dx)}{3(a^2-b^2)d(a+b \cos(c+dx))^3} + \frac{(2a^2+3b^2) \sin(c+dx)}{6(a^2-b^2)^2d(a+b \cos(c+dx))^2} + \frac{a(2a^2+13b^2) \sin(c+dx)}{6(a^2-b^2)^3d(a+b \cos(c+dx))}$$

```
[Out] -b*(4*a^2+b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d+1/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*(2*a^2+3*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*a*(2*a^2+13*b^2)*sin(d*x+c)/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used

= {2833, 12, 2738, 211}

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4} dx = -\frac{b(4a^2+b^2)\arctan\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} + \frac{a(2a^2+13b^2)\sin(c+dx)}{6d(a^2-b^2)^3(a+b\cos(c+dx))} + \frac{(2a^2+3b^2)\sin(c+dx)}{6d(a^2-b^2)^2(a+b\cos(c+dx))^2} + \frac{a\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^3}$$

[In] Int[Cos[c + d*x]/(a + b*cos[c + d*x])^4,x]

[Out] -((b*(4*a^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) + (a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*cos[c + d*x])^3) + ((2*a^2 + 3*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*cos[c + d*x])^2) + (a*(2*a^2 + 13*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2833

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} - \frac{\int \frac{3b - 2a \cos(c + dx)}{(a + b \cos(c + dx))^3} dx}{3(a^2 - b^2)} \\
&= \frac{a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} \\
&\quad + \frac{(2a^2 + 3b^2) \sin(c + dx)}{6(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} + \frac{\int \frac{-10ab + (2a^2 + 3b^2) \cos(c + dx)}{(a + b \cos(c + dx))^2} dx}{6(a^2 - b^2)^2} \\
&= \frac{a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} + \frac{(2a^2 + 3b^2) \sin(c + dx)}{6(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{a(2a^2 + 13b^2) \sin(c + dx)}{6(a^2 - b^2)^3 d(a + b \cos(c + dx))} - \frac{\int \frac{3b(4a^2 + b^2)}{a + b \cos(c + dx)} dx}{6(a^2 - b^2)^3} \\
&= \frac{a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} + \frac{(2a^2 + 3b^2) \sin(c + dx)}{6(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{a(2a^2 + 13b^2) \sin(c + dx)}{6(a^2 - b^2)^3 d(a + b \cos(c + dx))} - \frac{(b(4a^2 + b^2)) \int \frac{1}{a + b \cos(c + dx)} dx}{2(a^2 - b^2)^3} \\
&= \frac{a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} + \frac{(2a^2 + 3b^2) \sin(c + dx)}{6(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{a(2a^2 + 13b^2) \sin(c + dx)}{6(a^2 - b^2)^3 d(a + b \cos(c + dx))} \\
&\quad - \frac{(b(4a^2 + b^2)) \text{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{(a^2 - b^2)^3 d} \\
&= -\frac{b(4a^2 + b^2) \arctan\left(\frac{\sqrt{a - b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b}}\right)}{(a - b)^{7/2} (a + b)^{7/2} d} + \frac{a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} \\
&\quad + \frac{(2a^2 + 3b^2) \sin(c + dx)}{6(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} + \frac{a(2a^2 + 13b^2) \sin(c + dx)}{6(a^2 - b^2)^3 d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^4} dx \\
&= \frac{6b(4a^2 + b^2) \operatorname{arctanh}\left(\frac{(a - b) \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{-a^2 + b^2}}\right)}{(-a^2 + b^2)^{7/2}} + \frac{(6a^5 + 10a^3b^2 - ab^4 - 3b(-2a^4 - 9a^2b^2 + b^4) \cos(c + dx) + ab^2(2a^2 + 13b^2) \cos^2(c + dx)) \sin(c + dx)}{(a - b)^3 (a + b)^3 (a + b \cos(c + dx))^3} \\
&= \frac{\hspace{10em}}{6d}
\end{aligned}$$


```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
[Out] [1/12*(3*(4*a^5*b + a^3*b^3 + (4*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(4*a^3*b^3 + a*b^5)*cos(d*x + c)^2 + 3*(4*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6 + (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), -1/6*(3*(4*a^5*b + a^3*b^3 + (4*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(4*a^3*b^3 + a*b^5)*cos(d*x + c)^2 + 3*(4*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6 + (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```


$$\begin{aligned} & *a*b^2 - 3*a^2*b + a^3 - b^3)) / (d*(3*a*b^2 - \tan(c/2 + (d*x)/2)^4*(3*a*b^2 \\ & + 3*a^2*b - 3*a^3 - 3*b^3) - \tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a \\ & ^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + \tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b \\ & + a^3 - b^3))) - (b*\operatorname{atan}(b*\tan(c/2 + (d*x)/2)*(4*a^2 + b^2)*(2*a - 2*b)*(\\ & 3*a*b^2 - 3*a^2*b + a^3 - b^3)) / (2*(a + b)^{(1/2)}*(a - b)^{(7/2)}*(4*a^2*b + b \\ & ^3)))*(4*a^2 + b^2)) / (d*(a + b)^{(7/2)}*(a - b)^{(7/2)}) \end{aligned}$$

3.483 $\int \frac{1}{(a+b \cos(c+dx))^4} dx$

Optimal result	5132
Rubi [A] (verified)	5132
Mathematica [A] (verified)	5135
Maple [A] (verified)	5135
Fricas [B] (verification not implemented)	5136
Sympy [F(-1)]	5136
Maxima [F(-2)]	5137
Giac [B] (verification not implemented)	5137
Mupad [B] (verification not implemented)	5138

Optimal result

Integrand size = 12, antiderivative size = 184

$$\int \frac{1}{(a+b \cos(c+dx))^4} dx = \frac{a(2a^2+3b^2) \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b \sin(c+dx)}{3(a^2-b^2)d(a+b \cos(c+dx))^3} - \frac{5ab \sin(c+dx)}{6(a^2-b^2)^2 d(a+b \cos(c+dx))^2} - \frac{b(11a^2+4b^2) \sin(c+dx)}{6(a^2-b^2)^3 d(a+b \cos(c+dx))}$$

[Out] a*(2*a^2+3*b^2)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^3-5/6*a*b*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*sin(d*x+c)/(a^2-b^2)^3/d/(a+b*cos(d*x+c))

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used

= {2743, 2833, 12, 2738, 211}

$$\int \frac{1}{(a + b \cos(c + dx))^4} dx = \frac{a(2a^2 + 3b^2) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}} - \frac{b(11a^2 + 4b^2) \sin(c + dx)}{6d(a^2 - b^2)^3 (a + b \cos(c + dx))} - \frac{5ab \sin(c + dx)}{6d(a^2 - b^2)^2 (a + b \cos(c + dx))^2} - \frac{b \sin(c + dx)}{3d(a^2 - b^2) (a + b \cos(c + dx))^3}$$

[In] Int[(a + b*Cos[c + d*x])^(-4),x]

[Out] (a*(2*a^2 + 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) - (5*a*b*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) - (b*(11*a^2 + 4*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 211

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2833

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} - \frac{\int \frac{-3a + 2b \cos(c + dx)}{(a + b \cos(c + dx))^3} dx}{3(a^2 - b^2)} \\
&= -\frac{b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} \\
&\quad - \frac{5ab \sin(c + dx)}{6(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} + \frac{\int \frac{2(3a^2 + 2b^2) - 5ab \cos(c + dx)}{(a + b \cos(c + dx))^2} dx}{6(a^2 - b^2)^2} \\
&= -\frac{b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} - \frac{5ab \sin(c + dx)}{6(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad - \frac{b(11a^2 + 4b^2) \sin(c + dx)}{6(a^2 - b^2)^3 d(a + b \cos(c + dx))} - \frac{\int -\frac{3a(2a^2 + 3b^2)}{a + b \cos(c + dx)} dx}{6(a^2 - b^2)^3} \\
&= -\frac{b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} - \frac{5ab \sin(c + dx)}{6(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad - \frac{b(11a^2 + 4b^2) \sin(c + dx)}{6(a^2 - b^2)^3 d(a + b \cos(c + dx))} + \frac{(a(2a^2 + 3b^2)) \int \frac{1}{a + b \cos(c + dx)} dx}{2(a^2 - b^2)^3} \\
&= -\frac{b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} - \frac{5ab \sin(c + dx)}{6(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad - \frac{b(11a^2 + 4b^2) \sin(c + dx)}{6(a^2 - b^2)^3 d(a + b \cos(c + dx))} \\
&\quad + \frac{(a(2a^2 + 3b^2)) \text{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{(a^2 - b^2)^3 d} \\
&= \frac{a(2a^2 + 3b^2) \arctan\left(\frac{\sqrt{a - b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b}}\right)}{(a - b)^{7/2}(a + b)^{7/2} d} - \frac{b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^3} \\
&\quad - \frac{5ab \sin(c + dx)}{6(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} - \frac{b(11a^2 + 4b^2) \sin(c + dx)}{6(a^2 - b^2)^3 d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a + b \cos(c + dx))^4} dx$$

$$= \frac{6a(2a^2+3b^2) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right) - \frac{b(18a^4-5a^2b^2+2b^4+3ab(9a^2+b^2) \cos(c+dx)+b^2(11a^2+4b^2) \cos^2(c+dx)) \sin(c+dx)}{(a-b)^3(a+b)^3(a+b \cos(c+dx))^3}}{6d}$$

`[In] Integrate[(a + b*Cos[c + d*x])^(-4), x]`

```
[Out] ((6*a*(2*a^2 + 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
/(-a^2 + b^2)^(7/2) - (b*(18*a^4 - 5*a^2*b^2 + 2*b^4 + 3*a*b*(9*a^2 + b^2)*
Cos[c + d*x] + b^2*(11*a^2 + 4*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^
3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.52

method	result
derivativedivides	$\frac{\frac{(6a^2+3ab+2b^2)b \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 4(9a^2+b^2)b \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - (6a^2-3ab+2b^2)b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a-b)(a^3+3a^2b+3ab^2+b^3)} - \frac{4(9a^2+b^2)b \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - (6a^2-3ab+2b^2)b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a+b)(a^3-3a^2b+3ab^2-b^3)} + \frac{a(2a^2+3b^2) \operatorname{arctan}\left(\frac{(a-b)}{\sqrt{-a^2+b^2}}\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)}}{\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+a+b\right)^3} + \frac{d}{(a^6-3a^4b^2+3a^2b^4-b^6)}}$
default	$\frac{\frac{(6a^2+3ab+2b^2)b \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 4(9a^2+b^2)b \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - (6a^2-3ab+2b^2)b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a-b)(a^3+3a^2b+3ab^2+b^3)} - \frac{4(9a^2+b^2)b \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - (6a^2-3ab+2b^2)b \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{(a+b)(a^3-3a^2b+3ab^2-b^3)} + \frac{a(2a^2+3b^2) \operatorname{arctan}\left(\frac{(a-b)}{\sqrt{-a^2+b^2}}\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)}}{\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a-b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)+a+b\right)^3} + \frac{d}{(a^6-3a^4b^2+3a^2b^4-b^6)}}$
risch	$-\frac{i(6a^3b^2e^{5i(dx+c)}+9ab^4e^{5i(dx+c)}+30a^4be^{4i(dx+c)}+45a^2b^3e^{4i(dx+c)}+44a^5e^{3i(dx+c)}+82a^3b^2e^{3i(dx+c)}+24ab^4e^{3i(dx+c)}+3(a^2-b^2)^3d(b e^{2i(dx+c)}+2a e^{i(dx+c)}))}{3(a^2-b^2)^3d(b e^{2i(dx+c)}+2a e^{i(dx+c)})}$

`[In] int(1/(a+cos(d*x+c)*b)^4, x, method=_RETURNVERBOSE)`

```
[Out] 1/d*(2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5-2/3*(9*a^2+b^2)*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^3+a*(2*a^2+
3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1
/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(169) = 338.

Time = 0.31 (sec) , antiderivative size = 895, normalized size of antiderivative = 4.86

$$\int \frac{1}{(a + b \cos(c + dx))^4} dx$$

$$= \frac{3(2a^6 + 3a^4b^2 + (2a^3b^3 + 3ab^5) \cos(dx + c)^3 + 3(2a^4b^2 + 3a^2b^4) \cos(dx + c)^2 + 3(2a^5b + 3a^3b^3) \cos(dx + c)) \sqrt{-a^2 + b^2} \log((2a*b*\cos(dx + c) + (2a^2 - b^2)*\cos(dx + c)^2 - 2*\sqrt{-a^2 + b^2}*(a*\cos(dx + c) + b)*\sin(dx + c) - a^2 + 2*b^2)/(b^2*\cos(dx + c)^2 + 2*a*b*\cos(dx + c) + a^2)) - 2*(18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*\cos(dx + c)^2 + 3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6)*\cos(dx + c))*\sin(dx + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*\cos(dx + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*\cos(dx + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*\cos(dx + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), 1/6*(3*(2*a^6 + 3*a^4*b^2 + (2*a^3*b^3 + 3*a*b^5)*\cos(dx + c)^3 + 3*(2*a^4*b^2 + 3*a^2*b^4)*\cos(dx + c)^2 + 3*(2*a^5*b + 3*a^3*b^3)*\cos(dx + c))*\sqrt{a^2 - b^2}*\arctan(-(a*\cos(dx + c) + b)/(\sqrt{a^2 - b^2}*\sin(dx + c))) - (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*\cos(dx + c)^2 + 3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6)*\cos(dx + c))*\sin(dx + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*\cos(dx + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*\cos(dx + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*\cos(dx + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]$$

[In] integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="fricas")

[Out] [1/12*(3*(2*a^6 + 3*a^4*b^2 + (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^2 + 3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), 1/6*(3*(2*a^6 + 3*a^4*b^2 + (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^2 + 3*(9*a^5*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d)]

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^4} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))**4,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a + b \cos(c + dx))^4} dx = \text{Exception raised: ValueError}$$

[In] integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(169) = 338.

Time = 0.31 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.17

$$\int \frac{1}{(a + b \cos(c + dx))^4} dx = \frac{3(2a^3 + 3ab^2) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\sqrt{a^2 - b^2}} + \frac{18a^4b \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 27a^3b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)^4 + \dots}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)\sqrt{a^2 - b^2}}$$

[In] integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out]
$$\frac{-1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + \arctan(-(a*\tan(1/2*d*x + 1/2*c) - b*\tan(1/2*d*x + 1/2*c))/\sqrt{a^2 - b^2})))}{(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*\sqrt{a^2 - b^2}} + (18*a^4*b*\tan(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*\tan(1/2*d*x + 1/2*c)^4 + 6*a^2*b^3*\tan(1/2*d*x + 1/2*c)^3 - 3*a*b^4*\tan(1/2*d*x + 1/2*c)^2 + 6*b^5*\tan(1/2*d*x + 1/2*c) + 36*a^4*b*\tan(1/2*d*x + 1/2*c)^3 - 32*a^2*b^3*\tan(1/2*d*x + 1/2*c)^2 - 4*b^5*\tan(1/2*d*x + 1/2*c) + 18*a^4*b*\tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*\tan(1/2*d*x + 1/2*c) + 6*a^2*b^3*\tan(1/2*d*x + 1/2*c) + 3*a*b^4*\tan(1/2*d*x + 1/2*c) + 6*b^5*\tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 + a + b)^3)/d$$

Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.05

$$\int \frac{1}{(a + b \cos(c + dx))^4} dx$$

$$= \frac{a \operatorname{atan}\left(\frac{a \tan\left(\frac{c}{2} + \frac{dx}{2}\right) (2a^2 + 3b^2) (2a - 2b) (a^3 - 3a^2b + 3ab^2 - b^3)}{2(2a^3 + 3ab^2) \sqrt{a+b} (a-b)^{7/2}}\right) (2a^2 + 3b^2)}{d(a+b)^{7/2} (a-b)^{7/2}}$$

$$- \frac{\frac{4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 (9a^2b + b^3)}{3(a+b)^2 (a^2 - 2ab + b^2)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 (6a^2b + 3ab^2 + 2b^3)}{(a+b)^3 (a-b)} + \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{(a+b)}}{d \left(3ab^2 - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 (-3a^3 + 3a^2b + 3ab^2 - 3b^3) - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 (-3a^3 - 3a^2b + 3ab^2 + 3b^3)\right)}$$

`[In] int(1/(a + b*cos(c + d*x))^4,x)`

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(3*a*b^2 + 2*a^3)*(a + b)^(1/2)*(a - b)^(7/2))))*(2*a^2 + 3*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((4*tan(c/2 + (d*x)/2)^3*(9*a^2*b + b^3))/(3*(a + b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)^5*(3*a*b^2 + 6*a^2*b + 2*b^3))/((a + b)^3*(a - b)) + (tan(c/2 + (d*x)/2)*(6*a^2*b - 3*a*b^2 + 2*b^3))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(3*a*b^2 - tan(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))
```

$$3.484 \quad \int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^4} dx$$

Optimal result	5139
Rubi [A] (verified)	5139
Mathematica [A] (verified)	5142
Maple [A] (verified)	5143
Fricas [B] (verification not implemented)	5143
Sympy [F]	5145
Maxima [F(-2)]	5145
Giac [B] (verification not implemented)	5145
Mupad [B] (verification not implemented)	5146

Optimal result

Integrand size = 19, antiderivative size = 251

$$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^4} dx = -\frac{b(8a^6 - 8a^4b^2 + 7a^2b^4 - 2b^6) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d}$$

$$+ \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{b^2 \sin(c+dx)}{3a(a^2-b^2)d(a+b \cos(c+dx))^3}$$

$$+ \frac{b^2(8a^2-3b^2) \sin(c+dx)}{6a^2(a^2-b^2)^2 d(a+b \cos(c+dx))^2}$$

$$+ \frac{b^2(26a^4-17a^2b^2+6b^4) \sin(c+dx)}{6a^3(a^2-b^2)^3 d(a+b \cos(c+dx))}$$

```
[Out] -b*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/
(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2)/d+arctanh(sin(d*x+c))/a^4/d+1/3*b^
2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*b^2*(8*a^2-3*b^2)*sin(d*x
+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/6*b^2*(26*a^4-17*a^2*b^2+6*b^4)*
sin(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

= {2881, 3134, 3080, 3855, 2738, 211}

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4} dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4 d} + \frac{b^2(8a^2-3b^2)\sin(c+dx)}{6a^2 d(a^2-b^2)^2(a+b\cos(c+dx))^2} + \frac{b^2\sin(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^3} - \frac{b(8a^6-8a^4b^2+7a^2b^4-2b^6)\operatorname{arctan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4 d(a-b)^{7/2}(a+b)^{7/2}} + \frac{b^2(26a^4-17a^2b^2+6b^4)\sin(c+dx)}{6a^3 d(a^2-b^2)^3(a+b\cos(c+dx))}$$

[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^4,x]

[Out] -((b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d)) + ArcTanh[Sin[c + d*x]]/(a^4*d) + (b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (b^2*(8*a^2 - 3*b^2)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + (b^2*(26*a^4 - 17*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{b^2 \sin(c + dx)}{3a(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{\int \frac{(3(a^2 - b^2) - 3ab \cos(c + dx) + 2b^2 \cos^2(c + dx)) \sec(c + dx)}{(a + b \cos(c + dx))^3} dx}{3a(a^2 - b^2)} \\ &= \frac{b^2 \sin(c + dx)}{3a(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{b^2(8a^2 - 3b^2) \sin(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\ &\quad + \frac{\int \frac{(6(a^2 - b^2)^2 - 2ab(6a^2 - b^2) \cos(c + dx) + b^2(8a^2 - 3b^2) \cos^2(c + dx)) \sec(c + dx)}{(a + b \cos(c + dx))^2} dx}{6a^2(a^2 - b^2)^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{b^2 \sin(c + dx)}{3a(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{b^2(8a^2 - 3b^2) \sin(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{b^2(26a^4 - 17a^2b^2 + 6b^4) \sin(c + dx)}{6a^3(a^2 - b^2)^3 d(a + b \cos(c + dx))} \\
&\quad + \frac{\int \frac{(6(a^2 - b^2)^3 - 3ab(6a^4 - 2a^2b^2 + b^4) \cos(c + dx)) \sec(c + dx)}{a + b \cos(c + dx)} dx}{6a^3(a^2 - b^2)^3} \\
&= \frac{b^2 \sin(c + dx)}{3a(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{b^2(8a^2 - 3b^2) \sin(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\
&\quad + \frac{b^2(26a^4 - 17a^2b^2 + 6b^4) \sin(c + dx)}{6a^3(a^2 - b^2)^3 d(a + b \cos(c + dx))} + \frac{\int \sec(c + dx) dx}{a^4} \\
&\quad - \frac{(b(8a^6 - 8a^4b^2 + 7a^2b^4 - 2b^6)) \int \frac{1}{a + b \cos(c + dx)} dx}{2a^4(a^2 - b^2)^3} \\
&= \frac{\operatorname{arctanh}(\sin(c + dx))}{a^4 d} + \frac{b^2 \sin(c + dx)}{3a(a^2 - b^2)d(a + b \cos(c + dx))^3} \\
&\quad + \frac{b^2(8a^2 - 3b^2) \sin(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} + \frac{b^2(26a^4 - 17a^2b^2 + 6b^4) \sin(c + dx)}{6a^3(a^2 - b^2)^3 d(a + b \cos(c + dx))} \\
&\quad - \frac{(b(8a^6 - 8a^4b^2 + 7a^2b^4 - 2b^6)) \operatorname{Subst}\left(\int \frac{1}{a + b + (a - b)x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{a^4(a^2 - b^2)^3 d} \\
&= -\frac{b(8a^6 - 8a^4b^2 + 7a^2b^4 - 2b^6) \arctan\left(\frac{\sqrt{a - b} \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a + b}}\right)}{a^4(a - b)^{7/2}(a + b)^{7/2}d} \\
&\quad + \frac{\operatorname{arctanh}(\sin(c + dx))}{a^4 d} + \frac{b^2 \sin(c + dx)}{3a(a^2 - b^2)d(a + b \cos(c + dx))^3} \\
&\quad + \frac{b^2(8a^2 - 3b^2) \sin(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} + \frac{b^2(26a^4 - 17a^2b^2 + 6b^4) \sin(c + dx)}{6a^3(a^2 - b^2)^3 d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.09

$$\begin{aligned}
&\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^4} dx \\
&= \frac{6b(-8a^6 + 8a^4b^2 - 7a^2b^4 + 2b^6) \operatorname{arctanh}\left(\frac{(a - b) \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{-a^2 + b^2}}\right)}{(-a^2 + b^2)^{7/2}} - 6 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right) + 6 \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)
\end{aligned}$$

[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^4,x]

```
[Out] ((6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 6*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a^3*b^2*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + (a^2*b^2*(8*a^2 - 3*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + (a*b^2*(2*6*a^4 - 17*a^2*b^2 + 6*b^4)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Cos[c + d*x]))/(6*a^4*d)
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{a^4} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^4}}{2b \left(\frac{-\frac{(12a^4 + 4a^3b - 6a^2b^2 - ab^3 + 2b^4)ab \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2(a-b)(a^3 + 3a^2b + 3ab^2 + b^3)} - \frac{2(18a^4 - 11a^2b^2 + 3b^4)ab}{3(a^2 - 2ab + b^2)(a^2 + b^2)} \right)}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{a-b} \left(\tan^2\left(\frac{dx}{2}\right)\right)^d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{a^4} - \frac{\ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{a^4}}{2b \left(\frac{-\frac{(12a^4 + 4a^3b - 6a^2b^2 - ab^3 + 2b^4)ab \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2(a-b)(a^3 + 3a^2b + 3ab^2 + b^3)} - \frac{2(18a^4 - 11a^2b^2 + 3b^4)ab}{3(a^2 - 2ab + b^2)(a^2 + b^2)} \right)}{\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{a-b} \left(\tan^2\left(\frac{dx}{2}\right)\right)^d}$
risch	Expression too large to display

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^4*ln(tan(1/2*d*x+1/2*c)+1)-1/a^4*ln(tan(1/2*d*x+1/2*c)-1)-2*b/a^4*((-1/2*(12*a^4+4*a^3*b-6*a^2*b^2-a*b^3+2*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(18*a^4-11*a^2*b^2+3*b^4)*a*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(12*a^4-4*a^3*b-6*a^2*b^2+a*b^3+2*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^3+1/2*(8*a^6-8*a^4*b^2+7*a^2*b^4-2*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 873 vs. 2(236) = 472.

Time = 1.73 (sec) , antiderivative size = 1815, normalized size of antiderivative = 7.23

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(8*a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7 + (8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10)*cos(d*x + c)^3 + 3*(8*a^7*b^3 - 8*a^5*b^5 + 7*a^3*b^7 - 2*a*b^9)*cos(d*x + c)^2 + 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 6*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + 6*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 - 11*a^3*b^8 + (26*a^7*b^4 - 43*a^5*b^6 + 23*a^3*b^8 - 6*a*b^10)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d*cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d), -1/6*(3*(8*a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7 + (8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10)*cos(d*x + c)^3 + 3*(8*a^7*b^3 - 8*a^5*b^5 + 7*a^3*b^7 - 2*a*b^9)*cos(d*x + c)^2 + 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 - 11*a^3*b^8 + (26*a^7*b^4 - 43*a^5*b^6 + 23*a^3*b^8 - 6*a*b^10)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d*cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d)]
```

Sympy [F]

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^4} dx = \int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^4} dx$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**4,x)

[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**4, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Exception raised: ValueError}$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(236) = 472.

Time = 0.35 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.21

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^4} dx$$

$$= \frac{3(8a^6b - 8a^4b^3 + 7a^2b^5 - 2b^7) \left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(-2a+2b) + \arctan\left(\frac{-a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^{10} - 3a^8b^2 + 3a^6b^4 - a^4b^6)\sqrt{a^2 - b^2}} + \frac{36a^6b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 60a^5b^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 6a^4b^4 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 45a^3b^5 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 6a^2b^6 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 - 15ab^7 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 6b^8 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5 + 72a^6b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)^3 - 116a^4b^2 \tan(\frac{1}{2} dx + \frac{1}{2} c)}{(a^{10} - 3a^8b^2 + 3a^6b^4 - a^4b^6)\sqrt{a^2 - b^2}}$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out] 1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(a^2 - b^2)) + (36*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a*b^7*tan(1/2*d*x + 1/2*c)^5 + 6*b^8*tan(1/2*d*x + 1/2*c)^5 + 72*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 116*a^4b^2*tan(1/2*d*x + 1/2*c))

$$\begin{aligned} & *b^4 \tan(1/2*d*x + 1/2*c)^3 + 56*a^2*b^6 \tan(1/2*d*x + 1/2*c)^3 - 12*b^8 \tan(1/2*d*x + 1/2*c)^3 \\ & + 36*a^6*b^2 \tan(1/2*d*x + 1/2*c) + 60*a^5*b^3 \tan(1/2*d*x + 1/2*c) - 6*a^4*b^4 \tan(1/2*d*x + 1/2*c) \\ & - 45*a^3*b^5 \tan(1/2*d*x + 1/2*c) - 6*a^2*b^6 \tan(1/2*d*x + 1/2*c) + 15*a*b^7 \tan(1/2*d*x + 1/2*c) + 6*b^8 \tan(1/2*d*x + 1/2*c) \\ & / ((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6) * (a \tan(1/2*d*x + 1/2*c)^2 - b \tan(1/2*d*x + 1/2*c)^2 + a + b)^3) \\ & + 3 \log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1)) / a^4 - 3 \log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1)) / a^4 / d \end{aligned}$$

Mupad [B] (verification not implemented)

Time = 27.37 (sec) , antiderivative size = 7235, normalized size of antiderivative = 28.82

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Too large to display}$$

[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^4),x)

[Out] - (atan(((((((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2)))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (8*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)))/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))))/a^4 - (8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2)))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))*1i)/a^4 - (((8*(16*a^20*b - 4*a^21 + 4*a^8*b^13 - 2*a^9*b^12 - 26*a^10*b^11 + 14*a^11*b^10 + 70*a^12*b^9 - 30*a^13*b^8 - 110*a^14*b^7 + 30*a^15*b^6 + 110*a^16*b^5 - 20*a^17*b^4 - 64*a^18*b^3 + 12*a^19*b^2)))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (8*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)))/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2))))/a^4 + (8*tan(c/2 + (d*x)/2)*(4*a^14 - 8*a^13*b - 8*a*b^13 + 8*b^14 - 48*a^2*b^12 + 48*a^3*b^11 + 117*a^4*b^10 - 120*a^5*b^9 - 164*a^6*b^8 + 160*a^7*b^7 + 156*a^8*b^6 - 120*a^9*b^5 - 92*a^10*b^4 + 48*a^11*b^3 + 44*a^12*b^2)))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9

$$\begin{aligned}
& + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2)) * 1i) / a^4) / (((8(16a^{20}b - 4a^{21} + 4a^8b^{13} - 2a^9b^{12} - 26a^{10}b^{11} + 14a^{11}b^{10} + 70a^{12}b^9 - 30a^{13}b^8 - 110a^{14}b^7 + 30a^{15}b^6 + 110a^{16}b^5 - 20a^{17}b^4 - 64a^{18}b^3 + 12a^{19}b^2)) / (a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) - (8 \tan(c/2 + (d*x)/2) * (8a^{21}b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2)) / (a^4 * (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2))) / a^4 - (8 \tan(c/2 + (d*x)/2) * (4a^{14} - 8a^{13}b - 8a^2b^{13} + 8b^{14} - 48a^2b^{12} + 48a^3b^{11} + 117a^4b^{10} - 120a^5b^9 - 164a^6b^8 + 160a^7b^7 + 156a^8b^6 - 120a^9b^5 - 92a^{10}b^4 + 48a^{11}b^3 + 44a^{12}b^2)) / (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2)) / a^4 + (((8(16a^{20}b - 4a^{21} + 4a^8b^{13} - 2a^9b^{12} - 26a^{10}b^{11} + 14a^{11}b^{10} + 70a^{12}b^9 - 30a^{13}b^8 - 110a^{14}b^7 + 30a^{15}b^6 + 110a^{16}b^5 - 20a^{17}b^4 - 64a^{18}b^3 + 12a^{19}b^2)) / (a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) + (8 \tan(c/2 + (d*x)/2) * (8a^2b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2)) / (a^4 * (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2))) / a^4 + (8 \tan(c/2 + (d*x)/2) * (4a^{14} - 8a^{13}b - 8a^2b^{13} + 8b^{14} - 48a^2b^{12} + 48a^3b^{11} + 117a^4b^{10} - 120a^5b^9 - 164a^6b^8 + 160a^7b^7 + 156a^8b^6 - 120a^9b^5 - 92a^{10}b^4 + 48a^{11}b^3 + 44a^{12}b^2)) / (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2)) / a^4 - (16(16a^{12}b - 2a^2b^{12} + 4b^{13} - 26a^2b^{11} + 11a^3b^{10} + 70a^4b^9 - 34a^5b^8 - 110a^6b^7 + 66a^7b^6 + 110a^8b^5 - 64a^9b^4 - 64a^{10}b^3 + 48a^{11}b^2)) / (a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2))) * 2i) / (a^4 * d) - ((\tan(c/2 + (d*x)/2)^5 * (2b^6 - ab^5 - 6a^2b^4 + 4a^3b^3 + 12a^4b^2)) / ((a^3b - a^4) * (a + b)^3) - (4 \tan(c/2 + (d*x)/2)^3 * (3b^6 - 11a^2b^4 + 18a^4b^2)) / (3(a + b)^2 * (a^5 - 2a^4b + a^3b^2)) + (\tan(c/2 + (d*x)/2) * (a^2b^5 + 2b^6 - 6a^2b^4 - 4a^3b^3 + 12a^4b^2)) / ((a + b) * (3a^5b - a^6 + a^3b^3 - 3a^4b^2))) / (d * (3a^2b^2 - \tan(c/2 + (d*x)/2)^4 * (3a^2b^2 + 3a^2b - 3a^3 - 3b^3) - \tan(c/2 + (d*x)/2)^2 * (3a^2b^2 - 3a^2b - 3a^3 + 3b^3) + 3a^2b + a^3 + b^3 + \tan(c/2 + (d*x)/2)^6 * (3a^2b^2 - 3a^2b + a^3 - b^3))) - (b * \operatorname{atan}(((b * ((8 \tan(c/2 + (d*x)/2) * (4a^{14} - 8a^{13}b - 8a^2b^{13} + 8b^{14} - 48a^2b^{12} + 48a^3b^{11} + 117a^4b^{10} - 120a^5b^9 - 164a^6b^8 + 160a^7b^7 + 156a^8b^6 - 120a^9b^5 - 92a^{10}b^4 + 4
\end{aligned}$$

$$\begin{aligned}
& (8a^{11}b^3 + 44a^{12}b^2)/(a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 \\
& + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2) - (b \cdot (-a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot ((8(16a^{20}b - 4a^{21} \\
& + 4a^8b^{13} - 2a^9b^{12} - 26a^{10}b^{11} + 14a^{11}b^{10} + 70a^{12}b^9 - 30a^{13}b^8 - 110a^{14}b^7 + 30a^{15}b^6 + 110a^{16}b^5 - 20a^{17}b^4 - 64 \\
& a^{18}b^3 + 12a^{19}b^2))/(a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) - (4b \cdot \tan(c/2 + (d \cdot x)/2) \cdot (-a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot ((8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) \cdot (8a^{21}b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2))/((a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2) \cdot (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2)) \cdot (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) \cdot (8a^{21}b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2))/((a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) \cdot (-a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) \cdot i) / (2 \cdot (a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) + (b \cdot ((8 \cdot \tan(c/2 + (d \cdot x)/2) \cdot (4a^{14} - 8a^{13}b - 8a^8b^{13} + 8b^{14} - 48a^2b^{12} + 48a^3b^{11} + 117a^4b^{10} - 120a^5b^9 - 164a^6b^8 + 160a^7b^7 + 156a^8b^6 - 120a^9b^5 - 92a^{10}b^4 + 48a^{11}b^3 + 44a^{12}b^2)))/(a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2) + (b \cdot (-a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot ((8(16a^{20}b - 4a^{21} + 4a^8b^{13} - 2a^9b^{12} - 26a^{10}b^{11} + 14a^{11}b^{10} + 70a^{12}b^9 - 30a^{13}b^8 - 110a^{14}b^7 + 30a^{15}b^6 + 110a^{16}b^5 - 20a^{17}b^4 - 64a^{18}b^3 + 12a^{19}b^2))/(a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) + (4b \cdot \tan(c/2 + (d \cdot x)/2) \cdot (-a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) \cdot (8a^{21}b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2))/((a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2) \cdot (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2)) \cdot (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) / (2 \cdot (a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) \cdot (-a + b)^7 \cdot (a - b)^7)^{(1/2)} \cdot (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) \cdot i) / (2 \cdot (a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) / ((16(16a^{12}b - 2a^8b^{12} + 4b^{13} - 26a^2b^{11} + 11a^3b^{10} + 70a^4b^9 - 34a^5b^8 - 110a^6b^7 + 66a^7b^6 + 110a^8b^5 - 64a^9b^4 - 64a^{10}b^3 + 48a^{11}b^2)))/(a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) + (b \cdot ((8 \cdot \tan(c/2 + (d \cdot x)/2) \cdot (4
\end{aligned}$$

$$\begin{aligned}
& a^{14} - 8a^{13}b - 8a^8b^{13} + 8b^{14} - 48a^2b^{12} + 48a^3b^{11} + 117a^4b^{10} - 120a^5b^9 - 164a^6b^8 + 160a^7b^7 + 156a^8b^6 - 120a^9b^5 \\
& - 92a^{10}b^4 + 48a^{11}b^3 + 44a^{12}b^2) / (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 \\
& - 5a^{14}b^3 - 5a^{15}b^2) - (b^{17} - (a + b)^7(a - b)^7)^{1/2} \cdot ((8 * (16a^{20}b - 4a^{21} + 4a^8b^{13} - 2a^9b^{12} - 26a^{10}b^{11} + 14a^{11}b^{10} + 70a^{12}b^9 \\
& - 30a^{13}b^8 - 110a^{14}b^7 + 30a^{15}b^6 + 110a^{16}b^5 - 20a^{17}b^4 - 64a^{18}b^3 + 12a^{19}b^2)) / (a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 \\
& + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) - (4b \tan(c/2 + (d*x)/2) * (-(a + b)^7 * (a - b)^7)^{1/2} * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) * (8a^{21}b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2)) / ((a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2) * (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2))) * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) / (2 * (a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2))) * (-(a + b)^7 * (a - b)^7)^{1/2} * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) / (2 * (a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) - (b^{17} - (a + b)^7(a - b)^7)^{1/2} * ((8 * (16a^{20}b - 4a^{21} + 4a^8b^{13} - 2a^9b^{12} - 26a^{10}b^{11} + 14a^{11}b^{10} + 70a^{12}b^9 - 30a^{13}b^8 - 110a^{14}b^7 + 30a^{15}b^6 + 110a^{16}b^5 - 20a^{17}b^4 - 64a^{18}b^3 + 12a^{19}b^2)) / (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2) + (b^{17} - (a + b)^7(a - b)^7)^{1/2} * ((8 * (16a^{20}b - 4a^{21} + 4a^8b^{13} - 2a^9b^{12} - 26a^{10}b^{11} + 14a^{11}b^{10} + 70a^{12}b^9 - 30a^{13}b^8 - 110a^{14}b^7 + 30a^{15}b^6 + 110a^{16}b^5 - 20a^{17}b^4 - 64a^{18}b^3 + 12a^{19}b^2)) / (a^{19}b + a^{20} - a^9b^{11} - a^{10}b^{10} + 5a^{11}b^9 + 5a^{12}b^8 - 10a^{13}b^7 - 10a^{14}b^6 + 10a^{15}b^5 + 10a^{16}b^4 - 5a^{17}b^3 - 5a^{18}b^2) + (4b \tan(c/2 + (d*x)/2) * (-(a + b)^7 * (a - b)^7)^{1/2} * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) * (8a^{21}b - 8a^8b^{14} + 8a^9b^{13} + 48a^{10}b^{12} - 48a^{11}b^{11} - 120a^{12}b^{10} + 120a^{13}b^9 + 160a^{14}b^8 - 160a^{15}b^7 - 120a^{16}b^6 + 120a^{17}b^5 + 48a^{18}b^4 - 48a^{19}b^3 - 8a^{20}b^2)) / ((a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2) * (a^{16}b + a^{17} - a^6b^{11} - a^7b^{10} + 5a^8b^9 + 5a^9b^8 - 10a^{10}b^7 - 10a^{11}b^6 + 10a^{12}b^5 + 10a^{13}b^4 - 5a^{14}b^3 - 5a^{15}b^2))) * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) / (2 * (a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2))) * (-(a + b)^7 * (a - b)^7)^{1/2} * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) / (2 * (a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2))) * (-(a + b)^7 * (a - b)^7)^{1/2} * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) * i) / (d * (a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2))
\end{aligned}$$

3.485 $\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^4} dx$

Optimal result	5150
Rubi [A] (verified)	5151
Mathematica [A] (verified)	5154
Maple [A] (verified)	5155
Fricas [B] (verification not implemented)	5155
Sympy [F]	5157
Maxima [F(-2)]	5157
Giac [B] (verification not implemented)	5157
Mupad [B] (verification not implemented)	5158

Optimal result

Integrand size = 21, antiderivative size = 308

$$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^4} dx = \frac{b^2(20a^6 - 35a^4b^2 + 28a^2b^4 - 8b^6) \arctan\left(\frac{\sqrt{a-b} \tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4b \operatorname{arctanh}(\sin(c+dx))}{a^5d} + \frac{(6a^6 - 65a^4b^2 + 68a^2b^4 - 24b^6) \tan(c+dx)}{6a^4(a^2 - b^2)^3d} + \frac{b^2 \tan(c+dx)}{3a(a^2 - b^2)d(a+b \cos(c+dx))^3} + \frac{b^2(9a^2 - 4b^2) \tan(c+dx)}{6a^2(a^2 - b^2)^2d(a+b \cos(c+dx))^2} + \frac{b^2(12a^4 - 11a^2b^2 + 4b^4) \tan(c+dx)}{2a^3(a^2 - b^2)^3d(a+b \cos(c+dx))}$$

```
[Out] b^2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(7/2)/(a+b)^(7/2)/d-4*b*arctanh(sin(d*x+c))/a^5/d+1/6*(6*a^6-65*a^4*b^2+68*a^2*b^4-24*b^6)*tan(d*x+c)/a^4/(a^2-b^2)^3/d+1/3*b^2*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^3+1/6*b^2*(9*a^2-4*b^2)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^2+1/2*b^2*(12*a^4-11*a^2*b^2+4*b^4)*tan(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2881, 3134, 3080, 3855, 2738, 211}

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4} dx = -\frac{4b\operatorname{arctanh}(\sin(c+dx))}{a^5d} + \frac{b^2(9a^2-4b^2)\tan(c+dx)}{6a^2d(a^2-b^2)^2(a+b\cos(c+dx))^2} + \frac{b^2\tan(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^3} + \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\tan(c+dx)}{6a^4d(a^2-b^2)^3} + \frac{b^2(12a^4-11a^2b^2+4b^4)\tan(c+dx)}{2a^3d(a^2-b^2)^3(a+b\cos(c+dx))} + \frac{b^2(20a^6-35a^4b^2+28a^2b^4-8b^6)\operatorname{arctan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}$$

[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]

[Out] (b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) - (4*b*ArcTanh[Sin[c + d*x]]/(a^5*d) + ((6*a^6 - 65*a^4*b^2 + 68*a^2*b^4 - 24*b^6)*Tan[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) + (b^2*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (b^2*(9*a^2 - 4*b^2)*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + (b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x])))

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]

```
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*sin[(e_.) + (f
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{b^2 \tan(c + dx)}{3a(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{\int \frac{(3a^2 - 4b^2 - 3ab \cos(c + dx) + 3b^2 \cos^2(c + dx)) \sec^2(c + dx)}{(a + b \cos(c + dx))^3} dx}{3a(a^2 - b^2)} \\ &= \frac{b^2 \tan(c + dx)}{3a(a^2 - b^2)d(a + b \cos(c + dx))^3} + \frac{b^2(9a^2 - 4b^2) \tan(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \cos(c + dx))^2} \\ &\quad + \frac{\int \frac{(6a^4 - 23a^2b^2 + 12b^4 - 2ab(6a^2 - b^2) \cos(c + dx) + 2b^2(9a^2 - 4b^2) \cos^2(c + dx)) \sec^2(c + dx)}{(a + b \cos(c + dx))^2} dx}{6a^2(a^2 - b^2)^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{b^2 \tan(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^3} + \frac{b^2(9a^2-4b^2)\tan(c+dx)}{6a^2(a^2-b^2)^2d(a+b\cos(c+dx))^2} \\
&+ \frac{b^2(12a^4-11a^2b^2+4b^4)\tan(c+dx)}{2a^3(a^2-b^2)^3d(a+b\cos(c+dx))} \\
&+ \frac{\int \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6-ab(18a^4-7a^2b^2+4b^4)\cos(c+dx)+3b^2(12a^4-11a^2b^2+4b^4)\cos^2(c+dx))\sec^2(c+dx)}{a+b\cos(c+dx)} dx}{6a^3(a^2-b^2)^3} \\
&= \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\tan(c+dx)}{6a^4(a^2-b^2)^3d} + \frac{b^2 \tan(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^3} \\
&+ \frac{b^2(9a^2-4b^2)\tan(c+dx)}{6a^2(a^2-b^2)^2d(a+b\cos(c+dx))^2} + \frac{b^2(12a^4-11a^2b^2+4b^4)\tan(c+dx)}{2a^3(a^2-b^2)^3d(a+b\cos(c+dx))} \\
&+ \frac{\int \frac{(-24b(a^2-b^2)^3+3ab^2(12a^4-11a^2b^2+4b^4)\cos(c+dx))\sec(c+dx)}{a+b\cos(c+dx)} dx}{6a^4(a^2-b^2)^3} \\
&= \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\tan(c+dx)}{6a^4(a^2-b^2)^3d} + \frac{b^2 \tan(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^3} \\
&+ \frac{b^2(9a^2-4b^2)\tan(c+dx)}{6a^2(a^2-b^2)^2d(a+b\cos(c+dx))^2} + \frac{b^2(12a^4-11a^2b^2+4b^4)\tan(c+dx)}{2a^3(a^2-b^2)^3d(a+b\cos(c+dx))} \\
&- \frac{(4b)\int \sec(c+dx) dx}{a^5} + \frac{(b^2(20a^6-35a^4b^2+28a^2b^4-8b^6))\int \frac{1}{a+b\cos(c+dx)} dx}{2a^5(a^2-b^2)^3} \\
&= -\frac{4b\operatorname{arctanh}(\sin(c+dx))}{a^5d} + \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\tan(c+dx)}{6a^4(a^2-b^2)^3d} \\
&+ \frac{b^2 \tan(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^3} + \frac{b^2(9a^2-4b^2)\tan(c+dx)}{6a^2(a^2-b^2)^2d(a+b\cos(c+dx))^2} \\
&+ \frac{b^2(12a^4-11a^2b^2+4b^4)\tan(c+dx)}{2a^3(a^2-b^2)^3d(a+b\cos(c+dx))} \\
&+ \frac{(b^2(20a^6-35a^4b^2+28a^2b^4-8b^6))\operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^5(a^2-b^2)^3d} \\
&= \frac{b^2(20a^6-35a^4b^2+28a^2b^4-8b^6)\operatorname{arctan}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4b\operatorname{arctanh}(\sin(c+dx))}{a^5d} \\
&+ \frac{(6a^6-65a^4b^2+68a^2b^4-24b^6)\tan(c+dx)}{6a^4(a^2-b^2)^3d} + \frac{b^2 \tan(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^3} \\
&+ \frac{b^2(9a^2-4b^2)\tan(c+dx)}{6a^2(a^2-b^2)^2d(a+b\cos(c+dx))^2} + \frac{b^2(12a^4-11a^2b^2+4b^4)\tan(c+dx)}{2a^3(a^2-b^2)^3d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.58 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.35

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^4} dx = -\frac{b^2(20a^6 - 35a^4b^2 + 28a^2b^4 - 8b^6) \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{a^5(a^2 - b^2)^3\sqrt{-a^2 + b^2}d}$$

$$+ \frac{4b \log\left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{a^5 d}$$

$$- \frac{4b \log\left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}{a^5 d}$$

$$+ \frac{\sin\left(\frac{1}{2}(c + dx)\right)}{a^4 d \left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}$$

$$+ \frac{\sin\left(\frac{1}{2}(c + dx)\right)}{a^4 d \left(\cos\left(\frac{1}{2}(c + dx)\right) + \sin\left(\frac{1}{2}(c + dx)\right)\right)}$$

$$- \frac{b^3 \sin(c + dx)}{3a^2(a - b)(a + b)d(a + b \cos(c + dx))^3}$$

$$+ \frac{-11a^2b^3 \sin(c + dx) + 6b^5 \sin(c + dx)}{6a^3(a - b)^2(a + b)^2d(a + b \cos(c + dx))^2}$$

$$+ \frac{-47a^4b^3 \sin(c + dx) + 50a^2b^5 \sin(c + dx) - 18b^7 \sin(c + dx)}{6a^4(a - b)^3(a + b)^3d(a + b \cos(c + dx))}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(a^5*(a^2 - b^2)^3*Sqrt[-a^2 + b^2]*d) + (4*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a^5*d) - (4*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a^5*d) + Sin[(c + d*x)/2]/(a^4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) - (b^3*Sin[c + d*x])/(3*a^2*(a - b)*(a + b)*d*(a + b*Cos[c + d*x])^3) + (-11*a^2*b^3*Sin[c + d*x] + 6*b^5*Sin[c + d*x])/(6*a^3*(a - b)^2*(a + b)^2*d*(a + b*Cos[c + d*x])^2) + (-47*a^4*b^3*Sin[c + d*x] + 50*a^2*b^5*Sin[c + d*x] - 18*b^7*Sin[c + d*x])/(6*a^4*(a - b)^3*(a + b)^3*d*(a + b*Cos[c + d*x]))
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.37

method	result
derivativedivides	$2b^2 \left(\frac{-\frac{(20a^4+5a^3b-18a^2b^2-2ab^3+6b^4)ab(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{2(a-b)(a^3+3a^2b+3ab^2+b^3)} - \frac{2(30a^4-29a^2b^2+9b^4)ab(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{3(a^2-2ab+b^2)(a^2+2ab+b^2)} - \frac{(20a^4-5a^3b-18a^2b^2+2ab^3+6b^4)ab(\tan(\frac{dx}{2}+\frac{c}{2}))}{2(a+b)(a^3-3a^2b+3ab^2+b^3)}}{((\tan^2(\frac{dx}{2}+\frac{c}{2}))^{a-b}(\tan^2(\frac{dx}{2}+\frac{c}{2}))^{a+b})^3} \right) \frac{1}{a^5}$
default	$2b^2 \left(\frac{-\frac{(20a^4+5a^3b-18a^2b^2-2ab^3+6b^4)ab(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{2(a-b)(a^3+3a^2b+3ab^2+b^3)} - \frac{2(30a^4-29a^2b^2+9b^4)ab(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{3(a^2-2ab+b^2)(a^2+2ab+b^2)} - \frac{(20a^4-5a^3b-18a^2b^2+2ab^3+6b^4)ab(\tan(\frac{dx}{2}+\frac{c}{2}))}{2(a+b)(a^3-3a^2b+3ab^2+b^3)}}{((\tan^2(\frac{dx}{2}+\frac{c}{2}))^{a-b}(\tan^2(\frac{dx}{2}+\frac{c}{2}))^{a+b})^3} \right) \frac{1}{a^5}$
risch	Expression too large to display

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*b)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2/a^5*((-1/2*(20*a^4+5*a^3*b-18*a^2*b^2-2*a*b^3+6*b^4)*a*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(30*a^4-29*a^2*b^2+9*b^4)*a*b/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(20*a^4-5*a^3*b-18*a^2*b^2+2*a*b^3+6*b^4)*a*b/(a+b)/(a^3-3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)))/(tan(1/2*d*x+1/2*c)^2*a-b*tan(1/2*d*x+1/2*c)^2+a+b)^3+1/2*(20*a^6-35*a^4*b^2+28*a^2*b^4-8*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2))-1/a^4/(tan(1/2*d*x+1/2*c)+1)-4*b/a^5*ln(tan(1/2*d*x+1/2*c)+1)-1/a^4/(tan(1/2*d*x+1/2*c)-1)+4*b/a^5*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. 2(291) = 582.

Time = 1.69 (sec) , antiderivative size = 2048, normalized size of antiderivative = 6.65

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4} dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b^11)*cos(d*x + c))^4 + 3*(20*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c)^3 + 3*(20*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + (20*a^9*b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*
```

$$\begin{aligned}
& \cos(dx + c) + a^2)) + 24*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^{10} + \\
& b^{12})*\cos(dx + c)^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^{11})*\cos(dx + c)^3 + 3*(a^{10}*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2 \\
& *b^{10})*\cos(dx + c)^2 + (a^{11}*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*\cos(dx + c))*\log(\sin(dx + c) + 1) - 24*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^{10} + \\
& b^{12})*\cos(dx + c)^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^{11})*\cos(dx + c)^3 + 3*(a^{10}*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2 \\
& *b^{10})*\cos(dx + c)^2 + (a^{11}*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*\cos(dx + c))*\log(-\sin(dx + c) + 1) - 2*(6*a^{12} - 8 \\
& *4*a^{10}*b^2 + 36*a^8*b^4 - 24*a^6*b^6 + 6*a^4*b^8 + (6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a^3*b^9 + 24*a*b^{11})*\cos(dx + c)^3 + 3*(6*a^{10}*b^2 - 59 \\
& *a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 + 20*a^2*b^{10})*\cos(dx + c)^2 + (18*a^{11}*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*\cos(dx + c))*s \\
& \sin(dx + c))/((a^{13}*b^3 - 4*a^{11}*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^{11})*d* \\
& \cos(dx + c)^4 + 3*(a^{14}*b^2 - 4*a^{12}*b^4 + 6*a^{10}*b^6 - 4*a^8*b^8 + a^6*b^{10})*d*\cos(dx + c)^3 + 3*(a^{15}*b - 4*a^{13}*b^3 + 6*a^{11}*b^5 - 4*a^9*b^7 + a^7 \\
& *b^9)*d*\cos(dx + c)^2 + (a^{16} - 4*a^{14}*b^2 + 6*a^{12}*b^4 - 4*a^{10}*b^6 + a^8*b^8)*d*\cos(dx + c)), 1/6*(3*((20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b \\
& ^{11})*\cos(dx + c)^4 + 3*(20*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^{10})*c \\
& \cos(dx + c)^3 + 3*(20*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*\cos(dx \\
& + c)^2 + (20*a^9*b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*\cos(dx + c)) \\
& *sqrt(a^2 - b^2)*\arctan(-(a*\cos(dx + c) + b)/(sqrt(a^2 - b^2)*\sin(dx + c) \\
&)) - 12*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^{10} + b^{12})*\cos(dx + c) \\
& ^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^{11})*\cos(dx + c)^3 \\
& + 3*(a^{10}*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^{10})*\cos(dx + c) \\
&)^2 + (a^{11}*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*\cos(dx + c))* \\
& \log(\sin(dx + c) + 1) + 12*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^{10} + \\
& b^{12})*\cos(dx + c)^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a* \\
& b^{11})*\cos(dx + c)^3 + 3*(a^{10}*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2 \\
& *b^{10})*\cos(dx + c)^2 + (a^{11}*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3* \\
& b^9)*\cos(dx + c))*\log(-\sin(dx + c) + 1) + (6*a^{12} - 24*a^{10}*b^2 + 36*a^8* \\
& b^4 - 24*a^6*b^6 + 6*a^4*b^8 + (6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a \\
& ^3*b^9 + 24*a*b^{11})*\cos(dx + c)^3 + 3*(6*a^{10}*b^2 - 59*a^8*b^4 + 110*a^6*b^6 \\
& - 77*a^4*b^8 + 20*a^2*b^{10})*\cos(dx + c)^2 + (18*a^{11}*b - 132*a^9*b^3 + \\
& 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*\cos(dx + c))*\sin(dx + c))/((a^{13}* \\
& b^3 - 4*a^{11}*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^{11})*d*\cos(dx + c)^4 + 3*(\\
& a^{14}*b^2 - 4*a^{12}*b^4 + 6*a^{10}*b^6 - 4*a^8*b^8 + a^6*b^{10})*d*\cos(dx + c)^3 \\
& + 3*(a^{15}*b - 4*a^{13}*b^3 + 6*a^{11}*b^5 - 4*a^9*b^7 + a^7*b^9)*d*\cos(dx + c) \\
&)^2 + (a^{16} - 4*a^{14}*b^2 + 6*a^{12}*b^4 - 4*a^{10}*b^6 + a^8*b^8)*d*\cos(dx + c \\
&))]
\end{aligned}$$

SymPy [F]

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^4} dx = \int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^4} dx$$

[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**4,x)

[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**4, x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Exception raised: ValueError}$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. 2(291) = 582.

Time = 0.38 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.91

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^4} dx = \frac{3(20a^6b^2 - 35a^4b^4 + 28a^2b^6 - 8b^8) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(-2a+2b) + \arctan \left(-\frac{a \tan(\frac{1}{2} dx + \frac{1}{2} c) - b \tan(\frac{1}{2} dx + \frac{1}{2} c)}{\sqrt{a^2 - b^2}} \right) \right)}{(a^{11} - 3a^9b^2 + 3a^7b^4 - a^5b^6)\sqrt{a^2 - b^2}} + \frac{60a^6b^3 \tan(\frac{1}{2} dx + \frac{1}{2} c)^5}{\dots}$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out] -1/3*(3*(20*a^6*b^2 - 35*a^4*b^4 + 28*a^2*b^6 - 8*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(a^2 - b^2)) + (60*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 105*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 24*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 24*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*a*b^8*tan(1/2*d*x + 1/2*c)^5 + 18*b^9*tan(1/2*d*x + 1/2*c)^5 + 120*a^6*b^3*tan(1/2*d*x + 1/2*c)

$$\begin{aligned} &^3 - 236*a^4*b^5*\tan(1/2*d*x + 1/2*c)^3 + 152*a^2*b^7*\tan(1/2*d*x + 1/2*c)^3 \\ & - 36*b^9*\tan(1/2*d*x + 1/2*c)^3 + 60*a^6*b^3*\tan(1/2*d*x + 1/2*c) + 105*a^5*b^4*\tan(1/2*d*x + 1/2*c) \\ & - 24*a^4*b^5*\tan(1/2*d*x + 1/2*c) - 117*a^3*b^6*\tan(1/2*d*x + 1/2*c) - 24*a^2*b^7*\tan(1/2*d*x + 1/2*c) \\ & + 42*a*b^8*\tan(1/2*d*x + 1/2*c) + 18*b^9*\tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6) \\ & *(a*\tan(1/2*d*x + 1/2*c)^2 - b*\tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 12*b*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) + 1))/a^5 \\ & - 12*b*\log(\text{abs}(\tan(1/2*d*x + 1/2*c) - 1))/a^5 + 6*\tan(1/2*d*x + 1/2*c)/((\tan(1/2*d*x + 1/2*c)^2 - 1)*a^4))/d \end{aligned}$$

Mupad [B] (verification not implemented)

Time = 23.52 (sec) , antiderivative size = 7490, normalized size of antiderivative = 24.32

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^4} dx = \text{Too large to display}$$

[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^4),x)

[Out] (b*atan(((b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2)))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (4*b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2)))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (32*b*tan(c/2 + (d*x)/2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2)))/(a^5*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))))/a^5)*4i)/a^5 + (b*((8*tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2)))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (4*b*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2)))/(a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) + (32*b*tan(c/2 + (d*x)/2)*(8*a^23*b

$$\begin{aligned}
& - 8a^{10}b^{14} + 8a^{11}b^{13} + 48a^{12}b^{12} - 48a^{13}b^{11} - 120a^{14}b^{10} + \\
& 120a^{15}b^9 + 160a^{16}b^8 - 160a^{17}b^7 - 120a^{18}b^6 + 120a^{19}b^5 + \\
& 48a^{20}b^4 - 48a^{21}b^3 - 8a^{22}b^2) / (a^5(a^{18}b + a^{19} - a^8b^{11} - \\
& a^9b^{10} + 5a^{10}b^9 + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + 10a^{14}b^5 \\
& + 10a^{15}b^4 - 5a^{16}b^3 - 5a^{17}b^2))) / a^5) * 4i) / a^5) / ((32*(128b^{16} \\
& - 64ab^{15} - 832a^2b^{14} + 400a^3b^{13} + 2288a^4b^{12} - 1088a^5b^{11} - \\
& 3472a^6b^{10} + 1602a^7b^9 + 3088a^8b^8 - 1280a^9b^7 - 1520a^{10}b^6 \\
& + 480a^{11}b^5 + 320a^{12}b^4)) / (a^{22}b + a^{23} - a^{12}b^{11} - a^{13}b^{10} + 5 \\
& a^{14}b^9 + 5a^{15}b^8 - 10a^{16}b^7 - 10a^{17}b^6 + 10a^{18}b^5 + 10a^{19}b^4 \\
& - 5a^{20}b^3 - 5a^{21}b^2) - (4b*((8*\tan(c/2 + (d*x)/2)*(128b^{16} - 12 \\
& 8ab^{15} - 768a^2b^{14} + 768a^3b^{13} + 1920a^4b^{12} - 1920a^5b^{11} - 26 \\
& 00a^6b^{10} + 2560a^7b^9 + 2025a^8b^8 - 1920a^9b^7 - 824a^{10}b^6 + 7 \\
& 68a^{11}b^5 + 80a^{12}b^4 - 128a^{13}b^3 + 64a^{14}b^2)) / (a^{18}b + a^{19} - a \\
& ^8b^{11} - a^9b^{10} + 5a^{10}b^9 + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + \\
& 10a^{14}b^5 + 10a^{15}b^4 - 5a^{16}b^3 - 5a^{17}b^2) - (4b*((16*(8a^{23}b \\
& - 8a^{10}b^{14} + 4a^{11}b^{13} + 52a^{12}b^{12} - 25a^{13}b^{11} - 143a^{14}b^{10} + \\
& 63a^{15}b^9 + 217a^{16}b^8 - 87a^{17}b^7 - 193a^{18}b^6 + 73a^{19}b^5 + 95 \\
& a^{20}b^4 - 36a^{21}b^3 - 20a^{22}b^2)) / (a^{22}b + a^{23} - a^{12}b^{11} - a^{13}b \\
& ^{10} + 5a^{14}b^9 + 5a^{15}b^8 - 10a^{16}b^7 - 10a^{17}b^6 + 10a^{18}b^5 + 1 \\
& 0a^{19}b^4 - 5a^{20}b^3 - 5a^{21}b^2) - (32b*\tan(c/2 + (d*x)/2)*(8a^{23}b \\
& - 8a^{10}b^{14} + 8a^{11}b^{13} + 48a^{12}b^{12} - 48a^{13}b^{11} - 120a^{14}b^{10} + \\
& 120a^{15}b^9 + 160a^{16}b^8 - 160a^{17}b^7 - 120a^{18}b^6 + 120a^{19}b^5 + \\
& 48a^{20}b^4 - 48a^{21}b^3 - 8a^{22}b^2)) / (a^5(a^{18}b + a^{19} - a^8b^{11} - \\
& a^9b^{10} + 5a^{10}b^9 + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + 10a^{14}b^5 \\
& + 10a^{15}b^4 - 5a^{16}b^3 - 5a^{17}b^2))) / a^5) / a^5 + (4b*((8*\tan(c/2 \\
& + (d*x)/2)*(128b^{16} - 128ab^{15} - 768a^2b^{14} + 768a^3b^{13} + 1920a^4b^{12} \\
& - 1920a^5b^{11} - 2600a^6b^{10} + 2560a^7b^9 + 2025a^8b^8 - 1920a^9b^7 - 824a^{10}b^6 \\
& + 768a^{11}b^5 + 80a^{12}b^4 - 128a^{13}b^3 + 64a^{14}b^2)) / (a^{18}b + a^{19} - a^8b^{11} \\
& - a^9b^{10} + 5a^{10}b^9 + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + 10a^{14}b^5 + 10a^{15}b^4 \\
& - 5a^{16}b^3 - 5a^{17}b^2) + (4b*((16*(8a^{23}b - 8a^{10}b^{14} + 4a^{11}b^{13} + 52a^{12}b^{12} \\
& - 25a^{13}b^{11} - 143a^{14}b^{10} + 63a^{15}b^9 + 217a^{16}b^8 - 87a^{17}b^7 - 193a^{18}b^6 \\
& + 73a^{19}b^5 + 95a^{20}b^4 - 36a^{21}b^3 - 20a^{22}b^2)) / (a^{22}b + \\
& a^{23} - a^{12}b^{11} - a^{13}b^{10} + 5a^{14}b^9 + 5a^{15}b^8 - 10a^{16}b^7 - 10a^{17}b^6 \\
& + 10a^{18}b^5 + 10a^{19}b^4 - 5a^{20}b^3 - 5a^{21}b^2) + (32b*\tan(c/2 + (d*x)/2) \\
& *(8a^{23}b - 8a^{10}b^{14} + 8a^{11}b^{13} + 48a^{12}b^{12} - 48a^{13}b^{11} - 120a^{14}b^{10} \\
& + 120a^{15}b^9 + 160a^{16}b^8 - 160a^{17}b^7 - 120a^{18}b^6 + 120a^{19}b^5 + 48a^{20}b^4 \\
& - 48a^{21}b^3 - 8a^{22}b^2)) / (a^5(a^{18}b + a^{19} - a^8b^{11} - a^9b^{10} + 5a^{10}b^9 \\
& + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + 10a^{14}b^5 + 10a^{15}b^4 - 5a^{16}b^3 - 5a^{17}b^2))) \\
&) / a^5) * 8i) / (a^5*d) - ((\tan(c/2 + (d*x)/2)^3*(12ab^7 - 18a^8 - 72b^8 + \\
& 236a^2b^6 - 47a^3b^5 - 273a^4b^4 + 60a^5b^3 + 72a^6b^2)) / (3a^4*(\\
& a + b)^2*(a - b)^3) - (\tan(c/2 + (d*x)/2)^5*(12ab^7 + 18a^8 + 72b^8 - 2 \\
& 36a^2b^6 - 47a^3b^5 + 273a^4b^4 + 60a^5b^3 - 72a^6b^2)) / (3a^4*(a \\
& + b)^3*(a - b)^2) + (\tan(c/2 + (d*x)/2)*(4ab^6 - 2a^6b - 2a^7 + 8b^7
\end{aligned}$$

$$\begin{aligned}
& - 24*a^2*b^5 - 11*a^3*b^4 + 26*a^4*b^3 + 6*a^5*b^2) / (a^4*(a + b)*(a - b)^3) + (\tan(c/2 + (d*x)/2)^7*(4*a*b^6 + 2*a^6*b - 2*a^7 - 8*b^7 + 24*a^2*b^5 \\
& - 11*a^3*b^4 - 26*a^4*b^3 + 6*a^5*b^2) / (a^4*(a + b)^3*(a - b)) / (d*(3*a*b^2 + 3*a^2*b - \tan(c/2 + (d*x)/2)^4*(6*a^2*b - 6*b^3) - \tan(c/2 + (d*x)/2)^2 \\
& *(6*a*b^2 - 2*a^3 + 4*b^3) - \tan(c/2 + (d*x)/2)^6*(2*a^3 - 6*a*b^2 + 4*b^3) + a^3 + b^3 - \tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (b^2*atan(((b^2*((8*\tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2)) / (a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) - (b^2*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2)) / (a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (4*b^2*\tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2)) / ((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2))) * (-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)) / (2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2))) * (-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*i) / (2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)) + (b^2*((8*\tan(c/2 + (d*x)/2)*(128*b^16 - 128*a*b^15 - 768*a^2*b^14 + 768*a^3*b^13 + 1920*a^4*b^12 - 1920*a^5*b^11 - 2600*a^6*b^10 + 2560*a^7*b^9 + 2025*a^8*b^8 - 1920*a^9*b^7 - 824*a^10*b^6 + 768*a^11*b^5 + 80*a^12*b^4 - 128*a^13*b^3 + 64*a^14*b^2)) / (a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2) + (b^2*((16*(8*a^23*b - 8*a^10*b^14 + 4*a^11*b^13 + 52*a^12*b^12 - 25*a^13*b^11 - 143*a^14*b^10 + 63*a^15*b^9 + 217*a^16*b^8 - 87*a^17*b^7 - 193*a^18*b^6 + 73*a^19*b^5 + 95*a^20*b^4 - 36*a^21*b^3 - 20*a^22*b^2)) / (a^22*b + a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) + (4*b^2*\tan(c/2 + (d*x)/2)*(-(a + b)^7*(a - b)^7)^(1/2)*(20*a^6 - 8*b^6 + 28*a^2*b^4 - 35*a^4*b^2)*(8*a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^19*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2)) / ((a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 +
\end{aligned}$$

$$\begin{aligned}
& (10a^{14}b^5 + 10a^{15}b^4 - 5a^{16}b^3 - 5a^{17}b^2)) * (- (a + b)^7 * (a - b)^7)^{(1/2)} * (20a^6 - 8b^6 + 28a^2b^4 - 35a^4b^2)) / (2 * (a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2)) * (- (a + b)^7 * (a - b)^7)^{(1/2)} * (20a^6 - 8b^6 + 28a^2b^4 - 35a^4b^2) * i) / (2 * (a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2)) / ((32 * (128b^{16} - 64a * b^{15} - 832a^2 * b^{14} + 400a^3 * b^{13} + 2288a^4 * b^{12} - 1088a^5 * b^{11} - 3472a^6 * b^{10} + 1602a^7 * b^9 + 3088a^8 * b^8 - 1280a^9 * b^7 - 1520a^{10} * b^6 + 480a^{11} * b^5 + 320a^{12} * b^4)) / (a^{22} * b + a^{23} - a^{12} * b^{11} - a^{13} * b^{10} + 5a^{14} * b^9 + 5a^{15} * b^8 - 10a^{16} * b^7 - 10a^{17} * b^6 + 10a^{18} * b^5 + 10a^{19} * b^4 - 5a^{20} * b^3 - 5a^{21} * b^2) - (b^2 * ((8 * \tan(c/2 + (d * x)/2) * (128b^{16} - 128a * b^{15} - 768a^2 * b^{14} + 768a^3 * b^{13} + 1920a^4 * b^{12} - 1920a^5 * b^{11} - 2600a^6 * b^{10} + 2560a^7 * b^9 + 2025a^8 * b^8 - 1920a^9 * b^7 - 824a^{10} * b^6 + 768a^{11} * b^5 + 80a^{12} * b^4 - 128a^{13} * b^3 + 64a^{14} * b^2)) / (a^{18} * b + a^{19} - a^8 * b^{11} - a^9 * b^{10} + 5a^{10} * b^9 + 5a^{11} * b^8 - 10a^{12} * b^7 - 10a^{13} * b^6 + 10a^{14} * b^5 + 10a^{15} * b^4 - 5a^{16} * b^3 - 5a^{17} * b^2) - (b^2 * ((16 * (8a^{23} * b - 8a^{10} * b^{14} + 4a^{11} * b^{13} + 52a^{12} * b^{12} - 25a^{13} * b^{11} - 143a^{14} * b^{10} + 63a^{15} * b^9 + 217a^{16} * b^8 - 87a^{17} * b^7 - 193a^{18} * b^6 + 73a^{19} * b^5 + 95a^{20} * b^4 - 36a^{21} * b^3 - 20a^{22} * b^2)) / (a^{22} * b + a^{23} - a^{12} * b^{11} - a^{13} * b^{10} + 5a^{14} * b^9 + 5a^{15} * b^8 - 10a^{16} * b^7 - 10a^{17} * b^6 + 10a^{18} * b^5 + 10a^{19} * b^4 - 5a^{20} * b^3 - 5a^{21} * b^2) - (4 * b^2 * \tan(c/2 + (d * x)/2) * (- (a + b)^7 * (a - b)^7)^{(1/2)} * (20a^6 - 8b^6 + 28a^2b^4 - 35a^4b^2)) * (8a^{23} * b - 8a^{10} * b^{14} + 8a^{11} * b^{13} + 48a^{12} * b^{12} - 48a^{13} * b^{11} - 120a^{14} * b^{10} + 120a^{15} * b^9 + 160a^{16} * b^8 - 160a^{17} * b^7 - 120a^{18} * b^6 + 120a^{19} * b^5 + 48a^{20} * b^4 - 48a^{21} * b^3 - 8a^{22} * b^2)) / ((a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2)) * (- (a + b)^7 * (a - b)^7)^{(1/2)} * (20a^6 - 8b^6 + 28a^2b^4 - 35a^4b^2)) / (2 * (a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2)) * (- (a + b)^7 * (a - b)^7)^{(1/2)} * (20a^6 - 8b^6 + 28a^2b^4 - 35a^4b^2)) / (2 * (a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2)) + (b^2 * ((8 * \tan(c/2 + (d * x)/2) * (128b^{16} - 128a * b^{15} - 768a^2 * b^{14} + 768a^3 * b^{13} + 1920a^4 * b^{12} - 1920a^5 * b^{11} - 2600a^6 * b^{10} + 2560a^7 * b^9 + 2025a^8 * b^8 - 1920a^9 * b^7 - 824a^{10} * b^6 + 768a^{11} * b^5 + 80a^{12} * b^4 - 128a^{13} * b^3 + 64a^{14} * b^2)) / (a^{18} * b + a^{19} - a^8 * b^{11} - a^9 * b^{10} + 5a^{10} * b^9 + 5a^{11} * b^8 - 10a^{12} * b^7 - 10a^{13} * b^6 + 10a^{14} * b^5 + 10a^{15} * b^4 - 5a^{16} * b^3 - 5a^{17} * b^2) + (b^2 * ((16 * (8a^{23} * b - 8a^{10} * b^{14} + 4a^{11} * b^{13} + 52a^{12} * b^{12} - 25a^{13} * b^{11} - 143a^{14} * b^{10} + 63a^{15} * b^9 + 217a^{16} * b^8 - 87a^{17} * b^7 - 193a^{18} * b^6 + 73a^{19} * b^5 + 95a^{20} * b^4 - 36a^{21} * b^3 - 20a^{22} * b^2)) / (a^{22} * b + a^{23} - a^{12} * b^{11} - a^{13} * b^{10} + 5a^{14} * b^9 + 5a^{15} * b^8 - 10a^{16} * b^7 - 10a^{17} * b^6 + 10a^{18} * b^5 + 10a^{19} * b^4 - 5a^{20} * b^3 - 5a^{21} * b^2) + (4 * b^2 * \tan(c/2 + (d * x)/2) * (- (a + b)^7 * (a - b)^7)^{(1/2)} * (20a^6 - 8b^6 + 28a^2b^4 - 35a^4b^2)) * (8a^{23} * b - 8a^{10} * b^{14} + 8a^{11} * b^{13} + 48a^{12} * b^{12} - 48a^{13} * b^{11} - 120a^{14} * b^{10} + 120a^{15} *
\end{aligned}$$

$$\begin{aligned}
& b^9 + 160a^{16}b^8 - 160a^{17}b^7 - 120a^{18}b^6 + 120a^{19}b^5 + 48a^{20}b^4 \\
& - 48a^{21}b^3 - 8a^{22}b^2) / ((a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} \\
& + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2) * (a^{18}b + a^{19} - \\
& a^8b^{11} - a^9b^{10} + 5a^{10}b^9 + 5a^{11}b^8 - 10a^{12}b^7 - 10a^{13}b^6 + \\
& 10a^{14}b^5 + 10a^{15}b^4 - 5a^{16}b^3 - 5a^{17}b^2))) * (-(a + b)^7 * (a - b)^7)^{(1/2)} \\
& * (20a^6 - 8b^6 + 28a^2b^4 - 35a^4b^2) / (2 * (a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} \\
& + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2))) * (-(a + b)^7 * (a - b)^7)^{(1/2)} \\
& * (20a^6 - 8b^6 + 28a^2b^4 - 35a^4b^2) / (2 * (a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 \\
& + 21a^{15}b^4 - 7a^{17}b^2))) * (-(a + b)^7 * (a - b)^7)^{(1/2)} * (20a^6 - 8b^6 + 28a^2b^4 - 35a^4b^2) * i) \\
& / (d * (a^{19} - a^5b^{14} + 7a^7b^{12} - 21a^9b^{10} + 35a^{11}b^8 - 35a^{13}b^6 + 21a^{15}b^4 - 7a^{17}b^2))
\end{aligned}$$

3.486 $\int \cos^3(c + dx) \sqrt{a + b \cos(c + dx)} dx$

Optimal result	5163
Rubi [A] (verified)	5164
Mathematica [A] (verified)	5167
Maple [B] (verified)	5167
Fricas [C] (verification not implemented)	5168
Sympy [F(-1)]	5169
Maxima [F]	5169
Giac [F]	5169
Mupad [F(-1)]	5169

Optimal result

Integrand size = 23, antiderivative size = 264

$$\begin{aligned}
 & \int \cos^3(c + dx) \sqrt{a + b \cos(c + dx)} dx \\
 &= \frac{2a(8a^2 + 19b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{105b^3 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 & - \frac{2(8a^4 + 17a^2b^2 - 25b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{105b^3 d \sqrt{a + b \cos(c + dx)}} \\
 & + \frac{2(8a^2 + 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105b^2 d} - \frac{8a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35b^2 d} \\
 & + \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7bd}
 \end{aligned}$$

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[Out] -8/35*a*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*cos(d*x+c)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/105*(8*a^2+25*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/105*a*(8*a^2+19*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/105*(8*a^4+17*a^2*b^2-25*b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)

```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2872, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)} dx$$

$$= \frac{2(8a^2+25b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d} + \frac{2a(8a^2+19b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(8a^4+17a^2b^2-25b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\cos(c+dx)}} - \frac{8a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{35b^2d} + \frac{2\sin(c+dx)\cos(c+dx)(a+b\cos(c+dx))^{3/2}}{7bd}$$

[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*a*(8*a^2 + 19*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^4 + 17*a^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(105*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*a^2 + 25*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^2*d) - (8*a*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*b*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*SIN[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m/(f*(m + 1)))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2872

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & NeQ[c, 0])))

Rule 3102

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]

&& !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7bd} \\
 &+ \frac{2 \int \sqrt{a + b \cos(c + dx)} \left(a + \frac{5}{2} b \cos(c + dx) - 2a \cos^2(c + dx) \right) dx}{7b} \\
 &= -\frac{8a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35b^2d} \\
 &+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7bd} \\
 &+ \frac{4 \int \sqrt{a + b \cos(c + dx)} \left(-\frac{ab}{2} + \frac{1}{4}(8a^2 + 25b^2) \cos(c + dx) \right) dx}{35b^2} \\
 &= \frac{2(8a^2 + 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105b^2d} - \frac{8a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35b^2d} \\
 &+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7bd} + \frac{8 \int \frac{\frac{1}{8}b(2a^2 + 25b^2) + \frac{1}{8}a(8a^2 + 19b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{105b^2} \\
 &= \frac{2(8a^2 + 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105b^2d} - \frac{8a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35b^2d} \\
 &+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7bd} \\
 &+ \frac{(a(8a^2 + 19b^2)) \int \sqrt{a + b \cos(c + dx)} dx}{105b^3} \\
 &- \frac{(8a^4 + 17a^2b^2 - 25b^4) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{105b^3} \\
 &= \frac{2(8a^2 + 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105b^2d} - \frac{8a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35b^2d} \\
 &+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7bd} \\
 &+ \frac{\left(a(8a^2 + 19b^2) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{105b^3 \sqrt{\frac{a+b \cos(c + dx)}{a+b}}} \\
 &- \frac{\left((8a^4 + 17a^2b^2 - 25b^4) \sqrt{\frac{a+b \cos(c + dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{105b^3 \sqrt{a + b \cos(c + dx)}}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2a(8a^2 + 19b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{105b^3 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad - \frac{2(8a^4 + 17a^2b^2 - 25b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{105b^3 d \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{2(8a^2 + 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105b^2 d} \\
&\quad - \frac{8a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35b^2 d} \\
&\quad + \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.81

$$\int \cos^3(c + dx) \sqrt{a + b \cos(c + dx)} dx$$

$$= \frac{4a(8a^3 + 8a^2b + 19ab^2 + 19b^3) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) - 4(8a^4 + 17a^2b^2 - 25b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) + \dots}{\dots}$$

[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]],x]

[Out] (4*a*(8*a^3 + 8*a^2*b + 19*a*b^2 + 19*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(8*a^4 + 17*a^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-16*a^3 + 136*a*b^2 + (-4*a^2*b + 145*b^3)*Cos[c + d*x] + 36*a*b^2*Cos[2*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*b^3*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. 2(298) = 596.

Time = 7.53 (sec) , antiderivative size = 827, normalized size of antiderivative = 3.13

method	result	size
default	Expression too large to display	827

[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2*d*x+1/2*c)^9*b^4+144*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*c)^7*b^4-4*cos(1/2*d*x+1/2*c)^5*a^2*b^2-288*cos(1/2*d*x+1/2*c)^5*a*b^3+640*co

$$\begin{aligned} & s(1/2*d*x+1/2*c)^5*b^4-8*\cos(1/2*d*x+1/2*c)^3*a^3*b+6*\cos(1/2*d*x+1/2*c)^3* \\ & a^2*b^2+230*\cos(1/2*d*x+1/2*c)^3*a*b^3-360*\cos(1/2*d*x+1/2*c)^3*b^4-8*(\sin(\\ & 1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{Elliptic} \\ & \text{F}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)})*a^4-17*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & *((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2* \\ & c), (-2*b/(a-b))^{(1/2)})*a^2*b^2+25*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/ \\ & 2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) \\ & *b^4+8*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)} \\ & *\text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)})*a^4-8*(\sin(1/2 \\ & *d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE} \\ & (\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)})*a^3*b+19*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & *((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c \\ &), (-2*b/(a-b))^{(1/2)})*a^2*b^2-19*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2 \\ & *d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) \\ & *a*b^3+8*\cos(1/2*d*x+1/2*c)*a^3*b-2*\cos(1/2*d*x+1/2*c)*a^2*b^2-86*\cos(\\ & 1/2*d*x+1/2*c)*a*b^3+80*\cos(1/2*d*x+1/2*c)*b^4)/b^3/(-2*\sin(1/2*d*x+1/2*c)^ \\ & 4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+ \\ & 1/2*c)^2+a+b)^{(1/2)}/d \end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.80

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)} dx$$

$$= \frac{\sqrt{2}(16i a^4 + 32i a^2 b^2 - 75i b^4)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right)}{1}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/315*(sqrt(2)*(16*I*a^4 + 32*I*a^2*b^2 - 75*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-16*I*a^4 - 32*I*a^2*b^2 + 75*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-8*I*a^3*b - 19*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(8*I*a^3*b + 19*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(15*b^4*cos(d*x + c)^2 + 3*a*b^3*cos(d*x + c) - 4*a^2*b^2 + 25*b^4)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^4*d)

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c + dx) \sqrt{a + b \cos(c + dx)} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(1/2), x)

[Out] Timed out

Maxima [F]

$$\int \cos^3(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \cos(dx + c)^3 dx$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)

Giac [F]

$$\int \cos^3(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \cos(dx + c)^3 dx$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \cos(c + dx)^3 \sqrt{a + b \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2), x)

[Out] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2), x)

3.487 $\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} dx$

Optimal result	5170
Rubi [A] (verified)	5170
Mathematica [A] (verified)	5173
Maple [B] (verified)	5174
Fricas [C] (verification not implemented)	5174
Sympy [F]	5175
Maxima [F]	5175
Giac [F]	5175
Mupad [F(-1)]	5176

Optimal result

Integrand size = 23, antiderivative size = 207

$$\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} dx$$

$$= -\frac{2(2a^2 - 9b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{15b^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{4a(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{15b^2 d \sqrt{a + b \cos(c + dx)}} - \frac{4a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15bd} + \frac{2(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd}$$

```
[Out] 2/5*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d-4/15*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d-2/15*(2*a^2-9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+4/15*a*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {2870, 2832, 2831, 2742, 2740, 2734, 2732}

$$\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} dx = \frac{4a(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{15b^2 d \sqrt{a + b \cos(c + dx)}} - \frac{2(2a^2 - 9b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{15b^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2 \sin(c + dx)(a + b \cos(c + dx))^{3/2}}{5bd} - \frac{4a \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{15bd}$$

[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]],x]

[Out] (-2*(2*a^2 - 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b*d) + (2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]
```

Rule 2870

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
& \text{integral} \\
&= \frac{2(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} + \frac{2 \int \left(\frac{3b}{2} - a \cos(c + dx)\right) \sqrt{a + b \cos(c + dx)} dx}{5b} \\
&= -\frac{4a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15bd} \\
&\quad + \frac{2(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} + \frac{4 \int \frac{\frac{7ab}{4} - \frac{1}{4}(2a^2 - 9b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{15b} \\
&= -\frac{4a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15bd} + \frac{2(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} \\
&\quad + \frac{1}{15} \left(9 - \frac{2a^2}{b^2}\right) \int \sqrt{a + b \cos(c + dx)} dx + \frac{(2a(a^2 - b^2)) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{15b^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{4a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} \\
&+ \frac{\left(\left(9-\frac{2a^2}{b^2}\right)\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{15\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+ \frac{\left(2a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{15b^2\sqrt{a+b\cos(c+dx)}} \\
&= \frac{2\left(9-\frac{2a^2}{b^2}\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+ \frac{4a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}} \\
&- \frac{4a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.87

$$\begin{aligned}
&\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}dx \\
&= \frac{-2(2a^3+2a^2b-9ab^2-9b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+4a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]],x]

[Out] (-2*(2*a^3 + 2*a^2*b - 9*a*b^2 - 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 4*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(2*a^2 + 3*b^2 + 8*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*b^2*d*Sqrt[a + b*Cos[c + d*x]])


```
in(d*x + c) + 2*a)/b) + sqrt(2)*(4*I*a^3 + 3*I*a*b^2)*sqrt(b)*weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(2*I*a^2*b - 9*I*b^3)*sq
rt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(-2*I*a^2
*b + 9*I*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(
3*b^3*cos(d*x + c) + a*b^2)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^3*d)
```

Sympy [F]

$$\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{a + b \cos(c + dx)} \cos^2(c + dx) dx$$

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**2, x)
```

Maxima [F]

$$\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \cos(dx + c)^2 dx$$

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \cos(dx + c)^2 dx$$

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \cos(c + dx)^2 \sqrt{a + b \cos(c + dx)} dx$$

```
[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2), x)
```

3.488 $\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx$

Optimal result	5177
Rubi [A] (verified)	5177
Mathematica [A] (verified)	5180
Maple [B] (verified)	5180
Fricas [C] (verification not implemented)	5181
Sympy [F]	5181
Maxima [F]	5181
Giac [F]	5182
Mupad [F(-1)]	5182

Optimal result

Integrand size = 21, antiderivative size = 162

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx = \frac{2a \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3bd \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3bd \sqrt{a + b \cos(c + dx)}} + \frac{2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d}$$

```
[Out] 2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {2832, 2831, 2742, 2740, 2734, 2732}

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx = -\frac{2(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3bd \sqrt{a + b \cos(c + dx)}} + \frac{2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{3d} + \frac{2a \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3bd \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[In] Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]]

], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2}{3} \int \frac{\frac{b}{2} + \frac{1}{2}a \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{a \int \sqrt{a + b \cos(c + dx)} dx}{3b} - \frac{(a^2 - b^2) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{3b} \\
 &= \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{\left(a\sqrt{a + b \cos(c + dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{3b\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 &\quad - \frac{\left((a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{3b\sqrt{a + b \cos(c + dx)}} \\
 &= \frac{2a\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 &\quad - \frac{2(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3bd\sqrt{a + b \cos(c + dx)}} \\
 &\quad + \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.85

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx$$

$$= \frac{2a(a + b) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a + b}\right) - 2(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a + b}\right) + 2b(a + b \cos(c + dx))}{3bd \sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x]/(3*b*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(204) = 408.

Time = 5.45 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.79

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a - b\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(4\left(\cos^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b^2 + 2\left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)ab - 6\left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b^2 - a^2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right)}$

[In] int(cos(d*x+c)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2-a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+1/2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/b/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.46

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx$$

$$= \frac{3i \sqrt{2} ab^{\frac{3}{2}} \text{weierstrassZeta}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}\right), \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx)}{b^2}\right)}{b^2}$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/9*(3*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*sqrt(b*cos(d*x + c) + a)*b^2*sin(d*x + c) + sqrt(2)*(2*I*a^2 - 3*I*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-2*I*a^2 + 3*I*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b))/ (b^2*d)

Sympy [F]

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{a + b \cos(c + dx)} \cos(c + dx) dx$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a + b*cos(c + d*x))*cos(c + d*x), x)

Maxima [F]

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)

Giac [F]

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \cos(c + dx) \sqrt{a + b \cos(c + dx)} dx$$

[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2), x)

3.489 $\int \sqrt{a + b \cos(c + dx)} dx$

Optimal result	5183
Rubi [A] (verified)	5183
Mathematica [A] (verified)	5184
Maple [B] (verified)	5184
Fricas [C] (verification not implemented)	5185
Sympy [F]	5185
Maxima [F]	5186
Giac [F]	5186
Mupad [F(-1)]	5186

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \sqrt{a + b \cos(c + dx)} dx = \frac{2\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[Out] $2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b))^{(1/2)}*(a+b*\cos(d*x+c))^{(1/2)}/d/((a+b*\cos(d*x+c))/(a+b))^{(1/2)})$

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2734, 2732}

$$\int \sqrt{a + b \cos(c + dx)} dx = \frac{2\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[In] `Int[Sqrt[a + b*Cos[c + d*x]], x]`

[Out] $(2*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, (2*b)/(a + b)])/(d*\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/(a + b)])$

Rule 2732

`Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]`

Rule 2734

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{a + b \cos(c + dx)} \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\ &= \frac{2\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \sqrt{a + b \cos(c + dx)} dx = \frac{2\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(82) = 164.

Time = 3.00 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.98

method	result	size
default	$\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a - b\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{\frac{2b\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a - b}{a - b}} E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{-\frac{2b}{a-b}}\right)(a-b)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + (a+b)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a + b} d}$	170
risch	Expression too large to display	1046

```
[In] int((a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos
```

$(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}*(a-b)/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 6.23

$$\int \sqrt{a + b \cos(c + dx)} dx$$

$$= -i \sqrt{2a} \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3ib \sin(dx+c) + 2a}{3b} \right) + i \sqrt{2a} \sqrt{b} \text{weierstrassPInverse} \left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) - 3ib \sin(dx+c) + 2a}{3b} \right) / (b*d)$$

[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $1/3*(-I*\sqrt{2}*a*\sqrt{b}*\text{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) + 3*I*b*\sin(d*x + c) + 2*a)/b) + I*\sqrt{2}*a*\sqrt{b}*\text{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) - 3*I*b*\sin(d*x + c) + 2*a)/b) + 3*I*\sqrt{2}*b^{(3/2)}*\text{weierstrassZeta}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, \text{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) + 3*I*b*\sin(d*x + c) + 2*a)/b)) - 3*I*\sqrt{2}*b^{(3/2)}*\text{weierstrassZeta}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, \text{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) - 3*I*b*\sin(d*x + c) + 2*a)/b)))/(b*d)$

Sympy [F]

$$\int \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{a + b \cos(c + dx)} dx$$

[In] integrate((a+b*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a + b*cos(c + d*x)), x)

Maxima [F]

$$\int \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{a + b \cos(c + dx)} dx$$

[In] int((a + b*cos(c + d*x))^(1/2),x)

[Out] int((a + b*cos(c + d*x))^(1/2), x)

3.490 $\int \sqrt{a + b \cos(c + dx)} \sec(c + dx) dx$

Optimal result	5187
Rubi [A] (verified)	5187
Mathematica [A] (verified)	5189
Maple [A] (verified)	5189
Fricas [F(-1)]	5190
Sympy [F]	5190
Maxima [F]	5190
Giac [F]	5190
Mupad [F(-1)]	5191

Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \sqrt{a + b \cos(c + dx)} \sec(c + dx) dx = \frac{2b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} + \frac{2a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}}$$

[Out] $2*b*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b))^{(1/2)})*((a+b*\cos(d*x+c))/(a+b))^{(1/2)}/d/(a+b*\cos(d*x+c))^{(1/2)}+2*a*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2^{(1/2)}*(b/(a+b))^{(1/2)})*((a+b*\cos(d*x+c))/(a+b))^{(1/2)}/d/(a+b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2882, 2742, 2740, 2886, 2884}

$$\int \sqrt{a + b \cos(c + dx)} \sec(c + dx) dx = \frac{2b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} + \frac{2a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[a + b*\operatorname{Cos}[c + d*x]]*\operatorname{Sec}[c + d*x], x]$

```
[Out] (2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)
))/ (d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2882

```
Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
+ Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x])
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rubi steps

$$\text{integral} = a \int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx + b \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx$$

$$\begin{aligned}
&= \frac{\left(a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{\sqrt{a+b\cos(c+dx)}} + \frac{\left(b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{\sqrt{a+b\cos(c+dx)}} \\
&= \frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 13.94 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.69

$$\begin{aligned}
&\int \sqrt{a+b\cos(c+dx)} \sec(c+dx) dx \\
&= \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \left(b \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right) + a \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x], x]

[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt[a + b*Cos[c + d*x]])

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.64

method	result
default	$ -\frac{2\sqrt{\left(2b\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + a - b\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + a - b\right)}{a - b}}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + (a+b)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a + b d}} \left(F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{-\frac{2b}{a-b}}\right)b - \Pi\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{-\frac{2b}{a-b}}\right)\right) $

[In] int(sec(d*x+c)*(a+cos(d*x+c)*b)^(1/2), x, method=_RETURNVERBOSE)

[Out] -2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))*b-EllipticPi(cos(1/2*d*x+1/2*c), 2, (-2*b/(a-b))^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec(c + dx) dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec(c + dx) dx = \int \sqrt{a + b \cos(c + dx)} \sec(c + dx) dx$$

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c) dx$$

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Giac [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c) dx$$

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec(c + dx) dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos(c + dx)} dx$$

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.491 $\int \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) dx$

Optimal result	5192
Rubi [A] (verified)	5193
Mathematica [C] (verified)	5195
Maple [B] (verified)	5196
Fricas [F(-1)]	5197
Sympy [F]	5197
Maxima [F]	5197
Giac [F]	5197
Mupad [F(-1)]	5198

Optimal result

Integrand size = 23, antiderivative size = 197

$$\int \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) dx = -\frac{\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} + \frac{b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} + \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2875, 3139, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) dx = \frac{\tan(c + dx) \sqrt{a + b \cos(c + dx)}}{d} + \frac{a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} - \frac{\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2,x]

[Out] -((Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]/(a + b))) + (a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[a + b*Cos[c + d*x]]) + (b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2875

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(
n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
- b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
-1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)]))/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3139

```
Int[((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2/(Sqrt[(a_) + (b_)*sin[(e_)
+ (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c
```

+ d*Sin[e + f*x)), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d} + \int \frac{\left(\frac{b}{2} - \frac{1}{2}b \cos^2(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d} - \frac{1}{2} \int \sqrt{a + b \cos(c + dx)} dx \\
&\quad - \frac{\int \frac{\left(-\frac{b^2}{2} - \frac{1}{2}ab \cos(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{b} \\
&= \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d} + \frac{1}{2}a \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx \\
&\quad + \frac{1}{2}b \int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx - \frac{\sqrt{a + b \cos(c + dx)} \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{2\sqrt{\frac{a+b \cos(c + dx)}{a+b}}} \\
&= -\frac{\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b \cos(c + dx)}{a+b}}} + \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d} \\
&\quad + \frac{\left(a\sqrt{\frac{a+b \cos(c + dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{2\sqrt{a + b \cos(c + dx)}} + \frac{\left(b\sqrt{\frac{a+b \cos(c + dx)}{a+b}}\right) \int \frac{\sec(c + dx)}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{2\sqrt{a + b \cos(c + dx)}} \\
&= -\frac{\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b \cos(c + dx)}{a+b}}} + \frac{a\sqrt{\frac{a+b \cos(c + dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{b\sqrt{\frac{a+b \cos(c + dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}} + \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.87 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.56

$$\begin{aligned}
&\int \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) dx \\
&\frac{2b\sqrt{\frac{a+b \cos(c + dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{\sqrt{a + b \cos(c + dx)}} - \frac{2i\sqrt{-\frac{b(-1 + \cos(c + dx))}{a+b}} \sqrt{\frac{b(1 + \cos(c + dx))}{-a+b}} \csc(c + dx) \left(-2a(a-b)E\left(i \operatorname{arcsinh}\left(\sqrt{-\frac{1}{a+b}} \sqrt{a+b}\right)\right)\right)}{\sqrt{a + b \cos(c + dx)}} \\
&= \dots
\end{aligned}$$

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] ((2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(270) = 540.

Time = 4.96 (sec) , antiderivative size = 622, normalized size of antiderivative = 3.16

method	result
default	$-\frac{\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(-2a-2b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{-2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}}\right)$

```
[In] int(sec(d*x+c)^2*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b+(-2*a-2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b-EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b-b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```


Fricas [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) dx = \int \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) dx$$

```
[In] integrate(sec(d*x+c)**2*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^2 dx$$

```
[In] integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^2 dx$$

```
[In] integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos(c + dx)^2} dx$$

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.492 $\int \sqrt{a + b \cos(c + dx)} \sec^3(c + dx) dx$

Optimal result	5199
Rubi [A] (verified)	5200
Mathematica [C] (verified)	5203
Maple [B] (verified)	5204
Fricas [F(-1)]	5205
Sympy [F]	5205
Maxima [F]	5205
Giac [F]	5205
Mupad [F(-1)]	5206

Optimal result

Integrand size = 23, antiderivative size = 262

$$\begin{aligned}
 & \int \sqrt{a + b \cos(c + dx)} \sec^3(c + dx) dx \\
 &= -\frac{b\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{3b\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4d\sqrt{a + b \cos(c + dx)}} \\
 &+ \frac{(4a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4ad\sqrt{a + b \cos(c + dx)}} \\
 &+ \frac{b\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{4ad} + \frac{\sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d}
 \end{aligned}$$

```

[Out] -1/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+b*cos(d*x+
c))/(a+b))^(1/2)+3/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(
1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(b/(a+b))^(1/2)
)*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*b*(a+b*cos(
d*x+c))^(1/2)*tan(d*x+c)/a/d+1/2*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+
c)/d

```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2875, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int \sqrt{a + b \cos(c + dx)} \sec^3(c + dx) dx$$

$$= \frac{(4a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4ad \sqrt{a + b \cos(c + dx)}} + \frac{b \tan(c + dx) \sqrt{a + b \cos(c + dx)}}{4ad} + \frac{3b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4d \sqrt{a + b \cos(c + dx)}} - \frac{b \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{4ad \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{\tan(c + dx) \sec(c + dx) \sqrt{a + b \cos(c + dx)}}{2d}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^3,x]

[Out] -1/4*(b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (3*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(4*a*d*Sqrt[a + b*Cos[c + d*x]]) + (b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*d) + (Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2875

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(
n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x]
- b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m,
-1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)]))/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) +
```

```

+ (f_.)*(x_)^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))

```

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d} \\
&+ \frac{1}{2} \int \frac{\left(\frac{b}{2} + a \cos(c + dx) + \frac{1}{2}b \cos^2(c + dx)\right) \sec^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{b\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{4ad} + \frac{\sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d} \\
&+ \frac{\int \frac{\left(\frac{1}{4}(4a^2 - b^2) + \frac{1}{2}ab \cos(c + dx) - \frac{1}{4}b^2 \cos^2(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{2a} \\
&= \frac{b\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{4ad} + \frac{\sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d} \\
&- \frac{\int \frac{\left(-\frac{1}{4}b(4a^2 - b^2) - \frac{3}{4}ab^2 \cos(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{2ab} - \frac{b \int \sqrt{a + b \cos(c + dx)} dx}{8a}
\end{aligned}$$

$$\begin{aligned}
&= \frac{b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad + \frac{1}{8}(3b) \int \frac{1}{\sqrt{a+b\cos(c+dx)}} dx + \frac{(4a^2-b^2) \int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{8a} \\
&\quad - \frac{\left(b\sqrt{a+b\cos(c+dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}} dx}{8a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&= -\frac{b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} \\
&\quad + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad + \frac{\left(3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{8\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\left((4a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{8a\sqrt{a+b\cos(c+dx)}} \\
&= -\frac{b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad + \frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{(4a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.68 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.50

$$\begin{aligned}
&\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx) dx \\
&= \frac{8b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2(8a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{a\sqrt{a+b\cos(c+dx)}} - \frac{2i\sqrt{\frac{-b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b(1+\cos(c+dx))}{-a+b}}}{\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

$*b/(a-b)^{(1/2)}*b^2/a/(2*\cos(1/2*d*x+1/2*c)^2-1)^2/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/2)}/d$

Fricas [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec^3(c + dx) dx = \text{Timed out}$$

[In] `integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^3(c + dx) dx = \int \sqrt{a + b \cos(c + dx)} \sec^3(c + dx) dx$$

[In] `integrate(sec(d*x+c)**3*(a+b*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**3, x)`

Maxima [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^3(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^3 dx$$

[In] `integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)`

Giac [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^3(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^3 dx$$

[In] `integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)`

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec^3(c + dx) dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos(c + dx)^3} dx$$

```
[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.493 $\int \cos^3(c + dx)(a + b \cos(c + dx))^{3/2} dx$

Optimal result	5207
Rubi [A] (verified)	5208
Mathematica [A] (verified)	5212
Maple [B] (verified)	5212
Fricas [C] (verification not implemented)	5213
Sympy [F(-1)]	5214
Maxima [F]	5214
Giac [F]	5214
Mupad [F(-1)]	5214

Optimal result

Integrand size = 23, antiderivative size = 314

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{3/2} dx = \frac{2(8a^4 + 33a^2b^2 + 147b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) + 2a(8a^4 + 31a^2b^2 - 39b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right) - 2a(8a^2 + 39b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx) + \frac{2(8a^2 + 49b^2)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315b^2d} - \frac{8a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63b^2d} + \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{9bd}}{315b^3d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

```
[Out] 2/315*(8*a^2+49*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d-8/63*a*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/9*cos(d*x+c)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/315*a*(8*a^2+39*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/315*(8*a^4+33*a^2*b^2+147*b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/315*a*(8*a^4+31*a^2*b^2-39*b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2872, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{3/2} dx = \frac{2(8a^2 + 49b^2) \sin(c + dx)(a + b \cos(c + dx))^{3/2}}{315b^2d} + \frac{2a(8a^2 + 39b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{315b^2d} - \frac{2a(8a^4 + 31a^2b^2 - 39b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{315b^3d \sqrt{a + b \cos(c + dx)}} + \frac{2(8a^4 + 33a^2b^2 + 147b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{315b^3d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{8a \sin(c + dx)(a + b \cos(c + dx))^{5/2}}{63b^2d} + \frac{2 \sin(c + dx) \cos(c + dx)(a + b \cos(c + dx))^{5/2}}{9bd}$$

[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(3/2), x]

[Out] (2*(8*a^4 + 33*a^2*b^2 + 147*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(315*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(8*a^4 + 31*a^2*b^2 - 39*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(315*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*(8*a^2 + 39*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^2*d) + (2*(8*a^2 + 49*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^2*d) - (8*a*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b^2*d) + (2*Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*b*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m/(f*(m + 1))))], x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]
```

Rule 2872

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_)), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2)), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{9bd} \\
&+ \frac{2 \int (a + b \cos(c + dx))^{3/2} (a + \frac{7}{2}b \cos(c + dx) - 2a \cos^2(c + dx)) dx}{9b} \\
&= -\frac{8a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63b^2d} \\
&+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{9bd} \\
&+ \frac{4 \int (a + b \cos(c + dx))^{3/2} (-\frac{3ab}{2} + \frac{1}{4}(8a^2 + 49b^2) \cos(c + dx)) dx}{63b^2} \\
&= \frac{2(8a^2 + 49b^2)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315b^2d} \\
&- \frac{8a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63b^2d} \\
&+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{9bd} \\
&+ \frac{8 \int \sqrt{a + b \cos(c + dx)} (-\frac{3}{8}b(2a^2 - 49b^2) + \frac{3}{8}a(8a^2 + 39b^2) \cos(c + dx)) dx}{315b^2} \\
&= \frac{2a(8a^2 + 39b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315b^2d} \\
&+ \frac{2(8a^2 + 49b^2)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315b^2d} \\
&- \frac{8a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63b^2d} \\
&+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{9bd} \\
&+ \frac{16 \int \frac{\frac{3}{8}ab(a^2 + 93b^2) + \frac{3}{16}(8a^4 + 33a^2b^2 + 147b^4) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{945b^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2a(8a^2 + 39b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315b^2d} \\
&+ \frac{2(8a^2 + 49b^2) (a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315b^2d} \\
&- \frac{8a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63b^2d} \\
&+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{9bd} \\
&- \frac{(a(8a^4 + 31a^2b^2 - 39b^4)) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{315b^3} \\
&+ \frac{(8a^4 + 33a^2b^2 + 147b^4) \int \sqrt{a + b \cos(c + dx)} dx}{315b^3} \\
&= \frac{2a(8a^2 + 39b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315b^2d} \\
&+ \frac{2(8a^2 + 49b^2) (a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315b^2d} \\
&- \frac{8a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63b^2d} \\
&+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{9bd} \\
&+ \frac{\left((8a^4 + 33a^2b^2 + 147b^4) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{315b^3 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&- \frac{\left(a(8a^4 + 31a^2b^2 - 39b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{315b^3 \sqrt{a + b \cos(c + dx)}} \\
&= \frac{2(8a^4 + 33a^2b^2 + 147b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{315b^3d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&- \frac{2a(8a^4 + 31a^2b^2 - 39b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{315b^3d \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{2a(8a^2 + 39b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315b^2d} \\
&+ \frac{2(8a^2 + 49b^2) (a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315b^2d} \\
&- \frac{8a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63b^2d} \\
&+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{9bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.83

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{3/2} dx = \frac{8(8a^5 + 8a^4b + 33a^3b^2 + 33a^2b^3 + 147ab^4 + 147b^5) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) - 8a(8a^4 + 31a^3b + 1606a^2b^2 - 39b^4) \sqrt{a + b \cos(c + dx)} + 170ab^3 \cos(3(c + dx)) + 35b^4 \cos(4(c + dx)) \sin(c + dx)}{(1260b^3 \sqrt{a + b \cos(c + dx)})}$$

[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(3/2),x]

[Out] (8*(8*a^5 + 8*a^4*b + 33*a^3*b^2 + 33*a^2*b^3 + 147*a*b^4 + 147*b^5)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 8*a*(8*a^4 + 31*a^2*b^2 - 39*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-32*a^4 + 916*a^2*b^2 + 301*b^4 + (-8*a^3*b + 1606*a*b^3)*Cos[c + d*x] + 4*(53*a^2*b^2 + 84*b^4)*Cos[2*(c + d*x)] + 170*a*b^3*Cos[3*(c + d*x)] + 35*b^4*Cos[4*(c + d*x)])*Sin[c + d*x]/(1260*b^3*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(344) = 688.

Time = 8.67 (sec) , antiderivative size = 995, normalized size of antiderivative = 3.17

method	result	size
default	Expression too large to display	995

[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^5+(1360*a*b^4+2240*b^5)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-424*a^2*b^3-2040*a*b^4-2072*b^5)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-4*a^3*b^2+424*a^2*b^3+1568*a*b^4+952*b^5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(8*a^4*b+2*a^3*b^2-282*a^2*b^3-444*a*b^4-168*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-31*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+39*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4+8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+33*(s


```

in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-33*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^3/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/
2)/d

```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.64

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{3/2} dx =$$

$$4\sqrt{2}(-4i a^5 - 15i a^3 b^2 + 66i a b^4)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3ib \sin(dx+c)}{3b}\right)$$

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```

[Out] -1/945*(4*sqrt(2)*(-4*I*a^5 - 15*I*a^3*b^2 + 66*I*a*b^4)*sqrt(b)*weierstras
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*co
s(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + 4*sqrt(2)*(4*I*a^5 + 15*I*a^3*b
^2 - 66*I*a*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b
) + 3*sqrt(2)*(-8*I*a^4*b - 33*I*a^2*b^3 - 147*I*b^5)*sqrt(b)*weierstrassZe
ta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInvers
e(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x +
c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*sqrt(2)*(8*I*a^4*b + 33*I*a^2*b^3 +
147*I*b^5)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 -
9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) - 6*(35*
b^5*cos(d*x + c)^3 + 50*a*b^4*cos(d*x + c)^2 - 4*a^3*b^2 + 88*a*b^4 + (3*a^
2*b^3 + 49*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^4*d
)

```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{3/2} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^3 dx$$

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^3 dx$$

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int \cos(c + dx)^3 (a + b \cos(c + dx))^{3/2} dx$$

```
[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2), x)
```

3.494 $\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2} dx$

Optimal result	5215
Rubi [A] (verified)	5216
Mathematica [A] (verified)	5219
Maple [B] (verified)	5219
Fricas [C] (verification not implemented)	5220
Sympy [F(-1)]	5220
Maxima [F]	5221
Giac [F]	5221
Mupad [F(-1)]	5221

Optimal result

Integrand size = 23, antiderivative size = 258

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2} dx =$$

$$-\frac{4a(3a^2 - 41b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{105b^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

$$+ \frac{2(6a^4 - 31a^2b^2 + 25b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{105b^2 d \sqrt{a + b \cos(c + dx)}}$$

$$- \frac{2(6a^2 - 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105bd}$$

$$- \frac{4a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35bd} + \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd}$$

[Out] $-4/35*a*(a+b*\cos(d*x+c))^(3/2)*\sin(d*x+c)/b/d+2/7*(a+b*\cos(d*x+c))^(5/2)*\sin(d*x+c)/b/d-2/105*(6*a^2-25*b^2)*\sin(d*x+c)*(a+b*\cos(d*x+c))^(1/2)/b/d-4/105*a*(3*a^2-41*b^2)*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*\cos(d*x+c))^(1/2)/b^2/d/((a+b*\cos(d*x+c))/(a+b))^(1/2)+2/105*(6*a^4-31*a^2*b^2+25*b^4)*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*\cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*\cos(d*x+c))^(1/2)$

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2870, 2832, 2831, 2742, 2740, 2734, 2732}

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2} dx =$$

$$\frac{2(6a^2 - 25b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{105bd}$$

$$- \frac{4a(3a^2 - 41b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{105b^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

$$+ \frac{2(6a^4 - 31a^2b^2 + 25b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{105b^2 d \sqrt{a + b \cos(c + dx)}}$$

$$+ \frac{2 \sin(c + dx)(a + b \cos(c + dx))^{5/2}}{7bd} - \frac{4a \sin(c + dx)(a + b \cos(c + dx))^{3/2}}{35bd}$$

[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2), x]

[Out] (-4*a*(3*a^2 - 41*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(6*a^4 - 31*a^2*b^2 + 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(105*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(6*a^2 - 25*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b*d) - (4*a*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*b*d) + (2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m/(f*(m + 1)))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2870

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
 &+ \frac{2 \int \left(\frac{5b}{2} - a \cos(c + dx)\right) (a + b \cos(c + dx))^{3/2} dx}{7b} \\
 &= -\frac{4a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35bd} + \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
 &+ \frac{4 \int \sqrt{a + b \cos(c + dx)} \left(\frac{19ab}{4} - \frac{1}{4}(6a^2 - 25b^2) \cos(c + dx)\right) dx}{35b}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{2(6a^2 - 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105bd} - \frac{4a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35bd} \\
&+ \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} + \frac{8 \int \frac{\frac{1}{8}b(51a^2 + 25b^2) - \frac{1}{4}a(3a^2 - 41b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{105b} \\
&= -\frac{2(6a^2 - 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105bd} \\
&- \frac{4a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35bd} + \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
&+ \frac{1}{105} \left(2a \left(41 - \frac{3a^2}{b^2} \right) \right) \int \sqrt{a + b \cos(c + dx)} dx \\
&+ \frac{(6a^4 - 31a^2b^2 + 25b^4) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{105b^2} \\
&= -\frac{2(6a^2 - 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105bd} \\
&- \frac{4a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35bd} + \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
&+ \frac{\left(2a \left(41 - \frac{3a^2}{b^2} \right) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{105 \sqrt{\frac{a+b \cos(c + dx)}{a+b}}} \\
&+ \frac{\left((6a^4 - 31a^2b^2 + 25b^4) \sqrt{\frac{a+b \cos(c + dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{105b^2 \sqrt{a + b \cos(c + dx)}} \\
&= \frac{4a \left(41 - \frac{3a^2}{b^2} \right) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{105d \sqrt{\frac{a+b \cos(c + dx)}{a+b}}} \\
&+ \frac{2(6a^4 - 31a^2b^2 + 25b^4) \sqrt{\frac{a+b \cos(c + dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{105b^2 d \sqrt{a + b \cos(c + dx)}} \\
&- \frac{2(6a^2 - 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105bd} \\
&- \frac{4a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35bd} + \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd}
\end{aligned}$$

$$\frac{(1/2*d*x+1/2*c)^2+a-b}{(a-b)}^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)})*a*b^3-6*\cos(1/2*d*x+1/2*c)*a^3*b+54*\cos(1/2*d*x+1/2*c)*a^2*b^2-12*8*\cos(1/2*d*x+1/2*c)*a*b^3+80*\cos(1/2*d*x+1/2*c)*b^4)/b^2/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.84

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2} dx = \frac{\sqrt{2}(-12i a^4 + 11i a^2 b^2 - 75i b^4)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i}{3b}\right)}{\dots}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(sqrt(2)*(-12*I*a^4 + 11*I*a^2*b^2 - 75*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(12*I*a^4 - 11*I*a^2*b^2 + 75*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 6*sqrt(2)*(3*I*a^3*b - 41*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 6*sqrt(2)*(-3*I*a^3*b + 41*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(15*b^4*cos(d*x + c)^2 + 24*a*b^3*cos(d*x + c) + 3*a^2*b^2 + 25*b^4)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```


Maxima [F]

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{3/2} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)

Giac [F]

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{3/2} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int \cos(c + dx)^2 (a + b \cos(c + dx))^{3/2} dx$$

[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2), x)

3.495 $\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} dx$

Optimal result	5222
Rubi [A] (verified)	5222
Mathematica [A] (verified)	5225
Maple [B] (verified)	5225
Fricas [C] (verification not implemented)	5226
Sympy [F]	5227
Maxima [F]	5227
Giac [F]	5227
Mupad [F(-1)]	5227

Optimal result

Integrand size = 21, antiderivative size = 199

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} dx = \frac{2(a^2 + 3b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{5bd \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2a(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{5bd \sqrt{a + b \cos(c + dx)}} + \frac{2a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d} + \frac{2(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d}$$

[Out] $2/5*(a+b*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+2/5*a*\sin(d*x+c)*(a+b*\cos(d*x+c))^(1/2)/d+2/5*(a^2+3*b^2)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*\cos(d*x+c))^(1/2)/b/d/((a+b*\cos(d*x+c))/(a+b))^(1/2)-2/5*a*(a^2-b^2)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*\cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*\cos(d*x+c))^(1/2)$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {2832, 2831, 2742, 2740, 2734, 2732}

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} dx =$$

$$-\frac{2a(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{5bd \sqrt{a + b \cos(c + dx)}} +$$

$$+\frac{2(a^2 + 3b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{5bd \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} +$$

$$+\frac{2 \sin(c + dx)(a + b \cos(c + dx))^{3/2}}{5d} + \frac{2a \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{5d}$$

[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2), x]

[Out] (2*(a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(5*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(5*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*SIN[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*SIN[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&+ \frac{2}{5} \int \left(\frac{3b}{2} + \frac{3}{2} a \cos(c + dx) \right) \sqrt{a + b \cos(c + dx)} dx \\
&= \frac{2a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d} + \frac{2(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&+ \frac{4}{15} \int \frac{3ab + \frac{3}{4}(a^2 + 3b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d} + \frac{2(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&- \frac{(a(a^2 - b^2)) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{5b} + \frac{(a^2 + 3b^2) \int \sqrt{a + b \cos(c + dx)} dx}{5b} \\
&= \frac{2a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d} + \frac{2(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&+ \frac{\left((a^2 + 3b^2) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{5b \sqrt{\frac{a+b \cos(c + dx)}{a+b}}} \\
&- \frac{\left(a(a^2 - b^2) \sqrt{\frac{a+b \cos(c + dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{5b \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2(a^2 + 3b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{5bd \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad - \frac{2a(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{5bd \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{2a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d} + \frac{2(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.87

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} dx = \frac{2(a^3 + a^2b + 3ab^2 + 3b^3) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) - 2a(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right) + b(4a^2 + b^2 + 6ab \cos(c + dx) + b^2 \cos[2(c + dx)]) \sin(c + dx)}{5bd \sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2), x]

[Out] (2*(a^3 + a^2*b + 3*a*b^2 + 3*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(4*a^2 + b^2 + 6*a*b*Cos[c + d*x] + b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(5*b*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(237) = 474.

Time = 6.56 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.33

method	result
default	$ \frac{2 \sqrt{\left(2b \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + a - b\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(8 \left(\cos^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b^3 + 12 \left(\cos^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a b^2 - 16 \left(\cos^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b^3 + 4 \left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a^2 b - 18 \left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a^2 b^2 + 10 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(a^3 b^3 - a^3 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{1/2} \left((2b \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + a - b\right)^{1/2} \operatorname{EllipticF}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \left(-2b/(a-b)\right)^{1/2}\right) + a b^2 \sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{5bd \sqrt{a + b \cos(c + dx)}} $

[In] int(cos(d*x+c)*(a+cos(d*x+c)*b)^(3/2), x, method=_RETURNVERBOSE)

[Out] -2/5*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*cos(1/2*d*x+1/2*c)^7*b^3+12*cos(1/2*d*x+1/2*c)^5*a*b^2-16*cos(1/2*d*x+1/2*c)^5*b^3+4*cos(1/2*d*x+1/2*c)^3*a^2*b-18*cos(1/2*d*x+1/2*c)^3*a*b^2+10*cos(1/2*d*x+1/2*c)^3*b^3-a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))+a*b^2*(sin(1/2*d*x+1/2*c))

$$\begin{aligned} & \left(\frac{1}{2}dx + \frac{1}{2}c\right)^{1/2} \cdot \left(\frac{2b \cos(\frac{1}{2}dx + \frac{1}{2}c)^2 + a - b}{a - b}\right)^{1/2} \cdot \text{EllipticF}\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right), \left(-\frac{2b}{a - b}\right)^{1/2}\right) \\ & + \left(\frac{1}{2}dx + \frac{1}{2}c\right)^{1/2} \cdot \left(\frac{2b \cos(\frac{1}{2}dx + \frac{1}{2}c)^2 + a - b}{a - b}\right)^{1/2} \cdot \text{EllipticE}\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right), \left(-\frac{2b}{a - b}\right)^{1/2}\right) \\ & \cdot a^3 - \left(\frac{1}{2}dx + \frac{1}{2}c\right)^{1/2} \cdot \left(\frac{2b \cos(\frac{1}{2}dx + \frac{1}{2}c)^2 + a - b}{a - b}\right)^{1/2} \cdot \text{EllipticE}\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right), \left(-\frac{2b}{a - b}\right)^{1/2}\right) \\ & \cdot a^2 b + 3 \cdot \left(\frac{1}{2}dx + \frac{1}{2}c\right)^{1/2} \cdot \left(\frac{2b \cos(\frac{1}{2}dx + \frac{1}{2}c)^2 + a - b}{a - b}\right)^{1/2} \cdot \text{EllipticE}\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right), \left(-\frac{2b}{a - b}\right)^{1/2}\right) \\ & \cdot a \cdot b^2 - 3 \cdot \left(\frac{1}{2}dx + \frac{1}{2}c\right)^{1/2} \cdot \left(\frac{2b \cos(\frac{1}{2}dx + \frac{1}{2}c)^2 + a - b}{a - b}\right)^{1/2} \cdot \text{EllipticE}\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right), \left(-\frac{2b}{a - b}\right)^{1/2}\right) \\ & \cdot b^3 - 4 \cdot \cos\left(\frac{1}{2}dx + \frac{1}{2}c\right) \cdot a^2 b + 6 \cdot \cos\left(\frac{1}{2}dx + \frac{1}{2}c\right) \cdot a \cdot b^2 - 2 \cdot \cos\left(\frac{1}{2}dx + \frac{1}{2}c\right) \cdot b^3 \\ & / b / \left(-2 \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^4 b + (a + b) \cdot \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)^{1/2} / \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right) / \left(-2b \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)^2 + a + b)^{1/2} / d \end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.20

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} dx =$$

$$2\sqrt{2}(-i a^3 + 3i ab^2)\sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3ib \sin(dx+c) + 2a}{3b}\right) + 2\sqrt{2}(i a^3 + 3i ab^2)\sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) - 3ib \sin(dx+c) + 2a}{3b}\right)$$

[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] $-1/15 \cdot (2\sqrt{2}) \cdot (-I a^3 + 3I a b^2) \cdot \sqrt{b} \cdot \text{weierstrassPInverse}\left(\frac{4}{3} \cdot (4a^2 - 3b^2) / b^2, -8/27 \cdot (8a^3 - 9a b^2) / b^3, \frac{1}{3} \cdot (3b \cos(dx + c) + 3I b \sin(dx + c) + 2a) / b\right) + 2\sqrt{2} \cdot (I a^3 - 3I a b^2) \cdot \sqrt{b} \cdot \text{weierstrassPInverse}\left(\frac{4}{3} \cdot (4a^2 - 3b^2) / b^2, -8/27 \cdot (8a^3 - 9a b^2) / b^3, \frac{1}{3} \cdot (3b \cos(dx + c) - 3I b \sin(dx + c) + 2a) / b\right) + 3\sqrt{2} \cdot (-I a^2 b - 3I b^3) \cdot \sqrt{b} \cdot \text{weierstrassZeta}\left(\frac{4}{3} \cdot (4a^2 - 3b^2) / b^2, -8/27 \cdot (8a^3 - 9a b^2) / b^3, \text{weierstrassPInverse}\left(\frac{4}{3} \cdot (4a^2 - 3b^2) / b^2, -8/27 \cdot (8a^3 - 9a b^2) / b^3, \frac{1}{3} \cdot (3b \cos(dx + c) + 3I b \sin(dx + c) + 2a) / b\right) + 3\sqrt{2} \cdot (I a^2 b + 3I b^3) \cdot \sqrt{b} \cdot \text{weierstrassZeta}\left(\frac{4}{3} \cdot (4a^2 - 3b^2) / b^2, -8/27 \cdot (8a^3 - 9a b^2) / b^3, \text{weierstrassPInverse}\left(\frac{4}{3} \cdot (4a^2 - 3b^2) / b^2, -8/27 \cdot (8a^3 - 9a b^2) / b^3, \frac{1}{3} \cdot (3b \cos(dx + c) - 3I b \sin(dx + c) + 2a) / b\right) - 6 \cdot (b^3 \cos(dx + c) + 2a b^2) \cdot \sqrt{b \cos(dx + c) + a} \cdot \sin(dx + c)\right) / (b^2 d)$

Sympy [F]

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int (a + b \cos(c + dx))^{\frac{3}{2}} \cos(c + dx) dx$$

[In] `integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2),x)`

[Out] `Integral((a + b*cos(c + d*x))**(3/2)*cos(c + d*x), x)`

Maxima [F]

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \cos(dx + c) dx$$

[In] `integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)`

Giac [F]

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \cos(dx + c) dx$$

[In] `integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")`

[Out] `integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)`

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int \cos(c + dx) (a + b \cos(c + dx))^{3/2} dx$$

[In] `int(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2),x)`

[Out] `int(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2), x)`

3.496 $\int (a + b \cos(c + dx))^{3/2} dx$

Optimal result	5228
Rubi [A] (verified)	5228
Mathematica [A] (verified)	5230
Maple [B] (verified)	5231
Fricas [C] (verification not implemented)	5231
Sympy [F]	5232
Maxima [F]	5232
Giac [F]	5232
Mupad [F(-1)]	5232

Optimal result

Integrand size = 14, antiderivative size = 157

$$\int (a + b \cos(c + dx))^{3/2} dx = \frac{8a \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3d \sqrt{a + b \cos(c + dx)}} + \frac{2b \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d}$$

[Out] $2/3*b*\sin(d*x+c)*(a+b*\cos(d*x+c))^{(1/2)}/d+8/3*a*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b))^{(1/2)})*(a+b*\cos(d*x+c))^{(1/2)}/d/((a+b*\cos(d*x+c))/(a+b))^{(1/2)}-2/3*(a^2-b^2)*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b))^{(1/2)})*((a+b*\cos(d*x+c))/(a+b))^{(1/2)}/d/(a+b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2735, 2831, 2742, 2740, 2734, 2732}

$$\int (a + b \cos(c + dx))^{3/2} dx = -\frac{2(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3d \sqrt{a + b \cos(c + dx)}} + \frac{2b \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{3d} + \frac{8a \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2),x]

[Out] (8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*d*Sqrt[a + b*Cos[c + d*x]])) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2735

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3} \int \frac{\frac{1}{2}(3a^2+b^2)+2ab\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}(4a) \int \sqrt{a+b\cos(c+dx)} dx \\
 &\quad + \frac{1}{3}(-a^2+b^2) \int \frac{1}{\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} \\
 &\quad + \frac{\left(4a\sqrt{a+b\cos(c+dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}} dx}{3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
 &\quad + \frac{\left((-a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{3\sqrt{a+b\cos(c+dx)}} \\
 &= \frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
 &\quad - \frac{2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}} \\
 &\quad + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int (a+b\cos(c+dx))^{3/2} dx = \frac{8a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right) - 2(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2),x]

[Out] (8*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x]/(3*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(199) = 398.

Time = 4.68 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+2\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-6\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2-a^2\sqrt{\frac{1}{2}-\frac{\cos(dx+\frac{c}{2})}{2}}}\right)$

[In] `int((a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$-2/3*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(4*\cos(1/2*d*x+1/2*c)^5*b^2+2*\cos(1/2*d*x+1/2*c)^3*a*b-6*\cos(1/2*d*x+1/2*c)^3*b^2-a^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})+b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})+4*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^2-4*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a*b-2*\cos(1/2*d*x+1/2*c)*a*b+2*\cos(1/2*d*x+1/2*c)*b^2)/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.54

$$\int (a + b \cos(c + dx))^{3/2} dx = \frac{12i \sqrt{2} ab^3 \text{weierstrassZeta}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}\right)\right)}{\dots}$$

[In] `integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out]
$$1/9*(12*I*\sqrt{2}*a*b^3*\text{weierstrassZeta}(4/3*(4*a^2-3*b^2)/b^2, -8/27*(8*a^3-9*a*b^2)/b^3, \text{weierstrassPInverse}(4/3*(4*a^2-3*b^2)/b^2, -8/27*(8*a^3-9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x+c)+3*I*b*\sin(d*x+c)+2*a)/b)) - 12*I*\sqrt{2}*a*b^3*\text{weierstrassZeta}(4/3*(4*a^2-3*b^2)/b^2, -8/27*(8*a^3-9*a*b^2)/b^3, \text{weierstrassPInverse}(4/3*(4*a^2-3*b^2)/b^2, -8/27*(8*a^3-9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x+c)-3*I*b*\sin(d*x+c)+2*a)/b)) + 6*\sqrt{b*\cos(d*x+c)+a}*b^2*\sin(d*x+c) + \sqrt{2}*(-I*a^2-3*I*b^2)*s$$

```

qrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)
/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(I*a^2
+ 3*I*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a
^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b))/(b
*d)

```

Sympy [F]

$$\int (a + b \cos(c + dx))^{3/2} dx = \int (a + b \cos(c + dx))^{\frac{3}{2}} dx$$

```
[In] integrate((a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} dx = \int (a + b \cos(c + dx))^{\frac{3}{2}} dx$$

```
[In] int((a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(3/2), x)
```

3.497 $\int (a + b \cos(c + dx))^{3/2} \sec(c + dx) dx$

Optimal result	5233
Rubi [A] (verified)	5233
Mathematica [A] (verified)	5236
Maple [A] (verified)	5236
Fricas [F]	5237
Sympy [F]	5237
Maxima [F]	5237
Giac [F]	5237
Mupad [F(-1)]	5238

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int (a + b \cos(c + dx))^{3/2} \sec(c + dx) dx = \frac{2b\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2ab\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}} + \frac{2a^2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}}$$

```
[Out] 2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used

= {2883, 2734, 2732, 2882, 2742, 2740, 2886, 2884}

$$\int (a + b \cos(c + dx))^{3/2} \sec(c + dx) dx = \frac{2a^2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} + \frac{2ab \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} + \frac{2b \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x], x]

[Out] (2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[a + b*Cos[c + d*x]]))

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2882

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]

+ Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2883

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2886

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \sqrt{a + b \cos(c + dx)} \sec(c + dx) dx + b \int \sqrt{a + b \cos(c + dx)} dx \\
 &= a^2 \int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx + (ab) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx \\
 &\quad + \frac{\left(b \sqrt{a + b \cos(c + dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 &= \frac{2b \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{\left(a^2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{\sqrt{a + b \cos(c + dx)}} \\
 &\quad + \frac{\left(ab \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{\sqrt{a + b \cos(c + dx)}}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+ \frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{2a^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 26.93 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.60

$$\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)dx = \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}(b(a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right) + a(b\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right) + a\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right))}{d\sqrt{a+b\cos(c+dx)}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]

[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + a*(b*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]))/d*Sqrt[a + b*Cos[c + d*x]])

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.39

method	result
default	$ -\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}\sqrt{\frac{2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}{a-b}}\left(bF\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)+a+bE\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)\right)+\frac{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}\sqrt{-2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} $

[In] int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c),x,method=_RETURNVERBOSE)

[Out] -2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(b*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-b^2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

Fricas [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c) dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")

[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)

Sympy [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec(c + dx) dx = \int (a + b \cos(c + dx))^{3/2} \sec(c + dx) dx$$

[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c),x)

[Out] Integral((a + b*cos(c + d*x))**(3/2)*sec(c + d*x), x)

Maxima [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c) dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c) dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec(c + dx) dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos(c + dx)} dx$$

```
[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

```
[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

3.498 $\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx$

Optimal result	5239
Rubi [A] (verified)	5240
Mathematica [C] (verified)	5243
Maple [B] (verified)	5244
Fricas [F(-1)]	5244
Sympy [F(-1)]	5245
Maxima [F]	5245
Giac [F]	5245
Mupad [F(-1)]	5245

Optimal result

Integrand size = 23, antiderivative size = 209

$$\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx = -\frac{a\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid \frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{(a^2 + 2b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}} + \frac{3ab\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}} + \frac{a\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d}$$

```
[Out] -a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+(a^2+2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+a*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2878, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \frac{(a^2 + 2b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} + \frac{a \tan(c + dx) \sqrt{a + b \cos(c + dx)}}{d} - \frac{a \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{3ab \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]

[Out] -((a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + ((a^2 + 2*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[a + b*Cos[c + d*x]])) + (3*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[a + b*Cos[c + d*x]])) + (a*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2878

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*(a + b*Si
n[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))),
x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d
*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) +
(d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*
d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3081

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)])^n/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^
2)/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
```

x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} \\
&+ \int \frac{\left(\frac{3ab}{2} + b^2\cos(c+dx) - \frac{1}{2}ab\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{a\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{1}{2}a \int \sqrt{a+b\cos(c+dx)} dx \\
&\quad - \frac{\int \frac{\left(-\frac{3ab^2}{2} - \frac{1}{2}b(a^2+2b^2)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{b} \\
&= \frac{a\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \frac{1}{2}(3ab) \int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx \\
&\quad - \frac{1}{2}(-a^2 - 2b^2) \int \frac{1}{\sqrt{a+b\cos(c+dx)}} dx \\
&\quad - \frac{\left(a\sqrt{a+b\cos(c+dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}} dx}{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&= -\frac{a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{a\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} \\
&\quad + \frac{\left(3ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{2\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{\left((-a^2 - 2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{2\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+ \frac{(a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{3ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{a\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.28 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.74

$$\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)dx = \frac{b\left(\frac{8b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{10a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - 2i\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{b}{a+b}}\right)}{4d}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]

[Out] (b*((8*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (10*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(b^2*Sqrt[-(a + b)^(-1)]) + 4*a*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x]/(4*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. 2(282) = 564.

Time = 5.12 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.54

method	result
default	$-\frac{\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab+(-2a^2-2ab)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\sqrt{-}}$

[In] `int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)`

[Out]
$$-\left((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)*\sin(1/2*d*x+1/2*c)^2\right)^{(1/2)}*(4*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4*a*b+(-2*a^2-2*a*b)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)-2*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^2+2*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*b^2-\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^2+b*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a-3*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^{(1/2)})*a*b)*\sin(1/2*d*x+1/2*c)^2+(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^2+2*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})-(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^2+(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*b*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a-3*a*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^{(1/2)}))/\left(2*\cos(1/2*d*x+1/2*c)^2-1\right)/\left(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2\right)^{(1/2)}/\sin(1/2*d*x+1/2*c)/\left(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b\right)^{(1/2)}/d$$

Fricas [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \text{Timed out}$$

[In] `integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**2,x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^2 dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)

Giac [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^2 dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos(c + dx)^2} dx$$

[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)

[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)

3.499 $\int (a + b \cos(c + dx))^{3/2} \sec^3(c + dx) dx$

Optimal result	5246
Rubi [A] (verified)	5247
Mathematica [C] (verified)	5251
Maple [B] (verified)	5251
Fricas [F(-1)]	5252
Sympy [F(-1)]	5252
Maxima [F]	5253
Giac [F]	5253
Mupad [F(-1)]	5253

Optimal result

Integrand size = 23, antiderivative size = 255

$$\int (a + b \cos(c + dx))^{3/2} \sec^3(c + dx) dx = -\frac{5b\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{7ab\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4d\sqrt{a + b \cos(c + dx)}} + \frac{(4a^2 + 3b^2)\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4d\sqrt{a + b \cos(c + dx)}} + \frac{5b\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{4d} + \frac{a\sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d}$$

```
[Out] -5/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+7/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+5/4*b*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*a*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2878, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int (a + b \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \frac{(4a^2 + 3b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4d \sqrt{a + b \cos(c + dx)}} + \frac{5b \tan(c + dx) \sqrt{a + b \cos(c + dx)}}{4d} + \frac{7ab \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4d \sqrt{a + b \cos(c + dx)}} - \frac{5b \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{4d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{a \tan(c + dx) \sec(c + dx) \sqrt{a + b \cos(c + dx)}}{2d}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]

[Out] (-5*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (7*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(4*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2 + 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (5*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2878

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2886

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3081

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))

```

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Ssin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Ssin[e + f*x]]*(c + d*Ssin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&+ \frac{1}{2} \int \frac{\left(\frac{5ab}{2} + (a^2 + 2b^2)\cos(c+dx) + \frac{1}{2}ab\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{5b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&+ \frac{\int \frac{\left(\frac{1}{4}a(4a^2+3b^2) + \frac{1}{2}a^2b\cos(c+dx) - \frac{5}{4}ab^2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{2a} \\
&= \frac{5b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&- \frac{\int \frac{\left(-\frac{1}{4}ab(4a^2+3b^2) - \frac{7}{4}a^2b^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{2ab} - \frac{1}{8}(5b) \int \sqrt{a+b\cos(c+dx)} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{5b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&+ \frac{1}{8}(7ab) \int \frac{1}{\sqrt{a+b\cos(c+dx)}} dx - \frac{1}{8}(-4a^2-3b^2) \int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx \\
&- \frac{\left(5b\sqrt{a+b\cos(c+dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}} dx}{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&= -\frac{5b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{5b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} \\
&+ \frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&+ \frac{\left(7ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{8\sqrt{a+b\cos(c+dx)}} \\
&- \frac{\left((-4a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{8\sqrt{a+b\cos(c+dx)}} \\
&= -\frac{5b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+ \frac{7ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{(4a^2+3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{5b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} \\
&+ \frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.85 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.51

$$\int (a + b \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \frac{4ab \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b \cos(c+dx)}} + \frac{(8a^2+b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b \cos(c+dx)}} - \frac{5i \sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}}{\sqrt{a+b \cos(c+dx)}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]

[Out] ((4*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] + ((8*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] - ((5*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b))))/(a*Sqrt[-(a + b)^(-1)] + 2*Sqrt[a + b*Cos[c + d*x]])*(2*a + 5*b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(8*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 979 vs. 2(316) = 632.

Time = 6.47 (sec) , antiderivative size = 980, normalized size of antiderivative = 3.84

method	result	size
default	Expression too large to display	980

[In] int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] -1/4*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-40*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^2+(28*a*b+40*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-4*a^2-14*a*b-10*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2+3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2-7*b*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+5*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-5*b^2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))

```

^(1/2))*a^2+3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2-7*b*E
llipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+5*b*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a-5*b^2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*sin(1/2*d*x+1/2*c)^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2
*b/(a-b))^(1/2))*a^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))*b^2+7*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-5*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+5*b^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)^2/(-
2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

```

Fricas [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```


Maxima [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^3 dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)

Giac [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^3 dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^3(c + dx) dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos(c + dx)^3} dx$$

[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)

[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)

3.500 $\int \cos^3(c + dx)(a + b \cos(c + dx))^{5/2} dx$

Optimal result	5254
Rubi [A] (verified)	5255
Mathematica [A] (verified)	5260
Maple [B] (verified)	5260
Fricas [C] (verification not implemented)	5261
Sympy [F(-1)]	5262
Maxima [F]	5262
Giac [F]	5262
Mupad [F(-1)]	5262

Optimal result

Integrand size = 23, antiderivative size = 371

$$\begin{aligned}
 \int \cos^3(c + dx)(a + b \cos(c + dx))^{5/2} dx = & \frac{2a(8a^4 + 51a^2b^2 + 741b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{693b^3d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 & - \frac{2(8a^6 + 49a^4b^2 + 78a^2b^4 - 135b^6) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{693b^3d \sqrt{a + b \cos(c + dx)}} \\
 & + \frac{2(8a^4 + 57a^2b^2 + 135b^4) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{693b^2d} \\
 & + \frac{2a(8a^2 + 67b^2) (a + b \cos(c + dx))^{3/2} \sin(c + dx)}{693b^2d} \\
 & + \frac{2(8a^2 + 81b^2) (a + b \cos(c + dx))^{5/2} \sin(c + dx)}{693b^2d} \\
 & - \frac{8a(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{99b^2d} \\
 & + \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{11bd}
 \end{aligned}$$

[Out] 2/693*a*(8*a^2+67*b^2)*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/693*(8*a^2+81*b^2)*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d-8/99*a*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b^2/d+2/11*cos(d*x+c)*(a+b*cos(d*x+c))^(7/2)*sin(d*x+c)/b/d+2/693*(8*a^4+57*a^2*b^2+135*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/693*a*(8*a^4+51*a^2*b^2+741*b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/693*(8*a^6+49*a^4*b^2+78*a^2*b^4-135*b^6)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin

$(1/2*d*x+1/2*c), 2^{(1/2)*(b/(a+b))^{(1/2)}}*((a+b*\cos(d*x+c))/(a+b))^{(1/2)}/b^3$
 $/d/(a+b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00,
 number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used
 = {2872, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{5/2} dx = \frac{2(8a^2 + 81b^2) \sin(c + dx)(a + b \cos(c + dx))^{5/2}}{693b^2d} + \frac{2a(8a^2 + 67b^2) \sin(c + dx)(a + b \cos(c + dx))^{3/2}}{693b^2d} + \frac{2(8a^4 + 57a^2b^2 + 135b^4) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{693b^2d} + \frac{2a(8a^4 + 51a^2b^2 + 741b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{693b^3d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2(8a^6 + 49a^4b^2 + 78a^2b^4 - 135b^6) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{693b^3d \sqrt{a + b \cos(c + dx)}} - \frac{8a \sin(c + dx)(a + b \cos(c + dx))^{7/2}}{99b^2d} + \frac{2 \sin(c + dx) \cos(c + dx)(a + b \cos(c + dx))^{7/2}}{11bd}$$

[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(5/2), x]

[Out] (2*a*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(693*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^6 + 49*a^4*b^2 + 78*a^2*b^4 - 135*b^6)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(693*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*a^4 + 57*a^2*b^2 + 135*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*b^2*d) + (2*a*(8*a^2 + 67*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(693*b^2*d) + (2*(8*a^2 + 81*b^2)*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(693*b^2*d) - (8*a*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^2*d) + (2*Cos[c + d*x]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2872

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
```

|| IntegersQ[2*m, 2*n] && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & & NeQ[c, 0])))

Rule 3102

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{11bd} \\
 &+ \frac{2 \int (a + b \cos(c + dx))^{5/2} (a + \frac{9}{2}b \cos(c + dx) - 2a \cos^2(c + dx)) dx}{11b} \\
 &= -\frac{8a(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{99b^2d} \\
 &+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{11bd} \\
 &+ \frac{4 \int (a + b \cos(c + dx))^{5/2} (-\frac{5ab}{2} + \frac{1}{4}(8a^2 + 81b^2) \cos(c + dx)) dx}{99b^2} \\
 &= \frac{2(8a^2 + 81b^2)(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{693b^2d} \\
 &- \frac{8a(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{99b^2d} \\
 &+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{11bd} \\
 &+ \frac{8 \int (a + b \cos(c + dx))^{3/2} (-\frac{15}{8}b(2a^2 - 27b^2) + \frac{5}{8}a(8a^2 + 67b^2) \cos(c + dx)) dx}{693b^2} \\
 &= \frac{2a(8a^2 + 67b^2)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{693b^2d} \\
 &+ \frac{2(8a^2 + 81b^2)(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{693b^2d} \\
 &- \frac{8a(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{99b^2d} \\
 &+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{11bd} \\
 &+ \frac{16 \int \sqrt{a + b \cos(c + dx)} (-\frac{15}{8}ab(a^2 - 101b^2) + \frac{15}{16}(8a^4 + 57a^2b^2 + 135b^4) \cos(c + dx)) dx}{3465b^2}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2(8a^4 + 57a^2b^2 + 135b^4) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{693b^2d} \\
&+ \frac{2a(8a^2 + 67b^2) (a + b \cos(c + dx))^{3/2} \sin(c + dx)}{693b^2d} \\
&+ \frac{2(8a^2 + 81b^2) (a + b \cos(c + dx))^{5/2} \sin(c + dx)}{693b^2d} \\
&- \frac{8a(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{99b^2d} \\
&+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{11bd} \\
&+ \frac{32 \int \frac{\frac{15}{32}b(2a^4 + 663a^2b^2 + 135b^4) + \frac{15}{32}a(8a^4 + 51a^2b^2 + 741b^4) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{10395b^2} \\
&= \frac{2(8a^4 + 57a^2b^2 + 135b^4) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{693b^2d} \\
&+ \frac{2a(8a^2 + 67b^2) (a + b \cos(c + dx))^{3/2} \sin(c + dx)}{693b^2d} \\
&+ \frac{2(8a^2 + 81b^2) (a + b \cos(c + dx))^{5/2} \sin(c + dx)}{693b^2d} \\
&- \frac{8a(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{99b^2d} \\
&+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{11bd} \\
&+ \frac{(a(8a^4 + 51a^2b^2 + 741b^4)) \int \sqrt{a + b \cos(c + dx)} dx}{693b^3} \\
&- \frac{(8a^6 + 49a^4b^2 + 78a^2b^4 - 135b^6) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{693b^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2(8a^4 + 57a^2b^2 + 135b^4) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{693b^2d} \\
&+ \frac{2a(8a^2 + 67b^2) (a + b \cos(c + dx))^{3/2} \sin(c + dx)}{693b^2d} \\
&+ \frac{2(8a^2 + 81b^2) (a + b \cos(c + dx))^{5/2} \sin(c + dx)}{693b^2d} \\
&- \frac{8a(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{99b^2d} \\
&+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{11bd} \\
&+ \frac{\left(a(8a^4 + 51a^2b^2 + 741b^4) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{693b^3 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&- \frac{\left((8a^6 + 49a^4b^2 + 78a^2b^4 - 135b^6) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{693b^3 \sqrt{a + b \cos(c + dx)}} \\
&= \frac{2a(8a^4 + 51a^2b^2 + 741b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{693b^3d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&- \frac{2(8a^6 + 49a^4b^2 + 78a^2b^4 - 135b^6) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{693b^3d \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{2(8a^4 + 57a^2b^2 + 135b^4) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{693b^2d} \\
&+ \frac{2a(8a^2 + 67b^2) (a + b \cos(c + dx))^{3/2} \sin(c + dx)}{693b^2d} \\
&+ \frac{2(8a^2 + 81b^2) (a + b \cos(c + dx))^{5/2} \sin(c + dx)}{693b^2d} \\
&- \frac{8a(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{99b^2d} \\
&+ \frac{2 \cos(c + dx)(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{11bd}
\end{aligned}$$

$$\begin{aligned} & *c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^6 - 8 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \\ & (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^5 * b + 51 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \\ & \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^4 * b^2 - 51 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d \\ & *x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^3 * b^3 + 741 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c \\ &)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^2 * b^4 - 741 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b \\ &)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a * b^5 / b^3 / \\ & (-2 * \sin(1/2*d*x+1/2*c)^4 * b + (a+b) * \sin(1/2*d*x+1/2*c)^2)^{(1/2)} / \sin(1/2*d*x+1/2*c) / (-2 * b * \sin(1/2*d*x+1/2*c)^2 + a + b)^{(1/2)} / d \end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.51

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{5/2} dx = \frac{\sqrt{2}(16i a^6 + 96i a^4 b^2 - 507i a^2 b^4 - 405i b^6) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3}{27b^3}\right)}{\dots}$$

[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/2079*(sqrt(2)*(16*I*a^6 + 96*I*a^4*b^2 - 507*I*a^2*b^4 - 405*I*b^6)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-16*I*a^6 - 96*I*a^4*b^2 + 507*I*a^2*b^4 + 405*I*b^6)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-8*I*a^5*b - 51*I*a^3*b^3 - 741*I*a*b^5)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(8*I*a^5*b + 51*I*a^3*b^3 + 741*I*a*b^5)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(63*b^6*cos(d*x + c)^4 + 161*a*b^5*cos(d*x + c)^3 - 4*a^4*b^2 + 205*a^2*b^4 + 135*b^6 + (113*a^2*b^4 + 81*b^6)*cos(d*x + c)^2 + (3*a^3*b^3 + 229*a*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^4*d)

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{5/2} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{5/2} dx = \int (b \cos(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^3 dx$$

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{5/2} dx = \int (b \cos(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^3 dx$$

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c + dx)(a + b \cos(c + dx))^{5/2} dx = \int \cos(c + dx)^3 (a + b \cos(c + dx))^{5/2} dx$$

```
[In] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*(a + b*cos(c + d*x))^(5/2), x)
```

3.501 $\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} dx$

Optimal result	5263
Rubi [A] (verified)	5264
Mathematica [A] (verified)	5267
Maple [B] (verified)	5267
Fricas [C] (verification not implemented)	5268
Sympy [F(-1)]	5269
Maxima [F]	5269
Giac [F]	5269
Mupad [F(-1)]	5270

Optimal result

Integrand size = 23, antiderivative size = 308

$$\begin{aligned}
 & \int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} dx = \\
 & - \frac{2(10a^4 - 279a^2b^2 - 147b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{315b^2d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 & + \frac{4a(5a^4 - 62a^2b^2 + 57b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{315b^2d \sqrt{a + b \cos(c + dx)}} \\
 & - \frac{4a(5a^2 - 57b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315bd} \\
 & - \frac{2(10a^2 - 49b^2) (a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315bd} \\
 & - \frac{4a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63bd} + \frac{2(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd}
 \end{aligned}$$

[Out] $-2/315*(10*a^2-49*b^2)*(a+b*\cos(d*x+c))^(3/2)*\sin(d*x+c)/b/d-4/63*a*(a+b*\cos(d*x+c))^(5/2)*\sin(d*x+c)/b/d+2/9*(a+b*\cos(d*x+c))^(7/2)*\sin(d*x+c)/b/d-4/315*a*(5*a^2-57*b^2)*\sin(d*x+c)*(a+b*\cos(d*x+c))^(1/2)/b/d-2/315*(10*a^4-279*a^2*b^2-147*b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*\cos(d*x+c))^(1/2)/b^2/d/((a+b*\cos(d*x+c))/(a+b))^(1/2)+4/315*a*(5*a^4-62*a^2*b^2+57*b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*\cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*\cos(d*x+c))^(1/2)$

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2870, 2832, 2831, 2742, 2740, 2734, 2732}

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} dx =$$

$$\frac{2(10a^2 - 49b^2) \sin(c + dx)(a + b \cos(c + dx))^{3/2}}{315bd}$$

$$- \frac{4a(5a^2 - 57b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{315bd}$$

$$+ \frac{4a(5a^4 - 62a^2b^2 + 57b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{315b^2d \sqrt{a + b \cos(c + dx)}}$$

$$- \frac{2(10a^4 - 279a^2b^2 - 147b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{315b^2d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

$$+ \frac{2 \sin(c + dx)(a + b \cos(c + dx))^{7/2}}{9bd} - \frac{4a \sin(c + dx)(a + b \cos(c + dx))^{5/2}}{63bd}$$

[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2),x]

[Out] (-2*(10*a^4 - 279*a^2*b^2 - 147*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(315*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*(5*a^4 - 62*a^2*b^2 + 57*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*(5*a^2 - 57*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b*d) - (2*(10*a^2 - 49*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b*d) - (4*a*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b*d) + (2*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sine[c + d*x]]/Sqrt[(a + b*Sine[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2870

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} \\
 &+ \frac{2 \int \left(\frac{7b}{2} - a \cos(c + dx)\right) (a + b \cos(c + dx))^{5/2} dx}{9b} \\
 &= -\frac{4a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63bd} + \frac{2(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} \\
 &+ \frac{4 \int (a + b \cos(c + dx))^{3/2} \left(\frac{39ab}{4} - \frac{1}{4}(10a^2 - 49b^2) \cos(c + dx)\right) dx}{63b}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{2(10a^2 - 49b^2)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315bd} \\
&\quad - \frac{4a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63bd} + \frac{2(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} \\
&\quad + \frac{8 \int \sqrt{a + b \cos(c + dx)} \left(\frac{3}{8}b(55a^2 + 49b^2) - \frac{3}{4}a(5a^2 - 57b^2) \cos(c + dx) \right) dx}{315b} \\
&= -\frac{4a(5a^2 - 57b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315bd} \\
&\quad - \frac{2(10a^2 - 49b^2)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315bd} \\
&\quad - \frac{4a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63bd} + \frac{2(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} \\
&\quad + \frac{16 \int \frac{\frac{3}{16}ab(155a^2 + 261b^2) - \frac{3}{16}(10a^4 - 279a^2b^2 - 147b^4) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{945b} \\
&= -\frac{4a(5a^2 - 57b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315bd} \\
&\quad - \frac{2(10a^2 - 49b^2)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315bd} \\
&\quad - \frac{4a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63bd} + \frac{2(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} \\
&\quad - \frac{(10a^4 - 279a^2b^2 - 147b^4) \int \sqrt{a + b \cos(c + dx)} dx}{315b^2} \\
&\quad + \frac{(2a(5a^4 - 62a^2b^2 + 57b^4)) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{315b^2} \\
&= -\frac{4a(5a^2 - 57b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315bd} \\
&\quad - \frac{2(10a^2 - 49b^2)(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315bd} \\
&\quad - \frac{4a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63bd} + \frac{2(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} \\
&\quad - \frac{\left((10a^4 - 279a^2b^2 - 147b^4) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{315b^2 \sqrt{\frac{a + b \cos(c + dx)}{a+b}}} \\
&\quad + \frac{\left(2a(5a^4 - 62a^2b^2 + 57b^4) \sqrt{\frac{a + b \cos(c + dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{315b^2 \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2(10a^4 - 279a^2b^2 - 147b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{315b^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&+ \frac{4a(5a^4 - 62a^2b^2 + 57b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{315b^2 d \sqrt{a + b \cos(c + dx)}} \\
&- \frac{4a(5a^2 - 57b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315bd} \\
&- \frac{2(10a^2 - 49b^2) (a + b \cos(c + dx))^{3/2} \sin(c + dx)}{315bd} \\
&- \frac{4a(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{63bd} + \frac{2(a + b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.85

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} dx = \frac{-8(10a^5 + 10a^4b - 279a^3b^2 - 279a^2b^3 - 147ab^4 - 147b^5) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) + 1}{\dots}$$

[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2), x]

[Out] (-8*(10*a^5 + 10*a^4*b - 279*a^3*b^2 - 279*a^2*b^3 - 147*a*b^4 - 147*b^5)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 16*a*(5*a^4 - 62*a^2*b^2 + 57*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(40*a^4 + 1984*a^2*b^2 + 301*b^4 + 4*a*b*(160*a^2 + 619*b^2)*Cos[c + d*x] + 8*(85*a^2*b^2 + 42*b^4)*Cos[2*(c + d*x)] + 260*a*b^3*Cos[3*(c + d*x)] + 35*b^4*Cos[4*(c + d*x)]*Sin[c + d*x])/(1260*b^2*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(338) = 676.

Time = 10.02 (sec) , antiderivative size = 995, normalized size of antiderivative = 3.23

method	result	size
default	Expression too large to display	995

[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)

[Out] -2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^5+(2080*a*b^4+2240*b^5)*sin(1/2*d

```

*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1360*a^2*b^3-3120*a*b^4-2072*b^5)*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(320*a^3*b^2+1360*a^2*b^3+2408*a*b^4+952*b^
5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*a^4*b-160*a^3*b^2-666*a^2*b
^3-684*a*b^4-168*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-124*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+114*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a*b^4-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^5+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1
/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^4*b+279*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a
+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-2
79*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^2/(-2*sin(1/2*d*x+1/2*c)^4*b
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2
*c)^2+a+b)^(1/2)/d

```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.19 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.67

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} dx = \frac{\sqrt{2}(-20i a^5 + 93i a^3 b^2 - 489i a b^4) \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c)}{b^2}\right)}{\dots}$$

[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

```

[Out] 1/945*(sqrt(2)*(-20*I*a^5 + 93*I*a^3*b^2 - 489*I*a*b^4)*sqrt(b)*weierstrass
PInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos
(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(20*I*a^5 - 93*I*a^3*b^2
+ 489*I*a*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*
(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)
- 3*sqrt(2)*(10*I*a^4*b - 279*I*a^2*b^3 - 147*I*b^5)*sqrt(b)*weierstrassZe
ta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInvers
e(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x +

```


$c) + 3I*b*\sin(dx + c) + 2*a)/b)) - 3*\sqrt{2}*(-10*I*a^4*b + 279*I*a^2*b^3 + 147*I*b^5)*\sqrt{b}*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(dx + c) - 3*I*b*\sin(dx + c) + 2*a)/b)) + 6*(35*b^5*\cos(dx + c)^3 + 95*a*b^4*\cos(dx + c)^2 + 5*a^3*b^2 + 163*a*b^4 + (75*a^2*b^3 + 49*b^5)*\cos(dx + c))*\sqrt{b*\cos(dx + c) + a}*\sin(dx + c))/(b^3*d)$

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} dx = \text{Timed out}$$

[In] integrate(cos(dx+c)**2*(a+b*cos(dx+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} dx = \int (b \cos(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^2 dx$$

[In] integrate(cos(dx+c)^2*(a+b*cos(dx+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(dx + c) + a)^(5/2)*cos(dx + c)^2, x)

Giac [F]

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} dx = \int (b \cos(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^2 dx$$

[In] integrate(cos(dx+c)^2*(a+b*cos(dx+c))^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(dx + c) + a)^(5/2)*cos(dx + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(a + b \cos(c + dx))^{5/2} dx = \int \cos(c + dx)^2 (a + b \cos(c + dx))^{5/2} dx$$

```
[In] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2), x)
```

3.502 $\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} dx$

Optimal result	5271
Rubi [A] (verified)	5272
Mathematica [A] (verified)	5274
Maple [B] (verified)	5275
Fricas [C] (verification not implemented)	5275
Sympy [F(-1)]	5276
Maxima [F]	5276
Giac [F]	5276
Mupad [F(-1)]	5277

Optimal result

Integrand size = 21, antiderivative size = 249

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} dx = \frac{2a(3a^2 + 29b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{21bd \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2(3a^4 + 2a^2b^2 - 5b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{21bd \sqrt{a + b \cos(c + dx)}} + \frac{2(3a^2 + 5b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7d} + \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d}$$

```
[Out] 2/7*a*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*(a+b*cos(d*x+c))^(5/2)*sin(d*x+c)/d+2/21*(3*a^2+5*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/21*a*(3*a^2+29*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/21*(3*a^4+2*a^2*b^2-5*b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2} dx = \frac{2(3a^2+5b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d} + \frac{2a(3a^2+29b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{21bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2(3a^4+2a^2b^2-5b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{21bd\sqrt{a+b\cos(c+dx)}} + \frac{2\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{7d} + \frac{2a\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{7d}$$

[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2),x]

[Out] (2*a*(3*a^2 + 29*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(21*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a^4 + 2*a^2*b^2 - 5*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(21*b*d*Sqrt[a + b*Cos[c + d*x]])) + (2*(3*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\
&+ \frac{2}{7} \int \left(\frac{5b}{2} + \frac{5}{2} a \cos(c + dx) \right) (a + b \cos(c + dx))^{3/2} dx \\
&= \frac{2a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7d} + \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\
&+ \frac{4}{35} \int \sqrt{a + b \cos(c + dx)} \left(10ab + \frac{5}{4} (3a^2 + 5b^2) \cos(c + dx) \right) dx \\
&= \frac{2(3a^2 + 5b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{21d} \\
&+ \frac{2a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7d} + \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\
&+ \frac{8}{105} \int \frac{\frac{5}{8} b (27a^2 + 5b^2) + \frac{5}{8} a (3a^2 + 29b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2(3a^2 + 5b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7d} \\
&+ \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} + \frac{(a(3a^2 + 29b^2)) \int \sqrt{a + b \cos(c + dx)} dx}{21b} \\
&- \frac{(3a^4 + 2a^2b^2 - 5b^4) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{21b}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2(3a^2 + 5b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{21d} \\
&+ \frac{2a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7d} + \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\
&+ \frac{\left(a(3a^2 + 29b^2) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{21b \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&- \frac{\left((3a^4 + 2a^2b^2 - 5b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{21b \sqrt{a + b \cos(c + dx)}} \\
&= \frac{2a(3a^2 + 29b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{21bd \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&- \frac{2(3a^4 + 2a^2b^2 - 5b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{21bd \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{2(3a^2 + 5b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{21d} \\
&+ \frac{2a(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{7d} + \frac{2(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.86

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} dx = \frac{4a(3a^3 + 3a^2b + 29ab^2 + 29b^3) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) - 4(3a^4 + 2a^2b^2 - 5b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right) + b(36a^3 + 44a^2b + b(72a^2 + 29b^2) \cos(c + dx) + 24ab^2 \cos(2(c + dx)) + 3b^3 \cos(3(c + dx))) \sin(c + dx)}{(42bd \sqrt{a + b \cos(c + dx)})}$$

[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2), x]

[Out] (4*a*(3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(3*a^4 + 2*a^2*b^2 - 5*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(36*a^3 + 44*a^2*b + b*(72*a^2 + 29*b^2)*Cos[c + d*x] + 24*a*b^2*Cos[2*(c + d*x)] + 3*b^3*Cos[3*(c + d*x)])*Sin[c + d*x]/(42*b*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(283) = 566$.

Time = 7.88 (sec) , antiderivative size = 827, normalized size of antiderivative = 3.32

method	result	size
default	Expression too large to display	827

[In] `int(cos(d*x+c)*(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$-2/21*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(48*\cos(1/2*d*x+1/2*c)^9*b^4+96*\cos(1/2*d*x+1/2*c)^7*a*b^3-120*\cos(1/2*d*x+1/2*c)^7*b^4+72*\cos(1/2*d*x+1/2*c)^5*a^2*b^2-192*\cos(1/2*d*x+1/2*c)^5*a*b^3+128*\cos(1/2*d*x+1/2*c)^5*b^4+18*\cos(1/2*d*x+1/2*c)^3*a^3*b-108*\cos(1/2*d*x+1/2*c)^3*a^2*b^2+130*\cos(1/2*d*x+1/2*c)^3*a*b^3-72*\cos(1/2*d*x+1/2*c)^3*b^4-3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^4-2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^2*b^2+5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*b^4+3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^4-3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^3*b+29*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^2*b^2-29*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a*b^3-18*\cos(1/2*d*x+1/2*c)*a^3*b+36*\cos(1/2*d*x+1/2*c)*a^2*b^2-34*\cos(1/2*d*x+1/2*c)*a*b^3+16*\cos(1/2*d*x+1/2*c)*b^4)/b/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.90

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2} dx = \frac{\sqrt{2}(6i a^4 - 23i a^2 b^2 - 15i b^4)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3i b\sin(dx+c)}{3b}\right)}{b^2}$$

[In] `integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

```
[Out] 1/63*(sqrt(2)*(6*I*a^4 - 23*I*a^2*b^2 - 15*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-6*I*a^4 + 23*I*a^2*b^2 + 15*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*sqrt(2)*(-3*I*a^3*b - 29*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*sqrt(2)*(3*I*a^3*b + 29*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(3*b^4*cos(d*x + c)^2 + 9*a*b^3*cos(d*x + c) + 9*a^2*b^2 + 5*b^4)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} dx = \int (b \cos(dx + c) + a)^{5/2} \cos(dx + c) dx$$

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} dx = \int (b \cos(dx + c) + a)^{5/2} \cos(dx + c) dx$$

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```


Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx)(a + b \cos(c + dx))^{5/2} dx = \int \cos(c + dx) (a + b \cos(c + dx))^{5/2} dx$$

```
[In] int(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2), x)
```

3.503 $\int (a + b \cos(c + dx))^{5/2} dx$

Optimal result	5278
Rubi [A] (verified)	5278
Mathematica [A] (verified)	5281
Maple [B] (verified)	5282
Fricas [C] (verification not implemented)	5282
Sympy [F]	5283
Maxima [F]	5283
Giac [F]	5283
Mupad [F(-1)]	5284

Optimal result

Integrand size = 14, antiderivative size = 197

$$\int (a + b \cos(c + dx))^{5/2} dx = \frac{2(23a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{15d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{16a(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{15d \sqrt{a + b \cos(c + dx)}} + \frac{16ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2b(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d}$$

```
[Out] 2/5*b*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+16/15*a*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(23*a^2+9*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-16/15*a*(a^2-b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

= {2735, 2832, 2831, 2742, 2740, 2734, 2732}

$$\int (a + b \cos(c + dx))^{5/2} dx = -\frac{16a(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{15d \sqrt{a + b \cos(c + dx)}} + \frac{2(23a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{15d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2b \sin(c + dx)(a + b \cos(c + dx))^{3/2}}{5d} + \frac{16ab \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{15d}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2),x]

[Out] (2*(23*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (16*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(15*d*Sqrt[a + b*Cos[c + d*x]]) + (16*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2735

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&+ \frac{2}{5} \int \sqrt{a + b \cos(c + dx)} \left(\frac{1}{2} (5a^2 + 3b^2) + 4ab \cos(c + dx) \right) dx \\
&= \frac{16ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2b(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&+ \frac{4}{15} \int \frac{\frac{1}{4}a(15a^2 + 17b^2) + \frac{1}{4}b(23a^2 + 9b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{16ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15d} + \frac{2b(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&- \frac{1}{15} (8a(a^2 - b^2)) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx + \frac{1}{15} (23a^2 \\
&\qquad\qquad\qquad + 9b^2) \int \sqrt{a + b \cos(c + dx)} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} \\
&+ \frac{\left((23a^2+9b^2)\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{15\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&- \frac{\left(8a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{15\sqrt{a+b\cos(c+dx)}} \\
&= \frac{2(23a^2+9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&- \frac{16a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.90

$$\int (a+b\cos(c+dx))^{5/2} dx = \frac{2(23a^3+23a^2b+9ab^2+9b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-16a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)-16a(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)+b(22a^2+3b^2+28ab\cos(c+dx)+3b^2\cos(2(c+dx)))\sin(c+dx)}{15d\sqrt{a+b\cos(c+dx)}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2), x]

[Out] (2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 16*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(22*a^2 + 3*b^2 + 28*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x]/(15*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(235) = 470$.

Time = 6.55 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.36

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(24\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+56\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab^2-48\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+22\left(\cos^3\left(\frac{dx}{2}\right.\right.\right.$

[In] `int((a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$-2/15*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(24*\cos(1/2*d*x+1/2*c)^7*b^3+56*\cos(1/2*d*x+1/2*c)^5*a*b^2-48*\cos(1/2*d*x+1/2*c)^5*b^3+22*\cos(1/2*d*x+1/2*c)^3*a^2*b-84*\cos(1/2*d*x+1/2*c)^3*a*b^2+30*\cos(1/2*d*x+1/2*c)^3*b^3-8*a^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})+8*a*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})+23*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^3-23*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^2*b+9*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a*b^2-9*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*b^3-22*\cos(1/2*d*x+1/2*c)*a^2*b+28*\cos(1/2*d*x+1/2*c)*a*b^2-6*\cos(1/2*d*x+1/2*c)*b^3)/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.22

$$\int (a + b \cos(c + dx))^{5/2} dx = \frac{\sqrt{2}(i a^3 - 33i a b^2)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3i b \sin(dx+c) + 2a}{3b}\right)}{\dots}$$

[In] `integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out]
$$1/45*(\text{sqrt}(2)*(I*a^3 - 33*I*a*b^2)*\text{sqrt}(b)*\text{weierstrassPInverse}(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) + 3*I*b*\sin$$

$(d*x + c) + 2*a)/b) + \sqrt{2}*(-I*a^3 + 33*I*a*b^2)*\sqrt{b}*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) - 3*I*b*\sin(d*x + c) + 2*a)/b) - 3*\sqrt{2}*(-23*I*a^2*b - 9*I*b^3)*\sqrt{b}*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) + 3*I*b*\sin(d*x + c) + 2*a)/b)) - 3*\sqrt{2}*(23*I*a^2*b + 9*I*b^3)*\sqrt{b}*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) - 3*I*b*\sin(d*x + c) + 2*a)/b)) + 6*(3*b^3*\cos(d*x + c) + 11*a*b^2)*\sqrt{b*\cos(d*x + c) + a}*\sin(d*x + c))/(b*d)$

Sympy [F]

$$\int (a + b \cos(c + dx))^{5/2} dx = \int (a + b \cos(c + dx))^{\frac{5}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))**(5/2),x)

[Out] Integral((a + b*cos(c + d*x))**(5/2), x)

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} dx = \int (b \cos(dx + c) + a)^{\frac{5}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} dx = \int (b \cos(dx + c) + a)^{\frac{5}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} dx = \int (a + b \cos(c + dx))^{5/2} dx$$

```
[In] int((a + b*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + b*cos(c + d*x))^(5/2), x)
```


3.504 $\int (a + b \cos(c + dx))^{5/2} \sec(c + dx) dx$

Optimal result	5285
Rubi [A] (verified)	5286
Mathematica [C] (verified)	5289
Maple [A] (verified)	5290
Fricas [F(-1)]	5290
Sympy [F(-1)]	5290
Maxima [F]	5291
Giac [F]	5291
Mupad [F(-1)]	5291

Optimal result

Integrand size = 21, antiderivative size = 222

$$\int (a + b \cos(c + dx))^{5/2} \sec(c + dx) dx = \frac{14ab\sqrt{a + b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2b(2a^2 + b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3d\sqrt{a + b \cos(c + dx)}} + \frac{2a^3 \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}} + \frac{2b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d}$$

```
[Out] 2/3*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+14/3*a*b*(cos(1/2*d*x+1/2*c))^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*b*(2*a^2+b^2)*(cos(1/2*d*x+1/2*c))^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+2*a^3*(cos(1/2*d*x+1/2*c))^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2872, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int (a+b \cos(c+dx))^{5/2} \sec(c+dx) dx = \frac{2a^3 \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d \sqrt{a+b \cos(c+dx)}} + \frac{2b(2a^2+b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3d \sqrt{a+b \cos(c+dx)}} + \frac{2b^2 \sin(c+dx) \sqrt{a+b \cos(c+dx)}}{3d} + \frac{14ab \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x], x]

[Out] (14*a*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*b*(2*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a^3*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b)

+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2872

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2886

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Ssin[e + f*x])/(c + d)]/Sqrt[c + d*Ssin[e + f*x]], Int[1/((a + b*Ssin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3081

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Ssin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Ssin[e + f*x])^m/(c + d*Ssin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3138

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Ssin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e

+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
 [{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} \\
 &+ \frac{2}{3} \int \frac{\left(\frac{3a^3}{2} + \frac{1}{2}b(9a^2 + b^2) \cos(c + dx) + \frac{7}{2}ab^2 \cos^2(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{2b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} - \frac{2 \int \frac{\left(-\frac{3a^3b}{2} - \frac{1}{2}b^2(2a^2 + b^2) \cos(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{3b} \\
 &+ \frac{1}{3}(7ab) \int \sqrt{a + b \cos(c + dx)} dx \\
 &= \frac{2b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} + a^3 \int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
 &+ \frac{1}{3}(b(2a^2 + b^2)) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx \\
 &+ \frac{\left(7ab \sqrt{a + b \cos(c + dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{3 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 &= \frac{14ab \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} \\
 &+ \frac{\left(a^3 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{\sqrt{a + b \cos(c + dx)}} \\
 &+ \frac{\left(b(2a^2 + b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{3 \sqrt{a + b \cos(c + dx)}}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{14ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+ \frac{2b(2a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{2a^3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{2b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.71

$$\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)dx = \frac{4b(9a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2a(6a^2+7b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{14i\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}}{\sqrt{a+b\cos(c+dx)}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]

[Out] ((4*b*(9*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] + (2*a*(6*a^2 + 7*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] + ((14*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))/Sqrt[-(a + b)^(-1)] + 4*b^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(6*d)

Maple [A] (verified)

Time = 6.06 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.38

method	result
default	$2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+2\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab^2-6\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+2a^2b\sqrt{\frac{1}{2}-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}\right)$

```
[In] int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x+1/2*c)^5*b^3+2*cos(1/2*d*x+1/2*c)^3*a*b^2-6*cos(1/2*d*x+1/2*c)^3*b^3+2*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-3*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2*cos(1/2*d*x+1/2*c)*a*b^2+2*cos(1/2*d*x+1/2*c)*b^3)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c) dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c) dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec(c + dx) dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\cos(c + dx)} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x),x)

[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x), x)

3.505 $\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx$

Optimal result	5292
Rubi [A] (verified)	5293
Mathematica [C] (verified)	5296
Maple [B] (verified)	5297
Fricas [F(-1)]	5297
Sympy [F(-1)]	5298
Maxima [F]	5298
Giac [F]	5298
Mupad [F(-1)]	5298

Optimal result

Integrand size = 23, antiderivative size = 222

$$\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx =$$

$$-\frac{(a^2 - 2b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

$$+ \frac{a(a^2 + 4b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}}$$

$$+ \frac{5a^2 b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} + \frac{a^2 \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d}$$

```
[Out] -(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(
d*x+c))/(a+b))^(1/2)+a*(a^2+4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x
+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+5*a^2*b*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(
1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+a^2*(a+b*cos(
d*x+c))^(1/2)*tan(d*x+c)/d
```


Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2871, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \frac{a(a^2 + 4b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} - \frac{(a^2 - 2b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{a^2 \tan(c + dx) \sqrt{a + b \cos(c + dx)}}{d} + \frac{5a^2 b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]

[Out] -(((a^2 - 2*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + (a*(a^2 + 4*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (5*a^2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*SIN[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2871

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
+ a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)]))/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^
2)/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) +
```

```
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a^2 \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d} \\
&+ \int \frac{\left(\frac{5a^2b}{2} + 3ab^2 \cos(c + dx) - \frac{1}{2}b(a^2 - 2b^2) \cos^2(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{a^2 \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d} - \frac{\int \frac{\left(-\frac{5}{2}a^2b^2 - \frac{1}{2}ab(a^2 + 4b^2) \cos(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{b} \\
&+ \frac{1}{2}(-a^2 + 2b^2) \int \sqrt{a + b \cos(c + dx)} dx \\
&= \frac{a^2 \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d} + \frac{1}{2}(5a^2b) \int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
&+ \frac{1}{2}(a(a^2 + 4b^2)) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx \\
&+ \frac{\left((-a^2 + 2b^2) \sqrt{a + b \cos(c + dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&= -\frac{(a^2 - 2b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&+ \frac{a^2 \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d} + \frac{\left(5a^2b\sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{2\sqrt{a + b \cos(c + dx)}} \\
&+ \frac{\left(a(a^2 + 4b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{2\sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{(a^2 - 2b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&+ \frac{a(a^2 + 4b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{5a^2 b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{a^2 \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.76

$$\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \frac{24ab^2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b \cos(c+dx)}} + \frac{2b(9a^2+2b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b \cos(c+dx)}} + \frac{2i(a^2-2b^2) \sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}}}{\sqrt{a+b \cos(c+dx)}}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] ((24*a*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] + (2*b*(9*a^2 + 2*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(a^2 - 2*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))/(a*b*Sqrt[-(a + b)^(-1)]) + 4*a^2*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(295) = 590$.

Time = 10.75 (sec) , antiderivative size = 960, normalized size of antiderivative = 4.32

method	result	size
default	Expression too large to display	960

[In] `int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)`

[Out]
$$-\left((2b\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a-b\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(4\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4a^2b+(-2a^3-2a^2b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2b/(a-b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+(a+b)/(a-b)\right)^{1/2}\left(\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)a^3+4\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)a^2b-2\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)a^3+\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)a^2b+2\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)a^2b-2\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)b^3-5\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2,(-2b/(a-b))^{1/2}\right)a^2b\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2b/(a-b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+(a+b)/(a-b)\right)^{1/2}\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)a^3+4a^2b^2\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2b/(a-b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+(a+b)/(a-b)\right)^{1/2}\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)-\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2b/(a-b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+(a+b)/(a-b)\right)^{1/2}\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)a^3+\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2b/(a-b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+(a+b)/(a-b)\right)^{1/2}\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)a^2b+2\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2b/(a-b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+(a+b)/(a-b)\right)^{1/2}\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)a^2b-2\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2b/(a-b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+(a+b)/(a-b)\right)^{1/2}\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),(-2b/(a-b))^{1/2}\right)b^3-5a^2b\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(-2b/(a-b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+(a+b)/(a-b)\right)^{1/2}\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2,(-2b/(a-b))^{1/2}\right)\right)/\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)/\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4b+(a+b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}/\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)/\left(-2b\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)^{1/2}/d$$

Fricas [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \text{Timed out}$$

[In] `integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \text{Timed out}$$

[In] `integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)`

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^2 dx$$

[In] `integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)`

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^2 dx$$

[In] `integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")`

[Out] `integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)`

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\cos(c + dx)^2} dx$$

[In] `int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)`

[Out] `int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)`

3.506 $\int (a + b \cos(c + dx))^{5/2} \sec^3(c + dx) dx$

Optimal result	5299
Rubi [A] (verified)	5300
Mathematica [C] (verified)	5303
Maple [B] (verified)	5304
Fricas [F(-1)]	5305
Sympy [F(-1)]	5305
Maxima [F]	5305
Giac [F]	5306
Mupad [F(-1)]	5306

Optimal result

Integrand size = 23, antiderivative size = 270

$$\int (a + b \cos(c + dx))^{5/2} \sec^3(c + dx) dx = -\frac{9ab\sqrt{a + b \cos(c + dx)}E\left(\frac{1}{2}(c + dx)\left|\frac{2b}{a+b}\right.\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b(11a^2 + 8b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4d\sqrt{a + b \cos(c + dx)}} + \frac{a(4a^2 + 15b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4d\sqrt{a + b \cos(c + dx)}} + \frac{9ab\sqrt{a + b \cos(c + dx)}\tan(c + dx)}{4d} + \frac{a^2\sqrt{a + b \cos(c + dx)}\sec(c + dx)\tan(c + dx)}{2d}$$

```
[Out] -9/4*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/4*b*(11*a^2+8*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*a*(4*a^2+15*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+9/4*a*b*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2871, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int (a + b \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \frac{b(11a^2 + 8b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4d \sqrt{a + b \cos(c + dx)}} + \frac{a(4a^2 + 15b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4d \sqrt{a + b \cos(c + dx)}} + \frac{a^2 \tan(c + dx) \sec(c + dx) \sqrt{a + b \cos(c + dx)}}{2d} + \frac{9ab \tan(c + dx) \sqrt{a + b \cos(c + dx)}}{4d} - \frac{9ab \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{4d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]

[Out] (-9*a*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(4*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (b*(11*a^2 + 8*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (a*(4*a^2 + 15*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (9*a*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2871

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2886

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3081

Int((((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_))/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B

, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3138

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

integral

$$\begin{aligned}
 &= \frac{a^2 \sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d} \\
 &+ \frac{1}{2} \int \frac{\left(\frac{9a^2b}{2} + a(a^2 + 6b^2) \cos(c + dx) + \frac{1}{2}b(a^2 + 4b^2) \cos^2(c + dx)\right) \sec^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{9ab \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{4d} + \frac{a^2 \sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d} \\
 &+ \frac{\int \left(\frac{1}{4}a^2(4a^2 + 15b^2) + \frac{1}{2}ab(a^2 + 4b^2) \cos(c + dx) - \frac{9}{4}a^2b^2 \cos^2(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{2a} \\
 &= \frac{9ab \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{4d} + \frac{a^2 \sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d} \\
 &- \frac{\int \left(-\frac{1}{4}a^2b(4a^2 + 15b^2) - \frac{1}{4}ab^2(11a^2 + 8b^2) \cos(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{2ab} - \frac{1}{8}(9ab) \int \sqrt{a + b \cos(c + dx)} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{9ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{a^2\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&+ \frac{1}{8}(b(11a^2+8b^2)) \int \frac{1}{\sqrt{a+b\cos(c+dx)}} dx + \frac{1}{8}(a(4a^2+15b^2)) \int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx \\
&- \frac{\left(9ab\sqrt{a+b\cos(c+dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}} dx}{8\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&= -\frac{9ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{9ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} \\
&+ \frac{a^2\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&+ \frac{\left(b(11a^2+8b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{8\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{\left(a(4a^2+15b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{8\sqrt{a+b\cos(c+dx)}} \\
&= -\frac{9ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&+ \frac{b(11a^2+8b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{a(4a^2+15b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{9ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{a^2\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.46

$$\int (a+b\cos(c+dx))^{5/2} \sec^3(c+dx) dx = \frac{4b(a^2+4b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{a(8a^2+21b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - 9i\sqrt{-\frac{b(-1+...)}{...}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]

```
[Out] ((4*b*(a^2 + 4*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] + (a*(8*a^2 + 21*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] - ((9*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)))/Sqrt[-(a + b)^(-1)] + 2*a*Sqrt[a + b*Cos[c + d*x]]*(2*a + 9*b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(8*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(331) = 662$.

Time = 33.46 (sec) , antiderivative size = 1134, normalized size of antiderivative = 4.20

method	result	size
default	Expression too large to display	1134

```
[In] int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-72*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a*b^2+(44*a^2*b+72*a*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-4*a^3-22*a^2*b-18*a*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(11*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+8*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3-15*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b^2)*sin(1/2*d*x+1/2*c)^4-4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(11*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+8*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3-15*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b^2)*sin(1/2*d*x+1/2*c)^2+11*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+8*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*
```

$d*x+1/2*c), 2, (-2*b/(a-b))^{(1/2)}*a^3-15*a*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), 2, (-2*b/(a-b))^{(1/2)})/(2*\cos(1/2*d*x+1/2*c)^2-1)^2/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/2)}/d$

Fricas [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^3 dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^3 dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^3(c + dx) dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\cos(c + dx)^3} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)

[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)

3.507 $\int (a + b \cos(c + dx))^{5/2} \sec^4(c + dx) dx$

Optimal result	5307
Rubi [A] (verified)	5308
Mathematica [C] (verified)	5312
Maple [B] (verified)	5313
Fricas [F(-1)]	5314
Sympy [F(-1)]	5314
Maxima [F]	5315
Giac [F]	5315
Mupad [F(-1)]	5315

Optimal result

Integrand size = 23, antiderivative size = 323

$$\begin{aligned}
 & \int (a + b \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \\
 & \frac{(16a^2 + 33b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{24d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 & + \frac{a(16a^2 + 59b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{24d \sqrt{a + b \cos(c + dx)}} \\
 & + \frac{5b(4a^2 + b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{8d \sqrt{a + b \cos(c + dx)}} \\
 & + \frac{(16a^2 + 33b^2) \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{24d} \\
 & + \frac{13ab \sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{12d} \\
 & + \frac{a^2 \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) \tan(c + dx)}{3d}
 \end{aligned}$$

```
[Out] -1/24*(16*a^2+33*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/(
(a+b*cos(d*x+c))/(a+b))^(1/2)+1/24*a*(16*a^2+59*b^2)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1
/2))*(a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)+5/8*b*(4*a^2+b
^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos
(d*x+c))^(1/2)+1/24*(16*a^2+33*b^2)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+13/
12*a*b*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/3*a^2*sec(d*x+c)^2*
(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2871, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int (a + b \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \frac{(16a^2 + 33b^2) \tan(c + dx) \sqrt{a + b \cos(c + dx)}}{24d} + \frac{a(16a^2 + 59b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{24d \sqrt{a + b \cos(c + dx)}} - \frac{(16a^2 + 33b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{24d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{5b(4a^2 + b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{8d \sqrt{a + b \cos(c + dx)}} + \frac{a^2 \tan(c + dx) \sec^2(c + dx) \sqrt{a + b \cos(c + dx)}}{3d} + \frac{13ab \tan(c + dx) \sec(c + dx) \sqrt{a + b \cos(c + dx)}}{12d}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]

[Out] -1/24*((16*a^2 + 33*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (a*(16*a^2 + 59*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(24*d*Sqrt[a + b*Cos[c + d*x]]) + (5*b*(4*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c + d*x]]) + ((16*a^2 + 33*b^2)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*d) + (13*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*d) + (a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,

0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2871

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2886

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3138

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} & \text{integral} \\ &= \frac{a^2 \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) \tan(c + dx)}{3d} \\ &+ \frac{1}{3} \int \frac{\left(\frac{13a^2b}{2} + a(2a^2 + 9b^2) \cos(c + dx) + \frac{3}{2}b(a^2 + 2b^2) \cos^2(c + dx) \right) \sec^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{13ab\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12d} \\
&+ \frac{a^2\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d} \\
&+ \frac{\int \frac{(\frac{1}{4}a^2(16a^2+33b^2)+\frac{1}{2}ab(19a^2+12b^2)\cos(c+dx)+\frac{13}{4}a^2b^2\cos^2(c+dx))\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{6a} \\
&= \frac{(16a^2+33b^2)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d} \\
&+ \frac{13ab\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12d} \\
&+ \frac{a^2\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d} \\
&+ \frac{\int \frac{(\frac{15}{8}a^2b(4a^2+b^2)+\frac{13}{4}a^3b^2\cos(c+dx)-\frac{1}{8}a^2b(16a^2+33b^2)\cos^2(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{6a^2} \\
&= \frac{(16a^2+33b^2)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d} \\
&+ \frac{13ab\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12d} \\
&+ \frac{a^2\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d} \\
&- \frac{\int \frac{(-\frac{15}{8}a^2b^2(4a^2+b^2)-\frac{1}{8}a^3b(16a^2+59b^2)\cos(c+dx))\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{6a^2b} \\
&+ \frac{1}{48}(-16a^2-33b^2)\int\sqrt{a+b\cos(c+dx)}dx \\
&= \frac{(16a^2+33b^2)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d} \\
&+ \frac{13ab\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12d} \\
&+ \frac{a^2\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d} \\
&+ \frac{1}{16}(5b(4a^2+b^2))\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx \\
&+ \frac{1}{48}(a(16a^2+59b^2))\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx \\
&+ \frac{\left((-16a^2-33b^2)\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{48\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{(16a^2 + 33b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{24d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&+ \frac{(16a^2 + 33b^2) \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{24d} \\
&+ \frac{13ab \sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{12d} \\
&+ \frac{a^2 \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) \tan(c + dx)}{3d} \\
&+ \frac{\left(5b(4a^2 + b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{16 \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{\left(a(16a^2 + 59b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{48 \sqrt{a + b \cos(c + dx)}} \\
&= -\frac{(16a^2 + 33b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{24d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&+ \frac{a(16a^2 + 59b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{24d \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{5b(4a^2 + b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{8d \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{(16a^2 + 33b^2) \sqrt{a + b \cos(c + dx)} \tan(c + dx)}{24d} \\
&+ \frac{13ab \sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{12d} \\
&+ \frac{a^2 \sqrt{a + b \cos(c + dx)} \sec^2(c + dx) \tan(c + dx)}{3d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.34

$$\begin{aligned}
&\int (a + b \cos(c + dx))^{5/2} \sec^4(c \\
&+ dx) dx = \frac{104ab^2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b \cos(c+dx)}} + \frac{2b(104a^2 - 3b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b \cos(c+dx)}} - \frac{2i(16a^2 + 33b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{24d}
\end{aligned}$$

[In] Integrate[(a + b*cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]

[Out] $\left(\frac{104ab^2\sqrt{a+b\cos[c+dx]}}{(a+b)}\operatorname{EllipticF}\left[\frac{c+dx}{2}, \frac{2b}{a+b}\right]\right)\sqrt{a+b\cos[c+dx]} + (2b(104a^2-3b^2)\sqrt{a+b\cos[c+dx]})\operatorname{EllipticPi}\left[2, \frac{c+dx}{2}, \frac{2b}{a+b}\right]\sqrt{a+b\cos[c+dx]} - ((2I)(16a^2+33b^2)\sqrt{-(b(-1+\cos[c+dx]))/(a+b)})\sqrt{-(b(1+\cos[c+dx]))/(a-b)}\operatorname{Csc}[c+dx](-2a(a-b)\operatorname{EllipticE}[I\operatorname{ArcSinh}[\sqrt{-(a+b)^{-1}}]\sqrt{a+b\cos[c+dx]}], (a+b)/(a-b)] + b(-2a\operatorname{EllipticF}[I\operatorname{ArcSinh}[\sqrt{-(a+b)^{-1}}]\sqrt{a+b\cos[c+dx]}], (a+b)/(a-b)] + b\operatorname{EllipticPi}[(a+b)/a, I\operatorname{ArcSinh}[\sqrt{-(a+b)^{-1}}]\sqrt{a+b\cos[c+dx]}], (a+b)/(a-b)))/\left(a b\sqrt{-(a+b)^{-1}}\right) + 4\sqrt{a+b\cos[c+dx]}\operatorname{Sec}[c+dx]^2(26ab\sin[c+dx] + (8a^2 + (33b^2)/2)\sin[2(c+dx)] + 8a^2\tan[c+dx])/96d$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1741 vs. $2(380) = 760$.

Time = 103.95 (sec) , antiderivative size = 1742, normalized size of antiderivative = 5.39

method	result	size
default	Expression too large to display	1742

[In] int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] $-1/24*((2b\cos(1/2dx+1/2c)^2+a-b)\sin(1/2dx+1/2c)^2)^{(1/2)}*((256a^2b+528b^3)\cos(1/2dx+1/2c)\sin(1/2dx+1/2c)^8+(-128a^3-384a^2b-472ab^2-792b^3)\sin(1/2dx+1/2c)^6\cos(1/2dx+1/2c)+(128a^3+328a^2b+472ab^2+396b^3)\sin(1/2dx+1/2c)^4\cos(1/2dx+1/2c)+(-48a^3-100a^2b-118ab^2-66b^3)\sin(1/2dx+1/2c)^2\cos(1/2dx+1/2c)+8(-2b/(a-b)\sin(1/2dx+1/2c)^2+(a+b)/(a-b))^{(1/2)}(\sin(1/2dx+1/2c)^2)^{(1/2)}(60\operatorname{EllipticPi}(\cos(1/2dx+1/2c), 2, (-2b/(a-b))^{(1/2)})a^2b+15b^3\operatorname{EllipticPi}(\cos(1/2dx+1/2c), 2, (-2b/(a-b))^{(1/2)})-16\operatorname{EllipticF}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})a^3-59\operatorname{EllipticF}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})ab^2+16\operatorname{EllipticE}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})a^3-16\operatorname{EllipticE}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})a^2b+33\operatorname{EllipticE}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})ab^2-33\operatorname{EllipticE}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})b^3)\sin(1/2dx+1/2c)^6-12(-2b/(a-b)\sin(1/2dx+1/2c)^2+(a+b)/(a-b))^{(1/2)}(\sin(1/2dx+1/2c)^2)^{(1/2)}(60\operatorname{EllipticPi}(\cos(1/2dx+1/2c), 2, (-2b/(a-b))^{(1/2)})a^2b+15b^3\operatorname{EllipticPi}(\cos(1/2dx+1/2c), 2, (-2b/(a-b))^{(1/2)})-16\operatorname{EllipticF}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})a^3-59\operatorname{EllipticF}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})ab^2+16\operatorname{EllipticE}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})a^3-16\operatorname{EllipticE}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})a^2b+33\operatorname{EllipticE}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})ab^2-33\operatorname{EllipticE}(\cos(1/2dx+1/2c), (-2b/(a-b))^{(1/2)})b^3)\sin(1/2dx+1/2c)^4+6(-2b/(a-b)\sin(1/2dx+1/2c)^2+(a+b)/(a-b))^{(1/2)}(\sin(1/2dx+1/2c)^2)^{(1/2)}$

```
(1/2)*(60*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b+15*b^3*
EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-16*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-59*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a*b^2+16*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-16*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+33*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-33*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*b^3)*sin(1/2*d*x+1/2*c)^2-60*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-15*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))+16*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^3+59*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+16*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-33*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+33*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*b^3)/(2*cos(1/2*d*x+1/2*c)^2-1)^3/(-2*sin(1/2*d*x+1/2
*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*
d*x+1/2*c)^2+a+b)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^4 dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^4 dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^4(c + dx) dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\cos(c + dx)^4} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)

[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)

3.508 $\int (a + b \cos(c + dx))^{7/2} dx$

Optimal result	5316
Rubi [A] (verified)	5317
Mathematica [A] (verified)	5320
Maple [B] (verified)	5320
Fricas [C] (verification not implemented)	5321
Sympy [F(-1)]	5321
Maxima [F]	5322
Giac [F]	5322
Mupad [F(-1)]	5322

Optimal result

Integrand size = 14, antiderivative size = 246

$$\int (a + b \cos(c + dx))^{7/2} dx = \frac{32a(11a^2 + 13b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{105d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2(71a^4 - 46a^2b^2 - 25b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{105d \sqrt{a + b \cos(c + dx)}} + \frac{2b(71a^2 + 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105d} + \frac{24ab(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2b(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d}$$

```
[Out] 24/35*a*b*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*b*(a+b*cos(d*x+c))^(5/2)*
sin(d*x+c)/d+2/105*b*(71*a^2+25*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+32
/105*a*(11*a^2+13*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/
((a+b*cos(d*x+c))/(a+b))^(1/2)-2/105*(71*a^4-46*a^2*b^2-25*b^4)*(cos(1/2*d*
x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(
b/(a+b))^(1/2))*(a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)
```


Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2735, 2832, 2831, 2742, 2740, 2734, 2732}

$$\int (a + b \cos(c + dx))^{7/2} dx = \frac{2b(71a^2 + 25b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{105d} + \frac{32a(11a^2 + 13b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{105d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2(71a^4 - 46a^2b^2 - 25b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{105d \sqrt{a + b \cos(c + dx)}} + \frac{2b \sin(c + dx)(a + b \cos(c + dx))^{5/2}}{7d} + \frac{24ab \sin(c + dx)(a + b \cos(c + dx))^{3/2}}{35d}$$

[In] Int[(a + b*Cos[c + d*x])^(7/2), x]

[Out] (32*a*(11*a^2 + 13*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(105*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(71*a^4 - 46*a^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(105*d*Sqrt[a + b*Cos[c + d*x]])) + (2*b*(71*a^2 + 25*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (24*a*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*b*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2735

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&

IntegerQ[2*n]

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2b(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\ &+ \frac{2}{7} \int (a + b \cos(c + dx))^{3/2} \left(\frac{1}{2}(7a^2 + 5b^2) + 6ab \cos(c + dx) \right) dx \\ &= \frac{24ab(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2b(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\ &+ \frac{4}{35} \int \sqrt{a + b \cos(c + dx)} \left(\frac{1}{4}a(35a^2 + 61b^2) + \frac{1}{4}b(71a^2 + 25b^2) \cos(c + dx) \right) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2b(71a^2 + 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105d} \\
&\quad + \frac{24ab(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2b(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\
&\quad + \frac{8}{105} \int \frac{\frac{1}{8}(105a^4 + 254a^2b^2 + 25b^4) + 2ab(11a^2 + 13b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2b(71a^2 + 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105d} \\
&\quad + \frac{24ab(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2b(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\
&\quad + \frac{1}{105} (16a(11a^2 + 13b^2)) \int \sqrt{a + b \cos(c + dx)} dx \\
&\quad + \frac{1}{105} (-71a^4 + 46a^2b^2 + 25b^4) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2b(71a^2 + 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105d} \\
&\quad + \frac{24ab(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2b(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\
&\quad + \frac{\left(16a(11a^2 + 13b^2) \sqrt{a + b \cos(c + dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{105 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad + \frac{\left((-71a^4 + 46a^2b^2 + 25b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{105 \sqrt{a + b \cos(c + dx)}} \\
&= \frac{32a(11a^2 + 13b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{105d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad - \frac{2(71a^4 - 46a^2b^2 - 25b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{105d \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{2b(71a^2 + 25b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{105d} \\
&\quad + \frac{24ab(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{35d} + \frac{2b(a + b \cos(c + dx))^{5/2} \sin(c + dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.86

$$\int (a + b \cos(c + dx))^{7/2} dx = \frac{64a(11a^3 + 11a^2b + 13ab^2 + 13b^3) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right) - 4(71a^4 - 46a^2b^2 - 25b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right) + b(488a^3 + 262a^2b + b(752a^2 + 145b^2) \cos[c+dx] + 162a^2b^2 \cos[2(c+dx)] + 15b^3 \cos[3(c+dx)]) \sin[c+dx]}{(210d \sqrt{a+b \cos[c+dx]})}$$

[In] Integrate[(a + b*Cos[c + d*x])^(7/2),x]

[Out] (64*a*(11*a^3 + 11*a^2*b + 13*a*b^2 + 13*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(71*a^4 - 46*a^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(488*a^3 + 262*a*b^2 + b*(752*a^2 + 145*b^2)*Cos[c + d*x] + 162*a*b^2 *Cos[2*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. 2(280) = 560.

Time = 7.40 (sec) , antiderivative size = 824, normalized size of antiderivative = 3.35

method	result	size
default	Expression too large to display	824

[In] int((a+cos(d*x+c)*b)^(7/2),x,method=_RETURNVERBOSE)

[Out] -2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2*d*x+1/2*c)^9*b^4+648*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*c)^7*b^4+752*cos(1/2*d*x+1/2*c)^5*a^2*b^2-1296*cos(1/2*d*x+1/2*c)^5*a*b^3+640*cos(1/2*d*x+1/2*c)^5*b^4+244*cos(1/2*d*x+1/2*c)^3*a^3*b-1128*cos(1/2*d*x+1/2*c)^3*a^2*b^2+860*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^3*b^4-71*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+46*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^4+176*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-176*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+208*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-208*(sin(1/2*d*x+1/2*c)^2)^(1/2)

)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))*a*b^3-244*cos(1/2*d*x+1/2*c)*a^3*b+376*cos(1/2*d*x+1/2*c)*a^2*b^2-212*cos(1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+1/2*c)*b^4)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.93

$$\int (a + b \cos(c + dx))^{7/2} dx = \frac{\sqrt{2}(37i a^4 - 346i a^2 b^2 - 75i b^4) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c)+3a}{3}\right)}{\dots}$$

[In] integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")

[Out] 1/315*(sqrt(2)*(37*I*a^4 - 346*I*a^2*b^2 - 75*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(-37*I*a^4 + 346*I*a^2*b^2 + 75*I*b^4)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 4*8*sqrt(2)*(-11*I*a^3*b - 13*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 48*sqrt(2)*(11*I*a^3*b + 13*I*a*b^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*(15*b^4*cos(d*x + c)^2 + 66*a*b^3*cos(d*x + c) + 122*a^2*b^2 + 25*b^4)*sqrt(b*cos(d*x + c) + a)*sin(d*x + c)/(b*d)

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{7/2} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{7/2} dx = \int (b \cos(dx + c) + a)^{7/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(7/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{7/2} dx = \int (b \cos(dx + c) + a)^{7/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{7/2} dx = \int (a + b \cos(c + dx))^{7/2} dx$$

[In] int((a + b*cos(c + d*x))^(7/2),x)

[Out] int((a + b*cos(c + d*x))^(7/2), x)

3.509 $\int \cos^3(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$

Optimal result	5323
Rubi [A] (verified)	5323
Mathematica [A] (verified)	5326
Maple [A] (verified)	5326
Fricas [C] (verification not implemented)	5327
Sympy [F(-1)]	5327
Maxima [F]	5327
Giac [F]	5328
Mupad [F(-1)]	5328

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \cos^3(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \frac{47E\left(\frac{1}{2}(c + dx) \middle| \frac{8}{7}\right)}{20\sqrt{7}d} + \frac{59 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{60\sqrt{7}d} + \frac{59\sqrt{3 + 4 \cos(c + dx)} \sin(c + dx)}{105d} - \frac{3(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{70d} + \frac{\cos(c + dx)(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{14d}$$

[Out] $-3/70*(3+4*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+1/14*\cos(d*x+c)*(3+4*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+47/140*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+59/420*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+59/105*\sin(d*x+c)*(3+4*\cos(d*x+c))^(1/2)/d$

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {2872, 3102, 2832, 2831, 2740, 2732}

$$\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)} dx = \frac{59 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{60\sqrt{7}d} + \frac{47E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{20\sqrt{7}d} + \frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d} - \frac{3\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{70d} + \frac{59\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{105d}$$

[In] Int[Cos[c + d*x]^3*Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] (47*EllipticE[(c + d*x)/2, 8/7])/(20*Sqrt[7]*d) + (59*EllipticF[(c + d*x)/2, 8/7])/(60*Sqrt[7]*d) + (59*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(105*d) - (3*(3 + 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(70*d) + (Cos[c + d*x]*(3 + 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(14*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2872


```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))

```

Rule 3102

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cos(c + dx)(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{14d} \\
&+ \frac{1}{14} \int \sqrt{3 + 4 \cos(c + dx)} (3 + 10 \cos(c + dx) - 6 \cos^2(c + dx)) dx \\
&= -\frac{3(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{70d} \\
&+ \frac{\cos(c + dx)(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{14d} \\
&+ \frac{1}{140} \int \sqrt{3 + 4 \cos(c + dx)} (-6 + 118 \cos(c + dx)) dx \\
&= \frac{59\sqrt{3 + 4 \cos(c + dx)} \sin(c + dx)}{105d} - \frac{3(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{70d} \\
&+ \frac{\cos(c + dx)(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{14d} + \frac{1}{210} \int \frac{209 + 141 \cos(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx \\
&= \frac{59\sqrt{3 + 4 \cos(c + dx)} \sin(c + dx)}{105d} - \frac{3(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{70d} \\
&+ \frac{\cos(c + dx)(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{14d} \\
&+ \frac{47}{280} \int \sqrt{3 + 4 \cos(c + dx)} dx + \frac{59}{120} \int \frac{1}{\sqrt{3 + 4 \cos(c + dx)}} dx
\end{aligned}$$

$$= \frac{47E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{20\sqrt{7}d} + \frac{59\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{60\sqrt{7}d} + \frac{59\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{105d} - \frac{3(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} + \frac{\cos(c+dx)(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{14d}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.67

$$\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}dx$$

$$= \frac{141\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right) + 59\sqrt{7}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right) + \sqrt{3+4\cos(c+dx)}(212\sin(c+dx) + 9\sin(2(c+dx)) + 30\sin(3(c+dx)))}{420d}$$

[In] Integrate[Cos[c + d*x]^3*Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] (141*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + 59*Sqrt[7]*EllipticF[(c + d*x)/2, 8/7] + Sqrt[3 + 4*Cos[c + d*x]]*(212*Sin[c + d*x] + 9*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d)

Maple [A] (verified)

Time = 6.00 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.99

method	result
default	$-\frac{\sqrt{\left(8\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(7680\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-14976\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12344\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{420\sqrt{-8\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

[In] int(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/420*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7680*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-14976*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+12344*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4480*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+413*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-141*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

$$\int \cos^3(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$$

$$= \frac{4 (60 \cos(dx + c)^2 + 9 \cos(dx + c) + 91) \sqrt{4 \cos(dx + c) + 3} \sin(dx + c) - 277i \sqrt{2} \text{weierstrassPInverse}(\dots)}{d}$$

[In] integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/840*(4*(60*cos(d*x + c)^2 + 9*cos(d*x + c) + 91)*sqrt(4*cos(d*x + c) + 3)*sin(d*x + c) - 277*I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2) + 277*I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2) + 282*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2)) - 282*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2)))/d

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**3*(3+4*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \cos^3(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(dx + c) + 3} \cos(dx + c)^3 dx$$

[In] integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)

Giac [F]

$$\int \cos^3(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(dx + c) + 3} \cos(dx + c)^3 dx$$

[In] integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \cos(c + dx)^3 \sqrt{4 \cos(c + dx) + 3} dx$$

[In] int(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2),x)

[Out] int(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2), x)

3.510 $\int \cos^2(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$

Optimal result	5329
Rubi [A] (verified)	5329
Mathematica [A] (verified)	5331
Maple [A] (verified)	5331
Fricas [C] (verification not implemented)	5332
Sympy [F]	5332
Maxima [F]	5332
Giac [F]	5333
Mupad [F(-1)]	5333

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \cos^2(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \frac{21\sqrt{7}E\left(\frac{1}{2}(c + dx)\middle|\frac{8}{7}\right)}{20d} - \frac{\sqrt{7}\text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{20d} - \frac{\sqrt{3 + 4 \cos(c + dx)} \sin(c + dx)}{5d} + \frac{(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{10d}$$

[Out] $1/10*(3+4*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d+21/20*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/20*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/5*\sin(d*x+c)*(3+4*\cos(d*x+c))^(1/2)/d$

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2870, 2832, 2831, 2740, 2732}

$$\int \cos^2(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = -\frac{\sqrt{7}\text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{20d} + \frac{21\sqrt{7}E\left(\frac{1}{2}(c + dx)\middle|\frac{8}{7}\right)}{20d} + \frac{\sin(c + dx)(4 \cos(c + dx) + 3)^{3/2}}{10d} - \frac{\sin(c + dx) \sqrt{4 \cos(c + dx) + 3}}{5d}$$

[In] Int[Cos[c + d*x]^2*Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] (21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(20*d) - (Sqrt[7]*EllipticF[(c + d*x)/2, 8/7])/(20*d) - (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(5*d) + ((3 + 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2870

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rubi steps

$$\text{integral} = \frac{(3 + 4 \cos(c + dx))^{3/2} \sin(c + dx)}{10d} + \frac{1}{10} \int (6 - 3 \cos(c + dx)) \sqrt{3 + 4 \cos(c + dx)} dx$$

$$\begin{aligned}
&= -\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{5d} \\
&\quad + \frac{(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{10d} + \frac{1}{15} \int \frac{21+\frac{63}{2}\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{5d} + \frac{(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{10d} \\
&\quad - \frac{7}{40} \int \frac{1}{\sqrt{3+4\cos(c+dx)}} dx + \frac{21}{40} \int \sqrt{3+4\cos(c+dx)} dx \\
&= \frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{20d} - \frac{\sqrt{7}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{20d} \\
&\quad - \frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{5d} + \frac{(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{10d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.77

$$\begin{aligned}
&\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)} dx \\
&= \frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right) - \sqrt{7}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right) + 2\sqrt{3+4\cos(c+dx)}(\sin(c+dx) + 2\sin(2(c+dx)))}{20d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^2*Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] (21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] - Sqrt[7]*EllipticF[(c + d*x)/2, 8/7] + 2*Sqrt[3 + 4*Cos[c + d*x]]*(Sin[c + d*x] + 2*Sin[2*(c + d*x)]))/(20*d)

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.41

method	result
default	$-\frac{\sqrt{\left(8\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-256\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+384\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-140\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{20\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

[In] int(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/20*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-256*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+384*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-140*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE

$\cos(1/2*d*x+1/2*c), 2*2^{(1/2)})/(-8*\sin(1/2*d*x+1/2*c)^4+7*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(8*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int \cos^2(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$$

$$= \frac{4 \sqrt{4 \cos(dx + c) + 3} (4 \cos(dx + c) + 1) \sin(dx + c) - 7i \sqrt{2} \text{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) + 1/2) + 7i \sqrt{2} \text{weierstrassPInverse}(-1, 1, \cos(dx + c) - i \sin(dx + c) + 1/2) + 42i \sqrt{2} \text{weierstrassZeta}(-1, 1, \text{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) + 1/2)) - 42i \sqrt{2} \text{weierstrassZeta}(-1, 1, \text{weierstrassPInverse}(-1, 1, \cos(dx + c) - i \sin(dx + c) + 1/2))}{d}$$

[In] integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/40*(4*sqrt(4*cos(d*x + c) + 3)*(4*cos(d*x + c) + 1)*sin(d*x + c) - 7*I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2) + 7*I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2) + 42*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2)) - 42*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2)))/d

Sympy [F]

$$\int \cos^2(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(c + dx) + 3} \cos^2(c + dx) dx$$

[In] integrate(cos(d*x+c)**2*(3+4*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(4*cos(c + d*x) + 3)*cos(c + d*x)**2, x)

Maxima [F]

$$\int \cos^2(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(dx + c) + 3} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)

Giac [F]

$$\int \cos^2(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(dx + c) + 3} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \cos(c + dx)^2 \sqrt{4 \cos(c + dx) + 3} dx$$

[In] int(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2),x)

[Out] int(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2), x)

3.511 $\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$

Optimal result	5334
Rubi [A] (verified)	5334
Mathematica [A] (verified)	5336
Maple [A] (verified)	5336
Fricas [C] (verification not implemented)	5336
Sympy [F]	5337
Maxima [F]	5337
Giac [F]	5337
Mupad [F(-1)]	5338

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \frac{\sqrt{7} E\left(\frac{1}{2}(c + dx) \middle| \frac{8}{7}\right)}{2d} + \frac{\sqrt{7} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{6d} + \frac{2\sqrt{3 + 4 \cos(c + dx)} \sin(c + dx)}{3d}$$

[Out] $\frac{1}{2} * (\cos(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / \cos(1/2 * d * x + 1/2 * c) * \operatorname{EllipticE}(\sin(1/2 * d * x + 1/2 * c), 2/7 * 14 ^ (1/2)) / d * 7 ^ (1/2) + 1/6 * (\cos(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / \cos(1/2 * d * x + 1/2 * c) * \operatorname{EllipticF}(\sin(1/2 * d * x + 1/2 * c), 2/7 * 14 ^ (1/2)) / d * 7 ^ (1/2) + 2/3 * \sin(d * x + c) * (3 + 4 * \cos(d * x + c)) ^ (1/2) / d$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2832, 2831, 2740, 2732}

$$\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \frac{\sqrt{7} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{6d} + \frac{\sqrt{7} E\left(\frac{1}{2}(c + dx) \middle| \frac{8}{7}\right)}{2d} + \frac{2 \sin(c + dx) \sqrt{4 \cos(c + dx) + 3}}{3d}$$

[In] `Int[Cos[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]],x]`

[Out] $(\operatorname{Sqrt}[7] * \operatorname{EllipticE}[(c + d*x)/2, 8/7]) / (2*d) + (\operatorname{Sqrt}[7] * \operatorname{EllipticF}[(c + d*x)/2, 8/7]) / (6*d) + (2 * \operatorname{Sqrt}[3 + 4 * \operatorname{Cos}[c + d*x]] * \operatorname{Sin}[c + d*x]) / (3*d)$

Rule 2732

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a
+ b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3} \int \frac{2+\frac{3}{2}\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
&= \frac{2\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{3d} \\
&\quad + \frac{1}{4} \int \sqrt{3+4\cos(c+dx)} dx + \frac{7}{12} \int \frac{1}{\sqrt{3+4\cos(c+dx)}} dx \\
&= \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{2d} + \frac{\sqrt{7}\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{6d} + \frac{2\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$$

$$= \frac{3\sqrt{7}E\left(\frac{1}{2}(c + dx)\middle|\frac{8}{7}\right) + \sqrt{7}\operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right) + 4\sqrt{3 + 4 \cos(c + dx)} \sin(c + dx)}{6d}$$

[In] Integrate[Cos[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] (3*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + Sqrt[7]*EllipticF[(c + d*x)/2, 8/7] + 4*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(6*d)

Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.96

method	result
default	$-\frac{\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\left(64\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 56\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 7\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{8}\right)}{6\sqrt{-8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 7\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)}$

[In] int(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/6*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-56*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.64

$$\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx$$

$$= \frac{8\sqrt{4\cos(dx+c)+3}\sin(dx+c) - 5i\sqrt{2}\operatorname{weierstrassPInverse}(-1, 1, \cos(dx+c) + i\sin(dx+c) + \frac{1}{2})}{d}$$

[In] integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{12} \cdot (8 \cdot \sqrt{4 \cos(dx + c) + 3} \cdot \sin(dx + c) - 5 \cdot I \cdot \sqrt{2} \cdot \text{weierstrassPInverse}(-1, 1, \cos(dx + c) + I \cdot \sin(dx + c) + 1/2) + 5 \cdot I \cdot \sqrt{2} \cdot \text{weierstrassPInverse}(-1, 1, \cos(dx + c) - I \cdot \sin(dx + c) + 1/2) + 6 \cdot I \cdot \sqrt{2} \cdot \text{weierstrassZeta}(-1, 1, \text{weierstrassPInverse}(-1, 1, \cos(dx + c) + I \cdot \sin(dx + c) + 1/2)) - 6 \cdot I \cdot \sqrt{2} \cdot \text{weierstrassZeta}(-1, 1, \text{weierstrassPInverse}(-1, 1, \cos(dx + c) - I \cdot \sin(dx + c) + 1/2))) / d$

Sympy [F]

$$\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(c + dx) + 3} \cos(c + dx) dx$$

[In] `integrate(cos(d*x+c)*(3+4*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(4*cos(c + d*x) + 3)*cos(c + d*x), x)`

Maxima [F]

$$\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(dx + c) + 3} \cos(dx + c) dx$$

[In] `integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c), x)`

Giac [F]

$$\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(dx + c) + 3} \cos(dx + c) dx$$

[In] `integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c), x)`

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx) \sqrt{3 + 4 \cos(c + dx)} dx = \int \cos(c + dx) \sqrt{4 \cos(c + dx) + 3} dx$$

```
[In] int(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2),x)
```

```
[Out] int(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2), x)
```

3.512 $\int \sqrt{3 + 4 \cos(c + dx)} dx$

Optimal result	5339
Rubi [A] (verified)	5339
Mathematica [A] (verified)	5340
Maple [B] (verified)	5340
Fricas [C] (verification not implemented)	5341
Sympy [F]	5341
Maxima [F]	5341
Giac [F]	5342
Mupad [F(-1)]	5342

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \sqrt{3 + 4 \cos(c + dx)} dx = \frac{2\sqrt{7}E\left(\frac{1}{2}(c + dx)\middle|\frac{8}{7}\right)}{d}$$

[Out] $2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}$

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2732}

$$\int \sqrt{3 + 4 \cos(c + dx)} dx = \frac{2\sqrt{7}E\left(\frac{1}{2}(c + dx)\middle|\frac{8}{7}\right)}{d}$$

[In] `Int[Sqrt[3 + 4*Cos[c + d*x]],x]`

[Out] `(2*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d`

Rule 2732

`Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]`

Rubi steps

$$\text{integral} = \frac{2\sqrt{7}E\left(\frac{1}{2}(c + dx)\middle|\frac{8}{7}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sqrt{3 + 4 \cos(c + dx)} dx = \frac{2\sqrt{7}E\left(\frac{1}{2}(c + dx)\middle|\frac{8}{7}\right)}{d}$$

[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] (2*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(47) = 94.

Time = 3.66 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.96

method	result
default	$\frac{2\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{1 - 8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2\sqrt{2}\right)}{\sqrt{-8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 7\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}d}$
risch	$i \frac{6\left(\frac{3}{4} + \frac{i\sqrt{7}}{4}\right)\sqrt{\frac{e^{i(dx+c)} + \frac{3}{4} + \frac{i\sqrt{7}}{4}}{\frac{3}{4} + \frac{i\sqrt{7}}{4}}}\sqrt{14}\sqrt{i\left(e^{i(dx+c)} + \frac{3}{4} - \frac{i\sqrt{7}}{4}\right)}\sqrt{7}\sqrt{\frac{e^{i(dx+c)}}{-\frac{3}{4} - \frac{i\sqrt{7}}{4}}}}{7\sqrt{2e^{3i(dx+c)} + 3e^{2i(dx+c)} + 2e^{i(dx+c)}}}}{d}$

[In] int((3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2*2^(1/2))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.70

$$\int \sqrt{3 + 4 \cos(c + dx)} dx$$

$$= \frac{-i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) + \frac{1}{2}) + i \sqrt{2} \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) - i \sin(dx + c) + \frac{1}{2}) + 4 \operatorname{weierstrassZeta}(-1, 1, \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) + i \sin(dx + c) + \frac{1}{2})) - 4 \operatorname{weierstrassZeta}(-1, 1, \operatorname{weierstrassPInverse}(-1, 1, \cos(dx + c) - i \sin(dx + c) + \frac{1}{2}))}{d}$$

[In] integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/2*(-I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2) + I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2) + 4*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2)) - 4*I*sqrt(2)*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2)))/d

Sympy [F]

$$\int \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(c + dx) + 3} dx$$

[In] integrate((3+4*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(4*cos(c + d*x) + 3), x)

Maxima [F]

$$\int \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(dx + c) + 3} dx$$

[In] integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(dx + c) + 3} dx$$

[In] integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(4*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 + 4 \cos(c + dx)} dx = \int \sqrt{4 \cos(c + dx) + 3} dx$$

[In] int((4*cos(c + d*x) + 3)^(1/2),x)

[Out] int((4*cos(c + d*x) + 3)^(1/2), x)

3.513 $\int \sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) dx$

Optimal result	5343
Rubi [A] (verified)	5343
Mathematica [A] (verified)	5344
Maple [A] (verified)	5345
Fricas [F]	5345
Sympy [F]	5345
Maxima [F]	5346
Giac [F]	5346
Mupad [F(-1)]	5346

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) dx = \frac{8 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d} + \frac{6 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[Out] $8/7*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+6/7*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2/7*14^{(1/2)})/d*7^{(1/2)}$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2882, 2740, 2884}

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) dx = \frac{8 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d} + \frac{6 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[3 + 4*\operatorname{Cos}[c + d*x]]*\operatorname{Sec}[c + d*x], x]$

[Out] $(8*\operatorname{EllipticF}[(c + d*x)/2, 8/7])/(\operatorname{Sqrt}[7]*d) + (6*\operatorname{EllipticPi}[2, (c + d*x)/2, 8/7])/(\operatorname{Sqrt}[7]*d)$

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2882

```
Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 3 \int \frac{\sec(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx + 4 \int \frac{1}{\sqrt{3 + 4 \cos(c + dx)}} dx \\ &= \frac{8 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d} + \frac{6 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\begin{aligned} &\int \sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) dx \\ &= \frac{8 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right) + 6 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d} \end{aligned}$$

```
[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (8*EllipticF[(c + d*x)/2, 8/7] + 6*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.29

method	result
default	$-\frac{2\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\sqrt{2}\right)-3\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,2\right)\right)}{\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d$

```
[In] int(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)*(4*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-3*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2)))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [F]

$$\int \sqrt{3+4\cos(c+dx)} \sec(c+dx) dx = \int \sqrt{4\cos(dx+c)+3} \sec(dx+c) dx$$

```
[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

Sympy [F]

$$\int \sqrt{3+4\cos(c+dx)} \sec(c+dx) dx = \int \sqrt{4\cos(c+dx)+3} \sec(c+dx) dx$$

```
[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) dx = \int \sqrt{4 \cos(dx + c) + 3} \sec(dx + c) dx$$

[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)

Giac [F]

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) dx = \int \sqrt{4 \cos(dx + c) + 3} \sec(dx + c) dx$$

[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) dx = \int \frac{\sqrt{4 \cos(c + dx) + 3}}{\cos(c + dx)} dx$$

[In] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x),x)

[Out] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x), x)

3.514 $\int \sqrt{3 + 4 \cos(c + dx)} \sec^2(c + dx) dx$

Optimal result	5347
Rubi [A] (verified)	5347
Mathematica [C] (verified)	5349
Maple [B] (verified)	5350
Fricas [F]	5350
Sympy [F]	5350
Maxima [F]	5351
Giac [F]	5351
Mupad [F(-1)]	5351

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec^2(c + dx) dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c + dx)\middle|\frac{8}{7}\right)}{d} + \frac{3 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

$$+ \frac{4 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

$$+ \frac{\sqrt{3 + 4 \cos(c + dx)} \tan(c + dx)}{d}$$

[Out] $3/7*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+4/7*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2/7*14^{(1/2)})/d*7^{(1/2)}-(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+(3+4*\cos(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2875, 3139, 2732, 3081, 2740, 2884}

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec^2(c + dx) dx = \frac{3 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d} - \frac{\sqrt{7}E\left(\frac{1}{2}(c + dx)\middle|\frac{8}{7}\right)}{d}$$

$$+ \frac{4 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

$$+ \frac{\sqrt{4 \cos(c + dx) + 3} \tan(c + dx)}{d}$$

[In] Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]

[Out] -((Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d) + (3*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d) + (4*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Tan[c + d*x])/d

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2875

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]))/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3139


```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
+ (f_.)*(x_)]])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{(2-2\cos^2(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
&= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d} \\
&\quad - \frac{1}{4} \int \frac{(-8-6\cos(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx - \frac{1}{2} \int \sqrt{3+4\cos(c+dx)} dx \\
&= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{d} + \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d} \\
&\quad + \frac{3}{2} \int \frac{1}{\sqrt{3+4\cos(c+dx)}} dx + 2 \int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{d} + \frac{3\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d} \\
&\quad + \frac{4\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d} + \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.65

$$\begin{aligned}
&\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx) dx \\
&= \frac{6\sqrt{7}\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right) + \frac{i\sqrt{7}\left(21E\left(i\text{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right)\middle|-\frac{1}{7}\right) - 12\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right), -\frac{1}{7}\right)\right)}{\sqrt{\sin^2(c+dx)}}}{21d}
\end{aligned}$$

[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]

[Out] (6*sqrt(7)*EllipticPi[2, (c + d*x)/2, 8/7] + (I*sqrt(7)*(21*EllipticE[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2] + 21*sqrt(3 + 4*Cos[c + d*x])*Tan[c + d*x])/(21*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(165) = 330.

Time = 3.78 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.68

method	result
default	$\frac{\sqrt{-\left(1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}+3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$

[In] `int(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -\left(-\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right) \\ & \left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1 \\ & +3\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4 \\ & +7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2\right)^{\frac{1}{2}} \\ & +\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4 \\ & +7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\text{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2\right)^{\frac{1}{2}} \\ & -4\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4 \\ & +7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\text{EllipticPi}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2,2\right)^{\frac{1}{2}}\right) \\ & / \sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right) / \left(8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)^{\frac{1}{2}} / d \end{aligned}$$

Fricas [F]

$$\int \sqrt{3+4\cos(c+dx)} \sec^2(c+dx) dx = \int \sqrt{4\cos(dx+c)+3} \sec(dx+c)^2 dx$$

[In] `integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x,algorithm="fricas")`

[Out] `integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)`

Sympy [F]

$$\int \sqrt{3+4\cos(c+dx)} \sec^2(c+dx) dx = \int \sqrt{4\cos(c+dx)+3} \sec^2(c+dx) dx$$

[In] `integrate(sec(d*x+c)**2*(3+4*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x)**2, x)`

Maxima [F]

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec^2(c + dx) dx = \int \sqrt{4 \cos(dx + c) + 3} \sec(dx + c)^2 dx$$

[In] integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)

Giac [F]

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec^2(c + dx) dx = \int \sqrt{4 \cos(dx + c) + 3} \sec(dx + c)^2 dx$$

[In] integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec^2(c + dx) dx = \int \frac{\sqrt{4 \cos(c + dx) + 3}}{\cos(c + dx)^2} dx$$

[In] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^2,x)

[Out] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^2, x)

3.515 $\int \sqrt{3 + 4 \cos(c + dx)} \sec^3(c + dx) dx$

Optimal result	5352
Rubi [A] (verified)	5352
Mathematica [C] (verified)	5355
Maple [B] (verified)	5356
Fricas [F]	5356
Sympy [F]	5356
Maxima [F]	5357
Giac [F]	5357
Mupad [F(-1)]	5357

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec^3(c + dx) dx = -\frac{\sqrt{7} E\left(\frac{1}{2}(c + dx) \middle| \frac{8}{7}\right)}{3d} + \frac{3 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

$$+ \frac{5 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{8}{7}\right)}{3\sqrt{7}d}$$

$$+ \frac{\sqrt{3 + 4 \cos(c + dx)} \tan(c + dx)}{3d}$$

$$+ \frac{\sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d}$$

[Out] $3/7*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+5/21*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2/7*14^{(1/2)})/d*7^{(1/2)}-1/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+1/3*(3+4*\cos(d*x+c))^{(1/2)}*\tan(d*x+c)/d+1/2*\sec(d*x+c)*(3+4*\cos(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {2875, 3134, 3138, 2732, 3081, 2740, 2884}

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec^3(c + dx) dx = \frac{3 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d} - \frac{\sqrt{7}E\left(\frac{1}{2}(c + dx) \middle| \frac{8}{7}\right)}{3d} + \frac{5 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{8}{7}\right)}{3\sqrt{7}d} + \frac{\sqrt{4 \cos(c + dx) + 3 \tan(c + dx)}}{3d} + \frac{\sqrt{4 \cos(c + dx) + 3 \tan(c + dx)} \sec(c + dx)}{2d}$$

[In] Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]

[Out] -1/3*(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d + (3*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d) + (5*EllipticPi[2, (c + d*x)/2, 8/7])/(3*Sqrt[7]*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2875

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c

, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3138

Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\text{integral} = \frac{\sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2} \int \frac{(2 + 3 \cos(c + dx) + 2 \cos^2(c + dx)) \sec^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx$$

$$\begin{aligned}
&= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad + \frac{1}{6} \int \frac{(5+6\cos(c+dx)-4\cos^2(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
&= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad - \frac{1}{24} \int \frac{(-20-36\cos(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx - \frac{1}{6} \int \sqrt{3+4\cos(c+dx)} dx \\
&= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} \\
&\quad + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} \\
&\quad + \frac{5}{6} \int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx + \frac{3}{2} \int \frac{1}{\sqrt{3+4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{3\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d} + \frac{5\operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{3\sqrt{7}d} \\
&\quad + \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.44

$$\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx) dx$$

$$= \frac{12\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}} + \frac{6\operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}} + \frac{2i\left(21E\left(i\operatorname{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right)\middle|-\frac{1}{7}\right)-12\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right)\right)\right)}{3\sqrt{7}\sqrt{\sin^2(c+dx)}}$$

[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]

[Out] ((12*EllipticF[(c + d*x)/2, 8/7])/Sqrt[7] + (6*EllipticPi[2, (c + d*x)/2, 8/7])/Sqrt[7] + (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + (3 + 2*Cos[c + d*x])*Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(195) = 390.

Time = 4.08 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.02

method	result
default	$\frac{\sqrt{-\left(1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}\right)}$

[In] `int(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -\left(-\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right) \\ & \left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1 \\ & \left)^{-2}-\frac{2}{3}\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}} \\ & \left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1+3\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{\frac{1}{2}}\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2 \\ & \right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2*2^{\frac{1}{2}}\right) \\ & +\frac{1}{3}\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{\frac{1}{2}}\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4 \\ & +7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\text{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2*2^{\frac{1}{2}}\right)-\frac{5}{3}\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2 \\ & \right)^{\frac{1}{2}}\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right) \\ & \right)^{\frac{1}{2}}\text{EllipticPi}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2,2*2^{\frac{1}{2}}\right)\right)/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/\left(8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)^{\frac{1}{2}}/d \end{aligned}$$

Fricas [F]

$$\int \sqrt{3+4\cos(c+dx)} \sec^3(c+dx) dx = \int \sqrt{4\cos(dx+c)+3} \sec(dx+c)^3 dx$$

[In] `integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)`

Sympy [F]

$$\int \sqrt{3+4\cos(c+dx)} \sec^3(c+dx) dx = \int \sqrt{4\cos(c+dx)+3} \sec^3(c+dx) dx$$

[In] `integrate(sec(d*x+c)**3*(3+4*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x)**3, x)`

Maxima [F]

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec^3(c + dx) dx = \int \sqrt{4 \cos(dx + c) + 3} \sec(dx + c)^3 dx$$

[In] integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)

Giac [F]

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec^3(c + dx) dx = \int \sqrt{4 \cos(dx + c) + 3} \sec(dx + c)^3 dx$$

[In] integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 + 4 \cos(c + dx)} \sec^3(c + dx) dx = \int \frac{\sqrt{4 \cos(c + dx) + 3}}{\cos(c + dx)^3} dx$$

[In] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^3,x)

[Out] int((4*cos(c + d*x) + 3)^(1/2)/cos(c + d*x)^3, x)

3.516 $\int \sqrt{3 - 4 \cos(c + dx)} \cos^3(c + dx) dx$

Optimal result	5358
Rubi [A] (verified)	5358
Mathematica [A] (verified)	5361
Maple [A] (verified)	5361
Fricas [C] (verification not implemented)	5362
Sympy [F(-1)]	5362
Maxima [F]	5363
Giac [F]	5363
Mupad [F(-1)]	5363

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^3(c + dx) dx = -\frac{47E\left(\frac{1}{2}(c + \pi + dx) \middle| \frac{8}{7}\right)}{20\sqrt{7}d} - \frac{59 \operatorname{EllipticF}\left(\frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{60\sqrt{7}d} + \frac{59\sqrt{3 - 4 \cos(c + dx)} \sin(c + dx)}{105d} - \frac{3(3 - 4 \cos(c + dx))^{3/2} \sin(c + dx)}{70d} - \frac{(3 - 4 \cos(c + dx))^{3/2} \cos(c + dx) \sin(c + dx)}{14d}$$

```
[Out] -3/70*(3-4*cos(d*x+c))^(3/2)*sin(d*x+c)/d-1/14*(3-4*cos(d*x+c))^(3/2)*cos(d*x+c)*sin(d*x+c)/d+47/140*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+59/420*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+59/105*sin(d*x+c)*(3-4*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {2872, 3102, 2832, 2831, 2741, 2733}

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^3(c + dx) dx = -\frac{59 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx + \pi), \frac{8}{7}\right)}{60\sqrt{7}d} - \frac{47E\left(\frac{1}{2}(c + dx + \pi) \middle| \frac{8}{7}\right)}{20\sqrt{7}d} - \frac{\sin(c + dx) \cos(c + dx)(3 - 4 \cos(c + dx))^{3/2}}{14d} - \frac{3 \sin(c + dx)(3 - 4 \cos(c + dx))^{3/2}}{70d} + \frac{59 \sin(c + dx) \sqrt{3 - 4 \cos(c + dx)}}{105d}$$

[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^3,x]

[Out] (-47*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*Sqrt[7]*d) - (59*EllipticF[(c + Pi + d*x)/2, 8/7])/(60*Sqrt[7]*d) + (59*Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(105*d) - (3*(3 - 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(70*d) - ((3 - 4*Cos[c + d*x])^(3/2)*Cos[c + d*x]*Sin[c + d*x])/(14*d)

Rule 2733

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2741

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,

0] && IntegerQ[2*m]

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*
x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Ssin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{(3 - 4 \cos(c + dx))^{3/2} \cos(c + dx) \sin(c + dx)}{14d} \\
 &\quad - \frac{1}{14} \int \sqrt{3 - 4 \cos(c + dx)} (3 - 10 \cos(c + dx) - 6 \cos^2(c + dx)) dx \\
 &= -\frac{3(3 - 4 \cos(c + dx))^{3/2} \sin(c + dx)}{70d} \\
 &\quad - \frac{(3 - 4 \cos(c + dx))^{3/2} \cos(c + dx) \sin(c + dx)}{14d} \\
 &\quad + \frac{1}{140} \int \sqrt{3 - 4 \cos(c + dx)} (6 + 118 \cos(c + dx)) dx \\
 &= \frac{59 \sqrt{3 - 4 \cos(c + dx)} \sin(c + dx)}{105d} - \frac{3(3 - 4 \cos(c + dx))^{3/2} \sin(c + dx)}{70d} \\
 &\quad - \frac{(3 - 4 \cos(c + dx))^{3/2} \cos(c + dx) \sin(c + dx)}{14d} \\
 &\quad + \frac{1}{210} \int \frac{-209 + 141 \cos(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{59\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{105d} - \frac{3(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} \\
&\quad - \frac{(3-4\cos(c+dx))^{3/2}\cos(c+dx)\sin(c+dx)}{14d} \\
&\quad - \frac{47}{280} \int \sqrt{3-4\cos(c+dx)} dx - \frac{59}{120} \int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx \\
&= -\frac{47E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{20\sqrt{7}d} - \frac{59\operatorname{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{60\sqrt{7}d} \\
&\quad + \frac{59\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{105d} - \frac{3(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} \\
&\quad - \frac{(3-4\cos(c+dx))^{3/2}\cos(c+dx)\sin(c+dx)}{14d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81

$$\begin{aligned}
&\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx) dx \\
&= \frac{141\sqrt{-3+4\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|8\right) - 413\sqrt{-3+4\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),8\right) + 654\sin(c+dx)}{420d\sqrt{3-4\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^3,x]

[Out] (141*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] - 413*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] + 654*Sin[c + d*x] - 511*Sin[2*(c + d*x)] + 108*Sin[3*(c + d*x)] - 60*Sin[4*(c + d*x)])/(420*d*Sqrt[3 - 4*Cos[c + d*x]])

Maple [A] (verified)

Time = 7.31 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.97

method	result
default	$ \frac{\sqrt{-\left(8\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(7680\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8064\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+5432\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{420\sqrt{8\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-3}} $

[In] int(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/420*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7680*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-8064*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+5432*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-568*sin(1/2*d*x+1/2*c)^2*c

```
os(1/2*d*x+1/2*c)+59*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-
7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+141*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/
7*14^(1/2)))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^3(c + dx) dx$$

$$= \frac{4 (60 \cos(dx + c)^2 - 9 \cos(dx + c) + 91) \sqrt{-4 \cos(dx + c) + 3} \sin(dx + c) + 277 \sqrt{2} \text{weierstrassPInverse}(\dots)}{\dots}$$

```
[In] integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/840*(4*(60*cos(d*x + c)^2 - 9*cos(d*x + c) + 91)*sqrt(-4*cos(d*x + c) + 3
)*sin(d*x + c) + 277*sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I*s
in(d*x + c) - 1/2) + 277*sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) -
I*sin(d*x + c) - 1/2) + 282*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPIn
verse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2)) + 282*sqrt(2)*weierstra
ssZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) -
1/2)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^3(c + dx) dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^3(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \cos(dx + c)^3 dx$$

[In] integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)

Giac [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^3(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \cos(dx + c)^3 dx$$

[In] integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^3(c + dx) dx = \int \cos(c + dx)^3 \sqrt{3 - 4 \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2), x)

3.517 $\int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx$

Optimal result	5364
Rubi [A] (verified)	5364
Mathematica [A] (verified)	5366
Maple [A] (verified)	5366
Fricas [C] (verification not implemented)	5367
Sympy [F]	5367
Maxima [F]	5367
Giac [F]	5368
Mupad [F(-1)]	5368

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx = \frac{21\sqrt{7}E\left(\frac{1}{2}(c + \pi + dx)\middle|\frac{8}{7}\right)}{20d} - \frac{\sqrt{7} \operatorname{EllipticF}\left(\frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{20d} + \frac{\sqrt{3 - 4 \cos(c + dx)} \sin(c + dx)}{5d} - \frac{(3 - 4 \cos(c + dx))^{3/2} \sin(c + dx)}{10d}$$

[Out] $-1/10*(3-4*\cos(d*x+c))^{3/2}*\sin(d*x+c)/d-21/20*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c),2/7*14^{(1/2)})/d*7^{(1/2)}+1/20*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c),2/7*14^{(1/2)})/d*7^{(1/2)}+1/5*\sin(d*x+c)*(3-4*\cos(d*x+c))^{1/2}/d$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2870, 2832, 2831, 2741, 2733}

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx = -\frac{\sqrt{7} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx + \pi), \frac{8}{7}\right)}{20d} + \frac{21\sqrt{7}E\left(\frac{1}{2}(c + dx + \pi)\middle|\frac{8}{7}\right)}{20d} - \frac{\sin(c + dx)(3 - 4 \cos(c + dx))^{3/2}}{10d} + \frac{\sin(c + dx)\sqrt{3 - 4 \cos(c + dx)}}{5d}$$

[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^2,x]

[Out] (21*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*d) - (Sqrt[7]*EllipticF[(c + Pi + d*x)/2, 8/7])/(20*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(5*d) - ((3 - 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10*d)

Rule 2733

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2741

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2832

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m/(f*(m + 1))))], x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2870

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rubi steps

$$\text{integral} = -\frac{(3 - 4 \cos(c + dx))^{3/2} \sin(c + dx)}{10d} - \frac{1}{10} \int \sqrt{3 - 4 \cos(c + dx)} (-6 - 3 \cos(c + dx)) dx$$

$$\begin{aligned}
&= \frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{5d} - \frac{(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{10d} \\
&\quad - \frac{1}{15} \int \frac{-21 + \frac{63}{2}\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
&= \frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{5d} - \frac{(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{10d} \\
&\quad - \frac{7}{40} \int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx + \frac{21}{40} \int \sqrt{3-4\cos(c+dx)} dx \\
&= \frac{21\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{20d} - \frac{\sqrt{7}\operatorname{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{20d} \\
&\quad + \frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{5d} - \frac{(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{10d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx) dx = \frac{21\sqrt{-3+4\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|8\right) + 7\sqrt{-3+4\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 8\right) + 14\sin(c+dx)}{20d\sqrt{3-4\cos(c+dx)}}$$

[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^2, x]

[Out] -1/20*(21*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 7*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] + 14*Sin[c + d*x] - 16*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)])/(d*Sqrt[3 - 4*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.36

method	result
default	$\frac{\sqrt{-\left(8\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(-256\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{20\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$

[In] int(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2), x, method=_RETURNVERBOSE)

[Out] 1/20*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-256*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2/7*14^(1/2))

$$-21*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(56*\sin(1/2*d*x+1/2*c)^2-7)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2/7*14^{(1/2)})/(8*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(-8*\cos(1/2*d*x+1/2*c)^2+7)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx$$

$$= \frac{4(4 \cos(dx + c) - 1)\sqrt{-4 \cos(dx + c) + 3} \sin(dx + c) - 7\sqrt{2} \text{weierstrassPInverse}(-1, -1, \cos(dx + c))}{d}$$

[In] integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/40*(4*(4*cos(d*x + c) - 1)*sqrt(-4*cos(d*x + c) + 3)*sin(d*x + c) - 7*sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2) - 7*sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2) - 4*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2)) - 4*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2)))/d

Sympy [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx = \int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx$$

[In] integrate(cos(d*x+c)**2*(3-4*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(3 - 4*cos(c + d*x))*cos(c + d*x)**2, x)

Maxima [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)

Giac [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos^2(c + dx) dx = \int \cos(c + dx)^2 \sqrt{3 - 4 \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2), x)

3.518 $\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx$

Optimal result	5369
Rubi [A] (verified)	5369
Mathematica [A] (verified)	5371
Maple [A] (verified)	5371
Fricas [C] (verification not implemented)	5371
Sympy [F]	5372
Maxima [F]	5372
Giac [F]	5372
Mupad [F(-1)]	5373

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx = -\frac{\sqrt{7} E\left(\frac{1}{2}(c + \pi + dx) \middle| \frac{8}{7}\right)}{2d} - \frac{\sqrt{7} \operatorname{EllipticF}\left(\frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{6d} + \frac{2\sqrt{3 - 4 \cos(c + dx)} \sin(c + dx)}{3d}$$

[Out] $\frac{1}{2} \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} / \sin(\frac{1}{2} d x + \frac{1}{2} c) \cdot \operatorname{EllipticE}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2/7 \cdot 14^{\frac{1}{2}}) / d \cdot 7^{\frac{1}{2}} + 1/6 \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} / \sin(\frac{1}{2} d x + \frac{1}{2} c) \cdot \operatorname{EllipticF}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2/7 \cdot 14^{\frac{1}{2}}) / d \cdot 7^{\frac{1}{2}} + 2/3 \cdot \sin(dx + c) \cdot (3 - 4 \cdot \cos(dx + c))^{\frac{1}{2}} / d$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2832, 2831, 2741, 2733}

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx = -\frac{\sqrt{7} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx + \pi), \frac{8}{7}\right)}{6d} - \frac{\sqrt{7} E\left(\frac{1}{2}(c + dx + \pi) \middle| \frac{8}{7}\right)}{2d} + \frac{2 \sin(c + dx) \sqrt{3 - 4 \cos(c + dx)}}{3d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[3 - 4 \cdot \operatorname{Cos}[c + d \cdot x]] \cdot \operatorname{Cos}[c + d \cdot x], x]$

[Out] $-1/2*(\text{Sqrt}[7]*\text{EllipticE}[(c + \text{Pi} + d*x)/2, 8/7])/d - (\text{Sqrt}[7]*\text{EllipticF}[(c + \text{Pi} + d*x)/2, 8/7])/(6*d) + (2*\text{Sqrt}[3 - 4*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*d)$

Rule 2733

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\text{sin}[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[2*(\text{Sqrt}[a - b]/d)*\text{EllipticE}[(1/2)*(c + \text{Pi}/2 + d*x), -2*(b/(a - b))], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a - b, 0]$

Rule 2741

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*\text{sin}[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2/(d*\text{Sqrt}[a - b]))*\text{EllipticF}[(1/2)*(c + \text{Pi}/2 + d*x), -2*(b/(a - b))], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a - b, 0]$

Rule 2831

$\text{Int}[((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])/\text{Sqrt}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]], x_Symbol] \rightarrow \text{Dist}[(b*c - a*d)/b, \text{Int}[1/\text{Sqrt}[a + b*\text{Sin}[e + f*x]], x], x] + \text{Dist}[d/b, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2832

$\text{Int}[((a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)])^m*((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)]), x_Symbol] \rightarrow \text{Simp}[(-d)*\text{Cos}[e + f*x]*((a + b*\text{Sin}[e + f*x])^m/(f*(m + 1))), x] + \text{Dist}[1/(m + 1), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m-1}*\text{Simp}[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3} \int \frac{-2 + \frac{3}{2}\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\ &= \frac{2\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{3d} \\ &\quad - \frac{1}{4} \int \sqrt{3-4\cos(c+dx)} dx - \frac{7}{12} \int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx \\ &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{2d} - \frac{\sqrt{7}\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{6d} \\ &\quad + \frac{2\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx$$

$$= \frac{3\sqrt{-3 + 4 \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 8\right) - 7\sqrt{-3 + 4 \cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 8\right) + 12 \sin(c + dx)}{6d\sqrt{3 - 4 \cos(c + dx)}}$$

[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x],x]

[Out] (3*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] - 7*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] + 12*Sin[c + d*x] - 8*Sin[2*(c + d*x)]) / (6*d*Sqrt[3 - 4*Cos[c + d*x]])

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.89

method	result
default	$\frac{\sqrt{-\left(8\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 7\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(64\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 8\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{56\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sin\left(\frac{dx}{2} + \frac{c}{2}\right)}\right)}{6\sqrt{8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sin\left(\frac{dx}{2} + \frac{c}{2}\right)}}$

[In] int(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.60

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx$$

$$= \frac{8\sqrt{-4 \cos(dx + c) + 3 \sin(dx + c)} + 5\sqrt{2}\operatorname{weierstrassPInverse}(-1, -1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2})}{2}$$

[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/12*(8*sqrt(-4*cos(d*x + c) + 3)*sin(d*x + c) + 5*sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2) + 5*sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2) + 6*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2)) + 6*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2)))/d

Sympy [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx = \int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx$$

[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(3 - 4*cos(c + d*x))*cos(c + d*x), x)

Maxima [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c), x)

Giac [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 - 4 \cos(c + dx)} \cos(c + dx) dx = \int \cos(c + dx) \sqrt{3 - 4 \cos(c + dx)} dx$$

```
[In] int(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2), x)
```

3.519 $\int \sqrt{3 - 4 \cos(c + dx)} dx$

Optimal result	5374
Rubi [A] (verified)	5374
Mathematica [A] (verified)	5375
Maple [B] (verified)	5375
Fricas [C] (verification not implemented)	5376
Sympy [F]	5376
Maxima [F]	5376
Giac [F]	5377
Mupad [F(-1)]	5377

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \sqrt{3 - 4 \cos(c + dx)} dx = \frac{2\sqrt{7}E\left(\frac{1}{2}(c + \pi + dx)\middle|\frac{8}{7}\right)}{d}$$

[Out] $-2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}$

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2733}

$$\int \sqrt{3 - 4 \cos(c + dx)} dx = \frac{2\sqrt{7}E\left(\frac{1}{2}(c + dx + \pi)\middle|\frac{8}{7}\right)}{d}$$

[In] `Int[Sqrt[3 - 4*Cos[c + d*x]],x]`

[Out] `(2*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d`

Rule 2733

`Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]`

Rubi steps

$$\text{integral} = \frac{2\sqrt{7}E\left(\frac{1}{2}(c + \pi + dx)\middle|\frac{8}{7}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \sqrt{3 - 4 \cos(c + dx)} dx = -\frac{2\sqrt{-3 + 4 \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 8\right)}{d\sqrt{3 - 4 \cos(c + dx)}}$$

[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]],x]

[Out] (-2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8])/(d*Sqrt[3 - 4*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(47) = 94.

Time = 3.81 (sec) , antiderivative size = 138, normalized size of antiderivative = 5.75

method	result
default	$-\frac{2\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 7\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{56\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 7} E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \frac{2\sqrt{14}}{7}\right)}{\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 7} d}$
risch	$-\frac{2i\sqrt{-(2e^{2i(dx+c)} - 3e^{i(dx+c)} + 2)e^{-i(dx+c)}}}{d} + \left(\frac{6\left(-\frac{3}{4} + \frac{i\sqrt{7}}{4}\right) \sqrt{\frac{e^{i(dx+c)} - \frac{3}{4} + \frac{i\sqrt{7}}{4}}{-\frac{3}{4} + \frac{i\sqrt{7}}{4}}} \sqrt{14} \sqrt{i\left(e^{i(dx+c)} - \frac{3}{4} - \frac{i\sqrt{7}}{4}\right)} \sqrt{7} \sqrt{\frac{e^{i(dx+c)}}{\frac{3}{4} - \frac{i\sqrt{7}}{4}}} F\left(\sqrt{\frac{e^{i(dx+c)}}{\frac{3}{4} - \frac{i\sqrt{7}}{4}}}\right)}{7\sqrt{-2e^{3i(dx+c)} + 3e^{2i(dx+c)} - 2e^{i(dx+c)}}}\right)$

[In] int((3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2/7*14^(1/2))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.42

$$\int \sqrt{3 - 4 \cos(c + dx)} dx = \frac{\sqrt{2} \operatorname{weierstrassPInverse}(-1, -1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2}) + \sqrt{2} \operatorname{weierstrassPInverse}(-1, -1, \cos(dx + c) - i \sin(dx + c) - \frac{1}{2})}{d}$$

[In] integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -1/2*(sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2) + sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2) + 4*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2)) + 4*sqrt(2)*weierstrassZeta(-1, -1, weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2)))/d

Sympy [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} dx = \int \sqrt{3 - 4 \cos(c + dx)} dx$$

[In] integrate((3-4*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(3 - 4*cos(c + d*x)), x)

Maxima [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} dx = \int \sqrt{-4 \cos(dx + c) + 3} dx$$

[In] integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} dx = \int \sqrt{-4 \cos(dx + c) + 3} dx$$

[In] integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 - 4 \cos(c + dx)} dx = \int \sqrt{3 - 4 \cos(c + dx)} dx$$

[In] int((3 - 4*cos(c + d*x))^(1/2),x)

[Out] int((3 - 4*cos(c + d*x))^(1/2), x)

3.520 $\int \sqrt{3 - 4 \cos(c + dx)} \sec(c + dx) dx$

Optimal result	5378
Rubi [A] (verified)	5378
Mathematica [A] (verified)	5379
Maple [A] (verified)	5380
Fricas [F]	5380
Sympy [F]	5380
Maxima [F]	5381
Giac [F]	5381
Mupad [F(-1)]	5381

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec(c + dx) dx = -\frac{8 \operatorname{EllipticF}\left(\frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{\sqrt{7}d} - \frac{6 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[Out] $8/7*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+6/7*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\cos(1/2*d*x+1/2*c), 2, 2/7*14^{(1/2)})/d*7^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2882, 2741, 2885}

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec(c + dx) dx = -\frac{8 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx + \pi), \frac{8}{7}\right)}{\sqrt{7}d} - \frac{6 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx + \pi), \frac{8}{7}\right)}{\sqrt{7}d}$$

[In] `Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x], x]`

[Out] $(-8*\operatorname{EllipticF}[(c + \pi + d*x)/2, 8/7])/(\operatorname{Sqrt}[7]*d) - (6*\operatorname{EllipticPi}[2, (c + \pi + d*x)/2, 8/7])/(\operatorname{Sqrt}[7]*d)$

Rule 2741

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2882

```
Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2885

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c - d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 3 \int \frac{\sec(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx - 4 \int \frac{1}{\sqrt{3 - 4 \cos(c + dx)}} dx \\ &= -\frac{8 \operatorname{EllipticF}\left(\frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{\sqrt{7}d} - \frac{6 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{\sqrt{7}d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\begin{aligned} &\int \sqrt{3 - 4 \cos(c + dx)} \sec(c + dx) dx \\ &= \frac{2\sqrt{-3 + 4 \cos(c + dx)}(-4 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 8\right) + 3 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), 8\right))}{d\sqrt{3 - 4 \cos(c + dx)}} \end{aligned}$$

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[-3 + 4*Cos[c + d*x]]*(-4*EllipticF[(c + d*x)/2, 8] + 3*EllipticPi[2, (c + d*x)/2, 8]))/(d*Sqrt[3 - 4*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.18

method	result
default	$\frac{2\sqrt{-\left(8\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\left(4F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2\sqrt{14}}{7}\right)+3\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2,\frac{2\sqrt{14}}{7}\right)\right)}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7d}}$

```
[In] int(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*(4*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+3*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Fricas [F]

$$\int \sqrt{3-4\cos(c+dx)} \sec(c+dx) dx = \int \sqrt{-4\cos(dx+c)+3} \sec(dx+c) dx$$

```
[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

Sympy [F]

$$\int \sqrt{3-4\cos(c+dx)} \sec(c+dx) dx = \int \sqrt{3-4\cos(c+dx)} \sec(c+dx) dx$$

```
[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x), x)
```


Maxima [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \sec(dx + c) dx$$

[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)

Giac [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \sec(dx + c) dx$$

[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec(c + dx) dx = \int \frac{\sqrt{3 - 4 \cos(c + dx)}}{\cos(c + dx)} dx$$

[In] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x),x)

[Out] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x), x)

3.521 $\int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx$

Optimal result	5382
Rubi [A] (verified)	5382
Mathematica [C] (verified)	5385
Maple [B] (verified)	5385
Fricas [F]	5386
Sympy [F]	5386
Maxima [F]	5386
Giac [F]	5386
Mupad [F(-1)]	5387

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx = -\frac{\sqrt{7} E\left(\frac{1}{2}(c + \pi + dx) \middle| \frac{8}{7}\right)}{d} + \frac{3 \operatorname{EllipticF}\left(\frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{\sqrt{7}d} + \frac{4 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{\sqrt{7}d} + \frac{\sqrt{3 - 4 \cos(c + dx)} \tan(c + dx)}{d}$$

[Out] $-3/7*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}-4/7*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\cos(1/2*d*x+1/2*c), 2, 2/7*14^{(1/2)})/d*7^{(1/2)}+(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+(3-4*\cos(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2875,

3139, 2733, 3081, 2741, 2885}

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx = \frac{3 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx + \pi), \frac{8}{7}\right)}{\sqrt{7}d} - \frac{\sqrt{7}E\left(\frac{1}{2}(c + dx + \pi) \middle| \frac{8}{7}\right)}{d} + \frac{4 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx + \pi), \frac{8}{7}\right)}{\sqrt{7}d} + \frac{\sqrt{3 - 4 \cos(c + dx)} \tan(c + dx)}{d}$$

[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]

[Out] -((Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d) + (3*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d) + (4*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/d

Rule 2733

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2741

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2875

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2885

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

, 0] && GtQ[c - d, 0]

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3139

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)^2]/(Sqrt[(a_.) + (b_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{(-2+2\cos^2(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
&= \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d} \\
&\quad + \frac{1}{4} \int \frac{(-8+6\cos(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx - \frac{1}{2} \int \sqrt{3-4\cos(c+dx)} dx \\
&= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d} \\
&\quad + \frac{3}{2} \int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx - 2 \int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{d} + \frac{3\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{\sqrt{7}d} \\
&\quad + \frac{4\text{EllipticPi}\left(2, \frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{\sqrt{7}d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.82

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx$$

$$= \frac{42\sqrt{-3+4\cos(c+dx)} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), 8\right) - i\sqrt{7}\left(21E\left(i\operatorname{arcsinh}\left(\sqrt{3-4\cos(c+dx)}\right)\right) - \frac{1}{7}\right) - 12 \operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{3-4\cos(c+dx)}\right), \sqrt{\sin^2(c+dx)}\right)}{21d}$$

[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]

[Out] ((-42*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c + d*x]] - (I*Sqrt[7]*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2 + 21*Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(21*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(164) = 328.

Time = 4.61 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.58

method	result
default	$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 7\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(-\frac{2\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} + \frac{3\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{56\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 7}}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 7}}\right)}{\sin\left(\frac{dx}{2}\right)}$

[In] int(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+4/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)))/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d

Fricas [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \sec(dx + c)^2 dx$$

```
[In] integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

Sympy [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx = \int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx$$

```
[In] integrate(sec(d*x+c)**2*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \sec(dx + c)^2 dx$$

```
[In] integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \sec(dx + c)^2 dx$$

```
[In] integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^2(c + dx) dx = \int \frac{\sqrt{3 - 4 \cos(c + dx)}}{\cos(c + dx)^2} dx$$

```
[In] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

```
[Out] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.522 $\int \sqrt{3 - 4 \cos(c + dx)} \sec^3(c + dx) dx$

Optimal result	5388
Rubi [A] (verified)	5389
Mathematica [C] (verified)	5391
Maple [B] (verified)	5392
Fricas [F]	5392
Sympy [F]	5393
Maxima [F]	5393
Giac [F]	5393
Mupad [F(-1)]	5393

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^3(c + dx) dx = \frac{\sqrt{7} E\left(\frac{1}{2}(c + \pi + dx) \middle| \frac{8}{7}\right)}{3d} - \frac{3 \operatorname{EllipticF}\left(\frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{\sqrt{7}d} - \frac{5 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{3\sqrt{7}d} - \frac{\sqrt{3 - 4 \cos(c + dx)} \tan(c + dx)}{3d} + \frac{\sqrt{3 - 4 \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2d}$$

```
[Out] 3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/d*7^(1/2)-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/3*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/2*sec(d*x+c)*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```


Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2875, 3134, 3138, 2733, 3081, 2741, 2885}

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^3(c + dx) dx = -\frac{3 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx + \pi), \frac{8}{7}\right)}{\sqrt{7}d} + \frac{\sqrt{7}E\left(\frac{1}{2}(c + dx + \pi) \middle| \frac{8}{7}\right)}{3d} - \frac{5 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx + \pi), \frac{8}{7}\right)}{3\sqrt{7}d} - \frac{\sqrt{3 - 4 \cos(c + dx)} \tan(c + dx)}{3d} + \frac{\sqrt{3 - 4 \cos(c + dx)} \tan(c + dx) \sec(c + dx)}{2d}$$

[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]

[Out] (Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(3*d) - (3*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d) - (5*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3*Sqrt[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)

Rule 2733

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2741

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2875

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2885

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[
-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_))*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_))*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{3-4\cos(c+dx)} \sec(c+dx) \tan(c+dx)}{2d} \\
&+ \frac{1}{2} \int \frac{(-2+3\cos(c+dx)-2\cos^2(c+dx)) \sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{3-4\cos(c+dx)} \tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)} \sec(c+dx) \tan(c+dx)}{2d} \\
&+ \frac{1}{6} \int \frac{(5-6\cos(c+dx)-4\cos^2(c+dx)) \sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{3-4\cos(c+dx)} \tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)} \sec(c+dx) \tan(c+dx)}{2d} \\
&+ \frac{1}{24} \int \frac{(20-36\cos(c+dx)) \sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx + \frac{1}{6} \int \sqrt{3-4\cos(c+dx)} dx \\
&= \frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\left|\frac{8}{7}\right.\right)}{3d} - \frac{\sqrt{3-4\cos(c+dx)} \tan(c+dx)}{3d} \\
&+ \frac{\sqrt{3-4\cos(c+dx)} \sec(c+dx) \tan(c+dx)}{2d} \\
&+ \frac{5}{6} \int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx - \frac{3}{2} \int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx \\
&= \frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\left|\frac{8}{7}\right.\right)}{3d} - \frac{3 \operatorname{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{\sqrt{7}d} - \frac{5 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{3\sqrt{7}d} \\
&- \frac{\sqrt{3-4\cos(c+dx)} \tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)} \sec(c+dx) \tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.72

$$\begin{aligned}
&\int \sqrt{3-4\cos(c+dx)} \sec^3(c+dx) dx \\
&= \frac{-12\sqrt{-3+4\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 8\right)}{\sqrt{3-4\cos(c+dx)}} + \frac{6\sqrt{-3+4\cos(c+dx)} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), 8\right)}{\sqrt{3-4\cos(c+dx)}} + \frac{2i\left(21E\left(i \operatorname{arcsinh}\left(\sqrt{3-4\cos(c+dx)}\right)\right)\right)}{\sqrt{3-4\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]

[Out] ((-12*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c + d*x]] + (6*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c + d*x]] + (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c

```
+ d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] -
  8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x
  ]/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) - Sqrt[3 - 4*Cos[c + d*x]]*(-3 + 2*Cos[c
  + d*x])*Sec[c + d*x]*Tan[c + d*x]/(6*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(195) = 390.

Time = 5.86 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.96

method	result
default	$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}+\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}\right)}$

```
[In] int(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2
*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^2+2/3*cos(1/2*d*x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*s
in(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-5/21*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)
))/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Fricas [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^3(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \sec(dx + c)^3 dx$$

```
[In] integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

Sympy [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^3(c + dx) dx = \int \sqrt{3 - 4 \cos(c + dx)} \sec^3(c + dx) dx$$

[In] integrate(sec(d*x+c)**3*(3-4*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x)**3, x)

Maxima [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^3(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \sec(dx + c)^3 dx$$

[In] integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)

Giac [F]

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^3(c + dx) dx = \int \sqrt{-4 \cos(dx + c) + 3} \sec(dx + c)^3 dx$$

[In] integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3 - 4 \cos(c + dx)} \sec^3(c + dx) dx = \int \frac{\sqrt{3 - 4 \cos(c + dx)}}{\cos(c + dx)^3} dx$$

[In] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)

[Out] int((3 - 4*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)

$$3.523 \quad \int \frac{\cos^3(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	5394
Rubi [A] (verified)	5395
Mathematica [A] (verified)	5397
Maple [B] (verified)	5398
Fricas [C] (verification not implemented)	5398
Sympy [F(-1)]	5399
Maxima [F]	5399
Giac [F]	5399
Mupad [F(-1)]	5400

Optimal result

Integrand size = 23, antiderivative size = 215

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2(8a^2 + 9b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{15b^3 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2a(8a^2 + 7b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{15b^3 d \sqrt{a+b \cos(c+dx)}} - \frac{8a \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{15b^2 d} + \frac{2 \cos(c+dx) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5bd}$$

```
[Out] -8/15*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/d+2/5*cos(d*x+c)*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)/b/d+2/15*(8*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+
b*cos(d*x+c))^(1/2)/b^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/15*a*(8*a^2+7*b^
2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos
(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2872, 3102, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx = -\frac{2a(8a^2+7b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}} + \frac{2(8a^2+9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{8a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{15b^2d} + \frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5bd}$$

[In] Int[Cos[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]], x]

[Out] (2*(8*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(8*a^2 + 7*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(15*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (8*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^2*d) + (2*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2872

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2 \cos(c + dx) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5bd} + \frac{2 \int \frac{a + \frac{3}{2}b \cos(c + dx) - 2a \cos^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{5b} \\ &= -\frac{8a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15b^2d} \\ &\quad + \frac{2 \cos(c + dx) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5bd} + \frac{4 \int \frac{\frac{ab}{2} + \frac{1}{4}(8a^2 + 9b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{15b^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{8a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd} \\
&\quad - \frac{(a(8a^2+7b^2))\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{15b^3} + \frac{(8a^2+9b^2)\int\sqrt{a+b\cos(c+dx)}dx}{15b^3} \\
&= -\frac{8a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd} \\
&\quad + \frac{\left((8a^2+9b^2)\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{15b^3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad - \frac{\left(a(8a^2+7b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{15b^3\sqrt{a+b\cos(c+dx)}} \\
&= \frac{2(8a^2+9b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad - \frac{2a(8a^2+7b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{8a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

$$\begin{aligned}
&\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{2(8a^3+8a^2b+9ab^2+9b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right) - 2a(8a^2+7b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*(8*a^3 + 8*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(8*a^2 + 7*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-8*a^2 + 3*b^2 - 2*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x]/(15*b^3*d*Sqrt[a + b*Cos[c + d*x]])

$b \sin(dx + c) + 2a)/b) + 4\sqrt{2}*(4Ia^3 + 3Iab^2)*\sqrt{b}*\text{weierstrassPInverse}(4/3*(4a^2 - 3b^2)/b^2, -8/27*(8a^3 - 9ab^2)/b^3, 1/3*(3b*\cos(dx + c) - 3Ib*\sin(dx + c) + 2a)/b) + 3\sqrt{2}*(-8Ia^2*b - 9Ib^3)*\sqrt{b}*\text{weierstrassZeta}(4/3*(4a^2 - 3b^2)/b^2, -8/27*(8a^3 - 9ab^2)/b^3, \text{weierstrassPInverse}(4/3*(4a^2 - 3b^2)/b^2, -8/27*(8a^3 - 9ab^2)/b^3, 1/3*(3b*\cos(dx + c) + 3Ib*\sin(dx + c) + 2a)/b)) + 3\sqrt{2}*(8Ia^2*b + 9Ib^3)*\sqrt{b}*\text{weierstrassZeta}(4/3*(4a^2 - 3b^2)/b^2, -8/27*(8a^3 - 9ab^2)/b^3, \text{weierstrassPInverse}(4/3*(4a^2 - 3b^2)/b^2, -8/27*(8a^3 - 9ab^2)/b^3, 1/3*(3b*\cos(dx + c) - 3Ib*\sin(dx + c) + 2a)/b)) - 6*(3b^3*\cos(dx + c) - 4ab^2)*\sqrt{b*\cos(dx + c) + a}*\sin(dx + c))/(b^4*d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(cos(dx+c)**3/(a+b*cos(dx+c))**(1/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^3}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(dx+c)^3/(a+b*cos(dx+c))^(1/2), x, algorithm="maxima")

[Out] integrate(cos(dx + c)^3/sqrt(b*cos(dx + c) + a), x)

Giac [F]

$$\int \frac{\cos^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^3}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(dx+c)^3/(a+b*cos(dx+c))^(1/2), x, algorithm="giac")

[Out] integrate(cos(dx + c)^3/sqrt(b*cos(dx + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^3}{\sqrt{a + b \cos(c + dx)}} dx$$

```
[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(1/2), x)
```

$$3.524 \quad \int \frac{\cos^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	5401
Rubi [A] (verified)	5401
Mathematica [A] (verified)	5404
Maple [B] (verified)	5404
Fricas [C] (verification not implemented)	5405
Sympy [F]	5405
Maxima [F]	5405
Giac [F]	5406
Mupad [B] (verification not implemented)	5406

Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = -\frac{4a\sqrt{a+b \cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2(2a^2+b^2)\sqrt{\frac{a+b \cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b \cos(c+dx)}} + \frac{2\sqrt{a+b \cos(c+dx)}\sin(c+dx)}{3bd}$$

```
[Out] 2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d-4/3*a*(cos(1/2*d*x+1/2*c))^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(2*a^2+b^2)*(cos(1/2*d*x+1/2*c))^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {2870, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx = \frac{2(2a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}} - \frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

[In] Int[Cos[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (-4*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*SIN[e + f*x]]

], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2870

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3bd} + \frac{2 \int \frac{\frac{b}{2} - a \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{3b} \\
 &= \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3bd} \\
 &\quad + \frac{1}{3} \left(1 + \frac{2a^2}{b^2}\right) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx - \frac{(2a) \int \sqrt{a + b \cos(c + dx)} dx}{3b^2} \\
 &= \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3bd} - \frac{(2a\sqrt{a + b \cos(c + dx)}) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{3b^2 \sqrt{\frac{a+b \cos(c + dx)}{a+b}}} \\
 &\quad + \frac{\left(\left(1 + \frac{2a^2}{b^2}\right) \sqrt{\frac{a+b \cos(c + dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{3\sqrt{a + b \cos(c + dx)}} \\
 &= -\frac{4a\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3b^2 d \sqrt{\frac{a+b \cos(c + dx)}{a+b}}} \\
 &\quad + \frac{2\left(1 + \frac{2a^2}{b^2}\right) \sqrt{\frac{a+b \cos(c + dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3d\sqrt{a + b \cos(c + dx)}} \\
 &\quad + \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3bd}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

$$= \frac{-4a(a + b) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a + b}\right) + 2(2a^2 + b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a + b}\right) + 2b(a + b) \sqrt{\frac{a + b \cos(c + dx)}{a + b}}}{3b^2 d \sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[Cos[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]], x]

[Out] (-4*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 2*(2*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x]/(3*b^2*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(207) = 414.

Time = 4.11 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.75

method	result
default	$\frac{2 \sqrt{\left(2b \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a - b\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\dots} \left(4 \left(\cos^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b^2 + 2 \left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) ab - 6 \left(\cos^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b^2 + 2a^2 \sqrt{\frac{1}{2} - \frac{\cos(dx + c)}{2}}\right)$

[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^(1/2), x, method=_RETURNVERBOSE)

[Out] -2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2+2*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2))*EllipticF(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))+b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))*a^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+1/2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2/b^2/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.41

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

$$= -6i \sqrt{2ab^3} \operatorname{weierstrassZeta}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}\right), \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}}\right)$$

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/9*(-6*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*I*sqrt(2)*a*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) + 6*sqrt(b*cos(d*x + c) + a)*b^2*sin(d*x + c) + sqrt(2)*(-4*I*a^2 - 3*I*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + sqrt(2)*(4*I*a^2 + 3*I*b^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)) / (b^3*d)

Sympy [F]

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))^(1/2),x)

[Out] Integral(cos(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^2}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\cos^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^2}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)

Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.70

$$\begin{aligned} & \int \frac{\cos^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx \\ &= \frac{2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{3bd} \\ &+ \frac{2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \left(F\left(\frac{c}{2} + \frac{dx}{2} \middle| \frac{2b}{a+b}\right) (2a^2 + b^2) - 2a E\left(\frac{c}{2} + \frac{dx}{2} \middle| \frac{2b}{a+b}\right) (a + b) \right)}{3b^2 d \sqrt{a + b \cos(c + dx)}} \end{aligned}$$

[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(1/2),x)

[Out] (2*sin(c + d*x)*(a + b*cos(c + d*x))^(1/2))/(3*b*d) + (2*((a + b*cos(c + d*x))/(a + b))^(1/2)*(ellipticF(c/2 + (d*x)/2, (2*b)/(a + b))*(2*a^2 + b^2) - 2*a*ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a + b)))/(3*b^2*d*(a + b*cos(c + d*x))^(1/2))

$$3.525 \quad \int \frac{\cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	5407
Rubi [A] (verified)	5407
Mathematica [A] (verified)	5409
Maple [A] (verified)	5409
Fricas [C] (verification not implemented)	5410
Sympy [F]	5410
Maxima [F]	5411
Giac [F]	5411
Mupad [B] (verification not implemented)	5411

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{\cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2\sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2a\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{bd\sqrt{a+b \cos(c+dx)}}$$

[Out] $2*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(b/(a+b))^{1/2})*(a+b*\cos(d*x+c))^{1/2}/b/d/((a+b*\cos(d*x+c))/(a+b))^{1/2}-2*a*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(b/(a+b))^{1/2})*((a+b*\cos(d*x+c))/(a+b))^{1/2}/b/d/(a+b*\cos(d*x+c))^{1/2}$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2\sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2a\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{bd\sqrt{a+b \cos(c+dx)}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]/\operatorname{Sqrt}[a + b*\operatorname{Cos}[c + d*x]], x]$

[Out] $(2\sqrt{a + b\cos[c + dx]}\text{EllipticE}[(c + dx)/2, (2b)/(a + b)]/(b d \sqrt{(a + b\cos[c + dx])/(a + b)}) - (2a\sqrt{(a + b\cos[c + dx])/(a + b)}\text{EllipticF}[(c + dx)/2, (2b)/(a + b)]/(b d \sqrt{a + b\cos[c + dx]}))$

Rule 2732

$\text{Int}[\sqrt{(a_) + (b_)\sin[(c_) + (d_)(x_)]}, x_Symbol] \rightarrow \text{Simp}[2(\sqrt{a + b}/d)\text{EllipticE}[(1/2)(c - \text{Pi}/2 + dx), 2(b/(a + b))], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a + b, 0]$

Rule 2734

$\text{Int}[\sqrt{(a_) + (b_)\sin[(c_) + (d_)(x_)]}, x_Symbol] \rightarrow \text{Dist}[\sqrt{a + b\sin[c + dx]}/\sqrt{(a + b\sin[c + dx])/(a + b)}, \text{Int}[\sqrt{a/(a + b) + (b/(a + b))\sin[c + dx]}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& !\text{GtQ}[a + b, 0]$

Rule 2740

$\text{Int}[1/\sqrt{(a_) + (b_)\sin[(c_) + (d_)(x_)]}, x_Symbol] \rightarrow \text{Simp}[(2/(d\sqrt{a + b}))\text{EllipticF}[(1/2)(c - \text{Pi}/2 + dx), 2(b/(a + b))], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[a + b, 0]$

Rule 2742

$\text{Int}[1/\sqrt{(a_) + (b_)\sin[(c_) + (d_)(x_)]}, x_Symbol] \rightarrow \text{Dist}[\sqrt{(a + b\sin[c + dx])/(a + b)}/\sqrt{a + b\sin[c + dx]}, \text{Int}[1/\sqrt{a/(a + b) + (b/(a + b))\sin[c + dx]}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& !\text{GtQ}[a + b, 0]$

Rule 2831

$\text{Int}[(c_) + (d_)\sin[(e_) + (f_)(x_)]/\sqrt{(a_) + (b_)\sin[(e_) + (f_)(x_)]}, x_Symbol] \rightarrow \text{Dist}[(b*c - a*d)/b, \text{Int}[1/\sqrt{a + b\sin[e + f*x]}, x], x] + \text{Dist}[d/b, \text{Int}[\sqrt{a + b\sin[e + f*x]}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \sqrt{a + b \cos(c + dx)} dx}{b} - \frac{a \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{b} \\ &= \frac{\sqrt{a + b \cos(c + dx)} \int \sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a + b}} dx}{b \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} - \frac{\left(a \sqrt{\frac{a + b \cos(c + dx)}{a + b}} \right) \int \frac{1}{\sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a + b}}} dx}{b \sqrt{a + b \cos(c + dx)}} \end{aligned}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx$$

$$= \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right) - a\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

[In] Integrate[Cos[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(b*d*Sqrt[a + b*Cos[c + d*x]])

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.80

method	result
default	$\frac{2\sqrt{\left(2b\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}{a-b}}\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a-E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}b\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}}$
risch	$\frac{i\left(b e^{2i(dx+c)}+2a e^{i(dx+c)}+b\right)\sqrt{2}e^{-i(dx+c)}}{bd\sqrt{\left(b e^{2i(dx+c)}+2a e^{i(dx+c)}+b\right)e^{-i(dx+c)}}} + \frac{2\left(b e^{2i(dx+c)}+2a e^{i(dx+c)}+b\right)}{b\sqrt{\left(b e^{2i(dx+c)}+2a e^{i(dx+c)}+b\right)e^{i(dx+c)}}} + \frac{2\left(\sqrt{a^2-b^2}+a\right)\sqrt{\frac{\left(e^{i(dx+c)}+\sqrt{a^2-b^2}\right)}{b}}}{\sqrt{a^2-b^2}+a}$

[In] int(cos(d*x+c)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a

$(-b)^{(1/2)} * a + \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)} * b) / (-2*\sin(1/2*d*x+1/2*c)^4 * b + (a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} / b / \sin(1/2*d*x+1/2*c) / (-2*b*\sin(1/2*d*x+1/2*c)^2 + a + b)^{(1/2)} / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.91

$$\int \frac{\cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

$$= \frac{2i \sqrt{2a} \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3ib \sin(dx+c) + 2a}{3b}\right) - 2i \sqrt{2a} \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) - 3ib \sin(dx+c) + 2a}{3b}\right)}{b}$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{3} * (2 * I * \sqrt{2} * a * \sqrt{b} * \text{weierstrassPInverse}(4/3 * (4 * a^2 - 3 * b^2) / b^2, -8/27 * (8 * a^3 - 9 * a * b^2) / b^3, 1/3 * (3 * b * \cos(d * x + c) + 3 * I * b * \sin(d * x + c) + 2 * a) / b) - 2 * I * \sqrt{2} * a * \sqrt{b} * \text{weierstrassPInverse}(4/3 * (4 * a^2 - 3 * b^2) / b^2, -8/27 * (8 * a^3 - 9 * a * b^2) / b^3, 1/3 * (3 * b * \cos(d * x + c) - 3 * I * b * \sin(d * x + c) + 2 * a) / b) + 3 * I * \sqrt{2} * b^{(3/2)} * \text{weierstrassZeta}(4/3 * (4 * a^2 - 3 * b^2) / b^2, -8/27 * (8 * a^3 - 9 * a * b^2) / b^3, \text{weierstrassPInverse}(4/3 * (4 * a^2 - 3 * b^2) / b^2, -8/27 * (8 * a^3 - 9 * a * b^2) / b^3, 1/3 * (3 * b * \cos(d * x + c) + 3 * I * b * \sin(d * x + c) + 2 * a) / b)) - 3 * I * \sqrt{2} * b^{(3/2)} * \text{weierstrassZeta}(4/3 * (4 * a^2 - 3 * b^2) / b^2, -8/27 * (8 * a^3 - 9 * a * b^2) / b^3, \text{weierstrassPInverse}(4/3 * (4 * a^2 - 3 * b^2) / b^2, -8/27 * (8 * a^3 - 9 * a * b^2) / b^3, 1/3 * (3 * b * \cos(d * x + c) - 3 * I * b * \sin(d * x + c) + 2 * a) / b))} / (b^2 * d)$

Sympy [F]

$$\int \frac{\cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)

[Out] Integral(cos(c + d*x)/sqrt(a + b*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{\cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)

Mupad [B] (verification not implemented)

Time = 14.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{\cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \frac{2 \left(E\left(\frac{c}{2} + \frac{dx}{2} \middle| \frac{2b}{a+b}\right) (a + b) - a F\left(\frac{c}{2} + \frac{dx}{2} \middle| \frac{2b}{a+b}\right) \right) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}{bd \sqrt{a + b \cos(c + dx)}}$$

[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^(1/2),x)

[Out] (2*(ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a + b) - a*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b)))*((a + b*cos(c + d*x))/(a + b))^(1/2))/(b*d*(a + b*cos(c + d*x))^(1/2))

3.526 $\int \frac{1}{\sqrt{a+b \cos(c+dx)}} dx$

Optimal result	5412
Rubi [A] (verified)	5412
Mathematica [A] (verified)	5413
Maple [C] (verified)	5413
Fricas [C] (verification not implemented)	5414
Sympy [F]	5414
Maxima [F]	5414
Giac [F]	5415
Mupad [B] (verification not implemented)	5415

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{1}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d\sqrt{a+b \cos(c+dx)}}$$

[Out] $2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b))^{(1/2)})*((a+b*\cos(d*x+c))/(a+b))^{(1/2)}/d/(a+b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2742, 2740}

$$\int \frac{1}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d\sqrt{a+b \cos(c+dx)}}$$

[In] `Int[1/Sqrt[a + b*Cos[c + d*x]],x]`

[Out] $(2*\operatorname{Sqrt}[(a + b*\operatorname{Cos}[c + d*x])/(a + b)]*\operatorname{EllipticF}[(c + d*x)/2, (2*b)/(a + b)])/(d*\operatorname{Sqrt}[a + b*\operatorname{Cos}[c + d*x]])$

Rule 2740

`Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]`

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])]/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{\sqrt{a + b \cos(c + dx)}} \\ &= \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx = \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{d\sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[1/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*Sqrt[(a + b*Cos[c + d*x])]/(a + b))*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[a + b*Cos[c + d*x]])

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

method	result	size
default	$\frac{2\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - a - b}{a+b}} \operatorname{am}^{-1}\left(\frac{dx}{2} + \frac{c}{2} \mid \frac{\sqrt{2}\sqrt{b}}{\sqrt{a+b}}\right)}{d\sqrt{2b\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a - b}}$	78

[In] int(1/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/d/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^(1/2)*(-(2*b*sin(1/2*d*x+1/2*c)^2-a-b)/(a+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)/(a+b)^(1/2)*b^(1/2))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx$$

$$= \frac{-i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) + 3ib \sin(dx+c) + 2a}{3b}\right) + i \sqrt{2} \sqrt{b} \operatorname{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\frac{8(8a^3 - 9ab^2)}{27b^3}, \frac{3b \cos(dx+c) - 3ib \sin(dx+c) + 2a}{3b}\right)}{bd}$$

[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b))/(b*d)

Sympy [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))**(1/2),x)

[Out] Integral(1/sqrt(a + b*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*cos(d*x + c) + a), x)

Mupad [B] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx = \frac{2 F\left(\frac{c}{2} + \frac{dx}{2} \middle| \frac{2b}{a+b}\right) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}{d \sqrt{a + b \cos(c + dx)}}$$

[In] int(1/(a + b*cos(c + d*x))^(1/2),x)

[Out] (2*ellipticF(c/2 + (d*x)/2, (2*b)/(a + b))*((a + b*cos(c + d*x))/(a + b))^(1/2))/(d*(a + b*cos(c + d*x))^(1/2))

$$3.527 \quad \int \frac{\sec(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	5416
Rubi [A] (verified)	5416
Mathematica [A] (verified)	5417
Maple [A] (verified)	5417
Fricas [F(-1)]	5418
Sympy [F]	5418
Maxima [F]	5418
Giac [F]	5419
Mupad [F(-1)]	5419

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d\sqrt{a+b \cos(c+dx)}}$$

[Out] 2*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2886, 2884}

$$\int \frac{\sec(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d\sqrt{a+b \cos(c+dx)}}$$

[In] Int[Sec[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c

, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2886

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])]/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{\sqrt{a+b \cos(c+dx)}} \\ &= \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d\sqrt{a+b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d\sqrt{a+b \cos(c+dx)}}$$

[In] Integrate[Sec[c + d*x]/Sqrt[a + b*Cos[c + d*x]], x]

[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.86

method	result	size
default	$\frac{2\sqrt{(2b(\cos^2(\frac{dx}{2} + \frac{c}{2})) + a - b)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{\frac{2b(\cos^2(\frac{dx}{2} + \frac{c}{2})) + a - b}{a - b}} \Pi(\cos(\frac{dx}{2} + \frac{c}{2}), 2, \sqrt{-\frac{2b}{a-b}})}{\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2}))b + (a+b)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \sin(\frac{dx}{2} + \frac{c}{2}) \sqrt{-2b(\sin^2(\frac{dx}{2} + \frac{c}{2})) + a + b} d}$	166

[In] int(sec(d*x+c)/(a+cos(d*x+c)*b)^(1/2), x, method=_RETURNVERBOSE)

```
[Out] 2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2))/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(
1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)}{\sqrt{b \cos(dx + c) + a}} dx$$

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx) \sqrt{a + b \cos(c + dx)}} dx$$

[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(1/2)), x)

$$3.528 \quad \int \frac{\sec^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	5420
Rubi [A] (verified)	5421
Mathematica [C] (verified)	5424
Maple [A] (verified)	5424
Fricas [F(-1)]	5425
Sympy [F]	5425
Maxima [F]	5425
Giac [F]	5426
Mupad [F(-1)]	5426

Optimal result

Integrand size = 23, antiderivative size = 206

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = -\frac{\sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d\sqrt{a+b \cos(c+dx)}} - \frac{b\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{ad\sqrt{a+b \cos(c+dx)}} + \frac{\sqrt{a+b \cos(c+dx)} \tan(c+dx)}{ad}$$

```
[Out] -(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/d/(a+b*cos(d*x+c))^(1/2)-b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d
```


Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2881, 3139, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx = \frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad} + \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} - \frac{\sqrt{a+b\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

[In] Int[Sec[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]], x]

[Out] -((Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + (Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(a*d*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(a*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && (IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3081

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)])/(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3139

```

Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
+ (f_.)*(x_.)]]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{ad} + \frac{\int \frac{\left(-\frac{b}{2} - \frac{1}{2} b \cos^2(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{a} \\
&= \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{ad} - \frac{\int \sqrt{a + b \cos(c + dx)} dx}{2a} - \frac{\int \frac{\left(\frac{b^2}{2} - \frac{1}{2} ab \cos(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{ab} \\
&= \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{ad} + \frac{1}{2} \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx \\
&\quad - \frac{b \int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{2a} - \frac{\sqrt{a + b \cos(c + dx)} \int \sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a + b}} dx}{2a \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} \\
&= -\frac{\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a + b}\right)}{ad \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} + \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{ad} \\
&\quad + \frac{\sqrt{\frac{a + b \cos(c + dx)}{a + b}} \int \frac{1}{\sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a + b}}} dx}{2\sqrt{a + b \cos(c + dx)}} - \frac{\left(b \sqrt{\frac{a + b \cos(c + dx)}{a + b}}\right) \int \frac{\sec(c + dx)}{\sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a + b}}} dx}{2a \sqrt{a + b \cos(c + dx)}} \\
&= -\frac{\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a + b}\right)}{ad \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} + \frac{\sqrt{\frac{a + b \cos(c + dx)}{a + b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a + b}\right)}{d \sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{b \sqrt{\frac{a + b \cos(c + dx)}{a + b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a + b}\right)}{ad \sqrt{a + b \cos(c + dx)}} + \frac{\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{ad}
\end{aligned}$$


```
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2,(-2*b/(a-b))^(1/2))/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a
+b)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sec^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sec^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^2}{\sqrt{b \cos(dx + c) + a}} dx$$

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^2}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^2 \sqrt{a + b \cos(c + dx)}} dx$$

[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(1/2)), x)

$$3.529 \quad \int \frac{\sec^3(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	5427
Rubi [A] (verified)	5428
Mathematica [C] (verified)	5431
Maple [B] (verified)	5432
Fricas [F(-1)]	5433
Sympy [F]	5433
Maxima [F]	5433
Giac [F]	5433
Mupad [F(-1)]	5434

Optimal result

Integrand size = 23, antiderivative size = 268

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \frac{3b\sqrt{a+b \cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{b\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4ad\sqrt{a+b \cos(c+dx)}} + \frac{(4a^2+3b^2)\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b \cos(c+dx)}} - \frac{3b\sqrt{a+b \cos(c+dx)} \tan(c+dx)}{4a^2d} + \frac{\sqrt{a+b \cos(c+dx)} \sec(c+dx) \tan(c+dx)}{2ad}$$

```
[Out] 3/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-1/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)-3/4*b*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a^2/d+1/2*sec(d*x+c)*(a+b*cos(d*x+c))^(1/2)*tan(d*x+c)/a/d
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2881, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx = \frac{(4a^2+3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}} - \frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d} + \frac{3b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+dx)}}{2ad}$$

[In] Int[Sec[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]], x]

[Out] (3*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(4*a^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(4*a*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2 + 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) - (3*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*a*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2886

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3081

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_)/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)^2), x_Symbol] :> Simp[(- (A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))

```

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])], x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Ssin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Ssin[e + f*x])*(c + d*Ssin[e + f*x])], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2ad} \\
&+ \frac{\int \frac{\left(-\frac{3b}{2} + a \cos(c + dx) + \frac{1}{2}b \cos^2(c + dx)\right) \sec^2(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{2a} \\
&= -\frac{3b\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{4a^2d} + \frac{\sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2ad} \\
&+ \frac{\int \frac{\left(\frac{1}{4}(4a^2 + 3b^2) + \frac{1}{2}ab \cos(c + dx) + \frac{3}{4}b^2 \cos^2(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{2a^2} \\
&= -\frac{3b\sqrt{a + b \cos(c + dx)} \tan(c + dx)}{4a^2d} + \frac{\sqrt{a + b \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{2ad} \\
&- \frac{\int \frac{\left(-\frac{1}{4}b(4a^2 + 3b^2) + \frac{1}{4}ab^2 \cos(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{2a^2b} + \frac{(3b) \int \sqrt{a + b \cos(c + dx)} dx}{8a^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{3b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad} \\
&\quad - \frac{b\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{8a} + \frac{1}{8}\left(4+\frac{3b^2}{a^2}\right)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx \\
&\quad + \frac{\left(3b\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{8a^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&= \frac{3b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{3b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} \\
&\quad + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad} \\
&\quad - \frac{\left(b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{8a\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\left(\left(4+\frac{3b^2}{a^2}\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int\frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{8\sqrt{a+b\cos(c+dx)}} \\
&= \frac{3b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\left(4+\frac{3b^2}{a^2}\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{3b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.93

$$\begin{aligned}
&\int\frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx \\
&\quad \frac{8ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2(8a^2+9b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{6ib^2\sqrt{\frac{b-b\cos(c+dx)}{a+b}}\sqrt{-\frac{b+b\cos(c+dx)}{a-b}}}{\sqrt{a+b\cos(c+dx)}} \\
&= \frac{\sqrt{a+b\cos(c+dx)}\left(-\frac{3b\tan(c+dx)}{4a^2} + \frac{\sec(c+dx)\tan(c+dx)}{2a}\right)}{d}
\end{aligned}$$

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2 + 9*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] - ((6*I)*b^2*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(16*a^2*d + (Sqrt[a + b*Cos[c + d*x]]*((-3*b*Tan[c + d*x])/(4*a^2) + (Sec[c + d*x]*Tan[c + d*x])/(2*a)))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(329) = 658.

Time = 3.67 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.65

method	result
default	$-\frac{\sqrt{-(-2b(\cos^2(\frac{dx}{2} + \frac{c}{2})) - a + b)(\sin^2(\frac{dx}{2} + \frac{c}{2}))}}{\left(-\frac{\cos(\frac{dx}{2} + \frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2}))b + (a+b)(\sin^2(\frac{dx}{2} + \frac{c}{2}))}}{a(2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1)^2} + \frac{3b \cos(\frac{dx}{2} + \frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2}))}}{2a^2(2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1)} \right)}$

```
[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/2*b/a^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/4*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/4/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/4*b^2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3/4/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-3/4/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
```

2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)**3/sqrt(a + b*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^3}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^3}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^3 \sqrt{a + b \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(1/2)), x)
```

$$3.530 \quad \int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	5435
Rubi [A] (verified)	5436
Mathematica [A] (verified)	5439
Maple [B] (verified)	5439
Fricas [C] (verification not implemented)	5440
Sympy [F(-1)]	5441
Maxima [F]	5441
Giac [F]	5441
Mupad [F(-1)]	5442

Optimal result

Integrand size = 23, antiderivative size = 326

$$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = \frac{2(16a^4 - 8a^2b^2 - 3b^4) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{5b^4 (a^2 - b^2) d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{8a(4a^2 + b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{5b^4 d \sqrt{a+b \cos(c+dx)}} - \frac{2a^2 \cos^2(c+dx) \sin(c+dx)}{b(a^2 - b^2) d \sqrt{a+b \cos(c+dx)}} - \frac{2a(8a^2 - 3b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5b^3 (a^2 - b^2) d} + \frac{2(6a^2 - b^2) \cos(c+dx) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5b^2 (a^2 - b^2) d}$$

```
[Out] -2*a^2*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2/5*a*(
8*a^2-3*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d+2/5*(6*a^2-b
^2)*cos(d*x+c)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d+2/5*(16*a^
4-8*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^4/(a
^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-8/5*a*(4*a^2+b^2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+
b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^4/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2871, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = -\frac{2a^2 \sin(c+dx) \cos^2(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}} + \frac{2(6a^2-b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d(a^2-b^2)} - \frac{8a(4a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{5b^4d\sqrt{a+b\cos(c+dx)}} - \frac{2a(8a^2-3b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^3d(a^2-b^2)} + \frac{2(16a^4-8a^2b^2-3b^4)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{5b^4d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(3/2),x]

[Out] (2*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(5*b^4*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (8*a*(4*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(5*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a*(8*a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^3*(a^2 - b^2)*d) + (2*(6*a^2 - b^2)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^2*(a^2 - b^2)*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2871

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

Rule 3102

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 3128

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d

(m + n + 2) + C(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B))*(m + n + 2) - C*(a*c - b*d*(m + n + 1))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n + 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rubi steps

integral

$$\begin{aligned}
&= -\frac{2a^2 \cos^2(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{2 \int \frac{\cos(c+dx)(2a^2 - \frac{1}{2}ab \cos(c+dx) - \frac{1}{2}(6a^2 - b^2) \cos^2(c+dx))}{\sqrt{a+b \cos(c+dx)}} dx}{b(a^2 - b^2)} \\
&= -\frac{2a^2 \cos^2(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{2(6a^2 - b^2) \cos(c + dx) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5b^2(a^2 - b^2) d} \\
&\quad - \frac{4 \int \frac{-\frac{1}{2}a(6a^2 - b^2) + \frac{1}{4}b(2a^2 + 3b^2) \cos(c+dx) + \frac{3}{4}a(8a^2 - 3b^2) \cos^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx}{5b^2(a^2 - b^2)} \\
&= -\frac{2a^2 \cos^2(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{2a(8a^2 - 3b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5b^3(a^2 - b^2) d} \\
&\quad + \frac{2(6a^2 - b^2) \cos(c + dx) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5b^2(a^2 - b^2) d} \\
&\quad - \frac{8 \int \frac{-\frac{3}{8}ab(4a^2 + b^2) - \frac{3}{8}(16a^4 - 8a^2b^2 - 3b^4) \cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx}{15b^3(a^2 - b^2)} \\
&= -\frac{2a^2 \cos^2(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{2a(8a^2 - 3b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5b^3(a^2 - b^2) d} \\
&\quad + \frac{2(6a^2 - b^2) \cos(c + dx) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5b^2(a^2 - b^2) d} \\
&\quad - \frac{(4a(4a^2 + b^2)) \int \frac{1}{\sqrt{a+b \cos(c+dx)}} dx}{5b^4} + \frac{(16a^4 - 8a^2b^2 - 3b^4) \int \sqrt{a + b \cos(c + dx)} dx}{5b^4(a^2 - b^2)} \\
&= -\frac{2a^2 \cos^2(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{2a(8a^2 - 3b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5b^3(a^2 - b^2) d} \\
&\quad + \frac{2(6a^2 - b^2) \cos(c + dx) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5b^2(a^2 - b^2) d} \\
&\quad + \frac{\left((16a^4 - 8a^2b^2 - 3b^4) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{5b^4(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad - \frac{\left(4a(4a^2 + b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{5b^4 \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2(16a^4 - 8a^2b^2 - 3b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{5b^4 (a^2 - b^2) d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad - \frac{8a(4a^2 + b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{5b^4 d \sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{2a^2 \cos^2(c + dx) \sin(c + dx)}{b (a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{2a(8a^2 - 3b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5b^3 (a^2 - b^2) d} \\
&\quad + \frac{2(6a^2 - b^2) \cos(c + dx) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5b^2 (a^2 - b^2) d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.74

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \frac{2(16a^5 + 16a^4b - 8a^3b^2 - 8a^2b^3 - 3ab^4 - 3b^5) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{5b^4 (a^2 - b^2) d \sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (2*(16*a^5 + 16*a^4*b - 8*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4 - 3*b^5)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 8*a*(4*a^4 - 3*a^2*b^2 - b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - b*(16*a^4 - 7*a^2*b^2 + b^4 + 4*a*b*(a^2 - b^2)*Cos[c + d*x] + (-a^2*b^2) + b^4)*Cos[2*(c + d*x)]*Sin[c + d*x]/(5*(a - b)*b^4*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1284 vs. 2(362) = 724.

Time = 6.16 (sec) , antiderivative size = 1285, normalized size of antiderivative = 3.94

method	result	size
default	Expression too large to display	1285

[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*b)^(3/2), x, method=_RETURNVERBOSE)

[Out] -2/5*(-8*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(a^2-b^2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(a^3-a^2*b-a*b^2+b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(8*a^4+2*a^3*b-4*a^2*b^2-2*a*b^3+b^4)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-16*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)

$$\begin{aligned} &^2)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^5 + 12 * (-2 * \sin(1/2*d*x+1/2*c)^4 * b + (a+b) * \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^3 * b^2 + 4 * (-2 * \sin(1/2*d*x+1/2*c)^4 * b + (a+b) * \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a * b^4 + 16 * (-2 * \sin(1/2*d*x+1/2*c)^4 * b + (a+b) * \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^5 - 16 * (-2 * \sin(1/2*d*x+1/2*c)^4 * b + (a+b) * \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^4 * b - 8 * (-2 * \sin(1/2*d*x+1/2*c)^4 * b + (a+b) * \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^3 * b^2 + 8 * (-2 * \sin(1/2*d*x+1/2*c)^4 * b + (a+b) * \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a^2 * b^3 - 3 * (-2 * \sin(1/2*d*x+1/2*c)^4 * b + (a+b) * \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * a * b^4 + 3 * (-2 * \sin(1/2*d*x+1/2*c)^4 * b + (a+b) * \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*b/(a-b) * \sin(1/2*d*x+1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) * b^5 / b^4 / (a-b) / (a+b) / (-2 * \sin(1/2*d*x+1/2*c)^4 * b + (a+b) * \sin(1/2*d*x+1/2*c)^2)^{(1/2)} / \sin(1/2*d*x+1/2*c) / (-2 * b * \sin(1/2*d*x+1/2*c)^2 + a + b)^{(1/2)} / d \end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 687, normalized size of antiderivative = 2.11

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \frac{6(8a^4b^2 - 3a^2b^4 - (a^2b^4 - b^6) \cos(dx + c)^2 + 2(a^3b^3 - ab^5) \cos(dx + c)) \sqrt{b \cos(dx + c) + a} \sin(dx + c)}{\dots}$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] -1/15*(6*(8*a^4*b^2 - 3*a^2*b^4 - (a^2*b^4 - b^6)*cos(d*x + c)^2 + 2*(a^3*b^3 - a*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c) + (sqrt(2)*(-32*I*a^5*b + 28*I*a^3*b^3 + 9*I*a*b^5)*cos(d*x + c) + sqrt(2)*(-32*I*a^6 + 28*I*a^4*b^2 + 9*I*a^2*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x

+ c) + 2*a)/b) + (sqrt(2)*(32*I*a^5*b - 28*I*a^3*b^3 - 9*I*a*b^5)*cos(d*x + c) + sqrt(2)*(32*I*a^6 - 28*I*a^4*b^2 - 9*I*a^2*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(16*I*a^4*b^2 - 8*I*a^2*b^4 - 3*I*b^6)*cos(d*x + c) + sqrt(2)*(16*I*a^5*b - 8*I*a^3*b^3 - 3*I*a*b^5))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(sqrt(2)*(-16*I*a^4*b^2 + 8*I*a^2*b^4 + 3*I*b^6)*cos(d*x + c) + sqrt(2)*(-16*I*a^5*b + 8*I*a^3*b^3 + 3*I*a*b^5))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^2*b^6 - b^8)*d*cos(d*x + c) + (a^3*b^5 - a*b^7)*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)^4}{(b \cos(dx + c) + a)^{3/2}} dx$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)^4}{(b \cos(dx + c) + a)^{3/2}} dx$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^4}{(a + b \cos(c + dx))^{3/2}} dx$$

```
[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(3/2), x)
```

$$3.531 \quad \int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	5443
Rubi [A] (verified)	5443
Mathematica [A] (verified)	5446
Maple [B] (verified)	5447
Fricas [C] (verification not implemented)	5447
Sympy [F(-1)]	5448
Maxima [F]	5448
Giac [F]	5448
Mupad [F(-1)]	5449

Optimal result

Integrand size = 23, antiderivative size = 257

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = -\frac{2a(8a^2-5b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3b^3(a^2-b^2) d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2(8a^2+b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^3 d \sqrt{a+b \cos(c+dx)}} - \frac{2a^2 \cos(c+dx) \sin(c+dx)}{b(a^2-b^2) d \sqrt{a+b \cos(c+dx)}} + \frac{2(4a^2-b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{3b^2(a^2-b^2) d}$$

```
[Out] -2*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d-2/3*a*(8*a^2-5*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(8*a^2+b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {2871, 3102, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = -\frac{2a^2 \sin(c+dx) \cos(c+dx)}{bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}} + \frac{2(4a^2-b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^2d(a^2-b^2)} + \frac{2(8a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}} - \frac{2a(8a^2-5b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^3d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (-2*a*(8*a^2 - 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(8*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*SIN[c + d*x])/(a + b)]/Sqrt[a + b*SIN[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -

$b^2, 0] \&\& !\text{GtQ}[a + b, 0]$

Rule 2831

$\text{Int}[\frac{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}{\sqrt{a_. + (b_.)\sin[(e_.) + (f_.)x]}}, x_Symbol] \rightarrow \text{Dist}[\frac{b*c - a*d}{b}, \text{Int}[\frac{1}{\sqrt{a + b*\sin[e + f*x]}}, x], x] + \text{Dist}[\frac{d}{b}, \text{Int}[\sqrt{a + b*\sin[e + f*x]}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2871

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)x]^m * ((c_.) + (d_.)\sin[(e_.) + (f_.)x])^n, x_Symbol] \rightarrow \text{Simp}[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{m-2}*(c + d*\sin[e + f*x])^{n+1}/(d*f*(n+1)*(c^2 - d^2)), x] + \text{Dist}[1/(d*(n+1)*(c^2 - d^2)), \text{Int}[(a + b*\sin[e + f*x])^{m-3}*(c + d*\sin[e + f*x])^{n+1}*\text{Simp}[b*(m-2)*(b*c - a*d)^2 + a*d*(n+1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n+1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n+2)*(b*c - a*d)^2)*\sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*\sin[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 2] \&\& \text{LtQ}[n, -1] \&\& (\text{IntegerQ}[m] \parallel \text{IntegersQ}[2*m, 2*n])$

Rule 3102

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)x]^m * ((A_.) + (B_.)\sin[(e_.) + (f_.)x] + (C_.)\sin[(e_.) + (f_.)x]^2), x_Symbol] \rightarrow \text{Simp}[(-C)*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{m+1}/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a + b*\sin[e + f*x])^m*\text{Simp}[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C, m, x\} \&\& !\text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2a^2 \cos(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{2 \int \frac{a^2 - \frac{1}{2}ab \cos(c+dx) - \frac{1}{2}(4a^2 - b^2) \cos^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx}{b(a^2 - b^2)} \\ &= -\frac{2a^2 \cos(c + dx) \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{2(4a^2 - b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3b^2 (a^2 - b^2) d} \\ &\quad - \frac{4 \int \frac{\frac{1}{4}b(2a^2 + b^2) + \frac{1}{4}a(8a^2 - 5b^2) \cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx}{3b^2 (a^2 - b^2)} \end{aligned}$$

$$\begin{aligned}
&= -\frac{2a^2 \cos(c+dx) \sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)d} \\
&\quad - \frac{(a(8a^2-5b^2))\int\sqrt{a+b\cos(c+dx)}dx}{3b^3(a^2-b^2)} + \frac{(8a^2+b^2)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}dx}{3b^3} \\
&= -\frac{2a^2 \cos(c+dx) \sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)d} \\
&\quad - \frac{(a(8a^2-5b^2)\sqrt{a+b\cos(c+dx)})\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}dx}{3b^3(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad + \frac{\left((8a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}dx}{3b^3\sqrt{a+b\cos(c+dx)}} \\
&= -\frac{2a(8a^2-5b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^3(a^2-b^2)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad + \frac{2(8a^2+b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{2a^2 \cos(c+dx) \sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.77

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \frac{-2a(8a^3+8a^2b-5ab^2-5b^3)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)+2(8a^4-7a^2b^2-b^4)\sqrt{a+b\cos(c+dx)}}{3(a-b)b^3}$$

[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (-2*a*(8*a^3 + 8*a^2*b - 5*a*b^2 - 5*b^3)*Sqrt[(a + b*Cos[c + d*x])]/(a + b) + 2*(8*a^4 - 7*a^2*b^2 - b^4)*Sqrt[(a + b*Cos[c + d*x])]/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*b*(-4*a^3 + a*b^2 + (-a^2*b) + b^3)*Cos[c + d*x]*Sin[c + d*x]/(3*(a - b)*b^3*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])


```

sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos
s(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) - (sqrt(2)*(-16*I*a^4*b + 16*I*a^
2*b^3 + 3*I*b^5)*cos(d*x + c) + sqrt(2)*(-16*I*a^5 + 16*I*a^3*b^2 + 3*I*a*b
^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*
a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*(sqrt(
2)*(-8*I*a^3*b^2 + 5*I*a*b^4)*cos(d*x + c) + sqrt(2)*(-8*I*a^4*b + 5*I*a^2*
b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b
^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^
2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) + 3*(sqrt(2)*
(8*I*a^3*b^2 - 5*I*a*b^4)*cos(d*x + c) + sqrt(2)*(8*I*a^4*b - 5*I*a^2*b^3))
*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b
^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^
3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^2*b^5 - b^7)*
d*cos(d*x + c) + (a^3*b^4 - a*b^6)*d)

```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)^3}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)^3}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^3}{(a + b \cos(c + dx))^{3/2}} dx$$

```
[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(3/2), x)
```

$$3.532 \quad \int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	5450
Rubi [A] (verified)	5450
Mathematica [A] (verified)	5452
Maple [B] (verified)	5453
Fricas [C] (verification not implemented)	5453
Sympy [F]	5454
Maxima [F]	5454
Giac [F]	5454
Mupad [F(-1)]	5455

Optimal result

Integrand size = 23, antiderivative size = 186

$$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = \frac{2(2a^2 - b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{b^2 (a^2 - b^2) d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{4a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{b^2 d \sqrt{a+b \cos(c+dx)}} - \frac{2a^2 \sin(c+dx)}{b (a^2 - b^2) d \sqrt{a+b \cos(c+dx)}}$$

[Out] $-2*a^2*\sin(d*x+c)/b/(a^2-b^2)/d/(a+b*\cos(d*x+c))^{(1/2)}+2*(2*a^2-b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b))^{(1/2)})*(a+b*\cos(d*x+c))^{(1/2)}/b^2/(a^2-b^2)/d/((a+b*\cos(d*x+c))/(a+b))^{(1/2)}-4*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b))^{(1/2)})*((a+b*\cos(d*x+c))/(a+b))^{(1/2)}/b^2/d/(a+b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2869, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = -\frac{2a^2 \sin(c+dx)}{bd (a^2 - b^2) \sqrt{a+b \cos(c+dx)}} + \frac{2(2a^2 - b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{b^2 d (a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{4a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{b^2 d \sqrt{a+b \cos(c+dx)}}$$

[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (2*(2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2869

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1))

+ c^2*(m + 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{2a^2 \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{2 \int \frac{\frac{ab}{2} + \frac{1}{2}(2a^2 - b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{b(a^2 - b^2)} \\
 &= -\frac{2a^2 \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{(2a) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{b^2} \\
 &\quad + \frac{(2a^2 - b^2) \int \sqrt{a + b \cos(c + dx)} dx}{b^2(a^2 - b^2)} \\
 &= -\frac{2a^2 \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} \\
 &\quad + \frac{\left((2a^2 - b^2) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{b^2(a^2 - b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 &\quad - \frac{\left(2a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{b^2 \sqrt{a + b \cos(c + dx)}} \\
 &= \frac{2(2a^2 - b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{b^2(a^2 - b^2) d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 &\quad - \frac{4a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{b^2 d \sqrt{a + b \cos(c + dx)}} - \frac{2a^2 \sin(c + dx)}{b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \frac{2(2a^3 + 2a^2b - ab^2 - b^3) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) - 2a \left(2(a^2 - b^2) \sqrt{a + b \cos(c + dx)} \right)}{(a - b)b^2(a + b)d \sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (2*(2*a^3 + 2*a^2*b - a*b^2 - b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*b*Sin[c + d*x])/((a - b)*b^2*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(234) = 468$.

Time = 5.25 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.85

method	result
default	$-\frac{2\left(2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a-b}+\frac{a+b}{a-b}}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)a^3+2ab^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{1}$

[In] `int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$-2*(2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*a^2*b-2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^3+2*a*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})+2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^3-2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a^2*b-(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a*b^2+(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*b^3/b^2/(a-b)/(a+b)/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 567, normalized size of antiderivative = 3.05

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \frac{6\sqrt{b\cos(dx+c)+aa^2b^2\sin(dx+c)}+(\sqrt{2}(-4ia^3b+5iab^3)\cos(dx+c)+\sqrt{2}(-4ia^4+5ia^2b^2))\sqrt{b\cos(dx+c)+aa^2b^2\sin(dx+c)}}{1}$$

[In] `integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out]
$$-1/3*(6*\text{sqrt}(b*\cos(d*x+c)+a)*a^2*b^2*\sin(d*x+c)+(\text{sqrt}(2)*(-4*I*a^3*b+5*I*a*b^3)*\cos(d*x+c)+\text{sqrt}(2)*(-4*I*a^4+5*I*a^2*b^2))*\text{sqrt}(b)*\text{weierstrassPInverse}(4/3*(4*a^2-3*b^2)/b^2,-8/27*(8*a^3-9*a*b^2)/b^3,1/3*(3*b*\cos(d*x+c)+3*I*b*\sin(d*x+c)+2*a)/b)+(\text{sqrt}(2)*(4*I*a^3*b-5*I*a*b^3)*\cos(d*x+c)+\text{sqrt}(2)*(4*I*a^4-5*I*a^2*b^2))*\text{sqrt}(b)*\text{weierstrass}$$

```
sPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(2*I*a^2*b^2 - I*b^4)*cos(d*x + c) + sqrt(2)*(2*I*a^3*b - I*a*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(sqrt(2)*(-2*I*a^2*b^2 + I*b^4)*cos(d*x + c) + sqrt(2)*(-2*I*a^3*b + I*a*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^2*b^4 - b^6)*d*cos(d*x + c) + (a^3*b^3 - a*b^5)*d)
```

Sympy [F]

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx$$

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**(3/2), x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*cos(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)^2}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2), x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)^2}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2), x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^2}{(a + b \cos(c + dx))^{3/2}} dx$$

```
[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(3/2), x)
```

3.533 $\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$

Optimal result	5456
Rubi [A] (verified)	5456
Mathematica [A] (verified)	5458
Maple [A] (verified)	5459
Fricas [C] (verification not implemented)	5459
Sympy [F]	5460
Maxima [F]	5460
Giac [F]	5460
Mupad [F(-1)]	5460

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = -\frac{2a\sqrt{a+b \cos(c+dx)}E\left(\frac{1}{2}(c+dx)\left|\frac{2b}{a+b}\right.\right)}{b(a^2-b^2)d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{bd\sqrt{a+b \cos(c+dx)}} + \frac{2a \sin(c+dx)}{(a^2-b^2)d\sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*a*(cos(1/2*d*x+1/2*c))^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2*(cos(1/2*d*x+1/2*c))^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = \frac{2a \sin(c+dx)}{d(a^2-b^2)\sqrt{a+b \cos(c+dx)}} - \frac{2a\sqrt{a+b \cos(c+dx)}E\left(\frac{1}{2}(c+dx)\left|\frac{2b}{a+b}\right.\right)}{bd(a^2-b^2)\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{bd\sqrt{a+b \cos(c+dx)}}$$

[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]

[Out] (-2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(b*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2833

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -

$a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{2 \int \frac{\frac{b}{2} + \frac{1}{2} a \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{a^2 - b^2} \\
 &= \frac{2a \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{b} - \frac{a \int \sqrt{a + b \cos(c + dx)} dx}{b(a^2 - b^2)} \\
 &= \frac{2a \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{\left(a \sqrt{a + b \cos(c + dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{b(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a+b}}} \\
 &\quad + \frac{\sqrt{\frac{a + b \cos(c + dx)}{a+b}} \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{b \sqrt{a + b \cos(c + dx)}} \\
 &= -\frac{2a \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid \frac{2b}{a+b}\right)}{b(a^2 - b^2) d \sqrt{\frac{a + b \cos(c + dx)}{a+b}}} \\
 &\quad + \frac{2 \sqrt{\frac{a + b \cos(c + dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{bd \sqrt{a + b \cos(c + dx)}} + \frac{2a \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.81

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \frac{-2a(a + b) \sqrt{\frac{a + b \cos(c + dx)}{a+b}} E\left(\frac{1}{2}(c + dx) \mid \frac{2b}{a+b}\right) + 2(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{(a - b)b(a + b)d \sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (-2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*a*b*Sin[c + d*x])/((a - b)*b*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.19

method	result
default	$4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) ab - 2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a-b} + \frac{a+b}{a-b}} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{-\frac{2b}{a-b}}\right) a^2 + 2b^2 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a-b} + \frac{a+b}{a-b}}$

```
[In] int(cos(d*x+c)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a)/b/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.10

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \frac{6\sqrt{b\cos(dx+c)+aab^2}\sin(dx+c) - (\sqrt{2}(2ia^2b-3ib^3)\cos(dx+c) + \sqrt{2}}$$

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(6*sqrt(b*cos(d*x+c)+a)*a*b^2*sin(d*x+c) - (sqrt(2)*(2*I*a^2*b - 3*I*b^3)*cos(d*x+c) + sqrt(2)*(2*I*a^3 - 3*I*a*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x+c) + 3*I*b*sin(d*x+c) + 2*a)/b) - (sqrt(2)*(-2*I*a^2*b + 3*I*b^3)*cos(d*x+c) + sqrt(2)*(-2*I*a^3 + 3*I*a*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x+c) - 3*I*b*sin(d*x+c) + 2*a)/b) + 3*(-I*sqrt(2)*a*b^2*cos(d*x+c) - I*sqrt(2)*a^2*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x+c) + 3*I*b*sin(d*x+c) + 2*a)/b)) + 3*(I*sqrt(2)*a*b^2*cos(d*x+c) + I*sqrt(2)*a^2*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x+c) - 3*I*b*sin(d*x+c) + 2*a)/b)))/((a^2*b^3 - b^5)*d*cos(d*x+c) + (a^3*b^2 - a*b^4)*d)
```

Sympy [F]

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)

[Out] Integral(cos(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)}{(b \cos(dx + c) + a)^{3/2}} dx$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)}{(b \cos(dx + c) + a)^{3/2}} dx$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)/(a + b*cos(c + d*x))^(3/2), x)

$$3.534 \quad \int \frac{1}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	5461
Rubi [A] (verified)	5461
Mathematica [A] (verified)	5463
Maple [A] (verified)	5463
Fricas [C] (verification not implemented)	5463
Sympy [F]	5464
Maxima [F]	5464
Giac [F]	5464
Mupad [F(-1)]	5465

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{(a+b \cos(c+dx))^{3/2}} dx = \frac{2\sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{(a^2-b^2) d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2b \sin(c+dx)}{(a^2-b^2) d \sqrt{a+b \cos(c+dx)}}$$

[Out] $-2*b*\sin(d*x+c)/(a^2-b^2)/d/(a+b*\cos(d*x+c))^{(1/2)+2*(\cos(1/2*d*x+1/2*c))^2}^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b))^{(1/2)})*(a+b*\cos(d*x+c))^{(1/2)}/(a^2-b^2)/d/((a+b*\cos(d*x+c))/(a+b))^{(1/2)}$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2743, 21, 2734, 2732}

$$\int \frac{1}{(a+b \cos(c+dx))^{3/2}} dx = \frac{2\sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{d(a^2-b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2b \sin(c+dx)}{d(a^2-b^2) \sqrt{a+b \cos(c+dx)}}$$

[In] $\text{Int}[(a+b*\text{Cos}[c+d*x])^{-3/2}, x]$

[Out] $(2*\text{Sqrt}[a+b*\text{Cos}[c+d*x]]*\text{EllipticE}[(c+d*x)/2, (2*b)/(a+b)])/((a^2-b^2)*d*\text{Sqrt}[(a+b*\text{Cos}[c+d*x])/(a+b)]) - (2*b*\text{Sin}[c+d*x])/((a^2-b^2)*d*\text{Sqrt}[a+b*\text{Cos}[c+d*x]])$

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a
+ b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{2 \int \frac{-\frac{a}{2} - \frac{1}{2} b \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{a^2 - b^2} \\
&= -\frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\int \sqrt{a + b \cos(c + dx)} dx}{a^2 - b^2} \\
&= -\frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\sqrt{a + b \cos(c + dx)} \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a+b}}} \\
&= \frac{2\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{(a^2 - b^2) d \sqrt{\frac{a + b \cos(c + dx)}{a+b}}} - \frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx = \frac{2(a + b) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) - 2b \sin(c + dx)}{(a - b)(a + b)d \sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(-3/2),x]

[Out] (2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*b*Sin[c + d*x])/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.05

method	result
default	$-\frac{2 \left(2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b + E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{-\frac{2b}{a-b}}\right) \sqrt{-\frac{2b \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a-b}} + \frac{a+b}{a-b} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} a - E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{-\frac{2b}{a-b}}\right) \sqrt{-\frac{2b \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a-b}} \right)}{(a-b)(a+b) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2b \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + a+b} d}$

[In] int(1/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+EllipticE(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-EllipticE(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.55

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx = \frac{6 \sqrt{b \cos(dx + c) + ab^2 \sin(dx + c)} + (i \sqrt{2ab} \cos(dx + c) + i \sqrt{2a^2}) \sqrt{b} \text{weierstrassPInverse}\left(\frac{4(4a^2 - 3b^2)}{3b^2}, -\right)}{d}$$

[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

```
[Out] -1/3*(6*sqrt(b*cos(d*x + c) + a)*b^2*sin(d*x + c) + (I*sqrt(2)*a*b*cos(d*x
+ c) + I*sqrt(2)*a^2)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2
*a)/b) + (-I*sqrt(2)*a*b*cos(d*x + c) - I*sqrt(2)*a^2)*sqrt(b)*weierstrassP
Inverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(
d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(I*sqrt(2)*b^2*cos(d*x + c) + I
*sqrt(2)*a*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3
- 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 3*(
-I*sqrt(2)*b^2*cos(d*x + c) - I*sqrt(2)*a*b)*sqrt(b)*weierstrassZeta(4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*
a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*
b*sin(d*x + c) + 2*a)/b)))/((a^2*b^2 - b^4)*d*cos(d*x + c) + (a^3*b - a*b^3
)*d)
```

Sympy [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{1}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{1}{(a + b \cos(c + dx))^{3/2}} dx$$

```
[In] int(1/(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int(1/(a + b*cos(c + d*x))^(3/2), x)
```

$$3.535 \quad \int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	5466
Rubi [A] (verified)	5466
Mathematica [C] (verified)	5469
Maple [A] (verified)	5469
Fricas [F(-1)]	5470
Sympy [F]	5470
Maxima [F]	5470
Giac [F]	5470
Mupad [F(-1)]	5471

Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = -\frac{2b\sqrt{a+b \cos(c+dx)}E\left(\frac{1}{2}(c+dx)\left|\frac{2b}{a+b}\right.\right)}{a(a^2-b^2)d\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{ad\sqrt{a+b \cos(c+dx)}} + \frac{2b^2 \sin(c+dx)}{a(a^2-b^2)d\sqrt{a+b \cos(c+dx)}}$$

[Out] $2*b^2*\sin(d*x+c)/a/(a^2-b^2)/d/(a+b*\cos(d*x+c))^{(1/2)}-2*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b)))^{(1/2)}*(a+b*\cos(d*x+c))^{(1/2)}/a/(a^2-b^2)/d/((a+b*\cos(d*x+c))/(a+b))^{(1/2)}+2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2^{(1/2)}*(b/(a+b)))^{(1/2)}*(a+b*\cos(d*x+c))/(a+b)^{(1/2)}/a/d/(a+b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2881, 3138, 2734, 2732, 12, 2886, 2884}

$$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = \frac{2b^2 \sin(c+dx)}{ad(a^2-b^2)\sqrt{a+b \cos(c+dx)}} - \frac{2b\sqrt{a+b \cos(c+dx)}E\left(\frac{1}{2}(c+dx)\left|\frac{2b}{a+b}\right.\right)}{ad(a^2-b^2)\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{ad\sqrt{a+b \cos(c+dx)}}$$

[In] Int[Sec[c + d*x]/(a + b*cos[c + d*x])^(3/2), x]

[Out] (-2*b*Sqrt[a + b*cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*(a^2 - b^2)*d*Sqrt[(a + b*cos[c + d*x])/(a + b)] + (2*Sqrt[(a + b*cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(a*d*Sqrt[a + b*cos[c + d*x]]) + (2*b^2*sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*cos[c + d*x]]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*sin[c + d*x]]/Sqrt[(a + b*sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*sin[e + f*x])^(m + 1)*((c + d*sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*sin[e + f*x] - b^2*d*(m + n + 3)*sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2886

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])]/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3138

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{2 \int \frac{(\frac{1}{2}(a^2 - b^2) - \frac{1}{2}ab \cos(c + dx) - \frac{1}{2}b^2 \cos^2(c + dx)) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{a(a^2 - b^2)} \\
 &= \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} - \frac{2 \int -\frac{b(a^2 - b^2) \sec(c + dx)}{2\sqrt{a + b \cos(c + dx)}} dx}{ab(a^2 - b^2)} - \frac{b \int \sqrt{a + b \cos(c + dx)} dx}{a(a^2 - b^2)} \\
 &= \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{a} \\
 &\quad - \frac{\left(b \sqrt{a + b \cos(c + dx)}\right) \int \sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a + b}} dx}{a(a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} \\
 &= -\frac{2b \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a + b}\right)}{a(a^2 - b^2) d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} \\
 &\quad + \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\sqrt{\frac{a + b \cos(c + dx)}{a + b}} \int \frac{\sec(c + dx)}{\sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a + b}}} dx}{a \sqrt{a + b \cos(c + dx)}} \\
 &= -\frac{2b \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a + b}\right)}{a(a^2 - b^2) d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} \\
 &\quad + \frac{2 \sqrt{\frac{a + b \cos(c + dx)}{a + b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a + b}\right)}{ad \sqrt{a + b \cos(c + dx)}} + \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.28

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \frac{-\frac{4ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right) + 2(2a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \dots$$

[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^(3/2), x]

[Out] $(-(((-4*a*b*\sqrt{(a + b*\cos[c + d*x])/(a + b)})*\operatorname{EllipticF}[(c + d*x)/2, (2*b)/(a + b)]/\sqrt{a + b*\cos[c + d*x]} + (2*(2*a^2 - 3*b^2)*\sqrt{(a + b*\cos[c + d*x])/(a + b)})*\operatorname{EllipticPi}[2, (c + d*x)/2, (2*b)/(a + b)]/\sqrt{a + b*\cos[c + d*x]} - ((2*I)*\sqrt{-((b*(-1 + \cos[c + d*x]))/(a + b))})*\sqrt{(b*(1 + \cos[c + d*x]))/(-a + b)}*\operatorname{Csc}[c + d*x]*(-2*a*(a - b)*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\sqrt{-(a + b)^{-1}}]*\sqrt{a + b*\cos[c + d*x]}], (a + b)/(a - b)] + b*(-2*a*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\sqrt{-(a + b)^{-1}}]*\sqrt{a + b*\cos[c + d*x]}], (a + b)/(a - b)] + b*\operatorname{EllipticPi}[(a + b)/a, I*\operatorname{ArcSinh}[\sqrt{-(a + b)^{-1}}]*\sqrt{a + b*\cos[c + d*x]}], (a + b)/(a - b))))/(a*\sqrt{-(a + b)^{-1}})/((-a + b)*(a + b)) + (4*b^2*\sin[c + d*x])/((a^2 - b^2)*\sqrt{a + b*\cos[c + d*x]})/(2*a*d)$

Maple [A] (verified)

Time = 4.45 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.14

method	result
default	$\frac{4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b^2 + 2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{a-b} + \frac{a+b}{a-b} \Pi\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2, \sqrt{-\frac{2b}{a-b}}\right) a^2 - 2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-\dots}}{\dots}$

[In] int(sec(d*x+c)/(a+cos(d*x+c)*b)^(3/2), x, method=_RETURNVERBOSE)

[Out] $2*(2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*b^2+(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*\operatorname{EllipticPi}(\cos(1/2*d*x+1/2*c), 2, (-2*b/(a-b))^(1/2))*a^2-(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*\operatorname{EllipticPi}(\cos(1/2*d*x+1/2*c), 2, (-2*b/(a-b))^(1/2))*b^2+(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))*a-b^2*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2)))/a/(a-b)/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)

[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec(dx + c)}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec(dx + c)}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{1}{\cos(c + dx) (a + b \cos(c + dx))^{3/2}} dx$$

```
[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)), x)
```

```
[Out] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(3/2)), x)
```

3.536 $\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$

Optimal result	5472
Rubi [A] (verified)	5473
Mathematica [C] (verified)	5476
Maple [B] (verified)	5477
Fricas [F(-1)]	5478
Sympy [F]	5478
Maxima [F]	5478
Giac [F]	5478
Mupad [F(-1)]	5479

Optimal result

Integrand size = 23, antiderivative size = 277

$$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = -\frac{(a^2 - 3b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid \frac{2b}{a+b}\right)}{a^2 (a^2 - b^2) d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{ad \sqrt{a+b \cos(c+dx)}} - \frac{3b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{a^2 d \sqrt{a+b \cos(c+dx)}} + \frac{b(a^2 - 3b^2) \sin(c+dx)}{a^2 (a^2 - b^2) d \sqrt{a+b \cos(c+dx)}} + \frac{\tan(c+dx)}{ad \sqrt{a+b \cos(c+dx)}}$$

```
[Out] b*(a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-(a^2-3*b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*
c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((a+b*co
s(d*x+c))/(a+b))^(1/2)+(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(
1/2)/a/d/(a+b*cos(d*x+c))^(1/2)-3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*co
s(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)+tan(d*x+c)/a/d/(a+b*cos
(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2881, 3135, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \frac{b(a^2-3b^2)\sin(c+dx)}{a^2d(a^2-b^2)\sqrt{a+b\cos(c+dx)}} - \frac{(a^2-3b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{a^2d(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2,\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2), x]

[Out] -(((a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b*Cos[c + d*x]]) - (3*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(a^2 - 3*b^2)*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + Tan[c + d*x]/(a*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && (IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))
```

Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)]))/(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a,
0])))

```

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan(c + dx)}{ad\sqrt{a + b \cos(c + dx)}} + \frac{\int \frac{\left(-\frac{3b}{2} + \frac{1}{2}b \cos^2(c + dx)\right) \sec(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx}{a} \\
&= \frac{b(a^2 - 3b^2) \sin(c + dx)}{a^2(a^2 - b^2) d\sqrt{a + b \cos(c + dx)}} + \frac{\tan(c + dx)}{ad\sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{2 \int \frac{\left(-\frac{3}{4}b(a^2 - b^2) + \frac{1}{2}ab^2 \cos(c + dx) - \frac{1}{4}b(a^2 - 3b^2) \cos^2(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{a^2(a^2 - b^2)} \\
&= \frac{b(a^2 - 3b^2) \sin(c + dx)}{a^2(a^2 - b^2) d\sqrt{a + b \cos(c + dx)}} + \frac{\tan(c + dx)}{ad\sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{2 \int \frac{\left(\frac{3}{4}b^2(a^2 - b^2) - \frac{1}{4}ab(a^2 - b^2) \cos(c + dx)\right) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{a^2b(a^2 - b^2)} \\
&\quad - \frac{(a^2 - 3b^2) \int \sqrt{a + b \cos(c + dx)} dx}{2a^2(a^2 - b^2)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{b(a^2 - 3b^2) \sin(c + dx)}{a^2 (a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\tan(c + dx)}{ad \sqrt{a + b \cos(c + dx)}} + \frac{\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{2a} \\
&\quad - \frac{(3b) \int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{2a^2} - \frac{\left((a^2 - 3b^2) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a + b}} dx}{2a^2 (a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} \\
&= - \frac{(a^2 - 3b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid \frac{2b}{a + b}\right)}{a^2 (a^2 - b^2) d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} \\
&\quad + \frac{b(a^2 - 3b^2) \sin(c + dx)}{a^2 (a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\tan(c + dx)}{ad \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{\sqrt{\frac{a + b \cos(c + dx)}{a + b}} \int \frac{1}{\sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a + b}}} dx}{2a \sqrt{a + b \cos(c + dx)}} - \frac{\left(3b \sqrt{\frac{a + b \cos(c + dx)}{a + b}} \right) \int \frac{\sec(c + dx)}{\sqrt{\frac{a}{a + b} + \frac{b \cos(c + dx)}{a + b}}} dx}{2a^2 \sqrt{a + b \cos(c + dx)}} \\
&= - \frac{(a^2 - 3b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid \frac{2b}{a + b}\right)}{a^2 (a^2 - b^2) d \sqrt{\frac{a + b \cos(c + dx)}{a + b}}} \\
&\quad + \frac{\sqrt{\frac{a + b \cos(c + dx)}{a + b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a + b}\right)}{ad \sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{3b \sqrt{\frac{a + b \cos(c + dx)}{a + b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a + b}\right)}{a^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{b(a^2 - 3b^2) \sin(c + dx)}{a^2 (a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\tan(c + dx)}{ad \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.59

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \frac{b \left(-\frac{8ab \sqrt{\frac{a + b \cos(c + dx)}{a + b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a + b}\right)}{\sqrt{a + b \cos(c + dx)}} + \frac{2(7a^2 - 9b^2) \sqrt{\frac{a + b \cos(c + dx)}{a + b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a + b}\right)}{\sqrt{a + b \cos(c + dx)}} \right)}{a^2 (a^2 - 3b^2) \sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (-((b*((-8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(7*a^2 - 9*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(a^2 - 3*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))


```

]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b^2*Sqrt[-(a + b)^(-1)]))/((a - b)*(a + b))) + (4*(a^3 - a*b^2 + b*(a^2 - 3*b^2)*Cos[c + d*x])*Tan[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(4*a^2*d)

```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 897 vs. 2(348) = 696.

Time = 6.44 (sec) , antiderivative size = 898, normalized size of antiderivative = 3.24

method	result	size
default	Expression too large to display	898

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

```

[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(-cos(1/2*d*x+1/2*c)/a*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2))/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))+2/a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2*b^2/a^2/sin(1/2*d*x+1/2*c)^2/(2*b*sin(1/2*d*x+1/2*c)^2-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx$$

[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)

[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec(dx + c)^2}{(b \cos(dx + c) + a)^{3/2}} dx$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec(dx + c)^2}{(b \cos(dx + c) + a)^{3/2}} dx$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{1}{\cos(c + dx)^2 (a + b \cos(c + dx))^{3/2}} dx$$

```
[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(3/2)), x)
```

$$3.537 \quad \int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	5480
Rubi [A] (verified)	5481
Mathematica [C] (verified)	5485
Maple [B] (verified)	5485
Fricas [F(-1)]	5486
Sympy [F]	5487
Maxima [F]	5487
Giac [F]	5487
Mupad [F(-1)]	5487

Optimal result

Integrand size = 23, antiderivative size = 345

$$\int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = \frac{b(7a^2 - 15b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{4a^3 (a^2 - b^2) d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{5b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4a^2 d \sqrt{a+b \cos(c+dx)}} + \frac{(4a^2 + 15b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4a^3 d \sqrt{a+b \cos(c+dx)}} - \frac{b^2(7a^2 - 15b^2) \sin(c+dx)}{4a^3 (a^2 - b^2) d \sqrt{a+b \cos(c+dx)}} - \frac{5b \tan(c+dx)}{4a^2 d \sqrt{a+b \cos(c+dx)}} + \frac{\sec(c+dx) \tan(c+dx)}{2ad \sqrt{a+b \cos(c+dx)}}$$

```
[Out] -1/4*b^2*(7*a^2-15*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+1/4*b*(7*a^2-15*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-5/4*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/4*(4*a^2+15*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*cos(d*x+c))^(1/2)-5/4*b*tan(d*x+c)/a^2/d/(a+b*cos(d*x+c))^(1/2)+1/2*sec(d*x+c)*tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2881, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = -\frac{5b \tan(c+dx)}{4a^2 d \sqrt{a+b\cos(c+dx)}} - \frac{5b \sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4a^2 d \sqrt{a+b\cos(c+dx)}} - \frac{b^2(7a^2-15b^2) \sin(c+dx)}{4a^3 d (a^2-b^2) \sqrt{a+b\cos(c+dx)}} + \frac{b(7a^2-15b^2) \sqrt{a+b\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{4a^3 d (a^2-b^2) \sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(4a^2+15b^2) \sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{4a^3 d \sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx) \sec(c+dx)}{2ad \sqrt{a+b\cos(c+dx)}}$$

[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (b*(7*a^2 - 15*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(4*a^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (5*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2 + 15*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(4*a^3*d*Sqrt[a + b*Cos[c + d*x]]) - (b^2*(7*a^2 - 15*b^2)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (5*b*Tan[c + d*x])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*SIN[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2886

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3081

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)^2], x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))

```

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])], x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{5b}{2}+a\cos(c+dx)+\frac{3}{2}b\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx}{2a} \\
&= -\frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\int \frac{\left(\frac{1}{4}(4a^2+15b^2)+\frac{3}{2}ab\cos(c+dx)-\frac{5}{4}b^2\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx}{2a^2} \\
&= -\frac{b^2(7a^2-15b^2)\sin(c+dx)}{4a^3(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} - \frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{8}(4a^4+11a^2b^2-15b^4)+\frac{1}{4}ab(a^2-5b^2)\cos(c+dx)+\frac{1}{8}b^2(7a^2-15b^2)\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}}{a^3(a^2-b^2)}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{b^2(7a^2 - 15b^2) \sin(c + dx)}{4a^3 (a^2 - b^2) d\sqrt{a + b \cos(c + dx)}} - \frac{5b \tan(c + dx)}{4a^2 d\sqrt{a + b \cos(c + dx)}} \\
&+ \frac{\sec(c + dx) \tan(c + dx)}{2ad\sqrt{a + b \cos(c + dx)}} - \frac{\int \frac{(-\frac{1}{8}b(4a^4 + 11a^2b^2 - 15b^4) + \frac{5}{8}ab^2(a^2 - b^2) \cos(c + dx)) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{a^3b(a^2 - b^2)} \\
&+ \frac{(b(7a^2 - 15b^2)) \int \sqrt{a + b \cos(c + dx)} dx}{8a^3 (a^2 - b^2)} \\
&= -\frac{b^2(7a^2 - 15b^2) \sin(c + dx)}{4a^3 (a^2 - b^2) d\sqrt{a + b \cos(c + dx)}} \\
&- \frac{5b \tan(c + dx)}{4a^2 d\sqrt{a + b \cos(c + dx)}} + \frac{\sec(c + dx) \tan(c + dx)}{2ad\sqrt{a + b \cos(c + dx)}} \\
&- \frac{(5b) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{8a^2} + \frac{(4a^2 + 15b^2) \int \frac{\sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{8a^3} \\
&+ \frac{\left(b(7a^2 - 15b^2) \sqrt{a + b \cos(c + dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{8a^3 (a^2 - b^2) \sqrt{\frac{a + b \cos(c + dx)}{a+b}}} \\
&= \frac{b(7a^2 - 15b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{4a^3 (a^2 - b^2) d\sqrt{\frac{a + b \cos(c + dx)}{a+b}}} \\
&- \frac{b^2(7a^2 - 15b^2) \sin(c + dx)}{4a^3 (a^2 - b^2) d\sqrt{a + b \cos(c + dx)}} - \frac{5b \tan(c + dx)}{4a^2 d\sqrt{a + b \cos(c + dx)}} \\
&+ \frac{\sec(c + dx) \tan(c + dx)}{2ad\sqrt{a + b \cos(c + dx)}} - \frac{\left(5b\sqrt{\frac{a + b \cos(c + dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{8a^2 \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{\left((4a^2 + 15b^2) \sqrt{\frac{a + b \cos(c + dx)}{a+b}}\right) \int \frac{\sec(c + dx)}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{8a^3 \sqrt{a + b \cos(c + dx)}} \\
&= \frac{b(7a^2 - 15b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{4a^3 (a^2 - b^2) d\sqrt{\frac{a + b \cos(c + dx)}{a+b}}} \\
&- \frac{5b\sqrt{\frac{a + b \cos(c + dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4a^2 d\sqrt{a + b \cos(c + dx)}} \\
&+ \frac{(4a^2 + 15b^2) \sqrt{\frac{a + b \cos(c + dx)}{a+b}} \text{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{4a^3 d\sqrt{a + b \cos(c + dx)}} \\
&- \frac{b^2(7a^2 - 15b^2) \sin(c + dx)}{4a^3 (a^2 - b^2) d\sqrt{a + b \cos(c + dx)}} \\
&- \frac{5b \tan(c + dx)}{4a^2 d\sqrt{a + b \cos(c + dx)}} + \frac{\sec(c + dx) \tan(c + dx)}{2ad\sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.58 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.35

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \frac{8ab(a^2-5b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right) + 2(8a^4+29a^2b^2-45b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}\right)}{\sqrt{a+b\cos(c+dx)}}$$

[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2), x]

[Out]
$$\begin{aligned} & -\left(\frac{8ab(a^2-5b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{c+dx}{2}, \frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2(8a^4+29a^2b^2-45b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}\right) \\ & - \frac{((2*I)*(-7*a^2 + 15*b^2)*\sqrt{-(b*(1 + \cos[c + d*x]))/(a - b)}) * \operatorname{Csc}[c + d*x] * (-2*a*(a - b)*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\sqrt{-(a + b)^{-1}}]*\sqrt{a + b\cos[c + d*x]}], (a + b)/(a - b) + b*(-2*a*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\sqrt{-(a + b)^{-1}}]*\sqrt{a + b\cos[c + d*x]}], (a + b)/(a - b) + b*\operatorname{EllipticPi}[(a + b)/a, I*\operatorname{ArcSinh}[\sqrt{-(a + b)^{-1}}]*\sqrt{a + b\cos[c + d*x]}], (a + b)/(a - b)))/(a*\sqrt{-(a + b)^{-1}})}{((-a + b)*(a + b))} + 4*\sqrt{a + b\cos[c + d*x]} * ((8*b^4*\sin[c + d*x])/((a^2 - b^2)*(a + b\cos[c + d*x])) + (-7*b + 2*a*\operatorname{Sec}[c + d*x])*Tan[c + d*x])/(16*a^3*d) \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. 2(402) = 804.

Time = 8.82 (sec) , antiderivative size = 1546, normalized size of antiderivative = 4.48

method	result	size
default	Expression too large to display	1546

[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*b)^(3/2), x, method=_RETURNVERBOSE)

[Out]
$$\begin{aligned} & -\left(-(-2*b*\cos(1/2*d*x+1/2*c)^2-a+b)*\sin(1/2*d*x+1/2*c)^2\right)^{(1/2)} * (2/a*(-1/2*\cos(1/2*d*x+1/2*c)/a*(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}) \\ & / (2*\cos(1/2*d*x+1/2*c)^2-1)^2 + 3/4*b/a^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} / (2*\cos(1/2*d*x+1/2*c)^2-1) \\ & - 1/8*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) + 3/8/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & * ((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * b*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a- \end{aligned}$$

$$\begin{aligned}
& b))^{(1/2)} - 3/8 * b^2 / a^2 * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * ((2 * b * \cos(1/2 * d * x + 1/2 * c) \\
&)^2 + a - b) / (a - b))^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c)^2 \\
&)^{(1/2)} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), (-2 * b / (a - b))^{(1/2)}) - 1/2 * (\sin(1/2 * d * x + 1/2 * c) \\
&)^2)^{(1/2)} * ((2 * b * \cos(1/2 * d * x + 1/2 * c)^2 + a - b) / (a - b))^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c) \\
& + 1/2 * c)^4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2 * d * x + 1/2 * c) \\
& , 2, (-2 * b / (a - b))^{(1/2)}) - 3/8 / a^2 * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * ((2 * b * \cos(1/2 * d * x + 1/2 * c) \\
& * x + 1/2 * c)^2 + a - b) / (a - b))^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c) \\
& + 1/2 * c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2 * d * x + 1/2 * c), 2, (-2 * b / (a - b))^{(1/2)}) * b^2 - 2 * \\
& b^2 / a^3 * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * ((2 * b * \cos(1/2 * d * x + 1/2 * c)^2 + a - b) / (a - b)) \\
&)^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2 * d * x + 1/2 * c), 2, (-2 * b / (a - b))^{(1/2)}) - 2 / a^2 * b * (-\cos(1/2 * d * x + 1/2 * c) / a \\
& * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / (2 * \cos(1/2 * d * x + 1/2 * c)^2 - 1) + 1/2 * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * ((2 * b * \cos(1/2 * d * x + 1/2 * c)^2 + a - b) / (a - b))^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), (-2 * b / (a - b))^{(1/2)}) - 1/2 * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * ((2 * b * \cos(1/2 * d * x + 1/2 * c)^2 + a - b) / (a - b))^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), (-2 * b / (a - b))^{(1/2)}) + 1/2 / a * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * ((2 * b * \cos(1/2 * d * x + 1/2 * c)^2 + a - b) / (a - b))^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * b * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), (-2 * b / (a - b))^{(1/2)}) + 1/2 / a * b * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * ((2 * b * \cos(1/2 * d * x + 1/2 * c)^2 + a - b) / (a - b))^{(1/2)} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2 * d * x + 1/2 * c), 2, (-2 * b / (a - b))^{(1/2)}) + 2 / a^3 * b^3 / \sin(1/2 * d * x + 1/2 * c)^2 / (2 * b * \sin(1/2 * d * x + 1/2 * c)^2 - a - b) / (a^2 - b^2) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 * b + (a + b) * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (2 * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c)^2 * b + \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), (-2 * b / (a - b))^{(1/2)}) * (-2 * b / (a - b) * \sin(1/2 * d * x + 1/2 * c)^2 + (a + b) / (a - b))^{(1/2)} * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * a - \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), (-2 * b / (a - b))^{(1/2)}) * (-2 * b / (a - b) * \sin(1/2 * d * x + 1/2 * c)^2 + (a + b) / (a - b))^{(1/2)} * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * b) / \sin(1/2 * d * x + 1/2 * c) / (-2 * b * \sin(1/2 * d * x + 1/2 * c)^2 + a + b)^{(1/2)} / d
\end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**(3/2), x)

[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec(dx + c)^3}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec(dx + c)^3}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{1}{\cos(c + dx)^3 (a + b \cos(c + dx))^{3/2}} dx$$

[In] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)), x)

[Out] int(1/(cos(c + d*x)^3*(a + b*cos(c + d*x))^(3/2)), x)

$$3.538 \quad \int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	5488
Rubi [A] (verified)	5489
Mathematica [A] (verified)	5493
Maple [B] (verified)	5493
Fricas [C] (verification not implemented)	5495
Sympy [F(-1)]	5496
Maxima [F]	5496
Giac [F]	5496
Mupad [F(-1)]	5496

Optimal result

Integrand size = 23, antiderivative size = 436

$$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = \frac{2(128a^6 - 212a^4b^2 + 55a^2b^4 + 9b^6) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{15b^5 (a^2 - b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2a(128a^4 - 116a^2b^2 - 17b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{15b^5 (a^2 - b^2) d \sqrt{a+b \cos(c+dx)}} - \frac{2a^2 \cos^3(c+dx) \sin(c+dx)}{3b (a^2 - b^2) d (a+b \cos(c+dx))^{3/2}} - \frac{8a^2 (2a^2 - 3b^2) \cos^2(c+dx) \sin(c+dx)}{3b^2 (a^2 - b^2)^2 d \sqrt{a+b \cos(c+dx)}} - \frac{4a(32a^4 - 49a^2b^2 + 7b^4) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{15b^4 (a^2 - b^2)^2 d} + \frac{2(48a^4 - 71a^2b^2 + 3b^4) \cos(c+dx) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{15b^3 (a^2 - b^2)^2 d}$$

[Out] $-2/3*a^2*\cos(d*x+c)^3*\sin(d*x+c)/b/(a^2-b^2)/d/(a+b*\cos(d*x+c))^{3/2}-8/3*a^2*(2*a^2-3*b^2)*\cos(d*x+c)^2*\sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*\cos(d*x+c))^{1/2}-4/15*a*(32*a^4-49*a^2*b^2+7*b^4)*\sin(d*x+c)*(a+b*\cos(d*x+c))^{1/2}/b^4/(a^2-b^2)^2/d+2/15*(48*a^4-71*a^2*b^2+3*b^4)*\cos(d*x+c)*\sin(d*x+c)*(a+b*\cos(d*x+c))^{1/2}/b^3/(a^2-b^2)^2/d+2/15*(128*a^6-212*a^4*b^2+55*a^2*b^4+9*b^6)*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(b/(a+b))^{1/2})*(a+b*\cos(d*x+c))^{1/2}/b^5/(a^2-b^2)^2/d/((a+b*\cos(d*x+c))/(a+b))^{1/2}-2/15*a*(128*a^4-116*a^2*b^2-17*b^4)*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{1/2}*(b/(a+b))^{1/2})*((a+b*\cos(d*x+c))/(a+b))^{1/2}/b^5/(a^2-b^2)/d/(a+b*\cos(d*x+c))^{1/2}$

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {2871, 3126, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = -\frac{2a^2 \sin(c+dx) \cos^3(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}} - \frac{8a^2(2a^2-3b^2) \sin(c+dx) \cos^2(c+dx)}{3b^2d(a^2-b^2)^2 \sqrt{a+b\cos(c+dx)}} - \frac{4a(32a^4-49a^2b^2+7b^4) \sin(c+dx) \sqrt{a+b\cos(c+dx)}}{15b^4d(a^2-b^2)^2} - \frac{2a(128a^4-116a^2b^2-17b^4) \sqrt{\frac{a+b\cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{15b^5d(a^2-b^2) \sqrt{a+b\cos(c+dx)}} + \frac{2(48a^4-71a^2b^2+3b^4) \sin(c+dx) \cos(c+dx) \sqrt{a+b\cos(c+dx)}}{15b^3d(a^2-b^2)^2} + \frac{2(128a^6-212a^4b^2+55a^2b^4+9b^6) \sqrt{a+b\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{15b^5d(a^2-b^2)^2 \sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (2*(128*a^6 - 212*a^4*b^2 + 55*a^2*b^4 + 9*b^6)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(128*a^4 - 116*a^2*b^2 - 17*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(15*b^5*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (8*a^2*(2*a^2 - 3*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*(32*a^4 - 49*a^2*b^2 + 7*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^4*(a^2 - b^2)^2*d) + (2*(48*a^4 - 71*a^2*b^2 + 3*b^4)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)^2*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,

0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2871

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

Rule 3102

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 3126

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-c^2*C - B*c*d + A*d^2)*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
) - a*c*(n + 2))] - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))]*Sin[e + f*x
+ b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

Rule 3128

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2a^2 \cos^3(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} \\
&\quad - \frac{2 \int \frac{\cos^2(c+dx)(3a^2 - \frac{3}{2}ab \cos(c+dx) - \frac{1}{2}(8a^2 - 3b^2) \cos^2(c+dx))}{(a+b \cos(c+dx))^{3/2}} dx}{3b(a^2 - b^2)} \\
&= -\frac{2a^2 \cos^3(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{8a^2(2a^2 - 3b^2) \cos^2(c + dx) \sin(c + dx)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{4 \int \frac{\cos(c+dx)(-4a^2(2a^2 - 3b^2) + \frac{1}{2}ab(a^2 - 3b^2) \cos(c+dx) + \frac{1}{4}(48a^4 - 71a^2b^2 + 3b^4) \cos^2(c+dx))}{\sqrt{a+b \cos(c+dx)}} dx}{3b^2(a^2 - b^2)^2} \\
&= -\frac{2a^2 \cos^3(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{8a^2(2a^2 - 3b^2) \cos^2(c + dx) \sin(c + dx)}{3b^2(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{2(48a^4 - 71a^2b^2 + 3b^4) \cos(c + dx) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15b^3(a^2 - b^2)^2 d} \\
&\quad + \frac{8 \int \frac{\frac{1}{4}a(48a^4 - 71a^2b^2 + 3b^4) - \frac{1}{8}b(16a^4 - 27a^2b^2 - 9b^4) \cos(c+dx) - \frac{3}{4}a(32a^4 - 49a^2b^2 + 7b^4) \cos^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx}{15b^3(a^2 - b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2a^2 \cos^3(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2(2a^2-3b^2)\cos^2(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{4a(32a^4-49a^2b^2+7b^4)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^4(a^2-b^2)^2 d} \\
&\quad + \frac{2(48a^4-71a^2b^2+3b^4)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^3(a^2-b^2)^2 d} \\
&\quad + \frac{16 \int \frac{\frac{3}{4}ab(8a^4-11a^2b^2-2b^4) + \frac{3}{16}(128a^6-212a^4b^2+55a^2b^4+9b^6)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{45b^4(a^2-b^2)^2} \\
&= -\frac{2a^2 \cos^3(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2(2a^2-3b^2)\cos^2(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{4a(32a^4-49a^2b^2+7b^4)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^4(a^2-b^2)^2 d} \\
&\quad + \frac{2(48a^4-71a^2b^2+3b^4)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^3(a^2-b^2)^2 d} \\
&\quad - \frac{(a(128a^4-116a^2b^2-17b^4)) \int \frac{1}{\sqrt{a+b\cos(c+dx)}} dx}{15b^5(a^2-b^2)} \\
&\quad + \frac{(128a^6-212a^4b^2+55a^2b^4+9b^6) \int \sqrt{a+b\cos(c+dx)} dx}{15b^5(a^2-b^2)^2} \\
&= -\frac{2a^2 \cos^3(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2(2a^2-3b^2)\cos^2(c+dx)\sin(c+dx)}{3b^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{4a(32a^4-49a^2b^2+7b^4)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^4(a^2-b^2)^2 d} \\
&\quad + \frac{2(48a^4-71a^2b^2+3b^4)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^3(a^2-b^2)^2 d} \\
&\quad + \frac{\left((128a^6-212a^4b^2+55a^2b^4+9b^6)\sqrt{a+b\cos(c+dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}} dx}{15b^5(a^2-b^2)^2 \sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad - \frac{\left(a(128a^4-116a^2b^2-17b^4)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{15b^5(a^2-b^2)\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2(128a^6 - 212a^4b^2 + 55a^2b^4 + 9b^6) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{15b^5 (a^2 - b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad - \frac{2a(128a^4 - 116a^2b^2 - 17b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{15b^5 (a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{2a^2 \cos^3(c + dx) \sin(c + dx)}{3b (a^2 - b^2) d (a + b \cos(c + dx))^{3/2}} - \frac{8a^2 (2a^2 - 3b^2) \cos^2(c + dx) \sin(c + dx)}{3b^2 (a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{4a(32a^4 - 49a^2b^2 + 7b^4) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15b^4 (a^2 - b^2)^2 d} \\
&\quad + \frac{2(48a^4 - 71a^2b^2 + 3b^4) \cos(c + dx) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15b^3 (a^2 - b^2)^2 d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.62

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \frac{2\left(\frac{a+b \cos(c+dx)}{a+b}\right)^{3/2} \left((128a^6 - 212a^4b^2 + 55a^2b^4 + 9b^6) E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right) + a(-128a^5 + 128a^4b + 116a^3b^2 - \dots)\right)}{(a-b)^2}$$

[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^(5/2), x]

[Out] ((2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*((128*a^6 - 212*a^4*b^2 + 55*a^2*b^4 + 9*b^6)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + a*(-128*a^5 + 128*a^4*b + 116*a^3*b^2 - 116*a^2*b^3 + 17*a*b^4 - 17*b^5)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2 + b*((10*a^5*Sin[c + d*x])/(a^2 - b^2) - (10*a^4*(11*a^2 - 15*b^2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 28*a*(a + b*Cos[c + d*x])^2*Sin[c + d*x] + 3*b*(a + b*Cos[c + d*x])^2*Sin[2*(c + d*x)]))/(15*b^5*d*(a + b*Cos[c + d*x])^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1687 vs. 2(466) = 932.

Time = 12.06 (sec) , antiderivative size = 1688, normalized size of antiderivative = 3.87

method	result	size
default	Expression too large to display	1688

[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)

[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16/b^2*(-1/10/b*cos(1/2*d*x+1/2*c)^3*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c

$$\begin{aligned}
& c^2)^{(1/2)} - 1/60/b^2 * (-4*a + 12*b) * \cos(1/2*d*x + 1/2*c) * (-2*\sin(1/2*d*x + 1/2*c))^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} + 1/60/b^2 * (-4*a + 12*b) * (a-b) * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x + 1/2*c)^2 + a-b)/(a-b))^{(1/2)} / (-2*\sin(1/2*d*x + 1/2*c)^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)}) - 1/60 * (4*a^2 - 15*a*b + 27*b^2) / b^3 * (a-b) * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x + 1/2*c)^2 + a-b)/(a-b))^{(1/2)} / (-2*\sin(1/2*d*x + 1/2*c)^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} * (\text{EllipticF}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)}) - \text{EllipticE}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)})) - 8/b^3 * (2*a + 3*b) * (-1/6/b*\cos(1/2*d*x + 1/2*c) * (-2*\sin(1/2*d*x + 1/2*c))^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} + 1/6 * (a-b) / b * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x + 1/2*c)^2 + a-b)/(a-b))^{(1/2)} / (-2*\sin(1/2*d*x + 1/2*c)^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)}) - 1/12/b^2 * (-2*a + 6*b) * (a-b) * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x + 1/2*c)^2 + a-b)/(a-b))^{(1/2)} / (-2*\sin(1/2*d*x + 1/2*c)^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} * (\text{EllipticF}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)}) - \text{EllipticE}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)})) - 10/b^5 * a^4 / \sin(1/2*d*x + 1/2*c)^2 / (2*b*\sin(1/2*d*x + 1/2*c)^2 - a-b) / (a^2 - b^2) * (-2*\sin(1/2*d*x + 1/2*c))^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} * (2*\cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c)^{2*b} + \text{EllipticE}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)}) * (-2*b/(a-b) * \sin(1/2*d*x + 1/2*c)^2 + (a+b)/(a-b))^{(1/2)} * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * b) - 2/b^5 * a^5 * (1/6/b/(a-b)/(a+b) * \cos(1/2*d*x + 1/2*c) * (-2*\sin(1/2*d*x + 1/2*c))^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} / (\cos(1/2*d*x + 1/2*c)^2 + 1/2 * (a-b) / b)^2 + 8/3 * \sin(1/2*d*x + 1/2*c)^{2*b} / (a-b)^2 / (a+b)^2 * \cos(1/2*d*x + 1/2*c) * a / (-(-2*b*\cos(1/2*d*x + 1/2*c)^2 - a+b) * \sin(1/2*d*x + 1/2*c)^2)^{(1/2)} + (3*a-b) / (3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3) * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x + 1/2*c)^2 + a-b)/(a-b))^{(1/2)} / (-2*\sin(1/2*d*x + 1/2*c)^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)}) - 4/3 * a / (a-b) / (a+b)^2 * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x + 1/2*c)^2 + a-b)/(a-b))^{(1/2)} / (-2*\sin(1/2*d*x + 1/2*c)^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} * (\text{EllipticF}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)}) - \text{EllipticE}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)})) - 2/b^5 * (3*a^2 + 4*a*b + 3*b^2) * (a-b) * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x + 1/2*c)^2 + a-b)/(a-b))^{(1/2)} / (-2*\sin(1/2*d*x + 1/2*c)^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} * (\text{EllipticF}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)}) - \text{EllipticE}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)})) - 2 * (4*a^3 + 3*a^2*b + 2*a*b^2 + b^3) / b^5 * (\sin(1/2*d*x + 1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x + 1/2*c)^2 + a-b)/(a-b))^{(1/2)} / (-2*\sin(1/2*d*x + 1/2*c)^{4*b + (a+b)*\sin(1/2*d*x + 1/2*c)^2})^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x + 1/2*c), (-2*b/(a-b))^{(1/2)})) / \sin(1/2*d*x + 1/2*c) / (-2*b*\sin(1/2*d*x + 1/2*c)^2 + a+b)^{(1/2)} / d
\end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.39

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out]
$$-1/45*(6*(64*a^7*b^2 - 98*a^5*b^4 + 14*a^3*b^6 - 3*(a^4*b^5 - 2*a^2*b^7 + b^9)*\cos(d*x + c)^3 + 8*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*\cos(d*x + c)^2 + 5*(16*a^6*b^3 - 25*a^4*b^5 + 5*a^2*b^7)*\cos(d*x + c))*\sqrt{b*\cos(d*x + c) + a}*\sin(d*x + c) + 2*(\sqrt{2})*(-128*I*a^7*b^2 + 260*I*a^5*b^4 - 121*I*a^3*b^6 - 21*I*a*b^8)*\cos(d*x + c)^2 + 2*\sqrt{2})*(-128*I*a^8*b + 260*I*a^6*b^3 - 121*I*a^4*b^5 - 21*I*a^2*b^7)*\cos(d*x + c) + \sqrt{2})*(-128*I*a^9 + 260*I*a^7*b^2 - 121*I*a^5*b^4 - 21*I*a^3*b^6))*\sqrt{b}*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) + 3*I*b*\sin(d*x + c) + 2*a)/b) + 2*(\sqrt{2})*(128*I*a^7*b^2 - 260*I*a^5*b^4 + 121*I*a^3*b^6 + 21*I*a*b^8)*\cos(d*x + c)^2 + 2*\sqrt{2})*(128*I*a^8*b - 260*I*a^6*b^3 + 121*I*a^4*b^5 + 21*I*a^2*b^7)*\cos(d*x + c) + \sqrt{2})*(128*I*a^9 - 260*I*a^7*b^2 + 121*I*a^5*b^4 + 21*I*a^3*b^6))*\sqrt{b}*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) - 3*I*b*\sin(d*x + c) + 2*a)/b) + 3*(\sqrt{2})*(-128*I*a^6*b^3 + 212*I*a^4*b^5 - 55*I*a^2*b^7 - 9*I*b^9)*\cos(d*x + c)^2 + 2*\sqrt{2})*(-128*I*a^7*b^2 + 212*I*a^5*b^4 - 55*I*a^3*b^6 - 9*I*a*b^8)*\cos(d*x + c) + \sqrt{2})*(-128*I*a^8*b + 212*I*a^6*b^3 - 55*I*a^4*b^5 - 9*I*a^2*b^7))*\sqrt{b}*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) + 3*I*b*\sin(d*x + c) + 2*a)/b)) + 3*(\sqrt{2})*(128*I*a^6*b^3 - 212*I*a^4*b^5 + 55*I*a^2*b^7 + 9*I*b^9)*\cos(d*x + c)^2 + 2*\sqrt{2})*(128*I*a^7*b^2 - 212*I*a^5*b^4 + 55*I*a^3*b^6 + 9*I*a*b^8)*\cos(d*x + c) + \sqrt{2})*(128*I*a^8*b - 212*I*a^6*b^3 + 55*I*a^4*b^5 + 9*I*a^2*b^7))*\sqrt{b}*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x + c) - 3*I*b*\sin(d*x + c) + 2*a)/b)))/((a^4*b^8 - 2*a^2*b^10 + b^12)*d*\cos(d*x + c)^2 + 2*(a^5*b^7 - 2*a^3*b^9 + a*b^11)*d*\cos(d*x + c) + (a^6*b^6 - 2*a^4*b^8 + a^2*b^10)*d)$$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(dx + c)^5}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(dx + c)^5}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^5}{(a + b \cos(c + dx))^{5/2}} dx$$

[In] int(cos(c + d*x)^5/(a + b*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^5/(a + b*cos(c + d*x))^(5/2), x)

$$3.539 \quad \int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	5497
Rubi [A] (verified)	5498
Mathematica [A] (verified)	5501
Maple [B] (verified)	5502
Fricas [C] (verification not implemented)	5503
Sympy [F(-1)]	5503
Maxima [F]	5504
Giac [F]	5504
Mupad [F(-1)]	5504

Optimal result

Integrand size = 23, antiderivative size = 345

$$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx =$$

$$\frac{8a(4a^4 - 7a^2b^2 + 2b^4) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3b^4 (a^2 - b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

$$+ \frac{2(16a^4 - 16a^2b^2 - b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^4 (a^2 - b^2) d \sqrt{a+b \cos(c+dx)}}$$

$$- \frac{2a^2 \cos^2(c+dx) \sin(c+dx)}{3b (a^2 - b^2) d (a+b \cos(c+dx))^{3/2}} + \frac{4a^3 (3a^2 - 5b^2) \sin(c+dx)}{3b^3 (a^2 - b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

$$+ \frac{2(2a^2 - b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{3b^3 (a^2 - b^2) d}$$

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[Out] -2/3*a^2*cos(d*x+c)^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+4/3*a^3*(3*a^2-5*b^2)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(2*a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^3/(a^2-b^2)/d-8/3*a*(4*a^4-7*a^2*b^2+2*b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^4/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(16*a^4-16*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^4/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
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Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2871, 3110, 3102, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = -\frac{2a^2 \sin(c+dx) \cos^2(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}} + \frac{2(2a^2-b^2) \sin(c+dx) \sqrt{a+b\cos(c+dx)}}{3b^3d(a^2-b^2)} + \frac{2(16a^4-16a^2b^2-b^4) \sqrt{\frac{a+b\cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^4d(a^2-b^2) \sqrt{a+b\cos(c+dx)}} - \frac{8a(4a^4-7a^2b^2+2b^4) \sqrt{a+b\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3b^4d(a^2-b^2)^2 \sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{4a^3(3a^2-5b^2) \sin(c+dx)}{3b^3d(a^2-b^2)^2 \sqrt{a+b\cos(c+dx)}}$$

[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (-8*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*b^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(16*a^4 - 16*a^2*b^2 - b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (4*a^3*(3*a^2 - 5*b^2)*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2871

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3102

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 3110

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])*(A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m

+ 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))) *Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2a^2 \cos^2(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} \\
&\quad - \frac{2 \int \frac{\cos(c+dx)(2a^2 - \frac{3}{2}ab \cos(c+dx) - \frac{3}{2}(2a^2 - b^2) \cos^2(c+dx))}{(a+b \cos(c+dx))^{3/2}} dx}{3b(a^2 - b^2)} \\
&= -\frac{2a^2 \cos^2(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} + \frac{4a^3(3a^2 - 5b^2) \sin(c + dx)}{3b^3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{4 \int \frac{\frac{1}{2}a^2b(3a^2 - 5b^2) + \frac{1}{2}a(6a^4 - 11a^2b^2 + 3b^4) \cos(c+dx) - \frac{3}{4}b(a^2 - b^2)(2a^2 - b^2) \cos^2(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx}{3b^3(a^2 - b^2)^2} \\
&= -\frac{2a^2 \cos^2(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} + \frac{4a^3(3a^2 - 5b^2) \sin(c + dx)}{3b^3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{2(2a^2 - b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3b^3(a^2 - b^2) d} \\
&\quad - \frac{8 \int \frac{\frac{3}{8}b^2(4a^4 - 7a^2b^2 - b^4) + \frac{3}{2}ab(4a^4 - 7a^2b^2 + 2b^4) \cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx}{9b^4(a^2 - b^2)^2} \\
&= -\frac{2a^2 \cos^2(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} + \frac{4a^3(3a^2 - 5b^2) \sin(c + dx)}{3b^3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{2(2a^2 - b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3b^3(a^2 - b^2) d} \\
&\quad + \frac{(16a^4 - 16a^2b^2 - b^4) \int \frac{1}{\sqrt{a+b \cos(c+dx)}} dx}{3b^4(a^2 - b^2)} \\
&\quad - \frac{(4a(4a^4 - 7a^2b^2 + 2b^4)) \int \sqrt{a + b \cos(c + dx)} dx}{3b^4(a^2 - b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2a^2 \cos^2(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b \cos(c+dx))^{3/2}} + \frac{4a^3(3a^2-5b^2) \sin(c+dx)}{3b^3(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}} \\
&\quad + \frac{2(2a^2-b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{3b^3(a^2-b^2)d} \\
&\quad - \frac{\left(4a(4a^4-7a^2b^2+2b^4) \sqrt{a+b \cos(c+dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{3b^4(a^2-b^2)^2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad + \frac{\left((16a^4-16a^2b^2-b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{3b^4(a^2-b^2) \sqrt{a+b \cos(c+dx)}} \\
&= -\frac{8a(4a^4-7a^2b^2+2b^4) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3b^4(a^2-b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad + \frac{2(16a^4-16a^2b^2-b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^4(a^2-b^2) d \sqrt{a+b \cos(c+dx)}} \\
&\quad - \frac{2a^2 \cos^2(c+dx) \sin(c+dx)}{3b(a^2-b^2)d(a+b \cos(c+dx))^{3/2}} + \frac{4a^3(3a^2-5b^2) \sin(c+dx)}{3b^3(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}} \\
&\quad + \frac{2(2a^2-b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{3b^3(a^2-b^2)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.69

$$\int \frac{\cos^4(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = \frac{2 \left(\left(\frac{a+b \cos(c+dx)}{a+b} \right)^{3/2} \left(-4(4a^5-7a^3b^2+2ab^4) E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right) + (16a^5-16a^4b-16a^3b^2+16a^2b^3-ab^4) \right) \right)}{(a-b)^2}$$

[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (2*(((a + b*Cos[c + d*x])/(a + b))^(3/2)*(-4*(4*a^5 - 7*a^3*b^2 + 2*a*b^4)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (16*a^5 - 16*a^4*b - 16*a^3*b^2 + 16*a^2*b^3 - a*b^4 + b^5)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2 + (b*(16*a^6 - 25*a^4*b^2 + b^6 + 4*a*b*(5*a^4 - 8*a^2*b^2 + b^4)*Cos[c + d*x] + (-a^2*b + b^3)^2*Cos[2*(c + d*x)]*Sin[c + d*x])/(2*(a^2 - b^2)^2))/(3*b^4*d*(a + b*Cos[c + d*x])^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. $2(379) = 758$.

Time = 10.29 (sec) , antiderivative size = 1295, normalized size of antiderivative = 3.75

method	result	size
default	Expression too large to display	1295

[In] `int(cos(d*x+c)^4/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -(-(-2*b*\cos(1/2*d*x+1/2*c)^2-a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(8/b^2*(-1/6 \\ & /b*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2 \\ &)^{(1/2)}+1/6*(a-b)/b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2 \\ & +a-b)/(a-b))^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & *EllipticF(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})-1/12/b^2*(-2*a+6*b)*(\\ & a-b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)} \\ & /(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(EllipticF \\ & (\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})-EllipticE(\cos(1/2*d*x+1/2*c),(-2*b/ \\ & (a-b))^{(1/2)})))+4*(a+b)/b^4*(a-b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/ \\ & 2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d \\ & *x+1/2*c)^2)^{(1/2)}*(EllipticF(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})-Ellipti \\ & cE(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)}))+2/b^4*a^4*(1/6/b/(a-b)/(a+b)*co \\ & s(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & /(\cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*\sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/ \\ & (a+b)^2*\cos(1/2*d*x+1/2*c)*a/(-(-2*b*\cos(1/2*d*x+1/2*c)^2-a+b)*\sin(1/2*d*x+ \\ & 1/2*c)^2)^{(1/2)}+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(\sin(1/2*d*x+1/2*c)^2 \\ &)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c) \\ & ^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),(-2*b/(\\ & a-b))^{(1/2)})-4/3*a/(a-b)/(a+b)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2 \\ & *d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d* \\ & x+1/2*c)^2)^{(1/2)}*(EllipticF(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})-Ellipti \\ & cE(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)}))+2*(3*a^2+2*a*b+b^2)/b^4*(\sin(1/ \\ & 2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}/(-2*\sin \\ & (1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+ \\ & 1/2*c),(-2*b/(a-b))^{(1/2)})+8*a^3/b^4/\sin(1/2*d*x+1/2*c)^2/(2*b*\sin(1/2*d*x+ \\ & 1/2*c)^2-a-b)/(a^2-b^2)*(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c) \\ & ^2)^{(1/2)}*(2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*b+EllipticE(\cos(1/2*d* \\ & x+1/2*c),(-2*b/(a-b))^{(1/2)}))*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)} \\ & *(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a-EllipticE(\cos(1/2*d*x+1/2*c),(-2*b/(a- \\ & b))^{(1/2)}))*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*(\sin(1/2*d*x \\ & +1/2*c)^2)^{(1/2)}*b)/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/2)}/d \end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 954, normalized size of antiderivative = 2.77

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \text{Too large to display}$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/9*(6*(8*a^6*b^2 - 13*a^4*b^4 + a^2*b^6 + (a^4*b^4 - 2*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(5*a^5*b^3 - 8*a^3*b^5 + a*b^7)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c) + (sqrt(2)*(-32*I*a^6*b^2 + 68*I*a^4*b^4 - 37*I*a^2*b^6 - 3*I*b^8)*cos(d*x + c)^2 - 2*sqrt(2)*(32*I*a^7*b - 68*I*a^5*b^3 + 37*I*a^3*b^5 + 3*I*a*b^7)*cos(d*x + c) + sqrt(2)*(-32*I*a^8 + 68*I*a^6*b^2 - 37*I*a^4*b^4 - 3*I*a^2*b^6))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (sqrt(2)*(32*I*a^6*b^2 - 68*I*a^4*b^4 + 37*I*a^2*b^6 + 3*I*b^8)*cos(d*x + c)^2 - 2*sqrt(2)*(-32*I*a^7*b + 68*I*a^5*b^3 - 37*I*a^3*b^5 - 3*I*a*b^7)*cos(d*x + c) + sqrt(2)*(32*I*a^8 - 68*I*a^6*b^2 + 37*I*a^4*b^4 + 3*I*a^2*b^6))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 12*(sqrt(2)*(4*I*a^5*b^3 - 7*I*a^3*b^5 + 2*I*a*b^7)*cos(d*x + c)^2 + 2*sqrt(2)*(4*I*a^6*b^2 - 7*I*a^4*b^4 + 2*I*a^2*b^6)*cos(d*x + c) + sqrt(2)*(4*I*a^7*b - 7*I*a^5*b^3 + 2*I*a^3*b^5))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b)) - 12*(sqrt(2)*(-4*I*a^5*b^3 + 7*I*a^3*b^5 - 2*I*a*b^7)*cos(d*x + c)^2 + 2*sqrt(2)*(-4*I*a^6*b^2 + 7*I*a^4*b^4 - 2*I*a^2*b^6)*cos(d*x + c) + sqrt(2)*(-4*I*a^7*b + 7*I*a^5*b^3 - 2*I*a^3*b^5))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^4*b^7 - 2*a^2*b^9 + b^11)*d*cos(d*x + c)^2 + 2*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 2*a^4*b^7 + a^2*b^9)*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(dx + c)^4}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(dx + c)^4}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^4}{(a + b \cos(c + dx))^{5/2}} dx$$

[In] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^4/(a + b*cos(c + d*x))^(5/2), x)

$$3.540 \quad \int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	5505
Rubi [A] (verified)	5506
Mathematica [A] (verified)	5508
Maple [B] (verified)	5509
Fricas [C] (verification not implemented)	5509
Sympy [F(-1)]	5510
Maxima [F]	5510
Giac [F]	5511
Mupad [F(-1)]	5511

Optimal result

Integrand size = 23, antiderivative size = 281

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = \frac{2(8a^4 - 15a^2b^2 + 3b^4) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3b^3 (a^2 - b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

$$- \frac{2a(8a^2 - 9b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^3 (a^2 - b^2) d \sqrt{a+b \cos(c+dx)}}$$

$$- \frac{2a^2 \cos(c+dx) \sin(c+dx)}{3b (a^2 - b^2) d (a+b \cos(c+dx))^{3/2}} - \frac{8a^2 (a^2 - 2b^2) \sin(c+dx)}{3b^2 (a^2 - b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] -2/3*a^2*cos(d*x+c)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a^2
*(a^2-2*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+2/3*(8*a^4
-15*a^2*b^2+3*b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^3/(a
^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*a*(8*a^2-9*b^2)*(cos(1/2*d*x
+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b
/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^3/(a^2-b^2)/d/(a+b*cos(d*x+
c))^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2871, 3100, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = -\frac{8a^2(a^2-2b^2)\sin(c+dx)}{3b^2d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}} - \frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}} - \frac{2a(8a^2-9b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^3d(a^2-b^2)\sqrt{a+b\cos(c+dx)}} + \frac{2(8a^4-15a^2b^2+3b^4)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3b^3d(a^2-b^2)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(8*a^2 - 9*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (8*a^2*(a^2 - 2*b^2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*SIN[c + d*x]]/Sqrt[(a + b*SIN[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2871

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
+ a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-A*b^2
- a*b*B + a^2*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2a^2 \cos(c + dx) \sin(c + dx)}{3b(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{2 \int \frac{a^2 - \frac{3}{2}ab \cos(c + dx) - \frac{1}{2}(4a^2 - 3b^2) \cos^2(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx}{3b(a^2 - b^2)}$$

$$\begin{aligned}
&= -\frac{2a^2 \cos(c+dx) \sin(c+dx)}{3b(a^2-b^2) d(a+b \cos(c+dx))^{3/2}} - \frac{8a^2(a^2-2b^2) \sin(c+dx)}{3b^2(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}} \\
&\quad + \frac{4 \int \frac{\frac{1}{2}ab(a^2-3b^2) + \frac{1}{4}(8a^4-15a^2b^2+3b^4) \cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx}{3b^2(a^2-b^2)^2} \\
&= -\frac{2a^2 \cos(c+dx) \sin(c+dx)}{3b(a^2-b^2) d(a+b \cos(c+dx))^{3/2}} - \frac{8a^2(a^2-2b^2) \sin(c+dx)}{3b^2(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}} \\
&\quad - \frac{(a(8a^2-9b^2)) \int \frac{1}{\sqrt{a+b \cos(c+dx)}} dx}{3b^3(a^2-b^2)} + \frac{(8a^4-15a^2b^2+3b^4) \int \sqrt{a+b \cos(c+dx)} dx}{3b^3(a^2-b^2)^2} \\
&= -\frac{2a^2 \cos(c+dx) \sin(c+dx)}{3b(a^2-b^2) d(a+b \cos(c+dx))^{3/2}} - \frac{8a^2(a^2-2b^2) \sin(c+dx)}{3b^2(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}} \\
&\quad + \frac{\left((8a^4-15a^2b^2+3b^4) \sqrt{a+b \cos(c+dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{3b^3(a^2-b^2)^2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad - \frac{\left(a(8a^2-9b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{3b^3(a^2-b^2) \sqrt{a+b \cos(c+dx)}} \\
&= \frac{2(8a^4-15a^2b^2+3b^4) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3b^3(a^2-b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&\quad - \frac{2a(8a^2-9b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^3(a^2-b^2) d \sqrt{a+b \cos(c+dx)}} \\
&\quad - \frac{2a^2 \cos(c+dx) \sin(c+dx)}{3b(a^2-b^2) d(a+b \cos(c+dx))^{3/2}} - \frac{8a^2(a^2-2b^2) \sin(c+dx)}{3b^2(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.67

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = \frac{2 \left(\frac{(a+b \cos(c+dx))^{3/2}}{a+b} \left((8a^4-15a^2b^2+3b^4) E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right) + a(-8a^3+8a^2b+9ab^2-9b^3) \text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right) \right) \right)}{(a-b)^2} + \frac{2a^2 \cos(c+dx) \sin(c+dx)}{3b^3 d(a+b \cos(c+dx))^{3/2}}$$

[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (2*(((a + b*Cos[c + d*x])/(a + b))^(3/2)*((8*a^4 - 15*a^2*b^2 + 3*b^4)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + a*(-8*a^3 + 8*a^2*b + 9*a*b^2 - 9*b^3)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2 + (a^2*b*(-4*a^3 + 8*a*b^2 + (-5*a^2*b + 9*b^3)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2)/(3*b^3*d*(a + b*Cos[c + d*x])^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(319) = 638$.

Time = 9.85 (sec) , antiderivative size = 911, normalized size of antiderivative = 3.24

method	result	size
default	Expression too large to display	911

[In] `int(cos(d*x+c)^3/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$-(-(-2*b*\cos(1/2*d*x+1/2*c)^2-a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2/b^3/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(3*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a-\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*a+\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*b)-6/b^3*a^2/\sin(1/2*d*x+1/2*c)^2/(2*b*\sin(1/2*d*x+1/2*c)^2-a-b)/(a^2-b^2)*(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*b+\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*a-\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})*(-2*b/(a-b)*\sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*b)-2/b^3*a^3*(1/6/b/(a-b)/(a+b)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*\sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*\cos(1/2*d*x+1/2*c)*a/(-(-2*b*\cos(1/2*d*x+1/2*c)^2-a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})-4/3*a/(a-b)/(a+b)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})-\text{EllipticE}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 867, normalized size of antiderivative = 3.09

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \text{Too large to display}$$

[In] `integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out]
$$-1/9*(6*(4*a^5*b^2 - 8*a^3*b^4 + (5*a^4*b^3 - 9*a^2*b^5)*\cos(d*x + c))*\sqrt{b*\cos(d*x + c) + a}*\sin(d*x + c) + 4*(\sqrt{2})*(-4*I*a^5*b^2 + 9*I*a^3*b^4$$

```

- 6*I*a*b^6)*cos(d*x + c)^2 + 2*sqrt(2)*(-4*I*a^6*b + 9*I*a^4*b^3 - 6*I*a^2
*b^5)*cos(d*x + c) + sqrt(2)*(-4*I*a^7 + 9*I*a^5*b^2 - 6*I*a^3*b^4))*sqrt(b
)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
  1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + 4*(sqrt(2)*(4*I*a^5
*b^2 - 9*I*a^3*b^4 + 6*I*a*b^6)*cos(d*x + c)^2 + 2*sqrt(2)*(4*I*a^6*b - 9*I
*a^4*b^3 + 6*I*a^2*b^5)*cos(d*x + c) + sqrt(2)*(4*I*a^7 - 9*I*a^5*b^2 + 6*I
*a^3*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^
3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) + 3*
(sqrt(2)*(-8*I*a^4*b^3 + 15*I*a^2*b^5 - 3*I*b^7)*cos(d*x + c)^2 + 2*sqrt(2)
*(-8*I*a^5*b^2 + 15*I*a^3*b^4 - 3*I*a*b^6)*cos(d*x + c) + sqrt(2)*(-8*I*a^6
*b + 15*I*a^4*b^3 - 3*I*a^2*b^5))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^
2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x +
c) + 2*a)/b)) + 3*(sqrt(2)*(8*I*a^4*b^3 - 15*I*a^2*b^5 + 3*I*b^7)*cos(d*x
+ c)^2 + 2*sqrt(2)*(8*I*a^5*b^2 - 15*I*a^3*b^4 + 3*I*a*b^6)*cos(d*x + c) +
sqrt(2)*(8*I*a^6*b - 15*I*a^4*b^3 + 3*I*a^2*b^5))*sqrt(b)*weierstrassZeta(4
/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) -
3*I*b*sin(d*x + c) + 2*a)/b))) / ((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c
)^2 + 2*(a^5*b^5 - 2*a^3*b^7 + a*b^9)*d*cos(d*x + c) + (a^6*b^4 - 2*a^4*b^6
+ a^2*b^8)*d)

```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(dx + c)^3}{(b \cos(dx + c) + a)^{5/2}} dx$$

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(dx + c)^3}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^3}{(a + b \cos(c + dx))^{5/2}} dx$$

[In] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^3/(a + b*cos(c + d*x))^(5/2), x)

3.541 $\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$

Optimal result	5512
Rubi [A] (verified)	5512
Mathematica [A] (verified)	5515
Maple [B] (verified)	5515
Fricas [C] (verification not implemented)	5516
Sympy [F(-1)]	5517
Maxima [F]	5517
Giac [F]	5517
Mupad [F(-1)]	5518

Optimal result

Integrand size = 23, antiderivative size = 263

$$\int \frac{\cos^2(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = -\frac{4a(a^2-3b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid \frac{2b}{a+b}\right)}{3b^2(a^2-b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2(2a^2-3b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^2(a^2-b^2) d \sqrt{a+b \cos(c+dx)}} - \frac{2a^2 \sin(c+dx)}{3b(a^2-b^2) d (a+b \cos(c+dx))^{3/2}} + \frac{4a(a^2-3b^2) \sin(c+dx)}{3b(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] -2/3*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+4/3*a*(a^2-3*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-4/3*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*(2*a^2-3*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/b^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {2869, 2833, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = -\frac{2a^2 \sin(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}} + \frac{4a(a^2-3b^2)\sin(c+dx)}{3bd(a^2-b^2)^2 \sqrt{a+b\cos(c+dx)}} + \frac{2(2a^2-3b^2) \sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^2d(a^2-b^2) \sqrt{a+b\cos(c+dx)}} - \frac{4a(a^2-3b^2) \sqrt{a+b\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3b^2d(a^2-b^2)^2 \sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (-4*a*(a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2 - 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*b^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (4*a*(a^2 - 3*b^2)*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -

$b^2, 0] \&\& !\text{GtQ}[a + b, 0]$

Rule 2831

$\text{Int}[\frac{(c_.) + (d_.)\sin[e_.] + (f_.)x}{\sqrt{a_. + (b_.)\sin[e_.] + (f_.)x}}, x_Symbol] \rightarrow \text{Dist}[(b*c - a*d)/b, \text{Int}[1/\sqrt{a + b*\sin[e + f*x]}, x], x] + \text{Dist}[d/b, \text{Int}[\sqrt{a + b*\sin[e + f*x]}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2833

$\text{Int}[(a_.) + (b_.)\sin[e_.] + (f_.)x]^m * ((c_.) + (d_.)\sin[e_.] + (f_.)x), x_Symbol] \rightarrow \text{Simp}[(-b*c - a*d)*\text{Cos}[e + f*x] * (a + b*\sin[e + f*x])^{m+1} / (f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/((m+1)*(a^2 - b^2)), \text{Int}[(a + b*\sin[e + f*x])^{m+1} * \text{Simp}[a*c - b*d*(m+1) - (b*c - a*d)*(m+2)*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m]$

Rule 2869

$\text{Int}[(a_.) + (b_.)\sin[e_.] + (f_.)x]^2 * ((c_.) + (d_.)\sin[e_.] + (f_.)x)^2, x_Symbol] \rightarrow \text{Simp}[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\text{Cos}[e + f*x] * (a + b*\sin[e + f*x])^{m+1} / (b*f*(m+1)*(a^2 - b^2)), x] - \text{Dist}[1/(b*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\sin[e + f*x])^{m+1} * \text{Simp}[b*(m+1)*(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m+2) + b^2*(d^2*(m+1) + c^2*(m+2)))*\sin[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2a^2 \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^{3/2}} + \frac{2 \int \frac{\frac{3ab}{2} + \frac{1}{2}(2a^2 - 3b^2) \cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx}{3b(a^2 - b^2)} \\ &= -\frac{2a^2 \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^{3/2}} + \frac{4a(a^2 - 3b^2) \sin(c + dx)}{3b(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\ &\quad - \frac{4 \int \frac{-\frac{1}{4}b(a^2 + 3b^2) + \frac{1}{2}a(a^2 - 3b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{3b(a^2 - b^2)^2} \\ &= -\frac{2a^2 \sin(c + dx)}{3b(a^2 - b^2)d(a + b \cos(c + dx))^{3/2}} + \frac{4a(a^2 - 3b^2) \sin(c + dx)}{3b(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\ &\quad - \frac{(2a(a^2 - 3b^2)) \int \sqrt{a + b \cos(c + dx)} dx}{3b^2(a^2 - b^2)^2} + \frac{(2a^2 - 3b^2) \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{3b^2(a^2 - b^2)} \end{aligned}$$

$$\begin{aligned}
&= -\frac{2a^2 \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{4a(a^2-3b^2)\sin(c+dx)}{3b(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{\left(2a(a^2-3b^2)\sqrt{a+b\cos(c+dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}} dx}{3b^2(a^2-b^2)^2 \sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad + \frac{\left((2a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{3b^2(a^2-b^2)\sqrt{a+b\cos(c+dx)}} \\
&= -\frac{4a(a^2-3b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\left|\frac{2b}{a+b}\right.\right)}{3b^2(a^2-b^2)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad + \frac{2(2a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b^2(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{2a^2 \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{4a(a^2-3b^2)\sin(c+dx)}{3b(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = \frac{2\left(-\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(2(a^3-3ab^2)E\left(\frac{1}{2}(c+dx)\left|\frac{2b}{a+b}\right.\right)+(-2a^3+2a^2b+3ab^2-3b^3)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^2}\right)}{3b^2d(a+b\cos(c+dx))^{3/2}}$$

[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (2*(-(((a + b*Cos[c + d*x])/(a + b))^(3/2)*(2*(a^3 - 3*a*b^2)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (-2*a^3 + 2*a^2*b + 3*a*b^2 - 3*b^3)*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/(a - b)^2 + (a*b*(a^3 - 5*a*b^2 + 2*b*(a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2)/(3*b^2*d*(a + b*Cos[c + d*x])^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 849 vs. 2(301) = 602.

Time = 8.48 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.23

method	result	size
default	Expression too large to display	850

[In] `int(cos(d*x+c)^2/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$-\left(-2b\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a+b\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(\frac{2}{b^2}\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(\frac{2b\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a-b}{(a-b)}\right)^{1/2}\right)/\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4b+(a+b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\text{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),\left(-2b/(a-b)\right)^{1/2}\right)+2a^2/b^2\left(1/6/b/(a-b)/(a+b)\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4b+(a+b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}/\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1/2(a-b)/b\right)^2+8/3\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2b/(a-b)^2/(a+b)^2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)a/\left(-2b\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a+b\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}+(3a-b)/(3a^3+3a^2b-3ab^2-3b^3)\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(\frac{2b\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a-b}{(a-b)}\right)^{1/2}/\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4b+(a+b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\text{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),\left(-2b/(a-b)\right)^{1/2}\right)-4/3a/(a-b)/(a+b)^2\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(\frac{2b\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a-b}{(a-b)}\right)^{1/2}/\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4b+(a+b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(\text{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),\left(-2b/(a-b)\right)^{1/2}\right)-\text{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),\left(-2b/(a-b)\right)^{1/2}\right)\right)+4a/b^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2/(2b\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b)/(a^2-b^2)\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4b+(a+b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2b+\text{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),\left(-2b/(a-b)\right)^{1/2}\right)\right)\left(-2b/(a-b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+(a+b)/(a-b)\right)^{1/2}\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}a-\text{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),\left(-2b/(a-b)\right)^{1/2}\right)\left(-2b/(a-b)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+(a+b)/(a-b)\right)^{1/2}\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}b)/\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)/\left(-2b\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)^{1/2}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 814, normalized size of antiderivative = 3.10

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \text{Too large to display}$$

[In] `integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{9}\left(6(a^4b^2 - 5a^2b^4 + 2(a^3b^3 - 3ab^5))\cos(dx + c)\sqrt{b\cos(dx + c) + a}\sin(dx + c) + (\sqrt{2})(-4Ia^4b^2 + 9Ia^2b^4 - 9Ib^6)\cos(dx + c)^2 - 2\sqrt{2}(4Ia^5b - 9Ia^3b^3 + 9Iab^5)\cos(dx + c) + \sqrt{2}(-4Ia^6 + 9Ia^4b^2 - 9Ia^2b^4)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4}{3}(4a^2 - 3b^2)/b^2, -8/27(8a^3 - 9ab^2)/b^3, 1/3(3b\cos(dx + c) + 3Ib\sin(dx + c) + 2a)/b) + (\sqrt{2})(4Ia^4b^2 - 9Ia^2b^4 + 9Ib^6)\cos(dx + c)^2 - 2\sqrt{2}(-4Ia^5b + 9Ia^3b^3 - 9Iab^5)\cos(dx + c) + \sqrt{2}(4Ia^6 - 9Ia^4b^2 + 9Ia^2b^4)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4}{3}(4a^2 - 3b^2)/b^2, -8/27(8a^3 - 9ab^2)/b^3, 1/3(3b\cos(dx + c) - 3Ib\sin(dx + c) + 2a)/b) - 6(\sqrt{2})(Ia^3b^3 - 3Iab^5)\cos(dx + c)^2 + 2\sqrt{2}(Ia^4b^2 - 3Ia^2b^4)\cos(dx + c)\right)\right)$$

$x + c) + \sqrt{2}*(I*a^5*b - 3*I*a^3*b^3))*\sqrt{b}*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(dx + c) + 3*I*b*\sin(dx + c) + 2*a)/b)) - 6*(\sqrt{2}*(-I*a^3*b^3 + 3*I*a*b^5)*\cos(dx + c)^2 + 2*\sqrt{2}*(-I*a^4*b^2 + 3*I*a^2*b^4)*\cos(dx + c) + \sqrt{2}*(-I*a^5*b + 3*I*a^3*b^3))*\sqrt{b}*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*\cos(dx + c) - 3*I*b*\sin(dx + c) + 2*a)/b)))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*\cos(dx + c)^2 + 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*\cos(dx + c) + (a^6*b^3 - 2*a^4*b^5 + a^2*b^7)*d)$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(dx+c)**2/(a+b*cos(dx+c))**(5/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(dx + c)^2}{(b \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(dx+c)^2/(a+b*cos(dx+c))^(5/2), x, algorithm="maxima")

[Out] integrate(cos(dx + c)^2/(b*cos(dx + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(dx + c)^2}{(b \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(dx+c)^2/(a+b*cos(dx+c))^(5/2), x, algorithm="giac")

[Out] integrate(cos(dx + c)^2/(b*cos(dx + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^2}{(a + b \cos(c + dx))^{5/2}} dx$$

```
[In] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int(cos(c + d*x)^2/(a + b*cos(c + d*x))^(5/2), x)
```

$$3.542 \quad \int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	5519
Rubi [A] (verified)	5519
Mathematica [A] (verified)	5522
Maple [B] (verified)	5522
Fricas [C] (verification not implemented)	5523
Sympy [F]	5524
Maxima [F]	5524
Giac [F]	5524
Mupad [F(-1)]	5524

Optimal result

Integrand size = 21, antiderivative size = 243

$$\int \frac{\cos(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = -\frac{2(a^2+3b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3b(a^2-b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\ + \frac{2a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3b(a^2-b^2) d \sqrt{a+b \cos(c+dx)}} \\ + \frac{2a \sin(c+dx)}{3(a^2-b^2) d (a+b \cos(c+dx))^{3/2}} + \frac{2(a^2+3b^2) \sin(c+dx)}{3(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*(a^2+3*b^2)*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-2/3*(a^2+3*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b)))^(1/2)*(a+b*cos(d*x+c))^(1/2)/b/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*a*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b)))^(1/2)*((a+b*cos(d*x+c))/(a+b))^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {2833, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = \frac{2(a^2+3b^2)\sin(c+dx)}{3d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}} + \frac{2a\sin(c+dx)}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}} + \frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3bd(a^2-b^2)\sqrt{a+b\cos(c+dx)}} - \frac{2(a^2+3b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3bd(a^2-b^2)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (-2*(a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2 + 3*b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{2 \int \frac{\frac{3b}{2} - \frac{1}{2} a \cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx}{3(a^2 - b^2)} \\
&= \frac{2a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} \\
&\quad + \frac{2(a^2 + 3b^2) \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} + \frac{4 \int \frac{-ab - \frac{1}{4}(a^2 + 3b^2) \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{3(a^2 - b^2)^2} \\
&= \frac{2a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} + \frac{2(a^2 + 3b^2) \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{a \int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{3b(a^2 - b^2)} - \frac{(a^2 + 3b^2) \int \sqrt{a + b \cos(c + dx)} dx}{3b(a^2 - b^2)^2} \\
&= \frac{2a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} + \frac{2(a^2 + 3b^2) \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{\left((a^2 + 3b^2) \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}} dx}{3b(a^2 - b^2)^2 \sqrt{\frac{a + b \cos(c + dx)}{a+b}}} \\
&\quad + \frac{\left(a \sqrt{\frac{a + b \cos(c + dx)}{a+b}} \right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c + dx)}{a+b}}} dx}{3b(a^2 - b^2) \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

$$= -\frac{2(a^2 + 3b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3b(a^2 - b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{2a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3b(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{2a \sin(c + dx)}{3(a^2 - b^2) d (a + b \cos(c + dx))^{3/2}} + \frac{2(a^2 + 3b^2) \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.63

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \frac{2 \left(-\frac{\left(\frac{a+b \cos(c+dx)}{a+b}\right)^{3/2} \left((a^2+3b^2) E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right) + a(-a+b) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right) \right)}{(a-b)^2 b} \right) + \frac{2a(a^2+3b^2) \sin(c+dx)}{3(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}}}{3d(a + b \cos(c + dx))^{3/2}}$$

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^(5/2), x]
```

```
[Out] (2*(-(((a + b*Cos[c + d*x])/(a + b))^(3/2)*((a^2 + 3*b^2)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + a*(-a + b)*EllipticF[(c + d*x)/2, (2*b)/(a + b)])))/((a - b)^2*b)) + ((2*a*(a^2 + b^2) + b*(a^2 + 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2)/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(281) = 562.

Time = 7.80 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.07

method	result
default	$-\frac{\sqrt{-(-2b \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - a + b) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{2 \sqrt{-2 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b + (a+b) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b + E\left(\frac{1}{2}(c + dx) \middle \frac{2b}{a+b}\right) \right)$

```
[In] int(cos(d*x+c)/(a+cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b/sin(1/2*d*x+1/2*c)^2/(2*b*sin(1/2*d*x+1/2*c)^2-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)
```

```

+1/2*c)^2*b+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b-2/b*a*(1/6/b/(a-b)/(a+b
))*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*sin(1/2*d*x+1/2*c)^2*b/(a-b
)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos
(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*b*s
in(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 765, normalized size of antiderivative = 3.15

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = \frac{6(2a^3b^2+2ab^4+(a^2b^3+3b^5)\cos(dx+c))\sqrt{b\cos(dx+c)+a\sin(dx+c)}}{(a+b\cos(c+dx))^{5/2}}$$

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```

[Out] 1/9*(6*(2*a^3*b^2+2*a*b^4+(a^2*b^3+3*b^5)*cos(d*x+c))*sqrt(b*cos(d*
x+c)+a)*sin(d*x+c)-2*(sqrt(2)*(I*a^3*b^2-3*I*a*b^4)*cos(d*x+c)^
2+2*sqrt(2)*(I*a^4*b-3*I*a^2*b^3)*cos(d*x+c)+sqrt(2)*(I*a^5-3*I*a
^3*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2-3*b^2)/b^2,-8/27*(8*a^3
-9*a*b^2)/b^3,1/3*(3*b*cos(d*x+c)+3*I*b*sin(d*x+c)+2*a)/b)-2*(s
qrt(2)*(-I*a^3*b^2+3*I*a*b^4)*cos(d*x+c)^2+2*sqrt(2)*(-I*a^4*b+3*I
a^2*b^3)*cos(d*x+c)+sqrt(2)*(-I*a^5+3*I*a^3*b^2))*sqrt(b)*weierstrass
PInverse(4/3*(4*a^2-3*b^2)/b^2,-8/27*(8*a^3-9*a*b^2)/b^3,1/3*(3*b*cos
(d*x+c)-3*I*b*sin(d*x+c)+2*a)/b)-3*(sqrt(2)*(I*a^2*b^3+3*I*b^5)
*cos(d*x+c)^2+2*sqrt(2)*(I*a^3*b^2+3*I*a*b^4)*cos(d*x+c)+sqrt(2)*
(I*a^4*b+3*I*a^2*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2-3*b^2)/b^2,-
8/27*(8*a^3-9*a*b^2)/b^3,weierstrassPInverse(4/3*(4*a^2-3*b^2)/b^2,-8
/27*(8*a^3-9*a*b^2)/b^3,1/3*(3*b*cos(d*x+c)+3*I*b*sin(d*x+c)+2*a
)/b))-3*(sqrt(2)*(-I*a^2*b^3-3*I*b^5)*cos(d*x+c)^2+2*sqrt(2)*(-I*a^
3*b^2-3*I*a*b^4)*cos(d*x+c)+sqrt(2)*(-I*a^4*b-3*I*a^2*b^3))*sqrt(b)
*weierstrassZeta(4/3*(4*a^2-3*b^2)/b^2,-8/27*(8*a^3-9*a*b^2)/b^3,weie
rstrassPInverse(4/3*(4*a^2-3*b^2)/b^2,-8/27*(8*a^3-9*a*b^2)/b^3,1/3*(
3*b*cos(d*x+c)-3*I*b*sin(d*x+c)+2*a)/b)))/((a^4*b^4-2*a^2*b^6+b

```

$^8)*d*\cos(d*x + c)^2 + 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*\cos(d*x + c) + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*d)$

Sympy [F]

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)

[Out] Integral(cos(c + d*x)/(a + b*cos(c + d*x))**(5/2), x)

Maxima [F]

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(dx + c)}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(dx + c)}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

[In] int(cos(c + d*x)/(a + b*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)/(a + b*cos(c + d*x))^(5/2), x)

$$3.543 \quad \int \frac{1}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	5525
Rubi [A] (verified)	5525
Mathematica [A] (verified)	5528
Maple [A] (verified)	5528
Fricas [C] (verification not implemented)	5529
Sympy [F]	5530
Maxima [F]	5530
Giac [F]	5530
Mupad [F(-1)]	5530

Optimal result

Integrand size = 14, antiderivative size = 221

$$\int \frac{1}{(a+b \cos(c+dx))^{5/2}} dx = \frac{8a\sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3(a^2-b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3(a^2-b^2) d \sqrt{a+b \cos(c+dx)}} - \frac{2b \sin(c+dx)}{3(a^2-b^2) d (a+b \cos(c+dx))^{3/2}} - \frac{8ab \sin(c+dx)}{3(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] -2/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a*b*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+8/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

= {2743, 2833, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx = -\frac{8ab \sin(c + dx)}{3d(a^2 - b^2)^2 \sqrt{a + b \cos(c + dx)}} - \frac{2b \sin(c + dx)}{3d(a^2 - b^2)(a + b \cos(c + dx))^{3/2}} - \frac{2\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3d(a^2 - b^2) \sqrt{a + b \cos(c + dx)}} + \frac{8a\sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3d(a^2 - b^2)^2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[In] Int[(a + b*Cos[c + d*x])^(-5/2), x]

[Out] (8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (8*a*b*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2743

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{2 \int \frac{-\frac{3a}{2} + \frac{1}{2} b \cos(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx}{3(a^2 - b^2)} \\
&= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} \\
&\quad - \frac{8ab \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} + \frac{4 \int \frac{\frac{1}{4}(3a^2 + b^2) + ab \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{3(a^2 - b^2)^2} \\
&= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{8ab \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{(4a) \int \sqrt{a + b \cos(c + dx)} dx}{3(a^2 - b^2)^2} - \frac{\int \frac{1}{\sqrt{a + b \cos(c + dx)}} dx}{3(a^2 - b^2)}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{8ab \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{\left(4a \sqrt{a + b \cos(c + dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{3(a^2 - b^2)^2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
&- \frac{\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{3(a^2 - b^2) \sqrt{a + b \cos(c + dx)}} \\
&= \frac{8a \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3(a^2 - b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} \\
&- \frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} - \frac{8ab \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx = \frac{8a(a + b)^2 \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{3/2} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right) - 2(a - b)(a + b)^2 \left(\frac{a+b \cos(c+dx)}{a+b}\right)}{3(a - b)^2 (a + b)^2 d(a + b \cos(c + dx))^{3/2}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(-5/2), x]

[Out] (8*a*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*(a - b)*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cos[c + d*x])*Sin[c + d*x]/(3*(a - b)^2*(a + b)^2*d*(a + b*Cos[c + d*x])^(3/2))

Maple [A] (verified)

Time = 5.96 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.21

method	result
default	$ -\frac{\sqrt{-(-2b(\cos^2(\frac{dx}{2} + \frac{c}{2})) - a + b)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \left(\frac{\cos(\frac{dx}{2} + \frac{c}{2}) \sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2}))b + (a+b)(\sin^2(\frac{dx}{2} + \frac{c}{2}))}}{3b(a-b)(a+b)(\cos^2(\frac{dx}{2} + \frac{c}{2}) + \frac{a-b}{2b})^2} + \frac{16(\sin^2(\frac{dx}{2} + \frac{c}{2}))}{3(a-b)^2(a+b)^2 \sqrt{-(-2b(\cos^2(\frac{dx}{2} + \frac{c}{2})) - a + b)(\sin^2(\frac{dx}{2} + \frac{c}{2}))}} \right)}{3(a-b)^2(a+b)^2 \sqrt{-(-2b(\cos^2(\frac{dx}{2} + \frac{c}{2})) - a + b)(\sin^2(\frac{dx}{2} + \frac{c}{2}))}} $

[In] int(1/(a+cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/3/b/(a-b)
/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+16/3*sin(1/2*d*x+1/2*c)^2*
b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-8/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/
(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 692, normalized size of antiderivative = 3.13

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx =$$

$$\frac{6(4ab^3 \cos(dx + c) + 5a^2b^2 - b^4)\sqrt{b \cos(dx + c) + a \sin(dx + c)} - (\sqrt{2}(-ia^2b^2 - 3ib^4) \cos(dx + c))^2 - \dots}{\dots}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/9*(6*(4*a*b^3*cos(d*x + c) + 5*a^2*b^2 - b^4)*sqrt(b*cos(d*x + c) + a)*s
in(d*x + c) - (sqrt(2)*(-I*a^2*b^2 - 3*I*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(I
*a^3*b + 3*I*a*b^3)*cos(d*x + c) + sqrt(2)*(-I*a^4 - 3*I*a^2*b^2))*sqrt(b)*
weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1
/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) - (sqrt(2)*(I*a^2*b^2 +
3*I*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(-I*a^3*b - 3*I*a*b^3)*cos(d*x + c) +
sqrt(2)*(I*a^4 + 3*I*a^2*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b
^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x
+ c) + 2*a)/b) + 12*(-I*sqrt(2)*a*b^3*cos(d*x + c)^2 - 2*I*sqrt(2)*a^2*b^2
*cos(d*x + c) - I*sqrt(2)*a^3*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x +
c) + 2*a)/b)) + 12*(I*sqrt(2)*a*b^3*cos(d*x + c)^2 + 2*I*sqrt(2)*a^2*b^2*co
s(d*x + c) + I*sqrt(2)*a^3*b)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b
^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c)
+ 2*a)/b)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c)^2 + 2*(a^5*b^2 - 2*
a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)
```

Sympy [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))**(5/2),x)

[Out] Integral((a + b*cos(c + d*x))**(-5/2), x)

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(-5/2), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(-5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{1}{(a + b \cos(c + dx))^{5/2}} dx$$

[In] int(1/(a + b*cos(c + d*x))^(5/2),x)

[Out] int(1/(a + b*cos(c + d*x))^(5/2), x)

3.544 $\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$

Optimal result	5531
Rubi [A] (verified)	5532
Mathematica [C] (verified)	5536
Maple [B] (verified)	5536
Fricas [F(-1)]	5537
Sympy [F]	5537
Maxima [F]	5537
Giac [F]	5538
Mupad [F(-1)]	5538

Optimal result

Integrand size = 21, antiderivative size = 320

$$\int \frac{\sec(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = -\frac{2b(7a^2-3b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3a^2(a^2-b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\ + \frac{2b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3a(a^2-b^2) d \sqrt{a+b \cos(c+dx)}} \\ + \frac{2 \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{a^2 d \sqrt{a+b \cos(c+dx)}} \\ + \frac{2b^2 \sin(c+dx)}{3a(a^2-b^2) d (a+b \cos(c+dx))^{3/2}} + \frac{2b^2(7a^2-3b^2) \sin(c+dx)}{3a^2(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+2/3*b^2*(7*a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-2/3*b*(7*a^2-3*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+2/3*b*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2881, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = \frac{2b^2(7a^2-3b^2)\sin(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^{3/2}} + \frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}} - \frac{2b(7a^2-3b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3a^2d(a^2-b^2)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{a^2d\sqrt{a+b\cos(c+dx)}}$$

[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (-2*b*(7*a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*b^2*(7*a^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[

{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 2886

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 3081

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))

```

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Ssin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Ssin[e + f*x])*(c + d*Ssin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} \\
&+ \frac{2 \int \frac{(\frac{3}{2}(a^2 - b^2) - \frac{3}{2}ab \cos(c + dx) + \frac{1}{2}b^2 \cos^2(c + dx)) \sec(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx}{3a(a^2 - b^2)} \\
&= \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} + \frac{2b^2(7a^2 - 3b^2) \sin(c + dx)}{3a^2(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{4 \int \frac{(\frac{3}{4}(a^2 - b^2)^2 - \frac{1}{2}ab(3a^2 - b^2) \cos(c + dx) - \frac{1}{4}b^2(7a^2 - 3b^2) \cos^2(c + dx)) \sec(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx}{3a^2(a^2 - b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2b^2 \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{2b^2(7a^2-3b^2)\sin(c+dx)}{3a^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{4 \int \frac{\left(-\frac{3}{4}b(a^2-b^2)^2 - \frac{1}{4}ab^2(a^2-b^2)\cos(c+dx)\right) \sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{3a^2b(a^2-b^2)^2} \\
&\quad - \frac{(b(7a^2-3b^2)) \int \sqrt{a+b\cos(c+dx)} dx}{3a^2(a^2-b^2)^2} \\
&= \frac{2b^2 \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{2b^2(7a^2-3b^2)\sin(c+dx)}{3a^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{a^2} + \frac{b \int \frac{1}{\sqrt{a+b\cos(c+dx)}} dx}{3a(a^2-b^2)} \\
&\quad - \frac{\left(b(7a^2-3b^2)\sqrt{a+b\cos(c+dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}} dx}{3a^2(a^2-b^2)^2 \sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&= -\frac{2b(7a^2-3b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3a^2(a^2-b^2)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad + \frac{2b^2 \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{2b^2(7a^2-3b^2)\sin(c+dx)}{3a^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{a^2\sqrt{a+b\cos(c+dx)}} + \frac{\left(b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{3a(a^2-b^2)\sqrt{a+b\cos(c+dx)}} \\
&= -\frac{2b(7a^2-3b^2)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|\frac{2b}{a+b}\right)}{3a^2(a^2-b^2)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad + \frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3a(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{a^2 d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{2b^2 \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{2b^2(7a^2-3b^2)\sin(c+dx)}{3a^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.45

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \frac{-\frac{8ab(3a^2 - b^2)\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b \cos(c+dx)}} + \frac{2(6a^4 - 19a^2b^2 + 9b^4)\sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx)\right)}{\sqrt{a+b \cos(c+dx)}}}{\dots}$$

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^(5/2), x]
```

```
[Out] (((-8*a*b*(3*a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^4 - 19*a^2*b^2 + 9*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-7*a^2 + 3*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*Sqrt[-(a + b)^(-1)])/(a^2*(a - b)^2*(a + b)^2 + (4*b^2*(8*a^3 - 4*a*b^2 + b*(7*a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^3 - a*b^2)^2*(a + b*Cos[c + d*x])^(3/2)))/(6*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. 2(383) = 766.

Time = 9.02 (sec) , antiderivative size = 849, normalized size of antiderivative = 2.65

method	result	size
default	Expression too large to display	849

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)
```

```
[Out] -((-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c), 2, (-2*b/(a-b))^(1/2))+2/a^2*b/sin(1/2*d*x+1/2*c)^2/(2*b*sin(1/2*d*x+1/2*c)^2-a-b)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+EllipticE(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-EllipticE(cos(1/2*d*x+1/2*c), (-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2
```

$$\begin{aligned} & *d*x+1/2*c)^2)^{(1/2)*b}-2/a*b*(1/6/b/(a-b)/(a+b)*\cos(1/2*d*x+1/2*c)*(-2*\sin \\ & (1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)/(\cos(1/2*d*x+1/2*c)^2 \\ & +1/2*(a-b)/b)^2+8/3*\sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*\cos(1/2*d*x+1/2* \\ & c)*a/(-(-2*b*\cos(1/2*d*x+1/2*c)^2-a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)+(3*a-b)/ \\ & (3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*((2*b*\cos(1/2*d* \\ & x+1/2*c)^2+a-b)/(a-b))^{(1/2)/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1 \\ & /2*c)^2)^{(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})-4/3*a/(a-b) \\ & /((a+b)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b) \\ &)^{(1/2)/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(\text{Ellip} \\ & ticF(\cos(1/2*d*x+1/2*c),(-2*b/(a-b))^{(1/2)})-\text{EllipticE}(\cos(1/2*d*x+1/2*c),(- \\ & 2*b/(a-b))^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(-2*b*\sin(1/2*d*x+1/2*c)^2+a+b)^{(1/ \\ & 2)/d \end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)

[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**(5/2), x)

Maxima [F]

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\sec(dx + c)}{(b \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\sec(dx + c)}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{1}{\cos(c + dx) (a + b \cos(c + dx))^{5/2}} dx$$

[In] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)),x)

[Out] int(1/(cos(c + d*x)*(a + b*cos(c + d*x))^(5/2)), x)

$$3.545 \quad \int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	5539
Rubi [A] (verified)	5540
Mathematica [C] (verified)	5544
Maple [B] (verified)	5545
Fricas [F(-1)]	5546
Sympy [F]	5546
Maxima [F]	5546
Giac [F]	5547
Mupad [F(-1)]	5547

Optimal result

Integrand size = 23, antiderivative size = 380

$$\int \frac{\sec^2(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx =$$

$$\frac{(3a^4 - 26a^2b^2 + 15b^4) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid \frac{2b}{a+b}\right)}{3a^3 (a^2 - b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

$$+ \frac{(3a^2 - 5b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3a^2 (a^2 - b^2) d \sqrt{a+b \cos(c+dx)}}$$

$$- \frac{5b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{a^3 d \sqrt{a+b \cos(c+dx)}}$$

$$+ \frac{b(3a^2 - 5b^2) \sin(c+dx)}{3a^2 (a^2 - b^2) d (a+b \cos(c+dx))^{3/2}}$$

$$+ \frac{b(3a^4 - 26a^2b^2 + 15b^4) \sin(c+dx)}{3a^3 (a^2 - b^2)^2 d \sqrt{a+b \cos(c+dx)}} + \frac{\tan(c+dx)}{ad(a+b \cos(c+dx))^{3/2}}$$

[Out] 1/3*b*(3*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+1/3*b*(3*a^4-26*a^2*b^2+15*b^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)-1/3*(3*a^4-26*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/((a+b*cos(d*x+c))/(a+b))^(1/2)+1/3*(3*a^2-5*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^2/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-5*b*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*cos(d*x+c))^(1/2)+tan(d*x+c)/a/d/(a+b*cos(d*x+c))^(3/2)

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2881, 3135, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = -\frac{5b\sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{a^3 d \sqrt{a+b\cos(c+dx)}} + \frac{b(3a^2 - 5b^2) \sin(c+dx)}{3a^2 d (a^2 - b^2) (a+b\cos(c+dx))^{3/2}} + \frac{(3a^2 - 5b^2) \sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{3a^2 d (a^2 - b^2) \sqrt{a+b\cos(c+dx)}} + \frac{b(3a^4 - 26a^2b^2 + 15b^4) \sin(c+dx)}{3a^3 d (a^2 - b^2)^2 \sqrt{a+b\cos(c+dx)}} - \frac{(3a^4 - 26a^2b^2 + 15b^4) \sqrt{a+b\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{3a^3 d (a^2 - b^2)^2 \sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}}$$

[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2), x]

[Out] -1/3*((3*a^4 - 26*a^2*b^2 + 15*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(a^3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + ((3*a^2 - 5*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(3*a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (5*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]/(a^3*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(3*a^2 - 5*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (b*(3*a^4 - 26*a^2*b^2 + 15*b^4)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + Tan[c + d*x]/(a*d*(a + b*Cos[c + d*x])^(3/2))

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]
```

Rule 2881

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c + d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 3081

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^n/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[
```

$B/d, \text{Int}[(a + b*\sin[e + f*x])^m, x], x] - \text{Dist}[(B*c - A*d)/d, \text{Int}[(a + b*\sin[e + f*x])^m/(c + d*\sin[e + f*x]), x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]])^{(m_)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]])^{(n_)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)] + (C_.)*\sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> \text{Simp}[(-A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{(m+1)}*((c + d*\sin[e + f*x])^{(n+1)})/(f*(m+1)*(b*c - a*d)*(a^2 - b^2))], x] + \text{Dist}[1/((m+1)*(b*c - a*d)*(a^2 - b^2)), \text{Int}[(a + b*\sin[e + f*x])^{(m+1)}*(c + d*\sin[e + f*x])^n*\text{Simp}[(m+1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m+n+2) - (c*(A*b^2 - a*b*B + a^2*C) + (m+1)*(b*c - a*d)*(A*b - a*B + b*C))*\text{Sin}[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m+n+3)*\text{Sin}[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3135

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]])^{(m_)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]])^{(n_)}*((A_.) + (C_.)*\sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> \text{Simp}[(-A*b^2 + a^2*C)*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{(m+1)}*((c + d*\sin[e + f*x])^{(n+1)})/(f*(m+1)*(b*c - a*d)*(a^2 - b^2))], x] + \text{Dist}[1/((m+1)*(b*c - a*d)*(a^2 - b^2)), \text{Int}[(a + b*\sin[e + f*x])^{(m+1)}*(c + d*\sin[e + f*x])^n*\text{Simp}[a*(m+1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m+n+2) - (c*(A*b^2 + a^2*C) + b*(m+1)*(b*c - a*d)*(A + C))*\text{Sin}[e + f*x] - d*(A*b^2 + a^2*C)*(m+n+3)*\text{Sin}[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3138

$\text{Int}[(A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)] + (C_.)*\sin[(e_.) + (f_.)*(x_)]^2)/(\text{Sqrt}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]])*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> \text{Dist}[C/(b*d), \text{Int}[\text{Sqrt}[a + b*\sin[e + f*x]], x], x] - \text{Dist}[1/(b*d), \text{Int}[\text{Simp}[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*\text{Sin}[e + f*x], x]/(\text{Sqrt}[a + b*\sin[e + f*x]]*(c + d*\sin[e + f*x])), x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(-\frac{5b}{2} + \frac{3}{2}b\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx}{a} \\
&= \frac{b(3a^2-5b^2)\sin(c+dx)}{3a^2(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} \\
&\quad + \frac{2\int \frac{\left(-\frac{15}{4}b(a^2-b^2) + \frac{3}{2}ab^2\cos(c+dx) + \frac{1}{4}b(3a^2-5b^2)\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx}{3a^2(a^2-b^2)} \\
&= \frac{b(3a^2-5b^2)\sin(c+dx)}{3a^2(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} \\
&\quad + \frac{b(3a^4-26a^2b^2+15b^4)\sin(c+dx)}{3a^3(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} \\
&\quad + \frac{4\int \frac{\left(-\frac{15}{8}b(a^2-b^2)^2 + \frac{1}{4}ab^2(9a^2-5b^2)\cos(c+dx) - \frac{1}{8}b(3a^4-26a^2b^2+15b^4)\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{3a^3(a^2-b^2)^2} \\
&= \frac{b(3a^2-5b^2)\sin(c+dx)}{3a^2(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{b(3a^4-26a^2b^2+15b^4)\sin(c+dx)}{3a^3(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} - \frac{4\int \frac{\left(\frac{15}{8}b^2(a^2-b^2)^2 - \frac{1}{8}ab(3a^4-8a^2b^2+5b^4)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{3a^3b(a^2-b^2)^2} \\
&\quad - \frac{(3a^4-26a^2b^2+15b^4)\int \sqrt{a+b\cos(c+dx)} dx}{6a^3(a^2-b^2)^2} \\
&= \frac{b(3a^2-5b^2)\sin(c+dx)}{3a^2(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} \\
&\quad + \frac{b(3a^4-26a^2b^2+15b^4)\sin(c+dx)}{3a^3(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} \\
&\quad - \frac{(5b)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx}{2a^3} + \frac{(3a^2-5b^2)\int \frac{1}{\sqrt{a+b\cos(c+dx)}} dx}{6a^2(a^2-b^2)} \\
&\quad - \frac{\left((3a^4-26a^2b^2+15b^4)\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}} dx}{6a^3(a^2-b^2)^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}
\end{aligned}$$

$$\begin{aligned}
 &= -\frac{(3a^4 - 26a^2b^2 + 15b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3a^3 (a^2 - b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 &+ \frac{b(3a^2 - 5b^2) \sin(c + dx)}{3a^2 (a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} + \frac{b(3a^4 - 26a^2b^2 + 15b^4) \sin(c + dx)}{3a^3 (a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
 &+ \frac{\tan(c + dx)}{ad(a + b \cos(c + dx))^{3/2}} - \frac{\left(5b \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{2a^3 \sqrt{a + b \cos(c + dx)}} \\
 &+ \frac{\left((3a^2 - 5b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}}} dx}{6a^2 (a^2 - b^2) \sqrt{a + b \cos(c + dx)}} \\
 &= -\frac{(3a^4 - 26a^2b^2 + 15b^4) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{3a^3 (a^2 - b^2)^2 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\
 &+ \frac{(3a^2 - 5b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{3a^2 (a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} \\
 &- \frac{5b \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{a^3 d \sqrt{a + b \cos(c + dx)}} \\
 &+ \frac{b(3a^2 - 5b^2) \sin(c + dx)}{3a^2 (a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} \\
 &+ \frac{b(3a^4 - 26a^2b^2 + 15b^4) \sin(c + dx)}{3a^3 (a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} + \frac{\tan(c + dx)}{ad(a + b \cos(c + dx))^{3/2}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.22 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.36

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \frac{b \left(-\frac{8ab(9a^2 - 5b^2) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b \cos(c+dx)}} + \frac{2(33a^4 - 86a^2b^2 + 45b^4) \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{\sqrt{a+b \cos(c+dx)}} \right)}{a^3 d \sqrt{a + b \cos(c + dx)}}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2), x]
```

```
[Out] (-((b*((-8*a*b*(9*a^2 - 5*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(33*a^4 - 86*a^
2*b^2 + 45*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(3*a^4 - 26*a^2*b^2 +
15*b^4)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x
```

$$\left. \right) / (a - b) \Big] \cdot \text{Csc}[c + d*x] \cdot (-2*a*(a - b) * \text{EllipticE}[I * \text{ArcSinh}[\text{Sqrt}[-(a + b)^{-1}] * \text{Sqrt}[a + b * \text{Cos}[c + d*x]]], (a + b)/(a - b)] + b * (-2*a * \text{EllipticF}[I * \text{ArcSinh}[\text{Sqrt}[-(a + b)^{-1}] * \text{Sqrt}[a + b * \text{Cos}[c + d*x]]], (a + b)/(a - b)] + b * \text{EllipticPi}[(a + b)/a, I * \text{ArcSinh}[\text{Sqrt}[-(a + b)^{-1}] * \text{Sqrt}[a + b * \text{Cos}[c + d*x]]], (a + b)/(a - b)])) / (a * b^2 * \text{Sqrt}[-(a + b)^{-1}])) / ((a - b)^2 * (a + b)^2) + (2 * (4 * a * b * (3 * a^4 - 17 * a^2 * b^2 + 10 * b^4) * \text{Sin}[c + d*x] + (3 * a^4 * b^2 - 26 * a^2 * b^4 + 15 * b^6) * \text{Sin}[2 * (c + d*x)] + 6 * (a^3 - a * b^2)^2 * \text{Tan}[c + d*x])) / ((a^2 - b^2)^2 * (a + b * \text{Cos}[c + d*x])^{3/2})) / (12 * a^3 * d)$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. $2(441) = 882$.

Time = 12.07 (sec) , antiderivative size = 1324, normalized size of antiderivative = 3.48

method	result	size
default	Expression too large to display	1324

[In] `int(sec(d*x+c)^2/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -(-(-2*b*\cos(1/2*d*x+1/2*c)^2-a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2/a^2*(-\cos(1/2*d*x+1/2*c)/a*(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)})/(2*\cos(1/2*d*x+1/2*c)^2-1)+1/2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) - 1/2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}) / (-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) + 1/2/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}) / (-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * b * \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) + 1/2/a*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}) / (-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2, (-2*b/(a-b))^{(1/2)}) + 4/a^3*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}) / (-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2, (-2*b/(a-b))^{(1/2)}) + 2*b^2/a^2*(1/6/b/(a-b)/(a+b)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}) / (\cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2 + 8/3*\sin(1/2*d*x+1/2*c)^2*b/(a-b)^2/(a+b)^2*\cos(1/2*d*x+1/2*c)*a/(-(-2*b*\cos(1/2*d*x+1/2*c)^2-a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} + (3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}) / (-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) - 4/3*a/(a-b)/(a+b)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * ((2*b*\cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^{(1/2)}) / (-2*\sin(1/2*d*x+1/2*c)^4*b+(a+b)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (\text{EllipticF}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)}) - \text{EllipticE}(\cos(1/2*d*x+1/2*c), (-2*b/(a-b))^{(1/2)})) - 4*b^2/a^3/\sin(1/2*d*x+1/2*c)^2/(2*b*\sin(1/2*d*x+1/2*c)^2-a-b \end{aligned}$$

)/(a^2-b^2)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b))/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)

[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**(5/2), x)

Maxima [F]

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\sec(dx + c)^2}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\sec(dx + c)^2}{(b \cos(dx + c) + a)^{5/2}} dx$$

[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{1}{\cos(c + dx)^2 (a + b \cos(c + dx))^{5/2}} dx$$

[In] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)),x)

[Out] int(1/(cos(c + d*x)^2*(a + b*cos(c + d*x))^(5/2)), x)

3.546 $\int \frac{1}{(a+b \cos(c+dx))^{7/2}} dx$

Optimal result	5548
Rubi [A] (verified)	5549
Mathematica [A] (verified)	5551
Maple [A] (verified)	5552
Fricas [C] (verification not implemented)	5552
Sympy [F(-1)]	5553
Maxima [F]	5553
Giac [F]	5554
Mupad [F(-1)]	5554

Optimal result

Integrand size = 14, antiderivative size = 282

$$\int \frac{1}{(a+b \cos(c+dx))^{7/2}} dx = \frac{2(23a^2 + 9b^2) \sqrt{a+b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| \frac{2b}{a+b}\right)}{15(a^2 - b^2)^3 d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} - \frac{16a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{15(a^2 - b^2)^2 d \sqrt{a+b \cos(c+dx)}} - \frac{2b \sin(c+dx)}{5(a^2 - b^2) d (a+b \cos(c+dx))^{5/2}} - \frac{16ab \sin(c+dx)}{15(a^2 - b^2)^2 d (a+b \cos(c+dx))^{3/2}} - \frac{2b(23a^2 + 9b^2) \sin(c+dx)}{15(a^2 - b^2)^3 d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] -2/5*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(5/2)-16/15*a*b*sin(d*x+c)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(3/2)-2/15*b*(23*a^2+9*b^2)*sin(d*x+c)/(a^2-b^2)^3/d/(a+b*cos(d*x+c))^(1/2)+2/15*(23*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/(a^2-b^2)^3/d/((a+b*cos(d*x+c))/(a+b))^(1/2)-16/15*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(b/(a+b))^(1/2))*((a+b*cos(d*x+c))/(a+b))^(1/2)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)
```


Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2743, 2833, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{1}{(a + b \cos(c + dx))^{7/2}} dx = -\frac{2b(23a^2 + 9b^2) \sin(c + dx)}{15d(a^2 - b^2)^3 \sqrt{a + b \cos(c + dx)}} - \frac{16ab \sin(c + dx)}{15d(a^2 - b^2)^2 (a + b \cos(c + dx))^{3/2}} - \frac{2b \sin(c + dx)}{5d(a^2 - b^2) (a + b \cos(c + dx))^{5/2}} - \frac{16a \sqrt{\frac{a+b \cos(c+dx)}{a+b}} \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{2b}{a+b}\right)}{15d(a^2 - b^2)^2 \sqrt{a + b \cos(c + dx)}} + \frac{2(23a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{15d(a^2 - b^2)^3 \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[In] Int[(a + b*Cos[c + d*x])^(-7/2), x]

[Out] (2*(23*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)]/(15*(a^2 - b^2)^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (16*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]/(15*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*b*Sin[c + d*x])/(5*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(5/2)) - (16*a*b*Sin[c + d*x])/(15*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2)) - (2*b*(23*a^2 + 9*b^2)*Sin[c + d*x])/(15*(a^2 - b^2)^3*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2742

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[(a
+ b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e +
f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b \sin(c + dx)}{5(a^2 - b^2) d(a + b \cos(c + dx))^{5/2}} - \frac{2 \int \frac{-\frac{5a}{2} + \frac{3}{2}b \cos(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx}{5(a^2 - b^2)} \\
&= -\frac{2b \sin(c + dx)}{5(a^2 - b^2) d(a + b \cos(c + dx))^{5/2}} \\
&\quad - \frac{16ab \sin(c + dx)}{15(a^2 - b^2)^2 d(a + b \cos(c + dx))^{3/2}} + \frac{4 \int \frac{\frac{3}{4}(5a^2 + 3b^2) - 2ab \cos(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx}{15(a^2 - b^2)^2} \\
&= -\frac{2b \sin(c + dx)}{5(a^2 - b^2) d(a + b \cos(c + dx))^{5/2}} - \frac{16ab \sin(c + dx)}{15(a^2 - b^2)^2 d(a + b \cos(c + dx))^{3/2}} \\
&\quad - \frac{2b(23a^2 + 9b^2) \sin(c + dx)}{15(a^2 - b^2)^3 d\sqrt{a + b \cos(c + dx)}} - \frac{8 \int \frac{-\frac{1}{8}a(15a^2 + 17b^2) - \frac{1}{8}b(23a^2 + 9b^2) \cos(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx}{15(a^2 - b^2)^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2b \sin(c+dx)}{5(a^2-b^2)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab \sin(c+dx)}{15(a^2-b^2)^2 d(a+b\cos(c+dx))^{3/2}} \\
&\quad - \frac{2b(23a^2+9b^2) \sin(c+dx)}{15(a^2-b^2)^3 d\sqrt{a+b\cos(c+dx)}} - \frac{(8a) \int \frac{1}{\sqrt{a+b\cos(c+dx)}} dx}{15(a^2-b^2)^2} \\
&\quad + \frac{(23a^2+9b^2) \int \sqrt{a+b\cos(c+dx)} dx}{15(a^2-b^2)^3} \\
&= -\frac{2b \sin(c+dx)}{5(a^2-b^2)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab \sin(c+dx)}{15(a^2-b^2)^2 d(a+b\cos(c+dx))^{3/2}} \\
&\quad - \frac{2b(23a^2+9b^2) \sin(c+dx)}{15(a^2-b^2)^3 d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\left((23a^2+9b^2) \sqrt{a+b\cos(c+dx)}\right) \int \sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}} dx}{15(a^2-b^2)^3 \sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad - \frac{\left(8a \sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right) \int \frac{1}{\sqrt{\frac{a}{a+b} + \frac{b\cos(c+dx)}{a+b}}} dx}{15(a^2-b^2)^2 \sqrt{a+b\cos(c+dx)}} \\
&= \frac{2(23a^2+9b^2) \sqrt{a+b\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid \frac{2b}{a+b}\right)}{15(a^2-b^2)^3 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} \\
&\quad - \frac{16a \sqrt{\frac{a+b\cos(c+dx)}{a+b}} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right)}{15(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{2b \sin(c+dx)}{5(a^2-b^2)d(a+b\cos(c+dx))^{5/2}} \\
&\quad - \frac{16ab \sin(c+dx)}{15(a^2-b^2)^2 d(a+b\cos(c+dx))^{3/2}} - \frac{2b(23a^2+9b^2) \sin(c+dx)}{15(a^2-b^2)^3 d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}} dx = \frac{2 \left(\frac{(a+b\cos(c+dx))^{5/2} \left((23a^2+9b^2) E\left(\frac{1}{2}(c+dx) \mid \frac{2b}{a+b}\right) + 8a(-a+b) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{2b}{a+b}\right) \right)}{(a-b)^3} + \frac{b(34a^2+9b^2) \sin(c+dx)}{15d(a+b\cos(c+dx))^{3/2}} \right)}{15d(a+b\cos(c+dx))^{3/2}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(-7/2), x]

[Out] (2*(((a + b*Cos[c + d*x])/(a + b))^(5/2)*((23*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 8*a*(-a + b)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^3 + (b*(34*a^4 - 5*a^2*b^2 + 3*b^4 + 2*a*b*(27*a^2 + 5*b^2)*Cos[c + d*x] + b^2*(23*a^2 + 9*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/(-a^2 + b^2)^3)/(15*d*(a + b*Cos[c + d*x])^(5/2))

Maple [A] (verified)

Time = 7.51 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.18

method	result
default	$-\frac{\sqrt{-(-2b(\cos^2(\frac{dx}{2} + \frac{c}{2})) - a + b)(\sin^2(\frac{dx}{2} + \frac{c}{2}))}}{\left(\frac{\cos(\frac{dx}{2} + \frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2}))b + (a+b)(\sin^2(\frac{dx}{2} + \frac{c}{2}))}}{10b^2(a-b)(a+b)(\cos^2(\frac{dx}{2} + \frac{c}{2}) + \frac{a-b}{2b})^3} + \frac{8a \cos(\frac{dx}{2} + \frac{c}{2})\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2}))}}{15b(a-b)^2(a+b)^2} \right)}$

```
[In] int(1/(a+cos(d*x+c)*b)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/10/b^2/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^3+8/15*a/b/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+4/15*sin(1/2*d*x+1/2*c)^2*b/(a-b)^3/(a+b)^3*cos(1/2*d*x+1/2*c)*(23*a^2+9*b^2)/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(15*a^2-8*a*b+9*b^2)/(15*a^5+15*a^4*b-30*a^3*b^2-30*a^2*b^3+15*a*b^4+15*b^5)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2/15*(23*a^2+9*b^2)/(a-b)^2/(a+b)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4*b+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*b*sin(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 985, normalized size of antiderivative = 3.49

$$\int \frac{1}{(a + b \cos(c + dx))^{7/2}} dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/45*(6*(34*a^4*b^2 - 5*a^2*b^4 + 3*b^6 + (23*a^2*b^4 + 9*b^6)*cos(d*x + c)^2 + 2*(27*a^3*b^3 + 5*a*b^5)*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sin(d*x + c) + (sqrt(2)*(-I*a^3*b^3 + 33*I*a*b^5)*cos(d*x + c)^3 - 3*sqrt(2)*(I*a^4*b^2 - 33*I*a^2*b^4)*cos(d*x + c)^2 - 3*sqrt(2)*(I*a^5*b - 33*I*a^3*b^3)*cos(d*x + c) + sqrt(2)*(-I*a^6 + 33*I*a^4*b^2))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*a)/b) + (sqrt(2)*(I*a^3*b^3 - 33*I*a*b^5)*cos(d
```

```

*x + c)^3 - 3*sqrt(2)*(-I*a^4*b^2 + 33*I*a^2*b^4)*cos(d*x + c)^2 - 3*sqrt(2)
)*(-I*a^5*b + 33*I*a^3*b^3)*cos(d*x + c) + sqrt(2)*(I*a^6 - 33*I*a^4*b^2))*
sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2
)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b) - 3*(sqrt(2)*(2
3*I*a^2*b^4 + 9*I*b^6)*cos(d*x + c)^3 + 3*sqrt(2)*(23*I*a^3*b^3 + 9*I*a*b^5
)*cos(d*x + c)^2 + 3*sqrt(2)*(23*I*a^4*b^2 + 9*I*a^2*b^4)*cos(d*x + c) + sq
rt(2)*(23*I*a^5*b + 9*I*a^3*b^3))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^
2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x +
c) + 2*a)/b)) - 3*(sqrt(2)*(-23*I*a^2*b^4 - 9*I*b^6)*cos(d*x + c)^3 + 3*sq
rt(2)*(-23*I*a^3*b^3 - 9*I*a*b^5)*cos(d*x + c)^2 + 3*sqrt(2)*(-23*I*a^4*b^2
- 9*I*a^2*b^4)*cos(d*x + c) + sqrt(2)*(-23*I*a^5*b - 9*I*a^3*b^3))*sqrt(b)
*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weie
rstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(
3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) + 2*a)/b)))/((a^6*b^4 - 3*a^4*b^6 + 3
*a^2*b^8 - b^10)*d*cos(d*x + c)^3 + 3*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*
b^9)*d*cos(d*x + c)^2 + 3*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*d*cos
(d*x + c) + (a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d)

```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{7/2}} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{7/2}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{7/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(-7/2), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{7/2}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{7/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(-7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{7/2}} dx = \int \frac{1}{(a + b \cos(c + dx))^{7/2}} dx$$

[In] int(1/(a + b*cos(c + d*x))^(7/2),x)

[Out] int(1/(a + b*cos(c + d*x))^(7/2), x)

$$3.547 \quad \int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx$$

Optimal result	5555
Rubi [A] (verified)	5555
Mathematica [A] (verified)	5557
Maple [A] (verified)	5558
Fricas [C] (verification not implemented)	5558
Sympy [F(-1)]	5559
Maxima [F]	5559
Giac [F]	5559
Mupad [F(-1)]	5559

Optimal result

Integrand size = 23, antiderivative size = 111

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{20d} - \frac{23 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{20\sqrt{7}d}$$

$$- \frac{\sqrt{3+4\cos(c+dx)} \sin(c+dx)}{10d}$$

$$+ \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)} \sin(c+dx)}{10d}$$

```
[Out] -23/140*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+9/20*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/10*sin(d*x+c)*(3+4*cos(d*x+c))^(1/2)/d+1/10*cos(d*x+c)*sin(d*x+c)*(3+4*cos(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2872, 3102, 2831, 2740, 2732}

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = -\frac{23 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{20\sqrt{7}d} + \frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{20d}$$

$$+ \frac{\sin(c+dx) \cos(c+dx) \sqrt{4\cos(c+dx)+3}}{10d}$$

$$- \frac{\sin(c+dx) \sqrt{4\cos(c+dx)+3}}{10d}$$

[In] Int[Cos[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] (9*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(20*d) - (23*EllipticF[(c + d*x)/2, 8/7])/(20*Sqrt[7]*d) - (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(10*d) + (Cos[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(10*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2872

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & NeQ[c, 0])))

Rule 3102

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} \\
 &+ \frac{1}{10} \int \frac{3+6\cos(c+dx)-6\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
 &= -\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} \\
 &+ \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} + \frac{1}{60} \int \frac{6+54\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
 &= -\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} + \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} \\
 &+ \frac{9}{40} \int \sqrt{3+4\cos(c+dx)} dx - \frac{23}{40} \int \frac{1}{\sqrt{3+4\cos(c+dx)}} dx \\
 &= \frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{20d} - \frac{23\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right)}{20\sqrt{7}d} \\
 &- \frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} + \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.73

$$\begin{aligned}
 &\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
 &= \frac{63\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right) - 23\sqrt{7}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{8}{7}\right) + 7\sqrt{3+4\cos(c+dx)}(-2\sin(c+dx) + \sin(2(c+dx)))}{140d}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] (63*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] - 23*Sqrt[7]*EllipticF[(c + d*x)/2, 8/7] + 7*Sqrt[3 + 4*Cos[c + d*x]]*(-2*Sin[c + d*x] + Sin[2*(c + d*x)]))/(140*d)

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.08

method	result
default	$-\frac{\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-64\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+56\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-23\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{20\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sin\left(\frac{dx}{2}\right)}$

```
[In] int(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-64*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.23

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx$$

$$= \frac{4\sqrt{4\cos(dx+c)+3}(\cos(dx+c)-1)\sin(dx+c)+7i\sqrt{2}\text{weierstrassPInverse}(-1,1,\cos(dx+c)+i\sin(dx+c))}{d}$$

```
[In] integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(4*sqrt(4*cos(d*x+c)+3)*(cos(d*x+c)-1)*sin(d*x+c)+7*I*sqrt(2)*weierstrassPInverse(-1,1,cos(d*x+c)+I*sin(d*x+c)+1/2)-7*I*sqrt(2)*weierstrassPInverse(-1,1,cos(d*x+c)-I*sin(d*x+c)+1/2)+18*I*sqrt(2)*weierstrassZeta(-1,1,weierstrassPInverse(-1,1,cos(d*x+c)+I*sin(d*x+c)+1/2))-18*I*sqrt(2)*weierstrassZeta(-1,1,weierstrassPInverse(-1,1,cos(d*x+c)-I*sin(d*x+c)+1/2)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**3/(3+4*cos(d*x+c))**(1/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^3(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^3}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\cos^3(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^3}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(cos(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^3}{\sqrt{4 \cos(c + dx) + 3}} dx$$

[In] int(cos(c + d*x)^3/(4*cos(c + d*x) + 3)^(1/2), x)

[Out] int(cos(c + d*x)^3/(4*cos(c + d*x) + 3)^(1/2), x)

$$3.548 \quad \int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx$$

Optimal result	5560
Rubi [A] (verified)	5560
Mathematica [A] (verified)	5562
Maple [A] (verified)	5562
Fricas [C] (verification not implemented)	5562
Sympy [F]	5563
Maxima [F]	5563
Giac [F]	5563
Mupad [B] (verification not implemented)	5564

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{4d} + \frac{17 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{12\sqrt{7}d} + \frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{6d}$$

[Out] 17/84*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)-1/4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2/7*14^(1/2))/d*7^(1/2)+1/6*sin(d*x+c)*(3+4*cos(d*x+c))^(1/2)/d

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2870, 2831, 2740, 2732}

$$\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \frac{17 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{12\sqrt{7}d} - \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{4d} + \frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

[In] Int[Cos[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] -1/4*(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d + (17*EllipticF[(c + d*x)/2, 8/7])/(12*Sqrt[7]*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(6*d)

Rule 2732

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a
+ b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2870

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^2, x_Symbol] := Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{6d} + \frac{1}{6} \int \frac{2-3\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
&= \frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{6d} - \frac{1}{8} \int \sqrt{3+4\cos(c+dx)} dx + \frac{17}{24} \int \frac{1}{\sqrt{3+4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{4d} + \frac{17\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{12\sqrt{7}d} + \frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx$$

$$= \frac{-21\sqrt{7}E\left(\frac{1}{2}(c + dx)\middle|\frac{8}{7}\right) + 17\sqrt{7}\operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right) + 14\sqrt{3 + 4 \cos(c + dx)} \sin(c + dx)}{84d}$$

[In] Integrate[Cos[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]], x]

[Out] (-21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + 17*Sqrt[7]*EllipticF[(c + d*x)/2, 8/7] + 14*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(84*d)

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.96

method	result
default	$-\frac{\sqrt{\left(8\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\left(32\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 28\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 17\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{8}\right)}{12\sqrt{-8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 7\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)}$

[In] int(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2), x, method=_RETURNVERBOSE)

[Out]
$$-1/12 * \left((8 * \cos(1/2 * d * x + 1/2 * c) - 1) * \sin(1/2 * d * x + 1/2 * c) \right)^{1/2} * \left(32 * \sin(1/2 * d * x + 1/2 * c)^4 * \cos(1/2 * d * x + 1/2 * c) - 28 * \sin(1/2 * d * x + 1/2 * c)^2 * \cos(1/2 * d * x + 1/2 * c) + 17 * \left(\sin(1/2 * d * x + 1/2 * c) \right)^{1/2} * \left(8 * \sin(1/2 * d * x + 1/2 * c) - 7 \right)^{1/2} * \operatorname{EllipticF}\left(\cos(1/2 * d * x + 1/2 * c), 2 * 2^{1/2}\right) + 3 * \left(\sin(1/2 * d * x + 1/2 * c) \right)^{1/2} * \left(8 * \sin(1/2 * d * x + 1/2 * c) - 7 \right)^{1/2} * \operatorname{EllipticE}\left(\cos(1/2 * d * x + 1/2 * c), 2 * 2^{1/2}\right) \right) / \left(-8 * \sin(1/2 * d * x + 1/2 * c)^4 + 7 * \sin(1/2 * d * x + 1/2 * c)^2 \right)^{1/2} / \sin(1/2 * d * x + 1/2 * c) / \left(8 * \cos(1/2 * d * x + 1/2 * c) - 1 \right)^{1/2} / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx$$

$$= \frac{4\sqrt{4 \cos(dx + c) + 3} \sin(dx + c) - 7i\sqrt{2}\operatorname{weierstrassPInverse}\left(-1, 1, \cos(dx + c) + i \sin(dx + c) + \frac{1}{2}\right) + \dots}{\dots}$$

[In] integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] $1/24*(4*\sqrt{4*\cos(dx + c) + 3}*\sin(dx + c) - 7*I*\sqrt{2}*weierstrassPInverse(-1, 1, \cos(dx + c) + I*\sin(dx + c) + 1/2) + 7*I*\sqrt{2}*weierstrassPInverse(-1, 1, \cos(dx + c) - I*\sin(dx + c) + 1/2) - 6*I*\sqrt{2}*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, \cos(dx + c) + I*\sin(dx + c) + 1/2)) + 6*I*\sqrt{2}*weierstrassZeta(-1, 1, weierstrassPInverse(-1, 1, \cos(dx + c) - I*\sin(dx + c) + 1/2)))/d$

Sympy [F]

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\cos^2(c + dx)}{\sqrt{4 \cos(c + dx) + 3}} dx$$

[In] `integrate(cos(dx+c)**2/(3+4*cos(dx+c))**(1/2), x)`

[Out] `Integral(cos(c + dx)**2/sqrt(4*cos(c + dx) + 3), x)`

Maxima [F]

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^2}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] `integrate(cos(dx+c)^2/(3+4*cos(dx+c))^(1/2), x, algorithm="maxima")`

[Out] `integrate(cos(dx + c)^2/sqrt(4*cos(dx + c) + 3), x)`

Giac [F]

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^2}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] `integrate(cos(dx+c)^2/(3+4*cos(dx+c))^(1/2), x, algorithm="giac")`

[Out] `integrate(cos(dx + c)^2/sqrt(4*cos(dx + c) + 3), x)`

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \frac{\sin(c + dx) \sqrt{4 \cos(c + dx) + 3}}{6d} - \frac{\sqrt{\frac{4 \cos(c+dx)}{7} + \frac{3}{7}} (42 E(\frac{c}{2} + \frac{dx}{2} | \frac{8}{7}) - 34 F(\frac{c}{2} + \frac{dx}{2} | \frac{8}{7}))}{24d \sqrt{4 \cos(c + dx) + 3}}$$

[In] int(cos(c + d*x)^2/(4*cos(c + d*x) + 3)^(1/2),x)

[Out] (sin(c + d*x)*(4*cos(c + d*x) + 3)^(1/2))/(6*d) - (((4*cos(c + d*x))/7 + 3/7)^(1/2)*(42*ellipticE(c/2 + (d*x)/2, 8/7) - 34*ellipticF(c/2 + (d*x)/2, 8/7)))/(24*d*(4*cos(c + d*x) + 3)^(1/2))

$$3.549 \quad \int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx$$

Optimal result	5565
Rubi [A] (verified)	5565
Mathematica [A] (verified)	5566
Maple [A] (verified)	5566
Fricas [C] (verification not implemented)	5567
Sympy [F]	5568
Maxima [F]	5568
Giac [F]	5568
Mupad [B] (verification not implemented)	5568

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{2d} - \frac{3\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{2\sqrt{7}d}$$

[Out] $-3/14*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+1/2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2831, 2740, 2732}

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{2d} - \frac{3\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{2\sqrt{7}d}$$

[In] `Int[Cos[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]`

[Out] $(\text{Sqrt}[7]*\text{EllipticE}[(c + d*x)/2, 8/7])/(2*d) - (3*\text{EllipticF}[(c + d*x)/2, 8/7])/ (2*\text{Sqrt}[7]*d)$

Rule 2732

`Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]`

Rule 2740

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{4} \int \sqrt{3 + 4 \cos(c + dx)} dx - \frac{3}{4} \int \frac{1}{\sqrt{3 + 4 \cos(c + dx)}} dx \\ &= \frac{\sqrt{7} E\left(\frac{1}{2}(c + dx) \middle| \frac{8}{7}\right)}{2d} - \frac{3 \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{2\sqrt{7}d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{\cos(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \frac{7 E\left(\frac{1}{2}(c + dx) \middle| \frac{8}{7}\right) - 3 \text{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{2\sqrt{7}d}$$

```
[In] Integrate[Cos[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (7*EllipticE[(c + d*x)/2, 8/7] - 3*EllipticF[(c + d*x)/2, 8/7])/(2*Sqrt[7]*d)
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.04

method	result
default	$\frac{\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1-\cos(dx+c)}{2}}\sqrt{1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(3F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\sqrt{2}\right)+E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),2\sqrt{2}\right)\right)}{2\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d$
risch	$\frac{i\left(2e^{2i(dx+c)}+3e^{i(dx+c)}+2\right)e^{-i(dx+c)}}{2d\sqrt{\left(2e^{2i(dx+c)}+3e^{i(dx+c)}+2\right)e^{-i(dx+c)}}}-\left(\frac{2\left(\frac{3}{4}+\frac{i\sqrt{7}}{4}\right)\sqrt{\frac{e^{i(dx+c)}+\frac{3}{4}+\frac{i\sqrt{7}}{4}}{\frac{3}{4}+\frac{i\sqrt{7}}{4}}}\sqrt{14}\sqrt{i}}{\sqrt{\left(2e^{2i(dx+c)}+3e^{i(dx+c)}+2\right)e^{i(dx+c)}}}+\frac{2e^{2i(dx+c)}+3e^{i(dx+c)}+2}{\sqrt{\left(2e^{2i(dx+c)}+3e^{i(dx+c)}+2\right)e^{i(dx+c)}}}\right)$

[In] `int(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{2}*\left(\left(8*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2-1\right)*\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2\right)^{\frac{1}{2}}*\left(\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2\right)^{\frac{1}{2}}*\left(1-8*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2\right)^{\frac{1}{2}}*\left(3*EllipticF\left(\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right),2*2^{\frac{1}{2}}\right)+EllipticE\left(\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right),2*2^{\frac{1}{2}}\right)\right)/\left(-8*\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^4+7*\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2\right)^{\frac{1}{2}}/\sin\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)/\left(8*\cos\left(\frac{1}{2}*d*x+\frac{1}{2}*c\right)^2-1\right)^{\frac{1}{2}}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.12

$$\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx$$

$$= \frac{i\sqrt{2}\operatorname{weierstrassPInverse}\left(-1,1,\cos(dx+c)+i\sin(dx+c)+\frac{1}{2}\right)-i\sqrt{2}\operatorname{weierstrassPInverse}\left(-1,1,\cos(dx+c)+i\sin(dx+c)+\frac{1}{2}\right)}{d}$$

[In] `integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x,algorithm="fricas")`

[Out] $\frac{1}{4}*\left(I*\sqrt{2}*\operatorname{weierstrassPInverse}\left(-1,1,\cos(d*x+c)+I*\sin(d*x+c)+\frac{1}{2}\right)-I*\sqrt{2}*\operatorname{weierstrassPInverse}\left(-1,1,\cos(d*x+c)-I*\sin(d*x+c)+\frac{1}{2}\right)+2*I*\sqrt{2}*\operatorname{weierstrassZeta}\left(-1,1,\operatorname{weierstrassPInverse}\left(-1,1,\cos(d*x+c)+I*\sin(d*x+c)+\frac{1}{2}\right)\right)-2*I*\sqrt{2}*\operatorname{weierstrassZeta}\left(-1,1,\operatorname{weierstrassPInverse}\left(-1,1,\cos(d*x+c)-I*\sin(d*x+c)+\frac{1}{2}\right)\right)\right)/d$

Sympy [F]

$$\int \frac{\cos(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\cos(c + dx)}{\sqrt{4 \cos(c + dx) + 3}} dx$$

[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))**(1/2),x)

[Out] Integral(cos(c + d*x)/sqrt(4*cos(c + d*x) + 3), x)

Maxima [F]

$$\int \frac{\cos(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\cos(dx + c)}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\cos(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\cos(dx + c)}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)

Mupad [B] (verification not implemented)

Time = 14.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{\cos(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \frac{\sqrt{\frac{4 \cos(c+dx)}{7} + \frac{3}{7}} (7 E(\frac{c}{2} + \frac{dx}{2} | \frac{8}{7}) - 3 F(\frac{c}{2} + \frac{dx}{2} | \frac{8}{7}))}{2 d \sqrt{4 \cos(c + dx) + 3}}$$

[In] int(cos(c + d*x)/(4*cos(c + d*x) + 3)^(1/2),x)

[Out] (((4*cos(c + d*x))/7 + 3/7)^(1/2)*(7*ellipticE(c/2 + (d*x)/2, 8/7) - 3*ellipticF(c/2 + (d*x)/2, 8/7)))/(2*d*(4*cos(c + d*x) + 3)^(1/2))

$$3.550 \quad \int \frac{1}{\sqrt{3+4 \cos(c+dx)}} dx$$

Optimal result	5569
Rubi [A] (verified)	5569
Mathematica [A] (verified)	5570
Maple [C] (verified)	5570
Fricas [C] (verification not implemented)	5570
Sympy [F]	5571
Maxima [F]	5571
Giac [F]	5571
Mupad [B] (verification not implemented)	5571

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{1}{\sqrt{3+4 \cos(c+dx)}} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[Out] $2/7*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2740}

$$\int \frac{1}{\sqrt{3+4 \cos(c+dx)}} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[In] `Int[1/Sqrt[3 + 4*Cos[c + d*x]],x]`

[Out] `(2*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d)`

Rule 2740

`Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]`

Rubi steps

$$\text{integral} = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{3 + 4 \cos(c + dx)}} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[In] Integrate[1/Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] (2*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\sqrt{7} \operatorname{am}^{-1}\left(\frac{dx}{2} + \frac{c}{2} \mid \frac{2\sqrt{14}}{7}\right)}{7d}$	23

[In] int(1/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/7/d*7^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2/7*14^(1/2))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int \frac{1}{\sqrt{3 + 4 \cos(c + dx)}} dx = \frac{-i \sqrt{2} \operatorname{weierstrassPInverse}\left(-1, 1, \cos(dx + c) + i \sin(dx + c) + \frac{1}{2}\right) + i \sqrt{2} \operatorname{weierstrassPInverse}\left(-1, 1, \cos(dx + c) + i \sin(dx + c) - \frac{1}{2}\right)}{2d}$$

[In] integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/2*(-I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) + I*sin(d*x + c) + 1/2) + I*sqrt(2)*weierstrassPInverse(-1, 1, cos(d*x + c) - I*sin(d*x + c) + 1/2))/d

Sympy [F]

$$\int \frac{1}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{1}{\sqrt{4 \cos(c + dx) + 3}} dx$$

[In] integrate(1/(3+4*cos(d*x+c))**(1/2),x)

[Out] Integral(1/sqrt(4*cos(c + d*x) + 3), x)

Maxima [F]

$$\int \frac{1}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{1}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{1}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{1}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(4*cos(d*x + c) + 3), x)

Mupad [B] (verification not implemented)

Time = 14.82 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{3 + 4 \cos(c + dx)}} dx = \frac{2 \sqrt{\frac{4 \cos(c+dx)}{7} + \frac{3}{7}} F\left(\frac{c}{2} + \frac{dx}{2} \middle| \frac{8}{7}\right)}{d \sqrt{4 \cos(c + dx) + 3}}$$

[In] int(1/(4*cos(c + d*x) + 3)^(1/2),x)

[Out] (2*((4*cos(c + d*x))/7 + 3/7)^(1/2)*ellipticF(c/2 + (d*x)/2, 8/7))/(d*(4*cos(c + d*x) + 3)^(1/2))

$$3.551 \quad \int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx$$

Optimal result	5572
Rubi [A] (verified)	5572
Mathematica [A] (verified)	5573
Maple [B] (verified)	5573
Fricas [F]	5573
Sympy [F]	5574
Maxima [F]	5574
Giac [F]	5574
Mupad [F(-1)]	5574

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \frac{2 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[Out] $2/7*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2/7*14^{(1/2)})/d*7^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec), antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2884}

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \frac{2 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[In] `Int[Sec[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]], x]`

[Out] `(2*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d)`

Rule 2884

`Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]`

Rubi steps

$$\text{integral} = \frac{2 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \frac{2 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[In] Integrate[Sec[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]], x]

[Out] (2*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(48) = 96.

Time = 1.84 (sec) , antiderivative size = 138, normalized size of antiderivative = 5.75

method	result	size
default	$\frac{2\sqrt{\left(8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{1 - 8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\Pi\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2, 2\sqrt{2}\right)}{\sqrt{-8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 7\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}} d$	138

[In] int(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2), x, method=_RETURNVERBOSE)

[Out] 2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(1-8*cos(1/2*d*x+1/2*c)^2)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c), 2, 2*2^(1/2))/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [F]

$$\int \frac{\sec(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)

Sympy [F]

$$\int \frac{\sec(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\sec(c + dx)}{\sqrt{4 \cos(c + dx) + 3}} dx$$

[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)/sqrt(4*cos(c + d*x) + 3), x)

Maxima [F]

$$\int \frac{\sec(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\sec(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx) \sqrt{4 \cos(c + dx) + 3}} dx$$

[In] int(1/(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2)),x)

[Out] int(1/(cos(c + d*x)*(4*cos(c + d*x) + 3)^(1/2)), x)

$$3.552 \quad \int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx$$

Optimal result	5575
Rubi [A] (verified)	5575
Mathematica [C] (verified)	5577
Maple [B] (verified)	5578
Fricas [F]	5578
Sympy [F]	5578
Maxima [F]	5579
Giac [F]	5579
Mupad [F(-1)]	5579

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

$$- \frac{4 \text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{3\sqrt{7}d}$$

$$+ \frac{\sqrt{3+4\cos(c+dx)} \tan(c+dx)}{3d}$$

[Out] 1/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2/7*14^(1/2))/d*7^(1/2)-4/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2, 2/7*14^(1/2))/d*7^(1/2)-1/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2/7*14^(1/2))/d*7^(1/2)+1/3*(3+4*cos(d*x+c))^(1/2)*tan(d*x+c)/d

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2881, 3139, 2732, 3081, 2740, 2884}

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d} - \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d}$$

$$- \frac{4 \text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{3\sqrt{7}d}$$

$$+ \frac{\sqrt{4\cos(c+dx)+3} \tan(c+dx)}{3d}$$

[In] Int[Sec[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] $-\frac{1}{3} \frac{\sqrt{7} \operatorname{EllipticE}\left[\frac{c+d x}{2}, \frac{8}{7}\right]}{d} + \frac{\operatorname{EllipticF}\left[\frac{c+d x}{2}, \frac{8}{7}\right]}{\left(\sqrt{7} d\right) - \left(4 \operatorname{EllipticPi}\left[2, \frac{c+d x}{2}, \frac{8}{7}\right]\right) / \left(3 \sqrt{7} d\right) + \left(\sqrt{3+4 \cos [c+d x]}\right) \tan [c+d x]}{\left(3 d\right)}$

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*m] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]))/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3139

Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist [C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c *C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{1}{3} \int \frac{(-2-2\cos^2(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
 &= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} \\
 &\quad - \frac{1}{12} \int \frac{(8-6\cos(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx - \frac{1}{6} \int \sqrt{3+4\cos(c+dx)} dx \\
 &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} \\
 &\quad + \frac{1}{2} \int \frac{1}{\sqrt{3+4\cos(c+dx)}} dx - \frac{2}{3} \int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
 &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}d} \\
 &\quad - \frac{4\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{3\sqrt{7}d} + \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.56

$$\begin{aligned}
 &\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
 &= \frac{-\frac{6\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}} + \frac{i\left(21E\left(i\text{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right)\middle|-\frac{1}{7}\right) - 12\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right), -\frac{1}{7}\right) - 8\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)\right)}{3\sqrt{7}\sqrt{\sin^2(c+dx)}}}{3d}
 \end{aligned}$$

[In] Integrate[Sec[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]], x]

[Out] ((-6*EllipticPi[2, (c + d*x)/2, 8/7])/Sqrt[7] + ((1/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + Sqrt[3 + 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(166) = 332.

Time = 2.53 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.47

method	result
default	$\frac{\sqrt{-\left(1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\frac{3\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$

[In] int(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out]
$$\begin{aligned} & -\left(-\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-\frac{2}{3}\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right) \\ & \left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1 \\ & +\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4 \\ & +7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2\right)^{\frac{1}{2}} \\ & +\frac{1}{3}\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4 \\ & +7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\text{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2\right)^{\frac{1}{2}} \\ & +\frac{4}{3}\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4 \\ & +7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}} \\ & \text{EllipticPi}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2,2\right)^{\frac{1}{2}}\left.\right)/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/(8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1)^{\frac{1}{2}}/d \end{aligned}$$

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \int \frac{\sec(dx+c)^2}{\sqrt{4\cos(dx+c)+3}} dx$$

[In] integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \int \frac{\sec^2(c+dx)}{\sqrt{4\cos(c+dx)+3}} dx$$

[In] integrate(sec(d*x+c)**2/(3+4*cos(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)**2/sqrt(4*cos(c + d*x) + 3), x)

Maxima [F]

$$\int \frac{\sec^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^2}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\sec^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^2}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^2 \sqrt{4 \cos(c + dx) + 3}} dx$$

[In] int(1/(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2)),x)

[Out] int(1/(cos(c + d*x)^2*(4*cos(c + d*x) + 3)^(1/2)), x)

$$3.553 \quad \int \frac{\sec^3(c+dx)}{\sqrt{3+4 \cos(c+dx)}} dx$$

Optimal result	5580
Rubi [A] (verified)	5580
Mathematica [C] (verified)	5583
Maple [B] (verified)	5584
Fricas [F]	5584
Sympy [F]	5584
Maxima [F]	5585
Giac [F]	5585
Mupad [F(-1)]	5585

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4 \cos(c+dx)}} dx = \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{3\sqrt{7}d}$$

$$+ \frac{\sqrt{7} \text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{3d}$$

$$- \frac{\sqrt{3+4 \cos(c+dx)} \tan(c+dx)}{3d}$$

$$+ \frac{\sqrt{3+4 \cos(c+dx)} \sec(c+dx) \tan(c+dx)}{6d}$$

[Out] $-1/21*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+1/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+1/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticPi}(\sin(1/2*d*x+1/2*c), 2, 2/7*14^{(1/2)})/d*7^{(1/2)}-1/3*(3+4*\cos(d*x+c))^{(1/2)}*\tan(d*x+c)/d+1/6*\sec(d*x+c)*(3+4*\cos(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {2881, 3134, 3138, 2732, 3081, 2740, 2884}

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = -\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{3\sqrt{7}d} + \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d} \\ + \frac{\sqrt{7}\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{3d} \\ - \frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d} \\ + \frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)}{6d}$$

[In] Int[Sec[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]], x]

[Out] (Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(3*d) - EllipticF[(c + d*x)/2, 8/7]/(3*Sqrt[7]*d) + (Sqrt[7]*EllipticPi[2, (c + d*x)/2, 8/7])/(3*d) - (Sqrt[3 + 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d)

Rule 2732

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2740

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2884

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[

$2*(b/(a + b)), (1/2)*(e - \text{Pi}/2 + f*x), 2*(d/(c + d))], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3138

Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\text{integral} = \frac{\sqrt{3 + 4 \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{6d} + \frac{1}{6} \int \frac{(-6 + 3 \cos(c + dx) + 2 \cos^2(c + dx)) \sec^2(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx$$

$$\begin{aligned}
&= -\frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} \\
&\quad + \frac{1}{18} \int \frac{(21+6\cos(c+dx)+12\cos^2(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} \\
&\quad - \frac{1}{72} \int \frac{(-84+12\cos(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx + \frac{1}{6} \int \sqrt{3+4\cos(c+dx)} dx \\
&= \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d} - \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} \\
&\quad + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} \\
&\quad - \frac{1}{6} \int \frac{1}{\sqrt{3+4\cos(c+dx)}} dx + \frac{7}{6} \int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx \\
&= \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{3d} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{3\sqrt{7}d} + \frac{\sqrt{7}\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{3d} \\
&\quad - \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.42

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx$$

$$= \frac{4\text{EllipticF}\left(\frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}} + \frac{18\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), \frac{8}{7}\right)}{\sqrt{7}} - \frac{2i\left(21E\left(i\text{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right)\middle|-\frac{1}{7}\right)-12\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{3+4\cos(c+dx)}\right)\right)\right)}{3\sqrt{7}\sqrt{\sin^2(c+dx)}}$$

[In] Integrate[Sec[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]

[Out] ((4*EllipticF[(c + d*x)/2, 8/7])/Sqrt[7] + (18*EllipticPi[2, (c + d*x)/2, 8/7])/Sqrt[7] - (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) - (-1 + 2*Cos[c + d*x])*Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(195) = 390.

Time = 3.09 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.98

method	result
default	$\frac{\sqrt{-\left(1-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)^2}+\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}\right)}$

[In] `int(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -\left(-\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(-\frac{1}{3}\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right. \\ & \left.(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)^2+2/3\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right) \\ & (-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^{\frac{1}{2}}\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right) \\ & -1/3\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\left(\frac{1}{2}\right)\left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right) \\ & \right)^{\frac{1}{2}}\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2\right)^{\frac{1}{2}}-1/3\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{\frac{1}{2}} \\ & \left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right) \\ & \right)^{\frac{1}{2}}\text{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2\right)^{\frac{1}{2}}-7/3\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{\frac{1}{2}} \\ & \left(1-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{\frac{1}{2}}\left(-8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4+7\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right) \\ & \right)^{\frac{1}{2}}\text{EllipticPi}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2,2\right)^{\frac{1}{2}}\right)/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/\left(8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)^{\frac{1}{2}}/d \end{aligned}$$

Fricas [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \int \frac{\sec(dx+c)^3}{\sqrt{4\cos(dx+c)+3}} dx$$

[In] `integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)`

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}} dx = \int \frac{\sec^3(c+dx)}{\sqrt{4\cos(c+dx)+3}} dx$$

[In] `integrate(sec(d*x+c)**3/(3+4*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sec(c + d*x)**3/sqrt(4*cos(c + d*x) + 3), x)`

Maxima [F]

$$\int \frac{\sec^3(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^3}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\sec^3(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^3}{\sqrt{4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c + dx)}{\sqrt{3 + 4 \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^3 \sqrt{4 \cos(c + dx) + 3}} dx$$

[In] int(1/(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2)),x)

[Out] int(1/(cos(c + d*x)^3*(4*cos(c + d*x) + 3)^(1/2)), x)

$$3.554 \quad \int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx$$

Optimal result	5586
Rubi [A] (verified)	5586
Mathematica [A] (verified)	5588
Maple [A] (verified)	5589
Fricas [C] (verification not implemented)	5589
Sympy [F(-1)]	5590
Maxima [F]	5590
Giac [F]	5590
Mupad [F(-1)]	5590

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = -\frac{9\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{20d} + \frac{23 \operatorname{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{20\sqrt{7}d}$$

$$- \frac{\sqrt{3-4\cos(c+dx)} \sin(c+dx)}{10d}$$

$$- \frac{\sqrt{3-4\cos(c+dx)} \cos(c+dx) \sin(c+dx)}{10d}$$

[Out] $-23/140*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+9/20*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}-1/10*\sin(d*x+c)*(3-4*\cos(d*x+c))^{(1/2)}/d-1/10*\cos(d*x+c)*\sin(d*x+c)*(3-4*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2872, 3102, 2831, 2741, 2733}

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = \frac{23 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx+\pi), \frac{8}{7}\right)}{20\sqrt{7}d} - \frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{20d}$$

$$- \frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)} \cos(c+dx)}{10d}$$

$$- \frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{10d}$$

[In] Int[Cos[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]

[Out] (-9*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*d) + (23*EllipticF[(c + Pi + d*x)/2, 8/7])/(20*Sqrt[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(10*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]*Sin[c + d*x])/(10*d)

Rule 2733

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2741

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2831

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2872

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & NeQ[c, 0])))

Rule 3102

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\sqrt{3-4\cos(c+dx)}\cos(c+dx)\sin(c+dx)}{10d} \\
 &\quad -\frac{1}{10}\int\frac{3-6\cos(c+dx)-6\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx \\
 &= -\frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{10d} \\
 &\quad -\frac{\sqrt{3-4\cos(c+dx)}\cos(c+dx)\sin(c+dx)}{10d} + \frac{1}{60}\int\frac{-6+54\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx \\
 &= -\frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{10d} - \frac{\sqrt{3-4\cos(c+dx)}\cos(c+dx)\sin(c+dx)}{10d} \\
 &\quad -\frac{9}{40}\int\sqrt{3-4\cos(c+dx)}dx + \frac{23}{40}\int\frac{1}{\sqrt{3-4\cos(c+dx)}}dx \\
 &= -\frac{9\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{20d} + \frac{23\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx),\frac{8}{7}\right)}{20\sqrt{7}d} \\
 &\quad -\frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{10d} - \frac{\sqrt{3-4\cos(c+dx)}\cos(c+dx)\sin(c+dx)}{10d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\begin{aligned}
 &\int\frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx \\
 &= \frac{9\sqrt{-3+4\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|8\right) + 23\sqrt{-3+4\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),8\right) - 4\sin(c+dx) - 2\sin[3(c+dx)]}{20d\sqrt{3-4\cos(c+dx)}}
 \end{aligned}$$

[In] Integrate[Cos[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]

[Out] (9*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 23*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] - 4*Sin[c + d*x] + Sin[2*(c + d*x)] + 2*Sin[3*(c + d*x)])/(20*d*Sqrt[3 - 4*Cos[c + d*x]])

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.25

method	result
default	$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-448\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+504\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{140\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}$

```
[In] int(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/140*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-448*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+504*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-56*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.20

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = \frac{4(\cos(dx+c)+1)\sqrt{-4\cos(dx+c)+3}\sin(dx+c)+7\sqrt{2}\text{weierstrassPInverse}(-1,-1,\cos(dx+c))}{\dots}$$

```
[In] integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/40*(4*(cos(d*x+c)+1)*sqrt(-4*cos(d*x+c)+3)*sin(d*x+c)+7*sqrt(2)*weierstrassPInverse(-1,-1,cos(d*x+c)+I*sin(d*x+c)-1/2)+7*sqrt(2)*weierstrassPInverse(-1,-1,cos(d*x+c)-I*sin(d*x+c)-1/2)-18*sqrt(2)*weierstrassZeta(-1,-1,weierstrassPInverse(-1,-1,cos(d*x+c)+I*sin(d*x+c)-1/2))-18*sqrt(2)*weierstrassZeta(-1,-1,weierstrassPInverse(-1,-1,cos(d*x+c)-I*sin(d*x+c)-1/2)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^3}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

```
[In] integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

Giac [F]

$$\int \frac{\cos^3(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^3}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

```
[In] integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^3}{\sqrt{3 - 4 \cos(c + dx)}} dx$$

```
[In] int(cos(c + d*x)^3/(3 - 4*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3/(3 - 4*cos(c + d*x))^(1/2), x)
```

$$3.555 \quad \int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx$$

Optimal result	5591
Rubi [A] (verified)	5591
Mathematica [A] (verified)	5593
Maple [A] (verified)	5593
Fricas [C] (verification not implemented)	5593
Sympy [F]	5594
Maxima [F]	5594
Giac [F]	5594
Mupad [B] (verification not implemented)	5595

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{4d} + \frac{17 \operatorname{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{12\sqrt{7}d} - \frac{\sqrt{3-4\cos(c+dx)} \sin(c+dx)}{6d}$$

[Out] $-17/84*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+1/4*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}-1/6*\sin(d*x+c)*(3-4*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2870, 2831, 2741, 2733}

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = \frac{17 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx+\pi), \frac{8}{7}\right)}{12\sqrt{7}d} - \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{4d} - \frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^2/\operatorname{Sqrt}[3-4*\operatorname{Cos}[c+d*x]],x]$

[Out] $-1/4*(\operatorname{Sqrt}[7]*\operatorname{EllipticE}[(c+\operatorname{Pi}+d*x)/2, 8/7])/d + (17*\operatorname{EllipticF}[(c+\operatorname{Pi}+d*x)/2, 8/7])/(12*\operatorname{Sqrt}[7]*d) - (\operatorname{Sqrt}[3-4*\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(6*d)$

Rule 2733

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a
- b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2741

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*S
qrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (
f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2870

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^2, x_Symbol] := Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])
^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])
^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{6d} - \frac{1}{6} \int \frac{-2-3\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{6d} \\
&\quad - \frac{1}{8} \int \sqrt{3-4\cos(c+dx)} dx + \frac{17}{24} \int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{4d} + \frac{17\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{12\sqrt{7}d} - \frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx$$

$$= \frac{3\sqrt{-3 + 4 \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 8\right) + 17\sqrt{-3 + 4 \cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 8\right) - 6 \sin(c + dx)}{12d\sqrt{3 - 4 \cos(c + dx)}}$$

[In] Integrate[Cos[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]

[Out] (3*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 17*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] - 6*Sin[c + d*x] + 4*Sin[2*(c + d*x)])/(12*d*Sqrt[3 - 4*Cos[c + d*x]])

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.90

method	result
default	$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 7\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(224\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 28\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 17\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right)}{84\sqrt{8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} \sin\left(\frac{dx}{2} + \frac{c}{2}\right)$

[In] int(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/84*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(224*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-28*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+17*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.60

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx =$$

$$\frac{4\sqrt{-4 \cos(dx + c) + 3 \sin(dx + c)} + 7\sqrt{2} \operatorname{weierstrassPInverse}(-1, -1, \cos(dx + c) + i \sin(dx + c)) - \dots}{\dots}$$

[In] integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $-1/24*(4*\sqrt{-4*\cos(dx + c) + 3}*\sin(dx + c) + 7*\sqrt{2}*\text{weierstrassPInverse}(-1, -1, \cos(dx + c) + I*\sin(dx + c) - 1/2) + 7*\sqrt{2}*\text{weierstrassPInverse}(-1, -1, \cos(dx + c) - I*\sin(dx + c) - 1/2) - 6*\sqrt{2}*\text{weierstrassZeta}(-1, -1, \text{weierstrassPInverse}(-1, -1, \cos(dx + c) + I*\sin(dx + c) - 1/2)) - 6*\sqrt{2}*\text{weierstrassZeta}(-1, -1, \text{weierstrassPInverse}(-1, -1, \cos(dx + c) - I*\sin(dx + c) - 1/2)))/d$

Sympy [F]

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 - 4\cos(c + dx)}} dx = \int \frac{\cos^2(c + dx)}{\sqrt{3 - 4\cos(c + dx)}} dx$$

[In] integrate(cos(d*x+c)**2/(3-4*cos(d*x+c))**(1/2),x)

[Out] Integral(cos(c + d*x)**2/sqrt(3 - 4*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 - 4\cos(c + dx)}} dx = \int \frac{\cos(dx + c)^2}{\sqrt{-4\cos(dx + c) + 3}} dx$$

[In] integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 - 4\cos(c + dx)}} dx = \int \frac{\cos(dx + c)^2}{\sqrt{-4\cos(dx + c) + 3}} dx$$

[In] integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{\cos^2(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \frac{\sqrt{4 \cos(c + dx) - 3} (6 E(\frac{c}{2} + \frac{dx}{2} | 8) + 34 F(\frac{c}{2} + \frac{dx}{2} | 8))}{24 d \sqrt{3 - 4 \cos(c + dx)}} - \frac{\sin(c + dx) \sqrt{3 - 4 \cos(c + dx)}}{6 d}$$

[In] int(cos(c + d*x)^2/(3 - 4*cos(c + d*x))^(1/2),x)

```
[Out] ((4*cos(c + d*x) - 3)^(1/2)*(6*ellipticE(c/2 + (d*x)/2, 8) + 34*ellipticF(c/2 + (d*x)/2, 8)))/(24*d*(3 - 4*cos(c + d*x))^(1/2)) - (sin(c + d*x)*(3 - 4*cos(c + d*x))^(1/2))/(6*d)
```

3.556 $\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx$

Optimal result	5596
Rubi [A] (verified)	5596
Mathematica [A] (verified)	5597
Maple [A] (verified)	5597
Fricas [C] (verification not implemented)	5598
Sympy [F]	5599
Maxima [F]	5599
Giac [F]	5599
Mupad [B] (verification not implemented)	5599

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{2d} + \frac{3\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{2\sqrt{7}d}$$

[Out] $-3/14*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+1/2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2831, 2741, 2733}

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = \frac{3\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi), \frac{8}{7}\right)}{2\sqrt{7}d} - \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{2d}$$

[In] `Int[Cos[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]], x]`

[Out] $-1/2*(\text{Sqrt}[7]*\text{EllipticE}[(c + \text{Pi} + d*x)/2, 8/7])/d + (3*\text{EllipticF}[(c + \text{Pi} + d*x)/2, 8/7])/(2*\text{Sqrt}[7]*d)$

Rule 2733

`Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]`

Rule 2741

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2831

```
Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\left(\frac{1}{4} \int \sqrt{3 - 4 \cos(c + dx)} dx\right) + \frac{3}{4} \int \frac{1}{\sqrt{3 - 4 \cos(c + dx)}} dx \\ &= -\frac{\sqrt{7} E\left(\frac{1}{2}(c + \pi + dx) \middle| \frac{8}{7}\right)}{2d} + \frac{3 \text{EllipticF}\left(\frac{1}{2}(c + \pi + dx), \frac{8}{7}\right)}{2\sqrt{7}d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \frac{\cos(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \frac{\sqrt{-3 + 4 \cos(c + dx)} (E(\frac{1}{2}(c + dx) | 8) + 3 \text{EllipticF}(\frac{1}{2}(c + dx), 8))}{2d \sqrt{3 - 4 \cos(c + dx)}}$$

```
[In] Integrate[Cos[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[-3 + 4*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, 8] + 3*EllipticF[(c + d*x)/2, 8]))/(2*d*Sqrt[3 - 4*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.98

method	result
default	$-\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7}\left(3F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2\sqrt{14}}{7}\right)-7E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\frac{2\sqrt{14}}{7}\right)\right)+7d$ $i\left(\frac{-2e^{2i(dx+c)}+3e^{i(dx+c)}-2}{\sqrt{(-2e^{2i(dx+c)}+3e^{i(dx+c)}-2)e^{i(dx+c)}}}\right)+\frac{2\left(-\frac{3}{4}+\frac{i\sqrt{7}}{4}\right)\sqrt{\frac{e^{i(dx+c)}-\frac{3}{4}+\frac{i\sqrt{7}}{4}}{-\frac{3}{4}+\frac{i\sqrt{7}}{4}}}\sqrt{14}}{\sqrt{14}}$
risch	$-\frac{i(2e^{2i(dx+c)}-3e^{i(dx+c)}+2)e^{-i(dx+c)}}{2d\sqrt{-(2e^{2i(dx+c)}-3e^{i(dx+c)}+2)e^{-i(dx+c)}}}$

```
[In] int(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/14*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-7*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.00

$$\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = \frac{\sqrt{2}\text{weierstrassPInverse}(-1,-1,\cos(dx+c)+i\sin(dx+c)-\frac{1}{2})+\sqrt{2}\text{weierstrassPInverse}(-1,-1,\cos(dx+c)-i\sin(dx+c)-\frac{1}{2})}{2}$$

```
[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*weierstrassPInverse(-1,-1,cos(d*x+c)+I*sin(d*x+c)-1/2)+sqrt(2)*weierstrassPInverse(-1,-1,cos(d*x+c)-I*sin(d*x+c)-1/2)-2*sqrt(2)*weierstrassZeta(-1,-1,weierstrassPInverse(-1,-1,cos(d*x+c)+I*sin(d*x+c)-1/2))-2*sqrt(2)*weierstrassZeta(-1,-1,weierstrassPInverse(-1,-1,cos(d*x+c)-I*sin(d*x+c)-1/2)))/d
```

Sympy [F]

$$\int \frac{\cos(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\cos(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx$$

[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))**(1/2),x)

[Out] Integral(cos(c + d*x)/sqrt(3 - 4*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{\cos(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\cos(dx + c)}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\cos(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\cos(dx + c)}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)

Mupad [B] (verification not implemented)

Time = 15.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\cos(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \frac{\sqrt{4 \cos(c + dx) - 3} \left(E\left(\frac{c}{2} + \frac{dx}{2} \middle| 8\right) + 3 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 8\right) \right)}{2 d \sqrt{3 - 4 \cos(c + dx)}}$$

[In] int(cos(c + d*x)/(3 - 4*cos(c + d*x))^(1/2),x)

[Out] ((4*cos(c + d*x) - 3)^(1/2)*(ellipticE(c/2 + (d*x)/2, 8) + 3*ellipticF(c/2 + (d*x)/2, 8)))/(2*d*(3 - 4*cos(c + d*x))^(1/2))

$$3.557 \quad \int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx$$

Optimal result	5600
Rubi [A] (verified)	5600
Mathematica [A] (verified)	5601
Maple [C] (verified)	5601
Fricas [C] (verification not implemented)	5601
Sympy [F]	5602
Maxima [F]	5602
Giac [F]	5602
Mupad [B] (verification not implemented)	5602

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[Out] $-2/7*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2741}

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx+\pi), \frac{8}{7}\right)}{\sqrt{7}d}$$

[In] `Int[1/Sqrt[3 - 4*Cos[c + d*x]],x]`

[Out] `(2*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d)`

Rule 2741

`Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]`

Rubi steps

$$\text{integral} = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx = \frac{2\sqrt{-3+4\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

[In] Integrate[1/Sqrt[3 - 4*Cos[c + d*x]],x]

[Out] (2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/(d*Sqrt[3 - 4*Cos[c + d*x]])

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

method	result	size
default	$\frac{2\sqrt{8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7} \operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\mid 2\sqrt{2}\right)}{d\sqrt{-8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7}}$	54

[In] int(1/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/d/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)*(8*cos(1/2*d*x+1/2*c)^2-7)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2*2^(1/2))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx = \frac{\sqrt{2}\operatorname{weierstrassPInverse}\left(-1, -1, \cos(dx+c) + i\sin(dx+c) - \frac{1}{2}\right) + \sqrt{2}\operatorname{weierstrassPInverse}\left(-1, -1, \cos(dx+c) - i\sin(dx+c) - \frac{1}{2}\right)}{2d}$$

[In] integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -1/2*(sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) + I*sin(d*x + c) - 1/2) + sqrt(2)*weierstrassPInverse(-1, -1, cos(d*x + c) - I*sin(d*x + c) - 1/2))/d

Sympy [F]

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx = \int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx$$

```
[In] integrate(1/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3 - 4*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-4\cos(dx+c)+3}} dx$$

```
[In] integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-4*cos(d*x + c) + 3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-4\cos(dx+c)+3}} dx$$

```
[In] integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-4*cos(d*x + c) + 3), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx = \frac{2\sqrt{4\cos(c+dx)-3}F\left(\frac{c}{2} + \frac{dx}{2} \middle| 8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

```
[In] int(1/(3 - 4*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*(4*cos(c + d*x) - 3)^(1/2)*ellipticF(c/2 + (d*x)/2, 8))/(d*(3 - 4*cos(c + d*x))^(1/2))
```

$$3.558 \quad \int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx$$

Optimal result	5603
Rubi [A] (verified)	5603
Mathematica [A] (verified)	5604
Maple [B] (verified)	5604
Fricas [F]	5604
Sympy [F]	5605
Maxima [F]	5605
Giac [F]	5605
Mupad [F(-1)]	5605

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = -\frac{2 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

[Out] $2/7*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\cos(1/2*d*x+1/2*c), 2, 2/7*14^{(1/2)})/d*7^{(1/2)}$

Rubi [A] (verified)

Time = 0.04 (sec), antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2885}

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = -\frac{2 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+dx+\pi), \frac{8}{7}\right)}{\sqrt{7}d}$$

[In] `Int[Sec[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]], x]`

[Out] `(-2*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d)`

Rule 2885

`Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c - d, 0]`

Rubi steps

$$\text{integral} = -\frac{2 \operatorname{EllipticPi}\left(2, \frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{\sec(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \frac{2\sqrt{-3 + 4 \cos(c + dx)} \operatorname{EllipticPi}\left(2, \frac{1}{2}(c + dx), 8\right)}{d\sqrt{3 - 4 \cos(c + dx)}}$$

[In] Integrate[Sec[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]

[Out] (2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/(d*Sqrt[3 - 4*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(48) = 96.

Time = 1.74 (sec) , antiderivative size = 139, normalized size of antiderivative = 5.56

method	result	size
default	$\frac{2\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 7\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{56\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 7}\Pi\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), 2, \frac{2\sqrt{14}}{7}\right)}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-8\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 7}d}$	139

[In] int(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/7*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d

Fricas [F]

$$\int \frac{\sec(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)/(4*cos(d*x + c) - 3), x)

Sympy [F]

$$\int \frac{\sec(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\sec(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx$$

[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)/sqrt(3 - 4*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{\sec(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\sec(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx) \sqrt{3 - 4 \cos(c + dx)}} dx$$

[In] int(1/(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)*(3 - 4*cos(c + d*x))^(1/2)), x)

$$3.559 \quad \int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx$$

Optimal result	5606
Rubi [A] (verified)	5606
Mathematica [C] (verified)	5608
Maple [B] (verified)	5609
Fricas [F]	5609
Sympy [F]	5609
Maxima [F]	5610
Giac [F]	5610
Mupad [F(-1)]	5610

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{\sqrt{7}d}$$

$$- \frac{4\text{EllipticPi}\left(2, \frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{3\sqrt{7}d}$$

$$+ \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

[Out] $-1/7*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+4/21*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\text{EllipticPi}(\cos(1/2*d*x+1/2*c), 2, 2/7*14^{(1/2)})/d*7^{(1/2)}+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2/7*14^{(1/2)})/d*7^{(1/2)}+1/3*(3-4*\cos(d*x+c))^{(1/2)}*\tan(d*x+c)/d$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2881, 3139, 2733, 3081, 2741, 2885}

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi), \frac{8}{7}\right)}{\sqrt{7}d} - \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{3d}$$

$$- \frac{4\text{EllipticPi}\left(2, \frac{1}{2}(c+dx+\pi), \frac{8}{7}\right)}{3\sqrt{7}d}$$

$$+ \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

[In] Int[Sec[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]

[Out] $-1/3 \cdot (\sqrt{7} \operatorname{EllipticE}[(c + \pi + dx)/2, 8/7])/d + \operatorname{EllipticF}[(c + \pi + dx)/2, 8/7]/(\sqrt{7} \cdot d) - (4 \cdot \operatorname{EllipticPi}[2, (c + \pi + dx)/2, 8/7])/(3 \cdot \sqrt{7} \cdot d) + (\sqrt{3 - 4 \cdot \cos[c + dx]} \cdot \tan[c + dx])/(3 \cdot d)$

Rule 2733

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2741

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2885

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c - d, 0]

Rule 3081

Int((((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_))/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3139

Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{1}{3} \int \frac{(2+2\cos^2(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
 &= \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} \\
 &\quad + \frac{1}{12} \int \frac{(8+6\cos(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx - \frac{1}{6} \int \sqrt{3-4\cos(c+dx)} dx \\
 &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} \\
 &\quad + \frac{1}{2} \int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx + \frac{2}{3} \int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
 &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{\sqrt{7}d} \\
 &\quad - \frac{4\text{EllipticPi}\left(2, \frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{3\sqrt{7}d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.72

$$\begin{aligned}
 &\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
 &= \frac{6\sqrt{-3+4\cos(c+dx)}\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), 8\right)}{\sqrt{3-4\cos(c+dx)}} - \frac{i\left(21E\left(i\text{ArcSinh}\left(\sqrt{3-4\cos(c+dx)}\right)\middle|-\frac{1}{7}\right)-12\text{EllipticF}\left(i\text{ArcSinh}\left(\sqrt{3-4\cos(c+dx)}\right), -\frac{1}{7}\right)\right)}{3\sqrt{7}\sqrt{\sin^2(c+dx)}} \\
 &= \frac{\hspace{15em}}{3d}
 \end{aligned}$$

[In] Integrate[Sec[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]], x]

[Out] ((6*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c + d*x]] - ((I/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x]/(3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(166) = 332$.

Time = 2.89 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.38

method	result
default	$\frac{\sqrt{-\left(8\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-\frac{2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}+\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{56\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{7\sqrt{8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}\right)}$

[In] `int(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -\left(-\left(8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-7\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(-\frac{2}{3}\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right) \\ & \left(8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4-\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right) \\ & +\frac{1}{7}\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(56\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-7\right)^{\frac{1}{2}} \\ & \left(8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4-\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),\frac{2}{7}\sqrt{14}\right) \\ & -\frac{1}{3}\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(56\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-7\right)^{\frac{1}{2}} \\ & \left(8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4-\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\text{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),\frac{2}{7}\sqrt{14}\right) \\ & -\frac{4}{21}\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(56\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-7\right)^{\frac{1}{2}} \\ & \left(8\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4-\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\text{EllipticPi}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),\frac{2}{7}\sqrt{14}\right) \\ & \left. \right)/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/\left(-8\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+7\right)^{\frac{1}{2}}/d \end{aligned}$$

Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = \int \frac{\sec(dx+c)^2}{\sqrt{-4\cos(dx+c)+3}} dx$$

[In] `integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-4*cos(d*x+c)+3)*sec(d*x+c)^2/(4*cos(d*x+c)-3),x)`

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = \int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx$$

[In] `integrate(sec(d*x+c)**2/(3-4*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sec(c+d*x)**2/sqrt(3-4*cos(c+d*x)),x)`

Maxima [F]

$$\int \frac{\sec^2(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^2}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\sec^2(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^2}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^2 \sqrt{3 - 4 \cos(c + dx)}} dx$$

[In] int(1/(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^2*(3 - 4*cos(c + d*x))^(1/2)), x)

$$3.560 \quad \int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx$$

Optimal result	5611
Rubi [A] (verified)	5611
Mathematica [C] (verified)	5614
Maple [B] (verified)	5615
Fricas [F]	5615
Sympy [F]	5615
Maxima [F]	5616
Giac [F]	5616
Mupad [F(-1)]	5616

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{3\sqrt{7}d}$$

$$- \frac{\sqrt{7}\text{EllipticPi}\left(2, \frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{3d}$$

$$+ \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

$$+ \frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d}$$

```
[Out] -1/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticF(cos(1/2*d*x+1/2*c), 2/7*14^(1/2))/d*7^(1/2)+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticE(cos(1/2*d*x+1/2*c), 2/7*14^(1/2))/d*7^(1/2)+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)*EllipticPi(cos(1/2*d*x+1/2*c), 2, 2/7*14^(1/2))/d*7^(1/2)+1/3*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d+1/6*sec(d*x+c)*(3-4*cos(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {2881, 3134, 3138, 2733, 3081, 2741, 2885}

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx = \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi), \frac{8}{7}\right)}{3\sqrt{7}d} - \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{3d} - \frac{\sqrt{7}\text{EllipticPi}\left(2, \frac{1}{2}(c+dx+\pi), \frac{8}{7}\right)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)\sec(c+dx)}{6d}$$

[In] Int[Sec[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]], x]

[Out] -1/3*(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d + EllipticF[(c + Pi + d*x)/2, 8/7]/(3*Sqrt[7]*d) - (Sqrt[7]*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d)

Rule 2733

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a - b]/d)*EllipticE[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2741

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/(d*Sqrt[a - b]))*EllipticF[(1/2)*(c + Pi/2 + d*x), -2*(b/(a - b))], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]

Rule 2881

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2885

Int[1/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[(2/(f*(a - b)*Sqrt[c - d]))*EllipticPi[


```
-2*(b/(a - b)), (1/2)*(e + Pi/2 + f*x), -2*(d/(c - d)), x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(- (A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x])*(c + d*Sin[e + f*x]), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{3 - 4 \cos(c + dx)} \sec(c + dx) \tan(c + dx)}{6d} + \frac{1}{6} \int \frac{(6 + 3 \cos(c + dx) - 2 \cos^2(c + dx)) \sec^2(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx$$

$$\begin{aligned}
&= \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} \\
&\quad + \frac{1}{18} \int \frac{(21-6\cos(c+dx)+12\cos^2(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
&= \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} \\
&\quad + \frac{1}{72} \int \frac{(84+12\cos(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx - \frac{1}{6} \int \sqrt{3-4\cos(c+dx)} dx \\
&= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} \\
&\quad + \frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} \\
&\quad + \frac{1}{6} \int \frac{1}{\sqrt{3-4\cos(c+dx)}} dx + \frac{7}{6} \int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx \\
&= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{3d} + \frac{\text{EllipticF}\left(\frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{3\sqrt{7}d} \\
&\quad - \frac{\sqrt{7}\text{EllipticPi}\left(2, \frac{1}{2}(c+\pi+dx), \frac{8}{7}\right)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} \\
&\quad + \frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.69

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}} dx$$

$$= -\frac{4\sqrt{-3+4\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 8\right)}{\sqrt{3-4\cos(c+dx)}} + \frac{18\sqrt{-3+4\cos(c+dx)}\text{EllipticPi}\left(2, \frac{1}{2}(c+dx), 8\right)}{\sqrt{3-4\cos(c+dx)}} - \frac{2i\left(21E\left(i\text{arcsinh}\left(\sqrt{3-4\cos(c+dx)}\right)\middle|-\frac{1}{7}\right)\right)}{\sqrt{3-4\cos(c+dx)}}$$

[In] Integrate[Sec[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]], x]

[Out] ((-4*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c + d*x]] + (18*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c + d*x]] - (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + Sqrt[3 - 4*Cos[c + d*x]]*(1 + 2*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(6*d)

Maxima [F]

$$\int \frac{\sec^3(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^3}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\sec^3(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^3}{\sqrt{-4 \cos(dx + c) + 3}} dx$$

[In] integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c + dx)}{\sqrt{3 - 4 \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^3 \sqrt{3 - 4 \cos(c + dx)}} dx$$

[In] int(1/(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^3*(3 - 4*cos(c + d*x))^(1/2)), x)

3.561 $\int \cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx)) dx$

Optimal result	5617
Rubi [A] (verified)	5617
Mathematica [A] (verified)	5619
Maple [A] (verified)	5619
Fricas [C] (verification not implemented)	5620
Sympy [F(-1)]	5620
Maxima [F]	5620
Giac [F]	5621
Mupad [B] (verification not implemented)	5621

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx)) dx = \frac{6AE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{10B \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d}$$

$$+ \frac{10B \sqrt{\cos(c + dx)} \sin(c + dx)}{21d}$$

$$+ \frac{2A \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}$$

$$+ \frac{2B \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d}$$

[Out] $6/5*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+10/21*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*A*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/7*B*\cos(d*x+c)^{(5/2)}*\sin(d*x+c)/d+10/21*B*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

= {2827, 2715, 2719, 2720}

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))dx = \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d} \\ + \frac{10B\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} \\ + \frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d} \\ + \frac{10B\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

[In] Int[Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]),x]

[Out] (6*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*B*EllipticF[(c + d*x)/2, 2])/(21*d) + (10*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n-1)/(d*n)), x] + Dist[b^2*((n-1)/n), Int[(b*Ssin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\text{integral} = A \int \cos^{\frac{5}{2}}(c+dx)dx + B \int \cos^{\frac{7}{2}}(c+dx)dx \\ = \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2B\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} \\ + \frac{1}{5}(3A) \int \sqrt{\cos(c+dx)}dx + \frac{1}{7}(5B) \int \cos^{\frac{3}{2}}(c+dx)dx$$

$$\begin{aligned}
&= \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10B\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} \\
&\quad + \frac{2B\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{21}(5B)\int\frac{1}{\sqrt{\cos(c+dx)}}dx \\
&= \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10B\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{10B\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} \\
&\quad + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2B\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\begin{aligned}
&\int\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))dx \\
&= \frac{126AE\left(\frac{1}{2}(c+dx)\middle|2\right)+50B\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(65B+42A\cos(c+dx))+15B\cos(c+dx)}{105d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]),x]

[Out] (126*A*EllipticE[(c + d*x)/2, 2] + 50*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)

Maple [A] (verified)

Time = 9.74 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.61

method	result
default	$-\frac{2\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$
parts	$-\frac{2A\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))

$$)+25*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))dx$$

$$= \frac{2(15B\cos(dx+c)^2+21A\cos(dx+c)+25B)\sqrt{\cos(dx+c)}\sin(dx+c)-25i\sqrt{2}B\text{weierstrassPInverse}(\dots)}{d}$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/105*(2*(15*B*cos(d*x + c)^2 + 21*A*cos(d*x + c) + 25*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 25*I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))dx = \int (B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}dx$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A) \cos(dx + c)^{\frac{5}{2}} dx$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 15.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\begin{aligned} & \int \cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx)) dx \\ &= -\frac{2 A \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7 d \sqrt{\sin(c + dx)^2}} \\ & \quad - \frac{2 B \cos(c + dx)^{9/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c + dx)^2\right)}{9 d \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)),x)

[Out] - (2*A*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*B*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))

3.562 $\int \cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx)) dx$

Optimal result	5622
Rubi [A] (verified)	5622
Mathematica [A] (verified)	5624
Maple [B] (verified)	5624
Fricas [C] (verification not implemented)	5625
Sympy [F(-1)]	5625
Maxima [F]	5625
Giac [F]	5626
Mupad [B] (verification not implemented)	5626

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx)) dx = \frac{6BE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2A \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\ + \frac{2A \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} \\ + \frac{2B \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}$$

[Out] $6/5*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*B*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/3*A*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2827, 2715, 2720, 2719}

$$\int \cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx)) dx = \frac{2A \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\ + \frac{2A \sin(c + dx) \sqrt{\cos(c + dx)}}{3d} \\ + \frac{6BE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} \\ + \frac{2B \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d}$$

[In] Int[Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]),x]

[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*A*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= A \int \cos^{\frac{3}{2}}(c + dx) dx + B \int \cos^{\frac{5}{2}}(c + dx) dx \\
 &= \frac{2A \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2B \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} \\
 &\quad + \frac{1}{3} A \int \frac{1}{\sqrt{\cos(c + dx)}} dx + \frac{1}{5} (3B) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{6BE \left(\frac{1}{2}(c + dx) \middle| 2 \right)}{5d} + \frac{2A \text{EllipticF} \left(\frac{1}{2}(c + dx), 2 \right)}{3d} \\
 &\quad + \frac{2A \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2B \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))dx$$

$$= \frac{2\left(9BE\left(\frac{1}{2}(c+dx)\middle|2\right)+5A\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))\sin(c+dx)\right)}{15d}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]),x]

[Out] (2*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(127) = 254.

Time = 7.90 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.01

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-10A+6B)\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$

[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx)) dx$$

$$= \frac{2(3B \cos(dx + c) + 5A)\sqrt{\cos(dx + c)} \sin(dx + c) - 5i\sqrt{2}A \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i)}{d}$$

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/15*(2*(3*B*cos(d*x + c) + 5*A)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A) \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A) \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\begin{aligned} & \int \cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx)) dx \\ &= \frac{2 A F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{3 d} + \frac{2 A \sqrt{\cos(c + dx)} \sin(c + dx)}{3 d} \\ & \quad - \frac{2 B \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7 d \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)),x)

[Out] (2*A*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*A*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*B*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))

3.563 $\int \sqrt{\cos(c + dx)}(A + B \cos(c + dx)) dx$

Optimal result	5627
Rubi [A] (verified)	5627
Mathematica [A] (verified)	5628
Maple [B] (verified)	5629
Fricas [C] (verification not implemented)	5629
Sympy [F]	5630
Maxima [F]	5630
Giac [F]	5630
Mupad [B] (verification not implemented)	5630

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \sqrt{\cos(c + dx)}(A + B \cos(c + dx)) dx = \frac{2AE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2B \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2B \sqrt{\cos(c + dx)} \sin(c + dx)}{3d}$$

[Out] $2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*B*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2827, 2719, 2715, 2720}

$$\int \sqrt{\cos(c + dx)}(A + B \cos(c + dx)) dx = \frac{2AE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2B \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2B \sin(c + dx) \sqrt{\cos(c + dx)}}{3d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*(A + B*\operatorname{Cos}[c + d*x]), x]$

[Out] $(2*A*\operatorname{EllipticE}[(c + d*x)/2, 2])/d + (2*B*\operatorname{EllipticF}[(c + d*x)/2, 2])/(3*d) + (2*B*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sin}[c + d*x])/(3*d)$

Rule 2715

$\operatorname{Int}[(b_* \sin[(c_*) + (d_*)*(x_*)])^{(n_*)}, x_Symbol] \rightarrow \operatorname{Simp}[(-b)*\operatorname{Cos}[c + d*x]*((b*\operatorname{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \operatorname{Dist}[b^2*((n-1)/n), \operatorname{Int}[(b*\operatorname{Sin}[$

$c + d*x)^{(n - 2), x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2 * n]$

Rule 2719

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \ :> \ \text{Simp}[(2/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \ :> \ \text{Simp}[(2/d)*\text{EllipticF}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2827

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \ :> \ \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= A \int \sqrt{\cos(c + dx)} dx + B \int \cos^{\frac{3}{2}}(c + dx) dx \\ &= \frac{2AE(\frac{1}{2}(c + dx)|2)}{d} + \frac{2B\sqrt{\cos(c + dx)}\sin(c + dx)}{3d} + \frac{1}{3}B \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\ &= \frac{2AE(\frac{1}{2}(c + dx)|2)}{d} + \frac{2B \text{EllipticF}(\frac{1}{2}(c + dx), 2)}{3d} + \frac{2B\sqrt{\cos(c + dx)}\sin(c + dx)}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\begin{aligned} &\int \sqrt{\cos(c + dx)}(A + B \cos(c + dx)) dx \\ &= \frac{2\left(3AE\left(\frac{1}{2}(c + dx)|2\right) + B\left(\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sqrt{\cos(c + dx)}\sin(c + dx)\right)\right)}{3d} \end{aligned}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] (2*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(107) = 214$.

Time = 6.60 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.75

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right),\sqrt{2}}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$

[In] `int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{2}{3}*\left(\left(2*\cos\left(1/2*d*x+1/2*c\right)^2-1\right)*\sin\left(1/2*d*x+1/2*c\right)^2\right)^{1/2}*(-4*B*\cos\left(1/2*d*x+1/2*c\right)*\sin\left(1/2*d*x+1/2*c\right)^4+3*A*\left(\sin\left(1/2*d*x+1/2*c\right)^2\right)^{1/2}*\left(2*\sin\left(1/2*d*x+1/2*c\right)^2-1\right)^{1/2}*EllipticE\left(\cos\left(1/2*d*x+1/2*c\right),2^{1/2}\right)+2*B*\cos\left(1/2*d*x+1/2*c\right)*\sin\left(1/2*d*x+1/2*c\right)^2-B*\left(\sin\left(1/2*d*x+1/2*c\right)^2\right)^{1/2}*\left(2*\sin\left(1/2*d*x+1/2*c\right)^2-1\right)^{1/2}*EllipticF\left(\cos\left(1/2*d*x+1/2*c\right),2^{1/2}\right))/(-2*\sin\left(1/2*d*x+1/2*c\right)^4+\sin\left(1/2*d*x+1/2*c\right)^2)^{1/2}/\sin\left(1/2*d*x+1/2*c\right)/(2*\cos\left(1/2*d*x+1/2*c\right)^2-1)^{1/2}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.05

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))dx$$

$$= \frac{2B\sqrt{\cos(dx+c)}\sin(dx+c) - i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}B\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{d}$$

[In] `integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{3}*(2*B*\sqrt{\cos(d*x+c)}*\sin(d*x+c) - I*\sqrt{2}*B*\text{weierstrassPInverse}(-4,0,\cos(d*x+c)+I*\sin(d*x+c)) + I*\sqrt{2}*B*\text{weierstrassPInverse}(-4,0,\cos(d*x+c)-I*\sin(d*x+c)) + 3*I*\sqrt{2}*A*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d*x+c)+I*\sin(d*x+c))) - 3*I*\sqrt{2}*A*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d*x+c)-I*\sin(d*x+c))))/d$$

Sympy [F]

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx)) dx = \int (A+B\cos(c+dx)) \sqrt{\cos(c+dx)} dx$$

[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)

[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x)), x)

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx)) dx = \int (B\cos(dx+c)+A)\sqrt{\cos(dx+c)} dx$$

[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx)) dx = \int (B\cos(dx+c)+A)\sqrt{\cos(dx+c)} dx$$

[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx)) dx = \frac{2A E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2B F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{3d} + \frac{2B \sqrt{\cos(c+dx)} \sin(c+dx)}{3d}$$

[In] int(cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)),x)

[Out] (2*A*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*B*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d)

$$3.564 \quad \int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	5631
Rubi [A] (verified)	5631
Mathematica [A] (verified)	5632
Maple [A] (verified)	5632
Fricas [C] (verification not implemented)	5633
Sympy [F]	5633
Maxima [F]	5633
Giac [F]	5634
Mupad [B] (verification not implemented)	5634

Optimal result

Integrand size = 21, antiderivative size = 35

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \frac{2BE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2A \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

[Out] $2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d$

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2827, 2720, 2719}

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \frac{2A \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2BE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

[In] $\operatorname{Int}[(A + B*\operatorname{Cos}[c + d*x])/Sqrt[\operatorname{Cos}[c + d*x]], x]$

[Out] $(2*B*\operatorname{EllipticE}[(c + d*x)/2, 2])/d + (2*A*\operatorname{EllipticF}[(c + d*x)/2, 2])/d$

Rule 2719

$\operatorname{Int}[Sqrt[\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \rightarrow \operatorname{Simp}[(2/d)*\operatorname{EllipticE}[(1/2)*(c - \operatorname{Pi}/2 + d*x), 2], x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2720

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned} \text{integral} &= A \int \frac{1}{\sqrt{\cos(c + dx)}} dx + B \int \sqrt{\cos(c + dx)} dx \\ &= \frac{2BE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2A \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \frac{2BE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2A \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

`[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[Cos[c + d*x]], x]`

`[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/d`

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.34

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\left(AF\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - BE\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}d}$
parts	$\frac{2A \operatorname{am}^{-1}\left(\frac{dx}{2} + \frac{c}{2} \mid \sqrt{2}\right)}{d} + \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}d}$
risch	$-\frac{iB\left(e^{2i(dx+c)} + 1\right)\sqrt{2}e^{-i(dx+c)}}{d\sqrt{\left(e^{2i(dx+c)} + 1\right)e^{-i(dx+c)}}} - i\left(\frac{iA\sqrt{-i\left(e^{i(dx+c)} + i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)} - i\right)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i\left(e^{i(dx+c)} + i\right)}, \frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)} + e^{i(dx+c)}}}\right) + B\left(-\frac{2\left(e^{2i(dx+c)} + 1\right)\sqrt{2}e^{-i(dx+c)}}{\sqrt{\left(e^{2i(dx+c)} + 1\right)e^{-i(dx+c)}}}\right)$

`[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2), x, method=_RETURNVERBOSE)`

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.06

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{-i \sqrt{2} A \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + B \sqrt{2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - i \sqrt{2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{d}$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{\sqrt{\cos(dx + c)}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx = \frac{2 A F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{2 B E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d}$$

[In] int((A + B*cos(c + d*x))/cos(c + d*x)^(1/2),x)

[Out] (2*A*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*ellipticE(c/2 + (d*x)/2, 2))/d

$$3.565 \quad \int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	5635
Rubi [A] (verified)	5635
Mathematica [A] (verified)	5636
Maple [A] (verified)	5637
Fricas [C] (verification not implemented)	5637
Sympy [F]	5638
Maxima [F]	5638
Giac [F]	5638
Mupad [B] (verification not implemented)	5638

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx = -\frac{2AE\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2B \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} + \frac{2A \sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[Out] $-2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*A*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2827, 2716, 2719, 2720}

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx = -\frac{2AE\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2A \sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2B \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d}$$

[In] $\operatorname{Int}[(A+B*\operatorname{Cos}[c+d*x])/(\operatorname{Cos}[c+d*x])^{(3/2)}, x]$

[Out] $(-2*A*\operatorname{EllipticE}[(c+d*x)/2, 2])/d + (2*B*\operatorname{EllipticF}[(c+d*x)/2, 2])/d + (2*A*\operatorname{Sin}[c+d*x])/(d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])$

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= A \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + B \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2B \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2A \sin(c + dx)}{d \sqrt{\cos(c + dx)}} - A \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{2AE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2B \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{2A \sin(c + dx)}{d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2\left(-AE\left(\frac{1}{2}(c + dx) \mid 2\right) + B \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \frac{A \sin(c + dx)}{\sqrt{\cos(c + dx)}}\right)}{d}$$

```
[In] Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(3/2), x]
```

```
[Out] (2*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c + d*x)/2, 2] + (A*Sin[c
+ d*x])/Sqrt[Cos[c + d*x]]))/d
```


Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

method	result
default	$\frac{4A \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - 2B \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}{\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}$
parts	$\frac{2A \left(-2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}$

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.74

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{-i \sqrt{2} B \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + I \sqrt{2} B \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - I \sqrt{2} B \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + I \sqrt{2} A \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + I \sqrt{2} A \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2 A \sqrt{2} \cos(dx + c) \sin(dx + c) / (d \cos(dx + c))}{1}$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*A*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*A*sqrt(2)*cos(d*x + c)*sin(d*x + c)/(d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)

[Out] Integral((A + B*cos(c + d*x))/cos(c + d*x)**(3/2), x)

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{B \cos(dx + c) + A}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{B \cos(dx + c) + A}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 15.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2 B F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{2 A \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

[In] int((A + B*cos(c + d*x))/cos(c + d*x)^(3/2),x)

[Out] (2*B*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

$$3.566 \quad \int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	5639
Rubi [A] (verified)	5639
Mathematica [A] (verified)	5640
Maple [B] (verified)	5641
Fricas [C] (verification not implemented)	5641
Sympy [F(-1)]	5642
Maxima [F]	5642
Giac [F]	5642
Mupad [B] (verification not implemented)	5642

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} dx = -\frac{2BE\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2A \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2A \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2B \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

[Out] $-2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*A*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+2*B*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2827, 2716, 2720, 2719}

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2A \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{2A \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} \\ - \frac{2BE\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2B \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

[In] $\operatorname{Int}[(A+B*\operatorname{Cos}[c+d*x])/(\operatorname{Cos}[c+d*x]^{(5/2)}), x]$

[Out] $(-2*B*\operatorname{EllipticE}[(c+d*x)/2, 2])/d + (2*A*\operatorname{EllipticF}[(c+d*x)/2, 2])/(3*d) + (2*A*\operatorname{Sin}[c+d*x])/(3*d*\operatorname{Cos}[c+d*x]^{(3/2)}) + (2*B*\operatorname{Sin}[c+d*x])/(d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])$

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= A \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx + B \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2A \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2B \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{1}{3} A \int \frac{1}{\sqrt{\cos(c + dx)}} dx - B \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{2BE(\frac{1}{2}(c + dx)|2)}{d} + \frac{2A \text{EllipticF}(\frac{1}{2}(c + dx), 2)}{3d} + \frac{2A \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{2B \sin(c + dx)}{d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\begin{aligned}
&\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{-6BE(\frac{1}{2}(c + dx)|2) + 2A \text{EllipticF}(\frac{1}{2}(c + dx), 2) + \frac{2(A+3B \cos(c+dx)) \sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}}{3d}
\end{aligned}$$

```
[In] Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(5/2), x]
```

```
[Out] (-6*B*EllipticE[(c + d*x)/2, 2] + 2*A*EllipticF[(c + d*x)/2, 2] + (2*(A + 3
*B*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(127) = 254$.

Time = 6.82 (sec) , antiderivative size = 397, normalized size of antiderivative = 4.78

method	result
default	$2\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \left(2A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} (\sin^2(\frac{dx}{2} + \frac{c}{2})) - 12 \right.$
parts	$- \frac{2A\left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sin^2(\frac{dx}{2} + \frac{c}{2})) - 2(\sin^2(\frac{dx}{2} + \frac{c}{2})) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right.}{3\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})} (2(\cos^2(\frac{dx}{2} + \frac{c}{2})))}$

[In] `int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{2}{3} * (-(-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / (4 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 4 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 1) / \sin(1/2 * d * x + 1/2 * c) ^ 3 * (2 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 12 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 6 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 2 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) + 6 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 3 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2))) * (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.11

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{-i \sqrt{2} A \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3 * i \sqrt{2} * B * \cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3 * i \sqrt{2} * B * \cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{3 \sqrt{2} \cos(dx + c)^2}$$

[In] `integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{3} * (-I * \text{sqrt}(2) * A * \cos(d * x + c) ^ 2 * \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) + I * \sin(d * x + c)) + I * \text{sqrt}(2) * A * \cos(d * x + c) ^ 2 * \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) - I * \sin(d * x + c)) - 3 * I * \text{sqrt}(2) * B * \cos(d * x + c) ^ 2 * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) + I * \sin(d * x + c))) + 3 * I * \text{sqrt}(2) * B * \cos(d * x + c) ^ 2 * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) - I * \sin(d * x + c)))}{3 \sqrt{2} \cos(dx + c)^2}$$

$(d*x + c) - I*\sin(d*x + c))) + 2*(3*B*\cos(d*x + c) + A)*\sqrt{\cos(d*x + c)}*\sin(d*x + c))/(d*\cos(d*x + c)^2)$

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{B \cos(dx + c) + A}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{B \cos(dx + c) + A}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2 A \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{3 d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}} + \frac{2 B \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

[In] int((A + B*cos(c + d*x))/cos(c + d*x)^(5/2),x)

[Out] (2*A*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

$$3.567 \quad \int \frac{A+B \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	5643
Rubi [A] (verified)	5643
Mathematica [A] (verified)	5645
Maple [B] (verified)	5645
Fricas [C] (verification not implemented)	5646
Sympy [F(-1)]	5646
Maxima [F]	5646
Giac [F]	5647
Mupad [B] (verification not implemented)	5647

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)} dx = -\frac{6AE\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{2B \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2A \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{2B \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{6A \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[Out] $-6/5*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*A*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+2/3*B*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+6/5*A*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2827, 2716, 2719, 2720}

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)} dx = -\frac{6AE\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{2A \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{6A \sin(c+dx)}{5d \sqrt{\cos(c+dx)}} \\ + \frac{2B \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{2B \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)}$$

[In] $\operatorname{Int}[(A+B*\operatorname{Cos}[c+d*x])/(\operatorname{Cos}[c+d*x]^{(7/2)}), x]$

[Out] $(-6*A*\operatorname{EllipticE}[(c+d*x)/2, 2])/(5*d) + (2*B*\operatorname{EllipticF}[(c+d*x)/2, 2])/(3*d) + (2*A*\operatorname{Sin}[c+d*x])/(5*d*\operatorname{Cos}[c+d*x]^{(5/2)}) + (2*B*\operatorname{Sin}[c+d*x])/(3*d*\operatorname{Cos}[c+d*x]^{(3/2)}) + (6*A*\operatorname{Sin}[c+d*x])/(5*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])$

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= A \int \frac{1}{\cos^{\frac{7}{2}}(c + dx)} dx + B \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{2A \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2B \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{1}{5}(3A) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + \frac{1}{3}B \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2B \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2A \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2B \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} \\
&\quad + \frac{6A \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} - \frac{1}{5}(3A) \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{6AE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2B \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\
&\quad + \frac{2A \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2B \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{6A \sin(c + dx)}{5d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{-18A \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 10B \cos^{\frac{3}{2}}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 10B \sin(c + dx) + 9A}{15d \cos^{\frac{3}{2}}(c + dx)}$$

[In] Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(7/2), x]

[Out] (-18*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*B*Sin[c + d*x] + 9*A*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(147) = 294.

Time = 9.33 (sec) , antiderivative size = 502, normalized size of antiderivative = 4.52

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2B\left(-\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}{6\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2}\right)^2} + \frac{\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}\right)}$
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 12\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 10B\cos^{\frac{3}{2}}\left(\frac{dx}{2} + \frac{c}{2}\right)\text{EllipticF}\left(\frac{dx}{2} + \frac{c}{2}, 2\right) + 10B\sin\left(\frac{dx}{2} + \frac{c}{2}\right) + 9A\sin\left(2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 6A\tan\left(\frac{dx}{2} + \frac{c}{2}\right)}$

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))+2/5*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)

, $2^{(1/2)}) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} / \sin(1/2 * d * x + 1/2 * c) / (2 * \cos(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.69

$$\int \frac{A + B \cos(c + dx)}{\cos^{7/2}(c + dx)} dx$$

$$= \frac{-5i \sqrt{2} B \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 9A \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 9A \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2(9A \cos(dx + c)^2 + 5B \cos(dx + c) + 3A) \sqrt{\cos(dx + c)} \sin(dx + c)}{(d \cos(dx + c))^3}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 1/15*(-5*I*sqrt(2)*B*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{7/2}(c + dx)} dx = \int \frac{B \cos(dx + c) + A}{\cos(dx + c)^{7/2}} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{B \cos(dx + c) + A}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)

Mupad [B] (verification not implemented)

Time = 15.76 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx = \frac{2 A \sin(c + dx) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c + dx)^2\right)}{5 d \cos(c + dx)^{5/2} \sqrt{\sin(c + dx)^2}} + \frac{2 B \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{3 d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}}$$

[In] int((A + B*cos(c + d*x))/cos(c + d*x)^(7/2),x)

[Out] (2*A*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))

3.568 $\int \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^2 dx$

Optimal result	5648
Rubi [A] (verified)	5649
Mathematica [A] (verified)	5651
Maple [B] (verified)	5651
Fricas [C] (verification not implemented)	5652
Sympy [F(-1)]	5652
Maxima [F]	5652
Giac [F]	5653
Mupad [B] (verification not implemented)	5653

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^2 dx = \frac{2(9a^2 + 7b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{20ab \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{20ab \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{2(9a^2 + 7b^2) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} + \frac{4ab \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{2b^2 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d}$$

```
[Out] 2/15*(9*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+20/21*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*(9*a^2+7*b^2)*c
os(d*x+c)^(3/2)*sin(d*x+c)/d+4/7*a*b*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*b^2*
cos(d*x+c)^(7/2)*sin(d*x+c)/d+20/21*a*b*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00,
 number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used
 = {2868, 2715, 2720, 3093, 2719}

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2 dx = \frac{2(9a^2+7b^2)E\left(\frac{1}{2}(c+dx)|2\right)}{15d} + \frac{2(9a^2+7b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d} + \frac{20ab\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{4ab\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d} + \frac{20ab\sin(c+dx)\sqrt{\cos(c+dx)}}{21d} + \frac{2b^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}$$

[In] Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2,x]

[Out] (2*(9*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a*b*EllipticF[(c + d*x)/2, 2])/(21*d) + (20*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(9*a^2 + 7*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a*b*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*SIN[e + f*x])^(m + 1), x], x] + Int[(b*SIN[e + f*x])^m*(c^2 + d^2*SIN[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((b*SIN[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*SIN[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= (2ab) \int \cos^{\frac{7}{2}}(c + dx) dx + \int \cos^{\frac{5}{2}}(c + dx) (a^2 + b^2 \cos^2(c + dx)) dx \\
 &= \frac{4ab \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{2b^2 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d} \\
 &\quad + \frac{1}{7}(10ab) \int \cos^{\frac{3}{2}}(c + dx) dx + \frac{1}{9}(9a^2 + 7b^2) \int \cos^{\frac{5}{2}}(c + dx) dx \\
 &= \frac{20ab \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{2(9a^2 + 7b^2) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} \\
 &\quad + \frac{4ab \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{2b^2 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d} \\
 &\quad + \frac{1}{21}(10ab) \int \frac{1}{\sqrt{\cos(c + dx)}} dx + \frac{1}{15}(9a^2 + 7b^2) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{2(9a^2 + 7b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{20ab \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} \\
 &\quad + \frac{20ab \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{2(9a^2 + 7b^2) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} \\
 &\quad + \frac{4ab \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{2b^2 \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{9d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2 dx$$

$$= \frac{84(9a^2+7b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)+600ab\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(7(36a^2+43b^2)\cos(c+dx)+630d)}{630d}$$

[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2,x]

[Out] (84*(9*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2] + 600*a*b*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*a^2 + 43*b^2)*Cos[c + d*x] + 5*b*(156*a + 36*a*Cos[2*(c + d*x)] + 7*b*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(192) = 384.

Time = 17.91 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.49

method	result
default	$- \frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-1120\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+(1440ab+2240b^2)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)$
parts	$- \frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right)$

[In] int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^2+(1440*a*b+2240*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*a^2-2160*a*b-2072*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*a^2+1680*a*b+952*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*a^2-480*a*b-168*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+150*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.22

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^2 dx$$

$$= \frac{-150i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 150i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4,$$

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/315*(-150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(35*b^2*cos(d*x + c)^3 + 90*a*b*cos(d*x + c)^2 + 150*a*b + 7*(9*a^2 + 7*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 21*sqrt(2)*(-9*I*a^2 - 7*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(9*I*a^2 + 7*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^2 dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^2 dx = \int (b \cos(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}} dx$$

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```


Giac [F]

$$\int \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^2 dx = \int (b \cos(dx + c) + a)^2 \cos(dx + c)^{\frac{5}{2}} dx$$

[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 15.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\begin{aligned} & \int \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^2 dx \\ &= -\frac{2a^2 \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7d \sqrt{\sin(c + dx)^2}} \\ & \quad - \frac{2b^2 \cos(c + dx)^{11/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{4}; \frac{15}{4}; \cos(c + dx)^2\right)}{11d \sqrt{\sin(c + dx)^2}} \\ & \quad - \frac{4ab \cos(c + dx)^{9/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c + dx)^2\right)}{9d \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2,x)

[Out] - (2*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*b^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (4*a*b*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))

3.569 $\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2 dx$

Optimal result	5654
Rubi [A] (verified)	5654
Mathematica [A] (verified)	5656
Maple [B] (verified)	5657
Fricas [C] (verification not implemented)	5657
Sympy [F(-1)]	5658
Maxima [F]	5658
Giac [F]	5658
Mupad [B] (verification not implemented)	5658

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2 dx = \frac{12abE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2(7a^2 + 5b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{2(7a^2 + 5b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{4ab \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2b^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d}$$

```
[Out] 12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/5*a*b*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b^2*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*(7*a^2+5*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

= {2868, 2715, 2719, 3093, 2720}

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2 dx = \frac{2(7a^2 + 5b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{2(7a^2 + 5b^2) \sin(c + dx) \sqrt{\cos(c + dx)}}{21d} + \frac{12abE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{4ab \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d} + \frac{2b^2 \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)}{7d}$$

[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2,x]

[Out] (12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a^2 + 5*b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (4*a*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*b^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2868

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3093

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (2ab) \int \cos^{\frac{5}{2}}(c + dx) dx + \int \cos^{\frac{3}{2}}(c + dx) (a^2 + b^2 \cos^2(c + dx)) dx \\
&= \frac{4ab \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2b^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} \\
&\quad + \frac{1}{5}(6ab) \int \sqrt{\cos(c + dx)} dx + \frac{1}{7}(7a^2 + 5b^2) \int \cos^{\frac{3}{2}}(c + dx) dx \\
&= \frac{12abE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2(7a^2 + 5b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} \\
&\quad + \frac{4ab \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2b^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} \\
&\quad + \frac{1}{21}(7a^2 + 5b^2) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{12abE\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2(7a^2 + 5b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} \\
&\quad + \frac{2(7a^2 + 5b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} \\
&\quad + \frac{4ab \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2b^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\begin{aligned}
&\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2 dx \\
&= \frac{252abE\left(\frac{1}{2}(c + dx) \mid 2\right) + 10(7a^2 + 5b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sqrt{\cos(c + dx)}(70a^2 + 65b^2 + 84ab \cos(c + dx))}{105d}
\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (252*a*b*EllipticE[(c + d*x)/2, 2] + 10*(7*a^2 + 5*b^2)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*a^2 + 65*b^2 + 84*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(171) = 342$.

Time = 10.99 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.68

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+(-336ab-360b^2)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\dots}\right)$
parts	$-\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d$

[In] `int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)`

[Out]
$$-2/105*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(240*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^8*b^2+(-336*a*b-360*b^2)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+\dots)$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.33

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2 dx = \frac{126i\sqrt{2}ab\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 126i\sqrt{2}abw}{\dots}$$

[In] `integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")`

[Out]
$$1/105*(126*I*\sqrt{2}*a*b*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d*x+c)+I*\sin(d*x+c))) - 126*I*\sqrt{2}*a*b*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d*x+c)-I*\sin(d*x+c)))) + 2*(15*b^2*\cos(d*x+c)^2 + 42*a*b*\cos(d*x+c) + 35*a^2 + 25*b^2)*\sqrt{\cos(d*x+c)}*\sin(d*x+c) - 5*\sqrt{2}*(7*I*a^2 + 5*I*b^2)*\text{weierstrassPInverse}(-4,0,\cos(d*x+c)+I*\sin(d*x+c)) - 5*\sqrt{2}*(-7*I*a^2 - 5*I*b^2)*\text{weierstrassPInverse}(-4,0,\cos(d*x+c)-I*\sin(d*x+c)))/d$$

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2 dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2 dx = \int (b \cos(dx + c) + a)^2 \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2 dx = \int (b \cos(dx + c) + a)^2 \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 15.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.95

$$\begin{aligned} & \int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2 dx \\ &= \frac{2 \left(a^2 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + a^2 \sqrt{\cos(c + dx)} \sin(c + dx) \right)}{3d} \\ & \quad - \frac{2b^2 \cos(c + dx)^{9/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c + dx)^2\right)}{9d \sqrt{\sin(c + dx)}^2} \\ & \quad - \frac{4ab \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7d \sqrt{\sin(c + dx)}^2} \end{aligned}$$

```
[In] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] (2*(a^2*ellipticF(c/2 + (d*x)/2, 2) + a^2*cos(c + d*x)^(1/2)*sin(c + d*x))  
/(3*d) - (2*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4,  
cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (4*a*b*cos(c + d*x)^(7/2)*  
sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)  
)^2)^(1/2))
```

3.570 $\int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^2 dx$

Optimal result	5660
Rubi [A] (verified)	5660
Mathematica [A] (verified)	5662
Maple [B] (verified)	5662
Fricas [C] (verification not implemented)	5663
Sympy [F(-1)]	5663
Maxima [F]	5663
Giac [F]	5664
Mupad [B] (verification not implemented)	5664

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^2 dx = \frac{2(5a^2 + 3b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{4ab \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{4ab \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2b^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d}$$

[Out] $2/5*(5*a^2+3*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4/3*a*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/5*b^2*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d+4/3*a*b*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2868, 2715, 2720, 3093, 2719}

$$\int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^2 dx = \frac{2(5a^2 + 3b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{4ab \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{4ab \sin(c + dx) \sqrt{\cos(c + dx)}}{3d} + \frac{2b^2 \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)}{5d}$$

[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2,x]

[Out] (2*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c + d*x)/2, 2])/(3*d) + (4*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b^2 *Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2868

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3093

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \text{integral} &= (2ab) \int \cos^{\frac{3}{2}}(c + dx) dx + \int \sqrt{\cos(c + dx)}(a^2 + b^2 \cos^2(c + dx)) dx \\ &= \frac{4ab\sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2b^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} \\ &\quad + \frac{1}{3}(2ab) \int \frac{1}{\sqrt{\cos(c + dx)}} dx + \frac{1}{5}(5a^2 + 3b^2) \int \sqrt{\cos(c + dx)} dx \end{aligned}$$

$2*c), 2^{(1/2)}*b^2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.60

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2 dx$$

$$= \frac{-10i\sqrt{2}ab\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c)) + 10i\sqrt{2}ab\text{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\sin(dx+c))}{d}$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] 1/15*(-10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(3*b^2*cos(d*x + c) + 10*a*b)*sqrt(cos(d*x + c))*sin(d*x + c) - 3*sqrt(2)*(-5*I*a^2 - 3*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(5*I*a^2 + 3*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2 dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2 dx = \int (b\cos(dx+c) + a)^2 \sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^2 dx = \int (b \cos(dx + c) + a)^2 \sqrt{\cos(dx + c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 15.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\begin{aligned} & \int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^2 dx \\ &= \frac{2a^2 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{4ab F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{3d} + \frac{4ab \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} \\ & \quad - \frac{2b^2 \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7d \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2,x)

[Out] (2*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (4*a*b*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))

$$3.571 \quad \int \frac{(a+b \cos(c+dx))^2}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	5665
Rubi [A] (verified)	5665
Mathematica [A] (verified)	5666
Maple [B] (verified)	5667
Fricas [C] (verification not implemented)	5667
Sympy [F]	5668
Maxima [F]	5668
Giac [F]	5668
Mupad [B] (verification not implemented)	5668

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{(a+b \cos(c+dx))^2}{\sqrt{\cos(c+dx)}} dx = \frac{4abE\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2(3a^2+b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{2b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3d}$$

[Out] $4*a*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*(3*a^2+b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*b^2*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2868, 2719, 3093, 2720}

$$\int \frac{(a+b \cos(c+dx))^2}{\sqrt{\cos(c+dx)}} dx = \frac{2(3a^2+b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{4abE\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2b^2 \sin(c+dx) \sqrt{\cos(c+dx)}}{3d}$$

[In] $\operatorname{Int}[(a+b*\operatorname{Cos}[c+d*x])^2/\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]], x]$

[Out] $(4*a*b*\operatorname{EllipticE}[(c+d*x)/2, 2])/d + (2*(3*a^2+b^2)*\operatorname{EllipticF}[(c+d*x)/2, 2])/(3*d) + (2*b^2*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(3*d)$

Rule 2719

`Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2720

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2868

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3093

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= (2ab) \int \sqrt{\cos(c + dx)} dx + \int \frac{a^2 + b^2 \cos^2(c + dx)}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{4abE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2b^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3}(3a^2 + b^2) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{4abE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2(3a^2 + b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2b^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\begin{aligned}
 &\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{2\left(6abE\left(\frac{1}{2}(c + dx) \mid 2\right) + (3a^2 + b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + b^2 \sqrt{\cos(c + dx)} \sin(c + dx)\right)}{3d}
 \end{aligned}$$

`[In] Integrate[(a + b*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]], x]`

`[Out] (2*(6*a*b*EllipticE[(c + d*x)/2, 2] + (3*a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(118) = 236.

Time = 7.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.93

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+3a^2\sqrt{\frac{1}{2}-\frac{\cos(dx)}{2}}}\right)^{-1} \frac{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\dots}$
parts	$\frac{2a^2 \operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2} \sqrt{2}\right)}{d} - \frac{2b^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} \frac{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}\sqrt{2\left(\dots\right)}}{\dots}$

[In] `int((a+cos(d*x+c))*b^2/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-2/3*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(4*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4*b^2-2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*b^2+3*a^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-6*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a*b)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.04

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{2b^2\sqrt{\cos(dx+c)}\sin(dx+c) + 6i\sqrt{2}ab\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx+c) + I\sin(dx+c)))}{\dots}$$

[In] `integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]
$$1/3*(2*b^2*\sqrt{\cos(d*x+c)}*\sin(d*x+c) + 6*I*\sqrt{2}*a*b*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(d*x+c) + I*\sin(d*x+c))) - 6*I*\sqrt{2}*a*b*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(d*x+c) - I*\sin(d*x+c))) + \sqrt{2}*(-3*I*a^2 - I*b^2)*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x+c) + I*\sin(d*x+c)) + \sqrt{2}*(3*I*a^2 + I*b^2)*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x+c) - I*\sin(d*x+c)))/d$$

Sympy [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx$$

[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)

[Out] Integral((a + b*cos(c + d*x))**2/sqrt(cos(c + d*x)), x)

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^2}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^2}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\cos(c + dx)}} dx = \frac{2a^2 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{2b^2 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{3d} + \frac{2b^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{4ab E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d}$$

[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^(1/2),x)

[Out] (2*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^2*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*b^2*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (4*a*b*ellipticE(c/2 + (d*x)/2, 2))/d

$$3.572 \quad \int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	5669
Rubi [A] (verified)	5669
Mathematica [A] (verified)	5670
Maple [A] (verified)	5671
Fricas [C] (verification not implemented)	5671
Sympy [F(-1)]	5672
Maxima [F]	5672
Giac [F]	5672
Mupad [B] (verification not implemented)	5672

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)} dx = -\frac{2(a^2-b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{4ab \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} + \frac{2a^2 \sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[Out] $-2*(a^2-b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+4*a*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2868, 2720, 3091, 2719}

$$\int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)} dx = -\frac{2(a^2-b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2a^2 \sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{4ab \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d}$$

[In] $\operatorname{Int}[(a+b*\operatorname{Cos}[c+d*x])^2/\operatorname{Cos}[c+d*x]^{(3/2)}, x]$

[Out] $(-2*(a^2-b^2)*\operatorname{EllipticE}[(c+d*x)/2, 2])/d + (4*a*b*\operatorname{EllipticF}[(c+d*x)/2, 2])/d + (2*a^2*\operatorname{Sin}[c+d*x])/(d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])$

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] +
Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] := Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= (2ab) \int \frac{1}{\sqrt{\cos(c+dx)}} dx + \int \frac{a^2 + b^2 \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{4ab \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} + \frac{2a^2 \sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (a^2 - b^2) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{2(a^2 - b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{4ab \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} + \frac{2a^2 \sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\begin{aligned}
&\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2\left((-a^2 + b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right) + a\left(2b \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \frac{a \sin(c + dx)}{\sqrt{\cos(c + dx)}}\right)\right)}{d}
\end{aligned}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(3/2), x]
```

```
[Out] (2*((-a^2 + b^2)*EllipticE[(c + d*x)/2, 2] + a*(2*b*EllipticF[(c + d*x)/2,
2] + (a*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [A] (verified)

Time = 6.91 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.97

method	result
default	$\frac{4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a^2 - 4ab \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - 2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}{\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}$
parts	$\frac{2a^2 \left(-2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}$

[In] int((a+cos(d*x+c)*b)^2/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

```
[Out] 2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2-2*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.62

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{-2i \sqrt{2} ab \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2i \sqrt{2} ab \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 2a^2 \sqrt{2} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + \sqrt{2} (I a^2 - I b^2) \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - I \sin(dx + c)))}{(d \cos(dx + c))}$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")

```
[Out] (-2*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 2*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 2*a^2*sqrt(2)*cos(d*x + c)*sin(d*x + c) + sqrt(
2)*(-I*a^2 + I*b^2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(I*a^2 - I*b^2)*cos(d*x +
c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))))/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2b^2 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{4ab F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{2a^2 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^(3/2),x)

[Out] (2*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*a*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

$$3.573 \quad \int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	5673
Rubi [A] (verified)	5673
Mathematica [A] (verified)	5675
Maple [C] (verified)	5675
Fricas [C] (verification not implemented)	5676
Sympy [F(-1)]	5676
Maxima [F]	5676
Giac [F]	5677
Mupad [B] (verification not implemented)	5677

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)} dx = -\frac{4abE\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2(a^2+3b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\ + \frac{2a^2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{4ab \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

[Out] $-4*a*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*(a^2+3*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a^2*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+4*a*b*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2868, 2716, 2719, 3091, 2720}

$$\int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2(a^2+3b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{2a^2 \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} \\ - \frac{4abE\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{4ab \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

[In] $\operatorname{Int}[(a+b*\operatorname{Cos}[c+d*x])^2/\operatorname{Cos}[c+d*x]^{(5/2)}, x]$

[Out] $(-4*a*b*\operatorname{EllipticE}[(c+d*x)/2, 2])/d + (2*(a^2+3*b^2)*\operatorname{EllipticF}[(c+d*x)/2, 2])/(3*d) + (2*a^2*\operatorname{Sin}[c+d*x])/(3*d*\operatorname{Cos}[c+d*x]^{(3/2)}) + (4*a*b*\operatorname{Sin}[c+d*x])/(d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])$

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2868

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3091

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (2ab) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + \int \frac{a^2 + b^2 \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2a^2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{d \sqrt{\cos(c + dx)}} - (2ab) \int \sqrt{\cos(c + dx)} dx \\
 &\quad - \frac{1}{3}(-a^2 - 3b^2) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= -\frac{4abE\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2(a^2 + 3b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\
 &\quad + \frac{2a^2 \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{d \sqrt{\cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{2 \left(-6abE\left(\frac{1}{2}(c + dx) \mid 2\right) + (a^2 + 3b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \frac{a(a+6b \cos(c+dx)) \sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} \right)}{3d}$$

[In] Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(5/2), x]

[Out] (2*(-6*a*b*EllipticE[(c + d*x)/2, 2] + (a^2 + 3*b^2)*EllipticF[(c + d*x)/2, 2] + (a*(a + 6*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.41 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.42

method	result
parts	$\frac{2a^2 \left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right)}{3\sqrt{-2 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}$
default	$\frac{2\sqrt{-\left(-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(24 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) ab - 2F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}\right)}{d + 2b^2/d \operatorname{InverseJacobiAM}\left(\frac{1}{2}d*x + \frac{1}{2}c, 2^{\frac{1}{2}}\right) - 4*a*b*(-2*\cos(1/2*d*x + 1/2*c)*(-2*\sin(1/2*d*x + 1/2*c)^4 + \sin(1/2*d*x + 1/2*c)^2)^{\frac{1}{2}}*\sin(1/2*d*x + 1/2*c)^2 + (\sin(1/2*d*x + 1/2*c)^2)^{\frac{1}{2}}*(2*\sin(1/2*d*x + 1/2*c)^2 - 1)^{\frac{1}{2}}*(-2*\sin(1/2*d*x + 1/2*c)^4 + \sin(1/2*d*x + 1/2*c)^2)^{\frac{1}{2}}*\operatorname{EllipticE}(\cos(1/2*d*x + 1/2*c), 2^{\frac{1}{2}}) / (-2*\sin(1/2*d*x + 1/2*c)^4 + \sin(1/2*d*x + 1/2*c)^2)^{\frac{1}{2}}/\sin(1/2*d*x + 1/2*c)/(2*\cos(1/2*d*x + 1/2*c)^2 - 1)^{\frac{1}{2}}/d}$

[In] int((a+cos(d*x+c)*b)^2/cos(d*x+c)^(5/2), x, method=_RETURNVERBOSE)

[Out] -2/3*a^2*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d+2*b^2/d*InverseJacobiAM(1/2*d*x+1/2*c, 2^(1/2))-4*a*b*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.08

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{-6i \sqrt{2} ab \cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + \dots}{\dots}$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] 1/3*(-6*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 6*I*sqrt(2)*a*b*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + sqrt(2)*(-I*a^2 - 3*I*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 + 3*I*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6*a*b*cos(d*x + c) + a^2)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^2)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 15.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\begin{aligned} \int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{5}{2}}(c + dx)} dx &= \frac{2b^2 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} \\ &+ \frac{2a^2 \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{3d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}} \\ &+ \frac{4ab \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^(5/2),x)

[Out] (2*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (4*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))

$$3.574 \quad \int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	5678
Rubi [A] (verified)	5678
Mathematica [A] (verified)	5680
Maple [B] (verified)	5680
Fricas [C] (verification not implemented)	5681
Sympy [F(-1)]	5682
Maxima [F]	5682
Giac [F]	5682
Mupad [B] (verification not implemented)	5682

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)} dx = -\frac{2(3a^2+5b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{4ab \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d}$$

$$+ \frac{2a^2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{4ab \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2(3a^2+5b^2) \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}$$

[Out] -2/5*(3*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic E(sin(1/2*d*x+1/2*c), 2^(1/2))/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/d+2/5*a^2*sin(d*x+c)/d/cos(d*x+c)^(5/2)+4/3*a*b*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(3*a^2+5*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2868, 2716, 2720, 3091, 2719}

$$\int \frac{(a+b \cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)} dx = -\frac{2(3a^2+5b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d}$$

$$+ \frac{2(3a^2+5b^2) \sin(c+dx)}{5d \sqrt{\cos(c+dx)}} + \frac{2a^2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{4ab \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{4ab \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)}$$

[In] Int[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(7/2), x]

```
[Out] (-2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a*b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*a^2 + 5*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2868

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3091

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= (2ab) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx + \int \frac{a^2 + b^2 \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx \\ &= \frac{2a^2 \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} + \frac{1}{3}(2ab) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\ &\quad - \frac{1}{5}(-3a^2 - 5b^2) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{4ab \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{2a^2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{4ab \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{2(3a^2 + 5b^2) \sin(c+dx)}{5d \sqrt{\cos(c+dx)}} - \frac{1}{5}(3a^2 + 5b^2) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{2(3a^2 + 5b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{4ab \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} \\
&\quad + \frac{2a^2 \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{4ab \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{2(3a^2 + 5b^2) \sin(c+dx)}{5d \sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{-6(3a^2 + 5b^2) \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 20ab \cos^{\frac{3}{2}}(c + dx) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 20ab \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(7/2), x]

[Out] (-6*(3*a^2 + 5*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 20*a*b*Sin[c + d*x] + 9*a^2*Sin[2*(c + d*x)] + 15*b^2*Sin[2*(c + d*x)] + 6*a^2*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(171) = 342.

Time = 11.75 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.69

method	result
default	$\frac{\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))}}{\left(\frac{2a^2 \left(24 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 12 \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right) E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\right)}$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*b)^2/cos(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/5*a^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^4+6*(2*sin(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2)

$$\begin{aligned} & \frac{1}{2}c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c) \\ & ,2^{(1/2)})*\sin(1/2*d*x+1/2*c)^4-24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+1 \\ & 2*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(c \\ & \cos(1/2*d*x+1/2*c),2^{(1/2)})*\sin(1/2*d*x+1/2*c)^2+8*\sin(1/2*d*x+1/2*c)^2*\cos(\\ & 1/2*d*x+1/2*c)-3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1 \\ & /2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2 \\ & *d*x+1/2*c)^2)^{(1/2)}+2*b^2/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)* \\ & (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^ \\ & 2*\cos(1/2*d*x+1/2*c)-(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1 \\ &)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})))+4*a*b*(-1/6*\cos(1/2*d*x+1/2* \\ & c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c) \\ & ^2-1/2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2) \\ &)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d* \\ & x+1/2*c),2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.65

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{7/2}(c + dx)} dx$$

$$= \frac{-10i \sqrt{2} ab \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \cos(dx + c)}{\dots}$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 1/15*(-10*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*I*a^2 + 5*I*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*a^2 - 5*I*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(10*a*b*cos(d*x + c) + 3*(3*a^2 + 5*b^2)*cos(d*x + c)^2 + 3*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^2}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)

Mupad [B] (verification not implemented)

Time = 15.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int \frac{(a + b \cos(c + dx))^2}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \frac{6a^2 \sin(c + dx) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c + dx)^2\right) + 30b^2 \cos(c + dx)^2 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{15d \cos(c + dx)^{5/2} \sqrt{1 - \cos(c + dx)}}$$

[In] int((a + b*cos(c + d*x))^2/cos(c + d*x)^(7/2),x)

[Out] (6*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*b^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) + 20*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2))

3.575 $\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^3 dx$

Optimal result	5683
Rubi [A] (verified)	5684
Mathematica [A] (verified)	5686
Maple [B] (verified)	5686
Fricas [C] (verification not implemented)	5687
Sympy [F(-1)]	5687
Maxima [F]	5688
Giac [F]	5688
Mupad [B] (verification not implemented)	5688

Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^3 dx = \frac{2b(27a^2 + 7b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{2a(7a^2 + 15b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{2a(7a^2 + 15b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{2b(27a^2 + 7b^2) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} + \frac{40ab^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{63d} + \frac{2b^2 \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{9d}$$

```
[Out] 2/15*b*(27*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(7*a^2+15*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/45*
b*(27*a^2+7*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+40/63*a*b^2*cos(d*x+c)^(5/2)
*sin(d*x+c)/d+2/9*b^2*cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))*sin(d*x+c)/d+2/21*a
*(7*a^2+15*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2872, 3102, 2827, 2715, 2720, 2719}

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3 dx = \frac{2a(7a^2+15b^2)\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} + \frac{2b(27a^2+7b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2b(27a^2+7b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d} + \frac{2a(7a^2+15b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d} + \frac{2b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}{9d} + \frac{40ab^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}$$

[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3,x]

[Out] (2*b*(27*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(7*a^2 + 15*b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(27*a^2 + 7*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (40*a*b^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b^2*Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827


```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2b^2 \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{9d} + \frac{2}{9} \int \cos^{\frac{3}{2}}(c + dx) \left(\frac{1}{2}a(9a^2 + 5b^2) \right. \\
 &\quad \left. + \frac{1}{2}b(27a^2 + 7b^2) \cos(c + dx) + 10ab^2 \cos^2(c + dx) \right) dx \\
 &= \frac{40ab^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{63d} + \frac{2b^2 \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{9d} \\
 &\quad + \frac{4}{63} \int \cos^{\frac{3}{2}}(c + dx) \left(\frac{9}{4}a(7a^2 + 15b^2) + \frac{7}{4}b(27a^2 + 7b^2) \cos(c + dx) \right) dx \\
 &= \frac{40ab^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{63d} + \frac{2b^2 \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{9d} \\
 &\quad + \frac{1}{9}(b(27a^2 + 7b^2)) \int \cos^{\frac{5}{2}}(c + dx) dx + \frac{1}{7}(a(7a^2 + 15b^2)) \int \cos^{\frac{3}{2}}(c + dx) dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2a(7a^2 + 15b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{2b(27a^2 + 7b^2) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} \\
&+ \frac{40ab^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{63d} + \frac{2b^2 \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{9d} \\
&+ \frac{1}{15} (b(27a^2 + 7b^2)) \int \sqrt{\cos(c + dx)} dx + \frac{1}{21} (a(7a^2 + 15b^2)) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2b(27a^2 + 7b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{2a(7a^2 + 15b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} \\
&+ \frac{2a(7a^2 + 15b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{2b(27a^2 + 7b^2) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{45d} \\
&+ \frac{40ab^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{63d} + \frac{2b^2 \cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{9d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.71

$$\begin{aligned}
&\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^3 dx \\
&= \frac{84(27a^2b + 7b^3) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 60(7a^3 + 15ab^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sqrt{\cos(c + dx)}(7b(108a^2 + 43b^2) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + 5(84a^3 + 234ab^2 + 54a^2b^2 \cos(2(c + dx)) + 7b^3 \cos(3(c + dx)))) \sin(c + dx)}{630d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3,x]

[Out] (84*(27*a^2*b + 7*b^3)*EllipticE[(c + d*x)/2, 2] + 60*(7*a^3 + 15*a*b^2)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*b*(108*a^2 + 43*b^2)*Cos[c + d*x] + 5*(84*a^3 + 234*a*b^2 + 54*a^2*b^2*Cos[2*(c + d*x)] + 7*b^3*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(226) = 452.

Time = 13.53 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.42

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(-1120 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b^3 + (2160ab^2 + 2240b^3)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \dots}{630d}$
parts	Expression too large to display

[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^3+(2160*a*b^2+2240*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*a^2*b-3240*a*b^2-2072*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*a^3+1512*a^2*b+2520*a*b^2+952*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*a^3-378*a^2*b-720*a*b^2-168*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+225*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-567*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.17

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3 dx$$

$$= \frac{2(35b^3\cos(dx+c)^3 + 135ab^2\cos(dx+c)^2 + 105a^3 + 225ab^2 + 7(27a^2b + 7b^3)\cos(dx+c))\sqrt{\cos(dx+c)}}{1}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/315*(2*(35*b^3*cos(d*x + c)^3 + 135*a*b^2*cos(d*x + c)^2 + 105*a^3 + 225*a*b^2 + 7*(27*a^2*b + 7*b^3)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*(7*I*a^3 + 15*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 15*sqrt(2)*(-7*I*a^3 - 15*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-27*I*a^2*b - 7*I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(27*I*a^2*b + 7*I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3 dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^3 dx = \int (b \cos(dx + c) + a)^3 \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^3 dx = \int (b \cos(dx + c) + a)^3 \cos(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\begin{aligned} & \int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^3 dx \\ &= \frac{2 a^3 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{3 d} + \frac{2 a^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{3 d} \\ & - \frac{2 b^3 \cos(c + dx)^{11/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{4}; \frac{15}{4}; \cos(c + dx)^2\right)}{11 d \sqrt{\sin(c + dx)^2}} \\ & - \frac{6 a^2 b \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7 d \sqrt{\sin(c + dx)^2}} \\ & - \frac{2 a b^2 \cos(c + dx)^{9/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c + dx)^2\right)}{3 d \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3,x)

[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (2*a^3*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) - (2*b^3*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 11/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (6*a^2*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*a*b^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(3*d*(sin(c + d*x)^2)^(1/2))

3.576 $\int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^3 dx$

Optimal result	5689
Rubi [A] (verified)	5689
Mathematica [A] (verified)	5692
Maple [B] (verified)	5692
Fricas [C] (verification not implemented)	5693
Sympy [F(-1)]	5693
Maxima [F]	5693
Giac [F]	5694
Mupad [B] (verification not implemented)	5694

Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^3 dx = \frac{2a(5a^2 + 9b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2b(21a^2 + 5b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{2b(21a^2 + 5b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{32ab^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{35d} + \frac{2b^2 \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{7d}$$

```
[Out] 2/5*a*(5*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*b*(21*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/35*a*b^2*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*b^2*cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))*sin(d*x+c)/d+2/21*b*(21*a^2+5*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {2872, 3102, 2827, 2719, 2715, 2720}

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3 dx = \frac{2b(21a^2+5b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d} + \frac{2a(5a^2+9b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b(21a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d} + \frac{32ab^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d} + \frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}{7d}$$

[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3,x]

[Out] (2*a*(5*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(21*a^2 + 5*b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (32*a*b^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*b^2*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d)

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Ssin[c + d*x])^(n-1)/(d*n)), x] + Dist[b^2*((n-1)/n), Int[(b*Ssin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2872

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(n-1), x]

```
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b^2 \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{7d} \\
&+ \frac{2}{7} \int \sqrt{\cos(c + dx)} \left(\frac{1}{2}a(7a^2 + 3b^2) + \frac{1}{2}b(21a^2 + 5b^2) \cos(c + dx) \right. \\
&\quad \left. + 8ab^2 \cos^2(c + dx) \right) dx \\
&= \frac{32ab^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{35d} + \frac{2b^2 \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{7d} \\
&+ \frac{4}{35} \int \sqrt{\cos(c + dx)} \left(\frac{7}{4}a(5a^2 + 9b^2) + \frac{5}{4}b(21a^2 + 5b^2) \cos(c + dx) \right) dx \\
&= \frac{32ab^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{35d} + \frac{2b^2 \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{7d} \\
&+ \frac{1}{7}(b(21a^2 + 5b^2)) \int \cos^{\frac{3}{2}}(c + dx) dx + \frac{1}{5}(a(5a^2 + 9b^2)) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{2a(5a^2 + 9b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2b(21a^2 + 5b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} \\
&+ \frac{32ab^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{35d} + \frac{2b^2 \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{7d} \\
&+ \frac{1}{21}(b(21a^2 + 5b^2)) \int \frac{1}{\sqrt{\cos(c + dx)}} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{2a(5a^2 + 9b^2) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d} + \frac{2b(21a^2 + 5b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} \\
&+ \frac{2b(21a^2 + 5b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{21d} + \frac{32ab^2 \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{35d} \\
&+ \frac{2b^2 \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\begin{aligned}
&\int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^3 dx \\
&= \frac{42(5a^3 + 9ab^2) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 10(21a^2b + 5b^3) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + b\sqrt{\cos(c + dx)}(210a^2 + 65b^2)}{105d}
\end{aligned}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3,x]

[Out] (42*(5*a^3 + 9*a*b^2)*EllipticE[(c + d*x)/2, 2] + 10*(21*a^2*b + 5*b^3)*EllipticF[(c + d*x)/2, 2] + b*Sqrt[Cos[c + d*x]]*(210*a^2 + 65*b^2 + 126*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(195) = 390.

Time = 12.03 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.65

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(240\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b^3 + (-504ab^2 - 360b^3)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \dots}{105d}$
parts	Expression too large to display

[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^3+(-504*a*b^2-360*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*a^2*b+504*a*b^2+280*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*a^2*b-126*a*b^2-80*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s

$$\frac{\operatorname{in}(1/2*d*x+1/2*c)^2-1)^{1/2}*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c),2^{1/2})*a*b^2)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}/\sin(1/2*d*x+1/2*c)/(2*c\cos(1/2*d*x+1/2*c)^2-1)^{1/2}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.29

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3 dx$$

$$= \frac{2(15b^3\cos(dx+c)^2+63ab^2\cos(dx+c)+105a^2b+25b^3)\sqrt{\cos(dx+c)}\sin(dx+c)-5\sqrt{2}(21ia^2b+}$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] 1/105*(2*(15*b^3*cos(d*x + c)^2 + 63*a*b^2*cos(d*x + c) + 105*a^2*b + 25*b^3)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*(21*I*a^2*b + 5*I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-21*I*a^2*b - 5*I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-5*I*a^3 - 9*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(5*I*a^3 + 9*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3 dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3 dx = \int (b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3 dx = \int (b\cos(dx+c)+a)^3 \sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.92

$$\begin{aligned} & \int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3 dx \\ &= \frac{2 \left(a^3 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + a^2 b F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) + a^2 b \sqrt{\cos(c+dx)} \sin(c+dx) \right)}{d} \\ & \quad - \frac{2b^3 \cos(c+dx)^{9/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{9}{4}; \frac{13}{4}; \cos(c+dx)^2\right)}{9d \sqrt{\sin(c+dx)^2}} \\ & \quad - \frac{6ab^2 \cos(c+dx)^{7/2} \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c+dx)^2\right)}{7d \sqrt{\sin(c+dx)^2}} \end{aligned}$$

[In] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3,x)

[Out] (2*(a^3*ellipticE(c/2 + (d*x)/2, 2) + a^2*b*ellipticF(c/2 + (d*x)/2, 2) + a^2*b*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (2*b^3*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (6*a*b^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))

$$3.577 \quad \int \frac{(a+b \cos(c+dx))^3}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	5695
Rubi [A] (verified)	5695
Mathematica [A] (verified)	5697
Maple [B] (verified)	5698
Fricas [C] (verification not implemented)	5698
Sympy [F(-1)]	5699
Maxima [F]	5699
Giac [F]	5699
Mupad [B] (verification not implemented)	5699

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \frac{6b(5a^2 + b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2a(a^2 + b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{8ab^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{5d} + \frac{2b^2 \sqrt{\cos(c + dx)}(a + b \cos(c + dx)) \sin(c + dx)}{5d}$$

```
[Out] 6/5*b*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+8/5*a*b^2*sin(d*x
+c)*cos(d*x+c)^(1/2)/d+2/5*b^2*(a+b*cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

= {2872, 3102, 2827, 2720, 2719}

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \frac{2a(a^2 + b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{6b(5a^2 + b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2b^2 \sin(c + dx) \sqrt{\cos(c + dx)}(a + b \cos(c + dx))}{5d} + \frac{8ab^2 \sin(c + dx) \sqrt{\cos(c + dx)}}{5d}$$

[In] Int[(a + b*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]

[Out] (6*b*(5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2])/d + (8*a*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b^2*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2872

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & NeQ[c, 0])))

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b^2 \sqrt{\cos(c+dx)}(a+b \cos(c+dx)) \sin(c+dx)}{5d} \\
&+ \frac{2}{5} \int \frac{\frac{1}{2}a(5a^2+b^2) + \frac{3}{2}b(5a^2+b^2) \cos(c+dx) + 6ab^2 \cos^2(c+dx)}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{8ab^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{5d} + \frac{2b^2 \sqrt{\cos(c+dx)}(a+b \cos(c+dx)) \sin(c+dx)}{5d} \\
&+ \frac{4}{15} \int \frac{\frac{15}{4}a(a^2+b^2) + \frac{9}{4}b(5a^2+b^2) \cos(c+dx)}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{8ab^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{5d} + \frac{2b^2 \sqrt{\cos(c+dx)}(a+b \cos(c+dx)) \sin(c+dx)}{5d} \\
&+ (a(a^2+b^2)) \int \frac{1}{\sqrt{\cos(c+dx)}} dx + \frac{1}{5} (3b(5a^2+b^2)) \int \sqrt{\cos(c+dx)} dx \\
&= \frac{6b(5a^2+b^2) E(\frac{1}{2}(c+dx)|2)}{5d} + \frac{2a(a^2+b^2) \text{EllipticF}(\frac{1}{2}(c+dx), 2)}{d} \\
&+ \frac{8ab^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{5d} + \frac{2b^2 \sqrt{\cos(c+dx)}(a+b \cos(c+dx)) \sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\begin{aligned}
&\int \frac{(a+b \cos(c+dx))^3}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{2 \left(3(5a^2b+b^3) E\left(\frac{1}{2}(c+dx)|2\right) + 5a(a^2+b^2) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + b^2 \sqrt{\cos(c+dx)}(5a+b \cos(c+dx)) \right)}{5d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]

[Out] (2*(3*(5*a^2*b + b^3)*EllipticE[(c + d*x)/2, 2] + 5*a*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + b^2*Sqrt[Cos[c + d*x]]*(5*a + b*Cos[c + d*x])*Sin[c + d*x]))/(5*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(158) = 316.

Time = 9.48 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.55

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+20\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab^2+8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$
parts	$\frac{2a^3\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\mid\sqrt{2}\right)}{d}-\frac{2b^3\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\operatorname{si}$

[In] `int((a+cos(d*x+c)*b)^3/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -2/5*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6*b^3+20*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4*a*b^2+8*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*a*b^2-2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*b^3+5*a^3*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c),2^(1/2))+5*a*b^2*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c),2^(1/2))-15*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^(1/2)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d \end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.59

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{2(b^3 \cos(dx + c) + 5ab^2)\sqrt{\cos(dx + c)} \sin(dx + c) - 5\sqrt{2}(ia^3 + iab^2)\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c))}{d}$$

[In] `integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]
$$\begin{aligned} & 1/5*(2*(b^3*\cos(d*x + c) + 5*a*b^2)*\operatorname{sqrt}(\cos(d*x + c))*\sin(d*x + c) - 5*\operatorname{sqrt}(2)*(I*a^3 + I*a*b^2)*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) - 5*\operatorname{sqrt}(2)*(-I*a^3 - I*a*b^2)*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - 3*\operatorname{sqrt}(2)*(-5*I*a^2*b - I*b^3)*\operatorname{weierstrassZeta}(-4, 0, \cos(d*x + c))) \end{aligned}$$

```
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(5*
I*a^2*b + I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^3}{\sqrt{\cos(dx + c)}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^3}{\sqrt{\cos(dx + c)}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 14.75 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08

$$\begin{aligned} \int \frac{(a + b \cos(c + dx))^3}{\sqrt{\cos(c + dx)}} dx = & \frac{2 a^3 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{6 a^2 b E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} \\ & + \frac{2 a b^2 F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d} + \frac{2 a b^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{d} \\ & - \frac{2 b^3 \cos(c + dx)^{7/2} \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{4}; \frac{11}{4}; \cos(c + dx)^2\right)}{7 d \sqrt{\sin(c + dx)}^2} \end{aligned}$$

```
[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*a^3*ellipticF(c/2 + (d*x)/2, 2))/d + (6*a^2*b*ellipticE(c/2 + (d*x)/2, 2))/d + (2*a*b^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a*b^2*cos(c + d*x)^(1/2)*sin(c + d*x))/d - (2*b^3*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```


$$3.578 \quad \int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	5701
Rubi [A] (verified)	5701
Mathematica [A] (verified)	5703
Maple [A] (verified)	5704
Fricas [C] (verification not implemented)	5704
Sympy [F(-1)]	5705
Maxima [F]	5705
Giac [F]	5705
Mupad [B] (verification not implemented)	5706

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)} dx = -\frac{2a(a^2-3b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b(9a^2+b^2)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d} - \frac{2b(3a^2-b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2(a+b \cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a^2*(a+b*cos
(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(1/2)-2/3*b*(3*a^2-b^2)*sin(d*x+c)*cos(d*x
+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

= {2871, 3102, 2827, 2720, 2719}

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2b(9a^2 + b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} - \frac{2a(a^2 - 3b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} - \frac{2b(3a^2 - b^2) \sin(c + dx) \sqrt{\cos(c + dx)}}{3d} + \frac{2a^2 \sin(c + dx)(a + b \cos(c + dx))}{d \sqrt{\cos(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(3/2), x]

[Out] (-2*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(9*a^2 + b^2)*EllipticF[(c + d*x)/2, 2])/(3*d) - (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I

ntegersQ[2*m, 2*n])

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{d\sqrt{\cos(c + dx)}} \\
 &+ 2 \int \frac{2a^2b - \frac{1}{2}a(a^2 - 3b^2) \cos(c + dx) - \frac{1}{2}b(3a^2 - b^2) \cos^2(c + dx)}{\sqrt{\cos(c + dx)}} dx \\
 &= -\frac{2b(3a^2 - b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{d\sqrt{\cos(c + dx)}} \\
 &+ \frac{4}{3} \int \frac{\frac{1}{4}b(9a^2 + b^2) - \frac{3}{4}a(a^2 - 3b^2) \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx \\
 &= -\frac{2b(3a^2 - b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{d\sqrt{\cos(c + dx)}} \\
 &- (a(a^2 - 3b^2)) \int \sqrt{\cos(c + dx)} dx + \frac{1}{3}(b(9a^2 + b^2)) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= -\frac{2a(a^2 - 3b^2) E(\frac{1}{2}(c + dx)|2)}{d} + \frac{2b(9a^2 + b^2) \text{EllipticF}(\frac{1}{2}(c + dx), 2)}{3d} \\
 &- \frac{2b(3a^2 - b^2) \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{d\sqrt{\cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\begin{aligned}
 &\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2 \left(-3(a^3 - 3ab^2) E\left(\frac{1}{2}(c + dx)|2\right) + (9a^2b + b^3) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \frac{(3a^3 + b^3 \cos(c + dx)) \sin(c + dx)}{\sqrt{\cos(c + dx)}} \right)}{3d}
 \end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(3/2), x]

[Out] $(2*(-3*(a^3 - 3*a*b^2)*\text{EllipticE}[(c + d*x)/2, 2] + (9*a^2*b + b^3)*\text{EllipticF}[(c + d*x)/2, 2] + ((3*a^3 + b^3*\text{Cos}[c + d*x])*\text{Sin}[c + d*x])/\text{Sqrt}[\text{Cos}[c + d*x]]))/(3*d)$

Maple [A] (verified)

Time = 8.83 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

method	result
default	$-\frac{2\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+9a^2b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right)}{\dots}$
parts	$-\frac{2a^3\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d\right)}{\dots}$

[In] `int((a+cos(d*x+c)*b)^3/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $-2/3*(4*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4*b^3-6*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*a^3-2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*b^3+9*a^2*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^3-9*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a*b^2)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.73

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{\sqrt{2}(-9i a^2 b - i b^3) \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(9i a^2 b + i b^3) \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3\sqrt{2}(I*a^3 - 3*I*a*b^2)*\cos(d*x + c)*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0,$$

[In] `integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")`

[Out] $1/3*(\text{sqrt}(2)*(-9*I*a^2*b - I*b^3)*\cos(d*x + c)*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + \text{sqrt}(2)*(9*I*a^2*b + I*b^3)*\cos(d*x + c)*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - 3*\text{sqrt}(2)*(I*a^3 - 3*I*a*b^2)*\cos(d*x + c)*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0,$

```
cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*a^3 + 3*I*a*b^2)*cos(d*x +
c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(b^3*cos(d*x + c) + 3*a^3)*sqrt(cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 15.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2b^3 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{3d} + \frac{6ab^2 E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} \\ + \frac{6a^2 b F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} + \frac{2b^3 \sqrt{\cos(c + dx)} \sin(c + dx)}{3d} \\ + \frac{2a^3 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^(3/2),x)

```
[Out] (2*b^3*ellipticF(c/2 + (d*x)/2, 2))/(3*d) + (6*a*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (6*a^2*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*b^3*cos(c + d*x)^(1/2)*sin(c + d*x))/(3*d) + (2*a^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

$$3.579 \quad \int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	5707
Rubi [A] (verified)	5707
Mathematica [A] (verified)	5709
Maple [C] (verified)	5710
Fricas [C] (verification not implemented)	5710
Sympy [F(-1)]	5711
Maxima [F]	5711
Giac [F]	5711
Mupad [B] (verification not implemented)	5712

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)} dx = -\frac{2b(3a^2-b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2a(a^2+9b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} + \frac{16a^2b \sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2(a+b \cos(c+dx)) \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] $-2*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a*(a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^{(1/2)}/cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2/3*a^2*(a+b*cos(d*x+c))*\sin(d*x+c)/d/cos(d*x+c)^{(3/2)}+16/3*a^2*b*\sin(d*x+c)/d/cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2871, 3100, 2827, 2720, 2719}

$$\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{2a(a^2+9b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d} - \frac{2b(3a^2-b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2a^2 \sin(c+dx)(a+b \cos(c+dx))}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2b \sin(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]

[Out] (-2*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(a^2 + 9*b^2)*EllipticF[(c + d*x)/2, 2])/(3*d) + (16*a^2*b*Sin[c + d*x])/(3*d*sqrt[Cos[c + d*x]]) + (2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3100

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} \\
 &+ \frac{2}{3} \int \frac{4a^2b + \frac{1}{2}a(a^2 + 9b^2) \cos(c + dx) - \frac{1}{2}b(a^2 - 3b^2) \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{16a^2b \sin(c + dx)}{3d \sqrt{\cos(c + dx)}} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} \\
 &+ \frac{4}{3} \int \frac{\frac{1}{4}a(a^2 + 9b^2) - \frac{3}{4}b(3a^2 - b^2) \cos(c + dx)}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{16a^2b \sin(c + dx)}{3d \sqrt{\cos(c + dx)}} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} \\
 &- (b(3a^2 - b^2)) \int \sqrt{\cos(c + dx)} dx + \frac{1}{3}(a(a^2 + 9b^2)) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= -\frac{2b(3a^2 - b^2) E(\frac{1}{2}(c + dx) | 2)}{d} + \frac{2a(a^2 + 9b^2) \text{EllipticF}(\frac{1}{2}(c + dx), 2)}{3d} \\
 &+ \frac{16a^2b \sin(c + dx)}{3d \sqrt{\cos(c + dx)}} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\begin{aligned}
 &\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2 \left((-9a^2b + 3b^3) E\left(\frac{1}{2}(c + dx) \mid 2\right) + a \left((a^2 + 9b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \frac{a(a + 9b \cos(c + dx)) \sin(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} \right) \right)}{3d}
 \end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]

[Out] (2*((-9*a^2*b + 3*b^3)*EllipticE[(c + d*x)/2, 2] + a*((a^2 + 9*b^2)*EllipticF[(c + d*x)/2, 2] + (a*(a + 9*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))))/(3*d)

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.73 (sec) , antiderivative size = 558, normalized size of antiderivative = 4.65

method	result
parts	$\frac{2a^3 \left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \dots\right)}$
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(36\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a^2b - 2F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \right)}{\dots}$

```
[In] int((a+cos(d*x+c)*b)^3/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*
((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d+2*b^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d+6*a*b^2/d*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))-6*a^2*b*(-2*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.85

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{\sqrt{2}(-i a^3 - 9i ab^2) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(i a^3 + 9i ab^2)}{\dots}$$

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a^3 - 9*I*a*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^3 + 9*I*a*b^2)*cos(d*x + c)^
2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(3*
I*a^2*b - I*b^3)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-3*I*a^2*b + I*b^3)*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) + 2*(9*a^2*b*cos(d*x + c) + a^3)*sqrt(cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{5}{2}}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{5}{2}}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 15.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2 \left(E\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) b^3 + 3 a F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right) b^2 \right)}{d} + \frac{2 a^3 \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{3 d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}} + \frac{6 a^2 b \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}}$$

[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^(5/2),x)

```
[Out] (2*(b^3*ellipticE(c/2 + (d*x)/2, 2) + 3*a*b^2*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*a^3*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*
cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (6*a^2*b*sin(c + d*x)*hypergeo
m([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2
^(1/2))
```

$$3.580 \quad \int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	5713
Rubi [A] (verified)	5713
Mathematica [A] (verified)	5715
Maple [B] (verified)	5716
Fricas [C] (verification not implemented)	5717
Sympy [F(-1)]	5717
Maxima [F]	5717
Giac [F]	5718
Mupad [B] (verification not implemented)	5718

Optimal result

Integrand size = 23, antiderivative size = 149

$$\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)} dx = -\frac{6a(a^2+5b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{2b(a^2+b^2) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} + \frac{8a^2b \sin(c+dx)}{5d \cos^{\frac{3}{2}}(c+dx)} + \frac{6a(a^2+5b^2) \sin(c+dx)}{5d \sqrt{\cos(c+dx)}} + \frac{2a^2(a+b \cos(c+dx)) \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)}$$

[Out] $-6/5*a*(a^2+5*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+2*b*(a^2+b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/d+8/5*a^2*b*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}+2/5*a^2*(a+b*\cos(d*x+c))*\sin(d*x+c)/d/\cos(d*x+c)^{(5/2)}+6/5*a*(a^2+5*b^2)*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2871, 3100, 2827, 2716, 2719, 2720}

$$\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{2b(a^2+b^2) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d} - \frac{6a(a^2+5b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d} + \frac{6a(a^2+5b^2) \sin(c+dx)}{5d \sqrt{\cos(c+dx)}} + \frac{2a^2 \sin(c+dx)(a+b \cos(c+dx))}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{8a^2b \sin(c+dx)}{5d \cos^{\frac{3}{2}}(c+dx)}$$

[In] Int[(a + b*cos[c + d*x])^3/cos[c + d*x]^(7/2), x]

[Out] (-6*a*(a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2])/d + (8*a^2*b*sin[c + d*x])/(5*d*cos[c + d*x]^(3/2)) + (6*a*(a^2 + 5*b^2)*sin[c + d*x])/(5*d*sqrt[cos[c + d*x]]) + (2*a^2*(a + b*cos[c + d*x])*sin[c + d*x])/(5*d*cos[c + d*x]^(5/2))

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2871

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*sin[e + f*x])^(m - 2)*((c + d*sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*sin[e + f*x])^(m - 3)*(c + d*sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3100

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := Simp[(-A*b^2

- a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\
 &+ \frac{2}{5} \int \frac{6a^2b + \frac{3}{2}a(a^2 + 5b^2) \cos(c + dx) + \frac{1}{2}b(a^2 + 5b^2) \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{8a^2b \sin(c + dx)}{5d \cos^{\frac{3}{2}}(c + dx)} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\
 &+ \frac{4}{15} \int \frac{\frac{9}{4}a(a^2 + 5b^2) + \frac{15}{4}b(a^2 + b^2) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{8a^2b \sin(c + dx)}{5d \cos^{\frac{3}{2}}(c + dx)} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\
 &+ (b(a^2 + b^2)) \int \frac{1}{\sqrt{\cos(c + dx)}} dx + \frac{1}{5}(3a(a^2 + 5b^2)) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2b(a^2 + b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} + \frac{8a^2b \sin(c + dx)}{5d \cos^{\frac{3}{2}}(c + dx)} + \frac{6a(a^2 + 5b^2) \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} \\
 &+ \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} - \frac{1}{5}(3a(a^2 + 5b^2)) \int \sqrt{\cos(c + dx)} dx \\
 &= -\frac{6a(a^2 + 5b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2b(a^2 + b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} \\
 &+ \frac{8a^2b \sin(c + dx)}{5d \cos^{\frac{3}{2}}(c + dx)} + \frac{6a(a^2 + 5b^2) \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\begin{aligned}
 &\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{-6a(a^2 + 5b^2) \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 10b(a^2 + b^2) \cos^{\frac{3}{2}}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 10a^2b}{5d \cos^{\frac{3}{2}}(c + dx)}
 \end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(7/2), x]

```
[Out] (-6*a*(a^2 + 5*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*b*(a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a^2*b*Sin[c + d*x] + 3*(a^3 + 5*a*b^2)*Sin[2*(c + d*x)] + 2*a^3*Tan[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(187) = 374$.

Time = 12.05 (sec) , antiderivative size = 711, normalized size of antiderivative = 4.77

method	result	size
default	Expression too large to display	711
parts	Expression too large to display	783

```
[In] int((a+cos(d*x+c)*b)^3/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/5*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a*b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```


Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.64

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx = \frac{5\sqrt{2}(i a^2 b + i b^3) \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-i a^2 b$$

[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] -1/5*(5*sqrt(2)*(I*a^2*b + I*b^3)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^2*b - I*b^3)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(I*a^3 + 5*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-I*a^3 - 5*I*a*b^2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(5*a^2*b*cos(d*x + c) + a^3 + 3*(a^3 + 5*a*b^2)*cos(d*x + c)^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^3}{\cos^{\frac{7}{2}}(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)

Mupad [B] (verification not implemented)

Time = 15.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.05

$$\begin{aligned} \int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{7}{2}}(c + dx)} dx &= \frac{2b^3 F\left(\frac{c}{2} + \frac{dx}{2} \mid 2\right)}{d} \\ &+ \frac{2a^3 \sin(c + dx) {}_2F_1\left(-\frac{5}{4}, \frac{1}{2}; -\frac{1}{4}; \cos(c + dx)^2\right)}{5d \cos(c + dx)^{5/2} \sqrt{\sin(c + dx)^2}} \\ &+ \frac{6ab^2 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right)}{d \sqrt{\cos(c + dx)} \sqrt{\sin(c + dx)^2}} \\ &+ \frac{2a^2 b \sin(c + dx) {}_2F_1\left(-\frac{3}{4}, \frac{1}{2}; \frac{1}{4}; \cos(c + dx)^2\right)}{d \cos(c + dx)^{3/2} \sqrt{\sin(c + dx)^2}} \end{aligned}$$

[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^(7/2),x)

[Out] (2*b^3*ellipticF(c/2 + (d*x)/2, 2))/d + (2*a^3*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^(1/2)) + (6*a*b^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*a^2*b*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))

$$3.581 \quad \int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	5719
Rubi [A] (verified)	5720
Mathematica [A] (verified)	5722
Maple [B] (verified)	5722
Fricas [C] (verification not implemented)	5723
Sympy [F(-1)]	5724
Maxima [F]	5724
Giac [F]	5724
Mupad [B] (verification not implemented)	5724

Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)} dx = -\frac{2b(9a^2+5b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(5a^2+21b^2)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d} + \frac{32a^2b \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx)} + \frac{2a(5a^2+21b^2)\sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} + \frac{2b(9a^2+5b^2)\sin(c+dx)}{5d \sqrt{\cos(c+dx)}} + \frac{2a^2(a+b \cos(c+dx))\sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)}$$

```
[Out] -2/5*b*(9*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(5*a^2+21*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+32/35*a^2*b*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*a*(5*a^2+21*b^2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/7*a^2*(a+b*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/5*b*(9*a^2+5*b^2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2871, 3100, 2827, 2716, 2720, 2719}

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx = \frac{2a(5a^2 + 21b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} - \frac{2b(9a^2 + 5b^2) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2a(5a^2 + 21b^2) \sin(c + dx)}{21d \cos^{\frac{3}{2}}(c + dx)} + \frac{2b(9a^2 + 5b^2) \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} + \frac{2a^2 \sin(c + dx)(a + b \cos(c + dx))}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{32a^2 b \sin(c + dx)}{35d \cos^{\frac{5}{2}}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]

[Out] (-2*b*(9*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*a^2 + 21*b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (32*a^2*b*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*a*(5*a^2 + 21*b^2)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (2*b*(9*a^2 + 5*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2871

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
+ a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])

```

Rule 3100

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2
- a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} \\
&+ \frac{2}{7} \int \frac{8a^2b + \frac{1}{2}a(5a^2 + 21b^2) \cos(c + dx) + \frac{1}{2}b(3a^2 + 7b^2) \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)} dx \\
&= \frac{32a^2b \sin(c + dx)}{35d \cos^{\frac{5}{2}}(c + dx)} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} \\
&+ \frac{4}{35} \int \frac{\frac{5}{4}a(5a^2 + 21b^2) + \frac{7}{4}b(9a^2 + 5b^2) \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{32a^2b \sin(c + dx)}{35d \cos^{\frac{5}{2}}(c + dx)} + \frac{2a^2(a + b \cos(c + dx)) \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} \\
&+ \frac{1}{5}(b(9a^2 + 5b^2)) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)} dx + \frac{1}{7}(a(5a^2 + 21b^2)) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx)} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{32a^2b \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx)} + \frac{2a(5a^2+21b^2) \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{2b(9a^2+5b^2) \sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2a^2(a+b \cos(c+dx)) \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)} \\
&\quad - \frac{1}{5}(b(9a^2+5b^2)) \int \sqrt{\cos(c+dx)} dx + \frac{1}{21}(a(5a^2+21b^2)) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= -\frac{2b(9a^2+5b^2) E(\frac{1}{2}(c+dx)|2)}{5d} + \frac{2a(5a^2+21b^2) \text{EllipticF}(\frac{1}{2}(c+dx), 2)}{21d} \\
&\quad + \frac{32a^2b \sin(c+dx)}{35d \cos^{\frac{5}{2}}(c+dx)} + \frac{2a(5a^2+21b^2) \sin(c+dx)}{21d \cos^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{2b(9a^2+5b^2) \sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2a^2(a+b \cos(c+dx)) \sin(c+dx)}{7d \cos^{\frac{7}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\begin{aligned}
&\int \frac{(a+b \cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)} dx \\
&= \frac{-42b(9a^2+5b^2) \cos^{\frac{5}{2}}(c+dx) E(\frac{1}{2}(c+dx)|2) + 10a(5a^2+21b^2) \cos^{\frac{5}{2}}(c+dx) \text{EllipticF}(\frac{1}{2}(c+dx), 2) + 10a^2b \sin(c+dx) + 378a^2b \cos(c+dx) \sin(c+dx) + 210b^3 \cos^2(c+dx) \sin(c+dx) + 25a^3 \sin^2(c+dx) + 105ab^2 \sin^2(c+dx) + 30a^3 \tan(c+dx)}{105d \cos^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(9/2), x]

[Out] (-42*b*(9*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*a*(5*a^2 + 21*b^2)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 126*a^2*b*Sin[c + d*x] + 378*a^2*b*Cos[c + d*x]^2*Sin[c + d*x] + 210*b^3*Cos[c + d*x]^2*Sin[c + d*x] + 25*a^3*Sin[2*(c + d*x)] + 105*a*b^2*Sin[2*(c + d*x)] + 30*a^3*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. 2(226) = 452.

Time = 14.69 (sec) , antiderivative size = 820, normalized size of antiderivative = 4.23

method	result	size
default	Expression too large to display	820
parts	Expression too large to display	1008

[In] int((a+cos(d*x+c)*b)^3/cos(d*x+c)^(9/2), x, method=_RETURNVERBOSE)

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6/5*a^2*b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.39

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx =$$

$$5\sqrt{2}(5i a^3 + 21i ab^2) \cos(dx + c)^4 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-5$$

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*a^3 + 21*I*a*b^2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*a^3 - 21*I*a*b^2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(9*I*a^2*b + 5*I*b^3)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-9*I*a^2*b - 5*I*b^3)*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(63*a^2*b*cos(d*x + c) + 21*(9*a^2*b + 5*b^3)*cos(d*x + c)^3 + 15*a^3 + 5*(5*a^3 + 21*a*b^2)*cos(d*x + c)^2)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{9}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^3}{\cos(dx + c)^{\frac{9}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)

Mupad [B] (verification not implemented)

Time = 16.46 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.76

$$\int \frac{(a + b \cos(c + dx))^3}{\cos^{\frac{9}{2}}(c + dx)} dx$$

$$= \frac{2a^3 \sin(c+dx) {}_2F_1\left(-\frac{7}{4}, \frac{1}{2}; -\frac{3}{4}; \cos(c+dx)^2\right)}{7} + 2b^3 \cos(c + dx)^3 \sin(c + dx) {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cos(c + dx)^2\right) + \frac{6a^2 b \cos(c + dx)}{d \cos(c + dx)^{7/2} \sqrt{1 - \cos(c + dx)^2}}$$

[In] int((a + b*cos(c + d*x))^3/cos(c + d*x)^(9/2),x)

[Out] ((2*a^3*sin(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2))/7 + 2*b^3*cos(c + d*x)^3*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2) + (6*a^2*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/5 + 2*a*b^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))

$$3.582 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b \cos(c+dx)} dx$$

Optimal result	5725
Rubi [A] (verified)	5725
Mathematica [A] (verified)	5727
Maple [B] (verified)	5728
Fricas [F(-1)]	5728
Sympy [F(-1)]	5729
Maxima [F]	5729
Giac [F]	5729
Mupad [F(-1)]	5729

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b \cos(c+dx)} dx = -\frac{2aE\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^2d} + \frac{2(3a^2+b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^3d} \\ - \frac{2a^3 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{b^3(a+b)d} + \frac{2\sqrt{\cos(c+dx)} \sin(c+dx)}{3bd}$$

[Out] $-2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b^2/d+2/3*(3*a^2+b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b^3/d-2*a^3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^{(1/2)})/b^3/(a+b)/d+2/3*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/b/d$

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2872, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b \cos(c+dx)} dx = -\frac{2a^3 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{b^3d(a+b)} \\ + \frac{2(3a^2+b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^3d} \\ - \frac{2aE\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^2d} + \frac{2 \sin(c+dx) \sqrt{\cos(c+dx)}}{3bd}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^{(5/2)}/(a+b*\operatorname{Cos}[c+d*x]), x]$

[Out] $(-2*a*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(3*a^2 + b^2)*EllipticF[(c + d*x)/2, 2])/(3*b^3*d) - (2*a^3*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)$

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2872

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3138

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +

```
(f_.)*(x_)]), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2\int\frac{\frac{a}{2}+\frac{1}{2}b\cos(c+dx)-\frac{3}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b} \\
&= \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{2\int\frac{-\frac{ab}{2}-\frac{1}{2}(3a^2+b^2)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3b^2} - \frac{a\int\sqrt{\cos(c+dx)}dx}{b^2} \\
&= -\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{b^2d} + \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd} \\
&\quad - \frac{a^3\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{b^3} + \frac{(3a^2+b^2)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^3} \\
&= -\frac{2aE\left(\frac{1}{2}(c+dx)|2\right)}{b^2d} + \frac{2(3a^2+b^2)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^3d} \\
&\quad - \frac{2a^3\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^3(a+b)d} + \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.38 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.41

$$\begin{aligned}
&\int\frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}dx \\
&= \frac{4\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - \frac{6a\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + 4\sqrt{\cos(c+dx)}\sin(c+dx) - \frac{6(-2abE(\arcsin(\sqrt{\cos(c+dx)}))}{6bd}
\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (4*EllipticF[(c + d*x)/2, 2] - (6*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] - (6*(-2*a*b*EllipticE[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x
]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1
])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/(6*b*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(184) = 368.

Time = 5.14 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.93

method	result
default	$-\frac{2\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a b^2-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right)$

[In] `int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)`

[Out]
$$-\frac{2}{3}\left(\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(4\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a*b^2-4\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*b^3-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a*b^2+2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b^3+3*a^3\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}*\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)-3*a^2*b*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}*\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)+a*b^2*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}*\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)-b^3*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}*\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)+3*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}*\text{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)*a^2*b-3*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}*\text{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)*a*b^2-3*a^3*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}*\text{EllipticPi}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),-2*b/(a-b),2^{\frac{1}{2}}\right)\right)/b^3/(a-b)/\left(-2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/\left(2*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}/d$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x,algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{b \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{b \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(c + dx)^{5/2}}{a + b \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x)), x)

$$3.583 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b \cos(c+dx)} dx$$

Optimal result	5730
Rubi [A] (verified)	5730
Mathematica [A] (verified)	5732
Maple [A] (verified)	5732
Fricas [F(-1)]	5733
Sympy [F(-1)]	5733
Maxima [F]	5733
Giac [F]	5733
Mupad [F(-1)]	5734

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b \cos(c+dx)} dx = \frac{2E\left(\frac{1}{2}(c+dx) \mid 2\right)}{bd} - \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^2d} + \frac{2a^2 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{b^2(a+b)d}$$

[Out] $2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b/d - 2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b^2/d + 2*a^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^{(1/2)})/b^2/(a+b)/d$

Rubi [A] (verified)

Time = 0.18 (sec), antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2883, 2719, 2882, 2720, 2884}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b \cos(c+dx)} dx = \frac{2a^2 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{b^2d(a+b)} - \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^2d} + \frac{2E\left(\frac{1}{2}(c+dx) \mid 2\right)}{bd}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c+d*x]^{(3/2)}/(a+b*\operatorname{Cos}[c+d*x]),x]$

[Out] $(2*\operatorname{EllipticE}[(c+d*x)/2, 2])/(b*d) - (2*a*\operatorname{EllipticF}[(c+d*x)/2, 2])/(b^2*d) + (2*a^2*\operatorname{EllipticPi}[(2*b)/(a+b), (c+d*x)/2, 2])/(b^2*(a+b)*d)$

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2882

Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2883

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\int \sqrt{\cos(c + dx)} dx}{b} - \frac{a \int \frac{\sqrt{\cos(c+dx)}}{a+b \cos(c+dx)} dx}{b} \\
 &= \frac{2E\left(\frac{1}{2}(c + dx) \mid 2\right)}{bd} - \frac{a \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{b^2} + \frac{a^2 \int \frac{1}{\sqrt{\cos(c+dx)(a+b \cos(c+dx))}} dx}{b^2} \\
 &= \frac{2E\left(\frac{1}{2}(c + dx) \mid 2\right)}{bd} - \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{b^2 d} + \frac{2a^2 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{b^2(a + b)d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 10.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \frac{2\left(bE\left(\arcsin\left(\sqrt{\cos(c+dx)}\right)\middle| -1\right) - (a+b)\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\cos(c+dx)}\right), -1\right) + a\operatorname{EllipticPi}\left(\arcsin\left(\sqrt{\cos(c+dx)}\right), -1\right)\right)}{b^2 d \sqrt{\sin^2(c+dx)}}$$

[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]

```
[Out] (-2*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + a*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*d*Sqrt[Sin[c + d*x]^2])
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.03

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^2-F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)ab+b^2(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right)}{b^2 d \sqrt{\sin^2(c+dx)}}$

[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```


Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{b \cos(dx + c) + a} dx$$

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{b \cos(dx + c) + a} dx$$

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(c + dx)^{3/2}}{a + b \cos(c + dx)} dx$$

```
[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x)), x)
```

$$3.584 \quad \int \frac{\sqrt{\cos(c+dx)}}{a+b \cos(c+dx)} dx$$

Optimal result	5735
Rubi [A] (verified)	5735
Mathematica [A] (verified)	5736
Maple [A] (verified)	5736
Fricas [F(-1)]	5737
Sympy [F(-1)]	5737
Maxima [F]	5737
Giac [F]	5738
Mupad [F(-1)]	5738

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b \cos(c+dx)} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{bd} - \frac{2a \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{b(a+b)d}$$

[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^(1/2))/b/(a+b)/d

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2882, 2720, 2884}

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b \cos(c+dx)} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{bd} - \frac{2a \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{bd(a+b)}$$

[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x]),x]

[Out] (2*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b*(a + b)*d)

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
+ Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])], x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{b} - \frac{a \int \frac{1}{\sqrt{\cos(c+dx)(a+b \cos(c+dx))}} dx}{b} \\ &= \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{bd} - \frac{2a \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{b(a+b)d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b \cos(c+dx)} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - \frac{2a \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b}}{bd}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2]))/(a + b)/(b*d)
```

Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.55

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)b\right)}{(a-b)b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d$

[In] `int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)`

[Out] $-2*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}*(\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a-\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b-a*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})))/(a-b)/b/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)} dx = \int \frac{\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a} dx$$

[In] `integrate(sqrt(cos(d*x+c))/(b*cos(d*x+c)+a),x, algorithm="maxima")`

[Out] `integrate(sqrt(cos(d*x+c))/(b*cos(d*x+c)+a),x)`

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)} dx = \int \frac{\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)} dx = \int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x)), x)

$$3.585 \quad \int \frac{1}{\sqrt{\cos(c+dx)(a+b \cos(c+dx))}} dx$$

Optimal result	5739
Rubi [A] (verified)	5739
Mathematica [A] (verified)	5740
Maple [B] (verified)	5740
Fricas [F(-1)]	5740
Sympy [F(-1)]	5741
Maxima [F]	5741
Giac [F]	5741
Mupad [F(-1)]	5741

Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{1}{\sqrt{\cos(c+dx)(a+b \cos(c+dx))}} dx = \frac{2 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{(a+b)d}$$

[Out] $2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^{(1/2)})/(a+b)/d$

Rubi [A] (verified)

Time = 0.05 (sec), antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2884}

$$\int \frac{1}{\sqrt{\cos(c+dx)(a+b \cos(c+dx))}} dx = \frac{2 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{d(a+b)}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*(a+b*\operatorname{Cos}[c+d*x])), x]$

[Out] $(2*\operatorname{EllipticPi}[(2*b)/(a+b), (c+d*x)/2, 2])/((a+b)*d)$

Rule 2884

$\operatorname{Int}[1/(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])*\operatorname{Sqrt}[(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])), x_Symbol] \rightarrow \operatorname{Simp}[(2/(f*(a+b)*\operatorname{Sqrt}[c+d]))*\operatorname{EllipticPi}[2*(b/(a+b)), (1/2)*(e - \operatorname{Pi}/2 + f*x), 2*(d/(c+d))], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[a^2 - b^2, 0] \&\& \operatorname{NeQ}[c^2 - d^2, 0] \&\& \operatorname{GtQ}[c+d, 0]$

Rubi steps

$$\text{integral} = \frac{2 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{(a+b)d}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx = \frac{2 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{(a+b)d}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]

[Out] (2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(55) = 110.

Time = 2.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 5.17

method	result	size
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$	150

[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx = \int \frac{1}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx = \int \frac{1}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx = \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))),x)

[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))), x)

$$3.586 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} dx$$

Optimal result	5742
Rubi [A] (verified)	5742
Mathematica [B] (verified)	5744
Maple [B] (verified)	5744
Fricas [F(-1)]	5745
Sympy [F(-1)]	5745
Maxima [F]	5746
Giac [F]	5746
Mupad [F(-1)]	5746

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} dx = -\frac{2E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} - \frac{2b \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a(a+b)d} + \frac{2 \sin(c+dx)}{ad \sqrt{\cos(c+dx)}}$$

[Out] $-2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d-2*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^{(1/2)})/a/(a+b)/d+2*\sin(d*x+c)/a/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2881, 3138, 2719, 12, 2884}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} dx = -\frac{2b \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{ad(a+b)} - \frac{2E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad} + \frac{2 \sin(c+dx)}{ad \sqrt{\cos(c+dx)}}$$

[In] $\operatorname{Int}[1/(\operatorname{Cos}[c+d*x]^{(3/2)}*(a+b*\operatorname{Cos}[c+d*x])),x]$

[Out] $(-2*\text{EllipticE}[(c + d*x)/2, 2])/(a*d) - (2*b*\text{EllipticPi}[(2*b)/(a + b), (c + d*x)/2, 2])/(a*(a + b)*d) + (2*\text{Sin}[c + d*x])/(a*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 2719

$\text{Int}[\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2881

$\text{Int}[((a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_)])^{(m_)}*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-b^2)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m + 1)}*((c + d*\text{Sin}[e + f*x])^{(n + 1)})/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))], x] + \text{Dist}[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*(c + d*\text{Sin}[e + f*x])^n*\text{Simp}[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*\text{Sin}[e + f*x] - b^2*d*(m + n + 3)*\text{Sin}[e + f*x]^2, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegersQ}[2*m, 2*n] \ \&\& \ ((\text{EqQ}[a, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !\text{IntegerQ}[n]) \ || \ !(\text{IntegerQ}[2*n] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ ((\text{IntegerQ}[n] \ \&\& \ !\text{IntegerQ}[m]) \ || \ \text{EqQ}[a, 0])))$

Rule 2884

$\text{Int}[1/(((a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_)])*\text{Sqrt}[(c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_)]]), x_Symbol] \rightarrow \text{Simp}[(2/(f*(a + b)*\text{Sqrt}[c + d]))*\text{EllipticPi}[2*(b/(a + b)), (1/2)*(e - \text{Pi}/2 + f*x), 2*(d/(c + d))], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[c + d, 0]$

Rule 3138

$\text{Int}[((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_)])^2/(\text{Sqrt}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_)])), x_Symbol] \rightarrow \text{Dist}[C/(b*d), \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]], x], x] - \text{Dist}[1/(b*d), \text{Int}[\text{Simp}[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*\text{Sin}[e + f*x], x]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2 \sin(c + dx)}{ad\sqrt{\cos(c + dx)}} + \frac{2 \int \frac{-\frac{b}{2} - \frac{1}{2}a \cos(c+dx) - \frac{1}{2}b \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{a} \\
&= \frac{2 \sin(c + dx)}{ad\sqrt{\cos(c + dx)}} - \frac{\int \sqrt{\cos(c + dx)} dx}{a} - \frac{2 \int \frac{b^2}{2\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{ab} \\
&= -\frac{2E\left(\frac{1}{2}(c + dx) \mid 2\right)}{ad} + \frac{2 \sin(c + dx)}{ad\sqrt{\cos(c + dx)}} - \frac{b \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{a} \\
&= -\frac{2E\left(\frac{1}{2}(c + dx) \mid 2\right)}{ad} - \frac{2b \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{a(a + b)d} + \frac{2 \sin(c + dx)}{ad\sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 195 vs. $2(77) = 154$.

Time = 1.96 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.53

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))} dx = \frac{\frac{6b \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} + \frac{2a \left(2 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - \frac{2a \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b}\right)}{b} - \frac{4 \sin(c+dx)}{\sqrt{\cos(c+dx)}} + \frac{2(-2abE(\arcsin(\sqrt{\cos(c+dx)}))}{2ad}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]

[Out] -1/2*((6*b*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2])/(a*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(127) = 254$.

Time = 3.90 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.60

method	result
default	$\frac{2\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{d}$

[In] `int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -2*(-2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(a-b)*\cos(1/2*d \\ & *x+1/2*c)*\sin(1/2*d*x+1/2*c)^2+(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^ \\ & 2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*Elli \\ & pticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/ \\ & 2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)} \\ & *EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b-b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2* \\ & \sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1 \\ & /2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/a/(-2*\sin(1/2*d* \\ & x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(a-b)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2* \\ & d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} dx = \text{Timed out}$$

[In] `integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} dx = \text{Timed out}$$

[In] `integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} dx = \int \frac{1}{\cos(c+dx)^{3/2}(a+b\cos(c+dx))} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))),x)

[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))), x)

$$3.587 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))} dx$$

Optimal result	5747
Rubi [A] (verified)	5747
Mathematica [A] (verified)	5750
Maple [B] (verified)	5750
Fricas [F(-1)]	5751
Sympy [F(-1)]	5751
Maxima [F]	5751
Giac [F]	5752
Mupad [F(-1)]	5752

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))} dx = \frac{2bE\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2d} + \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3ad} \\ + \frac{2b^2 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a^2(a+b)d} \\ + \frac{2 \sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)} - \frac{2b \sin(c+dx)}{a^2d \sqrt{\cos(c+dx)}}$$

[Out] $2*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a^2/d+2/3*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/a/d+2*b^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^{(1/2)})/a^2/(a+b)/d+2/3*\sin(d*x+c)/a/d/\cos(d*x+c)^{(3/2)}-2*b*\sin(d*x+c)/a^2/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2881, 3134, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))} dx = \frac{2b^2 \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a^2d(a+b)} \\ + \frac{2bE\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2d} - \frac{2b \sin(c+dx)}{a^2d \sqrt{\cos(c+dx)}} \\ + \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3ad} + \frac{2 \sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]

[Out] (2*b*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*a*d) + (2*b^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)*d) + (2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*b*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134


```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))

```

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2 \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)} + \frac{2 \int \frac{-\frac{3b}{2} + \frac{1}{2}a \cos(c+dx) + \frac{1}{2}b \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} dx}{3a} \\
&= \frac{2 \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)} - \frac{2b \sin(c + dx)}{a^2 d \sqrt{\cos(c + dx)}} + \frac{4 \int \frac{\frac{1}{4}(a^2+3b^2) + ab \cos(c+dx) + \frac{3}{4}b^2 \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{3a^2} \\
&= \frac{2 \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)} - \frac{2b \sin(c + dx)}{a^2 d \sqrt{\cos(c + dx)}} \\
&\quad - \frac{4 \int \frac{-\frac{1}{4}b(a^2+3b^2) - \frac{1}{4}ab^2 \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{3a^2 b} + \frac{b \int \sqrt{\cos(c + dx)} dx}{a^2} \\
&= \frac{2bE\left(\frac{1}{2}(c + dx) \middle| 2\right)}{a^2 d} + \frac{2 \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)} - \frac{2b \sin(c + dx)}{a^2 d \sqrt{\cos(c + dx)}} \\
&\quad + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3a} + \frac{b^2 \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{a^2}
\end{aligned}$$

$$= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad} + \frac{2b^2\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a^2(a+b)d} + \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.64

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))} dx = \frac{2(2a^2+9b^2)\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + 8a\left(2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - \frac{2a\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b}\right) + \frac{4(a-3b\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]

[Out] ((2*(2*a^2 + 9*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + 8*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + (4*(a - 3*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2) + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/(6*a^2*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(198) = 396.

Time = 5.97 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.32

method	result
default	$\frac{\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{3\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2} + \frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}\right)a}$

[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] -((-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF

$(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 2/a^2*b/\sin(1/2*d*x+1/2*c)^2 / (2*\sin(1/2*d*x+1/2*c)^2 - 1) * (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (2*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c) - (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (2*\sin(1/2*d*x+1/2*c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})) - 4*b^3/a^2 / (-2*a*b + 2*b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2 - 1)^{(1/2)} / d$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{5/2}(c+dx)(a+b\cos(c+dx))} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{5/2}(c+dx)(a+b\cos(c+dx))} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\cos^{5/2}(c+dx)(a+b\cos(c+dx))} dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{5/2}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x+c)+a)*cos(d*x+c)^(5/2)),x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))} dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))} dx = \int \frac{1}{\cos(c+dx)^{5/2} (a+b\cos(c+dx))} dx$$

[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))),x)

[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))), x)

$$3.588 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	5753
Rubi [A] (verified)	5754
Mathematica [A] (verified)	5756
Maple [B] (verified)	5757
Fricas [F(-1)]	5758
Sympy [F(-1)]	5758
Maxima [F]	5758
Giac [F]	5758
Mupad [F(-1)]	5759

Optimal result

Integrand size = 23, antiderivative size = 245

$$\begin{aligned} \int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx = & -\frac{a(5a^2-4b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^3(a^2-b^2)d} \\ & + \frac{(15a^4-16a^2b^2-2b^4) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^4(a^2-b^2)d} \\ & - \frac{a^3(5a^2-7b^2) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{(a-b)b^4(a+b)^2d} \\ & + \frac{(5a^2-2b^2) \sqrt{\cos(c+dx)} \sin(c+dx)}{3b^2(a^2-b^2)d} \\ & - \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b \cos(c+dx))} \end{aligned}$$

```
[Out] -a*(5*a^2-4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c), 2^(1/2))/b^3/(a^2-b^2)/d+1/3*(15*a^4-16*a^2*b^2-2*b^4)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
, 2^(1/2))/b^4/(a^2-b^2)/d-a^3*(5*a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^(1/2))/(a-b)/b^4
/(a+b)^2/d-a^2*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))+1
/3*(5*a^2-2*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2871, 3128, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx = -\frac{a^2 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))} + \frac{(5a^2-2b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3b^2d(a^2-b^2)} - \frac{a(5a^2-4b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a^2-b^2)} + \frac{(15a^4-16a^2b^2-2b^4)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^4d(a^2-b^2)} - \frac{a^3(5a^2-7b^2)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{b^4d(a-b)(a+b)^2}$$

[In] Int[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^2,x]

[Out] -((a*(5*a^2 - 4*b^2)*EllipticE[(c + d*x)/2, 2])/(b^3*(a^2 - b^2)*d)) + ((15*a^4 - 16*a^2*b^2 - 2*b^4)*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d) - (a^3*(5*a^2 - 7*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a - b)*b^4*(a + b)^2*d + ((5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -

```
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_))*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*SIN[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*SIN[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_))*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e
_.) + (f_.)*(x_)])^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*SIN[e + f*x
])^m*((c + d*SIN[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*SIN[e + f*x])^(m - 1)*(c + d*SIN[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*SIN[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*SIN[e + f*x]]*(c + d*SIN[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

integral

$$\begin{aligned}
&= -\frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} - \frac{\int \frac{\sqrt{\cos(c+dx)} \left(\frac{3a^2}{2} - ab\cos(c+dx) - \frac{1}{2}(5a^2-2b^2)\cos^2(c+dx) \right)}{a+b\cos(c+dx)} dx}{b(a^2-b^2)} \\
&= \frac{(5a^2-2b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)d} - \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} \\
&\quad - \frac{2 \int \frac{-\frac{1}{4}a(5a^2-2b^2) + \frac{1}{2}b(2a^2+b^2)\cos(c+dx) + \frac{3}{4}a(5a^2-4b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{3b^2(a^2-b^2)} \\
&= \frac{(5a^2-2b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)d} - \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} \\
&\quad + \frac{2 \int \frac{\frac{1}{4}ab(5a^2-2b^2) + \frac{1}{4}(15a^4-16a^2b^2-2b^4)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{3b^3(a^2-b^2)} - \frac{(a(5a^2-4b^2)) \int \sqrt{\cos(c+dx)} dx}{2b^3(a^2-b^2)} \\
&= -\frac{a(5a^2-4b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a^2-b^2)d} + \frac{(5a^2-2b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)d} \\
&\quad - \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} - \frac{(a^3(5a^2-7b^2)) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2b^4(a^2-b^2)} \\
&\quad + \frac{(15a^4-16a^2b^2-2b^4) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{6b^4(a^2-b^2)} \\
&= -\frac{a(5a^2-4b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a^2-b^2)d} + \frac{(15a^4-16a^2b^2-2b^4)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^4(a^2-b^2)d} \\
&\quad - \frac{a^3(5a^2-7b^2)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{(a-b)b^4(a+b)^2d} \\
&\quad + \frac{(5a^2-2b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)d} - \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.09

$$\begin{aligned}
&\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx \\
&= \frac{4\sqrt{\cos(c+dx)} \left(2 + \frac{3a^3}{(a^2-b^2)(a+b\cos(c+dx))} \right) \sin(c+dx) - \frac{2(5a^3-8ab^2)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{8(2a^2+b^2)(a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2}}{\dots}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(7/2)/(a + b*cos[c + d*x])^2,x]


```
[Out] (4*sqrt(Cos[c + d*x])*(2 + (3*a^3)/((a^2 - b^2)*(a + b*cos[c + d*x])))*Sin[
c + d*x] - ((2*(5*a^3 - 8*a*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a + b) + (8*(2*a^2 + b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(5*a^2 - 4*b^2)*(-2*a*b*El
lipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt
[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c
+ d*x]]], -1])*Sin[c + d*x])/(b^2*sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b))
/(12*b^2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(315) = 630$.

Time = 17.42 (sec) , antiderivative size = 1070, normalized size of antiderivative = 4.37

method	result	size
default	Expression too large to display	1070

```
[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^2*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4*(a+b)/b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2/b^4*a^4*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/
2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2
*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2)))+2*(3*a^2+2*a*b+b^2)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*a^3/b^3/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
```

$(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})$
 $/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(c + dx)^{7/2}}{(a + b \cos(c + dx))^2} dx$$

```
[In] int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^2, x)
```

```
[Out] int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^2, x)
```

$$3.589 \quad \int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	5760
Rubi [A] (verified)	5760
Mathematica [A] (verified)	5763
Maple [B] (verified)	5763
Fricas [F(-1)]	5764
Sympy [F(-1)]	5764
Maxima [F]	5764
Giac [F]	5765
Mupad [F(-1)]	5765

Optimal result

Integrand size = 23, antiderivative size = 185

$$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^2} dx = \frac{(3a^2 - 2b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^2 (a^2 - b^2) d} - \frac{a(3a^2 - 4b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^3 (a^2 - b^2) d} + \frac{a^2(3a^2 - 5b^2) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{(a-b)b^3(a+b)^2 d} - \frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{b(a^2 - b^2) d(a+b \cos(c+dx))}$$

```
[Out] (3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d-a*(3*a^2-4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)/d+a^2*(3*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)/b^3/(a+b)^2/d-a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used

= {2871, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx = \frac{(3a^2-2b^2)E(\frac{1}{2}(c+dx)|2)}{b^2d(a^2-b^2)} - \frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd(a^2-b^2)(a+b\cos(c+dx))} - \frac{a(3a^2-4b^2)\text{EllipticF}(\frac{1}{2}(c+dx),2)}{b^3d(a^2-b^2)} + \frac{a^2(3a^2-5b^2)\text{EllipticPi}(\frac{2b}{a+b},\frac{1}{2}(c+dx),2)}{b^3d(a-b)(a+b)^2}$$

[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2, x]

[Out] ((3*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - (a*(3*a^2 - 4*b^2)*EllipticF[(c + d*x)/2, 2])/(b^3*(a^2 - b^2)*d) + (a^2*(3*a^2 - 5*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) - (a^2*sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c

, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3138

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} - \frac{\int \frac{\frac{a^2}{2} - ab \cos(c+dx) - \frac{1}{2}(3a^2-2b^2) \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{b(a^2-b^2)} \\
 &= -\frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{a^2b}{2} - \frac{1}{2}a(3a^2-4b^2) \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{b^2(a^2-b^2)} \\
 &\quad + \frac{(3a^2-2b^2) \int \sqrt{\cos(c+dx)} dx}{2b^2(a^2-b^2)} \\
 &= \frac{(3a^2-2b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^2(a^2-b^2)d} - \frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))} \\
 &\quad + \frac{(a^2(3a^2-5b^2)) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2b^3(a^2-b^2)} - \frac{(a(3a^2-4b^2)) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2b^3(a^2-b^2)} \\
 &= \frac{(3a^2-2b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^2(a^2-b^2)d} - \frac{a(3a^2-4b^2) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^3(a^2-b^2)d} \\
 &\quad + \frac{a^2(3a^2-5b^2) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{(a-b)b^3(a+b)^2d} - \frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{b(a^2-b^2)d(a+b\cos(c+dx))}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.36

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx$$

$$= \frac{4a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{(-a^2+b^2)(a+b\cos(c+dx))} + \frac{2(a^2-2b^2) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} + 4a \left(2 \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - \frac{2a \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} \right) + \frac{2(3a^2-2b^2)}{4bd}$$

[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]

[Out] ((4*a^2*sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x])) + ((2*(a^2 - 2*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + 4*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + (2*(3*a^2 - 2*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1]*Sin[c + d*x])/(a*b^2*sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b))/(4*b*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. 2(261) = 522.

Time = 17.08 (sec) , antiderivative size = 815, normalized size of antiderivative = 4.41

method	result	size
default	Expression too large to display	815

[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*a+b*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))-12/b^2*a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^(1/2))-2/b^3*a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)

$\ast c)^2)^{(1/2)} \ast \text{EllipticE}(\cos(1/2 \ast d \ast x + 1/2 \ast c), 2^{(1/2)}) - 3 \ast a / (a^2 - b^2) / (-2 \ast a \ast b + 2 \ast b^2) \ast b \ast (\sin(1/2 \ast d \ast x + 1/2 \ast c)^2)^{(1/2)} \ast (-2 \ast \cos(1/2 \ast d \ast x + 1/2 \ast c)^2 + 1)^{(1/2)} / (-2 \ast \sin(1/2 \ast d \ast x + 1/2 \ast c)^4 + \sin(1/2 \ast d \ast x + 1/2 \ast c)^2)^{(1/2)} \ast \text{EllipticPi}(\cos(1/2 \ast d \ast x + 1/2 \ast c), -2 \ast b / (a - b), 2^{(1/2)}) + 1 / a / (a^2 - b^2) / (-2 \ast a \ast b + 2 \ast b^2) \ast b^3 \ast (\sin(1/2 \ast d \ast x + 1/2 \ast c)^2)^{(1/2)} \ast (-2 \ast \cos(1/2 \ast d \ast x + 1/2 \ast c)^2 + 1)^{(1/2)} / (-2 \ast \sin(1/2 \ast d \ast x + 1/2 \ast c)^4 + \sin(1/2 \ast d \ast x + 1/2 \ast c)^2)^{(1/2)} \ast \text{EllipticPi}(\cos(1/2 \ast d \ast x + 1/2 \ast c), -2 \ast b / (a - b), 2^{(1/2)})) / \sin(1/2 \ast d \ast x + 1/2 \ast c) / (2 \ast \cos(1/2 \ast d \ast x + 1/2 \ast c)^2 - 1)^{(1/2)} / d$

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{5/2}(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{5/2}(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{5/2}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{5/2}}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(c + dx)^{5/2}}{(a + b \cos(c + dx))^2} dx$$

[In] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^2,x)

[Out] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^2, x)

$$3.590 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	5766
Rubi [A] (verified)	5766
Mathematica [A] (verified)	5768
Maple [B] (verified)	5769
Fricas [F(-1)]	5769
Sympy [F(-1)]	5770
Maxima [F]	5770
Giac [F]	5770
Mupad [F(-1)]	5770

Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx = -\frac{aE\left(\frac{1}{2}(c+dx) \mid 2\right)}{b(a^2-b^2)d} + \frac{(a^2-2b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^2(a^2-b^2)d} - \frac{a(a^2-3b^2) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{(a-b)b^2(a+b)^2d} + \frac{a\sqrt{\cos(c+dx)} \sin(c+dx)}{(a^2-b^2)d(a+b \cos(c+dx))}$$

[Out] $-a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b/(a^2-b^2)/d+(a^2-2*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})/b^2/(a^2-b^2)/d-a*(a^2-3*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^{(1/2)})/(a-b)/b^2/(a+b)^2/d+a*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/(a^2-b^2)/d/(a+b*\cos(d*x+c))$

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2878, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx = \frac{(a^2-2b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^2d(a^2-b^2)} - \frac{aE\left(\frac{1}{2}(c+dx) \mid 2\right)}{bd(a^2-b^2)} - \frac{a(a^2-3b^2) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{b^2d(a-b)(a+b)^2} + \frac{a \sin(c+dx) \sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b \cos(c+dx))}$$

[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]

[Out] -((a*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d)) + ((a^2 - 2*b^2)*EllipticF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - (a*(a^2 - 3*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^2*(a + b)^2*d) + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_.)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{a}{2}+b\cos(c+dx)+\frac{1}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{-a^2+b^2} \\
&= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{ab}{2}+\frac{1}{2}(a^2-2b^2)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{b(a^2-b^2)} - \frac{a\int \sqrt{\cos(c+dx)} dx}{2b(a^2-b^2)} \\
&= -\frac{aE\left(\frac{1}{2}(c+dx)\mid 2\right)}{b(a^2-b^2)d} + \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\cos(c+dx))} \\
&\quad - \frac{(a(a^2-3b^2))\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2b^2(a^2-b^2)} + \frac{(a^2-2b^2)\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2b^2(a^2-b^2)} \\
&= -\frac{aE\left(\frac{1}{2}(c+dx)\mid 2\right)}{b(a^2-b^2)d} + \frac{(a^2-2b^2)\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^2(a^2-b^2)d} \\
&\quad - \frac{a(a^2-3b^2)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{(a-b)b^2(a+b)^2d} + \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.19

$$\begin{aligned}
&\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx \\
&= \frac{4a\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))} - \frac{8\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - \frac{10a\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} + \frac{2(-2abE(\arcsin(\sqrt{\cos(c+dx)})) - 1) + 2a(a+b)\text{EllipticF}(\arcsin(\sqrt{\cos(c+dx)}))}{(a-b)(a+b)}}{4d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2, x]

[Out] ((4*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])) - (8*EllipticF[(c + d*x)/2, 2] - (10*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/(a - b)*(a + b))/(4*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(239) = 478$.

Time = 5.82 (sec) , antiderivative size = 794, normalized size of antiderivative = 4.87

method	result	size
default	Expression too large to display	794

[In] `int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)`

[Out]
$$-(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2/b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+2*a^2/b^2*(-1/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+8*a/b/(-2*a*b+2*b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)}))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(c + dx)^{\frac{3}{2}}}{(a + b \cos(c + dx))^2} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^2,x)

[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^2, x)

$$3.591 \quad \int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^2} dx$$

Optimal result	5771
Rubi [A] (verified)	5771
Mathematica [A] (verified)	5773
Maple [B] (verified)	5774
Fricas [F(-1)]	5774
Sympy [F(-1)]	5775
Maxima [F]	5775
Giac [F]	5775
Mupad [F(-1)]	5775

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^2} dx = \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{(a^2-b^2)d} + \frac{a \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b(a^2-b^2)d} - \frac{(a^2+b^2) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{(a-b)b(a+b)^2d} - \frac{b\sqrt{\cos(c+dx)} \sin(c+dx)}{(a^2-b^2)d(a+b \cos(c+dx))}$$

[Out] (cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/(a^2-b^2)/d+a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/b/(a^2-b^2)/d-(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^(1/2))/(a-b)/b/(a+b)^2/d-b*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2875, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^2} dx = \frac{a \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{bd(a^2-b^2)} + \frac{E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d(a^2-b^2)} - \frac{(a^2+b^2) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{bd(a-b)(a+b)^2} - \frac{b \sin(c+dx) \sqrt{\cos(c+dx)}}{d(a^2-b^2)(a+b \cos(c+dx))}$$

[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^2,x]

[Out] EllipticE[(c + d*x)/2, 2]/((a^2 - b^2)*d) + (a*EllipticF[(c + d*x)/2, 2])/((b*(a^2 - b^2)*d) - ((a^2 + b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/((a - b)*b*(a + b)^2*d - (b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x])))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2875

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3138

Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +


```
(f_.)*(x_)]), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{b}{2}-a\cos(c+dx)-\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{-a^2+b^2} \\
&= -\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)} dx}{2(a^2-b^2)} + \frac{\int \frac{-\frac{b^2}{2}+\frac{1}{2}ab\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{b(a^2-b^2)} \\
&= \frac{E\left(\frac{1}{2}(c+dx)\mid 2\right)}{(a^2-b^2)d} - \frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\cos(c+dx))} \\
&\quad + \frac{a\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2b(a^2-b^2)} - \frac{(a^2+b^2)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2b(a^2-b^2)} \\
&= \frac{E\left(\frac{1}{2}(c+dx)\mid 2\right)}{(a^2-b^2)d} + \frac{a\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b(a^2-b^2)d} \\
&\quad - \frac{(a^2+b^2)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{(a-b)b(a+b)^2d} - \frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{(a^2-b^2)d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2} dx$$

$$\frac{4b\sqrt{\cos(c+dx)}\sin(c+dx)}{(-a^2+b^2)(a+b\cos(c+dx))} - \frac{2\left(-\frac{b^2\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} + 2a\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - \frac{2a\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b}\right)\right)}{b(-a^2+b^2)} + \frac{(-2abE(\arcsin(\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{a+b})))}{b(-a^2+b^2)}$$

4d

[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^2, x]

```
[Out] (((b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x]))
- (2*(-((b^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + 2*a*(2*E
llipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
(a + b)) + ((-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)
*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/
a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2))
))/(b*(-a + b)*(a + b)))/(4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(224) = 448$.

Time = 5.60 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.82

method	result
default	$\frac{\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(\frac{4\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} \Pi\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), -\frac{2b}{a-b}, \sqrt{2}\right)}{(-2ab+2b^2)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} - \frac{2a}{a^2} \left(-\frac{b^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)}{a^2} \right) \right)}$

[In] `int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-4/(-2*a*b+2*b^2) \\ & *(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+ \\ & \sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) - 2/b*a*(-1/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+ \\ & \sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b) - 1/2/a/(a+b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) + 1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}* \\ & \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2} dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2} dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^2,x)

[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^2, x)

$$3.592 \quad \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^2} dx$$

Optimal result	5776
Rubi [A] (verified)	5776
Mathematica [A] (verified)	5778
Maple [B] (verified)	5779
Fricas [F(-1)]	5779
Sympy [F(-1)]	5780
Maxima [F]	5780
Giac [F]	5780
Mupad [F(-1)]	5780

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^2} dx = -\frac{bE\left(\frac{1}{2}(c+dx) \mid 2\right)}{a(a^2-b^2)d} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{(a^2-b^2)d}$$

$$+ \frac{(3a^2-b^2)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a(a-b)(a+b)^2d}$$

$$+ \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a(a^2-b^2)d(a+b \cos(c+dx))}$$

```
[Out] -b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))/a/(a^2-b^2)/d-(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/(a^2-b^2)/d+(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^(1/2))/a/(a-b)/(a+b)^2/d+b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2881, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^2} dx = -\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d(a^2-b^2)} - \frac{bE\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad(a^2-b^2)}$$

$$+ \frac{(3a^2-b^2)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{ad(a-b)(a+b)^2}$$

$$+ \frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad(a^2-b^2)(a+b \cos(c+dx))}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]

[Out] -((b*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d)) - EllipticF[(c + d*x)/2, 2]/((a^2 - b^2)*d) + ((3*a^2 - b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*(a + b)^2*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^n/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])], x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}(2a^2-b^2)-ab\cos(c+dx)-\frac{1}{2}b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{a(a^2-b^2)} \\
&= \frac{b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}b(2a^2-b^2)+\frac{1}{2}ab^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{ab(a^2-b^2)} - \frac{b \int \sqrt{\cos(c+dx)} dx}{2a(a^2-b^2)} \\
&= -\frac{bE\left(\frac{1}{2}(c+dx)|2\right)}{a(a^2-b^2)d} + \frac{b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(a+b\cos(c+dx))} \\
&\quad - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2(a^2-b^2)} + \frac{(3a^2-b^2) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2a(a^2-b^2)} \\
&= -\frac{bE\left(\frac{1}{2}(c+dx)|2\right)}{a(a^2-b^2)d} - \frac{\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{(a^2-b^2)d} \\
&\quad + \frac{(3a^2-b^2) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a(a-b)(a+b)^2d} + \frac{b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.52

$$\begin{aligned}
&\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} dx \\
&= \frac{4b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))} + \frac{2(4a^2-3b^2) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} + 8a \left(-\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \frac{a \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} \right) - \frac{2(-2abE(\arcsin(\frac{b\sqrt{\cos(c+dx)} \sin(c+dx)}{a+b\cos(c+dx)}))}{4ad}
\end{aligned}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2), x]

[Out] ((4*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])) + ((2*(4*a^2 - 3*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + 8*a*(-EllipticF[(c + d*x)/2, 2] + (a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) - (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/(a - b)*(a + b))/(4*a*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. 2(233) = 466.

Time = 4.75 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.90

method	result
default	$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-\frac{2b^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{\frac{a(a^2-b^2)}{2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}}-\frac{\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{a(a+b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}\right)}$

[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out]
$$\begin{aligned} & -\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\left(-2/a*b^2/(a^2-b^2)\right. \\ & \left.^*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\right. \\ & \left./\left(2*b*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+a-b\right)-1/a/(a+b)*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\left(-2*\right.\right. \\ & \left.\left.\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{(1/2)}\left(-2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}*EllipticF\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{(1/2)}\right)-1/\left(a^2-b^2\right)*b/a*\left(\sin\left(\frac{1}{2}d*x+\right.\right.\right. \\ & \left.\left.\left.\frac{1}{2}c\right)^2\right)^{(1/2)}\left(-2*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{(1/2)}\left(-2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\right.\right. \\ & \left.\left.\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}*EllipticF\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{(1/2)}\right)+1/\left(a^2-b^2\right)*b/a*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\left(-2*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{(1/2)}\left(-2*\right.\right. \\ & \left.\left.\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}*EllipticE\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{(1/2)}\right)-6*a/\left(a^2-b^2\right)/\left(-2*a*b+2*b^2\right)*b*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\left(-2*\right.\right. \\ & \left.\left.\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{(1/2)}\left(-2*\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}*EllipticPi\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),-2*b/(a-b),2^{(1/2)}\right)+2/a/\left(a^2-b^2\right)/\left(-2*\right.\right. \\ & \left.\left.*a*b+2*b^2\right)*b^3*\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}\left(-2*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{(1/2)}\left(-2*\right.\right. \\ & \left.\left.\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{(1/2)}*EllipticPi\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),-2*b/(a-b),2^{(1/2)}\right)\right)/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/\left(2*\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{(1/2)}/d \end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} dx = \int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}} dx$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} dx = \int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}} dx$$

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} dx = \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} dx$$

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^2), x)
```


$$3.593 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^2} dx$$

Optimal result	5781
Rubi [A] (verified)	5782
Mathematica [A] (verified)	5784
Maple [B] (verified)	5785
Fricas [F(-1)]	5786
Sympy [F(-1)]	5786
Maxima [F]	5786
Giac [F]	5786
Mupad [F(-1)]	5787

Optimal result

Integrand size = 23, antiderivative size = 217

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^2} dx = -\frac{(2a^2 - 3b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2 (a^2 - b^2) d} + \frac{b \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{a (a^2 - b^2) d} - \frac{b(5a^2 - 3b^2) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a^2 (a-b)(a+b)^2 d} + \frac{(2a^2 - 3b^2) \sin(c+dx)}{a^2 (a^2 - b^2) d \sqrt{\cos(c+dx)}} + \frac{b^2 \sin(c+dx)}{a (a^2 - b^2) d \sqrt{\cos(c+dx)}(a+b \cos(c+dx))}$$

```
[Out] -(2*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d+b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)/d-b*(5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^2/(a-b)/(a+b)^2/d+(2*a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)+b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2881, 3134, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \frac{b \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad(a^2-b^2)} - \frac{(2a^2-3b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2d(a^2-b^2)} - \frac{b(5a^2-3b^2) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a^2d(a-b)(a+b)^2} + \frac{b^2 \sin(c+dx)}{ad(a^2-b^2) \sqrt{\cos(c+dx)}(a+b\cos(c+dx))} + \frac{(2a^2-3b^2) \sin(c+dx)}{a^2d(a^2-b^2) \sqrt{\cos(c+dx)}}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2), x]

[Out] -(((2*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d)) + (b*EllipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) - (b*(5*a^2 - 3*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2 - 3*b^2)*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2

*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3138

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \sqrt{\cos(c+dx)}(a+b \cos(c+dx))} \\
&+ \frac{\int \frac{\frac{1}{2}(2a^2-3b^2)-ab \cos(c+dx)+\frac{1}{2}b^2 \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} dx}{a(a^2-b^2)} \\
&= \frac{(2a^2-3b^2) \sin(c+dx)}{a^2(a^2-b^2) d \sqrt{\cos(c+dx)}} + \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \sqrt{\cos(c+dx)}(a+b \cos(c+dx))} \\
&+ \frac{2 \int \frac{-\frac{1}{4}b(4a^2-3b^2)-\frac{1}{2}a(a^2-2b^2) \cos(c+dx)-\frac{1}{4}b(2a^2-3b^2) \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{a^2(a^2-b^2)} \\
&= \frac{(2a^2-3b^2) \sin(c+dx)}{a^2(a^2-b^2) d \sqrt{\cos(c+dx)}} + \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \sqrt{\cos(c+dx)}(a+b \cos(c+dx))} \\
&- \frac{2 \int \frac{\frac{1}{4}b^2(4a^2-3b^2)-\frac{1}{4}ab^3 \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{a^2b(a^2-b^2)} - \frac{(2a^2-3b^2) \int \sqrt{\cos(c+dx)} dx}{2a^2(a^2-b^2)} \\
&= -\frac{(2a^2-3b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2(a^2-b^2) d} + \frac{(2a^2-3b^2) \sin(c+dx)}{a^2(a^2-b^2) d \sqrt{\cos(c+dx)}} \\
&+ \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \sqrt{\cos(c+dx)}(a+b \cos(c+dx))} \\
&+ \frac{b \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2a(a^2-b^2)} - \frac{(b(5a^2-3b^2)) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{2a^2(a^2-b^2)} \\
&= -\frac{(2a^2-3b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2(a^2-b^2) d} + \frac{b \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{a(a^2-b^2) d} \\
&- \frac{b(5a^2-3b^2) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a^2(a-b)(a+b)^2 d} + \frac{(2a^2-3b^2) \sin(c+dx)}{a^2(a^2-b^2) d \sqrt{\cos(c+dx)}} \\
&+ \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \sqrt{\cos(c+dx)}(a+b \cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.28

$$\begin{aligned}
&\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^2} dx \\
&\frac{2(-10a^2b+9b^3) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} + \frac{(-4a^3+8ab^2) \left(2 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - \frac{2a \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b}\right)}{b} - \frac{2(2a^2-3b^2) (-2abE(\arcsin(\sqrt{\cos(c+dx)}))}{(-a+b)(a+b)}
\end{aligned}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]

[Out]
$$\begin{aligned} & -\left(\frac{((2*(-10*a^2*b + 9*b^3)*\text{EllipticPi}[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + ((-4*a^3 + 8*a*b^2)*(2*\text{EllipticF}[(c + d*x)/2, 2] - (2*a*\text{EllipticPi}[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b - (2*(2*a^2 - 3*b^2)*(-2*a*b*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[\text{Cos}[c + d*x]]], -1] + 2*a*(a + b)*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\text{Cos}[c + d*x]]], -1] + (-2*a^2 + b^2)*\text{EllipticPi}[-(b/a), \text{ArcSin}[\text{Sqrt}[\text{Cos}[c + d*x]]], -1])*\text{Sin}[c + d*x])/(a*b*\text{Sqrt}[\text{Sin}[c + d*x]^2]))/((-a + b)*(a + b)) + 4*\text{Sqrt}[\text{Cos}[c + d*x]]*(b^3*\text{Sin}[c + d*x])/((-a^2 + b^2)*(a + b*\text{Cos}[c + d*x])) + 2*\text{Tan}[c + d*x])/(4*a^2*d)} \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(291) = 582$.

Time = 6.61 (sec) , antiderivative size = 847, normalized size of antiderivative = 3.90

method	result	size
default	Expression too large to display	847

[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)

[Out]
$$\begin{aligned} & -\left(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2\right)^{(1/2)}*(2/a^2/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)-(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})) + 4*b^2/a^2/(-2*a*b+2*b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) - 2/a*b*(-1/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b) - 1/2/a/(a+b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) + 1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \int \frac{1}{(b\cos(dx+c)+a)^2 \cos(dx+c)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \int \frac{1}{(b\cos(dx+c)+a)^2 \cos(dx+c)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2} dx = \int \frac{1}{\cos(c + dx)^{\frac{3}{2}}(a + b \cos(c + dx))^2} dx$$

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2), x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^2), x)
```

$$3.594 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^2} dx$$

Optimal result	5788
Rubi [A] (verified)	5789
Mathematica [A] (verified)	5792
Maple [B] (verified)	5792
Fricas [F(-1)]	5793
Sympy [F(-1)]	5793
Maxima [F]	5793
Giac [F]	5794
Mupad [F(-1)]	5794

Optimal result

Integrand size = 23, antiderivative size = 281

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^2} dx = \frac{b(4a^2 - 5b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^3(a^2 - b^2)d} + \frac{(2a^2 - 5b^2) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2(a^2 - b^2)d} + \frac{b^2(7a^2 - 5b^2) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a^3(a-b)(a+b)^2d} + \frac{(2a^2 - 5b^2) \sin(c+dx)}{3a^2(a^2 - b^2)d \cos^{\frac{3}{2}}(c+dx)} - \frac{b(4a^2 - 5b^2) \sin(c+dx)}{a^3(a^2 - b^2)d \sqrt{\cos(c+dx)}} + \frac{b^2 \sin(c+dx)}{a(a^2 - b^2)d \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))}$$

```
[Out] b*(4*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+1/3*(2*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d+b^2*(7*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^2/d+1/3*(2*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-b^2)/d/cos(d*x+c)^(3/2)+b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))-b*(4*a^2-5*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```


Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2881, 3134, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \frac{(2a^2-5b^2)\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d(a^2-b^2)} + \frac{b^2\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{(2a^2-5b^2)\sin(c+dx)}{3a^2d(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)} + \frac{b(4a^2-5b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a^2-b^2)} + \frac{b^2(7a^2-5b^2)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a^3d(a-b)(a+b)^2} - \frac{b(4a^2-5b^2)\sin(c+dx)}{a^3d(a^2-b^2)\sqrt{\cos(c+dx)}}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2), x]

[Out] (b*(4*a^2 - 5*b^2)*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d) + ((2*a^2 - 5*b^2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*(a^2 - b^2)*d) + (b^2*(7*a^2 - 5*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + ((2*a^2 - 5*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)) - (b*(4*a^2 - 5*b^2)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d*sqrt[Cos[c + d*x]]) + (b^2*SIN[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m + 1)*((c + d*SIN[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*SIN[e + f*x]

```
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rule 3138

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
```

[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

Rubi steps

integral

$$\begin{aligned}
&= \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} + \frac{\int \frac{\frac{1}{2}(2a^2-5b^2)-ab \cos(c+dx)+\frac{3}{2}b^2 \cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))} dx}{a(a^2-b^2)} \\
&= \frac{(2a^2-5b^2) \sin(c+dx)}{3a^2(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)} + \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} \\
&\quad + \frac{2 \int \frac{-\frac{3}{4}b(4a^2-5b^2)+\frac{1}{2}a(a^2+2b^2) \cos(c+dx)+\frac{1}{4}b(2a^2-5b^2) \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} dx}{3a^2(a^2-b^2)} \\
&= \frac{(2a^2-5b^2) \sin(c+dx)}{3a^2(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)} - \frac{b(4a^2-5b^2) \sin(c+dx)}{a^3(a^2-b^2) d \sqrt{\cos(c+dx)}} \\
&\quad + \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} \\
&\quad + \frac{4 \int \frac{\frac{1}{8}(2a^4+16a^2b^2-15b^4)+\frac{1}{4}ab(7a^2-10b^2) \cos(c+dx)+\frac{3}{8}b^2(4a^2-5b^2) \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{3a^3(a^2-b^2)} \\
&= \frac{(2a^2-5b^2) \sin(c+dx)}{3a^2(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)} - \frac{b(4a^2-5b^2) \sin(c+dx)}{a^3(a^2-b^2) d \sqrt{\cos(c+dx)}} \\
&\quad + \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} \\
&\quad - \frac{4 \int \frac{-\frac{1}{8}b(2a^4+16a^2b^2-15b^4)-\frac{1}{8}ab^2(2a^2-5b^2) \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{3a^3b(a^2-b^2)} + \frac{(b(4a^2-5b^2)) \int \sqrt{\cos(c+dx)} dx}{2a^3(a^2-b^2)} \\
&= \frac{b(4a^2-5b^2) E(\frac{1}{2}(c+dx)|2)}{a^3(a^2-b^2) d} + \frac{(2a^2-5b^2) \sin(c+dx)}{3a^2(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{b(4a^2-5b^2) \sin(c+dx)}{a^3(a^2-b^2) d \sqrt{\cos(c+dx)}} + \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} \\
&\quad + \frac{(2a^2-5b^2) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{6a^2(a^2-b^2)} + \frac{(b^2(7a^2-5b^2)) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{2a^3(a^2-b^2)} \\
&= \frac{b(4a^2-5b^2) E(\frac{1}{2}(c+dx)|2)}{a^3(a^2-b^2) d} + \frac{(2a^2-5b^2) \text{EllipticF}(\frac{1}{2}(c+dx), 2)}{3a^2(a^2-b^2) d} \\
&\quad + \frac{b^2(7a^2-5b^2) \text{EllipticPi}(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2)}{a^3(a-b)(a+b)^2 d} + \frac{(2a^2-5b^2) \sin(c+dx)}{3a^2(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{b(4a^2-5b^2) \sin(c+dx)}{a^3(a^2-b^2) d \sqrt{\cos(c+dx)}} + \frac{b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))}
\end{aligned}$$

$2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2})+1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{1/2})-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{1/2})+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{1/2})))-8*b^3/a^3/(-2*a*b+2*b^2)*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{1/2}))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{1/2}/d$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \int \frac{1}{(b\cos(dx+c)+a)^2 \cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \int \frac{1}{(b\cos(dx+c)+a)^2 \cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2} dx = \int \frac{1}{\cos(c+dx)^{\frac{5}{2}}(a+b\cos(c+dx))^2} dx$$

[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2), x)

[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^2), x)

$$3.595 \quad \int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$$

Optimal result	5795
Rubi [A] (verified)	5796
Mathematica [A] (verified)	5800
Maple [B] (verified)	5800
Fricas [F]	5802
Sympy [F(-1)]	5802
Maxima [F]	5802
Giac [F]	5802
Mupad [F(-1)]	5803

Optimal result

Integrand size = 23, antiderivative size = 346

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx = -\frac{a(35a^4 - 65a^2b^2 + 24b^4) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4b^4 (a^2 - b^2)^2 d} + \frac{(105a^6 - 223a^4b^2 + 128a^2b^4 + 8b^6) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{12b^5 (a^2 - b^2)^2 d} - \frac{a^3(35a^4 - 86a^2b^2 + 63b^4) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4(a-b)^2 b^5 (a+b)^3 d} + \frac{(35a^4 - 61a^2b^2 + 8b^4) \sqrt{\cos(c+dx)} \sin(c+dx)}{12b^3 (a^2 - b^2)^2 d} - \frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2b (a^2 - b^2) d (a+b \cos(c+dx))^2} - \frac{a^2(7a^2 - 13b^2) \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4b^2 (a^2 - b^2)^2 d (a+b \cos(c+dx))}$$

```
[Out] -1/4*a*(35*a^4-65*a^2*b^2+24*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^4/(a^2-b^2)^2/d+1/12*(105*a^
6-223*a^4*b^2+128*a^2*b^4+8*b^6)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^5/(a^2-b^2)^2/d-1/4*a^3*(35*a
^4-86*a^2*b^2+63*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^5/(a+b)^3/d-1/2*a^2*c
os(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/4*a^2*(7*a^2-
13*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))+1/12
*(35*a^4-61*a^2*b^2+8*b^4)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2871, 3126, 3128, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3} dx = -\frac{a^2(7a^2-13b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4b^2d(a^2-b^2)^2(a+b\cos(c+dx))} - \frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2} - \frac{a(35a^4-65a^2b^2+24b^4)E\left(\frac{1}{2}(c+dx)\mid 2\right)}{4b^4d(a^2-b^2)^2} + \frac{(35a^4-61a^2b^2+8b^4)\sin(c+dx)\sqrt{\cos(c+dx)}}{12b^3d(a^2-b^2)^2} + \frac{(105a^6-223a^4b^2+128a^2b^4+8b^6)\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{12b^5d(a^2-b^2)^2} - \frac{a^3(35a^4-86a^2b^2+63b^4)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4b^5d(a-b)^2(a+b)^3}$$

[In] Int[Cos[c + d*x]^(9/2)/(a + b*Cos[c + d*x])^3,x]

[Out] -1/4*(a*(35*a^4 - 65*a^2*b^2 + 24*b^4)*EllipticE[(c + d*x)/2, 2])/(b^4*(a^2 - b^2)^2*d) + ((105*a^6 - 223*a^4*b^2 + 128*a^2*b^4 + 8*b^6)*EllipticF[(c + d*x)/2, 2])/(12*b^5*(a^2 - b^2)^2*d) - (a^3*(35*a^4 - 86*a^2*b^2 + 63*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^5*(a + b)^3*d) + ((35*a^4 - 61*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d) - (a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(7*a^2 - 13*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*


```
(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
+ a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
) - a*c*(n + 2)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))*Sin[e + f*x]
+ b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
```

```

*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

```

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_.)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Ssin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Ssin[e + f*x]]*(c + d*Ssin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} \\
&\quad - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx) \left(\frac{5a^2}{2} - 2ab\cos(c+dx) - \frac{1}{2}(7a^2-4b^2)\cos^2(c+dx) \right)}{(a+b\cos(c+dx))^2} dx}{2b(a^2-b^2)} \\
&= -\frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{a^2(7a^2-13b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4b^2(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad + \frac{\int \frac{\sqrt{\cos(c+dx)} \left(-\frac{3}{4}a^2(7a^2-13b^2) + ab(a^2-4b^2)\cos(c+dx) + \frac{1}{4}(35a^4-61a^2b^2+8b^4)\cos^2(c+dx) \right)}{a+b\cos(c+dx)} dx}{2b^2(a^2-b^2)^2} \\
&= \frac{(35a^4-61a^2b^2+8b^4)\sqrt{\cos(c+dx)}\sin(c+dx)}{12b^3(a^2-b^2)^2d} \\
&\quad - \frac{a^2 \cos^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{a^2(7a^2-13b^2)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4b^2(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad + \frac{\int \frac{\frac{1}{8}a(35a^4-61a^2b^2+8b^4) - \frac{1}{2}b(7a^4-14a^2b^2-2b^4)\cos(c+dx) - \frac{3}{8}a(35a^4-65a^2b^2+24b^4)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{3b^3(a^2-b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(35a^4 - 61a^2b^2 + 8b^4) \sqrt{\cos(c + dx)} \sin(c + dx)}{12b^3 (a^2 - b^2)^2 d} \\
&\quad - \frac{a^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2b (a^2 - b^2) d(a + b \cos(c + dx))^2} - \frac{a^2(7a^2 - 13b^2) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{4b^2 (a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad - \frac{\int \frac{-\frac{1}{8}ab(35a^4 - 61a^2b^2 + 8b^4) - \frac{1}{8}(105a^6 - 223a^4b^2 + 128a^2b^4 + 8b^6) \cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))} dx}{3b^4 (a^2 - b^2)^2} \\
&\quad - \frac{(a(35a^4 - 65a^2b^2 + 24b^4)) \int \sqrt{\cos(c + dx)} dx}{8b^4 (a^2 - b^2)^2} \\
&= - \frac{a(35a^4 - 65a^2b^2 + 24b^4) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{4b^4 (a^2 - b^2)^2 d} \\
&\quad + \frac{(35a^4 - 61a^2b^2 + 8b^4) \sqrt{\cos(c + dx)} \sin(c + dx)}{12b^3 (a^2 - b^2)^2 d} \\
&\quad - \frac{a^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2b (a^2 - b^2) d(a + b \cos(c + dx))^2} - \frac{a^2(7a^2 - 13b^2) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{4b^2 (a^2 - b^2)^2 d(a + b \cos(c + dx))} \\
&\quad - \frac{(a^3(35a^4 - 86a^2b^2 + 63b^4)) \int \frac{1}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))} dx}{8b^5 (a^2 - b^2)^2} \\
&\quad + \frac{(105a^6 - 223a^4b^2 + 128a^2b^4 + 8b^6) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{24b^5 (a^2 - b^2)^2} \\
&= - \frac{a(35a^4 - 65a^2b^2 + 24b^4) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{4b^4 (a^2 - b^2)^2 d} \\
&\quad + \frac{(105a^6 - 223a^4b^2 + 128a^2b^4 + 8b^6) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{12b^5 (a^2 - b^2)^2 d} \\
&\quad - \frac{a^3(35a^4 - 86a^2b^2 + 63b^4) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{4(a - b)^2 b^5 (a + b)^3 d} \\
&\quad + \frac{(35a^4 - 61a^2b^2 + 8b^4) \sqrt{\cos(c + dx)} \sin(c + dx)}{12b^3 (a^2 - b^2)^2 d} \\
&\quad - \frac{a^2 \cos^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2b (a^2 - b^2) d(a + b \cos(c + dx))^2} - \frac{a^2(7a^2 - 13b^2) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{4b^2 (a^2 - b^2)^2 d(a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.02

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3} dx$$

$$= \frac{4\sqrt{\cos(c+dx)}\left(35a^6-57a^4b^2+4b^6+ab(49a^4-83a^2b^2+16b^4)\cos(c+dx)+4(-a^2b+b^3)^2\cos(2(c+dx))\right)\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2} - \frac{2(35a^5-73a^3b^2+56ab^4)\operatorname{Elliptic}\pi\left(\frac{2b}{a+b}, \frac{c+dx}{2}, 2\right)}{a+b}$$

[In] Integrate[Cos[c + d*x]^(9/2)/(a + b*Cos[c + d*x])^3,x]

[Out] ((4*Sqrt[Cos[c + d*x]]*(35*a^6 - 57*a^4*b^2 + 4*b^6 + a*b*(49*a^4 - 83*a^2*b^2 + 16*b^4)*Cos[c + d*x] + 4*(-a^2*b) + b^3)^2*Cos[2*(c + d*x)]*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((2*(35*a^5 - 73*a^3*b^2 + 56*a*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(7*a^4 - 14*a^2*b^2 - 2*b^4)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(35*a^4 - 65*a^2*b^2 + 24*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2])/((a - b)^2*(a + b)^2))/(48*b^3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2193 vs. 2(406) = 812.

Time = 87.67 (sec) , antiderivative size = 2194, normalized size of antiderivative = 6.34

method	result	size
default	Expression too large to display	2194

[In] int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] -((-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/b^4*(3*a+2*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(6*a^2+3*a*b+b^2)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/b^5*a^5*(-1/2/a*b^2/(a^2-b

$$\begin{aligned}
& ^2) * \cos(1/2*d*x+1/2*c) * (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\
& / (2*b*\cos(1/2*d*x+1/2*c)^2 + a - b)^2 - 3/4*b^2*(3*a^2 - b^2)/a^2/(a^2 - b^2)^2 * \cos(1/2*d*x+1/2*c) \\
& * (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} / (2*b*\cos(1/2*d*x+1/2*c)^2 + a - b) \\
& - 7/8/(a+b)/(a^2 - b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\
& + 1/4/(a+b)/(a^2 - b^2)/a * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * b \\
& + 3/8/(a+b)/(a^2 - b^2)/a^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * b^2 \\
& - 9/8*b/(a^2 - b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\
& + 3/8*b^3/a^2/(a^2 - b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\
& + 9/8*b/(a^2 - b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\
& - 3/8*b^3/a^2/(a^2 - b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\
& - 15/4*a^2/(a^2 - b^2)^2 / (-2*a*b + 2*b^2) * b * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) \\
& + 3/2/(a^2 - b^2)^2 / (-2*a*b + 2*b^2) * b^3 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) \\
& - 3/4/a^2/(a^2 - b^2)^2 / (-2*a*b + 2*b^2) * b^5 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) \\
& + 10/b^5 * a^4 * (-1/a*b^2/(a^2 - b^2) * \cos(1/2*d*x+1/2*c) * (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\
& / (2*b*\cos(1/2*d*x+1/2*c)^2 + a - b) - 1/2/a/(a+b) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2/(a^2 - b^2) * b/a \\
& * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\
& * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 1/2/(a^2 - b^2) * b/a * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} \\
& / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3*a/(a^2 - b^2) \\
& / (-2*a*b + 2*b^2) * b * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\
& * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) + 1/a/(a^2 - b^2) / (-2*a*b + 2*b^2) * b^3 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\
& * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) \\
& + 40/b^4 * a^3 / (-2*a*b + 2*b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\
& * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2 - 1)^{(1/2)}
\end{aligned}$$

/2)/d

Fricas [F]

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{9}{2}}}{(b \cos(dx + c) + a)^3} dx$$

```
[In] integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(9/2)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(9/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{9}{2}}}{(b \cos(dx + c) + a)^3} dx$$

```
[In] integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c) + a)^3, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{9}{2}}}{(b \cos(dx + c) + a)^3} dx$$

```
[In] integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(c + dx)^{9/2}}{(a + b \cos(c + dx))^3} dx$$

```
[In] int(cos(c + d*x)^(9/2)/(a + b*cos(c + d*x))^3, x)
```

```
[Out] int(cos(c + d*x)^(9/2)/(a + b*cos(c + d*x))^3, x)
```

3.596 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$

Optimal result	5804
Rubi [A] (verified)	5805
Mathematica [A] (verified)	5808
Maple [B] (verified)	5808
Fricas [F]	5810
Sympy [F(-1)]	5810
Maxima [F]	5810
Giac [F]	5810
Mupad [F(-1)]	5811

Optimal result

Integrand size = 23, antiderivative size = 282

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx = \frac{(15a^4 - 29a^2b^2 + 8b^4) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4b^3 (a^2 - b^2)^2 d} - \frac{3a(5a^4 - 11a^2b^2 + 8b^4) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4b^4 (a^2 - b^2)^2 d} + \frac{a^2(15a^4 - 38a^2b^2 + 35b^4) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4(a-b)^2 b^4 (a+b)^3 d} - \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b (a^2 - b^2) d (a+b \cos(c+dx))^2} - \frac{a^2(5a^2 - 11b^2) \sqrt{\cos(c+dx)} \sin(c+dx)}{4b^2 (a^2 - b^2)^2 d (a+b \cos(c+dx))}$$

```
[Out] 1/4*(15*a^4-29*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d-3/4*a*(5*a^4-11*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^4/(a^2-b^2)^2/d+1/4*a^2*(15*a^4-38*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/(a-b)^2/b^4/(a+b)^3/d-1/2*a^2*cos(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/4*a^2*(5*a^2-11*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```


Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2871, 3126, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3} dx = -\frac{a^2 \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2} - \frac{a^2(5a^2-11b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{4b^2d(a^2-b^2)^2(a+b\cos(c+dx))} - \frac{3a(5a^4-11a^2b^2+8b^4)\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4b^4d(a^2-b^2)^2} + \frac{a^2(15a^4-38a^2b^2+35b^4)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4b^4d(a-b)^2(a+b)^3} + \frac{(15a^4-29a^2b^2+8b^4)E\left(\frac{1}{2}(c+dx)|2\right)}{4b^3d(a^2-b^2)^2}$$

[In] Int[Cos[c + d*x]^(7/2)/(a + b*cos[c + d*x])^3, x]

[Out] ((15*a^4 - 29*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) - (3*a*(5*a^4 - 11*a^2*b^2 + 8*b^4)*EllipticF[(c + d*x)/2, 2])/(4*b^4*(a^2 - b^2)^2*d) + (a^2*(15*a^4 - 38*a^2*b^2 + 35*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^4*(a + b)^3*d) - (a^2*cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*cos[c + d*x])^2) - (a^2*(5*a^2 - 11*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +

```

b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])

```

Rule 2884

```

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]

```

Rule 3081

```

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3126

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n +
1) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))*Sin[e + f*x]
+ b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} \\
&\quad - \frac{\int \frac{\sqrt{\cos(c+dx)} \left(\frac{3a^2}{2} - 2ab\cos(c+dx) - \frac{1}{2}(5a^2-4b^2)\cos^2(c+dx) \right)}{(a+b\cos(c+dx))^2} dx}{2b(a^2-b^2)} \\
&= -\frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{a^2(5a^2-11b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4b^2(a^2-b^2)^2 d(a+b\cos(c+dx))} \\
&\quad + \frac{\int \frac{-\frac{1}{4}a^2(5a^2-11b^2)+ab(a^2-4b^2)\cos(c+dx)+\frac{1}{4}(15a^4-29a^2b^2+8b^4)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2b^2(a^2-b^2)^2} \\
&= -\frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{a^2(5a^2-11b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4b^2(a^2-b^2)^2 d(a+b\cos(c+dx))} \\
&\quad - \frac{\int \frac{\frac{1}{4}a^2b(5a^2-11b^2)+\frac{3}{4}a(5a^4-11a^2b^2+8b^4)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2b^3(a^2-b^2)^2} + \frac{(15a^4-29a^2b^2+8b^4)\int \sqrt{\cos(c+dx)} dx}{8b^3(a^2-b^2)^2} \\
&= \frac{(15a^4-29a^2b^2+8b^4)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3(a^2-b^2)^2 d} - \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} \\
&\quad - \frac{a^2(5a^2-11b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4b^2(a^2-b^2)^2 d(a+b\cos(c+dx))} \\
&\quad - \frac{(3a(5a^4-11a^2b^2+8b^4))\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{8b^4(a^2-b^2)^2} \\
&\quad + \frac{(a^2(15a^4-38a^2b^2+35b^4))\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{8b^4(a^2-b^2)^2} \\
&= \frac{(15a^4-29a^2b^2+8b^4)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3(a^2-b^2)^2 d} \\
&\quad - \frac{3a(5a^4-11a^2b^2+8b^4)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^4(a^2-b^2)^2 d} \\
&\quad + \frac{a^2(15a^4-38a^2b^2+35b^4)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4(a-b)^2b^4(a+b)^3d} \\
&\quad - \frac{a^2 \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{a^2(5a^2-11b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4b^2(a^2-b^2)^2 d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.10

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3} dx$$

$$= \frac{-2a^2\sqrt{\cos(c+dx)}(5a^3-11ab^2+b(7a^2-13b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2} + \frac{(5a^4-7a^2b^2+8b^4)\operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) + 8(a^3-4ab^2)((a+b)\operatorname{EllipticF}\left(\frac{c+dx}{2}, 2\right))}{a+b}$$

[In] Integrate[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^3,x]

[Out] $((-2*a^2*\sqrt{\cos[c + d*x]}*(5*a^3 - 11*a*b^2 + b*(7*a^2 - 13*b^2)*\cos[c + d*x])* \sin[c + d*x])/((a^2 - b^2)^2*(a + b*\cos[c + d*x])^2) + (((5*a^4 - 7*a^2*b^2 + 8*b^4)*\operatorname{EllipticPi}[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(a^3 - 4*a*b^2)*((a + b)*\operatorname{EllipticF}[(c + d*x)/2, 2] - a*\operatorname{EllipticPi}[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*(-2*a*b*\operatorname{EllipticE}[\operatorname{ArcSin}[\sqrt{\cos[c + d*x]}], -1] + 2*a*(a + b)*\operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{\cos[c + d*x]}], -1] + (-2*a^2 + b^2)*\operatorname{EllipticPi}[-(b/a), \operatorname{ArcSin}[\sqrt{\cos[c + d*x]}], -1]))*\sin[c + d*x])/((a*b^2*\sqrt{\sin[c + d*x]^2}))/((a - b)^2*(a + b)^2))/(8*b^2*d)$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1934 vs. 2(346) = 692.

Time = 88.06 (sec) , antiderivative size = 1935, normalized size of antiderivative = 6.86

method	result	size
default	Expression too large to display	1935

[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] $-((-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2/b^4/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(3*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*a+b*\operatorname{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}))+2/b^4*a^4*(-1/2/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+1/4/(a+b)/(a^2-b^2)/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\operatorname{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*b+3/8/(a$

$$\begin{aligned}
& +b)/(a^2-b^2)/a^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b^2-9/8*b/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3/8*b^3/a^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+9/8*b/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3/8*b^3/a^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})-24*a^2/b^3/(-2*a*b+2*b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})-8*a^3/b^4*(-1/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)}))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d
\end{aligned}$$

Fricas [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(b \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] integral(cos(d*x + c)^(7/2)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(b \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{7}{2}}}{(b \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(c + dx)^{7/2}}{(a + b \cos(c + dx))^3} dx$$

```
[In] int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^3, x)
```

```
[Out] int(cos(c + d*x)^(7/2)/(a + b*cos(c + d*x))^3, x)
```

$$3.597 \quad \int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^3} dx$$

Optimal result	5812
Rubi [A] (verified)	5813
Mathematica [A] (verified)	5816
Maple [B] (verified)	5816
Fricas [F(-1)]	5818
Sympy [F(-1)]	5818
Maxima [F]	5818
Giac [F]	5818
Mupad [F(-1)]	5819

Optimal result

Integrand size = 23, antiderivative size = 264

$$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^3} dx = -\frac{3a(a^2-3b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4b^2(a^2-b^2)^2 d} + \frac{(3a^4-5a^2b^2+8b^4) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4b^3(a^2-b^2)^2 d} - \frac{3a(a^4-2a^2b^2+5b^4) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4(a-b)^2 b^3 (a+b)^3 d} - \frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2b(a^2-b^2) d(a+b \cos(c+dx))^2} + \frac{3a(a^2-3b^2) \sqrt{\cos(c+dx)} \sin(c+dx)}{4b(a^2-b^2)^2 d(a+b \cos(c+dx))}$$

```
[Out] -3/4*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d+1/4*(3*a^4-5*a^2*b^2+8*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2))/b^3/(a^2-b^2)^2/d-3/4*a*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1
/2))/(a-b)^2/b^3/(a+b)^3/d-1/2*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/
d/(a+b*cos(d*x+c))^2+3/4*a*(a^2-3*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b
^2)^2/d/(a+b*cos(d*x+c))
```


Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2871, 3134, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3} dx = -\frac{3a(a^2-3b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d(a^2-b^2)^2} - \frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd(a^2-b^2)(a+b\cos(c+dx))^2} + \frac{3a(a^2-3b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{4bd(a^2-b^2)^2(a+b\cos(c+dx))} + \frac{(3a^4-5a^2b^2+8b^4)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^3d(a^2-b^2)^2} - \frac{3a(a^4-2a^2b^2+5b^4)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4b^3d(a-b)^2(a+b)^3}$$

[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]

[Out] (-3*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4 - 5*a^2*b^2 + 8*b^4)*EllipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) - (3*a*(a^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) - (a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -

```
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])/(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*SIN[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*SIN[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)])^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*SIN[e + f*x])^(m + 1)*(c + d*SIN[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rule 3138

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]]), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*SIN[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*SIN[e + f*x]]*(c + d*SIN[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

&& NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{a^2}{2} - 2ab\cos(c+dx) - \frac{1}{2}(3a^2-4b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)(a+b\cos(c+dx))^2}} dx}{2b(a^2-b^2)} \\
&= -\frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4b(a^2-b^2)^2 d(a+b\cos(c+dx))} \\
&\quad - \frac{\int \frac{-\frac{1}{4}a^2(a^2-7b^2) - ab(a^2+2b^2)\cos(c+dx) + \frac{3}{4}a^2(a^2-3b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)(a+b\cos(c+dx))}} dx}{2ab(a^2-b^2)^2} \\
&= -\frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4b(a^2-b^2)^2 d(a+b\cos(c+dx))} \\
&\quad + \frac{\int \frac{\frac{1}{4}a^2b(a^2-7b^2) + \frac{1}{4}a(3a^4-5a^2b^2+8b^4)\cos(c+dx)}{\sqrt{\cos(c+dx)(a+b\cos(c+dx))}} dx}{2ab^2(a^2-b^2)^2} - \frac{(3a(a^2-3b^2)) \int \sqrt{\cos(c+dx)} dx}{8b^2(a^2-b^2)^2} \\
&= -\frac{3a(a^2-3b^2)E\left(\frac{1}{2}(c+dx)|2\right)}{4b^2(a^2-b^2)^2 d} - \frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} \\
&\quad + \frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4b(a^2-b^2)^2 d(a+b\cos(c+dx))} \\
&\quad - \frac{(3a(a^4-2a^2b^2+5b^4)) \int \frac{1}{\sqrt{\cos(c+dx)(a+b\cos(c+dx))}} dx}{8b^3(a^2-b^2)^2} \\
&\quad + \frac{(3a^4-5a^2b^2+8b^4) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{8b^3(a^2-b^2)^2} \\
&= -\frac{3a(a^2-3b^2)E\left(\frac{1}{2}(c+dx)|2\right)}{4b^2(a^2-b^2)^2 d} + \frac{(3a^4-5a^2b^2+8b^4)\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4b^3(a^2-b^2)^2 d} \\
&\quad - \frac{3a(a^4-2a^2b^2+5b^4)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4(a-b)^2 b^3 (a+b)^3 d} \\
&\quad - \frac{a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2b(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4b(a^2-b^2)^2 d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3} dx$$

$$= \frac{4a\sqrt{\cos(c+dx)}(a^3-7ab^2+3b(a^2-3b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2} - \frac{2(a^3+5ab^2)\operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} - \frac{16(a^2+2b^2)\left((a+b)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - a\right)}{a+b}$$

[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]

[Out] ((4*a*Sqrt[Cos[c + d*x]]*(a^3 - 7*a*b^2 + 3*b*(a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((2*(a^3 + 5*a*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (16*(a^2 + 2*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(a^2 - 3*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)/(16*b*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1913 vs. 2(328) = 656.

Time = 86.51 (sec) , antiderivative size = 1914, normalized size of antiderivative = 7.25

method	result	size
default	Expression too large to display	1914

[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] -((-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-2/b^3*a^3*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)

$$\begin{aligned}
& *c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) * b^2 - 9/8*b / (a^2 - b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\
&) + 3/8*b^3/a^2 / (a^2 - b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 9/8*b / (a^2 - b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\
&) * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3/8*b^3/a^2 / (a^2 - b^2)^2 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\
&) - 15/4*a^2 / (a^2 - b^2)^2 / (-2*a*b + 2*b^2) * b * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b / (a - b), 2^{(1/2)}) + 3/2 / (a^2 - b^2)^2 / (-2*a*b + 2*b^2) * b^3 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b / (a - b), 2^{(1/2)}) - 3/4/a^2 / (a^2 - b^2)^2 / (-2*a*b + 2*b^2) * b^5 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b / (a - b), 2^{(1/2)}) + 12/b^2*a / (-2*a*b + 2*b^2) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b / (a - b), 2^{(1/2)}) + 6/b^3*a^2 * (-1/a*b^2 / (a^2 - b^2) * \cos(1/2*d*x+1/2*c) * (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} / (2*b*\cos(1/2*d*x+1/2*c)^2 + a - b) - 1/2/a / (a + b) * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2 / (a^2 - b^2) * b/a * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) + 1/2 / (a^2 - b^2) * b/a * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3*a / (a^2 - b^2) / (-2*a*b + 2*b^2) * b * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b / (a - b), 2^{(1/2)}) + 1/a / (a^2 - b^2) / (-2*a*b + 2*b^2) * b^3 * (\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2 + 1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b / (a - b), 2^{(1/2)})) / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2 - 1)^{(1/2)} / d
\end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(c + dx)^{5/2}}{(a + b \cos(c + dx))^3} dx$$

```
[In] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^3, x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^3, x)
```

$$3.598 \quad \int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^3} dx$$

Optimal result	5820
Rubi [A] (verified)	5821
Mathematica [A] (verified)	5823
Maple [B] (verified)	5824
Fricas [F(-1)]	5825
Sympy [F(-1)]	5825
Maxima [F]	5826
Giac [F]	5826
Mupad [F(-1)]	5826

Optimal result

Integrand size = 23, antiderivative size = 244

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^3} dx = -\frac{(a^2+5b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4b(a^2-b^2)^2 d} + \frac{a(a^2-7b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4b^2(a^2-b^2)^2 d} - \frac{(a^4-10a^2b^2-3b^4) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4(a-b)^2 b^2 (a+b)^3 d} + \frac{a \sqrt{\cos(c+dx)} \sin(c+dx)}{2(a^2-b^2) d (a+b \cos(c+dx))^2} + \frac{(a^2+5b^2) \sqrt{\cos(c+dx)} \sin(c+dx)}{4(a^2-b^2)^2 d (a+b \cos(c+dx))}$$

```
[Out] -1/4*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c), 2^(1/2))/b/(a^2-b^2)^2/d+1/4*a*(a^2-7*b^2)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/b^
2/(a^2-b^2)^2/d-1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^(1/2))/(a-b)^2/b^
2/(a+b)^3/d+1/2*a*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^
2+1/4*(a^2+5*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*cos(d*x+c)
)
```


Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2878, 3134, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3} dx = \frac{a(a^2-7b^2)\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4b^2d(a^2-b^2)^2} - \frac{(a^2+5b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd(a^2-b^2)^2} + \frac{(a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a^2-b^2)^2(a+b\cos(c+dx))} + \frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a^2-b^2)(a+b\cos(c+dx))^2} - \frac{(a^4-10a^2b^2-3b^4)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4b^2d(a-b)^2(a+b)^3}$$

[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3, x]

[Out] -1/4*((a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)^2*d) + (a*(a^2 - 7*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) - ((a^4 - 10*a^2*b^2 - 3*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^2*(a + b)^3*d) + (a*sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + ((a^2 + 5*b^2)*sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &

& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
 && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3081

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3134

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3138

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)])^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{\int \frac{-\frac{a}{2}+2b\cos(c+dx)-\frac{1}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} dx}{2(a^2-b^2)} \\
&= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{(a^2+5b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad - \frac{\int \frac{-\frac{3}{4}a(a^2+b^2)+3a^2b\cos(c+dx)+\frac{1}{4}a(a^2+5b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2a(a^2-b^2)^2} \\
&= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{(a^2+5b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad + \frac{\int \frac{\frac{3}{4}ab(a^2+b^2)+\frac{1}{4}a^2(a^2-7b^2)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2ab(a^2-b^2)^2} - \frac{(a^2+5b^2)\int\sqrt{\cos(c+dx)}dx}{8b(a^2-b^2)^2} \\
&= -\frac{(a^2+5b^2)E(\frac{1}{2}(c+dx)|2)}{4b(a^2-b^2)^2d} + \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\cos(c+dx))^2} \\
&\quad + \frac{(a^2+5b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4(a^2-b^2)^2d(a+b\cos(c+dx))} + \frac{(a(a^2-7b^2))\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{8b^2(a^2-b^2)^2} \\
&\quad - \frac{(a^4-10a^2b^2-3b^4)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{8b^2(a^2-b^2)^2} \\
&= -\frac{(a^2+5b^2)E(\frac{1}{2}(c+dx)|2)}{4b(a^2-b^2)^2d} + \frac{a(a^2-7b^2)\text{EllipticF}(\frac{1}{2}(c+dx),2)}{4b^2(a^2-b^2)^2d} \\
&\quad - \frac{(a^4-10a^2b^2-3b^4)\text{EllipticPi}(\frac{2b}{a+b},\frac{1}{2}(c+dx),2)}{4(a-b)^2b^2(a+b)^3d} \\
&\quad + \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{(a^2+5b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4(a^2-b^2)^2d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.11

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3} dx$$

$$= \frac{4\sqrt{\cos(c+dx)}(3a(a^2+b^2)+b(a^2+5b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2} - \frac{2(5a^2+b^2)\text{EllipticPi}(\frac{2b}{a+b},\frac{1}{2}(c+dx),2)}{a+b} + 24a\left(2\text{EllipticF}(\frac{1}{2}(c+dx),2) - \frac{2a\text{EllipticF}(\frac{1}{2}(c+dx),2)}{a+b}\right)$$

[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]

```
[Out] ((4*sqrt[Cos[c + d*x]]*(3*a*(a^2 + b^2) + b*(a^2 + 5*b^2)*Cos[c + d*x])*Sin
[c + d*x])/((a^2 - b^2)^2*(a + b*cos[c + d*x])^2) - ((-2*(5*a^2 + b^2)*Elli
pticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + 24*a*(2*EllipticF[(c + d*x
)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + (2*(a^
2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*
EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a
), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*sqrt[Sin[c + d*x]^
2]))/((a - b)^2*(a + b)^2)/(16*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1835 vs. $2(308) = 616$.

Time = 9.60 (sec) , antiderivative size = 1836, normalized size of antiderivative = 7.52

method	result	size
default	Expression too large to display	1836

```
[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -((-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/b/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+2*a^2/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2
+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8
/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
```

,2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-4*a/b^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^3} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\cos(c + dx)^{3/2}}{(a + b \cos(c + dx))^3} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^3,x)

[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^3, x)

$$3.599 \quad \int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^3} dx$$

Optimal result	5827
Rubi [A] (verified)	5828
Mathematica [A] (verified)	5830
Maple [B] (verified)	5831
Fricas [F(-1)]	5832
Sympy [F(-1)]	5832
Maxima [F]	5832
Giac [F]	5833
Mupad [F(-1)]	5833

Optimal result

Integrand size = 23, antiderivative size = 250

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^3} dx = \frac{(5a^2+b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4a(a^2-b^2)^2 d} + \frac{3(a^2+b^2) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4b(a^2-b^2)^2 d} - \frac{(3a^4+10a^2b^2-b^4) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4a(a-b)^2 b(a+b)^3 d} - \frac{b\sqrt{\cos(c+dx)} \sin(c+dx)}{2(a^2-b^2) d(a+b \cos(c+dx))^2} - \frac{b(5a^2+b^2) \sqrt{\cos(c+dx)} \sin(c+dx)}{4a(a^2-b^2)^2 d(a+b \cos(c+dx))}$$

```
[Out] 1/4*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)^2/d+3/4*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)^2/d-1/4*(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a/(a-b)^2/b/(a+b)^3/d-1/2*b*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^2-1/4*b*(5*a^2+b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2875, 3134, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3} dx = \frac{3(a^2+b^2)\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4bd(a^2-b^2)^2} + \frac{(5a^2+b^2)E\left(\frac{1}{2}(c+dx)|2\right)}{4ad(a^2-b^2)^2} - \frac{b(5a^2+b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{4ad(a^2-b^2)^2(a+b\cos(c+dx))} - \frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a^2-b^2)(a+b\cos(c+dx))^2} - \frac{(3a^4+10a^2b^2-b^4)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4abd(a-b)^2(a+b)^3}$$

[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^3,x]

[Out] ((5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + (3*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) - ((3*a^4 + 10*a^2*b^2 - b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a*(a - b)^2*b*(a + b)^3*d) - (b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2875

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])], x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{b}{2}-2a\cos(c+dx)+\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} dx}{2(a^2-b^2)}$$

$$\begin{aligned}
&= -\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{b(5a^2+b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad - \frac{\int \frac{\frac{1}{4}b(7a^2-b^2)-a(2a^2+b^2)\cos(c+dx)-\frac{1}{4}b(5a^2+b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2a(a^2-b^2)^2} \\
&= -\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{b(5a^2+b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a(a^2-b^2)^2d(a+b\cos(c+dx))} \\
&\quad + \frac{\int \frac{-\frac{1}{4}b^2(7a^2-b^2)+\frac{3}{4}ab(a^2+b^2)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2ab(a^2-b^2)^2} + \frac{(5a^2+b^2)\int\sqrt{\cos(c+dx)}dx}{8a(a^2-b^2)^2} \\
&= \frac{(5a^2+b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a(a^2-b^2)^2d} - \frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\cos(c+dx))^2} \\
&\quad - \frac{b(5a^2+b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a(a^2-b^2)^2d(a+b\cos(c+dx))} + \frac{(3(a^2+b^2))\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{8b(a^2-b^2)^2} \\
&\quad - \frac{(3a^4+10a^2b^2-b^4)\int\frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{8ab(a^2-b^2)^2} \\
&= \frac{(5a^2+b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a(a^2-b^2)^2d} + \frac{3(a^2+b^2)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b(a^2-b^2)^2d} \\
&\quad - \frac{(3a^4+10a^2b^2-b^4)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{4a(a-b)^2b(a+b)^3d} \\
&\quad - \frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(a+b\cos(c+dx))^2} - \frac{b(5a^2+b^2)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a(a^2-b^2)^2d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3} dx$$

$$= -\frac{4b\sqrt{\cos(c+dx)}(7a^3-ab^2+b(5a^2+b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2} + \frac{2(-9a^2b+3b^3)\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)}{a+b} + \frac{8a(2a^2+b^2)}{b}\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2}{b}\right)$$

[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^3,x]

[Out] ((-4*b*Sqrt[Cos[c + d*x]]*(7*a^3 - a*b^2 + b*(5*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(-9*a^2*b + 3*b^3)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(2*a^2 + b^2)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))

)/(a + b))/b + (2*(5*a^2 + b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1]*Sin[c + d*x]))/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. 2(314) = 628.

Time = 8.23 (sec) , antiderivative size = 1736, normalized size of antiderivative = 6.94

method	result	size
default	Expression too large to display	1736

[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2/b*(-1/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)}))-2/b*a*(-1/2/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+ \end{aligned}$$

$$\begin{aligned} & 9/8*b/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\ & -3/8*b^3/a^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\ & -15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) \\ & +3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) \\ & -3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})) / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)} / d \end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3} dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3} dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^3,x)

[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^3, x)

$$3.600 \quad \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^3} dx$$

Optimal result	5834
Rubi [A] (verified)	5835
Mathematica [A] (verified)	5837
Maple [B] (verified)	5838
Fricas [F(-1)]	5839
Sympy [F(-1)]	5839
Maxima [F]	5839
Giac [F]	5840
Mupad [F(-1)]	5840

Optimal result

Integrand size = 23, antiderivative size = 261

$$\begin{aligned} & \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^3} dx \\ &= -\frac{3b(3a^2-b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4a^2(a^2-b^2)^2 d} - \frac{(7a^2-b^2) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4a(a^2-b^2)^2 d} \\ & \quad + \frac{3(5a^4-2a^2b^2+b^4) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4a^2(a-b)^2(a+b)^3 d} \\ & \quad + \frac{b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2a(a^2-b^2) d(a+b \cos(c+dx))^2} + \frac{3b^2(3a^2-b^2) \sqrt{\cos(c+dx)} \sin(c+dx)}{4a^2(a^2-b^2)^2 d(a+b \cos(c+dx))} \end{aligned}$$

```
[Out] -3/4*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c), 2^(1/2))/a^2/(a^2-b^2)^2/d-1/4*(7*a^2-b^2)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))/
a/(a^2-b^2)^2/d+3/4*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^(1/2))/a^2/(a-b)^2
/(a+b)^3/d+1/2*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c
))^2+3/4*b^2*(3*a^2-b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b
*cos(d*x+c))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2881, 3134, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3} dx$$

$$= -\frac{(7a^2-b^2)\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4ad(a^2-b^2)^2} - \frac{3b(3a^2-b^2)E\left(\frac{1}{2}(c+dx)|2\right)}{4a^2d(a^2-b^2)^2}$$

$$+ \frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{4a^2d(a^2-b^2)^2(a+b\cos(c+dx))} + \frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad(a^2-b^2)(a+b\cos(c+dx))^2}$$

$$+ \frac{3(5a^4-2a^2b^2+b^4)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4a^2d(a-b)^2(a+b)^3}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3), x]

[Out] (-3*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - ((7*a^2 - b^2)*EllipticF[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + (3*(5*a^4 - 2*a^2*b^2 + b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^2*(a - b)^2*(a + b)^3*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*b^2*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n]))

] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_))*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_))*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rule 3138

```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```


Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}(4a^2-3b^2)-2ab\cos(c+dx)+\frac{1}{2}b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} dx}{2a(a^2-b^2)} \\
&= \frac{b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{3b^2(3a^2-b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4a^2(a^2-b^2)^2 d(a+b\cos(c+dx))} \\
&\quad + \frac{\int \frac{\frac{1}{4}(8a^4-5a^2b^2+3b^4)-ab(4a^2-b^2)\cos(c+dx)-\frac{3}{4}b^2(3a^2-b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2a^2(a^2-b^2)^2} \\
&= \frac{b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{3b^2(3a^2-b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4a^2(a^2-b^2)^2 d(a+b\cos(c+dx))} \\
&\quad - \frac{\int \frac{-\frac{1}{4}b(8a^4-5a^2b^2+3b^4)+\frac{1}{4}ab^2(7a^2-b^2)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2a^2b(a^2-b^2)^2} - \frac{(3b(3a^2-b^2)) \int \sqrt{\cos(c+dx)} dx}{8a^2(a^2-b^2)^2} \\
&= -\frac{3b(3a^2-b^2)E(\frac{1}{2}(c+dx)|2)}{4a^2(a^2-b^2)^2 d} + \frac{b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} \\
&\quad + \frac{3b^2(3a^2-b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4a^2(a^2-b^2)^2 d(a+b\cos(c+dx))} - \frac{(7a^2-b^2) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{8a(a^2-b^2)^2} \\
&\quad + \frac{(3(5a^4-2a^2b^2+b^4)) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{8a^2(a^2-b^2)^2} \\
&= -\frac{3b(3a^2-b^2)E(\frac{1}{2}(c+dx)|2)}{4a^2(a^2-b^2)^2 d} - \frac{(7a^2-b^2)\text{EllipticF}(\frac{1}{2}(c+dx),2)}{4a(a^2-b^2)^2 d} \\
&\quad + \frac{3(5a^4-2a^2b^2+b^4)\text{EllipticPi}(\frac{2b}{a+b},\frac{1}{2}(c+dx),2)}{4a^2(a-b)^2(a+b)^3 d} \\
&\quad + \frac{b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{2a(a^2-b^2)d(a+b\cos(c+dx))^2} + \frac{3b^2(3a^2-b^2)\sqrt{\cos(c+dx)} \sin(c+dx)}{4a^2(a^2-b^2)^2 d(a+b\cos(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.15

$$\begin{aligned}
&\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3} dx \\
&= \frac{4b^2 \sqrt{\cos(c+dx)}(11a^3-5ab^2+(9a^2b-3b^3)\cos(c+dx)) \sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2} + \frac{2(16a^4-19a^2b^2+9b^4)\text{EllipticPi}(\frac{2b}{a+b},\frac{1}{2}(c+dx),2)}{a+b} + \frac{16(-4a^3+ab^2)((a+b)\text{EllipticF}(\frac{1}{2}(c+dx),2))}{(a+b)^2}
\end{aligned}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]

```
[Out] ((4*b^2*Sqrt[Cos[c + d*x]]*(11*a^3 - 5*a*b^2 + (9*a^2*b - 3*b^3)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(16*a^4 - 19*a^2*b^2 + 9*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(-4*a^3 + a*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) - (6*(3*a^2 - b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)/(16*a^2*d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(325) = 650$.

Time = 6.32 (sec) , antiderivative size = 1176, normalized size of antiderivative = 4.51

method	result	size
default	Expression too large to display	1176

```
[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/2*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/4/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/4*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/2*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
```

, $2^{(1/2)}$)- $3/2/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$)* $(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})})/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3} dx = \int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3} dx = \int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3} dx = \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3), x)

[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^3), x)

$$3.601 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^3} dx$$

Optimal result	5841
Rubi [A] (verified)	5842
Mathematica [A] (verified)	5845
Maple [B] (verified)	5846
Fricas [F(-1)]	5847
Sympy [F(-1)]	5847
Maxima [F(-2)]	5848
Giac [F]	5848
Mupad [F(-1)]	5848

Optimal result

Integrand size = 23, antiderivative size = 328

$$\begin{aligned} & \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^3} dx \\ &= -\frac{(8a^4 - 29a^2b^2 + 15b^4) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4a^3(a^2 - b^2)^2 d} + \frac{b(11a^2 - 5b^2) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4a^2(a^2 - b^2)^2 d} \\ & \quad - \frac{b(35a^4 - 38a^2b^2 + 15b^4) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4a^3(a-b)^2(a+b)^3 d} \\ & \quad + \frac{(8a^4 - 29a^2b^2 + 15b^4) \sin(c+dx)}{4a^3(a^2 - b^2)^2 d \sqrt{\cos(c+dx)}} + \frac{b^2 \sin(c+dx)}{2a(a^2 - b^2) d \sqrt{\cos(c+dx)}(a+b \cos(c+dx))^2} \\ & \quad + \frac{b^2(11a^2 - 5b^2) \sin(c+dx)}{4a^2(a^2 - b^2)^2 d \sqrt{\cos(c+dx)}(a+b \cos(c+dx))} \end{aligned}$$

```
[Out] -1/4*(8*a^4-29*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d+1/4*b*(11*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d-1/4*b*(35*a^4-38*a^2*b^2+15*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^3/(a-b)^2/(a+b)^3/d+1/4*(8*a^4-29*a^2*b^2+15*b^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2)+1/4*b^2*(11*a^2-5*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2881, 3134, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3} dx$$

$$= \frac{b(11a^2 - 5b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4a^2d(a^2 - b^2)^2} + \frac{b^2(11a^2 - 5b^2) \sin(c+dx)}{4a^2d(a^2 - b^2)^2 \sqrt{\cos(c+dx)(a+b\cos(c+dx))}}$$

$$+ \frac{b^2 \sin(c+dx)}{2ad(a^2 - b^2) \sqrt{\cos(c+dx)(a+b\cos(c+dx))^2}}$$

$$- \frac{(8a^4 - 29a^2b^2 + 15b^4) E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{4a^3d(a^2 - b^2)^2}$$

$$- \frac{b(35a^4 - 38a^2b^2 + 15b^4) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4a^3d(a-b)^2(a+b)^3}$$

$$+ \frac{(8a^4 - 29a^2b^2 + 15b^4) \sin(c+dx)}{4a^3d(a^2 - b^2)^2 \sqrt{\cos(c+dx)}}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]

[Out] -1/4*((8*a^4 - 29*a^2*b^2 + 15*b^4)*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)^2*d) + (b*(11*a^2 - 5*b^2)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - (b*(35*a^4 - 38*a^2*b^2 + 15*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) + (b^2*(11*a^2 - 5*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2

```

))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

```

Rule 2884

```

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]

```

Rule 3081

```

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3134

```

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(- (A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))

```

Rule 3138

```

Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] := Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e

```

+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
 [{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^2} \\
 &+ \frac{\int \frac{\frac{1}{2}(4a^2 - 5b^2) - 2ab \cos(c + dx) + \frac{3}{2}b^2 \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))^2} dx}{2a(a^2 - b^2)} \\
 &= \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^2} \\
 &+ \frac{b^2(11a^2 - 5b^2) \sin(c + dx)}{4a^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))} \\
 &+ \frac{\int \frac{\frac{1}{4}(8a^4 - 29a^2b^2 + 15b^4) - ab(4a^2 - b^2) \cos(c + dx) + \frac{1}{4}b^2(11a^2 - 5b^2) \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))} dx}{2a^2(a^2 - b^2)^2} \\
 &= \frac{(8a^4 - 29a^2b^2 + 15b^4) \sin(c + dx)}{4a^3(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} + \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^2} \\
 &+ \frac{b^2(11a^2 - 5b^2) \sin(c + dx)}{4a^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))} \\
 &+ \frac{\int \frac{-\frac{3}{8}b(8a^4 - 11a^2b^2 + 5b^4) - \frac{1}{2}a(2a^4 - 10a^2b^2 + 5b^4) \cos(c + dx) - \frac{1}{8}b(8a^4 - 29a^2b^2 + 15b^4) \cos^2(c + dx)}{\sqrt{\cos(c + dx)} (a + b \cos(c + dx))} dx}{a^3(a^2 - b^2)^2} \\
 &= \frac{(8a^4 - 29a^2b^2 + 15b^4) \sin(c + dx)}{4a^3(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} + \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^2} \\
 &+ \frac{b^2(11a^2 - 5b^2) \sin(c + dx)}{4a^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))} \\
 &- \frac{\int \frac{\frac{3}{8}b^2(8a^4 - 11a^2b^2 + 5b^4) - \frac{1}{8}ab^3(11a^2 - 5b^2) \cos(c + dx)}{\sqrt{\cos(c + dx)} (a + b \cos(c + dx))} dx}{a^3b(a^2 - b^2)^2} \\
 &- \frac{(8a^4 - 29a^2b^2 + 15b^4) \int \sqrt{\cos(c + dx)} dx}{8a^3(a^2 - b^2)^2}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{(8a^4 - 29a^2b^2 + 15b^4) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{4a^3 (a^2 - b^2)^2 d} + \frac{(8a^4 - 29a^2b^2 + 15b^4) \sin(c + dx)}{4a^3 (a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&\quad + \frac{b^2 \sin(c + dx)}{2a (a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^2} \\
&\quad + \frac{b^2 (11a^2 - 5b^2) \sin(c + dx)}{4a^2 (a^2 - b^2)^2 d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))} \\
&\quad + \frac{(b(11a^2 - 5b^2)) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{8a^2 (a^2 - b^2)^2} \\
&\quad - \frac{(b(35a^4 - 38a^2b^2 + 15b^4)) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx}{8a^3 (a^2 - b^2)^2} \\
&= -\frac{(8a^4 - 29a^2b^2 + 15b^4) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{4a^3 (a^2 - b^2)^2 d} + \frac{b(11a^2 - 5b^2) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{4a^2 (a^2 - b^2)^2 d} \\
&\quad - \frac{b(35a^4 - 38a^2b^2 + 15b^4) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{4a^3 (a - b)^2 (a + b)^3 d} \\
&\quad + \frac{(8a^4 - 29a^2b^2 + 15b^4) \sin(c + dx)}{4a^3 (a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&\quad + \frac{b^2 \sin(c + dx)}{2a (a^2 - b^2) d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^2} \\
&\quad + \frac{b^2 (11a^2 - 5b^2) \sin(c + dx)}{4a^2 (a^2 - b^2)^2 d \sqrt{\cos(c + dx)} (a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.02

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^3} dx$$

$$\frac{2(56a^4b - 95a^2b^3 + 45b^5) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} + \frac{8a(2a^4 - 10a^2b^2 + 5b^4)}{b} \left(2 \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - \frac{2a \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} \right) + \frac{2(8a^4 - 29a^2b^2 + 15b^4) \sin(c + dx)}{(a-b)^2}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3), x]

[Out] (-(((2*(56*a^4*b - 95*a^2*b^3 + 45*b^5)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(2*a^4 - 10*a^2*b^2 + 5*b^4)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(8*a^4 - 29*a^2*b^2 + 15*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqr

$$t[\text{Sin}[c + d*x]^2]]/((a - b)^2*(a + b)^2) + 4*\text{Sqrt}[\text{Cos}[c + d*x]]*((b^3*(-15*a^3 + 9*a*b^2 + (-13*a^2*b + 7*b^3)*\text{Cos}[c + d*x])*\text{Sin}[c + d*x])/((a^2 - b^2)^2*(a + b*\text{Cos}[c + d*x])^2) + 8*\text{Tan}[c + d*x]))/(16*a^3*d)$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. $2(388) = 776$.

Time = 11.48 (sec) , antiderivative size = 1965, normalized size of antiderivative = 5.99

method	result	size
default	Expression too large to display	1965

[In] `int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2/a^3/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)-(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})) \\ & -2/a*b*(-1/2/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\ & +1/4/(a+b)/(a^2-b^2)/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*b+3/8/(a+b)/(a^2-b^2)/a^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*b^2-9/8*b/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\ & +9/8*b/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-3/8*b^3/a^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\ & -15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(\end{aligned}$$

$$\begin{aligned}
& -2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+ \\
& 1/2*c),-2*b/(a-b),2^{(1/2)}))+4*b^2/a^3/(-2*a*b+2*b^2)*(\sin(1/2*d*x+1/2*c)^2) \\
& ^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d \\
& *x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})-2*b/a^ \\
& 2*(-1/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d \\
& *x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(\sin(1/2*d*x+ \\
& 1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+ \\
& \sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-1/2/(a^2- \\
& b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2 \\
& *\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2 \\
& *c),2^{(1/2)})+1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x \\
& +1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*Ell \\
& ipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(\sin(1/2* \\
& d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c \\
&)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos \\
& s(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})))/\sin(1/2*d*x+1/2*c \\
&)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d
\end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3} dx = \text{Exception raised: RuntimeError}$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3} dx = \int \frac{1}{(b\cos(dx+c)+a)^3 \cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3} dx = \int \frac{1}{\cos(c+dx)^{3/2} (a+b\cos(c+dx))^3} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3), x)

[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^3), x)

$$3.602 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^3} dx$$

Optimal result	5849
Rubi [A] (verified)	5850
Mathematica [A] (verified)	5854
Maple [B] (verified)	5854
Fricas [F(-1)]	5856
Sympy [F(-1)]	5856
Maxima [F(-1)]	5856
Giac [F]	5856
Mupad [F(-1)]	5857

Optimal result

Integrand size = 23, antiderivative size = 395

$$\begin{aligned} & \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^3} dx \\ &= \frac{b(24a^4 - 65a^2b^2 + 35b^4) E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4a^4(a^2 - b^2)^2 d} \\ & \quad + \frac{(8a^4 - 61a^2b^2 + 35b^4) \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{12a^3(a^2 - b^2)^2 d} \\ & \quad + \frac{b^2(63a^4 - 86a^2b^2 + 35b^4) \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4a^4(a-b)^2(a+b)^3 d} \\ & \quad + \frac{(8a^4 - 61a^2b^2 + 35b^4) \sin(c+dx)}{12a^3(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c+dx)} - \frac{b(24a^4 - 65a^2b^2 + 35b^4) \sin(c+dx)}{4a^4(a^2 - b^2)^2 d \sqrt{\cos(c+dx)}} \\ & \quad + \frac{b^2 \sin(c+dx)}{2a(a^2 - b^2) d \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^2} \\ & \quad + \frac{b^2(13a^2 - 7b^2) \sin(c+dx)}{4a^2(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))} \end{aligned}$$

```
[Out] 1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/12*(8*a^4-61*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d+1/4*b^2*(63*a^4-86*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))/a^4/(a-b)^2/(a+b)^3/d+1/12*(8*a^4-61*a^2*b^2+35*b^4)*sin(d*x+c)/a^3/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)+1/2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2+1/4*b^2*(13*a^2-7*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))-1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2881, 3134, 3138, 2719, 3081, 2720, 2884}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3} dx$$

$$= \frac{b^2(13a^2 - 7b^2)\sin(c+dx)}{4a^2d(a^2 - b^2)^2 \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{b^2\sin(c+dx)}{2ad(a^2 - b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{b(24a^4 - 65a^2b^2 + 35b^4)E\left(\frac{1}{2}(c+dx)\mid 2\right)}{4a^4d(a^2 - b^2)^2} + \frac{b^2(63a^4 - 86a^2b^2 + 35b^4)\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4a^4d(a-b)^2(a+b)^3} - \frac{b(24a^4 - 65a^2b^2 + 35b^4)\sin(c+dx)}{4a^4d(a^2 - b^2)^2\sqrt{\cos(c+dx)}} + \frac{(8a^4 - 61a^2b^2 + 35b^4)\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{12a^3d(a^2 - b^2)^2} + \frac{(8a^4 - 61a^2b^2 + 35b^4)\sin(c+dx)}{12a^3d(a^2 - b^2)^2\cos^{\frac{3}{2}}(c+dx)}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]

[Out] (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*EllipticF[(c + d*x)/2, 2])/(12*a^3*(a^2 - b^2)^2*d) + (b^2*(63*a^4 - 86*a^2*b^2 + 35*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)) - (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b^2*Ssin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2) + (b^2*(13*a^2 - 7*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2881

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*
x])^(m + 1)*((c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x]
)^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

```

Rule 2884

```

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]

```

Rule 3081

```

Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[
B/d, Int[(a + b*Ssin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Ssin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3134

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))

```

Rule 3138

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^

```

2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2} \\
 &+ \frac{\int \frac{\frac{1}{2}(4a^2 - 7b^2) - 2ab \cos(c + dx) + \frac{5}{2}b^2 \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^2} dx}{2a(a^2 - b^2)} \\
 &= \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2} \\
 &+ \frac{b^2(13a^2 - 7b^2) \sin(c + dx)}{4a^2(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))} \\
 &+ \frac{\int \frac{\frac{1}{4}(8a^4 - 61a^2b^2 + 35b^4) - ab(4a^2 - b^2) \cos(c + dx) + \frac{3}{4}b^2(13a^2 - 7b^2) \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))} dx}{2a^2(a^2 - b^2)^2} \\
 &= \frac{(8a^4 - 61a^2b^2 + 35b^4) \sin(c + dx)}{12a^3(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)} + \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2} \\
 &+ \frac{b^2(13a^2 - 7b^2) \sin(c + dx)}{4a^2(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))} \\
 &+ \frac{\int \frac{-\frac{3}{8}b(24a^4 - 65a^2b^2 + 35b^4) + \frac{1}{2}a(2a^4 + 14a^2b^2 - 7b^4) \cos(c + dx) + \frac{1}{8}b(8a^4 - 61a^2b^2 + 35b^4) \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))} dx}{3a^3(a^2 - b^2)^2} \\
 &= \frac{(8a^4 - 61a^2b^2 + 35b^4) \sin(c + dx)}{12a^3(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)} - \frac{b(24a^4 - 65a^2b^2 + 35b^4) \sin(c + dx)}{4a^4(a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
 &+ \frac{b^2 \sin(c + dx)}{2a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^2} \\
 &+ \frac{b^2(13a^2 - 7b^2) \sin(c + dx)}{4a^2(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))} \\
 &+ \frac{2 \int \frac{\frac{1}{16}(8a^6 + 128a^4b^2 - 223a^2b^4 + 105b^6) + \frac{1}{4}ab(20a^4 - 64a^2b^2 + 35b^4) \cos(c + dx) + \frac{3}{16}b^2(24a^4 - 65a^2b^2 + 35b^4) \cos^2(c + dx)}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))} dx}{3a^4(a^2 - b^2)^2}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{(8a^4 - 61a^2b^2 + 35b^4) \sin(c + dx)}{12a^3 (a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)} - \frac{b(24a^4 - 65a^2b^2 + 35b^4) \sin(c + dx)}{4a^4 (a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&\quad + \frac{b^2 \sin(c + dx)}{2a (a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))^2} \\
&\quad + \frac{b^2(13a^2 - 7b^2) \sin(c + dx)}{4a^2 (a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))} \\
&\quad - \frac{2 \int \frac{-\frac{1}{16}b(8a^6 + 128a^4b^2 - 223a^2b^4 + 105b^6) - \frac{1}{16}ab^2(8a^4 - 61a^2b^2 + 35b^4) \cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))} dx}{3a^4b (a^2 - b^2)^2} \\
&\quad + \frac{(b(24a^4 - 65a^2b^2 + 35b^4)) \int \sqrt{\cos(c + dx)} dx}{8a^4 (a^2 - b^2)^2} \\
&= \frac{b(24a^4 - 65a^2b^2 + 35b^4) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{4a^4 (a^2 - b^2)^2 d} \\
&\quad + \frac{(8a^4 - 61a^2b^2 + 35b^4) \sin(c + dx)}{12a^3 (a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)} - \frac{b(24a^4 - 65a^2b^2 + 35b^4) \sin(c + dx)}{4a^4 (a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&\quad + \frac{b^2 \sin(c + dx)}{2a (a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))^2} \\
&\quad + \frac{b^2(13a^2 - 7b^2) \sin(c + dx)}{4a^2 (a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))} \\
&\quad + \frac{(b^2(63a^4 - 86a^2b^2 + 35b^4)) \int \frac{1}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))} dx}{8a^4 (a^2 - b^2)^2} \\
&\quad + \frac{(8a^4 - 61a^2b^2 + 35b^4) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{24a^3 (a^2 - b^2)^2} \\
&= \frac{b(24a^4 - 65a^2b^2 + 35b^4) E\left(\frac{1}{2}(c + dx) \mid 2\right)}{4a^4 (a^2 - b^2)^2 d} \\
&\quad + \frac{(8a^4 - 61a^2b^2 + 35b^4) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{12a^3 (a^2 - b^2)^2 d} \\
&\quad + \frac{b^2(63a^4 - 86a^2b^2 + 35b^4) \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{4a^4(a - b)^2(a + b)^3 d} \\
&\quad + \frac{(8a^4 - 61a^2b^2 + 35b^4) \sin(c + dx)}{12a^3 (a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)} - \frac{b(24a^4 - 65a^2b^2 + 35b^4) \sin(c + dx)}{4a^4 (a^2 - b^2)^2 d \sqrt{\cos(c + dx)}} \\
&\quad + \frac{b^2 \sin(c + dx)}{2a (a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))^2} \\
&\quad + \frac{b^2(13a^2 - 7b^2) \sin(c + dx)}{4a^2 (a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.73 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.88

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3} dx$$

$$\frac{2(16a^6+328a^4b^2-641a^2b^4+315b^6)\operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a+b} + \frac{16(20a^5-64a^3b^2+35ab^4)\left((a+b)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) - a\operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)\right)}{a+b} + \frac{6(24a^4-65a^2b^2+35b^4)(-2ab\operatorname{EllipticE}[\operatorname{ArcSin}[\sqrt{\cos(c+dx)}]], -1) + 2a(a+b)\operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{\cos(c+dx)}]], -1) + (-2a^2+b^2)\operatorname{EllipticPi}[-(b/a), \operatorname{ArcSin}[\sqrt{\cos(c+dx)}]], -1)\operatorname{Sin}[c+dx]}{(a\sqrt{\sin(c+dx)^2})((a-b)^2(a+b)^2+4\sqrt{\cos(c+dx)}\operatorname{Sin}[c+dx])((3b^4(19a^3-13ab^2+b(17a^2-11b^2))\cos(c+dx))\operatorname{Sin}[c+dx])((a^2-b^2)^2(a+b\cos(c+dx))^2)+8(-9b+a\sec(c+dx))\tan[c+dx])/(48a^4d)}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3), x]

[Out] (((2*(16*a^6 + 328*a^4*b^2 - 641*a^2*b^4 + 315*b^6)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(20*a^5 - 64*a^3*b^2 + 35*a*b^4)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(24*a^4 - 65*a^2*b^2 + 35*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2 + 4*sqrt[Cos[c + d*x]]*((3*b^4*(19*a^3 - 13*a*b^2 + b*(17*a^2 - 11*b^2))*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*cos[c + d*x])^2) + 8*(-9*b + a*Sec[c + d*x])*Tan[c + d*x]))/(48*a^4*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2100 vs. 2(451) = 902.

Time = 15.66 (sec) , antiderivative size = 2101, normalized size of antiderivative = 5.32

method	result	size
default	Expression too large to display	2101

[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))-6/a^4*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))+2*b^2/a^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)

$$\begin{aligned}
& 1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c) \\
& ^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/ \\
& 2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(cos(1/2*d*x+1/2*c),2^{(1/2)})+1/4/(a+b)/(a^2- \\
& b^2)/a*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*s \\
& in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(cos(1/2*d*x+1/2*c \\
&),2^{(1/2)})*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1 \\
& /2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1 \\
& 2)}*EllipticF(cos(1/2*d*x+1/2*c),2^{(1/2)})*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x \\
& +1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4 \\
& +sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(cos(1/2*d*x+1/2*c),2^{(1/2)})+3/8*b^3/ \\
& a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1 \\
& /2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(cos(1/2* \\
& d*x+1/2*c),2^{(1/2)})+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(\\
& 1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1 \\
& /2)}*EllipticE(cos(1/2*d*x+1/2*c),2^{(1/2)})-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2* \\
& d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c \\
&)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(cos(1/2*d*x+1/2*c),2^{(1/2)})-15/4* \\
& a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d \\
& *x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*E \\
& llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+3/2/(a^2-b^2)^2/(-2*a*b+2* \\
& b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2 \\
& *sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(cos(1/2*d*x+1/ \\
& 2*c),-2*b/(a-b),2^{(1/2)})-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d* \\
& x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^ \\
& 4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1 \\
& /2)))-12*b^3/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d* \\
& x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*E \\
& llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+4*b^2/a^3*(-1/a*b^2/(a^2-b^ \\
& 2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}/ \\
& (2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2 \\
& *cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^ \\
& 2)^{(1/2)}*EllipticF(cos(1/2*d*x+1/2*c),2^{(1/2)})-1/2/(a^2-b^2)*b/a*(sin(1/2*d \\
& *x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c) \\
& ^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(cos(1/2*d*x+1/2*c),2^{(1/2)})+1/2/(a \\
& ^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/ \\
& (-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(cos(1/2*d*x+ \\
& 1/2*c),2^{(1/2)})-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\
& *(-2*cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2 \\
& *c)^2)^{(1/2)}*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+1/a/(a^2-b^2 \\
&)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*cos(1/2*d*x+1/2*c)^2+ \\
& 1)^{(1/2)}/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(co \\
& s(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/ \\
& 2*c)^2-1)^{(1/2)}/d
\end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3} dx = \int \frac{1}{(b\cos(dx+c)+a)^3 \cos(dx+c)^{\frac{5}{2}}} dx$$

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^3} dx = \int \frac{1}{\cos(c + dx)^{\frac{5}{2}}(a + b \cos(c + dx))^3} dx$$

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3), x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^3), x)
```

3.603 $\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)} dx$

Optimal result	5858
Rubi [A] (verified)	5859
Mathematica [A] (verified)	5862
Maple [B] (verified)	5863
Fricas [F]	5864
Sympy [F]	5864
Maxima [F]	5864
Giac [F(-1)]	5864
Mupad [F(-1)]	5865

Optimal result

Integrand size = 25, antiderivative size = 438

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)} dx =$$

$$\frac{(a - b)\sqrt{a + b} \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{4bd}$$

$$+ \frac{\sqrt{a + b}(a + 2b) \cot(c + dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{4bd}$$

$$+ \frac{\sqrt{a + b}(a^2 - 4b^2) \cot(c + dx) \operatorname{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{4b^2d}$$

$$+ \frac{a\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{4bd\sqrt{\cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{2d}$$

```
[Out] 1/4*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)+1/2*sin(d*x+c)
*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/4*(a-b)*cot(d*x+c)*EllipticE((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b)^(1/2)*(a*(1+sec(d*x+c))/(a-b)^(1/2)/b/d
+1/4*(a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d
x+c)^(1/2),((-a-b)/(a-b))^(1/2)*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b)^(1/2)
*(a*(1+sec(d*x+c))/(a-b)^(1/2)/b/d+1/4*(a^2-4*b^2)*cot(d*x+c)*EllipticPi((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2), (a+b)/b,((-a-b)/(a-b))^(
1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b)^(1/2)*(a*(1+sec(d*x+c))/(a-b)^(
1/2)/b^2/d
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00,
 number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used
 = {2900, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)} dx$$

$$= \frac{\sqrt{a+b}(a^2-4b^2) \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{4b^2d}$$

$$+ \frac{\sqrt{a+b}(a+2b) \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{4bd}$$

$$- \frac{(a-b) \sqrt{a+b} \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right)}{4bd}$$

$$+ \frac{\sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}{2d} + \frac{a \sin(c+dx) \sqrt{a+b \cos(c+dx)}}{4bd \sqrt{\cos(c+dx)}}$$

[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]],x]

[Out] -1/4*((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b*d) + (Sqrt[a + b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*b*d) + (Sqrt[a + b]*(a^2 - 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*b^2*d) + (a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 2888

Int[Sqrt[(b_)*sin[(e_.) + (f_)*(x_)]]/Sqrt[(c_.) + (d_)*sin[(e_.) + (f_)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_.) + (f_)*(x_)]]*Sqrt[(a_.) + (b_)*sin[(e_.) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr

```
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
```


NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3140

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\int \frac{\frac{ab}{2}+b^2\cos(c+dx)+\frac{1}{2}ab\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx}{2b} \\
 &= \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} \\
 &\quad + \frac{\int \frac{-\frac{a^2b}{2}+ab^2\cos(c+dx)-\frac{1}{2}b(a^2-4b^2)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{4b^2} \\
 &= \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} \\
 &\quad + \frac{\int \frac{-\frac{a^2b}{2}+ab^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{4b^2} - \frac{(a^2-4b^2)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{8b} \\
 &= \frac{\sqrt{a+b}(a^2-4b^2)\cot(c+dx)\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{4b^2d} \\
 &\quad + \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} \\
 &\quad - \frac{a^2\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{8b} + \frac{(a(a+2b))\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx}{8b}
 \end{aligned}$$

$$\begin{aligned}
&= \\
&\frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4bd} \\
&+ \frac{\sqrt{a+b}(a+2b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4bd} \\
&+ \frac{\sqrt{a+b}(a^2-4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4b^2d} \\
&+ \frac{a\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.18 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.98

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}dx$$

$$\sqrt{\cos(c+dx)}\left(4(a+b\cos(c+dx))\sin(c+dx) + \frac{2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)+4(a-2b)b\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}}{8d\sqrt{a+b\cos(c+dx)}}\right)$$

[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]],x]

[Out] (Sqrt[Cos[c + d*x]]*(4*(a + b*Cos[c + d*x])*Sin[c + d*x] + (2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*(a - 2*b)*b*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*a^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 16*b^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/(b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]))/(8*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1687 vs. 2(396) = 792.

Time = 7.06 (sec) , antiderivative size = 1688, normalized size of antiderivative = 3.85

method	result	size
default	Expression too large to display	1688

[In] `int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4}d \cdot (-\text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a^2 \cdot \cos(d*x+c)^2 - \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a \cdot b \cdot \cos(d*x+c)^2 + 2 \cdot \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a^2 \cdot \cos(d*x+c)^2 - 8 \cdot \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot b^2 \cdot \cos(d*x+c)^2 - 2 \cdot \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a \cdot b \cdot \cos(d*x+c)^2 + 4 \cdot \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot b^2 \cdot \cos(d*x+c)^2 + 2 \cdot b^2 \cdot \cos(d*x+c)^3 \cdot \sin(d*x+c) - 2 \cdot \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a^2 \cdot \cos(d*x+c)^2 - 2 \cdot \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a \cdot b \cdot \cos(d*x+c) + 4 \cdot \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a^2 \cdot \cos(d*x+c) - 16 \cdot \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot b^2 \cdot \cos(d*x+c) - 4 \cdot \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a \cdot b \cdot \cos(d*x+c) + 8 \cdot \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot b^2 \cdot \cos(d*x+c) + 3 \cdot \cos(d*x+c)^2 \cdot \sin(d*x+c) \cdot a \cdot b + 2 \cdot b^2 \cdot \cos(d*x+c)^2 \cdot \sin(d*x+c) - ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a^2 - ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a \cdot b + 2 \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot a^2 - 8 \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \cdot b^2 - 2 \cdot ((a + \cos(d*x+c)) \cdot b) / (1 + \cos(d*x+c)) / (a+b)^{1/2} \cdot \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2}$

$$\left. \right)^{1/2} * a * b + 4 * \left(\frac{a + \cos(dx+c) * b}{1 + \cos(dx+c)} \right) / (a+b)^{1/2} * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} * b^2 + a^2 * \cos(dx+c) * \sin(dx+c) + 2 * a * b * \cos(dx+c) * \sin(dx+c) / (1 + \cos(dx+c)) / (a + \cos(dx+c) * b)^{1/2} / \cos(dx+c)^{1/2} / b$$

Fricas [F]

$$\int \cos^{3/2}(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \cos(dx + c)^{3/2} dx$$

[In] integrate(cos(dx+c)^(3/2)*(a+b*cos(dx+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(dx + c) + a)*cos(dx + c)^(3/2), x)

Sympy [F]

$$\int \cos^{3/2}(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{a + b \cos(c + dx)} \cos^{3/2}(c + dx) dx$$

[In] integrate(cos(dx+c)**(3/2)*(a+b*cos(dx+c))^(1/2),x)

[Out] Integral(sqrt(a + b*cos(c + dx))*cos(c + dx)**(3/2), x)

Maxima [F]

$$\int \cos^{3/2}(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \cos(dx + c)^{3/2} dx$$

[In] integrate(cos(dx+c)^(3/2)*(a+b*cos(dx+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(dx + c) + a)*cos(dx + c)^(3/2), x)

Giac [F(-1)]

Timed out.

$$\int \cos^{3/2}(c + dx) \sqrt{a + b \cos(c + dx)} dx = \text{Timed out}$$

[In] integrate(cos(dx+c)^(3/2)*(a+b*cos(dx+c))^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)} dx = \int \cos(c + dx)^{3/2} \sqrt{a + b \cos(c + dx)} dx$$

```
[In] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2), x)
```

3.604 $\int \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)} dx$

Optimal result	5866
Rubi [A] (verified)	5867
Mathematica [A] (verified)	5870
Maple [B] (verified)	5870
Fricas [F]	5871
Sympy [F]	5871
Maxima [F]	5871
Giac [F]	5872
Mupad [F(-1)]	5872

Optimal result

Integrand size = 25, antiderivative size = 371

$$\begin{aligned}
 & \int \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)} dx = \\
 & \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad} \\
 & + \frac{\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\
 & - \frac{a\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd} \\
 & + \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
 \end{aligned}$$

```

[Out] sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(a-b)*cot(d*x+c)*Ellip
ticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a/d+cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(
1+sec(d*x+c))/(a-b))^(1/2)/d-a*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(
1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d

```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2900, 3133, 2888, 12, 2880, 2895, 3073}

$$\int \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)} dx$$

$$= \frac{\sqrt{a+b} \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{(a-b)\sqrt{a+b} \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)} - \frac{a\sqrt{a+b} \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{bd} + \frac{\sin(c+dx) \sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]],x]

[Out] -(((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d)) + (Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/d - (a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b*d) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2880

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2888

```
Int[Sqrt[(b_)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] := Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]^(n_)), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), In
t[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m
+ n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c -
b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && N
eQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3073

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)]
^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3133

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
+ Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
```


$x)^{3/2} \sqrt{c + d \sin(e + f x)}], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, C\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{\int \frac{-\frac{ab}{2} + \frac{1}{2} ab \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{b} \\
 &= \frac{\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{1}{2} a \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}} dx \\
 &\quad + \frac{\int -\frac{ab}{2 \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{b} \\
 &= \frac{a \sqrt{a + b} \cot(c + dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd} \\
 &\quad + \frac{\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} - \frac{1}{2} a \int \frac{1}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{a \sqrt{a + b} \cot(c + dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd} \\
 &\quad + \frac{\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{1}{2} a \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
 &\quad - \frac{1}{2} a \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{(a - b) \sqrt{a + b} \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad} \\
 &\quad + \frac{\sqrt{a + b} \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\
 &\quad - \frac{a \sqrt{a + b} \cot(c + dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd} \\
 &\quad + \frac{\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 5.87 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.85

$$\int \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)} dx$$

$$= \frac{\sqrt{\cos(c+dx)} \left(\frac{2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}} E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right) \frac{-a+b}{a+b}}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}} - \frac{4a\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}} \text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}} \right)}{1}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]],x]

[Out] (Sqrt[Cos[c + d*x]]*((2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (4*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (4*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])])]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + b*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Tan[(c + d*x)/2] - b*Tan[(c + d*x)/2]))/(2*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1084 vs. 2(343) = 686.

Time = 6.42 (sec) , antiderivative size = 1085, normalized size of antiderivative = 2.92

method	result	size
default	Expression too large to display	1085

[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/d*(-EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*a*cos(d*x+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*b*cos(d*x+c)^2-2*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*a*cos(d*x+c)^2+2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*a*cos(d*x+c)^2-2*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*a*cos(d*x+c)-2*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*b*cos

$(d*x+c)-4*EllipticPi(\cot(d*x+c)-\csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a*\cos(d*x+c)+4*EllipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a*\cos(d*x+c)+b*\cos(d*x+c)^2*\sin(d*x+c)-(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*EllipticE(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a-(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*EllipticE(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*b-2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*EllipticPi(\cot(d*x+c)-\csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a+2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*EllipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a+\sin(d*x+c)*\cos(d*x+c)*a)/(1+\cos(d*x+c))/\cos(d*x+c)^{(1/2)}/(a+\cos(d*x+c)*b)^{(1/2)}$

Fricas [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}dx = \int \sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)

Sympy [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}dx = \int \sqrt{a+b\cos(c+dx)}\sqrt{\cos(c+dx)}dx$$

[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)), x)

Maxima [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}dx = \int \sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)} dx = \int \sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)} dx = \int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)} dx$$

[In] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2), x)

$$3.605 \quad \int \frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	5873
Rubi [A] (verified)	5873
Mathematica [A] (verified)	5874
Maple [A] (verified)	5875
Fricas [F]	5875
Sympy [F]	5875
Maxima [F]	5876
Giac [F]	5876
Mupad [F(-1)]	5876

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx = \frac{2\sqrt{\frac{a(1-\cos(c+dx))}{a+b \cos(c+dx)}} \sqrt{\frac{a(1+\cos(c+dx))}{a+b \cos(c+dx)}} (a+b \cos(c+dx)) \operatorname{csc}(c+dx) \operatorname{EllipticPi}\left(\frac{b}{a+b}, \arcsin\left(\frac{\sqrt{a+b} \sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}}\right)\right)}{\sqrt{a+bd}}$$

[Out] $-2*(a+b*\cos(d*x+c))*\operatorname{csc}(d*x+c)*\operatorname{EllipticPi}((a+b)^{(1/2)}*\cos(d*x+c)^{(1/2)}/(a+b*\cos(d*x+c))^{(1/2)}, b/(a+b), ((-a+b)/(a+b))^{(1/2)}*(a*(1-\cos(d*x+c))/(a+b*\cos(d*x+c)))^{(1/2)}*a*(1+\cos(d*x+c))/(a+b*\cos(d*x+c)))^{(1/2)}/d/(a+b)^{(1/2)}$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2890}

$$\int \frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx = \frac{2 \operatorname{csc}(c+dx) \sqrt{\frac{a(1-\cos(c+dx))}{a+b \cos(c+dx)}} \sqrt{\frac{a(\cos(c+dx)+1)}{a+b \cos(c+dx)}} (a+b \cos(c+dx)) \operatorname{EllipticPi}\left(\frac{b}{a+b}, \arcsin\left(\frac{\sqrt{a+b} \sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}}\right)\right)}{d\sqrt{a+b}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[a+b*\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]],x]$

[Out] $(-2*\operatorname{Sqrt}[(a*(1-\operatorname{Cos}[c+d*x]))/(a+b*\operatorname{Cos}[c+d*x])]*\operatorname{Sqrt}[(a*(1+\operatorname{Cos}[c+d*x]))/(a+b*\operatorname{Cos}[c+d*x])]*(a+b*\operatorname{Cos}[c+d*x])* \operatorname{Csc}[c+d*x]*\operatorname{EllipticPi}[b$

$$\frac{1}{(a+b)} \operatorname{ArcSin}\left[\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right] - \frac{-(a-b)/(a+b)}{\sqrt{a+b}d}$$

Rule 2890

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]] , x_Symbol]
:> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e + f*x])))]*Sqrt[(-b*c - a*d)*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d))), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rubi steps

integral =

$$\frac{2\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(1+\cos(c+dx))}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\csc(c+dx)\operatorname{EllipticPi}\left(\frac{b}{a+b}, \arcsin\left(\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right)\right)}{\sqrt{a+bd}}$$

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}((a-b)\operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right), \frac{-a+b}{a+b}\right) + 2b\operatorname{EllipticPi}\left(-1, \arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{a+b\cos(c+dx)}}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]

[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c + d*x]]/(1 + Cos[c + d*x]))*Sqrt[a + b*Cos[c + d*x]])

Maple [A] (verified)

Time = 8.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.34

method	result
default	$-\frac{2\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}} \left(F\left(\cot(dx+c)-\operatorname{csc}(dx+c), \sqrt{-\frac{a-b}{a+b}}\right)a - F\left(\cot(dx+c)-\operatorname{csc}(dx+c), \sqrt{-\frac{a-b}{a+b}}\right)b + 2b\Pi\left(\cot(dx+c)-\operatorname{csc}(dx+c), -1, \sqrt{-\frac{a-b}{a+b}}\right) \right)}{d\sqrt{a+\cos(dx+c)}b\sqrt{\cos(dx+c)}}$

```
[In] int((a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((a+cos(d*x+c)*b)/(1+cos(d*x+c))^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a-EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*b+2*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-a-b)/(a+b))^(1/2))/(a+cos(d*x+c)*b)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx = \int \frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx = \int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} dx$$

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx$$

[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)

[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2), x)

$$3.606 \quad \int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	5877
Rubi [A] (verified)	5877
Mathematica [A] (verified)	5879
Maple [B] (verified)	5879
Fricas [F]	5880
Sympy [F]	5880
Maxima [F]	5881
Giac [F]	5881
Mupad [F(-1)]	5881

Optimal result

Integrand size = 25, antiderivative size = 229

$$\int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx$$

$$= \frac{2(a-b)\sqrt{a+b} \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

$$= \frac{2(a-b)\sqrt{a+b} \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

```
[Out] 2*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-2*(a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used

= {2874, 2895, 3073}

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{2(a - b)\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{ad}$$

$$= \frac{2(a - b)\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{ad}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]

[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d) - (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d)

Rule 2874

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]]/Rt[(c + d)/b, 2], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rubi steps

$$\begin{aligned}
\text{integral} &= a \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&\quad + (-a + b) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2(a - b) \sqrt{a + b} \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{ad} \\
&= \frac{2(a - b) \sqrt{a + b} \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{\sqrt{a + b \cos(c + dx)} \sec^2\left(\frac{1}{2}(c + dx)\right) \left(\sqrt{\cos(c + dx)} \sqrt{1 + \cos(c + dx)} E\left(\arcsin\left(\tan\left(\frac{1}{2}(c + dx)\right)\right)\right) \mid -\frac{a + b}{a + b}\right)}{d \sqrt{\cos(c + dx)}}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(3/2), x]

[Out] -((Sqrt[a + b*Cos[c + d*x]]*Sec[(c + d*x)/2]^2*(Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)] - Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*EllipticF[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)] - Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sin[c + d*x])/((d*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 769 vs. 2(213) = 426.

Time = 10.10 (sec) , antiderivative size = 770, normalized size of antiderivative = 3.36

method	result
default	$ \frac{2\left((\csc^2(dx+c))(1-\cos(dx+c))^2-1\right)\left(-\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1}\sqrt{\frac{(\csc^2(dx+c)a(1-\cos(dx+c))^2-(\csc^2(dx+c)b(1-\cos(dx+c)))}{a+b}}\right)}{ad} $

```
[In] int((a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+csc(d*x+c)^3*(1-cos(d*x+c))^3*a-csc(d*x+c)^3*(1-cos(d*x+c))^3*b+a*(csc(d*x+c)-cot(d*x+c))+b*(csc(d*x+c)-cot(d*x+c)))*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(-csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(3/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)
```

Fricas [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos^{\frac{3}{2}}(dx + c)} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx$$

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
[Out] Integral(sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos(c + dx)^{\frac{3}{2}}} dx$$

[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)

[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)

$$3.607 \quad \int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	5882
Rubi [A] (verified)	5883
Mathematica [A] (verified)	5885
Maple [B] (verified)	5885
Fricas [F]	5886
Sympy [F]	5886
Maxima [F]	5886
Giac [F]	5887
Mupad [F(-1)]	5887

Optimal result

Integrand size = 25, antiderivative size = 271

$$\int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)} dx$$

$$= \frac{2(a-b)b\sqrt{a+b} \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2d}$$

$$+ \frac{2(a-b)\sqrt{a+b} \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3ad}$$

$$+ \frac{2\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/3*(a-b)*b*cot(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)))/
(a-b)^(1/2)/a^2/d+2/3*(a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a
+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2875, 3077, 2895, 3073}

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{2b(a - b)\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right)}{3a^2 d}$$

$$+ \frac{2(a - b)\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{3ad}$$

$$+ \frac{2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{3d \cos^{\frac{3}{2}}(c + dx)}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(5/2), x]

[Out] (2*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^2*d) + (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rule 2875

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Ssin[e + f*x]]/Sqrt[d*Ssin[e + f*x]]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3} \int \frac{\frac{b}{2} + \frac{1}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}(a-b) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx \\
&\quad + \frac{1}{3}b \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{2(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2d} \\
&\quad + \frac{2(a-b)\sqrt{a+b}\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3ad} \\
&\quad + \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.47 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{-2b(a + b) \cos^{\frac{3}{2}}(c + dx) \sqrt{1 + \cos(c + dx)} \sqrt{\frac{a + b \cos(c + dx)}{(a + b)(1 + \cos(c + dx))}} E\left(\arcsin\left(\tan\left(\frac{1}{2}(c + dx)\right)\right) \middle| \frac{-a + b}{a + b}\right) + 2a(a + b)}{\dots}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]

[Out] $(-2*b*(a + b)*Cos[c + d*x]^{(3/2)}*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b) + 2*a*(a + b)*Cos[c + d*x]^{(3/2)}*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b) + (2*a^2 + a*b + b^2 + 2*a*(a + 2*b)*Cos[c + d*x] + b*(a + b)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2]/(3*a*d*Cos[c + d*x]^{(3/2)}*Sqrt[a + b*Cos[c + d*x]])$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. 2(245) = 490.

Time = 12.65 (sec) , antiderivative size = 1179, normalized size of antiderivative = 4.35

method	result	size
default	Expression too large to display	1179

[In] int((a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] $2/3/d*(-(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*a^2*\cos(d*x+c)^3-(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*a*b*\cos(d*x+c)^3+(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*a*b*\cos(d*x+c)^3+(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*b^2*\cos(d*x+c)^3-2*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*a^2*\cos(d*x+c)^2-2*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*a*b*\cos(d*x+c)^2+2*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}$

$$\begin{aligned} & /2) * a * b * \cos(d*x+c)^2 + 2 * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} * ((a + \cos(d*x+c) * b) / \\ & (1 + \cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b) / (a+b))^{1/2} \\ & (1/2)) * b^2 * \cos(d*x+c)^2 - (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} * ((a + \cos(d*x+c) * b) / \\ & (1 + \cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b) / (a+b))^{1/2} \\ & (1/2)) * a^2 * \cos(d*x+c) - \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b) / (a+b))^{1/2} \\ & (1/2)) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{1/2} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \\ & (1/2)) * a * b * \cos(d*x+c) + \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b) / (a+b))^{1/2} \\ & (1/2)) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{1/2} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} \\ & (1/2)) * a * b * \cos(d*x+c) + (\cos(d*x+c) / (1 + \cos(d*x+c)))^{1/2} * ((a + \cos(d*x+c) * b) / (1 \\ & + \cos(d*x+c)) / (a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b) / (a+b))^{1/2} \\ & (1/2)) * b^2 * \cos(d*x+c) + \cos(d*x+c)^2 * \sin(d*x+c) * a * b + b^2 * \cos(d*x+c)^2 * \sin(d*x+c) \\ & + a^2 * \cos(d*x+c) * \sin(d*x+c) + 2 * a * b * \cos(d*x+c) * \sin(d*x+c) + a^2 * \sin(d*x+c) / (1 + \cos(d*x+c)) / (a + \cos(d*x+c) * b)^{1/2} / \cos(d*x+c)^{3/2} / a \end{aligned}$$

Fricas [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos^{\frac{5}{2}}(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)

Sympy [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx$$

[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)

[Out] Integral(sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(5/2), x)

Maxima [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos^{\frac{5}{2}}(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos(c + dx)^{\frac{5}{2}}} dx$$

[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)

[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2), x)

$$3.608 \quad \int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	5888
Rubi [A] (verified)	5889
Mathematica [A] (verified)	5891
Maple [B] (verified)	5892
Fricas [F]	5893
Sympy [F(-1)]	5893
Maxima [F]	5894
Giac [F]	5894
Mupad [F(-1)]	5894

Optimal result

Integrand size = 25, antiderivative size = 329

$$\int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx$$

$$= \frac{2(a-b)\sqrt{a+b}(9a^2-2b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{15a^3d}$$

$$- \frac{2(a-b)\sqrt{a+b}(9a+2b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{15a^2d}$$

$$+ \frac{2\sqrt{a+b \cos(c+dx)}\sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b \cos(c+dx)}\sin(c+dx)}{15ad \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/5*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/15*b*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*a^2-2*b^2)*cot(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/a^3/d-2/15*(a-b)*(9*a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c
))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00,
 number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used
 = {2875, 3134, 3077, 2895, 3073}

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx =$$

$$\frac{2(a - b)\sqrt{a + b}(9a + 2b) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right),}{15a^2d}$$

$$+ \frac{2(a - b)\sqrt{a + b}(9a^2 - 2b^2) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{15a^3d}$$

$$+ \frac{2b \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{15ad \cos^{\frac{3}{2}}(c + dx)} + \frac{2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{5d \cos^{\frac{5}{2}}(c + dx)}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(7/2), x]

[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(15*a^2*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d*x]^(3/2))

Rule 2875

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Ssin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

&& PosQ[(a + b)/d]

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)]
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_)
+ (f_)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)
*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{2}{5} \int \frac{\frac{b}{2} + \frac{3}{2}a \cos(c + dx) + b \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx$$

$$\begin{aligned}
&= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15ad\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int\frac{\frac{1}{4}(9a^2-2b^2)+\frac{7}{4}ab\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{15a} \\
&= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15ad\cos^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{((a-b)(9a+2b))\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{15a} \\
&\quad - \frac{(-9a^2+2b^2)\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{15a} \\
&= \frac{2(a-b)\sqrt{a+b}(9a^2-2b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\mid-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{15a^3d} \\
&\quad - \frac{2(a-b)\sqrt{a+b}(9a+2b)\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{15a^2d} \\
&\quad + \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15ad\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.46 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.38

$$\begin{aligned}
&\int\frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}dx \\
&= \frac{8\cos^2\left(\frac{1}{2}(c+dx)\right)^{7/2}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(-2(9a^3+9a^2b-2ab^2-2b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\right)}{d} \\
&\quad + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)(9a^2\sin(c+dx)-2b^2\sin(c+dx))}{15a^2} + \frac{2b\sec(c+dx)\tan(c+dx)}{15a} + \frac{2}{5}\sec^2(c+dx)\right)}{d}
\end{aligned}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(7/2), x]

[Out] (8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(9*a^2 + 7*a*b - 2*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (9*a^2 - 2*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*S

$\cos(dx+c)^2 + 2 \operatorname{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * a * b^2 * \cos(dx+c)^2 + 9 \operatorname{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * a^2 * b * \cos(dx+c)^4 + 9 * a^2 * b * \cos(dx+c)^3 * \sin(dx+c) + a * b^2 * \cos(dx+c)^3 * \sin(dx+c) - 9 \operatorname{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * a^3 * \cos(dx+c)^2 + 9 \operatorname{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * a^3 * \cos(dx+c)^4 - 2 \operatorname{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * b^3 * \cos(dx+c)^4 - 9 \operatorname{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * a^3 * \cos(dx+c)^4 + 18 \operatorname{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * a^3 * \cos(dx+c)^3 - 4 \operatorname{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * b^3 * \cos(dx+c)^3 + 9 * a^3 * \cos(dx+c)^2 * \sin(dx+c) / (1+\cos(dx+c)) / (a+\cos(dx+c)*b)^{1/2} / \cos(dx+c)^{5/2} / a^2$

Fricas [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{7/2}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos^2(dx + c)} dx$$

[In] integrate((a+b*cos(dx+c))^(1/2)/cos(dx+c)^(7/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(dx + c) + a)/cos(dx + c)^(7/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(dx+c))**(1/2)/cos(dx+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)

Giac [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos(c + dx)^{\frac{7}{2}}} dx$$

[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)

[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2), x)

$$3.609 \quad \int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	5895
Rubi [A] (verified)	5896
Mathematica [C] (verified)	5899
Maple [B] (verified)	5900
Fricas [F]	5901
Sympy [F(-1)]	5902
Maxima [F]	5902
Giac [F]	5902
Mupad [F(-1)]	5902

Optimal result

Integrand size = 25, antiderivative size = 389

$$\int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)} dx$$

$$= \frac{2(a-b)b\sqrt{a+b}(19a^2+8b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{105a^4d}$$

$$+ \frac{2(a-b)\sqrt{a+b}(25a^2+6ab+8b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{105a^3d}$$

$$+ \frac{2\sqrt{a+b \cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b \cos(c+dx)}\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{2(25a^2-4b^2)\sqrt{a+b \cos(c+dx)}\sin(c+dx)}{105a^2d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/7*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/35*b*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/105*(25*a^2-4*b^2)*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/105*(a-b)*b*(19*a^2+8*b^2)
*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2), (
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a^4/d+2/105*(a-b)*(25*a^2+6*a*b+8*b^2)*cot(d*x+c)*Ellip
ticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2), ((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a^3/d
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2875, 3134, 3077, 2895, 3073}

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx = \frac{2(25a^2 - 4b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{105a^2 d \cos^{\frac{3}{2}}(c + dx)} + \frac{2b(a - b) \sqrt{a + b} (19a^2 + 8b^2) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right)}{105a^4 d} + \frac{2(a - b) \sqrt{a + b} (25a^2 + 6ab + 8b^2) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{105a^3 d} + \frac{2b \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{35ad \cos^{\frac{5}{2}}(c + dx)} + \frac{2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{7d \cos^{\frac{7}{2}}(c + dx)}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(9/2), x]

[Out] (2*(a - b)*b*Sqrt[a + b]*(19*a^2 + 8*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2 + 6*a*b + 8*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(105*a^3*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*a*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2))

Rule 2875

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sine[e + f*x]]/Sqrt[d*Sine[e + f*x]]]/Rt[(a + b)/d, 2]

], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3134

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)])^2), x_Symbol] :> Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rubi steps

$$\text{integral} = \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{2}{7} \int \frac{\frac{b}{2} + \frac{5}{2}a \cos(c + dx) + 2b \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx$$

$$\begin{aligned}
&= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{4\int\frac{\frac{1}{4}(25a^2-4b^2)+\frac{23}{4}ab\cos(c+dx)+\frac{1}{2}b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{35a} \\
&= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{2(25a^2-4b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105a^2d\cos^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{8\int\frac{\frac{1}{8}b(19a^2+8b^2)+\frac{1}{8}a(25a^2+2b^2)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{105a^2} \\
&= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{2(25a^2-4b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105a^2d\cos^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{((a-b)(25a^2+6ab+8b^2))\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{105a^2} \\
&\quad + \frac{1}{105}\left(b\left(19+\frac{8b^2}{a^2}\right)\right)\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx \\
&= \frac{2(a-b)b\sqrt{a+b}(19a^2+8b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\mid-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{105a^4d} \\
&\quad + \frac{2(a-b)\sqrt{a+b}(25a^2+6ab+8b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{105a^3d} \\
&\quad + \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{2(25a^2-4b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105a^2d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 1304, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx$$

$$\frac{4a(25a^4 - 17a^2b^2 - 8b^4) \sqrt{\frac{(a+b) \cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}} \sqrt{-\frac{(a+b) \cos(c+dx) \csc^2\left(\frac{1}{2}(c+dx)\right)}{a}} \sqrt{\frac{(a+b \cos(c+dx)) \csc^2\left(\frac{1}{2}(c+dx)\right)}{a}} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}{(a+b) \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}\right)\right)}{(a+b) \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

$$+ \frac{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)} \left(\frac{2 \sec^2(c+dx) (25a^2 \sin(c+dx) - 4b^2 \sin(c+dx))}{105a^2} + \frac{2 \sec(c+dx) (19a^2b \sin(c+dx) + 8b^3 \sin(c+dx))}{105a^3} \right)}{d}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(9/2), x]

[Out] ((-4*a*(25*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-19*a^3*b - 8*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 2*(-19*a^2*b^2 - 8*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]])*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]

$$\begin{aligned} & * \sin\left[\frac{c + dx}{2}\right]^4 / \left((a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} \right) \\ & - \left(a \sqrt{\left(\frac{a + b}{2} \cot\left[\frac{c + dx}{2}\right]^2 \right) / (-a + b)} \sqrt{-\left((a + b) \cos[c + dx] \right) \operatorname{Csc}\left[\frac{c + dx}{2}\right]^2 / a} \right. \\ & \left. * \operatorname{Csc}[c + dx] * \operatorname{EllipticPi}\left[-\frac{a}{b}, \operatorname{ArcSin}\left[\sqrt{\left((a + b \cos[c + dx]) \operatorname{Csc}\left[\frac{c + dx}{2}\right]^2 / a\right)} \right] \right] \right. \\ & \left. + \frac{(-2a) / (-a + b) * \sin\left[\frac{c + dx}{2}\right]^4}{(b \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]})} \right) / b + \left(\sqrt{a + b \cos[c + dx]} \sin[c + dx] \right) / (b \sqrt{\cos[c + dx]}) \\ & \left. \right) / (105a^3d) + \left(\sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]} * \left((2 \operatorname{Sec}[c + dx]^2 * (25a^2 \sin[c + dx] - 4b^2 \sin[c + dx])) / (105a^2) \right. \right. \\ & \left. \left. + (2 \operatorname{Sec}[c + dx] * (19a^2 b \sin[c + dx] + 8b^3 \sin[c + dx])) / (105a^3) + (2b \operatorname{Sec}[c + dx]^2 \operatorname{Tan}[c + dx]) / (35a) + (2 \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]) / 7 \right) \right) / d \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(351) = 702$.

Time = 16.38 (sec) , antiderivative size = 2499, normalized size of antiderivative = 6.42

method	result	size
default	Expression too large to display	2499

[In] `int((a+cos(dx+c)*b)^(1/2)/cos(dx+c)^(9/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & 2/105/d * (15a^4 \sin(dx+c) - a^2 b^2 \cos(dx+c)^2 \sin(dx+c) + 8b^4 \cos(dx+c)^4 \sin(dx+c) \\ & + 25a^4 \cos(dx+c)^2 \sin(dx+c) + 19a^2 b^2 \cos(dx+c)^4 \sin(dx+c) + 8 \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-\frac{a-b}{a+b})^{1/2}) * \\ & ((a + \cos(dx+c) * b) / (1 + \cos(dx+c))) / (a+b)^{1/2} * (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * b^4 \cos(dx+c)^5 \\ & - 25 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-\frac{a-b}{a+b})^{1/2}) * ((a + \cos(dx+c) * b) / (1 + \cos(dx+c))) / (a+b)^{1/2} * \\ & (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * a^4 \cos(dx+c)^5 + 16 \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-\frac{a-b}{a+b})^{1/2}) * \\ & ((a + \cos(dx+c) * b) / (1 + \cos(dx+c))) / (a+b)^{1/2} * (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * b^4 \cos(dx+c)^4 \\ & - 50 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-\frac{a-b}{a+b})^{1/2}) * ((a + \cos(dx+c) * b) / (1 + \cos(dx+c))) / (a+b)^{1/2} * \\ & (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * a^4 \cos(dx+c)^4 + 8 \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-\frac{a-b}{a+b})^{1/2}) * \\ & ((a + \cos(dx+c) * b) / (1 + \cos(dx+c))) / (a+b)^{1/2} * (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * b^4 \cos(dx+c)^3 \\ & - 25 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-\frac{a-b}{a+b})^{1/2}) * ((a + \cos(dx+c) * b) / (1 + \cos(dx+c))) / (a+b)^{1/2} * \\ & (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * a^4 \cos(dx+c)^3 + 16 \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-\frac{a-b}{a+b})^{1/2}) * \\ & ((a + \cos(dx+c) * b) / (1 + \cos(dx+c))) / (a+b)^{1/2} * (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * a^3 b^3 \cos(dx+c)^4 \\ & - 38 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-\frac{a-b}{a+b})^{1/2}) * ((a + \cos(dx+c) * b) / (1 + \cos(dx+c))) / (a+b)^{1/2} * \\ & (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * a^3 b^3 \cos(dx+c)^4 - 4 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-\frac{a-b}{a+b})^{1/2}) * \\ & ((a + \cos(dx+c) * b) / (1 + \cos(dx+c))) / (a+b)^{1/2} * (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * a^2 b^2 \cos(dx+c)^4 \\ & - 16 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-\frac{a-b}{a+b})^{1/2}) * ((a + \cos(dx+c) * b) / (1 + \cos(dx+c))) / (a+b)^{1/2} * \\ & (\cos(dx+c) / (1 + \cos(dx+c)))^{1/2} * a^2 b^2 \cos(dx+c)^4 + 19 \operatorname{EllipticE}(\cot(dx+c) \end{aligned}$$

$-\csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^3 * b * \cos(dx+c)^3 + 19 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^2 * \cos(dx+c)^3 + 8 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a * b^3 * \cos(dx+c)^3 - 19 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^3 * b * \cos(dx+c)^3 - 2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^2 * \cos(dx+c)^3 - 8 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a * b^3 * \cos(dx+c)^3 + 19 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^3 * b * \cos(dx+c)^5 + 19 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^2 * \cos(dx+c)^5 + 8 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a * b^3 * \cos(dx+c)^5 - 19 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^3 * b * \cos(dx+c)^5 - 2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^2 * \cos(dx+c)^5 - 8 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a * b^3 * \cos(dx+c)^5 + 38 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^3 * b * \cos(dx+c)^4 + 38 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)) / (a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^2 * \cos(dx+c)^4 + 25 * a^4 * \cos(dx+c)^3 * \sin(dx+c) + 15 * a^4 * \cos(dx+c) * \sin(dx+c) + 18 * a^3 * b * \cos(dx+c) * \sin(dx+c) + 18 * a^3 * b * \cos(dx+c)^2 * \sin(dx+c) + 44 * a^3 * b * \cos(dx+c)^3 * \sin(dx+c) - a^2 * b^2 * \cos(dx+c)^3 * \sin(dx+c) + 4 * a * b^3 * \cos(dx+c)^3 * \sin(dx+c) + 25 * a^3 * b * \cos(dx+c)^4 * \sin(dx+c) - 4 * a * b^3 * \cos(dx+c)^4 * \sin(dx+c)) / (1+\cos(dx+c)) / (a+\cos(dx+c)*b)^{1/2} / \cos(dx+c)^{7/2} / a^3$

Fricas [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos^{\frac{9}{2}}(dx + c)} dx$$

[In] integrate((a+b*cos(dx+c))^(1/2)/cos(dx+c)^(9/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(dx + c) + a)/cos(dx + c)^(9/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos(dx + c)^{\frac{9}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2), x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)

Giac [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\cos(dx + c)^{\frac{9}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2), x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos(c + dx)^{9/2}} dx$$

[In] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(9/2), x)

[Out] int((a + b*cos(c + d*x))^(1/2)/cos(c + d*x)^(9/2), x)

3.610 $\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2} dx$

Optimal result	5903
Rubi [A] (verified)	5904
Mathematica [C] (warning: unable to verify)	5908
Maple [B] (verified)	5909
Fricas [F]	5911
Sympy [F(-1)]	5911
Maxima [F]	5911
Giac [F]	5911
Mupad [F(-1)]	5912

Optimal result

Integrand size = 25, antiderivative size = 508

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2} dx =$$

$$\frac{(a - b)\sqrt{a + b}(3a^2 + 16b^2) \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{24abd}$$

$$+ \frac{\sqrt{a + b}(a + 2b)(3a + 8b) \cot(c + dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{24bd}$$

$$+ \frac{a\sqrt{a + b}(a^2 - 12b^2) \cot(c + dx) \operatorname{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{8b^2d}$$

$$+ \frac{(3a^2 + 16b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{24bd \sqrt{\cos(c + dx)}}$$

$$+ \frac{a \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{4d}$$

$$+ \frac{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{3d}$$

```
[Out] 1/3*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+1/24*(3*a^2+16*b^2)
)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)+1/4*a*sin(d*x+c)*c
os(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-1/24*(a-b)*(3*a^2+16*b^2)*cot(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a
-b))^(1/2)/a/b/d+1/24*(a+2*b)*(3*a+8*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c
))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*
(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+1/8*a*(a^2-1
2*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2))
```

$(1/2), (a+b)/b, ((-a-b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*(a*(1-\sec(d*x+c)))/(a+b)^{(1/2)}*(a*(1+\sec(d*x+c)))/(a-b)^{(1/2)}/b^{2/d}$

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2900, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \cos^{3/2}(c+dx)(a+b\cos(c+dx))^{3/2} dx =$$

$$\frac{(a-b)\sqrt{a+b}(3a^2+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{24abd}$$

$$+\frac{a\sqrt{a+b}(a^2-12b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right),-\frac{a+b}{a-b}}{8b^2d}$$

$$+\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}$$

$$+\frac{\sqrt{a+b}(a+2b)(3a+8b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right),-\frac{a+b}{a-b}}{24bd}$$

$$+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}{3d}$$

$$+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{4d}$$

[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2), x]

[Out] $-1/24*((a-b)*\text{Sqrt}[a+b]*(3*a^2+16*b^2)*\text{Cot}[c+d*x]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Cos}[c+d*x]]]/(\text{Sqrt}[a+b]*\text{Sqrt}[\text{Cos}[c+d*x]])], -((a+b)/(a-b)))*\text{Sqrt}[(a*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[(a*(1+\text{Sec}[c+d*x]))/(a-b)])/ (a*b*d) + (\text{Sqrt}[a+b]*(a+2*b)*(3*a+8*b)*\text{Cot}[c+d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a+b*\text{Cos}[c+d*x]]]/(\text{Sqrt}[a+b]*\text{Sqrt}[\text{Cos}[c+d*x]])], -((a+b)/(a-b)))*\text{Sqrt}[(a*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[(a*(1+\text{Sec}[c+d*x]))/(a-b)])/ (24*b*d) + (a*\text{Sqrt}[a+b]*(a^2-12*b^2)*\text{Cot}[c+d*x]*\text{EllipticPi}[(a+b)/b, \text{ArcSin}[\text{Sqrt}[a+b*\text{Cos}[c+d*x]]]/(\text{Sqrt}[a+b]*\text{Sqrt}[\text{Cos}[c+d*x]])], -((a+b)/(a-b)))*\text{Sqrt}[(a*(1-\text{Sec}[c+d*x]))/(a+b)]*\text{Sqrt}[(a*(1+\text{Sec}[c+d*x]))/(a-b)])/ (8*b^2*d) + ((3*a^2+16*b^2)*\text{Sqrt}[a+b*\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/ (24*b*d*\text{Sqrt}[\text{Cos}[c+d*x]]) + (a*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[a+b*\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/ (4*d) + (\text{Sqrt}[\text{Cos}[c+d*x]]*(a+b*\text{Cos}[c+d*x])^(3/2)*\text{Sin}[c+d*x])/ (3*d)$

Rule 2888

Int[Sqrt[(b_)*sin[(e_)+(f_)*(x_)]]/Sqrt[(c_)+(d_)*sin[(e_)+(f_)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e+f*x]/(d*f))*Rt[(c+d)/b, 2]*Sqrt[c

```
*((1 + Csc[e + f*x])/(c - d))*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)])*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 2900

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ[m + n, 0] && (IntegerQ[m] || IntegerQ[2*m, 2*n])
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3128

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)])^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B))*(m + n + 2) - C*(a*
c - b*d*(m + n + 1))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

```

Rule 3132

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]
]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3140

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x])/(d*f*Sqrt[a + b*Sin[e + f*x]))], x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x])))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&+ \frac{\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{ab}{2}+2b^2\cos(c+dx)+\frac{3}{2}ab\cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}} dx}{3b}
\end{aligned}$$

$$\begin{aligned}
&= \frac{a\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d} \\
&+ \frac{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&+ \frac{\int \frac{\frac{7a^2b}{4} + \frac{13}{2}ab^2\cos(c+dx) + \frac{1}{4}b(3a^2+16b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx}{6b} \\
&= \frac{(3a^2+16b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}} \\
&+ \frac{a\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d} \\
&+ \frac{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&+ \frac{\int \frac{-\frac{1}{4}ab(3a^2+16b^2) + \frac{7}{2}a^2b^2\cos(c+dx) - \frac{3}{4}ab(a^2-12b^2)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{12b^2} \\
&= \frac{(3a^2+16b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}} \\
&+ \frac{a\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d} \\
&+ \frac{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&+ \frac{\int \frac{-\frac{1}{4}ab(3a^2+16b^2) + \frac{7}{2}a^2b^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{12b^2} - \frac{(a(a^2-12b^2)) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{16b} \\
&= \frac{a\sqrt{a+b}(a^2-12b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{8b^2d} \\
&+ \frac{(3a^2+16b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}} \\
&+ \frac{a\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d} \\
&+ \frac{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&+ \frac{(a(a+2b)(3a+8b)) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx}{48b} \\
&- \frac{(a(3a^2+16b^2)) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{48b}
\end{aligned}$$

$$\begin{aligned}
 &= \frac{(a-b)\sqrt{a+b}(3a^2+16b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{24abd} \\
 &+ \frac{\sqrt{a+b}(a+2b)(3a+8b)\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{24bd} \\
 &+ \frac{a\sqrt{a+b}(a^2-12b^2)\cot(c+dx)\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{8b^2d} \\
 &+ \frac{(3a^2+16b^2)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}} \\
 &+ \frac{a\sqrt{\cos(c+dx)}\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{4d} \\
 &+ \frac{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d}
 \end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.90 (sec) , antiderivative size = 1189, normalized size of antiderivative = 2.34

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2} dx =$$

$$\frac{4a(17a^2+16b^2)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b}\cos(c+dx)}$$

$$\begin{aligned}
 &+ dx)^{3/2} dx = \frac{\sqrt{\cos(c+dx)}\sqrt{a+b}\cos(c+dx)\left(\frac{7}{12}a\sin(c+dx)+\frac{1}{6}b\sin(2(c+dx))\right)}{d}
 \end{aligned}$$

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2), x]
```

```
[Out] ((-4*a*(17*a^2 + 16*b^2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 208*a^2*b*((Sqrt[((a
```


Fricas [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}} dx$$

[In] `integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral((b*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)`

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(3/2),x)`

[Out] Timed out

Maxima [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}} dx$$

[In] `integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)`

Giac [F]

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}} dx$$

[In] `integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")`

[Out] `integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2} dx = \int \cos(c + dx)^{3/2} (a + b \cos(c + dx))^{3/2} dx$$

```
[In] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.611 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2} dx$

Optimal result	5913
Rubi [A] (verified)	5914
Mathematica [A] (verified)	5917
Maple [B] (verified)	5918
Fricas [F]	5919
Sympy [F]	5919
Maxima [F]	5920
Giac [F]	5920
Mupad [F(-1)]	5920

Optimal result

Integrand size = 25, antiderivative size = 433

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2} dx =$$

$$\frac{5(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d}$$

$$+\frac{\sqrt{a+b}(5a+2b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d}$$

$$-\frac{\sqrt{a+b}(3a^2+4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4bd}$$

$$+\frac{3a\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}}+\frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}}$$

```
[Out] 1/2*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+3/4*a*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-5/4*(a-b)*cot(d*x+c)*EllipticE((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b
)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d+1/4
*(5*a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+
c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(3*a^2+4*b^2)*cot(d*x+c)*EllipticPi((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/b/d
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2900, 3126, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2} dx =$$

$$\frac{\sqrt{a+b}(3a^2+4b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{4bd}$$

$$+ \frac{\sqrt{a+b}(5a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{4d}$$

$$- \frac{5(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{4d}$$

$$+ \frac{\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}} + \frac{3a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sqrt{\cos(c+dx)}}$$

[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2), x]

[Out] (-5*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*d) + (Sqrt[a + b]*(5*a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*d) - (Sqrt[a + b]*(3*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*b*d) + (3*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + ((a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])

Rule 2888

Int[Sqrt[(b_)*sin[(e_.) + (f_)*(x_)]]/Sqrt[(c_.) + (d_)*sin[(e_.) + (f_)*(x_)]], x_Symbol] := Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_.) + (f_)*(x_)]]*Sqrt[(a_.) + (b_)*sin[(e_.) + (f_)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr

```
t[a*((1 - Csc[e + f*x])/(a + b))*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), In
t[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m
+ n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c -
b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f
*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && N
eQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
])], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
```

) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

Rule 3132

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3140

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{2d \sqrt{\cos(c + dx)}} + \frac{\int \frac{\sqrt{a+b \cos(c+dx)} \left(-\frac{ab}{2} + b^2 \cos(c+dx) + \frac{3}{2} ab \cos^2(c+dx)\right)}{\cos^{\frac{3}{2}}(c+dx)} dx}{2b} \\
 &= -\frac{a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{2d \sqrt{\cos(c + dx)}} + \frac{(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{2d \sqrt{\cos(c + dx)}} \\
 &\quad + \frac{\int \frac{\frac{ab^2}{4} + \frac{1}{2} b(2a^2 + b^2) \cos(c + dx) + \frac{5}{4} ab^2 \cos^2(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx}{b} \\
 &= \frac{3a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{4d \sqrt{\cos(c + dx)}} + \frac{(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{2d \sqrt{\cos(c + dx)}} \\
 &\quad + \frac{\int \frac{-\frac{5}{4} a^2 b^2 + \frac{1}{2} ab^3 \cos(c + dx) + \frac{1}{4} b^2 (3a^2 + 4b^2) \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{2b^2}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{3a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{\int \frac{-\frac{5}{4}a^2b^2 + \frac{1}{2}ab^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{2b^2} + \frac{1}{8}(3a^2+4b^2) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{\sqrt{a+b}(3a^2+4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4bd} \\
&\quad + \frac{3a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} \\
&\quad - \frac{1}{8}(5a^2) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&\quad + \frac{1}{8}(a(5a+2b)) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{5(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d} \\
&\quad + \frac{\sqrt{a+b}(5a+2b)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d} \\
&\quad - \frac{\sqrt{a+b}(3a^2+4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4bd} \\
&\quad + \frac{3a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.43 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.01

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2} dx = \frac{\sqrt{\cos(c+dx)}\left(4b(a+b\cos(c+dx))\sin(c+dx) + \frac{10a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{(a+b)(1+\cos(c+dx))}\right)}{4bd}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2), x]

[Out] (Sqrt[Cos[c + d*x]]*(4*b*(a + b*Cos[c + d*x])*Sin[c + d*x] + (10*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(4*a^2 - a*b + 2*b^2)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]

], $(-a + b)/(a + b) + 12a^2\sqrt{(a + b\cos[c + dx])}/((a + b)(1 + \cos[c + dx])) * \text{EllipticPi}[-1, \text{ArcSin}[\text{Tan}[(c + dx)/2]], (-a + b)/(a + b) + 16b^2\sqrt{(a + b\cos[c + dx])}/((a + b)(1 + \cos[c + dx])) * \text{EllipticPi}[-1, \text{ArcSin}[\text{Tan}[(c + dx)/2]], (-a + b)/(a + b) + 5a*b\sqrt{\cos[c + dx]/(1 + \cos[c + dx])} * \text{Sec}[(c + dx)/2] * \text{Sin}[(3*(c + dx))/2] + 10a^2\sqrt{\cos[c + dx]/(1 + \cos[c + dx])} * \text{Tan}[(c + dx)/2] - 5a*b\sqrt{\cos[c + dx]/(1 + \cos[c + dx])} * \text{Tan}[(c + dx)/2] / \sqrt{\cos[c + dx]/(1 + \cos[c + dx])}) / (8 * \sqrt{a + b\cos[c + dx]})$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1945 vs. $2(391) = 782$.

Time = 7.20 (sec) , antiderivative size = 1946, normalized size of antiderivative = 4.49

method	result	size
default	Expression too large to display	1946

[In] `int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{1}{4}d^* \left(-5 \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} \right) * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * a*b*\cos(d*x+c)^2 - 2 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * a*b*\cos(d*x+c)^2 - 10 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * a*b*\cos(d*x+c) - 4 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * a*b*\cos(d*x+c) + 7*\cos(d*x+c)^2*\sin(d*x+c)*a*b+2*a*b*\cos(d*x+c)*\sin(d*x+c) - 8 * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2} * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * b^2*\cos(d*x+c)^2 + 4 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * b^2*\cos(d*x+c)^2 - 10 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * a^2*\cos(d*x+c) - 12 * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2} * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * a^2*\cos(d*x+c) - 16 * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2} * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * b^2*\cos(d*x+c) + 8 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * b^2*\cos(d*x+c) - 5 * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * a*b - 2 * \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))/(a+b)} \right)^{1/2} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * \left(\frac{\cos(d*x+c)}{(1+\cos(d*x+c))} \right)^{1/2} * a*b + 2*b^2*\cos(d*x+c)^2*\sin(d*x+c) - 5 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)$

```

)/(a+b)^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2-6*EllipticPi(cot(d*x+c)-csc(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2+2*b^2*cos(d*x+c)^3*sin(d*x+c)+
5*a^2*cos(d*x+c)*sin(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x
+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2-5*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticE
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*a^2-6*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticPi(cot(d*x+
c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^
2-8*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticPi(cot(d*x+c)-csc
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^2+4*((
a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c)
,(-(a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^2+8*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*Ellipti
cF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)^2+16*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*Ell
ipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)/(1+cos(d
*x+c))/(a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)

```

Fricas [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2} dx = \int (b\cos(dx+c)+a)^{\frac{3}{2}} \sqrt{\cos(dx+c)} dx$$

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Sympy [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2} dx = \int (a+b\cos(c+dx))^{\frac{3}{2}} \sqrt{\cos(c+dx)} dx$$

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \sqrt{\cos(dx + c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^{3/2} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \sqrt{\cos(dx + c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^{3/2} dx = \int \sqrt{\cos(c + dx)}(a + b \cos(c + dx))^{3/2} dx$$

[In] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2),x)

[Out] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2), x)

$$3.612 \quad \int \frac{(a+b \cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	5921
Rubi [A] (verified)	5922
Mathematica [A] (verified)	5924
Maple [B] (verified)	5925
Fricas [F]	5926
Sympy [F]	5926
Maxima [F]	5926
Giac [F]	5927
Mupad [F(-1)]	5927

Optimal result

Integrand size = 25, antiderivative size = 375

$$\int \frac{(a+b \cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}} dx =$$

$$\frac{(a-b)b\sqrt{a+b} \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

$$+ \frac{\sqrt{a+b}(2a+b) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

$$- \frac{3a\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

$$+ \frac{b\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(a-b)*b*cot(d*x+c)*E
llipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/a/d+(2*a+b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-3*a*cot(d*x+c)*EllipticPi((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2)*(a
+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2900, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \frac{\sqrt{a + b}(2a + b) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b}}{\sqrt{a + b \cos(c + dx)}}\right)\right) + \frac{b(a - b)\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right) + \frac{3a\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{d} + \frac{b \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]

[Out] -(((a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d) + (Sqrt[a + b]*(2*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/d - (3*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/d + (b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 2888

Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

&& PosQ[(a + b)/d]

Rule 2900

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ[m + n, 0] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 3073

Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3132

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \int \frac{-\frac{ab}{2} + a^2\cos(c+dx) + \frac{3}{2}ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}(3ab) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx \\
&\quad + \int \frac{-\frac{ab}{2} + a^2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{3a\sqrt{a+b}\cot(c+dx)\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\
&\quad + \frac{b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}(ab) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&\quad + \frac{1}{2}(a(2a+b)) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad} \\
&\quad + \frac{\sqrt{a+b}(2a+b)\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\
&\quad - \frac{3a\sqrt{a+b}\cot(c+dx)\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\
&\quad + \frac{b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}} dx = \frac{\sqrt{\cos(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2b(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\middle|\frac{-a+b}{a+b}\right)}{d}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]], x]

[Out] -((Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]^2*(2*b*(a + b)*Sqrt[Cos[c + d*x]]/(1 + Cos[c + d*x]))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*E1


```

lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a - 2*b)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(a + b*cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))] * EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 12*a*b*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(a + b*cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))] * EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] + b*cos[c + d*x]*(a + b*cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
])/((d*Sqrt[a + b*cos[c + d*x]]*(-1 + Tan[(c + d*x)/2]^4)))

```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. 2(347) = 694.

Time = 9.36 (sec) , antiderivative size = 1363, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	1363

```
[In] int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*
x+c)^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(
a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*b^2*cos(d
*x+c)^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (-a-b)/(a+b))^(1/2))*a
*b*cos(d*x+c)^2-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos
(d*x+c)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))
*a^2*cos(d*x+c)^2+4*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*
(a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*a*b*cos(d*x+c)^2-2*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2)
)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*a*b*cos(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b)
)^(1/2))*b^2*cos(d*x+c)-12*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (-a-b)/
(a+b))^(1/2))*a*b*cos(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*
x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)
/(a+b))^(1/2))*a^2*cos(d*x+c)+8*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+
b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*a*b*cos(d*x+c)+b^2*cos(d*x+c)^2*sin(d*x+c)-((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b)
)^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b-(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-cs
c(d*x+c), (-a-b)/(a+b))^(1/2))*b^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+
cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),
-1, (-a-b)/(a+b))^(1/2))*a*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*
```

$x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)*\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^2+4*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)*\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)})*a*b+a*b*\cos(d*x+c)*\sin(d*x+c))/(1+\cos(d*x+c))/(a+\cos(d*x+c)*b)^{(1/2)}/\cos(d*x+c)^{(1/2)}$

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)

Sympy [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx$$

[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)

[Out] Integral((a + b*cos(c + d*x))**(3/2)/sqrt(cos(c + d*x)), x)

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\cos(c + dx)}} dx$$

[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)

[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2), x)

$$3.613 \quad \int \frac{(a+b \cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	5928
Rubi [A] (verified)	5929
Mathematica [A] (verified)	5931
Maple [B] (verified)	5931
Fricas [F]	5932
Sympy [F]	5932
Maxima [F]	5933
Giac [F]	5933
Mupad [F(-1)]	5933

Optimal result

Integrand size = 25, antiderivative size = 337

$$\int \frac{(a+b \cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{2(a-b)\sqrt{a+b} \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d} \\ - \frac{2(a-2b)\sqrt{a+b} \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\ - \frac{2b\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

```
[Out] 2*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*(a-2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*b*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2877, 2888, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{\frac{3}{2}}(c + dx)} dx =$$

$$\frac{2(a - 2b)\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{d}$$

$$+ \frac{2(a - b)\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{d}$$

$$- \frac{2b\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{d}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2), x]

[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/d - (2*(a - 2*b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/d - (2*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/d

Rule 2877

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2)/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2888

Int[Sqrt[(b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_)+(f_)*(x_)])*Sqrt[(a_)+(b_)*sin[(e_)+(f_)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e+f*x]/(a*f))*Rt[(a+b)/d, 2]*Sqrt[a*((1-Csc[e+f*x])/(a+b))]*Sqrt[a*((1+Csc[e+f*x])/(a-b))]*EllipticF[ArcSin[Sqrt[a+b*Sin[e+f*x]]/Sqrt[d*Sin[e+f*x]]]/Rt[(a+b)/d, 2], -(a+b)/(a-b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2-b^2, 0] && PosQ[(a+b)/d]
```

Rule 3073

```
Int[((A_)+(B_)*sin[(e_)+(f_)*(x_)])/(((b_)*sin[(e_)+(f_)*(x_)])^(3/2)*Sqrt[(c_)+(d_)*sin[(e_)+(f_)*(x_)]]), x_Symbol] := Simp[-2*A*(c-d)*(Tan[e+f*x]/(f*b*c^2))*Rt[(c+d)/b, 2]*Sqrt[c*((1+Csc[e+f*x])/(c-d))]*Sqrt[c*((1-Csc[e+f*x])/(c+d))]*EllipticE[ArcSin[Sqrt[c+d*Sin[e+f*x]]/Sqrt[b*Sin[e+f*x]]]/Rt[(c+d)/b, 2], -(c+d)/(c-d), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2-d^2, 0] && EqQ[A, B] && PosQ[(c+d)/b]
```

Rule 3077

```
Int[((A_)+(B_)*sin[(e_)+(f_)*(x_)])/(((a_)+(b_)*sin[(e_)+(f_)*(x_)])^(3/2)*Sqrt[(c_)+(d_)*sin[(e_)+(f_)*(x_)]]), x_Symbol] := Dist[(A-B)/(a-b), Int[1/(Sqrt[a+b*Sin[e+f*x]]*Sqrt[c+d*Sin[e+f*x]]), x], x] - Dist[(A*b-a*B)/(a-b), Int[(1+Sin[e+f*x])/((a+b*Sin[e+f*x])^(3/2)*Sqrt[c+d*Sin[e+f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c-a*d, 0] && NeQ[a^2-b^2, 0] && NeQ[c^2-d^2, 0] && NeQ[A, B]
```

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \frac{a + 2b \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx + b^2 \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}} dx \\ &= \frac{2b\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\ &\quad + a^2 \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\ &\quad - (a(a - 2b)) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \end{aligned}$$

$$= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\ - \frac{2(a-2b)\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\ - \frac{2b\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

Mathematica [A] (verified)

Time = 8.60 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{3/2}(c+dx)} dx = \frac{2a(a+b\cos(c+dx))\sin(c+dx) + \cos(c+dx) \left(-\frac{2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}} \right)}{\cos^{3/2}(c+dx)}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2), x]

[Out] (2*a*(a + b*Cos[c + d*x])*Sin[c + d*x] + Cos[c + d*x]*((-2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (2*(a^2 + 2*a*b - b^2)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (4*b^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - a*b*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] - 2*a^2*Tan[(c + d*x)/2] + a*b*Tan[(c + d*x)/2]))/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. 2(313) = 626.

Time = 10.74 (sec) , antiderivative size = 1016, normalized size of antiderivative = 3.01

method	result	size
default	Expression too large to display	1016

[In] int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] -2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a*b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a^2-2

```

*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc
(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*
((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b
))^1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+(-csc(d*
x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+
c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x
+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*
EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-2*(-csc(d*x+c)^2*
(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*
(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*b^2+csc(d*x+c)^3*a^2*(1-cos(d*x+c))^3-csc(d*x+c)^3*a*b*(1
-cos(d*x+c))^3+a^2*(csc(d*x+c)-cot(d*x+c))+a*b*(csc(d*x+c)-cot(d*x+c))*((c
sc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*
x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+
c)^2*b*(1-cos(d*x+c))^2+a+b)/(-csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c
)^2*(1-cos(d*x+c))^2+1))^(3/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)

```

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{3/2}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx$$

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3/2), x)
```


Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{3/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{3/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos(c + dx)^{3/2}} dx$$

[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)

[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2), x)

$$3.614 \quad \int \frac{(a+b \cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	5934
Rubi [A] (verified)	5934
Mathematica [A] (verified)	5937
Maple [B] (verified)	5937
Fricas [F]	5938
Sympy [F]	5938
Maxima [F]	5939
Giac [F]	5939
Mupad [F(-1)]	5939

Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{(a+b \cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{8(a-b)b\sqrt{a+b} \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3ad} + \frac{2(a-3b)(a-b)\sqrt{a+b} \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3ad} + \frac{2a\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/3*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+8/3*(a-b)*b*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+2/3*(a-3*b)*(a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b)^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

= {2878, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \frac{2(a - 3b)(a - b)\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right)}{3ad} + \frac{8b(a - b)\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right)}{3ad} + \frac{2a \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{3d \cos^{3/2}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2), x]

[Out] (8*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*d) + (2*(a - 3*b)*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] & NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])

)/(c - d)]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]))^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]], x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3} \int \frac{2ab + \frac{1}{2}(a^2 + 3b^2)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} \\
 &\quad + \frac{1}{3}((a-3b)(a-b)) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx \\
 &\quad + \frac{1}{3}(4ab) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{8(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3ad} \\
 &\quad + \frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3ad} \\
 &\quad + \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}
 \end{aligned}$$

$x+c))^{1/2} * b^2 * \cos(d*x+c)^2 - 8 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * a*b*\cos(d*x+c)^2 - 8 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * b^2 * \cos(d*x+c)^2 + (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * a^2 * \cos(d*x+c) + 4 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * a*b*\cos(d*x+c) + 3 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * b^2 * \cos(d*x+c) - 4 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * a*b*\cos(d*x+c) - 4 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * b^2 * \cos(d*x+c) - \cos(d*x+c)^2 * \sin(d*x+c) * a*b - 4 * b^2 * \cos(d*x+c)^2 * \sin(d*x+c) - a^2 * \cos(d*x+c) * \sin(d*x+c) - 5 * a*b * \cos(d*x+c) * \sin(d*x+c) - a^2 * \sin(d*x+c) / (1+\cos(d*x+c)) / (a+\cos(d*x+c)*b)^{1/2} / \cos(d*x+c)^{3/2}$

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos^{5/2}(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)

Sympy [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx$$

[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)

[Out] Integral((a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(5/2), x)

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{5/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{5/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos(c + dx)^{5/2}} dx$$

[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)

[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2), x)

$$3.615 \quad \int \frac{(a+b \cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	5940
Rubi [A] (verified)	5940
Mathematica [A] (verified)	5943
Maple [B] (verified)	5944
Fricas [F]	5945
Sympy [F(-1)]	5945
Maxima [F]	5945
Giac [F]	5946
Mupad [F(-1)]	5946

Optimal result

Integrand size = 25, antiderivative size = 325

$$\int \frac{(a+b \cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{2(a-b)\sqrt{a+b}(3a^2+b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{5a^2d}$$

$$- \frac{2(a-b)(3a-b)\sqrt{a+b} \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{5ad}$$

$$+ \frac{2a\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{4b\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5d \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/5*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+4/5*b*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/5*(a-b)*(3*a^2+b^2)*cot(d*x+c)
*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b)
)^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/a^2/d-2/5*(a-b)*(3*a-b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

= {2878, 3134, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \frac{2(a - b)\sqrt{a + b}(3a^2 + b^2) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{5a^2 d} - \frac{2(a - b)(3a - b)\sqrt{a + b} \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right), -\frac{a + b}{a - b}}{5ad} + \frac{4b \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{5d \cos^{3/2}(c + dx)} + \frac{2a \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{5d \cos^{5/2}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2), x]

[Out] (2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))] * Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)] * Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)] / (5*a^2*d) - (2*(a - b)*(3*a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))] * Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)] * Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)] / (5*a*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]) / (5*d*Cos[c + d*x]^(5/2)) + (4*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]) / (5*d*Cos[c + d*x]^(3/2))

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Sine[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))] * Sqrt[a*((1 + Csc[e + f*x])/(a - b))] * EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*A*

$(c - d) \cdot (\tan[e + f \cdot x] / (f \cdot b \cdot c^2)) \cdot \text{Rt}[(c + d)/b, 2] \cdot \text{Sqrt}[c \cdot ((1 + \text{Csc}[e + f \cdot x]) / (c - d))] \cdot \text{Sqrt}[c \cdot ((1 - \text{Csc}[e + f \cdot x]) / (c + d))] \cdot \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[c + d \cdot \text{Sin}[e + f \cdot x]]] / \text{Sqrt}[b \cdot \text{Sin}[e + f \cdot x]]] / \text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d), x] /;$ FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

$\text{Int}[\frac{(A \cdot \sin[e + f \cdot x] + B \cdot \sin[e + f \cdot x])}{(a + b \cdot \sin[e + f \cdot x])^{3/2} \cdot \text{Sqrt}[c + d \cdot \sin[e + f \cdot x]]}, x_Symbol] \rightarrow \text{Dist}[(A - B)/(a - b), \text{Int}[1/(\text{Sqrt}[a + b \cdot \text{Sin}[e + f \cdot x]] \cdot \text{Sqrt}[c + d \cdot \text{Sin}[e + f \cdot x]])], x] - \text{Dist}[(A \cdot b - a \cdot B)/(a - b), \text{Int}[(1 + \text{Sin}[e + f \cdot x]) / ((a + b \cdot \text{Sin}[e + f \cdot x])^{3/2} \cdot \text{Sqrt}[c + d \cdot \text{Sin}[e + f \cdot x]])], x] /;$ FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b \cdot c - a \cdot d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3134

$\text{Int}[\frac{(a \cdot \sin[e + f \cdot x] + b \cdot \sin[e + f \cdot x])^m \cdot (c + d \cdot \sin[e + f \cdot x] + (f \cdot \sin[e + f \cdot x])^n)}{(A \cdot \sin[e + f \cdot x] + B \cdot \sin[e + f \cdot x] + C \cdot \sin[e + f \cdot x] + (f \cdot \sin[e + f \cdot x])^2)}, x_Symbol] \rightarrow \text{Simp}[(-A \cdot b^2 - a \cdot b \cdot B + a^2 \cdot C) \cdot \text{Cos}[e + f \cdot x] \cdot (a + b \cdot \text{Sin}[e + f \cdot x])^{m+1} \cdot (c + d \cdot \text{Sin}[e + f \cdot x])^{n+1} / (f \cdot (m+1) \cdot (b \cdot c - a \cdot d) \cdot (a^2 - b^2)), x] + \text{Dist}[1 / ((m+1) \cdot (b \cdot c - a \cdot d) \cdot (a^2 - b^2)), \text{Int}[(a + b \cdot \text{Sin}[e + f \cdot x])^{m+1} \cdot (c + d \cdot \text{Sin}[e + f \cdot x])^n \cdot \text{Simp}[(m+1) \cdot (b \cdot c - a \cdot d) \cdot (a \cdot A - b \cdot B + a \cdot C) + d \cdot (A \cdot b^2 - a \cdot b \cdot B + a^2 \cdot C) \cdot (m+n+2) - (c \cdot (A \cdot b^2 - a \cdot b \cdot B + a^2 \cdot C) + (m+1) \cdot (b \cdot c - a \cdot d) \cdot (A \cdot b - a \cdot B + b \cdot C)) \cdot \text{Sin}[e + f \cdot x] - d \cdot (A \cdot b^2 - a \cdot b \cdot B + a^2 \cdot C) \cdot (m+n+3) \cdot \text{Sin}[e + f \cdot x]^2], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b \cdot c - a \cdot d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2 \cdot n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\ &+ \frac{2}{5} \int \frac{3ab + \frac{1}{2}(3a^2 + 5b^2) \cos(c + dx) + ab \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\ &= \frac{2a \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\ &+ \frac{4b \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d \cos^{\frac{3}{2}}(c + dx)} + \frac{4 \int \frac{\frac{3}{4}a(3a^2 + b^2) + 3a^2 b \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{15a} \end{aligned}$$

$$\begin{aligned}
&= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} \\
&\quad - \frac{1}{5}((a-b)(3a-b)) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx \\
&\quad + \frac{1}{5}(3a^2+b^2) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{2(a-b)\sqrt{a+b}(3a^2+b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{5a^2d} \\
&\quad - \frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{5ad} \\
&\quad + \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.81 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.23

$$\int \frac{(a+b\cos(c+dx))^{\frac{3}{2}}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{8\cos^2\left(\frac{1}{2}(c+dx)\right)^{\frac{7}{2}}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\left(-2(3a^3+3a^2b+ab^2+b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\right)}{\dots}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2), x]

[Out] ((8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1 + Cos[c + d*x])^(3/2) + (a + b*Cos[c + d*x])*(5*a^2 + b^2 + 4*a*b*Cos[c + d*x] + (3*a^2 + b^2)*Cos[2*(c + d*x)])*Tan[c + d*x]/(5*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2103 vs. $2(293) = 586$.

Time = 13.86 (sec) , antiderivative size = 2104, normalized size of antiderivative = 6.47

method	result	size
default	Expression too large to display	2104

[In] `int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -2/5/d*(-\text{EllipticE}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/ \\ & (1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*a*b^2*c \\ & \text{os}(d*x+c)^4+4*\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d* \\ & x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^ \\ & 2*b*\cos(d*x+c)^4+\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos \\ & (d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ & *a*b^2*\cos(d*x+c)^4-6*\text{EllipticE}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+b))^{1/2} \\ & *(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b) \\ & ^{1/2}*a^2*b*\cos(d*x+c)^3-2*\text{EllipticE}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+b))^{1/2} \\ & *(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a \\ & +b))^{1/2}*a*b^2*\cos(d*x+c)^3+8*\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+ \\ & b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c) \\ &))/(a+b))^{1/2}*a^2*b*\cos(d*x+c)^3+2*\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b) \\ &)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos \\ & (d*x+c)))/(a+b))^{1/2}*a*b^2*\cos(d*x+c)^3-3*\text{EllipticE}(\cot(d*x+c)-\text{csc}(d*x+c), \\ & (-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1 \\ & +\cos(d*x+c)))/(a+b))^{1/2}*a^2*b*\cos(d*x+c)^2+6*\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x \\ & +c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)* \\ & b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*a^3*\cos(d*x+c)^3-3*\text{EllipticE}(\cot(d*x+c)-\text{csc} \\ & (d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+ \\ & c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*a^3*\cos(d*x+c)^2-\text{EllipticE}(\cot(d*x+c)-\text{csc} \\ & (d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x \\ & +c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*b^3*\cos(d*x+c)^2-3*a*b^2*\cos(d*x+c)^2*\text{si} \\ & \text{n}(d*x+c)-3*a^2*b*\cos(d*x+c)*\text{sin}(d*x+c)-\text{sin}(d*x+c)*\cos(d*x+c)*a^3-3*\text{sin}(d*x+ \\ & c)*\cos(d*x+c)^2*a^2*b-a^3*\text{sin}(d*x+c)-b^3*\cos(d*x+c)^3*\text{sin}(d*x+c)-\text{EllipticE} \\ & (\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} \\ & *((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*a*b^2*\cos(d*x+c)^2+4*\text{Ellip} \\ & \text{ticF}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+c)) \\ &)^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*a^2*b*\cos(d*x+c)^2+\text{El} \\ & \text{lipticF}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+ \\ & c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*a*b^2*\cos(d*x+c)^2 \\ & -3*\text{EllipticE}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos \\ & (d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*a^2*b*\cos(d*x \\ & +c)^4-3*a^2*b*\cos(d*x+c)^3*\text{sin}(d*x+c)-2*a*b^2*\cos(d*x+c)^3*\text{sin}(d*x+c)+3*\text{Ell} \\ & \text{ipticF}(\cot(d*x+c)-\text{csc}(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+c) \end{aligned}$$

$$\left. \right)^{1/2} \cdot \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right) / (a+b)^{1/2} \cdot a^3 \cos(dx+c)^2 - 3 \cdot \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} \cdot \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} \cdot \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right) / (a+b)^{1/2} \cdot a^3 \cos(dx+c)^4 - \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} \cdot \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} \cdot \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right) / (a+b)^{1/2} \cdot b^3 \cos(dx+c)^4 + 3 \cdot \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} \cdot \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} \cdot \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right) / (a+b)^{1/2} \cdot a^3 \cos(dx+c)^4 - 6 \cdot \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} \cdot \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} \cdot \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right) / (a+b)^{1/2} \cdot a^3 \cos(dx+c)^3 - 2 \cdot \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} \cdot \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} \cdot \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right) / (a+b)^{1/2} \cdot b^3 \cos(dx+c)^3 - 3 \cdot a^3 \cos(dx+c)^2 \sin(dx+c) / (1 + \cos(dx+c)) / (a + \cos(dx+c)b)^{1/2} / \cos(dx+c)^{5/2} / a$$

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{7/2}} dx$$

[In] integrate((a+b*cos(dx+c))^(3/2)/cos(dx+c)^(7/2),x, algorithm="fricas")

[Out] integral((b*cos(dx + c) + a)^(3/2)/cos(dx + c)^(7/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(dx+c))**(3/2)/cos(dx+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{7/2}} dx$$

[In] integrate((a+b*cos(dx+c))^(3/2)/cos(dx+c)^(7/2),x, algorithm="maxima")

[Out] integrate((b*cos(dx + c) + a)^(3/2)/cos(dx + c)^(7/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{7/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{7/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos(c + dx)^{7/2}} dx$$

[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2),x)

[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2), x)

$$3.616 \quad \int \frac{(a+b \cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	5947
Rubi [A] (verified)	5948
Mathematica [C] (verified)	5951
Maple [B] (verified)	5952
Fricas [F]	5953
Sympy [F(-1)]	5954
Maxima [F]	5954
Giac [F]	5954
Mupad [F(-1)]	5954

Optimal result

Integrand size = 25, antiderivative size = 387

$$\int \frac{(a+b \cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{4(a-b)b\sqrt{a+b}(41a^2-3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{105a^3d}$$

$$+ \frac{2(a-b)\sqrt{a+b}(25a^2-57ab-6b^2)\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{105a^2d}$$

$$+ \frac{2a\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{2(25a^2+3b^2)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{105ad\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/7*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+16/35*b*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/105*(25*a^2+3*b^2)*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+4/105*(a-b)*b*(41*a^2-3*b^2)*
cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*
x+c))/(a-b))^(1/2)/a^3/d+2/105*(a-b)*(25*a^2-57*a*b-6*b^2)*cot(d*x+c)*Ellip
ticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/
2)/a^2/d
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00,
 number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used
 = {2878, 3134, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \frac{2(a - b)\sqrt{a + b}(25a^2 - 57ab - 6b^2) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}}}{105a^2 d}$$

$$+ \frac{2(25a^2 + 3b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{105ad \cos^{3/2}(c + dx)}$$

$$+ \frac{4b(a - b)\sqrt{a + b}(41a^2 - 3b^2) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right)}{105a^3 d}$$

$$+ \frac{16b \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{35d \cos^{5/2}(c + dx)} + \frac{2a \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{7d \cos^{7/2}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2), x]

[Out] (4*(a - b)*b*Sqrt[a + b]*(41*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^3*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2 - 57*a*b - 6*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (16*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*a*d*Cos[c + d*x]^(3/2))

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr


```
t[a*((1 - Csc[e + f*x])/(a + b))*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_)
+ (f_)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} \\
&+ \frac{2}{7} \int \frac{4ab + \frac{1}{2}(5a^2 + 7b^2)\cos(c+dx) + 2ab\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{4 \int \frac{\frac{1}{4}a(25a^2+3b^2)+11a^2b\cos(c+dx)+4ab^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{35a} \\
&= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{2(25a^2+3b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105ad\cos^{\frac{3}{2}}(c+dx)} \\
&+ \frac{8 \int \frac{\frac{1}{4}ab(41a^2-3b^2)+\frac{1}{8}a^2(25a^2+51b^2)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{105a^2} \\
&= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{2(25a^2+3b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105ad\cos^{\frac{3}{2}}(c+dx)} \\
&+ \frac{((a-b)(25a^2-57ab-6b^2)) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx}{105a} \\
&+ \frac{(2b(41a^2-3b^2)) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{105a} \\
&= \frac{4(a-b)b\sqrt{a+b}(41a^2-3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{105a^3d} \\
&+ \frac{2(a-b)\sqrt{a+b}(25a^2-57ab-6b^2)\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{105a^2d} \\
&+ \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{2(25a^2+3b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105ad\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 1302, normalized size of antiderivative = 3.36

$$4a(25a^4 - 31a^2b^2 + 6b^4) \sqrt{\frac{(a+b) \cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}} \sqrt{-\frac{(a+b) \cos(c+dx) \csc^2\left(\frac{1}{2}(c+dx)\right)}{a}} \sqrt{\frac{(a+b \cos(c+dx)) \csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}$$

$$\frac{\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \frac{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)} \left(\frac{2 \sec^2(c+dx)(25a^2 \sin(c+dx) + 3b^2 \sin(c+dx))}{105a} + \frac{4 \sec(c+dx)(41a^2 b \sin(c+dx) - 3b^3 \sin(c+dx))}{105a^2} \right)}{d}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]

[Out] ((-4*a*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-82*a^3*b + 6*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 2*(-82*a^2*b^2 + 6*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]])*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x))/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

$$\begin{aligned} & x] * \text{Csc}[(c + d*x)/2]^2/a] * \text{Sqrt}[(a + b*\text{Cos}[c + d*x]) * \text{Csc}[(c + d*x)/2]^2/a \\ &] * \text{Csc}[c + d*x] * \text{EllipticPi}[-(a/b), \text{ArcSin}[\text{Sqrt}[(a + b*\text{Cos}[c + d*x]) * \text{Csc}[(c \\ & + d*x)/2]^2/a] / \text{Sqrt}[2]], (-2*a)/(-a + b)] * \text{Sin}[(c + d*x)/2]^4 / (b * \text{Sqrt}[\text{Cos}[\\ & c + d*x]] * \text{Sqrt}[a + b*\text{Cos}[c + d*x]]) / b + (\text{Sqrt}[a + b*\text{Cos}[c + d*x]] * \text{Sin}[c + \\ & d*x]) / (b * \text{Sqrt}[\text{Cos}[c + d*x]])) / (105*a^2*d) + (\text{Sqrt}[\text{Cos}[c + d*x]] * \text{Sqrt}[a + \\ & b*\text{Cos}[c + d*x]] * ((2*\text{Sec}[c + d*x]^2 * (25*a^2*\text{Sin}[c + d*x] + 3*b^2*\text{Sin}[c + d*x \\ &])) / (105*a) + (4*\text{Sec}[c + d*x] * (41*a^2*b*\text{Sin}[c + d*x] - 3*b^3*\text{Sin}[c + d*x])) \\ & / (105*a^2) + (16*b*\text{Sec}[c + d*x]^2 * \text{Tan}[c + d*x]) / 35 + (2*a*\text{Sec}[c + d*x]^3 * \text{Tan} \\ & [c + d*x]) / 7)) / d \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(349) = 698$.

Time = 15.87 (sec) , antiderivative size = 2499, normalized size of antiderivative = 6.46

method	result	size
default	Expression too large to display	2499

[In] `int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & 2/105/d * (15*a^4*\text{sin}(d*x+c) + 27*a^2*b^2*\text{cos}(d*x+c)^2*\text{sin}(d*x+c) - 6*b^4*\text{cos}(d*x \\ & +c)^4*\text{sin}(d*x+c) + 25*a^4*\text{cos}(d*x+c)^2*\text{sin}(d*x+c) + 82*a^2*b^2*\text{cos}(d*x+c)^4*\text{sin} \\ & (d*x+c) - 6*\text{EllipticE}(\text{cot}(d*x+c) - \text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * ((a+\text{cos}(d*x \\ & +c)*b)/(1+\text{cos}(d*x+c)) / (a+b))^{1/2} * (\text{cos}(d*x+c)/(1+\text{cos}(d*x+c)))^{1/2} * b^4*\text{co} \\ & \text{s}(d*x+c)^5 - 25*\text{EllipticF}(\text{cot}(d*x+c) - \text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * ((a+\text{cos} \\ & (d*x+c)*b)/(1+\text{cos}(d*x+c)) / (a+b))^{1/2} * (\text{cos}(d*x+c)/(1+\text{cos}(d*x+c)))^{1/2} * a^ \\ & 4*\text{cos}(d*x+c)^5 - 12*\text{EllipticE}(\text{cot}(d*x+c) - \text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * ((a \\ & +\text{cos}(d*x+c)*b)/(1+\text{cos}(d*x+c)) / (a+b))^{1/2} * (\text{cos}(d*x+c)/(1+\text{cos}(d*x+c)))^{1/2} \\ &) * b^4*\text{cos}(d*x+c)^4 - 50*\text{EllipticF}(\text{cot}(d*x+c) - \text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) \\ & * ((a+\text{cos}(d*x+c)*b)/(1+\text{cos}(d*x+c)) / (a+b))^{1/2} * (\text{cos}(d*x+c)/(1+\text{cos}(d*x+c)))^{1/2} \\ &) * a^4*\text{cos}(d*x+c)^4 - 6*\text{EllipticE}(\text{cot}(d*x+c) - \text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * ((a+\text{cos}(d*x+c)*b)/(1+\text{cos}(d*x+c)) / (a+b))^{1/2} * (\text{cos}(d*x+c)/(1+\text{cos}(d*x+c)) \\ &))^{1/2} * b^4*\text{cos}(d*x+c)^3 - 25*\text{EllipticF}(\text{cot}(d*x+c) - \text{csc}(d*x+c), (-a-b)/(a+b)) \\ & ^{1/2}) * ((a+\text{cos}(d*x+c)*b)/(1+\text{cos}(d*x+c)) / (a+b))^{1/2} * (\text{cos}(d*x+c)/(1+\text{cos}(d* \\ & x+c)))^{1/2} * a^4*\text{cos}(d*x+c)^3 - 12*\text{EllipticE}(\text{cot}(d*x+c) - \text{csc}(d*x+c), (-a-b)/(a \\ & +b))^{1/2}) * ((a+\text{cos}(d*x+c)*b)/(1+\text{cos}(d*x+c)) / (a+b))^{1/2} * (\text{cos}(d*x+c)/(1+\text{co} \\ & s(d*x+c)))^{1/2} * a*b^3*\text{cos}(d*x+c)^4 - 164*\text{EllipticF}(\text{cot}(d*x+c) - \text{csc}(d*x+c), (- \\ & a-b)/(a+b))^{1/2}) * ((a+\text{cos}(d*x+c)*b)/(1+\text{cos}(d*x+c)) / (a+b))^{1/2} * (\text{cos}(d*x+c) \\ &) / (1+\text{cos}(d*x+c)))^{1/2} * a^3*b*\text{cos}(d*x+c)^4 - 102*\text{EllipticF}(\text{cot}(d*x+c) - \text{csc}(d*x \\ & +c), (-a-b)/(a+b))^{1/2}) * ((a+\text{cos}(d*x+c)*b)/(1+\text{cos}(d*x+c)) / (a+b))^{1/2} * (\text{co} \\ & s(d*x+c)/(1+\text{cos}(d*x+c)))^{1/2} * a^2*b^2*\text{cos}(d*x+c)^4 + 12*\text{EllipticF}(\text{cot}(d*x+c) \\ & - \text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * ((a+\text{cos}(d*x+c)*b)/(1+\text{cos}(d*x+c)) / (a+b))^{1/2} * (\text{cos}(d*x+c)/(1+\text{cos}(d*x+c)) \\ &))^{1/2} * a*b^3*\text{cos}(d*x+c)^4 + 82*\text{EllipticE}(\text{cot}(d*x+c) - \text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * ((a+\text{cos}(d*x+c)*b)/(1+\text{cos}(d*x+c)) / (a \\ & +b))^{1/2} * (\text{cos}(d*x+c)/(1+\text{cos}(d*x+c)))^{1/2} * a^3*b*\text{cos}(d*x+c)^3 + 82*\text{Elliptic} \end{aligned}$$

$E(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^2 * \cos(dx+c)^3 - 6 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a * b^3 * \cos(dx+c)^3 - 82 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^3 * b * \cos(dx+c)^3 - 51 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^2 * \cos(dx+c)^3 + 6 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a * b^3 * \cos(dx+c)^3 + 82 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^3 * b * \cos(dx+c)^5 + 82 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^2 * \cos(dx+c)^5 - 6 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a * b^3 * \cos(dx+c)^5 - 82 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^3 * b * \cos(dx+c)^5 - 51 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^2 * \cos(dx+c)^5 + 6 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a * b^3 * \cos(dx+c)^5 + 164 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^3 * b * \cos(dx+c)^4 + 164 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^2 * \cos(dx+c)^4 + 25 * a^4 * \cos(dx+c)^3 * \sin(dx+c) + 15 * a^4 * \cos(dx+c) * \sin(dx+c) + 39 * a^3 * b * \cos(dx+c) * \sin(dx+c) + 39 * a^3 * b * \cos(dx+c)^2 * \sin(dx+c) + 107 * a^3 * b * \cos(dx+c)^3 * \sin(dx+c) + 27 * a^2 * b^2 * \cos(dx+c)^3 * \sin(dx+c) - 3 * a * b^3 * \cos(dx+c)^3 * \sin(dx+c) + 25 * a^3 * b * \cos(dx+c)^4 * \sin(dx+c) + 3 * a * b^3 * \cos(dx+c)^4 * \sin(dx+c)) / (1 + \cos(dx+c)) / (a + \cos(dx+c) * b)^{1/2} / \cos(dx+c)^{7/2} / a^2$

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{\frac{3}{2}}}{\cos^{\frac{9}{2}}(dx + c)} dx$$

[In] integrate((a+b*cos(dx+c))^(3/2)/cos(dx+c)^(9/2),x, algorithm="fricas")

[Out] integral((b*cos(dx + c) + a)^(3/2)/cos(dx + c)^(9/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{9/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2), x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{9/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2), x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{9/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos(c + dx)^{9/2}} dx$$

[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2), x)

[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2), x)

$$3.617 \quad \int \frac{(a+b \cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)} dx$$

Optimal result	5955
Rubi [A] (verified)	5956
Mathematica [C] (verified)	5959
Maple [B] (verified)	5960
Fricas [F]	5962
Sympy [F(-1)]	5963
Maxima [F]	5963
Giac [F]	5963
Mupad [F(-1)]	5963

Optimal result

Integrand size = 25, antiderivative size = 454

$$\int \frac{(a+b \cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)} dx = \frac{2(a-b)\sqrt{a+b}(147a^4+33a^2b^2+8b^4)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{315a^4d} - \frac{2(a-b)\sqrt{a+b}(147a^3-39a^2b-6ab^2-8b^3)\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{a(1-\cos(c+dx))}}{315a^3d} + \frac{2a\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2(49a^2+3b^2)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{315ad\cos^{\frac{5}{2}}(c+dx)} + \frac{8b(22a^2-b^2)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{315a^2d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/9*a*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+20/63*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/315*(49*a^2+3*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+8/315*b*(22*a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/315*(a-b)*(147*a^4+33*a^2*b^2+8*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d-2/315*(a-b)*(147*a^3-39*a^2*b-6*a*b^2-8*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2878, 3134, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{11/2}(c + dx)} dx = \frac{8b(22a^2 - b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{315a^2 d \cos^{3/2}(c + dx)} + \frac{2(49a^2 + 3b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{315ad \cos^{5/2}(c + dx)} + \frac{2(a - b) \sqrt{a + b} (147a^4 + 33a^2b^2 + 8b^4) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{315a^4 d} - \frac{2(a - b) \sqrt{a + b} (147a^3 - 39a^2b - 6ab^2 - 8b^3) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{315a^3 d} + \frac{20b \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{63d \cos^{7/2}(c + dx)} + \frac{2a \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{9d \cos^{9/2}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2), x]

[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 33*a^2*b^2 + 8*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(315*a^4*d) - (2*(a - b)*Sqrt[a + b]*(147*a^3 - 39*a^2*b - 6*a*b^2 - 8*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(315*a^3*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (20*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(5/2)) + (8*b*(22*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Cos[c + d*x]^(3/2))

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} \\
 &+ \frac{2}{9} \int \frac{5ab + \frac{1}{2}(7a^2 + 9b^2)\cos(c+dx) + 3ab\cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} \\
 &+ \frac{4 \int \frac{\frac{1}{4}a(49a^2+3b^2)+23a^2b\cos(c+dx)+10ab^2\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{63a} \\
 &= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} \\
 &+ \frac{2(49a^2 + 3b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315ad\cos^{\frac{5}{2}}(c+dx)} \\
 &+ \frac{8 \int \frac{\frac{3}{2}ab(22a^2-b^2)+\frac{1}{8}a^2(147a^2+209b^2)\cos(c+dx)+\frac{1}{4}ab(49a^2+3b^2)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{315a^2} \\
 &= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} \\
 &+ \frac{2(49a^2 + 3b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315ad\cos^{\frac{5}{2}}(c+dx)} \\
 &+ \frac{8b(22a^2 - b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315a^2d\cos^{\frac{3}{2}}(c+dx)} \\
 &+ \frac{16 \int \frac{\frac{3}{16}a(147a^4+33a^2b^2+8b^4)+\frac{3}{8}a^2b(93a^2+b^2)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{945a^3} \\
 &= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} \\
 &+ \frac{2(49a^2 + 3b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315ad\cos^{\frac{5}{2}}(c+dx)} \\
 &+ \frac{8b(22a^2 - b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315a^2d\cos^{\frac{3}{2}}(c+dx)} \\
 &- \frac{((a-b)(147a^3 - 39a^2b - 6ab^2 - 8b^3)) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx}{315a^2} \\
 &+ \frac{(147a^4 + 33a^2b^2 + 8b^4) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{315a^2}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2(a-b)\sqrt{a+b}(147a^4 + 33a^2b^2 + 8b^4) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{315a^4d} \\
&- \frac{2(a-b)\sqrt{a+b}(147a^3 - 39a^2b - 6ab^2 - 8b^3) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{315a^3d} \\
&+ \frac{2a\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} \\
&+ \frac{2(49a^2 + 3b^2)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{315ad\cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{8b(22a^2 - b^2)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{315a^2d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.43 (sec) , antiderivative size = 1368, normalized size of antiderivative = 3.01

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)} dx =$$

$$\begin{aligned}
&\frac{4a(-39a^4b+31a^2b^3+8b^5)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec^3(c+dx)(49a^2\sin(c+dx)+3b^2\sin(c+dx))}{315a} + \frac{8\sec^2(c+dx)(22a^2b\sin(c+dx)-b^3\sin(c+dx))}{315a^2}\right)}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2), x]

[Out] -1/315*((-4*a*(-39*a^4*b + 31*a^2*b^3 + 8*b^5)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5 + 33*a^3*b^2 + 8*a*b^4)*((Sqrt[(a + b)*Cot[(c + d*x)/2]^2)/

```
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*b + 33*a^2*b^3 + 8*b^5)*((I*Cos[(
c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/S
qrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*
((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^
2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*
Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]))/b + (
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(a^3*d) + (
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(49*a^2*Ssin[
c + d*x] + 3*b^2*Ssin[c + d*x]))/(315*a) + (8*Sec[c + d*x]^2*(22*a^2*b*Ssin[
c + d*x] - b^3*Ssin[c + d*x]))/(315*a^2) + (2*Sec[c + d*x]*(147*a^4*Ssin[c + d
*x] + 33*a^2*b^2*Ssin[c + d*x] + 8*b^4*Ssin[c + d*x]))/(315*a^3) + (20*b*Sec[
c + d*x]^3*Tan[c + d*x])/63 + (2*a*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3453 vs. $2(410) = 820$.

Time = 19.00 (sec) , antiderivative size = 3454, normalized size of antiderivative = 7.61

method	result	size
default	Expression too large to display	3454

```
[In] int((a+cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/d*(4*a*b^4*cos(d*x+c)^4*sin(d*x+c)+35*a^5*sin(d*x+c)-2*EllipticF(cot(
d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^3*cos(d*x+c)^4-8*Ellipti
cF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^4*cos(d*x+c)^4+147*
EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b*cos(d*x+c)
```

$$\begin{aligned}
& ^6+33*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+ \\
& \cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^3*b^2*\cos \\
& (d*x+c)^6+33*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d* \\
& x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^ \\
& 2*b^3*\cos(d*x+c)^6+53*a^3*b^2*\cos(d*x+c)^3*\sin(d*x+c)-a^2*b^3*\cos(d*x+c)^3* \\
& \sin(d*x+c)+147*a^4*b*\cos(d*x+c)^5*\sin(d*x+c)+88*a^3*b^2*\cos(d*x+c)^5*\sin(d* \\
& x+c)+33*a^2*b^3*\cos(d*x+c)^5*\sin(d*x+c)-4*a*b^4*\cos(d*x+c)^5*\sin(d*x+c)+85* \\
& a^4*b*\cos(d*x+c)*\sin(d*x+c)+85*a^4*b*\cos(d*x+c)^2*\sin(d*x+c)+53*a^3*b^2*\cos \\
& (d*x+c)^2*\sin(d*x+c)+137*a^4*b*\cos(d*x+c)^4*\sin(d*x+c)+121*a^3*b^2*\cos(d*x+ \\
& c)^4*\sin(d*x+c)-a^2*b^3*\cos(d*x+c)^4*\sin(d*x+c)+137*a^4*b*\cos(d*x+c)^3*\sin(\\
& d*x+c)+147*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c) \\
&)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^5*c \\
& \cos(d*x+c)^6+8*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d* \\
& x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*b^ \\
& 5*\cos(d*x+c)^6-147*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(c \\
& \cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/ \\
& 2)}*a^5*\cos(d*x+c)^6+294*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2 \\
&)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b) \\
&)^{(1/2)}*a^5*\cos(d*x+c)^5+16*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(\\
& 1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(\\
& a+b))^{(1/2)}*b^5*\cos(d*x+c)^5-294*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a \\
& +b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+ \\
& c))/(a+b))^{(1/2)}*a^5*\cos(d*x+c)^5+147*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a- \\
& b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos \\
& (d*x+c))/(a+b))^{(1/2)}*a^5*\cos(d*x+c)^4+8*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (- \\
& a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+ \\
& \cos(d*x+c))/(a+b))^{(1/2)}*b^5*\cos(d*x+c)^4-147*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+ \\
& c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b \\
&)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^5*\cos(d*x+c)^4+147*a^5*\cos(d*x+c)^4*\sin(d*x \\
& +c)+8*b^5*\cos(d*x+c)^5*\sin(d*x+c)+35*a^5*\cos(d*x+c)*\sin(d*x+c)+49*a^5*\cos(d \\
& *x+c)^2*\sin(d*x+c)+49*a^5*\cos(d*x+c)^3*\sin(d*x+c)+8*\text{EllipticE}(\cot(d*x+c)-\cs \\
& c(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d* \\
& x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a*b^4*\cos(d*x+c)^6-186*\text{EllipticF}(\cot(d* \\
& x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a \\
& +\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^4*b*\cos(d*x+c)^6-33*\text{EllipticF} \\
& (\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/ \\
& 2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^3*b^2*\cos(d*x+c)^6-2*\text{Ell \\
& ipsisF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c) \\
&)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^2*b^3*\cos(d*x+c)^ \\
& 6-8*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+co \\
& s(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a*b^4*\cos(d* \\
& x+c)^6+294*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c) \\
&)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^4*b \\
& *\cos(d*x+c)^5+66*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos \\
& (d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}
\end{aligned}$$

```

*a^3*b^2*cos(d*x+c)^5+66*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^2*b^3*cos(d*x+c)^5+16*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a*b^4*cos(d*x+c)^5-372*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^4*b*cos(d*x+c)^5-66*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^3*b^2*cos(d*x+c)^5-4*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^2*b^3*cos(d*x+c)^5-16*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a*b^4*cos(d*x+c)^5+147*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^4*b*cos(d*x+c)^4+33*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^3*b^2*cos(d*x+c)^4+33*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^2*b^3*cos(d*x+c)^4+8*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a*b^4*cos(d*x+c)^4-186*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^4*b*cos(d*x+c)^4-33*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^3*b^2*cos(d*x+c)^4)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(9/2)/a^3

```

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{11/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos^{11/2}(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")

[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{11/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(11/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{11/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{11/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2), x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{11/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\cos(dx + c)^{11/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2), x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{11/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos(c + dx)^{11/2}} dx$$

[In] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(11/2), x)

[Out] int((a + b*cos(c + d*x))^(3/2)/cos(c + d*x)^(11/2), x)

3.618 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2} dx$

Optimal result	5964
Rubi [A] (verified)	5965
Mathematica [C] (warning: unable to verify)	5969
Maple [B] (verified)	5970
Fricas [F]	5972
Sympy [F(-1)]	5972
Maxima [F]	5972
Giac [F]	5973
Mupad [F(-1)]	5973

Optimal result

Integrand size = 25, antiderivative size = 506

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2} dx =$$

$$\frac{(a-b)\sqrt{a+b}(33a^2+16b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{24ad}$$

$$+ \frac{\sqrt{a+b}(33a^2+26ab+16b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{24d}$$

$$- \frac{5a\sqrt{a+b}(a^2+4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{8bd}$$

$$+ \frac{(33a^2+16b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}}$$

$$+ \frac{13ab\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d}$$

$$+ \frac{b^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}$$

[Out] $\frac{1}{3}b^2\cos(dx+c)^{3/2}\sin(dx+c)(a+b\cos(dx+c))^{1/2}/d+1/24*(33a^2+16b^2)\sin(dx+c)(a+b\cos(dx+c))^{1/2}/d/\cos(dx+c)^{1/2}+13/12*a*b*\sin(dx+c)*\cos(dx+c)^{1/2}*(a+b\cos(dx+c))^{1/2}/d-1/24*(a-b)*(33a^2+16b^2)*\cot(dx+c)*\operatorname{EllipticE}((a+b\cos(dx+c))^{1/2}/(a+b)^{1/2}/\cos(dx+c)^{1/2},((-a-b)/(a-b))^{1/2})*(a+b)^{1/2}*(a*(1-\sec(dx+c)))/(a+b)^{1/2}*(a*(1+\sec(dx+c)))/(a-b)^{1/2}/a/d+1/24*(33a^2+26ab+16b^2)*\cot(dx+c)*\operatorname{EllipticF}((a+b\cos(dx+c))^{1/2}/(a+b)^{1/2}/\cos(dx+c)^{1/2},((-a-b)/(a-b))^{1/2})*(a+b)^{1/2}*(a*(1-\sec(dx+c)))/(a+b)^{1/2}*(a*(1+\sec(dx+c)))/(a-b)^{1/2}/d-5/8*a*(a^2+4b^2)*\cot(dx+c)*\operatorname{EllipticPi}((a+b\cos(dx+c))^{1/2}/(a+b)^{1/2}/\cos$

$$(d*x+c)^{(1/2)}, (a+b)/b, ((-a-b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*(a*(1-\sec(d*x+c)))/(a+b))^{(1/2)}*(a*(1+\sec(d*x+c)))/(a-b))^{(1/2)}/b/d$$

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2872, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2} dx = \frac{\sqrt{a+b}(33a^2+26ab+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{24d} - \frac{(a-b)\sqrt{a+b}(33a^2+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)\left|-\frac{a+b}{a-b}\right.)}{24ad} - \frac{5a\sqrt{a+b}(a^2+4b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{8bd} + \frac{(33a^2+16b^2)\sin(c+dx)\sqrt{a+b}\cos(c+dx)}{24d\sqrt{\cos(c+dx)}} + \frac{b^2\sin(c+dx)\cos^3(c+dx)\sqrt{a+b}\cos(c+dx)}{3d} + \frac{13ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b}\cos(c+dx)}{12d}$$

[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2), x]

[Out] -1/24*((a - b)*Sqrt[a + b]*(33*a^2 + 16*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d) + (Sqrt[a + b]*(33*a^2 + 26*a*b + 16*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(24*d) - (5*a*Sqrt[a + b]*(a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(8*b*d) + ((33*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (13*a*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (b^2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 2872

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x

```
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d),
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

&& NeQ[A, B]

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

integral

$$\begin{aligned}
 &= \frac{b^2 \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d} \\
 &+ \frac{1}{3} \int \frac{\sqrt{\cos(c + dx)} \left(\frac{3}{2} a (2a^2 + b^2) + b(9a^2 + 2b^2) \cos(c + dx) + \frac{13}{2} ab^2 \cos^2(c + dx) \right)}{\sqrt{a + b \cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{13ab\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d} \\
&+ \frac{b^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} \\
&+ \frac{\int \frac{\frac{13a^2b^2}{4} + \frac{1}{2}ab(12a^2+19b^2)\cos(c+dx) + \frac{1}{4}b^2(33a^2+16b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx}{6b} \\
&= \frac{(33a^2+16b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} \\
&+ \frac{13ab\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d} \\
&+ \frac{b^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} \\
&+ \frac{\int \frac{-\frac{1}{4}ab^2(33a^2+16b^2) + \frac{13}{2}a^2b^3\cos(c+dx) + \frac{15}{4}ab^2(a^2+4b^2)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{12b^2} \\
&= \frac{(33a^2+16b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} \\
&+ \frac{13ab\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d} \\
&+ \frac{b^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} \\
&+ \frac{\int \frac{-\frac{1}{4}ab^2(33a^2+16b^2) + \frac{13}{2}a^2b^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{12b^2} + \frac{1}{16}(5a(a^2+4b^2)) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{5a\sqrt{a+b}(a^2+4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{8bd} \\
&+ \frac{(33a^2+16b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} \\
&+ \frac{13ab\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d} \\
&+ \frac{b^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} \\
&- \frac{1}{48}(a(33a^2+16b^2)) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&+ \frac{1}{48}(a(33a^2+26ab+16b^2)) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx
\end{aligned}$$

=

$$\begin{aligned}
& - \frac{(a-b)\sqrt{a+b}(33a^2+16b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{24ad} \\
& + \frac{\sqrt{a+b}(33a^2+26ab+16b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{24d} \\
& - \frac{5a\sqrt{a+b}(a^2+4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{8bd} \\
& + \frac{(33a^2+16b^2)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} \\
& + \frac{13ab\sqrt{\cos(c+dx)}\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{12d} \\
& + \frac{b^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.79 (sec) , antiderivative size = 1203, normalized size of antiderivative = 2.38

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c$$

$$\frac{4a(59a^2b+16b^3)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)\operatorname{EllipticF}}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$\begin{aligned}
& + dx)^{5/2} dx = \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{13}{12}ab\sin(c+dx)+\frac{1}{6}b^2\sin(2(c+dx))\right)}{d} \\
& + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{13}{12}ab\sin(c+dx)+\frac{1}{6}b^2\sin(2(c+dx))\right)}{d}
\end{aligned}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2),x]

[Out] ((-4*a*(59*a^2*b + 16*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(48*a^3 + 76*a

$$\begin{aligned}
& *b^2*((\text{Sqrt}[(a+b)\text{Cot}[(c+dx)/2]^2]/(-a+b))\text{Sqrt}[-((a+b)\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a])\text{Sqrt}[(a+b\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a)\text{Csc}[c+dx]\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(a+b\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a]/\text{Sqrt}[2]], (-2a)/(-a+b)]\text{Sin}[(c+dx)/2]^4/((a+b)\text{Sqrt}[\text{Cos}[c+dx]])\text{Sqrt}[a+b\text{Cos}[c+dx]]) - (\text{Sqrt}[(a+b)\text{Cot}[(c+dx)/2]^2]/(-a+b))\text{Sqrt}[-((a+b)\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a])\text{Sqrt}[(a+b\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a)\text{Csc}[c+dx]\text{EllipticPi}[-(a/b), \text{ArcSin}[\text{Sqrt}[(a+b\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a]/\text{Sqrt}[2]], (-2a)/(-a+b)]\text{Sin}[(c+dx)/2]^4/(b\text{Sqrt}[\text{Cos}[c+dx]])\text{Sqrt}[a+b\text{Cos}[c+dx]]) + 2*(33a^2b + 16b^3)*((I\text{Cos}[(c+dx)/2]\text{Sqrt}[a+b\text{Cos}[c+dx]])\text{EllipticE}[I\text{ArcSinh}[\text{Sin}[(c+dx)/2]/\text{Sqrt}[\text{Cos}[c+dx]]], (-2a)/(-a-b)]\text{Sec}[c+dx])/(b\text{Sqrt}[\text{Cos}[(c+dx)/2]^2\text{Sec}[c+dx]])\text{Sqrt}[(a+b\text{Cos}[c+dx])\text{Sec}[c+dx])/(a+b)) + (2a*((a\text{Sqrt}[(a+b)\text{Cot}[(c+dx)/2]^2]/(-a+b))\text{Sqrt}[-((a+b)\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a])\text{Sqrt}[(a+b\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a)\text{Csc}[c+dx]\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(a+b\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a]/\text{Sqrt}[2]], (-2a)/(-a+b)]\text{Sin}[(c+dx)/2]^4/((a+b)\text{Sqrt}[\text{Cos}[c+dx]])\text{Sqrt}[a+b\text{Cos}[c+dx]]) - (a\text{Sqrt}[(a+b)\text{Cot}[(c+dx)/2]^2]/(-a+b))\text{Sqrt}[-((a+b)\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a])\text{Sqrt}[(a+b\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a)\text{Csc}[c+dx]\text{EllipticPi}[-(a/b), \text{ArcSin}[\text{Sqrt}[(a+b\text{Cos}[c+dx])\text{Csc}[(c+dx)/2]^2/a]/\text{Sqrt}[2]], (-2a)/(-a+b)]\text{Sin}[(c+dx)/2]^4/(b\text{Sqrt}[\text{Cos}[c+dx]])\text{Sqrt}[a+b\text{Cos}[c+dx]])/b + (\text{Sqrt}[a+b\text{Cos}[c+dx]]\text{Sin}[c+dx])/(b\text{Sqrt}[\text{Cos}[c+dx]])))/(48*d) + (\text{Sqrt}[\text{Cos}[c+dx]]\text{Sqrt}[a+b\text{Cos}[c+dx]])*((13a*b*\text{Sin}[c+dx])/12 + (b^2*\text{Sin}[2*(c+dx)]/6))/d
\end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2567 vs. $2(458) = 916$.

Time = 8.33 (sec) , antiderivative size = 2568, normalized size of antiderivative = 5.08

method	result	size
default	Expression too large to display	2568

[In] `int(cos(dx+c)^(1/2)*(a+cos(dx+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $1/24/d*(-33\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+c), -(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*a^2*b*\cos(dx+c)^2-33\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+c), -(a-b)/(a+b))^{1/2}*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*a^3*\cos(dx+c)^2-16\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+c), -(a-b)/(a+b))^{1/2}*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*b^3*\cos(dx+c)^2+34*a*b^2*\cos(dx+c)^2*\sin(dx+c)+26*a^2*b*\cos(dx+c)*\sin(dx+c)+16*\sin(dx+c)*\cos(dx+c)^2*b^3+33*\sin(dx+c)*\cos(dx+c)*a^3+59*\sin(dx+c)*\cos(dx+c)^2*a^2*b+16*\sin(dx+c)*\cos(dx+c)*a*b^2+8*b^3*\cos(dx+c)^3*\sin(dx+c)-16\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+c), -(a-b)/(a+b))^{1/2}*(c$

$\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*a^3*\cos(dx+c)^2+34*a*b^2*\cos(dx+c)^3*\sin(dx+c)+48*EllipticF(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*a^3*\cos(dx+c)^2/(1+\cos(dx+c))/(a+\cos(dx+c)*b)^{1/2}/\cos(dx+c)^{1/2}$

Fricas [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2} dx = \int (b\cos(dx+c)+a)^{5/2} \sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(dx+c)^(1/2)*(a+b*cos(dx+c))^(5/2),x, algorithm="fricas")

[Out] integral((b^2*cos(dx+c)^2 + 2*a*b*cos(dx+c) + a^2)*sqrt(b*cos(dx+c) + a)*sqrt(cos(dx+c)), x)

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2} dx = \text{Timed out}$$

[In] integrate(cos(dx+c)**(1/2)*(a+b*cos(dx+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2} dx = \int (b\cos(dx+c)+a)^{5/2} \sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(dx+c)^(1/2)*(a+b*cos(dx+c))^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(dx+c) + a)^(5/2)*sqrt(cos(dx+c)), x)

Giac [F]

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2} dx = \int (b\cos(dx+c)+a)^{5/2} \sqrt{\cos(dx+c)} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2} dx = \int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2} dx$$

[In] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2),x)

[Out] int(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2), x)

$$3.619 \quad \int \frac{(a+b \cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	5974
Rubi [A] (verified)	5975
Mathematica [A] (verified)	5978
Maple [B] (verified)	5978
Fricas [F]	5980
Sympy [F(-1)]	5980
Maxima [F]	5980
Giac [F]	5981
Mupad [F(-1)]	5981

Optimal result

Integrand size = 25, antiderivative size = 443

$$\int \frac{(a+b \cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}} dx =$$

$$\frac{9(a-b)b\sqrt{a+b} \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d}$$

$$+ \frac{\sqrt{a+b}(8a^2+9ab+2b^2) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d}$$

$$+ \frac{\sqrt{a+b}(15a^2+4b^2) \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d}$$

$$+ \frac{9ab\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{\cos(c+dx)}\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{2d}$$

```
[Out] 9/4*a*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2)/d-9/4*(a-b)*b*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d+1/4*(8*a^2+9*a*b+2*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(15*a^2+4*b^2)*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2872, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \frac{\sqrt{a+b}(8a^2 + 9ab + 2b^2) \cot(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a}{a-b}\right)}{4d} - \frac{\sqrt{a+b}(15a^2 + 4b^2) \cot(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a}{a-b}\right)}{4d} - \frac{9b(a-b)\sqrt{a+b} \cot(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{4d} + \frac{b^2 \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}}{2d} + \frac{9ab \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{4d \sqrt{\cos(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]], x]

[Out] (-9*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*d) + (Sqrt[a + b]*(8*a^2 + 9*a*b + 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*d) - (Sqrt[a + b]*(15*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*d) + (9*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (b^2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 2872

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]
]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]))]*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{2d} \\
&+ \frac{1}{2} \int \frac{\frac{1}{2} a(4a^2+b^2) + b(6a^2+b^2) \cos(c+dx) + \frac{9}{2} ab^2 \cos^2(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx \\
&= \frac{9ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{4d \sqrt{\cos(c+dx)}} \\
&+ \frac{b^2 \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{2d} \\
&+ \frac{\int \frac{-\frac{9}{2} a^2 b^2 + ab(4a^2+b^2) \cos(c+dx) + \frac{1}{2} b^2 (15a^2+4b^2) \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{4b} \\
&= \frac{9ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{4d \sqrt{\cos(c+dx)}} + \frac{b^2 \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{2d} \\
&+ \frac{\int \frac{-\frac{9}{2} a^2 b^2 + ab(4a^2+b^2) \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{4b} + \frac{1}{8} (b(15a^2+4b^2)) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx \\
&= \frac{\sqrt{a+b}(15a^2+4b^2) \cot(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4d} \\
&+ \frac{9ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{4d \sqrt{\cos(c+dx)}} \\
&+ \frac{b^2 \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{2d} \\
&- \frac{1}{8} (9a^2b) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx \\
&+ \frac{1}{8} (a(8a^2+9ab+2b^2)) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx
\end{aligned}$$

$$\begin{aligned}
&= \\
&\frac{9(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d} \\
&+ \frac{\sqrt{a+b}(8a^2+9ab+2b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4d} \\
&- \frac{\sqrt{a+b}(15a^2+4b^2)\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4d} \\
&+ \frac{9ab\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} \\
&+ \frac{b^2\sqrt{\cos(c+dx)}\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.36 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}} dx = \frac{2b^2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))\sin(c+dx) + \sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\left(9ab(a+b\cos(c+dx))\right)}{\sqrt{\cos(c+dx)}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]

[Out] (2*b^2*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*Sin[c + d*x] + Sqrt[Cos[(c + d*x)/2]^2]*(9*a*b*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b))*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 2*(4*a^3 - 12*a^2*b + a*b^2 - 2*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 2*b*(15*a^2 + 4*b^2)*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 9*a*b*(a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(4*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. 2(401) = 802.

Time = 10.17 (sec) , antiderivative size = 2232, normalized size of antiderivative = 5.04

method	result	size
default	Expression too large to display	2232

[In] int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

```

[Out] -1/4/d*(9*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*
cos(d*x+c)^2-11*a*b^2*cos(d*x+c)^2*sin(d*x+c)-9*a^2*b*cos(d*x+c)*sin(d*x+c)
-2*sin(d*x+c)*cos(d*x+c)^2*b^3-2*sin(d*x+c)*cos(d*x+c)*a*b^2-2*b^3*cos(d*x+
c)^3*sin(d*x+c)+9*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2
)*a*b^2*cos(d*x+c)^2-24*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2
))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b)
)^(1/2)*a^2*b*cos(d*x+c)^2+2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))
^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/
(a+b))^(1/2)*a*b^2*cos(d*x+c)^2+18*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/
(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*
x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)+18*EllipticE(cot(d*x+c)-csc(d*x+c), (-a
-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)-48*EllipticF(cot(d*x+c)-csc(d*x+c),
(-a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(
1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)+4*EllipticF(cot(d*x+c)-csc(d*x+
c), (-a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b
)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)+16*EllipticF(cot(d*x+c)-csc(
d*x+c), (-a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+
c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*cos(d*x+c)+8*EllipticF(cot(d*x+c)-csc
(d*x+c), (-a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x
+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3+8*EllipticPi(cot(d*x+c)-csc(d*x+c), -
1, (-a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b
)/(1+cos(d*x+c))/(a+b))^(1/2)*b^3*cos(d*x+c)^2-4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-c
sc(d*x+c), (-a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)^2+16*EllipticPi(cot(d*x+c)-c
sc(d*x+c), -1, (-a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+co
s(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b^3*cos(d*x+c)-8*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot
(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*b^3*cos(d*x+c)+30*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
cot(d*x+c)-csc(d*x+c), -1, (-a-b)/(a+b))^(1/2))*a^2*b+9*EllipticE(cot(d*x+c)
-csc(d*x+c), (-a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos
(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+9*EllipticE(cot(d*x+c)-csc(d*x
+c), (-a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2-24*EllipticF(cot(d*x+c)-csc(d*x+c), (-
a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+c
os(d*x+c))/(a+b))^(1/2)*a^2*b+2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+
b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c
)))/(a+b))^(1/2)*a*b^2+30*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (-a-b)/(a+b))
^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/
(a+b))^(1/2)*a^2*b*cos(d*x+c)^2+60*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (-a
-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)+8*EllipticF(cot(d*x+c)-csc(d*x+c), (

```

$$\begin{aligned}
 & -\frac{(a-b)}{(a+b)} \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{1/2} \left(\frac{(a+\cos(dx+c)b)}{1+\cos(dx+c)} \right) \left(\frac{1}{a+b} \right)^{1/2} a^3 \cos(dx+c)^2 + 8 \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{1/2} \\
 & \left(\frac{(a+\cos(dx+c)b)}{1+\cos(dx+c)} \right) \left(\frac{1}{a+b} \right)^{1/2} \text{EllipticPi}(\cot(dx+c) - \csc(dx+c), -1, \left(\frac{(a-b)}{(a+b)} \right)^{1/2}) b^3 - 4 \left(\frac{\cos(dx+c)}{1+\cos(dx+c)} \right)^{1/2} \left(\frac{(a+\cos(dx+c)b)}{1+\cos(dx+c)} \right) \left(\frac{1}{a+b} \right)^{1/2} \\
 & \text{EllipticF}(\cot(dx+c) - \csc(dx+c), \left(\frac{(a-b)}{(a+b)} \right)^{1/2}) b^3 \left(\frac{1}{1+\cos(dx+c)} \right) \left(\frac{1}{a+\cos(dx+c)b} \right)^{1/2} \left(\frac{1}{\cos(dx+c)} \right)^{1/2}
 \end{aligned}$$

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+b*cos(dx+c))^(5/2)/cos(dx+c)^(1/2),x, algorithm="fricas")

[Out] integral((b^2*cos(dx + c)^2 + 2*a*b*cos(dx + c) + a^2)*sqrt(b*cos(dx + c) + a)/sqrt(cos(dx + c)), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate((a+b*cos(dx+c))**(5/2)/cos(dx+c)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+b*cos(dx+c))^(5/2)/cos(dx+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(dx + c) + a)^(5/2)/sqrt(cos(dx + c)), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\cos(c + dx)}} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)

[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2), x)

$$3.620 \quad \int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	5982
Rubi [A] (verified)	5983
Mathematica [A] (verified)	5986
Maple [B] (verified)	5986
Fricas [F]	5988
Sympy [F(-1)]	5988
Maxima [F]	5988
Giac [F]	5989
Mupad [F(-1)]	5989

Optimal result

Integrand size = 25, antiderivative size = 445

$$\int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{(a-b)\sqrt{a+b}(2a^2-b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{ad} \\ - \frac{\sqrt{a+b}(2a^2-6ab-b^2) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\ - \frac{5ab\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\ + \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} - \frac{(2a^2-b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

```
[Out] 2*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-(2*a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+(a-b)*(2*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-(2*a^2-6*a*b-b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-5*a*b*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2871, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{3/2}(c + dx)} dx =$$

$$\frac{\sqrt{a+b}(2a^2 - 6ab - b^2) \cot(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{d}$$

$$+ \frac{(a-b)\sqrt{a+b}(2a^2 - b^2) \cot(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right)}{d}$$

$$- \frac{(2a^2 - b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}} + \frac{2a^2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}}$$

$$- \frac{5ab\sqrt{a+b} \cot(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{d}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2), x]

[Out] ((a - b)*Sqrt[a + b]*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b *Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d) - (Sqrt[a + b]*(2*a^2 - 6*a*b - b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/d - (5*a*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/d + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I

ntegersQ[2*m, 2*n])

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] := Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)]
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d),
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
```

- 2*a*C)*Sin[e + f*x])/((a + b*SIN[e + f*x])^(3/2)*Sqrt[c + d*SIN[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3140

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*SIN[e + f*x]]/(d*f*Sqrt[a + b*SIN[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*SIN[e + f*x])^(3/2)*Sqrt[c + d*SIN[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
 &+ 2 \int \frac{\frac{3a^2b}{2} - \frac{1}{2}a(a^2 - 3b^2) \cos(c + dx) - \frac{1}{2}b(2a^2 - b^2) \cos^2(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} - \frac{(2a^2 - b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
 &+ \frac{\int \frac{\frac{1}{2}ab(2a^2 - b^2) + 3a^2b^2 \cos(c + dx) + \frac{5}{2}ab^3 \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{b} \\
 &= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} - \frac{(2a^2 - b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
 &+ \frac{\int \frac{\frac{1}{2}ab(2a^2 - b^2) + 3a^2b^2 \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{b} + \frac{1}{2} (5ab^2) \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{5ab \sqrt{a + b} \cot(c + dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\
 &+ \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} - \frac{(2a^2 - b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\
 &+ \frac{1}{2} (a(2a^2 - b^2)) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
 &- \frac{1}{2} (a(2a^2 - 6ab - b^2)) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{(a-b)\sqrt{a+b}(2a^2-b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad} \\
&- \frac{\sqrt{a+b}(2a^2-6ab-b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\
&- \frac{5ab\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\
&+ \frac{2a^2\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{(2a^2-b^2)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.99 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{3/2}(c+dx)} dx = \frac{2\left(a^2\cos^2(c+dx)(a+b\cos(c+dx))\sin(c+dx) + \cos^2\left(\frac{1}{2}(c+dx)\right)^{5/2}\left(\frac{\cos(c+dx)}{1+\cos(c+dx)}\right)\right)}{\cos^{3/2}(c+dx)}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2), x]

[Out] (2*(a^2*Cos[c + d*x]^2*(a + b*Cos[c + d*x])*Sin[c + d*x] + (Cos[(c + d*x)/2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[1 + Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2*(-2*(2*a^3 + 2*a^2*b - a*b^2 - b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a^2 + 3*a*b - 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 20*a*b^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + ((2*a^2 - b^2)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/(d*Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. 2(413) = 826.

Time = 10.91 (sec) , antiderivative size = 2508, normalized size of antiderivative = 5.64

method	result	size
default	Expression too large to display	2508

[In] int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out]
$$-1/d*(\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2-1})*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1}))^{(1/2)}$$

$$*(2*a^3*(\csc(d*x+c)-\cot(d*x+c))+b^3*(\csc(d*x+c)-\cot(d*x+c))^{-2*\csc(d*x+c)^2*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a^3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}$$

$$*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*(1-\cos(d*x+c))^{-2+2*\csc(d*x+c)^2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a^3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}$$

$$*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*b^3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}$$

$$*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*(1-\cos(d*x+c))^{-2+a*b^2*(\csc(d*x+c)-\cot(d*x+c))+\csc(d*x+c)^5*b^3*(1-\cos(d*x+c))^{-5-6*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a^2*b+6*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a*b^2+2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a^2*b-(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a*b^2-10*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*\text{EllipticPi}(\cot(d*x+c)-\csc(d*x+c),-1,(-a-b)/(a+b))^{(1/2)})*a*b^2-\csc(d*x+c)^5*a*b^2*(1-\cos(d*x+c))^{-5-2*\csc(d*x+c)^5*a^2*b*(1-\cos(d*x+c))^{-5-2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a^3+2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a^3-(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*b^3+2*\csc(d*x+c)^5*a^3*(1-\cos(d*x+c))^{-5-2*\csc(d*x+c)^3*b^3*(1-\cos(d*x+c))^{-3+4*\csc(d*x+c)^3*a^3*(1-\cos(d*x+c))^{-3+2*a^2*b*(\csc(d*x+c)-\cot(d*x+c))-6*\csc(d*x+c)^2*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a^2*b*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*(1-\cos(d*x+c))^{-2+6*\csc(d*x+c)^2*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a*b^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}*(1-\cos(d*x+c))^{-2+2*\csc(d*x+c)^2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*a^2*b*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{-2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{-2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{-2+a+b}}/(a+b))^{(1/2)}$$

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{3/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{3/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\cos(c + dx)^{3/2}} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)

[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2), x)

$$3.621 \quad \int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	5990
Rubi [A] (verified)	5991
Mathematica [A] (verified)	5994
Maple [B] (verified)	5994
Fricas [F]	5995
Sympy [F(-1)]	5996
Maxima [F]	5996
Giac [F]	5996
Mupad [F(-1)]	5996

Optimal result

Integrand size = 25, antiderivative size = 392

$$\int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{14(a-b)b\sqrt{a+b} \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3d}$$

$$+ \frac{2\sqrt{a+b}(a^2 - 7ab + 9b^2) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3d}$$

$$+ \frac{2b^2 \sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

$$+ \frac{2a^2 \sqrt{a+b} \cos(c+dx) \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/3*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+14/3*(a-b)*b*c
ot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x
+c))/(a-b))^(1/2)/d+2/3*(a^2-7*a*b+9*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x
+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(
a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d-2*b^2*cot(d*
x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b
,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2871, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \frac{2\sqrt{a+b}(a^2 - 7ab + 9b^2) \cot(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) + \frac{2a^2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{3d \cos^{3/2}(c + dx)} - \frac{2b^2 \sqrt{a+b} \cot(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{d} + \frac{14b(a-b)\sqrt{a+b} \cot(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{3d}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]

[Out] (14*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*d) + (2*Sqrt[a + b]*(a^2 - 7*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*d) - (2*b^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/d + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]
]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} \\
&+ \frac{2}{3} \int \frac{\frac{7a^2b}{2} + \frac{1}{2}a(a^2 + 9b^2) \cos(c + dx) + \frac{3}{2}b^3 \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} \\
&+ \frac{2}{3} \int \frac{\frac{7a^2b}{2} + \frac{1}{2}a(a^2 + 9b^2) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx + b^3 \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \\
&\frac{2b^2 \sqrt{a + b} \cot(c + dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\
&+ \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)} \\
&+ \frac{1}{3}(7a^2b) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&+ \frac{1}{3}(a(a^2 - 7ab + 9b^2)) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{14(a - b)b \sqrt{a + b} \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3d} \\
&+ \frac{2\sqrt{a + b}(a^2 - 7ab + 9b^2) \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3d} \\
&- \frac{2b^2 \sqrt{a + b} \cot(c + dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d} \\
&+ \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d \cos^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.84

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \frac{\frac{2a(a+b \cos(c+dx))(a+7b \cos(c+dx)) \sin(c+dx)}{\cos^{3/2}(c+dx)} + 2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)} \left(-7ab(a+b)E(\arcsin(\cos\left(\frac{1}{2}(c+dx)\right))\right)}{\cos^{5/2}(c+dx)}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2), x]

[Out] ((2*a*(a + b*Cos[c + d*x])*(a + 7*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2) + 2*sqrt[Cos[(c + d*x)/2]^2]*(-7*a*b*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)]*sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (a^3 + 7*a^2*b + 9*a*b^2 - 3*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)]*sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 6*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)]*sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 7*a*b*(a + b*Cos[c + d*x])*sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2])/(3*d*sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2210 vs. 2(358) = 716.

Time = 12.26 (sec) , antiderivative size = 2211, normalized size of antiderivative = 5.64

method	result	size
default	Expression too large to display	2211

[In] int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(5/2), x, method=_RETURNVERBOSE)

[Out] 2/3/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-7*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a^2*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-9*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a*b^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+3*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*b^3*(1-cos(d*x+c))^2+7*csc(d*x+c)^2*EllipticE(cot(d*x+c)

$$\begin{aligned}
& -\csc(dx+c), (-\frac{a-b}{a+b})^{1/2}) * a^2 * b * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1}) \\
& ^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+}} \\
& b) / (a+b))^{1/2} * (1-\cos(dx+c))^{2+7 * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), \\
& (-\frac{a-b}{a+b})^{1/2}) * a * b^2 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * (\\
& (\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+}} \\
& b) / (a+b))^{1/2} * (1-\cos(dx+c))^{2-6 * \csc(dx+c)^2 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * \\
& ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+}} \\
& b) / (a+b))^{1/2} * \text{EllipticPi}(\cot(dx+c) - \csc(dx+c), -1, (-\frac{a-b}{a+b})^{1/2}) * b^3 * \\
& (1-\cos(dx+c))^{2+7 * \csc(dx+c)^5 * a^2 * b * (1-\cos(dx+c))^{5-7 * \csc(dx+c)^5 * a * b} \\
& ^2 * (1-\cos(dx+c))^{5+(-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * \\
& (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+}} \\
& b) / (a+b))^{1/2} * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2}) * a^3 + 7 * (-\csc(dx+c)^2 * (1-\cos \\
& (dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos \\
& (dx+c))^{2+a+}} \\
& b) / (a+b))^{1/2} * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2}) * a^2 * b + 9 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * \\
& ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+}} \\
& b) / (a+b))^{1/2} * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2}) * a * b^2 - 3 * (-\csc(dx+c)^2 * (1-\cos \\
& (dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos \\
& (dx+c))^{2+a+}} \\
& b) / (a+b))^{1/2} * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2}) * b^3 - 7 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * \\
& ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+}} \\
& b) / (a+b))^{1/2} * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2}) * a^2 * b - 7 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * \\
& ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+}} \\
& b) / (a+b))^{1/2} * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-\frac{a-b}{a+b})^{1/2}) * a * b^2 + 6 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * \\
& ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+}} \\
& b) / (a+b))^{1/2} * \text{EllipticPi}(\cot(dx+c) - \csc(dx+c), -1, (-\frac{a-b}{a+b})^{1/2}) * b^3 - 2 * \csc(dx+c)^3 * a^3 * (1-\cos \\
& (dx+c))^{3+2 * \csc(dx+c)^3 * a^2 * b * (1-\cos(dx+c))^{3+14 * \csc(dx+c)^3 * a * b^2 * (1-\cos \\
& (dx+c))^{3-2 * a^3 * (\csc(dx+c) - \cot(dx+c)) - 9 * a^2 * b * (\csc(dx+c) - \cot(dx+c)) - \\
& 7 * a * b^2 * (\csc(dx+c) - \cot(dx+c))} / (\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+}} \\
& b) / (\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{2/(-\csc(dx+c)^2 * (1-\cos(dx+c))^{2-1}) / (\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{5/2}}
\end{aligned}$$

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^5(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos^5(dx + c)} dx$$

[In] integrate((a+b*cos(dx+c))^(5/2)/cos(dx+c)^(5/2),x, algorithm="fricas")

[Out] integral((b^2*cos(dx + c)^2 + 2*a*b*cos(dx + c) + a^2)*sqrt(b*cos(dx + c) + a)/cos(dx + c)^(5/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{5/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2), x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{5/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2), x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{5/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\cos(c + dx)^{5/2}} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)

[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2), x)

$$3.622 \quad \int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	5997
Rubi [A] (verified)	5998
Mathematica [A] (verified)	6000
Maple [B] (verified)	6001
Fricas [F]	6002
Sympy [F(-1)]	6003
Maxima [F]	6003
Giac [F]	6003
Mupad [F(-1)]	6003

Optimal result

Integrand size = 25, antiderivative size = 338

$$\int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{2(a-b)\sqrt{a+b}(9a^2+23b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{15ad} - \frac{2(a-b)\sqrt{a+b}(9a^2-8ab+15b^2) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{15ad} + \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5d \cos^{\frac{5}{2}}(c+dx)} + \frac{22ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{15d \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/5*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+22/15*a*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/15*(a-b)*(9*a^2+23*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-2/15*(a-b)*(9*a^2-8*a*b+15*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00,
 number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used
 = {2871, 3134, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{\frac{7}{2}}(c + dx)} dx =$$

$$\frac{2(a - b)\sqrt{a + b}(9a^2 - 8ab + 15b^2) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{15ad}$$

$$+ \frac{2(a - b)\sqrt{a + b}(9a^2 + 23b^2) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{15ad}$$

$$+ \frac{2a^2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{22ab \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{15d \cos^{\frac{3}{2}}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2), x]

[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 + 23*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(15*a*d) - (2*(a - b)*Sqrt[a + b]*(9*a^2 - 8*a*b + 15*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(15*a*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (22*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(- (b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr

```
t[a*((1 - Csc[e + f*x])/(a + b))*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_)
+ (f_)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} \\
 &+ \frac{2}{5} \int \frac{\frac{11a^2b}{2} + \frac{3}{2}a(a^2 + 5b^2) \cos(c + dx) + \frac{1}{2}b(2a^2 + 5b^2) \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{22ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx)} \\
 &+ \frac{4 \int \frac{\frac{1}{4}a^2(9a^2 + 23b^2) + \frac{1}{4}ab(17a^2 + 15b^2) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{15a} \\
 &= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{22ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx)} \\
 &- \frac{1}{15} ((a - b)(9a^2 - 8ab + 15b^2)) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
 &+ \frac{1}{15} (a(9a^2 + 23b^2)) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{2(a - b) \sqrt{a + b} (9a^2 + 23b^2) \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{15ad} \\
 &- \frac{2(a - b) \sqrt{a + b} (9a^2 - 8ab + 15b^2) \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}}}{15ad} \\
 &+ \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{5d \cos^{\frac{5}{2}}(c + dx)} + \frac{22ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{15d \cos^{\frac{3}{2}}(c + dx)}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 7.80 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.14

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx = \frac{-4 \cos^2\left(\frac{1}{2}(c + dx)\right)^{5/2} \left(\frac{\cos(c + dx)}{1 + \cos(c + dx)}\right)^{3/2} \sqrt{1 + \cos(c + dx)} \left((9a^3 + 9a^2b + 23ab^2 + 23b^3) \text{EllipticE}\left[\text{ArcSin}\left[\frac{\tan\left(\frac{c + dx}{2}\right)}{2}\right], \frac{-a + b}{a + b}\right] \text{Sec}\left[\frac{c + dx}{2}\right]^2 \sqrt{\frac{a + b \cos(c + dx)}{a + b}}\right) \text{Sec}\left[\frac{c + dx}{2}\right]^2 / (a + b) - (9a^3 + 17a^2b + 23ab^2 + 15b^3) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left(\frac{c + dx}{2}\right)}{2}\right], \frac{-a + b}{a + b}\right] \text{Sec}\left[\frac{c + dx}{2}\right]^2}{15ad}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2), x]

[Out] (-4*(Cos[(c + d*x)/2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[1 + Cos[c + d*x]]*((9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[(a + b*Cos[c + d*x])/ (a + b)] - (9*a^3 + 17*a^2*b + 23*a*b^2 + 15*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2)/15ad

$$\begin{aligned} &))^{(1/2)} * a^2 * b * \cos(d*x+c)^2 + 23 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} \\ &))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) \\ &)/(a+b))^{(1/2)} * a * b^2 * \cos(d*x+c)^2 - 9 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} \\ &))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) \\ &)/(a+b))^{(1/2)} * a^2 * b * \cos(d*x+c)^4 + 15 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} \\ & * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} \\ &))^{(1/2)} * b^3 * \cos(d*x+c)^2 - 9 * a^2 * b * \cos(d*x+c)^3 * \sin(d*x+c) - 11 * a * b^2 * \cos(d*x+c)^3 \\ & * \sin(d*x+c) + 15 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} \\ & * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * b^3 * \cos(d*x+c)^4 + 30 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} \\ &))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * b^3 * \cos(d*x+c)^3 \\ & + 9 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\ & * a^3 * \cos(d*x+c)^2 - 9 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} \\ & * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * a^3 * \cos(d*x+c)^4 - 23 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} \\ &))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * b^3 * \cos(d*x+c)^4 \\ & + 9 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\ & * a^3 * \cos(d*x+c)^4 - 18 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} \\ & * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * a^3 * \cos(d*x+c)^3 - 46 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} \\ &))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * b^3 * \cos(d*x+c)^3 - 9 * a^3 * \cos(d*x+c)^2 * \sin(d*x+c) \\ &)/(1+\cos(d*x+c))/(a+\cos(d*x+c)*b)^{(1/2)}/\cos(d*x+c)^{(5/2)} \end{aligned}$$

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{7/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{7/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2), x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{7/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2), x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{7/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\cos(c + dx)^{7/2}} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2), x)

[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2), x)

$$3.623 \quad \int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	6004
Rubi [A] (verified)	6005
Mathematica [C] (verified)	6008
Maple [B] (verified)	6009
Fricas [F]	6010
Sympy [F(-1)]	6011
Maxima [F]	6011
Giac [F]	6011
Mupad [F(-1)]	6011

Optimal result

Integrand size = 25, antiderivative size = 387

$$\int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{2(a-b)b\sqrt{a+b}(29a^2+3b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\cos(c+dx)}}{21a^2d}$$

$$+ \frac{2(a-b)\sqrt{a+b}(5a^2-24ab+3b^2)\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\cos(c+dx)}}{21ad}$$

$$+ \frac{2a^2\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6ab\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{2(5a^2+9b^2)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/7*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+6/7*a*b*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/21*(5*a^2+9*b^2)*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+2/21*(a-b)*b*(29*a^2+3*b^2)*cot
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)
))/(a-b)^(1/2)/a^2/d+2/21*(a-b)*(5*a^2-24*a*b+3*b^2)*cot(d*x+c)*EllipticF(
(a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2)*(
a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/
d
```


Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00,
 number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used
 = {2871, 3134, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \frac{2(a - b)\sqrt{a + b}(5a^2 - 24ab + 3b^2) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}}}{21ad} E$$

$$+ \frac{2b(a - b)\sqrt{a + b}(29a^2 + 3b^2) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{21a^2d}$$

$$+ \frac{2(5a^2 + 9b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{21d \cos^{3/2}(c + dx)}$$

$$+ \frac{2a^2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{7d \cos^{7/2}(c + dx)} + \frac{6ab \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{7d \cos^{5/2}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2), x]

[Out] (2*(a - b)*b*Sqrt[a + b]*(29*a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(21*a^2*d) + (2*(a - b)*Sqrt[a + b]*(5*a^2 - 24*a*b + 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(21*a*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(5/2)) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2))

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)])^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} \\
&+ \frac{2}{7} \int \frac{\frac{15a^2b}{2} + \frac{1}{2}a(5a^2 + 21b^2) \cos(c + dx) + \frac{1}{2}b(4a^2 + 7b^2) \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{6ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{5}{2}}(c + dx)} \\
&+ \frac{4 \int \frac{\frac{5}{4}a^2(5a^2 + 9b^2) + \frac{5}{4}ab(13a^2 + 7b^2) \cos(c + dx) + \frac{15}{2}a^2b^2 \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{35a} \\
&= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{6ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{5}{2}}(c + dx)} \\
&+ \frac{2(5a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{21d \cos^{\frac{3}{2}}(c + dx)} \\
&+ \frac{8 \int \frac{\frac{5}{8}a^2b(29a^2 + 3b^2) + \frac{5}{8}a^3(5a^2 + 27b^2) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{105a^2} \\
&= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{6ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{5}{2}}(c + dx)} \\
&+ \frac{2(5a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{21d \cos^{\frac{3}{2}}(c + dx)} \\
&+ \frac{1}{21} (b(29a^2 + 3b^2)) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&+ \frac{1}{21} ((a - b)(5a^2 - 24ab + 3b^2)) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2(a - b)b \sqrt{a + b}(29a^2 + 3b^2) \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a + b}}}{21a^2d} \\
&+ \frac{2(a - b)\sqrt{a + b}(5a^2 - 24ab + 3b^2) \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a + b}}}{21ad} \\
&+ \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{7}{2}}(c + dx)} + \frac{6ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{7d \cos^{\frac{5}{2}}(c + dx)} \\
&+ \frac{2(5a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{21d \cos^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 1302, normalized size of antiderivative = 3.36

$$4a(5a^4 - 2a^2b^2 - 3b^4) \sqrt{\frac{(a+b) \cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}} \sqrt{-\frac{(a+b) \cos(c+dx) \csc^2\left(\frac{1}{2}(c+dx)\right)}{a}} \sqrt{\frac{(a+b \cos(c+dx)) \csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}$$

$$(a+b) \sqrt{\cos(c+dx)} \sqrt{a-}$$

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \frac{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)} \left(\frac{2}{21} \sec^2(c + dx) (5a^2 \sin(c + dx) + 9b^2 \sin(c + dx)) + \frac{2 \sec(c + dx) (29a^2 b \sin(c + dx) + 21a^2)}{21a} \right)}{d}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]

[Out] ((-4*a*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-29*a^3*b - 3*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-29*a^2*b^2 - 3*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]

$$\begin{aligned} & *Csc[(c + d*x)/2]^2/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a)* \\ & Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + \\ & d*x)/2]^2/a)/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c \\ & + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d \\ & *x])/(b*Sqrt[Cos[c + d*x]])))/(21*a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos \\ & [c + d*x]]*((2*Sec[c + d*x]^2*(5*a^2*Sin[c + d*x] + 9*b^2*Sin[c + d*x]))/21 \\ & + (2*Sec[c + d*x]*(29*a^2*b*Sin[c + d*x] + 3*b^3*Sin[c + d*x]))/(21*a) + (\\ & 6*a*b*Sec[c + d*x]^2*Tan[c + d*x])/7 + (2*a^2*Sec[c + d*x]^3*Tan[c + d*x])/ \\ & 7))/d \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. 2(349) = 698.

Time = 16.34 (sec) , antiderivative size = 2499, normalized size of antiderivative = 6.46

method	result	size
default	Expression too large to display	2499

[In] int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out]
$$\begin{aligned} & -2/21/d*(-3*a^4*\sin(d*x+c)-18*a^2*b^2*\cos(d*x+c)^2*\sin(d*x+c)-3*b^4*\cos(d*x+ \\ & c)^4*\sin(d*x+c)-5*a^4*\cos(d*x+c)^2*\sin(d*x+c)-29*a^2*b^2*\cos(d*x+c)^4*\sin(\\ & d*x+c)-3*EllipticE(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+ \\ & c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*b^4*\cos \\ & (d*x+c)^5+5*EllipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*((a+\cos(d \\ & *x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*a^4* \\ & \cos(d*x+c)^5-6*EllipticE(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*((a+\cos \\ & (d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*b \\ & ^4*\cos(d*x+c)^4+10*EllipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*((\\ & a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/ \\ & 2)}*a^4*\cos(d*x+c)^4-3*EllipticE(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)} \\ & *((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(\\ & 1/2)}*b^4*\cos(d*x+c)^3+5*EllipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/ \\ & 2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c) \\ &))^{(1/2)}*a^4*\cos(d*x+c)^3-6*EllipticE(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(\\ & 1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x \\ & +c)))^{(1/2)}*a*b^3*\cos(d*x+c)^4+58*EllipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(\\ & a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+c \\ & os(d*x+c)))^{(1/2)}*a^3*b*\cos(d*x+c)^4+54*EllipticF(\cot(d*x+c)-\csc(d*x+c), (- \\ & a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c \\ &)/(1+\cos(d*x+c)))^{(1/2)}*a^2*b^2*\cos(d*x+c)^4+6*EllipticF(\cot(d*x+c)-\csc(d*x \\ & +c), (-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos \\ & (d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*a*b^3*\cos(d*x+c)^4-29*EllipticE(\cot(d*x+c)-\csc \\ & (d*x+c), (-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/ \\ & 2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*a^3*b*\cos(d*x+c)^3-29*EllipticE(\cot(d*x+c) \end{aligned}$$

$x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2*b^2*\cos(d*x+c)^3-3*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b^3*\cos(d*x+c)^3+29*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^3*b*\cos(d*x+c)^3+27*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2*b^2*\cos(d*x+c)^3+3*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b^3*\cos(d*x+c)^3-29*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^3*b*\cos(d*x+c)^5-29*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2*b^2*\cos(d*x+c)^5-3*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b^3*\cos(d*x+c)^5+29*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^3*b*\cos(d*x+c)^5+27*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2*b^2*\cos(d*x+c)^5+3*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b^3*\cos(d*x+c)^5-58*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^3*b*\cos(d*x+c)^4-58*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2*b^2*\cos(d*x+c)^4-5*a^4*\cos(d*x+c)^3*\sin(d*x+c)-3*a^4*\cos(d*x+c)*\sin(d*x+c)-12*a^3*b*\cos(d*x+c)*\sin(d*x+c)-12*a^3*b*\cos(d*x+c)^2*\sin(d*x+c)-34*a^3*b*\cos(d*x+c)^3*\sin(d*x+c)-18*a^2*b^2*\cos(d*x+c)^3*\sin(d*x+c)-12*a*b^3*\cos(d*x+c)^3*\sin(d*x+c)-5*a^3*b*\cos(d*x+c)^4*\sin(d*x+c)-9*a*b^3*\cos(d*x+c)^4*\sin(d*x+c))/(1+\cos(d*x+c))/(a+\cos(d*x+c)*b)^{1/2}/\cos(d*x+c)^{7/2}/a$

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{\frac{5}{2}}}{\cos^{\frac{9}{2}}(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{9/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2), x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{9/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2), x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{9/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\cos(c + dx)^{9/2}} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2), x)

[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2), x)

$$3.624 \quad \int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)} dx$$

Optimal result	6012
Rubi [A] (verified)	6013
Mathematica [C] (verified)	6016
Maple [B] (verified)	6018
Fricas [F]	6020
Sympy [F(-1)]	6020
Maxima [F]	6020
Giac [F]	6021
Mupad [F(-1)]	6021

Optimal result

Integrand size = 25, antiderivative size = 454

$$\int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)} dx = \frac{2(a-b)\sqrt{a+b}(147a^4 + 279a^2b^2 - 10b^4) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right)\right)}{315a^3d}$$

$$- \frac{2(a-b)\sqrt{a+b}(147a^3 - 114a^2b + 165ab^2 + 10b^3) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{315a^2d}$$

$$+ \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{9d \cos^{\frac{9}{2}}(c+dx)} + \frac{38ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{63d \cos^{\frac{7}{2}}(c+dx)}$$

$$+ \frac{2(49a^2 + 75b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{315d \cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{2b(163a^2 + 5b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{315ad \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/9*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+38/63*a*b*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/315*(49*a^2+75*b^2)*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+2/315*b*(163*a^2+5*b^2)*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)+2/315*(a-b)*(147*a^4+2
79*a^2*b^2-10*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-2/315*(a-b)*(147*a^3-114*a^2*b+
165*a*b^2+10*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(
1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d
```


Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2871, 3134, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{11/2}(c + dx)} dx = \frac{2(49a^2 + 75b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{315d \cos^{5/2}(c + dx)} + \frac{2b(163a^2 + 5b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{315ad \cos^{3/2}(c + dx)} + \frac{2a^2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{9d \cos^{9/2}(c + dx)} - \frac{2(a - b) \sqrt{a + b} (147a^3 - 114a^2b + 165ab^2 + 10b^3) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{315a^2d} + \frac{2(a - b) \sqrt{a + b} (147a^4 + 279a^2b^2 - 10b^4) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{315a^3d} + \frac{38ab \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{63d \cos^{7/2}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2), x]

[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b))]/(315*a^3*d) - (2*(a - b)*Sqrt[a + b]*(147*a^3 - 114*a^2*b + 165*a*b^2 + 10*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b))]/(315*a^2*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (38*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*a^2 + 75*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5/2)) + (2*b*(163*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(3/2))

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I

ntegersQ[2*m, 2*n])

Rule 2895

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Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
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Rule 3073

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Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
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Rule 3077

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Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
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Rule 3134

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Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
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EqQ[a, 0])))

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{9d \cos^{\frac{9}{2}}(c + dx)} \\
&+ \frac{2}{9} \int \frac{\frac{19a^2b}{2} + \frac{1}{2}a(7a^2 + 27b^2) \cos(c + dx) + \frac{3}{2}b(2a^2 + 3b^2) \cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{9d \cos^{\frac{9}{2}}(c + dx)} + \frac{38ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{63d \cos^{\frac{7}{2}}(c + dx)} \\
&+ \frac{4 \int \frac{\frac{1}{4}a^2(49a^2 + 75b^2) + \frac{1}{4}ab(137a^2 + 63b^2) \cos(c + dx) + 19a^2b^2 \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{63a} \\
&= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{9d \cos^{\frac{9}{2}}(c + dx)} + \frac{38ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{63d \cos^{\frac{7}{2}}(c + dx)} \\
&+ \frac{2(49a^2 + 75b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315d \cos^{\frac{5}{2}}(c + dx)} \\
&+ \frac{8 \int \frac{\frac{3}{8}a^2b(163a^2 + 5b^2) + \frac{1}{8}a^3(147a^2 + 605b^2) \cos(c + dx) + \frac{1}{4}a^2b(49a^2 + 75b^2) \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{315a^2} \\
&= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{9d \cos^{\frac{9}{2}}(c + dx)} + \frac{38ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{63d \cos^{\frac{7}{2}}(c + dx)} \\
&+ \frac{2(49a^2 + 75b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315d \cos^{\frac{5}{2}}(c + dx)} \\
&+ \frac{2b(163a^2 + 5b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{315ad \cos^{\frac{3}{2}}(c + dx)} \\
&+ \frac{16 \int \frac{\frac{3}{16}a^2(147a^4 + 279a^2b^2 - 10b^4) + \frac{3}{16}a^3b(261a^2 + 155b^2) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{945a^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{9d \cos^{\frac{9}{2}}(c+dx)} + \frac{38ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{63d \cos^{\frac{7}{2}}(c+dx)} \\
&+ \frac{2(49a^2 + 75b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{315d \cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{2b(163a^2 + 5b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{315ad \cos^{\frac{3}{2}}(c+dx)} \\
&- \frac{((a-b)(147a^3 - 114a^2b + 165ab^2 + 10b^3)) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx}{315a} \\
&+ \frac{(147a^4 + 279a^2b^2 - 10b^4) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{315a} \\
&= \frac{2(a-b)\sqrt{a+b}(147a^4 + 279a^2b^2 - 10b^4) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{315a^3d} \\
&- \frac{2(a-b)\sqrt{a+b}(147a^3 - 114a^2b + 165ab^2 + 10b^3) \cot(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right)\right)}{315a^2d} \\
&+ \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{9d \cos^{\frac{9}{2}}(c+dx)} + \frac{38ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{63d \cos^{\frac{7}{2}}(c+dx)} \\
&+ \frac{2(49a^2 + 75b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{315d \cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{2b(163a^2 + 5b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{315ad \cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.48 (sec) , antiderivative size = 1368, normalized size of antiderivative = 3.01

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{11/2}(c + dx)} dx =$$

$$\frac{4a(-114a^4b + 124a^2b^3 - 10b^5) \sqrt{\frac{(a+b) \cot^2(\frac{1}{2}(c+dx))}{-a+b}} \sqrt{-\frac{(a+b) \cos(c+dx) \csc^2(\frac{1}{2}(c+dx))}{a}} \sqrt{\frac{(a+b \cos(c+dx)) \csc^2(\frac{1}{2}(c+dx))}{a}} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a+b) \cot^2(\frac{1}{2}(c+dx))}{-a+b}} \sqrt{-\frac{(a+b) \cos(c+dx) \csc^2(\frac{1}{2}(c+dx))}{a}} \sqrt{\frac{(a+b \cos(c+dx)) \csc^2(\frac{1}{2}(c+dx))}{a}}}}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}}\right)}{(a+b) \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}\right)}{+ \frac{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)} \left(\frac{2}{315} \sec^3(c+dx) (49a^2 \sin(c+dx) + 75b^2 \sin(c+dx)) + \frac{2 \sec^2(c+dx) (163a^2 + 10b^2)}{315} \right)}{(a+b) \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2), x]

[Out] -1/315*((-4*a*(-114*a^4*b + 124*a^2*b^3 - 10*b^5)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(147*a^5 + 279*a^3*b^2 - 10*a*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*b + 279*a^2*b^3 - 10*b^5)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b)*Cos[c + d*x])*Sec[c + d*x])/(a + b)) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a

$$+ b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a) \operatorname{Csc}[c + dx] \operatorname{EllipticPi}[-(a/b), \operatorname{ArcSin}[\operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a]/\operatorname{Sqrt}[2]], (-2a)/(-a + b)] \operatorname{Sin}[(c + dx)/2]^4/(b \operatorname{Sqrt}[\cos[c + dx]] \operatorname{Sqrt}[a + b \cos[c + dx]])] / b + (\operatorname{Sqrt}[a + b \cos[c + dx]] \operatorname{Sin}[c + dx]) / (b \operatorname{Sqrt}[\cos[c + dx]])) / (a^2 * d) + (\operatorname{Sqrt}[\cos[c + dx]] \operatorname{Sqrt}[a + b \cos[c + dx]] * ((2 \operatorname{Sec}[c + dx]^3 (49 a^2 \operatorname{Sin}[c + dx] + 75 b^2 \operatorname{Sin}[c + dx])) / 315 + (2 \operatorname{Sec}[c + dx]^2 (163 a^2 b \operatorname{Sin}[c + dx] + 5 b^3 \operatorname{Sin}[c + dx])) / (315 a) + (2 \operatorname{Sec}[c + dx] * (147 a^4 \operatorname{Sin}[c + dx] + 279 a^2 b^2 \operatorname{Sin}[c + dx] - 10 b^4 \operatorname{Sin}[c + dx])) / (315 a^2) + (38 a b \operatorname{Sec}[c + dx]^3 \operatorname{Tan}[c + dx]) / 63 + (2 a^2 \operatorname{Sec}[c + dx]^4 \operatorname{Tan}[c + dx]) / 9)) / d$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3453 vs. $2(410) = 820$.

Time = 18.32 (sec) , antiderivative size = 3454, normalized size of antiderivative = 7.61

method	result	size
default	Expression too large to display	3454

[In] `int((a+cos(dx+c)*b)^(5/2)/cos(dx+c)^(11/2),x,method=_RETURNVERBOSE)`

[Out]
$$-2/315/d*(5*a*b^4*\cos(dx+c)^4*\sin(dx+c)-35*a^5*\sin(dx+c)+155*\operatorname{EllipticF}(\cot(dx+c)-\operatorname{csc}(dx+c),(-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2})*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*a^2*b^3*\cos(dx+c)^4-10*\operatorname{EllipticF}(\cot(dx+c)-\operatorname{csc}(dx+c),(-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2})*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*a*b^4*\cos(dx+c)^4-147*\operatorname{EllipticE}(\cot(dx+c)-\operatorname{csc}(dx+c),(-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2})*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*a^4*b*\cos(dx+c)^6-279*\operatorname{EllipticE}(\cot(dx+c)-\operatorname{csc}(dx+c),(-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2})*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*a^3*b^2*\cos(dx+c)^6-279*\operatorname{EllipticE}(\cot(dx+c)-\operatorname{csc}(dx+c),(-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2})*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*a^2*b^3*\cos(dx+c)^6-170*a^3*b^2*\cos(dx+c)^3*\sin(dx+c)-80*a^2*b^3*\cos(dx+c)^3*\sin(dx+c)-147*a^4*b*\cos(dx+c)^5*\sin(dx+c)-163*a^3*b^2*\cos(dx+c)^5*\sin(dx+c)-279*a^2*b^3*\cos(dx+c)^5*\sin(dx+c)-5*a*b^4*\cos(dx+c)^5*\sin(dx+c)-130*a^4*b*\cos(dx+c)*\sin(dx+c)-130*a^4*b*\cos(dx+c)^2*\sin(dx+c)-170*a^3*b^2*\cos(dx+c)^2*\sin(dx+c)-212*a^4*b*\cos(dx+c)^4*\sin(dx+c)-442*a^3*b^2*\cos(dx+c)^4*\sin(dx+c)-80*a^2*b^3*\cos(dx+c)^4*\sin(dx+c)-212*a^4*b*\cos(dx+c)^3*\sin(dx+c)-147*\operatorname{EllipticE}(\cot(dx+c)-\operatorname{csc}(dx+c),(-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2})*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*a^5*\cos(dx+c)^6+10*\operatorname{EllipticE}(\cot(dx+c)-\operatorname{csc}(dx+c),(-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2})*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*b^5*\cos(dx+c)^6+147*\operatorname{EllipticF}(\cot(dx+c)-\operatorname{csc}(dx+c),(-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2})*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b))^{1/2}*a^5*\cos(dx+c)^6-294*\operatorname{EllipticE}(\cot(dx+c)-\operatorname{csc}(dx+c),$$

$$\frac{1}{(a+b)^{1/2}} a^3 b^2 \cos(d*x+c)^4 - 279 \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-\frac{a-b}{a+b})^{1/2}) * (\frac{\cos(d*x+c)}{1+\cos(d*x+c)})^{1/2} * ((\frac{a+\cos(d*x+c)*b}{1+\cos(d*x+c)}) / (a+b))^{1/2} a^2 b^3 \cos(d*x+c)^4 + 10 \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-\frac{a-b}{a+b})^{1/2}) * (\frac{\cos(d*x+c)}{1+\cos(d*x+c)})^{1/2} * ((\frac{a+\cos(d*x+c)*b}{1+\cos(d*x+c)}) / (a+b))^{1/2} a * b^4 \cos(d*x+c)^4 + 261 \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-\frac{a-b}{a+b})^{1/2}) * (\frac{\cos(d*x+c)}{1+\cos(d*x+c)})^{1/2} * ((\frac{a+\cos(d*x+c)*b}{1+\cos(d*x+c)}) / (a+b))^{1/2} a^4 b \cos(d*x+c)^4 + 279 \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-\frac{a-b}{a+b})^{1/2}) * (\frac{\cos(d*x+c)}{1+\cos(d*x+c)})^{1/2} * ((\frac{a+\cos(d*x+c)*b}{1+\cos(d*x+c)}) / (a+b))^{1/2} a^3 b^2 \cos(d*x+c)^4 / (1+\cos(d*x+c)) / (a+\cos(d*x+c)*b)^{1/2} / \cos(d*x+c)^{9/2} / a^2$$

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{11/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{11/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(11/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{11/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{11/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{11/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{11/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{11/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{11/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\cos(c + dx)^{11/2}} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2),x)

[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2), x)

$$3.625 \quad \int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)} dx$$

Optimal result	6022
Rubi [A] (verified)	6023
Mathematica [C] (verified)	6027
Maple [B] (verified)	6028
Fricas [F]	6030
Sympy [F(-1)]	6030
Maxima [F]	6031
Giac [F]	6031
Mupad [F(-1)]	6031

Optimal result

Integrand size = 25, antiderivative size = 522

$$\int \frac{(a+b \cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)} dx = \frac{2(a-b)b\sqrt{a+b}(741a^4 + 51a^2b^2 + 8b^4) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right)\right)}{693a^4d}$$

$$+ \frac{2(a-b)\sqrt{a+b}(135a^4 - 606a^3b + 57a^2b^2 + 6ab^3 + 8b^4) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{693a^3d}$$

$$+ \frac{2a^2\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{11d \cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{99d \cos^{\frac{9}{2}}(c+dx)}$$

$$+ \frac{2(81a^2 + 113b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693d \cos^{\frac{7}{2}}(c+dx)}$$

$$+ \frac{2b(229a^2 + 3b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693ad \cos^{\frac{5}{2}}(c+dx)}$$

$$+ \frac{2(135a^4 + 205a^2b^2 - 4b^4) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693a^2d \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/11*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(11/2)+46/99*a*b*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+2/693*(81*a^2+113*b^2)*s
in(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+2/693*b*(229*a^2+3*b^2)
*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(5/2)+2/693*(135*a^4+205*
a^2*b^2-4*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(3/2)+2/6
93*(a-b)*b*(741*a^4+51*a^2*b^2+8*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1
-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+2/693*(a-b)*
(135*a^4-606*a^3*b+57*a^2*b^2+6*a*b^3+8*b^4)*cot(d*x+c)*EllipticF((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 1.99 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2871, 3134, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{13/2}(c + dx)} dx = \frac{2(81a^2 + 113b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{693d \cos^{7/2}(c + dx)} + \frac{2b(229a^2 + 3b^2) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{693ad \cos^{5/2}(c + dx)} + \frac{2a^2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{11d \cos^{11/2}(c + dx)} + \frac{2b(a - b) \sqrt{a + b} (741a^4 + 51a^2b^2 + 8b^4) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{693a^4d} + \frac{2(135a^4 + 205a^2b^2 - 4b^4) \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{693a^2d \cos^{3/2}(c + dx)} + \frac{2(a - b) \sqrt{a + b} (135a^4 - 606a^3b + 57a^2b^2 + 6ab^3 + 8b^4) \cot(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}}{693a^3d} + \frac{46ab \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{99d \cos^{9/2}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2), x]

[Out] (2*(a - b)*b*Sqrt[a + b]*(741*a^4 + 51*a^2*b^2 + 8*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(693*a^4*d) + (2*(a - b)*Sqrt[a + b]*(135*a^4 - 606*a^3*b + 57*a^2*b^2 + 6*a*b^3 + 8*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(693*a^3*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2)) + (46*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2)) + (2*(81*a^2 + 113*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*d*Cos[c + d*x]^(7/2)) + (2*b*(229*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*a*d*Cos[c + d*x]^(5/2)) + (2*(135*a^4 + 205*a^2*b^2 - 4*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*a^2*d*Cos[c + d*x]^(3/2))

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +

$b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2*\sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*\sin[e + f*x]^2, x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)])*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3134

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,

c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{11d \cos^{\frac{11}{2}}(c + dx)} \\
&+ \frac{2}{11} \int \frac{\frac{23a^2b}{2} + \frac{3}{2}a(3a^2 + 11b^2) \cos(c + dx) + \frac{1}{2}b(8a^2 + 11b^2) \cos^2(c + dx)}{\cos^{\frac{11}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{11d \cos^{\frac{11}{2}}(c + dx)} + \frac{46ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{99d \cos^{\frac{9}{2}}(c + dx)} \\
&+ \frac{4 \int \frac{\frac{1}{4}a^2(81a^2 + 113b^2) + \frac{1}{4}ab(233a^2 + 99b^2) \cos(c + dx) + \frac{69}{2}a^2b^2 \cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{99a} \\
&= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{11d \cos^{\frac{11}{2}}(c + dx)} + \frac{46ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{99d \cos^{\frac{9}{2}}(c + dx)} \\
&+ \frac{2(81a^2 + 113b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{693d \cos^{\frac{7}{2}}(c + dx)} \\
&+ \frac{8 \int \frac{\frac{5}{8}a^2b(229a^2 + 3b^2) + \frac{1}{8}a^3(405a^2 + 1531b^2) \cos(c + dx) + \frac{1}{2}a^2b(81a^2 + 113b^2) \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{693a^2} \\
&= \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{11d \cos^{\frac{11}{2}}(c + dx)} + \frac{46ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{99d \cos^{\frac{9}{2}}(c + dx)} \\
&+ \frac{2(81a^2 + 113b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{693d \cos^{\frac{7}{2}}(c + dx)} \\
&+ \frac{2b(229a^2 + 3b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{693ad \cos^{\frac{5}{2}}(c + dx)} \\
&+ \frac{16 \int \frac{\frac{15}{16}a^2(135a^4 + 205a^2b^2 - 4b^4) + \frac{5}{16}a^3b(1011a^2 + 461b^2) \cos(c + dx) + \frac{5}{8}a^2b^2(229a^2 + 3b^2) \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3465a^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{11d \cos^{\frac{11}{2}}(c+dx)} + \frac{46ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{99d \cos^{\frac{9}{2}}(c+dx)} \\
&+ \frac{2(81a^2 + 113b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693d \cos^{\frac{7}{2}}(c+dx)} \\
&+ \frac{2b(229a^2 + 3b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693ad \cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{2(135a^4 + 205a^2b^2 - 4b^4) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693a^2d \cos^{\frac{3}{2}}(c+dx)} \\
&+ \frac{32 \int \frac{\frac{15}{32}a^2b(741a^4 + 51a^2b^2 + 8b^4) + \frac{15}{32}a^3(135a^4 + 663a^2b^2 + 2b^4) \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{10395a^4} \\
&= \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{11d \cos^{\frac{11}{2}}(c+dx)} + \frac{46ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{99d \cos^{\frac{9}{2}}(c+dx)} \\
&+ \frac{2(81a^2 + 113b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693d \cos^{\frac{7}{2}}(c+dx)} \\
&+ \frac{2b(229a^2 + 3b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693ad \cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{2(135a^4 + 205a^2b^2 - 4b^4) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693a^2d \cos^{\frac{3}{2}}(c+dx)} \\
&+ \frac{(b(741a^4 + 51a^2b^2 + 8b^4)) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{693a^2} \\
&+ \frac{((a-b)(135a^4 - 606a^3b + 57a^2b^2 + 6ab^3 + 8b^4)) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx}{693a^2} \\
&= \frac{2(a-b)b\sqrt{a+b}(741a^4 + 51a^2b^2 + 8b^4) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{693a^4d} \\
&+ \frac{2(a-b)\sqrt{a+b}(135a^4 - 606a^3b + 57a^2b^2 + 6ab^3 + 8b^4) \cot(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right)\right)}{693a^3d} \\
&+ \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{11d \cos^{\frac{11}{2}}(c+dx)} + \frac{46ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{99d \cos^{\frac{9}{2}}(c+dx)} \\
&+ \frac{2(81a^2 + 113b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693d \cos^{\frac{7}{2}}(c+dx)} \\
&+ \frac{2b(229a^2 + 3b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693ad \cos^{\frac{5}{2}}(c+dx)} \\
&+ \frac{2(135a^4 + 205a^2b^2 - 4b^4) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{693a^2d \cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.57 (sec) , antiderivative size = 1431, normalized size of antiderivative = 2.74

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{13/2}(c + dx)} dx = \frac{4a(135a^6 - 78a^4b^2 - 49a^2b^4 - 8b^6) \sqrt{\frac{(a+b) \cot^2(\frac{1}{2}(c+dx))}{-a+b}} \sqrt{-\frac{(a+b) \cos(c+dx) \csc^2(\frac{1}{2}(c+dx))}{a}} \sqrt{\frac{(a+b) \cos(c+dx)}{a+b}}}{(a+b) \sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)} \sqrt{a + b \cos(c+dx)} \left(\frac{2}{693} \sec^4(c+dx) (81a^2 \sin(c+dx) + 113b^2 \sin^3(c+dx)) + \frac{2 \sec^3(c+dx) (229a^2 + 113b^2) \sin(c+dx)}{693} \right)}{\cos^{13/2}(c+dx)}$$

[In] Integrate[(a + bCos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2), x]

[Out] ((-4*a*(135*a^6 - 78*a^4*b^2 - 49*a^2*b^4 - 8*b^6)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*cos[c + d*x]]) - 4*a*(-741*a^5*b - 51*a^3*b^3 - 8*a*b^5)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*cos[c + d*x]]) + 2*(-741*a^4*b^2 - 51*a^2*b^4 - 8*b^6)*((I*cos[(c + d*x)/2]*Sqrt[a + b*cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b)*Cos[c + d*x])*Sec[c + d*x])/(a + b)) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a)]*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a)]*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*cos[c + d*x]])

$$\begin{aligned} & 2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(\\ & (a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), \\ & ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/ \\ & (-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x] \\ &])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(\\ & 693*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^ \\ & 4*(81*a^2*Sin[c + d*x] + 113*b^2*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^3*(22 \\ & 9*a^2*b*Sin[c + d*x] + 3*b^3*Sin[c + d*x]))/(693*a) + (2*Sec[c + d*x]^2*(13 \\ & 5*a^4*Sin[c + d*x] + 205*a^2*b^2*Sin[c + d*x] - 4*b^4*Sin[c + d*x]))/(693*a \\ & ^2) + (2*Sec[c + d*x]*(741*a^4*b*Sin[c + d*x] + 51*a^2*b^3*Sin[c + d*x] + 8 \\ & *b^5*Sin[c + d*x]))/(693*a^3) + (46*a*b*Sec[c + d*x]^4*Tan[c + d*x])/99 + (\\ & 2*a^2*Sec[c + d*x]^5*Tan[c + d*x])/11))/d \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3890 vs. $2(472) = 944$.

Time = 21.01 (sec) , antiderivative size = 3891, normalized size of antiderivative = 7.45

method	result	size
default	Expression too large to display	3891

[In] `int((a+cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(13/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & 2/693/d*(135*a^6*\cos(d*x+c)^5*\sin(d*x+c)+135*a^6*\cos(d*x+c)^4*\sin(d*x+c)+81 \\ & *a^6*\cos(d*x+c)^3*\sin(d*x+c)+81*a^6*\cos(d*x+c)^2*\sin(d*x+c)+63*a^6*\cos(d*x+ \\ & c)*\sin(d*x+c)+8*b^6*\cos(d*x+c)^6*\sin(d*x+c)+63*a^6*\sin(d*x+c)+8*(\cos(d*x+c) \\ & /(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*Elliptic \\ & E(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*b^6*\cos(d*x+c)^5-135*Elliptic \\ & F(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos(d*x+c))) \\ & ^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^6*\cos(d*x+c)^7+8*(\cos \\ & (d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} \\ &)*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*b^6*\cos(d*x+c)^7-27 \\ & 0*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c)/(1+\cos \\ & (d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^6*\cos(d*x+c) \\ & ^6+16*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a \\ & +b))^{1/2}*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*b^6*\cos(d* \\ & x+c)^6-135*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d*x+c) \\ & /(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*a^6*c \\ & \cos(d*x+c)^5-8*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*(\cos(d* \\ & x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*a* \\ & b^5*\cos(d*x+c)^7+741*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c)*b)/(1 \\ & +\cos(d*x+c)))/(a+b)^{1/2}*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1 \\ & /2})*a^5*b*\cos(d*x+c)^7+741*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+\cos(d*x+c) \\ &)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a \\ & +b))^{1/2})*a^4*b^2*\cos(d*x+c)^7+51*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*((a+c \end{aligned}$$

$$\begin{aligned}
& \cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+c), (- \\
& (a-b)/(a+b))^{1/2})*a^3*b^3*\cos(dx+c)^7+51*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \\
& *((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c)-\text{csc}(d \\
& *x+c), (-a-b)/(a+b))^{1/2})*a^2*b^4*\cos(dx+c)^7+8*(\cos(dx+c)/(1+\cos(dx+c) \\
&)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c) \\
&)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*a*b^5*\cos(dx+c)^7-1482*\text{EllipticF}(\cot(d \\
& x+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a \\
& +\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*a^5*b*\cos(dx+c)^6-1326*\text{Elliptic} \\
& \text{F}(\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \\
& *((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*a^4*b^2*\cos(dx+c)^6-102 \\
& *\text{EllipticF}(\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(d \\
& *x+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*a^3*b^3*\cos(dx \\
& +c)^6-4*\text{EllipticF}(\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(\\
& 1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*a^2*b^4* \\
& \cos(dx+c)^6-16*\text{EllipticF}(\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*(\cos(\\
& dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}* \\
& a*b^5*\cos(dx+c)^6+1482*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b) \\
& / (1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b)) \\
& ^{1/2})*a^5*b*\cos(dx+c)^6+1482*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(d \\
& *x+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+c), (-a-b) \\
&)/(a+b))^{1/2})*a^4*b^2*\cos(dx+c)^6+102*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}* \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+ \\
& c), (-a-b)/(a+b))^{1/2})*a^3*b^3*\cos(dx+c)^6+102*(\cos(dx+c)/(1+\cos(dx+c) \\
&))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c) \\
& -\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*a^2*b^4*\cos(dx+c)^6+16*(\cos(dx+c)/(1+co \\
& s(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(co \\
& t(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*a*b^5*\cos(dx+c)^6-741*\text{EllipticF} \\
& (\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \\
& *((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*a^5*b*\cos(dx+c)^5-663*\text{Ell} \\
& \text{ipticF}(\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c) \\
&)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*a^4*b^2*\cos(dx+c)^ \\
& 5-51*\text{EllipticF}(\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+c \\
& os(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*a^3*b^3*\cos \\
& (dx+c)^5-2*\text{EllipticF}(\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*(\cos(dx+ \\
& c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*a^2* \\
& b^4*\cos(dx+c)^5-8*\text{EllipticF}(\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b))^{1/2})*(\cos(dx+c) \\
& / (1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2})*a*b^5*\cos(dx+c)^5+741*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2} \\
& *((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+c), (-a-b)/(a+b) \\
&))^{1/2})*a^5*b*\cos(dx+c)^5+741*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(\\
& dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+c), (-a-b) \\
&)/(a+b))^{1/2})*a^4*b^2*\cos(dx+c)^5+51*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}* \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c)-\text{csc}(dx+ \\
& c), (-a-b)/(a+b))^{1/2})*a^3*b^3*\cos(dx+c)^5+51*(\cos(dx+c)/(1+\cos(dx+c) \\
&))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{1/2}*\text{EllipticE}(\cot(dx+c)-
\end{aligned}$$

```

csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * a^2*b^4*cos(d*x+c)^5+224*a^5*b*cos(d*x+c)*
sin(d*x+c)+310*a^5*b*cos(d*x+c)^3*sin(d*x+c)+274*a^4*b^2*cos(d*x+c)^3*sin(d
*x+c)+116*a^3*b^3*cos(d*x+c)^3*sin(d*x+c)+224*a^5*b*cos(d*x+c)^2*sin(d*x+c)
+274*a^4*b^2*cos(d*x+c)^2*sin(d*x+c)+876*a^5*b*cos(d*x+c)^5*sin(d*x+c)+434*
a^4*b^2*cos(d*x+c)^5*sin(d*x+c)+256*a^3*b^3*cos(d*x+c)^5*sin(d*x+c)-a^2*b^4
*cos(d*x+c)^5*sin(d*x+c)+4*a*b^5*cos(d*x+c)^5*sin(d*x+c)+135*a^5*b*cos(d*x+
c)^6*sin(d*x+c)+741*a^4*b^2*cos(d*x+c)^6*sin(d*x+c)+205*a^3*b^3*cos(d*x+c)^
6*sin(d*x+c)+51*a^2*b^4*cos(d*x+c)^6*sin(d*x+c)-4*a*b^5*cos(d*x+c)^6*sin(d*
x+c)+310*a^5*b*cos(d*x+c)^4*sin(d*x+c)+434*a^4*b^2*cos(d*x+c)^4*sin(d*x+c)+
116*a^3*b^3*cos(d*x+c)^4*sin(d*x+c)-a^2*b^4*cos(d*x+c)^4*sin(d*x+c)+8*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*
EllipticE(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * a*b^5*cos(d*x+c)^5-74
1*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * (cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^5*b*cos(d*x+
c)^7-663*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * (cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*a^4*b^2
*cos(d*x+c)^7-51*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * (cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)
*a^3*b^3*cos(d*x+c)^7-2*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)
)) * (cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)
)^(1/2)*a^2*b^4*cos(d*x+c)^7)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/cos(d*x
+c)^(11/2)/a^3

```

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{13/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos(dx + c)^{13/2}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2), x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c)
) + a)/cos(d*x + c)^(13/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{13/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(13/2), x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{13/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos^{13/2}(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{13/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\cos^{13/2}(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{13/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\cos^{13/2}(c + dx)} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(13/2),x)

[Out] int((a + b*cos(c + d*x))^(5/2)/cos(c + d*x)^(13/2), x)

$$3.626 \quad \int \frac{\cos^3(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6032
Rubi [A] (verified)	6033
Mathematica [A] (verified)	6036
Maple [B] (verified)	6037
Fricas [F]	6037
Sympy [F]	6038
Maxima [F]	6038
Giac [F]	6038
Mupad [F(-1)]	6038

Optimal result

Integrand size = 25, antiderivative size = 379

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx =$$

$$\frac{(a-b)\sqrt{a+b} \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{abd}$$

$$+ \frac{\sqrt{a+b} \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd}$$

$$+ \frac{a\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2 d}$$

$$+ \frac{\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{bd \sqrt{\cos(c+dx)}}$$

```
[Out] sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)-(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+a*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```


$eQ[c^2 - d^2, 0]$

Rule 2888

```
Int[Sqrt[(b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*
(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f
_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 2899

```
Int[((d_)*sin[(e_) + (f_)*(x_)])^(3/2)/Sqrt[(a_) + (b_)*sin[(e_) + (f
_)*(x_)]], x_Symbol] :> Dist[(-a)*(d/(2*b)), Int[Sqrt[d*Sin[e + f*x]]/Sqrt
[a + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[Sqrt[d*Sin[e + f*x]]*((a +
2*b*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3072

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(d_)*sin[(e_) + (f_)*(
x_)]]*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d),
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
```

PosQ[(c + d)/b]

Rule 3082

```

Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a +
b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)
*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(
a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}} dx}{2b} - \frac{a \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{2b} \\
&= \frac{a\sqrt{a+b}\cot(c+dx)\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d} \\
&\quad + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{2ab+2a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx}{4b} \\
&= \frac{a\sqrt{a+b}\cot(c+dx)\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d} \\
&\quad + \frac{a\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\int \frac{-2a^3+2ab^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{4b(a^2-b^2)} \\
&= \frac{a\sqrt{a+b}\cot(c+dx)\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d} \\
&\quad + \frac{a\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}} \\
&\quad - \frac{a \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{2b}
\end{aligned}$$

$$\begin{aligned}
&= \frac{a\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d} \\
&+ \frac{a\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{a\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b} - \frac{a\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b} \\
&= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{abd} \\
&+ \frac{\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd} \\
&+ \frac{a\sqrt{a+b}\cot(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d} \\
&+ \frac{a\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.59

$$\begin{aligned}
&\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{\cos^{\frac{3}{2}}(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\left((a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right) - 2a\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\right)}{2bd\left(\frac{\cos(c+dx)}{1+\cos(c+dx)}\right)^{3/2}}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (Cos[c + d*x]^(3/2)*Sec[(c + d*x)/2]^2*((a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(a + b*Cos[c + d*x])*Tan[(c + d*x)/2])/((2*b*d*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt[a + b*Cos[c + d*x]]))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. 2(351) = 702.

Time = 8.93 (sec) , antiderivative size = 834, normalized size of antiderivative = 2.20

method	result
default	$\frac{-E\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}\right)a(\cos^2(dx+c))-E\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}\right)}{1}$

[In] `int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{d} \left(-\text{EllipticE}\left(\cot(d*x+c)-\csc(d*x+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) \left(\frac{\cos(d*x+c)}{1+\cos(d*x+c)}\right)^{1/2} \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))(a+b)}\right)^{1/2} a \cos(d*x+c)^2 - \text{EllipticE}\left(\cot(d*x+c)-\csc(d*x+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) \left(\frac{\cos(d*x+c)}{1+\cos(d*x+c)}\right)^{1/2} \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))(a+b)}\right)^{1/2} b \cos(d*x+c)^2 + 2 \text{EllipticPi}\left(\cot(d*x+c)-\csc(d*x+c), -1, \left(-\frac{a-b}{a+b}\right)^{1/2}\right) \left(\frac{\cos(d*x+c)}{1+\cos(d*x+c)}\right)^{1/2} \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))(a+b)}\right)^{1/2} a \cos(d*x+c)^2 - 2 \text{EllipticE}\left(\cot(d*x+c)-\csc(d*x+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) \left(\frac{\cos(d*x+c)}{1+\cos(d*x+c)}\right)^{1/2} \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))(a+b)}\right)^{1/2} a \cos(d*x+c) - 2 \text{EllipticE}\left(\cot(d*x+c)-\csc(d*x+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) \left(\frac{\cos(d*x+c)}{1+\cos(d*x+c)}\right)^{1/2} \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))(a+b)}\right)^{1/2} b \cos(d*x+c) + 4 \text{EllipticPi}\left(\cot(d*x+c)-\csc(d*x+c), -1, \left(-\frac{a-b}{a+b}\right)^{1/2}\right) \left(\frac{\cos(d*x+c)}{1+\cos(d*x+c)}\right)^{1/2} \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))(a+b)}\right)^{1/2} a \cos(d*x+c) + b \cos(d*x+c)^2 \sin(d*x+c) - \left(\frac{\cos(d*x+c)}{1+\cos(d*x+c)}\right)^{1/2} \text{EllipticE}\left(\cot(d*x+c)-\csc(d*x+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))(a+b)}\right)^{1/2} a - \left(\frac{\cos(d*x+c)}{1+\cos(d*x+c)}\right)^{1/2} \text{EllipticE}\left(\cot(d*x+c)-\csc(d*x+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))(a+b)}\right)^{1/2} b + 2 \left(\frac{\cos(d*x+c)}{1+\cos(d*x+c)}\right)^{1/2} \text{EllipticPi}\left(\cot(d*x+c)-\csc(d*x+c), -1, \left(-\frac{a-b}{a+b}\right)^{1/2}\right) \left(\frac{a+\cos(d*x+c)*b}{(1+\cos(d*x+c))(a+b)}\right)^{1/2} a + \sin(d*x+c) \cos(d*x+c) a / (1+\cos(d*x+c)) / (a+\cos(d*x+c)*b)^{1/2} / \cos(d*x+c)^{1/2} / b \right)$$

Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}} dx$$

[In] `integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)`

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Integral(cos(c + d*x)**(3/2)/sqrt(a + b*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{\frac{3}{2}}}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(1/2), x)

[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(1/2), x)

$$3.627 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6039
Rubi [A] (verified)	6039
Mathematica [A] (verified)	6040
Maple [A] (verified)	6040
Fricas [F]	6041
Sympy [F]	6041
Maxima [F]	6041
Giac [F]	6042
Mupad [F(-1)]	6042

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd}$$

[Out] $-2*\cot(d*x+c)*\operatorname{EllipticPi}((a+b*\cos(d*x+c))^{1/2}/(a+b)^{1/2}/\cos(d*x+c)^{1/2}), (a+b)/b, ((-a-b)/(a-b))^{1/2})*(a+b)^{1/2}*(a*(1-\sec(d*x+c))/(a+b))^{1/2}*(a*(1+\sec(d*x+c))/(a-b))^{1/2}/b/d$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2888}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2\sqrt{a+b} \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{bd}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[a+b*\operatorname{Cos}[c+d*x]],x]$

[Out] $(-2*\operatorname{Sqrt}[a+b]*\operatorname{Cot}[c+d*x]*\operatorname{EllipticPi}[(a+b)/b, \operatorname{ArcSin}[\operatorname{Sqrt}[a+b*\operatorname{Cos}[c+d*x]]]/(\operatorname{Sqrt}[a+b]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])], -((a+b)/(a-b))]*\operatorname{Sqrt}[(a*(1-\operatorname{Sec}[c+d*x]))/(a+b)]*\operatorname{Sqrt}[(a*(1+\operatorname{Sec}[c+d*x]))/(a-b)]/(b*d)$

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_.)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_.)], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

integral =

$$\frac{2\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx = \frac{2\sqrt{\cos(c+dx)} \sqrt{\frac{a+b \cos(c+dx)}{(a+b)(1+\cos(c+dx))}} \left(\operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right), \frac{-a+b}{a+b}\right) - 2 \operatorname{EllipticPi}\left(-1, \arcsin\left(\frac{d \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{a+b \cos(c+dx)}}{d \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{a+b \cos(c+dx)}}\right)\right)}{d \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{a+b \cos(c+dx)}}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Cos[c + d*x]], x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x]])*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.71 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.25

method	result
default	$\frac{2\left(F\left(\cot(dx+c)-\csc(dx+c), \sqrt{-\frac{a-b}{a+b}}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c), -1, \sqrt{-\frac{a-b}{a+b}}\right)\right) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}} (1+\cos(dx+c))}{d \sqrt{a+\cos(dx+c)b} \sqrt{\cos(dx+c)}}$

```
[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2), x, method=_RETURNVERBOSE)
```

```
[Out] 2/d*(EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))-2*EllipticPi(cot
(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)/(a+cos(d*x+c)*b)^(1/2)*(1
+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}} dx$$

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx$$

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}} dx$$

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(1/2), x)

$$3.628 \quad \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx$$

Optimal result	6043
Rubi [A] (verified)	6043
Mathematica [A] (verified)	6044
Maple [A] (verified)	6044
Fricas [F]	6045
Sympy [F]	6045
Maxima [F]	6045
Giac [F]	6045
Mupad [F(-1)]	6046

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx$$

$$= \frac{2\sqrt{a+b}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

[Out] 2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2), ((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2895}

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx$$

$$= \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{ad}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]

[Out] (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d)

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_.)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rubi steps

integral

$$= \frac{2\sqrt{a+b} \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx = \frac{4(a+b) \cos^{\frac{3}{2}}(c+dx) \sqrt{-\frac{(a+b) \cot^2(\frac{1}{2}(c+dx))}{a-b}} \sqrt{\frac{(a+b \cos(c+dx)) \operatorname{csc}^2(\frac{1}{2}(c+dx))}{a}} \operatorname{csc}(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\sqrt{-\frac{(a+b) \cos(c+dx) \operatorname{csc}^2(\frac{1}{2}(c+dx))}{a}}\right)\right)}{ad \sqrt{a+b \cos(c+dx)} \left(-\frac{(a+b) \cos(c+dx) \operatorname{csc}^2(\frac{1}{2}(c+dx))}{a}\right)^{3/2}}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]

```
[Out] (-4*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[-(((a + b)*Cot[(c + d*x)/2]^2)/(a - b))]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[-((a + b*Cos[c + d*x])/(a*(-1 + Cos[c + d*x])))]], (2*a)/(a - b))]/(a*d*Sqrt[a + b*Cos[c + d*x]]*(-((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a))^(3/2)
```

Maple [A] (verified)

Time = 8.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{2(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} F\left(\cot(dx+c)-\operatorname{csc}(dx+c), \sqrt{-\frac{a-b}{a+b}}\right) \sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}}{d\sqrt{a+\cos(dx+c)b} \sqrt{\cos(dx+c)}}$	111

[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] $-2/d*(1+\cos(d*x+c))*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}/(a+\cos(d*x+c)*b)^{(1/2)}$
 $*\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}/\cos(d*x+c)^{(1/2)}$

Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}} dx$$

[In] `integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^2 + a*cos(d*x + c)), x)`

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

[In] `integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)`

[Out] `Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x))), x)`

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}} dx$$

[In] `integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)`

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}} dx$$

[In] `integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

$$3.629 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6047
Rubi [A] (verified)	6047
Mathematica [A] (verified)	6049
Maple [B] (verified)	6049
Fricas [F]	6050
Sympy [F]	6050
Maxima [F]	6050
Giac [F]	6050
Mupad [F(-1)]	6051

Optimal result

Integrand size = 25, antiderivative size = 224

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx$$

$$= \frac{2(a-b)\sqrt{a+b} \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2 d}$$

$$= \frac{2\sqrt{a+b} \cot(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

```
[Out] 2*(a-b)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2880, 2895, 3073}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx$$

$$= \frac{2(a-b)\sqrt{a+b} \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{a^2 d}$$

$$= \frac{2\sqrt{a+b} \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{ad}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]

[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d)

Rule 2880

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rubi steps

$$\begin{aligned} \text{integral} &= - \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx + \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\ &= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2d} \\ &= \frac{2\sqrt{a+b}\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad} \end{aligned}$$

Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.94

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

$$= \frac{2\left(-\left((a+b)\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\right)+a\sqrt{c}}{ad\sqrt{c}}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]

[Out] (2*(-((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a + b*Cos[c + d*x])*Tan[(c + d*x)/2])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. 2(208) = 416.

Time = 11.28 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.94

method	result
default	$\frac{2\left(\left(\csc^2(dx+c)(1-\cos(dx+c))^2-1\right)\left(-\sqrt{-\left(\csc^2(dx+c)(1-\cos(dx+c))^2+1}\right)\sqrt{\frac{\left(\csc^2(dx+c)a(1-\cos(dx+c))^2-\left(\csc^2(dx+c)b(1-\cos(dx+c))^2\right)}{a+b}}}\right)\right)}{ad\sqrt{c}}$

[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b+csc(d*x+c)^3*(1-cos(d*x+c))^3*a-csc(d*x+c)^3*(1-cos(d*x+c))^3*b+a*(csc(d*x+c)-cot(d*x+c))+b*(csc(d*x+c)-cot(d*x+c)))*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-

$\cos(dx+c)^2+1)/(-(\csc(dx+c)^2*(1-\cos(dx+c))^2-1)/(\csc(dx+c)^2*(1-\cos(dx+c))^2+1))^{3/2}/a$

Fricas [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{3}{2}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{a+b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)} dx$$

[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2)), x)

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{3}{2}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{3}{2}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^{\frac{3}{2}} \sqrt{a + b \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

$$3.630 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6052
Rubi [A] (verified)	6053
Mathematica [A] (verified)	6055
Maple [B] (verified)	6055
Fricas [F]	6056
Sympy [F]	6056
Maxima [F]	6057
Giac [F]	6057
Mupad [F(-1)]	6057

Optimal result

Integrand size = 25, antiderivative size = 274

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx =$$

$$\frac{4(a-b)b\sqrt{a+b} \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^3 d}$$

$$+ \frac{2\sqrt{a+b}(a+2b) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2 d}$$

$$+ \frac{2\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{3ad \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/3*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d/cos(d*x+c)^(3/2)-4/3*(a-b)*b*cot(
d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b
)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c
))/(a-b))^(1/2)/a^3/d+2/3*(a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2
)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(
d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d
```


Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2881, 3077, 2895, 3073}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx =$$

$$\frac{4b(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{3a^3d}$$

$$+\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{3a^2d}$$

$$+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]

[Out] (-4*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^3*d) + (2*Sqrt[a + b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^2*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sine[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

&& PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)} + \frac{2 \int \frac{-b + \frac{1}{2}a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3a} \\
 &= \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)} - \frac{(2b) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3a} \\
 &\quad + \frac{(a + 2b) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx}{3a} \\
 &= \\
 &\quad - \frac{4(a - b)b\sqrt{a + b} \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{3a^3 d} \\
 &\quad + \frac{2\sqrt{a + b}(a + 2b) \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{3a^2 d} \\
 &\quad + \frac{2\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3ad \cos^{\frac{3}{2}}(c + dx)}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 9.65 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.24

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

$$= \frac{2\left((a-2b\cos(c+dx))(a+b\cos(c+dx))\sin(c+dx) + \frac{8\cos^2\left(\frac{1}{2}(c+dx)\right)^{7/2}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\right)}{\dots}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]

[Out] (2*((a - 2*b*Cos[c + d*x])*(a + b*Cos[c + d*x])*Sin[c + d*x] + (8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(a + b)*(1 + Cos[c + d*x])])*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a - 2*b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(a + b)*(1 + Cos[c + d*x])])*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(1 + Cos[c + d*x])^(3/2))/(3*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1183 vs. 2(248) = 496.

Time = 13.14 (sec) , antiderivative size = 1184, normalized size of antiderivative = 4.32

method	result	size
default	Expression too large to display	1184

[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a*b*cos(d*x+c)^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a*b*cos(d*x+c)^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*b^2*cos(d*x+c)^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)^2-4*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))

$$\begin{aligned} &^{(1/2)} * a * b * \cos(d * x + c) ^ 2 + 4 * \text{EllipticE}(\cot(d * x + c) - \csc(d * x + c), (- (a - b) / (a + b)) ^ (1 / 2)) * ((a + \cos(d * x + c) * b) / (1 + \cos(d * x + c)) / (a + b)) ^ (1 / 2) * (\cos(d * x + c) / (1 + \cos(d * x + c))) ^ (1 / 2) * a * b * \cos(d * x + c) ^ 2 + 4 * (\cos(d * x + c) / (1 + \cos(d * x + c))) ^ (1 / 2) * ((a + \cos(d * x + c) * b) / (1 + \cos(d * x + c)) / (a + b)) ^ (1 / 2) * \text{EllipticE}(\cot(d * x + c) - \csc(d * x + c), (- (a - b) / (a + b)) ^ (1 / 2)) * b ^ 2 * \cos(d * x + c) ^ 2 + (\cos(d * x + c) / (1 + \cos(d * x + c))) ^ (1 / 2) * ((a + \cos(d * x + c) * b) / (1 + \cos(d * x + c)) / (a + b)) ^ (1 / 2) * \text{EllipticF}(\cot(d * x + c) - \csc(d * x + c), (- (a - b) / (a + b)) ^ (1 / 2)) * a ^ 2 * \cos(d * x + c) - 2 * \text{EllipticF}(\cot(d * x + c) - \csc(d * x + c), (- (a - b) / (a + b)) ^ (1 / 2)) * ((a + \cos(d * x + c) * b) / (1 + \cos(d * x + c)) / (a + b)) ^ (1 / 2) * (\cos(d * x + c) / (1 + \cos(d * x + c))) ^ (1 / 2) * a * b * \cos(d * x + c) + 2 * \text{EllipticE}(\cot(d * x + c) - \csc(d * x + c), (- (a - b) / (a + b)) ^ (1 / 2)) * ((a + \cos(d * x + c) * b) / (1 + \cos(d * x + c)) / (a + b)) ^ (1 / 2) * (\cos(d * x + c) / (1 + \cos(d * x + c))) ^ (1 / 2) * a * b * \cos(d * x + c) + 2 * (\cos(d * x + c) / (1 + \cos(d * x + c))) ^ (1 / 2) * ((a + \cos(d * x + c) * b) / (1 + \cos(d * x + c)) / (a + b)) ^ (1 / 2) * \text{EllipticE}(\cot(d * x + c) - \csc(d * x + c), (- (a - b) / (a + b)) ^ (1 / 2)) * b ^ 2 * \cos(d * x + c) - \cos(d * x + c) ^ 2 * \sin(d * x + c) * a * b + 2 * b ^ 2 * \cos(d * x + c) ^ 2 * \sin(d * x + c) - a ^ 2 * \cos(d * x + c) * \sin(d * x + c) + a * b * \cos(d * x + c) * \sin(d * x + c) - a ^ 2 * \sin(d * x + c) / (1 + \cos(d * x + c)) / (a + \cos(d * x + c) * b) ^ (1 / 2) / \cos(d * x + c) ^ (3 / 2) / a ^ 2 \end{aligned}$$

Fricas [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\sqrt{b \cos(dx + c) + a \cos(dx + c)}^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)

Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\sqrt{a + b \cos(c + dx)} \cos^{\frac{5}{2}}(c + dx)} dx$$

[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(5/2)), x)

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{5}{2}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{5}{2}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(1/2)), x)

$$3.631 \quad \int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	6058
Rubi [A] (verified)	6059
Mathematica [C] (verified)	6062
Maple [B] (warning: unable to verify)	6063
Fricas [F]	6065
Sympy [F(-1)]	6065
Maxima [F]	6065
Giac [F]	6065
Mupad [F(-1)]	6066

Optimal result

Integrand size = 25, antiderivative size = 465

$$\int \frac{\cos^5(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx =$$

$$\frac{(3a^2 - b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ab^2 \sqrt{a+bd}}$$

$$+ \frac{(3a+b) \cot(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2 \sqrt{a+bd}}$$

$$+ \frac{3a\sqrt{a+b} \cot(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3 d}$$

$$- \frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{b(a^2 - b^2) d \sqrt{a+b \cos(c+dx)}} + \frac{(3a^2 - b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{b^2 (a^2 - b^2) d \sqrt{\cos(c+dx)}}$$

```
[Out] -2*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+(3*
a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/cos(d*x+c)^(1/2)
-(3*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c)))/(a-b))^(1/2)/a/b^2/d/(a+b)^(1/2)+(3*a+b)*cot(d*x+c)*EllipticF((a+b*c
os(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-s
ec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)+3*
a*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2)
,(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(
a*(1+sec(d*x+c)))/(a-b))^(1/2)/b^3/d
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2871, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx =$$

$$\frac{(3a^2 - b^2) \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right)}{ab^2 d \sqrt{a+b}}$$

$$- \frac{2a^2 \sin(c+dx) \sqrt{\cos(c+dx)}}{bd(a^2 - b^2) \sqrt{a+b\cos(c+dx)}} + \frac{(3a^2 - b^2) \sin(c+dx) \sqrt{a+b\cos(c+dx)}}{b^2 d (a^2 - b^2) \sqrt{\cos(c+dx)}}$$

$$+ \frac{3a\sqrt{a+b} \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{b^3 d}$$

$$+ \frac{(3a+b) \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{b^2 d \sqrt{a+b}}$$

[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2), x]

[Out] -(((3*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*b^2*Sqrt[a + b]*d) + ((3*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b^2*Sqrt[a + b]*d) + (3*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b^3*d) - (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]))

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b

$\wedge 2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 2] \&\& \text{LtQ}[n, -1] \&\& (\text{IntegerQ}[m] \mid \mid \text{IntegersQ}[2*m, 2*n])$

Rule 2888

$\text{Int}[\text{Sqrt}[(b_.) * \sin[(e_.) + (f_.) * (x_)]]/\text{Sqrt}[(c_.) + (d_.) * \sin[(e_.) + (f_.) * (x_)]], x_Symbol] \rightarrow \text{Simp}[2*b*(\text{Tan}[e + f*x]/(d*f))*\text{Rt}[(c + d)/b, 2]*\text{Sqrt}[c*((1 + \text{Csc}[e + f*x])/(c - d))]*\text{Sqrt}[c*((1 - \text{Csc}[e + f*x])/(c + d))]*\text{EllipticPi}[(c + d)/d, \text{ArcSin}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[b*\text{Sin}[e + f*x]]/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d), x] /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(c + d)/b]$

Rule 2895

$\text{Int}[1/(\text{Sqrt}[(d_.) * \sin[(e_.) + (f_.) * (x_)]]) * \text{Sqrt}[(a_.) + (b_.) * \sin[(e_.) + (f_.) * (x_)]], x_Symbol] \rightarrow \text{Simp}[-2*(\text{Tan}[e + f*x]/(a*f))*\text{Rt}[(a + b)/d, 2]*\text{Sqrt}[a*((1 - \text{Csc}[e + f*x])/(a + b))]*\text{Sqrt}[a*((1 + \text{Csc}[e + f*x])/(a - b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[d*\text{Sin}[e + f*x]]/\text{Rt}[(a + b)/d, 2]], -(a + b)/(a - b), x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{PosQ}[(a + b)/d]$

Rule 3073

$\text{Int}[(A_.) + (B_.) * \sin[(e_.) + (f_.) * (x_)]/((b_.) * \sin[(e_.) + (f_.) * (x_)])^{(3/2)} * \text{Sqrt}[(c_.) + (d_.) * \sin[(e_.) + (f_.) * (x_)]], x_Symbol] \rightarrow \text{Simp}[-2*A*(c - d)*(\text{Tan}[e + f*x]/(f*b*c^2))*\text{Rt}[(c + d)/b, 2]*\text{Sqrt}[c*((1 + \text{Csc}[e + f*x])/(c - d))]*\text{Sqrt}[c*((1 - \text{Csc}[e + f*x])/(c + d))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[b*\text{Sin}[e + f*x]]/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d), x] /; \text{FreeQ}\{b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{EqQ}[A, B] \&\& \text{PosQ}[(c + d)/b]$

Rule 3077

$\text{Int}[(A_.) + (B_.) * \sin[(e_.) + (f_.) * (x_)]/((a_.) + (b_.) * \sin[(e_.) + (f_.) * (x_)])^{(3/2)} * \text{Sqrt}[(c_.) + (d_.) * \sin[(e_.) + (f_.) * (x_)]], x_Symbol] \rightarrow \text{Dist}[(A - B)/(a - b), \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]] * \text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] - \text{Dist}[(A*b - a*B)/(a - b), \text{Int}[(1 + \text{Sin}[e + f*x])/(a + b*\text{Sin}[e + f*x])^{(3/2)} * \text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[A, B]$

Rule 3132

$\text{Int}[(A_.) + (B_.) * \sin[(e_.) + (f_.) * (x_)] + (C_.) * \sin[(e_.) + (f_.) * (x_)]^2/((a_.) + (b_.) * \sin[(e_.) + (f_.) * (x_)])^{(3/2)} * \text{Sqrt}[(c_.) + (d_.) * \sin[(e_.) + (f_.) * (x_)]], x_Symbol] \rightarrow \text{Dist}[C/b^2, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/$

Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3140

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int\frac{\frac{a^2}{2}-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}(3a^2-b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{b(a^2-b^2)} \\
 &= -\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} + \frac{(3a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d\sqrt{\cos(c+dx)}} \\
 &\quad - \frac{\int\frac{\frac{1}{2}a(3a^2-b^2)+a^2b\cos(c+dx)+\frac{3}{2}a(a^2-b^2)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{b^2(a^2-b^2)} \\
 &= -\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} + \frac{(3a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d\sqrt{\cos(c+dx)}} \\
 &\quad - \frac{(3a)\int\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx}{2b^2} - \frac{\int\frac{\frac{1}{2}a(3a^2-b^2)+a^2b\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{b^2(a^2-b^2)} \\
 &= \frac{3a\sqrt{a+b}\cot(c+dx)\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3d} \\
 &\quad - \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} + \frac{(3a^2-b^2)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d\sqrt{\cos(c+dx)}} \\
 &\quad + \frac{(a(3a+b))\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{2b^2(a+b)} \\
 &\quad - \frac{(a(3a^2-b^2))\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{2b^2(a^2-b^2)}
 \end{aligned}$$

$$\begin{aligned}
&= - \frac{(3a^2 - b^2) \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ab^2 \sqrt{a+bd}} \\
&+ \frac{(3a+b) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2 \sqrt{a+bd}} \\
&+ \frac{3a\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3 d} \\
&- \frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{b(a^2 - b^2) d \sqrt{a+b} \cos(c+dx)} + \frac{(3a^2 - b^2) \sqrt{a+b} \cos(c+dx) \sin(c+dx)}{b^2 (a^2 - b^2) d \sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 1201, normalized size of antiderivative = 2.58

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = \frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{b(-a^2 + b^2) d \sqrt{a+b} \cos(c+dx)}$$

$$\frac{4a(a^2 - b^2) \sqrt{\frac{(a+b) \cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}} \sqrt{-\frac{(a+b) \cos(c+dx) \csc^2\left(\frac{1}{2}(c+dx)\right)}{a}} \sqrt{\frac{(a+b \cos(c+dx)) \csc^2\left(\frac{1}{2}(c+dx)\right)}{a}} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{(a+b \cos(c+dx)) \csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\right)\right)}{(a+b) \sqrt{\cos(c+dx)} \sqrt{a+b} \cos(c+dx)}$$

+

[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(-a^2 + b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + ((-4*a*(a^2 - b^2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a]]*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 8*a^2*b*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a]]*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[

$$\begin{aligned}
& -(((a + b) \cos[c + dx] \operatorname{Csc}[(c + dx)/2]^2/a)] \operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a] \operatorname{Csc}[c + dx] \operatorname{EllipticPi}[-(a/b), \operatorname{ArcSin}[\operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a] / \operatorname{Sqrt}[2]], (-2a)/(-a + b)] \operatorname{Sin}[(c + dx)/2]^4 / (b \operatorname{Sqrt}[\cos[c + dx]] \operatorname{Sqrt}[a + b \cos[c + dx]]) + 2(3a^2 - b^2) \\
& * ((I \cos[(c + dx)/2] \operatorname{Sqrt}[a + b \cos[c + dx]] \operatorname{EllipticE}[I \operatorname{ArcSinh}[\operatorname{Sin}[(c + dx)/2] / \operatorname{Sqrt}[\cos[c + dx]]], (-2a)/(-a - b)] \operatorname{Sec}[c + dx]) / (b \operatorname{Sqrt}[\cos[(c + dx)/2]^2 \operatorname{Sec}[c + dx]] \operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Sec}[c + dx]) / (a + b)) \\
& + (2a * ((a \operatorname{Sqrt}[(a + b) \operatorname{Cot}[(c + dx)/2]^2 / (-a + b)] \operatorname{Sqrt}[-((a + b) \cos[c + dx] \operatorname{Csc}[(c + dx)/2]^2/a)] \operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a] \operatorname{Csc}[c + dx] \operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a] / \operatorname{Sqrt}[2]], (-2a)/(-a + b)] \operatorname{Sin}[(c + dx)/2]^4 / ((a + b) \operatorname{Sqrt}[\cos[c + dx]] \operatorname{Sqrt}[a + b \cos[c + dx]]) - (a \operatorname{Sqrt}[(a + b) \operatorname{Cot}[(c + dx)/2]^2 / (-a + b)] \operatorname{Sqrt}[-((a + b) \cos[c + dx] \operatorname{Csc}[(c + dx)/2]^2/a)] \operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a] \operatorname{Csc}[c + dx] \operatorname{EllipticPi}[-(a/b), \operatorname{ArcSin}[\operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a] / \operatorname{Sqrt}[2]], (-2a)/(-a + b)] \operatorname{Sin}[(c + dx)/2]^4 / (b \operatorname{Sqrt}[\cos[c + dx]] \operatorname{Sqrt}[a + b \cos[c + dx]])) / b + (\operatorname{Sqrt}[a + b \cos[c + dx]] \operatorname{Sin}[c + dx]) / (b \operatorname{Sqrt}[\cos[c + dx]]) / (2(a - b) * b * (a + b) * d)
\end{aligned}$$

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2525 vs. $2(433) = 866$.

Time = 9.31 (sec) , antiderivative size = 2526, normalized size of antiderivative = 5.43

method	result	size
default	Expression too large to display	2526

[In] $\int (\cos(dx+c))^{5/2} / (a+\cos(dx+c)*b)^{3/2}, x, \text{method}=_RETURNVERBOSE)$

[Out] $1/d * (-(\operatorname{csc}(dx+c))^{2*(1-\cos(dx+c))^{-2-1}} / (\operatorname{csc}(dx+c)^{2*(1-\cos(dx+c))^{-2+1}})^{5/2} * (\operatorname{csc}(dx+c)^{2*(1-\cos(dx+c))^{-2+1}})^{2*((\operatorname{csc}(dx+c)^{2*a*(1-\cos(dx+c))^{-2-\operatorname{csc}(dx+c)^{2*b*(1-\cos(dx+c))^{-2+a+b}} / (\operatorname{csc}(dx+c)^{2*(1-\cos(dx+c))^{-2+1}})^{(1/2)} * (-3*a^3*(\operatorname{csc}(dx+c)-\cot(dx+c))+b^3*(\operatorname{csc}(dx+c)-\cot(dx+c))-6*(-\operatorname{csc}(dx+c)^{2*(1-\cos(dx+c))^{-2+1}})^{(1/2)} * ((\operatorname{csc}(dx+c)^{2*a*(1-\cos(dx+c))^{-2-\operatorname{csc}(dx+c)^{2*b*(1-\cos(dx+c))^{-2+a+b}} / (a+b))^{(1/2)} * \operatorname{EllipticPi}(\cot(dx+c)-\operatorname{csc}(dx+c), -1, (-a-b)/(a+b))^{(1/2)}) * a^3 + 3*\operatorname{csc}(dx+c)^2 * \operatorname{EllipticE}(\cot(dx+c)-\operatorname{csc}(dx+c), (-a-b)/(a+b))^{(1/2)}) * a^3 * (-\operatorname{csc}(dx+c)^{2*(1-\cos(dx+c))^{-2+1}})^{(1/2)} * ((\operatorname{csc}(dx+c)^{2*a*(1-\cos(dx+c))^{-2-\operatorname{csc}(dx+c)^{2*b*(1-\cos(dx+c))^{-2+a+b}} / (a+b))^{(1/2)} * (1-\cos(dx+c))^{-2-\operatorname{csc}(dx+c)^2 * \operatorname{EllipticE}(\cot(dx+c)-\operatorname{csc}(dx+c), (-a-b)/(a+b))^{(1/2)}) * b^3 * (-\operatorname{csc}(dx+c)^{2*(1-\cos(dx+c))^{-2+1}})^{(1/2)} * ((\operatorname{csc}(dx+c)^{2*a*(1-\cos(dx+c))^{-2-\operatorname{csc}(dx+c)^{2*b*(1-\cos(dx+c))^{-2+a+b}} / (a+b))^{(1/2)} * (1-\cos(dx+c))^{-2-6*\operatorname{csc}(dx+c)^2 * (-\operatorname{csc}(dx+c)^{2*(1-\cos(dx+c))^{-2+1}})^{(1/2)} * ((\operatorname{csc}(dx+c)^{2*a*(1-\cos(dx+c))^{-2-\operatorname{csc}(dx+c)^{2*b*(1-\cos(dx+c))^{-2+a+b}} / (a+b))^{(1/2)} * \operatorname{EllipticPi}(\cot(dx+c)-\operatorname{csc}(dx+c), -1, (-a-b)/(a+b))^{(1/2)}) * a^3 * (1-\cos(dx+c))^{-2+a*b^2 * (\operatorname{csc}(dx+c)-\cot(dx+c))+\operatorname{csc}(dx+c)^5 * b^3 * (1-\cos(dx+c))^{-5+2*\operatorname{csc}(dx+c}}$

$$\begin{aligned}
&)^3 a^2 b (1 - \cos(dx+c))^{-3-2} (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^2 b^{-2} (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2}) * a b^2 + 3 (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^2 b - (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2}) * a b^2 + 6 (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * \text{EllipticPi}(\cot(dx+c) - \csc(dx+c), -1, (-a-b)/(a+b))^{1/2}) * a b^2 - \csc(dx+c)^5 a b^2 (1 - \cos(dx+c))^{-5-3} \csc(dx+c)^5 a^2 b (1 - \cos(dx+c))^{-5+3} (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^3 - (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2}) * b^3 + 3 \csc(dx+c)^5 a^3 (1 - \cos(dx+c))^{-5-2} \csc(dx+c)^3 b^3 (1 - \cos(dx+c))^{-3+a^2 b} (\csc(dx+c) - \cot(dx+c))^{-2} \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^2 b (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * (1 - \cos(dx+c))^{-2-2} \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2}) * a b^2 (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * (1 - \cos(dx+c))^{-2+3} \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^2 b (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * (1 - \cos(dx+c))^{-2} \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2}) * a b^2 (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * (1 - \cos(dx+c))^{-2+6} \csc(dx+c)^2 * \text{EllipticPi}(\cot(dx+c) - \csc(dx+c), -1, (-a-b)/(a+b))^{1/2}) * a b^2 (-\csc(dx+c)^2 (1 - \cos(dx+c))^{-2+1})^{1/2} * ((\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / (a+b))^{1/2} * (1 - \cos(dx+c))^{-2} / (\csc(dx+c)^2 (1 - \cos(dx+c))^{-2-1})^3 / (\csc(dx+c)^2 a (1 - \cos(dx+c))^{-2} - \csc(dx+c)^2 b (1 - \cos(dx+c))^{-2+a+b}) / b^2 / (a-b) / (a+b)
\end{aligned}$$

Fricas [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^{5/2}}{(a + b \cos(c + dx))^{3/2}} dx$$

```
[In] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(3/2), x)
```

$$3.632 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^{\frac{3}{2}}} dx$$

Optimal result	6067
Rubi [A] (verified)	6068
Mathematica [A] (verified)	6070
Maple [B] (warning: unable to verify)	6071
Fricas [F]	6071
Sympy [F]	6072
Maxima [F]	6072
Giac [F]	6072
Mupad [F(-1)]	6072

Optimal result

Integrand size = 25, antiderivative size = 387

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^{\frac{3}{2}}} dx = \frac{2 \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b \sqrt{a+bd}}$$

$$- \frac{2 \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b \sqrt{a+bd}}$$

$$- \frac{2 \sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2 d}$$

$$- \frac{2a^2 \sin(c+dx)}{b(a^2-b^2) d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

```
[Out] -2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b)^(1/2))*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b)^(1/2))*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/(a+b)^(1/2)-2*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b)^(1/2))*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2876, 2888, 2873, 2874, 2895, 3073}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = -\frac{2a^2 \sin(c+dx)}{bd(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} \\ - \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{b^2d} \\ - \frac{2\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}} \\ + \frac{2\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}$$

[In] Int[Cos[c + d*x]^(3/2)/(a + b*cos[c + d*x])^(3/2), x]

[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b*Sqrt[a + b]*d) - (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b*Sqrt[a + b]*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b^2*d) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*cos[c + d*x]])

Rule 2873

Int[Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 2874

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b

$^2, 0]$ && NeQ[$c^2 - d^2, 0]$

Rule 2876

Int[((d_)*sin[(e_.) + (f_)*(x_)])^(3/2)/((a_) + (b_)*sin[(e_.) + (f_)*(x_)])^(3/2), x_Symbol] :> Dist[d/b, Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[a*(d/b), Int[Sqrt[d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 2888

Int[Sqrt[(b_)*sin[(e_.) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_.) + (f_)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_.) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_.) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_.) + (f_)*(x_)])/(((b_)*sin[(e_.) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_.) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{b} - \frac{a \int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx}{b}$$

$$\begin{aligned}
&= \frac{2\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2 d} \\
&- \frac{2a^2 \sin(c+dx)}{b(a^2-b^2) d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} + \frac{a \int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx}{b(a^2-b^2)} \\
&= \frac{2\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2 d} \\
&- \frac{2a^2 \sin(c+dx)}{b(a^2-b^2) d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} \\
&- \frac{a \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx}{b(a+b)} + \frac{a^2 \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{b(a^2-b^2)} \\
&= \frac{2 \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+bd}} \\
&- \frac{2 \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+bd}} \\
&- \frac{2\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2 d} \\
&- \frac{2a^2 \sin(c+dx)}{b(a^2-b^2) d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.19 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.73

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = \frac{\sqrt{\cos(c+dx)} \left(-2a(a+b) \sqrt{\frac{a+b \cos(c+dx)}{(a+b)(1+\cos(c+dx))}} E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right) \middle| \frac{-a+b}{a+b}\right)\right)}{b^2 d}$$

[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (Sqrt[Cos[c + d*x]]*(-2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*b*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*(a - b)*(-2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2]))/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(359) = 718.

Time = 9.77 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.68

method	result	size
default	Expression too large to display	1036

[In] `int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{2}{d} \frac{(-\csc(d*x+c)^2(1-\cos(d*x+c))^{2-1}) / (\csc(d*x+c)^2(1-\cos(d*x+c))^{2+1})^{3/2} \cdot (\csc(d*x+c)^2(1-\cos(d*x+c))^{2+1})^{2+1} \cdot ((\csc(d*x+c)^2 a (1-\cos(d*x+c))^{2-1} - \csc(d*x+c)^2 b (1-\cos(d*x+c))^{2+a+b}) / (\csc(d*x+c)^2(1-\cos(d*x+c))^{2+1})^{1/2} \cdot (-(-\csc(d*x+c)^2(1-\cos(d*x+c))^{2+1})^{1/2} \cdot ((\csc(d*x+c)^2 a (1-\cos(d*x+c))^{2-1} - \csc(d*x+c)^2 b (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} \cdot \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot a \cdot b - (-\csc(d*x+c)^2(1-\cos(d*x+c))^{2+1})^{1/2} \cdot ((\csc(d*x+c)^2 a (1-\cos(d*x+c))^{2-1} - \csc(d*x+c)^2 b (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} \cdot \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot b^2 + (-\csc(d*x+c)^2(1-\cos(d*x+c))^{2+1})^{1/2} \cdot ((\csc(d*x+c)^2 a (1-\cos(d*x+c))^{2-1} - \csc(d*x+c)^2 b (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} \cdot \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot a^2 + (-\csc(d*x+c)^2(1-\cos(d*x+c))^{2+1})^{1/2} \cdot ((\csc(d*x+c)^2 a (1-\cos(d*x+c))^{2-1} - \csc(d*x+c)^2 b (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} \cdot \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) \cdot a \cdot b - 2 \cdot (-\csc(d*x+c)^2(1-\cos(d*x+c))^{2+1})^{1/2} \cdot ((\csc(d*x+c)^2 a (1-\cos(d*x+c))^{2-1} - \csc(d*x+c)^2 b (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} \cdot \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) \cdot a^2 + 2 \cdot (-\csc(d*x+c)^2(1-\cos(d*x+c))^{2+1})^{1/2} \cdot ((\csc(d*x+c)^2 a (1-\cos(d*x+c))^{2-1} - \csc(d*x+c)^2 b (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} \cdot \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) \cdot b^2 + \csc(d*x+c)^3 a^2 (1-\cos(d*x+c))^3 - \csc(d*x+c)^3 a \cdot b (1-\cos(d*x+c))^3 - a^2 (\csc(d*x+c) - \cot(d*x+c)) + a \cdot b (\csc(d*x+c) - \cot(d*x+c)) / (\csc(d*x+c)^2(1-\cos(d*x+c))^{2-1})^{1/2} \cdot 2 / (\csc(d*x+c)^2 a (1-\cos(d*x+c))^{2-1} - \csc(d*x+c)^2 b (1-\cos(d*x+c))^{2+a+b}) / b / (a-b) / (a+b)$$

Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}} dx$$

[In] `integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)`

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2), x)

[Out] Integral(cos(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{\cos(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{\cos(c + dx)^{\frac{3}{2}}}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx$$

[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(3/2), x)

[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(3/2), x)

$$3.633 \quad \int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	6073
Rubi [A] (verified)	6074
Mathematica [A] (verified)	6075
Maple [B] (verified)	6076
Fricas [F]	6076
Sympy [F]	6077
Maxima [F]	6077
Giac [F]	6077
Mupad [F(-1)]	6077

Optimal result

Integrand size = 25, antiderivative size = 266

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^{3/2}} dx =$$

$$\frac{2 \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+bd}}$$

$$+ \frac{2 \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+bd}}$$

$$+ \frac{2a \sin(c+dx)}{(a^2-b^2)d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)-2*cot(d*
x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/
(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)
/a/d/(a+b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1
+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2873, 2874, 2895, 3073}

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \frac{2a \sin(c+dx)}{d(a^2-b^2) \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} + \frac{2 \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}} - \frac{2 \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$

[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (-2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*Sqrt[a + b]*d) + (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*Sqrt[a + b]*d) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Rule 2873

Int[Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 2874

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a \sin(c + dx)}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} - \frac{\int \frac{\sqrt{a+b \cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx}{a^2 - b^2} \\
&= \frac{2a \sin(c + dx)}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{\int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx}{a + b} - \frac{a \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{a^2 - b^2} \\
&= -\frac{2 \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a \sqrt{a + bd}} \\
&\quad + \frac{2 \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a \sqrt{a + bd}} \\
&\quad + \frac{2a \sin(c + dx)}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\cos(c + dx)}}{(a + b \cos(c + dx))^{3/2}} dx = \frac{2 \left((a + b) \sqrt{1 + \cos(c + dx)} \sqrt{\frac{a+b \cos(c+dx)}{(a+b)(1+\cos(c+dx))}} E\left(\arcsin\left(\tan\left(\frac{1}{2}(c + dx)\right)\right) \middle| -\frac{a}{a+b}\right) \right)}{a^2 - b^2}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(3/2), x]
```

```
[Out] (2*((a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (a
```

+ b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a - b)*Sqrt[Cos[c + d*x]]*Tan[(c + d*x)/2])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(246) = 492.

Time = 6.68 (sec) , antiderivative size = 783, normalized size of antiderivative = 2.94

method	result
default	$2\sqrt{-\frac{(\csc^2(dx+c)(1-\cos(dx+c))^2-1)}{(\csc^2(dx+c)(1-\cos(dx+c))^2+1)}}\left((\csc^2(dx+c)(1-\cos(dx+c))^2+1)\sqrt{\frac{(\csc^2(dx+c)a(1-\cos(dx+c))^2-(\csc^2(dx+c)b(1-\cos(dx+c))^2+a)}{(\csc^2(dx+c)(1-\cos(dx+c))^2+1)}}}\right)$

[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)

[Out] $2/d*(-(\csc(d*x+c)^2*(1-\cos(d*x+c))^2-1)/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1))^{1/2}*(\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1))^{1/2}*(-(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{1/2}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{1/2}*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*a-(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{1/2}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{1/2}*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*b+(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{1/2}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{1/2}*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*a+(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{1/2}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{1/2}*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*b+csc(d*x+c)^3*(1-\cos(d*x+c))^3*a-\csc(d*x+c)^3*(1-\cos(d*x+c))^3*b-a*(\csc(d*x+c)-\cot(d*x+c))+b*(\csc(d*x+c)-\cot(d*x+c)))/(\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2-1)/(a-b)/(a+b)$

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{3/2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx$$

[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2), x)

[Out] Integral(sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{3/2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{3/2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx$$

[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(3/2), x)

[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(3/2), x)

$$3.634 \quad \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	6078
Rubi [A] (verified)	6078
Mathematica [A] (verified)	6080
Maple [B] (verified)	6081
Fricas [F]	6081
Sympy [F]	6082
Maxima [F]	6082
Giac [F]	6082
Mupad [F(-1)]	6082

Optimal result

Integrand size = 25, antiderivative size = 267

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^{3/2}} dx = \frac{2b \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{a^2 \sqrt{a+bd}} + \frac{2 \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a \sqrt{a+bd}} - \frac{2b \sin(c+dx)}{(a^2-b^2) d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

```
[Out] -2*b*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2*b*cot
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1
/2)/a^2/d/(a+b)^(1/2)+2*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*
(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

= {2879, 3077, 2895, 3073}

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx = \frac{2b \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{a^2 d \sqrt{a+b}} - \frac{2b \sin(c+dx)}{d(a^2-b^2) \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} + \frac{2 \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right), -\frac{a+b}{a-b}}{ad\sqrt{a+b}}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)), x]

[Out] (2*b*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a^2*Sqrt[a + b]*d) + (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*Sqrt[a + b]*d) - (2*b*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Rule 2879

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)), x_Symbol] :> Simp[2*b*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(b + a*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&

PosQ[(c + d)/b]

Rule 3077

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} + \frac{\int \frac{b+a \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{a^2 - b^2} \\ &= -\frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} \\ &\quad + \frac{\int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx}{a + b} + \frac{b \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{a^2 - b^2} \\ &= \frac{2b \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2 \sqrt{a + bd}} \\ &\quad + \frac{2 \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a \sqrt{a + bd}} \\ &\quad - \frac{2b \sin(c + dx)}{(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 4.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{\cos(c + dx)} (a + b \cos(c + dx))^{3/2}} dx = \frac{2 \left(-b(a + b) \sqrt{1 + \cos(c + dx)} \sqrt{\frac{a + b \cos(c + dx)}{(a + b)(1 + \cos(c + dx))}} E\left(\arcsin\left(\tan\left(\frac{c + dx}{2}\right)\right)\right) \right)}{(a + b)^2 \sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x]

[Out] (2*(-(b*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])])]*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)) + a*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])])]*EllipticF[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)] + b*(-a + b)*Sqrt[Cos[c + d*x]]*Tan[(c + d*x)/2])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(247) = 494$.

Time = 10.47 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.81

method	result
default	$2 \left(-\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{(\csc^2(dx+c))^a(1-\cos(dx+c))^2-(\csc^2(dx+c))^b(1-\cos(dx+c))^2+a+b}{a+b}} F\left(\cot(dx+c)-\csc(dx+c), \dots \right) \right)$

[In] `int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{2}{d} \left(-\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{(\csc^2(dx+c))^a(1-\cos(dx+c))^2-(\csc^2(dx+c))^b(1-\cos(dx+c))^2+a+b}{a+b}} F\left(\cot(dx+c)-\csc(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) a^2 - \sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{(\csc^2(dx+c))^a(1-\cos(dx+c))^2-(\csc^2(dx+c))^b(1-\cos(dx+c))^2+a+b}{a+b}} F\left(\cot(dx+c)-\csc(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) a*b + \sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{(\csc^2(dx+c))^a(1-\cos(dx+c))^2-(\csc^2(dx+c))^b(1-\cos(dx+c))^2+a+b}{a+b}} E\left(\cot(dx+c)-\csc(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) a*b + \sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{(\csc^2(dx+c))^a(1-\cos(dx+c))^2-(\csc^2(dx+c))^b(1-\cos(dx+c))^2+a+b}{a+b}} E\left(\cot(dx+c)-\csc(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) b^2 + \csc^3(dx+c) * a*b*(1-\cos(dx+c))^3 - \csc^3(dx+c) * b^2*(1-\cos(dx+c))^3 - a*b*(\csc(dx+c) - \cot(dx+c)) + b^2*(\csc(dx+c) - \cot(dx+c)) \right) * \left(\frac{\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{(\csc^2(dx+c))^a(1-\cos(dx+c))^2-(\csc^2(dx+c))^b(1-\cos(dx+c))^2+a+b}{a+b}}}{\csc^2(dx+c) * (1-\cos(dx+c))^2+1} \right)^{1/2} / \left(\frac{\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{(\csc^2(dx+c))^a(1-\cos(dx+c))^2-(\csc^2(dx+c))^b(1-\cos(dx+c))^2+a+b}{a+b}}}{\csc^2(dx+c) * (1-\cos(dx+c))^2-1} \right)^{1/2} / (a+b) / (a-b) / a$$

Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{3/2} \sqrt{\cos(dx+c)}} dx$$

[In] `integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*cos(d*x+c)+a)*sqrt(cos(d*x+c))/(b^2*cos(d*x+c)^3+2*a*b*cos(d*x+c)^2+a^2*cos(d*x+c)),x)`

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(a+b\cos(c+dx))^{3/2} \sqrt{\cos(c+dx)}} dx$$

[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2), x)

[Out] Integral(1/((a + b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{3/2} \sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{3/2} \sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)), x)

[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(3/2)), x)

$$3.635 \quad \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	6083
Rubi [A] (verified)	6083
Mathematica [C] (verified)	6086
Maple [B] (verified)	6087
Fricas [F]	6088
Sympy [F]	6088
Maxima [F]	6088
Giac [F]	6088
Mupad [F(-1)]	6089

Optimal result

Integrand size = 25, antiderivative size = 285

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^{3/2}} dx = \frac{2(a^2 - 2b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{a^3 \sqrt{a+bd}} - \frac{2(a+2b) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2 \sqrt{a+bd}} + \frac{2b^2 \sin(c+dx)}{a(a^2 - b^2) d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2*(a^2-2*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)-2*(a+2*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

= {2881, 3077, 2895, 3073}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{3}{2}}} dx =$$

$$\frac{2(a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}}$$

$$+ \frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$+ \frac{2(a^2-2b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{a^3d\sqrt{a+b}}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)), x]

[Out] (2*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a^3*Sqrt[a + b]*d) - (2*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a^2*Sqrt[a + b]*d) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} + \frac{2 \int \frac{\frac{1}{2}(a^2 - 2b^2) - \frac{1}{2}ab \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{a(a^2 - b^2)} \\
&= \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{(a + 2b) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx}{a(a + b)} + \frac{(a^2 - 2b^2) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{a(a^2 - b^2)} \\
&= \frac{2(a^2 - 2b^2) \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{a^3 \sqrt{a + bd}} \\
&\quad - \frac{2(a + 2b) \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{a^2 \sqrt{a + bd}} \\
&\quad + \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.36 (sec) , antiderivative size = 1233, normalized size of antiderivative = 4.33

$$4a(2a^2b-2b^3)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b)\cos(c+dx)}{(a+b)\sqrt{\dots}}}$$

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{3}{2}}} dx = \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{2b^3\sin(c+dx)}{a^2(a^2-b^2)(a+b\cos(c+dx))} + \frac{2\tan(c+dx)}{a^2}\right)}{d}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x]

[Out] ((-4*a*(2*a^2*b - 2*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(a^3 - 2*a*b^2)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[(a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 2*(a^2*b - 2*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b)*Cos[c + d*x])*Sec[c + d*x])/(a + b)) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a

$$\begin{aligned} &)] * \text{Sqrt}[\left(\frac{a + b \cos[c + dx]}{a}\right) * \text{Csc}\left[\frac{c + dx}{2}\right]^2 / a * \text{Csc}[c + dx] * \text{EllipticP} \\ & \text{i}\left[-\frac{a}{b}, \text{ArcSin}\left[\frac{\text{Sqrt}\left[\left(\frac{a + b \cos[c + dx]}{a}\right) * \text{Csc}\left[\frac{c + dx}{2}\right]^2 / a\right]}{\text{Sqrt}[2]}\right], \right. \\ & \left. \frac{-2a}{-a + b} * \text{Sin}\left[\frac{c + dx}{2}\right]^4 / \left(b * \text{Sqrt}[\cos[c + dx]] * \text{Sqrt}[a + b \cos[c + dx]]\right)\right) / b + \left(\frac{\text{Sqrt}[a + b \cos[c + dx]] * \text{Sin}[c + dx]}{b * \text{Sqrt}[\cos[c + dx]]}\right) \\ & \left. \right) / \left(a^2 * (-a + b) * (a + b) * d\right) + \left(\frac{\text{Sqrt}[\cos[c + dx]] * \text{Sqrt}[a + b \cos[c + dx]] * \left(-2 * b^3 * \text{Sin}[c + dx]\right)}{a^2 * (a^2 - b^2) * (a + b \cos[c + dx])}\right) + \left(\frac{2 * \text{Tan}[c + dx]}{a^2}\right) / d \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. 2(265) = 530.

Time = 12.92 (sec) , antiderivative size = 1228, normalized size of antiderivative = 4.31

method	result	size
default	Expression too large to display	1228

[In] `int(1/cos(dx+c)^(3/2)/(a+cos(dx+c)*b)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -2/d * \left(\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2-1} * \left(-(-\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2+1}\right)^{1/2} * \left(\text{csc}(dx+c)^2 * a * (1-\cos(dx+c))^{-2-\text{csc}(dx+c)^2 * b * (1-\cos(dx+c))^{-2+a+b}}\right)^{1/2} * \text{EllipticF}\left(\cot(dx+c) - \text{csc}(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) * a^3 + \right. \\ & \left. -\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2+1}\right)^{1/2} * \left(\text{csc}(dx+c)^2 * a * (1-\cos(dx+c))^{-2-\text{csc}(dx+c)^2 * b * (1-\cos(dx+c))^{-2+a+b}}\right)^{1/2} * \text{EllipticF}\left(\cot(dx+c) - \text{csc}(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) * a^2 * b^2 * \left(-\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2+1}\right)^{1/2} * \left(\text{csc}(dx+c)^2 * a * (1-\cos(dx+c))^{-2-\text{csc}(dx+c)^2 * b * (1-\cos(dx+c))^{-2+a+b}}\right)^{1/2} * \text{EllipticF}\left(\cot(dx+c) - \text{csc}(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) * a * b^2 + \left(-\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2+1}\right)^{1/2} * \left(\text{csc}(dx+c)^2 * a * (1-\cos(dx+c))^{-2-\text{csc}(dx+c)^2 * b * (1-\cos(dx+c))^{-2+a+b}}\right)^{1/2} * \text{EllipticE}\left(\cot(dx+c) - \text{csc}(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) * a^3 + \left(-\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2+1}\right)^{1/2} * \left(\text{csc}(dx+c)^2 * a * (1-\cos(dx+c))^{-2-\text{csc}(dx+c)^2 * b * (1-\cos(dx+c))^{-2+a+b}}\right)^{1/2} * \text{EllipticE}\left(\cot(dx+c) - \text{csc}(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) * a^2 * b^2 * \left(-\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2+1}\right)^{1/2} * \left(\text{csc}(dx+c)^2 * a * (1-\cos(dx+c))^{-2-\text{csc}(dx+c)^2 * b * (1-\cos(dx+c))^{-2+a+b}}\right)^{1/2} * \text{EllipticE}\left(\cot(dx+c) - \text{csc}(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) * b^3 + \text{csc}(dx+c)^3 * a^3 * (1-\cos(dx+c))^{-3-\text{csc}(dx+c)^3 * a^2 * b * (1-\cos(dx+c))^{-3-2 * \text{csc}(dx+c)^3 * a * b^2 * (1-\cos(dx+c))^{-3+2 * \text{csc}(dx+c)^3 * b^3 * (1-\cos(dx+c))^{-3+a^3 * (\text{csc}(dx+c) - \cot(dx+c))} + a^2 * b * (\text{csc}(dx+c) - \cot(dx+c)) - 2 * b^3 * (\text{csc}(dx+c) - \cot(dx+c))} * \left(\text{csc}(dx+c)^2 * a * (1-\cos(dx+c))^{-2-\text{csc}(dx+c)^2 * b * (1-\cos(dx+c))^{-2+a+b}}\right) / \left(\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2+1}\right)^{1/2} / \left(\text{csc}(dx+c)^2 * a * (1-\cos(dx+c))^{-2-\text{csc}(dx+c)^2 * b * (1-\cos(dx+c))^{-2+a+b}}\right) / \left(\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2+1}\right) / \left(-\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2-1}\right) / \left(\text{csc}(dx+c)^2 * (1-\cos(dx+c))^{-2+1}\right)^{3/2} / a^2 / (a-b) / (a+b) \end{aligned}$$

Fricas [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}} \cos^{\frac{3}{2}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(a+b\cos(c+dx))^{\frac{3}{2}} \cos^{\frac{3}{2}}(c+dx)} dx$$

[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)

[Out] Integral(1/((a + b*cos(c + d*x))**(3/2)*cos(c + d*x)**(3/2)), x)

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}} \cos^{\frac{3}{2}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}} \cos^{\frac{3}{2}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2}} dx = \int \frac{1}{\cos(c + dx)^{3/2} (a + b \cos(c + dx))^{3/2}} dx$$

```
[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

$$3.636 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	6090
Rubi [A] (verified)	6091
Mathematica [C] (verified)	6094
Maple [B] (verified)	6095
Fricas [F]	6096
Sympy [F(-1)]	6097
Maxima [F]	6097
Giac [F]	6097
Mupad [F(-1)]	6097

Optimal result

Integrand size = 25, antiderivative size = 357

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^{3/2}} dx =$$

$$\frac{2b(5a^2 - 8b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^4 \sqrt{a+bd}}$$

$$+ \frac{2(a+2b)(a+4b) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^3 \sqrt{a+bd}}$$

$$+ \frac{2b^2 \sin(c+dx)}{a(a^2 - b^2) d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} + \frac{2(a^2 - 4b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{3a^2 (a^2 - b^2) d \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(a^2-4*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/cos(d*x+c)^(3/2)-2/3*b*(5*a^2-8*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)+2/3*(a+2*b)*(a+4*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.00,
 number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used
 = {2881, 3134, 3077, 2895, 3073}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \frac{2(a+2b)(a+4b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{3a^3d\sqrt{a+b}} \text{Ellip}$$

$$+ \frac{2b^2\sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2(a^2-4b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)}$$

$$- \frac{2b(5a^2-8b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{3a^4d\sqrt{a+b}}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x]

[Out] (-2*b*(5*a^2 - 8*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^4*Sqrt[a + b]*d) + (2*(a + 2*b)*(a + 4*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^3*Sqrt[a + b]*d) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Ssin[e + f*x]]/Sqrt[d*Ssin[e + f*x]]/Rt[(a + b)/d, 2]

], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3134

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rubi steps

$$\text{integral} = \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} + \frac{2 \int \frac{\frac{1}{2}(a^2 - 4b^2) - \frac{1}{2}ab \cos(c + dx) + b^2 \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{a(a^2 - b^2)}$$

$$\begin{aligned}
&= \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{2(a^2 - 4b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3a^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)} \\
&+ \frac{4 \int \frac{-\frac{1}{4}b(5a^2 - 8b^2) + \frac{1}{4}a(a^2 + 2b^2) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3a^2(a^2 - b^2)} \\
&= \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{2(a^2 - 4b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3a^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)} \\
&+ \frac{((a + 2b)(a + 4b)) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx}{3a^2(a + b)} \\
&- \frac{(b(5a^2 - 8b^2)) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3a^2(a^2 - b^2)} \\
&= \frac{2b(5a^2 - 8b^2) \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{3a^4 \sqrt{a + bd}} \\
&+ \frac{2(a + 2b)(a + 4b) \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{3a^3 \sqrt{a + bd}} \\
&+ \frac{2b^2 \sin(c + dx)}{a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{2(a^2 - 4b^2) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3a^2(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

$$\frac{\begin{aligned} & (c + d*x)/2)^2/a)] * \text{Sqrt}[(a + b*\text{Cos}[c + d*x]) * \text{Csc}[(c + d*x)/2]^2/a] * \text{Csc}[c \\ & + d*x] * \text{EllipticPi}[-(a/b), \text{ArcSin}[\text{Sqrt}[(a + b*\text{Cos}[c + d*x]) * \text{Csc}[(c + d*x)/2 \\ &]^2/a] / \text{Sqrt}[2]], (-2*a)/(-a + b)] * \text{Sin}[(c + d*x)/2]^4 / (b * \text{Sqrt}[\text{Cos}[c + d*x] \\ &] * \text{Sqrt}[a + b*\text{Cos}[c + d*x]]) / b + (\text{Sqrt}[a + b*\text{Cos}[c + d*x]] * \text{Sin}[c + d*x]) / (\\ & b * \text{Sqrt}[\text{Cos}[c + d*x]]) / (3*a^3*(a - b)*(a + b)*d) + (\text{Sqrt}[\text{Cos}[c + d*x]] * \text{Sqr} \\ & t[a + b*\text{Cos}[c + d*x]] * ((2*b^4*\text{Sin}[c + d*x]) / (a^3*(a^2 - b^2)*(a + b*\text{Cos}[c + \\ & d*x])) - (10*b*\text{Tan}[c + d*x]) / (3*a^3) + (2*\text{Sec}[c + d*x]*\text{Tan}[c + d*x]) / (3*a^ \\ & 2))) / d \end{aligned}}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2617 vs. $2(327) = 654$.

Time = 14.29 (sec) , antiderivative size = 2618, normalized size of antiderivative = 7.33

method	result	size
default	Expression too large to display	2618

[In] `int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & 2/3/d*(\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^{-2-1})*(5*\text{csc}(d*x+c)^2*\text{EllipticF}(\text{cot}(d*x+c) \\ &)-\text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * a^3 * b * (-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^{-2+1}) \\ & ^{1/2} * ((\text{csc}(d*x+c)^2 * a * (1-\text{cos}(d*x+c))^{-2} - \text{csc}(d*x+c)^2 * b * (1-\text{cos}(d*x+c))^{-2+a} \\ & + b)/(a+b))^{1/2} * (1-\text{cos}(d*x+c))^{-2} - 5*\text{EllipticF}(\text{cot}(d*x+c)-\text{csc}(d*x+c), (-a-b)/ \\ & (a+b))^{1/2}) * a^3 * b * (-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^{-2+1})^{1/2} * ((\text{csc}(d*x+c)^2 \\ & * a * (1-\text{cos}(d*x+c))^{-2} - \text{csc}(d*x+c)^2 * b * (1-\text{cos}(d*x+c))^{-2+a} + b)/(a+b))^{1/2} + 2*\text{Ell} \\ & \text{ipticF}(\text{cot}(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * a^2 * b^2 * (-\text{csc}(d*x+c)^2*(\\ & 1-\text{cos}(d*x+c))^{-2+1})^{1/2} * ((\text{csc}(d*x+c)^2 * a * (1-\text{cos}(d*x+c))^{-2} - \text{csc}(d*x+c)^2 * b * (\\ & 1-\text{cos}(d*x+c))^{-2+a} + b)/(a+b))^{1/2} + 8*\text{EllipticF}(\text{cot}(d*x+c)-\text{csc}(d*x+c), (-a-b) \\ & / (a+b))^{1/2}) * a * b^3 * (-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^{-2+1})^{1/2} * ((\text{csc}(d*x+c)^2 \\ & * a * (1-\text{cos}(d*x+c))^{-2} - \text{csc}(d*x+c)^2 * b * (1-\text{cos}(d*x+c))^{-2+a} + b)/(a+b))^{1/2} + 5*\text{El} \\ & \text{lipticE}(\text{cot}(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * a^3 * b * (-\text{csc}(d*x+c)^2*(1 \\ & -\text{cos}(d*x+c))^{-2+1})^{1/2} * ((\text{csc}(d*x+c)^2 * a * (1-\text{cos}(d*x+c))^{-2} - \text{csc}(d*x+c)^2 * b * (1 \\ & -\text{cos}(d*x+c))^{-2+a} + b)/(a+b))^{1/2} + 5*\text{EllipticE}(\text{cot}(d*x+c)-\text{csc}(d*x+c), (-a-b)/ \\ & (a+b))^{1/2}) * a^2 * b^2 * (-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^{-2+1})^{1/2} * ((\text{csc}(d*x+c) \\ & ^2 * a * (1-\text{cos}(d*x+c))^{-2} - \text{csc}(d*x+c)^2 * b * (1-\text{cos}(d*x+c))^{-2+a} + b)/(a+b))^{1/2} - 8*\text{El} \\ & \text{lipticE}(\text{cot}(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * a * b^3 * (-\text{csc}(d*x+c)^2*(\\ & 1-\text{cos}(d*x+c))^{-2+1})^{1/2} * ((\text{csc}(d*x+c)^2 * a * (1-\text{cos}(d*x+c))^{-2} - \text{csc}(d*x+c)^2 * b * (\\ & 1-\text{cos}(d*x+c))^{-2+a} + b)/(a+b))^{1/2} - \text{csc}(d*x+c)^2 * \text{EllipticF}(\text{cot}(d*x+c)-\text{csc}(d*x \\ & +c), (-a-b)/(a+b))^{1/2}) * a^4 * (-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^{-2+1})^{1/2} * ((\text{cs} \\ & c(d*x+c)^2 * a * (1-\text{cos}(d*x+c))^{-2} - \text{csc}(d*x+c)^2 * b * (1-\text{cos}(d*x+c))^{-2+a} + b)/(a+b))^{1/2} \\ & * (1-\text{cos}(d*x+c))^{-2} - 8*\text{csc}(d*x+c)^5 * b^4 * (1-\text{cos}(d*x+c))^{-5} - 2*\text{csc}(d*x+c)^3 * a^ \\ & 4 * (1-\text{cos}(d*x+c))^{-3} + 16*\text{csc}(d*x+c)^3 * b^4 * (1-\text{cos}(d*x+c))^{-3} + 3*a^3 * b * (\text{csc}(d*x+c) \\ & - \text{cot}(d*x+c)) + 7*a^2 * b^2 * (\text{csc}(d*x+c) - \text{cot}(d*x+c)) - 2*\text{csc}(d*x+c)^2 * \text{EllipticF}(\text{cot} \\ & (d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{1/2}) * a^2 * b^2 * (-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+ \\ & c))^{-2+1})^{1/2} * ((\text{csc}(d*x+c)^2 * a * (1-\text{cos}(d*x+c))^{-2} - \text{csc}(d*x+c)^2 * b * (1-\text{cos}(d*x+ \end{aligned}$$

$c)^{2+a+b}/(a+b)^{1/2}*(1-\cos(dx+c))^{-2}-8*\csc(dx+c)^2*\text{EllipticF}(\cot(dx+c)$
 $-\csc(dx+c), (-a-b)/(a+b))^{1/2})*a*b^3*(-\csc(dx+c)^2*(1-\cos(dx+c))^{2+1})$
 $^{1/2}*((\csc(dx+c)^2*a*(1-\cos(dx+c))^{-2}-\csc(dx+c)^2*b*(1-\cos(dx+c))^{2+a+b})$
 $/ (a+b))^{1/2}*(1-\cos(dx+c))^{-2}-5*\csc(dx+c)^2*\text{EllipticE}(\cot(dx+c)-\csc(dx$
 $x+c), (-a-b)/(a+b))^{1/2})*a^3*b*(-\csc(dx+c)^2*(1-\cos(dx+c))^{2+1})^{1/2}*($
 $(\csc(dx+c)^2*a*(1-\cos(dx+c))^{-2}-\csc(dx+c)^2*b*(1-\cos(dx+c))^{2+a+b})/(a+b)$
 $)^{1/2}*(1-\cos(dx+c))^{-2}-5*\csc(dx+c)^2*\text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-$
 $a-b)/(a+b))^{1/2})*a^2*b^2*(-\csc(dx+c)^2*(1-\cos(dx+c))^{2+1})^{1/2}*((\csc(d$
 $*x+c)^2*a*(1-\cos(dx+c))^{-2}-\csc(dx+c)^2*b*(1-\cos(dx+c))^{2+a+b})/(a+b))^{1/2}$
 $)*(1-\cos(dx+c))^{-2}+8*\csc(dx+c)^2*\text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/($
 $a+b))^{1/2})*a*b^3*(-\csc(dx+c)^2*(1-\cos(dx+c))^{2+1})^{1/2}*((\csc(dx+c)^2*$
 $a*(1-\cos(dx+c))^{-2}-\csc(dx+c)^2*b*(1-\cos(dx+c))^{2+a+b})/(a+b))^{1/2}*(1-\cos$
 $(dx+c))^{-2}-5*\csc(dx+c)^5*a^3*b*(1-\cos(dx+c))^{-5}-8*\csc(dx+c)^3*a*b^3*(1-co$
 $s(dx+c))^{-3}+5*\csc(dx+c)^5*a^2*b^2*(1-\cos(dx+c))^{-5}+8*\csc(dx+c)^5*a*b^3*(1-$
 $-\cos(dx+c))^{-5}+2*\csc(dx+c)^3*a^3*b*(1-\cos(dx+c))^{-3}-8*\csc(dx+c)^3*a^2*b^2$
 $*(1-\cos(dx+c))^{-3}+\text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2})*a^4$
 $*(-\csc(dx+c)^2*(1-\cos(dx+c))^{2+1})^{1/2}*((\csc(dx+c)^2*a*(1-\cos(dx+c))^{-2}$
 $-\csc(dx+c)^2*b*(1-\cos(dx+c))^{2+a+b})/(a+b))^{1/2}-8*\text{EllipticE}(\cot(dx+c)-c$
 $sc(dx+c), (-a-b)/(a+b))^{1/2})*b^4*(-\csc(dx+c)^2*(1-\cos(dx+c))^{2+1})^{1/2}$
 $*((\csc(dx+c)^2*a*(1-\cos(dx+c))^{-2}-\csc(dx+c)^2*b*(1-\cos(dx+c))^{2+a+b})/(a$
 $+b))^{1/2}+8*\csc(dx+c)^2*\text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1$
 $/2))*b^4*(-\csc(dx+c)^2*(1-\cos(dx+c))^{2+1})^{1/2}*((\csc(dx+c)^2*a*(1-\cos(d$
 $*x+c))^{-2}-\csc(dx+c)^2*b*(1-\cos(dx+c))^{2+a+b})/(a+b))^{1/2}*(1-\cos(dx+c))^{-2}$
 $-2*a^4*(\csc(dx+c)-\cot(dx+c))-8*b^4*(\csc(dx+c)-\cot(dx+c))*((\csc(dx+c)^$
 $2*a*(1-\cos(dx+c))^{-2}-\csc(dx+c)^2*b*(1-\cos(dx+c))^{2+a+b})/(\csc(dx+c)^2*(1-$
 $\cos(dx+c))^{2+1})^{1/2}/(\csc(dx+c)^2*a*(1-\cos(dx+c))^{-2}-\csc(dx+c)^2*b*(1-$
 $\cos(dx+c))^{2+a+b})/(\csc(dx+c)^2*(1-\cos(dx+c))^{2+1})^{1/2}/(-(\csc(dx+c)^2*(1-$
 $\cos(dx+c))^{-2}-1)/(\csc(dx+c)^2*(1-\cos(dx+c))^{2+1})^{5/2}/(a+b)/(a-b)/a^3$

Fricas [F]

$$\int \frac{1}{\cos^{5/2}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{3/2}\cos(dx+c)^{5/2}} dx$$

[In] integrate(1/cos(dx+c)^(5/2)/(a+b*cos(dx+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(dx+c)+a)*sqrt(cos(dx+c))/(b^2*cos(dx+c)^5+2*a*b*cos(dx+c)^4+a^2*cos(dx+c)^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{3}{2}}} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2), x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{3}{2}}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}} dx$$

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2), x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{3}{2}}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}} dx$$

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2), x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{3}{2}}} dx = \int \frac{1}{\cos(c+dx)^{\frac{5}{2}}(a+b\cos(c+dx))^{\frac{3}{2}}} dx$$

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

$$3.637 \quad \int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	6098
Rubi [A] (verified)	6099
Mathematica [C] (verified)	6102
Maple [B] (verified)	6103
Fricas [F]	6105
Sympy [F(-1)]	6105
Maxima [F]	6106
Giac [F]	6106
Mupad [F(-1)]	6106

Optimal result

Integrand size = 25, antiderivative size = 433

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b \cos(c+dx))^{3/2}} dx = \frac{2(3a^4 + 8a^2b^2 - 16b^4) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right)}{5a^5 \sqrt{a+bd}} - \frac{2(3a+4b)(a^2+4b^2) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{5a^4 \sqrt{a+bd}} + \frac{2b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} + \frac{2(a^2-6b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5a^2 (a^2-b^2) d \cos^{\frac{5}{2}}(c+dx)} - \frac{2b(3a^2-8b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5a^3 (a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2)+2/5*(a^2-6*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/cos(d*x+c)^(5/2)-2/5*b*(3*a^2-8*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/cos(d*x+c)^(3/2)+2/5*(3*a^4+8*a^2*b^2-16*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^5/d/(a+b)^(1/2)-2/5*(3*a^4*b)*(a^2+4*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2881, 3134, 3077, 2895, 3073}

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \frac{2b^2 \sin(c+dx)}{ad(a^2-b^2)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2(a^2-6b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^2d(a^2-b^2)\cos^{\frac{5}{2}}(c+dx)} - \frac{2(3a+4b)(a^2+4b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{5a^4d\sqrt{a+b}} - \frac{2b(3a^2-8b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^3d(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)} + \frac{2(3a^4+8a^2b^2-16b^4)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{5a^5d\sqrt{a+b}}$$

[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^(3/2)),x]

[Out] (2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(5*a^5*Sqrt[a + b]*d) - (2*(3*a + 4*b)*(a^2 + 4*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(5*a^4*Sqrt[a + b]*d) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^2 - 6*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)) - (2*b*(3*a^2 - 8*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```


Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} \\
 &+ \frac{2 \int \frac{\frac{1}{2}(a^2-6b^2) - \frac{1}{2}ab \cos(c+dx) + 2b^2 \cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{a(a^2-b^2)} \\
 &= \frac{2b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} \\
 &+ \frac{2(a^2-6b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5a^2(a^2-b^2) d \cos^{\frac{5}{2}}(c+dx)} \\
 &+ \frac{4 \int \frac{-\frac{3}{4}b(3a^2-8b^2) + \frac{1}{4}a(3a^2+2b^2) \cos(c+dx) + \frac{1}{2}b(a^2-6b^2) \cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{5a^2(a^2-b^2)} \\
 &= \frac{2b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} \\
 &+ \frac{2(a^2-6b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5a^2(a^2-b^2) d \cos^{\frac{5}{2}}(c+dx)} \\
 &- \frac{2b(3a^2-8b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5a^3(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)} \\
 &+ \frac{8 \int \frac{\frac{3}{8}(3a^4+8a^2b^2-16b^4) - \frac{3}{8}ab(a^2+4b^2) \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{15a^3(a^2-b^2)} \\
 &= \frac{2b^2 \sin(c+dx)}{a(a^2-b^2) d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} \\
 &+ \frac{2(a^2-6b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5a^2(a^2-b^2) d \cos^{\frac{5}{2}}(c+dx)} \\
 &- \frac{2b(3a^2-8b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{5a^3(a^2-b^2) d \cos^{\frac{3}{2}}(c+dx)} \\
 &- \frac{((3a+4b)(a^2+4b^2)) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx}{5a^3(a+b)} \\
 &+ \frac{(3a^4+8a^2b^2-16b^4) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{5a^3(a^2-b^2)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{2(3a^4 + 8a^2b^2 - 16b^4) \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{5a^5\sqrt{a+bd}} \\
 &- \frac{2(3a+4b)(a^2+4b^2)\cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{5a^4\sqrt{a+bd}} \\
 &+ \frac{2b^2 \sin(c+dx)}{a(a^2-b^2)d \cos^{\frac{5}{2}}(c+dx) \sqrt{a+b} \cos(c+dx)} \\
 &+ \frac{2(a^2-6b^2) \sqrt{a+b} \cos(c+dx) \sin(c+dx)}{5a^2(a^2-b^2)d \cos^{\frac{5}{2}}(c+dx)} \\
 &- \frac{2b(3a^2-8b^2) \sqrt{a+b} \cos(c+dx) \sin(c+dx)}{5a^3(a^2-b^2)d \cos^{\frac{3}{2}}(c+dx)}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.42 (sec) , antiderivative size = 1314, normalized size of antiderivative = 3.03

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \frac{(a^2+4b^2) \left(\frac{4a(4a^2b-4b^3) \sqrt{\frac{(a+b)\cot^2(\frac{1}{2}(c+dx))}{-a+b}} \sqrt{-\frac{(a+b)\cos(c+dx)\csc^2(\frac{1}{2}(c+dx))}{a}}}{(a^2+4b^2)} \right)}{d \sqrt{\cos(c+dx)} \sqrt{a+b} \cos(c+dx) \left(-\frac{2b^5 \sin(c+dx)}{a^4(a^2-b^2)(a+b\cos(c+dx))} + \frac{2\sec(c+dx)(3a^2 \sin(c+dx)+11b^2 \sin(c+dx))}{5a^4} - \frac{6b \sec(c+dx) \tan(c+dx)}{5a^3} \right)}$$

[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^(3/2)),x]

[Out] ((a^2 + 4*b^2)*((-4*a*(4*a^2*b - 4*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^3 - 4*a*b^2)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs

$$\begin{aligned}
&) * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b) \\
& ^{1/2} * a^3 * b^2 * \cos(dx+c)^4 + 6 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), -(a-b)/(a+b)) \\
&)^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^5 * \cos(dx+c)^3 - 32 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), -(a-b)/(a+b))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * b^5 * \cos(dx+c)^3 - 6 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), -(a-b)/(a+b))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^5 * \cos(dx+c)^3 + 3 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), -(a-b)/(a+b))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^5 * \cos(dx+c)^2 - 16 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), -(a-b)/(a+b))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * b^5 * \cos(dx+c)^2 - 3 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), -(a-b)/(a+b))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^5 * \cos(dx+c)^2 + 8 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), -(a-b)/(a+b))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^2 * b^3 * \cos(dx+c)^2 - 16 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), -(a-b)/(a+b))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a * b^4 * \cos(dx+c)^2 + \text{EllipticF}(\cot(dx+c) - \csc(dx+c), -(a-b)/(a+b))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c)))^{1/2} * a^4 * b * \cos(dx+c)^2 / (1+\cos(dx+c)) / (a+\cos(dx+c)*b)^{1/2} / \cos(dx+c)^{5/2} / (a+b) / (a-b) / a^4
\end{aligned}$$

Fricas [F]

$$\int \frac{1}{\cos^{7/2}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{3/2} \cos(dx+c)^{7/2}} dx$$

[In] integrate(1/cos(dx+c)^(7/2)/(a+b*cos(dx+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(dx+c)+a)*sqrt(cos(dx+c))/(b^2*cos(dx+c)^6+2*a*b*cos(dx+c)^5+a^2*cos(dx+c)^4),x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{7/2}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(1/cos(dx+c)**(7/2)/(a+b*cos(dx+c))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}} \cos(dx+c)^{\frac{7}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}} \cos(dx+c)^{\frac{7}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx = \int \frac{1}{\cos(c+dx)^{7/2} (a+b\cos(c+dx))^{3/2}} dx$$

[In] int(1/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(3/2)),x)

[Out] int(1/(cos(c + d*x)^(7/2)*(a + b*cos(c + d*x))^(3/2)), x)

$$3.638 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^{\frac{5}{2}}} dx$$

Optimal result	6107
Rubi [A] (verified)	6108
Mathematica [C] (warning: unable to verify)	6111
Maple [B] (warning: unable to verify)	6112
Fricas [F]	6115
Sympy [F(-1)]	6115
Maxima [F]	6115
Giac [F]	6115
Mupad [F(-1)]	6116

Optimal result

Integrand size = 25, antiderivative size = 497

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^{\frac{5}{2}}} dx = \frac{2(3a^2 - 7b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{\frac{3}{2}}d} - \frac{2(3a^2 + ab - 6b^2) \cot(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{\frac{3}{2}}d} - \frac{2\sqrt{a+b} \cot(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3d} - \frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3b(a^2 - b^2)d(a+b \cos(c+dx))^{\frac{3}{2}}} - \frac{2a^2(3a^2 - 7b^2) \sin(c+dx)}{3b^2(a^2 - b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

```
[Out] -2/3*a^2*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-2/3*a^2*(3*a^2-7*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+2/3*(3*a^2-7*b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d-2/3*(3*a^2+a*b-6*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d-2*cot(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2871, 3130, 2888, 3072, 3077, 2895, 3073}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx =$$

$$\frac{2(3a^2+ab-6b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{3b^2d(a-b)(a+b)^{\frac{3}{2}}}$$

$$+ \frac{2(3a^2-7b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{3b^2d(a-b)(a+b)^{\frac{3}{2}}}$$

$$- \frac{2a^2(3a^2-7b^2)\sin(c+dx)}{3b^2d(a^2-b^2)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd(a^2-b^2)(a+b\cos(c+dx))^{\frac{3}{2}}}$$

$$- \frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{b^3d}$$

[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (2*(3*a^2 - 7*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*(a - b)*b^2*(a + b)^(3/2)*d) - (2*(3*a^2 + a*b - 6*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*(a - b)*b^2*(a + b)^(3/2)*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b^3*d) - (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b

$^2, 0]$ && $\text{NeQ}[c^2 - d^2, 0]$ && $\text{GtQ}[m, 2]$ && $\text{LtQ}[n, -1]$ && $(\text{IntegerQ}[m] \mid \mid \text{IntegersQ}[2*m, 2*n])$

Rule 2888

$\text{Int}[\text{Sqrt}[(b_.)*\sin[(e_.) + (f_.)*(x_)]]/\text{Sqrt}[(c_) + (d_.)*\sin[(e_.) + (f_.)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[2*b*(\text{Tan}[e + f*x]/(d*f))*\text{Rt}[(c + d)/b, 2]*\text{Sqrt}[c*((1 + \text{Csc}[e + f*x])/(c - d))]*\text{Sqrt}[c*((1 - \text{Csc}[e + f*x])/(c + d))]*\text{EllipticPi}[(c + d)/d, \text{ArcSin}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[b*\text{Sin}[e + f*x]]/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(c + d)/b]$

Rule 2895

$\text{Int}[1/(\text{Sqrt}[(d_.)*\sin[(e_.) + (f_.)*(x_)]]*\text{Sqrt}[(a_) + (b_.)*\sin[(e_.) + (f_.)*(x_)]]), x_Symbol] \rightarrow \text{Simp}[-2*(\text{Tan}[e + f*x]/(a*f))*\text{Rt}[(a + b)/d, 2]*\text{Sqrt}[a*((1 - \text{Csc}[e + f*x])/(a + b))]*\text{Sqrt}[a*((1 + \text{Csc}[e + f*x])/(a - b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[d*\text{Sin}[e + f*x]]/\text{Rt}[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{PosQ}[(a + b)/d]$

Rule 3072

$\text{Int}[(A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)]]/(\text{Sqrt}[(d_.)*\sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)])^{3/2}), x_Symbol] \rightarrow \text{Simp}[2*(A*b - a*B)*(\text{Cos}[e + f*x]/(f*(a^2 - b^2))*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[d*\text{Sin}[e + f*x]]), x] + \text{Dist}[d/(a^2 - b^2), \text{Int}[(A*b - a*B + (a*A - b*B)*\text{Sin}[e + f*x]]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(d*\text{Sin}[e + f*x])^{3/2}), x], x] /; \text{FreeQ}\{a, b, d, e, f, A, B\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3073

$\text{Int}[(A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)]]/(((b_.)*\sin[(e_.) + (f_.)*(x_)])^{3/2}*\text{Sqrt}[(c_) + (d_.)*\sin[(e_.) + (f_.)*(x_)]]), x_Symbol] \rightarrow \text{Simp}[-2*A*(c - d)*(\text{Tan}[e + f*x]/(f*b*c^2))*\text{Rt}[(c + d)/b, 2]*\text{Sqrt}[c*((1 + \text{Csc}[e + f*x])/(c - d))]*\text{Sqrt}[c*((1 - \text{Csc}[e + f*x])/(c + d))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[b*\text{Sin}[e + f*x]]/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; \text{FreeQ}\{b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{EqQ}[A, B] \&\& \text{PosQ}[(c + d)/b]$

Rule 3077

$\text{Int}[(A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)]]/(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])^{3/2}*\text{Sqrt}[(c_) + (d_.)*\sin[(e_.) + (f_.)*(x_)]]), x_Symbol] \rightarrow \text{Dist}[(A - B)/(a - b), \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] - \text{Dist}[(A*b - a*B)/(a - b), \text{Int}[(1 + \text{Sin}[e + f*x])/(a + b*\text{Sin}[e + f*x])], x]$

$(e + f*x)^{(3/2)*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]}, x, x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[A, B]$

Rule 3130

$\text{Int}[\{(A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^2\}/(\text{Sqrt}[(d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]*(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]))^{\{3/2\}}, x_Symbol] \text{:>} \text{Dist}[C/(b*d), \text{Int}[\text{Sqrt}[d*\text{Sin}[e + f*x]]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]], x], x] + \text{Dist}[1/b, \text{Int}[(A*b + (b*B - a*C)*\text{Sin}[e + f*x])/((a + b*\text{Sin}[e + f*x])^{\{3/2\}}*\text{Sqrt}[d*\text{Sin}[e + f*x]]), x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} - \frac{2 \int \frac{\frac{a^2}{2} - \frac{3}{2}ab \cos(c+dx) - \frac{3}{2}(a^2-b^2) \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx}{3b(a^2-b^2)} \\
 &= -\frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{b^2} \\
 &\quad - \frac{2 \int \frac{\frac{a^2b}{2} + \left(-\frac{3ab^2}{2} + \frac{3}{2}a(a^2-b^2)\right) \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx}{3b^2(a^2-b^2)} \\
 &= \\
 &\quad - \frac{2\sqrt{a+b} \cot(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3d} \\
 &\quad - \frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} \\
 &\quad - \frac{2a^2(3a^2-7b^2) \sin(c+dx)}{3b^2(a^2-b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} \\
 &\quad - \frac{2 \int \frac{\frac{a^2b^2}{2} - a\left(-\frac{3ab^2}{2} + \frac{3}{2}a(a^2-b^2)\right) + \left(\frac{a^3b}{2} - b\left(-\frac{3ab^2}{2} + \frac{3}{2}a(a^2-b^2)\right)\right) \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{3b^2(a^2-b^2)^2}
 \end{aligned}$$

$$\begin{aligned}
&= \\
&\frac{2\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3 d} \\
&\frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3b(a^2-b^2) d(a+b \cos(c+dx))^{3/2}} \\
&\frac{2a^2(3a^2-7b^2) \sin(c+dx)}{3b^2(a^2-b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} \\
&\frac{(a^2(3a^2+ab-6b^2)) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx}{3(a-b)b^2(a+b)^2} \\
&\frac{(a^2(3a^2-7b^2)) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{3b^2(a^2-b^2)^2} \\
&+ \\
&= \frac{2(3a^2-7b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{3/2} d} \\
&\frac{2(3a^2+ab-6b^2) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{3/2} d} \\
&\frac{2\sqrt{a+b} \cot(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3 d} \\
&\frac{2a^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3b(a^2-b^2) d(a+b \cos(c+dx))^{3/2}} - \frac{2a^2(3a^2-7b^2) \sin(c+dx)}{3b^2(a^2-b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 1282, normalized size of antiderivative = 2.58

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = \frac{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)} \left(\frac{2a^2 \sin(c+dx)}{3b(-a^2+b^2)(a+b \cos(c+dx))^2} + \frac{2(3a^3 \sin(c+dx)-7ab^2)}{3b(-a^2+b^2)^2(a+b \cos(c+dx))} \right)}{d}$$

$$\frac{4a(a^3-ab^2) \sqrt{\frac{(a+b) \cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}} \sqrt{-\frac{(a+b) \cos(c+dx) \operatorname{csc}^2\left(\frac{1}{2}(c+dx)\right)}{a}} \sqrt{\frac{(a+b \cos(c+dx)) \operatorname{csc}^2\left(\frac{1}{2}(c+dx)\right)}{a}} \operatorname{csc}(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{(a+b) \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*a^2*Sin[c + d*x])/(3*b*(-a
^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(3*a^3*Sin[c + d*x] - 7*a*b^2*Sin[c
+ d*x]))/(3*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x])))/d - (((-4*a*(a^3 - a*b^
2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-(a^2*b) - 3*b^3)*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 2*(3*a^3 - 7*a*b^2)*((I*Cos
[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]
/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*
a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b +
(Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*(a - b
)^2*b*(a + b)^2*d)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4444 vs. 2(457) = 914.

Time = 9.72 (sec) , antiderivative size = 4445, normalized size of antiderivative = 8.94

method	result	size
default	Expression too large to display	4445

```
[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1
```


$$\begin{aligned}
& 1 - \cos(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticPi} \\
& \text{i}(\cot(dx+c) - \csc(dx+c), -1, (-a-b)/(a+b) \Big)^{1/2}) * a^3 * b^2 * (1 - \cos(dx+c) \Big)^{-2} - \\
& 2 * \csc(dx+c) \Big)^2 * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 \\
& - \cos(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticPi} \\
& (\cot(dx+c) - \csc(dx+c), -1, (-a-b)/(a+b) \Big)^{1/2}) * a^2 * b^3 * (1 - \cos(dx+c) \Big)^{-2} - 6 * \\
& \csc(dx+c) \Big)^2 * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 - \c \\
& \text{os}(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticPi}(\c \\
& \text{ot}(dx+c) - \csc(dx+c), -1, (-a-b)/(a+b) \Big)^{1/2}) * a * b^4 * (1 - \cos(dx+c) \Big)^{-2} - 2 * \csc(\\
& dx+c) \Big)^2 * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 - \cos(d \\
& *x+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticF}(\cot(dx \\
& x+c) - \csc(dx+c), (-a-b)/(a+b) \Big)^{1/2}) * a^4 * b * (1 - \cos(dx+c) \Big)^{-2} + 3 * \csc(dx+c) \Big)^2 \\
& * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 - \cos(dx+c) \Big)^{-2} \\
& - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticF}(\cot(dx+c) - \csc \\
& (dx+c), (-a-b)/(a+b) \Big)^{1/2}) * a^3 * b^2 * (1 - \cos(dx+c) \Big)^{-2} + 5 * \csc(dx+c) \Big)^2 * (-\csc \\
& (dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 - \cos(dx+c) \Big)^{-2} - \csc(d \\
& *x+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticF}(\cot(dx+c) - \csc(dx+c \\
&), (-a-b)/(a+b) \Big)^{1/2}) * a^2 * b^3 * (1 - \cos(dx+c) \Big)^{-2} - 2 * a^4 * b * (\csc(dx+c) - \cot(dx \\
& x+c)) + 10 * a^3 * b^2 * (\csc(dx+c) - \cot(dx+c)) + 2 * a^2 * b^3 * (\csc(dx+c) - \cot(dx+c)) - \\
& 7 * a * b^4 * (\csc(dx+c) - \cot(dx+c)) + 3 * \csc(dx+c) \Big)^5 * a^5 * (1 - \cos(dx+c) \Big)^{-5} - 3 * a^5 * (\\
& \csc(dx+c) - \cot(dx+c)) - 6 * \csc(dx+c) \Big)^2 * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1 \\
& /2} * ((\csc(dx+c) \Big)^2 * a * (1 - \cos(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / \\
& (a+b) \Big)^{1/2} * \text{EllipticPi}(\cot(dx+c) - \csc(dx+c), -1, (-a-b)/(a+b) \Big)^{1/2}) * a^5 * \\
& (1 - \cos(dx+c) \Big)^{-2} + 6 * \csc(dx+c) \Big)^2 * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\\
& \csc(dx+c) \Big)^2 * a * (1 - \cos(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \\
& \Big)^{1/2} * \text{EllipticPi}(\cot(dx+c) - \csc(dx+c), -1, (-a-b)/(a+b) \Big)^{1/2}) * b^5 * (1 - \cos \\
& (dx+c) \Big)^{-2} + 14 * \csc(dx+c) \Big)^3 * a * b^4 * (1 - \cos(dx+c) \Big)^{-3} + 3 * (-\csc(dx+c) \Big)^2 * (1 - \cos(d \\
& *x+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 - \cos(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(d \\
& *x+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b) \Big)^{1/2}) * b^5 + 3 * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 - \cos(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b) \Big)^{1/2}) * a^5 - 6 * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 - \cos(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticPi}(\cot(dx+c) - \csc(dx+c), -1, (-a-b)/(a+b) \Big)^{1/2}) * a^5 - 6 * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 - \cos(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticPi}(\cot(dx+c) - \csc(dx+c), -1, (-a-b)/(a+b) \Big)^{1/2}) * b^5 - 6 * \csc(dx+c) \Big)^5 * a^4 * b * (1 - \cos(dx+c) \Big)^{-5} - 4 * \csc(dx+c) \Big)^5 * a^3 * b^2 * (1 - \cos(dx+c) \Big)^{-5} + 14 * \csc(dx+c) \Big)^5 * a^2 * b^3 * (1 - \cos(dx+c) \Big)^{-5} - 7 * \csc(dx+c) \Big)^5 * a * b^4 * (1 - \cos(dx+c) \Big)^{-5} + 8 * \csc(dx+c) \Big)^3 * a^4 * b * (1 - \cos(dx+c) \Big)^{-3} - 6 * \csc(dx+c) \Big)^3 * a^3 * b^2 * (1 - \cos(dx+c) \Big)^{-3} - 10 * \csc(dx+c) \Big)^2 * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 - \cos(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b) \Big)^{1/2}) * a^3 * b^2 * (1 - \cos(dx+c) \Big)^{-2} + 7 * \csc(dx+c) \Big)^2 * (-\csc(dx+c) \Big)^2 * (1 - \cos(dx+c) \Big)^{2+1} \Big)^{1/2} * ((\csc(dx+c) \Big)^2 * a * (1 - \cos(dx+c) \Big)^{-2} - \csc(dx+c) \Big)^2 * b * (1 - \cos(dx+c) \Big)^{2+a+b} / (a+b) \Big)^{1/2} * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b) \Big)^{1/2}) * a * b^4 * (1 - \cos(dx+c) \Big)^{-2} - 16 * \csc(dx+c) \Big)^3 * a^2 * b^3 * (1 - \cos
\end{aligned}$$

$(d*x+c)^3/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^3/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)^2/(a-b)^2/(a+b)^2/b^2$

Fricas [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\cos(c + dx)^{\frac{5}{2}}}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx$$

```
[In] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int(cos(c + d*x)^(5/2)/(a + b*cos(c + d*x))^(5/2), x)
```


$$3.639 \quad \int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	6117
Rubi [A] (verified)	6117
Mathematica [A] (verified)	6120
Maple [B] (warning: unable to verify)	6120
Fricas [F]	6122
Sympy [F]	6122
Maxima [F]	6122
Giac [F]	6122
Mupad [F(-1)]	6123

Optimal result

Integrand size = 25, antiderivative size = 342

$$\int \frac{\cos^3(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = \frac{8b \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d}$$

$$+ \frac{2(a-3b) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d}$$

$$+ \frac{2a \sqrt{\cos(c+dx)} \sin(c+dx)}{3(a^2-b^2)d(a+b \cos(c+dx))^{3/2}} - \frac{8ab \sin(c+dx)}{3(a^2-b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2/3*a*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-8/3*a*
b*sin(d*x+c)/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2)+8/3*b*co
t(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/a/(a-b)/(a+b)^(3/2)/d+2/3*(a-3*b)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c)
)/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

= {2878, 3072, 3077, 2895, 3073}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = -\frac{8ab\sin(c+dx)}{3d(a^2-b^2)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a^2-b^2)(a+b\cos(c+dx))^{3/2}} + \frac{2(a-3b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{3ad(a-b)(a+b)^{3/2}} + \frac{8b\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{3ad(a-b)(a+b)^{3/2}}$$

[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (8*b*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*(a - b)*(a + b)^(3/2)*d) + (2*(a - 3*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*(a - b)*(a + b)^(3/2)*d) + (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (8*a*b*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] := Simp[2*(A*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int\frac{-\frac{a}{2}+\frac{3}{2}b\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3(a^2-b^2)} \\
&= \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} \\
&\quad - \frac{8ab\sin(c+dx)}{3(a^2-b^2)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{2\int\frac{-2ab+\left(-\frac{a^2}{2}-\frac{3b^2}{2}\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3(a^2-b^2)^2} \\
&= \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3(a^2-b^2)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{(a-3b)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3(a-b)(a+b)^2} + \frac{(4ab)\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3(a^2-b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{8b \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d} \\
&+ \frac{2(a-3b) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d} \\
&+ \frac{2a \sqrt{\cos(c+dx)} \sin(c+dx)}{3(a^2-b^2)d(a+b \cos(c+dx))^{3/2}} - \frac{8ab \sin(c+dx)}{3(a^2-b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.81

$$\int \frac{\cos^{3/2}(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = \frac{2 \left(\sqrt{\cos(c+dx)} (a^3 + 3ab^2 + 4b^3 \cos(c+dx)) \sin(c+dx) - \sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)} \right)}{(a+b \cos(c+dx))^{5/2}}$$

[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (2*(Sqrt[Cos[c + d*x]]*(a^3 + 3*a*b^2 + 4*b^3*Cos[c + d*x])*Sin[c + d*x] - Sqrt[Cos[(c + d*x)/2]^2]*(a + b*Cos[c + d*x])*(4*b*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (a^2 + 4*a*b + 3*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2))

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2128 vs. 2(310) = 620.

Time = 9.31 (sec) , antiderivative size = 2129, normalized size of antiderivative = 6.23

method	result	size
default	Expression too large to display	2129

[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)

[Out] 2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^3/2*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^2*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^1/2*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^1/2)*a^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^1/2*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^1/2*(1-cos(d*x+c))^2-3*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^1/2)*a^2*b*(-cs

$$\begin{aligned}
& c(d*x+c)^2*(1-\cos(d*x+c))^{2+1}^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)} \\
& d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b}/(a+b))^{(1/2)}*(1-\cos(d*x+c))^{2+\csc(d*x+c)^2} \\
& *EllipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*a*b^2*(-\csc(d*x+c)^2 \\
& *(1-\cos(d*x+c))^{2+1}^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b} \\
& *(1-\cos(d*x+c))^{2+a+b}/(a+b))^{(1/2)}*(1-\cos(d*x+c))^{2+3*\csc(d*x+c)^2*(-\csc(d \\
& *x+c)^2*(1-\cos(d*x+c))^{2+1}^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x \\
& +c)^2*b*(1-\cos(d*x+c))^{2+a+b}/(a+b))^{(1/2)}*EllipticF(\cot(d*x+c)-\csc(d*x+c), \\
& (-a-b)/(a+b))^{(1/2)}*b^3*(1-\cos(d*x+c))^{2+4*\csc(d*x+c)^2*EllipticE(\cot(d*x \\
& +c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*a^2*b*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+ \\
& 1}^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+ \\
& a+b}/(a+b))^{(1/2)}*(1-\cos(d*x+c))^{2-4*\csc(d*x+c)^2*EllipticE(\cot(d*x+c)-\csc(\\
& d*x+c), (-a-b)/(a+b))^{(1/2)}*b^3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1}^{(1/2)}*(\\
& (\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b}/(a+b) \\
&)^{(1/2)}*(1-\cos(d*x+c))^{2+4*\csc(d*x+c)^5*a^2*b*(1-\cos(d*x+c))^{5-8*\csc(d*x+c) \\
& ^5*a*b^2*(1-\cos(d*x+c))^{5+4*\csc(d*x+c)^5*b^3*(1-\cos(d*x+c))^{5-(-\csc(d*x+c)^ \\
& 2*(1-\cos(d*x+c))^{2+1}^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b} \\
& b*(1-\cos(d*x+c))^{2+a+b}/(a+b))^{(1/2)}*EllipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b) \\
&)/(a+b))^{(1/2)}*a^3-5*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1}^{(1/2)}*((\csc(d*x+c) \\
& ^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b}/(a+b))^{(1/2)}*Ell \\
& ipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*a^2*b-7*(-\csc(d*x+c)^2*(\\
& 1-\cos(d*x+c))^{2+1}^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(\\
& 1-\cos(d*x+c))^{2+a+b}/(a+b))^{(1/2)}*EllipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(\\
& a+b))^{(1/2)}*a*b^2-3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1}^{(1/2)}*((\csc(d*x+c)^ \\
& 2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b}/(a+b))^{(1/2)}*Ell \\
& ipticF(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*b^3+4*(-\csc(d*x+c)^2*(1-c \\
& os(d*x+c))^{2+1}^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-c \\
& os(d*x+c))^{2+a+b}/(a+b))^{(1/2)}*EllipticE(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b \\
&))^{(1/2)}*a^2*b+8*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1}^{(1/2)}*((\csc(d*x+c)^2*a \\
& *(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b}/(a+b))^{(1/2)}*Ellipti \\
& cE(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)}*a*b^2+4*(-\csc(d*x+c)^2*(1-co \\
& s(d*x+c))^{2+1}^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-co \\
& s(d*x+c))^{2+a+b}/(a+b))^{(1/2)}*EllipticE(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b) \\
&)^{(1/2)}*b^3-2*\csc(d*x+c)^3*a^3*(1-\cos(d*x+c))^{3+10*\csc(d*x+c)^3*a*b^2*(1-c \\
& os(d*x+c))^{3-8*\csc(d*x+c)^3*b^3*(1-\cos(d*x+c))^{3+2*a^3*(\csc(d*x+c)-\cot(d*x+ \\
& c))-4*a^2*b*(\csc(d*x+c)-\cot(d*x+c))-2*a*b^2*(\csc(d*x+c)-\cot(d*x+c))+4*b^3*(\\
& \csc(d*x+c)-\cot(d*x+c)))/(\csc(d*x+c)^2*(1-\cos(d*x+c))^{2-1}^2/(\csc(d*x+c)^2*a \\
& *(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b}^2/(a-b)^2/(a+b)^2}
\end{aligned}$$

Fricas [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)

[Out] Integral(cos(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(5/2), x)

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\cos(c + dx)^{\frac{3}{2}}}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx$$

```
[In] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(a + b*cos(c + d*x))^(5/2), x)
```

$$3.640 \quad \int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	6124
Rubi [A] (verified)	6125
Mathematica [C] (verified)	6127
Maple [B] (verified)	6128
Fricas [F]	6130
Sympy [F]	6130
Maxima [F]	6130
Giac [F]	6130
Mupad [F(-1)]	6131

Optimal result

Integrand size = 25, antiderivative size = 359

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b \cos(c+dx))^{5/2}} dx =$$

$$\frac{2(3a^2 + b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d}$$

$$+ \frac{2(3a-b) \cot(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d}$$

$$- \frac{2b \sqrt{\cos(c+dx)} \sin(c+dx)}{3(a^2-b^2)d(a+b \cos(c+dx))^{3/2}} + \frac{2(3a^2+b^2) \sin(c+dx)}{3(a^2-b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

[Out] $-2/3*b*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/(a^2-b^2)/d/(a+b*\cos(d*x+c))^{(3/2)}+2/3*(3*a^2+b^2)*\sin(d*x+c)/(a^2-b^2)^2/d/\cos(d*x+c)^{(1/2)}/(a+b*\cos(d*x+c))^{(1/2)}$
 $-2/3*(3*a^2+b^2)*\cot(d*x+c)*\text{EllipticE}((a+b*\cos(d*x+c))^{(1/2)}/(a+b)^{(1/2)}/\cos(d*x+c)^{(1/2)},((-a-b)/(a-b))^{(1/2)})*(a*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(a*(1+\sec(d*x+c))/(a-b))^{(1/2)}/a^2/(a-b)/(a+b)^{(3/2)}/d+2/3*(3*a-b)*\cot(d*x+c)*\text{EllipticF}((a+b*\cos(d*x+c))^{(1/2)}/(a+b)^{(1/2)}/\cos(d*x+c)^{(1/2)},((-a-b)/(a-b))^{(1/2)})*(a*(1-\sec(d*x+c))/(a+b))^{(1/2)}*(a*(1+\sec(d*x+c))/(a-b))^{(1/2)}/a/(a-b)/(a+b)^{(3/2)}/d$

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2875, 3072, 3077, 2895, 3073}

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx =$$

$$\frac{2(3a^2 + b^2) \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right)}{3a^2d(a-b)(a+b)^{3/2}}$$

$$+ \frac{2(3a^2 + b^2) \sin(c+dx)}{3d(a^2 - b^2)^2 \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} - \frac{2b \sin(c+dx) \sqrt{\cos(c+dx)}}{3d(a^2 - b^2)(a+b\cos(c+dx))^{3/2}}$$

$$+ \frac{2(3a - b) \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{3ad(a-b)(a+b)^{3/2}}$$

[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]

[Out] (-2*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^2*(a - b)*(a + b)^(3/2)*d) + (2*(3*a - b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*(a - b)*(a + b)^(3/2)*d) - (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(3*a^2 + b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Rule 2875

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sine[e + f*x]]/Sqrt[d*Sine[e + f*x]]]/Rt[(a + b)/d, 2]

], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3072

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]

Rule 3073

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int\frac{\frac{b}{2}-\frac{3}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}dx}{3(a^2-b^2)} \\ &= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} \\ &\quad + \frac{2(3a^2+b^2)\sin(c+dx)}{3(a^2-b^2)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{2\int\frac{\frac{3a^2}{2}+\frac{b^2}{2}+2ab\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3(a^2-b^2)^2} \end{aligned}$$

$$\begin{aligned}
&= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{2(3a^2+b^2)\sin(c+dx)}{3(a^2-b^2)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} \\
&+ \frac{(3a-b)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3(a-b)(a+b)^2} - \frac{(3a^2+b^2)\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3(a^2-b^2)^2} \\
&= -\frac{2(3a^2+b^2)\cot(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d} \\
&+ \frac{2(3a-b)\cot(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d} \\
&- \frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{2(3a^2+b^2)\sin(c+dx)}{3(a^2-b^2)^2d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 1273, normalized size of antiderivative = 3.55

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{2b\sin(c+dx)}{3(a^2-b^2)(a+b\cos(c+dx))^2} - \frac{2(3a^2b\sin(c+dx)+b^3\sin(c+dx))}{3a(a^2-b^2)^2(a+b\cos(c+dx))}\right)}{d}$$

$$\frac{4a(-a^2b+b^3)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{a+b\cos(c+dx)}{a+b\cos(c+dx)}}\right)\right)}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

+

[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]

[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*b*Sin[c + d*x])/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(3*a^2*b*Sin[c + d*x] + b^3*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x])))/d + ((-4*a*(-a^2*b) + b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^3 + a*b^2)*((Sqrt[(a + b)*Cot[(c + d

```

*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 2*(3*a^2*b + b^3)*((I*Cos[(c + d*x
)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos
[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a*(a - b)^2*(a
+ b)^2*d)

```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2613 vs. $2(327) = 654$.

Time = 8.03 (sec) , antiderivative size = 2614, normalized size of antiderivative = 7.28

method	result	size
default	Expression too large to display	2614

```
[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

```

[Out] 2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)
)^(1/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1
/2)*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c)
))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-7*El
lipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1
-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1
-cos(d*x+c))^2+a+b)/(a+b))^(1/2)-5*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^2*b^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)
^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)-Ell

```

$$\begin{aligned} & \text{ipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a * b^3 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} + 6 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^3 * b * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} + 4 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^2 * b^2 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} + 2 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a * b^3 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} - 3 * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^4 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} * (1-\cos(dx+c))^{2+3} * \csc(dx+c)^2 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^4 * (1-\cos(dx+c))^{2+\csc(dx+c)^5 * b^4 * (1-\cos(dx+c))^{5-2} * \csc(dx+c)^3 * b^4 * (1-\cos(dx+c))^{3+2} * a^3 * b * (\csc(dx+c)-\cot(dx+c)) + 2 * a^2 * b^2 * (\csc(dx+c)-\cot(dx+c)) + 3 * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^2 * b^2 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} * (1-\cos(dx+c))^{2+\csc(dx+c)^2} * \text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a * b^3 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} * (1-\cos(dx+c))^{2-2} * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^2 * b^2 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} * (1-\cos(dx+c))^{2-2} * a * b^3 * (\csc(dx+c)-\cot(dx+c)) + 3 * \csc(dx+c)^5 * a^4 * (1-\cos(dx+c))^{5-6} * \csc(dx+c)^5 * a^3 * b * (1-\cos(dx+c))^{5+4} * \csc(dx+c)^3 * a * b^3 * (1-\cos(dx+c))^{3+4} * \csc(dx+c)^5 * a^2 * b^2 * (1-\cos(dx+c))^{5-2} * \csc(dx+c)^5 * a * b^3 * (1-\cos(dx+c))^{5+4} * \csc(dx+c)^3 * a^3 * b * (1-\cos(dx+c))^{3-6} * \csc(dx+c)^3 * a^2 * b^2 * (1-\cos(dx+c))^{3-3} * \text{EllipticF}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^4 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} + \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * b^4 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} - \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * b^4 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} * (1-\cos(dx+c))^{2+3} * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^4 * 3 * a^4 * (\csc(dx+c)-\cot(dx+c)) + b^4 * (\csc(dx+c)-\cot(dx+c)) / (\csc(dx+c)^2 * (1-\cos(dx+c))^{2-1}) / (\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})^2 / (a-b)^2 / (a+b)^2 / a \end{aligned}$$

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx$$

[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2), x)

[Out] Integral(sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(5/2), x)

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{5/2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{(a + b \cos(c + dx))^{5/2}} dx = \int \frac{\sqrt{\cos(c + dx)}}{(a + b \cos(c + dx))^{5/2}} dx$$

```
[In] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(a + b*cos(c + d*x))^(5/2), x)
```

$$3.641 \quad \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	6132
Rubi [A] (verified)	6133
Mathematica [C] (verified)	6135
Maple [B] (verified)	6136
Fricas [F]	6138
Sympy [F]	6138
Maxima [F]	6138
Giac [F]	6139
Mupad [F(-1)]	6139

Optimal result

Integrand size = 25, antiderivative size = 381

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))^{5/2}} dx = \frac{4b(3a^2 - b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a^3(a-b)(a+b)^{3/2}d}$$

$$+ \frac{2(3a^2 - 3ab - 2b^2) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d}$$

$$+ \frac{2b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3a(a^2 - b^2)d(a+b \cos(c+dx))^{3/2}} - \frac{4b(3a^2 - b^2) \sin(c+dx)}{3a(a^2 - b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2/3*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)-4/
3*b*(3*a^2-b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)
)^(1/2)+4/3*b*(3*a^2-b^2)*cot(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2
)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d+2/3*(3*a^2-3*a*b-2
*b^2)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1
/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d
```


Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00,
 number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used
 = {2881, 3072, 3077, 2895, 3073}

$$\int \frac{1}{\sqrt{\cos(c+dx)(a+b\cos(c+dx))^{5/2}}} dx = \frac{2(3a^2 - 3ab - 2b^2) \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{3a^2 d(a-b)(a+b)^{3/2}} + \frac{2b^2 \sin(c+dx) \sqrt{\cos(c+dx)}}{3ad(a^2 - b^2)(a+b\cos(c+dx))^{3/2}} - \frac{4b(3a^2 - b^2) \sin(c+dx)}{3ad(a^2 - b^2)^2 \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} + \frac{4b(3a^2 - b^2) \cot(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{3a^3 d(a-b)(a+b)^{3/2}}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]

[Out] (4*b*(3*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^3*(a - b)*(a + b)^(3/2)*d) + (2*(3*a^2 - 3*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^2*(a - b)*(a + b)^(3/2)*d) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (4*b*(3*a^2 - b^2)*Sin[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Ssin[e + f*x]]/Sqrt[d*Ssin[e + f*x]]]/Rt[(a + b)/d, 2]

], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3072

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]

Rule 3073

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2b^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3a(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} + \frac{2 \int \frac{\frac{1}{2}(3a^2 - 2b^2) - \frac{3}{2}ab \cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))^{3/2}} dx}{3a(a^2 - b^2)} \\ &= \frac{2b^2 \sqrt{\cos(c + dx)} \sin(c + dx)}{3a(a^2 - b^2) d(a + b \cos(c + dx))^{3/2}} \\ &\quad - \frac{4b(3a^2 - b^2) \sin(c + dx)}{3a(a^2 - b^2)^2 d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} \\ &\quad + \frac{2 \int \frac{\frac{3a^2 b}{2} + \frac{1}{2}b(3a^2 - 2b^2) + \left(\frac{3ab^2}{2} + \frac{1}{2}a(3a^2 - 2b^2)\right) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3a(a^2 - b^2)^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{2b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} \\
&\quad - \frac{4b(3a^2-b^2)\sin(c+dx)}{3a(a^2-b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{(3a^2-3ab-2b^2) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{3a(a-b)(a+b)^2} \\
&\quad + \frac{(2b(3a^2-b^2)) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{3a(a^2-b^2)^2} \\
&= \frac{4b(3a^2-b^2) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^3(a-b)(a+b)^{3/2}d} \\
&\quad + \frac{2(3a^2-3ab-2b^2) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d} \\
&\quad + \frac{2b^2 \sqrt{\cos(c+dx)} \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} - \frac{4b(3a^2-b^2)\sin(c+dx)}{3a(a^2-b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.29 (sec) , antiderivative size = 1296, normalized size of antiderivative = 3.40

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}} dx = \frac{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)} \left(\frac{2b^2 \sin(c+dx)}{3a(a^2-b^2)(a+b\cos(c+dx))^2} + \frac{4(3a^2-b^2)\sin(c+dx)}{3a^2} \right)}{d}$$

$$- \frac{4a(3a^4-5a^2b^2+2b^4) \sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}} \sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}} \sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{(a+b)\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}}$$

+

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]

[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (4*(3*a^2*b^2*Sin[c + d*x] - b^4*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x])))/d + ((-4*a*(3*a^4 - 5*

$$\begin{aligned}
& a^2 b^2 + 2 b^4) \sqrt{\frac{(a+b) \cot\left(\frac{c+dx}{2}\right)^2}{-a+b}} \sqrt{-\left(\frac{(a+b) \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)}{a}\right) \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)}{a} \csc\left(\frac{c+dx}{2}\right)^2)}{a}} \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)^2)}{a}} \sqrt{2}} \\
& \sqrt{2}], (-2a)/(-a+b)] \sin\left(\frac{c+dx}{2}\right)^4 / \left(\frac{(a+b) \sqrt{\cos\left(\frac{c+dx}{2}\right)} \sqrt{a+b \cos\left(\frac{c+dx}{2}\right)}}{a}\right) - 4 a^2 (-6 a^3 b + 2 a b^3) \left(\sqrt{\frac{(a+b) \cot\left(\frac{c+dx}{2}\right)^2}{-a+b}} \sqrt{-\left(\frac{(a+b) \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)}{a}\right) \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)}{a} \csc\left(\frac{c+dx}{2}\right)^2)}{a}} \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)^2)}{a}} \sqrt{2}} \right. \\
& \left. \sqrt{2}], (-2a)/(-a+b)] \sin\left(\frac{c+dx}{2}\right)^4 / \left(\frac{(a+b) \sqrt{\cos\left(\frac{c+dx}{2}\right)} \sqrt{a+b \cos\left(\frac{c+dx}{2}\right)}}{a}\right) - \left(\sqrt{\frac{(a+b) \cot\left(\frac{c+dx}{2}\right)^2}{-a+b}} \sqrt{-\left(\frac{(a+b) \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)}{a}\right) \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)}{a} \csc\left(\frac{c+dx}{2}\right)^2)}{a}} \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)^2)}{a}} \sqrt{2}} \right) \\
& \sqrt{2}], (-2a)/(-a+b)] \sin\left(\frac{c+dx}{2}\right)^4 / \left(\frac{(a+b) \sqrt{\cos\left(\frac{c+dx}{2}\right)} \sqrt{a+b \cos\left(\frac{c+dx}{2}\right)}}{a}\right) + 2 (-6 a^2 b^2 + 2 b^4) \left(\frac{I \cos\left(\frac{c+dx}{2}\right) \sqrt{a+b \cos\left(\frac{c+dx}{2}\right)} \operatorname{EllipticE}\left[\frac{I \operatorname{ArcSinh}\left(\frac{\sin\left(\frac{c+dx}{2}\right)}{\sqrt{\cos\left(\frac{c+dx}{2}\right)}}\right)}{b \sqrt{\cos\left(\frac{c+dx}{2}\right)^2 \sec\left(\frac{c+dx}{2}\right)}}\right]}{b \sqrt{\cos\left(\frac{c+dx}{2}\right)^2 \sec\left(\frac{c+dx}{2}\right)}} \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \sec\left(\frac{c+dx}{2}\right)}{a+b}} \right) + \right. \\
& \left. (2 a^2 \left(\frac{a \sqrt{\frac{(a+b) \cot\left(\frac{c+dx}{2}\right)^2}{-a+b}} \sqrt{-\left(\frac{(a+b) \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)}{a}\right) \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)}{a} \csc\left(\frac{c+dx}{2}\right)^2)}{a}} \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)^2)}{a}} \sqrt{2}} \right) \sqrt{2}], (-2a)/(-a+b)] \sin\left(\frac{c+dx}{2}\right)^4 / \left(\frac{(a+b) \sqrt{\cos\left(\frac{c+dx}{2}\right)} \sqrt{a+b \cos\left(\frac{c+dx}{2}\right)}}{a}\right) - \right. \\
& \left. \left(\frac{a \sqrt{\frac{(a+b) \cot\left(\frac{c+dx}{2}\right)^2}{-a+b}} \sqrt{-\left(\frac{(a+b) \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)}{a}\right) \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)}{a} \csc\left(\frac{c+dx}{2}\right)^2)}{a}} \sqrt{\frac{(a+b \cos\left(\frac{c+dx}{2}\right) \csc\left(\frac{c+dx}{2}\right)^2)}{a}} \sqrt{2}} \right) \sqrt{2}], (-2a)/(-a+b)] \sin\left(\frac{c+dx}{2}\right)^4 / \left(\frac{(a+b) \sqrt{\cos\left(\frac{c+dx}{2}\right)} \sqrt{a+b \cos\left(\frac{c+dx}{2}\right)}}{a}\right) \right) \right) / b + \left(\frac{\sqrt{a+b \cos\left(\frac{c+dx}{2}\right)} \sin\left(\frac{c+dx}{2}\right)}{b \sqrt{\cos\left(\frac{c+dx}{2}\right)}} \right) \right) / (3 a^2 (a-b)^2 (a+b)^2 d)
\end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2840 vs. $2(349) = 698$.

Time = 11.18 (sec) , antiderivative size = 2841, normalized size of antiderivative = 7.46

method	result	size
default	Expression too large to display	2841

[In] `int(1/cos(dx+c)^(1/2)/(a+cos(dx+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{2}{3} \frac{d}{dx} \left(-3 \csc(dx+c)^2 (-\csc(dx+c)^2 (1-\cos(dx+c))^2 + 1)^{1/2} \left(\csc(dx+c) \right)^2 a^2 (1-\cos(dx+c))^2 - \csc(dx+c)^2 b (1-\cos(dx+c))^2 + a+b \right) / (a+b)^{1/2} \operatorname{EllipticF}\left(\cot(dx+c) - \csc(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) a^5 (1-\cos(dx+c))^2 + 6 \csc(dx+c)^2 (-\csc(dx+c)^2 (1-\cos(dx+c))^2 + 1)^{1/2} \left(\csc(dx+c) \right)^2 a^2 (1-\cos(dx+c))^2 - \csc(dx+c)^2 b (1-\cos(dx+c))^2 + a+b \right) / (a+b)^{1/2} \operatorname{EllipticE}\left(\cot(dx+c) - \csc(dx+c), \left(-\frac{a-b}{a+b}\right)^{1/2}\right) a^4 b (1-\cos(dx+c))^2 - 2 \csc(dx+c)$

$$\frac{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{5/2}\sqrt{\cos(dx+c)}} dx$$

Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{5/2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^4 + 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c)^2 + a^3*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}} dx = \int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\cos(c+dx)}} dx$$

[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)

[Out] Integral(1/((a + b*cos(c + d*x))**(5/2)*sqrt(cos(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{5/2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{5/2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)),x)

[Out] int(1/(cos(c + d*x)^(1/2)*(a + b*cos(c + d*x))^(5/2)), x)

3.642 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^{5/2}} dx$

Optimal result	6140
Rubi [A] (verified)	6141
Mathematica [C] (verified)	6144
Maple [B] (verified)	6145
Fricas [F]	6147
Sympy [F(-1)]	6147
Maxima [F]	6147
Giac [F]	6148
Mupad [F(-1)]	6148

Optimal result

Integrand size = 25, antiderivative size = 398

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^{5/2}} dx = \frac{2(3a^4 - 15a^2b^2 + 8b^4) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{3a^4(a-b)(a+b)^{3/2}d} - \frac{2(3a^3 + 9a^2b - 6ab^2 - 8b^3) \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^3(a-b)(a+b)^{3/2}d} + \frac{2b^2 \sin(c+dx)}{3a(a^2 - b^2) d \sqrt{\cos(c+dx)} (a+b \cos(c+dx))^{3/2}} + \frac{8b^2(2a^2 - b^2) \sin(c+dx)}{3a^2(a^2 - b^2)^2 d \sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+8/
3*b^2*(2*a^2-b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)/(a+b*cos(d*
x+c))^(1/2)+2/3*(3*a^4-15*a^2*b^2+8*b^4)*cot(d*x+c)*EllipticE((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+
c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d-2/3
*(3*a^3+9*a^2*b-6*a*b^2-8*b^3)*cot(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/
(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))
^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d
```


Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2881, 3134, 3077, 2895, 3073}

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}} dx = \frac{8b^2(2a^2-b^2)\sin(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{2(3a^4-15a^2b^2+8b^4)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)\Big|_{-\frac{a+b}{a-b}}}{3a^4d(a-b)(a+b)^{3/2}} - \frac{2(3a^3+9a^2b-6ab^2-8b^3)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{3a^3d(a-b)(a+b)^{3/2}}$$

[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)), x]

[Out] (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^4*(a - b)*(a + b)^(3/2)*d) - (2*(3*a^3 + 9*a^2*b - 6*a*b^2 - 8*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^3*(a - b)*(a + b)^(3/2)*d) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (8*b^2*(2*a^2 - b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr

```
t[a*((1 - Csc[e + f*x])/(a + b))*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d \sqrt{\cos(c + dx)(a + b \cos(c + dx))^{3/2}} \\
 &\quad + \frac{2 \int \frac{\frac{1}{2}(3a^2 - 4b^2) - \frac{3}{2}ab \cos(c + dx) + b^2 \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2}} dx}{3a(a^2 - b^2)} \\
 &= \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d \sqrt{\cos(c + dx)(a + b \cos(c + dx))^{3/2}} \\
 &\quad + \frac{8b^2(2a^2 - b^2) \sin(c + dx)}{3a^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} \\
 &\quad + \frac{4 \int \frac{\frac{1}{4}(3a^4 - 15a^2b^2 + 8b^4) - \frac{1}{2}ab(3a^2 - b^2) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3a^2(a^2 - b^2)^2} \\
 &= \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d \sqrt{\cos(c + dx)(a + b \cos(c + dx))^{3/2}} \\
 &\quad + \frac{8b^2(2a^2 - b^2) \sin(c + dx)}{3a^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} \\
 &\quad - \frac{(3a^3 + 9a^2b - 6ab^2 - 8b^3) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx}{3a^2(a - b)(a + b)^2} \\
 &\quad + \frac{(3a^4 - 15a^2b^2 + 8b^4) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3a^2(a^2 - b^2)^2} \\
 &= \frac{2(3a^4 - 15a^2b^2 + 8b^4) \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{3a^4(a - b)(a + b)^{3/2}d} \\
 &\quad - \frac{2(3a^3 + 9a^2b - 6ab^2 - 8b^3) \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}}}{3a^3(a - b)(a + b)^{3/2}d} \\
 &\quad + \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d \sqrt{\cos(c + dx)(a + b \cos(c + dx))^{3/2}} \\
 &\quad + \frac{8b^2(2a^2 - b^2) \sin(c + dx)}{3a^2(a^2 - b^2)^2 d \sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.40 (sec) , antiderivative size = 1321, normalized size of antiderivative = 3.32

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}} dx =$$

$$\frac{4a(9a^4b-17a^2b^3+8b^5)\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\right)\right)}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

$$+\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{2b^3\sin(c+dx)}{3a^2(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{2(9a^2b^3\sin(c+dx)-5b^5\sin(c+dx))}{3a^3(a^2-b^2)^2(a+b\cos(c+dx))}+\frac{2\tan(c+dx)}{a^3}\right)}{d}$$

[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x]

[Out]
$$-1/3*((-4*a*(9*a^4*b - 17*a^2*b^3 + 8*b^5)*\operatorname{Sqrt}[\frac{(a+b)\operatorname{Cot}[(c+d*x)/2]^2}{-a+b}])*\operatorname{Sqrt}[-\frac{(a+b)\operatorname{Cos}[c+d*x]*\operatorname{Csc}[(c+d*x)/2]^2}{a}])*\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Csc}[(c+d*x)/2]^2}{a}])*\operatorname{Csc}[c+d*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Csc}[(c+d*x)/2]^2}{a}]]/\operatorname{Sqrt}[2]],(-2*a)/(-a+b)]*\operatorname{Sin}[(c+d*x)/2]^4/\frac{(a+b)*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+b\operatorname{Cos}[c+d*x]]}{(a+b)*\operatorname{Sqrt}[\frac{(a+b)\operatorname{Cot}[(c+d*x)/2]^2}{-a+b}])*\operatorname{Sqrt}[-\frac{(a+b)\operatorname{Cos}[c+d*x]*\operatorname{Csc}[(c+d*x)/2]^2}{a}])*\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Csc}[(c+d*x)/2]^2}{a}])*\operatorname{Csc}[c+d*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Csc}[(c+d*x)/2]^2}{a}]]/\operatorname{Sqrt}[2]],(-2*a)/(-a+b)]*\operatorname{Sin}[(c+d*x)/2]^4/\frac{(a+b)*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+b\operatorname{Cos}[c+d*x]]}{(a+b)*\operatorname{Sqrt}[\frac{(a+b)\operatorname{Cot}[(c+d*x)/2]^2}{-a+b}])*\operatorname{Sqrt}[-\frac{(a+b)\operatorname{Cos}[c+d*x]*\operatorname{Csc}[(c+d*x)/2]^2}{a}])*\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Csc}[(c+d*x)/2]^2}{a}])*\operatorname{Csc}[c+d*x]*\operatorname{EllipticPi}[-(a/b),\operatorname{ArcSin}[\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Csc}[(c+d*x)/2]^2}{a}]]/\operatorname{Sqrt}[2]],(-2*a)/(-a+b)]*\operatorname{Sin}[(c+d*x)/2]^4/(b*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+b\operatorname{Cos}[c+d*x]]) + 2*(3*a^4*b - 15*a^2*b^3 + 8*b^5)*((I*\operatorname{Cos}[(c+d*x)/2]*\operatorname{Sqrt}[a+b\operatorname{Cos}[c+d*x]]*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\operatorname{Sin}[(c+d*x)/2]/\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]],(-2*a)/(-a-b)]*\operatorname{Sec}[c+d*x])/(b*\operatorname{Sqrt}[\operatorname{Cos}[(c+d*x)/2]^2*\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Sec}[c+d*x]}{(a+b)}]) + (2*a*((a*\operatorname{Sqrt}[\frac{(a+b)\operatorname{Cot}[(c+d*x)/2]^2}{-a+b}])*\operatorname{Sqrt}[-\frac{(a+b)\operatorname{Cos}[c+d*x]*\operatorname{Csc}[(c+d*x)/2]^2}{a}])*\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Csc}[(c+d*x)/2]^2}{a}])*\operatorname{Csc}[c+d*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Csc}[(c+d*x)/2]^2}{a}]]/\operatorname{Sqrt}[2]],(-2*a)/(-a+b)]*\operatorname{Sin}[(c+d*x)/2]^4/(b*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+b\operatorname{Cos}[c+d*x]]) + 2*(3*a^4*b - 15*a^2*b^3 + 8*b^5)*((I*\operatorname{Cos}[(c+d*x)/2]*\operatorname{Sqrt}[a+b\operatorname{Cos}[c+d*x]]*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\operatorname{Sin}[(c+d*x)/2]/\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]],(-2*a)/(-a-b)]*\operatorname{Sec}[c+d*x])/(b*\operatorname{Sqrt}[\operatorname{Cos}[(c+d*x)/2]^2*\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Sec}[c+d*x]}{(a+b)}]) + (2*a*((a*\operatorname{Sqrt}[\frac{(a+b)\operatorname{Cot}[(c+d*x)/2]^2}{-a+b}])*\operatorname{Sqrt}[-\frac{(a+b)\operatorname{Cos}[c+d*x]*\operatorname{Csc}[(c+d*x)/2]^2}{a}])*\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Csc}[(c+d*x)/2]^2}{a}])*\operatorname{Csc}[c+d*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[\frac{(a+b\operatorname{Cos}[c+d*x])*\operatorname{Csc}[(c+d*x)/2]^2}{a}]]/\operatorname{Sqrt}[2]],(-2*a)/(-a+b)]*\operatorname{Sin}[(c+d*x)/2]^4/(b*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[a+b\operatorname{Cos}[c+d*x]])$$

$$\begin{aligned} & \text{rt}[2]], (-2*a)/(-a + b)]*\text{Sin}[(c + d*x)/2]^4)/((a + b)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]) - (a*\text{Sqrt}[(a + b)*\text{Cot}[(c + d*x)/2]^2]/(-a + b)]*\text{Sqrt}[-(((a + b)*\text{Cos}[c + d*x]*\text{Csc}[(c + d*x)/2]^2)/a)]*\text{Sqrt}[(a + b*\text{Cos}[c + d*x])*\text{Csc}[(c + d*x)/2]^2)/a]*\text{Csc}[c + d*x]*\text{EllipticPi}[-(a/b), \text{ArcSin}[\text{Sqrt}[(a + b*\text{Cos}[c + d*x])*\text{Csc}[(c + d*x)/2]^2)/a]/\text{Sqrt}[2]], (-2*a)/(-a + b)]*\text{Sin}[(c + d*x)/2]^4)/(b*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Cos}[c + d*x]])/b + (\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(b*\text{Sqrt}[\text{Cos}[c + d*x]]))/((a^3*(a - b)^2*(a + b)^2*d) + (\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*((-2*b^3*\text{Sin}[c + d*x])/((3*a^2*(a^2 - b^2)*(a + b*\text{Cos}[c + d*x])^2) - (2*(9*a^2*b^3*\text{Sin}[c + d*x] - 5*b^5*\text{Sin}[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a + b*\text{Cos}[c + d*x])) + (2*\text{Tan}[c + d*x])/a^3))/d \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3674 vs. $2(366) = 732$.

Time = 13.49 (sec) , antiderivative size = 3675, normalized size of antiderivative = 9.23

method	result	size
default	Expression too large to display	3675

[In] `int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -2/3/d*(\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^2-1)*((\text{csc}(d*x+c)^2*a*(1-\text{cos}(d*x+c))^2- \\ & \text{csc}(d*x+c)^2*b*(1-\text{cos}(d*x+c))^2+a*b)/(\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^2+1))^(1/2) \\ & *(-3*\text{csc}(d*x+c)^2*\text{EllipticF}(\text{cot}(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^(1/2))*a^6 \\ & *(-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^2+1)^(1/2)*((\text{csc}(d*x+c)^2*a*(1-\text{cos}(d*x+c))^2- \\ & \text{csc}(d*x+c)^2*b*(1-\text{cos}(d*x+c))^2+a*b)/(a+b))^(1/2)*(1-\text{cos}(d*x+c))^2+8*b^6 \\ & *(\text{csc}(d*x+c)-\text{cot}(d*x+c))-7*\text{csc}(d*x+c)^5*a^2*b^4*(1-\text{cos}(d*x+c))^5-16*\text{csc}(d*x+c)^5 \\ & *a*b^5*(1-\text{cos}(d*x+c))^5-18*\text{csc}(d*x+c)^3*a^4*b^2*(1-\text{cos}(d*x+c))^3-16*\text{csc}(d*x+c)^3 \\ & *a^3*b^3*(1-\text{cos}(d*x+c))^3+36*\text{csc}(d*x+c)^3*a^2*b^4*(1-\text{cos}(d*x+c))^3+8*\text{csc}(d*x+c)^3 \\ & *a*b^5*(1-\text{cos}(d*x+c))^3-3*\text{EllipticF}(\text{cot}(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^(1/2) \\ & *a^6*(-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^2+1)^(1/2)*((\text{csc}(d*x+c)^2*a*(1-\text{cos}(d*x+c))^2- \\ & \text{csc}(d*x+c)^2*b*(1-\text{cos}(d*x+c))^2+a*b)/(a+b))^(1/2)+3*\text{EllipticE}(\text{cot}(d*x+c)-\text{csc}(d*x+c), \\ & (-a-b)/(a+b))^(1/2))*a^6*(-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^2+1)^(1/2)*((\text{csc}(d*x+c)^2*a*(1-\text{cos}(d*x+c))^2- \\ & \text{csc}(d*x+c)^2*b*(1-\text{cos}(d*x+c))^2+a*b)/(a+b))^(1/2)+8*\text{EllipticE}(\text{cot}(d*x+c)-\text{csc}(d*x+c), \\ & (-a-b)/(a+b))^(1/2))*b^6*(-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^2+1)^(1/2)*((\text{csc}(d*x+c)^2 \\ & *a*(1-\text{cos}(d*x+c))^2- \\ & \text{csc}(d*x+c)^2*b*(1-\text{cos}(d*x+c))^2+a*b)/(a+b))^(1/2)-6*\text{csc}(d*x+c)^5*a^5*b*(1-\text{cos}(d*x+c))^5-12*\text{csc}(d*x+c)^5 \\ & *a^4*b^2*(1-\text{cos}(d*x+c))^5+30*\text{csc}(d*x+c)^5*a^3*b^3*(1-\text{cos}(d*x+c))^5+3*\text{EllipticF}(\text{cot}(d*x+c)-\text{csc}(d*x+c), \\ & (-a-b)/(a+b))^(1/2))*a^5*b*(-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^2+1)^(1/2)*((\text{csc}(d*x+c)^2*a*(1-\text{cos}(d*x+c))^2- \\ & \text{csc}(d*x+c)^2*b*(1-\text{cos}(d*x+c))^2+a*b)/(a+b))^(1/2)+21*\text{EllipticF}(\text{cot}(d*x+c)-\text{csc}(d*x+c), \\ & (-a-b)/(a+b))^(1/2))*a^4*b^2*(-\text{csc}(d*x+c)^2*(1-\text{cos}(d*x+c))^2+1)^(1/2)*((\text{csc}(d*x+c)^2*a*(1-\text{cos}(d*x+c))^2- \\ & \text{csc}(d*x+c)^2*b*(1-\text{cos}(d*x+c))^2+a*b)/(a+b))^(1/2)+13*\text{EllipticF}(\text{cot}(d*x+c)-\text{csc}(d*x+c) \end{aligned}$$

$$\begin{aligned}
&), (-a-b)/(a+b)^{(1/2)} * a^3 * b^3 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} \\
&- 10 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^2 * b^4 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} \\
&- 8 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a * b^5 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} \\
&+ 6 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^5 * b * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} \\
&- 12 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^4 * b^2 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} \\
&- 30 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^3 * b^3 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} \\
&- 7 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^2 * b^4 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} \\
&+ 16 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a * b^5 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} \\
&- 18 * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^4 * b^2 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2+23} * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^2 * b^4 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2+9} * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^5 * b * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2+9} * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^4 * b^2 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2-17} * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^3 * b^3 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2-6} * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a^2 * b^4 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2+8} * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b)^{(1/2)}) * a * b^5 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2+6} * \csc(dx+c)^3 * a^6 * (1-\cos(dx+c))^{3-16} * \csc(dx+c)^3 * b^6 * (1-\cos(dx+c))^{3-14} * a^3 * b^3 * (\csc(dx+c) - \cot(dx+c)) + 8 * a * b^5 * (\csc(dx+c) - \cot(dx+c)) + 6 * a^5 * b * (\csc(dx+c) - \cot(dx+c)) + 6 * a^4 * b^2 * (\csc(dx+c) - \cot(dx+c)) - 17 * a^2 * b^4 * (\csc(dx+c) - \cot(dx+c)) + 3 * \csc(dx+c)^5 * a^6 * (1-\cos(dx+c))^{5+8} * \csc(dx+c)^5 * b^6 * (1-\cos(dx+c))^{5+3} * a^6 * (\csc(dx+c) - \cot(dx+c)) + 3 * \csc(dx+c)^2 * \text{Ellipti}
\end{aligned}$$

$cE(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * a^6 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} * (1-\cos(dx+c))^{2-8 * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c), (-a-b)/(a+b))^{1/2}) * b^6 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{1/2} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{1/2} * (1-\cos(dx+c))^{2-2} / (\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})^{2-2} / (\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1}) / (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2-1}) / (\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{3/2} / (a-b)^{2/2} / (a+b)^2 / a^3$

Fricas [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}} \cos^{\frac{3}{2}}(dx+c)} dx$$

[In] integrate(1/cos(dx+c)^(3/2)/(a+b*cos(dx+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(dx + c) + a)*sqrt(cos(dx + c))/(b^3*cos(dx + c)^5 + 3*a*b^2*cos(dx + c)^4 + 3*a^2*b*cos(dx + c)^3 + a^3*cos(dx + c)^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(1/cos(dx+c)**(3/2)/(a+b*cos(dx+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}} \cos^{\frac{3}{2}}(dx+c)} dx$$

[In] integrate(1/cos(dx+c)^(3/2)/(a+b*cos(dx+c))^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(dx + c) + a)^(5/2)*cos(dx + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}} \cos(dx+c)^{\frac{3}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}} dx = \int \frac{1}{\cos(c+dx)^{3/2} (a+b\cos(c+dx))^{5/2}} dx$$

[In] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)),x)

[Out] int(1/(cos(c + d*x)^(3/2)*(a + b*cos(c + d*x))^(5/2)), x)

$$3.643 \quad \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	6149
Rubi [A] (verified)	6150
Mathematica [C] (verified)	6153
Maple [B] (verified)	6154
Fricas [F]	6155
Sympy [F(-1)]	6155
Maxima [F]	6155
Giac [F]	6155
Mupad [F(-1)]	6156

Optimal result

Integrand size = 25, antiderivative size = 473

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b \cos(c+dx))^{5/2}} dx =$$

$$\frac{8b(2a^4 - 7a^2b^2 + 4b^4) \cot(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^5(a-b)(a+b)^{3/2}d}$$

$$+ \frac{2(a^4 + 9a^3b + 16a^2b^2 - 12ab^3 - 16b^4) \cot(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a^4(a-b)(a+b)^{3/2}d}$$

$$+ \frac{2b^2 \sin(c+dx)}{3a(a^2 - b^2) d \cos^{\frac{3}{2}}(c+dx)(a+b \cos(c+dx))^{3/2}}$$

$$+ \frac{4b^2(5a^2 - 3b^2) \sin(c+dx)}{3a^2(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}}$$

$$+ \frac{2(a^4 - 13a^2b^2 + 8b^4) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{3a^3(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2)+4/
3*b^2*(5*a^2-3*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(a+b*cos(
d*x+c))^(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/
a^3/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*cot(d*x+c)
*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b)
)^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^5
/(a-b)/(a+b)^(3/2)/d+2/3*(a^4+9*a^3*b+16*a^2*b^2-12*a*b^3-16*b^4)*cot(d*x+c)
*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-
b))^(1/2))*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^
4/(a-b)/(a+b)^(3/2)/d
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2881, 3134, 3077, 2895, 3073}

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \frac{4b^2(5a^2-3b^2)\sin(c+dx)}{3a^2d(a^2-b^2)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{3ad(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{3}{2}}} - \frac{8b(2a^4-7a^2b^2+4b^4)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{3a^5d(a-b)(a+b)^{\frac{3}{2}}} + \frac{2(a^4+9a^3b+16a^2b^2-12ab^3-16b^4)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{3a^4d(a-b)(a+b)^{\frac{3}{2}}} + \frac{2(a^4-13a^2b^2+8b^4)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d(a^2-b^2)^2\cos^{\frac{3}{2}}(c+dx)}$$

[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]

[Out] (-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^5*(a - b)*(a + b)^(3/2)*d) + (2*(a^4 + 9*a^3*b + 16*a^2*b^2 - 12*a*b^3 - 16*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^4*(a - b)*(a + b)^(3/2)*d) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)) + (4*b^2*(5*a^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2))

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n]))

] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3134

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||

EqQ[a, 0]))))

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2}} \\
&+ \frac{2 \int \frac{\frac{3}{2}(a^2 - 2b^2) - \frac{3}{2}ab \cos(c + dx) + 2b^2 \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^{3/2}} dx}{3a(a^2 - b^2)} \\
&= \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2}} \\
&+ \frac{4b^2(5a^2 - 3b^2) \sin(c + dx)}{3a^2(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{4 \int \frac{\frac{3}{4}(a^4 - 13a^2b^2 + 8b^4) - \frac{1}{2}ab(3a^2 - b^2) \cos(c + dx) + b^2(5a^2 - 3b^2) \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3a^2(a^2 - b^2)^2} \\
&= \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2}} \\
&+ \frac{4b^2(5a^2 - 3b^2) \sin(c + dx)}{3a^2(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{2(a^4 - 13a^2b^2 + 8b^4) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3a^3(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)} \\
&+ \frac{8 \int \frac{-\frac{3}{2}b(2a^4 - 7a^2b^2 + 4b^4) + \frac{3}{8}a(a^4 + 7a^2b^2 - 4b^4) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{9a^3(a^2 - b^2)^2} \\
&= \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2}} \\
&+ \frac{4b^2(5a^2 - 3b^2) \sin(c + dx)}{3a^2(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{2(a^4 - 13a^2b^2 + 8b^4) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3a^3(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)} \\
&+ \frac{(a^4 + 9a^3b + 16a^2b^2 - 12ab^3 - 16b^4) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx}{3a^3(a - b)(a + b)^2} \\
&- \frac{(4b(2a^4 - 7a^2b^2 + 4b^4)) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3a^3(a^2 - b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= \\
&\frac{8b(2a^4 - 7a^2b^2 + 4b^4) \cot(c + dx) E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^5(a-b)(a+b)^{3/2}d} \\
&+ \frac{2(a^4 + 9a^3b + 16a^2b^2 - 12ab^3 - 16b^4) \cot(c + dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^4(a-b)(a+b)^{3/2}d} \\
&+ \frac{2b^2 \sin(c + dx)}{3a(a^2 - b^2) d \cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2}} \\
&+ \frac{4b^2(5a^2 - 3b^2) \sin(c + dx)}{3a^2(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} \\
&+ \frac{2(a^4 - 13a^2b^2 + 8b^4) \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3a^3(a^2 - b^2)^2 d \cos^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.44 (sec) , antiderivative size = 1351, normalized size of antiderivative = 2.86

$$\frac{4a(a^6 + 15a^4b^2 - 32a^2b^4 + 16b^6) \sqrt{\frac{(a+b) \cot^2\left(\frac{1}{2}(c+dx)\right)}{-a+b}} \sqrt{-\frac{(a+b) \cos(c+dx) \operatorname{csc}^2\left(\frac{1}{2}(c+dx)\right)}{a}}}{d}$$

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^{5/2}} dx = \frac{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)} \left(\frac{2b^4 \sin(c + dx)}{3a^3(a^2 - b^2)(a + b \cos(c + dx))^2} + \frac{8(3a^2b^4 \sin(c + dx) - 2b^6 \sin(c + dx))}{3a^4(a^2 - b^2)^2(a + b \cos(c + dx))} - \frac{16b \tan(c + dx)}{3a^4} + 2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}} \right)}{d}$$

[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]

[Out] ((-4*a*(a^6 + 15*a^4*b^2 - 32*a^2*b^4 + 16*b^6)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(8*a^5*b - 28*a^3*b^3 + 16*a*b^5)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a])*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[

$$\begin{aligned} & ((a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a) / \sqrt{2}], (-2a)/(-a + b)] \operatorname{Sin} \\ & [(c + dx)/2]^4 / ((a + b) \sqrt{\cos[c + dx]} \sqrt{a + b \cos[c + dx]}) - (\operatorname{Sqrt} \\ & [((a + b) \operatorname{Cot}[(c + dx)/2]^2)/(-a + b)] \operatorname{Sqrt}[-((a + b) \cos[c + dx] \operatorname{Csc} \\ & [(c + dx)/2]^2/a)] \operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a] \operatorname{Csc} \\ & [c + dx] \operatorname{EllipticPi}[-(a/b), \operatorname{ArcSin}[\operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx) \\ & /2]^2/a] / \sqrt{2}], (-2a)/(-a + b)] \operatorname{Sin}[(c + dx)/2]^4 / (b \sqrt{\cos[c + dx]} \\ & \operatorname{Sqrt}[a + b \cos[c + dx]])) + 2(8a^4b^2 - 28a^2b^4 + 16b^6) * ((I \operatorname{Cos} \\ & [(c + dx)/2] \operatorname{Sqrt}[a + b \cos[c + dx]] \operatorname{EllipticE}[I \operatorname{ArcSinh}[\operatorname{Sin}[(c + dx)/2] \\ &] / \sqrt{\cos[c + dx]}], (-2a)/(-a - b)] \operatorname{Sec}[c + dx]) / (b \sqrt{\cos[(c + dx) \\ & /2]^2} \operatorname{Sec}[c + dx]) \operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Sec}[c + dx]) / (a + b)) + (2 \\ & * a * ((a \operatorname{Sqrt}[(a + b) \operatorname{Cot}[(c + dx)/2]^2)/(-a + b)] \operatorname{Sqrt}[-((a + b) \cos[c + \\ & dx] \operatorname{Csc}[(c + dx)/2]^2/a)] \operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2 \\ & /a] \operatorname{Csc}[c + dx] \operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2] \\ & ^2)/a] / \sqrt{2}], (-2a)/(-a + b)] \operatorname{Sin}[(c + dx)/2]^4 / ((a + b) \sqrt{\cos[c + \\ & dx]} \operatorname{Sqrt}[a + b \cos[c + dx]]) - (a \operatorname{Sqrt}[(a + b) \operatorname{Cot}[(c + dx)/2]^2) / (- \\ & a + b)] \operatorname{Sqrt}[-((a + b) \cos[c + dx] \operatorname{Csc}[(c + dx)/2]^2/a)] \operatorname{Sqrt}[(a + b \cos[c + dx]) \\ & \operatorname{Csc}[(c + dx)/2]^2/a] \operatorname{Csc}[c + dx] \operatorname{EllipticPi}[-(a/b), \operatorname{ArcSin} \\ & [\operatorname{Sqrt}[(a + b \cos[c + dx]) \operatorname{Csc}[(c + dx)/2]^2/a] / \sqrt{2}], (-2a)/(-a + b) \\ &)) \operatorname{Sin}[(c + dx)/2]^4 / (b \sqrt{\cos[c + dx]} \operatorname{Sqrt}[a + b \cos[c + dx]])) / b \\ & + (\operatorname{Sqrt}[a + b \cos[c + dx]] \operatorname{Sin}[c + dx]) / (b \sqrt{\cos[c + dx]}) / (3a^4 * (\\ & a - b)^2 * (a + b)^2 * d) + (\operatorname{Sqrt}[\cos[c + dx]] \operatorname{Sqrt}[a + b \cos[c + dx]]) * ((2b^4 \\ & \operatorname{Sin}[c + dx]) / (3a^3 * (a^2 - b^2) * (a + b \cos[c + dx])^2) + (8 * (3a^2 * b^4 \\ & \operatorname{Sin}[c + dx] - 2b^6 * \operatorname{Sin}[c + dx])) / (3a^4 * (a^2 - b^2)^2 * (a + b \cos[c + dx] \\ &))) - (16b * \operatorname{Tan}[c + dx]) / (3a^4) + (2 * \operatorname{Sec}[c + dx] * \operatorname{Tan}[c + dx]) / (3a^3)) \\ & / d \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5223 vs. 2(435) = 870.

Time = 15.91 (sec) , antiderivative size = 5224, normalized size of antiderivative = 11.04

method	result	size
default	Expression too large to display	5224

[In] `int(1/cos(dx+c)^(5/2)/(a+cos(dx+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out] result too large to display

Fricas [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^6 + 3*a*b^2*cos(d*x + c)^5 + 3*a^2*b*cos(d*x + c)^4 + a^3*cos(d*x + c)^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c + dx)(a + b \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{1}{\cos(c + dx)^{\frac{5}{2}}(a + b \cos(c + dx))^{\frac{5}{2}}} dx$$

```
[In] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(5/2)*(a + b*cos(c + d*x))^(5/2)), x)
```


$$3.644 \quad \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx$$

Optimal result	6157
Rubi [A] (verified)	6157
Mathematica [B] (verified)	6158
Maple [B] (verified)	6158
Fricas [F]	6158
Sympy [F]	6159
Maxima [F]	6159
Giac [F]	6159
Mupad [F(-1)]	6159

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right), \frac{1}{5}\right)}{\sqrt{5}d}$$

[Out] 2/5*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),1/5*5^(1/2))/d*5^(1/2)

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2892}

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right), \frac{1}{5}\right)}{\sqrt{5}d}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]

[Out] (2*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 1/5])/(Sqrt[5]*d)

Rule 2892

Int[1/(Sqrt[(d_)*sin[e_] + (f_)*(x_)]*Sqrt[(a_) + (b_)*sin[e_] + (f_)*(x_)]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]

Rubi steps

$$\text{integral} = \frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right), \frac{1}{5}\right)}{\sqrt{5}d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 131 vs. $2(32) = 64$.

Time = 9.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.09

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{2+3\cos(c+dx)}\right)\right)}{d\sqrt{\frac{-2-3\cos(c+dx)}{-1+\cos(c+dx)}}\sqrt{\frac{\cos(c+dx)}{-1+\cos(c+dx)}}}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]

[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Cot[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4])/(d*Sqrt[(-2 - 3*Cos[c + d*x])/(-1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(31) = 62$.

Time = 7.88 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.22

method	result	size
default	$-\frac{(1+\cos(dx+c))\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{10}\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}F\left(\cot(dx+c)-\csc(dx+c),\frac{\sqrt{5}}{5}\right)}{5d\sqrt{2+3\cos(dx+c)}\sqrt{\cos(dx+c)}}$	103

[In] int(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/5/d*(1+cos(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(2+3*cos(d*x+c))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),1/5*5^(1/2))/cos(d*x+c)^(1/2)

Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{3\cos(dx+c)+2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 + 2*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{3\cos(c+dx)+2}\sqrt{\cos(c+dx)}} dx$$

[In] integrate(1/cos(d*x+c)**(1/2)/(2+3*cos(d*x+c))**(1/2), x)

[Out] Integral(1/(sqrt(3*cos(c + d*x) + 2)*sqrt(cos(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{3\cos(dx+c)+2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{3\cos(dx+c)+2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)+2}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2)^(1/2)), x)

[Out] int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) + 2)^(1/2)), x)

$$3.645 \quad \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx$$

Optimal result	6160
Rubi [A] (verified)	6160
Mathematica [B] (verified)	6161
Maple [B] (verified)	6161
Fricas [F]	6161
Sympy [F]	6162
Maxima [F]	6162
Giac [F]	6162
Mupad [F(-1)]	6162

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right), 5\right)}{d}$$

[Out] 2*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),5^(1/2))/d

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2892}

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right), 5\right)}{d}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]

[Out] (2*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 5])/d

Rule 2892

Int[1/(Sqrt[(d_)*sin[(e_)+(f_)*(x_)]]*Sqrt[(a_)+(b_)*sin[(e_)+(f_)*(x_)]]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]

Rubi steps

$$\text{integral} = \frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right), 5\right)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 156 vs. $2(25) = 50$.

Time = 0.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 6.24

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx$$

$$= \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)}\operatorname{csc}^2\left(\frac{1}{2}(c+dx)\right)\sqrt{-\left((-2+3\cos(c+dx))\operatorname{csc}^2\left(\frac{1}{2}(c+dx)\right)\right)}\operatorname{csc}(c+dx)}{\sqrt{5}d\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}$$

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]

[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]/2], 4/5]*Sin[(c + d*x)/2]^4)/(Sqrt[5]*d*Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(26) = 52$.

Time = 6.78 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.80

method	result	size
default	$-\frac{2(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}F\left(\cot(dx+c)-\operatorname{csc}(dx+c),\sqrt{5}\right)}{d\sqrt{-2+3\cos(dx+c)}\sqrt{\cos(dx+c)}}$	95

[In] int(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/d*(1+cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2+3*cos(d*x+c))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))/cos(d*x+c)^(1/2)

Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{3\cos(dx+c)-2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 - 2*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{3\cos(c+dx)-2}\sqrt{\cos(c+dx)}} dx$$

[In] integrate(1/cos(d*x+c)**(1/2)/(-2+3*cos(d*x+c))**(1/2), x)

[Out] Integral(1/(sqrt(3*cos(c + d*x) - 2)*sqrt(cos(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{3\cos(dx+c)-2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{3\cos(dx+c)-2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)-2}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2)^(1/2)), x)

[Out] int(1/(cos(c + d*x)^(1/2)*(3*cos(c + d*x) - 2)^(1/2)), x)

$$3.646 \quad \int \frac{1}{\sqrt{2-3 \cos(c+dx)} \sqrt{\cos(c+dx)}} dx$$

Optimal result	6163
Rubi [A] (verified)	6163
Mathematica [B] (verified)	6164
Maple [B] (verified)	6165
Fricas [F]	6165
Sympy [F]	6165
Maxima [F]	6166
Giac [F]	6166
Mupad [F(-1)]	6166

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{1}{\sqrt{2-3 \cos(c+dx)} \sqrt{\cos(c+dx)}} dx$$

$$= -\frac{2\sqrt{-\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), \frac{1}{5}\right)}{\sqrt{5}d\sqrt{\cos(c+dx)}}$$

[Out] $-2/5*\operatorname{EllipticF}(\sin(d*x+c)/(1-\cos(d*x+c)), 1/5*5^{(1/2)})*(-\cos(d*x+c))^{(1/2)}/d*5^{(1/2)}/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2893, 2892}

$$\int \frac{1}{\sqrt{2-3 \cos(c+dx)} \sqrt{\cos(c+dx)}} dx$$

$$= -\frac{2\sqrt{-\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), \frac{1}{5}\right)}{\sqrt{5}d\sqrt{\cos(c+dx)}}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[2-3*\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]),x]$

[Out] $(-2*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sin}[c+d*x]/(1-\operatorname{Cos}[c+d*x])], 1/5])/(\operatorname{Sqrt}[5]*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])$

Rule 2892

$\operatorname{Int}[1/(\operatorname{Sqrt}[(d_*)*\sin[(e_*) + (f_*)*(x_*)]]*\operatorname{Sqrt}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)])], x_Symbol] \rightarrow \operatorname{Simp}[-2*(d/(f*\operatorname{Sqrt}[a + b*d]))*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Co}$

```
s[e + f*x]/(1 + d*Sin[e + f*x]), -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b,
d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rule 2893

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] && !(EqQ[d^
2, 1] && GtQ[b*d, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{-\cos(c+dx)} \int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx}{\sqrt{\cos(c+dx)}} \\ &= -\frac{2\sqrt{-\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), \frac{1}{5}\right)}{\sqrt{5}d\sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(56) = 112.

Time = 1.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.55

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2-3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), \frac{1}{5}\right)}{d\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*
Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[Co
s[c + d*x]*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sqrt[2 - 3*Co
s[c + d*x]]*Sqrt[Cos[c + d*x]])
```


Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(51) = 102.

Time = 7.70 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{2(1+\cos(dx+c))\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2-3\cos(dx+c)}\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}F(\cot(dx+c)-\operatorname{csc}(dx+c),\sqrt{5})}{d(-2+3\cos(dx+c))\sqrt{\cos(dx+c)}}$	107

[In] `int(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `2/d*(1+cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2-3*cos(d*x+c))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))/(-2+3*cos(d*x+c))/cos(d*x+c)^(1/2)`

Fricas [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}} dx$$

[In] `integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,algorithm="fricas")`

[Out] `integral(-sqrt(-3*cos(d*x+c)+2)*sqrt(cos(d*x+c))/(3*cos(d*x+c)^2-2*cos(d*x+c)),x)`

Sympy [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

[In] `integrate(1/(2-3*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)`

[Out] `Integral(1/(sqrt(2-3*cos(c+d*x))*sqrt(cos(c+d*x))),x)`

Maxima [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)

Giac [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2-3\cos(c+dx)}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(2 - 3*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(1/2)*(2 - 3*cos(c + d*x))^(1/2)), x)

$$3.647 \quad \int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

Optimal result	6167
Rubi [A] (verified)	6167
Mathematica [A] (verified)	6168
Maple [B] (verified)	6168
Fricas [F]	6169
Sympy [F]	6169
Maxima [F]	6169
Giac [F]	6170
Mupad [F(-1)]	6170

Optimal result

Integrand size = 25, antiderivative size = 49

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

$$= -\frac{2\sqrt{-\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), 5\right)}{d\sqrt{\cos(c+dx)}}$$

[Out] $-2*\operatorname{EllipticF}(\sin(d*x+c)/(1-\cos(d*x+c)), 5^{(1/2)})*(-\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2893, 2892}

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

$$= -\frac{2\sqrt{-\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), 5\right)}{d\sqrt{\cos(c+dx)}}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[-2-3*\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]),x]$

[Out] $(-2*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sin}[c+d*x]/(1-\operatorname{Cos}[c+d*x])], 5])/d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]$

Rule 2892

$\operatorname{Int}[1/(\operatorname{Sqrt}[(d_*)\sin[(e_*) + (f_*)(x_*)]]*\operatorname{Sqrt}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_*)])]), x_Symbol] \rightarrow \operatorname{Simp}[-2*(d/(f*\operatorname{Sqrt}[a + b*d]))*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Co}$

```
s[e + f*x]/(1 + d*Sin[e + f*x]), -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b,
d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rule 2893

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] && !(EqQ[d^
2, 1] && GtQ[b*d, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{-\cos(c+dx)} \int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx}{\sqrt{\cos(c+dx)}} \\ &= -\frac{2\sqrt{-\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), 5\right)}{d\sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\begin{aligned} &\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx \\ &= \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right), \frac{1}{5}\right)}{\sqrt{5}d\sqrt{-2-3\cos(c+dx)}\sqrt{\frac{\cos(c+dx)}{2+3\cos(c+dx)}}} \end{aligned}$$

```
[In] Integrate[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Tan[(c + d*x)/2]], 1/5])/(Sqrt[5]*d*
Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]/(2 + 3*Cos[c + d*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(46) = 92.

Time = 7.79 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.47

method	result	size
default	$-\frac{(1+\cos(dx+c))\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-2-3\cos(dx+c)}\sqrt{10}\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}F\left(\frac{\csc(dx+c)-\cot(dx+c)\sqrt{5}}{5},\sqrt{5}\right)\sqrt{5}}{5d(2+3\cos(dx+c))\sqrt{\cos(dx+c)}}$	121

[In] `int(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/5/d*(1+\cos(d*x+c))^2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*(-2-3*\cos(d*x+c))^{(1/2)}*10^{(1/2)}*((2+3*\cos(d*x+c))/(1+\cos(d*x+c)))^{(1/2)}*EllipticF(1/5*(\csc(d*x+c)-\cot(d*x+c))*5^{(1/2)},5^{(1/2)})/(2+3*\cos(d*x+c))/\cos(d*x+c)^{(1/2)}*5^{(1/2)}$$

Fricas [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-3\cos(dx+c)-2}\sqrt{\cos(dx+c)}} dx$$

[In] `integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 + 2*cos(d*x + c)), x)`

Sympy [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-3\cos(c+dx)-2}\sqrt{\cos(c+dx)}} dx$$

[In] `integrate(1/(-2-3*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)`

[Out] `Integral(1/(sqrt(-3*cos(c + d*x) - 2)*sqrt(cos(c + d*x))), x)`

Maxima [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-3\cos(dx+c)-2}\sqrt{\cos(dx+c)}} dx$$

[In] `integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)`

Giac [F]

$$\int \frac{1}{\sqrt{-2 - 3 \cos(c + dx)} \sqrt{\cos(c + dx)}} dx = \int \frac{1}{\sqrt{-3 \cos(dx + c) - 2} \sqrt{\cos(dx + c)}} dx$$

[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2 - 3 \cos(c + dx)} \sqrt{\cos(c + dx)}} dx = \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{-3 \cos(c + dx) - 2}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(- 3*cos(c + d*x) - 2)^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(1/2)*(- 3*cos(c + d*x) - 2)^(1/2)), x)

$$3.648 \quad \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx$$

Optimal result	6171
Rubi [A] (verified)	6171
Mathematica [B] (verified)	6172
Maple [A] (verified)	6172
Fricas [F]	6173
Sympy [F]	6173
Maxima [F]	6173
Giac [F]	6173
Mupad [F(-1)]	6174

Optimal result

Integrand size = 25, antiderivative size = 58

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx$$

$$= \frac{2 \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right) \sqrt{-\tan^2(c+dx)}}{d}$$

[Out] 2*cot(d*x+c)*EllipticF(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2), I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2894}

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx$$

$$= \frac{2\sqrt{-\tan^2(c+dx)} \cot(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right)}{d}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]), x]

[Out] (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/d

Rule 2894

Int[1/(Sqrt[(d_)*sin[e_] + (f_)*(x_)])*Sqrt[(a_) + (b_)*sin[e_] + (f_)*(x_)]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqr

```
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] / ; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[a^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2 \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right) \sqrt{-\tan^2(c + dx)}}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(58) = 116.

Time = 1.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{3 + 2\cos(c + dx)}} dx$$

$$= \frac{4\sqrt{\cos(c + dx)}\sqrt{3 + 2\cos(c + dx)}\sqrt{-\cot^2\left(\frac{1}{2}(c + dx)\right)} \csc(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{(3+2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}}{\sqrt{6}}\right)}{d\sqrt{-\cos(c + dx)\csc^2\left(\frac{1}{2}(c + dx)\right)}\sqrt{(3 + 2\cos(c + dx))\csc^2\left(\frac{1}{2}(c + dx)\right)}}$$

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*Sqrt[-Cot[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/Sqrt[6]], 6])/(d*Sqrt[-(Cos[c + d*x])*Csc[(c + d*x)/2]^2])*Sqrt[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2])
```

Maple [A] (verified)

Time = 7.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

method	result	size
default	$-\frac{(1+\cos(dx+c))\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}F\left(\cot(dx+c)-\csc(dx+c), \frac{i\sqrt{5}}{5}\right)}{5d\sqrt{3+2\cos(dx+c)}\sqrt{\cos(dx+c)}}$	104

```
[In] int(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/d*(1+cos(d*x+c))^2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(3+2*cos(d*x+c))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),1/5*I*5^(1/2))/cos(d*x+c)^(1/2)
```


Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{2\cos(dx+c)+3}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 + 3*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{2\cos(c+dx)+3}\sqrt{\cos(c+dx)}} dx$$

[In] integrate(1/cos(d*x+c)**(1/2)/(3+2*cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(2*cos(c + d*x) + 3)*sqrt(cos(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{2\cos(dx+c)+3}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{2\cos(dx+c)+3}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2\cos(c+dx)+3}} dx$$

```
[In] int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) + 3)^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) + 3)^(1/2)), x)
```

$$3.649 \quad \int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

Optimal result	6175
Rubi [A] (verified)	6175
Mathematica [B] (verified)	6176
Maple [B] (verified)	6176
Fricas [F]	6177
Sympy [F]	6177
Maxima [F]	6177
Giac [F]	6177
Mupad [F(-1)]	6178

Optimal result

Integrand size = 25, antiderivative size = 60

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

$$= \frac{2\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d}$$

[Out] $2/5*\cot(d*x+c)*\operatorname{EllipticF}((3-2*\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)}, 1/5*I*5^{(1/2)})*(-\tan(d*x+c)^2)^{(1/2)}/d*5^{(1/2)}$

Rubi [A] (verified)

Time = 0.08 (sec), antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2894}

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

$$= \frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right)}{\sqrt{5}d}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[3-2*\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]),x]$

[Out] $(2*\operatorname{Cot}[c+d*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[3-2*\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]], -1/5]*\operatorname{Sqrt}[-\operatorname{Tan}[c+d*x]^2])/(\operatorname{Sqrt}[5]*d)$

Rule 2894

$\operatorname{Int}[1/(\operatorname{Sqrt}[(d_*)\sin[(e_*) + (f_*)(x_*)]]*\operatorname{Sqrt}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_*)]]), x_Symbol] \rightarrow \operatorname{Simp}[-2*\operatorname{Sqrt}[a^2]*(\operatorname{Sqrt}[-\operatorname{Cot}[e + f*x]^2]/(a*f*\operatorname{Sqr$

```
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] / ; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[a^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2 \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right) \sqrt{-\tan^2(c + dx)}}{\sqrt{5}d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(60) = 120.

Time = 1.44 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

$$= \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3-2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

[In] Integrate[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]

[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(3 - 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]/Sqrt[3]], 6]*Sin[(c + d*x)/2]^4)/(d*Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(54) = 108.

Time = 7.54 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.88

method	result	size
default	$\frac{(1+\cos(dx+c))\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{3-2\cos(dx+c)}\sqrt{\frac{-2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}F\left(\cot(dx+c)-\csc(dx+c),i\sqrt{5}\right)}{d(-3+2\cos(dx+c))\sqrt{\cos(dx+c)}}$	113

[In] int(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] $1/d*(1+\cos(d*x+c))*2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*(3-2*\cos(d*x+c))^{(1/2)}*(-2*(-3+2*\cos(d*x+c))/(1+\cos(d*x+c)))^{(1/2)}*EllipticF(\cot(d*x+c)-csc(d*x+c), I*5^{(1/2)})/(-3+2*\cos(d*x+c))/\cos(d*x+c)^{(1/2)}$

Fricas [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-2\cos(dx+c)+3}\sqrt{\cos(dx+c)}} dx$$

[In] `integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 - 3*cos(d*x + c)), x)`

Sympy [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

[In] `integrate(1/(3-2*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)`

[Out] `Integral(1/(sqrt(3 - 2*cos(c + d*x))*sqrt(cos(c + d*x))), x)`

Maxima [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-2\cos(dx+c)+3}\sqrt{\cos(dx+c)}} dx$$

[In] `integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)`

Giac [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-2\cos(dx+c)+3}\sqrt{\cos(dx+c)}} dx$$

[In] `integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3 - 2 \cos(c + dx)} \sqrt{\cos(c + dx)}} dx = \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{3 - 2 \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(1/2)*(3 - 2*cos(c + d*x))^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(3 - 2*cos(c + d*x))^(1/2)), x)
```

$$3.650 \quad \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx$$

Optimal result	6179
Rubi [A] (verified)	6179
Mathematica [A] (verified)	6180
Maple [A] (verified)	6181
Fricas [F]	6181
Sympy [F]	6181
Maxima [F]	6181
Giac [F]	6182
Mupad [F(-1)]	6182

Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d}$$

[Out] -2/5*csc(d*x+c)*EllipticF((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2), 1/5*I*5^(1/2))*(-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-tan(d*x+c)^2)^(1/2)/d*5^(1/2)

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2896, 2894}

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{-\tan^2(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right)}{\sqrt{5}d}$$

[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]

[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/ (Sqrt[5]*d)

Rule 2894

```
Int[1/(Sqrt[(d_)*sin[(e_)+(f_)*(x_)])*Sqrt[(a_)+(b_)*sin[(e_)+(f_)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e+f*x]^2]/(a*f*Sqrt[a^2-b^2]*Cot[e+f*x]))*Rt[(a+b)/d, 2]*EllipticF[ArcSin[Sqrt[a+b*Sin[e+f*x]]/Sqrt[d*Sin[e+f*x]]]/Rt[(a+b)/d, 2]], -(a+b)/(a-b)], x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2-b^2, 0] && PosQ[(a+b)/d] && GtQ[a^2, 0]
```

Rule 2896

```
Int[1/(Sqrt[(d_)*sin[(e_)+(f_)*(x_)])*Sqrt[(a_)+(b_)*sin[(e_)+(f_)*(x_)]]), x_Symbol] :> Dist[Sqrt[(-d)*Sin[e+f*x]]/Sqrt[d*Sin[e+f*x]], Int[1/(Sqrt[a+b*Sin[e+f*x]]*Sqrt[(-d)*Sin[e+f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2-b^2, 0] && NegQ[(a+b)/d]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{-\cos(c+dx)} \int \frac{1}{\sqrt{-\cos(c+dx)} \sqrt{-3+2\cos(c+dx)}} dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{2\sqrt{-\cos(c+dx)} \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right) \sqrt{-\tan^2(c+dx)}}{\sqrt{5}d} \end{aligned}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.71

$$\begin{aligned} &\int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{-3+2\cos(c+dx)}} dx \\ &= \frac{2\sqrt{\cos(c+dx)} \sqrt{\frac{-3+2\cos(c+dx)}{-1+\cos(c+dx)}} \sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{-3+2\cos(c+dx)}{-1+\cos(c+dx)}}}{\sqrt{3}}\right), \frac{6}{5}\right) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}d \sqrt{\frac{\cos(c+dx)}{-1+\cos(c+dx)}} \sqrt{-3+2\cos(c+dx)}} \end{aligned}$$

[In] Integrate[1/(Sqrt[Cos[c+d*x]]*Sqrt[-3+2*Cos[c+d*x]]),x]

[Out] (2*Sqrt[Cos[c+d*x]]*Sqrt[(-3+2*Cos[c+d*x])/(-1+Cos[c+d*x])]*Sqrt[-Cot[(c+d*x)/2]^2]*EllipticF[ArcSin[Sqrt[(-3+2*Cos[c+d*x])/(-1+Cos[c+d*x])]]/Sqrt[3]], 6/5]*Tan[(c+d*x)/2])/(Sqrt[5]*d*Sqrt[Cos[c+d*x]/(-1+Cos[c+d*x])]*Sqrt[-3+2*Cos[c+d*x]])

Maple [A] (verified)

Time = 7.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{i(1+\cos(dx+c))\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}F\left(i(\csc(dx+c)-\cot(dx+c))\sqrt{5},\frac{i\sqrt{5}}{5}\right)\sqrt{5}}{5d\sqrt{-3+2\cos(dx+c)}\sqrt{\cos(dx+c)}}$	112

[In] int(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] $-1/5*I/d*(1+\cos(d*x+c))^2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)/(-3+2*\cos(d*x+c))^{(1/2)}*(-2*(-3+2*\cos(d*x+c))/(1+\cos(d*x+c)))^{(1/2)}*EllipticF(I*(\csc(d*x+c)-\cot(d*x+c))*5^{(1/2)},1/5*I*5^{(1/2)})/\cos(d*x+c)^{(1/2)}*5^{(1/2)}$

Fricas [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{2\cos(dx+c)-3}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 - 3*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{2\cos(c+dx)-3}\sqrt{\cos(c+dx)}} dx$$

[In] integrate(1/cos(d*x+c)**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(2*cos(c + d*x) - 3)*sqrt(cos(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{2\cos(dx+c)-3}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)

Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{2\cos(dx+c)-3}\sqrt{\cos(dx+c)}} dx$$

[In] integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2\cos(c+dx)-3}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) - 3)^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(1/2)*(2*cos(c + d*x) - 3)^(1/2)), x)

$$3.651 \quad \int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx$$

Optimal result	6183
Rubi [A] (verified)	6183
Mathematica [A] (verified)	6184
Maple [A] (verified)	6185
Fricas [F]	6185
Sympy [F]	6185
Maxima [F]	6185
Giac [F]	6186
Mupad [F(-1)]	6186

Optimal result

Integrand size = 25, antiderivative size = 82

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right)\sqrt{-\tan^2(c+dx)}}{d}$$

[Out] $-2*\csc(d*x+c)*\operatorname{EllipticF}(1/5*(-3-2*\cos(d*x+c))^{(1/2)}*5^{(1/2)/(-\cos(d*x+c))^{(1/2)}, I*5^{(1/2)})*(-\cos(d*x+c))^{(1/2)}*\cos(d*x+c)^{(1/2)}*(-\tan(d*x+c)^2)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2896, 2894}

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{-\tan^2(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right)}{d}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[-3-2*\operatorname{Cos}[c+d*x]])*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]], x]$

[Out] $(-2*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Csc}[c+d*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[-3-2*\operatorname{Cos}[c+d*x]]/(\operatorname{Sqrt}[5]*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]])], -5]*\operatorname{Sqrt}[-\operatorname{Tan}[c+d*x]^2])/d$

Rule 2894

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[a^2, 0]
```

Rule 2896

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(-d)*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[(-d)*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{-\cos(c+dx)} \int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right)\sqrt{-\tan^2(c+dx)}}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.87

$$\begin{aligned} &\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx \\ &= \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)}\csc^2\left(\frac{1}{2}(c+dx)\right)\sqrt{(3+2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5d}\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} \end{aligned}$$

```
[In] Integrate[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]), x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[5/3]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]]], 6/5)*Sin[(c + d*x)/2]^4)/(Sqrt[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.54

method	result	size
default	$\frac{i(1+\cos(dx+c))\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-3-2\cos(dx+c)}\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}F\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},i\sqrt{5}\right)\sqrt{5}}{5d(3+2\cos(dx+c))\sqrt{\cos(dx+c)}}$	126

```
[In] int(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*I/d*(1+cos(d*x+c))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-3-2*cos(d*x+c))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),I*5^(1/2))/(3+2*cos(d*x+c))/cos(d*x+c)^(1/2)*5^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-2\cos(dx+c)-3}\sqrt{\cos(dx+c)}} dx$$

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 + 3*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-2\cos(c+dx)-3}\sqrt{\cos(c+dx)}} dx$$

```
[In] integrate(1/(-3-2*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-2*cos(c + d*x) - 3)*sqrt(cos(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-2\cos(dx+c)-3}\sqrt{\cos(dx+c)}} dx$$

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-3 - 2 \cos(c + dx)} \sqrt{\cos(c + dx)}} dx = \int \frac{1}{\sqrt{-2 \cos(dx + c) - 3} \sqrt{\cos(dx + c)}} dx$$

[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3 - 2 \cos(c + dx)} \sqrt{\cos(c + dx)}} dx = \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{-2 \cos(c + dx) - 3}} dx$$

[In] int(1/(cos(c + d*x)^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)), x)

$$3.652 \quad \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx$$

Optimal result	6187
Rubi [A] (verified)	6187
Mathematica [B] (verified)	6188
Maple [B] (verified)	6189
Fricas [F]	6189
Sympy [F]	6189
Maxima [F]	6190
Giac [F]	6190
Mupad [F(-1)]	6190

Optimal result

Integrand size = 27, antiderivative size = 54

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx$$

$$= \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right), \frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

[Out] 2/5*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),1/5*5^(1/2))*cos(d*x+c)^(1/2)/d*5^(1/2)/(-cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2893, 2892}

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx$$

$$= \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right), \frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]

[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 1/5))/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]])

Rule 2892

Int[1/(Sqrt[(d_)*sin[e_] + (f_)*(x_)]*Sqrt[(a_) + (b_)*sin[e_] + (f_)*(x_)]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Co

`s[e + f*x]/(1 + d*Sin[e + f*x]), -(a - b*d)/(a + b*d), x] /; FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]`

Rule 2893

`Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] && !(EqQ[d^2, 1] && GtQ[b*d, 0])`

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx}{\sqrt{-\cos(c + dx)}} \\ &= \frac{2\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right), \frac{1}{5}\right)}{\sqrt{5d}\sqrt{-\cos(c + dx)}} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 150 vs. 2(54) = 108.

Time = 0.46 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.78

$$\int \frac{1}{\sqrt{-\cos(c + dx)}\sqrt{2 + 3\cos(c + dx)}} dx = \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c + dx)\right)}\sqrt{-\cos(c + dx)}\csc^2\left(\frac{1}{2}(c + dx)\right)\sqrt{(2 + 3\cos(c + dx))\csc^2\left(\frac{1}{2}(c + dx)\right)}\csc(c + dx)}{d\sqrt{-\cos(c + dx)}\sqrt{2 + 3\cos(c + dx)}}$$

`[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]`

`[Out] (-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]])`

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(49) = 98$.

Time = 7.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.06

method	result	size
default	$\frac{(1+\cos(dx+c))F\left(\frac{\csc(dx+c)-\cot(dx+c)\sqrt{5}}{5},\sqrt{5}\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{10}\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{5}}{5d\sqrt{2+3\cos(dx+c)}\sqrt{-\cos(dx+c)}}$	111

[In] `int(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] `1/5/d*(1+cos(d*x+c))*EllipticF(1/5*(csc(d*x+c)-cot(d*x+c))*5^(1/2),5^(1/2))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(2+3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)*5^(1/2)`

Fricas [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)+2}} dx$$

[In] `integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)/(3*cos(d*x + c)^2 + 2*cos(d*x + c)), x)`

Sympy [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)+2}} dx$$

[In] `integrate(1/(-cos(d*x+c))**(1/2)/(2+3*cos(d*x+c))**(1/2),x)`

[Out] `Integral(1/(sqrt(-cos(c + d*x))*sqrt(3*cos(c + d*x) + 2)), x)`

Maxima [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)+2}} dx$$

[In] integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)), x)

Giac [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)+2}} dx$$

[In] integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)+2}} dx$$

[In] int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) + 2)^(1/2)),x)

[Out] int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) + 2)^(1/2)), x)

$$3.653 \quad \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx$$

Optimal result	6191
Rubi [A] (verified)	6191
Mathematica [B] (verified)	6192
Maple [B] (verified)	6192
Fricas [F]	6193
Sympy [F]	6193
Maxima [F]	6193
Giac [F]	6194
Mupad [F(-1)]	6194

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right), 5\right)}{d\sqrt{-\cos(c+dx)}}$$

[Out] 2*EllipticF(sin(d*x+c)/(1+cos(d*x+c)),5^(1/2))*cos(d*x+c)^(1/2)/d/(-cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2893, 2892}

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right), 5\right)}{d\sqrt{-\cos(c+dx)}}$$

[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]

[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 5)/(d*Sqrt[-Cos[c + d*x]])

Rule 2892

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_.)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]), x_Symbol] :> Simp[-2*(d/(f*Sqrt[a + b*d]))*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*Sin[e + f*x])], -(a - b*d)/(a + b*d)], x] /; FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rule 2893

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] && !(EqQ[d^2, 1] && GtQ[b*d, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{-2+3\cos(c+dx)}} dx}{\sqrt{-\cos(c+dx)}} \\ &= \frac{2\sqrt{\cos(c+dx)} \text{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right), 5\right)}{d\sqrt{-\cos(c+dx)}} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. $2(47) = 94$.

Time = 0.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.36

$$\begin{aligned} &\int \frac{1}{\sqrt{-\cos(c+dx)} \sqrt{-2+3\cos(c+dx)}} dx \\ &= \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)} \sqrt{\cos(c+dx) \csc^2\left(\frac{1}{2}(c+dx)\right)} \sqrt{-((-2+3\cos(c+dx)) \csc^2\left(\frac{1}{2}(c+dx)\right))} \csc(c+dx)}{\sqrt{5}d\sqrt{-\cos(c+dx)} \sqrt{-2+3\cos(c+dx)}} \end{aligned}$$

[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]

[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]/2], 4/5]*Sin[(c + d*x)/2]^4)/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.

Time = 6.59 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.06

method	result	size
default	$-\frac{2(1+\cos(dx+c))F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{d\sqrt{-2+3\cos(dx+c)}\sqrt{-\cos(dx+c)}}$	97

```
[In] int(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/d*(1+cos(d*x+c))*EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2+3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)-2}} dx$$

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)/(3*cos(d*x + c)^2 - 2*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)-2}} dx$$

```
[In] integrate(1/(-cos(d*x+c))**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(3*cos(c + d*x) - 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)-2}} dx$$

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)-2}} dx$$

[In] integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)-2}} dx$$

[In] int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) - 2)^(1/2)),x)

[Out] int(1/((-cos(c + d*x))^(1/2)*(3*cos(c + d*x) - 2)^(1/2)), x)

$$3.654 \quad \int \frac{1}{\sqrt{2-3 \cos(c+dx)} \sqrt{-\cos(c+dx)}} dx$$

Optimal result	6195
Rubi [A] (verified)	6195
Mathematica [B] (verified)	6196
Maple [B] (verified)	6196
Fricas [F]	6196
Sympy [F]	6197
Maxima [F]	6197
Giac [F]	6197
Mupad [F(-1)]	6197

Optimal result

Integrand size = 27, antiderivative size = 34

$$\int \frac{1}{\sqrt{2-3 \cos(c+dx)} \sqrt{-\cos(c+dx)}} dx = -\frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), \frac{1}{5}\right)}{\sqrt{5}d}$$

[Out] $-2/5*\operatorname{EllipticF}(\sin(d*x+c)/(1-\cos(d*x+c)), 1/5*5^{(1/2)})/d*5^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2892}

$$\int \frac{1}{\sqrt{2-3 \cos(c+dx)} \sqrt{-\cos(c+dx)}} dx = -\frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), \frac{1}{5}\right)}{\sqrt{5}d}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[2-3*\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]),x]$

[Out] $(-2*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sin}[c+d*x]/(1-\operatorname{Cos}[c+d*x])], 1/5])/(\operatorname{Sqrt}[5]*d)$

Rule 2892

$\operatorname{Int}[1/(\operatorname{Sqrt}[(d_*)\sin[(e_*) + (f_*)(x_*)]]*\operatorname{Sqrt}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_*)]]), x_Symbol] \rightarrow \operatorname{Simp}[-2*(d/(f*\operatorname{Sqrt}[a + b*d]))*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Cos}[e + f*x]/(1 + d*\operatorname{Sin}[e + f*x])], -(a - b*d)/(a + b*d)], x] /; \operatorname{FreeQ}\{a, b, d, e, f\}, x \ \&\& \operatorname{LtQ}[a^2 - b^2, 0] \ \&\& \operatorname{EqQ}[d^2, 1] \ \&\& \operatorname{GtQ}[b*d, 0]$

Rubi steps

$$\text{integral} = -\frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), \frac{1}{5}\right)}{\sqrt{5}d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 145 vs. $2(34) = 68$.

Time = 0.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.26

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)\sqrt{(2-3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{\cos(c+dx)}{2-3\cos(c+dx)}}\right]\right]}{d\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

[In] Integrate[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]

[Out] (-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(33) = 66$.

Time = 6.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.21

method	result	size
default	$\frac{2(1+\cos(dx+c))F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2-3\cos(dx+c)}}{d\sqrt{-\cos(dx+c)}(-2+3\cos(dx+c))}$	109

[In] int(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/d*(1+cos(d*x+c))*EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))

Fricas [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)+2}} dx$$

[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)/(3*cos(d*x + c)^2 - 2*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2-3\cos(c+dx)}} dx$$

[In] integrate(1/(2-3*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(2 - 3*cos(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)+2}} dx$$

[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)), x)

Giac [F]

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)+2}} dx$$

[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2-3\cos(c+dx)}} dx$$

[In] int(1/((-cos(c + d*x))^(1/2)*(2 - 3*cos(c + d*x))^(1/2)),x)

[Out] int(1/((-cos(c + d*x))^(1/2)*(2 - 3*cos(c + d*x))^(1/2)), x)

$$3.655 \quad \int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$$

Optimal result	6198
Rubi [A] (verified)	6198
Mathematica [B] (verified)	6199
Maple [B] (verified)	6199
Fricas [F]	6199
Sympy [F]	6200
Maxima [F]	6200
Giac [F]	6200
Mupad [F(-1)]	6200

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = -\frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), 5\right)}{d}$$

[Out] $-2*\operatorname{EllipticF}(\sin(d*x+c)/(1-\cos(d*x+c)), 5^{(1/2)})/d$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2892}

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = -\frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), 5\right)}{d}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[-2 - 3*\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[-\operatorname{Cos}[c + d*x]]), x]$

[Out] $(-2*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sin}[c + d*x]/(1 - \operatorname{Cos}[c + d*x])], 5])/d$

Rule 2892

$\operatorname{Int}[1/(\operatorname{Sqrt}[(d_*)*\sin[(e_*) + (f_*)(x_*)]]*\operatorname{Sqrt}[(a_*) + (b_*)*\sin[(e_*) + (f_*)(x_*)]]), x_Symbol] :> \operatorname{Simp}[-2*(d/(f*\operatorname{Sqrt}[a + b*d]))*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Cos}[e + f*x]/(1 + d*\operatorname{Sin}[e + f*x])], -(a - b*d)/(a + b*d)], x] /;$ $\operatorname{FreeQ}\{a, b, d, e, f\}, x$ && $\operatorname{LtQ}[a^2 - b^2, 0]$ && $\operatorname{EqQ}[d^2, 1]$ && $\operatorname{GtQ}[b*d, 0]$

Rubi steps

$$\text{integral} = -\frac{2 \operatorname{EllipticF}\left(\arcsin\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right), 5\right)}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 74 vs. 2(27) = 54.

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.74

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$$

$$= -\frac{2\sqrt{-\cos(c+dx)} \operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right), \frac{1}{5}\right)}{\sqrt{5}d\sqrt{-2-3\cos(c+dx)}\sqrt{\frac{\cos(c+dx)}{2+3\cos(c+dx)}}}$$

[In] Integrate[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]), x]

[Out] (-2*Sqrt[-Cos[c + d*x]]*EllipticF[ArcSin[Tan[(c + d*x)/2]], 1/5])/(Sqrt[5]*d*Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]/(2 + 3*Cos[c + d*x])])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(28) = 56.

Time = 6.84 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.33

method	result	size
default	$\frac{(1+\cos(dx+c))F\left(\cot(dx+c)-\operatorname{csc}(dx+c), \frac{\sqrt{5}}{5}\right)\sqrt{10}\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-2-3\cos(dx+c)}}{5d\sqrt{-\cos(dx+c)}(2+3\cos(dx+c))}$	117

[In] int(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2), x, method=_RETURNVERBOSE)

[Out] 1/5/d*(1+cos(d*x+c))*EllipticF(cot(d*x+c)-csc(d*x+c), 1/5*5^(1/2))*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))

Fricas [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)-2}} dx$$

[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)/(3*cos(d*x + c)^2 + 2*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3\cos(c+dx)-2}} dx$$

[In] integrate(1/(-2-3*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2), x)

[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(-3*cos(c + d*x) - 2)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)-2}} dx$$

[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)), x)

Giac [F]

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)-2}} dx$$

[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3\cos(c+dx)-2}} dx$$

[In] int(1/((-cos(c + d*x))^(1/2)*(-3*cos(c + d*x) - 2)^(1/2)), x)

[Out] int(1/((-cos(c + d*x))^(1/2)*(-3*cos(c + d*x) - 2)^(1/2)), x)

$$3.656 \quad \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx$$

Optimal result	6201
Rubi [A] (verified)	6201
Mathematica [A] (verified)	6202
Maple [A] (verified)	6203
Fricas [F]	6203
Sympy [F]	6203
Maxima [F]	6203
Giac [F]	6204
Mupad [F(-1)]	6204

Optimal result

Integrand size = 27, antiderivative size = 80

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right)\sqrt{-\tan^2(c+dx)}}{d\sqrt{-\cos(c+dx)}}$$

[Out] 2*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticF(1/5*(3+2*cos(d*x+c))^(1/2)*5^(1/2)/cos(d*x+c)^(1/2), I*5^(1/2))*(-tan(d*x+c)^2)^(1/2)/d/(-cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2896, 2894}

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right)}{d\sqrt{-\cos(c+dx)}}$$

[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]), x]

[Out] (2*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/(d*Sqrt[-Cos[c + d*x]])

Rule 2894

```
Int[1/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[a^2, 0]
```

Rule 2896

```
Int[1/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(-d)*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[(-d)*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx}{\sqrt{-\cos(c+dx)}} \\ &= \frac{2 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right) \sqrt{-\tan^2(c+dx)}}{d\sqrt{-\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \frac{4\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)}\csc^2\left(\frac{1}{2}(c+dx)\right)\sqrt{(3+2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)}{d\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}$$

```
[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[-Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/Sqrt[6]], 6]*Sin[(c + d*x)/2]^4)/(d*Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{i(1+\cos(dx+c))F\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5}, i\sqrt{5}\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{5}}{5d\sqrt{3+2\cos(dx+c)}\sqrt{-\cos(dx+c)}}$	116

[In] int(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out]
$$-1/5*I/d*(1+\cos(d*x+c))*\text{EllipticF}\left(\frac{1}{5}*I*(\csc(d*x+c)-\cot(d*x+c))*5^{(1/2)}, I*5^{(1/2)}\right)*2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*10^{(1/2)}*((3+2*\cos(d*x+c))/(1+\cos(d*x+c)))^{(1/2)}/(3+2*\cos(d*x+c))^{(1/2)}/(-\cos(d*x+c))^{(1/2)}*5^{(1/2)}$$

Fricas [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)+3}} dx$$

[In] integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)/(2*cos(d*x + c)^2 + 3*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)+3}} dx$$

[In] integrate(1/(-cos(d*x+c))**(1/2)/(3+2*cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(2*cos(c + d*x) + 3)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)+3}} dx$$

[In] integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)), x)

Giac [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)+3}} dx$$

[In] integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)+3}} dx$$

[In] int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) + 3)^(1/2)),x)

[Out] int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) + 3)^(1/2)), x)

$$3.657 \quad \int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$$

Optimal result	6205
Rubi [A] (verified)	6205
Mathematica [A] (verified)	6206
Maple [A] (verified)	6207
Fricas [F]	6207
Sympy [F]	6207
Maxima [F]	6208
Giac [F]	6208
Mupad [F(-1)]	6208

Optimal result

Integrand size = 27, antiderivative size = 82

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

[Out] $2/5*\cos(d*x+c)^{(3/2)}*\csc(d*x+c)*\operatorname{EllipticF}((3-2*\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)}, 1/5*I*5^{(1/2)})*(-\tan(d*x+c)^2)^{(1/2)}/d*5^{(1/2)}/(-\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2896, 2894}

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[3-2*\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]),x]$

[Out] $(2*\operatorname{Cos}[c+d*x]^{(3/2)}*\operatorname{Csc}[c+d*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[3-2*\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]], -1/5]*\operatorname{Sqrt}[-\operatorname{Tan}[c+d*x]^2])/(\operatorname{Sqrt}[5]*d*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]])$

Rule 2894

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Simp[-2*Sqrt[a^2]*(Sqrt[-Cot[e + f*x]^2]/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[a^2, 0]
```

Rule 2896

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Dist[Sqrt[(-d)*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[(-d)*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} dx}{\sqrt{-\cos(c+dx)}} \\ &= \frac{2 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right) \sqrt{-\tan^2(c+dx)}}{\sqrt{5}d\sqrt{-\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.78

$$\begin{aligned} &\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx \\ &= \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3-2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} \end{aligned}$$

```
[In] Integrate[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(3 - 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]/Sqrt[3]], 6]*Sin[(c + d*x)/2]^4)/(d*Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{2iF\left(i(\csc(dx+c)-\cot(dx+c))\sqrt{5}, \frac{i\sqrt{5}}{5}\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{3-2\cos(dx+c)}\sqrt{5}}{5d\sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\sqrt{-\cos(dx+c)}}$	106

```
[In] int(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I/d*EllipticF(I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),1/5*I*5^(1/2))*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)*5^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)+3}} dx$$

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)/(2*cos(d*x + c)^2 -
3*cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3-2\cos(c+dx)}} dx$$

```
[In] integrate(1/(3-2*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(3 - 2*cos(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)+3}} dx$$

[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)), x)

Giac [F]

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)+3}} dx$$

[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3-2\cos(c+dx)}} dx$$

[In] int(1/((-cos(c + d*x))^(1/2)*(3 - 2*cos(c + d*x))^(1/2)),x)

[Out] int(1/((-cos(c + d*x))^(1/2)*(3 - 2*cos(c + d*x))^(1/2)), x)

$$3.658 \quad \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx$$

Optimal result	6209
Rubi [A] (verified)	6209
Mathematica [B] (verified)	6210
Maple [A] (verified)	6210
Fricas [F]	6211
Sympy [F]	6211
Maxima [F]	6211
Giac [F]	6211
Mupad [F(-1)]	6212

Optimal result

Integrand size = 27, antiderivative size = 62

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx$$

$$= -\frac{2\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d}$$

[Out] $-2/5*\cot(d*x+c)*\operatorname{EllipticF}((-3+2*\cos(d*x+c))^{(1/2)/(-\cos(d*x+c))^{(1/2)}, 1/5*I$
 $*5^{(1/2)})*(-\tan(d*x+c)^2)^{(1/2)}/d*5^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec), antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2894}

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx$$

$$= -\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right)}{\sqrt{5}d}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[-3+2*\operatorname{Cos}[c+d*x]]), x]$

[Out] $(-2*\operatorname{Cot}[c+d*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[-3+2*\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]], -1/5)*\operatorname{Sqrt}[-\operatorname{Tan}[c+d*x]^2])/(\operatorname{Sqrt}[5]*d)$

Rule 2894

$\operatorname{Int}[1/(\operatorname{Sqrt}[(d_*)*\sin[(e_*)+(f_*)*(x_*)]]*\operatorname{Sqrt}[(a_*)+(b_*)*\sin[(e_*)+(f_*)*(x_*)]]), x_Symbol] \rightarrow \operatorname{Simp}[-2*\operatorname{Sqrt}[a^2]*(\operatorname{Sqrt}[-\operatorname{Cot}[e+f*x]^2]/(a*f*\operatorname{Sqr$

```
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] / ; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[a^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2 \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right) \sqrt{-\tan^2(c + dx)}}{\sqrt{5}d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(62) = 124.

Time = 0.56 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{-\cos(c + dx)}\sqrt{-3 + 2\cos(c + dx)}} dx$$

$$= \frac{4\sqrt{-\cot^2\left(\frac{1}{2}(c + dx)\right)} \cot(c + dx) \sqrt{-\cos(c + dx)} \csc^2\left(\frac{1}{2}(c + dx)\right) \sqrt{-((-3 + 2\cos(c + dx)) \csc^2\left(\frac{1}{2}(c + dx)\right))}}{\sqrt{5}d(-\cos(c + dx))^{3/2} \sqrt{-3 + 2\cos(c + dx)}}$$

```
[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[-Cot[(c + d*x)/2]^2]*Cot[c + d*x]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[-((-3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*EllipticF[ArcSin[Sqrt[(-3 + 2*Cos[c + d*x])/(-1 + Cos[c + d*x])]/Sqrt[3]], 6/5]*Sin[(c + d*x)/2]^4)/(Sqrt[5]*d*(-Cos[c + d*x])^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{2F\left(\cot(dx+c)-\csc(dx+c), i\sqrt{5}\right)\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-3+2\cos(dx+c)}}{d\sqrt{\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}}\sqrt{-\cos(dx+c)}}$	96

```
[In] int(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*EllipticF(cot(d*x+c)-csc(d*x+c),I*5^(1/2))/(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)-3}} dx$$

[In] integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)/(2*cos(d*x + c)^2 - 3*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)-3}} dx$$

[In] integrate(1/(-cos(d*x+c))**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(2*cos(c + d*x) - 3)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)-3}} dx$$

[In] integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)), x)

Giac [F]

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)-3}} dx$$

[In] integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)-3}} dx$$

```
[In] int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) - 3)^(1/2)),x)
```

```
[Out] int(1/((-cos(c + d*x))^(1/2)*(2*cos(c + d*x) - 3)^(1/2)), x)
```


$$3.659 \quad \int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$$

Optimal result	6213
Rubi [A] (verified)	6213
Mathematica [B] (verified)	6214
Maple [B] (verified)	6214
Fricas [F]	6215
Sympy [F]	6215
Maxima [F]	6215
Giac [F]	6215
Mupad [F(-1)]	6216

Optimal result

Integrand size = 27, antiderivative size = 60

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$$

$$= -\frac{2\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right)\sqrt{-\tan^2(c+dx)}}{d}$$

[Out] $-2*\cot(d*x+c)*\operatorname{EllipticF}(1/5*(-3-2*\cos(d*x+c))^{(1/2)}*5^{(1/2)/(-\cos(d*x+c))^{(1/2)}, I*5^{(1/2)})*(-\tan(d*x+c)^2)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.07 (sec), antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2894}

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$$

$$= -\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right)}{d}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[-3-2*\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]), x]$

[Out] $(-2*\operatorname{Cot}[c+d*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[-3-2*\operatorname{Cos}[c+d*x]]/(\operatorname{Sqrt}[5]*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]])], -5]*\operatorname{Sqrt}[-\operatorname{Tan}[c+d*x]^2])/d$

Rule 2894

$\operatorname{Int}[1/(\operatorname{Sqrt}[(d_*)\sin[(e_*) + (f_*)(x_*)]]*\operatorname{Sqrt}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_*)]]), x_Symbol] \rightarrow \operatorname{Simp}[-2*\operatorname{Sqrt}[a^2]*(\operatorname{Sqrt}[-\operatorname{Cot}[e + f*x]^2]/(a*f*\operatorname{Sqr$

```
t[a^2 - b^2]*Cot[e + f*x]))*Rt[(a + b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] / ; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] && GtQ[a^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2 \cot(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right) \sqrt{-\tan^2(c + dx)}}{d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 155 vs. $2(60) = 120$.

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.58

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx$$

$$= \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)}\csc^2\left(\frac{1}{2}(c+dx)\right)\sqrt{(3+2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right)}{\sqrt{5}d\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

```
[In] Integrate[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[5/3]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]]], 6/5)*Sin[(c + d*x)/2]^4)/(Sqrt[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.

Time = 6.84 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

method	result	size
default	$\frac{(1+\cos(dx+c))F\left(\cot(dx+c)-\csc(dx+c), \frac{i\sqrt{5}}{5}\right)\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-3-2\cos(dx+c)}}{5d\sqrt{-\cos(dx+c)}(3+2\cos(dx+c))}$	118

```
[In] int(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/d*(1+cos(d*x+c))*EllipticF(cot(d*x+c)-csc(d*x+c), 1/5*I*5^(1/2))*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))
```

Fricas [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)-3}} dx$$

[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)/(2*cos(d*x + c)^2 + 3*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2\cos(c+dx)-3}} dx$$

[In] integrate(1/(-3-2*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)

[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(-2*cos(c + d*x) - 3)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)-3}} dx$$

[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)), x)

Giac [F]

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}} dx = \int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)-3}} dx$$

[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3 - 2 \cos(c + dx)} \sqrt{-\cos(c + dx)}} dx = \int \frac{1}{\sqrt{-\cos(c + dx)} \sqrt{-2 \cos(c + dx) - 3}} dx$$

```
[In] int(1/((-cos(c + d*x))^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)),x)
```

```
[Out] int(1/((-cos(c + d*x))^(1/2)*(- 2*cos(c + d*x) - 3)^(1/2)), x)
```

$$3.660 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx$$

Optimal result	6217
Rubi [A] (verified)	6217
Mathematica [B] (verified)	6218
Maple [A] (verified)	6218
Fricas [F]	6219
Sympy [F]	6219
Maxima [F]	6219
Giac [F]	6220
Mupad [F(-1)]	6220

Optimal result

Integrand size = 25, antiderivative size = 77

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \frac{4 \cot(c+dx) \operatorname{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), 5\right) \sqrt{-1-\sec(c+dx)} \sqrt{1-\sec(c+dx)}}{3d}$$

[Out] $-4/3*\cot(d*x+c)*\operatorname{EllipticPi}(1/5*(2+3*\cos(d*x+c))^{(1/2)}*5^{(1/2)}/\cos(d*x+c)^{(1/2)}, 5/3, 5^{(1/2)})*(-1-\sec(d*x+c))^{(1/2)}*(1-\sec(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.05 (sec), antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2888}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \frac{4 \cot(c+dx) \sqrt{-\sec(c+dx)-1} \sqrt{1-\sec(c+dx)} \operatorname{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), 5\right)}{3d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[2+3*\operatorname{Cos}[c+d*x]], x]$

[Out] $(-4*\operatorname{Cot}[c+d*x]*\operatorname{EllipticPi}[5/3, \operatorname{ArcSin}[\operatorname{Sqrt}[2+3*\operatorname{Cos}[c+d*x]]]/(\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])], 5)*\operatorname{Sqrt}[-1-\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1-\operatorname{Sec}[c+d*x]]/(3*d)$

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

integral =

$$\frac{4 \cot(c + dx) \operatorname{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{2+3 \cos(c+dx)}}{\sqrt{5} \sqrt{\cos(c+dx)}}\right), 5\right) \sqrt{-1 - \sec(c + dx)} \sqrt{1 - \sec(c + dx)}}{3d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 175 vs. 2(77) = 154.

Time = 12.63 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.27

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{2 + 3 \cos(c + dx)}} dx$$

$$= \frac{2 \sqrt{\cos(c + dx)} \sqrt{2 + 3 \cos(c + dx)} \sqrt{\cot^2\left(\frac{1}{2}(c + dx)\right) \csc(c + dx)} \left(3 \operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2} \sqrt{(2 + 3 \cos(c + dx))}\right), \frac{5}{3}\right) - 5 \operatorname{EllipticPi}\left[-\frac{2}{3}, \arcsin\left(\frac{\sqrt{2 + 3 \cos(c + dx)} \csc\left(\frac{c + dx}{2}\right)}{\sqrt{2 + 3 \cos(c + dx)}}\right)\right]\right)}{3d \sqrt{\frac{-2 - 3 \cos(c + dx)}{-1 + \cos(c + dx)}}}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Cot[(c + d*x)/2]^2]*Csc
[c + d*x]*(3*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]
/2], -4] - 5*EllipticPi[-2/3, ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*
x)/2]^2]/2], -4]))/(3*d*Sqrt[(-2 - 3*Cos[c + d*x])/(-1 + Cos[c + d*x])]
*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])])
```

Maple [A] (verified)

Time = 6.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.66

method	result	size
default	$\frac{\sqrt{2} \sqrt{10} \left(F\left(\cot(dx+c) - \csc(dx+c), \frac{\sqrt{5}}{5}\right) - 2\Pi\left(\cot(dx+c) - \csc(dx+c), -1, \frac{\sqrt{5}}{5}\right) \right) \sqrt{\frac{2+3 \cos(dx+c)}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (1+\cos(dx+c))}{5d \sqrt{2+3 \cos(dx+c)} \sqrt{\cos(dx+c)}}$	128

```
[In] int(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

[Out] $1/5/d*2^{(1/2)}*10^{(1/2)}*(\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x+c), 1/5*5^{(1/2)})-2*\text{EllipticPi}(\cot(d*x+c)-\text{csc}(d*x+c), -1, 1/5*5^{(1/2)}))*((2+3*\cos(d*x+c))/(1+\cos(d*x+c)))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}/(2+3*\cos(d*x+c))^{(1/2)}*(1+\cos(d*x+c))/\cos(d*x+c)^{(1/2)}$

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}} dx$$

[In] `integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)`

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3\cos(c+dx)+2}} dx$$

[In] `integrate(cos(d*x+c)**(1/2)/(2+3*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(cos(c + d*x))/sqrt(3*cos(c + d*x) + 2), x)`

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}} dx$$

[In] `integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)`

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3\cos(c+dx)+2}} dx$$

[In] int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) + 2)^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) + 2)^(1/2), x)

$$3.661 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx$$

Optimal result	6221
Rubi [A] (verified)	6221
Mathematica [A] (verified)	6222
Maple [A] (verified)	6222
Fricas [F]	6223
Sympy [F]	6223
Maxima [F]	6223
Giac [F]	6223
Mupad [F(-1)]	6224

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx = \frac{4 \cot(c+dx) \operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), \frac{1}{5}\right) \sqrt{-1+\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{3\sqrt{5}d}$$

[Out] $-4/15*\cot(d*x+c)*\operatorname{EllipticPi}((-2+3*\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)}, 1/3, 1/5*5^{(1/2)})*(-1+\sec(d*x+c))^{(1/2)}*(1+\sec(d*x+c))^{(1/2)}/d*5^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec), antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2888}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx = \frac{4 \cot(c+dx) \sqrt{\sec(c+dx)-1} \sqrt{\sec(c+dx)+1} \operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right), \frac{1}{5}\right)}{3\sqrt{5}d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[-2+3*\operatorname{Cos}[c+d*x]], x]$

[Out] $(-4*\operatorname{Cot}[c+d*x]*\operatorname{EllipticPi}[1/3, \operatorname{ArcSin}[\operatorname{Sqrt}[-2+3*\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]], 1/5)*\operatorname{Sqrt}[-1+\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1+\operatorname{Sec}[c+d*x]]/(3*\operatorname{Sqrt}[5]*d)$

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

integral =

$$\frac{4 \cot(c + dx) \operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{-2+3 \cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), \frac{1}{5}\right) \sqrt{-1 + \sec(c + dx)} \sqrt{1 + \sec(c + dx)}}{3\sqrt{5}d}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.87

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-2 + 3 \cos(c + dx)}} dx = \frac{4 \cos^2\left(\frac{1}{2}(c + dx)\right) \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{\frac{-2+3 \cos(c+dx)}{1+\cos(c+dx)}} \left(\operatorname{EllipticF}\left(\arcsin\left(\sqrt{5} \tan\left(\frac{1}{2}(c + dx)\right)\right), \frac{1}{5}\right) - 2 \operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{-2+3 \cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), \frac{1}{5}\right)\right)}{\sqrt{5}d \sqrt{\cos(c + dx)} \sqrt{-2 + 3 \cos(c + dx)}}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]], x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(-2 + 3*Cos[c + d*x])/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Sqrt[5]*Tan[(c + d*x)/2]], 1/5] - 2*EllipticPi[-1/5, ArcSin[Sqrt[5]*Tan[(c + d*x)/2]], 1/5]))/(Sqrt[5]*d*Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 5.67 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.57

method	result	size
default	$\frac{2\left(F\left(\cot(dx+c)-\operatorname{csc}(dx+c), \sqrt{5}\right)-2\Pi\left(\cot(dx+c)-\operatorname{csc}(dx+c), -1, \sqrt{5}\right)\right) \sqrt{\frac{-2+3 \cos(dx+c)}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (1+\cos(dx+c))}{d \sqrt{-2+3 \cos(dx+c)} \sqrt{\cos(dx+c)}}$	118

```
[In] int(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2), x, method=_RETURNVERBOSE)
```

```
[Out] 2/d*(EllipticF(cot(d*x+c)-csc(d*x+c), 5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, 5^(1/2)))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2+3*cos(d*x+c))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-2 + 3 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(dx + c)}}{\sqrt{3 \cos(dx + c) - 2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)

Sympy [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-2 + 3 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{3 \cos(c + dx) - 2}} dx$$

[In] integrate(cos(d*x+c)**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(cos(c + d*x))/sqrt(3*cos(c + d*x) - 2), x)

Maxima [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-2 + 3 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(dx + c)}}{\sqrt{3 \cos(dx + c) - 2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)

Giac [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-2 + 3 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(dx + c)}}{\sqrt{3 \cos(dx + c) - 2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-2 + 3 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{3 \cos(c + dx) - 2}} dx$$

```
[In] int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) - 2)^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(3*cos(c + d*x) - 2)^(1/2), x)
```

$$3.662 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx$$

Optimal result	6225
Rubi [A] (verified)	6225
Mathematica [B] (verified)	6226
Maple [A] (verified)	6227
Fricas [F]	6227
Sympy [F]	6227
Maxima [F]	6228
Giac [F]	6228
Mupad [F(-1)]	6228

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \frac{4 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), \frac{1}{5}\right) \sqrt{-1+\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{3\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

[Out] $-4/15*\cos(d*x+c)^{(3/2)}*\csc(d*x+c)*\operatorname{EllipticPi}((2-3*\cos(d*x+c))^{(1/2)/(-\cos(d*x+c))^{(1/2)}, 1/3, 1/5*5^{(1/2)})*(-1+\sec(d*x+c))^{(1/2)}*(1+\sec(d*x+c))^{(1/2)}/d*5^{(1/2)/(-\cos(d*x+c))^{(1/2)}}$

Rubi [A] (verified)

Time = 0.13 (sec), antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2889, 2888}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \frac{4 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \sqrt{\sec(c+dx)-1} \sqrt{\sec(c+dx)+1} \operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), \frac{1}{5}\right)}{3\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[2-3*\operatorname{Cos}[c+d*x]],x]$

[Out] $(-4*\operatorname{Cos}[c+d*x]^{(3/2)}*\operatorname{Csc}[c+d*x]*\operatorname{EllipticPi}[1/3, \operatorname{ArcSin}[\operatorname{Sqrt}[2-3*\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]], 1/5]*\operatorname{Sqrt}[-1+\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1+\operatorname{Sec}[c+d*x]])/(3*\operatorname{Sqrt}[5]*d*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]])$

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2889

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]], In
t[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx}{\sqrt{-\cos(c+dx)}} \\ &= \frac{4 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \text{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), \frac{1}{5}\right) \sqrt{-1+\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{3\sqrt{5}d\sqrt{-\cos(c+dx)}} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(99) = 198.

Time = 1.62 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.03

$$\begin{aligned} &\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx \\ &= \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)}\csc^2\left(\frac{1}{2}(c+dx)\right)\sqrt{-((-2+3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right))}\csc(c+dx)}{3\sqrt{5}d\sqrt{-\cos(c+dx)}} \end{aligned}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((-
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*(3*EllipticF[ArcSin[S
qrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], 4/5] - EllipticPi[2/3, Arc
Sin[Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], 4/5])*Sin[(c + d*x)/2
]^4)/(3*Sqrt[5]*d*Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 6.71 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2\left(F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right)-2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{5}\right)\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2-3\cos(dx+c)}(1+\cos(dx+c))}{d\sqrt{\cos(dx+c)}(-2+3\cos(dx+c))}$

```
[In] int(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,5^(1/2)))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2-3*cos(d*x+c))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))
```

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}} dx$$

```
[In] integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c) - 2), x)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx$$

```
[In] integrate(cos(d*x+c)**(1/2)/(2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(2 - 3*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx$$

[In] int(cos(c + d*x)^(1/2)/(2 - 3*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)/(2 - 3*cos(c + d*x))^(1/2), x)

$$3.663 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx$$

Optimal result	6229
Rubi [A] (verified)	6229
Mathematica [A] (verified)	6230
Maple [A] (verified)	6231
Fricas [F]	6231
Sympy [F]	6231
Maxima [F]	6232
Giac [F]	6232
Mupad [F(-1)]	6232

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \frac{4 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \operatorname{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), 5\right) \sqrt{-1-\sec(c+dx)} \sqrt{1-\sec(c+dx)}}{3d\sqrt{-\cos(c+dx)}}$$

[Out] $-4/3*\cos(d*x+c)^{(3/2)}*\csc(d*x+c)*\operatorname{EllipticPi}(1/5*(-2-3*\cos(d*x+c))^{(1/2)}*5^{(1/2)}/(-\cos(d*x+c))^{(1/2)}, 5/3, 5^{(1/2)})*(-1-\sec(d*x+c))^{(1/2)}*(1-\sec(d*x+c))^{(1/2)}/d/(-\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2889, 2888}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \frac{4 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \sqrt{-\sec(c+dx)-1} \sqrt{1-\sec(c+dx)} \operatorname{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\right)}{3d\sqrt{-\cos(c+dx)}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[-2-3*\operatorname{Cos}[c+d*x]],x]$

[Out] $(-4*\operatorname{Cos}[c+d*x]^{(3/2)}*\operatorname{Csc}[c+d*x]*\operatorname{EllipticPi}[5/3, \operatorname{ArcSin}[\operatorname{Sqrt}[-2-3*\operatorname{Cos}[c+d*x]]/(\operatorname{Sqrt}[5]*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]])], 5]*\operatorname{Sqrt}[-1-\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1-\operatorname{Sec}[c+d*x]])/(3*d*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]])$

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2889

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]], In
t[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx}{\sqrt{-\cos(c+dx)}} \\ &= \frac{4 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \text{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), 5\right) \sqrt{-1-\sec(c+dx)} \sqrt{1-\sec(c+dx)}}{3d\sqrt{-\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \frac{4 \cos^2\left(\frac{1}{2}(c+dx)\right) \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{-\frac{(2+3\cos(c+dx))^2}{(1+\cos(c+dx))^2}} \left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right), \frac{1}{5}\right) - 2 \text{EllipticPi}\left(-1, \arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right), \frac{1}{5}\right)\right)}{\sqrt{5}d\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{-\frac{2+3\cos(c+dx)}{1+\cos(c+dx)}}}$$

[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[-((2 + 3*
Cos[c + d*x])^2/(1 + Cos[c + d*x])^2)]*(EllipticF[ArcSin[Tan[(c + d*x)/2]],
1/5] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], 1/5]))/(Sqrt[5]*d*Sqrt[-
2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Sqrt[-((2 + 3*Cos[c + d*x])/(1 + Cos
[c + d*x]))])
```

Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

method	result
default	$\frac{\left(F\left(\frac{\csc(dx+c)-\cot(dx+c)\sqrt{5}}{5},\sqrt{5}\right)-2\Pi\left(\frac{\csc(dx+c)-\cot(dx+c)\sqrt{5}}{5},-5,\sqrt{5}\right)\right)\sqrt{2}\sqrt{10}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-2-3\cos(dx+c)}}{5d\sqrt{\cos(dx+c)}(2+3\cos(dx+c))}$

```
[In] int(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/d*(EllipticF(1/5*(csc(d*x+c)-cot(d*x+c))*5^(1/2),5^(1/2))-2*EllipticPi(
1/5*(csc(d*x+c)-cot(d*x+c))*5^(1/2),-5,5^(1/2)))*2^(1/2)*10^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(-2-3*cos(
d*x+c))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))*5^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)-2}} dx$$

```
[In] integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c) + 2)
, x)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3\cos(c+dx)-2}} dx$$

```
[In] integrate(cos(d*x+c)**(1/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(-3*cos(c + d*x) - 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)-2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)-2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3\cos(c+dx)-2}} dx$$

[In] int(cos(c + d*x)^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2), x)

$$3.664 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx$$

Optimal result	6233
Rubi [A] (verified)	6233
Mathematica [C] (verified)	6234
Maple [A] (verified)	6234
Fricas [F]	6235
Sympy [F]	6235
Maxima [F]	6235
Giac [F]	6235
Mupad [F(-1)]	6236

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \frac{3 \cot(c+dx) \operatorname{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right) \sqrt{1-\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{d}$$

[Out] $-3*\cot(d*x+c)*\operatorname{EllipticPi}(1/5*(3+2*\cos(d*x+c))^{1/2}*5^{1/2}/\cos(d*x+c)^{1/2}, 5/2, I*5^{1/2})*(1-\sec(d*x+c))^{1/2}*(1+\sec(d*x+c))^{1/2}/d$

Rubi [A] (verified)

Time = 0.06 (sec), antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2887}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \frac{3 \cot(c+dx) \sqrt{1-\sec(c+dx)} \sqrt{\sec(c+dx)+1} \operatorname{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right)}{d}$$

[In] `Int[Sqrt[Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]`

[Out] `(-3*Cot[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d`

Rule 2887

`Int[Sqrt[(b_)*sin[(e_)+(f_)*(x_)]]/Sqrt[(c_)+(d_)*sin[(e_)+(f_)*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e`

+ f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] && PosQ[(c + d)/b] && GtQ[c^2, 0]

Rubi steps

integral =

$$\frac{3 \cot(c + dx) \operatorname{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right) \sqrt{1 - \sec(c + dx)} \sqrt{1 + \sec(c + dx)}}{d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{3 + 2\cos(c + dx)}} dx$$

$$= \frac{2i \sqrt{\cos(c + dx)} \sqrt{3 + 2\cos(c + dx)} \left(\operatorname{EllipticF}\left(i \operatorname{arcsinh}\left(\frac{\tan(\frac{1}{2}(c+dx))}{\sqrt{5}}\right), -5\right) - 2 \operatorname{EllipticPi}\left(5, i \operatorname{arcsinh}\left(\frac{\tan(\frac{1}{2}(c+dx))}{\sqrt{5}}\right)\right) \right)}{d \sqrt{(1 + 3\cos(c + dx) + \cos(2(c + dx)))} \sec^4\left(\frac{1}{2}(c + dx)\right)}$$

[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]

[Out] ((2*I)*Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*(EllipticF[I*ArcSinh[Tan[(c + d*x)/2]/Sqrt[5]], -5] - 2*EllipticPi[5, I*ArcSinh[Tan[(c + d*x)/2]/Sqrt[5]], -5])*Sec[(c + d*x)/2]^2)/(d*Sqrt[(1 + 3*Cos[c + d*x] + Cos[2*(c + d*x)])]*Sec[(c + d*x)/2]^4]

Maple [A] (verified)

Time = 6.73 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.78

method	result	si
default	$\frac{\left(F\left(\cot(dx+c)-\csc(dx+c), \frac{i\sqrt{5}}{5}\right) - 2\Pi\left(\cot(dx+c)-\csc(dx+c), -1, \frac{i\sqrt{5}}{5}\right)\right) \sqrt{10} \sqrt{2} \sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (1+\cos(dx+c))}{5d\sqrt{3+2\cos(dx+c)} \sqrt{\cos(dx+c)}}$	13

[In] int(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/5/d*(EllipticF(cot(d*x+c)-csc(d*x+c), 1/5*I*5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, 1/5*I*5^(1/2)))*10^(1/2)*2^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(3+2*cos(d*x+c))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/2)

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2\cos(c+dx)+3}} dx$$

[In] integrate(cos(d*x+c)**(1/2)/(3+2*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(cos(c + d*x))/sqrt(2*cos(c + d*x) + 3), x)

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{3 + 2 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{2 \cos(c + dx) + 3}} dx$$

```
[In] int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) + 3)^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) + 3)^(1/2), x)
```


$$3.665 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

Optimal result	6237
Rubi [A] (verified)	6237
Mathematica [C] (verified)	6238
Maple [B] (verified)	6238
Fricas [F]	6239
Sympy [F]	6239
Maxima [F]	6239
Giac [F]	6240
Mupad [F(-1)]	6240

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

$$= \frac{3 \cot(c+dx) \operatorname{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right) \sqrt{1-\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{\sqrt{5}d}$$

[Out] 3/5*cot(d*x+c)*EllipticPi((3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2), -1/2, 1/5*I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2887}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

$$= \frac{3 \cot(c+dx) \sqrt{1-\sec(c+dx)} \sqrt{\sec(c+dx)+1} \operatorname{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right)}{\sqrt{5}d}$$

[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]

[Out] (3*Cot[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d)

Rule 2887

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
+ f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

integral

$$= \frac{3 \cot(c + dx) \operatorname{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right) \sqrt{1 - \sec(c + dx)} \sqrt{1 + \sec(c + dx)}}{\sqrt{5}d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{3 - 2 \cos(c + dx)}} dx$$

$$= \frac{2i \sqrt{\cos(c + dx)} \left(\operatorname{EllipticF}\left(i \operatorname{arcsinh}\left(\sqrt{5} \tan\left(\frac{1}{2}(c + dx)\right)\right), -\frac{1}{5}\right) - 2 \operatorname{EllipticPi}\left(\frac{1}{5}, i \operatorname{arcsinh}\left(\sqrt{5} \tan\left(\frac{1}{2}(c + dx)\right)\right)\right) \right)}{d \sqrt{30 - 20 \cos(c + dx)} \sqrt{\frac{\cos(c + dx)}{1 + \cos(c + dx)}}}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[Cos[c + d*x]]*(EllipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -
1/5] - 2*EllipticPi[1/5, I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5])*Sqrt[1
+ 5*Tan[(c + d*x)/2]^2])/(d*Sqrt[30 - 20*Cos[c + d*x]]*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x]))]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(65) = 130.

Time = 5.62 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.87

method	result
default	$-\frac{\sqrt{2} \left(F\left(\cot(dx+c)-\csc(dx+c), i\sqrt{5}\right) - 2\Pi\left(\cot(dx+c)-\csc(dx+c), -1, i\sqrt{5}\right) \right) \sqrt{\frac{-2(-3+2\cos(dx+c))}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{3-2\cos(dx+c)}}{d \sqrt{\cos(dx+c)} (-3+2\cos(dx+c))}$

[In] `int(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)`

[Out] `-1/d*2^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),I*5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,I*5^(1/2)))*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(3-2*cos(d*x+c))^(1/2)*(1+cos(d*x+c))/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))`

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-2\cos(dx+c)+3}} dx$$

[In] `integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-2*cos(d*x+c)+3)*sqrt(cos(d*x+c))/(2*cos(d*x+c)-3),x)`

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

[In] `integrate(cos(d*x+c)**(1/2)/(3-2*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(cos(c+d*x))/sqrt(3-2*cos(c+d*x)),x)`

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-2\cos(dx+c)+3}} dx$$

[In] `integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(cos(d*x+c))/sqrt(-2*cos(d*x+c)+3),x)`

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-2\cos(dx+c)+3}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

[In] int(cos(c + d*x)^(1/2)/(3 - 2*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)/(3 - 2*cos(c + d*x))^(1/2), x)

$$3.666 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx$$

Optimal result	6241
Rubi [A] (verified)	6241
Mathematica [C] (verified)	6242
Maple [A] (verified)	6243
Fricas [F]	6243
Sympy [F]	6243
Maxima [F]	6243
Giac [F]	6244
Mupad [F(-1)]	6244

Optimal result

Integrand size = 25, antiderivative size = 99

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx$$

$$= \frac{3 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \operatorname{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right) \sqrt{1-\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

[Out] 3/5*cos(d*x+c)^(3/2)*csc(d*x+c)*EllipticPi((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2), -1/2, 1/5*I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)/(-cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.15 (sec), antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2889, 2887}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx$$

$$= \frac{3 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \sqrt{1-\sec(c+dx)} \sqrt{\sec(c+dx)+1} \operatorname{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]

[Out] (3*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]])

Rule 2887

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
+ f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rule 2889

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]], In
t[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{\cos(c+dx)} \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx}{\sqrt{-\cos(c+dx)}}}{\sqrt{5}d\sqrt{-\cos(c+dx)}} = \frac{3 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \text{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right) \sqrt{1-\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx = \frac{2i\sqrt{-3+2\cos(c+dx)}\sqrt{\frac{\cos(c+dx)}{5+5\cos(c+dx)}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right), -\frac{1}{5}\right) - 2\text{EllipticPi}\left(\frac{1}{5}, i\text{arcsinh}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{d\sqrt{\cos(c+dx)}\sqrt{\frac{3-2\cos(c+dx)}{1+\cos(c+dx)}}}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]], x]
```

```
[Out] ((-2*I)*Sqrt[-3 + 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]/(5 + 5*Cos[c + d*x])]*
EllipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5] - 2*EllipticPi[1/5, I*
ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5))/(d*Sqrt[Cos[c + d*x]]*Sqrt[(3 -
2*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

Maple [A] (verified)

Time = 7.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.45

method	result
default	$\frac{i \left(F \left(i(\csc(dx+c) - \cot(dx+c))\sqrt{5}, \frac{i\sqrt{5}}{5} \right) - 2\Pi \left(i(\csc(dx+c) - \cot(dx+c))\sqrt{5}, \frac{1}{5}, \frac{i\sqrt{5}}{5} \right) \right) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}} (1+\cos(dx+c))}{5d\sqrt{-3+2\cos(dx+c)} \sqrt{\cos(dx+c)}}$

[In] int(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] $\frac{1}{5}I/d*(\text{EllipticF}(I*(\csc(d*x+c)-\cot(d*x+c))*5^{(1/2)}, 1/5*I*5^{(1/2)})-2*\text{EllipticPi}(I*(\csc(d*x+c)-\cot(d*x+c))*5^{(1/2)}, 1/5, 1/5*I*5^{(1/2)}))*2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*(-2*(-3+2*\cos(d*x+c))/(1+\cos(d*x+c)))^{(1/2)}/(-3+2*\cos(d*x+c))^{(1/2)}*(1+\cos(d*x+c))/\cos(d*x+c)^{(1/2)}*5^{(1/2)}$

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2\cos(c+dx)-3}} dx$$

[In] integrate(cos(d*x+c)**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(cos(c + d*x))/sqrt(2*cos(c + d*x) - 3), x)

Maxima [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)

Giac [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-3 + 2 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(dx + c)}}{\sqrt{2 \cos(dx + c) - 3}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-3 + 2 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{2 \cos(c + dx) - 3}} dx$$

[In] int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) - 3)^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)/(2*cos(c + d*x) - 3)^(1/2), x)

$$3.667 \quad \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx$$

Optimal result	6245
Rubi [A] (verified)	6245
Mathematica [C] (verified)	6246
Maple [A] (verified)	6247
Fricas [F]	6247
Sympy [F]	6247
Maxima [F]	6248
Giac [F]	6248
Mupad [F(-1)]	6248

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \frac{3 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \operatorname{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right) \sqrt{1-\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{d \sqrt{-\cos(c+dx)}}$$

[Out] $-3*\cos(d*x+c)^{(3/2)}*\csc(d*x+c)*\operatorname{EllipticPi}(1/5*(-3-2*\cos(d*x+c))^{(1/2)}*5^{(1/2)/(-\cos(d*x+c))^{(1/2)}, 5/2, I*5^{(1/2)})*(1-\sec(d*x+c))^{(1/2)}*(1+\sec(d*x+c))^{(1/2)}/d/(-\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.17 (sec), antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2889, 2887}

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \frac{3 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \sqrt{1-\sec(c+dx)} \sqrt{\sec(c+dx)+1} \operatorname{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right)}{d \sqrt{-\cos(c+dx)}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[-3-2*\operatorname{Cos}[c+d*x]],x]$

[Out] $(-3*\operatorname{Cos}[c+d*x]^{(3/2)}*\operatorname{Csc}[c+d*x]*\operatorname{EllipticPi}[5/2, \operatorname{ArcSin}[\operatorname{Sqrt}[-3-2*\operatorname{Cos}[c+d*x]]]/(\operatorname{Sqrt}[5]*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]])], -5)*\operatorname{Sqrt}[1-\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1+\operatorname{Sec}[c+d*x]]/(d*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]])$

Rule 2887

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
+ f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rule 2889

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]], In
t[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx}{\sqrt{-\cos(c+dx)}} \\ &= \frac{3 \cos^{\frac{3}{2}}(c+dx) \csc(c+dx) \text{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right) \sqrt{1-\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{d\sqrt{-\cos(c+dx)}} \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.30

$$\begin{aligned} &\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx \\ &= \frac{2i \cos^2\left(\frac{1}{2}(c+dx)\right) \left(\text{EllipticF}\left(i \operatorname{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right), -5\right) - 2 \text{EllipticPi}\left(5, i \operatorname{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right), -5\right)\right)}{d\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}} \end{aligned}$$

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

```
[Out] ((2*I)*Cos[(c + d*x)/2]^2*(EllipticF[I*ArcSinh[Tan[(c + d*x)/2]/Sqrt[5]], -
5] - 2*EllipticPi[5, I*ArcSinh[Tan[(c + d*x)/2]/Sqrt[5]], -5])*Sqrt[Cos[c +
d*x]*(3 + 2*Cos[c + d*x])*Sec[(c + d*x)/2]^4)]/(d*Sqrt[-3 - 2*Cos[c + d*x]
]*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 6.82 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.63

method	result
default	$-\frac{i\sqrt{10}\sqrt{2}\left(F\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},i\sqrt{5}\right)-2\Pi\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},5,i\sqrt{5}\right)\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-3-2\cos(dx+c)}}{5d\sqrt{\cos(dx+c)}(3+2\cos(dx+c))}$

[In] int(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out]
$$-1/5*I/d*10^{(1/2)}*2^{(1/2)}*(\text{EllipticF}(1/5*I*(\csc(d*x+c)-\cot(d*x+c))*5^{(1/2)}, I*5^{(1/2)})-2*\text{EllipticPi}(1/5*I*(\csc(d*x+c)-\cot(d*x+c))*5^{(1/2)}, 5, I*5^{(1/2)}))$$

$$*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((3+2*\cos(d*x+c))/(1+\cos(d*x+c)))^{(1/2)}$$

$$*(-3-2*\cos(d*x+c))^{(1/2)}*(1+\cos(d*x+c))/\cos(d*x+c)^{(1/2)}/(3+2*\cos(d*x+c))*5^{(1/2)}$$

Fricas [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-2\cos(dx+c)-3}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c) + 3), x)

Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2\cos(c+dx)-3}} dx$$

[In] integrate(cos(d*x+c)**(1/2)/(-3-2*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(cos(c + d*x))/sqrt(-2*cos(c + d*x) - 3), x)

Maxima [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-3 - 2 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(dx + c)}}{\sqrt{-2 \cos(dx + c) - 3}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)

Giac [F]

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-3 - 2 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(dx + c)}}{\sqrt{-2 \cos(dx + c) - 3}} dx$$

[In] integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-3 - 2 \cos(c + dx)}} dx = \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{-2 \cos(c + dx) - 3}} dx$$

[In] int(cos(c + d*x)^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2),x)

[Out] int(cos(c + d*x)^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2), x)

$$3.668 \quad \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx$$

Optimal result	6249
Rubi [A] (verified)	6249
Mathematica [A] (verified)	6250
Maple [A] (verified)	6251
Fricas [F]	6251
Sympy [F]	6251
Maxima [F]	6251
Giac [F]	6252
Mupad [F(-1)]	6252

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), 5\right)\sqrt{-1-\sec(c+dx)}}{3d}$$

[Out] $-4/3*\csc(d*x+c)*\operatorname{EllipticPi}(1/5*(2+3*\cos(d*x+c))^{(1/2)}*5^{(1/2)}/\cos(d*x+c)^{(1/2)}, 5/3, 5^{(1/2)})*(-\cos(d*x+c))^{(1/2)}*\cos(d*x+c)^{(1/2)}*(-1-\sec(d*x+c))^{(1/2)}*(1-\sec(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.19 (sec), antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2889, 2888}

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\operatorname{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), 5\right)}{3d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[2+3*\operatorname{Cos}[c+d*x]], x]$

[Out] $(-4*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Csc}[c+d*x]*\operatorname{EllipticPi}[5/3, \operatorname{ArcSin}[\operatorname{Sqrt}[2+3*\operatorname{Cos}[c+d*x]]/(\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])], 5]*\operatorname{Sqrt}[-1-\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1-\operatorname{Sec}[c+d*x]])/(3*d)$

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_.)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_.)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2889

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_.)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_.)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]], In
t[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{-\cos(c+dx)} \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), 5\right)\sqrt{-1-\sec(c+dx)}}{3d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.96

$$\begin{aligned} &\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx \\ &= \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)}\csc^2\left(\frac{1}{2}(c+dx)\right)\sqrt{(2+3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\left(3\text{EllipticF}\left[\arcsin\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), 5\right]\right)}{3d} \end{aligned}$$

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]], x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*(3*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4] - 5*EllipticPi[-2/3, ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4])*Sin[(c + d*x)/2]^4)/(3*d*Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 6.91 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\sqrt{-\cos(dx+c)} \left(F\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5}, \sqrt{5}\right) - 2\Pi\left(\frac{(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5}, -5, \sqrt{5}\right) \right) \sqrt{10} \sqrt{\frac{2+3\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{5d\sqrt{2+3\cos(dx+c)}}$

```
[In] int((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/d*(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2)*(EllipticF(1/5*(csc(d*x+c)-cot(d*x+c))*5^(1/2),5^(1/2))-2*EllipticPi(1/5*(csc(d*x+c)-cot(d*x+c))*5^(1/2),-5,5^(1/2)))*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(1+sec(d*x+c))*5^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}} dx$$

```
[In] integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3\cos(c+dx)+2}} dx$$

```
[In] integrate((-cos(d*x+c))**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(3*cos(c + d*x) + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}} dx$$

```
[In] integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3\cos(c+dx)+2}} dx$$

[In] int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) + 2)^(1/2),x)

[Out] int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) + 2)^(1/2), x)

$$3.669 \quad \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx$$

Optimal result	6253
Rubi [A] (verified)	6253
Mathematica [A] (verified)	6254
Maple [A] (verified)	6255
Fricas [F]	6255
Sympy [F]	6255
Maxima [F]	6256
Giac [F]	6256
Mupad [F(-1)]	6256

Optimal result

Integrand size = 27, antiderivative size = 97

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx = \frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), \frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}}{3\sqrt{5}d}$$

[Out] $-4/15*\csc(d*x+c)*\operatorname{EllipticPi}((-2+3*\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)}, 1/3, 1/5*5^{(1/2)})*(-\cos(d*x+c))^{(1/2)}*\cos(d*x+c)^{(1/2)}*(-1+\sec(d*x+c))^{(1/2)}*(1+\sec(d*x+c))^{(1/2)}/d*5^{(1/2)}$

Rubi [A] (verified)

Time = 0.18 (sec), antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2889, 2888}

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx = \frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), \frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}}{3\sqrt{5}d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[-2+3*\operatorname{Cos}[c+d*x]], x]$

[Out] $(-4*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Csc}[c+d*x]*\operatorname{EllipticPi}[1/3, \operatorname{ArcSin}[\operatorname{Sqrt}[-2+3*\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]], 1/5]*\operatorname{Sqrt}[-1+\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1+\operatorname{Sec}[c+d*x]])/(3*\operatorname{Sqrt}[5]*d)$

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2889

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]], In
t[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{-\cos(c+dx)} \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), \frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}}{3\sqrt{5}d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.46

$$\begin{aligned} &\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx \\ &= \frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{-2+3\cos(c+dx)}{1+\cos(c+dx)}}\left(\text{EllipticF}\left(\arcsin\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right), \frac{1}{5}\right) - 2\text{EllipticPi}\left(-\frac{1}{5}, \text{ArcSin}\left[\frac{\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)}{2}\right], \frac{1}{5}\right)\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}} \end{aligned}$$

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(-2 + 3*Cos[c + d*x])/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Sqrt[5]*Tan[(c + d*x)/2]], 1/5] - 2*EllipticPi[-1/5, ArcSin[Sqrt[5]*Tan[(c + d*x)/2]], 1/5]))/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 6.78 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

method	result
default	$\frac{2\sqrt{-\cos(dx+c)} \left(F\left(\cot(dx+c)-\csc(dx+c),\sqrt{5}\right) - 2\Pi\left(\cot(dx+c)-\csc(dx+c),-1,\sqrt{5}\right) \right) \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}} (1+\sec(dx+c))}{d\sqrt{-2+3\cos(dx+c)}}$

```
[In] int((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,5^(1/2)))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(1+sec(d*x+c))
```

Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\cos(dx+c)-2}} dx$$

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3\cos(c+dx)-2}} dx$$

```
[In] integrate((-cos(d*x+c))**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(3*cos(c + d*x) - 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\cos(dx+c)-2}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)

Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\cos(dx+c)-2}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3\cos(c+dx)-2}} dx$$

[In] int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) - 2)^(1/2),x)

[Out] int((-cos(c + d*x))^(1/2)/(3*cos(c + d*x) - 2)^(1/2), x)

$$3.670 \quad \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx$$

Optimal result	6257
Rubi [A] (verified)	6257
Mathematica [B] (verified)	6258
Maple [B] (verified)	6258
Fricas [F]	6259
Sympy [F]	6259
Maxima [F]	6259
Giac [F]	6260
Mupad [F(-1)]	6260

Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \frac{4 \cot(c+dx) \operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), \frac{1}{5}\right) \sqrt{-1+\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{3\sqrt{5}d}$$

[Out] $-4/15*\cot(d*x+c)*\operatorname{EllipticPi}((2-3*\cos(d*x+c))^{(1/2)/(-\cos(d*x+c))^{(1/2)}, 1/3, 1/5*5^{(1/2)})*(-1+\sec(d*x+c))^{(1/2)}*(1+\sec(d*x+c))^{(1/2)}/d*5^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec), antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2888}

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \frac{4 \cot(c+dx) \sqrt{\sec(c+dx)-1} \sqrt{\sec(c+dx)+1} \operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), \frac{1}{5}\right)}{3\sqrt{5}d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[2-3*\operatorname{Cos}[c+d*x]],x]$

[Out] $(-4*\operatorname{Cot}[c+d*x]*\operatorname{EllipticPi}[1/3, \operatorname{ArcSin}[\operatorname{Sqrt}[2-3*\operatorname{Cos}[c+d*x]]]/\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]], 1/5)*\operatorname{Sqrt}[-1+\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1+\operatorname{Sec}[c+d*x]]/(3*\operatorname{Sqrt}[5]*d)$

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

integral =

$$\frac{4 \cot(c + dx) \operatorname{EllipticPi}\left(\frac{1}{3}, \arcsin\left(\frac{\sqrt{2-3 \cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), \frac{1}{5}\right) \sqrt{-1 + \sec(c + dx)} \sqrt{1 + \sec(c + dx)}}{3\sqrt{5}d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 203 vs. 2(77) = 154.

Time = 0.74 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.64

$$\int \frac{\sqrt{-\cos(c + dx)}}{\sqrt{2 - 3 \cos(c + dx)}} dx$$

$$= \frac{4\sqrt{\cot^2\left(\frac{1}{2}(c + dx)\right) \cot(c + dx) \sqrt{\cos(c + dx) \csc^2\left(\frac{1}{2}(c + dx)\right)} \sqrt{-((-2 + 3 \cos(c + dx)) \csc^2\left(\frac{1}{2}(c + dx)\right))}}{3\sqrt{5}d}$$

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Cot[c + d*x]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*(3*EllipticF[ArcSin[Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], 4/5] - EllipticPi[2/3, ArcSin[Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], 4/5])*Sin[(c + d*x)/2]^4)/(3*Sqrt[5]*d*Sqrt[2 - 3*Cos[c + d*x]]*(-Cos[c + d*x])^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(64) = 128.

Time = 5.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.71

method	result
default	$-\frac{2\sqrt{-\cos(dx+c)} \sqrt{2-3 \cos(dx+c)} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{\frac{-2+3 \cos(dx+c)}{1+\cos(dx+c)}} \left(F\left(\cot(dx+c)-\csc(dx+c), \sqrt{5}\right) - 2\Pi\left(\cot(dx+c)-\csc(dx+c), -\frac{1}{5}\right)\right)}{d(-2+3 \cos(dx+c))}$

```
[In] int((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(-cos(d*x+c))^(1/2)*(2-3*cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(EllipticF(cot(d*x+c)-csc(d
*x+c),5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,5^(1/2)))/(-2+3*cos(d*
x+c))*(1+sec(d*x+c))
```

Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}} dx$$

```
[In] integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)/(3*cos(d*x + c) - 2
), x)
```

Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx$$

```
[In] integrate((-cos(d*x+c))**(1/2)/(2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(2 - 3*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}} dx$$

```
[In] integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}} dx$$

[In] int((-cos(c + d*x))^(1/2)/(2 - 3*cos(c + d*x))^(1/2),x)

[Out] int((-cos(c + d*x))^(1/2)/(2 - 3*cos(c + d*x))^(1/2), x)

$$3.671 \quad \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx$$

Optimal result	6261
Rubi [A] (verified)	6261
Mathematica [A] (verified)	6262
Maple [B] (verified)	6262
Fricas [F]	6263
Sympy [F]	6263
Maxima [F]	6263
Giac [F]	6264
Mupad [F(-1)]	6264

Optimal result

Integrand size = 27, antiderivative size = 79

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \frac{4 \cot(c+dx) \operatorname{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), 5\right) \sqrt{-1-\sec(c+dx)} \sqrt{1-\sec(c+dx)}}{3d}$$

[Out] $-4/3*\cot(d*x+c)*\operatorname{EllipticPi}(1/5*(-2-3*\cos(d*x+c))^{(1/2)}*5^{(1/2)/(-\cos(d*x+c))^{(1/2)}, 5/3, 5^{(1/2)})*(-1-\sec(d*x+c))^{(1/2)}*(1-\sec(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.09 (sec), antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2888}

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \frac{4 \cot(c+dx) \sqrt{-\sec(c+dx)-1} \sqrt{1-\sec(c+dx)} \operatorname{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), 5\right)}{3d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[-2-3*\operatorname{Cos}[c+d*x]], x]$

[Out] $(-4*\operatorname{Cot}[c+d*x]*\operatorname{EllipticPi}[5/3, \operatorname{ArcSin}[\operatorname{Sqrt}[-2-3*\operatorname{Cos}[c+d*x]]/(\operatorname{Sqrt}[5]*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]])], 5]*\operatorname{Sqrt}[-1-\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1-\operatorname{Sec}[c+d*x]])/(3*d)$

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

integral =

$$\frac{4 \cot(c + dx) \operatorname{EllipticPi}\left(\frac{5}{3}, \arcsin\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), 5\right) \sqrt{-1 - \sec(c + dx)} \sqrt{1 - \sec(c + dx)}}{3d}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{-\cos(c + dx)}}{\sqrt{-2 - 3\cos(c + dx)}} dx$$

$$= \frac{4 \cos^2\left(\frac{1}{2}(c + dx)\right) \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{-\frac{(2+3\cos(c+dx))^2}{(1+\cos(c+dx))^2}} \left(\operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c + dx)\right)\right), \frac{1}{5}\right) - 2 \operatorname{EllipticPi}\left(-\frac{\sqrt{5}d\sqrt{-2 - 3\cos(c + dx)}\sqrt{-\cos(c + dx)}\sqrt{\frac{-2-3\cos(c+dx)}{1+\cos(c+dx)}}}{\sqrt{5}d\sqrt{-2 - 3\cos(c + dx)}\sqrt{-\cos(c + dx)}\sqrt{\frac{-2-3\cos(c+dx)}{1+\cos(c+dx)}}}\right)}{\sqrt{5}d\sqrt{-2 - 3\cos(c + dx)}\sqrt{-\cos(c + dx)}\sqrt{\frac{-2-3\cos(c+dx)}{1+\cos(c+dx)}}}$$

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[-((2 + 3*Cos[c + d*x])^2/(1 + Cos[c + d*x])^2)]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], 1/5] - 2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], 1/5]))/(Sqrt[5]*d*Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]*Sqrt[(-2 - 3*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.

Time = 6.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.80

method	result
default	$-\frac{\sqrt{-\cos(dx+c)} \sqrt{-2-3\cos(dx+c)} \left(F\left(\cot(dx+c)-\csc(dx+c), \frac{\sqrt{5}}{5}\right) - 2\Pi\left(\cot(dx+c)-\csc(dx+c), -1, \frac{\sqrt{5}}{5}\right) \right) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{10}}{5d(2+3\cos(dx+c))}$

```
[In] int((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/d*(-cos(d*x+c))^(1/2)*(-2-3*cos(d*x+c))^(1/2)*(EllipticF(cot(d*x+c)-cs
c(d*x+c),1/5*5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,1/5*5^(1/2)))*2
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)/(2+3*cos(d*x+c))*(1+sec(d*x+c))
```

Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\cos(dx+c)-2}} dx$$

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)/(3*cos(d*x + c) + 2
), x)
```

Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3\cos(c+dx)-2}} dx$$

```
[In] integrate((-cos(d*x+c))**(1/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(-3*cos(c + d*x) - 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\cos(dx+c)-2}} dx$$

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\cos(dx+c)-2}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3\cos(c+dx)-2}} dx$$

[In] int((-cos(c + d*x))^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2),x)

[Out] int((-cos(c + d*x))^(1/2)/(- 3*cos(c + d*x) - 2)^(1/2), x)

$$3.672 \quad \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx$$

Optimal result	6265
Rubi [A] (verified)	6265
Mathematica [C] (verified)	6266
Maple [A] (verified)	6267
Fricas [F]	6267
Sympy [F]	6267
Maxima [F]	6267
Giac [F]	6268
Mupad [F(-1)]	6268

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right)\sqrt{1-\sec(c+dx)}}{d}$$

[Out] $-3*\csc(d*x+c)*\operatorname{EllipticPi}(1/5*(3+2*\cos(d*x+c))^{1/2}*5^{1/2}/\cos(d*x+c)^{1/2}, 5/2, I*5^{1/2})*(-\cos(d*x+c))^{1/2}*\cos(d*x+c)^{1/2}*(1-\sec(d*x+c))^{1/2}*(1+\sec(d*x+c))^{1/2}/d$

Rubi [A] (verified)

Time = 0.18 (sec), antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2889, 2887}

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\operatorname{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right)\sqrt{1+\sec(c+dx)}}{d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[3+2*\operatorname{Cos}[c+d*x]],x]$

[Out] $(-3*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Csc}[c+d*x]*\operatorname{EllipticPi}[5/2, \operatorname{ArcSin}[\operatorname{Sqrt}[3+2*\operatorname{Cos}[c+d*x]]/(\operatorname{Sqrt}[5]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])], -5]*\operatorname{Sqrt}[1-\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1+\operatorname{Sec}[c+d*x]])/d$

Rule 2887

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
+ f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rule 2889

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]], In
t[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{-\cos(c+dx)} \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right), -5\right)\sqrt{1-\sec^2(c+dx)}}{d} \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$\begin{aligned} &\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx \\ &= \frac{2i\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right), -5\right) - 2\text{EllipticPi}\left(5, i\text{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right)\right)\right)}{d\sqrt{(1+3\cos(c+dx)+\cos(2(c+dx)))\sec^4\left(\frac{1}{2}(c+dx)\right)}} \end{aligned}$$

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]], x]
```

```
[Out] ((2*I)*Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*(EllipticF[I*ArcSinh[Ta
n[(c + d*x)/2]/Sqrt[5]], -5] - 2*EllipticPi[5, I*ArcSinh[Tan[(c + d*x)/2]/S
qrt[5]], -5])*Sec[(c + d*x)/2]^2)/(d*Sqrt[(1 + 3*Cos[c + d*x] + Cos[2*(c +
d*x)])*Sec[(c + d*x)/2]^4])
```

Maple [A] (verified)

Time = 6.62 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.56

method	result
default	$\frac{i\sqrt{-\cos(dx+c)}\sqrt{10}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(F\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},i\sqrt{5}\right)-2\Pi\left(\frac{i(\csc(dx+c)-\cot(dx+c))\sqrt{5}}{5},i\sqrt{5}\right)\right)}{5d\sqrt{3+2\cos(dx+c)}}$

[In] int((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] $\frac{1}{5}I/d*(-\cos(d*x+c))^{(1/2)}/(3+2*\cos(d*x+c))^{(1/2)}*10^{(1/2)}*((3+2*\cos(d*x+c))/(1+\cos(d*x+c)))^{(1/2)}*2^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*(\text{EllipticF}(1/5*I*(\csc(d*x+c)-\cot(d*x+c))*5^{(1/2)},I*5^{(1/2)})-2*\text{EllipticPi}(1/5*I*(\csc(d*x+c)-\cot(d*x+c))*5^{(1/2)},5,I*5^{(1/2)}))*(1+\sec(d*x+c))*5^{(1/2)}$

Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)

Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)+3}} dx$$

[In] integrate((-cos(d*x+c))**(1/2)/(3+2*cos(d*x+c))**(1/2),x)

[Out] Integral(sqrt(-cos(c + d*x))/sqrt(2*cos(c + d*x) + 3), x)

Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)+3}} dx$$

[In] int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) + 3)^(1/2),x)

[Out] int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) + 3)^(1/2), x)

$$3.673 \quad \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

Optimal result	6269
Rubi [A] (verified)	6269
Mathematica [C] (verified)	6270
Maple [A] (verified)	6271
Fricas [F]	6271
Sympy [F]	6271
Maxima [F]	6272
Giac [F]	6272
Mupad [F(-1)]	6272

Optimal result

Integrand size = 27, antiderivative size = 97

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

$$= \frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}}{\sqrt{5}d}$$

[Out] $3/5*\csc(d*x+c)*\operatorname{EllipticPi}((3-2*\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)}, -1/2, 1/5*I*5^{(1/2)})*(-\cos(d*x+c))^{(1/2)}*\cos(d*x+c)^{(1/2)}*(1-\sec(d*x+c))^{(1/2)}*(1+\sec(d*x+c))^{(1/2)}/d*5^{(1/2)}$

Rubi [A] (verified)

Time = 0.21 (sec), antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2889, 2887}

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

$$= \frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\operatorname{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}}{\sqrt{5}d}$$

[In] `Int[Sqrt[-Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]], x]`

[Out] `(3*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[-1/2, Arc Sin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d)`

Rule 2887

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
+ f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rule 2889

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[(-b)*Sin[e + f*x]], In
t[Sqrt[(-b)*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{-\cos(c+dx)} \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right), -\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}}{\sqrt{5}d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

$$= \frac{2i\sqrt{-\cos(c+dx)}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right), -\frac{1}{5}\right) - 2\text{EllipticPi}\left(\frac{1}{5}, i\text{arcsinh}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{d\sqrt{30-20\cos(c+dx)}\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}}$$

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[-Cos[c + d*x]]*(EllipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]],
-1/5] - 2*EllipticPi[1/5, I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5])*Sqrt[
1 + 5*Tan[(c + d*x)/2]^2])/(d*Sqrt[30 - 20*Cos[c + d*x]]*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])])
```

Maple [A] (verified)

Time = 7.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.65

method	result
default	$\frac{i \left(2\Pi \left(i(\csc(dx+c) - \cot(dx+c))\sqrt{5}, \frac{1}{5}, \frac{i\sqrt{5}}{5} \right) - F \left(i(\csc(dx+c) - \cot(dx+c))\sqrt{5}, \frac{i\sqrt{5}}{5} \right) \right) \sqrt{-\cos(dx+c)} \sqrt{3-2\cos(dx+c)} \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{5d(-3+2\cos(dx+c))}$

```
[In] int((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*I/d*(2*EllipticPi(I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),1/5,1/5*I*5^(1/2))-
EllipticF(I*(csc(d*x+c)-cot(d*x+c))*5^(1/2),1/5*I*5^(1/2)))*(-cos(d*x+c))^(
1/2)*(3-2*cos(d*x+c))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*(
-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(-3+2*cos(d*x+c))*(1+sec(d*x+c))*5^(
1/2)
```

Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)+3}} dx$$

```
[In] integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)/(2*cos(d*x + c) - 3
), x)
```

Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

```
[In] integrate((-cos(d*x+c))**(1/2)/(3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(3 - 2*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)+3}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)

Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)+3}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}} dx$$

[In] int((-cos(c + d*x))^(1/2)/(3 - 2*cos(c + d*x))^(1/2),x)

[Out] int((-cos(c + d*x))^(1/2)/(3 - 2*cos(c + d*x))^(1/2), x)

$$3.674 \quad \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx$$

Optimal result	6273
Rubi [A] (verified)	6273
Mathematica [C] (verified)	6274
Maple [A] (verified)	6274
Fricas [F]	6275
Sympy [F]	6275
Maxima [F]	6275
Giac [F]	6276
Mupad [F(-1)]	6276

Optimal result

Integrand size = 27, antiderivative size = 77

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx$$

$$= \frac{3 \cot(c+dx) \operatorname{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right) \sqrt{1-\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{\sqrt{5}d}$$

[Out] 3/5*cot(d*x+c)*EllipticPi((-3+2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2), -1/2, 1/5*I*5^(1/2))*(1-sec(d*x+c))^(1/2)*(1+sec(d*x+c))^(1/2)/d*5^(1/2)

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2887}

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx$$

$$= \frac{3 \cot(c+dx) \sqrt{1-\sec(c+dx)} \sqrt{\sec(c+dx)+1} \operatorname{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right)}{\sqrt{5}d}$$

[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]

[Out] (3*Cot[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d)

Rule 2887

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[e
+ f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

integral

$$= \frac{3 \cot(c + dx) \operatorname{EllipticPi}\left(-\frac{1}{2}, \arcsin\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right), -\frac{1}{5}\right) \sqrt{1 - \sec(c + dx)} \sqrt{1 + \sec(c + dx)}}{\sqrt{5}d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{-\cos(c + dx)}}{\sqrt{-3 + 2\cos(c + dx)}} dx$$

$$= \frac{2i \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{-3 + 2\cos(c + dx)} \left(\operatorname{EllipticF}\left(i \operatorname{arcsinh}\left(\sqrt{5} \tan\left(\frac{1}{2}(c + dx)\right)\right), -\frac{1}{5}\right) - 2 \operatorname{EllipticPi}\left(\frac{1}{5}, i \operatorname{arcsinh}\left(\sqrt{5} \tan\left(\frac{1}{2}(c + dx)\right)\right)\right) \right)}{\sqrt{5}d \sqrt{-\cos(c + dx)} \sqrt{\frac{3-2\cos(c+dx)}{1+\cos(c+dx)}}}$$

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[-3 + 2*Cos[c + d*x]]*(EllipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5] - 2*EllipticPi[1/5, I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5]))/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]]*Sqrt[(3 - 2*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

method	result
default	$\frac{\sqrt{-\cos(dx+c)} \sqrt{-\frac{2(-3+2\cos(dx+c))}{1+\cos(dx+c)}} \left(F\left(\cot(dx+c)-\csc(dx+c), i\sqrt{5}\right) - 2\Pi\left(\cot(dx+c)-\csc(dx+c), -1, i\sqrt{5}\right) \right) \sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} (1+\sec(dx+c))}{d \sqrt{-3+2\cos(dx+c)}}$

```
[In] int((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(-3+2*cos(d*x+c))^(1/2)*(-cos(d*x+c))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),I*5^(1/2))-2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,I*5^(1/2)))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1+sec(d*x+c))
```

Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}} dx$$

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)-3}} dx$$

```
[In] integrate((-cos(d*x+c))**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(2*cos(c + d*x) - 3), x)
```

Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}} dx$$

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)-3}} dx$$

[In] int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) - 3)^(1/2),x)

[Out] int((-cos(c + d*x))^(1/2)/(2*cos(c + d*x) - 3)^(1/2), x)

$$3.675 \quad \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx$$

Optimal result	6277
Rubi [A] (verified)	6277
Mathematica [C] (verified)	6278
Maple [B] (verified)	6278
Fricas [F]	6279
Sympy [F]	6279
Maxima [F]	6279
Giac [F]	6280
Mupad [F(-1)]	6280

Optimal result

Integrand size = 27, antiderivative size = 75

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \frac{3 \cot(c+dx) \operatorname{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right) \sqrt{1-\sec(c+dx)} \sqrt{1+\sec(c+dx)}}{d}$$

[Out] $-3*\cot(d*x+c)*\operatorname{EllipticPi}(1/5*(-3-2*\cos(d*x+c))^{(1/2)}*5^{(1/2)/(-\cos(d*x+c))}^{(1/2)}, 5/2, I*5^{(1/2)})*(1-\sec(d*x+c))^{(1/2)}*(1+\sec(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.06 (sec), antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2887}

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \frac{3 \cot(c+dx) \sqrt{1-\sec(c+dx)} \sqrt{\sec(c+dx)+1} \operatorname{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right)}{d}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]]/\operatorname{Sqrt}[-3-2*\operatorname{Cos}[c+d*x]], x]$

[Out] $(-3*\operatorname{Cot}[c+d*x]*\operatorname{EllipticPi}[5/2, \operatorname{ArcSin}[\operatorname{Sqrt}[-3-2*\operatorname{Cos}[c+d*x]]/(\operatorname{Sqrt}[5]*\operatorname{Sqrt}[-\operatorname{Cos}[c+d*x]])], -5]*\operatorname{Sqrt}[1-\operatorname{Sec}[c+d*x]]*\operatorname{Sqrt}[1+\operatorname{Sec}[c+d*x]])/d$

Rule 2887

$\operatorname{Int}[\operatorname{Sqrt}[(b_*)*\sin[(e_*)+(f_*)(x_*)]]/\operatorname{Sqrt}[(c_*)+(d_*)*\sin[(e_*)+(f_*)(x_*)]], x_Symbol] :> \operatorname{Simp}[2*c*Rt[b*(c+d), 2]*\operatorname{Tan}[e+f*x]*\operatorname{Sqrt}[1+\operatorname{Csc}[e$

```
+ f*x]]*(Sqrt[1 - Csc[e + f*x]]/(d*f*Sqrt[c^2 - d^2]))*EllipticPi[(c + d)/
d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]],
-(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0] &&
PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

integral =

$$\frac{3 \cot(c + dx) \operatorname{EllipticPi}\left(\frac{5}{2}, \arcsin\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right), -5\right) \sqrt{1 - \sec(c + dx)} \sqrt{1 + \sec(c + dx)}}{d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{-\cos(c + dx)}}{\sqrt{-3 - 2\cos(c + dx)}} dx = \frac{2i \cos^2\left(\frac{1}{2}(c + dx)\right) \left(\operatorname{EllipticF}\left(i \operatorname{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right), -5\right) - 2 \operatorname{EllipticPi}\left(5, i \operatorname{arcsinh}\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{5}}\right), -5\right)\right)}{d \sqrt{-3 - 2\cos(c + dx)} \sqrt{-\cos(c + dx)}}$$

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

```
[Out] ((-2*I)*Cos[(c + d*x)/2]^2*(EllipticF[I*ArcSinh[Tan[(c + d*x)/2]/Sqrt[5]],
-5] - 2*EllipticPi[5, I*ArcSinh[Tan[(c + d*x)/2]/Sqrt[5]], -5])*Sqrt[Cos[c
+ d*x]*(3 + 2*Cos[c + d*x])*Sec[(c + d*x)/2]^4])/(d*Sqrt[-3 - 2*Cos[c + d*x
]]*Sqrt[-Cos[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(68) = 136.

Time = 5.94 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.92

method	result
default	$-\frac{\sqrt{2} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} \sqrt{-\cos(dx+c)} \sqrt{-3-2\cos(dx+c)} \left(F\left(\cot(dx+c)-\csc(dx+c), \frac{i\sqrt{5}}{5}\right) - 2\Pi\left(\cot(dx+c)-\csc(dx+c), -1, \frac{i\sqrt{5}}{5}\right) \right) \sqrt{10}}{5d(3+2\cos(dx+c))}$

```
[In] int((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-cos(d*x+c))^(1/2)*(-3-2*
cos(d*x+c))^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),1/5*I*5^(1/2))-2*Ellipti
```

`cPi(cot(d*x+c)-csc(d*x+c),-1,1/5*I*5^(1/2))*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(3+2*cos(d*x+c))*(1+sec(d*x+c))`

Fricas [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)-3}} dx$$

[In] `integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(-sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)/(2*cos(d*x + c) + 3), x)`

Sympy [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2\cos(c+dx)-3}} dx$$

[In] `integrate((-cos(d*x+c))**(1/2)/(-3-2*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(-cos(c + d*x))/sqrt(-2*cos(c + d*x) - 3), x)`

Maxima [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)-3}} dx$$

[In] `integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)`

Giac [F]

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)-3}} dx$$

[In] integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}} dx = \int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2\cos(c+dx)-3}} dx$$

[In] int((-cos(c + d*x))^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2),x)

[Out] int((-cos(c + d*x))^(1/2)/(- 2*cos(c + d*x) - 3)^(1/2), x)

$$3.676 \quad \int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b \cos(c+dx)} dx$$

Optimal result	6281
Rubi [A] (verified)	6281
Mathematica [B] (warning: unable to verify)	6283
Maple [F]	6286
Fricas [F(-1)]	6286
Sympy [F(-1)]	6286
Maxima [F]	6286
Giac [F]	6287
Mupad [F(-1)]	6287

Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b \cos(c+dx)} dx$$

$$= -\frac{b \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \cos^{\frac{2}{3}}(c+dx) \sin(c+dx)}{(a^2-b^2) d \sqrt[3]{\cos^2(c+dx)}} + \frac{a \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \sqrt[6]{\cos^2(c+dx)} \sin(c+dx)}{(a^2-b^2) d \sqrt[3]{\cos(c+dx)}}$$

```
[Out] -b*AppellF1(1/2,-1/3,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d*x+c)^(2/3)*sin(d*x+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/3)+a*AppellF1(1/2,1/6,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used

= {2902, 3268, 440}

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)} dx$$

$$= \frac{a \sin(c+dx) \sqrt[6]{\cos^2(c+dx)} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2) \sqrt[3]{\cos(c+dx)}} - \frac{b \sin(c+dx) \cos^{\frac{2}{3}}(c+dx) \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2) \sqrt[3]{\cos^2(c+dx)}}$$

[In] Int[Cos[c + d*x]^(2/3)/(a + b*Cos[c + d*x]),x]

[Out] -((b*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(2/3)*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3))) + (a*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(1/3))

Rule 440

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 2902

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]

Rule 3268

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]

Rubi steps

$$\text{integral} = a \int \frac{\cos^{\frac{2}{3}}(c+dx)}{a^2 - b^2 \cos^2(c+dx)} dx - b \int \frac{\cos^{\frac{5}{3}}(c+dx)}{a^2 - b^2 \cos^2(c+dx)} dx$$

$$\begin{aligned}
&= -\frac{\left(b \cos^{\frac{2}{3}}(c+dx)\right) \text{Subst}\left(\int \frac{\sqrt[3]{1-x^2}}{a^2-b^2+b^2x^2} dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos^2(c+dx)}} \\
&+ \frac{\left(a\sqrt[6]{\cos^2(c+dx)}\right) \text{Subst}\left(\int \frac{1}{\sqrt[6]{1-x^2(a^2-b^2+b^2x^2)}} dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos(c+dx)}} \\
&= -\frac{b \text{AppellF1}\left(\frac{1}{2}, -\frac{1}{3}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \cos^{\frac{2}{3}}(c+dx) \sin(c+dx)}{(a^2-b^2)d\sqrt[3]{\cos^2(c+dx)}} \\
&+ \frac{a \text{AppellF1}\left(\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \sqrt[6]{\cos^2(c+dx)} \sin(c+dx)}{(a^2-b^2)d\sqrt[3]{\cos(c+dx)}}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4614 vs. 2(176) = 352.

Time = 34.42 (sec) , antiderivative size = 4614, normalized size of antiderivative = 26.22

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)} dx = \text{Result too large to show}$$

[In] Integrate[Cos[c + d*x]^(2/3)/(a + b*Cos[c + d*x]),x]

[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2 + (b*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2))/(d*Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(5/6)*(-b^2 + a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*((a*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2 + (b*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan

$$\begin{aligned}
& [c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*\text{AppellF1}[3/2, 5/ \\
& 6, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - \\
& b^2)*\text{AppellF1}[3/2, 11/6, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(\\
& a^2 - b^2))]*\text{Tan}[c + d*x]^2))/(-b^2 + a^2*\text{Sec}[c + d*x]^2) - (18*a^2*(a^2 \\
& - b^2)*(\text{Sec}[c + d*x]^2)^{(1/6)}*\text{Tan}[c + d*x]^2*((a*\text{AppellF1}[1/2, 1/3, 1, 3/2, \\
& -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]*\text{Sqrt}[\text{Sec}[c + d*x]^2] \\
&))/(9*(a^2 - b^2)*\text{AppellF1}[1/2, 1/3, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + \\
& d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*\text{AppellF1}[3/2, 1/3, 2, 5/2, -\text{Tan}[c + d*x]^ \\
& 2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*\text{AppellF1}[3/2, 4/3, 1, \\
& 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]*\text{Tan}[c + d*x]^2 \\
&) + (b*\text{AppellF1}[1/2, 5/6, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(\\
& a^2 - b^2)))]/(-9*(a^2 - b^2)*\text{AppellF1}[1/2, 5/6, 1, 3/2, -\text{Tan}[c + d*x]^2, - \\
& ((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*\text{AppellF1}[3/2, 5/6, 2, 5/2, -\text{Tan} \\
& [c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*\text{AppellF1} \\
& [3/2, 11/6, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2)))* \\
& \text{Tan}[c + d*x]^2))/(-b^2 + a^2*\text{Sec}[c + d*x]^2)^2 - (15*(a^2 - b^2)*\text{Tan}[c + d \\
& *x]^2*((a*\text{AppellF1}[1/2, 1/3, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2 \\
&))/(a^2 - b^2)))*\text{Sqrt}[\text{Sec}[c + d*x]^2])/ (9*(a^2 - b^2)*\text{AppellF1}[1/2, 1/3, 1, \\
& 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*\text{Appel} \\
& lF1[3/2, 1/3, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] \\
& + (a^2 - b^2)*\text{AppellF1}[3/2, 4/3, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d \\
& *x]^2)/(a^2 - b^2))]*\text{Tan}[c + d*x]^2) + (b*\text{AppellF1}[1/2, 5/6, 1, 3/2, -\text{Tan} \\
& [c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2)))]/(-9*(a^2 - b^2)*\text{AppellF1} \\
& [1/2, 5/6, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] + (\\
& 6*a^2*\text{AppellF1}[3/2, 5/6, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a \\
& ^2 - b^2))] + 5*(a^2 - b^2)*\text{AppellF1}[3/2, 11/6, 1, 5/2, -\text{Tan}[c + d*x]^2, -(\\
& (a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]*\text{Tan}[c + d*x]^2))/((\text{Sec}[c + d*x]^2)^{(5/ \\
& 6)}*(-b^2 + a^2*\text{Sec}[c + d*x]^2)) + (9*(a^2 - b^2)*\text{Tan}[c + d*x]*((a*\text{AppellF1} \\
& [1/2, 1/3, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2)))*\text{Sqr} \\
& t[\text{Sec}[c + d*x]^2]*\text{Tan}[c + d*x])/ (9*(a^2 - b^2)*\text{AppellF1}[1/2, 1/3, 1, 3/2, - \\
& \text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*\text{AppellF1}[3/ \\
& 2, 1/3, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] + (a^ \\
& 2 - b^2)*\text{AppellF1}[3/2, 4/3, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2) \\
& / (a^2 - b^2))]*\text{Tan}[c + d*x]^2) + (a*\text{Sqrt}[\text{Sec}[c + d*x]^2]*((-2*a^2*\text{AppellF1} \\
& [3/2, 1/3, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2)))*\text{Se} \\
& c[c + d*x]^2*\text{Tan}[c + d*x])/(3*(a^2 - b^2)) - (2*\text{AppellF1}[3/2, 4/3, 1, 5/2, \\
& -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]*\text{Sec}[c + d*x]^2*\text{Tan}[c \\
& + d*x])/9))/ (9*(a^2 - b^2)*\text{AppellF1}[1/2, 1/3, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a \\
& ^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*\text{AppellF1}[3/2, 1/3, 2, 5/2, -\text{Tan} \\
& [c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*\text{AppellF1}[3/ \\
& 2, 4/3, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]*\text{Tan} \\
& [c + d*x]^2) + (b*((-2*a^2*\text{AppellF1}[3/2, 5/6, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^ \\
& 2*\text{Tan}[c + d*x]^2)/(a^2 - b^2)))*\text{Sec}[c + d*x]^2*\text{Tan}[c + d*x])/(3*(a^2 - b^2) \\
&) - (5*\text{AppellF1}[3/2, 11/6, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/ \\
& (a^2 - b^2))]*\text{Sec}[c + d*x]^2*\text{Tan}[c + d*x])/9))/(-9*(a^2 - b^2)*\text{AppellF1}[1/2
\end{aligned}$$

$$\begin{aligned}
& , 5/6, 1, 3/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] + (6a^2 \operatorname{AppellF1}[3/2, 5/6, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] + 5(a^2 - b^2) \operatorname{AppellF1}[3/2, 11/6, 1, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))]) \tan[c + dx]^2 - (a \operatorname{AppellF1}[1/2, 1/3, 1, 3/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sqrt{\sec[c + dx]^2} * (-4(3a^2 \operatorname{AppellF1}[3/2, 1/3, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] + (a^2 - b^2) \operatorname{AppellF1}[3/2, 4/3, 1, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))]) \sec[c + dx]^2 \tan[c + dx] + 9(a^2 - b^2) * (-2a^2 \operatorname{AppellF1}[3/2, 1/3, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / (3(a^2 - b^2)) - (2 \operatorname{AppellF1}[3/2, 4/3, 1, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / 9) - 2 \tan[c + dx]^2 * (3a^2 * (-12a^2 \operatorname{AppellF1}[5/2, 1/3, 3, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / (5(a^2 - b^2)) - (2 \operatorname{AppellF1}[5/2, 4/3, 2, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / 5) + (a^2 - b^2) * (-6a^2 \operatorname{AppellF1}[5/2, 4/3, 2, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / (5(a^2 - b^2)) - (8 \operatorname{AppellF1}[5/2, 7/3, 1, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / 5)) / (9(a^2 - b^2) \operatorname{AppellF1}[1/2, 1/3, 1, 3/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] - 2(3a^2 \operatorname{AppellF1}[3/2, 1/3, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] + (a^2 - b^2) \operatorname{AppellF1}[3/2, 4/3, 1, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))]) \tan[c + dx]^2 - (b \operatorname{AppellF1}[1/2, 5/6, 1, 3/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] * (2(6a^2 \operatorname{AppellF1}[3/2, 5/6, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] + 5(a^2 - b^2) \operatorname{AppellF1}[3/2, 11/6, 1, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))]) \sec[c + dx]^2 \tan[c + dx] - 9(a^2 - b^2) * (-2a^2 \operatorname{AppellF1}[3/2, 5/6, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / (3(a^2 - b^2)) - (5 \operatorname{AppellF1}[3/2, 11/6, 1, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / 9) + \tan[c + dx]^2 * (6a^2 * (-12a^2 \operatorname{AppellF1}[5/2, 5/6, 3, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / (5(a^2 - b^2)) - \operatorname{AppellF1}[5/2, 11/6, 2, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) + 5(a^2 - b^2) * (-6a^2 \operatorname{AppellF1}[5/2, 11/6, 2, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / (5(a^2 - b^2)) - (11 \operatorname{AppellF1}[5/2, 17/6, 1, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] \sec[c + dx]^2 \tan[c + dx]) / 5)) / (-9(a^2 - b^2) \operatorname{AppellF1}[1/2, 5/6, 1, 3/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] + (6a^2 \operatorname{AppellF1}[3/2, 5/6, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] + 5(a^2 - b^2) \operatorname{AppellF1}[3/2, 11/6, 1, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))]) \tan[c + dx]^2) / ((\sec[c + dx]^2)^{(5/6)} * (-b^2 + a^2 \sec[c + dx]^2)))
\end{aligned}$$

Maple [F]

$$\int \frac{\cos^{\frac{2}{3}}(dx + c)}{a + \cos(dx + c)b} dx$$

[In] int(cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b),x)

[Out] int(cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{a + b \cos(c + dx)} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{a + b \cos(c + dx)} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(2/3)/(a+b*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{2}{3}}}{b \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(2/3)/(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{2}{3}}}{b \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(2/3)/(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(c + dx)^{2/3}}{a + b \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x)), x)

$$3.677 \quad \int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)} dx$$

Optimal result	6288
Rubi [A] (verified)	6288
Mathematica [B] (warning: unable to verify)	6290
Maple [F]	6293
Fricas [F(-1)]	6293
Sympy [F(-1)]	6293
Maxima [F]	6293
Giac [F]	6294
Mupad [F(-1)]	6294

Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)} dx$$

$$= -\frac{b \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \sqrt[3]{\cos(c+dx)} \sin(c+dx)}{(a^2-b^2) d \sqrt[6]{\cos^2(c+dx)}} + \frac{a \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \sqrt[3]{\cos^2(c+dx)} \sin(c+dx)}{(a^2-b^2) d \cos^{\frac{2}{3}}(c+dx)}$$

```
[Out] -b*AppellF1(1/2,-1/6,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d*x+c)^(1/3)*sin(d*x+c)/(a^2-b^2)/d/(cos(d*x+c)^2)^(1/6)+a*AppellF1(1/2,1/3,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(2/3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used

= {2902, 3268, 440}

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)} dx$$

$$= \frac{a \sin(c+dx) \sqrt[3]{\cos^2(c+dx)} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2) \cos^{\frac{2}{3}}(c+dx)}$$

$$- \frac{b \sin(c+dx) \sqrt[3]{\cos(c+dx)} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2) \sqrt[6]{\cos^2(c+dx)}}$$

[In] Int[Cos[c + d*x]^(1/3)/(a + b*Cos[c + d*x]),x]

[Out] -((b*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^(1/6))) + (a*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(2/3)))

Rule 440

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 2902

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]

Rule 3268

Int[((d_)*sin[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])/(f*(Sin[e + f*x]^2)^(FracPart[(m - 1)/2]))], Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]

Rubi steps

$$\text{integral} = a \int \frac{\sqrt[3]{\cos(c+dx)}}{a^2 - b^2 \cos^2(c+dx)} dx - b \int \frac{\cos^{\frac{4}{3}}(c+dx)}{a^2 - b^2 \cos^2(c+dx)} dx$$

$$\begin{aligned}
&= -\frac{\left(b\sqrt[3]{\cos(c+dx)}\right) \text{Subst}\left(\int \frac{\sqrt[6]{1-x^2}}{a^2-b^2+b^2x^2} dx, x, \sin(c+dx)\right)}{d\sqrt[6]{\cos^2(c+dx)}} \\
&+ \frac{\left(a\sqrt[3]{\cos^2(c+dx)}\right) \text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x^2(a^2-b^2+b^2x^2)}} dx, x, \sin(c+dx)\right)}{d\cos^{\frac{2}{3}}(c+dx)} \\
&= -\frac{b \text{AppellF1}\left(\frac{1}{2}, -\frac{1}{6}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \sqrt[3]{\cos(c+dx)} \sin(c+dx)}{(a^2-b^2)d\sqrt[6]{\cos^2(c+dx)}} \\
&+ \frac{a \text{AppellF1}\left(\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \sqrt[3]{\cos^2(c+dx)} \sin(c+dx)}{(a^2-b^2)d\cos^{\frac{2}{3}}(c+dx)}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4613 vs. 2(176) = 352.

Time = 34.33 (sec) , antiderivative size = 4613, normalized size of antiderivative = 26.21

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)} dx = \text{Result too large to show}$$

[In] Integrate[Cos[c + d*x]^(1/3)/(a + b*Cos[c + d*x]), x]

[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2 + (b*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)/(d*Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(2/3)*(-b^2 + a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/3)*((a*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2 + (b*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])

$$\begin{aligned}
& 2, 2/3, 1, 3/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))] + 2*(\\
& 3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a \\
& ^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -\tan[c + dx]^2, -((\\
& a^2 \tan[c + dx]^2)/(a^2 - b^2))]*\tan[c + dx]^2) - (a*AppellF1[1/2, 1/6, \\
& 1, 3/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2))]*\sqrt{\sec[c + \\
& dx]^2}*(2*(-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c \\
& + dx]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -\tan[c + \\
& dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2)))*\sec[c + dx]^2*\tan[c + dx] \\
& + 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 1/6, 2, 5/2, -\tan[c + dx]^2, -((a^2 \\
& * \tan[c + dx]^2)/(a^2 - b^2)))*\sec[c + dx]^2*\tan[c + dx])/(3*(a^2 - b^2)) \\
& - (AppellF1[3/2, 7/6, 1, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 \\
& - b^2)))*\sec[c + dx]^2*\tan[c + dx])/9) + \tan[c + dx]^2*(-6*a^2*((-12*a^ \\
& 2*AppellF1[5/2, 1/6, 3, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - \\
& b^2)))*\sec[c + dx]^2*\tan[c + dx])/(5*(a^2 - b^2)) - (AppellF1[5/2, 7/6, \\
& 2, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2)))*\sec[c + dx]^ \\
& 2*\tan[c + dx])/5) + (-a^2 + b^2)*((-6*a^2*AppellF1[5/2, 7/6, 2, 7/2, -\tan[\\
& c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2)))*\sec[c + dx]^2*\tan[c + dx \\
&])/(5*(a^2 - b^2)) - (7*AppellF1[5/2, 13/6, 1, 7/2, -\tan[c + dx]^2, -((a^2 \\
& * \tan[c + dx]^2)/(a^2 - b^2)))*\sec[c + dx]^2*\tan[c + dx])/5))))/(9*(a^2 - \\
& b^2)*AppellF1[1/2, 1/6, 1, 3/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a \\
& ^2 - b^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan \\
& [c + dx]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -\tan[\\
& c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2)))*\tan[c + dx]^2 - (b*Ap \\
& pellF1[1/2, 2/3, 1, 3/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2 \\
&)))*(4*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx] \\
& ^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -\tan[c + dx] \\
&]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2)))*\sec[c + dx]^2*\tan[c + dx] - 9* \\
& (a^2 - b^2)*((-2*a^2*AppellF1[3/2, 2/3, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan \\
& [c + dx]^2)/(a^2 - b^2)))*\sec[c + dx]^2*\tan[c + dx])/(3*(a^2 - b^2)) - (\\
& 4*AppellF1[3/2, 5/3, 1, 5/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - \\
& b^2)))*\sec[c + dx]^2*\tan[c + dx])/9) + 2*\tan[c + dx]^2*(3*a^2*((-12*a^2 \\
& *AppellF1[5/2, 2/3, 3, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - \\
& b^2)))*\sec[c + dx]^2*\tan[c + dx])/(5*(a^2 - b^2)) - (4*AppellF1[5/2, 5/3, \\
& 2, 7/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2)))*\sec[c + dx] \\
& ^2*\tan[c + dx])/5) + 2*(a^2 - b^2)*((-6*a^2*AppellF1[5/2, 5/3, 2, 7/2, -\tan \\
& [c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2)))*\sec[c + dx]^2*\tan[c + d \\
& *x])/(5*(a^2 - b^2)) - 2*AppellF1[5/2, 8/3, 1, 7/2, -\tan[c + dx]^2, -((a^2 \\
& * \tan[c + dx]^2)/(a^2 - b^2)))*\sec[c + dx]^2*\tan[c + dx])))/(-9*(a^2 - b \\
& ^2)*AppellF1[1/2, 2/3, 1, 3/2, -\tan[c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 \\
& - b^2))] + 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -\tan[c + dx]^2, -((a^2 \tan \\
& [c + dx]^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -\tan[\\
& c + dx]^2, -((a^2 \tan[c + dx]^2)/(a^2 - b^2)))*\tan[c + dx]^2))/((\sec \\
& [c + dx]^2)^(2/3)*(-b^2 + a^2*\sec[c + dx]^2)))
\end{aligned}$$

Maple [F]

$$\int \frac{\cos^{\frac{1}{3}}(dx + c)}{a + \cos(dx + c)b} dx$$

[In] `int(cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b),x)`

[Out] `int(cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b),x)`

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\cos(c + dx)}}{a + b \cos(c + dx)} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\cos(c + dx)}}{a + b \cos(c + dx)} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(1/3)/(a+b*cos(d*x+c)),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt[3]{\cos(c + dx)}}{a + b \cos(c + dx)} dx = \int \frac{\cos(dx + c)^{\frac{1}{3}}}{b \cos(dx + c) + a} dx$$

[In] `integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")`

[Out] `integrate(cos(d*x + c)^(1/3)/(b*cos(d*x + c) + a), x)`

Giac [F]

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)} dx = \int \frac{\cos(dx+c)^{\frac{1}{3}}}{b\cos(dx+c)+a} dx$$

[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(1/3)/(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)} dx = \int \frac{\cos(c+dx)^{1/3}}{a+b\cos(c+dx)} dx$$

[In] int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x)), x)

$$3.678 \quad \int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))} dx$$

Optimal result	6295
Rubi [A] (verified)	6295
Mathematica [B] (warning: unable to verify)	6297
Maple [F]	6299
Fricas [F(-1)]	6300
Sympy [F(-1)]	6300
Maxima [F]	6300
Giac [F]	6300
Mupad [F(-1)]	6301

Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))} dx$$

$$= -\frac{b \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \sqrt[6]{\cos^2(c+dx)} \sin(c+dx)}{(a^2-b^2) d \sqrt[3]{\cos(c+dx)}} + \frac{a \operatorname{AppellF1}\left(\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \cos^2(c+dx)^{2/3} \sin(c+dx)}{(a^2-b^2) d \cos^{4/3}(c+dx)}$$

[Out] -b*AppellF1(1/2,1/6,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/6)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/3)+a*AppellF1(1/2,2/3,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(2/3)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(4/3)

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2902, 3268, 440}

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))} dx$$

$$= \frac{a \sin(c+dx) \cos^2(c+dx)^{2/3} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d (a^2-b^2) \cos^{4/3}(c+dx)} - \frac{b \sin(c+dx) \sqrt[6]{\cos^2(c+dx)} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d (a^2-b^2) \sqrt[3]{\cos(c+dx)}}$$

[In] Int[1/(Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])),x]

[Out] -((b*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(1/3))) + (a*AppellF1[1/2, 2/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(2/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(4/3))

Rule 440

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 2902

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]

Rule 3268

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])/(f*(Sin[e + f*x]^2)^(FracPart[(m - 1)/2]))), Subst[Int[(1 - ff^2*x^2)^(m - 1)/2*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \frac{1}{\sqrt[3]{\cos(c + dx)} (a^2 - b^2 \cos^2(c + dx))} dx - b \int \frac{\cos^{\frac{2}{3}}(c + dx)}{a^2 - b^2 \cos^2(c + dx)} dx \\
 &= -\frac{\left(b \sqrt[6]{\cos^2(c + dx)}\right) \text{Subst}\left(\int \frac{1}{\sqrt[6]{1 - x^2(a^2 - b^2 + b^2 x^2)}} dx, x, \sin(c + dx)\right)}{d \sqrt[3]{\cos(c + dx)}} \\
 &\quad + \frac{(a \cos^2(c + dx)^{2/3}) \text{Subst}\left(\int \frac{1}{(1 - x^2)^{2/3}(a^2 - b^2 + b^2 x^2)} dx, x, \sin(c + dx)\right)}{d \cos^{\frac{4}{3}}(c + dx)} \\
 &= -\frac{b \text{AppellF1}\left(\frac{1}{2}, \frac{1}{6}, 1, \frac{3}{2}, \sin^2(c + dx), -\frac{b^2 \sin^2(c + dx)}{a^2 - b^2}\right) \sqrt[6]{\cos^2(c + dx)} \sin(c + dx)}{(a^2 - b^2) d \sqrt[3]{\cos(c + dx)}} \\
 &\quad + \frac{a \text{AppellF1}\left(\frac{1}{2}, \frac{2}{3}, 1, \frac{3}{2}, \sin^2(c + dx), -\frac{b^2 \sin^2(c + dx)}{a^2 - b^2}\right) \cos^2(c + dx)^{2/3} \sin(c + dx)}{(a^2 - b^2) d \cos^{\frac{4}{3}}(c + dx)}
 \end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4605 vs. 2(176) = 352.

Time = 34.38 (sec) , antiderivative size = 4605, normalized size of antiderivative = 26.16

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)(a+b\cos(c+dx))}} dx = \text{Result too large to show}$$

[In] Integrate[1/(Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])),x]

[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2 + (b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)/(d*Cos[c + d*x]^(4/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(1/3)*(-b^2 + a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(2/3)*((a*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2 + (b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2))/(-b^2 + a^2*Sec[c + d*x]^2) - (18*a^2*(a^2 - b^2)*(Sec[c + d*x]^2)^(2/3)*Tan[c + d*x]^2*((a*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2 + (b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])

$$\begin{aligned}
&]*\tan[c + d*x]^2))/(-b^2 + a^2*\sec[c + d*x]^2)^2 - (6*(a^2 - b^2)*\tan[c + \\
& d*x]^2*((a*\text{AppellF1}[1/2, -1/6, 1, 3/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sqrt{\sec[c + d*x]^2})/(9*(a^2 - b^2)*\text{AppellF1}[1/2, -1/6, \\
& 1, 3/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*\text{AppellF1}[3/2, -1/6, 2, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*\text{AppellF1}[3/2, 5/6, 1, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\tan[c + d*x]^2 + (b*\text{AppellF1}[1/2, 1/3, 1, 3/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])/(9*(a^2 - b^2)*\text{AppellF1}[1/2, 1/3, 1, 3/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*\text{AppellF1}[3/2, 1/3, 2, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*\text{AppellF1}[3/2, 4/3, 1, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\tan[c + d*x]^2))/((\sec[c + d*x]^2)^(1/3)*(-b^2 + a^2*\sec[c + d*x]^2)) + (9*(a^2 - b^2)*\tan[c + d*x]*((a*\text{AppellF1}[1/2, -1/6, 1, 3/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sqrt{\sec[c + d*x]^2}*\tan[c + d*x])/(9*(a^2 - b^2)*\text{AppellF1}[1/2, -1/6, 1, 3/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*\text{AppellF1}[3/2, -1/6, 2, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*\text{AppellF1}[3/2, 5/6, 1, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\tan[c + d*x]^2 + (a*\sqrt{\sec[c + d*x]^2}*((-2*a^2*\text{AppellF1}[3/2, -1/6, 2, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sec[c + d*x]^2*\tan[c + d*x])/(3*(a^2 - b^2)) + (\text{AppellF1}[3/2, 5/6, 1, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sec[c + d*x]^2*\tan[c + d*x])/9))/(9*(a^2 - b^2)*\text{AppellF1}[1/2, -1/6, 1, 3/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*\text{AppellF1}[3/2, -1/6, 2, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*\text{AppellF1}[3/2, 5/6, 1, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\tan[c + d*x]^2 + (b*((-2*a^2*\text{AppellF1}[3/2, 1/3, 2, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sec[c + d*x]^2*\tan[c + d*x])/(3*(a^2 - b^2)) - (2*\text{AppellF1}[3/2, 4/3, 1, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sec[c + d*x]^2*\tan[c + d*x])/9))/(9*(a^2 - b^2)*\text{AppellF1}[1/2, 1/3, 1, 3/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*\text{AppellF1}[3/2, 1/3, 2, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*\text{AppellF1}[3/2, 4/3, 1, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\tan[c + d*x]^2 - (a*\text{AppellF1}[1/2, -1/6, 1, 3/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sqrt{\sec[c + d*x]^2}*(2*(-6*a^2*\text{AppellF1}[3/2, -1/6, 2, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*\text{AppellF1}[3/2, 5/6, 1, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sec[c + d*x]^2*\tan[c + d*x] + 9*(a^2 - b^2)*((-2*a^2*\text{AppellF1}[3/2, -1/6, 2, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sec[c + d*x]^2*\tan[c + d*x])/(3*(a^2 - b^2)) + (\text{AppellF1}[3/2, 5/6, 1, 5/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sec[c + d*x]^2*\tan[c + d*x])/9) + \tan[c + d*x]^2*(-6*a^2*((-12*a^2*\text{AppellF1}[5/2, -1/6, 3, 7/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sec[c + d*x]^2*\tan[c + d*x])/(5*(a^2 - b^2)) + (\text{AppellF1}[5/2, 5/6, 2, 7/2, -\tan[c + d*x]^2, -((a^2*\tan[c + d*x]^2)/(a^2 - b^2))])*\sec[c + d
\end{aligned}$$

```

*x]^2*Tan[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 5/6, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - AppellF1[5/2, 11/6, 1, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])))/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2 - (b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])* (4*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] - 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x])/9) + 2*Tan[c + d*x]^2*(3*a^2*(-12*a^2*AppellF1[5/2, 1/3, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (2*AppellF1[5/2, 4/3, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 4/3, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (8*AppellF1[5/2, 7/3, 1, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x])/5)))/(-9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2)/((Sec[c + d*x]^2)^(1/3)*(-b^2 + a^2*Sec[c + d*x]^2)))

```

Maple [F]

$$\int \frac{1}{\cos(dx+c)^{\frac{1}{3}}(a+\cos(dx+c)b)} dx$$

```
[In] int(1/cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b),x)
```

```
[Out] int(1/cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)**(1/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))} dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{1}{3}}} dx$$

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))} dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{1}{3}}} dx$$

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```


Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{\cos(c + dx)(a + b \cos(c + dx))}} dx = \int \frac{1}{\cos(c + dx)^{1/3} (a + b \cos(c + dx))} dx$$

```
[In] int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))), x)
```

```
[Out] int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))), x)
```

$$3.679 \quad \int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b \cos(c+dx))} dx$$

Optimal result	6302
Rubi [A] (verified)	6302
Mathematica [B] (warning: unable to verify)	6304
Maple [F]	6306
Fricas [F(-1)]	6307
Sympy [F(-1)]	6307
Maxima [F]	6307
Giac [F]	6307
Mupad [F(-1)]	6308

Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b \cos(c+dx))} dx$$

$$= -\frac{b \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \sqrt[3]{\cos^2(c+dx)} \sin(c+dx)}{(a^2-b^2) d \cos^{\frac{2}{3}}(c+dx)}$$

$$+ \frac{a \operatorname{AppellF1}\left(\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \cos^2(c+dx)^{5/6} \sin(c+dx)}{(a^2-b^2) d \cos^{\frac{5}{3}}(c+dx)}$$

[Out] -b*AppellF1(1/2,1/3,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(1/3)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(2/3)+a*AppellF1(1/2,5/6,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*(cos(d*x+c)^2)^(5/6)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(5/3)

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2902, 3268, 440}

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b \cos(c+dx))} dx$$

$$= \frac{a \sin(c+dx) \cos^2(c+dx)^{5/6} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2) \cos^{\frac{5}{3}}(c+dx)}$$

$$- \frac{b \sin(c+dx) \sqrt[3]{\cos^2(c+dx)} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2) \cos^{\frac{2}{3}}(c+dx)}$$

[In] Int[1/(Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])),x]

[Out] -((b*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(2/3))) + (a*AppellF1[1/2, 5/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(5/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^(5/3))

Rule 440

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 2902

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]

Rule 3268

Int[((d_)*sin[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \frac{1}{\cos^{\frac{2}{3}}(c + dx) (a^2 - b^2 \cos^2(c + dx))} dx - b \int \frac{\sqrt[3]{\cos(c + dx)}}{a^2 - b^2 \cos^2(c + dx)} dx \\
 &= -\frac{\left(b \sqrt[3]{\cos^2(c + dx)}\right) \text{Subst}\left(\int \frac{1}{\sqrt[3]{1 - x^2} (a^2 - b^2 + b^2 x^2)} dx, x, \sin(c + dx)\right)}{d \cos^{\frac{2}{3}}(c + dx)} \\
 &\quad + \frac{(a \cos^2(c + dx)^{5/6}) \text{Subst}\left(\int \frac{1}{(1 - x^2)^{5/6} (a^2 - b^2 + b^2 x^2)} dx, x, \sin(c + dx)\right)}{d \cos^{\frac{5}{3}}(c + dx)} \\
 &= -\frac{b \text{AppellF1}\left(\frac{1}{2}, \frac{1}{3}, 1, \frac{3}{2}, \sin^2(c + dx), -\frac{b^2 \sin^2(c + dx)}{a^2 - b^2}\right) \sqrt[3]{\cos^2(c + dx)} \sin(c + dx)}{(a^2 - b^2) d \cos^{\frac{2}{3}}(c + dx)} \\
 &\quad + \frac{a \text{AppellF1}\left(\frac{1}{2}, \frac{5}{6}, 1, \frac{3}{2}, \sin^2(c + dx), -\frac{b^2 \sin^2(c + dx)}{a^2 - b^2}\right) \cos^2(c + dx)^{5/6} \sin(c + dx)}{(a^2 - b^2) d \cos^{\frac{5}{3}}(c + dx)}
 \end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4608 vs. $2(176) = 352$.

Time = 34.39 (sec) , antiderivative size = 4608, normalized size of antiderivative = 26.18

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))} dx = \text{Result too large to show}$$

[In] Integrate[1/(Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])),x]

[Out] $(9*(a^2 - b^2)*\text{Sin}[c + d*x]*((a*\text{AppellF1}[1/2, -1/3, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]*\text{Sqrt}[\text{Sec}[c + d*x]^2])/(9*(a^2 - b^2)*\text{AppellF1}[1/2, -1/3, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) - 2*(3*a^2*\text{AppellF1}[3/2, -1/3, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) + (-a^2 + b^2)*\text{AppellF1}[3/2, 2/3, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))])*\text{Tan}[c + d*x]^2 + (b*\text{AppellF1}[1/2, 1/6, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]))/(-9*(a^2 - b^2)*\text{AppellF1}[1/2, 1/6, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) + (6*a^2*\text{AppellF1}[3/2, 1/6, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*\text{AppellF1}[3/2, 7/6, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))])*\text{Tan}[c + d*x]^2)/(d*\text{Cos}[c + d*x]^(5/3)*(a + b*\text{Cos}[c + d*x])*(\text{Sec}[c + d*x]^2)^(1/6)*(-b^2 + a^2*\text{Sec}[c + d*x]^2)*((9*(a^2 - b^2)*(\text{Sec}[c + d*x]^2)^(5/6)*((a*\text{AppellF1}[1/2, -1/3, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))])*\text{Sqrt}[\text{Sec}[c + d*x]^2])/(9*(a^2 - b^2)*\text{AppellF1}[1/2, -1/3, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) - 2*(3*a^2*\text{AppellF1}[3/2, -1/3, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) + (-a^2 + b^2)*\text{AppellF1}[3/2, 2/3, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))])*\text{Tan}[c + d*x]^2 + (b*\text{AppellF1}[1/2, 1/6, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]))/(-9*(a^2 - b^2)*\text{AppellF1}[1/2, 1/6, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) + (6*a^2*\text{AppellF1}[3/2, 1/6, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*\text{AppellF1}[3/2, 7/6, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))])*\text{Tan}[c + d*x]^2))/(-b^2 + a^2*\text{Sec}[c + d*x]^2) - (18*a^2*(a^2 - b^2)*(\text{Sec}[c + d*x]^2)^(5/6)*\text{Tan}[c + d*x]^2*((a*\text{AppellF1}[1/2, -1/3, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))])*\text{Sqrt}[\text{Sec}[c + d*x]^2])/(9*(a^2 - b^2)*\text{AppellF1}[1/2, -1/3, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) - 2*(3*a^2*\text{AppellF1}[3/2, -1/3, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) + (-a^2 + b^2)*\text{AppellF1}[3/2, 2/3, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))])*\text{Tan}[c + d*x]^2 + (b*\text{AppellF1}[1/2, 1/6, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]))/(-9*(a^2 - b^2)*\text{AppellF1}[1/2, 1/6, 1, 3/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) + (6*a^2*\text{AppellF1}[3/2, 1/6, 2, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))]) + (a^2 - b^2)*\text{AppellF1}[3/2, 7/6, 1, 5/2, -\text{Tan}[c + d*x]^2, -((a^2*\text{Tan}[c + d*x]^2)/(a^2 - b^2))])$

$$\begin{aligned}
&) * \tan[c + d*x]^2)) / (-b^2 + a^2 * \sec[c + d*x]^2)^2 - (3*(a^2 - b^2) * \tan[c + \\
& d*x]^2 * ((a * \text{AppellF1}[1/2, -1/3, 1, 3/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x] \\
&]^2) / (a^2 - b^2))] * \sqrt{\sec[c + d*x]^2}) / (9*(a^2 - b^2) * \text{AppellF1}[1/2, -1/3, \\
& 1, 3/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))] - 2*(3*a^2 * A \\
& ppellF1[3/2, -1/3, 2, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b \\
& ^2))) + (-a^2 + b^2) * \text{AppellF1}[3/2, 2/3, 1, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan \\
& [c + d*x]^2) / (a^2 - b^2))]) * \tan[c + d*x]^2) + (b * \text{AppellF1}[1/2, 1/6, 1, 3/2, \\
& -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))]) / (-9*(a^2 - b^2) * \text{App} \\
& ellF1[1/2, 1/6, 1, 3/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2) \\
&)) + (6*a^2 * \text{AppellF1}[3/2, 1/6, 2, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x] \\
& ^2) / (a^2 - b^2))] + (a^2 - b^2) * \text{AppellF1}[3/2, 7/6, 1, 5/2, -\tan[c + d*x]^2, \\
& -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))]) * \tan[c + d*x]^2)) / ((\sec[c + d*x]^2)^ \\
& (1/6) * (-b^2 + a^2 * \sec[c + d*x]^2)) + (9*(a^2 - b^2) * \tan[c + d*x] * ((a * \text{Appell} \\
& F1[1/2, -1/3, 1, 3/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))] \\
& * \sqrt{\sec[c + d*x]^2} * \tan[c + d*x]) / (9*(a^2 - b^2) * \text{AppellF1}[1/2, -1/3, 1, 3 \\
& /2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))] - 2*(3*a^2 * \text{Appell} \\
& F1[3/2, -1/3, 2, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))] \\
& + (-a^2 + b^2) * \text{AppellF1}[3/2, 2/3, 1, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + \\
& d*x]^2) / (a^2 - b^2))]) * \tan[c + d*x]^2) + (a * \sqrt{\sec[c + d*x]^2} * ((-2*a^2 * A \\
& ppellF1[3/2, -1/3, 2, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b \\
& ^2))) * \sec[c + d*x]^2 * \tan[c + d*x]) / (3*(a^2 - b^2)) + (2 * \text{AppellF1}[3/2, 2/3, \\
& 1, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))] * \sec[c + d*x]^ \\
& 2 * \tan[c + d*x]) / 9) / (9*(a^2 - b^2) * \text{AppellF1}[1/2, -1/3, 1, 3/2, -\tan[c + d*x] \\
&]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))] - 2*(3*a^2 * \text{AppellF1}[3/2, -1/3, 2, \\
& 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))) + (-a^2 + b^2) * \\
& \text{AppellF1}[3/2, 2/3, 1, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b \\
& ^2))]) * \tan[c + d*x]^2) + (b * ((-2*a^2 * \text{AppellF1}[3/2, 1/6, 2, 5/2, -\tan[c + d* \\
& x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))] * \sec[c + d*x]^2 * \tan[c + d*x]) / (3* \\
& (a^2 - b^2)) - (\text{AppellF1}[3/2, 7/6, 1, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + \\
& d*x]^2) / (a^2 - b^2))] * \sec[c + d*x]^2 * \tan[c + d*x]) / 9) / (-9*(a^2 - b^2) * \text{Appe} \\
& llF1[1/2, 1/6, 1, 3/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2)) \\
&] + (6*a^2 * \text{AppellF1}[3/2, 1/6, 2, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^ \\
& 2) / (a^2 - b^2))] + (a^2 - b^2) * \text{AppellF1}[3/2, 7/6, 1, 5/2, -\tan[c + d*x]^2, \\
& -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))]) * \tan[c + d*x]^2) - (a * \text{AppellF1}[1/2, -1 \\
& /3, 1, 3/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))] * \sqrt{\sec[\\
& c + d*x]^2} * (-4*(3*a^2 * \text{AppellF1}[3/2, -1/3, 2, 5/2, -\tan[c + d*x]^2, -((a^2 * \\
& \tan[c + d*x]^2) / (a^2 - b^2))] + (-a^2 + b^2) * \text{AppellF1}[3/2, 2/3, 1, 5/2, -\tan \\
& [c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))]) * \sec[c + d*x]^2 * \tan[c + \\
& d*x] + 9*(a^2 - b^2) * ((-2*a^2 * \text{AppellF1}[3/2, -1/3, 2, 5/2, -\tan[c + d*x]^2, \\
& -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))] * \sec[c + d*x]^2 * \tan[c + d*x]) / (3*(a^2 - \\
& b^2)) + (2 * \text{AppellF1}[3/2, 2/3, 1, 5/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x] \\
& ^2) / (a^2 - b^2))] * \sec[c + d*x]^2 * \tan[c + d*x]) / 9) - 2 * \tan[c + d*x]^2 * (3*a^2 \\
& * ((-12*a^2 * \text{AppellF1}[5/2, -1/3, 3, 7/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x] \\
& ^2) / (a^2 - b^2))] * \sec[c + d*x]^2 * \tan[c + d*x]) / (5*(a^2 - b^2)) + (2 * \text{AppellF} \\
& 1[5/2, 2/3, 2, 7/2, -\tan[c + d*x]^2, -((a^2 * \tan[c + d*x]^2) / (a^2 - b^2))] * S
\end{aligned}$$

```

ec[c + d*x]^2*Tan[c + d*x])/5) + (-a^2 + b^2)*((-6*a^2*AppellF1[5/2, 2/3, 2
, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2
*Tan[c + d*x])/(5*(a^2 - b^2)) - (4*AppellF1[5/2, 5/3, 1, 7/2, -Tan[c + d*x
]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))
)/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c
+ d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x
]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3,
1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x
]^2)^2 - (b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))]*(2*(6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c
+ d*x] - 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2
, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2
- b^2)) - (AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(6*a^2*(
-12*a^2*AppellF1[5/2, 1/6, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (AppellF1[5/2
, 7/6, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c
+ d*x]^2*Tan[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 7/6, 2, 7/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c
+ d*x])/(5*(a^2 - b^2)) - (7*AppellF1[5/2, 13/6, 1, 7/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(-9
*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2))/
((Sec[c + d*x]^2)^(1/6)*(-b^2 + a^2*Sec[c + d*x]^2))))

```

Maple [F]

$$\int \frac{1}{\cos(dx+c)^{\frac{2}{3}}(a+\cos(dx+c)b)} dx$$

[In] int(1/cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b),x)

[Out] int(1/cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))} dx = \text{Timed out}$$

```
[In] integrate(1/cos(d*x+c)**(2/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))} dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{2}{3}}} dx$$

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))} dx = \int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{2}{3}}} dx$$

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{2}{3}}(c + dx)(a + b \cos(c + dx))} dx = \int \frac{1}{\cos(c + dx)^{\frac{2}{3}}(a + b \cos(c + dx))} dx$$

```
[In] int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))), x)
```

```
[Out] int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))), x)
```


$$3.680 \quad \int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6309
Rubi [N/A]	6309
Mathematica [N/A]	6310
Maple [N/A] (verified)	6310
Fricas [N/A]	6310
Sympy [F(-1)]	6311
Maxima [N/A]	6311
Giac [N/A]	6311
Mupad [N/A]	6311

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \text{Int}\left(\frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}}, x\right)$$

[Out] Unintegrable(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2), x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

[In] Int[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] Defer[Int][Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]], x]

Rubi steps

$$\text{integral} = \int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Mathematica [N/A]

Not integrable

Time = 122.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{7}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos^{\frac{7}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] Integrate[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] Integrate[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]], x]

Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{7}{3}}(dx + c)}{\sqrt{a + \cos(dx + c)} b} dx$$

[In] int(cos(d*x+c)^(7/3)/(a+cos(d*x+c)*b)^(1/2), x)

[Out] int(cos(d*x+c)^(7/3)/(a+cos(d*x+c)*b)^(1/2), x)

Fricas [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{7}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{7}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(7/3)/(a+b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{7}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{7}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)

Giac [N/A]

Not integrable

Time = 22.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{7}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{7}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)

Mupad [N/A]

Not integrable

Time = 16.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{7}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{\frac{7}{3}}}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] int(cos(c + d*x)^(7/3)/(a + b*cos(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^(7/3)/(a + b*cos(c + d*x))^(1/2), x)

$$3.681 \quad \int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6312
Rubi [N/A]	6312
Mathematica [N/A]	6313
Maple [N/A] (verified)	6313
Fricas [N/A]	6313
Sympy [F(-1)]	6314
Maxima [N/A]	6314
Giac [N/A]	6314
Mupad [N/A]	6314

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \text{Int}\left(\frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}}, x\right)$$

[Out] Unintegrable(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2), x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

[In] Int[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] Defer[Int][Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]], x]

Rubi steps

$$\text{integral} = \int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Mathematica [N/A]

Not integrable

Time = 55.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{5}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos^{\frac{5}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] Integrate[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] Integrate[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]], x]

Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{5}{3}}(dx + c)}{\sqrt{a + \cos(dx + c)} b} dx$$

[In] int(cos(d*x+c)^(5/3)/(a+cos(d*x+c)*b)^(1/2), x)

[Out] int(cos(d*x+c)^(5/3)/(a+cos(d*x+c)*b)^(1/2), x)

Fricas [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{5}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/3)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Timed out

Maxima [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{5}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)

Giac [N/A]

Not integrable

Time = 23.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{5}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)

Mupad [N/A]

Not integrable

Time = 15.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{\frac{5}{3}}}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] int(cos(c + d*x)^(5/3)/(a + b*cos(c + d*x))^(1/2), x)

[Out] int(cos(c + d*x)^(5/3)/(a + b*cos(c + d*x))^(1/2), x)

$$3.682 \quad \int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6315
Rubi [N/A]	6315
Mathematica [N/A]	6316
Maple [N/A] (verified)	6316
Fricas [N/A]	6316
Sympy [N/A]	6317
Maxima [N/A]	6317
Giac [N/A]	6317
Mupad [N/A]	6318

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \text{Int}\left(\frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}}, x\right)$$

[Out] Unintegrable(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2), x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

[In] Int[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] Defer[Int][Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]], x]

Rubi steps

$$\text{integral} = \int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Mathematica [N/A]

Not integrable

Time = 33.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{4}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos^{\frac{4}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] Integrate[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] Integrate[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]], x]

Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{4}{3}}(dx + c)}{\sqrt{a + \cos(dx + c)b}} dx$$

[In] int(cos(d*x+c)^(4/3)/(a+cos(d*x+c)*b)^(1/2), x)

[Out] int(cos(d*x+c)^(4/3)/(a+cos(d*x+c)*b)^(1/2), x)

Fricas [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{4}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{4}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)

Sympy [N/A]

Not integrable

Time = 102.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cos^{\frac{4}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos^{\frac{4}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] integrate(cos(d*x+c)**(4/3)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Integral(cos(c + d*x)**(4/3)/sqrt(a + b*cos(c + d*x)), x)

Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{4}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{4}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)

Giac [N/A]

Not integrable

Time = 20.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{4}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{4}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)

Mupad [N/A]

Not integrable

Time = 15.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{4}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{4/3}}{\sqrt{a + b \cos(c + dx)}} dx$$

```
[In] int(cos(c + d*x)^(4/3)/(a + b*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(4/3)/(a + b*cos(c + d*x))^(1/2), x)
```

$$3.683 \quad \int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6319
Rubi [N/A]	6319
Mathematica [N/A]	6320
Maple [N/A] (verified)	6320
Fricas [N/A]	6320
Sympy [N/A]	6321
Maxima [N/A]	6321
Giac [N/A]	6321
Mupad [N/A]	6322

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \text{Int}\left(\frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}}, x\right)$$

[Out] Unintegrable(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2), x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx = \int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

[In] Int[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] Defer[Int][Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]], x]

Rubi steps

$$\text{integral} = \int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Mathematica [N/A]

Not integrable

Time = 24.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos^{\frac{2}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] Integrate[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] Integrate[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]], x]

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{2}{3}}(dx + c)}{\sqrt{a + \cos(dx + c)b}} dx$$

[In] int(cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b)^(1/2), x)

[Out] int(cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b)^(1/2), x)

Fricas [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{2}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)

Sympy [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos^{\frac{2}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] integrate(cos(d*x+c)**(2/3)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Integral(cos(c + d*x)**(2/3)/sqrt(a + b*cos(c + d*x)), x)

Maxima [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{2}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)

Giac [N/A]

Not integrable

Time = 18.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(dx + c)^{\frac{2}{3}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)

Mupad [N/A]

Not integrable

Time = 15.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{2}{3}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^{2/3}}{\sqrt{a + b \cos(c + dx)}} dx$$

```
[In] int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(2/3)/(a + b*cos(c + d*x))^(1/2), x)
```

$$3.684 \quad \int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx$$

Optimal result	6323
Rubi [N/A]	6323
Mathematica [N/A]	6324
Maple [N/A] (verified)	6324
Fricas [N/A]	6324
Sympy [N/A]	6325
Maxima [N/A]	6325
Giac [N/A]	6325
Mupad [N/A]	6326

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \text{Int}\left(\frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}, x\right)$$

[Out] Unintegrable(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2), x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx$$

[In] Int[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] Defer[Int][Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]], x]

Rubi steps

$$\text{integral} = \int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx$$

Mathematica [N/A]

Not integrable

Time = 15.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx$$

[In] Integrate[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] Integrate[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]], x]

Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{1}{3}}(dx+c)}{\sqrt{a+\cos(dx+c)}b} dx$$

[In] int(cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b)^(1/2), x)

[Out] int(cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b)^(1/2), x)

Fricas [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\cos(dx+c)^{\frac{1}{3}}}{\sqrt{b\cos(dx+c)+a}} dx$$

[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx$$

[In] integrate(cos(d*x+c)**(1/3)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Integral(cos(c + d*x)**(1/3)/sqrt(a + b*cos(c + d*x)), x)

Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\cos(dx+c)^{\frac{1}{3}}}{\sqrt{b\cos(dx+c)+a}} dx$$

[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)

Giac [N/A]

Not integrable

Time = 18.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\cos(dx+c)^{\frac{1}{3}}}{\sqrt{b\cos(dx+c)+a}} dx$$

[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)

Mupad [N/A]

Not integrable

Time = 15.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\cos(c+dx)^{1/3}}{\sqrt{a+b\cos(c+dx)}} dx$$

```
[In] int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x))^(1/2), x)
```

```
[Out] int(cos(c + d*x)^(1/3)/(a + b*cos(c + d*x))^(1/2), x)
```

$$3.685 \quad \int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx$$

Optimal result	6327
Rubi [N/A]	6327
Mathematica [N/A]	6328
Maple [N/A] (verified)	6328
Fricas [N/A]	6328
Sympy [N/A]	6329
Maxima [N/A]	6329
Giac [N/A]	6329
Mupad [N/A]	6330

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \text{Int}\left(\frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}, x\right)$$

[Out] Unintegrable(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2), x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx$$

[In] Int[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]), x]

[Out] Defer[Int][1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]), x]

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx$$

Mathematica [N/A]

Not integrable

Time = 13.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx$$

[In] Integrate[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]), x]

[Out] Integrate[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]), x]

Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos(dx+c)^{\frac{1}{3}}\sqrt{a+\cos(dx+c)}b} dx$$

[In] int(1/cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b)^(1/2), x)

[Out] int(1/cos(d*x+c)^(1/3)/(a+cos(d*x+c)*b)^(1/2), x)

Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{1}{3}}} dx$$

[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^2 + a*cos(d*x + c)), x)

Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt[3]{\cos(c+dx)}} dx$$

[In] integrate(1/cos(d*x+c)**(1/3)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(1/3)), x)

Maxima [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{1}{3}}} dx$$

[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)

Giac [N/A]

Not integrable

Time = 19.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{1}{3}}} dx$$

[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)

Mupad [N/A]

Not integrable

Time = 16.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\cos(c+dx)^{1/3}\sqrt{a+b\cos(c+dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(1/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

$$3.686 \quad \int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

Optimal result	6331
Rubi [N/A]	6331
Mathematica [N/A]	6332
Maple [N/A] (verified)	6332
Fricas [N/A]	6332
Sympy [N/A]	6333
Maxima [N/A]	6333
Giac [N/A]	6333
Mupad [N/A]	6334

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \text{Int}\left(\frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}, x\right)$$

[Out] Unintegrable(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2), x)

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

[In] Int[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]), x]

[Out] Defer[Int][1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]), x]

Rubi steps

$$\text{integral} = \int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

[In] Integrate[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]), x]

[Out] Integrate[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]), x]

Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos(dx+c)^{\frac{2}{3}}\sqrt{a+\cos(dx+c)}b} dx$$

[In] int(1/cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b)^(1/2), x)

[Out] int(1/cos(d*x+c)^(2/3)/(a+cos(d*x+c)*b)^(1/2), x)

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{2}{3}}} dx$$

[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)/(b*cos(d*x + c)^2 + a*cos(d*x + c)), x)

Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{a+b\cos(c+dx)}\cos^{\frac{2}{3}}(c+dx)} dx$$

[In] integrate(1/cos(d*x+c)**(2/3)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(2/3)), x)

Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{2}{3}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)

Giac [N/A]

Not integrable

Time = 19.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{2}{3}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)

Mupad [N/A]

Not integrable

Time = 14.70 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{2}{3}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^{\frac{2}{3}} \sqrt{a + b \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(2/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

$$3.687 \quad \int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

Optimal result	6335
Rubi [N/A]	6335
Mathematica [N/A]	6336
Maple [N/A] (verified)	6336
Fricas [N/A]	6336
Sympy [N/A]	6337
Maxima [N/A]	6337
Giac [N/A]	6337
Mupad [N/A]	6338

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \text{Int}\left(\frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}, x\right)$$

[Out] Unintegrable(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2), x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

[In] Int[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]), x]

[Out] Defer[Int][1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]), x]

Rubi steps

$$\text{integral} = \int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

Mathematica [N/A]

Not integrable

Time = 101.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

[In] Integrate[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]), x]

[Out] Integrate[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]), x]

Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos(dx+c)^{\frac{4}{3}}\sqrt{a+\cos(dx+c)}b} dx$$

[In] int(1/cos(d*x+c)^(4/3)/(a+cos(d*x+c)*b)^(1/2), x)

[Out] int(1/cos(d*x+c)^(4/3)/(a+cos(d*x+c)*b)^(1/2), x)

Fricas [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{4}{3}}} dx$$

[In] integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)

Sympy [N/A]

Not integrable

Time = 9.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{a+b\cos(c+dx)}\cos^{\frac{4}{3}}(c+dx)} dx$$

[In] integrate(1/cos(d*x+c)**(4/3)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(4/3)), x)

Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{4}{3}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(4/3)), x)

Giac [N/A]

Not integrable

Time = 20.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{4}{3}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(4/3)), x)

Mupad [N/A]

Not integrable

Time = 16.78 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{4}{3}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^{\frac{4}{3}} \sqrt{a + b \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(4/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(4/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

$$3.688 \quad \int \frac{1}{\cos^{\frac{5}{3}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6339
Rubi [N/A]	6339
Mathematica [N/A]	6340
Maple [N/A] (verified)	6340
Fricas [N/A]	6340
Sympy [N/A]	6341
Maxima [N/A]	6341
Giac [N/A]	6341
Mupad [N/A]	6342

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx = \text{Int} \left(\frac{1}{\cos^{\frac{5}{3}}(c+dx) \sqrt{a+b \cos(c+dx)}}, x \right)$$

[Out] Unintegrable(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2), x)

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx = \int \frac{1}{\cos^{\frac{5}{3}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx$$

[In] Int[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]), x]

[Out] Defer[Int][1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]), x]

Rubi steps

$$\text{integral} = \int \frac{1}{\cos^{\frac{5}{3}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx$$

Mathematica [N/A]

Not integrable

Time = 83.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

[In] Integrate[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]), x]

[Out] Integrate[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]), x]

Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos(dx+c)^{\frac{5}{3}}\sqrt{a+\cos(dx+c)}b} dx$$

[In] int(1/cos(d*x+c)^(5/3)/(a+cos(d*x+c)*b)^(1/2), x)

[Out] int(1/cos(d*x+c)^(5/3)/(a+cos(d*x+c)*b)^(1/2), x)

Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{3}}} dx$$

[In] integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)/(b*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)

Sympy [N/A]

Not integrable

Time = 23.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{a+b\cos(c+dx)}\cos^{\frac{5}{3}}(c+dx)} dx$$

[In] integrate(1/cos(d*x+c)**(5/3)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(5/3)), x)

Maxima [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{5}{3}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/3)), x)

Giac [N/A]

Not integrable

Time = 19.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos^{\frac{5}{3}}(dx+c)} dx$$

[In] integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/3)), x)

Mupad [N/A]

Not integrable

Time = 14.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{5}{3}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx = \int \frac{1}{\cos(c + dx)^{\frac{5}{3}} \sqrt{a + b \cos(c + dx)}} dx$$

```
[In] int(1/(cos(c + d*x)^(5/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

```
[Out] int(1/(cos(c + d*x)^(5/3)*(a + b*cos(c + d*x))^(1/2)), x)
```

$$3.689 \quad \int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

Optimal result	6343
Rubi [N/A]	6343
Mathematica [N/A]	6344
Maple [N/A] (verified)	6344
Fricas [N/A]	6344
Sympy [F(-1)]	6345
Maxima [N/A]	6345
Giac [N/A]	6345
Mupad [N/A]	6345

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \text{Int}\left(\frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}, x\right)$$

[Out] Unintegrable(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2), x)

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

[In] Int[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]), x]

[Out] Defer[Int][1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]), x]

Rubi steps

$$\text{integral} = \int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

Mathematica [N/A]

Not integrable

Time = 103.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx$$

[In] Integrate[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]), x]

[Out] Integrate[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]), x]

Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\cos(dx+c)^{\frac{7}{3}}\sqrt{a+\cos(dx+c)}b} dx$$

[In] int(1/cos(d*x+c)^(7/3)/(a+cos(d*x+c)*b)^(1/2), x)

[Out] int(1/cos(d*x+c)^(7/3)/(a+cos(d*x+c)*b)^(1/2), x)

Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{3}}} dx$$

[In] integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \text{Timed out}$$

[In] integrate(1/cos(d*x+c)**(7/3)/(a+b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{3}}} dx$$

[In] integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/3)), x)

Giac [N/A]

Not integrable

Time = 19.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{3}}} dx$$

[In] integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/3)), x)

Mupad [N/A]

Not integrable

Time = 16.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx = \int \frac{1}{\cos(c+dx)^{7/3}\sqrt{a+b\cos(c+dx)}} dx$$

[In] int(1/(cos(c + d*x)^(7/3)*(a + b*cos(c + d*x))^(1/2)),x)

[Out] int(1/(cos(c + d*x)^(7/3)*(a + b*cos(c + d*x))^(1/2)), x)

3.690 $\int (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$

Optimal result	6346
Rubi [A] (verified)	6346
Mathematica [A] (verified)	6349
Maple [B] (verified)	6349
Fricas [C] (verification not implemented)	6350
Sympy [F(-1)]	6350
Maxima [F]	6350
Giac [F]	6351
Mupad [F(-1)]	6351

Optimal result

Integrand size = 21, antiderivative size = 151

$$\int (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

$$= -\frac{6A\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)\sqrt{\sec(c + dx)}}{5d}$$

$$+ \frac{2B\sqrt{\cos(c + dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)\sqrt{\sec(c + dx)}}{3d} + \frac{6A\sqrt{\sec(c + dx)}\sin(c + dx)}{5d}$$

$$+ \frac{2B\sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{3d} + \frac{2A\sec^{\frac{5}{2}}(c + dx)\sin(c + dx)}{5d}$$

[Out] $\frac{2}{3}B*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2/5*A*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)/d+6/5$
 $*A*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-6/5*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2$
 $*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+$
 $c)^{(1/2)}/d+2/3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}$
 $(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00,
 number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {3317, 3872, 3853, 3856, 2720, 2719}

$$\int (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{2A \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} + \frac{6A \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

$$- \frac{6A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2B \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d}$$

$$+ \frac{2B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

[In] Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2), x]

[Out] (-6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (6*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sec^{\frac{5}{2}}(c + dx)(B + A \sec(c + dx)) dx \\
&= A \int \sec^{\frac{7}{2}}(c + dx) dx + B \int \sec^{\frac{5}{2}}(c + dx) dx \\
&= \frac{2B \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2A \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad + \frac{1}{5}(3A) \int \sec^{\frac{3}{2}}(c + dx) dx + \frac{1}{3}B \int \sqrt{\sec(c + dx)} dx \\
&= \frac{6A \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{2B \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2A \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad - \frac{1}{5}(3A) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{3} \left(B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{6A \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\
&\quad + \frac{2B \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2A \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} \\
&\quad - \frac{1}{5} \left(3A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{6A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
&\quad + \frac{2B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{6A \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\
&\quad + \frac{2B \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2A \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{\sec^{\frac{5}{2}}(c + dx) \left(-36A \cos^{\frac{5}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 20B \cos^{\frac{5}{2}}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 21A \sin\right)}{30d}$$

[In] Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]

[Out] (Sec[c + d*x]^(5/2)*(-36*A*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 20*B*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*A*Sin[c + d*x] + 10*B*Sin[2*(c + d*x)] + 9*A*Sin[3*(c + d*x)])/(30*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(179) = 358.

Time = 22.51 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.32

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{2B \left(-\frac{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}{6\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2}\right)^2} + \frac{\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}} \right)}$
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 12\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}} E\left(\frac{1}{2}(c + dx) \mid 2\right) + \dots$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)

,2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{-5i \sqrt{2} B \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 9i \sqrt{2} A \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 9i \sqrt{2} A \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2(9A \cos(dx + c)^2 + 5B \cos(dx + c) + 3A) \sin(dx + c) / \sqrt{\cos(dx + c)}}{(d \cos(dx + c))^2}$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] 1/15*(-5*I*sqrt(2)*B*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)

Sympy [F(-1)]

Timed out.

$$\int (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx = \int (B \cos(dx + c) + A) \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)

Giac [F]

$$\int (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx = \int (B \cos(dx + c) + A) \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int (A + B \cos(c + dx)) \sec^{\frac{7}{2}}(c + dx) dx = \int (A + B \cos(c + dx)) \left(\frac{1}{\cos(c + dx)} \right)^{7/2} dx$$

[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2),x)

[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(7/2), x)

3.691 $\int (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$

Optimal result	6352
Rubi [A] (verified)	6352
Mathematica [A] (verified)	6354
Maple [B] (verified)	6355
Fricas [C] (verification not implemented)	6355
Sympy [F(-1)]	6356
Maxima [F]	6356
Giac [F]	6356
Mupad [F(-1)]	6356

Optimal result

Integrand size = 21, antiderivative size = 123

$$\begin{aligned} & \int (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx \\ &= -\frac{2B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{2A \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\ & \quad + \frac{2B \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2A \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \end{aligned}$$

[Out] $2/3*A*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)/d+2*B*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3317, 3872, 3853, 3856, 2719, 2720}

$$\begin{aligned} & \int (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx \\ &= \frac{2A \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d} + \frac{2A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} \\ & \quad + \frac{2B \sin(c + dx) \sqrt{\sec(c + dx)}}{d} - \frac{2B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} \end{aligned}$$

[In] Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2), x]

[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\text{integral} = \int \sec^{\frac{3}{2}}(c + dx)(B + A \sec(c + dx)) dx$$

$$\begin{aligned}
&= A \int \sec^{\frac{5}{2}}(c + dx) dx + B \int \sec^{\frac{3}{2}}(c + dx) dx \\
&= \frac{2B \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2A \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad + \frac{1}{3} A \int \sqrt{\sec(c + dx)} dx - B \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2B \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2A \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad + \frac{1}{3} \left(A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&\quad - \left(B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{2B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2A \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{2B \sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2A \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\begin{aligned}
&\int (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx \\
&= \frac{\sec^{\frac{3}{2}}(c + dx) \left(-6B \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 2A \cos^{\frac{3}{2}}(c + dx) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 2(A + 3B \cos(c + dx)) \sin(c + dx) \right)}{3d}
\end{aligned}$$

[In] Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2), x]

[Out] (Sec[c + d*x]^(3/2)*(-6*B*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*A*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(159) = 318$.

Time = 20.42 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.23

method	result
default	$2\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \left(2A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} (\sin^2(\frac{dx}{2} + \frac{c}{2})) - 12 \right.$
parts	$- \frac{2A\left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sin^2(\frac{dx}{2} + \frac{c}{2})) - 2(\sin^2(\frac{dx}{2} + \frac{c}{2})) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \right)}{3\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})} (2(\cos^2(\frac{dx}{2} + \frac{c}{2})))}$

[In] `int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{2}{3} * (-(-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / (4 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 4 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 1) / \sin(1/2 * d * x + 1/2 * c) ^ 3 * (2 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 12 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 6 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 2 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) + 6 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 3 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2))) * (-2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.36

$$\int (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{-i \sqrt{2} A \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3 * I * \sqrt{2} * B * \cos(dx + c) * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I * \sin(dx + c))) + 3 * I * \sqrt{2} * B * \cos(dx + c) * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I * \sin(dx + c)))}{3}$$

[In] `integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{3} * (-I * \sqrt{2} * A * \cos(d * x + c) * \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) + I * \sin(d * x + c)) + I * \sqrt{2} * A * \cos(d * x + c) * \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) - I * \sin(d * x + c)) - 3 * I * \sqrt{2} * B * \cos(d * x + c) * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) + I * \sin(d * x + c))) + 3 * I * \sqrt{2} * B * \cos(d * x + c) * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) - I * \sin(d * x + c)))$$

) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c) + A)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx = \int (B \cos(dx + c) + A) \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)

Giac [F]

$$\int (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx = \int (B \cos(dx + c) + A) \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int (A + B \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx) dx = \int (A + B \cos(c + dx)) \left(\frac{1}{\cos(c + dx)} \right)^{\frac{5}{2}} dx$$

[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)

[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)

3.692 $\int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	6357
Rubi [A] (verified)	6357
Mathematica [A] (verified)	6359
Maple [A] (verified)	6359
Fricas [C] (verification not implemented)	6360
Sympy [F]	6361
Maxima [F]	6361
Giac [F]	6361
Mupad [F(-1)]	6361

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

$$= -\frac{2A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2A \sqrt{\sec(c + dx)} \sin(c + dx)}{d}$$

[Out] $2A \sin(dx+c) \sec(dx+c)^{1/2} / d - 2A (\cos(1/2 dx + 1/2 c)^2)^{1/2} / \cos(1/2 dx + 1/2 c) * \operatorname{EllipticE}(\sin(1/2 dx + 1/2 c), 2^{1/2}) * \cos(dx+c)^{1/2} * \sec(dx+c)^{1/2} / d + 2B (\cos(1/2 dx + 1/2 c)^2)^{1/2} / \cos(1/2 dx + 1/2 c) * \operatorname{EllipticF}(\sin(1/2 dx + 1/2 c), 2^{1/2}) * \cos(dx+c)^{1/2} * \sec(dx+c)^{1/2} / d$

Rubi [A] (verified)

Time = 0.11 (sec), antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3317, 3872, 3856, 2720, 3853, 2719}

$$\int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{2A \sin(c + dx) \sqrt{\sec(c + dx)}}{d} - \frac{2A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

[In] $\operatorname{Int}[(A + B \cos[c + dx]) \sec[c + dx]^{3/2}, x]$

[Out] $(-2A\sqrt{\cos[c + dx]} \operatorname{EllipticE}[(c + dx)/2, 2] \sqrt{\sec[c + dx]})/d + (2B\sqrt{\cos[c + dx]} \operatorname{EllipticF}[(c + dx)/2, 2] \sqrt{\sec[c + dx]})/d + (2A\sqrt{\sec[c + dx]} \sin[c + dx])/d$

Rule 2719

$\operatorname{Int}[\sqrt{\sin[(c_.) + (d_.)x]}, x_Symbol] \rightarrow \operatorname{Simp}[(2/d) \operatorname{EllipticE}[(1/2)(c - \pi/2 + dx), 2], x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2720

$\operatorname{Int}[1/\sqrt{\sin[(c_.) + (d_.)x]}, x_Symbol] \rightarrow \operatorname{Simp}[(2/d) \operatorname{EllipticF}[(1/2)(c - \pi/2 + dx), 2], x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3317

$\operatorname{Int}[(\csc[(e_.) + (f_.)x] (d_.)^m ((a_.) + (b_.) \sin[(e_.) + (f_.)x])^{n_1})^{p_1}, x_Symbol] \rightarrow \operatorname{Dist}[d^{np}, \operatorname{Int}[(d \csc[e + fx])^{m - np} (b + a \csc[e + fx])^p, x], x] /; \operatorname{FreeQ}\{a, b, d, e, f, m, n, p\}, x] \&\amp; \operatorname{IntegerQ}[m] \&\amp; \operatorname{IntegersQ}[n, p]$

Rule 3853

$\operatorname{Int}[(\csc[(c_.) + (d_.)x] (b_.)^n), x_Symbol] \rightarrow \operatorname{Simp}[(-b) \cos[c + dx] ((b \csc[c + dx])^{n-1} / (d(n-1))), x] + \operatorname{Dist}[b^2 ((n-2)/(n-1)), \operatorname{Int}[(b \csc[c + dx])^{n-2}, x], x] /; \operatorname{FreeQ}\{b, c, d\}, x] \&\amp; \operatorname{GtQ}[n, 1] \&\amp; \operatorname{IntegerQ}[2n]$

Rule 3856

$\operatorname{Int}[(\csc[(c_.) + (d_.)x] (b_.)^n), x_Symbol] \rightarrow \operatorname{Dist}[(b \csc[c + dx])^n \sin[c + dx]^n, \operatorname{Int}[1/\sin[c + dx]^n, x], x] /; \operatorname{FreeQ}\{b, c, d\}, x] \&\amp; \operatorname{EqQ}[n^2, 1/4]$

Rule 3872

$\operatorname{Int}[(\csc[(e_.) + (f_.)x] (d_.)^n (\csc[(e_.) + (f_.)x] (b_.) + (a_))), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[(d \csc[e + fx])^n, x], x] + \operatorname{Dist}[b/d, \operatorname{Int}[(d \csc[e + fx])^{n+1}, x], x] /; \operatorname{FreeQ}\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \sqrt{\sec(c + dx)} (B + A \sec(c + dx)) dx \\ &= A \int \sec^{\frac{3}{2}}(c + dx) dx + B \int \sqrt{\sec(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - A \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\
&\quad + \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d} \\
&\quad - \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\begin{aligned}
&\int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx \\
&= \frac{2\sqrt{\sec(c+dx)}\left(-A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right) + B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + A\sin(c+dx)\right)}{d}
\end{aligned}$$

[In] Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2), x]

[Out] (2*Sqrt[Sec[c + d*x]]*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/d

Maple [A] (verified)

Time = 5.72 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

method	result
default	$\frac{4A \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - 2B \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}}{\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}} d$
parts	$-\frac{2A \left(-2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}} d$

[In] `int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $2*(2*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

$$\int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{-i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - i \sqrt{2} A \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + i \sqrt{2} A \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2*A*\sin(dx + c)/\sqrt{\cos(dx + c)}}{d}$$

[In] `integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")`

[Out] $(-I*\sqrt{2}*B*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + I*\sqrt{2}*B*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - I*\sqrt{2}*A*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) + I*\sqrt{2}*A*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) + 2*A*\sin(d*x + c)/\sqrt{\cos(d*x + c)})/d$

Sympy [F]

$$\int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx = \int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**(3/2), x)

Maxima [F]

$$\int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx = \int (B \cos(dx + c) + A) \sec(dx + c)^{\frac{3}{2}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)

Giac [F]

$$\int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx = \int (B \cos(dx + c) + A) \sec(dx + c)^{\frac{3}{2}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int (A + B \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx) dx = \int (A + B \cos(c + dx)) \left(\frac{1}{\cos(c + dx)} \right)^{\frac{3}{2}} dx$$

[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)

[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)

3.693 $\int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx$

Optimal result	6362
Rubi [A] (verified)	6362
Mathematica [A] (verified)	6364
Maple [A] (verified)	6364
Fricas [C] (verification not implemented)	6365
Sympy [F]	6365
Maxima [F]	6365
Giac [F]	6366
Mupad [F(-1)]	6366

Optimal result

Integrand size = 21, antiderivative size = 75

$$\begin{aligned} & \int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx \\ &= \frac{2B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\ &+ \frac{2A \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \end{aligned}$$

[Out] $2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.10 (sec), antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3317, 3872, 3856, 2719, 2720}

$$\begin{aligned} & \int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx \\ &= \frac{2A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d} \\ &+ \frac{2B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} \end{aligned}$$

[In] $\operatorname{Int}[(A + B*\operatorname{Cos}[c + d*x])*Sqrt[\operatorname{Sec}[c + d*x]], x]$

[Out] $(2*B*\sqrt{\cos[c + d*x]}*EllipticE[(c + d*x)/2, 2]*\sqrt{\sec[c + d*x]})/d + (2*A*\sqrt{\cos[c + d*x]}*EllipticF[(c + d*x)/2, 2]*\sqrt{\sec[c + d*x]})/d$

Rule 2719

$\text{Int}[\sqrt{\sin[(c_.) + (d_.)*(x_)]}], x_Symbol] \rightarrow \text{Simp}[(2/d)*EllipticE[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\sqrt{\sin[(c_.) + (d_.)*(x_)]}], x_Symbol] \rightarrow \text{Simp}[(2/d)*EllipticF[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3317

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)^{(m_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_.)})^{(p_.)}], x_Symbol] \rightarrow \text{Dist}[d^{(n*p)}, \text{Int}[(d*\text{Csc}[e + f*x])^{(m - n*p)}*(b + a*\text{Csc}[e + f*x])^p, x], x] /; \text{FreeQ}[\{a, b, d, e, f, m, n, p\}, x] \&\& !\text{IntegerQ}[m] \&\& \text{IntegersQ}[n, p]$

Rule 3856

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_)]*(b_.)^{(n_.)}], x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^n*\sin[c + d*x]^n, \text{Int}[1/\sin[c + d*x]^n, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{EqQ}[n^2, 1/4]$

Rule 3872

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)]*(d_.)^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)]), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{B + A \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx \\
 &= A \int \sqrt{\sec(c + dx)} dx + B \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &= \left(A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad + \left(B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{2B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
 &\quad + \frac{2A \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

$$= \frac{2\sqrt{\cos(c + dx)} \left(BE\left(\frac{1}{2}(c + dx) \mid 2\right) + A \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \right) \sqrt{\sec(c + dx)}}{d}$$

[In] Integrate[(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]

[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*x]])/d

Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.03

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\left(AF\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - BE\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}d}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}d} + \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}d}$
risch	$-\frac{iB(e^{2i(dx+c)} + 1)\sqrt{2}\sqrt{\frac{e^{i(dx+c)}}{e^{2i(dx+c)} + 1}}e^{-i(dx+c)}}{d} - \frac{i\left(\frac{iA\sqrt{-i(e^{i(dx+c)} + i)}\sqrt{2}\sqrt{i(e^{i(dx+c)} - i)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i(e^{i(dx+c)} + i)}, \frac{\sqrt{2}}{2}\right)}{\sqrt{e^{3i(dx+c)} + e^{i(dx+c)}}}\right)}{d}$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] $-2*\left(\left(2*\cos\left(1/2*d*x+1/2*c\right)^2-1\right)*\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*\left(\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*\left(-2*\cos\left(1/2*d*x+1/2*c\right)^2+1\right)^{(1/2)}*\left(A*\operatorname{EllipticF}\left(\cos\left(1/2*d*x+1/2*c\right), 2^{(1/2)}\right)-B*\operatorname{EllipticE}\left(\cos\left(1/2*d*x+1/2*c\right), 2^{(1/2)}\right)\right)/\left(-2*\sin\left(1/2*d*x+1/2*c\right)^4+\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}/\sin\left(1/2*d*x+1/2*c\right)/\left(2*\cos\left(1/2*d*x+1/2*c\right)^2-1\right)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

$$\int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

$$= \frac{-i \sqrt{2} A \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + \sqrt{2} B \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - \sqrt{2} B \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{d}$$

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F]

$$\int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx = \int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx$$

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx = \int (B \cos(dx + c) + A) \sqrt{\sec(dx + c)} dx$$

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx = \int (B \cos(dx + c) + A) \sqrt{\sec(dx + c)} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int (A + B \cos(c + dx)) \sqrt{\sec(c + dx)} dx = \int (A + B \cos(c + dx)) \sqrt{\frac{1}{\cos(c + dx)}} dx$$

[In] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)

[Out] int((A + B*cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)

$$3.694 \quad \int \frac{A+B \cos(c+dx)}{\sqrt{\sec(c+dx)}} dx$$

Optimal result	6367
Rubi [A] (verified)	6367
Mathematica [A] (verified)	6369
Maple [A] (verified)	6369
Fricas [C] (verification not implemented)	6370
Sympy [F]	6370
Maxima [F]	6371
Giac [F]	6371
Mupad [F(-1)]	6371

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{A+B \cos(c+dx)}{\sqrt{\sec(c+dx)}} dx = \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2B \sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

[Out] $2/3*B*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3317, 3872, 3854, 3856, 2720, 2719}

$$\int \frac{A+B \cos(c+dx)}{\sqrt{\sec(c+dx)}} dx = \frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d} + \frac{2B \sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d}$$

[In] $\operatorname{Int}[(A+B*\cos[c+d*x])/Sqrt[\sec[c+d*x]],x]$

[Out] $(2A\sqrt{\cos[c + dx]} \operatorname{EllipticE}[(c + dx)/2, 2] \sqrt{\sec[c + dx]})/d + (2B\sqrt{\cos[c + dx]} \operatorname{EllipticF}[(c + dx)/2, 2] \sqrt{\sec[c + dx]})/(3d) + (2B\sin[c + dx])/(3d\sqrt{\sec[c + dx]})$

Rule 2719

$\operatorname{Int}[\sqrt{\sin[(c_.) + (d_.)x]}, x_Symbol] \rightarrow \operatorname{Simp}[(2/d)\operatorname{EllipticE}[(1/2)(c - \pi/2 + dx), 2], x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 2720

$\operatorname{Int}[1/\sqrt{\sin[(c_.) + (d_.)x]}, x_Symbol] \rightarrow \operatorname{Simp}[(2/d)\operatorname{EllipticF}[(1/2)(c - \pi/2 + dx), 2], x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3317

$\operatorname{Int}[(\csc[(e_.) + (f_.)x] \cdot (d_.)^m) \cdot ((a_.) + (b_.)\sin[(e_.) + (f_.)x])^{n_1}]^{p_1}, x_Symbol] \rightarrow \operatorname{Dist}[d^{(n_1 p_1)}, \operatorname{Int}[(d \cdot \csc[e + fx])^{m - n_1 p_1} \cdot (b + a \cdot \csc[e + fx])^{n_1 p_1}, x], x] /; \operatorname{FreeQ}\{a, b, d, e, f, m, n, p\}, x] \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{IntegersQ}[n, p]$

Rule 3854

$\operatorname{Int}[(\csc[(c_.) + (d_.)x] \cdot (b_.)^n), x_Symbol] \rightarrow \operatorname{Simp}[\cos[c + dx] \cdot ((b \cdot \csc[c + dx])^{n+1} / (b \cdot d^n)), x] + \operatorname{Dist}[(n+1)/(b^{2n}), \operatorname{Int}[(b \cdot \csc[c + dx])^{n+2}, x], x] /; \operatorname{FreeQ}\{b, c, d\}, x] \&\& \operatorname{LtQ}[n, -1] \&\& \operatorname{IntegerQ}[2n]$

Rule 3856

$\operatorname{Int}[(\csc[(c_.) + (d_.)x] \cdot (b_.)^n), x_Symbol] \rightarrow \operatorname{Dist}[(b \cdot \csc[c + dx])^n \cdot \sin[c + dx]^n, \operatorname{Int}[1/\sin[c + dx]^n, x], x] /; \operatorname{FreeQ}\{b, c, d\}, x] \&\& \operatorname{EqQ}[n^2, 1/4]$

Rule 3872

$\operatorname{Int}[(\csc[(e_.) + (f_.)x] \cdot (d_.)^n) \cdot (\csc[(e_.) + (f_.)x] \cdot (b_.) + (a_)), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[(d \cdot \csc[e + fx])^n, x], x] + \operatorname{Dist}[b/d, \operatorname{Int}[(d \cdot \csc[e + fx])^{n+1}, x], x] /; \operatorname{FreeQ}\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{B + A \sec(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx \\ &= A \int \frac{1}{\sqrt{\sec(c + dx)}} dx + B \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2B \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{1}{3} B \int \sqrt{\sec(c + dx)} dx \\
&\quad + \left(A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{2A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} + \frac{2B \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{1}{3} \left(B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} + \frac{2B \sin(c + dx)}{3d \sqrt{\sec(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = \frac{\sqrt{\sec(c + dx)} \left(6A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + B \left(2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sin(2(c + dx)) \right) \right)}{3d}$$

[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]

[Out] (Sqrt[Sec[c + d*x]]*(6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + B*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d)

Maple [A] (verified)

Time = 7.76 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.27

method	result
default	$ \frac{2 \sqrt{\left(2 \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(-4B \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 3A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 3 \sqrt{-2 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{3d} $
parts	$ \frac{2A \sqrt{\left(2 \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) + 2B \sqrt{\left(2 \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{-2 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) d}}{3d} $

[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{2B\sqrt{\cos(dx + c)} \sin(dx + c) - i\sqrt{2}B\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i\sqrt{2}B\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{d}$$

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*B*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\sec(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(1/2),x)

[Out] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(1/2), x)

$$3.695 \quad \int \frac{A+B \cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	6372
Rubi [A] (verified)	6373
Mathematica [A] (verified)	6375
Maple [A] (verified)	6375
Fricas [C] (verification not implemented)	6376
Sympy [F]	6376
Maxima [F]	6376
Giac [F]	6377
Mupad [F(-1)]	6377

Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{A+B \cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx = \frac{6B \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} + \frac{2A \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} + \frac{2B \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{2A \sin(c+dx)}{3d \sqrt{\sec(c+dx)}}$$

```
[Out] 2/5*B*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*A*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5
*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*A*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d
```


Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3317, 3872, 3854, 3856, 2719, 2720}

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx = \frac{2A \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{2A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2B \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{6B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(A + B*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]

[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{B + A \sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &= A \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx + B \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2B \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2A \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{1}{3}A \int \sqrt{\sec(c + dx)} dx + \frac{1}{5}(3B) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{2B \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2A \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
 &\quad + \frac{1}{3} \left(A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad + \frac{1}{5} \left(3B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{6B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &\quad + \frac{2A \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
 &\quad + \frac{2B \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2A \sin(c + dx)}{3d \sqrt{\sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{\sqrt{\sec(c + dx)} \left(18B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 10A \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + (5A + 3B) \sin(2(c + dx)) \right)}{15d}$$

[In] Integrate[(A + B*Cos[c + d*x])/Sec[c + d*x]^(3/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(18*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 3*B*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)

Maple [A] (verified)

Time = 8.60 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.06

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(-24B \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (20A + 24B)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + (-10A + 6B)\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(4\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{\frac{1}{2} + \frac{\cos(dx+c)}{2}}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}d}$

[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out]
$$-\frac{2}{15} \left((2 \cos(1/2 d x + 1/2 c))^2 - 1 \right) \sin(1/2 d x + 1/2 c)^2 \sqrt{2 \cos(1/2 d x + 1/2 c) \sin(1/2 d x + 1/2 c)^6 + (20 A + 24 B) \sin(1/2 d x + 1/2 c)^4 \cos(1/2 d x + 1/2 c) + (-10 A - 6 B) \sin(1/2 d x + 1/2 c)^2 \cos(1/2 d x + 1/2 c) + 5 A \sin(1/2 d x + 1/2 c)^2} \sqrt{2 \cos(1/2 d x + 1/2 c) \sin(1/2 d x + 1/2 c)^2 - 1} \operatorname{EllipticF}\left(\cos(1/2 d x + 1/2 c), 2\right) - 9 B \sin(1/2 d x + 1/2 c)^2 \sqrt{2 \cos(1/2 d x + 1/2 c) \sin(1/2 d x + 1/2 c)^2 - 1} \operatorname{EllipticE}\left(\cos(1/2 d x + 1/2 c), 2\right) \right) / \left(-2 \sin(1/2 d x + 1/2 c)^4 + \sin(1/2 d x + 1/2 c)^2 \sqrt{2 \cos(1/2 d x + 1/2 c) \sin(1/2 d x + 1/2 c)^2 - 1} \right) / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.14

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{-5i \sqrt{2} A \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} A \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 9 \sqrt{2} B \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 9 \sqrt{2} B \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2(3B \cos(dx + c)^2 + 5A \cos(dx + c)) \sin(dx + c) / \sqrt{\cos(dx + c)}}{d}$$

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] 1/15*(-5*I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*A*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c)^2 + 5*A*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{A + B \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)

[Out] Integral((A + B*cos(c + d*x))/sec(c + d*x)**(3/2), x)

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{B \cos(dx + c) + A}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{B \cos(dx + c) + A}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{A + B \cos(c + dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

[In] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(3/2),x)

[Out] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(3/2), x)

$$3.696 \quad \int \frac{A+B \cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	6378
Rubi [A] (verified)	6379
Mathematica [A] (verified)	6381
Maple [A] (verified)	6381
Fricas [C] (verification not implemented)	6382
Sympy [F]	6382
Maxima [F]	6382
Giac [F]	6383
Mupad [F(-1)]	6383

Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{A+B \cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} dx = \frac{6A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2B \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{2A \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{10B \sin(c+dx)}{21d \sqrt{\sec(c+dx)}}$$

```
[Out] 2/7*B*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*A*sin(d*x+c)/d/sec(d*x+c)^(3/2)+10/21*B*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00,
 number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used
 = {3317, 3872, 3854, 3856, 2720, 2719}

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx = \frac{2A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{6A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2B \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{10B \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{10B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d}$$

[In] Int[(A + B*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]

[Out] (6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (10*B*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{B + A \sec(c + dx)}{\sec^{\frac{7}{2}}(c + dx)} dx \\
 &= A \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx + B \int \frac{1}{\sec^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2B \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{1}{5}(3A) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{7}(5B) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2B \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{10B \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{1}{21}(5B) \int \sqrt{\sec(c + dx)} dx \\
 &\quad + \frac{1}{5} \left(3A \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{6A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} + \frac{2B \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
 &\quad + \frac{10B \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{1}{21} \left(5B \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &= \frac{6A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &\quad + \frac{10B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{21d} \\
 &\quad + \frac{2B \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2A \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{10B \sin(c + dx)}{21d \sqrt{\sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.66

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{\sqrt{\sec(c + dx)} \left(252A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 100B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + (65B \right.}{210d}$$

[In] Integrate[(A + B*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]

[Out] (Sqrt[Sec[c + d*x]]*(252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)

Maple [A] (verified)

Time = 9.77 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(240B \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-168A - 360B)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \dots\right)}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(-8 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}{5\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}$

[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out]
$$-\frac{2}{105} \cdot \left((2 \cos(1/2 \cdot d \cdot x + 1/2 \cdot c))^{-2} - 1 \right) \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^{-2} \cdot \left(240 \cdot B \cdot \cos(1/2 \cdot d \cdot x + 1/2 \cdot c) \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^8 + (-168 \cdot A - 360 \cdot B) \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^6 \cdot \cos(1/2 \cdot d \cdot x + 1/2 \cdot c) + (168 \cdot A + 280 \cdot B) \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 \cdot \cos(1/2 \cdot d \cdot x + 1/2 \cdot c) + (-42 \cdot A - 80 \cdot B) \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot \cos(1/2 \cdot d \cdot x + 1/2 \cdot c) - 63 \cdot A \cdot (\sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^2)^{(1/2)} \cdot (2 \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^{-2} - 1)^{(1/2)} \cdot \operatorname{EllipticE}(\cos(1/2 \cdot d \cdot x + 1/2 \cdot c), 2^{(1/2)}) + 25 \cdot B \cdot (\sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^{-2})^{(1/2)} \cdot (2 \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^{-2} - 1)^{(1/2)} \cdot \operatorname{EllipticF}(\cos(1/2 \cdot d \cdot x + 1/2 \cdot c), 2^{(1/2)}) \right) / \left(-2 \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 + \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \right)^{(1/2)} / \sin(1/2 \cdot d \cdot x + 1/2 \cdot c) / (2 \cdot \cos(1/2 \cdot d \cdot x + 1/2 \cdot c)^{-2} - 1)^{(1/2)} / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{-25i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} B \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 63i \sqrt{2} A \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 63i \sqrt{2} A \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2*(15*B*\cos(dx + c)^3 + 21*A*\cos(dx + c)^2 + 25*B*\cos(dx + c))*\sin(dx + c)/\sqrt{\cos(dx + c))}}{d}$$

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] 1/105*(-25*I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*I*sqrt(2)*B*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*B*cos(d*x + c)^3 + 21*A*cos(d*x + c)^2 + 25*B*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{A + B \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx$$

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(5/2),x)

[Out] Integral((A + B*cos(c + d*x))/sec(c + d*x)**(5/2), x)

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{B \cos(dx + c) + A}{\sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{B \cos(dx + c) + A}{\sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{A + B \cos(c + dx)}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}} dx$$

[In] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(5/2),x)

[Out] int((A + B*cos(c + d*x))/(1/cos(c + d*x))^(5/2), x)

3.697 $\int (a + b \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx) dx$

Optimal result	6384
Rubi [A] (verified)	6385
Mathematica [A] (verified)	6387
Maple [B] (verified)	6387
Fricas [C] (verification not implemented)	6388
Sympy [F(-1)]	6389
Maxima [F]	6389
Giac [F]	6389
Mupad [F(-1)]	6389

Optimal result

Integrand size = 23, antiderivative size = 200

$$\begin{aligned}
 & \int (a + b \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx) dx \\
 &= -\frac{12ab\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)\sqrt{\sec(c + dx)}}{5d} \\
 &+ \frac{2(5a^2 + 7b^2)\sqrt{\cos(c + dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)\sqrt{\sec(c + dx)}}{21d} \\
 &+ \frac{12ab\sqrt{\sec(c + dx)}\sin(c + dx)}{5d} + \frac{2(5a^2 + 7b^2)\sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{21d} \\
 &+ \frac{4ab\sec^{\frac{5}{2}}(c + dx)\sin(c + dx)}{5d} + \frac{2a^2\sec^{\frac{7}{2}}(c + dx)\sin(c + dx)}{7d}
 \end{aligned}$$

```
[Out] 2/21*(5*a^2+7*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/5*a*b*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*a^2*sec(d*x+c)^(7/2)*sin(d*x+c)/d+12/5*a*b*sin(d*x+c)*sec(d*x+c)^(1/2)/d-12/5*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(5*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3317, 3873, 3853, 3856, 2719, 4131, 2720}

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{2(5a^2 + 7b^2) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{21d}$$

$$+ \frac{2(5a^2 + 7b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d}$$

$$+ \frac{2a^2 \sin(c + dx) \sec^{\frac{7}{2}}(c + dx)}{7d} + \frac{4ab \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d}$$

$$+ \frac{12ab \sin(c + dx) \sqrt{\sec(c + dx)}}{5d} - \frac{12ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(9/2), x]

[Out] (-12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (12*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(5*a^2 + 7*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (4*a*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegersQ[m] && IntegersQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*(n - 2)/(n - 1),

Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])
^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]

Rule 3873

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]

Rule 4131

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.
+ (A_.)), x_Symbol] := Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \sec^{\frac{5}{2}}(c + dx)(b + a \sec(c + dx))^2 dx \\
 &= (2ab) \int \sec^{\frac{7}{2}}(c + dx) dx + \int \sec^{\frac{5}{2}}(c + dx) (b^2 + a^2 \sec^2(c + dx)) dx \\
 &= \frac{4ab \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2a^2 \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{7d} \\
 &\quad + \frac{1}{5}(6ab) \int \sec^{\frac{3}{2}}(c + dx) dx + \frac{1}{7}(5a^2 + 7b^2) \int \sec^{\frac{5}{2}}(c + dx) dx \\
 &= \frac{12ab \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{2(5a^2 + 7b^2) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{21d} \\
 &\quad + \frac{4ab \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{2a^2 \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{7d} \\
 &\quad - \frac{1}{5}(6ab) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{21}(5a^2 + 7b^2) \int \sqrt{\sec(c + dx)} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{12ab\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(5a^2+7b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\
&\quad + \frac{4ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} \\
&\quad - \frac{1}{5}\left(6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\sqrt{\cos(c+dx)}dx \\
&\quad + \frac{1}{21}\left((5a^2+7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx \\
&= -\frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{12ab\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2(5a^2+7b^2)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} \\
&\quad + \frac{4ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{\sec^{\frac{7}{2}}(c + dx) \left(-504ab \cos^{\frac{7}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 20(5a^2 + 7b^2) \cos^{\frac{7}{2}}(c + dx) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \right)}{210d}$$

[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(9/2), x]

[Out] (Sec[c + d*x]^(7/2)*(-504*a*b*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2] + 20*(5*a^2 + 7*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(55*a^2 + 35*b^2 + 273*a*b*Cos[c + d*x] + 5*(5*a^2 + 7*b^2)*Cos[2*(c + d*x)] + 6*3*a*b*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(224) = 448.

Time = 112.04 (sec) , antiderivative size = 689, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	689
parts	Expression too large to display	821

[In] int((a+cos(d*x+c)*b)^2*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out]
$$-(-(-2\cos(1/2dx+1/2c)^2+1)\sin(1/2dx+1/2c)^2)^{1/2}(2b^2(-1/6\cos(1/2dx+1/2c)*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}/(\cos(1/2dx+1/2c)^2-1/2)^2+1/3(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))+2a^2(-1/56\cos(1/2dx+1/2c)*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}/(\cos(1/2dx+1/2c)^2-1/2)^4-5/42*\cos(1/2dx+1/2c)*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}/(\cos(1/2dx+1/2c)^2-1/2)^2+5/21(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))+4/5ab/\sin(1/2dx+1/2c)^2/(8\sin(1/2dx+1/2c)^6-12\sin(1/2dx+1/2c)^4+6\sin(1/2dx+1/2c)^2-1)*(24\cos(1/2dx+1/2c)*\sin(1/2dx+1/2c)^6-12*(2\sin(1/2dx+1/2c)^2-1)^{1/2}*(\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticE}(\cos(1/2dx+1/2c),2^{1/2}))*\sin(1/2dx+1/2c)^4-24\sin(1/2dx+1/2c)^4*\cos(1/2dx+1/2c)+12*(2\sin(1/2dx+1/2c)^2-1)^{1/2}*(\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticE}(\cos(1/2dx+1/2c),2^{1/2}))*\sin(1/2dx+1/2c)^2+8\sin(1/2dx+1/2c)^2*\cos(1/2dx+1/2c)-3*(\sin(1/2dx+1/2c)^2)^{1/2}*(2\sin(1/2dx+1/2c)^2-1)^{1/2}*\text{EllipticE}(\cos(1/2dx+1/2c),2^{1/2}))*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2})/\sin(1/2dx+1/2c)/(2\cos(1/2dx+1/2c)^2-1)^{1/2}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.18

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{-126i \sqrt{2} ab \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)))}{1}$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out]
$$1/105*(-126*I*\text{sqrt}(2)*a*b*\cos(dx + c)^3*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c))) + 126*I*\text{sqrt}(2)*a*b*\cos(dx + c)^3*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c))) - 5*\text{sqrt}(2)*(5*I*a^2 + 7*I*b^2)*\cos(dx + c)^3*\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c)) - 5*\text{sqrt}(2)*(-5*I*a^2 - 7*I*b^2)*\cos(dx + c)^3*\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c)) + 2*(126*a*b*\cos(dx + c)^3 + 42*a*b*\cos(dx + c) + 5*(5*a^2 + 7*b^2)*\cos(dx + c)^2 + 15*a^2)*\sin(dx + c)/\text{sqrt}(\cos(dx + c)))/(d*\cos(dx + c)^3)$$

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{9}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{9}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{9}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{9/2} (a + b \cos(c + dx))^2 dx$$

[In] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^2, x)

3.698 $\int (a + b \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx$

Optimal result	6390
Rubi [A] (verified)	.6391
Mathematica [A] (verified)	6393
Maple [B] (verified)	6393
Fricas [C] (verification not implemented)	6394
Sympy [F(-1)]	6395
Maxima [F]	6395
Giac [F]	6395
Mupad [F(-1)]	6395

Optimal result

Integrand size = 23, antiderivative size = 175

$$\begin{aligned}
 & \int (a + b \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx \\
 &= -\frac{2(3a^2 + 5b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &+ \frac{4ab \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
 &+ \frac{2(3a^2 + 5b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} \\
 &+ \frac{4ab \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d}
 \end{aligned}$$

```
[Out] 4/3*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)/d
+2/5*(3*a^2+5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*(3*a^2+5*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00,
 number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used
 = {3317, 3873, 3853, 3856, 2720, 4131, 2719}

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{2(3a^2 + 5b^2) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

$$- \frac{2(3a^2 + 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

$$+ \frac{2a^2 \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{5d} + \frac{4ab \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d}$$

$$+ \frac{4ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2), x]

[Out] (-2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(3*d) + (2*(3*a^2 + 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*a*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &

& IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3873

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4131

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^m*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \sec^{\frac{3}{2}}(c + dx)(b + a \sec(c + dx))^2 dx \\
 &= (2ab) \int \sec^{\frac{5}{2}}(c + dx) dx + \int \sec^{\frac{3}{2}}(c + dx) (b^2 + a^2 \sec^2(c + dx)) dx \\
 &= \frac{4ab \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} + \frac{2a^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} \\
 &\quad + \frac{1}{3}(2ab) \int \sqrt{\sec(c + dx)} dx + \frac{1}{5}(3a^2 + 5b^2) \int \sec^{\frac{3}{2}}(c + dx) dx \\
 &= \frac{2(3a^2 + 5b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{4ab \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
 &\quad + \frac{2a^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{5d} + \frac{1}{5}(-3a^2 - 5b^2) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &\quad + \frac{1}{3} \left(2ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{4ab\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} \\
&\quad + \frac{2(3a^2+5b^2)\sqrt{\sec(c+dx)} \sin(c+dx)}{5d} \\
&\quad + \frac{4ab \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d} + \frac{2a^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{1}{5} \left((-3a^2 - 5b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{2(3a^2+5b^2)\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{4ab\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3d} \\
&\quad + \frac{2(3a^2+5b^2)\sqrt{\sec(c+dx)} \sin(c+dx)}{5d} \\
&\quad + \frac{4ab \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3d} + \frac{2a^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.72

$$\begin{aligned}
&\int (a + b \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx \\
&= \frac{\sec^{\frac{5}{2}}(c + dx) \left(-12(3a^2 + 5b^2) \cos^{\frac{5}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 40ab \cos^{\frac{5}{2}}(c + dx) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \right)}{30d}
\end{aligned}$$

[In] Integrate[(a + b*cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]

[Out] (Sec[c + d*x]^(5/2)*(-12*(3*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 40*a*b*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a^2 + b^2) + 20*a*b*Cos[c + d*x] + 3*(3*a^2 + 5*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(30*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(203) = 406.

Time = 111.08 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.62

method	result
default	$\frac{\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)(\sin^2(\frac{dx}{2} + \frac{c}{2}))}}{2a^2 \left(24 \cos(\frac{dx}{2} + \frac{c}{2}) (\sin^6(\frac{dx}{2} + \frac{c}{2})) - 12 \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2}))} - 1 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} E(\cos(\frac{dx}{2} + \frac{c}{2})) \right)}$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*b)^2*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2/5*a^2/(8*\sin(\\ & 1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1)/\sin(1/2* \\ & d*x+1/2*c)^2*(24*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6-12*(2*\sin(1/2*d*x+ \\ & 1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c) \\ & ,2^{(1/2)})*\sin(1/2*d*x+1/2*c)^4-24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+1 \\ & 2*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(c \\ & os(1/2*d*x+1/2*c),2^{(1/2)})*\sin(1/2*d*x+1/2*c)^2+8*\sin(1/2*d*x+1/2*c)^2*\cos(\\ & 1/2*d*x+1/2*c)-3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1 \\ & /2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2 \\ & *d*x+1/2*c)^2)^{(1/2)}+2*b^2/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x+1/2*c)^2-1)* \\ & (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^ \\ & 2*\cos(1/2*d*x+1/2*c)-(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1 \\ &)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))+4*a*b*(-1/6*\cos(1/2*d*x+1/2* \\ & c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c) \\ & ^2-1/2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2 \\ &)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d* \\ & x+1/2*c),2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d \end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.27

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{-10i \sqrt{2} ab \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \cos(dx + c)^2}{\dots}$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/15*(-10*I*\sqrt{2}*a*b*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + \\ & c) + I*\sin(d*x + c)) + 10*I*\sqrt{2}*a*b*\cos(d*x + c)^2*\text{weierstrassPInverse} \\ & (-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - 3*\sqrt{2}*(3*I*a^2 + 5*I*b^2)*\cos(\\ & d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + \end{aligned}$$

$I \sin(dx + c)) - 3\sqrt{2}(-3Ia^2 - 5Ib^2)\cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I\sin(dx + c))) + 2(10ab\cos(dx + c) + 3(3a^2 + 5b^2)\cos(dx + c)^2 + 3a^2)\sin(dx + c)/\sqrt{\cos(dx + c)}/(d\cos(dx + c)^2)$

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{7}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{7/2} (a + b \cos(c + dx))^2 dx$$

[In] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^2, x)

3.699 $\int (a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx$

Optimal result	6396
Rubi [A] (verified)	6396
Mathematica [A] (verified)	6399
Maple [B] (verified)	6399
Fricas [C] (verification not implemented)	6400
Sympy [F(-1)]	6400
Maxima [F]	6400
Giac [F]	6401
Mupad [F(-1)]	6401

Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned} & \int (a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx \\ &= -\frac{4ab\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)\sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{2(a^2 + 3b^2)\sqrt{\cos(c + dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)\sqrt{\sec(c + dx)}}{3d} \\ & \quad + \frac{4ab\sqrt{\sec(c + dx)}\sin(c + dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c + dx)\sin(c + dx)}{3d} \end{aligned}$$

[Out] $2/3*a^2*\sec(d*x+c)^(3/2)*\sin(d*x+c)/d+4*a*b*\sin(d*x+c)*\sec(d*x+c)^(1/2)/d-4*a*b*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d+2/3*(a^2+3*b^2)*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2))*\cos(d*x+c)^(1/2)*\sec(d*x+c)^(1/2)/d$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3873, 3853, 3856, 2719, 4131, 2720}

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{2(a^2 + 3b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

$$+ \frac{2a^2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d} + \frac{4ab \sin(c + dx) \sqrt{\sec(c + dx)}}{d}$$

$$- \frac{4ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2), x]

[Out] (-4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_) + (f_)*(x_)]*(d_))^(m_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.)
+ (A_.)), x_Symbol] := Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{\sec(c + dx)}(b + a \sec(c + dx))^2 dx \\
&= (2ab) \int \sec^{\frac{3}{2}}(c + dx) dx + \int \sqrt{\sec(c + dx)}(b^2 + a^2 \sec^2(c + dx)) dx \\
&= \frac{4ab\sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad - (2ab) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{3}(a^2 + 3b^2) \int \sqrt{\sec(c + dx)} dx \\
&= \frac{4ab\sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
&\quad - \left(2ab\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx \\
&\quad + \frac{1}{3} \left((a^2 + 3b^2) \sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= -\frac{4ab\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2(a^2 + 3b^2) \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{4ab\sqrt{\sec(c + dx)} \sin(c + dx)}{d} + \frac{2a^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{2 \sec^{\frac{3}{2}}(c + dx) \left(-6ab \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + (a^2 + 3b^2) \cos^{\frac{3}{2}}(c + dx) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + a \right)}{3d}$$

[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]

[Out] (2*Sec[c + d*x]^(3/2)*(-6*a*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + (a^2 + 3*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + a*(a + 6*b*Cos[c + d*x])*Sin[c + d*x))/(3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(171) = 342.

Time = 106.69 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.80

method	result
default	$\frac{2\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(24\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)ab - 2F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) - 1$
parts	$\frac{2a^2\left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right) - 1$

[In] int((a+cos(d*x+c)*b)^2*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b-2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^2-6*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^2-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2*a*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2-12*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b+a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)*

$$\frac{-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2}{(2\cos(1/2dx+1/2c)^2-1)^{1/2}}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.41

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{-6i\sqrt{2}ab \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 6i\sqrt{2}ab \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + \sqrt{2}(-Ia^2 - 3Ib^2)\cos(dx + c)\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + I\sin(dx + c)) + \sqrt{2}(Ia^2 + 3Ib^2)\cos(dx + c)\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - I\sin(dx + c)) + 2(6ab\cos(dx + c) + a^2)\sin(dx + c)/\sqrt{\cos(dx + c)}}{d\cos(dx + c)}$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] 1/3*(-6*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 6*I*sqrt(2)*a*b*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + sqrt(2)*(-I*a^2 - 3*I*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*a^2 + 3*I*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(6*a*b*cos(d*x + c) + a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{5/2} (a + b \cos(c + dx))^2 dx$$

[In] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2, x)

3.700 $\int (a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	6402
Rubi [A] (verified)	6402
Mathematica [A] (verified)	6404
Maple [A] (verified)	6405
Fricas [C] (verification not implemented)	6405
Sympy [F(-1)]	6406
Maxima [F]	6406
Giac [F]	6406
Mupad [F(-1)]	6406

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$$

$$= -\frac{2(a^2 - b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d}$$

$$+ \frac{4ab \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d}$$

$$+ \frac{2a^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{d}$$

[Out] $2*a^2*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2*(a^2-b^2)*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+4*a*b*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3873, 3856, 2720, 4131, 2719}

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$$

$$= -\frac{2(a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2a^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{d}$$

$$+ \frac{4ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2), x]

[Out] (-2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3873

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4131

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\text{integral} = \int \frac{(b + a \sec(c + dx))^2}{\sqrt{\sec(c + dx)}} dx$$

$$\begin{aligned}
&= (2ab) \int \sqrt{\sec(c+dx)} dx + \int \frac{b^2 + a^2 \sec^2(c+dx)}{\sqrt{\sec(c+dx)}} dx \\
&= \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{d} + (-a^2 + b^2) \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\
&\quad + \left(2ab \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{4ab \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
&\quad + \left((-a^2 + b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{2(a^2 - b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{4ab \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\begin{aligned}
&\int (a + b \cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx) dx \\
&= \frac{2\sqrt{\sec(c+dx)} \left(-\left((a^2 - b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \right) + a \left(2b \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \right) \right)}{d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2), x]

[Out] (2*Sqrt[Sec[c + d*x]]*(-((a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + a*(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*Sin[c + d*x]))) / d

Maple [A] (verified)

Time = 8.49 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.87

method	result
default	$\frac{4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a^2 - 4ab \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - 2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}{\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}$
parts	$\frac{2a^2 \left(-2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d}$

```
[In] int((a+cos(d*x+c)*b)^2*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2-2*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.35

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{-2i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{\sin(dx + c) \sqrt{2(\cos^2(dx + c) - 1)}}$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (-2*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
+ 2*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
+ 2*a^2*sin(d*x + c)/sqrt(cos(d*x + c)) + sqrt(2)*(-I*a^2 + I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
+ sqrt(2)*(I*a^2 - I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Giac [F]

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{\frac{3}{2}} (a + b \cos(c + dx))^2 dx$$

```
[In] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2, x)
```

3.701 $\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$

Optimal result	6407
Rubi [A] (verified)	6407
Mathematica [A] (verified)	6409
Maple [A] (verified)	6409
Fricas [C] (verification not implemented)	6410
Sympy [F]	6411
Maxima [F]	6411
Giac [F]	6411
Mupad [F(-1)]	6411

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$$

$$= \frac{4ab \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d}$$

$$+ \frac{2(3a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} + \frac{2b^2 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}}$$

[Out] $2/3*b^2*\sin(d*x+c)/d/\sec(d*x+c)^{(1/2)}+4*a*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d+2/3*(3*a^2+b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3317, 3873, 3856, 2719, 4130, 2720}

$$\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$$

$$= \frac{2(3a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

$$+ \frac{4ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2b^2 \sin(c + dx)}{3d \sqrt{\sec(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]

[Out] (4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3873

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4130

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rubi steps

$$\text{integral} = \int \frac{(b + a \sec(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx$$

$$\begin{aligned}
&= (2ab) \int \frac{1}{\sqrt{\sec(c+dx)}} dx + \int \frac{b^2 + a^2 \sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{2b^2 \sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}(-3a^2 - b^2) \int \sqrt{\sec(c+dx)} dx \\
&\quad + \left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx \\
&= \frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b^2 \sin(c+dx)}{3d\sqrt{\sec(c+dx)}} \\
&\quad - \frac{1}{3}\left((-3a^2 - b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{2(3a^2 + b^2)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} \\
&\quad + \frac{2b^2 \sin(c+dx)}{3d\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\begin{aligned}
&\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx \\
&= \frac{\sqrt{\sec(c + dx)} \left(12ab\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx)|2\right) + 2(3a^2 + b^2)\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx),2\right) + \right)}{3d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]

[Out] (Sqrt[Sec[c + d*x]]*(12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b^2*Sin[2*(c + d*x)]))/(3*d)

Maple [A] (verified)

Time = 8.32 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.53

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+3a^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}$
parts	$\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}d-\frac{2b^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$

[In] `int((a+cos(d*x+c)*b)^2*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-2/3*\left(\left(2*\cos\left(1/2*d*x+1/2*c\right)^2-1\right)*\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*\left(4*\cos\left(1/2*d*x+1/2*c\right)*\sin\left(1/2*d*x+1/2*c\right)^4*b^2-2*\cos\left(1/2*d*x+1/2*c\right)*\sin\left(1/2*d*x+1/2*c\right)^2*b^2+3*a^2*\left(\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*\left(2*\sin\left(1/2*d*x+1/2*c\right)^2-1\right)^{(1/2)}*\text{EllipticF}\left(\cos\left(1/2*d*x+1/2*c\right),2^{(1/2)}\right)+b^2*\left(\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*\left(2*\sin\left(1/2*d*x+1/2*c\right)^2-1\right)^{(1/2)}*\text{EllipticF}\left(\cos\left(1/2*d*x+1/2*c\right),2^{(1/2)}\right)-6*\left(\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*\left(2*\sin\left(1/2*d*x+1/2*c\right)^2-1\right)^{(1/2)}*\text{EllipticE}\left(\cos\left(1/2*d*x+1/2*c\right),2^{(1/2)}\right)*a*b\right)/\left(-2*\sin\left(1/2*d*x+1/2*c\right)^4+\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}/\sin\left(1/2*d*x+1/2*c\right)/\left(2*\cos\left(1/2*d*x+1/2*c\right)^2-1\right)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$$

$$= \frac{2b^2 \sqrt{\cos(dx + c)} \sin(dx + c) + 6i \sqrt{2} ab \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 6i \sqrt{2} a^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + \sqrt{2} * (-3I*a^2 - I*b^2) * \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c)) + \sqrt{2} * (3I*a^2 + I*b^2) * \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c))}{d}$$

[In] `integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]
$$1/3*(2*b^2*\sqrt{\cos(dx + c)}*\sin(dx + c) + 6*I*\sqrt{2}*a*b*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c))) - 6*I*\sqrt{2}*a^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c))) + \sqrt{2}*(-3*I*a^2 - I*b^2)*\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c)) + \sqrt{2}*(3*I*a^2 + I*b^2)*\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c)))/d$$

Sympy [F]

$$\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx = \int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx$$

[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(1/2),x)

[Out] Integral((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x)), x)

Maxima [F]

$$\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx = \int (b \cos(dx + c) + a)^2 \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)

Giac [F]

$$\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx = \int (b \cos(dx + c) + a)^2 \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} (a + b \cos(c + dx))^2 dx$$

[In] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2, x)

$$3.702 \quad \int \frac{(a+b \cos(c+dx))^2}{\sqrt{\sec(c+dx)}} dx$$

Optimal result	6412
Rubi [A] (verified)	6413
Mathematica [A] (verified)	6415
Maple [B] (verified)	6415
Fricas [C] (verification not implemented)	6416
Sympy [F]	6416
Maxima [F]	6416
Giac [F]	6417
Mupad [F(-1)]	6417

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(a+b \cos(c+dx))^2}{\sqrt{\sec(c+dx)}} dx = \frac{2(5a^2+3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4ab\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2b^2 \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{4ab \sin(c+dx)}{3d \sqrt{\sec(c+dx)}}$$

```
[Out] 2/5*b^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/3*a*b*sin(d*x+c)/d/sec(d*x+c)^(1/2)
+2/5*(5*a^2+3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a*b*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```


Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3317, 3873, 3854, 3856, 2720, 4130, 2719}

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx = \frac{2(5a^2 + 3b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{4ab \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{4ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} + \frac{2b^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]], x]

[Out] (2*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a*b*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3873

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4130

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(b + a \sec(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &= (2ab) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx + \int \frac{b^2 + a^2 \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2b^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{1}{3}(2ab) \int \sqrt{\sec(c + dx)} dx \\
 &\quad - \frac{1}{5}(-5a^2 - 3b^2) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{2b^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
 &\quad + \frac{1}{3} \left(2ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad - \frac{1}{5} \left((-5a^2 - 3b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{2(5a^2 + 3b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &\quad + \frac{4ab \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
 &\quad + \frac{2b^2 \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{3d \sqrt{\sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx = \frac{\sqrt{\sec(c + dx)} \left(6(5a^2 + 3b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 20ab \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \right)}{15d}$$

[In] Integrate[(a + b*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]

[Out] (Sqrt[Sec[c + d*x]]*(6*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*(10*a + 3*b*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(173) = 346.

Time = 10.21 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.53

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}} \left(-24\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b^2 + 40\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)ab + 24\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a^2\right) + 20ab\sqrt{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\operatorname{EllipticF}\left(\frac{1}{2}\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) + b\sin\left(2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15d}$
parts	$\frac{2a^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - 2b^2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}d}$

[In] int((a+cos(d*x+c)*b)^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^2+40*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b+24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^2-20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^2+10*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{-10i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 10i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0,$$

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 10*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-5*I*a^2 - 3*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(5*I*a^2 + 3*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*b^2*cos(d*x + c)^2 + 10*a*b*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx$$

```
[In] integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^2}{\sqrt{\sec(dx + c)}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^2}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^2}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^2}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(1/2), x)

$$3.703 \quad \int \frac{(a+b \cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	6418
Rubi [A] (verified)	6418
Mathematica [A] (verified)	6421
Maple [A] (verified)	6421
Fricas [C] (verification not implemented)	6422
Sympy [F]	6422
Maxima [F]	6423
Giac [F]	6423
Mupad [F(-1)]	6423

Optimal result

Integrand size = 23, antiderivative size = 175

$$\begin{aligned} & \int \frac{(a+b \cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)} dx \\ &= \frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\ & \quad + \frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\ & \quad + \frac{2b^2 \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} + \frac{4ab \sin(c+dx)}{5d \sec^{\frac{3}{2}}(c+dx)} + \frac{2(7a^2+5b^2) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} \end{aligned}$$

```
[Out] 2/7*b^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/5*a*b*sin(d*x+c)/d/sec(d*x+c)^(3/2)
+2/21*(7*a^2+5*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+12/5*a*b*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(7*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3873, 3854, 3856, 2719, 4130, 2720}

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{2(7a^2 + 5b^2) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{2(7a^2 + 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{4ab \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)}$$

$$+ \frac{12ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2b^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(3/2), x]

[Out] (12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*a*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(7*a^2 + 5*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3873

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^2, x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4130

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^m*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(b + a \sec(c + dx))^2}{\sec^{\frac{7}{2}}(c + dx)} dx \\
 &= (2ab) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx + \int \frac{b^2 + a^2 \sec^2(c + dx)}{\sec^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2b^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{1}{5}(6ab) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &\quad - \frac{1}{7}(-7a^2 - 5b^2) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2b^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(7a^2 + 5b^2) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
 &\quad - \frac{1}{21}(-7a^2 - 5b^2) \int \sqrt{\sec(c + dx)} dx \\
 &\quad + \frac{1}{5}(6ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{12ab \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &\quad + \frac{2b^2 \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(7a^2 + 5b^2) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
 &\quad - \frac{1}{21} \left((-7a^2 - 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(7a^2+5b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\begin{aligned}
&\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{\sqrt{\sec(c+dx)}\left(504ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 20(7a^2+5b^2)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 20(7a^2+5b^2)\sin(c+dx)\right)}{210d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(3/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(504*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (70*a^2 + 65*b^2 + 84*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(210*d)

Maple [A] (verified)

Time = 12.08 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.07

method	result
default	$- \frac{2\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+(-336ab-360b^2)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+20(7a^2+5b^2)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{210d}$
parts	$- \frac{2a^2\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{3\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$

[In] int((a+cos(d*x+c)*b)^2/sec(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^2+(-336*a*b-360*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*a^2+336*a*b+280*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*a^2-84*a*b-80*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+25*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin

$$\frac{(1/2*d*x+1/2*c)^2-1)^{1/2}*EllipticF(\cos(1/2*d*x+1/2*c),2^{1/2})-126*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{1/2}*EllipticE(\cos(1/2*d*x+1/2*c),2^{1/2})*a*b)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}}{\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{1/2}/d}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{126i \sqrt{2} ab \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 126i \sqrt{2} ab \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) - 5\sqrt{2}*(7I*a^2 + 5I*b^2)*\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c)) - 5\sqrt{2}*(-7I*a^2 - 5I*b^2)*\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c)) + 2*(15*b^2*\cos(dx + c)^3 + 42*a*b*\cos(dx + c)^2 + 5*(7*a^2 + 5*b^2)*\cos(dx + c))*\sin(dx + c)/\sqrt{\cos(dx + c)}}}{d}$$

[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] 1/105*(126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 126*I*sqrt(2)*a*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 5*sqrt(2)*(7*I*a^2 + 5*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-7*I*a^2 - 5*I*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(15*b^2*cos(d*x + c)^3 + 42*a*b*cos(d*x + c)^2 + 5*(7*a^2 + 5*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d

Sympy [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)

[Out] Integral((a + b*cos(c + d*x))**2/sec(c + d*x)**(3/2), x)

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^2}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^2}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^2}{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{3}{2}}} dx$$

[In] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2),x)

[Out] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(3/2), x)

$$3.704 \quad \int \frac{(a+b \cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	6424
Rubi [A] (verified)	6425
Mathematica [A] (verified)	6427
Maple [A] (verified)	6427
Fricas [C] (verification not implemented)	6428
Sympy [F(-1)]	6428
Maxima [F]	6429
Giac [F]	6429
Mupad [F(-1)]	6429

Optimal result

Integrand size = 23, antiderivative size = 200

$$\int \frac{(a+b \cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)} dx = \frac{2(9a^2+7b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{15d} \\ + \frac{20ab \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} \\ + \frac{2b^2 \sin(c+dx)}{9d \sec^{\frac{7}{2}}(c+dx)} + \frac{4ab \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} \\ + \frac{2(9a^2+7b^2) \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)} + \frac{20ab \sin(c+dx)}{21d \sqrt{\sec(c+dx)}}$$

```
[Out] 2/9*b^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)+4/7*a*b*sin(d*x+c)/d/sec(d*x+c)^(5/2)
+2/45*(9*a^2+7*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+20/21*a*b*sin(d*x+c)/d/se
c(d*x+c)^(1/2)+2/15*(9*a^2+7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d+20/21*a*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3317, 3873, 3854, 3856, 2720, 4130, 2719}

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx = \frac{2(9a^2 + 7b^2) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{2(9a^2 + 7b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{4ab \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{20ab \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{20ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{2b^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(5/2), x]

[Out] (2*(9*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(15*d) + (20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(21*d) + (2*b^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (4*a*b*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(9*a^2 + 7*b^2)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (20*a*b*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +

$d*x])^{(n + 2), x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$
]

Rule 3856

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \text{ :> } \text{Dist}[(b*\text{Csc}[c + d*x])^{n*} \text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{EqQ}[n^2, 1/4]$

Rule 3873

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{2}, x_Symbol] \text{ :> } \text{Dist}[2*a*(b/d), \text{Int}[(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] + \text{Int}[(d*\text{Csc}[e + f*x])^{n*}*(a^2 + b^2*\text{Csc}[e + f*x]^2), x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 4130

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]^{2*(C_.) + (A_.)}, x_Symbol] \text{ :> } \text{Simp}[A*\text{Cot}[e + f*x]*(b*\text{Csc}[e + f*x])^{m/(f*m)}, x] + \text{Dist}[(C*m + A*(m + 1))/(b^2*m), \text{Int}[(b*\text{Csc}[e + f*x])^{(m + 2)}, x], x] /; \text{FreeQ}[\{b, e, f, A, C\}, x] \ \&\& \ \text{NeQ}[C*m + A*(m + 1), 0] \ \&\& \ \text{LeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(b + a \sec(c + dx))^2}{\sec^{\frac{9}{2}}(c + dx)} dx \\
 &= (2ab) \int \frac{1}{\sec^{\frac{7}{2}}(c + dx)} dx + \int \frac{b^2 + a^2 \sec^2(c + dx)}{\sec^{\frac{9}{2}}(c + dx)} dx \\
 &= \frac{2b^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{1}{7}(10ab) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &\quad - \frac{1}{9}(-9a^2 - 7b^2) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2b^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(9a^2 + 7b^2) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{20ab \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
 &\quad + \frac{1}{21}(10ab) \int \sqrt{\sec(c + dx)} dx - \frac{1}{15}(-9a^2 - 7b^2) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{2b^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{4ab \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(9a^2 + 7b^2) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} \\
 &\quad + \frac{20ab \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{1}{21} \left(10ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
 &\quad - \frac{1}{15} \left((-9a^2 - 7b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2(9a^2 + 7b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{15d} \\
&\quad + \frac{20ab \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{21d} + \frac{2b^2 \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
&\quad + \frac{4ab \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2(9a^2 + 7b^2) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{20ab \sin(c + dx)}{21d \sqrt{\sec(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.68

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{\sqrt{\sec(c + dx)} \left(168(9a^2 + 7b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 1200ab \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx) \mid 2\right) \right)}{126}$$

126

[In] Integrate[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(5/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(168*(9*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 1200*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*a^2 + 43*b^2)*Cos[c + d*x] + 5*b*(156*a + 36*a*Cos[2*(c + d*x)] + 7*b*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)])/(1260*d)

Maple [A] (verified)

Time = 13.89 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.99

method	result
default	$- \frac{2 \sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(-1120 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b^2 + (1440ab + 2240b^2) \left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \dots}{1260d}$
parts	$- \frac{2a^2 \sqrt{\left(2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(-8 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 8 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \dots}{5 \sqrt{-2 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}}$

[In] int((a+cos(d*x+c)*b)^2/sec(d*x+c)^(5/2), x, method=_RETURNVERBOSE)

[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^2+(1440*a*b+2240*b^2)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*a^2-2160*a*b-2072*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*a^2+1680*a*b+952*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*a^2-480*a*b-168*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+150*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El

```
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.02

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{-150i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 150i \sqrt{2} ab \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{\dots}$$

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(-150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) + 150*I*sqrt(2)*a*b*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 21*sqrt(2)*(-9*I*a^2 - 7*I*b^2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(9*I*a^2
+ 7*I*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(35*b^2*cos(d*x + c)^4 + 90*a*b*cos(d*x + c)^3 + 150*
a*b*cos(d*x + c) + 7*(9*a^2 + 7*b^2)*cos(d*x + c)^2*sin(d*x + c)/sqrt(cos(
d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```


Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^2}{\sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^2}{\sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^2}{\sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^2}{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{5}{2}}} dx$$

[In] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(5/2),x)

[Out] int((a + b*cos(c + d*x))^2/(1/cos(c + d*x))^(5/2), x)

3.705 $\int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx$

Optimal result	6430
Rubi [A] (verified)	.6431
Mathematica [A] (verified)	6434
Maple [B] (verified)	6434
Fricas [C] (verification not implemented)	6435
Sympy [F(-1)]	6435
Maxima [F]	6436
Giac [F]	6436
Mupad [F(-1)]	6436

Optimal result

Integrand size = 23, antiderivative size = 234

$$\begin{aligned}
 & \int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx \\
 &= -\frac{2b(9a^2 + 5b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &+ \frac{2a(5a^2 + 21b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{21d} \\
 &+ \frac{2b(9a^2 + 5b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{2a(5a^2 + 21b^2) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{21d} \\
 &+ \frac{32a^2 b \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{35d} + \frac{2a^2 \sec^{\frac{5}{2}}(c + dx) (b + a \sec(c + dx)) \sin(c + dx)}{7d}
 \end{aligned}$$

```
[Out] 2/21*a*(5*a^2+21*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+32/35*a^2*b*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*a^2*sec(d*x+c)^(5/2)*(b+a*sec(d*x+c))*sin(d*x+c)/d+2/5*b*(9*a^2+5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*b*(9*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(5*a^2+21*b^2)*(cos(1/2*d*x+1/2*c)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3317, 3927, 4132, 3853, 3856, 2720, 4131, 2719}

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{2a(5a^2 + 21b^2) \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{21d} + \frac{2b(9a^2 + 5b^2) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

$$+ \frac{2a(5a^2 + 21b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d}$$

$$- \frac{2b(9a^2 + 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

$$+ \frac{2a^2 \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)(a \sec(c + dx) + b)}{7d} + \frac{32a^2 b \sin(c + dx) \sec^{\frac{5}{2}}(c + dx)}{35d}$$

[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2), x]

[Out] (-2*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(21*d) + (2*b*(9*a^2 + 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]/(21*d) + (32*a^2*b*Sec[c + d*x]^(5/2)*Sin[c + d*x]/(35*d) + (2*a^2*Sec[c + d*x]^(5/2)*(b + a*Sec[c + d*x])*Sin[c + d*x]/(7*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegersQ[m] && IntegersQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*(n - 2)/(n - 1),

$\text{Int}[(b*\text{Csc}[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 3856

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \ :> \ \text{Dist}[(b*\text{Csc}[c + d*x])^{n*} \ \text{Sin}[c + d*x]^n, \ \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \ \text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{EqQ}[n^2, 1/4]$

Rule 3927

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)} * (\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] \ :> \ \text{Simp}[(-b^2)*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m - 2)} * ((d*\text{Csc}[e + f*x])^n / (f*(m + n - 1))), x] + \ \text{Dist}[1/(d*(m + n - 1)), \ \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m - 3)} * (d*\text{Csc}[e + f*x])^n * \ \text{Simp}[a^3*d*(m + n - 1) + a*b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*\text{Csc}[e + f*x] + a*b^2*d*(3*m + 2*n - 4)*\text{Csc}[e + f*x]^2, x], x], x] /; \ \text{FreeQ}\{a, b, d, e, f, n\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[m, 2] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{IntegersQ}[2*m, 2*n]) \ \&\& \ !(\text{IGtQ}[n, 2] \ \&\& \ !\text{IntegerQ}[m])$

Rule 4131

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)} * (\text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] \ :> \ \text{Simp}[(-C)*\text{Cot}[e + f*x]*((b*\text{Csc}[e + f*x])^m / (f*(m + 1))), x] + \ \text{Dist}[(C*m + A*(m + 1))/(m + 1), \ \text{Int}[(b*\text{Csc}[e + f*x])^m, x], x] /; \ \text{FreeQ}\{b, e, f, A, C, m\}, x] \ \&\& \ \text{NeQ}[C*m + A*(m + 1), 0] \ \&\& \ !\text{LeQ}[m, -1]$

Rule 4132

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)} * ((A_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] \ :> \ \text{Dist}[B/b, \ \text{Int}[(b*\text{Csc}[e + f*x])^{(m + 1)}, x], x] + \ \text{Int}[(b*\text{Csc}[e + f*x])^m * (A + C*\text{Csc}[e + f*x]^2), x] /; \ \text{FreeQ}\{b, e, f, A, B, C, m\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \sec^{\frac{3}{2}}(c + dx)(b + a \sec(c + dx))^3 dx \\ &= \frac{2a^2 \sec^{\frac{5}{2}}(c + dx)(b + a \sec(c + dx)) \sin(c + dx)}{7d} + \frac{2}{7} \int \sec^{\frac{3}{2}}(c + dx) \left(\frac{1}{2}b(3a^2 + 7b^2) \right. \\ &\quad \left. + \frac{1}{2}a(5a^2 + 21b^2) \sec(c + dx) + 8a^2b \sec^2(c + dx) \right) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2a^2 \sec^{\frac{5}{2}}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{7d} \\
&\quad + \frac{2}{7} \int \sec^{\frac{3}{2}}(c+dx) \left(\frac{1}{2}b(3a^2+7b^2) + 8a^2b \sec^2(c+dx) \right) dx \\
&\quad + \frac{1}{7}(a(5a^2+21b^2)) \int \sec^{\frac{5}{2}}(c+dx) dx \\
&= \frac{2a(5a^2+21b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{21d} + \frac{32a^2b \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{35d} \\
&\quad + \frac{2a^2 \sec^{\frac{5}{2}}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{7d} \\
&\quad + \frac{1}{5}(b(9a^2+5b^2)) \int \sec^{\frac{3}{2}}(c+dx) dx + \frac{1}{21}(a(5a^2+21b^2)) \int \sqrt{\sec(c+dx)} dx \\
&= \frac{2b(9a^2+5b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{2a(5a^2+21b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{21d} \\
&\quad + \frac{32a^2b \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{35d} + \frac{2a^2 \sec^{\frac{5}{2}}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{7d} \\
&\quad - \frac{1}{5}(b(9a^2+5b^2)) \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\
&\quad + \frac{1}{21} \left(a(5a^2+21b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&= \frac{2a(5a^2+21b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{2b(9a^2+5b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} \\
&\quad + \frac{2a(5a^2+21b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{21d} + \frac{32a^2b \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{35d} \\
&\quad + \frac{2a^2 \sec^{\frac{5}{2}}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{7d} \\
&\quad - \frac{1}{5} \left(b(9a^2+5b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{2b(9a^2+5b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{2a(5a^2+21b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{2b(9a^2+5b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} \\
&\quad + \frac{2a(5a^2+21b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{21d} + \frac{32a^2b \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{35d} \\
&\quad + \frac{2a^2 \sec^{\frac{5}{2}}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.82

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx$$

$$= \frac{\sec^{\frac{7}{2}}(c + dx) \left(-42b(9a^2 + 5b^2) \cos^{\frac{7}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 10a(5a^2 + 21b^2) \cos^{\frac{7}{2}}(c + dx) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx) \mid 2\right) + 30a^3 \sin(c + dx) + 50a^3 \cos(c + dx)^2 \sin(c + dx) + 210a^2 b \cos(c + dx)^2 \sin(c + dx) + 378a^2 b \cos(c + dx)^3 \sin(c + dx) + 210a b^3 \cos(c + dx)^3 \sin(c + dx) + 63a^2 b \sin[2(c + dx)] \right)}{105d}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]

[Out] (Sec[c + d*x]^(7/2)*(-42*b*(9*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2] + 10*a*(5*a^2 + 21*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 30*a^3*Sin[c + d*x] + 50*a^3*Cos[c + d*x]^2*Sin[c + d*x] + 210*a*b^2*Cos[c + d*x]^2*Sin[c + d*x] + 378*a^2*b*Cos[c + d*x]^3*Sin[c + d*x] + 210*b^3*Cos[c + d*x]^3*Sin[c + d*x] + 63*a^2*b*Sin[2*(c + d*x)]))/(105*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. 2(258) = 516.

Time = 506.96 (sec) , antiderivative size = 820, normalized size of antiderivative = 3.50

method	result	size
default	Expression too large to display	820
parts	Expression too large to display	1008

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6/5*a^2*b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)

$$\frac{1}{2} * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{1/2} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{1/2}) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{1/2} + 6 * a * b^2 * (-1/6 * \cos(1/2 * d * x + 1/2 * c) * (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{1/2} / (\cos(1/2 * d * x + 1/2 * c)^2 - 1/2)^{1/2} + 1/3 * (\sin(1/2 * d * x + 1/2 * c)^2)^{1/2} * (-2 * \cos(1/2 * d * x + 1/2 * c)^2 + 1)^{1/2} / (-2 * \sin(1/2 * d * x + 1/2 * c)^4 + \sin(1/2 * d * x + 1/2 * c)^2)^{1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{1/2})) / \sin(1/2 * d * x + 1/2 * c) / (2 * \cos(1/2 * d * x + 1/2 * c)^2 - 1)^{1/2} / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.15

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx =$$

$$\frac{5 \sqrt{2} (5i a^3 + 21i ab^2) \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5 \sqrt{2} (-5$$

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*a^3 + 21*I*a*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*a^3 - 21*I*a*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(9*I*a^2*b + 5*I*b^3)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-9*I*a^2*b - 5*I*b^3)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(63*a^2*b*cos(d*x + c) + 21*(9*a^2*b + 5*b^3)*cos(d*x + c)^3 + 15*a^3 + 5*(5*a^3 + 21*a*b^2)*cos(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{9}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{9}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{9}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{9/2} (a + b \cos(c + dx))^3 dx$$

[In] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^3, x)

3.706 $\int (a + b \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx$

Optimal result	6437
Rubi [A] (verified)	6438
Mathematica [A] (verified)	6440
Maple [B] (verified)	6441
Fricas [C] (verification not implemented)	6442
Sympy [F(-1)]	6442
Maxima [F]	6442
Giac [F]	6443
Mupad [F(-1)]	6443

Optimal result

Integrand size = 23, antiderivative size = 189

$$\begin{aligned}
 & \int (a + b \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx \\
 &= -\frac{6a(a^2 + 5b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &+ \frac{2b(a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \\
 &+ \frac{6a(a^2 + 5b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{5d} + \frac{8a^2 b \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{5d} \\
 &+ \frac{2a^2 \sec^{\frac{3}{2}}(c + dx) (b + a \sec(c + dx)) \sin(c + dx)}{5d}
 \end{aligned}$$

```
[Out] 8/5*a^2*b*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a^2*sec(d*x+c)^(3/2)*(b+a*sec(d
*x+c))*sin(d*x+c)/d+6/5*a*(a^2+5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-6/5*a*(
a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2*b*(a^2+b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3317, 3927, 4132, 3853, 3856, 2719, 4131, 2720}

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{6a(a^2 + 5b^2) \sin(c + dx) \sqrt{\sec(c + dx)}}{5d}$$

$$+ \frac{2b(a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

$$- \frac{6a(a^2 + 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

$$+ \frac{8a^2 b \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{5d} + \frac{2a^2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)(a \sec(c + dx) + b)}{5d}$$

[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]

[Out] (-6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(5*d) + (2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/d + (6*a*(a^2 + 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^2*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Sec[c + d*x]^(3/2)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &

& IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3927

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && !IntegerQ[m])

Rule 4131

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 4132

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \int \sqrt{\sec(c + dx)}(b + a \sec(c + dx))^3 dx \\ &= \frac{2a^2 \sec^{\frac{3}{2}}(c + dx)(b + a \sec(c + dx)) \sin(c + dx)}{5d} + \frac{2}{5} \int \sqrt{\sec(c + dx)} \left(\frac{1}{2}b(a^2 + 5b^2) \right. \\ &\quad \left. + \frac{3}{2}a(a^2 + 5b^2) \sec(c + dx) + 6a^2b \sec^2(c + dx) \right) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2a^2 \sec^{\frac{3}{2}}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{5d} \\
&\quad + \frac{2}{5} \int \sqrt{\sec(c+dx)} \left(\frac{1}{2}b(a^2+5b^2) + 6a^2b \sec^2(c+dx) \right) dx \\
&\quad + \frac{1}{5} (3a(a^2+5b^2)) \int \sec^{\frac{3}{2}}(c+dx) dx \\
&= \frac{6a(a^2+5b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{8a^2b \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{2a^2 \sec^{\frac{3}{2}}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{5d} \\
&\quad + (b(a^2+b^2)) \int \sqrt{\sec(c+dx)} dx - \frac{1}{5} (3a(a^2+5b^2)) \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\
&= \frac{6a(a^2+5b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{8a^2b \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{2a^2 \sec^{\frac{3}{2}}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{5d} \\
&\quad + \left(b(a^2+b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&\quad - \frac{1}{5} \left(3a(a^2+5b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&= -\frac{6a(a^2+5b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{2b(a^2+b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{d} \\
&\quad + \frac{6a(a^2+5b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{5d} + \frac{8a^2b \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{2a^2 \sec^{\frac{3}{2}}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\begin{aligned}
&\int (a+b \cos(c+dx))^3 \sec^{\frac{7}{2}}(c+dx) dx \\
&= \frac{2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \left(-3a(a^2+5b^2) E\left(\frac{1}{2}(c+dx) \mid 2\right) + 5b(a^2+b^2) \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \frac{a(5b^2+3a^2)}{2} \right)}{5d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2), x]

```
[Out] (2*sqrt[Cos[c + d*x]]*sqrt[Sec[c + d*x]]*(-3*a*(a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2] + 5*b*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + (a*(5*(a^2 + 3*b^2) + 10*a*b*cos[c + d*x] + 3*(a^2 + 5*b^2)*cos[2*(c + d*x)])*sin[c + d*x])/(2*cos[c + d*x]^(5/2)))/(5*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(219) = 438$.

Time = 482.10 (sec) , antiderivative size = 711, normalized size of antiderivative = 3.76

method	result	size
default	Expression too large to display	711
parts	Expression too large to display	898

```
[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2/5*a^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a*b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.29

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx =$$

$$\frac{5\sqrt{2}(i a^2 b + i b^3) \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-i a^2 b -$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] -1/5*(5*sqrt(2)*(I*a^2*b + I*b^3)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^2*b - I*b^3)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(I*a^3 + 5*I*a*b^2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-I*a^3 - 5*I*a*b^2)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(5*a^2*b*cos(d*x + c) + a^3 + 3*(a^3 + 5*a*b^2)*cos(d*x + c)^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{7/2} (a + b \cos(c + dx))^3 dx$$

[In] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^3, x)

3.707 $\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx$

Optimal result	6444
Rubi [A] (verified)	6444
Mathematica [A] (verified)	6447
Maple [B] (verified)	6447
Fricas [C] (verification not implemented)	6448
Sympy [F(-1)]	6449
Maxima [F]	6449
Giac [F]	6449
Mupad [F(-1)]	6449

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx$$

$$= -\frac{2b(3a^2 - b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d}$$

$$+ \frac{2a(a^2 + 9b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d}$$

$$+ \frac{16a^2b \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2a^2 \sqrt{\sec(c + dx)} (b + a \sec(c + dx)) \sin(c + dx)}{3d}$$

[Out] $16/3*a^2*b*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d+2/3*a^2*(b+a*\sec(d*x+c))*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/d-2*b*(3*a^2-b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/d+2/3*a*(a^2+9*b^2)*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)*\sec(d*x+c)^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3927, 4132, 3856, 2720, 4131, 2719}

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{2a(a^2 + 9b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d}$$

$$- \frac{2b(3a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d}$$

$$+ \frac{2a^2 \sin(c + dx) \sqrt{\sec(c + dx)} (a \sec(c + dx) + b)}{3d} + \frac{16a^2 b \sin(c + dx) \sqrt{\sec(c + dx)}}{3d}$$

[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2), x]

[Out] (-2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (16*a^2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_)^(p_), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x])^n]^p, x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3927

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b

$\wedge 2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d$
 $*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x] /; FreeQ[\{a, b, d, e, f, n\}, x]$
 $\&\& NeQ[a^2 - b^2, 0] \&\& GtQ[m, 2] \&\& (IntegerQ[m] || IntegersQ[2*m, 2*n])$
 $\&\& !(IGtQ[n, 2] \&\& !IntegerQ[m])$

Rule 4131

$Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.)$
 $+ (A_.)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)$
 $)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;$
 $FreeQ[\{b, e, f, A, C, m\}, x] \&\& NeQ[C*m + A*(m + 1), 0] \&\& !LeQ[m, -1]$

Rule 4132

$Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_.)]*$
 $(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc$
 $[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),$
 $x] /; FreeQ[\{b, e, f, A, B, C, m\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(b + a \sec(c + dx))^3}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{2a^2 \sqrt{\sec(c + dx)}(b + a \sec(c + dx)) \sin(c + dx)}{3d} \\
 &\quad + \frac{2}{3} \int \frac{-\frac{1}{2}b(a^2 - 3b^2) + \frac{1}{2}a(a^2 + 9b^2) \sec(c + dx) + 4a^2b \sec^2(c + dx)}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{2a^2 \sqrt{\sec(c + dx)}(b + a \sec(c + dx)) \sin(c + dx)}{3d} \\
 &\quad + \frac{2}{3} \int \frac{-\frac{1}{2}b(a^2 - 3b^2) + 4a^2b \sec^2(c + dx)}{\sqrt{\sec(c + dx)}} dx + \frac{1}{3} (a(a^2 + 9b^2)) \int \sqrt{\sec(c + dx)} dx \\
 &= \frac{16a^2b \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2a^2 \sqrt{\sec(c + dx)}(b + a \sec(c + dx)) \sin(c + dx)}{3d} \\
 &\quad - (b(3a^2 - b^2)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &\quad + \frac{1}{3} \left(a(a^2 + 9b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2a(a^2 + 9b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{16a^2b \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} \\
&\quad + \frac{2a^2 \sqrt{\sec(c + dx)} (b + a \sec(c + dx)) \sin(c + dx)}{3d} \\
&\quad - \left(b(3a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= -\frac{2b(3a^2 - b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2a(a^2 + 9b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{16a^2b \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} \\
&\quad + \frac{2a^2 \sqrt{\sec(c + dx)} (b + a \sec(c + dx)) \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\begin{aligned}
&\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx \\
&= \frac{\sec^{\frac{3}{2}}(c + dx) \left(6b(-3a^2 + b^2) \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + a \left(2(a^2 + 9b^2) \cos^{\frac{3}{2}}(c + dx) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \right. \right. \\
&\quad \left. \left. + 2a^2(a + 9b \cos(c + dx)) \sin(c + dx) \right) \right)}{3d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]

[Out] (Sec[c + d*x]^(3/2)*(6*b*(-3*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + a*(2*(a^2 + 9*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*a*(a + 9*b*Cos[c + d*x])*Sin[c + d*x]))/(3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(194) = 388.

Time = 483.02 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.94

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(36\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a^2b - 2\sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right) F\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d}$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out]
$$-2/3*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(4*\sin(1/2*d*x+1/2*c)^4-4*\sin(1/2*d*x+1/2*c)^2+1)/\sin(1/2*d*x+1/2*c)^3*(36*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4*a^2*b-2*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\sin(1/2*d*x+1/2*c)^2*a^3-18*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\sin(1/2*d*x+1/2*c)^2*a*b^2-18*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\sin(1/2*d*x+1/2*c)^2*a^2*b+6*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*\sin(1/2*d*x+1/2*c)^2*b^3-2*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*a^3-18*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2*a^2*b+a^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+9*a*b^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+9*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a^2*b-3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b^3*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.34

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{\sqrt{2}(-i a^3 - 9i ab^2) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(i a^3 + 9i ab^2) \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3\sqrt{2}*(3I*a^2*b - I*b^3)*\cos(dx + c)*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c))) - 3\sqrt{2}*(-3I*a^2*b + I*b^3)*\cos(dx + c)*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c))) + 2*(9*a^2*b*\cos(dx + c) + a^3)*\sin(dx + c)/\sqrt{\cos(dx + c)}}{(d*\cos(dx + c))}$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out]
$$1/3*(\sqrt{2}*(-I*a^3 - 9*I*a*b^2)*\cos(d*x + c)*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + \sqrt{2}*(I*a^3 + 9*I*a*b^2)*\cos(d*x + c)*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - 3*\sqrt{2}*(3*I*a^2*b - I*b^3)*\cos(d*x + c)*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) - 3*\sqrt{2}*(-3*I*a^2*b + I*b^3)*\cos(d*x + c)*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) + 2*(9*a^2*b*\cos(d*x + c) + a^3)*\sin(d*x + c)/\sqrt{\cos(d*x + c)}}/(d*\cos(d*x + c))$$

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{\frac{5}{2}} (a + b \cos(c + dx))^3 dx$$

[In] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3, x)

3.708 $\int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	6450
Rubi [A] (verified)	6450
Mathematica [A] (verified)	6453
Maple [A] (verified)	6453
Fricas [C] (verification not implemented)	6454
Sympy [F(-1)]	6454
Maxima [F]	6455
Giac [F]	6455
Mupad [F(-1)]	6455

Optimal result

Integrand size = 23, antiderivative size = 166

$$\begin{aligned} & \int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx \\ &= -\frac{2a(a^2 - 3b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{2b(9a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\ & \quad + \frac{2a(3a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \end{aligned}$$

```
[Out] 2/3*b^2*(b+a*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/3*a*(3*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*b*(9*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3926, 4132, 3856, 2720, 4131, 2719}

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{2a(3a^2 - b^2) \sin(c + dx) \sqrt{\sec(c + dx)}}{3d} + \frac{2b(9a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d} - \frac{2a(a^2 - 3b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d} + \frac{2b^2 \sin(c + dx)(a \sec(c + dx) + b)}{3d \sqrt{\sec(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2), x]

[Out] (-2*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(3*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3926

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegerQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_.)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(b + a \sec(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2}{3} \int \frac{4ab^2 + \frac{1}{2}b(9a^2 + b^2) \sec(c + dx) + \frac{1}{2}a(3a^2 - b^2) \sec^2(c + dx)}{\sqrt{\sec(c + dx)}} dx \\
&= \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} + \frac{2}{3} \int \frac{4ab^2 + \frac{1}{2}a(3a^2 - b^2) \sec^2(c + dx)}{\sqrt{\sec(c + dx)}} dx \\
&\quad + \frac{1}{3} (b(9a^2 + b^2)) \int \sqrt{\sec(c + dx)} dx \\
&= \frac{2a(3a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} \\
&\quad + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} - (a(a^2 - 3b^2)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
&\quad + \frac{1}{3} (b(9a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{2b(9a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{2a(3a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}} \\
&\quad - \left(a(a^2 - 3b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= - \frac{2a(a^2 - 3b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{d} \\
&\quad + \frac{2b(9a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3d} \\
&\quad + \frac{2a(3a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{3d} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{3d \sqrt{\sec(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

$$\begin{aligned}
&\int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx \\
&= \frac{\sqrt{\sec(c + dx)} \left(-6a(a^2 - 3b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 2b(9a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \right)}{3d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(-6*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(3*a^3 + b^3*Cos[c + d*x])*Sin[c + d*x]))/(3*d)

Maple [A] (verified)

Time = 8.46 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.83

method	result
default	$2 \left(4 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b^3 - 6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a^3 - 2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b^3 + 9a^2b \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2} \right)$
parts	$2a^3 \left(-2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{-2 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 1} \sqrt{-2 \left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) - 1} \right) - d$

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

```
[Out] -2/3*(4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+9*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.10

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{\sqrt{2}(-9i a^2 b - i b^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + \sqrt{2}(9i a^2 b + i b^3) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3 \sqrt{2}(I a^3 - 3 I a b^2) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I \sin(dx + c))) - 3 \sqrt{2}(-I a^3 + 3 I a b^2) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I \sin(dx + c))) + 2(b^3 \cos(dx + c) + 3 a^3) \sin(dx + c) / \sqrt{\cos(dx + c)}}{d}$$

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-9*I*a^2*b - I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(9*I*a^2*b + I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*a^3 - 3*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*a^3 + 3*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(b^3*cos(d*x + c) + 3*a^3)*sin(d*x + c)/sqrt(cos(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}} dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{\frac{3}{2}} (a + b \cos(c + dx))^3 dx$$

[In] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^3, x)

3.709 $\int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx$

Optimal result	6456
Rubi [A] (verified)	6456
Mathematica [A] (verified)	6459
Maple [B] (verified)	6459
Fricas [C] (verification not implemented)	6460
Sympy [F]	6460
Maxima [F]	6461
Giac [F]	6461
Mupad [F(-1)]	6461

Optimal result

Integrand size = 23, antiderivative size = 156

$$\begin{aligned} & \int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx \\ &= \frac{6b(5a^2 + b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\ & \quad + \frac{2a(a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{d} \\ & \quad + \frac{8ab^2 \sin(c + dx)}{5d \sqrt{\sec(c + dx)}} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \end{aligned}$$

```
[Out] 2/5*b^2*(b+a*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/5*a*b^2*sin(d*x+c)
/d/sec(d*x+c)^(1/2)+6/5*b*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
)^(1/2)/d+2*a*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

= {3317, 3926, 4132, 3856, 2719, 4130, 2720}

$$\int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx$$

$$= \frac{2a(a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d}$$

$$+ \frac{6b(5a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d}$$

$$+ \frac{2b^2 \sin(c + dx)(a \sec(c + dx) + b)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{8ab^2 \sin(c + dx)}{5d \sqrt{\sec(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]], x]

[Out] (6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(5*d) + (2*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/d + (8*a*b^2*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]) + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x])^n]^p, x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3926

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(

$n + 1)) * \text{Csc}[e + f*x] - b*(b^2*n + a^2*(m + n - 1)) * \text{Csc}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[m, 2] \&\& ((\text{IntegerQ}[m] \&\& \text{LtQ}[n, -1]) || (\text{IntegersQ}[m + 1/2, 2*n] \&\& \text{LeQ}[n, -1]))$

Rule 4130

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^m * (\text{csc}[(e_.) + (f_.)*(x_.)]^2 * (C_.) + (A_.)), x_Symbol] :> \text{Simp}[A * \text{Cot}[e + f*x] * ((b * \text{Csc}[e + f*x])^m / (f * m)), x] + \text{Dist}[(C * m + A * (m + 1)) / (b^2 * m), \text{Int}[(b * \text{Csc}[e + f*x])^{m + 2}], x], x] /; \text{FreeQ}\{b, e, f, A, C\}, x\} \&\& \text{NeQ}[C * m + A * (m + 1), 0] \&\& \text{LeQ}[m, -1]$

Rule 4132

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^m * ((A_.) + \text{csc}[(e_.) + (f_.)*(x_.)] * (B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2 * (C_.)), x_Symbol] :> \text{Dist}[B/b, \text{Int}[(b * \text{Csc}[e + f*x])^{m + 1}], x], x] + \text{Int}[(b * \text{Csc}[e + f*x])^m * (A + C * \text{Csc}[e + f*x]^2), x] /; \text{FreeQ}\{b, e, f, A, B, C, m\}, x\}$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(b + a \sec(c + dx))^3}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
 &\quad + \frac{2}{5} \int \frac{6ab^2 + \frac{3}{2}b(5a^2 + b^2) \sec(c + dx) + \frac{1}{2}a(5a^2 + b^2) \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &= \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} + \frac{2}{5} \int \frac{6ab^2 + \frac{1}{2}a(5a^2 + b^2) \sec^2(c + dx)}{\sec^{\frac{3}{2}}(c + dx)} dx \\
 &\quad + \frac{1}{5} (3b(5a^2 + b^2)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx \\
 &= \frac{8ab^2 \sin(c + dx)}{5d \sqrt{\sec(c + dx)}} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
 &\quad + (a(a^2 + b^2)) \int \sqrt{\sec(c + dx)} dx \\
 &\quad + \frac{1}{5} \left(3b(5a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
 &= \frac{6b(5a^2 + b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{\sec(c + dx)}}{5d} \\
 &\quad + \frac{8ab^2 \sin(c + dx)}{5d \sqrt{\sec(c + dx)}} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)} \\
 &\quad + \left(a(a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{6b(5a^2 + b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\
&\quad + \frac{2a(a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{5d} \\
&\quad + \frac{8ab^2 \sin(c + dx)}{5d \sqrt{\sec(c + dx)}} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{5d \sec^{\frac{3}{2}}(c + dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\begin{aligned}
&\int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx \\
&= \frac{\sqrt{\sec(c + dx)} \left(6b(5a^2 + b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 10a(a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + b^2(5a + b \cos(c + dx)) \sin(2(c + dx)) \right)}{5d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]

[Out] (Sqrt[Sec[c + d*x]]*(6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b^2*(5*a + b*Cos[c + d*x])*Sin[2*(c + d*x)])/(5*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(190) = 380.

Time = 9.71 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.64

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(-8\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b^3 + 20\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a b^2 + 8\cos\left(\frac{dx}{2} + \frac{c}{2}\right)a^2 b}\right)$
parts	$-\frac{2a^3\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)} - \frac{2b^3\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} d$

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*b^3+20*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a*b^2+8*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3-10*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a*b^2-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3+5*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)

$$\frac{1}{2}d^2x+1/2c)^2-1)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2})-15*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{1/2}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{1/2})*a^2*b-3*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{1/2}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{1/2})*b^3)/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{1/2}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.24

$$\int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx =$$

$$5\sqrt{2}(i a^3 + i ab^2)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-i a^3 - i ab^2)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 3\sqrt{2}(-5Ia^2b - Ib^3)\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I\sin(dx + c))) + 3\sqrt{2}(5Ia^2b + Ib^3)\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I\sin(dx + c))) - 2*(b^3*\cos(dx + c)^2 + 5*a*b^2*\cos(dx + c))*\sin(dx + c)/\sqrt{\cos(dx + c)}/d$$

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(5*sqrt(2)*(I*a^3 + I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*a^3 - I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-5*I*a^2*b - I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(5*I*a^2*b + I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(b^3*cos(d*x + c)^2 + 5*a*b^2*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]

$$\int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx = \int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx$$

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**3*sqrt(sec(c + d*x)), x)
```


Maxima [F]

$$\int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx = \int (b \cos(dx + c) + a)^3 \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)

Giac [F]

$$\int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx = \int (b \cos(dx + c) + a)^3 \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} (a + b \cos(c + dx))^3 dx$$

[In] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3, x)

$$3.710 \quad \int \frac{(a+b \cos(c+dx))^3}{\sqrt{\sec(c+dx)}} dx$$

Optimal result	6462
Rubi [A] (verified)	6462
Mathematica [A] (verified)	6465
Maple [A] (verified)	6466
Fricas [C] (verification not implemented)	6466
Sympy [F]	6467
Maxima [F]	6467
Giac [F]	6467
Mupad [F(-1)]	6467

Optimal result

Integrand size = 23, antiderivative size = 199

$$\begin{aligned} & \int \frac{(a + b \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx \\ &= \frac{2a(5a^2 + 9b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{5d} \\ &+ \frac{2b(21a^2 + 5b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{21d} \\ &+ \frac{32ab^2 \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx)} + \frac{2b(21a^2 + 5b^2) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \end{aligned}$$

[Out] 32/35*a*b^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*b^2*(b+a*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/21*b*(21*a^2+5*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/5*a*(5*a^2+9*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*b*(21*a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used

= {3317, 3926, 4132, 3854, 3856, 2720, 4130, 2719}

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{2b(21a^2 + 5b^2) \sin(c + dx)}{21d\sqrt{\sec(c + dx)}} + \frac{2b(21a^2 + 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{2a(5a^2 + 9b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d} + \frac{2b^2 \sin(c + dx)(a \sec(c + dx) + b)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{32ab^2 \sin(c + dx)}{35d \sec^{\frac{3}{2}}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]], x]

[Out] (2*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(21*d) + (32*a*b^2*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*b*(21*a^2 + 5*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d^n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3926

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((IntegerQ[m] && LtQ[n, -1]) || (IntegerQ[m + 1/2, 2*n] && LeQ[n, -1]))

Rule 4130

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_.)), x_Symbol] := Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 4132

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] := Dist[B/b, Int[(b*Csc[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2), x] /; FreeQ[{b, e, f, A, B, C, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{(b + a \sec(c + dx))^3}{\sec^{\frac{7}{2}}(c + dx)} dx \\
 &= \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} \\
 &\quad + \frac{2}{7} \int \frac{8ab^2 + \frac{1}{2}b(21a^2 + 5b^2) \sec(c + dx) + \frac{1}{2}a(7a^2 + 3b^2) \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{7d \sec^{\frac{5}{2}}(c + dx)} + \frac{2}{7} \int \frac{8ab^2 + \frac{1}{2}a(7a^2 + 3b^2) \sec^2(c + dx)}{\sec^{\frac{5}{2}}(c + dx)} dx \\
 &\quad + \frac{1}{7}(b(21a^2 + 5b^2)) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{32ab^2 \sin(c+dx)}{35d \sec^{\frac{3}{2}}(c+dx)} + \frac{2b(21a^2+5b^2) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{2b^2(b+a \sec(c+dx)) \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{1}{21} (b(21a^2+5b^2)) \int \sqrt{\sec(c+dx)} dx + \frac{1}{5} (a(5a^2+9b^2)) \int \frac{1}{\sqrt{\sec(c+dx)}} dx \\
&= \frac{32ab^2 \sin(c+dx)}{35d \sec^{\frac{3}{2}}(c+dx)} + \frac{2b(21a^2+5b^2) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{2b^2(b+a \sec(c+dx)) \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{1}{21} \left(b(21a^2+5b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\
&\quad + \frac{1}{5} \left(a(5a^2+9b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx \\
&= \frac{2a(5a^2+9b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{5d} \\
&\quad + \frac{2b(21a^2+5b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} \\
&\quad + \frac{32ab^2 \sin(c+dx)}{35d \sec^{\frac{3}{2}}(c+dx)} + \frac{2b(21a^2+5b^2) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} \\
&\quad + \frac{2b^2(b+a \sec(c+dx)) \sin(c+dx)}{7d \sec^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.66

$$\begin{aligned}
&\int \frac{(a+b \cos(c+dx))^3}{\sqrt{\sec(c+dx)}} dx \\
&= \frac{\sqrt{\sec(c+dx)} \left(84a(5a^2+9b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) + 20b(21a^2+5b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \right. \\
&\quad \left. + b(210a^2+65b^2+126ab \cos(c+dx)+15b^2 \cos(2(c+dx))) \sin(2(c+dx)) \right)}{210d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]

[Out] (Sqrt[Sec[c + d*x]]*(84*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*(210*a^2 + 65*b^2 + 126*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(210*d)

Maple [A] (verified)

Time = 11.24 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.12

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(240\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3+(-504ab^2-360b^3)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\dots}$
parts	Expression too large to display

[In] int((a+cos(d*x+c)*b)^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*b^3+(-504*a*b^2-360*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*a^2*b+504*a*b^2+280*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*a^2*b-126*a*b^2-80*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.09

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx =$$

$$-\frac{5\sqrt{2}(21ia^2b + 5ib^3)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5\sqrt{2}(-21ia^2b - 5ib^3)\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 21\sqrt{2}(-5Ia^3 - 9Ia^2b^2)\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I\sin(dx + c))) + 21\sqrt{2}(5Ia^3 + 9Ia^2b^2)\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I\sin(dx + c))) - 2*(15*b^3*\cos(dx + c)^3 + 63*a*b^2*\cos(dx + c)^2 + 5*(21*a^2*b + 5*b^3)*\cos(dx + c))*\sin(dx + c)/\sqrt{\cos(dx + c)))/d$$

[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")

```
[Out] -1/105*(5*sqrt(2)*(21*I*a^2*b + 5*I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*a^2*b - 5*I*b^3)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-5*I*a^3 - 9*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(5*I*a^3 + 9*I*a*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*b^3*cos(d*x + c)^3 + 63*a*b^2*cos(d*x + c)^2 + 5*(21*a^2*b + 5*b^3)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))/d
```

Sympy [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx$$

[In] integrate((a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)

[Out] Integral((a + b*cos(c + d*x))**3/sqrt(sec(c + d*x)), x)

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^3}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^3}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^3}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^3}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(1/2), x)

$$3.711 \quad \int \frac{(a+b \cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	6468
Rubi [A] (verified)	6469
Mathematica [A] (verified)	6472
Maple [A] (verified)	6472
Fricas [C] (verification not implemented)	6473
Sympy [F]	6473
Maxima [F]	6473
Giac [F]	6474
Mupad [F(-1)]	6474

Optimal result

Integrand size = 23, antiderivative size = 234

$$\begin{aligned} & \int \frac{(a+b \cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)} dx \\ &= \frac{2b(27a^2+7b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{15d} \\ &+ \frac{2a(7a^2+15b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{21d} \\ &+ \frac{40ab^2 \sin(c+dx)}{63d \sec^{\frac{5}{2}}(c+dx)} + \frac{2b(27a^2+7b^2) \sin(c+dx)}{45d \sec^{\frac{3}{2}}(c+dx)} \\ &+ \frac{2a(7a^2+15b^2) \sin(c+dx)}{21d \sqrt{\sec(c+dx)}} + \frac{2b^2(b+a \sec(c+dx)) \sin(c+dx)}{9d \sec^{\frac{7}{2}}(c+dx)} \end{aligned}$$

```
[Out] 40/63*a*b^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*b*(27*a^2+7*b^2)*sin(d*x+c)/
d/sec(d*x+c)^(3/2)+2/9*b^2*(b+a*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2
/21*a*(7*a^2+15*b^2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*b*(27*a^2+7*b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
, 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(7*a^2+15*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```


Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3317, 3926, 4132, 3854, 3856, 2719, 4130, 2720}

$$\int \frac{(a + b \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{2b(27a^2 + 7b^2) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a(7a^2 + 15b^2) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{2a(7a^2 + 15b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d} + \frac{2b(27a^2 + 7b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d} + \frac{2b^2 \sin(c + dx)(a \sec(c + dx) + b)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{40ab^2 \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)}$$

[In] Int[(a + b*Cos[c + d*x])^3/Sec[c + d*x]^(3/2), x]

[Out] (2*b*(27*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(15*d) + (2*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(21*d) + (40*a*b^2*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)) + (2*b*(27*a^2 + 7*b^2)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*a*(7*a^2 + 15*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3854

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +

$d*x])^{(n + 2), x], x] /; \text{FreeQ}\{b, c, d\}, x] \ \&\& \text{LtQ}[n, -1] \ \&\& \text{IntegerQ}[2*n]$

Rule 3856

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \ :> \text{Dist}[(b*\text{Csc}[c + d*x])^{n*} \text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x] \ \&\& \text{EqQ}[n^2, 1/4]$

Rule 3926

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] \ :> \text{Simp}[a^2*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m - 2)}*((d*\text{Csc}[e + f*x])^n/(f*n)), x] - \text{Dist}[1/(d*n), \text{Int}[(a + b*\text{Csc}[e + f*x])^{(m - 3)}*(d*\text{Csc}[e + f*x])^{(n + 1)}*\text{Simp}[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(n + 1))*\text{Csc}[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*\text{Csc}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \ \&\& \text{NeQ}[a^2 - b^2, 0] \ \&\& \text{GtQ}[m, 2] \ \&\& ((\text{IntegerQ}[m] \ \&\& \text{LtQ}[n, -1]) \ || \ (\text{IntegersQ}[m + 1/2, 2*n] \ \&\& \text{LeQ}[n, -1]))$

Rule 4130

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] \ :> \text{Simp}[A*\text{Cot}[e + f*x]*((b*\text{Csc}[e + f*x])^m/(f*m)), x] + \text{Dist}[(C*m + A*(m + 1))/(b^2*m), \text{Int}[(b*\text{Csc}[e + f*x])^{(m + 2)}, x], x] /; \text{FreeQ}\{b, e, f, A, C\}, x] \ \&\& \text{NeQ}[C*m + A*(m + 1), 0] \ \&\& \text{LeQ}[m, -1]$

Rule 4132

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*((A_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.)), x_Symbol] \ :> \text{Dist}[B/b, \text{Int}[(b*\text{Csc}[e + f*x])^{(m + 1)}, x], x] + \text{Int}[(b*\text{Csc}[e + f*x])^m*(A + C*\text{Csc}[e + f*x]^2), x] /; \text{FreeQ}\{b, e, f, A, B, C, m\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{(b + a \sec(c + dx))^3}{\sec^{\frac{9}{2}}(c + dx)} dx \\ &= \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\ &\quad + \frac{2}{9} \int \frac{10ab^2 + \frac{1}{2}b(27a^2 + 7b^2) \sec(c + dx) + \frac{1}{2}a(9a^2 + 5b^2) \sec^2(c + dx)}{\sec^{\frac{7}{2}}(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{2}{9} \int \frac{10ab^2 + \frac{1}{2}a(9a^2 + 5b^2) \sec^2(c + dx)}{\sec^{\frac{7}{2}}(c + dx)} dx \\
&\quad + \frac{1}{9}(b(27a^2 + 7b^2)) \int \frac{1}{\sec^{\frac{5}{2}}(c + dx)} dx \\
&= \frac{40ab^2 \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{2b(27a^2 + 7b^2) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
&\quad + \frac{1}{15}(b(27a^2 + 7b^2)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx + \frac{1}{7}(a(7a^2 + 15b^2)) \int \frac{1}{\sec^{\frac{3}{2}}(c + dx)} dx \\
&= \frac{40ab^2 \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{2b(27a^2 + 7b^2) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} + \frac{2a(7a^2 + 15b^2) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} + \frac{1}{21}(a(7a^2 + 15b^2)) \int \sqrt{\sec(c + dx)} dx \\
&\quad + \frac{1}{15} \left(b(27a^2 + 7b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx \\
&= \frac{2b(27a^2 + 7b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{15d} \\
&\quad + \frac{40ab^2 \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{2b(27a^2 + 7b^2) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} \\
&\quad + \frac{2a(7a^2 + 15b^2) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)} \\
&\quad + \frac{1}{21} \left(a(7a^2 + 15b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2b(27a^2 + 7b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{15d} \\
&\quad + \frac{2a(7a^2 + 15b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{21d} \\
&\quad + \frac{40ab^2 \sin(c + dx)}{63d \sec^{\frac{5}{2}}(c + dx)} + \frac{2b(27a^2 + 7b^2) \sin(c + dx)}{45d \sec^{\frac{3}{2}}(c + dx)} \\
&\quad + \frac{2a(7a^2 + 15b^2) \sin(c + dx)}{21d \sqrt{\sec(c + dx)}} + \frac{2b^2(b + a \sec(c + dx)) \sin(c + dx)}{9d \sec^{\frac{7}{2}}(c + dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.68

$$\int \frac{(a + b \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{\sqrt{\sec(c + dx)} \left(168b(27a^2 + 7b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 120a(7a^2 + 15b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{c + dx}{2}, 2\right) + (7b(108a^2 + 43b^2)\cos(c + dx) + 5(84a^3 + 234ab^2 + 54a^2b^2\cos[2(c + dx)] + 7b^3\cos[3(c + dx)])\sin[2(c + dx)]) \right)}{(1260d)}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Sec[c + d*x]^(3/2), x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*b*(27*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 120*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*b*(108*a^2 + 43*b^2)*Cos[c + d*x] + 5*(84*a^3 + 234*a*b^2 + 54*a^2*b^2*Cos[2*(c + d*x)] + 7*b^3*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)])/(1260*d)
```

Maple [A] (verified)

Time = 13.29 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.01

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(-1120\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b^3 + (2160ab^2 + 2240b^3)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \dots}{d}$
parts	Expression too large to display

```
[In] int((a+cos(d*x+c)*b)^3/sec(d*x+c)^(3/2), x, method=_RETURNVERBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*b^3+(2160*a*b^2+2240*b^3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*a^2*b-3240*a*b^2-2072*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(420*a^3+1512*a^2*b+2520*a*b^2+952*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*a^3-378*a^2*b-720*a*b^2-168*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+225*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-567*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*a^2*b-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02

$$\int \frac{(a + b \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx =$$

$$15 \sqrt{2}(7i a^3 + 15i ab^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 15 \sqrt{2}(-7i a^3 - 15i ab^2) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 21 \sqrt{2}(27a^2b + 7b^3) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 21 \sqrt{2}(27a^2b + 7b^3) \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) - 2(35b^3 \cos(dx + c)^4 + 135ab^2 \cos(dx + c)^3 + 7(27a^2b + 7b^3) \cos(dx + c)^2 + 15(7a^3 + 15ab^2) \cos(dx + c)) \sin(dx + c) / \sqrt{\cos(dx + c)}) / d$$

[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] -1/315*(15*sqrt(2)*(7*I*a^3 + 15*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-7*I*a^3 - 15*I*a*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-27*I*a^2*b - 7*I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(27*I*a^2*b + 7*I*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(35*b^3*cos(d*x + c)^4 + 135*a*b^2*cos(d*x + c)^3 + 7*(27*a^2*b + 7*b^3)*cos(d*x + c)^2 + 15*(7*a^3 + 15*a*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))/d

Sympy [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)

[Out] Integral((a + b*cos(c + d*x))**3/sec(c + d*x)**(3/2), x)

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^3}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^3}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^3}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^3}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

[In] int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2),x)

[Out] int((a + b*cos(c + d*x))^3/(1/cos(c + d*x))^(3/2), x)

$$3.712 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b \cos(c+dx)} dx$$

Optimal result	6475
Rubi [A] (verified)	6476
Mathematica [A] (verified)	6479
Maple [A] (verified)	6479
Fricas [F(-1)]	6480
Sympy [F(-1)]	6480
Maxima [F]	6480
Giac [F]	6481
Mupad [F(-1)]	6481

Optimal result

Integrand size = 23, antiderivative size = 188

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b \cos(c+dx)} dx = \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} + \frac{2b^2\sqrt{\cos(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} - \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}$$

```
[Out] 2/3*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d+2
*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2/3*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a^2/(a+b)/d
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3317, 3936, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a^2d(a+b)} - \frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{a^2d} + \frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad} + \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3ad}$$

[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]

[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d) + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)]

$(b + a \operatorname{Csc}[e + f x]^n)^p, x], x] /;$ FreeQ[{a, b, d, e, f, m, n, p}, x] &&
!IntegerQ[m] && IntegerQ[n, p]

Rule 3856

$\operatorname{Int}[(\operatorname{csc}[c] + (d)(x))(b)^n, x_Symbol] \rightarrow \operatorname{Dist}[(b \operatorname{Csc}[c + dx])^n \operatorname{Sin}[c + dx]^n, \operatorname{Int}[1/\operatorname{Sin}[c + dx]^n, x], x] /;$ FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]

Rule 3872

$\operatorname{Int}[(\operatorname{csc}[e] + (f)(x))(d)^n (\operatorname{csc}[e] + (f)(x))(b) + (a)], x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[(d \operatorname{Csc}[e + fx])^n, x], x] + \operatorname{Dist}[b/d, \operatorname{Int}[(d \operatorname{Csc}[e + fx])^{n+1}, x], x] /;$ FreeQ[{a, b, d, e, f, n}, x]

Rule 3934

$\operatorname{Int}[(\operatorname{csc}[e] + (f)(x))(d)^{3/2} / (\operatorname{csc}[e] + (f)(x))(b) + (a)], x_Symbol] \rightarrow \operatorname{Dist}[d \operatorname{Sqrt}[d \operatorname{Sin}[e + fx]] \operatorname{Sqrt}[d \operatorname{Csc}[e + fx]], \operatorname{Int}[1 / (\operatorname{Sqrt}[d \operatorname{Sin}[e + fx]] (b + a \operatorname{Sin}[e + fx])), x], x] /;$ FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3936

$\operatorname{Int}[(\operatorname{csc}[e] + (f)(x))(d)^n / (\operatorname{csc}[e] + (f)(x))(b) + (a)], x_Symbol] \rightarrow \operatorname{Simp}[(-d^3) \operatorname{Cot}[e + fx] ((d \operatorname{Csc}[e + fx])^{n-3} / (b f (n-2))), x] + \operatorname{Dist}[d^3 / (b(n-2)), \operatorname{Int}[(d \operatorname{Csc}[e + fx])^{n-3} (\operatorname{Simp}[a(n-3) + b(n-3) \operatorname{Csc}[e + fx] - a(n-2) \operatorname{Csc}[e + fx]^2, x] / (a + b \operatorname{Csc}[e + fx])), x], x] /;$ FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3]

Rule 4187

$\operatorname{Int}[(A) + \operatorname{csc}[e] + (f)(x)(B) + \operatorname{csc}[e] + (f)(x)]^2 (C) (\operatorname{csc}[e] + (f)(x))(d)^n (\operatorname{csc}[e] + (f)(x))(b) + (a))^m, x_Symbol] \rightarrow \operatorname{Simp}[(-C) d \operatorname{Cot}[e + fx] (a + b \operatorname{Csc}[e + fx])^{m+1} ((d \operatorname{Csc}[e + fx])^{n-1} / (b f (m+n+1))), x] + \operatorname{Dist}[d / (b(m+n+1)), \operatorname{Int}[(a + b \operatorname{Csc}[e + fx])^m (d \operatorname{Csc}[e + fx])^{n-1} \operatorname{Simp}[a C (n-1) + (A + b(m+n+1) + b C (m+n)) \operatorname{Csc}[e + fx] + (b B (m+n+1) - a C n) \operatorname{Csc}[e + fx]^2, x], x], x] /;$ FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]

Rule 4191

$\operatorname{Int}[(A) + \operatorname{csc}[e] + (f)(x)(B) + \operatorname{csc}[e] + (f)(x)]^2 (C) / (\operatorname{Sqrt}[\operatorname{csc}[e] + (f)(x)(d)] (\operatorname{csc}[e] + (f)(x))(b) + (a))$

$_))$, x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{\sec^{\frac{7}{2}}(c+dx)}{b+a\sec(c+dx)} dx \\
 &= \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{b}{2} + \frac{1}{2}a\sec(c+dx) - \frac{3}{2}b\sec^2(c+dx)\right)}{b+a\sec(c+dx)} dx}{3a} \\
 &= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} \\
 &\quad + \frac{4\int \frac{\frac{3b^2}{4} + ab\sec(c+dx) + \frac{1}{4}(a^2+3b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))} dx}{3a^2} \\
 &= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} \\
 &\quad + \frac{4\int \frac{\frac{3b^3}{4} + \frac{1}{4}ab^2\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{3a^2b^2} + \frac{b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)} dx}{a^2} \\
 &= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\int \sqrt{\sec(c+dx)} dx}{3a} \\
 &\quad + \frac{b\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{a^2} + \frac{\left(b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{a^2} \\
 &= \frac{2b^2\sqrt{\cos(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} \\
 &\quad - \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3a} \\
 &\quad + \frac{\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)} dx}{a^2}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} \\
&+ \frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3ad} \\
&+ \frac{2b^2\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} \\
&- \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 35.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.88

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \frac{\cot(c+dx)\left(-a^2\sec^{\frac{5}{2}}(c+dx)+a^2\cos(2(c+dx))\sec^{\frac{5}{2}}(c+dx)+6abE\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\middle| -1\right)\right)}{a^3}$$

[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]

[Out] $-1/3*(\cot[c + d*x]*(-a^2*\sec[c + d*x]^{(5/2)}) + a^2*\cos[2*(c + d*x)]*\sec[c + d*x]^{(5/2)} + 6*a*b*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[\sec[c + d*x]]], -1]*\operatorname{Sqrt}[-\tan[c + d*x]^2] - 2*(a^2 + 3*a*b + 3*b^2)*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[\sec[c + d*x]]], -1]*\operatorname{Sqrt}[-\tan[c + d*x]^2] + 6*b^2*\operatorname{EllipticPi}[-(a/b), \operatorname{ArcSin}[\operatorname{Sqrt}[\sec[c + d*x]]], -1]*\operatorname{Sqrt}[-\tan[c + d*x]^2]))/(a^3*d)$

Maple [A] (verified)

Time = 11.16 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.26

method	result
default	$ \frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}}{3\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2} + \frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{a\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}\right)} $

[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)

[Out] $-(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2/a*(-1/6*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c$

)²⁺¹)^(1/2)/(-2*sin(1/2*d*x+1/2*c)⁴+sin(1/2*d*x+1/2*c)²)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/a²*b/sin(1/2*d*x+1/2*c)²/(2*sin(1/2*d*x+1/2*c)²-1)*(-2*sin(1/2*d*x+1/2*c)⁴+sin(1/2*d*x+1/2*c)²)^(1/2)*(2*sin(1/2*d*x+1/2*c)²*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)²)^(1/2)*(2*sin(1/2*d*x+1/2*c)²-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4*b³/a²/(-2*a*b+2*b²)*(sin(1/2*d*x+1/2*c)²)^(1/2)*(-2*cos(1/2*d*x+1/2*c)²⁺¹)^(1/2)/(-2*sin(1/2*d*x+1/2*c)⁴+sin(1/2*d*x+1/2*c)²)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)²-1)^(1/2)/d

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{a+b\cos(c+dx)} dx$$

[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x)),x)

[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x)), x)

3.713 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	6482
Rubi [A] (verified)	6482
Mathematica [A] (verified)	6484
Maple [B] (verified)	6485
Fricas [F(-1)]	6485
Sympy [F]	6485
Maxima [F]	6486
Giac [F]	6486
Mupad [F(-1)]	6486

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b \cos(c+dx)} dx = -\frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{a(a+b)d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{ad}$$

[Out] 2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a+b)/d

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3317, 3935, 3853, 3856, 2719, 3934, 2884}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b \cos(c+dx)} dx = -\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{ad(a+b)} + \frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad} - \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{ad}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]

[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (2*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3935

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] :> Dist[d/b, Int[(d*Csc[e + f*x])^(3/2), x], x] - Dist[a*(
d/b), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sec^{\frac{5}{2}}(c + dx)}{b + a \sec(c + dx)} dx \\
&= \frac{\int \sec^{\frac{3}{2}}(c + dx) dx}{a} - \frac{b \int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a \sec(c+dx)} dx}{a} \\
&= \frac{2\sqrt{\sec(c + dx)} \sin(c + dx)}{ad} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{a} \\
&\quad - \frac{\left(b\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)(a+b \cos(c+dx))}} dx}{a} \\
&= -\frac{2b\sqrt{\cos(c + dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{a(a + b)d} \\
&\quad + \frac{2\sqrt{\sec(c + dx)} \sin(c + dx)}{ad} - \frac{\left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{a} \\
&= -\frac{2\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{ad} \\
&\quad - \frac{2b\sqrt{\cos(c + dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{a(a + b)d} \\
&\quad + \frac{2\sqrt{\sec(c + dx)} \sin(c + dx)}{ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 14.83 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.71

$$\begin{aligned}
&\int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + b \cos(c + dx)} dx \\
&= \frac{2 \cot(c + dx) \left(a E\left(\arcsin\left(\sqrt{\sec(c + dx)} \right) \mid -1 \right) - (a + b) \text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c + dx)} \right), -1 \right) + b \text{EllipticPi}\left(-\frac{a}{b}, \arcsin\left(\sqrt{\sec(c + dx)} \right) \right) \right)}{a^2 d}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]

[Out] (2*Cot[c + d*x]*(a*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1] - (a + b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + b*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(a^2*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(159) = 318$.

Time = 3.50 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.03

method	result
default	$-\frac{2\left(-2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}(a-b)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{d}$

[In] `int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)`

[Out]
$$-2*(-2*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(a-b)*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2+(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*a-(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b-b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)})/a/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(a-b)/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \text{Timed out}$$

[In] `integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)} dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)} dx$$

[In] `integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)`

[Out] `Integral(sec(c+d*x)**(3/2)/(a+b*cos(c+d*x)),x)`

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{b \cos(dx + c) + a} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{b \cos(dx + c) + a} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{a + b \cos(c + dx)} dx$$

[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x)),x)

[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x)), x)

3.714 $\int \frac{\sqrt{\sec(c+dx)}}{a+b \cos(c+dx)} dx$

Optimal result	6487
Rubi [A] (verified)	6487
Mathematica [A] (verified)	6488
Maple [B] (verified)	6489
Fricas [F(-1)]	6489
Sympy [F]	6489
Maxima [F]	6490
Giac [F]	6490
Mupad [F(-1)]	6490

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b \cos(c+dx)} dx = \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{(a+b)d}$$

[Out] $2*(\cos(1/2*d*x+1/2*c))^{1/2}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^{1/2})*\cos(d*x+c)^{1/2}*\sec(d*x+c)^{1/2}/(a+b)/d$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3317, 3934, 2884}

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b \cos(c+dx)} dx = \frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{d(a+b)}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]/(a+b*\operatorname{Cos}[c+d*x]),x]$

[Out] $(2*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{EllipticPi}[(2*b)/(a+b), (c+d*x)/2, 2]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]])/((a+b)*d)$

Rule 2884

$\operatorname{Int}[1/(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])*\operatorname{Sqrt}[(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])), x_Symbol] \rightarrow \operatorname{Simp}[(2/(f*(a+b)*\operatorname{Sqrt}[c+d]))*\operatorname{EllipticPi}[2*(b/(a+b)), (1/2)*(e - \operatorname{Pi}/2 + f*x), 2*(d/(c+d))], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[a^2 - b^2, 0] \&\& \operatorname{NeQ}[c^2 - d^2, 0] \&\& \operatorname{GtQ}[c + d, 0]$

Rule 3317

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{\sec^{\frac{3}{2}}(c + dx)}{b + a \sec(c + dx)} dx \\ &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))} dx \\ &= \frac{2\sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{(a + b)d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\begin{aligned} &\int \frac{\sqrt{\sec(c + dx)}}{a + b \cos(c + dx)} dx \\ &= \frac{2 \cot(c + dx) \left(\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\sec(c + dx)}\right), -1\right) - \operatorname{EllipticPi}\left(-\frac{a}{b}, \arcsin\left(\sqrt{\sec(c + dx)}\right), -1\right) \right)}{ad} \end{aligned}$$

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(a*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(71) = 142.

Time = 1.82 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.06

method	result	size
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(a-b\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\Pi\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),-\frac{2b}{a-b},\sqrt{2}\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d}$	150

[In] `int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)`

[Out] $-2\left(\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(-2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2+1\right)^{\frac{1}{2}}\text{EllipticPi}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),-2*b/(a-b),2^{\frac{1}{2}}\right)/(a-b)/\left(-2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}/d$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)} dx = \text{Timed out}$$

[In] `integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)} dx = \int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)} dx$$

[In] `integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)`

[Out] `Integral(sqrt(sec(c+d*x))/(a+b*cos(c+d*x)),x)`

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)} dx = \int \frac{\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)} dx = \int \frac{\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{a+b\cos(c+dx)} dx$$

[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x)),x)

[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x)), x)

$$3.715 \quad \int \frac{1}{(a+b \cos(c+dx)) \sqrt{\sec(c+dx)}} dx$$

Optimal result	6491
Rubi [A] (verified)	6491
Mathematica [A] (verified)	6493
Maple [A] (verified)	6493
Fricas [F(-1)]	6494
Sympy [F]	6494
Maxima [F]	6494
Giac [F]	6494
Mupad [F(-1)]	6495

Optimal result

Integrand size = 23, antiderivative size = 93

$$\begin{aligned} & \int \frac{1}{(a+b \cos(c+dx)) \sqrt{\sec(c+dx)}} dx \\ &= \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{bd} \\ & \quad - \frac{2a\sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{b(a+b)d} \end{aligned}$$

[Out] $2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/b/d-2*a*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticPi}(\sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}*\sec(d*x+c)^{(1/2)}/b/(a+b)/d$

Rubi [A] (verified)

Time = 0.22 (sec), antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3317, 3933, 2882, 2720, 2884}

$$\begin{aligned} & \int \frac{1}{(a+b \cos(c+dx)) \sqrt{\sec(c+dx)}} dx \\ &= \frac{2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{bd} \\ & \quad - \frac{2a\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{bd(a+b)} \end{aligned}$$

[In] $\operatorname{Int}[1/((a+b*\operatorname{Cos}[c+d*x])*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]),x]$

[Out] $(2\sqrt{\cos[c + dx]} \text{EllipticF}[(c + dx)/2, 2] \sqrt{\sec[c + dx]})/(b*d) - (2*a*\sqrt{\cos[c + dx]} \text{EllipticPi}[(2*b)/(a + b), (c + dx)/2, 2] \sqrt{\sec[c + dx]})/(b*(a + b)*d)$

Rule 2720

$\text{Int}[1/\sqrt{\sin[(c_.) + (d_.)*(x_)]}], x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticF}[(1/2)*(c - \text{Pi}/2 + dx), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2882

$\text{Int}[\sqrt{(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]}/((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] \rightarrow \text{Dist}[d/b, \text{Int}[1/\sqrt{c + d*\sin[e + f*x]}, x], x] + \text{Dist}[(b*c - a*d)/b, \text{Int}[1/((a + b*\sin[e + f*x])* \sqrt{c + d*\sin[e + f*x]})], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rule 2884

$\text{Int}[1/(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])*\sqrt{(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]})], x_Symbol] \rightarrow \text{Simp}[(2/(f*(a + b)*\sqrt{c + d}))*\text{EllipticPi}[2*(b/(a + b)), (1/2)*(e - \text{Pi}/2 + f*x), 2*(d/(c + d))], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[c + d, 0]$

Rule 3317

$\text{Int}[(\csc[(e_.) + (f_.)*(x_)]*(d_.))^m*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^n)^p, x_Symbol] \rightarrow \text{Dist}[d^{n*p}, \text{Int}[(d*\csc[e + f*x])^{m - n*p}*(b + a*\csc[e + f*x]^n)^p, x], x] /; \text{FreeQ}\{a, b, d, e, f, m, n, p\}, x] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n, p]$

Rule 3933

$\text{Int}[\sqrt{\csc[(e_.) + (f_.)*(x_)]*(d_.)}/(\csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] \rightarrow \text{Dist}[\sqrt{d*\sin[e + f*x]}*(\sqrt{d*\csc[e + f*x]}/d), \text{Int}[\sqrt{d*\sin[e + f*x]}/(b + a*\sin[e + f*x]), x], x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{\sqrt{\sec(c + dx)}}{b + a \sec(c + dx)} dx \\ &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\cos(c + dx)}}{a + b \cos(c + dx)} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{b} \\
&\quad - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)(a+b\cos(c+dx))}} dx}{b} \\
&= \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{bd} \\
&\quad - \frac{2a\sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{b(a+b)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} dx \\
&= \frac{2\cot(c+dx) \operatorname{EllipticPi}\left(-\frac{a}{b}, \arcsin\left(\sqrt{\sec(c+dx)}\right), -1\right) \sqrt{-\tan^2(c+dx)}}{bd}
\end{aligned}$$

[In] Integrate[1/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]), x]

[Out] (2*Cot[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2])/(b*d)

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.02

method	result
default	$ \frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a-F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{(a-b)b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}d $

[In] int(1/(a+cos(d*x+c)*b)/sec(d*x+c)^(1/2), x, method=_RETURNVERBOSE)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*b-a*EllipticPi(cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^(1/2)))/(a-b)/b/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx)) \sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(a + b \cos(c + dx)) \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a + b \cos(c + dx)) \sqrt{\sec(c + dx)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)

[Out] Integral(1/((a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx)) \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a) \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx)) \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a) \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx)) \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + b \cos(c + dx))} dx$$

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))), x)
```

$$3.716 \quad \int \frac{1}{(a+b \cos(c+dx)) \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	6496
Rubi [A] (verified)	6496
Mathematica [A] (verified)	6499
Maple [A] (verified)	6499
Fricas [F(-1)]	6500
Sympy [F]	6500
Maxima [F]	6500
Giac [F]	6500
Mupad [F(-1)]	6501

Optimal result

Integrand size = 23, antiderivative size = 135

$$\begin{aligned} & \int \frac{1}{(a+b \cos(c+dx)) \sec^{\frac{3}{2}}(c+dx)} dx \\ &= \frac{2\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{bd} \\ & \quad - \frac{2a\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{b^2d} \\ & \quad + \frac{2a^2\sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{b^2(a+b)d} \end{aligned}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/d+2*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a+b)/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used

= {3317, 3937, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{2a^2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{b^2 d (a + b)}$$

$$- \frac{2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{b^2 d}$$

$$+ \frac{2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{bd}$$

[In] Int[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]

[Out] (2*sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(b*d) - (2*a*sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(b^2*d) + (2*a^2*sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*sqrt[Sec[c + d*x]])/(b^2*(a + b)*d)

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&

EqQ[n^2, 1/4]

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3937

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))), x_Symbol] := Dist[b^2/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b
*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))} dx \\
&= \frac{\int \frac{b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{b^2} + \frac{a^2 \int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)} dx}{b^2} \\
&= -\frac{a \int \sqrt{\sec(c+dx)} dx}{b^2} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{b} \\
&\quad + \frac{\left(a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{b^2} \\
&= \frac{2a^2 \sqrt{\cos(c+dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{b^2(a+b)d} \\
&\quad - \frac{\left(a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{b^2} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{b}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{bd} \\
&\quad - \frac{2a\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{b^2d} \\
&\quad + \frac{2a^2\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 19.53 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)} dx$$

$$= \frac{\cot(c+dx)\left(-b\sec^{\frac{3}{2}}(c+dx) - b\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx) + b\sec^{\frac{7}{2}}(c+dx) + b\cos(2(c+dx))\sec^{\frac{7}{2}}(c+dx)\right)}{b^2(a+b)d}$$

[In] Integrate[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]

[Out] (Cot[c + d*x]*(-(b*Sec[c + d*x]^(3/2)) - b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + b*Sec[c + d*x]^(7/2) + b*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2) - 2*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*a*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(b^2*d)

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.68

method	result
default	$ \frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)a^2-F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{b^2(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}} $

[In] int(1/(a+cos(d*x+c)*b)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*cos(c + d*x))*sec(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```


Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + b \cos(c + dx))} dx$$

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))), x)
```

$$3.717 \quad \int \frac{1}{(a+b \cos(c+dx)) \sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	6502
Rubi [A] (verified)	6503
Mathematica [A] (verified)	6505
Maple [B] (verified)	6506
Fricas [F(-1)]	6506
Sympy [F(-1)]	6507
Maxima [F]	6507
Giac [F]	6507
Mupad [F(-1)]	6507

Optimal result

Integrand size = 23, antiderivative size = 172

$$\begin{aligned} & \int \frac{1}{(a+b \cos(c+dx)) \sec^{\frac{5}{2}}(c+dx)} dx \\ &= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{b^2d} \\ & \quad + \frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{3b^3d} \\ & \quad - \frac{2a^3\sqrt{\cos(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{b^3(a+b)d} + \frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} \end{aligned}$$

```
[Out] 2/3*sin(d*x+c)/b/d/sec(d*x+c)^(1/2)-2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/d+2/3*(3*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/d-2*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a+b)/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3317, 3938, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx$$

$$= -\frac{2a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{b^3 d(a + b)}$$

$$+ \frac{2(3a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3b^3 d}$$

$$- \frac{2a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{b^2 d} + \frac{2 \sin(c + dx)}{3bd \sqrt{\sec(c + dx)}}$$

[In] Int[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]

[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*d) + (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^3*d) - (2*a^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a + b)*d) + (2*Sin[c + d*x])/(3*b*d*Sqrt[Sec[c + d*x]])

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&

!IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3938

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dist[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n - a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 4191

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))} dx \\ &= \frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3a}{2} + \frac{1}{2}b\sec(c+dx) + \frac{1}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))} dx}{3b} \end{aligned}$$

$$\begin{aligned}
&= \frac{2 \sin(c + dx)}{3bd\sqrt{\sec(c + dx)}} + \frac{2 \int \frac{-\frac{3ab}{2} - \left(-\frac{3a^2}{2} - \frac{b^2}{2}\right) \sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{3b^3} - \frac{a^3 \int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a \sec(c+dx)} dx}{b^3} \\
&= \frac{2 \sin(c + dx)}{3bd\sqrt{\sec(c + dx)}} - \frac{a \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{b^2} + \frac{(3a^2 + b^2) \int \sqrt{\sec(c + dx)} dx}{3b^3} \\
&\quad - \frac{\left(a^3 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)(a+b \cos(c+dx))}} dx}{b^3} \\
&= -\frac{2a^3 \sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{b^3(a + b)d} \\
&\quad + \frac{2 \sin(c + dx)}{3bd\sqrt{\sec(c + dx)}} - \frac{\left(a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{b^2} \\
&\quad + \frac{\left((3a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b^3} \\
&= -\frac{2a \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{b^2 d} \\
&\quad + \frac{2(3a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3b^3 d} \\
&\quad - \frac{2a^3 \sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{b^3(a + b)d} + \frac{2 \sin(c + dx)}{3bd\sqrt{\sec(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 21.42 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx = \frac{\cot(c + dx) \left(-b^2 \sqrt{\sec(c + dx)} + 6ab \sec^{\frac{3}{2}}(c + dx) - 6ab \cos(2(c + dx)) \sec^{\frac{3}{2}}(c + dx) + b^2 \cos(3(c + dx)) \right)}{b^3(a + b)d}$$

[In] Integrate[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]

[Out] -1/6*(Cot[c + d*x]*(-b^2*sqrt[Sec[c + d*x]]) + 6*a*b*Sec[c + d*x]^(3/2) - 6*a*b*cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + b^2*cos[3*(c + d*x)]*Sec[c + d*x]^(3/2) - 12*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*sqrt[-Tan[c + d*x]^2] + 4*(3*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*sqrt[-Tan[c + d*x]^2] - 12*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*sqrt[-Tan[c + d*x]^2]))/(b^3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(232) = 464$.

Time = 3.93 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.21

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b^2-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^3-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}\right)$

[In] `int(1/(a+cos(d*x+c)*b)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$-\frac{2}{3} \left((2 \cos(1/2 d x + 1/2 c))^2 - 1 \right) \sin(1/2 d x + 1/2 c)^2)^{1/2} \left(4 \cos(1/2 d x + 1/2 c) \sin(1/2 d x + 1/2 c)^4 a^2 b^2 - 4 \cos(1/2 d x + 1/2 c) \sin(1/2 d x + 1/2 c)^4 b^3 - 2 \cos(1/2 d x + 1/2 c) \sin(1/2 d x + 1/2 c)^2 a^2 b^2 + 2 \cos(1/2 d x + 1/2 c) \sin(1/2 d x + 1/2 c)^2 b^3 + 3 a^3 \left(\sin(1/2 d x + 1/2 c)^2 \right)^{1/2} \left(2 \sin(1/2 d x + 1/2 c)^2 - 1 \right)^{1/2} \operatorname{EllipticF}\left(\cos(1/2 d x + 1/2 c), 2^{1/2}\right) - 3 a^2 b \left(\sin(1/2 d x + 1/2 c)^2 \right)^{1/2} \left(2 \sin(1/2 d x + 1/2 c)^2 - 1 \right)^{1/2} \operatorname{EllipticF}\left(\cos(1/2 d x + 1/2 c), 2^{1/2}\right) + a b^2 \left(\sin(1/2 d x + 1/2 c)^2 \right)^{1/2} \left(2 \sin(1/2 d x + 1/2 c)^2 - 1 \right)^{1/2} \operatorname{EllipticF}\left(\cos(1/2 d x + 1/2 c), 2^{1/2}\right) - b^3 \left(\sin(1/2 d x + 1/2 c)^2 \right)^{1/2} \left(2 \sin(1/2 d x + 1/2 c)^2 - 1 \right)^{1/2} \operatorname{EllipticF}\left(\cos(1/2 d x + 1/2 c), 2^{1/2}\right) + 3 \left(\sin(1/2 d x + 1/2 c)^2 \right)^{1/2} \left(2 \sin(1/2 d x + 1/2 c)^2 - 1 \right)^{1/2} \operatorname{EllipticE}\left(\cos(1/2 d x + 1/2 c), 2^{1/2}\right) a^2 b - 3 \left(\sin(1/2 d x + 1/2 c)^2 \right)^{1/2} \left(2 \sin(1/2 d x + 1/2 c)^2 - 1 \right)^{1/2} \operatorname{EllipticE}\left(\cos(1/2 d x + 1/2 c), 2^{1/2}\right) a b^2 - 3 a^3 \left(\sin(1/2 d x + 1/2 c)^2 \right)^{1/2} \left(2 \sin(1/2 d x + 1/2 c)^2 - 1 \right)^{1/2} \operatorname{EllipticPi}\left(\cos(1/2 d x + 1/2 c), -2 b / (a - b), 2^{1/2}\right) \right) / b^3 / (a - b) / \left(-2 \sin(1/2 d x + 1/2 c)^4 + \sin(1/2 d x + 1/2 c)^2 \right)^{1/2} / \sin(1/2 d x + 1/2 c) / \left(2 \cos(1/2 d x + 1/2 c)^2 - 1 \right)^{1/2} / d$$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] `integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a) \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx)) \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + b \cos(c + dx))} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))),x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))), x)

$$3.718 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	6508
Rubi [A] (verified)	6509
Mathematica [A] (verified)	6513
Maple [B] (verified)	6513
Fricas [F(-1)]	6514
Sympy [F(-1)]	6514
Maxima [F]	6514
Giac [F]	6515
Mupad [F(-1)]	6515

Optimal result

Integrand size = 23, antiderivative size = 341

$$\begin{aligned} & \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx \\ &= \frac{b(4a^2 - 5b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{a^3 (a^2 - b^2) d} \\ &+ \frac{(2a^2 - 5b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{3a^2 (a^2 - b^2) d} \\ &+ \frac{b^2(7a^2 - 5b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{a^3 (a-b)(a+b)^2 d} \\ &- \frac{b(4a^2 - 5b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{a^3 (a^2 - b^2) d} \\ &+ \frac{(2a^2 - 5b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2 (a^2 - b^2) d} + \frac{b^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{a (a^2 - b^2) d (b + a \sec(c+dx))} \end{aligned}$$

```
[Out] 1/3*(2*a^2-5*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)/d+b^2*sec(d*x+c)^(5/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))-b*(4*a^2-5*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+b*(4*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)/d+1/3*(2*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+b^2*(7*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)/(a+b)^2/d
```


Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3317, 3930, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\begin{aligned} & \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx \\ &= \frac{b^2 \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)} + \frac{(2a^2-5b^2) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{3a^2d(a^2-b^2)} \\ &+ \frac{(2a^2-5b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3a^2d(a^2-b^2)} \\ &- \frac{b(4a^2-5b^2) \sin(c+dx) \sqrt{\sec(c+dx)}}{a^3d(a^2-b^2)} \\ &+ \frac{b(4a^2-5b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^3d(a^2-b^2)} \\ &+ \frac{b^2(7a^2-5b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a^3d(a-b)(a+b)^2} \end{aligned}$$

[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]

[Out] (b*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) + ((2*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d) + (b^2*(7*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) - (b*(4*a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((2*a^2 - 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c

, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^m]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^p, x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3930

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^n]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m, x_Symbol] := Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegerQ[n + 1/2, 2*m] && GtQ[n, 2]))

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4187

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)) * (csc[(e_.) + (f_.)*(x_)]*(d_.))^n * (csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^m, x_Symbol] := Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1) * ((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))

, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 0]

Rule 4191

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{\sec^{\frac{9}{2}}(c + dx)}{(b + a \sec(c + dx))^2} dx \\
 &= \frac{b^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{a(a^2 - b^2) d(b + a \sec(c + dx))} + \frac{\int \frac{\sec^{\frac{3}{2}}(c + dx) \left(\frac{3b^2}{2} - ab \sec(c + dx) + \frac{1}{2}(2a^2 - 5b^2) \sec^2(c + dx) \right)}{b + a \sec(c + dx)} dx}{a(a^2 - b^2)} \\
 &= \frac{(2a^2 - 5b^2) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3a^2(a^2 - b^2) d} + \frac{b^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{a(a^2 - b^2) d(b + a \sec(c + dx))} \\
 &\quad + \frac{2 \int \frac{\sqrt{\sec(c + dx)} \left(\frac{1}{4}b(2a^2 - 5b^2) + \frac{1}{2}a(a^2 + 2b^2) \sec(c + dx) - \frac{3}{4}b(4a^2 - 5b^2) \sec^2(c + dx) \right)}{b + a \sec(c + dx)} dx}{3a^2(a^2 - b^2)} \\
 &= -\frac{b(4a^2 - 5b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{a^3(a^2 - b^2) d} \\
 &\quad + \frac{(2a^2 - 5b^2) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3a^2(a^2 - b^2) d} + \frac{b^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{a(a^2 - b^2) d(b + a \sec(c + dx))} \\
 &\quad + \frac{4 \int \frac{\frac{3}{8}b^3(4a^2 - 5b^2) + \frac{1}{4}ab(7a^2 - 10b^2) \sec(c + dx) + \frac{1}{8}(2a^4 + 16a^2b^2 - 15b^4) \sec^2(c + dx)}{\sqrt{\sec(c + dx)}(b + a \sec(c + dx))} dx}{3a^3(a^2 - b^2)} \\
 &= -\frac{b(4a^2 - 5b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{a^3(a^2 - b^2) d} \\
 &\quad + \frac{(2a^2 - 5b^2) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3a^2(a^2 - b^2) d} + \frac{b^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{a(a^2 - b^2) d(b + a \sec(c + dx))} \\
 &\quad + \frac{4 \int \frac{\frac{3}{8}b^3(4a^2 - 5b^2) - \left(-\frac{1}{4}ab^2(7a^2 - 10b^2) + \frac{3}{8}ab^2(4a^2 - 5b^2)\right) \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx}{3a^3b^2(a^2 - b^2)} \\
 &\quad + \frac{(b^2(7a^2 - 5b^2)) \int \frac{\sec^{\frac{3}{2}}(c + dx)}{b + a \sec(c + dx)} dx}{2a^3(a^2 - b^2)}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{b(4a^2 - 5b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{a^3 (a^2 - b^2) d} \\
&+ \frac{(2a^2 - 5b^2) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3a^2 (a^2 - b^2) d} + \frac{b^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{a (a^2 - b^2) d(b + a \sec(c + dx))} \\
&+ \frac{(2a^2 - 5b^2) \int \sqrt{\sec(c + dx)} dx}{6a^2 (a^2 - b^2)} + \frac{(b(4a^2 - 5b^2)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{2a^3 (a^2 - b^2)} \\
&+ \frac{\left(b^2(7a^2 - 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)(a + b \cos(c + dx))}} dx}{2a^3 (a^2 - b^2)} \\
&= \frac{b^2(7a^2 - 5b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{a^3(a - b)(a + b)^2 d} \\
&- \frac{b(4a^2 - 5b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{a^3 (a^2 - b^2) d} \\
&+ \frac{(2a^2 - 5b^2) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3a^2 (a^2 - b^2) d} + \frac{b^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{a (a^2 - b^2) d(b + a \sec(c + dx))} \\
&+ \frac{\left((2a^2 - 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{6a^2 (a^2 - b^2)} \\
&+ \frac{\left(b(4a^2 - 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx}{2a^3 (a^2 - b^2)} \\
&= \frac{b(4a^2 - 5b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{a^3 (a^2 - b^2) d} \\
&+ \frac{(2a^2 - 5b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{3a^2 (a^2 - b^2) d} \\
&+ \frac{b^2(7a^2 - 5b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{a^3(a - b)(a + b)^2 d} \\
&- \frac{b(4a^2 - 5b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{a^3 (a^2 - b^2) d} \\
&+ \frac{(2a^2 - 5b^2) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3a^2 (a^2 - b^2) d} + \frac{b^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{a (a^2 - b^2) d(b + a \sec(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.85 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx$$

$$= \frac{2a \left(\frac{3b^2(-4a^2+5b^2)\sin(c+dx)}{a^2-b^2} + 2a(-5b+a\sec(c+dx))\tan(c+dx) \right)}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\cot(c+dx)(-6ab(4a^2-5b^2)E(\arcsin(\sqrt{\sec(c+dx)})|-1)\sqrt{-\tan^2(c+dx)}}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*cos[c + d*x])^2,x]
```

```
[Out] ((2*a*((3*b^2*(-4*a^2 + 5*b^2)*Sin[c + d*x])/(a^2 - b^2) + 2*a*(-5*b + a*Sec[c + d*x])*Tan[c + d*x]))/(a + b*cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-6*a*b*(4*a^2 - 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*(2*a^4 + 12*a^3*b + 16*a^2*b^2 - 15*a*b^3 - 15*b^4)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*b*(a*(4*a^2 - 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 + b*(-7*a^2 + 5*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a - b)*(a + b))/(6*a^4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. 2(397) = 794.

Time = 37.46 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.88

method	result	size
default	Expression too large to display	981

```
[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/a^3*b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2/a^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
```

$2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*EllipticF(\cos(1/2*d*x+1/2*c),2^{1/2})+1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*EllipticE(\cos(1/2*d*x+1/2*c),2^{1/2})-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{1/2})+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{1/2})))-8*b^3/a^3/(-2*a*b+2*b^2)*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{1/2}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{1/2}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{1/2})))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{1/2}/d$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{(a + b \cos(c + dx))^2} dx$$

[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^2, x)

$$3.719 \quad \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx$$

Optimal result	6516
Rubi [A] (verified)	6517
Mathematica [A] (verified)	6520
Maple [B] (verified)	6521
Fricas [F(-1)]	6521
Sympy [F]	6522
Maxima [F]	6522
Giac [F]	6522
Mupad [F(-1)]	6522

Optimal result

Integrand size = 23, antiderivative size = 277

$$\begin{aligned} & \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^2} dx \\ &= -\frac{(2a^2-3b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{a^2(a^2-b^2)d} \\ & \quad + \frac{b \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{a(a^2-b^2)d} \\ & \quad - \frac{b(5a^2-3b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} \\ & \quad + \frac{(2a^2-3b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{a^2(a^2-b^2)d} + \frac{b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{a(a^2-b^2)d(b+a \sec(c+dx))} \end{aligned}$$

```
[Out] b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))+(2*a^2-3*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-(2*a^2-3*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d+b*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-b*(5*a^2-3*b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)/(a+b)^2/d
```


Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3317, 3930, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx$$

$$= \frac{b^2 \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)(a\sec(c+dx)+b)} + \frac{(2a^2-3b^2) \sin(c+dx) \sqrt{\sec(c+dx)}}{a^2d(a^2-b^2)}$$

$$+ \frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{ad(a^2-b^2)}$$

$$- \frac{(2a^2-3b^2) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{a^2d(a^2-b^2)}$$

$$- \frac{b(5a^2-3b^2) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{a^2d(a-b)(a+b)^2}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]

[Out] -(((2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) + (b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) - (b*(5*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3930

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegerQ[n + 1/2, 2*m] && GtQ[n, 2]))

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4187

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)) * (csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2

- b^2, 0] && GtQ[n, 0]

Rule 4191

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b+a\sec(c+dx))^2} dx \\
 &= \frac{b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))} + \frac{\int \frac{\sqrt{\sec(c+dx)} \left(\frac{b^2}{2} - ab \sec(c+dx) + \frac{1}{2}(2a^2-3b^2) \sec^2(c+dx) \right)}{b+a\sec(c+dx)} dx}{a(a^2-b^2)} \\
 &= \frac{(2a^2-3b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{a^2(a^2-b^2)d} + \frac{b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))} \\
 &\quad + \frac{2 \int \frac{-\frac{1}{4}b(2a^2-3b^2) - \frac{1}{2}a(a^2-2b^2) \sec(c+dx) - \frac{1}{4}b(4a^2-3b^2) \sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))} dx}{a^2(a^2-b^2)} \\
 &= \frac{(2a^2-3b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{a^2(a^2-b^2)d} + \frac{b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))} \\
 &\quad + \frac{2 \int \frac{-\frac{1}{4}b^2(2a^2-3b^2) - (-\frac{1}{4}ab(2a^2-3b^2) + \frac{1}{2}ab(a^2-2b^2)) \sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{a^2b^2(a^2-b^2)} \\
 &\quad - \frac{(b(5a^2-3b^2)) \int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)} dx}{2a^2(a^2-b^2)} \\
 &= \frac{(2a^2-3b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{a^2(a^2-b^2)d} + \frac{b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))} \\
 &\quad + \frac{b \int \sqrt{\sec(c+dx)} dx}{2a(a^2-b^2)} - \frac{(2a^2-3b^2) \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a^2(a^2-b^2)} \\
 &\quad - \frac{\left(b(5a^2-3b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2a^2(a^2-b^2)}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{b(5a^2 - 3b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{a^2(a - b)(a + b)^2 d} \\
&+ \frac{(2a^2 - 3b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{a^2(a^2 - b^2) d} + \frac{b^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{a(a^2 - b^2) d(b + a \sec(c + dx))} \\
&+ \frac{\left(b \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{2a(a^2 - b^2)} \\
&- \frac{\left((2a^2 - 3b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{2a^2(a^2 - b^2)} \\
&= -\frac{(2a^2 - 3b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{a^2(a^2 - b^2) d} \\
&+ \frac{b \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{a(a^2 - b^2) d} \\
&- \frac{b(5a^2 - 3b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{a^2(a - b)(a + b)^2 d} \\
&+ \frac{(2a^2 - 3b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{a^2(a^2 - b^2) d} + \frac{b^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{a(a^2 - b^2) d(b + a \sec(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.27

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx$$

$$= \frac{2a(2a^2b - 3b^3 + 2a(a^2 - b^2) \sec(c + dx)) \sin(c + dx)}{(a^2 - b^2)(a + b \cos(c + dx)) \sqrt{\sec(c + dx)}} + \frac{\cot(c + dx) \left(-2a^3 \sec^{\frac{3}{2}}(c + dx) + 3ab^2 \sec^{\frac{3}{2}}(c + dx) + 2a^3 \cos(2(c + dx)) \sec^{\frac{3}{2}}(c + dx) - 3ab^2 \cos(2(c + dx)) \sec^{\frac{3}{2}}(c + dx)\right)}{(a^2 - b^2)(a + b \cos(c + dx)) \sqrt{\sec(c + dx)}}$$

[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]

[Out] ((2*a*(2*a^2*b - 3*b^3 + 2*a*(a^2 - b^2)*Sec[c + d*x])*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-2*a^3*Sec[c + d*x]^(3/2) + 3*a*b^2*Sec[c + d*x]^(3/2) + 2*a^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) - 3*a*b^2*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*a*(2*a^2 - 3*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*(2*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 10*a^2*b*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*b^3*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*(a + b))/(2*a^3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(339) = 678$.

Time = 5.98 (sec) , antiderivative size = 847, normalized size of antiderivative = 3.06

method	result	size
default	Expression too large to display	847

[In] `int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)`

[Out]
$$-\left(-\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(\frac{2}{a^2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\left(2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^{\frac{1}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)\right)+4b^2/a^2\left(-2ab+2b^2\right)\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{\frac{1}{2}}\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),-2b/(a-b),2^{\frac{1}{2}}\right)-2ab\left(-1/a^2b^2/(a^2-b^2)\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(2b\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a-b\right)-1/2/a/(a+b)\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{\frac{1}{2}}\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)-1/2/(a^2-b^2)b/a\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{\frac{1}{2}}\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)+1/2/(a^2-b^2)b/a\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{\frac{1}{2}}\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2^{\frac{1}{2}}\right)-3a/(a^2-b^2)\left(-2ab+2b^2\right)b\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{\frac{1}{2}}\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),-2b/(a-b),2^{\frac{1}{2}}\right)+1/a/(a^2-b^2)\left(-2ab+2b^2\right)b^3\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{\frac{1}{2}}\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{\frac{1}{2}}\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),-2b/(a-b),2^{\frac{1}{2}}\right)\right)/\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)/(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1)^{\frac{1}{2}}/d$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] `integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")`

[Out] Timed out

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx$$

[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)

[Out] Integral(sec(c + d*x)**(3/2)/(a + b*cos(c + d*x))**2, x)

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{(a + b \cos(c + dx))^2} dx$$

[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^2, x)

$$3.720 \quad \int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^2} dx$$

Optimal result	6523
Rubi [A] (verified)	6524
Mathematica [A] (verified)	6526
Maple [B] (verified)	6527
Fricas [F(-1)]	6528
Sympy [F]	6528
Maxima [F]	6528
Giac [F]	6528
Mupad [F(-1)]	6529

Optimal result

Integrand size = 23, antiderivative size = 217

$$\begin{aligned} & \int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^2} dx \\ &= -\frac{b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}}{a(a^2-b^2)d} \\ & \quad -\frac{\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{(a^2-b^2)d} \\ & \quad +\frac{(3a^2-b^2)\sqrt{\cos(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(c+dx),2\right)\sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} \\ & \quad +\frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))} \end{aligned}$$

```
[Out] b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))-b*(cos(1/2*d
*x+1/2*c)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d-(cos(1/2*d*x+1/2*c)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/(a^2-b^2)/d+(3*a^2-b^2)*(cos(1/2*d*x+1/2*c)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/a/(a-b)/(a+b)^2/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3317, 3930, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2} dx$$

$$= \frac{b^2 \sin(c+dx) \sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a\sec(c+dx)+b)} - \frac{\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d(a^2-b^2)}$$

$$- \frac{b \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{ad(a^2-b^2)}$$

$$+ \frac{(3a^2-b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{ad(a-b)(a+b)^2}$$

[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^2,x]

[Out] -((b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) - (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a^2 - b^2)*d) + ((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*(a + b)^2*d) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^m)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^n)^p, x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&

!IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3930

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2]))

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4191

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\text{integral} = \int \frac{\sec^{\frac{5}{2}}(c + dx)}{(b + a \sec(c + dx))^2} dx$$

$$\begin{aligned}
&= \frac{b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{b^2}{2} - ab \sec(c+dx) + \frac{1}{2}(2a^2-b^2) \sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))} dx}{a(a^2-b^2)} \\
&= \frac{b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{b^3}{2} - \frac{1}{2}ab^2 \sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{ab^2(a^2-b^2)} + \frac{(3a^2-b^2) \int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)} dx}{2a(a^2-b^2)} \\
&= \frac{b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)} dx}{2(a^2-b^2)} - \frac{b \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2a(a^2-b^2)} \\
&\quad + \frac{\left((3a^2-b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2a(a^2-b^2)} \\
&= \frac{(3a^2-b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} \\
&\quad + \frac{b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2(a^2-b^2)} \\
&\quad - \frac{\left(b \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx}{2a(a^2-b^2)} \\
&= -\frac{b \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{a(a^2-b^2)d} \\
&\quad - \frac{\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{(a^2-b^2)d} \\
&\quad + \frac{(3a^2-b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} \\
&\quad + \frac{b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{a(a^2-b^2)d(b+a\sec(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.34 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2} dx$$

$$= \frac{2ab^2 \sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{\cot(c+dx) \left(ab \sec^{\frac{3}{2}}(c+dx) + ab \cos(2(c+dx)) \sec^{\frac{3}{2}}(c+dx) - ab \sec^{\frac{7}{2}}(c+dx) - ab \cos(2(c+dx)) \sec^{\frac{7}{2}}(c+dx) \right)}{(a^2-b^2)(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^2,x]

[Out] ((2*a*b^2*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(a*b*Sec[c + d*x]^(3/2) + a*b*Cos[2*(c + d*x)]*Sec[c + d

$$\begin{aligned} & *x]^{(3/2)} - a*b*\text{Sec}[c + d*x]^{(7/2)} - a*b*\text{Cos}[2*(c + d*x)]*\text{Sec}[c + d*x]^{(7/2)} \\ &) + 2*a*b*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]], -1]*\text{Sqrt}[-\text{Tan}[c + d*x]^2] + \\ & 2*(2*a^2 - a*b - b^2)*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]], -1]*\text{Sqrt}[-\text{Tan}[\\ & c + d*x]^2] - 6*a^2*\text{EllipticPi}[-(a/b), \text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]], -1]*\text{Sqrt} \\ & [-\text{Tan}[c + d*x]^2] + 2*b^2*\text{EllipticPi}[-(a/b), \text{ArcSin}[\text{Sqrt}[\text{Sec}[c + d*x]]], -1 \\ &]*\text{Sqrt}[-\text{Tan}[c + d*x]^2])/((a - b)*(a + b))/(2*a^2*d) \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(281) = 562$.

Time = 4.03 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.82

method	result
default	$-\frac{\sqrt{-(-2\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1}(\sin^2(\frac{dx}{2} + \frac{c}{2}))}{a(a^2 - b^2)(2b\cos^2(\frac{dx}{2} + \frac{c}{2}) + a - b)} \left(-\frac{2b^2 \cos(\frac{dx}{2} + \frac{c}{2}) \sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2})) + \sin^2(\frac{dx}{2} + \frac{c}{2})}}{a(a^2 - b^2)(2b\cos^2(\frac{dx}{2} + \frac{c}{2}) + a - b)} - \frac{\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2(\cos^2(\frac{dx}{2} + \frac{c}{2}))}}{a(a+b)\sqrt{-2(\sin^4(\frac{dx}{2} + \frac{c}{2}))}} \right)$

[In] `int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2/a*b^2/(a^2-b \\ & ^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} \\ & / (2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-1/a/(a+b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2* \\ & \cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2 \\ &)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-1/(a^2-b^2)*b/a*(\sin(1/2*d*x+ \\ & 1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+ \\ & \sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+1/(a^2-b^2) \\ &)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-6*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2* \\ & \cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2 \\ &)^{(1/2)}*\text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})+2/a/(a^2-b^2)/(-2 \\ & *a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1 \\ & /2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\text{EllipticPi}(\cos(1/2 \\ & *d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})/(\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2 \\ & -1)^{(1/2)}/d \end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2} dx = \int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2} dx$$

[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)

[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**2, x)

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2} dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^2} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2} dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^2} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{(a+b\cos(c+dx))^2} dx$$

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^2, x)
```

$$3.721 \quad \int \frac{1}{(a+b \cos(c+dx))^2 \sqrt{\sec(c+dx)}} dx$$

Optimal result	6530
Rubi [A] (verified)	6531
Mathematica [B] (warning: unable to verify)	6533
Maple [B] (verified)	6534
Fricas [F(-1)]	6535
Sympy [F]	6535
Maxima [F]	6535
Giac [F]	6536
Mupad [F(-1)]	6536

Optimal result

Integrand size = 23, antiderivative size = 208

$$\begin{aligned} & \int \frac{1}{(a+b \cos(c+dx))^2 \sqrt{\sec(c+dx)}} dx \\ &= \frac{\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{(a^2-b^2)d} \\ &+ \frac{a \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{b(a^2-b^2)d} \\ &- \frac{(a^2+b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{(a-b)b(a+b)^2d} \\ &- \frac{b \sqrt{\sec(c+dx)} \sin(c+dx)}{(a^2-b^2)d(b+a \sec(c+dx))} \end{aligned}$$

```
[Out] -b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))+cos(1/2*d*x+1/2*c)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a^2-b^2)/d+a*(cos(1/2*d*x+1/2*c)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d-(a^2+b^2)*(cos(1/2*d*x+1/2*c)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b/(a+b)^2/d
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3317, 3929, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx$$

$$= -\frac{b \sin(c + dx) \sqrt{\sec(c + dx)}}{d(a^2 - b^2)(a \sec(c + dx) + b)} + \frac{a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{bd(a^2 - b^2)}$$

$$+ \frac{\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d(a^2 - b^2)}$$

$$- \frac{(a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{bd(a - b)(a + b)^2}$$

[In] Int[1/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]

[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/((a^2 - b^2)*d) + (a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) - ((a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/((a - b)*b*(a + b)^2*d) - (b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&

!IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n, x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3929

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^n*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m, x_Symbol] := Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Dist[d^2/((m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4191

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\text{integral} = \int \frac{\sec^{\frac{3}{2}}(c + dx)}{(b + a \sec(c + dx))^2} dx$$

$$\begin{aligned}
&= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(b+a\sec(c+dx))} - \frac{\int \frac{-\frac{b}{2}-a\sec(c+dx)+\frac{1}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))} dx}{a^2-b^2} \\
&= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(b+a\sec(c+dx))} - \frac{\int \frac{-\frac{b^2}{2}-\frac{1}{2}ab\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{b^2(a^2-b^2)} - \frac{(a^2+b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)} dx}{2b(a^2-b^2)} \\
&= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(b+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2(a^2-b^2)} + \frac{a\int \sqrt{\sec(c+dx)} dx}{2b(a^2-b^2)} \\
&\quad - \frac{\left((a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2b(a^2-b^2)} \\
&= -\frac{(a^2+b^2)\sqrt{\cos(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{(a-b)b(a+b)^2d} \\
&\quad - \frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(b+a\sec(c+dx))} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)} dx}{2(a^2-b^2)} \\
&\quad + \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2b(a^2-b^2)} \\
&= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{(a^2-b^2)d} \\
&\quad + \frac{a\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{b(a^2-b^2)d} \\
&\quad - \frac{(a^2+b^2)\sqrt{\cos(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{(a-b)b(a+b)^2d} \\
&\quad - \frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(b+a\sec(c+dx))}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 574 vs. 2(208) = 416.

Time = 6.41 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.76

$$\begin{aligned}
\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} dx &= \frac{\sqrt{\sec(c+dx)}\left(-\frac{\sin(c+dx)}{a^2-b^2} + \frac{a\sin(c+dx)}{(a^2-b^2)(a+b\cos(c+dx))}\right)}{d} \\
&\quad - \frac{2b\cos^2(c+dx)\left(\text{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right), -1\right) - \text{EllipticPi}\left(-\frac{a}{b}, \arcsin\left(\sqrt{\sec(c+dx)}\right), -1\right)\right)(b+a\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{a(a+b\cos(c+dx))(1-\cos^2(c+dx))} \\
&\quad + \dots
\end{aligned}$$

[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]

[Out] (Sqrt[Sec[c + d*x]]*(-(Sin[c + d*x]/(a^2 - b^2)) + (a*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])))/d + ((-2*b*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (8*a*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x]/(a*b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*(a + b)*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(272) = 544.

Time = 5.33 (sec) , antiderivative size = 713, normalized size of antiderivative = 3.43

method	result
default	$\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}{\left(-2ab + 2b^2\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} \left(-\frac{4\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\Pi\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), -\frac{2b}{a-b}, \sqrt{2}\right)}{(-2ab + 2b^2)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} - \frac{2a\left(-\frac{b^2\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}{a\left(\frac{dx}{2} + \frac{c}{2}\right)}\right)}{(-2ab + 2b^2)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)}} \right)$

[In] int(1/(a+cos(d*x+c)*b)^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^(1/2))-2/b*a*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-3*a/(a^2-b^2)/(-2

$$\frac{a^2 b + 2 b^2}{\sin(1/2 d x + 1/2 c)} \cdot \frac{(-2 \cos(1/2 d x + 1/2 c)^2 + 1)^{1/2}}{(-2 \sin(1/2 d x + 1/2 c)^4 + \sin(1/2 d x + 1/2 c)^2)^{1/2}} \cdot \frac{1}{(a^2 - b^2)^{1/2}} \cdot \frac{1}{(-2 a b + 2 b^2)^{1/2}} \cdot \frac{1}{(-2 \sin(1/2 d x + 1/2 c)^4 + \sin(1/2 d x + 1/2 c)^2)^{1/2}} \cdot \frac{1}{(-2 \cos(1/2 d x + 1/2 c)^2 + 1)^{1/2}} \cdot \frac{1}{d} \cdot \text{EllipticPi}(\cos(1/2 d x + 1/2 c), -2 b / (a - b), 2^{1/2})$$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)

[Out] Integral(1/((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a)^2 \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a)^2 \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + b \cos(c + dx))^2} dx$$

[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2),x)

[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^2), x)

$$3.722 \quad \int \frac{1}{(a+b \cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	6537
Rubi [A] (verified)	6538
Mathematica [A] (verified)	6541
Maple [B] (verified)	6541
Fricas [F(-1)]	6542
Sympy [F]	6542
Maxima [F]	6542
Giac [F]	6543
Mupad [F(-1)]	6543

Optimal result

Integrand size = 23, antiderivative size = 223

$$\begin{aligned} & \int \frac{1}{(a+b \cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)} dx \\ &= -\frac{a \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{b(a^2-b^2)d} \\ & \quad + \frac{(a^2-2b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{b^2(a^2-b^2)d} \\ & \quad - \frac{a(a^2-3b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} \\ & \quad + \frac{a \sqrt{\sec(c+dx)} \sin(c+dx)}{(a^2-b^2)d(b+a \sec(c+dx))} \end{aligned}$$

```
[Out] a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))-a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^2/(a+b)^2/d
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3317, 3928, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{a \sin(c + dx) \sqrt{\sec(c + dx)}}{d(a^2 - b^2)(a \sec(c + dx) + b)}$$

$$+ \frac{(a^2 - 2b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{b^2 d(a^2 - b^2)}$$

$$- \frac{a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{bd(a^2 - b^2)}$$

$$- \frac{a(a^2 - 3b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{b^2 d(a - b)(a + b)^2}$$

[In] Int[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]

[Out] -((a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) + ((a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) - (a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^2*(a + b)^2*d) + (a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3928

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4191

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sqrt{\sec(c+dx)}}{(b+a\sec(c+dx))^2} dx \\
&= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{a}{2}-b\sec(c+dx)+\frac{1}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))} dx}{a^2-b^2} \\
&= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(b+a\sec(c+dx))} + \frac{\left(a\left(3-\frac{a^2}{b^2}\right)\right) \int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)} dx}{2(a^2-b^2)} + \frac{\int \frac{-\frac{ab}{2}-\left(-\frac{a^2}{2}+b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{b^2(a^2-b^2)} \\
&= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(b+a\sec(c+dx))} - \frac{a \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2b(a^2-b^2)} + \frac{(a^2-2b^2) \int \sqrt{\sec(c+dx)} dx}{2b^2(a^2-b^2)} \\
&\quad + \frac{\left(a\left(3-\frac{a^2}{b^2}\right) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2(a^2-b^2)} \\
&= -\frac{a(a^2-3b^2) \sqrt{\cos(c+dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} \\
&\quad + \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(b+a\sec(c+dx))} \\
&\quad - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{2b(a^2-b^2)} \\
&\quad + \frac{\left((a^2-2b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2b^2(a^2-b^2)} \\
&= -\frac{a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right) \sqrt{\sec(c+dx)}}{b(a^2-b^2)d} \\
&\quad + \frac{(a^2-2b^2) \sqrt{\cos(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{b^2(a^2-b^2)d} \\
&\quad - \frac{a(a^2-3b^2) \sqrt{\cos(c+dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} \\
&\quad + \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)d(b+a\sec(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.40 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{\cos(2(c + dx)) \csc(c + dx) \sec^{\frac{3}{2}}(c + dx) \left(-b(-a + b)(a + b \cos(c + dx)) \operatorname{EllipticF} \left(\arcsin \left(\sqrt{\sec(c + dx)} \right) \right)}{\right)}$$

[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]

[Out] (Cos[2*(c + d*x)]*Csc[c + d*x]*Sec[c + d*x]^(3/2)*(-(b*(-a + b)*(a + b*Cos[c + d*x])*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]) - (a^2 - 3*b^2)*(a + b*Cos[c + d*x])*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + a*b*(a*Tan[c + d*x]^2 - (a + b*Cos[c + d*x])*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])))/((a - b)*b^2*(a + b)*d*(b + a*Sec[c + d*x])*(-2 + Sec[c + d*x]^2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. 2(287) = 574.

Time = 5.53 (sec) , antiderivative size = 794, normalized size of antiderivative = 3.56

method	result	size
default	Expression too large to display	794

[In] int(1/(a+cos(d*x+c)*b)^2/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a^2/b^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2

*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+8*a/b/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)

[Out] Integral(1/((a + b*cos(c + d*x))**2*sec(c + d*x)**(3/2)), x)

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + b \cos(c + dx))^2} dx$$

[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2),x)

[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^2), x)

$$3.723 \quad \int \frac{1}{(a+b \cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	6544
Rubi [A] (verified)	6545
Mathematica [A] (verified)	6548
Maple [B] (verified)	6548
Fricas [F(-1)]	6549
Sympy [F(-1)]	6549
Maxima [F]	6549
Giac [F]	6550
Mupad [F(-1)]	6550

Optimal result

Integrand size = 23, antiderivative size = 245

$$\begin{aligned} & \int \frac{1}{(a+b \cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)} dx \\ &= \frac{(3a^2 - 2b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{b^2 (a^2 - b^2) d} \\ & \quad - \frac{a(3a^2 - 4b^2) \sqrt{\cos(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{b^3 (a^2 - b^2) d} \\ & \quad + \frac{a^2(3a^2 - 5b^2) \sqrt{\cos(c+dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2 d} \\ & \quad - \frac{a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{b(a^2 - b^2) d(b + a \sec(c+dx))} \end{aligned}$$

```
[Out] -a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+a*sec(d*x+c))+(3*a^2-2*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*(3*a^2-4*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)/d+a^2*(3*a^2-5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^3/(a+b)^2/d
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3317, 3932, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx$$

$$= -\frac{a^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{bd(a^2 - b^2)(a \sec(c + dx) + b)}$$

$$+ \frac{(3a^2 - 2b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{b^2 d(a^2 - b^2)}$$

$$- \frac{a(3a^2 - 4b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{b^3 d(a^2 - b^2)}$$

$$+ \frac{a^2(3a^2 - 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{b^3 d(a - b)(a + b)^2}$$

[In] Int[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]

[Out] ((3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(b^2*(a^2 - b^2)*d) - (a*(3*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(b^3*(a^2 - b^2)*d) + (a^2*(3*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/((a - b)*b^3*(a + b)^2*d) - (a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((b*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3932

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] := Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1) - b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4191

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2} dx \\
&= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(b+a\sec(c+dx))} + \frac{\int \frac{\frac{3a^2}{2}-b^2+ab\sec(c+dx)-\frac{1}{2}a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))} dx}{b(a^2-b^2)} \\
&= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(b+a\sec(c+dx))} + \frac{\int \frac{b(\frac{3a^2}{2}-b^2)-(-ab^2+a(\frac{3a^2}{2}-b^2))\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{b^3(a^2-b^2)} \\
&\quad + \frac{(a^2(3a^2-5b^2))\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)} dx}{2b^3(a^2-b^2)} \\
&= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(b+a\sec(c+dx))} \\
&\quad - \frac{(a(3a^2-4b^2))\int \sqrt{\sec(c+dx)} dx}{2b^3(a^2-b^2)} + \frac{(3a^2-2b^2)\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{2b^2(a^2-b^2)} \\
&\quad + \frac{(a^2(3a^2-5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{2b^3(a^2-b^2)} \\
&= \frac{a^2(3a^2-5b^2)\sqrt{\cos(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} \\
&\quad - \frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(b+a\sec(c+dx))} \\
&\quad - \frac{(a(3a^2-4b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{2b^3(a^2-b^2)} \\
&\quad + \frac{((3a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})\int \sqrt{\cos(c+dx)} dx}{2b^2(a^2-b^2)} \\
&= \frac{(3a^2-2b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{b^2(a^2-b^2)d} \\
&\quad - \frac{a(3a^2-4b^2)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{b^3(a^2-b^2)d} \\
&\quad + \frac{a^2(3a^2-5b^2)\sqrt{\cos(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} \\
&\quad - \frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b(a^2-b^2)d(b+a\sec(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.83 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{4a^2 \sin(c+dx)}{b(-a^2+b^2)(a+b \cos(c+dx))\sqrt{\sec(c+dx)}} - \frac{2 \cot(c+dx) \left(-3a^2 b \sec^{\frac{3}{2}}(c+dx) + 2b^3 \sec^{\frac{3}{2}}(c+dx) + 3a^2 b \cos(2(c+dx)) \sec^{\frac{3}{2}}(c+dx) - 2b^3 \cos(2(c+dx)) \right)}{b(-a^2+b^2)(a+b \cos(c+dx))\sqrt{\sec(c+dx)}}$$

[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]

[Out] ((4*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) - (2*Cot[c + d*x]*(-3*a^2*b*Sec[c + d*x]^(3/2) + 2*b^3*Sec[c + d*x]^(3/2) + 3*a^2*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) - 2*b^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*b*(3*a^2 - 2*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*b*(-3*a^2 + a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^3*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 10*a*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a - b)*b^3*(a + b))/(4*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. 2(309) = 618.

Time = 6.51 (sec) , antiderivative size = 815, normalized size of antiderivative = 3.33

method	result	size
default	Expression too large to display	815

[In] int(1/(a+cos(d*x+c)*b)^2/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+b*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-12/b^2*a^2/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/b^3*a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)

$$\begin{aligned} & *(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2 \\ & *c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3*a/(a^2-b^2) / (-2*a*b+2* \\ & b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*s \\ & \sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2* \\ & c), -2*b/(a-b), 2^{(1/2)}) + 1/a/(a^2-b^2) / (-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c) \\ & ^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4 + \sin(1/ \\ & 2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})) / s \\ & \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)} / d \end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^2 \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^2 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + b \cos(c + dx))^2} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2),x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^2), x)

$$3.724 \quad \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$$

Optimal result	6551
Rubi [A] (verified)	6552
Mathematica [A] (warning: unable to verify)	6557
Maple [B] (verified)	6558
Fricas [F(-1)]	6559
Sympy [F(-1)]	6559
Maxima [F(-1)]	6560
Giac [F]	6560
Mupad [F(-1)]	6560

Optimal result

Integrand size = 23, antiderivative size = 455

$$\begin{aligned} & \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx \\ &= \frac{b(24a^4 - 65a^2b^2 + 35b^4) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{4a^4 (a^2 - b^2)^2 d} \\ &+ \frac{(8a^4 - 61a^2b^2 + 35b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{12a^3 (a^2 - b^2)^2 d} \\ &+ \frac{b^2(63a^4 - 86a^2b^2 + 35b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a^4 (a-b)^2 (a+b)^3 d} \\ &- \frac{b(24a^4 - 65a^2b^2 + 35b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^4 (a^2 - b^2)^2 d} \\ &+ \frac{(8a^4 - 61a^2b^2 + 35b^4) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12a^3 (a^2 - b^2)^2 d} \\ &+ \frac{b^2 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{2a (a^2 - b^2) d (b + a \sec(c+dx))^2} + \frac{b^2 (13a^2 - 7b^2) \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{4a^2 (a^2 - b^2)^2 d (b + a \sec(c+dx))} \end{aligned}$$

[Out] 1/12*(8*a^4-61*a^2*b^2+35*b^4)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b^2*sec(d*x+c)^(7/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/4*b^2*(13*a^2-7*b^2)*sec(d*x+c)^(5/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/4*b*(24*a^4-65*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/12*(8*a^4-61*a^2*b^2+35*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/d+1/4*b^2*(63*a^4-86*a^2*b

$$\sqrt{2+35b^4} \cdot (\cos(1/2dx+1/2c))^2)^{1/2} / \cos(1/2dx+1/2c) \cdot \text{EllipticPi}(\sin(1/2dx+1/2c), 2b/(a+b), 2^{1/2}) \cdot \cos(dx+c)^{1/2} \cdot \sec(dx+c)^{1/2} / a^4 / (a-b)^2 / (a+b)^3 / d$$

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3317, 3930, 4183, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\begin{aligned} & \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3} dx \\ &= \frac{b^2 \sin(c+dx) \sec^{\frac{7}{2}}(c+dx)}{2ad(a^2-b^2)(a\sec(c+dx)+b)^2} + \frac{b^2(13a^2-7b^2) \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{4a^2d(a^2-b^2)^2(a\sec(c+dx)+b)} \\ & \quad - \frac{b(24a^4-65a^2b^2+35b^4) \sin(c+dx) \sqrt{\sec(c+dx)}}{4a^4d(a^2-b^2)^2} \\ & \quad + \frac{b(24a^4-65a^2b^2+35b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4a^4d(a^2-b^2)^2} \\ & \quad + \frac{b^2(63a^4-86a^2b^2+35b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4a^4d(a-b)^2(a+b)^3} \\ & \quad + \frac{(8a^4-61a^2b^2+35b^4) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{12a^3d(a^2-b^2)^2} \\ & \quad + \frac{(8a^4-61a^2b^2+35b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{12a^3d(a^2-b^2)^2} \end{aligned}$$

[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]

[Out] (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*a^3*(a^2 - b^2)^2*d) + (b^2*(63*a^4 - 86*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]])*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a - b)^2*(a + b)^3*d) - (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (b^2*(13*a^2 - 7*b^2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3930

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegerQ[n + 1/2, 2*m] && GtQ[n, 2]))

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1

$/(\text{Sqrt}[d \cdot \text{Sin}[e + f \cdot x]] \cdot (b + a \cdot \text{Sin}[e + f \cdot x])), x, x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 4183

$\text{Int}[((A_.) + \text{csc}[(e_.) + (f_.)(x_.)])(B_.) + \text{csc}[(e_.) + (f_.)(x_.)]^2(C_.)) \cdot (\text{csc}[(e_.) + (f_.)(x_.)](d_.))^n \cdot (\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.))^m, x_Symbol] \rightarrow \text{Simp}[(-d) \cdot (A \cdot b^2 - a \cdot b \cdot B + a^2 \cdot C) \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^{m+1} \cdot ((d \cdot \text{Csc}[e + f \cdot x])^{n-1} / (b \cdot f \cdot (a^2 - b^2) \cdot (m+1))), x] + \text{Dist}[d / (b \cdot (a^2 - b^2) \cdot (m+1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^{m+1} \cdot (d \cdot \text{Csc}[e + f \cdot x])^{n-1} \cdot \text{Simp}[A \cdot b^2 \cdot (n-1) - a \cdot (b \cdot B - a \cdot C) \cdot (n-1) + b \cdot (a \cdot A - b \cdot B + a \cdot C) \cdot (m+1) \cdot \text{Csc}[e + f \cdot x] - (b \cdot (A \cdot b - a \cdot B) \cdot (m+n+1) + C \cdot (a^2 \cdot n + b^2 \cdot (m+1))) \cdot \text{Csc}[e + f \cdot x]^2, x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[n, 0]$

Rule 4187

$\text{Int}[((A_.) + \text{csc}[(e_.) + (f_.)(x_.)])(B_.) + \text{csc}[(e_.) + (f_.)(x_.)]^2(C_.)) \cdot (\text{csc}[(e_.) + (f_.)(x_.)](d_.))^n \cdot (\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.))^m, x_Symbol] \rightarrow \text{Simp}[(-C) \cdot d \cdot \text{Cot}[e + f \cdot x] \cdot (a + b \cdot \text{Csc}[e + f \cdot x])^{m+1} \cdot ((d \cdot \text{Csc}[e + f \cdot x])^{n-1} / (b \cdot f \cdot (m+n+1))), x] + \text{Dist}[d / (b \cdot (m+n+1)), \text{Int}[(a + b \cdot \text{Csc}[e + f \cdot x])^m \cdot (d \cdot \text{Csc}[e + f \cdot x])^{n-1} \cdot \text{Simp}[a \cdot C \cdot (n-1) + (A \cdot b \cdot (m+n+1) + b \cdot C \cdot (m+n)) \cdot \text{Csc}[e + f \cdot x] + (b \cdot B \cdot (m+n+1) - a \cdot C \cdot n) \cdot \text{Csc}[e + f \cdot x]^2, x], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, C, m\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[n, 0]$

Rule 4191

$\text{Int}[((A_.) + \text{csc}[(e_.) + (f_.)(x_.)])(B_.) + \text{csc}[(e_.) + (f_.)(x_.)]^2(C_.)) / (\text{Sqrt}[\text{csc}[(e_.) + (f_.)(x_.)](d_.)] \cdot (\text{csc}[(e_.) + (f_.)(x_.)](b_.) + (a_.))), x_Symbol] \rightarrow \text{Dist}[(A \cdot b^2 - a \cdot b \cdot B + a^2 \cdot C) / (a^2 \cdot d^2), \text{Int}[(d \cdot \text{Csc}[e + f \cdot x])^{3/2} / (a + b \cdot \text{Csc}[e + f \cdot x]), x], x] + \text{Dist}[1/a^2, \text{Int}[(a \cdot A - (A \cdot b - a \cdot B) \cdot \text{Csc}[e + f \cdot x]) / \text{Sqrt}[d \cdot \text{Csc}[e + f \cdot x]], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, C\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{\sec^{\frac{11}{2}}(c + dx)}{(b + a \sec(c + dx))^3} dx \\ &= \frac{b^2 \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{2a(a^2 - b^2) d (b + a \sec(c + dx))^2} + \int \frac{\sec^{\frac{5}{2}}(c + dx) \left(\frac{5b^2}{2} - 2ab \sec(c + dx) + \frac{1}{2}(4a^2 - 7b^2) \sec^2(c + dx) \right)}{2a(a^2 - b^2) (b + a \sec(c + dx))^2} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{b^2 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{2a(a^2-b^2)d(b+a \sec(c+dx))^2} + \frac{b^2(13a^2-7b^2) \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2-b^2)^2 d(b+a \sec(c+dx))} \\
&\quad + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx) (\frac{3}{4}b^2(13a^2-7b^2) - ab(4a^2-b^2) \sec(c+dx) + \frac{1}{4}(8a^4-61a^2b^2+35b^4) \sec^2(c+dx))}{b+a \sec(c+dx)} dx}{2a^2(a^2-b^2)^2} \\
&= \frac{(8a^4-61a^2b^2+35b^4) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12a^3(a^2-b^2)^2 d} \\
&\quad + \frac{b^2 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{2a(a^2-b^2)d(b+a \sec(c+dx))^2} + \frac{b^2(13a^2-7b^2) \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2-b^2)^2 d(b+a \sec(c+dx))} \\
&\quad + \frac{\int \frac{\sqrt{\sec(c+dx)} (\frac{1}{8}b(8a^4-61a^2b^2+35b^4) + \frac{1}{2}a(2a^4+14a^2b^2-7b^4) \sec(c+dx) - \frac{3}{8}b(24a^4-65a^2b^2+35b^4) \sec^2(c+dx))}{b+a \sec(c+dx)} dx}{3a^3(a^2-b^2)^2} \\
&= -\frac{b(24a^4-65a^2b^2+35b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^4(a^2-b^2)^2 d} \\
&\quad + \frac{(8a^4-61a^2b^2+35b^4) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12a^3(a^2-b^2)^2 d} \\
&\quad + \frac{b^2 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{2a(a^2-b^2)d(b+a \sec(c+dx))^2} + \frac{b^2(13a^2-7b^2) \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2-b^2)^2 d(b+a \sec(c+dx))} \\
&\quad + \frac{2 \int \frac{\frac{3}{16}b^2(24a^4-65a^2b^2+35b^4) + \frac{1}{4}ab(20a^4-64a^2b^2+35b^4) \sec(c+dx) + \frac{1}{16}(8a^6+128a^4b^2-223a^2b^4+105b^6) \sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a \sec(c+dx))} dx}{3a^4(a^2-b^2)^2} \\
&= -\frac{b(24a^4-65a^2b^2+35b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^4(a^2-b^2)^2 d} \\
&\quad + \frac{(8a^4-61a^2b^2+35b^4) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12a^3(a^2-b^2)^2 d} \\
&\quad + \frac{b^2 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{2a(a^2-b^2)d(b+a \sec(c+dx))^2} + \frac{b^2(13a^2-7b^2) \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2-b^2)^2 d(b+a \sec(c+dx))} \\
&\quad + \frac{2 \int \frac{\frac{3}{16}b^3(24a^4-65a^2b^2+35b^4) - (\frac{3}{16}ab^2(24a^4-65a^2b^2+35b^4) - \frac{1}{4}ab^2(20a^4-64a^2b^2+35b^4)) \sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{3a^4b^2(a^2-b^2)^2} \\
&\quad + \frac{(b^2(63a^4-86a^2b^2+35b^4)) \int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a \sec(c+dx)} dx}{8a^4(a^2-b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{b(24a^4 - 65a^2b^2 + 35b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^4(a^2 - b^2)^2 d} \\
&+ \frac{(8a^4 - 61a^2b^2 + 35b^4) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12a^3(a^2 - b^2)^2 d} \\
&+ \frac{b^2 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{2a(a^2 - b^2) d(b+a \sec(c+dx))^2} + \frac{b^2(13a^2 - 7b^2) \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2 - b^2)^2 d(b+a \sec(c+dx))} \\
&+ \frac{(b(24a^4 - 65a^2b^2 + 35b^4)) \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{8a^4(a^2 - b^2)^2} \\
&+ \frac{(8a^4 - 61a^2b^2 + 35b^4) \int \sqrt{\sec(c+dx)} dx}{24a^3(a^2 - b^2)^2} \\
&+ \frac{\left(b^2(63a^4 - 86a^2b^2 + 35b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)(a+b \cos(c+dx))}} dx}{8a^4(a^2 - b^2)^2} \\
&= \frac{b^2(63a^4 - 86a^2b^2 + 35b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a^4(a-b)^2(a+b)^3 d} \\
&- \frac{b(24a^4 - 65a^2b^2 + 35b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^4(a^2 - b^2)^2 d} \\
&+ \frac{(8a^4 - 61a^2b^2 + 35b^4) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12a^3(a^2 - b^2)^2 d} \\
&+ \frac{b^2 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{2a(a^2 - b^2) d(b+a \sec(c+dx))^2} + \frac{b^2(13a^2 - 7b^2) \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2 - b^2)^2 d(b+a \sec(c+dx))} \\
&+ \frac{\left(b(24a^4 - 65a^2b^2 + 35b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{8a^4(a^2 - b^2)^2} \\
&+ \frac{\left((8a^4 - 61a^2b^2 + 35b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{24a^3(a^2 - b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{b(24a^4 - 65a^2b^2 + 35b^4) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{4a^4(a^2 - b^2)^2 d} \\
&+ \frac{(8a^4 - 61a^2b^2 + 35b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{12a^3(a^2 - b^2)^2 d} \\
&+ \frac{b^2(63a^4 - 86a^2b^2 + 35b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a^4(a-b)^2(a+b)^3 d} \\
&- \frac{b(24a^4 - 65a^2b^2 + 35b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^4(a^2 - b^2)^2 d} \\
&+ \frac{(8a^4 - 61a^2b^2 + 35b^4) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{12a^3(a^2 - b^2)^2 d} \\
&+ \frac{b^2 \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{2a(a^2 - b^2)d(b+a \sec(c+dx))^2} + \frac{b^2(13a^2 - 7b^2) \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2 - b^2)^2 d(b+a \sec(c+dx))}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 6.60 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.64

$$\begin{aligned}
&\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx \\
&\frac{2(16a^6 + 328a^4b^2 - 641a^2b^4 + 315b^6) \cos^2(c+dx) \left(\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\sec(c+dx)}\right), -1\right) - \operatorname{EllipticPi}\left(-\frac{a}{b}, \arcsin\left(\sqrt{\sec(c+dx)}\right), -1\right) \right) (b+a \sec(c+dx))}{a(a+b \cos(c+dx))(1-\cos^2(c+dx))} \\
&= \\
&+ \frac{\sqrt{\sec(c+dx)} \left(-\frac{b(24a^4 - 65a^2b^2 + 35b^4) \sin(c+dx)}{4a^4(a^2 - b^2)^2} - \frac{b^3 \sin(c+dx)}{2a^2(a^2 - b^2)(a+b \cos(c+dx))^2} - \frac{3(5a^2b^3 \sin(c+dx) - 3b^5 \sin(c+dx))}{4a^3(a^2 - b^2)^2(a+b \cos(c+dx))} + \frac{2 \tan^3}{3} \right)}{d}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]

[Out] ((2*(16*a^6 + 328*a^4*b^2 - 641*a^2*b^4 + 315*b^6)*Cos[c + d*x]^2*(Elliptic F[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(160*a^5*b - 512*a^3*b^3 + 280*a*b^5)*Cos[c + d*x]^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((72*a^4*b^2 - 195*a^2*b^4 + 105*b^6)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*a^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))/(48*a^4*(a - b)^2*(a + b

$$\begin{aligned} &)^2*d) + (\text{Sqrt}[\text{Sec}[c + d*x]]*(-1/4*(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*\text{Sin}[c \\ &+ d*x]))/(a^4*(a^2 - b^2)^2) - (b^3*\text{Sin}[c + d*x])/(2*a^2*(a^2 - b^2)*(a + b* \\ &\text{Cos}[c + d*x])^2) - (3*(5*a^2*b^3*\text{Sin}[c + d*x] - 3*b^5*\text{Sin}[c + d*x]))/(4*a^3 \\ &*(a^2 - b^2)^2*(a + b*\text{Cos}[c + d*x])) + (2*\text{Tan}[c + d*x])/(3*a^3))/d \end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2100 vs. $2(499) = 998$.

Time = 160.23 (sec) , antiderivative size = 2101, normalized size of antiderivative = 4.62

method	result	size
default	Expression too large to display	2101

[In] `int(sec(d*x+c)^(5/2)/(a*cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} &-(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2/a^3*(-1/6*\cos \\ &(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)})/(\cos(1 \\ &/2*d*x+1/2*c)^2-1/2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2 \\ &*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*Elli \\ &pticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})-6/a^4*b/\sin(1/2*d*x+1/2*c)^2/(2*\sin(1/2*d*x \\ &+1/2*c)^2-1)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/ \\ &2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)-(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d \\ &*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)}))+2*b^2/a^2*(-1/2 \\ &/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/ \\ &2*c)^2)^{(1/2)})/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2 \\ &-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(\\ &1/2)})/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(\sin(1/2*d*x+1/2*c) \\ &^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/ \\ &2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+1/4/(a+b)/(a^2- \\ &b^2)/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*s \\ &\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c \\ &),2^{(1/2)})*b+3/8/(a+b)/(a^2-b^2)/a^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1 \\ &/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/ \\ &2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b^2-9/8*b/(a^2-b^2)^2*(\sin(1/2*d*x \\ &+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4 \\ &+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3/8*b^3/ \\ &a^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1 \\ &/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2* \\ &d*x+1/2*c),2^{(1/2)})+9/8*b/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(\\ &1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1 \\ &/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3/8*b^3/a^2/(a^2-b^2)^2*(\sin(1/2* \\ &d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c \\ &)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-15/4* \\ &a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d \\ &*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*E \end{aligned}$$

```

l ellipticPi(cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*
b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c), -2*b/(a-b), 2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^(1
/2))) -12*b^3/a^4/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^(1/2))+4*b^2/a^3*(-1/a*b^2/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+1/2/(a
^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c), 2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^(1/2))+1/a/(a^2-b^2
)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c), -2*b/(a-b), 2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d

```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c) + a)^3} dx$$

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{(a + b \cos(c + dx))^3} dx$$

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^3, x)
```

$$3.725 \quad \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$$

Optimal result	6561
Rubi [A] (verified)	6562
Mathematica [A] (verified)	6566
Maple [B] (verified)	6567
Fricas [F(-1)]	6568
Sympy [F]	6568
Maxima [F(-1)]	6569
Giac [F]	6569
Mupad [F(-1)]	6569

Optimal result

Integrand size = 23, antiderivative size = 388

$$\begin{aligned} & \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx \\ &= -\frac{(8a^4 - 29a^2b^2 + 15b^4) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{4a^3 (a^2 - b^2)^2 d} \\ &+ \frac{b(11a^2 - 5b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a^2 (a^2 - b^2)^2 d} \\ &- \frac{b(35a^4 - 38a^2b^2 + 15b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a^3 (a-b)^2 (a+b)^3 d} \\ &+ \frac{(8a^4 - 29a^2b^2 + 15b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^3 (a^2 - b^2)^2 d} \\ &+ \frac{b^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2a (a^2 - b^2) d (b + a \sec(c+dx))^2} + \frac{b^2 (11a^2 - 5b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4a^2 (a^2 - b^2)^2 d (b + a \sec(c+dx))} \end{aligned}$$

[Out] $\frac{1}{2} b^2 \sec(d*x+c)^{(5/2)} \sin(d*x+c) / a / (a^2 - b^2) / d / (b + a \sec(d*x+c))^{2+1/4} b^2 (11 a^2 - 5 b^2) \sec(d*x+c)^{(3/2)} \sin(d*x+c) / a^2 / (a^2 - b^2)^2 / d / (b + a \sec(d*x+c))^{1/4} (8 a^4 - 29 a^2 b^2 + 15 b^4) \sin(d*x+c) \sec(d*x+c)^{(1/2)} / a^3 / (a^2 - b^2)^2 / d - 1/4 (8 a^4 - 29 a^2 b^2 + 15 b^4) (\cos(1/2 d*x + 1/2 c))^2)^{(1/2)} / \cos(1/2 d*x + 1/2 c) * \operatorname{EllipticE}(\sin(1/2 d*x + 1/2 c), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} \sec(d*x+c)^{(1/2)} / a^3 / (a^2 - b^2)^2 / d + 1/4 b (11 a^2 - 5 b^2) (\cos(1/2 d*x + 1/2 c))^2)^{(1/2)} / \cos(1/2 d*x + 1/2 c) * \operatorname{EllipticF}(\sin(1/2 d*x + 1/2 c), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} \sec(d*x+c)^{(1/2)} / a^2 / (a^2 - b^2)^2 / d - 1/4 b (35 a^4 - 38 a^2 b^2 + 15 b^4) (\cos(1/2 d*x + 1/2 c))^2)^{(1/2)} / \cos(1/2 d*x + 1/2 c) * \operatorname{EllipticPi}(\sin(1/2 d*x + 1/2 c), 2b / (a+b), 2^{(1/2)}) * \cos(d*x+c)^{(1/2)} \sec(d*x+c)^{(1/2)} / a^3 / (a-b)^2 / (a+b)^3 / d$

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3317, 3930, 4183, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3} dx$$

$$= \frac{b^2 \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)}{2ad(a^2-b^2)(a\sec(c+dx)+b)^2} + \frac{b^2(11a^2-5b^2) \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{4a^2d(a^2-b^2)^2(a\sec(c+dx)+b)}$$

$$+ \frac{b(11a^2-5b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4a^2d(a^2-b^2)^2}$$

$$+ \frac{(8a^4-29a^2b^2+15b^4) \sin(c+dx) \sqrt{\sec(c+dx)}}{4a^3d(a^2-b^2)^2}$$

$$- \frac{(8a^4-29a^2b^2+15b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4a^3d(a^2-b^2)^2}$$

$$- \frac{b(35a^4-38a^2b^2+15b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4a^3d(a-b)^2(a+b)^3}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]

[Out] -1/4*((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)^2*d) + (b*(11*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - (b*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (b^2*(11*a^2 - 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[

$2*(b/(a + b)), (1/2)*(e - \text{Pi}/2 + f*x), 2*(d/(c + d))], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)]^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n * Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3930

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2]))

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4183

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)) * (csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_) * (csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +

```

b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]

```

Rule 4187

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]

```

Rule 4191

```

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
C}, x] && NeQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\sec^{\frac{9}{2}}(c + dx)}{(b + a \sec(c + dx))^3} dx \\
&= \frac{b^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2a(a^2 - b^2) d(b + a \sec(c + dx))^2} + \frac{\int \frac{\sec^{\frac{3}{2}}(c + dx) \left(\frac{3b^2}{2} - 2ab \sec(c + dx) + \frac{1}{2}(4a^2 - 5b^2) \sec^2(c + dx) \right)}{(b + a \sec(c + dx))^2} dx}{2a(a^2 - b^2)} \\
&= \frac{b^2 \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{2a(a^2 - b^2) d(b + a \sec(c + dx))^2} + \frac{b^2(11a^2 - 5b^2) \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{4a^2(a^2 - b^2)^2 d(b + a \sec(c + dx))} \\
&\quad + \frac{\int \frac{\sqrt{\sec(c + dx)} \left(\frac{1}{4}b^2(11a^2 - 5b^2) - ab(4a^2 - b^2) \sec(c + dx) + \frac{1}{4}(8a^4 - 29a^2b^2 + 15b^4) \sec^2(c + dx) \right)}{b + a \sec(c + dx)} dx}{2a^2(a^2 - b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(8a^4 - 29a^2b^2 + 15b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^3(a^2 - b^2)^2 d} \\
&+ \frac{b^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2a(a^2 - b^2) d(b+a \sec(c+dx))^2} + \frac{b^2(11a^2 - 5b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2 - b^2)^2 d(b+a \sec(c+dx))} \\
&+ \frac{\int \frac{-\frac{1}{8}b(8a^4 - 29a^2b^2 + 15b^4) - \frac{1}{2}a(2a^4 - 10a^2b^2 + 5b^4) \sec(c+dx) - \frac{3}{8}b(8a^4 - 11a^2b^2 + 5b^4) \sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a \sec(c+dx))} dx}{a^3(a^2 - b^2)^2} \\
&= \frac{(8a^4 - 29a^2b^2 + 15b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^3(a^2 - b^2)^2 d} \\
&+ \frac{b^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2a(a^2 - b^2) d(b+a \sec(c+dx))^2} + \frac{b^2(11a^2 - 5b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2 - b^2)^2 d(b+a \sec(c+dx))} \\
&+ \frac{\int \frac{-\frac{1}{8}b^2(8a^4 - 29a^2b^2 + 15b^4) - (\frac{1}{2}ab(2a^4 - 10a^2b^2 + 5b^4) - \frac{1}{8}ab(8a^4 - 29a^2b^2 + 15b^4)) \sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{a^3b^2(a^2 - b^2)^2} \\
&- \frac{(b(35a^4 - 38a^2b^2 + 15b^4)) \int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a \sec(c+dx)} dx}{8a^3(a^2 - b^2)^2} \\
&= \frac{(8a^4 - 29a^2b^2 + 15b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^3(a^2 - b^2)^2 d} \\
&+ \frac{b^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2a(a^2 - b^2) d(b+a \sec(c+dx))^2} + \frac{b^2(11a^2 - 5b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2 - b^2)^2 d(b+a \sec(c+dx))} \\
&+ \frac{(b(11a^2 - 5b^2)) \int \sqrt{\sec(c+dx)} dx}{8a^2(a^2 - b^2)^2} - \frac{(8a^4 - 29a^2b^2 + 15b^4) \int \frac{1}{\sqrt{\sec(c+dx)}} dx}{8a^3(a^2 - b^2)^2} \\
&- \frac{\left((b(35a^4 - 38a^2b^2 + 15b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}(a+b \cos(c+dx))} dx \right)}{8a^3(a^2 - b^2)^2} \\
&= \frac{b(35a^4 - 38a^2b^2 + 15b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a^3(a-b)^2(a+b)^3 d} \\
&+ \frac{(8a^4 - 29a^2b^2 + 15b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^3(a^2 - b^2)^2 d} \\
&+ \frac{b^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2a(a^2 - b^2) d(b+a \sec(c+dx))^2} + \frac{b^2(11a^2 - 5b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2 - b^2)^2 d(b+a \sec(c+dx))} \\
&+ \frac{\left((b(11a^2 - 5b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \right)}{8a^2(a^2 - b^2)^2} \\
&- \frac{\left((8a^4 - 29a^2b^2 + 15b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx}{8a^3(a^2 - b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{(8a^4 - 29a^2b^2 + 15b^4) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{4a^3(a^2-b^2)^2 d} \\
&+ \frac{b(11a^2 - 5b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2 d} \\
&- \frac{b(35a^4 - 38a^2b^2 + 15b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a^3(a-b)^2(a+b)^3 d} \\
&+ \frac{(8a^4 - 29a^2b^2 + 15b^4) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^3(a^2-b^2)^2 d} \\
&+ \frac{b^2 \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{2a(a^2-b^2)d(b+a \sec(c+dx))^2} + \frac{b^2(11a^2 - 5b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{4a^2(a^2-b^2)^2 d(b+a \sec(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.05 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.37

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^3} dx$$

$$= \frac{2a(16a^6 - 24a^4b^2 - 13a^2b^4 + 15b^6 + (32a^5b - 94a^3b^3 + 50ab^5) \cos(c+dx) + (8a^4b^2 - 29a^2b^4 + 15b^6) \cos(2(c+dx))) \tan(c+dx)}{(a^2-b^2)^2} - \frac{4 \cos(c+dx)(a+b \cos(c+dx))}{(a^2-b^2)^2}$$

[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3, x]

[Out] ((2*a*(16*a^6 - 24*a^4*b^2 - 13*a^2*b^4 + 15*b^6 + (32*a^5*b - 94*a^3*b^3 + 50*a*b^5)*Cos[c + d*x] + (8*a^4*b^2 - 29*a^2*b^4 + 15*b^6)*Cos[2*(c + d*x)])*Tan[c + d*x])/(a^2 - b^2)^2 - (4*Cos[c + d*x]*(a + b*Cos[c + d*x])*Cot[c + d*x]*(b + a*Sec[c + d*x])*(-8*a^5 + 29*a^3*b^2 - 15*a*b^4 + 8*a^5*Sec[c + d*x]^2 - 29*a^3*b^2*Sec[c + d*x]^2 + 15*a*b^4*Sec[c + d*x]^2 - a*(8*a^4 - 29*a^2*b^2 + 15*b^4)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + (8*a^5 + 24*a^4*b - 29*a^3*b^2 - 33*a^2*b^3 + 15*a*b^4 + 15*b^5)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 35*a^4*b*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + 38*a^2*b^3*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 15*b^5*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2)/(16*a^4*d*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1964 vs. $2(436) = 872$.

Time = 10.03 (sec) , antiderivative size = 1965, normalized size of antiderivative = 5.06

method	result	size
default	Expression too large to display	1965

[In] `int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)`

[Out]
$$-\left(-\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(\frac{2}{a^3}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(\frac{2}{2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\right)\left(\frac{2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\left(\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\left(2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^{1/2}\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2^{1/2}\right)\right)-\frac{2}{ab}\left(-\frac{1}{2}ab^2/(a^2-b^2)\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}\right)\left(\frac{2b\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a-b}{2-3/4b^2}\right)^{1/2}\left(\frac{3a^2-b^2}{a^2}\right)^{1/2}\left(\frac{2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\left(-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)^{1/2}}{2b\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a-b}\right)^{1/2}\left(\frac{7}{8}\right)^{1/2}\left(\frac{1}{a+b}\right)^{1/2}\left(\frac{1}{a^2-b^2}\right)^{1/2}\left(\frac{\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{2}\right)^{1/2}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{1/2}\left(\frac{1}{-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)^{1/2}\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2^{1/2}\right)+\frac{1}{4}\left(\frac{1}{a+b}\right)^{1/2}\left(\frac{1}{a^2-b^2}\right)^{1/2}\left(\frac{\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}\right)^{1/2}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{1/2}\left(\frac{1}{-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)^{1/2}\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2^{1/2}\right)+\frac{3}{8}\left(\frac{1}{a+b}\right)^{1/2}\left(\frac{1}{a^2-b^2}\right)^{1/2}\left(\frac{1}{a^2}\right)^{1/2}\left(\frac{\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{2}\right)^{1/2}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{1/2}\left(\frac{1}{-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)^{1/2}\operatorname{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2^{1/2}\right)+\frac{9}{8}\left(\frac{1}{a+b}\right)^{1/2}\left(\frac{1}{a^2-b^2}\right)^{1/2}\left(\frac{1}{2}\right)^{1/2}\left(\frac{\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{2}\right)^{1/2}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{1/2}\left(\frac{1}{-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)^{1/2}\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2^{1/2}\right)-\frac{3}{8}\left(\frac{1}{a+b}\right)^{1/2}\left(\frac{1}{a^2-b^2}\right)^{1/2}\left(\frac{1}{2}\right)^{1/2}\left(\frac{\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{2}\right)^{1/2}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{1/2}\left(\frac{1}{-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)^{1/2}\operatorname{EllipticE}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),2^{1/2}\right)-\frac{15}{4}\left(\frac{1}{a^2}\right)^{1/2}\left(\frac{1}{a^2-b^2}\right)^{1/2}\left(\frac{1}{-2ab+2b^2}\right)^{1/2}\left(\frac{1}{b}\right)^{1/2}\left(\frac{\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{2}\right)^{1/2}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{1/2}\left(\frac{1}{-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)^{1/2}\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),-\frac{2b}{a-b},2^{1/2}\right)+\frac{3}{2}\left(\frac{1}{a^2-b^2}\right)^{1/2}\left(\frac{1}{-2ab+2b^2}\right)^{1/2}\left(\frac{1}{b^3}\right)^{1/2}\left(\frac{\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{2}\right)^{1/2}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{1/2}\left(\frac{1}{-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)^{1/2}\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),-\frac{2b}{a-b},2^{1/2}\right)-\frac{3}{4}\left(\frac{1}{a^2}\right)^{1/2}\left(\frac{1}{a^2-b^2}\right)^{1/2}\left(\frac{1}{-2ab+2b^2}\right)^{1/2}\left(\frac{1}{b^5}\right)^{1/2}\left(\frac{\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{2}\right)^{1/2}\left(-2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{1/2}\left(\frac{1}{-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}\right)^{1/2}\operatorname{EllipticPi}\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right),-\frac{2b}{a-b},2^{1/2}\right)-\frac{2b}{a^2}$$

```

2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b
^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))+1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(
1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx$$

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*cos(c + d*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^3} dx$$

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^3} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{(a + b \cos(c + dx))^3} dx$$

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^3, x)
```

$$3.726 \quad \int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^3} dx$$

Optimal result	6570
Rubi [A] (verified)	6571
Mathematica [A] (verified)	6574
Maple [B] (verified)	6575
Fricas [F(-1)]	6576
Sympy [F]	6576
Maxima [F]	6576
Giac [F]	6576
Mupad [F(-1)]	6577

Optimal result

Integrand size = 23, antiderivative size = 321

$$\begin{aligned} & \int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^3} dx \\ &= -\frac{3b(3a^2-b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2 d} \\ & \quad - \frac{(7a^2-b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a(a^2-b^2)^2 d} \\ & \quad + \frac{3(5a^4-2a^2b^2+b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a^2(a-b)^2(a+b)^3 d} \\ & \quad + \frac{b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2a(a^2-b^2)d(b+a \sec(c+dx))^2} + \frac{3b^2(3a^2-b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a^2(a^2-b^2)^2 d(b+a \sec(c+dx))} \end{aligned}$$

```
[Out] 1/2*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+3/4*b^2*(3*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-3/4*b*(3*a^2-b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d-1/4*(7*a^2-b^2)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d+3/4*(5*a^4-2*a^2*b^2+b^4)*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)^2/(a+b)^3/d
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3317, 3930, 4183, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3} dx$$

$$= \frac{b^2 \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{2ad(a^2-b^2)(a\sec(c+dx)+b)^2} + \frac{3b^2(3a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d(a^2-b^2)^2(a\sec(c+dx)+b)}$$

$$- \frac{(7a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{4ad(a^2-b^2)^2}$$

$$- \frac{3b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{4a^2d(a^2-b^2)^2}$$

$$+ \frac{3(5a^4-2a^2b^2+b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)}{4a^2d(a-b)^2(a+b)^3}$$

[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^3, x]

[Out] (-3*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(4*a^2*(a^2 - b^2)^2*d) - ((7*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(4*a*(a^2 - b^2)^2*d) + (3*(5*a^4 - 2*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(4*a^2*(a - b)^2*(a + b)^3*d) + (b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (3*b^2*(3*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,

0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3930

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_))^(m_), x_Symbol] := Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegerQ[n + 1/2, 2*m] && GtQ[n, 2]))

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4183

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))^(m_)*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_)), x_Symbol] := Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d

*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A - b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b^2*(m + 1)))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]

Rule 4191

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{\sec^{\frac{7}{2}}(c + dx)}{(b + a \sec(c + dx))^3} dx \\
 &= \frac{b^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2a(a^2 - b^2) d(b + a \sec(c + dx))^2} + \frac{\int \frac{\sqrt{\sec(c + dx)} \left(\frac{b^2}{2} - 2ab \sec(c + dx) + \frac{1}{2}(4a^2 - 3b^2) \sec^2(c + dx) \right)}{(b + a \sec(c + dx))^2} dx}{2a(a^2 - b^2)} \\
 &= \frac{b^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2a(a^2 - b^2) d(b + a \sec(c + dx))^2} + \frac{3b^2(3a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{4a^2(a^2 - b^2)^2 d(b + a \sec(c + dx))} \\
 &\quad + \frac{\int \frac{-\frac{3}{4}b^2(3a^2 - b^2) - ab(4a^2 - b^2) \sec(c + dx) + \frac{1}{4}(8a^4 - 5a^2b^2 + 3b^4) \sec^2(c + dx)}{\sqrt{\sec(c + dx)}(b + a \sec(c + dx))} dx}{2a^2(a^2 - b^2)^2} \\
 &= \frac{b^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2a(a^2 - b^2) d(b + a \sec(c + dx))^2} + \frac{3b^2(3a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{4a^2(a^2 - b^2)^2 d(b + a \sec(c + dx))} \\
 &\quad + \frac{\int \frac{-\frac{3}{4}b^3(3a^2 - b^2) - (-\frac{3}{4}ab^2(3a^2 - b^2) + ab^2(4a^2 - b^2)) \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx}{2a^2b^2(a^2 - b^2)^2} \\
 &\quad + \frac{(3(5a^4 - 2a^2b^2 + b^4)) \int \frac{\sec^{\frac{3}{2}}(c + dx)}{b + a \sec(c + dx)} dx}{8a^2(a^2 - b^2)^2} \\
 &= \frac{b^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2a(a^2 - b^2) d(b + a \sec(c + dx))^2} + \frac{3b^2(3a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{4a^2(a^2 - b^2)^2 d(b + a \sec(c + dx))} \\
 &\quad - \frac{(3b(3a^2 - b^2)) \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{8a^2(a^2 - b^2)^2} - \frac{(7a^2 - b^2) \int \sqrt{\sec(c + dx)} dx}{8a(a^2 - b^2)^2} \\
 &\quad + \frac{\left(3(5a^4 - 2a^2b^2 + b^4) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))} dx}{8a^2(a^2 - b^2)^2}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{3(5a^4 - 2a^2b^2 + b^4) \sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{4a^2(a - b)^2(a + b)^3d} \\
&+ \frac{b^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2a(a^2 - b^2)d(b + a \sec(c + dx))^2} + \frac{3b^2(3a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{4a^2(a^2 - b^2)^2d(b + a \sec(c + dx))} \\
&- \frac{\left(3b(3a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{8a^2(a^2 - b^2)^2} \\
&- \frac{\left((7a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{8a(a^2 - b^2)^2} \\
&= - \frac{3b(3a^2 - b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{4a^2(a^2 - b^2)^2d} \\
&- \frac{(7a^2 - b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{4a(a^2 - b^2)^2d} \\
&+ \frac{3(5a^4 - 2a^2b^2 + b^4) \sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{4a^2(a - b)^2(a + b)^3d} \\
&+ \frac{b^2 \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{2a(a^2 - b^2)d(b + a \sec(c + dx))^2} + \frac{3b^2(3a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{4a^2(a^2 - b^2)^2d(b + a \sec(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.36 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \frac{\sqrt{\sec(c + dx)}}{(a + b \cos(c + dx))^3} dx \\
&= \frac{4ab^2(11a^3 - 5ab^2 + (9a^2b - 3b^3)\cos(c + dx))\sin(c + dx)}{(a^2 - b^2)^2} + \frac{4\cos(c + dx)(a + b\cos(c + dx))\cot(c + dx)(b + a\sec(c + dx))(3ab(3a^2 - b^2)E(\arcsin(\sqrt{\sec(c + dx)})))}{(a^2 - b^2)^2}
\end{aligned}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^3,x]

[Out] ((4*a*b^2*(11*a^3 - 5*a*b^2 + (9*a^2*b - 3*b^3)*Cos[c + d*x])*Sin[c + d*x]) / (a^2 - b^2)^2 + (4*Cos[c + d*x]*(a + b*Cos[c + d*x])*Cot[c + d*x]*(b + a*Sec[c + d*x])*(3*a*b*(3*a^2 - b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + (8*a^4 - 9*a^3*b - 5*a^2*b^2 + 3*a*b^3 + 3*b^4)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 3*(a*b*(3*a^2 - b^2)*Tan[c + d*x]^2 + (5*a^4 - 2*a^2*b^2 + b^4)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])) / ((a - b)^2*(a + b)^2) / (16*a^3*d*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. 2(373) = 746.

Time = 5.52 (sec) , antiderivative size = 1176, normalized size of antiderivative = 3.66

method	result	size
default	Expression too large to display	1176

[In] `int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^3,x,method=_RETURNVERBOSE)`

[Out]
$$-(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-1/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)^2-3/2*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-7/4/(a+b)/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+1/2/(a+b)/(a^2-b^2)/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b+3/4/(a+b)/(a^2-b^2)/a^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*b^2-9/4*b/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+3/4*b^3/a^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+9/4*b/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3/4*b^3/a^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})-15/2*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+3/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})-3/2/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)}))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3} dx = \int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3} dx$$

[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)

[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**3, x)

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3} dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^3} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3, x)

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3} dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^3} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{(a+b\cos(c+dx))^3} dx$$

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3,x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^3, x)
```

$$3.727 \quad \int \frac{1}{(a+b \cos(c+dx))^3 \sqrt{\sec(c+dx)}} dx$$

Optimal result	6578
Rubi [A] (verified)	6579
Mathematica [A] (verified)	6582
Maple [B] (verified)	6583
Fricas [F(-1)]	6584
Sympy [F]	6584
Maxima [F]	6584
Giac [F]	6585
Mupad [F(-1)]	6585

Optimal result

Integrand size = 23, antiderivative size = 317

$$\begin{aligned} & \int \frac{1}{(a+b \cos(c+dx))^3 \sqrt{\sec(c+dx)}} dx \\ &= \frac{(5a^2+b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{4a(a^2-b^2)^2 d} \\ &+ \frac{3(a^2+b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4b(a^2-b^2)^2 d} \\ &- \frac{(3a^4+10a^2b^2-b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4a(a-b)^2 b(a+b)^3 d} \\ &+ \frac{b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{2a(a^2-b^2) d(b+a \sec(c+dx))^2} - \frac{b(7a^2-b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{4a(a^2-b^2)^2 d(b+a \sec(c+dx))} \end{aligned}$$

```
[Out] 1/2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(b+a*sec(d*x+c))^2-1/4*b*
(7*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(b+a*sec(d*x+c))+1/
4*(5*a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d+
3/4*(a^2+b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d-
1/4*(3*a^4+10*a^2*b^2-b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/a/(a-b)^2/b/(a+b)^3/d
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3317, 3930, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx$$

$$= \frac{b^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{2ad(a^2 - b^2)(a \sec(c + dx) + b)^2} - \frac{b(7a^2 - b^2) \sin(c + dx) \sqrt{\sec(c + dx)}}{4ad(a^2 - b^2)^2(a \sec(c + dx) + b)}$$

$$+ \frac{3(a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{4bd(a^2 - b^2)^2}$$

$$+ \frac{(5a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{4ad(a^2 - b^2)^2}$$

$$- \frac{(3a^4 + 10a^2b^2 - b^4) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{4abd(a - b)^2(a + b)^3}$$

[In] Int[1/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]

[Out] ((5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a^2 - b^2)^2*d) + (3*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b*(a^2 - b^2)^2*d) - ((3*a^4 + 10*a^2*b^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b*(a + b)^3*d) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) - (b*(7*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3930

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2]))

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4185

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)) * (csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +

$n + 2) * \text{Csc}[e + f*x]^2, x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, C, n\}, x]$
 $\&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& !(\text{ILtQ}[m + 1/2, 0] \&\& \text{ILtQ}[n, 0])$

Rule 4191

$\text{Int}[(A_.) + \text{csc}[(e_.) + (f_.)*(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.) / (\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.)]*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))), x_Symbol] :> \text{Dist}[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), \text{Int}[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + \text{Dist}[1/a^2, \text{Int}[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; \text{FreeQ}[\{a, b, d, e, f, A, B, C\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{\sec^{\frac{5}{2}}(c + dx)}{(b + a \sec(c + dx))^3} dx \\
 &= \frac{b^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{2a(a^2 - b^2) d(b + a \sec(c + dx))^2} + \frac{\int \frac{-\frac{b^2}{2} - 2ab \sec(c + dx) + \frac{1}{2}(4a^2 - b^2) \sec^2(c + dx)}{\sqrt{\sec(c + dx)}(b + a \sec(c + dx))^2} dx}{2a(a^2 - b^2)} \\
 &= \frac{b^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{2a(a^2 - b^2) d(b + a \sec(c + dx))^2} - \frac{b(7a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{4a(a^2 - b^2)^2 d(b + a \sec(c + dx))} \\
 &\quad + \frac{\int \frac{\frac{1}{4}b^2(5a^2 + b^2) + ab(2a^2 + b^2) \sec(c + dx) - \frac{1}{4}b^2(7a^2 - b^2) \sec^2(c + dx)}{\sqrt{\sec(c + dx)}(b + a \sec(c + dx))} dx}{2ab(a^2 - b^2)^2} \\
 &= \frac{b^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{2a(a^2 - b^2) d(b + a \sec(c + dx))^2} - \frac{b(7a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{4a(a^2 - b^2)^2 d(b + a \sec(c + dx))} \\
 &\quad + \frac{\int \frac{\frac{1}{4}b^3(5a^2 + b^2) - (-ab^2(2a^2 + b^2) + \frac{1}{4}ab^2(5a^2 + b^2)) \sec(c + dx)}{\sqrt{\sec(c + dx)}} dx}{2ab^3(a^2 - b^2)^2} \\
 &\quad - \frac{(3a^4 + 10a^2b^2 - b^4) \int \frac{\sec^{\frac{3}{2}}(c + dx)}{b + a \sec(c + dx)} dx}{8ab(a^2 - b^2)^2} \\
 &= \frac{b^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{2a(a^2 - b^2) d(b + a \sec(c + dx))^2} - \frac{b(7a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{4a(a^2 - b^2)^2 d(b + a \sec(c + dx))} \\
 &\quad + \frac{(3(a^2 + b^2)) \int \sqrt{\sec(c + dx)} dx}{8b(a^2 - b^2)^2} + \frac{(5a^2 + b^2) \int \frac{1}{\sqrt{\sec(c + dx)}} dx}{8a(a^2 - b^2)^2} \\
 &\quad - \frac{\left((3a^4 + 10a^2b^2 - b^4) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))} dx}{8ab(a^2 - b^2)^2}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{(3a^4 + 10a^2b^2 - b^4) \sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{4a(a - b)^2b(a + b)^3d} \\
&+ \frac{b^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{2a(a^2 - b^2)d(b + a \sec(c + dx))^2} - \frac{b(7a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{4a(a^2 - b^2)^2 d(b + a \sec(c + dx))} \\
&+ \frac{\left(3(a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{8b(a^2 - b^2)^2} \\
&+ \frac{\left((5a^2 + b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{8a(a^2 - b^2)^2} \\
&= \frac{(5a^2 + b^2) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{\sec(c + dx)}}{4a(a^2 - b^2)^2 d} \\
&+ \frac{3(a^2 + b^2) \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{4b(a^2 - b^2)^2 d} \\
&- \frac{(3a^4 + 10a^2b^2 - b^4) \sqrt{\cos(c + dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right) \sqrt{\sec(c + dx)}}{4a(a - b)^2b(a + b)^3d} \\
&+ \frac{b^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{2a(a^2 - b^2)d(b + a \sec(c + dx))^2} - \frac{b(7a^2 - b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{4a(a^2 - b^2)^2 d(b + a \sec(c + dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.71 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.25

$$\begin{aligned}
&\int \frac{1}{(a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx \\
&= \frac{4b(7a^3 - ab^2 + b(5a^2 + b^2) \cos(c + dx)) \sin(c + dx)}{a(a^2 - b^2)^2 (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)}} + \frac{2 \cot(c + dx) (5a^3 b \sec^{\frac{3}{2}}(c + dx) + ab^3 \sec^{\frac{3}{2}}(c + dx) - 5a^3 b \cos(2(c + dx)) \sec^{\frac{3}{2}}(c + dx) - ab^3 \cos(2(c + dx)))}{a(a^2 - b^2)^2 (a + b \cos(c + dx))^2 \sqrt{\sec(c + dx)}}
\end{aligned}$$

[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]

[Out] ((-4*b*(7*a^3 - a*b^2 + b*(5*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a*(a^2 - b^2)^2*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) + (2*Cot[c + d*x]*(5*a^3*b*Sec[c + d*x]^(3/2) + a*b^3*Sec[c + d*x]^(3/2) - 5*a^3*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) - a*b^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) - 2*a*b*(5*a^2 + b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*b*(5*a^3 - 7*a^2*b + a*b^2 + b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^4*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 20*a^2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b^4*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^2*(a - b)^2*b*(a + b)^2)/(16*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(369) = 738$.

Time = 8.23 (sec) , antiderivative size = 1736, normalized size of antiderivative = 5.48

method	result	size
default	Expression too large to display	1736

[In] `int(1/(a+cos(d*x+c)*b)^3/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-(-(-2\cos(1/2dx+1/2c)^2+1)\sin(1/2dx+1/2c)^2)^{1/2}(2/b(-1/ab^2/(a^2-b^2)\cos(1/2dx+1/2c)*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}/(2b\cos(1/2dx+1/2c)^2+a-b)-1/2/a/(a+b)(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))-1/2/(a^2-b^2)*b/a*(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))+1/2/(a^2-b^2)*b/a*(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticE}(\cos(1/2dx+1/2c),2^{1/2}))-3a/(a^2-b^2)/(-2ab+2b^2)*b*(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticPi}(\cos(1/2dx+1/2c),-2b/(a-b),2^{1/2}))+1/a/(a^2-b^2)/(-2ab+2b^2)*b^3*(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticPi}(\cos(1/2dx+1/2c),-2b/(a-b),2^{1/2}))-2/b*a*(-1/2/ab^2/(a^2-b^2)\cos(1/2dx+1/2c)*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}/(2b\cos(1/2dx+1/2c)^2+a-b)^2-3/4*b^2*(3a^2-b^2)/a^2/(a^2-b^2)^2\cos(1/2dx+1/2c)*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}/(2b\cos(1/2dx+1/2c)^2+a-b)-7/8/(a+b)/(a^2-b^2)(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))+1/4/(a+b)/(a^2-b^2)/a*(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))*b+3/8/(a+b)/(a^2-b^2)/a^2*(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))*b^2-9/8*b/(a^2-b^2)^2*(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))+3/8*b^3/a^2/(a^2-b^2)^2*(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))+9/8*b/(a^2-b^2)^2*(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticE}(\cos(1/2dx+1/2c),2^{1/2}))-3/8*b^3/a^2/(a^2-b^2)^2*(\sin(1/2dx+1/2c)^2)^{1/2}*(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}*\text{EllipticE}(\cos(1/2dx+1/2c),2^{1/2}))-15/4*a^2/(a^2-b^2)^2/(-2*$$

$$a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})+3/2/(a^2-b^2)^{2/(-2*a*b+2*b^2)}*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)})-3/4/a^2/(a^2-b^2)^{2/(-2*a*b+2*b^2)}*b^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c),-2*b/(a-b),2^{(1/2)}))/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)

[Out] Integral(1/((a + b*cos(c + d*x))**3*sqrt(sec(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a)^3 \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a)^3 \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + b \cos(c + dx))^3} dx$$

[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3),x)

[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^3), x)

$$3.728 \quad \int \frac{1}{(a+b \cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	6586
Rubi [A] (verified)	6587
Mathematica [A] (verified)	6590
Maple [B] (verified)	6591
Fricas [F(-1)]	6592
Sympy [F(-1)]	6592
Maxima [F]	6592
Giac [F]	6593
Mupad [F(-1)]	6593

Optimal result

Integrand size = 23, antiderivative size = 302

$$\begin{aligned} & \int \frac{1}{(a+b \cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)} dx \\ &= -\frac{(a^2+5b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{4b(a^2-b^2)^2 d} \\ & \quad + \frac{a(a^2-7b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2 d} \\ & \quad - \frac{(a^4-10a^2b^2-3b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4(a-b)^2 b^2 (a+b)^3 d} \\ & \quad - \frac{b \sqrt{\sec(c+dx)} \sin(c+dx)}{2(a^2-b^2) d (b+a \sec(c+dx))^2} + \frac{3(a^2+b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{4(a^2-b^2)^2 d (b+a \sec(c+dx))} \end{aligned}$$

```
[Out] -1/2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+3/4*(a^2+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-1/4*(a^2+5*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d+1/4*a*(a^2-7*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d-1/4*(a^4-10*a^2*b^2-3*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c), 2*b/(a+b), 2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^2/(a+b)^3/d
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3317, 3929, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{3(a^2 + b^2) \sin(c + dx) \sqrt{\sec(c + dx)}}{4d(a^2 - b^2)^2 (a \sec(c + dx) + b)} - \frac{b \sin(c + dx) \sqrt{\sec(c + dx)}}{2d(a^2 - b^2) (a \sec(c + dx) + b)^2}$$

$$+ \frac{a(a^2 - 7b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{4b^2 d (a^2 - b^2)^2}$$

$$- \frac{(a^2 + 5b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{4bd (a^2 - b^2)^2}$$

$$- \frac{(a^4 - 10a^2b^2 - 3b^4) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{4b^2 d (a - b)^2 (a + b)^3}$$

[In] Int[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]

[Out] -1/4*((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)^2*d) + (a*(a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) - ((a^4 - 10*a^2*b^2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^2*(a + b)^3*d) - (b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3929

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := Simp[a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(f*(m + 1)*(a^2 - b^2))), x] - Dist[d^2/((m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegerQ[2*m, 2*n]

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4185

Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]

&& NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rule 4191

Int[((A_.) + csc[(e_.) + (f_.)*(x_)])*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))), x_Symbol] := Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b+a\sec(c+dx))^3} dx \\
 &= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{b}{2}-2a\sec(c+dx)+\frac{3}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2} dx}{2(a^2-b^2)} \\
 &= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{3(a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4(a^2-b^2)^2d(b+a\sec(c+dx))} \\
 &\quad - \frac{\int \frac{\frac{1}{4}b(a^2+5b^2)+3ab^2\sec(c+dx)-\frac{3}{4}b(a^2+b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))} dx}{2b(a^2-b^2)^2} \\
 &= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{3(a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4(a^2-b^2)^2d(b+a\sec(c+dx))} \\
 &\quad - \frac{\int \frac{\frac{1}{4}b^2(a^2+5b^2)-(-3ab^3+\frac{1}{4}ab(a^2+5b^2))\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{2b^3(a^2-b^2)^2} \\
 &\quad - \frac{(a^4-10a^2b^2-3b^4)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)} dx}{8b^2(a^2-b^2)^2} \\
 &= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{3(a^2+b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4(a^2-b^2)^2d(b+a\sec(c+dx))} \\
 &\quad + \frac{(a(a^2-7b^2))\int \sqrt{\sec(c+dx)} dx}{8b^2(a^2-b^2)^2} - \frac{(a^2+5b^2)\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{8b(a^2-b^2)^2} \\
 &\quad - \frac{\left((a^4-10a^2b^2-3b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{8b^2(a^2-b^2)^2}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{(a^4 - 10a^2b^2 - 3b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4(a-b)^2b^2(a+b)^3d} \\
&\quad - \frac{b\sqrt{\sec(c+dx)} \sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{3(a^2+b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{4(a^2-b^2)^2d(b+a\sec(c+dx))} \\
&\quad + \frac{\left(a(a^2-7b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{8b^2(a^2-b^2)^2} \\
&\quad - \frac{\left((a^2+5b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{8b(a^2-b^2)^2} \\
&= -\frac{(a^2+5b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{4b(a^2-b^2)^2d} \\
&\quad + \frac{a(a^2-7b^2) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2d} \\
&\quad - \frac{(a^4 - 10a^2b^2 - 3b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4(a-b)^2b^2(a+b)^3d} \\
&\quad - \frac{b\sqrt{\sec(c+dx)} \sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{3(a^2+b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{4(a^2-b^2)^2d(b+a\sec(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.00 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.42

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)} dx \\
&= \frac{4b^2(3a(a^2+b^2)+b(a^2+5b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2} + \frac{4\cos(c+dx)(a+b\cos(c+dx))\cot(c+dx)(b+a\sec(c+dx))(a^3b+5ab^3-a^3b\sec^2(c+dx)-5ab^3)}{(a^2-b^2)^2}
\end{aligned}$$

[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)), x]

[Out] ((4*b^2*(3*a*(a^2 + b^2) + b*(a^2 + 5*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 + (4*Cos[c + d*x]*(a + b*Cos[c + d*x])*Cot[c + d*x]*(b + a*Sec[c + d*x]))*(a^3*b + 5*a*b^3 - a^3*b*Sec[c + d*x]^2 - 5*a*b^3*Sec[c + d*x]^2 + a*b*(a^2 + 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]])*Sqrt[-Tan[c + d*x]^2] + b*(-a^3 + 3*a^2*b - 5*a*b^2 + 3*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] + a^4*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 10*a^2*b^2*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] - 3*b^4*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])/(a*(a - b)^2*(a + b)^2)/(16*b^2*d*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1835 vs. 2(354) = 708.

Time = 8.52 (sec) , antiderivative size = 1836, normalized size of antiderivative = 6.08

method	result	size
default	Expression too large to display	1836

[In] `int(1/(a+cos(d*x+c)*b)^3/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$-(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-4/b/(-2*a*b+2*b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})+2*a^2/b^2*(-1/2/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)})/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)^{-2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)})/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+1/4/(a+b)/(a^2-b^2)/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*b+3/8/(a+b)/(a^2-b^2)/a^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*b^2-9/8*b/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+3/8*b^3/a^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c), 2^{(1/2)})+9/8*b/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-3/8*b^3/a^2/(a^2-b^2)^2*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticPi(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})-4*a/b^2*(-1/a*b^2/(a^2-b^2)*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)})/(2*b*\cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)})/(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}$$

```
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/(a^2-b^2)*b/a*(sin(1/2*d*x+1/2*c)
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a^2-b^2)*
b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + b \cos(c + dx))^3} dx$$

[In] int(1/((1/cos(c + d*x))^3/2)*(a + b*cos(c + d*x))^3,x)

[Out] int(1/((1/cos(c + d*x))^3/2)*(a + b*cos(c + d*x))^3), x)

$$3.729 \quad \int \frac{1}{(a+b \cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	6594
Rubi [A] (verified)	6595
Mathematica [A] (verified)	6598
Maple [B] (verified)	6599
Fricas [F(-1)]	6600
Sympy [F(-1)]	6600
Maxima [F]	6600
Giac [F]	6601
Mupad [F(-1)]	6601

Optimal result

Integrand size = 23, antiderivative size = 319

$$\begin{aligned} & \int \frac{1}{(a+b \cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)} dx \\ &= -\frac{3a(a^2-3b^2) \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2 d} \\ &+ \frac{(3a^4-5a^2b^2+8b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4b^3(a^2-b^2)^2 d} \\ &- \frac{3a(a^4-2a^2b^2+5b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4(a-b)^2 b^3 (a+b)^3 d} \\ &+ \frac{a \sqrt{\sec(c+dx)} \sin(c+dx)}{2(a^2-b^2) d (b+a \sec(c+dx))^2} + \frac{a(a^2-7b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{4b(a^2-b^2)^2 d (b+a \sec(c+dx))} \end{aligned}$$

```
[Out] 1/2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*sec(d*x+c))^2+1/4*a*(a^2-7*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)^2/d/(b+a*sec(d*x+c))-3/4*a*(a^2-3*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d+1/4*(3*a^4-5*a^2*b^2+8*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d-3/4*a*(a^4-2*a^2*b^2+5*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*b/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^3/(a+b)^3/d
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3317, 3928, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{a(a^2 - 7b^2) \sin(c + dx) \sqrt{\sec(c + dx)}}{4bd(a^2 - b^2)^2 (a \sec(c + dx) + b)} + \frac{a \sin(c + dx) \sqrt{\sec(c + dx)}}{2d(a^2 - b^2) (a \sec(c + dx) + b)^2}$$

$$- \frac{3a(a^2 - 3b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{4b^2 d (a^2 - b^2)^2}$$

$$+ \frac{(3a^4 - 5a^2 b^2 + 8b^4) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{4b^3 d (a^2 - b^2)^2}$$

$$- \frac{3a(a^4 - 2a^2 b^2 + 5b^4) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \text{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c + dx), 2\right)}{4b^3 d (a - b)^2 (a + b)^3}$$

[In] Int[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]

[Out] (-3*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4 - 5*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(4*b^3*(a^2 - b^2)^2*d) - (3*a*(a^4 - 2*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]/(4*(a - b)^2*b^3*(a + b)^3*d) + (a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (a*(a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2884

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,

0] && GtQ[c + d, 0]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 3856

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 3872

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3928

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] := Simp[(-b)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegerQ[2*m, 2*n]

Rule 3934

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 4185

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] := Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1

) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])

Rule 4191

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.)))/(Sqrt[csc[(e_.) + (f_.)*(x_.)]*(d_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \frac{\sqrt{\sec(c+dx)}}{(b+a\sec(c+dx))^3} dx \\
 &= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{a}{2}-2b\sec(c+dx)+\frac{3}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2} dx}{2(a^2-b^2)} \\
 &= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b(a^2-b^2)^2d(b+a\sec(c+dx))} \\
 &\quad + \frac{\int \frac{-\frac{3}{4}a(a^2-3b^2)+b(a^2+2b^2)\sec(c+dx)+\frac{1}{4}a(a^2-7b^2)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))} dx}{2b(a^2-b^2)^2} \\
 &= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b(a^2-b^2)^2d(b+a\sec(c+dx))} \\
 &\quad + \frac{\int \frac{-\frac{3}{4}ab(a^2-3b^2)-(-\frac{3}{4}a^2(a^2-3b^2)-b^2(a^2+2b^2))\sec(c+dx)}{\sqrt{\sec(c+dx)}} dx}{2b^3(a^2-b^2)^2} \\
 &\quad - \frac{(3a(a^4-2a^2b^2+5b^4))\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)} dx}{8b^3(a^2-b^2)^2} \\
 &= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b(a^2-b^2)^2d(b+a\sec(c+dx))} \\
 &\quad - \frac{(3a(a^2-3b^2))\int \frac{1}{\sqrt{\sec(c+dx)}} dx}{8b^2(a^2-b^2)^2} + \frac{(3a^4-5a^2b^2+8b^4)\int \sqrt{\sec(c+dx)} dx}{8b^3(a^2-b^2)^2} \\
 &\quad - \frac{(3a(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)})\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} dx}{8b^3(a^2-b^2)^2}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{3a(a^4 - 2a^2b^2 + 5b^4) \sqrt{\cos(c+dx)} \operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right) \sqrt{\sec(c+dx)}}{4(a-b)^2b^3(a+b)^3d} \\
&+ \frac{a\sqrt{\sec(c+dx)} \sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)} \sin(c+dx)}{4b(a^2-b^2)^2d(b+a\sec(c+dx))} \\
&- \frac{\left(3a(a^2-3b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{8b^2(a^2-b^2)^2} \\
&+ \frac{\left((3a^4-5a^2b^2+8b^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{8b^3(a^2-b^2)^2} \\
&= -\frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)\sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2d} \\
&+ \frac{(3a^4-5a^2b^2+8b^4)\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{4b^3(a^2-b^2)^2d} \\
&- \frac{3a(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\operatorname{EllipticPi}\left(\frac{2b}{a+b}, \frac{1}{2}(c+dx), 2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^3(a+b)^3d} \\
&+ \frac{a\sqrt{\sec(c+dx)} \sin(c+dx)}{2(a^2-b^2)d(b+a\sec(c+dx))^2} + \frac{a(a^2-7b^2)\sqrt{\sec(c+dx)} \sin(c+dx)}{4b(a^2-b^2)^2d(b+a\sec(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.88

$$\begin{aligned}
&\int \frac{1}{(a+b\cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)} dx \\
&= \frac{2ab^2(a^3-7ab^2+3b(a^2-3b^2)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}} + \frac{\cot(c+dx)\left(-6ab(a^2-3b^2)\sec^{\frac{3}{2}}(c+dx)\sin^2(c+dx)+6ab(a^2-3b^2)E\left(\arcsin\left(\sqrt{\sec(c+dx)}\right)\right)\right)}{(a-b)^2(a+b)^2}
\end{aligned}$$

[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]

[Out] ((2*a*b^2*(a^3 - 7*a*b^2 + 3*b*(a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-6*a*b*(a^2 - 3*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 + 6*a*b*(a^2 - 3*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b*(3*a^3 - a^2*b - 9*a*b^2 + 7*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*(a^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[-(a/b), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2)/(8*b^3*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1913 vs. 2(371) = 742.

Time = 9.08 (sec) , antiderivative size = 1914, normalized size of antiderivative = 6.00

method	result	size
default	Expression too large to display	1914

[In] `int(1/(a+cos(d*x+c)*b)^3/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$-(-(-2\cos(1/2dx+1/2c)^2+1)\sin(1/2dx+1/2c)^2)^{1/2}(2/b^3(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2})-2/b^3a^3(-1/2ab^2/(a^2-b^2)\cos(1/2dx+1/2c)*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}/(2b\cos(1/2dx+1/2c)^2+a-b)^2-3/4b^2(3a^2-b^2)/a^2/(a^2-b^2)^2\cos(1/2dx+1/2c)*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}/(2b\cos(1/2dx+1/2c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))+1/4/(a+b)/(a^2-b^2)/a(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2})*b+3/8/(a+b)/(a^2-b^2)/a^2(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2})*b^2-9/8b/(a^2-b^2)^2(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))+3/8b^3/a^2/(a^2-b^2)^2(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2}))+9/8b/(a^2-b^2)^2(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticE}(\cos(1/2dx+1/2c),2^{1/2})-3/8b^3/a^2/(a^2-b^2)^2(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticE}(\cos(1/2dx+1/2c),2^{1/2})-15/4a^2/(a^2-b^2)^2/(-2ab+2b^2)*b(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticPi}(\cos(1/2dx+1/2c),-2b/(a-b),2^{1/2}))+3/2/(a^2-b^2)^2/(-2ab+2b^2)*b^3(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticPi}(\cos(1/2dx+1/2c),-2b/(a-b),2^{1/2})-3/4/a^2/(a^2-b^2)^2/(-2ab+2b^2)*b^5(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticPi}(\cos(1/2dx+1/2c),-2b/(a-b),2^{1/2}))+12/b^2a/(-2ab+2b^2)(\sin(1/2dx+1/2c)^2)^{1/2}(-2\cos(1/2dx+1/2c)^2+1)^{1/2}/(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2)^{1/2}\text{EllipticPi}(\cos(1/2dx+1/2c),-2b/(a-b),2^{1/2}))+6/b^3a^2(-1/ab^2/(a^2-b^2)\cos(1/2dx+1/2c)*(-2\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2$$

$$\begin{aligned} &)^{(1/2)} / (2*b*\cos(1/2*d*x+1/2*c)^2+a-b) - 1/2/a/(a+b)*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) \\ &)+ 1/2/(a^2-b^2)*b/a*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)}) + 1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * (-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)} / (-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)} * \text{EllipticPi}(\cos(1/2*d*x+1/2*c), -2*b/(a-b), 2^{(1/2)})) / \sin(1/2*d*x+1/2*c) / (2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)} / d \end{aligned}$$

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^3 \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + b \cos(c + dx))^3} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3),x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^3), x)

3.730 $\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx$

Optimal result	6602
Rubi [A] (verified)	6603
Mathematica [A] (verified)	6606
Maple [B] (verified)	6606
Fricas [F]	6608
Sympy [F(-1)]	6608
Maxima [F]	6608
Giac [F]	6608
Mupad [F(-1)]	6609

Optimal result

Integrand size = 25, antiderivative size = 369

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{2(a-b)\sqrt{a+b}(9a^2-2b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a}{a+b}}}{15a^3d\sqrt{\sec(c+dx)}} + \frac{2(a-b)\sqrt{a+b}(9a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{15a^2d\sqrt{\sec(c+dx)}} + \frac{2b\sqrt{a+b}\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad} + \frac{2\sqrt{a+b}\cos(c+dx)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

```
[Out] 2/15*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d+2/5*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*a^2-2*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a^3/d/sec(d*x+c)^(1/2)-2/15*(a-b)*(9*a+2*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00,
 number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used
 = {4307, 2875, 3134, 3077, 2895, 3073}

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx =$$

$$\frac{2(a - b)\sqrt{a + b}(9a + 2b)\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b}}\right)\right)}{15a^2 d \sqrt{\sec(c + dx)}} +$$

$$\frac{2(a - b)\sqrt{a + b}(9a^2 - 2b^2)\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{15a^3 d \sqrt{\sec(c + dx)}} +$$

$$\frac{2 \sin(c + dx) \sec^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}}{5d} +$$

$$\frac{2b \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}}{15ad}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]

[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(9*a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(15*a^2*d*Sqrt[Sec[c + d*x]]) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rule 2875

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr

$t[a*((1 - \text{Csc}[e + f*x])/(a + b))] * \text{Sqrt}[a*((1 + \text{Csc}[e + f*x])/(a - b))] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[d*\text{Sin}[e + f*x]]/\text{Rt}[(a + b)/d, 2], -(a + b)/(a - b)], x] /;$
 $\text{FreeQ}\{a, b, d, e, f\}, x \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{PosQ}[(a + b)/d]$

Rule 3073

$\text{Int}[(A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)]/((b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(3/2)}*\text{Sqrt}[(c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[-2*A*(c - d)*(\text{Tan}[e + f*x]/(f*b*c^2))*\text{Rt}[(c + d)/b, 2]*\text{Sqrt}[c*((1 + \text{Csc}[e + f*x])/(c - d))]*\text{Sqrt}[c*((1 - \text{Csc}[e + f*x])/(c + d))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[b*\text{Sin}[e + f*x]]/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d)], x] /;$
 $\text{FreeQ}\{b, c, d, e, f, A, B\}, x \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{EqQ}[A, B] \ \&\& \ \text{PosQ}[(c + d)/b]$

Rule 3077

$\text{Int}[(A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)]/((a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(3/2)}*\text{Sqrt}[(c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]], x_Symbol] \rightarrow \text{Dist}[(A - B)/(a - b), \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] - \text{Dist}[(A*b - a*B)/(a - b), \text{Int}[(1 + \text{Sin}[e + f*x])/(a + b*\text{Sin}[e + f*x])^{(3/2)}*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, f, A, B\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{NeQ}[A, B]$

Rule 3134

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_)}*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(n_)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^2), x_Symbol] \rightarrow \text{Simp}[(-A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m + 1)}*((c + d*\text{Sin}[e + f*x])^{(n + 1)})/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + \text{Dist}[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*(c + d*\text{Sin}[e + f*x])^n*\text{Simp}[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*\text{Sin}[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*\text{Sin}[e + f*x]^2, x], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, f, A, B, C, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ ((\text{EqQ}[a, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !\text{IntegerQ}[n]) \ || \ !(\text{IntegerQ}[2*n] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ ((\text{IntegerQ}[n] \ \&\& \ !\text{IntegerQ}[m]) \ || \ \text{EqQ}[a, 0])))$

Rule 4307

$\text{Int}[(\text{csc}[(a_.) + (b_.)*(x_.)]*(c_.))^{(m_.)}*(u_), x_Symbol] \rightarrow \text{Dist}[(c*\text{Csc}[a + b*x])^m*(c*\text{Sin}[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u]/(c*\text{Sin}[a + b*x])^m, x], x$

] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)} dx \\
 &= \frac{2\sqrt{a+b\cos(c+dx)} \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} \\
 &\quad + \frac{1}{5} \left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{b}{2} + \frac{3}{2}a \cos(c+dx) + b \cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{2b\sqrt{a+b\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15ad} \\
 &\quad + \frac{2\sqrt{a+b\cos(c+dx)} \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} \\
 &\quad + \frac{\left(4\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{4}(9a^2-2b^2) + \frac{7}{4}ab \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{15a} \\
 &= \frac{2b\sqrt{a+b\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15ad} \\
 &\quad + \frac{2\sqrt{a+b\cos(c+dx)} \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d} \\
 &\quad - \frac{\left((a-b)(9a+2b) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{15a} \\
 &\quad - \frac{\left((-9a^2+2b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{15a} \\
 &= \frac{2(a-b)\sqrt{a+b}(9a^2-2b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{15a^3d\sqrt{\sec(c+dx)}} \\
 &\quad - \frac{2(a-b)\sqrt{a+b}(9a+2b) \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{15a^2d\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{2b\sqrt{a+b\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{15ad} \\
 &\quad + \frac{2\sqrt{a+b\cos(c+dx)} \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 23.41 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.96

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx$$

$$= \frac{2 \left(\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right) \sec(c+dx)} \left(-2(9a^3+9a^2b-2ab^2-2b^3)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right) \sqrt{\frac{1}{1+\sec(c+dx)}} \sqrt{\frac{b+a \sec(c+dx)}{(a+b)(1+\sec(c+dx))}} + 2a(9a^2+7ab-\right. \right.}{\sqrt{s}}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2), x]

[Out] (2*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*a*(9*a^2 + 7*a*b - 2*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (9*a^2 - 2*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2 + (a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]*((9*a^2 - 2*b^2)*Sin[c + d*x] + a*(b + 3*a*Sec[c + d*x])*Tan[c + d*x])])/(15*a^2*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2114 vs. 2(329) = 658.

Time = 10.36 (sec) , antiderivative size = 2115, normalized size of antiderivative = 5.73

method	result	size
default	Expression too large to display	2115

[In] int(sec(d*x+c)^(7/2)*(a+cos(d*x+c)*b)^(1/2), x, method=_RETURNVERBOSE)

[Out] -2/15/d*sec(d*x+c)^(7/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(4*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^4+14*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^4-4*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^4-9*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3+2*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3+7*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos

$$\begin{aligned}
& ((d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) / (a+b))^{1/2} \\
& * a^2 * b * \cos(d*x+c)^3 - 2 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} \\
& * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) / (a+b))^{1/2} \\
& * a * b^2 * \cos(d*x+c)^3 + 9 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} \\
& * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) / (a+b))^{1/2} \\
& * a^3 * \cos(d*x+c)^3 - 3 * \sin(d*x+c) * \cos(d*x+c) * a^3 - 4 * \sin(d*x+c) * \cos(d*x+c) \\
& * x+c)^2 * a^2 * b - 18 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * (\cos(d*x+c) \\
& / (1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) / (a+b))^{1/2} * a^2 * b * \cos(d*x+c) \\
& ^4 + 9 * \cos(d*x+c)^5 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * ((a+\cos(d*x+c)*b) \\
& / (1+\cos(d*x+c)) / (a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * a^3 - 9 * \cos(d*x+c)^5 * ((a+\cos(d*x+c)*b) \\
& / (1+\cos(d*x+c)) / (a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), \\
& (-a-b)/(a+b))^{1/2} * a^3 + 2 * b^3 * \cos(d*x+c)^4 * \sin(d*x+c) - 4 * a^2 * b * \cos(d*x+c)^3 \\
& * \sin(d*x+c) + a * b^2 * \cos(d*x+c)^3 * \sin(d*x+c) + 7 * \cos(d*x+c)^5 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), \\
& (-a-b)/(a+b))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) / (a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} \\
& * a^2 * b - 2 * \cos(d*x+c)^5 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * ((a+\cos(d*x+c)*b) \\
& / (1+\cos(d*x+c)) / (a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * a * b^2 - 9 * \cos(d*x+c)^5 * ((a+\cos(d*x+c)*b) \\
& / (1+\cos(d*x+c)) / (a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), \\
& (-a-b)/(a+b))^{1/2} * a^2 * b + 2 * \cos(d*x+c)^5 * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) / (a+b))^{1/2} * (\cos(d*x+c) \\
& / (1+\cos(d*x+c)))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * a * b^2 - 18 * \text{EllipticE} \\
& (\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b) \\
& / (1+\cos(d*x+c)) / (a+b))^{1/2} * a^3 * \cos(d*x+c)^4 + 4 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} \\
& * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) / (a+b))^{1/2} * b^3 * \cos(d*x+c)^4 + 18 * \text{EllipticF} \\
& (\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b) \\
& / (1+\cos(d*x+c)) / (a+b))^{1/2} * a^3 * \cos(d*x+c)^4 - 9 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} \\
& * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) / (a+b))^{1/2} * a^3 * \cos(d*x+c)^3 \\
& + 2 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b) \\
& / (1+\cos(d*x+c)) / (a+b))^{1/2} * a^3 * \cos(d*x+c)^3 - 3 * a^3 * \cos(d*x+c)^2 * \sin(d*x+c) + 2 * \cos(d*x+c)^5 * ((a+\cos(d*x+c)*b) \\
& / (1+\cos(d*x+c)) / (a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), \\
& (-a-b)/(a+b))^{1/2} * b^3 - 9 * \cos(d*x+c)^3 * a^3 * \sin(d*x+c) - 9 * \cos(d*x+c)^4 * a^2 * b * \sin(d*x+c) - \cos(d*x+c)^4 * a * b^2 * \sin(d*x+c) / a^2
\end{aligned}$$

Fricas [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(7/2)*(a+b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)

Giac [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{7}{2}} dx$$

[In] integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{7/2} \sqrt{a + b \cos(c + dx)} dx$$

```
[In] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2), x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(1/2), x)
```

3.731 $\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx$

Optimal result	6610
Rubi [A] (verified)	6611
Mathematica [A] (verified)	6613
Maple [B] (verified)	6613
Fricas [F]	6614
Sympy [F(-1)]	6614
Maxima [F]	6615
Giac [F]	6615
Mupad [F(-1)]	6615

Optimal result

Integrand size = 25, antiderivative size = 311

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{2(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2d\sqrt{\sec(c+dx)}} + \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3ad\sqrt{\sec(c+dx)}} + \frac{2\sqrt{a+b}\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

```
[Out] 2/3*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/3*(a-b)*b*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/3*(a-b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00,
 number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used
 = {4307, 2875, 3077, 2895, 3073}

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx$$

$$= \frac{2b(a - b)\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b}\sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right)}{3a^2 d \sqrt{\sec(c + dx)}} + \frac{2(a - b)\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b}\sqrt{\cos(c + dx)}}\right)\right)}{3ad \sqrt{\sec(c + dx)}} + \frac{2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}}{3d}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2), x]

[Out] (2*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)

Rule 2875

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])*(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Ssin[e + f*x]]/Sqrt[d*Ssin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

&& PosQ[(a + b)/d]

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)]
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{5}{2}}(c + dx)} dx \\
 &= \frac{2\sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
 &\quad + \frac{1}{3} \left(2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\frac{b}{2} + \frac{1}{2}a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{2\sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3d} \\
 &\quad + \frac{1}{3} \left((a - b) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
 &\quad + \frac{1}{3} \left(b \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{3a^2d\sqrt{\sec(c+dx)}} \\
&+ \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3ad\sqrt{\sec(c+dx)}} \\
&+ \frac{2\sqrt{a+b}\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 14.47 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.84

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)dx$$

$$= \frac{\sqrt{\sec(c+dx)}\left(-4b(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\right)}{3a^2d\sqrt{\sec(c+dx)}}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]

[Out] (Sqrt[Sec[c + d*x]]*(-4*b*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)*Sqrt[(1 + Sec[c + d*x])^(-1)] + 4*a*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)*Sqrt[(1 + Sec[c + d*x])^(-1)] + (2*a^2 + a*b + b^2 + 2*a*(a + 2*b)*Cos[c + d*x] + b*(a + b)*Cos[2*(c + d*x)])*Sec[c + d*x]*Tan[(c + d*x)/2])/(3*a*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. 2(277) = 554.

Time = 11.34 (sec) , antiderivative size = 1201, normalized size of antiderivative = 3.86

method	result	size
default	Expression too large to display	1201

[In] int(sec(d*x+c)^(5/2)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/3/d*sec(d*x+c)^(5/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(cos(d*x+c)^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2+cos(d*x+c)^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(a+cos(d*x+c)*b)/(

```

1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b-cos(d*x+c)
^4*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
)^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-cos(d*x+c)
)^4*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(d*x+c)^3+2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*cos(d*x+c)^3-2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*cos(d*x+c)^3
-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*cos(d*x+c
)^3+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(d*x+
c)^2+EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b
)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*x
+c)^2-EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*
x+c)^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*cos(d
*x+c)^2-a*b*cos(d*x+c)^3*sin(d*x+c)-b^2*cos(d*x+c)^3*sin(d*x+c)-a^2*cos(d*x
+c)^2*sin(d*x+c)-2*cos(d*x+c)^2*sin(d*x+c)*a*b-a^2*cos(d*x+c)*sin(d*x+c))/a

```

Fricas [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{5}{2}} dx$$

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**(5/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)

Giac [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{5}{2}} dx$$

[In] integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{5/2} \sqrt{a + b \cos(c + dx)} dx$$

[In] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2),x)

[Out] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2), x)

3.732 $\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	6616
Rubi [A] (verified)	6616
Mathematica [A] (verified)	6618
Maple [B] (warning: unable to verify)	6619
Fricas [F]	6619
Sympy [F(-1)]	6620
Maxima [F]	6620
Giac [F]	6620
Mupad [F(-1)]	6620

Optimal result

Integrand size = 25, antiderivative size = 269

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

$$= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] 2*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a/d/sec(d*x+c)^(1/2)-2*(a-b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

= {4307, 2874, 2895, 3073}

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx$$

$$= \frac{2(a - b)\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b}\sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{ad\sqrt{\sec(c + dx)}} - \frac{2(a - b)\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b}\sqrt{\cos(c + dx)}}\right)\right)}{ad\sqrt{\sec(c + dx)}}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2), x]

[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d*Sqrt[Sec[c + d*x]])

Rule 2874

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&

PosQ[(c + d)/b]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + b \cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)} dx \\
 &= \left(a \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
 &\quad + \left((-a + b) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
 &= \frac{2(a - b) \sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a + b}}}{ad \sqrt{\sec(c + dx)}} \\
 &\quad - \frac{2(a - b) \sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}}}{ad \sqrt{\sec(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 4.53 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.80

$$\begin{aligned}
 &\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx \\
 &= \frac{2 \left((a + b \cos(c + dx)) \sqrt{\sec(c + dx)} \sin(c + dx) + \frac{(a + b) \sqrt{\frac{a + b \cos(c + dx)}{(a + b)(1 + \cos(c + dx))}} \left(E\left(\arcsin\left(\tan\left(\frac{1}{2}(c + dx)\right)\right) \middle| -\frac{a + b}{a + b}\right) - \text{EllipticF}\left(\arcsin\left(\frac{\cos(c + dx)}{1 + \cos(c + dx)}\right) \middle| \frac{\cos(c + dx)}{1 + \cos(c + dx)}\right) \right)}{\sqrt{\sec^2\left(\frac{1}{2}(c + dx)\right)} \sqrt{\cos^2\left(\frac{1}{2}(c + dx)\right)}} \right)}{d \sqrt{a + b \cos(c + dx)}}
 \end{aligned}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]

[Out] (2*((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (-(((a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*(EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) - (a + b*Cos[c + d*x])*Tan[(c + d*x)/2])/(Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]))/(d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(245) = 490$.

Time = 9.09 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.86

method	result
default	$-2 \left(-\frac{\csc^2(dx+c)(1-\cos(dx+c))^2+1}{\csc^2(dx+c)(1-\cos(dx+c))^2-1} \right)^{\frac{3}{2}} \left((\csc^2(dx+c)(1-\cos(dx+c))^2-1) \left(-\sqrt{-(\csc^2(dx+c)(1-\cos(dx+c))^2+1}} \sqrt{\csc^2(dx+c)} \right) \right)$

[In] `int(sec(d*x+c)^(3/2)*(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -2/d * (-\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 + 1) / (\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 - 1) \\ & ^{(3/2)} * (\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 - 1) * (-(-\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 + \\ & 1)^{(1/2)} * ((\csc(d*x+c)^2 * a * (1-\cos(d*x+c))^2 - \csc(d*x+c)^2 * b * (1-\cos(d*x+c))^2 + \\ & a+b)/(a+b))^{(1/2)} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)}) * a - \\ & (-\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 + 1)^{(1/2)} * ((\csc(d*x+c)^2 * a * (1-\cos(d*x+c))^2 - \csc \\ & \csc(d*x+c)^2 * b * (1-\cos(d*x+c))^2 + a+b)/(a+b))^{(1/2)} * \text{EllipticF}(\cot(d*x+c) - \csc(d \\ & *x+c), (-a-b)/(a+b))^{(1/2)}) * b + (-\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 + 1)^{(1/2)} * ((\csc \\ & c(d*x+c)^2 * a * (1-\cos(d*x+c))^2 - \csc(d*x+c)^2 * b * (1-\cos(d*x+c))^2 + a+b)/(a+b))^{(\\ & 1/2)} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)}) * a + (-\csc(d*x+c)^2 \\ & * (1-\cos(d*x+c))^2 + 1)^{(1/2)} * ((\csc(d*x+c)^2 * a * (1-\cos(d*x+c))^2 - \csc(d*x+c)^2 * b \\ & * (1-\cos(d*x+c))^2 + a+b)/(a+b))^{(1/2)} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b) \\ & / (a+b))^{(1/2)}) * b + \csc(d*x+c)^3 * (1-\cos(d*x+c))^3 * a - \csc(d*x+c)^3 * (1-\cos(d*x+c) \\ &)^3 * b + a * (\csc(d*x+c) - \cot(d*x+c)) + b * (\csc(d*x+c) - \cot(d*x+c)) * ((\csc(d*x+c)^2 * a \\ & * (1-\cos(d*x+c))^2 - \csc(d*x+c)^2 * b * (1-\cos(d*x+c))^2 + a+b) / (\csc(d*x+c)^2 * (1-\cos \\ & (d*x+c))^2 + 1))^{(1/2)} / (\csc(d*x+c)^2 * a * (1-\cos(d*x+c))^2 - \csc(d*x+c)^2 * b * (1-\cos \\ & (d*x+c))^2 + a+b) / (\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 + 1) \end{aligned}$$

Fricas [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{3}{2}} dx$$

[In] `integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)`

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)

Giac [F]

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \int \sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{3}{2}} dx$$

[In] integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{\frac{3}{2}} \sqrt{a + b \cos(c + dx)} dx$$

[In] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2),x)

[Out] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2), x)

3.733 $\int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx$

Optimal result	6621
Rubi [A] (verified)	6621
Mathematica [A] (verified)	6622
Maple [A] (verified)	6623
Fricas [F]	6623
Sympy [F]	6623
Maxima [F]	6623
Giac [F]	6624
Mupad [F(-1)]	6624

Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx =$$

$$\frac{2\sqrt{\cos(c + dx)} \sqrt{\frac{a(1 - \cos(c + dx))}{a + b \cos(c + dx)}} \sqrt{\frac{a(1 + \cos(c + dx))}{a + b \cos(c + dx)}} (a + b \cos(c + dx)) \operatorname{csc}(c + dx) \operatorname{EllipticPi}\left(\frac{b}{a + b}, \arcsin\left(\frac{\sqrt{a}}{\sqrt{a + b}}\right)\right)}{\sqrt{a + b}d}$$

[Out] $-2*(a+b*\cos(d*x+c))*\operatorname{csc}(d*x+c)*\operatorname{EllipticPi}((a+b)^{(1/2)}*\cos(d*x+c)^{(1/2)}/(a+b)*\cos(d*x+c)^{(1/2)}, b/(a+b), ((-a+b)/(a+b))^{(1/2)}*\cos(d*x+c)^{(1/2)}*(a*(1-\cos(d*x+c))/(a+b*\cos(d*x+c)))^{(1/2)}*(a*(1+\cos(d*x+c))/(a+b*\cos(d*x+c)))^{(1/2)}*\sec(d*x+c)^{(1/2)}/d/(a+b)^{(1/2)})$

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4307, 2890}

$$\int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx =$$

$$\frac{2\sqrt{\cos(c + dx)} \operatorname{csc}(c + dx) \sqrt{\sec(c + dx)} \sqrt{\frac{a(1 - \cos(c + dx))}{a + b \cos(c + dx)}} \sqrt{\frac{a(\cos(c + dx) + 1)}{a + b \cos(c + dx)}} (a + b \cos(c + dx)) \operatorname{EllipticPi}\left(\frac{b}{a + b}, \arcsin\left(\frac{\sqrt{a}}{\sqrt{a + b}}\right)\right)}{d\sqrt{a + b}}$$

[In] $\operatorname{Int}[\operatorname{Sqrt}[a + b*\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c + d*x]], x]$

[Out] $(-2*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]]*\operatorname{Sqrt}[(a*(1 - \operatorname{Cos}[c + d*x]))/(a + b*\operatorname{Cos}[c + d*x])]*\operatorname{Sqrt}[(a*(1 + \operatorname{Cos}[c + d*x]))/(a + b*\operatorname{Cos}[c + d*x])]*(a + b*\operatorname{Cos}[c + d*x])* \operatorname{Csc}[c + d*x]*\operatorname{EllipticPi}[b/(a + b), \operatorname{ArcSin}[(\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[\operatorname{Cos}[c + d*x]])/\operatorname{Sqrt}[a + b]])/d\operatorname{Sqrt}[a + b])$

+ b*cos[c + d*x]], -(a - b)/(a + b))*Sqrt[Sec[c + d*x]]/(Sqrt[a + b]*d
)

Rule 2890

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d))), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} dx \\ &= \frac{2 \sqrt{\cos(c + dx)} \sqrt{\frac{a(1 - \cos(c + dx))}{a + b \cos(c + dx)}} \sqrt{\frac{a(1 + \cos(c + dx))}{a + b \cos(c + dx)}} (a + b \cos(c + dx)) \csc(c + dx) \text{EllipticPi} \left(\frac{b}{a + b}, \arcsin \left(\frac{b}{a + b} \right) \right)}{\sqrt{a + bd}} \end{aligned}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\begin{aligned} &\int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx \\ &= \frac{2 \sqrt{a + b \cos(c + dx)} \left((a - b) \text{EllipticF} \left(\arcsin \left(\tan \left(\frac{1}{2}(c + dx) \right) \right), \frac{-a + b}{a + b} \right) + 2b \text{EllipticPi} \left(-1, \arcsin \left(\tan \left(\frac{1}{2}(c + dx) \right) \right) \right) \right)}{(a + b)d \sqrt{\frac{(a + b \cos(c + dx)) \sec^2 \left(\frac{1}{2}(c + dx) \right)}{a + b}}} \end{aligned}$$

[In] Integrate[Sqrt[a + b*cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]

[Out] (2*Sqrt[a + b*cos[c + d*x]]*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]/((a + b)*d*Sqrt[((a + b*cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))

Maple [A] (verified)

Time = 8.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.17

method	result
default	$-\frac{2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}}{d\sqrt{a+\cos(dx+c)b}}\left(F\left(\cot(dx+c)-\operatorname{csc}(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)a-F\left(\cot(dx+c)-\operatorname{csc}(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)b+2b\Pi\left(\cot(dx+c)-\operatorname{csc}(dx+c),-1,\sqrt{-\frac{a-b}{a+b}}\right)\right)$

```
[In] int((a+cos(d*x+c)*b)^(1/2)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a-EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*b+2*b*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-a-b)/(a+b))^(1/2))/(a+cos(d*x+c)*b)^(1/2)*sec(d*x+c)^(1/2)*(1+cos(d*x+c))
```

Fricas [F]

$$\int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \sqrt{\sec(dx + c)} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Sympy [F]

$$\int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx = \int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx$$

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \sqrt{\sec(dx + c)} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx = \int \sqrt{b \cos(dx + c) + a} \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} \sqrt{a + b \cos(c + dx)} dx$$

[In] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2),x)

[Out] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2), x)

$$3.734 \quad \int \frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{\sec(c+dx)}} dx$$

Optimal result	6625
Rubi [A] (verified)	6626
Mathematica [B] (warning: unable to verify)	6629
Maple [B] (verified)	6631
Fricas [F]	6632
Sympy [F]	6632
Maxima [F]	6633
Giac [F]	6633
Mupad [F(-1)]	6633

Optimal result

Integrand size = 25, antiderivative size = 431

$$\int \frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{\sec(c+dx)}} dx =$$

$$\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d\sqrt{\sec(c+dx)}} +$$

$$\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b \cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

```
[Out] sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(a-b)*csc(d*x+c)*Ellip
ticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/
2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d
*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+csc(d*x+c)*EllipticF((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*co
s(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2
)/d/sec(d*x+c)^(1/2)-a*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(
1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec
(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00,
 number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used
 = {4307, 2900, 3133, 2888, 12, 2880, 2895, 3073}

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx$$

$$= \frac{\sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{d \sqrt{\sec(c + dx)}} - \frac{(a - b) \sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{ad \sqrt{\sec(c + dx)}} - \frac{a \sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{bd \sqrt{\sec(c + dx)}} + \frac{\sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \cos(c + dx)}}{d}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]

[Out] -(((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b))]/(d*Sqrt[Sec[c + d*x]]) - (a*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b))]/(b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2880

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N

$eQ[c^2 - d^2, 0]$

Rule 2888

$\text{Int}[\text{Sqrt}[(b_)\sin[(e_)] + (f_)(x_)]/\text{Sqrt}[(c_)] + (d_)\sin[(e_)] + (f_)(x_)], x_Symbol] \rightarrow \text{Simp}[2*b*(\text{Tan}[e + f*x]/(d*f))*\text{Rt}[(c + d)/b, 2]*\text{Sqrt}[c*((1 + \text{Csc}[e + f*x])/(c - d))]*\text{Sqrt}[c*((1 - \text{Csc}[e + f*x])/(c + d))]*\text{EllipticPi}[(c + d)/d, \text{ArcSin}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[b*\text{Sin}[e + f*x]]/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; \text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(c + d)/b]$

Rule 2895

$\text{Int}[1/(\text{Sqrt}[(d_)\sin[(e_)] + (f_)(x_)]*\text{Sqrt}[(a_)] + (b_)\sin[(e_)] + (f_)(x_)]), x_Symbol] \rightarrow \text{Simp}[-2*(\text{Tan}[e + f*x]/(a*f))*\text{Rt}[(a + b)/d, 2]*\text{Sqrt}[a*((1 - \text{Csc}[e + f*x])/(a + b))]*\text{Sqrt}[a*((1 + \text{Csc}[e + f*x])/(a - b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[d*\text{Sin}[e + f*x]]/\text{Rt}[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; \text{FreeQ}\{a, b, d, e, f\}, x\} \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{PosQ}[(a + b)/d]$

Rule 2900

$\text{Int}[(a_)] + (b_)\sin[(e_)] + (f_)(x_)]^{(m_)}*((c_)] + (d_)\sin[(e_)] + (f_)(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m - 1)}*((c + d*\text{Sin}[e + f*x])^n/(f*(m + n))), x] + \text{Dist}[1/(d*(m + n)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m - 2)}*(c + d*\text{Sin}[e + f*x])^{(n - 1)}*\text{Simp}[a^2*c*d*(m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b*d*(m + n - 1)))*\text{Sin}[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*\text{Sin}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[0, m, 2] \&\& \text{LtQ}[-1, n, 2] \&\& \text{NeQ}[m + n, 0] \&\& (\text{IntegerQ}[m] \mid\mid \text{IntegersQ}[2*m, 2*n])$

Rule 3073

$\text{Int}[(A_)] + (B_)\sin[(e_)] + (f_)(x_)]/(((b_)\sin[(e_)] + (f_)(x_)]^{(3/2)}*\text{Sqrt}[(c_)] + (d_)\sin[(e_)] + (f_)(x_)]), x_Symbol] \rightarrow \text{Simp}[-2*A*(c - d)*(\text{Tan}[e + f*x]/(f*b*c^2))*\text{Rt}[(c + d)/b, 2]*\text{Sqrt}[c*((1 + \text{Csc}[e + f*x])/(c - d))]*\text{Sqrt}[c*((1 - \text{Csc}[e + f*x])/(c + d))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[b*\text{Sin}[e + f*x]]/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; \text{FreeQ}\{b, c, d, e, f, A, B\}, x\} \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{EqQ}[A, B] \&\& \text{PosQ}[(c + d)/b]$

Rule 3133

$\text{Int}[(A_)] + (C_)\sin[(e_)] + (f_)(x_)]^2/(((a_)] + (b_)\sin[(e_)] + (f_)(x_)]^{(3/2)}*\text{Sqrt}[(c_)] + (d_)\sin[(e_)] + (f_)(x_)]), x_Symbol] :$

```
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
+ Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e, f, A,
C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)} dx \\
&= \frac{\sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{ab}{2} + \frac{1}{2}ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{b} \\
&= \frac{\sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
&\quad + \frac{1}{2} \left(a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int -\frac{ab}{2\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{b} \\
&= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{bd\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
&\quad - \frac{1}{2} \left(a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx
\end{aligned}$$

$$\begin{aligned}
&= \\
&\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{bd\sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{d} \\
&+ \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b}\cos(c+dx)}dx \\
&- \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}\cos(c+dx)}dx \\
&= \\
&\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ad\sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} \\
&- \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{bd\sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2995 vs. 2(431) = 862.

Time = 19.00 (sec) , antiderivative size = 2995, normalized size of antiderivative = 6.95

$$\int \frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{\sec(c+dx)}} dx = \text{Result too large to show}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]

[Out] (Sec[(c + d*x)/2]^2*Sqrt[1 + Sec[c + d*x]]*(8*(a + b)*Sqrt[Cos[c + d*x]]/(1 + Cos[c + d*x]))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 16*a*Sqrt[Cos[c + d*x]]/(1 + Cos[c + d*x])*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 16*a*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(-Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(16*d*((1 + Cos[c + d*x])^(-1))^3/2)*Sqrt[Sec[c + d*x]]*((b*Sec[(c + d*x)/2]^2*Sqrt[1 + Sec[c + d*x]]*Sin[c + d*x]*(8*(a + b)*Sqrt[Cos[c + d*x]]/(1 + Cos

$$\begin{aligned}
& + d*x))/((a + b)*(1 + \text{Cos}[c + d*x])) + ((a + b*\text{Cos}[c + d*x])* \text{Sin}[c + d*x] \\
&)/((a + b)*(1 + \text{Cos}[c + d*x]^2)))/\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/((a + b)*(1 + \\
& \text{Cos}[c + d*x]))] + (8*a*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{EllipticPi}[-1, \\
& \text{ArcSin}[\text{Tan}[(c + d*x)/2]], (-a + b)/(a + b)] * (-((b*\text{Sin}[c + d*x])/((a + b)*(\\
& 1 + \text{Cos}[c + d*x]))) + ((a + b*\text{Cos}[c + d*x])* \text{Sin}[c + d*x])/((a + b)*(1 + \text{Cos} \\
& [c + d*x]^2)))/\text{Sqrt}[(a + b*\text{Cos}[c + d*x])/((a + b)*(1 + \text{Cos}[c + d*x]))] - 2 \\
& *b*\text{Sec}[(c + d*x)/2]^3*\text{Sin}[c + d*x]*(-\text{Sin}[(c + d*x)/2] + \text{Sin}[(3*(c + d*x))/2 \\
&]) + 3*(a + b*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^3*(-\text{Sin}[(c + d*x)/2] + \text{Sin}[(3* \\
& (c + d*x))/2])* \text{Tan}[(c + d*x)/2] - (8*a*\text{Sec}[(c + d*x)/2]^2*\text{Sqrt}[(1 + \text{Sec}[c + \\
& d*x])^{-1}]*\text{Sqrt}[(b + a*\text{Sec}[c + d*x])/((a + b)*(1 + \text{Sec}[c + d*x]))])/(\text{Sqrt} \\
& [1 - \text{Tan}[(c + d*x)/2]^2]*\text{Sqrt}[1 - ((-a + b)*\text{Tan}[(c + d*x)/2]^2)/(a + b)]) + \\
& (8*a*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(a + b*\text{Cos}[c + d*x])/((a + \\
& b)*(1 + \text{Cos}[c + d*x]))] * \text{Sec}[(c + d*x)/2]^2)/(\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2]* \\
& (1 + \text{Tan}[(c + d*x)/2]^2)*\text{Sqrt}[1 - ((-a + b)*\text{Tan}[(c + d*x)/2]^2)/(a + b)]) + \\
& (4*(a + b)*\text{Sqrt}[\text{Cos}[c + d*x]/(1 + \text{Cos}[c + d*x])] * \text{Sqrt}[(a + b*\text{Cos}[c + d*x]) \\
&]/((a + b)*(1 + \text{Cos}[c + d*x]))] * \text{Sec}[(c + d*x)/2]^2*\text{Sqrt}[1 - ((-a + b)*\text{Tan}[(c \\
& + d*x)/2]^2)/(a + b)]/\text{Sqrt}[1 - \text{Tan}[(c + d*x)/2]^2] + 8*a*\text{EllipticF}[\text{ArcSin} \\
& [\text{Tan}[(c + d*x)/2]], (-a + b)/(a + b)] * \text{Sec}[c + d*x]*((1 + \text{Sec}[c + d*x])^{-1}) \\
&)^{(3/2)}*\text{Sqrt}[(b + a*\text{Sec}[c + d*x])/((a + b)*(1 + \text{Sec}[c + d*x]))] * \text{Tan}[c + d*x] \\
&] - (8*a*\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d*x)/2]], (-a + b)/(a + b)] * \text{Sqrt}[(1 + \text{Se} \\
& c[c + d*x])^{-1}]*((a*\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/((a + b)*(1 + \text{Sec}[c + d*x] \\
&)) - (\text{Sec}[c + d*x]*(b + a*\text{Sec}[c + d*x])* \text{Tan}[c + d*x])/((a + b)*(1 + \text{Sec}[c + \\
& d*x])^2)))/\text{Sqrt}[(b + a*\text{Sec}[c + d*x])/((a + b)*(1 + \text{Sec}[c + d*x]))])/((16*(\\
& (1 + \text{Cos}[c + d*x])^{-1})^{(3/2)}*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]))
\end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(391) = 782$.

Time = 6.16 (sec) , antiderivative size = 1069, normalized size of antiderivative = 2.48

method	result	size
default	Expression too large to display	1069

[In] $\text{int}((a+\cos(d*x+c))*b)^{(1/2)}/\text{sec}(d*x+c)^{(1/2)}, x, \text{method}=_RETURNVERBOSE)$

[Out] $1/d/(1+\cos(d*x+c))/(a+\cos(d*x+c))*b)^{(1/2)}/\text{sec}(d*x+c)^{(1/2)}*(2*\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c))*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a*\cos(d*x+c)-\text{EllipticE}(\cot(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c))*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a*\cos(d*x+c)-\text{EllipticE}(\cot(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c))*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*b*\cos(d*x+c)-2*\text{EllipticPi}(\cot(d*x+c)-\text{csc}(d*x+c), -1, (-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c))*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a*\cos(d*x+c)+4*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*\text{EllipticF}(\cot(d*x+c)-\text{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}*((a$

```

cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b
)/(1+cos(d*x+c))/(a+b))^(1/2)*a-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ellipti
cE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x
+c))/(a+b))^(1/2)*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi(cot(d*x+
c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+
b))^(1/2)*a+2*sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(cot(d*
x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2)*a-sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c
)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(
1/2)*a-sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-c
sc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/
2)*b-2*sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi(cot(d*x+c)-c
sc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(
1/2)*a+sin(d*x+c)*cos(d*x+c)*b+a*sin(d*x+c))

```

Fricas [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\sqrt{\sec(dx + c)}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx$$

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)

Giac [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\sec(c + dx)}} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(1/2), x)

$$3.735 \quad \int \frac{\sqrt{a+b \cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	6634
Rubi [A] (verified)	6635
Mathematica [A] (warning: unable to verify)	6638
Maple [B] (verified)	6639
Fricas [F(-1)]	6640
Sympy [F]	6640
Maxima [F]	6641
Giac [F]	6641
Mupad [F(-1)]	6641

Optimal result

Integrand size = 25, antiderivative size = 498

$$\int \frac{\sqrt{a+b \cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)} dx =$$

$$\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4bd\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4bd\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}(a^2-4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4b^2d\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{a\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{4bd}$$

```
[Out] 1/2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*a*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/4*(a-b)*csc(d*x+c)*EllipticE((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/4*(a+2*b)*csc(d*x+c)*EllipticF((a+b*co
s(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1
/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c))/(a-b
))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/4*(a^2-4*b^2)*csc(d*x+c)*EllipticPi((a+b*cos
(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4307, 2900, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{\sqrt{a + b}(a^2 - 4b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{4b^2 d \sqrt{\sec(c + dx)}} + \frac{\sqrt{a + b}(a + 2b) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{4bd \sqrt{\sec(c + dx)}} - \frac{(a - b) \sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right) \Big| - \frac{a + b}{a - b}}{4bd \sqrt{\sec(c + dx)}} + \frac{a \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \cos(c + dx)}}{4bd} + \frac{\sin(c + dx) \sqrt{a + b \cos(c + dx)}}{2d \sqrt{\sec(c + dx)}}$$

[In] Int[Sqrt[a + b*Cos[c + d*x]]/Sec[c + d*x]^(3/2), x]

[Out] -1/4*((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*d)

Rule 2888

Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)])*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 2900

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3132

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2)/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
```


$\text{Sqrt}[c + d*\text{Sin}[e + f*x]], x], x] + \text{Dist}[1/b^2, \text{Int}[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*\text{Sin}[e + f*x])/((a + b*\text{Sin}[e + f*x])^{3/2}*\text{Sqrt}[c + d*\text{Sin}[e + f*x])], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3140

$\text{Int}[(A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^2)/(\text{Sqrt}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]]*\text{Sqrt}[(c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]]), x_Symbol] := \text{Simp}[(-C)*\text{Cos}[e + f*x]*(\text{Sqrt}[c + d*\text{Sin}[e + f*x]])/(d*f*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] + \text{Dist}[1/(2*d), \text{Int}[(1/((a + b*\text{Sin}[e + f*x])^{3/2}*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]))*\text{Simp}[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*\text{Sin}[e + f*x] + (2*b*B*d - C*(b*c + a*d))*\text{Sin}[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 4307

$\text{Int}[(\text{csc}[(a_.) + (b_.)*(x_.)]*(c_.))^{(m_.)}*(u_.), x_Symbol] := \text{Dist}[(c*\text{Csc}[a + b*x])^m*(c*\text{Sin}[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u]/(c*\text{Sin}[a + b*x])^m, x], x] /;$ FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)} dx \\
 &= \frac{\sqrt{a+b\cos(c+dx)} \sin(c+dx)}{2d\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{ab}{2} + b^2 \cos(c+dx) + \frac{1}{2} ab \cos^2(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{2b} \\
 &= \frac{\sqrt{a+b\cos(c+dx)} \sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{a\sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{4bd} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{a^2b}{2} + ab^2 \cos(c+dx) - \frac{1}{2} b(a^2 - 4b^2) \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{4b^2} \\
 &= \frac{\sqrt{a+b\cos(c+dx)} \sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{a\sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{4bd} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{a^2b}{2} + ab^2 \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{4b^2} \\
 &\quad - \frac{\left((a^2 - 4b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{8b}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{\sqrt{a+b}(a^2 - 4b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4b^2 d \sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b\cos(c+dx)} \sin(c+dx)}{2d \sqrt{\sec(c+dx)}} + \frac{a \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{4bd} \\
&- \frac{\left(a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{8b} \\
&+ \frac{\left(a(a+2b) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{8b} \\
&= \frac{(a-b) \sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{4bd \sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}(a+2b) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4bd \sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}(a^2 - 4b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4b^2 d \sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b\cos(c+dx)} \sin(c+dx)}{2d \sqrt{\sec(c+dx)}} + \frac{a \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{4bd}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 15.95 (sec) , antiderivative size = 837, normalized size of antiderivative = 1.68

$$\begin{aligned}
\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)} dx &= \frac{\sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(2(c+dx))}{4d} \\
&+ \frac{-a^2 \tan\left(\frac{1}{2}(c+dx)\right) - ab \tan\left(\frac{1}{2}(c+dx)\right) + 2ab \tan^3\left(\frac{1}{2}(c+dx)\right) + a^2 \tan^5\left(\frac{1}{2}(c+dx)\right) - ab \tan^5\left(\frac{1}{2}(c+dx)\right)}{4d}
\end{aligned}$$

[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Sec[c + d*x]^(3/2), x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)]/(4*d) + (-a^2*Tan[(c + d*x)/2]) - a*b*Tan[(c + d*x)/2] + 2*a*b*Tan[(c + d*x)/2]^3 + a^2*Tan[(c + d*x)/2]^5 - a*b*Tan[(c + d*x)/2]^5 + 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b]) - 8*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2])

$$\begin{aligned}
& 2]^{2} * \text{Sqrt}[(a + b + a * \text{Tan}[(c + d * x) / 2]^{2} - b * \text{Tan}[(c + d * x) / 2]^{2}) / (a + b)] + \\
& 2 * a^{2} * \text{EllipticPi}[-1, \text{ArcSin}[\text{Tan}[(c + d * x) / 2]], (-a + b) / (a + b)] * \text{Tan}[(c + \\
& d * x) / 2]^{2} * \text{Sqrt}[1 - \text{Tan}[(c + d * x) / 2]^{2}] * \text{Sqrt}[(a + b + a * \text{Tan}[(c + d * x) / 2]^{2} - \\
& b * \text{Tan}[(c + d * x) / 2]^{2}) / (a + b)] - 8 * b^{2} * \text{EllipticPi}[-1, \text{ArcSin}[\text{Tan}[(c + d * x) \\
& / 2]], (-a + b) / (a + b)] * \text{Tan}[(c + d * x) / 2]^{2} * \text{Sqrt}[1 - \text{Tan}[(c + d * x) / 2]^{2}] * \text{Sqr} \\
& t[(a + b + a * \text{Tan}[(c + d * x) / 2]^{2} - b * \text{Tan}[(c + d * x) / 2]^{2}) / (a + b)] - a * (a + b \\
&) * \text{EllipticE}[\text{ArcSin}[\text{Tan}[(c + d * x) / 2]], (-a + b) / (a + b)] * \text{Sqrt}[1 - \text{Tan}[(c + d \\
& * x) / 2]^{2}] * (1 + \text{Tan}[(c + d * x) / 2]^{2}) * \text{Sqrt}[(a + b + a * \text{Tan}[(c + d * x) / 2]^{2} - b * \text{T} \\
& \text{an}[(c + d * x) / 2]^{2}) / (a + b)] - 2 * (a - 2 * b) * b * \text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d * x) / \\
& 2]], (-a + b) / (a + b)] * \text{Sqrt}[1 - \text{Tan}[(c + d * x) / 2]^{2}] * (1 + \text{Tan}[(c + d * x) / 2]^{2} \\
&) * \text{Sqrt}[(a + b + a * \text{Tan}[(c + d * x) / 2]^{2} - b * \text{Tan}[(c + d * x) / 2]^{2}) / (a + b)] / (4 * b \\
& * d * \text{Sqrt}[(1 - \text{Tan}[(c + d * x) / 2]^{2})^{-1}] * (-1 + \text{Tan}[(c + d * x) / 2]^{2}) * (1 + \text{Tan}[(c \\
& + d * x) / 2]^{2})^{3/2} * \text{Sqrt}[(a + b + a * \text{Tan}[(c + d * x) / 2]^{2} - b * \text{Tan}[(c + d * x) / 2 \\
&]^{2}) / (1 + \text{Tan}[(c + d * x) / 2]^{2})]
\end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1654 vs. 2(444) = 888.

Time = 7.52 (sec) , antiderivative size = 1655, normalized size of antiderivative = 3.32

method	result	size
default	Expression too large to display	1655

[In] int((a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out]
$$\begin{aligned}
& -1/4/d/(1+\cos(d*x+c))/(a+\cos(d*x+c)*b)^{(1/2)}/\sec(d*x+c)^{(3/2)}*(2*((a+\cos(d* \\
& x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b) \\
& / (a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*a*b-4*((a+\cos(d*x+c)*b)/(1 \\
& +\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1 \\
& /2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*b^2+((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) \\
& / (a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d* \\
& x+c)/(1+\cos(d*x+c)))^{(1/2)}*a^2+((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\
&)*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(\\
& d*x+c)))^{(1/2)}*a*b-2*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{Elliptic} \\
& \text{Pi}(\cot(d*x+c)-\csc(d*x+c),-1,(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c) \\
&))^{(1/2)}*a^2+8*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*\text{EllipticPi}(\cot \\
& (d*x+c)-\csc(d*x+c),-1,(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/ \\
& 2)}*b^2+4*\sec(d*x+c)*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\\
& (a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1 \\
& /2)}*a*b-8*\sec(d*x+c)*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}* \\
& ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(\\
& 1/2)}*b^2+2*\sec(d*x+c)*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)} \\
&)*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{ \\
& (1/2)}*a^2+2*\sec(d*x+c)*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)} \\
&)*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))
\end{aligned}$$

```

^(1/2)*a*b-4*sec(d*x+c)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2+16*sec(d*x+c)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^2-2*cos(d*x+c)*b^2*sin(d*x+c)+2*sec(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b-4*sec(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^2+sec(d*x+c)^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2+sec(d*x+c)^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b-2*sec(d*x+c)^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2+8*sec(d*x+c)^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^2-3*a*b*sin(d*x+c)-2*sin(d*x+c)*b^2-tan(d*x+c)*a^2-2*tan(d*x+c)*a*b)/b

```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)

[Out] Integral(sqrt(a + b*cos(c + d*x))/sec(c + d*x)**(3/2), x)

Maxima [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(dx + c) + a}}{\sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b \cos(c + dx)}}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{a + b \cos(c + dx)}}{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{3}{2}}} dx$$

[In] int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2),x)

[Out] int((a + b*cos(c + d*x))^(1/2)/(1/cos(c + d*x))^(3/2), x)

3.736 $\int (a + b \cos(c + dx))^{3/2} \sec^{\frac{9}{2}}(c + dx) dx$

Optimal result	6642
Rubi [A] (verified)	6643
Mathematica [A] (verified)	6646
Maple [B] (verified)	6647
Fricas [F]	6648
Sympy [F(-1)]	6648
Maxima [F]	6649
Giac [F]	6649
Mupad [F(-1)]	6649

Optimal result

Integrand size = 25, antiderivative size = 427

$$\int (a + b \cos(c + dx))^{3/2} \sec^{\frac{9}{2}}(c + dx) dx = \frac{4(a-b)b\sqrt{a+b}(41a^2 - 3b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{105a^3 d \sqrt{\sec(c+dx)}} + \frac{2(a-b)\sqrt{a+b}(25a^2 - 57ab - 6b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{105a^2 d \sqrt{\sec(c+dx)}} + \frac{2(25a^2 + 3b^2) \sqrt{a+b}\cos(c+dx) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{105ad} + \frac{16b\sqrt{a+b}\cos(c+dx) \sec^{\frac{5}{2}}(c+dx) \sin(c+dx)}{35d} + \frac{2a\sqrt{a+b}\cos(c+dx) \sec^{\frac{7}{2}}(c+dx) \sin(c+dx)}{7d}$$

```
[Out] 2/105*(25*a^2+3*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d
+16/35*b*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/7*a*sec(d*x
+c)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+4/105*(a-b)*b*(41*a^2-3*b^2)*
csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((
-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b))^(
1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a^3/d/sec(d*x+c)^(1/2)+2/105*(a-b)*(25*
a^2-57*a*b-6*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/c
os(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-se
c(d*x+c)))/(a+b))^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a^2/d/sec(d*x+c)^(1/2
)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00,
 number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used
 = {4307, 2878, 3134, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \frac{2(a-b)\sqrt{a+b}(25a^2 - 57ab - 6b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticE}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right) + \frac{2(25a^2 + 3b^2) \sin(c+dx) \sec^{3/2}(c+dx) \sqrt{a+b\cos(c+dx)}}{105ad} + \frac{4b(a-b)\sqrt{a+b}(41a^2 - 3b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right) + \frac{2a \sin(c+dx) \sec^{7/2}(c+dx) \sqrt{a+b\cos(c+dx)}}{7d} + \frac{16b \sin(c+dx) \sec^{5/2}(c+dx) \sqrt{a+b\cos(c+dx)}}{35d}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2), x]

[Out] (4*(a - b)*b*Sqrt[a + b]*(41*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(105*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(25*a^2 - 57*a*b - 6*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(105*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a*d) + (16*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] & NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]], x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{9/2}(c+dx)} dx \\
&= \frac{2a\sqrt{a+b\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{1}{7} \left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{4ab + \frac{1}{2}(5a^2 + 7b^2) \cos(c+dx) + 2ab \cos^2(c+dx)}{\cos^{7/2}(c+dx) \sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{16b\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{35d} \\
&\quad + \frac{2a\sqrt{a+b\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{\left(4\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{4}a(25a^2+3b^2)+11a^2b\cos(c+dx)+4ab^2\cos^2(c+dx)}{\cos^{5/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{35a} \\
&= \frac{2(25a^2 + 3b^2) \sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{105ad} \\
&\quad + \frac{16b\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{35d} \\
&\quad + \frac{2a\sqrt{a+b\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{\left(8\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{4}ab(41a^2-3b^2)+\frac{1}{8}a^2(25a^2+51b^2)\cos(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{105a^2} \\
&= \frac{2(25a^2 + 3b^2) \sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{105ad} \\
&\quad + \frac{16b\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{35d} \\
&\quad + \frac{2a\sqrt{a+b\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{\left((a-b)(25a^2 - 57ab - 6b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{105a} \\
&\quad + \frac{\left(2b(41a^2 - 3b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{3/2}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{105a}
\end{aligned}$$

$$\begin{aligned}
&= \frac{4(a-b)b\sqrt{a+b}(41a^2-3b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{105a^3d\sqrt{\sec(c+dx)}} \\
&+ \frac{2(a-b)\sqrt{a+b}(25a^2-57ab-6b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{105a^2d\sqrt{\sec(c+dx)}} \\
&+ \frac{2(25a^2+3b^2)\sqrt{a+b}\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105ad} \\
&+ \frac{16b\sqrt{a+b}\cos(c+dx)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} \\
&+ \frac{2a\sqrt{a+b}\cos(c+dx)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.05 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.03

$$\begin{aligned}
&\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c \\
&+dx)dx = \frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\sec(c+dx)\left(2b(-41a^3-41a^2b+3ab^2+3b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\right)}{\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}} \\
&+ \frac{\left(-\frac{4b(-41a^2+3b^2)\sin(c+dx)}{105a^2} + \frac{2\sec(c+dx)(25a^2\sin(c+dx)+3b^2\sin(c+dx))}{105a} + \frac{16}{35}b\sec(c+dx)\right)}{d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2), x]

[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(-41*a^3 - 41*a^2*b + 3*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(a + b)*(1 + Cos[c + d*x])])*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(25*a^3 + 82*a^2*b + 51*a*b^2 - 6*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(-41*a^2 + 3*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(105*a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-4*b*(-41*a^2 + 3*b^2)*Sin[c + d*x])/(105*a^2) + (2*Sec[c + d*x]*(25*a^2*Sin[c + d*x] + 3*b^2*Sin[c + d*x]))/(105*a) + (16*b*Sec[c + d*x]*Tan[c + d*x])/35 + (2*a*Sec[c + d*x]^2*Tan[c + d*x])/7))/d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2508 vs. 2(381) = 762.

Time = 15.28 (sec) , antiderivative size = 2509, normalized size of antiderivative = 5.88

method	result	size
default	Expression too large to display	2509

[In] $\text{int}((a+\cos(dx+c)*b)^{(3/2)}*\sec(dx+c)^{(9/2)},x,\text{method}=_RETURNVERBOSE)$

[Out] $\frac{2}{105}d*\sec(dx+c)^{(9/2)}/(1+\cos(dx+c))/(a+\cos(dx+c)*b)^{(1/2)}*(25*a^4*\cos(dx+c)^4*\sin(dx+c)+15*a^4*\cos(dx+c)^2*\sin(dx+c)+27*a^2*b^2*\cos(dx+c)^4*\sin(dx+c)-6*\cos(dx+c)^5*b^4*\sin(dx+c)-12*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*b^4*\cos(dx+c)^5-50*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a^4*\cos(dx+c)^5-6*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*b^4*\cos(dx+c)^4-25*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a^4*\cos(dx+c)^4-6*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a*b^3*\cos(dx+c)^4-82*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a^3*b*\cos(dx+c)^4-51*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a^2*b^2*\cos(dx+c)^4+6*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a*b^3*\cos(dx+c)^4+164*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a^3*b*\cos(dx+c)^5+164*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a^2*b^2*\cos(dx+c)^5-12*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a*b^3*\cos(dx+c)^5-164*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a^3*b*\cos(dx+c)^5-102*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a^2*b^2*\cos(dx+c)^5+12*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a*b^3*\cos(dx+c)^5+82*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a^3*b*\cos(dx+c)^4+82*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(dx+c)*b)/(1+\cos(dx+c))/(a+b))^{(1/2)}*(\cos(dx+c)/(1+\cos(dx+c)))^{(1/2)}*a^2*b^2*\cos(dx+c)^4$

```

)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b^2*cos(d*x+c)^4-6*cos(d*x+c)
^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)
)^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-25*cos(d*
x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(
a+b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-82*co
s(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
))/(a+b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b
-51*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos
(d*x+c)))/(a+b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b^2+6*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)
/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^3+82*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x
+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^3*b+82*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+c
os(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^2-6*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*EllipticE(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+25*a^4*cos(d*x+c)^3*sin(d*x+c)+15*a^4*cos(
d*x+c)*sin(d*x+c)+25*cos(d*x+c)^5*a^3*b*sin(d*x+c)+82*cos(d*x+c)^5*a^2*b^2*
sin(d*x+c)+3*cos(d*x+c)^5*a*b^3*sin(d*x+c)+39*a^3*b*cos(d*x+c)^2*sin(d*x+c)
+39*a^3*b*cos(d*x+c)^3*sin(d*x+c)+27*a^2*b^2*cos(d*x+c)^3*sin(d*x+c)+107*a^
3*b*cos(d*x+c)^4*sin(d*x+c)-3*a*b^3*cos(d*x+c)^4*sin(d*x+c))/a^2

```

Fricas [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{9/2} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{9/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{9/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^{9/2}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{9/2} (a + b \cos(c + dx))^{3/2} dx$$

[In] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2),x)

[Out] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(3/2), x)

3.737 $\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx$

Optimal result	6650
Rubi [A] (verified)	6651
Mathematica [A] (verified)	6654
Maple [B] (verified)	6654
Fricas [F]	6656
Sympy [F(-1)]	6656
Maxima [F]	6656
Giac [F]	6656
Mupad [F(-1)]	6657

Optimal result

Integrand size = 25, antiderivative size = 365

$$\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \frac{2(a-b)\sqrt{a+b}(3a^2+b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{5a^2d\sqrt{\sec(c+dx)}} + \frac{2(a-b)(3a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{5ad\sqrt{\sec(c+dx)}} + \frac{4b\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}$$

```
[Out] 4/5*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5*a*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5*(a-b)*(3*a^2+b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2/5*(a-b)*(3*a-b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00,
 number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used
 = {4307, 2878, 3134, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \frac{2(a-b)\sqrt{a+b}(3a^2+b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{5a^2d\sqrt{\sec(c+dx)}} - \frac{2(a-b)(3a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{5ad\sqrt{\sec(c+dx)}} + \frac{2a\sin(c+dx)\sec^{5/2}(c+dx)\sqrt{a+b\cos(c+dx)}}{5d} + \frac{4b\sin(c+dx)\sec^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}}{5d}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2), x]

[Out] (2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(5*a^2*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*(3*a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(5*a*d*Sqrt[Sec[c + d*x]]) + (4*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_)+(f_)*(x_)])*Sqrt[(a_)+(b_)*sin[(e_)+(f_)*(x_)])], x_Symbol] :> Simp[-2*(Tan[e+f*x]/(a*f))*Rt[(a+b)/d, 2]*Sqrt[a*((1-Csc[e+f*x])/(a+b))*Sqrt[a*((1+Csc[e+f*x])/(a-b))]*EllipticF[ArcSin[Sqrt[a+b*Sin[e+f*x]]/Sqrt[d*Sin[e+f*x]]/Rt[(a+b)/d, 2]], -(a+b)/(a-b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2-b^2, 0] && PosQ[(a+b)/d]
```

Rule 3073

```
Int[((A_)+(B_)*sin[(e_)+(f_)*(x_)])/(((b_)*sin[(e_)+(f_)*(x_)])^(3/2)*Sqrt[(c_)+(d_)*sin[(e_)+(f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c-d)*(Tan[e+f*x]/(f*b*c^2))*Rt[(c+d)/b, 2]*Sqrt[c*((1+Csc[e+f*x])/(c-d))*Sqrt[c*((1-Csc[e+f*x])/(c+d))]*EllipticE[ArcSin[Sqrt[c+d*Sin[e+f*x]]/Sqrt[b*Sin[e+f*x]]/Rt[(c+d)/b, 2]], -(c+d)/(c-d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2-d^2, 0] && EqQ[A, B] && PosQ[(c+d)/b]
```

Rule 3077

```
Int[((A_)+(B_)*sin[(e_)+(f_)*(x_)])/(((a_)+(b_)*sin[(e_)+(f_)*(x_)])^(3/2)*Sqrt[(c_)+(d_)*sin[(e_)+(f_)*(x_)]]), x_Symbol] :> Dist[(A-B)/(a-b), Int[1/(Sqrt[a+b*Sin[e+f*x]]*Sqrt[c+d*Sin[e+f*x]]), x], x] - Dist[(A*b-a*B)/(a-b), Int[(1+Sin[e+f*x])/((a+b*Sin[e+f*x])^(3/2)*Sqrt[c+d*Sin[e+f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c-a*d, 0] && NeQ[a^2-b^2, 0] && NeQ[c^2-d^2, 0] && NeQ[A, B]
```

Rule 3134

```
Int[((a_)+(b_)*sin[(e_)+(f_)*(x_)])^(m_)*((c_)+(d_)*sin[(e_)+(f_)*(x_)])^(n_)*((A_)+(B_)*sin[(e_)+(f_)*(x_)]+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2-a*b*B+a^2*C))*Cos[e+f*x]*(a+b*Sin[e+f*x])^(m+1)*((c+d*Sin[e+f*x])^(n+1)/(f*(m+1)*(b*c-a*d)*(a^2-b^2))), x] + Dist[1/((m+1)*(b*c-a*d)*(a^2-b^2)), Int[(a+b*Sin[e+f*x])^(m+1)*(c+d*Sin[e+f*x])^n*Simp[(m+1)*(b*c-a*d)*(a*A-b*B+a*C)+d*(A*b^2-a*b*B+a^2*C)*(m+n+2)-(c*(A*b^2-a*b*B+a^2*C)+(m+1)*(b*c-a*d)*(A*b-a*B+b*C))*Sin[e+f*x]-d*(A*b^2-a*b*B+a^2*C)*(m+n+3)*Sin[e+f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c-a*d, 0] && NeQ[a^2-b^2, 0] && NeQ[c^2-d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 4307

```
Int[(csc[(a_)+(b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Csc[a
```


+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{7/2}(c+dx)} dx \\
 &= \frac{2a\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{5d} \\
 &\quad + \frac{1}{5} \left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{3ab + \frac{1}{2}(3a^2 + 5b^2) \cos(c+dx) + ab \cos^2(c+dx)}{\cos^{5/2}(c+dx) \sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{4b\sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{5d} \\
 &\quad + \frac{2a\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{5d} \\
 &\quad + \frac{\left(4\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{3}{4}a(3a^2+b^2)+3a^2b\cos(c+dx)}{\cos^{3/2}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{15a} \\
 &= \frac{4b\sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{5d} \\
 &\quad + \frac{2a\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{5d} - \frac{1}{5} \left((a-b)(3a \right. \\
 &\quad \left. - b) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx \\
 &\quad + \frac{1}{5} \left((3a^2 \right. \\
 &\quad \left. + b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1 + \cos(c+dx)}{\cos^{3/2}(c+dx) \sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{2(a-b)\sqrt{a+b}(3a^2+b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{5a^2d\sqrt{\sec(c+dx)}} \\
 &\quad - \frac{2(a-b)(3a-b)\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{5ad\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{4b\sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{5d} \\
 &\quad + \frac{2a\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 8.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.95

$$\int (a + b \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \frac{2 \left(\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right) \sec(c+dx)} \left(-2(3a^3 + 3a^2b + ab^2 + b^3) E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right) \middle| \frac{-a+b}{a+b}\right) \sqrt{\frac{1}{1+\sec(c+dx)}} \sqrt{\frac{b+a \sec(c+dx)}{(a+b)(1+\sec(c+dx))}} + 2a(3a^2 + 4ab + b^2) \operatorname{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right) \middle| \frac{-a+b}{a+b}\right) \sqrt{(1+\sec(c+dx))^{-1}} \sqrt{\frac{b+a \sec(c+dx)}{(a+b)(1+\sec(c+dx))}} - (3a^2 + b^2) \cos(c+dx) (a + b \cos(c+dx)) \sec\left(\frac{c+dx}{2}\right)^2 \tan\left(\frac{c+dx}{2}\right) \right) / \sqrt{\sec\left(\frac{c+dx}{2}\right)^2 + (a + b \cos(c+dx)) \operatorname{Sqrt}[\sec(c+dx)] * ((3a^2 + b^2) \sin(c+dx) + a(2b + a \sec(c+dx)) \tan(c+dx))} \right)}{5a d \operatorname{Sqrt}[a + b \cos(c + dx)]}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2), x]

[Out] (2*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*a*(3*a^2 + 4*a*b + b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/Sqrt[Sec[(c + d*x)/2]^2 + (a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((3*a^2 + b^2)*Sin[c + d*x] + a*(2*b + a*Sec[c + d*x])*Tan[c + d*x])))/(5*a*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2113 vs. 2(325) = 650.

Time = 13.08 (sec) , antiderivative size = 2114, normalized size of antiderivative = 5.79

method	result	size
default	Expression too large to display	2114

[In] int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] -2/5/d*sec(d*x+c)^(7/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(-2*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^4+8*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^4+2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^4-3*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3-EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^3+4*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*

$$\begin{aligned}
& x+c)/(1+\cos(d*x+c))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^2*b*\cos(d*x+c)^3+\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)} \\
& *a*b^2*\cos(d*x+c)^3+3*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)} \\
& *(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^3*\cos(d*x+c)^3-\sin(d*x+c)*\cos(d*x+c)*a^3-3*\sin(d*x+c)*\cos(d*x+c)^2* \\
& a^2*b-6*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^2*b*\cos(d*x+c)^4+3*\cos(d*x+c)^5*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)} \\
& *((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*a^3-3*\cos(d*x+c)^5*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}* \\
& (\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*a^3-b^3*\cos(d*x+c)^4*\sin(d*x+c)-3*a^2*b*\cos(d*x+c)^3*\sin(d*x+c) \\
&)-3*a*b^2*\cos(d*x+c)^3*\sin(d*x+c)+4*\cos(d*x+c)^5*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*a^2*b+\cos(d*x+c)^5*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*a*b^2-3*\cos(d*x+c)^5*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*a^2*b-\cos(d*x+c)^5*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*a*b^2-6*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^3*\cos(d*x+c)^4-2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*b^3*\cos(d*x+c)^4+6*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^3*\cos(d*x+c)^4-3*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*a^3*\cos(d*x+c)^3-\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*b^3*\cos(d*x+c)^3-a^3*\cos(d*x+c)^2*\sin(d*x+c)-\cos(d*x+c)^5*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*b^3-3*\cos(d*x+c)^3*a^3*\sin(d*x+c)-3*\cos(d*x+c)^4*a^2*b*\sin(d*x+c)-2*\cos(d*x+c)^4*a*b^2*\sin(d*x+c))/a
\end{aligned}$$

Fricas [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{7/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{7/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{7/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^{7/2}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{7/2} (a + b \cos(c + dx))^{3/2} dx$$

```
[In] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.738 $\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx$

Optimal result	6658
Rubi [A] (verified)	6659
Mathematica [A] (verified)	6661
Maple [B] (verified)	6661
Fricas [F]	6662
Sympy [F(-1)]	6663
Maxima [F]	6663
Giac [F]	6663
Mupad [F(-1)]	6663

Optimal result

Integrand size = 25, antiderivative size = 317

$$\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \frac{8(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{3ad\sqrt{\sec(c+dx)}} + \frac{2(a-3b)(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3ad\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$

```
[Out] 2/3*a*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+8/3*(a-b)*b*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/3*(a-3*b)*(a-b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00,
 number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used
 = {4307, 2878, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{3/2} \sec^{\frac{5}{2}}(c + dx) dx = \frac{2(a - 3b)(a - b)\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b}\sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{3ad\sqrt{\sec(c + dx)}} + \frac{8b(a - b)\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b}\sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{3ad\sqrt{\sec(c + dx)}} + \frac{2a \sin(c + dx) \sec^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}}{3d}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]

[Out] (8*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*(a - 3*b)*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*(a + b*Sint[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] & NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)])]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

&& PosQ[(a + b)/d]

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)]
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{5/2}(c + dx)} dx \\
&= \frac{2a \sqrt{a + b \cos(c + dx)} \sec^{3/2}(c + dx) \sin(c + dx)}{3d} \\
&\quad + \frac{1}{3} \left(2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{2ab + \frac{1}{2}(a^2 + 3b^2) \cos(c + dx)}{\cos^{3/2}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2a \sqrt{a + b \cos(c + dx)} \sec^{3/2}(c + dx) \sin(c + dx)}{3d} + \frac{1}{3} \left((a - 3b)(a \right. \\
&\quad \left. - b) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
&\quad + \frac{1}{3} \left(4ab \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1 + \cos(c + dx)}{\cos^{3/2}(c + dx) \sqrt{a + b \cos(c + dx)}} dx
\end{aligned}$$

$$\begin{aligned}
&= \frac{8(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{3ad\sqrt{\sec(c+dx)}} \\
&+ \frac{2(a-3b)(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{3ad\sqrt{\sec(c+dx)}} \\
&+ \frac{2a\sqrt{a+b}\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.90 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.92

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)dx = \frac{\sqrt{\sec(c+dx)}\left(4\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-4b(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\right)}{3d}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(4*Cos[(c + d*x)/2]^2*(-4*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])) + 2*(a + b*Cos[c + d*x])*(a + 4*b*Cos[c + d*x])*Tan[c + d*x]))/(3*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1468 vs. 2(283) = 566.

Time = 11.98 (sec) , antiderivative size = 1469, normalized size of antiderivative = 4.63

method	result	size
default	Expression too large to display	1469

[In] int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^(5/2), x, method=_RETURNVERBOSE)

[Out] 2/3/d*sec(d*x+c)^(5/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(4*cos(d*x+c)^4*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a*b+4*cos(d*x+c)

```

c)^4*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2-cos(d*x
+c)^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2-4*cos(
d*x+c)^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d*x+
c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b-3*c
os(d*x+c)^4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos(d
*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^2+
8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*cos(d*x+c)
^3+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*cos(d*x
+c)^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/
(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(
d*x+c)^3-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*c
os(d*x+c)^3-6*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+cos
(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*b^
2*cos(d*x+c)^3+4*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*((a+
cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*a*b*cos(d*x+c)^2+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/
2))*b^2*cos(d*x+c)^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^2*cos(d*x+c)^2-4*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2
))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*a*b*cos(d*x+c)^2-3*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(
1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*b^2*cos(d*x+c)^2+a*b*cos(d*x+c)^3*sin(d*x+c)+4*b^2*cos(d*x+c)^3*
sin(d*x+c)+a^2*cos(d*x+c)^2*sin(d*x+c)+5*cos(d*x+c)^2*sin(d*x+c)*a*b+a^2*co
s(d*x+c)*sin(d*x+c))

```

Fricas [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{5/2} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{5/2} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

Giac [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{5/2} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^{5/2}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{5/2} (a + b \cos(c + dx))^{3/2} dx$$

```
[In] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2),x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.739 $\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx$

Optimal result	6664
Rubi [A] (verified)	6665
Mathematica [A] (verified)	6667
Maple [B] (warning: unable to verify)	6668
Fricas [F]	6669
Sympy [F(-1)]	6669
Maxima [F]	6669
Giac [F]	6669
Mupad [F(-1)]	6670

Optimal result

Integrand size = 25, antiderivative size = 397

$$\int (a + b \cos(c + dx))^{3/2} \sec^2(c + dx) dx = \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} - \frac{2(a-2b)\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} - \frac{2b\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}}$$

```
[Out] 2*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/d/sec(d*x+c)^(1/2)-2*(a-2*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/d/sec(d*x+c)^(1/2)-2*b*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2877, 2888, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx =$$

$$\frac{2(a - 2b)\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{d\sqrt{\sec(c + dx)}} +$$

$$\frac{2(a - b)\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{d\sqrt{\sec(c + dx)}} -$$

$$\frac{2b\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{d\sqrt{\sec(c + dx)}}.$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]

[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(d*Sqrt[Sec[c + d*x]]) - (2*(a - 2*b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(d*Sqrt[Sec[c + d*x]])

Rule 2877

Int[((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(3/2)/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2888

Int[Sqrt[(b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]]/Rt[(c +

d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 4307

Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\text{integral} = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{(a + b \cos(c + dx))^{3/2}}{\cos^{3/2}(c + dx)} dx$$

$$\begin{aligned}
&= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{a+2b\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&\quad + \left(b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{2b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} \\
&\quad + \left(a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&\quad + \left(a(-a+2b)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} \\
&\quad - \frac{2(a-2b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} \\
&\quad - \frac{2b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 15.74 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.61

$$\int (a+b\cos(c+dx))^{\frac{3}{2}}\sec^{\frac{3}{2}}(c+dx) dx = \frac{2a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} \\
+ \frac{2\left(a^2\tan\left(\frac{1}{2}(c+dx)\right) + ab\tan\left(\frac{1}{2}(c+dx)\right) - 2ab\tan^3\left(\frac{1}{2}(c+dx)\right) - a^2\tan^5\left(\frac{1}{2}(c+dx)\right) + ab\tan^5\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2), x]

[Out] (2*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*(a^2*Tan[(c + d*x)/2] + a*b*Tan[(c + d*x)/2] - 2*a*b*Tan[(c + d*x)/2]^3 - a^2*Tan[(c + d*x)/2]^5 + a*b*Tan[(c + d*x)/2]^5 - 2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)

$$\begin{aligned} &]^2/(a+b)] + a*(a+b)*\text{EllipticE}[\text{ArcSin}[\text{Tan}[(c+d*x)/2]], (-a+b)/(a+b)] \\ & * \text{Sqrt}[1 - \text{Tan}[(c+d*x)/2]^2*(1 + \text{Tan}[(c+d*x)/2]^2)*\text{Sqrt}[(a+b+a*\text{Tan}[(c+d*x)/2]^2 - b*\text{Tan}[(c+d*x)/2]^2)/(a+b)] - (a^2 + 2*a*b - b^2)*\text{E} \\ & \text{llipticF}[\text{ArcSin}[\text{Tan}[(c+d*x)/2]], (-a+b)/(a+b)]*\text{Sqrt}[1 - \text{Tan}[(c+d*x)/2]^2*(1 + \text{Tan}[(c+d*x)/2]^2)*\text{Sqrt}[(a+b+a*\text{Tan}[(c+d*x)/2]^2 - b*\text{Tan}[(c+d*x)/2]^2)/(a+b))] \\ &)/(d*\text{Sqrt}[(1 - \text{Tan}[(c+d*x)/2]^2)^{-1}]*(-1 + \text{Tan}[(c+d*x)/2]^2*(1 + \text{Tan}[(c+d*x)/2]^2)^{(3/2)}*\text{Sqrt}[(a+b+a*\text{Tan}[(c+d*x)/2]^2 - b*\text{Tan}[(c+d*x)/2]^2)/(1 + \text{Tan}[(c+d*x)/2]^2)]) \end{aligned}$$

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(361) = 722$.

Time = 10.27 (sec) , antiderivative size = 1016, normalized size of antiderivative = 2.56

method	result	size
default	Expression too large to display	1016

[In] `int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -2/d*(-(\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})/(\csc(d*x+c)^2*(1-\cos(d*x+c))^{2-1})) \\ & ^{(3/2)}*(\csc(d*x+c)^2*(1-\cos(d*x+c))^{2-1})*(-(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{(1/2)}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^2 \\ & -2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{(1/2)}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a*b+(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{(1/2)} \\ & *((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{(1/2)}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*b^2+(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^2+(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a*b-2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{(1/2)}*\text{EllipticPi}(\cot(d*x+c)-\csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)})*b^2+\csc(d*x+c)^3*a^2*(1-\cos(d*x+c))^{3-\csc(d*x+c)^3*a*b*(1-\cos(d*x+c))^{3+a^2*(\csc(d*x+c)-\cot(d*x+c))+a*b*(\csc(d*x+c)-\cot(d*x+c))}*(\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1}))^{(1/2)}/(\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1}) \end{aligned}$$

Fricas [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{3/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{3/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{3/2} \sec^{3/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{3/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sec^{\frac{3}{2}}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{3/2} (a + b \cos(c + dx))^{3/2} dx$$

```
[In] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2), x)
```

3.740 $\int (a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx$

Optimal result	6671
Rubi [A] (verified)	6672
Mathematica [A] (verified)	6675
Maple [B] (verified)	6675
Fricas [F]	6676
Sympy [F(-1)]	6677
Maxima [F]	6677
Giac [F]	6677
Mupad [F(-1)]	6677

Optimal result

Integrand size = 25, antiderivative size = 435

$$\int (a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx =$$

$$\frac{(a - b)b\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad\sqrt{\sec(c + dx)}} +$$

$$\frac{\sqrt{a + b}(2a + b)\sqrt{\cos(c + dx)} \csc(c + dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d\sqrt{\sec(c + dx)}} +$$

$$\frac{3a\sqrt{a + b}\sqrt{\cos(c + dx)} \csc(c + dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d\sqrt{\sec(c + dx)}} +$$

$$\frac{b\sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{d}$$

```
[Out] b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(a-b)*b*csc(d*x+c)*E
llipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))
^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+s
ec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+(2*a+b)*csc(d*x+c)*EllipticF((
a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a
+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))
/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-3*a*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))
^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/
2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))
^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4307, 2900, 3132, 2888, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \frac{\sqrt{a+b}(2a+b)\sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) + b(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) + 3a\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right) + \frac{b \sin(c+dx) \sqrt{\sec(c+dx)} \sqrt{a+b \cos(c+dx)}}{d}}{d \sqrt{\sec(c+dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]

[Out] -(((a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(2*a + b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(d*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(d*Sqrt[Sec[c + d*x]]) + (b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d

Rule 2888

Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Ssin[e + f*x]]/Sqrt[d*Ssin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 2900

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 1)*((c + d*Ssin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Ssin[e + f*x])^(m - 2)*(c + d*Ssin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ[m + n, 0] && (IntegerQ[m] || IntegerQ[2*m, 2*n])
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Ssin[e + f*x]]/Sqrt[b*Ssin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Ssin[e + f*x]]*Sqrt[c + d*Ssin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Ssin[e + f*x])^(3/2)*Sqrt[c + d*Ssin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3132

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)])^2/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Ssin[e + f*x]]/Sqrt[c + d*Ssin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Ssin[e + f*x])^(3/2)*Sqrt[c + d*Ssin[e + f*x]]), x], x]
```

), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^m*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}} dx \\
 &= \frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} \\
 &\quad + \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{ab}{2} + a^2\cos(c+dx) + \frac{3}{2}ab\cos^2(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} \\
 &\quad + \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{ab}{2} + a^2\cos(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
 &\quad + \frac{1}{2} \left(3ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx \\
 &= \\
 &\quad - \frac{3a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} \\
 &\quad - \frac{1}{2} \left(ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
 &\quad + \frac{1}{2} \left(a(2a+b)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \\
&\frac{(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a}{a+b}}}{ad\sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}(2a+b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} \\
&- \frac{3a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} \\
&+ \frac{b\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.74

$$\int (a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \frac{\cos^2\left(\frac{1}{2}(c + dx)\right) \sqrt{\sec(c + dx)} \left(2b(a + b) \sqrt{\frac{\cos(c + dx)}{1 + \cos(c + dx)}} \sqrt{\frac{a + b \cos(c + dx)}{(a + b)(1 + \cos(c + dx))}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}}\right)\right)\right)}{d}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]

[Out] (Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a - 2*b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 12*a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1361 vs. 2(395) = 790.

Time = 8.99 (sec) , antiderivative size = 1362, normalized size of antiderivative = 3.13

method	result	size
default	Expression too large to display	1362

[In] int((a+cos(d*x+c)*b)^(3/2)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] -1/d*(EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b*cos(d*
x+c)^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*b^2*cos(d
*x+c)^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (-a-b)/(a+b))^(1/2))*a*
b*cos(d*x+c)^2+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2)
*a^2*cos(d*x+c)^2-4*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2)*
(a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*a*b*cos(d*x+c)^2+2*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2)
)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*a*b*cos(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b)
)^(1/2))*b^2*cos(d*x+c)+12*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (-a-b)/
(a+b))^(1/2))*a*b*cos(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*
x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)
/(a+b))^(1/2))*a^2*cos(d*x+c)-8*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a
+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*a*b*cos(d*x+c)+((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)
)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*a*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2)
))*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (-a-b)/(a+b))^(1/2))*a*b
+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a^2-4*((a+cos
(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a
-b)/(a+b))^(1/2))*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a*b-b^2*cos(d*x+c)^2*si
n(d*x+c)-a*b*cos(d*x+c)*sin(d*x+c))*sec(d*x+c)^(1/2)/(1+cos(d*x+c))/(a+cos(
d*x+c)*b)^(1/2)
```

Fricas [F]

$$\int (a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \int (b \cos(dx + c) + a)^{3/2} \sqrt{\sec(dx + c)} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```


Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \int (b \cos(dx + c) + a)^{\frac{3}{2}} \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} (a + b \cos(c + dx))^{3/2} dx$$

[In] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2),x)

[Out] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2), x)

$$3.741 \quad \int \frac{(a+b \cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}} dx$$

Optimal result	6678
Rubi [A] (verified)	6679
Mathematica [A] (verified)	6683
Maple [B] (verified)	6684
Fricas [F]	6685
Sympy [F]	6685
Maxima [F]	6686
Giac [F]	6686
Mupad [F(-1)]	6686

Optimal result

Integrand size = 25, antiderivative size = 493

$$\int \frac{(a+b \cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}} dx =$$

$$\frac{5(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d\sqrt{\sec(c+dx)}} + \frac{\sqrt{a+b}(5a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d\sqrt{\sec(c+dx)}} - \frac{\sqrt{a+b}(3a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4bd\sqrt{\sec(c+dx)}} + \frac{3a\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d} + \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{2d}$$

```
[Out] 1/2*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+3/4*a*sin(d*x+c)*(
a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-5/4*(a-b)*csc(d*x+c)*EllipticE((a+
b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b
)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(
a-b))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*(5*a+2*b)*csc(d*x+c)*EllipticF((a+b*cos(
d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2
)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(
1/2)/d/sec(d*x+c)^(1/2)-1/4*(3*a^2+4*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d
*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+
b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/
(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4307, 2900, 3126, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx =$$

$$\frac{\sqrt{a + b}(3a^2 + 4b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{4bd \sqrt{\sec(c + dx)}} +$$

$$\frac{\sqrt{a + b}(5a + 2b) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{4d \sqrt{\sec(c + dx)}} +$$

$$\frac{5(a - b) \sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{4d \sqrt{\sec(c + dx)}} +$$

$$\frac{\sin(c + dx) \sqrt{\sec(c + dx)} (a + b \cos(c + dx))^{3/2}}{2d} +$$

$$\frac{3a \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \cos(c + dx)}}{4d}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]

[Out] (-5*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(5*a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(3*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*b*d*Sqrt[Sec[c + d*x]]) + (3*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d) + ((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 2888

Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -

$d^2, 0] \&\& \text{PosQ}[(c + d)/b]$

Rule 2895

$\text{Int}[1/(\text{Sqrt}[(d_*)\sin[(e_*) + (f_*)(x_*)])*\text{Sqrt}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_*)])], x_Symbol] \rightarrow \text{Simp}[-2*(\text{Tan}[e + f*x]/(a*f))*\text{Rt}[(a + b)/d, 2]*\text{Sqrt}[a*((1 - \text{Csc}[e + f*x])/(a + b))]*\text{Sqrt}[a*((1 + \text{Csc}[e + f*x])/(a - b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[d*\text{Sin}[e + f*x]]/\text{Rt}[(a + b)/d, 2]], -(a + b)/(a - b), x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{PosQ}[(a + b)/d]$

Rule 2900

$\text{Int}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_*)])^{(m_*)}((c_*) + (d_*)\sin[(e_*) + (f_*)(x_*)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(-b)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m - 1)}*(c + d*\text{Sin}[e + f*x])^n/(f*(m + n)), x] + \text{Dist}[1/(d*(m + n)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m - 2)}*(c + d*\text{Sin}[e + f*x])^{(n - 1)}*\text{Simp}[a^2*c*d*(m + n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b*d*(m + n - 1)))*\text{Sin}[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*\text{Sin}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[0, m, 2] \&\& \text{LtQ}[-1, n, 2] \&\& \text{NeQ}[m + n, 0] \&\& (\text{IntegerQ}[m] \mid\mid \text{IntegersQ}[2*m, 2*n])$

Rule 3073

$\text{Int}[(A_*) + (B_*)\sin[(e_*) + (f_*)(x_*)])/((b_*)\sin[(e_*) + (f_*)(x_*)])^{(3/2)}*\text{Sqrt}[(c_*) + (d_*)\sin[(e_*) + (f_*)(x_*)]), x_Symbol] \rightarrow \text{Simp}[-2*A*(c - d)*(\text{Tan}[e + f*x]/(f*b*c^2))*\text{Rt}[(c + d)/b, 2]*\text{Sqrt}[c*((1 + \text{Csc}[e + f*x])/(c - d))]*\text{Sqrt}[c*((1 - \text{Csc}[e + f*x])/(c + d))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[b*\text{Sin}[e + f*x]]/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d), x] /; \text{FreeQ}\{b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{EqQ}[A, B] \&\& \text{PosQ}[(c + d)/b]$

Rule 3077

$\text{Int}[(A_*) + (B_*)\sin[(e_*) + (f_*)(x_*)])/((a_*) + (b_*)\sin[(e_*) + (f_*)(x_*)])^{(3/2)}*\text{Sqrt}[(c_*) + (d_*)\sin[(e_*) + (f_*)(x_*)]), x_Symbol] \rightarrow \text{Dist}[(A - B)/(a - b), \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] - \text{Dist}[(A*b - a*B)/(a - b), \text{Int}[(1 + \text{Sin}[e + f*x])/((a + b*\text{Sin}[e + f*x])^{(3/2)}*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[A, B]$

Rule 3126

$\text{Int}[(a_*) + (b_*)\sin[(e_*) + (f_*)(x_*)])^{(m_*)}((c_*) + (d_*)\sin[(e_*) + (f_*)(x_*)])^{(n_*)}((A_*) + (B_*)\sin[(e_*) + (f_*)(x_*)] + (C_*)\sin[(e_*)$

```

+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2)))] - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1)))]*Sin[e + f*x]
+ b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))]*Sin[e + f*
x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]

```

Rule 3132

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3140

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])]*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 4307

```

Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \sqrt{\cos(c+dx)} (a + b \cos(c+dx))^{3/2} dx \\
&= \frac{(a + b \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)} \sin(c+dx)}{2d} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+b \cos(c+dx)} \left(-\frac{ab}{2} + b^2 \cos(c+dx) + \frac{3}{2} ab \cos^2(c+dx) \right)}{\cos^{\frac{3}{2}}(c+dx)} dx}{2b}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{2d} \\
&+ \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{2d} \\
&+ \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{ab^2}{4} + \frac{1}{2}b(2a^2+b^2)\cos(c+dx) + \frac{5}{4}ab^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx}{b} \\
&= \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4d} \\
&+ \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{2d} \\
&+ \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{-\frac{5}{4}a^2b^2 + \frac{1}{2}ab^3\cos(c+dx) + \frac{1}{4}b^2(3a^2+4b^2)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{2b^2} \\
&= \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4d} \\
&+ \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{2d} \\
&+ \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{-\frac{5}{4}a^2b^2 + \frac{1}{2}ab^3\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{2b^2} \\
&+ \frac{1}{8} \left((3a^2 + 4b^2) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{\sqrt{a+b}(3a^2 + 4b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{a+b}}{4bd\sqrt{\sec(c+dx)}} \\
&+ \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4d} \\
&+ \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{2d} \\
&- \frac{1}{8} \left(5a^2 \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{1 + \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&+ \frac{1}{8} \left(a(5a+2b) \sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx
\end{aligned}$$

$$\begin{aligned}
&= \\
&\frac{5(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a}{a+b}}}{4d\sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}(5a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4d\sqrt{\sec(c+dx)}} \\
&- \frac{\sqrt{a+b}(3a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a}{a+b}}}{4bd\sqrt{\sec(c+dx)}} \\
&+ \frac{3a\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d} \\
&+ \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 15.85 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.71

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}} dx = \frac{b\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(2(c+dx))}{4d} \\
- 5a^2 \tan\left(\frac{1}{2}(c+dx)\right) + 5ab \tan\left(\frac{1}{2}(c+dx)\right) - 10ab \tan^3\left(\frac{1}{2}(c+dx)\right) - 5a^2 \tan^5\left(\frac{1}{2}(c+dx)\right) + 5ab \tan^5\left(\frac{1}{2}(c+dx)\right)$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]

[Out] (b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) - (5*a^2*Tan[(c + d*x)/2] + 5*a*b*Tan[(c + d*x)/2] - 10*a*b*Tan[(c + d*x)/2]^3 - 5*a^2*Tan[(c + d*x)/2]^5 + 5*a*b*Tan[(c + d*x)/2]^5 + 6*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 5*a*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(4*a^2 - a*b + 2*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1

$$+ \text{Tan}[(c + d*x)/2]^2 * \text{Sqrt}[(a + b + a*\text{Tan}[(c + d*x)/2]^2 - b*\text{Tan}[(c + d*x)/2]^2)/(a + b)] / (4*d*\text{Sqrt}[(1 - \text{Tan}[(c + d*x)/2]^2)^{-1}] * (-1 + \text{Tan}[(c + d*x)/2]^2) * (1 + \text{Tan}[(c + d*x)/2]^2)^{3/2} * \text{Sqrt}[(a + b + a*\text{Tan}[(c + d*x)/2]^2 - b*\text{Tan}[(c + d*x)/2]^2)/(1 + \text{Tan}[(c + d*x)/2]^2)])$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1915 vs. $2(439) = 878$.

Time = 7.58 (sec) , antiderivative size = 1916, normalized size of antiderivative = 3.89

method	result	size
default	Expression too large to display	1916

[In] `int((a*cos(d*x+c)*b)^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$-1/4/d/(1+\cos(d*x+c))/(a+\cos(d*x+c)*b)^{1/2}/\sec(d*x+c)^{1/2}*(5*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b*\cos(d*x+c)+2*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b*\cos(d*x+c)-2*a*b*\sin(d*x+c)-4*\sec(d*x+c)*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*b^2+5*\sec(d*x+c)*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2+6*\sec(d*x+c)*\text{EllipticPi}(\cot(d*x+c)-\csc(d*x+c),-1,(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2+8*\sec(d*x+c)*\text{EllipticPi}(\cot(d*x+c)-\csc(d*x+c),-1,(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*b^2-7*a*b*\cos(d*x+c)*\sin(d*x+c)+5*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2*\cos(d*x+c)+6*\text{EllipticPi}(\cot(d*x+c)-\csc(d*x+c),-1,(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2*\cos(d*x+c)+8*\text{EllipticPi}(\cot(d*x+c)-\csc(d*x+c),-1,(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*b^2*\cos(d*x+c)-4*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*b^2*\cos(d*x+c)+10*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b+4*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b-2*b^2*\cos(d*x+c)^2*\sin(d*x+c)-5*a^2*\sin(d*x+c)-8*\sec(d*x+c)*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*a^2+2*\sec(d*x+c)*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c))$$


```

)))^(1/2)*a*b+5*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(
1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*a*b-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2-2*cos(d*x+c)*b^2*sin(d*x+c)+10*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*a^2+12*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*a^2+16*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*b^2-8*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(co
t(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)*b^2-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*cos(d
*x+c))

```

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\sqrt{\sec(dx + c)}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx$$

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2)/sqrt(sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(1/2), x)

$$3.742 \quad \int \frac{(a+b \cos(c+dx))^{3/2}}{\sec^2(c+dx)} dx$$

Optimal result	6687
Rubi [A] (verified)	6688
Mathematica [A] (verified)	6692
Maple [B] (verified)	6693
Fricas [F(-1)]	6694
Sympy [F]	6695
Maxima [F]	6695
Giac [F]	6695
Mupad [F(-1)]	6695

Optimal result

Integrand size = 25, antiderivative size = 568

$$\int \frac{(a+b \cos(c+dx))^{3/2}}{\sec^2(c+dx)} dx =$$

$$\frac{(a-b)\sqrt{a+b}(3a^2+16b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{24abd\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}(a+2b)(3a+8b)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{24bd\sqrt{\sec(c+dx)}} +$$

$$\frac{a\sqrt{a+b}(a^2-12b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{8b^2d\sqrt{\sec(c+dx)}} +$$

$$\frac{a\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} +$$

$$\frac{(3a^2+16b^2)\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{24bd}$$

```
[Out] 1/3*(a+b*cos(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/4*a*sin(d*x+c)*
(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/24*(3*a^2+16*b^2)*sin(d*x+c)*(a+
b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/24*(a-b)*(3*a^2+16*b^2)*csc(d*x+
c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a
-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*
(1+sec(d*x+c)))/(a-b)^(1/2)/a/b/d/sec(d*x+c)^(1/2)+1/24*(a+2*b)*(3*a+8*b)*c
sc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1
/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/b/d/sec(d*x+c)^(1/2)+1/8*a*(a^2-12*b^2)*
```

$\text{csc}(d*x+c)*\text{EllipticPi}((a+b*\cos(d*x+c))^{1/2}/(a+b)^{1/2}/\cos(d*x+c)^{1/2}, (a+b)/b, ((-a-b)/(a-b))^{1/2})*(a+b)^{1/2}*\cos(d*x+c)^{1/2}*(a*(1-\sec(d*x+c)))/(a+b))^{1/2}*(a*(1+\sec(d*x+c)))/(a-b))^{1/2}/b^2/d/\sec(d*x+c)^{1/2}$

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4307, 2900, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sec^3(c + dx)} dx =$$

$$\frac{(a - b)\sqrt{a + b}(3a^2 + 16b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{24abd \sqrt{\sec(c + dx)}} +$$

$$\frac{a\sqrt{a + b}(a^2 - 12b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{8b^2 d \sqrt{\sec(c + dx)}} +$$

$$\frac{(3a^2 + 16b^2) \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \cos(c + dx)}}{24bd} +$$

$$\frac{\sqrt{a + b}(a + 2b)(3a + 8b) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{24bd \sqrt{\sec(c + dx)}} +$$

$$\frac{\sin(c + dx)(a + b \cos(c + dx))^{3/2}}{3d \sqrt{\sec(c + dx)}} + \frac{a \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{4d \sqrt{\sec(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2), x]

[Out] -1/24*((a - b)*Sqrt[a + b]*(3*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(a + 2*b)*(3*a + 8*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(24*b*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b]*(a^2 - 12*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(8*b^2*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + ((a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((3*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b*d)

Rule 2888

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 2900

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[1/(d*(m + n)), In
t[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m
+ n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c -
b*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && N
eQ[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]
]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

&& NeQ[A, B]

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]
]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x])/(d*f*Sqrt[a + b*Sin[e + f*x]))], x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x])))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \cos^{\frac{3}{2}}(c + dx) (a + b \cos(c + dx))^{3/2} dx$$

$$\begin{aligned}
&= \frac{(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{\sqrt{a+b \cos(c+dx)}\left(\frac{ab}{2}+2b^2 \cos(c+dx)+\frac{3}{2}ab \cos^2(c+dx)\right)}{\sqrt{\cos(c+dx)}} dx}{3b} \\
&= \frac{a\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{4d\sqrt{\sec(c + dx)}} + \frac{(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{\frac{7a^2b}{4}+\frac{13}{2}ab^2 \cos(c+dx)+\frac{1}{4}b(3a^2+16b^2) \cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b \cos(c+dx)}} dx}{6b} \\
&= \frac{a\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{4d\sqrt{\sec(c + dx)}} + \frac{(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} \\
&\quad + \frac{(3a^2 + 16b^2) \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{24bd} \\
&\quad + \frac{\left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{-\frac{1}{4}ab(3a^2+16b^2)+\frac{7}{2}a^2b^2 \cos(c+dx)-\frac{3}{4}ab(a^2-12b^2) \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b \cos(c+dx)}} dx}{12b^2} \\
&= \frac{a\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{4d\sqrt{\sec(c + dx)}} + \frac{(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} \\
&\quad + \frac{(3a^2 + 16b^2) \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{24bd} \\
&\quad + \frac{\left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{-\frac{1}{4}ab(3a^2+16b^2)+\frac{7}{2}a^2b^2 \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b \cos(c+dx)}} dx}{12b^2} \\
&\quad - \frac{\left(a(a^2 - 12b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx}{16b} \\
&= \frac{a\sqrt{a + b}(a^2 - 12b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\sec(c + dx)}}{8b^2d\sqrt{\sec(c + dx)}} \\
&\quad + \frac{a\sqrt{a + b \cos(c + dx)} \sin(c + dx)}{4d\sqrt{\sec(c + dx)}} + \frac{(a + b \cos(c + dx))^{3/2} \sin(c + dx)}{3d\sqrt{\sec(c + dx)}} \\
&\quad + \frac{(3a^2 + 16b^2) \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{24bd} \\
&\quad + \frac{\left(a(a + 2b)(3a + 8b) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b \cos(c+dx)}} dx}{48b} \\
&\quad - \frac{\left(a(3a^2 + 16b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b \cos(c+dx)}} dx}{48b}
\end{aligned}$$

=

$$\begin{aligned}
& - \frac{(a-b)\sqrt{a+b}(3a^2+16b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{24abd\sqrt{\sec(c+dx)}} \\
& + \frac{\sqrt{a+b}(a+2b)(3a+8b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{24bd\sqrt{\sec(c+dx)}} \\
& + \frac{a\sqrt{a+b}(a^2-12b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{8b^2d\sqrt{\sec(c+dx)}} \\
& + \frac{a\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} \\
& + \frac{(3a^2+16b^2)\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{24bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 14.80 (sec) , antiderivative size = 961, normalized size of antiderivative = 1.69

$$\begin{aligned}
& \int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^3(c+dx)} dx = \frac{\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\left(\frac{1}{12}b\sin(c+dx) + \frac{7}{24}a\sin(2(c+dx)) + \frac{1}{12}b\sin(3(c+dx))\right)}{d} \\
& + \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(3a^3\tan\left(\frac{1}{2}(c+dx)\right) + 3a^2b\tan\left(\frac{1}{2}(c+dx)\right) + 16ab^2\tan\left(\frac{1}{2}(c+dx)\right) + 16b^3\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*Sin[c + d*x])/12 + (7*a*Sin[2*(c + d*x)]/24 + (b*Sin[3*(c + d*x)]/12))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(3*a^3*Tan[(c + d*x)/2] + 3*a^2*b*Tan[(c + d*x)/2] + 16*a*b^2*Tan[(c + d*x)/2] + 16*b^3*Tan[(c + d*x)/2] - 6*a^2*b*Tan[(c + d*x)/2]^3 - 3*2*b^3*Tan[(c + d*x)/2]^3 - 3*a^3*Tan[(c + d*x)/2]^5 + 3*a^2*b*Tan[(c + d*x)/2]^5 - 16*a*b^2*Tan[(c + d*x)/2]^5 + 16*b^3*Tan[(c + d*x)/2]^5 - 6*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 72*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 72*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (3*a^3 + 3*a^2*b + 16*a*b^2 + 16*b^3)*EllipticE[ArcSin[Tan[

$$(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*(7*a - 26*b)*b*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b))]/(24*b*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2264 vs. 2(508) = 1016.

Time = 8.30 (sec) , antiderivative size = 2265, normalized size of antiderivative = 3.99

method	result	size
default	Expression too large to display	2265

[In] int((a+cos(d*x+c)*b)^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out]
$$\begin{aligned} & -1/24/d/(1+\cos(d*x+c))/(a+\cos(d*x+c)*b)^{(1/2)}/\sec(d*x+c)^{(3/2)}*(-8*\sin(d*x+c)*\cos(d*x+c)^2*b^3-22*\sin(d*x+c)*\cos(d*x+c)*a*b^2-22*\sin(d*x+c)*a*b^2-17*\sin(d*x+c)*a^2*b-16*b^3*\sin(d*x+c)-3*a^3*\tan(d*x+c)-8*b^3*\cos(d*x+c)*\sin(d*x+c)-104*\sec(d*x+c)*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a*b^2+14*\sec(d*x+c)^2*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a^2*b-52*\sec(d*x+c)^2*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a*b^2+3*\sec(d*x+c)^2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a^2*b+16*\sec(d*x+c)^2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a*b^2+72*\sec(d*x+c)^2*\text{EllipticPi}(\cot(d*x+c)-\csc(d*x+c),-1,(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a*b^2+6*\sec(d*x+c)*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a^2*b-14*a^2*b*\tan(d*x+c)+6*\sec(d*x+c)*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a^3+32*\sec(d*x+c)*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*b^3-12*\sec(d*x+c)*\text{EllipticPi}(\cot(d*x+c)-\csc(d*x+c),-1,(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a^3+3*\sec(d*x+c)^2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a^3+16*\sec(d*x+c)^2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)})*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{(1/2)}*a^3 \end{aligned}$$

```

s(d*x+c)/(1+cos(d*x+c))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)
)*b^3-6*sec(d*x+c)^2*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(1+cos(d*x+c))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))
)^(1/2)*a^3+3*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(1+cos(d*x+c))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a
^3+16*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b^3-6*Elli
pticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3+3*EllipticE(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))^(1/2)
)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+16*EllipticE(cot(d*x
+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))^(1/2)*((a+
cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+72*EllipticPi(cot(d*x+c)-cs
c(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))^(1/2)*((a+cos
(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+14*EllipticF(cot(d*x+c)-csc(d*
x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))^(1/2)*((a+cos(d*x+c)
)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b-52*EllipticF(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))^(1/2)*((a+cos(d*x+c)*b)/(1+
cos(d*x+c))/(a+b))^(1/2)*a*b^2-16*tan(d*x+c)*a*b^2+32*sec(d*x+c)*EllipticE(
cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c))^(1/2)
)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+144*sec(d*x+c)*Ellip
ticPi(cot(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+28*sec(d*x+
c)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos
(d*x+c))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b)/b

```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sec^{3/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\sec^{3/2}(c + dx)} dx$$

[In] integrate((a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2), x)

[Out] Integral((a + b*cos(c + d*x))**(3/2)/sec(c + d*x)**(3/2), x)

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\sec(dx + c)^{3/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2), x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{3/2}}{\sec(dx + c)^{3/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2), x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{3/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{3/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

[In] int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)

[Out] int((a + b*cos(c + d*x))^(3/2)/(1/cos(c + d*x))^(3/2), x)

3.743 $\int (a + b \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx$

Optimal result	6696
Rubi [A] (verified)	6697
Mathematica [A] (verified)	6701
Maple [B] (verified)	6701
Fricas [F]	6703
Sympy [F(-1)]	6704
Maxima [F]	6704
Giac [F]	6704
Mupad [F(-1)]	6704

Optimal result

Integrand size = 25, antiderivative size = 494

$$\int (a + b \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \frac{2(a-b)\sqrt{a+b}(147a^4 + 279a^2b^2 - 10b^4) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right) - \frac{2(a-b)\sqrt{a+b}(147a^3 - 114a^2b + 165ab^2 + 10b^3) \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{315a^3d\sqrt{\sec(c+dx)}}}{315a^2d\sqrt{\sec(c+dx)}} + \frac{2b(163a^2 + 5b^2) \sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{315ad} + \frac{2(49a^2 + 75b^2) \sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{315d} + \frac{38ab\sqrt{a+b\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{63d} + \frac{2a^2\sqrt{a+b\cos(c+dx)} \sec^{9/2}(c+dx) \sin(c+dx)}{9d}$$

[Out] $\frac{2}{315}b*(163*a^2+5*b^2)*\sec(d*x+c)^{(3/2)}*\sin(d*x+c)*(a+b*\cos(d*x+c))^{(1/2)}/a/d+2/315*(49*a^2+75*b^2)*\sec(d*x+c)^{(5/2)}*\sin(d*x+c)*(a+b*\cos(d*x+c))^{(1/2)}/d+38/63*a*b*\sec(d*x+c)^{(7/2)}*\sin(d*x+c)*(a+b*\cos(d*x+c))^{(1/2)}/d+2/9*a^2*\sec(d*x+c)^{(9/2)}*\sin(d*x+c)*(a+b*\cos(d*x+c))^{(1/2)}/d+2/315*(a-b)*(147*a^4+279*a^2*b^2-10*b^4)*\csc(d*x+c)*\text{EllipticE}((a+b*\cos(d*x+c))^{(1/2)}/(a+b)^{(1/2)}/\cos(d*x+c)^{(1/2)},((-a-b)/(a-b))^{(1/2)})*(a+b)^{(1/2)}*\cos(d*x+c)^{(1/2)}*(a*(1-\sec(d*x+c)))/(a+b)^{(1/2)}*(a*(1+\sec(d*x+c)))/(a-b)^{(1/2)}/a^3/d/\sec(d*x+c)^{(1/2)}-2/315*(a-b)*(147*a^3-114*a^2*b+165*a*b^2+10*b^3)*\csc(d*x+c)*\text{EllipticF}((a+b*\cos(d*x+c))^{(1/2)}/(a+b)^{(1/2)}/\cos(d*x+c)^{(1/2)},((-a-b)/(a-b))^{(1/2)})*(a+$

$$(a+b)^{1/2} \cos(dx+c)^{1/2} (a(1-\sec(dx+c)))/(a+b)^{1/2} (a(1+\sec(dx+c)))/(a-b)^{1/2} / a^2/d/\sec(dx+c)^{1/2}$$

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2871, 3134, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \frac{2(49a^2 + 75b^2) \sin(c + dx) \sec^{5/2}(c + dx) \sqrt{a + b \cos(c + dx)}}{315d} + \frac{2b(163a^2 + 5b^2) \sin(c + dx) \sec^{3/2}(c + dx) \sqrt{a + b \cos(c + dx)}}{315ad} + \frac{2a^2 \sin(c + dx) \sec^{9/2}(c + dx) \sqrt{a + b \cos(c + dx)}}{9d} - \frac{2(a - b) \sqrt{a + b} (147a^3 - 114a^2b + 165ab^2 + 10b^3) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}}}{315a^2d \sqrt{\sec(c + dx)}} + \frac{2(a - b) \sqrt{a + b} (147a^4 + 279a^2b^2 - 10b^4) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E(\arcsin)}{315a^3d \sqrt{\sec(c + dx)}} + \frac{38ab \sin(c + dx) \sec^{7/2}(c + dx) \sqrt{a + b \cos(c + dx)}}{63d}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2), x]

[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(315*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(147*a^3 - 114*a^2*b + 165*a*b^2 + 10*b^3)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*b*(163*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*a*d) + (2*(49*a^2 + 75*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(315*d) + (38*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*d)

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Co

```
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)])*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] :> Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
```

```
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x]
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{1/2}(c+dx)} dx \\
&= \frac{2a^2 \sqrt{a+b\cos(c+dx)} \sec^{9/2}(c+dx) \sin(c+dx)}{9d} \\
&\quad + \frac{1}{9} \left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{19a^2b}{2} + \frac{1}{2}a(7a^2+27b^2)\cos(c+dx) + \frac{3}{2}b(2a^2+3b^2)\cos^2(c+dx)}{\cos^{9/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{38ab\sqrt{a+b\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{63d} \\
&\quad + \frac{2a^2 \sqrt{a+b\cos(c+dx)} \sec^{9/2}(c+dx) \sin(c+dx)}{9d} \\
&\quad + \frac{\left(4\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{4}a^2(49a^2+75b^2) + \frac{1}{4}ab(137a^2+63b^2)\cos(c+dx) + 19a^2b^2\cos^2(c+dx)}{\cos^{7/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{63a} \\
&= \frac{2(49a^2+75b^2)\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{315d} \\
&\quad + \frac{38ab\sqrt{a+b\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{63d} \\
&\quad + \frac{2a^2 \sqrt{a+b\cos(c+dx)} \sec^{9/2}(c+dx) \sin(c+dx)}{9d} \\
&\quad + \frac{\left(8\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{3}{8}a^2b(163a^2+5b^2) + \frac{1}{8}a^3(147a^2+605b^2)\cos(c+dx) + \frac{1}{4}a^2b(49a^2+75b^2)\cos^2(c+dx)}{\cos^{5/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{315a^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2b(163a^2 + 5b^2) \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{315ad} \\
&+ \frac{2(49a^2 + 75b^2) \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{315d} \\
&+ \frac{38ab \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{63d} \\
&+ \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) \sin(c + dx)}{9d} \\
&+ \frac{\left(16 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{\frac{3}{16} a^2 (147a^4 + 279a^2 b^2 - 10b^4) + \frac{3}{16} a^3 b (261a^2 + 155b^2) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{945a^3} \\
&= \frac{2b(163a^2 + 5b^2) \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{315ad} \\
&+ \frac{2(49a^2 + 75b^2) \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{315d} \\
&+ \frac{38ab \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{63d} \\
&+ \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) \sin(c + dx)}{9d} \\
&- \frac{\left((a - b)(147a^3 - 114a^2 b + 165ab^2 + 10b^3) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx}{315a} \\
&+ \frac{\left((147a^4 + 279a^2 b^2 - 10b^4) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{315a} \\
&= \frac{2(a - b) \sqrt{a + b} (147a^4 + 279a^2 b^2 - 10b^4) \sqrt{\cos(c + dx)} \csc(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{315a^3 d \sqrt{\sec(c + dx)}} \\
&- \frac{2(a - b) \sqrt{a + b} (147a^3 - 114a^2 b + 165ab^2 + 10b^3) \sqrt{\cos(c + dx)} \csc(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{315a^2 d \sqrt{\sec(c + dx)}} \\
&+ \frac{2b(163a^2 + 5b^2) \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{315ad} \\
&+ \frac{2(49a^2 + 75b^2) \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{315d} \\
&+ \frac{38ab \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{63d} \\
&+ \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sec^{\frac{9}{2}}(c + dx) \sin(c + dx)}{9d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 12.64 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.05

$$\int (a + b \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c + dx)\right)} \sec(c + dx) \left(-2(147a^5 + 147a^4b + 279a^3b^2 + 279a^2b^3 - 10ab^4 - 10b^5)\sqrt{\frac{\cos(c + dx)}{1 + \cos(c + dx)}}\right)}{\sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \left(\frac{2(147a^4 + 279a^2b^2 - 10b^4) \sin(c + dx)}{315a^2} + \frac{2}{315} \sec^2(c + dx) (49a^2 \sin(c + dx) + 75b^2 \sin(c + dx))\right)} + \dots$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]

[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a^2*b^3 - 10*a*b^4 - 10*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(147*a^4 + 261*a^3*b + 279*a^2*b^2 + 155*a*b^3 - 10*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (147*a^4 + 279*a^2*b^2 - 10*b^4)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((315*a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sin[c + d*x])/(315*a^2) + (2*Sec[c + d*x]^2*(49*a^2*Ssin[c + d*x] + 75*b^2*Ssin[c + d*x]))/315 + (2*Sec[c + d*x]*(163*a^2*b*Ssin[c + d*x] + 5*b^3*Ssin[c + d*x]))/(315*a) + (38*a*b*Sec[c + d*x]^2*Tan[c + d*x])/63 + (2*a^2*Sec[c + d*x]^3*Tan[c + d*x])/9))/d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3463 vs. 2(442) = 884.

Time = 1052.19 (sec) , antiderivative size = 3464, normalized size of antiderivative = 7.01

method	result	size
default	Expression too large to display	3464

[In] int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(11/2),x,method=_RETURNVERBOSE)

[Out] 2/315/d*sec(d*x+c)^(11/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(294*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b*cos(d*x+c)^6+558*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^2*cos(d*x

$$\begin{aligned} & +c)^6+558*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c) \\ & /(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*a^2*b^ \\ & 3*\cos(d*x+c)^6+170*a^3*b^2*\cos(d*x+c)^3*\sin(d*x+c)+212*a^4*b*\cos(d*x+c)^5*s \\ & \sin(d*x+c)+442*a^3*b^2*\cos(d*x+c)^5*\sin(d*x+c)+80*a^2*b^3*\cos(d*x+c)^5*\sin(d \\ & *x+c)-5*a*b^4*\cos(d*x+c)^5*\sin(d*x+c)+130*a^4*b*\cos(d*x+c)^2*\sin(d*x+c)+212 \\ & *a^4*b*\cos(d*x+c)^4*\sin(d*x+c)+170*a^3*b^2*\cos(d*x+c)^4*\sin(d*x+c)+80*a^2*b \\ & ^3*\cos(d*x+c)^4*\sin(d*x+c)+130*a^4*b*\cos(d*x+c)^3*\sin(d*x+c)+147*\cos(d*x+c) \\ & ^7*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)) \\ &)^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*a^5-10*\cos(d* \\ & x+c)^7*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x \\ & +c)))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*b^5-147*c \\ & \cos(d*x+c)^7*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+co \\ & s(d*x+c)))^{(1/2)}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*a^5+ \\ & 294*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+co \\ & s(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*a^5*\cos(d*x+ \\ & c)^6-20*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(\\ & 1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*b^5*\cos(\\ & d*x+c)^6-294*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x \\ & +c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*a^5 \\ & *\cos(d*x+c)^6+147*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(co \\ & s(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} \\ &)*a^5*\cos(d*x+c)^5-10*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)} \\ & *(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(\\ & 1/2)}*b^5*\cos(d*x+c)^5-147*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(\\ & 1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a \\ & +b))^{(1/2)}*a^5*\cos(d*x+c)^5+49*a^5*\cos(d*x+c)^4*\sin(d*x+c)+35*a^5*\cos(d*x+c \\ &)*\sin(d*x+c)+35*a^5*\cos(d*x+c)^2*\sin(d*x+c)+49*a^5*\cos(d*x+c)^3*\sin(d*x+c)- \\ & 10*\cos(d*x+c)^6*b^5*\sin(d*x+c)+147*\cos(d*x+c)^5*a^5*\sin(d*x+c)-20*\text{EllipticE} \\ & (\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1 \\ & /2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*a*b^4*\cos(d*x+c)^6-522*El \\ & lpticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+ \\ & c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*a^4*b*\cos(d*x+c)^6 \\ & -558*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+c \\ & os(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*a^3*b^2*\cos \\ & (d*x+c)^6-310*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)}*(\cos(d* \\ & x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)}*a^ \\ & 2*b^3*\cos(d*x+c)^6+20*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{(1/2)} \\ & *(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(\\ & 1/2)}*a*b^4*\cos(d*x+c)^6+147*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b)) \\ & ^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/ \\ & (a+b))^{(1/2)}*a^4*b*\cos(d*x+c)^5+279*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b) \\ & /(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b)/(1+\cos(d \\ & *x+c)))/(a+b))^{(1/2)}*a^3*b^2*\cos(d*x+c)^5+279*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c \\ &),(-a-b)/(a+b))^{(1/2)}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)}*((a+\cos(d*x+c)*b) \\ & /(1+\cos(d*x+c)))/(a+b))^{(1/2)}*a^2*b^3*\cos(d*x+c)^5-10*\text{EllipticE}(\cot(d*x+c)-c \end{aligned}$$

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sc(d*x+c), (- (a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d
*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^4*cos(d*x+c)^5-261*EllipticF(cot(d
*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^4*b*cos(d*x+c)^5-279*Elliptic
F(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b^2*cos(d*x+c)^5-155
*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^3*cos(d*x
+c)^5+10*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^4*c
os(d*x+c)^5+147*cos(d*x+c)^7*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (- (a-b)/(
a+b))^(1/2))*a^4*b+279*cos(d*x+c)^7*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-
(a-b)/(a+b))^(1/2))*a^3*b^2+279*cos(d*x+c)^7*((a+cos(d*x+c)*b)/(1+cos(d*x+c
)))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(
d*x+c), (- (a-b)/(a+b))^(1/2))*a^2*b^3-10*cos(d*x+c)^7*((a+cos(d*x+c)*b)/(1+c
os(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x
+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2))*a*b^4-261*cos(d*x+c)^7*((a+cos(d*x+c)*
b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(
cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2))*a^4*b-279*cos(d*x+c)^7*((a+cos(
d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*Ell
ipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2))*a^3*b^2-155*cos(d*x+c)^7
*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2))*a^2*b^3+10*cos(
d*x+c)^7*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2))*a*b^4+1
47*cos(d*x+c)^6*a^4*b*sin(d*x+c)+163*cos(d*x+c)^6*a^3*b^2*sin(d*x+c)+279*cos
(d*x+c)^6*a^2*b^3*sin(d*x+c)+5*cos(d*x+c)^6*a*b^4*sin(d*x+c))/a^2

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Fricas [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{11/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(11/2), x)

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(11/2), x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{11/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2), x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{11/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2), x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^{11/2}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{11/2} (a + b \cos(c + dx))^{5/2} dx$$

[In] int((1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2), x)

[Out] int((1/cos(c + d*x))^(11/2)*(a + b*cos(c + d*x))^(5/2), x)

3.744 $\int (a + b \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx$

Optimal result	6705
Rubi [A] (verified)	6706
Mathematica [A] (verified)	6709
Maple [B] (verified)	6710
Fricas [F]	6711
Sympy [F(-1)]	6712
Maxima [F]	6712
Giac [F]	6712
Mupad [F(-1)]	6712

Optimal result

Integrand size = 25, antiderivative size = 427

$$\int (a + b \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \frac{2(a-b)b\sqrt{a+b}(29a^2+3b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sin(c+dx))}{a+b}}}{21a^2d\sqrt{\sec(c+dx)}} + \frac{2(a-b)\sqrt{a+b}(5a^2-24ab+3b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sin(c+dx))}{a+b}}}{21ad\sqrt{\sec(c+dx)}} + \frac{2(5a^2+9b^2)\sqrt{a+b\cos(c+dx)}\sec^{3/2}(c+dx)\sin(c+dx)}{21d} + \frac{6ab\sqrt{a+b\cos(c+dx)}\sec^{5/2}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{7/2}(c+dx)\sin(c+dx)}{7d}$$

```
[Out] 2/21*(5*a^2+9*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+6/7
*a*b*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/7*a^2*sec(d*x+c)
)^(7/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/21*(a-b)*b*(29*a^2+3*b^2)*csc
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2
)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/21*(a-b)*(5*a^2-2
4*a*b+3*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x
+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00,
 number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used
 = {4307, 2871, 3134, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \frac{2(a-b)\sqrt{a+b}(5a^2 - 24ab + 3b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticE}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right) + 2b(a-b)\sqrt{a+b}(29a^2 + 3b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right) + \frac{2(5a^2 + 9b^2) \sin(c+dx) \sec^{3/2}(c+dx) \sqrt{a+b \cos(c+dx)}}{21d} + \frac{2a^2 \sin(c+dx) \sec^{7/2}(c+dx) \sqrt{a+b \cos(c+dx)}}{7d} + \frac{6ab \sin(c+dx) \sec^{5/2}(c+dx) \sqrt{a+b \cos(c+dx)}}{7d}}{21ad\sqrt{\sec(c+dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2), x]

[Out] (2*(a - b)*b*Sqrt[a + b]*(29*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(21*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*(5*a^2 - 24*a*b + 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(21*a*d*Sqrt[Sec[c + d*x]]) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol] :> Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b

$\wedge 2, 0]$ && NeQ[$c^2 - d^2, 0]$ && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegerQ[2*m, 2*n])

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_)] + (f_)*(x_)])*Sqrt[(a_)] + (b_)*sin[(e_)] + (f_)*(x_)]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_)] + (B_)*sin[(e_)] + (f_)*(x_)]/(((b_)*sin[(e_)] + (f_)*(x_)])^3/2)*Sqrt[(c_)] + (d_)*sin[(e_)] + (f_)*(x_)]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_)] + (B_)*sin[(e_)] + (f_)*(x_)]/(((a_)] + (b_)*sin[(e_)] + (f_)*(x_)])^3/2)*Sqrt[(c_)] + (d_)*sin[(e_)] + (f_)*(x_)]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/(a + b*Sin[e + f*x])^3/2)*Sqrt[c + d*Sin[e + f*x]]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3134

Int[((a_)] + (b_)*sin[(e_)] + (f_)*(x_)]^m)*((c_)] + (d_)*sin[(e_)] + (f_)*(x_)]^n)*((A_)] + (B_)*sin[(e_)] + (f_)*(x_)] + (C_)*sin[(e_)] + (f_)*(x_)]^2), x_Symbol] :> Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||

EqQ[a, 0]))))

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{9/2}(c+dx)} dx \\
&= \frac{2a^2 \sqrt{a+b\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{1}{7} \left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{15a^2b}{2} + \frac{1}{2}a(5a^2+21b^2)\cos(c+dx) + \frac{1}{2}b(4a^2+7b^2)\cos^2(c+dx)}{\cos^{7/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{6ab\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{2a^2\sqrt{a+b\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{\left(4\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{5}{4}a^2(5a^2+9b^2) + \frac{5}{4}ab(13a^2+7b^2)\cos(c+dx) + \frac{15}{2}a^2b^2\cos^2(c+dx)}{\cos^{5/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{35a} \\
&= \frac{2(5a^2+9b^2)\sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{21d} \\
&\quad + \frac{6ab\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{2a^2\sqrt{a+b\cos(c+dx)} \sec^{7/2}(c+dx) \sin(c+dx)}{7d} \\
&\quad + \frac{\left(8\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{5}{8}a^2b(29a^2+3b^2) + \frac{5}{8}a^3(5a^2+27b^2)\cos(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{105a^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2(5a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{21d} \\
&\quad + \frac{6ab \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} \\
&\quad + \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{7d} + \frac{1}{21} \left(b(29a^2 \right. \\
&\quad \left. + 3b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&\quad + \frac{1}{21} \left((a - b)(5a^2 - 24ab \right. \\
&\quad \left. + 3b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{2(a - b)b\sqrt{a + b}(29a^2 + 3b^2) \sqrt{\cos(c + dx)} \csc(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}}}{21a^2 d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2(a - b)\sqrt{a + b}(5a^2 - 24ab + 3b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{21ad \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2(5a^2 + 9b^2) \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{21d} \\
&\quad + \frac{6ab \sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx) \sin(c + dx)}{7d} \\
&\quad + \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sec^{\frac{7}{2}}(c + dx) \sin(c + dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.85 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.91

$$\int (a + b \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \frac{2 \left(\sqrt{\cos^2\left(\frac{1}{2}(c + dx)\right)} \sec(c + dx) \left(-2b(29a^3 + 29a^2b + 3ab^2 + 3b^3) E\left(\arcsin\left(\tan\left(\frac{1}{2}(c + dx)\right)\right) \middle| -\frac{a + b}{a + b}\right) \sqrt{\frac{1}{1 + \sec(c + dx)}} \sqrt{\frac{b + a \sec(c + dx)}{(a + b)(1 + \sec(c + dx))}} \right) \right)}{\dots}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2), x]

[Out] (2*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*b*(29*a^3 + 29*a^2*b + 3*a*b^2 + 3*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*a*(5*a^3 + 29*a^2*b + 27*a*b^2 + 3*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))])

$$\frac{d*x]}{(a + b)*(1 + \text{Sec}[c + d*x])}] - b*(29*a^2 + 3*b^2)*\text{Cos}[c + d*x]*(a + b*\text{Cos}[c + d*x])* \text{Sec}[(c + d*x)/2]^2*\text{Tan}[(c + d*x)/2])/ \text{Sqrt}[\text{Sec}[(c + d*x)/2]^2 + (a + b*\text{Cos}[c + d*x])* \text{Sqrt}[\text{Sec}[c + d*x]]*(b*(29*a^2 + 3*b^2)*\text{Sin}[c + d*x] + a*(5*a^2 + 9*b^2 + 9*a*b*\text{Sec}[c + d*x] + 3*a^2*\text{Sec}[c + d*x]^2)*\text{Tan}[c + d*x]))]/(21*a*d*\text{Sqrt}[a + b*\text{Cos}[c + d*x]])$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2508 vs. $2(381) = 762$.

Time = 1590.09 (sec) , antiderivative size = 2509, normalized size of antiderivative = 5.88

method	result	size
default	Expression too large to display	2509

[In] `int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)`

[Out] $\frac{2}{21} \frac{d \sec(d*x+c)^{9/2}}{(1+\cos(d*x+c))} \frac{1}{(a+\cos(d*x+c)*b)^{1/2}} * (5*a^4*\cos(d*x+c)^4*\sin(d*x+c)+3*a^4*\cos(d*x+c)^2*\sin(d*x+c)+18*a^2*b^2*\cos(d*x+c)^4*\sin(d*x+c)+3*\cos(d*x+c)^5*b^4*\sin(d*x+c)+6*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*b^4*\cos(d*x+c)^5-10*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^4*\cos(d*x+c)^5+3*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*b^4*\cos(d*x+c)^4-5*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^4*\cos(d*x+c)^4+3*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b^3*\cos(d*x+c)^4-29*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^3*b*\cos(d*x+c)^4-27*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2*b^2*\cos(d*x+c)^4-3*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b^3*\cos(d*x+c)^4+58*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^3*b*\cos(d*x+c)^5+58*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^2*b^2*\cos(d*x+c)^5+6*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a*b^3*\cos(d*x+c)^5-58*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*a^3*b*\cos(d*x+c)^5-54*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-\frac{a-b}{a+b})^{1/2})*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*$

```

+c)))^(1/2)*a^2*b^2*cos(d*x+c)^5-6*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/
(a+b))^(1/2))*((a*cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*a*b^3*cos(d*x+c)^5+29*EllipticE(cot(d*x+c)-csc(d*x+c),(-
(a-b)/(a+b))^(1/2))*((a*cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*a^3*b*cos(d*x+c)^4+29*EllipticE(cot(d*x+c)-csc(d*x
+c),(-(a-b)/(a+b))^(1/2))*((a*cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*a^2*b^2*cos(d*x+c)^4+3*cos(d*x+c)^6*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2))*((a*cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*Ell
ipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4-5*cos(d*x+c)^6*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2))*((a*cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*
EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-29*cos(d*x+c)^6*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*((a*cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1
/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b-27*cos(d*x+
c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*((a*cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+
b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-3*c
os(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*((a*cos(d*x+c)*b)/(1+cos(d*x+
c)))/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^
3+29*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*((a*cos(d*x+c)*b)/(1+co
s(d*x+c)))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^3*b+29*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*((a*cos(d*x+c)*b)
/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b^2+3*cos(d*x+c)^6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*((a*cos(d*
x+c)*b)/(1+cos(d*x+c)))/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)
/(a+b))^(1/2))*a*b^3+5*a^4*cos(d*x+c)^3*sin(d*x+c)+3*a^4*cos(d*x+c)*sin(d*x
+c)+5*cos(d*x+c)^5*a^3*b*sin(d*x+c)+29*cos(d*x+c)^5*a^2*b^2*sin(d*x+c)+9*co
s(d*x+c)^5*a*b^3*sin(d*x+c)+12*a^3*b*cos(d*x+c)^2*sin(d*x+c)+12*a^3*b*cos(d
*x+c)^3*sin(d*x+c)+18*a^2*b^2*cos(d*x+c)^3*sin(d*x+c)+34*a^3*b*cos(d*x+c)^4
*sin(d*x+c)+12*a*b^3*cos(d*x+c)^4*sin(d*x+c))/a

```

Fricas [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{9/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(9/2), x)

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(9/2), x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{9/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2), x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{9/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2), x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^{9/2}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{9/2} (a + b \cos(c + dx))^{5/2} dx$$

[In] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2), x)

[Out] int((1/cos(c + d*x))^(9/2)*(a + b*cos(c + d*x))^(5/2), x)

3.745 $\int (a + b \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx$

Optimal result	6713
Rubi [A] (verified)	6714
Mathematica [A] (verified)	6717
Maple [B] (verified)	6717
Fricas [F]	6719
Sympy [F(-1)]	6719
Maxima [F]	6719
Giac [F]	6719
Mupad [F(-1)]	6720

Optimal result

Integrand size = 25, antiderivative size = 378

$$\int (a + b \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \frac{2(a-b)\sqrt{a+b}(9a^2 + 23b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{15ad\sqrt{\sec(c+dx)}} - \frac{2(a-b)\sqrt{a+b}(9a^2 - 8ab + 15b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{15ad\sqrt{\sec(c+dx)}} + \frac{22ab\sqrt{a+b}\cos(c+dx)\sec^{3/2}(c+dx)\sin(c+dx)}{15d} + \frac{2a^2\sqrt{a+b}\cos(c+dx)\sec^{5/2}(c+dx)\sin(c+dx)}{5d}$$

```
[Out] 22/15*a*b*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/5*a^2*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+2/15*(a-b)*(9*a^2+23*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a/d/sec(d*x+c)^(1/2)-2/15*(a-b)*(9*a^2-8*a*b+15*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a/d/sec(d*x+c)^(1/2)
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Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00,
 number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used
 = {4307, 2871, 3134, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx =$$

$$\frac{2(a - b)\sqrt{a + b}(9a^2 - 8ab + 15b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{15ad\sqrt{\sec(c + dx)}} +$$

$$\frac{2(a - b)\sqrt{a + b}(9a^2 + 23b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{15ad\sqrt{\sec(c + dx)}} +$$

$$\frac{2a^2 \sin(c + dx) \sec^{5/2}(c + dx) \sqrt{a + b \cos(c + dx)}}{5d} +$$

$$\frac{22ab \sin(c + dx) \sec^{3/2}(c + dx) \sqrt{a + b \cos(c + dx)}}{15d}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2), x]

[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 + 23*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b))]/(15*a*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(9*a^2 - 8*a*b + 15*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b))]/(15*a*d*Sqrt[Sec[c + d*x]]) + (22*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)])^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{7/2}(c+dx)} dx \\
&= \frac{2a^2 \sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{1}{5} \left(2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{11a^2b}{2} + \frac{3}{2}a(a^2+5b^2)\cos(c+dx) + \frac{1}{2}b(2a^2+5b^2)\cos^2(c+dx)}{\cos^{5/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{22ab\sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{15d} \\
&\quad + \frac{2a^2\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{5d} \\
&\quad + \frac{\left(4 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{4}a^2(9a^2+23b^2) + \frac{1}{4}ab(17a^2+15b^2)\cos(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{15a} \\
&= \frac{22ab\sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{15d} \\
&\quad + \frac{2a^2\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{5d} - \frac{1}{15} \left((a-b)(9a^2-8ab \right. \\
&\quad \left. + 15b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx \\
&\quad + \frac{1}{15} \left(a(9a^2 \right. \\
&\quad \left. + 23b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{2(a-b)\sqrt{a+b}(9a^2+23b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{15ad\sqrt{\sec(c+dx)}} \\
&\quad - \frac{2(a-b)\sqrt{a+b}(9a^2-8ab+15b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{15ad\sqrt{\sec(c+dx)}} \\
&\quad + \frac{22ab\sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{15d} \\
&\quad + \frac{2a^2\sqrt{a+b\cos(c+dx)} \sec^{5/2}(c+dx) \sin(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.55 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.99

$$\int (a + b \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \frac{2(2(9a^3 + 9a^2b + 23ab^2 + 23b^3)E(\arcsin(\tan(\frac{1}{2}(c+dx))) | \frac{-a+b}{a+b}) \sqrt{\frac{1}{1+\sec(c+dx)}} \sqrt{\frac{b+a\sec(c+dx)}{(a+b)(1+\sec(c+dx))}} - 2(9a^3 + 17a^2b + 23ab^2 + 15b^3)E(\arcsin(\tan(\frac{1}{2}(c+dx))) | \frac{-a+b}{a+b}) \sqrt{\frac{1}{1+\sec(c+dx)}} \sqrt{\frac{b+a\sec(c+dx)}{(a+b)(1+\sec(c+dx))}} + (9a^2 + 23b^2)\cos(c+dx)(a+b\cos(c+dx))\sec^2(\frac{c+dx}{2})\tan(\frac{c+dx}{2}))}{\sqrt{\sec^2(\frac{1}{2}(c+dx))}\sqrt{\cos^2(\frac{1}{2}(c+dx))}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2), x]

[Out] ((2*(2*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - 2*(9*a^3 + 17*a^2*b + 23*a*b^2 + 15*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + (9*a^2 + 23*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) + 2*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((9*a^2 + 23*b^2)*Sin[c + d*x] + a*(11*b + 3*a*Sec[c + d*x])*Tan[c + d*x]))/(15*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2382 vs. 2(338) = 676.

Time = 1036.66 (sec) , antiderivative size = 2383, normalized size of antiderivative = 6.30

method	result	size
default	Expression too large to display	2383

[In] int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] 2/15/d*sec(d*x+c)^(7/2)/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)*(46*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^4-34*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^4-46*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^4+9*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^3+23*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*

$$\begin{aligned}
& b^2 \cos(dx+c)^3 - 17 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * \\
& \cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * \\
& a^2 * b * \cos(dx+c)^3 - 23 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * \\
& (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * \\
& a^2 * b^2 * \cos(dx+c)^3 - 9 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * \\
& (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * \\
& a^3 * \cos(dx+c)^3 + 3 \sin(dx+c) * \cos(dx+c) * a^3 + 14 \sin(dx+c) * \cos(dx+c)^2 * \\
& a^2 * b + 18 \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * \\
& (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * \\
& a^2 * b * \cos(dx+c)^4 - 9 \cos(dx+c)^5 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& a^3 + 9 \cos(dx+c)^5 * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * a^3 + 23 * b^3 * \cos(dx+c)^4 * \sin(dx+c) + \\
& 14 * a^2 * b * \cos(dx+c)^3 * \sin(dx+c) + 34 * a * b^2 * \cos(dx+c)^3 * \sin(dx+c) - 30 * \\
& \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * b^3 * \cos(dx+c)^4 - 15 * \\
& \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c))^{1/2} * \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& a^2 * b - 23 * \cos(dx+c)^5 \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& a * b^2 + 9 * \cos(dx+c)^5 * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * a^2 * b + 23 * \cos(dx+c)^5 * \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * a * b^2 + 18 * \\
& \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * a^3 * \cos(dx+c)^4 + 46 * \\
& \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * b^3 * \cos(dx+c)^4 - 18 * \\
& \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * a^3 * \cos(dx+c)^4 + 9 * \\
& \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * a^3 * \cos(dx+c)^3 + 23 * \\
& \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \\
& ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * b^3 * \cos(dx+c)^3 + 3 * a^3 * \cos(dx+c)^2 * \\
& \sin(dx+c) + 23 * \cos(dx+c)^5 * ((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2} * \\
& (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * \\
& b^3 + 9 * \cos(dx+c)^3 * a^3 * \sin(dx+c) + 9 * \cos(dx+c)^4 * a^2 * b * \sin(dx+c) + \\
& 11 * \cos(dx+c)^4 * a * b^2 * \sin(dx+c) - 15 * \cos(dx+c)^5 * \\
& \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), (-a-b)/(a+b))^{1/2} * ((a+\cos(dx+c)*b)/(1+\cos(dx+c))^{1/2} * \\
& (\cos(dx+c)/(1+\cos(dx+c))^{1/2} * b^3)
\end{aligned}$$

Fricas [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{7/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{7/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{7/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^{7/2}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{7/2} (a + b \cos(c + dx))^{5/2} dx$$

```
[In] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int((1/cos(c + d*x))^(7/2)*(a + b*cos(c + d*x))^(5/2), x)
```

3.746 $\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx$

Optimal result	6721
Rubi [A] (verified)	6722
Mathematica [A] (verified)	6725
Maple [B] (warning: unable to verify)	6725
Fricas [F]	6727
Sympy [F(-1)]	6727
Maxima [F]	6727
Giac [F]	6727
Mupad [F(-1)]	6728

Optimal result

Integrand size = 25, antiderivative size = 452

$$\begin{aligned}
 & \int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx \\
 &= \frac{14(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{3d\sqrt{\sec(c+dx)}} \\
 &+ \frac{2\sqrt{a+b}(a^2-7ab+9b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{3d\sqrt{\sec(c+dx)}} \\
 &- \frac{2b^2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} \\
 &+ \frac{2a^2\sqrt{a+b}\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}
 \end{aligned}$$

```

[Out] 2/3*a^2*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d+14/3*(a-b)*b*c
sc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1
/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/d/sec(d*x+c)^(1/2)+2/3*(a^2-7*a*b+9*b^2)
*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(-
(-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(
1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/d/sec(d*x+c)^(1/2)-2*b^2*csc(d*x+c)*El
lipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)
)/(a-b)^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)
*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/d/sec(d*x+c)^(1/2)

```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4307, 2871, 3132, 2888, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{5/2} \sec^5(c + dx) dx = \frac{2\sqrt{a+b}(a^2 - 7ab + 9b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right) + \frac{2a^2 \sin(c + dx) \sec^3(c + dx) \sqrt{a + b \cos(c + dx)}}{3d} - \frac{2b^2 \sqrt{a+b} \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{d\sqrt{\sec(c + dx)}} + \frac{14b(a-b)\sqrt{a+b}\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right) \Big| - \frac{a+b}{a-b}}{3d\sqrt{\sec(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2), x]

[Out] (14*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(a^2 - 7*a*b + 9*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*d*Sqrt[Sec[c + d*x]]) - (2*b^2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(d*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b

$^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 2] \&\& \text{LtQ}[n, -1] \&\& (\text{IntegerQ}[m] \mid \mid \text{IntegersQ}[2*m, 2*n])$

Rule 2888

$\text{Int}[\text{Sqrt}[(b_.)\sin[(e_.) + (f_.)x]]/\text{Sqrt}[(c_.) + (d_.)\sin[(e_.) + (f_.)x]], x_Symbol] \rightarrow \text{Simp}[2*b*(\text{Tan}[e + f*x]/(d*f))*\text{Rt}[(c + d)/b, 2]*\text{Sqrt}[c*((1 + \text{Csc}[e + f*x])/(c - d))]*\text{Sqrt}[c*((1 - \text{Csc}[e + f*x])/(c + d))]*\text{EllipticPi}[(c + d)/d, \text{ArcSin}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[b*\text{Sin}[e + f*x]]/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d), x] /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(c + d)/b]$

Rule 2895

$\text{Int}[1/(\text{Sqrt}[(d_.)\sin[(e_.) + (f_.)x]]*\text{Sqrt}[(a_.) + (b_.)\sin[(e_.) + (f_.)x]]), x_Symbol] \rightarrow \text{Simp}[-2*(\text{Tan}[e + f*x]/(a*f))*\text{Rt}[(a + b)/d, 2]*\text{Sqrt}[a*((1 - \text{Csc}[e + f*x])/(a + b))]*\text{Sqrt}[a*((1 + \text{Csc}[e + f*x])/(a - b))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[d*\text{Sin}[e + f*x]]/\text{Rt}[(a + b)/d, 2]], -(a + b)/(a - b), x] /; \text{FreeQ}\{a, b, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{PosQ}[(a + b)/d]$

Rule 3073

$\text{Int}[(A_.) + (B_.)\sin[(e_.) + (f_.)x]]/(((b_.)\sin[(e_.) + (f_.)x]))^{3/2}*\text{Sqrt}[(c_.) + (d_.)\sin[(e_.) + (f_.)x]], x_Symbol] \rightarrow \text{Simp}[-2*A*(c - d)*(\text{Tan}[e + f*x]/(f*b*c^2))*\text{Rt}[(c + d)/b, 2]*\text{Sqrt}[c*((1 + \text{Csc}[e + f*x])/(c - d))]*\text{Sqrt}[c*((1 - \text{Csc}[e + f*x])/(c + d))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[b*\text{Sin}[e + f*x]]/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d), x] /; \text{FreeQ}\{b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{EqQ}[A, B] \&\& \text{PosQ}[(c + d)/b]$

Rule 3077

$\text{Int}[(A_.) + (B_.)\sin[(e_.) + (f_.)x]]/(((a_.) + (b_.)\sin[(e_.) + (f_.)x]))^{3/2}*\text{Sqrt}[(c_.) + (d_.)\sin[(e_.) + (f_.)x]], x_Symbol] \rightarrow \text{Dist}[(A - B)/(a - b), \text{Int}[1/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] - \text{Dist}[(A*b - a*B)/(a - b), \text{Int}[(1 + \text{Sin}[e + f*x])/(a + b*\text{Sin}[e + f*x])^{3/2}*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[A, B]$

Rule 3132

$\text{Int}[(A_.) + (B_.)\sin[(e_.) + (f_.)x] + (C_.)\sin[(e_.) + (f_.)x]]^{2/3}/(((a_.) + (b_.)\sin[(e_.) + (f_.)x]))^{3/2}*\text{Sqrt}[(c_.) + (d_.)\sin[(e_.) + (f_.)x]], x_Symbol] \rightarrow \text{Dist}[C/b^2, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/$

Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{5/2}(c+dx)} dx \\
 &= \frac{2a^2 \sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{3d} \\
 &\quad + \frac{1}{3} \left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{7a^2b}{2} + \frac{1}{2}a(a^2+9b^2)\cos(c+dx) + \frac{3}{2}b^3\cos^2(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{2a^2 \sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{3d} \\
 &\quad + \frac{1}{3} \left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{7a^2b}{2} + \frac{1}{2}a(a^2+9b^2)\cos(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
 &\quad + \left(b^3 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{2b^2 \sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{2a^2 \sqrt{a+b\cos(c+dx)} \sec^{3/2}(c+dx) \sin(c+dx)}{3d} \\
 &\quad + \frac{1}{3} \left(7a^2b \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx \\
 &\quad + \frac{1}{3} \left(a(a^2-7ab \right. \\
 &\quad \quad \left. + 9b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx
 \end{aligned}$$

$$\begin{aligned}
&= \frac{14(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{3d\sqrt{\sec(c+dx)}} \\
&+ \frac{2\sqrt{a+b}(a^2-7ab+9b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{3d\sqrt{\sec(c+dx)}} \\
&- \frac{2b^2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d\sqrt{\sec(c+dx)}} \\
&+ \frac{2a^2\sqrt{a+b}\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 9.62 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.83

$$\int (a+b\cos(c+dx))^{\frac{5}{2}}\sec^{\frac{5}{2}}(c+dx)dx = \frac{\sqrt{\sec(c+dx)}\left(-\cos^2\left(\frac{1}{2}(c+dx)\right)\left(28ab(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\right)}{3d}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]

[Out] (Sqrt[Sec[c + d*x]]*(-(Cos[(c + d*x)/2]^2*(28*a*b*(a + b)*Sqrt[Cos[c + d*x]]/(1 + Cos[c + d*x]))*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(a^3 + 7*a^2*b + 9*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 24*b^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 14*a*b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + 2*a*(a + b*Cos[c + d*x])*(a + 7*b*Cos[c + d*x])*Tan[c + d*x]))/(3*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2210 vs. 2(406) = 812.

Time = 1044.13 (sec) , antiderivative size = 2211, normalized size of antiderivative = 4.89

method	result	size
default	Expression too large to display	2211

$$-\cos(dx+c)^3 - 2a^3(\csc(dx+c) - \cot(dx+c)) - 9a^2b(\csc(dx+c) - \cot(dx+c)) - 7ab^2(\csc(dx+c) - \cot(dx+c)) / (\csc(dx+c)^2 a(1 - \cos(dx+c))^2 - \csc(dx+c)^2 b(1 - \cos(dx+c))^2 + a + b) / (\csc(dx+c)^2 (1 - \cos(dx+c))^{2+1})^2$$

Fricas [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{5/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{5/2} dx$$

[In] integrate((a+b*cos(dx+c))^(5/2)*sec(dx+c)^(5/2),x, algorithm="fricas")

[Out] integral((b^2*cos(dx + c)^2 + 2*a*b*cos(dx + c) + a^2)*sqrt(b*cos(dx + c) + a)*sec(dx + c)^(5/2), x)

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^{5/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(dx+c))**(5/2)*sec(dx+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{5/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{5/2} dx$$

[In] integrate((a+b*cos(dx+c))^(5/2)*sec(dx+c)^(5/2),x, algorithm="maxima")

[Out] integrate((b*cos(dx + c) + a)^(5/2)*sec(dx + c)^(5/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{5/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{5/2} dx$$

[In] integrate((a+b*cos(dx+c))^(5/2)*sec(dx+c)^(5/2),x, algorithm="giac")

[Out] integrate((b*cos(dx + c) + a)^(5/2)*sec(dx + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^2(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{5/2} (a + b \cos(c + dx))^{5/2} dx$$

```
[In] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2), x)
```

3.747 $\int (a + b \cos(c + dx))^{5/2} \sec^{\frac{3}{2}}(c + dx) dx$

Optimal result	6729
Rubi [A] (verified)	6730
Mathematica [A] (verified)	6734
Maple [B] (warning: unable to verify)	6734
Fricas [F]	6736
Sympy [F(-1)]	6736
Maxima [F]	6736
Giac [F]	6736
Mupad [F(-1)]	6737

Optimal result

Integrand size = 25, antiderivative size = 505

$$\int (a + b \cos(c + dx))^{5/2} \sec^{\frac{3}{2}}(c + dx) dx = \frac{(a - b)\sqrt{a + b}(2a^2 - b^2) \sqrt{\cos(c + dx)} \csc(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}}}{ad\sqrt{\sec(c + dx)}} - \frac{\sqrt{a + b}(2a^2 - 6ab - b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}}}{d\sqrt{\sec(c + dx)}} - \frac{5ab\sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a + b}}}{d\sqrt{\sec(c + dx)}} + \frac{2a^2 \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{d} - \frac{(2a^2 - b^2) \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{d}$$

```
[Out] 2*a^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-(2*a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d+(a-b)*(2*a^2-b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a+b))^(1/2)/a/d/sec(d*x+c)^(1/2)-(2*a^2-6*a*b-b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a+b))^(1/2)/d/sec(d*x+c)^(1/2)-5*a*b*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a+b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4307, 2871, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx =$$

$$\frac{\sqrt{a+b}(2a^2 - 6ab - b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{d\sqrt{\sec(c+dx)}} +$$

$$\frac{(a-b)\sqrt{a+b}(2a^2 - b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{ad\sqrt{\sec(c+dx)}} -$$

$$\frac{(2a^2 - b^2) \sin(c+dx) \sqrt{\sec(c+dx)} \sqrt{a+b\cos(c+dx)}}{d} +$$

$$\frac{2a^2 \sin(c+dx) \sqrt{\sec(c+dx)} \sqrt{a+b\cos(c+dx)}}{d} -$$

$$\frac{5ab\sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{d\sqrt{\sec(c+dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2), x]

[Out] ((a - b)*Sqrt[a + b]*(2*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*a^2 - 6*a*b - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(d*Sqrt[Sec[c + d*x]]) - (5*a*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(d*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +

$b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2*\sin[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*\sin[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

Rule 2888

Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{3/2}(c+dx)} dx \\
 &= \frac{2a^2 \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
 &\quad + \left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{3a^2b}{2} - \frac{1}{2}a(a^2 - 3b^2) \cos(c+dx) - \frac{1}{2}b(2a^2 - b^2) \cos^2(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{2a^2 \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
 &\quad - \frac{(2a^2 - b^2) \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{2}ab(2a^2 - b^2) + 3a^2b^2 \cos(c+dx) + \frac{5}{2}ab^3 \cos^2(c+dx)}{\cos^{3/2}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{b}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
&\quad - \frac{(2a^2 - b^2) \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{2}ab(2a^2 - b^2) + 3a^2b^2 \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{b} \\
&\quad + \frac{1}{2} \left(5ab^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx \\
&= \frac{5ab \sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi} \left(\frac{a+b}{b}, \arcsin \left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}} \right), -\frac{a+b}{a-b} \right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d \sqrt{\sec(c+dx)}} \\
&\quad + \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
&\quad - \frac{(2a^2 - b^2) \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} + \frac{1}{2} \left(a(2a^2 \right. \\
&\quad \quad \left. - b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1 + \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx \\
&\quad - \frac{1}{2} \left(a(2a^2 - 6ab \right. \\
&\quad \quad \left. - b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx \\
&= \frac{(a-b) \sqrt{a+b} (2a^2 - b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E \left(\arcsin \left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}} \right) \mid -\frac{a+b}{a-b} \right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{ad \sqrt{\sec(c+dx)}} \\
&\quad - \frac{\sqrt{a+b} (2a^2 - 6ab - b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF} \left(\arcsin \left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}} \right), -\frac{a+b}{a-b} \right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d \sqrt{\sec(c+dx)}} \\
&\quad - \frac{5ab \sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi} \left(\frac{a+b}{b}, \arcsin \left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}} \right), -\frac{a+b}{a-b} \right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{d \sqrt{\sec(c+dx)}} \\
&\quad + \frac{2a^2 \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d} \\
&\quad - \frac{(2a^2 - b^2) \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 11.19 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.83

$$\int (a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx = \frac{2a^2(a + b \cos(c + dx)) \sqrt{\sec(c + dx)} \sin(c + dx) + \frac{\sqrt{\cos^2(\frac{1}{2}(c + dx)) \sec(c + dx)} (-2(2a^3 + 2a^2b - ab^2 - b^3)) \sqrt{\frac{\cos(c + dx)}{1 + \cos(c + dx)}}}{\dots}}{\dots}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2), x]

[Out] (2*a^2*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(2*a^3 + 2*a^2*b - a*b^2 - b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a^2 + 3*a*b - 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 20*a*b^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + ((2*a^2 - b^2)*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]))/2)/Sqrt[Sec[(c + d*x)/2]^2)/(d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2507 vs. 2(461) = 922.

Time = 10.07 (sec) , antiderivative size = 2508, normalized size of antiderivative = 4.97

method	result	size
default	Expression too large to display	2508

[In] int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(3/2), x, method=_RETURNVERBOSE)

[Out] -1/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^3/2*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(2*a^3*(csc(d*x+c)-cot(d*x+c))+b^3*(csc(d*x+c)-cot(d*x+c))-2*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+2*csc(d*x+c)^2*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-csc(d*x+c)^2*EllipticE(cot(d*x+c)

Fricas [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{3/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{3/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx = \int (b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{3/2} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^{3/2} (a + b \cos(c + dx))^{5/2} dx$$

```
[In] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2), x)
```

3.748 $\int (a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx$

Optimal result	6738
Rubi [A] (verified)	6739
Mathematica [A] (verified)	6743
Maple [B] (verified)	6743
Fricas [F]	6745
Sympy [F(-1)]	6745
Maxima [F]	6745
Giac [F]	6745
Mupad [F(-1)]	6746

Optimal result

Integrand size = 25, antiderivative size = 503

$$\int (a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx =$$

$$\frac{9(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4d\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}(8a^2+9ab+2b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4d\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}(15a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4d\sqrt{\sec(c+dx)}} +$$

$$\frac{b^2\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{9ab\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d}$$

```
[Out] 1/2*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+9/4*a*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-9/4*(a-b)*b*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*((a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)+1/4*(8*a^2+9*a*b+2*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*((a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-1/4*(15*a^2+4*b^2)*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2))*((a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4307, 2872, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int (a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \frac{\sqrt{a+b}(8a^2 + 9ab + 2b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{4d \sqrt{\sec(c + dx)}} - \frac{\sqrt{a+b}(15a^2 + 4b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b \cos(c+dx)}}\right)\right)}{4d \sqrt{\sec(c + dx)}} - \frac{9b(a-b) \sqrt{a+b} \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right)}{4d \sqrt{\sec(c + dx)}} + \frac{b^2 \sin(c + dx) \sqrt{a+b \cos(c + dx)}}{2d \sqrt{\sec(c + dx)}} + \frac{9ab \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a+b \cos(c + dx)}}{4d}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]], x]

[Out] (-9*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a^2 + 9*a*b + 2*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(15*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*d*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (9*a*b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d)

Rule 2872

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m])

|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & & NeQ[c, 0])))

Rule 2888

Int[Sqrt[(b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3132

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2)/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/

Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3140

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}} dx \\
 &= \frac{b^2 \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{2d \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{1}{2} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{2}a(4a^2+b^2) + b(6a^2+b^2)\cos(c+dx) + \frac{9}{2}ab^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{b^2 \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{2d \sqrt{\sec(c+dx)}} + \frac{9ab \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{4d} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{9}{2}a^2b^2 + ab(4a^2+b^2)\cos(c+dx) + \frac{1}{2}b^2(15a^2+4b^2)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{4b}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{2d \sqrt{\sec(c + dx)}} \\
&+ \frac{9ab \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{4d} \\
&+ \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{-\frac{9}{2} a^2 b^2 + ab(4a^2 + b^2) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{4b} \\
&+ \frac{1}{8} \left(b(15a^2 + 4b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{\sqrt{a + b}(15a^2 + 4b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \operatorname{EllipticPi} \left(\frac{a+b}{b}, \arcsin \left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}} \right), -\frac{a+b}{a-b} \right) \sqrt{a}}{4d \sqrt{\sec(c + dx)}} \\
&+ \frac{b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{2d \sqrt{\sec(c + dx)}} \\
&+ \frac{9ab \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{4d} \\
&- \frac{1}{8} \left(9a^2 b \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&+ \frac{1}{8} \left(a(8a^2 + 9ab \right. \\
&\quad \left. + 2b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{9(a - b)b \sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) E \left(\arcsin \left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}} \right) \mid -\frac{a+b}{a-b} \right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a}{a+b}}}{4d \sqrt{\sec(c + dx)}} \\
&+ \frac{\sqrt{a + b}(8a^2 + 9ab + 2b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \operatorname{EllipticF} \left(\arcsin \left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}} \right), -\frac{a+b}{a-b} \right) \sqrt{a}}{4d \sqrt{\sec(c + dx)}} \\
&- \frac{\sqrt{a + b}(15a^2 + 4b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \operatorname{EllipticPi} \left(\frac{a+b}{b}, \arcsin \left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}} \right), -\frac{a+b}{a-b} \right) \sqrt{a}}{4d \sqrt{\sec(c + dx)}} \\
&+ \frac{b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{2d \sqrt{\sec(c + dx)}} \\
&+ \frac{9ab \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 8.07 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.84

$$\int (a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \frac{b^2(a + b \cos(c + dx)) \sqrt{\sec(c + dx)} \sin(2(c + dx)) + \frac{-18ab(a+b) \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{\frac{a+b \cos(c+dx)}{(a+b)(1+\cos(c+dx))}}}{(a+b) \sqrt{\sec(c+dx)}}}{(a+b) \sqrt{\sec(c+dx)}}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]

[Out] (b^2*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)] + (-18*a*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(4*a^3 - 12*a^2*b + a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*b*(15*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 9*a*b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)))/(4*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2231 vs. 2(449) = 898.

Time = 9.31 (sec) , antiderivative size = 2232, normalized size of antiderivative = 4.44

method	result	size
default	Expression too large to display	2232

[In] int((a+cos(d*x+c)*b)^(5/2)*sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4/d*(-9*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^2+11*a*b^2*cos(d*x+c)^2*sin(d*x+c)+9*a^2*b*cos(d*x+c)*sin(d*x+c)+2*sin(d*x+c)*cos(d*x+c)^2*b^3+2*sin(d*x+c)*cos(d*x+c)*a*b^2+2*b^3*cos(d*x+c)^3*sin(d*x+c)-9*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)^2+24*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)^2-2*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/

$$\begin{aligned}
& (a+b)^{(1/2)} * a * b^2 * \cos(d*x+c)^2 - 18 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/ \\
& (a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d* \\
& x+c)))/(a+b))^{(1/2)} * a^2 * b * \cos(d*x+c) - 18 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a \\
& -b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+co \\
& s(d*x+c)))/(a+b))^{(1/2)} * a * b^2 * \cos(d*x+c) + 48 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), \\
& (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(\\
& 1+\cos(d*x+c)))/(a+b))^{(1/2)} * a^2 * b * \cos(d*x+c) - 4 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+ \\
& c), (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b) \\
&)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * a * b^2 * \cos(d*x+c) - 16 * \text{EllipticF}(\cot(d*x+c) - \csc(\\
& d*x+c), (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+ \\
& c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * a^3 * \cos(d*x+c) - 8 * \text{EllipticF}(\cot(d*x+c) - \csc \\
& (d*x+c), (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x \\
& +c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * a^3 - 8 * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), - \\
& 1, (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b) \\
&)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * b^3 * \cos(d*x+c)^2 + 4 * (\cos(d*x+c)/(1+\cos(d*x+c))) \\
& ^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}(\cot(d*x+c) - c \\
& sc(d*x+c), (-a-b)/(a+b))^{(1/2)} * b^3 * \cos(d*x+c)^2 - 16 * \text{EllipticPi}(\cot(d*x+c) - c \\
& sc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+co \\
& s(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * b^3 * \cos(d*x+c) + 8 * (\cos(d*x+c)/(1+\cos \\
& (d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}(\cot \\
& (d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * b^3 * \cos(d*x+c) - 30 * (\cos(d*x+c)/(1+c \\
& os(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticPi} \\
& (\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)} * a^2 * b - 9 * \text{EllipticE}(\cot(d*x+c) \\
& - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos \\
& (d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * a^2 * b - 9 * \text{EllipticE}(\cot(d*x+c) - \csc(d*x \\
& +c), (-a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)* \\
& b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * a * b^2 + 24 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (- \\
& a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+c \\
& os(d*x+c)))/(a+b))^{(1/2)} * a^2 * b - 2 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+ \\
& b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c) \\
&))/(a+b))^{(1/2)} * a * b^2 - 30 * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b)) \\
& ^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/ \\
& (a+b))^{(1/2)} * a^2 * b * \cos(d*x+c)^2 - 60 * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a \\
& -b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1+co \\
& s(d*x+c)))/(a+b))^{(1/2)} * a^2 * b * \cos(d*x+c) - 8 * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (\\
& -a-b)/(a+b))^{(1/2)} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((a+\cos(d*x+c)*b)/(1 \\
& +\cos(d*x+c)))/(a+b))^{(1/2)} * a^3 * \cos(d*x+c)^2 - 8 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1 \\
& /2)} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticPi}(\cot(d*x+c) - \csc \\
& (d*x+c), -1, (-a-b)/(a+b))^{(1/2)} * b^3 + 4 * (\cos(d*x+c)/(1+\cos(d*x+c)))^{(1/2)} * ((\\
& a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b))^{(1/2)} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c) \\
& , (-a-b)/(a+b))^{(1/2)} * b^3 * \sec(d*x+c)^{(1/2)} / (1+\cos(d*x+c)) / (a+\cos(d*x+c)*b \\
&)^{(1/2)}
\end{aligned}$$

Fricas [F]

$$\int (a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \int (b \cos(dx + c) + a)^{5/2} \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)

Sympy [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int (a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \int (b \cos(dx + c) + a)^{5/2} \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \int (b \cos(dx + c) + a)^{5/2} \sqrt{\sec(dx + c)} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)} dx = \int \sqrt{\frac{1}{\cos(c + dx)}} (a + b \cos(c + dx))^{5/2} dx$$

```
[In] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2), x)
```

$$3.749 \quad \int \frac{(a+b \cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}} dx$$

Optimal result	6747
Rubi [A] (verified)	6748
Mathematica [A] (verified)	6752
Maple [B] (verified)	6753
Fricas [F]	6754
Sympy [F(-1)]	6755
Maxima [F]	6755
Giac [F]	6755
Mupad [F(-1)]	6755

Optimal result

Integrand size = 25, antiderivative size = 566

$$\int \frac{(a+b \cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}} dx =$$

$$\frac{(a-b)\sqrt{a+b}(33a^2+16b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{24ad\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}(33a^2+26ab+16b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{24d\sqrt{\sec(c+dx)}} +$$

$$\frac{5a\sqrt{a+b}(a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{8bd\sqrt{\sec(c+dx)}} +$$

$$\frac{b^2\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{13ab\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{12d\sqrt{\sec(c+dx)}} +$$

$$\frac{(33a^2+16b^2)\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{24d}$$

```
[Out] 1/3*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+13/12*a*b*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/24*(33*a^2+16*b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/d-1/24*(a-b)*(33*a^2+16*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)+1/24*(33*a^2+26*a*b+16*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/d/sec(d*x+c)^(1/2)-5/8*a*(a^2
```

$+4*b^2)*\csc(d*x+c)*\text{EllipticPi}((a+b*\cos(d*x+c))^{1/2}/(a+b)^{1/2}/\cos(d*x+c)^{1/2}, (a+b)/b, ((-a-b)/(a-b))^{1/2})*(a+b)^{1/2}*\cos(d*x+c)^{1/2}*(a*(1-\sec(d*x+c)))/(a+b))^{1/2}*(a*(1+\sec(d*x+c)))/(a-b))^{1/2}/b/d/\sec(d*x+c)^{1/2}$

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4307, 2872, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \frac{\sqrt{a + b}(33a^2 + 26ab + 16b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{24d \sqrt{\sec(c + dx)}} \\ - \frac{(a - b) \sqrt{a + b}(33a^2 + 16b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{24ad \sqrt{\sec(c + dx)}} \\ - \frac{5a \sqrt{a + b}(a^2 + 4b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{8bd \sqrt{\sec(c + dx)}} \\ + \frac{(33a^2 + 16b^2) \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \cos(c + dx)}}{24d} \\ + \frac{b^2 \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{3d \sec^{3/2}(c + dx)} + \frac{13ab \sin(c + dx) \sqrt{a + b \cos(c + dx)}}{12d \sqrt{\sec(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]], x]

[Out] $-1/24*((a - b)*\text{Sqrt}[a + b]*(33*a^2 + 16*b^2)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Csc}[c + d*x] \\ * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Cos}[c + d*x]]/(\text{Sqrt}[a + b]*\text{Sqrt}[\text{Cos}[c + d*x]])], -((a + b)/(a - b))*\text{Sqrt}[(a*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[(a*(1 + \text{Sec}[c + d*x]))/(a - b))]/(a*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (\text{Sqrt}[a + b]*(33*a^2 + 26*a*b + 16*b^2)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Csc}[c + d*x]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[a + b*\text{Cos}[c + d*x]]/(\text{Sqrt}[a + b]*\text{Sqrt}[\text{Cos}[c + d*x]])], -((a + b)/(a - b))*\text{Sqrt}[(a*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[(a*(1 + \text{Sec}[c + d*x]))/(a - b))]/(24*d*\text{Sqrt}[\text{Sec}[c + d*x]]) - (5*a*\text{Sqrt}[a + b]*(a^2 + 4*b^2)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Csc}[c + d*x]*\text{EllipticPi}[(a + b)/b, \text{ArcSin}[\text{Sqrt}[a + b*\text{Cos}[c + d*x]]/(\text{Sqrt}[a + b]*\text{Sqrt}[\text{Cos}[c + d*x]])], -((a + b)/(a - b))*\text{Sqrt}[(a*(1 - \text{Sec}[c + d*x]))/(a + b)]*\text{Sqrt}[(a*(1 + \text{Sec}[c + d*x]))/(a - b))]/(8*b*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + (b^2*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(3*d*\text{Sec}[c + d*x]^(3/2)) + (13*a*b*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(12*d*\text{Sqrt}[\text{Sec}[c + d*x]]) + ((33*a^2 + 16*b^2)*\text{Sqrt}[a + b*\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sec}[c + d*x]]*\text{Sin}[c + d*x])/(24*d)$

Rule 2872

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sine + f


```
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2888

```
Int[Sqrt[(b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f
_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

&& NeQ[A, B]

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C))*Sin[e + f*x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]
]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x],
x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \sqrt{\cos(c + dx)} (a + b \cos(c + dx))^{5/2} dx$$

$$\begin{aligned}
&= \frac{b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)} \\
&+ \frac{1}{3} \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\cos(c + dx)} \left(\frac{3}{2} a(2a^2 + b^2) + b(9a^2 + 2b^2) \cos(c + dx) + \frac{13}{2} \right)}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)} + \frac{13ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{12d \sqrt{\sec(c + dx)}} \\
&+ \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\frac{13a^2 b^2}{4} + \frac{1}{2} ab(12a^2 + 19b^2) \cos(c + dx) + \frac{1}{4} b^2(33a^2 + 16b^2) \cos^2(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx}{6b} \\
&= \frac{b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)} + \frac{13ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{12d \sqrt{\sec(c + dx)}} \\
&+ \frac{(33a^2 + 16b^2) \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{24d} \\
&+ \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{-\frac{1}{4} ab^2(33a^2 + 16b^2) + \frac{13}{2} a^2 b^3 \cos(c + dx) + \frac{15}{4} ab^2(a^2 + 4b^2) \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{12b^2} \\
&= \frac{b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)} + \frac{13ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{12d \sqrt{\sec(c + dx)}} \\
&+ \frac{(33a^2 + 16b^2) \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{24d} \\
&+ \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{-\frac{1}{4} ab^2(33a^2 + 16b^2) + \frac{13}{2} a^2 b^3 \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{12b^2} \\
&+ \frac{1}{16} \left(5a(a^2 + 4b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}} dx \\
&= \frac{5a \sqrt{a + b}(a^2 + 4b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \operatorname{EllipticPi} \left(\frac{a+b}{b}, \arcsin \left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}} \right), -\frac{a+b}{a-b} \right)}{8bd \sqrt{\sec(c + dx)}} \\
&+ \frac{b^2 \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{3d \sec^{\frac{3}{2}}(c + dx)} + \frac{13ab \sqrt{a + b \cos(c + dx)} \sin(c + dx)}{12d \sqrt{\sec(c + dx)}} \\
&+ \frac{(33a^2 + 16b^2) \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \sin(c + dx)}{24d} - \frac{1}{48} \left(a(33a^2 \right. \\
&\quad \left. + 16b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\
&+ \frac{1}{48} \left(a(33a^2 + 26ab \right. \\
&\quad \left. + 16b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx
\end{aligned}$$

=

$$\begin{aligned}
& \frac{(a-b)\sqrt{a+b}(33a^2+16b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{24ad\sqrt{\sec(c+dx)}} \\
& + \frac{\sqrt{a+b}(33a^2+26ab+16b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{24d\sqrt{\sec(c+dx)}} \\
& - \frac{5a\sqrt{a+b}(a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{8bd\sqrt{\sec(c+dx)}} \\
& + \frac{b^2\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{13ab\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{12d\sqrt{\sec(c+dx)}} \\
& + \frac{(33a^2+16b^2)\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{24d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 14.79 (sec) , antiderivative size = 970, normalized size of antiderivative = 1.71

$$\begin{aligned}
& \int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}} dx = \frac{\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\left(\frac{1}{12}b^2\sin(c+dx) + \frac{13}{24}ab\sin(2(c+dx))\right) + \frac{1}{12}b^2\sqrt{a+b}\cos(c+dx)}{d} \\
& + \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(33a^3\tan\left(\frac{1}{2}(c+dx)\right) + 33a^2b\tan\left(\frac{1}{2}(c+dx)\right) + 16ab^2\tan\left(\frac{1}{2}(c+dx)\right) + 16b^3\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sin[c + d*x])/12 + (13*a*b*Sin[2*(c + d*x)])/24 + (b^2*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(33*a^3*Tan[(c + d*x)/2] + 33*a^2*b*Tan[(c + d*x)/2] + 16*a*b^2*Tan[(c + d*x)/2] + 16*b^3*Tan[(c + d*x)/2] - 66*a^2*b*Tan[(c + d*x)/2]^3 - 32*b^3*Tan[(c + d*x)/2]^3 - 33*a^3*Tan[(c + d*x)/2]^5 + 33*a^2*b*Tan[(c + d*x)/2]^5 - 16*a*b^2*Tan[(c + d*x)/2]^5 + 16*b^3*Tan[(c + d*x)/2]^5 + 30*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 120*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 120*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (33*a^3 + 33*a^2*b + 16*a*b^2 + 16*b^3)*Ell

```

ipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(24*a^2 - 13*a*b + 38*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)))/(24*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))]

```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2525 vs. 2(506) = 1012.

Time = 9.10 (sec) , antiderivative size = 2526, normalized size of antiderivative = 4.46

method	result	size
default	Expression too large to display	2526

```
[In] int((a+cos(d*x+c)*b)^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/d/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(1/2)*(-34*a*b^2*cos(d*x+c)^2*sin(d*x+c)-59*a^2*b*cos(d*x+c)*sin(d*x+c)-8*sin(d*x+c)*cos(d*x+c)^2*b^3-34*sin(d*x+c)*cos(d*x+c)*a*b^2-16*sin(d*x+c)*a*b^2-26*sin(d*x+c)*a^2*b-33*a^3*sin(d*x+c)-8*b^3*cos(d*x+c)^3*sin(d*x+c)-16*b^3*cos(d*x+c)*sin(d*x+c)-76*sec(d*x+c)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2+33*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b+33*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3+16*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b^3+30*sec(d*x+c)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3+33*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)+16*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)+120*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)+26*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b*cos(d*x+c)-76*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^2*cos(d*x+c)-48*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*cos(d*x+c)-96*EllipticF(cot(d*x+c)
```

```

-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * (cos(d*x+c)/(1+cos(d*x+c)))^(1/2) * ((a+cos
(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2) * a^3-48*sec(d*x+c) * (cos(d*x+c)/(1+cos
(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2) * EllipticF(cot
(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * a^3+66*EllipticE(cot(d*x+c)-csc(d*
x+c), (- (a-b)/(a+b))^(1/2)) * (cos(d*x+c)/(1+cos(d*x+c)))^(1/2) * ((a+cos(d*x+c)
*b)/(1+cos(d*x+c))/(a+b))^(1/2) * a^3+32*EllipticE(cot(d*x+c)-csc(d*x+c), (- (a
-b)/(a+b))^(1/2)) * (cos(d*x+c)/(1+cos(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+co
s(d*x+c))/(a+b))^(1/2) * b^3+60*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (- (a-b)/(
a+b))^(1/2)) * (cos(d*x+c)/(1+cos(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x
+c))/(a+b))^(1/2) * a^3+66*EllipticE(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/
2)) * (cos(d*x+c)/(1+cos(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b
))^(1/2) * a^2*b+32*EllipticE(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * (co
s(d*x+c)/(1+cos(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)
)*a*b^2+240*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (- (a-b)/(a+b))^(1/2)) * (cos(
d*x+c)/(1+cos(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2) *
a*b^2+52*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * (cos(d*x+c)/
(1+cos(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2) * a^2*b-1
52*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * (cos(d*x+c)/(1+cos
(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2) * a*b^2+33*Elli
pticE(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * (cos(d*x+c)/(1+cos(d*x+c)
))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2) * a^3*cos(d*x+c)+16*El
lipticE(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * (cos(d*x+c)/(1+cos(d*x+
c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2) * b^3*cos(d*x+c)+30*
EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (- (a-b)/(a+b))^(1/2)) * (cos(d*x+c)/(1+co
s(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2) * a^3*cos(d*x+
c)+16*sec(d*x+c)*EllipticE(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b))^(1/2)) * (cos
(d*x+c)/(1+cos(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)
*a*b^2+120*sec(d*x+c)*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (- (a-b)/(a+b))^(1
/2)) * (cos(d*x+c)/(1+cos(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+
b))^(1/2) * a*b^2+26*sec(d*x+c)*EllipticF(cot(d*x+c)-csc(d*x+c), (- (a-b)/(a+b)
))^(1/2)) * (cos(d*x+c)/(1+cos(d*x+c)))^(1/2) * ((a+cos(d*x+c)*b)/(1+cos(d*x+c)
)/(a+b))^(1/2) * a^2*b)

```

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\sqrt{\sec(dx + c)}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\sec(c + dx)}} dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\sqrt{\frac{1}{\cos(c+dx)}}} dx$$

[In] int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2),x)

[Out] int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(1/2), x)

$$3.750 \quad \int \frac{(a+b \cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	6756
Rubi [A] (verified)	6757
Mathematica [A] (verified)	6762
Maple [B] (verified)	6763
Fricas [F]	6765
Sympy [F(-1)]	6765
Maxima [F]	6765
Giac [F]	6765
Mupad [F(-1)]	6766

Optimal result

Integrand size = 25, antiderivative size = 638

$$\int \frac{(a+b \cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)} dx =$$

$$\frac{(a-b)\sqrt{a+b}(15a^2+284b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{192bd\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}(15a^3+118a^2b+284ab^2+72b^3)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{192bd\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}(5a^4-120a^2b^2-48b^4)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{64b^2d\sqrt{\sec(c+dx)}} +$$

$$\frac{b^2\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{17ab\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} +$$

$$\frac{(59a^2+36b^2)\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{96d\sqrt{\sec(c+dx)}} +$$

$$\frac{a(15a^2+284b^2)\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{192bd}$$

```
[Out] 1/4*b^2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+17/24*a*b*sin(
d*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+1/96*(59*a^2+36*b^2)*sin(d
*x+c)*(a+b*cos(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+1/192*a*(15*a^2+284*b^2)*si
n(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b/d-1/192*(a-b)*(15*a^2+28
4*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+1/192*(15*a
```


$$\begin{aligned} & \sqrt{3+118a^2b+284a^2b^2+72b^3} \operatorname{csc}(dx+c) \operatorname{EllipticF}\left(\frac{a+b\cos(dx+c)}{(a+b)^{1/2}/\cos(dx+c)^{1/2}}, \left(\frac{-a-b}{a-b}\right)^{1/2}\right) \frac{(a+b)^{1/2}\cos(dx+c)^{1/2}}{(a+b)^{1/2}} \frac{(a(1-\sec(dx+c)))/(a+b)^{1/2}}{(a+b)^{1/2}} \frac{(a(1+\sec(dx+c)))/(a-b)^{1/2}}{(a-b)^{1/2}} \frac{b/d/\sec(dx+c)^{1/2}}{b/d/\sec(dx+c)^{1/2}} \\ & + \frac{1}{64} (5a^4 - 120a^2b^2 - 48b^4) \operatorname{csc}(dx+c) \operatorname{EllipticPi}\left(\frac{a+b\cos(dx+c)}{(a+b)^{1/2}/\cos(dx+c)^{1/2}}, \frac{a+b}{b}, \left(\frac{-a-b}{a-b}\right)^{1/2}\right) \frac{(a+b)^{1/2}\cos(dx+c)^{1/2}}{(a+b)^{1/2}} \frac{(a(1-\sec(dx+c)))/(a+b)^{1/2}}{(a+b)^{1/2}} \frac{(a(1+\sec(dx+c)))/(a-b)^{1/2}}{(a-b)^{1/2}} \frac{b^2/d/\sec(dx+c)^{1/2}}{b^2/d/\sec(dx+c)^{1/2}} \end{aligned}$$

Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4307, 2872, 3128, 3140, 3132, 2888, 3077, 2895, 3073}

$$\begin{aligned} & \int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^3(c+dx)} dx = \\ & \frac{(a-b)\sqrt{a+b}(15a^2+284b^2)\sqrt{\cos(c+dx)}\operatorname{csc}(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{192bd\sqrt{\sec(c+dx)}} \\ & + \frac{(59a^2+36b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{96d\sqrt{\sec(c+dx)}} \\ & + \frac{a(15a^2+284b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192bd} \\ & + \frac{\sqrt{a+b}(5a^4-120a^2b^2-48b^4)\sqrt{\cos(c+dx)}\operatorname{csc}(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{64b^2d\sqrt{\sec(c+dx)}} \\ & + \frac{\sqrt{a+b}(15a^3+118a^2b+284ab^2+72b^3)\sqrt{\cos(c+dx)}\operatorname{csc}(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{192bd\sqrt{\sec(c+dx)}} \\ & + \frac{b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{4d\sec^5(c+dx)} + \frac{17ab\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sec^3(c+dx)} \end{aligned}$$

[In] Int[(a + b*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2), x]

[Out] -1/192*((a - b)*Sqrt[a + b]*(15*a^2 + 284*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(15*a^3 + 118*a^2*b + 284*a*b^2 + 72*b^3)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(192*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(5*a^4 - 120*a^2*b^2 - 48*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sq

```

rt[a + b*cos[c + d*x]]/(sqrt[a + b]*sqrt[cos[c + d*x]]), -((a + b)/(a - b)
)]*sqrt[(a*(1 - sec[c + d*x]))/(a + b)]*sqrt[(a*(1 + sec[c + d*x]))/(a - b)
)]/(64*b^2*d*sqrt[sec[c + d*x]]) + (b^2*sqrt[a + b*cos[c + d*x]]*sin[c + d*
x])/(4*d*sec[c + d*x]^(5/2)) + (17*a*b*sqrt[a + b*cos[c + d*x]]*sin[c + d*x
])/(24*d*sec[c + d*x]^(3/2)) + ((59*a^2 + 36*b^2)*sqrt[a + b*cos[c + d*x]]*
sin[c + d*x])/(96*d*sqrt[sec[c + d*x]]) + (a*(15*a^2 + 284*b^2)*sqrt[a + b*
cos[c + d*x]]*sqrt[sec[c + d*x]]*sin[c + d*x])/(192*b*d)

```

Rule 2872

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*cos[e + f*x]*(a + b*sin[e + f*
x])^(m - 2)*((c + d*sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*sin[e + f*x])^(m - 3)*(c + d*sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))

```

Rule 2888

```

Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*sin[e + f*x]]/sqrt[b*sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]

```

Rule 2895

```

Int[1/(sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*sin[e + f*x]]/sqrt[d*sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]

```

Rule 3073

```

Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*sin[e + f*x]]/sqrt[b*sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&

```

PosQ[(c + d)/b]

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x])], x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3132

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3140

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^(m_.)*(u_.), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \cos^{\frac{3}{2}}(c+dx) (a+b\cos(c+dx))^{5/2} dx \\
&= \frac{b^2 \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{1}{4} \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx) \left(\frac{1}{2}a(8a^2+5b^2) + 3b(4a^2+b^2)\cos(c+dx) + \frac{17}{2}c \right)}{\sqrt{a+b\cos(c+dx)}} dx \\
&= \frac{b^2 \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} + \frac{17ab \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{24d \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)} \left(\frac{51a^2b^2}{4} + \frac{1}{2}ab(24a^2+49b^2)\cos(c+dx) + \frac{1}{4}b^2(59a^2+36b^2)\cos^2(c+dx) \right)}{\sqrt{a+b\cos(c+dx)}} dx}{12b} \\
&= \frac{b^2 \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} + \frac{17ab \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{24d \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{(59a^2+36b^2) \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{96d \sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{8}ab^2(59a^2+36b^2) + \frac{1}{4}b^3(161a^2+36b^2)\cos(c+dx) + \frac{1}{8}ab^2(15a^2+284b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{24b^2} \\
&= \frac{b^2 \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} + \frac{17ab \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{24d \sec^{\frac{3}{2}}(c+dx)} \\
&\quad + \frac{(59a^2+36b^2) \sqrt{a+b\cos(c+dx)} \sin(c+dx)}{96d \sqrt{\sec(c+dx)}} \\
&\quad + \frac{a(15a^2+284b^2) \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{192bd} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{1}{8}a^2b^2(15a^2+284b^2) + \frac{1}{4}ab^3(59a^2+36b^2)\cos(c+dx) - \frac{3}{8}b^2(5a^4-120a^2b^2-48b^4)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{48b^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{b^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} + \frac{17ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{24d \sec^{\frac{3}{2}}(c+dx)} \\
&+ \frac{(59a^2 + 36b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{96d \sqrt{\sec(c+dx)}} \\
&+ \frac{a(15a^2 + 284b^2) \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{192bd} \\
&+ \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{-\frac{1}{8}a^2b^2(15a^2+284b^2) + \frac{1}{4}ab^3(59a^2+36b^2) \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{48b^3} \\
&- \frac{\left((5a^4 - 120a^2b^2 - 48b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx}{128b} \\
&= \frac{\sqrt{a+b}(5a^4 - 120a^2b^2 - 48b^4) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right)\right)}{64b^2d \sqrt{\sec(c+dx)}} \\
&+ \frac{b^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} + \frac{17ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{24d \sec^{\frac{3}{2}}(c+dx)} \\
&+ \frac{(59a^2 + 36b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{96d \sqrt{\sec(c+dx)}} \\
&+ \frac{a(15a^2 + 284b^2) \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{192bd} \\
&- \frac{\left(a^2(15a^2 + 284b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{384b} \\
&+ \frac{\left(a(15a^3 + 118a^2b + 284ab^2 + 72b^3) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx}{384b} \\
&= \frac{(a-b)\sqrt{a+b}(15a^2 + 284b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(15a^3 + 118a^2b + 284ab^2 + 72b^3)}{a+b}}}{192bd \sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}(15a^3 + 118a^2b + 284ab^2 + 72b^3) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right)\right)}{192bd \sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}(5a^4 - 120a^2b^2 - 48b^4) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right)\right)}{64b^2d \sqrt{\sec(c+dx)}} \\
&+ \frac{b^2 \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{4d \sec^{\frac{5}{2}}(c+dx)} + \frac{17ab \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{24d \sec^{\frac{3}{2}}(c+dx)} \\
&+ \frac{(59a^2 + 36b^2) \sqrt{a+b \cos(c+dx)} \sin(c+dx)}{96d \sqrt{\sec(c+dx)}} \\
&+ \frac{a(15a^2 + 284b^2) \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{192bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 13.88 (sec) , antiderivative size = 1226, normalized size of antiderivative = 1.92

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sec^3(c + dx)} dx = \frac{\sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \left(\frac{17}{96} ab \sin(c + dx) + \frac{1}{192} (59a^2 + 48b^2) \sin(2(c + dx)) \right)}{d} - \frac{15a^4 \tan\left(\frac{1}{2}(c + dx)\right) - 15a^3 b \tan\left(\frac{1}{2}(c + dx)\right) - 284a^2 b^2 \tan\left(\frac{1}{2}(c + dx)\right) - 284ab^3 \tan\left(\frac{1}{2}(c + dx)\right) + 30a^3 b^4 \tan\left(\frac{1}{2}(c + dx)\right)}{d}$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2), x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((17*a*b*Sin[c + d*x])/96 + ((59*a^2 + 48*b^2)*Sin[2*(c + d*x)]/192 + (17*a*b*Sin[3*(c + d*x)]/96 + (b^2*Sin[4*(c + d*x)]/32))/d + (-15*a^4*Tan[(c + d*x)/2] - 15*a^3*b*Tan[(c + d*x)/2] - 284*a^2*b^2*Tan[(c + d*x)/2] - 284*a*b^3*Tan[(c + d*x)/2] + 30*a^3*b^4*Tan[(c + d*x)/2]^3 + 568*a*b^3*Tan[(c + d*x)/2]^3 + 15*a^4*Tan[(c + d*x)/2]^5 - 15*a^3*b*Tan[(c + d*x)/2]^5 + 284*a^2*b^2*Tan[(c + d*x)/2]^5 - 284*a*b^3*Tan[(c + d*x)/2]^5 + 30*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b))*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 720*a^2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b))*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 288*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b))*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b))*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 720*a^2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b))*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 288*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b))*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - a*(15*a^3 + 15*a^2*b + 284*a*b^2 + 284*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b))*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b*(-59*a^3 + 322*a^2*b - 36*a*b^2 + 72*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b))*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]/(192*b*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3158 vs. $2(572) = 1144$.

Time = 10.01 (sec) , antiderivative size = 3159, normalized size of antiderivative = 4.95

method	result	size
default	Expression too large to display	3159

[In] $\text{int}((a+\cos(dx+c)*b)^{5/2}/\sec(dx+c)^{3/2},x,\text{method}=_RETURNVERBOSE)$

[Out]
$$-1/192/d/(1+\cos(dx+c))/(a+\cos(dx+c)*b)^{1/2}/\sec(dx+c)^{3/2}*(-254*\sin(dx+c)*a^2*b^2-15*a^4*\tan(dx+c)-48*b^4*\cos(dx+c)^2*\sin(dx+c)-118*a^3*b*\tan(dx+c)-72*\tan(dx+c)*a*b^3-133*\sin(dx+c)*a^3*b-356*\sin(dx+c)*a*b^3-72*\sin(dx+c)*b^4-30*\sec(dx+c)^2*\text{EllipticPi}(\cot(dx+c)-\csc(dx+c),-1,(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a^4+288*\sec(dx+c)^2*\text{EllipticPi}(\cot(dx+c)-\csc(dx+c),-1,(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*b^4+15*\sec(dx+c)^2*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a^4-60*\sec(dx+c)*\text{EllipticPi}(\cot(dx+c)-\csc(dx+c),-1,(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a^4+576*\sec(dx+c)*\text{EllipticPi}(\cot(dx+c)-\csc(dx+c),-1,(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*b^4+30*\sec(dx+c)*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a^4-144*\sec(dx+c)^2*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*b^4-288*\sec(dx+c)*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*b^4+118*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a^3*b-644*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a^2*b^2+72*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a*b^3+720*\text{EllipticPi}(\cot(dx+c)-\csc(dx+c),-1,(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a^2*b^2+15*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a^3*b+284*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a^2*b^2+284*\text{EllipticE}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/(a+b)^{1/2}*a*b^3-144*\text{EllipticF}(\cot(dx+c)-\csc(dx+c),(-(a-b)/(a+b))^{1/2})*(\cos(dx+c)/(1+\cos(dx+c)))^{1/2}*((a+\cos(dx+c)*b)/(1+\cos(dx+c)))/($$

$$\begin{aligned}
& (a+b)^{1/2} b^4 - 184 a b^3 \cos(d*x+c)^2 \sin(d*x+c) - 254 a^2 b^2 \cos(d*x+c) \sin(d*x+c) - 184 a^3 b \cos(d*x+c) \sin(d*x+c) - 30 \operatorname{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^4 + 288 \operatorname{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} b^4 + 15 \operatorname{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^4 - 48 b^4 \cos(d*x+c)^3 \sin(d*x+c) - 284 \tan(d*x+c) a^2 b^2 - 72 \cos(d*x+c) b^4 \sin(d*x+c) - 1288 \sec(d*x+c) \operatorname{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^2 b^2 + 144 \sec(d*x+c) \operatorname{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^3 b + 1440 \sec(d*x+c) \operatorname{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^2 b^2 + 30 \sec(d*x+c) \operatorname{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^3 b + 568 \sec(d*x+c) \operatorname{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^2 b^2 + 118 \sec(d*x+c)^2 \operatorname{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^3 b - 644 \sec(d*x+c)^2 \operatorname{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^2 b^2 + 72 \sec(d*x+c)^2 \operatorname{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^3 b + 568 \sec(d*x+c) \operatorname{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^3 b + 236 \sec(d*x+c) \operatorname{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^3 b + 284 \sec(d*x+c)^2 \operatorname{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^3 b + 720 \sec(d*x+c)^2 \operatorname{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^2 b^2 + 15 \sec(d*x+c)^2 \operatorname{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^3 b + 284 \sec(d*x+c)^2 \operatorname{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2} (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} a^2 b^2 / b
\end{aligned}$$

Fricas [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\sec(dx + c)^{3/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sec^{3/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\sec(dx + c)^{3/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sec^{3/2}(c + dx)} dx = \int \frac{(b \cos(dx + c) + a)^{5/2}}{\sec(dx + c)^{3/2}} dx$$

[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \cos(c + dx))^{5/2}}{\sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{(a + b \cos(c + dx))^{5/2}}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}} dx$$

```
[In] int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

```
[Out] int((a + b*cos(c + d*x))^(5/2)/(1/cos(c + d*x))^(3/2), x)
```

$$3.751 \quad \int \frac{\sec^5(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6767
Rubi [A] (verified)	6768
Mathematica [A] (warning: unable to verify)	6770
Maple [B] (verified)	6771
Fricas [F]	6772
Sympy [F(-1)]	6772
Maxima [F]	6772
Giac [F]	6772
Mupad [F(-1)]	6773

Optimal result

Integrand size = 25, antiderivative size = 314

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx =$$

$$\frac{4(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^3d\sqrt{\sec(c+dx)}} +$$

$$\frac{2\sqrt{a+b}(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2d\sqrt{\sec(c+dx)}} +$$

$$\frac{2\sqrt{a+b \cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}$$

```
[Out] 2/3*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a/d-4/3*(a-b)*b*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a+b))^(1/2)/a^3/d/sec(d*x+c)^(1/2)+2/3*(a+2*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a+b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2881, 3077, 2895, 3073}

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx =$$

$$-\frac{4b(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{3a^3d\sqrt{\sec(c+dx)}} - \frac{a}{a-b}$$

$$+\frac{2\sqrt{a+b}(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{3a^2d\sqrt{\sec(c+dx)}} + \frac{a}{a-b}$$

$$+\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3ad}$$

[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (-4*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d)

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*m] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_.)])*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\cos^{\frac{5}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx \\ &= \frac{2 \sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx) \sin(c + dx)}{3ad} \\ &\quad + \frac{\left(2 \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{-b + \frac{1}{2}a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3a} \end{aligned}$$

$$\begin{aligned}
&= \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} \\
&\quad - \frac{\left(2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a} \\
&\quad + \frac{\left((a+2b)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3a} \\
&= \frac{4(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\mid-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{3a^3d\sqrt{\sec(c+dx)}} \\
&\quad + \frac{2\sqrt{a+b}(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a^2d\sqrt{\sec(c+dx)}} \\
&\quad + \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 10.70 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.03

$$\begin{aligned}
&\int\frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx \\
&= \frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2b(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\mid\frac{-a+b}{a+b}\right)\right)}{d} \\
&\quad + \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{4b\sin(c+dx)}{3a^2}+\frac{2\tan(c+dx)}{3a}\right)}{d}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + b*Cos[c + d*x]], x]

[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a - 2*b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*a^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-4*b*Sin[c + d*x])/(3*a^2) + (2*Tan[c + d*x])/(3*a)))/d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. 2(280) = 560.

Time = 12.19 (sec) , antiderivative size = 1202, normalized size of antiderivative = 3.83

method	result	size
default	Expression too large to display	1202

[In] `int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & 2/3/d*\sec(d*x+c)^(5/2)/(1+\cos(d*x+c))/(a+\cos(d*x+c)*b)^(1/2)*(-\cos(d*x+c))^4 \\ & *EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*((a+\cos(d*x+c)*b)/(1 \\ & +\cos(d*x+c))/(a+b))^(1/2)*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*a^2+2*\cos(d*x+c) \\ &)^4*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*((a+\cos(d*x+c)*b) \\ & /(1+\cos(d*x+c))/(a+b))^(1/2)*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*a*b-2*\cos(d* \\ & x+c)^4*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^(1/2)*(\cos(d*x+c)/(1+\cos(d*x \\ & +c)))^(1/2)*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*a*b-2*\cos \\ & (d*x+c)^4*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^(1/2)*(\cos(d*x+c)/(1+\cos(\\ & d*x+c)))^(1/2)*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*b^2-2* \\ & (\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^(\\ & 1/2)*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^2*\cos(d*x+c)^3 \\ & +4*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a+b) \\ &)^(1/2)*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*a*b*\cos(d*x+c) \\ &)^3-4*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c))/(a \\ & +b))^(1/2)*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*a*b*\cos(d* \\ & x+c)^3-4*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)) \\ & /(a+b))^(1/2)*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*b^2*\cos \\ & (d*x+c)^3-(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*((a+\cos(d*x+c)*b)/(1+\cos(d*x+c) \\ &))/(a+b))^(1/2)*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^2*co \\ & s(d*x+c)^2+2*EllipticF(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*((a+\cos(\\ & d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^(1/2)*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*a*b \\ & *\cos(d*x+c)^2-2*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2))*((a+c \\ & os(d*x+c)*b)/(1+\cos(d*x+c))/(a+b))^(1/2)*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)* \\ & a*b*\cos(d*x+c)^2-2*(\cos(d*x+c)/(1+\cos(d*x+c)))^(1/2)*((a+\cos(d*x+c)*b)/(1+c \\ & os(d*x+c))/(a+b))^(1/2)*EllipticE(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^(1/2) \\ &))*b^2*\cos(d*x+c)^2+a*b*\cos(d*x+c)^3*\sin(d*x+c)-2*b^2*\cos(d*x+c)^3*\sin(d*x+ \\ & c)+a^2*\cos(d*x+c)^2*\sin(d*x+c)-\cos(d*x+c)^2*\sin(d*x+c)*a*b+a^2*\cos(d*x+c)*s \\ & in(d*x+c))/a^2 \end{aligned}$$

Fricas [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{5/2}}{\sqrt{a + b \cos(c + dx)}} dx$$

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(1/2), x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(1/2), x)
```

$$3.752 \quad \int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6774
Rubi [A] (verified)	6774
Mathematica [A] (verified)	6776
Maple [B] (warning: unable to verify)	6777
Fricas [F]	6777
Sympy [F]	6778
Maxima [F]	6778
Giac [F]	6778
Mupad [F(-1)]	6778

Optimal result

Integrand size = 25, antiderivative size = 264

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b \cos(c+dx)}} dx$$

$$= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)} \operatorname{csc}(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2 d \sqrt{\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)} \operatorname{csc}(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad \sqrt{\sec(c+dx)}}$$

```
[Out] 2*(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/sec(d*x+c)^(1/2)-2*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

= {4307, 2880, 2895, 3073}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx$$

$$= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*d*Sqrt[Sec[c + d*x]])

Rule 2880

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],

`x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]`

Rule 4307

`Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sine[a + b*x])^m, Int[ActivateTrig[u]/(c*Sine[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]`

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx \\ &= - \left(\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx \right) \\ &\quad + \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx \\ &= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{a^2 d \sqrt{\sec(c+dx)}} \\ &\quad - \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{ad \sqrt{\sec(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 11.24 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.12

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx = \frac{2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\sec(c+dx)\left(2(a+b)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\middle|\frac{-a+b}{a+b}\right)}{ad\sqrt{\sec(c+dx)}}$$

`[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]], x]`

`[Out] (2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])`

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. 2(240) = 480.

Time = 10.20 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.49

method	result
default	$-2 \left(-\frac{(\csc^2(dx+c))(1-\cos(dx+c))^2+1}{(\csc^2(dx+c))(1-\cos(dx+c))^2-1} \right)^{\frac{3}{2}} \left((\csc^2(dx+c))(1-\cos(dx+c))^2-1 \right) \left(-\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \sqrt{\frac{\csc^2(dx+c)}{(\csc^2(dx+c))(1-\cos(dx+c))^2-1}} \right)$

[In] `int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & -2/d * (-\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 + 1) / (\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 - 1) \\ & ^{(3/2)} * (\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 - 1) * (-(-\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 + \\ & 1)^{(1/2)} * ((\csc(d*x+c)^2 * a * (1-\cos(d*x+c))^2 - \csc(d*x+c)^2 * b * (1-\cos(d*x+c))^2 + \\ & a + b) / (a + b))^{(1/2)} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a - b) / (a + b))^{(1/2)}) * a + \\ & (-\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 + 1)^{(1/2)} * ((\csc(d*x+c)^2 * a * (1-\cos(d*x+c))^2 - \csc \\ & \csc(d*x+c)^2 * b * (1-\cos(d*x+c))^2 + a + b) / (a + b))^{(1/2)} * \text{EllipticE}(\cot(d*x+c) - \csc(d \\ & *x+c), (-a - b) / (a + b))^{(1/2)}) * a + (-\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 + 1)^{(1/2)} * ((\csc \\ & \csc(d*x+c)^2 * a * (1-\cos(d*x+c))^2 - \csc(d*x+c)^2 * b * (1-\cos(d*x+c))^2 + a + b) / (a + b))^{(\\ & 1/2)} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a - b) / (a + b))^{(1/2)}) * b + \csc(d*x+c)^3 * (\\ & 1 - \cos(d*x+c))^3 * a - \csc(d*x+c)^3 * (1 - \cos(d*x+c))^3 * b + a * (\csc(d*x+c) - \cot(d*x+c)) \\ & + b * (\csc(d*x+c) - \cot(d*x+c)) * ((\csc(d*x+c)^2 * a * (1-\cos(d*x+c))^2 - \csc(d*x+c)^2 * \\ & b * (1-\cos(d*x+c))^2 + a + b) / (\csc(d*x+c)^2 * (1-\cos(d*x+c))^2 + 1))^{(1/2)} / (\csc(d*x+c) \\ &)^2 * (1-\cos(d*x+c))^2 + 1) / (\csc(d*x+c)^2 * a * (1-\cos(d*x+c))^2 - \csc(d*x+c)^2 * b * (1- \\ & \cos(d*x+c))^2 + a + b) / a \end{aligned}$$

Fricas [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] `integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)`

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2), x)

[Out] Integral(sec(c + d*x)**(3/2)/sqrt(a + b*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2), x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{3/2}}{\sqrt{a + b \cos(c + dx)}} dx$$

[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(1/2), x)

[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(1/2), x)

$$3.753 \quad \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx$$

Optimal result	6779
Rubi [A] (verified)	6779
Mathematica [A] (verified)	6780
Maple [A] (verified)	6781
Fricas [F]	6781
Sympy [F]	6781
Maxima [F]	6781
Giac [F]	6782
Mupad [F(-1)]	6782

Optimal result

Integrand size = 25, antiderivative size = 129

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx$$

$$= \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

[Out] 2*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2), ((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/d/sec(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4307, 2895}

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b \cos(c+dx)}} dx$$

$$= \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Cos[c + d*x]], x]

[Out] (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)))*Sqrt[(

$a*(1 - \text{Sec}[c + d*x])/(a + b)*\text{Sqrt}[(a*(1 + \text{Sec}[c + d*x]))/(a - b)]/(a*d*\text{Sqrt}[\text{Sec}[c + d*x]])$

Rule 2895

`Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)])*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]`

Rule 4307

`Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]`

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx \\ &= \frac{2\sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \text{EllipticF} \left(\arcsin \left(\frac{\sqrt{a + b} \cos(c + dx)}{\sqrt{a + b} \sqrt{\cos(c + dx)}} \right), -\frac{a + b}{a - b} \right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{ad \sqrt{\sec(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{\sec(c + dx)}}{\sqrt{a + b \cos(c + dx)}} dx = \frac{2\sqrt{\frac{a + b \cos(c + dx)}{(a + b)(1 + \cos(c + dx))}} \text{EllipticF} \left(\arcsin \left(\tan \left(\frac{1}{2}(c + dx) \right) \right), \frac{-a + b}{a + b} \right)}{d \sqrt{\frac{\cos(c + dx)}{1 + \cos(c + dx)}} \sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)}}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])

Maple [A] (verified)

Time = 7.81 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2(1+\cos(dx+c))(\sqrt{\sec(dx+c)})F\left(\cot(dx+c)-\csc(dx+c),\sqrt{-\frac{a-b}{a+b}}\right)\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}{d\sqrt{a+\cos(dx+c)b}}$	111

[In] `int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)`

[Out] `-2/d*(1+cos(d*x+c))*sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)`

Fricas [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}} dx$$

[In] `integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x,algorithm="fricas")`

[Out] `integral(sqrt(sec(d*x+c))/sqrt(b*cos(d*x+c)+a),x)`

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx$$

[In] `integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(sec(c+d*x))/sqrt(a+b*cos(c+d*x)),x)`

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}} dx$$

[In] `integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x,algorithm="maxima")`

[Out] `integrate(sqrt(sec(d*x+c))/sqrt(b*cos(d*x+c)+a),x)`

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{\sqrt{a+b\cos(c+dx)}} dx$$

[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(1/2),x)

[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(1/2), x)

$$3.754 \quad \int \frac{1}{\sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)}} dx$$

Optimal result	6783
Rubi [A] (verified)	6783
Mathematica [A] (verified)	6784
Maple [A] (verified)	6785
Fricas [F]	6785
Sympy [F]	6785
Maxima [F]	6785
Giac [F]	6786
Mupad [F(-1)]	6786

Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \frac{1}{\sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)}} dx = \frac{2\sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{bd \sqrt{\sec(c+dx)}}$$

[Out] $-2*\csc(d*x+c)*\operatorname{EllipticPi}((a+b*\cos(d*x+c))^{1/2}/(a+b)^{1/2}/\cos(d*x+c)^{1/2}), (a+b)/b, ((-a-b)/(a-b))^{1/2})*(a+b)^{1/2}*\cos(d*x+c)^{1/2}*(a*(1-\sec(d*x+c)))/(a+b)^{1/2}*(a*(1+\sec(d*x+c)))/(a-b)^{1/2}/b/d/\sec(d*x+c)^{1/2}$

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4307, 2888}

$$\int \frac{1}{\sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)}} dx = \frac{2\sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right)\right)}{bd \sqrt{\sec(c+dx)}}$$

[In] $\operatorname{Int}[1/(\operatorname{Sqrt}[a+b*\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Sec}[c+d*x]]),x]$

[Out] $(-2*\operatorname{Sqrt}[a+b]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Csc}[c+d*x]*\operatorname{EllipticPi}[(a+b)/b, \operatorname{ArcSin}[\operatorname{Sqrt}[a+b*\operatorname{Cos}[c+d*x]]/(\operatorname{Sqrt}[a+b]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])], -(a+b)/(a$

$- b)] * \text{Sqrt}[(a * (1 - \text{Sec}[c + d * x])) / (a + b)] * \text{Sqrt}[(a * (1 + \text{Sec}[c + d * x])) / (a - b)] / (b * d * \text{Sqrt}[\text{Sec}[c + d * x]])$

Rule 2888

$\text{Int}[\text{Sqrt}[(b \cdot) * \sin[(e \cdot) + (f \cdot) * (x \cdot)]] / \text{Sqrt}[(c \cdot) + (d \cdot) * \sin[(e \cdot) + (f \cdot) * (x \cdot)]]], x_Symbol] \rightarrow \text{Simp}[2 * b * (\text{Tan}[e + f * x] / (d * f)) * \text{Rt}[(c + d) / b, 2] * \text{Sqrt}[c * ((1 + \text{Csc}[e + f * x]) / (c - d))] * \text{Sqrt}[c * ((1 - \text{Csc}[e + f * x]) / (c + d))] * \text{EllipticPi}[(c + d) / d, \text{ArcSin}[\text{Sqrt}[c + d * \text{Sin}[e + f * x]]] / \text{Sqrt}[b * \text{Sin}[e + f * x]]] / \text{Rt}[(c + d) / b, 2]], -(c + d) / (c - d)], x] /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(c + d) / b]$

Rule 4307

$\text{Int}[(\text{csc}[a \cdot) + (b \cdot) * (x \cdot)] * (c \cdot)^{(m \cdot)} * (u \cdot), x_Symbol] \rightarrow \text{Dist}[(c * \text{Csc}[a + b * x])^m * (c * \text{Sin}[a + b * x])^m, \text{Int}[\text{ActivateTrig}[u] / (c * \text{Sin}[a + b * x])^m, x], x] /; \text{FreeQ}\{a, b, c, m\}, x] \&\& !\text{IntegerQ}[m] \&\& \text{KnownSineIntegrandQ}[u, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}} dx \\ &= \frac{2\sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b} \cos(c+dx)}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{bd \sqrt{\sec(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)}} dx = \frac{2 \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{\frac{a+b \cos(c+dx)}{(a+b)(1+\cos(c+dx))}} \left(\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(c + dx)\right)\right), \frac{-a+b}{a+b}\right) - 2 \text{EllipticPi}\left(-1, \arcsin\left(\tan\left(\frac{1}{2}(c + dx)\right)\right)\right) \right)}{d \sqrt{\frac{1}{1+\cos(c+dx)}} \sqrt{a + b \cos(c + dx)}}$$

[In] Integrate[1/(Sqrt[a + b * Cos[c + d * x]] * Sqrt[Sec[c + d * x]]), x]

[Out] $(-2 * \text{Sqrt}[\text{Cos}[c + d * x] / (1 + \text{Cos}[c + d * x])] * \text{Sqrt}[(a + b * \text{Cos}[c + d * x]) / ((a + b) * (1 + \text{Cos}[c + d * x]))] * (\text{EllipticF}[\text{ArcSin}[\text{Tan}[(c + d * x) / 2]], (-a + b) / (a + b)] - 2 * \text{EllipticPi}[-1, \text{ArcSin}[\text{Tan}[(c + d * x) / 2]], (-a + b) / (a + b)]) * \text{Sqrt}[1 + \text{Sec}[c + d * x]]) / (d * \text{Sqrt}[(1 + \text{Cos}[c + d * x])^{-1}] * \text{Sqrt}[a + b * \text{Cos}[c + d * x]])$

Maple [A] (verified)

Time = 7.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{2\sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}} \left(F\left(\cot(dx+c)-\csc(dx+c), \sqrt{-\frac{a-b}{a+b}}\right) - 2\Pi\left(\cot(dx+c)-\csc(dx+c), -1, \sqrt{-\frac{a-b}{a+b}}\right) \right)}{d\sqrt{a+\cos(dx+c)b} \sqrt{\sec(dx+c)} \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}$	137

```
[In] int(1/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/(a+cos(d*x+c)*b)^(1/2)*((a+cos(d*x+c)*b)/(1+cos(d*x+c)))/(a+b)^(1/2)*(E
llipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))-2*EllipticPi(cot(d*x+c
)-csc(d*x+c),-1,(-a-b)/(a+b))^(1/2))/sec(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} dx = \int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} dx = \int \frac{1}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{b \cos(dx + c) + a} \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} \sqrt{a + b \cos(c + dx)}} dx$$

[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)),x)

[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(1/2)), x)

$$3.755 \quad \int \frac{1}{\sqrt{a+b \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	6787
Rubi [A] (verified)	6788
Mathematica [A] (warning: unable to verify)	6792
Maple [A] (verified)	6792
Fricas [F]	6793
Sympy [F]	6793
Maxima [F]	6793
Giac [F]	6793
Mupad [F(-1)]	6794

Optimal result

Integrand size = 25, antiderivative size = 474

$$\int \frac{1}{\sqrt{a+b \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} dx =$$

$$\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{abd\sqrt{\sec(c+dx)}} +$$

$$\frac{\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd\sqrt{\sec(c+dx)}} +$$

$$\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d\sqrt{\sec(c+dx)}} +$$

$$\frac{\sin(c+dx)}{d\sqrt{a+b \cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a\sqrt{\sec(c+dx)} \sin(c+dx)}{bd\sqrt{a+b \cos(c+dx)}}$$

```
[Out] sin(d*x+c)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+a*sin(d*x+c)*sec(d*x+c)^(1/2)/b/d/(a+b*cos(d*x+c))^(1/2)-(a-b)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d/sec(d*x+c)^(1/2)+csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b/d/sec(d*x+c)^(1/2)+a*csc(d*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2), (a+b)/b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00,
 number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used
 = {4307, 2899, 2888, 3082, 3072, 12, 2880, 2895, 3073}

$$\int \frac{1}{\sqrt{a+b \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx)} dx$$

$$= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) - b^2 d \sqrt{\sec(c+dx)}}{bd\sqrt{\sec(c+dx)}} + \frac{\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) + bd\sqrt{\sec(c+dx)}}{bd\sqrt{\sec(c+dx)}} - \frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) + abd\sqrt{\sec(c+dx)}}{abd\sqrt{\sec(c+dx)}} + \frac{a \sin(c+dx) \sqrt{\sec(c+dx)}}{bd\sqrt{a+b \cos(c+dx)}} + \frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a+b \cos(c+dx)}}$$

[In] Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]

[Out] -(((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[Sec[c + d*x]]) + Sin[c + d*x]/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Cos[c + d*x]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2880

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;

FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2888

Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 2899

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Dist[(-a)*(d/(2*b)), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[Sqrt[d*Sin[e + f*x]]*((a + 2*b*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3072

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]

Rule 3073

Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&

PosQ[(c + d)/b]

Rule 3082

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((c + d*Sin[e + f*x])^n/(f*(2*
n + 3))), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)/Sqrt[a +
b*Sin[e + f*x]])*Simp[a*A*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)
*(2*n + 1) + A*(b*c + a*d)*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(
a*d + 2*b*c*n))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)(a+2b\cos(c+dx))}}{\sqrt{a+b\cos(c+dx)}} dx}{2b} \\
 &\quad - \frac{\left(a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{2b} \\
 &= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2 d \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\sin(c+dx)}{d \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)}} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{2ab+2a^2\cos(c+dx)}{\sqrt{\cos(c+dx)(a+b\cos(c+dx))^{3/2}}} dx}{4b}
 \end{aligned}$$

$$\begin{aligned}
& \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d\sqrt{\sec(c+dx)}} \\
& + \frac{\sin(c+dx)}{d\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}} + \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{a+b}\cos(c+dx)} \\
& + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{-2a^3+2ab^2}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}\cos(c+dx)}dx}{4b(a^2-b^2)} \\
& \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d\sqrt{\sec(c+dx)}} \\
& + \frac{\sin(c+dx)}{d\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}} + \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{a+b}\cos(c+dx)} \\
& - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}\cos(c+dx)}dx}{2b} \\
& \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d\sqrt{\sec(c+dx)}} \\
& + \frac{\sin(c+dx)}{d\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}} + \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{a+b}\cos(c+dx)} \\
& + \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b}\cos(c+dx)}dx}{2b} \\
& - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}\cos(c+dx)}dx}{2b} \\
& \frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(1-\sec(c+dx))}}{abd\sqrt{\sec(c+dx)}} \\
& + \frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a(1-\sec(c+dx))}}{bd\sqrt{\sec(c+dx)}} \\
& + \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2d\sqrt{\sec(c+dx)}} \\
& + \frac{\sin(c+dx)}{d\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}} + \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{bd\sqrt{a+b}\cos(c+dx)}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 5.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{2 \cos(c + dx) \sqrt{\frac{\cos(c+dx)}{(1+\cos(c+dx))^2}} (\cos^2(\frac{1}{2}(c + dx)) \sec(c + dx))^{3/2} \left((a + b) E(\arcsin(\tan(\frac{1}{2}(c + dx)))) \Big|_{\frac{-a+b}{a+b}} \right) \sqrt{\frac{-a+b}{a+b}}}{\dots}$$

```
[In] Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*Cos[c + d*x]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 2*a*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2))/(b*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.86 (sec) , antiderivative size = 820, normalized size of antiderivative = 1.73

method	result
default	$-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} E(\cot(dx+c)-\csc(dx+c), \sqrt{-\frac{a-b}{a+b}}) \sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}} a - \sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} E(\cot(dx+c)-\csc(dx+c), \sqrt{-\frac{a-b}{a+b}}) \sqrt{\frac{a+\cos(dx+c)b}{(1+\cos(dx+c))(a+b)}}$

```
[In] int(1/sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(1+cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2)/sec(d*x+c)^(3/2)*(-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (-a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-2*sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-2*sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*b+4*sec(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c), -1, (-a-b)/(a+b))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a+b*sin(d*x+c)-sec(d*x+c)^2*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*((a+cos(d*x+c)*b)/(1+cos(d*x+c))/(a+b))^(1/2)*a-sec(d*x+c)^2*EllipticE(cot(d*x+c)-csc
```

$(d*x+c), (-a-b)/(a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} * b + 2 * \sec(d*x+c)^2 * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2} * (\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2} * ((a+\cos(d*x+c)*b)/(1+\cos(d*x+c)))/(a+b)^{1/2} * a + \tan(d*x+c) * a) / b$

Fricas [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

Sympy [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))^(1/2),x)

[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**(3/2)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{3}{2}} \sqrt{a + b \cos(c + dx)}} dx$$

```
[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(1/2)), x)
```

$$3.756 \quad \int \frac{1}{\sqrt{a+b \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	6795
Rubi [A] (verified)	6796
Mathematica [A] (verified)	6799
Maple [B] (verified)	6800
Fricas [F]	6801
Sympy [F(-1)]	6801
Maxima [F]	6802
Giac [F]	6802
Mupad [F(-1)]	6802

Optimal result

Integrand size = 25, antiderivative size = 505

$$\int \frac{1}{\sqrt{a+b \cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} dx$$

$$= \frac{3(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4b^2 d \sqrt{\sec(c+dx)}} - \frac{(3a-2b)\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4b^2 d \sqrt{\sec(c+dx)}} - \frac{\sqrt{a+b}(3a^2+4b^2)\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{4b^3 d \sqrt{\sec(c+dx)}} + \frac{\sqrt{a+b \cos(c+dx)} \sin(c+dx)}{2bd \sqrt{\sec(c+dx)}} - \frac{3a\sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{4b^2 d}$$

```
[Out] 1/2*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/b/d/sec(d*x+c)^(1/2)-3/4*a*sin(d*x+c)
*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/d+3/4*(a-b)*csc(d*x+c)*Ellipti
cE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2)
)*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x
+c)))/(a-b)^(1/2)/b^2/d/sec(d*x+c)^(1/2)-1/4*(3*a-2*b)*csc(d*x+c)*EllipticF
((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*
(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c
)))/(a-b)^(1/2)/b^2/d/sec(d*x+c)^(1/2)-1/4*(3*a^2+4*b^2)*csc(d*x+c)*Ellipti
cPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-
b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(
1+sec(d*x+c)))/(a-b)^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.00,
 number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used
 = {4307, 2872, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)} \sec^{\frac{5}{2}}(c+dx)} dx =$$

$$\frac{\sqrt{a+b}(3a^2+4b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\cos(c+dx)}}\right)\right) - (3a-2b)\sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right) - 3(a-b)\sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right) - \frac{3a \sin(c+dx) \sqrt{\sec(c+dx)} \sqrt{a+b\cos(c+dx)}}{4b^2d} + \frac{\sin(c+dx) \sqrt{a+b\cos(c+dx)}}{2bd\sqrt{\sec(c+dx)}}$$

[In] Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)), x]

[Out] (3*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*b^2*d*Sqrt[Sec[c + d*x]]) - ((3*a - 2*b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(3*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(4*b^3*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*b*d*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d)

Rule 2872

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 2)*((c + d*Ssin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Ssin[e + f*x])^(m - 3)*(c + d*Ssin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]

|| IntegersQ[2*m, 2*n] && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & & NeQ[c, 0])))

Rule 2888

Int[Sqrt[(b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3132

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)])^2/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/

Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x])], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 3140

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^m*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}} dx \\
 &= \frac{\sqrt{a+b\cos(c+dx)} \sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{a}{2} + b\cos(c+dx) - \frac{3}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{2b} \\
 &= \frac{\sqrt{a+b\cos(c+dx)} \sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} - \frac{3a\sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{4b^2d} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{3a^2}{2} + ab\cos(c+dx) + \frac{1}{2}(3a^2+4b^2)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{4b^2} \\
 &= \frac{\sqrt{a+b\cos(c+dx)} \sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} - \frac{3a\sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{4b^2d} \\
 &\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{3a^2}{2} + ab\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{4b^2} \\
 &\quad + \frac{\left((3a^2 + 4b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{8b^2}
 \end{aligned}$$

$$\begin{aligned}
&= \\
&\frac{\sqrt{a+b}(3a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{a+b}}{4b^3d\sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} - \frac{3a\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^2d} \\
&+ \frac{\left(3a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}\cos(c+dx)}dx}{8b^2} \\
&- \frac{\left(a(3a-2b)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b}\cos(c+dx)}dx}{8b^2} \\
&= \frac{3(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{4b^2d\sqrt{\sec(c+dx)}} \\
&- \frac{(3a-2b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4b^2d\sqrt{\sec(c+dx)}} \\
&- \frac{\sqrt{a+b}(3a^2+4b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{a+b}}{4b^3d\sqrt{\sec(c+dx)}} \\
&+ \frac{\sqrt{a+b}\cos(c+dx)\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} - \frac{3a\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 12.71 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{a+b}\cos(c+dx)\sec^{\frac{5}{2}}(c+dx)} dx = \frac{\sqrt{a+b}\cos(c+dx)\sqrt{\sec(c+dx)}\sin(2(c+dx))}{4bd} \\
- \frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(24a(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\arcsin\left(\tan\left(\frac{c+dx}{2}\right)\right)\middle|-\frac{a+b}{a+b}\right)\right)}{4bd}$$

[In] Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)),x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*b*d) - (Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(24*a*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 16*(a - 2*b)*b*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 36*a^2*Cos[(c + d*x)/2]*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b +

$$\begin{aligned}
& a \operatorname{Sec}[c + d*x] / ((a + b) * (1 + \operatorname{Sec}[c + d*x])) - 48*b^2 * \operatorname{Cos}[(c + d*x)/2] * \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c + d*x)/2]], (-a + b)/(a + b)] * \operatorname{Sqrt}[(1 + \operatorname{Sec}[c + d*x])^{-1}] * \operatorname{Sqrt}[(b + a * \operatorname{Sec}[c + d*x]) / ((a + b) * (1 + \operatorname{Sec}[c + d*x]))] - 12*a^2 * \operatorname{Cos}[(3*(c + d*x))/2] * \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c + d*x)/2]], (-a + b)/(a + b)] * \operatorname{Sqrt}[(1 + \operatorname{Sec}[c + d*x])^{-1}] * \operatorname{Sqrt}[(b + a * \operatorname{Sec}[c + d*x]) / ((a + b) * (1 + \operatorname{Sec}[c + d*x]))] - 16*b^2 * \operatorname{Cos}[(3*(c + d*x))/2] * \operatorname{EllipticPi}[-1, \operatorname{ArcSin}[\operatorname{Tan}[(c + d*x)/2]], (-a + b)/(a + b)] * \operatorname{Sqrt}[(1 + \operatorname{Sec}[c + d*x])^{-1}] * \operatorname{Sqrt}[(b + a * \operatorname{Sec}[c + d*x]) / ((a + b) * (1 + \operatorname{Sec}[c + d*x]))] - 6*a^2 * \operatorname{Sin}[(c + d*x)/2] + 6*a*b * \operatorname{Sin}[(c + d*x)/2] + 6*a^2 * \operatorname{Sin}[(3*(c + d*x))/2] - 3*a*b * \operatorname{Sin}[(3*(c + d*x))/2] + 3*a*b * \operatorname{Sin}[(5*(c + d*x))/2]) / (16*b^2 * d * \operatorname{Sqrt}[a + b * \operatorname{Cos}[c + d*x]])
\end{aligned}$$

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1688 vs. $2(451) = 902$.

Time = 8.53 (sec) , antiderivative size = 1689, normalized size of antiderivative = 3.34

method	result	size
default	Expression too large to display	1689

[In] `int(1/sec(d*x+c)^(5/2)/(a*cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $\begin{aligned}
& 1/4/d * \operatorname{sec}(d*x+c)^{(1/2)} * (-2 * \operatorname{EllipticF}(\cot(d*x+c) - \operatorname{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * a * b * \cos(d*x+c)^2 + 4 * \operatorname{EllipticF}(\cot(d*x+c) - \operatorname{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * b^2 * \cos(d*x+c)^2 + 3 * \operatorname{EllipticE}(\cot(d*x+c) - \operatorname{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * a^2 * \cos(d*x+c)^2 + 3 * \operatorname{EllipticE}(\cot(d*x+c) - \operatorname{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * a * b * \cos(d*x+c)^2 - 6 * \operatorname{EllipticPi}(\cot(d*x+c) - \operatorname{csc}(d*x+c), -1, (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * a^2 * \cos(d*x+c)^2 - 8 * \operatorname{EllipticPi}(\cot(d*x+c) - \operatorname{csc}(d*x+c), -1, (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * b^2 * \cos(d*x+c)^2 + 2 * b^2 * \cos(d*x+c)^3 * \sin(d*x+c) - 4 * \operatorname{EllipticF}(\cot(d*x+c) - \operatorname{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * a * b * \cos(d*x+c) + 8 * \operatorname{EllipticF}(\cot(d*x+c) - \operatorname{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * b^2 * \cos(d*x+c) + 6 * \operatorname{EllipticE}(\cot(d*x+c) - \operatorname{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * a^2 * \cos(d*x+c) + 6 * \operatorname{EllipticE}(\cot(d*x+c) - \operatorname{csc}(d*x+c), (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * a * b * \cos(d*x+c) - 12 * \operatorname{EllipticPi}(\cot(d*x+c) - \operatorname{csc}(d*x+c), -1, (-a-b)/(a+b))^{(1/2)}) * ((a + \cos(d*x+c) * b) / (1 + \cos(d*x+c)) / (a+b))^{(1/2)} * (\cos(d*x+c) / (1 + \cos(d*x+c)))^{(1/2)} * a^2 * \cos(d*x+c) - 16 * \operatorname{EllipticPi}(\cot(d*x+c) - \operatorname{csc}(d*x+c), -1, (-a-b)/(a+b))^{(1/2)}
\end{aligned}$

$$\begin{aligned}
 & \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right)^{1/2} \frac{\cos(dx+c)}{1 + \cos(dx+c)} \\
 & \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right)^{1/2} b^2 \cos(dx+c) - \cos(dx+c)^2 \sin(dx+c) a b + 2 b^2 \cos(dx+c)^2 \sin(dx+c) \\
 & - 2 \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right)^{1/2} \frac{\cos(dx+c)}{1 + \cos(dx+c)} \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), \\
 & \left(-\frac{a-b}{a+b} \right)^{1/2}) \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} a b + 4 \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right)^{1/2} \\
 & \operatorname{EllipticF}(\cot(dx+c) - \operatorname{csc}(dx+c), \left(-\frac{a-b}{a+b} \right)^{1/2}) \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} b^2 + 3 \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right)^{1/2} \\
 & \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), \left(-\frac{a-b}{a+b} \right)^{1/2}) \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} a^2 + 3 \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right)^{1/2} \\
 & \operatorname{EllipticE}(\cot(dx+c) - \operatorname{csc}(dx+c), \left(-\frac{a-b}{a+b} \right)^{1/2}) \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} a b - 6 \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right)^{1/2} \\
 & \operatorname{EllipticPi}(\cot(dx+c) - \operatorname{csc}(dx+c), -1, \left(-\frac{a-b}{a+b} \right)^{1/2}) \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} a^2 - 8 \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right)^{1/2} \\
 & \operatorname{EllipticPi}(\cot(dx+c) - \operatorname{csc}(dx+c), -1, \left(-\frac{a-b}{a+b} \right)^{1/2}) \left(\frac{\cos(dx+c)}{1 + \cos(dx+c)} \right)^{1/2} b^2 - 3 a^2 \cos(dx+c) \sin(dx+c) + 2 a b \cos(dx+c) \sin(dx+c) \\
 & \left(\frac{a + \cos(dx+c)b}{1 + \cos(dx+c)} \right)^{1/2} / b^2
 \end{aligned}$$

Fricas [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{5/2}(c + dx)} dx = \int \frac{1}{\sqrt{b \cos(dx + c) + a} \sec(dx + c)^{5/2}} dx$$

[In] integrate(1/sec(dx+c)^(5/2)/(a+b*cos(dx+c))^(1/2),x, algorithm="fricas")

[Out] integral(1/(sqrt(b*cos(dx + c) + a)*sec(dx + c)^(5/2)), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate(1/sec(dx+c)**(5/2)/(a+b*cos(dx+c))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{\sqrt{b \cos(dx + c) + a} \sec(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + b \cos(c + dx)} \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} \sqrt{a + b \cos(c + dx)}} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2)),x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(1/2)), x)

$$3.757 \quad \int \frac{\sec^5(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	6803
Rubi [A] (verified)	6804
Mathematica [A] (warning: unable to verify)	6807
Maple [B] (warning: unable to verify)	6807
Fricas [F]	6809
Sympy [F(-1)]	6809
Maxima [F]	6809
Giac [F]	6809
Mupad [F(-1)]	6810

Optimal result

Integrand size = 25, antiderivative size = 397

$$\int \frac{\sec^5(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx =$$

$$\frac{2b(5a^2 - 8b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^4 \sqrt{a+bd} \sqrt{\sec(c+dx)}} +$$

$$\frac{2(a+2b)(a+4b) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^3 \sqrt{a+bd} \sqrt{\sec(c+dx)}} +$$

$$\frac{2b^2 \sec^3(c+dx) \sin(c+dx)}{a(a^2 - b^2) d \sqrt{a+b \cos(c+dx)}} + \frac{2(a^2 - 4b^2) \sqrt{a+b \cos(c+dx)} \sec^3(c+dx) \sin(c+dx)}{3a^2 (a^2 - b^2) d}$$

```
[Out] 2*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2/3*(a^2-4*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)/a^2/(a^2-b^2)/d-2/3*b*(5*a^2-8*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a^4/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2/3*(a+2*b)*(a+4*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a^3/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00,
 number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used
 = {4307, 2881, 3134, 3077, 2895, 3073}

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \frac{2(a+2b)(a+4b)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}}\right], -\frac{(a+b)}{(a-b)}\right]}{3a^3d\sqrt{a+b}\sqrt{\sec(c+dx)}} + \frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}} + \frac{2(a^2-4b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d(a^2-b^2)} - \frac{2b(5a^2-8b^2)\sqrt{a+b\cos(c+dx)}}{3a^2d(a^2-b^2)}$$

[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (-2*b*(5*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(a + 2*b)*(a + 4*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Ssin[e + f*x]]/Sqrt[d*Ssin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

&& PosQ[(a + b)/d]

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 3134

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_)
+ (f_)*(x_)]^2), x_Symbol] := Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
EqQ[a, 0])))
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{\frac{3}{2}}} dx \\
&= \frac{2b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{a(a^2-b^2) d \sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{2}(a^2-4b^2) - \frac{1}{2}ab\cos(c+dx) + b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{a(a^2-b^2)} \\
&= \frac{2b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{a(a^2-b^2) d \sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{2(a^2-4b^2) \sqrt{a+b\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2(a^2-b^2) d} \\
&\quad + \frac{\left(4\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{1}{4}b(5a^2-8b^2) + \frac{1}{4}a(a^2+2b^2)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{3a^2(a^2-b^2)} \\
&= \frac{2b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{a(a^2-b^2) d \sqrt{a+b\cos(c+dx)}} + \frac{2(a^2-4b^2) \sqrt{a+b\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2(a^2-b^2) d} \\
&\quad + \frac{\left((a-b)(a+2b)(a+4b) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{3a^2(a^2-b^2)} \\
&\quad - \frac{\left(b(5a^2-8b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{3a^2(a^2-b^2)} \\
&= \frac{2b(5a^2-8b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a+b}}}{3a^4 \sqrt{a+bd} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{2(a+2b)(a+4b) \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a^3 \sqrt{a+bd} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{2b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{a(a^2-b^2) d \sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{2(a^2-4b^2) \sqrt{a+b\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2(a^2-b^2) d}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 11.41 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.11

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}} dx =$$

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)} \sec(c+dx) \left(2b(-5a^3 - 5a^2b + 8ab^2 + 8b^3) \sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}} \sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}} E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)\right) + \frac{\sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \left(-\frac{2b(5a^2-8b^2)\sin(c+dx)}{3a^3(a^2-b^2)} - \frac{2b^3\sin(c+dx)}{a^2(a^2-b^2)(a+b\cos(c+dx))} + \frac{2\tan(c+dx)}{3a^2}\right)}{d}$$

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(a^3 - 5*a^2*b + 2*a*b^2 + 8*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(-5*a^2 + 8*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*a^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*(5*a^2 - 8*b^2)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)) - (2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*a^2)))/d
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2617 vs. 2(359) = 718.

Time = 12.46 (sec) , antiderivative size = 2618, normalized size of antiderivative = 6.59

method	result	size
default	Expression too large to display	2618

```
[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^5/2*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(5*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-5*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)+2*E
```

$$\begin{aligned}
& \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * a^2 * b^2 * (-\csc(dx+c))^{2+1} \\
& * (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b \\
& * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} + 8 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b) \\
& / (a+b))^{1/2} * a * b^3 * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c) \\
&)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} + 5 * \\
& \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * a^3 * b * (-\csc(dx+c)^2 * \\
& (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * \\
& (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} + 5 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b) \\
& / (a+b))^{1/2} * a^2 * b^2 * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c) \\
&)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} - 8 \\
& * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * a * b^3 * (-\csc(dx+c)^2 \\
& * (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b \\
& * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} - \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b) \\
& / (a+b))^{1/2} * a^4 * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c) \\
&)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} \\
& * (1 - \cos(dx+c))^{2+1} * (1 - \cos(dx+c))^{2+1} - 8 * \csc(dx+c)^5 * b^4 * (1 - \cos(dx+c))^{5+2} * \csc(dx+c)^3 * \\
& a^4 * (1 - \cos(dx+c))^{3+16} * \csc(dx+c)^3 * b^4 * (1 - \cos(dx+c))^{3+3} * a^3 * b * (\csc(dx+c) \\
& - \cot(dx+c)) + 7 * a^2 * b^2 * (\csc(dx+c) - \cot(dx+c)) - 2 * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c) \\
& - \csc(dx+c), (-a-b)/(a+b))^{1/2} * a^2 * b^2 * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c) \\
&)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} \\
& * (1 - \cos(dx+c))^{2+1} * (1 - \cos(dx+c))^{2+1} - 8 * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b) \\
& / (a+b))^{1/2} * a * b^3 * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c) \\
&)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} \\
& * (1 - \cos(dx+c))^{2+1} * (1 - \cos(dx+c))^{2+1} - 5 * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b) \\
& / (a+b))^{1/2} * a^3 * b * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c) \\
&)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} \\
& * (1 - \cos(dx+c))^{2+1} * (1 - \cos(dx+c))^{2+1} - 5 * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b) \\
& / (a+b))^{1/2} * a^2 * b^2 * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c) \\
&)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} \\
& * (1 - \cos(dx+c))^{2+1} + 8 * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b) \\
& / (a+b))^{1/2} * a * b^3 * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} * ((\csc(dx+c) \\
&)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} * (1 - \cos(dx+c))^{2+1} \\
& - 5 * \csc(dx+c)^5 * a^3 * b * (1 - \cos(dx+c))^{5+8} * \csc(dx+c)^3 * a * b^3 * (1 - \cos(dx+c))^{3+5} \\
& * \csc(dx+c)^5 * a^2 * b^2 * (1 - \cos(dx+c))^{5+8} * \csc(dx+c)^5 * a * b^3 * (1 - \cos(dx+c))^{5+2} \\
& * \csc(dx+c)^3 * a^3 * b * (1 - \cos(dx+c))^{3+8} * \csc(dx+c)^3 * a^2 * b^2 * (1 - \cos(dx+c))^{3+5} \\
& + \text{EllipticF}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * a^4 * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} \\
& * ((\csc(dx+c)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} \\
& - 8 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * b^4 * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} \\
& * ((\csc(dx+c)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} \\
& + 8 * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c) - \csc(dx+c), (-a-b)/(a+b))^{1/2} * b^4 * (-\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1} \\
& * ((\csc(dx+c)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (a+b))^{1/2} \\
& * (1 - \cos(dx+c))^{2+1} - 2 * a^4 * (\csc(dx+c) - \cot(dx+c)) - 8 * b^4 * (\csc(dx+c) - \cot(dx+c)) * ((\csc(dx+c) \\
&)^2 * a * (1 - \cos(dx+c))^{2+1} - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}) / (\csc(dx+c)^2 * (
\end{aligned}$$

$(1 - \cos(dx+c))^{2+1})^{1/2} / (\csc(dx+c)^2 * a * (1 - \cos(dx+c))^{2 - \csc(dx+c)^2 * b * (1 - \cos(dx+c))^{2+a+b}} / (\csc(dx+c)^2 * (1 - \cos(dx+c))^{2+1})^2 / (a+b) / (a-b) / a^3$

Fricas [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(dx+c)^(5/2)/(a+b*cos(dx+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(dx + c) + a)*sec(dx + c)^(5/2)/(b^2*cos(dx + c)^2 + 2*a*b*cos(dx + c) + a^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(sec(dx+c)**(5/2)/(a+b*cos(dx+c))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(dx+c)^(5/2)/(a+b*cos(dx+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(dx + c)^(5/2)/(b*cos(dx + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(dx+c)^(5/2)/(a+b*cos(dx+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(dx + c)^(5/2)/(b*cos(dx + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{5}{2}}}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx$$

```
[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(3/2), x)
```

$$3.758 \quad \int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	6811
Rubi [A] (verified)	6812
Mathematica [A] (verified)	6814
Maple [B] (warning: unable to verify)	6815
Fricas [F]	6816
Sympy [F]	6816
Maxima [F]	6816
Giac [F]	6816
Mupad [F(-1)]	6817

Optimal result

Integrand size = 25, antiderivative size = 325

$$\int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = \frac{2(a^2 - 2b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a}{a-b}}}{a^3 \sqrt{a+bd} \sqrt{\sec(c+dx)}} - \frac{2(a+2b) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2 \sqrt{a+bd} \sqrt{\sec(c+dx)}} + \frac{2b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{a(a^2 - b^2) d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*(a^2-2*b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*(a+2*b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00,
 number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used
 = {4307, 2881, 3077, 2895, 3073}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx =$$

$$\frac{2(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}\sqrt{\sec(c+dx)}} +$$

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)\sqrt{a+b\cos(c+dx)}} +$$

$$\frac{2(a^2-2b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)}{a^3d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (2*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a^3*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])]

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr


```
t[a*((1 - Csc[e + f*x])/(a + b))*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_
_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\cos^{\frac{3}{2}}(c + dx)(a + b \cos(c + dx))^{3/2}} dx \\ &= \frac{2b^2 \sqrt{\sec(c + dx)} \sin(c + dx)}{a(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\left(2\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\frac{1}{2}(a^2 - 2b^2) - \frac{1}{2}ab \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{a(a^2 - b^2)} \end{aligned}$$

$$\begin{aligned}
&= \frac{2b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{a(a^2-b^2) d \sqrt{a+b \cos(c+dx)}} \\
&\quad - \frac{\left((a-b)(a+2b) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b \cos(c+dx)}} dx}{a(a^2-b^2)} \\
&\quad - \frac{\left((-a^2+2b^2) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b \cos(c+dx)}} dx}{a(a^2-b^2)} \\
&= \frac{2(a^2-2b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^3 \sqrt{a+bd} \sqrt{\sec(c+dx)}} \\
&\quad - \frac{2(a+2b) \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2 \sqrt{a+bd} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{2b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{a(a^2-b^2) d \sqrt{a+b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 7.42 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.14

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b \cos(c+dx))^{3/2}} dx = \frac{2\left((ab^2+(a^2-2b^2)(a+b \cos(c+dx))) \sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)} \sqrt{\sec(c+dx)} \sin(c+dx)\right)}{a^3 \sqrt{a+bd} \sqrt{\sec(c+dx)}}$$

[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (2*((a*b^2 + (a^2 - 2*b^2)*(a + b*Cos[c + d*x]))*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*Sin[c + d*x] - Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(a^2 - a*b - 2*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a^2 - 2*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. $2(297) = 594$.

Time = 11.52 (sec) , antiderivative size = 1228, normalized size of antiderivative = 3.78

method	result	size
default	Expression too large to display	1228

[In] `int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)`

[Out]
$$-2/d*(-(\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2-1))^{\frac{3}{2}}*(\csc(d*x+c)^2*(1-\cos(d*x+c))^2-1)*(-(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{1}{2}}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{\frac{1}{2}}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{\frac{1}{2}})*a^3+(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{1}{2}}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{\frac{1}{2}}*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{\frac{1}{2}})*a^2*b+2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{1}{2}}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{\frac{1}{2}}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{\frac{1}{2}})*a^3+(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{1}{2}}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{\frac{1}{2}}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{\frac{1}{2}})*a^2*b-2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{1}{2}}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{\frac{1}{2}}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{\frac{1}{2}})*a*b^2-2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{\frac{1}{2}}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{\frac{1}{2}}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c),(-a-b)/(a+b))^{\frac{1}{2}})*b^3+\csc(d*x+c)^3*a^3*(1-\cos(d*x+c))^3-\csc(d*x+c)^3*a^2*b*(1-\cos(d*x+c))^3-2*\csc(d*x+c)^3*a*b^2*(1-\cos(d*x+c))^3+2*\csc(d*x+c)^3*b^3*(1-\cos(d*x+c))^3+a^3*(\csc(d*x+c)-\cot(d*x+c))+a^2*b*(\csc(d*x+c)-\cot(d*x+c))-2*b^3*(\csc(d*x+c)-\cot(d*x+c))*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1))^{\frac{1}{2}}/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)/(\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/a^2/(a-b)/(a+b)$$

Fricas [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)

[Out] Integral(sec(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{3/2}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^{\frac{3}{2}}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{3}{2}}}{(a + b \cos(c + dx))^{\frac{3}{2}}} dx$$

```
[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(3/2), x)
```

$$3.759 \quad \int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^{3/2}} dx$$

Optimal result	6818
Rubi [A] (verified)	6818
Mathematica [A] (verified)	6821
Maple [B] (warning: unable to verify)	6821
Fricas [F]	6822
Sympy [F]	6822
Maxima [F]	6822
Giac [F]	6822
Mupad [F(-1)]	6823

Optimal result

Integrand size = 25, antiderivative size = 307

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^{3/2}} dx = \frac{2b\sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{a^2 \sqrt{a+bd} \sqrt{\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a \sqrt{a+bd} \sqrt{\sec(c+dx)}} - \frac{2b\sqrt{\sec(c+dx)} \sin(c+dx)}{(a^2-b^2) d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] -2*b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*b*csc
(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-
b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(
d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2*csc(d*x+c)*Ellipt
icF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b)
)^(1/2)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

= {4307, 2879, 3077, 2895, 3073}

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \frac{2b\sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{a^2 d \sqrt{a+b} \sqrt{\sec(c+dx)}} - \frac{2b \sin(c+dx) \sqrt{\sec(c+dx)}}{d(a^2-b^2) \sqrt{a+b\cos(c+dx)}} + \frac{2\sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right), -\frac{a+b}{a-b}}{ad\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(3/2), x]

[Out] (2*b*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2879

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)), x_Symbol] :> Simp[2*b*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(b + a*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],

$x]$ /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x]] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} (a + b \cos(c + dx))^{3/2}} dx \\
 &= -\frac{2b \sqrt{\sec(c + dx)} \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} + \frac{\left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{b + a \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{a^2 - b^2} \\
 &= -\frac{2b \sqrt{\sec(c + dx)} \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}} \\
 &\quad + \frac{\left((a - b) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} \sqrt{a + b \cos(c + dx)}} dx}{a^2 - b^2} \\
 &\quad + \frac{\left(b \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{a^2 - b^2} \\
 &= \frac{2b \sqrt{\cos(c + dx)} \csc(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{a^2 \sqrt{a + b} d \sqrt{\sec(c + dx)}} \\
 &\quad + \frac{2 \sqrt{\cos(c + dx)} \csc(c + dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{a \sqrt{a + b} d \sqrt{\sec(c + dx)}} \\
 &\quad - \frac{2b \sqrt{\sec(c + dx)} \sin(c + dx)}{(a^2 - b^2) d \sqrt{a + b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 6.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \frac{2\sqrt{\sec(c+dx)} \left(-2b(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right) \sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}} E\left(\arcsin\left(\tan\left(\frac{c+dx}{2}\right)\right)\right) \right)}{(a+b\cos(c+dx))^{3/2}}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]

[Out] (2*Sqrt[Sec[c + d*x]]*(-2*b*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)*Sqrt[(1 + Sec[c + d*x])^(-1)] + 2*a*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)*Sqrt[(1 + Sec[c + d*x])^(-1)] + b*(-a + b)*Cos[c + d*x]*Tan[(c + d*x)/2))/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. 2(279) = 558.

Time = 9.05 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.44

method	result
default	$2\sqrt{-\frac{(\csc^2(dx+c)(1-\cos(dx+c))^2+1)}{(\csc^2(dx+c)(1-\cos(dx+c))^2-1)}} \sqrt{\frac{(\csc^2(dx+c))^a(1-\cos(dx+c))^2-(\csc^2(dx+c))^b(1-\cos(dx+c))^2+a+b}{(\csc^2(dx+c)(1-\cos(dx+c))^2+1)}} \left(-\sqrt{-(\csc^2(dx+c)(1-\cos(dx+c)))} \right)$

[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^2-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*b^2+csc(d*x+c)^3*a*b*(1-cos(d*x+c))^3-csc(d*x+c)^3*b^2*(1-cos(d*x+c))^3-a*b*(csc(d*x+c)-cot(d*x+c))+b^2*(csc(d*x+c)-cot(d*x+c))

)/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/a/(a-b)/(a+b)

Fricas [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{3/2}} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)

Sympy [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx$$

[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)

[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{3/2}} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{3/2}} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{(a+b\cos(c+dx))^{3/2}} dx$$

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(3/2), x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(3/2), x)
```

$$3.760 \quad \int \frac{1}{(a+b \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} dx$$

Optimal result	6824
Rubi [A] (verified)	6825
Mathematica [A] (verified)	6827
Maple [B] (warning: unable to verify)	6827
Fricas [F]	6828
Sympy [F]	6828
Maxima [F]	6828
Giac [F]	6828
Mupad [F(-1)]	6829

Optimal result

Integrand size = 25, antiderivative size = 306

$$\int \frac{1}{(a+b \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} dx =$$

$$\frac{2\sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+bd} \sqrt{\sec(c+dx)}} +$$

$$\frac{2\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+bd} \sqrt{\sec(c+dx)}} +$$

$$\frac{2a\sqrt{\sec(c+dx)} \sin(c+dx)}{(a^2-b^2)d\sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2*a*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)-2*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+2*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4307, 2873, 2874, 2895, 3073}

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \frac{2a \sin(c + dx) \sqrt{\sec(c + dx)}}{d(a^2 - b^2) \sqrt{a + b \cos(c + dx)}} + \frac{2\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{ad\sqrt{a + b} \sqrt{\sec(c + dx)}} - \frac{2\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{ad\sqrt{a + b} \sqrt{\sec(c + dx)}}$$

[In] Int[1/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]

[Out] (-2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2873

Int[Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 2874

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr

```
t[a*((1 - Csc[e + f*x])/(a + b))*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
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Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 4307

```
Int[(csc[(a_) + (b_)*(x_)]*(c_))^(m_)*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx \\
 &= \frac{2a\sqrt{\sec(c+dx)} \sin(c+dx)}{(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{3/2}(c+dx)} dx}{a^2-b^2} \\
 &= \frac{2a\sqrt{\sec(c+dx)} \sin(c+dx)}{(a^2-b^2)d\sqrt{a+b\cos(c+dx)}} \\
 &\quad - \frac{\left(a\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{3/2}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{a^2-b^2} \\
 &\quad - \frac{\left((-a+b)\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx}{a^2-b^2} \\
 &= \frac{2\sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+bd}\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{2\sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+bd}\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{2a\sqrt{\sec(c+dx)} \sin(c+dx)}{(a^2-b^2)d\sqrt{a+b\cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \frac{\sec^2\left(\frac{1}{2}(c + dx)\right) \left((a + b \cos(c + dx)) E\left(\arcsin\left(\tan\left(\frac{1}{2}(c + dx)\right)\right)\right) \right)}{(a^2 - b^2) \sqrt{\sec(c + dx)}}$$

[In] Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]

[Out] (Sec[(c + d*x)/2]^2*((a + b*Cos[c + d*x])*EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)) - (a + b*Cos[c + d*x])*EllipticF[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)) + (a - b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))])*Sqrt[Sec[c + d*x]])

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(278) = 556.

Time = 6.09 (sec) , antiderivative size = 783, normalized size of antiderivative = 2.56

method	result
default	$2 \left((\csc^2(dx+c))(1-\cos(dx+c))^2+1 \right) \sqrt{\frac{\csc^2(dx+c) a(1-\cos(dx+c))^2 - (\csc^2(dx+c) b(1-\cos(dx+c))^2 + a+b}{(\csc^2(dx+c))(1-\cos(dx+c))^2+1}} \left(-\sqrt{-(\csc^2(dx+c))(1-\cos(dx+c))^2+1} \right)$

[In] int(1/(a+cos(d*x+c)*b)^(3/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*b+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*b+csc(d*x+c)^3*(1-cos(d*x+c))^3*a-csc(d*x+c)^3*(1-cos(d*x+c))^3*b-a*(csc(d*x+c)-cot(d*x+c))+b*(csc(d*x+c)-cot(d*x+c)))/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)

$(d*x+c)^2-1)^{1/2}/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2-1)/(\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a-b)/(a+b)$

Fricas [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{3/2} \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))), x)

Sympy [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2), x)

[Out] Integral(1/((a + b*cos(c + d*x))**(3/2)*sqrt(sec(c + d*x))), x)

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{3/2} \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{3/2} \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + b \cos(c + dx))^{3/2}} dx$$

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(3/2)), x)
```

$$3.761 \quad \int \frac{1}{(a+b \cos(c+dx))^{3/2} \sec^2(c+dx)} dx$$

Optimal result	6830
Rubi [A] (verified)	6831
Mathematica [A] (verified)	6834
Maple [B] (warning: unable to verify)	6834
Fricas [F]	6835
Sympy [F(-1)]	6835
Maxima [F]	6836
Giac [F]	6836
Mupad [F(-1)]	6836

Optimal result

Integrand size = 25, antiderivative size = 447

$$\int \frac{1}{(a+b \cos(c+dx))^{3/2} \sec^2(c+dx)} dx = \frac{2\sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right) \sqrt{a+b} + b d \sqrt{\sec(c+dx)}}{b\sqrt{a+b} d \sqrt{\sec(c+dx)}} - \frac{2\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b} d \sqrt{\sec(c+dx)}} - \frac{2\sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2 d \sqrt{\sec(c+dx)}} - \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{b(a^2 - b^2) d \sqrt{a+b} \cos(c+dx)}$$

```
[Out] -2*a^2*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)+2*c
sc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-
a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+se
c(d*x+c)))/(a-b)^(1/2)/b/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*csc(d*x+c)*Ellipt
icF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2
))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)
^(1/2)/b/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)-2*csc(d*x+c)*EllipticPi((a+b*cos(d*
x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),(a+b)/b,((-a-b)/(a-b))^(1/2)*(a+b
)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(
a-b)^(1/2)/b^2/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4307, 2876, 2888, 2873, 2874, 2895, 3073}

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{\frac{3}{2}}(c + dx)} dx = -\frac{2a^2 \sin(c + dx) \sqrt{\sec(c + dx)}}{bd(a^2 - b^2) \sqrt{a + b \cos(c + dx)}} - \frac{2\sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{b^2 d \sqrt{\sec(c + dx)}} - \frac{2\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{bd\sqrt{a + b} \sqrt{\sec(c + dx)}} + \frac{2\sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{bd\sqrt{a + b} \sqrt{\sec(c + dx)}}$$

[In] Int[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]

[Out] (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b^2*d*Sqrt[Sec[c + d*x]]) - (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2873

Int[Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Simp[-2*a*d*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 2874

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In

$t[(1 + \sin[e + f*x])/((a + b*\sin[e + f*x])^{3/2}*\sqrt{c + d*\sin[e + f*x]}), x, x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2876

$\text{Int}[(d_*\sin[e_*] + (f_*)(x_*))^{3/2}/((a_*) + (b_*)\sin[e_*] + (f_*)(x_*))^{3/2}, x_Symbol] \rightarrow \text{Dist}[d/b, \text{Int}[\sqrt{d*\sin[e + f*x]}/\sqrt{a + b*\sin[e + f*x]}, x], x] - \text{Dist}[a*(d/b), \text{Int}[\sqrt{d*\sin[e + f*x]}/(a + b*\sin[e + f*x])^{3/2}, x], x] /;$ FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 2888

$\text{Int}[\sqrt{(b_*\sin[e_*] + (f_*)(x_*))}/\sqrt{(c_*) + (d_*)\sin[e_*] + (f_*)(x_*)}], x_Symbol] \rightarrow \text{Simp}[2*b*(\tan[e + f*x]/(d*f))*\text{Rt}[(c + d)/b, 2]*\sqrt{c*((1 + \text{Csc}[e + f*x])/(c - d))}*\sqrt{c*((1 - \text{Csc}[e + f*x])/(c + d))}*\text{EllipticPi}[(c + d)/d, \text{ArcSin}[\sqrt{c + d*\sin[e + f*x]}/\sqrt{b*\sin[e + f*x]}/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d)], x] /;$ FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

$\text{Int}[1/(\sqrt{(d_*\sin[e_*] + (f_*)(x_*))}*\sqrt{(a_*) + (b_*)\sin[e_*] + (f_*)(x_*)})], x_Symbol] \rightarrow \text{Simp}[-2*(\tan[e + f*x]/(a*f))*\text{Rt}[(a + b)/d, 2]*\sqrt{a*((1 - \text{Csc}[e + f*x])/(a + b))}*\sqrt{a*((1 + \text{Csc}[e + f*x])/(a - b))}*\text{EllipticF}[\text{ArcSin}[\sqrt{a + b*\sin[e + f*x]}/\sqrt{d*\sin[e + f*x]}/\text{Rt}[(a + b)/d, 2]], -(a + b)/(a - b)], x] /;$ FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

$\text{Int}[(A_* + (B_*)\sin[e_*] + (f_*)(x_*))^{3/2}/((b_*)\sin[e_*] + (f_*)(x_*))^{3/2}*\sqrt{(c_*) + (d_*)\sin[e_*] + (f_*)(x_*)}], x_Symbol] \rightarrow \text{Simp}[-2*A*(c - d)*(\tan[e + f*x]/(f*b*c^2))*\text{Rt}[(c + d)/b, 2]*\sqrt{c*((1 + \text{Csc}[e + f*x])/(c - d))}*\sqrt{c*((1 - \text{Csc}[e + f*x])/(c + d))}*\text{EllipticE}[\text{ArcSin}[\sqrt{c + d*\sin[e + f*x]}/\sqrt{b*\sin[e + f*x]}/\text{Rt}[(c + d)/b, 2]], -(c + d)/(c - d)], x] /;$ FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 4307

$\text{Int}[(\text{csc}[(a_*) + (b_*)(x_*)]*(c_*))^{(m_*)}*(u_*), x_Symbol] \rightarrow \text{Dist}[(c*\text{Csc}[a + b*x])^m*(c*\sin[a + b*x])^m, \text{Int}[\text{ActivateTrig}[u]/(c*\sin[a + b*x])^m, x], x] /;$ FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx \\
&= \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{b} \\
&\quad - \frac{\left(a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}} dx}{b} \\
&= \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2 d \sqrt{\sec(c+dx)}} \\
&\quad - \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{b(a^2-b^2) d \sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\left(a \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} dx}{b(a^2-b^2)} \\
&= \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2 d \sqrt{\sec(c+dx)}} \\
&\quad - \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{b(a^2-b^2) d \sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\left(a^2 \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{b(a^2-b^2)} \\
&\quad + \frac{\left(a(-a+b) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{b(a^2-b^2)} \\
&= \frac{2\sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+bd}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{2\sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+bd}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^2 d \sqrt{\sec(c+dx)}} \\
&\quad - \frac{2a^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{b(a^2-b^2) d \sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 15.62 (sec) , antiderivative size = 893, normalized size of antiderivative = 2.00

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{3/2}(c + dx)} dx = \frac{\sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \left(\frac{2a \sin(c + dx)}{b(a^2 - b^2)} + \frac{2a^2 \sin(c + dx)}{b(-a^2 + b^2)(a + b \cos(c + dx))} \right)}{d} + 2 \left(-a^2 \tan\left(\frac{1}{2}(c + dx)\right) - ab \tan\left(\frac{1}{2}(c + dx)\right) + 2ab \tan^3\left(\frac{1}{2}(c + dx)\right) + a^2 \tan^5\left(\frac{1}{2}(c + dx)\right) - ab \tan^5\left(\frac{1}{2}(c + dx)\right) \right)$$

[In] Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(b*(a^2 - b^2)) + (2*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x])))/d - (2*(-(a^2*Tan[(c + d*x)/2]) - a*b*Tan[(c + d*x)/2] + 2*a*b*Tan[(c + d*x)/2]^3 + a^2*Tan[(c + d*x)/2]^5 - a*b*Tan[(c + d*x)/2]^5 + 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - a*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + b*(a + b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)))/(b*(a^2 - b^2)*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2))

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(407) = 814.

Time = 8.92 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.32

method	result	size
default	Expression too large to display	1036

```
[In] int(1/(a+cos(d*x+c)*b)^(3/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^2*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a*b-(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*b^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a*b-2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-a-b)/(a+b))^(1/2))*a^2+2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticPi(cot(d*x+c)-csc(d*x+c),-1,(-a-b)/(a+b))^(1/2))*b^2+csc(d*x+c)^3*a^2*(1-cos(d*x+c))^3-csc(d*x+c)^3*a*b*(1-cos(d*x+c))^3-a^2*(csc(d*x+c)-cot(d*x+c))+a*b*(csc(d*x+c)-cot(d*x+c)))/(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(3/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^2/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/b/(a-b)/(a+b)
```

Fricas [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{\frac{3}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^{\frac{3}{2}} \sec^{\frac{3}{2}}(dx + c)} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{\frac{3}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{3/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{3/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + b \cos(c + dx))^{3/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)),x)

[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(3/2)), x)

$$3.762 \quad \int \frac{1}{(a+b \cos(c+dx))^{3/2} \sec^2(c+dx)} dx$$

Optimal result	6837
Rubi [A] (verified)	6838
Mathematica [A] (verified)	6842
Maple [B] (warning: unable to verify)	6843
Fricas [F]	6844
Sympy [F(-1)]	6844
Maxima [F]	6845
Giac [F]	6845
Mupad [F(-1)]	6845

Optimal result

Integrand size = 25, antiderivative size = 525

$$\int \frac{1}{(a+b \cos(c+dx))^{3/2} \sec^2(c+dx)} dx =$$

$$\frac{(3a^2 - b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ab^2 \sqrt{a+bd} \sqrt{\sec(c+dx)}} +$$

$$\frac{(3a+b) \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2 \sqrt{a+bd} \sqrt{\sec(c+dx)}} +$$

$$\frac{3a \sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3 d \sqrt{\sec(c+dx)}} -$$

$$\frac{2a^2 \sin(c+dx)}{b(a^2 - b^2) d \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)}} +$$

$$\frac{(3a^2 - b^2) \sqrt{a+b \cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{b^2 (a^2 - b^2) d}$$

```
[Out] -2*a^2*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2)+(3*
a^2-b^2)*sin(d*x+c)*(a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d
-(3*a^2-b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*
x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(
1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a/b^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+(
3*a+b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(
1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*
(a*(1+sec(d*x+c)))/(a-b)^(1/2)/b^2/d/(a+b)^(1/2)/sec(d*x+c)^(1/2)+3*a*csc(d
*x+c)*EllipticPi((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2), (a+b)/
b,((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b
))^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/b^3/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4307, 2871, 3140, 3132, 2888, 3077, 2895, 3073}

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^5(c + dx)} dx =$$

$$\frac{(3a^2 - b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{ab^2 d \sqrt{a + b} \sqrt{\sec(c + dx)}} -$$

$$\frac{2a^2 \sin(c + dx)}{bd(a^2 - b^2) \sqrt{\sec(c + dx)} \sqrt{a + b \cos(c + dx)}} +$$

$$\frac{(3a^2 - b^2) \sin(c + dx) \sqrt{\sec(c + dx)} \sqrt{a + b \cos(c + dx)}}{b^2 d (a^2 - b^2)} +$$

$$\frac{3a \sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right), -}{b^3 d \sqrt{\sec(c + dx)}} +$$

$$\frac{(3a + b) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{b^2 d \sqrt{a + b} \sqrt{\sec(c + dx)}}$$

[In] Int[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]

[Out] -(((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(a*b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]])) + ((3*a + b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (3*a*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/b^3*d*Sqrt[Sec[c + d*x]] - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +

$b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*\text{Sin}[e + f*x] + b*(b^2*(c^2 - d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*\text{Sin}[e + f*x]^2, x], x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || IntegersQ[2*m, 2*n])

Rule 2888

Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/b]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3132

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_.)])), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 3140

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]*Sqrt[(c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_.)])), x_Symbol] := Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[1/(2*d), Int[(1/((a + b*Sin
[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]])*Simp[2*a*A*d - C*(b*c - a*d) -
2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

```

Rule 4307

```

Int[(csc[(a_.) + (b_.)*(x_.)]*(c_.))^m*(u_), x_Symbol] := Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}} dx \\
&= -\frac{2a^2 \sin(c+dx)}{b(a^2-b^2) d \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{a^2}{2} - \frac{1}{2}ab\cos(c+dx) - \frac{1}{2}(3a^2-b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{b(a^2-b^2)} \\
&= -\frac{2a^2 \sin(c+dx)}{b(a^2-b^2) d \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{(3a^2-b^2) \sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \sin(c+dx)}{b^2(a^2-b^2) d} \\
&\quad - \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{2}a(3a^2-b^2) + a^2b\cos(c+dx) + \frac{3}{2}a(a^2-b^2)\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{a+b\cos(c+dx)}} dx}{b^2(a^2-b^2)}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2a^2 \sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&+ \frac{(3a^2-b^2)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d} \\
&- \frac{\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{2b^2} \\
&- \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{1}{2}a(3a^2-b^2)+a^2b\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{b^2(a^2-b^2)} \\
&= \frac{3a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^3d\sqrt{\sec(c+dx)}} \\
&- \frac{2a^2 \sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&+ \frac{(3a^2-b^2)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d} \\
&+ \frac{\left(a(a-b)(3a+b)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} dx}{2b^2(a^2-b^2)} \\
&- \frac{\left(a(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{2b^2(a^2-b^2)} \\
&= \frac{(3a^2-b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ab^2\sqrt{a+bd}\sqrt{\sec(c+dx)}} \\
&+ \frac{(3a+b)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+bd}\sqrt{\sec(c+dx)}} \\
&+ \frac{3a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^3d\sqrt{\sec(c+dx)}} \\
&- \frac{2a^2 \sin(c+dx)}{b(a^2-b^2)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} \\
&+ \frac{(3a^2-b^2)\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2(a^2-b^2)d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 13.21 (sec) , antiderivative size = 1025, normalized size of antiderivative = 1.95

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{\frac{5}{2}}(c + dx)} dx = \frac{\sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \left(-\frac{2a^2 \sin(c + dx)}{b^2(a^2 - b^2)} - \frac{2a^3 \sin(c + dx)}{b^2(-a^2 + b^2)(a + b \cos(c + dx))} \right)}{d}$$

$$\sqrt{\frac{1}{1 - \tan^2\left(\frac{1}{2}(c + dx)\right)}} \sqrt{\frac{a + b + a \tan^2\left(\frac{1}{2}(c + dx)\right) - b \tan^2\left(\frac{1}{2}(c + dx)\right)}{1 + \tan^2\left(\frac{1}{2}(c + dx)\right)}} \left(-3a^3 \tan\left(\frac{1}{2}(c + dx)\right) - 3a^2 b \tan\left(\frac{1}{2}(c + dx)\right) + ab^2 \tan\left(\frac{1}{2}(c + dx)\right) \right)$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sin[c + d*x])/(b^2*(a^2 - b^2)) - (2*a^3*Sin[c + d*x])/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])))/d - (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-3*a^3*Tan[(c + d*x)/2] - 3*a^2*b*Tan[(c + d*x)/2] + a*b^2*Tan[(c + d*x)/2] + b^3*Tan[(c + d*x)/2] + 6*a^2*b*Tan[(c + d*x)/2]^3 - 2*b^3*Tan[(c + d*x)/2]^3 + 3*a^3*Tan[(c + d*x)/2]^5 - 3*a^2*b*Tan[(c + d*x)/2]^5 - a*b^2*Tan[(c + d*x)/2]^5 + b^3*Tan[(c + d*x)/2]^5 + 6*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (3*a^3 + 3*a^2*b - a*b^2 - b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*b*(a + b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)))/(b^2*(-a^2 + b^2)*d*Sqrt[1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2525 vs. $2(481) = 962$.

Time = 8.87 (sec) , antiderivative size = 2526, normalized size of antiderivative = 4.81

method	result	size
default	Expression too large to display	2526

[In] `int(1/(a+cos(d*x+c)*b)^(3/2)/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{1}{d} \frac{\csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+1} \left(\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \right)}{\csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b} (1-\cos(d*x+c))^{2+1} \left(\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \right)}$$

$$\frac{(-3*a^3 (\csc(d*x+c) - \cot(d*x+c)) + b^3 (\csc(d*x+c) - \cot(d*x+c)) - 6 (-\csc(d*x+c))^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) * a^3 + 3 * \csc(d*x+c)^{2*} \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^3 * (-\csc(d*x+c)^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*} \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) * b^3 * (-\csc(d*x+c)^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * (1-\cos(d*x+c))^{2-6} \csc(d*x+c)^{2*} (-\csc(d*x+c)^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) * a^3 * (1-\cos(d*x+c))^{2+a} b^2 * (\csc(d*x+c) - \cot(d*x+c)) + \csc(d*x+c)^5 * b^3 * (1-\cos(d*x+c))^{5+2} \csc(d*x+c)^3 * a^2 * b * (1-\cos(d*x+c))^{3-2} * (-\csc(d*x+c)^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^2 * b - 2 * (-\csc(d*x+c))^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a * b^2 + 3 * (-\csc(d*x+c)^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^2 * b - (-\csc(d*x+c))^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a * b^2 + 6 * (-\csc(d*x+c)^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{1/2}) * a * b^2 - \csc(d*x+c)^5 * a * b^2 * (1-\cos(d*x+c))^{5-3} \csc(d*x+c)^5 * a^2 * b * (1-\cos(d*x+c))^{5+3} * (-\csc(d*x+c))^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^3 - (-\csc(d*x+c)^{2*(1-\cos(d*x+c))^{2+1}} (1/2) * ((\csc(d*x+c)^{2*a} (1-\cos(d*x+c))^{2-c} \csc(d*x+c)^{2*b} (1-\cos(d*x+c))^{2+a+b}) / (a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{1/2}) * b^3 + 3 * \csc(d*x+c)^5 * a^3 * (1-\cos(d*x+c))^{5-2} \csc(d*x+c)^3 * b^3 * (1-\cos(d*x+c))^{3+a} * a^2 * b * (\csc(d*x+c) - \cot(d*x+c))$$

```

x+c))-2*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^2*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+
c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-2*
csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*(-
csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-cs
c(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+3*csc(d*x+
c)^2*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*(-csc(d*x+
c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)
^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-csc(d*x+c)^2*Ellip
ticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*(-csc(d*x+c)^2*(1-co
s(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-co
s(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+6*csc(d*x+c)^2*EllipticPi(co
t(d*x+c)-csc(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*(-csc(d*x+c)^2*(1-cos(d*
x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*
x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2)/(-(csc(d*x+c)^2*(1-cos(d*x+c))^
2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(5/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^
2-1)^3/(csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b
)/b^2/(a-b)/(a+b)

```

Fricas [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{\frac{5}{2}}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^{\frac{3}{2}} \sec^{\frac{5}{2}}(dx + c)} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```


Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{5/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^{3/2} \sec(dx + c)^{5/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{3/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + b \cos(c + dx))^{3/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2)),x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(3/2)), x)

3.763 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^{\frac{5}{2}}} dx$

Optimal result	6846
Rubi [A] (verified)	6847
Mathematica [A] (warning: unable to verify)	6850
Maple [B] (warning: unable to verify)	6851
Fricas [F]	6851
Sympy [F(-1)]	6851
Maxima [F]	6851
Giac [F]	6852
Mupad [F(-1)]	6852

Optimal result

Integrand size = 25, antiderivative size = 513

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b \cos(c+dx))^{\frac{5}{2}}} dx =$$

$$\frac{8b(2a^4 - 7a^2b^2 + 4b^4) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^5(a-b)(a+b)^{\frac{3}{2}} d \sqrt{\sec(c+dx)}} +$$

$$\frac{2(a^4 + 9a^3b + 16a^2b^2 - 12ab^3 - 16b^4) \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right)}{3a^4(a-b)(a+b)^{\frac{3}{2}} d \sqrt{\sec(c+dx)}} +$$

$$\frac{2b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a(a^2 - b^2) d(a+b \cos(c+dx))^{\frac{3}{2}}} + \frac{4b^2(5a^2 - 3b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2(a^2 - b^2)^2 d \sqrt{a+b \cos(c+dx)}} +$$

$$\frac{2(a^4 - 13a^2b^2 + 8b^4) \sqrt{a+b \cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^3(a^2 - b^2)^2 d}$$

```
[Out] 2/3*b^2*sec(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+4/
3*b^2*(5*a^2-3*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*cos(
d*x+c))^(1/2)+2/3*(a^4-13*a^2*b^2+8*b^4)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*c
os(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d-8/3*b*(2*a^4-7*a^2*b^2+4*b^4)*csc(d*x+c)
*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a^5/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+2/3*(a^4+9*a^3*b+16*
a^2*b^2-12*a*b^3-16*b^4)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(
1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x
+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a^4/(a-b)/(a+b)^(3/2)/d/se
c(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2881, 3134, 3077, 2895, 3073}

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \frac{4b^2(5a^2-3b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}} + \frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad(a^2-b^2)(a+b\cos(c+dx))^{\frac{3}{2}}} - \frac{8b(2a^4-7a^2b^2+4b^4)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{3a^5d(a-b)(a+b)^{\frac{3}{2}}\sqrt{\sec(c+dx)}} + \frac{2(a^4+9a^3b+16a^2b^2-12ab^3-16b^4)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{3a^4d(a-b)(a+b)^{\frac{3}{2}}\sqrt{\sec(c+dx)}} + \frac{2(a^4-13a^2b^2+8b^4)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^3d(a^2-b^2)^2}$$

[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^5*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a^4 + 9*a^3*b + 16*a^2*b^2 - 12*a*b^3 - 16*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (4*b^2*(5*a^2 - 3*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2

*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)])*Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]

Rule 3073

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]

Rule 3077

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3134

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||

EqQ[a, 0]))

Rule 4307

Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}} dx \\
&= \frac{2b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} \\
&\quad + \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{3}{2}(a^2-2b^2) - \frac{3}{2}ab\cos(c+dx) + 2b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx}{3a(a^2-b^2)} \\
&= \frac{2b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2(a^2-b^2)^2 d \sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\left(4\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{3}{4}(a^4-13a^2b^2+8b^4) - \frac{1}{2}ab(3a^2-b^2)\cos(c+dx) + b^2(5a^2-3b^2)\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{3a^2(a^2-b^2)^2} \\
&= \frac{2b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2(a^2-b^2)^2 d \sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{2(a^4-13a^2b^2+8b^4) \sqrt{a+b\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^3(a^2-b^2)^2 d} \\
&\quad + \frac{\left(8\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{-\frac{3}{2}b(2a^4-7a^2b^2+4b^4) + \frac{3}{8}a(a^4+7a^2b^2-4b^4)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{9a^3(a^2-b^2)^2} \\
&= \frac{2b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{4b^2(5a^2-3b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2(a^2-b^2)^2 d \sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{2(a^4-13a^2b^2+8b^4) \sqrt{a+b\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^3(a^2-b^2)^2 d} \\
&\quad + \frac{\left((a-b)(a^4+9a^3b+16a^2b^2-12ab^3-16b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{3a^3(a^2-b^2)^2} \\
&\quad - \frac{\left(4b(2a^4-7a^2b^2+4b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{3a^3(a^2-b^2)^2}
\end{aligned}$$

=

$$\begin{aligned}
& \frac{8b(2a^4 - 7a^2b^2 + 4b^4) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a^5(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}} \\
& + \frac{2(a^4 + 9a^3b + 16a^2b^2 - 12ab^3 - 16b^4) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{3a^4(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}} \\
& + \frac{2b^2 \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a(a^2 - b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{4b^2(5a^2 - 3b^2) \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^2(a^2 - b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
& + \frac{2(a^4 - 13a^2b^2 + 8b^4) \sqrt{a+b\cos(c+dx)} \sec^{\frac{3}{2}}(c+dx) \sin(c+dx)}{3a^3(a^2 - b^2)^2 d}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 13.24 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.06

$$\begin{aligned}
& \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = \frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)} \sec(c+dx) (4b(2a^5 + 2a^4b - 7a^3b^2 - 7a^2b^3 + 4ab^4 + 4b^5))}{(a+b\cos(c+dx))^{5/2}} \\
& + \frac{\sqrt{a+b\cos(c+dx)} \sqrt{\sec(c+dx)} \left(-\frac{8b(2a^4 - 7a^2b^2 + 4b^4) \sin(c+dx)}{3a^4(a^2 - b^2)^2} - \frac{2b^3 \sin(c+dx)}{3a^2(a^2 - b^2)(a+b\cos(c+dx))^2} - \frac{2(11a^2b^3 \sin(c+dx) - 7b^5 \sin(c+dx))}{3a^3(a^2 - b^2)^2(a+b\cos(c+dx))} \right)}{d}
\end{aligned}$$

[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(4*b*(2*a^5 + 2*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 4*a*b^4 + 4*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * EllipticE[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)] + a*(a^5 - 8*a^4*b + 7*a^3*b^2 + 28*a^2*b^3 - 4*a*b^4 - 16*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * EllipticF[ArcSin[Tan[(c + d*x)/2]]], (-a + b)/(a + b)] + 2*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) / (3*a^4*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]] * Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]] * Sqrt[Sec[c + d*x]] * ((-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sin[c + d*x]) / (3*a^4*(a^2 - b^2)^2) - (2*b^3*Sin[c + d*x]) / (3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(11*a^2*b^3*Sin[c + d*x] - 7*b^5*Sin[c + d*x])) / (3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*Tan[c + d*x]) / (3*a^3))) / d

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5848 vs. $2(467) = 934$.

Time = 13.99 (sec) , antiderivative size = 5849, normalized size of antiderivative = 11.40

method	result	size
default	Expression too large to display	5849

[In] `int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out] result too large to display

Fricas [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}} dx$$

[In] `integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] `integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}} dx$$

[In] `integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)`

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\sec(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{5}{2}}}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx$$

[In] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(5/2),x)

[Out] int((1/cos(c + d*x))^(5/2)/(a + b*cos(c + d*x))^(5/2), x)

$$3.764 \quad \int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	6853
Rubi [A] (verified)	6854
Mathematica [A] (warning: unable to verify)	6857
Maple [B] (warning: unable to verify)	6857
Fricas [F]	6859
Sympy [F(-1)]	6860
Maxima [F]	6860
Giac [F]	6860
Mupad [F(-1)]	6860

Optimal result

Integrand size = 25, antiderivative size = 438

$$\int \frac{\sec^3(c+dx)}{(a+b \cos(c+dx))^{5/2}} dx = \frac{2(3a^4 - 15a^2b^2 + 8b^4) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right)\right) - 2(3a^3 + 9a^2b - 6ab^2 - 8b^3) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a-b}}}{3a^4(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)} - 3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}} + \frac{2b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{3a(a^2 - b^2)d(a+b \cos(c+dx))^{3/2}} + \frac{8b^2(2a^2 - b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{3a^2(a^2 - b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2/3*b^2*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)+8/
3*b^2*(2*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d/(a+b*cos(d*
x+c))^(1/2)+2/3*(3*a^4-15*a^2*b^2+8*b^4)*csc(d*x+c)*EllipticE((a+b*cos(d*x+
c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/
2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)))/(a-b)^(1/2)/a^4/(a-b)/
(a+b)^(3/2)/d/sec(d*x+c)^(1/2)-2/3*(3*a^3+9*a^2*b-6*a*b^2-8*b^3)*csc(d*x+c)
*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b
))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c)))/(a+b)^(1/2)*(a*(1+sec(d*x+c)
)/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2881, 3134, 3077, 2895, 3073}

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \frac{8b^2(2a^2-b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}} + \frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad(a^2-b^2)(a+b\cos(c+dx))^{\frac{3}{2}}} + \frac{2(3a^4-15a^2b^2+8b^4)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{3a^4d(a-b)(a+b)^{\frac{3}{2}}\sqrt{\sec(c+dx)}} - \frac{2(3a^3+9a^2b-6ab^2-8b^3)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right)}{3a^3d(a-b)(a+b)^{\frac{3}{2}}\sqrt{\sec(c+dx)}}$$

[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*a^3 + 9*a^2*b - 6*a*b^2 - 8*b^3)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(a + b)/(a - b)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (8*b^2*(2*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1)*((c + d*Ssin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_)] + (f_)*(x_)])*Sqrt[(a_)] + (b_)*sin[(e_)] + (f_)*(x_)], x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_)] + (f_)*(x_)]/(((b_)*sin[(e_)] + (f_)*(x_))]^(3/2)*Sqrt[(c_)] + (d_)*sin[(e_)] + (f_)*(x_)], x_Symbol] :> Simp[-2*A*(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_) + (B_)*sin[(e_)] + (f_)*(x_)]/(((a_)] + (b_)*sin[(e_)] + (f_)*(x_))]^(3/2)*Sqrt[(c_)] + (d_)*sin[(e_)] + (f_)*(x_)], x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rule 3134

```
Int[((a_)] + (b_)*sin[(e_)] + (f_)*(x_))]^(m_)*((c_)] + (d_)*sin[(e_)] + (f_)*(x_))]^(n_)*((A_)] + (B_)*sin[(e_)] + (f_)*(x_)] + (C_)*sin[(e_)] + (f_)*(x_)]^2), x_Symbol] :> Simp[(-A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 4307

```
Int[(csc[(a_)] + (b_)*(x_)]*(c_)]^(m_)]*(u_)]], x_Symbol] :> Dist[(c*Csc[a
```

+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}} dx \\
 &= \frac{2b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} \\
 &\quad + \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{2}(3a^2-4b^2) - \frac{3}{2}ab\cos(c+dx) + b^2\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} dx}{3a(a^2-b^2)} \\
 &= \frac{2b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{3a^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
 &\quad + \frac{\left(4\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{1}{4}(3a^4-15a^2b^2+8b^4) - \frac{1}{2}ab(3a^2-b^2)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{3a^2(a^2-b^2)^2} \\
 &= \frac{2b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{3a^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
 &\quad - \frac{\left((a-b)(3a^3+9a^2b-6ab^2-8b^3) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)} \sqrt{a+b\cos(c+dx)}} dx}{3a^2(a^2-b^2)^2} \\
 &\quad - \frac{\left((-3a^4+15a^2b^2-8b^4) \sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{3a^2(a^2-b^2)^2} \\
 &= \frac{2(3a^4-15a^2b^2+8b^4) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b\sqrt{\cos(c+dx)}}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a^4(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}} \\
 &\quad - \frac{2(3a^3+9a^2b-6ab^2-8b^3) \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b\sqrt{\cos(c+dx)}}}\right), -\frac{a+b}{a-b}\right)}{3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}} \\
 &\quad + \frac{2b^2 \sqrt{\sec(c+dx)} \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{8b^2(2a^2-b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{3a^2(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 12.97 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.20

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx = \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2(3a^4-15a^2b^2+8b^4)\sin(c+dx)}{3a^3(a^2-b^2)^2} + \frac{2b^2\sin(c+dx)}{3a(a^2-b^2)(a+b\cos(c+dx))}\right)}{d} + \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\sec(c+dx)\left(-2(3a^5+3a^4b-15a^3b^2-15a^2b^3+8ab^4+8b^5)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\right)}{d}$$

[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2), x]

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (8*(2*a^2*b^2*Sin[c + d*x] - b^4*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x])))/d + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(3*a^5 + 3*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 8*a*b^4 + 8*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(3*a^4 - 6*a^3*b - 15*a^2*b^2 + 2*a*b^3 + 8*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] * Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] * EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^4 - 15*a^2*b^2 + 8*b^4)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3674 vs. 2(398) = 796.

Time = 11.88 (sec) , antiderivative size = 3675, normalized size of antiderivative = 8.39

method	result	size
default	Expression too large to display	3675

[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)

```
[Out] -2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^3/2*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^1/2*(-3*csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^1/2)*a^6*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^1/2*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^1/2*(1-cos(d*x+c))^2+8*b^6*(csc(d*x+c)-cot(d*x+c))-7*csc(d*x+c)^5*a^2*b^4*(1-cos(d*x+c))^5-16*csc(d*x+c)^5*a*b^5*(1-cos(d*x+c))^5-18*csc(d*x+c)^3*a^4*b^2*(1-cos(d*x+c))^3-16*
```


$c(dx+c), (-a-b)/(a+b)^{(1/2)} * a^4 * b^2 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2-17 * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c))}, (-a-b)/(a+b)^{(1/2)} * a^3 * b^3 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2-6 * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c))}, (-a-b)/(a+b)^{(1/2)} * a^2 * b^4 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2+8 * \csc(dx+c)^2 * \text{EllipticF}(\cot(dx+c)-\csc(dx+c))}, (-a-b)/(a+b)^{(1/2)} * a * b^5 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2+6 * \csc(dx+c)^3 * a^6 * (1-\cos(dx+c))^{3-16 * \csc(dx+c)^3 * b^6 * (1-\cos(dx+c))^{3-14 * a^3 * b^3 * (\csc(dx+c)-\cot(dx+c))+8 * a * b^5 * (\csc(dx+c)-\cot(dx+c))+6 * a^5 * b * (\csc(dx+c)-\cot(dx+c))+6 * a^4 * b^2 * (\csc(dx+c)-\cot(dx+c))-17 * a^2 * b^4 * (\csc(dx+c)-\cot(dx+c))+3 * \csc(dx+c)^5 * a^6 * (1-\cos(dx+c))^{5+8 * \csc(dx+c)^5 * b^6 * (1-\cos(dx+c))^{5+3 * a^6 * (\csc(dx+c)-\cot(dx+c))+3 * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c))}, (-a-b)/(a+b)^{(1/2)} * a^6 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2-8 * \csc(dx+c)^2 * \text{EllipticE}(\cot(dx+c)-\csc(dx+c))}, (-a-b)/(a+b)^{(1/2)} * b^6 * (-\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})^{(1/2)} * ((\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})/(a+b))^{(1/2)} * (1-\cos(dx+c))^{2}/(\csc(dx+c)^2 * (1-\cos(dx+c))^{2+1})/(\csc(dx+c)^2 * a * (1-\cos(dx+c))^{2-\csc(dx+c)^2 * b * (1-\cos(dx+c))^{2+a+b}})^2/(a-b)^2/a^3/(a+b)^2$

Fricas [F]

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{5}{2}}} dx = \int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}} dx$$

[In] integrate(sec(dx+c)^(3/2)/(a+b*cos(dx+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(dx+c)+a)*sec(dx+c)^(3/2)/(b^3*cos(dx+c)^3+3*a*b^2*cos(dx+c)^2+3*a^2*b*cos(dx+c)+a^3),x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\sec(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c) + a)^{\frac{5}{2}}} dx$$

[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2), x, algorithm="giac")

[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{\left(\frac{1}{\cos(c+dx)}\right)^{\frac{3}{2}}}{(a + b \cos(c + dx))^{\frac{5}{2}}} dx$$

[In] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(5/2), x)

[Out] int((1/cos(c + d*x))^(3/2)/(a + b*cos(c + d*x))^(5/2), x)

$$3.765 \quad \int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^{5/2}} dx$$

Optimal result	6861
Rubi [A] (verified)	6862
Mathematica [A] (warning: unable to verify)	6864
Maple [B] (warning: unable to verify)	6865
Fricas [F]	6867
Sympy [F(-1)]	6867
Maxima [F]	6867
Giac [F]	6867
Mupad [F(-1)]	6868

Optimal result

Integrand size = 25, antiderivative size = 421

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b \cos(c+dx))^{5/2}} dx = \frac{4b(3a^2 - b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a}{a-b}}}{3a^3(a-b)(a+b)^{3/2} d \sqrt{\sec(c+dx)}} + \frac{2(3a^2 - 3ab - 2b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a}{a-b}}}{3a^2(a-b)(a+b)^{3/2} d \sqrt{\sec(c+dx)}} + \frac{2b^2 \sin(c+dx)}{3a(a^2 - b^2) d (a+b \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} - \frac{4b(3a^2 - b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{3a(a^2 - b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2/3*b^2*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)-4/
3*b*(3*a^2-b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(a+b*cos(d*x+c)
)^(1/2)+4/3*b*(3*a^2-b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)
^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*
x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/(a-b)/(a+b)^(3/2)/d/s
ec(d*x+c)^(1/2)+2/3*(3*a^2-3*a*b-2*b^2)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c)
)^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2
)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(
a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.00,
 number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used
 = {4307, 2881, 3072, 3077, 2895, 3073}

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \frac{2(3a^2 - 3ab - 2b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} \operatorname{Elli}}{3a^2 d(a-b)(a+b)^{3/2} \sqrt{\sec(c+dx)}} + \frac{2b^2 \sin(c+dx)}{3ad(a^2 - b^2) \sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}} - \frac{4b(3a^2 - b^2) \sin(c+dx) \sqrt{\sec(c+dx)}}{3ad(a^2 - b^2)^2 \sqrt{a+b\cos(c+dx)}} + \frac{4b(3a^2 - b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(\sec(c+dx)+1)}{a-b}} E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right) \mid -\frac{a+b}{a-b}\right)}{3a^3 d(a-b)(a+b)^{3/2} \sqrt{\sec(c+dx)}}$$

[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(5/2), x]

[Out] (4*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(3*a^2 - 3*a*b - 2*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^2*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) - (4*b*(3*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2881

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)])*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)} (a + b \cos(c + dx))^{5/2}} dx$$

$$\begin{aligned}
&= \frac{2b^2 \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{1}{2}(3a^2-2b^2)-\frac{3}{2}ab\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx}{3a(a^2-b^2)} \\
&= \frac{2b^2 \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{4b(3a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} \\
&\quad + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{\frac{3a^2b}{2}+\frac{1}{2}b(3a^2-2b^2)+\left(\frac{3ab^2}{2}+\frac{1}{2}a(3a^2-2b^2)\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{3a(a^2-b^2)^2} \\
&= \frac{2b^2 \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{4b(3a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\left((a-b)(3a^2-3ab-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{3a(a^2-b^2)^2} \\
&\quad + \frac{\left(2b(3a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right) \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} dx}{3a(a^2-b^2)^2} \\
&= \frac{4b(3a^2-b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b\sqrt{\cos(c+dx)}}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}} \\
&\quad + \frac{2(3a^2-3ab-2b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b\sqrt{\cos(c+dx)}}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a^2(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}} \\
&\quad + \frac{2b^2 \sin(c+dx)}{3a(a^2-b^2)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} \\
&\quad - \frac{4b(3a^2-b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a(a^2-b^2)^2 d\sqrt{a+b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 11.20 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.12

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx &= \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{4b(3a^2-b^2)\sin(c+dx)}{3a^2(a^2-b^2)^2} - \frac{2b\sin(c+dx)}{3(a^2-b^2)(a+b\cos(c+dx))^2} - \frac{d}{d}\right)}{d} \\
&+ \frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\sec(c+dx)\left(2b(-3a^3-3a^2b+ab^2+b^3)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\right)E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}
\end{aligned}$$

[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(5/2), x]

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*b*(3*a^2 - b^2)*Sin[c + d*x])/
(3*a^2*(a^2 - b^2)^2) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) -
(2*(5*a^2*b*Sin[c + d*x] - b^3*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x])))/d +
(4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(-3*a^3 - 3*a^2*b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)] + a*(3*a^3 + 6*a^2*b + a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
(-a + b)/(a + b)] + b*(-3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2]))/(3*(a^3 - a*b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2840 vs. 2(381) = 762.

Time = 9.68 (sec) , antiderivative size = 2841, normalized size of antiderivative = 6.75

method	result	size
default	Expression too large to display	2841

```
[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)
)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a
+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-3*csc(d*x+c)^2*(-csc(d*x+c)^
2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*
b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)
)/(a+b))^(1/2)*a^5*(1-cos(d*x+c))^2+6*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d
*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d
*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(
1/2))*a^4*b*(1-cos(d*x+c))^2-2*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2
+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2
+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a*
b^4*(1-cos(d*x+c))^2+2*csc(d*x+c)^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)
)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a
+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*b^5*(1-cos
(d*x+c))^2-9*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-c
os(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(co
t(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^4*b-7*(-csc(d*x+c)^2*(1-cos(d*x
+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x
+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/
2))*a^3*b^2+(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-co
s(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot
(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^2*b^3+2*(-csc(d*x+c)^2*(1-cos(d*
```

$$\begin{aligned}
& x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a*b^4+6*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^4*b+12*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^3*b^2+4*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^2*b^3-4*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a*b^4-2*\csc(d*x+c)^5*b^5*(1-\cos(d*x+c))^{5+4*\csc(d*x+c)^3*b^5*(1-\cos(d*x+c))^{3-3*\csc(d*x+c)^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^4*b*(1-\cos(d*x+c))^{2+5*\csc(d*x+c)^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^3*b^2*(1-\cos(d*x+c))^{2+3*\csc(d*x+c)^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^2*b^3*(1-\cos(d*x+c))^{2-6*a^4*b*(\csc(d*x+c)-\cot(d*x+c))+2*a^3*b^2*(\csc(d*x+c)-\cot(d*x+c))+8*a^2*b^3*(\csc(d*x+c)-\cot(d*x+c))-2*a*b^4*(\csc(d*x+c)-\cot(d*x+c))-8*\csc(d*x+c)^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^2*b^3*(1-\cos(d*x+c))^{2-2*\csc(d*x+c)^3*a*b^4*(1-\cos(d*x+c))^{3+6*\csc(d*x+c)^5*a^4*b*(1-\cos(d*x+c))^{5-12*\csc(d*x+c)^5*a^3*b^2*(1-\cos(d*x+c))^{5+4*\csc(d*x+c)^5*a^2*b^3*(1-\cos(d*x+c))^{5+4*\csc(d*x+c)^5*a*b^4*(1-\cos(d*x+c))^{5+10*\csc(d*x+c)^3*a^3*b^2*(1-\cos(d*x+c))^{3-2*b^5*(\csc(d*x+c)-\cot(d*x+c))-12*\csc(d*x+c)^3*a^2*b^3*(1-\cos(d*x+c))^{3-3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * a^5-2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{1/2} * ((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{1/2} * \text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{1/2}) * b^5/(csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})^2/(a-b)^2/(a+b)^2/a^2
\end{aligned}$$

Fricas [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{5/2}} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{5/2}} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{5/2}} dx$$

[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\frac{1}{\cos(c+dx)}}}{(a+b\cos(c+dx))^{5/2}} dx$$

```
[In] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(5/2), x)
```

```
[Out] int((1/cos(c + d*x))^(1/2)/(a + b*cos(c + d*x))^(5/2), x)
```


$$3.766 \quad \int \frac{1}{(a+b \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} dx$$

Optimal result	6869
Rubi [A] (verified)	6870
Mathematica [A] (warning: unable to verify)	6873
Maple [B] (warning: unable to verify)	6873
Fricas [F]	6875
Sympy [F(-1)]	6875
Maxima [F]	6875
Giac [F]	6875
Mupad [F(-1)]	6876

Optimal result

Integrand size = 25, antiderivative size = 399

$$\int \frac{1}{(a+b \cos(c+dx))^{5/2} \sqrt{\sec(c+dx)}} dx =$$

$$\frac{2(3a^2 + b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2} d \sqrt{\sec(c+dx)}} +$$

$$\frac{2(3a-b) \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2} d \sqrt{\sec(c+dx)}} -$$

$$\frac{2b \sin(c+dx)}{3(a^2 - b^2) d (a+b \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} + \frac{2(3a^2 + b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{3(a^2 - b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] -2/3*b*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)+2/3*(
3*a^2+b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)
-2/3*(3*a^2+b^2)*csc(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/co
s(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+
b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)
^(1/2)+2/3*(3*a-b)*csc(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/
cos(d*x+c)^(1/2),((-a-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(
a+b))^(1/2)*(a*(1+sec(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)
^(1/2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4307, 2875, 3072, 3077, 2895, 3073}

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx =$$

$$\frac{2(3a^2 + b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{3a^2 d(a - b)(a + b)^{3/2} \sqrt{\sec(c + dx)}} +$$

$$\frac{2(3a^2 + b^2) \sin(c + dx) \sqrt{\sec(c + dx)}}{3d(a^2 - b^2)^2 \sqrt{a + b \cos(c + dx)}} - \frac{2b \sin(c + dx)}{3d(a^2 - b^2) \sqrt{\sec(c + dx)}(a + b \cos(c + dx))^{3/2}} +$$

$$\frac{2(3a - b) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{3ad(a - b)(a + b)^{3/2} \sqrt{\sec(c + dx)}}$$

[In] Int[1/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]

[Out] (-2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a^2*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(3*a - b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) + (2*(3*a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2875

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli

```
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)])*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)])*(c_.)^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\sqrt{\cos(c + dx)}}{(a + b \cos(c + dx))^{5/2}} dx$$

$$\begin{aligned}
&= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} \\
&\quad - \frac{\left(2\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{\frac{b}{2} - \frac{3}{2}a \cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))^{3/2}} dx}{3(a^2 - b^2)} \\
&= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2(3a^2 + b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{\left(2\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{\frac{3a^2}{2} + \frac{b^2}{2} + 2ab \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3(a^2 - b^2)^2} \\
&= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2(3a^2 + b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}} + \frac{\left((a - b)(3a - b) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3(a^2 - b^2)^2} \\
&\quad + \frac{\left((-3a^2 - b^2) \sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)}\right) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3(a^2 - b^2)^2} \\
&= -\frac{2(3a^2 + b^2) \sqrt{\cos(c + dx)} \csc(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{3a^2(a - b)(a + b)^{3/2} d \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2(3a - b) \sqrt{\cos(c + dx)} \csc(c + dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{3a(a - b)(a + b)^{3/2} d \sqrt{\sec(c + dx)}} \\
&= -\frac{2b \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} \\
&\quad + \frac{2(3a^2 + b^2) \sqrt{\sec(c + dx)} \sin(c + dx)}{3(a^2 - b^2)^2 d \sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 10.78 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \frac{\sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \left(-\frac{2(3a^2 + b^2) \sin(c + dx)}{3a(a^2 - b^2)^2} + \frac{2a}{3(a^2 - b^2)} \right) + \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c + dx)\right) \sec(c + dx)} \left(-2(3a^3 + 3a^2b + ab^2 + b^3) \sqrt{\frac{\cos(c + dx)}{1 + \cos(c + dx)}} \sqrt{\frac{a + b \cos(c + dx)}{(a + b)(1 + \cos(c + dx))}} E(\arcsin(\tan$$

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(3*a^2 + b^2)*Sin[c + d*x])/(3*a*(a^2 - b^2)^2) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (4*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])))/d - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] *Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] *Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2))/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2613 vs. 2(359) = 718.

Time = 7.94 (sec) , antiderivative size = 2614, normalized size of antiderivative = 6.55

method	result	size
default	Expression too large to display	2614

```
[In] int(1/(a+cos(d*x+c)*b)^(5/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1/2)*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-7*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^3*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)-5*EllipticF(cot(d*x+c)-csc(d*x+c),(-a-b)/(a+b))^(1/2))*a^2*b^2*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2
```

$$\begin{aligned}
& *a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}-\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a*b^3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}+6*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^3*b*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}+4*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^2*b^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}+2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a*b^3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}-3*\csc(d*x+c)^2*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^4*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}*(1-\cos(d*x+c))^2+3*\csc(d*x+c)^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^4*(1-\cos(d*x+c))^2+\csc(d*x+c)^5*b^4*(1-\cos(d*x+c))^5-2*\csc(d*x+c)^3*b^4*(1-\cos(d*x+c))^3+2*a^3*b*(\csc(d*x+c)-\cot(d*x+c))+2*a^2*b^2*(\csc(d*x+c)-\cot(d*x+c))+3*\csc(d*x+c)^2*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^2*b^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}*(1-\cos(d*x+c))^2+\csc(d*x+c)^2*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a*b^3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}*(1-\cos(d*x+c))^2-2*\csc(d*x+c)^2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^2*b^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}*(1-\cos(d*x+c))^2-2*a*b^3*(\csc(d*x+c)-\cot(d*x+c))+3*\csc(d*x+c)^5*a^4*(1-\cos(d*x+c))^5-6*\csc(d*x+c)^5*a^3*b*(1-\cos(d*x+c))^5+4*\csc(d*x+c)^3*a*b^3*(1-\cos(d*x+c))^3+4*\csc(d*x+c)^5*a^2*b^2*(1-\cos(d*x+c))^5-2*\csc(d*x+c)^5*a*b^3*(1-\cos(d*x+c))^5+4*\csc(d*x+c)^3*a^3*b*(1-\cos(d*x+c))^3-6*\csc(d*x+c)^3*a^2*b^2*(1-\cos(d*x+c))^3-3*\text{EllipticF}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^4*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}+*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*b^4*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}-\csc(d*x+c)^2*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*b^4*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}*(1-\cos(d*x+c))^2+3*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)/(a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-a-b)/(a+b))^{(1/2)})*a^4-3*a^4*(\csc(d*x+c)-\cot(d*x+c))+b^4*(\csc(d*x+c)-\cot(d*x+c)))/(-(\csc(d*x+c)^2*(1-\cos(d*x+c))^2+1)/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2-1))^{(1/2)}/(\csc(d*x+c)^2*(1-\cos(d*x+c))^2-1)/(\csc(d*x+c)^2*a*(1-\cos(d*x+c))^2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^2+a+b)^2/(a-b)^2/(a+b)^2/a
\end{aligned}$$

Fricas [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{5/2} \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sqrt(sec(d*x + c))), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{5/2} \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{(b \cos(dx + c) + a)^{5/2} \sqrt{\sec(dx + c)}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sqrt{\sec(c + dx)}} dx = \int \frac{1}{\sqrt{\frac{1}{\cos(c+dx)}} (a + b \cos(c + dx))^{5/2}} dx$$

```
[In] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/((1/cos(c + d*x))^(1/2)*(a + b*cos(c + d*x))^(5/2)), x)
```


$$3.767 \quad \int \frac{1}{(a+b \cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	6877
Rubi [A] (verified)	6878
Mathematica [A] (verified)	6880
Maple [B] (warning: unable to verify)	6881
Fricas [F]	6882
Sympy [F(-1)]	6882
Maxima [F]	6883
Giac [F]	6883
Mupad [F(-1)]	6883

Optimal result

Integrand size = 25, antiderivative size = 382

$$\int \frac{1}{(a+b \cos(c+dx))^{5/2} \sec^{\frac{3}{2}}(c+dx)} dx = \frac{8b\sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right) \middle| -\frac{a+b}{a-b}\right)}{3a(a-b)(a+b)^{3/2} d \sqrt{\sec(c+dx)}} + \frac{2(a-3b)\sqrt{\cos(c+dx)} \csc(c+dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2} d \sqrt{\sec(c+dx)}} + \frac{2a \sin(c+dx)}{3(a^2-b^2) d (a+b \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} - \frac{8ab \sqrt{\sec(c+dx)} \sin(c+dx)}{3(a^2-b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

```
[Out] 2/3*a*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2)-8/3*a*
b*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)^2/d/(a+b*cos(d*x+c))^(1/2)+8/3*b*cs
c(d*x+c)*EllipticE((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)+2/3*(a-3*b)*cs
c(d*x+c)*EllipticF((a+b*cos(d*x+c))^(1/2)/(a+b)^(1/2)/cos(d*x+c)^(1/2),((-a
-b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*(a*(1-sec(d*x+c))/(a+b))^(1/2)*(a*(1+sec
(d*x+c))/(a-b))^(1/2)/a/(a-b)/(a+b)^(3/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00,
 number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used
 = {4307, 2878, 3072, 3077, 2895, 3073}

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = -\frac{8ab \sin(c + dx) \sqrt{\sec(c + dx)}}{3d(a^2 - b^2)^2 \sqrt{a + b \cos(c + dx)}} + \frac{2a \sin(c + dx)}{3d(a^2 - b^2) \sqrt{\sec(c + dx)} (a + b \cos(c + dx))^{3/2}} + \frac{2(a - 3b) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right), -\frac{a + b}{a - b}\right)}{3ad(a - b)(a + b)^{3/2} \sqrt{\sec(c + dx)}} + \frac{8b \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \middle| -\frac{a + b}{a - b}\right)}{3ad(a - b)(a + b)^{3/2} \sqrt{\sec(c + dx)}}$$

[In] Int[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)), x]

[Out] (8*b*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a - 3*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*a*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) - (8*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])

Rule 2878

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(- (b*c - a*d))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n - 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] & & NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && LtQ[1, n, 2] && IntegersQ[2*m, 2*n]

Rule 2895

Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr

```
t[a*((1 - Csc[e + f*x])/(a + b))*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)])*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 4307

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\cos(c + dx)} \sqrt{\sec(c + dx)} \right) \int \frac{\cos^{\frac{3}{2}}(c + dx)}{(a + b \cos(c + dx))^{5/2}} dx$$

$$\begin{aligned}
&= \frac{2a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} \\
&\quad - \frac{\left(2\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{-\frac{a}{2} + \frac{3}{2}b \cos(c + dx)}{\sqrt{\cos(c + dx)}(a + b \cos(c + dx))^{3/2}} dx}{3(a^2 - b^2)} \\
&= \frac{2a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} - \frac{8ab\sqrt{\sec(c + dx)} \sin(c + dx)}{3(a^2 - b^2)^2 d\sqrt{a + b \cos(c + dx)}} \\
&\quad - \frac{\left(2\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{-2ab + \left(-\frac{a^2}{2} - \frac{3b^2}{2}\right) \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3(a^2 - b^2)^2} \\
&= \frac{2a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} - \frac{8ab\sqrt{\sec(c + dx)} \sin(c + dx)}{3(a^2 - b^2)^2 d\sqrt{a + b \cos(c + dx)}} \\
&\quad + \frac{\left((a - 3b)(a - b)\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}\sqrt{a + b \cos(c + dx)}} dx}{3(a^2 - b^2)^2} \\
&\quad + \frac{\left(4ab\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right) \int \frac{1 + \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{a + b \cos(c + dx)}} dx}{3(a^2 - b^2)^2} \\
&= \frac{8b\sqrt{\cos(c + dx)} \csc(c + dx) E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{3a(a - b)(a + b)^{3/2} d\sqrt{\sec(c + dx)}} \\
&\quad + \frac{2(a - 3b)\sqrt{\cos(c + dx)} \csc(c + dx) \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b \cos(c + dx)}}\right), -\frac{a + b}{a - b}\right) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(1 + \sec(c + dx))}{a - b}}}{3a(a - b)(a + b)^{3/2} d\sqrt{\sec(c + dx)}} \\
&\quad + \frac{2a \sin(c + dx)}{3(a^2 - b^2) d(a + b \cos(c + dx))^{3/2} \sqrt{\sec(c + dx)}} \\
&\quad - \frac{8ab\sqrt{\sec(c + dx)} \sin(c + dx)}{3(a^2 - b^2)^2 d\sqrt{a + b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.65 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{\frac{3}{2}}(c + dx)} dx = \frac{2\sqrt{\sec(c + dx)} \left(a^2(a^2 - b^2) \sin(c + dx) - a(a^2 - 5b^2) (a + b \cos(c + dx)) \sin(c + dx) - 4b^2(a + b \cos(c + dx)) \right)}{\dots}$$

[In] Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]

```
[Out] (-2*Sqrt[Sec[c + d*x]]*(a^2*(a^2 - b^2)*Sin[c + d*x] - a*(a^2 - 5*b^2)*(a +
b*Cos[c + d*x])*Sin[c + d*x] - 4*b^2*(a + b*Cos[c + d*x])^2*Ssin[c + d*x] +
2*b*Cos[(c + d*x)/2]^2*(a + b*Cos[c + d*x])*(4*b*(a + b)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (a^2 + 4*a*b + 3*
b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] - b*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(
c + d*x)/2])))/(3*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2128 vs. 2(342) = 684.

Time = 8.40 (sec) , antiderivative size = 2129, normalized size of antiderivative = 5.57

method	result	size
default	Expression too large to display	2129

```
[In] int(1/(a+cos(d*x+c)*b)^(5/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^2*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2
-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1))^(1
/2)*(-csc(d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a^
3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^
2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-3*csc(
d*x+c)^2*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a^2*b*(-csc(
d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*
x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+csc(d*x+c)^2*E
llipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a*b^2*(-csc(d*x+c)^2*(
1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(
1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*(1-cos(d*x+c))^2+3*csc(d*x+c)^2*(-csc(d*x
+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c
)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-
a-b)/(a+b))^(1/2))*b^3*(1-cos(d*x+c))^2+4*csc(d*x+c)^2*EllipticE(cot(d*x+c
)-csc(d*x+c), (-a-b)/(a+b))^(1/2))*a^2*b*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)
^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+
b)/(a+b))^(1/2)*(1-cos(d*x+c))^2-4*csc(d*x+c)^2*EllipticE(cot(d*x+c)-csc(d*
x+c), (-a-b)/(a+b))^(1/2))*b^3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((c
sc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(
1/2)*(1-cos(d*x+c))^2+4*csc(d*x+c)^5*a^2*b*(1-cos(d*x+c))^5-8*csc(d*x+c)^5
*a*b^2*(1-cos(d*x+c))^5+4*csc(d*x+c)^5*b^3*(1-cos(d*x+c))^5-(-csc(d*x+c)^2*
(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*
(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c), (-a-b)/
(a+b))^(1/2))*a^3-5*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2
*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*Ellip
```

```

ticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b-7*(-csc(d*x+c)^2*(1-
cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-
cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b^2-3*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*
a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*Ellipt
icF(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3+4*(-csc(d*x+c)^2*(1-cos
(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos
(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b+8*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(
1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE
(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2+4*(-csc(d*x+c)^2*(1-cos(
d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*a*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(
d*x+c))^2+a+b)/(a+b))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),(-(a-b)/(a+b))^(
1/2))*b^3-2*csc(d*x+c)^3*a^3*(1-cos(d*x+c))^3+10*csc(d*x+c)^3*a*b^2*(1-cos
(d*x+c))^3-8*csc(d*x+c)^3*b^3*(1-cos(d*x+c))^3+2*a^3*(csc(d*x+c)-cot(d*x+c)
)-4*a^2*b*(csc(d*x+c)-cot(d*x+c))-2*a*b^2*(csc(d*x+c)-cot(d*x+c))+4*b^3*(cs
c(d*x+c)-cot(d*x+c))/(-(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-
cos(d*x+c))^2-1))^(3/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^2/(csc(d*x+c)^2*a
*(1-cos(d*x+c))^2-csc(d*x+c)^2*b*(1-cos(d*x+c))^2+a+b)^2/(a-b)^2/(a+b)^2

```

Fricas [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{3/2}} dx$$

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x +
c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{3/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{3/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{3/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{3/2} (a + b \cos(c + dx))^{5/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)),x)

[Out] int(1/((1/cos(c + d*x))^(3/2)*(a + b*cos(c + d*x))^(5/2)), x)

$$3.768 \quad \int \frac{1}{(a+b \cos(c+dx))^{5/2} \sec^2(c+dx)} dx$$

Optimal result	6884
Rubi [A] (verified)	6885
Mathematica [B] (verified)	6889
Maple [B] (warning: unable to verify)	6890
Fricas [F]	6892
Sympy [F(-1)]	6892
Maxima [F]	6892
Giac [F]	6893
Mupad [F(-1)]	6893

Optimal result

Integrand size = 25, antiderivative size = 557

$$\int \frac{1}{(a+b \cos(c+dx))^{5/2} \sec^2(c+dx)} dx = \frac{2(3a^2 - 7b^2) \sqrt{\cos(c+dx)} \csc(c+dx) E\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right)\right)}{3(a-b)b^2(a+b)^{3/2} d \sqrt{\sec(c+dx)}} - \frac{2(3a^2 + ab - 6b^2) \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{3/2} d \sqrt{\sec(c+dx)}} - \frac{2\sqrt{a+b} \sqrt{\cos(c+dx)} \csc(c+dx) \text{EllipticPi}\left(\frac{a+b}{b}, \arcsin\left(\frac{\sqrt{a+b \cos(c+dx)}}{\sqrt{a+b} \sqrt{\cos(c+dx)}}\right), -\frac{a+b}{a-b}\right) \sqrt{\frac{a(1-\sec(c+dx))}{a+b}} \sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3 d \sqrt{\sec(c+dx)}} - \frac{2a^2 \sin(c+dx)}{3b(a^2 - b^2) d(a+b \cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} - \frac{2a^2(3a^2 - 7b^2) \sqrt{\sec(c+dx)} \sin(c+dx)}{3b^2(a^2 - b^2)^2 d \sqrt{a+b \cos(c+dx)}}$$

[Out] $-2/3*a^2*\sin(d*x+c)/b/(a^2-b^2)/d/(a+b*\cos(d*x+c))^{3/2}/\sec(d*x+c)^{(1/2)}-2/3*a^2*(3*a^2-7*b^2)*\sin(d*x+c)*\sec(d*x+c)^{(1/2)}/b^2/(a^2-b^2)^2/d/(a+b*\cos(d*x+c))^{(1/2)}+2/3*(3*a^2-7*b^2)*\csc(d*x+c)*\text{EllipticE}((a+b*\cos(d*x+c))^{(1/2)})/(a+b)^{(1/2)}/\cos(d*x+c)^{(1/2)},((-a-b)/(a-b))^{(1/2)}*\cos(d*x+c)^{(1/2)}*(a*(1-\sec(d*x+c)))/(a+b)^{(1/2)}*(a*(1+\sec(d*x+c)))/(a-b))^{(1/2)}/(a-b)/b^2/(a+b)^{(3/2)}/d/\sec(d*x+c)^{(1/2)}-2/3*(3*a^2+a*b-6*b^2)*\csc(d*x+c)*\text{EllipticF}((a+b*\cos(d*x+c))^{(1/2)})/(a+b)^{(1/2)}/\cos(d*x+c)^{(1/2)},((-a-b)/(a-b))^{(1/2)}*\cos(d*x+c)^{(1/2)}*(a*(1-\sec(d*x+c)))/(a+b)^{(1/2)}*(a*(1+\sec(d*x+c)))/(a-b))^{(1/2)}/(a-b)/b^2/(a+b)^{(3/2)}/d/\sec(d*x+c)^{(1/2)}-2*\csc(d*x+c)*\text{EllipticPi}((a+b*\cos(d*x+c))^{(1/2)})/(a+b)^{(1/2)}/\cos(d*x+c)^{(1/2)},(a+b)/b,((-a-b)/(a-b))^{(1/2)}*(a+b)^{(1/2)}*\cos(d*x+c)^{(1/2)}*(a*(1-\sec(d*x+c)))/(a+b)^{(1/2)}*(a*(1+\sec(d*x+c)))/(a-b))^{(1/2)}/b^3/d/\sec(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4307, 2871, 3130, 2888, 3072, 3077, 2895, 3073}

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx =$$

$$\frac{2(3a^2 + ab - 6b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{3b^2 d (a - b) (a + b)^{3/2} \sqrt{\sec(c + dx)}} +$$

$$\frac{2(3a^2 - 7b^2) \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} E\left(\arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right) \mid -\frac{a + b}{a - b}\right)}{3b^2 d (a - b) (a + b)^{3/2} \sqrt{\sec(c + dx)}} -$$

$$\frac{2a^2(3a^2 - 7b^2) \sin(c + dx) \sqrt{\sec(c + dx)}}{3b^2 d (a^2 - b^2)^2 \sqrt{a + b \cos(c + dx)}} -$$

$$\frac{2a^2 \sin(c + dx)}{3bd (a^2 - b^2) \sqrt{\sec(c + dx)} (a + b \cos(c + dx))^{3/2}} -$$

$$\frac{2\sqrt{a + b} \sqrt{\cos(c + dx)} \csc(c + dx) \sqrt{\frac{a(1 - \sec(c + dx))}{a + b}} \sqrt{\frac{a(\sec(c + dx) + 1)}{a - b}} \text{EllipticPi}\left(\frac{a + b}{b}, \arcsin\left(\frac{\sqrt{a + b \cos(c + dx)}}{\sqrt{a + b} \sqrt{\cos(c + dx)}}\right)\right)}{b^3 d \sqrt{\sec(c + dx)}}$$

[In] Int[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]

[Out] (2*(3*a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*a^2 + a*b - 6*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(3*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*a^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) - (2*a^2*(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])]

Rule 2871

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[

```
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
+ a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 2888

```
Int[Sqrt[(b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)
*(x_)]], x_Symbol] := Simp[2*b*(Tan[e + f*x]/(d*f))*Rt[(c + d)/b, 2]*Sqrt[c
*((1 + Csc[e + f*x])/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*Ellipti
cPi[(c + d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c +
d)/b, 2]], -(c + d)/(c - d)], x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c^2 -
d^2, 0] && PosQ[(c + d)/b]
```

Rule 2895

```
Int[1/(Sqrt[(d_)*sin[(e_) + (f_)*(x_)]]*Sqrt[(a_) + (b_)*sin[(e_) + (f
_)*(x_)]]), x_Symbol] := Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 3072

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(d_)*sin[(e_) + (f_)*(
x_)]]*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)), x_Symbol] := Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((b_)*sin[(e_) + (f_)*(x_)])
^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

```

Rule 3130

```

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] := Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]

```

Rule 4307

```

Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] := Dist[(c*Csc[a + b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}} dx \\
&= -\frac{2a^2 \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
&\quad - \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{a^2}{2} - \frac{3}{2}ab\cos(c+dx) - \frac{3}{2}(a^2-b^2)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx}{3b(a^2-b^2)} \\
&= -\frac{2a^2 \sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2} \sqrt{\sec(c+dx)}} \\
&\quad + \frac{\left(\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}} dx}{b^2} \\
&\quad - \frac{\left(2\sqrt{\cos(c+dx)} \sqrt{\sec(c+dx)} \right) \int \frac{\frac{a^2b}{2} + \left(-\frac{3ab^2}{2} + \frac{3}{2}a(a^2-b^2) \right) \cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} dx}{3b^2(a^2-b^2)}
\end{aligned}$$

=

$$\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^3d\sqrt{\sec(c+dx)}}$$

$$\frac{2a^2\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{2a^2(3a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{\frac{a^2b^2}{2}-a\left(-\frac{3ab^2}{2}+\frac{3}{2}a(a^2-b^2)\right)+\left(\frac{a^3b}{2}-b\left(-\frac{3ab^2}{2}+\frac{3}{2}a(a^2-b^2)\right)\right)\cos(c+dx}}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3b^2(a^2-b^2)^2}$$

=

$$\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^3d\sqrt{\sec(c+dx)}}$$

$$\frac{2a^2\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{2a^2(3a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}} + \frac{\left(a^2(3a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3b^2(a^2-b^2)^2} + \frac{\left(a(a-b)(3a^2+ab-6b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx}{3b^2(a^2-b^2)^2}$$

=

$$\frac{2(3a^2-7b^2)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$\frac{2(3a^2+ab-6b^2)\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3(a-b)b^2(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

$$\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{b},\arcsin\left(\frac{\sqrt{a+b}\cos(c+dx)}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right),-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{b^3d\sqrt{\sec(c+dx)}}$$

$$\frac{2a^2\sin(c+dx)}{3b(a^2-b^2)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{2a^2(3a^2-7b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1335 vs. $2(557) = 1114$.

Time = 12.43 (sec) , antiderivative size = 1335, normalized size of antiderivative = 2.40

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx = \frac{\sqrt{a + b \cos(c + dx)} \sqrt{\sec(c + dx)} \left(\frac{2a(3a^2 - 7b^2) \sin(c + dx)}{3b^2(a^2 - b^2)^2} - \frac{2a}{3b^2(-a^2 + b^2)} \right)}{d} + \frac{2 \sqrt{\frac{a + b + a \tan^2(\frac{1}{2}(c + dx)) - b \tan^2(\frac{1}{2}(c + dx))}{1 + \tan^2(\frac{1}{2}(c + dx))}} \left(-3a^4 \tan\left(\frac{1}{2}(c + dx)\right) - 3a^3 b \tan\left(\frac{1}{2}(c + dx)\right) + 7a^2 b^2 \tan\left(\frac{1}{2}(c + dx)\right) \right)}{d}$$

[In] Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]

[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(3*a^2 - 7*b^2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2) - (2*a^3*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) - (8*(a^4*Sin[c + d*x] - 2*a^2*b^2*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x])))/d + (2*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-3*a^4*Tan[(c + d*x)/2] - 3*a^3*b*Tan[(c + d*x)/2] + 7*a^2*b^2*Tan[(c + d*x)/2] + 7*a*b^3*Tan[(c + d*x)/2] + 6*a^3*b*Tan[(c + d*x)/2]^3 - 14*a*b^3*Tan[(c + d*x)/2]^3 + 3*a^4*Tan[(c + d*x)/2]^5 - 3*a^3*b*Tan[(c + d*x)/2]^5 - 7*a^2*b^2*Tan[(c + d*x)/2]^5 + 7*a*b^3*Tan[(c + d*x)/2]^5 + 6*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a^2*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*b^4*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - a*(3*a^3 + 3*a^2*b - 7*a*b^2 - 7*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - b*(-2*a^3 + a^2*b + 6*a*b^2 + 3*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)))/(3*b^2*(a^2 - b^2)^2*d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 + Tan[(c + d*x)/2]^2)))

$$\begin{aligned}
& t(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * a^2 * b^3 - 7 * (-\csc(d*x+c))^{2 * (1 - \cos(d \\
& *x+c))^{2+1}}^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d \\
& *x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} \\
&)^{(1/2)} * a * b^4 + 3 * \csc(d*x+c)^2 * (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d \\
& *x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} \\
&) * \text{EllipticE}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * a^5 * (1 - \cos(d*x+c))^{2+6} * \csc(d*x+c)^2 * \\
& (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{Elliptic} \\
& \text{Pi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)} * a^4 * b * (1 - \cos(d*x+c))^{2+12} * \csc(d*x+c)^2 * \\
& (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticPi} \\
& (\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)} * a^3 * b^2 * (1 - \cos(d*x+c))^{2-12} * \csc(d*x+c)^2 * \\
& (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticPi}(c \\
& \cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)} * a^2 * b^3 * (1 - \cos(d*x+c))^{2-6} * \csc \\
& (d*x+c)^2 * (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticPi}(\cot \\
& (d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)} * a * b^4 * (1 - \cos(d*x+c))^{2-2} * \csc(d* \\
& x+c)^2 * (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x \\
& +c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticF}(\cot(d*x+ \\
& c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * a^4 * b * (1 - \cos(d*x+c))^{2+3} * \csc(d*x+c)^2 * \\
& (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc \\
& (d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticF}(\cot(d*x+c) - \csc(d \\
& *x+c), (-a-b)/(a+b))^{(1/2)} * a^3 * b^2 * (1 - \cos(d*x+c))^{2+5} * \csc(d*x+c)^2 * (-\csc(d \\
& *x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x \\
& +c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), \\
& (-a-b)/(a+b))^{(1/2)} * a^2 * b^3 * (1 - \cos(d*x+c))^{2-2} * a^4 * b * (\csc(d*x+c) - \cot(d*x+ \\
& c)) + 10 * a^3 * b^2 * (\csc(d*x+c) - \cot(d*x+c)) + 2 * a^2 * b^3 * (\csc(d*x+c) - \cot(d*x+c)) - 7 * \\
& a * b^4 * (\csc(d*x+c) - \cot(d*x+c)) + 3 * \csc(d*x+c)^5 * a^5 * (1 - \cos(d*x+c))^{5-3} * a^5 * (\csc \\
& (d*x+c) - \cot(d*x+c)) - 6 * \csc(d*x+c)^2 * (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} \\
&) * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a \\
& +b))^{(1/2)} * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)} * a^5 * (1 \\
& - \cos(d*x+c))^{2+6} * \csc(d*x+c)^2 * (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc \\
& (d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)} * b^5 * (1 - \cos(d \\
& *x+c))^{2+14} * \csc(d*x+c)^3 * a * b^4 * (1 - \cos(d*x+c))^{3+3} * (-\csc(d*x+c)^2 * (1 - \cos(d*x \\
& +c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x \\
& +c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticF}(\cot(d*x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} \\
&) * b^5 + 3 * (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(\\
& d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticE}(\cot(d \\
& *x+c) - \csc(d*x+c), (-a-b)/(a+b))^{(1/2)} * a^5 - 6 * (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticPi}(\cot(d*x+c) - \csc(d*x+c), -1, (-a-b)/(a+b))^{(1/2)} * a^5 - 6 * (-\csc(d*x+c)^2 * (1 - \cos(d*x+c))^{2+1})^{(1/2)} * ((\csc(d*x+c)^2 * a * (1 - \cos(d*x+c))^{2 - \csc(d*x+c)^2 * b * (1 - \cos(d*x+c))^{2+a+b}} / (a+b))^{(1/2)} * \text{EllipticPi}(\cot(d
\end{aligned}$$

$x+c)-\csc(d*x+c), -1, (-(a-b)/(a+b))^{(1/2)}*b^5-6*\csc(d*x+c)^5*a^4*b*(1-\cos(d*x+c))^{5-4*\csc(d*x+c)^5*a^3*b^2*(1-\cos(d*x+c))^{5+14*\csc(d*x+c)^5*a^2*b^3*(1-\cos(d*x+c))^{5-7*\csc(d*x+c)^5*a*b^4*(1-\cos(d*x+c))^{5+8*\csc(d*x+c)^3*a^4*b*(1-\cos(d*x+c))^{3-6*\csc(d*x+c)^3*a^3*b^2*(1-\cos(d*x+c))^{3-10*\csc(d*x+c)^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-(a-b)/(a+b))^{(1/2)})*a^3*b^2*(1-\cos(d*x+c))^{2+7*\csc(d*x+c)^2*(-\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})^{(1/2)}*((\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})/(a+b))^{(1/2)}*\text{EllipticE}(\cot(d*x+c)-\csc(d*x+c), (-(a-b)/(a+b))^{(1/2)})*a*b^4*(1-\cos(d*x+c))^{2-16*\csc(d*x+c)^3*a^2*b^3*(1-\cos(d*x+c))^{3-10*\csc(d*x+c)^2*(1-\cos(d*x+c))^{2+1})/(a+b))^{(1/2)}*((\csc(d*x+c)^2*(1-\cos(d*x+c))^{2-1})^{(5/2)}/(\csc(d*x+c)^2*(1-\cos(d*x+c))^{2-1})^{3/2}/(\csc(d*x+c)^2*a*(1-\cos(d*x+c))^{2-\csc(d*x+c)^2*b*(1-\cos(d*x+c))^{2+a+b})^{2/2}/(a-b)^{2/2}/(a+b)^{2/b^2}$

Fricas [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{5/2}(c+dx)} dx = \int \frac{1}{(b\cos(dx+c)+a)^{5/2}\sec^{5/2}(dx+c)} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(5/2)), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{5/2}(c+dx)} dx = \text{Timed out}$$

[In] integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{5/2}(c+dx)} dx = \int \frac{1}{(b\cos(dx+c)+a)^{5/2}\sec^{5/2}(dx+c)} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)

Giac [F]

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{(b \cos(dx + c) + a)^{5/2} \sec(dx + c)^{5/2}} dx$$

[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \cos(c + dx))^{5/2} \sec^{5/2}(c + dx)} dx = \int \frac{1}{\left(\frac{1}{\cos(c+dx)}\right)^{5/2} (a + b \cos(c + dx))^{5/2}} dx$$

[In] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)),x)

[Out] int(1/((1/cos(c + d*x))^(5/2)*(a + b*cos(c + d*x))^(5/2)), x)

3.769 $\int \cos^m(c + dx)(a + b \cos(c + dx))^4 dx$

Optimal result	6894
Rubi [A] (verified)	6895
Mathematica [A] (verified)	6897
Maple [F]	6898
Fricas [F]	6898
Sympy [F(-1)]	6898
Maxima [F]	6899
Giac [F]	6899
Mupad [F(-1)]	6899

Optimal result

Integrand size = 21, antiderivative size = 330

$$\begin{aligned}
 & \int \cos^m(c + dx)(a + b \cos(c + dx))^4 dx \\
 = & \frac{b^2(b^2(3 + m) + a^2(22 + 5m)) \cos^{1+m}(c + dx) \sin(c + dx)}{d(2 + m)(4 + m)} \\
 & + \frac{2ab^3(5 + m) \cos^{2+m}(c + dx) \sin(c + dx)}{d(3 + m)(4 + m)} \\
 & + \frac{b^2 \cos^{1+m}(c + dx)(a + b \cos(c + dx))^2 \sin(c + dx)}{d(4 + m)} \\
 & - \frac{(b^4(3 + 4m + m^2) + 6a^2b^2(4 + 5m + m^2) + a^4(8 + 6m + m^2)) \cos^{1+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \cos^2(c + dx)\right)}{d(1 + m)(2 + m)(4 + m)\sqrt{\sin^2(c + dx)}} \\
 & - \frac{4ab(b^2(2 + m) + a^2(3 + m)) \cos^{2+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 + m)(3 + m)\sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

```

[Out] b^2*(b^2*(3+m)+a^2*(22+5*m))*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)/(4+m)+2*a*
b^3*(5+m)*cos(d*x+c)^(2+m)*sin(d*x+c)/d/(3+m)/(4+m)+b^2*cos(d*x+c)^(1+m)*(a
+b*cos(d*x+c))^2*sin(d*x+c)/d/(4+m)-(b^4*(m^2+4*m+3)+6*a^2*b^2*(m^2+5*m+4)+
a^4*(m^2+6*m+8))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m], [3/2+1/2*m], co
s(d*x+c)^2)*sin(d*x+c)/d/(4+m)/(m^2+3*m+2)/(sin(d*x+c)^2)^(1/2)-4*a*b*(b^2*
(2+m)+a^2*(3+m))*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m], [2+1/2*m], cos(d*
x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m)/(sin(d*x+c)^2)^(1/2)

```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00,
 number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used
 = {2872, 3112, 3102, 2827, 2722}

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^4 dx =$$

$$\frac{4ab(a^2(m+3) + b^2(m+2)) \sin(c + dx) \cos^{m+2}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \cos^2(c + dx)\right)}{d(m+2)(m+3)\sqrt{\sin^2(c + dx)}} +$$

$$\frac{b^2(a^2(5m+22) + b^2(m+3)) \sin(c + dx) \cos^{m+1}(c + dx)}{d(m+2)(m+4)} -$$

$$\frac{(a^4(m^2 + 6m + 8) + 6a^2b^2(m^2 + 5m + 4) + b^4(m^2 + 4m + 3)) \sin(c + dx) \cos^{m+1}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \cos^2(c + dx)\right)}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c + dx)}} +$$

$$\frac{2ab^3(m+5) \sin(c + dx) \cos^{m+2}(c + dx)}{d(m+3)(m+4)} +$$

$$\frac{b^2 \sin(c + dx) \cos^{m+1}(c + dx)(a + b \cos(c + dx))^2}{d(m+4)}$$

[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^4,x]

[Out] (b^2*(b^2*(3 + m) + a^2*(22 + 5*m))*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(4 + m)) + (2*a*b^3*(5 + m)*Cos[c + d*x]^(2 + m)*Sin[c + d*x])/(d*(3 + m)*(4 + m)) + (b^2*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])^2*SIN[c + d*x])/(d*(4 + m)) - ((b^4*(3 + 4*m + m^2) + 6*a^2*b^2*(4 + 5*m + m^2) + a^4*(8 + 6*m + m^2))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[SIN[c + d*x]^2]) - (4*a*b*(b^2*(2 + m) + a^2*(3 + m))*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[SIN[c + d*x]^2])

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*SIN[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2872

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))

```

Rule 3102

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]

```

Rule 3112

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d))*(m + 3))*Sin[e + f*x]^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \cos^{1+m}(c + dx)(a + b \cos(c + dx))^2 \sin(c + dx)}{d(4 + m)} \\
&+ \frac{\int \cos^m(c + dx)(a + b \cos(c + dx))(a(b^2(1 + m) + a^2(4 + m)) + b(b^2(3 + m) + 3a^2(4 + m)) \cos(c + dx) - a^2 \cos^2(c + dx)) dx}{4 + m} \\
&= \frac{2ab^3(5 + m) \cos^{2+m}(c + dx) \sin(c + dx)}{d(3 + m)(4 + m)} \\
&+ \frac{b^2 \cos^{1+m}(c + dx)(a + b \cos(c + dx))^2 \sin(c + dx)}{d(4 + m)} \\
&+ \frac{\int \cos^m(c + dx)(a^2(3 + m)(b^2(1 + m) + a^2(4 + m)) + 4ab(4 + m)(b^2(2 + m) + a^2(3 + m)) \cos(c + dx) - a^2 \cos^2(c + dx)) dx}{12 + 7m + m^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{b^2(b^2(3+m) + a^2(22+5m)) \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)(4+m)} \\
&+ \frac{2ab^3(5+m) \cos^{2+m}(c+dx) \sin(c+dx)}{d(3+m)(4+m)} \\
&+ \frac{b^2 \cos^{1+m}(c+dx)(a+b \cos(c+dx))^2 \sin(c+dx)}{d(4+m)} \\
&+ \frac{\int \cos^m(c+dx) ((3+m)(b^4(3+4m+m^2) + 6a^2b^2(4+5m+m^2) + a^4(8+6m+m^2)) + 4ab(24+26m+9m^2+m^3))}{24+26m+9m^2+m^3} \\
&= \frac{b^2(b^2(3+m) + a^2(22+5m)) \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)(4+m)} \\
&+ \frac{2ab^3(5+m) \cos^{2+m}(c+dx) \sin(c+dx)}{d(3+m)(4+m)} \\
&+ \frac{b^2 \cos^{1+m}(c+dx)(a+b \cos(c+dx))^2 \sin(c+dx)}{d(4+m)} \\
&+ \left(4ab \left(a^2 + \frac{b^2(2+m)}{3+m} \right) \right) \int \cos^{1+m}(c+dx) dx \\
&+ \frac{(b^4(3+4m+m^2) + 6a^2b^2(4+5m+m^2) + a^4(8+6m+m^2)) \int \cos^m(c+dx) dx}{8+6m+m^2} \\
&= \frac{b^2(b^2(3+m) + a^2(22+5m)) \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)(4+m)} \\
&+ \frac{2ab^3(5+m) \cos^{2+m}(c+dx) \sin(c+dx)}{d(3+m)(4+m)} \\
&+ \frac{b^2 \cos^{1+m}(c+dx)(a+b \cos(c+dx))^2 \sin(c+dx)}{d(4+m)} \\
&- \frac{(b^4(3+4m+m^2) + 6a^2b^2(4+5m+m^2) + a^4(8+6m+m^2)) \cos^{1+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c+dx)\right) \sqrt{\sin^2(c+dx)}}{d(1+m)(8+6m+m^2)} \\
&- \frac{4ab \left(a^2 + \frac{b^2(2+m)}{3+m} \right) \cos^{2+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c+dx)\right) \sin(c+dx)}{d(2+m)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.73

$$\begin{aligned}
&\int \cos^m(c+dx)(a+b \cos(c+dx))^4 dx \\
&= \frac{\cos^{1+m}(c+dx) \csc(c+dx) \left(-\frac{a^4 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right)}{1+m} + b \cos(c+dx) \left(-\frac{4a^3 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c+dx)\right)}{2} \right) \right)}{d}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^4,x]

```
[Out] (Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-(a^4*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2)]/(1 + m)) + b*Cos[c + d*x]*((-4*a^3*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2)]/(2 + m) + b*Cos[c + d*x]*((-6*a^2*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2)]/(3 + m) + b*Cos[c + d*x]*((-4*a*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[c + d*x]^2)]/(4 + m) - (b*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, Cos[c + d*x]^2)]/(5 + m))))*Sqrt[Sin[c + d*x]^2])/d
```

Maple [F]

$$\int (\cos^m(dx + c))(a + \cos(dx + c)b)^4 dx$$

```
[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^4,x)
```

```
[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^4,x)
```

Fricas [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^4 dx = \int (b \cos(dx + c) + a)^4 \cos(dx + c)^m dx$$

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + 4*a*b^3*cos(d*x + c)^3 + 6*a^2*b^2*cos(d*x + c)^2 + 4*a^3*b*cos(d*x + c) + a^4)*cos(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^4 dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^4 dx = \int (b \cos(dx + c) + a)^4 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)

Giac [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^4 dx = \int (b \cos(dx + c) + a)^4 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^4 dx = \int \cos(c + dx)^m (a + b \cos(c + dx))^4 dx$$

[In] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^4,x)

[Out] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^4, x)

3.770 $\int \cos^m(c + dx)(a + b \cos(c + dx))^3 dx$

Optimal result	6900
Rubi [A] (verified)	6900
Mathematica [A] (verified)	6903
Maple [F]	6903
Fricas [F]	6903
Sympy [F(-1)]	6904
Maxima [F]	6904
Giac [F]	6904
Mupad [F(-1)]	6904

Optimal result

Integrand size = 21, antiderivative size = 250

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^3 dx = \frac{ab^2(7 + 2m) \cos^{1+m}(c + dx) \sin(c + dx)}{d(2 + m)(3 + m)} + \frac{b^2 \cos^{1+m}(c + dx)(a + b \cos(c + dx)) \sin(c + dx)}{d(3 + m)} - \frac{a(3b^2(1 + m) + a^2(2 + m)) \cos^{1+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1 + m)(2 + m) \sqrt{\sin^2(c + dx)}} - \frac{b(b^2(2 + m) + 3a^2(3 + m)) \cos^{2+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 + m)(3 + m) \sqrt{\sin^2(c + dx)}}$$

```
[Out] a*b^2*(7+2*m)*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)/(3+m)+b^2*cos(d*x+c)^(1+m)
*(a+b*cos(d*x+c))*sin(d*x+c)/d/(3+m)-a*(3*b^2*(1+m)+a^2*(2+m))*cos(d*x+c)^(1+m)
*hypergeom([1/2, 1/2+1/2*m], [3/2+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/d/(1+m)
/(2+m)/(sin(d*x+c)^2)^(1/2)-b*(b^2*(2+m)+3*a^2*(3+m))*cos(d*x+c)^(2+m)*hy
pergeom([1/2, 1+1/2*m], [2+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(3+m)/(si
n(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

= {2872, 3102, 2827, 2722}

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^3 dx =$$

$$\frac{a(a^2(m+2) + 3b^2(m+1)) \sin(c + dx) \cos^{m+1}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \cos^2(c + dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c + dx)}} -$$

$$\frac{b(3a^2(m+3) + b^2(m+2)) \sin(c + dx) \cos^{m+2}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \cos^2(c + dx)\right)}{d(m+2)(m+3)\sqrt{\sin^2(c + dx)}} +$$

$$\frac{ab^2(2m+7) \sin(c + dx) \cos^{m+1}(c + dx)}{d(m+2)(m+3)} +$$

$$\frac{b^2 \sin(c + dx) \cos^{m+1}(c + dx)(a + b \cos(c + dx))}{d(m+3)}$$

[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^3,x]

[Out] (a*b^2*(7 + 2*m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (b^2*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(d*(3 + m)) - (a*(3*b^2*(1 + m) + a^2*(2 + m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (b*(b^2*(2 + m) + 3*a^2*(3 + m))*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2872

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m

|| IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && (!IntegerQ[m] || (EqQ[a, 0] & & NeQ[c, 0])))

Rule 3102

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{b^2 \cos^{1+m}(c+dx)(a+b \cos(c+dx)) \sin(c+dx)}{d(3+m)} \\
 &+ \frac{\int \cos^m(c+dx) (a(b^2(1+m) + a^2(3+m)) + b(b^2(2+m) + 3a^2(3+m)) \cos(c+dx) + ab^2(7+2m) \cos^2(c+dx)) dx}{3+m} \\
 &= \frac{ab^2(7+2m) \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)(3+m)} + \frac{b^2 \cos^{1+m}(c+dx)(a+b \cos(c+dx)) \sin(c+dx)}{d(3+m)} \\
 &+ \frac{\int \cos^m(c+dx) (a(3+m) (3b^2(1+m) + a^2(2+m)) + b(2+m) (b^2(2+m) + 3a^2(3+m)) \cos(c+dx)) dx}{6+5m+m^2} \\
 &= \frac{ab^2(7+2m) \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)(3+m)} \\
 &+ \frac{b^2 \cos^{1+m}(c+dx)(a+b \cos(c+dx)) \sin(c+dx)}{d(3+m)} \\
 &+ \left(a \left(a^2 + \frac{3b^2(1+m)}{2+m} \right) \right) \int \cos^m(c+dx) dx \\
 &+ \left(b \left(3a^2 + \frac{b^2(2+m)}{3+m} \right) \right) \int \cos^{1+m}(c+dx) dx \\
 &= \frac{ab^2(7+2m) \cos^{1+m}(c+dx) \sin(c+dx)}{d(2+m)(3+m)} + \frac{b^2 \cos^{1+m}(c+dx)(a+b \cos(c+dx)) \sin(c+dx)}{d(3+m)} \\
 &- \frac{a \left(a^2 + \frac{3b^2(1+m)}{2+m} \right) \cos^{1+m}(c+dx) \text{Hypergeometric2F1} \left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx) \right) \sin(c+dx)}{d(1+m) \sqrt{\sin^2(c+dx)}} \\
 &- \frac{b \left(3a^2 + \frac{b^2(2+m)}{3+m} \right) \cos^{2+m}(c+dx) \text{Hypergeometric2F1} \left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c+dx) \right) \sin(c+dx)}{d(2+m) \sqrt{\sin^2(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.79

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^3 dx$$

$$= \frac{\cos^{1+m}(c + dx) \csc(c + dx) \left(-\frac{a^3 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c+dx)\right)}{1+m} + b \cos(c + dx) \left(-\frac{3a^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c+dx)\right)}{2+m} + b \cos(c + dx) \left(-\frac{3a \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \cos^2(c+dx)\right)}{3+m} + b \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4+m}{2}, \frac{6+m}{2}, \cos^2(c+dx)\right) \right) \right)}{d}$$

[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^3,x]

[Out] (Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-(a^3*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2)]/(1 + m)) + b*Cos[c + d*x]*((-3*a^2*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2)]/(2 + m) + b*Cos[c + d*x]*((-3*a*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2)]/(3 + m) - (b*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[c + d*x]^2)]/(4 + m)))*Sqrt[Sin[c + d*x]^2])/d

Maple [F]

$$\int (\cos^m(dx + c))(a + \cos(dx + c)b)^3 dx$$

[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^3,x)

[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^3,x)

Fricas [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^3 dx = \int (b \cos(dx + c) + a)^3 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*cos(d*x + c)^m, x)

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^3 dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**3,x)

[Out] Timed out

Maxima [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^3 dx = \int (b \cos(dx + c) + a)^3 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)

Giac [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^3 dx = \int (b \cos(dx + c) + a)^3 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^3 dx = \int \cos(c + dx)^m (a + b \cos(c + dx))^3 dx$$

[In] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^3,x)

[Out] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^3, x)

3.771 $\int \cos^m(c + dx)(a + b \cos(c + dx))^2 dx$

Optimal result	6905
Rubi [A] (verified)	6905
Mathematica [A] (verified)	6907
Maple [F]	6907
Fricas [F]	6907
Sympy [F]	6907
Maxima [F]	6908
Giac [F]	6908
Mupad [F(-1)]	6908

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^2 dx = \frac{b^2 \cos^{1+m}(c + dx) \sin(c + dx)}{d(2 + m)} - \frac{(b^2(1 + m) + a^2(2 + m)) \cos^{1+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1 + m)(2 + m) \sqrt{\sin^2(c + dx)}} - \frac{2ab \cos^{2+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 + m) \sqrt{\sin^2(c + dx)}}$$

```
[Out] b^2*cos(d*x+c)^(1+m)*sin(d*x+c)/d/(2+m)-(b^2*(1+m)+a^2*(2+m))*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/2+1/2*m], [3/2+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/d/(1+m)/(2+m)/(sin(d*x+c)^2)^(1/2)-2*a*b*cos(d*x+c)^(2+m)*hypergeom([1/2, 1+1/2*m], [2+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/d/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2868, 2722, 3093}

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^2 dx = \frac{(a^2(m + 2) + b^2(m + 1)) \sin(c + dx) \cos^{m+1}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \cos^2(c + dx)\right)}{d(m + 1)(m + 2) \sqrt{\sin^2(c + dx)}} - \frac{2ab \sin(c + dx) \cos^{m+2}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \cos^2(c + dx)\right)}{d(m + 2) \sqrt{\sin^2(c + dx)}} + \frac{b^2 \sin(c + dx) \cos^{m+1}(c + dx)}{d(m + 2)}$$

[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2,x]

[Out] (b^2*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) - ((b^2*(1 + m) + a^2*(2 + m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*a*b*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2868

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Dist[2*c*(d/b), Int[(b*Sin[e + f*x])^(m + 1), x], x] + Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3093

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(-C)*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (2ab) \int \cos^{1+m}(c + dx) dx + \int \cos^m(c + dx) (a^2 + b^2 \cos^2(c + dx)) dx \\
 &= \frac{b^2 \cos^{1+m}(c + dx) \sin(c + dx)}{d(2 + m)} \\
 &\quad - \frac{2ab \cos^{2+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 + m) \sqrt{\sin^2(c + dx)}} \\
 &\quad + \left(a^2 + \frac{b^2(1 + m)}{2 + m}\right) \int \cos^m(c + dx) dx \\
 &= \frac{b^2 \cos^{1+m}(c + dx) \sin(c + dx)}{d(2 + m)} \\
 &\quad - \frac{\left(a^2 + \frac{b^2(1+m)}{2+m}\right) \cos^{1+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1 + m) \sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{2ab \cos^{2+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 + m) \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.94

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^2 dx = \frac{\cos^{1+m}(c + dx) \csc(c + dx) (a^2(6 + 5m + m^2) \text{Hypergeometric2F1}(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)) + b(1 +$$

[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2,x]

[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(a^2*(6 + 5*m + m^2)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2] + b*(1 + m)*Cos[c + d*x]*(2*a*(3 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2] + b*(2 + m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2]/(d*(1 + m)*(2 + m)*(3 + m)))

Maple [F]

$$\int (\cos^m(dx + c))(a + \cos(dx + c)b)^2 dx$$

[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^2,x)

[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*b)^2,x)

Fricas [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^2 dx = \int (b \cos(dx + c) + a)^2 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*cos(d*x + c)^m, x)

Sympy [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^2 dx = \int (a + b \cos(c + dx))^2 \cos^m(c + dx) dx$$

[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**2,x)

[Out] Integral((a + b*cos(c + d*x))**2*cos(c + d*x)**m, x)

Maxima [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^2 dx = \int (b \cos(dx + c) + a)^2 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)

Giac [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^2 dx = \int (b \cos(dx + c) + a)^2 \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + b \cos(c + dx))^2 dx = \int \cos(c + dx)^m (a + b \cos(c + dx))^2 dx$$

[In] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^2,x)

[Out] int(cos(c + d*x)^m*(a + b*cos(c + d*x))^2, x)

3.772 $\int \cos^m(c + dx)(a + b \cos(c + dx)) dx$

Optimal result	6909
Rubi [A] (verified)	6909
Mathematica [A] (verified)	6910
Maple [F]	6911
Fricas [F]	6911
Sympy [F]	6911
Maxima [F]	6911
Giac [F]	6912
Mupad [F(-1)]	6912

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \cos^m(c + dx)(a + b \cos(c + dx)) dx$$

$$= -\frac{a \cos^{1+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1+m)\sqrt{\sin^2(c + dx)}} - \frac{b \cos^{2+m}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2+m)\sqrt{\sin^2(c + dx)}}$$

[Out] $-a \cos(d*x+c)^{(1+m)} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2} + \frac{1}{2} * m\right], \left[\frac{3}{2} + \frac{1}{2} * m\right], \cos(d*x+c)^2\right) * \sin(d*x+c) / d / (1+m) / (\sin(d*x+c)^2)^{(1/2)} - b \cos(d*x+c)^{(2+m)} \operatorname{hypergeom}\left(\left[\frac{1}{2}, 1 + \frac{1}{2} * m\right], \left[2 + \frac{1}{2} * m\right], \cos(d*x+c)^2\right) * \sin(d*x+c) / d / (2+m) / (\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2827, 2722}

$$\int \cos^m(c + dx)(a + b \cos(c + dx)) dx$$

$$= -\frac{a \sin(c + dx) \cos^{m+1}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \cos^2(c + dx)\right)}{d(m+1)\sqrt{\sin^2(c + dx)}} - \frac{b \sin(c + dx) \cos^{m+2}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \cos^2(c + dx)\right)}{d(m+2)\sqrt{\sin^2(c + dx)}}$$

[In] $\operatorname{Int}[\operatorname{Cos}[c + d*x]^m * (a + b * \operatorname{Cos}[c + d*x]), x]$

```
[Out] -((a*cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (b*cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*SIN[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)])], x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= a \int \cos^m(c + dx) dx + b \int \cos^{1+m}(c + dx) dx \\ &= -\frac{a \cos^{1+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1+m)\sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{b \cos^{2+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2+m)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \cos^m(c + dx)(a + b \cos(c + dx)) dx = -\frac{\cos^{1+m}(c + dx) \csc(c + dx) (a(2 + m) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, \cos^2(c + dx)\right) + b(1 + m) \cos(c + dx))}{d(1 + m)(2 + m)}$$

```
[In] Integrate[Cos[c + d*x]^m*(a + b*cos[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(a*(2 + m)*Hypergeometric2F1[1/2, (1 +
m)/2, (3 + m)/2, Cos[c + d*x]^2] + b*(1 + m)*Cos[c + d*x]*Hypergeometric2F
1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 +
m)*(2 + m))
```

Maple [F]

$$\int (\cos^m(dx + c))(a + \cos(dx + c)b) dx$$

```
[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*b),x)
```

```
[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*b),x)
```

Fricas [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx)) dx = \int (b \cos(dx + c) + a) \cos(dx + c)^m dx$$

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

Sympy [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx)) dx = \int (a + b \cos(c + dx)) \cos^m(c + dx) dx$$

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((a + b*cos(c + d*x))*cos(c + d*x)**m, x)
```

Maxima [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx)) dx = \int (b \cos(dx + c) + a) \cos(dx + c)^m dx$$

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

Giac [F]

$$\int \cos^m(c + dx)(a + b \cos(c + dx)) dx = \int (b \cos(dx + c) + a) \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(a + b \cos(c + dx)) dx = \int \cos(c + dx)^m (a + b \cos(c + dx)) dx$$

[In] int(cos(c + d*x)^m*(a + b*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^m*(a + b*cos(c + d*x)), x)

3.773 $\int \frac{\cos^m(c+dx)}{a+b \cos(c+dx)} dx$

Optimal result	6913
Rubi [A] (verified)	6913
Mathematica [B] (warning: unable to verify)	6915
Maple [F]	6915
Fricas [F]	6915
Sympy [F(-1)]	6915
Maxima [F]	6916
Giac [F]	6916
Mupad [F(-1)]	6916

Optimal result

Integrand size = 21, antiderivative size = 190

$$\int \frac{\cos^m(c+dx)}{a+b \cos(c+dx)} dx$$

$$= \frac{a \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1-m}{2}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \cos^{-1+m}(c+dx) \cos^2(c+dx)^{\frac{1-m}{2}} \sin(c+dx)}{(a^2-b^2)d} - \frac{b \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \cos^m(c+dx) \cos^2(c+dx)^{-m/2} \sin(c+dx)}{(a^2-b^2)d}$$

[Out] a*AppellF1(1/2,1/2-1/2*m,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d*x+c)^(-1+m)*(cos(d*x+c)^2)^(1/2-1/2*m)*sin(d*x+c)/(a^2-b^2)/d-b*AppellF1(1/2,-1/2*m,1,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d*x+c)^m*sin(d*x+c)/(a^2-b^2)/d/((cos(d*x+c)^2)^(1/2*m))

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2902, 3268, 440}

$$\int \frac{\cos^m(c+dx)}{a+b \cos(c+dx)} dx$$

$$= \frac{a \sin(c+dx) \cos^{m-1}(c+dx) \cos^2(c+dx)^{\frac{1-m}{2}} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1-m}{2}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)} - \frac{b \sin(c+dx) \cos^m(c+dx) \cos^2(c+dx)^{-m/2} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)}$$

[In] Int[Cos[c + d*x]^m/(a + b*Cos[c + d*x]),x]

[Out] (a*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 - m)/2)*Sin[c + d*x])/((a^2 - b^2)*d) - (b*AppellF1[1/2, -1/2*m, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^m*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(m/2))

Rule 440

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 2902

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]

Rule 3268

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sin[e + f*x])^(2*FracPart[(m - 1)/2])/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2])), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x], Cos[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]

Rubi steps

$$\begin{aligned}
 \text{integral} &= a \int \frac{\cos^m(c + dx)}{a^2 - b^2 \cos^2(c + dx)} dx - b \int \frac{\cos^{1+m}(c + dx)}{a^2 - b^2 \cos^2(c + dx)} dx \\
 &= \frac{\left(a \cos^{2(-\frac{1}{2} + \frac{m}{2})}(c + dx) \cos^2(c + dx)^{\frac{1}{2} - \frac{m}{2}} \right) \text{Subst} \left(\int \frac{(1-x^2)^{\frac{1}{2}(-1+m)}}{a^2 - b^2 + b^2 x^2} dx, x, \sin(c + dx) \right)}{d} \\
 &\quad - \frac{\left(b \cos^m(c + dx) \cos^2(c + dx)^{-m/2} \right) \text{Subst} \left(\int \frac{(1-x^2)^{m/2}}{a^2 - b^2 + b^2 x^2} dx, x, \sin(c + dx) \right)}{d} \\
 &= \frac{a \text{AppellF1} \left(\frac{1}{2}, \frac{1-m}{2}, 1, \frac{3}{2}, \sin^2(c + dx), -\frac{b^2 \sin^2(c+dx)}{a^2 - b^2} \right) \cos^{-1+m}(c + dx) \cos^2(c + dx)^{\frac{1-m}{2}} \sin(c + dx)}{(a^2 - b^2) d} \\
 &\quad - \frac{b \text{AppellF1} \left(\frac{1}{2}, -\frac{m}{2}, 1, \frac{3}{2}, \sin^2(c + dx), -\frac{b^2 \sin^2(c+dx)}{a^2 - b^2} \right) \cos^m(c + dx) \cos^2(c + dx)^{-m/2} \sin(c + dx)}{(a^2 - b^2) d}
 \end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6534 vs. 2(190) = 380.

Time = 23.38 (sec) , antiderivative size = 6534, normalized size of antiderivative = 34.39

$$\int \frac{\cos^m(c + dx)}{a + b \cos(c + dx)} dx = \text{Result too large to show}$$

[In] Integrate[Cos[c + d*x]^m/(a + b*Cos[c + d*x]),x]

[Out] Result too large to show

Maple [F]

$$\int \frac{\cos^m(dx + c)}{a + \cos(dx + c)b} dx$$

[In] int(cos(d*x+c)^m/(a+cos(d*x+c)*b),x)

[Out] int(cos(d*x+c)^m/(a+cos(d*x+c)*b),x)

Fricas [F]

$$\int \frac{\cos^m(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(dx + c)^m}{b \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="fricas")

[Out] integral(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^m(c + dx)}{a + b \cos(c + dx)} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**m/(a+b*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^m(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(dx + c)^m}{b \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)

Giac [F]

$$\int \frac{\cos^m(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(dx + c)^m}{b \cos(dx + c) + a} dx$$

[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c + dx)}{a + b \cos(c + dx)} dx = \int \frac{\cos(c + dx)^m}{a + b \cos(c + dx)} dx$$

[In] int(cos(c + d*x)^m/(a + b*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^m/(a + b*cos(c + d*x)), x)

3.774 $\int \frac{\cos^m(c+dx)}{(a+b \cos(c+dx))^2} dx$

Optimal result	6917
Rubi [A] (verified)	6918
Mathematica [B] (warning: unable to verify)	6920
Maple [F]	6920
Fricas [F]	6920
Sympy [F(-1)]	6920
Maxima [F]	6921
Giac [F]	6921
Mupad [F(-1)]	6921

Optimal result

Integrand size = 21, antiderivative size = 294

$$\int \frac{\cos^m(c+dx)}{(a+b \cos(c+dx))^2} dx$$

$$= \frac{b^2 \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}(-1-m), 2, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \cos^{1+m}(c+dx) \cos^2(c+dx)^{\frac{1}{2}(-1-m)} \sin(c+dx)}{(a^2-b^2)^2 d}$$

$$+ \frac{a^2 \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1-m}{2}, 2, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \cos^{-1+m}(c+dx) \cos^2(c+dx)^{\frac{1-m}{2}} \sin(c+dx)}{(a^2-b^2)^2 d}$$

$$- \frac{2ab \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{m}{2}, 2, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right) \cos^m(c+dx) \cos^2(c+dx)^{-m/2} \sin(c+dx)}{(a^2-b^2)^2 d}$$

```
[Out] b^2*AppellF1(1/2,-1/2-1/2*m,2,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))
*cos(d*x+c)^(1+m)*(cos(d*x+c)^2)^(-1/2-1/2*m)*sin(d*x+c)/(a^2-b^2)^2/d+a^2*
AppellF1(1/2,1/2-1/2*m,2,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(
d*x+c)^(-1+m)*(cos(d*x+c)^2)^(1/2-1/2*m)*sin(d*x+c)/(a^2-b^2)^2/d-2*a*b*App
ellF1(1/2,-1/2*m,2,3/2,sin(d*x+c)^2,-b^2*sin(d*x+c)^2/(a^2-b^2))*cos(d*x+c)
^m*sin(d*x+c)/(a^2-b^2)^2/d/((cos(d*x+c)^2)^(1/2*m))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00,
 number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used
 = {2903, 3268, 440}

$$\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2} dx$$

$$= \frac{b^2 \sin(c+dx) \cos^{m+1}(c+dx) \cos^2(c+dx)^{\frac{1}{2}(-m-1)} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}(-m-1), 2, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)^2}$$

$$+ \frac{a^2 \sin(c+dx) \cos^{m-1}(c+dx) \cos^2(c+dx)^{\frac{1-m}{2}} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1-m}{2}, 2, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)^2}$$

$$- \frac{2abs \sin(c+dx) \cos^m(c+dx) \cos^2(c+dx)^{-m/2} \operatorname{AppellF1}\left(\frac{1}{2}, -\frac{m}{2}, 2, \frac{3}{2}, \sin^2(c+dx), -\frac{b^2 \sin^2(c+dx)}{a^2-b^2}\right)}{d(a^2-b^2)^2}$$

[In] Int[Cos[c + d*x]^m/(a + b*Cos[c + d*x])^2,x]

[Out] (b^2*AppellF1[1/2, (-1 - m)/2, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(1 + m)*(Cos[c + d*x]^2)^((-1 - m)/2)*Sin[c + d*x])/((a^2 - b^2)^2*d) + (a^2*AppellF1[1/2, (1 - m)/2, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 - m)/2)*Sin[c + d*x])/((a^2 - b^2)^2*d) - (2*a*b*AppellF1[1/2, -1/2*m, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^m*Sin[c + d*x])/((a^2 - b^2)^2*d*(Cos[c + d*x]^2)^(m/2))

Rule 440

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
 := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 2903

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Int[ExpandTrig[(d*sin[e + f*x])^n*(1/((a - b*sin[e + f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m)), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]

Rule 3268

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[(-ff)*d^(2*IntPart[(m - 1)/2] + 1)*((d*Sine + f*x))^(2*FracPart[(m - 1)/2])

$$/(f*(\text{Sin}[e + f*x]^2)^{\text{FracPart}[(m - 1)/2]}), \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m - 1)/2}*(a + b - b*ff^2*x^2)^p, x], x, \text{Cos}[e + f*x]/ff], x] /; \text{FreeQ}\{a, b, d, e, f, m, p\}, x\} \&\amp; \text{!IntegerQ}[m]$$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{a^2 \cos^m(c + dx)}{(a^2 - b^2 \cos^2(c + dx))^2} - \frac{2ab \cos^{1+m}(c + dx)}{(a^2 - b^2 \cos^2(c + dx))^2} \right. \\
 &\quad \left. + \frac{b^2 \cos^{2+m}(c + dx)}{(-a^2 + b^2 \cos^2(c + dx))^2} \right) dx \\
 &= a^2 \int \frac{\cos^m(c + dx)}{(a^2 - b^2 \cos^2(c + dx))^2} dx - (2ab) \int \frac{\cos^{1+m}(c + dx)}{(a^2 - b^2 \cos^2(c + dx))^2} dx \\
 &\quad + b^2 \int \frac{\cos^{2+m}(c + dx)}{(-a^2 + b^2 \cos^2(c + dx))^2} dx \\
 &= \frac{\left(b^2 \cos^{2(\frac{1}{2} + \frac{m}{2})}(c + dx) \cos^2(c + dx)^{-\frac{1}{2} - \frac{m}{2}} \right) \text{Subst} \left(\int \frac{(1-x^2)^{\frac{1+m}{2}}}{(-a^2 + b^2 - b^2 x^2)^2} dx, x, \sin(c + dx) \right)}{d} \\
 &\quad + \frac{\left(a^2 \cos^{2(-\frac{1}{2} + \frac{m}{2})}(c + dx) \cos^2(c + dx)^{\frac{1}{2} - \frac{m}{2}} \right) \text{Subst} \left(\int \frac{(1-x^2)^{\frac{1}{2}(-1+m)}}{(a^2 - b^2 + b^2 x^2)^2} dx, x, \sin(c + dx) \right)}{d} \\
 &\quad - \frac{\left(2ab \cos^m(c + dx) \cos^2(c + dx)^{-m/2} \right) \text{Subst} \left(\int \frac{(1-x^2)^{m/2}}{(a^2 - b^2 + b^2 x^2)^2} dx, x, \sin(c + dx) \right)}{d} \\
 &= \frac{b^2 \text{AppellF1} \left(\frac{1}{2}, \frac{1}{2}(-1 - m), 2, \frac{3}{2}, \sin^2(c + dx), -\frac{b^2 \sin^2(c + dx)}{a^2 - b^2} \right) \cos^{1+m}(c + dx) \cos^2(c + dx)^{\frac{1}{2}(-1-m)}}{(a^2 - b^2)^2 d} \\
 &\quad + \frac{a^2 \text{AppellF1} \left(\frac{1}{2}, \frac{1-m}{2}, 2, \frac{3}{2}, \sin^2(c + dx), -\frac{b^2 \sin^2(c + dx)}{a^2 - b^2} \right) \cos^{-1+m}(c + dx) \cos^2(c + dx)^{\frac{1-m}{2}} \sin(c + dx)}{(a^2 - b^2)^2 d} \\
 &\quad - \frac{2ab \text{AppellF1} \left(\frac{1}{2}, -\frac{m}{2}, 2, \frac{3}{2}, \sin^2(c + dx), -\frac{b^2 \sin^2(c + dx)}{a^2 - b^2} \right) \cos^m(c + dx) \cos^2(c + dx)^{-m/2} \sin(c + dx)}{(a^2 - b^2)^2 d}
 \end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7214 vs. $2(294) = 588$.

Time = 28.60 (sec) , antiderivative size = 7214, normalized size of antiderivative = 24.54

$$\int \frac{\cos^m(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Result too large to show}$$

[In] Integrate[Cos[c + d*x]^m/(a + b*Cos[c + d*x])^2,x]

[Out] Result too large to show

Maple [F]

$$\int \frac{\cos^m(dx + c)}{(a + \cos(dx + c)b)^2} dx$$

[In] int(cos(d*x+c)^m/(a+cos(d*x+c)*b)^2,x)

[Out] int(cos(d*x+c)^m/(a+cos(d*x+c)*b)^2,x)

Fricas [F]

$$\int \frac{\cos^m(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^m}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="fricas")

[Out] integral(cos(d*x + c)^m/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^m(c + dx)}{(a + b \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**m/(a+b*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^m(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^m}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a)^2, x)

Giac [F]

$$\int \frac{\cos^m(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(dx + c)^m}{(b \cos(dx + c) + a)^2} dx$$

[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c + dx)}{(a + b \cos(c + dx))^2} dx = \int \frac{\cos(c + dx)^m}{(a + b \cos(c + dx))^2} dx$$

[In] int(cos(c + d*x)^m/(a + b*cos(c + d*x))^2,x)

[Out] int(cos(c + d*x)^m/(a + b*cos(c + d*x))^2, x)

3.775 $\int (a + b \cos(c + dx))^3 \sec^m(c + dx) dx$

Optimal result	6922
Rubi [A] (verified)	6922
Mathematica [A] (verified)	6925
Maple [F]	6926
Fricas [F]	6926
Sympy [F]	6926
Maxima [F]	6926
Giac [F]	6927
Mupad [F(-1)]	6927

Optimal result

Integrand size = 21, antiderivative size = 282

$$\int (a + b \cos(c + dx))^3 \sec^m(c + dx) dx = -\frac{a^2 b (1 - 2m) \sec^{-2+m}(c + dx) \sin(c + dx)}{d(1 - m)(2 - m)} - \frac{a^2 \sec^{-2+m}(c + dx) (b + a \sec(c + dx)) \sin(c + dx)}{d(1 - m)} - \frac{b(b^2(2 - m) + 3a^2(3 - m)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4-m}{2}, \frac{6-m}{2}, \cos^2(c + dx)\right) \sec^{-4+m}(c + dx) \sin(c + dx)}{d(2 - m)(4 - m) \sqrt{\sin^2(c + dx)}} - \frac{a(3b^2(1 - m) + a^2(2 - m)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3-m}{2}, \frac{5-m}{2}, \cos^2(c + dx)\right) \sec^{-3+m}(c + dx) \sin(c + dx)}{d(1 - m)(3 - m) \sqrt{\sin^2(c + dx)}}$$

```
[Out] -a^2*b*(1-2*m)*sec(d*x+c)^(-2+m)*sin(d*x+c)/d/(m^2-3*m+2)-a^2*sec(d*x+c)^(-2+m)*(b+a*sec(d*x+c))*sin(d*x+c)/d/(1-m)-b*(b^2*(2-m)+3*a^2*(3-m))*hypergeom([1/2, 2-1/2*m],[3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-4+m)*sin(d*x+c)/d/(m^2-6*m+8)/(sin(d*x+c)^2)^(1/2)-a*(3*b^2*(1-m)+a^2*(2-m))*hypergeom([1/2, 3/2-1/2*m],[5/2-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-3+m)*sin(d*x+c)/d/(m^2-4*m+3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

= {3317, 3927, 4132, 3857, 2722, 4131}

$$\int (a + b \cos(c + dx))^3 \sec^m(c + dx) dx =$$

$$\frac{b(3a^2(3 - m) + b^2(2 - m)) \sin(c + dx) \sec^{m-4}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4-m}{2}, \frac{6-m}{2}, \cos^2(c + dx)\right)}{d(2 - m)(4 - m)\sqrt{\sin^2(c + dx)}} -$$

$$\frac{a(a^2(2 - m) + 3b^2(1 - m)) \sin(c + dx) \sec^{m-3}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3-m}{2}, \frac{5-m}{2}, \cos^2(c + dx)\right)}{d(1 - m)(3 - m)\sqrt{\sin^2(c + dx)}} -$$

$$\frac{a^2 \sin(c + dx) \sec^{m-2}(c + dx)(a \sec(c + dx) + b)}{d(1 - m)} -$$

$$\frac{a^2 b(1 - 2m) \sin(c + dx) \sec^{m-2}(c + dx)}{d(1 - m)(2 - m)}$$

[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^m,x]

[Out] -((a^2*b*(1 - 2*m)*Sec[c + d*x]^(-2 + m)*Sin[c + d*x])/(d*(1 - m)*(2 - m)))
 - (a^2*Sec[c + d*x]^(-2 + m)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(d*(1 - m))
) - (b*(b^2*(2 - m) + 3*a^2*(3 - m))*Hypergeometric2F1[1/2, (4 - m)/2, (6 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-4 + m)*Sin[c + d*x])/(d*(2 - m)*(4 - m)*Sqrt[Sin[c + d*x]^2]) - (a*(3*b^2*(1 - m) + a^2*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-3 + m)*Sin[c + d*x])/(d*(1 - m)*(3 - m)*Sqrt[Sin[c + d*x]^2])

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.)^(m_))*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_)]^(p_), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3857

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.)^(n_)), x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3927

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.)^(n_))*((csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^(m_)), x_Symbol] := Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -

```

2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b
^2*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d
*(3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x]
&& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
&& !(IGtQ[n, 2] && !IntegerQ[m])

```

Rule 4131

```

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_.)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

```

Rule 4132

```

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
x] /; FreeQ[{b, e, f, A, B, C, m}, x]

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sec^{-3+m}(c+dx)(b+a\sec(c+dx))^3 dx \\
&= -\frac{a^2 \sec^{-2+m}(c+dx)(b+a\sec(c+dx)) \sin(c+dx)}{d(1-m)} \\
&\quad + \frac{\int \sec^{-3+m}(c+dx) (-b(b^2(1-m) + a^2(3-m)) - a(3b^2(1-m) + a^2(2-m)) \sec(c+dx) - a^2b)}{-1+m} dx \\
&= -\frac{a^2 \sec^{-2+m}(c+dx)(b+a\sec(c+dx)) \sin(c+dx)}{d(1-m)} \\
&\quad + \left(a \left(3b^2 + \frac{a^2(2-m)}{1-m} \right) \right) \int \sec^{-2+m}(c+dx) dx \\
&\quad + \frac{\int \sec^{-3+m}(c+dx) (-b(b^2(1-m) + a^2(3-m)) - a^2b(1-2m) \sec^2(c+dx)) dx}{-1+m} \\
&= -\frac{a^2b(1-2m) \sec^{-2+m}(c+dx) \sin(c+dx)}{d(1-m)(2-m)} \\
&\quad - \frac{a^2 \sec^{-2+m}(c+dx)(b+a\sec(c+dx)) \sin(c+dx)}{d(1-m)} \\
&\quad + \left(b \left(b^2 + \frac{3a^2(3-m)}{2-m} \right) \right) \int \sec^{-3+m}(c+dx) dx \\
&\quad + \left(a \left(3b^2 + \frac{a^2(2-m)}{1-m} \right) \cos^m(c+dx) \sec^m(c+dx) \right) \int \cos^{2-m}(c+dx) dx
\end{aligned}$$

$$\begin{aligned}
&= -\frac{a^2 b(1-2m) \sec^{-2+m}(c+dx) \sin(c+dx)}{d(1-m)(2-m)} \\
&\quad - \frac{a^2 \sec^{-2+m}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{d(1-m)} \\
&\quad - \frac{a \left(3b^2 + \frac{a^2(2-m)}{1-m}\right) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3-m}{2}, \frac{5-m}{2}, \cos^2(c+dx)\right) \sec^{-3+m}(c+dx) \sin(c+dx)}{d(3-m)\sqrt{\sin^2(c+dx)}} \\
&\quad + \left(b \left(b^2 + \frac{3a^2(3-m)}{2-m}\right) \cos^m(c+dx) \sec^m(c+dx)\right) \int \cos^{3-m}(c+dx) dx \\
&= -\frac{a^2 b(1-2m) \sec^{-2+m}(c+dx) \sin(c+dx)}{d(1-m)(2-m)} \\
&\quad - \frac{a^2 \sec^{-2+m}(c+dx)(b+a \sec(c+dx)) \sin(c+dx)}{d(1-m)} \\
&\quad - \frac{b \left(b^2 + \frac{3a^2(3-m)}{2-m}\right) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4-m}{2}, \frac{6-m}{2}, \cos^2(c+dx)\right) \sec^{-4+m}(c+dx) \sin(c+dx)}{d(4-m)\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{a \left(3b^2 + \frac{a^2(2-m)}{1-m}\right) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3-m}{2}, \frac{5-m}{2}, \cos^2(c+dx)\right) \sec^{-3+m}(c+dx) \sin(c+dx)}{d(3-m)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.79

$$\begin{aligned}
&\int (a + b \cos(c + dx))^3 \sec^m(c + dx) dx \\
&= \frac{\csc(c + dx) \sec^{-4+m}(c + dx) (b^3 m(2 - 3m + m^2) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-3 + m), \frac{1}{2}(-1 + m), \sec^2(c + dx)\right) + (a + b \cos(c + dx))^3 \sec^m(c + dx))}{d(3 - m)\sqrt{\sin^2(c + dx)}}
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^m,x]

[Out] (Csc[c + d*x]*Sec[c + d*x]^(-4 + m)*(b^3*m*(2 - 3*m + m^2)*Hypergeometric2F1[1/2, (-3 + m)/2, (-1 + m)/2, Sec[c + d*x]^2] + (a*(-3 + m)*(6*b^2*(-1 + m)*m*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[c + d*x]^2] + 2*a*(-2 + m)*(3*b*m*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2]))*Sec[c + d*x]^3/2)*Sqrt[-Tan[c + d*x]^2])/(d*(-3 + m)*(-2 + m)*(-1 + m)*m)

Maple [F]

$$\int (a + \cos(dx + c)b)^3 (\sec^m(dx + c)) dx$$

[In] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^m,x)

[Out] int((a+cos(d*x+c)*b)^3*sec(d*x+c)^m,x)

Fricas [F]

$$\int (a + b \cos(c + dx))^3 \sec^m(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^m dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="fricas")

[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^m, x)

Sympy [F]

$$\int (a + b \cos(c + dx))^3 \sec^m(c + dx) dx = \int (a + b \cos(c + dx))^3 \sec^m(c + dx) dx$$

[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**m,x)

[Out] Integral((a + b*cos(c + d*x))**3*sec(c + d*x)**m, x)

Maxima [F]

$$\int (a + b \cos(c + dx))^3 \sec^m(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^m dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^m, x)

Giac [F]

$$\int (a + b \cos(c + dx))^3 \sec^m(c + dx) dx = \int (b \cos(dx + c) + a)^3 \sec(dx + c)^m dx$$

[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^3 \sec^m(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^m (a + b \cos(c + dx))^3 dx$$

[In] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^3,x)

[Out] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^3, x)

3.776 $\int (a + b \cos(c + dx))^2 \sec^m(c + dx) dx$

Optimal result	6928
Rubi [A] (verified)	6928
Mathematica [A] (verified)	6930
Maple [F]	6931
Fricas [F]	6931
Sympy [F]	6931
Maxima [F]	6931
Giac [F]	6932
Mupad [F(-1)]	6932

Optimal result

Integrand size = 21, antiderivative size = 200

$$\int (a + b \cos(c + dx))^2 \sec^m(c + dx) dx = -\frac{a^2 \sec^{-1+m}(c + dx) \sin(c + dx)}{d(1 - m)} - \frac{(b^2(1 - m) + a^2(2 - m)) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3-m}{2}, \frac{5-m}{2}, \cos^2(c + dx)\right) \sec^{-3+m}(c + dx) \sin(c + dx)}{d(1 - m)(3 - m)\sqrt{\sin^2(c + dx)}} - \frac{2ab \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2-m}{2}, \frac{4-m}{2}, \cos^2(c + dx)\right) \sec^{-2+m}(c + dx) \sin(c + dx)}{d(2 - m)\sqrt{\sin^2(c + dx)}}$$

```
[Out] -a^2*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(1-m)-(b^2*(1-m)+a^2*(2-m))*hypergeom([1/2, 3/2-1/2*m], [5/2-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^(-3+m)*sin(d*x+c)/d/(m^2-4*m+3)/(sin(d*x+c)^2)^(1/2)-2*a*b*hypergeom([1/2, 1-1/2*m], [2-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^(-2+m)*sin(d*x+c)/d/(2-m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3317, 3873, 3857, 2722, 4131}

$$\int (a + b \cos(c + dx))^2 \sec^m(c + dx) dx = \frac{(a^2(2 - m) + b^2(1 - m)) \sin(c + dx) \sec^{m-3}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3-m}{2}, \frac{5-m}{2}, \cos^2(c + dx)\right)}{d(1 - m)(3 - m)\sqrt{\sin^2(c + dx)}} - \frac{a^2 \sin(c + dx) \sec^{m-1}(c + dx)}{d(1 - m)} - \frac{2ab \sin(c + dx) \sec^{m-2}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2-m}{2}, \frac{4-m}{2}, \cos^2(c + dx)\right)}{d(2 - m)\sqrt{\sin^2(c + dx)}}$$

[In] Int[(a + b*cos[c + d*x])^2*sec[c + d*x]^m,x]

[Out] -((a^2*sec[c + d*x]^(-1 + m)*sin[c + d*x])/(d*(1 - m))) - ((b^2*(1 - m) + a^2*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-3 + m)*sin[c + d*x])/(d*(1 - m)*(3 - m)*Sqrt[Sin[c + d*x]^2]) - (2*a*b*Hypergeometric2F1[1/2, (2 - m)/2, (4 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-2 + m)*sin[c + d*x])/(d*(2 - m)*Sqrt[Sin[c + d*x]^2])

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^n_], x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 3317

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.)^(m_))*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_)], x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 3857

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.)^(n_)], x_Symbol] := Simp[(b*Csc[c + d*x])^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[n]

Rule 3873

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.)^(n_))*((csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)^2), x_Symbol] := Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x] + Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 4131

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.)^(m_))*((csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)), x_Symbol] := Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rubi steps

$$\text{integral} = \int \sec^{-2+m}(c + dx)(b + a \sec(c + dx))^2 dx$$

$$\begin{aligned}
&= (2ab) \int \sec^{-1+m}(c+dx) dx + \int \sec^{-2+m}(c+dx) (b^2 + a^2 \sec^2(c+dx)) dx \\
&= -\frac{a^2 \sec^{-1+m}(c+dx) \sin(c+dx)}{d(1-m)} + \left(b^2 + \frac{a^2(2-m)}{1-m}\right) \int \sec^{-2+m}(c+dx) dx \\
&\quad + (2ab \cos^m(c+dx) \sec^m(c+dx)) \int \cos^{1-m}(c+dx) dx \\
&= -\frac{a^2 \sec^{-1+m}(c+dx) \sin(c+dx)}{d(1-m)} \\
&\quad - \frac{2ab \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2-m}{2}, \frac{4-m}{2}, \cos^2(c+dx)\right) \sec^{-2+m}(c+dx) \sin(c+dx)}{d(2-m)\sqrt{\sin^2(c+dx)}} \\
&\quad + \left(\left(b^2 + \frac{a^2(2-m)}{1-m}\right) \cos^m(c+dx) \sec^m(c+dx)\right) \int \cos^{2-m}(c+dx) dx \\
&= -\frac{a^2 \sec^{-1+m}(c+dx) \sin(c+dx)}{d(1-m)} \\
&\quad - \frac{\left(b^2 + \frac{a^2(2-m)}{1-m}\right) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3-m}{2}, \frac{5-m}{2}, \cos^2(c+dx)\right) \sec^{-3+m}(c+dx) \sin(c+dx)}{d(3-m)\sqrt{\sin^2(c+dx)}} \\
&\quad - \frac{2ab \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2-m}{2}, \frac{4-m}{2}, \cos^2(c+dx)\right) \sec^{-2+m}(c+dx) \sin(c+dx)}{d(2-m)\sqrt{\sin^2(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.80

$$\begin{aligned}
&\int (a + b \cos(c+dx))^2 \sec^m(c+dx) dx \\
&= \frac{\csc(c+dx) \sec^{-3+m}(c+dx) (b^2(-1+m)m \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-2+m), \frac{m}{2}, \sec^2(c+dx)\right) + a(-2 +
\end{aligned}$$

[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^m,x]

[Out] (Csc[c + d*x]*Sec[c + d*x]^(-3 + m)*(b^2*(-1 + m)*m*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[c + d*x]^2] + a*(-2 + m)*(2*b*m*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2])*Sec[c + d*x]^2)*Sqrt[-Tan[c + d*x]^2])/(d*(-2 + m)*(-1 + m)*m)

Maple [F]

$$\int (a + \cos(dx + c)b)^2 (\sec^m(dx + c)) dx$$

[In] int((a+cos(d*x+c)*b)^2*sec(d*x+c)^m,x)

[Out] int((a+cos(d*x+c)*b)^2*sec(d*x+c)^m,x)

Fricas [F]

$$\int (a + b \cos(c + dx))^2 \sec^m(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^m dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="fricas")

[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^m, x)

Sympy [F]

$$\int (a + b \cos(c + dx))^2 \sec^m(c + dx) dx = \int (a + b \cos(c + dx))^2 \sec^m(c + dx) dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x)

[Out] Integral((a + b*cos(c + d*x))^2*sec(c + d*x)^m, x)

Maxima [F]

$$\int (a + b \cos(c + dx))^2 \sec^m(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^m dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^m, x)

Giac [F]

$$\int (a + b \cos(c + dx))^2 \sec^m(c + dx) dx = \int (b \cos(dx + c) + a)^2 \sec(dx + c)^m dx$$

[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^2 \sec^m(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^m (a + b \cos(c + dx))^2 dx$$

[In] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^2,x)

[Out] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x))^2, x)

3.777 $\int (a + b \cos(c + dx)) \sec^m(c + dx) dx$

Optimal result	6933
Rubi [A] (verified)	6933
Mathematica [A] (verified)	6935
Maple [F]	6935
Fricas [F]	6935
Sympy [F]	6935
Maxima [F]	6936
Giac [F]	6936
Mupad [F(-1)]	6936

Optimal result

Integrand size = 19, antiderivative size = 143

$$\int (a + b \cos(c + dx)) \sec^m(c + dx) dx$$

$$= -\frac{b \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2-m}{2}, \frac{4-m}{2}, \cos^2(c + dx)\right) \sec^{-2+m}(c + dx) \sin(c + dx)}{d(2-m)\sqrt{\sin^2(c + dx)}} - \frac{a \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-m}{2}, \frac{3-m}{2}, \cos^2(c + dx)\right) \sec^{-1+m}(c + dx) \sin(c + dx)}{d(1-m)\sqrt{\sin^2(c + dx)}}$$

[Out] -b*hypergeom([1/2, 1-1/2*m], [2-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^(-2+m)*sin(d*x+c)/d/(2-m)/(sin(d*x+c)^2)^(1/2)-a*hypergeom([1/2, 1/2-1/2*m], [3/2-1/2*m], cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(1-m)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3317, 3872, 3857, 2722}

$$\int (a + b \cos(c + dx)) \sec^m(c + dx) dx$$

$$= -\frac{a \sin(c + dx) \sec^{m-1}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-m}{2}, \frac{3-m}{2}, \cos^2(c + dx)\right)}{d(1-m)\sqrt{\sin^2(c + dx)}} - \frac{b \sin(c + dx) \sec^{m-2}(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2-m}{2}, \frac{4-m}{2}, \cos^2(c + dx)\right)}{d(2-m)\sqrt{\sin^2(c + dx)}}$$

[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^m,x]

[Out] $-\left(\frac{b \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2-m}{2}, \frac{4-m}{2}, \cos^2(c+dx)\right] \sec(c+dx) \sin^{-(2+m)}(c+dx)}{d(2-m)\sqrt{\sin^2(c+dx)}}\right) - \left(\frac{a \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1-m}{2}, \frac{3-m}{2}, \cos^2(c+dx)\right] \sec(c+dx) \sin^{-(1+m)}(c+dx)}{d(1-m)\sqrt{\sin^2(c+dx)}}\right)$

Rule 2722

$\operatorname{Int}[(b \sin(c) + d x)^\alpha, x_Symbol] \rightarrow \operatorname{Simp}[\cos(c+dx) \left(\frac{b \sin(c+dx)^{\alpha+1}}{b d (\alpha+1) \sqrt{\cos^2(c+dx)}}\right) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{\alpha+1}{2}, \frac{\alpha+3}{2}, \sin^2(c+dx)\right], x] /; \operatorname{FreeQ}\{b, c, d, \alpha\}, x \&\& \operatorname{IntegerQ}[2\alpha]$

Rule 3317

$\operatorname{Int}[(\csc(e) + (f x) d)^\alpha (a + (b \sin(e) + f x)^\beta)^\gamma, x_Symbol] \rightarrow \operatorname{Dist}[d^{\alpha \beta}, \operatorname{Int}[(d \csc(e+fx))^{\alpha(\beta-m)} (b + a \csc(e+fx)^\beta)^\gamma, x] /; \operatorname{FreeQ}\{a, b, d, e, f, m, n, p\}, x \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{IntegersQ}[n, p]$

Rule 3857

$\operatorname{Int}[(\csc(c) + (d x) b)^\alpha, x_Symbol] \rightarrow \operatorname{Simp}[(b \csc(c+dx))^{\alpha-1} \left(\frac{\sin(c+dx)}{b}\right)^{\alpha-1} \operatorname{Int}\left[\frac{1}{(\sin(c+dx)/b)^\alpha}, x\right], x] /; \operatorname{FreeQ}\{b, c, d, \alpha\}, x \&\& \operatorname{IntegerQ}[n]$

Rule 3872

$\operatorname{Int}[(\csc(e) + (f x) d)^\alpha (a + (b \sin(e) + f x)^\beta)^\gamma, x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[(d \csc(e+fx))^\alpha, x] + \operatorname{Dist}[b/d, \operatorname{Int}[(d \csc(e+fx))^{\alpha+1}, x] /; \operatorname{FreeQ}\{a, b, d, e, f, n\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \int \sec^{-1+m}(c+dx)(b+a \sec(c+dx)) dx \\ &= a \int \sec^m(c+dx) dx + b \int \sec^{-1+m}(c+dx) dx \\ &= (a \cos^m(c+dx) \sec^m(c+dx)) \int \cos^{-m}(c+dx) dx \\ &\quad + (b \cos^m(c+dx) \sec^m(c+dx)) \int \cos^{1-m}(c+dx) dx \\ &= -\frac{b \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2-m}{2}, \frac{4-m}{2}, \cos^2(c+dx)\right) \sec^{-2+m}(c+dx) \sin(c+dx)}{d(2-m)\sqrt{\sin^2(c+dx)}} \\ &\quad - \frac{a \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1-m}{2}, \frac{3-m}{2}, \cos^2(c+dx)\right) \sec^{-1+m}(c+dx) \sin(c+dx)}{d(1-m)\sqrt{\sin^2(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int (a + b \cos(c + dx)) \sec^m(c + dx) dx$$

$$= \frac{\csc(c + dx) (bm \cos(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1 + m), \frac{1+m}{2}, \sec^2(c + dx)\right) + a(-1 + m) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1 + m), \frac{1+m}{2}, \sec^2(c + dx)\right))}{d(-1 + m)m}$$

[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^m,x]

[Out] (Csc[c + d*x]*(b*m*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2])*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + m)*m)

Maple [F]

$$\int (a + \cos(dx + c) b) (\sec^m(dx + c)) dx$$

[In] int((a+cos(d*x+c)*b)*sec(d*x+c)^m,x)

[Out] int((a+cos(d*x+c)*b)*sec(d*x+c)^m,x)

Fricas [F]

$$\int (a + b \cos(c + dx)) \sec^m(c + dx) dx = \int (b \cos(dx + c) + a) \sec(dx + c)^m dx$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="fricas")

[Out] integral((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)

Sympy [F]

$$\int (a + b \cos(c + dx)) \sec^m(c + dx) dx = \int (a + b \cos(c + dx)) \sec^m(c + dx) dx$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**m,x)

[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**m, x)

Maxima [F]

$$\int (a + b \cos(c + dx)) \sec^m(c + dx) dx = \int (b \cos(dx + c) + a) \sec(dx + c)^m dx$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="maxima")

[Out] integrate((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)

Giac [F]

$$\int (a + b \cos(c + dx)) \sec^m(c + dx) dx = \int (b \cos(dx + c) + a) \sec(dx + c)^m dx$$

[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="giac")

[Out] integrate((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx)) \sec^m(c + dx) dx = \int \left(\frac{1}{\cos(c + dx)} \right)^m (a + b \cos(c + dx)) dx$$

[In] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x)),x)

[Out] int((1/cos(c + d*x))^m*(a + b*cos(c + d*x)), x)

$$3.778 \quad \int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}} dx$$

Optimal result	6937
Rubi [A] (verified)	6937
Mathematica [C] (verified)	6938
Maple [B] (verified)	6938
Fricas [A] (verification not implemented)	6939
Sympy [F]	6939
Maxima [F(-2)]	6939
Giac [B] (verification not implemented)	6940
Mupad [F(-1)]	6940

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}} dx = -2 \arctan \left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}} \right)$$

[Out] $-2*\arctan(\sin(x)/(1-\cos(x))^{(1/2)}/(a-\cos(x))^{(1/2)})$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2854, 210}

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}} dx = -2 \arctan \left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}} \right)$$

[In] `Int[Sqrt[1 - Cos[x]]/Sqrt[a - Cos[x]],x]`

[Out] `-2*ArcTan[Sin[x]/(Sqrt[1 - Cos[x]]*Sqrt[a - Cos[x]])]`

Rule 210

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(-Rt[-a, 2]*Rt[-b, 2])^(-1)*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])`

Rule 2854

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x`

```
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, \frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right) \\ &= -2 \arctan\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right) \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}} dx = i\sqrt{2-2\cos(x)} \csc\left(\frac{x}{2}\right) \log\left(i\sqrt{2}\cos\left(\frac{x}{2}\right) + \sqrt{a-\cos(x)}\right)$$

```
[In] Integrate[Sqrt[1 - Cos[x]]/Sqrt[a - Cos[x]], x]
```

```
[Out] I*Sqrt[2 - 2*Cos[x]]*Csc[x/2]*Log[I*Sqrt[2]*Cos[x/2] + Sqrt[a - Cos[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(22) = 44.

Time = 2.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

method	result	size
default	$-\frac{\csc(x)(2-2\cos(x))^{\frac{3}{2}}\sqrt{a-\cos(x)}\arctan\left(\frac{\sqrt{-\frac{2(-a+\cos(x))}{\cos(x)+1}}\sqrt{2}}{2}\right)}{(\cos(x)-1)\sqrt{-\frac{2(-a+\cos(x))}{\cos(x)+1}}}$	65

```
[In] int((1-cos(x))^(1/2)/(a-cos(x))^(1/2), x, method=_RETURNVERBOSE)
```

```
[Out] -csc(x)*(2-2*cos(x))^(3/2)*(a-cos(x))^(1/2)*arctan(1/2*(-2*(-a+cos(x)))/(cos(x)+1))^(1/2)*2^(1/2)/(cos(x)-1)/(-2*(-a+cos(x))/(cos(x)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1 - \cos(x)}}{\sqrt{a - \cos(x)}} dx = \arctan \left(\frac{(a - 2 \cos(x) - 1)\sqrt{-\cos(x) + 1}}{2 \sqrt{a - \cos(x)} \sin(x)} \right)$$

[In] integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="fricas")

[Out] arctan(1/2*(a - 2*cos(x) - 1)*sqrt(-cos(x) + 1)/(sqrt(a - cos(x))*sin(x)))

Sympy [F]

$$\int \frac{\sqrt{1 - \cos(x)}}{\sqrt{a - \cos(x)}} dx = \int \frac{\sqrt{1 - \cos(x)}}{\sqrt{a - \cos(x)}} dx$$

[In] integrate((1-cos(x))**(1/2)/(a-cos(x))**(1/2),x)

[Out] Integral(sqrt(1 - cos(x))/sqrt(a - cos(x)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1 - \cos(x)}}{\sqrt{a - \cos(x)}} dx = \text{Exception raised: ValueError}$$

[In] integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-1>0)', see 'assume?' for more details)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(22) = 44.

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{1 - \cos(x)}}{\sqrt{a - \cos(x)}} dx$$

$$= 4 \arctan \left(-\frac{1}{4} \sqrt{2} \left(\sqrt{a-1} \tan \left(\frac{1}{4} x \right)^2 - \sqrt{a \tan \left(\frac{1}{4} x \right)^4 - \tan \left(\frac{1}{4} x \right)^4 + 2a \tan \left(\frac{1}{4} x \right)^2 + 6 \tan \left(\frac{1}{4} x \right)} \right) \right)$$

[In] integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="giac")

[Out] 4*arctan(-1/4*sqrt(2)*(sqrt(a - 1)*tan(1/4*x)^2 - sqrt(a*tan(1/4*x)^4 - tan(1/4*x)^4 + 2*a*tan(1/4*x)^2 + 6*tan(1/4*x)^2 + a - 1) + sqrt(a - 1)))*sgn(sin(1/2*x))

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1 - \cos(x)}}{\sqrt{a - \cos(x)}} dx = \int \frac{\sqrt{1 - \cos(x)}}{\sqrt{a - \cos(x)}} dx$$

[In] int((1 - cos(x))^(1/2)/(a - cos(x))^(1/2),x)

[Out] int((1 - cos(x))^(1/2)/(a - cos(x))^(1/2), x)

$$3.779 \quad \int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}} dx$$

Optimal result	6941
Rubi [A] (verified)	6941
Mathematica [A] (verified)	6942
Maple [A] (verified)	6943
Fricas [A] (verification not implemented)	6943
Sympy [F]	6943
Maxima [F(-2)]	6944
Giac [A] (verification not implemented)	6944
Mupad [F(-1)]	6944

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}} dx = -\frac{2 \arctan\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right) \sqrt{\frac{1-\cos(x)}{a-\cos(x)}} \sqrt{a-\cos(x)}}{\sqrt{1-\cos(x)}}$$

[Out] $-2*\arctan(\sin(x)/(1-\cos(x))^{(1/2)/(a-\cos(x))^{(1/2)}*((1-\cos(x))/(a-\cos(x)))^{(1/2)}*(a-\cos(x))^{(1/2)/(1-\cos(x))^{(1/2)})}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4485, 2854, 210}

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}} dx = -\frac{2\sqrt{\frac{1-\cos(x)}{a-\cos(x)}} \sqrt{a-\cos(x)} \arctan\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)}{\sqrt{1-\cos(x)}}$$

[In] Int[Sqrt[(1 - Cos[x])/(a - Cos[x])],x]

[Out] $(-2*\text{ArcTan}[\text{Sin}[x]/(\text{Sqrt}[1 - \text{Cos}[x]]*\text{Sqrt}[a - \text{Cos}[x]])]*\text{Sqrt}[(1 - \text{Cos}[x])/(a - \text{Cos}[x])]*\text{Sqrt}[a - \text{Cos}[x]])/\text{Sqrt}[1 - \text{Cos}[x]]$

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] & (LtQ[a, 0] || LtQ[b, 0])

Rule 2854

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_)
+ (f_)*(x_)]], x_Symbol] := Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 4485

```
Int[(u_)*((v_)^(m_)*(w_)^(n_))^(p_), x_Symbol] := With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))], Int[uu*vv^(m*p)*ww^(n*p), x],
x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && (!InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\right) \int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}} dx}{\sqrt{1-\cos(x)}} \\ &= \frac{\left(2\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\right) \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, \frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)}{\sqrt{1-\cos(x)}} \\ &= -\frac{2 \arctan\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right) \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}}{\sqrt{1-\cos(x)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}} dx = -\sqrt{2}\sqrt{\frac{-1+\cos(x)}{-a+\cos(x)}}\sqrt{-a+\cos(x)}\csc\left(\frac{x}{2}\right)\log\left(\sqrt{2}\cos\left(\frac{x}{2}\right) + \sqrt{-a+\cos(x)}\right)$$

```
[In] Integrate[Sqrt[(1 - Cos[x])/(a - Cos[x])], x]
```

```
[Out] -(Sqrt[2]*Sqrt[(-1 + Cos[x])/(-a + Cos[x])]*Sqrt[-a + Cos[x]]*Csc[x/2]*Log[
Sqrt[2]*Cos[x/2] + Sqrt[-a + Cos[x]]])
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result	size
default	$\sqrt{2} \sqrt{\frac{\cos(x)-1}{-a+\cos(x)}} \sqrt{\frac{2(-a+\cos(x))}{\cos(x)+1}} \arctan\left(\frac{\sqrt{\frac{-2(-a+\cos(x))}{\cos(x)+1}} \sqrt{2}}{2}\right) (\cot(x) + \csc(x))$	63

[In] `int(((1-cos(x))/(a-cos(x)))^(1/2),x,method=_RETURNVERBOSE)`

[Out] `2^(1/2)*((cos(x)-1)/(-a+cos(x)))^(1/2)*(-2*(-a+cos(x))/(cos(x)+1))^(1/2)*arctan(1/2*(-2*(-a+cos(x))/(cos(x)+1))^(1/2)*2^(1/2))*(cot(x)+csc(x))`

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

$$\int \sqrt{\frac{1 - \cos(x)}{a - \cos(x)}} dx = -\arctan\left(-\frac{(a - 2 \cos(x) - 1) \sqrt{\frac{-\cos(x) - 1}{a - \cos(x)}}}{2 \sin(x)}\right)$$

[In] `integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="fricas")`

[Out] `-arctan(-1/2*(a - 2*cos(x) - 1)*sqrt(-(cos(x) - 1)/(a - cos(x)))/sin(x))`

Sympy [F]

$$\int \sqrt{\frac{1 - \cos(x)}{a - \cos(x)}} dx = \int \sqrt{\frac{1 - \cos(x)}{a - \cos(x)}} dx$$

[In] `integrate(((1-cos(x))/(a-cos(x)))**(1/2),x)`

[Out] `Integral(sqrt((1 - cos(x))/(a - cos(x))), x)`

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\frac{1 - \cos(x)}{a - \cos(x)}} dx = \text{Exception raised: ValueError}$$

[In] integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a-1>0)', see 'assume?' for more details)

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \sqrt{\frac{1 - \cos(x)}{a - \cos(x)}} dx$$

$$= 2 \arctan \left(\frac{1}{2} \sqrt{2} \sqrt{a \tan \left(\frac{1}{2} x \right)^2 + \tan \left(\frac{1}{2} x \right)^2 + a - 1} \right) \operatorname{sgn} \left(\tan \left(\frac{1}{2} x \right)^3 + \tan \left(\frac{1}{2} x \right) \right) \operatorname{sgn}(a - \cos(x))$$

[In] integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="giac")

[Out] 2*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*x)^2 + tan(1/2*x)^2 + a - 1))*sgn(tan(1/2*x)^3 + tan(1/2*x))*sgn(a - cos(x))

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{1 - \cos(x)}{a - \cos(x)}} dx = \int \sqrt{-\frac{\cos(x) - 1}{a - \cos(x)}} dx$$

[In] int((-cos(x) - 1)/(a - cos(x)))^(1/2),x)

[Out] int((-cos(x) - 1)/(a - cos(x)))^(1/2), x)

3.780 $\int (a + a \cos(c + dx)) \left(-\frac{B}{2} + B \cos(c + dx)\right) dx$

Optimal result	6945
Rubi [A] (verified)	6945
Mathematica [A] (verified)	6946
Maple [A] (verified)	6946
Fricas [A] (verification not implemented)	6947
Sympy [B] (verification not implemented)	6947
Maxima [A] (verification not implemented)	6947
Giac [A] (verification not implemented)	6948
Mupad [B] (verification not implemented)	6948

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int (a + a \cos(c + dx)) \left(-\frac{B}{2} + B \cos(c + dx)\right) dx$$

$$= \frac{aB \sin(c + dx)}{2d} + \frac{aB \cos(c + dx) \sin(c + dx)}{2d}$$

[Out] $1/2*a*B*\sin(d*x+c)/d+1/2*a*B*\cos(d*x+c)*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2813}

$$\int (a + a \cos(c + dx)) \left(-\frac{B}{2} + B \cos(c + dx)\right) dx$$

$$= \frac{aB \sin(c + dx)}{2d} + \frac{aB \sin(c + dx) \cos(c + dx)}{2d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])*(-1/2*B + B*\text{Cos}[c + d*x]),x]$

[Out] $(a*B*\text{Sin}[c + d*x])/(2*d) + (a*B*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(2*d)$

Rule 2813

$\text{Int}[(a + (b_*)*\sin[(e_*) + (f_*)*(x_*)])*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)]), x_Symbol] :> \text{Simp}[(2*a*c + b*d)*(x/2), x] + (-\text{Simp}[(b*c + a*d)*(Cos[e + f*x]/f), x] - \text{Simp}[b*d*\text{Cos}[e + f*x]*(\text{Sin}[e + f*x]/(2*f)), x]) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

Rubi steps

$$\text{integral} = \frac{aB \sin(c + dx)}{2d} + \frac{aB \cos(c + dx) \sin(c + dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int (a + a \cos(c + dx)) \left(-\frac{B}{2} + B \cos(c + dx) \right) dx = \frac{aB(2c + 2 \sin(c + dx) + \sin(2(c + dx)))}{4d}$$

[In] Integrate[(a + a*Cos[c + d*x])*(-1/2*B + B*Cos[c + d*x]),x]

[Out] (a*B*(2*c + 2*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
parallelrisc	$\frac{Ba(2 \sin(dx+c) + \sin(2dx+2c))}{4d}$	26
risc	$\frac{aB \sin(dx+c)}{2d} + \frac{Ba \sin(2dx+2c)}{4d}$	31
norman	$\frac{2Ba \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}$	32
parts	$\frac{Ba\left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d} - \frac{aBx}{2} + \frac{aB \sin(dx+c)}{2d}$	48
derivativedivides	$\frac{2Ba\left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + Ba \sin(dx+c) - Ba(dx+c)}{2d}$	51
default	$\frac{2Ba\left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + Ba \sin(dx+c) - Ba(dx+c)}{2d}$	51

[In] int((a+cos(d*x+c)*a)*(-1/2*B+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/4*B*a*(2*sin(d*x+c)+sin(2*d*x+2*c))/d

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int (a + a \cos(c + dx)) \left(-\frac{B}{2} + B \cos(c + dx) \right) dx = \frac{(Ba \cos(dx + c) + Ba) \sin(dx + c)}{2d}$$

[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(B*a*cos(d*x + c) + B*a)*sin(d*x + c)/d

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(32) = 64.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.35

$$\int (a + a \cos(c + dx)) \left(-\frac{B}{2} + B \cos(c + dx) \right) dx$$

$$= \begin{cases} \frac{Bax \sin^2(c+dx)}{2} + \frac{Bax \cos^2(c+dx)}{2} - \frac{Bax}{2} + \frac{Ba \sin(c+dx) \cos(c+dx)}{2d} + \frac{Ba \sin(c+dx)}{2d} & \text{for } d \neq 0 \\ x(B \cos(c) - \frac{B}{2})(a \cos(c) + a) & \text{otherwise} \end{cases}$$

[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x)

[Out] Piecewise((B*a*x*sin(c + d*x)**2/2 + B*a*x*cos(c + d*x)**2/2 - B*a*x/2 + B*a*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*sin(c + d*x)/(2*d), Ne(d, 0)), (x*(B*cos(c) - B/2)*(a*cos(c) + a), True))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int (a + a \cos(c + dx)) \left(-\frac{B}{2} + B \cos(c + dx) \right) dx$$

$$= \frac{(2dx + 2c + \sin(2dx + 2c))Ba - 2(dx + c)Ba + 2Ba \sin(dx + c)}{4d}$$

[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 2*(d*x + c)*B*a + 2*B*a*sin(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int (a + a \cos(c + dx)) \left(-\frac{B}{2} + B \cos(c + dx) \right) dx = \frac{Ba \sin(2dx + 2c)}{4d} + \frac{Ba \sin(dx + c)}{2d}$$

[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="giac")

[Out] 1/4*B*a*sin(2*d*x + 2*c)/d + 1/2*B*a*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int (a + a \cos(c + dx)) \left(-\frac{B}{2} + B \cos(c + dx) \right) dx = \frac{Ba(2 \sin(c + dx) + \sin(2c + 2dx))}{4d}$$

[In] int(-(B/2 - B*cos(c + d*x))*(a + a*cos(c + d*x)),x)

[Out] (B*a*(2*sin(c + d*x) + sin(2*c + 2*d*x)))/(4*d)

3.781 $\int (a + a \cos(c + dx))^4 \left(-\frac{4B}{5} + B \cos(c + dx)\right) dx$

Optimal result	6949
Rubi [A] (verified)	6949
Mathematica [A] (verified)	6950
Maple [A] (verified)	6950
Fricas [B] (verification not implemented)	6951
Sympy [B] (verification not implemented)	6951
Maxima [B] (verification not implemented)	6952
Giac [B] (verification not implemented)	6952
Mupad [B] (verification not implemented)	6952

Optimal result

Integrand size = 27, antiderivative size = 26

$$\int (a + a \cos(c + dx))^4 \left(-\frac{4B}{5} + B \cos(c + dx)\right) dx = \frac{B(a + a \cos(c + dx))^4 \sin(c + dx)}{5d}$$

[Out] 1/5*B*(a+a*cos(d*x+c))^4*sin(d*x+c)/d

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2828}

$$\int (a + a \cos(c + dx))^4 \left(-\frac{4B}{5} + B \cos(c + dx)\right) dx = \frac{B \sin(c + dx)(a \cos(c + dx) + a)^4}{5d}$$

[In] Int[(a + a*Cos[c + d*x])^4*((-4*B)/5 + B*Cos[c + d*x]),x]

[Out] (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)

Rule 2828

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]

Rubi steps

$$\text{integral} = \frac{B(a + a \cos(c + dx))^4 \sin(c + dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int (a + a \cos(c + dx))^4 \left(-\frac{4B}{5} + B \cos(c + dx) \right) dx = \frac{a^4 B (1 + \cos(c + dx))^4 \sin(c + dx)}{5d}$$

[In] Integrate[(a + a*Cos[c + d*x])^4*((-4*B)/5 + B*Cos[c + d*x]),x]

[Out] (a^4*B*(1 + Cos[c + d*x])^4*Sin[c + d*x])/(5*d)

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

method	result
norman	$\frac{32B a^4 \tan\left(\frac{dx+c}{2}\right)}{5d \left(1 + \tan^2\left(\frac{dx+c}{2}\right)\right)^5}$
parallelrisc	$\frac{B a^4 (42 \sin(dx+c) + \sin(5dx+5c) + 8 \sin(4dx+4c) + 27 \sin(3dx+3c) + 48 \sin(2dx+2c))}{80d}$
risc	$\frac{21B a^4 \sin(dx+c)}{40d} + \frac{B a^4 \sin(5dx+5c)}{80d} + \frac{B a^4 \sin(4dx+4c)}{10d} + \frac{27B a^4 \sin(3dx+3c)}{80d} + \frac{3B a^4 \sin(2dx+2c)}{5d}$
derivativedivides	$16B a^4 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + B a^4 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4 \left(\cos^2(dx+c) \right)}{3} \right) \sin(dx+c) + \frac{14B a^4 (2+)}{5d}$
default	$16B a^4 \left(\frac{\left(\cos^3(dx+c) + \frac{3 \cos(dx+c)}{2} \right) \sin(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + B a^4 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4 \left(\cos^2(dx+c) \right)}{3} \right) \sin(dx+c) + \frac{14B a^4 (2+)}{5d}$
parts	$\frac{B a^4 \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4 \left(\cos^2(dx+c) \right)}{3} \right) \sin(dx+c)}{5d} - \frac{4B a^4 x}{5} - \frac{11B a^4 \sin(dx+c)}{5d} - \frac{4B a^4 \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} \right)}{5d}$

[In] int((a+cos(d*x+c)*a)^4*(-4/5*B+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 32/5*B*a^4/d*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)^5

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(24) = 48$.

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.69

$$\int (a + a \cos(c + dx))^4 \left(-\frac{4B}{5} + B \cos(c + dx) \right) dx$$

$$= \frac{(Ba^4 \cos(dx + c))^4 + 4Ba^4 \cos(dx + c)^3 + 6Ba^4 \cos(dx + c)^2 + 4Ba^4 \cos(dx + c) + Ba^4 \sin(dx + c)}{5d}$$

```
[In] integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/5*(B*a^4*cos(d*x + c)^4 + 4*B*a^4*cos(d*x + c)^3 + 6*B*a^4*cos(d*x + c)^2 + 4*B*a^4*cos(d*x + c) + B*a^4)*sin(d*x + c)/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(22) = 44$.

Time = 0.34 (sec) , antiderivative size = 333, normalized size of antiderivative = 12.81

$$\int (a + a \cos(c + dx))^4 \left(-\frac{4B}{5} + B \cos(c + dx) \right) dx$$

$$= \begin{cases} \frac{6Ba^4x \sin^4(c+dx)}{5} + \frac{12Ba^4x \sin^2(c+dx) \cos^2(c+dx)}{5} - \frac{2Ba^4x \sin^2(c+dx)}{5} + \frac{6Ba^4x \cos^4(c+dx)}{5} - \frac{2Ba^4x \cos^2(c+dx)}{5} - \frac{4Ba^4x}{5} + \\ x(B \cos(c) - \frac{4B}{5})(a \cos(c) + a)^4 \end{cases}$$

```
[In] integrate((a+a*cos(d*x+c))**4*(-4/5*B+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((6*B*a**4*x*sin(c + d*x)**4/5 + 12*B*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/5 - 2*B*a**4*x*sin(c + d*x)**2/5 + 6*B*a**4*x*cos(c + d*x)**4/5 - 2*B*a**4*x*cos(c + d*x)**2/5 - 4*B*a**4*x/5 + 8*B*a**4*sin(c + d*x)**5/(15*d) + 4*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 6*B*a**4*sin(c + d*x)**3*cos(c + d*x)/(5*d) + 28*B*a**4*sin(c + d*x)**3/(15*d) + B*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 2*B*a**4*sin(c + d*x)*cos(c + d*x)**3/d + 14*B*a**4*sin(c + d*x)*cos(c + d*x)**2/(5*d) - 2*B*a**4*sin(c + d*x)*cos(c + d*x)/(5*d) - 11*B*a**4*sin(c + d*x)/(5*d), Ne(d, 0)), (x*(B*cos(c) - 4*B/5)*(a*cos(c) + a)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(24) = 48$.

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 5.54

$$\int (a + a \cos(c + dx))^4 \left(-\frac{4B}{5} + B \cos(c + dx) \right) dx$$

$$= \frac{2(3 \sin(dx + c)^5 - 10 \sin(dx + c)^3 + 15 \sin(dx + c))Ba^4 - 28(\sin(dx + c)^3 - 3 \sin(dx + c))Ba^4 + 3(1$$

[In] integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/30*(2*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 - 28*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 24*(d*x + c)*B*a^4 - 66*B*a^4*sin(d*x + c))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(24) = 48$.

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int (a + a \cos(c + dx))^4 \left(-\frac{4B}{5} + B \cos(c + dx) \right) dx$$

$$= \frac{Ba^4 \sin(5dx + 5c)}{80d} + \frac{Ba^4 \sin(4dx + 4c)}{10d} + \frac{27Ba^4 \sin(3dx + 3c)}{80d}$$

$$+ \frac{3Ba^4 \sin(2dx + 2c)}{5d} + \frac{21Ba^4 \sin(dx + c)}{40d}$$

[In] integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="giac")

[Out] 1/80*B*a^4*sin(5*d*x + 5*c)/d + 1/10*B*a^4*sin(4*d*x + 4*c)/d + 27/80*B*a^4*sin(3*d*x + 3*c)/d + 3/5*B*a^4*sin(2*d*x + 2*c)/d + 21/40*B*a^4*sin(d*x + c)/d

Mupad [B] (verification not implemented)

Time = 14.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int (a + a \cos(c + dx))^4 \left(-\frac{4B}{5} + B \cos(c + dx) \right) dx = \frac{32Ba^4 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^9 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{5d}$$

[In] int(-((4*B)/5 - B*cos(c + d*x))*(a + a*cos(c + d*x))^4,x)

[Out] (32*B*a^4*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2))/(5*d)

$$3.782 \quad \int (a + a \cos(c + dx))^n \left(-\frac{Bn}{1+n} + B \cos(c + dx) \right) dx$$

Optimal result	6953
Rubi [A] (verified)	6953
Mathematica [A] (verified)	6954
Maple [A] (verified)	6954
Fricas [A] (verification not implemented)	6954
Sympy [B] (verification not implemented)	6955
Maxima [B] (verification not implemented)	6955
Giac [B] (verification not implemented)	6956
Mupad [B] (verification not implemented)	6957

Optimal result

Integrand size = 31, antiderivative size = 28

$$\int (a + a \cos(c + dx))^n \left(-\frac{Bn}{1+n} + B \cos(c + dx) \right) dx = \frac{B(a + a \cos(c + dx))^n \sin(c + dx)}{d(1+n)}$$

[Out] B*(a+a*cos(d*x+c))^n*sin(d*x+c)/d/(1+n)

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2828}

$$\int (a + a \cos(c + dx))^n \left(-\frac{Bn}{1+n} + B \cos(c + dx) \right) dx = \frac{B \sin(c + dx)(a \cos(c + dx) + a)^n}{d(n+1)}$$

[In] Int[(a + a*Cos[c + d*x])^n*(-((B*n)/(1 + n)) + B*Cos[c + d*x]),x]

[Out] (B*(a + a*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n))

Rule 2828

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]

Rubi steps

$$\text{integral} = \frac{B(a + a \cos(c + dx))^n \sin(c + dx)}{d(1+n)}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx))^n \left(-\frac{Bn}{1+n} + B \cos(c + dx) \right) dx = \frac{B(a(1 + \cos(c + dx)))^n \sin(c + dx)}{d(1+n)}$$

[In] Integrate[(a + a*Cos[c + d*x])^n*(-((B*n)/(1 + n)) + B*Cos[c + d*x]),x]

[Out] (B*(a*(1 + Cos[c + d*x]))^n*Sin[c + d*x])/(d*(1 + n))

Maple [A] (verified)

Time = 5.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{B(a(1+\cos(dx+c)))^n \sin(dx+c)}{d(1+n)}$
norman	$\frac{2B \tan\left(\frac{dx}{2} + \frac{c}{2}\right) e^{n \ln\left(a + \frac{1 - \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}\right) a}{d(1+n) \left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$
risch	$\frac{B(e^{i(dx+c)})^{-n} (\sin(dx) \cos(c) + \cos(dx) \sin(c)) (e^{i(dx+c)} + 1)^{2n} a^n \left(\frac{1}{2}\right)^n e^{-\frac{i\pi n}{2} (\text{csgn}(i(e^{2i(dx+c)} + 2e^{i(dx+c)} + 1)) \text{csgn}(i(e^{i(dx+c)} + 1)))}}{d(1+n)}$

[In] int((a+cos(d*x+c)*a)^n*(-B*n/(1+n)+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] B/d/(1+n)*(a*(1+cos(d*x+c)))^n*sin(d*x+c)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (a + a \cos(c + dx))^n \left(-\frac{Bn}{1+n} + B \cos(c + dx) \right) dx = \frac{(a \cos(dx + c) + a)^n B \sin(dx + c)}{dn + d}$$

[In] integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="fricas")

[Out] (a*cos(d*x + c) + a)^n*B*sin(d*x + c)/(d*n + d)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(24) = 48$.

Time = 1.69 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.07

$$\int (a + a \cos(c + dx))^n \left(-\frac{Bn}{1+n} + B \cos(c + dx) \right) dx$$

$$= \begin{cases} \frac{2B \left(a - \frac{a \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right)}{\tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 1} + \frac{a}{\tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 1} \right)^n \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{dn \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + dn + d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + d} & \text{for } d \neq 0 \\ x(a \cos(c) + a)^n \left(-\frac{Bn}{n+1} + B \cos(c) \right) & \text{otherwise} \end{cases}$$

[In] integrate((a+a*cos(d*x+c))**n*(-B*n/(1+n)+B*cos(d*x+c)),x)

[Out] Piecewise((2*B*(a - a*tan(c/2 + d*x/2)**2/(tan(c/2 + d*x/2)**2 + 1) + a/(tan(c/2 + d*x/2)**2 + 1))**n*tan(c/2 + d*x/2)/(d*n*tan(c/2 + d*x/2)**2 + d*n + d*tan(c/2 + d*x/2)**2 + d), Ne(d, 0)), (x*(a*cos(c) + a)**n*(-B*n/(n + 1) + B*cos(c)), True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(28) = 56$.

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.11

$$\int (a + a \cos(c + dx))^n \left(-\frac{Bn}{1+n} + B \cos(c + dx) \right) dx =$$

$$\frac{(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1)^n B a^n \sin(-(dx + c)(n + 1) + 2n \arctan(\sin(dx + c), \cos(dx + c) + 1)) - (\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \cos(dx + c) + 1)^n B a^n \sin(-(dx + c)(n - 1) + 2n \arctan(\sin(dx + c), \cos(dx + c) + 1))}{2^n d (n + 1)}$$

[In] integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="maxima")

[Out] -1/2*((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*B*a^n*sin(-(d*x + c)*(n + 1) + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1)) - (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*B*a^n*sin(-(d*x + c)*(n - 1) + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1)))/(2^n*d*(n + 1))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1370 vs. 2(28) = 56.

Time = 7.95 (sec) , antiderivative size = 1370, normalized size of antiderivative = 48.93

$$\int (a + a \cos(c + dx))^n \left(-\frac{Bn}{1+n} + B \cos(c + dx) \right) dx = \text{Too large to display}$$

```
[In] integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="giac")
[Out] -2*(B*(sqrt(-tan(d*x + c)^4*tan(1/2*d*x + 1/2*c)^4 + 2*tan(d*x + c)^4*tan(1/2*d*x + 1/2*c)^2 - tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^4 + 3*tan(d*x + c)^4 + 6*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 7*tan(d*x + c)^2 + 4*tan(1/2*d*x + 1/2*c)^2 + 4)*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))^n*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c) - B*(sqrt(-tan(d*x + c)^4*tan(1/2*d*x + 1/2*c)^4 + 2*tan(d*x + c)^4*tan(1/2*d*x + 1/2*c)^2 - tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^4 + 3*tan(d*x + c)^4 + 6*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 7*tan(d*x + c)^2 + 4*tan(1/2*d*x + 1/2*c)^2 + 4)*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))^n*tan(1/2*d*x + 1/2*c))/(d*n*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2 + d*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2 + d*n*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi
```



```

n*sgn(tan(1/2*d*x + 1/2*c)))^2 + d*n*tan(1/2*d*x + 1/2*c)^2 + d*tan(-1/4*pi
*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) +
  1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi
i*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x +
  1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c
)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*
c)))^2 + d*tan(1/2*d*x + 1/2*c)^2 + d*n + d)

```

Mupad [B] (verification not implemented)

Time = 14.65 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a + a \cos(c + dx))^n \left(-\frac{Bn}{1+n} + B \cos(c + dx) \right) dx = \frac{B \sin(c + dx) (a (\cos(c + dx) + 1))^n}{d (n + 1)}$$

```
[In] int((B*cos(c + d*x) - (B*n)/(n + 1))*(a + a*cos(c + d*x))^n,x)
```

```
[Out] (B*sin(c + d*x)*(a*(cos(c + d*x) + 1))^n)/(d*(n + 1))
```

$$3.783 \quad \int \frac{-\frac{3B}{2} + B \cos(c+dx)}{(a+a \cos(c+dx))^3} dx$$

Optimal result	6958
Rubi [A] (verified)	6958
Mathematica [A] (verified)	6959
Maple [A] (verified)	6959
Fricas [B] (verification not implemented)	6959
Sympy [B] (verification not implemented)	6960
Maxima [B] (verification not implemented)	6960
Giac [A] (verification not implemented)	6960
Mupad [B] (verification not implemented)	6961

Optimal result

Integrand size = 27, antiderivative size = 26

$$\int \frac{-\frac{3B}{2} + B \cos(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{B \sin(c+dx)}{2d(a+a \cos(c+dx))^3}$$

[Out] $-1/2*B*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^3$

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2828}

$$\int \frac{-\frac{3B}{2} + B \cos(c+dx)}{(a+a \cos(c+dx))^3} dx = -\frac{B \sin(c+dx)}{2d(a \cos(c+dx) + a)^3}$$

[In] $\text{Int}[((-3*B)/2 + B*\text{Cos}[c + d*x])/(a + a*\text{Cos}[c + d*x])^3, x]$

[Out] $-1/2*(B*\text{Sin}[c + d*x])/(d*(a + a*\text{Cos}[c + d*x])^3)$

Rule 2828

$\text{Int}[(a_+ + (b_+)*\sin[(e_+) + (f_+)*(x_+)])^{(m_+)}*((c_+) + (d_+)*\sin[(e_+) + (f_+)*(x_+)])], x_Symbol] \rightarrow \text{Simp}[(-d)*\text{Cos}[e + f*x]*((a + b*\text{Sin}[e + f*x])^m/(f*(m + 1))), x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[a*d*m + b*c*(m + 1), 0]$

Rubi steps

$$\text{integral} = -\frac{B \sin(c+dx)}{2d(a+a \cos(c+dx))^3}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{-\frac{3B}{2} + B \cos(c + dx)}{(a + a \cos(c + dx))^3} dx = -\frac{B \sin(c + dx)}{2a^3 d (1 + \cos(c + dx))^3}$$

[In] Integrate[((-3*B)/2 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]

[Out] -1/2*(B*Sin[c + d*x])/(a^3*d*(1 + Cos[c + d*x])^3)

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

method	result	size
parallelrisch	$-\frac{\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sec^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) B}{8a^3 d}$	30
risch	$\frac{2iB(e^{3i(dx+c)} - e^{2i(dx+c)})}{d a^3 (e^{i(dx+c)} + 1)^5}$	45
derivativedivides	$\frac{B\left(-\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d a^3}$	48
default	$\frac{B\left(-\left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d a^3}$	48
norman	$-\frac{B \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8ad} - \frac{3B \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8ad} - \frac{3B \left(\tan^5\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8ad} - \frac{B \left(\tan^7\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8ad}$ $\frac{1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{a^2}$	99

[In] int((-3/2*B+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x,method=_RETURNVERBOSE)

[Out] -1/8*tan(1/2*d*x+1/2*c)*sec(1/2*d*x+1/2*c)^4*B/a^3/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(24) = 48.

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int \frac{-\frac{3B}{2} + B \cos(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= -\frac{B \sin(dx + c)}{2(a^3 d \cos(dx + c)^3 + 3a^3 d \cos(dx + c)^2 + 3a^3 d \cos(dx + c) + a^3 d)}$$

[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")

[Out] -1/2*B*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(24) = 48$.

Time = 0.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.08

$$\int \frac{-\frac{3B}{2} + B \cos(c + dx)}{(a + a \cos(c + dx))^3} dx = \begin{cases} -\frac{B \tan^5\left(\frac{c}{2} + \frac{dx}{2}\right)}{8a^3d} - \frac{B \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{4a^3d} - \frac{B \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8a^3d} & \text{for } d \neq 0 \\ \frac{x\left(B \cos(c) - \frac{3B}{2}\right)}{(a \cos(c) + a)^3} & \text{otherwise} \end{cases}$$

[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)

[Out] Piecewise((-B*tan(c/2 + d*x/2)**5/(8*a**3*d) - B*tan(c/2 + d*x/2)**3/(4*a**3*d) - B*tan(c/2 + d*x/2)/(8*a**3*d), Ne(d, 0)), (x*(B*cos(c) - 3*B/2)/(a*cos(c) + a)**3, True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(24) = 48$.

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 4.42

$$\int \frac{-\frac{3B}{2} + B \cos(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= -\frac{B \left(\frac{15 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{a^3} - \frac{2B \left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{a^3}$$

$$40d$$

[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")

[Out] -1/40*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 2*B*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{-\frac{3B}{2} + B \cos(c + dx)}{(a + a \cos(c + dx))^3} dx$$

$$= -\frac{B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 2B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + B \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{8a^3d}$$

[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac")

[Out] -1/8*(B*tan(1/2*d*x + 1/2*c)^5 + 2*B*tan(1/2*d*x + 1/2*c)^3 + B*tan(1/2*d*x + 1/2*c))/(a^3*d)

Mupad [B] (verification not implemented)

Time = 14.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{-\frac{3B}{2} + B \cos(c + dx)}{(a + a \cos(c + dx))^3} dx = -\frac{B \tan\left(\frac{c}{2} + \frac{dx}{2}\right) \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1\right)^2}{8 a^3 d}$$

[In] int(-((3*B)/2 - B*cos(c + d*x))/(a + a*cos(c + d*x))^3,x)

[Out] -(B*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 + 1)^2)/(8*a^3*d)

3.784 $\int (a + a \cos(c + dx))^{3/2} \left(-\frac{3B}{5} + B \cos(c + dx)\right) dx$

Optimal result	6962
Rubi [A] (verified)	6962
Mathematica [A] (verified)	6963
Maple [A] (verified)	6963
Fricas [A] (verification not implemented)	6963
Sympy [F]	6964
Maxima [B] (verification not implemented)	6964
Giac [B] (verification not implemented)	6964
Mupad [F(-1)]	6965

Optimal result

Integrand size = 29, antiderivative size = 28

$$\int (a + a \cos(c + dx))^{3/2} \left(-\frac{3B}{5} + B \cos(c + dx)\right) dx = \frac{2B(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d}$$

[Out] $2/5*B*(a+a*\cos(d*x+c))^(3/2)*\sin(d*x+c)/d$

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2828}

$$\int (a + a \cos(c + dx))^{3/2} \left(-\frac{3B}{5} + B \cos(c + dx)\right) dx = \frac{2B \sin(c + dx)(a \cos(c + dx) + a)^{3/2}}{5d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^(3/2)*((-3*B)/5 + B*\text{Cos}[c + d*x]),x]$

[Out] $(2*B*(a + a*\text{Cos}[c + d*x])^(3/2)*\text{Sin}[c + d*x])/(5*d)$

Rule 2828

$\text{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_)]))^(m_)*((c_ + (d_)*\sin[(e_ + (f_)*(x_)]))^(m_ + 1))], x_Symbol] \rightarrow \text{Simp}[(-d)*\text{Cos}[e + f*x]*((a + b*\text{Sin}[e + f*x])^(m/(m + 1))), x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, m\}, x$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{EqQ}[a^2 - b^2, 0]$ && $\text{EqQ}[a*d*m + b*c*(m + 1), 0]$

Rubi steps

$$\text{integral} = \frac{2B(a + a \cos(c + dx))^{3/2} \sin(c + dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int (a + a \cos(c + dx))^{3/2} \left(-\frac{3B}{5} + B \cos(c + dx) \right) dx = \frac{8aB \cos^3\left(\frac{1}{2}(c + dx)\right) \sqrt{a(1 + \cos(c + dx))} \sin\left(\frac{1}{2}(c + dx)\right)}{5d}$$

[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*((-3*B)/5 + B*Cos[c + d*x]),x]

[Out] (8*a*B*Cos[(c + d*x)/2]^3*Sqrt[a*(1 + Cos[c + d*x])]*Sin[(c + d*x)/2])/(5*d)

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

method	result
default	$\frac{8 \left(\cos^5\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a^2 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) B \sqrt{2}}{5 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)} d}$
parts	$\frac{4B \cos\left(\frac{dx}{2} + \frac{c}{2}\right) a^2 \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(2 \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) + \cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 2 \right) \sqrt{2}}{5 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)} d} - \frac{4B a^2 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) + 2 \right) \sqrt{2}}{5 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)} d}$

[In] int((a+cos(d*x+c)*a)^(3/2)*(-3/5*B+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 8/5*cos(1/2*d*x+1/2*c)^5*a^2*sin(1/2*d*x+1/2*c)*B*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int (a + a \cos(c + dx))^{3/2} \left(-\frac{3B}{5} + B \cos(c + dx) \right) dx = \frac{2 (Ba \cos(dx + c) + Ba) \sqrt{a \cos(dx + c) + a} \sin(dx + c)}{5d}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="fricas")

[Out] 2/5*(B*a*cos(d*x + c) + B*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/d

Sympy [F]

$$\int (a + a \cos(c + dx))^{3/2} \left(-\frac{3B}{5} + B \cos(c + dx) \right) dx = \frac{B \left(\int \left(-3a \sqrt{a \cos(c + dx) + a} \right) dx + \int 2a \sqrt{a \cos(c + dx) + a} \cos(c + dx) dx + \int 5a \right)}{5}$$

[In] integrate((a+a*cos(d*x+c))**(3/2)*(-3/5*B+B*cos(d*x+c)),x)

[Out] B*(Integral(-3*a*sqrt(a*cos(c + d*x) + a), x) + Integral(2*a*sqrt(a*cos(c + d*x) + a)*cos(c + d*x), x) + Integral(5*a*sqrt(a*cos(c + d*x) + a)*cos(c + d*x)**2, x))/5

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(24) = 48.

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.29

$$\int (a + a \cos(c + dx))^{3/2} \left(-\frac{3B}{5} + B \cos(c + dx) \right) dx = \frac{(\sqrt{2}a \sin(\frac{5}{2} dx + \frac{5}{2} c) + 5 \sqrt{2}a \sin(\frac{3}{2} dx + \frac{3}{2} c) + 20 \sqrt{2}a \sin(\frac{1}{2} dx + \frac{1}{2} c)) B \sqrt{a} - 2(\sqrt{2}a \sin(\frac{5}{2} dx + \frac{5}{2} c) + 5 \sqrt{2}a \sin(\frac{3}{2} dx + \frac{3}{2} c) + 20 \sqrt{2}a \sin(\frac{1}{2} dx + \frac{1}{2} c))}{10 d}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/10*((sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a) - 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(24) = 48.

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int (a + a \cos(c + dx))^{3/2} \left(-\frac{3B}{5} + B \cos(c + dx) \right) dx = \frac{\sqrt{2}(B \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{5}{2} dx + \frac{5}{2} c) + 3 B \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{3}{2} dx + \frac{3}{2} c) + 2 B \operatorname{sgn}(\cos(\frac{1}{2} dx + \frac{1}{2} c)) \sin(\frac{1}{2} dx + \frac{1}{2} c)) \sqrt{a}}{10 d}$$

[In] integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="giac")

[Out] 1/10*sqrt(2)*(B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c) + 3*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + 2*B*a*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*sqrt(a)/d

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{3/2} \left(-\frac{3B}{5} + B \cos(c + dx) \right) dx = \int -\left(\frac{3B}{5} - B \cos(c + dx) \right) (a + a \cos(c + dx))^{3/2} dx$$

```
[In] int(-((3*B)/5 - B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

```
[Out] int(-((3*B)/5 - B*cos(c + d*x))*(a + a*cos(c + d*x))^(3/2), x)
```

$$3.785 \quad \int \frac{B+B \cos(c+dx)}{\sqrt{a+a \cos(c+dx)}} dx$$

Optimal result	6966
Rubi [A] (verified)	6966
Mathematica [A] (verified)	6967
Maple [A] (verified)	6967
Fricas [A] (verification not implemented)	6968
Sympy [F]	6968
Maxima [B] (verification not implemented)	6968
Giac [A] (verification not implemented)	6981
Mupad [B] (verification not implemented)	6981

Optimal result

Integrand size = 25, antiderivative size = 26

$$\int \frac{B + B \cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{2B \sin(c + dx)}{d\sqrt{a + a \cos(c + dx)}}$$

[Out] $2*B*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {21, 2725}

$$\int \frac{B + B \cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{2B \sin(c + dx)}{d\sqrt{a \cos(c + dx) + a}}$$

[In] `Int[(B + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]`

[Out] `(2*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])`

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

Q[a² - b², 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{B \int \sqrt{a + a \cos(c + dx)} dx}{a} \\ &= \frac{2B \sin(c + dx)}{d \sqrt{a + a \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{B + B \cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{2B \sqrt{a(1 + \cos(c + dx))} \tan\left(\frac{1}{2}(c + dx)\right)}{ad}$$

[In] Integrate[(B + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]

[Out] (2*B*Sqrt[a*(1 + Cos[c + d*x]])*Tan[(c + d*x)/2])/(a*d)

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result
default	$\frac{2B \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2}}{\sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$
risch	$-\frac{iB\sqrt{2} (e^{i(dx+c)} - 1)(1 + e^{-i(dx+c)})}{\sqrt{a(e^{i(dx+c)} + 1)^2 e^{-i(dx+c)}} d}$
parts	$\frac{B\sqrt{2} \operatorname{am}^{-1}\left(\frac{dx}{2} + \frac{c}{2} 1\right)}{d \operatorname{sec}\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \operatorname{csgn}\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - \frac{B \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(\sqrt{2} \ln\left(\frac{4\sqrt{a} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4a}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right)\right)}{a^{\frac{3}{2}} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d}$

[In] int((B+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{B + B \cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{2 \sqrt{a \cos(dx + c) + a} B \sin(dx + c)}{ad \cos(dx + c) + ad}$$

[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c)/(a*d*cos(d*x + c) + a*d)

Sympy [F]

$$\int \frac{B + B \cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = B \left(\int \frac{\cos(c + dx)}{\sqrt{a \cos(c + dx) + a}} dx + \int \frac{1}{\sqrt{a \cos(c + dx) + a}} dx \right)$$

[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)

[Out] B*(Integral(cos(c + d*x)/sqrt(a*cos(c + d*x) + a), x) + Integral(1/sqrt(a*cos(c + d*x) + a), x))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19040 vs. 2(24) = 48.

Time = 0.60 (sec) , antiderivative size = 19040, normalized size of antiderivative = 732.31

$$\int \frac{B + B \cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \text{Too large to display}$$

[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

```
[Out] 1/12*(6*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/sqrt(a) - (12*sqrt(2)*cos(3/2*d*x + 3/2*c)^3*sin(d*x + c) - 12*(sqrt(2)*cos(d*x + c) + sqrt(2))*sin(3/2*d*x + 3/2*c)^3 - 8*sqrt(2)*sin(1/2*d*x + 1/2*c)^3 + ((3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*cos(d*x + c)^2 + (3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*

```

$$\begin{aligned}
& \sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 \\
& + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\
& * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + \\
& 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2* \\
& c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\
& 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d* \\
& x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + \\
& 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c)^2 + (12*\sqrt{2}*\cos(3/2*d*x + 3/2*c))^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos \\
& (d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c))^3 - 8*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c))^3 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + \\
& 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(\\
& 1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x \\
& + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2 \\
& *\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2 \\
& *d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2 \\
& *c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/ \\
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))^3 - \\
& 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2 \\
& *c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(\\
& 1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + \\
& 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(c \\
& \cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + \\
& 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})
\end{aligned}$$

$$\begin{aligned} & * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\ & c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos \\ & (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\ &) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1 \\ & /2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12* \\ & \sqrt{2}*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + \\ & 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2} \\ & *\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2* \\ & c) + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos \\ & (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\ & - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\ & *d*x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 \\ & - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\ & ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos \\ & (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\ & *\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\ & *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2 \\ & *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + \\ & 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/ \\ & 2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\ & + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c) \\ & ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\ & 2*c)^2 - 2*((8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2} \\ &)*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/ \\ & 2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2 \\ & *\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2} \\ &)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + \\ & 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})* \\ & \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c \\ &) + 1))*\cos(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2* \\ & c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1 \\ & /2*c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2* \\ & \sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\ & x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + \\ & c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\ & (1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\ & + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2} \\ &)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c)^2))*\sin(d*x + c))*\cos \\ & (3/2*d*x + 3/2*c) - 2*(8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos \\ & (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) \\ & - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\ & d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/ \\ & 2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log \\ & (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + \\ & 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \end{aligned}$$

$$\begin{aligned}
&) * \sin(1/2*d*x + 1/2*c) * \cos(d*x + c) - 2*(6*(\sqrt{2})*\cos(d*x + c) + \sqrt{2}) \\
&) * \cos(3/2*d*x + 3/2*c)^2 + (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c)^2 + \\
& (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\cos(d*x + c) - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2}))*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\cos(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + ((3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32*\sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2}))*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 3*(\sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 12*((\sqrt{2})*\cos(d*x + c)^2 + \sqrt{2})*\sin(d*x + c)^2 + 2*\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
& 2\sqrt{2}\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3\sqrt{2})\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 20\sqrt{2}\sin(1/2*d*x + 1/2*c)\cos(d*x + c) + 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3\sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 32\sqrt{2}\sin(1/2*d*x + 1/2*c)\sin(3/2*d*x + 3/2*c)^2 - (8\sqrt{2}\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)\sin(d*x + c)^2 + 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 - 2*((8\sqrt{2})\cos(1/2*d*x + 1/2*c)\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)\cos(d*x + c)^2 + (8\sqrt{2})\cos(1/2*d*x + 1/2*c)\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)\cos(d*x + c) - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)^2)\sin(d*x + c)\cos(3/2*d*x + 3/2*c) - 2*(8\sqrt{2})\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2\sqrt{2})\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})\sin(1/2*d*x + 1/2*c)\cos(d*x + c) - 2*(6*(\sqrt{2})\cos(d*x + c) + \sqrt{2})\cos(3/2*d*x + 3/2*c)^2 + (8\sqrt{2})\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2\sqrt{2})\cos(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& + (8\sqrt{2})\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/ \\
& /2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*s \\
& \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(d*x + c)^2 + 6*\sqrt{2})*\cos(1/2*d*x + 1 \\
& /2*c)^2 + 14*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2})*\cos(d*x + c)*\cos(\\
& 1/2*d*x + 1/2*c) - \sqrt{2})*\sin(d*x + c)*\sin(1/2*d*x + 1/2*c) + \sqrt{2})*\cos(\\
& 1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^ \\
& 2 + 11*\sqrt{2})*\sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos \\
& (1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) \\
& *\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\cos(d*x + c) - 3*(\sqrt{2})\log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (2)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/ \\
& 2*c) + 1))*\sin(1/2*d*x + 1/2*c) + 2*\sqrt{2})*\sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\sin(2/3*\arctan \\
& 2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 - 2*((8*\sqrt{2})*\cos(1/2*d \\
& *x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(\\
& 1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2* \\
& d*x + 1/2*c) - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin \\
& (1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\sin \\
& (d*x + c)^2 + 8*\sqrt{2})*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2} \\
&)*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})\log(\cos(1/2*d \\
& *x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (2)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1 \\
& /2*c) + 1))*\cos(1/2*d*x + 1/2*c))*\cos(d*x + c) - 3*(\sqrt{2})\log(\cos(1/2*d*x \\
& + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} \\
& (2)\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2 \\
& *c) + 1))*\cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} \\
& (2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c))*\cos(3/2*d*x + 3/2*c) - 2*(8*\sqrt{2} \\
&)*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d \\
& *x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + \\
& 1/2*c)^2 - 3*(\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 \\
& + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/ \\
& 2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4* \\
& (2*\sqrt{2})*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x \\
& + c) + 2*(12*\sqrt{2})*\cos(3/2*d*x + 3/2*c)^3*\sin(d*x + c) - 12*(\sqrt{2})*\cos(\\
& d*x + c) + \sqrt{2})*\sin(3/2*d*x + 3/2*c)^3 - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c) \\
& ^3 + ((3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2})*\sin(1/2*d*x + 1/2*c) \\
&)*\cos(d*x + c)^2 + (3*\sqrt{2})\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/
\end{aligned}$$

$$\begin{aligned}
& 2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2 \\
& *d*x + 1/2*c))*\sin(d*x + c)^2 + 24*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c \\
&) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d* \\
& x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c) \\
&)*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c \\
&)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \\
& \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d* \\
& x + 1/2*c))*\cos(3/2*d*x + 3/2*c)^2 - (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3* \\
& (\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c) \\
& ^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(c \\
& os(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1 \\
&) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2 \\
& *d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1))*\cos(1/2*d*x + 1/2*c)^2 + ((3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c)^2 + (3*\sqrt{2}*\log(\cos(\\
& 1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& 3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2* \\
& d*x + 1/2*c) + 1) - 8*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(d*x + c)^2 + 12*\sqrt{2} \\
& *t(2)*\cos(3/2*d*x + 3/2*c)*\sin(d*x + c) + 2*(3*\sqrt{2}*\log(\cos(1/2*d*x + 1/2 \\
& *c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3*\sqrt{2}*\lo \\
& g(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1) - 20*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\cos(d*x + c) + 3*\sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - 3 \\
& *\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d* \\
& x + 1/2*c) + 1) - 32*\sqrt{2}*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c)^2 - \\
& (8*\sqrt{2}*\sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/ \\
& 2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\co \\
& s(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x \\
& + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c) \\
& ^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/ \\
& 2*c)^2 + 4*(2*\sqrt{2}*\cos(1/2*d*x + 1/2*c)^2 + \sqrt{2})*\sin(1/2*d*x + 1/2*c) \\
&))*\sin(d*x + c)^2 + 3*(\sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2}*\log(\cos(1/2*d*x + 1/2*c)^2 \\
& + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1))*\sin(1/2*d*x + 1/2*c \\
&)^2 - 2*((8*\sqrt{2}*\cos(1/2*d*x + 1/2*c)*\sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2})* \\
& \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c)
\end{aligned}$$

$$\begin{aligned}
&) + 1) - \sqrt{2} \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c)^2 + (8*\sqrt{2}) * \\
& \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log \\
& (\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \sin(d*x + c)^2 + 8*\sqrt{2} * \cos(1/2*d*x + 1/2*c) * \\
& \sin(1/2*d*x + 1/2*c) + 2*(8*\sqrt{2}) * \cos(1/2*d*x + 1/2*c) * \sin(1/2*d*x + 1/2*c) - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin \\
& (1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)) * \cos(d*x + c) \\
& - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1 \\
& /2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c) - 6*(\sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 * \sin(d*x + c)) * \cos(3/ \\
& 2*d*x + 3/2*c) - 2*(8*\sqrt{2}) * \sin(1/2*d*x + 1/2*c)^3 - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \\
& \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \cos(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c) \\
&)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1) \\
&) * \sin(1/2*d*x + 1/2*c)^2 + 4*(2*\sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c)) * \cos(d*x + c) - 2*(6*(\sqrt{2}) * \cos(d*x + c) + \sqrt{2}) * \cos \\
& (3/2*d*x + 3/2*c)^2 + (8*\sqrt{2}) * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + \\
& 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \cos(d*x + c)^2 + (8* \\
& \sqrt{2}) * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d* \\
& x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \sin(d*x + c)^2 + 6*\sqrt{2} * \cos(1/2*d*x + 1/2*c) \\
& ^2 + 14*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 + 12*(\sqrt{2}) * \cos(d*x + c) * \cos(1/2*d*x + 1/2*c) - \sqrt{2} * \sin(d*x + c) * \sin(1/2*d*x + 1/2*c) + \sqrt{2} * \cos(1/2*d \\
& *x + 1/2*c)) * \cos(3/2*d*x + 3/2*c) + 2*(3*\sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + 1 \\
& 1*\sqrt{2} * \sin(1/2*d*x + 1/2*c)^2 - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d* \\
& d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \cos(d*x + c) - 3*(\sqrt{2}) * \log(\cos(1/2*d*x + 1 \\
& /2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 + 2*\sin(1/2*d*x + 1/2*c) + 1) - \sqrt{2} * \log(\cos(1/2*d*x + 1/2*c)^2 + \sin(1/2*d*x + 1/2*c)^2 - 2*\sin(1/2*d*x + 1/2*c) \\
& + 1)) * \sin(1/2*d*x + 1/2*c) + 2*\sqrt{2} * \sin(3/2*d*x + 3/2*c) - 4*(2*\sqrt{2}) \\
& * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x + 1/2*c) - 6*((\sqrt{2}) * \cos(d \\
& *x + c)^2 + \sqrt{2} * \sin(d*x + c)^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2}) * \cos(\\
& 3/2*d*x + 3/2*c)^2 + (\sqrt{2}) * \cos(1/2*d*x + 1/2*c)^2 + \sqrt{2} * \sin(1/2*d*x \\
& + 1/2*c)^2 * \cos(d*x + c)^2 + (\sqrt{2}) * \cos(d*x + c)^2 + \sqrt{2} * \sin(d*x + c) \\
& ^2 + 2*\sqrt{2} * \cos(d*x + c) + \sqrt{2} * \sin(3/2*d*x + 3/2*c)^2 + (\sqrt{2}) * \cos
\end{aligned}$$

$$\begin{aligned}
& s(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2*\sin(d*x + c)^2 + \text{sqr} \\
& t(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*c \\
& \cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + \\
& c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + \\
& 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2) \\
& *\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d \\
& *x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d \\
& *x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + \\
& 3/2*c))*\sin(1/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))*\cos(2 \\
& /3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(3/2*d*x + 3/2*c))) + 3*((\text{sqrt}(2)*\cos(d \\
& *x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(\\
& 3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2)*\cos(d*x + c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c \\
&)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*c \\
& \cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (s \\
& \text{qrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + s \\
& \text{qrt}(2))*\sin(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)* \\
& \sin(1/2*d*x + 1/2*c)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + s \\
& \text{qrt}(2)*\sin(1/2*d*x + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2 \\
& *c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)* \\
& \cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + \\
& 2*(\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d* \\
& x + c) + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + \\
& c)^2*\sin(1/2*d*x + 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \\
& \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\cos(2/3*\arctan2(\sin(3/2 \\
& *d*x + 3/2*c), \cos(3/2*d*x + 3/2*c)))^2 + (\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2) \\
& *\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x + 3/2*c)^2 \\
& + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\sin(d*x \\
& + c)^2 + ((\text{sqrt}(2)*\cos(d*x + c)^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos \\
& (d*x + c) + \text{sqrt}(2))*\cos(3/2*d*x + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c) \\
& ^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c)^2 + (\text{sqrt}(2)*\cos(d*x + c) \\
& ^2 + \text{sqrt}(2)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c) + \text{sqrt}(2))*\sin(3/2*d*x \\
& + 3/2*c)^2 + (\text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c \\
&)^2)*\sin(d*x + c)^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(1 \\
& /2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2*c \\
&) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/2 \\
& *d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x + 1/2*c)^2)*\cos(d*x + c) + 2*(\text{sqrt}(2) \\
& *\cos(d*x + c)^2*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(d*x + c)^2*\sin(1/2*d*x + \\
& 1/2*c) + 2*\text{sqrt}(2)*\cos(d*x + c)*\sin(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c))*\sin(3/2*d*x + 3/2*c))*\sin(2/3*\arctan2(\sin(3/2*d*x + 3/2*c), \cos(\\
& 3/2*d*x + 3/2*c)))^2 + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c)^2 + \text{sqrt}(2)*\sin(1/2*d*x \\
& + 1/2*c)^2 + 2*(\text{sqrt}(2)*\cos(d*x + c)^2*\cos(1/2*d*x + 1/2*c) + \text{sqrt}(2)*\cos(\\
& 1/2*d*x + 1/2*c)*\sin(d*x + c)^2 + 2*\text{sqrt}(2)*\cos(d*x + c)*\cos(1/2*d*x + 1/2* \\
& c) + \text{sqrt}(2)*\cos(1/2*d*x + 1/2*c))*\cos(3/2*d*x + 3/2*c) + 2*(\text{sqrt}(2)*\cos(1/
\end{aligned}$$

$$\begin{aligned}
& 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c) \\
&)\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/ \\
& 2dx + 1/2c)^2)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/ \\
& 2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c) \\
&)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c) \\
&)\sin(2/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 + \sqrt{2}\cos \\
& (1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx \\
& + c)^2\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 \\
& + 2\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c \\
&))\cos(3/2dx + 3/2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1 \\
& /2dx + 1/2c)^2)\cos(dx + c) + 2((\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx \\
& + c)^2 + 2\sqrt{2}\cos(dx + c) + \sqrt{2}))\cos(3/2dx + 3/2c)^2 + (\sqrt{2} \\
&)\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2)\cos(dx + c) \\
& ^2 + (\sqrt{2}\cos(dx + c)^2 + \sqrt{2}\sin(dx + c)^2 + 2\sqrt{2}\cos(dx + \\
& c) + \sqrt{2})\sin(3/2dx + 3/2c)^2 + (\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2} \\
&)\sin(1/2dx + 1/2c)^2)\sin(dx + c)^2 + \sqrt{2}\cos(1/2dx + 1/2c \\
&)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2 + 2(\sqrt{2}\cos(dx + c)^2\cos(1/2dx \\
& + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c)\sin(dx + c)^2 + 2\sqrt{2}\cos(dx \\
& + c)\cos(1/2dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3 \\
& /2c) + 2(\sqrt{2}\cos(1/2dx + 1/2c)^2 + \sqrt{2}\sin(1/2dx + 1/2c)^2) \\
&)\cos(dx + c) + 2(\sqrt{2}\cos(dx + c)^2\sin(1/2dx + 1/2c) + \sqrt{2}\sin \\
& (dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2}\cos(dx + c)\sin(1/2dx + 1/ \\
& 2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2dx + 3/2c))\cos(2/3\arctan2(\\
& \sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + 2(\sqrt{2}\cos(dx + c)^2\sin \\
& (1/2dx + 1/2c) + \sqrt{2}\sin(dx + c)^2\sin(1/2dx + 1/2c) + 2\sqrt{2} \\
&)\cos(dx + c)\sin(1/2dx + 1/2c) + \sqrt{2}\sin(1/2dx + 1/2c))\sin(3/2 \\
& dx + 3/2c))\log(\cos(1/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2 \\
& c)))^2 + \sin(1/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c)))^2 - 2 \\
&)\sin(1/3\arctan2(\sin(3/2dx + 3/2c), \cos(3/2dx + 3/2c))) + 1) - 2(6(\\
& \sqrt{2}\cos(dx + c) + \sqrt{2})\cos(3/2dx + 3/2c)^2 + (8\sqrt{2}\sin(1/2 \\
& dx + 1/2c)^2 - 3(\sqrt{2}\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2 \\
& c)^2 + 2\sin(1/2dx + 1/2c) + 1) - \sqrt{2}\log(\cos(1/2dx + 1/2c)^2 + \\
& \sin(1/2dx + 1/2c)^2 - 2\sin(1/2dx + 1/2c) + 1))\sin(1/2dx + 1/2c) \\
& + 2\sqrt{2})\cos(dx + c)^2 + (8\sqrt{2}\sin(1/2dx + 1/2c)^2 - 3(\sqrt{2} \\
&)\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1/2 \\
& c) + 1) - \sqrt{2}\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - 2 \\
&)\sin(1/2dx + 1/2c) + 1))\sin(1/2dx + 1/2c) + 2\sqrt{2})\sin(dx + c)^2 \\
& + 6\sqrt{2}\cos(1/2dx + 1/2c)^2 + 14\sqrt{2}\sin(1/2dx + 1/2c)^2 + 1 \\
& 2(\sqrt{2}\cos(dx + c)\cos(1/2dx + 1/2c) - \sqrt{2}\sin(dx + c)\sin(1/2 \\
& dx + 1/2c) + \sqrt{2}\cos(1/2dx + 1/2c))\cos(3/2dx + 3/2c) + 2(3\sqrt{2} \\
&)\cos(1/2dx + 1/2c)^2 + 11\sqrt{2}\sin(1/2dx + 1/2c)^2 - 3(\sqrt{2} \\
&)\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1/2dx + 1 \\
& /2c) + 1) - \sqrt{2}\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 - \\
& 2\sin(1/2dx + 1/2c) + 1))\sin(1/2dx + 1/2c) + 2\sqrt{2})\cos(dx + c) \\
& - 3(\sqrt{2}\log(\cos(1/2dx + 1/2c)^2 + \sin(1/2dx + 1/2c)^2 + 2\sin(1
\end{aligned}$$

)^2 + 2*((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*cos(3/2*d*x + 3/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c)^2 + (cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c)^2 + (cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*sin(d*x + c)^2 + 2*(cos(d*x + c)^2*cos(1/2*d*x + 1/2*c) + cos(1/2*d*x + 1/2*c)*sin(d*x + c)^2 + 2*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + cos(1/2*d*x + 1/2*c)*cos(3/2*d*x + 3/2*c) + 2*(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2)*cos(d*x + c) + cos(1/2*d*x + 1/2*c)^2 + 2*(cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + sin(1/2*d*x + 1/2*c)^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(cos(d*x + c)^2*sin(1/2*d*x + 1/2*c) + sin(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 2*cos(d*x + c)*sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + sin(1/2*d*x + 1/2*c)^2)*sqrt(a))/d

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{B + B \cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{2\sqrt{2}B \sin\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{a}d \operatorname{sgn}\left(\cos\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}$$

[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] 2*sqrt(2)*B*sin(1/2*d*x + 1/2*c)/(sqrt(a)*d*sgn(cos(1/2*d*x + 1/2*c)))

Mupad [B] (verification not implemented)

Time = 14.71 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{B + B \cos(c + dx)}{\sqrt{a + a \cos(c + dx)}} dx = \frac{2B \sin(c + dx) \sqrt{a (\cos(c + dx) + 1)}}{a d (\cos(c + dx) + 1)}$$

[In] int((B + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/2),x)

[Out] (2*B*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(a*d*(cos(c + d*x) + 1))

$$3.786 \quad \int \frac{-\frac{5B}{3} + B \cos(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx$$

Optimal result	6982
Rubi [A] (verified)	6982
Mathematica [A] (verified)	6983
Maple [A] (verified)	6983
Fricas [B] (verification not implemented)	6983
Sympy [F]	6984
Maxima [F(-1)]	6984
Giac [A] (verification not implemented)	6984
Mupad [B] (verification not implemented)	6985

Optimal result

Integrand size = 29, antiderivative size = 28

$$\int \frac{-\frac{5B}{3} + B \cos(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = -\frac{2B \sin(c+dx)}{3d(a+a \cos(c+dx))^{5/2}}$$

[Out] $-2/3*B*\sin(d*x+c)/d/(a+a*\cos(d*x+c))^{(5/2)}$

Rubi [A] (verified)

Time = 0.05 (sec), antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2828}

$$\int \frac{-\frac{5B}{3} + B \cos(c+dx)}{(a+a \cos(c+dx))^{5/2}} dx = -\frac{2B \sin(c+dx)}{3d(a \cos(c+dx) + a)^{5/2}}$$

[In] $\text{Int}[((-5*B)/3 + B*\text{Cos}[c + d*x])/(a + a*\text{Cos}[c + d*x])^{(5/2)}, x]$

[Out] $(-2*B*\text{Sin}[c + d*x])/(3*d*(a + a*\text{Cos}[c + d*x])^{(5/2)})$

Rule 2828

$\text{Int}[(a + (b_*)*\sin[(e_*) + (f_*)*(x_*)])^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)]), x_Symbol] \rightarrow \text{Simp}[(-d)*\text{Cos}[e + f*x]*((a + b*\sin[e + f*x])^{(m+1)})/(f*(m+1)), x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[a*d*m + b*c*(m+1), 0]$

Rubi steps

$$\text{integral} = -\frac{2B \sin(c+dx)}{3d(a+a \cos(c+dx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{-\frac{5B}{3} + B \cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = -\frac{2B \sin(c + dx)}{3d(a(1 + \cos(c + dx)))^{5/2}}$$

[In] Integrate[((-5*B)/3 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]

[Out] (-2*B*Sin[c + d*x])/(3*d*(a*(1 + Cos[c + d*x]))^(5/2))

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

method	result
default	$-\frac{\sin\left(\frac{dx}{2} + \frac{c}{2}\right) B \sqrt{2}}{6 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)^3 a^2 \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d$
parts	$\frac{B \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(5\sqrt{2} \ln\left(\frac{4\sqrt{a} \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 4a}}{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)}\right) a \left(\cos^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 5 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \sqrt{a \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \sqrt{2} \sqrt{a} - 2\sqrt{2}}{32 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)^3 a^{\frac{7}{2}} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{a \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} d$

[In] int((-5/3*B+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/6/cos(1/2*d*x+1/2*c)^3/a^2*sin(1/2*d*x+1/2*c)*B*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \frac{-\frac{5B}{3} + B \cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx =$$

$$-\frac{2 \sqrt{a \cos(dx + c) + a} B \sin(dx + c)}{3 (a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 + 3 a^3 d \cos(dx + c) + a^3 d)}$$

[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -2/3*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F]

$$\int \frac{-\frac{5B}{3} + B \cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \frac{B \left(\int \frac{3 \cos(c+dx)}{a^2 \sqrt{a \cos(c+dx)+a} \cos^2(c+dx)+2a^2 \sqrt{a \cos(c+dx)+a} \cos(c+dx)+a^2 \sqrt{a \cos(c+dx)+a}} dx + \int \right)}{3}$$

[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)

[Out] B*(Integral(3*cos(c + d*x)/(a**2*sqrt(a*cos(c + d*x) + a)*cos(c + d*x)**2 + 2*a**2*sqrt(a*cos(c + d*x) + a)*cos(c + d*x) + a**2*sqrt(a*cos(c + d*x) + a)), x) + Integral(-5/(a**2*sqrt(a*cos(c + d*x) + a)*cos(c + d*x)**2 + 2*a**2*sqrt(a*cos(c + d*x) + a)*cos(c + d*x) + a**2*sqrt(a*cos(c + d*x) + a)), x))/3

Maxima [F(-1)]

Timed out.

$$\int \frac{-\frac{5B}{3} + B \cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

Giac [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{-\frac{5B}{3} + B \cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = -\frac{\sqrt{2}B \sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{6 \left(\sin\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1\right)^2 a^{5/2} \operatorname{dsgn}\left(\cos\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}$$

[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] -1/6*sqrt(2)*B*sin(1/2*d*x + 1/2*c)/((sin(1/2*d*x + 1/2*c)^2 - 1)^2*a^(5/2)*d*sgn(cos(1/2*d*x + 1/2*c)))

Mupad [B] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.04

$$\int \frac{-\frac{5B}{3} + B \cos(c + dx)}{(a + a \cos(c + dx))^{5/2}} dx = \frac{8 B e^{c2i+dx2i} \sqrt{a + a \left(\frac{e^{-c1i-dx1i}}{2} + \frac{e^{c1i+dx1i}}{2} \right)} (e^{c1i+dx1i} 1i - i)}{3 a^3 d (e^{c1i+dx1i} + 1)^5}$$

[In] int(-((5*B)/3 - B*cos(c + d*x))/(a + a*cos(c + d*x))^(5/2),x)

[Out] (8*B*exp(c*2i + d*x*2i)*(a + a*(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*1i - 1i))/(3*a^3*d*(exp(c*1i + d*x*1i) + 1)^5)

3.787 $\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx$

Optimal result	6986
Rubi [A] (verified)	6986
Mathematica [A] (verified)	6988
Maple [F]	6988
Fricas [F]	6988
Sympy [F]	6989
Maxima [F]	6989
Giac [F]	6989
Mupad [F(-1)]	6989

Optimal result

Integrand size = 25, antiderivative size = 104

$$\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \frac{3B(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5d} + \frac{2\sqrt[6]{2}(5A + 2B)(a + a \cos(c + dx))^{2/3} \text{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{5d(1 + \cos(c + dx))^{7/6}}$$

[Out] $3/5*B*(a+a*\cos(d*x+c))^{(2/3)}*\sin(d*x+c)/d+2/5*2^{(1/6)}*(5*A+2*B)*(a+a*\cos(d*x+c))^{(2/3)}*\text{hypergeom}([-1/6, 1/2], [3/2], 1/2-1/2*\cos(d*x+c))*\sin(d*x+c)/d/(1+\cos(d*x+c))^{(7/6)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2731, 2730}

$$\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \frac{2\sqrt[6]{2}(5A + 2B) \sin(c + dx)(a \cos(c + dx) + a)^{2/3} \text{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right)}{5d(\cos(c + dx) + 1)^{7/6}} + \frac{3B \sin(c + dx)(a \cos(c + dx) + a)^{2/3}}{5d}$$

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(2/3)}*(A + B*\text{Cos}[c + d*x]), x]$

[Out] $(3*B*(a + a*\text{Cos}[c + d*x])^{(2/3)}*\text{Sin}[c + d*x])/(5*d) + (2*2^{(1/6)}*(5*A + 2*B)*(a + a*\text{Cos}[c + d*x])^{(2/3)}*\text{Hypergeometric2F1}[-1/6, 1/2, 3/2, (1 - \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/(5*d*(1 + \text{Cos}[c + d*x])^{(7/6)})$

Rule 2730

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-2^(n +
1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3B(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5d} + \frac{1}{5}(5A + 2B) \int (a + a \cos(c + dx))^{2/3} dx \\
&= \frac{3B(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5d} \\
&\quad + \frac{((5A + 2B)(a + a \cos(c + dx))^{2/3}) \int (1 + \cos(c + dx))^{2/3} dx}{5(1 + \cos(c + dx))^{2/3}} \\
&= \frac{3B(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5d} \\
&\quad + \frac{2\sqrt[6]{2}(5A + 2B)(a + a \cos(c + dx))^{2/3} \text{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{5d(1 + \cos(c + dx))^{7/6}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.58

$$\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \frac{(a(1 + \cos(c + dx)))^{2/3} \sec^2\left(\frac{1}{2}(c + dx)\right) \left(3 \cdot 2^{5/6} (5A + 4B + 2B \cos(c + dx)) \sqrt[6]{1 - \cos(c + dx)}\right)}{}$$

[In] Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]

[Out] ((a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*(3*2^(5/6)*(5*A + 4*B + 2*B*Cos[c + d*x])*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*Sin[c + d*x] - 2*(5*A + 2*B)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(d*x)/2 - ArcTan[Cot[c/2]]]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]]))/(20*2^(5/6)*d*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6))

Maple [F]

$$\int (a + \cos(dx + c) a)^{2/3} (A + B \cos(dx + c)) dx$$

[In] int((a+cos(d*x+c)*a)^(2/3)*(A+B*cos(d*x+c)),x)

[Out] int((a+cos(d*x+c)*a)^(2/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(a \cos(dx + c) + a)^{2/3} dx$$

[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)

Sympy [F]

$$\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \int (a(\cos(c + dx) + 1))^{2/3} (A + B \cos(c + dx)) dx$$

[In] integrate((a+a*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)

[Out] Integral((a*(cos(c + d*x) + 1))**(2/3)*(A + B*cos(c + d*x)), x)

Maxima [F]

$$\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(a \cos(dx + c) + a)^{2/3} dx$$

[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)

Giac [F]

$$\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(a \cos(dx + c) + a)^{2/3} dx$$

[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \int (A + B \cos(c + dx)) (a + a \cos(c + dx))^{2/3} dx$$

[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(2/3),x)

[Out] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(2/3), x)

3.788 $\int \sqrt[3]{a + a \cos(c + dx)}(A + B \cos(c + dx)) dx$

Optimal result	6990
Rubi [A] (verified)	6990
Mathematica [B] (verified)	6991
Maple [F]	6992
Fricas [F]	6992
Sympy [F]	6992
Maxima [F]	6993
Giac [F]	6993
Mupad [F(-1)]	6993

Optimal result

Integrand size = 25, antiderivative size = 102

$$\int \sqrt[3]{a + a \cos(c + dx)}(A + B \cos(c + dx)) dx = \frac{3B \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{4d} + \frac{(4A + B) \sqrt[3]{a + a \cos(c + dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{2\sqrt[6]{2d}(1 + \cos(c + dx))^{5/6}}$$

[Out] $3/4*B*(a+a*\cos(d*x+c))^{(1/3)}*\sin(d*x+c)/d+1/4*(4*A+B)*(a+a*\cos(d*x+c))^{(1/3)}*\operatorname{hypergeom}([1/6, 1/2], [3/2], 1/2-1/2*\cos(d*x+c))*\sin(d*x+c)*2^{(5/6)}/d/(1+\cos(d*x+c))^{(5/6)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2731, 2730}

$$\int \sqrt[3]{a + a \cos(c + dx)}(A + B \cos(c + dx)) dx = \frac{(4A + B) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right)}{2\sqrt[6]{2d}(\cos(c + dx) + 1)^{5/6}} + \frac{3B \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a}}{4d}$$

[In] $\operatorname{Int}[(a + a*\operatorname{Cos}[c + d*x])^{(1/3)}*(A + B*\operatorname{Cos}[c + d*x]), x]$

[Out] $(3*B*(a + a*\operatorname{Cos}[c + d*x])^{(1/3)}*\operatorname{Sin}[c + d*x])/(4*d) + ((4*A + B)*(a + a*\operatorname{Cos}[c + d*x])^{(1/3)}*\operatorname{Hypergeometric2F1}[1/6, 1/2, 3/2, (1 - \operatorname{Cos}[c + d*x])/2]*\operatorname{Sin}[c + d*x])/(2*2^{(1/6)}*d*(1 + \operatorname{Cos}[c + d*x])^{(5/6)})$

Rule 2730

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-2^(n +
1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2830

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3B\sqrt[3]{a + a\cos(c + dx)}\sin(c + dx)}{4d} + \frac{1}{4}(4A + B) \int \sqrt[3]{a + a\cos(c + dx)} dx \\
&= \frac{3B\sqrt[3]{a + a\cos(c + dx)}\sin(c + dx)}{4d} + \frac{\left((4A + B)\sqrt[3]{a + a\cos(c + dx)}\right) \int \sqrt[3]{1 + \cos(c + dx)} dx}{4\sqrt[3]{1 + \cos(c + dx)}} \\
&= \frac{3B\sqrt[3]{a + a\cos(c + dx)}\sin(c + dx)}{4d} \\
&\quad + \frac{(4A + B)\sqrt[3]{a + a\cos(c + dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{2^{\frac{5}{6}}\sqrt{2}d(1 + \cos(c + dx))^{\frac{5}{6}}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 304 vs. 2(102) = 204.

Time = 3.49 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.98

$$\begin{aligned}
&\int \sqrt[3]{a + a\cos(c + dx)}(A + B\cos(c + dx)) dx \\
&= \frac{\sqrt[3]{a(1 + \cos(c + dx))} \left(-6(4A + B) \cot\left(\frac{c}{2}\right) \sqrt{\sec^2\left(\frac{c}{2}\right)} + 5(4A + B) \cos\left(\frac{1}{2}(c - dx - 2\arctan\left(\tan\left(\frac{c}{2}\right)\right))\right) \right)}{2^{\frac{5}{6}}\sqrt{2}d(1 + \cos(c + dx))^{\frac{5}{6}}}
\end{aligned}$$

[In] Integrate[(a + a*cos[c + d*x])^(1/3)*(A + B*cos[c + d*x]),x]

[Out] ((a*(1 + Cos[c + d*x]))^(1/3)*(-6*(4*A + B)*Cot[c/2]*Sqrt[Sec[c/2]^2] + 5*(4*A + B)*Cos[(c - d*x - 2*ArcTan[Tan[c/2]])/2]*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2] + 4*A*cos[(c + d*x + 2*ArcTan[Tan[c/2]])/2]*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2] + B*cos[(c + d*x + 2*ArcTan[Tan[c/2]])/2]*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2] + 6*B*Sqrt[Sec[c/2]^2]*Sin[c + d*x] - (4*(4*A + B)*HypergeometricPFQ[{-1/2, -1/6}, {5/6}, Cos[(d*x)/2 + ArcTan[Tan[c/2]]]^2]*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2 + ArcTan[Tan[c/2]]])/Sqrt[Sin[(d*x)/2 + ArcTan[Tan[c/2]]]^2))/(8*d*Sqrt[Sec[c/2]^2])

Maple [F]

$$\int (a + \cos(dx + c)) a^{\frac{1}{3}} (A + B \cos(dx + c)) dx$$

[In] int((a+cos(d*x+c)*a)^(1/3)*(A+B*cos(d*x+c)),x)

[Out] int((a+cos(d*x+c)*a)^(1/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\begin{aligned} & \int \sqrt[3]{a + a \cos(c + dx)} (A + B \cos(c + dx)) dx \\ &= \int (B \cos(dx + c) + A) (a \cos(dx + c) + a)^{\frac{1}{3}} dx \end{aligned}$$

[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)

Sympy [F]

$$\begin{aligned} & \int \sqrt[3]{a + a \cos(c + dx)} (A + B \cos(c + dx)) dx \\ &= \int \sqrt[3]{a (\cos(c + dx) + 1)} (A + B \cos(c + dx)) dx \end{aligned}$$

[In] integrate((a+a*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)

[Out] Integral((a*(cos(c + d*x) + 1))**(1/3)*(A + B*cos(c + d*x)), x)

Maxima [F]

$$\int \sqrt[3]{a + a \cos(c + dx)}(A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(a \cos(dx + c) + a)^{\frac{1}{3}} dx$$

[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)

Giac [F]

$$\int \sqrt[3]{a + a \cos(c + dx)}(A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(a \cos(dx + c) + a)^{\frac{1}{3}} dx$$

[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a + a \cos(c + dx)}(A + B \cos(c + dx)) dx$$

$$= \int (A + B \cos(c + dx)) (a + a \cos(c + dx))^{1/3} dx$$

[In] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/3),x)

[Out] int((A + B*cos(c + d*x))*(a + a*cos(c + d*x))^(1/3), x)

$$3.789 \quad \int \frac{A+B \cos(c+dx)}{\sqrt[3]{a+a \cos(c+dx)}} dx$$

Optimal result	6994
Rubi [A] (verified)	6994
Mathematica [A] (verified)	6996
Maple [F]	6996
Fricas [F]	6996
Sympy [F]	6997
Maxima [F]	6997
Giac [F]	6997
Mupad [F(-1)]	6997

Optimal result

Integrand size = 25, antiderivative size = 101

$$\begin{aligned} & \int \frac{A+B \cos(c+dx)}{\sqrt[3]{a+a \cos(c+dx)}} dx \\ &= \frac{3B \sin(c+dx)}{2d \sqrt[3]{a+a \cos(c+dx)}} \\ &+ \frac{(2A-B) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, \frac{1}{2}(1-\cos(c+dx))\right) \sin(c+dx)}{2^{5/6} d \sqrt[6]{1+\cos(c+dx)} \sqrt[3]{a+a \cos(c+dx)}} \end{aligned}$$

[Out] 3/2*B*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/3)+1/2*(2*A-B)*hypergeom([1/2, 5/6], [3/2], 1/2-1/2*cos(d*x+c))*sin(d*x+c)*2^(1/6)/d/(1+cos(d*x+c))^(1/6)/(a+a*cos(d*x+c))^(1/3)

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2830, 2731, 2730}

$$\begin{aligned} & \int \frac{A+B \cos(c+dx)}{\sqrt[3]{a+a \cos(c+dx)}} dx \\ &= \frac{(2A-B) \sin(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, \frac{1}{2}(1-\cos(c+dx))\right)}{2^{5/6} d \sqrt[6]{\cos(c+dx)+1} \sqrt[3]{a \cos(c+dx)+a}} \\ &+ \frac{3B \sin(c+dx)}{2d \sqrt[3]{a \cos(c+dx)+a}} \end{aligned}$$

[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(1/3), x]

[Out] (3*B*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(1/3)) + ((2*A - B)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2^(5/6)*d*(1 + Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))

Rule 2730

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-2^(n + 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 2731

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[a^IntPart[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2830

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{3B \sin(c + dx)}{2d \sqrt[3]{a + a \cos(c + dx)}} + \frac{1}{2}(2A - B) \int \frac{1}{\sqrt[3]{a + a \cos(c + dx)}} dx \\
 &= \frac{3B \sin(c + dx)}{2d \sqrt[3]{a + a \cos(c + dx)}} + \frac{\left((2A - B) \sqrt[3]{1 + \cos(c + dx)} \right) \int \frac{1}{\sqrt[3]{1 + \cos(c + dx)}} dx}{2 \sqrt[3]{a + a \cos(c + dx)}} \\
 &= \frac{3B \sin(c + dx)}{2d \sqrt[3]{a + a \cos(c + dx)}} \\
 &\quad + \frac{(2A - B) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{2^{5/6} d \sqrt[6]{1 + \cos(c + dx)} \sqrt[3]{a + a \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + a \cos(c + dx)}} dx$$

$$= \frac{3 \cdot 2^{5/6} B \sqrt[6]{1 - \cos\left(dx - 2 \arctan\left(\cot\left(\frac{c}{2}\right)\right)\right)} \sin(c + dx) - 2(2A - B) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, \cos^2\left(\frac{dx}{2} - \arctan\left(\cot\left(\frac{c}{2}\right)\right)\right)\right)}{4d \sqrt[3]{a(1 + \cos(c + dx))} \sqrt[6]{\sin^2\left(\frac{dx}{2} - \arctan\left(\cot\left(\frac{c}{2}\right)\right)\right)}}$$

[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(1/3), x]

[Out] (3*2^(5/6)*B*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*Sin[c + d*x] - 2*(2*A - B)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(d*x)/2 - ArcTan[Cot[c/2]]]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]])/(4*d*(a*(1 + Cos[c + d*x]))^(1/3)*(Sin[(d*x)/2 - ArcTan[Cot[c/2]]]^2)^(1/6))

Maple [F]

$$\int \frac{A + B \cos(dx + c)}{(a + \cos(dx + c)a)^{\frac{1}{3}}} dx$$

[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/3), x)

[Out] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/3), x)

Fricas [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + a \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3), x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + a \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt[3]{a (\cos(c + dx) + 1)}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/3),x)

[Out] Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(1/3), x)

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + a \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + a \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + a \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^{1/3}} dx$$

[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/3),x)

[Out] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(1/3), x)

$$3.790 \quad \int \frac{A+B \cos(c+dx)}{(a+a \cos(c+dx))^{2/3}} dx$$

Optimal result	6998
Rubi [A] (verified)	6998
Mathematica [B] (verified)	6999
Maple [F]	7000
Fricas [F]	7000
Sympy [F]	7000
Maxima [F]	7001
Giac [F]	7001
Mupad [F(-1)]	7001

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx = \frac{3(A - B) \sin(c + dx)}{d(a + a \cos(c + dx))^{2/3}} - \frac{2^{5/6}(A - 2B) \sqrt[3]{a + a \cos(c + dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{ad(1 + \cos(c + dx))^{5/6}}$$

[Out] 3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^(2/3)-2^(5/6)*(A-2*B)*(a+a*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[3/2],1/2-1/2*cos(d*x+c))*sin(d*x+c)/a/d/(1+cos(d*x+c))^(5/6)

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2829, 2731, 2730}

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx = \frac{3(A - B) \sin(c + dx)}{d(a \cos(c + dx) + a)^{2/3}} - \frac{2^{5/6}(A - 2B) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right)}{ad(\cos(c + dx) + 1)^{5/6}}$$

[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(2/3),x]

[Out] (3*(A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) - (2^(5/6)*(A - 2*B)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(a*d*(1 + Cos[c + d*x])^(5/6))

Rule 2730

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-2^(n + 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[a^IntPart[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2829

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{3(A - B) \sin(c + dx)}{d(a + a \cos(c + dx))^{2/3}} - \frac{(A - 2B) \int \sqrt[3]{a + a \cos(c + dx)} dx}{a} \\ &= \frac{3(A - B) \sin(c + dx)}{d(a + a \cos(c + dx))^{2/3}} - \frac{\left((A - 2B) \sqrt[3]{a + a \cos(c + dx)} \right) \int \sqrt[3]{1 + \cos(c + dx)} dx}{a \sqrt[3]{1 + \cos(c + dx)}} \\ &= \frac{3(A - B) \sin(c + dx)}{d(a + a \cos(c + dx))^{2/3}} \\ &\quad - \frac{2^{5/6} (A - 2B) \sqrt[3]{a + a \cos(c + dx)} \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right) \sin(c + dx)}{ad(1 + \cos(c + dx))^{5/6}} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 254 vs. 2(105) = 210.

Time = 1.99 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.42

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx = \frac{\cos\left(\frac{1}{2}(c + dx)\right) \left(4(A - 2B) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{6}; \frac{5}{6}; \cos^2\left(\frac{dx}{2} + \arctan\left(\tan\left(\frac{c}{2}\right)\right)\right)\right)\right) \sec\left(\frac{1}{2}(c + dx)\right)}{ad(1 + \cos(c + dx))^{5/6}}$$

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(2/3), x]
```

```
[Out] (Cos[(c + d*x)/2]*(4*(A - 2*B)*HypergeometricPFQ[{-1/2, -1/6}, {5/6}, Cos[(d*x)/2 + ArcTan[Tan[c/2]]]^2]*Sec[c/2]*Sin[(d*x)/2 + ArcTan[Tan[c/2]]] - Cs
c[c/2]*(5*(A - 2*B)*Cos[(c - d*x - 2*ArcTan[Tan[c/2]])/2]*Sec[c/2] + (A - 2
*B)*Cos[(c + d*x + 2*ArcTan[Tan[c/2]])/2]*Sec[c/2] + 3*((-2*A + 3*B)*Cos[(d
*x)/2] + B*Cos[c + (d*x)/2])*Sqrt[Sec[c/2]^2])*Sqrt[Sin[(d*x)/2 + ArcTan[Ta
n[c/2]]]^2)))/(d*(a*(1 + Cos[c + d*x]))^(2/3)*Sqrt[Sec[c/2]^2]*Sqrt[Sin[(d*
x)/2 + ArcTan[Tan[c/2]]]^2])
```

Maple [F]

$$\int \frac{A + B \cos(dx + c)}{(a + \cos(dx + c)a)^{\frac{2}{3}}} dx$$

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx = \int \frac{B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx = \int \frac{A + B \cos(c + dx)}{(a(\cos(c + dx) + 1))^{\frac{2}{3}}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(2/3), x)
```

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx = \int \frac{B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{2/3}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx = \int \frac{B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{2/3}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx = \int \frac{A + B \cos(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx$$

[In] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(2/3),x)

[Out] int((A + B*cos(c + d*x))/(a + a*cos(c + d*x))^(2/3), x)

$$3.791 \quad \int \frac{\frac{bB}{a} + B \cos(c+dx)}{a+b \cos(c+dx)} dx$$

Optimal result	7002
Rubi [A] (verified)	7002
Mathematica [A] (verified)	7003
Maple [A] (verified)	7003
Fricas [A] (verification not implemented)	7004
Sympy [B] (verification not implemented)	7005
Maxima [F(-2)]	7005
Giac [B] (verification not implemented)	7006
Mupad [B] (verification not implemented)	7006

Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \frac{\frac{bB}{a} + B \cos(c+dx)}{a+b \cos(c+dx)} dx = \frac{Bx}{b} - \frac{2\sqrt{a-b}\sqrt{a+b}B \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

[Out] B*x/b-2*B*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a/b/d

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2814, 2738, 211}

$$\int \frac{\frac{bB}{a} + B \cos(c+dx)}{a+b \cos(c+dx)} dx = \frac{Bx}{b} - \frac{2B\sqrt{a-b}\sqrt{a+b} \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

[In] Int[((b*B)/a + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]

[Out] (B*x)/b - (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*b*d)

Rule 211

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2738

```
Int[((a_) + (b_)*sin[Pi/2 + (c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{Bx}{b} - \frac{\left(aB - \frac{b^2B}{a}\right) \int \frac{1}{a+b \cos(c+dx)} dx}{b} \\ &= \frac{Bx}{b} - \frac{\left(2\left(a - \frac{b^2}{a}\right) B\right) \text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2} dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd} \\ &= \frac{Bx}{b} - \frac{2\sqrt{a-b}\sqrt{a+b}B \arctan\left(\frac{\sqrt{a-b} \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{\frac{bB}{a} + B \cos(c + dx)}{a + b \cos(c + dx)} dx = \frac{B\left(a(c + dx) + 2\sqrt{-a^2 + b^2} \operatorname{arctanh}\left(\frac{(-a+b) \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)\right)}{abd}$$

```
[In] Integrate[((b*B)/a + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*(a*(c + d*x) + 2*sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/sqrt[-a^2 + b^2]])/(a*b*d)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{2B \left(\frac{a \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{b} - \frac{(a-b)(a+b) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}} \right)}{da}$	77
default	$\frac{2B \left(\frac{a \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{b} - \frac{(a-b)(a+b) \arctan\left(\frac{(a-b) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}} \right)}{da}$	77
risch	$\frac{Bx}{b} + \frac{\sqrt{-a^2+b^2} B \ln\left(e^{i(dx+c)} + \frac{i\sqrt{-a^2+b^2}+a}{b}\right)}{dba} - \frac{\sqrt{-a^2+b^2} B \ln\left(e^{i(dx+c)} - \frac{i\sqrt{-a^2+b^2}-a}{b}\right)}{dba}$	118

```
[In] int((b*B/a+B*cos(d*x+c))/(a*cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*B/a*(1/b*a*arctan(tan(1/2*d*x+1/2*c))-(a-b)*(a+b)/b/((a-b)*(a+b))^(1/2)
*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.08

$$\int \frac{\frac{bB}{a} + B \cos(c + dx)}{a + b \cos(c + dx)} dx$$

$$= \left[\frac{2 B a d x + \sqrt{-a^2 + b^2} B \log\left(\frac{2 a b \cos(dx+c) + (2 a^2 - b^2) \cos(dx+c)^2 + 2 \sqrt{-a^2 + b^2} (a \cos(dx+c) + b) \sin(dx+c) - a^2 + 2 b^2}{b^2 \cos(dx+c)^2 + 2 a b \cos(dx+c) + a^2}\right)}{2 a b d}, \frac{B a d x - \sqrt{-a^2 + b^2} B \arctan\left(\frac{-(a \cos(dx+c) + b)}{\sqrt{a^2 - b^2} \sin(dx+c)}\right)}{a b d} \right]$$

```
[In] integrate((b*B/a+B*cos(d*x+c))/(a*b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*d*x + sqrt(-a^2 + b^2)*B*log(((2*a*b*cos(d*x + c) + (2*a^2 - b^2)
)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/(a*b*d), (B*a
*d*x - sqrt(a^2 - b^2)*B*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(
d*x + c))))/(a*b*d)]
```


Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(51) = 102.

Time = 17.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.70

$$\int \frac{\frac{bB}{a} + B \cos(c + dx)}{a + b \cos(c + dx)} dx$$

$$= \begin{cases} \text{NaN} \\ \frac{B \sin(c+dx)}{ad} \\ \frac{x \left(B \cos(c) + \frac{Bb}{a} \right)}{a + b \cos(c)} \\ \frac{Bx}{b} \\ \frac{Bx}{b} - \frac{B \log \left(-\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan \left(\frac{c}{2} + \frac{dx}{2} \right) \right)}{bd \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} + \frac{B \log \left(\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan \left(\frac{c}{2} + \frac{dx}{2} \right) \right)}{bd \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} - \frac{B \log \left(-\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan \left(\frac{c}{2} + \frac{dx}{2} \right) \right)}{ad \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} + \frac{B \log \left(\sqrt{-\frac{a}{a-b} - \frac{b}{a-b}} + \tan \left(\frac{c}{2} + \frac{dx}{2} \right) \right)}{ad \sqrt{-\frac{a}{a-b} - \frac{b}{a-b}}} \end{cases}$$

[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)

[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (B*sin(c + d*x)/(a*d), Eq(b, 0)), (x*(B*cos(c) + B*b/a)/(a + b*cos(c)), Eq(d, 0)), (B*x/b, Eq(a, b) | Eq(a, -b)), (B*x/b - B*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(b*d*sqrt(-a/(a - b) - b/(a - b))) + B*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(b*d*sqrt(-a/(a - b) - b/(a - b))) - B*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a - b))) + B*log(sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a - b))), True))

Maxima [F(-2)]

Exception generated.

$$\int \frac{\frac{bB}{a} + B \cos(c + dx)}{a + b \cos(c + dx)} dx = \text{Exception raised: ValueError}$$

[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(54) = 108.

Time = 0.31 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.46

$$\int \frac{\frac{bB}{a} + B \cos(c + dx)}{a + b \cos(c + dx)} dx =$$

$$\frac{(\sqrt{a^2 - b^2} B |a - b| |a| |b| + (2a^2 + ab) \sqrt{a^2 - b^2} B |a - b|) \left(\pi \left[\frac{dx + c}{2\pi} + \frac{1}{2} \right] + \arctan \left(\frac{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\sqrt{\frac{a^2 + \sqrt{a^4 - (a^2 + ab)(a^2 - ab)}}{a^2 - ab}}} \right) \right)}{(a - b)a^2 b^2 + (a^3 - a^2 b) |a| |b|} + \frac{(2Ba^3 - Ba^2 b - Bab^2 - Ba|a|)}{d}$$

[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] -((sqrt(a^2 - b^2)*B*abs(a - b)*abs(a)*abs(b) + (2*a^2 + a*b)*sqrt(a^2 - b^2)*B*abs(a - b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt((a^2 + sqrt(a^4 - (a^2 + a*b)*(a^2 - a*b)))/(a^2 - a*b))))/(a - b)*a^2*b^2 + (a^3 - a^2*b)*abs(a)*abs(b)) + (2*B*a^3 - B*a^2*b - B*a*b^2 - B*a*abs(a)*abs(b) + B*b*abs(a)*abs(b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt((a^2 - sqrt(a^4 - (a^2 + a*b)*(a^2 - a*b)))/(a^2 - a*b))))/(a^2*b^2 - a^2*abs(a)*abs(b))/d

Mupad [B] (verification not implemented)

Time = 14.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.48

$$\int \frac{\frac{bB}{a} + B \cos(c + dx)}{a + b \cos(c + dx)} dx = \frac{2B \operatorname{atan}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{bd} + \frac{2B \operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{b^2 - a^2}}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right) (a+b)}\right) \sqrt{b^2 - a^2}}{abd}$$

[In] int((B*cos(c + d*x) + (B*b)/a)/(a + b*cos(c + d*x)),x)

[Out] (2*B*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d) + (2*B*atanh((sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a + b)))*(b^2 - a^2)^(1/2))/(a*b*d)

$$3.792 \quad \int \frac{a+b \cos(c+dx)}{(b+a \cos(c+dx))^2} dx$$

Optimal result	7007
Rubi [A] (verified)	7007
Mathematica [A] (verified)	7008
Maple [A] (verified)	7008
Fricas [A] (verification not implemented)	7009
Sympy [F(-1)]	7009
Maxima [F(-2)]	7009
Giac [B] (verification not implemented)	7009
Mupad [B] (verification not implemented)	7010

Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{a + b \cos(c + dx)}{(b + a \cos(c + dx))^2} dx = \frac{\sin(c + dx)}{d(b + a \cos(c + dx))}$$

[Out] sin(d*x+c)/d/(b+a*cos(d*x+c))

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2833, 8}

$$\int \frac{a + b \cos(c + dx)}{(b + a \cos(c + dx))^2} dx = \frac{\sin(c + dx)}{d(a \cos(c + dx) + b)}$$

[In] Int[(a + b*Cos[c + d*x])/(b + a*Cos[c + d*x])^2,x]

[Out] Sin[c + d*x]/(d*(b + a*Cos[c + d*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2833

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -

`a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]`

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sin(c + dx)}{d(b + a \cos(c + dx))} + \frac{\int 0 dx}{a^2 - b^2} \\ &= \frac{\sin(c + dx)}{d(b + a \cos(c + dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{a + b \cos(c + dx)}{(b + a \cos(c + dx))^2} dx = \frac{\sin(c + dx)}{d(b + a \cos(c + dx))}$$

[In] Integrate[(a + b*Cos[c + d*x])/(b + a*Cos[c + d*x])^2,x]

[Out] Sin[c + d*x]/(d*(b + a*Cos[c + d*x]))

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$\frac{\sin(dx+c)}{d(b+\cos(dx+c)a)}$	23
risch	$\frac{2i(b e^{i(dx+c)}+a)}{ad(a e^{2i(dx+c)}+2b e^{i(dx+c)}+a)}$	50
derivativedivides	$-\frac{2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - a - b\right)}$	51
default	$-\frac{2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - a - b\right)}$	51
norman	$\frac{-\frac{2 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{d} - \frac{2\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d}}{\left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a - b\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - a - b\right)}$	84

[In] int((a+cos(d*x+c)*b)/(b+cos(d*x+c)*a)^2,x,method=_RETURNVERBOSE)

[Out] sin(d*x+c)/d/(b+cos(d*x+c)*a)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{a + b \cos(c + dx)}{(b + a \cos(c + dx))^2} dx = \frac{\sin(dx + c)}{ad \cos(dx + c) + bd}$$

[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="fricas")

[Out] sin(d*x + c)/(a*d*cos(d*x + c) + b*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \cos(c + dx)}{(b + a \cos(c + dx))^2} dx = \text{Timed out}$$

[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))**2,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \cos(c + dx)}{(b + a \cos(c + dx))^2} dx = \text{Exception raised: ValueError}$$

[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more de

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(22) = 44.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{a + b \cos(c + dx)}{(b + a \cos(c + dx))^2} dx = -\frac{2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - a - b\right) d}$$

[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="giac")

[Out] -2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*d)

Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{a + b \cos(c + dx)}{(b + a \cos(c + dx))^2} dx = \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d \left((b - a) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + a + b \right)}$$

[In] int((a + b*cos(c + d*x))/(b + a*cos(c + d*x))^2,x)

[Out] (2*tan(c/2 + (d*x)/2))/(d*(a + b - tan(c/2 + (d*x)/2)^2*(a - b))

3.793 $\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)} dx$

Optimal result	7011
Rubi [A] (verified)	7011
Mathematica [A] (verified)	7012
Maple [A] (verified)	7012
Fricas [A] (verification not implemented)	7013
Sympy [A] (verification not implemented)	7013
Maxima [A] (verification not implemented)	7013
Giac [A] (verification not implemented)	7014
Mupad [B] (verification not implemented)	7014

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{3 + \cos(c + dx)}{2 - \cos(c + dx)} dx = -x + \frac{5x}{\sqrt{3}} + \frac{10 \arctan\left(\frac{\sin(c+dx)}{2+\sqrt{3}-\cos(c+dx)}\right)}{\sqrt{3}d}$$

[Out] $-x+5/3*x*3^{(1/2)}+10/3*\arctan(\sin(d*x+c)/(2-\cos(d*x+c)+3^{(1/2)}))/d*3^{(1/2)}$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2814, 2736}

$$\int \frac{3 + \cos(c + dx)}{2 - \cos(c + dx)} dx = \frac{10 \arctan\left(\frac{\sin(c+dx)}{-\cos(c+dx)+\sqrt{3}+2}\right)}{\sqrt{3}d} + \frac{5x}{\sqrt{3}} - x$$

[In] $\text{Int}[(3 + \text{Cos}[c + d*x])/(2 - \text{Cos}[c + d*x]), x]$

[Out] $-x + (5*x)/\text{Sqrt}[3] + (10*\text{ArcTan}[\text{Sin}[c + d*x]/(2 + \text{Sqrt}[3] - \text{Cos}[c + d*x])]) / (\text{Sqrt}[3]*d)$

Rule 2736

$\text{Int}[(a + (b_*)\sin[(c_*) + (d_*)(x_*)])^{-1}, x_Symbol] := \text{With}[\{q = \text{Rt}[a^2 - b^2, 2]\}, \text{Simp}[x/q, x] + \text{Simp}[(2/(d*q))*\text{ArcTan}[b*(\text{Cos}[c + d*x]/(a + q + b*\text{Sin}[c + d*x]))], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{GtQ}[a^2 - b^2, 0] \ \&\& \ \text{PosQ}[a]$

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -x + 5 \int \frac{1}{2 - \cos(c + dx)} dx \\ &= -x + \frac{5x}{\sqrt{3}} + \frac{10 \arctan\left(\frac{\sin(c+dx)}{2+\sqrt{3}-\cos(c+dx)}\right)}{\sqrt{3}d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{3 + \cos(c + dx)}{2 - \cos(c + dx)} dx = -x + \frac{10 \arctan\left(\sqrt{3} \tan\left(\frac{1}{2}(c + dx)\right)\right)}{\sqrt{3}d}$$

```
[In] Integrate[(3 + Cos[c + d*x])/(2 - Cos[c + d*x]),x]
```

```
[Out] -x + (10*ArcTan[Sqrt[3]*Tan[(c + d*x)/2]])/(Sqrt[3]*d)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{10\sqrt{3} \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{3}\right)}{3}}{d}$	37
default	$\frac{-2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{10\sqrt{3} \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{3}\right)}{3}}{d}$	37
risch	$-x + \frac{5i\sqrt{3} \ln\left(e^{i(dx+c)} - \sqrt{3} - 2\right)}{3d} - \frac{5i\sqrt{3} \ln\left(e^{i(dx+c)} + \sqrt{3} - 2\right)}{3d}$	55

```
[In] int((3+cos(d*x+c))/(2-cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*arctan(tan(1/2*d*x+1/2*c))+10/3*3^(1/2)*arctan(tan(1/2*d*x+1/2*c)*3
^(1/2)))
```


Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{3 + \cos(c + dx)}{2 - \cos(c + dx)} dx = -\frac{3 dx + 5 \sqrt{3} \arctan\left(\frac{2\sqrt{3} \cos(dx+c) - \sqrt{3}}{3 \sin(dx+c)}\right)}{3 d}$$

[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="fricas")

[Out] -1/3*(3*d*x + 5*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(d*x + c) - sqrt(3))/sin(d*x + c)))/d

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \frac{3 + \cos(c + dx)}{2 - \cos(c + dx)} dx = \begin{cases} -x + \frac{10\sqrt{3}\left(\operatorname{atan}\left(\sqrt{3}\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) + \pi\left\lfloor\frac{\frac{c}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi}\right\rfloor\right)}{3d} & \text{for } d \neq 0 \\ \frac{x(\cos(c)+3)}{2-\cos(c)} & \text{otherwise} \end{cases}$$

[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x)

[Out] Piecewise((-x + 10*sqrt(3)*(atan(sqrt(3)*tan(c/2 + d*x/2)) + pi*floor((c/2 + d*x/2 - pi/2)/pi))/(3*d), Ne(d, 0)), (x*(cos(c) + 3)/(2 - cos(c)), True))

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \frac{3 + \cos(c + dx)}{2 - \cos(c + dx)} dx = \frac{2\left(5\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(dx+c)}{\cos(dx+c)+1}\right) - 3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)\right)}{3 d}$$

[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="maxima")

[Out] 2/3*(5*sqrt(3)*arctan(sqrt(3)*sin(d*x + c)/(cos(d*x + c) + 1)) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \frac{3 + \cos(c + dx)}{2 - \cos(c + dx)} dx$$

$$= -\frac{3 dx - 5 \sqrt{3} \left(dx + c + 2 \arctan \left(-\frac{\sqrt{3} \sin(dx+c) - 3 \sin(dx+c)}{\sqrt{3} \cos(dx+c) + \sqrt{3} - 3 \cos(dx+c) + 3} \right) \right) + 3c}{3d}$$

[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="giac")

[Out] -1/3*(3*d*x - 5*sqrt(3)*(d*x + c + 2*arctan(-(sqrt(3)*sin(d*x + c) - 3*sin(d*x + c))/(sqrt(3)*cos(d*x + c) + sqrt(3) - 3*cos(d*x + c) + 3))) + 3*c)/d

Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \frac{3 + \cos(c + dx)}{2 - \cos(c + dx)} dx = \frac{\left(\frac{\pi - 5\pi\sqrt{3}}{d} - \frac{\pi + 5\pi\sqrt{3}}{d} \right) \left(\operatorname{atan}\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) - \frac{dx}{2} \right)}{\pi}$$

$$- \frac{dx - \frac{10\sqrt{3} \operatorname{atan}\left(\sqrt{3} \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{3}}{d}$$

[In] int(-(cos(c + d*x) + 3)/(cos(c + d*x) - 2),x)

[Out] (((pi - (5*3^(1/2)*pi)/3)/d - (pi + (5*3^(1/2)*pi)/3)/d)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)) - (d*x - (10*3^(1/2)*atan(3^(1/2)*tan(c/2 + (d*x)/2)))/3)/d

$$3.794 \quad \int \frac{aB + bB \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

Optimal result	7015
Rubi [A] (verified)	7015
Mathematica [A] (verified)	7016
Maple [B] (verified)	7017
Fricas [C] (verification not implemented)	7017
Sympy [F]	7018
Maxima [F]	7018
Giac [F]	7018
Mupad [B] (verification not implemented)	7018

Optimal result

Integrand size = 28, antiderivative size = 58

$$\int \frac{aB + bB \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \frac{2B \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[Out] 2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2)*(b/(a+b))^(1/2))*(a+b*cos(d*x+c))^(1/2)/d/((a+b*cos(d*x+c))/(a+b))^(1/2)

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {21, 2734, 2732}

$$\int \frac{aB + bB \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \frac{2B \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

[In] Int[(a*B + b*B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]

[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,

$a + b*x))$

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_) + (d_.)*(x_)]], x_Symbol] := Simp[2*(Sqrt[a
+ b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
0] && !GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= B \int \sqrt{a + b \cos(c + dx)} dx \\ &= \frac{\left(B \sqrt{a + b \cos(c + dx)} \right) \int \sqrt{\frac{a}{a+b} + \frac{b \cos(c+dx)}{a+b}} dx}{\sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \\ &= \frac{2B \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{aB + bB \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \frac{2B \sqrt{a + b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \frac{2b}{a+b}\right)}{d \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}$$

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 5.17 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.95

method	result
default	$-\frac{2\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}B\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}{a-b}}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{-\frac{2b}{a-b}}\right)(a-b)$
parts	$\frac{2Ba\sqrt{-\frac{2b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}{a+b}}}{d\sqrt{2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}}\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\left \frac{\sqrt{2}\sqrt{b}}{\sqrt{a+b}}\right.\right)+\frac{2B\sqrt{\left(2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+(a+b)}}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\frac{2b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a-b}{a-b}}$
risch	Expression too large to display

[In] int((B*a+b*B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out]
$$-2*\left(\left(2*b*\cos\left(1/2*d*x+1/2*c\right)^2+a-b\right)*\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*B*\left(\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}*\left(\left(2*b*\cos\left(1/2*d*x+1/2*c\right)^2+a-b\right)/\left(a-b\right)\right)^{(1/2)}*EllipticE\left(\cos\left(1/2*d*x+1/2*c\right),\left(-2*b/\left(a-b\right)\right)^{(1/2)}\right)*(a-b)/\left(-2*\sin\left(1/2*d*x+1/2*c\right)^4*b+(a+b)*\sin\left(1/2*d*x+1/2*c\right)^2\right)^{(1/2)}/\sin\left(1/2*d*x+1/2*c\right)/\left(-2*b*\sin\left(1/2*d*x+1/2*c\right)^2+a+b\right)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 359, normalized size of antiderivative = 6.19

$$\int \frac{aB + bB \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx$$

$$= -i\sqrt{2}Ba\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)+3ib\sin(dx+c)+2a}{3b}\right) + i\sqrt{2}Ba\sqrt{b}\operatorname{weierstrassPInverse}\left(\frac{4(4a^2-3b^2)}{3b^2}, -\frac{8(8a^3-9ab^2)}{27b^3}, \frac{3b\cos(dx+c)-3ib\sin(dx+c)+2a}{3b}\right)$$

[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out]
$$1/3*(-I*\sqrt{2}*B*a*\sqrt{b}*weierstrassPInverse(4/3*(4*a^2-3*b^2)/b^2, -8/27*(8*a^3-9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x+c)+3*I*b*\sin(d*x+c)+2*a)/b) + I*\sqrt{2}*B*a*\sqrt{b}*weierstrassPInverse(4/3*(4*a^2-3*b^2)/b^2, -8/27*(8*a^3-9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x+c)-3*I*b*\sin(d*x+c)+2*a)/b) + 3*I*\sqrt{2}*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2-3*b^2)/b^2, -8/27*(8*a^3-9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2-3*b^2)/b^2, -8/27*(8*a^3-9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x+c)+3*I*b*\sin(d*x+c)+2*a)/b)) - 3*I*\sqrt{2}*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2-3*b^2)/b^2, -8/27*(8*a^3-9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2-3*b^2)/b^2, -8/27*(8*a^3-9*a*b^2)/b^3, 1/3*(3*b*\cos(d*x+c)-3*I*b*\sin(d*x+c)+2*a)/b))$$

$8a^3 - 9ab^2)/b^3$, $\text{weierstrassPInverse}(4/3*(4a^2 - 3b^2)/b^2, -8/27*(8a^3 - 9ab^2)/b^3, 1/3*(3b*\cos(dx + c) - 3I*b*\sin(dx + c) + 2a)/b))$
 $/(b*d)$

Sympy [F]

$$\int \frac{aB + bB \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = B \int \sqrt{a + b \cos(c + dx)} dx$$

[In] `integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)`

[Out] `B*Integral(sqrt(a + b*cos(c + d*x)), x)`

Maxima [F]

$$\int \frac{aB + bB \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{Bb \cos(dx + c) + Ba}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] `integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*b*cos(d*x + c) + B*a)/sqrt(b*cos(d*x + c) + a), x)`

Giac [F]

$$\int \frac{aB + bB \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \int \frac{Bb \cos(dx + c) + Ba}{\sqrt{b \cos(dx + c) + a}} dx$$

[In] `integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate((B*b*cos(d*x + c) + B*a)/sqrt(b*cos(d*x + c) + a), x)`

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{aB + bB \cos(c + dx)}{\sqrt{a + b \cos(c + dx)}} dx = \frac{2B E\left(\frac{c}{2} + \frac{dx}{2} \middle| \frac{2b}{a+b}\right) (a+b) \sqrt{\frac{a+b \cos(c+dx)}{a+b}}}{d \sqrt{a + b \cos(c + dx)}}$$

[In] `int((B*a + B*b*cos(c + d*x))/(a + b*cos(c + d*x))^(1/2),x)`

[Out] `(2*B*ellipticE(c/2 + (d*x)/2, (2*b)/(a + b))*(a + b)*((a + b*cos(c + d*x))/(a + b))^(1/2))/(d*(a + b*cos(c + d*x))^(1/2))`

3.795 $\int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx$

Optimal result	7019
Rubi [A] (verified)	7019
Mathematica [A] (verified)	7022
Maple [F]	7022
Fricas [F]	7022
Sympy [F]	7023
Maxima [F]	7023
Giac [F]	7023
Mupad [F(-1)]	7023

Optimal result

Integrand size = 25, antiderivative size = 229

$$\int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \frac{\sqrt{2}(a + b)B \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{5}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) (a + b \cos(c + dx))^{2/3}}{bd\sqrt{1 + \cos(c + dx)} \left(\frac{a + b \cos(c + dx)}{a + b}\right)^{2/3}} + \frac{\sqrt{2}(Ab - aB) \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) (a + b \cos(c + dx))^{2/3} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)} \left(\frac{a + b \cos(c + dx)}{a + b}\right)^{2/3}}$$

```
[Out] (a+b)*B*AppellF1(1/2,-5/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))
*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(2
/3)/(1+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c
))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/b/d/
((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

= {2835, 2744, 144, 143}

$$\int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \frac{\sqrt{2}(Ab - aB) \sin(c + dx)(a + b \cos(c + dx))^{2/3} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right)}{bd \sqrt{\cos(c + dx) + 1} \left(\frac{a + b \cos(c + dx)}{a + b}\right)^{2/3}} + \frac{\sqrt{2}B(a + b) \sin(c + dx)(a + b \cos(c + dx))^{2/3} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{5}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx))\right), \frac{b(1 - \cos(c + dx))}{a + b}}{bd \sqrt{\cos(c + dx) + 1} \left(\frac{a + b \cos(c + dx)}{a + b}\right)^{2/3}}$$

[In] Int[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]), x]

[Out] (Sqrt[2]*(a + b)*B*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(A*b - a*B)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))

Rule 143

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f)))^p)*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])

Rule 144

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b/(b*e - a*f) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d

, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 2835

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{B \int (a + b \cos(c + dx))^{5/3} dx}{b} + \frac{(Ab - aB) \int (a + b \cos(c + dx))^{2/3} dx}{b} \\
 &= -\frac{(B \sin(c + dx)) \text{Subst}\left(\int \frac{(a+bx)^{5/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\
 &\quad - \frac{((Ab - aB) \sin(c + dx)) \text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\
 &= \frac{((-a - b)B(a + b \cos(c + dx))^{2/3} \sin(c + dx)) \text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b} - \frac{bx}{-a-b}\right)^{5/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}\left(-\frac{a+b \cos(c+dx)}{-a-b}\right)^{2/3}} \\
 &\quad - \frac{((Ab - aB)(a + b \cos(c + dx))^{2/3} \sin(c + dx)) \text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b} - \frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}\left(-\frac{a+b \cos(c+dx)}{-a-b}\right)^{2/3}} \\
 &= \frac{\sqrt{2}(a + b)B \text{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{5}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right) (a + b \cos(c + dx))^{2/3} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)}\left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3}} \\
 &\quad + \frac{\sqrt{2}(Ab - aB) \text{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right) (a + b \cos(c + dx))^{2/3} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)}\left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.13

$$\int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \frac{3(a + b \cos(c + dx))^{2/3} \left(5(a^2 - b^2) B \operatorname{AppellF1} \left(\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, \frac{a+b \cos(c+dx)}{a-b}, \frac{a+b \cos(c+dx)}{a+b} \right) \sqrt{-\frac{b(-1 + \cos(c+dx))}{a+b}} \right) + 5A^2 b^2 \operatorname{Sin}[c + dx]}{25b^2 d}$$

[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]

[Out] (3*(a + b*Cos[c + d*x])^(2/3)*(5*(a^2 - b^2)*B*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x] - (5*A*b + 2*a*B)*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])*Csc[c + d*x] + 5*b^2*B*Sin[c + d*x]))/(25*b^2*d)

Maple [F]

$$\int (a + \cos(dx + c) b)^{2/3} (A + B \cos(dx + c)) dx$$

[In] int((a+cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)),x)

[Out] int((a+cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c) + a)^{2/3} dx$$

[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)

Sympy [F]

$$\int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \int (A + B \cos(c + dx)) (a + b \cos(c + dx))^{2/3} dx$$

[In] integrate((a+b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)

[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(2/3), x)

Maxima [F]

$$\int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c) + a)^{2/3} dx$$

[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)

Giac [F]

$$\int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c) + a)^{2/3} dx$$

[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx = \int (A + B \cos(c + dx)) (a + b \cos(c + dx))^{2/3} dx$$

[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(2/3),x)

[Out] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(2/3), x)

3.796 $\int \sqrt[3]{a + b \cos(c + dx)}(A + B \cos(c + dx)) dx$

Optimal result	7024
Rubi [A] (verified)	7024
Mathematica [A] (verified)	7027
Maple [F]	7027
Fricas [F]	7027
Sympy [F]	7028
Maxima [F]	7028
Giac [F]	7028
Mupad [F(-1)]	7028

Optimal result

Integrand size = 25, antiderivative size = 229

$$\int \sqrt[3]{a + b \cos(c + dx)}(A + B \cos(c + dx)) dx$$

$$= \frac{\sqrt{2}(a + b)B \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}} + \frac{\sqrt{2}(Ab - aB) \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

```
[Out] (a+b)*B*AppellF1(1/2, -4/3, 1/2, 3/2, b*(1-cos(d*x+c))/(a+b), 1/2-1/2*cos(d*x+c))
*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1
/3)/(1+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2, -1/3, 1/2, 3/2, b*(1-cos(d*x+c
))/(a+b), 1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/b/d/
((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

= {2835, 2744, 144, 143}

$$\int \sqrt[3]{a + b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{\sqrt{2}(Ab - aB) \sin(c + dx) \sqrt[3]{a + b \cos(c + dx)} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{bd \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}} + \frac{\sqrt{2}B(a + b) \sin(c + dx) \sqrt[3]{a + b \cos(c + dx)} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{bd \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

[In] Int[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]), x]

[Out] (Sqrt[2]*(a + b)*B*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x]/(b*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(A*b - a*B)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x]/(b*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)))

Rule 143

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^(n*(b/(b*e - a*f)))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rule 144

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d

, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 2835

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{B \int (a + b \cos(c + dx))^{4/3} dx}{b} + \frac{(Ab - aB) \int \sqrt[3]{a + b \cos(c + dx)} dx}{b} \\
 &= -\frac{(B \sin(c + dx)) \text{Subst}\left(\int \frac{(a+bx)^{4/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\
 &\quad - \frac{((Ab - aB) \sin(c + dx)) \text{Subst}\left(\int \frac{\sqrt[3]{a + bx}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\
 &= \frac{\left((-a - b)B \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)\right) \text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b} - \frac{bx}{-a-b}\right)^{4/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)} \sqrt[3]{-\frac{a + b \cos(c + dx)}{-a - b}}} \\
 &\quad - \frac{\left((Ab - aB) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)\right) \text{Subst}\left(\int \frac{\sqrt[3]{-\frac{a}{-a-b} - \frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)} \sqrt[3]{-\frac{a + b \cos(c + dx)}{-a - b}}} \\
 &= \frac{\sqrt{2}(a + b)B \text{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{4}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}} \\
 &\quad + \frac{\sqrt{2}(Ab - aB) \text{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.10

$$\int \sqrt[3]{a + b \cos(c + dx)} (A + B \cos(c + dx)) dx =$$

$$3 \sqrt[3]{a + b \cos(c + dx)} \csc(c + dx) \left(4(-a^2 + b^2) B \operatorname{AppellF1} \left(\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{a+b \cos(c+dx)}{a-b}, \frac{a+b \cos(c+dx)}{a+b} \right) \sqrt{-\frac{b(-a^2 + b^2)}{a^2 - b^2}} \right)$$

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*B*AppellF1[1/3,
  1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]
 *Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a
 - b))] + (4*A*b + a*B)*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a
 - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b
 ))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 4*b^2*B*Sin
 [c + d*x]^2))/(16*b^2*d)
```

Maple [F]

$$\int (a + \cos(dx + c)b)^{\frac{1}{3}} (A + B \cos(dx + c)) dx$$

```
[In] int((a+cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int((a+cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)
```

Fricas [F]

$$\int \sqrt[3]{a + b \cos(c + dx)} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c) + a)^{\frac{1}{3}} dx$$

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{a + b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int (A + B \cos(c + dx)) \sqrt[3]{a + b \cos(c + dx)} dx$$

[In] integrate((a+b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)

[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(1/3), x)

Maxima [F]

$$\int \sqrt[3]{a + b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c) + a)^{\frac{1}{3}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)

Giac [F]

$$\int \sqrt[3]{a + b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c) + a)^{\frac{1}{3}} dx$$

[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a + b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int (A + B \cos(c + dx)) (a + b \cos(c + dx))^{1/3} dx$$

[In] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/3),x)

[Out] int((A + B*cos(c + d*x))*(a + b*cos(c + d*x))^(1/3), x)

$$3.797 \quad \int \frac{A+B \cos(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx$$

Optimal result	7029
Rubi [A] (verified)	7029
Mathematica [A] (verified)	7032
Maple [F]	7032
Fricas [F]	7032
Sympy [F]	7033
Maxima [F]	7033
Giac [F]	7033
Mupad [F(-1)]	7033

Optimal result

Integrand size = 25, antiderivative size = 226

$$\int \frac{A+B \cos(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx$$

$$= \frac{\sqrt{2}B \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2}(1-\cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right) (a+b \cos(c+dx))^{2/3} \sin(c+dx)}{bd\sqrt{1+\cos(c+dx)} \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3}}$$

$$+ \frac{\sqrt{2}(Ab-aB) \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2}(1-\cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} \sin(c+dx)}{bd\sqrt{1+\cos(c+dx)} \sqrt[3]{a+b \cos(c+dx)}}$$

[Out] B*AppellF1(1/2,-2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(2/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(2/3)/(1+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(1/3)*sin(d*x+c)*2^(1/2)/b/d/(a+b*cos(d*x+c))^(1/3)/(1+cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

= {2835, 2744, 144, 143}

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx$$

$$= \frac{\sqrt{2}(Ab - aB) \sin(c + dx) \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{bd \sqrt{\cos(c + dx) + 1} \sqrt[3]{a + b \cos(c + dx)}} + \frac{\sqrt{2}B \sin(c + dx)(a + b \cos(c + dx))^{2/3} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{bd \sqrt{\cos(c + dx) + 1} \left(\frac{a + b \cos(c + dx)}{a + b}\right)^{2/3}}$$

[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(1/3), x]

[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x]/(b*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(A*b - a*B)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x]/(b*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))

Rule 143

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^(n*(b/(b*e - a*f)))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplerQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rule 144

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d

, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 2835

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{B \int (a + b \cos(c + dx))^{2/3} dx}{b} + \frac{(Ab - aB) \int \frac{1}{\sqrt[3]{a + b \cos(c + dx)}} dx}{b} \\
 &= -\frac{(B \sin(c + dx)) \text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\
 &\quad - \frac{((Ab - aB) \sin(c + dx)) \text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a + bx}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\
 &= -\frac{(B(a + b \cos(c + dx))^{2/3} \sin(c + dx)) \text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b} - \frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}\left(-\frac{a+b \cos(c+dx)}{-a-b}\right)^{2/3}} \\
 &\quad - \frac{\left((Ab - aB) \sqrt[3]{-\frac{a + b \cos(c + dx)}{-a - b}} \sin(c + dx)\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{-\frac{a}{-a-b} - \frac{bx}{-a-b}}} dx, x, \cos(c + dx)\right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}\sqrt[3]{a + b \cos(c + dx)}} \\
 &= \frac{\sqrt{2}B \text{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) (a + b \cos(c + dx))^{2/3} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)}\left(\frac{a + b \cos(c + dx)}{a + b}\right)^{2/3}} \\
 &\quad + \frac{\sqrt{2}(Ab - aB) \text{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)}\sqrt[3]{a + b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.84

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx = \frac{3 \sqrt{-\frac{b(-1 + \cos(c + dx))}{a + b}} \sqrt{\frac{b(1 + \cos(c + dx))}{-a + b}} (a + b \cos(c + dx))^{2/3} \left(5(Ab - aB) \operatorname{AppellF1} \left(\frac{2}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, \frac{a + b \cos(c + dx)}{a - b}, \frac{a + b \cos(c + dx)}{a + b} \right) + 2B \operatorname{AppellF1} \left[\frac{5}{3}, \frac{1}{2}, \frac{1}{2}, \frac{8}{3}, \frac{a + b \cos(c + dx)}{a - b}, \frac{a + b \cos(c + dx)}{a + b} \right] \right) \operatorname{Csc}[c + dx]}{10b^2d}$$

[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(1/3),x]

[Out] (-3*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(2/3)*(5*(A*b - a*B)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)] + 2*B*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*(a + b*Cos[c + d*x]))*Csc[c + d*x])/(10*b^2*d)

Maple [F]

$$\int \frac{A + B \cos(dx + c)}{(a + \cos(dx + c)b)^{\frac{1}{3}}} dx$$

[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/3),x)

[Out] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(1/3),x)

Fricas [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/3), x)

[Out] Integral((A + B*cos(c + d*x))/(a + b*cos(c + d*x))**(1/3), x)

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3), x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{1/3}} dx$$

[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(1/3), x)

[Out] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(1/3), x)

$$3.798 \quad \int \frac{A+B \cos(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx$$

Optimal result	7034
Rubi [A] (verified)	7034
Mathematica [A] (verified)	7036
Maple [F]	7037
Fricas [F]	7037
Sympy [F]	7037
Maxima [F]	7037
Giac [F]	7038
Mupad [F(-1)]	7038

Optimal result

Integrand size = 25, antiderivative size = 226

$$\int \frac{A+B \cos(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx = \frac{\sqrt{2}B \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2}(1-\cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right) \sqrt[3]{a+b \cos(c+dx)}}{bd\sqrt{1+\cos(c+dx)} \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}}} + \frac{\sqrt{2}(Ab-aB) \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{1}{2}(1-\cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} \sin(c+dx)}{bd\sqrt{1+\cos(c+dx)}(a+b \cos(c+dx))^{2/3}}$$

[Out] B*AppellF1(1/2,-1/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*(a+b*cos(d*x+c))^(1/3)*sin(d*x+c)*2^(1/2)/b/d/((a+b*cos(d*x+c))/(a+b))^(1/3)/(1+cos(d*x+c))^(1/2)+(A*b-B*a)*AppellF1(1/2,2/3,1/2,3/2,b*(1-cos(d*x+c))/(a+b),1/2-1/2*cos(d*x+c))*((a+b*cos(d*x+c))/(a+b))^(2/3)*sin(d*x+c)*2^(1/2)/b/d/(a+b*cos(d*x+c))^(2/3)/(1+cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2835, 2744, 144, 143}

$$\int \frac{A+B \cos(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx = \frac{\sqrt{2}(Ab-aB) \sin(c+dx) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{1}{2}(1-\cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}(a+b \cos(c+dx))^{2/3}} + \frac{\sqrt{2}B \sin(c+dx) \sqrt[3]{a+b \cos(c+dx)} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2}(1-\cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1} \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}}}$$

[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(2/3), x]

[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x]/(b*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(A*b - a*B)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x]/(b*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))

Rule 143

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])

Rule 144

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2744

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 2835

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\text{integral} = \frac{B \int \sqrt[3]{a + b \cos(c + dx)} dx}{b} + \frac{(Ab - aB) \int \frac{1}{(a + b \cos(c + dx))^{2/3}} dx}{b}$$

$$\begin{aligned}
&= \frac{(B \sin(c + dx)) \text{Subst} \left(\int \frac{\sqrt[3]{a + bx}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx) \right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\
&= \frac{((Ab - aB) \sin(c + dx)) \text{Subst} \left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{2/3}} dx, x, \cos(c + dx) \right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}} \\
&= \frac{\left(B \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx) \right) \text{Subst} \left(\int \frac{\sqrt[3]{\frac{a}{-a-b} - \frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx) \right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)} \sqrt[3]{\frac{a + b \cos(c + dx)}{-a - b}}} \\
&= \frac{\left((Ab - aB) \left(-\frac{a+b \cos(c+dx)}{-a-b} \right)^{2/3} \sin(c + dx) \right) \text{Subst} \left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x} \left(-\frac{a}{-a-b} - \frac{bx}{-a-b} \right)^{2/3}} dx, x, \cos(c + dx) \right)}{bd\sqrt{1 - \cos(c + dx)}\sqrt{1 + \cos(c + dx)}(a + b \cos(c + dx))^{2/3}} \\
&= \frac{\sqrt{2} B \text{AppellF1} \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b} \right) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}} \\
&+ \frac{\sqrt{2}(Ab - aB) \text{AppellF1} \left(\frac{1}{2}, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b} \right) \left(\frac{a + b \cos(c + dx)}{a + b} \right)^{2/3} \sin(c + dx)}{bd\sqrt{1 + \cos(c + dx)}(a + b \cos(c + dx))^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.83

$$\int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{2/3}} dx = \frac{3\sqrt{-\frac{b(-1+\cos(c+dx))}{a+b}} \sqrt{\frac{b(1+\cos(c+dx))}{-a+b}} \sqrt[3]{a + b \cos(c + dx)} \left(4(Ab - aB) \text{AppellF1} \left(\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}, \frac{a+b \cos(c+dx)}{a-b}, \frac{a+b \cos(c+dx)}{a+b} \right) \right)}{4b^2d}$$

[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(2/3), x]

[Out] (-3*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])^(1/3)*(4*(A*b - a*B)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)] + B*AppellF1[1/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*(a + b*Cos[c + d*x]))*Csc[c + d*x])/(4*b^2*d)

Maple [F]

$$\int \frac{A + B \cos(dx + c)}{(a + \cos(dx + c)b)^{\frac{2}{3}}} dx$$

[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(2/3),x)

[Out] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*b)^(2/3),x)

Fricas [F]

$$\int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{\frac{2}{3}}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{\frac{2}{3}}} dx = \int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(2/3),x)

[Out] Integral((A + B*cos(c + d*x))/(a + b*cos(c + d*x))**(2/3), x)

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{\frac{2}{3}}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{2/3}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{2/3}} dx$$

[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{2/3}} dx = \int \frac{A + B \cos(c + dx)}{(a + b \cos(c + dx))^{2/3}} dx$$

[In] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(2/3),x)

[Out] int((A + B*cos(c + d*x))/(a + b*cos(c + d*x))^(2/3), x)

$$3.799 \quad \int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

Optimal result	7039
Rubi [A] (verified)	7040
Mathematica [A] (verified)	7042
Maple [A] (verified)	7042
Fricas [C] (verification not implemented)	7043
Sympy [F(-1)]	7043
Maxima [F]	7043
Giac [F]	7044
Mupad [F(-1)]	7044

Optimal result

Integrand size = 31, antiderivative size = 168

$$\begin{aligned} & \int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx \\ &= \frac{6A \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d \sqrt{\cos(c + dx)}} + \frac{10bB \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d \sqrt{b \cos(c + dx)}} \\ &+ \frac{10B \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2A (b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} \\ &+ \frac{2B (b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^2d} \end{aligned}$$

```
[Out] 2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+10/21*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2715, 2721, 2719, 2720}

$$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{2A \sin(c + dx) (b \cos(c + dx))^{3/2}}{5bd} + \frac{6AE \left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5d \sqrt{\cos(c + dx)}} + \frac{2B \sin(c + dx) (b \cos(c + dx))^{5/2}}{7b^2d} + \frac{10B \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d} + \frac{10bB \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d \sqrt{b \cos(c + dx)}}$$

[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (10*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx}{b^2} \\
&= \frac{A \int (b \cos(c + dx))^{5/2} dx}{b^2} + \frac{B \int (b \cos(c + dx))^{7/2} dx}{b^3} \\
&= \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^2d} \\
&\quad + \frac{1}{5}(3A) \int \sqrt{b \cos(c + dx)} dx + \frac{(5B) \int (b \cos(c + dx))^{3/2} dx}{7b} \\
&= \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} \\
&\quad + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^2d} \\
&\quad + \frac{1}{21}(5bB) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx + \frac{\left(3A\sqrt{b \cos(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}} \\
&= \frac{6A\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} \\
&\quad + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^2d} \\
&\quad + \frac{\left(5bB\sqrt{\cos(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{21\sqrt{b \cos(c + dx)}} \\
&= \frac{6A\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{10bB\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d\sqrt{b \cos(c + dx)}} \\
&\quad + \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} \\
&\quad + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.60

$$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{\sqrt{b \cos(c + dx)} \left(252AE \left(\frac{1}{2}(c + dx) \mid 2 \right) + 100B \operatorname{EllipticF} \left(\frac{1}{2}(c + dx), 2 \right) + 2\sqrt{\cos(c + dx)} (65B + 42A \cos(c + dx)) \right)}{210d\sqrt{\cos(c + dx)}}$$

[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] (Sqrt[b*Cos[c + d*x]]*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)]*Sin[c + d*x]))/(210*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 8.57 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.78

method	result
default	$-\frac{2\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\dots}{\dots}$
parts	$-\frac{2A\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}$

[In] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(240*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.96

$$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{-25i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(\dots)}{\dots}$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(-25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(15*B*cos(d*x + c)^2 + 21*A*cos(d*x + c) + 25*B)*sqrt(b*cos(
d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \cos(dx + c)^2 dx$$

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int \cos(c + dx)^2 \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)), x)

3.800 $\int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$

Optimal result	7045
Rubi [A] (verified)	7045
Mathematica [A] (verified)	7047
Maple [A] (verified)	7047
Fricas [C] (verification not implemented)	7048
Sympy [F(-1)]	7048
Maxima [F]	7049
Giac [F]	7049
Mupad [F(-1)]	7049

Optimal result

Integrand size = 29, antiderivative size = 139

$$\int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{6B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d \sqrt{\cos(c + dx)}} + \frac{2Ab \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d \sqrt{b \cos(c + dx)}}$$

$$+ \frac{2A \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2B (b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd}$$

[Out] $2/5*B*(b*\cos(d*x+c))^{(3/2)}*\sin(d*x+c)/b/d+2/3*A*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}+2/3*A*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d+6/5*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {16, 2827, 2715, 2721, 2720, 2719}

$$\int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{2A \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d} + \frac{2Ab \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d \sqrt{b \cos(c + dx)}}$$

$$+ \frac{2B \sin(c + dx) (b \cos(c + dx))^{3/2}}{5bd} + \frac{6BE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5d \sqrt{\cos(c + dx)}}$$

[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx}{b} \\ &= \frac{A \int (b \cos(c + dx))^{3/2} dx}{b} + \frac{B \int (b \cos(c + dx))^{5/2} dx}{b^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} \\
&\quad + \frac{1}{3}(Ab) \int \frac{1}{\sqrt{b\cos(c+dx)}} dx + \frac{1}{5}(3B) \int \sqrt{b\cos(c+dx)} dx \\
&= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} \\
&\quad + \frac{\left(Ab\sqrt{\cos(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3\sqrt{b\cos(c+dx)}} + \frac{\left(3B\sqrt{b\cos(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{5\sqrt{\cos(c+dx)}} \\
&= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\begin{aligned}
&\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)) dx \\
&= \frac{2(b\cos(c+dx))^{3/2}\left(9BE\left(\frac{1}{2}(c+dx)\mid 2\right) + 5A\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))\right)}{15bd\cos^{3/2}(c+dx)}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]), x]

[Out] (2*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*b*d*Cos[c + d*x]^(3/2))

Maple [A] (verified)

Time = 6.96 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.95

method	result
default	$ \frac{2\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-15\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right)}{15bd\cos^{3/2}(c+dx)} $
parts	$ \frac{2A\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)} $

[In] `int(cos(d*x+c)*(cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)`

[Out]
$$-2/15*((2*\cos(1/2*d*x+1/2*c)^2-1)*b*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*b*(-24*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-10*A-6*B)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+5*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-9*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/((2*\cos(1/2*d*x+1/2*c)^2-1)*b)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))dx$$

$$= \frac{-5i\sqrt{2}A\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}A\sqrt{b}\text{weierstrassPInverse}(-4,$$

[In] `integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")`

[Out]
$$1/15*(-5*I*\sqrt{2}*A*\sqrt{b}*\text{weierstrassPInverse}(-4,0,\cos(d*x+c)+I*\sin(d*x+c))+5*I*\sqrt{2}*A*\sqrt{b}*\text{weierstrassPInverse}(-4,0,\cos(d*x+c)-I*\sin(d*x+c))+9*I*\sqrt{2}*B*\sqrt{b}*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d*x+c)+I*\sin(d*x+c)))-9*I*\sqrt{2}*B*\sqrt{b}*\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d*x+c)-I*\sin(d*x+c)))+2*(3*B*\cos(d*x+c)+5*A)*\sqrt{b*\cos(d*x+c)}*\sin(d*x+c))/d$$

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)`

[Out] Timed out

Maxima [F]

$$\int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)

Giac [F]

$$\int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

[In] int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)

[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)), x)

3.801 $\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx$

Optimal result	7050
Rubi [A] (verified)	7050
Mathematica [A] (verified)	7052
Maple [A] (verified)	7052
Fricas [C] (verification not implemented)	7053
Sympy [F]	7053
Maxima [F]	7053
Giac [F]	7054
Mupad [F(-1)]	7054

Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \frac{2A\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d\sqrt{\cos(c + dx)}} + \frac{2bB\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2B\sqrt{b \cos(c + dx)}\sin(c + dx)}{3d}$$

[Out] $2/3*b*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}+2/3*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2827, 2721, 2719, 2715, 2720}

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \frac{2AE\left(\frac{1}{2}(c + dx) \mid 2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}} + \frac{2B\sin(c + dx)\sqrt{b \cos(c + dx)}}{3d} + \frac{2bB\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}}$$

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= A \int \sqrt{b \cos(c + dx)} dx + \frac{B \int (b \cos(c + dx))^{3/2} dx}{b} \\ &= \frac{2B \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3}(bB) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\ &\quad + \frac{\left(A \sqrt{b \cos(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} \\
&\quad + \frac{\left(bB\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2bB\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\begin{aligned}
&\int\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))dx \\
&= \frac{2\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\mid 2\right)+B\left(\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)\right)\right)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.20

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}{3d}$

[In] int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2)^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*c

$$d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/((2*\cos(1/2*d*x+1/2*c)^2-1)*b)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.29

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx$$

$$= \frac{-i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{2}$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] 1/3*(-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*B*sin(d*x + c))/d

Sympy [F]

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx$$

[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)

[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x)), x)

Maxima [F]

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} dx$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)

Giac [F]

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)\sqrt{b \cos(dx + c)} dx$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx$$

[In] int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)

[Out] int((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)), x)

3.802 $\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$

Optimal result	7055
Rubi [A] (verified)	7055
Mathematica [A] (verified)	7057
Maple [A] (verified)	7057
Fricas [C] (verification not implemented)	7058
Sympy [F]	7058
Maxima [F]	7058
Giac [F]	7059
Mupad [F(-1)]	7059

Optimal result

Integrand size = 29, antiderivative size = 80

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \frac{2B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d\sqrt{\cos(c + dx)}} + \frac{2Ab\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d\sqrt{b \cos(c + dx)}}$$

[Out] $2*A*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}+2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {16, 2827, 2721, 2720, 2719}

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \frac{2Ab\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d\sqrt{b \cos(c + dx)}} + \frac{2BE\left(\frac{1}{2}(c + dx) \mid 2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

[In] $\text{Int}[\text{Sqrt}[b*\text{Cos}[c + d*x]]*(A + B*\text{Cos}[c + d*x])*Sec[c + d*x], x]$

[Out] $(2*B*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/d*\text{Sqrt}[\text{Cos}[c + d*x]] + (2*A*b*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/d*\text{Sqrt}[b*\text{Cos}[c + d*x]]$

Rule 16

`Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]`

Rule 2719

`Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2720

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2721

`Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]`

Rule 2827

`Int[((b_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= b \int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\
 &= (Ab) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx + B \int \sqrt{b \cos(c + dx)} dx \\
 &= \frac{\left(Ab \sqrt{\cos(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{\sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{b \cos(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{2B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d \sqrt{\cos(c + dx)}} + \frac{2Ab \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d \sqrt{b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \frac{2b \sqrt{\cos(c + dx)} (BE(\frac{1}{2}(c + dx) | 2) + A \text{EllipticF}(\frac{1}{2}(c + dx), 2))}{d \sqrt{b \cos(c + dx)}}$$

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] (2*b*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/2, 2]))/(d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.01

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}\left(AF\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) - BE\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)bd}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1}F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)bd}} + \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)b}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}}$
risch	$-\frac{iB\sqrt{2}\sqrt{\left(e^{2i(dx+c)} + 1\right)be^{-i(dx+c)}}}{d} - \frac{i\left(\frac{iA\sqrt{-i\left(e^{i(dx+c)} + i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)} - i\right)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i\left(e^{i(dx+c)} + i\right)}, \frac{\sqrt{2}}{2}\right)}{\sqrt{be^{3i(dx+c)} + be^{i(dx+c)}}}\right)}{d} + B\left(-\frac{1}{b\sqrt{e^{i(dx+c)}}}\right)$

[In] int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c), x, method=_RETURNVERBOSE)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.49

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \frac{-i \sqrt{2} A \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0,$$

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F]

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$$

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x))*sec(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c) dx$$

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Giac [F]

$$\begin{aligned} & \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec(c + dx) dx \\ &= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c) dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\begin{aligned} & \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec(c + dx) dx \\ &= \int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx))}{\cos(c + dx)} dx \end{aligned}$$

[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x),x)

[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x), x)

3.803 $\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^2(c + dx) dx$

Optimal result	7060
Rubi [A] (verified)	7060
Mathematica [A] (verified)	7062
Maple [A] (verified)	7062
Fricas [C] (verification not implemented)	7063
Sympy [F]	7063
Maxima [F]	7064
Giac [F]	7064
Mupad [F(-1)]	7064

Optimal result

Integrand size = 31, antiderivative size = 105

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= -\frac{2A\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d\sqrt{\cos(c + dx)}} + \frac{2bB\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d\sqrt{b \cos(c + dx)}} + \frac{2Ab \sin(c + dx)}{d\sqrt{b \cos(c + dx)}}$$

[Out] 2*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*b*B*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2716, 2721, 2719, 2720}

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{2Ab \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2AE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}} + \frac{2bB\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d\sqrt{b \cos(c + dx)}}$$

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= b^2 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx \\ &= (Ab^2) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + (bB) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2Ab \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} - A \int \sqrt{b \cos(c+dx)} dx + \frac{(bB \sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{\sqrt{b \cos(c+dx)}} \\
&= \frac{2bB \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d\sqrt{b \cos(c+dx)}} + \frac{2Ab \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} \\
&\quad - \frac{(A\sqrt{b \cos(c+dx)}) \int \sqrt{\cos(c+dx)} dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2A\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2bB \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d\sqrt{b \cos(c+dx)}} + \frac{2Ab \sin(c+dx)}{d\sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\begin{aligned}
&\int \sqrt{b \cos(c+dx)} (A + B \cos(c+dx)) \sec^2(c+dx) dx \\
&= \frac{2\sqrt{b \cos(c+dx)} \left(-AE\left(\frac{1}{2}(c+dx) \mid 2\right) + B \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \frac{A \sin(c+dx)}{\sqrt{\cos(c+dx)}} \right)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (2*Sqrt[b*Cos[c + d*x]]*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c + d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.05

method	result
default	$ \frac{2b\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}} $
parts	$ -\frac{2Ab\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}} $

[In] int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERBOSE)

```
[Out] 2*b*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.62

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{-i \sqrt{2} B \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \cos(dx + c)}{}$$

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x))*sec(c + d*x)**2, x)
```

Maxima [F]

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)

Giac [F]

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx))}{\cos(c + dx)^2} dx$$

[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)

[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)

3.804 $\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^3(c + dx) dx$

Optimal result	7065
Rubi [A] (verified)	7065
Mathematica [A] (verified)	7067
Maple [B] (verified)	7067
Fricas [C] (verification not implemented)	7068
Sympy [F(-1)]	7069
Maxima [F]	7069
Giac [F]	7069
Mupad [F(-1)]	7070

Optimal result

Integrand size = 31, antiderivative size = 136

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^3(c + dx) dx$$

$$= -\frac{2B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{d\sqrt{\cos(c + dx)}} + \frac{2Ab\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}}$$

$$+ \frac{2Ab^2 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2bB \sin(c + dx)}{d\sqrt{b \cos(c + dx)}}$$

[Out] $2/3*A*b^2*\sin(d*x+c)/d/(b*\cos(d*x+c))^{(3/2)}+2*b*B*\sin(d*x+c)/d/(b*\cos(d*x+c))^{(1/2)}+2/3*A*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}-2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2716, 2721, 2720, 2719}

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \frac{2Ab^2 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2Ab\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}}$$

$$+ \frac{2bB \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2BE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
+ (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
+ d*x]]) + (2*A*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b*B*Sin
[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= b^3 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\ &= (Ab^3) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx + (b^2 B) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2Ab^2 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2bB \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} \\
&\quad + \frac{1}{3}(Ab) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx - B \int \sqrt{b \cos(c+dx)} dx \\
&= \frac{2Ab^2 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2bB \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + \frac{\left(Ab\sqrt{\cos(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3\sqrt{b \cos(c+dx)}} \\
&\quad - \frac{\left(B\sqrt{b \cos(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2B\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b \cos(c+dx)}} \\
&\quad + \frac{2Ab^2 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2bB \sin(c+dx)}{d\sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int \sqrt{b \cos(c+dx)}(A+B \cos(c+dx)) \sec^3(c+dx) dx$$

$$= \frac{2b\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right) + A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 3B \sin(c+dx) + A \tan(c+dx)\right)}{3d\sqrt{b \cos(c+dx)}}$$

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] (2*b*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(172) = 344.

Time = 6.51 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.96

method	result
default	$2\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)b(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \left(2A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} (\sin^2(\frac{dx}{2} + \frac{c}{2})) - 12F\right.$
parts	$\left. - \frac{2A\left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sin^2(\frac{dx}{2} + \frac{c}{2})) - 2(\sin^2(\frac{dx}{2} + \frac{c}{2})) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right.}{3\sqrt{-b(2(\sin^4(\frac{dx}{2} + \frac{c}{2})) - (\sin^2(\frac{dx}{2} + \frac{c}{2})))} (2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1) \sin(\frac{dx}{2} + \frac{c}{2})}$

[In] `int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERB OSE)`

[Out]
$$\frac{2/3*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*b*\sin(1/2*d*x+1/2*c)^2)^(1/2)/\sin(1/2*d*x+1/2*c)^3/(4*\sin(1/2*d*x+1/2*c)^4-4*\sin(1/2*d*x+1/2*c)^2+1)*(2*A*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^(1/2))*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\sin(1/2*d*x+1/2*c)^2-12*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+6*B*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^(1/2))*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\sin(1/2*d*x+1/2*c)^2+2*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-A*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^(1/2))+6*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-3*B*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*\sin(1/2*d*x+1/2*c)^4*b+b*\sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*\cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.39

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \frac{-i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \sqrt{b} \cos(dx + c)^2}{\dots}$$

[In] `integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")`

[Out]
$$\frac{1/3*(-I*\sqrt{2}*A*\sqrt{b}*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + I*\sqrt{2}*A*\sqrt{b}*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - 3*I*\sqrt{2}*B*\sqrt{b}*\cos(d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) + 3*I*\sqrt{2}*B*\sqrt{b}*\cos(d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) + 2*(3*B*\cos(d*x + c) + A)*\sqrt{b*\cos(d*x + c)}*\sin(d*x + c))/(d*\cos(d*x + c)^2)}$$

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^3(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)

[Out] Timed out

Maxima [F]

$$\begin{aligned} & \int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^3(c + dx) dx \\ &= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3 dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

Giac [F]

$$\begin{aligned} & \int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^3(c + dx) dx \\ &= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3 dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$
$$= \int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx))}{\cos(c + dx)^3} dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

3.805 $\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$

Optimal result	.7071
Rubi [A] (verified)	.7071
Mathematica [A] (verified)	.7074
Maple [B] (verified)	.7074
Fricas [C] (verification not implemented)	.7075
Sympy [F(-1)]	.7075
Maxima [F]	.7076
Giac [F]	.7076
Mupad [F(-1)]	.7076

Optimal result

Integrand size = 31, antiderivative size = 169

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

$$= -\frac{6A\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2bB\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}}$$

$$+ \frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^2 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6Ab \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}}$$

[Out] $2/5*A*b^3*\sin(d*x+c)/d/(b*\cos(d*x+c))^(5/2)+2/3*b^2*B*\sin(d*x+c)/d/(b*\cos(d*x+c))^(3/2)+6/5*A*b*\sin(d*x+c)/d/(b*\cos(d*x+c))^(1/2)+2/3*b*B*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2))*\cos(d*x+c)^(1/2)/d/(b*\cos(d*x+c))^(1/2)-6/5*A*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2))*(b*\cos(d*x+c))^(1/2)/d/\cos(d*x+c)^(1/2)$

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used

= {16, 2827, 2716, 2721, 2719, 2720}

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{6Ab \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{6AE(\frac{1}{2}(c + dx)|2) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}}$$

$$+ \frac{2b^2 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2bB\sqrt{\cos(c + dx)} \operatorname{EllipticF}(\frac{1}{2}(c + dx), 2)}{3d\sqrt{b \cos(c + dx)}}$$

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]

[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^2*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^4 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\
&= (Ab^4) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx + (b^3 B) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\
&= \frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^2 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} \\
&\quad + \frac{1}{5}(3Ab^2) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + \frac{1}{3}(bB) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^2 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6Ab \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{1}{5}(3A) \int \sqrt{b \cos(c + dx)} dx + \frac{(bB \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} \\
&= \frac{2bB \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\
&\quad + \frac{2b^2 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6Ab \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{(3A\sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}} \\
&= -\frac{6A\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2bB \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} \\
&\quad + \frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^2 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6Ab \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{2\sqrt{b \cos(c + dx)} \sec^2(c + dx) \left(-9A \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \cos^{\frac{3}{2}}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \right)}{15d}$$

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]

[Out] (2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(-9*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 5*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*B*Sin[c + d*x] + (9*A*Sin[2*(c + d*x)])/2 + 3*A*Tan[c + d*x]))/(15*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(197) = 394.

Time = 8.33 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.41

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(72A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 36A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1\right) E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 5B \cos^{\frac{3}{2}}\left(\frac{dx}{2} + \frac{c}{2}\right) \text{EllipticF}\left(\frac{dx}{2} + \frac{c}{2}, 2\right) + 5B \sin\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{9A \sin\left(2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + 3A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{15d}$
parts	$\frac{2A \sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(24 \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 12 \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right) E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 5B \cos^{\frac{3}{2}}\left(\frac{dx}{2} + \frac{c}{2}\right) \text{EllipticF}\left(\frac{dx}{2} + \frac{c}{2}, 2\right) + 5B \sin\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{9A \sin\left(2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2} + 3A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{15d}$

[In] int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERB OSE)

[Out] -2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)

$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$
 $= \frac{-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 9i \sqrt{2} A \sqrt{b} \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 9i \sqrt{2} A \sqrt{b} \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2(9A \cos(dx + c)^2 + 5B \cos(dx + c) + 3A) \sqrt{b \cos(dx + c)} \sin(dx + c)}{(d \cos(dx + c))^3}$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.20

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 9i \sqrt{2} A \sqrt{b} \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 9i \sqrt{2} A \sqrt{b} \cos(dx + c)^3 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2(9A \cos(dx + c)^2 + 5B \cos(dx + c) + 3A) \sqrt{b \cos(dx + c)} \sin(dx + c)}{(d \cos(dx + c))^3}$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")

[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3)

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)

[Out] Timed out

Maxima [F]

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^4 dx$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)

Giac [F]

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \int (B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^4 dx$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx))}{\cos(c + dx)^4} dx$$

[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)

[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)

$$3.806 \quad \int \cos(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx$$

Optimal result	.7077
Rubi [A] (verified)	.7078
Mathematica [A] (verified)	.7080
Maple [A] (verified)	.7080
Fricas [C] (verification not implemented)	.7081
Sympy [F(-1)]	.7081
Maxima [F]	.7081
Giac [F]	.7082
Mupad [F(-1)]	.7082

Optimal result

Integrand size = 29, antiderivative size = 169

$$\int \cos(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx = \frac{6Ab\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{10b^2B\sqrt{\cos(c + dx)}\text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{10bB\sqrt{b \cos(c + dx)}\sin(c + dx)}{21d} + \frac{2A(b \cos(c + dx))^{3/2}\sin(c + dx)}{5d} + \frac{2B(b \cos(c + dx))^{5/2}\sin(c + dx)}{7bd}$$

```
[Out] 2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+10/21*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {16, 2827, 2715, 2721, 2719, 2720}

$$\int \cos(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx = \frac{2A \sin(c + dx)(b \cos(c + dx))^{3/2}}{5d} + \frac{6AbE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5d \sqrt{\cos(c + dx)}} + \frac{10b^2 B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d \sqrt{b \cos(c + dx)}} + \frac{2B \sin(c + dx)(b \cos(c + dx))^{5/2}}{7bd} + \frac{10bB \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d}$$

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]

[Out] (6*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (10*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx}{b} \\
&= \frac{A \int (b \cos(c + dx))^{5/2} dx}{b} + \frac{B \int (b \cos(c + dx))^{7/2} dx}{b^2} \\
&= \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
&\quad + \frac{1}{5}(3Ab) \int \sqrt{b \cos(c + dx)} dx + \frac{1}{7}(5B) \int (b \cos(c + dx))^{3/2} dx \\
&= \frac{10bB \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&\quad + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
&\quad + \frac{1}{21}(5b^2B) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx + \frac{\left(3Ab \sqrt{b \cos(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}} \\
&= \frac{6Ab \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d \sqrt{\cos(c + dx)}} + \frac{10bB \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} \\
&\quad + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
&\quad + \frac{\left(5b^2B \sqrt{\cos(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{21 \sqrt{b \cos(c + dx)}} \\
&= \frac{6Ab \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d \sqrt{\cos(c + dx)}} \\
&\quad + \frac{10b^2B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d \sqrt{b \cos(c + dx)}} + \frac{10bB \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} \\
&\quad + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.61

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A + B\cos(c+dx)) dx = \frac{(b\cos(c+dx))^{5/2} \left(252AE\left(\frac{1}{2}(c+dx) \mid 2\right) + 100B \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 2\sqrt{\cos(c+dx)} \right)}{210bd \cos^{\frac{5}{2}}(c+dx)}$$

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]

```
[Out] ((b*Cos[c + d*x])^(5/2)*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[c + d*x]))/(210*b*d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 8.39 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.78

method	result
default	$\frac{2\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right)}{\dots}$
parts	$\frac{2A\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right)}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}$

[In] int(cos(d*x+c)*(cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(240*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2)^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2)^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

$$\int \cos(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx = \frac{-25i \sqrt{2} B b^{3/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} B b^{3/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 63i \sqrt{2} A b^{3/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 63i \sqrt{2} A b^{3/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2(15 B b \cos(dx + c)^2 + 21 A b \cos(dx + c) + 25 B b) \sqrt{b \cos(dx + c)} \sin(dx + c)}{d}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(-25*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*B*b*cos(d*x + c)^2 + 21*A*b*cos(d*x + c) + 25*B*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{3/2} \cos(dx + c) dx$$

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{3/2} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx = \int \cos(c + dx) (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx$$

[In] int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)

[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)), x)

3.807 $\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx$

Optimal result	7083
Rubi [A] (verified)	7084
Mathematica [A] (verified)	7085
Maple [A] (verified)	7086
Fricas [C] (verification not implemented)	7086
Sympy [F(-1)]	7087
Maxima [F]	7087
Giac [F]	7087
Mupad [F(-1)]	7087

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx = \frac{6bB \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d \sqrt{\cos(c + dx)}} + \frac{2Ab^2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d \sqrt{b \cos(c + dx)}} + \frac{2Ab \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2B(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d}$$

[Out] $2/5*B*(b*\cos(d*x+c))^{3/2}*\sin(d*x+c)/d+2/3*A*b^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}+2/3*A*b*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d+6/5*b*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2827, 2715, 2721, 2720, 2719}

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx = \frac{2Ab^2 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d \sqrt{b \cos(c + dx)}} + \frac{2Ab \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d} + \frac{2B \sin(c + dx) (b \cos(c + dx))^{3/2}}{5d} + \frac{6bBE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5d \sqrt{\cos(c + dx)}}$$

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]

[Out] (6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827


```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= A \int (b \cos(c + dx))^{3/2} dx + \frac{B \int (b \cos(c + dx))^{5/2} dx}{b} \\
&= \frac{2Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2B(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&\quad + \frac{1}{3}(Ab^2) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx + \frac{1}{5}(3bB) \int \sqrt{b \cos(c + dx)} dx \\
&= \frac{2Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2B(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
&\quad + \frac{(Ab^2 \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} + \frac{(3bB \sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}} \\
&= \frac{6bB \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d \sqrt{\cos(c + dx)}} + \frac{2Ab^2 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d \sqrt{b \cos(c + dx)}} \\
&\quad + \frac{2Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2B(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx = \frac{2(b \cos(c + dx))^{3/2} \left(9BE\left(\frac{1}{2}(c + dx) \mid 2\right) + 5A \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sqrt{\cos(c + dx)} \right)}{15d \cos^{3/2}(c + dx)}$$

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 6.96 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.95

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}b^2\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(20A+24B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+(-15\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right)}{15\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right)}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}b^2\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}d}$

[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx = \frac{-5i \sqrt{2} A b^{3/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} A b^{3/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 9i \sqrt{2} B b^{3/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 9i \sqrt{2} B b^{3/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2*(3*B*b*cos(dx + c) + 5*A*b)*sqrt(b*cos(dx + c))*sin(dx + c))/d}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")

```
[Out] 1/15*(-5*I*sqrt(2)*A*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*A*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*b*cos(d*x + c) + 5*A*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{3/2} dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{3/2} dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx = \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx$$

```
[In] int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] int((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)), x)
```

3.808 $\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec(c+dx) dx$

Optimal result	7088
Rubi [A] (verified)	7088
Mathematica [A] (verified)	7090
Maple [A] (verified)	7090
Fricas [C] (verification not implemented)	7091
Sympy [F(-1)]	7091
Maxima [F]	7092
Giac [F]	7092
Mupad [F(-1)]	7092

Optimal result

Integrand size = 29, antiderivative size = 112

$$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec(c+dx) dx = \frac{2Ab\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2bB \sqrt{b \cos(c+dx)} \sin(c+dx)}{3d}$$

[Out] $2/3*b^2*B*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}+2/3*b*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d+2*A*b*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {16, 2827, 2721, 2719, 2715, 2720}

$$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec(c+dx) dx = \frac{2AbE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2b^2 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2bB \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d}$$

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] (2*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= b \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx \\ &= (Ab) \int \sqrt{b \cos(c + dx)} dx + B \int (b \cos(c + dx))^{3/2} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}(b^2B) \int \frac{1}{\sqrt{b\cos(c+dx)}} dx \\
&\quad + \frac{\left(Ab\sqrt{b\cos(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{2Ab\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} \\
&\quad + \frac{\left(b^2B\sqrt{\cos(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2Ab\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2b^2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx) dx = \frac{2b\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx) \mid 2\right) + B\left(\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \sqrt{\cos(c+dx)}\sin(c+dx)\right)\right)}{3d\sqrt{\cos(c+dx)}}$$

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x], x]

[Out] (2*b*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 5.90 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.14

method	result
default	$ \frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)} $
parts	$ \frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)bd}} - \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)} $

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.25

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \frac{-i \sqrt{2} B b^{3/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B b^{3/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{2 \sqrt{2} B b^{3/2}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*B*b*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{3/2} \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)

Giac [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{3/2} \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos(c + dx)} dx$$

[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x),x)

[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x), x)

$$3.809 \quad \int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^2(c+dx) dx$$

Optimal result	7093
Rubi [A] (verified)	7093
Mathematica [A] (verified)	7095
Maple [A] (verified)	7095
Fricas [C] (verification not implemented)	7096
Sympy [F(-1)]	7096
Maxima [F]	7096
Giac [F]	7097
Mupad [F(-1)]	7097

Optimal result

Integrand size = 31, antiderivative size = 83

$$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^2(c+dx) dx = \frac{2bB \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d \sqrt{\cos(c+dx)}} + \frac{2Ab^2 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d \sqrt{b \cos(c+dx)}}$$

[Out] 2*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))* (b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 2827, 2721, 2720, 2719}

$$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^2(c+dx) dx = \frac{2Ab^2 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d \sqrt{b \cos(c+dx)}} + \frac{2bBE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^2 \int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\
 &= (Ab^2) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx + (bB) \int \sqrt{b \cos(c + dx)} dx \\
 &= \frac{(Ab^2 \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{\sqrt{b \cos(c + dx)}} + \frac{(bB \sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{2bB \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) | 2\right)}{d \sqrt{\cos(c + dx)}} + \frac{2Ab^2 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d \sqrt{b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \frac{2b^2 \sqrt{\cos(c + dx)} (BE(\frac{1}{2}(c + dx)|2) + A \text{EllipticF}(\frac{1}{2}(c + dx), 2))}{d \sqrt{b \cos(c + dx)}}$$

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (2*b^2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/2, 2]))/(d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.96

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(AF\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-BE\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}bd}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}bd} + \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}bd}$
risch	$-\frac{iBb\sqrt{2}\sqrt{\left(e^{2i(dx+c)}+1\right)}be^{-i(dx+c)}}{d} - \frac{i\left(\frac{iA\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{be^{3i(dx+c)}+be^{i(dx+c)}}}\right)}{d} + B\left(-\frac{1}{b\sqrt{\cos(c+dx)}}\right)$

[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERB OSE)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.43

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \frac{-i \sqrt{2} A b^{3/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A b^{3/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + i \sqrt{2} B b^{3/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - i \sqrt{2} B b^{3/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{d}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*A*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(2)*B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{3/2} \sec(dx + c)^2 dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Giac [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{3/2} \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos(c + dx)^2} dx$$

[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)

[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)

3.810 $\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^3(c+dx) dx$

Optimal result	7098
Rubi [A] (verified)	7098
Mathematica [A] (verified)	7100
Maple [A] (verified)	7100
Fricas [C] (verification not implemented)	7101
Sympy [F(-1)]	7101
Maxima [F]	7102
Giac [F]	7102
Mupad [F(-1)]	7102

Optimal result

Integrand size = 31, antiderivative size = 110

$$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^3(c+dx) dx =$$

$$\frac{2Ab\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d\sqrt{b \cos(c+dx)}} + \frac{2Ab^2 \sin(c+dx)}{d\sqrt{b \cos(c+dx)}}$$

[Out] $2*A*b^2*\sin(d*x+c)/d/(b*\cos(d*x+c))^{(1/2)}+2*b^2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}-2*A*b*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2716, 2721, 2719, 2720}

$$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^3(c+dx) dx = \frac{2Ab^2 \sin(c+dx)}{d\sqrt{b \cos(c+dx)}}$$

$$- \frac{2AbE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2b^2 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d\sqrt{b \cos(c+dx)}}$$

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] (-2*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= b^3 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx \\ &= (Ab^3) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + (b^2 B) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2Ab^2 \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} - (Ab) \int \sqrt{b \cos(c+dx)} dx + \frac{(b^2 B \sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{\sqrt{b \cos(c+dx)}} \\
&= \frac{2b^2 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d\sqrt{b \cos(c+dx)}} + \frac{2Ab^2 \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} \\
&\quad - \frac{(Ab \sqrt{b \cos(c+dx)}) \int \sqrt{\cos(c+dx)} dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2Ab \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2b^2 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d\sqrt{b \cos(c+dx)}} + \frac{2Ab^2 \sin(c+dx)}{d\sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

$$\int (b \cos(c+dx))^{3/2} (A + B \cos(c+dx)) \sec^3(c+dx) dx = \frac{2(b \cos(c+dx))^{3/2} \left(-AE\left(\frac{1}{2}(c+dx) \mid 2\right) + B \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \frac{A \sin(c+dx)}{\sqrt{\cos(c+dx)}} \right)}{d \cos^{3/2}(c+dx)}$$

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] (2*(b*Cos[c + d*x])^(3/2)*(-A*EllipticE[(c + d*x)/2, 2] + B*EllipticF[(c + d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]])/(d*Cos[c + d*x]^(3/2))

Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.97

method	result
default	$ \frac{2b^2 \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + b \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d \cos^{3/2}(c+dx)} \left(2A \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} E \left(\cos \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \right. $ $ \left. + B \operatorname{EllipticF} \left(\frac{dx}{2} + \frac{c}{2} \right) + \frac{A \sin \left(\frac{dx}{2} + \frac{c}{2} \right)}{\sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1}} \right) $
parts	$ -\frac{2Ab^2 \left(-2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + b \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1} \sqrt{-2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \right)}{d \cos^{3/2}(c+dx)} \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \frac{A \sin \left(\frac{dx}{2} + \frac{c}{2} \right)}{\sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - 1}} $

[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERB OSE)


```
[Out] 2*b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.55

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \frac{-i \sqrt{2} B b^{3/2} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B b^{3/2} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 2 \sqrt{b \cos(dx + c)} (A \sin(dx + c) + B \cos(dx + c))}{(d \cos(dx + c))^{3/2}}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*A*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*b*sin(d*x + c)/(d*cos(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{3/2} \sec(dx + c)^3 dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)

Giac [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{3/2} \sec(dx + c)^3 dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos(c + dx)^3} dx$$

[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)

[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)

3.811 $\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^4(c+dx) dx$

Optimal result	7103
Rubi [A] (verified)	7103
Mathematica [A] (verified)	7105
Maple [B] (verified)	7105
Fricas [C] (verification not implemented)	7106
Sympy [F(-1)]	7107
Maxima [F]	7107
Giac [F]	7107
Mupad [F(-1)]	7108

Optimal result

Integrand size = 31, antiderivative size = 141

$$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^4(c+dx) dx =$$

$$-\frac{2bB\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b \cos(c+dx)}}$$

$$+ \frac{2Ab^3 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2b^2B \sin(c+dx)}{d\sqrt{b \cos(c+dx)}}$$

[Out] $2/3*A*b^3*\sin(d*x+c)/d/(b*\cos(d*x+c))^(3/2)+2*b^2*B*\sin(d*x+c)/d/(b*\cos(d*x+c))^(1/2)+2/3*A*b^2*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2))*\cos(d*x+c)^(1/2)/d/(b*\cos(d*x+c))^(1/2)-2*b*B*(\cos(1/2*d*x+1/2*c)^2)^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2))*(b*\cos(d*x+c))^(1/2)/d/\cos(d*x+c)^(1/2)$

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2716, 2721, 2720, 2719}

$$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^4(c+dx) dx = \frac{2Ab^3 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}}$$

$$+ \frac{2Ab^2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b \cos(c+dx)}}$$

$$+ \frac{2b^2B \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} - \frac{2bBE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]

[Out] (-2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^2*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= b^4 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\ &= (Ab^4) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx + (b^3 B) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2Ab^3 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2b^2 B \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} \\
&\quad + \frac{1}{3}(Ab^2) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx - (bB) \int \sqrt{b \cos(c+dx)} dx \\
&= \frac{2Ab^3 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2b^2 B \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + \frac{(Ab^2 \sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3\sqrt{b \cos(c+dx)}} \\
&\quad - \frac{(bB \sqrt{b \cos(c+dx)}) \int \sqrt{\cos(c+dx)} dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2bB \sqrt{b \cos(c+dx)} E(\frac{1}{2}(c+dx)|2)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^2 \sqrt{\cos(c+dx)} \text{EllipticF}(\frac{1}{2}(c+dx), 2)}{3d\sqrt{b \cos(c+dx)}} \\
&\quad + \frac{2Ab^3 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2b^2 B \sin(c+dx)}{d\sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int (b \cos(c+dx))^{3/2} (A + B \cos(c+dx)) \sec^4(c+dx) dx = \frac{2b^2 \left(-3B \sqrt{\cos(c+dx)} E(\frac{1}{2}(c+dx)|2) + A \sqrt{\cos(c+dx)} \text{EllipticF}(\frac{1}{2}(c+dx), 2) + 3B \sin(c+dx) \right)}{3d\sqrt{b \cos(c+dx)}}$$

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]

[Out] (2*b^2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(177) = 354.

Time = 6.39 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.87

method	result
default	$2\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)b(\sin^2(\frac{dx}{2} + \frac{c}{2}))} b \left(2A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} (\sin^2(\frac{dx}{2} + \frac{c}{2})) - 12 \right)$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) (\sin^2(\frac{dx}{2} + \frac{c}{2})) - 2(\sin^2(\frac{dx}{2} + \frac{c}{2})) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right)}{3\sqrt{-b\left(2(\sin^4(\frac{dx}{2} + \frac{c}{2})) - (\sin^2(\frac{dx}{2} + \frac{c}{2}))\right) \left(2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right)}}$

[In] `int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERBOSE)`

[Out]
$$\frac{2/3*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*b*\sin(1/2*d*x+1/2*c)^2)^{(1/2)*b/\sin(1/2*d*x+1/2*c)^3/(4*\sin(1/2*d*x+1/2*c)^4-4*\sin(1/2*d*x+1/2*c)^2+1)*(2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)*\sin(1/2*d*x+1/2*c)^2-12*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+6*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)*\sin(1/2*d*x+1/2*c)^2+2*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})+6*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4*b+b*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}}{(2*\cos(1/2*d*x+1/2*c)^2-1)*b)^{(1/2)}/d}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.36

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \frac{-i \sqrt{2} A b^{3/2} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A b^{3/2} \cos(dx + c) \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3 i \sqrt{2} B b^{3/2} \cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3 i \sqrt{2} B b^{3/2} \cos(dx + c)^2 \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2*(3*B*b*\cos(dx + c) + A*b)*\sqrt{b*\cos(dx + c)}*\sin(dx + c))/(d*\cos(dx + c)^2}$$

[In] `integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")`

[Out]
$$\frac{1/3*(-I*\sqrt{2}*A*b^{3/2}*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + I*\sqrt{2}*A*b^{3/2}*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - 3*I*\sqrt{2}*B*b^{3/2}*\cos(d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) + 3*I*\sqrt{2}*B*b^{3/2}*\cos(d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) + 2*(3*B*b*\cos(dx + c) + A*b)*\sqrt{b*\cos(dx + c)}*\sin(dx + c))/(d*\cos(dx + c)^2)}$$

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)

[Out] Timed out

Maxima [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^4 dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)

Giac [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^4 dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos(c + dx)^4} dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```


3.812 $\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^5(c+dx) dx$

Optimal result	7109
Rubi [A] (verified)	7109
Mathematica [A] (verified)	7112
Maple [B] (verified)	7112
Fricas [C] (verification not implemented)	7113
Sympy [F(-1)]	7113
Maxima [F]	7114
Giac [F]	7114
Mupad [F(-1)]	7114

Optimal result

Integrand size = 31, antiderivative size = 174

$$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)) \sec^5(c+dx) dx =$$

$$-\frac{6Ab\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b \cos(c+dx)}}$$

$$+ \frac{2Ab^4 \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2b^3 B \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{6Ab^2 \sin(c+dx)}{5d\sqrt{b \cos(c+dx)}}$$

```
[Out] 2/5*A*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b^3*B*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*A*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used

= {16, 2827, 2716, 2721, 2719, 2720}

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{6Ab^2 \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{6AbE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2b^3 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b^2 B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}}$$

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]

[Out] (-6*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^3*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b^2*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^5 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\
&= (Ab^5) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx + (b^4 B) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\
&= \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^3 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} \\
&\quad + \frac{1}{5} (3Ab^3) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + \frac{1}{3} (b^2 B) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^3 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6Ab^2 \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{1}{5} (3Ab) \int \sqrt{b \cos(c + dx)} dx + \frac{(b^2 B \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} \\
&= \frac{2b^2 B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} \\
&\quad + \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^3 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} \\
&\quad + \frac{6Ab^2 \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{(3Ab\sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}} \\
&= -\frac{6Ab\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2b^2 B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} \\
&\quad + \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^3 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6Ab^2 \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.61

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \frac{2(b \cos(c + dx))^{3/2} \sec^3(c + dx) \left(-9A \cos^{3/2}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx) \mid 2\right) \right) + 5B \cos^{3/2}(c + dx) \text{EllipticE}\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticE}\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d}$$

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]

[Out] (2*(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(-9*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 5*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*B*Sin[c + d*x] + (9*A*Sin[2*(c + d*x)]/2 + 3*A*Tan[c + d*x]))/(15*d)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(202) = 404.

Time = 8.30 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.32

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)} b \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b \left(72A \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 36A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} E\left(\frac{1}{2}\left(\frac{dx}{2} + \frac{c}{2}\right) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticE}\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticE}\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d}$
parts	$\frac{2A \sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)} b \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b \left(24 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 12 \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} E\left(\frac{1}{2}\left(\frac{dx}{2} + \frac{c}{2}\right) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticE}\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticE}\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \cos^{3/2}(c + dx) \text{EllipticF}\left(\frac{1}{2}(c + dx) \mid 2\right)}{15d}$

[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x,method=_RETURNVERBOSE)

[Out] -2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)

```
*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \frac{-5i \sqrt{2} B b^{3/2} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B b^{3/2} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 9i \sqrt{2} A b^{3/2} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 9i \sqrt{2} A b^{3/2} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2(9A^2 b^2 \cos(dx + c)^2 + 5B^2 b^2 \cos(dx + c) + 3A^2 b) \sqrt{b \cos(dx + c)} \sin(dx + c)}{(d \cos(dx + c))^3}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*b^(3/2)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*b^(3/2)*co
s(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 9*I*sqrt(2)*A*b^(3/2)*cos(d*x + c)^3*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*A*
b*cos(d*x + c)^2 + 5*B*b*cos(d*x + c) + 3*A*b)*sqrt(b*cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{3/2} \sec(dx + c)^5 dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)

Giac [F]

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{3/2} \sec(dx + c)^5 dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos(c + dx)^5} dx$$

[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5,x)

[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5, x)

3.813 $\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx$

Optimal result	7115
Rubi [A] (verified)	7115
Mathematica [A] (verified)	7117
Maple [A] (verified)	7118
Fricas [C] (verification not implemented)	7118
Sympy [F(-1)]	7119
Maxima [F]	7119
Giac [F]	7119
Mupad [F(-1)]	7119

Optimal result

Integrand size = 23, antiderivative size = 171

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx = \frac{6Ab^2 \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d \sqrt{\cos(c + dx)}} + \frac{10b^3 B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d \sqrt{b \cos(c + dx)}} + \frac{10b^2 B \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2Ab(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7d}$$

```
[Out] 2/5*A*b*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/d+10/21*b^3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+6/5*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2827, 2715, 2721, 2719, 2720}

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx = \frac{6Ab^2 E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5d \sqrt{\cos(c + dx)}} + \frac{2Ab \sin(c + dx) (b \cos(c + dx))^{3/2}}{5d} + \frac{10b^3 B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d \sqrt{b \cos(c + dx)}} + \frac{10b^2 B \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d} + \frac{2B \sin(c + dx) (b \cos(c + dx))^{5/2}}{7d}$$

[In] Int[(b*cos[c + d*x])^(5/2)*(A + B*cos[c + d*x]),x]

[Out] (6*A*b^2*Sqrt[b*cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (10*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*cos[c + d*x]]) + (10*b^2*B*Sqrt[b*cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*b*(b*cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*B*(b*cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*SIN[c + d*x])^n/SIN[c + d*x]^n, Int[SIN[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= A \int (b \cos(c + dx))^{5/2} dx + \frac{B \int (b \cos(c + dx))^{7/2} dx}{b} \\ &= \frac{2Ab(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} \\ &\quad + \frac{1}{5}(3Ab^2) \int \sqrt{b \cos(c + dx)} dx + \frac{1}{7}(5bB) \int (b \cos(c + dx))^{3/2} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{10b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{21d} + \frac{2Ab(b \cos(c+dx))^{3/2} \sin(c+dx)}{5d} \\
&\quad + \frac{2B(b \cos(c+dx))^{5/2} \sin(c+dx)}{7d} \\
&\quad + \frac{1}{21} (5b^3 B) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx + \frac{\left(3Ab^2 \sqrt{b \cos(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{5\sqrt{\cos(c+dx)}} \\
&= \frac{6Ab^2 \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{10b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{21d} \\
&\quad + \frac{2Ab(b \cos(c+dx))^{3/2} \sin(c+dx)}{5d} + \frac{2B(b \cos(c+dx))^{5/2} \sin(c+dx)}{7d} \\
&\quad + \frac{\left(5b^3 B \sqrt{\cos(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{21\sqrt{b \cos(c+dx)}} \\
&= \frac{6Ab^2 \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{10b^3 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d\sqrt{b \cos(c+dx)}} + \frac{10b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{21d} \\
&\quad + \frac{2Ab(b \cos(c+dx))^{3/2} \sin(c+dx)}{5d} + \frac{2B(b \cos(c+dx))^{5/2} \sin(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.58

$$\int (b \cos(c+dx))^{5/2} (A + B \cos(c+dx)) dx = \frac{(b \cos(c+dx))^{5/2} \left(252AE\left(\frac{1}{2}(c+dx) \mid 2\right) + 100B \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 2\sqrt{\cos(c+dx)}\right)}{210d \cos^{5/2}(c+dx)}$$

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]

[Out] ((b*Cos[c + d*x])^(5/2)*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[c + d*x]))/(210*d*Cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 9.01 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.76

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}b^3\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$
parts	$\left. -\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}b^3\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}\right)$

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(240*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx = \frac{-25i \sqrt{2} B b^{5/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} B b^{5/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 63i \sqrt{2} A b^{5/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 63i \sqrt{2} A b^{5/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2*(15*B*b^2*\cos(d*x + c)^2 + 21*A*b^2*\cos(d*x + c) + 25*B*b^2)*\sqrt{b*\cos(d*x + c)}*\sin(d*x + c))/d$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/105*(-25*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*I*sqrt(2)*B*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*B*b^2*cos(d*x + c)^2 + 21*A*b^2*cos(d*x + c) + 25*B*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{5/2} dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{5/2} dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx = \int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx$$

```
[In] int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)),x)
```

```
[Out] int((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)), x)
```

3.814 $\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx$

Optimal result	7120
Rubi [A] (verified)	7120
Mathematica [A] (verified)	7122
Maple [A] (verified)	7123
Fricas [C] (verification not implemented)	7123
Sympy [F(-1)]	7124
Maxima [F]	7124
Giac [F]	7124
Mupad [F(-1)]	7125

Optimal result

Integrand size = 29, antiderivative size = 145

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \frac{6b^2 B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d \sqrt{\cos(c + dx)}} + \frac{2Ab^3 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d \sqrt{b \cos(c + dx)}} + \frac{2Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2bB(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d}$$

[Out] $\frac{2}{5} b^2 B (b \cos(dx+c))^{3/2} \sin(dx+c)/d + \frac{2}{3} A b^3 (\cos(1/2 dx + 1/2 c))^{1/2} / \cos(1/2 dx + 1/2 c) \operatorname{EllipticF}(\sin(1/2 dx + 1/2 c), 2^{1/2}) \cos(dx+c)^{1/2} / d + \frac{2}{3} A b^2 \sin(dx+c) (b \cos(dx+c))^{1/2} / d + \frac{6}{5} b^2 B (\cos(1/2 dx + 1/2 c))^{1/2} / \cos(1/2 dx + 1/2 c) \operatorname{EllipticE}(\sin(1/2 dx + 1/2 c), 2^{1/2}) (b \cos(dx+c))^{1/2} / d \cos(dx+c)^{1/2}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used

= {16, 2827, 2715, 2721, 2720, 2719}

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \frac{2Ab^3 \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d \sqrt{b \cos(c + dx)}} + \frac{2Ab^2 \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d} + \frac{6b^2 B E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5d \sqrt{\cos(c + dx)}} + \frac{2bB \sin(c + dx) (b \cos(c + dx))^{3/2}}{5d}$$

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] (6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx \\
 &= (Ab) \int (b \cos(c + dx))^{3/2} dx + B \int (b \cos(c + dx))^{5/2} dx \\
 &= \frac{2Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2bB(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
 &\quad + \frac{1}{3} (Ab^3) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx + \frac{1}{5} (3b^2 B) \int \sqrt{b \cos(c + dx)} dx \\
 &= \frac{2Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2bB(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
 &\quad + \frac{\left(Ab^3 \sqrt{\cos(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} \\
 &\quad + \frac{\left(3b^2 B \sqrt{b \cos(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}} \\
 &= \frac{6b^2 B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2Ab^3 \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} \\
 &\quad + \frac{2Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2bB(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \frac{2b(b \cos(c + dx))^{3/2} \left(9BE\left(\frac{1}{2}(c + dx) \mid 2\right) + 5A \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sqrt{\cos(c + dx)}(5A + 3B) \right)}{15d \cos^{3/2}(c + dx)}$$

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] (2*b*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2))

Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.88

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}b^3\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(20A+24B\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(15\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{15\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}b^3\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)+\left(3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\right)}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}$

[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x,method=_RETURNVERBOS E)

[Out]
$$-\frac{2}{15}\left(\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)b\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}b^3\left(-24B\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^6+\left(20A+24B\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)+\left(-10A-6B\right)\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)+5A\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{1/2}\left(2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)^{1/2}\text{EllipticF}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{1/2}\right)-9B\left(\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2\right)^{1/2}\left(2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)^{1/2}\text{EllipticE}\left(\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right),2^{1/2}\right)\right)/\left(-b\left(2\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^4-\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2\right)^{1/2}/\sin\left(\frac{1}{2}d*x+\frac{1}{2}c\right)/\left(\left(2\cos\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)^2-1\right)b\right)^{1/2}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.08

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \frac{-5i \sqrt{2} A b^{5/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} A b^{5/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 9i \sqrt{2} B b^{5/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 9i \sqrt{2} B b^{5/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2*(3*B*b^2*\cos(d*x + c) + 5*A*b^2)*\sqrt{b*\cos(d*x + c)}*\sin(d*x + c)}{d}$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")

[Out]
$$\frac{1}{15}\left(-5I\sqrt{2}A b^{5/2}\text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I\sin(dx + c)) + 5I\sqrt{2}A b^{5/2}\text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I\sin(dx + c)) + 9I\sqrt{2}B b^{5/2}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I\sin(dx + c))) - 9I\sqrt{2}B b^{5/2}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I\sin(dx + c))) + 2*(3*B*b^2*\cos(d*x + c) + 5*A*b^2)*\sqrt{b*\cos(d*x + c)}*\sin(d*x + c)\right)/d$$

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)

[Out] Timed out

Maxima [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{5/2} \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)

Giac [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{5/2} \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec(c + dx) dx = \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos(c + dx)} dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

3.815 $\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^2(c+dx) dx$

Optimal result	7126
Rubi [A] (verified)	7126
Mathematica [A] (verified)	7128
Maple [A] (verified)	7128
Fricas [C] (verification not implemented)	7129
Sympy [F(-1)]	7130
Maxima [F]	7130
Giac [F]	7130
Mupad [F(-1)]	7131

Optimal result

Integrand size = 31, antiderivative size = 116

$$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^2(c+dx) dx = \frac{2Ab^2 \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d \sqrt{\cos(c+dx)}} + \frac{2b^3 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d \sqrt{b \cos(c+dx)}} + \frac{2b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{3d}$$

[Out] $2/3*b^3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}+2/3*b^2*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d+2*A*b^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2721, 2719, 2715, 2720}

$$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^2(c+dx) dx = \frac{2Ab^2 E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{2b^3 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d \sqrt{b \cos(c+dx)}} + \frac{2b^2 B \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d}$$

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (2*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= b^2 \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx \\ &= (Ab^2) \int \sqrt{b \cos(c + dx)} dx + (bB) \int (b \cos(c + dx))^{3/2} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{3d} + \frac{1}{3} (b^3 B) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx \\
&\quad + \frac{\left(Ab^2 \sqrt{b \cos(c+dx)} \right) \int \sqrt{\cos(c+dx)} dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{2Ab^2 \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d \sqrt{\cos(c+dx)}} + \frac{2b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{3d} \\
&\quad + \frac{\left(b^3 B \sqrt{\cos(c+dx)} \right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3 \sqrt{b \cos(c+dx)}} \\
&= \frac{2Ab^2 \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d \sqrt{\cos(c+dx)}} \\
&\quad + \frac{2b^3 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d \sqrt{b \cos(c+dx)}} + \frac{2b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int (b \cos(c+dx))^{5/2} (A + B \cos(c+dx)) \sec^2(c+dx) dx = \frac{2b^2 \sqrt{b \cos(c+dx)} \left(3AE\left(\frac{1}{2}(c+dx) \mid 2\right) + B \left(\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \sqrt{\cos(c+dx)} \sin(c+dx) \right) \right)}{3d \sqrt{\cos(c+dx)}}$$

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x]))) / (3*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 15.67 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.07

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}b^3\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}d}$

[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x,method=_RETURNVERB OSE)

[Out] $\frac{2}{3} * \left((2 * \cos(1/2 * d * x + 1/2 * c))^2 - 1 \right) * b * \sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * b^3 * (-4 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c)^4 + 3 * A * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) + 2 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c)^2 - B * (\sin(1/2 * d * x + 1/2 * c)^2)^{(1/2)} * (2 * \sin(1/2 * d * x + 1/2 * c)^2 - 1)^{(1/2)} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2^{(1/2)}) / (-b * (2 * \sin(1/2 * d * x + 1/2 * c)^4 - \sin(1/2 * d * x + 1/2 * c)^2))^{(1/2)} / \sin(1/2 * d * x + 1/2 * c) / ((2 * \cos(1/2 * d * x + 1/2 * c)^2 - 1) * b)^{(1/2)} / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.22

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \frac{-i \sqrt{2} B b^{5/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B b^{5/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{2} + \frac{A b^{5/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) - A b^{5/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{2} + \frac{2 * \sqrt{2} * b^{5/2} * \cos(c + dx) * \sin(c + dx)}{2} + \frac{2 * \sqrt{2} * b^{5/2} * \cos(c + dx) * \sin(c + dx)}{2} / d$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")

[Out] $\frac{1}{3} * (-I * \sqrt{2} * B * b^{5/2} * \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) + I * \sin(d * x + c)) + I * \sqrt{2} * B * b^{5/2} * \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) - I * \sin(d * x + c)) + 3 * I * \sqrt{2} * A * b^{5/2} * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) + I * \sin(d * x + c))) - 3 * I * \sqrt{2} * A * b^{5/2} * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) - I * \sin(d * x + c))) + 2 * \sqrt{2} * (b * \cos(d * x + c)) * B * b^2 * \sin(d * x + c)) / d$

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)

[Out] Timed out

Maxima [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{5/2} \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)

Giac [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{5/2} \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos(c + dx)^2} dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

3.816 $\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^3(c+dx) dx$

Optimal result	7132
Rubi [A] (verified)	7132
Mathematica [A] (verified)	7134
Maple [A] (verified)	7134
Fricas [C] (verification not implemented)	7135
Sympy [F(-1)]	7135
Maxima [F]	7135
Giac [F]	7136
Mupad [F(-1)]	7136

Optimal result

Integrand size = 31, antiderivative size = 85

$$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^3(c+dx) dx = \frac{2b^2 B \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d \sqrt{\cos(c+dx)}} + \frac{2Ab^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d \sqrt{b \cos(c+dx)}}$$

[Out] $2A*b^3*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}+2*b^2*B*(\cos(1/2*d*x+1/2*c))^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 2827, 2721, 2720, 2719}

$$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^3(c+dx) dx = \frac{2Ab^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d \sqrt{b \cos(c+dx)}} + \frac{2b^2 B E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] (2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] :=> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] :=> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :=> Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :=> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^3 \int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\
 &= (Ab^3) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx + (b^2B) \int \sqrt{b \cos(c + dx)} dx \\
 &= \frac{(Ab^3 \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{\sqrt{b \cos(c + dx)}} + \frac{(b^2B \sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{2b^2B \sqrt{b \cos(c + dx)} E(\frac{1}{2}(c + dx) | 2)}{d \sqrt{\cos(c + dx)}} + \frac{2Ab^3 \sqrt{\cos(c + dx)} \text{EllipticF}(\frac{1}{2}(c + dx), 2)}{d \sqrt{b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \frac{2(b \cos(c + dx))^{5/2} (BE(\frac{1}{2}(c + dx) | 2) + A \text{EllipticF}(\frac{1}{2}(c + dx), 2))}{d \cos^{\frac{5}{2}}(c + dx)}$$

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] (2*(b*Cos[c + d*x])^(5/2)*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/2, 2]))/(d*Cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 53.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.92

method	result
default	$\frac{2\sqrt{2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1} b (\sin^2(\frac{dx}{2} + \frac{c}{2})) b^3 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1} (AF(\cos(\frac{dx}{2} + \frac{c}{2}), \sqrt{2}) - BE(\cos(\frac{dx}{2} + \frac{c}{2})))}{\sqrt{-b(2(\sin^4(\frac{dx}{2} + \frac{c}{2})) - (\sin^2(\frac{dx}{2} + \frac{c}{2})))} \sin(\frac{dx}{2} + \frac{c}{2}) \sqrt{2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1} b d}$
parts	$\frac{2A\sqrt{2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1} b (\sin^2(\frac{dx}{2} + \frac{c}{2})) b^3 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1} F(\cos(\frac{dx}{2} + \frac{c}{2}), \sqrt{2})}{\sqrt{-b(2(\sin^4(\frac{dx}{2} + \frac{c}{2})) - (\sin^2(\frac{dx}{2} + \frac{c}{2})))} \sin(\frac{dx}{2} + \frac{c}{2}) \sqrt{2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1} b d} + \frac{2B\sqrt{2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1} b (\sin^2(\frac{dx}{2} + \frac{c}{2})) b^3 \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1} F(\cos(\frac{dx}{2} + \frac{c}{2}), \sqrt{2})}{\sqrt{-b(2(\sin^4(\frac{dx}{2} + \frac{c}{2})) - (\sin^2(\frac{dx}{2} + \frac{c}{2})))} \sin(\frac{dx}{2} + \frac{c}{2}) \sqrt{2(\cos^2(\frac{dx}{2} + \frac{c}{2})) - 1} b d}$
risch	$\frac{i B b^2 \sqrt{2} \sqrt{(e^{2i(dx+c)} + 1) b e^{-i(dx+c)}}}{d} - \frac{i \left(\frac{i A \sqrt{-i(e^{i(dx+c)} + i)} \sqrt{2} \sqrt{i(e^{i(dx+c)} - i)} \sqrt{i e^{i(dx+c)}} F(\sqrt{-i(e^{i(dx+c)} + i)}, \frac{\sqrt{2}}{2})}{\sqrt{b e^{3i(dx+c)} + b e^{i(dx+c)}}} \right)}{d} + B \left(-\frac{1}{b \sqrt{e^{i(dx+c)}}} \right)$

[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x,method=_RETURNVERBOSE)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \frac{-i \sqrt{2} A b^{5/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A b^{5/2} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + B b^{5/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - B b^{5/2} \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{d}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*A*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(2)*B*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{5/2} \sec(dx + c)^3 dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Giac [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{5/2} \sec(dx + c)^3 dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos(c + dx)^3} dx$$

[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)

[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)

$$3.817 \quad \int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^4(c+dx) dx$$

Optimal result	.7137
Rubi [A] (verified)	.7137
Mathematica [A] (verified)	.7139
Maple [A] (verified)	.7139
Fricas [C] (verification not implemented)	.7140
Sympy [F(-1)]	.7140
Maxima [F]	.7141
Giac [F]	.7141
Mupad [F(-1)]	.7141

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^4(c+dx) dx =$$

$$\frac{2Ab^2 \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d \sqrt{\cos(c+dx)}} + \frac{2b^3 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d \sqrt{b \cos(c+dx)}} + \frac{2Ab^3 \sin(c+dx)}{d \sqrt{b \cos(c+dx)}}$$

[Out] $2*A*b^3*\sin(d*x+c)/d/(b*\cos(d*x+c))^{(1/2)}+2*b^3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}-2*A*b^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2716, 2721, 2719, 2720}

$$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^4(c+dx) dx = \frac{2Ab^3 \sin(c+dx)}{d \sqrt{b \cos(c+dx)}}$$

$$- \frac{2Ab^2 E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{2b^3 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d \sqrt{b \cos(c+dx)}}$$

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]

[Out] (-2*A*b^2*Sqrt[b*Cos[c + d*x])*EllipticE[(c + d*x)/2, 2]/(d*Sqrt[Cos[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x])*EllipticF[(c + d*x)/2, 2]/(d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= b^4 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx \\ &= (Ab^4) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + (b^3 B) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2Ab^3 \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} - (Ab^2) \int \sqrt{b \cos(c+dx)} dx + \frac{(b^3 B \sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{\sqrt{b \cos(c+dx)}} \\
&= \frac{2b^3 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d\sqrt{b \cos(c+dx)}} + \frac{2Ab^3 \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} \\
&\quad - \frac{(Ab^2 \sqrt{b \cos(c+dx)}) \int \sqrt{\cos(c+dx)} dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2Ab^2 \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2b^3 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{d\sqrt{b \cos(c+dx)}} + \frac{2Ab^3 \sin(c+dx)}{d\sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

$$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^4(c+dx) dx = \frac{2(b \cos(c+dx))^{5/2} \left(-AE\left(\frac{1}{2}(c+dx) \mid 2\right) + B \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \frac{A \sin(c+dx)}{\sqrt{\cos(c+dx)}} \right)}{d \cos^5(c+dx)}$$

[In] Integrate[(b*cos[c + d*x])^(5/2)*(A + B*cos[c + d*x])*Sec[c + d*x]^4,x]

[Out] (2*(b*cos[c + d*x])^(5/2)*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c + d*x)/2, 2] + (A*sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*cos[c + d*x]^5)

Maple [A] (verified)

Time = 158.89 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.94

method	result
default	$ \frac{2b^3 \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + b \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{\sqrt{-b \left(2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \right)} \left(2A \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} - 1 E \left(\cos \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \right) \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} $
parts	$ -\frac{2Ab^3 \left(-2 \cos \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \sqrt{-2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + b \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{\sqrt{-b \left(2 \left(\sin^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) - \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) \right)} \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} - 1 \sqrt{-2 \left(\sin^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} \right) \sin \left(\frac{dx}{2} + \frac{c}{2} \right) \sqrt{2 \left(\cos^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)} $

[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x,method=_RETURNVERB OSE)

```
[Out] 2*b^3*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \frac{-i \sqrt{2} B b^{5/2} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B b^{5/2} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 2 \sqrt{2} b^{5/2} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2 \sqrt{2} b^{5/2} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 2 \sqrt{2} b^{5/2} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 2 \sqrt{2} b^{5/2} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{(d \cos(c + dx))^2}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] (-I*sqrt(2)*B*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*B*b^(5/2)*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*b^(5/2)*cos(d*x + c)*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + I*sqrt(2)*A*b^(5/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*b^2
*sin(d*x + c))/(d*cos(d*x + c))^2
```

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```


Maxima [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{5/2} \sec(dx + c)^4 dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)

Giac [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{5/2} \sec(dx + c)^4 dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^4(c + dx) dx = \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos(c + dx)^4} dx$$

[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4,x)

[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)

3.818 $\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^5(c+dx) dx$

Optimal result	7142
Rubi [A] (verified)	7142
Mathematica [A] (verified)	7144
Maple [B] (verified)	7144
Fricas [C] (verification not implemented)	7145
Sympy [F(-1)]	7145
Maxima [F]	7146
Giac [F]	7146
Mupad [F(-1)]	7146

Optimal result

Integrand size = 31, antiderivative size = 143

$$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^5(c+dx) dx =$$

$$-\frac{2b^2 B \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d \sqrt{\cos(c+dx)}} + \frac{2Ab^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d \sqrt{b \cos(c+dx)}}$$

$$+ \frac{2Ab^4 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2b^3 B \sin(c+dx)}{d \sqrt{b \cos(c+dx)}}$$

[Out] $2/3*A*b^4*\sin(d*x+c)/d/(b*\cos(d*x+c))^(3/2)+2*b^3*B*\sin(d*x+c)/d/(b*\cos(d*x+c))^(1/2)+2/3*A*b^3*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^(1/2))*\cos(d*x+c)^(1/2)/d/(b*\cos(d*x+c))^(1/2)-2*b^2*B*(\cos(1/2*d*x+1/2*c))^2^(1/2)/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^(1/2))*(b*\cos(d*x+c))^(1/2)/d/\cos(d*x+c)^(1/2)$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2716, 2721, 2720, 2719}

$$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^5(c+dx) dx = \frac{2Ab^4 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}}$$

$$+ \frac{2Ab^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d \sqrt{b \cos(c+dx)}}$$

$$+ \frac{2b^3 B \sin(c+dx)}{d \sqrt{b \cos(c+dx)}} - \frac{2b^2 B E\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]

[Out] (-2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^3*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= b^5 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\ &= (Ab^5) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx + (b^4 B) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2Ab^4 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2b^3 B \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} \\
&\quad + \frac{1}{3}(Ab^3) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx - (b^2 B) \int \sqrt{b \cos(c+dx)} dx \\
&= \frac{2Ab^4 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2b^3 B \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + \frac{(Ab^3 \sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3\sqrt{b \cos(c+dx)}} \\
&\quad - \frac{(b^2 B \sqrt{b \cos(c+dx)}) \int \sqrt{\cos(c+dx)} dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2b^2 B \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2Ab^3 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b \cos(c+dx)}} \\
&\quad + \frac{2Ab^4 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2b^3 B \sin(c+dx)}{d\sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int (b \cos(c+dx))^{5/2} (A + B \cos(c+dx)) \sec^5(c+dx) dx = \frac{2b^3 \left(-3B \sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) + A \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + 3B \sin(c+dx) \right)}{3d\sqrt{b \cos(c+dx)}}$$

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]

[Out] (2*b^3*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(179) = 358.

Time = 2.43 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.84

$$2\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)b(\sin^2(\frac{dx}{2} + \frac{c}{2}))} b^2 \left(2A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \right)$$

[In] `int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)`

[Out] $2/3*(-(-2*\cos(1/2*d*x+1/2*c)^2+1)*b*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*b^2/\sin(1/2*d*x+1/2*c)^3/(4*\sin(1/2*d*x+1/2*c)^4-4*\sin(1/2*d*x+1/2*c)^2+1)*(2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2-12*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+6*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2+2*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+6*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})))*(-2*\sin(1/2*d*x+1/2*c)^4*b+b*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/((2*\cos(1/2*d*x+1/2*c)^2-1)*b)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.37

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \frac{-i \sqrt{2} A b^{5/2} \cos(dx + c)^2 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A b^{5/2} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 3 i \sqrt{2} B b^{5/2} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 3 i \sqrt{2} B b^{5/2} \cos(dx + c)^2 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2*(3*B*b^2*\cos(dx + c) + A*b^2)*\sqrt{b*\cos(dx + c)}*\sin(dx + c)/(d*\cos(dx + c)^2)}$$

[In] `integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")`

[Out] $1/3*(-I*\sqrt{2}*A*b^{5/2}*\cos(dx + c)^2*\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c)) + I*\sqrt{2}*A*b^{5/2}*\cos(dx + c)^2*\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c)) - 3*I*\sqrt{2}*B*b^{5/2}*\cos(dx + c)^2*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + I*\sin(dx + c))) + 3*I*\sqrt{2}*B*b^{5/2}*\cos(dx + c)^2*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - I*\sin(dx + c))) + 2*(3*B*b^2*\cos(dx + c) + A*b^2)*\sqrt{b*\cos(dx + c)}*\sin(dx + c))/(d*\cos(dx + c)^2)$

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \text{Timed out}$$

[In] `integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)`

[Out] Timed out

Maxima [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{5/2} \sec(dx + c)^5 dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)

Giac [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{5/2} \sec(dx + c)^5 dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^5(c + dx) dx = \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos(c + dx)^5} dx$$

[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5,x)

[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^5, x)

3.819 $\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^6(c+dx) dx$

Optimal result	.7147
Rubi [A] (verified)	.7147
Mathematica [A] (verified)	.7150
Maple [B] (verified)	.7150
Fricas [C] (verification not implemented)	.7151
Sympy [F(-1)]	.7151
Maxima [F]	.7151
Giac [F]	.7152
Mupad [F(-1)]	.7152

Optimal result

Integrand size = 31, antiderivative size = 176

$$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)) \sec^6(c+dx) dx =$$

$$\frac{6Ab^2 \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5d \sqrt{\cos(c+dx)}} + \frac{2b^3 B \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d \sqrt{b \cos(c+dx)}}$$

$$+ \frac{2Ab^5 \sin(c+dx)}{5d (b \cos(c+dx))^{5/2}} + \frac{2b^4 B \sin(c+dx)}{3d (b \cos(c+dx))^{3/2}} + \frac{6Ab^3 \sin(c+dx)}{5d \sqrt{b \cos(c+dx)}}$$

[Out] $2/5*A*b^5*\sin(d*x+c)/d/(b*\cos(d*x+c))^{(5/2)}+2/3*b^4*B*\sin(d*x+c)/d/(b*\cos(d*x+c))^{(3/2)}+6/5*A*b^3*\sin(d*x+c)/d/(b*\cos(d*x+c))^{(1/2)}+2/3*b^3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}-6/5*A*b^2*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used

= {16, 2827, 2716, 2721, 2719, 2720}

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^6(c + dx) dx = \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{6Ab^3 \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{6Ab^2 E\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2b^4 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b^3 B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}}$$

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]

[Out] (-6*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^4*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b^3*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827


```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^6 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\
&= (Ab^6) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx + (b^5 B) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\
&= \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^4 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} \\
&\quad + \frac{1}{5} (3Ab^4) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + \frac{1}{3} (b^3 B) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^4 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6Ab^3 \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{1}{5} (3Ab^2) \int \sqrt{b \cos(c + dx)} dx + \frac{(b^3 B \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} \\
&= \frac{2b^3 B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\
&\quad + \frac{2b^4 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6Ab^3 \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{(3Ab^2 \sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}} \\
&= -\frac{6Ab^2 \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5d\sqrt{\cos(c + dx)}} \\
&\quad + \frac{2b^3 B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} \\
&\quad + \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^4 B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6Ab^3 \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

$$\frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^6(c + dx) dx = 2b^4 \left(9A \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \mid 2\right) - 5B \cos^{\frac{3}{2}}(c + dx) \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) - 5B \sin(c + dx) - \frac{9}{2}A \sin(c + dx) \right)}{15d(b \cos(c + dx))^{3/2}}$$

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]

[Out] (-2*b^4*(9*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - 5*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] - 5*B*Sin[c + d*x] - (9*A*Sin[2*(c + d*x)]/2 - 3*A*Tan[c + d*x]))/(15*d*(b*Cos[c + d*x])^(3/2))

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(204) = 408.

Time = 3.17 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.29

$$\frac{2\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)b(\sin^2(\frac{dx}{2} + \frac{c}{2}))}b^2\left(72A\cos(\frac{dx}{2} + \frac{c}{2})(\sin^6(\frac{dx}{2} + \frac{c}{2})) - 36A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right)}{15d(b\cos(c+dx))^{3/2}}$$

[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)

[Out] -2/15*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+36*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+20*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+b*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.20

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^6(c + dx) dx = \frac{-5i \sqrt{2} B b^{5/2} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B b^{5/2} \cos(dx + c)^3 \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) - 9i \sqrt{2} A b^{5/2} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 9i \sqrt{2} A b^{5/2} \cos(dx + c)^3 \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2(9A b^2 \cos(dx + c)^2 + 5B b^2 \cos(dx + c) + 3A b^2) \sqrt{b \cos(dx + c)} \sin(dx + c)}{(d \cos(dx + c))^3}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*A*b^2*cos(d*x + c)^2 + 5*B*b^2*cos(d*x + c) + 3*A*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^6(c + dx) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^6(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{5/2} \sec(dx + c)^6 dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Giac [F]

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^6(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{5/2} \sec(dx + c)^6 dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) \sec^6(c + dx) dx = \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos(c + dx)^6} dx$$

[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^6,x)

[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^6, x)

$$3.820 \quad \int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal result	7153
Rubi [A] (verified)	7154
Mathematica [A] (verified)	7156
Maple [A] (verified)	7156
Fricas [C] (verification not implemented)	7157
Sympy [F(-1)]	7157
Maxima [F]	7157
Giac [F]	7158
Mupad [F(-1)]	7158

Optimal result

Integrand size = 31, antiderivative size = 173

$$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{6A\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21d\sqrt{b \cos(c+dx)}} + \frac{10B\sqrt{b \cos(c+dx)} \sin(c+dx)}{21bd} + \frac{2A(b \cos(c+dx))^{3/2} \sin(c+dx)}{5b^2d} + \frac{2B(b \cos(c+dx))^{5/2} \sin(c+dx)}{7b^3d}$$

```
[Out] 2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2715, 2721, 2719, 2720}

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx = \frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d} + \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}} + \frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d} + \frac{10B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21bd} + \frac{10B\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]

[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d) + (2*A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n-1)/(d*n)), x] + Dist[b^2*((n-1)/n), Int[(b*SIN[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])ⁿ/Sin[c + d*x]ⁿ, Int[Sin[c + d*x]ⁿ, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^{m+1}, x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx}{b^3} \\
 &= \frac{A \int (b \cos(c + dx))^{5/2} dx}{b^3} + \frac{B \int (b \cos(c + dx))^{7/2} dx}{b^4} \\
 &= \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^2 d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^3 d} \\
 &\quad + \frac{(3A) \int \sqrt{b \cos(c + dx)} dx}{5b} + \frac{(5B) \int (b \cos(c + dx))^{3/2} dx}{7b^2} \\
 &= \frac{10B \sqrt{b \cos(c + dx)} \sin(c + dx)}{21bd} + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^2 d} \\
 &\quad + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^3 d} \\
 &\quad + \frac{1}{21} (5B) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx + \frac{\left(3A \sqrt{b \cos(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{5b \sqrt{\cos(c + dx)}} \\
 &= \frac{6A \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5bd \sqrt{\cos(c + dx)}} + \frac{10B \sqrt{b \cos(c + dx)} \sin(c + dx)}{21bd} \\
 &\quad + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^2 d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^3 d} \\
 &\quad + \frac{\left(5B \sqrt{\cos(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{21 \sqrt{b \cos(c + dx)}} \\
 &= \frac{6A \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5bd \sqrt{\cos(c + dx)}} + \frac{10B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21d \sqrt{b \cos(c + dx)}} \\
 &\quad + \frac{10B \sqrt{b \cos(c + dx)} \sin(c + dx)}{21bd} + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^2 d} \\
 &\quad + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^3 d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.58

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{252A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 100B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + (65B + 42A \cos(c + dx)) \sin\left(\frac{1}{2}(c + dx)\right)}{210d \sqrt{b \cos(c + dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]/(210*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 7.52 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.72

method	result
default	$-\frac{2\sqrt{\left(2\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(240B\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + (-168A - 360B)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) + (168A + 280B)\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right) - 42A - 80B\right)}{\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{\left(2\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}$
parts	$-\frac{2A\sqrt{\left(2\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(-8\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 8\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\cos\left(\frac{dx}{2} + \frac{c}{2}\right) - 2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{\left(2\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}}$

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```


Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{-25i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 25i \sqrt{2} B \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 63i \sqrt{2} A \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - 63i \sqrt{2} A \sqrt{b} \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))) + 2(15B \cos(dx + c)^2 + 21A \cos(dx + c) + 25B) \sqrt{b \cos(dx + c)}}{b d}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(-25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(15*B*cos(d*x + c)^2 + 21*A*cos(d*x + c) + 25*B)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^3 (A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)

[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2), x)

$$3.821 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal result	7159
Rubi [A] (verified)	7159
Mathematica [A] (verified)	7161
Maple [A] (verified)	7162
Fricas [C] (verification not implemented)	7162
Sympy [F(-1)]	7163
Maxima [F]	7163
Giac [F]	7163
Mupad [F(-1)]	7163

Optimal result

Integrand size = 31, antiderivative size = 144

$$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{6B\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2A\sqrt{b \cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2B(b \cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}$$

```
[Out] 2/5*B*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d+6/5*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used

= {16, 2827, 2715, 2721, 2720, 2719}

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \frac{2A \sin(c + dx) \sqrt{b \cos(c + dx)}}{3bd} + \frac{2A \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d \sqrt{b \cos(c + dx)}} + \frac{2B \sin(c + dx) (b \cos(c + dx))^{3/2}}{5b^2 d} + \frac{6BE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5bd \sqrt{\cos(c + dx)}}$$

[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*SIN[c + d*x])^n/SIN[c + d*x]^n, Int[SIN[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

$\text{Int}[(b \cdot \sin[e \cdot x] + f \cdot x)^m \cdot (c + d \cdot \sin[e \cdot x] + f \cdot x)], x_{\text{Symbol}}] \rightarrow \text{Dist}[c, \text{Int}[(b \cdot \sin[e \cdot x] + f \cdot x)^m, x], x] + \text{Dist}[d/b, \text{Int}[(b \cdot \sin[e \cdot x] + f \cdot x)^{m+1}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx}{b^2} \\
 &= \frac{A \int (b \cos(c + dx))^{3/2} dx}{b^2} + \frac{B \int (b \cos(c + dx))^{5/2} dx}{b^3} \\
 &= \frac{2A \sqrt{b \cos(c + dx)} \sin(c + dx)}{3bd} + \frac{2B (b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^2d} \\
 &\quad + \frac{1}{3} A \int \frac{1}{\sqrt{b \cos(c + dx)}} dx + \frac{(3B) \int \sqrt{b \cos(c + dx)} dx}{5b} \\
 &= \frac{2A \sqrt{b \cos(c + dx)} \sin(c + dx)}{3bd} + \frac{2B (b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^2d} \\
 &\quad + \frac{(A \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} + \frac{(3B \sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{5b\sqrt{\cos(c + dx)}} \\
 &= \frac{6B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5bd\sqrt{\cos(c + dx)}} + \frac{2A \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} \\
 &\quad + \frac{2A \sqrt{b \cos(c + dx)} \sin(c + dx)}{3bd} + \frac{2B (b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^2d}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\begin{aligned}
 &\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx \\
 &= \frac{2\sqrt{\cos(c + dx)} \left(9BE\left(\frac{1}{2}(c + dx) \mid 2\right) + 5A \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + \sqrt{\cos(c + dx)}(5A + 3B \cos(c + dx)) \right)}{15d\sqrt{b \cos(c + dx)}}
 \end{aligned}$$

[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*Sqrt[Cos[c + d*x]]*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 6.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.88

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(20A+24B\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(-10A+24B\right)\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(-10A+24B\right)\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(-10A+24B\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{15\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)\sqrt{2}}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)bd}}$

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

$$= \frac{-5i\sqrt{2}A\sqrt{b}\text{weierstrassPInverse}(-4, 0, \cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}A\sqrt{b}\text{weierstrassPInverse}(-4, 0, \cos(dx+c)-i\sin(dx+c))}{\sqrt{b\cos(c+dx)}}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c) + 5*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Giac [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \int \frac{\cos(c + dx)^2 (A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

```
[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2), x)
```

$$3.822 \quad \int \frac{\cos(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal result	7164
Rubi [A] (verified)	7164
Mathematica [A] (verified)	7166
Maple [A] (verified)	7166
Fricas [C] (verification not implemented)	7167
Sympy [F(-1)]	7167
Maxima [F]	7168
Giac [F]	7168
Mupad [B] (verification not implemented)	7168

Optimal result

Integrand size = 29, antiderivative size = 113

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{2A\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2B\sqrt{b \cos(c+dx)} \sin(c+dx)}{3bd}$$

[Out] $2/3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}+2/3*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/b/d+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/b/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {16, 2827, 2721, 2719, 2715, 2720}

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{2AE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b \cos(c+dx)}}{bd\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)\sqrt{b \cos(c+dx)}}{3bd} + \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b \cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx}{b} \\ &= \frac{A \int \sqrt{b \cos(c + dx)} dx}{b} + \frac{B \int (b \cos(c + dx))^{3/2} dx}{b^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{1}{3}B \int \frac{1}{\sqrt{b\cos(c+dx)}} dx \\
&\quad + \frac{\left(A\sqrt{b\cos(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{b\sqrt{\cos(c+dx)}} \\
&= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} \\
&\quad + \frac{\left(B\sqrt{\cos(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{bd\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.69

$$\begin{aligned}
&\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx \\
&= \frac{2\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\mid 2\right) + B\left(\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + \sqrt{\cos(c+dx)}\sin(c+dx)\right)\right)}{3bd\sqrt{\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*b*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.10

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$
parts	$\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)$

[In] `int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{2}{3} * ((2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * b * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (-4 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 3 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) + 2 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2))) / (-b * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - \sin(1/2 * d * x + 1/2 * c) ^ 2)) ^ (1/2) / \sin(1/2 * d * x + 1/2 * c) / ((2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * b) ^ (1/2) / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.26

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{-i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c))}{2}$$

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{3} * (-I * \sqrt{2} * B * \sqrt{b} * \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I * \sin(dx + c)) + I * \sqrt{2} * B * \sqrt{b} * \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I * \sin(dx + c)) + 3 * I * \sqrt{2} * A * \sqrt{b} * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + I * \sin(dx + c))) - 3 * I * \sqrt{2} * A * \sqrt{b} * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx + c) - I * \sin(dx + c))) + 2 * \sqrt{b * \cos(dx + c)} * B * \sin(dx + c)) / (b * d)$$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)

Giac [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \frac{2 B \sin(c + dx) \sqrt{b \cos(c + dx)}}{3 b d} + \frac{2 A \sqrt{\cos(c + dx)} E\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{d \sqrt{b \cos(c + dx)}} + \frac{2 B \sqrt{\cos(c + dx)} F\left(\frac{c}{2} + \frac{dx}{2} \middle| 2\right)}{3 d \sqrt{b \cos(c + dx)}}$$

[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)

[Out] (2*B*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d) + (2*A*cos(c + d*x)^(1/2)*ellipticE(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2)) + (2*B*cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1/2))

3.823 $\int \frac{A+B \cos(c+dx)}{\sqrt{b \cos(c+dx)}} dx$

Optimal result	7169
Rubi [A] (verified)	7169
Mathematica [A] (verified)	7170
Maple [A] (verified)	7171
Fricas [C] (verification not implemented)	7171
Sympy [F]	7172
Maxima [F]	7172
Giac [F]	7172
Mupad [B] (verification not implemented)	7172

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \frac{2B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{bd\sqrt{\cos(c + dx)}} + \frac{2A\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d\sqrt{b \cos(c + dx)}}$$

[Out] $2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}+2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/b/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2827, 2721, 2720, 2719}

$$\int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \frac{2A\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d\sqrt{b \cos(c + dx)}} + \frac{2BE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{bd\sqrt{\cos(c + dx)}}$$

[In] $\operatorname{Int}[(A + B*\operatorname{Cos}[c + d*x])/Sqrt[b*\operatorname{Cos}[c + d*x]], x]$

[Out] $(2*B*Sqrt[b*\operatorname{Cos}[c + d*x]]*\operatorname{EllipticE}[(c + d*x)/2, 2])/(b*d*Sqrt[\operatorname{Cos}[c + d*x]]) + (2*A*Sqrt[\operatorname{Cos}[c + d*x]]*\operatorname{EllipticF}[(c + d*x)/2, 2])/(d*Sqrt[b*\operatorname{Cos}[c + d*x]])$

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= A \int \frac{1}{\sqrt{b \cos(c + dx)}} dx + \frac{B \int \sqrt{b \cos(c + dx)} dx}{b} \\
&= \frac{\left(A \sqrt{\cos(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{\sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{b \cos(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{b \sqrt{\cos(c + dx)}} \\
&= \frac{2B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{bd \sqrt{\cos(c + dx)}} + \frac{2A \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \frac{2 \sqrt{\cos(c + dx)} \left(B E\left(\frac{1}{2}(c + dx) \mid 2\right) + A \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \right)}{d \sqrt{b \cos(c + dx)}}$$

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[b*Cos[c + d*x]], x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/
2, 2]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(AF\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-BE\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)bd}}$
parts	$\frac{2A\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2} \sqrt{2}\right)}{d\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}+\frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)bd}}$
risch	$-\frac{iB\left(e^{2i(dx+c)}+1\right)\sqrt{2}e^{-i(dx+c)}}{d\sqrt{\left(e^{2i(dx+c)}+1\right)be^{-i(dx+c)}}}-\frac{i\left(\frac{iA\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{i e^{i(dx+c)}}F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{be^{3i(dx+c)}+be^{i(dx+c)}}}\right)}{b\sqrt{e^{i(dx+c)}}}+B\left(-\frac{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}{b\sqrt{e^{i(dx+c)}}}\right)}$

[In] int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] $-2*\left(\left(2*\cos\left(1/2*d*x+1/2*c\right)^2-1\right)*b*\sin\left(1/2*d*x+1/2*c\right)^2\right)^{1/2}*\left(\sin\left(1/2*d*x+1/2*c\right)^2\right)^{1/2}*\left(-2*\cos\left(1/2*d*x+1/2*c\right)^2+1\right)^{1/2}*\left(A*\operatorname{EllipticF}\left(\cos\left(1/2*d*x+1/2*c\right),2^{1/2}\right)-B*\operatorname{EllipticE}\left(\cos\left(1/2*d*x+1/2*c\right),2^{1/2}\right)\right)/\left(-b*\left(2*\sin\left(1/2*d*x+1/2*c\right)^4-\sin\left(1/2*d*x+1/2*c\right)^2\right)\right)^{1/2}/\sin\left(1/2*d*x+1/2*c\right)/\left(\left(2*\cos\left(1/2*d*x+1/2*c\right)^2-1\right)*b\right)^{1/2}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{-i\sqrt{2}A\sqrt{b}\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i\sqrt{2}A\sqrt{b}\operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + i\sqrt{2}B\sqrt{b}\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) - i\sqrt{2}B\sqrt{b}\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{(b*d)}$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] $\left(-I*\sqrt{2}*A*\sqrt{b}*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + I*\sqrt{2}*A*\sqrt{b}*\operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) + I*\sqrt{2}*B*\sqrt{b}*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) - I*\sqrt{2}*B*\sqrt{b}*\operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)))\right)/(b*d)$

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)

[Out] Integral((A + B*cos(c + d*x))/sqrt(b*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \frac{2 \sqrt{\cos(c + dx)} (A F(\frac{c}{2} + \frac{dx}{2} | 2) + B E(\frac{c}{2} + \frac{dx}{2} | 2))}{d \sqrt{b \cos(c + dx)}}$$

[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(1/2),x)

[Out] (2*cos(c + d*x)^(1/2)*(A*ellipticF(c/2 + (d*x)/2, 2) + B*ellipticE(c/2 + (d*x)/2, 2)))/(d*(b*cos(c + d*x))^(1/2))

$$3.824 \quad \int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal result	7173
Rubi [A] (verified)	7173
Mathematica [A] (verified)	7175
Maple [A] (verified)	7175
Fricas [C] (verification not implemented)	7176
Sympy [F]	7176
Maxima [F]	7177
Giac [F]	7177
Mupad [F(-1)]	7177

Optimal result

Integrand size = 29, antiderivative size = 106

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx = -\frac{2A\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{bd\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d\sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{d\sqrt{b \cos(c + dx)}}$$

[Out] 2*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {16, 2827, 2716, 2721, 2719, 2720}

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \frac{2A \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2AE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{bd\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d\sqrt{b \cos(c + dx)}}$$

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]

[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= b \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx \\ &= (Ab) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + B \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2A \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{A \int \sqrt{b \cos(c + dx)} dx}{b} + \frac{\left(B \sqrt{\cos(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{\sqrt{b \cos(c + dx)}} \\
&= \frac{2B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d\sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{\left(A \sqrt{b \cos(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{b\sqrt{\cos(c + dx)}} \\
&= -\frac{2A \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{bd\sqrt{\cos(c + dx)}} \\
&\quad + \frac{2B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{d\sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.69

$$\begin{aligned}
&\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2\left(-A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + A \sin(c + dx)\right)}{d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.02

method	result
default	$ \frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d} \left(2A \cos\left(\frac{dx}{2} + \frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1 E\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \right) $
parts	$ -\frac{2A\left(-2\cos\left(\frac{dx}{2} + \frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} - 1 \sqrt{-2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)} \sin\left(\frac{dx}{2} + \frac{c}{2}\right) \sqrt{2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1}} $

[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOS E)

```
[Out] 2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.63

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{-i \sqrt{2} B \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} B \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)) + 2 \sqrt{b} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))) + 2 \sqrt{b} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) - i \sin(dx + c)))}{(b \cos(dx + c))^{3/2}}$$

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c)/(b*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx) \sqrt{b \cos(c + dx)}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)), x)

$$3.825 \quad \int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal result	7178
Rubi [A] (verified)	7179
Mathematica [A] (verified)	7180
Maple [B] (verified)	7181
Fricas [C] (verification not implemented)	7181
Sympy [F]	7182
Maxima [F]	7182
Giac [F]	7182
Mupad [F(-1)]	7183

Optimal result

Integrand size = 31, antiderivative size = 135

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx = -\frac{2B\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{bd\sqrt{\cos(c + dx)}} + \frac{2A\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}}$$

```
[Out] 2/3*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2716, 2721, 2720, 2719}

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2A \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d \sqrt{b \cos(c + dx)}} + \frac{2B \sin(c + dx)}{d \sqrt{b \cos(c + dx)}} - \frac{2BE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{bd \sqrt{\cos(c + dx)}}$$

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]

[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)
]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\
&= (Ab^2) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx + (bB) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx \\
&= \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} + \frac{1}{3}A \int \frac{1}{\sqrt{b \cos(c + dx)}} dx - \frac{B \int \sqrt{b \cos(c + dx)} dx}{b} \\
&= \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} + \frac{\left(A\sqrt{\cos(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{\left(B\sqrt{b \cos(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{b\sqrt{\cos(c + dx)}} \\
&= -\frac{2B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{bd\sqrt{\cos(c + dx)}} + \frac{2A\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} \\
&\quad + \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.62

$$\begin{aligned}
&\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2\left(-3B\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right) + A\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 3B \sin(c + dx) + A \tan\right)}{3d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]]
)*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x])/(3*d*Sqrt
[b*Cos[c + d*x]])
```


Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(171) = 342$.

Time = 6.36 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.01

method	result
default	$2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$

[In] `int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERB
OSE)`

[Out]
$$\frac{2}{3}\left(-(-2\cos(1/2*d*x+1/2*c)^2+1)*b*\sin(1/2*d*x+1/2*c)^2\right)^{(1/2)}/b/\sin(1/2*d*x+1/2*c)^3/(4*\sin(1/2*d*x+1/2*c)^4-4*\sin(1/2*d*x+1/2*c)^2+1)*(2*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2-12*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+6*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2+2*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)})+6*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-3*B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(-2*\sin(1/2*d*x+1/2*c)^4+b+b*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/((2*\cos(1/2*d*x+1/2*c)^2-1)*b)^{(1/2)}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.42

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{-i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + i \sqrt{2} A \sqrt{b} \cos(dx + c)}{\dots}$$

[In] `integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{3}\left(-I*\sqrt{2}*A*\sqrt{b}*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + I*\sqrt{2}*A*\sqrt{b}*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - 3*I*\sqrt{2}*B*\sqrt{b}*\cos(d*x + c)\right)$$

+ c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^2)

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/sqrt(b*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^2 \sqrt{b \cos(c + dx)}} dx$$

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)), x)
```

$$3.826 \quad \int \frac{(A+B \cos(c+dx)) \sec^3(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal result	7184
Rubi [A] (verified)	7184
Mathematica [A] (verified)	7187
Maple [B] (verified)	7187
Fricas [C] (verification not implemented)	7188
Sympy [F]	7188
Maxima [F]	7189
Giac [F]	7189
Mupad [F(-1)]	7189

Optimal result

Integrand size = 31, antiderivative size = 168

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx = -\frac{6A\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5bd\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2bB \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}}$$

[Out] 2/5*A*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*b*B*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used

= {16, 2827, 2716, 2721, 2719, 2720}

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{6A \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{6AE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5bd\sqrt{\cos(c + dx)}} + \frac{2bB \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}}$$

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]

[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ

`[-1, n, 1] && IntegerQ[2*n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^3 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\
 &= (Ab^3) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx + (b^2 B) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\
 &= \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2bB \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} \\
 &\quad + \frac{1}{5}(3Ab) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + \frac{1}{3}B \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\
 &= \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2bB \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} \\
 &\quad - \frac{(3A) \int \sqrt{b \cos(c + dx)} dx}{5b} + \frac{\left(B\sqrt{\cos(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} \\
 &= \frac{2B\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\
 &\quad + \frac{2bB \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} \\
 &\quad - \frac{\left(3A\sqrt{b \cos(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx}{5b\sqrt{\cos(c + dx)}} \\
 &= -\frac{6A\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5bd\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3d\sqrt{b \cos(c + dx)}} \\
 &\quad + \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2bB \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.60

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{2 \left(-9A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) + 9A \sin(c + dx) + 5B \tan(c + dx) \right)}{15d \sqrt{b \cos(c + dx)}}$$

[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(196) = 392.

Time = 8.49 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.45

method	result
default	$- \frac{2 \sqrt{-\left(-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right) b \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(72A \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 36A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} E\left(\frac{1}{2}\left(\frac{dx}{2} + \frac{c}{2}\right) \mid 2\right) + 5B \sqrt{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)} \operatorname{EllipticF}\left(\frac{1}{2}\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) + 9A \sin\left(\frac{dx}{2} + \frac{c}{2}\right) + 5B \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 3A \sec\left(\frac{dx}{2} + \frac{c}{2}\right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{15d \sqrt{b \cos\left(\frac{dx}{2} + \frac{c}{2}\right)}}$
parts	$- \frac{2A \sqrt{-\left(-2 \left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right) b \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{\left(24 \cos\left(\frac{dx}{2} + \frac{c}{2}\right) \left(\sin^6\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 12 \sqrt{2 \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} E\left(\frac{1}{2}\left(\frac{dx}{2} + \frac{c}{2}\right) \mid 2\right) + 5B \sqrt{\cos\left(\frac{dx}{2} + \frac{c}{2}\right)} \operatorname{EllipticF}\left(\frac{1}{2}\left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) + 9A \sin\left(\frac{dx}{2} + \frac{c}{2}\right) + 5B \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 3A \sec\left(\frac{dx}{2} + \frac{c}{2}\right) \tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERB OSE)

[Out]
$$-2/15 * \left(-(-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) * b * \sin(1/2 * d * x + 1/2 * c) ^ 2 \right) ^ (1/2) / b / \sin(1/2 * d * x + 1/2 * c) ^ 3 / \left(8 * \sin(1/2 * d * x + 1/2 * c) ^ 6 - 12 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 6 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1 \right) * \left(72 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 6 - 36 * A * \left(\sin(1/2 * d * x + 1/2 * c) ^ 2 \right) ^ (1/2) * \left(2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1 \right) ^ (1/2) * \operatorname{EllipticE}\left(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)\right) * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 20 * B * \left(\sin(1/2 * d * x + 1/2 * c) ^ 2 \right) ^ (1/2) * \left(2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1 \right) ^ (1/2) * \operatorname{EllipticF}\left(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)\right) * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 72 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 36 * A * \left(\sin(1/2 * d * x + 1/2 * c) ^ 2 \right) ^ (1/2) * \left(2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1 \right) ^ (1/2) * \operatorname{EllipticE}\left(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)\right) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 20 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 20 * B * \left(\sin(1/2 * d * x + 1/2 * c) ^ 2 \right) ^ (1/2) * \left(2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1 \right) ^ (1/2) * \operatorname{EllipticF}\left(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)\right) * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 24 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 9 * A * \left(\sin(1/2 * d * x + 1/2 * c) ^ 2 \right) ^ (1/2) * \left(2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1 \right) ^ (1/2) * \operatorname{EllipticE}\left(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)\right) * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 5 * B * \sqrt{\cos(1/2 * d * x + 1/2 * c)} * \operatorname{EllipticF}\left(\frac{1}{2} * \left(\frac{dx}{2} + \frac{c}{2}\right), 2\right) + 9 * A * \sin(1/2 * d * x + 1/2 * c) + 5 * B * \tan(1/2 * d * x + 1/2 * c) + 3 * A * \sec(1/2 * d * x + 1/2 * c) * \tan(1/2 * d * x + 1/2 * c) \right) / (15 * d * \sqrt{b * \cos(1/2 * d * x + 1/2 * c)})$$

$$c)^2-1)^{1/2}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{1/2})+10*B*\cos(1/2*d*x+1/2*c) \\ * \sin(1/2*d*x+1/2*c)^2-5*B*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c) \\)^2-1)^{1/2}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{1/2}))*(-2*\sin(1/2*d*x+1/2*c)^ \\ 4*b+b*\sin(1/2*d*x+1/2*c)^2)^{1/2}/((2*\cos(1/2*d*x+1/2*c)^2-1)*b)^{1/2}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.22

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\ = \frac{-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c)) + 5i \sqrt{2} B \sqrt{b} \cos(dx + c)}{\dots}$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^3)

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/sqrt(b*cos(c + d*x)), x)

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^3 \sqrt{b \cos(c + dx)}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)), x)

$$3.827 \quad \int \frac{\cos^4(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7190
Rubi [A] (verified)	7190
Mathematica [A] (verified)	7192
Maple [A] (verified)	7193
Fricas [C] (verification not implemented)	7193
Sympy [F(-1)]	7194
Maxima [F]	7194
Giac [F]	7194
Mupad [F(-1)]	7194

Optimal result

Integrand size = 31, antiderivative size = 176

$$\int \frac{\cos^4(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{6A\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21bd\sqrt{b \cos(c+dx)}} + \frac{10B\sqrt{b \cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2A(b \cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{2B(b \cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}$$

[Out] $2/5*A*(b*\cos(d*x+c))^{(3/2)}*\sin(d*x+c)/b^3/d+2/7*B*(b*\cos(d*x+c))^{(5/2)}*\sin(d*x+c)/b^4/d+10/21*B*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}+10/21*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/b^2/d+6/5*A*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/b^2/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2715, 2721, 2719, 2720}

$$\int \frac{\cos^4(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{2A \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^3d} + \frac{6AE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{5/2}}{7b^4d} + \frac{10B \sin(c+dx)\sqrt{b \cos(c+dx)}}{21b^2d} + \frac{10B\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21bd\sqrt{b \cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2), x]

[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx}{b^4} \\ &= \frac{A \int (b \cos(c + dx))^{5/2} dx}{b^4} + \frac{B \int (b \cos(c + dx))^{7/2} dx}{b^5} \end{aligned}$$

$$\begin{aligned}
&= \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^3d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^4d} \\
&\quad + \frac{(3A) \int \sqrt{b \cos(c + dx)} dx}{5b^2} + \frac{(5B) \int (b \cos(c + dx))^{3/2} dx}{7b^3} \\
&= \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^2d} + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^3d} \\
&\quad + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^4d} + \frac{(5B) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{21b} \\
&\quad + \frac{\left(3A\sqrt{b \cos(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{5b^2 \sqrt{\cos(c + dx)}} \\
&= \frac{6A\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5b^2d\sqrt{\cos(c + dx)}} + \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^2d} \\
&\quad + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^3d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^4d} \\
&\quad + \frac{\left(5B\sqrt{\cos(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{21b\sqrt{b \cos(c + dx)}} \\
&= \frac{6A\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5b^2d\sqrt{\cos(c + dx)}} + \frac{10B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21bd\sqrt{b \cos(c + dx)}} \\
&\quad + \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^2d} + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^3d} \\
&\quad + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.59

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \frac{252A\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right) + 100B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{210bd\sqrt{b \cos(c + dx)}}$$

[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]

[Out] (252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]/(210*b*d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 8.57 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.71

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(240B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-168A-360B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\dots}\right)$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\frac{5b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}$

[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERB OSE)

[Out]
$$-\frac{2}{105} * \left((2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * b * \sin(1/2 * d * x + 1/2 * c) ^ 2 \right) ^ {1/2} / b * (240 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 8 + (-168 * A - 360 * B) * \sin(1/2 * d * x + 1/2 * c) ^ 6 * \cos(1/2 * d * x + 1/2 * c) + (168 * A + 280 * B) * \sin(1/2 * d * x + 1/2 * c) ^ 4 * \cos(1/2 * d * x + 1/2 * c) + (-42 * A - 80 * B) * \sin(1/2 * d * x + 1/2 * c) ^ 2 * \cos(1/2 * d * x + 1/2 * c) - 63 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2}) + 25 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) / (-b * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} / \sin(1/2 * d * x + 1/2 * c) / ((2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * b) ^ {1/2} / d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \frac{-25i \sqrt{2} B \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{(b \cos(c + dx))^{3/2}}$$

[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out]
$$\frac{1}{105} * (-25 * I * \sqrt{2} * B * \sqrt{b} * \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) + I * \sin(d * x + c)) + 25 * I * \sqrt{2} * B * \sqrt{b} * \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) - I * \sin(d * x + c)) + 63 * I * \sqrt{2} * A * \sqrt{b} * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) + I * \sin(d * x + c))) - 63 * I * \sqrt{2} * A * \sqrt{b} * \text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d * x + c) - I * \sin(d * x + c))) + 2 * (15 * B * \cos(d * x + c) ^ 2 + 21 * A * \cos(d * x + c) + 25 * B) * \sqrt{b * \cos(d * x + c)} * \sin(d * x + c) / (b ^ 2 * d)$$

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^4}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)

Giac [F]

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^4}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2), x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^4 (A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)

[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)

$$3.828 \quad \int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7195
Rubi [A] (verified)	7195
Mathematica [A] (verified)	7197
Maple [A] (verified)	7197
Fricas [C] (verification not implemented)	7198
Sympy [F(-1)]	7198
Maxima [F]	7199
Giac [F]	7199
Mupad [F(-1)]	7199

Optimal result

Integrand size = 31, antiderivative size = 147

$$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{6B\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3bd\sqrt{b \cos(c+dx)}} + \frac{2A\sqrt{b \cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2B(b \cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}$$

[Out] $2/5*B*(b*\cos(d*x+c))^{3/2}*\sin(d*x+c)/b^3/d+2/3*A*(\cos(1/2*d*x+1/2*c))^{2*(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}+2/3*A*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/b^2/d+6/5*B*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\text{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/b^2/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2715, 2721, 2720, 2719}

$$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{2A \sin(c+dx) \sqrt{b \cos(c+dx)}}{3b^2d} + \frac{2A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3bd\sqrt{b \cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^3d} + \frac{6BE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2), x]

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx}{b^3} \\ &= \frac{A \int (b \cos(c + dx))^{3/2} dx}{b^3} + \frac{B \int (b \cos(c + dx))^{5/2} dx}{b^4} \end{aligned}$$

$$\begin{aligned}
&= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} \\
&\quad + \frac{A\int\frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b} + \frac{(3B)\int\sqrt{b\cos(c+dx)}dx}{5b^2} \\
&= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} \\
&\quad + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b\sqrt{b\cos(c+dx)}} + \frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}dx}{5b^2\sqrt{\cos(c+dx)}} \\
&= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3bd\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \frac{2\cos^{3/2}(c+dx)\left(9BE\left(\frac{1}{2}(c+dx)\mid 2\right)+5A\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\right)}{15d(b\cos(c+dx))^{3/2}}$$

[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*Cos[c + d*x]^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d*(b*Cos[c + d*x])^(3/2))

Maple [A] (verified)

Time = 6.76 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.86

method	result
default	$ \frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(20A+24B\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(-15b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)} $
parts	$ \frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\cos\left(\frac{dx}{2}+\frac{c}{2}\right)} $

[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2), x, method=_RETURNVERB OSE)

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \frac{-5i \sqrt{2} A \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{(b \cos(c + dx))^{3/2}}$$

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c) + 5*A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)

Giac [F]

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^3 (A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)

[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)

$$3.829 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7200
Rubi [A] (verified)	7200
Mathematica [A] (verified)	7202
Maple [A] (verified)	7202
Fricas [C] (verification not implemented)	7203
Sympy [F(-1)]	7203
Maxima [F]	7203
Giac [F]	7204
Mupad [F(-1)]	7204

Optimal result

Integrand size = 31, antiderivative size = 116

$$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{2A\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^2 d \sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3bd\sqrt{b \cos(c+dx)}} + \frac{2B\sqrt{b \cos(c+dx)} \sin(c+dx)}{3b^2 d}$$

[Out] $2/3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}+2/3*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/b^2/d+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/b^2/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2721, 2719, 2715, 2720}

$$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{2AE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{b^2 d \sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx) \sqrt{b \cos(c+dx)}}{3b^2 d} + \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3bd\sqrt{b \cos(c+dx)}}$$

[In] $\operatorname{Int}[(\operatorname{Cos}[c+d*x]^2*(A+B*\operatorname{Cos}[c+d*x]))/(b*\operatorname{Cos}[c+d*x])^{(3/2)}, x]$

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :=> Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] :=> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] :=> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :=> Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :=> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \sqrt{b \cos(c + dx)}(A + B \cos(c + dx)) dx}{b^2} \\ &= \frac{A \int \sqrt{b \cos(c + dx)} dx}{b^2} + \frac{B \int (b \cos(c + dx))^{3/2} dx}{b^3} \\ &= \frac{2B \sqrt{b \cos(c + dx)} \sin(c + dx)}{3b^2 d} + \frac{B \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{3b} + \frac{(A \sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{b^2 \sqrt{\cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} \\
&\quad + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b\sqrt{b\cos(c+dx)}} \\
&= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.65

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \frac{2\cos^{\frac{3}{2}}(c+dx)\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\operatorname{EllipticE}\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\right)}{3d(b\cos(c+dx))^{3/2}}$$

[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*Cos[c + d*x]^(3/2)*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*(b*Cos[c + d*x])^(3/2))

Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.07

method	result
default	$ \frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)} $
parts	$ \frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)bd}} - \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right) $

[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2), x, method=_RETURNVERBOSE)

[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2

$*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/((2*\cos(1/2*d*x+1/2*c)^2-1)*b)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.22

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \frac{-i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\sin(dx+c))}{(b\cos(c+dx))^{3/2}}$$

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/3*(-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*B*sin(d*x + c))/(b^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

Giac [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx)^2 (A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)

[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)

$$3.830 \quad \int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7205
Rubi [A] (verified)	7205
Mathematica [A] (verified)	7207
Maple [A] (verified)	7207
Fricas [C] (verification not implemented)	7208
Sympy [F(-1)]	7208
Maxima [F]	7208
Giac [F]	7209
Mupad [F(-1)]	7209

Optimal result

Integrand size = 29, antiderivative size = 85

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{2B\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^2 d \sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{bd \sqrt{b \cos(c+dx)}}$$

[Out] $2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}+2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/b^2/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {16, 2827, 2721, 2720, 2719}

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{2A\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{bd \sqrt{b \cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{b^2 d \sqrt{\cos(c+dx)}}$$

[In] $\operatorname{Int}[(\operatorname{Cos}[c+d*x]*(A+B*\operatorname{Cos}[c+d*x]))/(b*\operatorname{Cos}[c+d*x])^{(3/2)}, x]$

[Out] $(2*B*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]]*\operatorname{EllipticE}[(c+d*x)/2, 2])/ (b^2*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]) + (2*A*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{EllipticF}[(c+d*x)/2, 2])/ (b*d*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]])$

Rule 16

$\text{Int}[(u_)*(v_)^{(m_)*((b_)*(v_))^{(n_)}}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

Rule 2719

$\text{Int}[\text{Sqrt}[\sin[(c_)+(d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\text{Sqrt}[\sin[(c_)+(d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticF}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2721

$\text{Int}[(b_)*\sin[(c_)+(d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(b*\text{Sin}[c + d*x])^n/\text{Sin}[c + d*x]^n, \text{Int}[\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{LtQ}[-1, n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2827

$\text{Int}[(b_)*\sin[(e_)+(f_)*(x_)]^{(m_)*((c_)+(d_)*\sin[(e_)+(f_)*(x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\int \frac{A+B \cos(c+dx)}{\sqrt{b \cos(c+dx)}} dx}{b} \\
 &= \frac{A \int \frac{1}{\sqrt{b \cos(c+dx)}} dx}{b} + \frac{B \int \sqrt{b \cos(c+dx)} dx}{b^2} \\
 &= \frac{\left(A \sqrt{\cos(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{b \sqrt{b \cos(c+dx)}} + \frac{\left(B \sqrt{b \cos(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{b^2 \sqrt{\cos(c+dx)}} \\
 &= \frac{2B \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^2 d \sqrt{\cos(c+dx)}} + \frac{2A \sqrt{\cos(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{bd \sqrt{b \cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \frac{2\sqrt{\cos(c+dx)}(BE(\frac{1}{2}(c+dx)|2) + A\text{EllipticF}(\frac{1}{2}(c+dx),2))}{bd\sqrt{b\cos(c+dx)}}$$

[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]

[Out] (2*sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/2, 2]))/(b*d*sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(AF\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-BE\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}bd}$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}bd} + \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}$
risch	$\frac{iB\left(e^{2i(dx+c)}+1\right)\sqrt{2}e^{-i(dx+c)}}{db\sqrt{\left(e^{2i(dx+c)}+1\right)}be^{-i(dx+c)}} - \frac{i\left(\frac{iA\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{be^{3i(dx+c)}+be^{i(dx+c)}}}\right)}{b\sqrt{e^{i(dx+c)}}} + B\left(-\frac{2\left(b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}\right)$

[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/b/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \frac{-i \sqrt{2} A \sqrt{b} \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{(b \cos(c + dx))^{3/2}}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{3/2}} dx$$

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{\cos(c + dx) (A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx$$

[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)

[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)

$$3.831 \quad \int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7210
Rubi [A] (verified)	7210
Mathematica [A] (verified)	7212
Maple [A] (verified)	7212
Fricas [C] (verification not implemented)	7213
Sympy [F(-1)]	7213
Maxima [F]	7213
Giac [F]	7214
Mupad [F(-1)]	7214

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx = -\frac{2A\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{b^2 d \sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{bd\sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{bd\sqrt{b \cos(c + dx)}}$$

[Out] 2*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2827, 2716, 2721, 2719, 2720}

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx = -\frac{2AE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{b^2 d \sqrt{\cos(c + dx)}} + \frac{2A \sin(c + dx)}{bd\sqrt{b \cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{bd\sqrt{b \cos(c + dx)}}$$

[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(3/2),x]

[Out] $(-2*A*\sqrt{b*\cos[c + d*x]}*EllipticE[(c + d*x)/2, 2])/(b^2*d*\sqrt{\cos[c + d*x]}) + (2*B*\sqrt{\cos[c + d*x]}*EllipticF[(c + d*x)/2, 2])/(b*d*\sqrt{b*\cos[c + d*x]}) + (2*A*\sin[c + d*x])/(b*d*\sqrt{b*\cos[c + d*x]})$

Rule 2716

$\text{Int}[(b_* \sin[c_*] + (d_*) \cdot (x_*))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[\cos[c + d*x] * ((b*\sin[c + d*x])^{(n + 1)} / (b*d*(n + 1))), x] + \text{Dist}[(n + 2) / (b^2*(n + 1)), \text{Int}[(b*\sin[c + d*x])^{(n + 2)}, x], x] /; \text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2719

$\text{Int}[\sqrt{\sin[c_*] + (d_*) \cdot (x_*)}, x_Symbol] \rightarrow \text{Simp}[(2/d)*EllipticE[(1/2)*(c - \pi/2 + d*x), 2], x] /; \text{FreeQ}\{c, d, x\}$

Rule 2720

$\text{Int}[1/\sqrt{\sin[c_*] + (d_*) \cdot (x_*)}, x_Symbol] \rightarrow \text{Simp}[(2/d)*EllipticF[(1/2)*(c - \pi/2 + d*x), 2], x] /; \text{FreeQ}\{c, d, x\}$

Rule 2721

$\text{Int}[(b_* \sin[c_*] + (d_*) \cdot (x_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(b*\sin[c + d*x])^n / \sin[c + d*x]^n, \text{Int}[\sin[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{LtQ}[-1, n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2827

$\text{Int}[(b_* \sin[e_*] + (f_*) \cdot (x_*))^{(m_*)} * ((c_*) + (d_*) \cdot \sin[e_*] + (f_*) \cdot (x_*)), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m, x\}$

Rubi steps

$$\begin{aligned} \text{integral} &= A \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + \frac{B \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{b} \\ &= \frac{2A \sin(c + dx)}{bd \sqrt{b \cos(c + dx)}} - \frac{A \int \sqrt{b \cos(c + dx)} dx}{b^2} + \frac{\left(B \sqrt{\cos(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{b \sqrt{b \cos(c + dx)}} \\ &= \frac{2B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{bd \sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{bd \sqrt{b \cos(c + dx)}} \\ &\quad - \frac{\left(A \sqrt{b \cos(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx}{b^2 \sqrt{\cos(c + dx)}} \end{aligned}$$

$$= -\frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int \frac{A + B\cos(c+dx)}{(b\cos(c+dx))^{3/2}} dx = \frac{2\left(-A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right) + B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) + A\sin(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}$$

[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*(-(A*Sqrt[Cos[c + d*x]])*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 4.83 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.94

method	result
default	$\frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$
parts	$-\frac{2A\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

[In] int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2), x, method=_RETURNVERBOSE)

[Out] 2/b*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \frac{-i \sqrt{2} B \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{(b \cos(c + dx))^{3/2}}$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^2*d*cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{3/2}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(3/2),x)

[Out] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(3/2), x)

$$3.832 \quad \int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7215
Rubi [A] (verified)	7215
Mathematica [A] (verified)	7217
Maple [B] (verified)	7217
Fricas [C] (verification not implemented)	7218
Sympy [F]	7218
Maxima [F]	7219
Giac [F]	7219
Mupad [F(-1)]	7219

Optimal result

Integrand size = 29, antiderivative size = 140

$$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{3/2}} dx = -\frac{2B\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^2 d \sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3bd\sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2B \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}}$$

[Out] $2/3*A*\sin(d*x+c)/d/(b*\cos(d*x+c))^{(3/2)}+2*B*\sin(d*x+c)/b/d/(b*\cos(d*x+c))^{(1/2)}+2/3*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}-2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/b^2/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {16, 2827, 2716, 2721, 2720, 2719}

$$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{3/2}} dx = \frac{2A \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2A\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3bd\sqrt{b \cos(c+dx)}} - \frac{2BE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{b^2 d \sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}}$$

[In] $\operatorname{Int}[\frac{(A+B*\cos[c+d*x])*Sec[c+d*x]}{(b*\cos[c+d*x])^{(3/2)}}, x]$

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d
*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c
+ d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_)^(n_)), x_Symbol] := Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)
*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= b \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\ &= (Ab) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx + B \int \frac{1}{(b \cos(c + dx))^{3/2}} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{2A \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2B \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} + \frac{A \int \frac{1}{\sqrt{b \cos(c+dx)}} dx}{3b} - \frac{B \int \sqrt{b \cos(c+dx)} dx}{b^2} \\
&= \frac{2A \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2B \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b\sqrt{b \cos(c+dx)}} \\
&\quad - \frac{\left(B\sqrt{b \cos(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{b^2\sqrt{\cos(c+dx)}} \\
&= -\frac{2B\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3bd\sqrt{b \cos(c+dx)}} \\
&\quad + \frac{2A \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2B \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \frac{(A + B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{3/2}} dx = \frac{2\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right) + A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\right)}{3bd\sqrt{b \cos(c+dx)}}$$

[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*b*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(176) = 352.

Time = 6.60 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.90

method	result
default	$2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 1\right)b\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} \left(2A\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right)$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1} F\left(\cos\left(\frac{dx}{2} + \frac{c}{2}\right), \sqrt{2}\right) \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2\left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) \cos\left(\frac{dx}{2} + \frac{c}{2}\right) + \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}}\right)}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \left(\sin^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)\right)} \left(2\left(\cos^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 1\right) \sin\left(\frac{dx}{2} + \frac{c}{2}\right)}$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(3/2), x, method=_RETURNVERBOSE)

E)

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2
*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*
c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.37

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \frac{-i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c)) + i \sin(dx + c)}{(b \cos(c + dx))^{3/2}}$$

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/3*(-I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^2*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*B*sqrt(b)*cos(d*x
+ c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + 3*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^2*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d
*x + c) + A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(b*cos(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx) (b \cos(c + dx))^{3/2}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)), x)

$$3.833 \quad \int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7220
Rubi [A] (verified)	7220
Mathematica [A] (verified)	7223
Maple [B] (verified)	7223
Fricas [C] (verification not implemented)	7224
Sympy [F]	7224
Maxima [F]	7224
Giac [F]	7225
Mupad [F(-1)]	7225

Optimal result

Integrand size = 31, antiderivative size = 171

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx = -\frac{6A\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5b^2 d \sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3bd\sqrt{b \cos(c + dx)}} + \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5bd\sqrt{b \cos(c + dx)}}$$

[Out] 2/5*A*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+6/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used

= {16, 2827, 2716, 2721, 2719, 2720}

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx = -\frac{6AE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5b^2 d \sqrt{\cos(c + dx)}} + \frac{6A \sin(c + dx)}{5bd \sqrt{b \cos(c + dx)}} + \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3bd \sqrt{b \cos(c + dx)}}$$

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]

[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b^2 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\
&= (Ab^2) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx + (bB) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\
&= \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} \\
&\quad + \frac{1}{5}(3A) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + \frac{B \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{3b} \\
&= \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5bd\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{(3A) \int \sqrt{b \cos(c + dx)} dx}{5b^2} + \frac{(B\sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3b\sqrt{b \cos(c + dx)}} \\
&= \frac{2B\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3bd\sqrt{b \cos(c + dx)}} + \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\
&\quad + \frac{2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5bd\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{(3A\sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{5b^2\sqrt{\cos(c + dx)}} \\
&= -\frac{6A\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5b^2 d \sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3bd\sqrt{b \cos(c + dx)}} \\
&\quad + \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5bd\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.61

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \frac{2 \left(-9A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx) \mid 2\right) \right)}{(b \cos(c + dx))^{3/2}}$$

[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]

[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(199) = 398.

Time = 8.14 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.39

method	result
default	$- \frac{2 \sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)b(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \left(72A \cos(\frac{dx}{2} + \frac{c}{2}) \left(\sin^6(\frac{dx}{2} + \frac{c}{2}) \right) - 36A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} E\left(\frac{1}{2}(c + dx) \mid 2\right) \right)}{(b \cos(c + dx))^{3/2}}$
parts	$- \frac{2A \sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)b(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \left(24 \cos(\frac{dx}{2} + \frac{c}{2}) \left(\sin^6(\frac{dx}{2} + \frac{c}{2}) \right) - 12 \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} E\left(\frac{1}{2}(c + dx) \mid 2\right) \right)}{(b \cos(c + dx))^{3/2}}$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERB OSE)

[Out]
$$-2/15 * (-(-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) * b * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / b ^ 2 / \sin(1/2 * d * x + 1/2 * c) ^ 3 / (8 * \sin(1/2 * d * x + 1/2 * c) ^ 6 - 12 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 6 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) * (72 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 6 - 36 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 20 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 72 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 36 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 20 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 20 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 24 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 9 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) + 10 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 5 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2))$$

$\ast c)^2 - 1)^{1/2} \ast \text{EllipticF}(\cos(1/2 \ast d \ast x + 1/2 \ast c), 2^{1/2})) \ast (-2 \ast \sin(1/2 \ast d \ast x + 1/2 \ast c))^4 \ast b + b \ast \sin(1/2 \ast d \ast x + 1/2 \ast c)^2)^{1/2} / ((2 \ast \cos(1/2 \ast d \ast x + 1/2 \ast c)^2 - 1) \ast b)^{1/2} / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.20

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \frac{-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c)) + \dots}{(b \cos(c + dx))^{3/2}}$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(b*cos(c + d*x))**(3/2), x)

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{3/2}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^2 (b \cos(c + dx))^{3/2}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)), x)

$$3.834 \quad \int \frac{\cos^5(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	7226
Rubi [A] (verified)	7226
Mathematica [A] (verified)	7228
Maple [A] (verified)	7229
Fricas [C] (verification not implemented)	7229
Sympy [F(-1)]	7230
Maxima [F]	7230
Giac [F]	7230
Mupad [F(-1)]	7230

Optimal result

Integrand size = 31, antiderivative size = 176

$$\int \frac{\cos^5(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{6A\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21b^2d\sqrt{b \cos(c+dx)}} + \frac{10B\sqrt{b \cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2A(b \cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{2B(b \cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}$$

[Out] 2/5*A*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/7*B*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^5/d+10/21*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c), 2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+10/21*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c), 2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2715, 2721, 2719, 2720}

$$\int \frac{\cos^5(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{2A \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^4d} + \frac{6AE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{5/2}}{7b^5d} + \frac{10B \sin(c+dx)\sqrt{b \cos(c+dx)}}{21b^3d} + \frac{10B\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{21b^2d\sqrt{b \cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]

[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (2*A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx)) dx}{b^5} \\ &= \frac{A \int (b \cos(c + dx))^{5/2} dx}{b^5} + \frac{B \int (b \cos(c + dx))^{7/2} dx}{b^6} \end{aligned}$$

$$\begin{aligned}
&= \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^4d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^5d} \\
&\quad + \frac{(3A) \int \sqrt{b \cos(c + dx)} dx}{5b^3} + \frac{(5B) \int (b \cos(c + dx))^{3/2} dx}{7b^4} \\
&= \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^3d} + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^4d} \\
&\quad + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^5d} + \frac{(5B) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{21b^2} \\
&\quad + \frac{\left(3A\sqrt{b \cos(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{5b^3 \sqrt{\cos(c + dx)}} \\
&= \frac{6A\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5b^3d\sqrt{\cos(c + dx)}} + \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^3d} \\
&\quad + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^4d} + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^5d} \\
&\quad + \frac{\left(5B\sqrt{\cos(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{21b^2\sqrt{b \cos(c + dx)}} \\
&= \frac{6A\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5b^3d\sqrt{\cos(c + dx)}} + \frac{10B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{21b^2d\sqrt{b \cos(c + dx)}} \\
&\quad + \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^3d} + \frac{2A(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^4d} \\
&\quad + \frac{2B(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.59

$$\int \frac{\cos^5(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \frac{252A\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right) + 100B\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{210b^2}$$

[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]

[Out] (252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]/(210*b^2*d*Sqrt[b*Cos[c + d*x]])

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^5(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^5}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)

Giac [F]

$$\int \frac{\cos^5(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^5}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2), x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^5 (A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

[In] int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)

[Out] int((cos(c + d*x)^5*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)

$$3.835 \quad \int \frac{\cos^4(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	.7231
Rubi [A] (verified)	.7231
Mathematica [A] (verified)	.7233
Maple [A] (verified)	.7233
Fricas [C] (verification not implemented)	.7234
Sympy [F(-1)]	.7234
Maxima [F]	.7235
Giac [F]	.7235
Mupad [F(-1)]	.7235

Optimal result

Integrand size = 31, antiderivative size = 147

$$\int \frac{\cos^4(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{6B\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2A\sqrt{b \cos(c+dx)} \sin(c+dx)}{3b^3d} + \frac{2B(b \cos(c+dx))^{3/2} \sin(c+dx)}{5b^4d}$$

[Out] $2/5*B*(b*\cos(d*x+c))^{(3/2)}*\sin(d*x+c)/b^4/d+2/3*A*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/b^2/d/(b*\cos(d*x+c))^{(1/2)}+2/3*A*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/b^3/d+6/5*B*(\cos(1/2*d*x+1/2*c))^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/b^3/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2715, 2721, 2720, 2719}

$$\int \frac{\cos^4(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{2A \sin(c+dx) \sqrt{b \cos(c+dx)}}{3b^3d} + \frac{2A\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^4d} + \frac{6BE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2715

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx}{b^4} \\ &= \frac{A \int (b \cos(c + dx))^{3/2} dx}{b^4} + \frac{B \int (b \cos(c + dx))^{5/2} dx}{b^5} \end{aligned}$$

$$\begin{aligned}
&= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} \\
&\quad + \frac{A\int\frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2} + \frac{(3B)\int\sqrt{b\cos(c+dx)}dx}{5b^3} \\
&= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} \\
&\quad + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}} + \frac{\left(3B\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}dx}{5b^3\sqrt{\cos(c+dx)}} \\
&= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^2d\sqrt{b\cos(c+dx)}} \\
&\quad + \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.62

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{2\sqrt{\cos(c+dx)}\left(9BE\left(\frac{1}{2}(c+dx)\mid 2\right) + 5A\text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\right)}{15b^2d\sqrt{b\cos(c+dx)}}$$

[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*sqrt[Cos[c + d*x]]*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d*x)/2, 2] + sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*b^2*d*sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 7.05 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.86

method	result
default	$ -\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(-24B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(20A+24B\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(-15b^2d\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15b^2d\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)} $
parts	$ -\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}} $

[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2), x, method=_RETURNVERB OSE)

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-24*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{-5i\sqrt{2}A\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{(b\cos(c+dx))^{5/2}}$$

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*B*cos(d*x + c) + 5*A)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^4}{(b \cos(dx + c))^{5/2}} dx$$

[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)

Giac [F]

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^4}{(b \cos(dx + c))^{5/2}} dx$$

[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^4 (A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

[In] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)

[Out] int((cos(c + d*x)^4*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)

$$3.836 \quad \int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	7236
Rubi [A] (verified)	7236
Mathematica [A] (verified)	7238
Maple [A] (verified)	7238
Fricas [C] (verification not implemented)	7239
Sympy [F(-1)]	7239
Maxima [F]	7239
Giac [F]	7240
Mupad [F(-1)]	7240

Optimal result

Integrand size = 31, antiderivative size = 116

$$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{2A\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^3 d \sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^2 d \sqrt{b \cos(c+dx)}} + \frac{2B\sqrt{b \cos(c+dx)} \sin(c+dx)}{3b^3 d}$$

[Out] $2/3*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/b^2/d/(b*\cos(d*x+c))^{(1/2)}+2/3*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/b^3/d+2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/b^3/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {16, 2827, 2721, 2719, 2715, 2720}

$$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{2AE\left(\frac{1}{2}(c+dx) \mid 2\right) \sqrt{b \cos(c+dx)}}{b^3 d \sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx) \sqrt{b \cos(c+dx)}}{3b^3 d} + \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^2 d \sqrt{b \cos(c+dx)}}$$

[In] $\operatorname{Int}[(\operatorname{Cos}[c+d*x]^3*(A+B*\operatorname{Cos}[c+d*x]))/(b*\operatorname{Cos}[c+d*x])^{(5/2)}, x]$


```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```

Rule 16

```
Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :=> Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] :=> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] :=> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :=> Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :=> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx}{b^3} \\ &= \frac{A \int \sqrt{b \cos(c + dx)} dx}{b^3} + \frac{B \int (b \cos(c + dx))^{3/2} dx}{b^4} \\ &= \frac{2B \sqrt{b \cos(c + dx)} \sin(c + dx)}{3b^3 d} + \frac{B \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{3b^2} + \frac{(A \sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{b^3 \sqrt{\cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} \\
&\quad + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{b^3d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{2\sqrt{\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\mid 2\right) + B\left(\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\right)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x]^(5/2)), x]

[Out] (2*Sqrt[Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.07

method	result
default	$ \frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(-4B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)} $
parts	$ \frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)bd}} - \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right) $

[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)

[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-4*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2)^(1/2))+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)

$2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticF(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/((2*\cos(1/2*d*x+1/2*c)^2-1)*b)^{(1/2)}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.22

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{-i\sqrt{2}B\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))}{(b\cos(c+dx))^{5/2}}$$

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/3*(-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*B*sin(d*x + c))/(b^3*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{5/2}} dx$$

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)

Giac [F]

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^3}{(b \cos(dx + c))^{5/2}} dx$$

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^3 (A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

[In] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)

[Out] int((cos(c + d*x)^3*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)

$$3.837 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	.7241
Rubi [A] (verified)	.7241
Mathematica [A] (verified)	.7243
Maple [A] (verified)	.7243
Fricas [C] (verification not implemented)	.7244
Sympy [F(-1)]	.7244
Maxima [F]	.7244
Giac [F]	.7245
Mupad [F(-1)]	.7245

Optimal result

Integrand size = 31, antiderivative size = 85

$$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{2B\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d\sqrt{b \cos(c+dx)}}$$

[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 2827, 2721, 2720, 2719}

$$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{2A\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d\sqrt{b \cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b \cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]

[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 16

`Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]`

Rule 2719

`Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2720

`Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]`

Rule 2721

`Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]`

Rule 2827

`Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\int \frac{A+B \cos(c+dx)}{\sqrt{b \cos(c+dx)}} dx}{b^2} \\
 &= \frac{A \int \frac{1}{\sqrt{b \cos(c+dx)}} dx}{b^2} + \frac{B \int \sqrt{b \cos(c+dx)} dx}{b^3} \\
 &= \frac{\left(A \sqrt{\cos(c+dx)}\right) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{b^2 \sqrt{b \cos(c+dx)}} + \frac{\left(B \sqrt{b \cos(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{b^3 \sqrt{\cos(c+dx)}} \\
 &= \frac{2B \sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^3 d \sqrt{\cos(c+dx)}} + \frac{2A \sqrt{\cos(c+dx)} \text{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^2 d \sqrt{b \cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{2\sqrt{\cos(c+dx)}(BE(\frac{1}{2}(c+dx)|2) + A\text{EllipticF}(\frac{1}{2}(c+dx),2))}{b^2 d \sqrt{b\cos(c+dx)}}$$

[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]

[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/2, 2]))/(b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\left(AF\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-BE\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}bd$
parts	$-\frac{2A\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}bd} + \frac{2B\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}$
risch	$-\frac{iB\left(e^{2i(dx+c)}+1\right)\sqrt{2}e^{-i(dx+c)}}{db^2\sqrt{\left(e^{2i(dx+c)}+1\right)}be^{-i(dx+c)}} - \frac{i\left(\frac{iA\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{be^{3i(dx+c)}+be^{i(dx+c)}}}\right)}{b^2\sqrt{\left(e^{2i(dx+c)}+1\right)}be^{-i(dx+c)}} + B\left(-\frac{2}{b\sqrt{e^{i(dx+c)}}}\right)$

[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERB OSE)

[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/b^2/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \frac{-i \sqrt{2} A \sqrt{b} \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{(b \cos(c + dx))^{5/2}}$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] (-I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*B*sqrt(b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```


Giac [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{5/2}} dx$$

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx)^2 (A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)

[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)

$$3.838 \quad \int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	7246
Rubi [A] (verified)	7246
Mathematica [A] (verified)	7248
Maple [A] (verified)	7248
Fricas [C] (verification not implemented)	7249
Sympy [F(-1)]	7249
Maxima [F]	7249
Giac [F]	7250
Mupad [F(-1)]	7250

Optimal result

Integrand size = 29, antiderivative size = 112

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = -\frac{2A\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx)|2\right)}{b^3 d \sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}}$$

[Out] $2*A*\sin(d*x+c)/b^2/d/(b*\cos(d*x+c))^{(1/2)}+2*B*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticF}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*\cos(d*x+c)^{(1/2)}/b^2/d/(b*\cos(d*x+c))^{(1/2)}-2*A*(\cos(1/2*d*x+1/2*c)^2)^{(1/2)}/\cos(1/2*d*x+1/2*c)*\operatorname{EllipticE}(\sin(1/2*d*x+1/2*c), 2^{(1/2)})*(b*\cos(d*x+c))^{(1/2)}/b^3/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {16, 2827, 2716, 2721, 2719, 2720}

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = -\frac{2AE\left(\frac{1}{2}(c+dx)|2\right) \sqrt{b \cos(c+dx)}}{b^3 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^2 d \sqrt{b \cos(c+dx)}}$$

[In] $\operatorname{Int}[(\operatorname{Cos}[c+d*x]*(A+B*\operatorname{Cos}[c+d*x]))/(b*\operatorname{Cos}[c+d*x])^{(5/2)}, x]$

[Out] $(-2A\sqrt{b\cos[c + dx]}\text{EllipticE}[(c + dx)/2, 2])/(b^3d\sqrt{\cos[c + dx]}) + (2B\sqrt{\cos[c + dx]}\text{EllipticF}[(c + dx)/2, 2])/(b^2d\sqrt{b\cos[c + dx]}) + (2A\sin[c + dx])/(b^2d\sqrt{b\cos[c + dx]})$

Rule 16

$\text{Int}[(u_.)*(v_.)^{(m_.)}*((b_.)*(v_.)^{(n_.)}), x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{b, n\}, x] \&\& \text{IntegerQ}[m]$

Rule 2716

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[\cos[c + dx]*((b*\sin[c + dx])^{(n+1)})/(b*d*(n+1)), x] + \text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(b*\sin[c + dx])^{(n+2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2719

$\text{Int}[\sqrt{\sin[(c_.) + (d_.)*(x_)]}, x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + dx), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\sqrt{\sin[(c_.) + (d_.)*(x_)]}, x_Symbol] \rightarrow \text{Simp}[(2/d)*\text{EllipticF}[(1/2)*(c - \text{Pi}/2 + dx), 2], x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2721

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\sin[c + dx])^n/\sin[c + dx]^n, \text{Int}[\sin[c + dx]^n, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{LtQ}[-1, n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2827

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_.)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{3/2}} dx}{b} \\ &= \frac{A \int \frac{1}{(b \cos(c+dx))^{3/2}} dx}{b} + \frac{B \int \frac{1}{\sqrt{b \cos(c+dx)}} dx}{b^2} \\ &= \frac{2A \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} - \frac{A \int \sqrt{b \cos(c+dx)} dx}{b^3} + \frac{(B \sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{b^2 \sqrt{b \cos(c+dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} \\
&\quad - \frac{\left(A\sqrt{b \cos(c+dx)}\right) \int \sqrt{\cos(c+dx)} dx}{b^3 \sqrt{\cos(c+dx)}} \\
&= -\frac{2A\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{b^3 d \sqrt{\cos(c+dx)}} \\
&\quad + \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{2\left(-A\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right) + B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\right)}{b^2 d \sqrt{b \cos(c+dx)}}$$

[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.94

method	result
default	$ \frac{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}} $
parts	$ -\frac{2A\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}} $

[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)

[Out] 2/b^2*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))

$c), 2^{(1/2)})) / (-b * (2 * \sin(1/2 * d * x + 1/2 * c)^4 - \sin(1/2 * d * x + 1/2 * c)^2))^{(1/2)} / \sin(1/2 * d * x + 1/2 * c) / ((2 * \cos(1/2 * d * x + 1/2 * c)^2 - 1) * b)^{(1/2)} / d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.54

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \frac{-i \sqrt{2} B \sqrt{b} \cos(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)) + i \sqrt{2} A \sqrt{b} \sin(dx + c) \operatorname{weierstrassPInverse}(-4, 0, \cos(dx + c)) - i \sqrt{2} B \sqrt{b} \cos(dx + c) \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c)) + i \sqrt{2} A \sqrt{b} \sin(dx + c) \operatorname{weierstrassZeta}(-4, 0, \cos(dx + c)) + 2 \sqrt{b \cos(dx + c)} A \sin(dx + c)}{(b^3 d \cos(dx + c))}$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] (-I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*B*sqrt(b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*A*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*A*sin(d*x + c))/(b^3*d*cos(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)

Giac [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{5/2}} dx$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{\cos(c + dx) (A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx$$

[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)

[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)

$$3.839 \quad \int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	.7251
Rubi [A] (verified)	.7251
Mathematica [A] (verified)	.7253
Maple [B] (verified)	.7253
Fricas [C] (verification not implemented)	.7254
Sympy [F(-1)]	.7254
Maxima [F]	.7254
Giac [F]	.7255
Mupad [F(-1)]	.7255

Optimal result

Integrand size = 23, antiderivative size = 143

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx = -\frac{2B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \mid 2\right)}{b^3 d \sqrt{\cos(c + dx)}} + \frac{2A\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3b^2 d \sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{b^2 d \sqrt{b \cos(c + dx)}}$$

[Out] 2/3*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2*B*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2827, 2716, 2721, 2720, 2719}

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \frac{2A\sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3b^2 d \sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} - \frac{2BE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{b^3 d \sqrt{\cos(c + dx)}} + \frac{2B \sin(c + dx)}{b^2 d \sqrt{b \cos(c + dx)}}$$

[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(5/2),x]

[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*

$\text{Cos}[c + d*x]] + (2*A*\text{Sin}[c + d*x])/(3*b*d*(b*\text{Cos}[c + d*x])^{3/2}) + (2*B*\text{Sin}[c + d*x])/(b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 2716

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] := \text{Simp}[\text{Cos}[c + d*x]*((b*\text{Sin}[c + d*x])^{(n + 1)})/(b*d*(n + 1)), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n + 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2719

$\text{Int}[\text{Sqrt}[\text{sin}[(c_*) + (d_*)*(x_)]], x_Symbol] := \text{Simp}[(2/d)*\text{EllipticE}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2720

$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_*) + (d_*)*(x_)]], x_Symbol] := \text{Simp}[(2/d)*\text{EllipticF}[(1/2)*(c - \text{Pi}/2 + d*x), 2], x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2721

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] := \text{Dist}[(b*\text{Sin}[c + d*x])^n/\text{Sin}[c + d*x]^n, \text{Int}[\text{Sin}[c + d*x]^n, x], x] /; \text{FreeQ}\{b, c, d\}, x \&\& \text{LtQ}[-1, n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2827

$\text{Int}[(b_*)*\text{sin}[(e_*) + (f_*)*(x_)]^{(m_)*((c_*) + (d_*)*\text{sin}[(e_*) + (f_*)*(x_)]), x_Symbol] := \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= A \int \frac{1}{(b \cos(c + dx))^{5/2}} dx + \frac{B \int \frac{1}{(b \cos(c + dx))^{3/2}} dx}{b} \\ &= \frac{2A \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{b^2 d \sqrt{b \cos(c + dx)}} + \frac{A \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{3b^2} - \frac{B \int \sqrt{b \cos(c + dx)} dx}{b^3} \\ &= \frac{2A \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{b^2 d \sqrt{b \cos(c + dx)}} \\ &\quad + \frac{\left(A \sqrt{\cos(c + dx)}\right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3b^2 \sqrt{b \cos(c + dx)}} - \frac{\left(B \sqrt{b \cos(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{b^3 \sqrt{\cos(c + dx)}} \end{aligned}$$

$$= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^2d\sqrt{b\cos(c+dx)}} \\ + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int \frac{A + B\cos(c+dx)}{(b\cos(c+dx))^{5/2}} dx = \frac{2\left(-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\mid 2\right) + A\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(179) = 358.

Time = 6.41 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.84

method	result
default	$\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}$
parts	$-\frac{2A\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right)}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)}$

[In] int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)

[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+6*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2)

$2*c)^2-1)^{1/2}*EllipticE(\cos(1/2*d*x+1/2*c),2^{1/2}))*(-2*\sin(1/2*d*x+1/2*c)^4*b+b*\sin(1/2*d*x+1/2*c)^2)^{1/2}/((2*\cos(1/2*d*x+1/2*c)^2-1)*b)^{1/2}/d$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.34

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \frac{-i \sqrt{2} A \sqrt{b} \cos(dx + c)^2 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{(b \cos(c + dx))^{5/2}}$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] $\frac{1}{3}*(-I*\sqrt{2}*A*\sqrt{b}*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + I*\sqrt{2}*A*\sqrt{b}*\cos(d*x + c)^2*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c)) - 3*I*\sqrt{2}*B*\sqrt{b}*\cos(d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c))) + 3*I*\sqrt{2}*B*\sqrt{b}*\cos(d*x + c)^2*\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(d*x + c) - I*\sin(d*x + c))) + 2*(3*B*\cos(d*x + c) + A)*\sqrt{b*\cos(d*x + c)}*\sin(d*x + c))/(b^3*d*\cos(d*x + c)^2)$

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{5/2}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(5/2),x)

[Out] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(5/2), x)

$$3.840 \quad \int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	7256
Rubi [A] (verified)	7256
Mathematica [A] (verified)	7258
Maple [B] (verified)	7259
Fricas [C] (verification not implemented)	7259
Sympy [F(-1)]	7260
Maxima [F]	7260
Giac [F]	7260
Mupad [F(-1)]	7261

Optimal result

Integrand size = 29, antiderivative size = 173

$$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{5/2}} dx = -\frac{6A\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5b^3 d \sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^2 d \sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2B \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}} + \frac{6A \sin(c+dx)}{5b^2 d \sqrt{b \cos(c+dx)}}$$

```
[Out] 2/5*A*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+6/5*A*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used

= {16, 2827, 2716, 2721, 2719, 2720}

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx = -\frac{6AE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5b^3 d \sqrt{\cos(c + dx)}} \\ + \frac{6A \sin(c + dx)}{5b^2 d \sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\ + \frac{2B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3b^2 d \sqrt{b \cos(c + dx)}} + \frac{2B \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}}$$

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]

[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\
&= (Ab) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx + B \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\
&= \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{(3A) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx}{5b} + \frac{B \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{3b^2} \\
&= \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5b^2 d \sqrt{b \cos(c + dx)}} \\
&\quad - \frac{(3A) \int \sqrt{b \cos(c + dx)} dx}{5b^3} + \frac{\left(B \sqrt{\cos(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3b^2 \sqrt{b \cos(c + dx)}} \\
&= \frac{2B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3b^2 d \sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\
&\quad + \frac{2B \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5b^2 d \sqrt{b \cos(c + dx)}} \\
&\quad - \frac{\left(3A \sqrt{b \cos(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx}{5b^3 \sqrt{\cos(c + dx)}} \\
&= -\frac{6A \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5b^3 d \sqrt{\cos(c + dx)}} + \frac{2B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3b^2 d \sqrt{b \cos(c + dx)}} \\
&\quad + \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5b^2 d \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.60

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \frac{2 \left(-9A \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B \sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right) \right)}{15b^2}$$

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2), x]
```

```
[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(201) = 402$.

Time = 8.42 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.35

method	result
default	$\frac{2\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)b(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \left(72A \cos(\frac{dx}{2} + \frac{c}{2}) (\sin^6(\frac{dx}{2} + \frac{c}{2})) - 36A \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} \sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} E \right)}{\dots}$
parts	$\frac{2A\sqrt{-(-2(\cos^2(\frac{dx}{2} + \frac{c}{2})) + 1)b(\sin^2(\frac{dx}{2} + \frac{c}{2}))} \left(24 \cos(\frac{dx}{2} + \frac{c}{2}) (\sin^6(\frac{dx}{2} + \frac{c}{2})) - 12\sqrt{2(\sin^2(\frac{dx}{2} + \frac{c}{2})) - 1} \sqrt{\frac{1}{2} - \frac{\cos(dx+c)}{2}} E \right)}{\dots}$

[In] `int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOS E)`

[Out]
$$\begin{aligned} & -2/15 * (-(-2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 + 1) * b * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / b ^ 3 / \sin(1 \\ & / 2 * d * x + 1/2 * c) ^ 3 / (8 * \sin(1/2 * d * x + 1/2 * c) ^ 6 - 12 * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 6 * \sin(1/2 * d \\ & * x + 1/2 * c) ^ 2 - 1) * (72 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 6 - 36 * A * (\sin(1/2 * \\ & d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + \\ & 1/2 * c), 2 ^ (1/2)) * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 20 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * s \\ & \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \sin(1/2 * \\ & d * x + 1/2 * c) ^ 4 - 72 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 36 * A * (\sin(1/2 * d * x \\ & + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 \\ & * c), 2 ^ (1/2)) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 20 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c \\ &) ^ 4 + 20 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ (1/2) * \text{Elli \\ & pticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 24 * A * \cos(1/2 * d * x + 1/2 \\ & * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 9 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/ \\ & 2 * c) ^ 2 - 1) ^ (1/2) * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) + 10 * B * \cos(1/2 * d * x + 1/2 * \\ & c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 5 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) * (2 * \sin(1/2 * d * x + 1/2 \\ & * c) ^ 2 - 1) ^ (1/2) * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ (1/2)) * (-2 * \sin(1/2 * d * x + 1/2 * c \\ &) ^ 4 * b + b * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ (1/2) / ((2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * b) ^ (1/2) / d \end{aligned}$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \frac{-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c))}{\dots}$$

[In] `integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] $\frac{1}{15}(-5I\sqrt{2}B\sqrt{b}\cos(dx+c)^3\text{weierstrassPInverse}(-4, 0, \cos(dx+c) + I\sin(dx+c)) + 5I\sqrt{2}B\sqrt{b}\cos(dx+c)^3\text{weierstrassPInverse}(-4, 0, \cos(dx+c) - I\sin(dx+c)) - 9I\sqrt{2}A\sqrt{b}\cos(dx+c)^3\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) + I\sin(dx+c))) + 9I\sqrt{2}A\sqrt{b}\cos(dx+c)^3\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-4, 0, \cos(dx+c) - I\sin(dx+c))) + 2(9A\cos(dx+c)^2 + 5B\cos(dx+c) + 3A)\sqrt{b\cos(dx+c)}\sin(dx+c)) / (b^3d\cos(dx+c)^3)$

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] `integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(5/2), x)`

[Out] Timed out

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{5/2}} dx$$

[In] `integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")`

[Out] `integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)`

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{5/2}} dx$$

[In] `integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2), x, algorithm="giac")`

[Out] `integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx) (b \cos(c + dx))^{5/2}} dx$$

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)), x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)), x)
```

$$3.841 \quad \int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{7/2}} dx$$

Optimal result	7262
Rubi [A] (verified)	7262
Mathematica [A] (verified)	7264
Maple [B] (verified)	7265
Fricas [C] (verification not implemented)	7265
Sympy [F(-1)]	7266
Maxima [F]	7266
Giac [F]	7266
Mupad [F(-1)]	7267

Optimal result

Integrand size = 23, antiderivative size = 176

$$\begin{aligned} \int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{7/2}} dx &= -\frac{6A\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \mid 2\right)}{5b^4d\sqrt{\cos(c+dx)}} \\ &+ \frac{2B\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right)}{3b^3d\sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{5bd(b \cos(c+dx))^{5/2}} \\ &+ \frac{2B \sin(c+dx)}{3b^2d(b \cos(c+dx))^{3/2}} + \frac{6A \sin(c+dx)}{5b^3d\sqrt{b \cos(c+dx)}} \end{aligned}$$

[Out] 2/5*A*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+2/3*B*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(3/2)+6/5*A*sin(d*x+c)/b^3/d/(b*cos(d*x+c))^(1/2)+2/3*B*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^3/d/(b*cos(d*x+c))^(1/2)-6/5*A*(cos(1/2*d*x+1/2*c))^2^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^4/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

= {2827, 2716, 2721, 2719, 2720}

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx = -\frac{6AE\left(\frac{1}{2}(c + dx) \mid 2\right) \sqrt{b \cos(c + dx)}}{5b^4 d \sqrt{\cos(c + dx)}} + \frac{6A \sin(c + dx)}{5b^3 d \sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2B \sqrt{\cos(c + dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3b^3 d \sqrt{b \cos(c + dx)}} + \frac{2B \sin(c + dx)}{3b^2 d (b \cos(c + dx))^{3/2}}$$

[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(7/2), x]

[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (2*B*Sin[c + d*x])/(3*b^2*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])

Rule 2716

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2720

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[(b*Sin[c + d*x])^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ[-1, n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
\text{integral} &= A \int \frac{1}{(b \cos(c + dx))^{7/2}} dx + \frac{B \int \frac{1}{(b \cos(c + dx))^{5/2}} dx}{b} \\
&= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3b^2d(b \cos(c + dx))^{3/2}} + \frac{(3A) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx}{5b^2} + \frac{B \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{3b^3} \\
&= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3b^2d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5b^3d\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{(3A) \int \sqrt{b \cos(c + dx)} dx}{5b^4} + \frac{\left(B\sqrt{\cos(c + dx)} \right) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3b^3\sqrt{b \cos(c + dx)}} \\
&= \frac{2B\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3b^3d\sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} \\
&\quad + \frac{2B \sin(c + dx)}{3b^2d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5b^3d\sqrt{b \cos(c + dx)}} \\
&\quad - \frac{\left(3A\sqrt{b \cos(c + dx)} \right) \int \sqrt{\cos(c + dx)} dx}{5b^4\sqrt{\cos(c + dx)}} \\
&= -\frac{6A\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right)}{5b^4d\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)}{3b^3d\sqrt{b \cos(c + dx)}} \\
&\quad + \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3b^2d(b \cos(c + dx))^{3/2}} + \frac{6A \sin(c + dx)}{5b^3d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.59

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx = \frac{2\left(-9A\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \mid 2\right) + 5B\sqrt{\cos(c + dx)} \text{EllipticF}\left(\frac{1}{2}(c + dx), 2\right)\right)}{15b^3d\sqrt{b \cos(c + dx)}}$$

[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(7/2), x]

[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b^3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(204) = 408$.

Time = 8.38 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.29

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(72A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-36A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}E\right)}{\dots}$
parts	$-\frac{2A\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}E\right)}{\dots}$

[In] `int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(7/2),x,method=_RETURNVERBOSE)`

[Out]
$$-\frac{2}{15}\left(-(-2\cos(1/2*d*x+1/2*c)^2+1)*b*\sin(1/2*d*x+1/2*c)^2\right)^{1/2}/b^4/\sin(1/2*d*x+1/2*c)^3/(8*\sin(1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1)*(72*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^6-36*A*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{1/2}*EllipticE(\cos(1/2*d*x+1/2*c),2^{1/2})*\sin(1/2*d*x+1/2*c)^4-20*B*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{1/2}*EllipticF(\cos(1/2*d*x+1/2*c),2^{1/2})*\sin(1/2*d*x+1/2*c)^4-72*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+36*A*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{1/2}*EllipticE(\cos(1/2*d*x+1/2*c),2^{1/2})*\sin(1/2*d*x+1/2*c)^2-20*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^4+20*B*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{1/2}*EllipticF(\cos(1/2*d*x+1/2*c),2^{1/2})*\sin(1/2*d*x+1/2*c)^2+24*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-9*A*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{1/2}*EllipticE(\cos(1/2*d*x+1/2*c),2^{1/2}))+10*B*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2-5*B*(\sin(1/2*d*x+1/2*c)^2)^{1/2}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{1/2}*EllipticF(\cos(1/2*d*x+1/2*c),2^{1/2}))*(-2*\sin(1/2*d*x+1/2*c)^4*b+b*\sin(1/2*d*x+1/2*c)^2)^{1/2}/((2*\cos(1/2*d*x+1/2*c)^2-1)*b)^{1/2}/d$$

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.16

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx = \frac{-5i \sqrt{2} B \sqrt{b} \cos(dx + c)^3 \text{weierstrassPInverse}(-4, 0, \cos(dx + c) + i \sin(dx + c))}{\dots}$$

[In] `integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")`

[Out]
$$1/15*(-5*I*\sqrt{2}*B*\sqrt{b}*\cos(d*x + c)^3*\text{weierstrassPInverse}(-4, 0, \cos(d*x + c) + I*\sin(d*x + c)) + 5*I*\sqrt{2}*B*\sqrt{b}*\cos(d*x + c)^3*\text{weierstra}$$

```
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 9*I*sqrt(2)*A*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*A*cos(d*x + c)^2 + 5*B*cos(d*x + c) + 3*A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^4*d*cos(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{7/2}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{7/2}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx = \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx$$

```
[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(7/2), x)
```

```
[Out] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(7/2), x)
```

$$3.842 \quad \int \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

Optimal result	7268
Rubi [A] (verified)	7268
Mathematica [A] (verified)	.7271
Maple [A] (verified)	.7271
Fricas [A] (verification not implemented)	.7271
Sympy [F(-1)]	7272
Maxima [A] (verification not implemented)	7272
Giac [A] (verification not implemented)	7273
Mupad [B] (verification not implemented)	7273

Optimal result

Integrand size = 33, antiderivative size = 172

$$\begin{aligned} & \int \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx \\ &= \frac{3Bx \sqrt{b \cos(c + dx)}}{8\sqrt{\cos(c + dx)}} + \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{d\sqrt{\cos(c + dx)}} \\ &+ \frac{3B \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{8d} \\ &+ \frac{B \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} - \frac{A \sqrt{b \cos(c + dx)} \sin^3(c + dx)}{3d\sqrt{\cos(c + dx)}} \end{aligned}$$

[Out] 1/4*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*A*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+3/8*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used

= {17, 2827, 2713, 2715, 8}

$$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$$

$$= -\frac{A \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{3Bx \sqrt{b \cos(c+dx)}}{8 \sqrt{\cos(c+dx)}} + \frac{B \sin(c+dx) \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}}{4d} + \frac{3B \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{8d}$$

[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] (3*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[[(

$b \cdot \text{Sin}[e + f \cdot x]^{(m + 1), x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{b \cos(c + dx)} \int \cos^3(c + dx)(A + B \cos(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{\left(A \sqrt{b \cos(c + dx)}\right) \int \cos^3(c + dx) dx}{\sqrt{\cos(c + dx)}} + \frac{\left(B \sqrt{b \cos(c + dx)}\right) \int \cos^4(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{B \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} \\
 &\quad + \frac{\left(3B \sqrt{b \cos(c + dx)}\right) \int \cos^2(c + dx) dx}{4\sqrt{\cos(c + dx)}} \\
 &\quad - \frac{\left(A \sqrt{b \cos(c + dx)}\right) \text{Subst}\left(\int (1 - x^2) dx, x, -\sin(c + dx)\right)}{d\sqrt{\cos(c + dx)}} \\
 &= \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + \frac{3B \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{8d} \\
 &\quad + \frac{B \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} \\
 &\quad - \frac{A \sqrt{b \cos(c + dx)} \sin^3(c + dx)}{3d\sqrt{\cos(c + dx)}} + \frac{\left(3B \sqrt{b \cos(c + dx)}\right) \int 1 dx}{8\sqrt{\cos(c + dx)}} \\
 &= \frac{3Bx \sqrt{b \cos(c + dx)}}{8\sqrt{\cos(c + dx)}} + \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{d\sqrt{\cos(c + dx)}} \\
 &\quad + \frac{3B \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{8d} \\
 &\quad + \frac{B \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} - \frac{A \sqrt{b \cos(c + dx)} \sin^3(c + dx)}{3d\sqrt{\cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.47

$$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$$

$$= \frac{\sqrt{b \cos(c+dx)} (36Bc + 36Bdx + 72A \sin(c+dx) + 24B \sin(2(c+dx)) + 8A \sin(3(c+dx)) + 3B \sin(4(c+dx)))}{96d \sqrt{\cos(c+dx)}}$$

[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] (Sqrt[b*Cos[c + d*x]]*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.53

method	result
default	$\frac{\sqrt{\cos(dx+c)} b (6B \sin(dx+c) (\cos^3(dx+c)) + 8A \sin(dx+c) (\cos^2(dx+c)) + 9B \sin(dx+c) \cos(dx+c) + 16A \sin(dx+c) + 9B(dx+c))}{24d \sqrt{\cos(dx+c)}}$
parts	$\frac{A(2+\cos^2(dx+c)) \sin(dx+c) \sqrt{\cos(dx+c)} b}{3d \sqrt{\cos(dx+c)}} + \frac{B \sqrt{\cos(dx+c)} b (2 \sin(dx+c) (\cos^3(dx+c)) + 3 \cos(dx+c) \sin(dx+c) + 3dx + 3c)}{8d \sqrt{\cos(dx+c)}}$
risch	$\frac{3 \sqrt{\cos(dx+c)} b (\sqrt{\cos(dx+c)}) e^{i(dx+c)} x B}{4(e^{2i(dx+c)}+1)} - \frac{i \sqrt{\cos(dx+c)} b (\sqrt{\cos(dx+c)}) e^{5i(dx+c)} B}{32(e^{2i(dx+c)}+1)d} - \frac{i \sqrt{\cos(dx+c)} b (\sqrt{\cos(dx+c)}) e^{4i(dx+c)} A}{12(e^{2i(dx+c)}+1)d}$

[In] int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURN VERBOSE)

[Out] 1/24/d*(cos(d*x+c)*b)^(1/2)*(6*B*sin(d*x+c)*cos(d*x+c)^3+8*A*sin(d*x+c)*cos(d*x+c)^2+9*B*sin(d*x+c)*cos(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.47

$$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$$

$$= \left[\frac{9B \sqrt{-b} \cos(dx+c) \log\left(2b \cos(dx+c)^2 - 2 \sqrt{b \cos(dx+c)} \sqrt{-b} \sqrt{\cos(dx+c)} \sin(dx+c) - b\right) + 2}{48d \cos(dx+c)} \right]$$

[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] [1/48*(9*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*cos(d*x + c)^3 + 8*A*cos(d*x + c)^2 + 9*B*cos(d*x + c) + 16*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*B*cos(d*x + c)^3 + 8*A*cos(d*x + c)^2 + 9*B*cos(d*x + c) + 16*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.54

$$\int \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{3 (12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin(\frac{1}{2} \arctan(\sin(4 dx + 4 c), \cos(4 dx + 4 c)))) B \sqrt{b} + 8 A \sqrt{b} (\sin(3 dx + 3 c) + 9 \sin(\frac{1}{3} \arctan(\sin(3 dx + 3 c), \cos(3 dx + 3 c))))}{96 d}$$

[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 8*A*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d

Giac [A] (verification not implemented)

none

Time = 3.40 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.62

$$\int \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{9 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 + 36 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 48 A \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 30 B \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 54 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 80 A \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 18 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 9 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 48 A \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 + 4 d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 6 d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 4 d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + d}$$

[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] 1/24*(9*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^8 + 36*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 48*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 - 30*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 54*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 80*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 - 18*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 9*B*sqrt(b)*d*x + 48*A*sqrt(b)*tan(1/2*d*x + 1/2*c))/ (d*tan(1/2*d*x + 1/2*c)^8 + 4*d*tan(1/2*d*x + 1/2*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(1/2*d*x + 1/2*c)^2 + d)

Mupad [B] (verification not implemented)

Time = 16.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.61

$$\int \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (24 B \sin(c + dx) + 80 A \sin(2c + 2dx) + 8 A \sin(4c + 4dx) + 27 B \sin(3c + 3dx) + 3 B \sin(5c + 5dx) + 72 B dx \cos(c + dx))}{96 d (\cos(2c + 2dx) + 1)}$$

[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*B*sin(c + d*x) + 80*A*sin(2*c + 2*d*x) + 8*A*sin(4*c + 4*d*x) + 27*B*sin(3*c + 3*d*x) + 3*B*sin(5*c + 5*d*x) + 72*B*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))

$$3.843 \quad \int \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$$

Optimal result	7274
Rubi [A] (verified)	7274
Mathematica [A] (verified)	7276
Maple [A] (verified)	7276
Fricas [A] (verification not implemented)	7277
Sympy [A] (verification not implemented)	7277
Maxima [A] (verification not implemented)	7278
Giac [A] (verification not implemented)	7278
Mupad [B] (verification not implemented)	7279

Optimal result

Integrand size = 33, antiderivative size = 136

$$\begin{aligned} & \int \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx \\ &= \frac{Ax \sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{B \sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\ &+ \frac{A\sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d} - \frac{B \sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} \end{aligned}$$

[Out] $\frac{1}{2}Ax(b \cos(dx+c))^{1/2} / \cos(dx+c)^{1/2} + B \sin(dx+c) (b \cos(dx+c))^{1/2} / d \cos(dx+c)^{1/2} - \frac{1}{3}B \sin(dx+c)^3 (b \cos(dx+c))^{1/2} / d \cos(dx+c)^{1/2} + \frac{1}{2}A \sin(dx+c) \cos(dx+c)^{1/2} (b \cos(dx+c))^{1/2} / d$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 2715, 8, 2713}

$$\begin{aligned} & \int \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx \\ &= \frac{Ax \sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{2d} \\ &- \frac{B \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{B \sin(c+dx) \sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} \end{aligned}$$

[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] (A*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) - (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{b \cos(c + dx)} \int \cos^2(c + dx) (A + B \cos(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{\left(A \sqrt{b \cos(c + dx)}\right) \int \cos^2(c + dx) dx}{\sqrt{\cos(c + dx)}} + \frac{\left(B \sqrt{b \cos(c + dx)}\right) \int \cos^3(c + dx) dx}{\sqrt{\cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{A\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int 1 dx}{2\sqrt{\cos(c+dx)}} \\
&\quad - \frac{\left(B\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int(1-x^2) dx, x, -\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}} \\
&= \frac{Ax\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{A\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} - \frac{B\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\begin{aligned}
&\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx)) dx \\
&= \frac{\sqrt{b\cos(c+dx)}(6Ac+6Adx+9B\sin(c+dx)+3A\sin(2(c+dx))+B\sin(3(c+dx)))}{12d\sqrt{\cos(c+dx)}}
\end{aligned}$$

[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] (Sqrt[b*Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])

Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

method	result
default	$\frac{\sqrt{\cos(dx+c)}b(2B\sin(dx+c)(\cos^2(dx+c))+3A\sin(dx+c)\cos(dx+c)+3A(dx+c)+4B\sin(dx+c))}{6d\sqrt{\cos(dx+c)}}$
parts	$\frac{A\sqrt{\cos(dx+c)}b(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}} + \frac{B(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)}b}{3d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{\cos(dx+c)}b(\sqrt{\cos(dx+c)})e^{i(dx+c)}xA}{e^{2i(dx+c)}+1} - \frac{i\sqrt{\cos(dx+c)}b(\sqrt{\cos(dx+c)})e^{4i(dx+c)}B}{12(e^{2i(dx+c)}+1)d} - \frac{i\sqrt{\cos(dx+c)}b(\sqrt{\cos(dx+c)})e^{3i(dx+c)}A}{4(e^{2i(dx+c)}+1)d}$

[In] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURN VERBOSE)

[Out] 1/6/d*(cos(d*x+c)*b)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*cos(d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.69

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \left[\frac{3 A \sqrt{-b} \cos(dx + c) \log\left(2 b \cos(dx + c)^2 - 2 \sqrt{b \cos(dx + c)} \sqrt{-b} \sqrt{\cos(dx + c)} \sin(dx + c) - b\right) + 2}{12 d \cos(dx + c)} \right]$$

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] [1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*cos(d*x + c)^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [A] (verification not implemented)

Time = 81.20 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.49

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \begin{cases} x \sqrt{b \cos(c)} (A + B \cos(c)) \cos^{\frac{3}{2}}(c) \\ 0 \\ \frac{Ax \sqrt{b \cos(c+dx)} \sin^2(c+dx)}{2 \sqrt{\cos(c+dx)}} + \frac{Ax \sqrt{b \cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)}{2} + \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx) \sqrt{\cos(c+dx)}}{2d} + \frac{2B \sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d \sqrt{\cos(c+dx)}} \end{cases}$$

[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)

[Out] Piecewise((x*sqrt(b*cos(c))*(A + B*cos(c))*cos(c)**(3/2), Eq(d, 0)), (0, Eq(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (A*x*sqrt(b*cos(c + d*x))*sin(c + d*x)**2/(2*sqrt(cos(c + d*x))) + A*x*sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)/2 + A*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d) + 2*B*sqrt(b*cos(c + d*x))*sin(c + d*x)**3/(3*d*sqrt(cos(c + d*x))) + B*sqrt(b*cos(c + d*x))*sin(c + d*x)*cos(c + d*x)**(3/2)/d, True))

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{3(2dx + 2c + \sin(2dx + 2c))A\sqrt{b} + B\sqrt{b}(\sin(3dx + 3c) + 9 \sin(\frac{1}{3} \arctan(\sin(3dx + 3c)), \cos(3dx + 3c)))}{12d}$$

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + B*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/d

Giac [A] (verification not implemented)

none

Time = 2.58 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.43

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{3A\sqrt{b}dx \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^6 + 9A\sqrt{b}dx \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 - 6A\sqrt{b} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 12B\sqrt{b} \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{6\left(d \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^6 + 3d\right)}$$

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 9*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 - 6*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 12*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 9*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 8*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 3*A*sqrt(b)*d*x + 6*A*sqrt(b)*tan(1/2*d*x + 1/2*c) + 12*B*sqrt(b)*tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^6 + 3*d*tan(1/2*d*x + 1/2*c)^4 + 3*d*tan(1/2*d*x + 1/2*c)^2 + d)

Mupad [B] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \frac{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (3A \sin(c + dx) + 3A \sin(3c + 3dx) + 10B \sin(2c + 2dx) + B \sin(4c + 4dx))}{12d (\cos(2c + 2dx) + 1)}$$

[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c + d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))

$$3.844 \quad \int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx$$

Optimal result	7280
Rubi [A] (verified)	7280
Mathematica [A] (verified)	7281
Maple [A] (verified)	7282
Fricas [A] (verification not implemented)	7282
Sympy [A] (verification not implemented)	7283
Maxima [A] (verification not implemented)	7283
Giac [A] (verification not implemented)	7283
Mupad [B] (verification not implemented)	7284

Optimal result

Integrand size = 33, antiderivative size = 98

$$\begin{aligned} & \int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx \\ &= \frac{Bx \sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} \\ & \quad + \frac{B \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d} \end{aligned}$$

[Out] 1/2*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2813}

$$\begin{aligned} & \int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)) dx \\ &= \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{Bx \sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} \\ & \quad + \frac{B \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{2d} \end{aligned}$$

[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] $(B*x*\sqrt{b*\cos[c + d*x]})/(2*\sqrt{\cos[c + d*x]}) + (A*\sqrt{b*\cos[c + d*x]}* \sin[c + d*x])/(d*\sqrt{\cos[c + d*x]}) + (B*\sqrt{\cos[c + d*x]}*\sqrt{b*\cos[c + d*x]}*\sin[c + d*x])/(2*d)$

Rule 17

$\text{Int}[(u_.)*((a_.)*(v_))^{(m)}*((b_.)*(v_))^{(n)}, x_Symbol] \rightarrow \text{Dist}[a^{(m + 1/2)}*b^{(n - 1/2)}*(\sqrt{b*v}/\sqrt{a*v}), \text{Int}[u*v^{(m + n)}, x], x] /;$ FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2813

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] \rightarrow \text{Simp}[(2*a*c + b*d)*(x/2), x] + (-\text{Simp}[(b*c + a*d)*(\cos[e + f*x]/f), x] - \text{Simp}[b*d*\cos[e + f*x]*(\sin[e + f*x]/(2*f)), x]) /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{b \cos(c + dx)} \int \cos(c + dx)(A + B \cos(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{Bx \sqrt{b \cos(c + dx)}}{2\sqrt{\cos(c + dx)}} + \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\ &\quad + \frac{B \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\begin{aligned} &\int \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (A + B \cos(c + dx)) dx \\ &= \frac{\sqrt{b \cos(c + dx)} (4A \sin(c + dx) + B(2(c + dx) + \sin(2(c + dx))))}{4d \sqrt{\cos(c + dx)}} \end{aligned}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]

[Out] $(\sqrt{b*\cos[c + d*x]}*(4*A*\sin[c + d*x] + B*(2*(c + d*x) + \sin[2*(c + d*x)])))/(4*d*\sqrt{\cos[c + d*x]})$

Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{\cos(dx+c)b} (B \sin(dx+c) \cos(dx+c) + 2A \sin(dx+c) + B(dx+c))}{2d\sqrt{\cos(dx+c)}}$
parts	$\frac{A \sin(dx+c) \sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}} + \frac{B \sqrt{\cos(dx+c)b} (\cos(dx+c) \sin(dx+c) + dx+c)}{2d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{\cos(dx+c)b} (\sqrt{\cos(dx+c)}) e^{i(dx+c)} x B}{e^{2i(dx+c)+1}} - \frac{i \sqrt{\cos(dx+c)b} (\sqrt{\cos(dx+c)}) e^{3i(dx+c)} B}{4(e^{2i(dx+c)+1})d} - \frac{i \sqrt{\cos(dx+c)b} (\sqrt{\cos(dx+c)}) e^{2i(dx+c)} A}{(e^{2i(dx+c)+1})d} +$

```
[In] int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c)),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/2/d*(cos(d*x+c)*b)^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c
))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.08

$$\int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A + B \cos(c+dx)) dx$$

$$= \left[\frac{B \sqrt{-b} \cos(dx+c) \log\left(2b \cos(dx+c)^2 - 2\sqrt{b \cos(dx+c)} \sqrt{-b} \sqrt{\cos(dx+c)} \sin(dx+c) - b\right) + 2(B \sqrt{\cos(dx+c)} \sin(dx+c) - b)}{4d \cos(dx+c)} \right]$$

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c
))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*cos(d*x + c) + 2*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/
2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)
^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [A] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.68

$$\int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A + B \cos(c+dx)) dx$$

$$= \begin{cases} x \sqrt{b \cos(c)} (A + B \cos(c)) \sqrt{\cos(c)} \\ 0 \\ \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \frac{B x \sqrt{b \cos(c+dx)} \sin^2(c+dx)}{2 \sqrt{\cos(c+dx)}} + \frac{B x \sqrt{b \cos(c+dx)} \cos^{\frac{3}{2}}(c+dx)}{2} + \frac{B \sqrt{b \cos(c+dx)} \sin(c+dx) \sqrt{\cos(c+dx)}}{2d} \end{cases}$$

[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)

[Out] Piecewise((x*sqrt(b*cos(c))*(A + B*cos(c))*sqrt(cos(c)), Eq(d, 0)), (0, Eq(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (A*sqrt(b*cos(c + d*x))*sin(c + d*x)/(d*sqrt(cos(c + d*x))) + B*x*sqrt(b*cos(c + d*x))*sin(c + d*x)**2/(2*sqrt(cos(c + d*x))) + B*x*sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)/2 + B*sqrt(b*cos(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d), True))

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A + B \cos(c+dx)) dx$$

$$= \frac{(2 dx + 2 c + \sin(2 dx + 2 c)) B \sqrt{b} + 4 A \sqrt{b} \sin(dx + c)}{4 d}$$

[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*sqrt(b) + 4*A*sqrt(b)*sin(d*x + c))/d

Giac [A] (verification not implemented)

none

Time = 1.88 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.45

$$\int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A + B \cos(c+dx)) dx$$

$$= \frac{B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 2 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 4 A \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 2 B \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{2 \left(d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 2 d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) \right)^2}$$

[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{2}(B\sqrt{b})dxtan(1/2dx + 1/2c)^4 + 2B\sqrt{b})dxtan(1/2dx + 1/2c)^2 + 4A\sqrt{b})tan(1/2dx + 1/2c)^3 - 2B\sqrt{b})tan(1/2dx + 1/2c)^3 + B\sqrt{b})dx + 4A\sqrt{b})tan(1/2dx + 1/2c) + 2B\sqrt{b})tan(1/2dx + 1/2c))/(d\tan(1/2dx + 1/2c)^4 + 2d\tan(1/2dx + 1/2c)^2 + d)$

Mupad [B] (verification not implemented)

Time = 14.95 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))dx$$

$$= \frac{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(B\sin(c+dx) + 4A\sin(2c+2dx) + B\sin(3c+3dx) + 4Bdx\cos(c+dx))}{4d(\cos(2c+2dx)+1)}$$

[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)),x)

[Out] $(\cos(c + d*x)^{(1/2)}*(b*\cos(c + d*x))^{(1/2)}*(B*\sin(c + d*x) + 4*A*\sin(2*c + 2*d*x) + B*\sin(3*c + 3*d*x) + 4*B*d*x*\cos(c + d*x)))/(4*d*(\cos(2*c + 2*d*x) + 1))$

$$3.845 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	7285
Rubi [A] (verified)	7285
Mathematica [A] (verified)	7286
Maple [A] (verified)	7286
Fricas [A] (verification not implemented)	7287
Sympy [A] (verification not implemented)	7287
Maxima [A] (verification not implemented)	7288
Giac [F]	7288
Mupad [B] (verification not implemented)	7288

Optimal result

Integrand size = 33, antiderivative size = 59

$$\begin{aligned} & \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx \\ &= \frac{Ax\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} \end{aligned}$$

[Out] A*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2717}

$$\begin{aligned} & \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx \\ &= \frac{Ax\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} \end{aligned}$$

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]

[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b

, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{b \cos(c + dx)} \int (A + B \cos(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ax \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{\left(B \sqrt{b \cos(c + dx)} \right) \int \cos(c + dx) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ax \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{B \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \frac{\sqrt{b \cos(c + dx)} (A(c + dx) + B \sin(c + dx))}{d \sqrt{\cos(c + dx)}}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\sqrt{\cos(dx+c)b} (A(dx+c)+B \sin(dx+c))}{d \sqrt{\cos(dx+c)}}$	39
risch	$\frac{Ax \sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}} + \frac{B \sin(dx+c) \sqrt{\cos(dx+c)b}}{d \sqrt{\cos(dx+c)}}$	52
parts	$\frac{A \sqrt{\cos(dx+c)b} (dx+c)}{d \sqrt{\cos(dx+c)}} + \frac{B \sin(dx+c) \sqrt{\cos(dx+c)b}}{d \sqrt{\cos(dx+c)}}$	59

[In] `int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)`

[Out] $1/d*(\cos(dx+c)*b)^{1/2}*(A*(dx+c)+B*\sin(dx+c))/\cos(dx+c)^{1/2}$

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.07

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

$$= \left[\frac{A\sqrt{-b} \cos(dx+c) \log\left(2b \cos(dx+c)^2 - 2\sqrt{b \cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)} \sin(dx+c) - b\right) + 2\sqrt{b \cos(dx+c)} \sin(dx+c)}{2d \cos(dx+c)} \right]$$

[In] `integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,algorithm="fricas")`

[Out] $[1/2*(A*\sqrt{-b}*\cos(dx+c)*\log(2*b*\cos(dx+c)^2 - 2*\sqrt{b*\cos(dx+c)}*\sqrt{-b}*\sqrt{\cos(dx+c)}*\sin(dx+c) - b) + 2*\sqrt{b*\cos(dx+c)}*B*\sqrt{\cos(dx+c)}*\sin(dx+c))/(d*\cos(dx+c)), (A*\sqrt{b}*\arctan(\sqrt{b*\cos(dx+c)}*\sin(dx+c)/(\sqrt{b}*\cos(dx+c)^{3/2}))*\cos(dx+c) + \sqrt{b*\cos(dx+c)}*B*\sqrt{\cos(dx+c)}*\sin(dx+c))/(d*\cos(dx+c))]$

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

$$= \begin{cases} \frac{Ax\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} & \text{for } d \neq 0 \\ \frac{x\sqrt{b \cos(c)}(A+B \cos(c))}{\sqrt{\cos(c)}} & \text{otherwise} \end{cases}$$

[In] `integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)`

[Out] `Piecewise((A*x*sqrt(b*cos(c+d*x))/sqrt(cos(c+d*x))+B*sqrt(b*cos(c+d*x))*sin(c+d*x)/(d*sqrt(cos(c+d*x))), Ne(d,0)),(x*sqrt(b*cos(c))*(A+B*cos(c))/sqrt(cos(c)), True))`

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \frac{2 A \sqrt{b} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + B \sqrt{b} \sin(dx + c)}{d}$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] (2*A*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*sqrt(b)*sin(d*x + c))/d

Giac [F]

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \frac{\sqrt{b \cos(c + dx)}(B \sin(c + dx) + A dx)}{d \sqrt{\cos(c + dx)}}$$

[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)

[Out] ((b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + A*d*x))/(d*cos(c + d*x)^(1/2))

$$3.846 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	7289
Rubi [A] (verified)	7289
Mathematica [A] (verified)	7290
Maple [A] (verified)	7291
Fricas [A] (verification not implemented)	7291
Sympy [F]	7292
Maxima [A] (verification not implemented)	7292
Giac [F]	7292
Mupad [F(-1)]	7293

Optimal result

Integrand size = 33, antiderivative size = 60

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{Bx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A \operatorname{arctanh}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[Out] B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {17, 2814, 3855}

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{A \operatorname{arctanh}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Bx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]

[Out] (B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{b \cos(c + dx)} \int (A + B \cos(c + dx)) \sec(c + dx) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{Bx \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{\left(A \sqrt{b \cos(c + dx)} \right) \int \sec(c + dx) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{Bx \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{A \operatorname{arctanh}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{(Bdx + A \operatorname{arctanh}(\sin(c + dx))) \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}}$$

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*
x]])
```

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\sqrt{\cos(dx+c)}b(2A \operatorname{arctanh}(\cot(dx+c)-\operatorname{csc}(dx+c))-B(dx+c))}{d\sqrt{\cos(dx+c)}}$	52
parts	$-\frac{2A \operatorname{arctanh}(\cot(dx+c)-\operatorname{csc}(dx+c))\sqrt{\cos(dx+c)}b}{d\sqrt{\cos(dx+c)}} + \frac{B\sqrt{\cos(dx+c)}b(dx+c)}{d\sqrt{\cos(dx+c)}}$	70
risch	$\frac{Bx\sqrt{\cos(dx+c)}b}{\sqrt{\cos(dx+c)}} + \frac{\sqrt{\cos(dx+c)}b A \ln(e^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}d} - \frac{\sqrt{\cos(dx+c)}b A \ln(e^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}d}$	96

[In] `int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURN
VERBOSE)`

[Out] $-1/d*(\cos(d*x+c)*b)^{(1/2)}*(2*A*\operatorname{arctanh}(\cot(d*x+c)-\operatorname{csc}(d*x+c))-B*(d*x+c))/\cos(d*x+c)^{(1/2)}$

Fricas [A] (verification not implemented)Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(52) = 104$.

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

$$= \left[-\frac{2A\sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)}\sqrt{-b} \sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) - B\sqrt{-b} \log\left(2b \cos(dx+c)^2 - 2\sqrt{b \cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\right)}{2d} \right]$$

[In] `integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")`

[Out] $[-1/2*(2*A*\sqrt{-b}*\arctan(\sqrt{b*\cos(d*x+c)}*\sqrt{-b}*\sin(d*x+c)/(b*\sqrt{\cos(d*x+c)})) - B*\sqrt{-b}*\log(2*b*\cos(d*x+c)^2 - 2*\sqrt{b*\cos(d*x+c)}*\sqrt{-b}\sqrt{\cos(dx+c)})/d, 1/2*(2*B*\sqrt{b}*\arctan(\sqrt{b*\cos(d*x+c)}*\sin(d*x+c)/(\sqrt{b}*\cos(d*x+c)^{(3/2)})) + A*\sqrt{b}*\log(-(b*\cos(d*x+c))^3 - 2*\sqrt{b*\cos(d*x+c)}*\sqrt{b}*\sqrt{\cos(d*x+c)}*\sin(d*x+c) - 2*b*\cos(d*x+c))/\cos(d*x+c)^3)/d]$

Sympy [F]

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2), x)

[Out] Integral(sqrt(b*cos(c + d*x))*(A + B*cos(c + d*x))/cos(c + d*x)**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

$$= \frac{A\sqrt{b}(\log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \sin(dx + c) + 1) - \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2 \sin(dx + c) + 1)) + 4B\sqrt{b} \operatorname{arctan}(\frac{\sin(dx + c)}{\cos(dx + c) + 1})}{2d}$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] 1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*B*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d

Giac [F]

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}}{\cos^{\frac{3}{2}}(dx + c)} dx$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos(c + dx)^{3/2}} dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2), x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2), x)
```

$$3.847 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	7294
Rubi [A] (verified)	7294
Mathematica [A] (verified)	7296
Maple [A] (verified)	7296
Fricas [A] (verification not implemented)	7296
Sympy [F(-1)]	7297
Maxima [A] (verification not implemented)	7297
Giac [F]	7298
Mupad [F(-1)]	7298

Optimal result

Integrand size = 33, antiderivative size = 68

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{\text{Barctanh}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 3852, 8, 3855}

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx = \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{\text{Barctanh}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2), x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 17

`Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{b \cos(c + dx)} \int (A + B \cos(c + dx)) \sec^2(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{\left(A \sqrt{b \cos(c + dx)}\right) \int \sec^2(c + dx) dx}{\sqrt{\cos(c + dx)}} + \frac{\left(B \sqrt{b \cos(c + dx)}\right) \int \sec(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{\text{Barctanh}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}} - \frac{\left(A \sqrt{b \cos(c + dx)}\right) \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d \sqrt{\cos(c + dx)}} \\
 &= \frac{\text{Barctanh}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}} + \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{\sqrt{b \cos(c + dx)}(B \operatorname{Arctanh}(\sin(c + dx)) \cos(c + dx) + A \sin(c + dx))}{d \cos^{\frac{3}{2}}(c + dx)}$$

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]

[Out] (Sqrt[b*Cos[c + d*x]]*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(3/2))

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{(-2B \cos(dx+c) \operatorname{arctanh}(\cot(dx+c) - \operatorname{csc}(dx+c)) + A \sin(dx+c)) \sqrt{\cos(dx+c)b}}{d \cos(dx+c)^{\frac{3}{2}}}$	57
parts	$\frac{A \sin(dx+c) \sqrt{\cos(dx+c)b}}{d \cos(dx+c)^{\frac{3}{2}}} - \frac{2B \operatorname{arctanh}(\cot(dx+c) - \operatorname{csc}(dx+c)) \sqrt{\cos(dx+c)b}}{d \sqrt{\cos(dx+c)}}$	71
risch	$\frac{2i \sqrt{\cos(dx+c)b} A}{\sqrt{\cos(dx+c)} d (e^{2i(dx+c)} + 1)} + \frac{\sqrt{\cos(dx+c)b} B \ln(e^{i(dx+c)} + i)}{\sqrt{\cos(dx+c)} d} - \frac{\sqrt{\cos(dx+c)b} B \ln(e^{i(dx+c)} - i)}{\sqrt{\cos(dx+c)} d}$	113

[In] int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURN VERBOSE)

[Out] 1/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.01

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \left[\frac{B\sqrt{b} \cos(dx + c)^2 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2\sqrt{b \cos(dx+c)}A\sqrt{\cos(dx+c)} \sin(dx+c)}{2d \cos(dx+c)^2} - \frac{B\sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)}\sqrt{-b} \sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) \cos(dx+c)^2 - \sqrt{b \cos(dx+c)}A\sqrt{\cos(dx+c)} \sin(dx+c)}{d \cos(dx+c)^2} \right]$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx$$

$$= \frac{B\sqrt{b}(\log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \sin(dx + c) + 1) - \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2 \sin(dx + c) + 1))}{2d}$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] $\frac{1}{2} * (B * \sqrt{b}) * (\log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 * \sin(dx + c) + 1) - \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2 * \sin(dx + c) + 1)) + 4 * A * \sqrt{b} * \sin(2 * dx + 2 * c) / (\cos(2 * dx + 2 * c)^2 + \sin(2 * dx + 2 * c)^2 + 2 * \cos(2 * dx + 2 * c) + 1) / d$

Giac [F]

$$\int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] `integrate((b*cos(dx+c))^(1/2)*(A+B*cos(dx+c))/cos(dx+c)^(5/2),x, algorithm="giac")`

[Out] `integrate((B*cos(dx + c) + A)*sqrt(b*cos(dx + c))/cos(dx + c)^(5/2), x)`

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx))}{\cos(c + dx)^{\frac{5}{2}}} dx$$

[In] `int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2),x)`

[Out] `int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)`

$$3.848 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	7299
Rubi [A] (verified)	7299
Mathematica [A] (verified)	.7301
Maple [A] (verified)	.7301
Fricas [A] (verification not implemented)	7302
Sympy [F(-1)]	7302
Maxima [B] (verification not implemented)	7303
Giac [F]	7303
Mupad [F(-1)]	.7304

Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{A \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{B \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] 1/2*A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*A*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {17, 2827, 3853, 3855, 3852, 8}

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx = \frac{A \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{B \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)}$$

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]

[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2827

Int[((b_)*sin[(e_.) + (f_)*(x_)]))^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3852

Int[csc[(c_.) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{b \cos(c + dx)} \int (A + B \cos(c + dx)) \sec^3(c + dx) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{\left(A \sqrt{b \cos(c + dx)} \right) \int \sec^3(c + dx) dx}{\sqrt{\cos(c + dx)}} + \frac{\left(B \sqrt{b \cos(c + dx)} \right) \int \sec^2(c + dx) dx}{\sqrt{\cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)dx}{2\sqrt{\cos(c+dx)}} \\
&\quad - \frac{\left(B\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int 1 dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}} \\
&= \frac{A\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\begin{aligned}
&\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx \\
&= \frac{\sqrt{b\cos(c+dx)}(A\text{arctanh}(\sin(c+dx))\cos^2(c+dx) + (A+2B\cos(c+dx))\sin(c+dx))}{2d\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

method	result
default	$\frac{(A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+2B\sin(dx+c)\cos(dx+c)+A\sin(dx+c))\sqrt{b\cos(dx+c)}}{2d\cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c))\sqrt{\cos(dx+c)b}}{2d\cos(dx+c)^{\frac{5}{2}}} + \frac{B\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}(Ae^{3i(dx+c)}-2Be^{2i(dx+c)}-Ae^{i(dx+c)}-2B)}{\sqrt{\cos(dx+c)}d(e^{2i(dx+c)}+1)^2} + \frac{\sqrt{\cos(dx+c)b}A\ln(e^{i(dx+c)}+i)}{2\sqrt{\cos(dx+c)}d} - \frac{\sqrt{\cos(dx+c)b}A\ln(e^{i(dx+c)}-i)}{2\sqrt{\cos(dx+c)}d}$

[In] int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2), x, method=_RETURNVERBOSE)

[Out] 1/2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx$$

$$= \left[\frac{A\sqrt{b} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(2B \cos(dx + c) + A)}{4d \cos(dx + c)^3} \right.$$

$$\left. - \frac{A\sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)}\sqrt{-b} \sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) \cos(dx + c)^3 - (2B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}\sqrt{\cos(dx + c)}}{2d \cos(dx + c)^3} \right]$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. 2(91) = 182.

Time = 0.62 (sec) , antiderivative size = 716, normalized size of antiderivative = 6.69

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/4*((4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*A*\sqrt{b}/(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1) - 8*B*\sqrt{b}*\sin(2*d*x + 2*c)/(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1))/d \end{aligned}$$

Giac [F]

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}}{\cos(dx + c)^{\frac{7}{2}}} dx$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos(c + dx)^{7/2}} dx$$

```
[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

```
[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

$$3.849 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	7305
Rubi [A] (verified)	7305
Mathematica [A] (verified)	7307
Maple [A] (verified)	7308
Fricas [A] (verification not implemented)	7308
Sympy [F(-1)]	7309
Maxima [B] (verification not implemented)	7309
Giac [F]	7310
Mupad [F(-1)]	7310

Optimal result

Integrand size = 33, antiderivative size = 145

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{\text{Barctanh}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b \cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{A\sqrt{b \cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{A\sqrt{b \cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

[Out] 1/2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+A*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*A*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)+1/2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used

= {17, 2827, 3852, 3853, 3855}

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \frac{A \sin^3(c + dx) \sqrt{b \cos(c + dx)}}{3d \cos^{\frac{7}{2}}(c + dx)} + \frac{A \sin(c + dx) \sqrt{b \cos(c + dx)}}{d \cos^{\frac{3}{2}}(c + dx)} + \frac{B \operatorname{ArcTanh}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}} + \frac{B \sin(c + dx) \sqrt{b \cos(c + dx)}}{2d \cos^{\frac{5}{2}}(c + dx)}$$

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{b \cos(c + dx)} \int (A + B \cos(c + dx)) \sec^4(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{\left(A \sqrt{b \cos(c + dx)}\right) \int \sec^4(c + dx) dx}{\sqrt{\cos(c + dx)}} + \frac{\left(B \sqrt{b \cos(c + dx)}\right) \int \sec^3(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{B \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)} + \frac{\left(B \sqrt{b \cos(c + dx)}\right) \int \sec(c + dx) dx}{2 \sqrt{\cos(c + dx)}} \\
 &\quad - \frac{\left(A \sqrt{b \cos(c + dx)}\right) \text{Subst}\left(\int (1 + x^2) dx, x, -\tan(c + dx)\right)}{d \sqrt{\cos(c + dx)}} \\
 &= \frac{\text{Barctanh}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}} + \frac{B \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)} \\
 &\quad + \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)} + \frac{A \sqrt{b \cos(c + dx)} \sin^3(c + dx)}{3d \cos^{\frac{7}{2}}(c + dx)}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\begin{aligned}
 &\int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx \\
 &= \frac{\sqrt{b \cos(c + dx)} (3 \text{Barctanh}(\sin(c + dx)) \cos^2(c + dx) + 3B \sin(c + dx) + 2A(2 + \cos(2(c + dx)))) \tan(c + dx)}{6d \cos^{\frac{5}{2}}(c + dx)}
 \end{aligned}$$

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]

[Out] (Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)]))*Tan[c + d*x])/(6*d*Cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84

method	result
default	$\frac{(-3B(\cos^3(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+3B(\cos^3(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1)+4A \sin(dx+c)(\cos^2(dx+c))+3)}{6d \cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{A(2(\cos^2(dx+c)+1)\sqrt{\cos(dx+c)b} \sin(dx+c)}{3d \cos(dx+c)^{\frac{7}{2}}} + \frac{B(-(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1))}{2d \cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}(3B e^{5i(dx+c)}-12A e^{2i(dx+c)}-3B e^{i(dx+c)}-4A)}{3\sqrt{\cos(dx+c)} d (e^{2i(dx+c)}+1)^3} + \frac{\sqrt{\cos(dx+c)b} B \ln(e^{i(dx+c)}+i)}{2\sqrt{\cos(dx+c)} d} - \frac{\sqrt{\cos(dx+c)b} B \ln(e^{i(dx+c)}-i)}{2\sqrt{\cos(dx+c)} d}$

[In] int((cos(d*x+c)*b)^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x,method=_RETURN
VERBOSE)

[Out] 1/6/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.74

$$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

$$= \left[\frac{3 B \sqrt{b} \cos(dx+c)^4 \log\left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2 b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2 (4 A \cos(dx+c)^2 + 3 B \cos(dx+c) + 2 A) \sqrt{b}}{12 d \cos(dx+c)^4} \right. \\ \left. - \frac{3 B \sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx+c)^4 - (4 A \cos(dx+c)^2 + 3 B \cos(dx+c) + 2 A) \sqrt{b}}{6 d \cos(dx+c)^4} \right]$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="fricas")

[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(123) = 246.

Time = 0.72 (sec) , antiderivative size = 957, normalized size of antiderivative = 6.60

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="maxima")

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*
```

$\cos(2dx + 2c) + 1) \cdot \cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4\cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4\sin(4dx + 4c) \cdot \sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1) / d$

Giac [F]

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}}{\cos(dx + c)^{\frac{9}{2}}} dx$$

[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{\sqrt{b \cos(c + dx)}(A + B \cos(c + dx))}{\cos(c + dx)^{9/2}} dx$$

[In] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2),x)

[Out] int(((b*cos(c + d*x))^(1/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)

$$3.850 \quad \int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) dx$$

Optimal result	7311
Rubi [A] (verified)	7311
Mathematica [A] (verified)	7314
Maple [A] (verified)	7314
Fricas [A] (verification not implemented)	7314
Sympy [F(-1)]	7315
Maxima [A] (verification not implemented)	7315
Giac [A] (verification not implemented)	7316
Mupad [B] (verification not implemented)	7316

Optimal result

Integrand size = 33, antiderivative size = 177

$$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) dx = \frac{3bBx\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3bB\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)} \sin(c+dx)}{8d} + \frac{bB \cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)} \sin(c+dx)}{4d} - \frac{Ab\sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

[Out] 1/4*b*B*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*A*b*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+3/8*b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used

= {17, 2827, 2713, 2715, 8}

$$\int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx =$$

$$-\frac{Ab \sin^3(c + dx) \sqrt{b \cos(c + dx)}}{3d \sqrt{\cos(c + dx)}} + \frac{Ab \sin(c + dx) \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}}$$

$$+ \frac{3bBx \sqrt{b \cos(c + dx)}}{8 \sqrt{\cos(c + dx)}} + \frac{bB \sin(c + dx) \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}}{4d}$$

$$+ \frac{3bB \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}{8d}$$

[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]

[Out] (3*b*B*x*Sqrt[b*Cos[c + d*x]]/(8*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*b*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b \cdot \sin[e + f \cdot x]^{(m + 1)}, x, x] / ; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right) \int \cos^3(c+dx)(A+B\cos(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\
 &= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right) \int \cos^3(c+dx) dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right) \int \cos^4(c+dx) dx}{\sqrt{\cos(c+dx)}} \\
 &= \frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} \\
 &\quad + \frac{\left(3bB\sqrt{b\cos(c+dx)}\right) \int \cos^2(c+dx) dx}{4\sqrt{\cos(c+dx)}} \\
 &\quad - \frac{\left(Ab\sqrt{b\cos(c+dx)}\right) \text{Subst}\left(\int (1-x^2) dx, x, -\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}} \\
 &= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} \\
 &\quad + \frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} \\
 &\quad - \frac{Ab\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{\left(3bB\sqrt{b\cos(c+dx)}\right) \int 1 dx}{8\sqrt{\cos(c+dx)}} \\
 &= \frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\
 &\quad + \frac{3bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} \\
 &\quad + \frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} - \frac{Ab\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.46

$$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}(A + B \cos(c+dx)) dx = \frac{(b \cos(c+dx))^{3/2}(36Bc + 36Bdx + 72A \sin(c+dx) + 24B \sin(2(c+dx)) + 8A \sin(3(c+dx)))}{96d \cos^{\frac{3}{2}}(c+dx)}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]

[Out] ((b*Cos[c + d*x])^(3/2)*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Cos[c + d*x]^(3/2))

Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.52

method	result
default	$\frac{b\sqrt{\cos(dx+c)}b(6B \sin(dx+c)(\cos^3(dx+c))+8A \sin(dx+c)(\cos^2(dx+c))+9B \sin(dx+c) \cos(dx+c)+16A \sin(dx+c)+9B(dx+c))}{24d\sqrt{\cos(dx+c)}}$
parts	$\frac{Ab(2+\cos^2(dx+c)) \sin(dx+c)\sqrt{\cos(dx+c)}b}{3d\sqrt{\cos(dx+c)}} + \frac{Bb\sqrt{\cos(dx+c)}b(2 \sin(dx+c)(\cos^3(dx+c))+3 \cos(dx+c) \sin(dx+c)+3dx+3c)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{3b\sqrt{\cos(dx+c)}b(\sqrt{\cos(dx+c)})e^{i(dx+c)}xB}{4(e^{2i(dx+c)}+1)} - \frac{ib\sqrt{\cos(dx+c)}b(\sqrt{\cos(dx+c)})e^{5i(dx+c)}B}{32(e^{2i(dx+c)}+1)d} - \frac{ib\sqrt{\cos(dx+c)}b(\sqrt{\cos(dx+c)})e^{4i(dx+c)}}{12(e^{2i(dx+c)}+1)d}$

[In] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURN VERBOSE)

[Out] 1/24*b/d*(cos(d*x+c)*b)^(1/2)*(6*B*sin(d*x+c)*cos(d*x+c)^3+8*A*sin(d*x+c)*cos(d*x+c)^2+9*B*sin(d*x+c)*cos(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.47

$$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}(A + B \cos(c+dx)) dx = \left[\frac{9B\sqrt{-bb} \cos(dx+c) \log\left(2b \cos(dx+c)^2 - 2\sqrt{b \cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)} \sin(dx+c)\right)}{\dots} \right]$$

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] [1/48*(9*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*b*cos(d*x + c)^3 + 8*A*b*cos(d*x + c)^2 + 9*B*b*cos(d*x + c) + 16*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*B*b*cos(d*x + c)^3 + 8*A*b*cos(d*x + c)^2 + 9*B*b*cos(d*x + c) + 16*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{3/2}(A + B \cos(c + dx)) dx = \frac{8(b \sin(3 dx + 3 c) + 9 b \sin(\frac{1}{3} \arctan(\sin(3 dx + 3 c), \cos(3 dx + 3 c)))) A \sqrt{b} + 3(12 \dots}{9}$$

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/96*(8*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*A*sqrt(b) + 3*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b))/d

Giac [A] (verification not implemented)

none

Time = 3.46 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.58

$$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}(A + B \cos(c+dx)) dx = \frac{\left(9 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 + 36 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 48 A \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 30 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 54 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 80 A \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 18 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 36 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 80 A \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 18 B \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 9 B \sqrt{b} dx + 48 A \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 30 B \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) b / (d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 + 4 d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 6 d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 4 d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + d)}{96 d (\cos(2c + 2dx) + 1)}$$

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] 1/24*(9*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^8 + 36*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 48*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 - 30*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 54*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 80*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 18*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 36*B*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 80*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 - 18*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 9*B*sqrt(b)*d*x + 48*A*sqrt(b)*tan(1/2*d*x + 1/2*c) + 30*B*sqrt(b)*tan(1/2*d*x + 1/2*c))*b/(d*tan(1/2*d*x + 1/2*c)^8 + 4*d*tan(1/2*d*x + 1/2*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(1/2*d*x + 1/2*c)^2 + d)

Mupad [B] (verification not implemented)

Time = 16.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}(A + B \cos(c+dx)) dx = \frac{b \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (24 B \sin(c+dx) + 80 A \sin(2c+2dx) + 8 A \sin(4c+4dx) + 8 A \sin(2c+2dx) + 8 A \sin(4c+4dx) + 27 B \sin(3c+3dx) + 3 B \sin(5c+5dx) + 72 B d x \cos(c+dx))}{96 d (\cos(2c+2dx) + 1)}$$

[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)

[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*B*sin(c + d*x) + 80*A*sin(2*c + 2*d*x) + 8*A*sin(4*c + 4*d*x) + 27*B*sin(3*c + 3*d*x) + 3*B*sin(5*c + 5*d*x) + 72*B*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))

3.851 $\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) dx$

Optimal result	.7317
Rubi [A] (verified)	.7317
Mathematica [A] (verified)	.7319
Maple [A] (verified)	.7319
Fricas [A] (verification not implemented)	.7320
Sympy [F(-1)]	.7320
Maxima [A] (verification not implemented)	.7320
Giac [A] (verification not implemented)	.7321
Mupad [B] (verification not implemented)	.7321

Optimal result

Integrand size = 33, antiderivative size = 140

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) dx = \frac{Abx \sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bB \sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{Ab \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d} - \frac{bB \sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

[Out] $1/2*A*b*x*(b*\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)}+b*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}-1/3*b*B*\sin(d*x+c)^3*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}+1/2*A*b*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}*(b*\cos(d*x+c))^{(1/2)}/d$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 2715, 8, 2713}

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}(A+B \cos(c+dx)) dx = \frac{Abx \sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{Ab \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{2d} - \frac{bB \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{bB \sin(c+dx) \sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]

[Out] (A*b*x*Sqrt[b*Cos[c + d*x]]/(2*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) - (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right) \int \cos^2(c+dx)(A+B\cos(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right) \int \cos^2(c+dx) dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right) \int \cos^3(c+dx) dx}{\sqrt{\cos(c+dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{Ab\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int 1 dx}{2\sqrt{\cos(c+dx)}} \\
&\quad - \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int(1-x^2) dx, x, -\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}} \\
&= \frac{Abx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{Ab\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} - \frac{bB\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A + B\cos(c+dx)) dx = \frac{(b\cos(c+dx))^{3/2}(6Ac + 6Adx + 9B\sin(c+dx) + 3A\sin(2(c+dx)) + B\sin(3(c+dx)))}{12d\cos^{\frac{3}{2}}(c+dx)}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]

[Out] ((b*Cos[c + d*x])^(3/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Cos[c + d*x]^(3/2))

Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

method	result
default	$\frac{b\sqrt{\cos(dx+c)}b(2B\sin(dx+c)(\cos^2(dx+c))+3A\sin(dx+c)\cos(dx+c)+3A(dx+c)+4B\sin(dx+c))}{6d\sqrt{\cos(dx+c)}}$
parts	$\frac{Ab\sqrt{\cos(dx+c)}b(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}} + \frac{Bb(2+\cos^2(dx+c))\sin(dx+c)\sqrt{\cos(dx+c)}b}{3d\sqrt{\cos(dx+c)}}$
risch	$\frac{b\sqrt{\cos(dx+c)}b(\sqrt{\cos(dx+c)})e^{i(dx+c)}xA}{e^{2i(dx+c)}+1} - \frac{ib\sqrt{\cos(dx+c)}b(\sqrt{\cos(dx+c)})e^{4i(dx+c)}B}{12(e^{2i(dx+c)}+1)d} - \frac{ib\sqrt{\cos(dx+c)}b(\sqrt{\cos(dx+c)})e^{3i(dx+c)}}{4(e^{2i(dx+c)}+1)d}$

[In] int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/6*b/d*(cos(d*x+c)*b)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*cos(d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.69

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}(A + B \cos(c+dx)) dx = \left[\frac{3 A \sqrt{-bb} \cos(dx+c) \log\left(2b \cos(dx+c)^2 - 2\sqrt{b \cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)} \sin(dx+c) - b\right) + 2(2Bb \cos(dx+c)^2 + 3Ab \cos(dx+c) + 4B^2b) \sqrt{b \cos(dx+c)} \sqrt{\cos(dx+c)} \sin(dx+c)}{d \cos(dx+c)}, \frac{1}{6} (3A^2 b^{3/2} \arctan(\sqrt{b \cos(dx+c)}) \sin(dx+c) / (\sqrt{b} \cos(dx+c)^{3/2})) \cos(dx+c) + (2B^2 b \cos(dx+c)^2 + 3A^2 b \cos(dx+c) + 4B^2 b) \sqrt{b \cos(dx+c)} \sqrt{\cos(dx+c)} \sin(dx+c)}{d \cos(dx+c)} \right]$$

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*b*cos(d*x + c)^2 + 3*A*b*cos(d*x + c) + 4*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*A*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*b*cos(d*x + c)^2 + 3*A*b*cos(d*x + c) + 4*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}(A + B \cos(c+dx)) dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.53

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}(A + B \cos(c+dx)) dx = \frac{3(2(dx+c)b + b \sin(2dx+2c))A\sqrt{b} + (b \sin(3dx+3c) + 9b \sin(\frac{1}{3} \arctan(\sin(3dx+3c))))}{12d}$$

[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] 1/12*(3*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + (b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqrt(b))/d

Giac [A] (verification not implemented)

none

Time = 2.59 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}(A + B \cos(c+dx)) dx = \frac{\left(3 A \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 9 A \sqrt{b} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 6 A \sqrt{b} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + \dots\right)}{6 \left(dt\right)}$$

[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] 1/6*(3*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^6 + 9*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 - 6*A*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 12*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 9*A*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^2 + 8*B*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 3*A*sqrt(b)*d*x + 6*A*sqrt(b)*tan(1/2*d*x + 1/2*c) + 12*B*sqrt(b)*tan(1/2*d*x + 1/2*c))*b/(d*tan(1/2*d*x + 1/2*c)^6 + 3*d*tan(1/2*d*x + 1/2*c)^4 + 3*d*tan(1/2*d*x + 1/2*c)^2 + d)

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}(A + B \cos(c+dx)) dx = \frac{b \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (3 A \sin(c+dx) + 3 A \sin(3c+3dx) + 10 B \sin(2c+2dx))}{12 d (\cos(2c+2dx) + 1)}$$

[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)),x)

[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c + d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))

$$3.852 \quad \int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	7322
Rubi [A] (verified)	7322
Mathematica [A] (verified)	7323
Maple [A] (verified)	7324
Fricas [A] (verification not implemented)	7324
Sympy [A] (verification not implemented)	7325
Maxima [A] (verification not implemented)	7325
Giac [F]	7325
Mupad [B] (verification not implemented)	7326

Optimal result

Integrand size = 33, antiderivative size = 101

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx = \frac{bBx \sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{Ab \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \frac{bB \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d}$$

[Out] 1/2*b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*b*B*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2813}

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx = \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{bBx \sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bB \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{2d}$$

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]

[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] := Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right) \int \cos(c+dx)(A+B\cos(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\ &\quad + \frac{bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}} dx = \frac{b\sqrt{b\cos(c+dx)}(4A\sin(c+dx) + B(2(c+dx) + \sin(2(c+dx))))}{4d\sqrt{\cos(c+dx)}}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)
]])))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{b\sqrt{\cos(dx+c)}b(B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c))}{2d\sqrt{\cos(dx+c)}}$	56
parts	$\frac{Ab\sin(dx+c)\sqrt{\cos(dx+c)}b}{d\sqrt{\cos(dx+c)}} + \frac{Bb\sqrt{\cos(dx+c)}b(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}$	75
risch	$\frac{bBx\sqrt{\cos(dx+c)}b}{2\sqrt{\cos(dx+c)}} + \frac{Ab\sin(dx+c)\sqrt{\cos(dx+c)}b}{d\sqrt{\cos(dx+c)}} + \frac{b\sqrt{\cos(dx+c)}bB\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}d}$	89

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/2*b/d*(cos(d*x+c)*b)^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x
+c))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.07

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \left[\frac{B\sqrt{-bb} \cos(dx + c) \log\left(2b \cos(dx + c)^2 - 2\sqrt{b \cos(dx + c)}\right)}{\dots} \right]$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*b*cos(d*x + c) +
2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c
)), 1/2*(B*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2)))*cos(d*x + c) + (B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```


Sympy [A] (verification not implemented)

Time = 31.58 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \begin{cases} \frac{A(b \cos(c + dx))^{\frac{3}{2}} \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)} + \frac{Bx(b \cos(c + dx))^{\frac{3}{2}} \sin^2(c + dx)}{2 \cos^{\frac{3}{2}}(c + dx)} + \frac{Bx(b \cos(c + dx))^{\frac{3}{2}} \sin^2(c + dx)}{2 \cos^{\frac{3}{2}}(c + dx)} \\ \frac{x(b \cos(c))^{\frac{3}{2}} (A + B \cos(c))}{\sqrt{\cos(c)}} \end{cases}$$

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)

[Out] Piecewise((A*(b*cos(c + d*x))**(3/2)*sin(c + d*x)/(d*cos(c + d*x)**(3/2)) + B*x*(b*cos(c + d*x))**(3/2)*sin(c + d*x)**2/(2*cos(c + d*x)**(3/2)) + B*x*(b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x))/2 + B*(b*cos(c + d*x))**(3/2)*sin(c + d*x)/(2*d*sqrt(cos(c + d*x))), Ne(d, 0)), (x*(b*cos(c))**(3/2)*(A + B*cos(c))/sqrt(cos(c)), True))

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.43

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \frac{4Ab^{\frac{3}{2}} \sin(dx + c) + (2(dx + c)b + b \sin(2dx + 2c))B\sqrt{b}}{4d}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/4*(4*A*b^(3/2)*sin(d*x + c) + (2*(d*x + c)*b + b*sin(2*d*x + 2*c))*B*sqrt(b))/d

Giac [F]

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{3}{2}}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \frac{b \sqrt{b \cos(c + dx)} (4 A \sin(c + dx) + B \sin(2c + 2dx) + 2 B dx)}{4 d \sqrt{\cos(c + dx)}}$$

[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)

[Out] (b*(b*cos(c + d*x))^(1/2)*(4*A*sin(c + d*x) + B*sin(2*c + 2*d*x) + 2*B*d*x))/(4*d*cos(c + d*x)^(1/2))

$$3.853 \quad \int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	7327
Rubi [A] (verified)	7327
Mathematica [A] (verified)	7328
Maple [A] (verified)	7328
Fricas [A] (verification not implemented)	7329
Sympy [A] (verification not implemented)	7329
Maxima [A] (verification not implemented)	7330
Giac [F]	7330
Mupad [B] (verification not implemented)	7330

Optimal result

Integrand size = 33, antiderivative size = 61

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{Abx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}}$$

[Out] A*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2717}

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{Abx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]

[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right) \int (A + B\cos(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right) \int \cos(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{3/2}(c+dx)} dx = \frac{(b\cos(c+dx))^{3/2}(A(c+dx)+B\sin(c+dx))}{d\cos^{3/2}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(A*(c + d*x) + B*SIN[c + d*x]))/(d*Cos[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{b\sqrt{\cos(dx+c)}b(A(dx+c)+B\sin(dx+c))}{d\sqrt{\cos(dx+c)}}$	40
risch	$\frac{Abx\sqrt{\cos(dx+c)}b}{\sqrt{\cos(dx+c)}} + \frac{bB\sin(dx+c)\sqrt{\cos(dx+c)}b}{d\sqrt{\cos(dx+c)}}$	54
parts	$\frac{Ab\sqrt{\cos(dx+c)}b(dx+c)}{d\sqrt{\cos(dx+c)}} + \frac{bB\sin(dx+c)\sqrt{\cos(dx+c)}b}{d\sqrt{\cos(dx+c)}}$	61

[In] `int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $b/d*(\cos(d*x+c)*b)^{(1/2)}*(A*(d*x+c)+B*\sin(d*x+c))/\cos(d*x+c)^{(1/2)}$

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.02

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \left[\frac{A\sqrt{-bb} \cos(dx + c) \log\left(2b \cos(dx + c)^2 - 2\sqrt{b \cos(dx + c)}\right)}{\dots} \right]$$

[In] `integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm hm="fricas")`

[Out] `[1/2*(A*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))*B*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), (A*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + sqrt(b*cos(d*x + c))*B*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)))]`

Sympy [A] (verification not implemented)

Time = 31.98 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \begin{cases} \frac{Ax(b \cos(c+dx))^{3/2}}{\cos^{3/2}(c+dx)} + \frac{B(b \cos(c+dx))^{3/2} \sin(c+dx)}{d \cos^{3/2}(c+dx)} & \text{for } d \neq 0 \\ \frac{x(b \cos(c))^{3/2} (A+B \cos(c))}{\cos^{3/2}(c)} & \text{otherwise} \end{cases}$$

[In] `integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)`

[Out] `Piecewise((A*x*(b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3/2) + B*(b*cos(c + d*x))**(3/2)*sin(c + d*x)/(d*cos(c + d*x)**(3/2)), Ne(d, 0)), (x*(b*cos(c))**(3/2)*(A + B*cos(c))/cos(c)**(3/2), True))`

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \frac{2 A b^{3/2} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + B b^{3/2} \sin(dx + c)}{d}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] (2*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*b^(3/2)*sin(d*x + c))/d

Giac [F]

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{3/2}}{\cos(dx + c)^{3/2}} dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 14.77 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \frac{b \sqrt{b \cos(c + dx)} (B \sin(c + dx) + A dx)}{d \sqrt{\cos(c + dx)}}$$

[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2),x)

[Out] (b*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + A*d*x))/(d*cos(c + d*x)^(1/2))

$$3.854 \quad \int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{5/2}(c+dx)} dx$$

Optimal result	7331
Rubi [A] (verified)	7331
Mathematica [A] (verified)	7332
Maple [A] (verified)	7333
Fricas [A] (verification not implemented)	7333
Sympy [F(-1)]	7334
Maxima [A] (verification not implemented)	7334
Giac [F]	7334
Mupad [F(-1)]	7335

Optimal result

Integrand size = 33, antiderivative size = 62

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{5/2}(c+dx)} dx = \frac{bBx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A b \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

[Out] b*B*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+A*b*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {17, 2814, 3855}

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{5/2}(c+dx)} dx = \frac{A b \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{bBx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]

[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right) \int (A+B\cos(c+dx)) \sec(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{\left(Ab\sqrt{b\cos(c+dx)}\right) \int \sec(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{5/2}(c+dx)} dx = \frac{(Bdx + A\text{arctanh}(\sin(c+dx)))(b\cos(c+dx))^{3/2}}{d\cos^{3/2}(c+dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*(b*Cos[c + d*x])^(3/2))/(d*Cos[c + d*x]^(
3/2))
```


Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{b\sqrt{\cos(dx+c)}(2A \operatorname{arctanh}(\cot(dx+c)) - \csc(dx+c)) - B(dx+c)}{d\sqrt{\cos(dx+c)}}$	53
parts	$-\frac{2A\sqrt{\cos(dx+c)}b \operatorname{arctanh}(\cot(dx+c)) - \csc(dx+c)}{d\sqrt{\cos(dx+c)}} + \frac{Bb\sqrt{\cos(dx+c)}(dx+c)}{d\sqrt{\cos(dx+c)}}$	72
risch	$\frac{bBx\sqrt{\cos(dx+c)}}{\sqrt{\cos(dx+c)}} + \frac{b\sqrt{\cos(dx+c)}A \ln(e^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}d} - \frac{b\sqrt{\cos(dx+c)}A \ln(e^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}d}$	99

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -b/d*(cos(d*x+c)*b)^(1/2)*(2*A*arctanh(cot(d*x+c))-csc(d*x+c))-B*(d*x+c))/co
s(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.42

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \left[-\frac{2A\sqrt{-bb} \arctan\left(\frac{\sqrt{b \cos(dx+c)}\sqrt{-b} \sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) - B\sqrt{-bb} \log\left(\dots\right)}{\dots} \right]$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*
sqrt(cos(d*x + c)))) - B*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*b^(3/2)
*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + A
*b^(3/2)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d
*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \frac{4 B b^{3/2} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + (b \log(\cos(dx+c)^2 + \sin(dx+c)^2 + 1) - b \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1)) A \sqrt{b}}{d}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] 1/2*(4*B*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d

Giac [F]

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{3/2}}{\cos^{5/2}(dx + c)} dx$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos(c + dx)^{5/2}} dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)
```

$$3.855 \quad \int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{7/2}(c+dx)} dx$$

Optimal result	7336
Rubi [A] (verified)	7336
Mathematica [A] (verified)	7338
Maple [A] (verified)	7338
Fricas [A] (verification not implemented)	7338
Sympy [F(-1)]	7339
Maxima [A] (verification not implemented)	7339
Giac [F]	7339
Mupad [F(-1)]	7340

Optimal result

Integrand size = 33, antiderivative size = 70

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{7/2}(c+dx)} dx = \frac{b \operatorname{Barctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{Ab \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{3/2}(c+dx)}$$

[Out] A*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 3852, 8, 3855}

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{7/2}(c+dx)} dx = \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{3/2}(c+dx)} + \frac{b \operatorname{Barctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]

[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 17

`Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right) \int (A + B\cos(c+dx)) \sec^2(c+dx) dx}{\sqrt{\cos(c+dx)}} \\
 &= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right) \int \sec^2(c+dx) dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right) \int \sec(c+dx) dx}{\sqrt{\cos(c+dx)}} \\
 &= \frac{b\text{Barctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} \\
 &\quad - \frac{\left(Ab\sqrt{b\cos(c+dx)}\right) \text{Subst}\left(\int 1 dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}} \\
 &= \frac{b\text{Barctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \frac{(b \cos(c + dx))^{3/2} (B \operatorname{arctanh}(\sin(c + dx)) \cos(c + dx) + A \sin(c + dx))}{d \cos^{5/2}(c + dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2), x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{b(-2B \cos(dx+c) \operatorname{arctanh}(\cot(dx+c) - \csc(dx+c)) + A \sin(dx+c)) \sqrt{\cos(dx+c)b}}{d \cos(dx+c)^{3/2}}$	58
parts	$\frac{Ab \sin(dx+c) \sqrt{\cos(dx+c)b}}{d \cos(dx+c)^{3/2}} - \frac{2B \sqrt{\cos(dx+c)b} \operatorname{arctanh}(\cot(dx+c) - \csc(dx+c))}{d \sqrt{\cos(dx+c)}}$	73
risch	$\frac{2ib \sqrt{\cos(dx+c)b} A}{\sqrt{\cos(dx+c)} d (e^{2i(dx+c)} + 1)} + \frac{b \sqrt{\cos(dx+c)b} B \ln(e^{i(dx+c)} + i)}{\sqrt{\cos(dx+c)} d} - \frac{b \sqrt{\cos(dx+c)b} B \ln(e^{i(dx+c)} - i)}{\sqrt{\cos(dx+c)} d}$	116

```
[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2), x, method=_RETURN VERBOSE)
```

```
[Out] b/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.97

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \left[\frac{B b^{3/2} \cos(dx+c)^2 \log\left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c)}{\cos(dx+c)^3}\right)}{2 d \cos(dx+c)^2} - \frac{B \sqrt{-b} b \operatorname{arctan}\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx+c)^2 - \sqrt{b \cos(dx+c)} A b \sqrt{\cos(dx+c)} \sin(dx+c)}{d \cos(dx+c)^2} \right]$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2), x, algorithm="fricas")
```

```
[Out] [1/2*(B*b^(3/2)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), -(B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c))/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \frac{(b \log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \sin(dx + c) + 1) - b \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2 \sin(dx + c) + 1)) * B * \sqrt{b} + 4 * A * b^{3/2} * \sin(2 * dx + 2 * c) / (\cos(2 * dx + 2 * c)^2 + \sin(2 * dx + 2 * c)^2 + 2 * \cos(2 * dx + 2 * c) + 1)}{d}$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*((b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b) + 4*A*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{3/2}}{\cos(dx + c)^{7/2}} dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos(c + dx)^{7/2}} dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```


$$3.856 \quad \int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	7341
Rubi [A] (verified)	7341
Mathematica [A] (verified)	7343
Maple [A] (verified)	7343
Fricas [A] (verification not implemented)	7344
Sympy [F(-1)]	7344
Maxima [B] (verification not implemented)	7344
Giac [F]	7345
Mupad [F(-1)]	7345

Optimal result

Integrand size = 33, antiderivative size = 110

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{A b \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{A b \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{b B \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] $1/2*A*b*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(5/2)}+b*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(3/2)}+1/2*A*b*\operatorname{arctanh}(\sin(d*x+c))*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {17, 2827, 3853, 3855, 3852, 8}

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{A b \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{A b \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{b B \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)}$$

[In] $\operatorname{Int}[(b*\operatorname{Cos}[c+d*x])^{(3/2)}*(A+B*\operatorname{Cos}[c+d*x])/ \operatorname{Cos}[c+d*x]^{(9/2)},x]$

[Out] $(A*b*\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]]*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]])/(2*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]) + (A*b*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(2*d*\operatorname{Cos}[c+d*x]^{(5/2)}) + (b*B*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]]*\operatorname{Sin}[c+d*x])/(d*\operatorname{Cos}[c+d*x]^{(3/2)})$

Rule 8

$\text{Int}[a_ , x_ \text{Symbol}] \text{:> Simp}[a*x, x] \text{/; FreeQ}[a, x]$

Rule 17

$\text{Int}[(u_)*(a_)*(v_)]^{(m_)}*((b_)*(v_)]^{(n_)} , x_ \text{Symbol}] \text{:> Dist}[a^{(m + 1/2)}*b^{(n - 1/2)}*(\text{Sqrt}[b*v]/\text{Sqrt}[a*v]), \text{Int}[u*v^{(m + n)}, x], x] \text{/; FreeQ}[\{a, b, m\}, x] \&\& \text{!IntegerQ}[m] \&\& \text{IGtQ}[n + 1/2, 0] \&\& \text{IntegerQ}[m + n]$

Rule 2827

$\text{Int}[(b_)*\sin[(e_)+(f_)*(x_)]^{(m_)}*((c_)+(d_)*\sin[(e_)+(f_)*(x_)]), x_ \text{Symbol}] \text{:> Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] \text{/; FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 3852

$\text{Int}[\text{csc}[(c_)+(d_)*(x_)]^{(n_)} , x_ \text{Symbol}] \text{:> Dist}[-d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] \text{/; FreeQ}[\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rule 3853

$\text{Int}[(\text{csc}[(c_)+(d_)*(x_)]*(b_)]^{(n_)} , x_ \text{Symbol}] \text{:> Simp}[(-b)*\text{Cos}[c + d*x]*(b*\text{Csc}[c + d*x])^{(n - 1)}/(d*(n - 1)), x] + \text{Dist}[b^2*((n - 2)/(n - 1)), \text{Int}[(b*\text{Csc}[c + d*x])^{(n - 2)}, x], x] \text{/; FreeQ}[\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \& \& \text{IntegerQ}[2*n]$

Rule 3855

$\text{Int}[\text{csc}[(c_)+(d_)*(x_)] , x_ \text{Symbol}] \text{:> Simp}[-\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] \text{/; FreeQ}[\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right) \int (A + B\cos(c+dx)) \sec^3(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right) \int \sec^3(c+dx) dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right) \int \sec^2(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Ab\sqrt{b\cos(c+dx)} \sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(Ab\sqrt{b\cos(c+dx)}\right) \int \sec(c+dx) dx}{2\sqrt{\cos(c+dx)}} \\ &\quad - \frac{\left(bB\sqrt{b\cos(c+dx)}\right) \text{Subst}(\int 1 dx, x, -\tan(c+dx))}{d\sqrt{\cos(c+dx)}} \end{aligned}$$

$$= \frac{A b \operatorname{arctanh}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}} + \frac{A b \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)} + \frac{b B \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.59

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \frac{(b \cos(c + dx))^{3/2} (A \operatorname{arctanh}(\sin(c + dx)) \cos^2(c + dx) + (A + 2B \cos(c + dx)) \sin(c + dx))}{2d \cos^{7/2}(c + dx)}$$

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2), x]

[Out] ((b*Cos[c + d*x])^(3/2)*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))

Maple [A] (verified)

Time = 4.99 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

method	result
default	$\frac{b(A(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1) - A(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1) + 2B \sin(dx+c) \cos(dx+c) + A \sin^2(dx+c))}{2d \cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A b (-\cos^2(dx+c) \ln(-\cot(dx+c)+\csc(dx+c)-1) + (\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1) + \sin(dx+c) \sqrt{\cos(dx+c)} b)}{2d \cos(dx+c)^{\frac{5}{2}}} + \frac{b \sin(dx+c)}{d}$
risch	$-\frac{i b \sqrt{\cos(dx+c)} b (A e^{3i(dx+c)} - 2B e^{2i(dx+c)} - A e^{i(dx+c)} - 2B)}{\sqrt{\cos(dx+c)} d (e^{2i(dx+c)} + 1)^2} + \frac{b \sqrt{\cos(dx+c)} b A \ln(e^{i(dx+c)} + i)}{2 \sqrt{\cos(dx+c)} d} - \frac{b \sqrt{\cos(dx+c)} b A \ln(e^{i(dx+c)} - i)}{2 \sqrt{\cos(dx+c)} d}$

[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2), x, method=_RETURN VERBOSE)

[Out] 1/2*b/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.11

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \frac{\left[\frac{Ab^{3/2} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c)}{\cos(dx+c)^3}\right) + A\sqrt{-bb} \arctan\left(\frac{\sqrt{b \cos(dx+c)}\sqrt{-b \sin(dx+c)}}{b\sqrt{\cos(dx+c)}}\right) \cos(dx + c)^3 - (2Bb \cos(dx + c) + Ab)\sqrt{b \cos(dx + c)}\sqrt{\cos(dx + c)}}{2d \cos(dx + c)^3} \right]}{2d \cos(dx + c)^3}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="fricas")

[Out] [1/4*(A*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3)]

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. 2(94) = 188.

Time = 0.55 (sec) , antiderivative size = 747, normalized size of antiderivative = 6.79

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="maxima")

```
[Out] 1/4*(8*B*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) - (4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{3/2}}{\cos(dx + c)^{9/2}} dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos(c + dx)^{9/2}} dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

$$3.857 \quad \int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$$

Optimal result	7346
Rubi [A] (verified)	7346
Mathematica [A] (verified)	7348
Maple [A] (verified)	7348
Fricas [A] (verification not implemented)	7349
Sympy [F(-1)]	7349
Maxima [B] (verification not implemented)	7349
Giac [F]	7350
Mupad [F(-1)]	7351

Optimal result

Integrand size = 33, antiderivative size = 149

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx = \frac{bB \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{bB \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{Ab \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{Ab \sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d \cos^{\frac{7}{2}}(c+dx)}$$

[Out] $1/2*b*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(5/2)}+A*b*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(3/2)}+1/3*A*b*\sin(d*x+c)^3*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(7/2)}+1/2*b*B*\operatorname{arctanh}(\sin(d*x+c))*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 3852, 3853, 3855}

$$\int \frac{(b \cos(c+dx))^{3/2}(A+B \cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx = \frac{Ab \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)} + \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{bB \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{bB \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)}$$

[In] Int[((b*cos[c + d*x])^(3/2)*(A + B*cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
 [Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (b*B*Sqrt[b*cos[c + d*x]]*Sin[c + d*x])/(2*d*cos[c + d*x]^(5/2)) + (A*b*Sq
 rt[b*cos[c + d*x]]*Sin[c + d*x])/(d*cos[c + d*x]^(3/2)) + (A*b*Sqrt[b*cos[c
 + d*x]]*Sin[c + d*x]^3)/(3*d*cos[c + d*x]^(7/2))

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
 , m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
 _)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(
 b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expa
 ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*
 x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
 & IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b\sqrt{b\cos(c+dx)}\right) \int (A+B\cos(c+dx)) \sec^4(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right) \int \sec^4(c+dx) dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right) \int \sec^3(c+dx) dx}{\sqrt{\cos(c+dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{(bB\sqrt{b\cos(c+dx)})\int\sec(c+dx)dx}{2\sqrt{\cos(c+dx)}} \\
&\quad - \frac{(Ab\sqrt{b\cos(c+dx)})\text{Subst}(\int(1+x^2)dx,x,-\tan(c+dx))}{d\sqrt{\cos(c+dx)}} \\
&= \frac{bB\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{Ab\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{11/2}(c+dx)} dx = \frac{b\sqrt{b\cos(c+dx)}(3B\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+3B\sin(c+dx))}{6d\cos^{5/2}(c+dx)}$$

[In] Integrate[((b*cos[c + d*x])^(3/2)*(A + B*cos[c + d*x]))/Cos[c + d*x]^(11/2),x]

[Out] (b*Sqrt[b*cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83

method	result
default	$\frac{b(-3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3B(\cos^3(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+4A\sin(dx+c)(\cos^2(dx+c)))+3B\sin(dx+c)}{6d\cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{Ab(2(\cos^2(dx+c))+1)\sqrt{\cos(dx+c)}b\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}} + \frac{Bb(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1))}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib\sqrt{\cos(dx+c)}b(3Be^{5i(dx+c)}-12Ae^{2i(dx+c)}-3Be^{i(dx+c)}-4A)}{3\sqrt{\cos(dx+c)}d(e^{2i(dx+c)}+1)^3} + \frac{b\sqrt{\cos(dx+c)}bB\ln(e^{i(dx+c)}+i)}{2\sqrt{\cos(dx+c)}d} - \frac{b\sqrt{\cos(dx+c)}bB\ln(e^{i(dx+c)}-i)}{2\sqrt{\cos(dx+c)}d}$

[In] int((cos(d*x+c)*b)^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)

[Out] 1/6*b/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.74

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx = \frac{\left[3 B b^{3/2} \cos(dx + c)^4 \log\left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b} \sqrt{\cos(dx+c)}}{\cos(dx+c)^3}\right) + 3 B \sqrt{-bb} \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx + c)^4 - (4 A b \cos(dx + c)^2 + 3 B b \cos(dx + c) + 2 A b) \sqrt{b \cos(dx + c)} \right]}{6 d \cos(dx + c)^4}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorithm="fricas")

[Out] [1/12*(3*B*b^(3/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(4*A*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (4*A*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(127) = 254.

Time = 0.54 (sec) , antiderivative size = 992, normalized size of antiderivative = 6.66

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorithm="maxima")

```
[Out] -1/12*(16*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3*(3*b*cos(2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*(4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{3/2}}{\cos^{11/2}(dx + c)} dx$$

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx = \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx))}{\cos(c + dx)^{11/2}} dx$$

```
[In] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2), x)
```

```
[Out] int(((b*cos(c + d*x))^(3/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2), x)
```

3.858 $\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) dx$

Optimal result	7352
Rubi [A] (verified)	7352
Mathematica [A] (verified)	7355
Maple [A] (verified)	7355
Fricas [A] (verification not implemented)	7355
Sympy [F(-1)]	7356
Maxima [A] (verification not implemented)	7356
Giac [A] (verification not implemented)	7357
Mupad [B] (verification not implemented)	7357

Optimal result

Integrand size = 33, antiderivative size = 187

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) dx = \frac{3b^2 Bx \sqrt{b \cos(c+dx)}}{8 \sqrt{\cos(c+dx)}} + \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \frac{3b^2 B \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{8d} + \frac{b^2 B \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} \sin(c+dx)}{4d} - \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d \sqrt{\cos(c+dx)}}$$

[Out] $\frac{1}{4}b^2B \cos(dx+c)^{5/2} \sin(dx+c) (b \cos(dx+c))^{1/2} / d + \frac{3}{8}b^2Bx (b \cos(dx+c))^{1/2} / \cos(dx+c)^{1/2} + \frac{A b^2 \sin(dx+c) (b \cos(dx+c))^{1/2}}{d \cos(dx+c)^{1/2}} - \frac{1}{3}A b^2 \sin^3(dx+c) (b \cos(dx+c))^{1/2} / \cos(dx+c)^{1/2} + \frac{3}{8}b^2B \sin(dx+c) \cos(dx+c)^{1/2} (b \cos(dx+c))^{1/2} / d$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used

= {17, 2827, 2713, 2715, 8}

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) dx =$$

$$-\frac{Ab^2 \sin^3(c+dx)\sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{Ab^2 \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

$$+ \frac{3b^2 Bx\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2 B \sin(c+dx) \cos^{5/2}(c+dx)\sqrt{b \cos(c+dx)}}{4d}$$

$$+ \frac{3b^2 B \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{8d}$$

[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]

[Out] (3*b^2*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*b^2*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[

$b \cdot \sin[e + f \cdot x]^{(m + 1)}, x] / ; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\left(b^2 \sqrt{b \cos(c + dx)}\right) \int \cos^3(c + dx)(A + B \cos(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{\left(Ab^2 \sqrt{b \cos(c + dx)}\right) \int \cos^3(c + dx) dx}{\sqrt{\cos(c + dx)}} + \frac{\left(b^2 B \sqrt{b \cos(c + dx)}\right) \int \cos^4(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{b^2 B \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} \\
 &\quad + \frac{\left(3b^2 B \sqrt{b \cos(c + dx)}\right) \int \cos^2(c + dx) dx}{4\sqrt{\cos(c + dx)}} \\
 &\quad - \frac{\left(Ab^2 \sqrt{b \cos(c + dx)}\right) \text{Subst}\left(\int (1 - x^2) dx, x, -\sin(c + dx)\right)}{d\sqrt{\cos(c + dx)}} \\
 &= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + \frac{3b^2 B \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{8d} \\
 &\quad + \frac{b^2 B \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} \\
 &\quad - \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin^3(c + dx)}{3d\sqrt{\cos(c + dx)}} + \frac{\left(3b^2 B \sqrt{b \cos(c + dx)}\right) \int 1 dx}{8\sqrt{\cos(c + dx)}} \\
 &= \frac{3b^2 B x \sqrt{b \cos(c + dx)}}{8\sqrt{\cos(c + dx)}} + \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{d\sqrt{\cos(c + dx)}} \\
 &\quad + \frac{3b^2 B \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{8d} \\
 &\quad + \frac{b^2 B \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} - \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin^3(c + dx)}{3d\sqrt{\cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.43

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}(A + B \cos(c+dx)) dx = \frac{(b \cos(c+dx))^{5/2}(36Bc + 36Bdx + 72A \sin(c+dx) + 24B \sin(2(c+dx)) + 8A \sin(3(c+dx)))}{96d \cos^{5/2}(c+dx)}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]

[Out] ((b*Cos[c + d*x])^(5/2)*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[2*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.50

method	result
default	$\frac{b^2 \sqrt{\cos(dx+c)} b (6B \sin(dx+c) (\cos^3(dx+c)) + 8A \sin(dx+c) (\cos^2(dx+c)) + 9B \sin(dx+c) \cos(dx+c) + 16A \sin(dx+c) + 9B(dx+c))}{24d \sqrt{\cos(dx+c)}}$
parts	$\frac{A b^2 (2 + \cos^2(dx+c)) \sqrt{\cos(dx+c)} b \sin(dx+c)}{3d \sqrt{\cos(dx+c)}} + \frac{B b^2 \sqrt{\cos(dx+c)} b (2 \sin(dx+c) (\cos^3(dx+c)) + 3 \cos(dx+c) \sin(dx+c) + 3dx + 3c)}{8d \sqrt{\cos(dx+c)}}$
risch	$\frac{3b^2 \sqrt{\cos(dx+c)} b (\sqrt{\cos(dx+c)}) e^{i(dx+c)} x B}{4(e^{2i(dx+c)} + 1)} - \frac{ib^2 \sqrt{\cos(dx+c)} b (\sqrt{\cos(dx+c)}) e^{5i(dx+c)} B}{32(e^{2i(dx+c)} + 1)d} - \frac{ib^2 \sqrt{\cos(dx+c)} b (\sqrt{\cos(dx+c)}) e^{4i(dx+c)}}{12(e^{2i(dx+c)} + 1)d}$

[In] int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/24*b^2/d*(cos(d*x+c)*b)^(1/2)*(6*B*sin(d*x+c)*cos(d*x+c)^3+8*A*sin(d*x+c)*cos(d*x+c)^2+9*B*sin(d*x+c)*cos(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.49

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}(A + B \cos(c+dx)) dx = \left[\frac{9B\sqrt{-bb^2} \cos(dx+c) \log\left(2b \cos(dx+c)^2 - 2\sqrt{b \cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\right)}{\dots} \right]$$

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*b^2*cos(d*x + c)^3 + 8*A*b^2*cos(d*x + c)^2 + 9*B*b^2*cos(d*x + c) + 16*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*B*b^2*cos(d*x + c)^3 + 8*A*b^2*cos(d*x + c)^2 + 9*B*b^2*cos(d*x + c) + 16*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}(A+B \cos(c+dx)) dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.59

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}(A + B \cos(c+dx)) dx = \frac{8(b^2 \sin(3dx+3c) + 9b^2 \sin(\frac{1}{3} \arctan(\sin(3dx+3c), \cos(3dx+3c))))A\sqrt{b} + 3(12(d*x+c)*b^2 + b^2*\sin(4*d*x + 4*c) + 8*b^2*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))))*B*\sqrt{b}}{d}$$

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(8*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*A*sqrt(b) + 3*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b))/d
```


Giac [A] (verification not implemented)

none

Time = 3.70 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.49

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}(A + B \cos(c+dx)) dx = \frac{9 B b^{5/2} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 + 36 B b^{5/2} dx \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 48 A b^{5/2} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 30 A b^{5/2} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 54 A b^{5/2} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 9 B b^{5/2} dx + 48 A b^{5/2} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 30 B b^{5/2} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 + 4 d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 6 d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 4 d \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + d}$$

[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] 1/24*(9*B*b^(5/2)*d*x*tan(1/2*d*x + 1/2*c)^8 + 36*B*b^(5/2)*d*x*tan(1/2*d*x + 1/2*c)^6 + 48*A*b^(5/2)*tan(1/2*d*x + 1/2*c)^7 - 30*B*b^(5/2)*tan(1/2*d*x + 1/2*c)^5 + 54*A*b^(5/2)*d*x*tan(1/2*d*x + 1/2*c)^4 + 80*A*b^(5/2)*tan(1/2*d*x + 1/2*c)^3 + 9*B*b^(5/2)*d*x + 48*A*b^(5/2)*tan(1/2*d*x + 1/2*c) + 30*B*b^(5/2)*tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^8 + 4*d*tan(1/2*d*x + 1/2*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(1/2*d*x + 1/2*c)^2 + d)

Mupad [B] (verification not implemented)

Time = 15.84 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.58

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}(A + B \cos(c+dx)) dx = \frac{b^2 \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (24 B \sin(c+dx) + 80 A \sin(2c+2dx) + 8 A \sin(3c+3dx) + 27 B \sin(4c+4dx) + 3 B \sin(5c+5dx) + 72 B dx \cos(c+dx))}{96 d (\cos(2c+2dx) + 1)}$$

[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)),x)

[Out] (b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(24*B*sin(c + dx) + 80*A*sin(2*c + 2*d*x) + 8*A*sin(4*c + 4*d*x) + 27*B*sin(3*c + 3*d*x) + 3*B*sin(5*c + 5*d*x) + 72*B*d*x*cos(c + d*x)))/(96*d*(cos(2*c + 2*d*x) + 1))

$$3.859 \quad \int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	7358
Rubi [A] (verified)	7358
Mathematica [A] (verified)	7360
Maple [A] (verified)	7360
Fricas [A] (verification not implemented)	7361
Sympy [F(-1)]	7361
Maxima [A] (verification not implemented)	7361
Giac [F]	7362
Mupad [B] (verification not implemented)	7362

Optimal result

Integrand size = 33, antiderivative size = 148

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx = \frac{Ab^2x\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{b \cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}\sin(c+dx)}{2d} - \frac{b^2B\sqrt{b \cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

[Out] 1/2*A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*B*sin(d*x+c)^3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/2*A*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 2715, 8, 2713}

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx = \frac{Ab^2x\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{Ab^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d} - \frac{b^2B\sin^3(c+dx)\sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{b^2B\sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[In] Int[((b*cos[c + d*x])^(5/2)*(A + B*cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]

[Out] (A*b^2*x*Sqrt[b*cos[c + d*x]]/(2*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[Cos[c + d*x]]*Sqrt[b*cos[c + d*x]]*Sin[c + d*x])/(2*d) - (b^2*B*Sqrt[b*cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b^2 \sqrt{b \cos(c + dx)}\right) \int \cos^2(c + dx)(A + B \cos(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{\left(A b^2 \sqrt{b \cos(c + dx)}\right) \int \cos^2(c + dx) dx}{\sqrt{\cos(c + dx)}} + \frac{\left(b^2 B \sqrt{b \cos(c + dx)}\right) \int \cos^3(c + dx) dx}{\sqrt{\cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{Ab^2 \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d} + \frac{\left(Ab^2 \sqrt{b \cos(c+dx)} \right) \int 1 dx}{2\sqrt{\cos(c+dx)}} \\
&\quad - \frac{\left(b^2 B \sqrt{b \cos(c+dx)} \right) \text{Subst}\left(\int (1-x^2) dx, x, -\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}} \\
&= \frac{Ab^2 x \sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} \\
&\quad + \frac{Ab^2 \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d} - \frac{b^2 B \sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

$$\int \frac{(b \cos(c+dx))^{5/2} (A + B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx = \frac{(b \cos(c+dx))^{5/2} (6Ac + 6Adx + 9B \sin(c+dx) + 3A \sin(2(c+dx)))}{12d \cos^{5/2}(c+dx)}$$

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + B*cos[c + d*x]))/Sqrt[Cos[c + d*x]], x]

[Out] ((b*cos[c + d*x])^(5/2)*(6*A*c + 6*A*d*x + 9*B*sin[c + d*x] + 3*A*sin[2*(c + d*x)] + B*sin[3*(c + d*x)]))/(12*d*cos[c + d*x]^(5/2))

Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{b^2 \sqrt{\cos(dx+c)} b (2B \sin(dx+c) (\cos^2(dx+c)) + 3A \sin(dx+c) \cos(dx+c) + 3A(dx+c) + 4B \sin(dx+c))}{6d \sqrt{\cos(dx+c)}}$	77
parts	$\frac{A b^2 \sqrt{\cos(dx+c)} b (\cos(dx+c) \sin(dx+c) + dx+c)}{2d \sqrt{\cos(dx+c)}} + \frac{B b^2 (2 + \cos^2(dx+c)) \sqrt{\cos(dx+c)} b \sin(dx+c)}{3d \sqrt{\cos(dx+c)}}$	90
risch	$\frac{A b^2 x \sqrt{\cos(dx+c)} b}{2\sqrt{\cos(dx+c)}} + \frac{3b^2 B \sin(dx+c) \sqrt{\cos(dx+c)} b}{4d \sqrt{\cos(dx+c)}} + \frac{b^2 \sqrt{\cos(dx+c)} b B \sin(3dx+3c)}{12\sqrt{\cos(dx+c)} d} + \frac{b^2 \sqrt{\cos(dx+c)} b A \sin(2dx+2c)}{4\sqrt{\cos(dx+c)} d}$	132

[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2), x, method=_RETURN VERBOSE)

[Out] 1/6*b^2/d*(cos(d*x+c)*b)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*cos(d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.70

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \left[\frac{3 A \sqrt{-bb^2} \cos(dx + c) \log\left(2b \cos(dx + c)^2 - 2\sqrt{b \cos(dx + c)}\right)}{\dots} \right]$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*A*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*b^2*cos(d*x + c)^2 + 3*A*b^2*cos(d*x + c) + 4*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)), 1/6*(3*A*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*b^2*cos(d*x + c)^2 + 3*A*b^2*cos(d*x + c) + 4*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c))]

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \frac{3(2(dx + c)b^2 + b^2 \sin(2dx + 2c))A\sqrt{b} + (b^2 \sin(3dx + 3c) + \dots)}{12}$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/12*(3*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + (b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqrt(b))/d

Giac [F]

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{5/2}}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.43

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \frac{b^2 \sqrt{b \cos(c + dx)} (9 B \sin(c + dx) + 3 A \sin(2c + 2dx) + \dots)}{12 d \sqrt{\cos(c + dx)}}$$

[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)

[Out] (b^2*(b*cos(c + d*x))^(1/2)*(9*B*sin(c + d*x) + 3*A*sin(2*c + 2*d*x) + B*sin(3*c + 3*d*x) + 6*A*d*x))/(12*d*cos(c + d*x)^(1/2))

$$3.860 \quad \int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	7363
Rubi [A] (verified)	7363
Mathematica [A] (verified)	7364
Maple [A] (verified)	7365
Fricas [A] (verification not implemented)	7365
Sympy [F(-1)]	7366
Maxima [A] (verification not implemented)	7366
Giac [F]	7366
Mupad [B] (verification not implemented)	7367

Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{b^2 B x \sqrt{b \cos(c+dx)}}{2 \sqrt{\cos(c+dx)}} + \frac{A b^2 \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \frac{b^2 B \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d}$$

[Out] $1/2*b^2*B*x*(b*\cos(d*x+c))^{(1/2)}/\cos(d*x+c)^{(1/2)}+A*b^2*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}+1/2*b^2*B*\sin(d*x+c)*\cos(d*x+c)^{(1/2)*(b*\cos(d*x+c))^{(1/2)}/d}$

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2813}

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{A b^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{b^2 B x \sqrt{b \cos(c+dx)}}{2 \sqrt{\cos(c+dx)}} + \frac{b^2 B \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{2d}$$

[In] $\text{Int}[\frac{(b*\text{Cos}[c+d*x])^{(5/2)}*(A+B*\text{Cos}[c+d*x])}{\text{Cos}[c+d*x]^{(3/2)}},x]$

[Out] $(b^2*B*x*\text{Sqrt}[b*\text{Cos}[c+d*x]])/(2*\text{Sqrt}[\text{Cos}[c+d*x]]) + (A*b^2*\text{Sqrt}[b*\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(d*\text{Sqrt}[\text{Cos}[c+d*x]]) + (b^2*B*\text{Sqrt}[\text{Cos}[c+d*x]]*\text{Sqrt}[b*\text{Cos}[c+d*x]]*\text{Sin}[c+d*x])/(2*d)$

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] := Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b^2 \sqrt{b \cos(c + dx)}\right) \int \cos(c + dx)(A + B \cos(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{b^2 B x \sqrt{b \cos(c + dx)}}{2 \sqrt{\cos(c + dx)}} + \frac{A b^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\ &\quad + \frac{b^2 B \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \frac{(b \cos(c + dx))^{5/2} (4A \sin(c + dx) + B(2(c + dx) + \sin(2(c + dx))))}{4d \cos^{5/2}(c + dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)
]))) / (4*d*Cos[c + d*x]^(5/2))
```


Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{b^2 \sqrt{\cos(dx+c)b} (B \sin(dx+c) \cos(dx+c) + 2A \sin(dx+c) + B(dx+c))}{2d \sqrt{\cos(dx+c)}}$	58
parts	$\frac{A b^2 \sin(dx+c) \sqrt{\cos(dx+c)b}}{d \sqrt{\cos(dx+c)}} + \frac{B b^2 \sqrt{\cos(dx+c)b} (\cos(dx+c) \sin(dx+c) + dx+c)}{2d \sqrt{\cos(dx+c)}}$	79
risch	$\frac{b^2 B x \sqrt{\cos(dx+c)b}}{2 \sqrt{\cos(dx+c)}} + \frac{A b^2 \sin(dx+c) \sqrt{\cos(dx+c)b}}{d \sqrt{\cos(dx+c)}} + \frac{b^2 \sqrt{\cos(dx+c)b} B \sin(2dx+2c)}{4 \sqrt{\cos(dx+c)} d}$	95

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/2*b^2/d*(cos(d*x+c)*b)^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d
*x+c))/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.05

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \left[\frac{B \sqrt{-bb^2} \cos(dx + c) \log(2b \cos(dx + c)^2 - 2 \sqrt{b \cos(dx + c)} \dots)}{\dots} \right]$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*b^2*cos(d*x + c
) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)), 1/2*(B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*
cos(d*x + c)^(3/2)))*cos(d*x + c) + (B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2), x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \frac{4 A b^{5/2} \sin(dx + c) + (2(dx + c)b^2 + b^2 \sin(2dx + 2c))B\sqrt{b}}{4d}$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] 1/4*(4*A*b^(5/2)*sin(d*x + c) + (2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*B*sqrt(b))/d

Giac [F]

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{5/2}}{\cos(dx + c)^{3/2}} dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 14.91 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.49

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{3/2}(c + dx)} dx = \frac{b^2 \sqrt{b \cos(c + dx)} (4 A \sin(c + dx) + B \sin(2c + 2dx) + 2 B dx)}{4 d \sqrt{\cos(c + dx)}}$$

[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2),x)

[Out] (b^2*(b*cos(c + d*x))^(1/2)*(4*A*sin(c + d*x) + B*sin(2*c + 2*d*x) + 2*B*d*x))/(4*d*cos(c + d*x)^(1/2))

$$3.861 \quad \int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{5/2}(c+dx)} dx$$

Optimal result	7368
Rubi [A] (verified)	7368
Mathematica [A] (verified)	7369
Maple [A] (verified)	7369
Fricas [A] (verification not implemented)	7370
Sympy [F(-1)]	7370
Maxima [A] (verification not implemented)	7371
Giac [F]	7371
Mupad [B] (verification not implemented)	7371

Optimal result

Integrand size = 33, antiderivative size = 65

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{5/2}(c+dx)} dx = \frac{Ab^2x\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

[Out] A*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2717}

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{5/2}(c+dx)} dx = \frac{Ab^2x\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2B \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]

[Out] (A*b^2*x*sqrt[b*cos[c + d*x]])/sqrt[Cos[c + d*x]] + (b^2*B*sqrt[b*cos[c + d*x]]*sin[c + d*x])/(d*sqrt[Cos[c + d*x]])

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b^2 \sqrt{b \cos(c + dx)}\right) \int (A + B \cos(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab^2 x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{\left(b^2 B \sqrt{b \cos(c + dx)}\right) \int \cos(c + dx) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab^2 x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{b^2 B \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \frac{(b \cos(c + dx))^{5/2} (A(c + dx) + B \sin(c + dx))}{d \cos^{5/2}(c + dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(A*(c + d*x) + B*SIN[c + d*x]))/(d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{b^2 \sqrt{\cos(dx+c)} b (A(dx+c) + B \sin(dx+c))}{d \sqrt{\cos(dx+c)}}$	42
risch	$\frac{A b^2 x \sqrt{\cos(dx+c)} b}{\sqrt{\cos(dx+c)}} + \frac{b^2 B \sin(dx+c) \sqrt{\cos(dx+c)} b}{d \sqrt{\cos(dx+c)}}$	58
parts	$\frac{A b^2 \sqrt{\cos(dx+c)} b (dx+c)}{d \sqrt{\cos(dx+c)}} + \frac{b^2 B \sin(dx+c) \sqrt{\cos(dx+c)} b}{d \sqrt{\cos(dx+c)}}$	65

[In] `int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $b^2/d*(\cos(d*x+c)*b)^{(1/2)}*(A*(d*x+c)+B*\sin(d*x+c))/\cos(d*x+c)^{(1/2)}$

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.92

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \left[\frac{A \sqrt{-b} b^2 \cos(dx + c) \log(2b \cos(dx + c)^2 - 2 \sqrt{b} \cos(dx + c) + b)}{\dots} \right]$$

[In] `integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] $[1/2*(A*\sqrt{-b})*b^2*\cos(d*x + c)*\log(2*b*\cos(d*x + c)^2 - 2*\sqrt{b}*\cos(d*x + c))*\sqrt{-b}*\sqrt{\cos(d*x + c)}*\sin(d*x + c) - b) + 2*\sqrt{b}*\cos(d*x + c))*B*b^2*\sqrt{\cos(d*x + c)}*\sin(d*x + c))/(d*\cos(d*x + c)), (A*b^{(5/2)}*\arctan(\sqrt{b}*\cos(d*x + c))*\sin(d*x + c)/(\sqrt{b}*\cos(d*x + c)^{(3/2)}))*\cos(d*x + c) + \sqrt{b}*\cos(d*x + c))*B*b^2*\sqrt{\cos(d*x + c)}*\sin(d*x + c))/(d*\cos(d*x + c))]$

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \text{Timed out}$$

[In] `integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)`

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \frac{2 A b^{5/2} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + B b^{5/2} \sin(dx + c)}{d}$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] (2*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*b^(5/2)*sin(d*x + c))/d

Giac [F]

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{5/2}}{\cos(dx + c)^{5/2}} dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{5/2}(c + dx)} dx = \frac{b^2 \sqrt{b \cos(c + dx)} (B \sin(c + dx) + A dx)}{d \sqrt{\cos(c + dx)}}$$

[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2),x)

[Out] (b^2*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + A*d*x))/(d*cos(c + d*x)^(1/2))

$$3.862 \quad \int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{7/2}(c+dx)} dx$$

Optimal result	7372
Rubi [A] (verified)	7372
Mathematica [A] (verified)	7373
Maple [A] (verified)	7374
Fricas [A] (verification not implemented)	7374
Sympy [F(-1)]	7375
Maxima [A] (verification not implemented)	7375
Giac [F]	7375
Mupad [F(-1)]	7376

Optimal result

Integrand size = 33, antiderivative size = 66

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{7/2}(c+dx)} dx = \frac{b^2 B x \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A b^2 \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

[Out] $b^2 B x (b \cos(d x+c))^{1/2} / \cos(d x+c)^{1/2} + A b^2 \operatorname{arctanh}(\sin(d x+c)) * (b \cos(d x+c))^{1/2} / d / \cos(d x+c)^{1/2}$

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {17, 2814, 3855}

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{7/2}(c+dx)} dx = \frac{A b^2 \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{b^2 B x \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

[In] $\text{Int}[(b \cos[c+dx])^{5/2} (A+B \cos[c+dx]) / \cos[c+dx]^{7/2}, x]$

[Out] $(b^2 B x \sqrt{b \cos[c+dx]}) / \sqrt{\cos[c+dx]} + (A b^2 \operatorname{ArcTanh}[\sin[c+dx]] * \sqrt{b \cos[c+dx]}) / (d \sqrt{\cos[c+dx]})$

Rule 17


```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b^2 \sqrt{b \cos(c + dx)}\right) \int (A + B \cos(c + dx)) \sec(c + dx) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{b^2 B x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{\left(A b^2 \sqrt{b \cos(c + dx)}\right) \int \sec(c + dx) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{b^2 B x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{A b^2 \operatorname{arctanh}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \frac{(B dx + A \operatorname{arctanh}(\sin(c + dx))) (b \cos(c + dx))^{5/2}}{d \cos^{5/2}(c + dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*(b*Cos[c + d*x])^(5/2))/(d*Cos[c + d*x]^(5/2))
```

Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b^2 \sqrt{\cos(dx+c)} b (2A \operatorname{arctanh}(\cot(dx+c) - \csc(dx+c)) - B(dx+c))}{d \sqrt{\cos(dx+c)}}$	55
parts	$-\frac{2A \operatorname{arctanh}(\cot(dx+c) - \csc(dx+c)) b^2 \sqrt{\cos(dx+c)} b}{d \sqrt{\cos(dx+c)}} + \frac{B b^2 \sqrt{\cos(dx+c)} b (dx+c)}{d \sqrt{\cos(dx+c)}}$	76
risch	$\frac{b^2 B x \sqrt{\cos(dx+c)} b}{\sqrt{\cos(dx+c)}} + \frac{b^2 \sqrt{\cos(dx+c)} b A \ln(e^{i(dx+c)} + i)}{\sqrt{\cos(dx+c)} d} - \frac{b^2 \sqrt{\cos(dx+c)} b A \ln(e^{i(dx+c)} - i)}{\sqrt{\cos(dx+c)} d}$	105

```
[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -b^2/d*(cos(d*x+c)*b)^(1/2)*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c))/
cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.27

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \left[-\frac{2 A \sqrt{-b} b^2 \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) - B \sqrt{-b} b^2 \log\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right)}{\cos^{7/2}(c + dx)} \right]$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(
b*sqrt(cos(d*x + c)))) - B*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*c
os(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*b^(
5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))
+ A*b^(5/2)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \frac{4 B b^{5/2} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + (b^2 \log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - b^2 \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1)) A \sqrt{b}}{d}$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 1/2*(4*B*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d

Giac [F]

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{5/2}}{\cos(dx + c)^{7/2}} dx$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(7/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{7/2}(c + dx)} dx = \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos(c + dx)^{7/2}} dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

$$3.863 \quad \int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	7377
Rubi [A] (verified)	7377
Mathematica [A] (verified)	7379
Maple [A] (verified)	7379
Fricas [A] (verification not implemented)	7379
Sympy [F(-1)]	7380
Maxima [A] (verification not implemented)	7380
Giac [F]	7380
Mupad [F(-1)]	7381

Optimal result

Integrand size = 33, antiderivative size = 74

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{b^2 \text{Barctanh}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+b^2*B*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 3852, 8, 3855}

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx = \frac{Ab^2 \sin(c+dx)\sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{b^2 \text{Barctanh}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]

[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\left(b^2 \sqrt{b \cos(c + dx)}\right) \int (A + B \cos(c + dx)) \sec^2(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{\left(Ab^2 \sqrt{b \cos(c + dx)}\right) \int \sec^2(c + dx) dx}{\sqrt{\cos(c + dx)}} + \frac{\left(b^2 B \sqrt{b \cos(c + dx)}\right) \int \sec(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{b^2 \text{Barctanh}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}} \\
 &\quad - \frac{\left(Ab^2 \sqrt{b \cos(c + dx)}\right) \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d \sqrt{\cos(c + dx)}} \\
 &= \frac{b^2 \text{Barctanh}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}} + \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \cos^{\frac{3}{2}}(c + dx)}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{(b \cos(c + dx))^{5/2}(A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \frac{(b \cos(c + dx))^{5/2}(B \operatorname{arctanh}(\sin(c + dx)) \cos(c + dx) + A \sin(c + dx))}{d \cos^{7/2}(c + dx)}$$

[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2), x]

[Out] ((b*Cos[c + d*x])^(5/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(7/2))

Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{b^2(-2B \cos(dx+c) \operatorname{arctanh}(\cot(dx+c) - \operatorname{csc}(dx+c)) + A \sin(dx+c)) \sqrt{\cos(dx+c)b}}{d \cos(dx+c)^{3/2}}$	60
parts	$\frac{A b^2 \sin(dx+c) \sqrt{\cos(dx+c)b}}{d \cos(dx+c)^{3/2}} - \frac{2B \operatorname{arctanh}(\cot(dx+c) - \operatorname{csc}(dx+c)) b^2 \sqrt{\cos(dx+c)b}}{d \sqrt{\cos(dx+c)}}$	77
risch	$\frac{2ib^2 \sqrt{\cos(dx+c)b} A}{\sqrt{\cos(dx+c)} d (e^{2i(dx+c)} + 1)} + \frac{b^2 \sqrt{\cos(dx+c)b} B \ln(e^{i(dx+c)} + i)}{\sqrt{\cos(dx+c)} d} - \frac{b^2 \sqrt{\cos(dx+c)b} B \ln(e^{i(dx+c)} - i)}{\sqrt{\cos(dx+c)} d}$	122

[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2), x, method=_RETURNVERBOSE)

[Out] b^2/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.89

$$\int \frac{(b \cos(c + dx))^{5/2}(A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \left[\frac{B b^{5/2} \cos(dx+c)^2 \log\left(\frac{-b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b} \sqrt{\cos(dx+c)}}{\cos(dx+c)^3}\right) + 2 d B \sqrt{-b} b^2 \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx+c)^2 - \sqrt{b \cos(dx+c)} A b^2 \sqrt{\cos(dx+c)} \sin(dx+c)}{d \cos(dx+c)^2} \right]$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2), x, algorithm="fricas")

```
[Out] [1/2*(B*b^(5/2)*cos(d*x + c)^2*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^2), -(B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \frac{4 A b^{5/2} \sin(2 dx + 2 c)}{\cos(2 dx + 2 c)^2 + \sin(2 dx + 2 c)^2 + 2 \cos(2 dx + 2 c) + 1} + \frac{(b^2 \log(\cos(dx + c))^2)}{\cos^{9/2}(c + dx)}$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*A*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + (b^2*log(cos(d*x + c))^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b))/d
```

Giac [F]

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{5/2}}{\cos(dx + c)^{9/2}} dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(9/2), x)
```


Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{9/2}(c + dx)} dx = \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos(c + dx)^{9/2}} dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

$$3.864 \quad \int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{11/2}(c+dx)} dx$$

Optimal result	7382
Rubi [A] (verified)	7382
Mathematica [A] (verified)	7384
Maple [A] (verified)	7384
Fricas [A] (verification not implemented)	7385
Sympy [F(-1)]	7385
Maxima [B] (verification not implemented)	7385
Giac [F]	7386
Mupad [F(-1)]	7386

Optimal result

Integrand size = 33, antiderivative size = 116

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{11/2}(c+dx)} dx = \frac{Ab^2 \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{5/2}(c+dx)} + \frac{b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{3/2}(c+dx)}$$

[Out] 1/2*A*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+b^2*B*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/2*A*b^2*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {17, 2827, 3853, 3855, 3852, 8}

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{11/2}(c+dx)} dx = \frac{Ab^2 \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{5/2}(c+dx)} + \frac{b^2 B \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{3/2}(c+dx)}$$

[In] Int[((b*cos[c + d*x])^(5/2)*(A + B*cos[c + d*x]))/Cos[c + d*x]^(11/2),x]

[Out] (A*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*cos[c + d*x]]*Sin[c + d*x])/(2*d*cos[c + d*x]^(5/2)) + (b^2*B*Sqrt[b*cos[c + d*x]]*Sin[c + d*x])/(d*cos[c + d*x]^(3/2))

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 17

`Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3853

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\left(b^2 \sqrt{b \cos(c + dx)}\right) \int (A + B \cos(c + dx)) \sec^3(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{\left(Ab^2 \sqrt{b \cos(c + dx)}\right) \int \sec^3(c + dx) dx}{\sqrt{\cos(c + dx)}} + \frac{\left(b^2 B \sqrt{b \cos(c + dx)}\right) \int \sec^2(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
 &= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)} + \frac{\left(Ab^2 \sqrt{b \cos(c + dx)}\right) \int \sec(c + dx) dx}{2\sqrt{\cos(c + dx)}} \\
 &\quad - \frac{\left(b^2 B \sqrt{b \cos(c + dx)}\right) \text{Subst}\left(\int 1 dx, x, -\tan(c + dx)\right)}{d\sqrt{\cos(c + dx)}}
 \end{aligned}$$

$$= \frac{Ab^2 \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{(b \cos(c+dx))^{5/2} (A + B \cos(c+dx))}{\cos^{11/2}(c+dx)} dx = \frac{(b \cos(c+dx))^{5/2} (A \operatorname{Arctanh}(\sin(c+dx)) \cos^2(c+dx) + (A + 2B \cos(c+dx)) \sin(c+dx))}{2d \cos^{9/2}(c+dx)}$$

[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]

[Out] ((b*Cos[c + d*x])^(5/2)*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(9/2))

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
default	$\frac{b^2 (A (\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1) - A (\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1) + 2B \sin(dx+c) \cos(dx+c) + A \sin(dx+c))}{2d \cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{Ab^2 (-\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1) + (\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1) + \sin(dx+c) \sqrt{\cos(dx+c)b}}{2d \cos(dx+c)^{\frac{5}{2}}} + \frac{b^2 \sin(dx+c)}{2d \cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib^2 \sqrt{\cos(dx+c)b} (A e^{3i(dx+c)} - 2B e^{2i(dx+c)} - A e^{i(dx+c)} - 2B)}{\sqrt{\cos(dx+c)} d (e^{2i(dx+c)} + 1)^2} + \frac{b^2 \sqrt{\cos(dx+c)b} A \ln(e^{i(dx+c)} + i)}{2 \sqrt{\cos(dx+c)} d} - \frac{b^2 \sqrt{\cos(dx+c)b} A \ln(e^{i(dx+c)} - i)}{2 \sqrt{\cos(dx+c)} d}$

[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)

[Out] 1/2*b^2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.09

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx = \frac{\left[\frac{Ab^{5/2} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)}\sqrt{\cos(dx+c)}}{\cos(dx+c)^3}\right)}{2d \cos(dx + c)^3} \right.}{A\sqrt{-bb^2} \arctan\left(\frac{\sqrt{b \cos(dx+c)}\sqrt{-b \sin(dx+c)}}{b\sqrt{\cos(dx+c)}}\right) \cos(dx + c)^3 - (2Bb^2 \cos(dx + c) + Ab^2) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)}}{2d \cos(dx + c)^3}$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorithm="fricas")

[Out] [1/4*(A*b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. 2(100) = 200.

Time = 0.44 (sec) , antiderivative size = 803, normalized size of antiderivative = 6.92

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algorithm="maxima")

```
[Out] 1/4*(8*B*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) - (4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*
c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x
+ 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(
4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x
+ 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*
cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4
*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 +
2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) - 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*
cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c
)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^{5/2}}{\cos^{11/2}(dx + c)} dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx = \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{11/2}(c + dx)} dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2),x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(11/2), x)
```

$$3.865 \quad \int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)} dx$$

Optimal result	.7387
Rubi [A] (verified)	.7387
Mathematica [A] (verified)	.7389
Maple [A] (verified)	.7389
Fricas [A] (verification not implemented)	.7390
Sympy [F(-1)]	.7390
Maxima [B] (verification not implemented)	.7390
Giac [F]	.7391
Mupad [F(-1)]	.7392

Optimal result

Integrand size = 33, antiderivative size = 157

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)} dx = \frac{b^2 B \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d \cos^{\frac{7}{2}}(c+dx)}$$

[Out] $1/2*b^2*B*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(5/2)}+A*b^2*\sin(d*x+c)*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(3/2)}+1/3*A*b^2*\sin(d*x+c)^3*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(7/2)}+1/2*b^2*B*\operatorname{arctanh}(\sin(d*x+c))*(b*\cos(d*x+c))^{(1/2)}/d/\cos(d*x+c)^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 3852, 3853, 3855}

$$\int \frac{(b \cos(c+dx))^{5/2}(A+B \cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)} dx = \frac{Ab^2 \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)} + \frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{b^2 B \operatorname{arctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d \sqrt{\cos(c+dx)}} + \frac{b^2 B \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)}$$

[In] Int[((b*cos[c + d*x])^(5/2)*(A + B*cos[c + d*x]))/cos[c + d*x]^(13/2), x]

[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*cos[c + d*x]]/(2*d*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*cos[c + d*x]]*Sin[c + d*x])/(2*d*cos[c + d*x]^(5/2)) + (A*b^2*Sqrt[b*cos[c + d*x]]*Sin[c + d*x])/(d*cos[c + d*x]^(3/2)) + (A*b^2*Sqrt[b*cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*cos[c + d*x]^(7/2))

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2827

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3852

Int[csc[(c_.) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right) \int (A+B\cos(c+dx))\sec^4(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right) \int \sec^4(c+dx) dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right) \int \sec^3(c+dx) dx}{\sqrt{\cos(c+dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b^2 B \sqrt{b \cos(c+dx)}\right) \int \sec(c+dx) dx}{2\sqrt{\cos(c+dx)}} \\
&\quad - \frac{\left(Ab^2 \sqrt{b \cos(c+dx)}\right) \text{Subst}\left(\int (1+x^2) dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}} \\
&= \frac{b^2 \text{Barctanh}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{b^2 B \sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} \\
&\quad + \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d \cos^{\frac{7}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{(b \cos(c+dx))^{\frac{5}{2}}(A+B \cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)} dx = \frac{(b \cos(c+dx))^{\frac{5}{2}}(3 \text{Barctanh}(\sin(c+dx)) \cos^2(c+dx) + 3B \cos(c+dx) + 2A \cos(c+dx))}{6d \cos^{\frac{9}{2}}(c+dx)}$$

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + B*cos[c + d*x]))/Cos[c + d*x]^(13/2), x]

[Out] ((b*cos[c + d*x])^(5/2)*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*cos[c + d*x]^(9/2))

Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

method	result
default	$\frac{b^2(-3B(\cos^3(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+3B(\cos^3(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1)+4A \sin(dx+c)(\cos^2(dx+c)-1))}{6d \cos(dx+c)^{\frac{7}{2}}}$
parts	$\frac{A b^2(2(\cos^2(dx+c)+1)\sqrt{\cos(dx+c)b} \sin(dx+c)}{3d \cos(dx+c)^{\frac{7}{2}}} + \frac{B b^2(-(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1))}{2d \cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i b^2 \sqrt{\cos(dx+c)b} (3B e^{5i(dx+c)} - 12A e^{2i(dx+c)} - 3B e^{i(dx+c)} - 4A)}{3\sqrt{\cos(dx+c)} d (e^{2i(dx+c)} + 1)^3} + \frac{b^2 \sqrt{\cos(dx+c)b} B \ln(e^{i(dx+c)} + i)}{2\sqrt{\cos(dx+c)} d} - \frac{b^2 \sqrt{\cos(dx+c)b} B \ln(e^{-i(dx+c)} - i)}{2\sqrt{\cos(dx+c)} d}$

[In] int((cos(d*x+c)*b)^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2), x, method=_RETURNVERBOSE)

[Out] 1/6*b^2/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.75

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{13/2}(c + dx)} dx = \frac{\left[3 B b^{5/2} \cos(dx + c)^4 \log\left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b} \sqrt{\cos(dx+c)}}{\cos(dx+c)^3}\right) + 3 B \sqrt{-b} b^2 \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx + c)^4 - (4 A b^2 \cos(dx + c)^2 + 3 B b^2 \cos(dx + c) + 2 A b^2) \right]}{6 d \cos(dx + c)^4}$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algorithm="fricas")

[Out] [1/12*(3*B*b^(5/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(4*A*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (4*A*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{13/2}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. 2(135) = 270.

Time = 0.44 (sec) , antiderivative size = 1060, normalized size of antiderivative = 6.75

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{13/2}(c + dx)} dx = \text{Too large to display}$$

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algorithm="maxima")

```
[Out] -1/12*(16*(3*b^2*cos(6*d*x + 6*c))*sin(2*d*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)
*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c) - 3*(3*
b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4
*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
+ 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*(4*(b^2*sin(4*d*x +
4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*co
s(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b
^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) +
(b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2
+ 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b
^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*
c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))) + 1) - 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x
+ 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^
2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d
*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
+ 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
+ 2*c) + 1))/d
```

Giac [F]

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{13/2}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A) (b \cos(dx + c))^{5/2}}{\cos(dx + c)^{13/2}} dx$$

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algori
thm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(13/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos^{13/2}(c + dx)} dx = \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx))}{\cos(c + dx)^{13/2}} dx$$

```
[In] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(13/2), x)
```

```
[Out] int(((b*cos(c + d*x))^(5/2)*(A + B*cos(c + d*x)))/cos(c + d*x)^(13/2), x)
```

$$3.866 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal result	7393
Rubi [A] (verified)	7393
Mathematica [A] (verified)	7395
Maple [A] (verified)	7395
Fricas [A] (verification not implemented)	7396
Sympy [F(-1)]	7396
Maxima [A] (verification not implemented)	7396
Giac [F]	7397
Mupad [B] (verification not implemented)	7397

Optimal result

Integrand size = 33, antiderivative size = 136

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{2\sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \sin(c+dx)}{d \sqrt{b \cos(c+dx)}} + \frac{A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d \sqrt{b \cos(c+dx)}} - \frac{B \sqrt{\cos(c+dx)} \sin^3(c+dx)}{3d \sqrt{b \cos(c+dx)}}$$

[Out] $1/2*A*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/d/(b*\cos(d*x+c))^{(1/2)}+1/2*A*x*\cos(d*x+c)^{(1/2)}/(b*\cos(d*x+c))^{(1/2)}+B*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}-1/3*B*\sin(d*x+c)^3*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 2715, 8, 2713}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{2\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2d \sqrt{b \cos(c+dx)}} - \frac{B \sin^3(c+dx) \sqrt{\cos(c+dx)}}{3d \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \sqrt{\cos(c+dx)}}{d \sqrt{b \cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]

[Out] (A*x*Sqrt[Cos[c + d*x]]/(2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*SIN[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int \cos^2(c + dx)(A + B \cos(c + dx)) dx}{\sqrt{b \cos(c + dx)}} \\ &= \frac{\left(A \sqrt{\cos(c + dx)}\right) \int \cos^2(c + dx) dx}{\sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{\cos(c + dx)}\right) \int \cos^3(c + dx) dx}{\sqrt{b \cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{b \cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right) \int 1 dx}{2\sqrt{b \cos(c+dx)}} \\
&\quad - \frac{\left(B\sqrt{\cos(c+dx)}\right) \text{Subst}\left(\int (1-x^2) dx, x, -\sin(c+dx)\right)}{d\sqrt{b \cos(c+dx)}} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{2\sqrt{b \cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} \\
&\quad + \frac{A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{b \cos(c+dx)}} - \frac{B\sqrt{\cos(c+dx)} \sin^3(c+dx)}{3d\sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\begin{aligned}
&\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx \\
&= \frac{\sqrt{\cos(c+dx)}(6Ac+6Adx+9B \sin(c+dx)+3A \sin(2(c+dx))+B \sin(3(c+dx)))}{12d\sqrt{b \cos(c+dx)}}
\end{aligned}$$

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]

[Out] (Sqrt[Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)})(2B \sin(dx+c)(\cos^2(dx+c))+3A \sin(dx+c) \cos(dx+c)+3A(dx+c)+4B \sin(dx+c))}{6d\sqrt{\cos(dx+c)}b}$	74
parts	$\frac{A(\cos(dx+c) \sin(dx+c)+dx+c)(\sqrt{\cos(dx+c)})}{2d\sqrt{\cos(dx+c)}b} + \frac{B(2+\cos^2(dx+c)) \sin(dx+c)(\sqrt{\cos(dx+c)})}{3d\sqrt{\cos(dx+c)}b}$	84
risch	$\frac{Ax(\sqrt{\cos(dx+c)})}{2\sqrt{\cos(dx+c)}b} + \frac{3B \sin(dx+c)(\sqrt{\cos(dx+c)})}{4d\sqrt{\cos(dx+c)}b} + \frac{(\sqrt{\cos(dx+c)})B \sin(3dx+3c)}{12\sqrt{\cos(dx+c)}bd} + \frac{(\sqrt{\cos(dx+c)})A \sin(2dx+2c)}{4\sqrt{\cos(dx+c)}bd}$	120

[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6/d*cos(d*x+c)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*cos(d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.74

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

$$= \left[-\frac{3A\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2}{12bd\cos(dx+c)} \right]$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/12*(3*A*sqrt(-b)*cos(d*x+c)*log(2*b*cos(d*x+c)^2+2*sqrt(b*cos(d*x+c))*sqrt(-b)*sqrt(cos(d*x+c))*sin(d*x+c)-b)-2*(2*B*cos(d*x+c)^2+3*A*cos(d*x+c)+4*B)*sqrt(b*cos(d*x+c))*sqrt(cos(d*x+c))*sin(d*x+c))/(b*d*cos(d*x+c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x+c))*sin(d*x+c)/(sqrt(b)*cos(d*x+c)^(3/2)))*cos(d*x+c)+(2*B*cos(d*x+c)^2+3*A*cos(d*x+c)+4*B)*sqrt(b*cos(d*x+c))*sqrt(cos(d*x+c))*sin(d*x+c))/(b*d*cos(d*x+c))]

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

$$= \frac{3(2dx+2c+\sin(2dx+2c))A}{\sqrt{b}} + \frac{B(\sin(3dx+3c)+9\sin(\frac{1}{3}\arctan(\frac{\sin(3dx+3c)}{\cos(3dx+3c)}))}{\sqrt{b}}$$

12 d

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/sqrt(b))/d

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{\sqrt{b \cos(dx + c)}} dx$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 16.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (3 A \sin(c + dx) + 3 A \sin(3 c + 3 d x) + 10 B \sin(2 c + 2 d x) + B \sin(4 c + 4 d x) + 12 A d x \cos(c + d x))}{12 b d (\cos(2 c + 2 d x) + 1)}$$

[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c + d*x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))

$$3.867 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal result	7398
Rubi [A] (verified)	7398
Mathematica [A] (verified)	7399
Maple [A] (verified)	7399
Fricas [A] (verification not implemented)	7400
Sympy [A] (verification not implemented)	7401
Maxima [A] (verification not implemented)	7401
Giac [F]	7401
Mupad [B] (verification not implemented)	7402

Optimal result

Integrand size = 33, antiderivative size = 98

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b \cos(c+dx)}} + \frac{A\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + \frac{B \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2d\sqrt{b \cos(c+dx)}}$$

[Out] $\frac{1}{2}B \cos(d*x+c)^{(3/2)} \sin(d*x+c) / d / (b \cos(d*x+c))^{(1/2)} + \frac{1}{2}B*x \cos(d*x+c)^{(1/2)} / (b \cos(d*x+c))^{(1/2)} + A \sin(d*x+c) \cos(d*x+c)^{(1/2)} / d / (b \cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2813}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{A \sin(c+dx) \sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b \cos(c+dx)}}$$

[In] $\text{Int}[(\text{Cos}[c + d*x])^{(3/2)} * (A + B * \text{Cos}[c + d*x])] / \text{Sqrt}[b * \text{Cos}[c + d*x]], x]$

[Out] $(B*x*\text{Sqrt}[\text{Cos}[c + d*x]]) / (2*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (A*\text{Sqrt}[\text{Cos}[c + d*x]] * \text{Sin}[c + d*x]) / (d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (B*\text{Cos}[c + d*x]^{(3/2)} * \text{Sin}[c + d*x]) / (2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_) + (f_.)
*(x_)]), x_Symbol] := Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)(A+B\cos(c+dx)) dx}{\sqrt{b\cos(c+dx)}} \\ &= \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\begin{aligned} &\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx \\ &= \frac{\sqrt{\cos(c+dx)}(4A\sin(c+dx) + B(2(c+dx) + \sin(2(c+dx))))}{4d\sqrt{b\cos(c+dx)}} \end{aligned}$$

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]))
)/(4*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)})(B \sin(dx+c) \cos(dx+c) + 2A \sin(dx+c) + B(dx+c))}{2d\sqrt{\cos(dx+c)b}}$	55
parts	$\frac{A \sin(dx+c)(\sqrt{\cos(dx+c)})}{d\sqrt{\cos(dx+c)b}} + \frac{B(\cos(dx+c) \sin(dx+c) + dx+c)(\sqrt{\cos(dx+c)})}{2d\sqrt{\cos(dx+c)b}}$	73
risch	$\frac{Bx(\sqrt{\cos(dx+c)})}{2\sqrt{\cos(dx+c)b}} + \frac{A \sin(dx+c)(\sqrt{\cos(dx+c)})}{d\sqrt{\cos(dx+c)b}} + \frac{(\sqrt{\cos(dx+c)})B \sin(2dx+2c)}{4\sqrt{\cos(dx+c)b}d}$	86

[In] `int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURN
VERBOSE)`

[Out] $1/2/d*\cos(d*x+c)^(1/2)*(B*\sin(d*x+c)*\cos(d*x+c)+2*A*\sin(d*x+c)+B*(d*x+c))/(\cos(d*x+c)*b)^(1/2)$

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.14

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

$$= \left[-\frac{B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2}{4bd\cos(dx+c)} \right]$$

[In] `integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x,algorit
hm="fricas")`

[Out] `[-1/4*(B*sqrt(-b)*cos(d*x+c)*log(2*b*cos(d*x+c)^2+2*sqrt(b*cos(d*x+c))
)*sqrt(-b)*sqrt(cos(d*x+c))*sin(d*x+c)-b)-2*(B*cos(d*x+c)+2*A
)*sqrt(b*cos(d*x+c))*sqrt(cos(d*x+c))*sin(d*x+c))/(b*d*cos(d*x+c)),
1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x+c))*sin(d*x+c)/(sqrt(b)*cos(d*x+c)
)^(3/2)))*cos(d*x+c)+(B*cos(d*x+c)+2*A)*sqrt(b*cos(d*x+c))*sqrt
(cos(d*x+c))*sin(d*x+c)/(b*d*cos(d*x+c))]`

Sympy [A] (verification not implemented)

Time = 36.74 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.54

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

$$= \begin{cases} \frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{Bx\sin^2(c+dx)\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{Bx\cos^{\frac{5}{2}}(c+dx)}{2\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))\cos^{\frac{3}{2}}(c)}{\sqrt{b\cos(c)}} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)

[Out] Piecewise((A*sin(c + d*x)*sqrt(cos(c + d*x))/(d*sqrt(b*cos(c + d*x))) + B*x*sin(c + d*x)**2*sqrt(cos(c + d*x))/(2*sqrt(b*cos(c + d*x))) + B*x*cos(c + d*x)**(5/2)/(2*sqrt(b*cos(c + d*x))) + B*sin(c + d*x)*cos(c + d*x)**(3/2)/(2*d*sqrt(b*cos(c + d*x))), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**(3/2)/sqrt(b*cos(c)), True))

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx = \frac{\frac{(2dx+2c+\sin(2dx+2c))B}{\sqrt{b}} + \frac{4A\sin(dx+c)}{\sqrt{b}}}{4d}$$

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/sqrt(b) + 4*A*sin(d*x + c)/sqrt(b))/d

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (B \sin(c + dx) + 4A \sin(2c + 2dx) + B \sin(3c + 3dx) + 4B dx \cos(c + dx))}{4bd (\cos(2c + 2dx) + 1)}$$

[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c + 2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*b*d*(cos(2*c + 2*d*x) + 1))

$$3.868 \quad \int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal result	7403
Rubi [A] (verified)	7403
Mathematica [A] (verified)	7404
Maple [A] (verified)	7404
Fricas [A] (verification not implemented)	7405
Sympy [A] (verification not implemented)	7405
Maxima [A] (verification not implemented)	7406
Giac [F]	7406
Mupad [B] (verification not implemented)	7406

Optimal result

Integrand size = 33, antiderivative size = 59

$$\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \sin(c+dx)}{d \sqrt{b \cos(c+dx)}}$$

[Out] $A*x*\cos(d*x+c)^{(1/2)}/(b*\cos(d*x+c))^{(1/2)}+B*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2717}

$$\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \sqrt{\cos(c+dx)}}{d \sqrt{b \cos(c+dx)}}$$

[In] $\text{Int}[(\text{Sqrt}[\text{Cos}[c + d*x]]*(A + B*\text{Cos}[c + d*x]))/\text{Sqrt}[b*\text{Cos}[c + d*x]],x]$

[Out] $(A*x*\text{Sqrt}[\text{Cos}[c + d*x]])/\text{Sqrt}[b*\text{Cos}[c + d*x]] + (B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 17

$\text{Int}[(u_*)*((a_*)*(v_))^{(m)}*((b_*)*(v_))^{(n)}, x_Symbol] \rightarrow \text{Dist}[a^{(m + 1/2)}*b^{(n - 1/2)}*(\text{Sqrt}[b*v]/\text{Sqrt}[a*v]), \text{Int}[u*v^{(m + n)}, x], x] /;$ FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :=> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int (A + B \cos(c+dx)) dx}{\sqrt{b \cos(c+dx)}} \\ &= \frac{Ax \sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{\left(B \sqrt{\cos(c+dx)} \right) \int \cos(c+dx) dx}{\sqrt{b \cos(c+dx)}} \\ &= \frac{Ax \sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \sin(c+dx)}{d \sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{\cos(c+dx)}(A + B \cos(c+dx))}{\sqrt{b \cos(c+dx)}} dx = \frac{\sqrt{\cos(c+dx)}(A(c+dx) + B \sin(c+dx))}{d \sqrt{b \cos(c+dx)}}$$

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)})(A(dx+c)+B \sin(dx+c))}{d \sqrt{\cos(dx+c)b}}$	39
risch	$\frac{Ax(\sqrt{\cos(dx+c)})}{\sqrt{\cos(dx+c)b}} + \frac{B \sin(dx+c)(\sqrt{\cos(dx+c)})}{d \sqrt{\cos(dx+c)b}}$	52
parts	$\frac{A(\sqrt{\cos(dx+c)})(dx+c)}{d \sqrt{\cos(dx+c)b}} + \frac{B \sin(dx+c)(\sqrt{\cos(dx+c)})}{d \sqrt{\cos(dx+c)b}}$	59

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] 1/d*cos(d*x+c)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)
```


Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

$$= \left[-\frac{A\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2}{2bd\cos(dx+c)} \right]$$

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)), (A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))] ]
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx = \begin{cases} \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))\sqrt{\cos(c)}}{\sqrt{b\cos(c)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((A*x*sqrt(cos(c + d*x))/sqrt(b*cos(c + d*x)) + B*sin(c + d*x)*sqrt(cos(c + d*x))/(d*sqrt(b*cos(c + d*x))), Ne(d, 0)), (x*(A + B*cos(c))*sqrt(cos(c))/sqrt(b*cos(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx = \frac{\frac{2A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + B\sin(dx+c)}{\sqrt{b}}}{d}$$

[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] (2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + B*sin(d*x + c)/sqrt(b))/d

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)}} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x)

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\begin{aligned} & \int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}} dx \\ &= \frac{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(B\sin(2c+2dx)+2Adx\cos(c+dx))}{bd(\cos(2c+2dx)+1)} \end{aligned}$$

[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(2*c + 2*d*x) + 2*A*d*x*cos(c + d*x)))/(b*d*(cos(2*c + 2*d*x) + 1))

$$3.869 \quad \int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} dx$$

Optimal result	.7407
Rubi [A] (verified)	.7407
Mathematica [A] (verified)	.7408
Maple [A] (verified)	.7409
Fricas [B] (verification not implemented)	.7409
Sympy [F]	.7410
Maxima [A] (verification not implemented)	.7410
Giac [F]	.7410
Mupad [F(-1)]	.7411

Optimal result

Integrand size = 33, antiderivative size = 60

$$\begin{aligned} & \int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} dx \\ &= \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{A \operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}} \end{aligned}$$

[Out] B*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {18, 2814, 3855}

$$\begin{aligned} & \int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} dx \\ &= \frac{A\sqrt{\cos(c+dx)}\operatorname{arctanh}(\sin(c+dx))}{d\sqrt{b \cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} \end{aligned}$$

[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]

[Out] (B*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m - 1/2)
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx)) \sec(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\ &= \frac{Bx \sqrt{\cos(c + dx)}}{\sqrt{b \cos(c + dx)}} + \frac{\left(A \sqrt{\cos(c + dx)} \right) \int \sec(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\ &= \frac{Bx \sqrt{\cos(c + dx)}}{\sqrt{b \cos(c + dx)}} + \frac{A \operatorname{arctanh}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{d \sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} dx = \frac{(Bdx + A \operatorname{arctanh}(\sin(c + dx))) \sqrt{\cos(c + dx)}}{d \sqrt{b \cos(c + dx)}}$$

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*
x]])
```

Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{(2A \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c))(\sqrt{\cos(dx+c)})}{d\sqrt{\cos(dx+c)}b}$	52
parts	$-\frac{2A \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))(\sqrt{\cos(dx+c)})}{d\sqrt{\cos(dx+c)}b} + \frac{B(\sqrt{\cos(dx+c)})(dx+c)}{d\sqrt{\cos(dx+c)}b}$	70
risch	$\frac{Bx(\sqrt{\cos(dx+c)})}{\sqrt{\cos(dx+c)}b} + \frac{(\sqrt{\cos(dx+c)})A \ln(e^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}bd} - \frac{(\sqrt{\cos(dx+c)})A \ln(e^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}bd}$	96

[In] `int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURN
VERBOSE)`

[Out] `-1/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c))*cos(d*x+c)^(1/2)/(cos(d
*x+c)*b)^(1/2)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(52) = 104.

Time = 0.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.58

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} dx$$

$$= \left[\frac{2A\sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) + B\sqrt{-b} \log\left(2b \cos(dx+c)^2 + 2\sqrt{b \cos(dx+c)} \sqrt{-b} \sqrt{\cos(dx+c)}\right)}{2bd} \right]$$

[In] `integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")`

[Out] `[-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/2*(2*B*sqrt(b)
*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + A
*sqrt(b)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d
*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b*d)]`

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)} \sqrt{\cos(c + dx)}} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)

[Out] Integral((A + B*cos(c + d*x))/(sqrt(b*cos(c + d*x))*sqrt(cos(c + d*x))), x)

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} dx$$

$$= \frac{A \left(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1) \right)}{\sqrt{b}} + \frac{4 B \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}$$

$$= \frac{\hspace{10em}}{2d}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*B*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b))/d

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)}} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} dx$$

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)), x)
```

$$3.870 \quad \int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx$$

Optimal result	7412
Rubi [A] (verified)	7412
Mathematica [A] (verified)	7414
Maple [A] (verified)	7414
Fricas [A] (verification not implemented)	7414
Sympy [F]	7415
Maxima [B] (verification not implemented)	7415
Giac [F]	7416
Mupad [F(-1)]	7416

Optimal result

Integrand size = 33, antiderivative size = 68

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx = \frac{B \operatorname{arctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

[Out] A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {18, 2827, 3852, 8, 3855}

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx = \frac{A \sin(c+dx)}{d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \operatorname{arctanh}(\sin(c+dx))}{d \sqrt{b \cos(c+dx)}}$$

[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 18

`Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m - 1/2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx)) \sec^2(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\
 &= \frac{\left(A \sqrt{\cos(c + dx)}\right) \int \sec^2(c + dx) dx}{\sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{\cos(c + dx)}\right) \int \sec(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\
 &= \frac{\text{Barctanh}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{d \sqrt{b \cos(c + dx)}} - \frac{\left(A \sqrt{\cos(c + dx)}\right) \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{d \sqrt{b \cos(c + dx)}} \\
 &= \frac{\text{Barctanh}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{d \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \frac{B \operatorname{arctanh}(\sin(c + dx)) \cos(c + dx) + A \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}$$

[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{-2B \cos(dx+c) \operatorname{arctanh}(\cot(dx+c) - \operatorname{csc}(dx+c)) + A \sin(dx+c)}{d \sqrt{\cos(dx+c)} b \sqrt{\cos(dx+c)}}$	57
parts	$\frac{A \sin(dx+c)}{d \sqrt{\cos(dx+c)} \sqrt{\cos(dx+c)} b} - \frac{2B \operatorname{arctanh}(\cot(dx+c) - \operatorname{csc}(dx+c)) (\sqrt{\cos(dx+c)})}{d \sqrt{\cos(dx+c)} b}$	71
risch	$\frac{ie^{-i(dx+c)} A}{\sqrt{\cos(dx+c)} b \sqrt{\cos(dx+c)} d} + \frac{(\sqrt{\cos(dx+c)}) B \ln(e^{i(dx+c)} + i)}{\sqrt{\cos(dx+c)} b d} - \frac{(\sqrt{\cos(dx+c)}) B \ln(e^{i(dx+c)} - i)}{\sqrt{\cos(dx+c)} b d}$	109

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURN VERBOSE)

[Out] 1/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.10

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx$$

$$= \left[\frac{B \sqrt{b} \cos(dx + c)^2 \log \left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b \cos(dx+c)} \sin(dx+c) - 2 b \cos(dx+c)}{\cos(dx+c)^3} \right) + 2 \sqrt{b \cos(dx+c)} A \sqrt{\cos(dx+c)}}{2 b d \cos(dx+c)^2} \right. \\ \left. - \frac{B \sqrt{-b} \arctan \left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}} \right) \cos(dx+c)^2 - \sqrt{b \cos(dx+c)} A \sqrt{\cos(dx+c)} \sin(dx+c)}{b d \cos(dx+c)^2} \right]$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^2)]

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)} \cos^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))^(1/2),x)

[Out] Integral((A + B*cos(c + d*x))/(sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(60) = 120.

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.84

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \frac{B \left(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1) \right)}{\sqrt{b}} + \frac{4 A \sqrt{b} \sin(2 dx + 2 c)}{b \cos(2 dx + 2 c)^2 + b \sin(2 dx + 2 c)^2 + 2 b}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/2*(B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*A*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c) + b))/d

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)} \cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(3/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^{3/2} \sqrt{b \cos(c + dx)}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)), x)

$$3.871 \quad \int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx$$

Optimal result	7417
Rubi [A] (verified)	7417
Mathematica [A] (verified)	7419
Maple [A] (verified)	7419
Fricas [A] (verification not implemented)	7420
Sympy [F(-1)]	7420
Maxima [B] (verification not implemented)	7421
Giac [F]	7421
Mupad [F(-1)]	7422

Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx = \frac{A \operatorname{arctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{2d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

[Out] $1/2*A*\sin(d*x+c)/d/\cos(d*x+c)^{(3/2)}/(b*\cos(d*x+c))^{(1/2)}+B*\sin(d*x+c)/d/\cos(d*x+c)^{(1/2)}/(b*\cos(d*x+c))^{(1/2)}+1/2*A*\operatorname{arctanh}(\sin(d*x+c))*\cos(d*x+c)^{(1/2)}/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {18, 2827, 3853, 3855, 3852, 8}

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx = \frac{A \sqrt{\cos(c+dx)} \operatorname{arctanh}(\sin(c+dx))}{2d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 18

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m - 1/2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx)) \sec^3(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\ &= \frac{\left(A \sqrt{\cos(c + dx)}\right) \int \sec^3(c + dx) dx}{\sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{\cos(c + dx)}\right) \int \sec^2(c + dx) dx}{\sqrt{b \cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{A \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{\left(A \sqrt{\cos(c + dx)}\right) \int \sec(c + dx) dx}{2 \sqrt{b \cos(c + dx)}} \\
&\quad - \frac{\left(B \sqrt{\cos(c + dx)}\right) \text{Subst}\left(\int 1 dx, x, -\tan(c + dx)\right)}{d \sqrt{b \cos(c + dx)}} \\
&= \frac{A \operatorname{arctanh}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2d \sqrt{b \cos(c + dx)}} \\
&\quad + \frac{A \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{B \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\begin{aligned}
&\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx \\
&= \frac{A \operatorname{arctanh}(\sin(c + dx)) \cos^2(c + dx) + (A + 2B \cos(c + dx)) \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}}
\end{aligned}$$

[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] (A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x])/ (2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

method	result
default	$\frac{A(\cos^2(dx+c) \ln(-\cot(dx+c)+\csc(dx+c)+1)-A(\cos^2(dx+c) \ln(-\cot(dx+c)+\csc(dx+c)-1)+2B \sin(dx+c) \cos(dx+c)+A \sin(dx+c))}{2d \sqrt{\cos(dx+c)} b \cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A(-(\cos^2(dx+c) \ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c) \ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c))}{2d \sqrt{\cos(dx+c)} b \cos(dx+c)^{\frac{3}{2}}} + \frac{B \sin(dx+c)}{d \sqrt{\cos(dx+c)} \sqrt{b \cos(dx+c)}}$
risch	$-\frac{i(A e^{2i(dx+c)} - A - 4B \cos(dx+c))}{2 \sqrt{\cos(dx+c)} b \sqrt{\cos(dx+c)} (e^{2i(dx+c)} + 1) d} + \frac{(\sqrt{\cos(dx+c)}) A \ln(e^{i(dx+c)} + i)}{2 \sqrt{\cos(dx+c)} b d} - \frac{(\sqrt{\cos(dx+c)}) A \ln(e^{i(dx+c)} - i)}{2 \sqrt{\cos(dx+c)} b d}$

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.16

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx$$

$$= \frac{\left[A\sqrt{b} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(2B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \right]}{4bd \cos(dx + c)^3} - \frac{A\sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)}\sqrt{-b \sin(dx+c)}}{b\sqrt{\cos(dx+c)}}\right) \cos(dx + c)^3 - (2B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)}}{2bd \cos(dx + c)^3}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)]

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. 2(91) = 182.

Time = 0.42 (sec) , antiderivative size = 722, normalized size of antiderivative = 6.75

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c) + b) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b))/d
```

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)} \cos(dx + c)^{\frac{5}{2}}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^{5/2} \sqrt{b \cos(c + dx)}} dx$$

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)), x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)), x)
```

$$3.872 \quad \int \frac{A+B \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx$$

Optimal result	7423
Rubi [A] (verified)	7423
Mathematica [A] (verified)	7425
Maple [A] (verified)	7426
Fricas [A] (verification not implemented)	7426
Sympy [F(-1)]	7427
Maxima [B] (verification not implemented)	7427
Giac [F]	7428
Mupad [F(-1)]	7428

Optimal result

Integrand size = 33, antiderivative size = 145

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx) \sqrt{b \cos(c+dx)}} dx = \frac{\text{Barctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{2d \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{A \sin^3(c+dx)}{3d \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}}$$

[Out] 1/2*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/3*A*sin(d*x+c)^3/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+1/2*B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used

= {18, 2827, 3852, 3853, 3855}

$$\int \frac{A + B \cos(c + dx)}{\cos^{7/2}(c + dx) \sqrt{b \cos(c + dx)}} dx = \frac{A \sin^3(c + dx)}{3d \cos^{5/2}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} + \frac{B \sqrt{\cos(c + dx)} \operatorname{arctanh}(\sin(c + dx))}{2d \sqrt{b \cos(c + dx)}} + \frac{B \sin(c + dx)}{2d \cos^{3/2}(c + dx) \sqrt{b \cos(c + dx)}}$$

[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]]/(2*d*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m - 1/2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3852

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int (A+B\cos(c+dx)) \sec^4(c+dx) dx}{\sqrt{b\cos(c+dx)}} \\
 &= \frac{\left(A\sqrt{\cos(c+dx)}\right) \int \sec^4(c+dx) dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right) \int \sec^3(c+dx) dx}{\sqrt{b\cos(c+dx)}} \\
 &= \frac{B \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx) \sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right) \int \sec(c+dx) dx}{2\sqrt{b\cos(c+dx)}} \\
 &\quad - \frac{\left(A\sqrt{\cos(c+dx)}\right) \text{Subst}\left(\int (1+x^2) dx, x, -\tan(c+dx)\right)}{d\sqrt{b\cos(c+dx)}} \\
 &= \frac{\text{Barctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{B \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx) \sqrt{b\cos(c+dx)}} \\
 &\quad + \frac{A \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A \sin^3(c+dx)}{3d \cos^{\frac{5}{2}}(c+dx) \sqrt{b\cos(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\begin{aligned}
 &\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} dx \\
 &= \frac{3\text{Barctanh}(\sin(c+dx))\cos^2(c+dx) + 3B\sin(c+dx) + 2A(2+\cos(2(c+dx)))\tan(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}
 \end{aligned}$$

[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x])/(6*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84

method	result
default	$\frac{-3B(\cos^3(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+3B(\cos^3(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1)+4A \sin(dx+c)(\cos^2(dx+c))+3B}{6d\sqrt{\cos(dx+c)b} \cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A(2(\cos^2(dx+c)+1) \sin(dx+c)}{3d\sqrt{\cos(dx+c)b} \cos(dx+c)^{\frac{5}{2}}} + \frac{B(-(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1))}{2d\sqrt{\cos(dx+c)b} \cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(3B e^{4i(dx+c)}-3B-16A \cos(dx+c)-8iA \sin(dx+c))}{6\sqrt{\cos(dx+c)b} \sqrt{\cos(dx+c)} (e^{2i(dx+c)}+1)^2 d} + \frac{(\sqrt{\cos(dx+c)})B \ln(e^{i(dx+c)}+i)}{2\sqrt{\cos(dx+c)b} d} - \frac{(\sqrt{\cos(dx+c)})B \ln(e^{i(dx+c)}-i)}{2\sqrt{\cos(dx+c)b} d}$

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURN VERBOSE)

[Out] 1/6/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.79

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx$$

$$= \left[\frac{3 B \sqrt{b} \cos(dx + c)^4 \log\left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2 b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2 (4 A \cos(dx + c)^2 + 3 B \cos(dx + c) + 2 A) \sqrt{b}}{12 b d \cos(dx + c)^4} \right. \\ \left. - \frac{3 B \sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx + c)^4 - (4 A \cos(dx + c)^2 + 3 B \cos(dx + c) + 2 A) \sqrt{b}}{6 b d \cos(dx + c)^4} \right]$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(123) = 246.

Time = 0.42 (sec) , antiderivative size = 957, normalized size of antiderivative = 6.60

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
```

$$\frac{\sin^2(4dx + 4c) + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin^2(2dx + 2c) + 4\cos(2dx + 2c) + 1}{\sqrt{b}} \cdot \frac{1}{d}$$

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \int \frac{B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)} \cos^{\frac{7}{2}}(dx + c)} dx$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \int \frac{A + B \cos(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)), x)

$$3.873 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7429
Rubi [A] (verified)	7429
Mathematica [A] (verified)	7431
Maple [A] (verified)	7431
Fricas [A] (verification not implemented)	7432
Sympy [F(-1)]	7432
Maxima [A] (verification not implemented)	7432
Giac [F]	7433
Mupad [B] (verification not implemented)	7433

Optimal result

Integrand size = 33, antiderivative size = 148

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{2b \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \sin(c+dx)}{bd \sqrt{b \cos(c+dx)}} + \frac{A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2bd \sqrt{b \cos(c+dx)}} - \frac{B \sqrt{\cos(c+dx)} \sin^3(c+dx)}{3bd \sqrt{b \cos(c+dx)}}$$

[Out] $1/2*A*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/b/d/(b*\cos(d*x+c))^{(1/2)}+1/2*A*x*\cos(d*x+c)^{(1/2)}/b/(b*\cos(d*x+c))^{(1/2)}+B*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}-1/3*B*\sin(d*x+c)^3*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 2715, 8, 2713}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{2b \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2bd \sqrt{b \cos(c+dx)}} - \frac{B \sin^3(c+dx) \sqrt{\cos(c+dx)}}{3bd \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \sqrt{\cos(c+dx)}}{bd \sqrt{b \cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]

[Out] (A*x*Sqrt[Cos[c + d*x]]/(2*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int \cos^2(c + dx)(A + B \cos(c + dx)) dx}{b\sqrt{b} \cos(c + dx)} \\ &= \frac{\left(A\sqrt{\cos(c + dx)}\right) \int \cos^2(c + dx) dx}{b\sqrt{b} \cos(c + dx)} + \frac{\left(B\sqrt{\cos(c + dx)}\right) \int \cos^3(c + dx) dx}{b\sqrt{b} \cos(c + dx)} \end{aligned}$$

$$\begin{aligned}
&= \frac{A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2bd\sqrt{b \cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right) \int 1 dx}{2b\sqrt{b \cos(c+dx)}} \\
&\quad - \frac{\left(B\sqrt{\cos(c+dx)}\right) \text{Subst}\left(\int (1-x^2) dx, x, -\sin(c+dx)\right)}{bd\sqrt{b \cos(c+dx)}} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{2b\sqrt{b \cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} \\
&\quad + \frac{A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2bd\sqrt{b \cos(c+dx)}} - \frac{B\sqrt{\cos(c+dx)} \sin^3(c+dx)}{3bd\sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.47

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{\cos^{\frac{3}{2}}(c+dx)(6Ac+6Adx+9B \sin(c+dx)+3A \sin(2(c+dx)))}{12d(b \cos(c+dx))^{3/2}}$$

[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2), x]

[Out] (Cos[c + d*x]^(3/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*x)]) + B*Sin[3*(c + d*x)])/(12*d*(b*Cos[c + d*x])^(3/2))

Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)})(2B \sin(dx+c)(\cos^2(dx+c))+3A \sin(dx+c) \cos(dx+c)+3A(dx+c)+4B \sin(dx+c))}{6bd\sqrt{\cos(dx+c)}b}$	77
parts	$\frac{A(\sqrt{\cos(dx+c)})(\cos(dx+c) \sin(dx+c)+dx+c)}{2db\sqrt{\cos(dx+c)}b} + \frac{B(2+\cos^2(dx+c)) \sin(dx+c)(\sqrt{\cos(dx+c)})}{3db\sqrt{\cos(dx+c)}b}$	90
risch	$\frac{Ax(\sqrt{\cos(dx+c)})}{2b\sqrt{\cos(dx+c)}b} + \frac{3B \sin(dx+c)(\sqrt{\cos(dx+c)})}{4bd\sqrt{\cos(dx+c)}b} + \frac{(\sqrt{\cos(dx+c)})B \sin(3dx+3c)}{12b\sqrt{\cos(dx+c)}b d} + \frac{(\sqrt{\cos(dx+c)})A \sin(2dx+2c)}{4b\sqrt{\cos(dx+c)}b d}$	132

[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/6/b/d*cos(d*x+c)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*cos(d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.59

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \left[-\frac{3A\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{b\cos(dx+c)}\right)}{(b\cos(c+dx))^{3/2}} \right]$$

[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/12*(3*A*sqrt(-b)*cos(d*x+c)*log(2*b*cos(d*x+c)^2+2*sqrt(b*cos(d*x+c))*sqrt(-b)*sqrt(cos(d*x+c))*sin(d*x+c)-b)-2*(2*B*cos(d*x+c)^2+3*A*cos(d*x+c)+4*B)*sqrt(b*cos(d*x+c))*sqrt(cos(d*x+c))*sin(d*x+c))/(b^2*d*cos(d*x+c)),1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x+c))*sin(d*x+c)/(sqrt(b)*cos(d*x+c)^(3/2)))*cos(d*x+c)+(2*B*cos(d*x+c)^2+3*A*cos(d*x+c)+4*B)*sqrt(b*cos(d*x+c))*sqrt(cos(d*x+c))*sin(d*x+c))/(b^2*d*cos(d*x+c))]

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.46

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \frac{\frac{3(2dx+2c+\sin(2dx+2c))A}{b^{\frac{3}{2}}} + \frac{B(\sin(3dx+3c)+9\sin(\frac{1}{3}\arctan(\frac{\sin(3dx+3c)}{\cos(3dx+3c)}))}{b^{\frac{3}{2}}}}{12d}$$

[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/12*(3*(2*d*x+2*c+sin(2*d*x+2*c))*A/b^(3/2)+B*(sin(3*d*x+3*c)+9*sin(1/3*arctan2(sin(3*d*x+3*c),cos(3*d*x+3*c))))/b^(3/2)/d

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^{\frac{7}{2}}}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2), x)

Mupad [B] (verification not implemented)

Time = 15.81 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \frac{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (3 A \sin(c + dx) + 3 A \sin(3c + 3d*x) + 10*B*\sin(2*c + 2*d*x) + B*\sin(4*c + 4*d*x) + 12*A*d*x*\cos(c + d*x))}{12 b^2 d (c + d*x)}$$

[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c + d*x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))

$$3.874 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7434
Rubi [A] (verified)	7434
Mathematica [A] (verified)	7435
Maple [A] (verified)	7435
Fricas [A] (verification not implemented)	7436
Sympy [F(-1)]	7436
Maxima [A] (verification not implemented)	7437
Giac [F]	7437
Mupad [B] (verification not implemented)	7437

Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b \cos(c+dx)}} + \frac{A\sqrt{\cos(c+dx)} \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} + \frac{B \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2bd\sqrt{b \cos(c+dx)}}$$

[Out] $1/2*B*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/b/d/(b*\cos(d*x+c))^{(1/2)}+1/2*B*x*\cos(d*x+c)^{(1/2)}/b/(b*\cos(d*x+c))^{(1/2)}+A*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2813}

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{A \sin(c+dx) \sqrt{\cos(c+dx)}}{bd\sqrt{b \cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b \cos(c+dx)}}$$

[In] $\text{Int}[(\text{Cos}[c + d*x])^{(5/2)}*(A + B*\text{Cos}[c + d*x])]/(b*\text{Cos}[c + d*x])^{(3/2)}, x]$

[Out] $(B*x*\text{Sqrt}[\text{Cos}[c + d*x]])/(2*b*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (A*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (B*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(2*b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_) + (f_.)
*(x_)]), x_Symbol] := Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)(A+B\cos(c+dx)) dx}{b\sqrt{b\cos(c+dx)}} \\ &= \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \frac{\cos^{\frac{3}{2}}(c+dx)(4A\sin(c+dx) + B(2(c+dx) + \sin(2(c+dx))))}{4d(b\cos(c+dx))^{3/2}}$$

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]))
)/(4*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)})(B \sin(dx+c) \cos(dx+c) + 2A \sin(dx+c) + B(dx+c))}{2bd\sqrt{\cos(dx+c)b}}$	58
parts	$\frac{A \sin(dx+c)(\sqrt{\cos(dx+c)})}{bd\sqrt{\cos(dx+c)b}} + \frac{B(\sqrt{\cos(dx+c)})(\cos(dx+c) \sin(dx+c) + dx+c)}{2db\sqrt{\cos(dx+c)b}}$	79
risch	$\frac{Bx(\sqrt{\cos(dx+c)})}{2b\sqrt{\cos(dx+c)b}} + \frac{A \sin(dx+c)(\sqrt{\cos(dx+c)})}{bd\sqrt{\cos(dx+c)b}} + \frac{(\sqrt{\cos(dx+c)})B \sin(2dx+2c)}{4b\sqrt{\cos(dx+c)b}d}$	95

[In] `int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)`

[Out] $1/2/b/d*\cos(d*x+c)^(1/2)*(B*\sin(d*x+c)*\cos(d*x+c)+2*A*\sin(d*x+c)+B*(d*x+c))/(\cos(d*x+c)*b)^(1/2)$

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.96

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \left[-\frac{B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\right)}{(b\cos(c+dx))^{3/2}} \right]$$

[In] `integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `[-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)), 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]`

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)`

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \frac{\frac{(2dx+2c+\sin(2dx+2c))B}{b^{\frac{3}{2}}} + \frac{4A\sin(dx+c)}{b^{\frac{3}{2}}}}{4d}$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(3/2) + 4*A*sin(d*x + c)/b^(3/2))/d

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2), x)

Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \frac{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(B\sin(c+dx)+4A\sin(2c+2dx))}{4b^2d(\cos(2c+2dx))}$$

[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c + 2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*b^2*d*(cos(2*c + 2*d*x) + 1))

$$3.875 \quad \int \frac{\cos^3(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7438
Rubi [A] (verified)	7438
Mathematica [A] (verified)	7439
Maple [A] (verified)	7439
Fricas [A] (verification not implemented)	7440
Sympy [A] (verification not implemented)	7440
Maxima [A] (verification not implemented)	7440
Giac [F]	.7441
Mupad [B] (verification not implemented)	.7441

Optimal result

Integrand size = 33, antiderivative size = 65

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{b \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \sin(c+dx)}{bd \sqrt{b \cos(c+dx)}}$$

[Out] $A*x*\cos(d*x+c)^{(1/2)}/b/(b*\cos(d*x+c))^{(1/2)}+B*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2717}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{b \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \sqrt{\cos(c+dx)}}{bd \sqrt{b \cos(c+dx)}}$$

[In] $\text{Int}[(\text{Cos}[c + d*x])^{(3/2)}*(A + B*\text{Cos}[c + d*x])]/(b*\text{Cos}[c + d*x])^{(3/2)}, x]$

[Out] $(A*x*\text{Sqrt}[\text{Cos}[c + d*x]])/(b*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (B*\text{Sqrt}[\text{Cos}[c + d*x]]*S \text{in}[c + d*x])/(b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 17

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_)}*((b_*)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[a^{(m + 1/2)}*b^{(n - 1/2)}*(\text{Sqrt}[b*v]/\text{Sqrt}[a*v]), \text{Int}[u*v^{(m + n)}, x], x] /; \text{FreeQ}[\{a, b, m\}, x] \&\& !\text{IntegerQ}[m] \&\& \text{IGTQ}[n + 1/2, 0] \&\& \text{IntegerQ}[m + n]$

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int (A + B \cos(c+dx)) dx}{b\sqrt{b \cos(c+dx)}} \\ &= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}} + \frac{(B\sqrt{\cos(c+dx)}) \int \cos(c+dx) dx}{b\sqrt{b \cos(c+dx)}} \\ &= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{\cos^{\frac{3}{2}}(c+dx)(A(c+dx)+B \sin(c+dx))}{d(b \cos(c+dx))^{3/2}}$$

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2), x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(A*(c + d*x) + B*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)})(A(dx+c)+B \sin(dx+c))}{bd\sqrt{\cos(dx+c)b}}$	42
risch	$\frac{Ax(\sqrt{\cos(dx+c)})}{b\sqrt{\cos(dx+c)b}} + \frac{B \sin(dx+c)(\sqrt{\cos(dx+c)})}{bd\sqrt{\cos(dx+c)b}}$	58
parts	$\frac{A(\sqrt{\cos(dx+c)})(dx+c)}{db\sqrt{\cos(dx+c)b}} + \frac{B \sin(dx+c)(\sqrt{\cos(dx+c)})}{bd\sqrt{\cos(dx+c)b}}$	65

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2), x, method=_RETURN
VERBOSE)
```

```
[Out] 1/b/d*cos(d*x+c)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.88

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{\frac{3}{2}}} dx = \left[-\frac{A\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\right)}{(b\cos(c+dx))^{\frac{3}{2}}} + \frac{B\sqrt{b}\sin(dx+c)}{(b\cos(c+dx))^{\frac{3}{2}}} \right]$$

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)), (A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]

Sympy [A] (verification not implemented)

Time = 36.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{\frac{3}{2}}} dx = \begin{cases} \frac{Ax\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}} + \frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(b\cos(c+dx))^{\frac{3}{2}}} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))\cos^{\frac{3}{2}}(c)}{(b\cos(c))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)

[Out] Piecewise((A*x*cos(c + d*x)**(3/2)/(b*cos(c + d*x))**(3/2) + B*sin(c + d*x)*cos(c + d*x)**(3/2)/(d*(b*cos(c + d*x))**(3/2)), Ne(d, 0)), (x*(A + B*cos(c))*cos(c)**(3/2)/(b*cos(c))**(3/2), True))

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{\frac{3}{2}}} dx = \frac{2A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}} + \frac{B\sin(dx+c)}{b^{\frac{3}{2}}}$$

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] (2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2) + B*sin(d*x + c)/b^(3/2))/d

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2), x)

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{3}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \frac{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (B \sin(2c + 2dx) + 2A dx \cos(c + dx))}{b^2 d (\cos(2c + 2dx) + 1)}$$

[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(2*c + 2*d*x) + 2*A*d*x*cos(c + d*x)))/(b^2*d*(cos(2*c + 2*d*x) + 1))

$$3.876 \quad \int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7442
Rubi [A] (verified)	7442
Mathematica [A] (verified)	7443
Maple [A] (verified)	7444
Fricas [A] (verification not implemented)	7444
Sympy [F]	7445
Maxima [A] (verification not implemented)	7445
Giac [F]	7445
Mupad [F(-1)]	7446

Optimal result

Integrand size = 33, antiderivative size = 66

$$\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}} + \frac{A \operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b \cos(c+dx)}}$$

[Out] B*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {17, 2814, 3855}

$$\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{A\sqrt{\cos(c+dx)}\operatorname{arctanh}(\sin(c+dx))}{bd\sqrt{b \cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}}$$

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]

[Out] (B*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]])

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int (A+B \cos(c+dx)) \sec(c+dx) dx}{b\sqrt{b \cos(c+dx)}} \\ &= \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right) \int \sec(c+dx) dx}{b\sqrt{b \cos(c+dx)}} \\ &= \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}} + \frac{A \operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{(b \cos(c+dx))^{3/2}} dx = \frac{(Bdx + A \operatorname{arctanh}(\sin(c+dx))) \cos^{3/2}(c+dx)}{d(b \cos(c+dx))^{3/2}}$$

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Cos[c + d*x]^(3/2))/(d*(b*Cos[c + d*x])^(
3/2))
```

Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{(2A \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c))(\sqrt{\cos(dx+c)})}{bd\sqrt{\cos(dx+c)}b}$	55
parts	$-\frac{2A(\sqrt{\cos(dx+c)}) \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}b} + \frac{B(\sqrt{\cos(dx+c)})(dx+c)}{db\sqrt{\cos(dx+c)}b}$	76
risch	$\frac{Bx(\sqrt{\cos(dx+c)})}{b\sqrt{\cos(dx+c)}b} + \frac{(\sqrt{\cos(dx+c)})A \ln(e^{i(dx+c)}+i)}{b\sqrt{\cos(dx+c)}bd} - \frac{(\sqrt{\cos(dx+c)})A \ln(e^{i(dx+c)}-i)}{b\sqrt{\cos(dx+c)}bd}$	105

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(3/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/b/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c))*cos(d*x+c)^(1/2)/(cos
(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \left[-\frac{2A\sqrt{-b} \arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) + B\sqrt{-b} \log\left(2b\cos\right)}{\dots} \right]$$

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/2*(2*B*sqrt(
b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) +
A*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos
(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b^2*d)]
```


Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \int \frac{(A+B\cos(c+dx))\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}} dx$$

[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)

[Out] Integral((A + B*cos(c + d*x))*sqrt(cos(c + d*x))/(b*cos(c + d*x))**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \frac{A(\log(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1))-\log(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1)}{b^{3/2}} \frac{1}{2d}$$

[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 4*B*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2))/d

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{3/2}} dx$$

[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}} dx$$

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(3/2), x)
```

$$3.877 \quad \int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}} dx$$

Optimal result	.7447
Rubi [A] (verified)	.7447
Mathematica [A] (verified)	.7449
Maple [A] (verified)	.7449
Fricas [A] (verification not implemented)	.7449
Sympy [F]	.7450
Maxima [B] (verification not implemented)	.7450
Giac [F]	.7450
Mupad [F(-1)]	.7451

Optimal result

Integrand size = 33, antiderivative size = 74

$$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}} dx = \frac{B \operatorname{arctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{bd \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{bd \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

[Out] A*sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {18, 2827, 3852, 8, 3855}

$$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}} dx = \frac{A \sin(c+dx)}{bd \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \operatorname{arctanh}(\sin(c+dx))}{bd \sqrt{b \cos(c+dx)}}$$

[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 18

`Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m - 1/2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx)) \sec^2(c + dx) dx}{b\sqrt{b \cos(c + dx)}} \\
 &= \frac{\left(A\sqrt{\cos(c + dx)}\right) \int \sec^2(c + dx) dx}{b\sqrt{b \cos(c + dx)}} + \frac{\left(B\sqrt{\cos(c + dx)}\right) \int \sec(c + dx) dx}{b\sqrt{b \cos(c + dx)}} \\
 &= \frac{\text{Barctanh}(\sin(c + dx))\sqrt{\cos(c + dx)}}{bd\sqrt{b \cos(c + dx)}} - \frac{\left(A\sqrt{\cos(c + dx)}\right) \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{bd\sqrt{b \cos(c + dx)}} \\
 &= \frac{\text{Barctanh}(\sin(c + dx))\sqrt{\cos(c + dx)}}{bd\sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{bd\sqrt{\cos(c + dx)}\sqrt{b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}} dx = \frac{\sqrt{\cos(c + dx)}(B \operatorname{arctanh}(\sin(c + dx)) \cos(c + dx) + A \sin(c + dx))}{d(b \cos(c + dx))^{3/2}}$$

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)), x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{-2B \cos(dx+c) \operatorname{arctanh}(\cot(dx+c) - \csc(dx+c)) + A \sin(dx+c)}{bd \sqrt{\cos(dx+c)} b \sqrt{\cos(dx+c)}}$	60
parts	$\frac{A \sin(dx+c)}{bd \sqrt{\cos(dx+c)} \sqrt{\cos(dx+c)} b} - \frac{2B(\sqrt{\cos(dx+c)}) \operatorname{arctanh}(\cot(dx+c) - \csc(dx+c))}{d \sqrt{\cos(dx+c)} b b}$	77
risch	$\frac{i e^{-i(dx+c)} A}{b \sqrt{\cos(dx+c)} b \sqrt{\cos(dx+c)} d} + \frac{(\sqrt{\cos(dx+c)}) B \ln(e^{i(dx+c)} + i)}{b \sqrt{\cos(dx+c)} b d} - \frac{(\sqrt{\cos(dx+c)}) B \ln(e^{i(dx+c)} - i)}{b \sqrt{\cos(dx+c)} b d}$	118

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(3/2), x, method=_RETURN VERBOSE)
```

```
[Out] 1/b/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.85

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}} dx = \left[\frac{B \sqrt{b} \cos(dx + c)^2 \log\left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c)}{\cos(dx+c)^3}\right)}{2 b^2 d \cos(dx+c)} - \frac{B \sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx+c)^2 - \sqrt{b \cos(dx+c)} A \sqrt{\cos(dx+c)} \sin(dx+c)}{b^2 d \cos(dx+c)^2} \right]$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2), x, algorithm="fricas")
```

```
[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)]
```

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}} dx = \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{\frac{3}{2}} \sqrt{\cos(c + dx)}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2), x)
```

```
[Out] Integral((A + B*cos(c + d*x))/((b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(66) = 132.

Time = 0.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}} dx = \frac{4 A \sqrt{b} \sin(2 dx + 2 c)}{b^2 \cos(2 dx + 2 c)^2 + b^2 \sin(2 dx + 2 c)^2 + 2 b^2 \cos(2 dx + 2 c) + b^2} + \frac{B (\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1))}{2 d}$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")
```

```
[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2))/d
```

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{3}{2}} \sqrt{\cos(dx + c)}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2), x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*sqrt(cos(d*x + c))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)} (b \cos(c + dx))^{3/2}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)} (b \cos(c + dx))^{3/2}} dx$$

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)), x)
```

$$3.878 \quad \int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7452
Rubi [A] (verified)	7452
Mathematica [A] (verified)	7454
Maple [A] (verified)	7454
Fricas [A] (verification not implemented)	7455
Sympy [F(-1)]	7455
Maxima [B] (verification not implemented)	7455
Giac [F]	7456
Mupad [F(-1)]	7456

Optimal result

Integrand size = 33, antiderivative size = 116

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx = \frac{A \operatorname{arctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{2bd \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2bd \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{bd \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

[Out] $1/2*A*\sin(d*x+c)/b/d/\cos(d*x+c)^{(3/2)}/(b*\cos(d*x+c))^{(1/2)}+B*\sin(d*x+c)/b/d/\cos(d*x+c)^{(1/2)}/(b*\cos(d*x+c))^{(1/2)}+1/2*A*\operatorname{arctanh}(\sin(d*x+c))*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {18, 2827, 3853, 3855, 3852, 8}

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx = \frac{A \sqrt{\cos(c+dx)} \operatorname{arctanh}(\sin(c+dx))}{2bd \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2bd \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{bd \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

[In] $\operatorname{Int}[(A+B*\operatorname{Cos}[c+d*x])/(\operatorname{Cos}[c+d*x]^{(3/2)}*(b*\operatorname{Cos}[c+d*x])^{(3/2)}),x]$

[Out] $(A*\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])/(2*b*d*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]]) + (A*\operatorname{Sin}[c+d*x])/(2*b*d*\operatorname{Cos}[c+d*x]^{(3/2)}*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]]) + (B*\operatorname{Sin}[c+d*x])/(b*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]])$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 18

`Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m - 1/2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3853

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx)) \sec^3(c + dx) dx}{b\sqrt{b} \cos(c + dx)} \\
 &= \frac{\left(A\sqrt{\cos(c + dx)}\right) \int \sec^3(c + dx) dx}{b\sqrt{b} \cos(c + dx)} + \frac{\left(B\sqrt{\cos(c + dx)}\right) \int \sec^2(c + dx) dx}{b\sqrt{b} \cos(c + dx)} \\
 &= \frac{A \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{\left(A\sqrt{\cos(c + dx)}\right) \int \sec(c + dx) dx}{2b\sqrt{b} \cos(c + dx)} \\
 &\quad - \frac{\left(B\sqrt{\cos(c + dx)}\right) \text{Subst}\left(\int 1 dx, x, -\tan(c + dx)\right)}{bd\sqrt{b} \cos(c + dx)}
 \end{aligned}$$

$$= \frac{\operatorname{Aarctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{A + B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}} dx = \frac{\operatorname{Aarctanh}(\sin(c+dx))\cos^2(c+dx) + (A + 2B\cos(c+dx))\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}$$

[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)), x]

[Out] (A*ArcTanh[Sin[c + d*x])*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))

Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
default	$\frac{A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)-A(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+2B\sin(dx+c)\cos(dx+c)+A\sin(dx+c)}{2bd\sqrt{\cos(dx+c)}b\cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A(-(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c))}{2db\sqrt{\cos(dx+c)}b\cos(dx+c)^{\frac{3}{2}}} + \frac{B\sin(dx+c)}{bd\sqrt{\cos(dx+c)}\sqrt{b\cos(dx+c)}}$
risch	$-\frac{i(Ae^{2i(dx+c)}-A-4B\cos(dx+c))}{2b\sqrt{\cos(dx+c)}b\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)d} + \frac{(\sqrt{\cos(dx+c)})A\ln(e^{i(dx+c)}+i)}{2b\sqrt{\cos(dx+c)}bd} - \frac{(\sqrt{\cos(dx+c)})A\ln(e^{i(dx+c)}-i)}{2b\sqrt{\cos(dx+c)}bd}$

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(3/2), x, method=_RETURN VERBOSE)

[Out] 1/2/b/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.99

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \frac{\left[\frac{A\sqrt{b} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c)}{\cos(dx+c)^3}\right) + 2(2B \cos(dx+c) + A)\sqrt{b \cos(dx+c)}\sqrt{\cos(dx+c)} \sin(dx+c)}{2b^2 d \cos(dx+c)^3} \right]}{2b^2 d \cos(dx+c)^3}$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. 2(100) = 200.

Time = 0.44 (sec) , antiderivative size = 739, normalized size of antiderivative = 6.37

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \text{Too large to display}$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*
c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x
+ 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b*co
s(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x +
2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b))/d
```

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{\frac{3}{2}}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^{\frac{3}{2}} (b \cos(c + dx))^{\frac{3}{2}}} dx$$

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)), x)
```

$$3.879 \quad \int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx$$

Optimal result	7457
Rubi [A] (verified)	7457
Mathematica [A] (verified)	7459
Maple [A] (verified)	7459
Fricas [A] (verification not implemented)	7460
Sympy [F(-1)]	7460
Maxima [B] (verification not implemented)	7460
Giac [F]	7461
Mupad [F(-1)]	7462

Optimal result

Integrand size = 33, antiderivative size = 157

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx = \frac{B \operatorname{arctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{2bd \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{2bd \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{bd \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{A \sin^3(c+dx)}{3bd \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}}$$

[Out] $1/2*B*\sin(d*x+c)/b/d/\cos(d*x+c)^{(3/2)}/(b*\cos(d*x+c))^{(1/2)}+1/3*A*\sin(d*x+c)^3/b/d/\cos(d*x+c)^{(5/2)}/(b*\cos(d*x+c))^{(1/2)}+A*\sin(d*x+c)/b/d/\cos(d*x+c)^{(1/2)}/(b*\cos(d*x+c))^{(1/2)}+1/2*B*\operatorname{arctanh}(\sin(d*x+c))*\cos(d*x+c)^{(1/2)}/b/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {18, 2827, 3852, 3853, 3855}

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx = \frac{A \sin^3(c+dx)}{3bd \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{bd \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \operatorname{arctanh}(\sin(c+dx))}{2bd \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{2bd \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}}$$

[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x]^3)/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[a^(m - 1/2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 2827

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3852

Int[csc[(c_.) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx)) \sec^4(c + dx) dx}{b\sqrt{b \cos(c + dx)}} \\ &= \frac{\left(A\sqrt{\cos(c + dx)}\right) \int \sec^4(c + dx) dx}{b\sqrt{b \cos(c + dx)}} + \frac{\left(B\sqrt{\cos(c + dx)}\right) \int \sec^3(c + dx) dx}{b\sqrt{b \cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{B \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{\cos(c + dx)} \right) \int \sec(c + dx) dx}{2b \sqrt{b \cos(c + dx)}} \\
&\quad - \frac{\left(A \sqrt{\cos(c + dx)} \right) \text{Subst}\left(\int (1 + x^2) dx, x, -\tan(c + dx)\right)}{bd \sqrt{b \cos(c + dx)}} \\
&= \frac{\text{Barctanh}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2bd \sqrt{b \cos(c + dx)}} + \frac{B \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} \\
&\quad + \frac{A \sin(c + dx)}{bd \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} + \frac{A \sin^3(c + dx)}{3bd \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx) (b \cos(c + dx))^{3/2}} dx = \frac{3 \text{Barctanh}(\sin(c + dx)) \cos^2(c + dx) + 3B \sin(c + dx) + 2A(2 + \cos(c + dx)) \tan(c + dx)}{6d \sqrt{\cos(c + dx)} (b \cos(c + dx))^{3/2}}$$

[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)), x]

[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)]*Tan[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))

Maple [A] (verified)

Time = 5.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

method	result
default	$\frac{-3B(\cos^3(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+3B(\cos^3(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1)+4A \sin(dx+c)(\cos^2(dx+c))+2A \cos(dx+c)}{6bd \sqrt{\cos(dx+c)} b \cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A(2(\cos^2(dx+c)+1) \sin(dx+c)}{3db \sqrt{\cos(dx+c)} b \cos(dx+c)^{\frac{5}{2}}} + \frac{B(-(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1))}{2db \sqrt{\cos(dx+c)} b \cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(3B e^{4i(dx+c)}-3B-16A \cos(dx+c)-8iA \sin(dx+c))}{6b \sqrt{\cos(dx+c)} b \sqrt{\cos(dx+c)} (e^{2i(dx+c)}+1)^2 d} + \frac{(\sqrt{\cos(dx+c)}) B \ln(e^{i(dx+c)}+i)}{2b \sqrt{\cos(dx+c)} b d} - \frac{(\sqrt{\cos(dx+c)}) B \ln(e^{i(dx+c)}-i)}{2b \sqrt{\cos(dx+c)} b d}$

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(3/2), x, method=_RETURNVERBOSE)

[Out] 1/6/b/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.65

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \frac{\left[3 B \sqrt{b} \cos(dx + c)^4 \log\left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b \cos(dx+c)} \sin(dx+c)}{\cos(dx+c)^3}\right) + 3 B \sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx + c)^4 - (4 A \cos(dx + c)^2 + 3 B \cos(dx + c) + 2 A) \sqrt{b \cos(dx + c)} \right]}{6 b^2 d \cos(dx + c)^4}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. 2(135) = 270.

Time = 0.44 (sec) , antiderivative size = 983, normalized size of antiderivative = 6.26

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \text{Too large to display}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")


```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
+ 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2
+ 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 +
18*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos
(4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*
d*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x +
4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) - 3*(4*(sin(4*d*x
+ 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) +
1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
+ 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c
) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x
+ 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*B/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x
+ 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2
+ 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)
*sqrt(b))/d
```

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^{5/2} (b \cos(c + dx))^{3/2}} dx$$

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)), x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)), x)
```

$$3.880 \quad \int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	7463
Rubi [A] (verified)	7463
Mathematica [A] (verified)	7465
Maple [A] (verified)	7465
Fricas [A] (verification not implemented)	7466
Sympy [F(-1)]	7466
Maxima [A] (verification not implemented)	7466
Giac [F]	7467
Mupad [B] (verification not implemented)	7467

Optimal result

Integrand size = 33, antiderivative size = 148

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{2b^2 \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b^2 d \sqrt{b \cos(c+dx)}} - \frac{B \sqrt{\cos(c+dx)} \sin^3(c+dx)}{3b^2 d \sqrt{b \cos(c+dx)}}$$

[Out] $\frac{1}{2}Ax \cos(d*x+c)^{(3/2)} \sin(d*x+c) / b^2/d / (b \cos(d*x+c))^{(1/2)} + \frac{1}{2}Ax \cos(d*x+c)^{(1/2)} / b^2 / (b \cos(d*x+c))^{(1/2)} + B \sin(d*x+c) \cos(d*x+c)^{(1/2)} / b^2/d / (b \cos(d*x+c))^{(1/2)} - \frac{1}{3}B \sin(d*x+c)^3 \cos(d*x+c)^{(1/2)} / b^2/d / (b \cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 2715, 8, 2713}

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{2b^2 \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2b^2 d \sqrt{b \cos(c+dx)}} - \frac{B \sin^3(c+dx) \sqrt{\cos(c+dx)}}{3b^2 d \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]

[Out] (A*x*Sqrt[Cos[c + d*x]]/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int \cos^2(c + dx)(A + B \cos(c + dx)) dx}{b^2 \sqrt{b \cos(c + dx)}} \\ &= \frac{\left(A \sqrt{\cos(c + dx)}\right) \int \cos^2(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{\cos(c + dx)}\right) \int \cos^3(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b^2 d \sqrt{b \cos(c+dx)}} + \frac{\left(A \sqrt{\cos(c+dx)}\right) \int 1 dx}{2b^2 \sqrt{b \cos(c+dx)}} \\
&\quad - \frac{\left(B \sqrt{\cos(c+dx)}\right) \text{Subst}\left(\int (1-x^2) dx, x, -\sin(c+dx)\right)}{b^2 d \sqrt{b \cos(c+dx)}} \\
&= \frac{Ax \sqrt{\cos(c+dx)}}{2b^2 \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} \\
&\quad + \frac{A \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b^2 d \sqrt{b \cos(c+dx)}} - \frac{B \sqrt{\cos(c+dx)} \sin^3(c+dx)}{3b^2 d \sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{\sqrt{\cos(c+dx)}(6Ac+6Adx+9B \sin(c+dx)+3A \sin(2(c+dx)))}{12b^2 d \sqrt{b \cos(c+dx)}}$$

[In] Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2), x]

[Out] (Sqrt[Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*x)]) + B*Sin[3*(c + d*x)])/(12*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] (verified)

Time = 5.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)})(2B \sin(dx+c)(\cos^2(dx+c))+3A \sin(dx+c) \cos(dx+c)+3A(dx+c)+4B \sin(dx+c))}{6b^2 d \sqrt{\cos(dx+c)}b}$	77
parts	$\frac{A(\sqrt{\cos(dx+c)})(\cos(dx+c) \sin(dx+c)+dx+c)}{2d b^2 \sqrt{\cos(dx+c)}b} + \frac{B(2+\cos^2(dx+c)) \sin(dx+c)(\sqrt{\cos(dx+c)})}{3d b^2 \sqrt{\cos(dx+c)}b}$	90
risch	$\frac{Ax(\sqrt{\cos(dx+c)})}{2b^2 \sqrt{\cos(dx+c)}b} + \frac{3B \sin(dx+c)(\sqrt{\cos(dx+c)})}{4b^2 d \sqrt{\cos(dx+c)}b} + \frac{(\sqrt{\cos(dx+c)})B \sin(3dx+3c)}{12b^2 \sqrt{\cos(dx+c)}b d} + \frac{(\sqrt{\cos(dx+c)})A \sin(2dx+2c)}{4b^2 \sqrt{\cos(dx+c)}b d}$	132

[In] int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)

[Out] 1/6/b^2/d*cos(d*x+c)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*sin(d*x+c)*cos(d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.59

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \left[-\frac{3A\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{b\cos(dx+c)}\right)}{(b\cos(c+dx))^{5/2}} \right]$$

[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*B*cos(d*x + c)^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.46

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{\frac{3(2dx+2c+\sin(2dx+2c))A}{b^{\frac{5}{2}}} + \frac{B(\sin(3dx+3c)+9\sin(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c)))}{b^{\frac{5}{2}}}}{12d}$$

[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(5/2))/d

Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^{\frac{9}{2}}}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2), x)

Mupad [B] (verification not implemented)

Time = 16.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{9}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \frac{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (3A \sin(c + dx) + 3A \sin(3c + 3d*x) + 10*B*\sin(2*c + 2*d*x) + B*\sin(4*c + 4*d*x) + 12*A*d*x*\cos(c + d*x))}{12 b^3 d (c + d*x)}$$

[In] int((cos(c + d*x)^(9/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(3*A*sin(c + d*x) + 3*A*sin(3*c + 3*d*x) + 10*B*sin(2*c + 2*d*x) + B*sin(4*c + 4*d*x) + 12*A*d*x*cos(c + d*x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))

$$3.881 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	7468
Rubi [A] (verified)	7468
Mathematica [A] (verified)	7469
Maple [A] (verified)	7469
Fricas [A] (verification not implemented)	7470
Sympy [F(-1)]	7470
Maxima [A] (verification not implemented)	7471
Giac [F]	7471
Mupad [B] (verification not implemented)	7471

Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{Bx \sqrt{\cos(c+dx)}}{2b^2 \sqrt{b \cos(c+dx)}} + \frac{A \sqrt{\cos(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{B \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b^2 d \sqrt{b \cos(c+dx)}}$$

[Out] $1/2*B*\cos(d*x+c)^{(3/2)}*\sin(d*x+c)/b^2/d/(b*\cos(d*x+c))^{(1/2)}+1/2*B*x*\cos(d*x+c)^{(1/2)}/b^2/(b*\cos(d*x+c))^{(1/2)}+A*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/b^2/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2813}

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{A \sin(c+dx) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{Bx \sqrt{\cos(c+dx)}}{2b^2 \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2b^2 d \sqrt{b \cos(c+dx)}}$$

[In] $\text{Int}[(\text{Cos}[c + d*x])^{(7/2)}*(A + B*\text{Cos}[c + d*x])]/(b*\text{Cos}[c + d*x])^{(5/2)}, x]$

[Out] $(B*x*\text{Sqrt}[\text{Cos}[c + d*x]])/(2*b^2*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (A*\text{Sqrt}[\text{Cos}[c + d*x]])*\text{Sin}[c + d*x]/(b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (B*\text{Cos}[c + d*x])^{(3/2)}*\text{Sin}[c + d*x]/(2*b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_) + (f_.)
*(x_)]), x_Symbol] := Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)(A+B\cos(c+dx)) dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{Bx \sqrt{\cos(c+dx)}}{2b^2 \sqrt{b \cos(c+dx)}} + \frac{A \sqrt{\cos(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{B \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2b^2 d \sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.56

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{\sqrt{\cos(c+dx)}(4A\sin(c+dx) + B(2(c+dx) + \sin(2(c+dx))))}{4b^2 d \sqrt{b \cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]))
)/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)})(B \sin(dx+c) \cos(dx+c) + 2A \sin(dx+c) + B(dx+c))}{2b^2 d \sqrt{\cos(dx+c)} b}$	58
parts	$\frac{A \sin(dx+c) (\sqrt{\cos(dx+c)})}{b^2 d \sqrt{\cos(dx+c)} b} + \frac{B (\sqrt{\cos(dx+c)}) (\cos(dx+c) \sin(dx+c) + dx+c)}{2d b^2 \sqrt{\cos(dx+c)} b}$	79
risch	$\frac{Bx (\sqrt{\cos(dx+c)})}{2b^2 \sqrt{\cos(dx+c)} b} + \frac{A \sin(dx+c) (\sqrt{\cos(dx+c)})}{b^2 d \sqrt{\cos(dx+c)} b} + \frac{(\sqrt{\cos(dx+c)}) B \sin(2dx+2c)}{4b^2 \sqrt{\cos(dx+c)} b d}$	95

[In] `int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)`

[Out] $1/2/b^2/d*\cos(d*x+c)^(1/2)*(B*\sin(d*x+c)*\cos(d*x+c)+2*A*\sin(d*x+c)+B*(d*x+c))/(\cos(d*x+c)*b)^(1/2)$

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.96

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \left[-\frac{B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\right)}{(b\cos(c+dx))^{5/2}} \right]$$

[In] `integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `[-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)), 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]`

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)`

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \frac{\frac{(2 dx + 2 c + \sin(2 dx + 2 c))B}{b^{\frac{5}{2}}} + \frac{4 A \sin(dx + c)}{b^{\frac{5}{2}}}}{4 d}$$

[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(5/2) + 4*A*sin(d*x + c)/b^(5/2))/d

Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^{\frac{7}{2}}}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2), x)

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{\cos^{\frac{7}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \frac{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (B \sin(c + dx) + 4 A \sin(2c + 2 dx))}{4 b^3 d (\cos(2c + 2 dx))}$$

[In] int((cos(c + d*x)^(7/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(c + d*x) + 4*A*sin(2*c + 2*d*x) + B*sin(3*c + 3*d*x) + 4*B*d*x*cos(c + d*x)))/(4*b^3*d*(cos(2*c + 2*d*x) + 1))

$$3.882 \quad \int \frac{\cos^5(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	7472
Rubi [A] (verified)	7472
Mathematica [A] (verified)	7473
Maple [A] (verified)	7473
Fricas [A] (verification not implemented)	7474
Sympy [F(-1)]	7474
Maxima [A] (verification not implemented)	7474
Giac [F]	7475
Mupad [B] (verification not implemented)	7475

Optimal result

Integrand size = 33, antiderivative size = 65

$$\int \frac{\cos^5(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}}$$

[Out] $A*x*\cos(d*x+c)^{(1/2)}/b^2/(b*\cos(d*x+c))^{(1/2)}+B*\sin(d*x+c)*\cos(d*x+c)^{(1/2)}/b^2/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {17, 2717}

$$\int \frac{\cos^5(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{Ax \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}}$$

[In] $\text{Int}[(\text{Cos}[c + d*x])^{(5/2)}*(A + B*\text{Cos}[c + d*x])]/(b*\text{Cos}[c + d*x])^{(5/2)}, x]$

[Out] $(A*x*\text{Sqrt}[\text{Cos}[c + d*x]])/(b^2*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/(b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 17

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_)}*((b_*)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[a^{(m + 1/2)}*b^{(n - 1/2)}*(\text{Sqrt}[b*v]/\text{Sqrt}[a*v]), \text{Int}[u*v^{(m + n)}, x], x] /; \text{FreeQ}[\{a, b, m\}, x] \&\& !\text{IntegerQ}[m] \&\& \text{IGTQ}[n + 1/2, 0] \&\& \text{IntegerQ}[m + n]$

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int (A + B \cos(c+dx)) dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{Ax \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}} + \frac{(B \sqrt{\cos(c+dx)}) \int \cos(c+dx) dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{Ax \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{\frac{5}{2}}} dx = \frac{\sqrt{\cos(c+dx)}(A(c+dx)+B\sin(c+dx))}{b^2 d \sqrt{b \cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*(c + d*x) + B*SIN[c + d*x]))/(b^2*d*Sqrt[b*Cos[c + d
*x]])
```

Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{(\sqrt{\cos(dx+c)})(A(dx+c)+B\sin(dx+c))}{b^2 d \sqrt{\cos(dx+c)b}}$	42
risch	$\frac{Ax(\sqrt{\cos(dx+c)})}{b^2 \sqrt{\cos(dx+c)b}} + \frac{B\sin(dx+c)(\sqrt{\cos(dx+c)})}{b^2 d \sqrt{\cos(dx+c)b}}$	58
parts	$\frac{A(\sqrt{\cos(dx+c)})(dx+c)}{d b^2 \sqrt{\cos(dx+c)b}} + \frac{B\sin(dx+c)(\sqrt{\cos(dx+c)})}{b^2 d \sqrt{\cos(dx+c)b}}$	65

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2), x, method=_RETURN
VERBOSE)
```

```
[Out] 1/b^2/d*cos(d*x+c)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.88

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{\frac{5}{2}}} dx = \left[-\frac{A\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\right)}{(b\cos(c+dx))^{\frac{5}{2}}} + \frac{B\sqrt{b}\sin(dx+c)}{(b\cos(c+dx))^{\frac{5}{2}}} + \frac{A\sqrt{b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)\cos(dx+c)}{(b\cos(c+dx))^{\frac{5}{2}}} + \frac{B\sqrt{b}\sin(dx+c)}{(b\cos(c+dx))^{\frac{5}{2}}}\right]$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)), (A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{\frac{5}{2}}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{\frac{5}{2}}} dx = \frac{2A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}} + \frac{B\sin(dx+c)}{b^{\frac{5}{2}}}$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] (2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2) + B*sin(d*x + c)/b^(5/2))/d

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2), x)

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{5}{2}}(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{\frac{5}{2}}} dx = \frac{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (B \sin(2c + 2dx) + 2A dx \cos(c + dx))}{b^3 d (\cos(2c + 2dx) + 1)}$$

[In] int((cos(c + d*x)^(5/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(B*sin(2*c + 2*d*x) + 2*A*d*x*cos(c + d*x)))/(b^3*d*(cos(2*c + 2*d*x) + 1))

$$3.883 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{\frac{5}{2}}} dx$$

Optimal result	7476
Rubi [A] (verified)	7476
Mathematica [A] (verified)	7477
Maple [A] (verified)	7478
Fricas [A] (verification not implemented)	7478
Sympy [F(-1)]	7479
Maxima [A] (verification not implemented)	7479
Giac [F]	7479
Mupad [F(-1)]	7480

Optimal result

Integrand size = 33, antiderivative size = 66

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{\frac{5}{2}}} dx = \frac{Bx \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}} + \frac{A \operatorname{arctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}}$$

[Out] B*x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)+A*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {17, 2814, 3855}

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{\frac{5}{2}}} dx = \frac{A \sqrt{\cos(c+dx)} \operatorname{arctanh}(\sin(c+dx))}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{Bx \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x]^(5/2)),x]

[Out] (B*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 17


```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] := Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c+dx)} \int (A + B \cos(c+dx)) \sec(c+dx) dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{Bx \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}} + \frac{\left(A \sqrt{\cos(c+dx)} \right) \int \sec(c+dx) dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{Bx \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}} + \frac{A \operatorname{arctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A + B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{(Bdx + A \operatorname{arctanh}(\sin(c+dx))) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}}$$

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c
+ d*x]])
```

Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{(2A \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))-B(dx+c))(\sqrt{\cos(dx+c)})}{b^2 d \sqrt{\cos(dx+c)} b}$	55
parts	$-\frac{2A(\sqrt{\cos(dx+c)}) \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d \sqrt{\cos(dx+c)} b b^2} + \frac{B(\sqrt{\cos(dx+c)})(dx+c)}{d b^2 \sqrt{\cos(dx+c)} b}$	76
risch	$\frac{Bx(\sqrt{\cos(dx+c)})}{b^2 \sqrt{\cos(dx+c)} b} + \frac{(\sqrt{\cos(dx+c)}) A \ln(e^{i(dx+c)}+i)}{b^2 \sqrt{\cos(dx+c)} b d} - \frac{(\sqrt{\cos(dx+c)}) A \ln(e^{i(dx+c)}-i)}{b^2 \sqrt{\cos(dx+c)} b d}$	105

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/b^2/d*(2*A*arctanh(cot(d*x+c)-csc(d*x+c))-B*(d*x+c))*cos(d*x+c)^(1/2)/(c
os(d*x+c)*b)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.26

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \left[-\frac{2A\sqrt{-b} \arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) + B\sqrt{-b} \log\left(2b\cos\right)}{2}$$

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/2*(2*B*sqrt(
b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) +
A*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos
(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b^3*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{A(\log(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1)-\log(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1))/b^{\frac{5}{2}}}{2d}$$

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 4*B*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2))/d

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \int \frac{\cos(c+dx)^{3/2}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx$$

```
[In] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

$$3.884 \quad \int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal result	7481
Rubi [A] (verified)	7481
Mathematica [A] (verified)	7483
Maple [A] (verified)	7483
Fricas [A] (verification not implemented)	7483
Sympy [F(-1)]	7484
Maxima [B] (verification not implemented)	7484
Giac [F]	7484
Mupad [F(-1)]	7485

Optimal result

Integrand size = 33, antiderivative size = 74

$$\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{B \operatorname{arctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

[Out] A*sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)+B*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {17, 2827, 3852, 8, 3855}

$$\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \operatorname{arctanh}(\sin(c+dx))}{b^2 d \sqrt{b \cos(c+dx)}}$$

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 17

`Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m + 1/2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx)) \sec^2(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} \\
 &= \frac{\left(A \sqrt{\cos(c + dx)}\right) \int \sec^2(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{\cos(c + dx)}\right) \int \sec(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} \\
 &= \frac{\text{Barctanh}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{b^2 d \sqrt{b \cos(c + dx)}} - \frac{\left(A \sqrt{\cos(c + dx)}\right) \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{b^2 d \sqrt{b \cos(c + dx)}} \\
 &= \frac{\text{Barctanh}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{b^2 d \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{b^2 d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{\cos^{3/2}(c+dx)(B\operatorname{arctanh}(\sin(c+dx))\cos(c+dx)+A\sin(c+dx))}{d(b\cos(c+dx))^{5/2}}$$

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2), x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{-2B\cos(dx+c)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))+A\sin(dx+c)}{b^2d\sqrt{\cos(dx+c)}b\sqrt{\cos(dx+c)}}$	60
parts	$\frac{A\sin(dx+c)}{b^2d\sqrt{\cos(dx+c)}\sqrt{\cos(dx+c)}b} - \frac{2B(\sqrt{\cos(dx+c)})\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}bb^2}$	77
risch	$\frac{2i(\sqrt{\cos(dx+c)})A}{b^2\sqrt{\cos(dx+c)}bd(e^{2i(dx+c)}+1)} + \frac{(\sqrt{\cos(dx+c)})B\ln(e^{i(dx+c)}+i)}{b^2\sqrt{\cos(dx+c)}bd} - \frac{(\sqrt{\cos(dx+c)})B\ln(e^{i(dx+c)}-i)}{b^2\sqrt{\cos(dx+c)}bd}$	122

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(5/2), x, method=_RETURN VERBOSE)
```

```
[Out] 1/b^2/d*(-2*B*cos(d*x+c)*arctanh(cot(d*x+c)-csc(d*x+c))+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \left[\frac{B\sqrt{b}\cos(dx+c)^2 \log\left(-\frac{b\cos(dx+c)^3 - 2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)}{\cos(dx+c)^3}\right)}{2b^3d\cos(dx+c)^2} - \frac{B\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^2 - \sqrt{b\cos(dx+c)}A\sqrt{\cos(dx+c)}\sin(dx+c)}{b^3d\cos(dx+c)^2} \right]$$

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2), x, algorithm="fricas")
```

```
[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(66) = 132.

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{4A\sqrt{b}\sin(2dx+2c)}{b^3\cos(2dx+2c)^2+b^3\sin(2dx+2c)^2+2b^3\cos(2dx+2c)+b^3} + \frac{B(\log(\cos(dx+c)^2+\sin(dx+c)^2)+1)}{2d}$$

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2))/d
```

Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{5/2}} dx$$

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(5/2), x)
```


Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx$$

```
[In] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(5/2), x)
```

$$3.885 \quad \int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}} dx$$

Optimal result	7486
Rubi [A] (verified)	7486
Mathematica [A] (verified)	7488
Maple [A] (verified)	7488
Fricas [A] (verification not implemented)	7489
Sympy [F(-1)]	7489
Maxima [B] (verification not implemented)	7489
Giac [F]	7490
Mupad [F(-1)]	7490

Optimal result

Integrand size = 33, antiderivative size = 116

$$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}} dx = \frac{A \operatorname{arctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{2b^2 d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2b^2 d \cos^{3/2}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

[Out] $1/2*A*\sin(d*x+c)/b^2/d/\cos(d*x+c)^{(3/2)}/(b*\cos(d*x+c))^{(1/2)}+B*\sin(d*x+c)/b^2/d/\cos(d*x+c)^{(1/2)}/(b*\cos(d*x+c))^{(1/2)}+1/2*A*\operatorname{arctanh}(\sin(d*x+c))*\cos(d*x+c)^{(1/2)}/b^2/d/(b*\cos(d*x+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {18, 2827, 3853, 3855, 3852, 8}

$$\int \frac{A+B \cos(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}} dx = \frac{A \sqrt{\cos(c+dx)} \operatorname{arctanh}(\sin(c+dx))}{2b^2 d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2b^2 d \cos^{3/2}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

[In] $\operatorname{Int}[(A+B*\operatorname{Cos}[c+d*x])/(\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*(b*\operatorname{Cos}[c+d*x])^{(5/2)}),x]$

[Out] $(A*\operatorname{ArcTanh}[\operatorname{Sin}[c+d*x]]*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]])/(2*b^2*d*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]]) + (A*\operatorname{Sin}[c+d*x])/(2*b^2*d*\operatorname{Cos}[c+d*x]^{(3/2)}*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]]) + (B*\operatorname{Sin}[c+d*x])/(b^2*d*\operatorname{Sqrt}[\operatorname{Cos}[c+d*x]]*\operatorname{Sqrt}[b*\operatorname{Cos}[c+d*x]])$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 18

`Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[a^(m - 1/2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rule 3852

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3853

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3855

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx)) \sec^3(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} \\
 &= \frac{\left(A \sqrt{\cos(c + dx)} \right) \int \sec^3(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{\cos(c + dx)} \right) \int \sec^2(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} \\
 &= \frac{A \sin(c + dx)}{2b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{\left(A \sqrt{\cos(c + dx)} \right) \int \sec(c + dx) dx}{2b^2 \sqrt{b \cos(c + dx)}} \\
 &\quad - \frac{\left(B \sqrt{\cos(c + dx)} \right) \text{Subst}\left(\int 1 dx, x, -\tan(c + dx)\right)}{b^2 d \sqrt{b \cos(c + dx)}}
 \end{aligned}$$

$$= \frac{A \operatorname{Arctanh}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2b^2 d \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{2b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{B \sin(c + dx)}{b^2 d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)} (b \cos(c + dx))^{5/2}} dx = \frac{\sqrt{\cos(c + dx)} (A \operatorname{Arctanh}(\sin(c + dx)) \cos^2(c + dx) + (A + 2B \cos(c + dx)) \sin(c + dx))}{2d (b \cos(c + dx))^{5/2}}$$

[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)), x]

[Out] (Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))

Maple [A] (verified)

Time = 5.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
default	$\frac{A(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1) - A(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1) + 2B \sin(dx+c) \cos(dx+c) + A \sin(dx+c)}{2b^2 d \sqrt{\cos(dx+c)} \cos(dx+c)^{\frac{3}{2}}}$
parts	$\frac{A(-(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1) + (\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1) + \sin(dx+c))}{2d b^2 \sqrt{\cos(dx+c)} \cos(dx+c)^{\frac{3}{2}}} + \frac{B \sin(dx+c)}{b^2 d \sqrt{\cos(dx+c)}}$
risch	$-\frac{i(A e^{2i(dx+c)} - A - 4B \cos(dx+c))}{2b^2 \sqrt{\cos(dx+c)} \sqrt{\cos(dx+c)} (e^{2i(dx+c)} + 1)d} + \frac{(\sqrt{\cos(dx+c)}) A \ln(e^{i(dx+c)} + i)}{2b^2 \sqrt{\cos(dx+c)} b d} - \frac{(\sqrt{\cos(dx+c)}) A \ln(e^{i(dx+c)} - i)}{2b^2 \sqrt{\cos(dx+c)} b d}$

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURN VERBOSE)

[Out] 1/2/b^2/d*(A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)+1)-A*cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.99

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{5/2}} dx = \frac{\left[\frac{A\sqrt{b} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c)}{\cos(dx+c)^3}\right) + A\sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)}\sqrt{-b} \sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{\cos(dx + c)^3 - (2B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}\sqrt{\cos(dx + c)}} \right]}{2b^3d \cos(dx + c)^3}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)]

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 757 vs. 2(100) = 200.

Time = 0.44 (sec) , antiderivative size = 757, normalized size of antiderivative = 6.53

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{5/2}} dx = \text{Too large to display}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*
c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x
+ 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b^2*
cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*
b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*co
s(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*s
qrt(b))/d
```

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{5/2}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{5/2} \sqrt{\cos(dx + c)}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*sqrt(cos(d*x + c))),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{5/2}} dx = \int \frac{A + B \cos(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{5/2}} dx$$

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)), x)
```

$$3.886 \quad \int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{5/2}} dx$$

Optimal result	.7491
Rubi [A] (verified)	.7491
Mathematica [A] (verified)	.7493
Maple [A] (verified)	.7493
Fricas [A] (verification not implemented)	.7494
Sympy [F(-1)]	.7494
Maxima [B] (verification not implemented)	.7494
Giac [F]	.7495
Mupad [F(-1)]	.7496

Optimal result

Integrand size = 33, antiderivative size = 157

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{5/2}} dx = \frac{B \operatorname{arctanh}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{2b^2 d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{A \sin^3(c+dx)}{3b^2 d \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}}$$

[Out] $\frac{1}{2} B \sin(dx+c) / b^2 / d / \cos(dx+c)^{(3/2)} / (b \cos(dx+c))^{(1/2)} + \frac{1}{3} A \sin(dx+c)^3 / b^2 / d / \cos(dx+c)^{(5/2)} / (b \cos(dx+c))^{(1/2)} + A \sin(dx+c) / b^2 / d / \cos(dx+c)^{(1/2)} / (b \cos(dx+c))^{(1/2)} + \frac{1}{2} B \operatorname{arctanh}(\sin(dx+c)) * \cos(dx+c)^{(1/2)} / b^2 / d / (b \cos(dx+c))^{(1/2)}$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {18, 2827, 3852, 3853, 3855}

$$\int \frac{A+B \cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{5/2}} dx = \frac{A \sin^3(c+dx)}{3b^2 d \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \operatorname{arctanh}(\sin(c+dx))}{2b^2 d \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{2b^2 d \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}}$$

[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x]^3)/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[a^(m - 1/2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 2827

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3852

Int[csc[(c_.) + (d_)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3853

Int[(csc[(c_.) + (d_)*(x_)]*(b_))^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3855

Int[csc[(c_.) + (d_)*(x_)], x_Symbol] := Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx)) \sec^4(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} \\ &= \frac{\left(A \sqrt{\cos(c + dx)} \right) \int \sec^4(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{\cos(c + dx)} \right) \int \sec^3(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} \end{aligned}$$

$$\begin{aligned}
&= \frac{B \sin(c + dx)}{2b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{\left(B \sqrt{\cos(c + dx)} \right) \int \sec(c + dx) dx}{2b^2 \sqrt{b \cos(c + dx)}} \\
&\quad - \frac{\left(A \sqrt{\cos(c + dx)} \right) \text{Subst}\left(\int (1 + x^2) dx, x, -\tan(c + dx)\right)}{b^2 d \sqrt{b \cos(c + dx)}} \\
&= \frac{\text{Barctanh}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2b^2 d \sqrt{b \cos(c + dx)}} + \frac{B \sin(c + dx)}{2b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} \\
&\quad + \frac{A \sin(c + dx)}{b^2 d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} + \frac{A \sin^3(c + dx)}{3b^2 d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx) (b \cos(c + dx))^{5/2}} dx = \frac{\sqrt{\cos(c + dx)} (3 \text{Barctanh}(\sin(c + dx)) \cos^2(c + dx) + 3B \sin(c + dx))}{6d (b \cos(c + dx))^{5/2}}$$

[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)), x]

[Out] (Sqrt[Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*(b*Cos[c + d*x])^(5/2))

Maple [A] (verified)

Time = 5.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

method	result
default	$\frac{-3B(\cos^3(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+3B(\cos^3(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)+1)+4A \sin(dx+c)(\cos^2(dx+c))+6b^2 d \sqrt{\cos(dx+c)b} \cos(dx+c)^{\frac{5}{2}}}{6b^2 d \sqrt{\cos(dx+c)b} \cos(dx+c)^{\frac{5}{2}}}$
parts	$\frac{A(2(\cos^2(dx+c)+1) \sin(dx+c)}{3d b^2 \sqrt{\cos(dx+c)b} \cos(dx+c)^{\frac{5}{2}}} + \frac{B(-(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c)-1)+(\cos^2(dx+c)) \ln(-\cot(dx+c)+\csc(dx+c))+2d b^2 \sqrt{\cos(dx+c)b} \cos(dx+c)^{\frac{3}{2}}}{2d b^2 \sqrt{\cos(dx+c)b} \cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(3B e^{4i(dx+c)} - 3B - 16A \cos(dx+c) - 8iA \sin(dx+c))}{6b^2 \sqrt{\cos(dx+c)b} \sqrt{\cos(dx+c)} (e^{2i(dx+c)} + 1)^2 d} + \frac{(\sqrt{\cos(dx+c)}) B \ln(e^{i(dx+c)} + i)}{2b^2 \sqrt{\cos(dx+c)b} d} - \frac{(\sqrt{\cos(dx+c)}) B \ln(e^{i(dx+c)} - i)}{2b^2 \sqrt{\cos(dx+c)b} d}$

[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(5/2), x, method=_RETURNVERBOSE)

[Out] 1/6/b^2/d*(-3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*B*cos(d*x+c)^3*ln(-cot(d*x+c)+csc(d*x+c)+1)+4*A*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.65

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{5/2}} dx = \frac{\left[\frac{3 B \sqrt{b} \cos(dx + c)^4 \log\left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b \cos(dx+c)} \sin(dx+c)}{\cos(dx+c)^3}\right)}{6 b^3 d \cos(dx + c)^4} + 3 B \sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx + c)^4 - (4 A \cos(dx + c)^2 + 3 B \cos(dx + c) + 2 A) \sqrt{b \cos(dx + c)} \right]}{6 b^3 d \cos(dx + c)^4}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^4)]

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{5/2}} dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. 2(135) = 270.

Time = 0.43 (sec) , antiderivative size = 1033, normalized size of antiderivative = 6.58

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{5/2}} dx = \text{Too large to display}$$

[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
+ 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos(6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c
)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x +
4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)
^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2
*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(
4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(6*d*x +
6*c))*sqrt(b) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x
+ 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - (2*(2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + (2*(2*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*
c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d
*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 4*(cos(
4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((b^2*cos(4*d*x + 4*c)^2 + 4*
b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 +
2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b))/d
```

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{5/2}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{5}{2}} \cos(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{\frac{5}{2}}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^{\frac{3}{2}}(b \cos(c + dx))^{\frac{5}{2}}} dx$$

```
[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)), x)
```

```
[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)), x)
```

$$3.887 \quad \int \cos^2(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

Optimal result	.7497
Rubi [A] (verified)	.7497
Mathematica [A] (verified)	.7498
Maple [F]	.7499
Fricas [F]	.7499
Sympy [F(-1)]	.7499
Maxima [F]	.7500
Giac [F]	.7500
Mupad [F(-1)]	.7500

Optimal result

Integrand size = 31, antiderivative size = 119

$$\int \cos^2(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= -\frac{3A(b \cos(c + dx))^{10/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{10b^3 d \sqrt{\sin^2(c + dx)}} - \frac{3B(b \cos(c + dx))^{13/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{13b^4 d \sqrt{\sin^2(c + dx)}}$$

[Out] $-3/10*A*(b*\cos(d*x+c))^{(10/3)}*\operatorname{hypergeom}([1/2, 5/3], [8/3], \cos(d*x+c)^2)*\sin(d*x+c)/b^3/d/(\sin(d*x+c)^2)^{(1/2)}-3/13*B*(b*\cos(d*x+c))^{(13/3)}*\operatorname{hypergeom}([1/2, 13/6], [19/6], \cos(d*x+c)^2)*\sin(d*x+c)/b^4/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int \cos^2(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= -\frac{3A \sin(c + dx) (b \cos(c + dx))^{10/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right)}{10b^3 d \sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx) (b \cos(c + dx))^{13/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c + dx)\right)}{13b^4 d \sqrt{\sin^2(c + dx)}}$$

[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]

[Out] (-3*A*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*b^4*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{7/3} (A + B \cos(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \cos(c + dx))^{7/3} dx}{b^2} + \frac{B \int (b \cos(c + dx))^{10/3} dx}{b^3} \\ &= -\frac{3A(b \cos(c + dx))^{10/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{10b^3 d \sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{3B(b \cos(c + dx))^{13/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{13b^4 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

$$\int \cos^2(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx = -\frac{3 \cos^2(c + dx) \sqrt[3]{b \cos(c + dx)} \cot(c + dx) (13A \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right) + 10B \cos(c + dx))}{130d}$$

[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]

[Out] $(-3*\text{Cos}[c + d*x]^2*(b*\text{Cos}[c + d*x])^{1/3}*\text{Cot}[c + d*x]*(13*A*\text{Hypergeometric}2F1[1/2, 5/3, 8/3, \text{Cos}[c + d*x]^2] + 10*B*\text{Cos}[c + d*x]*\text{Hypergeometric}2F1[1/2, 13/6, 19/6, \text{Cos}[c + d*x]^2])*\text{Sqrt}[\text{Sin}[c + d*x]^2])/(130*d)$

Maple [F]

$$\int (\cos^2(dx + c)) (\cos(dx + c) b)^{\frac{1}{3}} (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\begin{aligned} & \int \cos^2(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx \\ & = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^2 dx \end{aligned}$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \cos^2(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)

Giac [F]

$$\int \cos^2(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int \cos(c + dx)^2 (b \cos(c + dx))^{1/3} (A + B \cos(c + dx)) dx$$

[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)

$$3.888 \quad \int \cos(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

Optimal result	.7501
Rubi [A] (verified)	.7501
Mathematica [A] (verified)	.7502
Maple [F]	.7503
Fricas [F]	.7503
Sympy [F(-1)]	.7503
Maxima [F]	.7504
Giac [F]	.7504
Mupad [F(-1)]	.7504

Optimal result

Integrand size = 29, antiderivative size = 119

$$\begin{aligned} & \int \cos(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx \\ &= -\frac{3A(b \cos(c + dx))^{7/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7b^2 d \sqrt{\sin^2(c + dx)}} \\ & \quad - \frac{3B(b \cos(c + dx))^{10/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{10b^3 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

[Out] $-3/7*A*(b*\cos(d*x+c))^{(7/3)}*\operatorname{hypergeom}([1/2, 7/6], [13/6], \cos(d*x+c)^2)*\sin(d*x+c)/b^2/d/(\sin(d*x+c)^2)^{(1/2)}-3/10*B*(b*\cos(d*x+c))^{(10/3)}*\operatorname{hypergeom}([1/2, 5/3], [8/3], \cos(d*x+c)^2)*\sin(d*x+c)/b^3/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\begin{aligned} & \int \cos(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx \\ &= -\frac{3A \sin(c + dx) (b \cos(c + dx))^{7/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right)}{7b^2 d \sqrt{\sin^2(c + dx)}} \\ & \quad - \frac{3B \sin(c + dx) (b \cos(c + dx))^{10/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right)}{10b^3 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx}{b} \\ &= \frac{A \int (b \cos(c + dx))^{4/3} dx}{b} + \frac{B \int (b \cos(c + dx))^{7/3} dx}{b^2} \\ &= -\frac{3A(b \cos(c + dx))^{7/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7b^2 d \sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{3B(b \cos(c + dx))^{10/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{10b^3 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \cos(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx = \frac{3(b \cos(c + dx))^{4/3} \cot(c + dx) \left(10A \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) + 7B \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right)\right)}{70bd}$$

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]

[Out] $(-3*(b*\cos[c + d*x])^{(4/3)}*\cot[c + d*x]*(10*A*\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \cos[c + d*x]^2] + 7*B*\cos[c + d*x]*\text{Hypergeometric2F1}[1/2, 5/3, 8/3, \cos[c + d*x]^2])*\sqrt{\sin[c + d*x]^2})/(70*b*d)$

Maple [F]

$$\int \cos(dx + c) (\cos(dx + c) b)^{\frac{1}{3}} (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\begin{aligned} & \int \cos(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx \\ & = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c) dx \end{aligned}$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)]

Timed out.

$$\int \cos(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \cos(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)

Giac [F]

$$\int \cos(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int \cos(c + dx) (b \cos(c + dx))^{1/3} (A + B \cos(c + dx)) dx$$

[In] int(cos(c + d*x)*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)

[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)

3.889 $\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) dx$

Optimal result	7505
Rubi [A] (verified)	7505
Mathematica [A] (verified)	7506
Maple [F]	7507
Fricas [F]	7507
Sympy [F]	7507
Maxima [F]	7507
Giac [F]	7508
Mupad [F(-1)]	7508

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) dx$$

$$= -\frac{3A(b \cos(c + dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4bd\sqrt{\sin^2(c + dx)}} - \frac{3B(b \cos(c + dx))^{7/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7b^2d\sqrt{\sin^2(c + dx)}}$$

[Out] $-3/4*A*(b*\cos(d*x+c))^{(4/3)*\operatorname{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/b/d/(\sin(d*x+c)^2)^{(1/2)}-3/7*B*(b*\cos(d*x+c))^{(7/3)*\operatorname{hypergeom}([1/2, 7/6], [13/6], \cos(d*x+c)^2)*\sin(d*x+c)/b^2/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2827, 2722}

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) dx$$

$$= -\frac{3A \sin(c + dx)(b \cos(c + dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right)}{4bd\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{7/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right)}{7b^2d\sqrt{\sin^2(c + dx)}}$$

[In] $\operatorname{Int}[(b*\operatorname{Cos}[c + d*x])^{(1/3)}*(A + B*\operatorname{Cos}[c + d*x]), x]$

[Out] $(-3A(b\cos[c + dx])^{4/3}\text{Hypergeometric2F1}[1/2, 2/3, 5/3, \cos[c + dx]^2]\sin[c + dx]) / (4bd\sqrt{\sin[c + dx]^2}) - (3B(b\cos[c + dx])^{7/3}\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \cos[c + dx]^2]\sin[c + dx]) / (7b^2d\sqrt{\sin[c + dx]^2})$

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= A \int \sqrt[3]{b \cos(c + dx)} dx + \frac{B \int (b \cos(c + dx))^{4/3} dx}{b} \\ &= -\frac{3A(b \cos(c + dx))^{4/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4bd\sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{3B(b \cos(c + dx))^{7/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7b^2d\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \frac{3\sqrt[3]{b \cos(c + dx)} \cot(c + dx) (7A \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) + 4B \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right))}{28d}$$

[In] Integrate[(b*cos[c + d*x])^(1/3)*(A + B*cos[c + d*x]),x]

[Out] $(-3*(b\cos[c + d*x])^{1/3}\text{Cot}[c + d*x]*(7A\text{Hypergeometric2F1}[1/2, 2/3, 5/3, \cos[c + d*x]^2] + 4B\cos[c + d*x]\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \cos[c + d*x]^2])\sqrt{\sin[c + d*x]^2}) / (28*d)$

Maple [F]

$$\int (\cos(dx + c)b)^{\frac{1}{3}} (A + B \cos(dx + c)) dx$$

[In] int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)

[Out] int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{1}{3}} dx$$

[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)

Sympy [F]

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) dx$$

[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)

[Out] Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x)), x)

Maxima [F]

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{1}{3}} dx$$

[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)

Giac [F]

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{1}{3}} dx$$

[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) dx = \int (b \cos(c + dx))^{1/3} (A + B \cos(c + dx)) dx$$

[In] int((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)

[Out] int((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)

$$3.890 \quad \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec(c + dx) dx$$

Optimal result	7509
Rubi [A] (verified)	7509
Mathematica [A] (verified)	.7511
Maple [F]	.7511
Fricas [F]	.7511
Sympy [F]	7512
Maxima [F]	7512
Giac [F]	7512
Mupad [F(-1)]	7513

Optimal result

Integrand size = 29, antiderivative size = 114

$$\begin{aligned} & \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec(c + dx) dx \\ &= -\frac{3A \sqrt[3]{b \cos(c + dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{d \sqrt{\sin^2(c + dx)}} \\ & \quad - \frac{3B (b \cos(c + dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

[Out] $-3*A*(b*\cos(d*x+c))^{(1/3)}*\operatorname{hypergeom}([1/6, 1/2], [7/6], \cos(d*x+c)^2)*\sin(d*x+c)/d/(\sin(d*x+c)^2)^{(1/2)}-3/4*B*(b*\cos(d*x+c))^{(4/3)}*\operatorname{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/b/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\begin{aligned} & \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec(c + dx) dx \\ &= -\frac{3A \sin(c + dx) \sqrt[3]{b \cos(c + dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right)}{d \sqrt{\sin^2(c + dx)}} \\ & \quad - \frac{3B \sin(c + dx) (b \cos(c + dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right)}{4bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

[In] Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] (-3*A*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx \\
 &= (Ab) \int \frac{1}{(b \cos(c + dx))^{2/3}} dx + B \int \sqrt[3]{b \cos(c + dx)} dx \\
 &= -\frac{3A \sqrt[3]{b \cos(c + dx)} \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{d \sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{3B (b \cos(c + dx))^{4/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4bd \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec(c + dx) dx = \frac{3b \cot(c + dx) \left(4A \operatorname{Hypergeometric2F1} \left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx) \right) + B \cos(c + dx) \operatorname{Hypergeometric2F1} \left(\frac{1}{2}, \right. \right.}{4d(b \cos(c + dx))^{2/3}}$$

[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] (-3*b*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + B*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))

Maple [F]

$$\int (\cos(dx + c) b)^{\frac{1}{3}} (A + B \cos(dx + c)) \sec(dx + c) dx$$

[In] int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)

[Out] int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)

Fricas [F]

$$\int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)

Sympy [F]

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$$

[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)

[Out] Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x))*sec(c + d*x), x)

Maxima [F]

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)

Giac [F]

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec(c + dx) dx$$
$$= \int \frac{(b \cos(c + dx))^{1/3} (A + B \cos(c + dx))}{\cos(c + dx)} dx$$

```
[In] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x), x)
```

3.891 $\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^2(c + dx) dx$

Optimal result	7514
Rubi [A] (verified)	7514
Mathematica [A] (verified)	7516
Maple [F]	7516
Fricas [F]	7516
Sympy [F(-1)]	7517
Maxima [F]	7517
Giac [F]	7517
Mupad [F(-1)]	7518

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{3Ab \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{2d(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}} - \frac{3B \sqrt[3]{b \cos(c + dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{d \sqrt{\sin^2(c + dx)}}$$

[Out] 3/2*A*b*hypergeom([-1/3, 1/2], [2/3], cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*B*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2], [7/6], cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{3Ab \sin(c + dx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)\right)}{2d \sqrt{\sin^2(c + dx)}(b \cos(c + dx))^{2/3}} - \frac{3B \sin(c + dx) \sqrt[3]{b \cos(c + dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right)}{d \sqrt{\sin^2(c + dx)}}$$

[In] Int[(b*cos[c + d*x])^(1/3)*(A + B*cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (3*A*b*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^2 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/3}} dx \\
 &= (Ab^2) \int \frac{1}{(b \cos(c + dx))^{5/3}} dx + (bB) \int \frac{1}{(b \cos(c + dx))^{2/3}} dx \\
 &= \frac{3Ab \text{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{2d(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{3B \sqrt[3]{b \cos(c + dx)} \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{d \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{3b \csc(c + dx) \left(A \operatorname{Hypergeometric2F1} \left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx) \right) - 2B \cos(c + dx) \operatorname{Hypergeometric2F1} \left(\frac{1}{6}, \frac{1}{2}, \right. \right.}{2d(b \cos(c + dx))^{2/3}}$$

[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (3*b*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] - 2*B*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))

Maple [F]

$$\int (\cos(dx + c) b)^{\frac{1}{3}} (A + B \cos(dx + c)) (\sec^2(dx + c)) dx$$

[In] int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)

[Out] int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)

Fricas [F]

$$\int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^2(c + dx) dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\begin{aligned} & \int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^2(c + dx) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^2 dx \end{aligned}$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Giac [F]

$$\begin{aligned} & \int \sqrt[3]{b \cos(c + dx)}(A + B \cos(c + dx)) \sec^2(c + dx) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^2 dx \end{aligned}$$

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\begin{aligned} & \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^2(c + dx) dx \\ &= \int \frac{(b \cos(c + dx))^{1/3} (A + B \cos(c + dx))}{\cos(c + dx)^2} dx \end{aligned}$$

```
[In] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

$$3.892 \quad \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

Optimal result	7519
Rubi [A] (verified)	7519
Mathematica [A] (verified)	7521
Maple [F]	7521
Fricas [F]	7521
Sympy [F(-1)]	7522
Maxima [F]	7522
Giac [F]	7522
Mupad [F(-1)]	7523

Optimal result

Integrand size = 31, antiderivative size = 117

$$\begin{aligned} & \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx \\ &= \frac{3Ab^2 \operatorname{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{5d(b \cos(c + dx))^{5/3} \sqrt{\sin^2(c + dx)}} \\ & \quad + \frac{3bB \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{2d(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

[Out] 3/5*A*b^2*hypergeom([-5/6, 1/2], [1/6], cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)/(sin(d*x+c)^2)^(1/2)+3/2*b*B*hypergeom([-1/3, 1/2], [2/3], cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\begin{aligned} & \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx \\ &= \frac{3Ab^2 \sin(c + dx) \operatorname{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c + dx)\right)}{5d \sqrt{\sin^2(c + dx)} (b \cos(c + dx))^{5/3}} \\ & \quad + \frac{3bB \sin(c + dx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)\right)}{2d \sqrt{\sin^2(c + dx)} (b \cos(c + dx))^{2/3}} \end{aligned}$$

[In] Int[(b*cos[c + d*x])^(1/3)*(A + B*cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] (3*A*b^2*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*(b*cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^3 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{8/3}} dx \\
 &= (Ab^3) \int \frac{1}{(b \cos(c + dx))^{8/3}} dx + (b^2B) \int \frac{1}{(b \cos(c + dx))^{5/3}} dx \\
 &= \frac{3Ab^2 \text{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{5d(b \cos(c + dx))^{5/3} \sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{3bB \text{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{2d(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \frac{3 \sqrt[3]{b \cos(c + dx)} \csc(c + dx) (2A \operatorname{Hypergeometric2F1}(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c + dx)) + 5B \cos(c + dx) \operatorname{Hypergeometric2F1}(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)))}{10d}$$

[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] (3*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(2*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(10*d)

Maple [F]

$$\int (\cos(dx + c) b)^{\frac{1}{3}} (A + B \cos(dx + c)) (\sec^3(dx + c)) dx$$

[In] int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)

[Out] int((cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)

Fricas [F]

$$\int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^3 dx$$

[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)

[Out] Timed out

Maxima [F]

$$\begin{aligned} & \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx \\ &= \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^3 dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)

Giac [F]

$$\begin{aligned} & \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx \\ &= \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^3 dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \sec^3(c + dx) dx$$
$$= \int \frac{(b \cos(c + dx))^{1/3} (A + B \cos(c + dx))}{\cos(c + dx)^3} dx$$

```
[In] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

3.893 $\int \cos^2(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx$

Optimal result	7524
Rubi [A] (verified)	7524
Mathematica [A] (verified)	7525
Maple [F]	7526
Fricas [F]	7526
Sympy [F(-1)]	7526
Maxima [F]	7527
Giac [F]	7527
Mupad [F(-1)]	7527

Optimal result

Integrand size = 31, antiderivative size = 119

$$\int \cos^2(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx =$$

$$\frac{3A(b \cos(c+dx))^{13/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{13b^3 d \sqrt{\sin^2(c+dx)}} +$$

$$\frac{3B(b \cos(c+dx))^{16/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{8}{3}, \frac{11}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{16b^4 d \sqrt{\sin^2(c+dx)}}$$

[Out] -3/13*A*(b*cos(d*x+c))^(13/3)*hypergeom([1/2, 13/6], [19/6], cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)-3/16*B*(b*cos(d*x+c))^(16/3)*hypergeom([1/2, 8/3], [11/3], cos(d*x+c)^2)*sin(d*x+c)/b^4/d/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int \cos^2(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx =$$

$$\frac{3A \sin(c+dx)(b \cos(c+dx))^{13/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c+dx)\right)}{13b^3 d \sqrt{\sin^2(c+dx)}} +$$

$$\frac{3B \sin(c+dx)(b \cos(c+dx))^{16/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{8}{3}, \frac{11}{3}, \cos^2(c+dx)\right)}{16b^4 d \sqrt{\sin^2(c+dx)}}$$

[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]

[Out] (-3*A*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x]/(13*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(16/3)*Hypergeometric2F1[1/2, 8/3, 11/3, Cos[c + d*x]^2]*Sin[c + d*x]/(16*b^4*d*Sqrt[Sin[c + d*x]^2]))

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*SIN[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\int (b \cos(c + dx))^{10/3} (A + B \cos(c + dx)) dx}{b^2} \\
 &= \frac{A \int (b \cos(c + dx))^{10/3} dx}{b^2} + \frac{B \int (b \cos(c + dx))^{13/3} dx}{b^3} \\
 &= -\frac{3A(b \cos(c + dx))^{13/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{13b^3 d \sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{3B(b \cos(c + dx))^{16/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{8}{3}, \frac{11}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{16b^4 d \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

$$\int \cos^2(c + dx) (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx = \frac{3 \cos^2(c + dx) (b \cos(c + dx))^{4/3} \cot(c + dx) (16A \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c + dx)\right) + 13B \cos(2c + 2dx))}{208d}$$

[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]

[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(16*A*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2] + 13*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 8/3, 11/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(208*d)

Maple [F]

$$\int (\cos^2(dx + c)) (\cos(dx + c)b)^{\frac{4}{3}} (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)^2*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int \cos^2(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^4 + A*b*cos(d*x + c)^3)*(b*cos(d*x + c))^(1/3),x)

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \cos^2(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)

Giac [F]

$$\int \cos^2(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int \cos(c + dx)^2 (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx$$

[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)

$$3.894 \quad \int \cos(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx$$

Optimal result	7528
Rubi [A] (verified)	7528
Mathematica [A] (verified)	7529
Maple [F]	7530
Fricas [F]	7530
Sympy [F(-1)]	7530
Maxima [F]	7531
Giac [F]	7531
Mupad [F(-1)]	7531

Optimal result

Integrand size = 29, antiderivative size = 119

$$\int \cos(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx =$$

$$\frac{3A(b \cos(c+dx))^{10/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b \cos(c+dx))^{13/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

[Out] -3/10*A*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3], [8/3], cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)-3/13*B*(b*cos(d*x+c))^(13/3)*hypergeom([1/2, 13/6], [19/6], cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\int \cos(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx =$$

$$\frac{3A \sin(c+dx)(b \cos(c+dx))^{10/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B \sin(c+dx)(b \cos(c+dx))^{13/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]

[Out] (-3*A*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{7/3} (A + B \cos(c + dx)) dx}{b} \\ &= \frac{A \int (b \cos(c + dx))^{7/3} dx}{b} + \frac{B \int (b \cos(c + dx))^{10/3} dx}{b^2} \\ &= -\frac{3A(b \cos(c + dx))^{10/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{10b^2 d \sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{3B(b \cos(c + dx))^{13/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{13b^3 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \cos(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \frac{3(b \cos(c + dx))^{7/3} \cot(c + dx) \left(13A \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right) + 10B \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}, \cos^2(c + dx)\right)\right)}{130bd}$$

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]

[Out] $(-3*(b*\cos[c + d*x])^{7/3}*\cot[c + d*x]*(13*A*\text{Hypergeometric2F1}[1/2, 5/3, 8/3, \cos[c + d*x]^2] + 10*B*\cos[c + d*x]*\text{Hypergeometric2F1}[1/2, 13/6, 19/6, \cos[c + d*x]^2])*Sqrt[\sin[c + d*x]^2])/(130*b*d)$

Maple [F]

$$\int \cos(dx + c) (\cos(dx + c) b)^{\frac{4}{3}} (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int \cos(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3),x)

Sympy [F(-1)]

Timed out.

$$\int \cos(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \cos(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)

Giac [F]

$$\int \cos(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int \cos(c + dx) (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx$$

[In] int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)

[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)

3.895 $\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx$

Optimal result	7532
Rubi [A] (verified)	7532
Mathematica [A] (verified)	7533
Maple [F]	7534
Fricas [F]	7534
Sympy [F(-1)]	7534
Maxima [F]	7534
Giac [F]	7535
Mupad [F(-1)]	7535

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx =$$

$$\frac{3A(b \cos(c + dx))^{7/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7bd\sqrt{\sin^2(c + dx)}} -$$

$$\frac{3B(b \cos(c + dx))^{10/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{10b^2d\sqrt{\sin^2(c + dx)}}$$

[Out] $-3/7*A*(b*\cos(d*x+c))^{(7/3)}*\operatorname{hypergeom}([1/2, 7/6], [13/6], \cos(d*x+c)^2)*\sin(d*x+c)/b/d/(\sin(d*x+c)^2)^{(1/2)}-3/10*B*(b*\cos(d*x+c))^{(10/3)}*\operatorname{hypergeom}([1/2, 5/3], [8/3], \cos(d*x+c)^2)*\sin(d*x+c)/b^2/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2827, 2722}

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx =$$

$$\frac{3A \sin(c + dx)(b \cos(c + dx))^{7/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right)}{7bd\sqrt{\sin^2(c + dx)}} -$$

$$\frac{3B \sin(c + dx)(b \cos(c + dx))^{10/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right)}{10b^2d\sqrt{\sin^2(c + dx)}}$$

[In] $\operatorname{Int}[(b*\operatorname{Cos}[c + d*x])^{(4/3)}*(A + B*\operatorname{Cos}[c + d*x]), x]$

[Out] $(-3A(b\cos[c + dx])^{7/3} \text{Hypergeometric2F1}[1/2, 7/6, 13/6, \cos[c + dx]^2] \sin[c + dx]) / (7bd \sqrt{\sin[c + dx]^2}) - (3B(b\cos[c + dx])^{10/3} \text{Hypergeometric2F1}[1/2, 5/3, 8/3, \cos[c + dx]^2] \sin[c + dx]) / (10b^2 d \sqrt{\sin[c + dx]^2})$

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= A \int (b \cos(c + dx))^{4/3} dx + \frac{B \int (b \cos(c + dx))^{7/3} dx}{b} \\ &= -\frac{3A(b \cos(c + dx))^{7/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7bd \sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{3B(b \cos(c + dx))^{10/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{10b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx = \frac{3(b \cos(c + dx))^{4/3} \cot(c + dx) (10A \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) + 7B \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right))}{70d}$$

[In] Integrate[(b*cos[c + d*x])^(4/3)*(A + B*cos[c + d*x]),x]

[Out] $(-3*(b*\cos[c + d*x])^{4/3}*\cot[c + d*x]*(10*A*\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \cos[c + d*x]^2] + 7*B*\cos[c + d*x]*\text{Hypergeometric2F1}[1/2, 5/3, 8/3, \cos[c + d*x]^2])*Sqrt[\sin[c + d*x]^2])/(70*d)$

Maple [F]

$$\int (\cos(dx + c)b)^{\frac{4}{3}} (A + B \cos(dx + c)) dx$$

[In] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)

[Out] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int (b \cos(c + dx))^{\frac{4}{3}} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{4}{3}} dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{\frac{4}{3}} (A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int (b \cos(c + dx))^{\frac{4}{3}} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{4}{3}} dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{4/3} dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx = \int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx$$

[In] int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)

[Out] int((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)

3.896 $\int (b \cos(c+dx))^{4/3} (A+B \cos(c+dx)) \sec(c+dx) dx$

Optimal result	7536
Rubi [A] (verified)	7536
Mathematica [A] (verified)	7537
Maple [F]	7538
Fricas [F]	7538
Sympy [F(-1)]	7538
Maxima [F]	7539
Giac [F]	7539
Mupad [F(-1)]	7539

Optimal result

Integrand size = 29, antiderivative size = 116

$$\int (b \cos(c+dx))^{4/3} (A+B \cos(c+dx)) \sec(c+dx) dx =$$

$$\frac{3A(b \cos(c+dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}} - \frac{3B(b \cos(c+dx))^{7/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}$$

[Out] $-3/4*A*(b*\cos(d*x+c))^{(4/3)}*\operatorname{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(\sin(d*x+c)^2)^{(1/2)} - 3/7*B*(b*\cos(d*x+c))^{(7/3)}*\operatorname{hypergeom}([1/2, 7/6], [13/6], \cos(d*x+c)^2)*\sin(d*x+c)/b/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\int (b \cos(c+dx))^{4/3} (A+B \cos(c+dx)) \sec(c+dx) dx =$$

$$\frac{3A \sin(c+dx)(b \cos(c+dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}} - \frac{3B \sin(c+dx)(b \cos(c+dx))^{7/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] (-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*SIN[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx \\
 &= (Ab) \int \sqrt[3]{b \cos(c + dx)} dx + B \int (b \cos(c + dx))^{4/3} dx \\
 &= -\frac{3A(b \cos(c + dx))^{4/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4d\sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{3B(b \cos(c + dx))^{7/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7bd\sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.75

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec(c + dx) dx = \frac{3b^3 \sqrt[3]{b \cos(c + dx)} \cot(c + dx) (7A \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) + 4B \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right))}{28d}$$

[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*d)

Maple [F]

$$\int (\cos(dx + c) b)^{\frac{4}{3}} (A + B \cos(dx + c)) \sec(dx + c) dx$$

[In] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)

[Out] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)

Fricas [F]

$$\int (b \cos(c + dx))^{\frac{4}{3}} (A + B \cos(c + dx)) \sec(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{4}{3}} \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{\frac{4}{3}} (A + B \cos(c + dx)) \sec(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)

[Out] Timed out

Maxima [F]

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{4/3} \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)

Giac [F]

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec(c + dx) dx = \int (B \cos(dx + c) + A) (b \cos(dx + c))^{4/3} \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec(c + dx) dx = \int \frac{(b \cos(c + dx))^{4/3} (A + B \cos(c + dx))}{\cos(c + dx)} dx$$

[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x),x)

[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x), x)

$$3.897 \quad \int (b \cos(c+dx))^{4/3} (A+B \cos(c+dx)) \sec^2(c+dx) dx$$

Optimal result	7540
Rubi [A] (verified)	7540
Mathematica [A] (verified)	7542
Maple [F]	7542
Fricas [F]	7542
Sympy [F(-1)]	7543
Maxima [F]	7543
Giac [F]	7543
Mupad [F(-1)]	7544

Optimal result

Integrand size = 31, antiderivative size = 112

$$\int (b \cos(c+dx))^{4/3} (A+B \cos(c+dx)) \sec^2(c+dx) dx =$$

$$\frac{3Ab \sqrt[3]{b \cos(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{d \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B(b \cos(c+dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{4d \sqrt{\sin^2(c+dx)}}$$

[Out] $-3*A*b*(b*\cos(d*x+c))^{(1/3)}*\operatorname{hypergeom}([1/6, 1/2], [7/6], \cos(d*x+c)^2)*\sin(d*x+c)/d/(\sin(d*x+c)^2)^{(1/2)}-3/4*B*(b*\cos(d*x+c))^{(4/3)}*\operatorname{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int (b \cos(c+dx))^{4/3} (A+B \cos(c+dx)) \sec^2(c+dx) dx =$$

$$\frac{3Ab \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c+dx)\right)}{d \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B \sin(c+dx) (b \cos(c+dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c+dx)\right)}{4d \sqrt{\sin^2(c+dx)}}$$

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (-3*A*b*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^2 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx \\
 &= (Ab^2) \int \frac{1}{(b \cos(c + dx))^{2/3}} dx + (bB) \int \sqrt[3]{b \cos(c + dx)} dx \\
 &= -\frac{3Ab \sqrt[3]{b \cos(c + dx)} \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{d \sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{3B(b \cos(c + dx))^{4/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4d \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \frac{3b^2 \cot(c + dx) \left(4A \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) + B \cos(c + dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right)\right)}{4d(b \cos(c + dx))^{2/3}}$$

[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (-3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + B*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))

Maple [F]

$$\int (\cos(dx + c) b)^{4/3} (A + B \cos(dx + c)) (\sec^2(dx + c)) dx$$

[In] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)

[Out] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)

Fricas [F]

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)

[Out] Timed out

Maxima [F]

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)

Giac [F]

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^2(c + dx) dx = \int \frac{(b \cos(c + dx))^{4/3} (A + B \cos(c + dx))}{\cos(c + dx)^2} dx$$

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)
```

$$3.898 \quad \int (b \cos(c+dx))^{4/3} (A+B \cos(c+dx)) \sec^3(c+dx) dx$$

Optimal result	7545
Rubi [A] (verified)	7545
Mathematica [A] (verified)	7547
Maple [F]	7547
Fricas [F]	7547
Sympy [F(-1)]	7548
Maxima [F]	7548
Giac [F]	7548
Mupad [F(-1)]	7549

Optimal result

Integrand size = 31, antiderivative size = 115

$$\int (b \cos(c+dx))^{4/3} (A+B \cos(c+dx)) \sec^3(c+dx) dx = \frac{3Ab^2 \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{2d(b \cos(c+dx))^{2/3} \sqrt{\sin^2(c+dx)}} - \frac{3bB \sqrt[3]{b \cos(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{d \sqrt{\sin^2(c+dx)}}$$

[Out] 3/2*A*b^2*hypergeom([-1/3, 1/2], [2/3], cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*b*B*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2], [7/6], cos(d*x+c)^2)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int (b \cos(c+dx))^{4/3} (A+B \cos(c+dx)) \sec^3(c+dx) dx = \frac{3Ab^2 \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c+dx)\right)}{2d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{2/3}} - \frac{3bB \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c+dx)\right)}{d \sqrt{\sin^2(c+dx)}}$$

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] (3*A*b^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^3 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/3}} dx \\
 &= (Ab^3) \int \frac{1}{(b \cos(c + dx))^{5/3}} dx + (b^2 B) \int \frac{1}{(b \cos(c + dx))^{2/3}} dx \\
 &= \frac{3Ab^2 \text{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{2d(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{3bB \sqrt[3]{b \cos(c + dx)} \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{d \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \frac{3b^2 \csc(c + dx) (A \operatorname{Hypergeometric2F1}(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)) - 2B \cos(c + dx) \operatorname{Hypergeometric2F1}(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)))}{2d(b \cos(c + dx))^{2/3}}$$

[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] (3*b^2*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] - 2*B*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))

Maple [F]

$$\int (\cos(dx + c) b)^{4/3} (A + B \cos(dx + c)) (\sec^3(dx + c)) dx$$

[In] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)

[Out] int((cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)

Fricas [F]

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \sec(dx + c)^3 dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)

[Out] Timed out

Maxima [F]

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \sec(dx + c)^3 dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)

Giac [F]

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \sec(dx + c)^3 dx$$

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) \sec^3(c + dx) dx = \int \frac{(b \cos(c + dx))^{4/3} (A + B \cos(c + dx))}{\cos(c + dx)^3} dx$$

```
[In] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] int(((b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)
```

$$3.899 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx$$

Optimal result	7550
Rubi [A] (verified)	7550
Mathematica [A] (verified)	7551
Maple [F]	7552
Fricas [F]	7552
Sympy [F(-1)]	7552
Maxima [F]	7552
Giac [F]	7553
Mupad [F(-1)]	7553

Optimal result

Integrand size = 31, antiderivative size = 119

$$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx =$$

$$\frac{3A(b \cos(c+dx))^{7/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{7b^3 d \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B(b \cos(c+dx))^{10/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{10b^4 d \sqrt{\sin^2(c+dx)}}$$

[Out] $-3/7*A*(b*\cos(d*x+c))^{(7/3)}*\text{hypergeom}([1/2, 7/6], [13/6], \cos(d*x+c)^2)*\sin(d*x+c)/b^3/d/(\sin(d*x+c)^2)^{(1/2)}-3/10*B*(b*\cos(d*x+c))^{(10/3)}*\text{hypergeom}([1/2, 5/3], [8/3], \cos(d*x+c)^2)*\sin(d*x+c)/b^4/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx =$$

$$\frac{3A \sin(c+dx)(b \cos(c+dx))^{7/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c+dx)\right)}{7b^3 d \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B \sin(c+dx)(b \cos(c+dx))^{10/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c+dx)\right)}{10b^4 d \sqrt{\sin^2(c+dx)}}$$

[In] $\text{Int}[(\text{Cos}[c + d*x]^2*(A + B*\text{Cos}[c + d*x]))/(b*\text{Cos}[c + d*x])^{(2/3)}, x]$

```
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*SIN[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \cos(c + dx))^{4/3} dx}{b^2} + \frac{B \int (b \cos(c + dx))^{7/3} dx}{b^3} \\ &= -\frac{3A(b \cos(c + dx))^{7/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7b^3 d \sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{3B(b \cos(c + dx))^{10/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{10b^4 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \frac{3 \cos^2(c + dx) \cot(c + dx) (10A \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) + 7B \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{3}, \frac{8}{3}, \cos^2(c + dx)\right))}{70d(b \cos(c + dx))^{2/3}}$$

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3), x]
```

[Out] $(-3*\text{Cos}[c + d*x]^2*\text{Cot}[c + d*x]*(10*A*\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \text{Cos}[c + d*x]^2] + 7*B*\text{Cos}[c + d*x]*\text{Hypergeometric2F1}[1/2, 5/3, 8/3, \text{Cos}[c + d*x]^2])*\text{Sqrt}[\text{Sin}[c + d*x]^2])/(70*d*(b*\text{Cos}[c + d*x])^{(2/3)})$

Maple [F]

$$\int \frac{(\cos^2(dx + c))(A + B \cos(dx + c))}{(\cos(dx + c)b)^{\frac{2}{3}}} dx$$

[In] `int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)`

[Out] `int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)`

Fricas [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] `integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")`

[Out] `integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3)/b, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] `integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")`

[Out] `integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)`

Giac [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{2/3}} dx$$

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{\cos(c + dx)^2 (A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx$$

[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3),x)

[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3), x)

$$3.900 \quad \int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx$$

Optimal result	7554
Rubi [A] (verified)	7554
Mathematica [A] (verified)	7555
Maple [F]	7556
Fricas [F]	7556
Sympy [F(-1)]	7556
Maxima [F]	7556
Giac [F]	7557
Mupad [F(-1)]	7557

Optimal result

Integrand size = 29, antiderivative size = 119

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx =$$

$$\frac{3A(b \cos(c+dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{4b^2 d \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B(b \cos(c+dx))^{7/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{7b^3 d \sqrt{\sin^2(c+dx)}}$$

[Out] $-3/4*A*(b*\cos(d*x+c))^{(4/3)}*\operatorname{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/b^2/d/(\sin(d*x+c)^2)^{(1/2)}-3/7*B*(b*\cos(d*x+c))^{(7/3)}*\operatorname{hypergeom}([1/2, 7/6], [13/6], \cos(d*x+c)^2)*\sin(d*x+c)/b^3/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx =$$

$$\frac{3A \sin(c+dx)(b \cos(c+dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c+dx)\right)}{4b^2 d \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B \sin(c+dx)(b \cos(c+dx))^{7/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c+dx)\right)}{7b^3 d \sqrt{\sin^2(c+dx)}}$$

[In] $\operatorname{Int}[(\operatorname{Cos}[c+d*x]*(A+B*\operatorname{Cos}[c+d*x]))/(b*\operatorname{Cos}[c+d*x])^{(2/3)},x]$

[Out] $(-3A(b\cos[c + dx])^{4/3} \text{Hypergeometric2F1}[1/2, 2/3, 5/3, \cos[c + dx]^2] \sin[c + dx]) / (4b^2 d \sqrt{\sin[c + dx]^2}) - (3B(b\cos[c + dx])^{7/3} \text{Hypergeometric2F1}[1/2, 7/6, 13/6, \cos[c + dx]^2] \sin[c + dx]) / (7b^3 d \sqrt{\sin[c + dx]^2})$

Rule 16

$\text{Int}[(u_*)(v_)^{(m_*)}((b_*)(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \ \&\& \ \text{IntegerQ}[m]$

Rule 2722

$\text{Int}[(b_*)\sin[(c_*) + (d_*)(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[\cos[c + dx]*((b*\sin[c + dx])^{(n+1)} / (b*d*(n+1)*\sqrt{\cos[c + dx]^2})) * \text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \sin[c + dx]^2], x] /; \text{FreeQ}\{b, c, d, n, x\} \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 2827

$\text{Int}[(b_*)\sin[(e_*) + (f_*)(x_)]^{(m_*)}((c_*) + (d_*)\sin[(e_*) + (f_*)(x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m, x\}$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx}{b} \\ &= \frac{A \int \sqrt[3]{b \cos(c + dx)} dx}{b} + \frac{B \int (b \cos(c + dx))^{4/3} dx}{b^2} \\ &= -\frac{3A(b \cos(c + dx))^{4/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4b^2 d \sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{3B(b \cos(c + dx))^{7/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7b^3 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\frac{\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \frac{3\sqrt[3]{b \cos(c + dx)} \cot(c + dx) (7A \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) + 4B \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}, \cos^2(c + dx)\right))}{28bd}}$$

[In] $\text{Integrate}[(\cos[c + dx]*(A + B*\cos[c + dx]))/(b*\cos[c + dx])^{(2/3)}, x]$

[Out] $(-3*(b*\cos[c + d*x])^{(1/3)}*\cot[c + d*x]*(7*A*\text{Hypergeometric2F1}[1/2, 2/3, 5/3, \cos[c + d*x]^2] + 4*B*\cos[c + d*x]*\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \cos[c + d*x]^2])*\sqrt{\sin[c + d*x]^2})/(28*b*d)$

Maple [F]

$$\int \frac{\cos(dx + c)(A + B \cos(dx + c))}{(\cos(dx + c)b)^{\frac{2}{3}}} dx$$

[In] `int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)`

[Out] `int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)`

Fricas [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")`

[Out] `integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)/b, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")`

[Out] `integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)`

Giac [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{2/3}} dx$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{\cos(c + dx) (A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx$$

[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3),x)

[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3), x)

3.901 $\int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{2/3}} dx$

Optimal result	7558
Rubi [A] (verified)	7558
Mathematica [A] (verified)	7559
Maple [F]	7560
Fricas [F]	7560
Sympy [F]	7560
Maxima [F]	7560
Giac [F]	7561
Mupad [F(-1)]	7561

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx =$$

$$\frac{3A \sqrt[3]{b \cos(c + dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{bd \sqrt{\sin^2(c + dx)}} -$$

$$\frac{3B(b \cos(c + dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4b^2 d \sqrt{\sin^2(c + dx)}}$$

[Out] $-3*A*(b*\cos(d*x+c))^{(1/3)}*\operatorname{hypergeom}([1/6, 1/2], [7/6], \cos(d*x+c)^2)*\sin(d*x+c)/b/d/(\sin(d*x+c)^2)^{(1/2)}-3/4*B*(b*\cos(d*x+c))^{(4/3)}*\operatorname{hypergeom}([1/2, 2/3], [5/3], \cos(d*x+c)^2)*\sin(d*x+c)/b^2/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2827, 2722}

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx =$$

$$\frac{3A \sin(c + dx) \sqrt[3]{b \cos(c + dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right)}{bd \sqrt{\sin^2(c + dx)}} -$$

$$\frac{3B \sin(c + dx)(b \cos(c + dx))^{4/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right)}{4b^2 d \sqrt{\sin^2(c + dx)}}$$

[In] $\operatorname{Int}[(A + B*\operatorname{Cos}[c + d*x])/(b*\operatorname{Cos}[c + d*x])^{(2/3)}, x]$

[Out] $(-3A(b\cos[c + dx])^{1/3}\text{Hypergeometric2F1}[1/6, 1/2, 7/6, \cos[c + dx]^2] \sin[c + dx]) / (b d \sqrt{\sin[c + dx]^2}) - (3B(b\cos[c + dx])^{4/3}\text{Hypergeometric2F1}[1/2, 2/3, 5/3, \cos[c + dx]^2] \sin[c + dx]) / (4b^2 d \sqrt{\sin[c + dx]^2})$

Rule 2722

$\text{Int}[(b \sin[c + dx] + d(x))^{n-1}, x_Symbol] \rightarrow \text{Simp}[\cos[c + dx] * ((b \sin[c + dx])^{n+1} / (b d (n+1) \sqrt{\cos[c + dx]^2})) * \text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \sin[c + dx]^2], x] /;$ $\text{FreeQ}\{b, c, d, n\}, x$ && $! \text{IntegerQ}[2*n]$

Rule 2827

$\text{Int}[(b \sin[e + f x] + (f x))^{m-1} * ((c + d \sin[e + f x] + (f x))^{n-1}), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b \sin[e + f x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b \sin[e + f x])^{m+1}, x], x] /;$ $\text{FreeQ}\{b, c, d, e, f, m\}, x$

Rubi steps

$$\begin{aligned} \text{integral} &= A \int \frac{1}{(b \cos(c + dx))^{2/3}} dx + \frac{B \int \sqrt[3]{b \cos(c + dx)} dx}{b} \\ &= -\frac{3A \sqrt[3]{b \cos(c + dx)} \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{bd \sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{3B (b \cos(c + dx))^{4/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \frac{3 \cot(c + dx) (4A \text{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) + B \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \cos^2(c + dx)\right))}{4d(b \cos(c + dx))^{2/3}}$$

[In] $\text{Integrate}[(A + B \cos[c + dx]) / (b \cos[c + dx])^{2/3}, x]$

[Out] $(-3 \cot[c + dx] * (4A \text{Hypergeometric2F1}[1/6, 1/2, 7/6, \cos[c + dx]^2] + B \cos[c + dx] * \text{Hypergeometric2F1}[1/2, 2/3, 5/3, \cos[c + dx]^2]) * \sqrt{\sin[c + dx]^2}) / (4d * (b \cos[c + dx])^{2/3})$

Maple [F]

$$\int \frac{A + B \cos(dx + c)}{(\cos(dx + c)b)^{\frac{2}{3}}} dx$$

[In] int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)

[Out] int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3),x)

Fricas [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)

[Out] Integral((A + B*cos(c + d*x))/(b*cos(c + d*x))**(2/3), x)

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(2/3), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{2/3}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx$$

[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(2/3),x)

[Out] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(2/3), x)

$$3.902 \quad \int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{2/3}} dx$$

Optimal result	7562
Rubi [A] (verified)	7562
Mathematica [A] (verified)	7563
Maple [F]	7564
Fricas [F]	7564
Sympy [F]	7564
Maxima [F]	7564
Giac [F]	7565
Mupad [F(-1)]	7565

Optimal result

Integrand size = 29, antiderivative size = 114

$$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{2/3}} dx = \frac{3A \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{2d(b \cos(c+dx))^{2/3} \sqrt{\sin^2(c+dx)}} - \frac{3B \sqrt[3]{b \cos(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{bd \sqrt{\sin^2(c+dx)}}$$

[Out] 3/2*A*hypergeom([-1/3, 1/2], [2/3], cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*B*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2], [7/6], cos(d*x+c)^2)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{2/3}} dx = \frac{3A \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c+dx)\right)}{2d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{2/3}} - \frac{3B \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)}}$$

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3), x]

[Out] (3*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

`Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]`

Rule 2722

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= b \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{5/3}} dx \\
 &= (Ab) \int \frac{1}{(b \cos(c + dx))^{5/3}} dx + B \int \frac{1}{(b \cos(c + dx))^{2/3}} dx \\
 &= \frac{3A \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{2d(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{3B \sqrt[3]{b \cos(c + dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{bd \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \frac{3 \csc(c + dx) \left(A \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)\right) - 2B \cos(c + dx) \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos^2(c + dx)\right] \right) \sqrt{\sin^2(c + dx)}}{2d(b \cos(c + dx))^{2/3}}$$

`[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]`

`[Out] (3*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] - 2*B*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))`

Maple [F]

$$\int \frac{(A + B \cos(dx + c)) \sec(dx + c)}{(\cos(dx + c) b)^{\frac{2}{3}}} dx$$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(2/3), x)

[Out] int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(2/3), x)

Fricas [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3), x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(2/3), x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(b*cos(c + d*x))**(2/3), x)

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{2/3}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx) (b \cos(c + dx))^{2/3}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)), x)

$$3.903 \quad \int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{2/3}} dx$$

Optimal result	7566
Rubi [A] (verified)	7566
Mathematica [A] (verified)	7567
Maple [F]	7568
Fricas [F]	7568
Sympy [F]	7568
Maxima [F]	7568
Giac [F]	7569
Mupad [F(-1)]	7569

Optimal result

Integrand size = 31, antiderivative size = 114

$$\int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{2/3}} dx = \frac{3Ab \operatorname{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{5d(b \cos(c+dx))^{5/3} \sqrt{\sin^2(c+dx)}} + \frac{3B \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{2d(b \cos(c+dx))^{2/3} \sqrt{\sin^2(c+dx)}}$$

[Out] 3/5*A*b*hypergeom([-5/6, 1/2], [1/6], cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(5/3)/(sin(d*x+c)^2)^(1/2)+3/2*B*hypergeom([-1/3, 1/2], [2/3], cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{2/3}} dx = \frac{3Ab \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c+dx)\right)}{5d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{5/3}} + \frac{3B \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c+dx)\right)}{2d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{2/3}}$$

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3), x]

[Out] (3*A*b*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n+1)/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^2 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{8/3}} dx \\
 &= (Ab^2) \int \frac{1}{(b \cos(c + dx))^{8/3}} dx + (bB) \int \frac{1}{(b \cos(c + dx))^{5/3}} dx \\
 &= \frac{3Ab \operatorname{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{5d(b \cos(c + dx))^{5/3} \sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{3B \operatorname{Hypergeometric2F1}\left(-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{2d(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \frac{3b^2 \cot(c + dx) (2A \operatorname{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c + dx)\right) - 10d)}{10d}$$

[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]

[Out] (3*b^2*Cot[c + d*x]*(2*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(8/3))

Maple [F]

$$\int \frac{(A + B \cos(dx + c)) (\sec^2(dx + c))}{(\cos(dx + c) b)^{\frac{2}{3}}} dx$$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(2/3), x)

[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(2/3), x)

Fricas [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3), x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(2/3), x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(b*cos(c + d*x))**(2/3), x)

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{2/3}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^2 (b \cos(c + dx))^{2/3}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)), x)

$$3.904 \quad \int \frac{(A+B \cos(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{2/3}} dx$$

Optimal result	7570
Rubi [A] (verified)	7570
Mathematica [A] (verified)	7571
Maple [F]	7572
Fricas [F]	7572
Sympy [F]	7572
Maxima [F]	7572
Giac [F]	7573
Mupad [F(-1)]	7573

Optimal result

Integrand size = 31, antiderivative size = 117

$$\int \frac{(A+B \cos(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{2/3}} dx = \frac{3Ab^2 \operatorname{Hypergeometric2F1}\left(-\frac{4}{3}, \frac{1}{2}, -\frac{1}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{8d(b \cos(c+dx))^{8/3} \sqrt{\sin^2(c+dx)}} + \frac{3bB \operatorname{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{5d(b \cos(c+dx))^{5/3} \sqrt{\sin^2(c+dx)}}$$

[Out] $\frac{3}{8} A b^2 \operatorname{hypergeom}\left(\left[-\frac{4}{3}, \frac{1}{2}\right], \left[-\frac{1}{3}\right], \cos(d*x+c)^2\right) \sin(d*x+c) / d / (b \cos(d*x+c))^{8/3} / (\sin(d*x+c)^2)^{1/2} + \frac{3}{5} b B \operatorname{hypergeom}\left(\left[-\frac{5}{6}, \frac{1}{2}\right], \left[\frac{1}{6}\right], \cos(d*x+c)^2\right) \sin(d*x+c) / d / (b \cos(d*x+c))^{5/3} / (\sin(d*x+c)^2)^{1/2}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int \frac{(A+B \cos(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{2/3}} dx = \frac{3Ab^2 \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{4}{3}, \frac{1}{2}, -\frac{1}{3}, \cos^2(c+dx)\right)}{8d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{8/3}} + \frac{3bB \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c+dx)\right)}{5d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{5/3}}$$

[In] $\operatorname{Int}\left[\frac{(A+B \cos[c+dx]) \sec^3[c+dx]}{(b \cos[c+dx])^{2/3}}, x\right]$

[Out] $\frac{3 A b^2 \operatorname{Hypergeometric2F1}\left[-\frac{4}{3}, \frac{1}{2}, -\frac{1}{3}, \cos[c+dx]^2\right] \sin[c+dx]}{8 d (b \cos[c+dx])^{8/3} \sqrt{\sin[c+dx]^2}} + \frac{3 b B \operatorname{Hypergeometric2F1}\left[-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos[c+dx]^2\right] \sin[c+dx]}{5 d (b \cos[c+dx])^{5/3} \sqrt{\sin[c+dx]^2}}$

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n+1)/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= b^3 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{11/3}} dx \\ &= (Ab^3) \int \frac{1}{(b \cos(c + dx))^{11/3}} dx + (b^2B) \int \frac{1}{(b \cos(c + dx))^{8/3}} dx \\ &= \frac{3Ab^2 \text{Hypergeometric2F1}\left(-\frac{4}{3}, \frac{1}{2}, -\frac{1}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{8d(b \cos(c + dx))^{8/3} \sqrt{\sin^2(c + dx)}} \\ &\quad + \frac{3bB \text{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{5d(b \cos(c + dx))^{5/3} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \frac{3b^2 \csc(c + dx) (5A \text{Hypergeometric2F1}\left(-\frac{4}{3}, \frac{1}{2}, -\frac{1}{3}, \cos^2(c + dx)\right) + 8B \cos(c + dx) \text{Hypergeometric2F1}\left[-\frac{5}{6}, \frac{1}{2}, \frac{1}{6}, \cos^2(c + dx)\right]) \sqrt{\sin^2(c + dx)}}{40d(b \cos(c + dx))^{8/3}}$$

[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]

[Out] (3*b^2*Csc[c + d*x]*(5*A*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2] + 8*B*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(40*d*(b*Cos[c + d*x])^(8/3))

Maple [F]

$$\int \frac{(A + B \cos(dx + c)) (\sec^3(dx + c))}{(\cos(dx + c) b)^{\frac{2}{3}}} dx$$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(cos(d*x+c)*b)^(2/3), x)

[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(cos(d*x+c)*b)^(2/3), x)

Fricas [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3), x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(2/3), x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(b*cos(c + d*x))**(2/3), x)

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^3}{(b \cos(dx + c))^{2/3}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{2/3}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^3 (b \cos(c + dx))^{2/3}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)), x)

$$3.905 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx$$

Optimal result	7574
Rubi [A] (verified)	7574
Mathematica [A] (verified)	7575
Maple [F]	7576
Fricas [F]	7576
Sympy [F(-1)]	7576
Maxima [F]	7576
Giac [F]	7577
Mupad [F(-1)]	7577

Optimal result

Integrand size = 31, antiderivative size = 119

$$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx =$$

$$\frac{3A(b \cos(c+dx))^{5/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{5b^3 d \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B(b \cos(c+dx))^{8/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4}{3}, \frac{7}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{8b^4 d \sqrt{\sin^2(c+dx)}}$$

[Out] $-3/5*A*(b*\cos(d*x+c))^{(5/3)}*\operatorname{hypergeom}([1/2, 5/6], [11/6], \cos(d*x+c)^2)*\sin(d*x+c)/b^{3/d}/(\sin(d*x+c)^2)^{(1/2)}-3/8*B*(b*\cos(d*x+c))^{(8/3)}*\operatorname{hypergeom}([1/2, 4/3], [7/3], \cos(d*x+c)^2)*\sin(d*x+c)/b^{4/d}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx =$$

$$\frac{3A \sin(c+dx)(b \cos(c+dx))^{5/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c+dx)\right)}{5b^3 d \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B \sin(c+dx)(b \cos(c+dx))^{8/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4}{3}, \frac{7}{3}, \cos^2(c+dx)\right)}{8b^4 d \sqrt{\sin^2(c+dx)}}$$

[In] $\operatorname{Int}[(\operatorname{Cos}[c+d*x])^2*(A+B*\operatorname{Cos}[c+d*x])]/(b*\operatorname{Cos}[c+d*x])^{(4/3)},x]$

[Out] $(-3A(b\cos[c + dx])^{5/3}\text{Hypergeometric2F1}[1/2, 5/6, 11/6, \cos[c + dx]^2]\sin[c + dx]) / (5b^3d\sqrt{\sin[c + dx]^2}) - (3B(b\cos[c + dx])^{8/3}\text{Hypergeometric2F1}[1/2, 4/3, 7/3, \cos[c + dx]^2]\sin[c + dx]) / (8b^4d\sqrt{\sin[c + dx]^2})$

Rule 16

$\text{Int}[(u_*)(v_)^{(m_*)}((b_*)(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

Rule 2722

$\text{Int}[(b_*)\sin[(c_*) + (d_*)(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[\cos[c + dx]*((b*\sin[c + dx])^{(n+1)} / (b*d*(n+1)*\sqrt{\cos[c + dx]^2}))\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \sin[c + dx]^2], x] /; \text{FreeQ}[\{b, c, d, n\}, x] \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 2827

$\text{Int}[(b_*)\sin[(e_*) + (f_*)(x_)]^{(m_*)}((c_*) + (d_*)\sin[(e_*) + (f_*)(x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \cos(c + dx))^{2/3} dx}{b^2} + \frac{B \int (b \cos(c + dx))^{5/3} dx}{b^3} \\ &= -\frac{3A(b \cos(c + dx))^{5/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{5b^3d\sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{3B(b \cos(c + dx))^{8/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4}{3}, \frac{7}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{8b^4d\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \frac{3 \cos^2(c + dx) \cot(c + dx) (8A \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c + dx)\right) + 5B \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4}{3}, \frac{7}{3}, \cos^2(c + dx)\right))}{40d(b \cos(c + dx))^{4/3}}$$

[In] $\text{Integrate}[(\cos[c + dx]^2*(A + B*\cos[c + dx]))/(b*\cos[c + dx])^{4/3}, x]$

[Out] $(-3*\cos[c + d*x]^2*\cot[c + d*x]*(8*A*Hypergeometric2F1[1/2, 5/6, 11/6, \cos[c + d*x]^2] + 5*B*\cos[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, \cos[c + d*x]^2])*Sqrt[\sin[c + d*x]^2])/(40*d*(b*\cos[c + d*x])^{4/3})$

Maple [F]

$$\int \frac{(\cos^2(dx + c))(A + B \cos(dx + c))}{(\cos(dx + c)b)^{\frac{4}{3}}} dx$$

[In] `int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)`

[Out] `int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)`

Fricas [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] `integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")`

[Out] `integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b^2, x)`

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)`

[Out] Timed out

Maxima [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] `integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")`

[Out] `integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)`

Giac [F]

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{4/3}} dx$$

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \int \frac{\cos(c + dx)^2 (A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx$$

[In] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3),x)

[Out] int((cos(c + d*x)^2*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3), x)

$$3.906 \quad \int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx$$

Optimal result	7578
Rubi [A] (verified)	7578
Mathematica [A] (verified)	7579
Maple [F]	7580
Fricas [F]	7580
Sympy [F(-1)]	7580
Maxima [F]	7580
Giac [F]	7581
Mupad [F(-1)]	7581

Optimal result

Integrand size = 29, antiderivative size = 119

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx =$$

$$\frac{3A(b \cos(c+dx))^{2/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{2b^2 d \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B(b \cos(c+dx))^{5/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{5b^3 d \sqrt{\sin^2(c+dx)}}$$

[Out] $-3/2*A*(b*\cos(d*x+c))^{(2/3)}*\operatorname{hypergeom}([1/3, 1/2], [4/3], \cos(d*x+c)^2)*\sin(d*x+c)/b^2/d/(\sin(d*x+c)^2)^{(1/2)}-3/5*B*(b*\cos(d*x+c))^{(5/3)}*\operatorname{hypergeom}([1/2, 5/6], [11/6], \cos(d*x+c)^2)*\sin(d*x+c)/b^3/d/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx =$$

$$\frac{3A \sin(c+dx)(b \cos(c+dx))^{2/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c+dx)\right)}{2b^2 d \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B \sin(c+dx)(b \cos(c+dx))^{5/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c+dx)\right)}{5b^3 d \sqrt{\sin^2(c+dx)}}$$

[In] $\operatorname{Int}[(\operatorname{Cos}[c+d*x]*(A+B*\operatorname{Cos}[c+d*x]))/(b*\operatorname{Cos}[c+d*x])^{(4/3)},x]$

[Out] $(-3A*(b*\cos[c + d*x])^{(2/3)}*Hypergeometric2F1[1/3, 1/2, 4/3, \cos[c + d*x]^2]*\sin[c + d*x])/(2*b^2*d*\sqrt{\sin[c + d*x]^2}) - (3*B*(b*\cos[c + d*x])^{(5/3)}*Hypergeometric2F1[1/2, 5/6, 11/6, \cos[c + d*x]^2]*\sin[c + d*x])/(5*b^3*d*\sqrt{\sin[c + d*x]^2})$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*)^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

Rule 2722

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[\cos[c + d*x]*((b*\sin[c + d*x])^{(n+1)})/(b*d*(n+1)*\sqrt{\cos[c + d*x]^2})*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, \sin[c + d*x]^2], x] /; \text{FreeQ}[\{b, c, d, n\}, x] \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 2827

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)])], x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \frac{A+B \cos(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx}{b} \\ &= \frac{A \int \frac{1}{\sqrt[3]{b \cos(c+dx)}} dx}{b} + \frac{B \int (b \cos(c+dx))^{2/3} dx}{b^2} \\ &= -\frac{3A(b \cos(c+dx))^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{2b^2 d \sqrt{\sin^2(c+dx)}} \\ &\quad - \frac{3B(b \cos(c+dx))^{5/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{5b^3 d \sqrt{\sin^2(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{\cos(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx = \frac{3 \cot(c+dx) \left(5A \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c+dx)\right) + 2B \cos(c+dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \cos^2(c+dx)\right)\right)}{10bd^3 \sqrt[3]{b \cos(c+dx)}}$$

[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]

[Out] (-3*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(10*b*d*(b*Cos[c + d*x])^(1/3))

Maple [F]

$$\int \frac{\cos(dx + c)(A + B \cos(dx + c))}{(\cos(dx + c)b)^{\frac{4}{3}}} dx$$

[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)

[Out] int(cos(d*x+c)*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)

Fricas [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Maxima [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{4/3}} dx$$

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \int \frac{\cos(c + dx) (A + B \cos(c + dx))}{(b \cos(c + dx))^{4/3}} dx$$

[In] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3),x)

[Out] int((cos(c + d*x)*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3), x)

3.907 $\int \frac{A+B \cos(c+dx)}{(b \cos(c+dx))^{4/3}} dx$

Optimal result	7582
Rubi [A] (verified)	7582
Mathematica [A] (verified)	7583
Maple [F]	7584
Fricas [F]	7584
Sympy [F(-1)]	7584
Maxima [F]	7584
Giac [F]	7585
Mupad [F(-1)]	7585

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \frac{3A \operatorname{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}} - \frac{3B(b \cos(c + dx))^{2/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{2b^2 d \sqrt{\sin^2(c + dx)}}$$

[Out] 3*A*hypergeom([-1/6, 1/2], [5/6], cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3/2*B*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2], [4/3], cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2827, 2722}

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \frac{3A \sin(c + dx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(c + dx)\right)}{bd \sqrt{\sin^2(c + dx)} \sqrt[3]{b \cos(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{2/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c + dx)\right)}{2b^2 d \sqrt{\sin^2(c + dx)}}$$

[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(4/3),x]

[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= A \int \frac{1}{(b \cos(c + dx))^{4/3}} dx + \frac{B \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx}{b} \\ &= \frac{3A \text{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{3B(b \cos(c + dx))^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{2b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \frac{3 \cot(c + dx) \left(-2A \text{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(c + dx)\right) + B \cos(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \cos^2(c + dx)\right)\right)}{2d(b \cos(c + dx))^{4/3}}$$

[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(4/3),x]

[Out] (-3*Cot[c + d*x]*(-2*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] + B*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(4/3))

Maple [F]

$$\int \frac{A + B \cos(dx + c)}{(\cos(dx + c)b)^{\frac{4}{3}}} dx$$

[In] int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)

[Out] int((A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3),x)

Fricas [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{\frac{4}{3}}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{\frac{4}{3}}} dx = \text{Timed out}$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Maxima [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{\frac{4}{3}}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{B \cos(dx + c) + A}{(b \cos(dx + c))^{4/3}} dx$$

[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{4/3}} dx$$

[In] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(4/3),x)

[Out] int((A + B*cos(c + d*x))/(b*cos(c + d*x))^(4/3), x)

$$3.908 \quad \int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{4/3}} dx$$

Optimal result	7586
Rubi [A] (verified)	7586
Mathematica [A] (verified)	7587
Maple [F]	7588
Fricas [F]	7588
Sympy [F]	7588
Maxima [F]	7588
Giac [F]	7589
Mupad [F(-1)]	7589

Optimal result

Integrand size = 29, antiderivative size = 114

$$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{4/3}} dx = \frac{3A \operatorname{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{4d(b \cos(c+dx))^{4/3} \sqrt{\sin^2(c+dx)}} + \frac{3B \operatorname{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{bd^3 \sqrt[3]{b \cos(c+dx)} \sqrt{\sin^2(c+dx)}}$$

[Out] $\frac{3}{4}A*\operatorname{hypergeom}([-2/3, 1/2], [1/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\cos(d*x+c))^{4/3}/(\sin(d*x+c)^2)^{(1/2)}+3*B*\operatorname{hypergeom}([-1/6, 1/2], [5/6], \cos(d*x+c)^2)*\sin(d*x+c)/b/d/(b*\cos(d*x+c))^{1/3}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\int \frac{(A+B \cos(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{4/3}} dx = \frac{3A \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b \cos(c+dx))^{4/3}} + \frac{3B \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b \cos(c+dx)}}$$

[In] $\operatorname{Int}[(A+B*\operatorname{Cos}[c+d*x])*Sec[c+d*x]/(b*\operatorname{Cos}[c+d*x])^{4/3},x]$

[Out] $(3*A*\operatorname{Hypergeometric2F1}[-2/3, 1/2, 1/3, \operatorname{Cos}[c+d*x]^2]*\operatorname{Sin}[c+d*x])/(4*d*(b*\operatorname{Cos}[c+d*x])^{4/3}*\operatorname{Sqrt}[\operatorname{Sin}[c+d*x]^2])+(3*B*\operatorname{Hypergeometric2F1}[-1/6, 1/2, 5/6, \operatorname{Cos}[c+d*x]^2]*\operatorname{Sin}[c+d*x])/(b*d*(b*\operatorname{Cos}[c+d*x])^{1/3}*\operatorname{Sqrt}[\operatorname{Sin}[c+d*x]^2])$

Rule 16

`Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]`

Rule 2722

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]`

Rule 2827

`Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= b \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{7/3}} dx \\
 &= (Ab) \int \frac{1}{(b \cos(c + dx))^{7/3}} dx + B \int \frac{1}{(b \cos(c + dx))^{4/3}} dx \\
 &= \frac{3A \operatorname{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4d(b \cos(c + dx))^{4/3} \sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{3B \operatorname{Hypergeometric2F1}\left(-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{bd^3 \sqrt{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \frac{3b \cot(c + dx) \left(A \operatorname{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c + dx)\right) + 4 \right)}{4d(b \cos(c + dx))^{4/3}}$$

`[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]`

`[Out] (3*b*Cot[c + d*x]*(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(7/3))`

Maple [F]

$$\int \frac{(A + B \cos(dx + c)) \sec(dx + c)}{(\cos(dx + c)b)^{\frac{4}{3}}} dx$$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(4/3), x)

[Out] int((A+B*cos(d*x+c))*sec(d*x+c)/(cos(d*x+c)*b)^(4/3), x)

Fricas [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3), x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*cos(d*x + c)^2), x)

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(4/3), x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(b*cos(c + d*x))**(4/3), x)

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{4/3}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx) (b \cos(c + dx))^{4/3}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)), x)

$$3.909 \quad \int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx$$

Optimal result	7590
Rubi [A] (verified)	7590
Mathematica [A] (verified)	7591
Maple [F]	7592
Fricas [F]	7592
Sympy [F]	7592
Maxima [F]	7592
Giac [F]	7593
Mupad [F(-1)]	7593

Optimal result

Integrand size = 31, antiderivative size = 114

$$\int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx = \frac{3Ab \operatorname{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{7d(b \cos(c+dx))^{7/3} \sqrt{\sin^2(c+dx)}} + \frac{3B \operatorname{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{4d(b \cos(c+dx))^{4/3} \sqrt{\sin^2(c+dx)}}$$

[Out] $\frac{3/7*A*b*\operatorname{hypergeom}([-7/6, 1/2], [-1/6], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\cos(d*x+c))^{7/3}/(\sin(d*x+c)^2)^{1/2}+3/4*B*\operatorname{hypergeom}([-2/3, 1/2], [1/3], \cos(d*x+c)^2)*\sin(d*x+c)/d/(b*\cos(d*x+c))^{4/3}/(\sin(d*x+c)^2)^{1/2}}$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int \frac{(A+B \cos(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx = \frac{3Ab \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b \cos(c+dx))^{7/3}} + \frac{3B \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b \cos(c+dx))^{4/3}}$$

[In] $\operatorname{Int}[(A+B*\operatorname{Cos}[c+d*x])*Sec[c+d*x]^2/(b*\operatorname{Cos}[c+d*x])^{4/3},x]$

[Out] $(3*A*b*\operatorname{Hypergeometric2F1}[-7/6, 1/2, -1/6, \operatorname{Cos}[c+d*x]^2]*\operatorname{Sin}[c+d*x])/(7*d*(b*\operatorname{Cos}[c+d*x])^{7/3}*\operatorname{Sqrt}[\operatorname{Sin}[c+d*x]^2]) + (3*B*\operatorname{Hypergeometric2F1}[-2/3, 1/2, 1/3, \operatorname{Cos}[c+d*x]^2]*\operatorname{Sin}[c+d*x])/(4*d*(b*\operatorname{Cos}[c+d*x])^{4/3}*\operatorname{Sqrt}[\operatorname{Sin}[c+d*x]^2])$

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n+1)/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^2 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{10/3}} dx \\
 &= (Ab^2) \int \frac{1}{(b \cos(c + dx))^{10/3}} dx + (bB) \int \frac{1}{(b \cos(c + dx))^{7/3}} dx \\
 &= \frac{3Ab \operatorname{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7d(b \cos(c + dx))^{7/3} \sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{3B \operatorname{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{4d(b \cos(c + dx))^{4/3} \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \frac{3b^2 \cot(c + dx) (4A \operatorname{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos^2(c + dx)\right) + 7B \cos(c + dx) \operatorname{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c + dx)\right) \sqrt{\sin^2(c + dx)})}{28d}$$

[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]

[Out] (3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(10/3))

Maple [F]

$$\int \frac{(A + B \cos(dx + c)) (\sec^2(dx + c))}{(\cos(dx + c) b)^{\frac{4}{3}}} dx$$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(4/3), x)

[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(cos(d*x+c)*b)^(4/3), x)

Fricas [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3), x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*cos(d*x + c)^2), x)

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(4/3), x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**2/(b*cos(c + d*x))**(4/3), x)

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{4/3}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^2 (b \cos(c + dx))^{4/3}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)), x)

$$3.910 \quad \int \frac{(A+B \cos(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{4/3}} dx$$

Optimal result	7594
Rubi [A] (verified)	7594
Mathematica [A] (verified)	7595
Maple [F]	7596
Fricas [F]	7596
Sympy [F]	7596
Maxima [F]	7596
Giac [F]	7597
Mupad [F(-1)]	7597

Optimal result

Integrand size = 31, antiderivative size = 117

$$\int \frac{(A+B \cos(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{4/3}} dx = \frac{3Ab^2 \operatorname{Hypergeometric2F1}\left(-\frac{5}{3}, \frac{1}{2}, -\frac{2}{3}, \cos^2(c+dx)\right) \sin(c+dx)}{10d(b \cos(c+dx))^{10/3} \sqrt{\sin^2(c+dx)}} \\ + \frac{3bB \operatorname{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos^2(c+dx)\right) \sin(c+dx)}{7d(b \cos(c+dx))^{7/3} \sqrt{\sin^2(c+dx)}}$$

[Out] 3/10*A*b^2*hypergeom([-5/3, 1/2], [-2/3], cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(10/3)/(sin(d*x+c)^2)^(1/2)+3/7*b*B*hypergeom([-7/6, 1/2], [-1/6], cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(7/3)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 2827, 2722}

$$\int \frac{(A+B \cos(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{4/3}} dx = \frac{3Ab^2 \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{5}{3}, \frac{1}{2}, -\frac{2}{3}, \cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b \cos(c+dx))^{10/3}} \\ + \frac{3bB \sin(c+dx) \operatorname{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b \cos(c+dx))^{7/3}}$$

[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*A*b^2*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*d*(b*Cos[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

`Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]`

Rule 2722

`Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]`

Rule 2827

`Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^3 \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{13/3}} dx \\
 &= (Ab^3) \int \frac{1}{(b \cos(c + dx))^{13/3}} dx + (b^2 B) \int \frac{1}{(b \cos(c + dx))^{10/3}} dx \\
 &= \frac{3Ab^2 \text{Hypergeometric2F1}\left(-\frac{5}{3}, \frac{1}{2}, -\frac{2}{3}, \cos^2(c + dx)\right) \sin(c + dx)}{10d(b \cos(c + dx))^{10/3} \sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{3bB \text{Hypergeometric2F1}\left(-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos^2(c + dx)\right) \sin(c + dx)}{7d(b \cos(c + dx))^{7/3} \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \frac{3b^2 \csc(c + dx) (7A \text{Hypergeometric2F1}\left(-\frac{5}{3}, \frac{1}{2}, -\frac{2}{3}, \cos^2(c + dx)\right) + 10B \cos(c + dx) \text{Hypergeometric2F1}\left[-\frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \cos^2(c + dx)\right]) \sqrt{\sin^2(c + dx)}}{70d(b \cos(c + dx))^{10/3}}$$

`[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]`
`[Out] (3*b^2*Csc[c + d*x]*(7*A*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2] + 10*B*Cos[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(70*d*(b*Cos[c + d*x])^(10/3))`

Maple [F]

$$\int \frac{(A + B \cos(dx + c)) (\sec^3(dx + c))}{(\cos(dx + c) b)^{\frac{4}{3}}} dx$$

[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(cos(d*x+c)*b)^(4/3), x)

[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(cos(d*x+c)*b)^(4/3), x)

Fricas [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{\frac{4}{3}}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3), x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2*cos(d*x + c)^2), x)

Sympy [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{\frac{4}{3}}} dx = \int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(4/3), x)

[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)**3/(b*cos(c + d*x))**(4/3), x)

Maxima [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{\frac{4}{3}}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{(B \cos(dx + c) + A) \sec(dx + c)^3}{(b \cos(dx + c))^{4/3}} dx$$

[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B \cos(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{4/3}} dx = \int \frac{A + B \cos(c + dx)}{\cos(c + dx)^3 (b \cos(c + dx))^{4/3}} dx$$

[In] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)),x)

[Out] int((A + B*cos(c + d*x))/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)), x)

3.911 $\int \cos^m(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$

Optimal result	7598
Rubi [A] (verified)	7598
Mathematica [A] (verified)	7600
Maple [F]	7600
Fricas [F]	7600
Sympy [F]	7601
Maxima [F]	7601
Giac [F]	7601
Mupad [F(-1)]	7602

Optimal result

Integrand size = 29, antiderivative size = 157

$$\int \cos^m(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx =$$

$$\frac{A \cos^{1+m}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1+m+n), \frac{1}{2}(3+m+n), \cos^2(c+dx)\right) \sin(c+dx)}{d(1+m+n)\sqrt{\sin^2(c+dx)}} +$$

$$\frac{B \cos^{2+m}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(2+m+n), \frac{1}{2}(4+m+n), \cos^2(c+dx)\right) \sin(c+dx)}{d(2+m+n)\sqrt{\sin^2(c+dx)}}$$

[Out] $-A*\cos(d*x+c)^{(1+m)}*(b*\cos(d*x+c))^n*\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}+1/2*m+1/2*n\right], \left[\frac{3}{2}+1/2*m+1/2*n\right], \cos(d*x+c)^2\right)*\sin(d*x+c)/d/(1+m+n)/(\sin(d*x+c)^2)^{(1/2)}-B*\cos(d*x+c)^{(2+m)}*(b*\cos(d*x+c))^n*\operatorname{hypergeom}\left(\left[\frac{1}{2}, 1+1/2*m+1/2*n\right], \left[2+1/2*m+1/2*n\right], \cos(d*x+c)^2\right)*\sin(d*x+c)/d/(2+m+n)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {20, 2827, 2722}

$$\int \cos^m(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx =$$

$$\frac{A \sin(c+dx) \cos^{m+1}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(m+n+1), \frac{1}{2}(m+n+3), \cos^2(c+dx)\right)}{d(m+n+1)\sqrt{\sin^2(c+dx)}} +$$

$$\frac{B \sin(c+dx) \cos^{m+2}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(m+n+2), \frac{1}{2}(m+n+4), \cos^2(c+dx)\right)}{d(m+n+2)\sqrt{\sin^2(c+dx)}}$$

[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] -((A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2*Sin[c + d*x]]/(d*(1 + m + n)*Sqrt[Sin[c + d*x]^2])) - (B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2*Sin[c + d*x]]/(d*(2 + m + n)*Sqrt[Sin[c + d*x]^2]))

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{m+n}(c + dx)(A + B \cos(c + dx)) dx \\
 &= (A \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{m+n}(c + dx) dx \\
 &\quad + (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{1+m+n}(c + dx) dx \\
 &= \frac{A \cos^{1+m}(c + dx)(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1 + m + n), \frac{1}{2}(3 + m + n), \cos^2(c + dx)\right)}{d(1 + m + n)\sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{B \cos^{2+m}(c + dx)(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(2 + m + n), \frac{1}{2}(4 + m + n), \cos^2(c + dx)\right)}{d(2 + m + n)\sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \cos^m(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx =$$

$$\frac{\cos^{1+m}(c + dx)(b \cos(c + dx))^n \csc(c + dx) (A(2 + m + n) \text{Hypergeometric2F1}(\frac{1}{2}, \frac{1}{2}(1 + m + n), \frac{1}{2}(3 +$$

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] -((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(2 + m + n)*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2] + B*(1 + m + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m + n)*(2 + m + n))

Maple [F]

$$\int (\cos^m(dx + c)) (\cos(dx + c)b)^n (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)

Sympy [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= \int (b \cos(c + dx))^n(A + B \cos(c + dx)) \cos^m(c + dx) dx$$

[In] `integrate(cos(d*x+c)**m*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)`

[Out] `Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*cos(c + d*x)**m, x)`

Maxima [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^m dx$$

[In] `integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)`

Giac [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^m dx$$

[In] `integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)`

Mupad [F(-1)]

Timed out.

$$\begin{aligned} & \int \cos^m(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx \\ &= \int \cos(c + dx)^m (b \cos(c + dx))^n (A + B \cos(c + dx)) dx \end{aligned}$$

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

$$3.912 \quad \int \cos^2(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

Optimal result	7603
Rubi [A] (verified)	7603
Mathematica [A] (verified)	7604
Maple [F]	7605
Fricas [F]	7605
Sympy [F(-1)]	7605
Maxima [F]	7606
Giac [F]	7606
Mupad [F(-1)]	7606

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \cos^2(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= -\frac{A(b \cos(c + dx))^{3+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{b^3 d(3+n) \sqrt{\sin^2(c + dx)}} - \frac{B(b \cos(c + dx))^{4+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4+n}{2}, \frac{6+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{b^4 d(4+n) \sqrt{\sin^2(c + dx)}}$$

[Out] $-A*(b*\cos(d*x+c))^{(3+n)}*\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{2}+1/2*n\right], \left[\frac{5}{2}+1/2*n\right], \cos(d*x+c)^2\right)*\sin(d*x+c)/b^3/d/(3+n)/(\sin(d*x+c)^2)^{(1/2)}-B*(b*\cos(d*x+c))^{(4+n)}*\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{2}{2}+1/2*n\right], \left[\frac{3}{2}+1/2*n\right], \cos(d*x+c)^2\right)*\sin(d*x+c)/b^4/d/(4+n)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\int \cos^2(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= -\frac{A \sin(c + dx)(b \cos(c + dx))^{n+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+3}{2}, \frac{n+5}{2}, \cos^2(c + dx)\right)}{b^3 d(n+3) \sqrt{\sin^2(c + dx)}} - \frac{B \sin(c + dx)(b \cos(c + dx))^{n+4} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+4}{2}, \frac{n+6}{2}, \cos^2(c + dx)\right)}{b^4 d(n+4) \sqrt{\sin^2(c + dx)}}$$

[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] -((A*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int (b \cos(c + dx))^{2+n} (A + B \cos(c + dx)) dx}{b^2} \\ &= \frac{A \int (b \cos(c + dx))^{2+n} dx}{b^2} + \frac{B \int (b \cos(c + dx))^{3+n} dx}{b^3} \\ &= -\frac{A(b \cos(c + dx))^{3+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{b^3 d (3 + n) \sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{B(b \cos(c + dx))^{4+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4+n}{2}, \frac{6+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{b^4 d (4 + n) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int \cos^2(c + dx) (b \cos(c + dx))^n (A + B \cos(c + dx)) dx = \frac{\cos^2(c + dx) (b \cos(c + dx))^n \cot(c + dx) (A(4 + n) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos^2(c + dx)\right) + B(4 + n) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{4+n}{2}, \frac{6+n}{2}, \cos^2(c + dx)\right))}{d(3 + n)(4 + n)}$$

[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(4 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + B*(3 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(3 + n)*(4 + n))

Maple [F]

$$\int (\cos^2(dx + c)) (\cos(dx + c)b)^n (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)^2*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)^2*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\begin{aligned} & \int \cos^2(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx)) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^2 dx \end{aligned}$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\int \cos^2(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)

Giac [F]

$$\int \cos^2(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^2 dx$$

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= \int \cos(c + dx)^2 (b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)

[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)

3.913 $\int \cos(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$

Optimal result	7607
Rubi [A] (verified)	7607
Mathematica [A] (verified)	7608
Maple [F]	7609
Fricas [F]	7609
Sympy [F]	7609
Maxima [F]	7610
Giac [F]	7610
Mupad [F(-1)]	7610

Optimal result

Integrand size = 27, antiderivative size = 141

$$\int \cos(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= -\frac{A(b \cos(c + dx))^{2+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{b^2 d(2+n) \sqrt{\sin^2(c + dx)}} - \frac{B(b \cos(c + dx))^{3+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{b^3 d(3+n) \sqrt{\sin^2(c + dx)}}$$

[Out] -A*(b*cos(d*x+c))^(2+n)*hypergeom([1/2, 1+1/2*n], [2+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(3+n)*hypergeom([1/2, 3/2+1/2*n], [5/2+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {16, 2827, 2722}

$$\int \cos(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= -\frac{A \sin(c + dx)(b \cos(c + dx))^{n+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+2}{2}, \frac{n+4}{2}, \cos^2(c + dx)\right)}{b^2 d(n+2) \sqrt{\sin^2(c + dx)}} - \frac{B \sin(c + dx)(b \cos(c + dx))^{n+3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+3}{2}, \frac{n+5}{2}, \cos^2(c + dx)\right)}{b^3 d(n+3) \sqrt{\sin^2(c + dx)}}$$

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] -((A*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\int (b \cos(c + dx))^{1+n} (A + B \cos(c + dx)) dx}{b} \\
 &= \frac{A \int (b \cos(c + dx))^{1+n} dx}{b} + \frac{B \int (b \cos(c + dx))^{2+n} dx}{b^2} \\
 &= -\frac{A(b \cos(c + dx))^{2+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{b^2 d(2 + n) \sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{B(b \cos(c + dx))^{3+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{b^3 d(3 + n) \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int \cos(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx)) dx = \frac{\cos(c + dx)(b \cos(c + dx))^n \cot(c + dx) (A(3 + n) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \cos^2(c + dx)\right) + B(2+n) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos^2(c + dx)\right))}{d(2 + n)(3 + n)}$$

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + B*(2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + n)*(3 + n))

Maple [F]

$$\int \cos(dx + c) (\cos(dx + c) b)^n (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\begin{aligned} & \int \cos(c + dx) (b \cos(c + dx))^n (A + B \cos(c + dx)) dx \\ &= \int (B \cos(dx + c) + A) (b \cos(dx + c))^n \cos(dx + c) dx \end{aligned}$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^n, x)

Sympy [F]

$$\begin{aligned} & \int \cos(c + dx) (b \cos(c + dx))^n (A + B \cos(c + dx)) dx \\ &= \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \cos(c + dx) dx \end{aligned}$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)

[Out] Integral((b*cos(c + d*x))^n*(A + B*cos(c + d*x))*cos(c + d*x), x)

Maxima [F]

$$\int \cos(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)

Giac [F]

$$\int \cos(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c) dx$$

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int \cos(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

$$= \int \cos(c + dx) (b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

[In] int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)

[Out] int(cos(c + d*x)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)

3.914 $\int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx$

Optimal result	7611
Rubi [A] (verified)	7611
Mathematica [A] (verified)	7612
Maple [F]	7613
Fricas [F]	7613
Sympy [F]	7613
Maxima [F]	7613
Giac [F]	7614
Mupad [F(-1)]	7614

Optimal result

Integrand size = 21, antiderivative size = 141

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

$$= -\frac{A(b \cos(c + dx))^{1+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{bd(1+n)\sqrt{\sin^2(c + dx)}} - \frac{B(b \cos(c + dx))^{2+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{b^2d(2+n)\sqrt{\sin^2(c + dx)}}$$

```
[Out] -A*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n], [3/2+1/2*n], cos(d*x+c)^2)
)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)-B*(b*cos(d*x+c))^(2+n)*hypergeo
m([1/2, 1+1/2*n], [2+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2827, 2722}

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

$$= -\frac{A \sin(c + dx) (b \cos(c + dx))^{n+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+1}{2}, \frac{n+3}{2}, \cos^2(c + dx)\right)}{bd(n+1)\sqrt{\sin^2(c + dx)}} - \frac{B \sin(c + dx) (b \cos(c + dx))^{n+2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+2}{2}, \frac{n+4}{2}, \cos^2(c + dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c + dx)}}$$

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((A*(b*cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= A \int (b \cos(c + dx))^n dx + \frac{B \int (b \cos(c + dx))^{1+n} dx}{b} \\ &= -\frac{A(b \cos(c + dx))^{1+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{bd(1+n)\sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{B(b \cos(c + dx))^{2+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{b^2d(2+n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx = \frac{(b \cos(c + dx))^n \cot(c + dx) (A(2 + n) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos^2(c + dx)\right) + B(1 + n) \cos(c + dx))}{d(1 + n)(2 + n)}$$

```
[In] Integrate[(b*cos[c + d*x])^n*(A + B*cos[c + d*x]),x]
```

```
[Out] -(((b*cos[c + d*x])^n*Cot[c + d*x]*(A*(2 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2] + B*(1 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + n)*(2 + n)))
```


Maple [F]

$$\int (\cos(dx + c)b)^n (A + B \cos(dx + c)) dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)
```

Fricas [F]

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^n dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Sympy [F]

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx = \int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x)), x)
```

Maxima [F]

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^n dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Giac [F]

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^n dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx = \int (b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

[In] int((b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)

[Out] int((b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)

3.915 $\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec(c + dx) dx$

Optimal result	7615
Rubi [A] (verified)	7615
Mathematica [A] (verified)	7616
Maple [F]	7617
Fricas [F]	7617
Sympy [F]	7617
Maxima [F]	7618
Giac [F]	7618
Mupad [F(-1)]	7618

Optimal result

Integrand size = 27, antiderivative size = 132

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= -\frac{A(b \cos(c + dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{dn \sqrt{\sin^2(c + dx)}} - \frac{B(b \cos(c + dx))^{1+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{bd(1+n) \sqrt{\sin^2(c + dx)}}$$

[Out] $-A*(b*\cos(d*x+c))^n*\operatorname{hypergeom}([1/2, 1/2*n], [1+1/2*n], \cos(d*x+c)^2)*\sin(d*x+c)/d/n/(\sin(d*x+c)^2)^{(1/2)}-B*(b*\cos(d*x+c))^{(1+n)}*\operatorname{hypergeom}([1/2, 1/2+1/2*n], [3/2+1/2*n], \cos(d*x+c)^2)*\sin(d*x+c)/b/d/(1+n)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {16, 2827, 2722}

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= -\frac{A \sin(c + dx) (b \cos(c + dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{n+2}{2}, \cos^2(c + dx)\right)}{dn \sqrt{\sin^2(c + dx)}} - \frac{B \sin(c + dx) (b \cos(c + dx))^{n+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n+1}{2}, \frac{n+3}{2}, \cos^2(c + dx)\right)}{bd(n+1) \sqrt{\sin^2(c + dx)}}$$

[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] -((A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])) - (B*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned} \text{integral} &= b \int (b \cos(c + dx))^{-1+n} (A + B \cos(c + dx)) dx \\ &= (Ab) \int (b \cos(c + dx))^{-1+n} dx + B \int (b \cos(c + dx))^n dx \\ &= -\frac{A(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{dn \sqrt{\sin^2(c + dx)}} \\ &\quad - \frac{B(b \cos(c + dx))^{1+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+n}{2}, \frac{3+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{bd(1+n) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec(c + dx) dx = -\frac{b(b \cos(c + dx))^{-1+n} \cot(c + dx) (A(1 + n) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos^2(c + dx)\right) + Bn \cos(c + dx))}{dn(1 + n)}$$

[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x],x]

[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(A*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2] + B*n*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(1 + n))

Maple [F]

$$\int (\cos(dx + c)b)^n (A + B \cos(dx + c)) \sec(dx + c) dx$$

[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c),x)

[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c),x)

Fricas [F]

$$\begin{aligned} & \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec(c + dx) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c) dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)

Sympy [F]

$$\begin{aligned} & \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec(c + dx) dx \\ &= \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec(c + dx) dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x)

[Out] Integral((b*cos(c + d*x))^n*(A + B*cos(c + d*x))*sec(c + d*x), x)

Maxima [F]

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)

Giac [F]

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c) dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec(c + dx) dx$$

$$= \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos(c + dx)} dx$$

[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x),x)

[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x), x)

3.916 $\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^2(c + dx) dx$

Optimal result	7619
Rubi [A] (verified)	7619
Mathematica [A] (verified)	7620
Maple [F]	7621
Fricas [F]	7621
Sympy [F]	7621
Maxima [F]	7622
Giac [F]	7622
Mupad [F(-1)]	7622

Optimal result

Integrand size = 29, antiderivative size = 131

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{Ab(b \cos(c + dx))^{-1+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1 + n), \frac{1+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1 - n)\sqrt{\sin^2(c + dx)}} - \frac{B(b \cos(c + dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{dn\sqrt{\sin^2(c + dx)}}$$

[Out] A*b*(b*cos(d*x+c))⁽⁻¹⁺ⁿ⁾*hypergeom([1/2, -1/2+1/2*n], [1/2+1/2*n], cos(d*x+c)²)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)²)^(1/2)-B*(b*cos(d*x+c))ⁿ*hypergeom([1/2, 1/2*n], [1+1/2*n], cos(d*x+c)²)*sin(d*x+c)/d/n/(sin(d*x+c)²)^(1/2)

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \frac{Ab \sin(c + dx)(b \cos(c + dx))^{n-1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n-1}{2}, \frac{n+1}{2}, \cos^2(c + dx)\right)}{d(1 - n)\sqrt{\sin^2(c + dx)}} - \frac{B \sin(c + dx)(b \cos(c + dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{n+2}{2}, \cos^2(c + dx)\right)}{dn\sqrt{\sin^2(c + dx)}}$$

[In] Int[(b*cos[c + d*x])^n*(A + B*cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] (A*b*(b*cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^2 \int (b \cos(c + dx))^{-2+n} (A + B \cos(c + dx)) dx \\
 &= (Ab^2) \int (b \cos(c + dx))^{-2+n} dx + (bB) \int (b \cos(c + dx))^{-1+n} dx \\
 &= \frac{Ab(b \cos(c + dx))^{-1+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1 + n), \frac{1+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1 - n)\sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{B(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n}{2}, \frac{2+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{dn\sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^2(c + dx) dx = \frac{b(b \cos(c + dx))^{-1+n} \csc(c + dx) \left(An \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1 + n), \frac{1+n}{2}, \cos^2(c + dx)\right) + B(-1 + n) \right)}{d(-1 + n)n}$$

[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]

[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(A*n*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + B*(-1 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + n)*n))

Maple [F]

$$\int (\cos(dx + c) b)^n (A + B \cos(dx + c)) (\sec^2(dx + c)) dx$$

[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)

[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)

Fricas [F]

$$\begin{aligned} & \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^2(c + dx) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec^2(dx + c) dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Sympy [F]

$$\begin{aligned} & \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^2(c + dx) dx \\ &= \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^2(c + dx) dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)

[Out] Integral((b*cos(c + d*x))^n*(A + B*cos(c + d*x))*sec(c + d*x)**2, x)

Maxima [F]

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Giac [F]

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c)^2 dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^2(c + dx) dx$$

$$= \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos(c + dx)^2} dx$$

[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^2,x)

[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^2, x)

3.917 $\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^3(c + dx) dx$

Optimal result	7623
Rubi [A] (verified)	7623
Mathematica [A] (verified)	7624
Maple [F]	7625
Fricas [F]	7625
Sympy [F(-1)]	7625
Maxima [F]	7626
Giac [F]	7626
Mupad [F(-1)]	7626

Optimal result

Integrand size = 29, antiderivative size = 139

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \frac{Ab^2 (b \cos(c + dx))^{-2+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-2 + n), \frac{n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 - n) \sqrt{\sin^2(c + dx)}} + \frac{bB (b \cos(c + dx))^{-1+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1 + n), \frac{1+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1 - n) \sqrt{\sin^2(c + dx)}}$$

[Out] A*b^2*(b*cos(d*x+c))^(2-n)*hypergeom([1/2, -1+1/2*n], [1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)+b*B*(b*cos(d*x+c))^(1+n)*hypergeom([1/2, -1/2+1/2*n], [1/2+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \frac{Ab^2 \sin(c + dx) (b \cos(c + dx))^{n-2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n-2}{2}, \frac{n}{2}, \cos^2(c + dx)\right)}{d(2 - n) \sqrt{\sin^2(c + dx)}} + \frac{bB \sin(c + dx) (b \cos(c + dx))^{n-1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n-1}{2}, \frac{n+1}{2}, \cos^2(c + dx)\right)}{d(1 - n) \sqrt{\sin^2(c + dx)}}$$

[In] Int[(b*cos[c + d*x])^n*(A + B*cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] (A*b^2*(b*cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]) + (b*B*(b*cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^3 \int (b \cos(c + dx))^{-3+n} (A + B \cos(c + dx)) dx \\
 &= (Ab^3) \int (b \cos(c + dx))^{-3+n} dx + (b^2 B) \int (b \cos(c + dx))^{-2+n} dx \\
 &= \frac{Ab^2 (b \cos(c + dx))^{-2+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-2 + n), \frac{n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 - n) \sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{bB (b \cos(c + dx))^{-1+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-1 + n), \frac{1+n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(1 - n) \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.85

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^3(c + dx) dx = \frac{(b \cos(c + dx))^n \csc(c + dx) (A(-1 + n) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-2 + n), \frac{n}{2}, \cos^2(c + dx)\right) + B(-2 + n) \csc(c + dx))}{d(-2 + n) \csc(c + dx)}$$

[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]

[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2] + B*(-2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(d*(-2 + n)*(-1 + n)))

Maple [F]

$$\int (\cos(dx + c) b)^n (A + B \cos(dx + c)) (\sec^3(dx + c)) dx$$

[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)

[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)

Fricas [F]

$$\begin{aligned} & \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^3(c + dx) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c)^3 dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^3(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)

[Out] Timed out

Maxima [F]

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c)^3 dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)

Giac [F]

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c)^3 dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^3(c + dx) dx$$

$$= \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos(c + dx)^3} dx$$

[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^3,x)

[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^3, x)

3.918 $\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^4(c + dx) dx$

Optimal result	.7627
Rubi [A] (verified)	.7627
Mathematica [A] (verified)	.7629
Maple [F]	.7629
Fricas [F]	.7629
Sympy [F(-1)]	.7630
Maxima [F]	.7630
Giac [F]	.7630
Mupad [F(-1)]	.7631

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{Ab^3(b \cos(c + dx))^{-3+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-3 + n), \frac{1}{2}(-1 + n), \cos^2(c + dx)\right) \sin(c + dx)}{d(3 - n)\sqrt{\sin^2(c + dx)}} + \frac{b^2B(b \cos(c + dx))^{-2+n} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-2 + n), \frac{n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 - n)\sqrt{\sin^2(c + dx)}}$$

[Out] A*b^3*(b*cos(d*x+c))^(n-3)*hypergeom([1/2, -3/2+1/2*n], [-1/2+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(3-n)/(sin(d*x+c)^2)^(1/2)+b^2*B*(b*cos(d*x+c))^(n-2)*hypergeom([1/2, -1+1/2*n], [1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 2827, 2722}

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^4(c + dx) dx$$

$$= \frac{Ab^3 \sin(c + dx)(b \cos(c + dx))^{n-3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n-3}{2}, \frac{n-1}{2}, \cos^2(c + dx)\right)}{d(3 - n)\sqrt{\sin^2(c + dx)}} + \frac{b^2B \sin(c + dx)(b \cos(c + dx))^{n-2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{n-2}{2}, \frac{n}{2}, \cos^2(c + dx)\right)}{d(2 - n)\sqrt{\sin^2(c + dx)}}$$

[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]

[Out] (A*b^3*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(3 - n)*Sqrt[Sin[c + d*x]^2]) + (b^2*B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/d*(2 - n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= b^4 \int (b \cos(c + dx))^{-4+n} (A + B \cos(c + dx)) dx \\
 &= (Ab^4) \int (b \cos(c + dx))^{-4+n} dx + (b^3 B) \int (b \cos(c + dx))^{-3+n} dx \\
 &= \frac{Ab^3 (b \cos(c + dx))^{-3+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-3 + n), \frac{1}{2}(-1 + n), \cos^2(c + dx)\right) \sin(c + dx)}{d(3 - n) \sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{b^2 B (b \cos(c + dx))^{-2+n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(-2 + n), \frac{n}{2}, \cos^2(c + dx)\right) \sin(c + dx)}{d(2 - n) \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^4(c + dx) dx = \frac{(b \cos(c + dx))^n \csc(c + dx) \left(A(-2 + n) \operatorname{Hypergeometric2F1} \left(\frac{1}{2}, \frac{1}{2}(-3 + n), \frac{1}{2}(-1 + n), \cos^2(c + dx) \right) - \right)}{d(-3 + n)}$$

[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]

[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-2 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/2, Cos[c + d*x]^2] + B*(-3 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2])*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2]))/(d*(-3 + n)*(-2 + n))

Maple [F]

$$\int (\cos(dx + c) b)^n (A + B \cos(dx + c)) (\sec^4(dx + c)) dx$$

[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)

[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)

Fricas [F]

$$\begin{aligned} & \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^4(c + dx) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec^4(dx + c) dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)

Sympy [F(-1)]

Timed out.

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^4(c + dx) dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)

[Out] Timed out

Maxima [F]

$$\begin{aligned} & \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^4(c + dx) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c)^4 dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)

Giac [F]

$$\begin{aligned} & \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^4(c + dx) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c)^4 dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)

Mupad [F(-1)]

Timed out.

$$\begin{aligned} & \int (b \cos(c + dx))^n (A + B \cos(c + dx)) \sec^4(c + dx) dx \\ &= \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos(c + dx)^4} dx \end{aligned}$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^4, x)
```

$$3.919 \quad \int \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$$

Optimal result	7632
Rubi [A] (verified)	7632
Mathematica [A] (verified)	7634
Maple [F]	7634
Fricas [F]	7634
Sympy [F(-1)]	7635
Maxima [F]	7635
Giac [F]	7635
Mupad [F(-1)]	7636

Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx =$$

$$\frac{2A \cos^{\frac{7}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(7+2n), \frac{1}{4}(11+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(7+2n)\sqrt{\sin^2(c+dx)}} -$$

$$\frac{2B \cos^{\frac{9}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(9+2n), \frac{1}{4}(13+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(9+2n)\sqrt{\sin^2(c+dx)}}$$

[Out] $-2*A*\cos(d*x+c)^{(7/2)}*(b*\cos(d*x+c))^n*\operatorname{hypergeom}([1/2, 7/4+1/2*n], [11/4+1/2*n], \cos(d*x+c)^2)*\sin(d*x+c)/d/(7+2*n)/(\sin(d*x+c)^2)^{(1/2)}-2*B*\cos(d*x+c)^{(9/2)}*(b*\cos(d*x+c))^n*\operatorname{hypergeom}([1/2, 9/4+1/2*n], [13/4+1/2*n], \cos(d*x+c)^2)*\sin(d*x+c)/d/(9+2*n)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx =$$

$$\frac{2A \sin(c+dx) \cos^{\frac{7}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n+7), \frac{1}{4}(2n+11), \cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}} -$$

$$\frac{2B \sin(c+dx) \cos^{\frac{9}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n+9), \frac{1}{4}(2n+13), \cos^2(c+dx)\right)}{d(2n+9)\sqrt{\sin^2(c+dx)}}$$

[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] (-2*A*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x])/(d*(9 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{5}{2}+n}(c + dx)(A + B \cos(c + dx)) dx \\
 &= (A \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{5}{2}+n}(c + dx) dx \\
 &\quad + (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{7}{2}+n}(c + dx) dx \\
 &= \\
 &\quad \frac{2A \cos^{\frac{7}{2}}(c + dx)(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(7 + 2n), \frac{1}{4}(11 + 2n), \cos^2(c + dx)\right)}{d(7 + 2n)\sqrt{\sin^2(c + dx)}} \\
 &\quad \frac{2B \cos^{\frac{9}{2}}(c + dx)(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(9 + 2n), \frac{1}{4}(13 + 2n), \cos^2(c + dx)\right)}{d(9 + 2n)\sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx =$$

$$\frac{2 \cos^{\frac{7}{2}}(c + dx)(b \cos(c + dx))^n \csc(c + dx) \left(A(9 + 2n) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(7 + 2n), \frac{1}{4}(11 + 2n), \cos^2(c + dx)\right) + B(7 + 2n) \operatorname{CosineIntegral}\left(\frac{1}{2}, \frac{1}{4}(7 + 2n), \frac{1}{4}(11 + 2n), \cos^2(c + dx)\right) \right)}{d(7 + 2n)(9 + 2n)}$$

[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(9 + 2*n)*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + B*(7 + 2*n)*CosineIntegral[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(7 + 2*n)*(9 + 2*n))

Maple [F]

$$\int \left(\cos^{\frac{5}{2}}(dx + c) \right) (\cos(dx + c) b)^n (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^{\frac{5}{2}} dx$$

[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\begin{aligned} & \int \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^{\frac{5}{2}} dx \end{aligned}$$

[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)

Giac [F]

$$\begin{aligned} & \int \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^{\frac{5}{2}} dx \end{aligned}$$

[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$
$$= \int \cos(c + dx)^{5/2} (b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```


$$3.920 \quad \int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n (A+B \cos(c+dx)) dx$$

Optimal result	7637
Rubi [A] (verified)	7637
Mathematica [A] (verified)	7639
Maple [F]	7639
Fricas [F]	7639
Sympy [F(-1)]	7640
Maxima [F]	7640
Giac [F]	7640
Mupad [F(-1)]	7641

Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n (A+B \cos(c+dx)) dx =$$

$$\frac{2A \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(5+2n), \frac{1}{4}(9+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(5+2n)\sqrt{\sin^2(c+dx)}} -$$

$$\frac{2B \cos^{\frac{7}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(7+2n), \frac{1}{4}(11+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

[Out] -2*A*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n], [9/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)-2*B*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n], [11/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n (A+B \cos(c+dx)) dx =$$

$$\frac{2A \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n+5), \frac{1}{4}(2n+9), \cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}} -$$

$$\frac{2B \sin(c+dx) \cos^{\frac{7}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n+7), \frac{1}{4}(2n+11), \cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$

[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] (-2*A*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*(b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{3}{2}+n}(c + dx)(A + B \cos(c + dx)) dx \\
 &= (A \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{3}{2}+n}(c + dx) dx \\
 &\quad + (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{5}{2}+n}(c + dx) dx \\
 &= \\
 &\quad - \frac{2A \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(5 + 2n), \frac{1}{4}(9 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(5 + 2n)\sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{2B \cos^{\frac{7}{2}}(c + dx)(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(7 + 2n), \frac{1}{4}(11 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(7 + 2n)\sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx =$$

$$\frac{2 \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n \csc(c+dx) \left(A(7+2n) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(5+2n), \frac{1}{4}(9+2n), \cos^2(c+dx)\right) + B(5+2n) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(7+2n), \frac{1}{4}(11+2n), \cos^2(c+dx)\right) \right)}{d(5+2n)(7+2n)}$$

[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(7 + 2*n)*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + B*(5 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(5 + 2*n)*(7 + 2*n))

Maple [F]

$$\int \left(\cos^{\frac{3}{2}}(dx+c) \right) (\cos(dx+c)b)^n (A+B \cos(dx+c)) dx$$

[In] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$$

$$= \int (B \cos(dx+c) + A)(b \cos(dx+c))^n \cos(dx+c)^{\frac{3}{2}} dx$$

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)

[Out] Timed out

Maxima [F]

$$\begin{aligned} & \int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^{\frac{3}{2}} dx \end{aligned}$$

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)

Giac [F]

$$\begin{aligned} & \int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx \\ &= \int (B \cos(dx + c) + A)(b \cos(dx + c))^n \cos(dx + c)^{\frac{3}{2}} dx \end{aligned}$$

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$
$$= \int \cos(c + dx)^{\frac{3}{2}}(b \cos(c + dx))^n(A + B \cos(c + dx)) dx$$

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

3.921 $\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$

Optimal result	7642
Rubi [A] (verified)	7642
Mathematica [A] (verified)	7644
Maple [F]	7644
Fricas [F]	7644
Sympy [F]	7645
Maxima [F]	7645
Giac [F]	7645
Mupad [F(-1)]	7646

Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n(A+B \cos(c+dx)) dx =$$

$$\frac{2A \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(3+2n)\sqrt{\sin^2(c+dx)}} +$$

$$\frac{2B \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(5+2n), \frac{1}{4}(9+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

[Out] $-2*A*\cos(d*x+c)^{(3/2)}*(b*\cos(d*x+c))^n*\operatorname{hypergeom}([1/2, 3/4+1/2*n], [7/4+1/2*n], \cos(d*x+c)^2)*\sin(d*x+c)/d/(3+2*n)/(\sin(d*x+c)^2)^{(1/2)}-2*B*\cos(d*x+c)^{(5/2)}*(b*\cos(d*x+c))^n*\operatorname{hypergeom}([1/2, 5/4+1/2*n], [9/4+1/2*n], \cos(d*x+c)^2)*\sin(d*x+c)/d/(5+2*n)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n(A+B \cos(c+dx)) dx =$$

$$\frac{2A \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n+3), \frac{1}{4}(2n+7), \cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}} +$$

$$\frac{2B \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n+5), \frac{1}{4}(2n+9), \cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}$$

[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] $(-2*A*\text{Cos}[c + d*x]^{(3/2)}*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (3 + 2*n)/4, (7 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(3 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (2*B*\text{Cos}[c + d*x]^{(5/2)}*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (5 + 2*n)/4, (9 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(5 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{1}{2}+n}(c + dx)(A + B \cos(c + dx)) dx \\
 &= (A \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{1}{2}+n}(c + dx) dx \\
 &\quad + (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{3}{2}+n}(c + dx) dx \\
 &= \\
 &\quad \frac{2A \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(3 + 2n), \frac{1}{4}(7 + 2n), \cos^2(c + dx)\right) \sin}{d(3 + 2n)\sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{2B \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(5 + 2n), \frac{1}{4}(9 + 2n), \cos^2(c + dx)\right) \sin}{d(5 + 2n)\sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n(A+B \cos(c+dx)) dx = \frac{2 \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n \csc(c+dx) (A(5+2n) \operatorname{Hypergeometric2F1}(\frac{1}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \cos^2(c+dx)))}{d(3+2n)(5+2n)}$$

[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]

[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(5 + 2*n)*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + B*(3 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(3 + 2*n)*(5 + 2*n))

Maple [F]

$$\int (\cos(dx+c)b)^n (A+B \cos(dx+c)) (\sqrt{\cos(dx+c)}) dx$$

[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)

[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)

Fricas [F]

$$\begin{aligned} & \int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n(A+B \cos(c+dx)) dx \\ &= \int (B \cos(dx+c) + A)(b \cos(dx+c))^n \sqrt{\cos(dx+c)} dx \end{aligned}$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Sympy [F]

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$$

$$= \int (b \cos(c+dx))^n(A+B \cos(c+dx)) \sqrt{\cos(c+dx)} dx$$

[In] `integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)`

[Out] `Integral((b*cos(c+d*x))**n*(A+B*cos(c+d*x))*sqrt(cos(c+d*x)), x)`

Maxima [F]

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$$

$$= \int (B \cos(dx+c)+A)(b \cos(dx+c))^n \sqrt{\cos(dx+c)} dx$$

[In] `integrate((b*cos(d*x+c))n*(A+B*cos(d*x+c))*cos(d*x+c)(1/2),x, algorithm="maxima")`

[Out] `integrate((B*cos(d*x+c)+A)*(b*cos(d*x+c))n*sqrt(cos(d*x+c)), x)`

Giac [F]

$$\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n(A+B \cos(c+dx)) dx$$

$$= \int (B \cos(dx+c)+A)(b \cos(dx+c))^n \sqrt{\cos(dx+c)} dx$$

[In] `integrate((b*cos(d*x+c))n*(A+B*cos(d*x+c))*cos(d*x+c)(1/2),x, algorithm="giac")`

[Out] `integrate((B*cos(d*x+c)+A)*(b*cos(d*x+c))n*sqrt(cos(d*x+c)), x)`

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c + dx)} (b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$
$$= \int \sqrt{\cos(c + dx)} (b \cos(c + dx))^n (A + B \cos(c + dx)) dx$$

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n*(A + B*cos(c + d*x)), x)
```

$$3.922 \quad \int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal result	7647
Rubi [A] (verified)	7647
Mathematica [A] (verified)	7649
Maple [F]	7649
Fricas [F]	7649
Sympy [F]	7650
Maxima [F]	7650
Giac [F]	7650
Mupad [F(-1)]	7650

Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx =$$

$$\frac{2A \sqrt{\cos(c+dx)} (b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(1+2n), \frac{1}{4}(5+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(1+2n) \sqrt{\sin^2(c+dx)}} -$$

$$\frac{2B \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(3+2n), \frac{1}{4}(7+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(3+2n) \sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*B*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n], [7/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)-2*A*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n], [5/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\sqrt{\cos(c+dx)}} dx =$$

$$\frac{2A \sin(c+dx) \sqrt{\cos(c+dx)} (b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n+1), \frac{1}{4}(2n+5), \cos^2(c+dx)\right)}{d(2n+1) \sqrt{\sin^2(c+dx)}} -$$

$$\frac{2B \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n+3), \frac{1}{4}(2n+7), \cos^2(c+dx)\right)}{d(2n+3) \sqrt{\sin^2(c+dx)}}$$

[In] Int[((b*cos[c + d*x])^n*(A + B*cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]

[Out] (-2*A*Sqrt[Cos[c + d*x]]*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*cos[c + d*x]^(3/2)*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*(b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{1}{2}+n}(c + dx)(A + B \cos(c + dx)) dx \\
 &= (A \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{1}{2}+n}(c + dx) dx \\
 &\quad + (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{1}{2}+n}(c + dx) dx \\
 &= \\
 &\quad - \frac{2A \sqrt{\cos(c + dx)}(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(1 + 2n), \frac{1}{4}(5 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(1 + 2n) \sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{2B \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(3 + 2n), \frac{1}{4}(7 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(3 + 2n) \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \frac{2\sqrt{\cos(c + dx)}(b \cos(c + dx))^n \csc(c + dx) (A(3 + 2n) \text{Hypergeometric2F1}(\frac{1}{2}, \frac{1}{4}(1 + 2n), \frac{1}{4}(5 + 2n), \cos(c + dx)))}{d(1 + 2n)(3 + 2n)}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(3 + 2*n)*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + B*(1 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(3 + 2*n))
```

Maple [F]

$$\int \frac{(\cos(dx + c)b)^n (A + B \cos(dx + c))}{\sqrt{\cos(dx + c)}} dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)

[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))/sqrt(cos(c + d*x)), x)

Maxima [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((b*cos(d*x+c))ⁿ*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))ⁿ/sqrt(cos(d*x + c)), x)

Giac [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}} dx$$

[In] integrate((b*cos(d*x+c))ⁿ*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))ⁿ/sqrt(cos(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx = \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

[In] int(((b*cos(c + d*x))ⁿ*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2),x)

[Out] int(((b*cos(c + d*x))ⁿ*(A + B*cos(c + d*x)))/cos(c + d*x)^(1/2), x)

$$3.923 \quad \int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal result	.7651
Rubi [A] (verified)	.7651
Mathematica [A] (verified)	.7653
Maple [F]	.7653
Fricas [F]	.7653
Sympy [F]	.7654
Maxima [F]	.7654
Giac [F]	.7654
Mupad [F(-1)]	.7654

Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

$$= \frac{2A(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-1+2n), \frac{1}{4}(3+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}} - \frac{2B\sqrt{\cos(c+dx)}(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(1+2n), \frac{1}{4}(5+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

[Out] 2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n], [3/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)-2*B*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n], [5/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

$$= \frac{2A \sin(c+dx)(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n-1), \frac{1}{4}(2n+3), \cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}} - \frac{2B \sin(c+dx)\sqrt{\cos(c+dx)}(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n+1), \frac{1}{4}(2n+5), \cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

[In] Int[((b*cos[c + d*x])^n*(A + B*cos[c + d*x]))/Cos[c + d*x]^(3/2),x]

[Out] (2*A*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2]) - (2*B*Sqrt[Cos[c + d*x]]*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*(b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*Ssin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx)(A + B \cos(c + dx)) dx \\
 &= (A \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx) dx \\
 &\quad + (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{1}{2}+n}(c + dx) dx \\
 &= \frac{2A(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-1 + 2n), \frac{1}{4}(3 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(1 - 2n)\sqrt{\cos(c + dx)}\sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{2B\sqrt{\cos(c + dx)}(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(1 + 2n), \frac{1}{4}(5 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(1 + 2n)\sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{2(b \cos(c + dx))^n \csc(c + dx) (A(1 + 2n) \operatorname{Hypergeometric2F1}(\frac{1}{2}, \frac{1}{4}(-1 + 2n), \frac{1}{4}(3 + 2n), \cos^2(c + dx)))}{d(-1 + 4n^2)}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + B*(-1 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + 4*n^2)*Sqrt[Cos[c + d*x]])
```

Maple [F]

$$\int \frac{(\cos(dx + c)b)^n (A + B \cos(dx + c))}{\cos(dx + c)^{\frac{3}{2}}} dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Sympy [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx$$

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2), x)

[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))/cos(c + d*x)**(3/2), x)

Maxima [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)

Giac [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}} dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2), x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos(c + dx)^{3/2}} dx$$

[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2), x)

[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(3/2), x)

$$3.924 \quad \int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal result	7655
Rubi [A] (verified)	7655
Mathematica [A] (verified)	7657
Maple [F]	7657
Fricas [F]	7657
Sympy [F(-1)]	7658
Maxima [F]	7658
Giac [F]	7658
Mupad [F(-1)]	7658

Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

$$= \frac{2A(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-3+2n), \frac{1}{4}(1+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(3-2n) \cos^{\frac{3}{2}}(c+dx) \sqrt{\sin^2(c+dx)}} + \frac{2B(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-1+2n), \frac{1}{4}(3+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(1-2n) \sqrt{\cos(c+dx)} \sqrt{\sin^2(c+dx)}}$$

[Out] 2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n], [1/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n], [3/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

$$= \frac{2A \sin(c+dx) (b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n-3), \frac{1}{4}(2n+1), \cos^2(c+dx)\right)}{d(3-2n) \sqrt{\sin^2(c+dx)} \cos^{\frac{3}{2}}(c+dx)} + \frac{2B \sin(c+dx) (b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n-1), \frac{1}{4}(2n+3), \cos^2(c+dx)\right)}{d(1-2n) \sqrt{\sin^2(c+dx)} \sqrt{\cos(c+dx)}}$$

[In] Int[((b*cos[c + d*x])^n*(A + B*cos[c + d*x]))/Cos[c + d*x]^(5/2),x]

[Out] (2*A*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2]) + (2*B*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*(b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{5}{2}+n}(c + dx)(A + B \cos(c + dx)) dx \\
 &= (A \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{5}{2}+n}(c + dx) dx \\
 &\quad + (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx) dx \\
 &= \frac{2A(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-3 + 2n), \frac{1}{4}(1 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(3 - 2n) \cos^{\frac{3}{2}}(c + dx) \sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{2B(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-1 + 2n), \frac{1}{4}(3 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(1 - 2n) \sqrt{\cos(c + dx)} \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{2(b \cos(c + dx))^n \csc(c + dx) (A(-1 + 2n) \operatorname{Hypergeometric2F1}(\frac{1}{2}, \frac{1}{4}(-3 + 2n), \frac{1}{4}(1 + 2n), \cos^2(c + dx)) + B(-3 + 2n) \operatorname{Hypergeometric2F1}(\frac{1}{2}, \frac{1}{4}(-3 + 2n), \frac{1}{4}(1 + 2n), \cos^2(c + dx)))}{d(-3 + 2n)(-1 + 2n) \cos^{\frac{3}{2}}(c + dx)}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + B*(-3 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/((d*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2))
```

Maple [F]

$$\int \frac{(\cos(dx + c)b)^n (A + B \cos(dx + c))}{\cos(dx + c)^{\frac{5}{2}}} dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = \text{Timed out}$$

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2), x)

[Out] Timed out

Maxima [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

Giac [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}} dx$$

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2), x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos(c + dx)^{5/2}} dx$$

[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)

[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(5/2), x)

$$3.925 \quad \int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal result	7659
Rubi [A] (verified)	7659
Mathematica [A] (verified)	7661
Maple [F]	7661
Fricas [F]	7661
Sympy [F(-1)]	7662
Maxima [F]	7662
Giac [F]	7662
Mupad [F(-1)]	7662

Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

$$= \frac{2A(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-5+2n), \frac{1}{4}(-1+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(5-2n) \cos^{\frac{5}{2}}(c+dx) \sqrt{\sin^2(c+dx)}} + \frac{2B(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-3+2n), \frac{1}{4}(1+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(3-2n) \cos^{\frac{3}{2}}(c+dx) \sqrt{\sin^2(c+dx)}}$$

[Out] 2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n], [-1/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n], [1/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

$$= \frac{2A \sin(c+dx) (b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n-5), \frac{1}{4}(2n-1), \cos^2(c+dx)\right)}{d(5-2n) \sqrt{\sin^2(c+dx)} \cos^{\frac{5}{2}}(c+dx)} + \frac{2B \sin(c+dx) (b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n-3), \frac{1}{4}(2n+1), \cos^2(c+dx)\right)}{d(3-2n) \sqrt{\sin^2(c+dx)} \cos^{\frac{3}{2}}(c+dx)}$$

[In] Int[((b*cos[c + d*x])^n*(A + B*cos[c + d*x]))/Cos[c + d*x]^(7/2),x]

[Out] (2*A*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2]) + (2*B*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*(b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{7}{2}+n}(c + dx)(A + B \cos(c + dx)) dx \\
 &= (A \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{7}{2}+n}(c + dx) dx \\
 &\quad + (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{5}{2}+n}(c + dx) dx \\
 &= \frac{2A(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-5 + 2n), \frac{1}{4}(-1 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(5 - 2n) \cos^{\frac{5}{2}}(c + dx) \sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{2B(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-3 + 2n), \frac{1}{4}(1 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(3 - 2n) \cos^{\frac{3}{2}}(c + dx) \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx =$$

$$\frac{2(b \cos(c + dx))^n \csc(c + dx) (A(-3 + 2n) \text{Hypergeometric2F1}(\frac{1}{2}, \frac{1}{4}(-5 + 2n), \frac{1}{4}(-1 + 2n), \cos^2(c + dx)))}{d(-5 + 2n)(-1 + 2n)}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-3 + 2*n)*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + B*(-5 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2))
```

Maple [F]

$$\int \frac{(\cos(dx + c)b)^n (A + B \cos(dx + c))}{\cos(dx + c)^{\frac{7}{2}}} dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

Fricas [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2), x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2), x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2), x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos(c + dx)^{7/2}} dx$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(7/2), x)
```

$$3.926 \quad \int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal result	7663
Rubi [A] (verified)	7663
Mathematica [A] (verified)	7665
Maple [F]	7665
Fricas [F]	7665
Sympy [F(-1)]	7666
Maxima [F]	7666
Giac [F]	7666
Mupad [F(-1)]	7666

Optimal result

Integrand size = 31, antiderivative size = 163

$$\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

$$= \frac{2A(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-7+2n), \frac{1}{4}(-3+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(7-2n) \cos^{\frac{7}{2}}(c+dx) \sqrt{\sin^2(c+dx)}} + \frac{2B(b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-5+2n), \frac{1}{4}(-1+2n), \cos^2(c+dx)\right) \sin(c+dx)}{d(5-2n) \cos^{\frac{5}{2}}(c+dx) \sqrt{\sin^2(c+dx)}}$$

[Out] 2*A*(b*cos(d*x+c))^n*hypergeom([1/2, -7/4+1/2*n], [-3/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(7-2*n)/cos(d*x+c)^(7/2)/(sin(d*x+c)^2)^(1/2)+2*B*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n], [-1/4+1/2*n], cos(d*x+c)^2)*sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

$$= \frac{2A \sin(c+dx) (b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n-7), \frac{1}{4}(2n-3), \cos^2(c+dx)\right)}{d(7-2n) \sqrt{\sin^2(c+dx)} \cos^{\frac{7}{2}}(c+dx)} + \frac{2B \sin(c+dx) (b \cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(2n-5), \frac{1}{4}(2n-1), \cos^2(c+dx)\right)}{d(5-2n) \sqrt{\sin^2(c+dx)} \cos^{\frac{5}{2}}(c+dx)}$$

[In] Int[((b*cos[c + d*x])^n*(A + B*cos[c + d*x]))/Cos[c + d*x]^(9/2),x]

[Out] (2*A*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 - 2*n)*Cos[c + d*x]^(7/2)*Sqrt[Sin[c + d*x]^2]) + (2*B*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*(b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{9}{2}+n}(c + dx)(A + B \cos(c + dx)) dx \\
 &= (A \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{9}{2}+n}(c + dx) dx \\
 &\quad + (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{7}{2}+n}(c + dx) dx \\
 &= \frac{2A(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-7 + 2n), \frac{1}{4}(-3 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(7 - 2n) \cos^{\frac{7}{2}}(c + dx) \sqrt{\sin^2(c + dx)}} \\
 &\quad + \frac{2B(b \cos(c + dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4}(-5 + 2n), \frac{1}{4}(-1 + 2n), \cos^2(c + dx)\right) \sin(c + dx)}{d(5 - 2n) \cos^{\frac{5}{2}}(c + dx) \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.85

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx =$$

$$\frac{2(b \cos(c + dx))^n \csc(c + dx) (A(-5 + 2n) \text{Hypergeometric2F1}(\frac{1}{2}, \frac{1}{4}(-7 + 2n), \frac{1}{4}(-3 + 2n), \cos^2(c + dx)))}{d(-7 + 2n)(-}$$

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-5 + 2*n)*Hypergeometric2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2] + B*(-7 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/((d*(-7 + 2*n)*(-5 + 2*n)*Cos[c + d*x]^(7/2))
```

Maple [F]

$$\int \frac{(\cos(dx + c)b)^n (A + B \cos(dx + c))}{\cos(dx + c)^{\frac{9}{2}}} dx$$

```
[In] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
[Out] int((cos(d*x+c)*b)^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

Fricas [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{9}{2}}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="fricas")
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \text{Timed out}$$

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2), x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{9}{2}}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2), x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{9}{2}}} dx$$

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2), x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx))}{\cos(c + dx)^{9/2}} dx$$

```
[In] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

```
[Out] int(((b*cos(c + d*x))^n*(A + B*cos(c + d*x)))/cos(c + d*x)^(9/2), x)
```

$$3.927 \quad \int \cos^m(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx$$

Optimal result	.7667
Rubi [A] (verified)	.7667
Mathematica [A] (verified)	.7669
Maple [F]	.7669
Fricas [F]	.7669
Sympy [F(-1)]	.7670
Maxima [F]	.7670
Giac [F]	.7670
Mupad [F(-1)]	.7671

Optimal result

Integrand size = 31, antiderivative size = 169

$$\int \cos^m(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx =$$

$$\frac{3Ab \cos^{2+m}(c+dx) \sqrt[3]{b \cos(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(7+3m), \frac{1}{6}(13+3m), \cos^2(c+dx)\right) \sin(c+dx)}{d(7+3m) \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3bB \cos^{3+m}(c+dx) \sqrt[3]{b \cos(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(10+3m), \frac{1}{6}(16+3m), \cos^2(c+dx)\right) \sin(c+dx)}{d(10+3m) \sqrt{\sin^2(c+dx)}}$$

[Out] $-3A*b*\cos(d*x+c)^{(2+m)}*(b*\cos(d*x+c))^{(1/3)}*\operatorname{hypergeom}([1/2, 7/6+1/2*m], [13/6+1/2*m], \cos(d*x+c)^2)*\sin(d*x+c)/d/(7+3*m)/(\sin(d*x+c)^2)^{(1/2)}-3*b*B*\cos(d*x+c)^{(3+m)}*(b*\cos(d*x+c))^{(1/3)}*\operatorname{hypergeom}([1/2, 5/3+1/2*m], [8/3+1/2*m], \cos(d*x+c)^2)*\sin(d*x+c)/d/(10+3*m)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \cos^m(c+dx)(b \cos(c+dx))^{4/3}(A+B \cos(c+dx)) dx =$$

$$\frac{3Ab \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+2}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+7), \frac{1}{6}(3m+13), \cos^2(c+dx)\right)}{d(3m+7) \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3bB \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+3}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+10), \frac{1}{6}(3m+16), \cos^2(c+dx)\right)}{d(3m+10) \sqrt{\sin^2(c+dx)}}$$

[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]

[Out] (-3*A*b*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(7 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*(b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*SIN[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{4}{3}+m}(c+dx)(A+B\cos(c+dx))dx}{\sqrt[3]{\cos(c+dx)}} \\
 &= \frac{\left(Ab\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{4}{3}+m}(c+dx)dx}{\sqrt[3]{\cos(c+dx)}} + \frac{\left(bB\sqrt[3]{b\cos(c+dx)}\right)\int\cos^{\frac{7}{3}+m}(c+dx)dx}{\sqrt[3]{\cos(c+dx)}} \\
 &= \frac{3Ab\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)}{d(7+3m)\sqrt{\sin^2(c+dx)}} \\
 &\quad - \frac{3bB\cos^{3+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(10+3m),\frac{1}{6}(16+3m),\cos^2(c+dx)\right)}{d(10+3m)\sqrt{\sin^2(c+dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.83

$$\int \cos^m(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \frac{3 \cos^{1+m}(c + dx)(b \cos(c + dx))^{4/3} \csc(c + dx) (B(7 + 3m) \cos(c + dx) \operatorname{Hypergeometric2F1}(\frac{1}{2}, \frac{5}{3} + \frac{m}{2}, \frac{8}{3} + \frac{m}{2}, \cos(c + dx))^2 + A(10 + 3m) \operatorname{Hypergeometric2F1}(\frac{1}{2}, \frac{7 + 3m}{6}, \frac{13 + 3m}{6}, \cos(c + dx))^2) \sqrt{\sin(c + dx)^2}}{d(7 + 3m)(10 + 3m)}$$

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]

[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(B*(7 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] + A*(10 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(10 + 3*m))

Maple [F]

$$\int (\cos^m(dx + c)) (\cos(dx + c) b)^{4/3} (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Sympy [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \text{Timed out}$$

[In] `integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)`

[Out] Timed out

Maxima [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c)^m dx$$

[In] `integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")`

[Out] `integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)`

Giac [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c)^m dx$$

[In] `integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="giac")`

[Out] `integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)`

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(b \cos(c + dx))^{4/3}(A + B \cos(c + dx)) dx = \int \cos(c + dx)^m (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx$$

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3)*(A + B*cos(c + d*x)), x)
```

$$3.928 \quad \int \cos^m(c+dx)(b \cos(c+dx))^{2/3}(A+B \cos(c+dx)) dx$$

Optimal result	7672
Rubi [A] (verified)	7672
Mathematica [A] (verified)	7674
Maple [F]	7674
Fricas [F]	7674
Sympy [F]	7675
Maxima [F]	7675
Giac [F]	7675
Mupad [F(-1)]	7676

Optimal result

Integrand size = 31, antiderivative size = 167

$$\int \cos^m(c+dx)(b \cos(c+dx))^{2/3}(A+B \cos(c+dx)) dx =$$

$$\frac{3A \cos^{1+m}(c+dx)(b \cos(c+dx))^{2/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(5+3m), \frac{1}{6}(11+3m), \cos^2(c+dx)\right) \sin(c+dx)}{d(5+3m)\sqrt{\sin^2(c+dx)}} +$$

$$\frac{3B \cos^{2+m}(c+dx)(b \cos(c+dx))^{2/3} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(8+3m), \frac{1}{6}(14+3m), \cos^2(c+dx)\right) \sin(c+dx)}{d(8+3m)\sqrt{\sin^2(c+dx)}}$$

[Out] $-3*A*\cos(d*x+c)^{(1+m)}*(b*\cos(d*x+c))^{(2/3)}*\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{6}+1/2*m\right], \left[\frac{11}{6}+1/2*m\right], \cos(d*x+c)^2\right)*\sin(d*x+c)/d/(5+3*m)/(\sin(d*x+c)^2)^{(1/2)}-3*B*\cos(d*x+c)^{(2+m)}*(b*\cos(d*x+c))^{(2/3)}*\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{4}{3}+1/2*m\right], \left[\frac{7}{3}+1/2*m\right], \cos(d*x+c)^2\right)*\sin(d*x+c)/d/(8+3*m)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \cos^m(c+dx)(b \cos(c+dx))^{2/3}(A+B \cos(c+dx)) dx =$$

$$\frac{3A \sin(c+dx)(b \cos(c+dx))^{2/3} \cos^{m+1}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+5), \frac{1}{6}(3m+11), \cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}} +$$

$$\frac{3B \sin(c+dx)(b \cos(c+dx))^{2/3} \cos^{m+2}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+8), \frac{1}{6}(3m+14), \cos^2(c+dx)\right)}{d(3m+8)\sqrt{\sin^2(c+dx)}}$$

[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]

[Out] (-3*A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(5 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (8 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(8 + 3*m)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*SIN[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{(b \cos(c + dx))^{2/3} \int \cos^{\frac{2}{3}+m}(c + dx)(A + B \cos(c + dx)) dx}{\cos^{\frac{2}{3}}(c + dx)} \\
 &= \frac{(A(b \cos(c + dx))^{2/3}) \int \cos^{\frac{2}{3}+m}(c + dx) dx}{\cos^{\frac{2}{3}}(c + dx)} + \frac{(B(b \cos(c + dx))^{2/3}) \int \cos^{\frac{5}{3}+m}(c + dx) dx}{\cos^{\frac{2}{3}}(c + dx)} \\
 &= \\
 &\quad - \frac{3A \cos^{1+m}(c + dx)(b \cos(c + dx))^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(5 + 3m), \frac{1}{6}(11 + 3m), \cos^2(c + dx)\right)}{d(5 + 3m)\sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{3B \cos^{2+m}(c + dx)(b \cos(c + dx))^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(8 + 3m), \frac{1}{6}(14 + 3m), \cos^2(c + dx)\right)}{d(8 + 3m)\sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \cos^m(c + dx)(b \cos(c + dx))^{2/3}(A + B \cos(c + dx)) dx = \frac{3 \cos^{1+m}(c + dx)(b \cos(c + dx))^{2/3} \csc(c + dx) (A(8 + 3m) \text{Hypergeometric2F1}(\frac{1}{2}, \frac{1}{6}(5 + 3m), \frac{1}{6}(11 + 3m), d(5 + 3m)))}{d(5 + 3m)}$$

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]

[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(A*(8 + 3*m)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + B*(5 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(5 + 3*m)*(8 + 3*m))

Maple [F]

$$\int (\cos^m(dx + c)) (\cos(dx + c) b)^{\frac{2}{3}} (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^{2/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

Sympy [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^{2/3}(A + B \cos(c + dx)) dx = \int (b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) \cos^m(c + dx) dx$$

[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)

[Out] Integral((b*cos(c + d*x))**(2/3)*(A + B*cos(c + d*x))*cos(c + d*x)**m, x)

Maxima [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^{2/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{2/3} \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

Giac [F]

$$\int \cos^m(c + dx)(b \cos(c + dx))^{2/3}(A + B \cos(c + dx)) dx = \int (B \cos(dx + c) + A)(b \cos(dx + c))^{2/3} \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(b \cos(c + dx))^{2/3}(A + B \cos(c + dx)) dx = \int \cos(c + dx)^m (b \cos(c + dx))^{2/3} (A + B \cos(c + dx)) dx$$

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3)*(A + B*cos(c + d*x)), x)
```


$$3.929 \quad \int \cos^m(c+dx) \sqrt[3]{b \cos(c+dx)} (A+B \cos(c+dx)) dx$$

Optimal result	.7677
Rubi [A] (verified)	.7677
Mathematica [A] (verified)	.7679
Maple [F]	.7679
Fricas [F]	.7679
Sympy [F]	.7680
Maxima [F]	.7680
Giac [F]	.7680
Mupad [F(-1)]	.7681

Optimal result

Integrand size = 31, antiderivative size = 167

$$\int \cos^m(c+dx) \sqrt[3]{b \cos(c+dx)} (A+B \cos(c+dx)) dx =$$

$$\frac{3A \cos^{1+m}(c+dx) \sqrt[3]{b \cos(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(4+3m), \frac{1}{6}(10+3m), \cos^2(c+dx)\right) \sin(c+dx)}{d(4+3m) \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B \cos^{2+m}(c+dx) \sqrt[3]{b \cos(c+dx)} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(7+3m), \frac{1}{6}(13+3m), \cos^2(c+dx)\right) \sin(c+dx)}{d(7+3m) \sqrt{\sin^2(c+dx)}}$$

[Out] $-3*A*\cos(d*x+c)^{(1+m)}*(b*\cos(d*x+c))^{(1/3)}*\operatorname{hypergeom}([1/2, 2/3+1/2*m], [5/3+1/2*m], \cos(d*x+c)^2)*\sin(d*x+c)/d/(4+3*m)/(\sin(d*x+c)^2)^{(1/2)}-3*B*\cos(d*x+c)^{(2+m)}*(b*\cos(d*x+c))^{(1/3)}*\operatorname{hypergeom}([1/2, 7/6+1/2*m], [13/6+1/2*m], \cos(d*x+c)^2)*\sin(d*x+c)/d/(7+3*m)/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \cos^m(c+dx) \sqrt[3]{b \cos(c+dx)} (A+B \cos(c+dx)) dx =$$

$$\frac{3A \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+1}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+4), \frac{1}{6}(3m+10), \cos^2(c+dx)\right)}{d(3m+4) \sqrt{\sin^2(c+dx)}} -$$

$$\frac{3B \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+2}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+7), \frac{1}{6}(3m+13), \cos^2(c+dx)\right)}{d(3m+7) \sqrt{\sin^2(c+dx)}}$$

[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]

[Out] (-3*A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(4 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*(b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*SIN[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt[3]{b \cos(c + dx)} \int \cos^{\frac{1}{3}+m}(c + dx)(A + B \cos(c + dx)) dx}{\sqrt[3]{\cos(c + dx)}} \\
 &= \frac{\left(A \sqrt[3]{b \cos(c + dx)}\right) \int \cos^{\frac{1}{3}+m}(c + dx) dx}{\sqrt[3]{\cos(c + dx)}} + \frac{\left(B \sqrt[3]{b \cos(c + dx)}\right) \int \cos^{\frac{4}{3}+m}(c + dx) dx}{\sqrt[3]{\cos(c + dx)}} \\
 &= \\
 &= \frac{3A \cos^{1+m}(c + dx) \sqrt[3]{b \cos(c + dx)} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(4 + 3m), \frac{1}{6}(10 + 3m), \cos^2(c + dx)\right)}{d(4 + 3m) \sqrt{\sin^2(c + dx)}} \\
 &= \frac{3B \cos^{2+m}(c + dx) \sqrt[3]{b \cos(c + dx)} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(7 + 3m), \frac{1}{6}(13 + 3m), \cos^2(c + dx)\right)}{d(7 + 3m) \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \cos^m(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx =$$

$$\frac{3 \cos^{1+m}(c + dx) \sqrt[3]{b \cos(c + dx)} \csc(c + dx) (A(7 + 3m) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(4 + 3m), \frac{5}{3} + \frac{m}{2}, \cos^2(c + dx)\right) + B(4 + 3m) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7 + 3m}{6}, \frac{13 + 3m}{6}, \cos^2(c + dx)\right)) \sqrt{\sin^2(c + dx)}}{d(4 + 3m)(7 + 3m)}$$

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]

[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(A*(7 + 3*m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] + B*(4 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(4 + 3*m)*(7 + 3*m))

Maple [F]

$$\int (\cos^m(dx + c)) (\cos(dx + c) b)^{\frac{1}{3}} (A + B \cos(dx + c)) dx$$

[In] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)

[Out] int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3)*(A+B*cos(d*x+c)),x)

Fricas [F]

$$\int \cos^m(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Sympy [F]

$$\int \cos^m(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) \cos^m(c + dx) dx$$

[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)

[Out] Integral((b*cos(c + d*x))**(1/3)*(A + B*cos(c + d*x))*cos(c + d*x)**m, x)

Maxima [F]

$$\int \cos^m(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Giac [F]

$$\int \cos^m(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$

$$= \int (B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m dx$$

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^m(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx)) dx$$
$$= \int \cos(c + dx)^m (b \cos(c + dx))^{1/3} (A + B \cos(c + dx)) dx$$

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3)*(A + B*cos(c + d*x)), x)
```

$$3.930 \quad \int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal result	7682
Rubi [A] (verified)	7682
Mathematica [A] (verified)	7684
Maple [F]	7684
Fricas [F]	7684
Sympy [F]	7685
Maxima [F]	7685
Giac [F]	7685
Mupad [F(-1)]	7685

Optimal result

Integrand size = 31, antiderivative size = 167

$$\int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx =$$

$$\frac{3A \cos^{1+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(2+3m), \frac{1}{6}(8+3m), \cos^2(c+dx)\right) \sin(c+dx)}{d(2+3m) \sqrt[3]{b \cos(c+dx)} \sqrt{\sin^2(c+dx)}} +$$

$$\frac{3B \cos^{2+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(5+3m), \frac{1}{6}(11+3m), \cos^2(c+dx)\right) \sin(c+dx)}{d(5+3m) \sqrt[3]{b \cos(c+dx)} \sqrt{\sin^2(c+dx)}}$$

[Out] $-3*A*\cos(d*x+c)^{(1+m)}*\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{3}+\frac{1}{2}*m\right], \left[\frac{4}{3}+\frac{1}{2}*m\right], \cos(d*x+c)^2\right)*\sin(d*x+c)/d/(2+3*m)/(b*\cos(d*x+c))^{(1/3)}/(\sin(d*x+c)^2)^{(1/2)}-3*B*\cos(d*x+c)^{(2+m)}*\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{6}+\frac{1}{2}*m\right], \left[\frac{11}{6}+\frac{1}{2}*m\right], \cos(d*x+c)^2\right)*\sin(d*x+c)/d/(5+3*m)/(b*\cos(d*x+c))^{(1/3)}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx =$$

$$\frac{3A \sin(c+dx) \cos^{m+1}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+2), \frac{1}{6}(3m+8), \cos^2(c+dx)\right)}{d(3m+2) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}} +$$

$$\frac{3B \sin(c+dx) \cos^{m+2}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+5), \frac{1}{6}(3m+11), \cos^2(c+dx)\right)}{d(3m+5) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(1/3),x]

[Out] (-3*A*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*SIN[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\sqrt[3]{\cos(c + dx)} \int \cos^{-\frac{1}{3}+m}(c + dx)(A + B \cos(c + dx)) dx}{\sqrt[3]{b \cos(c + dx)}} \\
 &= \frac{\left(A \sqrt[3]{\cos(c + dx)}\right) \int \cos^{-\frac{1}{3}+m}(c + dx) dx}{\sqrt[3]{b \cos(c + dx)}} + \frac{\left(B \sqrt[3]{\cos(c + dx)}\right) \int \cos^{\frac{2}{3}+m}(c + dx) dx}{\sqrt[3]{b \cos(c + dx)}} \\
 &= \\
 &= \frac{3A \cos^{1+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(2 + 3m), \frac{1}{6}(8 + 3m), \cos^2(c + dx)\right) \sin(c + dx)}{d(2 + 3m) \sqrt[3]{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}} \\
 &= \frac{3B \cos^{2+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(5 + 3m), \frac{1}{6}(11 + 3m), \cos^2(c + dx)\right) \sin(c + dx)}{d(5 + 3m) \sqrt[3]{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{\cos^m(c + dx)(A + B \cos(c + dx))}{\sqrt[3]{b \cos(c + dx)}} dx = \frac{3 \cos^{1+m}(c + dx) \csc(c + dx) (A(5 + 3m) \text{Hypergeometric2F1}(\frac{1}{2}, \frac{1}{6}(2 + 3m), \frac{1}{6}(8 + 3m), \cos^2(c + dx)) + d(2 + 3m)(5 + 3m))}{d(2 + 3m)(5 + 3m)}$$

[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(1/3),x]

[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(5 + 3*m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + B*(2 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(1/3))

Maple [F]

$$\int \frac{(\cos^m(dx + c))(A + B \cos(dx + c))}{(\cos(dx + c)b)^{\frac{1}{3}}} dx$$

[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/3),x)

[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(1/3),x)

Fricas [F]

$$\int \frac{\cos^m(c + dx)(A + B \cos(c + dx))}{\sqrt[3]{b \cos(c + dx)}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}} dx = \int \frac{(A+B\cos(c+dx))\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}} dx$$

[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/3), x)

[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x)

Maxima [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}} dx$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)

Giac [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}} dx$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3), x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}} dx = \int \frac{\cos(c+dx)^m(A+B\cos(c+dx))}{(b\cos(c+dx))^{1/3}} dx$$

[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/3), x)

[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(1/3), x)

$$3.931 \quad \int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx$$

Optimal result	7686
Rubi [A] (verified)	7686
Mathematica [A] (verified)	7688
Maple [F]	7688
Fricas [F]	7688
Sympy [F]	7689
Maxima [F]	7689
Giac [F]	7689
Mupad [F(-1)]	7689

Optimal result

Integrand size = 31, antiderivative size = 167

$$\int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx =$$

$$\frac{3A \cos^{1+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(1+3m), \frac{1}{6}(7+3m), \cos^2(c+dx)\right) \sin(c+dx)}{d(1+3m)(b \cos(c+dx))^{2/3} \sqrt{\sin^2(c+dx)}} +$$

$$\frac{3B \cos^{2+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(4+3m), \frac{1}{6}(10+3m), \cos^2(c+dx)\right) \sin(c+dx)}{d(4+3m)(b \cos(c+dx))^{2/3} \sqrt{\sin^2(c+dx)}}$$

[Out] $-3*A*\cos(d*x+c)^{(1+m)}*\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{6}+1/2*m\right], \left[\frac{7}{6}+1/2*m\right], \cos(d*x+c)^2\right)*\sin(d*x+c)/d/(1+3*m)/(b*\cos(d*x+c))^{(2/3)}/(\sin(d*x+c)^2)^{(1/2)}-3*B*\cos(d*x+c)^{(2+m)}*\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{2}{3}+1/2*m\right], \left[\frac{5}{3}+1/2*m\right], \cos(d*x+c)^2\right)*\sin(d*x+c)/d/(4+3*m)/(b*\cos(d*x+c))^{(2/3)}/(\sin(d*x+c)^2)^{(1/2)}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{2/3}} dx =$$

$$\frac{3A \sin(c+dx) \cos^{m+1}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+1), \frac{1}{6}(3m+7), \cos^2(c+dx)\right)}{d(3m+1) \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{2/3}} +$$

$$\frac{3B \sin(c+dx) \cos^{m+2}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+4), \frac{1}{6}(3m+10), \cos^2(c+dx)\right)}{d(3m+4) \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{2/3}}$$

[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]

[Out] (-3*A*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[b^IntPart[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 2722

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((b*SIN[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2827

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\cos^{\frac{2}{3}}(c + dx) \int \cos^{-\frac{2}{3}+m}(c + dx)(A + B \cos(c + dx)) dx}{(b \cos(c + dx))^{2/3}} \\
 &= \frac{\left(A \cos^{\frac{2}{3}}(c + dx)\right) \int \cos^{-\frac{2}{3}+m}(c + dx) dx}{(b \cos(c + dx))^{2/3}} + \frac{\left(B \cos^{\frac{2}{3}}(c + dx)\right) \int \cos^{\frac{1}{3}+m}(c + dx) dx}{(b \cos(c + dx))^{2/3}} \\
 &= \\
 &\quad \frac{3A \cos^{1+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(1 + 3m), \frac{1}{6}(7 + 3m), \cos^2(c + dx)\right) \sin(c + dx)}{d(1 + 3m)(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}} \\
 &\quad - \frac{3B \cos^{2+m}(c + dx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(4 + 3m), \frac{1}{6}(10 + 3m), \cos^2(c + dx)\right) \sin(c + dx)}{d(4 + 3m)(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}} dx = \frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(4+3m)\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(1+3m), \frac{1}{6}(7+3m), \cos^2(c+dx)\right) + B\right)}{d(1+3m)(4+3m)(b\cos(c+dx))^{2/3}}$$

[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(4 + 3*m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2] + B*(1 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(1 + 3*m)*(4 + 3*m)*(b*Cos[c + d*x])^(2/3))

Maple [F]

$$\int \frac{(\cos^m(dx+c))(A+B\cos(dx+c))}{(\cos(dx+c)b)^{2/3}} dx$$

[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3), x)

[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(2/3), x)

Fricas [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{2/3}} dx$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3), x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)

Sympy [F]

$$\int \frac{\cos^m(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(A + B \cos(c + dx)) \cos^m(c + dx)}{(b \cos(c + dx))^{2/3}} dx$$

[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3), x)

[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x)

Maxima [F]

$$\int \frac{\cos^m(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^m}{(b \cos(dx + c))^{2/3}} dx$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)

Giac [F]

$$\int \frac{\cos^m(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{(B \cos(dx + c) + A) \cos(dx + c)^m}{(b \cos(dx + c))^{2/3}} dx$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3), x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c + dx)(A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx = \int \frac{\cos(c + dx)^m (A + B \cos(c + dx))}{(b \cos(c + dx))^{2/3}} dx$$

[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3), x)

[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(2/3), x)

$$3.932 \quad \int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx$$

Optimal result	7690
Rubi [A] (verified)	7690
Mathematica [A] (verified)	7692
Maple [F]	7692
Fricas [F]	7692
Sympy [F]	7693
Maxima [F]	7693
Giac [F]	7693
Mupad [F(-1)]	7693

Optimal result

Integrand size = 31, antiderivative size = 171

$$\int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx = \frac{3A \cos^m(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(-1+3m), \frac{1}{6}(5+3m), \cos^2(c+dx)\right) \sin(c+dx)}{bd(1-3m) \sqrt[3]{b \cos(c+dx)} \sqrt{\sin^2(c+dx)}} + \frac{3B \cos^{1+m}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(2+3m), \frac{1}{6}(8+3m), \cos^2(c+dx)\right) \sin(c+dx)}{bd(2+3m) \sqrt[3]{b \cos(c+dx)} \sqrt{\sin^2(c+dx)}}$$

[Out] 3*A*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m], [5/6+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/b/d/(1-3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*B*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m], [4/3+1/2*m], cos(d*x+c)^2)*sin(d*x+c)/b/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {20, 2827, 2722}

$$\int \frac{\cos^m(c+dx)(A+B \cos(c+dx))}{(b \cos(c+dx))^{4/3}} dx = \frac{3A \sin(c+dx) \cos^m(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m-1), \frac{1}{6}(3m+1), \cos^2(c+dx)\right)}{bd(1-3m) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}} + \frac{3B \sin(c+dx) \cos^{m+1}(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(3m+2), \frac{1}{6}(3m+8), \cos^2(c+dx)\right)}{bd(3m+2) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}}$$

[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*A*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c +

$d*x]^2]) - (3*B*\text{Cos}[c + d*x]^{(1 + m)}*\text{Hypergeometric2F1}[1/2, (2 + 3*m)/6, (8 + 3*m)/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(b*d*(2 + 3*m)*(b*\text{Cos}[c + d*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

$\text{Int}[(u_.)*((a_.)*(v_))^{(m_)}*((b_.)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[b^{\text{IntPart}[n]}*((b*v)^{\text{FracPart}[n]}/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, m, n\}, x \&\& \text{!IntegerQ}[m] \&\& \text{!IntegerQ}[n] \&\& \text{!IntegerQ}[m+n]$

Rule 2722

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[\text{Cos}[c + d*x]*((b*\text{Sin}[c + d*x])^{(n+1)}/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]))*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2], x] /; \text{FreeQ}\{b, c, d, n\}, x \&\& \text{!IntegerQ}[2*n]$

Rule 2827

$\text{Int}[(b_.)*\text{sin}[(e_.) + (f_.)*(x_)]^{(m_)}*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\sqrt[3]{\cos(c+dx)} \int \cos^{-\frac{4}{3}+m}(c+dx)(A+B\cos(c+dx)) dx}{b\sqrt[3]{b\cos(c+dx)}} \\ &= \frac{\left(A\sqrt[3]{\cos(c+dx)}\right) \int \cos^{-\frac{4}{3}+m}(c+dx) dx}{b\sqrt[3]{b\cos(c+dx)}} + \frac{\left(B\sqrt[3]{\cos(c+dx)}\right) \int \cos^{-\frac{1}{3}+m}(c+dx) dx}{b\sqrt[3]{b\cos(c+dx)}} \\ &= \frac{3A\cos^m(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(-1+3m), \frac{1}{6}(5+3m), \cos^2(c+dx)\right)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} \\ &\quad + \frac{3B\cos^{1+m}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(2+3m), \frac{1}{6}(8+3m), \cos^2(c+dx)\right)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \frac{3\cos^{1+m}(c+dx)\csc(c+dx)\left(A(2+3m)\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{6}(-1+3m), \frac{1}{6}(5+3m), \cos^2(c+dx)\right) + d(-1+3m)(2+3m)\right)}{d(-1+3m)(2+3m)}$$

[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(2 + 3*m)*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2] + B*(-1 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(-1 + 3*m)*(2 + 3*m)*(b*Cos[c + d*x])^(4/3))

Maple [F]

$$\int \frac{(\cos^m(dx+c))(A+B\cos(dx+c))}{(\cos(dx+c)b)^{4/3}} dx$$

[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3), x)

[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(cos(d*x+c)*b)^(4/3), x)

Fricas [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{4/3}} dx$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3), x, algorithm="fricas")

[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*cos(d*x + c)^2), x)

Sympy [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \int \frac{(A+B\cos(c+dx))\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}} dx$$

[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3), x)

[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(4/3), x)

Maxima [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{4/3}} dx$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)

Giac [F]

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{4/3}} dx$$

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3), x, algorithm="giac")

[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \int \frac{\cos(c+dx)^m(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}} dx$$

[In] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3), x)

[Out] int((cos(c + d*x)^m*(A + B*cos(c + d*x)))/(b*cos(c + d*x))^(4/3), x)

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 7695

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*      Small rewrite of logic in main function to make it*)
(*      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```

(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCo
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A",""}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count is
        ]
      ,(*ELSE*)
      finalresult={"C","Result contains complex when optimal does not."}
    ]
    ,(*ELSE*)(*result does not contains complex*)
    If[leafCountResult<=2*leafCountOptimal,
      finalresult={"A",""}
      ,(*ELSE*)
      finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"}
    ]
  ]
  ,(*ELSE*) (*expnResult>expnOptimal*)
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    finalresult={"C","Result contains higher order function than in optimal. Order "<>
    ,
    finalresult={"F","Contains unresolved integral."}
  ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)

```

```

(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
          If[ElementaryFunctionQ[Head[expn]],
            Max[3, ExpnType[expn[[1]]],
          If[SpecialFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
          If[HypergeometricFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
          If[AppellFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
          If[Head[expn]===RootSum,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
          If[Head[expn]===Integrate || Head[expn]===Int,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
          9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,

```

```

    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result, optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);

```

```

#do NOT call ExpnType() if leaf size is too large. Recursion problem
if leaf_count_result > 500000 then
    return "B","result has leaf size over 500,000. Avoiding possible recursion issues
fi;

leaf_count_optimal := leafcount(optimal);
ExpnType_result := ExpnType(result);
ExpnType_optimal := ExpnType(optimal);

if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 ("
```

```

                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
    end if
else #result contains complex but optimal is not
    if debug then
        print("result contains complex but optimal is not");
    fi;
    return "C","Result contains complex when optimal does not.";
fi;
else # result do not contain complex
    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well")
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A"," ";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),"="),convert(2*leaf_count
    fi;
fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
        convert(ExpnType_result,string)," vs. order ",
        convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

```



```

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+`) or type(expn,'*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else

```

```

9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func, [
    exp, log, ln,
    sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf, erfc, erfi,
    FresnelS, FresnelC,
    Ei, Ei, Li, Si, Ci, Shi, Chi,
    GAMMA, lnGAMMA, Psi, Zeta, polylog, dilog, LambertW,
    EllipticF, EllipticE, EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u), op(2..nops(u), u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

```

```

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:

```

```

    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is large"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
            else:
                grade = "C"
                grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

```

```

# print("Before returning. grade=", grade, " grade_annotation=", grade_annotation)

return grade, grade_annotation

```

SageMath grading function

```

# Dec 24, 2019. Nasser: Ported original Maple grading function by
# Albert Rich to use with Sagemath. This is used to
# grade Fricas, Giac and Maxima results.
# Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
# 'arctan2', 'floor', 'abs', 'log_integral'
# June 4, 2022 Made default grade_annotation "none" instead of "" due
# issue later when reading the file.
# July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    # print("Enter tree_size, expr is ", expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1] == 1/2: # expr.args[1] == Rational(1,2):
            if debug: print("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

```

```

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in is_atom")
        print ("caught exception" , type(error).__name__ )
        return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)

```



```

    return 1
  else:
    return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
  else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger than"
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. " + str(leaf_c

else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order " + str(expnType_result)

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```